Computer Algebra Independent Integration Tests

Summer 2024

5-Inverse-trig-functions/5.2-Inverse-cosine/273-5.2.4-arcsin

Nasser M. Abbasi

May 18, 2024 Compiled on May 18, 2024 at 2:34am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**705**]. This is test number [273].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.72 (703)	0.28 (2)
Mathematica	97.73 (689)	2.27 (16)
Maple	92.20 (650)	7.80 (55)
Fricas	37.45 (264)	62.55 (441)
Giac	36.31 (256)	63.69 (449)
Maxima	35.04 (247)	64.96 (458)
Sympy	29.36 (207)	70.64 (498)
Reduce	29.22 (206)	70.78 (499)
Mupad	20.71 (146)	79.29 (559)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

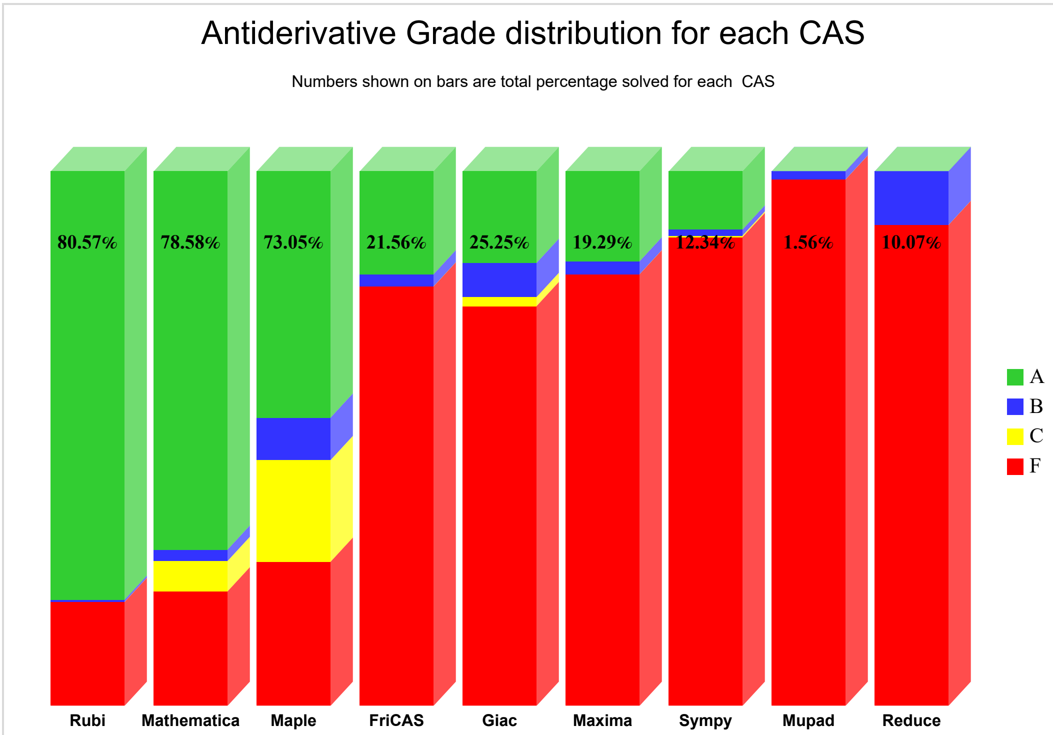
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

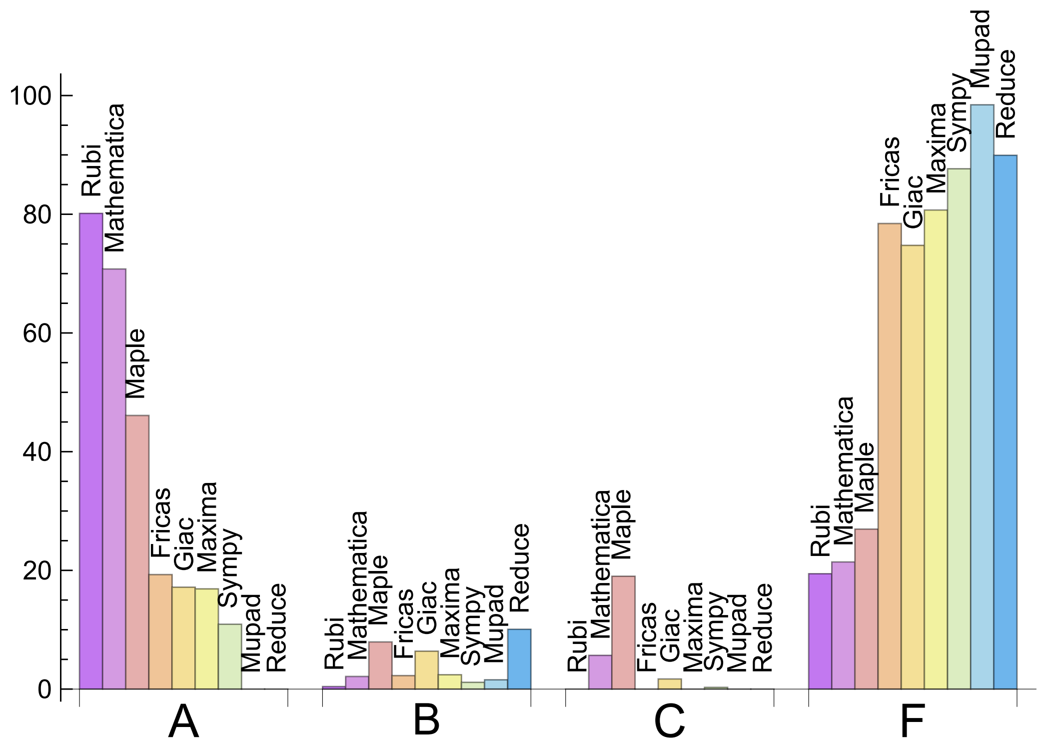
System	% A grade	% B grade	% C grade	% F grade
Rubi	80.142	0.426	0.000	19.433
Mathematica	70.780	2.128	5.674	21.418
Maple	46.099	7.943	19.007	26.950
Fricas	19.291	2.270	0.000	78.440
Giac	17.163	6.383	1.702	74.752
Maxima	16.879	2.411	0.000	80.709
Sympy	10.922	1.135	0.284	87.660
Mupad	0.000	1.560	0.000	98.440
Reduce	0.000	10.071	0.000	89.929

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	2	100.00	0.00	0.00
Mathematica	16	100.00	0.00	0.00
Maple	55	100.00	0.00	0.00
Giac	449	36.30	2.67	61.02
Fricas	441	85.26	0.00	14.74
Maxima	458	73.58	0.00	26.42
Sympy	498	79.12	20.48	0.40
Reduce	499	100.00	0.00	0.00
Mupad	559	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.12
Mupad	0.37
Maxima	0.39
Rubi	0.94
Giac	1.69
Mathematica	2.48
Maple	2.83
Reduce	7.11
Sympy	7.53

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	26.25	1.01	28.00	1.00
Reduce	118.07	2.92	85.00	1.76
Sympy	127.06	1.30	29.00	1.07
Maxima	161.94	2.36	136.00	1.06
Fricas	172.72	1.86	102.50	1.46
Rubi	204.84	0.94	159.00	1.00
Mathematica	241.29	1.06	171.00	1.06
Giac	434.29	2.74	104.00	1.00
Maple	479.76	1.83	262.50	1.26

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

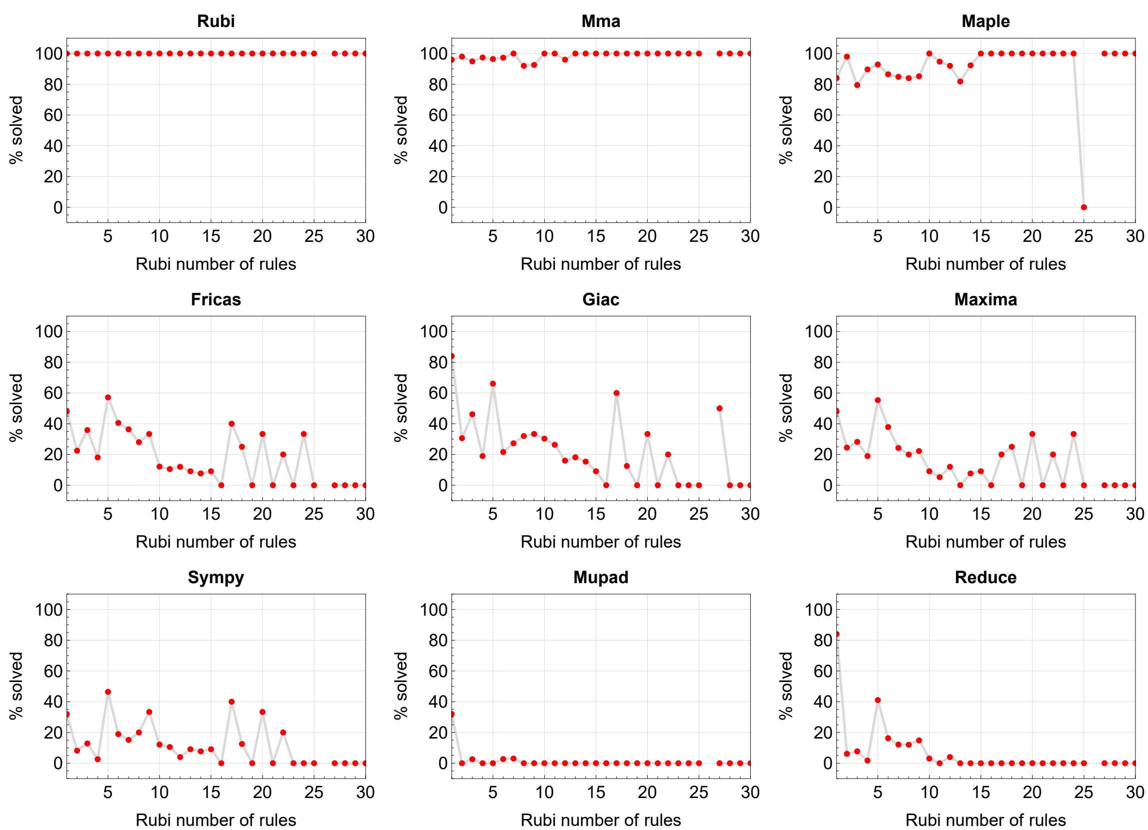


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

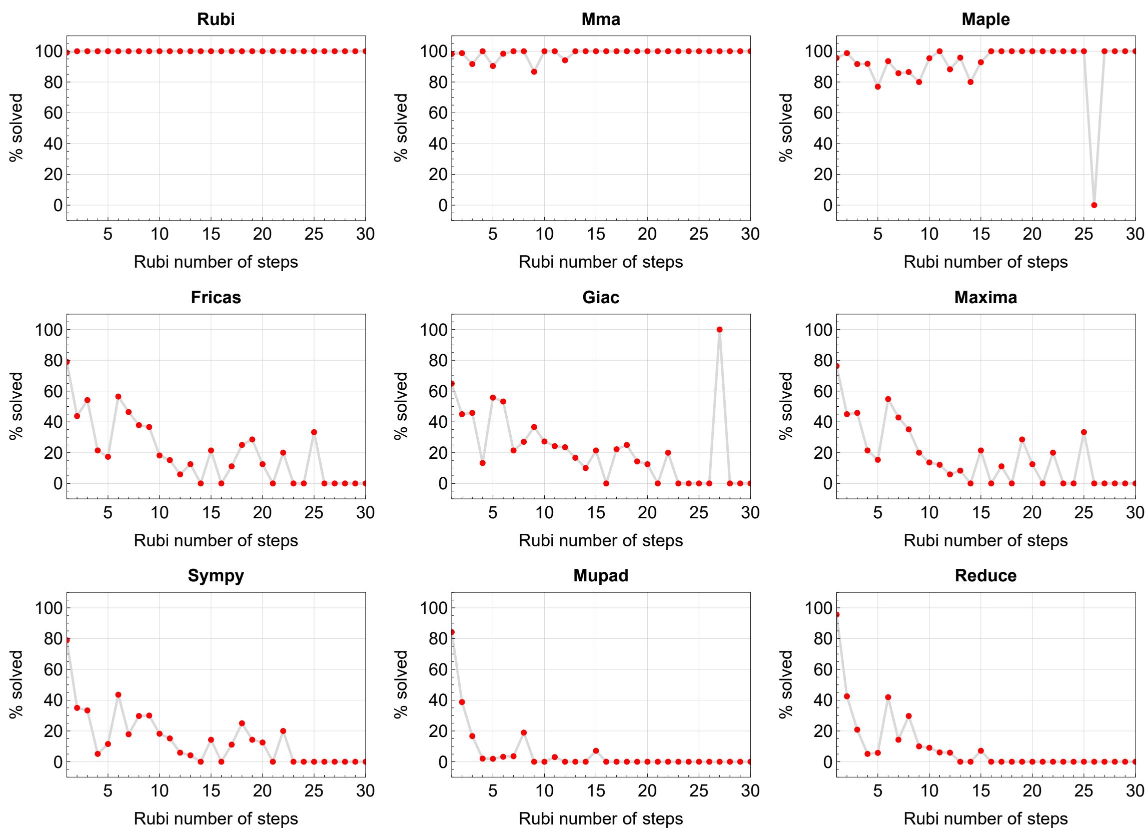


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

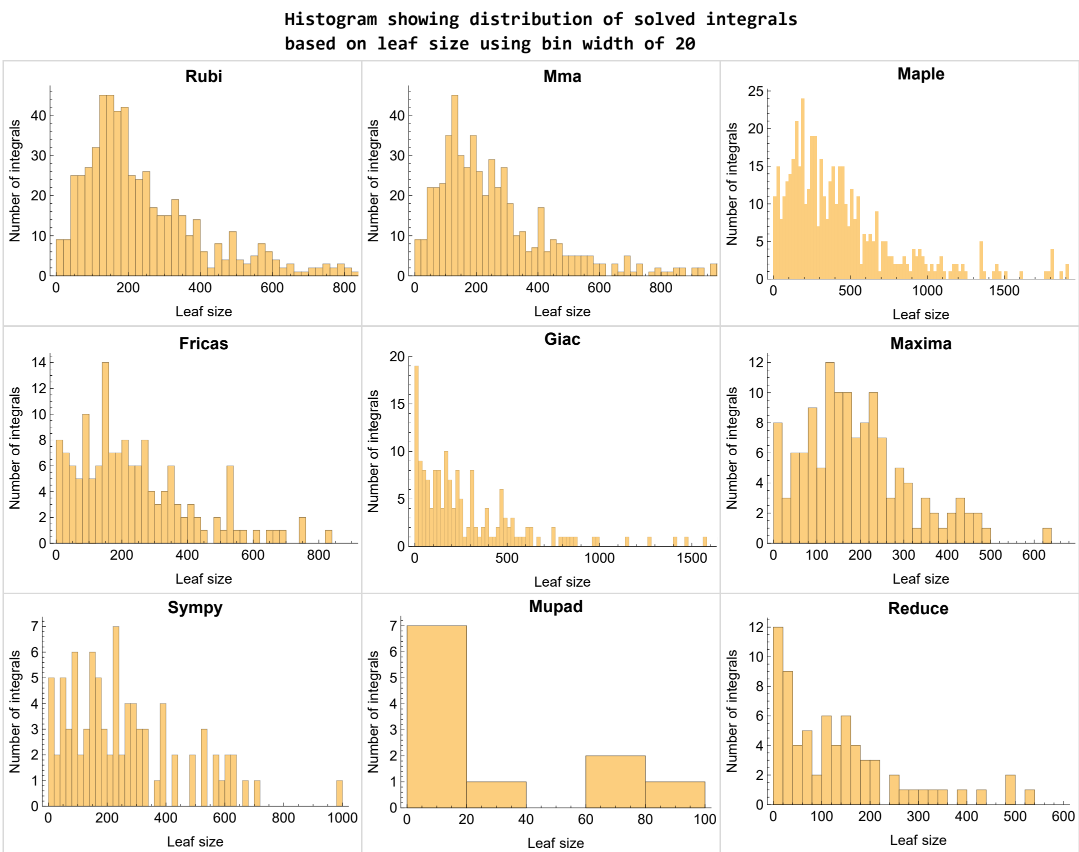


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

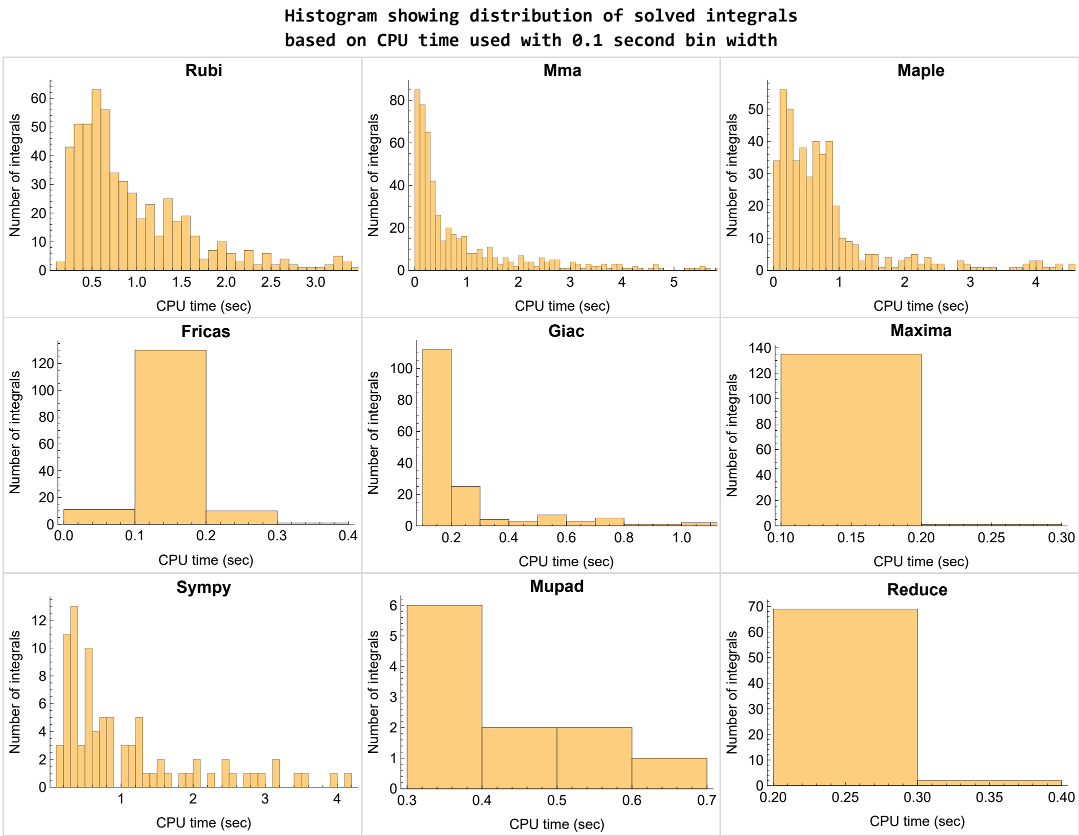


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

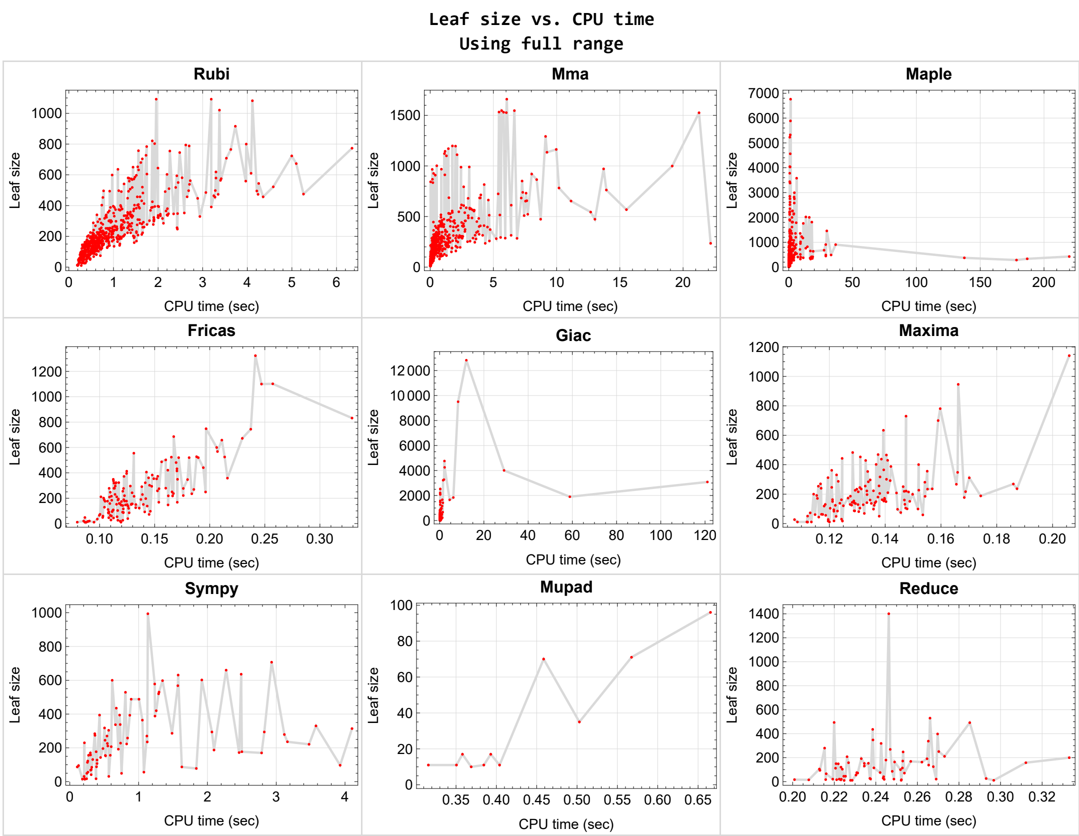


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{148, 149, 150, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 304, 316, 317, 323, 324, 325, 326, 331, 332, 333, 334, 339, 340, 341, 342, 349, 350, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 382, 387, 388, 389, 390, 391, 396, 397, 398, 399, 400, 405, 406, 407, 408, 409, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 437, 442, 447, 448, 452, 456, 460, 461, 465, 471, 472, 477, 478, 482, 483, 487, 488, 492, 493, 497, 498, 499, 504, 505, 651, 652, 659, 660, 666, 667, 668, 669, 673, 674, 675, 676, 677, 678, 682, 683, 684, 685, 686, 687, 691, 692, 695, 696, 700, 701, 704, 705}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{18, 20, 29, 151, 156, 234, 278, 279, 348, 356, 613, 615, 624\}$

Mathematica $\{154, 202, 209, 211, 254, 262, 264, 484, 485, 489, 490, 494, 495, 500, 501, 558, 559, 566, 573, 576, 627, 635, 637, 638, 639, 640, 641, 642, 643, 646, 647, 648, 649, 650, 654, 655\}$

Maple $\{627, 631, 633, 634, 635, 637, 638, 639, 640, 641, 642, 643, 646, 647, 648, 649, 650, 664\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
```

```
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
```

```
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

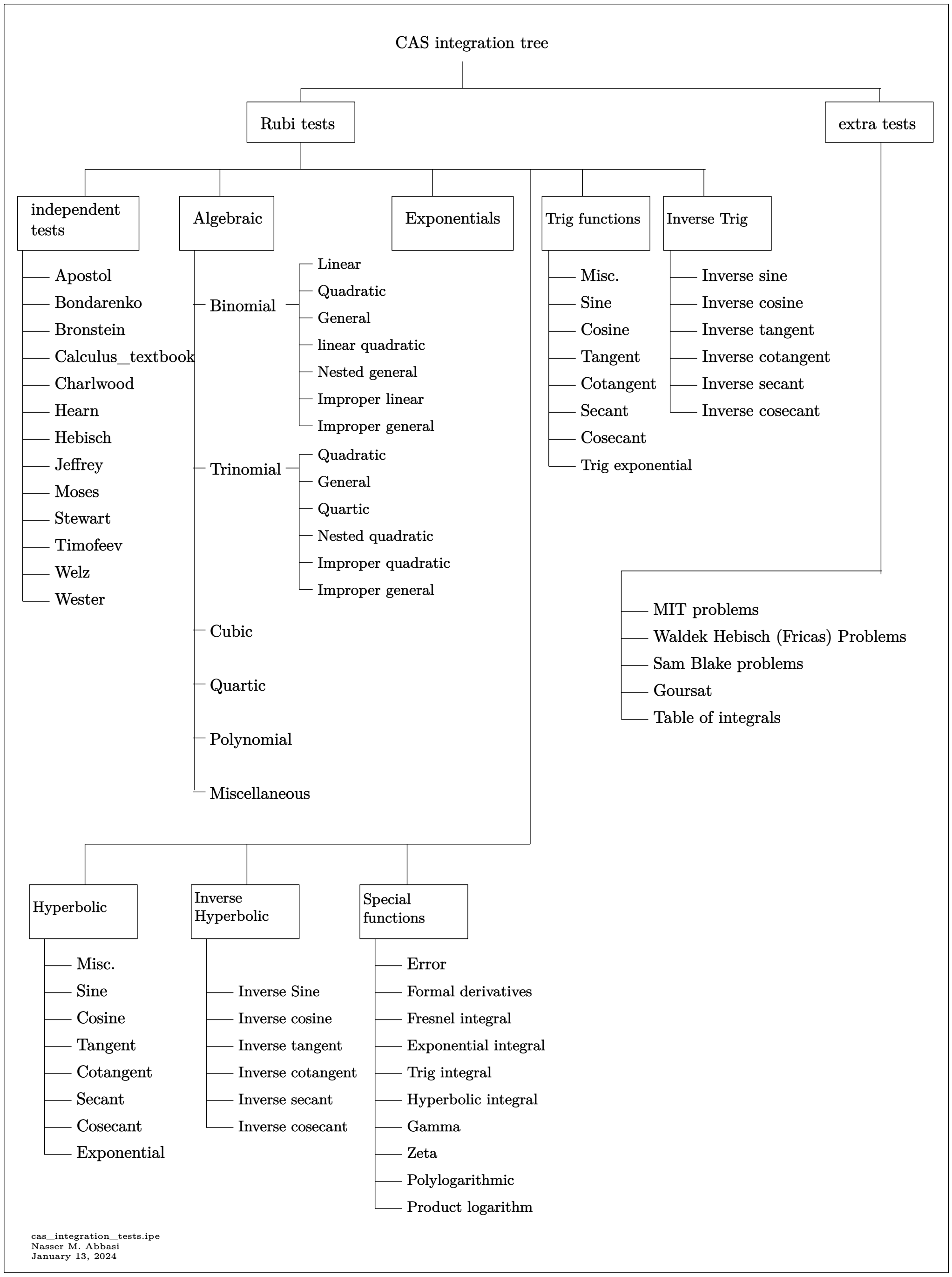
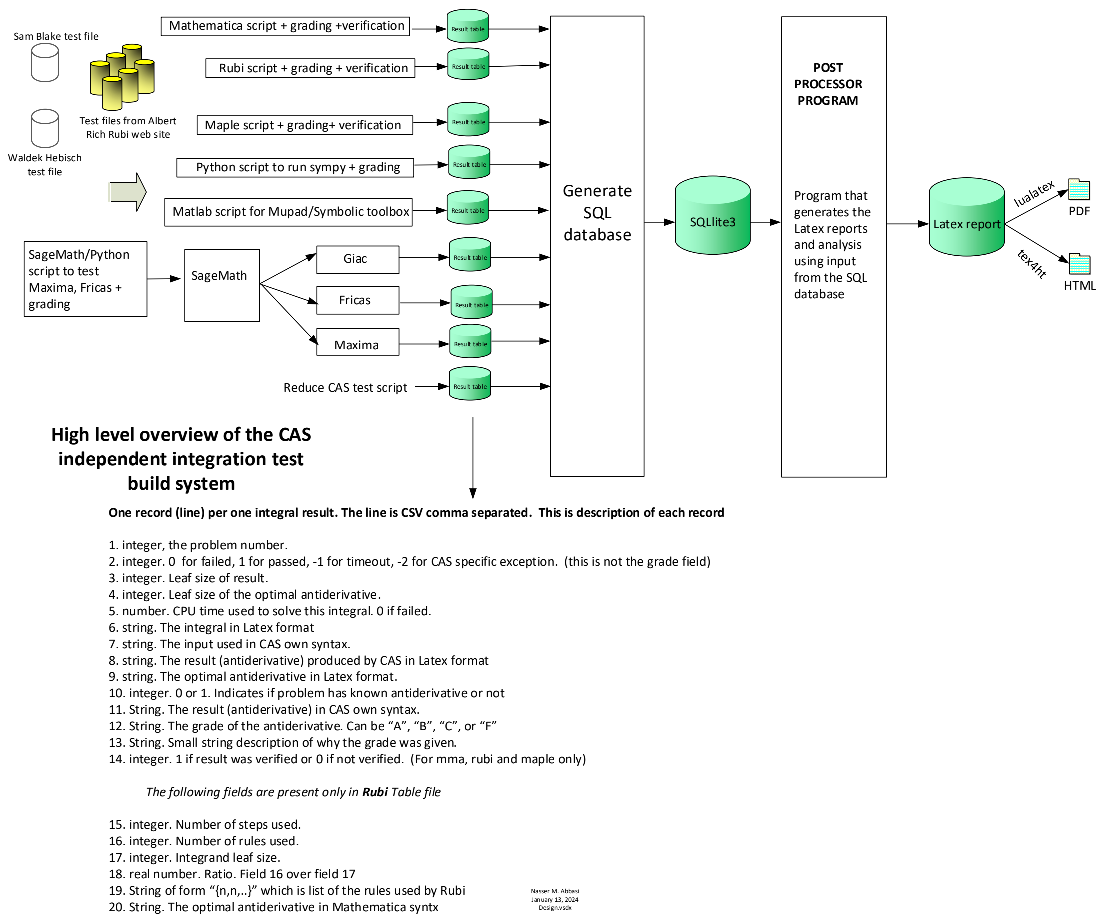


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 318, 319, 320, 321, 322, 327, 328, 329, 330, 335, 336, 337, 338, 343, 344, 345, 346, 347, 348, 351, 352, 353, 354, 355, 356, 376, 377, 378, 383, 385, 386, 392, 393, 394, 395, 401, 402, 403, 404, 410, 411, 412, 413, 414, 415, 432, 434, 435, 436, 438, 439, 440, 441, 443, 444, 445, 446, 449, 450, 451, 453, 454, 455, 457, 458, 459, 462, 463, 464, 466, 467, 468, 469, 470, 473, 474, 475, 476, 479, 480, 481, 484, 485, 486, 489, 490, 491, 494, 495, 496, 500, 501, 502, 503, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 653, 654, 655, 656, 657, 658, 661, 662, 663, 664, 665, 670, 671, 672, 679, 680, 681, 688, 689, 690, 693, 694, 697, 698, 699, 702, 703 }

B grade { 177, 384, 433 }

C grade { }

F normal fail { 264, 381 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 124, 126, 127, 128, 129, 130, 131, 133, 135, 137, 138, 139, 140, 141, 142, 145, 146, 147, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 194, 195, 196, 197, 198, 200, 201, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 318, 319, 320, 321, 322, 327, 328, 329, 330, 335, 336, 337, 338, 343, 344, 345, 346, 347, 348, 351, 352, 353, 354, 355, 356, 376, 377, 378, 383, 384, 385, 386, 392, 393, 394, 395, 401, 402, 403, 404, 410, 411, 412, 413, 414, 415, 432, 445, 446, 450, 451, 454, 455, 458, 459, 463, 464, 469, 470, 475, 476, 480, 481, 484, 485, 489, 490, 494, 495, 500, 501, 502, 503, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 560, 561, 562, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 644, 645, 648, 649, 650, 656, 657, 658, 661, 662, 663, 664, 670, 671, 672, 679, 680, 681 }

B grade { 1, 35, 190, 192, 193, 199, 202, 209, 527, 559, 563, 590, 643, 646, 647 }

C grade { 121, 123, 125, 132, 134, 136, 143, 144, 154, 157, 433, 434, 436, 438, 439, 441, 444, 449, 453, 457, 462, 466, 467, 468, 473, 474, 479, 653, 654, 655, 688, 689, 690, 693, 694, 697, 698, 699, 702, 703 }

F normal fail { 151, 152, 153, 155, 156, 278, 279, 280, 381, 435, 440, 443, 486, 491, 496, 665 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 85, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 116, 117, 119, 127, 129, 138, 140, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 191, 193, 194, 196, 198, 200, 202, 203, 205, 206, 207, 209, 211, 212, 216, 218, 220, 222, 224, 225, 226, 228, 230, 232, 233, 234, 236, 238, 242, 244, 248, 250, 251, 252, 253, 254, 256, 258, 260, 262, 264, 266, 267, 268, 269, 270, 272, 273, 274, 275, 276, 277, 291, 292, 293, 294, 295, 296, 300, 301, 302, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 318, 319, 320, 321, 322, 327, 328, 329, 330, 335, 336, 337, 338, 343, 344, 345, 346, 347, 348, 351, 352, 353, 354, 355, 356, 376, 377, 378, 383, 384, 385, 386, 392, 393, 394, 395, 401, 402, 403, 404, 410, 411, 412, 413, 414, 415, 432, 433, 434, 435, 436, 438, 439, 440, 441, 446, 451, 455, 459, 464, 466, 470, 476, 481, 503, 546, 552, 553, 558, 559, 564, 566, 567, 568, 569, 572, 573, 581, 586, 587, 591, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 629, 661, 662, 663, 664, 670, 671, 672, 679, 680, 681, 688, 689, 690, 697, 698, 699, 702, 703 }

B grade { 51, 106, 112, 114, 115, 188, 190, 192, 195, 197, 199, 201, 208, 210, 214, 217, 219, 227, 235, 237, 239, 241, 243, 245, 246, 247, 249, 255, 257, 259, 261, 263, 265, 303, 527, 547, 563, 565, 570, 571, 574, 575, 576, 577, 579, 582, 584, 590, 592, 593, 595, 636, 644, 645, 693, 694 }

C grade { 57, 58, 59, 60, 61, 62, 63, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 87, 88, 89, 90, 91, 92, 93, 94, 118, 120, 121, 122, 123, 124, 125, 126, 128, 130, 131, 132, 133, 134, 135, 136, 137, 139, 141, 142, 204, 213, 215, 221, 223, 229, 231, 240, 297, 298, 299, 443, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 548, 549, 550, 551, 554, 555, 556, 557, 560, 561, 562, 578, 580, 583, 585, 588, 589, 626, 627, 628, 630, 631, 632, 633, 634, 635, 637, 638, 639, 640, 641, 642, 643, 646, 647, 648, 649, 650 }

F normal fail { 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 271, 278, 279, 280, 310, 381, 444, 445, 449, 450, 453, 454, 457, 458, 462, 463, 467, 468, 469, 473, 474, 475, 479, 480, 484, 485, 486, 489, 490, 491, 494, 495, 496, 500, 501, 502, 653, 654, 655, 656, 657, 658, 665 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 3, 4, 5, 6, 7, 12, 13, 14, 15, 16, 18, 21, 22, 23, 24, 25, 27, 29, 42, 49, 51, 62, 63, 64, 65, 66, 75, 76, 77, 78, 79, 80, 81, 82, 92, 93, 94, 95, 96, 97, 101, 103, 104, 105, 106, 107, 109, 111, 113, 115, 118, 120, 121, 123, 125, 132, 134, 136, 158, 159, 160, 161, 162, 167, 168, 169, 170, 171, 176, 177, 178, 179, 180, 197, 204, 206, 212, 214, 220, 222, 228, 230, 236, 238, 240, 266, 267, 268, 269, 270, 291, 292, 293, 305, 306, 307, 308, 309, 348, 356, 415, 432, 459, 464, 476, 481, 503, 511, 528, 529, 533, 538, 539, 579, 584, 589, 598, 599, 600, 601, 602, 607, 608, 609, 610, 611, 616, 617, 618, 619, 620, 625, 661, 662, 663, 664 }

B grade { 9, 11, 20, 61, 604, 606, 613, 615, 622, 624, 636, 644, 645, 653, 654, 655 }

C grade { }

F normal fail { 1, 2, 8, 10, 17, 19, 26, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 67, 68, 69, 70, 71, 72, 73, 74, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 102, 108, 110, 112, 114, 116, 117, 119, 122, 124, 126, 127, 128, 129, 130, 131, 133, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 163, 164, 165, 166, 172, 173, 174, 175, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 205, 207, 208, 209, 210, 211, 213, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 227, 229, 231, 232, 233, 234, 235, 237, 239, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 310, 311, 312, 313, 314, 315, 318, 319, 320, 321, 322, 327, 328, 329, 330, 335, 336, 337, 338, 343, 344, 345, 346, 347, 351, 352, 353, 354, 355, 376, 377, 378, 381, 383, 384, 385, 386, 392, 393, 394, 395, 401, 402, 403, 404, 410, 411, 412, 413, 414, 484, 485, 486, 489, 490, 491, 494, 495, 496, 500, 501, 502, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 530, 531, 532, 534, 535, 536, 537, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 585, 586, 587, 588, 590, 591, 592, 593, 594, 595, 596, 597, 603, 605, 612, 614, 621, 623, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 637, 638, 639, 640, 641, 642, 643, 646, 647, 648, 649, 650, 656, 657, 658, 665, 670, 671, 672, 679, 680, 681 }

F(-1) timedout fail { }

F(-2) exception fail { 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 460, 461, 462, 463, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 477, 478, 479, 480, 482, 483, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705 }

Maxima

A grade { 3, 4, 5, 6, 7, 9, 11, 13, 14, 16, 18, 20, 27, 29, 61, 62, 63, 64, 65, 66, 75, 76, 77, 78, 79, 80, 81, 82, 92, 93, 94, 95, 96, 97, 101, 103, 104, 105, 106, 107, 109, 111, 113, 115, 116, 118, 120, 123, 126, 128, 134, 135, 137, 141, 142, 158, 160, 212, 214, 220, 222, 228, 230, 236, 238, 240, 267, 269, 270, 274, 291, 292, 293, 300, 306, 308, 309, 348, 356, 415, 432, 503, 511, 527, 528, 529, 533, 534, 535, 538, 539, 540, 541, 598, 599, 600, 601, 602, 604, 606, 607, 608, 609, 610, 611, 613, 615, 616, 617, 618, 619, 620, 622, 624, 625, 661, 662, 663, 664 }

B grade { 12, 15, 21, 22, 23, 24, 25, 42, 162, 167, 169, 171, 176, 178, 180, 197, 241 }

C grade { }

F normal fail { 1, 2, 8, 10, 17, 19, 26, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 67, 68, 69, 70, 71, 72, 73, 74, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 102, 108, 110, 112, 114, 117, 119, 121, 122, 124, 125, 127, 129, 130, 131, 132, 133, 136, 138, 139, 140, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 159, 161, 163, 164, 165, 166, 168, 170, 172, 173, 174, 175, 177, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 227, 229, 231, 232, 233, 234, 235, 237, 239, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 268, 271, 272, 273, 275, 276, 277, 278, 279, 280, 294, 295, 296, 297, 298, 299, 301, 302, 303, 305, 307, 310, 311, 312, 313, 314, 315, 318, 319, 320, 321, 322, 327, 328, 329, 330, 335, 336, 337, 338, 343, 344, 345, 346, 347, 351, 352, 353, 354, 355, 376, 377, 378, 381, 383, 384, 385, 386, 392, 393, 394, 395, 401, 402, 403, 404, 410, 411, 412, 413, 414, 433, 434, 435, 436, 438, 439, 440, 441, 484, 485, 486, 489, 490, 491, 494, 495, 496, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 530, 531, 532, 536, 537, 570, 576, 577, 594, 595, 603, 605, 612, 614, 621, 623, 627, 629, 631, 633, 635, 637, 638, 643, 644, 645, 646, 647, 654, 655, 656, 657, 658, 670, 671, 672, 679, 680, 681, 688, 689, 690, 693, 694, 697, 698, 699, 702, 703 }

F(-1) timedout fail { }

F(-2) exception fail { 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 499, 500, 501, 502, 504, 505, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 571, 572, 573, 574, 575, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 596, 597, 626, 628, 630, 632, 634, 636, 639, 640, 641, 642, 648, 649, 650, 651, 652, 653, 665, 666, 667, 668, 691, 695, 700 }

Giac

A grade { 3, 4, 5, 6, 7, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 42, 49, 57, 58, 70, 71, 86, 87, 101, 103, 105, 106, 107, 112, 114, 116, 124, 133, 135, 142, 158, 159, 160, 161, 162, 167, 168, 169, 170, 171, 176, 177, 178, 180, 213, 221, 229, 237, 239, 241, 266, 268, 269, 270, 274, 291, 292, 293, 300, 305, 307, 308, 309, 313, 314, 315, 321, 322, 330, 343, 345, 346, 347, 348, 352, 354, 355, 356, 376, 377, 378, 415, 432, 443, 446, 451, 455, 459, 464, 470, 476, 481, 503, 598, 599, 600, 601, 602, 607, 608, 609, 610, 611, 616, 617, 618, 619, 620, 625, 661, 662, 663, 664, 670, 671, 672 }

B grade { 9, 11, 18, 20, 27, 51, 109, 122, 131, 179, 197, 204, 206, 318, 320, 327, 328, 329, 335, 336, 337, 338, 384, 385, 386, 392, 393, 394, 395, 401, 402, 403, 404, 411, 413, 414, 604, 606, 613, 615, 622, 624, 679, 680, 681 }

C grade { 466, 467, 468, 469, 688, 689, 690, 693, 694, 697, 698, 699 }

F normal fail { 1, 2, 30, 32, 39, 41, 48, 50, 108, 110, 143, 144, 145, 146, 147, 157, 185, 187, 194, 196, 203, 205, 247, 249, 257, 259, 271, 272, 273, 275, 278, 279, 280, 294, 295, 296, 301, 310, 311, 312, 381, 433, 434, 435, 436, 438, 439, 440, 441, 457, 458, 462, 463, 473, 474, 475, 479, 480, 484, 489, 494, 501, 502, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 640, 648, 649, 656, 657, 658, 702, 703 }

F(-1) timedout fail { 29, 175, 182, 184, 202, 211, 637, 638, 639, 642, 646, 647 }

F(-2) exception fail { 8, 10, 17, 19, 26, 28, 31, 33, 34, 35, 36, 37, 38, 40, 43, 44, 45, 46, 47, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 104, 111, 113, 115, 117, 118, 119, 120, 121, 123, 125, 126, 127, 128, 129, 130, 132, 134, 136, 137, 138, 139, 140, 141, 148, 149, 150, 151, 152, 153, 154, 155, 156, 163, 164, 165, 166, 172, 173, 174, 181, 183, 186, 188, 189, 190, 191, 192, 193, 195, 198, 199, 200, 201, 207, 208, 209, 210, 212, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 238, 240, 242, 243, 244, 245, 246, 248, 250, 251, 252, 253, 254, 255, 256, 258, 260, 261, 262, 263, 264, 265, 267, 276, 277, 281, 282, 283, 284, 285, 286, 287, 288, 289, 297, 298, 299, 302, 303, 306, 319, 323, 325, 331, 333, 339, 341, 344, 351, 353, 357, 360, 362, 365, 367, 369, 370, 371, 382, 383, 387, 389, 391, 396, 398, 400, 405, 407, 410, 412, 416, 419, 421, 423, 426, 428, 430, 437, 442, 444, 445, 449, 450, 453, 454, 485, 486, 487, 488, 490, 491, 492, 493, 495, 496, 497, 498, 500, 603, 605, 612, 614, 621, 623, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 641, 643, 644, 645, 650, 652, 653, 654, 655, 659, 660, 665, 667, 668, 669, 683 }

Mupad

A grade { }

B grade { 9, 107, 270, 309, 348, 356, 415, 432, 503, 604, 664 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 318, 319, 320, 321, 322, 327, 328, 329, 330, 335, 336, 337, 338, 343, 344, 345, 346, 347, 351, 352, 353, 354, 355, 376, 377, 378, 381, 383, 384, 385, 386, 392, 393, 394, 395, 401, 402, 403, 404, 410, 411, 412, 413, 414, 433, 434, 435, 436, 438, 439, 440, 441, 443, 444, 445, 446, 449, 450, 451, 453, 454, 455, 457, 458, 459, 462, 463, 464, 466, 467, 468, 469, 470, 473, 474, 475, 476, 479, 480, 481, 484, 485, 486, 489, 490, 491, 494, 495, 496, 500, 501, 502, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 653, 654, 655, 656, 657, 658, 661, 662, 663, 665, 670, 671, 672, 679, 680, 681, 688, 689, 690, 693, 694, 697, 698, 699, 702, 703 }

F(-2) exception fail { }

Sympy

A grade { 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15, 16, 18, 20, 21, 22, 23, 24, 25, 27, 29, 101, 102, 103, 104, 105, 106, 107, 158, 159, 160, 162, 167, 168, 169, 171, 176, 177, 178, 180, 266, 267, 268, 269, 291, 292, 293, 305, 306, 307, 308, 348, 432, 598, 599, 600, 601, 602, 604, 606, 607, 608, 609, 610, 611, 613, 615, 616, 617, 618, 620, 622, 624, 625, 661, 662, 663 }

B grade { 161, 170, 179, 270, 309, 503, 619, 664 }

C grade { 356, 415 }

F normal fail { 1, 2, 8, 10, 17, 19, 26, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 81, 82, 83, 84, 85, 90, 91, 100, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 145, 146, 147, 153, 154, 155, 157, 163, 164, 165, 166, 172, 173, 174, 175, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 230, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 294, 295, 296, 298, 299, 300, 301, 302, 303, 310, 311, 312, 313, 314, 315, 318, 319, 320, 321, 322, 327, 328, 329, 330, 335, 336, 337, 338, 343, 344, 345, 346, 347, 351, 352, 353, 354, 355, 376, 377, 378, 381, 383, 384, 385, 386, 392, 393, 394, 395, 401, 402, 403, 404, 410, 411, 412, 413, 414, 433, 434, 435, 436, 438, 439, 440, 441, 443, 444, 445, 446, 450, 451, 457, 458, 459, 463, 464, 466, 468, 469, 470, 474, 475, 476, 479, 480, 481, 484, 485, 486, 500, 501, 502, 507, 508, 509, 510, 511, 514, 515, 516, 517, 525, 526, 527, 528, 529, 531, 532, 533, 534, 537, 538, 539, 543, 544, 545, 546, 547, 550, 551, 552, 553, 561, 562, 563, 564, 565, 567, 568, 569, 570, 573, 574, 575, 578, 579, 580, 581, 582, 589, 590, 591, 592, 593, 594, 595, 596, 603, 605, 612, 614, 621, 623, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 638, 639, 640, 641, 642, 643, 644, 645, 646, 648, 650, 653, 654, 657, 658, 665, 670, 671, 672, 679, 680, 681, 688, 689, 690, 693, 694, 697, 698, 699, 702, 703 }

F(-1) timedout fail { 52, 53, 70, 71, 77, 78, 79, 80, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 141, 144, 151, 152, 156, 207, 221, 228, 229, 231, 232, 284, 285, 289, 297, 369, 400, 449, 453, 454, 455, 456, 462, 467, 473, 478, 482, 483, 489, 490, 491, 493, 494, 495, 496, 497, 498, 506, 512, 513, 518, 519, 520, 521, 522, 523, 524, 530, 535, 536, 540, 541, 542, 548, 549, 554, 555, 556, 557, 558, 559, 560, 566, 571, 572, 576, 577, 583, 584, 585, 586, 587, 588, 597, 637, 647, 649, 655, 656, 660, 683, 705 }

F(-2) exception fail { 115, 240 }

Reduce

A grade { }

B grade { 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15, 16, 18, 20, 21, 22, 23, 24, 25, 27, 29, 106, 107, 115, 116, 240, 241, 269, 270, 274, 300, 308, 309, 348, 356, 415, 432, 443, 446, 451, 455, 459, 464, 470, 476, 481, 503, 598, 599, 600, 601, 602, 604, 606, 607, 608, 609, 610, 611, 613, 615, 616, 617, 618, 619, 620, 622, 624, 625, 636, 664 }

C grade { }

F normal fail { 1, 2, 8, 10, 17, 19, 26, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 271, 272, 273, 275, 276, 277, 278, 279, 280, 291, 292, 293, 294, 295, 296, 297, 298, 299, 301, 302, 303, 305, 306, 307, 310, 311, 312, 313, 314, 315, 318, 319, 320, 321, 322, 327, 328, 329, 330, 335, 336, 337, 338, 343, 344, 345, 346, 347, 351, 352, 353, 354, 355, 376, 377, 378, 381, 383, 384, 385, 386, 392, 393, 394, 395, 401, 402, 403, 404, 410, 411, 412, 413, 414, 433, 434, 435, 436, 438, 439, 440, 441, 444, 445, 449, 450, 453, 454, 457, 458, 462, 463, 466, 467, 468, 469, 473, 474, 475, 479, 480, 484, 485, 486, 489, 490, 491, 494, 495, 496, 500, 501, 502, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 603, 605, 612, 614, 621, 623, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 653, 654, 655, 656, 657, 658, 661, 662, 663, 665, 670, 671, 672, 679, 680, 681, 688, 689, 690, 693, 694, 697, 698, 699, 702, 703 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	63	213	59	0	0	0	0	19	0
N.S.	1	1.17	3.94	1.09	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.332	0.029	0.378	0.000	0.000	0.000	0.000	0.228	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	245	213	169	0	0	0	0	17	0
N.S.	1	1.01	0.88	0.70	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.699	0.059	0.504	0.000	0.000	0.000	0.000	0.236	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	126	87	126	191	102	156	126	125	0
N.S.	1	0.98	0.68	0.98	1.49	0.80	1.22	0.98	0.98	0.00
time (sec)	N/A	0.341	0.110	0.281	0.115	0.112	0.588	0.140	0.268	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	135	150	114	171	97	143	116	114	0
N.S.	1	1.10	1.22	0.93	1.39	0.79	1.16	0.94	0.93	0.00
time (sec)	N/A	0.311	0.088	0.206	0.116	0.146	0.446	0.138	0.253	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	85	106	150	92	131	104	105	0
N.S.	1	1.00	0.81	1.01	1.43	0.88	1.25	0.99	1.00	0.00
time (sec)	N/A	0.341	0.081	0.223	0.121	0.141	0.333	0.131	0.213	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	93	82	98	130	87	122	94	94	0
N.S.	1	1.03	0.91	1.09	1.44	0.97	1.36	1.04	1.04	0.00
time (sec)	N/A	0.260	0.099	0.339	0.119	0.122	0.261	0.143	0.213	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	67	80	100	72	95	76	76	0
N.S.	1	1.03	0.87	1.04	1.30	0.94	1.23	0.99	0.99	0.00
time (sec)	N/A	0.299	0.042	0.000	0.148	0.145	0.127	0.134	0.242	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	136	122	131	0	0	0	0	66	0
N.S.	1	1.12	1.01	1.08	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.591	0.068	0.604	0.000	0.000	0.000	0.000	0.237	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	85	68	84	147	87	475	66	70
N.S.	1	1.03	1.23	0.99	1.22	2.13	1.26	6.88	0.96	1.01
time (sec)	N/A	0.320	0.037	0.137	0.122	0.162	1.628	1.004	0.216	0.459

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	137	122	130	0	0	0	0	62	0
N.S.	1	0.99	0.88	0.94	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.637	0.066	0.427	0.000	0.000	0.000	0.000	0.207	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	80	106	87	125	168	177	1654	71	0
N.S.	1	0.99	1.31	1.07	1.54	2.07	2.19	20.42	0.88	0.00
time (sec)	N/A	0.310	0.056	0.168	0.128	0.171	2.501	4.513	0.254	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	174	120	168	331	154	235	194	169	0
N.S.	1	0.94	0.65	0.90	1.78	0.83	1.26	1.04	0.91	0.00
time (sec)	N/A	0.511	0.109	0.267	0.123	0.129	1.126	0.143	0.257	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	191	117	156	301	150	223	184	158	0
N.S.	1	1.04	0.64	0.85	1.64	0.82	1.21	1.00	0.86	0.00
time (sec)	N/A	0.475	0.152	0.240	0.140	0.125	0.826	0.138	0.227	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	153	112	148	270	142	207	170	149	0
N.S.	1	0.95	0.70	0.92	1.68	0.88	1.29	1.06	0.93	0.00
time (sec)	N/A	0.424	0.097	0.250	0.117	0.121	0.602	0.138	0.222	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	121	108	133	240	138	196	160	138	0
N.S.	1	0.98	0.87	1.07	1.94	1.11	1.58	1.29	1.11	0.00
time (sec)	N/A	0.289	0.105	0.237	0.117	0.126	0.482	0.142	0.265	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	127	95	119	200	122	170	140	121	0
N.S.	1	0.97	0.73	0.91	1.53	0.93	1.30	1.07	0.92	0.00
time (sec)	N/A	0.364	0.047	0.127	0.114	0.136	0.299	0.138	0.223	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	226	166	180	0	0	0	0	111	0
N.S.	1	1.23	0.90	0.98	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.838	0.135	0.568	0.000	0.000	0.000	0.000	0.226	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	116	127	113	163	221	187	1837	112	0
N.S.	1	0.94	1.03	0.92	1.33	1.80	1.52	14.93	0.91	0.00
time (sec)	N/A	0.438	0.089	0.174	0.121	0.176	2.096	6.182	0.221	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	234	186	206	0	0	0	0	119	0
N.S.	1	1.16	0.93	1.02	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.923	0.135	0.645	0.000	0.000	0.000	0.000	0.271	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	137	135	115	173	237	236	1899	115	0
N.S.	1	1.07	1.05	0.90	1.35	1.85	1.84	14.84	0.90	0.00
time (sec)	N/A	0.432	0.095	0.178	0.127	0.167	3.163	58.856	0.239	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	216	143	210	483	190	294	246	211	0
N.S.	1	0.93	0.62	0.91	2.08	0.82	1.27	1.06	0.91	0.00
time (sec)	N/A	0.649	0.140	0.309	0.128	0.137	2.065	0.150	0.273	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	199	144	198	443	186	286	236	200	0
N.S.	1	0.97	0.70	0.96	2.15	0.90	1.39	1.15	0.97	0.00
time (sec)	N/A	0.414	0.207	0.263	0.125	0.117	1.486	0.188	0.333	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	195	135	190	402	178	270	222	191	0
N.S.	1	0.94	0.65	0.92	1.94	0.86	1.30	1.07	0.92	0.00
time (sec)	N/A	0.604	0.155	0.261	0.137	0.123	1.120	0.138	0.265	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	145	132	166	362	174	258	212	180	0
N.S.	1	0.97	0.88	1.11	2.41	1.16	1.72	1.41	1.20	0.00
time (sec)	N/A	0.300	0.193	0.230	0.121	0.120	0.839	0.147	0.245	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	167	119	162	312	158	226	192	163	0
N.S.	1	0.95	0.68	0.93	1.78	0.90	1.29	1.10	0.93	0.00
time (sec)	N/A	0.495	0.081	0.133	0.120	0.109	0.545	0.139	0.262	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	335	207	221	0	0	0	0	153	0
N.S.	1	1.43	0.88	0.94	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	1.276	0.201	0.635	0.000	0.000	0.000	0.000	0.242	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	150	135	154	254	269	294	4007	153	0
N.S.	1	0.91	0.82	0.94	1.55	1.64	1.79	24.43	0.93	0.00
time (sec)	N/A	0.599	0.186	0.177	0.116	0.186	2.827	29.112	0.236	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	343	226	245	0	0	0	0	162	0
N.S.	1	1.30	0.86	0.93	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	1.368	0.243	0.782	0.000	0.000	0.000	0.000	0.234	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	160	175	159	246	277	330	0	158	0
N.S.	1	0.90	0.98	0.89	1.38	1.56	1.85	0.00	0.89	0.00
time (sec)	N/A	0.654	0.169	0.184	0.121	0.176	3.578	0.000	0.312	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	178	184	219	0	0	0	0	74	0
N.S.	1	1.03	1.07	1.27	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.854	0.228	0.697	0.000	0.000	0.000	0.000	0.257	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	146	180	176	0	0	0	0	108	0
N.S.	1	1.01	1.25	1.22	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.659	0.182	0.534	0.000	0.000	0.000	0.000	0.252	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	106	138	180	0	0	0	0	82	0
N.S.	1	0.85	1.11	1.45	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.528	0.159	0.535	0.000	0.000	0.000	0.000	0.257	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	79	115	142	0	0	0	0	58	0
N.S.	1	0.96	1.40	1.73	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.431	0.127	0.332	0.000	0.000	0.000	0.000	0.275	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	66	107	140	0	0	0	0	54	0
N.S.	1	0.79	1.27	1.67	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.339	0.017	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	67	143	196	0	0	0	0	58	0
N.S.	1	0.94	2.01	2.76	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.433	0.198	0.483	0.000	0.000	0.000	0.000	0.234	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	102	158	146	0	0	0	0	65	0
N.S.	1	0.88	1.36	1.26	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.548	0.213	0.651	0.000	0.000	0.000	0.000	0.307	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	115	220	248	0	0	0	0	87	0
N.S.	1	0.93	1.77	2.00	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.621	0.254	0.588	0.000	0.000	0.000	0.000	0.236	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	168	232	210	0	0	0	0	85	0
N.S.	1	0.97	1.34	1.21	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.838	0.260	1.032	0.000	0.000	0.000	0.000	0.262	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	191	294	250	0	0	0	0	167	0
N.S.	1	1.02	1.57	1.34	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.848	0.216	0.730	0.000	0.000	0.000	0.000	0.267	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	152	203	207	0	0	0	0	160	0
N.S.	1	0.98	1.31	1.34	0.00	0.00	0.00	0.00	1.03	0.00
time (sec)	N/A	0.653	0.346	0.473	0.000	0.000	0.000	0.000	0.241	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	125	251	189	0	0	0	0	156	0
N.S.	1	0.87	1.74	1.31	0.00	0.00	0.00	0.00	1.08	0.00
time (sec)	N/A	0.536	0.217	0.508	0.000	0.000	0.000	0.000	0.264	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	98	136	54	0	100	89	0
N.S.	1	1.00	0.86	1.72	2.39	0.95	0.00	1.75	1.56	0.00
time (sec)	N/A	0.233	0.107	0.236	0.129	0.116	0.000	0.141	0.263	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	122	220	189	0	0	0	0	148	0
N.S.	1	0.87	1.56	1.34	0.00	0.00	0.00	0.00	1.05	0.00
time (sec)	N/A	0.507	0.109	0.309	0.000	0.000	0.000	0.000	0.229	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	119	233	255	0	0	0	0	164	0
N.S.	1	0.98	1.91	2.09	0.00	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	0.623	0.361	0.650	0.000	0.000	0.000	0.000	0.238	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	193	251	332	0	0	0	0	165	0
N.S.	1	1.04	1.35	1.78	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.901	0.334	1.053	0.000	0.000	0.000	0.000	0.280	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	198	309	281	0	0	0	0	195	0
N.S.	1	1.25	1.94	1.77	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.952	0.438	0.754	0.000	0.000	0.000	0.000	0.243	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	295	319	250	0	0	0	0	184	0
N.S.	1	1.14	1.23	0.97	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	1.222	0.594	0.941	0.000	0.000	0.000	0.000	0.251	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	212	294	252	0	0	0	0	273	0
N.S.	1	1.04	1.44	1.24	0.00	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	0.961	0.781	0.738	0.000	0.000	0.000	0.000	0.268	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	99	79	177	0	90	0	124	162	0
N.S.	1	0.99	0.79	1.77	0.00	0.90	0.00	1.24	1.62	0.00
time (sec)	N/A	0.292	0.181	0.240	0.000	0.121	0.000	0.150	0.277	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	180	363	252	0	0	0	0	271	0
N.S.	1	0.89	1.80	1.25	0.00	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	0.741	0.272	0.727	0.000	0.000	0.000	0.000	0.274	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	81	60	151	0	87	0	172	158	0
N.S.	1	0.98	0.72	1.82	0.00	1.05	0.00	2.07	1.90	0.00
time (sec)	N/A	0.268	0.161	0.198	0.000	0.122	0.000	0.152	0.236	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	174	321	252	0	0	0	0	262	0
N.S.	1	0.89	1.64	1.29	0.00	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	0.663	0.267	0.359	0.000	0.000	0.000	0.000	0.254	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	190	343	328	0	0	0	0	289	0
N.S.	1	1.10	1.98	1.90	0.00	0.00	0.00	0.00	1.67	0.00
time (sec)	N/A	0.858	0.531	0.816	0.000	0.000	0.000	0.000	0.260	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	263	346	268	0	0	0	0	285	0
N.S.	1	1.09	1.43	1.11	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	1.074	0.964	0.875	0.000	0.000	0.000	0.000	0.236	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	291	408	389	0	0	0	0	328	0
N.S.	1	1.17	1.65	1.57	0.00	0.00	0.00	0.00	1.32	0.00
time (sec)	N/A	1.246	0.656	0.987	0.000	0.000	0.000	0.000	0.259	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	383	438	311	0	0	0	0	304	0
N.S.	1	1.21	1.38	0.98	0.00	0.00	0.00	0.00	0.96	0.00
time (sec)	N/A	1.512	0.928	1.140	0.000	0.000	0.000	0.000	0.232	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	215	188	672	0	0	0	218	98	0
N.S.	1	0.82	0.72	2.56	0.00	0.00	0.00	0.83	0.37	0.00
time (sec)	N/A	0.761	0.707	0.720	0.000	0.000	0.000	0.335	0.248	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	164	141	367	0	0	0	162	77	0
N.S.	1	0.87	0.75	1.94	0.00	0.00	0.00	0.86	0.41	0.00
time (sec)	N/A	0.584	0.511	0.437	0.000	0.000	0.000	0.330	0.276	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	133	280	0	0	0	0	51	0
N.S.	1	1.00	1.15	2.41	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.363	0.364	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	111	128	298	0	0	0	0	68	0
N.S.	1	1.01	1.16	2.71	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.397	0.373	0.695	0.000	0.000	0.000	0.000	0.238	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	84	122	159	137	415	0	0	68	0
N.S.	1	0.76	1.10	1.43	1.23	3.74	0.00	0.00	0.61	0.00
time (sec)	N/A	0.352	0.224	0.745	0.135	0.170	0.000	0.000	0.260	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	130	172	1903	140	502	0	0	88	0
N.S.	1	0.70	0.92	10.18	0.75	2.68	0.00	0.00	0.47	0.00
time (sec)	N/A	0.424	0.218	0.830	0.129	0.160	0.000	0.000	0.305	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	174	213	2751	199	568	0	0	109	0
N.S.	1	0.66	0.81	10.46	0.76	2.16	0.00	0.00	0.41	0.00
time (sec)	N/A	0.502	0.231	0.916	0.150	0.207	0.000	0.000	0.231	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	171	146	196	197	177	0	0	109	0
N.S.	1	0.67	0.57	0.77	0.77	0.69	0.00	0.00	0.43	0.00
time (sec)	N/A	0.497	0.158	0.940	0.134	0.124	0.000	0.000	0.238	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	128	122	180	138	150	0	0	89	0
N.S.	1	0.70	0.67	0.98	0.75	0.82	0.00	0.00	0.49	0.00
time (sec)	N/A	0.426	0.144	1.505	0.124	0.133	0.000	0.000	0.233	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	83	94	154	75	116	0	0	66	0
N.S.	1	0.75	0.85	1.40	0.68	1.05	0.00	0.00	0.60	0.00
time (sec)	N/A	0.320	0.101	0.404	0.146	0.122	0.000	0.000	0.248	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	155	198	330	0	0	0	0	53	0
N.S.	1	0.76	0.98	1.63	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.710	0.434	0.619	0.000	0.000	0.000	0.000	0.256	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	171	243	303	0	0	0	0	66	0
N.S.	1	0.76	1.08	1.35	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.704	1.244	0.712	0.000	0.000	0.000	0.000	0.248	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	217	286	364	0	0	0	0	85	0
N.S.	1	0.72	0.95	1.21	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.902	2.012	0.804	0.000	0.000	0.000	0.000	0.294	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	305	227	770	0	0	0	314	146	0
N.S.	1	0.90	0.67	2.26	0.00	0.00	0.00	0.92	0.43	0.00
time (sec)	N/A	1.175	0.757	0.582	0.000	0.000	0.000	0.533	0.220	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	254	266	679	0	0	0	251	126	0
N.S.	1	0.96	1.00	2.56	0.00	0.00	0.00	0.95	0.48	0.00
time (sec)	N/A	0.923	0.571	0.621	0.000	0.000	0.000	0.508	0.243	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	204	211	479	0	0	0	0	101	0
N.S.	1	1.10	1.14	2.59	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.549	0.379	0.000	0.000	0.000	0.000	0.000	0.293	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	198	208	249	0	0	0	0	114	0
N.S.	1	1.07	1.12	1.35	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.617	0.838	0.755	0.000	0.000	0.000	0.000	0.246	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	199	211	277	0	0	0	0	130	0
N.S.	1	1.04	1.10	1.45	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.722	0.795	0.799	0.000	0.000	0.000	0.000	0.262	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	94	156	2349	172	524	0	0	120	0
N.S.	1	0.61	1.01	15.25	1.12	3.40	0.00	0.00	0.78	0.00
time (sec)	N/A	0.376	0.349	0.887	0.140	0.165	0.000	0.000	0.274	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	139	198	3384	151	600	0	0	140	0
N.S.	1	0.60	0.86	14.65	0.65	2.60	0.00	0.00	0.61	0.00
time (sec)	N/A	0.404	0.343	0.997	0.149	0.206	0.000	0.000	0.251	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	187	238	4563	210	672	0	0	160	0
N.S.	1	0.61	0.77	14.81	0.68	2.18	0.00	0.00	0.52	0.00
time (sec)	N/A	0.508	0.369	1.145	0.141	0.230	0.000	0.000	0.251	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	227	278	5886	269	744	0	0	180	0
N.S.	1	0.59	0.72	15.29	0.70	1.93	0.00	0.00	0.47	0.00
time (sec)	N/A	0.731	0.394	1.314	0.186	0.237	0.000	0.000	0.246	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	225	180	256	267	249	0	0	177	0
N.S.	1	0.60	0.48	0.68	0.71	0.66	0.00	0.00	0.47	0.00
time (sec)	N/A	0.662	0.225	1.378	0.166	0.196	0.000	0.000	0.276	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	182	156	240	208	219	0	0	157	0
N.S.	1	0.60	0.52	0.80	0.69	0.73	0.00	0.00	0.52	0.00
time (sec)	N/A	0.518	0.190	1.021	0.144	0.162	0.000	0.000	0.259	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	139	132	224	149	189	0	0	137	0
N.S.	1	0.61	0.58	0.99	0.66	0.83	0.00	0.00	0.60	0.00
time (sec)	N/A	0.425	0.174	0.797	0.140	0.122	0.000	0.000	0.265	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	94	103	199	87	159	0	0	115	0
N.S.	1	0.61	0.67	1.30	0.57	1.04	0.00	0.00	0.75	0.00
time (sec)	N/A	0.311	0.128	1.006	0.129	0.118	0.000	0.000	0.289	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	233	279	541	0	0	0	0	103	0
N.S.	1	0.84	1.00	1.95	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	1.001	0.922	0.768	0.000	0.000	0.000	0.000	0.248	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	242	397	451	0	0	0	0	126	0
N.S.	1	0.81	1.34	1.52	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.984	1.835	0.720	0.000	0.000	0.000	0.000	0.226	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	261	547	386	0	0	0	0	117	0
N.S.	1	0.85	1.78	1.26	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	1.041	2.267	0.826	0.000	0.000	0.000	0.000	0.265	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	430	407	352	1102	0	0	0	387	195	0
N.S.	1	0.95	0.82	2.56	0.00	0.00	0.00	0.90	0.45	0.00
time (sec)	N/A	1.559	1.200	0.944	0.000	0.000	0.000	0.702	0.255	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	356	310	1080	0	0	0	322	175	0
N.S.	1	1.01	0.88	3.08	0.00	0.00	0.00	0.92	0.50	0.00
time (sec)	N/A	1.316	0.910	0.830	0.000	0.000	0.000	0.667	0.239	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	278	266	689	0	0	0	0	150	0
N.S.	1	1.06	1.02	2.63	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.761	0.351	0.600	0.000	0.000	0.000	0.000	0.244	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	300	256	307	0	0	0	0	165	0
N.S.	1	0.98	0.84	1.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.895	0.963	0.929	0.000	0.000	0.000	0.000	0.274	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	297	242	344	0	0	0	0	179	0
N.S.	1	1.07	0.87	1.24	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.960	1.253	0.882	0.000	0.000	0.000	0.000	0.248	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	298	242	2615	0	0	0	0	182	0
N.S.	1	1.08	0.87	9.44	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	1.056	1.295	0.945	0.000	0.000	0.000	0.000	0.259	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	107	190	4031	205	658	0	0	171	0
N.S.	1	0.53	0.94	19.86	1.01	3.24	0.00	0.00	0.84	0.00
time (sec)	N/A	0.346	0.433	1.025	0.138	0.211	0.000	0.000	0.302	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	168	232	5324	162	748	0	0	191	0
N.S.	1	0.60	0.82	18.88	0.57	2.65	0.00	0.00	0.68	0.00
time (sec)	N/A	0.440	0.441	1.161	0.139	0.197	0.000	0.000	0.250	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	199	272	6761	221	832	0	0	212	0
N.S.	1	0.55	0.75	18.73	0.61	2.30	0.00	0.00	0.59	0.00
time (sec)	N/A	0.513	0.448	1.413	0.147	0.329	0.000	0.000	0.259	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	194	165	256	219	291	0	0	206	0
N.S.	1	0.55	0.47	0.72	0.62	0.82	0.00	0.00	0.58	0.00
time (sec)	N/A	0.541	0.214	1.257	0.147	0.138	0.000	0.000	0.278	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	149	141	240	160	255	0	0	186	0
N.S.	1	0.54	0.51	0.86	0.58	0.92	0.00	0.00	0.67	0.00
time (sec)	N/A	0.454	0.182	0.894	0.135	0.121	0.000	0.000	0.285	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	104	113	215	98	215	0	0	163	0
N.S.	1	0.51	0.56	1.06	0.49	1.06	0.00	0.00	0.81	0.00
time (sec)	N/A	0.311	0.140	0.985	0.136	0.131	0.000	0.000	0.277	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	323	395	668	0	0	0	0	152	0
N.S.	1	0.89	1.09	1.85	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	1.414	2.053	0.857	0.000	0.000	0.000	0.000	0.243	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	332	463	673	0	0	0	0	176	0
N.S.	1	0.86	1.20	1.74	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	1.387	1.364	0.839	0.000	0.000	0.000	0.000	0.236	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	340	615	544	0	0	0	0	178	0
N.S.	1	0.87	1.58	1.40	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	1.405	2.481	0.840	0.000	0.000	0.000	0.000	0.329	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	33	30	26	31	27	13	0
N.S.	1	1.00	0.88	0.97	0.88	0.76	0.91	0.79	0.38	0.00
time (sec)	N/A	0.237	0.006	0.009	0.124	0.121	0.567	0.131	0.303	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	71	98	0	0	97	0	51	0
N.S.	1	1.00	1.04	1.44	0.00	0.00	1.43	0.00	0.75	0.00
time (sec)	N/A	0.307	0.098	0.436	0.000	0.000	3.929	0.000	0.322	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	96	64	76	85	60	88	79	23	0
N.S.	1	1.09	0.73	0.86	0.97	0.68	1.00	0.90	0.26	0.00
time (sec)	N/A	0.495	0.046	0.615	0.123	0.111	0.388	0.146	0.318	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	49	96	61	44	71	0	23	0
N.S.	1	1.06	0.68	1.33	0.85	0.61	0.99	0.00	0.32	0.00
time (sec)	N/A	0.385	0.037	0.340	0.115	0.113	0.311	0.000	0.255	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	43	41	56	39	49	47	23	0
N.S.	1	1.00	0.86	0.82	1.12	0.78	0.98	0.94	0.46	0.00
time (sec)	N/A	0.340	0.025	0.316	0.119	0.126	0.248	0.135	0.302	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	30	30	63	28	26	31	28	26	0
N.S.	1	1.03	1.03	2.17	0.97	0.90	1.07	0.97	0.90	0.00
time (sec)	N/A	0.228	0.021	0.286	0.120	0.117	0.207	0.135	0.293	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	15	11	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	1.15	0.85	0.85	0.85
time (sec)	N/A	0.203	0.006	0.107	0.120	0.119	0.183	0.116	0.297	0.404

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	60	56	119	0	0	0	0	23	0
N.S.	1	1.15	1.08	2.29	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.405	0.050	0.352	0.000	0.000	0.000	0.000	0.273	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	29	29	31	27	28	0	68	23	0
N.S.	1	1.04	1.04	1.11	0.96	1.00	0.00	2.43	0.82	0.00
time (sec)	N/A	0.256	0.025	0.380	0.108	0.110	0.000	0.141	0.326	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	101	126	186	0	0	0	0	23	0
N.S.	1	1.03	1.29	1.90	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.585	0.445	1.119	0.000	0.000	0.000	0.000	0.236	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	236	133	178	180	150	0	0	93	0
N.S.	1	1.05	0.59	0.79	0.80	0.67	0.00	0.00	0.42	0.00
time (sec)	N/A	0.756	0.252	0.875	0.137	0.129	0.000	0.000	0.233	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	208	161	377	0	0	0	135	82	0
N.S.	1	1.04	0.80	1.88	0.00	0.00	0.00	0.68	0.41	0.00
time (sec)	N/A	0.713	0.966	0.514	0.000	0.000	0.000	0.406	0.229	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	152	106	161	121	120	0	0	73	0
N.S.	1	1.03	0.72	1.09	0.82	0.81	0.00	0.00	0.49	0.00
time (sec)	N/A	0.459	0.230	0.751	0.125	0.126	0.000	0.000	0.270	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	131	268	0	0	0	80	61	0
N.S.	1	1.00	1.06	2.16	0.00	0.00	0.00	0.65	0.49	0.00
time (sec)	N/A	0.441	0.861	0.447	0.000	0.000	0.000	0.372	0.216	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-2)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	68	78	133	59	92	0	0	45	0
N.S.	1	1.01	1.16	1.99	0.88	1.37	0.00	0.00	0.67	0.00
time (sec)	N/A	0.260	0.168	0.559	0.154	0.108	0.000	0.000	0.225	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	50	86	44	0	0	24	27	0
N.S.	1	1.00	1.02	1.76	0.90	0.00	0.00	0.49	0.55	0.00
time (sec)	N/A	0.214	0.014	0.000	0.116	0.000	0.000	0.190	0.237	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	97	130	196	0	0	0	0	41	0
N.S.	1	0.67	0.90	1.35	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.431	0.325	0.596	0.000	0.000	0.000	0.000	0.199	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	67	80	204	104	218	0	0	49	0
N.S.	1	1.02	1.21	3.09	1.58	3.30	0.00	0.00	0.74	0.00
time (sec)	N/A	0.281	0.213	0.620	0.120	0.166	0.000	0.000	0.257	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	174	246	292	0	0	0	0	69	0
N.S.	1	0.76	1.07	1.28	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.675	1.590	0.717	0.000	0.000	0.000	0.000	0.193	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	146	135	851	124	432	0	0	73	0
N.S.	1	0.99	0.92	5.79	0.84	2.94	0.00	0.00	0.50	0.00
time (sec)	N/A	0.460	0.253	0.730	0.122	0.170	0.000	0.000	0.241	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	158	158	425	0	441	0	0	108	0
N.S.	1	0.71	0.71	1.92	0.00	2.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.468	0.466	0.876	0.000	0.194	0.000	0.000	0.228	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	231	199	432	0	0	0	969	189	0
N.S.	1	1.08	0.93	2.02	0.00	0.00	0.00	4.53	0.88	0.00
time (sec)	N/A	0.775	0.459	0.737	0.000	0.000	0.000	0.869	0.232	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	108	137	308	142	383	0	0	98	0
N.S.	1	0.76	0.96	2.17	1.00	2.70	0.00	0.00	0.69	0.00
time (sec)	N/A	0.387	0.282	0.672	0.133	0.148	0.000	0.000	0.233	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	160	274	0	0	0	111	124	0
N.S.	1	1.00	1.19	2.03	0.00	0.00	0.00	0.82	0.92	0.00
time (sec)	N/A	0.459	0.315	0.630	0.000	0.000	0.000	0.722	0.255	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	72	98	194	0	279	0	0	85	0
N.S.	1	0.99	1.34	2.66	0.00	3.82	0.00	0.00	1.16	0.00
time (sec)	N/A	0.283	0.211	0.563	0.000	0.160	0.000	0.000	0.235	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	177	62	0	0	0	80	0
N.S.	1	1.00	0.96	2.21	0.78	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.249	0.055	0.492	0.118	0.000	0.000	0.000	0.239	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	166	279	298	0	0	0	0	114	0
N.S.	1	0.75	1.27	1.35	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.730	0.786	0.759	0.000	0.000	0.000	0.000	0.237	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	118	171	229	129	0	0	0	96	0
N.S.	1	0.79	1.14	1.53	0.86	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.419	0.275	0.697	0.130	0.000	0.000	0.000	0.222	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	257	396	361	0	0	0	0	147	0
N.S.	1	0.81	1.25	1.14	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	1.223	4.532	0.879	0.000	0.000	0.000	0.000	0.225	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	173	224	1049	0	0	0	0	108	0
N.S.	1	0.73	0.94	4.41	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.566	0.286	0.852	0.000	0.000	0.000	0.000	0.225	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	358	253	429	0	0	0	1418	261	0
N.S.	1	1.22	0.86	1.46	0.00	0.00	0.00	4.84	0.89	0.00
time (sec)	N/A	1.380	0.659	1.113	0.000	0.000	0.000	1.447	0.210	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	171	170	413	0	481	0	0	217	0
N.S.	1	0.78	0.78	1.89	0.00	2.20	0.00	0.00	0.99	0.00
time (sec)	N/A	0.503	0.394	0.824	0.000	0.169	0.000	0.000	0.272	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	252	219	1511	0	0	0	217	252	0
N.S.	1	1.19	1.03	7.13	0.00	0.00	0.00	1.02	1.19	0.00
time (sec)	N/A	0.832	0.493	0.835	0.000	0.000	0.000	1.198	0.232	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	134	142	251	160	421	0	0	208	0
N.S.	1	0.89	0.95	1.67	1.07	2.81	0.00	0.00	1.39	0.00
time (sec)	N/A	0.389	0.310	0.720	0.147	0.162	0.000	0.000	0.246	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	104	101	1243	153	0	0	118	196	0
N.S.	1	0.83	0.81	9.94	1.22	0.00	0.00	0.94	1.57	0.00
time (sec)	N/A	0.357	0.259	0.810	0.140	0.000	0.000	0.727	0.235	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	103	120	222	0	373	0	0	195	0
N.S.	1	0.87	1.01	1.87	0.00	3.13	0.00	0.00	1.64	0.00
time (sec)	N/A	0.293	0.235	0.690	0.000	0.153	0.000	0.000	0.240	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	159	113	472	141	0	0	0	197	0
N.S.	1	1.03	0.73	3.06	0.92	0.00	0.00	0.00	1.28	0.00
time (sec)	N/A	0.391	0.102	0.608	0.136	0.000	0.000	0.000	0.236	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	268	352	350	0	0	0	0	284	0
N.S.	1	0.92	1.21	1.20	0.00	0.00	0.00	0.00	0.98	0.00
time (sec)	N/A	1.026	3.772	0.869	0.000	0.000	0.000	0.000	0.258	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	169	238	1347	0	0	0	0	217	0
N.S.	1	0.75	1.06	6.01	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.491	0.292	0.880	0.000	0.000	0.000	0.000	0.262	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	383	486	428	0	0	0	0	315	0
N.S.	1	0.88	1.12	0.99	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	1.584	3.309	0.996	0.000	0.000	0.000	0.000	0.245	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	222	277	1878	255	0	0	0	226	0
N.S.	1	0.72	0.89	6.06	0.82	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.658	0.355	0.893	0.133	0.000	0.000	0.000	0.232	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	223	111	410	149	0	0	122	88	0
N.S.	1	1.06	0.53	1.95	0.71	0.00	0.00	0.58	0.42	0.00
time (sec)	N/A	0.534	0.148	1.057	0.137	0.000	0.000	0.175	0.219	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	234	0	0	0	0	0	120	0
N.S.	1	1.00	2.96	0.00	0.00	0.00	0.00	0.00	1.52	0.00
time (sec)	N/A	0.299	0.750	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	235	0	0	0	0	0	125	0
N.S.	1	1.00	1.72	0.00	0.00	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	0.349	22.180	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	315	323	276	0	0	0	0	0	613	0
N.S.	1	1.03	0.88	0.00	0.00	0.00	0.00	0.00	1.95	0.00
time (sec)	N/A	1.904	0.454	0.000	0.000	0.000	0.000	0.000	0.292	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	216	204	0	0	0	0	0	346	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	1.59	0.00
time (sec)	N/A	0.556	0.259	0.000	0.000	0.000	0.000	0.000	0.261	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	126	127	0	0	0	0	0	155	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.00	0.00	1.20	0.00
time (sec)	N/A	0.354	0.218	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	30	29	37	0	49	27
N.S.	1	1.00	1.08	1.00	1.20	1.16	1.48	0.00	1.96	1.08
time (sec)	N/A	0.260	2.298	1.716	0.424	0.111	2.685	0.000	0.224	0.558

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	28	41	54	0	63	27
N.S.	1	1.00	1.08	1.00	1.12	1.64	2.16	0.00	2.52	1.08
time (sec)	N/A	0.442	8.260	1.510	0.436	0.107	15.712	0.000	0.236	0.529

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	30	55	73	0	81	27
N.S.	1	1.00	1.08	1.00	1.20	2.20	2.92	0.00	3.24	1.08
time (sec)	N/A	0.678	11.697	0.965	0.452	0.105	106.722	0.000	0.223	0.565

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	635	448	0	0	0	0	0	0	159	0
N.S.	1	0.71	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.317	0.000	0.000	0.000	0.000	0.000	0.000	0.274	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	399	324	0	0	0	0	0	0	103	0
N.S.	1	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.843	0.000	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	216	0	0	0	0	0	0	46	0
N.S.	1	0.88	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.536	0.000	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	162	180	0	0	0	0	0	52	0
N.S.	1	0.99	1.10	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.359	0.613	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	272	269	0	0	0	0	0	0	99	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.595	0.000	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	408	388	0	0	0	0	0	0	131	0
N.S.	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.932	0.000	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	99	114	0	0	0	0	0	23	0
N.S.	1	0.99	1.14	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.272	0.919	0.000	0.000	0.000	0.000	0.000	0.284	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	332	203	275	453	229	393	319	175	0
N.S.	1	1.14	0.70	0.95	1.56	0.79	1.36	1.10	0.60	0.00
time (sec)	N/A	1.433	0.220	0.350	0.131	0.139	0.872	0.168	0.245	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	334	195	329	0	211	337	296	164	0
N.S.	1	1.65	0.97	1.63	0.00	1.04	1.67	1.47	0.81	0.00
time (sec)	N/A	1.394	0.217	0.273	0.000	0.101	0.663	0.167	0.276	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	242	179	279	354	194	318	256	154	0
N.S.	1	1.15	0.85	1.32	1.68	0.92	1.51	1.21	0.73	0.00
time (sec)	N/A	1.071	0.187	0.271	0.134	0.117	0.505	0.152	0.264	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	153	158	198	0	176	274	232	184	0
N.S.	1	1.04	1.07	1.35	0.00	1.20	1.86	1.58	1.25	0.00
time (sec)	N/A	0.582	0.247	0.247	0.000	0.105	0.366	0.160	0.240	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	141	138	173	234	146	230	185	145	0
N.S.	1	1.10	1.08	1.35	1.83	1.14	1.80	1.45	1.13	0.00
time (sec)	N/A	0.536	0.116	0.146	0.130	0.120	0.212	0.160	0.239	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	197	246	327	0	0	0	0	151	0
N.S.	1	1.11	1.38	1.84	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	1.131	0.256	0.482	0.000	0.000	0.000	0.000	0.257	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	183	215	270	0	0	0	0	142	0
N.S.	1	1.23	1.44	1.81	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	1.135	0.338	0.442	0.000	0.000	0.000	0.000	0.256	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	198	258	331	0	0	0	0	113	0
N.S.	1	1.03	1.34	1.72	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	1.469	0.293	0.387	0.000	0.000	0.000	0.000	0.232	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	242	274	276	0	0	0	0	122	0
N.S.	1	1.38	1.56	1.57	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	1.177	0.641	0.405	0.000	0.000	0.000	0.000	0.238	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	518	253	530	781	337	568	475	243	0
N.S.	1	1.31	0.64	1.34	1.98	0.85	1.44	1.20	0.62	0.00
time (sec)	N/A	2.395	0.257	0.507	0.160	0.113	1.570	0.176	0.244	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	594	246	439	0	319	520	452	232	0
N.S.	1	1.97	0.81	1.45	0.00	1.06	1.72	1.50	0.77	0.00
time (sec)	N/A	2.420	0.250	0.423	0.000	0.115	1.290	0.171	0.274	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	389	229	399	634	296	488	406	222	0
N.S.	1	1.25	0.74	1.29	2.05	0.95	1.57	1.31	0.72	0.00
time (sec)	N/A	1.875	1.278	0.283	0.139	0.114	0.892	0.172	0.264	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	209	210	265	0	278	435	382	252	0
N.S.	1	0.96	0.96	1.22	0.00	1.28	2.00	1.75	1.16	0.00
time (sec)	N/A	0.788	1.259	0.269	0.000	0.109	0.676	0.174	0.267	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	245	193	275	466	247	394	329	213	0
N.S.	1	1.12	0.88	1.26	2.13	1.13	1.80	1.50	0.97	0.00
time (sec)	N/A	0.904	0.112	0.000	0.140	0.110	0.432	0.167	0.272	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	347	345	424	0	0	0	0	221	0
N.S.	1	1.25	1.25	1.53	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	2.233	0.485	0.513	0.000	0.000	0.000	0.000	0.243	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	321	349	386	0	0	0	0	213	0
N.S.	1	1.29	1.40	1.55	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	2.006	0.707	0.489	0.000	0.000	0.000	0.000	0.244	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	346	405	503	0	0	0	0	246	0
N.S.	1	1.21	1.41	1.75	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	2.348	0.409	0.603	0.000	0.000	0.000	0.000	0.282	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	380	384	396	0	0	0	0	224	0
N.S.	1	1.42	1.43	1.48	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	2.209	0.637	0.520	0.000	0.000	0.000	0.000	0.276	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	708	301	618	1141	413	707	595	309	0
N.S.	1	1.49	0.63	1.30	2.40	0.87	1.49	1.25	0.65	0.00
time (sec)	N/A	3.531	0.308	0.508	0.206	0.125	2.933	0.180	0.281	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	916	293	518	0	395	660	572	298	0
N.S.	1	2.39	0.76	1.35	0.00	1.03	1.72	1.49	0.78	0.00
time (sec)	N/A	3.732	0.339	0.459	0.000	0.124	2.272	0.185	0.279	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	538	277	524	946	372	631	526	288	0
N.S.	1	1.38	0.71	1.34	2.42	0.95	1.61	1.35	0.74	0.00
time (sec)	N/A	2.703	1.362	0.303	0.166	0.122	1.575	0.184	0.247	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	261	258	342	0	354	578	501	318	0
N.S.	1	0.94	0.93	1.23	0.00	1.28	2.09	1.81	1.15	0.00
time (sec)	N/A	0.981	1.478	0.276	0.000	0.149	1.234	0.174	0.267	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	351	241	384	730	323	529	449	279	0
N.S.	1	1.18	0.81	1.29	2.45	1.08	1.78	1.51	0.94	0.00
time (sec)	N/A	1.325	0.195	0.000	0.147	0.115	0.809	0.169	0.249	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	544	440	499	0	0	0	0	287	0
N.S.	1	1.51	1.22	1.39	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	4.255	0.855	0.589	0.000	0.000	0.000	0.000	0.224	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	473	462	434	0	0	0	0	280	0
N.S.	1	1.44	1.40	1.32	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	3.346	0.928	0.576	0.000	0.000	0.000	0.000	0.238	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	559	517	579	0	0	0	0	315	0
N.S.	1	1.41	1.31	1.46	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	3.965	0.853	0.749	0.000	0.000	0.000	0.000	0.227	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	564	492	504	0	0	0	0	294	0
N.S.	1	1.62	1.41	1.45	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	3.399	0.858	0.748	0.000	0.000	0.000	0.000	0.250	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	305	389	533	0	0	0	0	114	0
N.S.	1	1.03	1.31	1.79	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	1.906	0.486	0.574	0.000	0.000	0.000	0.000	0.229	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	208	327	391	0	0	0	0	214	0
N.S.	1	0.99	1.56	1.86	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	1.550	0.392	0.467	0.000	0.000	0.000	0.000	0.245	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	186	293	438	0	0	0	0	157	0
N.S.	1	0.85	1.34	2.01	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	1.176	0.722	0.482	0.000	0.000	0.000	0.000	0.246	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	209	327	0	0	0	0	92	0
N.S.	1	0.97	1.79	2.79	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.579	0.591	0.304	0.000	0.000	0.000	0.000	0.252	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	122	215	314	0	0	0	0	85	0
N.S.	1	0.78	1.38	2.01	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.534	0.054	0.332	0.000	0.000	0.000	0.000	0.222	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	126	313	460	0	0	0	0	92	0
N.S.	1	0.96	2.39	3.51	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.632	0.488	0.420	0.000	0.000	0.000	0.000	0.213	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	212	410	452	0	0	0	0	102	0
N.S.	1	0.89	1.72	1.90	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	1.634	0.789	0.621	0.000	0.000	0.000	0.000	0.262	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	191	453	602	0	0	0	0	128	0
N.S.	1	0.91	2.16	2.87	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	1.460	0.737	0.559	0.000	0.000	0.000	0.000	0.250	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	352	739	561	0	0	0	0	126	0
N.S.	1	1.06	2.22	1.68	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	2.588	7.323	0.918	0.000	0.000	0.000	0.000	0.226	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	299	513	599	0	0	0	0	262	0
N.S.	1	1.00	1.71	2.00	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	1.906	4.671	0.639	0.000	0.000	0.000	0.000	0.243	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	223	344	506	0	0	0	0	253	0
N.S.	1	0.98	1.52	2.23	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	1.653	0.924	0.460	0.000	0.000	0.000	0.000	0.230	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	202	408	423	0	0	0	0	249	0
N.S.	1	0.87	1.75	1.82	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	1.229	3.969	0.473	0.000	0.000	0.000	0.000	0.223	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	85	106	173	292	151	0	226	167	0
N.S.	1	0.96	1.19	1.94	3.28	1.70	0.00	2.54	1.88	0.00
time (sec)	N/A	0.345	0.583	0.258	0.142	0.140	0.000	0.189	0.237	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	197	401	423	0	0	0	0	233	0
N.S.	1	0.86	1.74	1.84	0.00	0.00	0.00	0.00	1.01	0.00
time (sec)	N/A	1.158	2.614	0.293	0.000	0.000	0.000	0.000	0.258	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	206	474	637	0	0	0	0	252	0
N.S.	1	0.98	2.25	3.02	0.00	0.00	0.00	0.00	1.19	0.00
time (sec)	N/A	1.104	1.089	0.589	0.000	0.000	0.000	0.000	0.244	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	329	612	727	0	0	0	0	256	0
N.S.	1	1.02	1.89	2.24	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	2.937	6.413	0.956	0.000	0.000	0.000	0.000	0.233	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	323	529	673	0	0	0	0	292	0
N.S.	1	1.20	1.96	2.49	0.00	0.00	0.00	0.00	1.08	0.00
time (sec)	N/A	1.994	1.486	0.711	0.000	0.000	0.000	0.000	0.246	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	522	920	703	0	0	0	0	279	0
N.S.	1	1.19	2.10	1.60	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	4.583	8.030	0.811	0.000	0.000	0.000	0.000	0.253	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	344	683	549	0	0	0	0	439	0
N.S.	1	1.00	1.99	1.60	0.00	0.00	0.00	0.00	1.28	0.00
time (sec)	N/A	2.052	7.203	0.669	0.000	0.000	0.000	0.000	0.254	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	187	192	387	0	243	0	318	311	0
N.S.	1	1.09	1.12	2.25	0.00	1.41	0.00	1.85	1.81	0.00
time (sec)	N/A	0.886	0.341	0.494	0.000	0.130	0.000	0.207	0.243	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	310	656	549	0	0	0	0	437	0
N.S.	1	0.91	1.92	1.61	0.00	0.00	0.00	0.00	1.28	0.00
time (sec)	N/A	1.910	7.643	0.712	0.000	0.000	0.000	0.000	0.252	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	140	162	270	0	235	0	395	304	0
N.S.	1	0.93	1.08	1.80	0.00	1.57	0.00	2.63	2.03	0.00
time (sec)	N/A	0.465	0.688	0.288	0.000	0.184	0.000	0.206	0.234	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	302	652	549	0	0	0	0	417	0
N.S.	1	0.91	1.96	1.65	0.00	0.00	0.00	0.00	1.26	0.00
time (sec)	N/A	1.807	7.541	0.603	0.000	0.000	0.000	0.000	0.221	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	336	584	789	0	0	0	0	455	0
N.S.	1	1.14	1.97	2.67	0.00	0.00	0.00	0.00	1.54	0.00
time (sec)	N/A	2.002	3.122	0.713	0.000	0.000	0.000	0.000	0.233	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	495	865	731	0	0	0	0	454	0
N.S.	1	1.15	2.02	1.70	0.00	0.00	0.00	0.00	1.06	0.00
time (sec)	N/A	4.213	8.425	0.824	0.000	0.000	0.000	0.000	0.255	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	498	714	935	0	0	0	0	505	0
N.S.	1	1.24	1.77	2.32	0.00	0.00	0.00	0.00	1.25	0.00
time (sec)	N/A	3.282	4.006	0.812	0.000	0.000	0.000	0.000	0.254	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	572	773	1135	820	0	0	0	0	477	0
N.S.	1	1.35	1.98	1.43	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	6.347	9.238	0.968	0.000	0.000	0.000	0.000	0.246	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	365	229	503	311	278	0	0	127	0
N.S.	1	0.98	0.61	1.34	0.83	0.74	0.00	0.00	0.34	0.00
time (sec)	N/A	1.449	0.267	1.530	0.170	0.114	0.000	0.000	0.275	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	304	221	678	0	0	0	368	115	0
N.S.	1	1.00	0.73	2.24	0.00	0.00	0.00	1.21	0.38	0.00
time (sec)	N/A	1.218	1.212	0.379	0.000	0.000	0.000	0.516	0.233	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	149	173	439	188	209	0	0	100	0
N.S.	1	0.79	0.92	2.34	1.00	1.11	0.00	0.00	0.53	0.00
time (sec)	N/A	0.457	0.449	0.356	0.174	0.109	0.000	0.000	0.232	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	168	219	531	0	0	0	0	82	0
N.S.	1	0.88	1.14	2.77	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.569	0.661	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	250	420	582	0	0	0	0	88	0
N.S.	1	0.66	1.11	1.54	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.082	1.045	1.143	0.000	0.000	0.000	0.000	0.238	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	182	250	525	0	0	0	0	118	0
N.S.	1	0.80	1.10	2.31	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	1.116	1.137	0.563	0.000	0.000	0.000	0.000	0.229	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	262	453	630	0	0	0	0	102	0
N.S.	1	0.66	1.14	1.58	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.336	1.607	0.653	0.000	0.000	0.000	0.000	0.220	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	205	259	1910	0	0	0	0	104	0
N.S.	1	0.65	0.82	6.08	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.888	0.871	0.659	0.000	0.000	0.000	0.000	0.229	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	562	254	528	356	361	0	0	209	0
N.S.	1	1.12	0.50	1.05	0.71	0.72	0.00	0.00	0.42	0.00
time (sec)	N/A	2.721	0.340	0.803	0.155	0.144	0.000	0.000	0.269	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	510	408	1432	0	0	0	515	198	0
N.S.	1	1.21	0.97	3.40	0.00	0.00	0.00	1.22	0.47	0.00
time (sec)	N/A	2.246	2.192	0.613	0.000	0.000	0.000	0.941	0.263	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	189	200	465	236	296	0	0	183	0
N.S.	1	0.68	0.72	1.67	0.85	1.06	0.00	0.00	0.66	0.00
time (sec)	N/A	0.569	1.338	0.983	0.157	0.128	0.000	0.000	0.255	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	331	328	987	0	0	0	0	166	0
N.S.	1	1.09	1.08	3.24	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	1.017	0.712	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	394	585	843	0	0	0	0	170	0
N.S.	1	0.72	1.07	1.55	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	1.927	2.415	1.105	0.000	0.000	0.000	0.000	0.220	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	369	401	472	0	0	0	0	197	0
N.S.	1	0.87	0.95	1.11	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	1.830	3.873	0.656	0.000	0.000	0.000	0.000	0.232	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	590	393	794	924	0	0	0	0	201	0
N.S.	1	0.67	1.35	1.57	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	2.014	2.651	0.712	0.000	0.000	0.000	0.000	0.235	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	391	492	2117	0	0	0	0	223	0
N.S.	1	0.98	1.23	5.29	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	3.193	1.634	0.709	0.000	0.000	0.000	0.000	0.252	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	651	800	280	594	401	487	0	0	292	0
N.S.	1	1.23	0.43	0.91	0.62	0.75	0.00	0.00	0.45	0.00
time (sec)	N/A	3.977	0.378	0.846	0.152	0.156	0.000	0.000	0.278	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	765	486	2228	0	0	0	666	281	0
N.S.	1	1.38	0.87	4.01	0.00	0.00	0.00	1.20	0.51	0.00
time (sec)	N/A	3.631	3.022	0.841	0.000	0.000	0.000	1.386	0.259	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	228	226	531	281	406	0	0	265	0
N.S.	1	0.60	0.59	1.39	0.74	1.06	0.00	0.00	0.69	0.00
time (sec)	N/A	0.699	1.388	1.208	0.154	0.143	0.000	0.000	0.286	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	520	407	1463	0	0	0	0	249	0
N.S.	1	1.19	0.93	3.34	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	1.647	2.540	0.674	0.000	0.000	0.000	0.000	0.264	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	687	579	816	1173	0	0	0	0	253	0
N.S.	1	0.84	1.19	1.71	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	3.297	4.273	1.402	0.000	0.000	0.000	0.000	0.272	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	561	619	587	603	0	0	0	0	282	0
N.S.	1	1.10	1.05	1.07	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	3.297	2.850	0.856	0.000	0.000	0.000	0.000	0.276	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	740	575	988	1365	0	0	0	0	286	0
N.S.	1	0.78	1.34	1.84	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	3.415	2.439	0.850	0.000	0.000	0.000	0.000	0.250	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	673	680	2513	0	0	0	0	306	0
N.S.	1	1.14	1.15	4.25	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	5.098	3.002	0.884	0.000	0.000	0.000	0.000	0.254	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	413	244	507	365	277	0	0	133	0
N.S.	1	1.03	0.61	1.27	0.91	0.69	0.00	0.00	0.33	0.00
time (sec)	N/A	1.686	0.436	0.810	0.139	0.122	0.000	0.000	0.269	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	348	282	722	0	0	0	295	122	0
N.S.	1	1.03	0.84	2.14	0.00	0.00	0.00	0.88	0.36	0.00
time (sec)	N/A	1.326	1.431	0.506	0.000	0.000	0.000	0.742	0.239	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	246	190	479	251	211	0	0	111	0
N.S.	1	0.89	0.69	1.73	0.91	0.76	0.00	0.00	0.40	0.00
time (sec)	N/A	0.885	0.380	0.684	0.146	0.116	0.000	0.000	0.276	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	177	208	517	0	0	0	169	99	0
N.S.	1	0.86	1.01	2.51	0.00	0.00	0.00	0.82	0.48	0.00
time (sec)	N/A	0.648	1.469	0.421	0.000	0.000	0.000	0.643	0.253	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-2)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	100	138	316	131	147	0	0	100	0
N.S.	1	0.68	0.95	2.16	0.90	1.01	0.00	0.00	0.68	0.00
time (sec)	N/A	0.347	0.655	0.489	0.147	0.123	0.000	0.000	0.252	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	64	142	105	0	0	38	41	0
N.S.	1	1.00	1.31	2.90	2.14	0.00	0.00	0.78	0.84	0.00
time (sec)	N/A	0.236	0.017	0.000	0.131	0.000	0.000	0.223	0.244	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	159	273	394	0	0	0	0	74	0
N.S.	1	0.62	1.06	1.53	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.678	0.638	0.799	0.000	0.000	0.000	0.000	0.233	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	138	158	387	0	0	0	0	83	0
N.S.	1	0.75	0.86	2.11	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.667	0.733	0.552	0.000	0.000	0.000	0.000	0.245	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	265	528	625	0	0	0	0	107	0
N.S.	1	0.66	1.31	1.55	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	1.395	1.586	0.681	0.000	0.000	0.000	0.000	0.262	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	255	269	2186	0	0	0	0	111	0
N.S.	1	0.80	0.84	6.85	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	1.314	0.654	0.648	0.000	0.000	0.000	0.000	0.246	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	549	497	419	1077	0	0	0	0	180	0
N.S.	1	0.91	0.76	1.96	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	2.685	0.817	0.805	0.000	0.000	0.000	0.000	0.243	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	396	312	945	0	0	0	0	397	0
N.S.	1	0.93	0.74	2.23	0.00	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	1.997	1.837	0.750	0.000	0.000	0.000	0.000	0.275	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	279	336	658	0	0	0	0	168	0
N.S.	1	0.68	0.82	1.60	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	1.362	0.681	0.614	0.000	0.000	0.000	0.000	0.220	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	195	284	550	0	0	0	0	212	0
N.S.	1	0.78	1.14	2.20	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.993	1.108	0.579	0.000	0.000	0.000	0.000	0.247	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	129	242	391	0	0	0	0	151	0
N.S.	1	0.62	1.16	1.88	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.598	0.891	0.526	0.000	0.000	0.000	0.000	0.264	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	140	167	399	0	0	0	0	142	0
N.S.	1	0.72	0.86	2.05	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.646	0.285	0.484	0.000	0.000	0.000	0.000	0.230	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	467	285	686	640	0	0	0	0	181	0
N.S.	1	0.61	1.47	1.37	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	2.120	1.482	0.755	0.000	0.000	0.000	0.000	0.222	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	277	323	517	0	0	0	0	164	0
N.S.	1	0.83	0.97	1.55	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	1.893	1.209	0.653	0.000	0.000	0.000	0.000	0.218	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	457	1163	766	0	0	0	0	221	0
N.S.	1	0.72	1.83	1.21	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	4.355	9.973	0.843	0.000	0.000	0.000	0.000	0.246	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	464	460	2850	0	0	0	0	180	0
N.S.	1	0.96	0.95	5.90	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	3.250	0.928	0.786	0.000	0.000	0.000	0.000	0.226	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	550	532	803	0	0	0	0	392	0
N.S.	1	1.01	0.97	1.47	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	2.450	1.456	0.790	0.000	0.000	0.000	0.000	0.246	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	425	349	4047	0	0	0	0	429	0
N.S.	1	1.01	0.83	9.61	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	1.872	1.324	0.802	0.000	0.000	0.000	0.000	0.262	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	334	294	493	0	0	0	0	381	0
N.S.	1	1.01	0.89	1.48	0.00	0.00	0.00	0.00	1.15	0.00
time (sec)	N/A	2.052	2.467	0.671	0.000	0.000	0.000	0.000	0.230	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	217	305	3439	0	0	0	0	364	0
N.S.	1	0.65	0.92	10.36	0.00	0.00	0.00	0.00	1.10	0.00
time (sec)	N/A	0.988	1.373	0.756	0.000	0.000	0.000	0.000	0.215	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	189	403	456	0	0	0	0	362	0
N.S.	1	0.64	1.37	1.55	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.779	1.594	0.622	0.000	0.000	0.000	0.000	0.202	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	270	322	2227	0	0	0	0	361	0
N.S.	1	0.87	1.04	7.16	0.00	0.00	0.00	0.00	1.16	0.00
time (sec)	N/A	1.345	0.371	0.625	0.000	0.000	0.000	0.000	0.211	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	473	782	741	0	0	0	0	460	0
N.S.	1	0.82	1.36	1.28	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	4.241	10.188	0.869	0.000	0.000	0.000	0.000	0.239	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	455	434	3779	0	0	0	0	392	0
N.S.	1	1.01	0.96	8.36	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	3.275	1.947	0.757	0.000	0.000	0.000	0.000	0.231	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	752	0	1292	905	0	0	0	0	500	0
N.S.	1	0.00	1.72	1.20	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.000	9.126	0.895	0.000	0.000	0.000	0.000	0.242	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	538	723	441	5230	0	0	0	0	405	0
N.S.	1	1.34	0.82	9.72	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	4.996	2.485	0.791	0.000	0.000	0.000	0.000	0.230	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	226	105	129	0	85	153	133	25	0
N.S.	1	1.44	0.67	0.82	0.00	0.54	0.97	0.85	0.16	0.00
time (sec)	N/A	0.963	0.075	0.331	0.000	0.127	0.520	0.150	0.225	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	160	81	127	105	65	128	0	25	0
N.S.	1	1.27	0.64	1.01	0.83	0.52	1.02	0.00	0.20	0.00
time (sec)	N/A	0.712	0.046	0.277	0.147	0.100	0.395	0.000	0.217	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	98	76	71	0	59	85	75	25	0
N.S.	1	1.10	0.85	0.80	0.00	0.66	0.96	0.84	0.28	0.00
time (sec)	N/A	0.479	0.038	0.343	0.000	0.103	0.319	0.153	0.256	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	57	51	80	51	35	56	51	45	0
N.S.	1	1.04	0.93	1.45	0.93	0.64	1.02	0.93	0.82	0.00
time (sec)	N/A	0.289	0.027	0.243	0.112	0.098	0.263	0.147	0.225	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	17	11	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	1.31	0.85	0.85	0.85
time (sec)	N/A	0.188	0.007	0.124	0.109	0.087	0.214	0.125	0.228	0.384

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	112	106	0	0	0	0	0	25	0
N.S.	1	1.22	1.15	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.520	0.080	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	88	73	100	0	0	0	0	25	0
N.S.	1	1.16	0.96	1.32	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.506	0.149	0.355	0.000	0.000	0.000	0.000	0.233	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	177	176	279	0	0	0	0	25	0
N.S.	1	1.09	1.08	1.71	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.003	0.177	0.400	0.000	0.000	0.000	0.000	0.213	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	52	49	0	0	14	16	0
N.S.	1	1.00	1.00	1.24	1.17	0.00	0.00	0.33	0.38	0.00
time (sec)	N/A	0.219	0.062	0.190	0.122	0.000	0.000	0.173	0.221	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	123	111	213	0	0	0	0	52	0
N.S.	1	0.69	0.62	1.19	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.589	0.243	0.463	0.000	0.000	0.000	0.000	0.235	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	243	143	409	0	0	0	0	68	0
N.S.	1	0.86	0.51	1.45	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.188	0.524	0.525	0.000	0.000	0.000	0.000	0.230	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	387	170	600	0	0	0	0	90	0
N.S.	1	0.99	0.44	1.54	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.851	0.695	0.629	0.000	0.000	0.000	0.000	0.247	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1312	1021	0	0	0	0	0	0	1135	0
N.S.	1	0.78	0.00	0.00	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	3.376	0.000	0.000	0.000	0.000	0.000	0.000	0.322	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	756	644	0	0	0	0	0	0	649	0
N.S.	1	0.85	0.00	0.00	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	1.997	0.000	0.000	0.000	0.000	0.000	0.000	0.270	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	351	0	0	0	0	0	0	297	0
N.S.	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	1.002	0.000	0.000	0.000	0.000	0.000	0.000	0.259	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	32	43	66	0	80	29
N.S.	1	1.00	1.07	1.00	1.19	1.59	2.44	0.00	2.96	1.07
time (sec)	N/A	0.281	3.884	0.500	0.582	0.122	3.567	0.000	0.220	0.469

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	30	55	92	0	102	29
N.S.	1	1.00	1.07	1.00	1.11	2.04	3.41	0.00	3.78	1.07
time (sec)	N/A	1.147	10.773	0.533	0.622	0.111	14.632	0.000	0.259	0.361

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	32	69	119	0	128	29
N.S.	1	1.00	1.07	1.00	1.19	2.56	4.41	0.00	4.74	1.07
time (sec)	N/A	1.983	15.016	0.586	0.602	0.102	98.588	0.000	0.223	0.363

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	134	0	0	264	29
N.S.	1	1.00	1.07	0.93	1.00	4.62	0.00	0.00	9.10	1.00
time (sec)	N/A	3.223	1.278	9.342	0.763	0.133	0.000	0.000	0.313	0.442

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	85	0	0	171	29
N.S.	1	1.00	1.07	0.93	1.00	2.93	0.00	0.00	5.90	1.00
time (sec)	N/A	1.676	0.708	3.965	0.572	0.122	0.000	0.000	0.294	0.436

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	41	31	0	77	29
N.S.	1	1.00	1.07	0.93	1.00	1.41	1.07	0.00	2.66	1.00
time (sec)	N/A	0.876	0.546	1.588	0.420	0.098	18.077	0.000	0.232	0.534

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	56	31	0	85	29
N.S.	1	1.00	1.07	0.93	1.00	1.93	1.07	0.00	2.93	1.00
time (sec)	N/A	0.333	3.470	0.937	0.435	0.115	7.178	0.000	0.232	0.531

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	68	31	0	153	29
N.S.	1	1.00	1.07	0.93	1.00	2.34	1.07	0.00	5.28	1.00
time (sec)	N/A	0.344	2.279	0.802	0.485	0.108	10.799	0.000	0.258	0.437

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	82	0	0	202	29
N.S.	1	1.00	1.07	0.93	1.00	2.83	0.00	0.00	6.97	1.00
time (sec)	N/A	0.340	2.447	0.878	0.498	0.103	0.000	0.000	0.246	0.421

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	36	26	24	25	24
N.S.	1	1.00	1.08	0.92	1.00	1.50	1.08	1.00	1.04	1.00
time (sec)	N/A	0.262	0.639	0.930	0.281	0.109	1.069	0.360	0.222	0.319

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	581	171	278	284	201	364	316	108	0
N.S.	1	1.57	0.46	0.75	0.77	0.54	0.98	0.85	0.29	0.00
time (sec)	N/A	2.538	0.352	0.334	0.134	0.104	1.055	0.163	0.239	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	378	139	206	216	157	270	232	90	0
N.S.	1	1.38	0.51	0.75	0.79	0.58	0.99	0.85	0.33	0.00
time (sec)	N/A	1.459	0.092	0.270	0.130	0.118	0.540	0.160	0.264	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	202	101	132	128	94	156	132	72	0
N.S.	1	1.28	0.64	0.84	0.81	0.59	0.99	0.84	0.46	0.00
time (sec)	N/A	0.812	0.078	0.250	0.132	0.111	0.272	0.149	0.221	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	158	186	253	0	0	0	0	25	0
N.S.	1	0.79	0.93	1.26	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.616	0.166	0.316	0.000	0.000	0.000	0.000	0.208	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	277	320	414	0	0	0	0	32	0
N.S.	1	0.82	0.95	1.23	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.751	1.508	0.401	0.000	0.000	0.000	0.000	0.234	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	447	481	519	0	0	0	0	41	0
N.S.	1	0.98	1.06	1.14	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	2.887	3.843	0.457	0.000	0.000	0.000	0.000	0.236	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	610	179	875	0	0	0	0	82	0
N.S.	1	1.15	0.34	1.65	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	4.084	0.647	0.622	0.000	0.000	0.000	0.000	0.224	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	365	138	533	0	0	0	0	53	0
N.S.	1	1.01	0.38	1.47	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	2.178	0.244	0.450	0.000	0.000	0.000	0.000	0.222	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	169	87	260	0	0	0	0	23	0
N.S.	1	0.79	0.40	1.21	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.762	0.043	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	52	69	0	0	14	16	0
N.S.	1	1.00	1.00	1.24	1.64	0.00	0.00	0.33	0.38	0.00
time (sec)	N/A	0.225	0.011	0.000	0.133	0.000	0.000	0.185	0.252	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	152	181	278	0	0	0	0	52	0
N.S.	1	0.64	0.76	1.17	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.743	0.181	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	292	216	673	0	0	0	0	68	0
N.S.	1	0.75	0.56	1.73	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.671	0.561	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	547	485	294	1029	0	0	0	0	90	0
N.S.	1	0.89	0.54	1.88	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	3.071	0.955	0.870	0.000	0.000	0.000	0.000	0.253	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	36	26	24	25	24
N.S.	1	1.00	1.08	0.92	1.00	1.50	1.08	1.00	1.04	1.00
time (sec)	N/A	0.258	0.641	0.539	0.277	0.121	2.562	0.388	0.223	0.317

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	275	125	160	0	111	192	168	25	0
N.S.	1	1.44	0.65	0.84	0.00	0.58	1.01	0.88	0.13	0.00
time (sec)	N/A	1.530	0.060	0.430	0.000	0.108	0.705	0.168	0.222	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	211	100	180	130	85	155	0	25	0
N.S.	1	1.34	0.64	1.15	0.83	0.54	0.99	0.00	0.16	0.00
time (sec)	N/A	1.251	0.050	0.383	0.127	0.108	0.516	0.000	0.254	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	115	87	86	0	73	107	96	25	0
N.S.	1	1.07	0.81	0.80	0.00	0.68	1.00	0.90	0.23	0.00
time (sec)	N/A	0.719	0.040	0.440	0.000	0.100	0.395	0.172	0.250	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	73	61	107	63	46	68	62	55	0
N.S.	1	1.09	0.91	1.60	0.94	0.69	1.01	0.93	0.82	0.00
time (sec)	N/A	0.397	0.026	0.338	0.117	0.108	0.303	0.163	0.230	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	17	11	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	1.31	0.85	0.85	0.85
time (sec)	N/A	0.196	0.006	0.184	0.112	0.095	0.244	0.123	0.229	0.351

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	170	156	0	0	0	0	0	25	0
N.S.	1	1.23	1.13	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.692	0.102	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	119	98	135	0	0	0	0	25	0
N.S.	1	1.20	0.99	1.36	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.634	0.217	0.412	0.000	0.000	0.000	0.000	0.225	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	285	246	439	0	0	0	0	25	0
N.S.	1	1.08	0.93	1.66	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.413	0.552	0.503	0.000	0.000	0.000	0.000	0.221	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	51	43	42	0	0	0	59	64	0
N.S.	1	0.76	0.64	0.63	0.00	0.00	0.00	0.88	0.96	0.00
time (sec)	N/A	0.341	0.138	0.067	0.000	0.000	0.000	0.137	0.238	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	40	34	35	0	0	0	44	46	0
N.S.	1	0.80	0.68	0.70	0.00	0.00	0.00	0.88	0.92	0.00
time (sec)	N/A	0.325	0.013	0.050	0.000	0.000	0.000	0.136	0.259	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	27	23	22	0	0	0	25	28	0
N.S.	1	0.93	0.79	0.76	0.00	0.00	0.00	0.86	0.97	0.00
time (sec)	N/A	0.291	0.046	0.033	0.000	0.000	0.000	0.132	0.267	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	25	24	22	24	27	22
N.S.	1	1.00	1.10	1.00	1.25	1.20	1.10	1.20	1.35	1.10
time (sec)	N/A	0.208	0.079	0.007	0.208	0.082	0.713	0.509	0.220	0.328

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	23	36	36	23	36	22
N.S.	1	1.00	1.10	1.00	1.15	1.80	1.80	1.15	1.80	1.10
time (sec)	N/A	0.204	0.145	0.000	0.222	0.093	0.956	1.998	0.221	0.325

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	172	152	157	0	0	0	472	27	0
N.S.	1	0.83	0.74	0.76	0.00	0.00	0.00	2.29	0.13	0.00
time (sec)	N/A	0.632	0.285	0.210	0.000	0.000	0.000	0.158	0.258	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	155	133	140	0	0	0	0	27	0
N.S.	1	0.85	0.73	0.77	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.611	0.249	0.152	0.000	0.000	0.000	0.000	0.218	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	72	66	68	0	0	0	168	27	0
N.S.	1	0.88	0.80	0.83	0.00	0.00	0.00	2.05	0.33	0.00
time (sec)	N/A	0.442	0.184	0.266	0.000	0.000	0.000	0.159	0.213	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	105	91	92	0	0	0	171	25	0
N.S.	1	0.87	0.75	0.76	0.00	0.00	0.00	1.41	0.21	0.00
time (sec)	N/A	0.449	0.199	0.136	0.000	0.000	0.000	0.158	0.218	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	72	64	65	0	0	0	102	24	0
N.S.	1	0.88	0.78	0.79	0.00	0.00	0.00	1.24	0.29	0.00
time (sec)	N/A	0.393	0.162	0.161	0.000	0.000	0.000	0.157	0.219	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	28	26	0	27	28
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.93	0.00	0.96	1.00
time (sec)	N/A	0.577	2.420	0.659	0.223	0.095	0.703	0.000	0.230	0.344

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	32	27	28	59	28
N.S.	1	1.00	1.07	0.93	1.00	1.14	0.96	1.00	2.11	1.00
time (sec)	N/A	0.488	2.438	0.600	0.237	0.101	0.619	0.410	0.219	0.336

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	32	27	0	31	28
N.S.	1	1.00	1.07	0.93	1.00	1.14	0.96	0.00	1.11	1.00
time (sec)	N/A	0.294	6.825	1.148	0.215	0.091	0.718	0.000	0.240	0.346

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	32	27	28	31	28
N.S.	1	1.00	1.07	0.93	1.00	1.14	0.96	1.00	1.11	1.00
time (sec)	N/A	0.302	1.571	1.029	0.225	0.085	0.821	1.644	0.234	0.340

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	205	179	184	0	0	0	615	60	0
N.S.	1	0.84	0.73	0.75	0.00	0.00	0.00	2.51	0.24	0.00
time (sec)	N/A	0.678	0.531	0.173	0.000	0.000	0.000	0.160	0.269	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	172	165	157	0	0	0	473	60	0
N.S.	1	0.83	0.80	0.76	0.00	0.00	0.00	2.30	0.29	0.00
time (sec)	N/A	0.577	0.425	0.148	0.000	0.000	0.000	0.163	0.262	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	155	135	140	0	0	0	361	58	0
N.S.	1	0.85	0.74	0.77	0.00	0.00	0.00	1.97	0.32	0.00
time (sec)	N/A	0.524	0.425	0.137	0.000	0.000	0.000	0.163	0.238	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	122	121	111	0	0	0	253	57	0
N.S.	1	0.85	0.84	0.77	0.00	0.00	0.00	1.76	0.40	0.00
time (sec)	N/A	0.443	0.285	0.167	0.000	0.000	0.000	0.162	0.228	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	28	26	0	58	28
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.93	0.00	2.07	1.00
time (sec)	N/A	0.907	2.369	0.324	0.250	0.100	2.029	0.000	0.229	0.337

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	32	27	28	89	28
N.S.	1	1.00	1.07	0.93	1.00	1.14	0.96	1.00	3.18	1.00
time (sec)	N/A	0.740	2.769	0.281	0.242	0.093	1.556	0.406	0.240	0.341

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	32	27	0	64	28
N.S.	1	1.00	1.07	0.93	1.00	1.14	0.96	0.00	2.29	1.00
time (sec)	N/A	0.318	6.816	1.030	0.325	0.105	1.991	0.000	0.246	0.337

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	32	27	28	109	28
N.S.	1	1.00	1.07	0.93	1.00	1.14	0.96	1.00	3.89	1.00
time (sec)	N/A	0.314	1.546	1.245	0.267	0.088	2.428	1.545	0.255	0.346

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	205	179	186	0	0	0	745	91	0
N.S.	1	0.84	0.73	0.76	0.00	0.00	0.00	3.04	0.37	0.00
time (sec)	N/A	0.689	0.821	0.174	0.000	0.000	0.000	0.179	0.248	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	222	209	203	0	0	0	755	91	0
N.S.	1	0.83	0.78	0.76	0.00	0.00	0.00	2.82	0.34	0.00
time (sec)	N/A	0.671	0.751	0.152	0.000	0.000	0.000	0.168	0.272	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	205	179	186	0	0	0	613	89	0
N.S.	1	0.84	0.73	0.76	0.00	0.00	0.00	2.50	0.36	0.00
time (sec)	N/A	0.600	0.700	0.144	0.000	0.000	0.000	0.168	0.229	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	172	165	159	0	0	0	472	88	0
N.S.	1	0.83	0.80	0.77	0.00	0.00	0.00	2.29	0.43	0.00
time (sec)	N/A	0.515	0.514	0.170	0.000	0.000	0.000	0.169	0.233	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	45	26	0	89	28
N.S.	1	1.00	1.07	0.93	1.00	1.61	0.93	0.00	3.18	1.00
time (sec)	N/A	1.201	2.402	0.305	0.314	0.092	4.859	0.000	0.275	0.330

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	49	27	28	121	28
N.S.	1	1.00	1.07	0.93	1.00	1.75	0.96	1.00	4.32	1.00
time (sec)	N/A	1.078	2.573	0.289	0.318	0.088	3.820	0.458	0.253	0.339

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	49	27	0	93	28
N.S.	1	1.00	1.07	0.93	1.00	1.75	0.96	0.00	3.32	1.00
time (sec)	N/A	0.309	6.910	0.951	0.299	0.079	3.972	0.000	0.237	0.339

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	49	27	28	128	28
N.S.	1	1.00	1.07	0.93	1.00	1.75	0.96	1.00	4.57	1.00
time (sec)	N/A	0.325	1.657	1.187	0.322	0.083	4.302	1.766	0.276	0.341

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	37	31	30	0	0	0	35	25	0
N.S.	1	0.90	0.76	0.73	0.00	0.00	0.00	0.85	0.61	0.00
time (sec)	N/A	0.394	0.069	0.329	0.000	0.000	0.000	0.144	0.225	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	22	21	0	0	0	0	25	0
N.S.	1	0.96	0.81	0.78	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.372	0.053	0.216	0.000	0.000	0.000	0.000	0.201	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	20	19	0	0	0	23	25	0
N.S.	1	0.96	0.74	0.70	0.00	0.00	0.00	0.85	0.93	0.00
time (sec)	N/A	0.347	0.055	0.171	0.000	0.000	0.000	0.137	0.212	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	20	19	0	0	0	23	25	0
N.S.	1	0.96	0.74	0.70	0.00	0.00	0.00	0.85	0.93	0.00
time (sec)	N/A	0.340	0.012	0.000	0.000	0.000	0.000	0.161	0.228	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	10	10	11	0	0	0	10	23	0
N.S.	1	1.11	1.11	1.22	0.00	0.00	0.00	1.11	2.56	0.00
time (sec)	N/A	0.278	0.039	0.154	0.000	0.000	0.000	0.140	0.212	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	10	10	11	10	10	14	11	10	10
N.S.	1	1.11	1.11	1.22	1.11	1.11	1.56	1.22	1.11	1.11
time (sec)	N/A	0.197	0.010	0.065	0.112	0.088	0.217	0.123	0.225	0.369

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	24	24	25	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	1.00	1.00	1.04	1.00
time (sec)	N/A	0.259	1.527	0.408	0.222	0.096	0.417	0.138	0.209	0.314

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	37	26	24	25	24
N.S.	1	1.00	1.08	0.92	1.00	1.54	1.08	1.00	1.04	1.00
time (sec)	N/A	0.254	0.096	0.221	0.215	0.082	0.418	0.140	0.251	0.312

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	155	135	138	0	0	0	0	39	0
N.S.	1	0.85	0.74	0.75	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.587	0.412	0.218	0.000	0.000	0.000	0.000	0.229	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	122	108	111	0	0	0	255	39	0
N.S.	1	0.85	0.75	0.77	0.00	0.00	0.00	1.77	0.27	0.00
time (sec)	N/A	0.541	0.325	0.160	0.000	0.000	0.000	0.161	0.236	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	105	91	92	0	0	0	0	39	0
N.S.	1	0.87	0.75	0.76	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.506	0.311	0.165	0.000	0.000	0.000	0.000	0.225	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	72	62	63	0	0	0	104	39	0
N.S.	1	0.88	0.76	0.77	0.00	0.00	0.00	1.27	0.48	0.00
time (sec)	N/A	0.438	0.263	0.163	0.000	0.000	0.000	0.157	0.209	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	49	45	46	0	0	0	51	37	0
N.S.	1	0.91	0.83	0.85	0.00	0.00	0.00	0.94	0.69	0.00
time (sec)	N/A	0.477	0.172	0.133	0.000	0.000	0.000	0.155	0.197	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	17	17	18	17	17	49	18	17	17
N.S.	1	1.06	1.06	1.12	1.06	1.06	3.06	1.12	1.06	1.06
time (sec)	N/A	0.213	0.048	0.065	0.125	0.089	0.752	0.124	0.201	0.393

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	49	27	0	37	28
N.S.	1	1.00	1.07	0.93	1.00	1.75	0.96	0.00	1.32	1.00
time (sec)	N/A	0.299	4.109	0.227	0.242	0.103	1.134	0.000	0.244	0.339

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	53	29	28	41	28
N.S.	1	1.00	1.07	0.93	1.00	1.89	1.04	1.00	1.46	1.00
time (sec)	N/A	0.289	0.063	0.236	0.230	0.086	0.963	0.385	0.213	0.338

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	63	27	28	100	28
N.S.	1	1.00	1.07	0.93	1.00	2.25	0.96	1.00	3.57	1.00
time (sec)	N/A	0.310	9.914	0.322	0.258	0.083	1.768	0.640	0.224	0.349

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	61	26	0	83	26
N.S.	1	1.00	1.08	0.92	1.00	2.35	1.00	0.00	3.19	1.00
time (sec)	N/A	0.278	0.083	0.444	0.237	0.086	1.776	0.000	0.211	0.347

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	60	26	25	81	25
N.S.	1	1.00	1.08	0.92	1.00	2.40	1.04	1.00	3.24	1.00
time (sec)	N/A	0.224	0.095	0.178	0.249	0.088	2.041	0.445	0.265	0.351

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	64	27	0	83	28
N.S.	1	1.00	1.07	0.93	1.00	2.29	0.96	0.00	2.96	1.00
time (sec)	N/A	0.295	10.074	1.503	0.268	0.095	3.819	0.000	0.223	0.441

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	68	29	28	87	28
N.S.	1	1.00	1.07	0.93	1.00	2.43	1.04	1.00	3.11	1.00
time (sec)	N/A	0.317	10.157	0.861	0.258	0.092	2.601	3.800	0.222	0.502

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	86	27	28	125	28
N.S.	1	1.00	1.07	0.93	1.00	3.07	0.96	1.00	4.46	1.00
time (sec)	N/A	0.308	5.115	1.548	0.275	0.087	1.897	2.920	0.227	0.375

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	84	26	0	123	26
N.S.	1	1.00	1.08	0.92	1.00	3.23	1.00	0.00	4.73	1.00
time (sec)	N/A	0.264	0.091	1.540	0.256	0.088	1.921	0.000	0.213	0.314

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	83	26	25	121	25
N.S.	1	1.00	1.08	0.92	1.00	3.32	1.04	1.00	4.84	1.00
time (sec)	N/A	0.229	0.092	0.919	0.265	0.087	2.104	1.106	0.193	0.317

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	85	27	0	123	28
N.S.	1	1.00	1.07	0.93	1.00	3.04	0.96	0.00	4.39	1.00
time (sec)	N/A	0.296	7.969	2.868	0.270	0.093	4.591	0.000	0.212	0.350

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	89	29	28	127	28
N.S.	1	1.00	1.07	0.93	1.00	3.18	1.04	1.00	4.54	1.00
time (sec)	N/A	0.302	9.750	2.340	0.277	0.100	3.001	7.768	0.237	0.338

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	45	0	0	97	28
N.S.	1	1.00	1.07	0.93	1.00	1.61	0.00	0.00	3.46	1.00
time (sec)	N/A	0.326	1.240	1.363	0.364	0.096	0.000	0.000	0.257	0.315

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	38	27	0	63	28
N.S.	1	1.00	1.07	0.93	1.00	1.36	0.96	0.00	2.25	1.00
time (sec)	N/A	0.299	0.697	1.266	0.333	0.101	13.123	0.000	0.219	0.310

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	28	27	0	27	28
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.96	0.00	0.96	1.00
time (sec)	N/A	0.287	0.159	2.218	0.258	0.087	0.664	0.000	0.224	0.321

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	50	27	28	39	28
N.S.	1	1.00	1.07	0.93	1.00	1.79	0.96	1.00	1.39	1.00
time (sec)	N/A	0.290	1.010	0.931	0.236	0.110	0.879	0.266	0.251	0.312

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	63	27	28	85	28
N.S.	1	1.00	1.07	0.93	1.00	2.25	0.96	1.00	3.04	1.00
time (sec)	N/A	0.299	1.562	0.866	0.277	0.091	9.517	0.555	0.236	0.311

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	28	86	27	28	125	28
N.S.	1	1.00	1.07	0.93	1.00	3.07	0.96	1.00	4.46	1.00
time (sec)	N/A	0.299	2.489	0.879	0.290	0.107	16.050	1.280	0.225	0.316

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	36	24	24	25	24
N.S.	1	1.00	1.08	0.92	1.00	1.50	1.00	1.00	1.04	1.00
time (sec)	N/A	0.251	0.411	0.454	0.257	0.112	0.646	0.244	0.219	0.313

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	79	83	107	0	0	0	169	64	0
N.S.	1	0.83	0.87	1.13	0.00	0.00	0.00	1.78	0.67	0.00
time (sec)	N/A	0.474	0.172	0.152	0.000	0.000	0.000	0.164	0.225	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	68	70	83	0	0	0	124	46	0
N.S.	1	0.87	0.90	1.06	0.00	0.00	0.00	1.59	0.59	0.00
time (sec)	N/A	0.446	0.104	0.096	0.000	0.000	0.000	0.151	0.198	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	53	55	61	0	0	0	73	28	0
N.S.	1	0.96	1.00	1.11	0.00	0.00	0.00	1.33	0.51	0.00
time (sec)	N/A	0.403	0.110	0.065	0.000	0.000	0.000	0.153	0.217	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	154	24	26	24	31	22
N.S.	1	1.00	1.10	1.00	7.70	1.20	1.30	1.20	1.55	1.10
time (sec)	N/A	0.362	0.258	0.007	0.550	0.088	0.904	0.629	0.226	0.330

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	201	36	41	23	42	22
N.S.	1	1.00	1.10	1.00	10.05	1.80	2.05	1.15	2.10	1.10
time (sec)	N/A	0.337	0.221	0.000	0.509	0.101	1.494	2.240	0.214	0.334

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	62	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.65	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	139	42	29	0	41	28
N.S.	1	1.00	1.07	0.93	4.96	1.50	1.04	0.00	1.46	1.00
time (sec)	N/A	0.296	0.719	1.539	1.233	0.095	1.488	0.000	0.218	0.357

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	291	175	341	0	0	0	0	41	0
N.S.	1	1.36	0.82	1.59	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.939	0.394	0.184	0.000	0.000	0.000	0.000	0.242	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	193	82	136	0	0	0	556	41	0
N.S.	1	2.05	0.87	1.45	0.00	0.00	0.00	5.91	0.44	0.00
time (sec)	N/A	1.070	0.283	0.280	0.000	0.000	0.000	0.230	0.207	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	183	126	224	0	0	0	630	39	0
N.S.	1	1.22	0.84	1.49	0.00	0.00	0.00	4.20	0.26	0.00
time (sec)	N/A	1.087	0.257	0.156	0.000	0.000	0.000	0.223	0.215	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	82	73	136	0	0	0	308	38	0
N.S.	1	0.95	0.85	1.58	0.00	0.00	0.00	3.58	0.44	0.00
time (sec)	N/A	0.617	0.146	0.182	0.000	0.000	0.000	0.218	0.225	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	129	43	27	0	42	28
N.S.	1	1.00	1.07	0.93	4.61	1.54	0.96	0.00	1.50	1.00
time (sec)	N/A	0.882	13.271	0.761	0.839	0.090	1.004	0.000	0.256	0.346

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	127	49	29	28	170	28
N.S.	1	1.00	1.07	0.93	4.54	1.75	1.04	1.00	6.07	1.00
time (sec)	N/A	0.375	2.768	0.731	0.540	0.105	0.949	0.759	0.225	0.352

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	136	49	29	0	48	28
N.S.	1	1.00	1.07	0.93	4.86	1.75	1.04	0.00	1.71	1.00
time (sec)	N/A	0.296	19.737	1.229	0.784	0.089	1.078	0.000	0.228	0.355

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	137	49	29	28	48	28
N.S.	1	1.00	1.07	0.93	4.89	1.75	1.04	1.00	1.71	1.00
time (sec)	N/A	0.291	3.425	1.471	0.791	0.105	1.346	2.411	0.247	0.353

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	160	52	29	0	91	28
N.S.	1	1.00	1.07	0.93	5.71	1.86	1.04	0.00	3.25	1.00
time (sec)	N/A	0.302	0.771	1.602	1.852	0.110	31.034	0.000	0.239	0.339

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	393	399	454	0	0	0	2080	88	0
N.S.	1	1.41	1.44	1.63	0.00	0.00	0.00	7.48	0.32	0.00
time (sec)	N/A	1.168	0.678	0.190	0.000	0.000	0.000	0.261	0.234	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	251	306	364	0	0	0	1574	88	0
N.S.	1	1.14	1.39	1.65	0.00	0.00	0.00	7.15	0.40	0.00
time (sec)	N/A	0.972	0.622	0.162	0.000	0.000	0.000	0.263	0.234	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	290	295	341	0	0	0	1264	86	0
N.S.	1	1.36	1.38	1.59	0.00	0.00	0.00	5.91	0.40	0.00
time (sec)	N/A	1.321	0.426	0.155	0.000	0.000	0.000	0.242	0.243	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	139	122	250	0	0	0	788	85	0
N.S.	1	0.93	0.81	1.67	0.00	0.00	0.00	5.25	0.57	0.00
time (sec)	N/A	0.599	0.382	0.184	0.000	0.000	0.000	0.241	0.248	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	145	43	27	0	87	28
N.S.	1	1.00	1.07	0.93	5.18	1.54	0.96	0.00	3.11	1.00
time (sec)	N/A	0.972	10.851	0.495	0.891	0.090	2.950	0.000	0.239	0.333

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	145	49	29	28	265	28
N.S.	1	1.00	1.07	0.93	5.18	1.75	1.04	1.00	9.46	1.00
time (sec)	N/A	0.498	4.880	0.668	1.111	0.080	2.856	0.743	0.271	0.329

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	152	49	29	0	96	28
N.S.	1	1.00	1.07	0.93	5.43	1.75	1.04	0.00	3.43	1.00
time (sec)	N/A	0.309	19.882	0.882	0.883	0.089	3.767	0.000	0.237	0.337

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	145	49	29	28	330	28
N.S.	1	1.00	1.07	0.93	5.18	1.75	1.04	1.00	11.79	1.00
time (sec)	N/A	0.421	2.884	1.513	0.758	0.098	5.123	1.713	0.262	0.330

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	187	59	0	0	139	28
N.S.	1	1.00	1.07	0.93	6.68	2.11	0.00	0.00	4.96	1.00
time (sec)	N/A	0.292	0.803	1.588	2.201	0.080	0.000	0.000	0.244	0.342

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	493	408	455	0	0	0	2528	133	0
N.S.	1	1.77	1.47	1.64	0.00	0.00	0.00	9.09	0.48	0.00
time (sec)	N/A	1.337	0.896	0.191	0.000	0.000	0.000	0.262	0.234	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	401	413	478	0	0	0	2512	133	0
N.S.	1	1.42	1.46	1.70	0.00	0.00	0.00	8.91	0.47	0.00
time (sec)	N/A	1.158	0.928	0.164	0.000	0.000	0.000	0.262	0.230	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	390	404	455	0	0	0	2108	131	0
N.S.	1	1.41	1.46	1.65	0.00	0.00	0.00	7.64	0.47	0.00
time (sec)	N/A	1.571	0.695	0.160	0.000	0.000	0.000	0.268	0.225	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	189	311	364	0	0	0	1464	130	0
N.S.	1	0.87	1.43	1.68	0.00	0.00	0.00	6.75	0.60	0.00
time (sec)	N/A	0.612	0.494	0.202	0.000	0.000	0.000	0.249	0.246	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	162	60	27	0	132	28
N.S.	1	1.00	1.07	0.93	5.79	2.14	0.96	0.00	4.71	1.00
time (sec)	N/A	1.152	13.787	0.582	1.013	0.092	7.256	0.000	0.244	0.329

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	163	66	29	28	364	28
N.S.	1	1.00	1.07	0.93	5.82	2.36	1.04	1.00	13.00	1.00
time (sec)	N/A	0.546	4.364	0.543	1.119	0.088	6.764	0.818	0.263	0.333

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	170	66	29	0	139	28
N.S.	1	1.00	1.07	0.93	6.07	2.36	1.04	0.00	4.96	1.00
time (sec)	N/A	0.293	20.038	1.118	1.108	0.080	6.998	0.000	0.278	0.326

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	162	66	29	28	381	28
N.S.	1	1.00	1.07	0.93	5.79	2.36	1.04	1.00	13.61	1.00
time (sec)	N/A	0.290	3.225	1.464	1.098	0.087	10.454	2.530	0.297	0.322

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	113	80	29	28	71	28
N.S.	1	1.00	1.07	0.93	4.04	2.86	1.04	1.00	2.54	1.00
time (sec)	N/A	0.381	1.034	0.921	0.923	0.092	2.748	0.386	0.232	0.335

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	176	157	340	0	0	0	0	64	0
N.S.	1	0.86	0.77	1.67	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.670	0.373	0.214	0.000	0.000	0.000	0.000	0.225	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	130	119	250	0	0	0	811	64	0
N.S.	1	0.92	0.84	1.77	0.00	0.00	0.00	5.75	0.45	0.00
time (sec)	N/A	0.596	0.220	0.182	0.000	0.000	0.000	0.254	0.244	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	126	112	226	0	0	0	0	64	0
N.S.	1	0.89	0.79	1.59	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.588	0.202	0.178	0.000	0.000	0.000	0.000	0.214	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	74	69	134	0	0	0	317	64	0
N.S.	1	0.94	0.87	1.70	0.00	0.00	0.00	4.01	0.81	0.00
time (sec)	N/A	0.707	0.176	0.172	0.000	0.000	0.000	0.235	0.226	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	68	59	109	0	0	0	201	62	0
N.S.	1	0.94	0.82	1.51	0.00	0.00	0.00	2.79	0.86	0.00
time (sec)	N/A	0.556	0.129	0.156	0.000	0.000	0.000	0.224	0.235	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	17	17	18	17	16	56	17	22	17
N.S.	1	0.94	0.94	1.00	0.94	0.89	3.11	0.94	1.22	0.94
time (sec)	N/A	0.212	0.011	0.069	0.120	0.089	1.077	0.133	0.237	0.358

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	110	80	29	0	63	28
N.S.	1	1.00	1.07	0.93	3.93	2.86	1.04	0.00	2.25	1.00
time (sec)	N/A	0.372	7.772	0.293	0.706	0.086	1.689	0.000	0.218	0.323

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	119	86	31	28	69	28
N.S.	1	1.00	1.07	0.93	4.25	3.07	1.11	1.00	2.46	1.00
time (sec)	N/A	0.376	1.595	0.207	0.699	0.105	1.515	0.753	0.226	0.329

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	217	106	29	28	141	28
N.S.	1	1.00	1.07	0.93	7.75	3.79	1.04	1.00	5.04	1.00
time (sec)	N/A	0.314	1.606	1.056	1.251	0.108	72.442	1.043	0.260	0.334

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	206	106	29	0	141	28
N.S.	1	1.00	1.07	0.93	7.36	3.79	1.04	0.00	5.04	1.00
time (sec)	N/A	0.321	60.439	0.887	0.929	0.095	3.497	0.000	0.238	0.328

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	194	106	29	28	307	28
N.S.	1	1.00	1.07	0.93	6.93	3.79	1.04	1.00	10.96	1.00
time (sec)	N/A	0.468	6.047	0.547	0.704	0.090	3.447	1.972	0.245	0.321

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	199	104	27	0	139	26
N.S.	1	1.00	1.08	0.92	7.65	4.00	1.04	0.00	5.35	1.00
time (sec)	N/A	0.286	46.885	0.476	0.804	0.097	3.563	0.000	0.226	0.331

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	193	103	27	25	137	25
N.S.	1	1.00	1.08	0.92	7.72	4.12	1.08	1.00	5.48	1.00
time (sec)	N/A	0.390	2.676	0.197	0.657	0.088	3.947	0.935	0.252	0.322

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	210	108	29	0	140	28
N.S.	1	1.00	1.07	0.93	7.50	3.86	1.04	0.00	5.00	1.00
time (sec)	N/A	0.321	46.965	2.228	0.945	0.097	6.340	0.000	0.237	0.332

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	219	114	31	28	146	28
N.S.	1	1.00	1.07	0.93	7.82	4.07	1.11	1.00	5.21	1.00
time (sec)	N/A	0.324	35.541	0.970	0.956	0.117	5.517	18.497	0.251	0.320

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	279	144	29	28	212	28
N.S.	1	1.00	1.07	0.93	9.96	5.14	1.04	1.00	7.57	1.00
time (sec)	N/A	0.331	2.527	1.033	1.430	0.111	73.795	2.411	0.246	0.346

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	265	144	29	0	212	28
N.S.	1	1.00	1.07	0.93	9.46	5.14	1.04	0.00	7.57	1.00
time (sec)	N/A	0.321	100.898	6.789	1.160	0.102	3.980	0.000	0.245	0.328

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	262	144	29	28	212	28
N.S.	1	1.00	1.07	0.93	9.36	5.14	1.04	1.00	7.57	1.00
time (sec)	N/A	0.326	14.957	4.243	0.999	0.089	3.672	6.635	0.231	0.322

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	260	142	27	0	210	26
N.S.	1	1.00	1.08	0.92	10.00	5.46	1.04	0.00	8.08	1.00
time (sec)	N/A	0.276	93.855	4.910	1.172	0.094	3.617	0.000	0.223	0.302

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	254	141	27	25	208	25
N.S.	1	1.00	1.08	0.92	10.16	5.64	1.08	1.00	8.32	1.00
time (sec)	N/A	0.386	5.914	1.556	0.829	0.087	3.911	2.229	0.261	0.330

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	270	144	29	0	211	28
N.S.	1	1.00	1.07	0.93	9.64	5.14	1.04	0.00	7.54	1.00
time (sec)	N/A	0.310	80.990	9.010	1.098	0.091	8.140	0.000	0.253	0.324

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	26	279	150	31	28	217	28
N.S.	1	1.00	1.07	0.93	9.96	5.36	1.11	1.00	7.75	1.00
time (sec)	N/A	0.306	23.376	9.349	1.112	0.088	6.086	36.007	0.246	0.331

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	17	11	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	1.31	0.85	0.85	0.85
time (sec)	N/A	0.201	0.002	0.090	0.114	0.080	0.381	0.124	0.252	0.316

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	507	287	304	0	0	0	0	86	0
N.S.	1	2.02	1.14	1.21	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	1.375	0.939	0.612	0.000	0.000	0.000	0.000	0.306	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	585	420	447	0	0	0	0	86	0
N.S.	1	0.99	0.71	0.76	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.481	0.847	0.704	0.000	0.000	0.000	0.000	0.272	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	285	0	305	0	0	0	0	84	0
N.S.	1	1.18	0.00	1.27	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	1.594	0.000	0.565	0.000	0.000	0.000	0.000	0.281	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	256	287	305	0	0	0	0	1522	0
N.S.	1	1.01	1.13	1.21	0.00	0.00	0.00	0.00	6.02	0.00
time (sec)	N/A	0.728	0.463	0.552	0.000	0.000	0.000	0.000	0.409	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	25	30	0	87	0	85	27
N.S.	1	1.00	1.07	0.93	1.11	0.00	3.22	0.00	3.15	1.00
time (sec)	N/A	1.815	0.585	0.799	0.797	0.000	4.005	0.000	0.253	0.363

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	605	540	589	0	0	0	0	131	0
N.S.	1	1.25	1.11	1.21	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	1.749	1.755	0.815	0.000	0.000	0.000	0.000	0.340	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	803	555	592	0	0	0	0	131	0
N.S.	1	1.57	1.09	1.16	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.922	1.739	0.789	0.000	0.000	0.000	0.000	0.283	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	603	0	449	0	0	0	0	129	0
N.S.	1	1.62	0.00	1.20	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	2.200	0.000	0.635	0.000	0.000	0.000	0.000	0.306	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	366	422	450	0	0	0	0	0	0
N.S.	1	0.94	1.08	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.906	1.095	0.683	0.000	0.000	0.000	0.000	0.468	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	30	0	133	0	130	29
N.S.	1	1.00	1.07	0.93	1.03	0.00	4.59	0.00	4.48	1.00
time (sec)	N/A	2.324	1.174	0.885	1.110	0.000	5.273	0.000	0.255	0.329

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F(-2)	F(-2)	F	A	B	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	47	0	0	0	46	29	0
N.S.	1	1.00	0.00	1.12	0.00	0.00	0.00	1.10	0.69	0.00
time (sec)	N/A	0.300	0.000	0.982	0.000	0.000	0.000	0.277	0.244	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	260	148	0	0	0	0	0	51	0
N.S.	1	1.15	0.65	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.686	0.250	0.000	0.000	0.000	0.000	0.000	0.263	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	87	0	0	0	0	0	22	0
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.866	0.182	0.000	0.000	0.000	0.000	0.000	0.263	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	0	0	0	14	19	0
N.S.	1	1.00	1.00	0.86	0.00	0.00	0.00	0.32	0.43	0.00
time (sec)	N/A	0.232	0.052	0.203	0.000	0.000	0.000	0.255	0.244	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	26	22	44	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.08	0.92	1.83	0.92
time (sec)	N/A	0.373	2.745	0.786	0.000	0.000	2.553	0.419	0.236	0.303

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	26	22	53	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.08	0.92	2.21	0.92
time (sec)	N/A	0.636	2.188	1.216	0.000	0.000	21.141	0.453	0.259	0.310

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	363	339	186	0	0	0	0	0	59	0
N.S.	1	0.93	0.51	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	2.200	0.373	0.000	0.000	0.000	0.000	0.000	0.272	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	219	164	103	0	0	0	0	0	26	0
N.S.	1	0.75	0.47	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.894	0.219	0.000	0.000	0.000	0.000	0.000	0.255	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	0	0	0	14	21	0
N.S.	1	1.00	1.00	0.86	0.00	0.00	0.00	0.32	0.48	0.00
time (sec)	N/A	0.237	0.060	0.200	0.000	0.000	0.000	0.389	0.269	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	26	22	93	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.08	0.92	3.88	0.92
time (sec)	N/A	0.355	3.202	0.978	0.000	0.000	14.213	0.587	0.245	0.309

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	431	475	180	0	0	0	0	0	63	0
N.S.	1	1.10	0.42	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	5.260	0.317	0.000	0.000	0.000	0.000	0.000	0.275	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	247	202	112	0	0	0	0	0	28	0
N.S.	1	0.82	0.45	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.444	0.237	0.000	0.000	0.000	0.000	0.000	0.307	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	0	0	0	14	21	0
N.S.	1	1.00	1.00	0.86	0.00	0.00	0.00	0.32	0.48	0.00
time (sec)	N/A	0.220	0.060	0.203	0.000	0.000	0.000	0.407	0.219	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	0	22	103	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.00	0.92	4.29	0.92
time (sec)	N/A	0.353	3.204	1.083	0.000	0.000	0.000	0.618	0.243	0.324

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	259	165	0	0	0	0	0	50	0
N.S.	1	1.15	0.73	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.657	0.211	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	90	0	0	0	0	0	20	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.858	0.154	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	0	20	0	15	15	0
N.S.	1	1.00	1.00	0.90	0.00	0.48	0.00	0.36	0.36	0.00
time (sec)	N/A	0.229	0.022	0.330	0.000	0.086	0.000	0.120	0.208	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	20	22	37	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.83	0.92	1.54	0.92
time (sec)	N/A	0.346	4.843	1.267	0.000	0.000	2.274	0.517	0.213	0.285

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	20	22	47	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.83	0.92	1.96	0.92
time (sec)	N/A	0.583	2.189	1.334	0.000	0.000	21.276	0.588	0.212	0.287

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	359	348	209	0	0	0	0	0	62	0
N.S.	1	0.97	0.58	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	2.438	0.335	0.000	0.000	0.000	0.000	0.000	0.264	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	168	109	0	0	0	0	0	26	0
N.S.	1	0.78	0.51	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.971	0.192	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	0	20	0	15	17	0
N.S.	1	1.00	1.00	0.90	0.00	0.48	0.00	0.36	0.40	0.00
time (sec)	N/A	0.221	0.026	0.324	0.000	0.091	0.000	0.133	0.216	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	20	22	97	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.83	0.92	4.04	0.92
time (sec)	N/A	0.360	5.251	1.175	0.000	0.000	13.960	0.647	0.254	0.283

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	26	56	21	0	0	0	37	30	0
N.S.	1	1.04	2.24	0.84	0.00	0.00	0.00	1.48	1.20	0.00
time (sec)	N/A	0.297	0.055	0.193	0.000	0.000	0.000	0.160	0.231	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	244	141	279	0	0	0	0	159	97	0
N.S.	1	0.58	1.14	0.00	0.00	0.00	0.00	0.65	0.40	0.00
time (sec)	N/A	0.430	0.496	0.000	0.000	0.000	0.000	0.712	0.243	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	106	155	0	0	0	0	107	63	0
N.S.	1	0.62	0.91	0.00	0.00	0.00	0.00	0.63	0.37	0.00
time (sec)	N/A	0.385	0.266	0.000	0.000	0.000	0.000	0.595	0.250	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	68	71	0	0	0	0	51	28	0
N.S.	1	0.69	0.72	0.00	0.00	0.00	0.00	0.52	0.28	0.00
time (sec)	N/A	0.347	0.135	0.000	0.000	0.000	0.000	0.518	0.248	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	0	0	0	14	15	0
N.S.	1	1.00	1.00	0.90	0.00	0.00	0.00	0.33	0.36	0.00
time (sec)	N/A	0.235	0.065	0.201	0.000	0.000	0.000	0.236	0.223	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	27	22	55	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.12	0.92	2.29	0.92
time (sec)	N/A	0.217	3.142	1.131	0.000	0.000	5.563	0.303	0.219	0.280

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	27	22	70	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.12	0.92	2.92	0.92
time (sec)	N/A	0.214	2.853	1.579	0.000	0.000	53.906	0.328	0.240	0.292

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	237	172	404	0	0	0	0	0	97	0
N.S.	1	0.73	1.70	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.527	0.794	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	137	211	0	0	0	0	0	63	0
N.S.	1	0.84	1.29	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.497	0.321	0.000	0.000	0.000	0.000	0.000	0.243	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	83	0	0	0	0	0	28	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.534	0.172	0.000	0.000	0.000	0.000	0.000	0.255	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	0	48	0	14	21	0
N.S.	1	1.00	1.00	0.90	0.00	1.14	0.00	0.33	0.50	0.00
time (sec)	N/A	0.219	0.055	0.209	0.000	0.087	0.000	0.256	0.222	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	27	22	61	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	1.12	0.92	2.54	0.92
time (sec)	N/A	0.354	3.152	1.155	0.000	0.000	36.580	0.337	0.222	0.284

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	0	22	78	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.00	0.92	3.25	0.92
time (sec)	N/A	0.351	2.635	1.289	0.000	0.000	0.000	0.351	0.246	0.290

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	199	268	0	0	0	0	0	63	0
N.S.	1	0.97	1.30	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	1.213	0.439	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	134	112	0	0	0	0	0	28	0
N.S.	1	1.03	0.86	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.518	0.254	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	0	48	0	14	21	0
N.S.	1	1.00	1.00	0.86	0.00	1.09	0.00	0.32	0.48	0.00
time (sec)	N/A	0.244	0.060	0.214	0.000	0.103	0.000	0.258	0.249	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	0	22	61	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.00	0.92	2.54	0.92
time (sec)	N/A	0.357	3.146	1.146	0.000	0.000	0.000	0.342	0.247	0.286

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	0	0	0	22	78	22
N.S.	1	1.00	1.08	0.83	0.00	0.00	0.00	0.92	3.25	0.92
time (sec)	N/A	0.375	2.316	1.267	0.000	0.000	0.000	0.359	0.273	0.286

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	199	191	0	0	0	0	0	30	0
N.S.	1	0.77	0.74	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.604	0.652	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	391	312	273	0	0	0	0	0	28	0
N.S.	1	0.80	0.70	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.690	0.604	0.000	0.000	0.000	0.000	0.000	0.243	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	199	0	0	0	0	0	0	27	0
N.S.	1	0.77	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.526	0.000	0.000	0.000	0.000	0.000	0.000	0.273	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	29	29	0	30	29
N.S.	1	1.00	1.07	0.93	1.00	1.00	1.00	0.00	1.03	1.00
time (sec)	N/A	0.816	0.308	0.717	0.656	0.111	2.207	0.000	0.257	0.316

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	29	31	0	105	29
N.S.	1	1.00	1.07	0.93	1.00	1.00	1.07	0.00	3.62	1.00
time (sec)	N/A	0.578	0.318	0.744	0.552	0.171	3.039	0.000	0.257	0.300

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	684	494	509	0	0	0	0	0	64	0
N.S.	1	0.72	0.74	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.914	3.012	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	595	453	468	0	0	0	0	0	62	0
N.S.	1	0.76	0.79	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.763	1.497	0.000	0.000	0.000	0.000	0.000	0.272	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	466	340	0	0	0	0	0	0	61	0
N.S.	1	0.73	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.629	0.000	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	29	29	0	62	29
N.S.	1	1.00	1.07	0.93	1.00	1.00	1.00	0.00	2.14	1.00
time (sec)	N/A	1.359	0.329	0.742	0.650	0.108	102.409	0.000	0.231	0.301

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	29	0	0	167	29
N.S.	1	1.00	1.07	0.93	1.00	1.00	0.00	0.00	5.76	1.00
time (sec)	N/A	1.107	0.350	0.730	0.636	0.112	0.000	0.000	0.266	0.304

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	906	636	990	0	0	0	0	0	97	0
N.S.	1	0.70	1.09	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.100	3.093	0.000	0.000	0.000	0.000	0.000	0.260	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	815	599	608	0	0	0	0	0	95	0
N.S.	1	0.73	0.75	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.970	2.653	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	698	496	0	0	0	0	0	0	94	0
N.S.	1	0.71	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.778	0.000	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	54	0	0	95	29
N.S.	1	1.00	1.07	0.93	1.00	1.86	0.00	0.00	3.28	1.00
time (sec)	N/A	2.122	0.347	0.789	0.764	0.119	0.000	0.000	0.246	0.307

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	27	29	54	0	0	234	29
N.S.	1	1.00	1.07	0.93	1.00	1.86	0.00	0.00	8.07	1.00
time (sec)	N/A	1.672	0.344	0.791	0.749	0.135	0.000	0.000	0.576	0.312

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	36	26	24	25	24
N.S.	1	1.00	1.08	0.92	0.00	1.50	1.08	1.00	1.04	1.00
time (sec)	N/A	0.278	0.582	0.656	0.000	0.112	3.546	0.408	0.223	0.290

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	164	137	0	0	0	0	0	25	0
N.S.	1	1.01	0.84	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.509	0.176	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	105	110	0	0	0	0	0	25	0
N.S.	1	0.96	1.01	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.454	0.272	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	78	73	0	0	0	0	0	23	0
N.S.	1	1.04	0.97	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.363	0.080	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	18	18	19	18	19	41	18	20	35
N.S.	1	1.06	1.06	1.12	1.06	1.12	2.41	1.06	1.18	2.06
time (sec)	N/A	0.209	0.012	0.135	0.122	0.119	0.311	0.130	0.223	0.503

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	35	24	24	25	24
N.S.	1	1.00	1.08	0.92	0.00	1.46	1.00	1.00	1.04	1.00
time (sec)	N/A	0.279	2.757	0.332	0.000	0.103	0.614	0.171	0.231	0.293

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	37	26	24	25	24
N.S.	1	1.00	1.08	0.92	0.00	1.54	1.08	1.00	1.04	1.00
time (sec)	N/A	0.278	0.847	0.389	0.000	0.119	0.991	0.171	0.224	0.284

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	185	293	961	0	0	0	0	187	0
N.S.	1	0.49	0.78	2.56	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.847	2.289	1.212	0.000	0.000	0.000	0.000	0.316	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	140	260	726	0	0	0	0	134	0
N.S.	1	0.51	0.95	2.66	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.600	1.779	0.924	0.000	0.000	0.000	0.000	0.268	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	93	209	366	0	0	0	0	67	0
N.S.	1	0.69	1.56	2.73	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.500	0.949	0.815	0.000	0.000	0.000	0.000	0.255	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	87	200	308	0	0	0	0	67	0
N.S.	1	0.62	1.42	2.18	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.569	1.462	0.939	0.000	0.000	0.000	0.000	0.258	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	105	195	323	0	0	0	0	95	0
N.S.	1	0.65	1.20	1.99	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.629	2.374	9.105	0.000	0.000	0.000	0.000	0.234	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	106	114	449	215	520	0	0	166	0
N.S.	1	0.65	0.70	2.75	1.32	3.19	0.00	0.00	1.02	0.00
time (sec)	N/A	0.554	1.290	2.162	0.148	0.181	0.000	0.000	0.239	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	189	305	1191	0	0	0	0	239	0
N.S.	1	0.46	0.74	2.90	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.691	2.532	1.211	0.000	0.000	0.000	0.000	0.335	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	149	248	617	0	0	0	0	121	0
N.S.	1	0.67	1.11	2.77	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.673	2.192	0.989	0.000	0.000	0.000	0.000	0.256	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	140	260	722	0	0	0	0	134	0
N.S.	1	0.51	0.95	2.64	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.579	2.063	0.999	0.000	0.000	0.000	0.000	0.253	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	127	258	714	0	0	0	0	118	0
N.S.	1	0.52	1.07	2.95	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.685	4.686	2.048	0.000	0.000	0.000	0.000	0.294	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	136	281	488	0	0	0	0	153	0
N.S.	1	0.54	1.12	1.94	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.687	5.229	29.033	0.000	0.000	0.000	0.000	0.247	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	170	293	348	0	0	0	0	333	0
N.S.	1	0.52	0.90	1.07	0.00	0.00	0.00	0.00	1.03	0.00
time (sec)	N/A	0.626	5.571	17.132	0.000	0.000	0.000	0.000	0.263	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	201	303	895	0	0	0	0	176	0
N.S.	1	0.64	0.97	2.87	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.837	2.585	1.716	0.000	0.000	0.000	0.000	0.318	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	189	305	1185	0	0	0	0	239	0
N.S.	1	0.46	0.74	2.88	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.662	2.447	1.864	0.000	0.000	0.000	0.000	0.320	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	185	293	952	0	0	0	0	187	0
N.S.	1	0.49	0.78	2.53	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.804	2.238	1.711	0.000	0.000	0.000	0.000	0.259	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	172	286	947	0	0	0	0	173	0
N.S.	1	0.50	0.83	2.74	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.880	5.937	9.134	0.000	0.000	0.000	0.000	0.258	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	187	314	907	0	0	0	0	219	0
N.S.	1	0.47	0.78	2.27	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.681	6.402	28.958	0.000	0.000	0.000	0.000	0.268	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	207	509	414	0	0	0	0	474	0
N.S.	1	0.49	1.21	0.99	0.00	0.00	0.00	0.00	1.13	0.00
time (sec)	N/A	0.701	7.432	17.715	0.000	0.000	0.000	0.000	0.269	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	172	286	946	0	0	0	0	173	0
N.S.	1	0.50	0.83	2.74	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.862	3.386	2.141	0.000	0.000	0.000	0.000	0.252	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	127	256	714	0	0	0	0	118	0
N.S.	1	0.52	1.06	2.95	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.688	3.157	4.013	0.000	0.000	0.000	0.000	0.243	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	89	199	309	0	0	0	0	68	0
N.S.	1	0.63	1.41	2.19	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.537	1.445	1.013	0.000	0.000	0.000	0.000	0.239	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	112	132	50	0	0	0	56	0
N.S.	1	1.00	2.04	2.40	0.91	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.397	0.983	0.917	0.138	0.000	0.000	0.000	0.263	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	83	80	169	97	347	0	0	83	0
N.S.	1	0.84	0.81	1.71	0.98	3.51	0.00	0.00	0.84	0.00
time (sec)	N/A	0.462	0.739	2.062	0.144	0.180	0.000	0.000	0.237	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	160	118	276	223	525	0	0	199	0
N.S.	1	0.60	0.45	1.04	0.84	1.98	0.00	0.00	0.75	0.00
time (sec)	N/A	0.564	0.788	3.766	0.136	0.214	0.000	0.000	0.242	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	187	314	906	0	0	0	0	231	0
N.S.	1	0.47	0.78	2.26	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.658	4.150	36.715	0.000	0.000	0.000	0.000	0.288	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	134	278	488	0	0	0	0	162	0
N.S.	1	0.53	1.10	1.94	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.657	3.627	33.142	0.000	0.000	0.000	0.000	0.286	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	105	195	323	0	0	0	0	101	0
N.S.	1	0.65	1.20	1.99	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.606	2.151	17.100	0.000	0.000	0.000	0.000	0.221	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	82	105	230	99	353	0	0	85	0
N.S.	1	0.84	1.07	2.35	1.01	3.60	0.00	0.00	0.87	0.00
time (sec)	N/A	0.454	0.798	2.120	0.152	0.152	0.000	0.000	0.249	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	80	107	225	86	0	0	0	97	0
N.S.	1	0.83	1.11	2.34	0.90	0.00	0.00	0.00	1.01	0.00
time (sec)	N/A	0.414	0.863	1.260	0.129	0.000	0.000	0.000	0.225	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	158	180	499	234	0	0	0	262	0
N.S.	1	0.62	0.71	1.96	0.92	0.00	0.00	0.00	1.03	0.00
time (sec)	N/A	0.550	1.024	1.422	0.155	0.000	0.000	0.000	0.236	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	208	284	414	0	0	0	0	492	0
N.S.	1	0.50	0.68	0.99	0.00	0.00	0.00	0.00	1.17	0.00
time (sec)	N/A	0.691	6.869	18.767	0.000	0.000	0.000	0.000	0.274	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	170	235	348	0	0	0	0	346	0
N.S.	1	0.52	0.73	1.07	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	0.619	4.356	18.481	0.000	0.000	0.000	0.000	0.261	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	106	127	451	217	520	0	0	172	0
N.S.	1	0.65	0.77	2.75	1.32	3.17	0.00	0.00	1.05	0.00
time (sec)	N/A	0.539	1.124	3.966	0.169	0.190	0.000	0.000	0.226	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	158	130	275	227	527	0	0	199	0
N.S.	1	0.60	0.49	1.04	0.86	1.99	0.00	0.00	0.75	0.00
time (sec)	N/A	0.583	0.839	3.931	0.152	0.188	0.000	0.000	0.231	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	156	184	502	237	0	0	0	262	0
N.S.	1	0.61	0.72	1.97	0.93	0.00	0.00	0.00	1.03	0.00
time (sec)	N/A	0.571	1.087	4.538	0.187	0.000	0.000	0.000	0.263	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	132	179	374	177	0	0	0	226	0
N.S.	1	0.70	0.95	1.99	0.94	0.00	0.00	0.00	1.20	0.00
time (sec)	N/A	0.575	1.031	11.335	0.168	0.000	0.000	0.000	0.226	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	613	334	574	1815	0	0	0	0	292	0
N.S.	1	0.54	0.94	2.96	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	1.306	3.141	2.928	0.000	0.000	0.000	0.000	0.355	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	252	457	1358	0	0	0	0	203	0
N.S.	1	0.55	1.00	2.98	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.880	2.759	2.436	0.000	0.000	0.000	0.000	0.292	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	145	288	634	0	0	0	0	100	0
N.S.	1	0.65	1.30	2.86	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.776	1.558	1.915	0.000	0.000	0.000	0.000	0.288	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	123	312	542	0	0	0	0	102	0
N.S.	1	0.53	1.36	2.36	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.741	2.320	2.135	0.000	0.000	0.000	0.000	0.242	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	255	410	318	0	0	0	0	146	0
N.S.	1	0.48	0.77	0.60	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	1.229	4.261	2.849	0.000	0.000	0.000	0.000	0.241	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	223	315	2821	0	0	0	0	295	0
N.S.	1	0.46	0.65	5.80	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	1.385	3.552	4.164	0.000	0.000	0.000	0.000	0.264	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	697	374	684	2264	0	0	0	0	380	0
N.S.	1	0.54	0.98	3.25	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	1.122	3.962	2.945	0.000	0.000	0.000	0.000	0.410	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	276	372	1099	0	0	0	0	190	0
N.S.	1	0.76	1.03	3.04	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	1.198	2.764	2.232	0.000	0.000	0.000	0.000	0.318	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	252	457	1354	0	0	0	0	203	0
N.S.	1	0.55	1.00	2.98	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.869	2.794	2.378	0.000	0.000	0.000	0.000	0.297	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	220	522	1357	0	0	0	0	189	0
N.S.	1	0.55	1.31	3.41	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.833	7.718	8.947	0.000	0.000	0.000	0.000	0.261	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	714	318	653	540	0	0	0	0	256	0
N.S.	1	0.45	0.91	0.76	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	1.332	11.141	6.178	0.000	0.000	0.000	0.000	0.274	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	243	762	647	0	0	0	0	595	0
N.S.	1	0.45	1.40	1.19	0.00	0.00	0.00	0.00	1.09	0.00
time (sec)	N/A	1.419	13.935	3.114	0.000	0.000	0.000	0.000	0.317	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	502	431	450	1613	0	0	0	0	281	0
N.S.	1	0.86	0.90	3.21	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	1.632	3.691	2.497	0.000	0.000	0.000	0.000	0.435	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	697	374	684	2258	0	0	0	0	380	0
N.S.	1	0.54	0.98	3.24	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	1.046	3.908	2.863	0.000	0.000	0.000	0.000	0.426	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	613	334	577	1806	0	0	0	0	292	0
N.S.	1	0.54	0.94	2.95	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	1.238	3.219	2.571	0.000	0.000	0.000	0.000	0.355	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	559	303	473	1812	0	0	0	0	282	0
N.S.	1	0.54	0.85	3.24	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.942	13.029	18.079	0.000	0.000	0.000	0.000	0.302	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	918	403	999	1076	0	0	0	0	376	0
N.S.	1	0.44	1.09	1.17	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	1.539	19.142	3.983	0.000	0.000	0.000	0.000	0.294	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	729	308	1526	877	0	0	0	0	871	0
N.S.	1	0.42	2.09	1.20	0.00	0.00	0.00	0.00	1.19	0.00
time (sec)	N/A	1.582	21.263	3.362	0.000	0.000	0.000	0.000	0.333	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	559	303	473	1810	0	0	0	0	282	0
N.S.	1	0.54	0.85	3.24	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.910	8.737	3.013	0.000	0.000	0.000	0.000	0.344	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	220	515	1357	0	0	0	0	189	0
N.S.	1	0.55	1.29	3.41	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.805	5.513	5.235	0.000	0.000	0.000	0.000	0.284	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	124	311	543	0	0	0	0	103	0
N.S.	1	0.54	1.35	2.35	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.707	2.084	1.410	0.000	0.000	0.000	0.000	0.247	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	159	196	0	0	0	0	91	0
N.S.	1	1.00	2.89	3.56	0.00	0.00	0.00	0.00	1.65	0.00
time (sec)	N/A	0.460	2.032	1.211	0.000	0.000	0.000	0.000	0.253	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	232	189	379	0	0	0	0	140	0
N.S.	1	0.51	0.42	0.83	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.959	1.719	4.049	0.000	0.000	0.000	0.000	0.248	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	896	457	324	2580	0	0	0	0	358	0
N.S.	1	0.51	0.36	2.88	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	1.595	3.872	4.273	0.000	0.000	0.000	0.000	0.236	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	918	403	971	1172	0	0	0	0	400	0
N.S.	1	0.44	1.06	1.28	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	1.471	13.716	11.055	0.000	0.000	0.000	0.000	0.314	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	713	317	731	636	0	0	0	0	273	0
N.S.	1	0.44	1.03	0.89	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	1.288	7.585	19.095	0.000	0.000	0.000	0.000	0.278	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	255	342	415	0	0	0	0	156	0
N.S.	1	0.48	0.65	0.78	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	1.170	3.658	2.818	0.000	0.000	0.000	0.000	0.240	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	231	245	483	0	0	0	0	144	0
N.S.	1	0.51	0.54	1.06	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.955	3.530	3.874	0.000	0.000	0.000	0.000	0.249	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	140	285	460	0	0	0	0	165	0
N.S.	1	0.65	1.31	2.12	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.912	2.535	1.704	0.000	0.000	0.000	0.000	0.272	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	709	378	563	2000	0	0	0	0	479	0
N.S.	1	0.53	0.79	2.82	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	1.194	3.489	2.505	0.000	0.000	0.000	0.000	0.290	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	730	309	568	974	0	0	0	0	907	0
N.S.	1	0.42	0.78	1.33	0.00	0.00	0.00	0.00	1.24	0.00
time (sec)	N/A	1.545	15.522	3.620	0.000	0.000	0.000	0.000	0.306	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	243	544	743	0	0	0	0	620	0
N.S.	1	0.45	1.00	1.37	0.00	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	1.356	12.693	3.228	0.000	0.000	0.000	0.000	0.276	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	223	371	2995	0	0	0	0	307	0
N.S.	1	0.46	0.76	6.16	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	1.315	4.746	4.383	0.000	0.000	0.000	0.000	0.260	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	896	457	405	2751	0	0	0	0	360	0
N.S.	1	0.51	0.45	3.07	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	1.534	2.662	4.388	0.000	0.000	0.000	0.000	0.267	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	709	378	566	2020	0	0	0	0	479	0
N.S.	1	0.53	0.80	2.85	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	1.187	3.340	13.385	0.000	0.000	0.000	0.000	0.262	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	231	554	2003	0	0	0	0	408	0
N.S.	1	0.63	1.51	5.47	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	1.514	3.465	16.052	0.000	0.000	0.000	0.000	0.259	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	281	298	759	0	0	0	0	135	0
N.S.	1	0.80	0.85	2.16	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	1.638	1.056	13.326	0.000	0.000	0.000	0.000	0.321	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	152	179	649	0	197	0	0	110	0
N.S.	1	0.68	0.80	2.88	0.00	0.88	0.00	0.00	0.49	0.00
time (sec)	N/A	0.782	1.181	16.936	0.000	0.142	0.000	0.000	0.296	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	145	288	634	0	0	0	0	100	0
N.S.	1	0.65	1.30	2.86	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.748	0.506	1.522	0.000	0.000	0.000	0.000	0.253	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	224	461	771	0	0	0	0	185	0
N.S.	1	0.52	1.07	1.78	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	1.515	1.912	5.209	0.000	0.000	0.000	0.000	0.259	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	159	368	584	0	0	0	0	105	0
N.S.	1	0.62	1.43	2.27	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	1.465	1.197	5.105	0.000	0.000	0.000	0.000	0.276	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	457	452	1458	0	0	0	0	224	0
N.S.	1	0.90	0.89	2.86	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	2.651	1.826	29.799	0.000	0.000	0.000	0.000	0.368	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	193	207	679	0	302	0	0	200	0
N.S.	1	0.57	0.61	2.01	0.00	0.89	0.00	0.00	0.59	0.00
time (sec)	N/A	0.959	2.026	27.660	0.000	0.145	0.000	0.000	0.365	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	276	372	1099	0	0	0	0	190	0
N.S.	1	0.76	1.03	3.04	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	1.181	0.639	2.112	0.000	0.000	0.000	0.000	0.303	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	647	336	663	1201	0	0	0	0	275	0
N.S.	1	0.52	1.02	1.86	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	2.213	4.602	5.487	0.000	0.000	0.000	0.000	0.305	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	316	538	516	0	0	0	0	192	0
N.S.	1	0.63	1.07	1.02	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	2.099	2.735	4.920	0.000	0.000	0.000	0.000	0.328	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	151	243	1016	0	0	0	0	123	0
N.S.	1	0.60	0.97	4.06	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	1.144	1.466	9.227	0.000	0.000	0.000	0.000	0.292	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	103	150	363	0	138	0	0	97	0
N.S.	1	0.58	0.85	2.05	0.00	0.78	0.00	0.00	0.55	0.00
time (sec)	N/A	0.663	1.682	16.713	0.000	0.143	0.000	0.000	0.269	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	159	196	0	0	0	0	91	0
N.S.	1	1.00	2.89	3.56	0.00	0.00	0.00	0.00	1.65	0.00
time (sec)	N/A	0.480	0.232	1.033	0.000	0.000	0.000	0.000	0.248	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	165	367	402	0	0	0	0	180	0
N.S.	1	0.57	1.28	1.40	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	1.090	2.216	9.304	0.000	0.000	0.000	0.000	0.244	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	141	188	436	0	0	0	0	97	0
N.S.	1	0.66	0.88	2.04	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	1.137	1.734	2.068	0.000	0.000	0.000	0.000	0.239	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	169	352	711	0	0	0	0	212	0
N.S.	1	0.57	1.19	2.41	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	1.516	2.746	8.749	0.000	0.000	0.000	0.000	0.302	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	135	251	445	0	0	0	0	174	0
N.S.	1	0.55	1.03	1.82	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	1.013	2.955	29.005	0.000	0.000	0.000	0.000	0.258	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	140	285	460	0	0	0	0	165	0
N.S.	1	0.65	1.31	2.12	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.898	0.131	1.595	0.000	0.000	0.000	0.000	0.245	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	256	724	961	0	0	0	0	323	0
N.S.	1	0.47	1.32	1.75	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	2.425	5.379	2.203	0.000	0.000	0.000	0.000	0.300	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	247	429	544	0	0	0	0	187	0
N.S.	1	0.62	1.08	1.37	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	2.433	2.635	1.516	0.000	0.000	0.000	0.000	0.270	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	153	145	194	185	129	228	185	192	0
N.S.	1	1.01	0.95	1.28	1.22	0.85	1.50	1.22	1.26	0.00
time (sec)	N/A	0.382	0.077	0.412	0.154	0.104	0.737	0.130	0.233	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	145	153	170	165	125	211	165	172	0
N.S.	1	0.97	1.03	1.14	1.11	0.84	1.42	1.11	1.15	0.00
time (sec)	N/A	0.356	0.085	0.382	0.140	0.133	0.552	0.133	0.253	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	122	125	154	144	108	177	141	150	0
N.S.	1	1.02	1.04	1.28	1.20	0.90	1.48	1.18	1.25	0.00
time (sec)	N/A	0.348	0.070	0.396	0.128	0.112	0.383	0.133	0.221	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	136	131	161	124	103	158	121	130	0
N.S.	1	1.13	1.09	1.34	1.03	0.86	1.32	1.01	1.08	0.00
time (sec)	N/A	0.322	0.069	0.360	0.136	0.116	0.301	0.132	0.234	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	84	91	100	94	83	114	91	104	0
N.S.	1	1.04	1.12	1.23	1.16	1.02	1.41	1.12	1.28	0.00
time (sec)	N/A	0.289	0.040	0.158	0.122	0.108	0.256	0.134	0.224	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	142	121	128	0	0	0	0	80	0
N.S.	1	1.08	0.92	0.97	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.565	0.097	1.080	0.000	0.000	0.000	0.000	0.238	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	71	80	79	81	155	78	859	72	71
N.S.	1	1.08	1.21	1.20	1.23	2.35	1.18	13.02	1.09	1.08
time (sec)	N/A	0.282	0.035	0.171	0.131	0.130	1.839	0.564	0.231	0.567

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	127	108	137	0	0	0	0	60	0
N.S.	1	1.07	0.91	1.15	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.548	0.066	1.037	0.000	0.000	0.000	0.000	0.241	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	84	130	113	119	168	170	3082	86	0
N.S.	1	0.99	1.53	1.33	1.40	1.98	2.00	36.26	1.01	0.00
time (sec)	N/A	0.301	0.042	0.179	0.118	0.154	2.787	121.072	0.248	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	188	329	317	220	420	340	348	0
N.S.	1	1.00	0.78	1.37	1.32	0.91	1.74	1.41	1.44	0.00
time (sec)	N/A	0.556	0.135	0.409	0.138	0.123	1.256	0.137	0.239	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	225	196	293	287	216	388	310	319	0
N.S.	1	0.93	0.81	1.22	1.19	0.90	1.61	1.29	1.32	0.00
time (sec)	N/A	0.482	0.161	0.389	0.123	0.111	1.238	0.137	0.243	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	199	159	269	256	187	338	269	280	0
N.S.	1	1.01	0.80	1.36	1.29	0.94	1.71	1.36	1.41	0.00
time (sec)	N/A	0.492	0.124	0.403	0.138	0.104	0.732	0.134	0.215	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	205	165	253	226	184	304	239	251	0
N.S.	1	1.16	0.93	1.43	1.28	1.04	1.72	1.35	1.42	0.00
time (sec)	N/A	0.439	0.135	0.396	0.129	0.111	0.564	0.134	0.270	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	152	126	194	186	152	245	192	208	0
N.S.	1	1.01	0.84	1.29	1.24	1.01	1.63	1.28	1.39	0.00
time (sec)	N/A	0.399	0.087	0.183	0.128	0.115	0.512	0.135	0.226	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	232	217	234	0	0	0	0	174	0
N.S.	1	1.01	0.95	1.02	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.718	0.205	1.120	0.000	0.000	0.000	0.000	0.230	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	114	131	157	154	258	172	4251	164	0
N.S.	1	0.90	1.04	1.25	1.22	2.05	1.37	33.74	1.30	0.00
time (sec)	N/A	0.499	0.105	0.204	0.129	0.152	2.461	2.243	0.249	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	194	183	196	0	0	0	0	147	0
N.S.	1	1.05	0.99	1.06	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.749	0.260	1.246	0.000	0.000	0.000	0.000	0.257	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	142	141	157	162	250	221	4761	152	0
N.S.	1	1.13	1.12	1.25	1.29	1.98	1.75	37.79	1.21	0.00
time (sec)	N/A	0.466	0.125	0.212	0.124	0.166	3.477	2.165	0.234	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	340	272	484	469	323	636	522	530	0
N.S.	1	1.00	0.80	1.42	1.38	0.95	1.87	1.53	1.55	0.00
time (sec)	N/A	0.838	0.192	0.438	0.137	0.117	2.491	0.153	0.266	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	403	276	436	429	319	602	482	492	0
N.S.	1	1.20	0.82	1.30	1.28	0.95	1.80	1.44	1.47	0.00
time (sec)	N/A	0.738	0.271	0.413	0.133	0.113	1.918	0.150	0.285	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	232	404	388	278	530	424	436	0
N.S.	1	1.00	0.81	1.41	1.35	0.97	1.85	1.48	1.52	0.00
time (sec)	N/A	0.723	0.189	0.427	0.142	0.113	1.297	0.143	0.238	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	288	238	365	348	275	488	384	398	0
N.S.	1	1.15	0.95	1.45	1.39	1.10	1.94	1.53	1.59	0.00
time (sec)	N/A	0.548	0.186	0.492	0.166	0.154	1.005	0.139	0.270	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	226	188	305	297	230	394	318	338	0
N.S.	1	1.00	0.84	1.36	1.32	1.02	1.75	1.41	1.50	0.00
time (sec)	N/A	0.632	0.130	0.187	0.135	0.114	0.724	0.137	0.265	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	356	320	385	0	0	0	0	295	0
N.S.	1	1.00	0.90	1.08	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.878	0.378	2.087	0.000	0.000	0.000	0.000	0.233	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	183	250	245	349	279	12827	269	0
N.S.	1	1.00	0.96	1.32	1.29	1.84	1.47	67.51	1.42	0.00
time (sec)	N/A	0.658	0.155	0.278	0.131	0.171	3.118	12.060	0.247	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	266	266	303	0	0	0	0	246	0
N.S.	1	1.02	1.02	1.16	0.00	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	1.171	0.304	2.325	0.000	0.000	0.000	0.000	0.256	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	172	194	235	235	358	314	9504	248	0
N.S.	1	0.92	1.04	1.26	1.26	1.92	1.69	51.10	1.33	0.00
time (sec)	N/A	0.742	0.197	0.221	0.131	0.216	4.102	8.333	0.253	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	261	440	430	322	598	475	494	0
N.S.	1	1.00	0.82	1.39	1.36	1.02	1.89	1.50	1.56	0.00
time (sec)	N/A	0.801	0.150	0.195	0.141	0.141	1.348	0.143	0.220	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	652	968	373	0	0	0	0	65	0
N.S.	1	1.00	1.48	0.57	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.544	0.890	137.576	0.000	0.000	0.000	0.000	0.227	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	559	559	926	2128	0	0	0	0	106	0
N.S.	1	1.00	1.66	3.81	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	1.354	0.269	3.832	0.000	0.000	0.000	0.000	0.224	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	579	580	909	284	0	0	0	0	79	0
N.S.	1	1.00	1.57	0.49	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.328	0.407	178.458	0.000	0.000	0.000	0.000	0.233	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	491	491	835	389	0	0	0	0	37	0
N.S.	1	1.00	1.70	0.79	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.176	0.214	1.284	0.000	0.000	0.000	0.000	0.223	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	541	843	234	0	0	0	0	46	0
N.S.	1	1.00	1.56	0.43	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.237	0.035	1.367	0.000	0.000	0.000	0.000	0.246	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	868	387	0	0	0	0	44	0
N.S.	1	1.00	1.68	0.75	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.420	0.182	1.904	0.000	0.000	0.000	0.000	0.223	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	579	578	912	334	0	0	0	0	58	0
N.S.	1	1.00	1.58	0.58	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.350	1.160	186.835	0.000	0.000	0.000	0.000	0.213	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	573	573	966	470	0	0	0	0	65	0
N.S.	1	1.00	1.69	0.82	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.458	0.164	1.947	0.000	0.000	0.000	0.000	0.231	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	649	650	1002	426	0	0	0	0	75	0
N.S.	1	1.00	1.54	0.66	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.439	0.523	219.782	0.000	0.000	0.000	0.000	0.275	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	1100	2151	0	0	0	0	123	0
N.S.	1	1.00	1.92	3.75	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.396	1.289	4.004	0.000	0.000	0.000	0.000	0.228	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	83	86	404	0	520	0	0	1401	0
N.S.	1	0.97	1.00	4.70	0.00	6.05	0.00	0.00	16.29	0.00
time (sec)	N/A	0.255	0.129	7.109	0.000	0.172	0.000	0.000	0.246	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	597	597	1145	512	0	0	0	0	138	0
N.S.	1	1.00	1.92	0.86	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	1.513	1.180	9.070	0.000	0.000	0.000	0.000	0.257	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	632	632	1196	640	0	0	0	0	168	0
N.S.	1	1.00	1.89	1.01	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	1.563	2.008	2.334	0.000	0.000	0.000	0.000	0.287	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	787	788	1171	920	0	0	0	0	149	0
N.S.	1	1.00	1.49	1.17	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	2.696	1.441	2.318	0.000	0.000	0.000	0.000	0.267	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	745	745	1122	819	0	0	0	0	144	0
N.S.	1	1.00	1.51	1.10	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	2.485	1.129	14.581	0.000	0.000	0.000	0.000	0.289	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	757	757	1111	834	0	0	0	0	137	0
N.S.	1	1.00	1.47	1.10	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.560	2.126	4.592	0.000	0.000	0.000	0.000	0.234	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	795	794	1198	907	0	0	0	0	155	0
N.S.	1	1.00	1.51	1.14	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	2.621	1.760	5.762	0.000	0.000	0.000	0.000	0.260	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	705	704	1547	3580	0	0	0	0	238	0
N.S.	1	1.00	2.19	5.08	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	1.647	6.662	6.145	0.000	0.000	0.000	0.000	0.268	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	178	152	1020	0	1102	0	0	180	0
N.S.	1	1.16	0.99	6.67	0.00	7.20	0.00	0.00	1.18	0.00
time (sec)	N/A	0.406	0.352	0.238	0.000	0.257	0.000	0.000	0.295	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	137	143	998	0	1100	0	0	172	0
N.S.	1	1.03	1.08	7.50	0.00	8.27	0.00	0.00	1.29	0.00
time (sec)	N/A	0.320	0.372	0.204	0.000	0.247	0.000	0.000	0.255	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	727	727	1533	1238	0	0	0	0	265	0
N.S.	1	1.00	2.11	1.70	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	1.730	5.446	1.448	0.000	0.000	0.000	0.000	0.242	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	783	784	1660	1459	0	0	0	0	299	0
N.S.	1	1.00	2.12	1.86	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	1.744	6.059	1.702	0.000	0.000	0.000	0.000	0.314	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1082	1082	1527	1760	0	0	0	0	272	0
N.S.	1	1.00	1.41	1.63	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	4.114	6.023	11.377	0.000	0.000	0.000	0.000	0.250	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1092	1092	1531	1226	0	0	0	0	271	0
N.S.	1	1.00	1.40	1.12	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	3.192	5.805	10.096	0.000	0.000	0.000	0.000	0.257	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1092	1092	1547	1786	0	0	0	0	263	0
N.S.	1	1.00	1.42	1.64	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.958	5.651	5.822	0.000	0.000	0.000	0.000	0.274	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	61	20
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	3.05	1.00
time (sec)	N/A	0.189	0.278	0.014	0.000	0.099	10.614	0.278	0.268	0.624

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	0	48	20
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	0.00	2.40	1.00
time (sec)	N/A	0.192	0.102	0.000	0.000	0.086	5.435	0.000	0.285	0.581

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	71	73	0	0	294	0	0	123	0
N.S.	1	1.01	1.04	0.00	0.00	4.20	0.00	0.00	1.76	0.00
time (sec)	N/A	0.268	0.045	0.000	0.000	0.147	0.000	0.000	0.277	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	147	190	0	0	686	0	0	276	0
N.S.	1	1.01	1.30	0.00	0.00	4.70	0.00	0.00	1.89	0.00
time (sec)	N/A	0.334	0.129	0.000	0.000	0.167	0.000	0.000	0.324	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	242	189	0	0	1324	0	0	461	0
N.S.	1	1.07	0.84	0.00	0.00	5.86	0.00	0.00	2.04	0.00
time (sec)	N/A	1.028	0.241	0.000	0.000	0.241	0.000	0.000	0.371	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	484	491	308	0	0	0	0	0	656	0
N.S.	1	1.01	0.64	0.00	0.00	0.00	0.00	0.00	1.36	0.00
time (sec)	N/A	2.159	0.381	0.000	0.000	0.000	0.000	0.000	0.300	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	293	295	222	0	0	0	0	0	360	0
N.S.	1	1.01	0.76	0.00	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.649	0.216	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	169	120	0	0	0	0	0	149	0
N.S.	1	1.05	0.75	0.00	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.396	0.147	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	20	0	43	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.87	0.00	1.87	1.09
time (sec)	N/A	0.247	3.356	1.215	0.396	0.108	10.662	0.000	0.278	0.459

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	36	0	0	65	25
N.S.	1	1.00	1.09	1.00	1.09	1.57	0.00	0.00	2.83	1.09
time (sec)	N/A	0.246	5.275	0.812	0.406	0.106	0.000	0.000	0.255	0.498

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	569	445	702	700	555	994	826	487	0
N.S.	1	1.00	0.78	1.23	1.23	0.98	1.75	1.45	0.86	0.00
time (sec)	N/A	1.419	0.256	1.654	0.159	0.131	1.134	0.165	0.246	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	291	443	438	349	600	491	324	0
N.S.	1	1.00	0.87	1.32	1.31	1.04	1.79	1.47	0.97	0.00
time (sec)	N/A	0.833	0.193	0.329	0.139	0.112	0.616	0.153	0.256	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	166	221	222	177	284	232	184	0
N.S.	1	1.00	1.06	1.42	1.42	1.13	1.82	1.49	1.18	0.00
time (sec)	N/A	0.495	0.101	0.225	0.133	0.107	0.363	0.160	0.231	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	47	52	76	74	73	65	87	75	75	96
N.S.	1	1.11	1.62	1.57	1.55	1.38	1.85	1.60	1.60	2.04
time (sec)	N/A	0.281	0.028	0.224	0.113	0.110	0.108	0.165	0.231	0.665

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	821	821	0	0	0	0	0	0	74	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.870	0.000	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	34	20	22	89	22
N.S.	1	1.00	1.09	0.91	0.00	1.55	0.91	1.00	4.05	1.00
time (sec)	N/A	0.201	0.052	0.000	0.000	0.096	10.221	0.384	0.316	0.556

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	34	20	0	76	22
N.S.	1	1.00	1.09	0.91	0.00	1.55	0.91	0.00	3.45	1.00
time (sec)	N/A	0.212	0.058	0.009	0.000	0.110	7.759	0.000	0.327	0.533

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	54	20	0	222	22
N.S.	1	1.00	1.09	0.91	0.00	2.45	0.91	0.00	10.09	1.00
time (sec)	N/A	0.207	0.306	0.000	0.000	0.112	5.835	0.000	0.317	0.476

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	130	65	20	0	488	22
N.S.	1	1.00	1.09	0.91	5.91	2.95	0.91	0.00	22.18	1.00
time (sec)	N/A	0.213	0.537	0.000	0.376	0.132	67.230	0.000	0.369	0.500

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	388	254	311	0	0	0	635	57	0
N.S.	1	1.00	0.66	0.80	0.00	0.00	0.00	1.64	0.15	0.00
time (sec)	N/A	1.021	0.438	0.000	0.000	0.000	0.000	0.173	0.244	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	180	125	152	0	0	0	229	33	0
N.S.	1	1.01	0.70	0.85	0.00	0.00	0.00	1.28	0.18	0.00
time (sec)	N/A	0.539	0.042	0.068	0.000	0.000	0.000	0.163	0.251	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	50	46	49	0	0	0	50	12	0
N.S.	1	0.94	0.87	0.92	0.00	0.00	0.00	0.94	0.23	0.00
time (sec)	N/A	0.391	0.010	0.000	0.000	0.000	0.000	0.139	0.239	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	29	17	22	31	22
N.S.	1	1.00	1.10	1.00	1.10	1.45	0.85	1.10	1.55	1.10
time (sec)	N/A	0.211	0.019	0.002	0.209	0.088	3.655	0.441	0.250	0.431

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	53	19	22	59	22
N.S.	1	1.00	1.10	1.00	1.10	2.65	0.95	1.10	2.95	1.10
time (sec)	N/A	0.219	0.022	0.000	0.211	0.093	109.962	6.934	0.303	0.426

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	22	19	22	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.00	0.86	1.00	1.00	1.00
time (sec)	N/A	0.219	0.020	0.000	0.200	0.084	0.443	0.252	200.039	0.403

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	39	20	22	29	22
N.S.	1	1.00	1.09	0.91	1.00	1.77	0.91	1.00	1.32	1.00
time (sec)	N/A	0.220	0.024	0.000	0.212	0.100	0.773	0.210	0.510	0.411

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	63	20	22	63	22
N.S.	1	1.00	1.09	0.91	1.00	2.86	0.91	1.00	2.86	1.00
time (sec)	N/A	0.220	0.025	0.000	0.230	0.094	1.974	0.254	0.577	0.399

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	87	20	22	107	22
N.S.	1	1.00	1.09	0.91	1.00	3.95	0.91	1.00	4.86	1.00
time (sec)	N/A	0.221	0.024	0.000	0.244	0.099	9.163	0.288	0.751	0.403

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	497	359	796	0	0	0	2213	99	0
N.S.	1	1.00	0.72	1.60	0.00	0.00	0.00	4.44	0.20	0.00
time (sec)	N/A	1.118	1.835	0.000	0.000	0.000	0.000	0.219	0.188	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	248	190	375	0	0	0	860	61	0
N.S.	1	1.00	0.76	1.51	0.00	0.00	0.00	3.45	0.24	0.00
time (sec)	N/A	0.689	0.825	0.149	0.000	0.000	0.000	0.201	0.189	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	81	72	74	0	0	0	193	26	0
N.S.	1	0.94	0.84	0.86	0.00	0.00	0.00	2.24	0.30	0.00
time (sec)	N/A	0.617	0.146	0.070	0.000	0.000	0.000	0.138	0.172	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	319	57	19	22	64	22
N.S.	1	1.00	1.10	1.00	15.95	2.85	0.95	1.10	3.20	1.10
time (sec)	N/A	0.206	0.225	0.000	2.234	0.099	45.256	0.841	0.242	0.426

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	438	98	0	0	115	22
N.S.	1	1.00	1.10	1.00	21.90	4.90	0.00	0.00	5.75	1.10
time (sec)	N/A	0.209	2.170	0.000	3.108	0.104	0.000	0.000	0.262	0.420

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	237	36	20	22	22	22
N.S.	1	1.00	1.09	0.91	10.77	1.64	0.91	1.00	1.00	1.00
time (sec)	N/A	0.218	0.161	0.000	1.301	0.097	0.744	0.396	200.017	0.441

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	365	67	22	22	22	22
N.S.	1	1.00	1.09	0.91	16.59	3.05	1.00	1.00	1.00	1.00
time (sec)	N/A	0.227	0.134	0.000	1.580	0.096	1.387	0.286	200.024	0.429

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	456	108	22	22	22	22
N.S.	1	1.00	1.09	0.91	20.73	4.91	1.00	1.00	1.00	1.00
time (sec)	N/A	0.224	0.170	0.001	3.237	0.102	5.405	0.383	200.017	0.423

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	578	149	22	22	187	22
N.S.	1	1.00	1.09	0.91	26.27	6.77	1.00	1.00	8.50	1.00
time (sec)	N/A	0.227	0.174	0.000	4.918	0.137	30.429	0.416	0.722	0.433

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	754	755	402	1155	0	0	0	3222	54	0
N.S.	1	1.00	0.53	1.53	0.00	0.00	0.00	4.27	0.07	0.00
time (sec)	N/A	2.264	0.934	1.180	0.000	0.000	0.000	1.560	0.237	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	370	246	556	0	0	0	1661	31	0
N.S.	1	1.00	0.67	1.51	0.00	0.00	0.00	4.50	0.08	0.00
time (sec)	N/A	1.149	0.352	0.346	0.000	0.000	0.000	1.194	0.232	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	117	122	186	0	0	0	531	11	0
N.S.	1	0.98	1.02	1.55	0.00	0.00	0.00	4.42	0.09	0.00
time (sec)	N/A	0.700	0.056	0.071	0.000	0.000	0.000	0.426	0.190	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	19	22	21	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	0.86	1.00	0.95	1.00
time (sec)	N/A	0.220	2.041	0.439	0.000	0.000	0.914	0.935	12.346	0.396

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	20	22	22	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	0.91	1.00	1.00	1.00
time (sec)	N/A	0.224	20.231	0.683	0.603	0.000	14.640	1.158	200.021	0.424

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	851	851	0	0	0	3270	74	0
N.S.	1	1.00	1.77	1.77	0.00	0.00	0.00	6.78	0.15	0.00
time (sec)	N/A	1.660	7.269	0.217	0.000	0.000	0.000	1.934	0.276	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	156	289	278	0	0	0	1159	32	0
N.S.	1	0.98	1.82	1.75	0.00	0.00	0.00	7.29	0.20	0.00
time (sec)	N/A	0.895	1.510	0.053	0.000	0.000	0.000	1.099	0.222	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	19	22	22	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	0.86	1.00	1.00	1.00
time (sec)	N/A	0.230	0.517	0.274	0.000	0.000	11.669	0.983	200.017	0.407

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	20	22	22	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	0.91	1.00	1.00	1.00
time (sec)	N/A	0.230	12.305	0.513	0.595	0.000	92.590	1.256	200.019	0.411

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	679	680	398	664	0	0	0	981	85	0
N.S.	1	1.00	0.59	0.98	0.00	0.00	0.00	1.44	0.13	0.00
time (sec)	N/A	1.613	1.075	0.260	0.000	0.000	0.000	0.698	0.221	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	330	242	310	0	0	0	481	52	0
N.S.	1	1.00	0.74	0.94	0.00	0.00	0.00	1.46	0.16	0.00
time (sec)	N/A	0.831	0.362	0.131	0.000	0.000	0.000	0.545	0.221	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	104	118	89	0	0	0	159	22	0
N.S.	1	1.03	1.17	0.88	0.00	0.00	0.00	1.57	0.22	0.00
time (sec)	N/A	0.512	0.056	0.039	0.000	0.000	0.000	0.230	0.198	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	0	0	20	22	41	22
N.S.	1	1.00	1.09	0.91	0.00	0.00	0.91	1.00	1.86	1.00
time (sec)	N/A	0.221	0.155	0.283	0.000	0.000	2.403	0.531	0.243	0.408

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	22	22	69	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	1.00	1.00	3.14	1.00
time (sec)	N/A	0.223	0.209	0.491	0.711	0.000	39.134	0.774	0.363	0.426

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	396	322	459	0	0	0	0	0	0
N.S.	1	1.01	0.82	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.995	0.576	0.200	0.000	0.000	0.000	0.000	0.359	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	138	150	157	0	0	0	0	170	0
N.S.	1	1.01	1.09	1.15	0.00	0.00	0.00	0.00	1.24	0.00
time (sec)	N/A	0.748	0.037	0.074	0.000	0.000	0.000	0.000	0.206	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	20	22	74	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	0.91	1.00	3.36	1.00
time (sec)	N/A	0.231	0.157	0.267	0.546	0.000	9.672	0.704	0.273	0.403

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	0	0	22	125	22
N.S.	1	1.00	1.09	0.91	1.00	0.00	0.00	1.00	5.68	1.00
time (sec)	N/A	0.246	0.202	0.502	0.564	0.000	0.000	3.172	1.630	0.411

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [183] had the largest ratio of [1.11111000000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.17	15	0.400
2	A	2	2	1.01	15	0.133
3	A	6	5	0.98	23	0.217
4	A	6	6	1.10	23	0.261
5	A	6	5	1.00	23	0.217
6	A	4	4	1.03	21	0.190
7	A	6	5	1.03	20	0.250
8	A	10	9	1.12	23	0.391
9	A	8	7	1.03	23	0.304
10	A	10	9	0.99	23	0.391
11	A	7	6	0.99	23	0.261
12	A	6	5	0.94	25	0.200
13	A	9	9	1.04	25	0.360
14	A	6	5	0.95	25	0.200
15	A	5	5	0.98	23	0.217
16	A	6	5	0.97	22	0.227
17	A	15	14	1.23	25	0.560
18	A	8	7	0.94	25	0.280
19	A	15	14	1.16	25	0.560
20	A	11	10	1.07	25	0.400
21	A	6	5	0.93	25	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	8	8	0.97	25	0.320
23	A	6	5	0.94	25	0.200
24	A	6	6	0.97	23	0.261
25	A	6	5	0.95	22	0.227
26	A	20	19	1.43	25	0.760
27	A	6	5	0.91	25	0.200
28	A	20	19	1.30	25	0.760
29	A	10	9	0.90	25	0.360
30	A	13	12	1.03	25	0.480
31	A	13	12	1.01	25	0.480
32	A	9	8	0.85	25	0.320
33	A	9	8	0.96	23	0.348
34	A	6	5	0.79	22	0.227
35	A	7	6	0.94	25	0.240
36	A	11	10	0.88	25	0.400
37	A	10	9	0.93	25	0.360
38	A	16	15	0.97	25	0.600
39	A	13	12	1.02	25	0.480
40	A	13	12	0.98	25	0.480
41	A	9	8	0.87	25	0.320
42	A	2	2	1.00	23	0.087
43	A	9	8	0.87	22	0.364
44	A	10	9	0.98	25	0.360
45	A	14	13	1.04	25	0.520
46	A	13	12	1.25	25	0.480
47	A	20	19	1.14	25	0.760
48	A	13	12	1.04	25	0.480
49	A	4	4	0.99	25	0.160
50	A	11	10	0.89	25	0.400
51	A	3	3	0.98	23	0.130
52	A	11	10	0.89	22	0.455
53	A	13	12	1.10	25	0.480

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	17	16	1.09	25	0.640
55	A	17	16	1.17	25	0.640
56	A	24	23	1.21	25	0.920
57	A	7	7	0.82	27	0.259
58	A	5	5	0.87	27	0.185
59	A	3	3	1.00	24	0.125
60	A	3	3	1.01	27	0.111
61	A	3	3	0.76	27	0.111
62	A	4	4	0.70	27	0.148
63	A	4	4	0.66	27	0.148
64	A	3	3	0.67	27	0.111
65	A	3	3	0.70	27	0.111
66	A	2	2	0.75	25	0.080
67	A	8	7	0.76	27	0.259
68	A	8	7	0.76	27	0.259
69	A	10	9	0.72	27	0.333
70	A	10	10	0.90	27	0.370
71	A	8	8	0.96	27	0.296
72	A	6	6	1.10	24	0.250
73	A	6	6	1.07	27	0.222
74	A	6	6	1.04	27	0.222
75	A	5	4	0.61	27	0.148
76	A	6	5	0.60	27	0.185
77	A	6	5	0.61	27	0.185
78	A	6	5	0.59	27	0.185
79	A	4	4	0.60	27	0.148
80	A	4	4	0.60	27	0.148
81	A	4	4	0.61	27	0.148
82	A	3	3	0.61	25	0.120
83	A	10	9	0.84	27	0.333
84	A	11	10	0.81	27	0.370
85	A	11	10	0.85	27	0.370

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	15	14	0.95	27	0.519
87	A	13	12	1.01	27	0.444
88	A	8	8	1.06	24	0.333
89	A	11	10	0.98	27	0.370
90	A	11	10	1.07	27	0.370
91	A	11	10	1.08	27	0.370
92	A	5	4	0.53	27	0.148
93	A	7	6	0.60	27	0.222
94	A	6	5	0.55	27	0.185
95	A	4	4	0.55	27	0.148
96	A	4	4	0.54	27	0.148
97	A	3	3	0.51	25	0.120
98	A	13	12	0.89	27	0.444
99	A	13	12	0.86	27	0.444
100	A	14	13	0.87	27	0.481
101	A	3	3	1.00	14	0.214
102	A	3	3	1.00	24	0.125
103	A	5	5	1.09	22	0.227
104	A	4	4	1.06	22	0.182
105	A	3	3	1.00	22	0.136
106	A	2	2	1.03	20	0.100
107	A	1	1	1.00	19	0.053
108	A	6	5	1.15	22	0.227
109	A	2	2	1.04	22	0.091
110	A	8	7	1.03	22	0.318
111	A	6	6	1.05	27	0.222
112	A	5	5	1.04	27	0.185
113	A	4	4	1.03	27	0.148
114	A	3	3	1.00	27	0.111
115	A	2	2	1.01	25	0.080
116	A	1	1	1.00	24	0.042
117	A	6	5	0.67	27	0.185

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.02	27	0.074
119	A	8	7	0.76	27	0.259
120	A	4	4	0.99	27	0.148
121	A	4	4	0.71	27	0.148
122	A	8	7	1.08	27	0.259
123	A	4	4	0.76	27	0.148
124	A	3	3	1.00	27	0.111
125	A	2	2	0.99	25	0.080
126	A	2	2	1.00	24	0.083
127	A	8	7	0.75	27	0.259
128	A	7	6	0.79	27	0.222
129	A	11	10	0.81	27	0.370
130	A	6	5	0.73	27	0.185
131	A	12	11	1.22	27	0.407
132	A	6	6	0.78	27	0.222
133	A	8	7	1.19	27	0.259
134	A	4	4	0.89	27	0.148
135	A	5	4	0.83	27	0.148
136	A	3	3	0.87	25	0.120
137	A	4	4	1.03	24	0.167
138	A	11	10	0.92	27	0.370
139	A	6	5	0.75	27	0.185
140	A	15	14	0.88	27	0.519
141	A	6	5	0.72	27	0.185
142	A	6	6	1.06	20	0.300
143	A	1	1	1.00	30	0.033
144	A	1	1	1.00	31	0.032
145	A	9	9	1.03	25	0.360
146	A	7	7	1.00	25	0.280
147	A	4	4	0.98	23	0.174
148	N/A	1	0	1.00	25	0.000
149	N/A	4	0	1.00	25	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	N/A	6	0	1.00	25	0.000
151	A	9	9	0.71	27	0.333
152	A	6	6	0.81	27	0.222
153	A	3	3	0.88	27	0.111
154	A	1	1	0.99	27	0.037
155	A	3	3	0.99	27	0.111
156	A	5	5	0.95	27	0.185
157	A	1	1	0.99	22	0.045
158	A	11	11	1.14	25	0.440
159	A	9	9	1.65	25	0.360
160	A	9	9	1.15	25	0.360
161	A	7	7	1.04	23	0.304
162	A	5	5	1.10	22	0.227
163	A	12	11	1.11	25	0.440
164	A	12	11	1.23	25	0.440
165	A	12	11	1.03	25	0.440
166	A	10	9	1.38	25	0.360
167	A	17	17	1.31	27	0.630
168	A	13	13	1.97	27	0.481
169	A	15	15	1.25	27	0.556
170	A	9	9	0.96	25	0.360
171	A	8	8	1.12	24	0.333
172	A	20	19	1.25	27	0.704
173	A	17	16	1.29	27	0.593
174	A	20	19	1.21	27	0.704
175	A	16	15	1.42	27	0.556
176	A	22	22	1.49	27	0.815
177	B	18	17	2.39	27	0.630
178	A	20	20	1.38	27	0.741
179	A	11	11	0.94	25	0.440
180	A	9	9	1.18	24	0.375
181	A	29	28	1.51	27	1.037

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	22	21	1.44	27	0.778
183	A	31	30	1.41	27	1.111
184	A	20	19	1.62	27	0.704
185	A	13	12	1.03	27	0.444
186	A	15	14	0.99	27	0.519
187	A	11	10	0.85	27	0.370
188	A	10	9	0.97	25	0.360
189	A	7	6	0.78	24	0.250
190	A	8	7	0.96	27	0.259
191	A	14	13	0.89	27	0.481
192	A	12	11	0.91	27	0.407
193	A	16	15	1.06	27	0.556
194	A	16	15	1.00	27	0.556
195	A	15	14	0.98	27	0.519
196	A	11	10	0.87	27	0.370
197	A	3	3	0.96	25	0.120
198	A	11	10	0.86	24	0.417
199	A	12	11	0.98	27	0.407
200	A	19	18	1.02	27	0.667
201	A	18	17	1.20	27	0.630
202	A	22	21	1.19	27	0.778
203	A	16	15	1.00	27	0.556
204	A	9	8	1.09	27	0.296
205	A	13	12	0.91	27	0.444
206	A	5	5	0.93	25	0.200
207	A	13	12	0.91	24	0.500
208	A	17	16	1.14	27	0.593
209	A	24	23	1.15	27	0.852
210	A	23	22	1.24	27	0.815
211	A	28	27	1.35	27	1.000
212	A	13	12	0.98	29	0.414
213	A	10	10	1.00	29	0.345

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
214	A	7	6	0.79	27	0.222
215	A	5	5	0.88	26	0.192
216	A	9	8	0.66	29	0.276
217	A	9	8	0.80	29	0.276
218	A	12	11	0.66	29	0.379
219	A	11	10	0.65	29	0.345
220	A	19	18	1.12	29	0.621
221	A	17	17	1.21	29	0.586
222	A	7	6	0.68	27	0.222
223	A	10	10	1.09	26	0.385
224	A	15	14	0.72	29	0.483
225	A	16	15	0.87	29	0.517
226	A	16	15	0.67	29	0.517
227	A	19	18	0.98	29	0.621
228	A	25	24	1.23	29	0.828
229	A	27	27	1.38	29	0.931
230	A	7	6	0.60	27	0.222
231	A	12	12	1.19	26	0.462
232	A	21	20	0.84	29	0.690
233	A	25	24	1.10	29	0.828
234	A	23	22	0.78	29	0.759
235	A	30	29	1.14	29	1.000
236	A	13	12	1.03	29	0.414
237	A	10	10	1.03	29	0.345
238	A	8	7	0.89	29	0.241
239	A	5	5	0.86	29	0.172
240	A	2	2	0.68	27	0.074
241	A	1	1	1.00	26	0.038
242	A	7	6	0.62	29	0.207
243	A	8	7	0.75	29	0.241
244	A	12	11	0.66	29	0.379
245	A	11	10	0.80	29	0.345

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	19	18	0.91	29	0.621
247	A	17	16	0.93	29	0.552
248	A	11	10	0.68	29	0.345
249	A	11	10	0.78	29	0.345
250	A	7	6	0.62	27	0.222
251	A	10	9	0.72	26	0.346
252	A	13	12	0.61	29	0.414
253	A	17	16	0.83	29	0.552
254	A	23	22	0.72	29	0.759
255	A	19	18	0.96	29	0.621
256	A	15	14	1.01	29	0.483
257	A	14	13	1.01	29	0.448
258	A	15	14	1.01	29	0.483
259	A	13	12	0.65	29	0.414
260	A	9	8	0.64	27	0.296
261	A	13	12	0.87	26	0.462
262	A	21	20	0.82	29	0.690
263	A	22	21	1.01	29	0.724
264	F	0	0	N/A	0.000	N/A
265	A	25	24	1.34	29	0.828
266	A	10	10	1.44	24	0.417
267	A	9	8	1.27	24	0.333
268	A	5	5	1.10	24	0.208
269	A	3	3	1.04	22	0.136
270	A	1	1	1.00	21	0.048
271	A	7	6	1.22	24	0.250
272	A	8	7	1.16	24	0.292
273	A	12	11	1.09	24	0.458
274	A	1	1	1.00	22	0.045
275	A	10	9	0.69	22	0.409
276	A	13	12	0.86	22	0.545
277	A	16	15	0.99	22	0.682

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	12	12	0.78	27	0.444
279	A	9	9	0.85	27	0.333
280	A	5	5	0.95	25	0.200
281	N/A	1	0	1.00	27	0.000
282	N/A	6	0	1.00	27	0.000
283	N/A	8	0	1.00	27	0.000
284	N/A	15	0	1.00	29	0.000
285	N/A	8	0	1.00	29	0.000
286	N/A	4	0	1.00	29	0.000
287	N/A	1	0	1.00	29	0.000
288	N/A	1	0	1.00	29	0.000
289	N/A	1	0	1.00	29	0.000
290	N/A	1	0	1.00	24	0.000
291	A	19	18	1.57	20	0.900
292	A	15	14	1.38	20	0.700
293	A	11	10	1.28	18	0.556
294	A	8	7	0.79	20	0.350
295	A	16	15	0.82	20	0.750
296	A	19	18	0.98	20	0.900
297	A	22	22	1.15	22	1.000
298	A	14	14	1.01	22	0.636
299	A	6	6	0.79	22	0.273
300	A	1	1	1.00	22	0.045
301	A	11	10	0.64	22	0.455
302	A	15	14	0.75	22	0.636
303	A	20	19	0.89	22	0.864
304	N/A	1	0	1.00	24	0.000
305	A	9	9	1.44	24	0.375
306	A	10	10	1.34	24	0.417
307	A	6	6	1.07	24	0.250
308	A	4	4	1.09	22	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
309	A	1	1	1.00	21	0.048
310	A	8	7	1.23	24	0.292
311	A	9	8	1.20	24	0.333
312	A	12	11	1.08	24	0.458
313	A	5	4	0.76	20	0.200
314	A	5	4	0.80	20	0.200
315	A	5	4	0.93	18	0.222
316	N/A	1	0	1.00	20	0.000
317	N/A	1	0	1.00	20	0.000
318	A	4	3	0.83	28	0.107
319	A	4	3	0.85	28	0.107
320	A	4	3	0.88	28	0.107
321	A	4	3	0.87	26	0.115
322	A	5	4	0.88	25	0.160
323	N/A	2	0	1.00	28	0.000
324	N/A	2	0	1.00	28	0.000
325	N/A	1	0	1.00	28	0.000
326	N/A	1	0	1.00	28	0.000
327	A	4	3	0.84	28	0.107
328	A	4	3	0.83	28	0.107
329	A	4	3	0.85	26	0.115
330	A	5	4	0.85	25	0.160
331	N/A	2	0	1.00	28	0.000
332	N/A	2	0	1.00	28	0.000
333	N/A	1	0	1.00	28	0.000
334	N/A	1	0	1.00	28	0.000
335	A	4	3	0.84	28	0.107
336	A	4	3	0.83	28	0.107
337	A	4	3	0.84	26	0.115
338	A	5	4	0.83	25	0.160
339	N/A	2	0	1.00	28	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	N/A	2	0	1.00	28	0.000
341	N/A	1	0	1.00	28	0.000
342	N/A	1	0	1.00	28	0.000
343	A	5	4	0.90	24	0.167
344	A	5	4	0.96	24	0.167
345	A	5	4	0.96	24	0.167
346	A	5	4	0.96	24	0.167
347	A	4	3	1.11	22	0.136
348	A	1	1	1.11	21	0.048
349	N/A	1	0	1.00	24	0.000
350	N/A	1	0	1.00	24	0.000
351	A	5	4	0.85	28	0.143
352	A	5	4	0.85	28	0.143
353	A	5	4	0.87	28	0.143
354	A	5	4	0.88	28	0.143
355	A	8	7	0.91	26	0.269
356	A	1	1	1.06	25	0.040
357	N/A	1	0	1.00	28	0.000
358	N/A	1	0	1.00	28	0.000
359	N/A	1	0	1.00	28	0.000
360	N/A	1	0	1.00	26	0.000
361	N/A	1	0	1.00	25	0.000
362	N/A	1	0	1.00	28	0.000
363	N/A	1	0	1.00	28	0.000
364	N/A	1	0	1.00	28	0.000
365	N/A	1	0	1.00	26	0.000
366	N/A	1	0	1.00	25	0.000
367	N/A	1	0	1.00	28	0.000
368	N/A	1	0	1.00	28	0.000
369	N/A	1	0	1.00	28	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
370	N/A	1	0	1.00	28	0.000
371	N/A	1	0	1.00	28	0.000
372	N/A	1	0	1.00	28	0.000
373	N/A	1	0	1.00	28	0.000
374	N/A	1	0	1.00	28	0.000
375	N/A	1	0	1.00	24	0.000
376	A	5	4	0.83	20	0.200
377	A	5	4	0.87	20	0.200
378	A	5	4	0.96	18	0.222
379	N/A	2	0	1.00	20	0.000
380	N/A	2	0	1.00	20	0.000
381	F	0	0	N/A	0.000	N/A
382	N/A	1	0	1.00	28	0.000
383	A	6	5	1.36	28	0.179
384	B	13	12	2.05	28	0.429
385	A	14	13	1.22	26	0.500
386	A	12	11	0.95	25	0.440
387	N/A	11	0	1.00	28	0.000
388	N/A	2	0	1.00	28	0.000
389	N/A	1	0	1.00	28	0.000
390	N/A	1	0	1.00	28	0.000
391	N/A	1	0	1.00	28	0.000
392	A	6	5	1.41	28	0.179
393	A	6	5	1.14	28	0.179
394	A	11	10	1.36	26	0.385
395	A	6	5	0.93	25	0.200
396	N/A	8	0	1.00	28	0.000
397	N/A	2	0	1.00	28	0.000
398	N/A	1	0	1.00	28	0.000
399	N/A	2	0	1.00	28	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
400	N/A	1	0	1.00	28	0.000
401	A	6	5	1.77	28	0.179
402	A	6	5	1.42	28	0.179
403	A	11	10	1.41	26	0.385
404	A	6	5	0.87	25	0.200
405	N/A	8	0	1.00	28	0.000
406	N/A	2	0	1.00	28	0.000
407	N/A	1	0	1.00	28	0.000
408	N/A	1	0	1.00	28	0.000
409	N/A	2	0	1.00	28	0.000
410	A	6	5	0.86	28	0.179
411	A	6	5	0.92	28	0.179
412	A	6	5	0.89	28	0.179
413	A	12	11	0.94	28	0.393
414	A	10	9	0.94	26	0.346
415	A	1	1	0.94	25	0.040
416	N/A	2	0	1.00	28	0.000
417	N/A	2	0	1.00	28	0.000
418	N/A	1	0	1.00	28	0.000
419	N/A	1	0	1.00	28	0.000
420	N/A	2	0	1.00	28	0.000
421	N/A	1	0	1.00	26	0.000
422	N/A	2	0	1.00	25	0.000
423	N/A	1	0	1.00	28	0.000
424	N/A	1	0	1.00	28	0.000
425	N/A	1	0	1.00	28	0.000
426	N/A	1	0	1.00	28	0.000
427	N/A	1	0	1.00	28	0.000
428	N/A	1	0	1.00	26	0.000
429	N/A	2	0	1.00	25	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
430	N/A	1	0	1.00	28	0.000
431	N/A	1	0	1.00	28	0.000
432	A	1	1	1.00	21	0.048
433	B	5	4	2.02	27	0.148
434	A	5	4	0.99	27	0.148
435	A	9	8	1.18	25	0.320
436	A	5	4	1.01	24	0.167
437	N/A	8	0	1.00	27	0.000
438	A	5	4	1.25	29	0.138
439	A	5	4	1.57	29	0.138
440	A	9	8	1.62	27	0.296
441	A	5	4	0.94	26	0.154
442	N/A	8	0	1.00	29	0.000
443	A	1	1	1.00	38	0.026
444	A	13	12	1.15	24	0.500
445	A	9	8	1.00	24	0.333
446	A	1	1	1.00	24	0.042
447	N/A	2	0	1.00	24	0.000
448	N/A	3	0	1.00	24	0.000
449	A	14	13	0.93	24	0.542
450	A	8	7	0.75	24	0.292
451	A	1	1	1.00	24	0.042
452	N/A	2	0	1.00	24	0.000
453	A	26	25	1.10	24	1.042
454	A	12	11	0.82	24	0.458
455	A	1	1	1.00	24	0.042
456	N/A	2	0	1.00	24	0.000
457	A	14	13	1.15	24	0.542
458	A	9	8	1.00	24	0.333
459	A	1	1	1.00	24	0.042
460	N/A	3	0	1.00	24	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
461	N/A	5	0	1.00	24	0.000
462	A	15	14	0.97	24	0.583
463	A	8	7	0.78	24	0.292
464	A	1	1	1.00	24	0.042
465	N/A	3	0	1.00	24	0.000
466	A	5	4	1.04	19	0.211
467	A	5	4	0.58	24	0.167
468	A	5	4	0.62	24	0.167
469	A	5	4	0.69	24	0.167
470	A	1	1	1.00	24	0.042
471	N/A	1	0	1.00	24	0.000
472	N/A	1	0	1.00	24	0.000
473	A	5	4	0.73	24	0.167
474	A	5	4	0.84	24	0.167
475	A	8	7	1.00	24	0.292
476	A	1	1	1.00	24	0.042
477	N/A	2	0	1.00	24	0.000
478	N/A	2	0	1.00	24	0.000
479	A	10	9	0.97	24	0.375
480	A	7	6	1.03	24	0.250
481	A	1	1	1.00	24	0.042
482	N/A	2	0	1.00	24	0.000
483	N/A	2	0	1.00	24	0.000
484	A	4	3	0.77	29	0.103
485	A	4	3	0.80	27	0.111
486	A	5	4	0.77	26	0.154
487	N/A	2	0	1.00	29	0.000
488	N/A	2	0	1.00	29	0.000
489	A	4	3	0.72	29	0.103
490	A	4	3	0.76	27	0.111
491	A	5	4	0.73	26	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
492	N/A	2	0	1.00	29	0.000
493	N/A	2	0	1.00	29	0.000
494	A	4	3	0.70	29	0.103
495	A	4	3	0.73	27	0.111
496	A	5	4	0.71	26	0.154
497	N/A	2	0	1.00	29	0.000
498	N/A	2	0	1.00	29	0.000
499	N/A	1	0	1.00	24	0.000
500	A	5	4	1.01	24	0.167
501	A	5	4	0.96	24	0.167
502	A	6	5	1.04	22	0.227
503	A	1	1	1.06	21	0.048
504	N/A	1	0	1.00	24	0.000
505	N/A	1	0	1.00	24	0.000
506	A	4	4	0.49	30	0.133
507	A	4	4	0.51	30	0.133
508	A	4	4	0.69	30	0.133
509	A	4	4	0.62	30	0.133
510	A	4	4	0.65	30	0.133
511	A	7	7	0.65	30	0.233
512	A	4	4	0.46	30	0.133
513	A	7	7	0.67	30	0.233
514	A	4	4	0.51	30	0.133
515	A	4	4	0.52	30	0.133
516	A	4	4	0.54	30	0.133
517	A	4	4	0.52	30	0.133
518	A	9	9	0.64	30	0.300
519	A	4	4	0.46	30	0.133
520	A	4	4	0.49	30	0.133
521	A	4	4	0.50	30	0.133
522	A	4	4	0.47	30	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
523	A	4	4	0.49	30	0.133
524	A	4	4	0.50	30	0.133
525	A	4	4	0.52	30	0.133
526	A	4	4	0.63	30	0.133
527	A	2	2	1.00	30	0.067
528	A	7	7	0.84	30	0.233
529	A	4	4	0.60	30	0.133
530	A	4	4	0.47	30	0.133
531	A	4	4	0.53	30	0.133
532	A	4	4	0.65	30	0.133
533	A	6	6	0.84	30	0.200
534	A	3	3	0.83	30	0.100
535	A	4	4	0.62	30	0.133
536	A	4	4	0.50	30	0.133
537	A	4	4	0.52	30	0.133
538	A	7	7	0.65	30	0.233
539	A	4	4	0.60	30	0.133
540	A	4	4	0.61	30	0.133
541	A	5	5	0.70	30	0.167
542	A	4	4	0.54	32	0.125
543	A	4	4	0.55	32	0.125
544	A	6	6	0.65	32	0.188
545	A	4	4	0.53	32	0.125
546	A	4	4	0.48	32	0.125
547	A	4	4	0.46	32	0.125
548	A	4	4	0.54	32	0.125
549	A	11	11	0.76	32	0.344
550	A	4	4	0.55	32	0.125
551	A	7	6	0.55	32	0.188
552	A	4	4	0.45	32	0.125
553	A	4	4	0.45	32	0.125
554	A	13	13	0.86	32	0.406

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
555	A	4	4	0.54	32	0.125
556	A	4	4	0.54	32	0.125
557	A	7	6	0.54	32	0.188
558	A	4	4	0.44	32	0.125
559	A	4	4	0.42	32	0.125
560	A	7	6	0.54	32	0.188
561	A	7	6	0.55	32	0.188
562	A	4	4	0.54	32	0.125
563	A	2	2	1.00	32	0.062
564	A	4	4	0.51	32	0.125
565	A	4	4	0.51	32	0.125
566	A	4	4	0.44	32	0.125
567	A	4	4	0.44	32	0.125
568	A	4	4	0.48	32	0.125
569	A	4	4	0.51	32	0.125
570	A	11	10	0.65	32	0.312
571	A	4	4	0.53	32	0.125
572	A	4	4	0.42	32	0.125
573	A	4	4	0.45	32	0.125
574	A	4	4	0.46	32	0.125
575	A	4	4	0.51	32	0.125
576	A	4	4	0.53	32	0.125
577	A	14	13	0.63	32	0.406
578	A	11	11	0.80	35	0.314
579	A	8	7	0.68	33	0.212
580	A	6	6	0.65	32	0.188
581	A	10	9	0.52	35	0.257
582	A	10	9	0.62	35	0.257
583	A	18	18	0.90	35	0.514
584	A	8	7	0.57	33	0.212
585	A	11	11	0.76	32	0.344
586	A	16	15	0.52	35	0.429

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
587	A	17	16	0.63	35	0.457
588	A	6	6	0.60	35	0.171
589	A	3	3	0.58	33	0.091
590	A	2	2	1.00	32	0.062
591	A	8	7	0.57	35	0.200
592	A	9	8	0.66	35	0.229
593	A	12	11	0.57	35	0.314
594	A	8	7	0.55	33	0.212
595	A	11	10	0.65	32	0.312
596	A	14	13	0.47	35	0.371
597	A	18	17	0.62	35	0.486
598	A	6	5	1.01	19	0.263
599	A	6	6	0.97	19	0.316
600	A	6	5	1.02	19	0.263
601	A	5	5	1.13	17	0.294
602	A	6	5	1.04	16	0.312
603	A	4	4	1.08	19	0.211
604	A	7	6	1.08	19	0.316
605	A	4	4	1.07	19	0.211
606	A	7	6	0.99	19	0.316
607	A	6	5	1.00	21	0.238
608	A	8	8	0.93	21	0.381
609	A	6	5	1.01	21	0.238
610	A	7	7	1.16	19	0.368
611	A	6	5	1.01	18	0.278
612	A	4	4	1.01	21	0.190
613	A	8	7	0.90	21	0.333
614	A	4	4	1.05	21	0.190
615	A	9	8	1.13	21	0.381
616	A	6	5	1.00	21	0.238
617	A	12	12	1.20	21	0.571
618	A	6	5	1.00	21	0.238

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
619	A	9	9	1.15	19	0.474
620	A	6	5	1.00	18	0.278
621	A	4	4	1.00	21	0.190
622	A	6	5	1.00	21	0.238
623	A	4	4	1.02	21	0.190
624	A	10	9	0.92	21	0.429
625	A	6	5	1.00	18	0.278
626	A	2	2	1.00	21	0.095
627	A	2	2	1.00	21	0.095
628	A	2	2	1.00	21	0.095
629	A	2	2	1.00	19	0.105
630	A	2	2	1.00	18	0.111
631	A	2	2	1.00	21	0.095
632	A	2	2	1.00	21	0.095
633	A	2	2	1.00	21	0.095
634	A	2	2	1.00	21	0.095
635	A	2	2	1.00	21	0.095
636	A	4	3	0.97	19	0.158
637	A	2	2	1.00	21	0.095
638	A	2	2	1.00	21	0.095
639	A	2	2	1.00	21	0.095
640	A	2	2	1.00	21	0.095
641	A	2	2	1.00	18	0.111
642	A	2	2	1.00	21	0.095
643	A	2	2	1.00	21	0.095
644	A	8	7	1.16	21	0.333
645	A	5	4	1.03	19	0.211
646	A	2	2	1.00	21	0.095
647	A	2	2	1.00	21	0.095
648	A	2	2	1.00	21	0.095
649	A	2	2	1.00	21	0.095
650	A	2	2	1.00	18	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
651	N/A	1	0	1.00	20	0.000
652	N/A	1	0	1.00	20	0.000
653	A	6	5	1.01	20	0.250
654	A	7	6	1.01	20	0.300
655	A	9	8	1.07	20	0.400
656	A	8	8	1.01	23	0.348
657	A	6	6	1.01	23	0.261
658	A	4	4	1.05	21	0.190
659	N/A	1	0	1.00	23	0.000
660	N/A	1	0	1.00	23	0.000
661	A	2	2	1.00	20	0.100
662	A	2	2	1.00	20	0.100
663	A	2	2	1.00	18	0.111
664	A	3	3	1.11	10	0.300
665	A	2	2	1.00	20	0.100
666	N/A	1	0	1.00	22	0.000
667	N/A	1	0	1.00	22	0.000
668	N/A	1	0	1.00	22	0.000
669	N/A	1	0	1.00	22	0.000
670	A	2	2	1.00	20	0.100
671	A	2	2	1.01	18	0.111
672	A	9	8	0.94	10	0.800
673	N/A	1	0	1.00	20	0.000
674	N/A	1	0	1.00	20	0.000
675	N/A	1	0	1.00	22	0.000
676	N/A	1	0	1.00	22	0.000
677	N/A	1	0	1.00	22	0.000
678	N/A	1	0	1.00	22	0.000
679	A	2	2	1.00	20	0.100
680	A	2	2	1.00	18	0.111
681	A	9	8	0.94	10	0.800

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
682	N/A	1	0	1.00	20	0.000
683	N/A	1	0	1.00	20	0.000
684	N/A	1	0	1.00	22	0.000
685	N/A	1	0	1.00	22	0.000
686	N/A	1	0	1.00	22	0.000
687	N/A	1	0	1.00	22	0.000
688	A	2	2	1.00	22	0.091
689	A	2	2	1.00	20	0.100
690	A	11	10	0.98	12	0.833
691	N/A	1	0	1.00	22	0.000
692	N/A	1	0	1.00	22	0.000
693	A	2	2	1.00	20	0.100
694	A	13	12	0.98	12	1.000
695	N/A	1	0	1.00	22	0.000
696	N/A	1	0	1.00	22	0.000
697	A	2	2	1.00	22	0.091
698	A	2	2	1.00	20	0.100
699	A	11	10	1.03	12	0.833
700	N/A	1	0	1.00	22	0.000
701	N/A	1	0	1.00	22	0.000
702	A	2	2	1.01	20	0.100
703	A	11	10	1.01	12	0.833
704	N/A	1	0	1.00	22	0.000
705	N/A	1	0	1.00	22	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{x \arcsin(2x)}{1-4x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{x \arcsin(2x)}{1-4x^2} \, dx = \frac{1}{8} i \arcsin(2x)^2 - \frac{1}{4} \arcsin(2x) \log\left(1+e^{2i \arcsin(2x)}\right) + \frac{1}{8} i \operatorname{PolyLog}\left(2,-e^{2i \arcsin(2x)}\right)$$

output
```
1/8*I*arcsin(2*x)^2-1/4*arcsin(2*x)*ln(1+(2*I*x+(-4*x^2+1)^(1/2))^2)+1/8*I
*polylog(2,-(2*I*x+(-4*x^2+1)^(1/2))^2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 213 vs. 2(54) = 108.

Time = 0.03 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.94

$$\int \frac{x \arcsin(2x)}{1-4x^2}\,dx = \frac{1}{8}\Bigg(-2i\pi\arcsin(2x) + i\arcsin(2x)^2 - 4\pi\log\left(1+e^{-i\arcsin(2x)}\right) - \pi\log\left(1-ie^{i\arcsin(2x)}\right) - 2\arcsin(2x)\log\left(1-ie^{i\arcsin(2x)}\right) + \pi\log\left(1+ie^{i\arcsin(2x)}\right) - 2\arcsin(2x)\log\left(1+ie^{i\arcsin(2x)}\right) + 4\pi\log\left(\cos\left(\frac{1}{2}\arcsin(2x)\right)\right) - \pi\log\left(-\cos\left(\frac{1}{4}(\pi+2\arcsin(2x))\right)\right) + \pi\log\left(\sin\left(\frac{1}{4}(\pi+2\arcsin(2x))\right)\right) + 2i\,\mathrm{PolyLog}\left(2,-ie^{i\arcsin(2x)}\right) + 2i\,\mathrm{PolyLog}\left(2,ie^{i\arcsin(2x)}\right)\Bigg)$$

input
```
Integrate[(x*ArcSin[2*x])/(1 - 4*x^2),x]
```

output
```
((-2*I)*Pi*ArcSin[2*x] + I*ArcSin[2*x]^2 - 4*Pi*Log[1 + E^((-I)*ArcSin[2*x
])] - Pi*Log[1 - I*E^(I*ArcSin[2*x])] - 2*ArcSin[2*x]*Log[1 - I*E^(I*ArcSi
n[2*x])] + Pi*Log[1 + I*E^(I*ArcSin[2*x])] - 2*ArcSin[2*x]*Log[1 + I*E^(I*
ArcSin[2*x])] + 4*Pi*Log[Cos[ArcSin[2*x]/2]] - Pi*Log[-Cos[(Pi + 2*ArcSin[
2*x])/4]] + Pi*Log[Sin[(Pi + 2*ArcSin[2*x])/4]] + (2*I)*PolyLog[2, (-I)*E^
(I*ArcSin[2*x])] + (2*I)*PolyLog[2, I*E^(I*ArcSin[2*x])])/8
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5180, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arcsin(2x)}{1-4x^2}dx$$

$\Big\downarrow$ 5180

$$\frac{1}{4}\int \frac{2x \arcsin(2x)}{\sqrt{1-4x^2}}d\arcsin(2x)$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\int \arcsin(2x)\tan(\arcsin(2x))d\arcsin(2x)$$

$\Big\downarrow$ 4202

$$\frac{1}{4}\left(\frac{1}{2}i\arcsin(2x)^2-2i\int \frac{e^{2i\arcsin(2x)}\arcsin(2x)}{1+e^{2i\arcsin(2x)}}d\arcsin(2x)\right)$$

$\Big\downarrow$ 2620

$$\frac{1}{4}\left(\frac{1}{2}i\arcsin(2x)^2-2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\arcsin(2x)}\right)d\arcsin(2x)-\frac{1}{2}i\arcsin(2x)\log\left(1+e^{2i\arcsin(2x)}\right)\right)\right)$$

$\Big\downarrow$ 2715

$$\frac{1}{4}\left(\frac{1}{2}i\arcsin(2x)^2-2i\left(\frac{1}{4}\int e^{-2i\arcsin(2x)}\log\left(1+e^{2i\arcsin(2x)}\right)de^{2i\arcsin(2x)}-\frac{1}{2}i\arcsin(2x)\log\left(1+e^{2i\arcsin(2x)}\right.\right.\right.$$

$\Big\downarrow$ 2838

$$\frac{1}{4}\left(\frac{1}{2}i\arcsin(2x)^2-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2i\arcsin(2x)}\right)-\frac{1}{2}i\arcsin(2x)\log\left(1+e^{2i\arcsin(2x)}\right)\right)\right)$$

input
```
Int[(x*ArcSin[2*x])/(1 - 4*x^2),x]
```

output
```
((I/2)*ArcSin[2*x]^2 - (2*I)*((-1/2*I)*ArcSin[2*x]*Log[1 + E^((2*I)*ArcSin
[2*x])] - PolyLog[2, -E^((2*I)*ArcSin[2*x])]/4))/4
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5180
```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[-e^(-1)   Subst[Int[(a + b*x)^n*Tan[x], x], x, ArcSin[c*x
]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{i\arcsin(2x)^2}{8}-\frac{\arcsin(2x)\ln\left(1+\left(2ix+\sqrt{-4x^2+1}\right)^2\right)}{4}+\frac{i\,\mathrm{polylog}\left(2,-\left(2ix+\sqrt{-4x^2+1}\right)^2\right)}{8}$	59
default	$\frac{i\arcsin(2x)^2}{8}-\frac{\arcsin(2x)\ln\left(1+\left(2ix+\sqrt{-4x^2+1}\right)^2\right)}{4}+\frac{i\,\mathrm{polylog}\left(2,-\left(2ix+\sqrt{-4x^2+1}\right)^2\right)}{8}$	59

input
```
int(x*arcsin(2*x)/(-4*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*I*arcsin(2*x)^2-1/4*arcsin(2*x)*ln(1+(2*I*x+(-4*x^2+1)^(1/2))^2)+1/8*I
*polylog(2,-(2*I*x+(-4*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{x\arcsin(2x)}{1-4x^2}\,dx=\int -\frac{x\arcsin(2x)}{4x^2-1}\,dx$$

input
```
integrate(x*arcsin(2*x)/(-4*x^2+1),x, algorithm="fricas")
```

output
```
integral(-x*arcsin(2*x)/(4*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x\arcsin(2x)}{1-4x^2}\,dx=-\int \frac{x\,\mathrm{asin}\,(2x)}{4x^2-1}\,dx$$

input
```
integrate(x*asin(2*x)/(-4*x**2+1),x)
```

output
```
-Integral(x*asin(2*x)/(4*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{x \arcsin(2x)}{1-4x^2}\,dx = \int -\frac{x \arcsin(2\,x)}{4\,x^2-1}\,dx$$

input
```
integrate(x*arcsin(2*x)/(-4*x^2+1),x, algorithm="maxima")
```

output
```
-1/8*arctan2(2*x, sqrt(2*x + 1)*sqrt(-2*x + 1))*log(2*x + 1) - 1/8*arctan2
(2*x, sqrt(2*x + 1)*sqrt(-2*x + 1))*log(-2*x + 1) - integrate(1/4*(e^(1/2*
log(2*x + 1) + 1/2*log(-2*x + 1))*log(2*x + 1) + e^(1/2*log(2*x + 1) + 1/2
*log(-2*x + 1))*log(-2*x + 1))/(16*x^4 - 4*x^2 + (4*x^2 - 1)*e^(log(2*x +
1) + log(-2*x + 1))), x)
```

Giac [F]

$$\int \frac{x \arcsin(2x)}{1-4x^2}\,dx = \int -\frac{x \arcsin(2\,x)}{4\,x^2-1}\,dx$$

input
```
integrate(x*arcsin(2*x)/(-4*x^2+1),x, algorithm="giac")
```

output
```
integrate(-x*arcsin(2*x)/(4*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \arcsin(2x)}{1-4x^2}\,dx = -\int \frac{x\,\mathrm{asin}(2\,x)}{4\,x^2-1}\,dx$$

input
```
int(-(x*asin(2*x))/(4*x^2 - 1),x)
```

output
```
-int((x*asin(2*x))/(4*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{x \arcsin(2x)}{1-4x^2}\,dx = -\left(\int \frac{asin(2x)\,x}{4x^2-1}dx\right)$$

input
```
int(x*asin(2*x)/(-4*x^2+1),x)
```

output
```
- int((asin(2*x)*x)/(4*x**2 - 1),x)
```

3.2 $\int \frac{x \arcsin(2x)}{1+4x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 243

$$
\begin{aligned}
\int \frac{x \arcsin(2x)}{1+4x^2} \, dx = & -\frac{1}{8} i \arcsin(2x)^2 + \frac{1}{8} \arcsin(2x) \log\left(1 - \left(1 - \sqrt{2}\right) e^{i \arcsin(2x)}\right) \\
& + \frac{1}{8} \arcsin(2x) \log\left(1 + \left(1 - \sqrt{2}\right) e^{i \arcsin(2x)}\right) \\
& + \frac{1}{8} \arcsin(2x) \log\left(1 - \left(1 + \sqrt{2}\right) e^{i \arcsin(2x)}\right) \\
& + \frac{1}{8} \arcsin(2x) \log\left(1 + \left(1 + \sqrt{2}\right) e^{i \arcsin(2x)}\right) \\
& - \frac{1}{8} i \operatorname{PolyLog}\left(2, \left(-1 - \sqrt{2}\right) e^{i \arcsin(2x)}\right) \\
& - \frac{1}{8} i \operatorname{PolyLog}\left(2, \left(1 - \sqrt{2}\right) e^{i \arcsin(2x)}\right) \\
& - \frac{1}{8} i \operatorname{PolyLog}\left(2, \left(-1 + \sqrt{2}\right) e^{i \arcsin(2x)}\right) \\
& - \frac{1}{8} i \operatorname{PolyLog}\left(2, \left(1 + \sqrt{2}\right) e^{i \arcsin(2x)}\right)
\end{aligned}
$$

output
```
-1/8*I*arcsin(2*x)^2+1/8*arcsin(2*x)*ln(1-(1-2^(1/2))*(2*I*x+(-4*x^2+1)^(1
/2)))+1/8*arcsin(2*x)*ln(1+(1-2^(1/2))*(2*I*x+(-4*x^2+1)^(1/2)))+1/8*arcsi
n(2*x)*ln(1-(1+2^(1/2))*(2*I*x+(-4*x^2+1)^(1/2)))+1/8*arcsin(2*x)*ln(1+(1+
2^(1/2))*(2*I*x+(-4*x^2+1)^(1/2)))-1/8*I*polylog(2,(-1-2^(1/2))*(2*I*x+(-4
*x^2+1)^(1/2)))-1/8*I*polylog(2,(1-2^(1/2))*(2*I*x+(-4*x^2+1)^(1/2)))-1/8*
I*polylog(2,(2^(1/2)-1)*(2*I*x+(-4*x^2+1)^(1/2)))-1/8*I*polylog(2,(1+2^(1/
2))*(2*I*x+(-4*x^2+1)^(1/2)))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.88

$$\int \frac{x \arcsin(2x)}{1+4x^2} \, dx = -\frac{1}{8} i \Big(\arcsin(2x)^2 + i \arcsin(2x) \log \left(1 - \left(-1+\sqrt{2}\right) e^{i \arcsin(2x)}\right) + i \arcsin(2x) \log \left(1 + \left(-1+\sqrt{2}\right) e^{i \arcsin(2x)}\right) + i \arcsin(2x) \log \left(1 - \left(1+\sqrt{2}\right) e^{i \arcsin(2x)}\right) + i \arcsin(2x) \log \left(1 + \left(1+\sqrt{2}\right) e^{i \arcsin(2x)}\right) + \text{PolyLog}\left(2, -\left(\left(-1+\sqrt{2}\right) e^{i \arcsin(2x)}\right)\right) + \text{PolyLog}\left(2, \left(-1+\sqrt{2}\right) e^{i \arcsin(2x)}\right) + \text{PolyLog}\left(2, -\left(\left(1+\sqrt{2}\right) e^{i \arcsin(2x)}\right)\right) + \text{PolyLog}\left(2, \left(1+\sqrt{2}\right) e^{i \arcsin(2x)}\right) \Big)$$

input
```
Integrate[(x*ArcSin[2*x])/(1 + 4*x^2),x]
```

output
```
(-1/8*I)*(ArcSin[2*x]^2 + I*ArcSin[2*x]*Log[1 - (-1 + Sqrt[2])*E^(I*ArcSin
[2*x])] + I*ArcSin[2*x]*Log[1 + (-1 + Sqrt[2])*E^(I*ArcSin[2*x])] + I*ArcS
in[2*x]*Log[1 - (1 + Sqrt[2])*E^(I*ArcSin[2*x])] + I*ArcSin[2*x]*Log[1 + (
1 + Sqrt[2])*E^(I*ArcSin[2*x])] + PolyLog[2, -((-1 + Sqrt[2])*E^(I*ArcSin[
2*x]))] + PolyLog[2, (-1 + Sqrt[2])*E^(I*ArcSin[2*x])] + PolyLog[2, -((1 +
 Sqrt[2])*E^(I*ArcSin[2*x]))] + PolyLog[2, (1 + Sqrt[2])*E^(I*ArcSin[2*x])
])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arcsin(2x)}{4x^2+1} \, dx$$
$$\Big\downarrow \text{5232}$$
$$\int \left(\frac{\arcsin(2x)}{4(2x+i)} - \frac{\arcsin(2x)}{4(-2x+i)}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{8} i \operatorname{PolyLog}\left(2,-\left(\left(1-\sqrt{2}\right) e^{i \arcsin(2x)}\right)\right)-\frac{1}{8} i \operatorname{PolyLog}\left(2,\left(1-\sqrt{2}\right) e^{i \arcsin(2x)}\right)-\frac{1}{8} i \operatorname{PolyLog}\left(2,-\left(\left(1+\sqrt{2}\right) e^{i \arcsin(2x)}\right)\right)-\frac{1}{8} i \operatorname{PolyLog}\left(2,\left(1+\sqrt{2}\right) e^{i \arcsin(2x)}\right)-\frac{1}{8} i \arcsin(2x)^2+\frac{1}{8} \arcsin(2x) \log \left(1-\left(1-\sqrt{2}\right) e^{i \arcsin(2x)}\right)+\frac{1}{8} \arcsin(2x) \log \left(1+\left(1-\sqrt{2}\right) e^{i \arcsin(2x)}\right)+\frac{1}{8} \arcsin(2x) \log \left(1-\left(1+\sqrt{2}\right) e^{i \arcsin(2x)}\right)+\frac{1}{8} \arcsin(2x) \log \left(1+\left(1+\sqrt{2}\right) e^{i \arcsin(2x)}\right)$$

input
```
Int[(x*ArcSin[2*x])/(1 + 4*x^2),x]
```

output
```
(-1/8*I)*ArcSin[2*x]^2 + (ArcSin[2*x]*Log[1 - (1 - Sqrt[2])*E^(I*ArcSin[2*
x])])/8 + (ArcSin[2*x]*Log[1 + (1 - Sqrt[2])*E^(I*ArcSin[2*x])])/8 + (ArcS
in[2*x]*Log[1 - (1 + Sqrt[2])*E^(I*ArcSin[2*x])])/8 + (ArcSin[2*x]*Log[1 +
 (1 + Sqrt[2])*E^(I*ArcSin[2*x])])/8 - (I/8)*PolyLog[2, -((1 - Sqrt[2])*E^
(I*ArcSin[2*x]))] - (I/8)*PolyLog[2, (1 - Sqrt[2])*E^(I*ArcSin[2*x])] - (I
/8)*PolyLog[2, -((1 + Sqrt[2])*E^(I*ArcSin[2*x]))] - (I/8)*PolyLog[2, (1 +
 Sqrt[2])*E^(I*ArcSin[2*x])]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{i \arcsin(2x)^2}{8}+\frac{\arcsin(2x)\ln\left(\frac{3+2\sqrt{2}-\left(2ix+\sqrt{-4x^2+1}\right)^2}{3+2\sqrt{2}}\right)}{8}+\frac{\arcsin(2x)\ln\left(\frac{-3+2\sqrt{2}+\left(2ix+\sqrt{-4x^2+1}\right)^2}{-3+2\sqrt{2}}\right)}{8}-$
default	$-\frac{i \arcsin(2x)^2}{8}+\frac{\arcsin(2x)\ln\left(\frac{3+2\sqrt{2}-\left(2ix+\sqrt{-4x^2+1}\right)^2}{3+2\sqrt{2}}\right)}{8}+\frac{\arcsin(2x)\ln\left(\frac{-3+2\sqrt{2}+\left(2ix+\sqrt{-4x^2+1}\right)^2}{-3+2\sqrt{2}}\right)}{8}-$

input
```
int(x*arcsin(2*x)/(4*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/8*I*arcsin(2*x)^2+1/8*arcsin(2*x)*ln((3+2*2^(1/2)-(2*I*x+(-4*x^2+1)^(1/
2))^2)/(3+2*2^(1/2)))+1/8*arcsin(2*x)*ln((-3+2*2^(1/2)+(2*I*x+(-4*x^2+1)^(
1/2))^2)/(-3+2*2^(1/2)))-1/16*I*dilog((3+2*2^(1/2)-(2*I*x+(-4*x^2+1)^(1/2)
)^2)/(3+2*2^(1/2)))-1/16*I*dilog((-3+2*2^(1/2)+(2*I*x+(-4*x^2+1)^(1/2))^2)
/(-3+2*2^(1/2)))
```

Fricas [F]

$$\int \frac{x \arcsin(2x)}{1+4x^2} \, dx = \int \frac{x \arcsin{(2\,x)}}{4\,x^2+1} \, dx$$

input
```
integrate(x*arcsin(2*x)/(4*x^2+1),x, algorithm="fricas")
```

output
```
integral(x*arcsin(2*x)/(4*x^2 + 1), x)
```

Sympy [F]

$$\int \frac{x \arcsin(2x)}{1+4x^2} \, dx = \int \frac{x \operatorname{asin}{(2x)}}{4x^2+1} \, dx$$

input
```
integrate(x*asin(2*x)/(4*x**2+1),x)
```

output
```
Integral(x*asin(2*x)/(4*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{x \arcsin(2x)}{1+4x^2} \, dx = \int \frac{x \arcsin{(2\,x)}}{4\,x^2+1} \, dx$$

input
```
integrate(x*arcsin(2*x)/(4*x^2+1),x, algorithm="maxima")
```

output
```
integrate(x*arcsin(2*x)/(4*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x\arcsin(2x)}{1+4x^2}\,dx = \int \frac{x\arcsin\left(2\,x\right)}{4\,x^2+1}\,dx$$

input
```
integrate(x*arcsin(2*x)/(4*x^2+1),x, algorithm="giac")
```

output
```
integrate(x*arcsin(2*x)/(4*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\arcsin(2x)}{1+4x^2}\,dx = \int \frac{x\,\mathrm{asin}(2\,x)}{4\,x^2+1}\,dx$$

input
```
int((x*asin(2*x))/(4*x^2 + 1),x)
```

output
```
int((x*asin(2*x))/(4*x^2 + 1), x)
```

Reduce [F]

$$\int \frac{x\arcsin(2x)}{1+4x^2}\,dx = \int \frac{asin(2x)\,x}{4x^2+1}dx$$

input
```
int(x*asin(2*x)/(4*x^2+1),x)
```

output
```
int((asin(2*x)*x)/(4*x**2 + 1),x)
```

3.3 $\int x^4(d-c^2dx^2)\,(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 128

$$\int x^4(d-c^2dx^2)\,(a+b\arccos(cx))\,dx = \frac{2bd\sqrt{1-c^2x^2}}{35c^5}+\frac{bd(1-c^2x^2)^{3/2}}{105c^5}-\frac{8bd(1-c^2x^2)^{5/2}}{175c^5}+\frac{bd(1-c^2x^2)^{7/2}}{49c^5}+\frac{1}{5}dx^5(a+b\arccos(cx))-\frac{1}{7}c^2dx^7(a+b\arccos(cx))$$

output
```
2/35*b*d*(-c^2*x^2+1)^(1/2)/c^5+1/105*b*d*(-c^2*x^2+1)^(3/2)/c^5-8/175*b*d
*(-c^2*x^2+1)^(5/2)/c^5+1/49*b*d*(-c^2*x^2+1)^(7/2)/c^5+1/5*d*x^5*(a+b*arc
cos(c*x))-1/7*c^2*d*x^7*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.68

$$\int x^4(d-c^2dx^2)\,(a+b\arccos(cx))\,dx = \frac{d\left(-105ax^5(-7+5c^2x^2)+\frac{b\sqrt{1-c^2x^2}(-152-76c^2x^2-57c^4x^4+75c^6x^6)}{c^5}-105bx^5(-7+5c^2x^2)\arccos(cx)\right)}{3675}$$

input
```
Integrate[x^4*(d - c^2*d*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d*(-105*a*x^5*(-7 + 5*c^2*x^2) + (b*Sqrt[1 - c^2*x^2]*(-152 - 76*c^2*x^2
- 57*c^4*x^4 + 75*c^6*x^6))/c^5 - 105*b*x^5*(-7 + 5*c^2*x^2)*ArcCos[c*x]))
/3675
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5193, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (d - c^2 d x^2) (a + b \arccos(cx)) \, dx$$

$$\Big\downarrow \text{5193}$$

$$bc \int \frac{d x^5 (7 - 5c^2x^2)}{35\sqrt{1 - c^2x^2}} dx - \frac{1}{7}c^2 d x^7 (a + b \arccos(cx)) + \frac{1}{5} d x^5 (a + b \arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{35} bcd \int \frac{x^5 (7 - 5c^2x^2)}{\sqrt{1 - c^2x^2}} dx - \frac{1}{7}c^2 d x^7 (a + b \arccos(cx)) + \frac{1}{5} d x^5 (a + b \arccos(cx))$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{70} bcd \int \frac{x^4 (7 - 5c^2x^2)}{\sqrt{1 - c^2x^2}} dx^2 - \frac{1}{7}c^2 d x^7 (a + b \arccos(cx)) + \frac{1}{5} d x^5 (a + b \arccos(cx))$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{70} bcd \int \left(\frac{5(1 - c^2x^2)^{5/2}}{c^4} - \frac{8(1 - c^2x^2)^{3/2}}{c^4} + \frac{\sqrt{1 - c^2x^2}}{c^4} + \frac{2}{c^4\sqrt{1 - c^2x^2}} \right) dx^2 - \frac{1}{7}c^2 d x^7 (a + b \arccos(cx)) + \frac{1}{5} d x^5 (a + b \arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{7}c^2dx^7(a+b\arccos(cx))+\frac{1}{5}dx^5(a+b\arccos(cx))+$$
$$\frac{1}{70}bcd\left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6}+\frac{16(1-c^2x^2)^{5/2}}{5c^6}-\frac{2(1-c^2x^2)^{3/2}}{3c^6}-\frac{4\sqrt{1-c^2x^2}}{c^6}\right)$$

input
```
Int[x^4*(d - c^2*d*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d*((-4*Sqrt[1 - c^2*x^2])/c^6 - (2*(1 - c^2*x^2)^(3/2))/(3*c^6) + (16
*(1 - c^2*x^2)^(5/2))/(5*c^6) - (10*(1 - c^2*x^2)^(7/2))/(7*c^6)))/70 + (d
*x^5*(a + b*ArcCos[c*x]))/5 - (c^2*d*x^7*(a + b*ArcCos[c*x]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

method	result
parts	$-da\left(\frac{1}{7}c^2x^7-\frac{1}{5}x^5\right)-\frac{db\left(\frac{\arccos(cx)c^7x^7}{7}-\frac{\arccos(cx)c^5x^5}{5}+\frac{19c^4x^4\sqrt{-c^2x^2+1}}{1225}+\frac{76c^2x^2\sqrt{-c^2x^2+1}}{3675}+\frac{152\sqrt{-c^2x^2+}}{3675}\right.}{c^5}$
derivativedivides	$\frac{-da\left(\frac{1}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-db\left(\frac{\arccos(cx)c^7x^7}{7}-\frac{\arccos(cx)c^5x^5}{5}+\frac{19c^4x^4\sqrt{-c^2x^2+1}}{1225}+\frac{76c^2x^2\sqrt{-c^2x^2+1}}{3675}+\frac{152\sqrt{-c^2x^2+1}}{3675}-\frac{c^6x}{}\right.}{c^5}$
default	$\frac{-da\left(\frac{1}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-db\left(\frac{\arccos(cx)c^7x^7}{7}-\frac{\arccos(cx)c^5x^5}{5}+\frac{19c^4x^4\sqrt{-c^2x^2+1}}{1225}+\frac{76c^2x^2\sqrt{-c^2x^2+1}}{3675}+\frac{152\sqrt{-c^2x^2+1}}{3675}-\frac{c^6x}{}\right.}{c^5}$
orering	$\frac{(975c^8x^8-1377c^6x^6-228c^4x^4-608c^2x^2+608)(-c^2d\,x^2+d)(a+b\arccos(cx))}{3675c^6x(c^2x^2-1)}-\frac{(75c^6x^6-57c^4x^4-76c^2x^2-152)\left(4\right.}{}$

input
```
int(x^4*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d*a*(1/7*c^2*x^7-1/5*x^5)-d*b/c^5*(1/7*arccos(c*x)*c^7*x^7-1/5*arccos(c*x
)*c^5*x^5+19/1225*c^4*x^4*(-c^2*x^2+1)^(1/2)+76/3675*c^2*x^2*(-c^2*x^2+1)^
(1/2)+152/3675*(-c^2*x^2+1)^(1/2)-1/49*c^6*x^6*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int x^4\left(d-c^2dx^2\right)(a+b\arccos(cx))\,dx =$$
$$-\frac{525\,ac^7dx^7-735\,ac^5dx^5+105\,(5\,bc^7dx^7-7\,bc^5dx^5)\arccos(cx)-(75\,bc^6dx^6-57\,bc^4dx^4-76\,bc^2dx^2}{3675\,c^5}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-1/3675*(525*a*c^7*d*x^7 - 735*a*c^5*d*x^5 + 105*(5*b*c^7*d*x^7 - 7*b*c^5*
d*x^5)*arccos(c*x) - (75*b*c^6*d*x^6 - 57*b*c^4*d*x^4 - 76*b*c^2*d*x^2 - 1
52*b*d)*sqrt(-c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.22

$$\int x^4(d-c^2dx^2)\,(a+b\arccos(cx))\,dx$$
$$=\begin{cases} -\frac{ac^2dx^7}{7}+\frac{adx^5}{5}-\frac{bc^2dx^7\operatorname{acos}(cx)}{7}+\frac{bcdx^6\sqrt{-c^2x^2+1}}{49}+\frac{bdx^5\operatorname{acos}(cx)}{5}-\frac{19bdx^4\sqrt{-c^2x^2+1}}{1225c}-\frac{76bdx^2\sqrt{-c^2x^2+1}}{3675c^3}-\frac{152bd\sqrt{}}{36} \\ \frac{dx^5\left(a+\frac{\pi b}{2}\right)}{5} \end{cases}$$

input
```
integrate(x**4*(-c**2*d*x**2+d)*(a+b*acos(c*x)),x)
```

output
```
Piecewise((-a*c**2*d*x**7/7 + a*d*x**5/5 - b*c**2*d*x**7*acos(c*x)/7 + b*c
*d*x**6*sqrt(-c**2*x**2 + 1)/49 + b*d*x**5*acos(c*x)/5 - 19*b*d*x**4*sqrt(
-c**2*x**2 + 1)/(1225*c) - 76*b*d*x**2*sqrt(-c**2*x**2 + 1)/(3675*c**3) -
152*b*d*sqrt(-c**2*x**2 + 1)/(3675*c**5), Ne(c, 0)), (d*x**5*(a + pi*b/2)/
5, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.49

$$\int x^4(d-c^2dx^2)\,(a+b\arccos(cx))\,dx=-\frac{1}{7}\,ac^2dx^7+\frac{1}{5}\,adx^5$$
$$-\frac{1}{245}\left(35\,x^7\arccos(cx)-\left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2}+\frac{6\sqrt{-c^2x^2+1}x^4}{c^4}+\frac{8\sqrt{-c^2x^2+1}x^2}{c^6}+\frac{16\sqrt{-c^2x^2+1}}{c^8}\right.\right.$$
$$\left.+\frac{1}{75}\left(15\,x^5\arccos(cx)-\left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2}+\frac{4\sqrt{-c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bd$$

input
```
integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-1/7*a*c^2*d*x^7 + 1/5*a*d*x^5 - 1/245*(35*x^7*arccos(c*x) - (5*sqrt(-c^2*
x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2
/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*c^2*d + 1/75*(15*x^5*arccos(c*x) -
(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2
*x^2 + 1)/c^6)*c)*b*d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int x^4(d-c^2dx^2)(a+b\arccos(cx))\,dx = -\frac{1}{7}bc^2dx^7\arccos(cx) - \frac{1}{7}ac^2dx^7 + \frac{1}{49}\sqrt{-c^2x^2+1}bcdx^6 + \frac{1}{5}bdx^5\arccos(cx) + \frac{1}{5}adx^5 - \frac{19\sqrt{-c^2x^2+1}bdx^4}{1225\,c} - \frac{76\sqrt{-c^2x^2+1}bdx^2}{3675\,c^3} - \frac{152\sqrt{-c^2x^2+1}bd}{3675\,c^5}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-1/7*b*c^2*d*x^7*arccos(c*x) - 1/7*a*c^2*d*x^7 + 1/49*sqrt(-c^2*x^2 + 1)*b
*c*d*x^6 + 1/5*b*d*x^5*arccos(c*x) + 1/5*a*d*x^5 - 19/1225*sqrt(-c^2*x^2 +
 1)*b*d*x^4/c - 76/3675*sqrt(-c^2*x^2 + 1)*b*d*x^2/c^3 - 152/3675*sqrt(-c^
2*x^2 + 1)*b*d/c^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d - c^2 d x^2) (a + b \arccos(cx)) \, dx = \int x^4 (a + b \operatorname{acos}(c\,x)) \left(d - c^2\,d\,x^2\right) \, dx$$

input
```
int(x^4*(a + b*acos(c*x))*(d - c^2*d*x^2),x)
```

output
```
int(x^4*(a + b*acos(c*x))*(d - c^2*d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

$$\int x^4 (d - c^2 d x^2) (a + b \arccos(cx)) \, dx$$
$$= \frac{d\left(-525 acos(cx)\, b\, c^7 x^7 + 735 acos(cx)\, b\, c^5 x^5 + 75\sqrt{-c^2x^2+1}\, b\, c^6 x^6 - 57\sqrt{-c^2x^2+1}\, b\, c^4 x^4 - 76\sqrt{-c^2x^2}\right.}{3675 c^5}$$

input
```
int(x^4*(-c^2*d*x^2+d)*(a+b*acos(c*x)),x)
```

output
```
(d*( - 525*acos(c*x)*b*c**7*x**7 + 735*acos(c*x)*b*c**5*x**5 + 75*sqrt( -
c**2*x**2 + 1)*b*c**6*x**6 - 57*sqrt( - c**2*x**2 + 1)*b*c**4*x**4 - 76*sq
rt( - c**2*x**2 + 1)*b*c**2*x**2 - 152*sqrt( - c**2*x**2 + 1)*b - 525*a*c*
*7*x**7 + 735*a*c**5*x**5))/(3675*c**5)
```

3.4 $\int x^3(d-c^2dx^2)(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int x^3(d-c^2dx^2)(a+b\arccos(cx))\,dx = \frac{bdx\sqrt{1-c^2x^2}}{24c^3}+\frac{bdx^3\sqrt{1-c^2x^2}}{36c} - \frac{1}{36}bcdx^5\sqrt{1-c^2x^2}-\frac{bd\arccos(cx)}{24c^4} + \frac{1}{4}dx^4(a+b\arccos(cx)) - \frac{1}{6}c^2dx^6(a+b\arccos(cx))$$

output
```
1/24*b*d*x*(-c^2*x^2+1)^(1/2)/c^3+1/36*b*d*x^3*(-c^2*x^2+1)^(1/2)/c-1/36*b
*c*d*x^5*(-c^2*x^2+1)^(1/2)-1/24*b*d*arccos(c*x)/c^4+1/4*d*x^4*(a+b*arccos
(c*x))-1/6*c^2*d*x^6*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.22

$$\int x^3(d-c^2dx^2)(a+b\arccos(cx))\,dx = \frac{1}{4}adx^4-\frac{1}{6}ac^2dx^6+bd\sqrt{1-c^2x^2}\left(-\frac{3x}{32c^3}-\frac{x^3}{16c}\right) - bc^2d\sqrt{1-c^2x^2}\left(-\frac{5x}{96c^5}-\frac{5x^3}{144c^3}-\frac{x^5}{36c}\right) + \frac{1}{4}bdx^4\arccos(cx) - \frac{1}{6}bc^2dx^6\arccos(cx)+\frac{bd\arcsin(cx)}{24c^4}$$

input
```
Integrate[x^3*(d - c^2*d*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(a*d*x^4)/4 - (a*c^2*d*x^6)/6 + b*d*Sqrt[1 - c^2*x^2]*((-3*x)/(32*c^3) - x
^3/(16*c)) - b*c^2*d*Sqrt[1 - c^2*x^2]*((-5*x)/(96*c^5) - (5*x^3)/(144*c^3
) - x^5/(36*c)) + (b*d*x^4*ArcCos[c*x])/4 - (b*c^2*d*x^6*ArcCos[c*x])/6 +
(b*d*ArcSin[c*x])/(24*c^4)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5193, 27, 363, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d-c^2dx^2)(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5193}$$

$$bc\int \frac{dx^4(3-2c^2x^2)}{12\sqrt{1-c^2x^2}}dx-\frac{1}{6}c^2dx^6(a+b\arccos(cx))+\frac{1}{4}dx^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}bcd\int\frac{x^4(3-2c^2x^2)}{\sqrt{1-c^2x^2}}dx-\frac{1}{6}c^2dx^6(a+b\arccos(cx))+\frac{1}{4}dx^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{363}$$

$$\frac{1}{12}bcd\left(\frac{4}{3}\int\frac{x^4}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^5\sqrt{1-c^2x^2}\right)-\frac{1}{6}c^2dx^6(a+b\arccos(cx))+\frac{1}{4}dx^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{12}bcd\left(\frac{4}{3}\left(\frac{3\int\frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{3}x^5\sqrt{1-c^2x^2}\right)-\frac{1}{6}c^2dx^6(a+b\arccos(cx))+\frac{1}{4}dx^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{12}bcd\left(\frac{4}{3}\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{3}x^5\sqrt{1-c^2x^2}\right)-\frac{1}{6}c^2dx^6(a+b\arccos(cx))+\frac{1}{4}dx^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{223}$$

$$-\frac{1}{6}c^2dx^6(a+b\arccos(cx))+\frac{1}{4}dx^4(a+b\arccos(cx))+\frac{1}{12}bcd\left(\frac{4}{3}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{3}x^5\sqrt{1-c^2x^2}\right)$$

input
```
Int[x^3*(d - c^2*d*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d*x^4*(a + b*ArcCos[c*x]))/4 - (c^2*d*x^6*(a + b*ArcCos[c*x]))/6 + (b*c*d
*((x^5*Sqrt[1 - c^2*x^2])/3 + (4*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-
1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/3))/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

method	result
parts	$-da\left(\frac{1}{6}c^2x^6-\frac{1}{4}x^4\right)-\frac{db\left(\frac{\arccos(cx)c^6x^6}{6}-\frac{c^4x^4\arccos(cx)}{4}+\frac{c^3x^3\sqrt{-c^2x^2+1}}{36}+\frac{cx\sqrt{-c^2x^2+1}}{24}-\frac{\arcsin(cx)}{24}-\frac{c^5x^5\sqrt{\ }}{}\right.}{c^4}$
derivativedivides	$\frac{-da\left(\frac{1}{6}c^6x^6-\frac{1}{4}c^4x^4\right)-db\left(\frac{\arccos(cx)c^6x^6}{6}-\frac{c^4x^4\arccos(cx)}{4}+\frac{c^3x^3\sqrt{-c^2x^2+1}}{36}+\frac{cx\sqrt{-c^2x^2+1}}{24}-\frac{\arcsin(cx)}{24}-\frac{c^5x^5\sqrt{-c^2x^2+}}{36}\right.}{c^4}$
default	$\frac{-da\left(\frac{1}{6}c^6x^6-\frac{1}{4}c^4x^4\right)-db\left(\frac{\arccos(cx)c^6x^6}{6}-\frac{c^4x^4\arccos(cx)}{4}+\frac{c^3x^3\sqrt{-c^2x^2+1}}{36}+\frac{cx\sqrt{-c^2x^2+1}}{24}-\frac{\arcsin(cx)}{24}-\frac{c^5x^5\sqrt{-c^2x^2+}}{36}\right.}{c^4}$
orering	$\frac{(22c^6x^6-34c^4x^4-9c^2x^2+12)(-c^2dx^2+d)(a+b\arccos(cx))}{72c^4(c^2x^2-1)}-\frac{(2c^4x^4-2c^2x^2-3)\left(3x^2(-c^2dx^2+d)(a+b\arccos(cx\right.}{72x^2}$

input
```
int(x^3*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d*a*(1/6*c^2*x^6-1/4*x^4)-d*b/c^4*(1/6*arccos(c*x)*c^6*x^6-1/4*c^4*x^4*ar
ccos(c*x)+1/36*c^3*x^3*(-c^2*x^2+1)^(1/2)+1/24*c*x*(-c^2*x^2+1)^(1/2)-1/24
*arcsin(c*x)-1/36*c^5*x^5*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.79

$$\int x^3(d-c^2dx^2)(a+b\arccos(cx))\,dx = -\frac{12\,ac^6dx^6-18\,ac^4dx^4+3\,(4\,bc^6dx^6-6\,bc^4dx^4+bd)\arccos(cx)-(2\,bc^5dx^5-2\,bc^3dx^3-3\,bcdx)\sqrt{-c}}{72\,c^4}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-1/72*(12*a*c^6*d*x^6 - 18*a*c^4*d*x^4 + 3*(4*b*c^6*d*x^6 - 6*b*c^4*d*x^4
+ b*d)*arccos(c*x) - (2*b*c^5*d*x^5 - 2*b*c^3*d*x^3 - 3*b*c*d*x)*sqrt(-c^2
*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.16

$$\int x^3(d-c^2dx^2)\,(a+b\arccos(cx))\,dx$$
$$=\begin{cases} -\frac{ac^2dx^6}{6}+\frac{adx^4}{4}-\frac{bc^2dx^6\,\mathrm{acos}\,(cx)}{6}+\frac{bcdx^5\sqrt{-c^2x^2+1}}{36}+\frac{bdx^4\,\mathrm{acos}\,(cx)}{4}-\frac{bdx^3\sqrt{-c^2x^2+1}}{36c}-\frac{bdx\sqrt{-c^2x^2+1}}{24c^3}-\frac{bd\,\mathrm{acos}\,(cx)}{24c^4} \\ \frac{dx^4\left(a+\frac{\pi b}{2}\right)}{4} \end{cases}$$

input
```
integrate(x**3*(-c**2*d*x**2+d)*(a+b*acos(c*x)),x)
```

output
```
Piecewise((-a*c**2*d*x**6/6 + a*d*x**4/4 - b*c**2*d*x**6*acos(c*x)/6 + b*c
*d*x**5*sqrt(-c**2*x**2 + 1)/36 + b*d*x**4*acos(c*x)/4 - b*d*x**3*sqrt(-c*
*2*x**2 + 1)/(36*c) - b*d*x*sqrt(-c**2*x**2 + 1)/(24*c**3) - b*d*acos(c*x)
/(24*c**4), Ne(c, 0)), (d*x**4*(a + pi*b/2)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.39

$$\int x^3(d-c^2dx^2)\,(a+b\arccos(cx))\,dx=-\frac{1}{6}\,ac^2dx^6+\frac{1}{4}\,adx^4$$
$$-\frac{1}{288}\left(48\,x^6\arccos\,(cx)-\left(\frac{8\,\sqrt{-c^2x^2+1}x^5}{c^2}+\frac{10\,\sqrt{-c^2x^2+1}x^3}{c^4}+\frac{15\,\sqrt{-c^2x^2+1}x}{c^6}-\frac{15\,\arcsin\,(cx)}{c^7}\right.\right.$$
$$+\frac{1}{32}\left(8\,x^4\arccos\,(cx)-\left(\frac{2\,\sqrt{-c^2x^2+1}x^3}{c^2}+\frac{3\,\sqrt{-c^2x^2+1}x}{c^4}-\frac{3\,\arcsin\,(cx)}{c^5}\right)c\right)bd$$

input
```
integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-1/6*a*c^2*d*x^6 + 1/4*a*d*x^4 - 1/288*(48*x^6*arccos(c*x) - (8*sqrt(-c^2*
x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x
/c^6 - 15*arcsin(c*x)/c^7)*c)*b*c^2*d + 1/32*(8*x^4*arccos(c*x) - (2*sqrt(
-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)
*b*d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int x^3(d-c^2dx^2)(a+b\arccos(cx))\,dx = -\frac{1}{6}bc^2dx^6\arccos(cx) - \frac{1}{6}ac^2dx^6 + \frac{1}{36}\sqrt{-c^2x^2+1}bcdx^5 + \frac{1}{4}bdx^4\arccos(cx) + \frac{1}{4}adx^4 - \frac{\sqrt{-c^2x^2+1}bdx^3}{36\,c} - \frac{\sqrt{-c^2x^2+1}bdx}{24\,c^3} - \frac{bd\arccos(cx)}{24\,c^4}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-1/6*b*c^2*d*x^6*arccos(c*x) - 1/6*a*c^2*d*x^6 + 1/36*sqrt(-c^2*x^2 + 1)*b
*c*d*x^5 + 1/4*b*d*x^4*arccos(c*x) + 1/4*a*d*x^4 - 1/36*sqrt(-c^2*x^2 + 1)
*b*d*x^3/c - 1/24*sqrt(-c^2*x^2 + 1)*b*d*x/c^3 - 1/24*b*d*arccos(c*x)/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d-c^2dx^2)(a+b\arccos(cx))\,dx = \int x^3\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)\,dx$$

input
```
int(x^3*(a + b*acos(c*x))*(d - c^2*d*x^2),x)
```

output
```
int(x^3*(a + b*acos(c*x))*(d - c^2*d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int x^3(d-c^2dx^2)\,(a+b\arccos(cx))\,dx$$
$$=\frac{d\left(-12acos(cx)\,b\,c^6x^6+18acos(cx)\,b\,c^4x^4+3asin(cx)\,b+2\sqrt{-c^2x^2+1}\,b\,c^5x^5-2\sqrt{-c^2x^2+1}\,b\,c^3x^3-3\right.}{72c^4}$$

input
```
int(x^3*(-c^2*d*x^2+d)*(a+b*acos(c*x)),x)
```

output
```
(d*( - 12*acos(c*x)*b*c**6*x**6 + 18*acos(c*x)*b*c**4*x**4 + 3*asin(c*x)*b
 + 2*sqrt( - c**2*x**2 + 1)*b*c**5*x**5 - 2*sqrt( - c**2*x**2 + 1)*b*c**3*
x**3 - 3*sqrt( - c**2*x**2 + 1)*b*c*x - 12*a*c**6*x**6 + 18*a*c**4*x**4))/
(72*c**4)
```

3.5 $\int x^2(d-c^2dx^2)(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int x^2\left(d-c^2dx^2\right)(a+b\arccos(cx))\,dx=\frac{2bd\sqrt{1-c^2x^2}}{15c^3}+\frac{bd\left(1-c^2x^2\right)^{3/2}}{45c^3}-\frac{bd\left(1-c^2x^2\right)^{5/2}}{25c^3}+\frac{1}{3}dx^3(a+b\arccos(cx))-\frac{1}{5}c^2dx^5(a+b\arccos(cx))$$

output
```
2/15*b*d*(-c^2*x^2+1)^(1/2)/c^3+1/45*b*d*(-c^2*x^2+1)^(3/2)/c^3-1/25*b*d*(
-c^2*x^2+1)^(5/2)/c^3+1/3*d*x^3*(a+b*arccos(c*x))-1/5*c^2*d*x^5*(a+b*arcco
s(c*x))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

$$\int x^2\left(d-c^2dx^2\right)(a+b\arccos(cx))\,dx=-\frac{d\left(15ac^3x^3\left(-5+3c^2x^2\right)+b\sqrt{1-c^2x^2}\left(26+13c^2x^2-9c^4x^4\right)+15bc^3x^3\left(-5+3c^2x^2\right)\arccos(cx)\right)}{225c^3}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/225*(d*(15*a*c^3*x^3*(-5 + 3*c^2*x^2) + b*Sqrt[1 - c^2*x^2]*(26 + 13*c^
2*x^2 - 9*c^4*x^4) + 15*b*c^3*x^3*(-5 + 3*c^2*x^2)*ArcCos[c*x]))/c^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5193, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d-c^2dx^2\right)(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5193}$$

$$bc\int \frac{dx^3\left(5-3c^2x^2\right)}{15\sqrt{1-c^2x^2}}dx-\frac{1}{5}c^2dx^5(a+b\arccos(cx))+\frac{1}{3}dx^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{15}bcd\int \frac{x^3\left(5-3c^2x^2\right)}{\sqrt{1-c^2x^2}}dx-\frac{1}{5}c^2dx^5(a+b\arccos(cx))+\frac{1}{3}dx^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{30}bcd\int \frac{x^2\left(5-3c^2x^2\right)}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{5}c^2dx^5(a+b\arccos(cx))+\frac{1}{3}dx^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{30}bcd\int \left(-\frac{3\left(1-c^2x^2\right)^{3/2}}{c^2}+\frac{\sqrt{1-c^2x^2}}{c^2}+\frac{2}{c^2\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{5}c^2dx^5(a+b\arccos(cx))+\frac{1}{3}dx^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{5}c^2dx^5(a+b\arccos(cx))+\frac{1}{3}dx^3(a+b\arccos(cx))+\frac{1}{30}bcd\left(\frac{6(1-c^2x^2)^{5/2}}{5c^4}-\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\frac{4\sqrt{1-c^2x^2}}{c^4}\right)$$

input
```
Int[x^2*(d - c^2*d*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d*((-4*Sqrt[1 - c^2*x^2])/c^4 - (2*(1 - c^2*x^2)^(3/2))/(3*c^4) + (6*
(1 - c^2*x^2)^(5/2))/(5*c^4)))/30 + (d*x^3*(a + b*ArcCos[c*x]))/3 - (c^2*d
*x^5*(a + b*ArcCos[c*x]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.01

method	result
parts	$-da\left(\frac{1}{5}c^2x^5-\frac{1}{3}x^3\right)-\frac{db\left(\frac{\arccos(cx)c^5x^5}{5}-\frac{c^3x^3\arccos(cx)}{3}+\frac{13c^2x^2\sqrt{-c^2x^2+1}}{225}+\frac{26\sqrt{-c^2x^2+1}}{225}-\frac{c^4x^4\sqrt{-c^2x^2+1}}{25}\right)}{c^3}$
derivativedivides	$\frac{-da\left(\frac{1}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-db\left(\frac{\arccos(cx)c^5x^5}{5}-\frac{c^3x^3\arccos(cx)}{3}+\frac{13c^2x^2\sqrt{-c^2x^2+1}}{225}+\frac{26\sqrt{-c^2x^2+1}}{225}-\frac{c^4x^4\sqrt{-c^2x^2+1}}{25}\right)}{c^3}$
default	$\frac{-da\left(\frac{1}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-db\left(\frac{\arccos(cx)c^5x^5}{5}-\frac{c^3x^3\arccos(cx)}{3}+\frac{13c^2x^2\sqrt{-c^2x^2+1}}{225}+\frac{26\sqrt{-c^2x^2+1}}{225}-\frac{c^4x^4\sqrt{-c^2x^2+1}}{25}\right)}{c^3}$
orering	$\frac{(81c^6x^6-145c^4x^4-78c^2x^2+52)(-c^2dx^2+d)(a+b\arccos(cx))}{225c^4x(c^2x^2-1)}-\frac{(9c^4x^4-13c^2x^2-26)\left(2x(-c^2dx^2+d)(a+b\arccos\right.}{225}$

input
```
int(x^2*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d*a*(1/5*c^2*x^5-1/3*x^3)-d*b/c^3*(1/5*arccos(c*x)*c^5*x^5-1/3*c^3*x^3*ar
ccos(c*x)+13/225*c^2*x^2*(-c^2*x^2+1)^(1/2)+26/225*(-c^2*x^2+1)^(1/2)-1/25
*c^4*x^4*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int x^2(d-c^2dx^2)(a+b\arccos(cx))\,dx =$$
$$-\frac{45\,ac^5dx^5-75\,ac^3dx^3+15\,(3\,bc^5dx^5-5\,bc^3dx^3)\arccos(cx)-(9\,bc^4dx^4-13\,bc^2dx^2-26\,bd)\sqrt{-c^2x^2}}{225\,c^3}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-1/225*(45*a*c^5*d*x^5 - 75*a*c^3*d*x^3 + 15*(3*b*c^5*d*x^5 - 5*b*c^3*d*x^
3)*arccos(c*x) - (9*b*c^4*d*x^4 - 13*b*c^2*d*x^2 - 26*b*d)*sqrt(-c^2*x^2 +
 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int x^2(d-c^2dx^2)\,(a+b\arccos(cx))\,dx$$
$$=\begin{cases} -\frac{ac^2dx^5}{5}+\frac{adx^3}{3}-\frac{bc^2dx^5\operatorname{acos}(cx)}{5}+\frac{bcdx^4\sqrt{-c^2x^2+1}}{25}+\frac{bdx^3\operatorname{acos}(cx)}{3}-\frac{13bdx^2\sqrt{-c^2x^2+1}}{225c}-\frac{26bd\sqrt{-c^2x^2+1}}{225c^3} & \text{for } c\neq 0 \\ \frac{dx^3\left(a+\frac{\pi b}{2}\right)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)*(a+b*acos(c*x)),x)
```

output
```
Piecewise((-a*c**2*d*x**5/5 + a*d*x**3/3 - b*c**2*d*x**5*acos(c*x)/5 + b*c
*d*x**4*sqrt(-c**2*x**2 + 1)/25 + b*d*x**3*acos(c*x)/3 - 13*b*d*x**2*sqrt(
-c**2*x**2 + 1)/(225*c) - 26*b*d*sqrt(-c**2*x**2 + 1)/(225*c**3), Ne(c, 0)
), (d*x**3*(a + pi*b/2)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.43

$$\int x^2(d-c^2dx^2)\,(a+b\arccos(cx))\,dx=-\frac{1}{5}\,ac^2dx^5$$
$$-\frac{1}{75}\left(15\,x^5\arccos(cx)-\left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2}+\frac{4\sqrt{-c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bc^2d$$
$$+\frac{1}{3}\,adx^3+\frac{1}{9}\left(3\,x^3\arccos(cx)-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bd$$

input
```
integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-1/5*a*c^2*d*x^5 - 1/75*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^
2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*c^2*d +
1/3*a*d*x^3 + 1/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*s
qrt(-c^2*x^2 + 1)/c^4))*b*d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int x^2 (d-c^2dx^2) (a+b\arccos(cx))\,dx = -\frac{1}{5}\,bc^2dx^5\arccos(cx) - \frac{1}{5}\,ac^2dx^5 + \frac{1}{25}\,\sqrt{-c^2x^2+1}bcdx^4 + \frac{1}{3}\,bdx^3\arccos(cx) + \frac{1}{3}\,adx^3 - \frac{13\,\sqrt{-c^2x^2+1}bdx^2}{225\,c} - \frac{26\,\sqrt{-c^2x^2+1}bd}{225\,c^3}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-1/5*b*c^2*d*x^5*arccos(c*x) - 1/5*a*c^2*d*x^5 + 1/25*sqrt(-c^2*x^2 + 1)*b
*c*d*x^4 + 1/3*b*d*x^3*arccos(c*x) + 1/3*a*d*x^3 - 13/225*sqrt(-c^2*x^2 +
1)*b*d*x^2/c - 26/225*sqrt(-c^2*x^2 + 1)*b*d/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d-c^2dx^2) (a+b\arccos(cx))\,dx = \int x^2\,\left(a+b\,\mathrm{acos}(c\,x)\right)\,\left(d-c^2\,d\,x^2\right)\,dx$$

input
```
int(x^2*(a + b*acos(c*x))*(d - c^2*d*x^2),x)
```

output
```
int(x^2*(a + b*acos(c*x))*(d - c^2*d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int x^2(d-c^2dx^2)(a+b\arccos(cx))\,dx$$
$$=\frac{d\left(-45acos(cx)\,b\,c^5x^5+75acos(cx)\,b\,c^3x^3+9\sqrt{-c^2x^2+1}\,b\,c^4x^4-13\sqrt{-c^2x^2+1}\,b\,c^2x^2-26\sqrt{-c^2x^2+\right.}{225c^3}$$

input
```
int(x^2*(-c^2*d*x^2+d)*(a+b*acos(c*x)),x)
```

output
```
(d*( - 45*acos(c*x)*b*c**5*x**5 + 75*acos(c*x)*b*c**3*x**3 + 9*sqrt( - c**
2*x**2 + 1)*b*c**4*x**4 - 13*sqrt( - c**2*x**2 + 1)*b*c**2*x**2 - 26*sqrt(
 - c**2*x**2 + 1)*b - 45*a*c**5*x**5 + 75*a*c**3*x**3))/(225*c**3)
```

3.6 $\int x(d-c^2dx^2)(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int x(d-c^2dx^2)(a+b\arccos(cx))\,dx=\frac{3bdx\sqrt{1-c^2x^2}}{32c}+\frac{bdx(1-c^2x^2)^{3/2}}{16c}+\frac{3bd\arccos(cx)}{32c^2}-\frac{d(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}$$

output
```
3/32*b*d*x*(-c^2*x^2+1)^(1/2)/c+1/16*b*d*x*(-c^2*x^2+1)^(3/2)/c+3/32*b*d*a
rccos(c*x)/c^2-1/4*d*(-c^2*x^2+1)^2*(a+b*arccos(c*x))/c^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int x(d-c^2dx^2)(a+b\arccos(cx))\,dx=\frac{d\left(cx\left(-8acx(-2+c^2x^2)+b\sqrt{1-c^2x^2}(-5+2c^2x^2)\right)-8bc^2x^2(-2+c^2x^2)\arccos(cx)+5b\arcsin(cx)\right)}{32c^2}$$

input
```
Integrate[x*(d - c^2*d*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d*(c*x*(-8*a*c*x*(-2 + c^2*x^2) + b*Sqrt[1 - c^2*x^2]*(-5 + 2*c^2*x^2)) -
 8*b*c^2*x^2*(-2 + c^2*x^2)*ArcCos[c*x] + 5*b*ArcSin[c*x]))/(32*c^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5183, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d-c^2dx^2\right)(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5183}$$

$$-\frac{bd\int \left(1-c^2x^2\right)^{3/2}dx}{4c}-\frac{d\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}$$

$$\Big\downarrow \text{211}$$

$$-\frac{bd\left(\frac{3}{4}\int \sqrt{1-c^2x^2}dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)}{4c}-\frac{d\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}$$

$$\Big\downarrow \text{211}$$

$$-\frac{bd\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)}{4c}-\frac{d\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}$$

$$\Big\downarrow \text{223}$$

$$-\frac{d\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}-\frac{bd\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)}{4c}$$

input
```
Int[x*(d - c^2*d*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/4*(d*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x]))/c^2 - (b*d*((x*(1 - c^2*x^2)^
(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/(4*c)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-\frac{da\left(c^2x^2-1\right)^2}{4}-db\left(\frac{c^4x^4\arccos(cx)}{4}-\frac{c^2x^2\arccos(cx)}{2}+\frac{\arccos(cx)}{4}+\frac{3\arcsin(cx)}{32}-\frac{c^3x^3\sqrt{-c^2x^2+1}}{16}+\frac{5cx\sqrt{-c^2x^2+1}}{32}\right)}{c^2}$
default	$\frac{-\frac{da\left(c^2x^2-1\right)^2}{4}-db\left(\frac{c^4x^4\arccos(cx)}{4}-\frac{c^2x^2\arccos(cx)}{2}+\frac{\arccos(cx)}{4}+\frac{3\arcsin(cx)}{32}-\frac{c^3x^3\sqrt{-c^2x^2+1}}{16}+\frac{5cx\sqrt{-c^2x^2+1}}{32}\right)}{c^2}$
parts	$-\frac{da(c^2x^2-1)^2}{4c^2}-\frac{db\left(\frac{c^4x^4\arccos(cx)}{4}-\frac{c^2x^2\arccos(cx)}{2}+\frac{\arccos(cx)}{4}+\frac{3\arcsin(cx)}{32}-\frac{c^3x^3\sqrt{-c^2x^2+1}}{16}+\frac{5cx\sqrt{-c^2x^2+1}}{32}\right)}{c^2}$
orering	$\frac{(14c^4x^4-33c^2x^2+10)(-c^2dx^2+d)(a+b\arccos(cx))}{32c^2(c^2x^2-1)}-\frac{(2c^2x^2-5)\left((-c^2dx^2+d)(a+b\arccos(cx))-2x^2dc^2(a+b\,\mathrm{ar}\right.}{32c^2}$

input
```
int(x*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/4*d*a*(c^2*x^2-1)^2-d*b*(1/4*c^4*x^4*arccos(c*x)-1/2*c^2*x^2*arc
cos(c*x)+1/4*arccos(c*x)+3/32*arcsin(c*x)-1/16*c^3*x^3*(-c^2*x^2+1)^(1/2)+
5/32*c*x*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int x(d-c^2dx^2)(a+b\arccos(cx))\,dx = -\frac{8\,ac^4dx^4-16\,ac^2dx^2+(8\,bc^4dx^4-16\,bc^2dx^2+5\,bd)\arccos(cx)-(2\,bc^3dx^3-5\,bcdx)\sqrt{-c^2x^2+1}}{32\,c^2}$$

input
```
integrate(x*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-1/32*(8*a*c^4*d*x^4 - 16*a*c^2*d*x^2 + (8*b*c^4*d*x^4 - 16*b*c^2*d*x^2 +
5*b*d)*arccos(c*x) - (2*b*c^3*d*x^3 - 5*b*c*d*x)*sqrt(-c^2*x^2 + 1))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.36

$$\int x(d-c^2dx^2)(a+b\arccos(cx))\,dx$$
$$=\begin{cases}-\frac{ac^2dx^4}{4}+\frac{adx^2}{2}-\frac{bc^2dx^4\operatorname{acos}(cx)}{4}+\frac{bcdx^3\sqrt{-c^2x^2+1}}{16}+\frac{bdx^2\operatorname{acos}(cx)}{2}-\frac{5bdx\sqrt{-c^2x^2+1}}{32c}-\frac{5bd\operatorname{acos}(cx)}{32c^2} & \text{for } c\neq 0\\ \frac{dx^2\left(a+\frac{\pi b}{2}\right)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(-c**2*d*x**2+d)*(a+b*acos(c*x)),x)
```

output
```
Piecewise((-a*c**2*d*x**4/4 + a*d*x**2/2 - b*c**2*d*x**4*acos(c*x)/4 + b*c
*d*x**3*sqrt(-c**2*x**2 + 1)/16 + b*d*x**2*acos(c*x)/2 - 5*b*d*x*sqrt(-c**
2*x**2 + 1)/(32*c) - 5*b*d*acos(c*x)/(32*c**2), Ne(c, 0)), (d*x**2*(a + pi
*b/2)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.44

$$\int x(d-c^2dx^2)(a+b\arccos(cx))\,dx = -\frac{1}{4}ac^2dx^4 - \frac{1}{32}\left(8x^4\arccos(cx) - \left(\frac{2\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\sqrt{-c^2x^2+1}x}{c^4} - \frac{3\arcsin(cx)}{c^5}\right)c\right)bc^2d + \frac{1}{2}adx^2 + \frac{1}{4}\left(2x^2\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x}{c^2} - \frac{\arcsin(cx)}{c^3}\right)\right)bd$$

input
```
integrate(x*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-1/4*a*c^2*d*x^4 - 1/32*(8*x^4*arccos(c*x) - (2*sqrt(-c^2*x^2 + 1)*x^3/c^2
 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*c^2*d + 1/2*a*d*x^
2 + 1/4*(2*x^2*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*x)/c^3
))*b*d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.04

$$\int x(d-c^2dx^2)(a+b\arccos(cx))\,dx = -\frac{1}{4}bc^2dx^4\arccos(cx) - \frac{1}{4}ac^2dx^4 + \frac{1}{16}\sqrt{-c^2x^2+1}bcdx^3 + \frac{1}{2}bdx^2\arccos(cx) + \frac{1}{2}adx^2 - \frac{5\sqrt{-c^2x^2+1}bdx}{32c} - \frac{5bd\arccos(cx)}{32c^2}$$

input
```
integrate(x*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-1/4*b*c^2*d*x^4*arccos(c*x) - 1/4*a*c^2*d*x^4 + 1/16*sqrt(-c^2*x^2 + 1)*b
*c*d*x^3 + 1/2*b*d*x^2*arccos(c*x) + 1/2*a*d*x^2 - 5/32*sqrt(-c^2*x^2 + 1)
*b*d*x/c - 5/32*b*d*arccos(c*x)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)\,(a+b\arccos(cx))\,dx=\int x\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)\,dx$$

input
```
int(x*(a + b*acos(c*x))*(d - c^2*d*x^2),x)
```

output
```
int(x*(a + b*acos(c*x))*(d - c^2*d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.04

$$\int x(d-c^2dx^2)\,(a+b\arccos(cx))\,dx$$
$$=\frac{d\left(-8acos(cx)\,b\,c^4x^4+16acos(cx)\,b\,c^2x^2+5asin(cx)\,b+2\sqrt{-c^2x^2+1}\,b\,c^3x^3-5\sqrt{-c^2x^2+1}\,bcx-8a\,c\right.}{32c^2}$$

input
```
int(x*(-c^2*d*x^2+d)*(a+b*acos(c*x)),x)
```

output
```
(d*( - 8*acos(c*x)*b*c**4*x**4 + 16*acos(c*x)*b*c**2*x**2 + 5*asin(c*x)*b
+ 2*sqrt( - c**2*x**2 + 1)*b*c**3*x**3 - 5*sqrt( - c**2*x**2 + 1)*b*c*x -
8*a*c**4*x**4 + 16*a*c**2*x**2))/(32*c**2)
```

3.7 $\int (d - c^2 dx^2) (a + b \arccos(cx)) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 77

$$\int (d - c^2 dx^2) (a + b \arccos(cx)) \, dx = \frac{2bd\sqrt{1-c^2x^2}}{3c} + \frac{bd(1-c^2x^2)^{3/2}}{9c} + dx(a + b\arccos(cx)) - \frac{1}{3}c^2dx^3(a + b\arccos(cx))$$

output
```
2/3*b*d*(-c^2*x^2+1)^(1/2)/c+1/9*b*d*(-c^2*x^2+1)^(3/2)/c+d*x*(a+b*arccos(
c*x))-1/3*c^2*d*x^3*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int (d - c^2 dx^2) (a + b \arccos(cx)) \, dx = \frac{d\left(b\sqrt{1-c^2x^2}(-7 + c^2x^2) + a(9cx - 3c^3x^3) - 3bcx(-3 + c^2x^2)\arccos(cx)\right)}{9c}$$

input
```
Integrate[(d - c^2*d*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d*(b*Sqrt[1 - c^2*x^2]*(-7 + c^2*x^2) + a*(9*c*x - 3*c^3*x^3) - 3*b*c*x*(
-3 + c^2*x^2)*ArcCos[c*x]))/(9*c)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5155, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d-c^2dx^2)(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5155}$$

$$bc\int \frac{dx(3-c^2x^2)}{3\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2dx^3(a+b\arccos(cx))+dx(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bcd\int \frac{x(3-c^2x^2)}{\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2dx^3(a+b\arccos(cx))+dx(a+b\arccos(cx))$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{6}bcd\int \frac{3-c^2x^2}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{3}c^2dx^3(a+b\arccos(cx))+dx(a+b\arccos(cx))$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{6}bcd\int \left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2dx^3(a+b\arccos(cx))+dx(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}c^2dx^3(a+b\arccos(cx))+dx(a+b\arccos(cx))+\frac{1}{6}bcd\left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2}-\frac{4\sqrt{1-c^2x^2}}{c^2}\right)$$

input
```
Int[(d - c^2*d*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d*((-4*Sqrt[1 - c^2*x^2])/c^2 - (2*(1 - c^2*x^2)^(3/2))/(3*c^2)))/6 +
 d*x*(a + b*ArcCos[c*x]) - (c^2*d*x^3*(a + b*ArcCos[c*x]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

method	result	size
parts	$-da\left(\frac{1}{3}c^2x^3-x\right)-\frac{db\left(\frac{c^3x^3\arccos(cx)}{3}-cx\arccos(cx)-\frac{c^2x^2\sqrt{-c^2x^2+1}}{9}+\frac{7\sqrt{-c^2x^2+1}}{9}\right)}{c}$	80
derivativedivides	$\frac{-da\left(\frac{1}{3}c^3x^3-cx\right)-db\left(\frac{c^3x^3\arccos(cx)}{3}-cx\arccos(cx)-\frac{c^2x^2\sqrt{-c^2x^2+1}}{9}+\frac{7\sqrt{-c^2x^2+1}}{9}\right)}{c}$	82
default	$\frac{-da\left(\frac{1}{3}c^3x^3-cx\right)-db\left(\frac{c^3x^3\arccos(cx)}{3}-cx\arccos(cx)-\frac{c^2x^2\sqrt{-c^2x^2+1}}{9}+\frac{7\sqrt{-c^2x^2+1}}{9}\right)}{c}$	82
orering	$\frac{x(5c^2x^2-23)(-c^2d\,x^2+d)(a+b\arccos(cx))}{9c^2x^2-9}-\frac{(c^2x^2-7)\left(-2d\,c^2x(a+b\arccos(cx))-\frac{(-c^2d\,x^2+d)bc}{\sqrt{-c^2x^2+1}}\right)}{9c^2}$	102

input
```
int((-c^2*d*x^2+d)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d*a*(1/3*c^2*x^3-x)-d*b/c*(1/3*c^3*x^3*arccos(c*x)-c*x*arccos(c*x)-1/9*c^
2*x^2*(-c^2*x^2+1)^(1/2)+7/9*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \left(d-c^2dx^2\right)(a+b\arccos(cx))\,dx = -\frac{3\,ac^3dx^3-9\,acdx+3\,(bc^3dx^3-3\,bcdx)\arccos{(cx)}-(bc^2dx^2-7\,bd)\sqrt{-c^2x^2+1}}{9\,c}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-1/9*(3*a*c^3*d*x^3 - 9*a*c*d*x + 3*(b*c^3*d*x^3 - 3*b*c*d*x)*arccos(c*x)
- (b*c^2*d*x^2 - 7*b*d)*sqrt(-c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.23

$$\int (d-c^2dx^2)(a+b\arccos(cx))\,dx$$
$$=\begin{cases} -\frac{ac^2dx^3}{3}+adx-\frac{bc^2dx^3\operatorname{acos}(cx)}{3}+\frac{bcdx^2\sqrt{-c^2x^2+1}}{9}+bdx\operatorname{acos}(cx)-\frac{7bd\sqrt{-c^2x^2+1}}{9c} & \text{for } c\neq 0 \\ dx\left(a+\frac{\pi b}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acos(c*x)),x)
```

output
```
Piecewise((-a*c**2*d*x**3/3 + a*d*x - b*c**2*d*x**3*acos(c*x)/3 + b*c*d*x*
*2*sqrt(-c**2*x**2 + 1)/9 + b*d*x*acos(c*x) - 7*b*d*sqrt(-c**2*x**2 + 1)/(
9*c), Ne(c, 0)), (d*x*(a + pi*b/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.30

$$\int (d-c^2dx^2)(a+b\arccos(cx))\,dx$$
$$=-\frac{1}{3}ac^2dx^3-\frac{1}{9}\left(3x^3\arccos(cx)-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bc^2d$$
$$+adx+\frac{\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)bd}{c}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-1/3*a*c^2*d*x^3 - 1/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2
+ 2*sqrt(-c^2*x^2 + 1)/c^4))*b*c^2*d + a*d*x + (c*x*arccos(c*x) - sqrt(-c^
2*x^2 + 1))*b*d/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (d-c^2dx^2)(a+b\arccos(cx))\,dx = -\frac{1}{3}bc^2dx^3\arccos(cx) - \frac{1}{3}ac^2dx^3 + \frac{1}{9}\sqrt{-c^2x^2+1}bcdx^2 + bdx\arccos(cx) + adx - \frac{7\sqrt{-c^2x^2+1}bd}{9c}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-1/3*b*c^2*d*x^3*arccos(c*x) - 1/3*a*c^2*d*x^3 + 1/9*sqrt(-c^2*x^2 + 1)*b*
c*d*x^2 + b*d*x*arccos(c*x) + a*d*x - 7/9*sqrt(-c^2*x^2 + 1)*b*d/c
```

Mupad [F(-1)]

Timed out.

$$\int (d-c^2dx^2)(a+b\arccos(cx))\,dx = \begin{cases} b\,c^2\,d\left(\frac{\sqrt{\frac{1}{c^2}-x^2}\left(\frac{2}{c^2}+x^2\right)}{9}-\frac{x^3\,\mathrm{acos}(c\,x)}{3}\right)-\frac{b\,d\left(\sqrt{1-c^2\,x^2}-c\,x\,\mathrm{acos}(c\,x)\right)}{c}-\frac{a\,d\,x\,(c^2\,x^2-3)}{3} & \text{if } 0<c \\ \int (a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)\,dx & \text{if } \neg 0<c \end{cases}$$

input
```
int((a + b*acos(c*x))*(d - c^2*d*x^2),x)
```

output
```
piecewise(0 < c, b*c^2*d*(((1/c^2 - x^2)^(1/2)*(2/c^2 + x^2))/9 - (x^3*aco
s(c*x))/3) - (b*d*((- c^2*x^2 + 1)^(1/2) - c*x*acos(c*x)))/c - (a*d*x*(c^2
*x^2 - 3))/3, ~0 < c, int((a + b*acos(c*x))*(d - c^2*d*x^2), x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (d - c^2 d x^2) (a + b \arccos(cx)) \, dx$$
$$= \frac{d\left(-3 acos(cx)\, b c^3 x^3 + 9 acos(cx)\, bcx + \sqrt{-c^2 x^2 + 1}\, b c^2 x^2 - 7\sqrt{-c^2 x^2 + 1}\, b - 3a\, c^3 x^3 + 9acx\right)}{9c}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acos(c*x)),x)
```

output
```
(d*( - 3*acos(c*x)*b*c**3*x**3 + 9*acos(c*x)*b*c*x + sqrt( - c**2*x**2 + 1
)*b*c**2*x**2 - 7*sqrt( - c**2*x**2 + 1)*b - 3*a*c**3*x**3 + 9*a*c*x))/(9*
c)
```

3.8 $\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\begin{aligned}\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x}\,dx = &-\frac{1}{4}bcdx\sqrt{1-c^2x^2}-\frac{1}{4}bd\arccos(cx)\\ &+\frac{1}{2}d(1-c^2x^2)(a+b\arccos(cx))\\ &-\frac{id(a+b\arccos(cx))^2}{2b}\\ &+d(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)\\ &-\frac{1}{2}ibd\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\end{aligned}$$

output
```
-1/4*b*c*d*x*(-c^2*x^2+1)^(1/2)-1/4*b*d*arccos(c*x)+1/2*d*(-c^2*x^2+1)*(a+
b*arccos(c*x))-1/2*I*d*(a+b*arccos(c*x))^2/b+d*(a+b*arccos(c*x))*ln(1-(c*x
+I*(-c^2*x^2+1)^(1/2))^2)-1/2*I*b*d*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2
)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.01

$$\int \frac{(d-c^2dx^2)\,(a+b\arccos(cx))}{x}\,dx = -\frac{1}{2}ac^2dx^2+\frac{1}{4}bcdx\sqrt{1-c^2x^2} -\frac{1}{2}bc^2dx^2\arccos(cx) -\frac{1}{2}ibd\arccos(cx)^2-\frac{1}{4}bd\arcsin(cx) +bd\arccos(cx)\log\left(1+e^{2i\arccos(cx)}\right) +ad\log(x)-\frac{1}{2}ibd\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)$$

input
```
Integrate[((d - c^2*d*x^2)*(a + b*ArcCos[c*x]))/x,x]
```

output
```
-1/2*(a*c^2*d*x^2) + (b*c*d*x*Sqrt[1 - c^2*x^2])/4 - (b*c^2*d*x^2*ArcCos[c
*x])/2 - (I/2)*b*d*ArcCos[c*x]^2 - (b*d*ArcSin[c*x])/4 + b*d*ArcCos[c*x]*L
og[1 + E^((2*I)*ArcCos[c*x])] + a*d*Log[x] - (I/2)*b*d*PolyLog[2, -E^((2*I
)*ArcCos[c*x])]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5189, 211, 223, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)\,(a+b\arccos(cx))}{x}\,dx$$

$$\Big\downarrow \text{5189}$$

$$d\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bcd\int \sqrt{1-c^2x^2}dx+\frac{1}{2}d(1-c^2x^2)\,(a+b\arccos(cx))$$

$$\Big\downarrow \text{211}$$

$$d\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bcd\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{2}d\left(1-c^2x^2\right)(a+ b\arccos(cx))$$

$$\Big\downarrow \text{223}$$

$$d\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}d\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bcd\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)$$

$$\Big\downarrow \text{5137}$$

$$-d\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)+\frac{1}{2}d\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bcd\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$-d\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)+\frac{1}{2}d\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bcd\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)$$

$$\Big\downarrow \text{4202}$$

$$-d\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)\right)+\frac{1}{2}d\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bcd\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)$$

$$\Big\downarrow \text{2620}$$

$$-d\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos\right.\right.$$
$$\frac{1}{2}d\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bcd\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)$$

$$\Big\downarrow \text{2715}$$

$$-d\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)\right.\right.$$
$$\frac{1}{2}d\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bcd\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}d(1-c^2x^2)(a+b\arccos(cx)) - d\left(\frac{i(a+b\arccos(cx))^2}{2b} - 2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx)) - \frac{1}{4}b\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\right)\right) + \frac{1}{2}bcd\left(\frac{\arcsin(cx)}{2c} + \frac{1}{2}x\sqrt{1-c^2x^2}\right)$$

input
```
Int[((d - c^2*d*x^2)*(a + b*ArcCos[c*x]))/x,x]
```

output
```
(d*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/2 + (b*c*d*((x*Sqrt[1 - c^2*x^2])/2
+ ArcSin[c*x]/(2*c)))/2 - d*(((I/2)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((-1/
2*I)*(a + b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCos[c*x])] - (b*PolyLog[2, -E
^((2*I)*ArcCos[c*x])])/4))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5189
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCos[c*x])/(2*p)), x] + (Simp[d
 Int[(d + e*x^2)^(p - 1)*((a + b*ArcCos[c*x])/x), x], x] + Simp[b*c*(d^p/(2
*p))   Int[(1 - c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.08

method	result
parts	$-da\left(\frac{c^{2}x^{2}}{2}-\ln\left(x\right)\right)-\frac{ibd\arccos(cx)^{2}}{2}+\frac{bcdx\sqrt{-c^{2}x^{2}+1}}{4}-\frac{\arccos(cx)b\,c^{2}d\,x^{2}}{2}+\frac{bd\arccos(cx)}{4}+db\,\mathrm{a}$
derivativedivides	$-da\left(\frac{c^{2}x^{2}}{2}-\ln\left(cx\right)\right)-\frac{ibd\arccos(cx)^{2}}{2}+\frac{bcdx\sqrt{-c^{2}x^{2}+1}}{4}-\frac{\arccos(cx)b\,c^{2}d\,x^{2}}{2}+\frac{bd\arccos(cx)}{4}+db$
default	$-da\left(\frac{c^{2}x^{2}}{2}-\ln\left(cx\right)\right)-\frac{ibd\arccos(cx)^{2}}{2}+\frac{bcdx\sqrt{-c^{2}x^{2}+1}}{4}-\frac{\arccos(cx)b\,c^{2}d\,x^{2}}{2}+\frac{bd\arccos(cx)}{4}+db$

input
```
int((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
-d*a*(1/2*c^2*x^2-ln(x))-1/2*I*b*d*arccos(c*x)^2+1/4*b*c*d*x*(-c^2*x^2+1)^
(1/2)-1/2*arccos(c*x)*b*c^2*d*x^2+1/4*b*d*arccos(c*x)+d*b*arccos(c*x)*ln(1
+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1/2*I*d*b*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1
/2))^2)
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)\,(a+b\arccos(cx))}{x}\,dx = \int -\frac{(c^2dx^2-d)(b\arccos{(cx)}+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x,x, algorithm="fricas")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccos(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)\,(a+b\arccos(cx))}{x}\,dx = -d\left(\int \left(-\frac{a}{x}\right)dx + \int ac^2x\,dx + \int \left(-\frac{b\operatorname{acos}{(cx)}}{x}\right)dx + \int bc^2x\operatorname{acos}{(cx)}\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acos(c*x))/x,x)
```

output
```
-d*(Integral(-a/x, x) + Integral(a*c**2*x, x) + Integral(-b*acos(c*x)/x, x
) + Integral(b*c**2*x*acos(c*x), x))
```

Maxima [F]

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))}{x} \, dx = \int -\frac{(c^2dx^2 - d)(b\arccos(cx) + a)}{x} \, dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x,x, algorithm="maxima")
```

output
```
-1/2*a*c^2*d*x^2 + a*d*log(x) - integrate((b*c^2*d*x^2 - b*d)*arctan2(sqrt
(c*x + 1)*sqrt(-c*x + 1), c*x)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))}{x} \, dx = \int \frac{(a + b\,\mathrm{acos}(c\,x))\,(d - c^2\,d\,x^2)}{x} \, dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2))/x,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2))/x, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)\,(a+b\arccos(cx))}{x}\,dx$$
$$=\frac{d\left(-2acos(cx)\,b\,c^2x^2-asin(cx)\,b+\sqrt{-c^2x^2+1}\,bcx+4\left(\int\frac{acos(cx)}{x}dx\right)b+4\log(x)\,a-2a\,c^2x^2\right)}{4}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acos(c*x))/x,x)
```

output
```
(d*( - 2*acos(c*x)*b*c**2*x**2 - asin(c*x)*b + sqrt( - c**2*x**2 + 1)*b*c*
x + 4*int(acos(c*x)/x,x)*b + 4*log(x)*a - 2*a*c**2*x**2))/4
```

3.9 $\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^2}\,dx = -bcd\sqrt{1-c^2x^2} - \frac{d(a+b\arccos(cx))}{x} - c^2dx(a+b\arccos(cx)) - bcd\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)$$

output
```
-b*c*d*(-c^2*x^2+1)^(1/2)-d*(a+b*arccos(c*x))/x-c^2*d*x*(a+b*arccos(c*x))-
b*c*d*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^2}\,dx = -\frac{ad}{x} - ac^2dx + bcd\sqrt{1-c^2x^2} - \frac{bd\arccos(cx)}{x} - bc^2dx\arccos(cx) - bcd\log(x) + bcd\log\left(1+\sqrt{1-c^2x^2}\right)$$

input
```
Integrate[((d - c^2*d*x^2)*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
-((a*d)/x) - a*c^2*d*x + b*c*d*Sqrt[1 - c^2*x^2] - (b*d*ArcCos[c*x])/x - b
*c^2*d*x*ArcCos[c*x] - b*c*d*Log[x] + b*c*d*Log[1 + Sqrt[1 - c^2*x^2]]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5193, 25, 27, 354, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^2}dx$$

$$\Big\downarrow \text{5193}$$

$$bc\int -\frac{d(c^2x^2+1)}{x\sqrt{1-c^2x^2}}dx+c^2(-d)x(a+b\arccos(cx))-\frac{d(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{25}$$

$$-bc\int \frac{d(c^2x^2+1)}{x\sqrt{1-c^2x^2}}dx+c^2(-d)x(a+b\arccos(cx))-\frac{d(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{27}$$

$$-bcd\int \frac{c^2x^2+1}{x\sqrt{1-c^2x^2}}dx+c^2(-d)x(a+b\arccos(cx))-\frac{d(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{2}bcd\int \frac{c^2x^2+1}{x^2\sqrt{1-c^2x^2}}dx^2+c^2(-d)x(a+b\arccos(cx))-\frac{d(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{90}$$

$$-\frac{1}{2}bcd\left(\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2-2\sqrt{1-c^2x^2}\right)+c^2(-d)x(a+b\arccos(cx))-\frac{d(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{73}$$

$$-\frac{1}{2}bcd\left(-\frac{2\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2}-2\sqrt{1-c^2x^2}\right)+c^2(-d)x(a+b\arccos(cx))-\frac{d(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{221}$$

$$c^2(-d)x(a+b\arccos(cx))-\frac{d(a+b\arccos(cx))}{x}-\frac{1}{2}bcd\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-2\sqrt{1-c^2x^2}\right)$$

input
```
Int[((d - c^2*d*x^2)*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
-((d*(a + b*ArcCos[c*x]))/x) - c^2*d*x*(a + b*ArcCos[c*x]) - (b*c*d*(-2*Sq
rt[1 - c^2*x^2] - 2*ArcTanh[Sqrt[1 - c^2*x^2]]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

method	result
parts	$-da\left(c^2x+\frac{1}{x}\right)-dbc\left(cx\arccos\left(cx\right)+\frac{\arccos(cx)}{cx}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)-\sqrt{-c^2x^2+1}\right.$
derivativedivides	$c\left(-da\left(cx+\frac{1}{cx}\right)-db\left(cx\arccos\left(cx\right)+\frac{\arccos(cx)}{cx}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)-\sqrt{-c^2x^2+}\right.\right.$
default	$c\left(-da\left(cx+\frac{1}{cx}\right)-db\left(cx\arccos\left(cx\right)+\frac{\arccos(cx)}{cx}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)-\sqrt{-c^2x^2+}\right.\right.$

input
```
int((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-d*a*(c^2*x+1/x)-d*b*c*(c*x*arccos(c*x)+arccos(c*x)/c/x-arctanh(1/(-c^2*x^
2+1)^(1/2))-(-c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(65) = 130$.

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.13

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))}{x^2}\,dx = -\frac{2\,ac^2dx^2 - bcdx\log\left(\sqrt{-c^2x^2+1}+1\right) + bcdx\log\left(\sqrt{-c^2x^2+1}-1\right) - 2\sqrt{-c^2x^2+1}bcdx + 2\,(bc^2+}{2\,x}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*c^2*d*x^2 - b*c*d*x*log(sqrt(-c^2*x^2 + 1) + 1) + b*c*d*x*log(sq
rt(-c^2*x^2 + 1) - 1) - 2*sqrt(-c^2*x^2 + 1)*b*c*d*x + 2*(b*c^2 + b)*d*x*a
rctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + 2*a*d + 2*(b*c^2*d*x^2 - (b*
c^2 + b)*d*x + b*d)*arccos(c*x))/x
```

Sympy [A] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))}{x^2}\,dx = -ac^2dx - \frac{ad}{x} - bc^2d\left(\begin{cases} \frac{\pi x}{2} & \text{for}\: c = 0 \\ x\operatorname{acos}(cx) - \frac{\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases}\right) - bcd\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for}\: \frac{1}{|c^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases}\right) - \frac{bd\operatorname{acos}(cx)}{x}$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acos(c*x))/x**2,x)
```

output
```
-a*c**2*d*x - a*d/x - b*c**2*d*Piecewise((pi*x/2, Eq(c, 0)), (x*acos(c*x)
- sqrt(-c**2*x**2 + 1)/c, True)) - b*c*d*Piecewise((-acosh(1/(c*x)), 1/Abs
(c**2*x**2) > 1), (I*asin(1/(c*x)), True)) - b*d*acos(c*x)/x
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^2}\,dx$$
$$= -ac^2dx - \left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)bcd + \left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{\arccos(cx)}{x}\right)bd - \frac{ad}{x}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x^2,x, algorithm="maxima")
```

output
```
-a*c^2*d*x - (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b*c*d + (c*log(2*sqrt(
-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*b*d - a*d/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(65) = 130$.

Time = 1.00 (sec) , antiderivative size = 475, normalized size of antiderivative = 6.88

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))}{x^2}\,dx = \frac{2\,bcd\arccos(cx)}{\frac{(c^2x^2-1)^2}{(cx+1)^4} - 1} - \frac{bcd\log\left(\left|cx + \sqrt{-c^2x^2+1} + 1\right|\right)}{\frac{(c^2x^2-1)^2}{(cx+1)^4} - 1} + \frac{bcd\log\left(\left|-cx + \sqrt{-c^2x^2+1} - 1\right|\right)}{\frac{(c^2x^2-1)^2}{(cx+1)^4} - 1} + \frac{2\,acd}{\frac{(c^2x^2-1)^2}{(cx+1)^4} - 1} - \frac{2\sqrt{-c^2x^2+1}bcd}{(cx+1)\left(\frac{(c^2x^2-1)^2}{(cx+1)^4} - 1\right)} + \frac{2\,(c^2x^2-1)^2bcd\arccos(cx)}{(cx+1)^4\left(\frac{(c^2x^2-1)^2}{(cx+1)^4} - 1\right)} + \frac{(c^2x^2-1)^2bcd\log\left(\left|cx + \sqrt{-c^2x^2+1} + 1\right|\right)}{(cx+1)^4\left(\frac{(c^2x^2-1)^2}{(cx+1)^4} - 1\right)} - \frac{(c^2x^2-1)^2bcd\log\left(\left|-cx + \sqrt{-c^2x^2+1} - 1\right|\right)}{(cx+1)^4\left(\frac{(c^2x^2-1)^2}{(cx+1)^4} - 1\right)} + \frac{2\,(-c^2x^2+1)^{\frac{3}{2}}bcd}{(cx+1)^3\left(\frac{(c^2x^2-1)^2}{(cx+1)^4} - 1\right)} + \frac{2\,(c^2x^2-1)^2acd}{(cx+1)^4\left(\frac{(c^2x^2-1)^2}{(cx+1)^4} - 1\right)}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x^2,x, algorithm="giac")
```

output
```
2*b*c*d*arccos(c*x)/((c^2*x^2 - 1)^2/(c*x + 1)^4 - 1) - b*c*d*log(abs(c*x
+ sqrt(-c^2*x^2 + 1) + 1))/((c^2*x^2 - 1)^2/(c*x + 1)^4 - 1) + b*c*d*log(a
bs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/((c^2*x^2 - 1)^2/(c*x + 1)^4 - 1) + 2*a
*c*d/((c^2*x^2 - 1)^2/(c*x + 1)^4 - 1) - 2*sqrt(-c^2*x^2 + 1)*b*c*d/((c*x
+ 1)*((c^2*x^2 - 1)^2/(c*x + 1)^4 - 1)) + 2*(c^2*x^2 - 1)^2*b*c*d*arccos(c
*x)/((c*x + 1)^4*((c^2*x^2 - 1)^2/(c*x + 1)^4 - 1)) + (c^2*x^2 - 1)^2*b*c*
d*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/((c*x + 1)^4*((c^2*x^2 - 1)^2/(c*
x + 1)^4 - 1)) - (c^2*x^2 - 1)^2*b*c*d*log(abs(-c*x + sqrt(-c^2*x^2 + 1) -
 1))/((c*x + 1)^4*((c^2*x^2 - 1)^2/(c*x + 1)^4 - 1)) + 2*(-c^2*x^2 + 1)^(3
/2)*b*c*d/((c*x + 1)^3*((c^2*x^2 - 1)^2/(c*x + 1)^4 - 1)) + 2*(c^2*x^2 - 1
)^2*a*c*d/((c*x + 1)^4*((c^2*x^2 - 1)^2/(c*x + 1)^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{(d-c^2dx^2)\,(a+b\arccos(cx))}{x^2}\,dx = b\,c\,d\,\left(\sqrt{1-c^2\,x^2}-c\,x\,\mathrm{acos}(c\,x)\right)-\frac{a\,d\,(c^2\,x^2+1)}{x} - \frac{b\,d\,\mathrm{acos}(c\,x)}{x}+b\,c\,d\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-c^2\,x^2}}\right)$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2))/x^2,x)
```

output
```
b*c*d*((1 - c^2*x^2)^(1/2) - c*x*acos(c*x)) - (a*d*(c^2*x^2 + 1))/x - (b*d
*acos(c*x))/x + b*c*d*atanh(1/(1 - c^2*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{(d-c^2dx^2)\,(a+b\arccos(cx))}{x^2}\,dx = \frac{d\left(-acos(cx)\,b\,c^2x^2-acos(cx)\,b+\sqrt{-c^2x^2+1}\,bcx-\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)bcx-a\,c^2x^2-a\right)}{x}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acos(c*x))/x^2,x)
```

output
```
(d*( - acos(c*x)*b*c**2*x**2 - acos(c*x)*b + sqrt( - c**2*x**2 + 1)*b*c*x
- log(tan(asin(c*x)/2))*b*c*x - a*c**2*x**2 - a))/x
```

3.10 $\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 139

$$\begin{aligned}\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^3}\,dx = & -\frac{bcd\sqrt{1-c^2x^2}}{2x}-\frac{1}{2}bc^2d\arccos(cx)\\ & -\frac{d(1-c^2x^2)(a+b\arccos(cx))}{2x^2}\\ & +\frac{ic^2d(a+b\arccos(cx))^2}{2b}\\ & -c^2d(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)\\ & +\frac{1}{2}ibc^2d\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\end{aligned}$$

output
```
-1/2*b*c*d*(-c^2*x^2+1)^(1/2)/x-1/2*b*c^2*d*arccos(c*x)-1/2*d*(-c^2*x^2+1)
*(a+b*arccos(c*x))/x^2+1/2*I*c^2*d*(a+b*arccos(c*x))^2/b-c^2*d*(a+b*arccos
(c*x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/2*I*b*c^2*d*polylog(2,(c*x+I*(
-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.88

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^3}\,dx = -\frac{ad}{2x^2}+\frac{bcd\sqrt{1-c^2x^2}}{2x} - \frac{bd\arccos(cx)}{2x^2}+\frac{1}{2}ibc^2d\arccos(cx)^2 - bc^2d\arccos(cx)\log\left(1+e^{2i\arccos(cx)}\right) - ac^2d\log(x)+\frac{1}{2}ibc^2d\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)$$

input
```
Integrate[((d - c^2*d*x^2)*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
-1/2*(a*d)/x^2 + (b*c*d*Sqrt[1 - c^2*x^2])/(2*x) - (b*d*ArcCos[c*x])/(2*x^
2) + (I/2)*b*c^2*d*ArcCos[c*x]^2 - b*c^2*d*ArcCos[c*x]*Log[1 + E^((2*I)*Ar
cCos[c*x])] - a*c^2*d*Log[x] + (I/2)*b*c^2*d*PolyLog[2, -E^((2*I)*ArcCos[c
*x])]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5191, 247, 223, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{5191}$$

$$c^2(-d)\int \frac{a+b\arccos(cx)}{x}dx-\frac{1}{2}bcd\int \frac{\sqrt{1-c^2x^2}}{x^2}dx-\frac{d(1-c^2x^2)(a+b\arccos(cx))}{2x^2}$$

$$\Big\downarrow \text{247}$$

$$c^2(-d)\int \frac{a+b\arccos(cx)}{x}dx-\frac{1}{2}bcd\left(c^2\left(-\int \frac{1}{\sqrt{1-c^2x^2}}dx\right)-\frac{\sqrt{1-c^2x^2}}{x}\right)-\frac{d(1-c^2x^2)\,(a+b\arccos(cx))}{2x^2}$$

$$\Big\downarrow \text{ 223}$$

$$c^2(-d)\int \frac{a+b\arccos(cx)}{x}dx-\frac{d(1-c^2x^2)\,(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd\left(-c\arcsin(cx)-\frac{\sqrt{1-c^2x^2}}{x}\right)$$

$$\Big\downarrow \text{ 5137}$$

$$c^2d\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)-\frac{d(1-c^2x^2)\,(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd\left(-c\arcsin(cx)-\frac{\sqrt{1-c^2x^2}}{x}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$c^2d\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)-\frac{d(1-c^2x^2)\,(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd\left(-c\arcsin(cx)-\frac{\sqrt{1-c^2x^2}}{x}\right)$$

$$\Big\downarrow \text{ 4202}$$

$$c^2d\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)\right)-\frac{d(1-c^2x^2)\,(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd\left(-c\arcsin(cx)-\frac{\sqrt{1-c^2x^2}}{x}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$c^2d\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos\right.\right.$$
$$\frac{d(1-c^2x^2)\,(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd\left(-c\arcsin(cx)-\frac{\sqrt{1-c^2x^2}}{x}\right)$$

$$\Big\downarrow \text{ 2715}$$

$$c^2d\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)\right.\right.$$
$$\frac{d(1-c^2x^2)(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd\left(-c\arcsin(cx)-\frac{\sqrt{1-c^2x^2}}{x}\right)$$

$$\Big\downarrow \text{2838}$$

$$c^2d\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\right)\right)$$
$$\frac{d(1-c^2x^2)(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd\left(-c\arcsin(cx)-\frac{\sqrt{1-c^2x^2}}{x}\right)$$

input
```
Int[((d - c^2*d*x^2)*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
-1/2*(d*(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/x^2 - (b*c*d*(-(Sqrt[1 - c^2*x^
2]/x) - c*ArcSin[c*x]))/2 + c^2*d*(((I/2)*(a + b*ArcCos[c*x])^2)/b - (2*I)
*((-1/2*I)*(a + b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCos[c*x])] - (b*PolyLog
[2, -E^((2*I)*ArcCos[c*x])])/4))
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5191
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCos[c*x
])/(f*(m + 1))), x] + (Simp[b*c*(d^p/(f*(m + 1)))   Int[(f*x)^(m + 1)*(1 -
c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m + 2)
*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x]), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

method	result
parts	$-da\,c^2\ln(x)-\frac{da}{2x^2}-db\,c^2\left(-\frac{i\arccos(cx)^2}{2}+\frac{-ic^2x^2-cx\sqrt{-c^2x^2+1}+\arccos(cx)}{2c^2x^2}+\arccos(cx)\ln\right.$
derivativedivides	$c^2\left(-da\left(\frac{1}{2c^2x^2}+\ln(cx)\right)-db\left(-\frac{i\arccos(cx)^2}{2}+\frac{-ic^2x^2-cx\sqrt{-c^2x^2+1}+\arccos(cx)}{2c^2x^2}+\arccos(cx\right.\right.$
default	$c^2\left(-da\left(\frac{1}{2c^2x^2}+\ln(cx)\right)-db\left(-\frac{i\arccos(cx)^2}{2}+\frac{-ic^2x^2-cx\sqrt{-c^2x^2+1}+\arccos(cx)}{2c^2x^2}+\arccos(cx\right.\right.$

input
```
int((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-d*a*c^2*ln(x)-1/2*d*a/x^2-d*b*c^2*(-1/2*I*arccos(c*x)^2+1/2*(-I*c^2*x^2-c
*x*(-c^2*x^2+1)^(1/2)+arccos(c*x))/c^2/x^2+arccos(c*x)*ln(1+(c*x+I*(-c^2*x
^2+1)^(1/2))^2)-1/2*I*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2))
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)\,(a+b\arccos(cx))}{x^3}\,dx=\int -\frac{(c^2dx^2-d)(b\arccos(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccos(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))}{x^3} dx = -d\left(\int \left(-\frac{a}{x^3}\right) dx + \int \frac{ac^2}{x} dx + \int \left(-\frac{b\,\mathrm{acos}\,(cx)}{x^3}\right) dx + \int \frac{bc^2\,\mathrm{acos}\,(cx)}{x} dx\right)$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acos(c*x))/x**3,x)
```

output
```
-d*(Integral(-a/x**3, x) + Integral(a*c**2/x, x) + Integral(-b*acos(c*x)/x
**3, x) + Integral(b*c**2*acos(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))}{x^3} dx = \int -\frac{(c^2dx^2 - d)(b\arccos(cx) + a)}{x^3} dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x^3,x, algorithm="maxima")
```

output
```
-b*c^2*d*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x, x) - a*c^
2*d*log(x) + 1/2*b*d*(sqrt(-c^2*x^2 + 1)*c/x - arccos(c*x)/x^2) - 1/2*a*d/
x^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))}{x^3} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^3}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)}{x^3}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2))/x^3,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^3}\,dx = \frac{d\left(-acos(cx)\,b+\sqrt{-c^2x^2+1}\,bcx-2\left(\int \frac{acos(cx)}{x}dx\right)b\,c^2x^2-2\log(x)\,a\,c^2x^2-a\right)}{2x^2}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acos(c*x))/x^3,x)
```

output
```
(d*( - acos(c*x)*b + sqrt( - c**2*x**2 + 1)*b*c*x - 2*int(acos(c*x)/x,x)*b
*c**2*x**2 - 2*log(x)*a*c**2*x**2 - a))/(2*x**2)
```

3.11 $\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^4}\,dx = -\frac{bcd\sqrt{1-c^2x^2}}{6x^2}-\frac{d(a+b\arccos(cx))}{3x^3}+\frac{c^2d(a+b\arccos(cx))}{x}+\frac{5}{6}bc^3d\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)$$

output
```
-1/6*b*c*d*(-c^2*x^2+1)^(1/2)/x^2-1/3*d*(a+b*arccos(c*x))/x^3+c^2*d*(a+b*a
rccos(c*x))/x+5/6*b*c^3*d*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.31

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^4}\,dx = -\frac{ad}{3x^3}+\frac{ac^2d}{x}+\frac{bcd\sqrt{1-c^2x^2}}{6x^2}-\frac{bd\arccos(cx)}{3x^3}+\frac{bc^2d\arccos(cx)}{x}+\frac{5}{6}bc^3d\log(x)-\frac{5}{6}bc^3d\log\left(1+\sqrt{1-c^2x^2}\right)$$

input
```
Integrate[((d - c^2*d*x^2)*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
-1/3*(a*d)/x^3 + (a*c^2*d)/x + (b*c*d*Sqrt[1 - c^2*x^2])/(6*x^2) - (b*d*Ar
cCos[c*x])/(3*x^3) + (b*c^2*d*ArcCos[c*x])/x + (5*b*c^3*d*Log[x])/6 - (5*b
*c^3*d*Log[1 + Sqrt[1 - c^2*x^2]])/6
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5193, 27, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^4}dx$$

$$\Big\downarrow \; 5193$$

$$bc\int -\frac{d(1-3c^2x^2)}{3x^3\sqrt{1-c^2x^2}}dx+\frac{c^2d(a+b\arccos(cx))}{x}-\frac{d(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{3}bcd\int \frac{1-3c^2x^2}{x^3\sqrt{1-c^2x^2}}dx+\frac{c^2d(a+b\arccos(cx))}{x}-\frac{d(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \; 354$$

$$-\frac{1}{6}bcd\int \frac{1-3c^2x^2}{x^4\sqrt{1-c^2x^2}}dx^2+\frac{c^2d(a+b\arccos(cx))}{x}-\frac{d(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \; 87$$

$$-\frac{1}{6}bcd\left(-\frac{5}{2}c^2\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{\sqrt{1-c^2x^2}}{x^2}\right)+\frac{c^2d(a+b\arccos(cx))}{x}-\frac{d(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \; 73$$

$$-\frac{1}{6}bcd\left(5\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}-\frac{\sqrt{1-c^2x^2}}{x^2}\right)+\frac{c^2d(a+b\arccos(cx))}{x}-\frac{d(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{c^2d(a+b\arccos(cx))}{x}-\frac{d(a+b\arccos(cx))}{3x^3}-\frac{1}{6}bcd\left(5c^2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{\sqrt{1-c^2x^2}}{x^2}\right)$$

input
```
Int[((d - c^2*d*x^2)*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
-1/3*(d*(a + b*ArcCos[c*x]))/x^3 + (c^2*d*(a + b*ArcCos[c*x]))/x - (b*c*d*
(-(Sqrt[1 - c^2*x^2]/x^2) + 5*c^2*ArcTanh[Sqrt[1 - c^2*x^2]]))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

method	result
parts	$-da\left(-\frac{c^{2}}{x}+\frac{1}{3x^{3}}\right)-db\,c^{3}\left(\frac{\arccos\left(cx\right)}{3c^{3}x^{3}}-\frac{\arccos\left(cx\right)}{cx}-\frac{\sqrt{-c^{2}x^{2}+1}}{6c^{2}x^{2}}+\frac{5\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^{2}x^{2}+1}}\right)}{6}\right)$
derivativedivides	$c^{3}\left(-da\left(\frac{1}{3c^{3}x^{3}}-\frac{1}{cx}\right)-db\left(\frac{\arccos\left(cx\right)}{3c^{3}x^{3}}-\frac{\arccos\left(cx\right)}{cx}-\frac{\sqrt{-c^{2}x^{2}+1}}{6c^{2}x^{2}}+\frac{5\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^{2}x^{2}+1}}\right)}{6}\right)\right)$
default	$c^{3}\left(-da\left(\frac{1}{3c^{3}x^{3}}-\frac{1}{cx}\right)-db\left(\frac{\arccos\left(cx\right)}{3c^{3}x^{3}}-\frac{\arccos\left(cx\right)}{cx}-\frac{\sqrt{-c^{2}x^{2}+1}}{6c^{2}x^{2}}+\frac{5\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^{2}x^{2}+1}}\right)}{6}\right)\right)$

input
```
int((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-d*a*(-c^2/x+1/3/x^3)-d*b*c^3*(1/3*arccos(c*x)/c^3/x^3-arccos(c*x)/c/x-1/6
/c^2/x^2*(-c^2*x^2+1)^(1/2)+5/6*arctanh(1/(-c^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(71) = 142$.

Time = 0.17 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.07

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))}{x^4}\,dx = -\frac{5\,bc^3dx^3\log\left(\sqrt{-c^2x^2+1}+1\right) - 5\,bc^3dx^3\log\left(\sqrt{-c^2x^2+1}-1\right) - 12\,ac^2dx^2 - 4\,(3\,bc^2-b)dx^3\arctan}{12\,x^3}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/12*(5*b*c^3*d*x^3*log(sqrt(-c^2*x^2 + 1) + 1) - 5*b*c^3*d*x^3*log(sqrt(
-c^2*x^2 + 1) - 1) - 12*a*c^2*d*x^2 - 4*(3*b*c^2 - b)*d*x^3*arctan(sqrt(-c
^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) - 2*sqrt(-c^2*x^2 + 1)*b*c*d*x + 4*a*d - 4*
(3*b*c^2*d*x^2 - (3*b*c^2 - b)*d*x^3 - b*d)*arccos(c*x))/x^3
```

Sympy [A] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.19

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))}{x^4}\,dx$$

$$= \frac{ac^2d}{x} - \frac{ad}{3x^3} + bc^3d\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \frac{1}{|c^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise}\end{cases}\right) + \frac{bc^2d\operatorname{acos}(cx)}{x} - \frac{bcd\left(\begin{cases} -\frac{c^2\operatorname{acosh}\left(\frac{1}{cx}\right)}{2} + \frac{c}{2x\sqrt{-1+\frac{1}{c^2x^2}}} - \frac{1}{2cx^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \frac{1}{|c^2x^2|} > 1 \\ \frac{ic^2\operatorname{asin}\left(\frac{1}{cx}\right)}{2} - \frac{ic\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)}{3} - \frac{bd\operatorname{acos}(cx)}{3x^3}$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acos(c*x))/x**4,x)
```

output
```
a*c**2*d/x - a*d/(3*x**3) + b*c**3*d*Piecewise((-acosh(1/(c*x)), 1/Abs(c**
2*x**2) > 1), (I*asin(1/(c*x)), True)) + b*c**2*d*acos(c*x)/x - b*c*d*Piec
ewise((-c**2*acosh(1/(c*x))/2 + c/(2*x*sqrt(-1 + 1/(c**2*x**2))) - 1/(2*c*
x**3*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (I*c**2*asin(1/(c*x
))/2 - I*c*sqrt(1 - 1/(c**2*x**2))/(2*x), True))/3 - b*d*acos(c*x)/(3*x**3
)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))}{x^4}\,dx$$
$$= -\left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{\arccos(cx)}{x}\right)bc^2d$$
$$+ \frac{1}{6}\left(\left(c^2\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{\sqrt{-c^2x^2+1}}{x^2}\right)c - \frac{2\arccos(cx)}{x^3}\right)bd$$
$$+ \frac{ac^2d}{x} - \frac{ad}{3x^3}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x^4,x, algorithm="maxima")
```

output
```
-(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*b*c^2*d +
 1/6*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1
)/x^2)*c - 2*arccos(c*x)/x^3)*b*d + a*c^2*d/x - 1/3*a*d/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1654 vs. 2(71) = 142.

Time = 4.51 (sec) , antiderivative size = 1654, normalized size of antiderivative = 20.42

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))/x^4,x, algorithm="giac")
```

output
```
2/3*b*c^3*d*arccos(c*x)/(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(
c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) - 5/6*b*c^3*d*log(abs(c*x +
sqrt(-c^2*x^2 + 1) + 1))/(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/
(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) + 5/6*b*c^3*d*log(abs(-c*x
+ sqrt(-c^2*x^2 + 1) - 1))/(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^
2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) + 2/3*a*c^3*d/(3*(c^2*x^2
 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x +
 1)^6 + 1) + 2*(c^2*x^2 - 1)*b*c^3*d*arccos(c*x)/((c*x + 1)^2*(3*(c^2*x^2
- 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x +
1)^6 + 1)) - 5/2*(c^2*x^2 - 1)*b*c^3*d*log(abs(c*x + sqrt(-c^2*x^2 + 1) +
1))/((c*x + 1)^2*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1
)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + 5/2*(c^2*x^2 - 1)*b*c^3*d*log(ab
s(-c*x + sqrt(-c^2*x^2 + 1) - 1))/((c*x + 1)^2*(3*(c^2*x^2 - 1)/(c*x + 1)^
2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + 1/
3*sqrt(-c^2*x^2 + 1)*b*c^3*d/((c*x + 1)*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(
c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + 2*(c^2*x^
2 - 1)*a*c^3*d/((c*x + 1)^2*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)
^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) - 2*(c^2*x^2 - 1)^2*b*c
^3*d*arccos(c*x)/((c*x + 1)^4*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 -
1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) - 5/2*(c^2*x^2 - 1...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2dx^2)\,(a + b\arccos(cx))}{x^4}\,dx = \int \frac{(a + b\,\mathrm{acos}(c\,x))\,\left(d - c^2\,d\,x^2\right)}{x^4}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2))/x^4,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))}{x^4}\,dx$$
$$=\frac{d\left(6acos(cx)\,b\,c^2x^2-2acos(cx)\,b+\sqrt{-c^2x^2+1}\,bcx+5\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)b\,c^3x^3+6a\,c^2x^2-2a\right)}{6x^3}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acos(c*x))/x^4,x)
```

output
```
(d*(6*acos(c*x)*b*c**2*x**2 - 2*acos(c*x)*b + sqrt( - c**2*x**2 + 1)*b*c*x
 + 5*log(tan(asin(c*x)/2))*b*c**3*x**3 + 6*a*c**2*x**2 - 2*a))/(6*x**3)
```

3.12 $\int x^4(d-c^2dx^2)^2 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 186

$$\int x^4(d-c^2dx^2)^2 (a+b\arccos(cx))\,dx = \frac{8bd^2\sqrt{1-c^2x^2}}{315c^5}+\frac{4bd^2(1-c^2x^2)^{3/2}}{945c^5}+\frac{bd^2(1-c^2x^2)^{5/2}}{525c^5} - \frac{10bd^2(1-c^2x^2)^{7/2}}{441c^5}+\frac{bd^2(1-c^2x^2)^{9/2}}{81c^5} + \frac{1}{5}d^2x^5(a+b\arccos(cx))-\frac{2}{7}c^2d^2x^7(a+b\arccos(cx))+\frac{1}{9}c^4d^2x^9(a+b\arccos(cx))$$

output
```
8/315*b*d^2*(-c^2*x^2+1)^(1/2)/c^5+4/945*b*d^2*(-c^2*x^2+1)^(3/2)/c^5+1/52
5*b*d^2*(-c^2*x^2+1)^(5/2)/c^5-10/441*b*d^2*(-c^2*x^2+1)^(7/2)/c^5+1/81*b*
d^2*(-c^2*x^2+1)^(9/2)/c^5+1/5*d^2*x^5*(a+b*arccos(c*x))-2/7*c^2*d^2*x^7*(
a+b*arccos(c*x))+1/9*c^4*d^2*x^9*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.65

$$\int x^4 \left(d-c^2 d x^2\right)^2 (a+b \arccos(cx)) \, dx$$
$$= \frac{d^2\left(315ac^5x^5(63-90c^2x^2+35c^4x^4)-b\sqrt{1-c^2x^2}(2104+1052c^2x^2+789c^4x^4-2650c^6x^6+1225c^8x^8)\right.}{99225c^5}$$

input
```
Integrate[x^4*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*(315*a*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4) - b*Sqrt[1 - c^2*x^2]*(
2104 + 1052*c^2*x^2 + 789*c^4*x^4 - 2650*c^6*x^6 + 1225*c^8*x^8) + 315*b*c
^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4)*ArcCos[c*x]))/(99225*c^5)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5193, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(d-c^2 d x^2\right)^2 (a+b \arccos(cx)) \, dx$$

$$\Big\downarrow \; 5193$$

$$bc\int \frac{d^2x^5\left(35c^4x^4-90c^2x^2+63\right)}{315\sqrt{1-c^2x^2}}dx+\frac{1}{9}c^4d^2x^9(a+b\arccos(cx))-\frac{2}{7}c^2d^2x^7(a+b\arccos(cx))+\frac{1}{5}d^2x^5(a+b\arccos(cx))$$

$$\Big\downarrow \; 27$$

$$\frac{1}{315}bcd^2\int \frac{x^5\left(35c^4x^4-90c^2x^2+63\right)}{\sqrt{1-c^2x^2}}dx+\frac{1}{9}c^4d^2x^9(a+b\arccos(cx))-\frac{2}{7}c^2d^2x^7(a+b\arccos(cx))+\frac{1}{5}d^2x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{630}bcd^2\int \frac{x^4(35c^4x^4-90c^2x^2+63)}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{9}c^4d^2x^9(a+b\arccos(cx))-\frac{2}{7}c^2d^2x^7(a+b\arccos(cx))+\frac{1}{5}d^2x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{1195}$$

$$\frac{1}{630}bcd^2\int \left(\frac{35(1-c^2x^2)^{7/2}}{c^4}-\frac{50(1-c^2x^2)^{5/2}}{c^4}+\frac{3(1-c^2x^2)^{3/2}}{c^4}+\frac{4\sqrt{1-c^2x^2}}{c^4}+\frac{8}{c^4\sqrt{1-c^2x^2}}\right)dx^2+\frac{1}{9}c^4d^2x^9(a+b\arccos(cx))-\frac{2}{7}c^2d^2x^7(a+b\arccos(cx))+\frac{1}{5}d^2x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}c^4d^2x^9(a+b\arccos(cx))-\frac{2}{7}c^2d^2x^7(a+b\arccos(cx))+\frac{1}{5}d^2x^5(a+b\arccos(cx))+\frac{1}{630}bcd^2\left(-\frac{70(1-c^2x^2)^{9/2}}{9c^6}+\frac{100(1-c^2x^2)^{7/2}}{7c^6}-\frac{6(1-c^2x^2)^{5/2}}{5c^6}-\frac{8(1-c^2x^2)^{3/2}}{3c^6}-\frac{16\sqrt{1-c^2x^2}}{c^6}\right)$$

input
```
Int[x^4*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d^2*((-16*Sqrt[1 - c^2*x^2])/c^6 - (8*(1 - c^2*x^2)^(3/2))/(3*c^6) -
(6*(1 - c^2*x^2)^(5/2))/(5*c^6) + (100*(1 - c^2*x^2)^(7/2))/(7*c^6) - (70*
(1 - c^2*x^2)^(9/2))/(9*c^6)))/630 + (d^2*x^5*(a + b*ArcCos[c*x]))/5 - (2*
c^2*d^2*x^7*(a + b*ArcCos[c*x]))/7 + (c^4*d^2*x^9*(a + b*ArcCos[c*x]))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.90

method	result
parts	$d^2a\left(\frac{1}{9}c^4x^9-\frac{2}{7}c^2x^7+\frac{1}{5}x^5\right)+\frac{d^2b\left(\frac{\arccos(cx)c^9x^9}{9}-\frac{2\arccos(cx)c^7x^7}{7}+\frac{\arccos(cx)c^5x^5}{5}-\frac{263c^4x^4\sqrt{-c^2x^2+1}}{33075}-1\right.}{c^5}$
derivativedivides	$\frac{d^2a\left(\frac{1}{9}c^9x^9-\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b\left(\frac{\arccos(cx)c^9x^9}{9}-\frac{2\arccos(cx)c^7x^7}{7}+\frac{\arccos(cx)c^5x^5}{5}-\frac{263c^4x^4\sqrt{-c^2x^2+1}}{33075}-\frac{1052c^2x^2\sqrt{-}}{99225}\right.}{c^5}$
default	$\frac{d^2a\left(\frac{1}{9}c^9x^9-\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b\left(\frac{\arccos(cx)c^9x^9}{9}-\frac{2\arccos(cx)c^7x^7}{7}+\frac{\arccos(cx)c^5x^5}{5}-\frac{263c^4x^4\sqrt{-c^2x^2+1}}{33075}-\frac{1052c^2x^2\sqrt{-}}{99225}\right.}{c^5}$
orering	$\frac{(20825c^{10}x^{10}-54450c^8x^8+36757c^6x^6+5260c^4x^4+12624c^2x^2-8416)(-c^2dx^2+d)^2(a+b\arccos(cx))}{99225c^6x(cx-1)(cx+1)(c^2x^2-1)}-\frac{(1225c^8x^8-}{}$

input
```
int(x^4*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/9*c^4*x^9-2/7*c^2*x^7+1/5*x^5)+d^2*b/c^5*(1/9*arccos(c*x)*c^9*x^9
-2/7*arccos(c*x)*c^7*x^7+1/5*arccos(c*x)*c^5*x^5-263/33075*c^4*x^4*(-c^2*x
^2+1)^(1/2)-1052/99225*c^2*x^2*(-c^2*x^2+1)^(1/2)-2104/99225*(-c^2*x^2+1)^
(1/2)+106/3969*c^6*x^6*(-c^2*x^2+1)^(1/2)-1/81*c^8*x^8*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.83

$$\int x^4 (d - c^2 d x^2)^2 (a + b \arccos(cx))\, dx = \frac{11025\, ac^9 d^2 x^9 - 28350\, ac^7 d^2 x^7 + 19845\, ac^5 d^2 x^5 + 315\, (35\, bc^9 d^2 x^9 - 90\, bc^7 d^2 x^7 + 63\, bc^5 d^2 x^5) \arccos{(cx}}{99225\, c^5}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/99225*(11025*a*c^9*d^2*x^9 - 28350*a*c^7*d^2*x^7 + 19845*a*c^5*d^2*x^5 +
 315*(35*b*c^9*d^2*x^9 - 90*b*c^7*d^2*x^7 + 63*b*c^5*d^2*x^5)*arccos(c*x)
- (1225*b*c^8*d^2*x^8 - 2650*b*c^6*d^2*x^6 + 789*b*c^4*d^2*x^4 + 1052*b*c^
2*d^2*x^2 + 2104*b*d^2)*sqrt(-c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.26

$$\int x^4 (d - c^2 d x^2)^2 (a + b \arccos(cx))\, dx = \begin{cases} \frac{ac^4 d^2 x^9}{9} - \frac{2ac^2 d^2 x^7}{7} + \frac{ad^2 x^5}{5} + \frac{bc^4 d^2 x^9 \operatorname{acos}{(cx)}}{9} - \frac{bc^3 d^2 x^8 \sqrt{-c^2 x^2 + 1}}{81} - \frac{2bc^2 d^2 x^7 \operatorname{acos}{(cx)}}{7} + \frac{106bcd^2 x^6 \sqrt{-c^2 x^2 + 1}}{3969} + \frac{bd^2 x^5}{} \\ \frac{d^2 x^5 \left(a + \frac{\pi b}{2}\right)}{5} \end{cases}$$

input
```
integrate(x**4*(-c**2*d*x**2+d)**2*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*c**4*d**2*x**9/9 - 2*a*c**2*d**2*x**7/7 + a*d**2*x**5/5 + b*c
**4*d**2*x**9*acos(c*x)/9 - b*c**3*d**2*x**8*sqrt(-c**2*x**2 + 1)/81 - 2*b
*c**2*d**2*x**7*acos(c*x)/7 + 106*b*c*d**2*x**6*sqrt(-c**2*x**2 + 1)/3969
+ b*d**2*x**5*acos(c*x)/5 - 263*b*d**2*x**4*sqrt(-c**2*x**2 + 1)/(33075*c)
 - 1052*b*d**2*x**2*sqrt(-c**2*x**2 + 1)/(99225*c**3) - 2104*b*d**2*sqrt(-
c**2*x**2 + 1)/(99225*c**5), Ne(c, 0)), (d**2*x**5*(a + pi*b/2)/5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(160) = 320$.

Time = 0.12 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.78

$$\int x^4(d-c^2dx^2)^2(a+b\arccos(cx))\,dx = \frac{1}{9}ac^4d^2x^9 - \frac{2}{7}ac^2d^2x^7 + \frac{1}{2835}\left(315\,x^9\arccos(cx) - \left(\frac{35\sqrt{-c^2x^2+1}x^8}{c^2} + \frac{40\sqrt{-c^2x^2+1}x^6}{c^4} + \frac{48\sqrt{-c^2x^2+1}x^4}{c^6} + \frac{64\sqrt{-c^2x^2+1}x^2}{c^8}\right.\right.$$

$$+\frac{1}{5}ad^2x^5$$

$$-\frac{2}{245}\left(35\,x^7\arccos(cx) - \left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\sqrt{-c^2x^2+1}}{c^8}\right.\right.$$

$$+\frac{1}{75}\left(15\,x^5\arccos(cx) - \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bd^2$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/9*a*c^4*d^2*x^9 - 2/7*a*c^2*d^2*x^7 + 1/2835*(315*x^9*arccos(c*x) - (35*
sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*
x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/
c^10)*c)*b*c^4*d^2 + 1/5*a*d^2*x^5 - 2/245*(35*x^7*arccos(c*x) - (5*sqrt(-
c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)
*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*c^2*d^2 + 1/75*(15*x^5*arccos(c
*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqr
t(-c^2*x^2 + 1)/c^6)*c)*b*d^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.04

$$\int x^4 (d - c^2 dx^2)^2 (a + b \arccos(cx)) dx$$
$$= \frac{1}{9} bc^4 d^2 x^9 \arccos(cx) + \frac{1}{9} ac^4 d^2 x^9 - \frac{1}{81} \sqrt{-c^2 x^2 + 1} bc^3 d^2 x^8 - \frac{2}{7} bc^2 d^2 x^7 \arccos(cx) - \frac{2}{7} ac^2 d^2 x^7 + \frac{106}{3969} \sqrt{-c^2 x^2 + 1} bcd^2 x^6 + \frac{1}{5} bd^2 x^5 \arccos(cx) + \frac{1}{5} ad^2 x^5 - \frac{263 \sqrt{-c^2 x^2 + 1} bd^2 x^4}{33075 c} - \frac{1052 \sqrt{-c^2 x^2 + 1} bd^2 x^2}{99225 c^3} - \frac{2104 \sqrt{-c^2 x^2 + 1} bd^2}{99225 c^5}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/9*b*c^4*d^2*x^9*arccos(c*x) + 1/9*a*c^4*d^2*x^9 - 1/81*sqrt(-c^2*x^2 + 1
)*b*c^3*d^2*x^8 - 2/7*b*c^2*d^2*x^7*arccos(c*x) - 2/7*a*c^2*d^2*x^7 + 106/
3969*sqrt(-c^2*x^2 + 1)*b*c*d^2*x^6 + 1/5*b*d^2*x^5*arccos(c*x) + 1/5*a*d^
2*x^5 - 263/33075*sqrt(-c^2*x^2 + 1)*b*d^2*x^4/c - 1052/99225*sqrt(-c^2*x^
2 + 1)*b*d^2*x^2/c^3 - 2104/99225*sqrt(-c^2*x^2 + 1)*b*d^2/c^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d - c^2 dx^2)^2 (a + b \arccos(cx)) dx = \int x^4 (a + b \operatorname{acos}(c x)) (d - c^2 d x^2)^2 dx$$

input
```
int(x^4*(a + b*acos(c*x))*(d - c^2*d*x^2)^2,x)
```

output
```
int(x^4*(a + b*acos(c*x))*(d - c^2*d*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

$$\int x^4 (d - c^2 d x^2)^2 (a + b \arccos(cx)) \, dx$$
$$= \frac{d^2 (11025 acos(cx) \, b c^9 x^9 - 28350 acos(cx) \, b c^7 x^7 + 19845 acos(cx) \, b c^5 x^5 - 1225 \sqrt{-c^2 x^2 + 1} \, b c^8 x^8 + 2650}{}$$

input
```
int(x^4*(-c^2*d*x^2+d)^2*(a+b*acos(c*x)),x)
```

output
```
(d**2*(11025*acos(c*x)*b*c**9*x**9 - 28350*acos(c*x)*b*c**7*x**7 + 19845*a
cos(c*x)*b*c**5*x**5 - 1225*sqrt( - c**2*x**2 + 1)*b*c**8*x**8 + 2650*sqrt
( - c**2*x**2 + 1)*b*c**6*x**6 - 789*sqrt( - c**2*x**2 + 1)*b*c**4*x**4 -
1052*sqrt( - c**2*x**2 + 1)*b*c**2*x**2 - 2104*sqrt( - c**2*x**2 + 1)*b +
11025*a*c**9*x**9 - 28350*a*c**7*x**7 + 19845*a*c**5*x**5))/(99225*c**5)
```

3.13 $\int x^3(d-c^2dx^2)^2\,(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 184

$$\int x^3(d-c^2dx^2)^2\,(a+b\arccos(cx))\,dx = \frac{73bd^2x\sqrt{1-c^2x^2}}{3072c^3}+\frac{73bd^2x^3\sqrt{1-c^2x^2}}{4608c} - \frac{43bcd^2x^5\sqrt{1-c^2x^2}}{1152}+\frac{1}{64}bc^3d^2x^7\sqrt{1-c^2x^2} - \frac{73bd^2\arccos(cx)}{3072c^4}+\frac{1}{4}d^2x^4(a+b\arccos(cx)) - \frac{1}{3}c^2d^2x^6(a+b\arccos(cx)) + \frac{1}{8}c^4d^2x^8(a+b\arccos(cx))$$

output
```
73/3072*b*d^2*x*(-c^2*x^2+1)^(1/2)/c^3+73/4608*b*d^2*x^3*(-c^2*x^2+1)^(1/2
)/c-43/1152*b*c*d^2*x^5*(-c^2*x^2+1)^(1/2)+1/64*b*c^3*d^2*x^7*(-c^2*x^2+1)
^(1/2)-73/3072*b*d^2*arccos(c*x)/c^4+1/4*d^2*x^4*(a+b*arccos(c*x))-1/3*c^2
*d^2*x^6*(a+b*arccos(c*x))+1/8*c^4*d^2*x^8*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.64

$$\int x^3 \left(d-c^2 d x^2\right)^2 (a+b \arccos (c x)) \, dx$$
$$= \frac{d^2\left(2304ax^4-3072ac^2x^6+1152ac^4x^8-\frac{bx\sqrt{1-c^2x^2}\left(219+146c^2x^2-344c^4x^4+144c^6x^6\right)}{c^3}+384bx^4(6-8c^2x^2+3c^4x\right.}{9216}$$

input
```
Integrate[x^3*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*(2304*a*x^4 - 3072*a*c^2*x^6 + 1152*a*c^4*x^8 - (b*x*Sqrt[1 - c^2*x^2
]*(219 + 146*c^2*x^2 - 344*c^4*x^4 + 144*c^6*x^6))/c^3 + 384*b*x^4*(6 - 8*
c^2*x^2 + 3*c^4*x^4)*ArcCos[c*x] + (219*b*ArcSin[c*x])/c^4))/9216
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5193, 27, 1590, 25, 27, 363, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d-c^2 d x^2\right)^2 (a+b \arccos (c x)) \, dx$$
$$\Big\downarrow \text{5193}$$
$$bc\int \frac{d^2x^4\left(3c^4x^4-8c^2x^2+6\right)}{24\sqrt{1-c^2x^2}}dx+\frac{1}{8}c^4d^2x^8(a+b\arccos(cx))-\frac{1}{3}c^2d^2x^6(a+b\arccos(cx))+\frac{1}{4}d^2x^4(a+b\arccos(cx))$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{24}bcd^2\int \frac{x^4\left(3c^4x^4-8c^2x^2+6\right)}{\sqrt{1-c^2x^2}}dx+\frac{1}{8}c^4d^2x^8(a+b\arccos(cx))-\frac{1}{3}c^2d^2x^6(a+b\arccos(cx))+\frac{1}{4}d^2x^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{1590}$$

$$\frac{1}{24}bcd^2\left(-\frac{\int -\frac{c^2x^4(48-43c^2x^2)}{\sqrt{1-c^2x^2}}dx}{8c^2}-\frac{3}{8}c^2x^7\sqrt{1-c^2x^2}\right)+\frac{1}{8}c^4d^2x^8(a+b\arccos(cx))-\frac{1}{3}c^2d^2x^6(a+b\arccos(cx))+\frac{1}{4}d^2x^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{24}bcd^2\left(\frac{\int \frac{c^2x^4(48-43c^2x^2)}{\sqrt{1-c^2x^2}}dx}{8c^2}-\frac{3}{8}c^2x^7\sqrt{1-c^2x^2}\right)+\frac{1}{8}c^4d^2x^8(a+b\arccos(cx))-\frac{1}{3}c^2d^2x^6(a+b\arccos(cx))+\frac{1}{4}d^2x^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}bcd^2\left(\frac{1}{8}\int \frac{x^4(48-43c^2x^2)}{\sqrt{1-c^2x^2}}dx-\frac{3}{8}c^2x^7\sqrt{1-c^2x^2}\right)+\frac{1}{8}c^4d^2x^8(a+b\arccos(cx))-\frac{1}{3}c^2d^2x^6(a+b\arccos(cx))+\frac{1}{4}d^2x^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{363}$$

$$\frac{1}{24}bcd^2\left(\frac{1}{8}\left(\frac{73}{6}\int \frac{x^4}{\sqrt{1-c^2x^2}}dx+\frac{43}{6}x^5\sqrt{1-c^2x^2}\right)-\frac{3}{8}c^2x^7\sqrt{1-c^2x^2}\right)+\frac{1}{8}c^4d^2x^8(a+b\arccos(cx))-\frac{1}{3}c^2d^2x^6(a+b\arccos(cx))+\frac{1}{4}d^2x^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{24}bcd^2\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3\int \frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{43}{6}x^5\sqrt{1-c^2x^2}\right)-\frac{3}{8}c^2x^7\sqrt{1-c^2x^2}\right)+\frac{1}{8}c^4d^2x^8(a+b\arccos(cx))-\frac{1}{3}c^2d^2x^6(a+b\arccos(cx))+\frac{1}{4}d^2x^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{24}bcd^2\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{43}{6}x^5\sqrt{1-c^2x^2}\right)-\frac{3}{8}c^2x^7\sqrt{1-c^2x^2}\right)+\frac{1}{8}c^4d^2x^8(a+b\arccos(cx))-\frac{1}{3}c^2d^2x^6(a+b\arccos(cx))+\frac{1}{4}d^2x^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{8}c^4d^2x^8(a+b\arccos(cx))-\frac{1}{3}c^2d^2x^6(a+b\arccos(cx))+\frac{1}{4}d^2x^4(a+b\arccos(cx))+\frac{1}{24}bcd^2\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{43}{6}x^5\sqrt{1-c^2x^2}\right)-\frac{3}{8}c^2x^7\sqrt{1-c^2x^2}\right)$$

input
```
Int[x^3*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*x^4*(a + b*ArcCos[c*x]))/4 - (c^2*d^2*x^6*(a + b*ArcCos[c*x]))/3 + (c
^4*d^2*x^8*(a + b*ArcCos[c*x]))/8 + (b*c*d^2*((-3*c^2*x^7*Sqrt[1 - c^2*x^2
])/8 + ((43*x^5*Sqrt[1 - c^2*x^2])/6 + (73*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c
^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/
6)/8))/24
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.85

method	result
parts	$d^2 a\left(\frac{1}{8}c^4x^8-\frac{1}{3}c^2x^6+\frac{1}{4}x^4\right)+\frac{d^2 b\left(\frac{\arccos(cx)c^8x^8}{8}-\frac{\arccos(cx)c^6x^6}{3}+\frac{c^4x^4\arccos(cx)}{4}-\frac{73c^3x^3\sqrt{-c^2x^2+1}}{4608}-\ldots\right.}{c^4}$
derivativedivides	$\frac{d^2 a\left(\frac{1}{8}c^8x^8-\frac{1}{3}c^6x^6+\frac{1}{4}c^4x^4\right)+d^2 b\left(\frac{\arccos(cx)c^8x^8}{8}-\frac{\arccos(cx)c^6x^6}{3}+\frac{c^4x^4\arccos(cx)}{4}-\frac{73c^3x^3\sqrt{-c^2x^2+1}}{4608}-\frac{73cx\sqrt{-c^2x^2+\ldots}}{3072}\right.}{c^4}$
default	$\frac{d^2 a\left(\frac{1}{8}c^8x^8-\frac{1}{3}c^6x^6+\frac{1}{4}c^4x^4\right)+d^2 b\left(\frac{\arccos(cx)c^8x^8}{8}-\frac{\arccos(cx)c^6x^6}{3}+\frac{c^4x^4\arccos(cx)}{4}-\frac{73c^3x^3\sqrt{-c^2x^2+1}}{4608}-\frac{73cx\sqrt{-c^2x^2+\ldots}}{3072}\right.}{c^4}$
orering	$\frac{(2160c^8x^8-5912c^6x^6+4358c^4x^4+1095c^2x^2-876)(-c^2dx^2+d)^2(a+b\arccos(cx))}{9216c^4(cx-1)(cx+1)(c^2x^2-1)}-\frac{(144c^6x^6-344c^4x^4+146c^2x^2+\ldots}{\ldots}$

input
```
int(x^3*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/8*c^4*x^8-1/3*c^2*x^6+1/4*x^4)+d^2*b/c^4*(1/8*arccos(c*x)*c^8*x^8
-1/3*arccos(c*x)*c^6*x^6+1/4*c^4*x^4*arccos(c*x)-73/4608*c^3*x^3*(-c^2*x^2
+1)^(1/2)-73/3072*c*x*(-c^2*x^2+1)^(1/2)+73/3072*arcsin(c*x)+43/1152*c^5*x
^5*(-c^2*x^2+1)^(1/2)-1/64*c^7*x^7*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.82

$$\int x^3 \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx$$
$$=\frac{1152\,ac^8d^2x^8-3072\,ac^6d^2x^6+2304\,ac^4d^2x^4+3\left(384\,bc^8d^2x^8-1024\,bc^6d^2x^6+768\,bc^4d^2x^4-73\,bd^2\right)\text{arc}}{9216\,c^4}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/9216*(1152*a*c^8*d^2*x^8 - 3072*a*c^6*d^2*x^6 + 2304*a*c^4*d^2*x^4 + 3*(
384*b*c^8*d^2*x^8 - 1024*b*c^6*d^2*x^6 + 768*b*c^4*d^2*x^4 - 73*b*d^2)*arc
cos(c*x) - (144*b*c^7*d^2*x^7 - 344*b*c^5*d^2*x^5 + 146*b*c^3*d^2*x^3 + 21
9*b*c*d^2*x)*sqrt(-c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.21

$$\int x^3 \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx$$
$$=\begin{cases}\frac{ac^4d^2x^8}{8}-\frac{ac^2d^2x^6}{3}+\frac{ad^2x^4}{4}+\frac{bc^4d^2x^8\operatorname{acos}(cx)}{8}-\frac{bc^3d^2x^7\sqrt{-c^2x^2+1}}{64}-\frac{bc^2d^2x^6\operatorname{acos}(cx)}{3}+\frac{43bcd^2x^5\sqrt{-c^2x^2+1}}{1152}+\frac{bd^2x^4\operatorname{aco}}{4}\\ \frac{d^2x^4\left(a+\frac{\pi b}{2}\right)}{4}\end{cases}$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**2*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*c**4*d**2*x**8/8 - a*c**2*d**2*x**6/3 + a*d**2*x**4/4 + b*c**
4*d**2*x**8*acos(c*x)/8 - b*c**3*d**2*x**7*sqrt(-c**2*x**2 + 1)/64 - b*c**
2*d**2*x**6*acos(c*x)/3 + 43*b*c*d**2*x**5*sqrt(-c**2*x**2 + 1)/1152 + b*d
**2*x**4*acos(c*x)/4 - 73*b*d**2*x**3*sqrt(-c**2*x**2 + 1)/(4608*c) - 73*b
*d**2*x*sqrt(-c**2*x**2 + 1)/(3072*c**3) - 73*b*d**2*acos(c*x)/(3072*c**4)
, Ne(c, 0)), (d**2*x**4*(a + pi*b/2)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.64

$$\int x^3 \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx = \frac{1}{8}\,ac^4d^2x^8 - \frac{1}{3}\,ac^2d^2x^6 + \frac{1}{3072}\left(384\,x^8\arccos\left(cx\right) - \left(\frac{48\,\sqrt{-c^2x^2+1}x^7}{c^2} + \frac{56\,\sqrt{-c^2x^2+1}x^5}{c^4} + \frac{70\,\sqrt{-c^2x^2+1}x^3}{c^6} + \frac{105\,\sqrt{-c^2}}{c^8}\right.\right.$$
$$+ \frac{1}{4}\,ad^2x^4 - \frac{1}{144}\left(48\,x^6\arccos\left(cx\right) - \left(\frac{8\,\sqrt{-c^2x^2+1}x^5}{c^2} + \frac{10\,\sqrt{-c^2x^2+1}x^3}{c^4} + \frac{15\,\sqrt{-c^2x^2+1}x}{c^6} - \frac{15\,\arcsin\left(cx\right)}{c^7}\right.\right.$$
$$+ \frac{1}{32}\left(8\,x^4\arccos\left(cx\right) - \left(\frac{2\,\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\,\sqrt{-c^2x^2+1}x}{c^4} - \frac{3\,\arcsin\left(cx\right)}{c^5}\right)c\right)bd^2$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*c^4*d^2*x^8 - 1/3*a*c^2*d^2*x^6 + 1/3072*(384*x^8*arccos(c*x) - (48*
sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(-c^2*
x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9)*c)*
b*c^4*d^2 + 1/4*a*d^2*x^4 - 1/144*(48*x^6*arccos(c*x) - (8*sqrt(-c^2*x^2 +
 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/c^6
- 15*arcsin(c*x)/c^7)*c)*b*c^2*d^2 + 1/32*(8*x^4*arccos(c*x) - (2*sqrt(-c^
2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*
d^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00

$$\int x^3 (d - c^2 d x^2)^2 (a + b \arccos(cx)) \, dx$$
$$= \frac{1}{8} b c^4 d^2 x^8 \arccos(cx) + \frac{1}{8} a c^4 d^2 x^8 - \frac{1}{64} \sqrt{-c^2 x^2 + 1} b c^3 d^2 x^7 - \frac{1}{3} b c^2 d^2 x^6 \arccos(cx) - \frac{1}{3} a c^2 d^2 x^6 + \frac{43}{1152} \sqrt{-c^2 x^2 + 1} b c d^2 x^5 + \frac{1}{4} b d^2 x^4 \arccos(cx) + \frac{1}{4} a d^2 x^4 - \frac{73 \sqrt{-c^2 x^2 + 1} b d^2 x^3}{4608 c} - \frac{73 \sqrt{-c^2 x^2 + 1} b d^2 x}{3072 c^3} - \frac{73 b d^2 \arccos(cx)}{3072 c^4}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/8*b*c^4*d^2*x^8*arccos(c*x) + 1/8*a*c^4*d^2*x^8 - 1/64*sqrt(-c^2*x^2 + 1
)*b*c^3*d^2*x^7 - 1/3*b*c^2*d^2*x^6*arccos(c*x) - 1/3*a*c^2*d^2*x^6 + 43/1
152*sqrt(-c^2*x^2 + 1)*b*c*d^2*x^5 + 1/4*b*d^2*x^4*arccos(c*x) + 1/4*a*d^2
*x^4 - 73/4608*sqrt(-c^2*x^2 + 1)*b*d^2*x^3/c - 73/3072*sqrt(-c^2*x^2 + 1)
*b*d^2*x/c^3 - 73/3072*b*d^2*arccos(c*x)/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d - c^2 d x^2)^2 (a + b \arccos(cx)) \, dx = \int x^3 (a + b \operatorname{acos}(c x)) (d - c^2 d x^2)^2 \, dx$$

input
```
int(x^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^2,x)
```

output
```
int(x^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.86

$$\int x^3 \left(d-c^2 d x^2\right)^2 (a+b \arccos (c x)) \, dx$$
$$= \frac{d^2 \left(1152 acos(cx)\, b\, c^8 x^8 - 3072 acos(cx)\, b\, c^6 x^6 + 2304 acos(cx)\, b\, c^4 x^4 + 219 asin(cx)\, b - 144 \sqrt{-c^2 x^2 + 1}\, b\right.}{}$$

input
```
int(x^3*(-c^2*d*x^2+d)^2*(a+b*acos(c*x)),x)
```

output
```
(d**2*(1152*acos(c*x)*b*c**8*x***8 - 3072*acos(c*x)*b*c**6*x***6 + 2304*acos
(c*x)*b*c**4*x***4 + 219*asin(c*x)*b - 144*sqrt( - c**2*x***2 + 1)*b*c**7*x*
*7 + 344*sqrt( - c**2*x***2 + 1)*b*c**5*x***5 - 146*sqrt( - c**2*x***2 + 1)*b
*c**3*x***3 - 219*sqrt( - c**2*x***2 + 1)*b*c*x + 1152*a*c**8*x***8 - 3072*a*
c**6*x***6 + 2304*a*c**4*x***4))/(9216*c**4)
```

3.14 $\int x^2(d-c^2dx^2)^2(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int x^2(d-c^2dx^2)^2(a+b\arccos(cx))\,dx = \frac{8bd^2\sqrt{1-c^2x^2}}{105c^3}+\frac{4bd^2(1-c^2x^2)^{3/2}}{315c^3}+\frac{bd^2(1-c^2x^2)^{5/2}}{175c^3}-\frac{bd^2(1-c^2x^2)^{7/2}}{49c^3}+\frac{1}{3}d^2x^3(a+b\arccos(cx))-\frac{2}{5}c^2d^2x^5(a+b\arccos(cx))+\frac{1}{7}c^4d^2x^7(a+b\arccos(cx))$$

output
```
8/105*b*d^2*(-c^2*x^2+1)^(1/2)/c^3+4/315*b*d^2*(-c^2*x^2+1)^(3/2)/c^3+1/17
5*b*d^2*(-c^2*x^2+1)^(5/2)/c^3-1/49*b*d^2*(-c^2*x^2+1)^(7/2)/c^3+1/3*d^2*x
^3*(a+b*arccos(c*x))-2/5*c^2*d^2*x^5*(a+b*arccos(c*x))+1/7*c^4*d^2*x^7*(a+
b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\int x^2(d-c^2dx^2)^2(a+b\arccos(cx))\,dx = \frac{d^2\left(105ac^3x^3(35-42c^2x^2+15c^4x^4)-b\sqrt{1-c^2x^2}(818+409c^2x^2-612c^4x^4+225c^6x^6)+105bc^3x^3(35-\right.}{11025c^3}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*(105*a*c^3*x^3*(35 - 42*c^2*x^2 + 15*c^4*x^4) - b*Sqrt[1 - c^2*x^2]*(
818 + 409*c^2*x^2 - 612*c^4*x^4 + 225*c^6*x^6) + 105*b*c^3*x^3*(35 - 42*c^
2*x^2 + 15*c^4*x^4)*ArcCos[c*x]))/(11025*c^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5193, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d-c^2 d x^2\right)^2 (a+b \arccos (c x)) \, dx$$

$$\Big\downarrow \text{5193}$$

$$bc \int \frac{d^2 x^3 \left(15 c^4 x^4-42 c^2 x^2+35\right)}{105 \sqrt{1-c^2 x^2}}dx+\frac{1}{7} c^4 d^2 x^7 (a+b \arccos (c x))-\frac{2}{5} c^2 d^2 x^5 (a+b \arccos (c x))+\frac{1}{3} d^2 x^3 (a+b \arccos (c x))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{105} b c d^2 \int \frac{x^3 \left(15 c^4 x^4-42 c^2 x^2+35\right)}{\sqrt{1-c^2 x^2}}dx+\frac{1}{7} c^4 d^2 x^7 (a+b \arccos (c x))-\frac{2}{5} c^2 d^2 x^5 (a+b \arccos (c x))+\frac{1}{3} d^2 x^3 (a+b \arccos (c x))$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{210} b c d^2 \int \frac{x^2 \left(15 c^4 x^4-42 c^2 x^2+35\right)}{\sqrt{1-c^2 x^2}}dx^2+\frac{1}{7} c^4 d^2 x^7 (a+b \arccos (c x))-\frac{2}{5} c^2 d^2 x^5 (a+b \arccos (c x))+\frac{1}{3} d^2 x^3 (a+b \arccos (c x))$$

$$\Big\downarrow \text{1195}$$

$$\frac{1}{210}bcd^2\int\left(-\frac{15(1-c^2x^2)^{5/2}}{c^2}+\frac{3(1-c^2x^2)^{3/2}}{c^2}+\frac{4\sqrt{1-c^2x^2}}{c^2}+\frac{8}{c^2\sqrt{1-c^2x^2}}\right)dx^2+\frac{1}{7}c^4d^2x^7(a+b\arccos(cx))-\frac{2}{5}c^2d^2x^5(a+b\arccos(cx))+\frac{1}{3}d^2x^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}c^4d^2x^7(a+b\arccos(cx))-\frac{2}{5}c^2d^2x^5(a+b\arccos(cx))+\frac{1}{3}d^2x^3(a+b\arccos(cx))+\frac{1}{210}bcd^2\left(\frac{30(1-c^2x^2)^{7/2}}{7c^4}-\frac{6(1-c^2x^2)^{5/2}}{5c^4}-\frac{8(1-c^2x^2)^{3/2}}{3c^4}-\frac{16\sqrt{1-c^2x^2}}{c^4}\right)$$

input
```
Int[x^2*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d^2*((-16*Sqrt[1 - c^2*x^2])/c^4 - (8*(1 - c^2*x^2)^(3/2))/(3*c^4) -
(6*(1 - c^2*x^2)^(5/2))/(5*c^4) + (30*(1 - c^2*x^2)^(7/2))/(7*c^4)))/210 +
 (d^2*x^3*(a + b*ArcCos[c*x]))/3 - (2*c^2*d^2*x^5*(a + b*ArcCos[c*x]))/5 +
 (c^4*d^2*x^7*(a + b*ArcCos[c*x]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

method	result
parts	$d^2a\left(\frac{1}{7}c^4x^7-\frac{2}{5}c^2x^5+\frac{1}{3}x^3\right)+\frac{d^2b\left(\frac{\arccos(cx)c^7x^7}{7}-\frac{2\arccos(cx)c^5x^5}{5}+\frac{c^3x^3\arccos(cx)}{3}-\frac{409c^2x^2\sqrt{-c^2x^2+1}}{11025}-\frac{8}{}\right.}{c^3}$
derivativedivides	$\frac{d^2a\left(\frac{1}{7}c^7x^7-\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b\left(\frac{\arccos(cx)c^7x^7}{7}-\frac{2\arccos(cx)c^5x^5}{5}+\frac{c^3x^3\arccos(cx)}{3}-\frac{409c^2x^2\sqrt{-c^2x^2+1}}{11025}-\frac{818\sqrt{-c^2x^2}}{11025}\right.}{c^3}$
default	$\frac{d^2a\left(\frac{1}{7}c^7x^7-\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b\left(\frac{\arccos(cx)c^7x^7}{7}-\frac{2\arccos(cx)c^5x^5}{5}+\frac{c^3x^3\arccos(cx)}{3}-\frac{409c^2x^2\sqrt{-c^2x^2+1}}{11025}-\frac{818\sqrt{-c^2x^2}}{11025}\right.}{c^3}$
orering	$\frac{(2925c^8x^8-8532c^6x^6+7353c^4x^4+4090c^2x^2-1636)\left(-c^2d\,x^2+d\right)^2(a+b\arccos(cx))}{11025c^4x(cx-1)(cx+1)(c^2x^2-1)}-\frac{(225c^6x^6-612c^4x^4+409c^2x^2}{}$

input
```
int(x^2*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/7*c^4*x^7-2/5*c^2*x^5+1/3*x^3)+d^2*b/c^3*(1/7*arccos(c*x)*c^7*x^7
-2/5*arccos(c*x)*c^5*x^5+1/3*c^3*x^3*arccos(c*x)-409/11025*c^2*x^2*(-c^2*x
^2+1)^(1/2)-818/11025*(-c^2*x^2+1)^(1/2)+68/1225*c^4*x^4*(-c^2*x^2+1)^(1/2
)-1/49*c^6*x^6*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.88

$$\int x^2\left(d-c^2dx^2\right)^2(a+b\arccos(cx))\,dx$$
$$=\frac{1575\,ac^7d^2x^7-4410\,ac^5d^2x^5+3675\,ac^3d^2x^3+105\left(15\,bc^7d^2x^7-42\,bc^5d^2x^5+35\,bc^3d^2x^3\right)\arccos(cx)-}{11025\,c^3}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/11025*(1575*a*c^7*d^2*x^7 - 4410*a*c^5*d^2*x^5 + 3675*a*c^3*d^2*x^3 + 10
5*(15*b*c^7*d^2*x^7 - 42*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3)*arccos(c*x) - (
225*b*c^6*d^2*x^6 - 612*b*c^4*d^2*x^4 + 409*b*c^2*d^2*x^2 + 818*b*d^2)*sqr
t(-c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.29

$$\int x^2 \left(d-c^2 d x^2\right)^2 (a+b \arccos (c x)) \, dx$$

$$= \begin{cases} \frac{ac^4d^2x^7}{7} - \frac{2ac^2d^2x^5}{5} + \frac{ad^2x^3}{3} + \frac{bc^4d^2x^7 \operatorname{acos}(cx)}{7} - \frac{bc^3d^2x^6\sqrt{-c^2x^2+1}}{49} - \frac{2bc^2d^2x^5 \operatorname{acos}(cx)}{5} + \frac{68bcd^2x^4\sqrt{-c^2x^2+1}}{1225} + \frac{bd^2x^3 \operatorname{a}}{} \\ \frac{d^2x^3\left(a+\frac{\pi b}{2}\right)}{3} \end{cases}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**2*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*c**4*d**2*x**7/7 - 2*a*c**2*d**2*x**5/5 + a*d**2*x**3/3 + b*c
**4*d**2*x**7*acos(c*x)/7 - b*c**3*d**2*x**6*sqrt(-c**2*x**2 + 1)/49 - 2*b
*c**2*d**2*x**5*acos(c*x)/5 + 68*b*c*d**2*x**4*sqrt(-c**2*x**2 + 1)/1225 +
 b*d**2*x**3*acos(c*x)/3 - 409*b*d**2*x**2*sqrt(-c**2*x**2 + 1)/(11025*c)
- 818*b*d**2*sqrt(-c**2*x**2 + 1)/(11025*c**3), Ne(c, 0)), (d**2*x**3*(a +
 pi*b/2)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.68

$$\int x^2(d-c^2dx^2)^2(a+b\arccos(cx))\,dx = \frac{1}{7}ac^4d^2x^7 - \frac{2}{5}ac^2d^2x^5 + \frac{1}{245}\left(35\,x^7\arccos(cx) - \left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\sqrt{-c^2x^2+1}}{c^8}\right.\right.$$
$$-\frac{2}{75}\left(15\,x^5\arccos(cx) - \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bc^2d^2$$
$$+\frac{1}{3}ad^2x^3 + \frac{1}{9}\left(3\,x^3\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bd^2$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*c^4*d^2*x^7 - 2/5*a*c^2*d^2*x^5 + 1/245*(35*x^7*arccos(c*x) - (5*sqr
t(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 +
 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*c^4*d^2 - 2/75*(15*x^5*arcco
s(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*
sqrt(-c^2*x^2 + 1)/c^6)*c)*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arccos(c
*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int x^2(d-c^2dx^2)^2(a+b\arccos(cx))\,dx = \frac{1}{7}\,bc^4d^2x^7\arccos(cx)+\frac{1}{7}\,ac^4d^2x^7 - \frac{1}{49}\,\sqrt{-c^2x^2+1}bc^3d^2x^6 - \frac{2}{5}\,bc^2d^2x^5\arccos(cx)-\frac{2}{5}\,ac^2d^2x^5 + \frac{68}{1225}\,\sqrt{-c^2x^2+1}bcd^2x^4 + \frac{1}{3}\,bd^2x^3\arccos(cx)+\frac{1}{3}\,ad^2x^3 - \frac{409\,\sqrt{-c^2x^2+1}bd^2x^2}{11025\,c} - \frac{818\,\sqrt{-c^2x^2+1}bd^2}{11025\,c^3}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/7*b*c^4*d^2*x^7*arccos(c*x) + 1/7*a*c^4*d^2*x^7 - 1/49*sqrt(-c^2*x^2 + 1
)*b*c^3*d^2*x^6 - 2/5*b*c^2*d^2*x^5*arccos(c*x) - 2/5*a*c^2*d^2*x^5 + 68/1
225*sqrt(-c^2*x^2 + 1)*b*c*d^2*x^4 + 1/3*b*d^2*x^3*arccos(c*x) + 1/3*a*d^2
*x^3 - 409/11025*sqrt(-c^2*x^2 + 1)*b*d^2*x^2/c - 818/11025*sqrt(-c^2*x^2
+ 1)*b*d^2/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d-c^2dx^2)^2(a+b\arccos(cx))\,dx = \int x^2\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2\,dx$$

input
```
int(x^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^2,x)
```

output
```
int(x^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93

$$\int x^2(d-c^2dx^2)^2(a+b\arccos(cx))\,dx$$
$$=\frac{d^2\left(1575acos(cx)\,b\,c^7x^7-4410acos(cx)\,b\,c^5x^5+3675acos(cx)\,b\,c^3x^3-225\sqrt{-c^2x^2+1}\,b\,c^6x^6+612\sqrt{-c}\right.}{11025}$$

input
```
int(x^2*(-c^2*d*x^2+d)^2*(a+b*acos(c*x)),x)
```

output
```
(d**2*(1575*acos(c*x)*b*c**7*x**7 - 4410*acos(c*x)*b*c**5*x**5 + 3675*acos
(c*x)*b*c**3*x**3 - 225*sqrt( - c**2*x**2 + 1)*b*c**6*x**6 + 612*sqrt( - c
**2*x**2 + 1)*b*c**4*x**4 - 409*sqrt( - c**2*x**2 + 1)*b*c**2*x**2 - 818*s
qrt( - c**2*x**2 + 1)*b + 1575*a*c**7*x**7 - 4410*a*c**5*x**5 + 3675*a*c**
3*x**3))/(11025*c**3)
```

3.15 $\int x(d-c^2dx^2)^2 (a+b\arccos(cx)) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int x(d-c^2dx^2)^2 (a+b\arccos(cx)) \, dx = \frac{5bd^2x\sqrt{1-c^2x^2}}{96c} + \frac{5bd^2x(1-c^2x^2)^{3/2}}{144c} + \frac{bd^2x(1-c^2x^2)^{5/2}}{36c} + \frac{5bd^2\arccos(cx)}{96c^2} - \frac{d^2(1-c^2x^2)^3 (a+b\arccos(cx))}{6c^2}$$

output
```
5/96*b*d^2*x*(-c^2*x^2+1)^(1/2)/c+5/144*b*d^2*x*(-c^2*x^2+1)^(3/2)/c+1/36*
b*d^2*x*(-c^2*x^2+1)^(5/2)/c+5/96*b*d^2*arccos(c*x)/c^2-1/6*d^2*(-c^2*x^2+
1)^3*(a+b*arccos(c*x))/c^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int x(d-c^2dx^2)^2 (a+b\arccos(cx)) \, dx = \frac{d^2\left(cx\left(b\sqrt{1-c^2x^2}(-33+26c^2x^2-8c^4x^4)+48acx(3-3c^2x^2+c^4x^4)\right)+48bc^2x^2(3-3c^2x^2+c^4x^4)\,\mathrm{arc}}{288c^2}$$

input
```
Integrate[x*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*(c*x*(b*Sqrt[1 - c^2*x^2]*(-33 + 26*c^2*x^2 - 8*c^4*x^4) + 48*a*c*x*(
3 - 3*c^2*x^2 + c^4*x^4)) + 48*b*c^2*x^2*(3 - 3*c^2*x^2 + c^4*x^4)*ArcCos[
c*x] + 33*b*ArcSin[c*x]))/(288*c^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5183, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d-c^2dx^2\right)^2(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5183}$$

$$-\frac{bd^2\int \left(1-c^2x^2\right)^{5/2}dx}{6c}-\frac{d^2\left(1-c^2x^2\right)^3(a+b\arccos(cx))}{6c^2}$$

$$\Big\downarrow \text{211}$$

$$-\frac{bd^2\left(\frac{5}{6}\int \left(1-c^2x^2\right)^{3/2}dx+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)}{6c}-\frac{d^2\left(1-c^2x^2\right)^3(a+b\arccos(cx))}{6c^2}$$

$$\Big\downarrow \text{211}$$

$$-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-c^2x^2}dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)}{6c}-\frac{d^2\left(1-c^2x^2\right)^3(a+b\arccos(cx))}{6c^2}$$

$$\Big\downarrow \text{211}$$

$$-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)}{6c}-\frac{d^2\left(1-c^2x^2\right)^3(a+b\arccos(cx))}{6c^2}$$

$$\Big\downarrow \; 223$$

$$-\frac{d^2(1-c^2x^2)^3(a+b\arccos(cx))}{6c^2}-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)}{6c}$$

input
```
Int[x*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
-1/6*(d^2*(1 - c^2*x^2)^3*(a + b*ArcCos[c*x]))/c^2 - (b*d^2*((x*(1 - c^2*x
^2)^(5/2))/6 + (5*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2
 + ArcSin[c*x]/(2*c)))/4))/6))/(6*c)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{\frac{d^2a(c^2x^2-1)^3}{6}+d^2b\left(\frac{\arccos(cx)c^6x^6}{6}-\frac{c^4x^4\arccos(cx)}{2}+\frac{c^2x^2\arccos(cx)}{2}-\frac{\arccos(cx)}{6}-\frac{c^5x^5\sqrt{-c^2x^2+1}}{36}+\frac{13c^3x^3\sqrt{-c^2x^2+1}}{144}\right.}{c^2}$
default	$\frac{\frac{d^2a(c^2x^2-1)^3}{6}+d^2b\left(\frac{\arccos(cx)c^6x^6}{6}-\frac{c^4x^4\arccos(cx)}{2}+\frac{c^2x^2\arccos(cx)}{2}-\frac{\arccos(cx)}{6}-\frac{c^5x^5\sqrt{-c^2x^2+1}}{36}+\frac{13c^3x^3\sqrt{-c^2x^2+1}}{144}\right.}{c^2}$
parts	$\frac{d^2a(c^2x^2-1)^3}{6c^2}+\frac{d^2b\left(\frac{\arccos(cx)c^6x^6}{6}-\frac{c^4x^4\arccos(cx)}{2}+\frac{c^2x^2\arccos(cx)}{2}-\frac{\arccos(cx)}{6}-\frac{c^5x^5\sqrt{-c^2x^2+1}}{36}+\frac{13c^3x^3\sqrt{-c^2x^2}}{144}\right.}{c^2}$
orering	$\frac{(88c^6x^6-282c^4x^4+335c^2x^2-66)(-c^2dx^2+d)^2(a+b\arccos(cx))}{288c^2(cx-1)(cx+1)(c^2x^2-1)}-\frac{(8c^4x^4-26c^2x^2+33)\left((-c^2dx^2+d)^2(a+b\arccos\right.}{2}$

input
```
int(x*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/6*d^2*a*(c^2*x^2-1)^3+d^2*b*(1/6*arccos(c*x)*c^6*x^6-1/2*c^4*x^4*
arccos(c*x)+1/2*c^2*x^2*arccos(c*x)-1/6*arccos(c*x)-1/36*c^5*x^5*(-c^2*x^2
+1)^(1/2)+13/144*c^3*x^3*(-c^2*x^2+1)^(1/2)-11/96*c*x*(-c^2*x^2+1)^(1/2)-5
/96*arcsin(c*x)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.11

$$\int x(d-c^2dx^2)^2(a+b\arccos(cx))\,dx$$
$$=\frac{48\,ac^6d^2x^6-144\,ac^4d^2x^4+144\,ac^2d^2x^2+3\,(16\,bc^6d^2x^6-48\,bc^4d^2x^4+48\,bc^2d^2x^2-11\,bd^2)\arccos{(cx)}+}{288\,c^2}$$

input
```
integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/288*(48*a*c^6*d^2*x^6 - 144*a*c^4*d^2*x^4 + 144*a*c^2*d^2*x^2 + 3*(16*b*
c^6*d^2*x^6 - 48*b*c^4*d^2*x^4 + 48*b*c^2*d^2*x^2 - 11*b*d^2)*arccos(c*x)
- (8*b*c^5*d^2*x^5 - 26*b*c^3*d^2*x^3 + 33*b*c*d^2*x)*sqrt(-c^2*x^2 + 1))/
c^2
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.58

$$\int x\left(d-c^2dx^2\right)^2\left(a+b\arccos(cx)\right)dx$$
$$=\begin{cases}\frac{ac^4d^2x^6}{6}-\frac{ac^2d^2x^4}{2}+\frac{ad^2x^2}{2}+\frac{bc^4d^2x^6\operatorname{acos}(cx)}{6}-\frac{bc^3d^2x^5\sqrt{-c^2x^2+1}}{36}-\frac{bc^2d^2x^4\operatorname{acos}(cx)}{2}+\frac{13bcd^2x^3\sqrt{-c^2x^2+1}}{144}+\frac{bd^2x^2\operatorname{aco}}{2} \\ \frac{d^2x^2\left(a+\frac{\pi b}{2}\right)}{2}\end{cases}$$

input
```
integrate(x*(-c**2*d*x**2+d)**2*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*c**4*d**2*x**6/6 - a*c**2*d**2*x**4/2 + a*d**2*x**2/2 + b*c**
4*d**2*x**6*acos(c*x)/6 - b*c**3*d**2*x**5*sqrt(-c**2*x**2 + 1)/36 - b*c**
2*d**2*x**4*acos(c*x)/2 + 13*b*c*d**2*x**3*sqrt(-c**2*x**2 + 1)/144 + b*d*
*2*x**2*acos(c*x)/2 - 11*b*d**2*x*sqrt(-c**2*x**2 + 1)/(96*c) - 11*b*d**2*
acos(c*x)/(96*c**2), Ne(c, 0)), (d**2*x**2*(a + pi*b/2)/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(107) = 214$.

Time = 0.12 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.94

$$\int x\left(d-c^2dx^2\right)^2\left(a+b\arccos(cx)\right)dx=\frac{1}{6}ac^4d^2x^6-\frac{1}{2}ac^2d^2x^4$$
$$+\frac{1}{288}\left(48x^6\arccos(cx)-\left(\frac{8\sqrt{-c^2x^2+1}x^5}{c^2}+\frac{10\sqrt{-c^2x^2+1}x^3}{c^4}+\frac{15\sqrt{-c^2x^2+1}x}{c^6}-\frac{15\arcsin(cx)}{c^7}\right.\right.$$
$$-\frac{1}{16}\left(8x^4\arccos(cx)-\left(\frac{2\sqrt{-c^2x^2+1}x^3}{c^2}+\frac{3\sqrt{-c^2x^2+1}x}{c^4}-\frac{3\arcsin(cx)}{c^5}\right)c\right)bc^2d^2$$
$$+\frac{1}{2}ad^2x^2+\frac{1}{4}\left(2x^2\arccos(cx)-c\left(\frac{\sqrt{-c^2x^2+1}x}{c^2}-\frac{\arcsin(cx)}{c^3}\right)\right)bd^2$$

input
```
integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*c^4*d^2*x^6 - 1/2*a*c^2*d^2*x^4 + 1/288*(48*x^6*arccos(c*x) - (8*sqr
t(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2
 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*b*c^4*d^2 - 1/16*(8*x^4*arccos(c*x) -
 (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x
)/c^5)*c)*b*c^2*d^2 + 1/2*a*d^2*x^2 + 1/4*(2*x^2*arccos(c*x) - c*(sqrt(-c^
2*x^2 + 1)*x/c^2 - arcsin(c*x)/c^3))*b*d^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.29

$$\int x\left(d-c^2dx^2\right)^2(a+b\arccos(cx))\,dx = \frac{1}{6}\,bc^4d^2x^6\arccos\left(cx\right)+\frac{1}{6}\,ac^4d^2x^6 - \frac{1}{36}\,\sqrt{-c^2x^2+1}bc^3d^2x^5 - \frac{1}{2}\,bc^2d^2x^4\arccos\left(cx\right)-\frac{1}{2}\,ac^2d^2x^4 + \frac{13}{144}\,\sqrt{-c^2x^2+1}bcd^2x^3 + \frac{1}{2}\,bd^2x^2\arccos\left(cx\right)+\frac{1}{2}\,ad^2x^2 - \frac{11\,\sqrt{-c^2x^2+1}bd^2x}{96\,c} - \frac{11\,bd^2\arccos\left(cx\right)}{96\,c^2}$$

input
```
integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/6*b*c^4*d^2*x^6*arccos(c*x) + 1/6*a*c^4*d^2*x^6 - 1/36*sqrt(-c^2*x^2 + 1
)*b*c^3*d^2*x^5 - 1/2*b*c^2*d^2*x^4*arccos(c*x) - 1/2*a*c^2*d^2*x^4 + 13/1
44*sqrt(-c^2*x^2 + 1)*b*c*d^2*x^3 + 1/2*b*d^2*x^2*arccos(c*x) + 1/2*a*d^2*
x^2 - 11/96*sqrt(-c^2*x^2 + 1)*b*d^2*x/c - 11/96*b*d^2*arccos(c*x)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x\left(d-c^2dx^2\right)^2(a+b\arccos(cx))\,dx=\int x\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2\,dx$$

input
```
int(x*(a + b*acos(c*x))*(d - c^2*d*x^2)^2,x)
```

output
```
int(x*(a + b*acos(c*x))*(d - c^2*d*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.11

$$\int x\left(d-c^2dx^2\right)^2(a+b\arccos(cx))\,dx$$
$$=\frac{d^2\left(48acos(cx)\,b\,c^6x^6-144acos(cx)\,b\,c^4x^4+144acos(cx)\,b\,c^2x^2+33asin(cx)\,b-8\sqrt{-c^2x^2+1}\,b\,c^5x^5+\right.}{288c^2}$$

input
```
int(x*(-c^2*d*x^2+d)^2*(a+b*acos(c*x)),x)
```

output
```
(d**2*(48*acos(c*x)*b*c**6*x**6 - 144*acos(c*x)*b*c**4*x**4 + 144*acos(c*x
)*b*c**2*x**2 + 33*asin(c*x)*b - 8*sqrt( - c**2*x**2 + 1)*b*c**5*x**5 + 26
*sqrt( - c**2*x**2 + 1)*b*c**3*x**3 - 33*sqrt( - c**2*x**2 + 1)*b*c*x + 48
*a*c**6*x**6 - 144*a*c**4*x**4 + 144*a*c**2*x**2))/(288*c**2)
```

3.16 $\int \left(d-c^2 d x^2\right)^2 (a+b \arccos (c x)) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 131

$$\begin{aligned}\int \left(d-c^2 d x^2\right)^2 (a+b \arccos (c x)) \, dx = {} & \frac{8 b d^2 \sqrt{1-c^2 x^2}}{15 c}+\frac{4 b d^2 \left(1-c^2 x^2\right)^{3/2}}{45 c} \\ & +\frac{b d^2 \left(1-c^2 x^2\right)^{5/2}}{25 c}+d^2 x (a+b \arccos (c x)) \\ & -\frac{2}{3} c^2 d^2 x^3 (a+b \arccos (c x)) \\ & +\frac{1}{5} c^4 d^2 x^5 (a+b \arccos (c x))\end{aligned}$$

output
```
8/15*b*d^2*(-c^2*x^2+1)^(1/2)/c+4/45*b*d^2*(-c^2*x^2+1)^(3/2)/c+1/25*b*d^2
*(-c^2*x^2+1)^(5/2)/c+d^2*x*(a+b*arccos(c*x))-2/3*c^2*d^2*x^3*(a+b*arccos(
c*x))+1/5*c^4*d^2*x^5*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx$$
$$= \frac{d^2\left(b\sqrt{1-c^2x^2}(-149+38c^2x^2-9c^4x^4)+15acx(15-10c^2x^2+3c^4x^4)+15bcx(15-10c^2x^2+3c^4x^4)\text{ ar}\right.}{225c}$$

input
```
Integrate[(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*(b*Sqrt[1 - c^2*x^2]*(-149 + 38*c^2*x^2 - 9*c^4*x^4) + 15*a*c*x*(15 -
 10*c^2*x^2 + 3*c^4*x^4) + 15*b*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4)*ArcCos[c
*x]))/(225*c)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5155, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5155}$$

$$bc\int \frac{d^2x\left(3c^4x^4-10c^2x^2+15\right)}{15\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4d^2x^5(a+b\arccos(cx))-\frac{2}{3}c^2d^2x^3(a+b\arccos(cx))+ d^2x(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}bcd^2\int \frac{x\left(3c^4x^4-10c^2x^2+15\right)}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4d^2x^5(a+b\arccos(cx))-\frac{2}{3}c^2d^2x^3(a+ b\arccos(cx))+d^2x(a+b\arccos(cx))$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{30}bcd^2\int \frac{3c^4x^4-10c^2x^2+15}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{5}c^4d^2x^5(a+b\arccos(cx))-\frac{2}{3}c^2d^2x^3(a+b\arccos(cx))+ d^2x(a+b\arccos(cx))$$

$$\Big\downarrow \text{1140}$$

$$\frac{1}{30}bcd^2\int \left(3\left(1-c^2x^2\right)^{3/2}+4\sqrt{1-c^2x^2}+\frac{8}{\sqrt{1-c^2x^2}}\right)dx^2+\frac{1}{5}c^4d^2x^5(a+b\arccos(cx))- \frac{2}{3}c^2d^2x^3(a+b\arccos(cx))+d^2x(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}c^4d^2x^5(a+b\arccos(cx))-\frac{2}{3}c^2d^2x^3(a+b\arccos(cx))+d^2x(a+b\arccos(cx))+ \frac{1}{30}bcd^2\left(-\frac{6\left(1-c^2x^2\right)^{5/2}}{5c^2}-\frac{8\left(1-c^2x^2\right)^{3/2}}{3c^2}-\frac{16\sqrt{1-c^2x^2}}{c^2}\right)$$

input
```
Int[(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d^2*((-16*Sqrt[1 - c^2*x^2])/c^2 - (8*(1 - c^2*x^2)^(3/2))/(3*c^2) -
(6*(1 - c^2*x^2)^(5/2))/(5*c^2)))/30 + d^2*x*(a + b*ArcCos[c*x]) - (2*c^2*
d^2*x^3*(a + b*ArcCos[c*x]))/3 + (c^4*d^2*x^5*(a + b*ArcCos[c*x]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91

method	result
parts	$a\,d^2\left(\frac{1}{5}c^4x^5 - \frac{2}{3}c^2x^3 + x\right) + \frac{b\,d^2\left(\frac{\arccos(cx)c^5x^5}{5} - \frac{2c^3x^3\arccos(cx)}{3} + cx\arccos(cx) - \frac{149\sqrt{-c^2x^2+1}}{225} + \frac{38c^2x^2\sqrt{-\cdots}}{22\cdots}\right.}{c}$ …
derivativedivides	$\frac{d^2a\left(\frac{1}{5}c^5x^5 - \frac{2}{3}c^3x^3 + cx\right) + d^2b\left(\frac{\arccos(cx)c^5x^5}{5} - \frac{2c^3x^3\arccos(cx)}{3} + cx\arccos(cx) - \frac{149\sqrt{-c^2x^2+1}}{225} + \frac{38c^2x^2\sqrt{-c^2x^2+1}}{225} - c^{\cdots}\right.}{c}$ …
default	$\frac{d^2a\left(\frac{1}{5}c^5x^5 - \frac{2}{3}c^3x^3 + cx\right) + d^2b\left(\frac{\arccos(cx)c^5x^5}{5} - \frac{2c^3x^3\arccos(cx)}{3} + cx\arccos(cx) - \frac{149\sqrt{-c^2x^2+1}}{225} + \frac{38c^2x^2\sqrt{-c^2x^2+1}}{225} - c^{\cdots}\right.}{c}$ …
orering	$\frac{x(81c^4x^4 - 302c^2x^2 + 821)(-c^2dx^2+d)^2(a+b\arccos(cx))}{225(cx-1)(cx+1)(c^2x^2-1)} - \frac{(9c^4x^4 - 38c^2x^2 + 149)\left(-4(-c^2dx^2+d)(a+b\arccos(cx\cdots\right.}{225c^2(cx-1)(cx+1)\cdots}$ …

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*d^2*(1/5*c^4*x^5-2/3*c^2*x^3+x)+b*d^2/c*(1/5*arccos(c*x)*c^5*x^5-2/3*c^3
*x^3*arccos(c*x)+c*x*arccos(c*x)-149/225*(-c^2*x^2+1)^(1/2)+38/225*c^2*x^2
*(-c^2*x^2+1)^(1/2)-1/25*c^4*x^4*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.93

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx$$
$$=\frac{45\,ac^5d^2x^5-150\,ac^3d^2x^3+225\,acd^2x+15\,(3\,bc^5d^2x^5-10\,bc^3d^2x^3+15\,bcd^2x)\arccos\,(cx)-(9\,bc^4d^2x^4}{225\,c}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/225*(45*a*c^5*d^2*x^5 - 150*a*c^3*d^2*x^3 + 225*a*c*d^2*x + 15*(3*b*c^5*
d^2*x^5 - 10*b*c^3*d^2*x^3 + 15*b*c*d^2*x)*arccos(c*x) - (9*b*c^4*d^2*x^4
- 38*b*c^2*d^2*x^2 + 149*b*d^2)*sqrt(-c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.30

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx$$
$$=\begin{cases}\frac{ac^4d^2x^5}{5}-\frac{2ac^2d^2x^3}{3}+ad^2x+\frac{bc^4d^2x^5\,\mathrm{acos}\,(cx)}{5}-\frac{bc^3d^2x^4\sqrt{-c^2x^2+1}}{25}-\frac{2bc^2d^2x^3\,\mathrm{acos}\,(cx)}{3}+\frac{38bcd^2x^2\sqrt{-c^2x^2+1}}{225}+bd^2x\,a\\ d^2x\left(a+\frac{\pi b}{2}\right)\end{cases}$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*c**4*d**2*x**5/5 - 2*a*c**2*d**2*x**3/3 + a*d**2*x + b*c**4*d
**2*x**5*acos(c*x)/5 - b*c**3*d**2*x**4*sqrt(-c**2*x**2 + 1)/25 - 2*b*c**2
*d**2*x**3*acos(c*x)/3 + 38*b*c*d**2*x**2*sqrt(-c**2*x**2 + 1)/225 + b*d**
2*x*acos(c*x) - 149*b*d**2*sqrt(-c**2*x**2 + 1)/(225*c), Ne(c, 0)), (d**2*
x*(a + pi*b/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.53

$$\int \left(d-c^2dx^2\right)^2(a+b\arccos(cx))\,dx = \frac{1}{5}\,ac^4d^2x^5 + \frac{1}{75}\left(15\,x^5\arccos\left(cx\right) - \left(\frac{3\,\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\,\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bc^4d^2 - \frac{2}{3}\,ac^2d^2x^3 - \frac{2}{9}\left(3\,x^3\arccos\left(cx\right) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\,\sqrt{-c^2x^2+1}}{c^4}\right)\right)bc^2d^2 + ad^2x + \frac{\left(cx\arccos\left(cx\right) - \sqrt{-c^2x^2+1}\right)bd^2}{c}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*c^4*d^2*x^5 + 1/75*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c
^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*c^4*d^2
 - 2/3*a*c^2*d^2*x^3 - 2/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/
c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*c^2*d^2 + a*d^2*x + (c*x*arccos(c*x) -
sqrt(-c^2*x^2 + 1))*b*d^2/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07

$$\int \left(d-c^2dx^2\right)^2(a+b\arccos(cx))\,dx = \frac{1}{5}\,bc^4d^2x^5\arccos\left(cx\right) + \frac{1}{5}\,ac^4d^2x^5 - \frac{1}{25}\,\sqrt{-c^2x^2+1}bc^3d^2x^4 - \frac{2}{3}\,bc^2d^2x^3\arccos\left(cx\right) - \frac{2}{3}\,ac^2d^2x^3 + \frac{38}{225}\,\sqrt{-c^2x^2+1}bcd^2x^2 + bd^2x\arccos\left(cx\right) + ad^2x - \frac{149\,\sqrt{-c^2x^2+1}bd^2}{225\,c}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/5*b*c^4*d^2*x^5*arccos(c*x) + 1/5*a*c^4*d^2*x^5 - 1/25*sqrt(-c^2*x^2 + 1
)*b*c^3*d^2*x^4 - 2/3*b*c^2*d^2*x^3*arccos(c*x) - 2/3*a*c^2*d^2*x^3 + 38/2
25*sqrt(-c^2*x^2 + 1)*b*c*d^2*x^2 + b*d^2*x*arccos(c*x) + a*d^2*x - 149/22
5*sqrt(-c^2*x^2 + 1)*b*d^2/c
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx = \int (a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2\,dx$$

input
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx$$
$$= \frac{d^2\left(45acos(cx)\,b\,c^5x^5-150acos(cx)\,b\,c^3x^3+225acos(cx)\,bcx-9\sqrt{-c^2x^2+1}\,b\,c^4x^4+38\sqrt{-c^2x^2+1}\,b\,c^2\cdots\right.}{225c}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acos(c*x)),x)
```

output
```
(d**2*(45*acos(c*x)*b*c**5*x**5 - 150*acos(c*x)*b*c**3*x**3 + 225*acos(c*x
)*b*c*x - 9*sqrt( - c**2*x**2 + 1)*b*c**4*x**4 + 38*sqrt( - c**2*x**2 + 1)
*b*c**2*x**2 - 149*sqrt( - c**2*x**2 + 1)*b + 45*a*c**5*x**5 - 150*a*c**3*
x**3 + 225*a*c*x))/(225*c)
```

3.17 $\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 184

$$\begin{aligned}\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))}{x}\,dx = &-\frac{11}{32}bcd^2x\sqrt{1-c^2x^2} \\ &-\frac{1}{16}bcd^2x(1-c^2x^2)^{3/2}-\frac{11}{32}bd^2\arccos(cx) \\ &+\frac{1}{2}d^2(1-c^2x^2)(a+b\arccos(cx)) \\ &+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\arccos(cx)) \\ &-\frac{id^2(a+b\arccos(cx))^2}{2b} \\ &+d^2(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right) \\ &-\frac{1}{2}ibd^2\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\end{aligned}$$

output
```
-11/32*b*c*d^2*x*(-c^2*x^2+1)^(1/2)-1/16*b*c*d^2*x*(-c^2*x^2+1)^(3/2)-11/3
2*b*d^2*arccos(c*x)+1/2*d^2*(-c^2*x^2+1)*(a+b*arccos(c*x))+1/4*d^2*(-c^2*x
^2+1)^2*(a+b*arccos(c*x))-1/2*I*d^2*(a+b*arccos(c*x))^2/b+d^2*(a+b*arccos(
c*x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1/2*I*b*d^2*polylog(2,(c*x+I*(-c^
2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.90

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x} \, dx = \frac{1}{32}d^2\left(-32ac^2x^2+8ac^4x^4+13bcx\sqrt{1-c^2x^2} - 2bc^3x^3\sqrt{1-c^2x^2}-16ib\arccos(cx)^2 - 26b\arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right) + 8b\arccos(cx)\left(-4c^2x^2+c^4x^4 + 4\log\left(1+e^{2i\arccos(cx)}\right)\right)+32a\log(x) - 16ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\right)$$

input
```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]))/x,x]
```

output
```
(d^2*(-32*a*c^2*x^2 + 8*a*c^4*x^4 + 13*b*c*x*Sqrt[1 - c^2*x^2] - 2*b*c^3*x
^3*Sqrt[1 - c^2*x^2] - (16*I)*b*ArcCos[c*x]^2 - 26*b*ArcTan[(c*x)/(-1 + Sq
rt[1 - c^2*x^2])] + 8*b*ArcCos[c*x]*(-4*c^2*x^2 + c^4*x^4 + 4*Log[1 + E^((
2*I)*ArcCos[c*x])]) + 32*a*Log[x] - (16*I)*b*PolyLog[2, -E^((2*I)*ArcCos[c
*x])]))/32
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.23, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {5189, 27, 211, 211, 223, 5189, 211, 223, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x} \, dx$$

$$\Big\downarrow \text{5189}$$

$$d\int \frac{d(1-c^2x^2)(a+b\arccos(cx))}{x}dx+\frac{1}{4}bcd^2\int (1-c^2x^2)^{3/2}dx+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\arccos(cx))$$

$$\Big\downarrow \; 27$$

$$d^2\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx+\frac{1}{4}bcd^2\int (1-c^2x^2)^{3/2}dx+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\arccos(cx))$$

$$\Big\downarrow \; 211$$

$$d^2\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx+\frac{1}{4}bcd^2\left(\frac{3}{4}\int \sqrt{1-c^2x^2}dx+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\arccos(cx))$$

$$\Big\downarrow \; 211$$

$$d^2\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx+\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\arccos(cx))$$

$$\Big\downarrow \; 223$$

$$d^2\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\arccos(cx))+\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)$$

$$\Big\downarrow \; 5189$$

$$d^2\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\int \sqrt{1-c^2x^2}dx+\frac{1}{2}(1-c^2x^2)(a+b\arccos(cx))\right)+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\arccos(cx))+\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)$$

$$\Big\downarrow \; 211$$

$$d^2\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{2}(1-c^2x^2)(a+b\arccos(cx))\right)+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\arccos(cx))+\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)$$

$$\Big\downarrow \text{223}$$

$$d^2\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)+ \frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))+ \frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)$$

$$\Big\downarrow \text{5137}$$

$$d^2\left(-\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1\right.\right. \frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))+ \frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^2\left(-\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}\right.\right. \frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))+ \frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)$$

$$\Big\downarrow \text{4202}$$

$$d^2\left(2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))-\frac{i(a+b\arccos(cx))^2}{2b}+\frac{1}{2}\right. \frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))+ \frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)$$

$$\Big\downarrow \text{2620}$$

$$d^2\left(2i\left(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{2}\left(1-c^2x^2\right)\right. \frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))+ \frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)$$

$$\Big\downarrow \text{2715}$$

$$d^2\left(2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{2}\right.$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))+$$
$$\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)$$

$$\Big\downarrow \text{2838}$$

$$d^2\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right.\right.\right.$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))+$$
$$\frac{1}{4}bcd^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)$$

input
```
Int[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]))/x,x]
```

output
```
(d^2*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x]))/4 + (b*c*d^2*((x*(1 - c^2*x^2)^(
3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/4 + d^2*((
(1 - c^2*x^2)*(a + b*ArcCos[c*x]))/2 - ((I/2)*(a + b*ArcCos[c*x])^2)/b + (
b*c*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 + (2*I)*((-1/2*I)*(a
+ b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCos[c*x])] - (b*PolyLog[2, -E^((2*I)*
ArcCos[c*x])])/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5189
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCos[c*x])/(2*p)), x] + (Simp[d
 Int[(d + e*x^2)^(p - 1)*((a + b*ArcCos[c*x])/x), x], x] + Simp[b*c*(d^p/(2
*p))   Int[(1 - c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.98

method	result
parts	$d^2a\left(\frac{c^4x^4}{4}-c^2x^2+\ln\left(x\right)\right)-\frac{ib\,d^2\arccos(cx)^2}{2}+\frac{3bc\,d^2x\sqrt{-c^2x^2+1}}{8}-\frac{3d^2b\arccos(cx)c^2x^2}{4}+\frac{3b\,d^2\,\mathrm{arc}}{8}$
derivativedivides	$d^2a\left(\frac{c^4x^4}{4}-c^2x^2+\ln\left(cx\right)\right)-\frac{ib\,d^2\arccos(cx)^2}{2}+\frac{3bc\,d^2x\sqrt{-c^2x^2+1}}{8}-\frac{3d^2b\arccos(cx)c^2x^2}{4}+\frac{3b\,d^2\,\mathrm{a}}{}$
default	$d^2a\left(\frac{c^4x^4}{4}-c^2x^2+\ln\left(cx\right)\right)-\frac{ib\,d^2\arccos(cx)^2}{2}+\frac{3bc\,d^2x\sqrt{-c^2x^2+1}}{8}-\frac{3d^2b\arccos(cx)c^2x^2}{4}+\frac{3b\,d^2\,\mathrm{a}}{}$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/4*c^4*x^4-c^2*x^2+ln(x))-1/2*I*b*d^2*arccos(c*x)^2+3/8*b*c*d^2*x*
(-c^2*x^2+1)^(1/2)-3/4*d^2*b*arccos(c*x)*c^2*x^2+3/8*b*d^2*arccos(c*x)+d^2
*b*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1/2*I*d^2*b*polylog(2,-(
c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/32*d^2*b*arccos(c*x)*cos(4*arccos(c*x))-1/1
28*d^2*b*sin(4*arccos(c*x))
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^2\,(a+b\arccos(cx))}{x}\,dx=\int \frac{(c^2dx^2-d)^2(b\arccos\left(cx\right)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccos(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))}{x}\,dx = d^2\left(\int \frac{a}{x}\,dx + \int \left(-2ac^2x\right)\,dx + \int ac^4x^3\,dx + \int \frac{b\,\mathrm{acos}\,(cx)}{x}\,dx + \int \left(-2bc^2x\,\mathrm{acos}\,(cx)\right)\,dx + \int bc^4x^3\,\mathrm{acos}\,(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acos(c*x))/x,x)
```

output
```
d**2*(Integral(a/x, x) + Integral(-2*a*c**2*x, x) + Integral(a*c**4*x**3,
x) + Integral(b*acos(c*x)/x, x) + Integral(-2*b*c**2*x*acos(c*x), x) + Int
egral(b*c**4*x**3*acos(c*x), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))}{x}\,dx = \int \frac{(c^2dx^2-d)^2(b\arccos(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*a*c^4*d^2*x^4 - a*c^2*d^2*x^2 + a*d^2*log(x) + integrate((b*c^4*d^2*x^
4 - 2*b*c^2*d^2*x^2 + b*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x,
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^2}{x}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^2)/x,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^2)/x, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x}\,dx$$
$$= \frac{d^2\Big(8acos(cx)\,b\,c^4x^4 - 32acos(cx)\,b\,c^2x^2 - 13asin(cx)\,b - 2\sqrt{-c^2x^2+1}\,b\,c^3x^3 + 13\sqrt{-c^2x^2+1}\,bcx + 32}{32}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acos(c*x))/x,x)
```

output
```
(d**2*(8*acos(c*x)*b*c**4*x**4 - 32*acos(c*x)*b*c**2*x**2 - 13*asin(c*x)*b
 - 2*sqrt( - c**2*x**2 + 1)*b*c**3*x**3 + 13*sqrt( - c**2*x**2 + 1)*b*c*x
+ 32*int(acos(c*x)/x,x)*b + 32*log(x)*a + 8*a*c**4*x**4 - 32*a*c**2*x**2))
/32
```

3.18 $\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))}{x^2}\,dx = -\frac{5}{3}bcd^2\sqrt{1-c^2x^2}-\frac{1}{9}bcd^2(1-c^2x^2)^{3/2} - \frac{d^2(a+b\arccos(cx))}{x}-2c^2d^2x(a+b\arccos(cx)) + \frac{1}{3}c^4d^2x^3(a+b\arccos(cx)) - bcd^2\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)$$

output

```
-5/3*b*c*d^2*(-c^2*x^2+1)^(1/2)-1/9*b*c*d^2*(-c^2*x^2+1)^(3/2)-d^2*(a+b*ar
ccos(c*x))/x-2*c^2*d^2*x*(a+b*arccos(c*x))+1/3*c^4*d^2*x^3*(a+b*arccos(c*x
))-b*c*d^2*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03

$$\int \frac{(d - c^2 d x^2)^2 (a + b \arccos(cx))}{x^2} \, dx$$
$$= \frac{d^2 (-9a - 18ac^2x^2 + 3ac^4x^4 + 16bcx\sqrt{1 - c^2x^2} - bc^3x^3\sqrt{1 - c^2x^2} + 3b(-3 - 6c^2x^2 + c^4x^4) \arccos(cx) -}{9x}$$

input
```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
(d^2*(-9*a - 18*a*c^2*x^2 + 3*a*c^4*x^4 + 16*b*c*x*Sqrt[1 - c^2*x^2] - b*c
^3*x^3*Sqrt[1 - c^2*x^2] + 3*b*(-3 - 6*c^2*x^2 + c^4*x^4)*ArcCos[c*x] - 9*
b*c*x*Log[x] + 9*b*c*x*Log[1 + Sqrt[1 - c^2*x^2]]))/(9*x)
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5193, 27, 1578, 1192, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d - c^2 d x^2)^2 (a + b \arccos(cx))}{x^2} \, dx$$

$$\Big\downarrow \text{5193}$$

$$bc \int -\frac{d^2(-c^4x^4 + 6c^2x^2 + 3)}{3x\sqrt{1 - c^2x^2}} dx + \frac{1}{3}c^4d^2x^3(a + b\arccos(cx)) - 2c^2d^2x(a + b\arccos(cx)) - \frac{d^2(a + b\arccos(cx))}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}bcd^2 \int \frac{-c^4x^4 + 6c^2x^2 + 3}{x\sqrt{1 - c^2x^2}} dx + \frac{1}{3}c^4d^2x^3(a + b\arccos(cx)) - 2c^2d^2x(a + b\arccos(cx)) - \frac{d^2(a + b\arccos(cx))}{x}$$

$$\Big\downarrow \text{1578}$$

$$-\frac{1}{6}bcd^2\int \frac{-c^4x^4+6c^2x^2+3}{x^2\sqrt{1-c^2x^2}}dx^2+\frac{1}{3}c^4d^2x^3(a+b\arccos(cx))-2c^2d^2x(a+b\arccos(cx))-\frac{d^2(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{1192}$$

$$-\frac{bd^2\int -\frac{-c^4x^8-4c^4x^4+8c^4}{1-x^4}d\sqrt{1-c^2x^2}}{3c^3}+\frac{1}{3}c^4d^2x^3(a+b\arccos(cx))-2c^2d^2x(a+b\arccos(cx))-\frac{d^2(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{25}$$

$$\frac{bd^2\int \frac{-c^4x^8-4c^4x^4+8c^4}{1-x^4}d\sqrt{1-c^2x^2}}{3c^3}+\frac{1}{3}c^4d^2x^3(a+b\arccos(cx))-2c^2d^2x(a+b\arccos(cx))-\frac{d^2(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{1467}$$

$$\frac{bd^2\int \left(x^4c^4+\frac{3c^4}{1-x^4}+5c^4\right)d\sqrt{1-c^2x^2}}{3c^3}+\frac{1}{3}c^4d^2x^3(a+b\arccos(cx))-2c^2d^2x(a+b\arccos(cx))-\frac{d^2(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}c^4d^2x^3(a+b\arccos(cx))-2c^2d^2x(a+b\arccos(cx))-\frac{d^2(a+b\arccos(cx))}{x}-\frac{bd^2\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c^4x^6-5c^4\sqrt{1-c^2x^2}\right)}{3c^3}$$

input
```
Int[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
-((d^2*(a + b*ArcCos[c*x]))/x) - 2*c^2*d^2*x*(a + b*ArcCos[c*x]) + (c^4*d^
2*x^3*(a + b*ArcCos[c*x]))/3 - (b*d^2*(-1/3*(c^4*x^6) - 5*c^4*Sqrt[1 - c^2
*x^2] - 3*c^4*ArcTanh[Sqrt[1 - c^2*x^2]]))/(3*c^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

method	result
parts	$d^2a\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)+d^2bc\left(\frac{c^3x^3\arccos(cx)}{3}-2cx\arccos(cx)-\frac{\arccos(cx)}{cx}-\frac{c^2x^2\sqrt{-c^2x^2+}}{9}\right.$
derivativedivides	$c\left(d^2a\left(\frac{c^3x^3}{3}-2cx-\frac{1}{cx}\right)+d^2b\left(\frac{c^3x^3\arccos(cx)}{3}-2cx\arccos(cx)-\frac{\arccos(cx)}{cx}-\frac{c^2x^2\sqrt{-c^2x^2}}{9}\right.\right.$
default	$c\left(d^2a\left(\frac{c^3x^3}{3}-2cx-\frac{1}{cx}\right)+d^2b\left(\frac{c^3x^3\arccos(cx)}{3}-2cx\arccos(cx)-\frac{\arccos(cx)}{cx}-\frac{c^2x^2\sqrt{-c^2x^2}}{9}\right.\right.$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/3*c^4*x^3-2*c^2*x-1/x)+d^2*b*c*(1/3*c^3*x^3*arccos(c*x)-2*c*x*arc
cos(c*x)-arccos(c*x)/c/x-1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)+16/9*(-c^2*x^2+1)^
(1/2)+arctanh(1/(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.80

$$\int \frac{(d-c^2dx^2)^2\,(a+b\arccos(cx))}{x^2}\,dx$$
$$=\frac{6\,ac^4d^2x^4-36\,ac^2d^2x^2+9\,bcd^2x\log\left(\sqrt{-c^2x^2+1}+1\right)-9\,bcd^2x\log\left(\sqrt{-c^2x^2+1}-1\right)+6\,(bc^4-6\,bc^2}{}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/18*(6*a*c^4*d^2*x^4 - 36*a*c^2*d^2*x^2 + 9*b*c*d^2*x*log(sqrt(-c^2*x^2 +
 1) + 1) - 9*b*c*d^2*x*log(sqrt(-c^2*x^2 + 1) - 1) + 6*(b*c^4 - 6*b*c^2 -
3*b)*d^2*x*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) - 18*a*d^2 + 6*(b*
c^4*d^2*x^4 - 6*b*c^2*d^2*x^2 - (b*c^4 - 6*b*c^2 - 3*b)*d^2*x - 3*b*d^2)*a
rccos(c*x) - 2*(b*c^3*d^2*x^3 - 16*b*c*d^2*x)*sqrt(-c^2*x^2 + 1))/x
```

Sympy [A] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.52

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x^2} dx$$
$$= \frac{ac^4d^2x^3}{3} - 2ac^2d^2x - \frac{ad^2}{x} + \frac{bc^5d^2\left(\begin{cases} -\frac{x^2\sqrt{-c^2x^2+1}}{3c^2} - \frac{2\sqrt{-c^2x^2+1}}{3c^4} & \text{for } c^2 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}\right)}{3} + \frac{bc^4d^2x^3\operatorname{acos}(cx)}{3} - 2bc^2d^2\left(\begin{cases} \frac{\pi x}{2} & \text{for } c = 0 \\ x\operatorname{acos}(cx) - \frac{\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases}\right) - bcd^2\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \frac{1}{|c^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases}\right) - \frac{bd^2\operatorname{acos}(cx)}{x}$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acos(c*x))/x**2,x)
```

output
```
a*c**4*d**2*x**3/3 - 2*a*c**2*d**2*x - a*d**2/x + b*c**5*d**2*Piecewise((-
x**2*sqrt(-c**2*x**2 + 1)/(3*c**2) - 2*sqrt(-c**2*x**2 + 1)/(3*c**4), Ne(c
**2, 0)), (x**4/4, True))/3 + b*c**4*d**2*x**3*acos(c*x)/3 - 2*b*c**2*d**2
*Piecewise((pi*x/2, Eq(c, 0)), (x*acos(c*x) - sqrt(-c**2*x**2 + 1)/c, True
)) - b*c*d**2*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1
/(c*x)), True)) - b*d**2*acos(c*x)/x
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.33

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x^2} dx$$
$$= \frac{1}{3}ac^4d^2x^3 + \frac{1}{9}\left(3x^3\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bc^4d^2 - 2ac^2d^2x - 2\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)bcd^2 + \left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{\arccos(cx)}{x}\right)bd^2 - \frac{ad^2}{x}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/3*a*c^4*d^2*x^3 + 1/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2
 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*c^4*d^2 - 2*a*c^2*d^2*x - 2*(c*x*arccos(c*
x) - sqrt(-c^2*x^2 + 1))*b*c*d^2 + (c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/
abs(x)) - arccos(c*x)/x)*b*d^2 - a*d^2/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1837 vs. $2(111) = 222$.

Time = 6.18 (sec) , antiderivative size = 1837, normalized size of antiderivative = 14.93

$$\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x^2,x, algorithm="giac")
```

output
```
8/3*b*c*d^2*arccos(c*x)/(2*(c^2*x^2 - 1)/(c*x + 1)^2 - 2*(c^2*x^2 - 1)^3/(
c*x + 1)^6 + (c^2*x^2 - 1)^4/(c*x + 1)^8 - 1) - b*c*d^2*log(abs(c*x + sqrt
(-c^2*x^2 + 1) + 1))/(2*(c^2*x^2 - 1)/(c*x + 1)^2 - 2*(c^2*x^2 - 1)^3/(c*x
 + 1)^6 + (c^2*x^2 - 1)^4/(c*x + 1)^8 - 1) + b*c*d^2*log(abs(-c*x + sqrt(-
c^2*x^2 + 1) - 1))/(2*(c^2*x^2 - 1)/(c*x + 1)^2 - 2*(c^2*x^2 - 1)^3/(c*x +
 1)^6 + (c^2*x^2 - 1)^4/(c*x + 1)^8 - 1) + 8/3*a*c*d^2/(2*(c^2*x^2 - 1)/(c
*x + 1)^2 - 2*(c^2*x^2 - 1)^3/(c*x + 1)^6 + (c^2*x^2 - 1)^4/(c*x + 1)^8 -
1) - 16/3*(c^2*x^2 - 1)*b*c*d^2*arccos(c*x)/((c*x + 1)^2*(2*(c^2*x^2 - 1)/
(c*x + 1)^2 - 2*(c^2*x^2 - 1)^3/(c*x + 1)^6 + (c^2*x^2 - 1)^4/(c*x + 1)^8
- 1)) + 2*(c^2*x^2 - 1)*b*c*d^2*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/((c
*x + 1)^2*(2*(c^2*x^2 - 1)/(c*x + 1)^2 - 2*(c^2*x^2 - 1)^3/(c*x + 1)^6 + (
c^2*x^2 - 1)^4/(c*x + 1)^8 - 1)) - 2*(c^2*x^2 - 1)*b*c*d^2*log(abs(-c*x +
sqrt(-c^2*x^2 + 1) - 1))/((c*x + 1)^2*(2*(c^2*x^2 - 1)/(c*x + 1)^2 - 2*(c^
2*x^2 - 1)^3/(c*x + 1)^6 + (c^2*x^2 - 1)^4/(c*x + 1)^8 - 1)) - 10/3*sqrt(-
c^2*x^2 + 1)*b*c*d^2/((c*x + 1)*(2*(c^2*x^2 - 1)/(c*x + 1)^2 - 2*(c^2*x^2
- 1)^3/(c*x + 1)^6 + (c^2*x^2 - 1)^4/(c*x + 1)^8 - 1)) - 16/3*(c^2*x^2 - 1
)*a*c*d^2/((c*x + 1)^2*(2*(c^2*x^2 - 1)/(c*x + 1)^2 - 2*(c^2*x^2 - 1)^3/(c
*x + 1)^6 + (c^2*x^2 - 1)^4/(c*x + 1)^8 - 1)) - 38/9*(-c^2*x^2 + 1)^(3/2)*
b*c*d^2/((c*x + 1)^3*(2*(c^2*x^2 - 1)/(c*x + 1)^2 - 2*(c^2*x^2 - 1)^3/(c*x
 + 1)^6 + (c^2*x^2 - 1)^4/(c*x + 1)^8 - 1)) - 16/3*(c^2*x^2 - 1)^3*b*c*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x^2}\,dx$$
$$= \begin{cases} b\,c\,d^2\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-c^2\,x^2}}\right)-\frac{a\,d^2\,(-c^4\,x^4+6\,c^2\,x^2+3)}{3\,x}+2\,b\,c\,d^2\,\left(\sqrt{1-c^2\,x^2}-c\,x\,\mathrm{acos}(c\,x)\right)-b\,c^4\,d^2\,\left(\frac{\sqrt{\frac{1}{c^2}-x^2}\,(}{9}\right. \\ \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2}{x^2}\,dx \end{cases}$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^2)/x^2,x)
```

output
```
piecewise(0 < c, b*c*d^2*atanh(1/(- c^2*x^2 + 1)^(1/2)) - (a*d^2*(6*c^2*x^
2 - c^4*x^4 + 3))/(3*x) + 2*b*c*d^2*((- c^2*x^2 + 1)^(1/2) - c*x*acos(c*x)
) - b*c^4*d^2*(((1/c^2 - x^2)^(1/2)*(2/c^2 + x^2))/9 - (x^3*acos(c*x))/3)
- (b*d^2*acos(c*x))/x, ~0 < c, int(((a + b*acos(c*x))*(d - c^2*d*x^2)^2)/x
^2, x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.91

$$\int \frac{(d - c^2dx^2)^2 (a + b\arccos(cx))}{x^2} dx$$
$$= \frac{d^2\left(3acos(cx)\,b\,c^4x^4 - 18acos(cx)\,b\,c^2x^2 - 9acos(cx)\,b - \sqrt{-c^2x^2+1}\,b\,c^3x^3 + 16\sqrt{-c^2x^2+1}\,bcx - 9\log\right.}{9x}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acos(c*x))/x^2,x)
```

output
```
(d**2*(3*acos(c*x)*b*c**4*x**4 - 18*acos(c*x)*b*c**2*x**2 - 9*acos(c*x)*b
- sqrt( - c**2*x**2 + 1)*b*c**3*x**3 + 16*sqrt( - c**2*x**2 + 1)*b*c*x - 9
*log(tan(asin(c*x)/2))*b*c*x + 3*a*c**4*x**4 - 18*a*c**2*x**2 - 9*a))/(9*x
)
```

3.19 $\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 201

$$\begin{aligned}\int \frac{(d-c^2dx^2)^2\,(a+b\arccos(cx))}{x^3}\,dx = &-\frac{1}{4}bc^3d^2x\sqrt{1-c^2x^2} \\ &-\frac{bcd^2(1-c^2x^2)^{3/2}}{2x}-\frac{1}{4}bc^2d^2\arccos(cx) \\ &-c^2d^2\left(1-c^2x^2\right)(a+b\arccos(cx)) \\ &-\frac{d^2(1-c^2x^2)^2\,(a+b\arccos(cx))}{2x^2} \\ &+\frac{ic^2d^2(a+b\arccos(cx))^2}{b} \\ &-2c^2d^2(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right) \\ &+ibc^2d^2\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\end{aligned}$$

output
```
-1/4*b*c^3*d^2*x*(-c^2*x^2+1)^(1/2)-1/2*b*c*d^2*(-c^2*x^2+1)^(3/2)/x-1/4*b
*c^2*d^2*arccos(c*x)-c^2*d^2*(-c^2*x^2+1)*(a+b*arccos(c*x))-1/2*d^2*(-c^2*
x^2+1)^2*(a+b*arccos(c*x))/x^2+I*c^2*d^2*(a+b*arccos(c*x))^2/b-2*c^2*d^2*(
a+b*arccos(c*x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+I*b*c^2*d^2*polylog(2,
(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.93

$$\int \frac{(d-c^2dx^2)^2\,(a+b\arccos(cx))}{x^3}\,dx$$
$$= \frac{d^2\left(-2a+2ac^4x^4+2bcx\sqrt{1-c^2x^2}-bc^3x^3\sqrt{1-c^2x^2}+4ibc^2x^2\arccos(cx)^2+2bc^2x^2\arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right)\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
(d^2*(-2*a + 2*a*c^4*x^4 + 2*b*c*x*Sqrt[1 - c^2*x^2] - b*c^3*x^3*Sqrt[1 -
c^2*x^2] + (4*I)*b*c^2*x^2*ArcCos[c*x]^2 + 2*b*c^2*x^2*ArcTan[(c*x)/(-1 +
Sqrt[1 - c^2*x^2])] + 2*b*ArcCos[c*x]*(-1 + c^4*x^4 - 4*c^2*x^2*Log[1 + E^
((2*I)*ArcCos[c*x])]) - 8*a*c^2*x^2*Log[x] + (4*I)*b*c^2*x^2*PolyLog[2, -E
^((2*I)*ArcCos[c*x])]))/(4*x^2)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.16, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {5191, 27, 247, 211, 223, 5189, 211, 223, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2\,(a+b\arccos(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{5191}$$

$$-2c^2d\int \frac{d(1-c^2x^2)\,(a+b\arccos(cx))}{x}dx-\frac{1}{2}bcd^2\int \frac{(1-c^2x^2)^{3/2}}{x^2}dx-\frac{d^2(1-c^2x^2)^2\,(a+b\arccos(cx))}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$-2c^2d^2\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx-\frac{1}{2}bcd^2\int \frac{(1-c^2x^2)^{3/2}}{x^2}dx-\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))}{2x^2}$$

$\downarrow$ 247

$$-2c^2d^2\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx-\frac{1}{2}bcd^2\left(-3c^2\int \sqrt{1-c^2x^2}dx-\frac{(1-c^2x^2)^{3/2}}{x}\right)-\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))}{2x^2}$$

$\downarrow$ 211

$$-2c^2d^2\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx-\frac{1}{2}bcd^2\left(-3c^2\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)-\frac{(1-c^2x^2)^{3/2}}{x}\right)-\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))}{2x^2}$$

$\downarrow$ 223

$$-2c^2d^2\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx-\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd^2\left(-3c^2\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)-\frac{(1-c^2x^2)^{3/2}}{x}\right)$$

$\downarrow$ 5189

$$-2c^2d^2\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\int \sqrt{1-c^2x^2}dx+\frac{1}{2}(1-c^2x^2)(a+b\arccos(cx))\right)-\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd^2\left(-3c^2\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)-\frac{(1-c^2x^2)^{3/2}}{x}\right)$$

$\downarrow$ 211

$$-2c^2d^2\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{2}(1-c^2x^2)(a+b\arccos(cx))\right)-\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd^2\left(-3c^2\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)-\frac{(1-c^2x^2)^{3/2}}{x}\right)$$

$$\Big\downarrow \ 223$$

$$-2c^2d^2\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)-\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd^2\left(-3c^2\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)-\frac{\left(1-c^2x^2\right)^{3/2}}{x}\right)$$

$$\Big\downarrow \ 5137$$

$$-2c^2d^2\left(-\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}\right.\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd^2\left(-3c^2\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)-\frac{\left(1-c^2x^2\right)^{3/2}}{x}\right)$$

$$\Big\downarrow \ 3042$$

$$-2c^2d^2\left(-\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}\right.\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd^2\left(-3c^2\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)-\frac{\left(1-c^2x^2\right)^{3/2}}{x}\right)$$

$$\Big\downarrow \ 4202$$

$$-2c^2d^2\left(2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))-\frac{i(a+b\arccos(cx))^2}{2b}\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd^2\left(-3c^2\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)-\frac{\left(1-c^2x^2\right)^{3/2}}{x}\right)$$

$$\Big\downarrow \ 2620$$

$$-2c^2d^2\left(2i\left(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{2}(1-c^2\right.$$
$$\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))}{2x^2}-$$
$$\frac{1}{2}bcd^2\left(-3c^2\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)-\frac{(1-c^2x^2)^{3/2}}{x}\right)$$

$\downarrow$ 2715

$$-2c^2d^2\left(2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)\right.$$
$$\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))}{2x^2}-$$
$$\frac{1}{2}bcd^2\left(-3c^2\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)-\frac{(1-c^2x^2)^{3/2}}{x}\right)$$

$\downarrow$ 2838

$$-2c^2d^2\left(\frac{1}{2}(1-c^2x^2)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\operatorname{PolyLog}\left(2,-e\right.\right.\right.$$
$$\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))}{2x^2}-$$
$$\frac{1}{2}bcd^2\left(-3c^2\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)-\frac{(1-c^2x^2)^{3/2}}{x}\right)$$

input
```
Int[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
-1/2*(d^2*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x]))/x^2 - (b*c*d^2*(-((1 - c^2*
x^2)^(3/2)/x) - 3*c^2*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c))))/2 -
2*c^2*d^2*(((1 - c^2*x^2)*(a + b*ArcCos[c*x]))/2 - ((I/2)*(a + b*ArcCos[c*
x])^2)/b + (b*c*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 + (2*I)*(
(-1/2*I)*(a + b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCos[c*x])] - (b*PolyLog[2
, -E^((2*I)*ArcCos[c*x])])/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5189
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCos[c*x])/(2*p)), x] + (Simp[d
 Int[(d + e*x^2)^(p - 1)*((a + b*ArcCos[c*x])/x), x], x] + Simp[b*c*(d^p/(2
*p))   Int[(1 - c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5191
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCos[c*x
])/(f*(m + 1))), x] + (Simp[b*c*(d^p/(f*(m + 1)))   Int[(f*x)^(m + 1)*(1 -
c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m + 2)
*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x]), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.02

method	result
derivativedivides	$c^2\left(d^2a\left(\frac{c^2x^2}{2}-\frac{1}{2c^2x^2}-2\ln\left(cx\right)\right)+ib\,d^2\arccos\left(cx\right)^2-\frac{bc\,d^2x\sqrt{-c^2x^2+1}}{4}+\frac{d^2b\arccos(cx)c^2x^2}{2}\right.$
default	$c^2\left(d^2a\left(\frac{c^2x^2}{2}-\frac{1}{2c^2x^2}-2\ln\left(cx\right)\right)+ib\,d^2\arccos\left(cx\right)^2-\frac{bc\,d^2x\sqrt{-c^2x^2+1}}{4}+\frac{d^2b\arccos(cx)c^2x^2}{2}\right.$
parts	$d^2a\left(\frac{c^4x^2}{2}-2c^2\ln\left(x\right)-\frac{1}{2x^2}\right)+id^2b\,c^2\arccos\left(cx\right)^2-\frac{b\,c^3d^2x\sqrt{-c^2x^2+1}}{4}+\frac{d^2b\,c^4\arccos(cx)x^2}{2}$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(d^2*a*(1/2*c^2*x^2-1/2/c^2/x^2-2*ln(c*x))+I*d^2*b*arccos(c*x)^2-1/4*b
*c*d^2*x*(-c^2*x^2+1)^(1/2)+1/2*d^2*b*arccos(c*x)*c^2*x^2-1/4*b*d^2*arccos
(c*x)+1/2*I*d^2*b+1/2*d^2*b/c/x*(-c^2*x^2+1)^(1/2)-1/2*d^2*b*arccos(c*x)/c
^2/x^2-2*d^2*b*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+I*d^2*b*poly
log(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2))
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x^3}\,dx = \int \frac{(c^2dx^2-d)^2(b\arccos(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccos(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x^3}\,dx = d^2\left(\int \frac{a}{x^3}\,dx + \int \left(-\frac{2ac^2}{x}\right)dx + \int ac^4x\,dx + \int \frac{b\operatorname{acos}(cx)}{x^3}\,dx + \int \left(-\frac{2bc^2\operatorname{acos}(cx)}{x}\right)dx + \int bc^4x\operatorname{acos}(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acos(c*x))/x**3,x)
```

output
```
d**2*(Integral(a/x**3, x) + Integral(-2*a*c**2/x, x) + Integral(a*c**4*x,
x) + Integral(b*acos(c*x)/x**3, x) + Integral(-2*b*c**2*acos(c*x)/x, x) +
Integral(b*c**4*x*acos(c*x), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x^3} dx = \int \frac{(c^2dx^2-d)^2 (b\arccos(cx)+a)}{x^3} dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*a*c^4*d^2*x^2 - 2*a*c^2*d^2*log(x) + 1/2*b*d^2*(sqrt(-c^2*x^2 + 1)*c/x
 - arccos(c*x)/x^2) - 1/2*a*d^2/x^2 + integrate((b*c^4*d^2*x^2 - 2*b*c^2*d
^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x^3} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x^3} dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^2}{x^3} dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^2)/x^3,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x^3}\,dx$$
$$= \frac{d^2\left(2acos(cx)\,b\,c^4x^4 - 2acos(cx)\,b + asin(cx)\,b\,c^2x^2 - \sqrt{-c^2x^2+1}\,b\,c^3x^3 + 2\sqrt{-c^2x^2+1}\,bcx - 8\left(\int \frac{acos}{x}\right.\right.}{4x^2}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acos(c*x))/x^3,x)
```

output
```
(d**2*(2*acos(c*x)*b*c**4*x**4 - 2*acos(c*x)*b + asin(c*x)*b*c**2*x**2 - s
qrt( - c**2*x**2 + 1)*b*c**3*x**3 + 2*sqrt( - c**2*x**2 + 1)*b*c*x - 8*int
(acos(c*x)/x,x)*b*c**2*x**2 - 8*log(x)*a*c**2*x**2 + 2*a*c**4*x**4 - 2*a))
/(4*x**2)
```

3.20 $\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))}{x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))}{x^4}\,dx = bc^3d^2\sqrt{1-c^2x^2}-\frac{bcd^2\sqrt{1-c^2x^2}}{6x^2}-\frac{d^2(a+b\arccos(cx))}{3x^3}+\frac{2c^2d^2(a+b\arccos(cx))}{x}+c^4d^2x(a+b\arccos(cx))+\frac{11}{6}bc^3d^2\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)$$

output
```
b*c^3*d^2*(-c^2*x^2+1)^(1/2)-1/6*b*c*d^2*(-c^2*x^2+1)^(1/2)/x^2-1/3*d^2*(a
+b*arccos(c*x))/x^3+2*c^2*d^2*(a+b*arccos(c*x))/x+c^4*d^2*x*(a+b*arccos(c*
x))+11/6*b*c^3*d^2*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x^4} \, dx = \frac{d^2\left(-2a+12ac^2x^2+6ac^4x^4+bcx\sqrt{1-c^2x^2}-6bc^3x^3\sqrt{1-c^2x^2}+2b(-1+6c^2x^2+3c^4x^4)\arccos(cx)+\right.}{6x^3}$$

input
```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
(d^2*(-2*a + 12*a*c^2*x^2 + 6*a*c^4*x^4 + b*c*x*Sqrt[1 - c^2*x^2] - 6*b*c^
3*x^3*Sqrt[1 - c^2*x^2] + 2*b*(-1 + 6*c^2*x^2 + 3*c^4*x^4)*ArcCos[c*x] + 1
1*b*c^3*x^3*Log[x] - 11*b*c^3*x^3*Log[1 + Sqrt[1 - c^2*x^2]]))/(6*x^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5193, 27, 1578, 1192, 25, 1471, 25, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x^4} \, dx$$

$$\Big\downarrow \text{5193}$$

$$bc\int -\frac{d^2(-3c^4x^4-6c^2x^2+1)}{3x^3\sqrt{1-c^2x^2}}dx+c^4d^2x(a+b\arccos(cx))+\frac{2c^2d^2(a+b\arccos(cx))}{x}-\frac{d^2(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}bcd^2\int \frac{-3c^4x^4-6c^2x^2+1}{x^3\sqrt{1-c^2x^2}}dx+c^4d^2x(a+b\arccos(cx))+\frac{2c^2d^2(a+b\arccos(cx))}{x}-\frac{d^2(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow 1578$$

$$-\frac{1}{6}bcd^2\int\frac{-3c^4x^4-6c^2x^2+1}{x^4\sqrt{1-c^2x^2}}dx^2+c^4d^2x(a+b\arccos(cx))+\frac{2c^2d^2(a+b\arccos(cx))}{x}-\frac{d^2(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow 1192$$

$$\frac{bd^2\int-\frac{3c^4x^8-12c^4x^4+8c^4}{(1-x^4)^2}d\sqrt{1-c^2x^2}}{3c}+c^4d^2x(a+b\arccos(cx))+\frac{2c^2d^2(a+b\arccos(cx))}{x}-\frac{d^2(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow 25$$

$$-\frac{bd^2\int\frac{3c^4x^8-12c^4x^4+8c^4}{(1-x^4)^2}d\sqrt{1-c^2x^2}}{3c}+c^4d^2x(a+b\arccos(cx))+\frac{2c^2d^2(a+b\arccos(cx))}{x}-\frac{d^2(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow 1471$$

$$\frac{bd^2\left(\frac{1}{2}\int-\frac{c^4(17-6x^4)}{1-x^4}d\sqrt{1-c^2x^2}+\frac{c^4\sqrt{1-c^2x^2}}{2(1-x^4)}\right)}{3c}+c^4d^2x(a+b\arccos(cx))+\frac{2c^2d^2(a+b\arccos(cx))}{x}-\frac{d^2(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow 25$$

$$\frac{bd^2\left(\frac{c^4\sqrt{1-c^2x^2}}{2(1-x^4)}-\frac{1}{2}\int\frac{c^4(17-6x^4)}{1-x^4}d\sqrt{1-c^2x^2}\right)}{3c}+c^4d^2x(a+b\arccos(cx))+\frac{2c^2d^2(a+b\arccos(cx))}{x}-\frac{d^2(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow 27$$

$$\frac{bd^2\left(\frac{c^4\sqrt{1-c^2x^2}}{2(1-x^4)}-\frac{1}{2}c^4\int\frac{17-6x^4}{1-x^4}d\sqrt{1-c^2x^2}\right)}{3c}+c^4d^2x(a+b\arccos(cx))+\frac{2c^2d^2(a+b\arccos(cx))}{x}-\frac{d^2(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow 299$$

$$\frac{bd^2\left(\frac{c^4\sqrt{1-c^2x^2}}{2(1-x^4)}-\frac{1}{2}c^4\left(11\int\frac{1}{1-x^4}d\sqrt{1-c^2x^2}+6\sqrt{1-c^2x^2}\right)\right)}{3c}+c^4d^2x(a+b\arccos(cx))+\frac{2c^2d^2(a+b\arccos(cx))}{x}-\frac{d^2(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow 219$$

$$c^4d^2x(a+b\arccos(cx))+\frac{2c^2d^2(a+b\arccos(cx))}{x}-\frac{d^2(a+b\arccos(cx))}{3x^3}+\frac{bd^2\left(\frac{c^4\sqrt{1-c^2x^2}}{2(1-x^4)}-\frac{1}{2}c^4\left(11\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+6\sqrt{1-c^2x^2}\right)\right)}{3c}$$

input
```
Int[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
-1/3*(d^2*(a + b*ArcCos[c*x]))/x^3 + (2*c^2*d^2*(a + b*ArcCos[c*x]))/x + c
^4*d^2*x*(a + b*ArcCos[c*x]) + (b*d^2*((c^4*Sqrt[1 - c^2*x^2])/(2*(1 - x^4
)) - (c^4*(6*Sqrt[1 - c^2*x^2] + 11*ArcTanh[Sqrt[1 - c^2*x^2]]))/2))/(3*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.90

method	result
parts	$d^2a\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)+d^2b\,c^3\left(cx\arccos(cx)-\frac{\arccos(cx)}{3c^3x^3}+\frac{2\arccos(cx)}{cx}+\frac{\sqrt{-c^2x^2+1}}{6c^2x^2}-\frac{11}{}\right.$
derivativedivides	$c^3\left(d^2a\left(cx-\frac{1}{3c^3x^3}+\frac{2}{cx}\right)+d^2b\left(cx\arccos(cx)-\frac{\arccos(cx)}{3c^3x^3}+\frac{2\arccos(cx)}{cx}+\frac{\sqrt{-c^2x^2+1}}{6c^2x^2}-\right.\right.$
default	$c^3\left(d^2a\left(cx-\frac{1}{3c^3x^3}+\frac{2}{cx}\right)+d^2b\left(cx\arccos(cx)-\frac{\arccos(cx)}{3c^3x^3}+\frac{2\arccos(cx)}{cx}+\frac{\sqrt{-c^2x^2+1}}{6c^2x^2}-\right.\right.$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(c^4*x+2*c^2/x-1/3/x^3)+d^2*b*c^3*(c*x*arccos(c*x)-1/3*arccos(c*x)/c
^3/x^3+2*arccos(c*x)/c/x+1/6/c^2/x^2*(-c^2*x^2+1)^(1/2)-11/6*arctanh(1/(-c
^2*x^2+1)^(1/2))-(-c^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(116) = 232$.

Time = 0.17 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.85

$$\int \frac{(d-c^2dx^2)^2\,(a+b\arccos(cx))}{x^4}\,dx$$
$$= \frac{12\,ac^4d^2x^4-11\,bc^3d^2x^3\log\left(\sqrt{-c^2x^2+1}+1\right)+11\,bc^3d^2x^3\log\left(\sqrt{-c^2x^2+1}-1\right)+24\,ac^2d^2x^2+4\,(3\,bc}{}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/12*(12*a*c^4*d^2*x^4 - 11*b*c^3*d^2*x^3*log(sqrt(-c^2*x^2 + 1) + 1) + 11
*b*c^3*d^2*x^3*log(sqrt(-c^2*x^2 + 1) - 1) + 24*a*c^2*d^2*x^2 + 4*(3*b*c^4
 + 6*b*c^2 - b)*d^2*x^3*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) - 4*a
*d^2 + 4*(3*b*c^4*d^2*x^4 + 6*b*c^2*d^2*x^2 - (3*b*c^4 + 6*b*c^2 - b)*d^2*
x^3 - b*d^2)*arccos(c*x) - 2*(6*b*c^3*d^2*x^3 - b*c*d^2*x)*sqrt(-c^2*x^2 +
 1))/x^3
```

Sympy [A] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.84

$$\int \frac{(d-c^2dx^2)^2\,(a+b\arccos(cx))}{x^4}\,dx$$
$$= ac^4d^2x + \frac{2ac^2d^2}{x} - \frac{ad^2}{3x^3} + bc^4d^2\left(\begin{cases} \frac{\pi x}{2} & \text{for } c=0 \\ x\operatorname{acos}(cx) - \frac{\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases}\right)$$
$$+ 2bc^3d^2\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \frac{1}{|c^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases}\right) + \frac{2bc^2d^2\operatorname{acos}(cx)}{x}$$
$$- \frac{bcd^2\left(\begin{cases} -\frac{c^2\operatorname{acosh}\left(\frac{1}{cx}\right)}{2} + \frac{c}{2x\sqrt{-1+\frac{1}{c^2x^2}}} - \frac{1}{2cx^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \frac{1}{|c^2x^2|} > 1 \\ \frac{ic^2\operatorname{asin}\left(\frac{1}{cx}\right)}{2} - \frac{ic\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise} \end{cases}\right)}{3}$$
$$- \frac{bd^2\operatorname{acos}(cx)}{3x^3}$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acos(c*x))/x**4,x)
```

output
```
a*c**4*d**2*x + 2*a*c**2*d**2/x - a*d**2/(3*x**3) + b*c**4*d**2*Piecewise(
(pi*x/2, Eq(c, 0)), (x*acos(c*x) - sqrt(-c**2*x**2 + 1)/c, True)) + 2*b*c*
*3*d**2*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)
), True)) + 2*b*c**2*d**2*acos(c*x)/x - b*c*d**2*Piecewise((-c**2*acosh(1/
(c*x))/2 + c/(2*x*sqrt(-1 + 1/(c**2*x**2))) - 1/(2*c*x**3*sqrt(-1 + 1/(c**
2*x**2))), 1/Abs(c**2*x**2) > 1), (I*c**2*asin(1/(c*x))/2 - I*c*sqrt(1 - 1
/(c**2*x**2))/(2*x), True))/3 - b*d**2*acos(c*x)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.35

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x^4} dx$$
$$= ac^4d^2x + \left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)bc^3d^2$$
$$- 2\left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{\arccos(cx)}{x}\right)bc^2d^2$$
$$+ \frac{1}{6}\left(\left(c^2\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{\sqrt{-c^2x^2+1}}{x^2}\right)c - \frac{2\arccos(cx)}{x^3}\right)bd^2$$
$$+ \frac{2ac^2d^2}{x} - \frac{ad^2}{3x^3}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x^4,x, algorithm="maxima")
```

output
```
a*c^4*d^2*x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b*c^3*d^2 - 2*(c*log(
2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*b*c^2*d^2 + 1/6*(
(c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)/x^2)
*c - 2*arccos(c*x)/x^3)*b*d^2 + 2*a*c^2*d^2/x - 1/3*a*d^2/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1899 vs. 2(116) = 232.

Time = 58.86 (sec) , antiderivative size = 1899, normalized size of antiderivative = 14.84

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x^4} dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))/x^4,x, algorithm="giac")
```

output
```
8/3*b*c^3*d^2*arccos(c*x)/(2*(c^2*x^2 - 1)/(c*x + 1)^2 - 2*(c^2*x^2 - 1)^3
/(c*x + 1)^6 - (c^2*x^2 - 1)^4/(c*x + 1)^8 + 1) - 11/6*b*c^3*d^2*log(abs(c
*x + sqrt(-c^2*x^2 + 1) + 1))/(2*(c^2*x^2 - 1)/(c*x + 1)^2 - 2*(c^2*x^2 -
1)^3/(c*x + 1)^6 - (c^2*x^2 - 1)^4/(c*x + 1)^8 + 1) + 11/6*b*c^3*d^2*log(a
bs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/(2*(c^2*x^2 - 1)/(c*x + 1)^2 - 2*(c^2*x
^2 - 1)^3/(c*x + 1)^6 - (c^2*x^2 - 1)^4/(c*x + 1)^8 + 1) + 8/3*a*c^3*d^2/(
2*(c^2*x^2 - 1)/(c*x + 1)^2 - 2*(c^2*x^2 - 1)^3/(c*x + 1)^6 - (c^2*x^2 - 1
)^4/(c*x + 1)^8 + 1) + 16/3*(c^2*x^2 - 1)*b*c^3*d^2*arccos(c*x)/((c*x + 1)
^2*(2*(c^2*x^2 - 1)/(c*x + 1)^2 - 2*(c^2*x^2 - 1)^3/(c*x + 1)^6 - (c^2*x^2
 - 1)^4/(c*x + 1)^8 + 1)) - 11/3*(c^2*x^2 - 1)*b*c^3*d^2*log(abs(c*x + sqr
t(-c^2*x^2 + 1) + 1))/((c*x + 1)^2*(2*(c^2*x^2 - 1)/(c*x + 1)^2 - 2*(c^2*x
^2 - 1)^3/(c*x + 1)^6 - (c^2*x^2 - 1)^4/(c*x + 1)^8 + 1)) + 11/3*(c^2*x^2
- 1)*b*c^3*d^2*log(abs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/((c*x + 1)^2*(2*(c^
2*x^2 - 1)/(c*x + 1)^2 - 2*(c^2*x^2 - 1)^3/(c*x + 1)^6 - (c^2*x^2 - 1)^4/(
c*x + 1)^8 + 1)) - 5/3*sqrt(-c^2*x^2 + 1)*b*c^3*d^2/((c*x + 1)*(2*(c^2*x^2
 - 1)/(c*x + 1)^2 - 2*(c^2*x^2 - 1)^3/(c*x + 1)^6 - (c^2*x^2 - 1)^4/(c*x +
 1)^8 + 1)) + 16/3*(c^2*x^2 - 1)*a*c^3*d^2/((c*x + 1)^2*(2*(c^2*x^2 - 1)/(
c*x + 1)^2 - 2*(c^2*x^2 - 1)^3/(c*x + 1)^6 - (c^2*x^2 - 1)^4/(c*x + 1)^8 +
 1)) + 19/3*(-c^2*x^2 + 1)^(3/2)*b*c^3*d^2/((c*x + 1)^3*(2*(c^2*x^2 - 1)/(
c*x + 1)^2 - 2*(c^2*x^2 - 1)^3/(c*x + 1)^6 - (c^2*x^2 - 1)^4/(c*x + 1)^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2\,(a+b\arccos(cx))}{x^4}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2}{x^4}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^2)/x^4,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.90

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))}{x^4}\,dx$$
$$= \frac{d^2\Big(6acos(cx)\,b\,c^4x^4+12acos(cx)\,b\,c^2x^2-2acos(cx)\,b-6\sqrt{-c^2x^2+1}\,b\,c^3x^3+\sqrt{-c^2x^2+1}\,bcx+11\log}{6x^3}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acos(c*x))/x^4,x)
```

output
```
(d**2*(6*acos(c*x)*b*c**4*x**4 + 12*acos(c*x)*b*c**2*x**2 - 2*acos(c*x)*b
- 6*sqrt( - c**2*x**2 + 1)*b*c**3*x**3 + sqrt( - c**2*x**2 + 1)*b*c*x + 11
*log(tan(asin(c*x)/2))*b*c**3*x**3 + 6*a*c**4*x**4 + 12*a*c**2*x**2 - 2*a)
)/(6*x**3)
```

3.21 $\int x^4(d-c^2dx^2)^3 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 232

$$\int x^4 \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx = \frac{16bd^3\sqrt{1-c^2x^2}}{1155c^5} + \frac{8bd^3\left(1-c^2x^2\right)^{3/2}}{3465c^5} + \frac{2bd^3\left(1-c^2x^2\right)^{5/2}}{1925c^5} + \frac{bd^3\left(1-c^2x^2\right)^{7/2}}{1617c^5} - \frac{4bd^3\left(1-c^2x^2\right)^{9/2}}{297c^5} + \frac{bd^3\left(1-c^2x^2\right)^{11/2}}{121c^5} + \frac{1}{5}d^3x^5(a+b\arccos(cx)) - \frac{3}{7}c^2d^3x^7(a+b\arccos(cx)) + \frac{1}{3}c^4d^3x^9(a+b\arccos(cx)) - \frac{1}{11}c^6d^3x^{11}(a+b\arccos$$

output

```
16/1155*b*d^3*(-c^2*x^2+1)^(1/2)/c^5+8/3465*b*d^3*(-c^2*x^2+1)^(3/2)/c^5+2
/1925*b*d^3*(-c^2*x^2+1)^(5/2)/c^5+1/1617*b*d^3*(-c^2*x^2+1)^(7/2)/c^5-4/2
97*b*d^3*(-c^2*x^2+1)^(9/2)/c^5+1/121*b*d^3*(-c^2*x^2+1)^(11/2)/c^5+1/5*d^
3*x^5*(a+b*arccos(c*x))-3/7*c^2*d^3*x^7*(a+b*arccos(c*x))+1/3*c^4*d^3*x^9*
(a+b*arccos(c*x))-1/11*c^6*d^3*x^11*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.62

$$\int x^4\left(d-c^2dx^2\right)^3(a+b\arccos(cx))\,dx =$$
$$-\frac{d^3\left(3465ac^5x^5(-231+495c^2x^2-385c^4x^4+105c^6x^6)+b\sqrt{1-c^2x^2}(50488+25244c^2x^2+18933c^4x^4\right.}{40}$$

input
```
Integrate[x^4*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
-1/4002075*(d^3*(3465*a*c^5*x^5*(-231 + 495*c^2*x^2 - 385*c^4*x^4 + 105*c^
6*x^6) + b*Sqrt[1 - c^2*x^2]*(50488 + 25244*c^2*x^2 + 18933*c^4*x^4 - 1176
25*c^6*x^6 + 111475*c^8*x^8 - 33075*c^10*x^10) + 3465*b*c^5*x^5*(-231 + 49
5*c^2*x^2 - 385*c^4*x^4 + 105*c^6*x^6)*ArcCos[c*x]))/c^5
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5193, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(d-c^2dx^2\right)^3(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5193}$$

$$bc\int \frac{d^3x^5\left(-105c^6x^6+385c^4x^4-495c^2x^2+231\right)}{1155\sqrt{1-c^2x^2}}dx-\frac{1}{11}c^6d^3x^{11}(a+b\arccos(cx))+\frac{1}{3}c^4d^3x^9(a+b\arccos(cx))-\frac{3}{7}c^2d^3x^7(a+b\arccos(cx))+\frac{1}{5}d^3x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{bcd^3\int \frac{x^5\left(-105c^6x^6+385c^4x^4-495c^2x^2+231\right)}{\sqrt{1-c^2x^2}}dx}{1155}-\frac{1}{11}c^6d^3x^{11}(a+b\arccos(cx))+\frac{1}{3}c^4d^3x^9(a+b\arccos(cx))-\frac{3}{7}c^2d^3x^7(a+b\arccos(cx))+\frac{1}{5}d^3x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{2331}$$

$$\frac{bcd^3\int \frac{x^4(-105c^6x^6+385c^4x^4-495c^2x^2+231)}{\sqrt{1-c^2x^2}}dx^2}{2310}-\frac{1}{11}c^6d^3x^{11}(a+b\arccos(cx))+\frac{1}{3}c^4d^3x^9(a+b\arccos(cx))-\frac{3}{7}c^2d^3x^7(a+b\arccos(cx))+\frac{1}{5}d^3x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{2123}$$

$$\frac{bcd^3\int \left(\frac{105(1-c^2x^2)^{9/2}}{c^4}-\frac{140(1-c^2x^2)^{7/2}}{c^4}+\frac{5(1-c^2x^2)^{5/2}}{c^4}+\frac{6(1-c^2x^2)^{3/2}}{c^4}+\frac{8\sqrt{1-c^2x^2}}{c^4}+\frac{16}{c^4\sqrt{1-c^2x^2}}\right)dx^2}{2310}-\frac{1}{11}c^6d^3x^{11}(a+b\arccos(cx))+\frac{1}{3}c^4d^3x^9(a+b\arccos(cx))-\frac{3}{7}c^2d^3x^7(a+b\arccos(cx))+\frac{1}{5}d^3x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{11}c^6d^3x^{11}(a+b\arccos(cx))+\frac{1}{3}c^4d^3x^9(a+b\arccos(cx))-\frac{3}{7}c^2d^3x^7(a+b\arccos(cx))+\frac{1}{5}d^3x^5(a+b\arccos(cx))+\frac{bcd^3\left(-\frac{210(1-c^2x^2)^{11/2}}{11c^6}+\frac{280(1-c^2x^2)^{9/2}}{9c^6}-\frac{10(1-c^2x^2)^{7/2}}{7c^6}-\frac{12(1-c^2x^2)^{5/2}}{5c^6}-\frac{16(1-c^2x^2)^{3/2}}{3c^6}-\frac{32\sqrt{1-c^2x^2}}{c^6}\right)}{2310}$$

input
```
Int[x^4*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d^3*((-32*Sqrt[1 - c^2*x^2])/c^6 - (16*(1 - c^2*x^2)^(3/2))/(3*c^6) -
 (12*(1 - c^2*x^2)^(5/2))/(5*c^6) - (10*(1 - c^2*x^2)^(7/2))/(7*c^6) + (28
0*(1 - c^2*x^2)^(9/2))/(9*c^6) - (210*(1 - c^2*x^2)^(11/2))/(11*c^6)))/231
0 + (d^3*x^5*(a + b*ArcCos[c*x]))/5 - (3*c^2*d^3*x^7*(a + b*ArcCos[c*x]))/
7 + (c^4*d^3*x^9*(a + b*ArcCos[c*x]))/3 - (c^6*d^3*x^11*(a + b*ArcCos[c*x]
))/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.91

method	result
parts	$-d^3a\left(\frac{1}{11}c^6x^{11}-\frac{1}{3}c^4x^9+\frac{3}{7}c^2x^7-\frac{1}{5}x^5\right)-\frac{d^3b\left(\frac{\arccos(cx)c^{11}x^{11}}{11}-\frac{\arccos(cx)c^9x^9}{3}+\frac{3\arccos(cx)c^7x^7}{7}-\text{arc}\right.}{}$
derivativedivides	$\frac{-d^3a\left(\frac{1}{11}c^{11}x^{11}-\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-d^3b\left(\frac{\arccos(cx)c^{11}x^{11}}{11}-\frac{\arccos(cx)c^9x^9}{3}+\frac{3\arccos(cx)c^7x^7}{7}-\frac{\arccos(cx)c^5x^5}{5}+\right.}{}$
default	$\frac{-d^3a\left(\frac{1}{11}c^{11}x^{11}-\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-d^3b\left(\frac{\arccos(cx)c^{11}x^{11}}{11}-\frac{\arccos(cx)c^9x^9}{3}+\frac{3\arccos(cx)c^7x^7}{7}-\frac{\arccos(cx)c^5x^5}{5}+\right.}{}$
orering	$\frac{(694575c^{12}x^{12}-2581075c^{10}x^{10}+3337325c^8x^8-1460245c^6x^6-176708c^4x^4-403904c^2x^2+201952)\left(-c^2dx^2+d\right)^3(a+}{4002075c^6x(cx-1)^2(cx+1)^2(c^2x^2-1)}$

input
```
int(x^4*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d^3*a*(1/11*c^6*x^11-1/3*c^4*x^9+3/7*c^2*x^7-1/5*x^5)-d^3*b/c^5*(1/11*arc
cos(c*x)*c^11*x^11-1/3*arccos(c*x)*c^9*x^9+3/7*arccos(c*x)*c^7*x^7-1/5*arc
cos(c*x)*c^5*x^5+6311/1334025*c^4*x^4*(-c^2*x^2+1)^(1/2)+25244/4002075*c^2
*x^2*(-c^2*x^2+1)^(1/2)+50488/4002075*(-c^2*x^2+1)^(1/2)-4705/160083*c^6*x
^6*(-c^2*x^2+1)^(1/2)+91/3267*c^8*x^8*(-c^2*x^2+1)^(1/2)-1/121*c^10*x^10*(
-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.82

$$\int x^4 \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx = -\frac{363825\,ac^{11}d^3x^{11}-1334025\,ac^9d^3x^9+1715175\,ac^7d^3x^7-800415\,ac^5d^3x^5+3465\,(105\,bc^{11}d^3x^{11}-38}{}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-1/4002075*(363825*a*c^11*d^3*x^11 - 1334025*a*c^9*d^3*x^9 + 1715175*a*c^7
*d^3*x^7 - 800415*a*c^5*d^3*x^5 + 3465*(105*b*c^11*d^3*x^11 - 385*b*c^9*d^
3*x^9 + 495*b*c^7*d^3*x^7 - 231*b*c^5*d^3*x^5)*arccos(c*x) - (33075*b*c^10
*d^3*x^10 - 111475*b*c^8*d^3*x^8 + 117625*b*c^6*d^3*x^6 - 18933*b*c^4*d^3*
x^4 - 25244*b*c^2*d^3*x^2 - 50488*b*d^3)*sqrt(-c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.27

$$\int x^4 \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx = \begin{cases} -\frac{ac^6d^3x^{11}}{11}+\frac{ac^4d^3x^9}{3}-\frac{3ac^2d^3x^7}{7}+\frac{ad^3x^5}{5}-\frac{bc^6d^3x^{11}\operatorname{acos}(cx)}{11}+\frac{bc^5d^3x^{10}\sqrt{-c^2x^2+1}}{121}+\frac{bc^4d^3x^9\operatorname{acos}(cx)}{3}-\frac{91bc^3d^3x^8\sqrt{-c}}{3267} \\ \frac{d^3x^5\left(a+\frac{\pi b}{2}\right)}{5} \end{cases}$$

input
```
integrate(x**4*(-c**2*d*x**2+d)**3*(a+b*acos(c*x)),x)
```

output
```
Piecewise((-a*c**6*d**3*x**11/11 + a*c**4*d**3*x**9/3 - 3*a*c**2*d**3*x**7
/7 + a*d**3*x**5/5 - b*c**6*d**3*x**11*acos(c*x)/11 + b*c**5*d**3*x**10*sq
rt(-c**2*x**2 + 1)/121 + b*c**4*d**3*x**9*acos(c*x)/3 - 91*b*c**3*d**3*x**
8*sqrt(-c**2*x**2 + 1)/3267 - 3*b*c**2*d**3*x**7*acos(c*x)/7 + 4705*b*c*d*
*3*x**6*sqrt(-c**2*x**2 + 1)/160083 + b*d**3*x**5*acos(c*x)/5 - 6311*b*d**
3*x**4*sqrt(-c**2*x**2 + 1)/(1334025*c) - 25244*b*d**3*x**2*sqrt(-c**2*x**
2 + 1)/(4002075*c**3) - 50488*b*d**3*sqrt(-c**2*x**2 + 1)/(4002075*c**5),
Ne(c, 0)), (d**3*x**5*(a + pi*b/2)/5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(200) = 400$.

Time = 0.13 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.08

$$\int x^4 (d - c^2 d x^2)^3 (a + b \arccos(cx)) \, dx = -\frac{1}{11} a c^6 d^3 x^{11} + \frac{1}{3} a c^4 d^3 x^9 - \frac{3}{7} a c^2 d^3 x^7$$
$$-\frac{1}{7623} \left(693 x^{11} \arccos(cx) - \left(\frac{63 \sqrt{-c^2x^2+1} x^{10}}{c^2} + \frac{70 \sqrt{-c^2x^2+1} x^8}{c^4} + \frac{80 \sqrt{-c^2x^2+1} x^6}{c^6} + \frac{96 \sqrt{-c\cdots}}{\cdots} \right.\right.$$
$$+\frac{1}{945} \left(315 x^9 \arccos(cx) - \left(\frac{35 \sqrt{-c^2x^2+1} x^8}{c^2} + \frac{40 \sqrt{-c^2x^2+1} x^6}{c^4} + \frac{48 \sqrt{-c^2x^2+1} x^4}{c^6} + \frac{64 \sqrt{-c^2x^2\cdots}}{c^8} \right.\right.$$
$$+\frac{1}{5} a d^3 x^5$$
$$-\frac{3}{245} \left(35 x^7 \arccos(cx) - \left(\frac{5 \sqrt{-c^2x^2+1} x^6}{c^2} + \frac{6 \sqrt{-c^2x^2+1} x^4}{c^4} + \frac{8 \sqrt{-c^2x^2+1} x^2}{c^6} + \frac{16 \sqrt{-c^2x^2+1\cdots}}{c^8} \right.\right.$$
$$+\frac{1}{75} \left(15 x^5 \arccos(cx) - \left(\frac{3 \sqrt{-c^2x^2+1} x^4}{c^2} + \frac{4 \sqrt{-c^2x^2+1} x^2}{c^4} + \frac{8 \sqrt{-c^2x^2+1}}{c^6} \right) c \right) b d^3$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-1/11*a*c^6*d^3*x^11 + 1/3*a*c^4*d^3*x^9 - 3/7*a*c^2*d^3*x^7 - 1/7623*(693
*x^11*arccos(c*x) - (63*sqrt(-c^2*x^2 + 1)*x^10/c^2 + 70*sqrt(-c^2*x^2 + 1
)*x^8/c^4 + 80*sqrt(-c^2*x^2 + 1)*x^6/c^6 + 96*sqrt(-c^2*x^2 + 1)*x^4/c^8
+ 128*sqrt(-c^2*x^2 + 1)*x^2/c^10 + 256*sqrt(-c^2*x^2 + 1)/c^12)*c)*b*c^6*
d^3 + 1/945*(315*x^9*arccos(c*x) - (35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqr
t(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2
 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/c^10)*c)*b*c^4*d^3 + 1/5*a*d^3*x^5
- 3/245*(35*x^7*arccos(c*x) - (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*
x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^
8)*c)*b*c^2*d^3 + 1/75*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2
 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.06

$$\begin{aligned}\int x^4(d-c^2dx^2)^3(a+b\arccos(cx))\,dx &= -\frac{1}{11}\,bc^6d^3x^{11}\arccos(cx)-\frac{1}{11}\,ac^6d^3x^{11}+\frac{1}{121}\,\sqrt{-c^2x^2+1}bc^5d^3x^{10}\\&\quad+\frac{1}{3}\,bc^4d^3x^9\arccos(cx)+\frac{1}{3}\,ac^4d^3x^9-\frac{91}{3267}\,\sqrt{-c^2x^2+1}bc^3d^3x^8\\&\quad-\frac{3}{7}\,bc^2d^3x^7\arccos(cx)-\frac{3}{7}\,ac^2d^3x^7+\frac{4705}{160083}\,\sqrt{-c^2x^2+1}bcd^3x^6\\&\quad+\frac{1}{5}\,bd^3x^5\arccos(cx)+\frac{1}{5}\,ad^3x^5-\frac{6311\,\sqrt{-c^2x^2+1}bd^3x^4}{1334025\,c}\\&\quad-\frac{25244\,\sqrt{-c^2x^2+1}bd^3x^2}{4002075\,c^3}-\frac{50488\,\sqrt{-c^2x^2+1}bd^3}{4002075\,c^5}\end{aligned}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-1/11*b*c^6*d^3*x^11*arccos(c*x) - 1/11*a*c^6*d^3*x^11 + 1/121*sqrt(-c^2*x
^2 + 1)*b*c^5*d^3*x^10 + 1/3*b*c^4*d^3*x^9*arccos(c*x) + 1/3*a*c^4*d^3*x^9
 - 91/3267*sqrt(-c^2*x^2 + 1)*b*c^3*d^3*x^8 - 3/7*b*c^2*d^3*x^7*arccos(c*x
) - 3/7*a*c^2*d^3*x^7 + 4705/160083*sqrt(-c^2*x^2 + 1)*b*c*d^3*x^6 + 1/5*b
*d^3*x^5*arccos(c*x) + 1/5*a*d^3*x^5 - 6311/1334025*sqrt(-c^2*x^2 + 1)*b*d
^3*x^4/c - 25244/4002075*sqrt(-c^2*x^2 + 1)*b*d^3*x^2/c^3 - 50488/4002075*
sqrt(-c^2*x^2 + 1)*b*d^3/c^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d-c^2dx^2)^3(a+b\arccos(cx))\,dx=\int x^4\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

input
```
int(x^4*(a + b*acos(c*x))*(d - c^2*d*x^2)^3,x)
```

output
```
int(x^4*(a + b*acos(c*x))*(d - c^2*d*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.91

$$\int x^4(d-c^2dx^2)^3(a+b\arccos(cx))\,dx$$
$$=\frac{d^3\left(-363825acos(cx)\,b\,c^{11}x^{11}+1334025acos(cx)\,b\,c^9x^9-1715175acos(cx)\,b\,c^7x^7+800415acos(cx)\,b\,c^5x\right.}{}$$

input
```
int(x^4*(-c^2*d*x^2+d)^3*(a+b*acos(c*x)),x)
```

output
```
(d**3*( - 363825*acos(c*x)*b*c**11*x**11 + 1334025*acos(c*x)*b*c**9*x**9 -
 1715175*acos(c*x)*b*c**7*x**7 + 800415*acos(c*x)*b*c**5*x**5 + 33075*sqrt
( - c**2*x**2 + 1)*b*c**10*x**10 - 111475*sqrt( - c**2*x**2 + 1)*b*c**8*x*
*8 + 117625*sqrt( - c**2*x**2 + 1)*b*c**6*x**6 - 18933*sqrt( - c**2*x**2 +
 1)*b*c**4*x**4 - 25244*sqrt( - c**2*x**2 + 1)*b*c**2*x**2 - 50488*sqrt( -
 c**2*x**2 + 1)*b - 363825*a*c**11*x**11 + 1334025*a*c**9*x**9 - 1715175*a
*c**7*x**7 + 800415*a*c**5*x**5))/(4002075*c**5)
```

3.22 $\int x^3(d-c^2dx^2)^3 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 206

$$\int x^3(d-c^2dx^2)^3 (a+b\arccos(cx))\,dx = \frac{49bd^3x\sqrt{1-c^2x^2}}{5120c^3}+\frac{49bd^3x(1-c^2x^2)^{3/2}}{7680c^3}+\frac{49bd^3x(1-c^2x^2)^{5/2}}{9600c^3}+\frac{7bd^3x(1-c^2x^2)^{7/2}}{1600c^3}-\frac{bd^3x(1-c^2x^2)^{9/2}}{100c^3}+\frac{49bd^3\arccos(cx)}{5120c^4}-\frac{d^3(1-c^2x^2)^4(a+b\arccos(cx))}{8c^4}+\frac{d^3(1-c^2x^2)^5(a+b\arccos(cx))}{10c^4}$$

output
```
49/5120*b*d^3*x*(-c^2*x^2+1)^(1/2)/c^3+49/7680*b*d^3*x*(-c^2*x^2+1)^(3/2)/
c^3+49/9600*b*d^3*x*(-c^2*x^2+1)^(5/2)/c^3+7/1600*b*d^3*x*(-c^2*x^2+1)^(7/
2)/c^3-1/100*b*d^3*x*(-c^2*x^2+1)^(9/2)/c^3+49/5120*b*d^3*arccos(c*x)/c^4-
1/8*d^3*(-c^2*x^2+1)^4*(a+b*arccos(c*x))/c^4+1/10*d^3*(-c^2*x^2+1)^5*(a+b*
arccos(c*x))/c^4
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.70

$$\int x^3 \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx = -\frac{d^3\left(1920ac^4x^4(-10+20c^2x^2-15c^4x^4+4c^6x^6)+bcx\sqrt{1-c^2x^2}(1185+790c^2x^2-3208c^4x^4+2736c^6\right.}{76800c^4}$$

input
```
Integrate[x^3*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
-1/76800*(d^3*(1920*a*c^4*x^4*(-10 + 20*c^2*x^2 - 15*c^4*x^4 + 4*c^6*x^6)
+ b*c*x*Sqrt[1 - c^2*x^2]*(1185 + 790*c^2*x^2 - 3208*c^4*x^4 + 2736*c^6*x^
6 - 768*c^8*x^8) + 1920*b*c^4*x^4*(-10 + 20*c^2*x^2 - 15*c^4*x^4 + 4*c^6*x
^6)*ArcCos[c*x] - 1185*b*ArcSin[c*x]))/c^4
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5193, 27, 299, 211, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5193}$$

$$bc\int -\frac{d^3\left(1-c^2x^2\right)^{7/2}\left(4c^2x^2+1\right)}{40c^4}dx+\frac{d^3\left(1-c^2x^2\right)^5(a+b\arccos(cx))}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))}{8c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bd^3\int\left(1-c^2x^2\right)^{7/2}\left(4c^2x^2+1\right)dx}{40c^3}+\frac{d^3\left(1-c^2x^2\right)^5(a+b\arccos(cx))}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))}{8c^4}$$

$$\Big\downarrow \text{299}$$

$$-\frac{bd^3\left(\frac{7}{5}\int\left(1-c^2x^2\right)^{7/2}dx-\frac{2}{5}x\left(1-c^2x^2\right)^{9/2}\right)}{40c^3}+\frac{d^3\left(1-c^2x^2\right)^5(a+b\arccos(cx))}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))}{8c^4}$$

$$\Big\downarrow \text{211}$$

$$-\frac{bd^3\left(\frac{7}{5}\left(\frac{7}{8}\int\left(1-c^2x^2\right)^{5/2}dx+\frac{1}{8}x\left(1-c^2x^2\right)^{7/2}\right)-\frac{2}{5}x\left(1-c^2x^2\right)^{9/2}\right)}{40c^3}+\frac{d^3\left(1-c^2x^2\right)^5(a+b\arccos(cx))}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))}{8c^4}$$

$$\Big\downarrow \text{211}$$

$$-\frac{bd^3\left(\frac{7}{5}\left(\frac{7}{8}\left(\frac{5}{6}\int\left(1-c^2x^2\right)^{3/2}dx+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)+\frac{1}{8}x\left(1-c^2x^2\right)^{7/2}\right)-\frac{2}{5}x\left(1-c^2x^2\right)^{9/2}\right)}{40c^3}+\frac{d^3\left(1-c^2x^2\right)^5(a+b\arccos(cx))}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))}{8c^4}$$

$$\Big\downarrow \text{211}$$

$$-\frac{bd^3\left(\frac{7}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{1-c^2x^2}dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)+\frac{1}{8}x\left(1-c^2x^2\right)^{7/2}\right)-\frac{2}{5}x\left(1-c^2x^2\right)^{9/2}}{40c^3}$$

$$\frac{d^3\left(1-c^2x^2\right)^5(a+b\arccos(cx))}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))}{8c^4}$$

$$\Big\downarrow \text{211}$$

$$-\frac{bd^3\left(\frac{7}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)+\frac{1}{8}x\left(1-c^2x^2\right)^{7/2}\right)}{40c^3}$$

$$\frac{d^3\left(1-c^2x^2\right)^5(a+b\arccos(cx))}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))}{8c^4}$$

$$\Big\downarrow \text{223}$$

$$\frac{d^3(1-c^2x^2)^5(a+b\arccos(cx))}{10c^4}-\frac{d^3(1-c^2x^2)^4(a+b\arccos(cx))}{8c^4}-$$
$$\frac{bd^3\left(\frac{7}{5}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)+\frac{1}{8}x(1-c^2x^2)^{7/2}\right)-\frac{2}{5}x(1\right.}{40c^3}$$

input
```
Int[x^3*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
-1/8*(d^3*(1 - c^2*x^2)^4*(a + b*ArcCos[c*x]))/c^4 + (d^3*(1 - c^2*x^2)^5*
(a + b*ArcCos[c*x]))/(10*c^4) - (b*d^3*((-2*x*(1 - c^2*x^2)^(9/2))/5 + (7*
((x*(1 - c^2*x^2)^(7/2))/8 + (7*((x*(1 - c^2*x^2)^(5/2))/6 + (5*((x*(1 - c
^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/6
))/8))/5))/(40*c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.96

method	result
parts	$-d^3a\left(\frac{1}{10}c^6x^{10}-\frac{3}{8}c^4x^8+\frac{1}{2}c^2x^6-\frac{1}{4}x^4\right)-\frac{d^3b\left(\frac{\arccos(cx)c^{10}x^{10}}{10}-\frac{3\arccos(cx)c^8x^8}{8}+\frac{\arccos(cx)c^6x^6}{2}-\frac{c^4x\cdots}{}\right.}{}$
derivativedivides	$\frac{-d^3a\left(\frac{1}{10}c^{10}x^{10}-\frac{3}{8}c^8x^8+\frac{1}{2}c^6x^6-\frac{1}{4}c^4x^4\right)-d^3b\left(\frac{\arccos(cx)c^{10}x^{10}}{10}-\frac{3\arccos(cx)c^8x^8}{8}+\frac{\arccos(cx)c^6x^6}{2}-\frac{c^4x^4\arccos(cx)}{4}+\cdots\right.}{c^4}$
default	$\frac{-d^3a\left(\frac{1}{10}c^{10}x^{10}-\frac{3}{8}c^8x^8+\frac{1}{2}c^6x^6-\frac{1}{4}c^4x^4\right)-d^3b\left(\frac{\arccos(cx)c^{10}x^{10}}{10}-\frac{3\arccos(cx)c^8x^8}{8}+\frac{\arccos(cx)c^6x^6}{2}-\frac{c^4x^4\arccos(cx)}{4}+\cdots\right.}{c^4}$
orering	$\frac{(4864c^{10}x^{10}-18576c^8x^8+25160c^6x^6-11978c^4x^4-2765c^2x^2+1580)(-c^2dx^2+d)^3(a+b\arccos(cx))}{25600c^4(cx-1)^2(cx+1)^2(c^2x^2-1)}-\frac{(768c^8x^8-2\cdots}{}$

input
```
int(x^3*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d^3*a*(1/10*c^6*x^10-3/8*c^4*x^8+1/2*c^2*x^6-1/4*x^4)-d^3*b/c^4*(1/10*arc
cos(c*x)*c^10*x^10-3/8*arccos(c*x)*c^8*x^8+1/2*arccos(c*x)*c^6*x^6-1/4*c^4
*x^4*arccos(c*x)+79/7680*c^3*x^3*(-c^2*x^2+1)^(1/2)+79/5120*c*x*(-c^2*x^2+
1)^(1/2)-79/5120*arcsin(c*x)-401/9600*c^5*x^5*(-c^2*x^2+1)^(1/2)+57/1600*c
^7*x^7*(-c^2*x^2+1)^(1/2)-1/100*c^9*x^9*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.90

$$\int x^3(d-c^2dx^2)^3(a+b\arccos(cx))\,dx =$$
$$-\frac{7680\,ac^{10}d^3x^{10}-28800\,ac^8d^3x^8+38400\,ac^6d^3x^6-19200\,ac^4d^3x^4+15\,(512\,bc^{10}d^3x^{10}-1920\,bc^8d^3x^8}{}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-1/76800*(7680*a*c^10*d^3*x^10 - 28800*a*c^8*d^3*x^8 + 38400*a*c^6*d^3*x^6
 - 19200*a*c^4*d^3*x^4 + 15*(512*b*c^10*d^3*x^10 - 1920*b*c^8*d^3*x^8 + 25
60*b*c^6*d^3*x^6 - 1280*b*c^4*d^3*x^4 + 79*b*d^3)*arccos(c*x) - (768*b*c^9
*d^3*x^9 - 2736*b*c^7*d^3*x^7 + 3208*b*c^5*d^3*x^5 - 790*b*c^3*d^3*x^3 - 1
185*b*c*d^3*x)*sqrt(-c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.39

$$\int x^3(d-c^2dx^2)^3(a+b\arccos(cx))\,dx$$
$$=\begin{cases}-\frac{ac^6d^3x^{10}}{10}+\frac{3ac^4d^3x^8}{8}-\frac{ac^2d^3x^6}{2}+\frac{ad^3x^4}{4}-\frac{bc^6d^3x^{10}\operatorname{acos}(cx)}{10}+\frac{bc^5d^3x^9\sqrt{-c^2x^2+1}}{100}+\frac{3bc^4d^3x^8\operatorname{acos}(cx)}{8}-\frac{57bc^3d^3x^7\sqrt{-c}}{1600} \\ \frac{d^3x^4\left(a+\frac{\pi b}{2}\right)}{4}\end{cases}$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**3*(a+b*acos(c*x)),x)
```

output
```
Piecewise((-a*c**6*d**3*x**10/10 + 3*a*c**4*d**3*x**8/8 - a*c**2*d**3*x**6
/2 + a*d**3*x**4/4 - b*c**6*d**3*x**10*acos(c*x)/10 + b*c**5*d**3*x**9*sqr
t(-c**2*x**2 + 1)/100 + 3*b*c**4*d**3*x**8*acos(c*x)/8 - 57*b*c**3*d**3*x*
*7*sqrt(-c**2*x**2 + 1)/1600 - b*c**2*d**3*x**6*acos(c*x)/2 + 401*b*c*d**3
*x**5*sqrt(-c**2*x**2 + 1)/9600 + b*d**3*x**4*acos(c*x)/4 - 79*b*d**3*x**3
*sqrt(-c**2*x**2 + 1)/(7680*c) - 79*b*d**3*x*sqrt(-c**2*x**2 + 1)/(5120*c*
*3) - 79*b*d**3*acos(c*x)/(5120*c**4), Ne(c, 0)), (d**3*x**4*(a + pi*b/2)/
4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(178) = 356$.

Time = 0.12 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.15

$$\int x^3 \left(d - c^2 d x^2\right)^3 (a + b \arccos(cx))\, dx = -\frac{1}{10}\, a c^6 d^3 x^{10} + \frac{3}{8}\, a c^4 d^3 x^8 - \frac{1}{2}\, a c^2 d^3 x^6$$
$$- \frac{1}{12800} \left(1280\, x^{10} \arccos(cx) - \left(\frac{128\, \sqrt{-c^2 x^2 + 1} x^9}{c^2} + \frac{144\, \sqrt{-c^2 x^2 + 1} x^7}{c^4} + \frac{168\, \sqrt{-c^2 x^2 + 1} x^5}{c^6} + 21\right.\right.$$
$$+ \frac{1}{1024} \left(384\, x^8 \arccos(cx) - \left(\frac{48\, \sqrt{-c^2 x^2 + 1} x^7}{c^2} + \frac{56\, \sqrt{-c^2 x^2 + 1} x^5}{c^4} + \frac{70\, \sqrt{-c^2 x^2 + 1} x^3}{c^6} + \frac{105\, \sqrt{-c^2}}{c^8}\right.\right.$$
$$+ \frac{1}{4}\, a d^3 x^4$$
$$- \frac{1}{96} \left(48\, x^6 \arccos(cx) - \left(\frac{8\, \sqrt{-c^2 x^2 + 1} x^5}{c^2} + \frac{10\, \sqrt{-c^2 x^2 + 1} x^3}{c^4} + \frac{15\, \sqrt{-c^2 x^2 + 1} x}{c^6} - \frac{15\, \arcsin(cx)}{c^7}\right)\right.$$
$$+ \frac{1}{32} \left(8\, x^4 \arccos(cx) - \left(\frac{2\, \sqrt{-c^2 x^2 + 1} x^3}{c^2} + \frac{3\, \sqrt{-c^2 x^2 + 1} x}{c^4} - \frac{3\, \arcsin(cx)}{c^5}\right) c\right) b d^3$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-1/10*a*c^6*d^3*x^10 + 3/8*a*c^4*d^3*x^8 - 1/2*a*c^2*d^3*x^6 - 1/12800*(12
80*x^10*arccos(c*x) - (128*sqrt(-c^2*x^2 + 1)*x^9/c^2 + 144*sqrt(-c^2*x^2
+ 1)*x^7/c^4 + 168*sqrt(-c^2*x^2 + 1)*x^5/c^6 + 210*sqrt(-c^2*x^2 + 1)*x^3
/c^8 + 315*sqrt(-c^2*x^2 + 1)*x/c^10 - 315*arcsin(c*x)/c^11)*c)*b*c^6*d^3
+ 1/1024*(384*x^8*arccos(c*x) - (48*sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*sqrt(-
c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(-c^2*x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2*x^2 +
 1)*x/c^8 - 105*arcsin(c*x)/c^9)*c)*b*c^4*d^3 + 1/4*a*d^3*x^4 - 1/96*(48*x
^6*arccos(c*x) - (8*sqrt(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3
/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*b*c^2*d^3 + 1/
32*(8*x^4*arccos(c*x) - (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 +
1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*d^3
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.15

$$\int x^3\left(d-c^2dx^2\right)^3\left(a+b\arccos(cx)\right)dx$$
$$=-\frac{1}{10}\,bc^6d^3x^{10}\arccos\left(cx\right)-\frac{1}{10}\,ac^6d^3x^{10}+\frac{1}{100}\,\sqrt{-c^2x^2+1}bc^5d^3x^9+\frac{3}{8}\,bc^4d^3x^8\arccos\left(cx\right)+\frac{3}{8}\,ac^4d^3x^8-\frac{57}{1600}\,\sqrt{-c^2x^2+1}bc^3d^3x^7-\frac{1}{2}\,bc^2d^3x^6\arccos\left(cx\right)-\frac{1}{2}\,ac^2d^3x^6+\frac{401}{9600}\,\sqrt{-c^2x^2+1}bcd^3x^5+\frac{1}{4}\,bd^3x^4\arccos\left(cx\right)+\frac{1}{4}\,ad^3x^4-\frac{79\,\sqrt{-c^2x^2+1}bd^3x^3}{7680\,c}-\frac{79\,\sqrt{-c^2x^2+1}bd^3x}{5120\,c^3}-\frac{79\,bd^3\arccos\left(cx\right)}{5120\,c^4}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-1/10*b*c^6*d^3*x^10*arccos(c*x) - 1/10*a*c^6*d^3*x^10 + 1/100*sqrt(-c^2*x
^2 + 1)*b*c^5*d^3*x^9 + 3/8*b*c^4*d^3*x^8*arccos(c*x) + 3/8*a*c^4*d^3*x^8
- 57/1600*sqrt(-c^2*x^2 + 1)*b*c^3*d^3*x^7 - 1/2*b*c^2*d^3*x^6*arccos(c*x)
 - 1/2*a*c^2*d^3*x^6 + 401/9600*sqrt(-c^2*x^2 + 1)*b*c*d^3*x^5 + 1/4*b*d^3
*x^4*arccos(c*x) + 1/4*a*d^3*x^4 - 79/7680*sqrt(-c^2*x^2 + 1)*b*d^3*x^3/c
- 79/5120*sqrt(-c^2*x^2 + 1)*b*d^3*x/c^3 - 79/5120*b*d^3*arccos(c*x)/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(d-c^2dx^2\right)^3\left(a+b\arccos(cx)\right)dx=\int x^3\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^3\,dx$$

input
```
int(x^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^3,x)
```

output
```
int(x^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.97

$$\int x^3(d-c^2dx^2)^3(a+b\arccos(cx))\,dx$$
$$=\frac{d^3(-7680acos(cx)\,b\,c^{10}x^{10}+28800acos(cx)\,b\,c^8x^8-38400acos(cx)\,b\,c^6x^6+19200acos(cx)\,b\,c^4x^4+118}{}$$

input
```
int(x^3*(-c^2*d*x^2+d)^3*(a+b*acos(c*x)),x)
```

output
```
(d**3*( - 7680*acos(c*x)*b*c**10*x**10 + 28800*acos(c*x)*b*c**8*x**8 - 384
00*acos(c*x)*b*c**6*x**6 + 19200*acos(c*x)*b*c**4*x**4 + 1185*asin(c*x)*b
+ 768*sqrt( - c**2*x**2 + 1)*b*c**9*x**9 - 2736*sqrt( - c**2*x**2 + 1)*b*c
**7*x**7 + 3208*sqrt( - c**2*x**2 + 1)*b*c**5*x**5 - 790*sqrt( - c**2*x**2
 + 1)*b*c**3*x**3 - 1185*sqrt( - c**2*x**2 + 1)*b*c*x - 7680*a*c**10*x**10
 + 28800*a*c**8*x**8 - 38400*a*c**6*x**6 + 19200*a*c**4*x**4))/(76800*c**4
)
```

3.23 $\int x^2(d-c^2dx^2)^3\,(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 207

$$\int x^2(d-c^2dx^2)^3\,(a+b\arccos(cx))\,dx=\frac{16bd^3\sqrt{1-c^2x^2}}{315c^3}+\frac{8bd^3(1-c^2x^2)^{3/2}}{945c^3}+\frac{2bd^3(1-c^2x^2)^{5/2}}{525c^3}+\frac{bd^3(1-c^2x^2)^{7/2}}{441c^3}-\frac{bd^3(1-c^2x^2)^{9/2}}{81c^3}+\frac{1}{3}d^3x^3(a+b\arccos(cx))-\frac{3}{5}c^2d^3x^5(a+b\arccos(cx))+\frac{3}{7}c^4d^3x^7(a+b\arccos(cx))-\frac{1}{9}c^6d^3x^9(a+b\arccos(c$$

output
```
16/315*b*d^3*(-c^2*x^2+1)^(1/2)/c^3+8/945*b*d^3*(-c^2*x^2+1)^(3/2)/c^3+2/5
25*b*d^3*(-c^2*x^2+1)^(5/2)/c^3+1/441*b*d^3*(-c^2*x^2+1)^(7/2)/c^3-1/81*b*
d^3*(-c^2*x^2+1)^(9/2)/c^3+1/3*d^3*x^3*(a+b*arccos(c*x))-3/5*c^2*d^3*x^5*(
a+b*arccos(c*x))+3/7*c^4*d^3*x^7*(a+b*arccos(c*x))-1/9*c^6*d^3*x^9*(a+b*ar
ccos(c*x))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.65

$$\int x^2 (d - c^2 d x^2)^3 (a + b \arccos(cx)) \, dx =$$
$$-\frac{d^3 \big(315 a c^3 x^3 (-105 + 189 c^2 x^2 - 135 c^4 x^4 + 35 c^6 x^6) + b\sqrt{1 - c^2 x^2}(5258 + 2629 c^2 x^2 - 6297 c^4 x^4 + 467}{99225 c^3}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
-1/99225*(d^3*(315*a*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^4 + 35*c^6*x^
6) + b*Sqrt[1 - c^2*x^2]*(5258 + 2629*c^2*x^2 - 6297*c^4*x^4 + 4675*c^6*x^
6 - 1225*c^8*x^8) + 315*b*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^4 + 35*c
^6*x^6)*ArcCos[c*x]))/c^3
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5193, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d - c^2 d x^2)^3 (a + b \arccos(cx)) \, dx$$

$$\Big\downarrow \text{ 5193}$$

$$bc \int \frac{d^3 x^3 (-35 c^6 x^6 + 135 c^4 x^4 - 189 c^2 x^2 + 105)}{315\sqrt{1 - c^2 x^2}} dx - \frac{1}{9} c^6 d^3 x^9 (a + b \arccos(cx)) + \frac{3}{7} c^4 d^3 x^7 (a + b \arccos(cx)) - \frac{3}{5} c^2 d^3 x^5 (a + b \arccos(cx)) + \frac{1}{3} d^3 x^3 (a + b \arccos(cx))$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{315} bcd^3 \int \frac{x^3 (-35 c^6 x^6 + 135 c^4 x^4 - 189 c^2 x^2 + 105)}{\sqrt{1 - c^2 x^2}} dx - \frac{1}{9} c^6 d^3 x^9 (a + b \arccos(cx)) + \frac{3}{7} c^4 d^3 x^7 (a + b \arccos(cx)) - \frac{3}{5} c^2 d^3 x^5 (a + b \arccos(cx)) + \frac{1}{3} d^3 x^3 (a + b \arccos(cx))$$

$$\Big\downarrow \; 2331$$

$$\frac{1}{630}bcd^3\int \frac{x^2(-35c^6x^6+135c^4x^4-189c^2x^2+105)}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{9}c^6d^3x^9(a+b\arccos(cx))+\frac{3}{7}c^4d^3x^7(a+b\arccos(cx))-\frac{3}{5}c^2d^3x^5(a+b\arccos(cx))+\frac{1}{3}d^3x^3(a+b\arccos(cx))$$

$$\Big\downarrow \; 2123$$

$$\frac{1}{630}bcd^3\int \left(-\frac{35(1-c^2x^2)^{7/2}}{c^2}+\frac{5(1-c^2x^2)^{5/2}}{c^2}+\frac{6(1-c^2x^2)^{3/2}}{c^2}+\frac{8\sqrt{1-c^2x^2}}{c^2}+\frac{16}{c^2\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{9}c^6d^3x^9(a+b\arccos(cx))+\frac{3}{7}c^4d^3x^7(a+b\arccos(cx))-\frac{3}{5}c^2d^3x^5(a+b\arccos(cx))+\frac{1}{3}d^3x^3(a+b\arccos(cx))$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{9}c^6d^3x^9(a+b\arccos(cx))+\frac{3}{7}c^4d^3x^7(a+b\arccos(cx))-\frac{3}{5}c^2d^3x^5(a+b\arccos(cx))+\frac{1}{3}d^3x^3(a+b\arccos(cx))+\frac{1}{630}bcd^3\left(\frac{70(1-c^2x^2)^{9/2}}{9c^4}-\frac{10(1-c^2x^2)^{7/2}}{7c^4}-\frac{12(1-c^2x^2)^{5/2}}{5c^4}-\frac{16(1-c^2x^2)^{3/2}}{3c^4}-\frac{32\sqrt{1-c^2x^2}}{c^4}\right)$$

input
```
Int[x^2*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d^3*((-32*Sqrt[1 - c^2*x^2])/c^4 - (16*(1 - c^2*x^2)^(3/2))/(3*c^4) -
 (12*(1 - c^2*x^2)^(5/2))/(5*c^4) - (10*(1 - c^2*x^2)^(7/2))/(7*c^4) + (70
*(1 - c^2*x^2)^(9/2))/(9*c^4)))/630 + (d^3*x^3*(a + b*ArcCos[c*x]))/3 - (3
*c^2*d^3*x^5*(a + b*ArcCos[c*x]))/5 + (3*c^4*d^3*x^7*(a + b*ArcCos[c*x]))/
7 - (c^6*d^3*x^9*(a + b*ArcCos[c*x]))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.92

method	result
parts	$-d^3 a\left(\frac{1}{9}c^6x^9-\frac{3}{7}c^4x^7+\frac{3}{5}c^2x^5-\frac{1}{3}x^3\right)-\frac{d^3 b\left(\frac{\arccos(cx)c^9x^9}{9}-\frac{3\arccos(cx)c^7x^7}{7}+\frac{3\arccos(cx)c^5x^5}{5}-\frac{c^3x^3 a\cdots}{}\right.}{\cdots}$
derivativedivides	$\frac{-d^3 a\left(\frac{1}{9}c^9x^9-\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-d^3 b\left(\frac{\arccos(cx)c^9x^9}{9}-\frac{3\arccos(cx)c^7x^7}{7}+\frac{3\arccos(cx)c^5x^5}{5}-\frac{c^3x^3\arccos(cx)}{3}+2629\cdots\right.}{c^3}$
default	$\frac{-d^3 a\left(\frac{1}{9}c^9x^9-\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-d^3 b\left(\frac{\arccos(cx)c^9x^9}{9}-\frac{3\arccos(cx)c^7x^7}{7}+\frac{3\arccos(cx)c^5x^5}{5}-\frac{c^3x^3\arccos(cx)}{3}+2629\cdots\right.}{c^3}$
orering	$\frac{\left(20825c^{10}x^{10}-82375c^8x^8+119261c^6x^6-66701c^4x^4-36806c^2x^2+10516\right)\left(-c^2d\,x^2+d\right)^3\left(a+b\arccos(cx)\right)}{99225c^4x(cx-1)^2(cx+1)^2(c^2x^2-1)}-\frac{\left(1225c^8\cdots\right.}{\cdots}$

input
```
int(x^2*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-d^3*a*(1/9*c^6*x^9-3/7*c^4*x^7+3/5*c^2*x^5-1/3*x^3)-d^3*b/c^3*(1/9*arccos
(c*x)*c^9*x^9-3/7*arccos(c*x)*c^7*x^7+3/5*arccos(c*x)*c^5*x^5-1/3*c^3*x^3*
arccos(c*x)+2629/99225*c^2*x^2*(-c^2*x^2+1)^(1/2)+5258/99225*(-c^2*x^2+1)^
(1/2)-2099/33075*c^4*x^4*(-c^2*x^2+1)^(1/2)+187/3969*c^6*x^6*(-c^2*x^2+1)^
(1/2)-1/81*c^8*x^8*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.86

$$\int x^2\left(d-c^2dx^2\right)^3\left(a+b\arccos(cx)\right)dx =$$
$$-\frac{11025\,ac^9d^3x^9-42525\,ac^7d^3x^7+59535\,ac^5d^3x^5-33075\,ac^3d^3x^3+315\left(35\,bc^9d^3x^9-135\,bc^7d^3x^7+1\right.}{}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-1/99225*(11025*a*c^9*d^3*x^9 - 42525*a*c^7*d^3*x^7 + 59535*a*c^5*d^3*x^5
- 33075*a*c^3*d^3*x^3 + 315*(35*b*c^9*d^3*x^9 - 135*b*c^7*d^3*x^7 + 189*b*
c^5*d^3*x^5 - 105*b*c^3*d^3*x^3)*arccos(c*x) - (1225*b*c^8*d^3*x^8 - 4675*
b*c^6*d^3*x^6 + 6297*b*c^4*d^3*x^4 - 2629*b*c^2*d^3*x^2 - 5258*b*d^3)*sqrt
(-c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.30

$$\int x^2\left(d-c^2dx^2\right)^3\left(a+b\arccos(cx)\right)dx$$
$$=\begin{cases}-\frac{ac^6d^3x^9}{9}+\frac{3ac^4d^3x^7}{7}-\frac{3ac^2d^3x^5}{5}+\frac{ad^3x^3}{3}-\frac{bc^6d^3x^9\operatorname{acos}(cx)}{9}+\frac{bc^5d^3x^8\sqrt{-c^2x^2+1}}{81}+\frac{3bc^4d^3x^7\operatorname{acos}(cx)}{7}-\frac{187bc^3d^3x^6\sqrt{-c}}{3969}\\ \frac{d^3x^3\left(a+\frac{\pi b}{2}\right)}{3}\end{cases}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**3*(a+b*acos(c*x)),x)
```

output
```
Piecewise((-a*c**6*d**3*x**9/9 + 3*a*c**4*d**3*x**7/7 - 3*a*c**2*d**3*x**5
/5 + a*d**3*x**3/3 - b*c**6*d**3*x**9*acos(c*x)/9 + b*c**5*d**3*x**8*sqrt(
-c**2*x**2 + 1)/81 + 3*b*c**4*d**3*x**7*acos(c*x)/7 - 187*b*c**3*d**3*x**6
*sqrt(-c**2*x**2 + 1)/3969 - 3*b*c**2*d**3*x**5*acos(c*x)/5 + 2099*b*c*d**
3*x**4*sqrt(-c**2*x**2 + 1)/33075 + b*d**3*x**3*acos(c*x)/3 - 2629*b*d**3*
x**2*sqrt(-c**2*x**2 + 1)/(99225*c) - 5258*b*d**3*sqrt(-c**2*x**2 + 1)/(99
225*c**3), Ne(c, 0)), (d**3*x**3*(a + pi*b/2)/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(179) = 358$.

Time = 0.14 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.94

$$\int x^2 (d - c^2 d x^2)^3 (a + b \arccos(cx)) \, dx = -\frac{1}{9} a c^6 d^3 x^9 + \frac{3}{7} a c^4 d^3 x^7$$

$$-\frac{1}{2835} \left(315 x^9 \arccos(cx) - \left(\frac{35 \sqrt{-c^2x^2+1} x^8}{c^2} + \frac{40 \sqrt{-c^2x^2+1} x^6}{c^4} + \frac{48 \sqrt{-c^2x^2+1} x^4}{c^6} + \frac{64 \sqrt{-c^2x}}{c^8} \right. \right.$$

$$-\frac{3}{5} a c^2 d^3 x^5$$

$$+\frac{3}{245} \left(35 x^7 \arccos(cx) - \left(\frac{5 \sqrt{-c^2x^2+1} x^6}{c^2} + \frac{6 \sqrt{-c^2x^2+1} x^4}{c^4} + \frac{8 \sqrt{-c^2x^2+1} x^2}{c^6} + \frac{16 \sqrt{-c^2x^2+1}}{c^8} \right. \right.$$

$$-\frac{1}{25} \left(15 x^5 \arccos(cx) - \left(\frac{3 \sqrt{-c^2x^2+1} x^4}{c^2} + \frac{4 \sqrt{-c^2x^2+1} x^2}{c^4} + \frac{8 \sqrt{-c^2x^2+1}}{c^6} \right) c \right) b c^2 d^3$$

$$+\frac{1}{3} a d^3 x^3 + \frac{1}{9} \left(3 x^3 \arccos(cx) - c \left(\frac{\sqrt{-c^2x^2+1} x^2}{c^2} + \frac{2 \sqrt{-c^2x^2+1}}{c^4} \right) \right) b d^3$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-1/9*a*c^6*d^3*x^9 + 3/7*a*c^4*d^3*x^7 - 1/2835*(315*x^9*arccos(c*x) - (35
*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2
*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)
/c^10)*c)*b*c^6*d^3 - 3/5*a*c^2*d^3*x^5 + 3/245*(35*x^7*arccos(c*x) - (5*s
qrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2
 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*c^4*d^3 - 1/25*(15*x^5*arc
cos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 +
8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/9*(3*x^3*arccos
(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.07

$$\int x^2 \left(d-c^2 d x^2\right)^3 (a+b \arccos(cx))\,dx$$
$$= -\frac{1}{9}\,bc^6d^3x^9 \arccos\left(cx\right) - \frac{1}{9}\,ac^6d^3x^9 + \frac{1}{81}\,\sqrt{-c^2x^2+1}bc^5d^3x^8 + \frac{3}{7}\,bc^4d^3x^7 \arccos\left(cx\right) + \frac{3}{7}\,ac^4d^3x^7 - \frac{187}{3969}\,\sqrt{-c^2x^2+1}bc^3d^3x^6 - \frac{3}{5}\,bc^2d^3x^5 \arccos\left(cx\right) - \frac{3}{5}\,ac^2d^3x^5 + \frac{2099}{33075}\,\sqrt{-c^2x^2+1}bcd^3x^4 + \frac{1}{3}\,bd^3x^3 \arccos\left(cx\right) + \frac{1}{3}\,ad^3x^3 - \frac{2629\,\sqrt{-c^2x^2+1}bd^3x^2}{99225\,c} - \frac{5258\,\sqrt{-c^2x^2+1}bd^3}{99225\,c^3}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-1/9*b*c^6*d^3*x^9*arccos(c*x) - 1/9*a*c^6*d^3*x^9 + 1/81*sqrt(-c^2*x^2 +
1)*b*c^5*d^3*x^8 + 3/7*b*c^4*d^3*x^7*arccos(c*x) + 3/7*a*c^4*d^3*x^7 - 187
/3969*sqrt(-c^2*x^2 + 1)*b*c^3*d^3*x^6 - 3/5*b*c^2*d^3*x^5*arccos(c*x) - 3
/5*a*c^2*d^3*x^5 + 2099/33075*sqrt(-c^2*x^2 + 1)*b*c*d^3*x^4 + 1/3*b*d^3*x
^3*arccos(c*x) + 1/3*a*d^3*x^3 - 2629/99225*sqrt(-c^2*x^2 + 1)*b*d^3*x^2/c
 - 5258/99225*sqrt(-c^2*x^2 + 1)*b*d^3/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d - c^2dx^2)^3 (a + b \arccos(cx)) \, dx = \int x^2 (a + b \operatorname{acos}(c\,x)) \left(d - c^2\,d\,x^2\right)^3 \, dx$$

input
```
int(x^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^3,x)
```

output
```
int(x^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.92

$$\int x^2 (d - c^2dx^2)^3 (a + b \arccos(cx)) \, dx$$
$$= \frac{d^3 (-11025 acos(cx)\, b\, c^9 x^9 + 42525 acos(cx)\, b\, c^7 x^7 - 59535 acos(cx)\, b\, c^5 x^5 + 33075 acos(cx)\, b\, c^3 x^3 + 1225}{}$$

input
```
int(x^2*(-c^2*d*x^2+d)^3*(a+b*acos(c*x)),x)
```

output
```
(d**3*( - 11025*acos(c*x)*b*c**9*x**9 + 42525*acos(c*x)*b*c**7*x**7 - 5953
5*acos(c*x)*b*c**5*x**5 + 33075*acos(c*x)*b*c**3*x**3 + 1225*sqrt( - c**2*
x**2 + 1)*b*c**8*x**8 - 4675*sqrt( - c**2*x**2 + 1)*b*c**6*x**6 + 6297*sqr
t( - c**2*x**2 + 1)*b*c**4*x**4 - 2629*sqrt( - c**2*x**2 + 1)*b*c**2*x**2
- 5258*sqrt( - c**2*x**2 + 1)*b - 11025*a*c**9*x**9 + 42525*a*c**7*x**7 -
59535*a*c**5*x**5 + 33075*a*c**3*x**3))/(99225*c**3)
```

3.24 $\int x(d-c^2dx^2)^3 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 150

$$\int x(d-c^2dx^2)^3 (a+b\arccos(cx))\,dx = \frac{35bd^3x\sqrt{1-c^2x^2}}{1024c} + \frac{35bd^3x(1-c^2x^2)^{3/2}}{1536c} + \frac{7bd^3x(1-c^2x^2)^{5/2}}{384c} + \frac{bd^3x(1-c^2x^2)^{7/2}}{64c} + \frac{35bd^3\arccos(cx)}{1024c^2} - \frac{d^3(1-c^2x^2)^4(a+b\arccos(cx))}{8c^2}$$

output
```
35/1024*b*d^3*x*(-c^2*x^2+1)^(1/2)/c+35/1536*b*d^3*x*(-c^2*x^2+1)^(3/2)/c+
7/384*b*d^3*x*(-c^2*x^2+1)^(5/2)/c+1/64*b*d^3*x*(-c^2*x^2+1)^(7/2)/c+35/10
24*b*d^3*arccos(c*x)/c^2-1/8*d^3*(-c^2*x^2+1)^4*(a+b*arccos(c*x))/c^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.88

$$\int x\left(d-c^2dx^2\right)^3(a+b\arccos(cx))\,dx$$
$$=\frac{d^3\left(cx\left(-384acx(-4+6c^2x^2-4c^4x^4+c^6x^6)+b\sqrt{1-c^2x^2}(-279+326c^2x^2-200c^4x^4+48c^6x^6)\right)-38\right.}{3072c^2}$$

input
```
Integrate[x*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
(d^3*(c*x*(-384*a*c*x*(-4 + 6*c^2*x^2 - 4*c^4*x^4 + c^6*x^6) + b*Sqrt[1 -
c^2*x^2]*(-279 + 326*c^2*x^2 - 200*c^4*x^4 + 48*c^6*x^6)) - 384*b*c^2*x^2*
(-4 + 6*c^2*x^2 - 4*c^4*x^4 + c^6*x^6)*ArcCos[c*x] + 279*b*ArcSin[c*x]))/(
3072*c^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5183, 211, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d-c^2dx^2\right)^3(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5183}$$

$$-\frac{bd^3\int\left(1-c^2x^2\right)^{7/2}dx}{8c}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))}{8c^2}$$

$$\Big\downarrow \text{211}$$

$$-\frac{bd^3\left(\frac{7}{8}\int\left(1-c^2x^2\right)^{5/2}dx+\frac{1}{8}x\left(1-c^2x^2\right)^{7/2}\right)}{8c}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))}{8c^2}$$

$$\Big\downarrow \text{211}$$

$$-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\int \left(1-c^2x^2\right)^{3/2}dx+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)+\frac{1}{8}x\left(1-c^2x^2\right)^{7/2}\right)}{8c}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))}{8c^2}$$

$$\Big\downarrow \; 211$$

$$-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-c^2x^2}dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)+\frac{1}{8}x\left(1-c^2x^2\right)^{7/2}\right)}{8c}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))}{8c^2}$$

$$\Big\downarrow \; 211$$

$$-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)+\frac{1}{8}x\left(1-c^2x^2\right)^{7/2}\right)}{8c}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))}{8c^2}$$

$$\Big\downarrow \; 223$$

$$-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))}{8c^2}-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)+\frac{1}{8}x\left(1-c^2x^2\right)^{7/2}\right)}{8c}$$

input
```
Int[x*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
-1/8*(d^3*(1 - c^2*x^2)^4*(a + b*ArcCos[c*x]))/c^2 - (b*d^3*((x*(1 - c^2*x
^2)^(7/2))/8 + (7*((x*(1 - c^2*x^2)^(5/2))/6 + (5*((x*(1 - c^2*x^2)^(3/2))
/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/6))/8))/(8*c)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{d^3 a \left(c^2 x^2-1\right)^4}{8}-d^3 b\left(\frac{\arccos(cx) c^8 x^8}{8}-\frac{\arccos(cx) c^6 x^6}{2}+\frac{3c^4x^4\arccos(cx)}{4}-\frac{c^2x^2\arccos(cx)}{2}+\frac{\arccos(cx)}{8}+\frac{35\arcsin(cx)}{1024}-c^7\right.}{c^2}$
default	$\frac{-\frac{d^3 a \left(c^2 x^2-1\right)^4}{8}-d^3 b\left(\frac{\arccos(cx) c^8 x^8}{8}-\frac{\arccos(cx) c^6 x^6}{2}+\frac{3c^4x^4\arccos(cx)}{4}-\frac{c^2x^2\arccos(cx)}{2}+\frac{\arccos(cx)}{8}+\frac{35\arcsin(cx)}{1024}-c^7\right.}{c^2}$
parts	$-\frac{d^3 a (c^2x^2-1)^4}{8c^2}-\frac{d^3 b\left(\frac{\arccos(cx) c^8 x^8}{8}-\frac{\arccos(cx) c^6 x^6}{2}+\frac{3c^4x^4\arccos(cx)}{4}-\frac{c^2x^2\arccos(cx)}{2}+\frac{\arccos(cx)}{8}+\frac{35\arcsin(cx)}{1024}\right.}{c^2}$
orering	$\frac{(720c^8x^8-2984c^6x^6+4786c^4x^4-3815c^2x^2+558)\left(-c^2dx^2+d\right)^3(a+b\arccos(cx))}{3072c^2(cx-1)^2(cx+1)^2(c^2x^2-1)}-\frac{(48c^6x^6-200c^4x^4+326c^2x^2-27}{}$

input
```
int(x*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/8*d^3*a*(c^2*x^2-1)^4-d^3*b*(1/8*arccos(c*x)*c^8*x^8-1/2*arccos(
c*x)*c^6*x^6+3/4*c^4*x^4*arccos(c*x)-1/2*c^2*x^2*arccos(c*x)+1/8*arccos(c*
x)+35/1024*arcsin(c*x)-1/64*c^7*x^7*(-c^2*x^2+1)^(1/2)+25/384*c^5*x^5*(-c^
2*x^2+1)^(1/2)-163/1536*c^3*x^3*(-c^2*x^2+1)^(1/2)+93/1024*c*x*(-c^2*x^2+1
)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.16

$$\int x\left(d-c^2dx^2\right)^3(a+b\arccos(cx))\,dx =$$
$$-\frac{384\,ac^8d^3x^8-1536\,ac^6d^3x^6+2304\,ac^4d^3x^4-1536\,ac^2d^3x^2+3\,(128\,bc^8d^3x^8-512\,bc^6d^3x^6+768\,bc^4d}{3}$$

input
```
integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-1/3072*(384*a*c^8*d^3*x^8 - 1536*a*c^6*d^3*x^6 + 2304*a*c^4*d^3*x^4 - 153
6*a*c^2*d^3*x^2 + 3*(128*b*c^8*d^3*x^8 - 512*b*c^6*d^3*x^6 + 768*b*c^4*d^3
*x^4 - 512*b*c^2*d^3*x^2 + 93*b*d^3)*arccos(c*x) - (48*b*c^7*d^3*x^7 - 200
*b*c^5*d^3*x^5 + 326*b*c^3*d^3*x^3 - 279*b*c*d^3*x)*sqrt(-c^2*x^2 + 1))/c^
2
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.72

$$\int x\left(d-c^2dx^2\right)^3(a+b\arccos(cx))\,dx$$
$$=\begin{cases}-\frac{ac^6d^3x^8}{8}+\frac{ac^4d^3x^6}{2}-\frac{3ac^2d^3x^4}{4}+\frac{ad^3x^2}{2}-\frac{bc^6d^3x^8\operatorname{acos}(cx)}{8}+\frac{bc^5d^3x^7\sqrt{-c^2x^2+1}}{64}+\frac{bc^4d^3x^6\operatorname{acos}(cx)}{2}-\frac{25bc^3d^3x^5\sqrt{-c^2x^2}}{384} \\ \frac{d^3x^2\left(a+\frac{\pi b}{2}\right)}{2}\end{cases}$$

input
```
integrate(x*(-c**2*d*x**2+d)**3*(a+b*acos(c*x)),x)
```

output
```
Piecewise((-a*c**6*d**3*x**8/8 + a*c**4*d**3*x**6/2 - 3*a*c**2*d**3*x**4/4
 + a*d**3*x**2/2 - b*c**6*d**3*x**8*acos(c*x)/8 + b*c**5*d**3*x**7*sqrt(-c
**2*x**2 + 1)/64 + b*c**4*d**3*x**6*acos(c*x)/2 - 25*b*c**3*d**3*x**5*sqrt
(-c**2*x**2 + 1)/384 - 3*b*c**2*d**3*x**4*acos(c*x)/4 + 163*b*c*d**3*x**3*
sqrt(-c**2*x**2 + 1)/1536 + b*d**3*x**2*acos(c*x)/2 - 93*b*d**3*x*sqrt(-c*
*2*x**2 + 1)/(1024*c) - 93*b*d**3*acos(c*x)/(1024*c**2), Ne(c, 0)), (d**3*
x**2*(a + pi*b/2)/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(129) = 258$.

Time = 0.12 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.41

$$\int x(d-c^2dx^2)^3(a+b\arccos(cx))\,dx = -\frac{1}{8}ac^6d^3x^8 + \frac{1}{2}ac^4d^3x^6$$
$$-\frac{1}{3072}\left(384\,x^8\arccos(cx) - \left(\frac{48\sqrt{-c^2x^2+1}x^7}{c^2} + \frac{56\sqrt{-c^2x^2+1}x^5}{c^4} + \frac{70\sqrt{-c^2x^2+1}x^3}{c^6} + \frac{105\sqrt{-c^2\cdots}}{c^8\cdots}\right.\right.$$
$$-\frac{3}{4}ac^2d^3x^4$$
$$+\frac{1}{96}\left(48\,x^6\arccos(cx) - \left(\frac{8\sqrt{-c^2x^2+1}x^5}{c^2} + \frac{10\sqrt{-c^2x^2+1}x^3}{c^4} + \frac{15\sqrt{-c^2x^2+1}x}{c^6} - \frac{15\arcsin(cx)}{c^7}\right)\cdots\right.$$
$$-\frac{3}{32}\left(8\,x^4\arccos(cx) - \left(\frac{2\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\sqrt{-c^2x^2+1}x}{c^4} - \frac{3\arcsin(cx)}{c^5}\right)c\right)bc^2d^3$$
$$+\frac{1}{2}ad^3x^2 + \frac{1}{4}\left(2\,x^2\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x}{c^2} - \frac{\arcsin(cx)}{c^3}\right)\right)bd^3$$

input
```
integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-1/8*a*c^6*d^3*x^8 + 1/2*a*c^4*d^3*x^6 - 1/3072*(384*x^8*arccos(c*x) - (48
*sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(-c^2
*x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9)*c)
*b*c^6*d^3 - 3/4*a*c^2*d^3*x^4 + 1/96*(48*x^6*arccos(c*x) - (8*sqrt(-c^2*x
^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/
c^6 - 15*arcsin(c*x)/c^7)*c)*b*c^4*d^3 - 3/32*(8*x^4*arccos(c*x) - (2*sqrt
(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c
)*b*c^2*d^3 + 1/2*a*d^3*x^2 + 1/4*(2*x^2*arccos(c*x) - c*(sqrt(-c^2*x^2 +
1)*x/c^2 - arcsin(c*x)/c^3))*b*d^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.41

$$\int x(d-c^2dx^2)^3(a+b\arccos(cx))\,dx = -\frac{1}{8}bc^6d^3x^8\arccos(cx) - \frac{1}{8}ac^6d^3x^8 + \frac{1}{64}\sqrt{-c^2x^2+1}bc^5d^3x^7 + \frac{1}{2}bc^4d^3x^6\arccos(cx) + \frac{1}{2}ac^4d^3x^6 - \frac{25}{384}\sqrt{-c^2x^2+1}bc^3d^3x^5 - \frac{3}{4}bc^2d^3x^4\arccos(cx) - \frac{3}{4}ac^2d^3x^4 + \frac{163}{1536}\sqrt{-c^2x^2+1}bcd^3x^3 + \frac{1}{2}bd^3x^2\arccos(cx) + \frac{1}{2}ad^3x^2 - \frac{93\sqrt{-c^2x^2+1}bd^3x}{1024\,c} - \frac{93\,bd^3\arccos(cx)}{1024\,c^2}$$

input
```
integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-1/8*b*c^6*d^3*x^8*arccos(c*x) - 1/8*a*c^6*d^3*x^8 + 1/64*sqrt(-c^2*x^2 +
1)*b*c^5*d^3*x^7 + 1/2*b*c^4*d^3*x^6*arccos(c*x) + 1/2*a*c^4*d^3*x^6 - 25/
384*sqrt(-c^2*x^2 + 1)*b*c^3*d^3*x^5 - 3/4*b*c^2*d^3*x^4*arccos(c*x) - 3/4
*a*c^2*d^3*x^4 + 163/1536*sqrt(-c^2*x^2 + 1)*b*c*d^3*x^3 + 1/2*b*d^3*x^2*a
rccos(c*x) + 1/2*a*d^3*x^2 - 93/1024*sqrt(-c^2*x^2 + 1)*b*d^3*x/c - 93/102
4*b*d^3*arccos(c*x)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)^3(a+b\arccos(cx))\,dx = \int x\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

input
```
int(x*(a + b*acos(c*x))*(d - c^2*d*x^2)^3,x)
```

output
```
int(x*(a + b*acos(c*x))*(d - c^2*d*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.20

$$\int x(d-c^2dx^2)^3(a+b\arccos(cx))\,dx$$
$$= \frac{d^3(-384acos(cx)\,b\,c^8x^8+1536acos(cx)\,b\,c^6x^6-2304acos(cx)\,b\,c^4x^4+1536acos(cx)\,b\,c^2x^2+279asin(c$$

input
```
int(x*(-c^2*d*x^2+d)^3*(a+b*acos(c*x)),x)
```

output
```
(d**3*( - 384*acos(c*x)*b*c**8*x**8 + 1536*acos(c*x)*b*c**6*x**6 - 2304*ac
os(c*x)*b*c**4*x**4 + 1536*acos(c*x)*b*c**2*x**2 + 279*asin(c*x)*b + 48*sq
rt( - c**2*x**2 + 1)*b*c**7*x**7 - 200*sqrt( - c**2*x**2 + 1)*b*c**5*x**5
+ 326*sqrt( - c**2*x**2 + 1)*b*c**3*x**3 - 279*sqrt( - c**2*x**2 + 1)*b*c*
x - 384*a*c**8*x**8 + 1536*a*c**6*x**6 - 2304*a*c**4*x**4 + 1536*a*c**2*x*
*2))/(3072*c**2)
```

3.25 $\int (d-c^2dx^2)^3 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 175

$$\int (d-c^2dx^2)^3 (a+b\arccos(cx))\,dx$$
$$= \frac{16bd^3\sqrt{1-c^2x^2}}{35c} + \frac{8bd^3(1-c^2x^2)^{3/2}}{105c} + \frac{6bd^3(1-c^2x^2)^{5/2}}{175c} + \frac{bd^3(1-c^2x^2)^{7/2}}{49c} + d^3x(a+b\arccos(cx)) - c^2d^3x^3(a+b\arccos(cx)) + \frac{3}{5}c^4d^3x^5(a+b\arccos(cx)) - \frac{1}{7}c^6d^3x^7(a+b\arccos(cx))$$

output
```
16/35*b*d^3*(-c^2*x^2+1)^(1/2)/c+8/105*b*d^3*(-c^2*x^2+1)^(3/2)/c+6/175*b*
d^3*(-c^2*x^2+1)^(5/2)/c+1/49*b*d^3*(-c^2*x^2+1)^(7/2)/c+d^3*x*(a+b*arccos
(c*x))-c^2*d^3*x^3*(a+b*arccos(c*x))+3/5*c^4*d^3*x^5*(a+b*arccos(c*x))-1/7
*c^6*d^3*x^7*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.68

$$\int (d-c^2dx^2)^3 (a+b\arccos(cx))\,dx =$$
$$-\frac{d^3\left(b\sqrt{1-c^2x^2}(2161-757c^2x^2+351c^4x^4-75c^6x^6)+105acx(-35+35c^2x^2-21c^4x^4+5c^6x^6)+10\right.}{3675c}$$

input
```
Integrate[(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
-1/3675*(d^3*(b*Sqrt[1 - c^2*x^2]*(2161 - 757*c^2*x^2 + 351*c^4*x^4 - 75*c
^6*x^6) + 105*a*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) + 105*b*c*
x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6)*ArcCos[c*x]))/c
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5155, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5155}$$

$$bc\int \frac{d^3x\left(-5c^6x^6+21c^4x^4-35c^2x^2+35\right)}{35\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^6d^3x^7(a+b\arccos(cx))+\frac{3}{5}c^4d^3x^5(a+b\arccos(cx))-c^2d^3x^3(a+b\arccos(cx))+d^3x(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{35}bcd^3\int \frac{x\left(-5c^6x^6+21c^4x^4-35c^2x^2+35\right)}{\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^6d^3x^7(a+b\arccos(cx))+\frac{3}{5}c^4d^3x^5(a+b\arccos(cx))-c^2d^3x^3(a+b\arccos(cx))+d^3x(a+b\arccos(cx))$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{70}bcd^3\int \frac{-5c^6x^6+21c^4x^4-35c^2x^2+35}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{7}c^6d^3x^7(a+b\arccos(cx))+\frac{3}{5}c^4d^3x^5(a+b\arccos(cx))-c^2d^3x^3(a+b\arccos(cx))+d^3x(a+b\arccos(cx))$$

$$\Big\downarrow \text{2389}$$

$$\frac{1}{70}bcd^3\int \left(5\left(1-c^2x^2\right)^{5/2}+6\left(1-c^2x^2\right)^{3/2}+8\sqrt{1-c^2x^2}+\frac{16}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{7}c^6d^3x^7(a+b\arccos(cx))+\frac{3}{5}c^4d^3x^5(a+b\arccos(cx))-c^2d^3x^3(a+b\arccos(cx))+d^3x(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{7}c^6d^3x^7(a+b\arccos(cx))+\frac{3}{5}c^4d^3x^5(a+b\arccos(cx))-c^2d^3x^3(a+b\arccos(cx))+d^3x(a+b\arccos(cx))+\frac{1}{70}bcd^3\left(-\frac{10(1-c^2x^2)^{7/2}}{7c^2}-\frac{12(1-c^2x^2)^{5/2}}{5c^2}-\frac{16(1-c^2x^2)^{3/2}}{3c^2}-\frac{32\sqrt{1-c^2x^2}}{c^2}\right)$$

input
```
Int[(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d^3*((-32*Sqrt[1 - c^2*x^2])/c^2 - (16*(1 - c^2*x^2)^(3/2))/(3*c^2) -
 (12*(1 - c^2*x^2)^(5/2))/(5*c^2) - (10*(1 - c^2*x^2)^(7/2))/(7*c^2)))/70
+ d^3*x*(a + b*ArcCos[c*x]) - c^2*d^3*x^3*(a + b*ArcCos[c*x]) + (3*c^4*d^3
*x^5*(a + b*ArcCos[c*x]))/5 - (c^6*d^3*x^7*(a + b*ArcCos[c*x]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.93

method	result
parts	$-a\,d^3\left(\frac{1}{7}c^6x^7-\frac{3}{5}c^4x^5+c^2x^3-x\right)-\frac{b\,d^3\left(\frac{\arccos(cx)c^7x^7}{7}-\frac{3\arccos(cx)c^5x^5}{5}+c^3x^3\arccos(cx)-cx\arccos(c\cdots\right.}{}$
derivativedivides	$\frac{-d^3a\left(\frac{1}{7}c^7x^7-\frac{3}{5}c^5x^5+c^3x^3-cx\right)-d^3b\left(\frac{\arccos(cx)c^7x^7}{7}-\frac{3\arccos(cx)c^5x^5}{5}+c^3x^3\arccos(cx)-cx\arccos(cx)+\frac{2161\sqrt{-c^2x^2}\cdots}{3675}\right.}{c}$
default	$\frac{-d^3a\left(\frac{1}{7}c^7x^7-\frac{3}{5}c^5x^5+c^3x^3-cx\right)-d^3b\left(\frac{\arccos(cx)c^7x^7}{7}-\frac{3\arccos(cx)c^5x^5}{5}+c^3x^3\arccos(cx)-cx\arccos(cx)+\frac{2161\sqrt{-c^2x^2}\cdots}{3675}\right.}{c}$
orering	$\frac{x\left(325c^6x^6-1437c^4x^4+2739c^2x^2-5547\right)\left(-c^2d\,x^2+d\right)^3\left(a+b\arccos(cx)\right)}{1225(cx-1)^2(cx+1)^2(c^2x^2-1)}-\frac{\left(75c^6x^6-351c^4x^4+757c^2x^2-2161\right)\left(-6\cdots\right.}{3675\cdots}$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-a*d^3*(1/7*c^6*x^7-3/5*c^4*x^5+c^2*x^3-x)-b*d^3/c*(1/7*arccos(c*x)*c^7*x^
7-3/5*arccos(c*x)*c^5*x^5+c^3*x^3*arccos(c*x)-c*x*arccos(c*x)+2161/3675*(-
c^2*x^2+1)^(1/2)-757/3675*c^2*x^2*(-c^2*x^2+1)^(1/2)+117/1225*c^4*x^4*(-c^
2*x^2+1)^(1/2)-1/49*c^6*x^6*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.90

$$\int \left(d-c^2dx^2\right)^3\left(a+b\arccos(cx)\right)dx =$$

$$-\frac{525\,ac^7d^3x^7-2205\,ac^5d^3x^5+3675\,ac^3d^3x^3-3675\,acd^3x+105\left(5\,bc^7d^3x^7-21\,bc^5d^3x^5+35\,bc^3d^3x^3\cdots\right.}{3675\,c}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-1/3675*(525*a*c^7*d^3*x^7 - 2205*a*c^5*d^3*x^5 + 3675*a*c^3*d^3*x^3 - 367
5*a*c*d^3*x + 105*(5*b*c^7*d^3*x^7 - 21*b*c^5*d^3*x^5 + 35*b*c^3*d^3*x^3 -
 35*b*c*d^3*x)*arccos(c*x) - (75*b*c^6*d^3*x^6 - 351*b*c^4*d^3*x^4 + 757*b
*c^2*d^3*x^2 - 2161*b*d^3)*sqrt(-c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.29

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx$$
$$=\begin{cases} -\frac{ac^6d^3x^7}{7}+\frac{3ac^4d^3x^5}{5}-ac^2d^3x^3+ad^3x-\frac{bc^6d^3x^7\operatorname{acos}(cx)}{7}+\frac{bc^5d^3x^6\sqrt{-c^2x^2+1}}{49}+\frac{3bc^4d^3x^5\operatorname{acos}(cx)}{5}-\frac{117bc^3d^3x^4\sqrt{-}}{1225} \\ d^3x\left(a+\frac{\pi b}{2}\right) \end{cases}$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acos(c*x)),x)
```

output
```
Piecewise((-a*c**6*d**3*x**7/7 + 3*a*c**4*d**3*x**5/5 - a*c**2*d**3*x**3 +
 a*d**3*x - b*c**6*d**3*x**7*acos(c*x)/7 + b*c**5*d**3*x**6*sqrt(-c**2*x**
2 + 1)/49 + 3*b*c**4*d**3*x**5*acos(c*x)/5 - 117*b*c**3*d**3*x**4*sqrt(-c*
*2*x**2 + 1)/1225 - b*c**2*d**3*x**3*acos(c*x) + 757*b*c*d**3*x**2*sqrt(-c
**2*x**2 + 1)/3675 + b*d**3*x*acos(c*x) - 2161*b*d**3*sqrt(-c**2*x**2 + 1)
/(3675*c), Ne(c, 0)), (d**3*x*(a + pi*b/2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(155) = 310.

Time = 0.12 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.78

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx = -\frac{1}{7}\,ac^6d^3x^7+\frac{3}{5}\,ac^4d^3x^5$$
$$-\frac{1}{245}\left(35\,x^7\arccos(cx)-\left(\frac{5\,\sqrt{-c^2x^2+1}x^6}{c^2}+\frac{6\,\sqrt{-c^2x^2+1}x^4}{c^4}+\frac{8\,\sqrt{-c^2x^2+1}x^2}{c^6}+\frac{16\,\sqrt{-c^2x^2+1}}{c^8}\right.\right.$$
$$+\frac{1}{25}\left(15\,x^5\arccos(cx)-\left(\frac{3\,\sqrt{-c^2x^2+1}x^4}{c^2}+\frac{4\,\sqrt{-c^2x^2+1}x^2}{c^4}+\frac{8\,\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bc^4d^3$$
$$-ac^2d^3x^3-\frac{1}{3}\left(3\,x^3\arccos(cx)-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\,\sqrt{-c^2x^2+1}}{c^4}\right)\right)bc^2d^3$$
$$+ad^3x+\frac{\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)bd^3}{c}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 - 1/245*(35*x^7*arccos(c*x) - (5*sq
rt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2
+ 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*c^6*d^3 + 1/25*(15*x^5*arcc
os(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8
*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*c^4*d^3 - a*c^2*d^3*x^3 - 1/3*(3*x^3*arccos(
c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*c^2*d^
3 + a*d^3*x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b*d^3/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx = -\frac{1}{7}\,bc^6d^3x^7\arccos\left(cx\right)-\frac{1}{7}\,ac^6d^3x^7+\frac{1}{49}\,\sqrt{-c^2x^2+1}bc^5d^3x^6+\frac{3}{5}\,bc^4d^3x^5\arccos\left(cx\right)+\frac{3}{5}\,ac^4d^3x^5-\frac{117}{1225}\,\sqrt{-c^2x^2+1}bc^3d^3x^4-bc^2d^3x^3\arccos\left(cx\right)-ac^2d^3x^3+\frac{757}{3675}\,\sqrt{-c^2x^2+1}bcd^3x^2+bd^3x\arccos\left(cx\right)+ad^3x-\frac{2161\,\sqrt{-c^2x^2+1}bd^3}{3675\,c}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-1/7*b*c^6*d^3*x^7*arccos(c*x) - 1/7*a*c^6*d^3*x^7 + 1/49*sqrt(-c^2*x^2 +
1)*b*c^5*d^3*x^6 + 3/5*b*c^4*d^3*x^5*arccos(c*x) + 3/5*a*c^4*d^3*x^5 - 117
/1225*sqrt(-c^2*x^2 + 1)*b*c^3*d^3*x^4 - b*c^2*d^3*x^3*arccos(c*x) - a*c^2
*d^3*x^3 + 757/3675*sqrt(-c^2*x^2 + 1)*b*c*d^3*x^2 + b*d^3*x*arccos(c*x) +
 a*d^3*x - 2161/3675*sqrt(-c^2*x^2 + 1)*b*d^3/c
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx=\int \left(a+b\,\mathrm{acos}(c\,x)\right)\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

input
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^3,x)
```

output
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx = \frac{d^3\left(-525acos(cx)\,b\,c^7x^7+2205acos(cx)\,b\,c^5x^5-3675acos(cx)\,b\,c^3x^3+3675acos(cx)\,bcx+75\sqrt{-c^2x^2+}\right.}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acos(c*x)),x)
```

output
```
(d**3*( - 525*acos(c*x)*b*c**7*x**7 + 2205*acos(c*x)*b*c**5*x**5 - 3675*ac
os(c*x)*b*c**3*x**3 + 3675*acos(c*x)*b*c*x + 75*sqrt( - c**2*x**2 + 1)*b*c
**6*x**6 - 351*sqrt( - c**2*x**2 + 1)*b*c**4*x**4 + 757*sqrt( - c**2*x**2
+ 1)*b*c**2*x**2 - 2161*sqrt( - c**2*x**2 + 1)*b - 525*a*c**7*x**7 + 2205*
a*c**5*x**5 - 3675*a*c**3*x**3 + 3675*a*c*x))/(3675*c)
```

3.26 $\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 235

$$\begin{aligned}\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x}\,dx = &-\frac{19}{48}bcd^3x\sqrt{1-c^2x^2}-\frac{7}{72}bcd^3x(1-c^2x^2)^{3/2}\\&-\frac{1}{36}bcd^3x(1-c^2x^2)^{5/2}-\frac{19}{48}bd^3\arccos(cx)\\&+\frac{1}{2}d^3(1-c^2x^2)(a+b\arccos(cx))\\&+\frac{1}{4}d^3(1-c^2x^2)^2(a+b\arccos(cx))\\&+\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))\\&-\frac{id^3(a+b\arccos(cx))^2}{2b}\\&+d^3(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)\\&-\frac{1}{2}ibd^3\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\end{aligned}$$

output
```
-19/48*b*c*d^3*x*(-c^2*x^2+1)^(1/2)-7/72*b*c*d^3*x*(-c^2*x^2+1)^(3/2)-1/36
*b*c*d^3*x*(-c^2*x^2+1)^(5/2)-19/48*b*d^3*arccos(c*x)+1/2*d^3*(-c^2*x^2+1)
*(a+b*arccos(c*x))+1/4*d^3*(-c^2*x^2+1)^2*(a+b*arccos(c*x))+1/6*d^3*(-c^2*
x^2+1)^3*(a+b*arccos(c*x))-1/2*I*d^3*(a+b*arccos(c*x))^2/b+d^3*(a+b*arccos
(c*x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1/2*I*b*d^3*polylog(2,(c*x+I*(-c
^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.88

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x} dx = -\frac{1}{144}d^3\Big(216ac^2x^2 - 108ac^4x^4 + 24ac^6x^6 - 75bcx\sqrt{1-c^2x^2} + 22bc^3x^3\sqrt{1-c^2x^2} - 4bc^5x^5\sqrt{1-c^2x^2} + 72ib\arccos(cx)^2 + 150b\arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right) + 12b\arccos(cx)\left(18c^2x^2 - 9c^4x^4 + 2c^6x^6 - 12\log\left(1+e^{2i\arccos(cx)}\right)\right) - 144a\log(x) + 72ib\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\Big)$$

input
```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]))/x,x]
```

output
```
-1/144*(d^3*(216*a*c^2*x^2 - 108*a*c^4*x^4 + 24*a*c^6*x^6 - 75*b*c*x*Sqrt[
1 - c^2*x^2] + 22*b*c^3*x^3*Sqrt[1 - c^2*x^2] - 4*b*c^5*x^5*Sqrt[1 - c^2*x
^2] + (72*I)*b*ArcCos[c*x]^2 + 150*b*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])
] + 12*b*ArcCos[c*x]*(18*c^2*x^2 - 9*c^4*x^4 + 2*c^6*x^6 - 12*Log[1 + E^((
2*I)*ArcCos[c*x])]) - 144*a*Log[x] + (72*I)*b*PolyLog[2, -E^((2*I)*ArcCos[
c*x])]))
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.43, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {5189, 27, 211, 211, 211, 223, 5189, 211, 211, 223, 5189, 211, 223, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x} dx$$

$$\Big\downarrow \text{5189}$$

$$d\int \frac{d^2\left(1-c^2x^2\right)^2 (a+b\arccos(cx))}{x}dx+\frac{1}{6}bcd^3\int \left(1-c^2x^2\right)^{5/2}dx+\frac{1}{6}d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))}{x}dx+\frac{1}{6}bcd^3\int \left(1-c^2x^2\right)^{5/2}dx+\frac{1}{6}d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))$$

$$\Big\downarrow \text{211}$$

$$d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))}{x}dx+\frac{1}{6}bcd^3\left(\frac{5}{6}\int \left(1-c^2x^2\right)^{3/2}dx+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)+\frac{1}{6}d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))$$

$$\Big\downarrow \text{211}$$

$$d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))}{x}dx+\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-c^2x^2}dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)+\frac{1}{6}d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))$$

$$\Big\downarrow \text{211}$$

$$d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))}{x}dx+\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)+\frac{1}{6}d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))$$

$$\Big\downarrow \text{223}$$

$$d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))}{x}dx+\frac{1}{6}d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))+\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)$$

$$\Big\downarrow \text{5189}$$

$$d^3\left(\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx+\frac{1}{4}bc\int (1-c^2x^2)^{3/2}\,dx+\frac{1}{4}(1-c^2x^2)^2(a+b\arccos(cx))\right)+$$
$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)$$

$\downarrow$ 211

$$d^3\left(\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx+\frac{1}{4}bc\left(\frac{3}{4}\int \sqrt{1-c^2x^2}dx+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{4}(1-c^2x^2)^2(a+b\,\text{arc}$$
$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)$$

$\downarrow$ 211

$$d^3\left(\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx+\frac{1}{4}bc\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{4}$$
$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)$$

$\downarrow$ 223

$$d^3\left(\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx+\frac{1}{4}(1-c^2x^2)^2(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right.\right.\right.$$
$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)$$

$\downarrow$ 5189

$$d^3\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\int \sqrt{1-c^2x^2}dx+\frac{1}{4}(1-c^2x^2)^2(a+b\arccos(cx))+\frac{1}{2}(1-c^2x^2)(a+b\arccos(\right.$$
$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)$$

$\downarrow$ 211

$$d^3\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{2}(1\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)$$

$\downarrow$ 223

$$d^3\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}\right.\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)$$

$\downarrow$ 5137

$$d^3\left(-\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)$$

$\downarrow$ 3042

$$d^3\left(-\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{2}\left(1-c^2x^2\right)(a+b\,a\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)$$

$\downarrow$ 4202

$$d^3\left(2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))+$$
$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}\right)$$

$\downarrow$ 2620

$$d^3\left(2i\left(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}(1-c^2x^2)^2$$

$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))+$$

$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)$$

$$\Big\downarrow \text{2715}$$

$$d^3\left(2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}$$

$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))+$$

$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)$$

$$\Big\downarrow \text{2838}$$

$$d^3\left(\frac{1}{4}(1-c^2x^2)^2(a+b\arccos(cx))+\frac{1}{2}(1-c^2x^2)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\,\mathrm{arcc}\right.\right.$$

$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))+$$

$$\frac{1}{6}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)$$

input
```
Int[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]))/x,x]
```

output
```
(d^3*(1 - c^2*x^2)^3*(a + b*ArcCos[c*x]))/6 + (b*c*d^3*((x*(1 - c^2*x^2)^(
5/2))/6 + (5*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + Ar
cSin[c*x]/(2*c)))/4))/6))/6 + d^3*(((1 - c^2*x^2)*(a + b*ArcCos[c*x]))/2 +
 ((1 - c^2*x^2)^2*(a + b*ArcCos[c*x]))/4 - ((I/2)*(a + b*ArcCos[c*x])^2)/b
 + (b*c*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 + (b*c*((x*(1 - c
^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/4
 + (2*I)*((-1/2*I)*(a + b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCos[c*x])] - (b
*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5189
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCos[c*x])/(2*p)), x] + (Simp[d
 Int[(d + e*x^2)^(p - 1)*((a + b*ArcCos[c*x])/x), x], x] + Simp[b*c*(d^p/(2
*p))   Int[(1 - c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.94

method	result
parts	$-d^{3}a\left(\frac{c^{6}x^{6}}{6}-\frac{3c^{4}x^{4}}{4}+\frac{3c^{2}x^{2}}{2}-\ln\left(x\right)\right)-\frac{ib\,d^{3}\arccos\left(cx\right)^{2}}{2}-\frac{29d^{3}b\arccos\left(cx\right)c^{2}x^{2}}{32}+\frac{29bc\,d^{3}x\sqrt{-c^{2}x^{2}}}{64}$
derivativedivides	$-d^{3}a\left(\frac{c^{6}x^{6}}{6}-\frac{3c^{4}x^{4}}{4}+\frac{3c^{2}x^{2}}{2}-\ln\left(cx\right)\right)-\frac{ib\,d^{3}\arccos\left(cx\right)^{2}}{2}+\frac{29bc\,d^{3}x\sqrt{-c^{2}x^{2}+1}}{64}-\frac{29d^{3}b\arccos\left(cx\right)}{32}$
default	$-d^{3}a\left(\frac{c^{6}x^{6}}{6}-\frac{3c^{4}x^{4}}{4}+\frac{3c^{2}x^{2}}{2}-\ln\left(cx\right)\right)-\frac{ib\,d^{3}\arccos\left(cx\right)^{2}}{2}+\frac{29bc\,d^{3}x\sqrt{-c^{2}x^{2}+1}}{64}-\frac{29d^{3}b\arccos\left(cx\right)}{32}$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
-d^3*a*(1/6*c^6*x^6-3/4*c^4*x^4+3/2*c^2*x^2-ln(x))-1/2*I*b*d^3*arccos(c*x)
^2-29/32*d^3*b*arccos(c*x)*c^2*x^2+29/64*b*c*d^3*x*(-c^2*x^2+1)^(1/2)+29/6
4*b*d^3*arccos(c*x)+d^3*b*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1
/2*I*d^3*b*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1/192*d^3*b*arccos(c*x
)*cos(6*arccos(c*x))+1/1152*d^3*b*sin(6*arccos(c*x))+1/16*d^3*b*arccos(c*x
)*cos(4*arccos(c*x))-1/64*d^3*b*sin(4*arccos(c*x))
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x}\,dx=\int -\frac{(c^2dx^2-d)^3(b\arccos(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x,x, algorithm="fricas")
```

output
```
integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*
c^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arccos(c*x))/x, x
)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x}\,dx=-d^3\left(\int\left(-\frac{a}{x}\right)dx+\int 3ac^2x\,dx+\int\left(-3ac^4x^3\right)dx+\int ac^6x^5\,dx+\int\left(-\frac{b\,\mathrm{acos}\,(cx)}{x}\right)dx+\int 3bc^2x\,\mathrm{acos}\,(cx)\,dx+\int\left(-3bc^4x^3\,\mathrm{acos}\,(cx)\right)dx+\int bc^6x^5\,\mathrm{acos}\,(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acos(c*x))/x,x)
```

output
```
-d**3*(Integral(-a/x, x) + Integral(3*a*c**2*x, x) + Integral(-3*a*c**4*x*
*3, x) + Integral(a*c**6*x**5, x) + Integral(-b*acos(c*x)/x, x) + Integral
(3*b*c**2*x*acos(c*x), x) + Integral(-3*b*c**4*x**3*acos(c*x), x) + Integr
al(b*c**6*x**5*acos(c*x), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x} dx = \int -\frac{(c^2dx^2-d)^3(b\arccos(cx)+a)}{x} dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x,x, algorithm="maxima")
```

output
```
-1/6*a*c^6*d^3*x^6 + 3/4*a*c^4*d^3*x^4 - 3/2*a*c^2*d^3*x^2 + a*d^3*log(x)
- integrate((b*c^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*ar
ctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x} dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^3}{x} dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^3)/x,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^3)/x, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x}\,dx$$
$$= \frac{d^3\Big(-24acos(cx)\,b\,c^6x^6+108acos(cx)\,b\,c^4x^4-216acos(cx)\,b\,c^2x^2-75asin(cx)\,b+4\sqrt{-c^2x^2+1}\,b\,c^5x^5\,+}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acos(c*x))/x,x)
```

output
```
(d**3*( - 24*acos(c*x)*b*c**6*x**6 + 108*acos(c*x)*b*c**4*x**4 - 216*acos(
c*x)*b*c**2*x**2 - 75*asin(c*x)*b + 4*sqrt( - c**2*x**2 + 1)*b*c**5*x**5 -
 22*sqrt( - c**2*x**2 + 1)*b*c**3*x**3 + 75*sqrt( - c**2*x**2 + 1)*b*c*x +
 144*int(acos(c*x)/x,x)*b + 144*log(x)*a - 24*a*c**6*x**6 + 108*a*c**4*x**
4 - 216*a*c**2*x**2))/144
```

3.27 $\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 164

$$\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x^2}\,dx = -\frac{11}{5}bcd^3\sqrt{1-c^2x^2}-\frac{1}{5}bcd^3\left(1-c^2x^2\right)^{3/2} -\frac{1}{25}bcd^3\left(1-c^2x^2\right)^{5/2}-\frac{d^3(a+b\arccos(cx))}{x} -3c^2d^3x(a+b\arccos(cx)) +c^4d^3x^3(a+b\arccos(cx)) -\frac{1}{5}c^6d^3x^5(a+b\arccos(cx)) -bcd^3\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)$$

output
```
-11/5*b*c*d^3*(-c^2*x^2+1)^(1/2)-1/5*b*c*d^3*(-c^2*x^2+1)^(3/2)-1/25*b*c*d
^3*(-c^2*x^2+1)^(5/2)-d^3*(a+b*arccos(c*x))/x-3*c^2*d^3*x*(a+b*arccos(c*x)
)+c^4*d^3*x^3*(a+b*arccos(c*x))-1/5*c^6*d^3*x^5*(a+b*arccos(c*x))-b*c*d^3*
arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.82

$$\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x^2}\,dx = \frac{1}{25}d^3\left(-\frac{25a}{x}-75ac^2x+25ac^4x^3-5ac^6x^5+bc\sqrt{1-c^2x^2}(61-7c^2x^2+c^4x^4)-\frac{5b(5+15c^2x^2-5c^4x^4+c^6x^6)\arccos(cx)}{x}-25bc\log(x)+25bc\log\left(1+\sqrt{1-c^2x^2}\right)\right)$$

input
```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
(d^3*((-25*a)/x - 75*a*c^2*x + 25*a*c^4*x^3 - 5*a*c^6*x^5 + b*c*Sqrt[1 - c
^2*x^2]*(61 - 7*c^2*x^2 + c^4*x^4) - (5*b*(5 + 15*c^2*x^2 - 5*c^4*x^4 + c^
6*x^6)*ArcCos[c*x])/x - 25*b*c*Log[x] + 25*b*c*Log[1 + Sqrt[1 - c^2*x^2]])
)/25
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5193, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x^2}\,dx$$

$$\Big\downarrow \text{5193}$$

$$bc\int -\frac{d^3(c^6x^6-5c^4x^4+15c^2x^2+5)}{5x\sqrt{1-c^2x^2}}dx-\frac{1}{5}c^6d^3x^5(a+b\arccos(cx))+c^4d^3x^3(a+b\arccos(cx))-3c^2d^3x(a+b\arccos(cx))-\frac{d^3(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}bcd^3\int \frac{c^6x^6-5c^4x^4+15c^2x^2+5}{x\sqrt{1-c^2x^2}}dx-\frac{1}{5}c^6d^3x^5(a+b\arccos(cx))+c^4d^3x^3(a+b\arccos(cx))-3c^2d^3x(a+b\arccos(cx))-\frac{d^3(a+b\arccos(cx))}{x}$$

$\Big\downarrow$ 2331

$$-\frac{1}{10}bcd^3\int \frac{c^6x^6-5c^4x^4+15c^2x^2+5}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{1}{5}c^6d^3x^5(a+b\arccos(cx))+c^4d^3x^3(a+b\arccos(cx))-3c^2d^3x(a+b\arccos(cx))-\frac{d^3(a+b\arccos(cx))}{x}$$

$\Big\downarrow$ 2123

$$-\frac{1}{10}bcd^3\int \left(\left(1-c^2x^2\right)^{3/2}c^2+3\sqrt{1-c^2x^2}c^2+\frac{11c^2}{\sqrt{1-c^2x^2}}+\frac{5}{x^2\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{5}c^6d^3x^5(a+b\arccos(cx))+c^4d^3x^3(a+b\arccos(cx))-3c^2d^3x(a+b\arccos(cx))-\frac{d^3(a+b\arccos(cx))}{x}$$

$\Big\downarrow$ 2009

$$-\frac{1}{5}c^6d^3x^5(a+b\arccos(cx))+c^4d^3x^3(a+b\arccos(cx))-3c^2d^3x(a+b\arccos(cx))-\frac{d^3(a+b\arccos(cx))}{x}-\frac{1}{10}bcd^3\left(-10\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{2}{5}\left(1-c^2x^2\right)^{5/2}-2\left(1-c^2x^2\right)^{3/2}-22\sqrt{1-c^2x^2}\right)$$

input
```
Int[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
-((d^3*(a + b*ArcCos[c*x]))/x) - 3*c^2*d^3*x*(a + b*ArcCos[c*x]) + c^4*d^3
*x^3*(a + b*ArcCos[c*x]) - (c^6*d^3*x^5*(a + b*ArcCos[c*x]))/5 - (b*c*d^3*
(-22*Sqrt[1 - c^2*x^2] - 2*(1 - c^2*x^2)^(3/2) - (2*(1 - c^2*x^2)^(5/2))/5
 - 10*ArcTanh[Sqrt[1 - c^2*x^2]]))/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.94

method	result
parts	$-d^3a\left(\frac{c^6x^5}{5}-c^4x^3+3c^2x+\frac{1}{x}\right)-d^3bc\left(\frac{\arccos(cx)c^5x^5}{5}-c^3x^3\arccos(cx)+3cx\arccos(c\right.$
derivativedivides	$c\left(-d^3a\left(\frac{c^5x^5}{5}-c^3x^3+3cx+\frac{1}{cx}\right)-d^3b\left(\frac{\arccos(cx)c^5x^5}{5}-c^3x^3\arccos(cx)+3cx\arccos(\right.\right.$
default	$c\left(-d^3a\left(\frac{c^5x^5}{5}-c^3x^3+3cx+\frac{1}{cx}\right)-d^3b\left(\frac{\arccos(cx)c^5x^5}{5}-c^3x^3\arccos(cx)+3cx\arccos(\right.\right.$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-d^3*a*(1/5*c^6*x^5-c^4*x^3+3*c^2*x+1/x)-d^3*b*c*(1/5*arccos(c*x)*c^5*x^5-
c^3*x^3*arccos(c*x)+3*c*x*arccos(c*x)+arccos(c*x)/c/x-1/25*c^4*x^4*(-c^2*x
^2+1)^(1/2)+7/25*c^2*x^2*(-c^2*x^2+1)^(1/2)-61/25*(-c^2*x^2+1)^(1/2)-arcta
nh(1/(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.64

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x^2} dx = -\frac{10\,ac^6d^3x^6 - 50\,ac^4d^3x^4 + 150\,ac^2d^3x^2 - 25\,bcd^3x\log\left(\sqrt{-c^2x^2+1}+1\right) + 25\,bcd^3x\log\left(\sqrt{-c^2x^2+1}\right.}{}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x^2,x, algorithm="fricas")
```

output
```
-1/50*(10*a*c^6*d^3*x^6 - 50*a*c^4*d^3*x^4 + 150*a*c^2*d^3*x^2 - 25*b*c*d^
3*x*log(sqrt(-c^2*x^2 + 1) + 1) + 25*b*c*d^3*x*log(sqrt(-c^2*x^2 + 1) - 1)
 + 10*(b*c^6 - 5*b*c^4 + 15*b*c^2 + 5*b)*d^3*x*arctan(sqrt(-c^2*x^2 + 1)*c
*x/(c^2*x^2 - 1)) + 50*a*d^3 + 10*(b*c^6*d^3*x^6 - 5*b*c^4*d^3*x^4 + 15*b*
c^2*d^3*x^2 - (b*c^6 - 5*b*c^4 + 15*b*c^2 + 5*b)*d^3*x + 5*b*d^3)*arccos(c
*x) - 2*(b*c^5*d^3*x^5 - 7*b*c^3*d^3*x^3 + 61*b*c*d^3*x)*sqrt(-c^2*x^2 + 1
))/x
```

Sympy [A] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.79

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x^2}\,dx$$
$$= -\frac{ac^6d^3x^5}{5} + ac^4d^3x^3 - 3ac^2d^3x - \frac{ad^3}{x} - \frac{bc^7d^3\left(\begin{cases} -\frac{x^4\sqrt{-c^2x^2+1}}{5c^2} - \frac{4x^2\sqrt{-c^2x^2+1}}{15c^4} - \frac{8\sqrt{-c^2x^2+1}}{15c^6} & \text{for}\ c^2 \neq 0 \\ \frac{x^6}{6} & \text{otherwise} \end{cases}\right)}{5}$$
$$- \frac{bc^6d^3x^5\operatorname{acos}(cx)}{5} + bc^5d^3\left(\begin{cases} -\frac{x^2\sqrt{-c^2x^2+1}}{3c^2} - \frac{2\sqrt{-c^2x^2+1}}{3c^4} & \text{for}\ c^2 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}\right)$$
$$+ bc^4d^3x^3\operatorname{acos}(cx) - 3bc^2d^3\left(\begin{cases} \frac{\pi x}{2} & \text{for}\ c = 0 \\ x\operatorname{acos}(cx) - \frac{\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases}\right)$$
$$- bcd^3\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for}\ \frac{1}{|c^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases}\right) - \frac{bd^3\operatorname{acos}(cx)}{x}$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acos(c*x))/x**2,x)
```

output
```
-a*c**6*d**3*x**5/5 + a*c**4*d**3*x**3 - 3*a*c**2*d**3*x - a*d**3/x - b*c*
*7*d**3*Piecewise((-x**4*sqrt(-c**2*x**2 + 1)/(5*c**2) - 4*x**2*sqrt(-c**2
*x**2 + 1)/(15*c**4) - 8*sqrt(-c**2*x**2 + 1)/(15*c**6), Ne(c**2, 0)), (x*
*6/6, True))/5 - b*c**6*d**3*x**5*acos(c*x)/5 + b*c**5*d**3*Piecewise((-x*
*2*sqrt(-c**2*x**2 + 1)/(3*c**2) - 2*sqrt(-c**2*x**2 + 1)/(3*c**4), Ne(c**
2, 0)), (x**4/4, True)) + b*c**4*d**3*x**3*acos(c*x) - 3*b*c**2*d**3*Piece
wise((pi*x/2, Eq(c, 0)), (x*acos(c*x) - sqrt(-c**2*x**2 + 1)/c, True)) - b
*c*d**3*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)
), True)) - b*d**3*acos(c*x)/x
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.55

$$\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x^2}\,dx = -\frac{1}{5}\,ac^6d^3x^5$$
$$-\frac{1}{75}\left(15\,x^5\arccos{(cx)}-\left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2}+\frac{4\sqrt{-c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bc^6d^3$$
$$+ac^4d^3x^3+\frac{1}{3}\left(3\,x^3\arccos{(cx)}-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bc^4d^3$$
$$-3\,ac^2d^3x-3\left(cx\arccos{(cx)}-\sqrt{-c^2x^2+1}\right)bcd^3$$
$$+\left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\arccos{(cx)}}{x}\right)bd^3-\frac{ad^3}{x}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x^2,x, algorithm="maxima")
```

output
```
-1/5*a*c^6*d^3*x^5 - 1/75*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/
c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*c^6*d^
3 + a*c^4*d^3*x^3 + 1/3*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2
 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*c^4*d^3 - 3*a*c^2*d^3*x - 3*(c*x*arccos(c*
x) - sqrt(-c^2*x^2 + 1))*b*c*d^3 + (c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/
abs(x)) - arccos(c*x)/x)*b*d^3 - a*d^3/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4007 vs. 2(148) = 296.

Time = 29.11 (sec) , antiderivative size = 4007, normalized size of antiderivative = 24.43

$$\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x^2,x, algorithm="giac")
```

output
```
16/5*b*c*d^3*arccos(c*x)/(4*(c^2*x^2 - 1)/(c*x + 1)^2 - 5*(c^2*x^2 - 1)^2/
(c*x + 1)^4 + 5*(c^2*x^2 - 1)^4/(c*x + 1)^8 - 4*(c^2*x^2 - 1)^5/(c*x + 1)^
10 + (c^2*x^2 - 1)^6/(c*x + 1)^12 - 1) - b*c*d^3*log(abs(c*x + sqrt(-c^2*x
^2 + 1) + 1))/(4*(c^2*x^2 - 1)/(c*x + 1)^2 - 5*(c^2*x^2 - 1)^2/(c*x + 1)^4
 + 5*(c^2*x^2 - 1)^4/(c*x + 1)^8 - 4*(c^2*x^2 - 1)^5/(c*x + 1)^10 + (c^2*x
^2 - 1)^6/(c*x + 1)^12 - 1) + b*c*d^3*log(abs(-c*x + sqrt(-c^2*x^2 + 1) -
1))/(4*(c^2*x^2 - 1)/(c*x + 1)^2 - 5*(c^2*x^2 - 1)^2/(c*x + 1)^4 + 5*(c^2*
x^2 - 1)^4/(c*x + 1)^8 - 4*(c^2*x^2 - 1)^5/(c*x + 1)^10 + (c^2*x^2 - 1)^6/
(c*x + 1)^12 - 1) + 16/5*a*c*d^3/(4*(c^2*x^2 - 1)/(c*x + 1)^2 - 5*(c^2*x^2
 - 1)^2/(c*x + 1)^4 + 5*(c^2*x^2 - 1)^4/(c*x + 1)^8 - 4*(c^2*x^2 - 1)^5/(c
*x + 1)^10 + (c^2*x^2 - 1)^6/(c*x + 1)^12 - 1) - 64/5*(c^2*x^2 - 1)*b*c*d^
3*arccos(c*x)/((c*x + 1)^2*(4*(c^2*x^2 - 1)/(c*x + 1)^2 - 5*(c^2*x^2 - 1)^
2/(c*x + 1)^4 + 5*(c^2*x^2 - 1)^4/(c*x + 1)^8 - 4*(c^2*x^2 - 1)^5/(c*x + 1
)^10 + (c^2*x^2 - 1)^6/(c*x + 1)^12 - 1)) + 4*(c^2*x^2 - 1)*b*c*d^3*log(ab
s(c*x + sqrt(-c^2*x^2 + 1) + 1))/((c*x + 1)^2*(4*(c^2*x^2 - 1)/(c*x + 1)^2
 - 5*(c^2*x^2 - 1)^2/(c*x + 1)^4 + 5*(c^2*x^2 - 1)^4/(c*x + 1)^8 - 4*(c^2*
x^2 - 1)^5/(c*x + 1)^10 + (c^2*x^2 - 1)^6/(c*x + 1)^12 - 1)) - 4*(c^2*x^2
- 1)*b*c*d^3*log(abs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/((c*x + 1)^2*(4*(c^2*
x^2 - 1)/(c*x + 1)^2 - 5*(c^2*x^2 - 1)^2/(c*x + 1)^4 + 5*(c^2*x^2 - 1)^4/(
c*x + 1)^8 - 4*(c^2*x^2 - 1)^5/(c*x + 1)^10 + (c^2*x^2 - 1)^6/(c*x + 1)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3\,(a+b\arccos(cx))}{x^2}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^3)/x^2,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^3)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.93

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x^2}\,dx$$

$$= \frac{d^3\left(-5acos(cx)\,b\,c^6x^6+25acos(cx)\,b\,c^4x^4-75acos(cx)\,b\,c^2x^2-25acos(cx)\,b+\sqrt{-c^2x^2+1}\,b\,c^5x^5-7\sqrt{}\right.}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acos(c*x))/x^2,x)
```

output
```
(d**3*( - 5*acos(c*x)*b*c**6*x**6 + 25*acos(c*x)*b*c**4*x**4 - 75*acos(c*x
)*b*c**2*x**2 - 25*acos(c*x)*b + sqrt( - c**2*x**2 + 1)*b*c**5*x**5 - 7*sq
rt( - c**2*x**2 + 1)*b*c**3*x**3 + 61*sqrt( - c**2*x**2 + 1)*b*c*x - 25*lo
g(tan(asin(c*x)/2))*b*c*x - 5*a*c**6*x**6 + 25*a*c**4*x**4 - 75*a*c**2*x**
2 - 25*a))/(25*x)
```

3.28 $\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 263

$$
\begin{aligned}
\int \frac{(d-c^2dx^2)^3\,(a+b\arccos(cx))}{x^3}\,dx = {} & \frac{3}{32}bc^3d^3x\sqrt{1-c^2x^2}-\frac{7}{16}bc^3d^3x\left(1-c^2x^2\right)^{3/2} \\
& -\frac{bcd^3(1-c^2x^2)^{5/2}}{2x}+\frac{3}{32}bc^2d^3\arccos(cx) \\
& -\frac{3}{2}c^2d^3\left(1-c^2x^2\right)(a+b\arccos(cx)) \\
& -\frac{3}{4}c^2d^3\left(1-c^2x^2\right)^2(a+b\arccos(cx)) \\
& -\frac{d^3(1-c^2x^2)^3\,(a+b\arccos(cx))}{2x^2} \\
& +\frac{3ic^2d^3(a+b\arccos(cx))^2}{2b} \\
& -3c^2d^3(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right) \\
& +\frac{3}{2}ibc^2d^3\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)
\end{aligned}
$$

output
```
3/32*b*c^3*d^3*x*(-c^2*x^2+1)^(1/2)-7/16*b*c^3*d^3*x*(-c^2*x^2+1)^(3/2)-1/
2*b*c*d^3*(-c^2*x^2+1)^(5/2)/x+3/32*b*c^2*d^3*arccos(c*x)-3/2*c^2*d^3*(-c^
2*x^2+1)*(a+b*arccos(c*x))-3/4*c^2*d^3*(-c^2*x^2+1)^2*(a+b*arccos(c*x))-1/
2*d^3*(-c^2*x^2+1)^3*(a+b*arccos(c*x))/x^2+3/2*I*c^2*d^3*(a+b*arccos(c*x))
^2/b-3*c^2*d^3*(a+b*arccos(c*x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3/2*I*
b*c^2*d^3*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.86

$$\int \frac{(d-c^2dx^2)^3\,(a+b\arccos(cx))}{x^3}\,dx$$
$$= \frac{d^3\Big(-16a+48ac^4x^4-8ac^6x^6+16bcx\sqrt{1-c^2x^2}-21bc^3x^3\sqrt{1-c^2x^2}+2bc^5x^5\sqrt{1-c^2x^2}+48ibc^2x^2\,\text{arc}}{}$$

input
```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
(d^3*(-16*a + 48*a*c^4*x^4 - 8*a*c^6*x^6 + 16*b*c*x*Sqrt[1 - c^2*x^2] - 21
*b*c^3*x^3*Sqrt[1 - c^2*x^2] + 2*b*c^5*x^5*Sqrt[1 - c^2*x^2] + (48*I)*b*c^
2*x^2*ArcCos[c*x]^2 + 42*b*c^2*x^2*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]
- 8*b*ArcCos[c*x]*(2 - 6*c^4*x^4 + c^6*x^6 + 12*c^2*x^2*Log[1 + E^((2*I)*A
rcCos[c*x])]) - 96*a*c^2*x^2*Log[x] + (48*I)*b*c^2*x^2*PolyLog[2, -E^((2*I
)*ArcCos[c*x])]))/(32*x^2)
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.30, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {5191, 27, 247, 211, 211, 223, 5189, 211, 211, 223, 5189, 211, 223, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^3 (a+b\arccos(cx))}{x^3}\,dx$$

$$\Big\downarrow \; 5191$$

$$-3c^2d\int \frac{d^2\left(1-c^2x^2\right)^2 (a+b\arccos(cx))}{x}dx-\frac{1}{2}bcd^3\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^2}dx-\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))}{2x^2}$$

$$\Big\downarrow \; 27$$

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))}{x}dx-\frac{1}{2}bcd^3\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^2}dx-\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))}{2x^2}$$

$$\Big\downarrow \; 247$$

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))}{x}dx-\frac{1}{2}bcd^3\left(-5c^2\int \left(1-c^2x^2\right)^{3/2}dx-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)-\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))}{2x^2}$$

$$\Big\downarrow \; 211$$

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))}{x}dx-\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\int \sqrt{1-c^2x^2}dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)-\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))}{2x^2}$$

$$\Big\downarrow \; 211$$

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))}{x}dx-\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)-\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))}{2x^2}$$

$$\Big\downarrow \; 223$$

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))}{x}dx-\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)$$

$\big\downarrow$ 5189

$$-3c^2d^3\left(\int \frac{\left(1-c^2x^2\right) (a+b\arccos(cx))}{x}dx+\frac{1}{4}bc\int \left(1-c^2x^2\right)^{3/2}dx+\frac{1}{4}\left(1-c^2x^2\right)^2 (a+b\arccos(cx))\right)-\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)$$

$\big\downarrow$ 211

$$-3c^2d^3\left(\int \frac{\left(1-c^2x^2\right) (a+b\arccos(cx))}{x}dx+\frac{1}{4}bc\left(\frac{3}{4}\int \sqrt{1-c^2x^2}dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{4}\left(1-c^2x^2\right)^2 (a+\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)$$

$\big\downarrow$ 211

$$-3c^2d^3\left(\int \frac{\left(1-c^2x^2\right) (a+b\arccos(cx))}{x}dx+\frac{1}{4}bc\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)$$

$\big\downarrow$ 223

$$-3c^2d^3\left(\int \frac{\left(1-c^2x^2\right) (a+b\arccos(cx))}{x}dx+\frac{1}{4}\left(1-c^2x^2\right)^2 (a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-\right.\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))}{2x^2}-\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)$$

$\Big\downarrow$ 5189

$$-3c^2d^3\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\int \sqrt{1-c^2x^2}dx+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{2}\left(1-c^2x^2\right)(a+b\text{ar}\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))}{2x^2}-$$
$$\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)$$

$\Big\downarrow$ 211

$$-3c^2d^3\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))}{2x^2}-$$
$$\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)$$

$\Big\downarrow$ 223

$$-3c^2d^3\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\text{arcsi}}{2}\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))}{2x^2}-$$
$$\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)$$

$\Big\downarrow$ 5137

$$-3c^2d^3\left(-\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{2}\left(1-c^2x^2\right)(a+b\text{a}\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))}{2x^2}-$$
$$\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)$$

$\Big\downarrow$ 3042

$$-3c^2d^3\left(-\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{2}\left(1-c^2x^2\right)(a$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))}{2x^2}-$$
$$\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)$$

$$\Big\downarrow \text{4202}$$

$$-3c^2d^3\left(2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{2}\left(1-c^2x^2\right)(a+b$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))}{2x^2}-$$
$$\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)$$

$$\Big\downarrow \text{2620}$$

$$-3c^2d^3\left(2i\left(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}\left(1-c^2\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))}{2x^2}-$$
$$\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)$$

$$\Big\downarrow \text{2715}$$

$$-3c^2d^3\left(2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))}{2x^2}-$$
$$\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)$$

$$\Big\downarrow \text{2838}$$

$$-3c^2d^3\left(\frac{1}{4}\left(1-c^2x^2\right)^2\left(a+b\arccos(cx)\right)+\frac{1}{2}\left(1-c^2x^2\right)\left(a+b\arccos(cx)\right)+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+\right.\right.$$

$$\frac{d^3\left(1-c^2x^2\right)^3\left(a+b\arccos(cx)\right)}{2x^2}-$$

$$\frac{1}{2}bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)-\frac{\left(1-c^2x^2\right)^{5/2}}{x}\right)$$

input
```
Int[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
-1/2*(d^3*(1 - c^2*x^2)^3*(a + b*ArcCos[c*x]))/x^2 - (b*c*d^3*(-((1 - c^2*
x^2)^(5/2)/x) - 5*c^2*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2
])/2 + ArcSin[c*x]/(2*c)))/4)))/2 - 3*c^2*d^3*(((1 - c^2*x^2)*(a + b*ArcCo
s[c*x]))/2 + ((1 - c^2*x^2)^2*(a + b*ArcCos[c*x]))/4 - ((I/2)*(a + b*ArcCo
s[c*x])^2)/b + (b*c*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 + (b*
c*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(
2*c)))/4))/4 + (2*I)*((-1/2*I)*(a + b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCos
[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5189
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCos[c*x])/(2*p)), x] + (Simp[d
 Int[(d + e*x^2)^(p - 1)*((a + b*ArcCos[c*x])/x), x], x] + Simp[b*c*(d^p/(2
*p))   Int[(1 - c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5191
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCos[c*x
])/(f*(m + 1))), x] + (Simp[b*c*(d^p/(f*(m + 1)))   Int[(f*x)^(m + 1)*(1 -
c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m + 2)
*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x]), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.93

method	result
derivativedivides	$c^2\left(-d^3 a\left(\frac{c^4x^4}{4}-\frac{3c^2x^2}{2}+\frac{1}{2c^2x^2}+3\ln(cx)\right)+\frac{3ib\,d^3\arccos(cx)^2}{2}-\frac{5bc\,d^3x\sqrt{-c^2x^2+1}}{8}+\frac{5d^3b\,\mathrm{arcc}}{}\right.$
default	$c^2\left(-d^3 a\left(\frac{c^4x^4}{4}-\frac{3c^2x^2}{2}+\frac{1}{2c^2x^2}+3\ln(cx)\right)+\frac{3ib\,d^3\arccos(cx)^2}{2}-\frac{5bc\,d^3x\sqrt{-c^2x^2+1}}{8}+\frac{5d^3b\,\mathrm{arcc}}{}\right.$
parts	$-d^3 a\left(\frac{c^6x^4}{4}-\frac{3c^4x^2}{2}+3c^2\ln(x)+\frac{1}{2x^2}\right)+\frac{3id^3b\,c^2\arccos(cx)^2}{2}+\frac{5d^3b\,c^4\arccos(cx)x^2}{4}-\frac{5b\,c^3d^3x\sqrt{-}}{8}$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-d^3*a*(1/4*c^4*x^4-3/2*c^2*x^2+1/2/c^2/x^2+3*ln(c*x))+3/2*I*d^3*b*ar
ccos(c*x)^2-5/8*b*c*d^3*x*(-c^2*x^2+1)^(1/2)+5/4*d^3*b*arccos(c*x)*c^2*x^2
-5/8*b*d^3*arccos(c*x)+1/2*I*d^3*b+1/2*d^3*b/c/x*(-c^2*x^2+1)^(1/2)-1/2*d^
3*b*arccos(c*x)/c^2/x^2-3*d^3*b*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2)
)^2)+3/2*I*d^3*b*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1/32*d^3*b*arcco
s(c*x)*cos(4*arccos(c*x))+1/128*d^3*b*sin(4*arccos(c*x)))
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x^3}\,dx = \int -\frac{(c^2dx^2-d)^3(b\arccos(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*
c^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arccos(c*x))/x^3,
 x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x^3}\,dx = -d^3\left(\int \left(-\frac{a}{x^3}\right)dx + \int \frac{3ac^2}{x}\,dx + \int \left(-3ac^4x\right)dx + \int ac^6x^3\,dx + \int \left(-\frac{b\,\mathrm{acos}\,(cx)}{x^3}\right)dx + \int \frac{3bc^2\,\mathrm{acos}\,(cx)}{x}\,dx + \int \left(-3bc^4x\,\mathrm{acos}\,(cx)\right)dx + \int bc^6x^3\,\mathrm{acos}\,(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acos(c*x))/x**3,x)
```

output
```
-d**3*(Integral(-a/x**3, x) + Integral(3*a*c**2/x, x) + Integral(-3*a*c**4
*x, x) + Integral(a*c**6*x**3, x) + Integral(-b*acos(c*x)/x**3, x) + Integ
ral(3*b*c**2*acos(c*x)/x, x) + Integral(-3*b*c**4*x*acos(c*x), x) + Integr
al(b*c**6*x**3*acos(c*x), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x^3}\,dx = \int -\frac{(c^2dx^2-d)^3(b\arccos(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x^3,x, algorithm="maxima")
```

output
```
-1/4*a*c^6*d^3*x^4 + 3/2*a*c^4*d^3*x^2 - 3*a*c^2*d^3*log(x) + 1/2*b*d^3*(s
qrt(-c^2*x^2 + 1)*c/x - arccos(c*x)/x^2) - 1/2*a*d^3/x^2 - integrate((b*c^
6*d^3*x^4 - 3*b*c^4*d^3*x^2 + 3*b*c^2*d^3)*arctan2(sqrt(c*x + 1)*sqrt(-c*x
 + 1), c*x)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^3}{x^3}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^3)/x^3,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^3)/x^3, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x^3} dx$$
$$= \frac{d^3\left(-8acos(cx)\,b\,c^6x^6+48acos(cx)\,b\,c^4x^4-16acos(cx)\,b+21asin(cx)\,b\,c^2x^2+2\sqrt{-c^2x^2+1}\,b\,c^5x^5-21\right.}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acos(c*x))/x^3,x)
```

output
```
(d**3*( - 8*acos(c*x)*b*c**6*x**6 + 48*acos(c*x)*b*c**4*x**4 - 16*acos(c*x
)*b + 21*asin(c*x)*b*c**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*b*c**5*x**5 - 21
*sqrt( - c**2*x**2 + 1)*b*c**3*x**3 + 16*sqrt( - c**2*x**2 + 1)*b*c*x - 96
*int(acos(c*x)/x,x)*b*c**2*x**2 - 96*log(x)*a*c**2*x**2 - 8*a*c**6*x**6 +
48*a*c**4*x**4 - 16*a))/(32*x**2)
```

3.29 $\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 178

$$\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x^4}\,dx = \frac{8}{3}bc^3d^3\sqrt{1-c^2x^2}-\frac{bcd^3\sqrt{1-c^2x^2}}{6x^2}+\frac{1}{9}bc^3d^3(1-c^2x^2)^{3/2}-\frac{d^3(a+b\arccos(cx))}{3x^3}+\frac{3c^2d^3(a+b\arccos(cx))}{x}+3c^4d^3x(a+b\arccos(cx))-\frac{1}{3}c^6d^3x^3(a+b\arccos(cx))+\frac{17}{6}bc^3d^3\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)$$

output
```
8/3*b*c^3*d^3*(-c^2*x^2+1)^(1/2)-1/6*b*c*d^3*(-c^2*x^2+1)^(1/2)/x^2+1/9*b*
c^3*d^3*(-c^2*x^2+1)^(3/2)-1/3*d^3*(a+b*arccos(c*x))/x^3+3*c^2*d^3*(a+b*ar
ccos(c*x))/x+3*c^4*d^3*x*(a+b*arccos(c*x))-1/3*c^6*d^3*x^3*(a+b*arccos(c*x
))+17/6*b*c^3*d^3*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.98

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x^4} dx = \frac{d^3\left(-6a+54ac^2x^2+54ac^4x^4-6ac^6x^6+3bcx\sqrt{1-c^2x^2}-50bc^3x^3\sqrt{1-c^2x^2}+2bc^5x^5\sqrt{1-c^2x^2}-6b(\right.}{18x^3}$$

input

```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]))/x^4,x]
```

output

```
(d^3*(-6*a + 54*a*c^2*x^2 + 54*a*c^4*x^4 - 6*a*c^6*x^6 + 3*b*c*x*Sqrt[1 -
c^2*x^2] - 50*b*c^3*x^3*Sqrt[1 - c^2*x^2] + 2*b*c^5*x^5*Sqrt[1 - c^2*x^2]
- 6*b*(1 - 9*c^2*x^2 - 9*c^4*x^4 + c^6*x^6)*ArcCos[c*x] + 51*b*c^3*x^3*Log
[x] - 51*b*c^3*x^3*Log[1 + Sqrt[1 - c^2*x^2]]))/(18*x^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5193, 27, 2331, 2124, 27, 1192, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x^4} dx$$

$$\Big\downarrow \text{5193}$$

$$bc\int -\frac{d^3(c^6x^6-9c^4x^4-9c^2x^2+1)}{3x^3\sqrt{1-c^2x^2}}dx - \frac{1}{3}c^6d^3x^3(a+b\arccos(cx))+3c^4d^3x(a+b\arccos(cx))+\frac{3c^2d^3(a+b\arccos(cx))}{x}-\frac{d^3(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}bcd^3\int \frac{c^6x^6-9c^4x^4-9c^2x^2+1}{x^3\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^6d^3x^3(a+b\arccos(cx))+3c^4d^3x(a+b\arccos(cx))+\frac{3c^2d^3(a+b\arccos(cx))}{x}-\frac{d^3(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \text{2331}$$

$$-\frac{1}{6}bcd^3\int \frac{c^6x^6-9c^4x^4-9c^2x^2+1}{x^4\sqrt{1-c^2x^2}}dx^2-\frac{1}{3}c^6d^3x^3(a+b\arccos(cx))+3c^4d^3x(a+b\arccos(cx))+\frac{3c^2d^3(a+b\arccos(cx))}{x}-\frac{d^3(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \text{2124}$$

$$-\frac{1}{6}bcd^3\left(-\int \frac{-2x^4c^6+18x^2c^4+17c^2}{2x^2\sqrt{1-c^2x^2}}dx^2-\frac{\sqrt{1-c^2x^2}}{x^2}\right)-\frac{1}{3}c^6d^3x^3(a+b\arccos(cx))+3c^4d^3x(a+b\arccos(cx))+\frac{3c^2d^3(a+b\arccos(cx))}{x}-\frac{d^3(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}bcd^3\left(-\frac{1}{2}\int \frac{-2x^4c^6+18x^2c^4+17c^2}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{\sqrt{1-c^2x^2}}{x^2}\right)-\frac{1}{3}c^6d^3x^3(a+b\arccos(cx))+3c^4d^3x(a+b\arccos(cx))+\frac{3c^2d^3(a+b\arccos(cx))}{x}-\frac{d^3(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \text{1192}$$

$$-\frac{1}{6}bcd^3\left(-\frac{\int -\frac{-2c^6x^8-14c^6x^4+33c^6}{1-x^4}d\sqrt{1-c^2x^2}}{c^4}-\frac{\sqrt{1-c^2x^2}}{x^2}\right)-\frac{1}{3}c^6d^3x^3(a+b\arccos(cx))+3c^4d^3x(a+b\arccos(cx))+\frac{3c^2d^3(a+b\arccos(cx))}{x}-\frac{d^3(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{6}bcd^3\left(\frac{\int \frac{-2c^6x^8-14c^6x^4+33c^6}{1-x^4}d\sqrt{1-c^2x^2}}{c^4}-\frac{\sqrt{1-c^2x^2}}{x^2}\right)-\frac{1}{3}c^6d^3x^3(a+b\arccos(cx))+3c^4d^3x(a+b\arccos(cx))+\frac{3c^2d^3(a+b\arccos(cx))}{x}-\frac{d^3(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \text{1467}$$

$$-\frac{1}{6}bcd^3\left(\frac{\int \left(2x^4c^6+\frac{17c^6}{1-x^4}+16c^6\right)d\sqrt{1-c^2x^2}}{c^4}-\frac{\sqrt{1-c^2x^2}}{x^2}\right)-\frac{1}{3}c^6d^3x^3(a+b\arccos(cx))+3c^4d^3x(a+b\arccos(cx))+\frac{3c^2d^3(a+b\arccos(cx))}{x}-\frac{d^3(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}c^6d^3x^3(a+b\arccos(cx))+3c^4d^3x(a+b\arccos(cx))+\frac{3c^2d^3(a+b\arccos(cx))}{x}-\frac{d^3(a+b\arccos(cx))}{3x^3}-\frac{1}{6}bcd^3\left(-\frac{-17c^6\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{2}{3}c^6x^6-16c^6\sqrt{1-c^2x^2}}{c^4}-\frac{\sqrt{1-c^2x^2}}{x^2}\right)$$

input
```
Int[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
-1/3*(d^3*(a + b*ArcCos[c*x]))/x^3 + (3*c^2*d^3*(a + b*ArcCos[c*x]))/x + 3
*c^4*d^3*x*(a + b*ArcCos[c*x]) - (c^6*d^3*x^3*(a + b*ArcCos[c*x]))/3 - (b*
c*d^3*(-(Sqrt[1 - c^2*x^2]/x^2) - ((-2*c^6*x^6)/3 - 16*c^6*Sqrt[1 - c^2*x^
2] - 17*c^6*ArcTanh[Sqrt[1 - c^2*x^2]])/c^4))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.89

method	result
parts	$-d^3 a\left(\frac{c^6x^3}{3}-3c^4x-\frac{3c^2}{x}+\frac{1}{3x^3}\right)-d^3 b\, c^3\left(\frac{c^3x^3\arccos(cx)}{3}-3cx\arccos\left(cx\right)+\frac{\arccos(cx)}{3c^3x^3}-\right.$
derivativedivides	$c^3\left(-d^3 a\left(\frac{c^3x^3}{3}-3cx+\frac{1}{3c^3x^3}-\frac{3}{cx}\right)-d^3 b\left(\frac{c^3x^3\arccos(cx)}{3}-3cx\arccos\left(cx\right)+\frac{\arccos(cx)}{3c^3x^3}-\right.\right.$
default	$c^3\left(-d^3 a\left(\frac{c^3x^3}{3}-3cx+\frac{1}{3c^3x^3}-\frac{3}{cx}\right)-d^3 b\left(\frac{c^3x^3\arccos(cx)}{3}-3cx\arccos\left(cx\right)+\frac{\arccos(cx)}{3c^3x^3}-\right.\right.$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-d^3*a*(1/3*c^6*x^3-3*c^4*x-3*c^2/x+1/3/x^3)-d^3*b*c^3*(1/3*c^3*x^3*arccos
(c*x)-3*c*x*arccos(c*x)+1/3*arccos(c*x)/c^3/x^3-3*arccos(c*x)/c/x-1/6/c^2/
x^2*(-c^2*x^2+1)^(1/2)+17/6*arctanh(1/(-c^2*x^2+1)^(1/2))-1/9*c^2*x^2*(-c^
2*x^2+1)^(1/2)+25/9*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.56

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x^4}\,dx =$$
$$-\frac{12\,ac^6d^3x^6 - 108\,ac^4d^3x^4 + 51\,bc^3d^3x^3\log\left(\sqrt{-c^2x^2+1}+1\right) - 51\,bc^3d^3x^3\log\left(\sqrt{-c^2x^2+1}-1\right) - 10}{}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/36*(12*a*c^6*d^3*x^6 - 108*a*c^4*d^3*x^4 + 51*b*c^3*d^3*x^3*log(sqrt(-c
^2*x^2 + 1) + 1) - 51*b*c^3*d^3*x^3*log(sqrt(-c^2*x^2 + 1) - 1) - 108*a*c^
2*d^3*x^2 + 12*(b*c^6 - 9*b*c^4 - 9*b*c^2 + b)*d^3*x^3*arctan(sqrt(-c^2*x^
2 + 1)*c*x/(c^2*x^2 - 1)) + 12*a*d^3 + 12*(b*c^6*d^3*x^6 - 9*b*c^4*d^3*x^4
 - 9*b*c^2*d^3*x^2 - (b*c^6 - 9*b*c^4 - 9*b*c^2 + b)*d^3*x^3 + b*d^3)*arcc
os(c*x) - 2*(2*b*c^5*d^3*x^5 - 50*b*c^3*d^3*x^3 + 3*b*c*d^3*x)*sqrt(-c^2*x
^2 + 1))/x^3
```

Sympy [A] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.85

$$\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x^4}\,dx$$
$$= -\frac{ac^6d^3x^3}{3}+3ac^4d^3x+\frac{3ac^2d^3}{x}-\frac{ad^3}{3x^3} - \frac{bc^7d^3\left(\begin{cases}-\frac{x^2\sqrt{-c^2x^2+1}}{3c^2}-\frac{2\sqrt{-c^2x^2+1}}{3c^4} & \text{for } c^2\neq 0\\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right)}{3}$$
$$-\frac{bc^6d^3x^3\operatorname{acos}(cx)}{3}+3bc^4d^3\left(\begin{cases}\frac{\pi x}{2} & \text{for } c=0\\ x\operatorname{acos}(cx)-\frac{\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)$$
$$+3bc^3d^3\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \frac{1}{|c^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise}\end{cases}\right)+\frac{3bc^2d^3\operatorname{acos}(cx)}{x}$$
$$-\frac{bcd^3\left(\begin{cases}-\frac{c^2\operatorname{acosh}\left(\frac{1}{cx}\right)}{2}+\frac{c}{2x\sqrt{-1+\frac{1}{c^2x^2}}}-\frac{1}{2cx^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \frac{1}{|c^2x^2|}>1\\ \frac{ic^2\operatorname{asin}\left(\frac{1}{cx}\right)}{2}-\frac{ic\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)}{3}$$
$$-\frac{bd^3\operatorname{acos}(cx)}{3x^3}$$

input

```
integrate((-c**2*d*x**2+d)**3*(a+b*acos(c*x))/x**4,x)
```

output

```
-a*c**6*d**3*x**3/3 + 3*a*c**4*d**3*x + 3*a*c**2*d**3/x - a*d**3/(3*x**3)
- b*c**7*d**3*Piecewise((-x**2*sqrt(-c**2*x**2 + 1)/(3*c**2) - 2*sqrt(-c**
2*x**2 + 1)/(3*c**4), Ne(c**2, 0)), (x**4/4, True))/3 - b*c**6*d**3*x**3*a
cos(c*x)/3 + 3*b*c**4*d**3*Piecewise((pi*x/2, Eq(c, 0)), (x*acos(c*x) - sq
rt(-c**2*x**2 + 1)/c, True)) + 3*b*c**3*d**3*Piecewise((-acosh(1/(c*x)), 1
/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)), True)) + 3*b*c**2*d**3*acos(c*x)/x
 - b*c*d**3*Piecewise((-c**2*acosh(1/(c*x))/2 + c/(2*x*sqrt(-1 + 1/(c**2*x
**2))) - 1/(2*c*x**3*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (I*
c**2*asin(1/(c*x))/2 - I*c*sqrt(1 - 1/(c**2*x**2))/(2*x), True))/3 - b*d**
3*acos(c*x)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.38

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x^4}\,dx$$
$$= -\frac{1}{3}ac^6d^3x^3 - \frac{1}{9}\left(3x^3\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bc^6d^3$$
$$+ 3ac^4d^3x + 3\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)bc^3d^3$$
$$- 3\left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{\arccos(cx)}{x}\right)bc^2d^3$$
$$+ \frac{1}{6}\left(\left(c^2\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{\sqrt{-c^2x^2+1}}{x^2}\right)c - \frac{2\arccos(cx)}{x^3}\right)bd^3$$
$$+ \frac{3ac^2d^3}{x} - \frac{ad^3}{3x^3}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x^4,x, algorithm="maxima")
```

output
```
-1/3*a*c^6*d^3*x^3 - 1/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^
2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*c^6*d^3 + 3*a*c^4*d^3*x + 3*(c*x*arccos(c
*x) - sqrt(-c^2*x^2 + 1))*b*c^3*d^3 - 3*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x)
 + 2/abs(x)) - arccos(c*x)/x)*b*c^2*d^3 + 1/6*((c^2*log(2*sqrt(-c^2*x^2 +
1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)/x^2)*c - 2*arccos(c*x)/x^3)*b*d
^3 + 3*a*c^2*d^3/x - 1/3*a*d^3/x^3
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))/x^4,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^3}{x^4}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^3)/x^4,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^3)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.89

$$\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))}{x^4}\,dx$$
$$= \frac{d^3\left(-6acos(cx)\,b\,c^6x^6+54acos(cx)\,b\,c^4x^4+54acos(cx)\,b\,c^2x^2-6acos(cx)\,b+2\sqrt{-c^2x^2+1}\,b\,c^5x^5-50\right.}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acos(c*x))/x^4,x)
```

output
```
(d**3*( - 6*acos(c*x)*b*c**6*x**6 + 54*acos(c*x)*b*c**4*x**4 + 54*acos(c*x
)*b*c**2*x**2 - 6*acos(c*x)*b + 2*sqrt( - c**2*x**2 + 1)*b*c**5*x**5 - 50*
sqrt( - c**2*x**2 + 1)*b*c**3*x**3 + 3*sqrt( - c**2*x**2 + 1)*b*c*x + 51*l
og(tan(asin(c*x)/2))*b*c**3*x**3 - 6*a*c**6*x**6 + 54*a*c**4*x**4 + 54*a*c
**2*x**2 - 6*a))/(18*x**3)
```

3.30 $\int \frac{x^4(a+b\arccos(cx))}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{x^4(a+b\arccos(cx))}{d-c^2dx^2}\,dx = -\frac{4b\sqrt{1-c^2x^2}}{3c^5d}+\frac{b(1-c^2x^2)^{3/2}}{9c^5d} - \frac{x(a+b\arccos(cx))}{c^4d}-\frac{x^3(a+b\arccos(cx))}{3c^2d} - \frac{2i(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c^5d} + \frac{ib\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c^5d} - \frac{ib\,\mathrm{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c^5d}$$

output
```
-4/3*b*(-c^2*x^2+1)^(1/2)/c^5/d+1/9*b*(-c^2*x^2+1)^(3/2)/c^5/d-x*(a+b*arcc
os(c*x))/c^4/d-1/3*x^3*(a+b*arccos(c*x))/c^2/d-2*I*(a+b*arccos(c*x))*arcta
n(c*x+I*(-c^2*x^2+1)^(1/2))/c^5/d+I*b*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/
2)))/c^5/d-I*b*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^5/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.07

$$\int \frac{x^4(a+b\arccos(cx))}{d-c^2dx^2}\,dx =$$
$$-\frac{18acx+6ac^3x^3-22b\sqrt{1-c^2x^2}-2bc^2x^2\sqrt{1-c^2x^2}+18bcx\arccos(cx)+6bc^3x^3\arccos(cx)+18b\operatorname{arc}}{}$$

input
```
Integrate[(x^4*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2),x]
```

output
```
-1/18*(18*a*c*x + 6*a*c^3*x^3 - 22*b*Sqrt[1 - c^2*x^2] - 2*b*c^2*x^2*Sqrt[
1 - c^2*x^2] + 18*b*c*x*ArcCos[c*x] + 6*b*c^3*x^3*ArcCos[c*x] + 18*b*ArcCo
s[c*x]*Log[1 - E^(I*ArcCos[c*x])] - 18*b*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c
*x])] + 9*a*Log[1 - c*x] - 9*a*Log[1 + c*x] + (18*I)*b*PolyLog[2, -E^(I*Ar
cCos[c*x])] - (18*I)*b*PolyLog[2, E^(I*ArcCos[c*x])])/(c^5*d)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5211, 27, 243, 53, 2009, 5211, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arccos(cx))}{d-c^2dx^2}dx$$
$$\Big\downarrow \text{5211}$$
$$\frac{\int \frac{x^2(a+b\arccos(cx))}{d(1-c^2x^2)}dx}{c^2}-\frac{b\int \frac{x^3}{\sqrt{1-c^2x^2}}dx}{3cd}-\frac{x^3(a+b\arccos(cx))}{3c^2d}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2d}-\frac{b\int \frac{x^3}{\sqrt{1-c^2x^2}}dx}{3cd}-\frac{x^3(a+b\arccos(cx))}{3c^2d}$$
$$\Big\downarrow \text{243}$$

$$\frac{\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2d}-\frac{b\int \frac{x^2}{\sqrt{1-c^2x^2}}dx^2}{6cd}-\frac{x^3(a+b\arccos(cx))}{3c^2d}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2d}-\frac{b\int \left(\frac{1}{c^2\sqrt{1-c^2x^2}}-\frac{\sqrt{1-c^2x^2}}{c^2}\right)dx^2}{6cd}-\frac{x^3(a+b\arccos(cx))}{3c^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2d}-\frac{x^3(a+b\arccos(cx))}{3c^2d}-\frac{b\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6cd}$$

$$\Big\downarrow \text{5211}$$

$$\frac{\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c^2}-\frac{b\int \frac{x}{\sqrt{1-c^2x^2}}dx}{c}-\frac{x(a+b\arccos(cx))}{c^2}}{c^2d}-\frac{x^3(a+b\arccos(cx))}{3c^2d}-\frac{b\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6cd}$$

$$\Big\downarrow \text{241}$$

$$\frac{\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}}{c^2d}-\frac{x^3(a+b\arccos(cx))}{3c^2d}-\frac{b\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6cd}$$

$$\Big\downarrow \text{5165}$$

$$\frac{-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}}{c^2d}-\frac{x^3(a+b\arccos(cx))}{3c^2d}-\frac{b\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6cd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}}{c^2d}-\frac{x^3(a+b\arccos(cx))}{3c^2d}-\frac{b\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6cd}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{\frac{-b\int \log(1-e^{i\arccos(cx)})d\arccos(cx)+b\int \log(1+e^{i\arccos(cx)})d\arccos(cx)-2\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}}{c^2d}\frac{x^3(a+b\arccos(cx))}{3c^2d}-\frac{b\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6cd}$$

$\Big\downarrow$ 2715

$$-\frac{\frac{ib\int e^{-i\arccos(cx)}\log(1-e^{i\arccos(cx)})de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log(1+e^{i\arccos(cx)})de^{i\arccos(cx)}-2\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(}{c^3}}{c^2d}\frac{x^3(a+b\arccos(cx))}{3c^2d}-\frac{b\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6cd}$$

$\Big\downarrow$ 2838

$$-\frac{\frac{-2\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))+ib\,\text{PolyLog}(2,-e^{i\arccos(cx)})-ib\,\text{PolyLog}(2,e^{i\arccos(cx)})}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}}{c^2d}-\frac{x^3(a+b\arccos(cx))}{3c^2d}-\frac{b\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6cd}$$

input
```
Int[(x^4*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2),x]
```

output
```
-1/6*(b*((-2*Sqrt[1 - c^2*x^2])/c^4 + (2*(1 - c^2*x^2)^(3/2))/(3*c^4)))/(c
*d) - (x^3*(a + b*ArcCos[c*x]))/(3*c^2*d) + ((b*Sqrt[1 - c^2*x^2])/c^3 - (
x*(a + b*ArcCos[c*x]))/c^2 - (-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c
*x])] + I*b*PolyLog[2, -E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*
x])])/c^3)/(c^2*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.27

method	result
derivativedivides	$-\frac{a\left(\frac{c^3x^3}{3}+cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}+\frac{5b\sqrt{-c^2x^2+1}}{4d}-\frac{5b\arccos(cx)cx}{4d}-\frac{b\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)}{d}+\frac{b\arccos(cx)\ln\left(\right.}{}$
default	$-\frac{a\left(\frac{c^3x^3}{3}+cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}+\frac{5b\sqrt{-c^2x^2+1}}{4d}-\frac{5b\arccos(cx)cx}{4d}-\frac{b\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)}{d}+\frac{b\arccos(cx)\ln\left(\right.}{}$
parts	$-\frac{a\left(\frac{\frac{1}{3}c^2x^3+x}{c^4}+\frac{\ln(cx-1)}{2c^5}-\frac{\ln(cx+1)}{2c^5}\right)}{d}+\frac{5b\sqrt{-c^2x^2+1}}{4c^5d}-\frac{5b\arccos(cx)x}{4d\,c^4}-\frac{b\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)}{d\,c^5}$

input
```
int(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(-a/d*(1/3*c^3*x^3+c*x+1/2*ln(c*x-1)-1/2*ln(c*x+1))+5/4*b/d*(-c^2*x^
2+1)^(1/2)-5/4*b/d*arccos(c*x)*c*x-b/d*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)
^(1/2))+b/d*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+I*b/d*polylog(2,c*x
+I*(-c^2*x^2+1)^(1/2))-I*b/d*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-1/12*b/d
*arccos(c*x)*cos(3*arccos(c*x))+1/36*b/d*sin(3*arccos(c*x)))
```

Fricas [F]

$$\int \frac{x^4(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos(cx)+a)x^4}{c^2dx^2-d}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*x^4*arccos(c*x) + a*x^4)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\arccos(cx))}{d-c^2dx^2}\,dx = -\frac{\int \frac{ax^4}{c^2x^2-1}\,dx + \int \frac{bx^4\operatorname{acos}(cx)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate(x**4*(a+b*acos(c*x))/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a*x**4/(c**2*x**2 - 1), x) + Integral(b*x**4*acos(c*x)/(c**2*x*
*2 - 1), x))/d
```

Maxima [F]

$$\int \frac{x^4(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos(cx)+a)x^4}{c^2dx^2-d}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/6*a*(2*(c^2*x^3 + 3*x)/(c^4*d) - 3*log(c*x + 1)/(c^5*d) + 3*log(c*x - 1
)/(c^5*d)) - 1/6*(6*c^5*d*integrate(-1/6*(2*c^3*x^3 + 6*c*x - 3*log(c*x +
1) + 3*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d*x^2 - c^4*d), x)
 + (2*c^3*x^3 + 6*c*x - 3*log(c*x + 1) + 3*log(-c*x + 1))*arctan2(sqrt(c*x
 + 1)*sqrt(-c*x + 1), c*x))*b/(c^5*d)
```

Giac [F]

$$\int \frac{x^4(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos(cx)+a)x^4}{c^2dx^2-d}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate(-(b*arccos(c*x) + a)*x^4/(c^2*d*x^2 - d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \int \frac{x^4\,(a+b\,\mathrm{acos}(c\,x))}{d-c^2\,d\,x^2}\,dx$$

input
```
int((x^4*(a + b*acos(c*x)))/(d - c^2*d*x^2),x)
```

output
```
int((x^4*(a + b*acos(c*x)))/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \frac{-6\left(\int \frac{acos(cx)x^4}{c^2x^2-1}dx\right)b\,c^5-3\log(c^2x-c)\,a+3\log(c^2x+c)\,a-2a\,c^3x^3-6acx}{6c^5d}$$

input
```
int(x^4*(a+b*acos(c*x))/(-c^2*d*x^2+d),x)
```

output
```
( - 6*int((acos(c*x)*x**4)/(c**2*x**2 - 1),x)*b*c**5 - 3*log(c**2*x - c)*a
 + 3*log(c**2*x + c)*a - 2*a*c**3*x**3 - 6*a*c*x)/(6*c**5*d)
```

3.31 $\int \frac{x^3(a+b\arccos(cx))}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 144

$$\int \frac{x^3(a+b\arccos(cx))}{d-c^2dx^2}\,dx = -\frac{bx\sqrt{1-c^2x^2}}{4c^3d}+\frac{b\arccos(cx)}{4c^4d} - \frac{x^2(a+b\arccos(cx))}{2c^2d}+\frac{i(a+b\arccos(cx))^2}{2bc^4d} - \frac{(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c^4d} + \frac{ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{2c^4d}$$

output
```
-1/4*b*x*(-c^2*x^2+1)^(1/2)/c^3/d+1/4*b*arccos(c*x)/c^4/d-1/2*x^2*(a+b*arc
cos(c*x))/c^2/d+1/2*I*(a+b*arccos(c*x))^2/b/c^4/d-(a+b*arccos(c*x))*ln(1+(
c*x+I*(-c^2*x^2+1)^(1/2))^2)/c^4/d+1/2*I*b*polylog(2,-(c*x+I*(-c^2*x^2+1)^
(1/2))^2)/c^4/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.25

$$\int \frac{x^3(a+b\arccos(cx))}{d-c^2dx^2}\,dx =$$
$$-\frac{2ac^2x^2 - bcx\sqrt{1-c^2x^2} + 2bc^2x^2\arccos(cx) - 2ib\arccos(cx)^2 + 2b\arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right) + 4b\arccos(c$$

input
```
Integrate[(x^3*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2),x]
```

output
```
-1/4*(2*a*c^2*x^2 - b*c*x*Sqrt[1 - c^2*x^2] + 2*b*c^2*x^2*ArcCos[c*x] - (2
*I)*b*ArcCos[c*x]^2 + 2*b*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])] + 4*b*Arc
Cos[c*x]*Log[1 - E^(I*ArcCos[c*x])] + 4*b*ArcCos[c*x]*Log[1 + E^(I*ArcCos[
c*x])] + 2*a*Log[1 - c^2*x^2] - (4*I)*b*PolyLog[2, -E^(I*ArcCos[c*x])] - (
4*I)*b*PolyLog[2, E^(I*ArcCos[c*x])])/(c^4*d)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5211, 27, 262, 223, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arccos(cx))}{d-c^2dx^2}\,dx$$
$$\Big\downarrow \ 5211$$
$$\frac{\int \frac{x(a+b\arccos(cx))}{d(1-c^2x^2)}dx}{c^2} - \frac{b\int \frac{x^2}{\sqrt{1-c^2x^2}}dx}{2cd} - \frac{x^2(a+b\arccos(cx))}{2c^2d}$$
$$\Big\downarrow \ 27$$
$$\frac{\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2d} - \frac{b\int \frac{x^2}{\sqrt{1-c^2x^2}}dx}{2cd} - \frac{x^2(a+b\arccos(cx))}{2c^2d}$$
$$\Big\downarrow \ 262$$

$$\frac{\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2d}-\frac{b\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2cd}-\frac{x^2(a+b\arccos(cx))}{2c^2d}$$

$$\Big\downarrow \text{223}$$

$$\frac{\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2d}-\frac{x^2(a+b\arccos(cx))}{2c^2d}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2cd}$$

$$\Big\downarrow \text{5181}$$

$$-\frac{\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^4d}-\frac{x^2(a+b\arccos(cx))}{2c^2d}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2cd}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{c^4d}-\frac{x^2(a+b\arccos(cx))}{2c^2d}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2cd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c^4d}-\frac{x^2(a+b\arccos(cx))}{2c^2d}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2cd}$$

$$\Big\downarrow \text{4200}$$

$$-\frac{2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4d}-\frac{x^2(a+b\arccos(cx))}{2c^2d}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2cd}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4d}-\frac{x^2(a+b\arccos(cx))}{2c^2d}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2cd}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4d}-\frac{x^2(a+b\arccos(cx))}{2c^2d}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2cd}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4d}-\frac{x^2(a+b\arccos(cx))}{2c^2d}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2cd}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4d}-\frac{x^2(a+b\arccos(cx))}{2c^2d}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2cd}$$

input
```
Int[(x^3*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2),x]
```

output
```
-1/2*(x^2*(a + b*ArcCos[c*x]))/(c^2*d) - (b*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^
2 + ArcSin[c*x]/(2*c^3)))/(2*c*d) - (((-1/2*I)*(a + b*ArcCos[c*x])^2)/b -
(2*I)*((I/2)*(a + b*ArcCos[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])] + (b*PolyL
og[2, E^((2*I)*ArcCos[c*x])])/4))/(c^4*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.22

method	result
parts	$-\frac{a\,x^2}{2d\,c^2}-\frac{a\ln\left(c^2x^2-1\right)}{2d\,c^4}-\frac{b\left(-\frac{i\arccos(cx)^2}{2}+\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)-i\,\mathrm{polylog}\left(2,-cx-i\sqrt{-c^2x^2+1}\right)\right.}{}$
derivativedivides	$\frac{-\frac{a\left(\frac{c^2x^2}{2}+\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(-\frac{i\arccos(cx)^2}{2}+\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)-i\,\mathrm{polylog}\left(2,-cx-i\sqrt{-c^2x^2+1}\right)+\mathrm{arcc}\right.}{}}{c^4}$
default	$\frac{-\frac{a\left(\frac{c^2x^2}{2}+\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(-\frac{i\arccos(cx)^2}{2}+\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)-i\,\mathrm{polylog}\left(2,-cx-i\sqrt{-c^2x^2+1}\right)+\mathrm{arcc}\right.}{}}{c^4}$

input
```
int(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/c^2*x^2-1/2*a/d/c^4*ln(c^2*x^2-1)-b/d/c^4*(-1/2*I*arccos(c*x)^2+a
rccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-I*polylog(2,-c*x-I*(-c^2*x^2+1)^
(1/2))+arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-I*polylog(2,c*x+I*(-c^2*
x^2+1)^(1/2))+1/4*arccos(c*x)*cos(2*arccos(c*x))-1/8*sin(2*arccos(c*x)))
```

Fricas [F]

$$\int \frac{x^3(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos(cx)+a)x^3}{c^2dx^2-d}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*x^3*arccos(c*x) + a*x^3)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\arccos(cx))}{d-c^2dx^2}\,dx = -\frac{\int \frac{ax^3}{c^2x^2-1}\,dx + \int \frac{bx^3\operatorname{acos}(cx)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate(x**3*(a+b*acos(c*x))/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a*x**3/(c**2*x**2 - 1), x) + Integral(b*x**3*acos(c*x)/(c**2*x*
*2 - 1), x))/d
```

Maxima [F]

$$\int \frac{x^3(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos(cx)+a)x^3}{c^2dx^2-d}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a*(x^2/(c^2*d) + log(c^2*x^2 - 1)/(c^4*d)) + 1/2*(2*c^4*d*integrate(1
/2*(c^2*x^2*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)) + e^(1/2*log(c*x + 1)
 + 1/2*log(-c*x + 1))*log(c*x + 1) + e^(1/2*log(c*x + 1) + 1/2*log(-c*x +
1))*log(-c*x + 1))/(c^7*d*x^4 - c^5*d*x^2 + (c^5*d*x^2 - c^3*d)*e^(log(c*x
 + 1) + log(-c*x + 1))), x) - (c^2*x^2 + log(c*x + 1) + log(-c*x + 1))*arc
tan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*b/(c^4*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\arccos(cx))}{d - c^2dx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\arccos(cx))}{d - c^2dx^2}\,dx = \int \frac{x^3\,(a + b\,\mathrm{acos}(c\,x))}{d - c^2\,d\,x^2}\,dx$$

input
```
int((x^3*(a + b*acos(c*x)))/(d - c^2*d*x^2),x)
```

output
```
int((x^3*(a + b*acos(c*x)))/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{x^3(a + b\arccos(cx))}{d - c^2dx^2}\,dx = \frac{-2acos(cx)\,b\,c^2x^2 + 2acos(cx)\,b + asin(cx)\,b + \sqrt{-c^2x^2+1}\,bcx - 4\left(\int \frac{acos(cx)x}{c^2x^2-1}dx\right)b\,c^2 - 2\log(c^2x - c)}{4c^4d}$$

input
```
int(x^3*(a+b*acos(c*x))/(-c^2*d*x^2+d),x)
```

output
```
( - 2*acos(c*x)*b*c**2*x**2 + 2*acos(c*x)*b + asin(c*x)*b + sqrt( - c**2*x
**2 + 1)*b*c*x - 4*int((acos(c*x)*x)/(c**2*x**2 - 1),x)*b*c**2 - 2*log(c**
2*x - c)*a - 2*log(c**2*x + c)*a - 2*a*c**2*x**2)/(4*c**4*d)
```

3.32 $\int \frac{x^2(a+b\arccos(cx))}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{x^2(a+b\arccos(cx))}{d-c^2dx^2}\,dx = -\frac{b\sqrt{1-c^2x^2}}{c^3d}-\frac{x(a+b\arccos(cx))}{c^2d} - \frac{2i(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c^3d} + \frac{ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c^3d} - \frac{ib\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c^3d}$$

output
```
-b*(-c^2*x^2+1)^(1/2)/c^3/d-x*(a+b*arccos(c*x))/c^2/d-2*I*(a+b*arccos(c*x)
)*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c^3/d+I*b*polylog(2,-I*(c*x+I*(-c^2*x^2
+1)^(1/2)))/c^3/d-I*b*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^3/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.11

$$\int \frac{x^2(a+b\arccos(cx))}{d-c^2dx^2}\,dx = -\frac{2acx-2b\sqrt{1-c^2x^2}+2bcx\arccos(cx)+2b\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)-2b\arccos(cx)\log\left(1+e^{i\cdots}\right.}{2c\cdots}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2),x]
```

output
```
-1/2*(2*a*c*x - 2*b*Sqrt[1 - c^2*x^2] + 2*b*c*x*ArcCos[c*x] + 2*b*ArcCos[c
*x]*Log[1 - E^(I*ArcCos[c*x])] - 2*b*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])
] + a*Log[1 - c*x] - a*Log[1 + c*x] + (2*I)*b*PolyLog[2, -E^(I*ArcCos[c*x]
)] - (2*I)*b*PolyLog[2, E^(I*ArcCos[c*x])])/(c^3*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5211, 27, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))}{d-c^2dx^2}\,dx$$

$$\Big\downarrow \text{5211}$$

$$\frac{\int \frac{a+b\arccos(cx)}{d(1-c^2x^2)}dx}{c^2}-\frac{b\int \frac{x}{\sqrt{1-c^2x^2}}dx}{cd}-\frac{x(a+b\arccos(cx))}{c^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c^2d}-\frac{b\int \frac{x}{\sqrt{1-c^2x^2}}dx}{cd}-\frac{x(a+b\arccos(cx))}{c^2d}$$

$$\Big\downarrow \text{241}$$

$$\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c^2d}-\frac{x(a+b\arccos(cx))}{c^2d}+\frac{b\sqrt{1-c^2x^2}}{c^3d}$$

$\Big\downarrow$ 5165

$$-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^3d}-\frac{x(a+b\arccos(cx))}{c^2d}+\frac{b\sqrt{1-c^2x^2}}{c^3d}$$

$\Big\downarrow$ 3042

$$-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{c^3d}-\frac{x(a+b\arccos(cx))}{c^2d}+\frac{b\sqrt{1-c^2x^2}}{c^3d}$$

$\Big\downarrow$ 4671

$$-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\text{arc}}{c^3d}$$
$$\frac{x(a+b\arccos(cx))}{c^2d}+\frac{b\sqrt{1-c^2x^2}}{c^3d}$$

$\Big\downarrow$ 2715

$$-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}}{c^3d}$$
$$\frac{x(a+b\arccos(cx))}{c^2d}+\frac{b\sqrt{1-c^2x^2}}{c^3d}$$

$\Big\downarrow$ 2838

$$-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{c^3d}-$$
$$\frac{x(a+b\arccos(cx))}{c^2d}+\frac{b\sqrt{1-c^2x^2}}{c^3d}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2),x]
```

output
```
(b*Sqrt[1 - c^2*x^2])/(c^3*d) - (x*(a + b*ArcCos[c*x]))/(c^2*d) - (-2*(a +
 b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(I*ArcCos[c
*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/(c^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{-\frac{a\left(cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}+\frac{b\sqrt{-c^2x^2+1}}{d}+\frac{b\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)}{d}-\frac{b\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)}{d}-\frac{b\,a}{}}{c^3}$
default	$\frac{-\frac{a\left(cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}+\frac{b\sqrt{-c^2x^2+1}}{d}+\frac{b\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)}{d}-\frac{b\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)}{d}-\frac{b\,a}{}}{c^3}$
parts	$-\frac{a\left(\frac{x}{c^2}+\frac{\ln(cx-1)}{2c^3}-\frac{\ln(cx+1)}{2c^3}\right)}{d}+\frac{b\sqrt{-c^2x^2+1}}{c^3d}-\frac{b\arccos(cx)x}{d\,c^2}+\frac{ib\,\mathrm{polylog}\left(2,cx+i\sqrt{-c^2x^2+1}\right)}{d\,c^3}-\frac{ib\,\mathrm{polylog}(}{}$

input
```
int(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-a/d*(c*x+1/2*ln(c*x-1)-1/2*ln(c*x+1))+b/d*(-c^2*x^2+1)^(1/2)+b/d*a
rccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-b/d*arccos(c*x)*ln(1-c*x-I*(-c^2
*x^2+1)^(1/2))-b/d*arccos(c*x)*c*x-I*b/d*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/
2))+I*b/d*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int\frac{x^2(a+b\arccos(cx))}{d-c^2dx^2}\,dx=\int-\frac{(b\arccos(cx)+a)x^2}{c^2dx^2-d}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*x^2*arccos(c*x) + a*x^2)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\arccos(cx))}{d-c^2dx^2}\,dx = -\frac{\int \frac{ax^2}{c^2x^2-1}\,dx + \int \frac{bx^2\operatorname{acos}(cx)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate(x**2*(a+b*acos(c*x))/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a*x**2/(c**2*x**2 - 1), x) + Integral(b*x**2*acos(c*x)/(c**2*x*
*2 - 1), x))/d
```

Maxima [F]

$$\int \frac{x^2(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos(cx)+a)x^2}{c^2dx^2-d}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a*(2*x/(c^2*d) - log(c*x + 1)/(c^3*d) + log(c*x - 1)/(c^3*d)) - 1/2*(
2*c^3*d*integrate(-1/2*(2*c*x - log(c*x + 1) + log(-c*x + 1))*sqrt(c*x + 1
)*sqrt(-c*x + 1)/(c^4*d*x^2 - c^2*d), x) + (2*c*x - log(c*x + 1) + log(-c*
x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*b/(c^3*d)
```

Giac [F]

$$\int \frac{x^2(a+b\arccos(cx))}{d-c^2dx^2}\,dx=\int -\frac{(b\arccos(cx)+a)x^2}{c^2dx^2-d}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate(-(b*arccos(c*x) + a)*x^2/(c^2*d*x^2 - d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))}{d-c^2dx^2}\,dx=\int \frac{x^2\,(a+b\,\mathrm{acos}(c\,x))}{d-c^2\,d\,x^2}\,dx$$

input
```
int((x^2*(a + b*acos(c*x)))/(d - c^2*d*x^2),x)
```

output
```
int((x^2*(a + b*acos(c*x)))/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))}{d-c^2dx^2}\,dx$$
$$=\frac{-2acos(cx)\,bcx+2\sqrt{-c^2x^2+1}\,b-2\left(\int \frac{acos(cx)}{c^2x^2-1}dx\right)bc-\log(c^2x-c)\,a+\log(c^2x+c)\,a-2acx}{2c^3d}$$

input
```
int(x^2*(a+b*acos(c*x))/(-c^2*d*x^2+d),x)
```

output
```
( - 2*acos(c*x)*b*c*x + 2*sqrt( - c**2*x**2 + 1)*b - 2*int(acos(c*x)/(c**2
*x**2 - 1),x)*b*c - log(c**2*x - c)*a + log(c**2*x + c)*a - 2*a*c*x)/(2*c*
*3*d)
```

3.33 $\int \frac{x(a+b\arccos(cx))}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{x(a+b\arccos(cx))}{d-c^2dx^2}\,dx=\frac{i(a+b\arccos(cx))^2}{2bc^2d}-\frac{(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c^2d}+\frac{ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{2c^2d}$$

output
```
1/2*I*(a+b*arccos(c*x))^2/b/c^2/d-(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+
1)^(1/2))^2)/c^2/d+1/2*I*b*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c^2/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.40

$$\int \frac{x(a+b\arccos(cx))}{d-c^2dx^2}\,dx=\frac{i\left(b\arccos(cx)^2+2ib\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)+2ib\arccos(cx)\log\left(1+e^{i\arccos(cx)}\right)+ia\log\left(1-c^2\right.\right.}{2c^2d}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2),x]
```

output
```
((I/2)*(b*ArcCos[c*x]^2 + (2*I)*b*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] +
 (2*I)*b*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] + I*a*Log[1 - c^2*x^2] + 2
*b*PolyLog[2, -E^(I*ArcCos[c*x])] + 2*b*PolyLog[2, E^(I*ArcCos[c*x])]))/(c
^2*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5181, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))}{d-c^2dx^2}dx$$

$$\Big\downarrow \text{5181}$$

$$-\frac{\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^2d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{c^2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c^2d}$$

$$\Big\downarrow \text{4200}$$

$$-\frac{2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^2d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^2d}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^2d}$$

$\Big\downarrow$ 2715

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right)-\frac{i(a+b}{}}{c^2d}$$

$\Big\downarrow$ 2838

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^2d}$$

input
```
Int[(x*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2),x]
```

output
```
-((((-1/2*I)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])*L
og[1 - E^((2*I)*ArcCos[c*x])] + (b*PolyLog[2, E^((2*I)*ArcCos[c*x])])/4))/
(c^2*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.73

method	result
parts	$-\frac{a\ln\left(c^{2}x^{2}-1\right)}{2d\,c^{2}}-\frac{b\left(-\frac{i\arccos\left(cx\right)^{2}}{2}+\arccos\left(cx\right)\ln\left(1+cx+i\sqrt{-c^{2}x^{2}+1}\right)-i\operatorname{polylog}\left(2,-cx-i\sqrt{-c^{2}x^{2}+1}\right)+\arccos(\right.}{d\,c^{2}}$
derivativedivides	$\frac{-\frac{a\left(\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(-\frac{i\arccos(cx)^{2}}{2}+\arccos(cx)\ln\left(1+cx+i\sqrt{-c^{2}x^{2}+1}\right)-i\operatorname{polylog}\left(2,-cx-i\sqrt{-c^{2}x^{2}+1}\right)+\arccos(cx)\right.}{d}}{c^{2}}$
default	$\frac{-\frac{a\left(\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(-\frac{i\arccos(cx)^{2}}{2}+\arccos(cx)\ln\left(1+cx+i\sqrt{-c^{2}x^{2}+1}\right)-i\operatorname{polylog}\left(2,-cx-i\sqrt{-c^{2}x^{2}+1}\right)+\arccos(cx)\right.}{d}}{c^{2}}$

input
```
int(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/c^2*ln(c^2*x^2-1)-b/d/c^2*(-1/2*I*arccos(c*x)^2+arccos(c*x)*ln(1+
c*x+I*(-c^2*x^2+1)^(1/2))-I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+arccos(c*
x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-I*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{x(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos(cx)+a)x}{c^2dx^2-d}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*x*arccos(c*x) + a*x)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x(a+b\arccos(cx))}{d-c^2dx^2}\,dx = -\frac{\int \frac{ax}{c^2x^2-1}\,dx + \int \frac{bx\,\mathrm{acos}\,(cx)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate(x*(a+b*acos(c*x))/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a*x/(c**2*x**2 - 1), x) + Integral(b*x*acos(c*x)/(c**2*x**2 - 1
), x))/d
```

Maxima [F]

$$\int \frac{x(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos(cx)+a)x}{c^2dx^2-d}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/2*(2*c^2*d*integrate(1/2*(e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))*log(c
*x + 1) + e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))*log(-c*x + 1))/(c^5*d*x
^4 - c^3*d*x^2 + (c^3*d*x^2 - c*d)*e^(log(c*x + 1) + log(-c*x + 1))), x) -
 (log(c*x + 1) + log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)
)*b/(c^2*d) - 1/2*a*log(c^2*d*x^2 - d)/(c^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b \arccos(cx))}{d - c^2 dx^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b \arccos(cx))}{d - c^2 dx^2} \, dx = \int \frac{x (a + b \operatorname{acos}(c x))}{d - c^2 d x^2} \, dx$$

input
```
int((x*(a + b*acos(c*x)))/(d - c^2*d*x^2),x)
```

output
```
int((x*(a + b*acos(c*x)))/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{x(a + b \arccos(cx))}{d - c^2 dx^2} \, dx = \frac{-2\left(\int \frac{acos(cx)x}{c^2x^2-1} dx\right) b c^2 - \log(c^2x - c) a - \log(c^2x + c) a}{2c^2d}$$

input
```
int(x*(a+b*acos(c*x))/(-c^2*d*x^2+d),x)
```

output
```
( - 2*int((acos(c*x)*x)/(c**2*x**2 - 1),x)*b*c**2 - log(c**2*x - c)*a - lo
g(c**2*x + c)*a)/(2*c**2*d)
```

3.34 $\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx = -\frac{2i(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{cd} + \frac{ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{cd} - \frac{ib\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{cd}$$

output
```
-2*I*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c/d+I*b*polylog(2,
-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/d-I*b*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/
2)))/c/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx = \frac{-2b\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)+2b\arccos(cx)\log\left(1+e^{i\arccos(cx)}\right)-a\log(1-cx)+a\log(1+cx)}{2cd}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d - c^2*d*x^2),x]
```

output
```
(-2*b*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] + 2*b*ArcCos[c*x]*Log[1 + E^(
I*ArcCos[c*x])] - a*Log[1 - c*x] + a*Log[1 + c*x] - (2*I)*b*PolyLog[2, -E^
(I*ArcCos[c*x])] + (2*I)*b*PolyLog[2, E^(I*ArcCos[c*x])])/(2*c*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}dx$$

$$\Big\downarrow \; 5165$$

$$-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{cd}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{cd}$$

$$\Big\downarrow \; 4671$$

$$-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\text{arc}}{cd}$$

$$\Big\downarrow \; 2715$$

$$-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}}{cd}$$

$$\Big\downarrow \; 2838$$

$$-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{cd}$$

input
```
Int[(a + b*ArcCos[c*x])/(d - c^2*d*x^2),x]
```

output
```
-((-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(
I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/(c*d))
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.67

method	result
derivativedivides	$\frac{\frac{a\,\mathrm{arctanh}(cx)}{d}-\frac{b\left(-\mathrm{arctanh}(cx)\arccos(cx)-i\,\mathrm{arctanh}(cx)\left(\ln\left(1-\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)-\ln\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)\right)+i\,\mathrm{dilog}\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)}{d}}{c}$
default	$\frac{\frac{a\,\mathrm{arctanh}(cx)}{d}-\frac{b\left(-\mathrm{arctanh}(cx)\arccos(cx)-i\,\mathrm{arctanh}(cx)\left(\ln\left(1-\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)-\ln\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)\right)+i\,\mathrm{dilog}\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)}{d}}{c}$
parts	$-\frac{a\ln(cx-1)}{2dc}+\frac{a\ln(cx+1)}{2dc}-\frac{b\left(-\mathrm{arctanh}(cx)\arccos(cx)-i\,\mathrm{arctanh}(cx)\left(\ln\left(1-\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right)-\ln\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+1}}\right.\right.\right.}{dc}$

input
```
int((a+b*arccos(c*x))/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c*(a/d*arctanh(c*x)-b/d*(-arctanh(c*x)*arccos(c*x)-I*arctanh(c*x)*(ln(1-
I*(c*x+1)/(-c^2*x^2+1)^(1/2))-ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2)))+I*dilog(
1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-I*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx=\int -\frac{b\arccos(cx)+a}{c^2dx^2-d}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*arccos(c*x) + a)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx = -\frac{\int \frac{a}{c^2x^2-1}\,dx + \int \frac{b\,\mathrm{acos}\,(cx)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate((a+b*acos(c*x))/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a/(c**2*x**2 - 1), x) + Integral(b*acos(c*x)/(c**2*x**2 - 1), x
))/d
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx = \int -\frac{b\arccos(cx)+a}{c^2dx^2-d}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a*(log(c*x + 1)/(c*d) - log(c*x - 1)/(c*d)) - 1/2*(2*c*d*integrate(1/2
*sqrt(c*x + 1)*sqrt(-c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/(c^2*d*x^2 -
d), x) - (log(c*x + 1) - log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x))*b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{d-c^2\,d\,x^2}\,dx$$

input
```
int((a + b*acos(c*x))/(d - c^2*d*x^2),x)
```

output
```
int((a + b*acos(c*x))/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{d-c^2dx^2}\,dx=\frac{-2\left(\int \frac{acos(cx)}{c^2x^2-1}dx\right)bc-\log(c^2x-c)\,a+\log(c^2x+c)\,a}{2cd}$$

input
```
int((a+b*acos(c*x))/(-c^2*d*x^2+d),x)
```

output
```
( - 2*int(acos(c*x)/(c**2*x**2 - 1),x)*b*c - log(c**2*x - c)*a + log(c**2*
x + c)*a)/(2*c*d)
```

3.35 $\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 71

$$\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)}\,dx = -\frac{2(a+b\arccos(cx))\text{arctanh}\left(e^{2i\arccos(cx)}\right)}{d} + \frac{ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{2d} - \frac{ib\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{2d}$$

output
```
-2*(a+b*arccos(c*x))*arctanh((c*x+I*(-c^2*x^2+1)^(1/2))^2)/d+1/2*I*b*polyl
og(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d-1/2*I*b*polylog(2,(c*x+I*(-c^2*x^2+1
)^(1/2))^2)/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 143 vs. 2(71) = 142.

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.01

$$\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)}\,dx = -\frac{2b\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)+2b\arccos(cx)\log\left(1+e^{i\arccos(cx)}\right)-2b\arccos(cx)\log\left(1+e^{2i\arccos}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x*(d - c^2*d*x^2)),x]
```

output
```
-1/2*(2*b*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] + 2*b*ArcCos[c*x]*Log[1 +
 E^(I*ArcCos[c*x])] - 2*b*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] - 2*a
*Log[x] + a*Log[1 - c^2*x^2] - (2*I)*b*PolyLog[2, -E^(I*ArcCos[c*x])] - (2
*I)*b*PolyLog[2, E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^((2*I)*ArcCos[c*x]
)])/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {5185, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)}\,dx$$

$$\Big\downarrow \text{5185}$$

$$-\frac{\int \frac{a+b\arccos(cx)}{cx\sqrt{1-c^2x^2}}d\arccos(cx)}{d}$$

$$\Big\downarrow \text{4919}$$

$$-\frac{2\int (a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int (a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)}{d}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{2\left(-\frac{1}{2}b\int \log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+\frac{1}{2}b\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\left(\text{arctanh}\left(e^{2i\arccos(cx)}\right)\right)(\right.}{d}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2\left(\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right.}{d}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2\left(-\left(\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}ib\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)-\frac{1}{4}ib\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)}{d}$$

input
```
Int[(a + b*ArcCos[c*x])/(x*(d - c^2*d*x^2)),x]
```

output
```
(-2*(-((a + b*ArcCos[c*x])*ArcTanh[E^((2*I)*ArcCos[c*x])]) + (I/4)*b*PolyL
og[2, -E^((2*I)*ArcCos[c*x])] - (I/4)*b*PolyLog[2, E^((2*I)*ArcCos[c*x])])
)/d
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.76

method	result
parts	$-\frac{a\left(-\ln(x)+\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)-i\,\text{polylog}\left(2,-cx-i\sqrt{-c^2x^2+1}\right)-\text{arcc}\right.}{}$
derivativedivides	$-\frac{a\left(\frac{\ln(cx-1)}{2}-\ln(cx)+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)-i\,\text{polylog}\left(2,-cx-i\sqrt{-c^2x^2+1}\right)-\text{arcco}\right.}{}$
default	$-\frac{a\left(\frac{\ln(cx-1)}{2}-\ln(cx)+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)-i\,\text{polylog}\left(2,-cx-i\sqrt{-c^2x^2+1}\right)-\text{arcco}\right.}{}$

input
```
int((a+b*arccos(c*x))/x/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-a/d*(-ln(x)+1/2*ln(c*x-1)+1/2*ln(c*x+1))-b/d*(arccos(c*x)*ln(1+c*x+I*(-c^
2*x^2+1)^(1/2))-I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-arccos(c*x)*ln(1+(c
*x+I*(-c^2*x^2+1)^(1/2))^2)+1/2*I*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)
+arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-I*polylog(2,c*x+I*(-c^2*x^2+1)
^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)}\,dx=\int -\frac{b\arccos\left(cx\right)+a}{(c^2dx^2-d)x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*arccos(c*x) + a)/(c^2*d*x^3 - d*x), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)}\,dx=-\frac{\int \frac{a}{c^2x^3-x}\,dx+\int \frac{b\operatorname{acos}\left(cx\right)}{c^2x^3-x}\,dx}{d}$$

input
```
integrate((a+b*acos(c*x))/x/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a/(c**2*x**3 - x), x) + Integral(b*acos(c*x)/(c**2*x**3 - x), x
))/d
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)}\,dx=\int -\frac{b\arccos\left(cx\right)+a}{(c^2dx^2-d)x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a*(log(c*x + 1)/d + log(c*x - 1)/d - 2*log(x)/d) - b*integrate(arctan
2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^2*d*x^3 - d*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x\,(d-c^2dx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x\,(d-c^2dx^2)}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x\,\left(d-c^2\,d\,x^2\right)}\,dx$$

input
```
int((a + b*acos(c*x))/(x*(d - c^2*d*x^2)),x)
```

output
```
int((a + b*acos(c*x))/(x*(d - c^2*d*x^2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x\,(d-c^2dx^2)}\,dx = \frac{-2\left(\int \frac{acos(cx)}{c^2x^3-x}dx\right)b-\log(c^2x-c)\,a-\log(c^2x+c)\,a+2\log(x)\,a}{2d}$$

input
```
int((a+b*acos(c*x))/x/(-c^2*d*x^2+d),x)
```

output
```
( - 2*int(acos(c*x)/(c**2*x**3 - x),x)*b - log(c**2*x - c)*a - log(c**2*x
+ c)*a + 2*log(x)*a)/(2*d)
```

3.36 $\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)}\,dx = -\frac{a+b\arccos(cx)}{dx} - \frac{2ic(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{d} - \frac{bc\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d} + \frac{ibc\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{d} - \frac{ibc\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{d}$$

output
```
-(a+b*arccos(c*x))/d/x-2*I*c*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(
1/2))/d-b*c*arctanh((-c^2*x^2+1)^(1/2))/d+I*b*c*polylog(2,-I*(c*x+I*(-c^2*
x^2+1)^(1/2)))/d-I*b*c*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.36

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)}\,dx =$$
$$-\frac{2a+2b\arccos(cx)+2bcx\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)-2bcx\arccos(cx)\log\left(1+e^{i\arccos(cx)}\right)+2bc}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^2*(d - c^2*d*x^2)),x]
```

output
```
-1/2*(2*a + 2*b*ArcCos[c*x] + 2*b*c*x*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x]
)] - 2*b*c*x*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] + 2*b*c*x*Log[x] + a*c
*x*Log[1 - c*x] - a*c*x*Log[1 + c*x] - 2*b*c*x*Log[1 + Sqrt[1 - c^2*x^2]]
+ (2*I)*b*c*x*PolyLog[2, -E^(I*ArcCos[c*x])] - (2*I)*b*c*x*PolyLog[2, E^(I
*ArcCos[c*x])])/(d*x)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5205, 27, 243, 73, 221, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)}dx$$

$$\Big\downarrow \text{5205}$$

$$c^2\int \frac{a+b\arccos(cx)}{d(1-c^2x^2)}dx-\frac{bc\int \frac{1}{x\sqrt{1-c^2x^2}}dx}{d}-\frac{a+b\arccos(cx)}{dx}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{d}-\frac{bc\int \frac{1}{x\sqrt{1-c^2x^2}}dx}{d}-\frac{a+b\arccos(cx)}{dx}$$

$$\Big\downarrow \text{243}$$

$$\frac{c^2\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{d}-\frac{bc\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2}{2d}-\frac{a+b\arccos(cx)}{dx}$$

$$\Big\downarrow \text{73}$$

$$\frac{c^2\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{d}+\frac{b\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{cd}-\frac{a+b\arccos(cx)}{dx}$$

$$\Big\downarrow \text{221}$$

$$\frac{c^2\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{d}-\frac{a+b\arccos(cx)}{dx}+\frac{bc\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d}$$

$$\Big\downarrow \text{ 5165}$$

$$-\frac{c\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{d}-\frac{a+b\arccos(cx)}{dx}+\frac{bc\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{c\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{d}-\frac{a+b\arccos(cx)}{dx}+\frac{bc\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d}$$

$$\Big\downarrow \text{ 4671}$$

$$-\frac{c\left(-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\,\text{a}\right.}{d}$$
$$\frac{a+b\arccos(cx)}{dx}+\frac{bc\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{c\left(ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arcta}\right.}{d}$$
$$\frac{a+b\arccos(cx)}{dx}+\frac{bc\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right)}{d}-$$
$$\frac{a+b\arccos(cx)}{dx}+\frac{bc\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^2*(d - c^2*d*x^2)),x]
```

output
```
-((a + b*ArcCos[c*x])/(d*x)) + (b*c*ArcTanh[Sqrt[1 - c^2*x^2]])/d - (c*(-2
*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(I*Arc
Cos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])]))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.26

method	result
parts	$-\frac{a\left(\frac{c\ln(cx-1)}{2}+\frac{1}{x}-\frac{c\ln(cx+1)}{2}\right)}{d}-\frac{bc\left(\frac{\arccos(cx)}{cx}+i\,\mathrm{dilog}\left(1+cx+i\sqrt{-c^2x^2+1}\right)+2i\arctan\left(cx+i\sqrt{-c^2x^2+1}\right)+i\,\mathrm{dilo}\right.}{d}$
derivativedivides	$c\left(-\frac{a\left(\frac{\ln(cx-1)}{2}+\frac{1}{cx}-\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(\frac{\arccos(cx)}{cx}+i\,\mathrm{dilog}\left(1+cx+i\sqrt{-c^2x^2+1}\right)+2i\arctan\left(cx+i\sqrt{-c^2x^2+1}\right)+i\,\mathrm{dil}\right.}{d}\right.$
default	$c\left(-\frac{a\left(\frac{\ln(cx-1)}{2}+\frac{1}{cx}-\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(\frac{\arccos(cx)}{cx}+i\,\mathrm{dilog}\left(1+cx+i\sqrt{-c^2x^2+1}\right)+2i\arctan\left(cx+i\sqrt{-c^2x^2+1}\right)+i\,\mathrm{dil}\right.}{d}\right.$

input
```
int((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-a/d*(1/2*c*ln(c*x-1)+1/x-1/2*c*ln(c*x+1))-b/d*c*(arccos(c*x)/c/x+I*dilog(
1+c*x+I*(-c^2*x^2+1)^(1/2))+2*I*arctan(c*x+I*(-c^2*x^2+1)^(1/2))+I*dilog(c
*x+I*(-c^2*x^2+1)^(1/2))-arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^2\,(d-c^2dx^2)}\,dx=\int -\frac{b\arccos{(cx)}+a}{(c^2dx^2-d)x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*arccos(c*x) + a)/(c^2*d*x^4 - d*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^2\,(d-c^2dx^2)}\,dx=-\frac{\int \frac{a}{c^2x^4-x^2}\,dx+\int \frac{b\,\mathrm{acos}\,(cx)}{c^2x^4-x^2}\,dx}{d}$$

input
```
integrate((a+b*acos(c*x))/x**2/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a/(c**2*x**4 - x**2), x) + Integral(b*acos(c*x)/(c**2*x**4 - x*
*2), x))/d
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x^2\,(d-c^2dx^2)}\,dx=\int -\frac{b\arccos{(cx)}+a}{(c^2dx^2-d)x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a*(c*log(c*x + 1)/d - c*log(c*x - 1)/d - 2/(d*x)) - 1/2*(2*d*x*integra
te(1/2*(c^2*x*log(c*x + 1) - c^2*x*log(-c*x + 1) - 2*c)*sqrt(c*x + 1)*sqrt
(-c*x + 1)/(c^2*d*x^3 - d*x), x) - (c*x*log(c*x + 1) - c*x*log(-c*x + 1) -
 2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*b/(d*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x^2\,(d-c^2\,d\,x^2)}\,dx$$

input
```
int((a + b*acos(c*x))/(x^2*(d - c^2*d*x^2)),x)
```

output
```
int((a + b*acos(c*x))/(x^2*(d - c^2*d*x^2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)}\,dx = \frac{-2\left(\int \frac{acos(cx)}{c^2x^4-x^2}dx\right)bx-\log(c^2x-c)\,acx+\log(c^2x+c)\,acx-2a}{2dx}$$

input
```
int((a+b*acos(c*x))/x^2/(-c^2*d*x^2+d),x)
```

output
```
( - 2*int(acos(c*x)/(c**2*x**4 - x**2),x)*b*x - log(c**2*x - c)*a*c*x + lo
g(c**2*x + c)*a*c*x - 2*a)/(2*d*x)
```

3.37 $\int \frac{a+b \arccos(cx)}{x^3(d-c^2dx^2)} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)}\,dx = -\frac{bc\sqrt{1-c^2x^2}}{2dx}-\frac{a+b\arccos(cx)}{2dx^2} - \frac{2c^2(a+b\arccos(cx))\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)}{d} + \frac{ibc^2\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{2d}-\frac{ibc^2\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{2d}$$

output

```
-1/2*b*c*(-c^2*x^2+1)^(1/2)/d/x-1/2*(a+b*arccos(c*x))/d/x^2-2*c^2*(a+b*arc
cos(c*x))*arctanh((c*x+I*(-c^2*x^2+1)^(1/2))^2)/d+1/2*I*b*c^2*polylog(2,-(
c*x+I*(-c^2*x^2+1)^(1/2))^2)/d-1/2*I*b*c^2*polylog(2,(c*x+I*(-c^2*x^2+1)^(
1/2))^2)/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.77

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)}\,dx =$$
$$-\frac{a-bcx\sqrt{1-c^2x^2}+b\arccos(cx)+2bc^2x^2\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)+2bc^2x^2\arccos(cx)\log\left(1+\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^3*(d - c^2*d*x^2)),x]
```

output
```
-1/2*(a - b*c*x*Sqrt[1 - c^2*x^2] + b*ArcCos[c*x] + 2*b*c^2*x^2*ArcCos[c*x
]*Log[1 - E^(I*ArcCos[c*x])] + 2*b*c^2*x^2*ArcCos[c*x]*Log[1 + E^(I*ArcCos
[c*x])] - 2*b*c^2*x^2*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] - 2*a*c^2
*x^2*Log[x] + a*c^2*x^2*Log[1 - c^2*x^2] - (2*I)*b*c^2*x^2*PolyLog[2, -E^(
I*ArcCos[c*x])] - (2*I)*b*c^2*x^2*PolyLog[2, E^(I*ArcCos[c*x])] + I*b*c^2*
x^2*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/(d*x^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5205, 27, 242, 5185, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)}\,dx$$

$$\Big\downarrow \text{5205}$$

$$c^2\int \frac{a+b\arccos(cx)}{dx(1-c^2x^2)}dx-\frac{bc\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx}{2d}-\frac{a+b\arccos(cx)}{2dx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d}-\frac{bc\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx}{2d}-\frac{a+b\arccos(cx)}{2dx^2}$$

$$\Big\downarrow \text{242}$$

$$\frac{c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d}-\frac{a+b\arccos(cx)}{2dx^2}+\frac{bc\sqrt{1-c^2x^2}}{2dx}$$

$$\Big\downarrow \text{5185}$$

$$-\frac{c^2\int \frac{a+b\arccos(cx)}{cx\sqrt{1-c^2x^2}}d\arccos(cx)}{d}-\frac{a+b\arccos(cx)}{2dx^2}+\frac{bc\sqrt{1-c^2x^2}}{2dx}$$

$$\Big\downarrow \text{4919}$$

$$-\frac{2c^2\int (a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)}{d}-\frac{a+b\arccos(cx)}{2dx^2}+\frac{bc\sqrt{1-c^2x^2}}{2dx}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2c^2\int (a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)}{d}-\frac{a+b\arccos(cx)}{2dx^2}+\frac{bc\sqrt{1-c^2x^2}}{2dx}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{2c^2\left(-\frac{1}{2}b\int \log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+\frac{1}{2}b\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\left(\text{arctanh}\left(e^{2i\arccos(cx)}\right)\right.}{d}$$
$$\frac{a+b\arccos(cx)}{2dx^2}+\frac{bc\sqrt{1-c^2x^2}}{2dx}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2c^2\left(\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(c}\right.}{d}$$
$$\frac{a+b\arccos(cx)}{2dx^2}+\frac{bc\sqrt{1-c^2x^2}}{2dx}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2c^2\left(-\left(\text{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right.\right.}{d}$$
$$\frac{a+b\arccos(cx)}{2dx^2}+\frac{bc\sqrt{1-c^2x^2}}{2dx}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^3*(d - c^2*d*x^2)),x]
```

output
```
(b*c*Sqrt[1 - c^2*x^2])/(2*d*x) - (a + b*ArcCos[c*x])/(2*d*x^2) - (2*c^2*(
-((a + b*ArcCos[c*x])*ArcTanh[E^((2*I)*ArcCos[c*x])]) + (I/4)*b*PolyLog[2,
 -E^((2*I)*ArcCos[c*x])] - (I/4)*b*PolyLog[2, E^((2*I)*ArcCos[c*x])]))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.00

method	result
derivativedivides	$c^2\left(-\frac{a\left(\frac{\ln(cx-1)}{2}+\frac{1}{2c^2x^2}-\ln(cx)+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(\frac{-ic^2x^2-cx\sqrt{-c^2x^2+1}+\arccos(cx)}{2c^2x^2}+\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2\right.\right.}}{}\right.$
default	$c^2\left(-\frac{a\left(\frac{\ln(cx-1)}{2}+\frac{1}{2c^2x^2}-\ln(cx)+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b\left(\frac{-ic^2x^2-cx\sqrt{-c^2x^2+1}+\arccos(cx)}{2c^2x^2}+\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2\right.\right.}}{}\right.$
parts	$-\frac{a\left(\frac{1}{2x^2}-c^2\ln(x)+\frac{c^2\ln(cx-1)}{2}+\frac{c^2\ln(cx+1)}{2}\right)}{d}-\frac{b\,c^2\left(\frac{-ic^2x^2-cx\sqrt{-c^2x^2+1}+\arccos(cx)}{2c^2x^2}+\arccos(cx)\ln\left(1+cx+i\sqrt{-\right.\right.}}{}$

input
```
int((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
c^2*(-a/d*(1/2*ln(c*x-1)+1/2/c^2/x^2-ln(c*x)+1/2*ln(c*x+1))-b/d*(1/2*(-I*c
^2*x^2-c*x*(-c^2*x^2+1)^(1/2)+arccos(c*x))/c^2/x^2+arccos(c*x)*ln(1+c*x+I*
(-c^2*x^2+1)^(1/2))-I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-arccos(c*x)*ln(
1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/2*I*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2)
)^2)+arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-I*polylog(2,c*x+I*(-c^2*x^
2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)}\,dx = \int -\frac{b\arccos(cx)+a}{(c^2dx^2-d)x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*arccos(c*x) + a)/(c^2*d*x^5 - d*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)}\,dx = -\frac{\int \frac{a}{c^2x^5-x^3}\,dx + \int \frac{b\,\mathrm{acos}\,(cx)}{c^2x^5-x^3}\,dx}{d}$$

input
```
integrate((a+b*acos(c*x))/x**3/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a/(c**2*x**5 - x**3), x) + Integral(b*acos(c*x)/(c**2*x**5 - x*
*3), x))/d
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)}\,dx = \int -\frac{b\arccos(cx)+a}{(c^2dx^2-d)x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*(c^2*log(c*x + 1)/d + c^2*log(c*x - 1)/d - 2*c^2*log(x)/d + 1/(d*x^2)
)*a - b*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^2*d*x^5 -
d*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x^3\,(d-c^2\,d\,x^2)}\,dx$$

input
```
int((a + b*acos(c*x))/(x^3*(d - c^2*d*x^2)),x)
```

output
```
int((a + b*acos(c*x))/(x^3*(d - c^2*d*x^2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)}\,dx$$
$$=\frac{-2\left(\int \frac{acos(cx)}{c^2x^5-x^3}dx\right)b\,x^2-\log(c^2x-c)\,a\,c^2x^2-\log(c^2x+c)\,a\,c^2x^2+2\log(x)\,a\,c^2x^2-a}{2d\,x^2}$$

input
```
int((a+b*acos(c*x))/x^3/(-c^2*d*x^2+d),x)
```

output
```
( - 2*int(acos(c*x)/(c**2*x**5 - x**3),x)*b*x**2 - log(c**2*x - c)*a*c**2*
x**2 - log(c**2*x + c)*a*c**2*x**2 + 2*log(x)*a*c**2*x**2 - a)/(2*d*x**2)
```

3.38 $\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 173

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)}\,dx = -\frac{bc\sqrt{1-c^2x^2}}{6dx^2}-\frac{a+b\arccos(cx)}{3dx^3}-\frac{c^2(a+b\arccos(cx))}{dx} - \frac{2ic^3(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{d} - \frac{7bc^3\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{6d}+\frac{ibc^3\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{d} - \frac{ibc^3\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{d}$$

output

```
-1/6*b*c*(-c^2*x^2+1)^(1/2)/d/x^2-1/3*(a+b*arccos(c*x))/d/x^3-c^2*(a+b*arc
cos(c*x))/d/x-2*I*c^3*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/d
-7/6*b*c^3*arctanh((-c^2*x^2+1)^(1/2))/d+I*b*c^3*polylog(2,-I*(c*x+I*(-c^2
*x^2+1)^(1/2)))/d-I*b*c^3*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.34

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)}\,dx =$$
$$-\frac{2a+6ac^2x^2-bcx\sqrt{1-c^2x^2}+2b\arccos(cx)+6bc^2x^2\arccos(cx)+6bc^3x^3\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^4*(d - c^2*d*x^2)),x]
```

output
```
-1/6*(2*a + 6*a*c^2*x^2 - b*c*x*Sqrt[1 - c^2*x^2] + 2*b*ArcCos[c*x] + 6*b*
c^2*x^2*ArcCos[c*x] + 6*b*c^3*x^3*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] -
 6*b*c^3*x^3*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] + 7*b*c^3*x^3*Log[x] +
 3*a*c^3*x^3*Log[1 - c*x] - 3*a*c^3*x^3*Log[1 + c*x] - 7*b*c^3*x^3*Log[1 +
 Sqrt[1 - c^2*x^2]] + (6*I)*b*c^3*x^3*PolyLog[2, -E^(I*ArcCos[c*x])] - (6*
I)*b*c^3*x^3*PolyLog[2, E^(I*ArcCos[c*x])])/(d*x^3)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5205, 27, 243, 52, 73, 221, 5205, 243, 73, 221, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)}\,dx$$

$$\Big\downarrow \text{ 5205}$$

$$c^2\int \frac{a+b\arccos(cx)}{dx^2(1-c^2x^2)}dx-\frac{bc\int \frac{1}{x^3\sqrt{1-c^2x^2}}dx}{3d}-\frac{a+b\arccos(cx)}{3dx^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)}dx}{d}-\frac{bc\int \frac{1}{x^3\sqrt{1-c^2x^2}}dx}{3d}-\frac{a+b\arccos(cx)}{3dx^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)}dx}{d}-\frac{bc\int \frac{1}{x^4\sqrt{1-c^2x^2}}dx^2}{6d}-\frac{a+b\arccos(cx)}{3dx^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)}dx}{d}-\frac{bc\left(\frac{1}{2}c^2\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{\sqrt{1-c^2x^2}}{x^2}\right)}{6d}-\frac{a+b\arccos(cx)}{3dx^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)}dx}{d}-\frac{bc\left(-\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}-\frac{\sqrt{1-c^2x^2}}{x^2}\right)}{6d}-\frac{a+b\arccos(cx)}{3dx^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)}dx}{d}-\frac{a+b\arccos(cx)}{3dx^3}-\frac{bc\left(c^2\left(-\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{\sqrt{1-c^2x^2}}{x^2}\right)}{6d}$$

$$\Big\downarrow \text{5205}$$

$$\frac{c^2\left(c^2\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx-bc\int \frac{1}{x\sqrt{1-c^2x^2}}dx-\frac{a+b\arccos(cx)}{x}\right)}{d}-\frac{a+b\arccos(cx)}{3dx^3}-\frac{bc\left(c^2\left(-\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{\sqrt{1-c^2x^2}}{x^2}\right)}{6d}$$

$$\Big\downarrow \text{243}$$

$$\frac{c^2\left(c^2\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx-\frac{1}{2}bc\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{a+b\arccos(cx)}{x}\right)}{d}-\frac{a+b\arccos(cx)}{3dx^3}-\frac{bc\left(c^2\left(-\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{\sqrt{1-c^2x^2}}{x^2}\right)}{6d}$$

$$\Big\downarrow \text{73}$$

$$\frac{c^2\left(c^2\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{b\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c}-\frac{a+b\arccos(cx)}{x}\right)}{d}-\frac{a+b\arccos(cx)}{3dx^3}-\frac{bc\left(c^2\left(-\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{\sqrt{1-c^2x^2}}{x^2}\right)}{6d}$$

$$\Big\downarrow \text{221}$$

$$\frac{c^2\left(c^2\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx-\frac{a+b\arccos(cx)}{x}+bc\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{d}-\frac{a+b\arccos(cx)}{3dx^3}-\frac{bc\left(c^2\left(-\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{\sqrt{1-c^2x^2}}{x^2}\right)}{6d}$$

$$\Big\downarrow \text{ 5165}$$

$$\frac{c^2\left(-c\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)-\frac{a+b\arccos(cx)}{x}+bc\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{d}-\frac{a+b\arccos(cx)}{3dx^3}-\frac{bc\left(c^2\left(-\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{\sqrt{1-c^2x^2}}{x^2}\right)}{6d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{c^2\left(-c\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)-\frac{a+b\arccos(cx)}{x}+bc\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{d}-\frac{a+b\arccos(cx)}{3dx^3}-\frac{bc\left(c^2\left(-\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{\sqrt{1-c^2x^2}}{x^2}\right)}{6d}$$

$$\Big\downarrow \text{ 4671}$$

$$\frac{c^2\left(-c\left(-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+\right.\right.}{d}$$
$$\frac{a+b\arccos(cx)}{3dx^3}-\frac{bc\left(c^2\left(-\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{\sqrt{1-c^2x^2}}{x^2}\right)}{6d}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{c^2\left(-c\left(ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{ar}\right.\right.}{d}$$
$$\frac{a+b\arccos(cx)}{3dx^3}-\frac{bc\left(c^2\left(-\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{\sqrt{1-c^2x^2}}{x^2}\right)}{6d}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{c^2\left(-c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right)-\right.}{d}$$
$$\frac{a+b\arccos(cx)}{3dx^3}-\frac{bc\left(c^2\left(-\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{\sqrt{1-c^2x^2}}{x^2}\right)}{6d}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^4*(d - c^2*d*x^2)),x]
```

output
```
-1/3*(a + b*ArcCos[c*x])/(d*x^3) - (b*c*(-(Sqrt[1 - c^2*x^2]/x^2) - c^2*Ar
cTanh[Sqrt[1 - c^2*x^2]]))/(6*d) + (c^2*(-((a + b*ArcCos[c*x])/x) + b*c*Ar
cTanh[Sqrt[1 - c^2*x^2]] - c*(-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c
*x])] + I*b*PolyLog[2, -E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*
x])])))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.21

method	result
parts	$-\frac{a\left(\frac{1}{3x^3}+\frac{c^2}{x}+\frac{c^3\ln(cx-1)}{2}-\frac{c^3\ln(cx+1)}{2}\right)}{d}-\frac{ib\left(6i\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)c^3x^3-6i\arccos(cx)c^2x^2+14\,\mathrm{a}\right.}{}$
derivativedivides	$c^3\left(-\frac{a\left(\frac{\ln(cx-1)}{2}+\frac{1}{3c^3x^3}+\frac{1}{cx}-\frac{\ln(cx+1)}{2}\right)}{d}-\frac{ib\left(6i\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)c^3x^3-6i\arccos(cx)c^2x^2+14\right.}{}\right.$
default	$c^3\left(-\frac{a\left(\frac{\ln(cx-1)}{2}+\frac{1}{3c^3x^3}+\frac{1}{cx}-\frac{\ln(cx+1)}{2}\right)}{d}-\frac{ib\left(6i\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)c^3x^3-6i\arccos(cx)c^2x^2+14\right.}{}\right.$

input
```
int((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-a/d*(1/3/x^3+c^2/x+1/2*c^3*ln(c*x-1)-1/2*c^3*ln(c*x+1))-1/6*I*b/d/x^3*(6*
I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*c^3*x^3-6*I*arccos(c*x)*c^2*x
^2+14*arctan(c*x+I*(-c^2*x^2+1)^(1/2))*c^3*x^3+6*dilog(1+c*x+I*(-c^2*x^2+1
)^(1/2))*c^3*x^3+6*dilog(c*x+I*(-c^2*x^2+1)^(1/2))*c^3*x^3+I*(-c^2*x^2+1)^
(1/2)*c*x-2*I*arccos(c*x))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^4\,(d-c^2dx^2)}\,dx = \int -\frac{b\arccos{(cx)}+a}{(c^2dx^2-d)x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*arccos(c*x) + a)/(c^2*d*x^6 - d*x^4), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)}\,dx = -\frac{\int \frac{a}{c^2x^6-x^4}\,dx + \int \frac{b\operatorname{acos}(cx)}{c^2x^6-x^4}\,dx}{d}$$

input
```
integrate((a+b*acos(c*x))/x**4/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a/(c**2*x**6 - x**4), x) + Integral(b*acos(c*x)/(c**2*x**6 - x*
*4), x))/d
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)}\,dx = \int -\frac{b\arccos(cx)+a}{(c^2dx^2-d)x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/6*(3*c^3*log(c*x + 1)/d - 3*c^3*log(c*x - 1)/d - 2*(3*c^2*x^2 + 1)/(d*x^
3))*a - 1/6*(6*d*x^3*integrate(1/6*(3*c^4*x^3*log(c*x + 1) - 3*c^4*x^3*log
(-c*x + 1) - 6*c^3*x^2 - 2*c)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^2*d*x^5 - d*
x^3), x) - (3*c^3*x^3*log(c*x + 1) - 3*c^3*x^3*log(-c*x + 1) - 6*c^2*x^2 -
 2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*b/(d*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^4\,(d-c^2dx^2)}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x^4\,(d-c^2\,d\,x^2)}\,dx$$

input
```
int((a + b*acos(c*x))/(x^4*(d - c^2*d*x^2)),x)
```

output
```
int((a + b*acos(c*x))/(x^4*(d - c^2*d*x^2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^4\,(d-c^2dx^2)}\,dx = \frac{-6\left(\int \frac{acos(cx)}{c^2x^6-x^4}dx\right)b\,x^3-3\log(c^2x-c)\,a\,c^3x^3+3\log(c^2x+c)\,a\,c^3x^3-6a\,c^2x^2-2a}{6d\,x^3}$$

input
```
int((a+b*acos(c*x))/x^4/(-c^2*d*x^2+d),x)
```

output
```
( - 6*int(acos(c*x)/(c**2*x**6 - x**4),x)*b*x**3 - 3*log(c**2*x - c)*a*c**
3*x**3 + 3*log(c**2*x + c)*a*c**3*x**3 - 6*a*c**2*x**2 - 2*a)/(6*d*x**3)
```

3.39 $\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 187

$$\begin{aligned}\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx &= -\frac{b}{2c^5d^2\sqrt{1-c^2x^2}}+\frac{b\sqrt{1-c^2x^2}}{c^5d^2}\\ &\quad+\frac{3x(a+b\arccos(cx))}{2c^4d^2}+\frac{x^3(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}\\ &\quad+\frac{3i(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c^5d^2}\\ &\quad-\frac{3ib\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{2c^5d^2}\\ &\quad+\frac{3ib\,\mathrm{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{2c^5d^2}\end{aligned}$$

output
```
-1/2*b/c^5/d^2/(-c^2*x^2+1)^(1/2)+b*(-c^2*x^2+1)^(1/2)/c^5/d^2+3/2*x*(a+b*
arccos(c*x))/c^4/d^2+1/2*x^3*(a+b*arccos(c*x))/c^2/d^2/(-c^2*x^2+1)+3*I*(a
+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c^5/d^2-3/2*I*b*polylog(2
,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^5/d^2+3/2*I*b*polylog(2,I*(c*x+I*(-c^2*x
^2+1)^(1/2)))/c^5/d^2
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.57

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = \frac{ax}{c^4d^2} - \frac{ax}{2c^4d^2(-1+c^2x^2)} + \frac{3a\log(1-cx)}{4c^5d^2} - \frac{3a\log(1+cx)}{4c^5d^2} + \frac{b\left(\frac{\sqrt{1-c^2x^2}-\arccos(cx)}{4c^4(c+c^2x)} + \frac{\sqrt{1-c^2x^2}+\arccos(cx)}{4c^4(c-c^2x)} + \frac{-\sqrt{1-c^2x^2}+cx\arccos(cx)}{c^5} - \frac{3\left(-\frac{i\arccos(cx)^2}{2c} + \frac{2\arccos(cx)\log\left(1+e^{i\arccos(cx)}\right)}{c} - \right.}{4c^4}\right.}{d^2}$$

input
```
Integrate[(x^4*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
(a*x)/(c^4*d^2) - (a*x)/(2*c^4*d^2*(-1 + c^2*x^2)) + (3*a*Log[1 - c*x])/(4
*c^5*d^2) - (3*a*Log[1 + c*x])/(4*c^5*d^2) + (b*((Sqrt[1 - c^2*x^2] - ArcC
os[c*x])/(4*c^4*(c + c^2*x)) + (Sqrt[1 - c^2*x^2] + ArcCos[c*x])/(4*c^4*(c
 - c^2*x)) + (-Sqrt[1 - c^2*x^2] + c*x*ArcCos[c*x])/c^5 - (3*(((-1/2*I)*Ar
cCos[c*x]^2)/c + (2*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])])/c - ((2*I)*Pol
yLog[2, -E^(I*ArcCos[c*x])])/c))/(4*c^4) - (((3*I)/8)*(ArcCos[c*x]*(ArcCos
[c*x] + (4*I)*Log[1 - E^(I*ArcCos[c*x])]) + 4*PolyLog[2, E^(I*ArcCos[c*x])
]))/c^5))/d^2
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5207, 27, 243, 53, 2009, 5211, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx$$

$$\Big\downarrow \text{5207}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))}{d(1-c^2x^2)}dx}{2c^2d} + \frac{b\int \frac{x^3}{(1-c^2x^2)^{3/2}}dx}{2cd^2} + \frac{x^3(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{2c^2d^2}+\frac{b\int \frac{x^3}{(1-c^2x^2)^{3/2}}dx}{2cd^2}+\frac{x^3(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{243}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{2c^2d^2}+\frac{b\int \frac{x^2}{(1-c^2x^2)^{3/2}}dx^2}{4cd^2}+\frac{x^3(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{53}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{2c^2d^2}+\frac{b\int \left(\frac{1}{c^2(1-c^2x^2)^{3/2}}-\frac{1}{c^2\sqrt{1-c^2x^2}}\right)dx^2}{4cd^2}+\frac{x^3(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{2c^2d^2}+\frac{x^3(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{4cd^2}$$

$$\Big\downarrow \text{5211}$$

$$-\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c^2}-\frac{b\int \frac{x}{\sqrt{1-c^2x^2}}dx}{c}-\frac{x(a+b\arccos(cx))}{c^2}\right)}{2c^2d^2}+\frac{x^3(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{4cd^2}$$

$$\Big\downarrow \text{241}$$

$$-\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}\right)}{2c^2d^2}+\frac{x^3(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{4cd^2}$$

$$\Big\downarrow \text{5165}$$

$$-\frac{3\left(-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}\right)}{2c^2d^2}+\frac{x^3(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{4cd^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}\right)}{2c^2d^2}+\frac{x^3(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{4cd^2}$$

$\big\downarrow$ 4671

$$-\frac{3\left(-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{c^3}-\frac{x(a+b\arccos(}{c^2}\right.}{2c^2d^2}+\frac{x^3(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{4cd^2}$$

$\big\downarrow$ 2715

$$-\frac{3\left(-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\,\text{a}}{c^3}\right.}{2c^2d^2}+\frac{x^3(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{4cd^2}$$

$\big\downarrow$ 2838

$$-\frac{3\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c}}{c^3}\right.}{2c^2d^2}+\frac{x^3(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{4cd^2}$$

input
```
Int[(x^4*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
(b*(2/(c^4*Sqrt[1 - c^2*x^2]) + (2*Sqrt[1 - c^2*x^2])/c^4))/(4*c*d^2) + (x
^3*(a + b*ArcCos[c*x]))/(2*c^2*d^2*(1 - c^2*x^2)) - (3*((b*Sqrt[1 - c^2*x^
2])/c^3 - (x*(a + b*ArcCos[c*x]))/c^2 - (-2*(a + b*ArcCos[c*x])*ArcTanh[E^
(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(
I*ArcCos[c*x])])/c^3))/(2*c^2*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{a\left(cx-\frac{1}{4(cx-1)}+\frac{3\ln(cx-1)}{4}-\frac{1}{4(cx+1)}-\frac{3\ln(cx+1)}{4}\right)}{d^2}-\frac{b\sqrt{-c^2x^2+1}}{d^2}+\frac{b\arccos(cx)cx}{d^2}-\frac{b\arccos(cx)cx}{2d^2\left(c^2x^2-1\right)}-\frac{b\sqrt{-c^2x^2+1}}{2d^2\left(c^2x^2-1\right)}+\frac{3b\,\mathrm{arcco}}{}}{}$
default	$\frac{\frac{a\left(cx-\frac{1}{4(cx-1)}+\frac{3\ln(cx-1)}{4}-\frac{1}{4(cx+1)}-\frac{3\ln(cx+1)}{4}\right)}{d^2}-\frac{b\sqrt{-c^2x^2+1}}{d^2}+\frac{b\arccos(cx)cx}{d^2}-\frac{b\arccos(cx)cx}{2d^2\left(c^2x^2-1\right)}-\frac{b\sqrt{-c^2x^2+1}}{2d^2\left(c^2x^2-1\right)}+\frac{3b\,\mathrm{arcco}}{}}{}$
parts	$\frac{a\left(\frac{x}{c^4}-\frac{1}{4c^5(cx-1)}+\frac{3\ln(cx-1)}{4c^5}-\frac{1}{4c^5(cx+1)}-\frac{3\ln(cx+1)}{4c^5}\right)}{d^2}+\frac{b\arccos(cx)x}{d^2c^4}-\frac{b\sqrt{-c^2x^2+1}}{c^5d^2}-\frac{b\arccos(cx)x}{2d^2c^4(c^2x^2-1)}-\frac{b}{2d^2}$

input
```
int(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(a/d^2*(c*x-1/4/(c*x-1)+3/4*ln(c*x-1)-1/4/(c*x+1)-3/4*ln(c*x+1))-b/d
^2*(-c^2*x^2+1)^(1/2)+b/d^2*arccos(c*x)*c*x-1/2*b/d^2/(c^2*x^2-1)*arccos(c
*x)*c*x-1/2*b/d^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)+3/2*b/d^2*arccos(c*x)*ln(
1-c*x-I*(-c^2*x^2+1)^(1/2))-3/2*I*b/d^2*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)
)-3/2*b/d^2*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+3/2*I*b/d^2*polylog
(2,-c*x-I*(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx=\int \frac{(b\arccos(cx)+a)x^4}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4*arccos(c*x) + a*x^4)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F]

$$\int \frac{x^4(a + b\arccos(cx))}{(d - c^2dx^2)^2}\,dx = \frac{\int \frac{ax^4}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{bx^4\,\mathrm{acos}\,(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**4*(a+b*acos(c*x))/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*x**4/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**4*acos(
c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^4(a + b\arccos(cx))}{(d - c^2dx^2)^2}\,dx = \int \frac{(b\arccos{(cx)} + a)x^4}{(c^2dx^2 - d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/4*a*(2*x/(c^6*d^2*x^2 - c^4*d^2) - 4*x/(c^4*d^2) + 3*log(c*x + 1)/(c^5*
d^2) - 3*log(c*x - 1)/(c^5*d^2)) + 1/4*((4*c^3*x^3 - 6*c*x - 3*(c^2*x^2 -
1)*log(c*x + 1) + 3*(c^2*x^2 - 1)*log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x) + 4*(c^7*d^2*x^2 - c^5*d^2)*integrate(-1/4*(4*c^3*x^3 -
6*c*x - 3*(c^2*x^2 - 1)*log(c*x + 1) + 3*(c^2*x^2 - 1)*log(-c*x + 1))*sqrt
(c*x + 1)*sqrt(-c*x + 1)/(c^8*d^2*x^4 - 2*c^6*d^2*x^2 + c^4*d^2), x))*b/(c
^7*d^2*x^2 - c^5*d^2)
```

Giac [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx=\int \frac{(b\arccos(cx)+a)x^4}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)*x^4/(c^2*d*x^2 - d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx=\int \frac{x^4\,(a+b\,\mathrm{acos}(c\,x))}{(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((x^4*(a + b*acos(c*x)))/(d - c^2*d*x^2)^2,x)
```

output
```
int((x^4*(a + b*acos(c*x)))/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx$$
$$=\frac{4\left(\int \frac{acos(cx)x^4}{c^4x^4-2c^2x^2+1}dx\right)b\,c^7x^2-4\left(\int \frac{acos(cx)x^4}{c^4x^4-2c^2x^2+1}dx\right)b\,c^5+3\log(c^2x-c)\,a\,c^2x^2-3\log(c^2x-c)\,a-3\log(c}{4c^5d^2\,(c^2x^2-1)}$$

input
```
int(x^4*(a+b*acos(c*x))/(-c^2*d*x^2+d)^2,x)
```

output
```
(4*int((acos(c*x)*x**4)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**7*x**2 - 4*i
nt((acos(c*x)*x**4)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**5 + 3*log(c**2*x
 - c)*a*c**2*x**2 - 3*log(c**2*x - c)*a - 3*log(c**2*x + c)*a*c**2*x**2 +
3*log(c**2*x + c)*a + 4*a*c**3*x**3 - 6*a*c*x)/(4*c**5*d**2*(c**2*x**2 - 1
))
```

3.40 $\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = -\frac{bx}{2c^3d^2\sqrt{1-c^2x^2}}+\frac{b\arccos(cx)}{2c^4d^2}+\frac{x^2(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}-\frac{i(a+b\arccos(cx))^2}{2bc^4d^2}+\frac{(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c^4d^2}-\frac{ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{2c^4d^2}$$

output
```
-1/2*b*x/c^3/d^2/(-c^2*x^2+1)^(1/2)+1/2*b*arccos(c*x)/c^4/d^2+1/2*x^2*(a+b
*arccos(c*x))/c^2/d^2/(-c^2*x^2+1)-1/2*I*(a+b*arccos(c*x))^2/b/c^4/d^2+(a+
b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c^4/d^2-1/2*I*b*polylog(
2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c^4/d^2
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.31

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx$$
$$= \frac{\frac{b\sqrt{1-c^2x^2}}{1-cx}-\frac{b\sqrt{1-c^2x^2}}{1+cx}-\frac{2a}{-1+c^2x^2}+\frac{b\arccos(cx)}{1-cx}+\frac{b\arccos(cx)}{1+cx}-2ib\arccos(cx)^2+4b\arccos(cx)\log\left(1-e^{i\arccos(}\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
((b*Sqrt[1 - c^2*x^2])/(1 - c*x) - (b*Sqrt[1 - c^2*x^2])/(1 + c*x) - (2*a)
/(-1 + c^2*x^2) + (b*ArcCos[c*x])/(1 - c*x) + (b*ArcCos[c*x])/(1 + c*x) -
(2*I)*b*ArcCos[c*x]^2 + 4*b*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] + 4*b*A
rcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] + 2*a*Log[1 - c^2*x^2] - (4*I)*b*Pol
yLog[2, -E^(I*ArcCos[c*x])] - (4*I)*b*PolyLog[2, E^(I*ArcCos[c*x])])/(4*c^
4*d^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5207, 27, 252, 223, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx$$
$$\Big\downarrow \text{5207}$$
$$-\frac{\int \frac{x(a+b\arccos(cx))}{d(1-c^2x^2)}dx}{c^2d}+\frac{b\int \frac{x^2}{(1-c^2x^2)^{3/2}}dx}{2cd^2}+\frac{x^2(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2d^2}+\frac{b\int \frac{x^2}{(1-c^2x^2)^{3/2}}dx}{2cd^2}+\frac{x^2(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{ 252}$$

$$-\frac{\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2d^2}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{c^2}\right)}{2cd^2}+\frac{x^2(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2d^2}+\frac{x^2(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2cd^2}$$

$$\Big\downarrow \text{ 5181}$$

$$\frac{\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^4d^2}+\frac{x^2(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2cd^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{c^4d^2}+\frac{x^2(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2cd^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c^4d^2}+\frac{x^2(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2cd^2}$$

$$\Big\downarrow \text{ 4200}$$

$$\frac{2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4d^2}+\frac{x^2(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2cd^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4d^2}+\frac{x^2(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2cd^2}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4d^2}+\frac{x^2(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2cd^2}$$

$\Big\downarrow$ 2715

$$\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4d^2}+\frac{x^2(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2cd^2}$$

$\Big\downarrow$ 2838

$$\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4d^2}+\frac{x^2(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2cd^2}$$

input
```
Int[(x^3*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
(x^2*(a + b*ArcCos[c*x]))/(2*c^2*d^2*(1 - c^2*x^2)) + (b*(x/(c^2*Sqrt[1 -
c^2*x^2]) - ArcSin[c*x]/c^3))/(2*c*d^2) + (((-1/2*I)*(a + b*ArcCos[c*x])^2
)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])] + (b
*PolyLog[2, E^((2*I)*ArcCos[c*x])])/4))/(c^4*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{a\left(-\frac{1}{4(cx-1)}+\frac{\ln(cx-1)}{2}+\frac{1}{4cx+4}+\frac{\ln(cx+1)}{2}\right)}{d^2}+\frac{b\left(-\frac{i\arccos(cx)^2}{2}-\frac{ic^2x^2+cx\sqrt{-c^2x^2+1}+\arccos(cx)-i}{2\left(c^2x^2-1\right)}+\arccos(cx)\ln\left(1+cx+i\right.\right.}{c^4}$
default	$\frac{a\left(-\frac{1}{4(cx-1)}+\frac{\ln(cx-1)}{2}+\frac{1}{4cx+4}+\frac{\ln(cx+1)}{2}\right)}{d^2}+\frac{b\left(-\frac{i\arccos(cx)^2}{2}-\frac{ic^2x^2+cx\sqrt{-c^2x^2+1}+\arccos(cx)-i}{2\left(c^2x^2-1\right)}+\arccos(cx)\ln\left(1+cx+i\right.\right.}{c^4}$
parts	$\frac{a\left(-\frac{1}{4c^4(cx-1)}+\frac{\ln(cx-1)}{2c^4}+\frac{1}{4c^4(cx+1)}+\frac{\ln(cx+1)}{2c^4}\right)}{d^2}+\frac{b\left(-\frac{i\arccos(cx)^2}{2}-\frac{ic^2x^2+cx\sqrt{-c^2x^2+1}+\arccos(cx)-i}{2\left(c^2x^2-1\right)}+\arccos(cx)\right.}{}$

input
```
int(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a/d^2*(-1/4/(c*x-1)+1/2*ln(c*x-1)+1/4/(c*x+1)+1/2*ln(c*x+1))+b/d^2*
(-1/2*I*arccos(c*x)^2-1/2*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)+arccos(c*x)-I)
/(c^2*x^2-1)+arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+arccos(c*x)*ln(1-c
*x-I*(-c^2*x^2+1)^(1/2))-I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-I*polylog(
2,c*x+I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)x^3}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3*arccos(c*x) + a*x^3)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = \frac{\int \frac{ax^3}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{bx^3\operatorname{acos}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**3*(a+b*acos(c*x))/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*x**3/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**3*acos(
c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)x^3}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(1/(c^6*d^2*x^2 - c^4*d^2) - log(c^2*x^2 - 1)/(c^4*d^2)) + 1/2*(((c
^2*x^2 - 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(-c*x + 1) - 1)*arctan2(sqrt(c
*x + 1)*sqrt(-c*x + 1), c*x) - 2*(c^6*d^2*x^2 - c^4*d^2)*integrate(1/2*((c
^2*x^2 - 1)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))*log(c*x + 1) + (c^2*x
^2 - 1)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))*log(-c*x + 1) - e^(1/2*lo
g(c*x + 1) + 1/2*log(-c*x + 1)))/(c^9*d^2*x^6 - 2*c^7*d^2*x^4 + c^5*d^2*x^
2 + (c^7*d^2*x^4 - 2*c^5*d^2*x^2 + c^3*d^2)*e^(log(c*x + 1) + log(-c*x + 1
))), x))*b/(c^6*d^2*x^2 - c^4*d^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b \arccos(cx))}{(d - c^2dx^2)^2} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b \arccos(cx))}{(d - c^2dx^2)^2} dx = \int \frac{x^3 (a + b \operatorname{acos}(c x))}{(d - c^2 d x^2)^2} dx$$

input
```
int((x^3*(a + b*acos(c*x)))/(d - c^2*d*x^2)^2,x)
```

output
```
int((x^3*(a + b*acos(c*x)))/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx$$
$$= \frac{2\left(\int \frac{acos(cx)x^3}{c^4x^4-2c^2x^2+1}dx\right) b\,c^6x^2 - 2\left(\int \frac{acos(cx)x^3}{c^4x^4-2c^2x^2+1}dx\right) b\,c^4 + \log(c^2x-c)\,a\,c^2x^2 - \log(c^2x-c)\,a + \log(c^2x +}{2c^4d^2\,(c^2x^2-1)}$$

input
```
int(x^3*(a+b*acos(c*x))/(-c^2*d*x^2+d)^2,x)
```

output
```
(2*int((acos(c*x)*x**3)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**6*x**2 - 2*i
nt((acos(c*x)*x**3)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**4 + log(c**2*x -
 c)*a*c**2*x**2 - log(c**2*x - c)*a + log(c**2*x + c)*a*c**2*x**2 - log(c*
*2*x + c)*a - a*c**2*x**2)/(2*c**4*d**2*(c**2*x**2 - 1))
```

3.41 $\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 144

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = -\frac{b}{2c^3d^2\sqrt{1-c^2x^2}}+\frac{x(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)} + \frac{i(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c^3d^2} - \frac{ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{2c^3d^2} + \frac{ib\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{2c^3d^2}$$

output
```
-1/2*b/c^3/d^2/(-c^2*x^2+1)^(1/2)+1/2*x*(a+b*arccos(c*x))/c^2/d^2/(-c^2*x^
2+1)+I*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c^3/d^2-1/2*I*b*
polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^3/d^2+1/2*I*b*polylog(2,I*(c*x+
I*(-c^2*x^2+1)^(1/2)))/c^3/d^2
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.74

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = -\frac{2acx+2b\sqrt{1-c^2x^2}+2bcx\arccos(cx)+2b\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)-2bc^2x^2\arccos(cx)\log\left(1\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
-1/4*(2*a*c*x + 2*b*Sqrt[1 - c^2*x^2] + 2*b*c*x*ArcCos[c*x] + 2*b*ArcCos[c
*x]*Log[1 - E^(I*ArcCos[c*x])] - 2*b*c^2*x^2*ArcCos[c*x]*Log[1 - E^(I*ArcC
os[c*x])] - 2*b*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] + 2*b*c^2*x^2*ArcCo
s[c*x]*Log[1 + E^(I*ArcCos[c*x])] + a*Log[1 - c*x] - a*c^2*x^2*Log[1 - c*x
] - a*Log[1 + c*x] + a*c^2*x^2*Log[1 + c*x] - (2*I)*b*(-1 + c^2*x^2)*PolyL
og[2, -E^(I*ArcCos[c*x])] + (2*I)*b*(-1 + c^2*x^2)*PolyLog[2, E^(I*ArcCos[
c*x])])/(c^3*d^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5207, 27, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx$$

$$\Big\downarrow \text{5207}$$

$$-\frac{\int \frac{a+b\arccos(cx)}{d(1-c^2x^2)}dx}{2c^2d}+\frac{b\int \frac{x}{(1-c^2x^2)^{3/2}}dx}{2cd^2}+\frac{x(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{2c^2d^2}+\frac{b\int \frac{x}{(1-c^2x^2)^{3/2}}dx}{2cd^2}+\frac{x(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}$$

$\Big\downarrow$ 241

$$-\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{2c^2d^2}+\frac{x(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b}{2c^3d^2\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5165

$$\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c^3d^2}+\frac{x(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b}{2c^3d^2\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 3042

$$\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2c^3d^2}+\frac{x(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b}{2c^3d^2\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 4671

$$\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\,\text{arcc}}{2c^3d^2}$$
$$\frac{x(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b}{2c^3d^2\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 2715

$$\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}(}{2c^3d^2}$$
$$\frac{x(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b}{2c^3d^2\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 2838

$$\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c^3d^2}+$$
$$\frac{x(a+b\arccos(cx))}{2c^2d^2(1-c^2x^2)}+\frac{b}{2c^3d^2\sqrt{1-c^2x^2}}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
b/(2*c^3*d^2*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcCos[c*x]))/(2*c^2*d^2*(1 -
c^2*x^2)) + (-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyL
og[2, -E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/(2*c^3*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{\frac{a\left(-\frac{1}{4(cx-1)}+\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}-\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{cx\arccos(cx)+\sqrt{-c^2x^2+1}}{2\left(c^2x^2-1\right)}+\frac{\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)}{2}-\frac{i\,\mathrm{polylog}}{}\right.}{}}{c^3}$
default	$\frac{\frac{a\left(-\frac{1}{4(cx-1)}+\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}-\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{cx\arccos(cx)+\sqrt{-c^2x^2+1}}{2\left(c^2x^2-1\right)}+\frac{\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)}{2}-\frac{i\,\mathrm{polylog}}{}\right.}{}}{c^3}$
parts	$\frac{a\left(-\frac{1}{4c^3(cx-1)}+\frac{\ln(cx-1)}{4c^3}-\frac{1}{4c^3(cx+1)}-\frac{\ln(cx+1)}{4c^3}\right)}{d^2}+\frac{b\left(-\frac{cx\arccos(cx)+\sqrt{-c^2x^2+1}}{2\left(c^2x^2-1\right)}+\frac{\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)}{2}\right.}{}$

input
```
int(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a/d^2*(-1/4/(c*x-1)+1/4*ln(c*x-1)-1/4/(c*x+1)-1/4*ln(c*x+1))+b/d^2*
(-1/2*(c*x*arccos(c*x)+(-c^2*x^2+1)^(1/2))/(c^2*x^2-1)+1/2*arccos(c*x)*ln(
1-c*x-I*(-c^2*x^2+1)^(1/2))-1/2*I*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))-1/2*
arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+1/2*I*polylog(2,-c*x-I*(-c^2*x^
2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)x^2}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2*arccos(c*x) + a*x^2)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

Sympy [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = \frac{\int \frac{ax^2}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{bx^2\operatorname{acos}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**2*(a+b*acos(c*x))/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*x**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**2*acos(
c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)x^2}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/4*a*(2*x/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - log(c*x - 1
)/(c^3*d^2)) - 1/4*((2*c*x + (c^2*x^2 - 1)*log(c*x + 1) - (c^2*x^2 - 1)*lo
g(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - 4*(c^5*d^2*x^2 -
 c^3*d^2)*integrate(1/4*(2*c*x + (c^2*x^2 - 1)*log(c*x + 1) - (c^2*x^2 - 1
)*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d^2*x^4 - 2*c^4*d^2*x^2
 + c^2*d^2), x))*b/(c^5*d^2*x^2 - c^3*d^2)
```

Giac [F]

$$\int \frac{x^2(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^2}\,dx=\int \frac{(b\arccos{(cx)}+a)x^2}{\left(c^2dx^2-d\right)^2}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)*x^2/(c^2*d*x^2 - d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^2}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{acos}(c\,x)\right)}{\left(d-c^2\,d\,x^2\right)^2}\,dx$$

input
```
int((x^2*(a + b*acos(c*x)))/(d - c^2*d*x^2)^2,x)
```

output
```
int((x^2*(a + b*acos(c*x)))/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx$$
$$= \frac{4\left(\int \frac{acos(cx)x^2}{c^4x^4-2c^2x^2+1}dx\right) b\,c^5x^2 - 4\left(\int \frac{acos(cx)x^2}{c^4x^4-2c^2x^2+1}dx\right) b\,c^3 + \log(c^2x-c)\,a\,c^2x^2 - \log(c^2x-c)\,a - \log(c^2x+}{4c^3d^2\,(c^2x^2-1)}$$

input
```
int(x^2*(a+b*acos(c*x))/(-c^2*d*x^2+d)^2,x)
```

output
```
(4*int((acos(c*x)*x**2)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**5*x**2 - 4*i
nt((acos(c*x)*x**2)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**3 + log(c**2*x -
 c)*a*c**2*x**2 - log(c**2*x - c)*a - log(c**2*x + c)*a*c**2*x**2 + log(c*
*2*x + c)*a - 2*a*c*x)/(4*c**3*d**2*(c**2*x**2 - 1))
```

3.42 $\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = -\frac{bx}{2cd^2\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{2c^2d^2(1-c^2x^2)}$$

output
```
-1/2*b*x/c/d^2/(-c^2*x^2+1)^(1/2)+1/2*(a+b*arccos(c*x))/c^2/d^2/(-c^2*x^2+
1)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = \frac{a+bcx\sqrt{1-c^2x^2}+b\arccos(cx)}{2c^2d^2-2c^4d^2x^2}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
(a + b*c*x*Sqrt[1 - c^2*x^2] + b*ArcCos[c*x])/(2*c^2*d^2 - 2*c^4*d^2*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5183, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^2}dx$$

$$\Big\downarrow \text{5183}$$

$$\frac{b\int \frac{1}{(1-c^2x^2)^{3/2}}dx}{2cd^2}+\frac{a+b\arccos(cx)}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{208}$$

$$\frac{a+b\arccos(cx)}{2c^2d^2(1-c^2x^2)}+\frac{bx}{2cd^2\sqrt{1-c^2x^2}}$$

input
```
Int[(x*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
(b*x)/(2*c*d^2*Sqrt[1 - c^2*x^2]) + (a + b*ArcCos[c*x])/(2*c^2*d^2*(1 - c^
2*x^2))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{-\frac{a}{2d^2\left(c^2x^2-1\right)}+\frac{b\left(-\frac{\arccos(cx)}{2\left(c^2x^2-1\right)}-\frac{\sqrt{-(cx-1)^2-2cx+2}}{4(cx-1)}-\frac{\sqrt{-(cx+1)^2+2cx+2}}{4(cx+1)}\right)}{d^2}}{c^2}$
default	$\frac{-\frac{a}{2d^2\left(c^2x^2-1\right)}+\frac{b\left(-\frac{\arccos(cx)}{2\left(c^2x^2-1\right)}-\frac{\sqrt{-(cx-1)^2-2cx+2}}{4(cx-1)}-\frac{\sqrt{-(cx+1)^2+2cx+2}}{4(cx+1)}\right)}{d^2}}{c^2}$
parts	$-\frac{a}{2d^2c^2(c^2x^2-1)}+\frac{b\left(-\frac{\arccos(cx)}{2\left(c^2x^2-1\right)}-\frac{\sqrt{-(cx-1)^2-2cx+2}}{4(cx-1)}-\frac{\sqrt{-(cx+1)^2+2cx+2}}{4(cx+1)}\right)}{d^2c^2}$
orering	$-\frac{(cx-1)(cx+1)\left(3c^2x^2+2\right)(a+b\arccos(cx))}{2c^2(-c^2d\,x^2+d)^2}-\frac{(cx-1)^2(cx+1)^2\left(\frac{a+b\arccos(cx)}{\left(-c^2d\,x^2+d\right)^2}-\frac{xbc}{\sqrt{-c^2x^2+1}\left(-c^2d\,x^2+d\right)^2}+\frac{4x^2(a\cdots}{(\cdots}\right.}{2c^2}$

input
```
int(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/2*a/d^2/(c^2*x^2-1)+b/d^2*(-1/2/(c^2*x^2-1)*arccos(c*x)-1/4/(c*x
-1)*(-(c*x-1)^2-2*c*x+2)^(1/2)-1/4/(c*x+1)*(-(c*x+1)^2+2*c*x+2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{x(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^2}\,dx=-\frac{ac^2x^2+\sqrt{-c^2x^2+1}bcx+b\arccos\left(cx\right)}{2\left(c^4d^2x^2-c^2d^2\right)}$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/2*(a*c^2*x^2 + sqrt(-c^2*x^2 + 1)*b*c*x + b*arccos(c*x))/(c^4*d^2*x^2 -
 c^2*d^2)
```

Sympy [F]

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = \frac{\int \frac{ax}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{bx\operatorname{acos}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x*(a+b*acos(c*x))/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*x/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x*acos(c*x)/(
c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(50) = 100$.

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.39

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = -\frac{1}{4}\left(\left(\frac{\sqrt{-c^2x^2+1}c^2d^2}{c^7d^4x+c^6d^4} + \frac{\sqrt{-c^2x^2+1}c^2d^2}{c^7d^4x-c^6d^4}\right)c^2 + \frac{2\arccos(cx)}{c^4d^2x^2-c^2d^2}\right)b - \frac{a}{2(c^4d^2x^2-c^2d^2)}$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/4*((sqrt(-c^2*x^2 + 1)*c^2*d^2/(c^7*d^4*x + c^6*d^4) + sqrt(-c^2*x^2 +
1)*c^2*d^2/(c^7*d^4*x - c^6*d^4))*c^2 + 2*arccos(c*x)/(c^4*d^2*x^2 - c^2*d
^2))*b - 1/2*a/(c^4*d^2*x^2 - c^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = -\frac{bx^2\arccos(cx)}{2\,(c^2x^2-1)d^2} - \frac{ax^2}{2\,(c^2x^2-1)d^2} - \frac{\sqrt{-c^2x^2+1}bx}{2\,(c^2x^2-1)cd^2} + \frac{b\arccos(cx)}{2\,c^2d^2} + \frac{a}{2\,c^2d^2}$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/2*b*x^2*arccos(c*x)/((c^2*x^2 - 1)*d^2) - 1/2*a*x^2/((c^2*x^2 - 1)*d^2)
 - 1/2*sqrt(-c^2*x^2 + 1)*b*x/((c^2*x^2 - 1)*c*d^2) + 1/2*b*arccos(c*x)/(c
^2*d^2) + 1/2*a/(c^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = \int \frac{x\,(a+b\,\mathrm{acos}(c\,x))}{(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((x*(a + b*acos(c*x)))/(d - c^2*d*x^2)^2,x)
```

output
```
int((x*(a + b*acos(c*x)))/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = \frac{2\left(\int \frac{acos(cx)x}{c^4x^4-2c^2x^2+1}dx\right)b\,c^2x^2 - 2\left(\int \frac{acos(cx)x}{c^4x^4-2c^2x^2+1}dx\right)b - a\,x^2}{2d^2\,(c^2x^2-1)}$$

input
```
int(x*(a+b*acos(c*x))/(-c^2*d*x^2+d)^2,x)
```

output
```
(2*int((acos(c*x)*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**2*x**2 - 2*int(
(acos(c*x)*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b - a*x**2)/(2*d**2*(c**2*x
**2 - 1))
```

3.43 $\int \frac{a+b\arccos(cx)}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 141

$$\int \frac{a+b\arccos(cx)}{\left(d-c^2dx^2\right)^2}\,dx = -\frac{b}{2cd^2\sqrt{1-c^2x^2}}+\frac{x(a+b\arccos(cx))}{2d^2\left(1-c^2x^2\right)} - \frac{i(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{cd^2} + \frac{ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{2cd^2} - \frac{ib\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{2cd^2}$$

output
```
-1/2*b/c/d^2/(-c^2*x^2+1)^(1/2)+1/2*x*(a+b*arccos(c*x))/d^2/(-c^2*x^2+1)-I
*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c/d^2+1/2*I*b*polylog(
2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/d^2-1/2*I*b*polylog(2,I*(c*x+I*(-c^2*x^
2+1)^(1/2)))/c/d^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.56

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^2}\,dx = \frac{\frac{b\sqrt{1-c^2x^2}}{c-c^2x}+\frac{b\sqrt{1-c^2x^2}}{c+c^2x}-\frac{2ax}{-1+c^2x^2}+\frac{b\arccos(cx)}{c-c^2x}-\frac{b\arccos(cx)}{c+c^2x}-\frac{2b\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)}{c}+\frac{2b\arccos(cx)\log\left(1+e^{i\arccos(cx)}\right)}{c}}{4d^2}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^2,x]
```

output
```
((b*Sqrt[1 - c^2*x^2])/(c - c^2*x) + (b*Sqrt[1 - c^2*x^2])/(c + c^2*x) - (
2*a*x)/(-1 + c^2*x^2) + (b*ArcCos[c*x])/(c - c^2*x) - (b*ArcCos[c*x])/(c +
 c^2*x) - (2*b*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])])/c + (2*b*ArcCos[c*x
]*Log[1 + E^(I*ArcCos[c*x])])/c - (a*Log[1 - c*x])/c + (a*Log[1 + c*x])/c
- ((2*I)*b*PolyLog[2, -E^(I*ArcCos[c*x])])/c + ((2*I)*b*PolyLog[2, E^(I*Ar
cCos[c*x])])/c)/(4*d^2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5163, 27, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^2}\,dx$$

$$\Big\downarrow \; 5163$$

$$\frac{\int \frac{a+b\arccos(cx)}{d(1-c^2x^2)}dx}{2d}+\frac{bc\int \frac{x}{(1-c^2x^2)^{3/2}}dx}{2d^2}+\frac{x(a+b\arccos(cx))}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{2d^2}+\frac{bc\int \frac{x}{(1-c^2x^2)^{3/2}}dx}{2d^2}+\frac{x(a+b\arccos(cx))}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{241}$$

$$\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{2d^2}+\frac{x(a+b\arccos(cx))}{2d^2\left(1-c^2x^2\right)}+\frac{b}{2cd^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5165}$$

$$-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2cd^2}+\frac{x(a+b\arccos(cx))}{2d^2\left(1-c^2x^2\right)}+\frac{b}{2cd^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2cd^2}+\frac{x(a+b\arccos(cx))}{2d^2\left(1-c^2x^2\right)}+\frac{b}{2cd^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\text{arc}}{2cd^2}$$
$$\frac{x(a+b\arccos(cx))}{2d^2\left(1-c^2x^2\right)}+\frac{b}{2cd^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}}{2cd^2}$$
$$\frac{x(a+b\arccos(cx))}{2d^2\left(1-c^2x^2\right)}+\frac{b}{2cd^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2cd^2}+$$
$$\frac{x(a+b\arccos(cx))}{2d^2\left(1-c^2x^2\right)}+\frac{b}{2cd^2\sqrt{1-c^2x^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^2,x]
```

output
```
b/(2*c*d^2*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcCos[c*x]))/(2*d^2*(1 - c^2*x^
2)) - (-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2,
-E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/(2*c*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{a\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{cx\arccos(cx)+\sqrt{-c^2x^2+1}}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)}{2}+\frac{i\,\mathrm{polylog}}{}\right.}{}}{c}$
default	$\frac{\frac{a\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{cx\arccos(cx)+\sqrt{-c^2x^2+1}}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)}{2}+\frac{i\,\mathrm{polylog}}{}\right.}{}}{c}$
parts	$\frac{a\left(-\frac{1}{4c(cx-1)}-\frac{\ln(cx-1)}{4c}-\frac{1}{4c(cx+1)}+\frac{\ln(cx+1)}{4c}\right)}{d^2}+\frac{b\left(-\frac{cx\arccos(cx)+\sqrt{-c^2x^2+1}}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)}{2}+\frac{i}{}\right.}{}$

input
```
int((a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a/d^2*(-1/4/(c*x-1)-1/4*ln(c*x-1)-1/4/(c*x+1)+1/4*ln(c*x+1))+b/d^2*(-
1/2*(c*x*arccos(c*x)+(-c^2*x^2+1)^(1/2))/(c^2*x^2-1)-1/2*arccos(c*x)*ln(1-
c*x-I*(-c^2*x^2+1)^(1/2))+1/2*I*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+1/2*ar
ccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-1/2*I*polylog(2,-c*x-I*(-c^2*x^2+
1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^2}\,dx=\int \frac{b\arccos{(cx)}+a}{\left(c^2dx^2-d\right)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a + b\arccos(cx)}{(d - c^2dx^2)^2}\,dx = \frac{\int \frac{a}{c^4x^4 - 2c^2x^2 + 1}\,dx + \int \frac{b\,\mathrm{acos}\,(cx)}{c^4x^4 - 2c^2x^2 + 1}\,dx}{d^2}$$

input
```
integrate((a+b*acos(c*x))/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*acos(c*x)/(c**4
*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{a + b\arccos(cx)}{(d - c^2dx^2)^2}\,dx = \int \frac{b\arccos{(cx)} + a}{(c^2dx^2 - d)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/4*a*(2*x/(c^2*d^2*x^2 - d^2) - log(c*x + 1)/(c*d^2) + log(c*x - 1)/(c*d
^2)) - 1/4*((2*c*x - (c^2*x^2 - 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(-c*x +
 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 4*(c^3*d^2*x^2 - c*d^2)*
integrate(-1/4*(2*c*x - (c^2*x^2 - 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(-c*
x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x))*b/(c^3*d^2*x^2 - c*d^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\arccos(cx)}{(d - c^2dx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{\left(d-c^2dx^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{\left(d-c^2\,d\,x^2\right)^2}\,dx$$

input
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{\left(d-c^2dx^2\right)^2}\,dx$$
$$=\frac{4\left(\int \frac{acos(cx)}{c^4x^4-2c^2x^2+1}dx\right)b\,c^3x^2-4\left(\int \frac{acos(cx)}{c^4x^4-2c^2x^2+1}dx\right)bc-\log(c^2x-c)\,a\,c^2x^2+\log(c^2x-c)\,a+\log(c^2x+c}{4c\,d^2\left(c^2x^2-1\right)}$$

input
```
int((a+b*acos(c*x))/(-c^2*d*x^2+d)^2,x)
```

output
```
(4*int(acos(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c**3*x**2 - 4*int(acos
(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b*c - log(c**2*x - c)*a*c**2*x**2 +
 log(c**2*x - c)*a + log(c**2*x + c)*a*c**2*x**2 - log(c**2*x + c)*a - 2*a
*c*x)/(4*c*d**2*(c**2*x**2 - 1))
```

3.44 $\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^2}\,dx = -\frac{bcx}{2d^2\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{2d^2(1-c^2x^2)} - \frac{2(a+b\arccos(cx))\text{arctanh}\left(e^{2i\arccos(cx)}\right)}{d^2} + \frac{ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{2d^2} - \frac{ib\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{2d^2}$$

output
```
-1/2*b*c*x/d^2/(-c^2*x^2+1)^(1/2)+1/2*(a+b*arccos(c*x))/d^2/(-c^2*x^2+1)-2
*(a+b*arccos(c*x))*arctanh((c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^2+1/2*I*b*polyl
og(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^2-1/2*I*b*polylog(2,(c*x+I*(-c^2*x^2
+1)^(1/2))^2)/d^2
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.91

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^2}\,dx$$
$$= \frac{\frac{b\sqrt{1-c^2x^2}}{1-cx}-\frac{b\sqrt{1-c^2x^2}}{1+cx}-\frac{2a}{-1+c^2x^2}+\frac{b\arccos(cx)}{1-cx}+\frac{b\arccos(cx)}{1+cx}-4b\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)-4b\arccos(c}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x*(d - c^2*d*x^2)^2),x]
```

output
```
((b*Sqrt[1 - c^2*x^2])/(1 - c*x) - (b*Sqrt[1 - c^2*x^2])/(1 + c*x) - (2*a)
/(-1 + c^2*x^2) + (b*ArcCos[c*x])/(1 - c*x) + (b*ArcCos[c*x])/(1 + c*x) -
4*b*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] - 4*b*ArcCos[c*x]*Log[1 + E^(I*
ArcCos[c*x])] + 4*b*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] + 4*a*Log[x
] - 2*a*Log[1 - c^2*x^2] + (4*I)*b*PolyLog[2, -E^(I*ArcCos[c*x])] + (4*I)*
b*PolyLog[2, E^(I*ArcCos[c*x])] - (2*I)*b*PolyLog[2, -E^((2*I)*ArcCos[c*x]
)])/(4*d^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5209, 27, 208, 5185, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^2}\,dx$$

$$\Big\downarrow \ 5209$$

$$\frac{\int \frac{a+b\arccos(cx)}{dx(1-c^2x^2)}dx}{d}+\frac{bc\int \frac{1}{(1-c^2x^2)^{3/2}}dx}{2d^2}+\frac{a+b\arccos(cx)}{2d^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d^2}+\frac{bc\int \frac{1}{(1-c^2x^2)^{3/2}}dx}{2d^2}+\frac{a+b\arccos(cx)}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{208}$$

$$\frac{\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d^2}+\frac{a+b\arccos(cx)}{2d^2(1-c^2x^2)}+\frac{bcx}{2d^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5185}$$

$$-\frac{\int \frac{a+b\arccos(cx)}{cx\sqrt{1-c^2x^2}}d\arccos(cx)}{d^2}+\frac{a+b\arccos(cx)}{2d^2(1-c^2x^2)}+\frac{bcx}{2d^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{4919}$$

$$-\frac{2\int (a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)}{d^2}+\frac{a+b\arccos(cx)}{2d^2(1-c^2x^2)}+\frac{bcx}{2d^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int (a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)}{d^2}+\frac{a+b\arccos(cx)}{2d^2(1-c^2x^2)}+\frac{bcx}{2d^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{2\left(-\frac{1}{2}b\int \log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+\frac{1}{2}b\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\left(\text{arctanh}\left(e^{2i\arccos(cx)}\right)\right)(a\right.}{d^2}$$
$$\frac{a+b\arccos(cx)}{2d^2(1-c^2x^2)}+\frac{bcx}{2d^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2\left(\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right.}{d^2}$$
$$\frac{a+b\arccos(cx)}{2d^2(1-c^2x^2)}+\frac{bcx}{2d^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2\left(-\left(\text{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)}{d^2}$$
$$\frac{a+b\arccos(cx)}{2d^2(1-c^2x^2)}+\frac{bcx}{2d^2\sqrt{1-c^2x^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(x*(d - c^2*d*x^2)^2),x]
```

output
```
(b*c*x)/(2*d^2*Sqrt[1 - c^2*x^2]) + (a + b*ArcCos[c*x])/(2*d^2*(1 - c^2*x^
2)) - (2*(-((a + b*ArcCos[c*x])*ArcTanh[E^((2*I)*ArcCos[c*x])]) + (I/4)*b*
PolyLog[2, -E^((2*I)*ArcCos[c*x])] - (I/4)*b*PolyLog[2, E^((2*I)*ArcCos[c*
x])]))/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.09

method	result
parts	$\frac{a\left(\ln(x)-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{2}+\frac{1}{4cx+4}-\frac{\ln(cx+1)}{2}\right)}{d^2}+\frac{b\left(-\frac{ic^2x^2+cx\sqrt{-c^2x^2+1}+\arccos(cx)-i}{2\left(c^2x^2-1\right)}-\arccos(cx)\ln\left(1+cx+i\sqrt{\right.\right.}}{}$
derivativedivides	$\frac{a\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{2}+\ln(cx)+\frac{1}{4cx+4}-\frac{\ln(cx+1)}{2}\right)}{d^2}+\frac{b\left(-\frac{ic^2x^2+cx\sqrt{-c^2x^2+1}+\arccos(cx)-i}{2\left(c^2x^2-1\right)}-\arccos(cx)\ln\left(1+cx+\right.\right.}{}$
default	$\frac{a\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{2}+\ln(cx)+\frac{1}{4cx+4}-\frac{\ln(cx+1)}{2}\right)}{d^2}+\frac{b\left(-\frac{ic^2x^2+cx\sqrt{-c^2x^2+1}+\arccos(cx)-i}{2\left(c^2x^2-1\right)}-\arccos(cx)\ln\left(1+cx+\right.\right.}{}$

input
```
int((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a/d^2*(ln(x)-1/4/(c*x-1)-1/2*ln(c*x-1)+1/4/(c*x+1)-1/2*ln(c*x+1))+b/d^2*(-
1/2*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)+arccos(c*x)-I)/(c^2*x^2-1)-arccos(c*
x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+a
rccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1/2*I*polylog(2,-(c*x+I*(-c^
2*x^2+1)^(1/2))^2)-arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+I*polylog(2,
c*x+I*(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^2}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(c^2dx^2-d\right)^2x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^2}\,dx=\frac{\int \frac{a}{c^4x^5-2c^2x^3+x}\,dx+\int \frac{b\,\mathrm{acos}\left(cx\right)}{c^4x^5-2c^2x^3+x}\,dx}{d^2}$$

input
```
integrate((a+b*acos(c*x))/x/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**5 - 2*c**2*x**3 + x), x) + Integral(b*acos(c*x)/(c**4
*x**5 - 2*c**2*x**3 + x), x))/d**2
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^2}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(c^2dx^2-d\right)^2x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(1/(c^2*d^2*x^2 - d^2) + log(c*x + 1)/d^2 + log(c*x - 1)/d^2 - 2*lo
g(x)/d^2) + b*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^4*d^
2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^2}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x\left(d-c^2\,d\,x^2\right)^2}\,dx$$

input
```
int((a + b*acos(c*x))/(x*(d - c^2*d*x^2)^2),x)
```

output
```
int((a + b*acos(c*x))/(x*(d - c^2*d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^2}\,dx$$
$$= \frac{2\left(\int \frac{acos(cx)}{c^4x^5-2c^2x^3+x}dx\right)b\,c^2x^2-2\left(\int \frac{acos(cx)}{c^4x^5-2c^2x^3+x}dx\right)b-\log(c^2x-c)\,a\,c^2x^2+\log(c^2x-c)\,a-\log(c^2x+c}{2d^2\left(c^2x^2-1\right)}$$

input
```
int((a+b*acos(c*x))/x/(-c^2*d*x^2+d)^2,x)
```

output
```
(2*int(acos(c*x)/(c**4*x**5 - 2*c**2*x**3 + x),x)*b*c**2*x**2 - 2*int(acos
(c*x)/(c**4*x**5 - 2*c**2*x**3 + x),x)*b - log(c**2*x - c)*a*c**2*x**2 + l
og(c**2*x - c)*a - log(c**2*x + c)*a*c**2*x**2 + log(c**2*x + c)*a + 2*log
(x)*a*c**2*x**2 - 2*log(x)*a - a*c**2*x**2)/(2*d**2*(c**2*x**2 - 1))
```

3.45 $\int \frac{a+b\arccos(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 186

$$\int \frac{a+b\arccos(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx = -\frac{bc}{2d^2\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{d^2x\left(1-c^2x^2\right)}+\frac{3c^2x(a+b\arccos(cx))}{2d^2\left(1-c^2x^2\right)} - \frac{3ic(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{d^2} - \frac{b\text{carctanh}\left(\sqrt{1-c^2x^2}\right)}{d^2}+\frac{3ibc\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{2d^2} - \frac{3ibc\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{2d^2}$$

output

```
-1/2*b*c/d^2/(-c^2*x^2+1)^(1/2)-(a+b*arccos(c*x))/d^2/x/(-c^2*x^2+1)+3/2*c
^2*x*(a+b*arccos(c*x))/d^2/(-c^2*x^2+1)-3*I*c*(a+b*arccos(c*x))*arctan(c*x
+I*(-c^2*x^2+1)^(1/2))/d^2-b*c*arctanh((-c^2*x^2+1)^(1/2))/d^2+3/2*I*b*c*p
olylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^2-3/2*I*b*c*polylog(2,I*(c*x+I*(
-c^2*x^2+1)^(1/2)))/d^2
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.35

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^2}\,dx = \frac{-\frac{4a}{x}+\frac{bc\sqrt{1-c^2x^2}}{1-cx}+\frac{bc\sqrt{1-c^2x^2}}{1+cx}-\frac{2ac^2x}{-1+c^2x^2}-\frac{4b\arccos(cx)}{x}+\frac{bc\arccos(cx)}{1-cx}-\frac{bc\arccos(cx)}{1+cx}-6bc\arccos(cx)\log\left(1-e\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^2*(d - c^2*d*x^2)^2),x]
```

output
```
((-4*a)/x + (b*c*Sqrt[1 - c^2*x^2])/(1 - c*x) + (b*c*Sqrt[1 - c^2*x^2])/(1
 + c*x) - (2*a*c^2*x)/(-1 + c^2*x^2) - (4*b*ArcCos[c*x])/x + (b*c*ArcCos[c
*x])/(1 - c*x) - (b*c*ArcCos[c*x])/(1 + c*x) - 6*b*c*ArcCos[c*x]*Log[1 - E
^(I*ArcCos[c*x])] + 6*b*c*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] - 4*b*c*L
og[x] - 3*a*c*Log[1 - c*x] + 3*a*c*Log[1 + c*x] + 4*b*c*Log[1 + Sqrt[1 - c
^2*x^2]] - (6*I)*b*c*PolyLog[2, -E^(I*ArcCos[c*x])] + (6*I)*b*c*PolyLog[2,
 E^(I*ArcCos[c*x])])/(4*d^2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {5205, 27, 243, 61, 73, 221, 5163, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^2}\,dx$$

$$\Big\downarrow \; 5205$$

$$3c^2\int \frac{a+b\arccos(cx)}{d^2(1-c^2x^2)^2}dx-\frac{bc\int \frac{1}{x(1-c^2x^2)^{3/2}}dx}{d^2}-\frac{a+b\arccos(cx)}{d^2x(1-c^2x^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{3c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx}{d^2}-\frac{bc\int \frac{1}{x(1-c^2x^2)^{3/2}}dx}{d^2}-\frac{a+b\arccos(cx)}{d^2x(1-c^2x^2)}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{3c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx}{d^2}-\frac{bc\int \frac{1}{x^2(1-c^2x^2)^{3/2}}dx^2}{2d^2}-\frac{a+b\arccos(cx)}{d^2x(1-c^2x^2)}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{3c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx}{d^2}-\frac{bc\left(\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2+\frac{2}{\sqrt{1-c^2x^2}}\right)}{2d^2}-\frac{a+b\arccos(cx)}{d^2x(1-c^2x^2)}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{3c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx}{d^2}-\frac{bc\left(\frac{2}{\sqrt{1-c^2x^2}}-\frac{2\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2}\right)}{2d^2}-\frac{a+b\arccos(cx)}{d^2x(1-c^2x^2)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{3c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx}{d^2}-\frac{a+b\arccos(cx)}{d^2x(1-c^2x^2)}-\frac{bc\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{2d^2}$$

$$\Big\downarrow \text{ 5163}$$

$$\frac{3c^2\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{x}{(1-c^2x^2)^{3/2}}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}\right)}{d^2}-\frac{a+b\arccos(cx)}{d^2x(1-c^2x^2)}-\frac{bc\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{2d^2}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{3c^2\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{d^2}-\frac{a+b\arccos(cx)}{d^2x(1-c^2x^2)}-\frac{bc\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{2d^2}$$

$$\Big\downarrow \text{ 5165}$$

$$\frac{3c^2\left(-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{d^2}-\frac{a+b\arccos(cx)}{d^2x(1-c^2x^2)}-\frac{bc\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{2d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3c^2\left(-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{d^2}-\frac{a+b\arccos(cx)}{d^2x(1-c^2x^2)}-\frac{bc\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{2d^2}$$

$$\Big\downarrow \text{4671}$$

$$\frac{3c^2\left(-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{2c}+\frac{x(a+b\arccos}{2(1-c^2x^2}\right.}{d^2}$$

$$\frac{a+b\arccos(cx)}{d^2x(1-c^2x^2)}-\frac{bc\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{2d^2}$$

$$\Big\downarrow \text{2715}$$

$$\frac{3c^2\left(-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\,a}{2c}\right.}{d^2}$$

$$\frac{a+b\arccos(cx)}{d^2x(1-c^2x^2)}-\frac{bc\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{2d^2}$$

$$\Big\downarrow \text{2838}$$

$$\frac{3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-}}\right.}{d^2}$$

$$\frac{a+b\arccos(cx)}{d^2x(1-c^2x^2)}-\frac{bc\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{2d^2}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^2*(d - c^2*d*x^2)^2),x]
```

output
```
-((a + b*ArcCos[c*x])/(d^2*x*(1 - c^2*x^2))) - (b*c*(2/Sqrt[1 - c^2*x^2] -
 2*ArcTanh[Sqrt[1 - c^2*x^2]]))/(2*d^2) + (3*c^2*(b/(2*c*Sqrt[1 - c^2*x^2]
) + (x*(a + b*ArcCos[c*x]))/(2*(1 - c^2*x^2)) - (-2*(a + b*ArcCos[c*x])*Ar
cTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(I*ArcCos[c*x])] - I*b*PolyLo
g[2, E^(I*ArcCos[c*x])])/(2*c)))/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.78

method	result
parts	$\frac{a\left(-\frac{1}{x}-\frac{c}{4(cx-1)}-\frac{3c\ln(cx-1)}{4}-\frac{c}{4(cx+1)}+\frac{3c\ln(cx+1)}{4}\right)}{d^2}-\frac{ib\left(3i\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)c^3x^3-3i\arccos(cx)\right.}{}$
derivativedivides	$c\left(\frac{a\left(-\frac{1}{4(cx-1)}-\frac{3\ln(cx-1)}{4}-\frac{1}{cx}-\frac{1}{4(cx+1)}+\frac{3\ln(cx+1)}{4}\right)}{d^2}-\frac{ib\left(3i\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)c^3x^3-3i\arccos(\right.}{}\right.$
default	$c\left(\frac{a\left(-\frac{1}{4(cx-1)}-\frac{3\ln(cx-1)}{4}-\frac{1}{cx}-\frac{1}{4(cx+1)}+\frac{3\ln(cx+1)}{4}\right)}{d^2}-\frac{ib\left(3i\arccos(cx)\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)c^3x^3-3i\arccos(\right.}{}\right.$

input
```
int((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a/d^2*(-1/x-1/4*c/(c*x-1)-3/4*c*ln(c*x-1)-1/4*c/(c*x+1)+3/4*c*ln(c*x+1))-1
/2*I*b/d^2/x/(c^2*x^2-1)*(3*I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*c
^3*x^3-3*I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*c*x-3*I*arccos(c*x)*
c^2*x^2+3*dilog(1+c*x+I*(-c^2*x^2+1)^(1/2))*c^3*x^3+4*arctan(c*x+I*(-c^2*x
^2+1)^(1/2))*c^3*x^3+3*dilog(c*x+I*(-c^2*x^2+1)^(1/2))*c^3*x^3-I*(-c^2*x^2
+1)^(1/2)*x*c+2*I*arccos(c*x)-3*dilog(1+c*x+I*(-c^2*x^2+1)^(1/2))*c*x-4*ar
ctan(c*x+I*(-c^2*x^2+1)^(1/2))*c*x-3*dilog(c*x+I*(-c^2*x^2+1)^(1/2))*c*x)
```

Fricas [F]

$$\int \frac{a + b \arccos(cx)}{x^2 (d - c^2 dx^2)^2} \, dx = \int \frac{b \arccos(cx) + a}{(c^2 dx^2 - d)^2 x^2} \, dx$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{a + b \arccos(cx)}{x^2 (d - c^2 dx^2)^2} \, dx = \frac{\int \frac{a}{c^4 x^6 - 2c^2 x^4 + x^2} \, dx + \int \frac{b \operatorname{acos}(cx)}{c^4 x^6 - 2c^2 x^4 + x^2} \, dx}{d^2}$$

input
```
integrate((a+b*acos(c*x))/x**2/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**6 - 2*c**2*x**4 + x**2), x) + Integral(b*acos(c*x)/(c
**4*x**6 - 2*c**2*x**4 + x**2), x))/d**2
```

Maxima [F]

$$\int \frac{a + b \arccos(cx)}{x^2 (d - c^2 dx^2)^2} \, dx = \int \frac{b \arccos(cx) + a}{(c^2 dx^2 - d)^2 x^2} \, dx$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/4*a*(2*(3*c^2*x^2 - 2)/(c^2*d^2*x^3 - d^2*x) - 3*c*log(c*x + 1)/d^2 + 3
*c*log(c*x - 1)/d^2) - 1/4*((6*c^2*x^2 - 3*(c^3*x^3 - c*x)*log(c*x + 1) +
3*(c^3*x^3 - c*x)*log(-c*x + 1) - 4)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1),
 c*x) + 4*(c^2*d^2*x^3 - d^2*x)*integrate(-1/4*(6*c^3*x^2 - 3*(c^4*x^3 - c
^2*x)*log(c*x + 1) + 3*(c^4*x^3 - c^2*x)*log(-c*x + 1) - 4*c)*sqrt(c*x + 1
)*sqrt(-c*x + 1)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x))*b/(c^2*d^2*x^3
 - d^2*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^2}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x^2\,(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((a + b*acos(c*x))/(x^2*(d - c^2*d*x^2)^2),x)
```

output
```
int((a + b*acos(c*x))/(x^2*(d - c^2*d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx$$

$$= \frac{4\left(\int \frac{acos(cx)}{c^4x^6-2c^2x^4+x^2}dx\right)b\,c^2x^3-4\left(\int \frac{acos(cx)}{c^4x^6-2c^2x^4+x^2}dx\right)bx-3\log(c^2x-c)\,a\,c^3x^3+3\log(c^2x-c)\,acx+3\,\mathrm{lo}}{4d^2x\,(c^2x^2-1)}$$

input
```
int((a+b*acos(c*x))/x^2/(-c^2*d*x^2+d)^2,x)
```

output
```
(4*int(acos(c*x)/(c**4*x**6 - 2*c**2*x**4 + x**2),x)*b*c**2*x**3 - 4*int(a
cos(c*x)/(c**4*x**6 - 2*c**2*x**4 + x**2),x)*b*x - 3*log(c**2*x - c)*a*c**
3*x**3 + 3*log(c**2*x - c)*a*c*x + 3*log(c**2*x + c)*a*c**3*x**3 - 3*log(c
**2*x + c)*a*c*x - 6*a*c**2*x**2 + 4*a)/(4*d**2*x*(c**2*x**2 - 1))
```

3.46 $\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 159

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^2}\,dx = -\frac{bc}{2d^2x\sqrt{1-c^2x^2}}+\frac{c^2(a+b\arccos(cx))}{d^2(1-c^2x^2)}-\frac{a+b\arccos(cx)}{2d^2x^2(1-c^2x^2)} - \frac{4c^2(a+b\arccos(cx))\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)}{d^2} + \frac{ibc^2\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{d^2}-\frac{ibc^2\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{d^2}$$

output
```
-1/2*b*c/d^2/x/(-c^2*x^2+1)^(1/2)+c^2*(a+b*arccos(c*x))/d^2/(-c^2*x^2+1)-1
/2*(a+b*arccos(c*x))/d^2/x^2/(-c^2*x^2+1)-4*c^2*(a+b*arccos(c*x))*arctanh(
(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^2+I*b*c^2*polylog(2,-(c*x+I*(-c^2*x^2+1)^(
1/2))^2)/d^2-I*b*c^2*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^2
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.94

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^2}\,dx$$
$$= \frac{-\frac{2a}{x^2}+\frac{2bc\sqrt{1-c^2x^2}}{x}+\frac{bc^2\sqrt{1-c^2x^2}}{1-cx}-\frac{bc^2\sqrt{1-c^2x^2}}{1+cx}-\frac{2ac^2}{-1+c^2x^2}-\frac{2b\arccos(cx)}{x^2}+\frac{bc^2\arccos(cx)}{1-cx}+\frac{bc^2\arccos(cx)}{1+cx}-8bc^2\,\mathrm{arc}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^3*(d - c^2*d*x^2)^2),x]
```

output
```
((-2*a)/x^2 + (2*b*c*Sqrt[1 - c^2*x^2])/x + (b*c^2*Sqrt[1 - c^2*x^2])/(1 -
 c*x) - (b*c^2*Sqrt[1 - c^2*x^2])/(1 + c*x) - (2*a*c^2)/(-1 + c^2*x^2) - (
2*b*ArcCos[c*x])/x^2 + (b*c^2*ArcCos[c*x])/(1 - c*x) + (b*c^2*ArcCos[c*x])
/(1 + c*x) - 8*b*c^2*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] - 8*b*c^2*ArcC
os[c*x]*Log[1 + E^(I*ArcCos[c*x])] + 8*b*c^2*ArcCos[c*x]*Log[1 + E^((2*I)*
ArcCos[c*x])] + 8*a*c^2*Log[x] - 4*a*c^2*Log[1 - c^2*x^2] + (8*I)*b*c^2*Po
lyLog[2, -E^(I*ArcCos[c*x])] + (8*I)*b*c^2*PolyLog[2, E^(I*ArcCos[c*x])] -
 (4*I)*b*c^2*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/(4*d^2)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5205, 27, 245, 208, 5209, 208, 5185, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^2}\,dx$$

$$\Big\downarrow \; 5205$$

$$2c^2\int \frac{a+b\arccos(cx)}{d^2x(1-c^2x^2)^2}dx-\frac{bc\int \frac{1}{x^2(1-c^2x^2)^{3/2}}dx}{2d^2}-\frac{a+b\arccos(cx)}{2d^2x^2(1-c^2x^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{2c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2}-\frac{bc\int \frac{1}{x^2(1-c^2x^2)^{3/2}}dx}{2d^2}-\frac{a+b\arccos(cx)}{2d^2x^2(1-c^2x^2)}$$

$$\Big\downarrow \text{245}$$

$$\frac{2c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2}-\frac{bc\left(2c^2\int \frac{1}{(1-c^2x^2)^{3/2}}dx-\frac{1}{x\sqrt{1-c^2x^2}}\right)}{2d^2}-\frac{a+b\arccos(cx)}{2d^2x^2(1-c^2x^2)}$$

$$\Big\downarrow \text{208}$$

$$\frac{2c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2}-\frac{a+b\arccos(cx)}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)}{2d^2}$$

$$\Big\downarrow \text{5209}$$

$$\frac{2c^2\left(\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\int \frac{1}{(1-c^2x^2)^{3/2}}dx+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}\right)}{d^2}-\frac{a+b\arccos(cx)}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)}{2d^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{2c^2\left(\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)}{d^2}-\frac{a+b\arccos(cx)}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)}{2d^2}$$

$$\Big\downarrow \text{5185}$$

$$\frac{2c^2\left(-\int \frac{a+b\arccos(cx)}{cx\sqrt{1-c^2x^2}}d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)}{d^2}-\frac{a+b\arccos(cx)}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)}{2d^2}$$

$$\Big\downarrow \text{4919}$$

$$\frac{2c^2\left(-2\int (a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)}{d^2}-\frac{a+b\arccos(cx)}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)}{2d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c^2\left(-2\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)}{d^2}-\frac{a+b\arccos(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)}{2d^2}$$

$\Big\downarrow$ 4671

$$\frac{2c^2\left(-2\left(-\frac{1}{2}b\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+\frac{1}{2}b\int\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\left(\operatorname{arctanh}\left(e^{2i\arccos(c\right.\right.\right.\right.}{d^2}-\frac{a+b\arccos(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)}{2d^2}$$

$\Big\downarrow$ 2715

$$\frac{2c^2\left(-2\left(\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos}\right.\right.}{d^2}-\frac{a+b\arccos(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)}{2d^2}$$

$\Big\downarrow$ 2838

$$\frac{2c^2\left(-2\left(-\left(\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}ib\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)-\frac{1}{4}ib\operatorname{PolyLog}\left(2,e^{2i\arccos(}\right.\right.\right.}{d^2}-\frac{a+b\arccos(cx)}{2d^2x^2\left(1-c^2x^2\right)}-\frac{bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)}{2d^2}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^3*(d - c^2*d*x^2)^2),x]
```

output
```
-1/2*(b*c*(-(1/(x*Sqrt[1 - c^2*x^2])) + (2*c^2*x)/Sqrt[1 - c^2*x^2]))/d^2
- (a + b*ArcCos[c*x])/(2*d^2*x^2*(1 - c^2*x^2)) + (2*c^2*((b*c*x)/(2*Sqrt[
1 - c^2*x^2]) + (a + b*ArcCos[c*x])/(2*(1 - c^2*x^2)) - 2*(-((a + b*ArcCos
[c*x])*ArcTanh[E^((2*I)*ArcCos[c*x])]) + (I/4)*b*PolyLog[2, -E^((2*I)*ArcC
os[c*x])] - (I/4)*b*PolyLog[2, E^((2*I)*ArcCos[c*x])])))/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.77

method	result
derivativedivides	$c^2\left(\frac{a\left(-\frac{1}{4(cx-1)}-\ln(cx-1)-\frac{1}{2c^2x^2}+2\ln(cx)+\frac{1}{4cx+4}-\ln(cx+1)\right)}{d^2}+\frac{b\left(-\frac{2c^2x^2\arccos(cx)+cx\sqrt{-c^2x^2+1}-\arccos(cx)}{2c^2x^2\left(c^2x^2-1\right)}\right.}{}\right.$
default	$c^2\left(\frac{a\left(-\frac{1}{4(cx-1)}-\ln(cx-1)-\frac{1}{2c^2x^2}+2\ln(cx)+\frac{1}{4cx+4}-\ln(cx+1)\right)}{d^2}+\frac{b\left(-\frac{2c^2x^2\arccos(cx)+cx\sqrt{-c^2x^2+1}-\arccos(cx)}{2c^2x^2\left(c^2x^2-1\right)}\right.}{}\right.$
parts	$\frac{a\left(-\frac{1}{2x^2}+2c^2\ln(x)-\frac{c^2}{4(cx-1)}-c^2\ln(cx-1)+\frac{c^2}{4cx+4}-c^2\ln(cx+1)\right)}{d^2}+\frac{b\,c^2\left(-\frac{2c^2x^2\arccos(cx)+cx\sqrt{-c^2x^2+1}-\arccos(cx}{2c^2x^2\left(c^2x^2-1\right)}\right.}{}$

input
```
int((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(a/d^2*(-1/4/(c*x-1)-ln(c*x-1)-1/2/c^2/x^2+2*ln(c*x)+1/4/(c*x+1)-ln(c*
x+1))+b/d^2*(-1/2*(2*c^2*x^2*arccos(c*x)+c*x*(-c^2*x^2+1)^(1/2)-arccos(c*x
))/c^2/x^2/(c^2*x^2-1)-2*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+2*arcc
os(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-2*arccos(c*x)*ln(1-c*x-I*(-c^2*
x^2+1)^(1/2))+2*I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-I*polylog(2,-(c*x+I
*(-c^2*x^2+1)^(1/2))^2)+2*I*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^2}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(c^2dx^2-d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^2}\,dx=\frac{\int \frac{a}{c^4x^7-2c^2x^5+x^3}\,dx+\int \frac{b\operatorname{acos}\left(cx\right)}{c^4x^7-2c^2x^5+x^3}\,dx}{d^2}$$

input
```
integrate((a+b*acos(c*x))/x**3/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**7 - 2*c**2*x**5 + x**3), x) + Integral(b*acos(c*x)/(c
**4*x**7 - 2*c**2*x**5 + x**3), x))/d**2
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^2}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(c^2dx^2-d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(2*c^2*log(c*x + 1)/d^2 + 2*c^2*log(c*x - 1)/d^2 - 4*c^2*log(x)/d^2
 + (2*c^2*x^2 - 1)/(c^2*d^2*x^4 - d^2*x^2)) + b*integrate(arctan2(sqrt(c*x
 + 1)*sqrt(-c*x + 1), c*x)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{acos}\left(c\,x\right)}{x^3\,\left(d-c^2\,d\,x^2\right)^2}\,dx$$

input
```
int((a + b*acos(c*x))/(x^3*(d - c^2*d*x^2)^2),x)
```

output
```
int((a + b*acos(c*x))/(x^3*(d - c^2*d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^2}\,dx$$
$$= \frac{2\left(\int \frac{acos(cx)}{c^4x^7-2c^2x^5+x^3}dx\right)b\,c^2x^4 - 2\left(\int \frac{acos(cx)}{c^4x^7-2c^2x^5+x^3}dx\right)b\,x^2 - 2\log(c^2x-c)\,a\,c^4x^4 + 2\log(c^2x-c)\,a\,c^2x^2 -}{2d^2x^2\left(c^2x^2-\right.}$$

input
```
int((a+b*acos(c*x))/x^3/(-c^2*d*x^2+d)^2,x)
```

output
```
(2*int(acos(c*x)/(c**4*x**7 - 2*c**2*x**5 + x**3),x)*b*c**2*x**4 - 2*int(a
cos(c*x)/(c**4*x**7 - 2*c**2*x**5 + x**3),x)*b*x**2 - 2*log(c**2*x - c)*a*
c**4*x**4 + 2*log(c**2*x - c)*a*c**2*x**2 - 2*log(c**2*x + c)*a*c**4*x**4
+ 2*log(c**2*x + c)*a*c**2*x**2 + 4*log(x)*a*c**4*x**4 - 4*log(x)*a*c**2*x
**2 - 2*a*c**4*x**4 + a)/(2*d**2*x**2*(c**2*x**2 - 1))
```

3.47 $\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 259

$$\begin{aligned}\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)^2}\,dx = &-\frac{bc^3}{3d^2\sqrt{1-c^2x^2}}-\frac{bc}{6d^2x^2\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{3d^2x^3(1-c^2x^2)}\\&-\frac{5c^2(a+b\arccos(cx))}{3d^2x(1-c^2x^2)}+\frac{5c^4x(a+b\arccos(cx))}{2d^2(1-c^2x^2)}\\&-\frac{5ic^3(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{d^2}\\&-\frac{13bc^3\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{6d^2}+\frac{5ibc^3\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{2d^2}\\&-\frac{5ibc^3\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{2d^2}\end{aligned}$$

output
```
-1/3*b*c^3/d^2/(-c^2*x^2+1)^(1/2)-1/6*b*c/d^2/x^2/(-c^2*x^2+1)^(1/2)-1/3*(
a+b*arccos(c*x))/d^2/x^3/(-c^2*x^2+1)-5/3*c^2*(a+b*arccos(c*x))/d^2/x/(-c^
2*x^2+1)+5/2*c^4*x*(a+b*arccos(c*x))/d^2/(-c^2*x^2+1)-5*I*c^3*(a+b*arccos(
c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/d^2-13/6*b*c^3*arctanh((-c^2*x^2+1)
^(1/2))/d^2+5/2*I*b*c^3*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^2-5/2*I
*b*c^3*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^2
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.23

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)^2}\,dx = -\frac{\frac{4a}{x^3}+\frac{24ac^2}{x}-\frac{2bc\sqrt{1-c^2x^2}}{x^2}+\frac{3bc^3\sqrt{1-c^2x^2}}{-1+cx}-\frac{3bc^3\sqrt{1-c^2x^2}}{1+cx}+\frac{6ac^4x}{-1+c^2x^2}+\frac{4b\arccos(cx)}{x^3}+\frac{24bc^2\arccos(cx)}{x}+\frac{3bc^3\arccos(cx)}{-1+cx}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^4*(d - c^2*d*x^2)^2),x]
```

output
```
-1/12*((4*a)/x^3 + (24*a*c^2)/x - (2*b*c*Sqrt[1 - c^2*x^2])/x^2 + (3*b*c^3
*Sqrt[1 - c^2*x^2])/(-1 + c*x) - (3*b*c^3*Sqrt[1 - c^2*x^2])/(1 + c*x) + (
6*a*c^4*x)/(-1 + c^2*x^2) + (4*b*ArcCos[c*x])/x^3 + (24*b*c^2*ArcCos[c*x])
/x + (3*b*c^3*ArcCos[c*x])/(-1 + c*x) + (3*b*c^3*ArcCos[c*x])/(1 + c*x) +
30*b*c^3*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] - 30*b*c^3*ArcCos[c*x]*Log
[1 + E^(I*ArcCos[c*x])] + 26*b*c^3*Log[x] + 15*a*c^3*Log[1 - c*x] - 15*a*c
^3*Log[1 + c*x] - 26*b*c^3*Log[1 + Sqrt[1 - c^2*x^2]] + (30*I)*b*c^3*PolyL
og[2, -E^(I*ArcCos[c*x])] - (30*I)*b*c^3*PolyLog[2, E^(I*ArcCos[c*x])])/d^
2
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.14, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {5205, 27, 243, 52, 61, 73, 221, 5205, 243, 61, 73, 221, 5163, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)^2}\,dx$$

$$\Big\downarrow \; 5205$$

$$\frac{5}{3}c^2\int \frac{a+b\arccos(cx)}{d^2x^2(1-c^2x^2)^2}dx-\frac{bc\int \frac{1}{x^3(1-c^2x^2)^{3/2}}dx}{3d^2}-\frac{a+b\arccos(cx)}{3d^2x^3(1-c^2x^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)^2}dx}{3d^2}-\frac{bc\int \frac{1}{x^3(1-c^2x^2)^{3/2}}dx}{3d^2}-\frac{a+b\arccos(cx)}{3d^2x^3(1-c^2x^2)}$$

$$\Big\downarrow \text{243}$$

$$\frac{5c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)^2}dx}{3d^2}-\frac{bc\int \frac{1}{x^4(1-c^2x^2)^{3/2}}dx^2}{6d^2}-\frac{a+b\arccos(cx)}{3d^2x^3(1-c^2x^2)}$$

$$\Big\downarrow \text{52}$$

$$\frac{5c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)^2}dx}{3d^2}-\frac{bc\left(\frac{3}{2}c^2\int \frac{1}{x^2(1-c^2x^2)^{3/2}}dx^2-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}-\frac{a+b\arccos(cx)}{3d^2x^3(1-c^2x^2)}$$

$$\Big\downarrow \text{61}$$

$$\frac{5c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)^2}dx}{3d^2}-\frac{bc\left(\frac{3}{2}c^2\left(\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2+\frac{2}{\sqrt{1-c^2x^2}}\right)-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}-\frac{a+b\arccos(cx)}{3d^2x^3(1-c^2x^2)}$$

$$\Big\downarrow \text{73}$$

$$\frac{5c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)^2}dx}{3d^2}-\frac{bc\left(\frac{3}{2}c^2\left(\frac{2}{\sqrt{1-c^2x^2}}-\frac{2\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2}\right)-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}-\frac{a+b\arccos(cx)}{3d^2x^3(1-c^2x^2)}$$

$$\Big\downarrow \text{221}$$

$$\frac{5c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)^2}dx}{3d^2}-\frac{a+b\arccos(cx)}{3d^2x^3(1-c^2x^2)}-\frac{bc\left(\frac{3}{2}c^2\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}$$

$$\Big\downarrow \text{5205}$$

$$\frac{5c^2\left(3c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx-bc\int \frac{1}{x(1-c^2x^2)^{3/2}}dx-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}\right)}{3d^2}-\frac{a+b\arccos(cx)}{3d^2x^3(1-c^2x^2)}-\frac{bc\left(\frac{3}{2}c^2\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{5c^2\left(3c^2\int\frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx-\frac{1}{2}bc\int\frac{1}{x^2(1-c^2x^2)^{3/2}}dx^2-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}\right)}{3d^2}-\frac{a+b\arccos(cx)}{3d^2x^3(1-c^2x^2)}-\frac{bc\left(\frac{3}{2}c^2\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{5c^2\left(3c^2\int\frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx-\frac{1}{2}bc\left(\int\frac{1}{x^2\sqrt{1-c^2x^2}}dx^2+\frac{2}{\sqrt{1-c^2x^2}}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}\right)}{3d^2}-\frac{a+b\arccos(cx)}{3d^2x^3(1-c^2x^2)}-\frac{bc\left(\frac{3}{2}c^2\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{5c^2\left(3c^2\int\frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx-\frac{1}{2}bc\left(\frac{2}{\sqrt{1-c^2x^2}}-\frac{2\int\frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}\right)}{3d^2}-\frac{a+b\arccos(cx)}{3d^2x^3(1-c^2x^2)}-\frac{bc\left(\frac{3}{2}c^2\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{5c^2\left(3c^2\int\frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)\right)}{3d^2}-\frac{a+b\arccos(cx)}{3d^2x^3(1-c^2x^2)}-\frac{bc\left(\frac{3}{2}c^2\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}$$

$$\Big\downarrow \text{5163}$$

$$\frac{5c^2\left(3c^2\left(\frac{1}{2}\int\frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int\frac{x}{(1-c^2x^2)^{3/2}}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\right.\right.\right.}{3d^2}$$
$$\frac{a+b\arccos(cx)}{3d^2x^3(1-c^2x^2)}-\frac{bc\left(\frac{3}{2}c^2\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{5c^2\left(3c^2\left(\frac{1}{2}\int\frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x\right.\right.\right.}{3d^2}$$
$$\frac{a+b\arccos(cx)}{3d^2x^3(1-c^2x^2)}-\frac{bc\left(\frac{3}{2}c^2\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}$$

$\Big\downarrow$ 5165

$$\frac{5c^2\left(3c^2\left(-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{\right.\right.\right.}}{3d^2}$$

$$\frac{a+b\arccos(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{bc\left(\frac{3}{2}c^2\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}$$

$\Big\downarrow$ 3042

$$\frac{5c^2\left(3c^2\left(-\frac{\int(a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2}{\sqrt{1-c^2x^2}}-\right.\right.}{3d^2}$$

$$\frac{a+b\arccos(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{bc\left(\frac{3}{2}c^2\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}$$

$\Big\downarrow$ 4671

$$\frac{5c^2\left(3c^2\left(-\frac{-b\int\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{2c}+\frac{x(a+b}{2(1}\right.\right.}{3d^2}$$

$$\frac{a+b\arccos(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{bc\left(\frac{3}{2}c^2\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}$$

$\Big\downarrow$ 2715

$$\frac{5c^2\left(3c^2\left(-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos(cx)}\right)}{2c}\right.\right.}{3d^2}$$

$$\frac{a+b\arccos(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{bc\left(\frac{3}{2}c^2\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}$$

$\Big\downarrow$ 2838

$$\frac{5c^2\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{}{2}\right.\right.}{3d^2}$$

$$\frac{a+b\arccos(cx)}{3d^2x^3\left(1-c^2x^2\right)}-\frac{bc\left(\frac{3}{2}c^2\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)-\frac{1}{x^2\sqrt{1-c^2x^2}}\right)}{6d^2}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^4*(d - c^2*d*x^2)^2),x]
```

output
```
-1/3*(a + b*ArcCos[c*x])/(d^2*x^3*(1 - c^2*x^2)) - (b*c*(-(1/(x^2*Sqrt[1 -
 c^2*x^2])) + (3*c^2*(2/Sqrt[1 - c^2*x^2] - 2*ArcTanh[Sqrt[1 - c^2*x^2]]))
/2))/(6*d^2) + (5*c^2*(-((a + b*ArcCos[c*x])/(x*(1 - c^2*x^2))) - (b*c*(2/
Sqrt[1 - c^2*x^2] - 2*ArcTanh[Sqrt[1 - c^2*x^2]]))/2 + 3*c^2*(b/(2*c*Sqrt[
1 - c^2*x^2]) + (x*(a + b*ArcCos[c*x]))/(2*(1 - c^2*x^2)) - (-2*(a + b*Arc
Cos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(I*ArcCos[c*x])]
- I*b*PolyLog[2, E^(I*ArcCos[c*x])])/(2*c))))/(3*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.97

method	result
derivativedivides	$c^3\left(\frac{a\left(-\frac{1}{4(cx-1)}-\frac{5\ln(cx-1)}{4}-\frac{1}{3c^3x^3}-\frac{2}{cx}-\frac{1}{4(cx+1)}+\frac{5\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{15c^4x^4\arccos(cx)+2c^3x^3\sqrt{-c^2x^2+1}-10c^2x^2}{6c^3x^3\left(c^2x^2\right.}\right.}{}\right.$
default	$c^3\left(\frac{a\left(-\frac{1}{4(cx-1)}-\frac{5\ln(cx-1)}{4}-\frac{1}{3c^3x^3}-\frac{2}{cx}-\frac{1}{4(cx+1)}+\frac{5\ln(cx+1)}{4}\right)}{d^2}+\frac{b\left(-\frac{15c^4x^4\arccos(cx)+2c^3x^3\sqrt{-c^2x^2+1}-10c^2x^2}{6c^3x^3\left(c^2x^2\right.}\right.}{}\right.$
parts	$\frac{a\left(-\frac{1}{3x^3}-\frac{2c^2}{x}-\frac{c^3}{4(cx-1)}-\frac{5c^3\ln(cx-1)}{4}-\frac{c^3}{4(cx+1)}+\frac{5c^3\ln(cx+1)}{4}\right)}{d^2}+\frac{b\,c^3\left(-\frac{15c^4x^4\arccos(cx)+2c^3x^3\sqrt{-c^2x^2+1}-10c^2x^2}{6c^3x^3\left(c^2x\right.}\right.}{}$

input
```
int((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c^3*(a/d^2*(-1/4/(c*x-1)-5/4*ln(c*x-1)-1/3/c^3/x^3-2/c/x-1/4/(c*x+1)+5/4*l
n(c*x+1))+b/d^2*(-1/6*(15*c^4*x^4*arccos(c*x)+2*c^3*x^3*(-c^2*x^2+1)^(1/2)
-10*c^2*x^2*arccos(c*x)+c*x*(-c^2*x^2+1)^(1/2)-2*arccos(c*x))/c^3/x^3/(c^2
*x^2-1)-5/2*I*dilog(1+c*x+I*(-c^2*x^2+1)^(1/2))-13/3*I*arctan(c*x+I*(-c^2*
x^2+1)^(1/2))-5/2*I*dilog(c*x+I*(-c^2*x^2+1)^(1/2))+5/2*arccos(c*x)*ln(1+c
*x+I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)^2}\,dx = \int \frac{b\arccos(cx)+a}{(c^2dx^2-d)^2x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)^2}\,dx = \frac{\int \frac{a}{c^4x^8-2c^2x^6+x^4}\,dx + \int \frac{b\operatorname{acos}(cx)}{c^4x^8-2c^2x^6+x^4}\,dx}{d^2}$$

input
```
integrate((a+b*acos(c*x))/x**4/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a/(c**4*x**8 - 2*c**2*x**6 + x**4), x) + Integral(b*acos(c*x)/(c
**4*x**8 - 2*c**2*x**6 + x**4), x))/d**2
```

Maxima [F]

$$\int \frac{a + b \arccos(cx)}{x^4 (d - c^2 d x^2)^2} \, dx = \int \frac{b \arccos(cx) + a}{(c^2 d x^2 - d)^2 x^4} \, dx$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/12*(15*c^3*log(c*x + 1)/d^2 - 15*c^3*log(c*x - 1)/d^2 - 2*(15*c^4*x^4 -
10*c^2*x^2 - 2)/(c^2*d^2*x^5 - d^2*x^3))*a - 1/12*((30*c^4*x^4 - 20*c^2*x^
2 - 15*(c^5*x^5 - c^3*x^3)*log(c*x + 1) + 15*(c^5*x^5 - c^3*x^3)*log(-c*x
+ 1) - 4)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 12*(c^2*d^2*x^5 - d
^2*x^3)*integrate(-1/12*(30*c^5*x^4 - 20*c^3*x^2 - 15*(c^6*x^5 - c^4*x^3)*
log(c*x + 1) + 15*(c^6*x^5 - c^4*x^3)*log(-c*x + 1) - 4*c)*sqrt(c*x + 1)*s
qrt(-c*x + 1)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x))*b/(c^2*d^2*x^5
- d^2*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arccos(cx)}{x^4 (d - c^2 d x^2)^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^2}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x^4\,\left(d-c^2\,d\,x^2\right)^2}\,dx$$

input
```
int((a + b*acos(c*x))/(x^4*(d - c^2*d*x^2)^2),x)
```

output
```
int((a + b*acos(c*x))/(x^4*(d - c^2*d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^2}\,dx$$
$$= \frac{12\left(\int \frac{acos(cx)}{c^4x^8-2c^2x^6+x^4}dx\right)b\,c^2x^5-12\left(\int \frac{acos(cx)}{c^4x^8-2c^2x^6+x^4}dx\right)b\,x^3-15\log(c^2x-c)\,a\,c^5x^5+15\log(c^2x-c)\,a\,c^3}{12d^2x^3\left(c^2x^2-1\right)}$$

input
```
int((a+b*acos(c*x))/x^4/(-c^2*d*x^2+d)^2,x)
```

output
```
(12*int(acos(c*x)/(c**4*x**8 - 2*c**2*x**6 + x**4),x)*b*c**2*x**5 - 12*int
(acos(c*x)/(c**4*x**8 - 2*c**2*x**6 + x**4),x)*b*x**3 - 15*log(c**2*x - c)
*a*c**5*x**5 + 15*log(c**2*x - c)*a*c**3*x**3 + 15*log(c**2*x + c)*a*c**5*
x**5 - 15*log(c**2*x + c)*a*c**3*x**3 - 30*a*c**4*x**4 + 20*a*c**2*x**2 +
4*a)/(12*d**2*x**3*(c**2*x**2 - 1))
```

3.48 $\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 204

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = -\frac{b}{12c^5d^3(1-c^2x^2)^{3/2}}+\frac{5b}{8c^5d^3\sqrt{1-c^2x^2}}+\frac{x^3(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}-\frac{3x(a+b\arccos(cx))}{8c^4d^3(1-c^2x^2)}-\frac{3i(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{4c^5d^3}+\frac{3ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{8c^5d^3}-\frac{3ib\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{8c^5d^3}$$

output
```
-1/12*b/c^5/d^3/(-c^2*x^2+1)^(3/2)+5/8*b/c^5/d^3/(-c^2*x^2+1)^(1/2)+1/4*x^
3*(a+b*arccos(c*x))/c^2/d^3/(-c^2*x^2+1)^2-3/8*x*(a+b*arccos(c*x))/c^4/d^3
/(-c^2*x^2+1)-3/4*I*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c^5
/d^3+3/8*I*b*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^5/d^3-3/8*I*b*poly
log(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^5/d^3
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.44

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx$$
$$= \frac{\frac{24acx}{(-1+c^2x^2)^2}+\frac{60acx}{-1+c^2x^2}-\frac{2b\left((-2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)\right)}{(-1+cx)^2}+\frac{2b\left((2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)\right)}{(1+cx)^2}-\frac{30b\left(\sqrt{1-c^2x^2}-\arccos(cx)\right)}{1+cx}}{}$$

input
```
Integrate[(x^4*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
((24*a*c*x)/(-1 + c^2*x^2)^2 + (60*a*c*x)/(-1 + c^2*x^2) - (2*b*((-2 + c*x
)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x]))/(-1 + c*x)^2 + (2*b*((2 + c*x)*Sqrt[
1 - c^2*x^2] - 3*ArcCos[c*x]))/(1 + c*x)^2 - (30*b*(Sqrt[1 - c^2*x^2] - Ar
cCos[c*x]))/(1 + c*x) + (30*b*(Sqrt[1 - c^2*x^2] + ArcCos[c*x]))/(-1 + c*x
) - 18*a*Log[1 - c*x] + 18*a*Log[1 + c*x] - (9*I)*b*(ArcCos[c*x]*(ArcCos[c
*x] + (4*I)*Log[1 + E^(I*ArcCos[c*x])]) + 4*PolyLog[2, -E^(I*ArcCos[c*x])]
) + (9*I)*b*(ArcCos[c*x]*(ArcCos[c*x] + (4*I)*Log[1 - E^(I*ArcCos[c*x])])
+ 4*PolyLog[2, E^(I*ArcCos[c*x])]))/(96*c^5*d^3)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5207, 27, 243, 53, 2009, 5207, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx$$
$$\Big\downarrow \text{ 5207}$$
$$-\frac{3\int \frac{x^2(a+b\arccos(cx))}{d^2(1-c^2x^2)^2}dx}{4c^2d}+\frac{b\int \frac{x^3}{(1-c^2x^2)^{5/2}}dx}{4cd^3}+\frac{x^3(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}$$
$$\Big\downarrow \text{ 27}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{4c^2d^3}+\frac{b\int \frac{x^3}{(1-c^2x^2)^{5/2}}dx}{4cd^3}+\frac{x^3(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{243}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{4c^2d^3}+\frac{b\int \frac{x^2}{(1-c^2x^2)^{5/2}}dx^2}{8cd^3}+\frac{x^3(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{4c^2d^3}+\frac{b\int \left(\frac{1}{c^2(1-c^2x^2)^{5/2}}-\frac{1}{c^2(1-c^2x^2)^{3/2}}\right)dx^2}{8cd^3}+\frac{x^3(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{4c^2d^3}+\frac{x^3(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}+\frac{b\left(\frac{2}{3c^4(1-c^2x^2)^{3/2}}-\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{8cd^3}$$

$$\Big\downarrow \text{5207}$$

$$-\frac{3\left(-\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{2c^2}+\frac{b\int \frac{x}{(1-c^2x^2)^{3/2}}dx}{2c}+\frac{x(a+b\arccos(cx))}{2c^2(1-c^2x^2)}\right)}{4c^2d^3}+\frac{x^3(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}+\frac{b\left(\frac{2}{3c^4(1-c^2x^2)^{3/2}}-\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{8cd^3}$$

$$\Big\downarrow \text{241}$$

$$-\frac{3\left(-\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{2c^2}+\frac{x(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b}{2c^3\sqrt{1-c^2x^2}}\right)}{4c^2d^3}+\frac{x^3(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}+\frac{b\left(\frac{2}{3c^4(1-c^2x^2)^{3/2}}-\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{8cd^3}$$

$$\Big\downarrow \text{5165}$$

$$-\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c^3}+\frac{x(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b}{2c^3\sqrt{1-c^2x^2}}\right)}{4c^2d^3}+\frac{x^3(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}+\frac{b\left(\frac{2}{3c^4(1-c^2x^2)^{3/2}}-\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{8cd^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2c^3}+\frac{x(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b}{2c^3\sqrt{1-c^2x^2}}\right)}{4c^2d^3}+\frac{x^3(a+b\arccos(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}+\frac{b\left(\frac{2}{3c^4(1-c^2x^2)^{3/2}}-\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{8cd^3}$$

$$\Big\downarrow \; 4671$$

$$-\frac{3\left(\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{2c^3}+\frac{x(a+b\arccos(cx}{2c^2(1-c^2x^2)}\right.}{4c^2d^3}$$

$$\frac{x^3(a+b\arccos(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}+\frac{b\left(\frac{2}{3c^4(1-c^2x^2)^{3/2}}-\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{8cd^3}$$

$$\Big\downarrow \; 2715$$

$$-\frac{3\left(\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\,\text{arcc}}{2c^3}\right.}{4c^2d^3}$$

$$\frac{x^3(a+b\arccos(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}+\frac{b\left(\frac{2}{3c^4(1-c^2x^2)^{3/2}}-\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{8cd^3}$$

$$\Big\downarrow \; 2838$$

$$-\frac{3\left(\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c^3}+\frac{x(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b}{2c^3\sqrt{1-c}}\right.}{4c^2d^3}$$

$$\frac{x^3(a+b\arccos(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}+\frac{b\left(\frac{2}{3c^4(1-c^2x^2)^{3/2}}-\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{8cd^3}$$

input
```
Int[(x^4*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
(b*(2/(3*c^4*(1 - c^2*x^2)^(3/2)) - 2/(c^4*Sqrt[1 - c^2*x^2])))/(8*c*d^3)
+ (x^3*(a + b*ArcCos[c*x]))/(4*c^2*d^3*(1 - c^2*x^2)^2) - (3*(b/(2*c^3*Sqr
t[1 - c^2*x^2]) + (x*(a + b*ArcCos[c*x]))/(2*c^2*(1 - c^2*x^2)) + (-2*(a +
 b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(I*ArcCos[c
*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/(2*c^3)))/(4*c^2*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.24

method	result
derivativedivides	$-\frac{a\left(-\frac{1}{16(cx-1)^2}-\frac{5}{16(cx-1)}+\frac{3\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}-\frac{5}{16(cx+1)}-\frac{3\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(-\frac{15c^3x^3\arccos(cx)+15c^2x^2\sqrt{-c^2x^2+1}-\cdots}{24\left(c^4x^4-2c^2x^2\cdots\right.}\right.}{\cdots}$
default	$-\frac{a\left(-\frac{1}{16(cx-1)^2}-\frac{5}{16(cx-1)}+\frac{3\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}-\frac{5}{16(cx+1)}-\frac{3\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(-\frac{15c^3x^3\arccos(cx)+15c^2x^2\sqrt{-c^2x^2+1}-\cdots}{24\left(c^4x^4-2c^2x^2\cdots\right.}\right.}{\cdots}$
parts	$-\frac{a\left(-\frac{1}{16c^5(cx-1)^2}-\frac{5}{16c^5(cx-1)}+\frac{3\ln(cx-1)}{16c^5}+\frac{1}{16c^5(cx+1)^2}-\frac{5}{16c^5(cx+1)}-\frac{3\ln(cx+1)}{16c^5}\right)}{d^3}-\frac{b\left(-\frac{15c^3x^3\arccos(cx)+15c^2x^2\cdots}{2\cdots}\right.}{\cdots}$

input
```
int(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(-a/d^3*(-1/16/(c*x-1)^2-5/16/(c*x-1)+3/16*ln(c*x-1)+1/16/(c*x+1)^2-
5/16/(c*x+1)-3/16*ln(c*x+1))-b/d^3*(-1/24*(15*c^3*x^3*arccos(c*x)+15*c^2*x
^2*(-c^2*x^2+1)^(1/2)-9*c*x*arccos(c*x)-13*(-c^2*x^2+1)^(1/2))/(c^4*x^4-2*
c^2*x^2+1)+3/8*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-3/8*I*polylog(2,
c*x+I*(-c^2*x^2+1)^(1/2))-3/8*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+3
/8*I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)x^4}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*x^4*arccos(c*x) + a*x^4)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2
*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = -\frac{\int \frac{ax^4}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{bx^4\operatorname{acos}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

input
```
integrate(x**4*(a+b*acos(c*x))/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a*x**4/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integr
al(b*x**4*acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)x^4}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/16*a*(2*(5*c^2*x^3 - 3*x)/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 3*lo
g(c*x + 1)/(c^5*d^3) - 3*log(c*x - 1)/(c^5*d^3)) + 1/16*((10*c^3*x^3 - 6*c
*x + 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) - 3*(c^4*x^4 - 2*c^2*x^2 + 1
)*log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - 16*(c^9*d^3*
x^4 - 2*c^7*d^3*x^2 + c^5*d^3)*integrate(1/16*(10*c^3*x^3 - 6*c*x + 3*(c^4
*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(-c*x
+ 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^10*d^3*x^6 - 3*c^8*d^3*x^4 + 3*c^6*d
^3*x^2 - c^4*d^3), x))*b/(c^9*d^3*x^4 - 2*c^7*d^3*x^2 + c^5*d^3)
```

Giac [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)x^4}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(-(b*arccos(c*x) + a)*x^4/(c^2*d*x^2 - d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \int \frac{x^4\,(a+b\,\mathrm{acos}(c\,x))}{(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((x^4*(a + b*acos(c*x)))/(d - c^2*d*x^2)^3,x)
```

output
```
int((x^4*(a + b*acos(c*x)))/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^4(a + b\arccos(cx))}{(d - c^2dx^2)^3}\,dx$$
$$= \frac{-16\left(\int \frac{acos(cx)x^4}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^9x^4 + 32\left(\int \frac{acos(cx)x^4}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^7x^2 - 16\left(\int \frac{acos(cx)x^4}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c}{}$$

input
```
int(x^4*(a+b*acos(c*x))/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 16*int((acos(c*x)*x**4)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)
*b*c**9*x**4 + 32*int((acos(c*x)*x**4)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x
**2 - 1),x)*b*c**7*x**2 - 16*int((acos(c*x)*x**4)/(c**6*x**6 - 3*c**4*x**4
 + 3*c**2*x**2 - 1),x)*b*c**5 - 3*log(c**2*x - c)*a*c**4*x**4 + 6*log(c**2
*x - c)*a*c**2*x**2 - 3*log(c**2*x - c)*a + 3*log(c**2*x + c)*a*c**4*x**4
- 6*log(c**2*x + c)*a*c**2*x**2 + 3*log(c**2*x + c)*a + 10*a*c**3*x**3 - 6
*a*c*x)/(16*c**5*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.49 $\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 100

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = -\frac{bx^3}{12cd^3(1-c^2x^2)^{3/2}}+\frac{bx}{4c^3d^3\sqrt{1-c^2x^2}}-\frac{b\arccos(cx)}{4c^4d^3}+\frac{x^4(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}$$

output
```
-1/12*b*x^3/c/d^3/(-c^2*x^2+1)^(3/2)+1/4*b*x/c^3/d^3/(-c^2*x^2+1)^(1/2)-1/
4*b*arccos(c*x)/c^4/d^3+1/4*x^4*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \frac{bcx\sqrt{1-c^2x^2}(-3+4c^2x^2)+a(-3+6c^2x^2)+3b(-1+2c^2x^2)\arccos(cx)}{12c^4d^3(-1+c^2x^2)^2}$$

input
```
Integrate[(x^3*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
(b*c*x*Sqrt[1 - c^2*x^2]*(-3 + 4*c^2*x^2) + a*(-3 + 6*c^2*x^2) + 3*b*(-1 +
 2*c^2*x^2)*ArcCos[c*x])/(12*c^4*d^3*(-1 + c^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5187, 252, 252, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^3}dx$$

$$\Big\downarrow \text{5187}$$

$$\frac{bc\int \frac{x^4}{(1-c^2x^2)^{5/2}}dx}{4d^3}+\frac{x^4(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{252}$$

$$\frac{bc\left(\frac{x^3}{3c^2(1-c^2x^2)^{3/2}}-\frac{\int \frac{x^2}{(1-c^2x^2)^{3/2}}dx}{c^2}\right)}{4d^3}+\frac{x^4(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{252}$$

$$\frac{bc\left(\frac{x^3}{3c^2(1-c^2x^2)^{3/2}}-\frac{\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{c^2}}{c^2}\right)}{4d^3}+\frac{x^4(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{x^4(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}+\frac{bc\left(\frac{x^3}{3c^2(1-c^2x^2)^{3/2}}-\frac{\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}}{c^2}\right)}{4d^3}$$

input
```
Int[(x^3*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
(x^4*(a + b*ArcCos[c*x]))/(4*d^3*(1 - c^2*x^2)^2) + (b*c*(x^3/(3*c^2*(1 -
c^2*x^2)^(3/2)) - (x/(c^2*Sqrt[1 - c^2*x^2]) - ArcSin[c*x]/c^3)/c^2))/(4*d
^3)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.77

method	result
orering	$-\frac{(cx-1)(cx+1)(4c^4x^4+3c^2x^2-4)(a+b\arccos(cx))}{4c^4(-c^2d\,x^2+d)^3}-\frac{(4c^2x^2-3)(cx-1)^2(cx+1)^2\left(\frac{3x^2(a+b\arccos(cx))}{(-c^2d\,x^2+d)^3}-\frac{}{\sqrt{-c^2x^2+}}\right.}{12x^2c^4}$
derivativedivides	$\frac{-\frac{a\left(-\frac{1}{16(cx-1)^2}-\frac{3}{16(cx-1)}-\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}\right)}{d^3}-\frac{b\left(-\frac{\arccos(cx)}{16(cx-1)^2}-\frac{3\arccos(cx)}{16(cx-1)}-\frac{\arccos(cx)}{16(cx+1)^2}+\frac{3\arccos(cx)}{16(cx+1)}-\frac{\sqrt{-(cx-1)^2}}{48(cx-}\right.}{}}{c^4}$
default	$\frac{-\frac{a\left(-\frac{1}{16(cx-1)^2}-\frac{3}{16(cx-1)}-\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}\right)}{d^3}-\frac{b\left(-\frac{\arccos(cx)}{16(cx-1)^2}-\frac{3\arccos(cx)}{16(cx-1)}-\frac{\arccos(cx)}{16(cx+1)^2}+\frac{3\arccos(cx)}{16(cx+1)}-\frac{\sqrt{-(cx-1)^2}}{48(cx-}\right.}{}}{c^4}$
parts	$-\frac{a\left(-\frac{1}{16c^4(cx-1)^2}-\frac{3}{16c^4(cx-1)}-\frac{1}{16c^4(cx+1)^2}+\frac{3}{16c^4(cx+1)}\right)}{d^3}-\frac{b\left(-\frac{\arccos(cx)}{16(cx-1)^2}-\frac{3\arccos(cx)}{16(cx-1)}-\frac{\arccos(cx)}{16(cx+1)^2}+\frac{3\arccos(cx}{16(cx+1)}\right.}{}$

input
```
int(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x-1)*(c*x+1)*(4*c^4*x^4+3*c^2*x^2-4)/c^4*(a+b*arccos(c*x))/(-c^2*d
*x^2+d)^3-1/12/x^2*(4*c^2*x^2-3)/c^4*(c*x-1)^2*(c*x+1)^2*(3*x^2*(a+b*arcco
s(c*x))/(-c^2*d*x^2+d)^3-x^3*b*c/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^3+6*x^4
*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^4*d*c^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \frac{3\,ac^4x^4+3\,(2\,bc^2x^2-b)\arccos{(cx)}+(4\,bc^3x^3-3\,bcx)\sqrt{-c^2x^2+1}}{12\,(c^8d^3x^4-2\,c^6d^3x^2+c^4d^3)}$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
1/12*(3*a*c^4*x^4 + 3*(2*b*c^2*x^2 - b)*arccos(c*x) + (4*b*c^3*x^3 - 3*b*c
*x)*sqrt(-c^2*x^2 + 1))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)
```

Sympy [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = -\frac{\int \frac{ax^3}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{bx^3\operatorname{acos}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

input
```
integrate(x**3*(a+b*acos(c*x))/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a*x**3/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integr
al(b*x**3*acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)x^3}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*(2*c^2*x^2 - 1)*a/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 1/4*((2*c^
2*x^2 - 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - 4*(c^8*d^3*x^4 - 2
*c^6*d^3*x^2 + c^4*d^3)*integrate(1/4*(2*c^2*x^2 - 1)*e^(1/2*log(c*x + 1)
+ 1/2*log(-c*x + 1))/(c^11*d^3*x^8 - 3*c^9*d^3*x^6 + 3*c^7*d^3*x^4 - c^5*d
^3*x^2 + (c^9*d^3*x^6 - 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 - c^3*d^3)*e^(log(c*
x + 1) + log(-c*x + 1))), x))*b/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.24

$$\begin{aligned}\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx &= \frac{bx^4\arccos(cx)}{4(c^2x^2-1)^2d^3} + \frac{ax^4}{4(c^2x^2-1)^2d^3} \\ &\quad - \frac{bx^3}{12(c^2x^2-1)\sqrt{-c^2x^2+1}cd^3} \\ &\quad - \frac{bx}{4\sqrt{-c^2x^2+1}c^3d^3} - \frac{b\arccos(cx)}{4c^4d^3} - \frac{a}{4c^4d^3}\end{aligned}$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
1/4*b*x^4*arccos(c*x)/((c^2*x^2 - 1)^2*d^3) + 1/4*a*x^4/((c^2*x^2 - 1)^2*d
^3) - 1/12*b*x^3/((c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*c*d^3) - 1/4*b*x/(sqrt(
-c^2*x^2 + 1)*c^3*d^3) - 1/4*b*arccos(c*x)/(c^4*d^3) - 1/4*a/(c^4*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^3}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{acos}(c\,x)\right)}{{\left(d-c^2\,d\,x^2\right)}^3}\,dx$$

input
```
int((x^3*(a + b*acos(c*x)))/(d - c^2*d*x^2)^3,x)
```

output
```
int((x^3*(a + b*acos(c*x)))/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^3}\,dx$$
$$=\frac{-4\left(\int \frac{acos(cx)\,x^3}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^4x^4+8\left(\int \frac{acos(cx)\,x^3}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^2x^2-4\left(\int \frac{acos(cx)\,x^3}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b+a\,x}{4d^3\left(c^4x^4-2c^2x^2+1\right)}$$

input
```
int(x^3*(a+b*acos(c*x))/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 4*int((acos(c*x)*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*
b*c**4*x**4 + 8*int((acos(c*x)*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**
2 - 1),x)*b*c**2*x**2 - 4*int((acos(c*x)*x**3)/(c**6*x**6 - 3*c**4*x**4 +
3*c**2*x**2 - 1),x)*b + a*x**4)/(4*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.50 $\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 202

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = -\frac{b}{12c^3d^3(1-c^2x^2)^{3/2}}+\frac{b}{8c^3d^3\sqrt{1-c^2x^2}} +\frac{x(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}-\frac{x(a+b\arccos(cx))}{8c^2d^3(1-c^2x^2)} +\frac{i(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{4c^3d^3} -\frac{ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{8c^3d^3} +\frac{ib\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{8c^3d^3}$$

output
```
-1/12*b/c^3/d^3/(-c^2*x^2+1)^(3/2)+1/8*b/c^3/d^3/(-c^2*x^2+1)^(1/2)+1/4*x*
(a+b*arccos(c*x))/c^2/d^3/(-c^2*x^2+1)^2-1/8*x*(a+b*arccos(c*x))/c^2/d^3/(
-c^2*x^2+1)+1/4*I*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c^3/d
^3-1/8*I*b*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^3/d^3+1/8*I*b*polylo
g(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^3/d^3
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.80

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx$$

$$= \frac{ax}{4c^2d^3\left(-1+c^2x^2\right)^2}+\frac{ax}{8c^2d^3\left(-1+c^2x^2\right)}+\frac{a\log(1-cx)}{16c^3d^3}-\frac{a\log(1+cx)}{16c^3d^3}$$

$$-\frac{b\left(\frac{(-2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)}{48c^3(-1+cx)^2}-\frac{(2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)}{48c^3(1+cx)^2}+\frac{\sqrt{1-c^2x^2}-\arccos(cx)}{16c^2(c+c^2x)}+\frac{\sqrt{1-c^2x^2}+\arccos(cx)}{16c^2(c-c^2x)}+\frac{-\frac{i\arccos(}{2c}}{}\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
(a*x)/(4*c^2*d^3*(-1 + c^2*x^2)^2) + (a*x)/(8*c^2*d^3*(-1 + c^2*x^2)) + (a
*Log[1 - c*x])/(16*c^3*d^3) - (a*Log[1 + c*x])/(16*c^3*d^3) - (b*(((-2 + c
*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x])/(48*c^3*(-1 + c*x)^2) - ((2 + c*x)*
Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x])/(48*c^3*(1 + c*x)^2) + (Sqrt[1 - c^2*x^
2] - ArcCos[c*x])/(16*c^2*(c + c^2*x)) + (Sqrt[1 - c^2*x^2] + ArcCos[c*x])
/(16*c^2*(c - c^2*x)) + (((-1/2*I)*ArcCos[c*x]^2)/c + (2*ArcCos[c*x]*Log[1
 + E^(I*ArcCos[c*x])])/c - ((2*I)*PolyLog[2, -E^(I*ArcCos[c*x])])/c)/(16*c
^2) + ((I/32)*(ArcCos[c*x]*(ArcCos[c*x] + (4*I)*Log[1 - E^(I*ArcCos[c*x])]
) + 4*PolyLog[2, E^(I*ArcCos[c*x])]))/c^3))/d^3
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5207, 27, 241, 5163, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx$$

$$\Big\downarrow \; 5207$$

$$-\frac{\int \frac{a+b\arccos(cx)}{d^2(1-c^2x^2)^2}dx}{4c^2d}+\frac{b\int \frac{x}{(1-c^2x^2)^{5/2}}dx}{4cd^3}+\frac{x(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx}{4c^2d^3}+\frac{b\int \frac{x}{(1-c^2x^2)^{5/2}}dx}{4cd^3}+\frac{x(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{241}$$

$$-\frac{\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx}{4c^2d^3}+\frac{x(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}+\frac{b}{12c^3d^3(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{5163}$$

$$-\frac{\frac{1}{2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{x}{(1-c^2x^2)^{3/2}}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}}{4c^2d^3}+\frac{x(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}+\frac{b}{12c^3d^3(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$-\frac{\frac{1}{2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}}{4c^2d^3}+\frac{x(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}+\frac{b}{12c^3d^3(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{5165}$$

$$-\frac{-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}}{4c^2d^3}+\frac{x(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}+\frac{b}{12c^3d^3(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}}{4c^2d^3}+\frac{x(a+b\arccos(cx))}{4c^2d^3(1-c^2x^2)^2}+\frac{b}{12c^3d^3(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}}{4c^2d^3}$$

$$\frac{x(a+b\arccos(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}+\frac{b}{12c^3d^3\left(1-c^2x^2\right)^{3/2}}$$

$\Big\downarrow$ 2715

$$-\frac{-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{2c}}{4c^2d^3}$$

$$\frac{x(a+b\arccos(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}+\frac{b}{12c^3d^3\left(1-c^2x^2\right)^{3/2}}$$

$\Big\downarrow$ 2838

$$-\frac{-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}}{4c^2d^3}$$

$$\frac{x(a+b\arccos(cx))}{4c^2d^3\left(1-c^2x^2\right)^2}+\frac{b}{12c^3d^3\left(1-c^2x^2\right)^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
b/(12*c^3*d^3*(1 - c^2*x^2)^(3/2)) + (x*(a + b*ArcCos[c*x]))/(4*c^2*d^3*(1
 - c^2*x^2)^2) - (b/(2*c*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcCos[c*x]))/(2*(
1 - c^2*x^2)) - (-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*P
olyLog[2, -E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/(2*c))/
(4*c^2*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.25

method	result
derivativedivides	$-\frac{\frac{a\left(-\frac{1}{16(cx-1)^2}-\frac{1}{16(cx-1)}-\frac{\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}-\frac{1}{16(cx+1)}+\frac{\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(-\frac{3c^3x^3\arccos(cx)+3c^2x^2\sqrt{-c^2x^2+1}+3cx\,\mathrm{ar}}{24\left(c^4x^4-2c^2x^2+1\right)}\right.}{}}{}$
default	$-\frac{\frac{a\left(-\frac{1}{16(cx-1)^2}-\frac{1}{16(cx-1)}-\frac{\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}-\frac{1}{16(cx+1)}+\frac{\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(-\frac{3c^3x^3\arccos(cx)+3c^2x^2\sqrt{-c^2x^2+1}+3cx\,\mathrm{ar}}{24\left(c^4x^4-2c^2x^2+1\right)}\right.}{}}{}$
parts	$-\frac{a\left(-\frac{1}{16c^3(cx-1)^2}-\frac{1}{16c^3(cx-1)}-\frac{\ln(cx-1)}{16c^3}+\frac{1}{16c^3(cx+1)^2}-\frac{1}{16c^3(cx+1)}+\frac{\ln(cx+1)}{16c^3}\right)}{d^3}-\frac{b\left(-\frac{3c^3x^3\arccos(cx)+3c^2x^2\sqrt{-}}{24(c^4}\right.}{}$

input
```
int(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-a/d^3*(-1/16/(c*x-1)^2-1/16/(c*x-1)-1/16*ln(c*x-1)+1/16/(c*x+1)^2-
1/16/(c*x+1)+1/16*ln(c*x+1))-b/d^3*(-1/24*(3*c^3*x^3*arccos(c*x)+3*c^2*x^2
*(-c^2*x^2+1)^(1/2)+3*c*x*arccos(c*x)-(-c^2*x^2+1)^(1/2))/(c^4*x^4-2*c^2*x
^2+1)-1/8*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+1/8*I*polylog(2,c*x+I
*(-c^2*x^2+1)^(1/2))+1/8*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-1/8*I*
polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)x^2}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*x^2*arccos(c*x) + a*x^2)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2
*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = -\frac{\int \frac{ax^2}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{bx^2\operatorname{acos}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

input
```
integrate(x**2*(a+b*acos(c*x))/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a*x**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integr
al(b*x**2*acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)x^2}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/16*a*(2*(c^2*x^3 + x)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3) - log(c*x
+ 1)/(c^3*d^3) + log(c*x - 1)/(c^3*d^3)) + 1/16*((2*c^3*x^3 + 2*c*x - (c^4
*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) + (c^4*x^4 - 2*c^2*x^2 + 1)*log(-c*x +
1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 16*(c^7*d^3*x^4 - 2*c^5*d
^3*x^2 + c^3*d^3)*integrate(-1/16*(2*c^3*x^3 + 2*c*x - (c^4*x^4 - 2*c^2*x^
2 + 1)*log(c*x + 1) + (c^4*x^4 - 2*c^2*x^2 + 1)*log(-c*x + 1))*sqrt(c*x +
1)*sqrt(-c*x + 1)/(c^8*d^3*x^6 - 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 - c^2*d^3),
 x))*b/(c^7*d^3*x^4 - 2*c^5*d^3*x^2 + c^3*d^3)
```

Giac [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos{(cx)}+a)x^2}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(-(b*arccos(c*x) + a)*x^2/(c^2*d*x^2 - d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \int \frac{x^2\,(a+b\,\mathrm{acos}(c\,x))}{(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((x^2*(a + b*acos(c*x)))/(d - c^2*d*x^2)^3,x)
```

output
```
int((x^2*(a + b*acos(c*x)))/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx$$
$$= \frac{-16\left(\int \frac{acos(cx)x^2}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^7x^4+32\left(\int \frac{acos(cx)x^2}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^5x^2-16\left(\int \frac{acos(cx)x^2}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c}{}$$

input
```
int(x^2*(a+b*acos(c*x))/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 16*int((acos(c*x)*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)
*b*c**7*x**4 + 32*int((acos(c*x)*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x
**2 - 1),x)*b*c**5*x**2 - 16*int((acos(c*x)*x**2)/(c**6*x**6 - 3*c**4*x**4
 + 3*c**2*x**2 - 1),x)*b*c**3 + log(c**2*x - c)*a*c**4*x**4 - 2*log(c**2*x
 - c)*a*c**2*x**2 + log(c**2*x - c)*a - log(c**2*x + c)*a*c**4*x**4 + 2*lo
g(c**2*x + c)*a*c**2*x**2 - log(c**2*x + c)*a + 2*a*c**3*x**3 + 2*a*c*x)/(
16*c**3*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.51 $\int \frac{x(a+b \arccos(cx))}{(d-c^2dx^2)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = -\frac{bx}{12cd^3(1-c^2x^2)^{3/2}} - \frac{bx}{6cd^3\sqrt{1-c^2x^2}} + \frac{a+b\arccos(cx)}{4c^2d^3(1-c^2x^2)^2}$$

output
```
-1/12*b*x/c/d^3/(-c^2*x^2+1)^(3/2)-1/6*b*x/c/d^3/(-c^2*x^2+1)^(1/2)+1/4*(a
+b*arccos(c*x))/c^2/d^3/(-c^2*x^2+1)^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \frac{3a+bcx(3-2c^2x^2)\sqrt{1-c^2x^2}+3b\arccos(cx)}{12c^2d^3(-1+c^2x^2)^2}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
(3*a + b*c*x*(3 - 2*c^2*x^2)*Sqrt[1 - c^2*x^2] + 3*b*ArcCos[c*x])/(12*c^2*
d^3*(-1 + c^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5183, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^3}dx$$

$$\Big\downarrow \text{5183}$$

$$\frac{b\int \frac{1}{(1-c^2x^2)^{5/2}}dx}{4cd^3}+\frac{a+b\arccos(cx)}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{209}$$

$$\frac{b\left(\frac{2}{3}\int \frac{1}{(1-c^2x^2)^{3/2}}dx+\frac{x}{3(1-c^2x^2)^{3/2}}\right)}{4cd^3}+\frac{a+b\arccos(cx)}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{a+b\arccos(cx)}{4c^2d^3(1-c^2x^2)^2}+\frac{b\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)}{4cd^3}$$

input
```
Int[(x*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
(b*(x/(3*(1 - c^2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 - c^2*x^2])))/(4*c*d^3) +
(a + b*ArcCos[c*x])/(4*c^2*d^3*(1 - c^2*x^2)^2)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(73) = 146$.

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{\frac{a}{4d^3\left(c^2x^2-1\right)^2}-\frac{b\left(-\frac{\arccos(cx)}{4\left(c^2x^2-1\right)^2}-\frac{\sqrt{-(cx-1)^2-2cx+2}}{48(cx-1)^2}+\frac{\sqrt{-(cx-1)^2-2cx+2}}{12cx-12}+\frac{\sqrt{-(cx+1)^2+2cx+2}}{48(cx+1)^2}+\frac{\sqrt{-(cx+1)^2+2cx+2}}{12cx+12}\right)}{d^3}}{c^2}$
default	$\frac{\frac{a}{4d^3\left(c^2x^2-1\right)^2}-\frac{b\left(-\frac{\arccos(cx)}{4\left(c^2x^2-1\right)^2}-\frac{\sqrt{-(cx-1)^2-2cx+2}}{48(cx-1)^2}+\frac{\sqrt{-(cx-1)^2-2cx+2}}{12cx-12}+\frac{\sqrt{-(cx+1)^2+2cx+2}}{48(cx+1)^2}+\frac{\sqrt{-(cx+1)^2+2cx+2}}{12cx+12}\right)}{d^3}}{c^2}$
parts	$\frac{a}{4d^3c^2(c^2x^2-1)^2}-\frac{b\left(-\frac{\arccos(cx)}{4\left(c^2x^2-1\right)^2}-\frac{\sqrt{-(cx-1)^2-2cx+2}}{48(cx-1)^2}+\frac{\sqrt{-(cx-1)^2-2cx+2}}{12cx-12}+\frac{\sqrt{-(cx+1)^2+2cx+2}}{48(cx+1)^2}+\frac{\sqrt{-(cx+1)^2+2cx}}{12cx+12}\right.}{d^3c^2}$
orering	$\frac{(cx-1)(cx+1)\left(10c^4x^4-13c^2x^2-6\right)(a+b\arccos(cx))}{12c^2(-c^2d\,x^2+d)^3}+\frac{\left(2c^2x^2-3\right)(cx-1)^2(cx+1)^2\left(\frac{a+b\arccos(cx)}{\left(-c^2d\,x^2+d\right)^3}-\frac{xbc}{\sqrt{-c^2x^2+1}\,(-c}\right.}{12c^2}$

input
```
int(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/4*a/d^3/(c^2*x^2-1)^2-b/d^3*(-1/4/(c^2*x^2-1)^2*arccos(c*x)-1/48/
(c*x-1)^2*(-(c*x-1)^2-2*c*x+2)^(1/2)+1/12/(c*x-1)*(-(c*x-1)^2-2*c*x+2)^(1/
2)+1/48/(c*x+1)^2*(-(c*x+1)^2+2*c*x+2)^(1/2)+1/12/(c*x+1)*(-(c*x+1)^2+2*c*
x+2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = -\frac{3\,ac^4x^4-6\,ac^2x^2-3\,b\arccos{(cx)}+(2\,bc^3x^3-3\,bcx)\sqrt{-c^2x^2+1}}{12\,(c^6d^3x^4-2\,c^4d^3x^2+c^2d^3)}$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
-1/12*(3*a*c^4*x^4 - 6*a*c^2*x^2 - 3*b*arccos(c*x) + (2*b*c^3*x^3 - 3*b*c*
x)*sqrt(-c^2*x^2 + 1))/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3)
```

Sympy [F]

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = -\frac{\int \frac{ax}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int \frac{bx\,\mathrm{acos}\,(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

input
```
integrate(x*(a+b*acos(c*x))/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a*x/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(
b*x*acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Maxima [F]

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)x}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*(4*(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3)*integrate(1/4*e^(1/2*log(c
*x + 1) + 1/2*log(-c*x + 1))/(c^9*d^3*x^8 - 3*c^7*d^3*x^6 + 3*c^5*d^3*x^4
- c^3*d^3*x^2 + (c^7*d^3*x^6 - 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 - c*d^3)*e^(l
og(c*x + 1) + log(-c*x + 1))), x) - arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1),
c*x))*b/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3) + 1/4*a/(c^6*d^3*x^4 - 2*c
^4*d^3*x^2 + c^2*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(72) = 144$.

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.07

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \frac{bc^2x^4\arccos(cx)}{4\,(c^2x^2-1)^2d^3} + \frac{ac^2x^4}{4\,(c^2x^2-1)^2d^3} - \frac{bcx^3}{12\,(c^2x^2-1)\sqrt{-c^2x^2+1}d^3} - \frac{bx^2\arccos(cx)}{2\,(c^2x^2-1)d^3} - \frac{ax^2}{2\,(c^2x^2-1)d^3} + \frac{bx}{4\,\sqrt{-c^2x^2+1}cd^3} + \frac{b\arccos(cx)}{4\,c^2d^3} + \frac{a}{4\,c^2d^3}$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
1/4*b*c^2*x^4*arccos(c*x)/((c^2*x^2 - 1)^2*d^3) + 1/4*a*c^2*x^4/((c^2*x^2
- 1)^2*d^3) - 1/12*b*c*x^3/((c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*d^3) - 1/2*b*
x^2*arccos(c*x)/((c^2*x^2 - 1)*d^3) - 1/2*a*x^2/((c^2*x^2 - 1)*d^3) + 1/4*
b*x/(sqrt(-c^2*x^2 + 1)*c*d^3) + 1/4*b*arccos(c*x)/(c^2*d^3) + 1/4*a/(c^2*
d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \int \frac{x\,(a+b\,\mathrm{acos}(c\,x))}{(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((x*(a + b*acos(c*x)))/(d - c^2*d*x^2)^3,x)
```

output
```
int((x*(a + b*acos(c*x)))/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \frac{-4\left(\int \frac{acos(cx)x}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^6x^4+8\left(\int \frac{acos(cx)x}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^4x^2-4\left(\int \frac{acos(cx)x}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^2+}{4c^2d^3\left(c^4x^4-2c^2x^2+1\right)}$$

input
```
int(x*(a+b*acos(c*x))/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 4*int((acos(c*x)*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*c
**6*x**4 + 8*int((acos(c*x)*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1)
,x)*b*c**4*x**2 - 4*int((acos(c*x)*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x*
*2 - 1),x)*b*c**2 + a)/(4*c**2*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.52 $\int \frac{a+b\arccos(cx)}{\left(d-c^2dx^2\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 196

$$\int \frac{a+b\arccos(cx)}{\left(d-c^2dx^2\right)^3}\,dx = -\frac{b}{12cd^3\left(1-c^2x^2\right)^{3/2}}-\frac{3b}{8cd^3\sqrt{1-c^2x^2}}+\frac{x(a+b\arccos(cx))}{4d^3\left(1-c^2x^2\right)^2}+\frac{3x(a+b\arccos(cx))}{8d^3\left(1-c^2x^2\right)}-\frac{3i(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{4cd^3}+\frac{3ib\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{8cd^3}-\frac{3ib\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{8cd^3}$$

output
```
-1/12*b/c/d^3/(-c^2*x^2+1)^(3/2)-3/8*b/c/d^3/(-c^2*x^2+1)^(1/2)+1/4*x*(a+b
*arccos(c*x))/d^3/(-c^2*x^2+1)^2+3/8*x*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)-
3/4*I*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c/d^3+3/8*I*b*pol
ylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/d^3-3/8*I*b*polylog(2,I*(c*x+I*(-c
^2*x^2+1)^(1/2)))/c/d^3
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.64

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx$$
$$= \frac{\frac{4ax}{(-1+c^2x^2)^2}-\frac{6ax}{-1+c^2x^2}+\frac{b\left((2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)\right)}{3c(1+cx)^2}+\frac{3b\left(\sqrt{1-c^2x^2}-\arccos(cx)\right)}{c+c^2x}+\frac{3b\left(\sqrt{1-c^2x^2}+\arccos(cx)\right)}{c-c^2x}+\frac{b\left((2-cx\right.}{}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^3,x]
```

output
```
((4*a*x)/(-1 + c^2*x^2)^2 - (6*a*x)/(-1 + c^2*x^2) + (b*((2 + c*x)*Sqrt[1
- c^2*x^2] - 3*ArcCos[c*x]))/(3*c*(1 + c*x)^2) + (3*b*(Sqrt[1 - c^2*x^2] -
 ArcCos[c*x]))/(c + c^2*x) + (3*b*(Sqrt[1 - c^2*x^2] + ArcCos[c*x]))/(c -
c^2*x) + (b*((2 - c*x)*Sqrt[1 - c^2*x^2] + 3*ArcCos[c*x]))/(3*c*(-1 + c*x)
^2) - (3*a*Log[1 - c*x])/c + (3*a*Log[1 + c*x])/c - (((3*I)/2)*b*(ArcCos[c
*x]*(ArcCos[c*x] + (4*I)*Log[1 + E^(I*ArcCos[c*x])]) + 4*PolyLog[2, -E^(I*
ArcCos[c*x])]))/c + (((3*I)/2)*b*(ArcCos[c*x]*(ArcCos[c*x] + (4*I)*Log[1 -
 E^(I*ArcCos[c*x])]) + 4*PolyLog[2, E^(I*ArcCos[c*x])]))/c)/(16*d^3)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {5163, 27, 241, 5163, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx$$

$$\Big\downarrow \; 5163$$

$$\frac{3\int \frac{a+b\arccos(cx)}{d^2(1-c^2x^2)^2}dx}{4d}+\frac{bc\int \frac{x}{(1-c^2x^2)^{5/2}}dx}{4d^3}+\frac{x(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{3\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx}{4d^3}+\frac{bc\int \frac{x}{(1-c^2x^2)^{5/2}}dx}{4d^3}+\frac{x(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \; 241$$

$$\frac{3\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx}{4d^3}+\frac{x(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}+\frac{b}{12cd^3(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \; 5163$$

$$\frac{3\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{x}{(1-c^2x^2)^{3/2}}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}\right)}{4d^3}+\frac{x(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}+\frac{b}{12cd^3(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \; 241$$

$$\frac{3\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{4d^3}+\frac{x(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}+\frac{b}{12cd^3(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \; 5165$$

$$\frac{3\left(-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{4d^3}+\frac{x(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}+\frac{b}{12cd^3(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{4d^3}+\frac{x(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}+\frac{b}{12cd^3(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \; 4671$$

$$\frac{3\left(-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{2c}+\frac{x(a+b\arccos(cx}{2(1-c^2x^2)}\right.}{4d^3}$$
$$\frac{x(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}+\frac{b}{12cd^3(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{3\left(-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{2c}\right.}{4d^3}$$
$$\frac{x(a+b\arccos(cx))}{4d^3\left(1-c^2x^2\right)^2}+\frac{b}{12cd^3\left(1-c^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{3\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{4d^3}$$
$$\frac{x(a+b\arccos(cx))}{4d^3\left(1-c^2x^2\right)^2}+\frac{b}{12cd^3\left(1-c^2x^2\right)^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^3,x]
```

output
```
b/(12*c*d^3*(1 - c^2*x^2)^(3/2)) + (x*(a + b*ArcCos[c*x]))/(4*d^3*(1 - c^2
*x^2)^2) + (3*(b/(2*c*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcCos[c*x]))/(2*(1 -
 c^2*x^2)) - (-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*Poly
Log[2, -E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/(2*c)))/(4
*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.29

method	result
derivativedivides	$-\frac{a\left(-\frac{1}{16(cx-1)^2}+\frac{3}{16(cx-1)}+\frac{3\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}-\frac{3\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(\frac{9c^3x^3\arccos(cx)+9c^2x^2\sqrt{-c^2x^2+1}-15cx}{24c^4x^4-48c^2x^2+}\right.}{}$
default	$-\frac{a\left(-\frac{1}{16(cx-1)^2}+\frac{3}{16(cx-1)}+\frac{3\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}-\frac{3\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(\frac{9c^3x^3\arccos(cx)+9c^2x^2\sqrt{-c^2x^2+1}-15cx}{24c^4x^4-48c^2x^2+}\right.}{}$
parts	$-\frac{a\left(-\frac{1}{16c(cx-1)^2}+\frac{3}{16c(cx-1)}+\frac{3\ln(cx-1)}{16c}+\frac{1}{16c(cx+1)^2}+\frac{3}{16c(cx+1)}-\frac{3\ln(cx+1)}{16c}\right)}{d^3}-\frac{b\left(\frac{9c^3x^3\arccos(cx)+9c^2x^2\sqrt{-c^2x^2}}{24c^4x^4-}\right.}{}$

input
```
int((a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-a/d^3*(-1/16/(c*x-1)^2+3/16/(c*x-1)+3/16*ln(c*x-1)+1/16/(c*x+1)^2+3/
16/(c*x+1)-3/16*ln(c*x+1))-b/d^3*(1/24*(9*c^3*x^3*arccos(c*x)+9*c^2*x^2*(-
c^2*x^2+1)^(1/2)-15*c*x*arccos(c*x)-11*(-c^2*x^2+1)^(1/2))/(c^4*x^4-2*c^2*
x^2+1)+3/8*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-3/8*arccos(c*x)*ln(1
+c*x+I*(-c^2*x^2+1)^(1/2))-3/8*I*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+3/8*I
*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx=\int -\frac{b\arccos(cx)+a}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*arccos(c*x) + a)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2
 - d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acos(c*x))/(-c**2*d*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx = \int -\frac{b\arccos(cx)+a}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/16*a*(2*(3*c^2*x^3 - 5*x)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) - 3*log(c
*x + 1)/(c*d^3) + 3*log(c*x - 1)/(c*d^3)) - 1/16*((6*c^3*x^3 - 10*c*x - 3*
(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) + 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(-
c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 16*(c^5*d^3*x^4 - 2
*c^3*d^3*x^2 + c*d^3)*integrate(-1/16*(6*c^3*x^3 - 10*c*x - 3*(c^4*x^4 - 2
*c^2*x^2 + 1)*log(c*x + 1) + 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(-c*x + 1))*sq
rt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 -
d^3), x))*b/(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^3,x)
```

output
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^3}\,dx = \frac{-16\left(\int \frac{acos(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^5x^4+32\left(\int \frac{acos(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b\,c^3x^2-16\left(\int \frac{acos(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)bc$$

input
```
int((a+b*acos(c*x))/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 16*int(acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*c**5
*x**4 + 32*int(acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*
c**3*x**2 - 16*int(acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x
)*b*c - 3*log(c**2*x - c)*a*c**4*x**4 + 6*log(c**2*x - c)*a*c**2*x**2 - 3*
log(c**2*x - c)*a + 3*log(c**2*x + c)*a*c**4*x**4 - 6*log(c**2*x + c)*a*c*
*2*x**2 + 3*log(c**2*x + c)*a - 6*a*c**3*x**3 + 10*a*c*x)/(16*c*d**3*(c**4
*x**4 - 2*c**2*x**2 + 1))
```

3.53 $\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 173

$$\begin{aligned}\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^3}\,dx = &-\frac{bcx}{12d^3(1-c^2x^2)^{3/2}}-\frac{2bcx}{3d^3\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{4d^3(1-c^2x^2)^2}\\&+\frac{a+b\arccos(cx)}{2d^3(1-c^2x^2)}-\frac{2(a+b\arccos(cx))\text{arctanh}\left(e^{2i\arccos(cx)}\right)}{d^3}\\&+\frac{ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{2d^3}-\frac{ib\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{2d^3}\end{aligned}$$

output
```
-1/12*b*c*x/d^3/(-c^2*x^2+1)^(3/2)-2/3*b*c*x/d^3/(-c^2*x^2+1)^(1/2)+1/4*(a
+b*arccos(c*x))/d^3/(-c^2*x^2+1)^2+1/2*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)-
2*(a+b*arccos(c*x))*arctanh((c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^3+1/2*I*b*poly
log(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^3-1/2*I*b*polylog(2,(c*x+I*(-c^2*x^
2+1)^(1/2))^2)/d^3
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.98

$$\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^3}\,dx = \frac{\frac{a}{(-1+c^2x^2)^2}-\frac{2a}{-1+c^2x^2}-\frac{b\left((2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)\right)}{12(1+cx)^2}-\frac{5b\left(\sqrt{1-c^2x^2}-\arccos(cx)\right)}{4+4cx}-2ib\arccos(cx)^2+\frac{5b\left(\sqrt{1-c^2x^2}+a\right.}{4-4c}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x*(d - c^2*d*x^2)^3),x]
```

output
```
(a/(-1 + c^2*x^2)^2 - (2*a)/(-1 + c^2*x^2) - (b*((2 + c*x)*Sqrt[1 - c^2*x^
2] - 3*ArcCos[c*x]))/(12*(1 + c*x)^2) - (5*b*(Sqrt[1 - c^2*x^2] - ArcCos[c
*x]))/(4 + 4*c*x) - (2*I)*b*ArcCos[c*x]^2 + (5*b*(Sqrt[1 - c^2*x^2] + ArcC
os[c*x]))/(4 - 4*c*x) + (b*((2 - c*x)*Sqrt[1 - c^2*x^2] + 3*ArcCos[c*x]))/
(12*(-1 + c*x)^2) + 4*b*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] + 4*a*L
og[x] - 2*a*Log[1 - c^2*x^2] + I*b*(ArcCos[c*x]*(ArcCos[c*x] + (4*I)*Log[1
 + E^(I*ArcCos[c*x])]) + 4*PolyLog[2, -E^(I*ArcCos[c*x])]) + I*b*(ArcCos[c
*x]*(ArcCos[c*x] + (4*I)*Log[1 - E^(I*ArcCos[c*x])]) + 4*PolyLog[2, E^(I*A
rcCos[c*x])]) - (2*I)*b*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/(4*d^3)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {5209, 27, 209, 208, 5209, 208, 5185, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^3}\,dx$$

$$\Big\downarrow \text{5209}$$

$$\frac{\int \frac{a+b\arccos(cx)}{d^2x(1-c^2x^2)^2}dx}{d}+\frac{bc\int \frac{1}{(1-c^2x^2)^{5/2}}dx}{4d^3}+\frac{a+b\arccos(cx)}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^3}+\frac{bc\int \frac{1}{(1-c^2x^2)^{5/2}}dx}{4d^3}+\frac{a+b\arccos(cx)}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{209}$$

$$\frac{\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^3}+\frac{bc\left(\frac{2}{3}\int \frac{1}{(1-c^2x^2)^{3/2}}dx+\frac{x}{3(1-c^2x^2)^{3/2}}\right)}{4d^3}+\frac{a+b\arccos(cx)}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^3}+\frac{a+b\arccos(cx)}{4d^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \text{5209}$$

$$\frac{\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\int \frac{1}{(1-c^2x^2)^{3/2}}dx+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}}{d^3}+\frac{a+b\arccos(cx)}{4d^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \text{208}$$

$$\frac{\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}}{d^3}+\frac{a+b\arccos(cx)}{4d^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \text{5185}$$

$$\frac{-\int \frac{a+b\arccos(cx)}{cx\sqrt{1-c^2x^2}}d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}}{d^3}+\frac{a+b\arccos(cx)}{4d^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \text{4919}$$

$$\frac{-2\int (a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}}{d^3}+\frac{a+b\arccos(cx)}{4d^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)}{4d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-2\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}}{d^3}+\frac{a+b\arccos(cx)}{4d^3(1-c^2x^2)^2}+\frac{bc\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)}{4d^3}$$

$\Big\downarrow$ 4671

$$\frac{-2\left(-\frac{1}{2}b\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+\frac{1}{2}b\int\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\left(\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)(a\right.\right.}{d^3}$$
$$\frac{a+b\arccos(cx)}{4d^3(1-c^2x^2)^2}+\frac{bc\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)}{4d^3}$$

$\Big\downarrow$ 2715

$$\frac{-2\left(\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right.}{d^3}$$
$$\frac{a+b\arccos(cx)}{4d^3(1-c^2x^2)^2}+\frac{bc\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)}{4d^3}$$

$\Big\downarrow$ 2838

$$\frac{-2\left(-\left(\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}ib\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)-\frac{1}{4}ib\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)}{d^3}$$
$$\frac{a+b\arccos(cx)}{4d^3(1-c^2x^2)^2}+\frac{bc\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)}{4d^3}$$

input
```
Int[(a + b*ArcCos[c*x])/(x*(d - c^2*d*x^2)^3),x]
```

output
```
(b*c*(x/(3*(1 - c^2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 - c^2*x^2])))/(4*d^3) +
(a + b*ArcCos[c*x])/(4*d^3*(1 - c^2*x^2)^2) + ((b*c*x)/(2*Sqrt[1 - c^2*x^2
]) + (a + b*ArcCos[c*x])/(2*(1 - c^2*x^2)) - 2*(-((a + b*ArcCos[c*x])*ArcT
anh[E^((2*I)*ArcCos[c*x])]) + (I/4)*b*PolyLog[2, -E^((2*I)*ArcCos[c*x])] -
 (I/4)*b*PolyLog[2, E^((2*I)*ArcCos[c*x])]))/d^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.90

method	result
parts	$-\frac{a\left(-\ln(x)-\frac{1}{16(cx-1)^2}+\frac{5}{16(cx-1)}+\frac{\ln(cx-1)}{2}-\frac{1}{16(cx+1)^2}-\frac{5}{16(cx+1)}+\frac{\ln(cx+1)}{2}\right)}{d^3}-\frac{b\left(\frac{8ic^4x^4+8c^3x^3\sqrt{-c^2x^2+1}+6c^2}{}\right.}{}$
derivativedivides	$-\frac{a\left(-\frac{1}{16(cx-1)^2}+\frac{5}{16(cx-1)}+\frac{\ln(cx-1)}{2}-\ln(cx)-\frac{1}{16(cx+1)^2}-\frac{5}{16(cx+1)}+\frac{\ln(cx+1)}{2}\right)}{d^3}-\frac{b\left(\frac{8ic^4x^4+8c^3x^3\sqrt{-c^2x^2+1}+6c^2}{}\right.}{}$
default	$-\frac{a\left(-\frac{1}{16(cx-1)^2}+\frac{5}{16(cx-1)}+\frac{\ln(cx-1)}{2}-\ln(cx)-\frac{1}{16(cx+1)^2}-\frac{5}{16(cx+1)}+\frac{\ln(cx+1)}{2}\right)}{d^3}-\frac{b\left(\frac{8ic^4x^4+8c^3x^3\sqrt{-c^2x^2+1}+6c^2}{}\right.}{}$

input
```
int((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-a/d^3*(-ln(x)-1/16/(c*x-1)^2+5/16/(c*x-1)+1/2*ln(c*x-1)-1/16/(c*x+1)^2-5/
16/(c*x+1)+1/2*ln(c*x+1))-b/d^3*(1/12*(8*I*c^4*x^4+8*c^3*x^3*(-c^2*x^2+1)^
(1/2)+6*c^2*x^2*arccos(c*x)-16*I*c^2*x^2-9*c*x*(-c^2*x^2+1)^(1/2)-9*arccos
(c*x)+8*I)/(c^4*x^4-2*c^2*x^2+1)+arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2)
)-I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2
+1)^(1/2))^2)+1/2*I*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+arccos(c*x)*l
n(1-c*x-I*(-c^2*x^2+1)^(1/2))-I*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^3}\,dx = \int -\frac{b\arccos\left(cx\right)+a}{\left(c^2dx^2-d\right)^3x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*arccos(c*x) + a)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3
 - d^3*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acos(c*x))/x/(-c**2*d*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^3}\,dx = \int -\frac{b\arccos\left(cx\right)+a}{\left(c^2dx^2-d\right)^3x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*a*((2*c^2*x^2 - 3)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) + 2*log(c*x +
1)/d^3 + 2*log(c*x - 1)/d^3 - 4*log(x)/d^3) - b*integrate(arctan2(sqrt(c*x
 + 1)*sqrt(-c*x + 1), c*x)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 -
d^3*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^3}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x\,\left(d-c^2\,d\,x^2\right)^3}\,dx$$

input
```
int((a + b*acos(c*x))/(x*(d - c^2*d*x^2)^3),x)
```

output
```
int((a + b*acos(c*x))/(x*(d - c^2*d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^3}\,dx$$
$$= \frac{-4\left(\int \frac{acos(cx)}{c^6x^7-3c^4x^5+3c^2x^3-x}dx\right)b\,c^4x^4+8\left(\int \frac{acos(cx)}{c^6x^7-3c^4x^5+3c^2x^3-x}dx\right)b\,c^2x^2-4\left(\int \frac{acos(cx)}{c^6x^7-3c^4x^5+3c^2x^3-x}dx\right)b-2}{}$$

input
```
int((a+b*acos(c*x))/x/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 4*int(acos(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*b*c**4*
x**4 + 8*int(acos(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*b*c*
*2*x**2 - 4*int(acos(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*b
 - 2*log(c**2*x - c)*a*c**4*x**4 + 4*log(c**2*x - c)*a*c**2*x**2 - 2*log(c
**2*x - c)*a - 2*log(c**2*x + c)*a*c**4*x**4 + 4*log(c**2*x + c)*a*c**2*x*
*2 - 2*log(c**2*x + c)*a + 4*log(x)*a*c**4*x**4 - 8*log(x)*a*c**2*x**2 + 4
*log(x)*a - a*c**4*x**4 + 2*a)/(4*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.54 $\int \frac{a+b\arccos(cx)}{x^2\left(d-c^2dx^2\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 242

$$\int \frac{a+b\arccos(cx)}{x^2\left(d-c^2dx^2\right)^3}\,dx = -\frac{bc}{12d^3\left(1-c^2x^2\right)^{3/2}}-\frac{7bc}{8d^3\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{5c^2x(a+b\arccos(cx))}{4d^3\left(1-c^2x^2\right)^2}+\frac{15c^2x(a+b\arccos(cx))}{8d^3\left(1-c^2x^2\right)}-\frac{15ic(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{4d^3}-\frac{b\text{carctanh}\left(\sqrt{1-c^2x^2}\right)}{d^3}+\frac{15ibc\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{8d^3}-\frac{15ibc\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{8d^3}$$

output
```
-1/12*b*c/d^3/(-c^2*x^2+1)^(3/2)-7/8*b*c/d^3/(-c^2*x^2+1)^(1/2)-(a+b*arcco
s(c*x))/d^3/x/(-c^2*x^2+1)^2+5/4*c^2*x*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)^
2+15/8*c^2*x*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)-15/4*I*c*(a+b*arccos(c*x))
*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/d^3-b*c*arctanh((-c^2*x^2+1)^(1/2))/d^3+
15/8*I*b*c*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^3-15/8*I*b*c*polylog
(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^3
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.43

$$\int \frac{a+b\arccos(cx)}{x^2\,(d-c^2dx^2)^3}\,dx =$$

$$-\frac{\frac{96a}{x}-\frac{24ac^2x}{(-1+c^2x^2)^2}+\frac{84ac^2x}{-1+c^2x^2}+\frac{2bc\left((-2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)\right)}{(-1+cx)^2}-\frac{2bc\left((2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)\right)}{(1+cx)^2}-\frac{42bc\left(\sqrt{1-c^2x^2}\right.}{1+}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^2*(d - c^2*d*x^2)^3),x]
```

output
```
-1/96*((96*a)/x - (24*a*c^2*x)/(-1 + c^2*x^2)^2 + (84*a*c^2*x)/(-1 + c^2*x
^2) + (2*b*c*((-2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x]))/(-1 + c*x)^2
- (2*b*c*((2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x]))/(1 + c*x)^2 - (42*
b*c*(Sqrt[1 - c^2*x^2] - ArcCos[c*x]))/(1 + c*x) + (96*b*ArcCos[c*x])/x +
(42*b*c*(Sqrt[1 - c^2*x^2] + ArcCos[c*x]))/(-1 + c*x) + 96*b*c*Log[x] + 90
*a*c*Log[1 - c*x] - 90*a*c*Log[1 + c*x] - 96*b*c*Log[1 + Sqrt[1 - c^2*x^2]
] + (45*I)*b*c*(ArcCos[c*x]*(ArcCos[c*x] + (4*I)*Log[1 + E^(I*ArcCos[c*x])
]) + 4*PolyLog[2, -E^(I*ArcCos[c*x])]) - (45*I)*b*c*(ArcCos[c*x]*(ArcCos[c
*x] + (4*I)*Log[1 - E^(I*ArcCos[c*x])]) + 4*PolyLog[2, E^(I*ArcCos[c*x])])
)/d^3
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {5205, 27, 243, 61, 61, 73, 221, 5163, 241, 5163, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^2\,(d-c^2dx^2)^3}\,dx$$

$$\Big\downarrow \; 5205$$

$$5c^2\int \frac{a+b\arccos(cx)}{d^3\left(1-c^2x^2\right)^3}dx-\frac{bc\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}}dx}{d^3}-\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{5c^2\int \frac{a+b\arccos(cx)}{\left(1-c^2x^2\right)^3}dx}{d^3}-\frac{bc\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}}dx}{d^3}-\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \textbf{243}$$

$$\frac{5c^2\int \frac{a+b\arccos(cx)}{\left(1-c^2x^2\right)^3}dx}{d^3}-\frac{bc\int \frac{1}{x^2\left(1-c^2x^2\right)^{5/2}}dx^2}{2d^3}-\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \textbf{61}$$

$$\frac{5c^2\int \frac{a+b\arccos(cx)}{\left(1-c^2x^2\right)^3}dx}{d^3}-\frac{bc\left(\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}}dx^2+\frac{2}{3\left(1-c^2x^2\right)^{3/2}}\right)}{2d^3}-\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \textbf{61}$$

$$\frac{5c^2\int \frac{a+b\arccos(cx)}{\left(1-c^2x^2\right)^3}dx}{d^3}-\frac{bc\left(\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3\left(1-c^2x^2\right)^{3/2}}\right)}{2d^3}-\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{5c^2\int \frac{a+b\arccos(cx)}{\left(1-c^2x^2\right)^3}dx}{d^3}-\frac{bc\left(-\frac{2\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2}+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3\left(1-c^2x^2\right)^{3/2}}\right)}{2d^3}-\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{5c^2\int \frac{a+b\arccos(cx)}{\left(1-c^2x^2\right)^3}dx}{d^3}-\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}-\frac{bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3\left(1-c^2x^2\right)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \textbf{5163}$$

$$\frac{5c^2\left(\frac{3}{4}\int \frac{a+b\arccos(cx)}{\left(1-c^2x^2\right)^2}dx+\frac{1}{4}bc\int \frac{x}{\left(1-c^2x^2\right)^{5/2}}dx+\frac{x(a+b\arccos(cx))}{4\left(1-c^2x^2\right)^2}\right)}{d^3}-\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}-\frac{bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3\left(1-c^2x^2\right)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \textbf{241}$$

$$\frac{5c^2\left(\frac{3}{4}\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx+\frac{x(a+b\arccos(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(1-c^2x^2)^{3/2}}\right)}{d^3}-\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}-\frac{bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 5163

$$\frac{5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{x}{(1-c^2x^2)^{3/2}}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}\right)+\frac{x(a+b\arccos(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(1-c^2x^2)^{3/2}}\right)}{d^3}-\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}-\frac{bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 241

$$\frac{5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)+\frac{x(a+b\arccos(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(1-c^2x^2)^{3/2}}\right)}{d^3}-\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}-\frac{bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 5165

$$\frac{5c^2\left(\frac{3}{4}\left(-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)+\frac{x(a+b\arccos(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(1-c^2x^2)^{3/2}}\right)}{d^3}-\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}-\frac{bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 3042

$$\frac{5c^2\left(\frac{3}{4}\left(-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)+\frac{x(a+b\arccos(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(1-c^2x^2)^{3/2}}\right)}{d^3}-\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}-\frac{bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 4671

$$\frac{5c^2\left(\frac{3}{4}\left(-\frac{-b\int\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{2c}+\frac{x(a+b\arccos(cx))}{2(1-\ldots}\right.\right.}{d^3}$$

$$\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}-\frac{bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \text{2715}$$

$$\frac{5c^2\left(\frac{3}{4}\left(-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a\ldots}{2c}\right.\right.}{d^3}$$

$$\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}-\frac{bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$$\Big\downarrow \text{2838}$$

$$\frac{5c^2\left(\frac{3}{4}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{\ldots}{2c\ldots}\right.\right.}{d^3}$$

$$\frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^2}-\frac{bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^2*(d - c^2*d*x^2)^3),x]
```

output
```
-((a + b*ArcCos[c*x])/(d^3*x*(1 - c^2*x^2)^2)) - (b*c*(2/(3*(1 - c^2*x^2)^
(3/2)) + 2/Sqrt[1 - c^2*x^2] - 2*ArcTanh[Sqrt[1 - c^2*x^2]]))/(2*d^3) + (5
*c^2*(b/(12*c*(1 - c^2*x^2)^(3/2)) + (x*(a + b*ArcCos[c*x]))/(4*(1 - c^2*x
^2)^2) + (3*(b/(2*c*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcCos[c*x]))/(2*(1 - c
^2*x^2)) - (-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLo
g[2, -E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/(2*c)))/4))/
d^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.11

method	result
derivativedivides	$c\left(-\frac{a\left(-\frac{1}{16(cx-1)^2}+\frac{7}{16(cx-1)}+\frac{15\ln(cx-1)}{16}+\frac{1}{cx}+\frac{1}{16(cx+1)^2}+\frac{7}{16(cx+1)}-\frac{15\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(\frac{45c^4x^4\arccos(cx)+21c^3x}{}\right.}{}\right.$
default	$c\left(-\frac{a\left(-\frac{1}{16(cx-1)^2}+\frac{7}{16(cx-1)}+\frac{15\ln(cx-1)}{16}+\frac{1}{cx}+\frac{1}{16(cx+1)^2}+\frac{7}{16(cx+1)}-\frac{15\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(\frac{45c^4x^4\arccos(cx)+21c^3x}{}\right.}{}\right.$
parts	$-\frac{a\left(\frac{1}{x}-\frac{c}{16(cx-1)^2}+\frac{7c}{16(cx-1)}+\frac{15c\ln(cx-1)}{16}+\frac{c}{16(cx+1)^2}+\frac{7c}{16(cx+1)}-\frac{15c\ln(cx+1)}{16}\right)}{d^3}-\frac{bc\left(\frac{45c^4x^4\arccos(cx)+21c^3x^3\sqrt{}}{}\right.}{}$

input
```
int((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c*(-a/d^3*(-1/16/(c*x-1)^2+7/16/(c*x-1)+15/16*ln(c*x-1)+1/c/x+1/16/(c*x+1)
^2+7/16/(c*x+1)-15/16*ln(c*x+1))-b/d^3*(1/24*(45*c^4*x^4*arccos(c*x)+21*c^
3*x^3*(-c^2*x^2+1)^(1/2)-75*c^2*x^2*arccos(c*x)-23*c*x*(-c^2*x^2+1)^(1/2)+
24*arccos(c*x))/c/x/(c^4*x^4-2*c^2*x^2+1)+15/8*I*dilog(1+c*x+I*(-c^2*x^2+1
)^(1/2))+2*I*arctan(c*x+I*(-c^2*x^2+1)^(1/2))+15/8*I*dilog(c*x+I*(-c^2*x^2
+1)^(1/2))-15/8*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^2\left(d-c^2dx^2\right)^3}\,dx=\int -\frac{b\arccos\left(cx\right)+a}{\left(c^2dx^2-d\right)^3x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*arccos(c*x) + a)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4
 - d^3*x^2), x)
```

Sympy [F]

$$\int \frac{a + b\arccos(cx)}{x^2 (d - c^2dx^2)^3} \, dx = -\frac{\int \frac{a}{c^6x^8 - 3c^4x^6 + 3c^2x^4 - x^2} \, dx + \int \frac{b \operatorname{acos}(cx)}{c^6x^8 - 3c^4x^6 + 3c^2x^4 - x^2} \, dx}{d^3}$$

input
```
integrate((a+b*acos(c*x))/x**2/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2), x) + Integral
(b*acos(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2), x))/d**3
```

Maxima [F]

$$\int \frac{a + b\arccos(cx)}{x^2 (d - c^2dx^2)^3} \, dx = \int -\frac{b\arccos(cx) + a}{(c^2dx^2 - d)^3 x^2} \, dx$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/16*a*(2*(15*c^4*x^4 - 25*c^2*x^2 + 8)/(c^4*d^3*x^5 - 2*c^2*d^3*x^3 + d^
3*x) - 15*c*log(c*x + 1)/d^3 + 15*c*log(c*x - 1)/d^3) - 1/16*((30*c^4*x^4
- 50*c^2*x^2 - 15*(c^5*x^5 - 2*c^3*x^3 + c*x)*log(c*x + 1) + 15*(c^5*x^5 -
 2*c^3*x^3 + c*x)*log(-c*x + 1) + 16)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1)
, c*x) + 16*(c^4*d^3*x^5 - 2*c^2*d^3*x^3 + d^3*x)*integrate(-1/16*(30*c^5*
x^4 - 50*c^3*x^2 - 15*(c^6*x^5 - 2*c^4*x^3 + c^2*x)*log(c*x + 1) + 15*(c^6
*x^5 - 2*c^4*x^3 + c^2*x)*log(-c*x + 1) + 16*c)*sqrt(c*x + 1)*sqrt(-c*x +
1)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x))*b/(c^4*d^3*x
^5 - 2*c^2*d^3*x^3 + d^3*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^2\left(d-c^2dx^2\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^2\left(d-c^2dx^2\right)^3}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x^2\,\left(d-c^2\,d\,x^2\right)^3}\,dx$$

input
```
int((a + b*acos(c*x))/(x^2*(d - c^2*d*x^2)^3),x)
```

output
```
int((a + b*acos(c*x))/(x^2*(d - c^2*d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^2\left(d-c^2dx^2\right)^3}\,dx$$
$$= \frac{-16\left(\int \frac{acos(cx)}{c^6x^8-3c^4x^6+3c^2x^4-x^2}dx\right)bc^4x^5+32\left(\int \frac{acos(cx)}{c^6x^8-3c^4x^6+3c^2x^4-x^2}dx\right)bc^2x^3-16\left(\int \frac{acos(cx)}{c^6x^8-3c^4x^6+3c^2x^4-x^2}dx\right)}{}$$

input
```
int((a+b*acos(c*x))/x^2/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 16*int(acos(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2),x)*b*c
**4*x**5 + 32*int(acos(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2)
,x)*b*c**2*x**3 - 16*int(acos(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4
- x**2),x)*b*x - 15*log(c**2*x - c)*a*c**5*x**5 + 30*log(c**2*x - c)*a*c**
3*x**3 - 15*log(c**2*x - c)*a*c*x + 15*log(c**2*x + c)*a*c**5*x**5 - 30*lo
g(c**2*x + c)*a*c**3*x**3 + 15*log(c**2*x + c)*a*c*x - 30*a*c**4*x**4 + 50
*a*c**2*x**2 - 16*a)/(16*d**3*x*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.55 $\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 248

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^3}\,dx = -\frac{bc}{2d^3x(1-c^2x^2)^{3/2}}+\frac{5bc^3x}{12d^3(1-c^2x^2)^{3/2}} - \frac{2bc^3x}{3d^3\sqrt{1-c^2x^2}}+\frac{3c^2(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2} - \frac{a+b\arccos(cx)}{2d^3x^2(1-c^2x^2)^2}+\frac{3c^2(a+b\arccos(cx))}{2d^3(1-c^2x^2)} - \frac{6c^2(a+b\arccos(cx))\text{arctanh}\left(e^{2i\arccos(cx)}\right)}{d^3} + \frac{3ibc^2\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{2d^3} - \frac{3ibc^2\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{2d^3}$$

output

```
-1/2*b*c/d^3/x/(-c^2*x^2+1)^(3/2)+5/12*b*c^3*x/d^3/(-c^2*x^2+1)^(3/2)-2/3*
b*c^3*x/d^3/(-c^2*x^2+1)^(1/2)+3/4*c^2*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)^
2-1/2*(a+b*arccos(c*x))/d^3/x^2/(-c^2*x^2+1)^2+3/2*c^2*(a+b*arccos(c*x))/d
^3/(-c^2*x^2+1)-6*c^2*(a+b*arccos(c*x))*arctanh((c*x+I*(-c^2*x^2+1)^(1/2))
^2)/d^3+3/2*I*b*c^2*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^3-3/2*I*b*c
^2*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^3
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.65

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^3}\,dx$$

$$= \frac{-\frac{2a}{x^2}+\frac{ac^2}{(-1+c^2x^2)^2}-\frac{4ac^2}{-1+c^2x^2}-\frac{bc^2\left((-2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)\right)}{12(-1+cx)^2}-\frac{bc^2\left((2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)\right)}{12(1+cx)^2}-\frac{9bc^2\left(\sqrt{1-c^2x^2}-\right.}{4+4c}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^3*(d - c^2*d*x^2)^3),x]
```

output
```
((-2*a)/x^2 + (a*c^2)/(-1 + c^2*x^2)^2 - (4*a*c^2)/(-1 + c^2*x^2) - (b*c^2
*((-2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x]))/(12*(-1 + c*x)^2) - (b*c^
2*((2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x]))/(12*(1 + c*x)^2) - (9*b*c
^2*(Sqrt[1 - c^2*x^2] - ArcCos[c*x]))/(4 + 4*c*x) + (2*b*(c*x*Sqrt[1 - c^2
*x^2] - ArcCos[c*x]))/x^2 + (9*b*c^2*(Sqrt[1 - c^2*x^2] + ArcCos[c*x]))/(4
 - 4*c*x) + 12*a*c^2*Log[x] - 6*a*c^2*Log[1 - c^2*x^2] + (3*I)*b*c^2*(ArcC
os[c*x]*(ArcCos[c*x] + (4*I)*Log[1 + E^(I*ArcCos[c*x])]) + 4*PolyLog[2, -E
^(I*ArcCos[c*x])]) + (3*I)*b*c^2*(ArcCos[c*x]*(ArcCos[c*x] + (4*I)*Log[1 -
 E^(I*ArcCos[c*x])]) + 4*PolyLog[2, E^(I*ArcCos[c*x])]) - (6*I)*b*c^2*(Arc
Cos[c*x]*(ArcCos[c*x] + (2*I)*Log[1 + E^((2*I)*ArcCos[c*x])]) + PolyLog[2,
 -E^((2*I)*ArcCos[c*x])]))/(4*d^3)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.17, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {5205, 27, 245, 209, 208, 5209, 209, 208, 5209, 208, 5185, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^3}\,dx$$

$$\Big\downarrow \; 5205$$

$$3c^2\int \frac{a+b\arccos(cx)}{d^3x\left(1-c^2x^2\right)^3}dx-\frac{bc\int \frac{1}{x^2(1-c^2x^2)^{5/2}}dx}{2d^3}-\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{3c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^3}dx}{d^3}-\frac{bc\int \frac{1}{x^2(1-c^2x^2)^{5/2}}dx}{2d^3}-\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}$$

$\Big\downarrow$ 245

$$\frac{3c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^3}dx}{d^3}-\frac{bc\left(4c^2\int \frac{1}{(1-c^2x^2)^{5/2}}dx-\frac{1}{x(1-c^2x^2)^{3/2}}\right)}{2d^3}-\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}$$

$\Big\downarrow$ 209

$$\frac{3c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^3}dx}{d^3}-\frac{bc\left(4c^2\left(\frac{2}{3}\int \frac{1}{(1-c^2x^2)^{3/2}}dx+\frac{x}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x(1-c^2x^2)^{3/2}}\right)}{2d^3}-\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}$$

$\Big\downarrow$ 208

$$\frac{3c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^3}dx}{d^3}-\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}-\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\Big\downarrow$ 5209

$$\frac{3c^2\left(\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx+\frac{1}{4}bc\int \frac{1}{(1-c^2x^2)^{5/2}}dx+\frac{a+b\arccos(cx)}{4(1-c^2x^2)^2}\right)}{d^3}-\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}-\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\Big\downarrow$ 209

$$\frac{3c^2\left(\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx+\frac{1}{4}bc\left(\frac{2}{3}\int \frac{1}{(1-c^2x^2)^{3/2}}dx+\frac{x}{3(1-c^2x^2)^{3/2}}\right)+\frac{a+b\arccos(cx)}{4(1-c^2x^2)^2}\right)}{d^3}-\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}-\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\Big\downarrow$ 208

$$\frac{3c^2\left(\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx+\frac{a+b\arccos(cx)}{4(1-c^2x^2)^2}+\frac{1}{4}bc\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)\right)}{d^3}-\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}-\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 5209

$$\frac{3c^2\left(\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\int \frac{1}{(1-c^2x^2)^{3/2}}dx+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{a+b\arccos(cx)}{4(1-c^2x^2)^2}+\frac{1}{4}bc\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)\right)}{d^3}-\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}-\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 208

$$\frac{3c^2\left(\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{a+b\arccos(cx)}{4(1-c^2x^2)^2}+\frac{bcx}{2\sqrt{1-c^2x^2}}+\frac{1}{4}bc\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)\right)}{d^3}-\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}-\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 5185

$$\frac{3c^2\left(-\int \frac{a+b\arccos(cx)}{cx\sqrt{1-c^2x^2}}d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{a+b\arccos(cx)}{4(1-c^2x^2)^2}+\frac{bcx}{2\sqrt{1-c^2x^2}}+\frac{1}{4}bc\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)\right)}{d^3}-\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}-\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 4919

$$\frac{3c^2\left(-2\int (a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{a+b\arccos(cx)}{4(1-c^2x^2)^2}+\frac{bcx}{2\sqrt{1-c^2x^2}}+\frac{1}{4}bc\left(\frac{2x}{3\sqrt{1-c}}\right.\right.}{d^3}$$

$$\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}-\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\downarrow$ 3042

$$\frac{3c^2\left(-2\int (a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{a+b\arccos(cx)}{4(1-c^2x^2)^2}+\frac{bcx}{2\sqrt{1-c^2x^2}}+\frac{1}{4}bc\left(\frac{2x}{3\sqrt{1-c^2x^2}}\right.\right.}{d^3}$$

$$\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}-\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\Big\downarrow$ 4671

$$\frac{3c^2\left(-2\left(-\frac{1}{2}b\int \log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+\frac{1}{2}b\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\left(\text{arctanh}\left(e^{2i\arccos(c\right.\right.\right.\right.}{d^3}$$

$$\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}-\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\Big\downarrow$ 2715

$$\frac{3c^2\left(-2\left(\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos}\right.\right.}{}$$

$$\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}-\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

$\Big\downarrow$ 2838

$$\frac{3c^2\left(-2\left(-\left(\text{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,e^{2i\arccos(}\right.\right.\right.}{d^3}$$

$$\frac{a+b\arccos(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}-\frac{bc\left(4c^2\left(\frac{2x}{3\sqrt{1-c^2x^2}}+\frac{x}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x(1-c^2x^2)^{3/2}}\right)}{2d^3}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^3*(d - c^2*d*x^2)^3),x]
```

output
```
-1/2*(b*c*(-(1/(x*(1 - c^2*x^2)^(3/2))) + 4*c^2*(x/(3*(1 - c^2*x^2)^(3/2))
 + (2*x)/(3*Sqrt[1 - c^2*x^2]))))/d^3 - (a + b*ArcCos[c*x])/(2*d^3*x^2*(1
- c^2*x^2)^2) + (3*c^2*((b*c*x)/(2*Sqrt[1 - c^2*x^2]) + (b*c*(x/(3*(1 - c^
2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 - c^2*x^2])))/4 + (a + b*ArcCos[c*x])/(4*(
1 - c^2*x^2)^2) + (a + b*ArcCos[c*x])/(2*(1 - c^2*x^2)) - 2*(-((a + b*ArcC
os[c*x])*ArcTanh[E^((2*I)*ArcCos[c*x])]) + (I/4)*b*PolyLog[2, -E^((2*I)*Ar
cCos[c*x])] - (I/4)*b*PolyLog[2, E^((2*I)*ArcCos[c*x])])))/d^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.57

method	result
derivativedivides	$c^2 \left(-\frac{a\left(-\frac{1}{16(cx-1)^2}+\frac{9}{16(cx-1)}+\frac{3\ln(cx-1)}{2}+\frac{1}{2c^2x^2}-3\ln(cx)-\frac{1}{16(cx+1)^2}-\frac{9}{16(cx+1)}+\frac{3\ln(cx+1)}{2}\right)}{d^3} - \frac{b\left(\frac{8ix^6c^6+8c}{}\right.}{}\right.$
default	$c^2 \left(-\frac{a\left(-\frac{1}{16(cx-1)^2}+\frac{9}{16(cx-1)}+\frac{3\ln(cx-1)}{2}+\frac{1}{2c^2x^2}-3\ln(cx)-\frac{1}{16(cx+1)^2}-\frac{9}{16(cx+1)}+\frac{3\ln(cx+1)}{2}\right)}{d^3} - \frac{b\left(\frac{8ix^6c^6+8c}{}\right.}{}\right.$
parts	$-\frac{a\left(\frac{1}{2x^2}-3c^2\ln(x)-\frac{c^2}{16(cx-1)^2}+\frac{9c^2}{16(cx-1)}+\frac{3c^2\ln(cx-1)}{2}-\frac{c^2}{16(cx+1)^2}-\frac{9c^2}{16(cx+1)}+\frac{3c^2\ln(cx+1)}{2}\right)}{d^3} - \frac{b\,c^2\left(\frac{8ix^6c^6+8c^5}{}\right.}{}$

input
```
int((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-a/d^3*(-1/16/(c*x-1)^2+9/16/(c*x-1)+3/2*ln(c*x-1)+1/2/c^2/x^2-3*ln(c
*x)-1/16/(c*x+1)^2-9/16/(c*x+1)+3/2*ln(c*x+1))-b/d^3*(1/12/(c^4*x^4-2*c^2*
x^2+1)/c^2/x^2*(8*I*x^6*c^6+8*c^5*x^5*(-c^2*x^2+1)^(1/2)+18*c^4*x^4*arccos
(c*x)-16*I*c^4*x^4-3*c^3*x^3*(-c^2*x^2+1)^(1/2)-27*c^2*x^2*arccos(c*x)+8*I
*c^2*x^2-6*c*x*(-c^2*x^2+1)^(1/2)+6*arccos(c*x))+3*arccos(c*x)*ln(1+c*x+I*
(-c^2*x^2+1)^(1/2))-3*I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-3*arccos(c*x)
*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3/2*I*polylog(2,-(c*x+I*(-c^2*x^2+1)^(
1/2))^2)+3*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-3*I*polylog(2,c*x+I*
(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^3}\,dx = \int -\frac{b\arccos(cx)+a}{\left(c^2dx^2-d\right)^3x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*arccos(c*x) + a)/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5
 - d^3*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^3}\,dx = -\frac{\int \frac{a}{c^6x^9-3c^4x^7+3c^2x^5-x^3}\,dx + \int \frac{b\,\mathrm{acos}\,(cx)}{c^6x^9-3c^4x^7+3c^2x^5-x^3}\,dx}{d^3}$$

input
```
integrate((a+b*acos(c*x))/x**3/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3), x) + Integral
(b*acos(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3), x))/d**3
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^3}\,dx = \int -\frac{b\arccos\left(cx\right)+a}{\left(c^2dx^2-d\right)^3x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*a*((6*c^4*x^4 - 9*c^2*x^2 + 2)/(c^4*d^3*x^6 - 2*c^2*d^3*x^4 + d^3*x^2
) + 6*c^2*log(c*x + 1)/d^3 + 6*c^2*log(c*x - 1)/d^3 - 12*c^2*log(x)/d^3) -
 b*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^6*d^3*x^9 - 3*c
^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^3}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x^3\,\left(d-c^2\,d\,x^2\right)^3}\,dx$$

input
```
int((a + b*acos(c*x))/(x^3*(d - c^2*d*x^2)^3),x)
```

output
```
int((a + b*acos(c*x))/(x^3*(d - c^2*d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^3}\,dx = \frac{-4\left(\int \frac{acos(cx)}{c^6x^9-3c^4x^7+3c^2x^5-x^3}dx\right)b\,c^4x^6+8\left(\int \frac{acos(cx)}{c^6x^9-3c^4x^7+3c^2x^5-x^3}dx\right)b\,c^2x^4-4\left(\int \frac{acos(cx)}{c^6x^9-3c^4x^7+3c^2x^5-x^3}dx\right)b\,x^2}{}$$

input
```
int((a+b*acos(c*x))/x^3/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 4*int(acos(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3),x)*b*c*
*4*x**6 + 8*int(acos(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3),x
)*b*c**2*x**4 - 4*int(acos(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x
**3),x)*b*x**2 - 6*log(c**2*x - c)*a*c**6*x**6 + 12*log(c**2*x - c)*a*c**4
*x**4 - 6*log(c**2*x - c)*a*c**2*x**2 - 6*log(c**2*x + c)*a*c**6*x**6 + 12
*log(c**2*x + c)*a*c**4*x**4 - 6*log(c**2*x + c)*a*c**2*x**2 + 12*log(x)*a
*c**6*x**6 - 24*log(x)*a*c**4*x**4 + 12*log(x)*a*c**2*x**2 - 3*a*c**6*x**6
 + 6*a*c**2*x**2 - 2*a)/(4*d**3*x**2*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.56 $\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 317

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^3}\,dx = \frac{bc^3}{12d^3\left(1-c^2x^2\right)^{3/2}} - \frac{bc}{6d^3x^2\left(1-c^2x^2\right)^{3/2}} - \frac{29bc^3}{24d^3\sqrt{1-c^2x^2}} - \frac{a+b\arccos(cx)}{3d^3x^3\left(1-c^2x^2\right)^2} - \frac{7c^2(a+b\arccos(cx))}{3d^3x\left(1-c^2x^2\right)^2} + \frac{35c^4x(a+b\arccos(cx))}{12d^3\left(1-c^2x^2\right)^2} + \frac{35c^4x(a+b\arccos(cx))}{8d^3\left(1-c^2x^2\right)} - \frac{35ic^3(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{4d^3} - \frac{19bc^3\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{6d^3} + \frac{35ibc^3\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{8d^3} - \frac{35ibc^3\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{8d^3}$$

output
```
1/12*b*c^3/d^3/(-c^2*x^2+1)^(3/2)-1/6*b*c/d^3/x^2/(-c^2*x^2+1)^(3/2)-29/24
*b*c^3/d^3/(-c^2*x^2+1)^(1/2)-1/3*(a+b*arccos(c*x))/d^3/x^3/(-c^2*x^2+1)^2
-7/3*c^2*(a+b*arccos(c*x))/d^3/x/(-c^2*x^2+1)^2+35/12*c^4*x*(a+b*arccos(c*
x))/d^3/(-c^2*x^2+1)^2+35/8*c^4*x*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)-35/4*
I*c^3*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/d^3-19/6*b*c^3*ar
ctanh((-c^2*x^2+1)^(1/2))/d^3+35/8*I*b*c^3*polylog(2,-I*(c*x+I*(-c^2*x^2+1
)^(1/2)))/d^3-35/8*I*b*c^3*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^3
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.38

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^3}\,dx =$$

$$-\frac{\frac{16a}{x^3}+\frac{144ac^2}{x}-\frac{8bc\sqrt{1-c^2x^2}}{x^2}-\frac{12ac^4x}{\left(-1+c^2x^2\right)^2}+\frac{66ac^4x}{-1+c^2x^2}+\frac{bc^3\left((-2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)\right)}{(-1+cx)^2}-\frac{bc^3\left((2+cx)\sqrt{1-c^2x^2}-3\,\mathrm{ar}\right.}{(1+cx)^2}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^4*(d - c^2*d*x^2)^3),x]
```

output
```
-1/48*((16*a)/x^3 + (144*a*c^2)/x - (8*b*c*Sqrt[1 - c^2*x^2])/x^2 - (12*a*
c^4*x)/(-1 + c^2*x^2)^2 + (66*a*c^4*x)/(-1 + c^2*x^2) + (b*c^3*((-2 + c*x)
*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x]))/(-1 + c*x)^2 - (b*c^3*((2 + c*x)*Sqrt
[1 - c^2*x^2] - 3*ArcCos[c*x]))/(1 + c*x)^2 - (33*b*c^3*(Sqrt[1 - c^2*x^2]
 - ArcCos[c*x]))/(1 + c*x) + (16*b*ArcCos[c*x])/x^3 + (33*b*c^3*(Sqrt[1 -
c^2*x^2] + ArcCos[c*x]))/(-1 + c*x) + 8*b*c^3*Log[x] + 105*a*c^3*Log[1 - c
*x] - 105*a*c^3*Log[1 + c*x] + (144*b*c^2*(ArcCos[c*x] + c*x*(Log[x] - Log
[1 + Sqrt[1 - c^2*x^2]])))/x - 8*b*c^3*Log[1 + Sqrt[1 - c^2*x^2]] + ((105*
I)/2)*b*c^3*(ArcCos[c*x]*(ArcCos[c*x] + (4*I)*Log[1 + E^(I*ArcCos[c*x])])
+ 4*PolyLog[2, -E^(I*ArcCos[c*x])]) - ((105*I)/2)*b*c^3*(ArcCos[c*x]*(ArcC
os[c*x] + (4*I)*Log[1 - E^(I*ArcCos[c*x])]) + 4*PolyLog[2, E^(I*ArcCos[c*x
])]))/d^3
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.21, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {5205, 27, 243, 52, 61, 61, 73, 221, 5205, 243, 61, 61, 73, 221, 5163, 241, 5163, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^3}dx$$

$$\Big\downarrow \text{5205}$$

$$\frac{7}{3}c^2\int \frac{a+b\arccos(cx)}{d^3x^2\left(1-c^2x^2\right)^3}dx-\frac{bc\int \frac{1}{x^3(1-c^2x^2)^{5/2}}dx}{3d^3}-\frac{a+b\arccos(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{7c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)^3}dx}{3d^3}-\frac{bc\int \frac{1}{x^3(1-c^2x^2)^{5/2}}dx}{3d^3}-\frac{a+b\arccos(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{7c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)^3}dx}{3d^3}-\frac{bc\int \frac{1}{x^4(1-c^2x^2)^{5/2}}dx^2}{6d^3}-\frac{a+b\arccos(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{7c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)^3}dx}{3d^3}-\frac{bc\left(\frac{5}{2}c^2\int \frac{1}{x^2(1-c^2x^2)^{5/2}}dx^2-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}-\frac{a+b\arccos(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{7c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)^3}dx}{3d^3}-\frac{bc\left(\frac{5}{2}c^2\left(\int \frac{1}{x^2(1-c^2x^2)^{3/2}}dx^2+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}-\frac{a+b\arccos(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{7c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)^3}dx}{3d^3}-\frac{bc\left(\frac{5}{2}c^2\left(\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}-\frac{a+b\arccos(cx)}{3d^3x^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{7c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)^3}dx}{3d^3}-\frac{bc\left(\frac{5}{2}c^2\left(-\frac{2\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2}+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}-\frac{a+b\arccos(cx)}{3d^3x^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{7c^2\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)^3}dx}{3d^3}-\frac{a+b\arccos(cx)}{3d^3x^3(1-c^2x^2)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

$$\Big\downarrow \text{5205}$$

$$\frac{7c^2\left(5c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^3}dx-bc\int \frac{1}{x(1-c^2x^2)^{5/2}}dx-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}\right)}{3d^3}-\frac{a+b\arccos(cx)}{3d^3x^3(1-c^2x^2)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{7c^2\left(5c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^3}dx-\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)^{5/2}}dx^2-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}\right)}{3d^3}-\frac{a+b\arccos(cx)}{3d^3x^3(1-c^2x^2)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

$$\Big\downarrow \text{61}$$

$$\frac{7c^2\left(5c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^3}dx-\frac{1}{2}bc\left(\int \frac{1}{x^2(1-c^2x^2)^{3/2}}dx^2+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}\right)}{3d^3}-\frac{a+b\arccos(cx)}{3d^3x^3(1-c^2x^2)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

$$\Big\downarrow \; 61$$

$$\frac{7c^2\left(5c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^3}dx-\frac{1}{2}bc\left(\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}\right)}{3d^3}-\frac{a+b\arccos(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

$$\Big\downarrow \; 73$$

$$\frac{7c^2\left(5c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^3}dx-\frac{1}{2}bc\left(-\frac{2\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2}+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}\right)}{3d^3}-\frac{a+b\arccos(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

$$\Big\downarrow \; 221$$

$$\frac{7c^2\left(5c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^3}dx-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}-\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)\right)}{3d^3}-\frac{a+b\arccos(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

$$\Big\downarrow \; 5163$$

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx+\frac{1}{4}bc\int \frac{x}{(1-c^2x^2)^{5/2}}dx+\frac{x(a+b\arccos(cx))}{4(1-c^2x^2)^2}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}-\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x\right.\right.\right.}{3d^3}-\frac{a+b\arccos(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

$$\Big\downarrow \; 241$$

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\int\frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx+\frac{x(a+b\arccos(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(1-c^2x^2)^{3/2}}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}-\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\right.\right.}{3d^3}$$

$$\frac{a+b\arccos(cx)}{3d^3x^3(1-c^2x^2)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

$\downarrow$ 5163

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int\frac{x}{(1-c^2x^2)^{3/2}}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}\right)+\frac{x(a+b\arccos(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(1-c^2x^2)^{3/2}}\right)-\frac{a+b}{x(}\right.}{3d^3}$$

$$\frac{a+b\arccos(cx)}{3d^3x^3(1-c^2x^2)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

$\downarrow$ 241

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)+\frac{x(a+b\arccos(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(1-c^2x^2)^{3/2}}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}\right.}{3d^3}$$

$$\frac{a+b\arccos(cx)}{3d^3x^3(1-c^2x^2)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

$\downarrow$ 5165

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)+\frac{x(a+b\arccos(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(1-c^2x^2)^{3/2}}\right)-\frac{a+b\,a}{x(1-}\right.}{3d^3}$$

$$\frac{a+b\arccos(cx)}{3d^3x^3(1-c^2x^2)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

$\downarrow$ 3042

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{\int(a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)+\frac{x(a+b\arccos(cx))}{4(1-c^2x^2)^2}+\frac{b}{12c(1-c^2x^2)}\right.\right.}{3d^3}$$

$$\frac{a+b\arccos(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

$\downarrow$ 4671

$$7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{-b\int\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{2c}+\frac{x(\right.\right.\right.$$

$$\frac{a+b\arccos(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

$\downarrow$ 2715

$$7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos(\right.\right.}{2c}\right.\right.\right.$$

$$\frac{a+b\arccos(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

$\downarrow$ 2838

$$7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}\right.\right.\right.$$

$$\frac{a+b\arccos(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}-\frac{bc\left(\frac{5}{2}c^2\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{\sqrt{1-c^2x^2}}+\frac{2}{3(1-c^2x^2)^{3/2}}\right)-\frac{1}{x^2(1-c^2x^2)^{3/2}}\right)}{6d^3}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^4*(d - c^2*d*x^2)^3),x]
```

output

```
-1/3*(a + b*ArcCos[c*x])/(d^3*x^3*(1 - c^2*x^2)^2) - (b*c*(-(1/(x^2*(1 - c
^2*x^2)^(3/2))) + (5*c^2*(2/(3*(1 - c^2*x^2)^(3/2)) + 2/Sqrt[1 - c^2*x^2]
- 2*ArcTanh[Sqrt[1 - c^2*x^2]]))/2))/(6*d^3) + (7*c^2*(-((a + b*ArcCos[c*x
])/(x*(1 - c^2*x^2)^2)) - (b*c*(2/(3*(1 - c^2*x^2)^(3/2)) + 2/Sqrt[1 - c^2
*x^2] - 2*ArcTanh[Sqrt[1 - c^2*x^2]]))/2 + 5*c^2*(b/(12*c*(1 - c^2*x^2)^(3
/2)) + (x*(a + b*ArcCos[c*x]))/(4*(1 - c^2*x^2)^2) + (3*(b/(2*c*Sqrt[1 - c
^2*x^2]) + (x*(a + b*ArcCos[c*x]))/(2*(1 - c^2*x^2)) - (-2*(a + b*ArcCos[c
*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(I*ArcCos[c*x])] - I*b
*PolyLog[2, E^(I*ArcCos[c*x])])/(2*c)))/4)))/(3*d^3)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.98

method	result
derivativedivides	$c^3\left(-\frac{a\left(-\frac{1}{16(cx-1)^2}+\frac{11}{16(cx-1)}+\frac{35\ln(cx-1)}{16}+\frac{1}{3c^3x^3}+\frac{3}{cx}+\frac{1}{16(cx+1)^2}+\frac{11}{16(cx+1)}-\frac{35\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(\frac{105\arccos(cx)c}{}\right.}{}\right.$
default	$c^3\left(-\frac{a\left(-\frac{1}{16(cx-1)^2}+\frac{11}{16(cx-1)}+\frac{35\ln(cx-1)}{16}+\frac{1}{3c^3x^3}+\frac{3}{cx}+\frac{1}{16(cx+1)^2}+\frac{11}{16(cx+1)}-\frac{35\ln(cx+1)}{16}\right)}{d^3}-\frac{b\left(\frac{105\arccos(cx)c}{}\right.}{}\right.$
parts	$-\frac{a\left(\frac{1}{3x^3}+\frac{3c^2}{x}-\frac{c^3}{16(cx-1)^2}+\frac{11c^3}{16(cx-1)}+\frac{35c^3\ln(cx-1)}{16}+\frac{c^3}{16(cx+1)^2}+\frac{11c^3}{16(cx+1)}-\frac{35c^3\ln(cx+1)}{16}\right)}{d^3}-\frac{b\,c^3\left(\frac{105\arccos(cx)c^6}{}\right.}{}$

input
```
int((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-a/d^3*(-1/16/(c*x-1)^2+11/16/(c*x-1)+35/16*ln(c*x-1)+1/3/c^3/x^3+3/c
/x+1/16/(c*x+1)^2+11/16/(c*x+1)-35/16*ln(c*x+1))-b/d^3*(1/24*(105*arccos(c
*x)*c^6*x^6+29*c^5*x^5*(-c^2*x^2+1)^(1/2)-175*c^4*x^4*arccos(c*x)-27*c^3*x
^3*(-c^2*x^2+1)^(1/2)+56*c^2*x^2*arccos(c*x)-4*c*x*(-c^2*x^2+1)^(1/2)+8*ar
ccos(c*x))/(c^4*x^4-2*c^2*x^2+1)/c^3/x^3+35/8*I*dilog(1+c*x+I*(-c^2*x^2+1)
^(1/2))+19/3*I*arctan(c*x+I*(-c^2*x^2+1)^(1/2))+35/8*I*dilog(c*x+I*(-c^2*x
^2+1)^(1/2))-35/8*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a + b\arccos(cx)}{x^4 \left(d - c^2 d x^2\right)^3} \, dx = \int -\frac{b \arccos\left(cx\right) + a}{\left(c^2 d x^2 - d\right)^3 x^4} \, dx$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*arccos(c*x) + a)/(c^6*d^3*x^10 - 3*c^4*d^3*x^8 + 3*c^2*d^3*x^
6 - d^3*x^4), x)
```

Sympy [F]

$$\int \frac{a + b\arccos(cx)}{x^4 \left(d - c^2 d x^2\right)^3} \, dx = -\frac{\int \frac{a}{c^6x^{10} - 3c^4x^8 + 3c^2x^6 - x^4}\, dx + \int \frac{b \operatorname{acos}\left(cx\right)}{c^6x^{10} - 3c^4x^8 + 3c^2x^6 - x^4}\, dx}{d^3}$$

input
```
integrate((a+b*acos(c*x))/x**4/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4), x) + Integra
l(b*acos(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4), x))/d**3
```

Maxima [F]

$$\int \frac{a + b \arccos(cx)}{x^4 (d - c^2 d x^2)^3} \, dx = \int -\frac{b \arccos(cx) + a}{(c^2 d x^2 - d)^3 x^4} \, dx$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/48*a*(105*c^3*log(c*x + 1)/d^3 - 105*c^3*log(c*x - 1)/d^3 - 2*(105*c^6*x
^6 - 175*c^4*x^4 + 56*c^2*x^2 + 8)/(c^4*d^3*x^7 - 2*c^2*d^3*x^5 + d^3*x^3)
) - 1/48*((210*c^6*x^6 - 350*c^4*x^4 + 112*c^2*x^2 - 105*(c^7*x^7 - 2*c^5*
x^5 + c^3*x^3)*log(c*x + 1) + 105*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*log(-c*x
 + 1) + 16)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 48*(c^4*d^3*x^7 -
 2*c^2*d^3*x^5 + d^3*x^3)*integrate(-1/48*(210*c^7*x^6 - 350*c^5*x^4 + 112
*c^3*x^2 - 105*(c^8*x^7 - 2*c^6*x^5 + c^4*x^3)*log(c*x + 1) + 105*(c^8*x^7
 - 2*c^6*x^5 + c^4*x^3)*log(-c*x + 1) + 16*c)*sqrt(c*x + 1)*sqrt(-c*x + 1)
/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x))*b/(c^4*d^3*x
^7 - 2*c^2*d^3*x^5 + d^3*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arccos(cx)}{x^4 (d - c^2 d x^2)^3} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^3}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{x^4\,\left(d-c^2\,d\,x^2\right)^3}\,dx$$

input
```
int((a + b*acos(c*x))/(x^4*(d - c^2*d*x^2)^3),x)
```

output
```
int((a + b*acos(c*x))/(x^4*(d - c^2*d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^3}\,dx$$
$$=\frac{-48\left(\int \frac{acos(cx)}{c^6x^{10}-3c^4x^8+3c^2x^6-x^4}dx\right)b\,c^4x^7+96\left(\int \frac{acos(cx)}{c^6x^{10}-3c^4x^8+3c^2x^6-x^4}dx\right)b\,c^2x^5-48\left(\int \frac{acos(cx)}{c^6x^{10}-3c^4x^8+3c^2x^6-x^4}dx\right.}{}$$

input
```
int((a+b*acos(c*x))/x^4/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 48*int(acos(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4),x)*b*
c**4*x**7 + 96*int(acos(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**
4),x)*b*c**2*x**5 - 48*int(acos(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x*
*6 - x**4),x)*b*x**3 - 105*log(c**2*x - c)*a*c**7*x**7 + 210*log(c**2*x -
c)*a*c**5*x**5 - 105*log(c**2*x - c)*a*c**3*x**3 + 105*log(c**2*x + c)*a*c
**7*x**7 - 210*log(c**2*x + c)*a*c**5*x**5 + 105*log(c**2*x + c)*a*c**3*x*
*3 - 210*a*c**6*x**6 + 350*a*c**4*x**4 - 112*a*c**2*x**2 - 16*a)/(48*d**3*
x**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.57 $\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 262

$$\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\frac{bx^2\sqrt{d-c^2dx^2}}{32c^3\sqrt{1-c^2x^2}}+\frac{bx^4\sqrt{d-c^2dx^2}}{96c\sqrt{1-c^2x^2}}-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{1-c^2x^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{16c^4}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{24c^2}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{32bc^5\sqrt{1-c^2x^2}}$$

output
```
1/32*b*x^2*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/96*b*x^4*(-c^2*d*
x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/36*b*c*x^6*(-c^2*d*x^2+d)^(1/2)/(-c^2*
x^2+1)^(1/2)-1/16*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c^4-1/24*x^3*(-
c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c^2+1/6*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b
*arccos(c*x))+1/32*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/b/c^5/(-c^2*x^
2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.72

$$\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$=\frac{48acx\sqrt{d-c^2dx^2}(-3-2c^2x^2+8c^4x^4)-144a\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+\frac{b\sqrt{d-c^2dx^2}(-72\arccos(cx)^2-18\cos(2a}{}}{2304c^5}$$

input
```
Integrate[x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(48*a*c*x*Sqrt[d - c^2*d*x^2]*(-3 - 2*c^2*x^2 + 8*c^4*x^4) - 144*a*Sqrt[d]
*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (b*Sqrt[d -
c^2*d*x^2]*(-72*ArcCos[c*x]^2 - 18*Cos[2*ArcCos[c*x]] + 9*Cos[4*ArcCos[c*x
]] + 2*Cos[6*ArcCos[c*x]] + 12*ArcCos[c*x]*(-3*Sin[2*ArcCos[c*x]] + 3*Sin[
4*ArcCos[c*x]] + Sin[6*ArcCos[c*x]])))/Sqrt[1 - c^2*x^2])/(2304*c^5)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5199, 15, 5211, 15, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$\Big\downarrow \text{5199}$$
$$\frac{\sqrt{d-c^2dx^2}\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{6\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int x^5dx}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$\Big\downarrow \text{15}$$
$$\frac{\sqrt{d-c^2dx^2}\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5211}$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{b\int x^3dx}{4c}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}\right)}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5211}$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int xdx}{2c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{\sqrt{d-c^2dx^2}\left(-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}+\frac{3\left(-\frac{(a+b\arccos(cx))^2}{4bc^3}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{1-c^2x^2}}+\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{1-c^2x^2}}$$

input
```
Int[x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*x^6*Sqrt[d - c^2*d*x^2])/(36*Sqrt[1 - c^2*x^2]) + (x^5*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCos[c*x]))/6 + (Sqrt[d - c^2*d*x^2]*(-1/16*(b*x^4)/c - (x^
3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(4*c^2) + (3*(-1/4*(b*x^2)/c - (x
*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(2*c^2) - (a + b*ArcCos[c*x])^2/(4
*b*c^3)))/(4*c^2)))/(6*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.56

method	result
default	$-\frac{a\,x^{3}\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{6 c^{2} d}-\frac{a x \left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{8 c^{4} d}+\frac{a x \sqrt{-c^{2} d\,x^{2}+d}}{16 c^{4}}+\frac{a d \arctan\left(\frac{\sqrt{c^{2} d}\,x}{\sqrt{-c^{2} d\,x^{2}+d}}\right)}{16 c^{4} \sqrt{c^{2} d}}+b\left(\frac{\sqrt{-d\left(c^{2} x^{2}-1\right)}\,\sqrt{-c^{2} x^{2}+1}}{32 c^{5}\left(c^{2} x^{2}-1\right)}\right.$
parts	$-\frac{a\,x^{3}\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{6 c^{2} d}-\frac{a x \left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{8 c^{4} d}+\frac{a x \sqrt{-c^{2} d\,x^{2}+d}}{16 c^{4}}+\frac{a d \arctan\left(\frac{\sqrt{c^{2} d}\,x}{\sqrt{-c^{2} d\,x^{2}+d}}\right)}{16 c^{4} \sqrt{c^{2} d}}+b\left(\frac{\sqrt{-d\left(c^{2} x^{2}-1\right)}\,\sqrt{-c^{2} x^{2}+1}}{32 c^{5}\left(c^{2} x^{2}-1\right)}\right.$

input
```
int(x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*a*x^3*(-c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a/c^4*x*(-c^2*d*x^2+d)^(3/2)/d+1
/16*a/c^4*x*(-c^2*d*x^2+d)^(1/2)+1/16*a/c^4*d/(c^2*d)^(1/2)*arctan((c^2*d)
^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(1/32*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)
^(1/2)/c^5/(c^2*x^2-1)*arccos(c*x)^2+1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*c^7
*x^7-64*c^5*x^5+32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+38*c^3*x^3-48*I*(-c^2*x^2+
1)^(1/2)*x^4*c^4-6*c*x+18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2
))*(I+6*arccos(c*x))/c^5/(c^2*x^2-1)-1/256*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-
c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arcco
s(c*x))/c^5/(c^2*x^2-1)-1/4608*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/
2)*x*c+c^2*x^2-1)*(11*I+24*arccos(c*x))*cos(5*arccos(c*x))/c^5/(c^2*x^2-1)
-1/4608*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(7*I+4
8*arccos(c*x))*sin(5*arccos(c*x))/c^5/(c^2*x^2-1)+1/512*(-d*(c^2*x^2-1))^(
1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(I+8*arccos(c*x))*cos(3*arccos(
c*x))/c^5/(c^2*x^2-1)-3/512*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*
x*c+c^2*x^2-1)*sin(3*arccos(c*x))/c^5/(c^2*x^2-1))
```

Fricas [F]

$$\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int\sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)x^4\,dx$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
integral((b*x^4*arccos(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int x^4\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acos}{(cx)})\,dx$$

input
```
integrate(x**4*(-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
Integral(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int\sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)x^4\,dx$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
b*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^4*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x), x) - 1/48*(8*(-c^2*d*x^2 + d)^(3/2)*x^3/(c^2*d) - 3
*sqrt(-c^2*d*x^2 + d)*x/c^4 + 6*(-c^2*d*x^2 + d)^(3/2)*x/(c^4*d) - 3*sqrt(
d)*arcsin(c*x)/c^5)*a
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.83

$$\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\frac{1}{6}\sqrt{-c^2dx^2+d}ax^5-\frac{\sqrt{-c^2dx^2+d}ax^3}{24\,c^2}+\frac{\left(64\,c^5x^6+384\sqrt{-c^2x^2+1}c^4x^5\arccos{(cx)}-24\,c^3x^4-96\sqrt{-c^2x^2+1}c^2x^3\arccos{(cx)}-72\,cx^2-14\right.}{2304\,c^4}-\frac{\sqrt{-c^2dx^2+d}ax}{16\,c^4}-\frac{ad\log\left(\left|-c\sqrt{-d}x+\sqrt{c^2x^2-1}\sqrt{-d}\right|\right)}{16\,c^5\sqrt{-d}}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/6*sqrt(-c^2*d*x^2 + d)*a*x^5 - 1/24*sqrt(-c^2*d*x^2 + d)*a*x^3/c^2 + 1/2
304*(64*c^5*x^6 + 384*sqrt(-c^2*x^2 + 1)*c^4*x^5*arccos(c*x) - 24*c^3*x^4
- 96*sqrt(-c^2*x^2 + 1)*c^2*x^3*arccos(c*x) - 72*c*x^2 - 144*sqrt(-c^2*x^2
 + 1)*x*arccos(c*x) - 72*arccos(c*x)^2/c + 25/c)*b*sqrt(d)/c^4 - 1/16*sqrt
(-c^2*d*x^2 + d)*a*x/c^4 - 1/16*a*d*log(abs(-c*sqrt(-d)*x + sqrt(c^2*x^2 -
 1)*sqrt(-d)))/(c^5*sqrt(-d))
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int x^4\,(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^4*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^4*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$=\frac{\sqrt{d}\left(3asin(cx)\,a+8\sqrt{-c^2x^2+1}\,a\,c^5x^5-2\sqrt{-c^2x^2+1}\,a\,c^3x^3-3\sqrt{-c^2x^2+1}\,acx+48\left(\int\sqrt{-c^2x^2+1}\,a\right.\right.}{48c^5}$$

input
```
int(x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*(3*asin(c*x)*a + 8*sqrt( - c**2*x**2 + 1)*a*c**5*x**5 - 2*sqrt( -
 c**2*x**2 + 1)*a*c**3*x**3 - 3*sqrt( - c**2*x**2 + 1)*a*c*x + 48*int(sqrt
( - c**2*x**2 + 1)*acos(c*x)*x**4,x)*b*c**5))/(48*c**5)
```

3.58 $\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 189

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\frac{bx^2\sqrt{d-c^2dx^2}}{16c\sqrt{1-c^2x^2}}-\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{8c^2}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{16bc^3\sqrt{1-c^2x^2}}$$

output
```
1/16*b*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/16*b*c*x^4*(-c^2*d*
x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/8*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x
))/c^2+1/4*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))+1/16*(-c^2*d*x^2+d)^
(1/2)*(a+b*arccos(c*x))^2/b/c^3/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.75

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$=\frac{16acx(-1+2c^2x^2)\sqrt{d-c^2dx^2}-16a\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+\frac{b\sqrt{d-c^2dx^2}(-8\arccos(cx)^2+\cos(4\arccos(cx))+4\,\mathrm{ar}}{\sqrt{1-c^2x^2}}}{128c^3}$$

input
```
Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(16*a*c*x*(-1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2] - 16*a*Sqrt[d]*ArcTan[(c*x*
Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (b*Sqrt[d - c^2*d*x^2]*(-
8*ArcCos[c*x]^2 + Cos[4*ArcCos[c*x]] + 4*ArcCos[c*x]*Sin[4*ArcCos[c*x]]))/
Sqrt[1 - c^2*x^2])/(128*c^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5199, 15, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5199}$$

$$\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int x^3dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 5211}$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int xdx}{2c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}\right)}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{\sqrt{d-c^2dx^2}\left(-\frac{(a+b\arccos(cx))^2}{4bc^3}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4\sqrt{1-c^2x^2}}+\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$

input
```
Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (x^3*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCos[c*x]))/4 + (Sqrt[d - c^2*d*x^2]*(-1/4*(b*x^2)/c - (x*S
qrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(2*c^2) - (a + b*ArcCos[c*x])^2/(4*b
*c^3)))/(4*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.94

method	result
default	$-\frac{a x\left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}{4 c^{2} d}+\frac{a x \sqrt{-c^{2} d \,x^{2}+d}}{8 c^{2}}+\frac{a d \arctan\left(\frac{\sqrt{c^{2} d}\, x}{\sqrt{-c^{2} d \,x^{2}+d}}\right)}{8 c^{2} \sqrt{c^{2} d}}+b \left(\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \sqrt{-c^{2} x^{2}+1}\, \arccos \left(c x \right)^{2}}{16 c^{3} \left(c^{2} x^{2}-1\right)}+\frac{\sqrt{-d(}\cdots}{}\right.$
parts	$-\frac{a x\left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}{4 c^{2} d}+\frac{a x \sqrt{-c^{2} d \,x^{2}+d}}{8 c^{2}}+\frac{a d \arctan\left(\frac{\sqrt{c^{2} d}\, x}{\sqrt{-c^{2} d \,x^{2}+d}}\right)}{8 c^{2} \sqrt{c^{2} d}}+b \left(\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \sqrt{-c^{2} x^{2}+1}\, \arccos \left(c x \right)^{2}}{16 c^{3} \left(c^{2} x^{2}-1\right)}+\frac{\sqrt{-d(}\cdots}{}\right.$

input
```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a/c^2*x*(-c^2*d*x^2+d)^(1/2)+1/8*a
/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(1/16*
(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2+1/
256*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^
4*c^4+4*c*x-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(I+4*arcc
os(c*x))/c^3/(c^2*x^2-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(-8*I*(-c^2*x^2+1)^(
1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x
^2+1)^(1/2)+4*c*x)*(-I+4*arccos(c*x))/c^3/(c^2*x^2-1))
```

Fricas [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int\sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*x^2*arccos(c*x) + a*x^2), x)
```

Sympy [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int x^2\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\operatorname{acos}{(cx)})\,dx$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int \sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
b*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x), x) + 1/8*a*(sqrt(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*
x^2 + d)^(3/2)*x/(c^2*d) + sqrt(d)*arcsin(c*x)/c^3)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.86

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\frac{1}{4}\sqrt{-c^2dx^2+d}ax^3$$
$$+\frac{\left(8\,c^3x^4+32\,\sqrt{-c^2x^2+1}c^2x^3\arccos{(cx)}-8\,cx^2-16\,\sqrt{-c^2x^2+1}x\arccos{(cx)}-\frac{8\,\arccos{(cx)}^2}{c}+\frac{1}{c}\right)b\sqrt{}}{128\,c^2}$$
$$-\frac{\sqrt{-c^2dx^2+d}ax}{8\,c^2}-\frac{ad\log{\left(\left|-c\sqrt{-d}x+\sqrt{c^2x^2-1}\sqrt{-d}\right|\right)}}{8\,c^3\sqrt{-d}}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/4*sqrt(-c^2*d*x^2 + d)*a*x^3 + 1/128*(8*c^3*x^4 + 32*sqrt(-c^2*x^2 + 1)*
c^2*x^3*arccos(c*x) - 8*c*x^2 - 16*sqrt(-c^2*x^2 + 1)*x*arccos(c*x) - 8*ar
ccos(c*x)^2/c + 1/c)*b*sqrt(d)/c^2 - 1/8*sqrt(-c^2*d*x^2 + d)*a*x/c^2 - 1/
8*a*d*log(abs(-c*sqrt(-d)*x + sqrt(c^2*x^2 - 1)*sqrt(-d)))/(c^3*sqrt(-d))
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int x^2\,(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$=\frac{\sqrt{d}\,\left(asin(cx)\,a+2\sqrt{-c^2x^2+1}\,a\,c^3x^3-\sqrt{-c^2x^2+1}\,acx+8\left(\int\sqrt{-c^2x^2+1}\,acos(cx)\,x^2dx\right)b\,c^3\right)}{8c^3}$$

input
```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*(asin(c*x)*a + 2*sqrt( - c**2*x**2 + 1)*a*c**3*x**3 - sqrt( - c**
2*x**2 + 1)*a*c*x + 8*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**2,x)*b*c**3)
)/(8*c**3)
```

3.59 $\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = -\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx)) + \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4bc\sqrt{1-c^2x^2}}$$

output
```
-1/4*b*c*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/2*x*(-c^2*d*x^2+d)^
(1/2)*(a+b*arccos(c*x))+1/4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/b/c/(
-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \frac{1}{8}\left(4ax\sqrt{d-c^2dx^2} - \frac{4a\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{c} + \frac{b\sqrt{d-c^2dx^2}(-2\arccos(cx)^2+\cos(2\arccos(cx))+2\arccos(cx)\sin(2\arccos(cx)))}{c\sqrt{1-c^2x^2}}\right)$$

input
```
Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(4*a*x*Sqrt[d - c^2*d*x^2] - (4*a*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])
/(Sqrt[d]*(-1 + c^2*x^2))])/c + (b*Sqrt[d - c^2*d*x^2]*(-2*ArcCos[c*x]^2 +
 Cos[2*ArcCos[c*x]] + 2*ArcCos[c*x]*Sin[2*ArcCos[c*x]]))/(c*Sqrt[1 - c^2*x
^2]))/8
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5157}$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int xdx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4bc\sqrt{1-c^2x^2}}+\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

input
```
Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) + (x*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCos[c*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4
*b*c*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.00 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.41

method	result
default	$\frac{ax\sqrt{-c^2dx^2+d}}{2}+\frac{ad\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2}{4(c^2x^2-1)c}+\frac{\sqrt{-d(c^2x^2-1)}\left(2c^3x^3-2cx+2i\right.}{}\right.$
parts	$\frac{ax\sqrt{-c^2dx^2+d}}{2}+\frac{ad\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2}{4(c^2x^2-1)c}+\frac{\sqrt{-d(c^2x^2-1)}\left(2c^3x^3-2cx+2i\right.}{}\right.$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x*(-c^2*d*x^2+d)^(1/2)+1/2*a*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/
(-c^2*d*x^2+d)^(1/2))+b*(1/4*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^
2*x^2-1)/c*arccos(c*x)^2+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*I*
(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(I+2*arccos(c*x))/(c^2*x^
2-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*
x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))/(c^2*x^2-1)/c)
```

Fricas [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int \sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int \sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))\;dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int \sqrt{-c^2dx^2+d}(b\arccos(cx)+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
b*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x), x) + 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/
c)*a
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int (a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$=\frac{\sqrt{d}\,\left(asin(cx)\,a+\sqrt{-c^2x^2+1}\,acx+2\left(\int\sqrt{-c^2x^2+1}\,acos(cx)\,dx\right)bc\right)}{2c}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*(asin(c*x)*a + sqrt( - c**2*x**2 + 1)*a*c*x + 2*int(sqrt( - c**2*
x**2 + 1)*acos(c*x),x)*b*c))/(2*c)
```

3.60 $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 110

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}\,dx = -\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x} - \frac{c\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2b\sqrt{1-c^2x^2}} + \frac{bc\sqrt{d-c^2dx^2}\log(x)}{\sqrt{1-c^2x^2}}$$

output
```
-(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x-1/2*c*(-c^2*d*x^2+d)^(1/2)*(a+b*
arccos(c*x))^2/b/(-c^2*x^2+1)^(1/2)+b*c*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x
^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}\,dx = -\frac{a\sqrt{d-c^2dx^2}}{x} + ac\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right) - \frac{1}{2}bc\sqrt{d-c^2dx^2}\left(\frac{2\arccos(cx)}{cx} - \frac{\arccos(cx)^2-2\log(cx)}{\sqrt{1-c^2x^2}}\right)$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
-((a*Sqrt[d - c^2*d*x^2])/x) + a*c*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2]
)/(Sqrt[d]*(-1 + c^2*x^2))] - (b*c*Sqrt[d - c^2*d*x^2]*((2*ArcCos[c*x])/(c
*x) - (ArcCos[c*x]^2 - 2*Log[c*x])/Sqrt[1 - c^2*x^2]))/2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5197, 14, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}\,dx$$

$$\Big\downarrow \text{5197}$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}} - \frac{bc\sqrt{d-c^2dx^2}\int \frac{1}{x}dx}{\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{14}$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}-\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{c\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2b\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}-\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x) + (c*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCos[c*x])^2)/(2*b*Sqrt[1 - c^2*x^2]) - (b*c*Sqrt[d - c^2*d*x^2]*Lo
g[x])/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5197
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^2
]/Sqrt[1 - c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x
] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 2)*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.71

method	result
default	$-\frac{a(-c^2d\,x^2+d)^{\frac{3}{2}}}{dx}-a\,c^2x\sqrt{-c^2d\,x^2+d}-\frac{a\,c^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx\right.$
parts	$-\frac{a(-c^2d\,x^2+d)^{\frac{3}{2}}}{dx}-a\,c^2x\sqrt{-c^2d\,x^2+d}-\frac{a\,c^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx\right.$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/x*(-c^2*d*x^2+d)^(3/2)-a*c^2*x*(-c^2*d*x^2+d)^(1/2)-a*c^2*d/(c^2*d)^(
1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/2*(-d*(c^2*x^2-1))
^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^2*c-2*I*(-d*(c^2*x^2-1))
^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)*c-(-d*(c^2*x^2-1))^(1/2)
*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*arccos(c*x)/x/(c^2*x^2-1)+(-d*(c^2*
x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2
))^2)*c)
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}\,dx = \int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))}{x^2}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))/x**2,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\arccos\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^2,x, algorithm="maxima"
)
```

output
```
b*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x)/x^2, x) - (c*sqrt(d)*arcsin(c*x) + sqrt(-c^2*d*x^2 + d)/
x)*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}\,dx = \frac{\sqrt{d}\left(acos(cx)^2\,bcx-2asin(cx)\,acx-2\sqrt{-c^2x^2+1}\,a+2\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,x^2}dx\right)bx\right)}{2x}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))/x^2,x)
```

output
```
(sqrt(d)*(acos(c*x)**2*b*c*x - 2*asin(c*x)*a*c*x - 2*sqrt( - c**2*x**2 + 1
)*a + 2*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*x**2),x)*b*x))/(2*x)
```

3.61 $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 111

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^4}\,dx = -\frac{bc\sqrt{d-c^2dx^2}}{6x^2\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{3dx^3} - \frac{bc^3\sqrt{d-c^2dx^2}\log(x)}{3\sqrt{1-c^2x^2}}$$

output
```
-1/6*b*c*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/3*(-c^2*d*x^2+d)^(3
/2)*(a+b*arccos(c*x))/d/x^3-1/3*b*c^3*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2
+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^4}\,dx = \frac{\sqrt{d-c^2dx^2}\left(bcx-3bc^3x^3-2a\sqrt{1-c^2x^2}+2ac^2x^2\sqrt{1-c^2x^2}-2b(1-c^2x^2)^{3/2}\arccos(cx)+2bc^3x^3\log\right.}{6x^3\sqrt{1-c^2x^2}}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(b*c*x - 3*b*c^3*x^3 - 2*a*Sqrt[1 - c^2*x^2] + 2*a*c^
2*x^2*Sqrt[1 - c^2*x^2] - 2*b*(1 - c^2*x^2)^(3/2)*ArcCos[c*x] + 2*b*c^3*x^
3*Log[x]))/(6*x^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5187, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^4}\,dx$$

$$\Big\downarrow \text{5187}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \frac{1-c^2x^2}{x^3}dx}{3\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3dx^3}$$

$$\Big\downarrow \text{244}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \left(\frac{1}{x^3}-\frac{c^2}{x}\right)dx}{3\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3dx^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3dx^3}-\frac{bc\sqrt{d-c^2dx^2}\left(c^2(-\log(x))-\frac{1}{2x^2}\right)}{3\sqrt{1-c^2x^2}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
-1/3*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(d*x^3) - (b*c*Sqrt[d - c
^2*d*x^2]*(-1/2*1/x^2 - c^2*Log[x]))/(3*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.43

method	result
default	$-\frac{a\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3 d\,x^{3}}+\frac{b\sqrt{-d\left(c^{2} x^{2}-1\right)}\,\sqrt{-c^{2} x^{2}+1}\,\left(2 i \arccos\left(c x\right) x^{3} c^{3}-2\ln\left(1+\left(c x+i\sqrt{-c^{2} x^{2}+1}\right)^{2}\right) x^{3} c^{3}-2\sqrt{-c^{2} x^{2}+1}\,\arccos(\right.}{6\left(c^{2} x^{2}-1\right) x^{3}}$
parts	$-\frac{a\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3 d\,x^{3}}+\frac{b\sqrt{-d\left(c^{2} x^{2}-1\right)}\,\sqrt{-c^{2} x^{2}+1}\,\left(2 i \arccos\left(c x\right) x^{3} c^{3}-2\ln\left(1+\left(c x+i\sqrt{-c^{2} x^{2}+1}\right)^{2}\right) x^{3} c^{3}-2\sqrt{-c^{2} x^{2}+1}\,\arccos(\right.}{6\left(c^{2} x^{2}-1\right) x^{3}}$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/d/x^3*(-c^2*d*x^2+d)^(3/2)+1/6*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1
)^(1/2)*(2*I*arccos(c*x)*x^3*c^3-2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*x^3*
c^3-2*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^2+2*arccos(c*x)*(-c^2*x^2+1)^(1
/2)-c*x)/(c^2*x^2-1)/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(95) = 190$.

Time = 0.17 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.74

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^4}\,dx$$

$$= \left[\frac{(bc^5x^5-bc^3x^3)\sqrt{d}\log\left(\frac{c^2dx^6+c^2dx^2-dx^4-\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}(x^4-1)\sqrt{d}-d}{c^2x^4-x^2}\right)+\sqrt{-c^2dx^2+d}(bcx^3-bcx)\sqrt{-c}}{6\left(c^2x^5-x^3\right)}\right.$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^4,x, algorithm="fricas"
)
```

output
```
[1/6*((b*c^5*x^5 - b*c^3*x^3)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 -
 sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^2*x^4 -
 x^2)) + sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(-c^2*x^2 + 1) + 2*(a*
c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c^2*x^2 + b)*arccos(c*x) + a)*sqr
t(-c^2*d*x^2 + d))/(c^2*x^5 - x^3), 1/6*(2*(b*c^5*x^5 - b*c^3*x^3)*sqrt(-d
)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)/(c^2*d
*x^4 + (c^2 - 1)*d*x^2 - d)) + sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt
(-c^2*x^2 + 1) + 2*(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c^2*x^2 + b
)*arccos(c*x) + a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^5 - x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^4}\,dx = \int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))}{x^4}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))/x**4,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^4}\,dx = -\frac{\left((-1)^{-2\,c^2dx^2+2\,d}\,c^2d^{\frac{3}{2}}\log\left(-2\,c^2d+\frac{2\,d}{x^2}\right)+c^2d^{\frac{3}{2}}\log\left(x^2-\frac{1}{c^2}\right)-\frac{\sqrt{c^4dx^4-2\,c^2dx^2+d}d}{x^2}\right)bc}{6\,d} - \frac{(-c^2dx^2+d)^{\frac{3}{2}}b\arccos(cx)}{3\,dx^3} - \frac{(-c^2dx^2+d)^{\frac{3}{2}}a}{3\,dx^3}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^4,x, algorithm="maxima"
)
```

output
```
-1/6*((-1)^(-2*c^2*d*x^2 + 2*d)*c^2*d^(3/2)*log(-2*c^2*d + 2*d/x^2) + c^2*
d^(3/2)*log(x^2 - 1/c^2) - sqrt(c^4*d*x^4 - 2*c^2*d*x^2 + d)*d/x^2)*b*c/d
- 1/3*(-c^2*d*x^2 + d)^(3/2)*b*arccos(c*x)/(d*x^3) - 1/3*(-c^2*d*x^2 + d)^
(3/2)*a/(d*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^4}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/x^4,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^4}\,dx = \frac{\sqrt{d}\left(\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a+3\left(\int \frac{\sqrt{-c^2x^2+1}\,acos(cx)}{x^4}dx\right)b\,x^3\right)}{3x^3}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))/x^4,x)
```

output
```
(sqrt(d)*(sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a +
3*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**4,x)*b*x**3))/(3*x**3)
```

3.62 $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^6}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 187

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^6}\,dx = -\frac{bc\sqrt{d-c^2dx^2}}{20x^4\sqrt{1-c^2x^2}}+\frac{bc^3\sqrt{d-c^2dx^2}}{30x^2\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{5dx^5} - \frac{2c^2(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{15dx^3} - \frac{2bc^5\sqrt{d-c^2dx^2}\log(x)}{15\sqrt{1-c^2x^2}}$$

output
```
-1/20*b*c*(-c^2*d*x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)+1/30*b*c^3*(-c^2*d*x
^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*
x))/d/x^5-2/15*c^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/d/x^3-2/15*b*c^5
*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^6}\,dx$$
$$= \frac{\sqrt{d-c^2dx^2}\left(9bcx-6bc^3x^3-50bc^5x^5-36a\sqrt{1-c^2x^2}+12ac^2x^2\sqrt{1-c^2x^2}+24ac^4x^4\sqrt{1-c^2x^2}+12b\right.}{180x^5\sqrt{1-c^2x^2}}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x^6,x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(9*b*c*x - 6*b*c^3*x^3 - 50*b*c^5*x^5 - 36*a*Sqrt[1 -
 c^2*x^2] + 12*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 24*a*c^4*x^4*Sqrt[1 - c^2*x^2
] + 12*b*Sqrt[1 - c^2*x^2]*(-3 + c^2*x^2 + 2*c^4*x^4)*ArcCos[c*x] + 24*b*c
^5*x^5*Log[x]))/(180*x^5*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5195, 27, 1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^6}\,dx$$

$$\Big\downarrow \; 5195$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{-2c^4x^4-c^2x^2+3}{15x^5}dx}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{5dx^5}-\frac{2c^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{15dx^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \frac{-2c^4x^4-c^2x^2+3}{x^5}dx}{15\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{5dx^5}-\frac{2c^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{15dx^3}$$

$\Big\downarrow$ 1433

$$-\frac{bc\sqrt{d-c^2dx^2}\int \left(-\frac{2c^4}{x}-\frac{c^2}{x^3}+\frac{3}{x^5}\right)dx}{15\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{5dx^5}-\frac{2c^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{15dx^3}$$

$\Big\downarrow$ 2009

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{5dx^5}-\frac{2c^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{15dx^3}-\frac{bc\sqrt{d-c^2dx^2}\left(-2c^4\log(x)+\frac{c^2}{2x^2}-\frac{3}{4x^4}\right)}{15\sqrt{1-c^2x^2}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x^6,x]
```

output
```
-1/5*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(d*x^5) - (2*c^2*(d - c^2
*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(15*d*x^3) - (b*c*Sqrt[d - c^2*d*x^2]*(
-3/(4*x^4) + c^2/(2*x^2) - 2*c^4*Log[x]))/(15*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 1903, normalized size of antiderivative = 10.18

method	result	size
default	Expression too large to display	1903
parts	Expression too large to display	1903

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/(c^2*x^2
-1)*c^5*(-c^2*x^2+1)^(1/2)-2/15*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2
)/(c^2*x^2-1)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*c^5+9/5*b*(-d*(c^2*x^2-1)
)^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x^5/(c^2*x^2-1)*arccos(c*x)-2*
I*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^6/(c^2*x^
2-1)*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^11+2/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(1
5*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^4/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arcco
s(c*x)*c^9+2*I*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9
)*x^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^7+a*(-1/5/d/x^5*(-c^2*d
*x^2+d)^(3/2)-2/15*c^2/d/x^3*(-c^2*d*x^2+d)^(3/2))-17/3*b*(-d*(c^2*x^2-1))
^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^3/(c^2*x^2-1)*arccos(c*x)*c^8
-11/12*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^2/(c
^2*x^2-1)*c^7*(-c^2*x^2+1)^(1/2)+98/15*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^
6-5*c^4*x^4-15*c^2*x^2+9)*x/(c^2*x^2-1)*arccos(c*x)*c^6+12/5*b*(-d*(c^2*x^
2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x/(c^2*x^2-1)*arccos(c*x)*
c^4+21/20*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x^2
/(c^2*x^2-1)*c^3*(-c^2*x^2+1)^(1/2)+2*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6
-5*c^4*x^4-15*c^2*x^2+9)*x^7/(c^2*x^2-1)*arccos(c*x)*c^12-5/3*b*(-d*(c^2*x
^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^5/(c^2*x^2-1)*arccos(c*
x)*c^10+3/5*I*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2...
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^6}\,dx$$
$$= \left[\frac{4\left(bc^7x^7-bc^5x^5\right)\sqrt{d}\log\left(\frac{c^2dx^6+c^2dx^2-dx^4-\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}(x^4-1)\sqrt{d}-d}{c^2x^4-x^2}\right)+\left(2\,bc^3x^3-\left(2\,bc^3-3\,bc\right)x^5-\right.}{}\right.$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^6,x, algorithm="fricas"
)
```

output
```
[1/60*(4*(b*c^7*x^7 - b*c^5*x^5)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^
4 - sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^2*x^
4 - x^2)) + (2*b*c^3*x^3 - (2*b*c^3 - 3*b*c)*x^5 - 3*b*c*x)*sqrt(-c^2*d*x^
2 + d)*sqrt(-c^2*x^2 + 1) + 4*(2*a*c^6*x^6 - a*c^4*x^4 - 4*a*c^2*x^2 + (2*
b*c^6*x^6 - b*c^4*x^4 - 4*b*c^2*x^2 + 3*b)*arccos(c*x) + 3*a)*sqrt(-c^2*d*
x^2 + d))/(c^2*x^7 - x^5), 1/60*(8*(b*c^7*x^7 - b*c^5*x^5)*sqrt(-d)*arctan
(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)/(c^2*d*x^4 + (
c^2 - 1)*d*x^2 - d)) + (2*b*c^3*x^3 - (2*b*c^3 - 3*b*c)*x^5 - 3*b*c*x)*sqr
t(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 4*(2*a*c^6*x^6 - a*c^4*x^4 - 4*a*c^
2*x^2 + (2*b*c^6*x^6 - b*c^4*x^4 - 4*b*c^2*x^2 + 3*b)*arccos(c*x) + 3*a)*s
qrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5)]
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^6}\,dx=\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))}{x^6}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))/x**6,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.75

$$\begin{aligned}
&\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^6}\,dx\\
&=\frac{1}{60}\left(8\,c^4\sqrt{d}\log\left(x\right)-\frac{2\,c^2\sqrt{d}x^2-3\,\sqrt{d}}{x^4}\right)bc\\
&\quad-\frac{1}{15}\,b\left(\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2}{dx^3}+\frac{3\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{dx^5}\right)\arccos\left(cx\right)\\
&\quad-\frac{1}{15}\,a\left(\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2}{dx^3}+\frac{3\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{dx^5}\right)
\end{aligned}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^6,x, algorithm="maxima"
)
```

output
```
1/60*(8*c^4*sqrt(d)*log(x) - (2*c^2*sqrt(d)*x^2 - 3*sqrt(d))/x^4)*b*c - 1/
15*b*(2*(-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^3) + 3*(-c^2*d*x^2 + d)^(3/2)/(d*x
^5))*arccos(c*x) - 1/15*a*(2*(-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^3) + 3*(-c^2*
d*x^2 + d)^(3/2)/(d*x^5))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^6}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^6,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^6}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^6}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/x^6,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^6}\,dx$$
$$=\frac{\sqrt{d}\left(2\sqrt{-c^2x^2+1}\,a\,c^4x^4+\sqrt{-c^2x^2+1}\,a\,c^2x^2-3\sqrt{-c^2x^2+1}\,a+15\left(\int\frac{\sqrt{-c^2x^2+1}\,acos(cx)}{x^6}dx\right)b\,x^5\right)}{15x^5}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))/x^6,x)
```

output
```
(sqrt(d)*(2*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 + sqrt( - c**2*x**2 + 1)*a*
c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)*a + 15*int((sqrt( - c**2*x**2 + 1)*ac
os(c*x))/x**6,x)*b*x**5))/(15*x**5)
```

3.63 $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^8}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 263

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^8}\,dx = -\frac{bc\sqrt{d-c^2dx^2}}{42x^6\sqrt{1-c^2x^2}}+\frac{bc^3\sqrt{d-c^2dx^2}}{140x^4\sqrt{1-c^2x^2}}+\frac{2bc^5\sqrt{d-c^2dx^2}}{105x^2\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{7dx^7}-\frac{4c^2(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{35dx^5}-\frac{8c^4(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{105dx^3}-\frac{8bc^7\sqrt{d-c^2dx^2}\log(x)}{105\sqrt{1-c^2x^2}}$$

output
```
-1/42*b*c*(-c^2*d*x^2+d)^(1/2)/x^6/(-c^2*x^2+1)^(1/2)+1/140*b*c^3*(-c^2*d*
x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)+2/105*b*c^5*(-c^2*d*x^2+d)^(1/2)/x^2/(
-c^2*x^2+1)^(1/2)-1/7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/d/x^7-4/35*c^
2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/d/x^5-8/105*c^4*(-c^2*d*x^2+d)^(3
/2)*(a+b*arccos(c*x))/d/x^3-8/105*b*c^7*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x
^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^8}\,dx$$
$$= \frac{\sqrt{d-c^2dx^2}\left(50bcx-15bc^3x^3-40bc^5x^5-392bc^7x^7-300a\sqrt{1-c^2x^2}+60ac^2x^2\sqrt{1-c^2x^2}+80ac^4x^4\sqrt{\right.}{2100x}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x^8,x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(50*b*c*x - 15*b*c^3*x^3 - 40*b*c^5*x^5 - 392*b*c^7*x
^7 - 300*a*Sqrt[1 - c^2*x^2] + 60*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 80*a*c^4*x
^4*Sqrt[1 - c^2*x^2] + 160*a*c^6*x^6*Sqrt[1 - c^2*x^2] + 20*b*Sqrt[1 - c^2
*x^2]*(-15 + 3*c^2*x^2 + 4*c^4*x^4 + 8*c^6*x^6)*ArcCos[c*x] + 160*b*c^7*x^
7*Log[x]))/(2100*x^7*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.66, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5195, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^8}\,dx$$

$$\Big\downarrow \text{ 5195}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{-8c^6x^6-4c^4x^4-3c^2x^2+15}{105x^7}dx}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{7dx^7}-\frac{4c^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{35dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{105dx^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \frac{-8c^6x^6-4c^4x^4-3c^2x^2+15}{x^7}dx}{105\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{7dx^7}-\frac{4c^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{35dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{105dx^3}$$

$$\Big\downarrow \text{ 2010}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \left(-\frac{8c^6}{x}-\frac{4c^4}{x^3}-\frac{3c^2}{x^5}+\frac{15}{x^7}\right)dx}{105\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{7dx^7}-\frac{4c^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{35dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{105dx^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{7dx^7}-\frac{4c^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{35dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{105dx^3}-\frac{bc\sqrt{d-c^2dx^2}\left(-8c^6\log(x)+\frac{2c^4}{x^2}+\frac{3c^2}{4x^4}-\frac{5}{2x^6}\right)}{105\sqrt{1-c^2x^2}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x^8,x]
```

output
```
-1/7*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(d*x^7) - (4*c^2*(d - c^2
*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(35*d*x^5) - (8*c^4*(d - c^2*d*x^2)^(3/
2)*(a + b*ArcCos[c*x]))/(105*d*x^3) - (b*c*Sqrt[d - c^2*d*x^2]*(-5/(2*x^6)
 + (3*c^2)/(4*x^4) + (2*c^4)/x^2 - 8*c^6*Log[x]))/(105*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 2751, normalized size of antiderivative = 10.46

method	result	size
default	Expression too large to display	2751
parts	Expression too large to display	2751

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
-351/5*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^
2*x^2+225)*x^3/(c^2*x^2-1)*arccos(c*x)*c^10-469/60*b*(-d*(c^2*x^2-1))^(1/2
)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^2/(c^2*x^2-1)*c^9
*(-c^2*x^2+1)^(1/2)+3057/35*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*
x^6-21*c^4*x^4-315*c^2*x^2+225)*x/(c^2*x^2-1)*arccos(c*x)*c^8-594/35*b*(-d
*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x
/(c^2*x^2-1)*arccos(c*x)*c^6+71/28*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-1
05*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x^2/(c^2*x^2-1)*c^5*(-c^2*x^2+1)^(1
/2)+342/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315
*c^2*x^2+225)/x^3/(c^2*x^2-1)*arccos(c*x)*c^4+a*(-1/7/d/x^7*(-c^2*d*x^2+d)
^(3/2)+4/7*c^2*(-1/5/d/x^5*(-c^2*d*x^2+d)^(3/2)-2/15*c^2/d/x^3*(-c^2*d*x^2
+d)^(3/2)))+8/5*I*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4
*x^4-315*c^2*x^2+225)*x^4/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^11+
24*I*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*
x^2+225)*x^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^9-64/3*I*b*(-d*(
c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^8
/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^15+8*I*b*(-d*(c^2*x^2-1))^(1
/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^6/(c^2*x^2-1)*(
-c^2*x^2+1)^(1/2)*arccos(c*x)*c^13+152/105*I*b*(-d*(c^2*x^2-1))^(1/2)/(280
*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^5/(c^2*x^2-1)*c^12+3...
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^8}\,dx$$
$$= \left[\frac{16\,(bc^9x^9 - bc^7x^7)\sqrt{d}\log\left(\frac{c^2dx^6+c^2dx^2-dx^4-\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}(x^4-1)\sqrt{d}-d}{c^2x^4-x^2}\right) + (8\,bc^5x^5 - (8\,bc^5 + 3\,bc^3 - 1\right.$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^8,x, algorithm="fricas"
)
```

output
```
[1/420*(16*(b*c^9*x^9 - b*c^7*x^7)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 - d*
x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^2*
x^4 - x^2)) + (8*b*c^5*x^5 - (8*b*c^5 + 3*b*c^3 - 10*b*c)*x^7 + 3*b*c^3*x^
3 - 10*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 4*(8*a*c^8*x^8 - 4
*a*c^6*x^6 - a*c^4*x^4 - 18*a*c^2*x^2 + (8*b*c^8*x^8 - 4*b*c^6*x^6 - b*c^4
*x^4 - 18*b*c^2*x^2 + 15*b)*arccos(c*x) + 15*a)*sqrt(-c^2*d*x^2 + d))/(c^2
*x^9 - x^7), 1/420*(32*(b*c^9*x^9 - b*c^7*x^7)*sqrt(-d)*arctan(sqrt(-c^2*d
*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x
^2 - d)) + (8*b*c^5*x^5 - (8*b*c^5 + 3*b*c^3 - 10*b*c)*x^7 + 3*b*c^3*x^3 -
 10*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 4*(8*a*c^8*x^8 - 4*a*
c^6*x^6 - a*c^4*x^4 - 18*a*c^2*x^2 + (8*b*c^8*x^8 - 4*b*c^6*x^6 - b*c^4*x^
4 - 18*b*c^2*x^2 + 15*b)*arccos(c*x) + 15*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^
9 - x^7)]
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^8}\,dx = \int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))}{x^8}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))/x**8,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))/x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.76

$$\begin{aligned}
&\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^8}\,dx \\
&= \frac{1}{420}\left(32\,c^6\sqrt{d}\log\left(x\right)-\frac{8\,c^4\sqrt{d}x^4+3\,c^2\sqrt{d}x^2-10\,\sqrt{d}}{x^6}\right)bc \\
&\quad-\frac{1}{105}\left(\frac{8\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4}{dx^3}+\frac{12\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2}{dx^5}+\frac{15\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{dx^7}\right)b\arccos\left(cx\right) \\
&\quad-\frac{1}{105}\left(\frac{8\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4}{dx^3}+\frac{12\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2}{dx^5}+\frac{15\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{dx^7}\right)a
\end{aligned}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^8,x, algorithm="maxima"
)
```

output
```
1/420*(32*c^6*sqrt(d)*log(x) - (8*c^4*sqrt(d)*x^4 + 3*c^2*sqrt(d)*x^2 - 10
*sqrt(d))/x^6)*b*c - 1/105*(8*(-c^2*d*x^2 + d)^(3/2)*c^4/(d*x^3) + 12*(-c^
2*d*x^2 + d)^(3/2)*c^2/(d*x^5) + 15*(-c^2*d*x^2 + d)^(3/2)/(d*x^7))*b*arcc
os(c*x) - 1/105*(8*(-c^2*d*x^2 + d)^(3/2)*c^4/(d*x^3) + 12*(-c^2*d*x^2 + d
)^(3/2)*c^2/(d*x^5) + 15*(-c^2*d*x^2 + d)^(3/2)/(d*x^7))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^8}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^8,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^8}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^8}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/x^8,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^8}\,dx$$
$$= \frac{\sqrt{d}\left(8\sqrt{-c^2x^2+1}\,a\,c^6x^6+4\sqrt{-c^2x^2+1}\,a\,c^4x^4+3\sqrt{-c^2x^2+1}\,a\,c^2x^2-15\sqrt{-c^2x^2+1}\,a+105\left(\int \frac{\sqrt{-c^2}}{}\right.\right.}{105x^7}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))/x^8,x)
```

output
```
(sqrt(d)*(8*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 + 4*sqrt( - c**2*x**2 + 1)*
a*c**4*x**4 + 3*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 15*sqrt( - c**2*x**2
+ 1)*a + 105*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**8,x)*b*x**7))/(105*
x**7)
```

3.64 $\int x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 256

$$\int x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \frac{8bx\sqrt{d-c^2dx^2}}{105c^5\sqrt{1-c^2x^2}}+\frac{4bx^3\sqrt{d-c^2dx^2}}{315c^3\sqrt{1-c^2x^2}}+\frac{bx^5\sqrt{d-c^2dx^2}}{175c\sqrt{1-c^2x^2}}-\frac{bcx^7\sqrt{d-c^2dx^2}}{49\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{3c^6d}+\frac{2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{5c^6d^2}-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{7c^6d^3}$$

output
```
8/105*b*x*(-c^2*d*x^2+d)^(1/2)/c^5/(-c^2*x^2+1)^(1/2)+4/315*b*x^3*(-c^2*d*
x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/175*b*x^5*(-c^2*d*x^2+d)^(1/2)/c/(-c
^2*x^2+1)^(1/2)-1/49*b*c*x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/3*(
-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/c^6/d+2/5*(-c^2*d*x^2+d)^(5/2)*(a+b*
arccos(c*x))/c^6/d^2-1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccos(c*x))/c^6/d^3
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.57

$$\int x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$=\frac{\sqrt{d-c^2dx^2}\Big(105a(-1+c^2x^2)^2\,(8+12c^2x^2+15c^4x^4)+bcx\sqrt{1-c^2x^2}(840+140c^2x^2+63c^4x^4-225c^6}{11025c^6\,(-1+c^2x^2)}$$

input
```
Integrate[x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(105*a*(-1 + c^2*x^2)^2*(8 + 12*c^2*x^2 + 15*c^4*x^4)
 + b*c*x*Sqrt[1 - c^2*x^2]*(840 + 140*c^2*x^2 + 63*c^4*x^4 - 225*c^6*x^6)
+ 105*b*(-1 + c^2*x^2)^2*(8 + 12*c^2*x^2 + 15*c^4*x^4)*ArcCos[c*x]))/(1102
5*c^6*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.67, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5195, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5195}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{-15c^6x^6+3c^4x^4+4c^2x^2+8}{105c^6}dx}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^6d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3c^6d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{d-c^2dx^2}\int\left(-15c^6x^6+3c^4x^4+4c^2x^2+8\right)dx}{105c^5\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^6d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3c^6d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^6d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3c^6d}-\frac{b\left(-\frac{15}{7}c^6x^7+\frac{3c^4x^5}{5}+\frac{4c^2x^3}{3}+8x\right)\sqrt{d-c^2dx^2}}{105c^5\sqrt{1-c^2x^2}}$$

input
```
Int[x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
-1/105*(b*Sqrt[d - c^2*d*x^2]*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (15*c
^6*x^7)/7))/(c^5*Sqrt[1 - c^2*x^2]) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos
[c*x]))/(3*c^6*d) + (2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^6*d
^2) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(7*c^6*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.77

method	result
orering	$\frac{(2925c^8x^8-3393c^6x^6-630c^4x^4-4760c^2x^2+5040)\sqrt{-c^2d\,x^2+d}\,(a+b\arccos(cx))}{11025c^6(c^2x^2-1)} - \frac{(225c^6x^6-63c^4x^4-140c^2x^2-840)\left(5x^4\sqrt{-c\right.}$
default	$a\left(-\frac{x^4(-c^2d\,x^2+d)^{\frac{3}{2}}}{7c^2d}+\frac{-\frac{4x^2\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{35c^2d}-\frac{8\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{105d\,c^4}}{c^2}\right)+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\left(64c^8x^8-144c^6x^6+64i\sqrt{-c^2x^2+1}\,x\right.}{}\right.$
parts	$a\left(-\frac{x^4(-c^2d\,x^2+d)^{\frac{3}{2}}}{7c^2d}+\frac{-\frac{4x^2\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{35c^2d}-\frac{8\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{105d\,c^4}}{c^2}\right)+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\left(64c^8x^8-144c^6x^6+64i\sqrt{-c^2x^2+1}\,x\right.}{}\right.$

input
```
int(x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/11025*(2925*c^8*x^8-3393*c^6*x^6-630*c^4*x^4-4760*c^2*x^2+5040)/c^6/(c^2
*x^2-1)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))-1/11025/x^4*(225*c^6*x^6-63
*c^4*x^4-140*c^2*x^2-840)/c^6*(5*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)
)-x^6/(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*d*c^2-b*c*x^5*(-c^2*d*x^2+d)^
(1/2)/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.69

$$\int x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx =$$
$$-\frac{(225\,bc^7x^7-63\,bc^5x^5-140\,bc^3x^3-840\,bcx)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}-105\,(15\,ac^8x^8-18\,ac^6x^6-a}{11025\,(c^8x^2}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
-1/11025*((225*b*c^7*x^7 - 63*b*c^5*x^5 - 140*b*c^3*x^3 - 840*b*c*x)*sqrt(
-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 105*(15*a*c^8*x^8 - 18*a*c^6*x^6 - a*
c^4*x^4 - 4*a*c^2*x^2 + (15*b*c^8*x^8 - 18*b*c^6*x^6 - b*c^4*x^4 - 4*b*c^2
*x^2 + 8*b)*arccos(c*x) + 8*a)*sqrt(-c^2*d*x^2 + d))/(c^8*x^2 - c^6)
```

Sympy [F]

$$\int x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int x^5\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))\,dx$$

input
```
integrate(x**5*(-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
Integral(x**5*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.77

$$\int x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$=-\frac{1}{105}\left(\frac{15\,(-c^2dx^2+d)^{\frac{3}{2}}x^4}{c^2d}+\frac{12\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^4d}+\frac{8\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^6d}\right)b\arccos\,(cx)-\frac{1}{105}\left(\frac{15\,(-c^2dx^2+d)^{\frac{3}{2}}x^4}{c^2d}+\frac{12\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^4d}+\frac{8\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^6d}\right)a+\frac{\left(225\,c^6\sqrt{d}x^7-63\,c^4\sqrt{d}x^5-140\,c^2\sqrt{d}x^3-840\,\sqrt{d}x\right)b}{11025\,c^5}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
-1/105*(15*(-c^2*d*x^2 + d)^(3/2)*x^4/(c^2*d) + 12*(-c^2*d*x^2 + d)^(3/2)*
x^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(3/2)/(c^6*d))*b*arccos(c*x) - 1/105*(15*
(-c^2*d*x^2 + d)^(3/2)*x^4/(c^2*d) + 12*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^4*d)
 + 8*(-c^2*d*x^2 + d)^(3/2)/(c^6*d))*a + 1/11025*(225*c^6*sqrt(d)*x^7 - 63
*c^4*sqrt(d)*x^5 - 140*c^2*sqrt(d)*x^3 - 840*sqrt(d)*x)*b/c^5
```

Giac [F(-2)]

Exception generated.

$$\int x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int x^5\,(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^5*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^5*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$=\frac{\sqrt{d}\,\left(15\sqrt{-c^2x^2+1}\,a\,c^6x^6-3\sqrt{-c^2x^2+1}\,a\,c^4x^4-4\sqrt{-c^2x^2+1}\,a\,c^2x^2-8\sqrt{-c^2x^2+1}\,a+105\left(\int\sqrt{-c\right.\right.}{105c^6}$$

input
```
int(x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*(15*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - 3*sqrt( - c**2*x**2 + 1)
*a*c**4*x**4 - 4*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 8*sqrt( - c**2*x**2
+ 1)*a + 105*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**5,x)*b*c**6))/(105*c*
*6)
```

3.65 $\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 183

$$\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\frac{2bx\sqrt{d-c^2dx^2}}{15c^3\sqrt{1-c^2x^2}}+\frac{bx^3\sqrt{d-c^2dx^2}}{45c\sqrt{1-c^2x^2}}-\frac{bcx^5\sqrt{d-c^2dx^2}}{25\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3c^4d}+\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^4d^2}$$

output
```
2/15*b*x*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/45*b*x^3*(-c^2*d*x^
2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/25*b*c*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^
2+1)^(1/2)-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/c^4/d+1/5*(-c^2*d*x^
2+d)^(5/2)*(a+b*arccos(c*x))/c^4/d^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.67

$$\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$=\frac{\sqrt{d-c^2dx^2}\left(15a(-1+c^2x^2)^2\left(2+3c^2x^2\right)+bcx\sqrt{1-c^2x^2}(30+5c^2x^2-9c^4x^4)+15b(-1+c^2x^2)^2\left(2+\right.\right.}{225c^4\left(-1+c^2x^2\right)}$$

input
```
Integrate[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(15*a*(-1 + c^2*x^2)^2*(2 + 3*c^2*x^2) + b*c*x*Sqrt[1
 - c^2*x^2]*(30 + 5*c^2*x^2 - 9*c^4*x^4) + 15*b*(-1 + c^2*x^2)^2*(2 + 3*c^
2*x^2)*ArcCos[c*x]))/(225*c^4*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5195, 27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$\Big\downarrow \text{5195}$$
$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{-3c^4x^4+c^2x^2+2}{15c^4}dx}{\sqrt{1-c^2x^2}}+\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^4d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3c^4d}$$
$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{d-c^2dx^2}\int\left(-3c^4x^4+c^2x^2+2\right)dx}{15c^3\sqrt{1-c^2x^2}}+\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^4d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3c^4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^4d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3c^4d}-\frac{b\left(-\frac{3}{5}c^4x^5+\frac{c^2x^3}{3}+2x\right)\sqrt{d-c^2dx^2}}{15c^3\sqrt{1-c^2x^2}}$$

input
```
Int[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
-1/15*(b*Sqrt[d - c^2*d*x^2]*(2*x + (c^2*x^3)/3 - (3*c^4*x^5)/5))/(c^3*Sqr
t[1 - c^2*x^2]) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(3*c^4*d) +
((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^4*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.98

method	result
orering	$\frac{(81c^6x^6-107c^4x^4-120c^2x^2+120)\sqrt{-c^2d\,x^2+d}\,(a+b\arccos(cx))}{225c^4(c^2x^2-1)}-\frac{(9c^4x^4-5c^2x^2-30)\left(3x^2\sqrt{-c^2d\,x^2+d}\,(a+b\arccos(cx))-\frac{x^4}{}\right.}{225x^2c^4}$
default	$a\left(-\frac{x^2(-c^2d\,x^2+d)^{\frac{3}{2}}}{5c^2d}-\frac{2(-c^2d\,x^2+d)^{\frac{3}{2}}}{15d\,c^4}\right)+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\left(16c^6x^6-28c^4x^4+16i\sqrt{-c^2x^2+1}\,x^5c^5+13c^2x^2-20i\sqrt{-c^2}\right.}{800c^4(c^2x^2-1)}\right.$
parts	$a\left(-\frac{x^2(-c^2d\,x^2+d)^{\frac{3}{2}}}{5c^2d}-\frac{2(-c^2d\,x^2+d)^{\frac{3}{2}}}{15d\,c^4}\right)+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\left(16c^6x^6-28c^4x^4+16i\sqrt{-c^2x^2+1}\,x^5c^5+13c^2x^2-20i\sqrt{-c^2}\right.}{800c^4(c^2x^2-1)}\right.$

input
```
int(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/225*(81*c^6*x^6-107*c^4*x^4-120*c^2*x^2+120)/c^4/(c^2*x^2-1)*(-c^2*d*x^2
+d)^(1/2)*(a+b*arccos(c*x))-1/225/x^2*(9*c^4*x^4-5*c^2*x^2-30)/c^4*(3*x^2*
(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))-x^4/(-c^2*d*x^2+d)^(1/2)*(a+b*arcco
s(c*x))*d*c^2-b*c*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.82

$$\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx =$$
$$-\frac{(9\,bc^5x^5-5\,bc^3x^3-30\,bcx)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}-15\,(3\,ac^6x^6-4\,ac^4x^4-ac^2x^2+(3\,bc^6x^6-4\,b}{225\,(c^6x^2-c^4)}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
-1/225*((9*b*c^5*x^5 - 5*b*c^3*x^3 - 30*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-
c^2*x^2 + 1) - 15*(3*a*c^6*x^6 - 4*a*c^4*x^4 - a*c^2*x^2 + (3*b*c^6*x^6 -
4*b*c^4*x^4 - b*c^2*x^2 + 2*b)*arccos(c*x) + 2*a)*sqrt(-c^2*d*x^2 + d))/(c
^6*x^2 - c^4)
```

Sympy [F]

$$\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int x^3\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\operatorname{acos}{(cx)})\;dx$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
Integral(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.75

$$\begin{aligned}&\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx\\&=-\frac{1}{15}\,b\left(\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\arccos{(cx)}\\&\quad-\frac{1}{15}\,a\left(\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\\&\quad+\frac{\left(9\,c^4\sqrt{d}x^5-5\,c^2\sqrt{d}x^3-30\,\sqrt{d}x\right)b}{225\,c^3}\end{aligned}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
-1/15*b*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/(
c^4*d))*arccos(c*x) - 1/15*a*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c
^2*d*x^2 + d)^(3/2)/(c^4*d)) + 1/225*(9*c^4*sqrt(d)*x^5 - 5*c^2*sqrt(d)*x^
3 - 30*sqrt(d)*x)*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int x^3\,(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$= \frac{\sqrt{d}\,\left(3\sqrt{-c^2x^2+1}\,a\,c^4x^4-\sqrt{-c^2x^2+1}\,a\,c^2x^2-2\sqrt{-c^2x^2+1}\,a+15\left(\int\sqrt{-c^2x^2+1}\,acos(cx)\,x^3dx\right)b\,c^4}{15c^4}$$

input
```
int(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*(3*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 - sqrt( - c**2*x**2 + 1)*a*
c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a + 15*int(sqrt( - c**2*x**2 + 1)*aco
s(c*x)*x**3,x)*b*c**4))/(15*c**4)
```

3.66 $\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 110

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\frac{bx\sqrt{d-c^2dx^2}}{3c\sqrt{1-c^2x^2}}-\frac{bcx^3\sqrt{d-c^2dx^2}}{9\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3c^2d}$$

output
```
1/3*b*x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/9*b*c*x^3*(-c^2*d*x^2+
d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/c^2
/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$=\frac{\sqrt{d-c^2dx^2}\left(bcx\sqrt{1-c^2x^2}(3-c^2x^2)+3a(-1+c^2x^2)^2+3b(-1+c^2x^2)^2\arccos(cx)\right)}{9c^2\left(-1+c^2x^2\right)}$$

input
```
Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(b*c*x*Sqrt[1 - c^2*x^2]*(3 - c^2*x^2) + 3*a*(-1 + c^
2*x^2)^2 + 3*b*(-1 + c^2*x^2)^2*ArcCos[c*x]))/(9*c^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5183, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5183}$$

$$-\frac{b\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)dx}{3c\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3c^2d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3c^2d}-\frac{b\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3c\sqrt{1-c^2x^2}}$$

input
```
Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
-1/3*(b*Sqrt[d - c^2*d*x^2]*(x - (c^2*x^3)/3))/(c*Sqrt[1 - c^2*x^2]) - ((d
 - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(3*c^2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.40

method	result
orering	$\frac{(5c^4x^4-13c^2x^2+6)\sqrt{-c^2dx^2+d}\,(a+b\arccos(cx))}{9c^2(c^2x^2-1)}-\frac{(c^2x^2-3)\left(\sqrt{-c^2dx^2+d}\,(a+b\arccos(cx))-\frac{x^2(a+b\arccos(cx))dc^2}{\sqrt{-c^2dx^2+d}}-\frac{bcx\sqrt{-c^2}}{\sqrt{-c^2}}\right.}{9c^2}$ …
default	$-\frac{a(-c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\left(4c^4x^4-5c^2x^2+4i\sqrt{-c^2x^2+1}\,x^3c^3-3i\sqrt{-c^2x^2+1}\,xc+1\right)(i+3\arccos(cx))}{72c^2(c^2x^2-1)}-\frac{\sqrt{-d(c^2}}{}\right.$ …
parts	$-\frac{a(-c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\left(4c^4x^4-5c^2x^2+4i\sqrt{-c^2x^2+1}\,x^3c^3-3i\sqrt{-c^2x^2+1}\,xc+1\right)(i+3\arccos(cx))}{72c^2(c^2x^2-1)}-\frac{\sqrt{-d(c^2}}{}\right.$ …

input
```
int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/9*(5*c^4*x^4-13*c^2*x^2+6)/c^2/(c^2*x^2-1)*(-c^2*d*x^2+d)^(1/2)*(a+b*arc
cos(c*x))-1/9*(c^2*x^2-3)/c^2*((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))-x^2/
(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*d*c^2-b*c*x*(-c^2*d*x^2+d)^(1/2)/(-
c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx =$$
$$-\frac{(bc^3x^3-3\,bcx)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}-3\,(ac^4x^4-2\,ac^2x^2+(bc^4x^4-2\,bc^2x^2+b)\arccos{(cx)}+a}{9\,(c^4x^2-c^2)}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-1/9*((b*c^3*x^3 - 3*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 3*(a
*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c^2*x^2 + b)*arccos(c*x) + a)*sq
rt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int x\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))\,dx$$

input
```
integrate(x*(-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = -\frac{(-c^2dx^2+d)^{\frac{3}{2}}b\arccos{(cx)}}{3\,c^2d} + \frac{\left(c^2d^{\frac{3}{2}}x^3-3\,d^{\frac{3}{2}}x\right)b}{9\,cd} - \frac{(-c^2dx^2+d)^{\frac{3}{2}}a}{3\,c^2d}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-1/3*(-c^2*d*x^2 + d)^(3/2)*b*arccos(c*x)/(c^2*d) + 1/9*(c^2*d^(3/2)*x^3 -
 3*d^(3/2)*x)*b/(c*d) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a/(c^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int x\,(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$=\frac{\sqrt{d}\left(\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a+3\left(\int\sqrt{-c^2x^2+1}\,acos(cx)\,xdx\right)b\,c^2\right)}{3c^2}$$

input
```
int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*(sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a +
3*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x,x)*b*c**2))/(3*c**2)
```

3.67 $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 203

$$\begin{aligned}
&\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}\,dx \\
&= -\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arccos(cx)) \\
&\quad - \frac{2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&\quad + \frac{ib\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} - \frac{ib\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}
\end{aligned}$$

output
```
-b*c*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+(-c^2*d*x^2+d)^(1/2)*(a+b*a
rccos(c*x))-2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*arctanh(c*x+I*(-c^2*x
^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+I*b*(-c^2*d*x^2+d)^(1/2)*polylog(2,-c*x-I*
(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-I*b*(-c^2*d*x^2+d)^(1/2)*polylog(2,
c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}\,dx$$
$$= a\sqrt{d-c^2dx^2}+a\sqrt{d}\log(x)-a\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)$$
$$+\frac{b\sqrt{d-c^2dx^2}\left(cx+\sqrt{1-c^2x^2}\arccos(cx)-\arccos(cx)\log\left(1-ie^{i\arccos(cx)}\right)+\arccos(cx)\log\left(1+ie^{i\,\mathrm{ar}}\right.\right.}{\sqrt{1-c^2x^2}}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x,x]
```

output
```
a*Sqrt[d - c^2*d*x^2] + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[
d - c^2*d*x^2]] + (b*Sqrt[d - c^2*d*x^2]*(c*x + Sqrt[1 - c^2*x^2]*ArcCos[c
*x] - ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] + ArcCos[c*x]*Log[1 + I*E^(
I*ArcCos[c*x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I*PolyLog[2, I*E^
(I*ArcCos[c*x])]))/Sqrt[1 - c^2*x^2]
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.76, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5199, 24, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}\,dx$$
$$\Big\downarrow \text{5199}$$
$$\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int 1dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5219}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\sqrt{d-c^2dx^2}\left(-b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(e^{i\arccos}\right.\right.}{\sqrt{1-c^2x^2}}$$
$$\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\sqrt{d-c^2dx^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos}\right.}{\sqrt{1-c^2x^2}}$$
$$\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{\sqrt{d-c^2dx^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,ie^{i\arccos}\right.\right.}{\sqrt{1-c^2x^2}}$$
$$\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x,x]
```

output
```
(b*c*x*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2] + Sqrt[d - c^2*d*x^2]*(a + b
*ArcCos[c*x]) - (Sqrt[d - c^2*d*x^2]*((-2*I)*(a + b*ArcCos[c*x])*ArcTan[E^
(I*ArcCos[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*b*PolyLog[2,
 I*E^(I*ArcCos[c*x])]))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.63

method	result
default	$-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)a+\sqrt{-c^2d\,x^2+d}\,a+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)(\arccos(cx)+}{2c^2x^2-2}\right.$
parts	$-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)a+\sqrt{-c^2d\,x^2+d}\,a+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)(\arccos(cx)+}{2c^2x^2-2}\right.$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
-d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)*a+(-c^2*d*x^2+d)^(1/2)
*a+b*(1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arc
cos(c*x)+I)/(c^2*x^2-1)+1/2*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*
x*c+c^2*x^2-1)*(arccos(c*x)-I)/(c^2*x^2-1)-(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^
2+1)^(1/2)/(c^2*x^2-1)*(arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-arc
cos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-I*dilog(1+I*(c*x+I*(-c^2*x^2+1
)^(1/2)))+I*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))))
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}\,dx = \int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))}{x}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))/x,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\arccos\,(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x,x, algorithm="maxima")
```

output
```
b*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x)/x, x) - (sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(
x) + 2*d/abs(x)) - sqrt(-c^2*d*x^2 + d))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/x,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}\,dx=\sqrt{d}\left(\sqrt{-c^2x^2+1}\,a+\left(\int \frac{\sqrt{-c^2x^2+1}\,acos(cx)}{x}dx\right)b+\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a-a\right)$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))/x,x)
```

output
```
sqrt(d)*(sqrt( - c**2*x**2 + 1)*a + int((sqrt( - c**2*x**2 + 1)*acos(c*x))
/x,x)*b + log(tan(asin(c*x)/2))*a - a)
```

3.68 $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 225

$$\begin{aligned}\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^3}\,dx &= -\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2x^2}\\ &+\frac{c^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}\\ &-\frac{ibc^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{2\sqrt{1-c^2x^2}}\\ &+\frac{ibc^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-1/2*b*c*(-c^2*d*x^2+d)^(1/2)/x/(-c^2*x^2+1)^(1/2)-1/2*(-c^2*d*x^2+d)^(1/2
)*(a+b*arccos(c*x))/x^2+c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*arctanh
(c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-1/2*I*b*c^2*(-c^2*d*x^2+d)^(
1/2)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+1/2*I*b*c^2*(
-c^2*d*x^2+d)^(1/2)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^3}\,dx$$
$$=\frac{1}{4}\left(-\frac{2a\sqrt{d-c^2dx^2}}{x^2}-2ac^2\sqrt{d}\log(x)+2ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)\right.$$
$$-\frac{2bd\sqrt{1-c^2x^2}\left(-cx+\sqrt{1-c^2x^2}\arccos(cx)-c^2x^2\arccos(cx)\log\left(1-ie^{i\arccos(cx)}\right)+c^2x^2\arccos(cx)\right.}{x^2\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
((-2*a*Sqrt[d - c^2*d*x^2])/x^2 - 2*a*c^2*Sqrt[d]*Log[x] + 2*a*c^2*Sqrt[d]
*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (2*b*d*Sqrt[1 - c^2*x^2]*(-(c*x) +
 Sqrt[1 - c^2*x^2]*ArcCos[c*x] - c^2*x^2*ArcCos[c*x]*Log[1 - I*E^(I*ArcCos
[c*x])] + c^2*x^2*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] - I*c^2*x^2*Pol
yLog[2, (-I)*E^(I*ArcCos[c*x])] + I*c^2*x^2*PolyLog[2, I*E^(I*ArcCos[c*x])
]))/(x^2*Sqrt[d - c^2*d*x^2]))/4
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.76, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5197, 15, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^3}\,dx$$
$$\Big\downarrow \text{5197}$$
$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}-\frac{bc\sqrt{d-c^2dx^2}\int \frac{1}{x^2}dx}{2\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2x^2}$$
$$\Big\downarrow \text{15}$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2x^2}+\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5219

$$\frac{c^2\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{cx}d\arccos(cx)}{2\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2x^2}+\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 3042

$$\frac{c^2\sqrt{d-c^2dx^2}\int(a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{2\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2x^2}+\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 4669

$$\frac{c^2\sqrt{d-c^2dx^2}\left(-b\int\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(e^{i\arccos(cx)}\right)\right.}{2\sqrt{1-c^2x^2}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2x^2}+\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 2715

$$\frac{c^2\sqrt{d-c^2dx^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right.}{2\sqrt{1-c^2x^2}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2x^2}+\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 2838

$$\frac{c^2\sqrt{d-c^2dx^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\right.}{2\sqrt{1-c^2x^2}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2x^2}+\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{1-c^2x^2}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
(b*c*Sqrt[d - c^2*d*x^2])/(2*x*Sqrt[1 - c^2*x^2]) - (Sqrt[d - c^2*d*x^2]*(
a + b*ArcCos[c*x]))/(2*x^2) + (c^2*Sqrt[d - c^2*d*x^2]*((-2*I)*(a + b*ArcC
os[c*x])*ArcTan[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])
] - I*b*PolyLog[2, I*E^(I*ArcCos[c*x])]))/(2*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5197
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^2
]/Sqrt[1 - c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x
] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 2)*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.35

method	result
default	$a\left(-\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{2 d\,x^{2}}-\frac{c^{2}\left(\sqrt{-c^{2} d\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2 d+2 \sqrt{d}\,\sqrt{-c^{2} d\,x^{2}+d}}{x}\right)\right)}{2}\right)+b\left(-\frac{\left(c^{2} x^{2} \arccos(cx)+cx\sqrt{-c^{2} x^{2}+1}-\operatorname{arccc}\right.}{2\left(c^{2} x^{2}-1\right) x^{2}}\right.$
parts	$a\left(-\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{2 d\,x^{2}}-\frac{c^{2}\left(\sqrt{-c^{2} d\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2 d+2 \sqrt{d}\,\sqrt{-c^{2} d\,x^{2}+d}}{x}\right)\right)}{2}\right)+b\left(-\frac{\left(c^{2} x^{2} \arccos(cx)+cx\sqrt{-c^{2} x^{2}+1}-\operatorname{arccc}\right.}{2\left(c^{2} x^{2}-1\right) x^{2}}\right.$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/d/x^2*(-c^2*d*x^2+d)^(3/2)-1/2*c^2*((-c^2*d*x^2+d)^(1/2)-d^(1/2)*l
n((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)))+b*(-1/2*(c^2*x^2*arccos(c*x)+c
*x*(-c^2*x^2+1)^(1/2)-arccos(c*x))*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/x^2+
1/2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*(arccos(c*x)*ln(
1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/
2)))-I*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+I*dilog(1-I*(c*x+I*(-c^2*x^2+
1)^(1/2))))*c^2)
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^3}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^3,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^3}\,dx=\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))}{x^3}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))/x**3,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^3}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\arccos\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^3,x, algorithm="maxima"
)
```

output
```
b*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x)/x^3, x) + 1/2*(c^2*sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sq
rt(d)/abs(x) + 2*d/abs(x)) - sqrt(-c^2*d*x^2 + d)*c^2 - (-c^2*d*x^2 + d)^(
3/2)/(d*x^2))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^3}\,dx = \int \frac{(a+b\operatorname{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^3}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/x^3,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^3}\,dx = \frac{\sqrt{d}\left(-\sqrt{-c^2x^2+1}\,a+2\left(\int \frac{\sqrt{-c^2x^2+1}\,acos(cx)}{x^3}dx\right)b\,x^2-\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a\,c^2x^2\right)}{2x^2}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))/x^3,x)
```

output

```
(sqrt(d)*( - sqrt( - c**2*x**2 + 1)*a + 2*int((sqrt( - c**2*x**2 + 1)*acos
(c*x))/x**3,x)*b*x**2 - log(tan(asin(c*x)/2))*a*c**2*x**2))/(2*x**2)
```

3.69 $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 301

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^5}\,dx = -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}}+\frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4x^4}+\frac{c^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{8x^2} + \frac{c^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))\text{arctanh}\left(e^{i\arccos(cx)}\right)}{4\sqrt{1-c^2x^2}} - \frac{ibc^4\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{8\sqrt{1-c^2x^2}} + \frac{ibc^4\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{8\sqrt{1-c^2x^2}}$$

output
```
-1/12*b*c*(-c^2*d*x^2+d)^(1/2)/x^3/(-c^2*x^2+1)^(1/2)+1/8*b*c^3*(-c^2*d*x^
2+d)^(1/2)/x/(-c^2*x^2+1)^(1/2)-1/4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))
/x^4+1/8*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^2+1/4*c^4*(-c^2*d*x^
2+d)^(1/2)*(a+b*arccos(c*x))*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1
)^(1/2)-1/8*I*b*c^4*(-c^2*d*x^2+d)^(1/2)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/
2))/(-c^2*x^2+1)^(1/2)+1/8*I*b*c^4*(-c^2*d*x^2+d)^(1/2)*polylog(2,c*x+I*(-
c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^5}\,dx$$
$$=\frac{1}{24}\left(\frac{3a(-2+c^2x^2)\sqrt{d-c^2dx^2}}{x^4}-3ac^4\sqrt{d}\log(x)+3ac^4\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)\right.$$
$$+\frac{b\sqrt{d-c^2dx^2}\left(2cx-3c^3x^3-6\sqrt{1-c^2x^2}\arccos(cx)+3c^2x^2\sqrt{1-c^2x^2}\arccos(cx)+3c^4x^4\arccos(cx)\right.}{}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x^5,x]
```

output
```
((3*a*(-2 + c^2*x^2)*Sqrt[d - c^2*d*x^2])/x^4 - 3*a*c^4*Sqrt[d]*Log[x] + 3
*a*c^4*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*Sqrt[d - c^2*d*x^
2]*(2*c*x - 3*c^3*x^3 - 6*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + 3*c^2*x^2*Sqrt[1
 - c^2*x^2]*ArcCos[c*x] + 3*c^4*x^4*ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x]
)] - 3*c^4*x^4*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] + (3*I)*c^4*x^4*Po
lyLog[2, (-I)*E^(I*ArcCos[c*x])] - (3*I)*c^4*x^4*PolyLog[2, I*E^(I*ArcCos[
c*x])]))/(x^4*Sqrt[1 - c^2*x^2]))/24
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.72, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5197, 15, 5205, 15, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^5}\,dx$$
$$\Big\downarrow \text{5197}$$
$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{x^3\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}-\frac{bc\sqrt{d-c^2dx^2}\int \frac{1}{x^4}dx}{4\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4x^4}$$

$$\Big\downarrow 15$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{x^3\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4x^4}+\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow 5205$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\left(\frac{1}{2}c^2\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{1}{2}bc\int\frac{1}{x^2}dx-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}\right)}{4\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4x^4}+\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow 15$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\left(\frac{1}{2}c^2\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}+\frac{bc}{2x}\right)}{4\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4x^4}+\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow 5219$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\left(-\frac{1}{2}c^2\int\frac{a+b\arccos(cx)}{cx}d\arccos(cx)-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}+\frac{bc}{2x}\right)}{4\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4x^4}+\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow 3042$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\left(-\frac{1}{2}c^2\int(a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}+\frac{bc}{2x}\right)}{4\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4x^4}+\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow 4669$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\left(-\frac{1}{2}c^2\left(-b\int\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arcta\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4x^4}+\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow 2715$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\left(-\frac{1}{2}c^2\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4x^4}+\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\left(-\frac{1}{2}c^2\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4x^4}+\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x^5,x]
```

output
```
(b*c*Sqrt[d - c^2*d*x^2])/(12*x^3*Sqrt[1 - c^2*x^2]) - (Sqrt[d - c^2*d*x^2
]*(a + b*ArcCos[c*x]))/(4*x^4) - (c^2*Sqrt[d - c^2*d*x^2]*((b*c)/(2*x) - (
Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(2*x^2) - (c^2*((-2*I)*(a + b*ArcCo
s[c*x])*ArcTan[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])]
 - I*b*PolyLog[2, I*E^(I*ArcCos[c*x])]))/2))/(4*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5197
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^2
]/Sqrt[1 - c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x
] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 2)*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.21

method	result
default	$-\frac{a(-c^2 d\,x^2+d)^{\frac{3}{2}}}{4d\,x^4}-\frac{a\,c^2(-c^2 d\,x^2+d)^{\frac{3}{2}}}{8d\,x^2}+\frac{a\,c^4\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)}{8}-\frac{a\,c^4\sqrt{-c^2 d\,x^2+d}}{8}+b\Bigg(\frac{\left(3c^4x^4\arccos(cx)-\right.}{}$
parts	$-\frac{a(-c^2 d\,x^2+d)^{\frac{3}{2}}}{4d\,x^4}-\frac{a\,c^2(-c^2 d\,x^2+d)^{\frac{3}{2}}}{8d\,x^2}+\frac{a\,c^4\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)}{8}-\frac{a\,c^4\sqrt{-c^2 d\,x^2+d}}{8}+b\Bigg(\frac{\left(3c^4x^4\arccos(cx)-\right.}{}$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/d/x^4*(-c^2*d*x^2+d)^(3/2)-1/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(3/2)+1/8
*a*c^4*d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-1/8*a*c^4*(-c^2*
d*x^2+d)^(1/2)+b*(1/24*(3*c^4*x^4*arccos(c*x)+3*c^3*x^3*(-c^2*x^2+1)^(1/2)
-9*c^2*x^2*arccos(c*x)-2*c*x*(-c^2*x^2+1)^(1/2)+6*arccos(c*x))*(-d*(c^2*x^
2-1))^(1/2)/(c^2*x^2-1)/x^4+1/8*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/
(c^2*x^2-1)*(arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-arccos(c*x)*ln
(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-I*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+I
*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2))))*c^4)
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^5}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)}{x^5}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^5,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/x^5, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^5}\,dx = \int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\operatorname{acos}{(cx)})}{x^5}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))/x**5,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^5}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)}{x^5}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^5,x, algorithm="maxima"
)
```

output
```
b*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x)/x^5, x) + 1/8*(c^4*sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sq
rt(d)/abs(x) + 2*d/abs(x)) - sqrt(-c^2*d*x^2 + d)*c^4 - (-c^2*d*x^2 + d)^(
3/2)*c^2/(d*x^2) - 2*(-c^2*d*x^2 + d)^(3/2)/(d*x^4))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^5}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^5}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/x^5,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^5}\,dx = \frac{\sqrt{d}\left(\sqrt{-c^2x^2+1}\,a\,c^2x^2-2\sqrt{-c^2x^2+1}\,a+8\left(\int \frac{\sqrt{-c^2x^2+1}\,acos(cx)}{x^5}dx\right)b\,x^4-\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a\,c^4x^4\right)}{8x^4}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))/x^5,x)
```

output
```
(sqrt(d)*(sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a
+ 8*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**5,x)*b*x**4 - log(tan(asin(c
*x)/2))*a*c**4*x**4))/(8*x**4)
```

3.70 $\int x^4(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 340

$$\begin{aligned}\int x^4(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))\,dx &= \frac{3bdx^2\sqrt{d-c^2dx^2}}{256c^3\sqrt{1-c^2x^2}}+\frac{bdx^4\sqrt{d-c^2dx^2}}{256c\sqrt{1-c^2x^2}}\\ &-\frac{bcdx^6\sqrt{d-c^2dx^2}}{32\sqrt{1-c^2x^2}}+\frac{bc^3dx^8\sqrt{d-c^2dx^2}}{64\sqrt{1-c^2x^2}}-\frac{3dx\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{128c^4}\\ &-\frac{dx^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{64c^2}+\frac{1}{16}dx^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))\\ &+\frac{1}{8}x^5(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))+\frac{3d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{256bc^5\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
3/256*b*d*x^2*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/256*b*d*x^4*(-
c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/32*b*c*d*x^6*(-c^2*d*x^2+d)^(1/2
)/(-c^2*x^2+1)^(1/2)+1/64*b*c^3*d*x^8*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1
/2)-3/128*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c^4-1/64*d*x^3*(-c^2*
d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c^2+1/16*d*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*
arccos(c*x))+1/8*x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))+3/256*d*(-c^2*
d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/b/c^5/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.67

$$\int x^4 \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos(cx)) \, dx = \frac{-96bd\sqrt{d-c^2dx^2}\arccos(cx)^2 - 192ad^{3/2}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right) - d\sqrt{d-c^2 \cdots}}{\cdots}$$

input
```
Integrate[x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(-96*b*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 192*a*d^(3/2)*Sqrt[1 - c^2*x^
2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - d*Sqrt[d -
 c^2*d*x^2]*(64*a*c*x*Sqrt[1 - c^2*x^2]*(3 + 2*c^2*x^2 - 24*c^4*x^4 + 16*c
^6*x^6) - 16*b*Cos[4*ArcCos[c*x]] + b*Cos[8*ArcCos[c*x]]) - 8*b*d*Sqrt[d -
 c^2*d*x^2]*ArcCos[c*x]*(-8*Sin[4*ArcCos[c*x]] + Sin[8*ArcCos[c*x]]))/(819
2*c^5*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5203, 244, 2009, 5199, 15, 5211, 15, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx)) \, dx$$

$$\Big\downarrow \text{ 5203}$$

$$\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int x^5\left(1-c^2x^2\right)dx}{8\sqrt{1-c^2x^2}}+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 244}$$

$$\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int\left(x^5-c^2x^7\right)dx}{8\sqrt{1-c^2x^2}}+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))$$

$\Big\downarrow$ 2009

$$\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5199

$$\frac{3}{8}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{6\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int x^5dx}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 15

$$\frac{3}{8}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{1-c^2x^2}}\right)+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5211

$$\frac{3}{8}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{3\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{b\int x^3dx}{4c}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}\right)}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right.$$
$$\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 15

$$\frac{3}{8}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{3\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\right.$$

$$\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5211}$$

$$\frac{3}{8}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{3\left(\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int xdx}{2c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-\right.$$

$$\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{8}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{3\left(\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-c\right.$$

$$\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+$$

$$\frac{3}{8}d\left(\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{\sqrt{d-c^2dx^2}\left(-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}+\frac{3\left(-\frac{(a+b\arccos(cx))^2}{4bc^3}-\frac{x\sqrt{1-c^2x^2}(a+}{2c}\right.}{4c^2}\right.}{6\sqrt{1-c^2x^2}}\right.$$

$$\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c^2dx^2}}{8\sqrt{1-c^2x^2}}$$

input
```
Int[x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d*Sqrt[d - c^2*d*x^2]*(x^6/6 - (c^2*x^8)/8))/(8*Sqrt[1 - c^2*x^2]) +
(x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/8 + (3*d*((b*c*x^6*Sqrt[d
- c^2*d*x^2])/(36*Sqrt[1 - c^2*x^2]) + (x^5*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cos[c*x]))/6 + (Sqrt[d - c^2*d*x^2]*(-1/16*(b*x^4)/c - (x^3*Sqrt[1 - c^2*x
^2]*(a + b*ArcCos[c*x]))/(4*c^2) + (3*(-1/4*(b*x^2)/c - (x*Sqrt[1 - c^2*x^
2]*(a + b*ArcCos[c*x]))/(2*c^2) - (a + b*ArcCos[c*x])^2/(4*b*c^3)))/(4*c^2
)))/(6*Sqrt[1 - c^2*x^2])))/8
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.26

method	result
default	$-\frac{a\,x^{3}\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{8 c^{2} d}-\frac{a x \left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{16 c^{4} d}+\frac{a x \left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{64 c^{4}}+\frac{3 a d x \sqrt{-c^{2} d\,x^{2}+d}}{128 c^{4}}+\frac{3 a\,d^{2} \arctan\left(\frac{\sqrt{c^{2} d}\,x}{\sqrt{-c^{2} d\,x^{2}+d}}\right)}{128 c^{4} \sqrt{c^{2} d}}+b\Bigg($
parts	$-\frac{a\,x^{3}\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{8 c^{2} d}-\frac{a x \left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{16 c^{4} d}+\frac{a x \left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{64 c^{4}}+\frac{3 a d x \sqrt{-c^{2} d\,x^{2}+d}}{128 c^{4}}+\frac{3 a\,d^{2} \arctan\left(\frac{\sqrt{c^{2} d}\,x}{\sqrt{-c^{2} d\,x^{2}+d}}\right)}{128 c^{4} \sqrt{c^{2} d}}+b\Bigg($

input
```
int(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*a*x^3*(-c^2*d*x^2+d)^(5/2)/c^2/d-1/16*a/c^4*x*(-c^2*d*x^2+d)^(5/2)/d+
1/64*a/c^4*x*(-c^2*d*x^2+d)^(3/2)+3/128*a/c^4*d*x*(-c^2*d*x^2+d)^(1/2)+3/1
28*a/c^4*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*
(3/256*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^5/(c^2*x^2-1)*arccos(c*
x)^2*d-1/16384*(-d*(c^2*x^2-1))^(1/2)*(128*c^9*x^9-320*c^7*x^7+128*I*(-c^2
*x^2+1)^(1/2)*x^8*c^8+272*c^5*x^5-256*I*(-c^2*x^2+1)^(1/2)*x^6*c^6-88*c^3*
x^3+160*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c*x-32*I*(-c^2*x^2+1)^(1/2)*x^2*c^2
+I*(-c^2*x^2+1)^(1/2))*(I+8*arccos(c*x))*d/c^5/(c^2*x^2-1)+1/1024*(-d*(c^2
*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-
8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(I+4*arccos(c*x))*d/c
^5/(c^2*x^2-1)+1/1024*(-d*(c^2*x^2-1))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*
c^4+8*c^5*x^5+8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/
2)+4*c*x)*(-I+4*arccos(c*x))*d/c^5/(c^2*x^2-1)-1/16384*(-d*(c^2*x^2-1))^(1
/2)*(-128*I*(-c^2*x^2+1)^(1/2)*x^8*c^8+128*c^9*x^9+256*I*(-c^2*x^2+1)^(1/2
)*x^6*c^6-320*c^7*x^7-160*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+272*c^5*x^5+32*I*(-
c^2*x^2+1)^(1/2)*x^2*c^2-88*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+8*c*x)*(-I+8*arcc
os(c*x))*d/c^5/(c^2*x^2-1))
```

Fricas [F]

$$\int x^4 (d - c^2 d x^2)^{3/2} (a + b \arccos(cx)) \, dx = \int (-c^2 d x^2 + d)^{\frac{3}{2}} (b \arccos(cx) + a) x^4 \, dx$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
integral(-(a*c^2*d*x^6 - a*d*x^4 + (b*c^2*d*x^6 - b*d*x^4)*arccos(c*x))*sq
rt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^4 (d - c^2 d x^2)^{3/2} (a + b \arccos(cx)) \, dx = \text{Timed out}$$

input
```
integrate(x**4*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^4 (d - c^2 d x^2)^{3/2} (a + b \arccos(cx)) \, dx = \int (-c^2 d x^2 + d)^{\frac{3}{2}} (b \arccos(cx) + a) x^4 \, dx$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
b*sqrt(d)*integrate(-(c^2*d*x^6 - d*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arct
an2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x) - 1/128*(16*(-c^2*d*x^2 + d)^(5
/2)*x^3/(c^2*d) - 2*(-c^2*d*x^2 + d)^(3/2)*x/c^4 + 8*(-c^2*d*x^2 + d)^(5/2
)*x/(c^4*d) - 3*sqrt(-c^2*d*x^2 + d)*d*x/c^4 - 3*d^(3/2)*arcsin(c*x)/c^5)*
a
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.92

$$\int x^4 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = -\frac{1}{8}\sqrt{-c^2dx^2+d}ac^2dx^7 + \frac{3}{16}\sqrt{-c^2dx^2+d}adx^5 - \frac{\sqrt{-c^2dx^2+d}adx^3}{64\,c^2} - \frac{3\sqrt{-c^2dx^2+d}adx}{128\,c^4} - \frac{3\,ad^2\log\left(\left|-c\sqrt{-d}x+\sqrt{c^2x^2-1}\sqrt{-d}\right|\right)}{128\,c^5\sqrt{-d}} - \frac{128\,bc^7d^{\frac{3}{2}}x^8 + 1024\sqrt{-c^2x^2+1}bc^6d^{\frac{3}{2}}x^7\arccos(cx) - 256\,bc^5d^{\frac{3}{2}}x^6 - 1536\sqrt{-c^2x^2+1}bc^4d^{\frac{3}{2}}x^5\arccos(cx}{}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-1/8*sqrt(-c^2*d*x^2 + d)*a*c^2*d*x^7 + 3/16*sqrt(-c^2*d*x^2 + d)*a*d*x^5
- 1/64*sqrt(-c^2*d*x^2 + d)*a*d*x^3/c^2 - 3/128*sqrt(-c^2*d*x^2 + d)*a*d*x
/c^4 - 3/128*a*d^2*log(abs(-c*sqrt(-d)*x + sqrt(c^2*x^2 - 1)*sqrt(-d)))/(c
^5*sqrt(-d)) - 1/8192*(128*b*c^7*d^(3/2)*x^8 + 1024*sqrt(-c^2*x^2 + 1)*b*c
^6*d^(3/2)*x^7*arccos(c*x) - 256*b*c^5*d^(3/2)*x^6 - 1536*sqrt(-c^2*x^2 +
1)*b*c^4*d^(3/2)*x^5*arccos(c*x) + 32*b*c^3*d^(3/2)*x^4 + 128*sqrt(-c^2*x^
2 + 1)*b*c^2*d^(3/2)*x^3*arccos(c*x) + 96*b*c*d^(3/2)*x^2 + 192*sqrt(-c^2*
x^2 + 1)*b*d^(3/2)*x*arccos(c*x) + 96*b*d^(3/2)*arccos(c*x)^2/c - 15*b*d^(
3/2)/c)/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d - c^2 dx^2)^{3/2} (a + b \arccos(cx)) \, dx = \int x^4 (a + b \operatorname{acos}(c x)) \left(d - c^2 d x^2\right)^{3/2} dx$$

input
```
int(x^4*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^4*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^4 (d - c^2 dx^2)^{3/2} (a + b \arccos(cx)) \, dx = \frac{\sqrt{d}\, d \left(3 asin(cx)\, a - 16\sqrt{-c^2x^2+1}\, a\, c^7x^7 + 24\sqrt{-c^2x^2+1}\, a\, c^5x^5 - 2\sqrt{-c^2x^2+1}\, a\, c^3 x$$

input
```
int(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d*(3*asin(c*x)*a - 16*sqrt( - c**2*x**2 + 1)*a*c**7*x**7 + 24*sqr
t( - c**2*x**2 + 1)*a*c**5*x**5 - 2*sqrt( - c**2*x**2 + 1)*a*c**3*x**3 - 3
*sqrt( - c**2*x**2 + 1)*a*c*x - 128*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x
**6,x)*b*c**7 + 128*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**4,x)*b*c**5))/
(128*c**5)
```

3.71 $\int x^2(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 265

$$\begin{aligned}\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx &= \frac{bdx^2\sqrt{d-c^2dx^2}}{32c\sqrt{1-c^2x^2}}\\ &\quad-\frac{7bcdx^4\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}+\frac{bc^3dx^6\sqrt{d-c^2dx^2}}{36\sqrt{1-c^2x^2}}\\ &\quad-\frac{dx\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{16c^2}+\frac{1}{8}dx^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\\ &\quad+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{32bc^3\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
1/32*b*d*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-7/96*b*c*d*x^4*(-c^
2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/36*b*c^3*d*x^6*(-c^2*d*x^2+d)^(1/2)/
(-c^2*x^2+1)^(1/2)-1/16*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c^2+1/8
*d*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))+1/6*x^3*(-c^2*d*x^2+d)^(3/2)
*(a+b*arccos(c*x))+1/32*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/b/c^3/(
-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00

$$\int x^2(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx = \frac{-72bd\sqrt{d-c^2dx^2}\arccos(cx)^2-144ad^{3/2}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+d\sqrt{d-c^2}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(-72*b*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 144*a*d^(3/2)*Sqrt[1 - c^2*x^
2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + d*Sqrt[d -
 c^2*d*x^2]*(-144*a*c*x*Sqrt[1 - c^2*x^2] + 672*a*c^3*x^3*Sqrt[1 - c^2*x^2
] - 384*a*c^5*x^5*Sqrt[1 - c^2*x^2] + 18*b*Cos[2*ArcCos[c*x]] + 9*b*Cos[4*
ArcCos[c*x]] - 2*b*Cos[6*ArcCos[c*x]]) - 12*b*d*Sqrt[d - c^2*d*x^2]*ArcCos
[c*x]*(-3*Sin[2*ArcCos[c*x]] - 3*Sin[4*ArcCos[c*x]] + Sin[6*ArcCos[c*x]]))
/(2304*c^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5203, 244, 2009, 5199, 15, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int x^3(1-c^2x^2)\,dx}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^3(d-c^2dx^2)^{3/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int\left(x^3-c^2x^5\right)dx}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))$$

$\big\downarrow$ 2009

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^4}{4}-\frac{c^2x^6}{6}\right)\sqrt{d-c^2dx^2}}{6\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 5199

$$\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int x^3dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^4}{4}-\frac{c^2x^6}{6}\right)\sqrt{d-c^2dx^2}}{6\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 15

$$\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}\right)+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^4}{4}-\frac{c^2x^6}{6}\right)\sqrt{d-c^2dx^2}}{6\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 5211

$$\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int xdx}{2c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}\right)}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}\right)+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^4}{4}-\frac{c^2x^6}{6}\right)\sqrt{d-c^2dx^2}}{6\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 15

$$\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^4\sqrt{}}{16\sqrt{}}\right.$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^4}{4}-\frac{c^2x^6}{6}\right)\sqrt{d-c^2dx^2}}{6\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+$$

$$\frac{1}{2}d\left(\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{\sqrt{d-c^2dx^2}\left(-\frac{(a+b\arccos(cx))^2}{4bc^3}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4\sqrt{1-c^2x^2}}+\frac{bcx^4}{16\sqrt{}}\right.$$

$$\frac{bcd\left(\frac{x^4}{4}-\frac{c^2x^6}{6}\right)\sqrt{d-c^2dx^2}}{6\sqrt{1-c^2x^2}}$$

input
```
Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d*Sqrt[d - c^2*d*x^2]*(x^4/4 - (c^2*x^6)/6))/(6*Sqrt[1 - c^2*x^2]) +
(x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/6 + (d*((b*c*x^4*Sqrt[d -
c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCo
s[c*x]))/4 + (Sqrt[d - c^2*d*x^2]*(-1/4*(b*x^2)/c - (x*Sqrt[1 - c^2*x^2]*(
a + b*ArcCos[c*x]))/(2*c^2) - (a + b*ArcCos[c*x])^2/(4*b*c^3)))/(4*Sqrt[1
- c^2*x^2])))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 679, normalized size of antiderivative = 2.56

method	result
default	$-\frac{ax(-c^2d\,x^2+d)^{\frac{5}{2}}}{6c^2d}+\frac{ax(-c^2d\,x^2+d)^{\frac{3}{2}}}{24c^2}+\frac{adx\sqrt{-c^2d\,x^2+d}}{16c^2}+\frac{a\,d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{16c^2\sqrt{c^2d}}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+}}{32c^3(c^2x^2-}\right.$
parts	$-\frac{ax(-c^2d\,x^2+d)^{\frac{5}{2}}}{6c^2d}+\frac{ax(-c^2d\,x^2+d)^{\frac{3}{2}}}{24c^2}+\frac{adx\sqrt{-c^2d\,x^2+d}}{16c^2}+\frac{a\,d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{16c^2\sqrt{c^2d}}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+}}{32c^3(c^2x^2-}\right.$

input
```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*a*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/24*a/c^2*x*(-c^2*d*x^2+d)^(3/2)+1/16
*a/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/16*a/c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d
)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(1/32*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1
)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2*d-1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*
c^7*x^7-64*c^5*x^5+32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+38*c^3*x^3-48*I*(-c^2*x
^2+1)^(1/2)*x^4*c^4-6*c*x+18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(
1/2))*(I+6*arccos(c*x))*d/c^3/(c^2*x^2-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(-2
*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*
arccos(c*x))*d/c^3/(c^2*x^2-1)-1/4608*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2
+1)^(1/2)*x*c+c^2*x^2-1)*(7*I+48*arccos(c*x))*cos(5*arccos(c*x))*d/c^3/(c^
2*x^2-1)-1/4608*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I
)*(11*I+24*arccos(c*x))*sin(5*arccos(c*x))*d/c^3/(c^2*x^2-1)-3/512*(-d*(c^
2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*cos(3*arccos(c*x))*d/
c^3/(c^2*x^2-1)-1/512*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(
1/2)-I)*(I+8*arccos(c*x))*sin(3*arccos(c*x))*d/c^3/(c^2*x^2-1))
```

Fricas [F]

$$\int x^2(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx=\int(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
integral(-(a*c^2*d*x^4 - a*d*x^2 + (b*c^2*d*x^4 - b*d*x^2)*arccos(c*x))*sq
rt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx=\text{Timed out}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx=\int(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
b*sqrt(d)*integrate(-(c^2*d*x^4 - d*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arct
an2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x) + 1/48*a*(2*(-c^2*d*x^2 + d)^(3
/2)*x/c^2 - 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*d*
x/c^2 + 3*d^(3/2)*arcsin(c*x)/c^3)
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.95

$$\int x^2 (d - c^2dx^2)^{3/2} (a + b \arccos(cx)) \, dx = -\frac{1}{6} \sqrt{-c^2dx^2 + d}ac^2dx^5 + \frac{7}{24} \sqrt{-c^2dx^2 + d}adx^3 - \frac{\sqrt{-c^2dx^2 + d}adx}{16\,c^2} - \frac{ad^2 \log \left(\left| -c\sqrt{-d}x + \sqrt{c^2x^2 - 1}\sqrt{-d} \right| \right)}{16\,c^3\sqrt{-d}} - \frac{64\,bc^5d^{\frac{3}{2}}x^6 + 384\,\sqrt{-c^2x^2 + 1}bc^4d^{\frac{3}{2}}x^5 \arccos(cx) - 168\,bc^3d^{\frac{3}{2}}x^4 - 672\,\sqrt{-c^2x^2 + 1}bc^2d^{\frac{3}{2}}x^3 \arccos(cx) + \cdots}{2304\,c^2}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-1/6*sqrt(-c^2*d*x^2 + d)*a*c^2*d*x^5 + 7/24*sqrt(-c^2*d*x^2 + d)*a*d*x^3
- 1/16*sqrt(-c^2*d*x^2 + d)*a*d*x/c^2 - 1/16*a*d^2*log(abs(-c*sqrt(-d)*x +
 sqrt(c^2*x^2 - 1)*sqrt(-d)))/(c^3*sqrt(-d)) - 1/2304*(64*b*c^5*d^(3/2)*x^
6 + 384*sqrt(-c^2*x^2 + 1)*b*c^4*d^(3/2)*x^5*arccos(c*x) - 168*b*c^3*d^(3/
2)*x^4 - 672*sqrt(-c^2*x^2 + 1)*b*c^2*d^(3/2)*x^3*arccos(c*x) + 72*b*c*d^(
3/2)*x^2 + 144*sqrt(-c^2*x^2 + 1)*b*d^(3/2)*x*arccos(c*x) + 72*b*d^(3/2)*a
rccos(c*x)^2/c + 7*b*d^(3/2)/c)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d - c^2dx^2)^{3/2} (a + b \arccos(cx)) \, dx = \int x^2 \, (a + b \operatorname{acos}(c\,x)) \, \left(d - c^2\,d\,x^2\right)^{3/2} dx$$

input
```
int(x^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^2 \left(d - c^2 d x^2\right)^{3/2} (a + b \arccos(cx))\, dx = \frac{\sqrt{d}\, d \left(3 asin(cx)\, a - 8\sqrt{-c^2x^2+1}\, a\, c^5 x^5 + 14\sqrt{-c^2x^2+1}\, a\, c^3 x^3 - 3\sqrt{-c^2x^2+1}\, acx - \right.}{48c^3}$$

input
```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d*(3*asin(c*x)*a - 8*sqrt( - c**2*x**2 + 1)*a*c**5*x**5 + 14*sqrt
( - c**2*x**2 + 1)*a*c**3*x**3 - 3*sqrt( - c**2*x**2 + 1)*a*c*x - 48*int(s
qrt( - c**2*x**2 + 1)*acos(c*x)*x**4,x)*b*c**5 + 48*int(sqrt( - c**2*x**2
+ 1)*acos(c*x)*x**2,x)*b*c**3))/(48*c**3)
```

3.72 $\int (d-c^2dx^2)^{3/2} (a+b\arccos(cx)) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 185

$$\int (d-c^2dx^2)^{3/2} (a+b\arccos(cx)) \, dx = -\frac{3bcdx^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}} + \frac{bd(1-c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}{16c} + \frac{3}{8}dx\sqrt{d-c^2dx^2}(a+b\arccos(cx)) + \frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arccos(cx)) + \frac{3d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{16bc\sqrt{1-c^2x^2}}$$

output

```
-3/16*b*c*d*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/16*b*d*(-c^2*x^2
+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)/c+3/8*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(
c*x))+1/4*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))+3/16*d*(-c^2*d*x^2+d)^(
1/2)*(a+b*arccos(c*x))^2/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.14

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx = \frac{-24bd\sqrt{d-c^2dx^2}\arccos(cx)^2-48ad^{3/2}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+d\sqrt{d-c^2d\ldots}}{}$$

input
```
Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(-24*b*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 48*a*d^(3/2)*Sqrt[1 - c^2*x^2
]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + d*Sqrt[d -
c^2*d*x^2]*(16*a*c*x*(5 - 2*c^2*x^2)*Sqrt[1 - c^2*x^2] + 16*b*Cos[2*ArcCos
[c*x]] - b*Cos[4*ArcCos[c*x]]) - 4*b*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(-8
*Sin[2*ArcCos[c*x]] + Sin[4*ArcCos[c*x]]))/(128*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5159, 244, 2009, 5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{244}$$

$$\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int \left(x-c^2x^3\right)dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 5157}$$

$$\frac{3}{4}d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int xdx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{3}{4}d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}\right)+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 5153}$$

$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}d\left(\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4bc\sqrt{1-c^2x^2}}+\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}\right)+\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

input
```
Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d*Sqrt[d - c^2*d*x^2]*(x^2/2 - (c^2*x^4)/4))/(4*Sqrt[1 - c^2*x^2]) +
(x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 + (3*d*((b*c*x^2*Sqrt[d -
c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) + (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c
*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*
x^2])))/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.00 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.59

method	result
default	$\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{4}+\frac{3adx\sqrt{-c^2dx^2+d}}{8}+\frac{3a\,d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}}+b\left(\frac{3\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2d}{16(c^2x^2-1)c}-\sqrt{}\right.$
parts	$\frac{ax(-c^2dx^2+d)^{\frac{3}{2}}}{4}+\frac{3adx\sqrt{-c^2dx^2+d}}{8}+\frac{3a\,d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}}+b\left(\frac{3\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2d}{16(c^2x^2-1)c}-\sqrt{}\right.$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a*d^2/(c^2
*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(3/16*(-d*(c^2*x^
2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^2*d-1/256*(-d*(c^
2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x
-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(I+4*arccos(c*x))*d/
(c^2*x^2-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2
+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))*d/(c^2*x^2-1)/c-
3/256*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(5*I+12
*arccos(c*x))*cos(3*arccos(c*x))*d/(c^2*x^2-1)/c-1/256*(-d*(c^2*x^2-1))^(1
/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(17*I+28*arccos(c*x))*sin(3*arcco
s(c*x))*d/(c^2*x^2-1)/c)
```

Fricas [F]

$$\int \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \int \left(-c^2dx^2+d\right)^{\frac{3}{2}} (b\arccos(cx)+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccos(c*x))*sqrt(-c^2*
d*x^2 + d), x)
```

Sympy [F]

$$\int \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \int (-d(cx-1)(cx+1))^{\frac{3}{2}} (a+b\,\mathrm{acos}(cx))\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \int \left(-c^2dx^2+d\right)^{\frac{3}{2}} (b\arccos(cx)+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
b*sqrt(d)*integrate(-(c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(
sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x) + 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x +
 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*arcsin(c*x)/c)*a
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx = \int (a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx = \frac{\sqrt{d}\,d\big(3asin(cx)\,a-2\sqrt{-c^2x^2+1}\,a\,c^3x^3+5\sqrt{-c^2x^2+1}\,acx-8\big(\int\sqrt{-c^2x^2+1}\,acos(c}{8c}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d*(3*asin(c*x)*a - 2*sqrt( - c**2*x**2 + 1)*a*c**3*x**3 + 5*sqrt(
 - c**2*x**2 + 1)*a*c*x - 8*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**2,x)*b
*c**3 + 8*int(sqrt( - c**2*x**2 + 1)*acos(c*x),x)*b*c))/(8*c)
```

3.73 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 185

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^2}\,dx = \frac{bc^3dx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} - \frac{3}{2}c^2dx\sqrt{d-c^2dx^2}(a+b\arccos(cx)) - \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x} - \frac{3cd\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4b\sqrt{1-c^2x^2}} + \frac{bcd\sqrt{d-c^2dx^2}\log(x)}{\sqrt{1-c^2x^2}}$$

output
```
1/4*b*c^3*d*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-3/2*c^2*d*x*(-c^2*
d*x^2+d)^(1/2)*(a+b*arccos(c*x))-(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x-
3/4*c*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/b/(-c^2*x^2+1)^(1/2)+b*c*
d*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.12

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^2}\,dx =$$
$$-\frac{ad(2+c^2x^2)\sqrt{d-c^2dx^2}}{2x}+\frac{3}{2}acd^{3/2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)$$
$$-\frac{1}{2}bcd\sqrt{d-c^2dx^2}\left(\frac{2\arccos(cx)}{cx}-\frac{\arccos(cx)^2-2\log(cx)}{\sqrt{1-c^2x^2}}\right)$$
$$-\frac{bcd\sqrt{d-c^2dx^2}(\cos(2\arccos(cx))+2\arccos(cx)(-\arccos(cx)+\sin(2\arccos(cx))))}{8\sqrt{1-c^2x^2}}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
-1/2*(a*d*(2 + c^2*x^2)*Sqrt[d - c^2*d*x^2])/x + (3*a*c*d^(3/2)*ArcTan[(c*
x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/2 - (b*c*d*Sqrt[d - c^2*
d*x^2]*((2*ArcCos[c*x])/(c*x) - (ArcCos[c*x]^2 - 2*Log[c*x])/Sqrt[1 - c^2*
x^2]))/2 - (b*c*d*Sqrt[d - c^2*d*x^2]*(Cos[2*ArcCos[c*x]] + 2*ArcCos[c*x]*
(-ArcCos[c*x] + Sin[2*ArcCos[c*x]])))/(8*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5201, 244, 2009, 5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^2}\,dx$$
$$\Big\downarrow \; 5201$$

$$-3c^2d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int \frac{1-c^2x^2}{x}dx}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{244}$$

$$-3c^2d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{1}{x}-c^2x\right)dx}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{2009}$$

$$-3c^2d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(\log(x)-\frac{c^2x^2}{2}\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5157}$$

$$-3c^2d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int xdx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(\log(x)-\frac{c^2x^2}{2}\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{15}$$

$$-3c^2d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(\log(x)-\frac{c^2x^2}{2}\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5153}$$

$$-3c^2d\left(\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4bc\sqrt{1-c^2x^2}}+\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(\log(x)-\frac{c^2x^2}{2}\right)}{\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
-(((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x) - 3*c^2*d*((b*c*x^2*Sqrt[
d - c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2]) + (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
os[c*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 -
c^2*x^2])) - (b*c*d*Sqrt[d - c^2*d*x^2]*(-1/2*(c^2*x^2) + Log[x]))/Sqrt[1
- c^2*x^2]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.35

method	result
default	$-\frac{a(-c^2d\,x^2+d)^{\frac{5}{2}}}{dx} - a\,c^2x(-c^2d\,x^2+d)^{\frac{3}{2}} - \frac{3a\,c^2dx\sqrt{-c^2d\,x^2+d}}{2} - \frac{3a\,c^2d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}} - \frac{b\sqrt{-d(c^2x^2-}{}$
parts	$-\frac{a(-c^2d\,x^2+d)^{\frac{5}{2}}}{dx} - a\,c^2x(-c^2d\,x^2+d)^{\frac{3}{2}} - \frac{3a\,c^2dx\sqrt{-c^2d\,x^2+d}}{2} - \frac{3a\,c^2d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}} - \frac{b\sqrt{-d(c^2x^2-}{}$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/x*(-c^2*d*x^2+d)^(5/2)-a*c^2*x*(-c^2*d*x^2+d)^(3/2)-3/2*a*c^2*d*x*(-c
^2*d*x^2+d)^(1/2)-3/2*a*c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2
*d*x^2+d)^(1/2))-1/8*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-
1)/x*(-4*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^2-2*c^3*x^3+6*arccos(c*x)^2*
c*x+8*I*arccos(c*x)*x*c-8*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*x*c-8*arccos(
c*x)*(-c^2*x^2+1)^(1/2)+c*x)*d
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}{x^2}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^2,x, algorithm="fricas"
)
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccos(c*x))*sqrt(-c^2*
d*x^2 + d)/x^2, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}{x^2}\,dx = \int \frac{(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acos}\,(cx))}{x^2}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))/x**2,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))/x**2, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}{x^2}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^2,x, algorithm="maxima"
)
```

output
```
-b*sqrt(d)*integrate((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(
sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x^2, x) - 1/2*(3*sqrt(-c^2*d*x^2 + d)*c
^2*d*x + 3*c*d^(3/2)*arcsin(c*x) + 2*(-c^2*d*x^2 + d)^(3/2)/x)*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^{3/2}}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^2}\,dx = \frac{\sqrt{d}\,d\Big(acos(cx)^2\,bcx - 3asin(cx)\,acx - \sqrt{-c^2x^2+1}\,a\,c^2x^2 - 2\sqrt{-}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))/x^2,x)
```

output
```
(sqrt(d)*d*(acos(c*x)**2*b*c*x - 3*asin(c*x)*a*c*x - sqrt( - c**2*x**2 + 1
)*a*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a + 2*int(acos(c*x)/(sqrt( - c**2
*x**2 + 1)*x**2),x)*b*x - 2*int(sqrt( - c**2*x**2 + 1)*acos(c*x),x)*b*c**2
*x))/(2*x)
```

3.74 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 191

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^4}\,dx = -\frac{bcd\sqrt{d-c^2dx^2}}{6x^2\sqrt{1-c^2x^2}} + \frac{c^2d\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x} - \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{3x^3} + \frac{c^3d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2b\sqrt{1-c^2x^2}} - \frac{4bc^3d\sqrt{d-c^2dx^2}\log(x)}{3\sqrt{1-c^2x^2}}$$

output
```
-1/6*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)+c^2*d*(-c^2*d*x^2+d
)^(1/2)*(a+b*arccos(c*x))/x-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^3
+1/2*c^3*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/b/(-c^2*x^2+1)^(1/2)-4
/3*b*c^3*d*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.10

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^4}\,dx = \frac{bd(-1+4c^2x^2)\sqrt{d-c^2dx^2}\arccos(cx)}{3x^3} - \frac{bc^3d\sqrt{d-c^2dx^2}\arccos(cx)^2}{2\sqrt{1-c^2x^2}} - ac^3d^{3/2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right) + \frac{d\sqrt{d-c^2dx^2}\left(bcx+2a\sqrt{1-c^2x^2}(-1+4c^2x^2)+8bc^3x^3\log(cx)\right)}{6x^3\sqrt{1-c^2x^2}}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
(b*d*(-1 + 4*c^2*x^2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x])/(3*x^3) - (b*c^3*d*
Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2)/(2*Sqrt[1 - c^2*x^2]) - a*c^3*d^(3/2)*A
rcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (d*Sqrt[d - c^
2*d*x^2]*(b*c*x + 2*a*Sqrt[1 - c^2*x^2]*(-1 + 4*c^2*x^2) + 8*b*c^3*x^3*Log
[c*x]))/(6*x^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5201, 244, 2009, 5197, 14, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^4}dx$$

$$\Big\downarrow \text{5201}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}dx - \frac{bcd\sqrt{d-c^2dx^2}\int \frac{1-c^2x^2}{x^3}dx}{3\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \text{244}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}dx-\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{1}{x^3}-\frac{c^2}{x}\right)dx}{3\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3x^3}-\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-\log(x))-\frac{1}{2x^2}\right)}{3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5197}$$

$$c^2(-d)\left(-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}-\frac{bc\sqrt{d-c^2dx^2}\int \frac{1}{x}dx}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3x^3}-\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-\log(x))-\frac{1}{2x^2}\right)}{3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{14}$$

$$c^2(-d)\left(-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}-\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3x^3}-\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-\log(x))-\frac{1}{2x^2}\right)}{3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5153}$$

$$c^2(-d)\left(\frac{c\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2b\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}-\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3x^3}-\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-\log(x))-\frac{1}{2x^2}\right)}{3\sqrt{1-c^2x^2}}$$

input

```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
-1/3*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^3 - (b*c*d*Sqrt[d - c^2
*d*x^2]*(-1/2*1/x^2 - c^2*Log[x]))/(3*Sqrt[1 - c^2*x^2]) - c^2*d*(-((Sqrt[
d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/x) + (c*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cos[c*x])^2)/(2*b*Sqrt[1 - c^2*x^2]) - (b*c*Sqrt[d - c^2*d*x^2]*Log[x])/Sq
rt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5197
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^2
]/Sqrt[1 - c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x
] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 2)*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.45

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{5}{2}}}{3dx^3}+\frac{2ac^2(-c^2dx^2+d)^{\frac{5}{2}}}{3dx}+\frac{2ac^4x(-c^2dx^2+d)^{\frac{3}{2}}}{3}+ac^4dx\sqrt{-c^2dx^2+d}+\frac{ac^4d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{\sqrt{c^2d}}$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{5}{2}}}{3dx^3}+\frac{2ac^2(-c^2dx^2+d)^{\frac{5}{2}}}{3dx}+\frac{2ac^4x(-c^2dx^2+d)^{\frac{3}{2}}}{3}+ac^4dx\sqrt{-c^2dx^2+d}+\frac{ac^4d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{\sqrt{c^2d}}$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/d/x^3*(-c^2*d*x^2+d)^(5/2)+2/3*a*c^2/d/x*(-c^2*d*x^2+d)^(5/2)+2/3*a
*c^4*x*(-c^2*d*x^2+d)^(3/2)+a*c^4*d*x*(-c^2*d*x^2+d)^(1/2)+a*c^4*d^2/(c^2*
d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/6*b*(-d*(c^2*x^2-1
))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/x^3*(3*arccos(c*x)^2*c^3*x^3+8*I*a
rccos(c*x)*x^3*c^3-8*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*x^3*c^3-8*(-c^2*x^
2+1)^(1/2)*arccos(c*x)*c^2*x^2+2*arccos(c*x)*(-c^2*x^2+1)^(1/2)-c*x)*d
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^4}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^4,x, algorithm="fricas"
)
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccos(c*x))*sqrt(-c^2*
d*x^2 + d)/x^4, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^4}\,dx = \int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\,\mathrm{acos}(cx))}{x^4}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))/x**4,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))/x**4, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^4}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^4,x, algorithm="maxima"
)
```

output
```
-b*sqrt(d)*integrate((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(
sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x^4, x) + 1/3*(3*sqrt(-c^2*d*x^2 + d)*c
^4*d*x + 3*c^3*d^(3/2)*arcsin(c*x) + 2*(-c^2*d*x^2 + d)^(3/2)*c^2/x - (-c^
2*d*x^2 + d)^(5/2)/(d*x^3))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^{3/2}}{x^4}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^4,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^4}\,dx = \frac{\sqrt{d}\,d\Big(-3acos(cx)^2\,b\,c^3x^3+6asin(cx)\,a\,c^3x^3+8\sqrt{-c^2x^2+1}\,a\,c^2 $$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))/x^4,x)
```

output
```
(sqrt(d)*d*( - 3*acos(c*x)**2*b*c**3*x**3 + 6*asin(c*x)*a*c**3*x**3 + 8*sq
rt( - c**2*x**2 + 1)*a*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a - 6*int(acos
(c*x)/(sqrt( - c**2*x**2 + 1)*x**2),x)*b*c**2*x**3 + 6*int((sqrt( - c**2*x
**2 + 1)*acos(c*x))/x**4,x)*b*x**3))/(6*x**3)
```

3.75 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^6}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 154

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^6}\,dx = -\frac{bcd\sqrt{d-c^2dx^2}}{20x^4\sqrt{1-c^2x^2}}+\frac{bc^3d\sqrt{d-c^2dx^2}}{5x^2\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{5dx^5}+\frac{bc^5d\sqrt{d-c^2dx^2}\log(x)}{5\sqrt{1-c^2x^2}}$$

output
```
-1/20*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)+1/5*b*c^3*d*(-c^2*
d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos
(c*x))/d/x^5+1/5*b*c^5*d*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^6}\,dx = -\frac{d\sqrt{d-c^2dx^2}\left(-3bcx+12bc^3x^3-25bc^5x^5+12a\sqrt{1-c^2x^2}-24ac^2x^2\sqrt{1-c^2x^2}+12ac^4x^4\sqrt{1-c^2x^2}+\right.}{60x^5\sqrt{1-c^2x^2}}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^6,x]
```

output
```
-1/60*(d*Sqrt[d - c^2*d*x^2]*(-3*b*c*x + 12*b*c^3*x^3 - 25*b*c^5*x^5 + 12*
a*Sqrt[1 - c^2*x^2] - 24*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 12*a*c^4*x^4*Sqrt[1
 - c^2*x^2] + 12*b*(1 - c^2*x^2)^(5/2)*ArcCos[c*x] + 12*b*c^5*x^5*Log[x]))
/(x^5*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.61, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5187, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^6}dx$$

$$\Big\downarrow \text{5187}$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2}{x^5}dx}{5\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{5dx^5}$$

$$\Big\downarrow \text{243}$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2}{x^6}dx^2}{10\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{5dx^5}$$

$$\Big\downarrow \text{49}$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{c^4}{x^2}-\frac{2c^2}{x^4}+\frac{1}{x^6}\right)dx^2}{10\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{5dx^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{5dx^5}-\frac{bcd\sqrt{d-c^2dx^2}\left(c^4\log(x^2)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^6,x]
```

output
```
-1/5*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(d*x^5) - (b*c*d*Sqrt[d -
 c^2*d*x^2]*(-1/2*1/x^4 + (2*c^2)/x^2 + c^4*Log[x^2]))/(10*Sqrt[1 - c^2*x^
2])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 2349, normalized size of antiderivative = 15.25

method	result	size
default	Expression too large to display	2349
parts	Expression too large to display	2349

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/5*I*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^
2+1)/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^5+3/10*I*b*(-d*(c^2*x^2-
1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^3/(c^2*x^2-1)*
(-c^2*x^2+1)*c^8-1/20*I*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+1
0*c^4*x^4-5*c^2*x^2+1)*x/(c^2*x^2-1)*(-c^2*x^2+1)*c^6+1/5*I*b*(-d*(c^2*x^2
-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^7/(c^2*x^2-1)
*(-c^2*x^2+1)*c^12-9/20*I*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6
+10*c^4*x^4-5*c^2*x^2+1)*x^5/(c^2*x^2-1)*(-c^2*x^2+1)*c^10-1/5*a/d/x^5*(-c
^2*d*x^2+d)^(5/2)-56/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10
*c^4*x^4-5*c^2*x^2+1)*x/(c^2*x^2-1)*arccos(c*x)*c^6-11*b*(-d*(c^2*x^2-1))^
(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^5/(c^2*x^2-1)*arcc
os(c*x)*c^10-9/4*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x
^4-5*c^2*x^2+1)*x^4/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*c^9+14*b*(-d*(c^2*x^2-1
))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^3/(c^2*x^2-1)*a
rccos(c*x)*c^8+b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4
-5*c^2*x^2+1)*x^6/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*c^11-2/5*I*b*(-d*(c^2*x^2
-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)*d*c^5+1/5*I*b*(-d*(c
^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^9/(c^2*
x^2-1)*c^14-13/20*I*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^
4*x^4-5*c^2*x^2+1)*x^7/(c^2*x^2-1)*c^12+3/4*I*b*(-d*(c^2*x^2-1))^(1/2)*...
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.40

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x^6}\,dx = \left[\frac{2\left(bc^7dx^7-bc^5dx^5\right)\sqrt{d}\log\left(\frac{c^2dx^6+c^2dx^2-dx^4+\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}(x^{\cdots}}{c^2x^4-x^2}\right.}{\cdots}\right.$$

$$-\frac{4\left(bc^7dx^7-bc^5dx^5\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}(x^2-1)\sqrt{-d}}{c^2dx^4+(c^2-1)dx^2-d}\right)-\left(4bc^3dx^3-(4bc^3-bc)dx^5-bcdx\right)\sqrt{-\cdots}}{\cdots}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^6,x, algorithm="fricas"
)
```

output
```
[1/20*(2*(b*c^7*d*x^7 - b*c^5*d*x^5)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 -
d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^
2*x^4 - x^2)) + (4*b*c^3*d*x^3 - (4*b*c^3 - b*c)*d*x^5 - b*c*d*x)*sqrt(-c^
2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 4*(a*c^6*d*x^6 - 3*a*c^4*d*x^4 + 3*a*c^2
*d*x^2 - a*d + (b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*arccos(
c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5), -1/20*(4*(b*c^7*d*x^7 - b*c^5
*d*x^5)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*
sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) - (4*b*c^3*d*x^3 - (4*b*c^3 -
b*c)*d*x^5 - b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 4*(a*c^6*d
*x^6 - 3*a*c^4*d*x^4 + 3*a*c^2*d*x^2 - a*d + (b*c^6*d*x^6 - 3*b*c^4*d*x^4
+ 3*b*c^2*d*x^2 - b*d)*arccos(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5)]
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^6}\,dx = \int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\,\mathrm{acos}\,(cx))}{x^6}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))/x**6,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.12

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^6}\,dx = \frac{\left(2\,(-1)^{-2\,c^2dx^2+2\,d}\,c^4d^{\frac{5}{2}}\log\left(-2\,c^2d+\frac{2\,d}{x^2}\right)+2\,c^4d^{\frac{5}{2}}\log\left(x^2-\frac{1}{c^2}\right)\right.}{20\,d} - \frac{(-c^2dx^2+d)^{\frac{5}{2}}b\arccos(cx)}{5\,dx^5} - \frac{(-c^2dx^2+d)^{\frac{5}{2}}a}{5\,dx^5}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^6,x, algorithm="maxima"
)
```

output
```
1/20*(2*(-1)^(-2*c^2*d*x^2 + 2*d)*c^4*d^(5/2)*log(-2*c^2*d + 2*d/x^2) + 2*
c^4*d^(5/2)*log(x^2 - 1/c^2) - 3*sqrt(c^4*d*x^4 - 2*c^2*d*x^2 + d)*c^2*d^2
/x^2 + sqrt(c^4*d*x^4 - 2*c^2*d*x^2 + d)*d^2/x^4)*b*c/d - 1/5*(-c^2*d*x^2
+ d)^(5/2)*b*arccos(c*x)/(d*x^5) - 1/5*(-c^2*d*x^2 + d)^(5/2)*a/(d*x^5)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^6}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^6,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^6}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^{3/2}}{x^6}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^6,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^6, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}{x^6}\,dx = \frac{\sqrt{d}\,d\Big(-\sqrt{-c^2x^2+1}\,a\,c^4x^4+2\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))/x^6,x)
```

output
```
(sqrt(d)*d*( - sqrt( - c**2*x**2 + 1)*a*c**4*x**4 + 2*sqrt( - c**2*x**2 +
1)*a*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a + 5*int((sqrt( - c**2*x**2 + 1)*
acos(c*x))/x**6,x)*b*x**5 - 5*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**4,
x)*b*c**2*x**5))/(5*x**5)
```

3.76 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^8}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 231

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^8}\,dx = -\frac{bcd\sqrt{d-c^2dx^2}}{42x^6\sqrt{1-c^2x^2}} + \frac{2bc^3d\sqrt{d-c^2dx^2}}{35x^4\sqrt{1-c^2x^2}} - \frac{bc^5d\sqrt{d-c^2dx^2}}{70x^2\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{7dx^7} - \frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{35dx^5} + \frac{2bc^7d\sqrt{d-c^2dx^2}\log(x)}{35\sqrt{1-c^2x^2}}$$

output
```
-1/42*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^6/(-c^2*x^2+1)^(1/2)+2/35*b*c^3*d*(-c^2
*d*x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)-1/70*b*c^5*d*(-c^2*d*x^2+d)^(1/2)/x
^2/(-c^2*x^2+1)^(1/2)-1/7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/d/x^7-2/3
5*c^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/d/x^5+2/35*b*c^7*d*(-c^2*d*x^
2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.86

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^8}\,dx =$$
$$-\frac{d\sqrt{d-c^2dx^2}\Big(-25bcx+60bc^3x^3-15bc^5x^5-147bc^7x^7+150a\sqrt{1-c^2x^2}-240ac^2x^2\sqrt{1-c^2x^2}+30ac^4}{1050x^7\sqrt{1-c}}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^8,x]
```

output
```
-1/1050*(d*Sqrt[d - c^2*d*x^2]*(-25*b*c*x + 60*b*c^3*x^3 - 15*b*c^5*x^5 -
147*b*c^7*x^7 + 150*a*Sqrt[1 - c^2*x^2] - 240*a*c^2*x^2*Sqrt[1 - c^2*x^2]
+ 30*a*c^4*x^4*Sqrt[1 - c^2*x^2] + 60*a*c^6*x^6*Sqrt[1 - c^2*x^2] + 30*b*(
1 - c^2*x^2)^(5/2)*(5 + 2*c^2*x^2)*ArcCos[c*x] + 60*b*c^7*x^7*Log[x]))/(x^
7*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.60, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5195, 27, 354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^8}dx$$

$$\Big\downarrow \text{5195}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d(1-c^2x^2)^2(2c^2x^2+5)}{35x^7}dx}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{7dx^7}-\frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{35dx^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(2c^2x^2+5)}{x^7}dx}{35\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{7dx^7}-\frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{35dx^5}$$

$\Big\downarrow$ 354

$$-\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(2c^2x^2+5)}{x^8}dx^2}{70\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{7dx^7}-\frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{35dx^5}$$

$\Big\downarrow$ 85

$$-\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{2c^6}{x^2}+\frac{c^4}{x^4}-\frac{8c^2}{x^6}+\frac{5}{x^8}\right)dx^2}{70\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{7dx^7}-\frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{35dx^5}$$

$\Big\downarrow$ 2009

$$-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{7dx^7}-\frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{35dx^5}-\frac{bcd\sqrt{d-c^2dx^2}\left(2c^6\log(x^2)-\frac{c^4}{x^2}+\frac{4c^2}{x^4}-\frac{5}{3x^6}\right)}{70\sqrt{1-c^2x^2}}$$

input

```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^8,x]
```

output

```
-1/7*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(d*x^7) - (2*c^2*(d - c^2
*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(35*d*x^5) - (b*c*d*Sqrt[d - c^2*d*x^2]
*(-5/(3*x^6) + (4*c^2)/x^4 - c^4/x^2 + 2*c^6*Log[x^2]))/(70*Sqrt[1 - c^2*x
^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 3384, normalized size of antiderivative = 14.65

method	result	size
default	Expression too large to display	3384
parts	Expression too large to display	3384

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/5*I*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c
^4*x^4-105*c^2*x^2+25)*x^9/(c^2*x^2-1)*(-c^2*x^2+1)*c^16+26/105*I*b*(-d*(c
^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2
*x^2+25)*x^7/(c^2*x^2-1)*(-c^2*x^2+1)*c^14-116/105*I*b*(-d*(c^2*x^2-1))^(1
/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^5/
(c^2*x^2-1)*(-c^2*x^2+1)*c^12-2/35*I*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x
^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^11/(c^2*x^2-1)*(-c
^2*x^2+1)*c^18+10/7*I*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-
70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/(c^2*x^2-1)*arccos(c*x)*(-c^2*x^2+1
)^(1/2)*c^7+20/21*I*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70
*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^3/(c^2*x^2-1)*(-c^2*x^2+1)*c^10-5/2
1*I*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4
*x^4-105*c^2*x^2+25)*x/(c^2*x^2-1)*(-c^2*x^2+1)*c^8+44/5*I*b*(-d*(c^2*x^2-
1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25
)*x^4/(c^2*x^2-1)*arccos(c*x)*(-c^2*x^2+1)^(1/2)*c^11-6*I*b*(-d*(c^2*x^2-1
))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)
*x^2/(c^2*x^2-1)*arccos(c*x)*(-c^2*x^2+1)^(1/2)*c^9+2*I*b*(-d*(c^2*x^2-1))
^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x
^10/(c^2*x^2-1)*arccos(c*x)*(-c^2*x^2+1)^(1/2)*c^17+25/7*b*(-d*(c^2*x^2-1)
)^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+2...
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.60

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^8}\,dx = \left[\frac{6\,(bc^9dx^9-bc^7dx^7)\sqrt{d}\log\left(\frac{c^2dx^6+c^2dx^2-dx^4+\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}\,(x^4}{c^2x^4-x^2}\right.}{}\right.$$

$$-\frac{12\,(bc^9dx^9-bc^7dx^7)\sqrt{-d}\arctan\left(\frac{\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}\,(x^2-1)\sqrt{-d}}{c^2dx^4+(c^2-1)dx^2-d}\right)+(3\,bc^5dx^5-(3\,bc^5-12\,bc^3+5\,bc)dx^7}{}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^8,x, algorithm="fricas"
)
```

output
```
[1/210*(6*(b*c^9*d*x^9 - b*c^7*d*x^7)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 -
 d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c
^2*x^4 - x^2)) - (3*b*c^5*d*x^5 - (3*b*c^5 - 12*b*c^3 + 5*b*c)*d*x^7 - 12*
b*c^3*d*x^3 + 5*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 6*(2*a*
c^8*d*x^8 - a*c^6*d*x^6 - 9*a*c^4*d*x^4 + 13*a*c^2*d*x^2 - 5*a*d + (2*b*c^
8*d*x^8 - b*c^6*d*x^6 - 9*b*c^4*d*x^4 + 13*b*c^2*d*x^2 - 5*b*d)*arccos(c*x
))*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7), -1/210*(12*(b*c^9*d*x^9 - b*c^7*
d*x^7)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*s
qrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) + (3*b*c^5*d*x^5 - (3*b*c^5 - 1
2*b*c^3 + 5*b*c)*d*x^7 - 12*b*c^3*d*x^3 + 5*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*
sqrt(-c^2*x^2 + 1) + 6*(2*a*c^8*d*x^8 - a*c^6*d*x^6 - 9*a*c^4*d*x^4 + 13*a
*c^2*d*x^2 - 5*a*d + (2*b*c^8*d*x^8 - b*c^6*d*x^6 - 9*b*c^4*d*x^4 + 13*b*c
^2*d*x^2 - 5*b*d)*arccos(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^8}\,dx = \int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))}{x^8}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))/x**8,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))/x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.65

$$\begin{aligned}\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^8}\,dx = & -\frac{1}{210}\left(12\,c^6d^{\frac{3}{2}}\log(x) - \frac{3\,c^4d^{\frac{3}{2}}x^4 - 12\,c^2d^{\frac{3}{2}}x^2 + 5\,d^{\frac{3}{2}}}{x^6}\right)bc \\ & - \frac{1}{35}\,b\left(\frac{2\,(-c^2dx^2+d)^{\frac{5}{2}}c^2}{dx^5} + \frac{5\,(-c^2dx^2+d)^{\frac{5}{2}}}{dx^7}\right)\arccos(cx) \\ & - \frac{1}{35}\,a\left(\frac{2\,(-c^2dx^2+d)^{\frac{5}{2}}c^2}{dx^5} + \frac{5\,(-c^2dx^2+d)^{\frac{5}{2}}}{dx^7}\right)\end{aligned}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^8,x, algorithm="maxima"
)
```

output
```
-1/210*(12*c^6*d^(3/2)*log(x) - (3*c^4*d^(3/2)*x^4 - 12*c^2*d^(3/2)*x^2 +
5*d^(3/2))/x^6)*b*c - 1/35*b*(2*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^5) + 5*(-c
^2*d*x^2 + d)^(5/2)/(d*x^7))*arccos(c*x) - 1/35*a*(2*(-c^2*d*x^2 + d)^(5/2
)*c^2/(d*x^5) + 5*(-c^2*d*x^2 + d)^(5/2)/(d*x^7))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^8}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^8,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^8}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^{3/2}}{x^8}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^8,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^8, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^8}\,dx = \frac{\sqrt{d}\,d\Big(-2\sqrt{-c^2x^2+1}\,a\,c^6x^6-\sqrt{-c^2x^2+1}\,a\,c^4x^4+8\sqrt{-c^2x^2+$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))/x^8,x)
```

output
```
(sqrt(d)*d*( - 2*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - sqrt( - c**2*x**2 +
1)*a*c**4*x**4 + 8*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 5*sqrt( - c**2*x**
2 + 1)*a + 35*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**8,x)*b*x**7 - 35*i
nt((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**6,x)*b*c**2*x**7))/(35*x**7)
```

3.77 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{10}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 308

$$\begin{aligned}\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{10}}\,dx &= -\frac{bcd\sqrt{d-c^2dx^2}}{72x^8\sqrt{1-c^2x^2}} \\ &+ \frac{5bc^3d\sqrt{d-c^2dx^2}}{189x^6\sqrt{1-c^2x^2}} - \frac{bc^5d\sqrt{d-c^2dx^2}}{420x^4\sqrt{1-c^2x^2}} - \frac{2bc^7d\sqrt{d-c^2dx^2}}{315x^2\sqrt{1-c^2x^2}} \\ &- \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{9dx^9} - \frac{4c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{63dx^7} \\ &- \frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{315dx^5} + \frac{8bc^9d\sqrt{d-c^2dx^2}\log(x)}{315\sqrt{1-c^2x^2}}\end{aligned}$$

output

```
-1/72*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^8/(-c^2*x^2+1)^(1/2)+5/189*b*c^3*d*(-c^
2*d*x^2+d)^(1/2)/x^6/(-c^2*x^2+1)^(1/2)-1/420*b*c^5*d*(-c^2*d*x^2+d)^(1/2)
/x^4/(-c^2*x^2+1)^(1/2)-2/315*b*c^7*d*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1
)^(1/2)-1/9*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/d/x^9-4/63*c^2*(-c^2*d*
x^2+d)^(5/2)*(a+b*arccos(c*x))/d/x^7-8/315*c^4*(-c^2*d*x^2+d)^(5/2)*(a+b*a
rccos(c*x))/d/x^5+8/315*b*c^9*d*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.77

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{10}}\,dx =$$
$$-\frac{d\sqrt{d-c^2dx^2}\left(-3675bcx+7000bc^3x^3-630bc^5x^5-1680bc^7x^7-18264bc^9x^9+29400a\sqrt{1-c^2x^2}-4200\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^10,x]
```

output
```
-1/264600*(d*Sqrt[d - c^2*d*x^2]*(-3675*b*c*x + 7000*b*c^3*x^3 - 630*b*c^5
*x^5 - 1680*b*c^7*x^7 - 18264*b*c^9*x^9 + 29400*a*Sqrt[1 - c^2*x^2] - 4200
0*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 2520*a*c^4*x^4*Sqrt[1 - c^2*x^2] + 3360*a*
c^6*x^6*Sqrt[1 - c^2*x^2] + 6720*a*c^8*x^8*Sqrt[1 - c^2*x^2] + 840*b*(1 -
c^2*x^2)^(5/2)*(35 + 20*c^2*x^2 + 8*c^4*x^4)*ArcCos[c*x] + 6720*b*c^9*x^9*
Log[x]))/(x^9*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.61, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5195, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{10}}dx$$

$$\Big\downarrow \text{5195}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d(1-c^2x^2)^2(8c^4x^4+20c^2x^2+35)}{315x^9}dx}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{9dx^9}-\frac{4c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{63dx^7}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{315dx^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(8c^4x^4+20c^2x^2+35)}{x^9}dx}{315\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{9dx^9}-\frac{4c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{63dx^7}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{315dx^5}$$

$$\Big\downarrow \text{ 1578}$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(8c^4x^4+20c^2x^2+35)}{x^{10}}dx^2}{630\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{9dx^9}-\frac{4c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{63dx^7}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{315dx^5}$$

$$\Big\downarrow \text{ 1195}$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{8c^8}{x^2}+\frac{4c^6}{x^4}+\frac{3c^4}{x^6}-\frac{50c^2}{x^8}+\frac{35}{x^{10}}\right)dx^2}{630\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{9dx^9}-\frac{4c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{63dx^7}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{315dx^5}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{9dx^9}-\frac{4c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{63dx^7}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{315dx^5}-\frac{bcd\sqrt{d-c^2dx^2}\left(8c^8\log(x^2)-\frac{4c^6}{x^2}-\frac{3c^4}{2x^4}+\frac{50c^2}{3x^6}-\frac{35}{4x^8}\right)}{630\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^10,x]
```

output
```
-1/9*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(d*x^9) - (4*c^2*(d - c^2
*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(63*d*x^7) - (8*c^4*(d - c^2*d*x^2)^(5/
2)*(a + b*ArcCos[c*x]))/(315*d*x^5) - (b*c*d*Sqrt[d - c^2*d*x^2]*(-35/(4*x
^8) + (50*c^2)/(3*x^6) - (3*c^4)/(2*x^4) - (4*c^6)/x^2 + 8*c^8*Log[x^2]))/
(630*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 4563, normalized size of antiderivative = 14.81

method	result	size
default	Expression too large to display	4563
parts	Expression too large to display	4563

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^10,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/9/d/x^9*(-c^2*d*x^2+d)^(5/2)+4/9*c^2*(-1/7/d/x^7*(-c^2*d*x^2+d)^(5/2
)-2/35*c^2/d/x^5*(-c^2*d*x^2+d)^(5/2)))-24*I*b*(-d*(c^2*x^2-1))^(1/2)*d/(8
40*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*
x^2+1225)*x^10/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^19+24/5*I*b*(-
d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x
^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^8/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arcc
os(c*x)*c^17-208/3*I*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^
10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^6/(c^2*x^2-1
)*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^15+1104/7*I*b*(-d*(c^2*x^2-1))^(1/2)*d/
(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^
2*x^2+1225)*x^4/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^13-120*I*b*(-
d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x
^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arcc
os(c*x)*c^11+64/3*I*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^1
0+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^12/(c^2*x^2-1
)*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^21+8/315*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2
*x^2+1)^(1/2)/(c^2*x^2-1)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*d*c^9+1225/9*
b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c
^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)/x^9/(c^2*x^2-1)*arccos(c*x)-30055/5
04*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-...
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.18

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{10}}\,dx = \left[\frac{96\,(bc^{11}dx^{11}-bc^9dx^9)\sqrt{d}\log\left(\frac{c^2dx^6+c^2dx^2-dx^4+\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+}}{c^2x^4-x^2}\right.}{}\right.$$

$$-\frac{192\,(bc^{11}dx^{11}-bc^9dx^9)\sqrt{-d}\arctan\left(\frac{\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}(x^2-1)\sqrt{-d}}{c^2dx^4+(c^2-1)dx^2-d}\right)+(48\,bc^7dx^7+18\,bc^5dx^5-(48\,bc^7+}{}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^10,x, algorithm="fricas
")
```

output
```
[1/7560*(96*(b*c^11*d*x^11 - b*c^9*d*x^9)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x
^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d
)/(c^2*x^4 - x^2)) - (48*b*c^7*d*x^7 + 18*b*c^5*d*x^5 - (48*b*c^7 + 18*b*c
^5 - 200*b*c^3 + 105*b*c)*d*x^9 - 200*b*c^3*d*x^3 + 105*b*c*d*x)*sqrt(-c^2
*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 24*(8*a*c^10*d*x^10 - 4*a*c^8*d*x^8 - a*c
^6*d*x^6 - 53*a*c^4*d*x^4 + 85*a*c^2*d*x^2 - 35*a*d + (8*b*c^10*d*x^10 - 4
*b*c^8*d*x^8 - b*c^6*d*x^6 - 53*b*c^4*d*x^4 + 85*b*c^2*d*x^2 - 35*b*d)*arc
cos(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^9), -1/7560*(192*(b*c^11*d*x
^11 - b*c^9*d*x^9)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)
*(x^2 - 1)*sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) + (48*b*c^7*d*x^7 +
 18*b*c^5*d*x^5 - (48*b*c^7 + 18*b*c^5 - 200*b*c^3 + 105*b*c)*d*x^9 - 200*
b*c^3*d*x^3 + 105*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 24*(8
*a*c^10*d*x^10 - 4*a*c^8*d*x^8 - a*c^6*d*x^6 - 53*a*c^4*d*x^4 + 85*a*c^2*d
*x^2 - 35*a*d + (8*b*c^10*d*x^10 - 4*b*c^8*d*x^8 - b*c^6*d*x^6 - 53*b*c^4*
d*x^4 + 85*b*c^2*d*x^2 - 35*b*d)*arccos(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x
^11 - x^9)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{10}}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))/x**10,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.68

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{10}}\,dx = -\frac{1}{7560}\left(192\,c^8d^{\frac{3}{2}}\log(x)-\frac{48\,c^6d^{\frac{3}{2}}x^6+18\,c^4d^{\frac{3}{2}}x^4-200\,c^2d^{\frac{3}{2}}x^2+105\,d^{\frac{3}{2}}}{x^8}\right)bc - \frac{1}{315}b\left(\frac{8\,(-c^2dx^2+d)^{\frac{5}{2}}c^4}{dx^5}+\frac{20\,(-c^2dx^2+d)^{\frac{5}{2}}c^2}{dx^7}+\frac{35\,(-c^2dx^2+d)^{\frac{5}{2}}}{dx^9}\right)\arccos(cx) - \frac{1}{315}a\left(\frac{8\,(-c^2dx^2+d)^{\frac{5}{2}}c^4}{dx^5}+\frac{20\,(-c^2dx^2+d)^{\frac{5}{2}}c^2}{dx^7}+\frac{35\,(-c^2dx^2+d)^{\frac{5}{2}}}{dx^9}\right)$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^10,x, algorithm="maxima
")
```

output
```
-1/7560*(192*c^8*d^(3/2)*log(x) - (48*c^6*d^(3/2)*x^6 + 18*c^4*d^(3/2)*x^4
 - 200*c^2*d^(3/2)*x^2 + 105*d^(3/2))/x^8)*b*c - 1/315*b*(8*(-c^2*d*x^2 +
d)^(5/2)*c^4/(d*x^5) + 20*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^7) + 35*(-c^2*d*
x^2 + d)^(5/2)/(d*x^9))*arccos(c*x) - 1/315*a*(8*(-c^2*d*x^2 + d)^(5/2)*c^
4/(d*x^5) + 20*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^7) + 35*(-c^2*d*x^2 + d)^(5
/2)/(d*x^9))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{10}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^10,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{10}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^{3/2}}{x^{10}}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^10,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^10, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{10}}\,dx=\frac{\sqrt{d}\,d\Big(-8\sqrt{-c^2x^2+1}\,a\,c^8x^8-4\sqrt{-c^2x^2+1}\,a\,c^6x^6-3\sqrt{-c^2x^2+}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))/x^10,x)
```

output
```
(sqrt(d)*d*( - 8*sqrt( - c**2*x**2 + 1)*a*c**8*x**8 - 4*sqrt( - c**2*x**2
+ 1)*a*c**6*x**6 - 3*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 + 50*sqrt( - c**2*
x**2 + 1)*a*c**2*x**2 - 35*sqrt( - c**2*x**2 + 1)*a + 315*int((sqrt( - c**
2*x**2 + 1)*acos(c*x))/x**10,x)*b*x**9 - 315*int((sqrt( - c**2*x**2 + 1)*a
cos(c*x))/x**8,x)*b*c**2*x**9))/(315*x**9)
```

3.78 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{12}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 385

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{12}}\,dx = -\frac{bcd\sqrt{d-c^2dx^2}}{110x^{10}\sqrt{1-c^2x^2}} + \frac{bc^3d\sqrt{d-c^2dx^2}}{66x^8\sqrt{1-c^2x^2}} - \frac{bc^5d\sqrt{d-c^2dx^2}}{1386x^6\sqrt{1-c^2x^2}} - \frac{bc^7d\sqrt{d-c^2dx^2}}{770x^4\sqrt{1-c^2x^2}} - \frac{4bc^9d\sqrt{d-c^2dx^2}}{1155x^2\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{11dx^{11}} - \frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{33dx^9} - \frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{231dx^7} - \frac{16c^6(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{1155dx^5} + \frac{16bc^{11}d\sqrt{d-c^2dx^2}\log(x)}{1155\sqrt{1-c^2x^2}}$$

output
```
-1/110*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^10/(-c^2*x^2+1)^(1/2)+1/66*b*c^3*d*(-c
^2*d*x^2+d)^(1/2)/x^8/(-c^2*x^2+1)^(1/2)-1/1386*b*c^5*d*(-c^2*d*x^2+d)^(1/
2)/x^6/(-c^2*x^2+1)^(1/2)-1/770*b*c^7*d*(-c^2*d*x^2+d)^(1/2)/x^4/(-c^2*x^2
+1)^(1/2)-4/1155*b*c^9*d*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/11*
(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/d/x^11-2/33*c^2*(-c^2*d*x^2+d)^(5/2
)*(a+b*arccos(c*x))/d/x^9-8/231*c^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))
/d/x^7-16/1155*c^6*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/d/x^5+16/1155*b*
c^11*d*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.72

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{12}}\,dx =$$
$$-\frac{d\sqrt{d-c^2dx^2}\left(-6615bcx+11025bc^3x^3-525bc^5x^5-945bc^7x^7-2520bc^9x^9-29524bc^{11}x^{11}+66150a\sqrt{1}\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^12,x]
```

output
```
-1/727650*(d*Sqrt[d - c^2*d*x^2]*(-6615*b*c*x + 11025*b*c^3*x^3 - 525*b*c^
5*x^5 - 945*b*c^7*x^7 - 2520*b*c^9*x^9 - 29524*b*c^11*x^11 + 66150*a*Sqrt[
1 - c^2*x^2] - 88200*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 3150*a*c^4*x^4*Sqrt[1 -
 c^2*x^2] + 3780*a*c^6*x^6*Sqrt[1 - c^2*x^2] + 5040*a*c^8*x^8*Sqrt[1 - c^2
*x^2] + 10080*a*c^10*x^10*Sqrt[1 - c^2*x^2] + 630*b*(1 - c^2*x^2)^(5/2)*(1
05 + 70*c^2*x^2 + 40*c^4*x^4 + 16*c^6*x^6)*ArcCos[c*x] + 10080*b*c^11*x^11
*Log[x]))/(x^11*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.59, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5195, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{12}}dx$$
$$\Big\downarrow \text{5195}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d(1-c^2x^2)^2(16c^6x^6+40c^4x^4+70c^2x^2+105)}{1155x^{11}}dx}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{11dx^{11}}-\frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{33dx^9}-\frac{16c^6(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{1155dx^5}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{231dx^7}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(16c^6x^6+40c^4x^4+70c^2x^2+105)}{x^{11}}dx}{1155\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{11dx^{11}}-\frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{33dx^9}-\frac{16c^6(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{1155dx^5}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{231dx^7}$$

$$\Big\downarrow \text{2331}$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(16c^6x^6+40c^4x^4+70c^2x^2+105)}{x^{12}}dx^2}{2310\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{11dx^{11}}-\frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{33dx^9}-\frac{16c^6(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{1155dx^5}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{231dx^7}$$

$$\Big\downarrow \text{2123}$$

$$-\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{16c^{10}}{x^2}+\frac{8c^8}{x^4}+\frac{6c^6}{x^6}+\frac{5c^4}{x^8}-\frac{140c^2}{x^{10}}+\frac{105}{x^{12}}\right)dx^2}{2310\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{11dx^{11}}-\frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{33dx^9}-\frac{16c^6(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{1155dx^5}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{231dx^7}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{11dx^{11}}-\frac{2c^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{33dx^9}-\frac{16c^6(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{1155dx^5}-\frac{8c^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{231dx^7}-\frac{bcd\sqrt{d-c^2dx^2}\left(16c^{10}\log\left(x^2\right)-\frac{8c^8}{x^2}-\frac{3c^6}{x^4}-\frac{5c^4}{3x^6}+\frac{35c^2}{x^8}-\frac{21}{x^{10}}\right)}{2310\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^12,x]
```

output
```
-1/11*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(d*x^11) - (2*c^2*(d - c
^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(33*d*x^9) - (8*c^4*(d - c^2*d*x^2)^(
5/2)*(a + b*ArcCos[c*x]))/(231*d*x^7) - (16*c^6*(d - c^2*d*x^2)^(5/2)*(a +
 b*ArcCos[c*x]))/(1155*d*x^5) - (b*c*d*Sqrt[d - c^2*d*x^2]*(-21/x^10 + (35
*c^2)/x^8 - (5*c^4)/(3*x^6) - (3*c^6)/x^4 - (8*c^8)/x^2 + 16*c^10*Log[x^2]
))/(2310*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 5886, normalized size of antiderivative = 15.29

method	result	size
default	Expression too large to display	5886
parts	Expression too large to display	5886

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^12,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 744, normalized size of antiderivative = 1.93

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{12}}\,dx = \left[\frac{48\,(bc^{13}dx^{13}-bc^{11}dx^{11})\sqrt{d}\log\left(\frac{c^2dx^6+c^2dx^2-dx^4+\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2}}{c^2x^4-x^2}\right.}{}\right.$$

$$-\frac{96\,(bc^{13}dx^{13}-bc^{11}dx^{11})\sqrt{-d}\arctan\left(\frac{\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}(x^2-1)\sqrt{-d}}{c^2dx^4+(c^2-1)dx^2-d}\right)+(24\,bc^9dx^9+9\,bc^7dx^7-(24\,bc^9+9}{}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^12,x, algorithm="fricas
")
```

output
```
[1/6930*(48*(b*c^13*d*x^13 - b*c^11*d*x^11)*sqrt(d)*log((c^2*d*x^6 + c^2*d
*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) -
 d)/(c^2*x^4 - x^2)) - (24*b*c^9*d*x^9 + 9*b*c^7*d*x^7 - (24*b*c^9 + 9*b*c
^7 + 5*b*c^5 - 105*b*c^3 + 63*b*c)*d*x^11 + 5*b*c^5*d*x^5 - 105*b*c^3*d*x^
3 + 63*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 6*(16*a*c^12*d*x
^12 - 8*a*c^10*d*x^10 - 2*a*c^8*d*x^8 - a*c^6*d*x^6 - 145*a*c^4*d*x^4 + 24
5*a*c^2*d*x^2 - 105*a*d + (16*b*c^12*d*x^12 - 8*b*c^10*d*x^10 - 2*b*c^8*d*
x^8 - b*c^6*d*x^6 - 145*b*c^4*d*x^4 + 245*b*c^2*d*x^2 - 105*b*d)*arccos(c*
x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11), -1/6930*(96*(b*c^13*d*x^13 -
b*c^11*d*x^11)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^
2 - 1)*sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) + (24*b*c^9*d*x^9 + 9*b
*c^7*d*x^7 - (24*b*c^9 + 9*b*c^7 + 5*b*c^5 - 105*b*c^3 + 63*b*c)*d*x^11 +
5*b*c^5*d*x^5 - 105*b*c^3*d*x^3 + 63*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c
^2*x^2 + 1) + 6*(16*a*c^12*d*x^12 - 8*a*c^10*d*x^10 - 2*a*c^8*d*x^8 - a*c^
6*d*x^6 - 145*a*c^4*d*x^4 + 245*a*c^2*d*x^2 - 105*a*d + (16*b*c^12*d*x^12
- 8*b*c^10*d*x^10 - 2*b*c^8*d*x^8 - b*c^6*d*x^6 - 145*b*c^4*d*x^4 + 245*b*
c^2*d*x^2 - 105*b*d)*arccos(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2dx^2)^{3/2} (a + b\arccos(cx))}{x^{12}}\, dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))/x**12,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.70

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{12}}\,dx =$$
$$-\frac{1}{6930}\left(96\,c^{10}d^{\frac{3}{2}}\log(x)-\frac{24\,c^8d^{\frac{3}{2}}x^8+9\,c^6d^{\frac{3}{2}}x^6+5\,c^4d^{\frac{3}{2}}x^4-105\,c^2d^{\frac{3}{2}}x^2+63\,d^{\frac{3}{2}}}{x^{10}}\right)bc$$
$$-\frac{1}{1155}\left(\frac{16\,(-c^2dx^2+d)^{\frac{5}{2}}c^6}{dx^5}+\frac{40\,(-c^2dx^2+d)^{\frac{5}{2}}c^4}{dx^7}+\frac{70\,(-c^2dx^2+d)^{\frac{5}{2}}c^2}{dx^9}+\frac{105\,(-c^2dx^2+d)^{\frac{5}{2}}}{dx^{11}}\right)b\arccos$$
$$-\frac{1}{1155}\left(\frac{16\,(-c^2dx^2+d)^{\frac{5}{2}}c^6}{dx^5}+\frac{40\,(-c^2dx^2+d)^{\frac{5}{2}}c^4}{dx^7}+\frac{70\,(-c^2dx^2+d)^{\frac{5}{2}}c^2}{dx^9}+\frac{105\,(-c^2dx^2+d)^{\frac{5}{2}}}{dx^{11}}\right)a$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^12,x, algorithm="maxima
")
```

output
```
-1/6930*(96*c^10*d^(3/2)*log(x) - (24*c^8*d^(3/2)*x^8 + 9*c^6*d^(3/2)*x^6
+ 5*c^4*d^(3/2)*x^4 - 105*c^2*d^(3/2)*x^2 + 63*d^(3/2))/x^10)*b*c - 1/1155
*(16*(-c^2*d*x^2 + d)^(5/2)*c^6/(d*x^5) + 40*(-c^2*d*x^2 + d)^(5/2)*c^4/(d
*x^7) + 70*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^9) + 105*(-c^2*d*x^2 + d)^(5/2)
/(d*x^11))*b*arccos(c*x) - 1/1155*(16*(-c^2*d*x^2 + d)^(5/2)*c^6/(d*x^5) +
 40*(-c^2*d*x^2 + d)^(5/2)*c^4/(d*x^7) + 70*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*
x^9) + 105*(-c^2*d*x^2 + d)^(5/2)/(d*x^11))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{12}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^12,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{12}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^{3/2}}{x^{12}}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^12,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^12, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^{12}}\,dx = \frac{\sqrt{d}\,d\Big(-16\sqrt{-c^2x^2+1}\,a\,c^{10}x^{10}-8\sqrt{-c^2x^2+1}\,a\,c^8x^8-6\sqrt{-c^2x}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))/x^12,x)
```

output
```
(sqrt(d)*d*( - 16*sqrt( - c**2*x**2 + 1)*a*c**10*x**10 - 8*sqrt( - c**2*x*
*2 + 1)*a*c**8*x**8 - 6*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - 5*sqrt( - c**
2*x**2 + 1)*a*c**4*x**4 + 140*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 105*sqr
t( - c**2*x**2 + 1)*a + 1155*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**12,
x)*b*x**11 - 1155*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**10,x)*b*c**2*x
**11))/(1155*x**11)
```

3.79 $\int x^7(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 375

$$\int x^7(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx = \frac{16bdx\sqrt{d-c^2dx^2}}{1155c^7\sqrt{1-c^2x^2}} + \frac{8bdx^3\sqrt{d-c^2dx^2}}{3465c^5\sqrt{1-c^2x^2}} + \frac{2bdx^5\sqrt{d-c^2dx^2}}{1925c^3\sqrt{1-c^2x^2}} + \frac{bdx^7\sqrt{d-c^2dx^2}}{1617c\sqrt{1-c^2x^2}} - \frac{4bcdx^9\sqrt{d-c^2dx^2}}{297\sqrt{1-c^2x^2}} + \frac{bc^3dx^{11}\sqrt{d-c^2dx^2}}{121\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{5c^8d} + \frac{3(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{7c^8d^2} - \frac{(d-c^2dx^2)^{9/2}(a+b\arccos(cx))}{3c^8d^3} + \frac{(d-c^2dx^2)^{11/2}(a+b\arccos(cx))}{11c^8d^4}$$

output
```
16/1155*b*d*x*(-c^2*d*x^2+d)^(1/2)/c^7/(-c^2*x^2+1)^(1/2)+8/3465*b*d*x^3*(
-c^2*d*x^2+d)^(1/2)/c^5/(-c^2*x^2+1)^(1/2)+2/1925*b*d*x^5*(-c^2*d*x^2+d)^(
1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/1617*b*d*x^7*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^
2+1)^(1/2)-4/297*b*c*d*x^9*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/121*b
*c^3*d*x^11*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(5/
2)*(a+b*arccos(c*x))/c^8/d+3/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccos(c*x))/c^8/
d^2-1/3*(-c^2*d*x^2+d)^(9/2)*(a+b*arccos(c*x))/c^8/d^3+1/11*(-c^2*d*x^2+d)
^(11/2)*(a+b*arccos(c*x))/c^8/d^4
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.48

$$\int x^7\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx =$$

$$-\frac{d\sqrt{d-c^2dx^2}\Big(3465a(-1+c^2x^2)^3\left(16+40c^2x^2+70c^4x^4+105c^6x^6\right)-bcx\sqrt{1-c^2x^2}(55440+9240c^2x^2}{400}$$

input
```
Integrate[x^7*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/4002075*(d*Sqrt[d - c^2*d*x^2]*(3465*a*(-1 + c^2*x^2)^3*(16 + 40*c^2*x^
2 + 70*c^4*x^4 + 105*c^6*x^6) - b*c*x*Sqrt[1 - c^2*x^2]*(55440 + 9240*c^2*
x^2 + 4158*c^4*x^4 + 2475*c^6*x^6 - 53900*c^8*x^8 + 33075*c^10*x^10) + 346
5*b*(-1 + c^2*x^2)^3*(16 + 40*c^2*x^2 + 70*c^4*x^4 + 105*c^6*x^6)*ArcCos[c
*x]))/(c^8*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5195, 27, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \; 5195$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d(1-c^2x^2)^2(105c^6x^6+70c^4x^4+40c^2x^2+16)}{1155c^8}dx}{\sqrt{1-c^2x^2}}+\frac{\left(d-c^2dx^2\right)^{11/2}(a+b\arccos(cx))}{11c^8d^4}-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\arccos(cx))}{3c^8d^3}+\frac{3\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^8d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^8d}$$

$$\Big\downarrow \; 27$$

$$-\frac{bd\sqrt{d-c^2dx^2}\int (1-c^2x^2)^2 (105c^6x^6+70c^4x^4+40c^2x^2+16)\,dx}{1155c^7\sqrt{1-c^2x^2}}+\frac{(d-c^2dx^2)^{11/2}(a+b\arccos(cx))}{11c^8d^4}-\frac{(d-c^2dx^2)^{9/2}(a+b\arccos(cx))}{3c^8d^3}+\frac{3(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{7c^8d^2}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{5c^8d}$$

$$\Big\downarrow \text{ 2341}$$

$$-\frac{bd\sqrt{d-c^2dx^2}\int (105c^{10}x^{10}-140c^8x^8+5c^6x^6+6c^4x^4+8c^2x^2+16)\,dx}{1155c^7\sqrt{1-c^2x^2}}+\frac{(d-c^2dx^2)^{11/2}(a+b\arccos(cx))}{11c^8d^4}-\frac{(d-c^2dx^2)^{9/2}(a+b\arccos(cx))}{3c^8d^3}+\frac{3(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{7c^8d^2}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{5c^8d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(d-c^2dx^2)^{11/2}(a+b\arccos(cx))}{11c^8d^4}-\frac{(d-c^2dx^2)^{9/2}(a+b\arccos(cx))}{3c^8d^3}+\frac{3(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{7c^8d^2}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{5c^8d}-\frac{bd\left(\frac{105c^{10}x^{11}}{11}-\frac{140c^8x^9}{9}+\frac{5c^6x^7}{7}+\frac{6c^4x^5}{5}+\frac{8c^2x^3}{3}+16x\right)\sqrt{d-c^2dx^2}}{1155c^7\sqrt{1-c^2x^2}}$$

input
```
Int[x^7*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/1155*(b*d*Sqrt[d - c^2*d*x^2]*(16*x + (8*c^2*x^3)/3 + (6*c^4*x^5)/5 + (
5*c^6*x^7)/7 - (140*c^8*x^9)/9 + (105*c^10*x^11)/11))/(c^7*Sqrt[1 - c^2*x^
2]) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^8*d) + (3*(d - c^2*
d*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(7*c^8*d^2) - ((d - c^2*d*x^2)^(9/2)*(a
+ b*ArcCos[c*x]))/(3*c^8*d^3) + ((d - c^2*d*x^2)^(11/2)*(a + b*ArcCos[c*x]
))/(11*c^8*d^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.68

method	result
orering	$\frac{(694575x^{12}c^{12}-1619450c^{10}x^{10}+904475c^8x^8+27720c^6x^6+70224c^4x^4+517440c^2x^2-443520)\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}(a+b\arccos(cx))}{4002075c^8(cx-1)(cx+1)(c^2x^2-1)} -$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4002075*(694575*c^12*x^12-1619450*c^10*x^10+904475*c^8*x^8+27720*c^6*x^6
+70224*c^4*x^4+517440*c^2*x^2-443520)/c^8/(c*x-1)/(c*x+1)/(c^2*x^2-1)*(-c^
2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))-1/4002075/x^6*(33075*c^10*x^10-53900*c^
8*x^8+2475*c^6*x^6+4158*c^4*x^4+9240*c^2*x^2+55440)/c^8/(c*x-1)/(c*x+1)*(7
*x^6*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))-3*x^8*(-c^2*d*x^2+d)^(1/2)*(a+
b*arccos(c*x))*d*c^2-x^7*(-c^2*d*x^2+d)^(3/2)*b*c/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.66

$$\int x^7 (d - c^2dx^2)^{3/2} (a + b \arccos(cx)) \, dx = \frac{(33075\, bc^{11}dx^{11} - 53900\, bc^9dx^9 + 2475\, bc^7dx^7 + 4158\, bc^5dx^5 + 9240\, bc^3dx^3 + 55440\, bc \cdots}{\cdots}$$

input
```
integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
1/4002075*((33075*b*c^11*d*x^11 - 53900*b*c^9*d*x^9 + 2475*b*c^7*d*x^7 + 4
158*b*c^5*d*x^5 + 9240*b*c^3*d*x^3 + 55440*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*s
qrt(-c^2*x^2 + 1) - 3465*(105*a*c^12*d*x^12 - 245*a*c^10*d*x^10 + 145*a*c^
8*d*x^8 + a*c^6*d*x^6 + 2*a*c^4*d*x^4 + 8*a*c^2*d*x^2 - 16*a*d + (105*b*c^
12*d*x^12 - 245*b*c^10*d*x^10 + 145*b*c^8*d*x^8 + b*c^6*d*x^6 + 2*b*c^4*d*
x^4 + 8*b*c^2*d*x^2 - 16*b*d)*arccos(c*x))*sqrt(-c^2*d*x^2 + d))/(c^10*x^2
 - c^8)
```

Sympy [F(-1)]

Timed out.

$$\int x^7 (d - c^2dx^2)^{3/2} (a + b \arccos(cx)) \, dx = \text{Timed out}$$

input
```
integrate(x**7*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.71

$$\int x^7\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx =$$
$$-\frac{1}{1155}\left(\frac{105\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^6}{c^2d}+\frac{70\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^4}{c^4d}+\frac{40\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^6d}+\frac{16\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^8d}\right)b\arccos$$
$$-\frac{1}{1155}\left(\frac{105\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^6}{c^2d}+\frac{70\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^4}{c^4d}+\frac{40\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^6d}+\frac{16\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^8d}\right)a$$
$$-\frac{\left(33075\,c^{10}d^{\frac{3}{2}}x^{11}-53900\,c^8d^{\frac{3}{2}}x^9+2475\,c^6d^{\frac{3}{2}}x^7+4158\,c^4d^{\frac{3}{2}}x^5+9240\,c^2d^{\frac{3}{2}}x^3+55440\,d^{\frac{3}{2}}x\right)b}{4002075\,c^7}$$

input
```
integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
-1/1155*(105*(-c^2*d*x^2 + d)^(5/2)*x^6/(c^2*d) + 70*(-c^2*d*x^2 + d)^(5/2
)*x^4/(c^4*d) + 40*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^6*d) + 16*(-c^2*d*x^2 + d
)^(5/2)/(c^8*d))*b*arccos(c*x) - 1/1155*(105*(-c^2*d*x^2 + d)^(5/2)*x^6/(c
^2*d) + 70*(-c^2*d*x^2 + d)^(5/2)*x^4/(c^4*d) + 40*(-c^2*d*x^2 + d)^(5/2)*
x^2/(c^6*d) + 16*(-c^2*d*x^2 + d)^(5/2)/(c^8*d))*a - 1/4002075*(33075*c^10
*d^(3/2)*x^11 - 53900*c^8*d^(3/2)*x^9 + 2475*c^6*d^(3/2)*x^7 + 4158*c^4*d^
(3/2)*x^5 + 9240*c^2*d^(3/2)*x^3 + 55440*d^(3/2)*x)*b/c^7
```

Giac [F(-2)]

Exception generated.

$$\int x^7\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^7 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \int x^7\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int(x^7*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^7*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^7 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \frac{\sqrt{d}\,d\left(-105\sqrt{-c^2x^2+1}\,a\,c^{10}x^{10}+140\sqrt{-c^2x^2+1}\,a\,c^8x^8-5\sqrt{-c^2x^2+1}\,a\,c^6x^6-6\sqrt{-\right.}{}$$

input
```
int(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d*( - 105*sqrt( - c**2*x**2 + 1)*a*c**10*x**10 + 140*sqrt( - c**2
*x**2 + 1)*a*c**8*x**8 - 5*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - 6*sqrt( -
c**2*x**2 + 1)*a*c**4*x**4 - 8*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 16*sqr
t( - c**2*x**2 + 1)*a - 1155*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**9,x)*
b*c**10 + 1155*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**7,x)*b*c**8))/(1155
*c**8)
```

3.80 $\int x^5(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 301

$$\int x^5\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx=\frac{8bdx\sqrt{d-c^2dx^2}}{315c^5\sqrt{1-c^2x^2}}+\frac{4bdx^3\sqrt{d-c^2dx^2}}{945c^3\sqrt{1-c^2x^2}}+\frac{bdx^5\sqrt{d-c^2dx^2}}{525c\sqrt{1-c^2x^2}}-\frac{10bcdx^7\sqrt{d-c^2dx^2}}{441\sqrt{1-c^2x^2}}+\frac{bc^3dx^9\sqrt{d-c^2dx^2}}{81\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^6d}+\frac{2(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{7c^6d^2}-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\arccos(cx))}{9c^6d^3}$$

output
```
8/315*b*d*x*(-c^2*d*x^2+d)^(1/2)/c^5/(-c^2*x^2+1)^(1/2)+4/945*b*d*x^3*(-c^
2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/525*b*d*x^5*(-c^2*d*x^2+d)^(1/2)
/c/(-c^2*x^2+1)^(1/2)-10/441*b*c*d*x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(
1/2)+1/81*b*c^3*d*x^9*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/5*(-c^2*d*
x^2+d)^(5/2)*(a+b*arccos(c*x))/c^6/d+2/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccos(
c*x))/c^6/d^2-1/9*(-c^2*d*x^2+d)^(9/2)*(a+b*arccos(c*x))/c^6/d^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.52

$$\int x^5\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx =$$
$$-\frac{d\sqrt{d-c^2dx^2}\Big(315a(-1+c^2x^2)^3\left(8+20c^2x^2+35c^4x^4\right)-bcx\sqrt{1-c^2x^2}(2520+420c^2x^2+189c^4x^4-22}{99225c^6\left(-1+c^2x^2\right)}$$

input
```
Integrate[x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/99225*(d*Sqrt[d - c^2*d*x^2]*(315*a*(-1 + c^2*x^2)^3*(8 + 20*c^2*x^2 +
35*c^4*x^4) - b*c*x*Sqrt[1 - c^2*x^2]*(2520 + 420*c^2*x^2 + 189*c^4*x^4 -
2250*c^6*x^6 + 1225*c^8*x^8) + 315*b*(-1 + c^2*x^2)^3*(8 + 20*c^2*x^2 + 35
*c^4*x^4)*ArcCos[c*x]))/(c^6*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5195, 27, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5195}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d(1-c^2x^2)^2(35c^4x^4+20c^2x^2+8)}{315c^6}dx}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\arccos(cx))}{9c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^6d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^6d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bd\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)^2 \left(35c^4x^4+20c^2x^2+8\right)dx}{315c^5\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\arccos(cx))}{9c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^6d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^6d}$$

$$\Big\downarrow \text{1467}$$

$$-\frac{bd\sqrt{d-c^2dx^2}\int \left(35c^8x^8-50c^6x^6+3c^4x^4+4c^2x^2+8\right)dx}{315c^5\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\arccos(cx))}{9c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^6d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^6d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\arccos(cx))}{9c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^6d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^6d}-\frac{bd\left(\frac{35c^8x^9}{9}-\frac{50c^6x^7}{7}+\frac{3c^4x^5}{5}+\frac{4c^2x^3}{3}+8x\right)\sqrt{d-c^2dx^2}}{315c^5\sqrt{1-c^2x^2}}$$

input
```
Int[x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/315*(b*d*Sqrt[d - c^2*d*x^2]*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (50
*c^6*x^7)/7 + (35*c^8*x^9)/9))/(c^5*Sqrt[1 - c^2*x^2]) - ((d - c^2*d*x^2)^
(5/2)*(a + b*ArcCos[c*x]))/(5*c^6*d) + (2*(d - c^2*d*x^2)^(7/2)*(a + b*Arc
Cos[c*x]))/(7*c^6*d^2) - ((d - c^2*d*x^2)^(9/2)*(a + b*ArcCos[c*x]))/(9*c^
6*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.80

method	result
orering	$\frac{(20825c^{10}x^{10}-50900c^8x^8+29457c^6x^6+2730c^4x^4+19320c^2x^2-15120)\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}(a+b\arccos(cx))}{99225c^6(cx-1)(cx+1)(c^2x^2-1)} - \frac{(1225c^8x^8-2250c^6x^6-}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/99225*(20825*c^10*x^10-50900*c^8*x^8+29457*c^6*x^6+2730*c^4*x^4+19320*c^
2*x^2-15120)/c^6/(c*x-1)/(c*x+1)/(c^2*x^2-1)*(-c^2*d*x^2+d)^(3/2)*(a+b*arc
cos(c*x))-1/99225/x^4*(1225*c^8*x^8-2250*c^6*x^6+189*c^4*x^4+420*c^2*x^2+2
520)/c^6/(c*x-1)/(c*x+1)*(5*x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))-3*x
^6*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*d*c^2-x^5*(-c^2*d*x^2+d)^(3/2)*b
*c/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.73

$$\int x^5 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \frac{\left(1225\,bc^9dx^9 - 2250\,bc^7dx^7 + 189\,bc^5dx^5 + 420\,bc^3dx^3 + 2520\,bcdx\right)\sqrt{-c^2dx^2+d}\sqrt{-c}}{}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
1/99225*((1225*b*c^9*d*x^9 - 2250*b*c^7*d*x^7 + 189*b*c^5*d*x^5 + 420*b*c^
3*d*x^3 + 2520*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 315*(35*
a*c^10*d*x^10 - 85*a*c^8*d*x^8 + 53*a*c^6*d*x^6 + a*c^4*d*x^4 + 4*a*c^2*d*
x^2 - 8*a*d + (35*b*c^10*d*x^10 - 85*b*c^8*d*x^8 + 53*b*c^6*d*x^6 + b*c^4*
d*x^4 + 4*b*c^2*d*x^2 - 8*b*d)*arccos(c*x))*sqrt(-c^2*d*x^2 + d))/(c^8*x^2
 - c^6)
```

Sympy [F(-1)]

Timed out.

$$\int x^5 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \text{Timed out}$$

input
```
integrate(x**5*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.69

$$\int x^5 (d - c^2dx^2)^{3/2} (a + b\arccos(cx))\,dx =$$
$$-\frac{1}{315}\left(\frac{35\,(-c^2dx^2+d)^{\frac{5}{2}}x^4}{c^2d}+\frac{20\,(-c^2dx^2+d)^{\frac{5}{2}}x^2}{c^4d}+\frac{8\,(-c^2dx^2+d)^{\frac{5}{2}}}{c^6d}\right)b\arccos(cx)$$
$$-\frac{1}{315}\left(\frac{35\,(-c^2dx^2+d)^{\frac{5}{2}}x^4}{c^2d}+\frac{20\,(-c^2dx^2+d)^{\frac{5}{2}}x^2}{c^4d}+\frac{8\,(-c^2dx^2+d)^{\frac{5}{2}}}{c^6d}\right)a$$
$$-\frac{\left(1225\,c^8d^{\frac{3}{2}}x^9-2250\,c^6d^{\frac{3}{2}}x^7+189\,c^4d^{\frac{3}{2}}x^5+420\,c^2d^{\frac{3}{2}}x^3+2520\,d^{\frac{3}{2}}x\right)b}{99225\,c^5}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
-1/315*(35*(-c^2*d*x^2 + d)^(5/2)*x^4/(c^2*d) + 20*(-c^2*d*x^2 + d)^(5/2)*
x^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(5/2)/(c^6*d))*b*arccos(c*x) - 1/315*(35*
(-c^2*d*x^2 + d)^(5/2)*x^4/(c^2*d) + 20*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^4*d)
 + 8*(-c^2*d*x^2 + d)^(5/2)/(c^6*d))*a - 1/99225*(1225*c^8*d^(3/2)*x^9 - 2
250*c^6*d^(3/2)*x^7 + 189*c^4*d^(3/2)*x^5 + 420*c^2*d^(3/2)*x^3 + 2520*d^(
3/2)*x)*b/c^5
```

Giac [F(-2)]

Exception generated.

$$\int x^5 (d - c^2dx^2)^{3/2} (a + b\arccos(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx=\int x^5\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int(x^5*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^5*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^5 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx=\frac{\sqrt{d}\,d\big(-35\sqrt{-c^2x^2+1}\,a\,c^8x^8+50\sqrt{-c^2x^2+1}\,a\,c^6x^6-3\sqrt{-c^2x^2+1}\,a\,c^4x^4-4\sqrt{-c^2x}}{}$$

input
```
int(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d*( - 35*sqrt( - c**2*x**2 + 1)*a*c**8*x**8 + 50*sqrt( - c**2*x**
2 + 1)*a*c**6*x**6 - 3*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 - 4*sqrt( - c**2
*x**2 + 1)*a*c**2*x**2 - 8*sqrt( - c**2*x**2 + 1)*a - 315*int(sqrt( - c**2
*x**2 + 1)*acos(c*x)*x**7,x)*b*c**8 + 315*int(sqrt( - c**2*x**2 + 1)*acos(
c*x)*x**5,x)*b*c**6))/(315*c**6)
```

3.81 $\int x^3(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 227

$$\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx=\frac{2bdx\sqrt{d-c^2dx^2}}{35c^3\sqrt{1-c^2x^2}}+\frac{bdx^3\sqrt{d-c^2dx^2}}{105c\sqrt{1-c^2x^2}}-\frac{8bcdx^5\sqrt{d-c^2dx^2}}{175\sqrt{1-c^2x^2}}+\frac{bc^3dx^7\sqrt{d-c^2dx^2}}{49\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^4d}+\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^4d^2}$$

output
```
2/35*b*d*x*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/105*b*d*x^3*(-c^2
*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-8/175*b*c*d*x^5*(-c^2*d*x^2+d)^(1/2)/
(-c^2*x^2+1)^(1/2)+1/49*b*c^3*d*x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2
)-1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/c^4/d+1/7*(-c^2*d*x^2+d)^(7/2
)*(a+b*arccos(c*x))/c^4/d^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.58

$$\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx =$$
$$-\frac{d\sqrt{d-c^2dx^2}\Big(105a(-1+c^2x^2)^3\left(2+5c^2x^2\right)-bcx\sqrt{1-c^2x^2}(210+35c^2x^2-168c^4x^4+75c^6x^6)+105b(}{3675c^4\left(-1+c^2x^2\right)}$$

input
```
Integrate[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/3675*(d*Sqrt[d - c^2*d*x^2]*(105*a*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2) - b
*c*x*Sqrt[1 - c^2*x^2]*(210 + 35*c^2*x^2 - 168*c^4*x^4 + 75*c^6*x^6) + 105
*b*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2)*ArcCos[c*x]))/(c^4*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5195, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx$$
$$\Big\downarrow \text{ 5195}$$
$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d\left(1-c^2x^2\right)^2\left(5c^2x^2+2\right)}{35c^4}dx}{\sqrt{1-c^2x^2}}+\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^4d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^4d}$$
$$\Big\downarrow \text{ 27}$$

$$-\frac{bd\sqrt{d-c^2dx^2}\int\left(1-c^2x^2\right)^2\left(5c^2x^2+2\right)dx}{35c^3\sqrt{1-c^2x^2}}+\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^4d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^4d}$$

$\Big\downarrow$ 290

$$-\frac{bd\sqrt{d-c^2dx^2}\int\left(5c^6x^6-8c^4x^4+c^2x^2+2\right)dx}{35c^3\sqrt{1-c^2x^2}}+\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^4d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^4d}$$

$\Big\downarrow$ 2009

$$\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^4d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^4d}-\frac{bd\left(\frac{5c^6x^7}{7}-\frac{8c^4x^5}{5}+\frac{c^2x^3}{3}+2x\right)\sqrt{d-c^2dx^2}}{35c^3\sqrt{1-c^2x^2}}$$

input
```
Int[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/35*(b*d*Sqrt[d - c^2*d*x^2]*(2*x + (c^2*x^3)/3 - (8*c^4*x^5)/5 + (5*c^6
*x^7)/7))/(c^3*Sqrt[1 - c^2*x^2]) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c
*x]))/(5*c^4*d) + ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(7*c^4*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.99

method	result
orering	$\frac{(325c^8x^8-866c^6x^6+553c^4x^4+420c^2x^2-280)\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}(a+b\arccos(cx))}{1225c^4(cx-1)(cx+1)(c^2x^2-1)}-\frac{(75c^6x^6-168c^4x^4+35c^2x^2+210)\left(3x^2\left(-c^2d\,x^2\right.\right.}{}$
default	$a\left(-\frac{x^2\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{7c^2d}-\frac{2\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{35d\,c^4}\right)+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(64c^8x^8-144c^6x^6+64i\sqrt{-c^2x^2+1}\,x^7c^7+104c^4x^4-112\right.}{}\right.$
parts	$a\left(-\frac{x^2\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{7c^2d}-\frac{2\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{35d\,c^4}\right)+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(64c^8x^8-144c^6x^6+64i\sqrt{-c^2x^2+1}\,x^7c^7+104c^4x^4-112\right.}{}\right.$

input
```
int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/1225*(325*c^8*x^8-866*c^6*x^6+553*c^4*x^4+420*c^2*x^2-280)/c^4/(c*x-1)/(
c*x+1)/(c^2*x^2-1)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))-1/3675/x^2*(75*c
^6*x^6-168*c^4*x^4+35*c^2*x^2+210)/c^4/(c*x-1)/(c*x+1)*(3*x^2*(-c^2*d*x^2+
d)^(3/2)*(a+b*arccos(c*x))-3*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*d*
c^2-x^3*(-c^2*d*x^2+d)^(3/2)*b*c/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

$$\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx=\frac{(75\,bc^7dx^7-168\,bc^5dx^5+35\,bc^3dx^3+210\,bcdx)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}-105\,(5\,ac^8}{}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
1/3675*((75*b*c^7*d*x^7 - 168*b*c^5*d*x^5 + 35*b*c^3*d*x^3 + 210*b*c*d*x)*
sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 105*(5*a*c^8*d*x^8 - 13*a*c^6*d*
x^6 + 9*a*c^4*d*x^4 + a*c^2*d*x^2 - 2*a*d + (5*b*c^8*d*x^8 - 13*b*c^6*d*x^
6 + 9*b*c^4*d*x^4 + b*c^2*d*x^2 - 2*b*d)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)
)/(c^6*x^2 - c^4)
```

Sympy [F]

$$\int x^3(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx = \int x^3(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acos}\,(cx))\,\,dx$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Integral(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.66

$$\int x^3(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx = -\frac{1}{35}\left(\frac{5\,(-c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)b\arccos(cx) - \frac{1}{35}\left(\frac{5\,(-c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)a - \frac{\left(75\,c^6d^{\frac{3}{2}}x^7-168\,c^4d^{\frac{3}{2}}x^5+35\,c^2d^{\frac{3}{2}}x^3+210\,d^{\frac{3}{2}}x\right)b}{3675\,c^3}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
-1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^
4*d))*b*arccos(c*x) - 1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2
*d*x^2 + d)^(5/2)/(c^4*d))*a - 1/3675*(75*c^6*d^(3/2)*x^7 - 168*c^4*d^(3/2
)*x^5 + 35*c^2*d^(3/2)*x^3 + 210*d^(3/2)*x)*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx = \int x^3\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int(x^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^3 (d - c^2 d x^2)^{3/2} (a + b \arccos(cx)) \, dx = \frac{\sqrt{d}\, d\left(-5\sqrt{-c^2x^2+1}\, a\, c^6x^6 + 8\sqrt{-c^2x^2+1}\, a\, c^4x^4 - \sqrt{-c^2x^2+1}\, a\, c^2x^2 - 2\sqrt{-c^2x^2+\right.}{35c}$$

input
```
int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d*( - 5*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 + 8*sqrt( - c**2*x**2
+ 1)*a*c**4*x**4 - sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 2*sqrt( - c**2*x**
2 + 1)*a - 35*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**5,x)*b*c**6 + 35*int
(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**3,x)*b*c**4))/(35*c**4)
```

3.82 $\int x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx=\frac{bdx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}-\frac{2bcdx^3\sqrt{d-c^2dx^2}}{15\sqrt{1-c^2x^2}}+\frac{bc^3dx^5\sqrt{d-c^2dx^2}}{25\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^2d}$$

output
```
1/5*b*d*x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-2/15*b*c*d*x^3*(-c^2*d
*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/25*b*c^3*d*x^5*(-c^2*d*x^2+d)^(1/2)/(-c
^2*x^2+1)^(1/2)-1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/c^2/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.67

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx=-\frac{d\sqrt{d-c^2dx^2}\left(15a(-1+c^2x^2)^3+bcx\sqrt{1-c^2x^2}(-15+10c^2x^2-3c^4x^4)+15b(-1+c^2x^2)^3\arccos(cx)\right)}{75c^2\left(-1+c^2x^2\right)}$$

input
```
Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/75*(d*Sqrt[d - c^2*d*x^2]*(15*a*(-1 + c^2*x^2)^3 + b*c*x*Sqrt[1 - c^2*x
^2]*(-15 + 10*c^2*x^2 - 3*c^4*x^4) + 15*b*(-1 + c^2*x^2)^3*ArcCos[c*x]))/(
c^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5183, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5183}$$

$$-\frac{bd\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)^2dx}{5c\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^2d}$$

$$\Big\downarrow \text{210}$$

$$-\frac{bd\sqrt{d-c^2dx^2}\int \left(c^4x^4-2c^2x^2+1\right)dx}{5c\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5c^2d}-\frac{bd\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}$$

input
```
Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/5*(b*d*Sqrt[d - c^2*d*x^2]*(x - (2*c^2*x^3)/3 + (c^4*x^5)/5))/(c*Sqrt[1
 - c^2*x^2]) - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^2*d)
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.30

method	result
orering	$\frac{\left(27 c^{6} x^{6}-88 c^{4} x^{4}+115 c^{2} x^{2}-30\right) \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \arccos \left(c x \right)\right)}{75 c^{2} \left(c x -1\right) \left(c x +1\right) \left(c^{2} x^{2}-1\right)}-\frac{\left(3 c^{4} x^{4}-10 c^{2} x^{2}+15\right) \left(\left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \arccos \left(c x \right)\right)-3 x^{2} \cdots\right.}{75 c^{2} \left(c x -1\right) \left(\cdots\right.}$
default	$-\frac{a \left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{5 c^{2} d}+b \left(-\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \left(16 c^{6} x^{6}-28 c^{4} x^{4}+16 i \sqrt{-c^{2} x^{2}+1}\, x^{5} c^{5}+13 c^{2} x^{2}-20 i \sqrt{-c^{2} x^{2}+1}\, x^{3} c^{3}+5 i \sqrt{-c^{2} x^{2}+1} \cdots\right.}{800 c^{2} \left(c^{2} x^{2}-1\right)}\right.$
parts	$-\frac{a \left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{5 c^{2} d}+b \left(-\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \left(16 c^{6} x^{6}-28 c^{4} x^{4}+16 i \sqrt{-c^{2} x^{2}+1}\, x^{5} c^{5}+13 c^{2} x^{2}-20 i \sqrt{-c^{2} x^{2}+1}\, x^{3} c^{3}+5 i \sqrt{-c^{2} x^{2}+1} \cdots\right.}{800 c^{2} \left(c^{2} x^{2}-1\right)}\right.$

input
```
int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/75*(27*c^6*x^6-88*c^4*x^4+115*c^2*x^2-30)/c^2/(c*x-1)/(c*x+1)/(c^2*x^2-1
)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))-1/75*(3*c^4*x^4-10*c^2*x^2+15)/c^
2/(c*x-1)/(c*x+1)*((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))-3*x^2*(-c^2*d*x^
2+d)^(1/2)*(a+b*arccos(c*x))*d*c^2-x*(-c^2*d*x^2+d)^(3/2)*b*c/(-c^2*x^2+1)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.04

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx = \frac{(3\,bc^5dx^5-10\,bc^3dx^3+15\,bcdx)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}-15\,(ac^6dx^6-3\,ac^4dx^4+3}{75\,(c^4x^2-}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/75*((3*b*c^5*d*x^5 - 10*b*c^3*d*x^3 + 15*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*s
qrt(-c^2*x^2 + 1) - 15*(a*c^6*d*x^6 - 3*a*c^4*d*x^4 + 3*a*c^2*d*x^2 - a*d
+ (b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*arccos(c*x))*sqrt(-c
^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F]

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx = \int x(-d(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acos}\,(cx))\;dx$$

input
```
integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Integral(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.57

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx = -\frac{(-c^2dx^2+d)^{\frac{5}{2}}b\arccos\left(cx\right)}{5\,c^2d} - \frac{(-c^2dx^2+d)^{\frac{5}{2}}a}{5\,c^2d} - \frac{\left(3\,c^4d^{\frac{5}{2}}x^5-10\,c^2d^{\frac{5}{2}}x^3+15\,d^{\frac{5}{2}}x\right)b}{75\,cd}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-1/5*(-c^2*d*x^2 + d)^(5/2)*b*arccos(c*x)/(c^2*d) - 1/5*(-c^2*d*x^2 + d)^(
5/2)*a/(c^2*d) - 1/75*(3*c^4*d^(5/2)*x^5 - 10*c^2*d^(5/2)*x^3 + 15*d^(5/2)
*x)*b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx = \int x\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int(x*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\,dx = \frac{\sqrt{d}\,d\left(-\sqrt{-c^2x^2+1}\,a\,c^4x^4+2\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a-5\left(\int\sqrt{-c^2x^2+1}\right.\right.}{5c^2}$$

input
```
int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d*( - sqrt( - c**2*x**2 + 1)*a*c**4*x**4 + 2*sqrt( - c**2*x**2 +
1)*a*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a - 5*int(sqrt( - c**2*x**2 + 1)*a
cos(c*x)*x**3,x)*b*c**4 + 5*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x,x)*b*c*
*2))/(5*c**2)
```

3.83 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 278

$$\begin{aligned}\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x}\,dx = &-\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}}+\frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{1-c^2x^2}}\\ &+d\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{1}{3}(d-c^2dx^2)^{3/2}(a+b\arccos(cx))\\ &-\frac{2d\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}\\ &+\frac{ibd\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}\\ &-\frac{ibd\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-4/3*b*c*d*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/9*b*c^3*d*x^3*(-c^2
*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)
)+1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))-2*d*(-c^2*d*x^2+d)^(1/2)*(a+b
*arccos(c*x))*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+I*b*d*(
-c^2*d*x^2+d)^(1/2)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2
)-I*b*d*(-c^2*d*x^2+d)^(1/2)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2
+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x}\,dx = -\frac{1}{3}ad(-4+c^2x^2)\sqrt{d-c^2dx^2} - \frac{bd\sqrt{d-c^2dx^2}\left(-9cx-12(1-c^2x^2)^{3/2}\arccos(cx)+\cos(3\arccos(cx))\right)}{36\sqrt{1-c^2x^2}} + ad^{3/2}\log(x) - ad^{3/2}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right) + \frac{bd\sqrt{d-c^2dx^2}(cx+\sqrt{1-c^2x^2}\arccos(cx)-\arccos(cx)\log}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x,x]
```

output
```
-1/3*(a*d*(-4 + c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (b*d*Sqrt[d - c^2*d*x^2]*(
-9*c*x - 12*(1 - c^2*x^2)^(3/2)*ArcCos[c*x] + Cos[3*ArcCos[c*x]]))/(36*Sqr
t[1 - c^2*x^2]) + a*d^(3/2)*Log[x] - a*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d - c^
2*d*x^2]] + (b*d*Sqrt[d - c^2*d*x^2]*(c*x + Sqrt[1 - c^2*x^2]*ArcCos[c*x]
- ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] + ArcCos[c*x]*Log[1 + I*E^(I*Ar
cCos[c*x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I*PolyLog[2, I*E^(I*A
rcCos[c*x])]))/Sqrt[1 - c^2*x^2]
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.84, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5203, 2009, 5199, 24, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x}\,dx$$

$$\Big\downarrow \text{5203}$$

$$d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}dx + \frac{bcd\sqrt{d-c^2dx^2}\int(1-c^2x^2)\,dx}{3\sqrt{1-c^2x^2}} + \frac{1}{3}(d-c^2dx^2)^{3/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5199}$$

$$d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int 1dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{24}$$

$$d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}\right)+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5219}$$

$$d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}\right)+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$d\left(-\frac{\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}\right)+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{4669}$$

$$d\left(-\frac{\sqrt{d-c^2dx^2}\left(-b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(e^{i\,a\right.}\right.}{\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}}$$

$\downarrow$ 2715

$$d\left(-\frac{\sqrt{d-c^2dx^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\,\mathrm{arc}}\right.}{\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}}$$

$\downarrow$ 2838

$$d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\mathrm{PolyLog}\left(2,ie^{i\,a}\right.\right.}{\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x,x]
```

output
```
(b*c*d*Sqrt[d - c^2*d*x^2]*(x - (c^2*x^3)/3))/(3*Sqrt[1 - c^2*x^2]) + ((d
- c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/3 + d*((b*c*x*Sqrt[d - c^2*d*x^2])
/Sqrt[1 - c^2*x^2] + Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]) - (Sqrt[d - c
^2*d*x^2]*((-2*I)*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcCos[c*x])] + I*b*Poly
Log[2, (-I)*E^(I*ArcCos[c*x])] - I*b*PolyLog[2, I*E^(I*ArcCos[c*x])]))/Sqr
t[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.95

method	result
default	$\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} a}{3}-a\,d^{\frac{3}{2}} \ln\left(\frac{2 d+2 \sqrt{d}\,\sqrt{-c^{2} d\,x^{2}+d}}{x}\right)+a d \sqrt{-c^{2} d\,x^{2}+d}-\frac{4 b \sqrt{-d \left(c^{2} x^{2}-1\right)}\,d \arccos\left(c x\right)}{3 \left(c^{2} x^{2}-1\right)}+\frac{b\,c^{3} d\,x^{3}}{}$
parts	$\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} a}{3}-a\,d^{\frac{3}{2}} \ln\left(\frac{2 d+2 \sqrt{d}\,\sqrt{-c^{2} d\,x^{2}+d}}{x}\right)+a d \sqrt{-c^{2} d\,x^{2}+d}-\frac{4 b \sqrt{-d \left(c^{2} x^{2}-1\right)}\,d \arccos\left(c x\right)}{3 \left(c^{2} x^{2}-1\right)}+\frac{b\,c^{3} d\,x^{3}}{}$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-c^2*d*x^2+d)^(3/2)*a-a*d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2
))/x)+a*d*(-c^2*d*x^2+d)^(1/2)-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x^2-1)*
arccos(c*x)+1/9*b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*
x^3*c^3-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x*c-
b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*d*arccos(c*x)*ln(1
+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)
/(c^2*x^2-1)*d*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+I*b*(-d*(c^2
*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*d*dilog(1+I*(c*x+I*(-c^2*x^2
+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*d*di
log(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-1/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x^
2-1)*arccos(c*x)*x^4*c^4+5/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c^2*x^2-1)*arccos
(c*x)*x^2*c^2
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x,x, algorithm="fricas")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccos(c*x))*sqrt(-c^2*
d*x^2 + d)/x, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x}\,dx = \int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))}{x}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))/x,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))/x, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}{x}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x,x, algorithm="maxima")
```

output
```
-b*sqrt(d)*integrate((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(
sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x, x) - 1/3*(3*d^(3/2)*log(2*sqrt(-c^2*
d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - (-c^2*d*x^2 + d)^(3/2) - 3*sqrt(
-c^2*d*x^2 + d)*d)*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x, x)
```

Reduce [F]

$$\int \frac{(d - c^2dx^2)^{3/2} (a + b\arccos(cx))}{x}\,dx = \frac{\sqrt{d}\,d\Big(-\sqrt{-c^2x^2+1}\,a\,c^2x^2 + 4\sqrt{-c^2x^2+1}\,a + 3\Big(\int \frac{\sqrt{-c^2x^2+1}\,acos}{x}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))/x,x)
```

output
```
(sqrt(d)*d*( - sqrt( - c**2*x**2 + 1)*a*c**2*x**2 + 4*sqrt( - c**2*x**2 +
1)*a + 3*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x,x)*b - 3*int(sqrt( - c**
2*x**2 + 1)*acos(c*x)*x,x)*b*c**2 + 3*log(tan(asin(c*x)/2))*a - 4*a))/3
```

3.84 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 297

$$\begin{aligned}\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^3}\,dx &= -\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{1-c^2x^2}}+\frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \\ &\quad -\frac{3}{2}c^2d\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{2x^2} \\ &\quad +\frac{3c^2d\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\ &\quad -\frac{3ibc^2d\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{2\sqrt{1-c^2x^2}} \\ &\quad +\frac{3ibc^2d\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2\sqrt{1-c^2x^2}}\end{aligned}$$

output

```
-1/2*b*c*d*(-c^2*d*x^2+d)^(1/2)/x/(-c^2*x^2+1)^(1/2)+b*c^3*d*x*(-c^2*d*x^2
+d)^(1/2)/(-c^2*x^2+1)^(1/2)-3/2*c^2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*
x))-1/2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^2+3*c^2*d*(-c^2*d*x^2+d)^
(1/2)*(a+b*arccos(c*x))*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/
2)-3/2*I*b*c^2*d*(-c^2*d*x^2+d)^(1/2)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))
/(-c^2*x^2+1)^(1/2)+3/2*I*b*c^2*d*(-c^2*d*x^2+d)^(1/2)*polylog(2,c*x+I*(-c
^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.34

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^3}\,dx = \frac{1}{2}d\left(-\frac{a(1+2c^2x^2)\sqrt{d-c^2dx^2}}{x^2}\right.$$

$$-3ac^2\sqrt{d}\log(x)+3ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)$$

$$-\frac{2bc^2\sqrt{d-c^2dx^2}\left(cx+\sqrt{1-c^2x^2}\arccos(cx)-\arccos(cx)\log\left(1-ie^{i\arccos(cx)}\right)+\arccos(cx)\log\left(1+ie^{i\,\mathrm{a}}\right.\right.}{\sqrt{1-c^2x^2}}$$

$$-\frac{bd\sqrt{1-c^2x^2}\left(-cx+\sqrt{1-c^2x^2}\arccos(cx)-c^2x^2\arccos(cx)\log\left(1-ie^{i\arccos(cx)}\right)+c^2x^2\arccos(cx)\log\left(\right.\right.}{x^2\sqrt{d-c^2dx^2}}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
(d*(-((a*(1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2])/x^2) - 3*a*c^2*Sqrt[d]*Log[x
] + 3*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (2*b*c^2*Sqrt[d
 - c^2*d*x^2]*(c*x + Sqrt[1 - c^2*x^2]*ArcCos[c*x] - ArcCos[c*x]*Log[1 - I
*E^(I*ArcCos[c*x])] + ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] - I*PolyLog
[2, (-I)*E^(I*ArcCos[c*x])] + I*PolyLog[2, I*E^(I*ArcCos[c*x])]))/Sqrt[1 -
 c^2*x^2] - (b*d*Sqrt[1 - c^2*x^2]*(-(c*x) + Sqrt[1 - c^2*x^2]*ArcCos[c*x]
 - c^2*x^2*ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] + c^2*x^2*ArcCos[c*x]*
Log[1 + I*E^(I*ArcCos[c*x])] - I*c^2*x^2*PolyLog[2, (-I)*E^(I*ArcCos[c*x])
] + I*c^2*x^2*PolyLog[2, I*E^(I*ArcCos[c*x])]))/(x^2*Sqrt[d - c^2*d*x^2]))
)/2
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.81, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5201, 244, 2009, 5199, 24, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x^3}dx$$

$$\Big\downarrow \text{5201}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\int \frac{1-c^2x^2}{x^2}dx}{2\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}$$

$$\Big\downarrow \text{244}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{1}{x^2}-c^2\right)dx}{2\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5199}$$

$$-\frac{3}{2}c^2d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int 1dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{24}$$

$$-\frac{3}{2}c^2d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5219}$$

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}\right)-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\right.$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(-b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\right.}{\sqrt{1-c^2x^2}}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)\right.}{\sqrt{1-c^2x^2}}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.}{\sqrt{1-c^2x^2}}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d*(-x^(-1) - c^2*x)*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2] - ((d
 - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(2*x^2) - (3*c^2*d*((b*c*x*Sqrt[d
 - c^2*d*x^2])/Sqrt[1 - c^2*x^2] + Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])
 - (Sqrt[d - c^2*d*x^2]*((-2*I)*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcCos[c*x
])] + I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*b*PolyLog[2, I*E^(I*ArcCo
s[c*x])]))/Sqrt[1 - c^2*x^2]))/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.52

method	result
default	$a\left(-\frac{\left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{2 d \,x^{2}}-\frac{3 c^{2} \left(\frac{\left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}{3}+d \left(\sqrt{-c^{2} d \,x^{2}+d}-\sqrt{d}\, \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{-c^{2} d \,x^{2}+d}}{x}\right)\right)\right)}{2}\right)+b \left(-\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}}{}\right.$
parts	$a\left(-\frac{\left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{2 d \,x^{2}}-\frac{3 c^{2} \left(\frac{\left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}{3}+d \left(\sqrt{-c^{2} d \,x^{2}+d}-\sqrt{d}\, \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{-c^{2} d \,x^{2}+d}}{x}\right)\right)\right)}{2}\right)+b \left(-\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}}{}\right.$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/d/x^2*(-c^2*d*x^2+d)^(5/2)-3/2*c^2*(1/3*(-c^2*d*x^2+d)^(3/2)+d*((-
c^2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x))))+b
*(-1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos
(c*x)+I)*c^2*d/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/
2)*x*c+c^2*x^2-1)*(arccos(c*x)-I)*c^2*d/(c^2*x^2-1)-1/2*d*(c^2*x^2*arccos(
c*x)+c*x*(-c^2*x^2+1)^(1/2)-arccos(c*x))*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1
)/x^2+3/2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*(arccos(c*
x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+
1)^(1/2)))-I*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+I*dilog(1-I*(c*x+I*(-c^
2*x^2+1)^(1/2))))*c^2*d)
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^3}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^3,x, algorithm="fricas"
)
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccos(c*x))*sqrt(-c^2*
d*x^2 + d)/x^3, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^3}\,dx=\int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))}{x^3}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))/x**3,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))/x**3, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}{x^3}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^3,x, algorithm="maxima"
)
```

output
```
-b*sqrt(d)*integrate((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(
sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x^3, x) + 1/2*(3*c^2*d^(3/2)*log(2*sqrt
(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - (-c^2*d*x^2 + d)^(3/2)*c^2
 - 3*sqrt(-c^2*d*x^2 + d)*c^2*d - (-c^2*d*x^2 + d)^(5/2)/(d*x^2))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}{x^3}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}{x^3}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x^3}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^3,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}{x^3}\,dx = \frac{\sqrt{d}\,d\Big(-8\sqrt{-c^2x^2+1}\,a\,c^2x^2-4\sqrt{-c^2x^2+1}\,a+8\Big(\int \frac{\sqrt{-c^2x^2+1}\,aco}{x^3}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))/x^3,x)
```

output
```
(sqrt(d)*d*( - 8*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 4*sqrt( - c**2*x**2
+ 1)*a + 8*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**3,x)*b*x**2 - 8*int((
sqrt( - c**2*x**2 + 1)*acos(c*x))/x,x)*b*c**2*x**2 - 12*log(tan(asin(c*x)/
2))*a*c**2*x**2 + 9*a*c**2*x**2))/(8*x**2)
```

3.85 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 307

$$\begin{aligned}\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^5}\,dx = & -\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}}+\frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{1-c^2x^2}} \\ & +\frac{3c^2d\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{8x^2}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{4x^4} \\ & -\frac{3c^4d\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{4\sqrt{1-c^2x^2}} \\ & +\frac{3ibc^4d\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{8\sqrt{1-c^2x^2}} \\ & -\frac{3ibc^4d\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{8\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-1/12*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^3/(-c^2*x^2+1)^(1/2)+5/8*b*c^3*d*(-c^2*
d*x^2+d)^(1/2)/x/(-c^2*x^2+1)^(1/2)+3/8*c^2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*ar
ccos(c*x))/x^2-1/4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^4-3/4*c^4*d*(-
c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/(-c
^2*x^2+1)^(1/2)+3/8*I*b*c^4*d*(-c^2*d*x^2+d)^(1/2)*polylog(2,-c*x-I*(-c^2*
x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-3/8*I*b*c^4*d*(-c^2*d*x^2+d)^(1/2)*polylo
g(2,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.78

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^5}\,dx = \frac{2bcd^2x-17bc^3d^2x^3+15bc^5d^2x^5-6ad^2\sqrt{1-c^2x^2}+21ac^2d^2x^2\sqrt{}}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^5,x]
```

output
```
(2*b*c*d^2*x - 17*b*c^3*d^2*x^3 + 15*b*c^5*d^2*x^5 - 6*a*d^2*Sqrt[1 - c^2*
x^2] + 21*a*c^2*d^2*x^2*Sqrt[1 - c^2*x^2] - 15*a*c^4*d^2*x^4*Sqrt[1 - c^2*
x^2] - 6*b*d^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + 21*b*c^2*d^2*x^2*Sqrt[1 - c
^2*x^2]*ArcCos[c*x] - 15*b*c^4*d^2*x^4*Sqrt[1 - c^2*x^2]*ArcCos[c*x] - 9*b
*c^4*d^2*x^4*ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] + 9*b*c^6*d^2*x^6*Ar
cCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] + 9*b*c^4*d^2*x^4*ArcCos[c*x]*Log[1
 + I*E^(I*ArcCos[c*x])] - 9*b*c^6*d^2*x^6*ArcCos[c*x]*Log[1 + I*E^(I*ArcCo
s[c*x])] + 9*a*c^4*d^(3/2)*x^4*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*Log[x
] - 9*a*c^4*d^(3/2)*x^4*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*Log[d + Sqrt
[d]*Sqrt[d - c^2*d*x^2]] + (9*I)*b*c^4*d^2*x^4*(-1 + c^2*x^2)*PolyLog[2, (
-I)*E^(I*ArcCos[c*x])] - (9*I)*b*c^4*d^2*x^4*(-1 + c^2*x^2)*PolyLog[2, I*E
^(I*ArcCos[c*x])])/(24*x^4*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5201, 244, 2009, 5197, 15, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^5}\,dx$$

$$\Big\downarrow \ 5201$$

$$-\frac{3}{4}c^2d\int\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^3}dx-\frac{bcd\sqrt{d-c^2dx^2}\int\frac{1-c^2x^2}{x^4}dx}{4\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{4x^4}$$

$$\Big\downarrow \text{ 244}$$

$$-\frac{3}{4}c^2d\int\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^3}dx-\frac{bcd\sqrt{d-c^2dx^2}\int\left(\frac{1}{x^4}-\frac{c^2}{x^2}\right)dx}{4\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{4x^4}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3}{4}c^2d\int\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^3}dx-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 5197}$$

$$-\frac{3}{4}c^2d\left(-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}-\frac{bc\sqrt{d-c^2dx^2}\int\frac{1}{x^2}dx}{2\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2x^2}\right)-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{3}{4}c^2d\left(-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2x^2}+\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{1-c^2x^2}}\right)-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 5219}$$

$$-\frac{3}{4}c^2d\left(\frac{c^2\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{cx}d\arccos(cx)}{2\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2x^2}+\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{1-c^2x^2}}\right)-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{4}c^2d\left(\frac{c^2\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{2\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2x^2}+\right.$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{3}{4}c^2d\left(\frac{c^2\sqrt{d-c^2dx^2}\left(-b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\,\text{arcta}\right.}{2\sqrt{1-c^2x^2}}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{3}{4}c^2d\left(\frac{c^2\sqrt{d-c^2dx^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)\right.}{2\sqrt{1-c^2x^2}}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{3}{4}c^2d\left(\frac{c^2\sqrt{d-c^2dx^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(\right.\right.}{2\sqrt{1-c^2x^2}}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd\left(\frac{c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^5,x]
```

output
```
-1/4*(b*c*d*(-1/3*1/x^3 + c^2/x)*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2] -
((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(4*x^4) - (3*c^2*d*((b*c*Sqrt[
d - c^2*d*x^2])/(2*x*Sqrt[1 - c^2*x^2]) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcC
os[c*x]))/(2*x^2) + (c^2*Sqrt[d - c^2*d*x^2]*((-2*I)*(a + b*ArcCos[c*x])*A
rcTan[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*b*Po
lyLog[2, I*E^(I*ArcCos[c*x])]))/(2*Sqrt[1 - c^2*x^2])))/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5197
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^2
]/Sqrt[1 - c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x
] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 2)*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.26

method	result
default	$-\frac{a\left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{4 d \,x^{4}}+\frac{a \,c^{2} \left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{8 d \,x^{2}}+\frac{a \,c^{4} \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}{8}-\frac{3 a \,c^{4} d^{\frac{3}{2}} \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{-c^{2} d \,x^{2}+d}}{x}\right)}{8}+\frac{3 a \,c^{4} d \sqrt{-c^{2} d \,x^{2}+d}}{8}$
parts	$-\frac{a\left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{4 d \,x^{4}}+\frac{a \,c^{2} \left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{8 d \,x^{2}}+\frac{a \,c^{4} \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}{8}-\frac{3 a \,c^{4} d^{\frac{3}{2}} \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{-c^{2} d \,x^{2}+d}}{x}\right)}{8}+\frac{3 a \,c^{4} d \sqrt{-c^{2} d \,x^{2}+d}}{8}$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/d/x^4*(-c^2*d*x^2+d)^(5/2)+1/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(5/2)+1/8
*a*c^4*(-c^2*d*x^2+d)^(3/2)-3/8*a*c^4*d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^
2+d)^(1/2))/x)+3/8*a*c^4*d*(-c^2*d*x^2+d)^(1/2)+b*(1/24*d*(15*c^4*x^4*arcc
os(c*x)+15*c^3*x^3*(-c^2*x^2+1)^(1/2)-21*c^2*x^2*arccos(c*x)-2*c*x*(-c^2*x
^2+1)^(1/2)+6*arccos(c*x))*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/x^4-3/8*(-d*
(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*(arccos(c*x)*ln(1+I*(c*x
+I*(-c^2*x^2+1)^(1/2)))-arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-I*d
ilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+I*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)
)))*c^4*d)
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^5}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}{x^5}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^5,x, algorithm="fricas"
)
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccos(c*x))*sqrt(-c^2*
d*x^2 + d)/x^5, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^5}\,dx = \int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))}{x^5}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))/x**5,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))/x**5, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}{x^5}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)}{x^5}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^5,x, algorithm="maxima"
)
```

output
```
-b*sqrt(d)*integrate((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(
sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x^5, x) - 1/8*(3*c^4*d^(3/2)*log(2*sqrt
(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - (-c^2*d*x^2 + d)^(3/2)*c^4
 - 3*sqrt(-c^2*d*x^2 + d)*c^4*d - (-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^2) + 2*(
-c^2*d*x^2 + d)^(5/2)/(d*x^4))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))}{x^5}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^5}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^{3/2}}{x^5}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^5,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/x^5, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^5}\,dx = \frac{\sqrt{d}\,d\left(5\sqrt{-c^2x^2+1}\,a\,c^2x^2-2\sqrt{-c^2x^2+1}\,a+8\left(\int \frac{\sqrt{-c^2x^2+1}\,acos(c}{x^5}\right.\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))/x^5,x)
```

output
```
(sqrt(d)*d*(5*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1
)*a + 8*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**5,x)*b*x**4 - 8*int((sqr
t( - c**2*x**2 + 1)*acos(c*x))/x**3,x)*b*c**2*x**4 + 3*log(tan(asin(c*x)/2
))*a*c**4*x**4))/(8*x**4)
```

3.86 $\int x^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 430

$$\int x^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))\,dx=\frac{3bd^2x^2\sqrt{d-c^2dx^2}}{512c^3\sqrt{1-c^2x^2}}+\frac{bd^2x^4\sqrt{d-c^2dx^2}}{512c\sqrt{1-c^2x^2}}-\frac{31bcd^2x^6\sqrt{d-c^2dx^2}}{960\sqrt{1-c^2x^2}}+\frac{21bc^3d^2x^8\sqrt{d-c^2dx^2}}{640\sqrt{1-c^2x^2}}-\frac{bc^5d^2x^{10}\sqrt{d-c^2dx^2}}{100\sqrt{1-c^2x^2}}-\frac{3d^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{256c^4}-\frac{d^2x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{128c^2}+\frac{1}{32}d^2x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{1}{16}dx^5(d-c^2dx^2)^{3/2}(a+b\arccos(cx))+\frac{1}{10}x^5(d-c^2dx^2)^{5/2}(a+b\arccos(cx))+\frac{3d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{512bc^5\sqrt{1-c^2x^2}}$$

output
```
3/512*b*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/512*b*d^2*x^
4*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-31/960*b*c*d^2*x^6*(-c^2*d*x^2
+d)^(1/2)/(-c^2*x^2+1)^(1/2)+21/640*b*c^3*d^2*x^8*(-c^2*d*x^2+d)^(1/2)/(-c
^2*x^2+1)^(1/2)-1/100*b*c^5*d^2*x^10*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/
2)-3/256*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c^4-1/128*d^2*x^3*(-
c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c^2+1/32*d^2*x^5*(-c^2*d*x^2+d)^(1/2)
*(a+b*arccos(c*x))+1/16*d*x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))+1/10*
x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))+3/512*d^2*(-c^2*d*x^2+d)^(1/2)*
(a+b*arccos(c*x))^2/b/c^5/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.82

$$\int x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\frac{d^2\left(-7200b\sqrt{d-c^2dx^2}\arccos(cx)^2-14400a\sqrt{d}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+\sqrt{d}\right.}{}$$

input
```
Integrate[x^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*(-7200*b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 14400*a*Sqrt[d]*Sqrt[1 -
 c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqr
t[d - c^2*d*x^2]*(-14400*a*c*x*Sqrt[1 - c^2*x^2] - 9600*a*c^3*x^3*Sqrt[1 -
 c^2*x^2] + 238080*a*c^5*x^5*Sqrt[1 - c^2*x^2] - 322560*a*c^7*x^7*Sqrt[1 -
 c^2*x^2] + 122880*a*c^9*x^9*Sqrt[1 - c^2*x^2] + 1200*b*Cos[2*ArcCos[c*x]]
 + 1200*b*Cos[4*ArcCos[c*x]] - 200*b*Cos[6*ArcCos[c*x]] - 75*b*Cos[8*ArcCo
s[c*x]] + 24*b*Cos[10*ArcCos[c*x]]) + 120*b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x
]*(20*Sin[2*ArcCos[c*x]] + 40*Sin[4*ArcCos[c*x]] - 10*Sin[6*ArcCos[c*x]] -
 5*Sin[8*ArcCos[c*x]] + 2*Sin[10*ArcCos[c*x]])))/(1228800*c^5*Sqrt[1 - c^2
*x^2])
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {5203, 243, 49, 2009, 5203, 244, 2009, 5199, 15, 5211, 15, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{1}{2}d\int x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int x^5\left(1-c^2x^2\right)^2dx}{10\sqrt{1-c^2x^2}}+\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}d\int x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int x^4\left(1-c^2x^2\right)^2dx^2}{20\sqrt{1-c^2x^2}}+\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}d\int x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(c^4x^8-2c^2x^6+x^4\right)dx^2}{20\sqrt{1-c^2x^2}}+\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}d\int x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5203}$$

$$\frac{1}{2}d\left(\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int x^5\left(1-c^2x^2\right)dx}{8\sqrt{1-c^2x^2}}+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{a}\right.$$

$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{2}d\left(\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int\left(x^5-c^2x^7\right)dx}{8\sqrt{1-c^2x^2}}+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\text{arc}\right.$$
$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}d\left(\frac{3}{8}d\int x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\sqrt{d-c}}{8\sqrt{1-c^2x^2}}\right.$$
$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5199}$$

$$\frac{1}{2}d\left(\frac{3}{8}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{6\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int x^5dx}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)+\frac{1}{8}x^5\right.$$
$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}d\left(\frac{3}{8}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{1-c^2x^2}}\right)+\frac{1}{8}x^5(d-\right.$$
$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5211}$$

$$\frac{1}{2}d\left(\frac{3}{8}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{3\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{b\int x^3dx}{4c}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}\right)}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos\right.\right.$$
$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{1-c^2x^2}}$$

$\downarrow$ 15

$$\frac{1}{2}d\left(\frac{3}{8}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{3\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx)\right.\right.$$

$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{1-c^2x^2}}$$

$\downarrow$ 5211

$$\frac{1}{2}d\left(\frac{3}{8}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{3\left(\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int xdx}{2c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\right.\right.$$

$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{1-c^2x^2}}$$

$\downarrow$ 15

$$\frac{1}{2}d\left(\frac{3}{8}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{3\left(\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^5\sqrt{\right.\right.$$

$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{1-c^2x^2}}$$

$\downarrow$ 5153

$$\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+$$

$$\frac{1}{2}d\left(\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{8}d\left(\frac{1}{6}x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{\sqrt{d-c^2dx^2}\left(-\frac{x^3\sqrt{1-c^2x^2}}{}\right.}{}\right.\right.$$

$$\frac{bcd^2\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)\sqrt{d-c^2dx^2}}{20\sqrt{1-c^2x^2}}$$

input
```
Int[x^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d^2*Sqrt[d - c^2*d*x^2]*(x^6/3 - (c^2*x^8)/2 + (c^4*x^10)/5))/(20*Sqr
t[1 - c^2*x^2]) + (x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/10 + (d*
((b*c*d*Sqrt[d - c^2*d*x^2]*(x^6/6 - (c^2*x^8)/8))/(8*Sqrt[1 - c^2*x^2]) +
 (x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/8 + (3*d*((b*c*x^6*Sqrt[d
 - c^2*d*x^2])/(36*Sqrt[1 - c^2*x^2]) + (x^5*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCos[c*x]))/6 + (Sqrt[d - c^2*d*x^2]*(-1/16*(b*x^4)/c - (x^3*Sqrt[1 - c^2*
x^2]*(a + b*ArcCos[c*x]))/(4*c^2) + (3*(-1/4*(b*x^2)/c - (x*Sqrt[1 - c^2*x
^2]*(a + b*ArcCos[c*x]))/(2*c^2) - (a + b*ArcCos[c*x])^2/(4*b*c^3)))/(4*c^
2)))/(6*Sqrt[1 - c^2*x^2])))/8))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 1102, normalized size of antiderivative = 2.56

method	result	size
default	Expression too large to display	1102
parts	Expression too large to display	1102

input
```
int(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/10*a*x^3*(-c^2*d*x^2+d)^(7/2)/c^2/d-3/80*a/c^4*x*(-c^2*d*x^2+d)^(7/2)/d
+1/160*a/c^4*x*(-c^2*d*x^2+d)^(5/2)+1/128*a/c^4*d*x*(-c^2*d*x^2+d)^(3/2)+3
/256*a/c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)+3/256*a/c^4*d^3/(c^2*d)^(1/2)*arctan
((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(3/512*(-d*(c^2*x^2-1))^(1/2)*(-c
^2*x^2+1)^(1/2)/c^5/(c^2*x^2-1)*arccos(c*x)^2*d^2+1/102400*(-d*(c^2*x^2-1)
)^(1/2)*(1696*c^7*x^7-832*c^5*x^5+512*I*(-c^2*x^2+1)^(1/2)*x^10*c^10-I*(-c
^2*x^2+1)^(1/2)+170*c^3*x^3+512*c^11*x^11-1280*I*(-c^2*x^2+1)^(1/2)*x^8*c^
8-400*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+50*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+1120*I*
(-c^2*x^2+1)^(1/2)*x^6*c^6-1536*c^9*x^9-10*c*x)*(I+10*arccos(c*x))*d^2/c^5
/(c^2*x^2-1)+1/2048*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^
2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))*d^2/c^5/(c^2*x^
2-1)+1/819200*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)
*(17*I+280*arccos(c*x))*cos(9*arccos(c*x))*d^2/c^5/(c^2*x^2-1)+3/819200*(-
d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(11*I+40*arccos(
c*x))*sin(9*arccos(c*x))*d^2/c^5/(c^2*x^2-1)+1/98304*(-d*(c^2*x^2-1))^(1/2
)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(11*I+24*arccos(c*x))*cos(7*arccos
(c*x))*d^2/c^5/(c^2*x^2-1)+1/98304*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(
-c^2*x^2+1)^(1/2)-I)*(5*I+72*arccos(c*x))*sin(7*arccos(c*x))*d^2/c^5/(c^2*
x^2-1)-5/12288*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1
)*(I+6*arccos(c*x))*cos(5*arccos(c*x))*d^2/c^5/(c^2*x^2-1)-1/12288*(-d*...
```

Fricas [F]

$$\int x^4 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos (c x)) \, dx=\int (-c^2 d x^2+d)^{\frac{5}{2}} (b \arccos (c x)+a) x^4 \, dx$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
integral((a*c^4*d^2*x^8 - 2*a*c^2*d^2*x^6 + a*d^2*x^4 + (b*c^4*d^2*x^8 - 2
*b*c^2*d^2*x^6 + b*d^2*x^4)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^4 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos (c x)) \, dx=\text{Timed out}$$

input
```
integrate(x**4*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^4 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos (c x)) \, dx=\int (-c^2 d x^2+d)^{\frac{5}{2}} (b \arccos (c x)+a) x^4 \, dx$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
b*sqrt(d)*integrate((c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4)*sqrt(c*x + 1)*
sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x) - 1/1280*(12
8*(-c^2*d*x^2 + d)^(7/2)*x^3/(c^2*d) - 8*(-c^2*d*x^2 + d)^(5/2)*x/c^4 + 48
*(-c^2*d*x^2 + d)^(7/2)*x/(c^4*d) - 10*(-c^2*d*x^2 + d)^(3/2)*d*x/c^4 - 15
*sqrt(-c^2*d*x^2 + d)*d^2*x/c^4 - 15*d^(5/2)*arcsin(c*x)/c^5)*a
```

Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.90

$$\int x^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))\,dx = \frac{1}{10}\sqrt{-c^2dx^2+d}ac^4d^2x^9 - \frac{21}{80}\sqrt{-c^2dx^2+d}ac^2d^2x^7 + \frac{31}{160}\sqrt{-c^2dx^2+d}ad^2x^5 - \frac{\sqrt{-c^2dx^2+d}ad^2x^3}{128\,c^2} - \frac{3\sqrt{-c^2dx^2+d}ad^2x}{256\,c^4} - \frac{3\,ad^3\log\left(\left|-c\sqrt{-d}x+\sqrt{c^2x^2-1}\sqrt{-d}\right|\right)}{256\,c^5\sqrt{-d}} + \frac{12288\,bc^9d^{\frac{5}{2}}x^{10} + 122880\sqrt{-c^2x^2+1}bc^8d^{\frac{5}{2}}x^9\arccos(cx) - 40320\,bc^7d^{\frac{5}{2}}x^8 - 322560\sqrt{-c^2x^2+1}bc^6d^{\frac{5}{2}}x^7}{}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/10*sqrt(-c^2*d*x^2 + d)*a*c^4*d^2*x^9 - 21/80*sqrt(-c^2*d*x^2 + d)*a*c^2
*d^2*x^7 + 31/160*sqrt(-c^2*d*x^2 + d)*a*d^2*x^5 - 1/128*sqrt(-c^2*d*x^2 +
 d)*a*d^2*x^3/c^2 - 3/256*sqrt(-c^2*d*x^2 + d)*a*d^2*x/c^4 - 3/256*a*d^3*l
og(abs(-c*sqrt(-d)*x + sqrt(c^2*x^2 - 1)*sqrt(-d)))/(c^5*sqrt(-d)) + 1/122
8800*(12288*b*c^9*d^(5/2)*x^10 + 122880*sqrt(-c^2*x^2 + 1)*b*c^8*d^(5/2)*x
^9*arccos(c*x) - 40320*b*c^7*d^(5/2)*x^8 - 322560*sqrt(-c^2*x^2 + 1)*b*c^6
*d^(5/2)*x^7*arccos(c*x) + 39680*b*c^5*d^(5/2)*x^6 + 238080*sqrt(-c^2*x^2
+ 1)*b*c^4*d^(5/2)*x^5*arccos(c*x) - 2400*b*c^3*d^(5/2)*x^4 - 9600*sqrt(-c
^2*x^2 + 1)*b*c^2*d^(5/2)*x^3*arccos(c*x) - 7200*b*c*d^(5/2)*x^2 - 14400*s
qrt(-c^2*x^2 + 1)*b*d^(5/2)*x*arccos(c*x) - 7200*b*d^(5/2)*arccos(c*x)^2/c
 + 101*b*d^(5/2)/c)/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d - c^2 dx^2)^{5/2} (a + b \arccos(cx))\, dx = \int x^4 (a + b\,\mathrm{acos}(c\,x))\, \left(d - c^2\, d\, x^2\right)^{5/2} dx$$

input
```
int(x^4*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^4*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^4 (d - c^2 dx^2)^{5/2} (a + b \arccos(cx))\, dx = \frac{\sqrt{d}\, d^2 \left(15 asin(cx)\, a + 128\sqrt{-c^2x^2+1}\, a\, c^9 x^9 - 336\sqrt{-c^2x^2+1}\, a\, c^7 x^7 + 248\sqrt{-c^2x^2+\right.}{}$$

input
```
int(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d**2*(15*asin(c*x)*a + 128*sqrt( - c**2*x**2 + 1)*a*c**9*x**9 - 3
36*sqrt( - c**2*x**2 + 1)*a*c**7*x**7 + 248*sqrt( - c**2*x**2 + 1)*a*c**5*
x**5 - 10*sqrt( - c**2*x**2 + 1)*a*c**3*x**3 - 15*sqrt( - c**2*x**2 + 1)*a
*c*x + 1280*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**8,x)*b*c**9 - 2560*int
(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**6,x)*b*c**7 + 1280*int(sqrt( - c**2*x
**2 + 1)*acos(c*x)*x**4,x)*b*c**5))/(1280*c**5)
```

3.87 $\int x^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 351

$$\int x^2\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\frac{5bd^2x^2\sqrt{d-c^2dx^2}}{256c\sqrt{1-c^2x^2}}-\frac{59bcd^2x^4\sqrt{d-c^2dx^2}}{768\sqrt{1-c^2x^2}}+\frac{17bc^3d^2x^6\sqrt{d-c^2dx^2}}{288\sqrt{1-c^2x^2}}-\frac{bc^5d^2x^8\sqrt{d-c^2dx^2}}{64\sqrt{1-c^2x^2}}-\frac{5d^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{128c^2}+\frac{5}{64}d^2x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{5}{48}dx^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{5d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{256bc^3\sqrt{1-c^2x^2}}$$

output
```
5/256*b*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-59/768*b*c*d^2*x
^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+17/288*b*c^3*d^2*x^6*(-c^2*d*x^
2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/64*b*c^5*d^2*x^8*(-c^2*d*x^2+d)^(1/2)/(-c^
2*x^2+1)^(1/2)-5/128*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c^2+5/64
*d^2*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))+5/48*d*x^3*(-c^2*d*x^2+d)^
(3/2)*(a+b*arccos(c*x))+1/8*x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))+5/2
56*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/b/c^3/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.88

$$\int x^2\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\frac{d^2\left(-1440b\sqrt{d-c^2dx^2}\arccos(cx)^2-2880a\sqrt{d}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+\sqrt{d-\cdots}\right.}{\cdots}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*(-1440*b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 2880*a*Sqrt[d]*Sqrt[1 -
c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt
[d - c^2*d*x^2]*(-2880*a*c*x*Sqrt[1 - c^2*x^2] + 22656*a*c^3*x^3*Sqrt[1 -
c^2*x^2] - 26112*a*c^5*x^5*Sqrt[1 - c^2*x^2] + 9216*a*c^7*x^7*Sqrt[1 - c^2
*x^2] + 576*b*Cos[2*ArcCos[c*x]] + 144*b*Cos[4*ArcCos[c*x]] - 64*b*Cos[6*A
rcCos[c*x]] + 9*b*Cos[8*ArcCos[c*x]]) + 24*b*Sqrt[d - c^2*d*x^2]*ArcCos[c*
x]*(48*Sin[2*ArcCos[c*x]] + 24*Sin[4*ArcCos[c*x]] - 16*Sin[6*ArcCos[c*x]]
+ 3*Sin[8*ArcCos[c*x]])))/(73728*c^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5203, 243, 49, 2009, 5203, 244, 2009, 5199, 15, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int x^3\left(1-c^2x^2\right)^2dx}{8\sqrt{1-c^2x^2}}+\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{243}$$

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int x^2\left(1-c^2x^2\right)^2dx^2}{16\sqrt{1-c^2x^2}}+\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{49}$$

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int\left(c^4x^6-2c^2x^4+x^2\right)dx^2}{16\sqrt{1-c^2x^2}}+\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5203}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int x^3\left(1-c^2x^2\right)dx}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{ a}\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{244}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int\left(x^3-c^2x^5\right)dx}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\text{ arc}\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^4}{4}-\frac{c^2x^6}{6}\right)\sqrt{d-c}}{6\sqrt{1-c^2x^2}}\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5199}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int x^3dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)+\frac{1}{6}x^3\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$

$\downarrow$ 15

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}\right)+\frac{1}{6}x^3(d-\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$

$\downarrow$ 5211

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int xdx}{2c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}\right)}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right.\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$

$\downarrow$ 15

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\right.\right.$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$

$\downarrow$ 5153

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+$$

$$\frac{5}{8}d\left(\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{2}d\left(\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{\sqrt{d-c^2dx^2}\left(-\frac{(a+b\arccos(\cdots}{4bc^3}\right.}{\cdots}\right.\right.$$

$$\frac{bcd^2\left(\frac{c^4x^8}{4}-\frac{2c^2x^6}{3}+\frac{x^4}{2}\right)\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$

input
```
Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d^2*Sqrt[d - c^2*d*x^2]*(x^4/2 - (2*c^2*x^6)/3 + (c^4*x^8)/4))/(16*Sq
rt[1 - c^2*x^2]) + (x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/8 + (5*
d*((b*c*d*Sqrt[d - c^2*d*x^2]*(x^4/4 - (c^2*x^6)/6))/(6*Sqrt[1 - c^2*x^2])
 + (x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/6 + (d*((b*c*x^4*Sqrt[d
 - c^2*d*x^2])/(16*Sqrt[1 - c^2*x^2]) + (x^3*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCos[c*x]))/4 + (Sqrt[d - c^2*d*x^2]*(-1/4*(b*x^2)/c - (x*Sqrt[1 - c^2*x^2
]*(a + b*ArcCos[c*x]))/(2*c^2) - (a + b*ArcCos[c*x])^2/(4*b*c^3)))/(4*Sqrt
[1 - c^2*x^2])))/2))/8
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 1080, normalized size of antiderivative = 3.08

method	result	size
default	Expression too large to display	1080
parts	Expression too large to display	1080

input
```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*a*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/48*a/c^2*x*(-c^2*d*x^2+d)^(5/2)+5/19
2*a/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/128*a/c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/
128*a/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b
*(5/256*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c
*x)^2*d^2+1/16384*(-d*(c^2*x^2-1))^(1/2)*(128*c^9*x^9-320*c^7*x^7+128*I*(-
c^2*x^2+1)^(1/2)*x^8*c^8+272*c^5*x^5-256*I*(-c^2*x^2+1)^(1/2)*x^6*c^6-88*c
^3*x^3+160*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c*x-32*I*(-c^2*x^2+1)^(1/2)*x^2*
c^2+I*(-c^2*x^2+1)^(1/2))*(I+8*arccos(c*x))*d^2/c^3/(c^2*x^2-1)-1/2304*(-d
*(c^2*x^2-1))^(1/2)*(32*c^7*x^7-64*c^5*x^5+32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6
+38*c^3*x^3-48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-6*c*x+18*I*(-c^2*x^2+1)^(1/2)*
x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(I+6*arccos(c*x))*d^2/c^3/(c^2*x^2-1)+1/256*
(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*
x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))*d^2/c^3/(c^2*x^2-1)+1/16384*(-d*(c^
2*x^2-1))^(1/2)*(-128*I*(-c^2*x^2+1)^(1/2)*x^8*c^8+128*c^9*x^9+256*I*(-c^2
*x^2+1)^(1/2)*x^6*c^6-320*c^7*x^7-160*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+272*c^5
*x^5+32*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-88*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+8*c*x
)*(-I+8*arccos(c*x))*d^2/c^3/(c^2*x^2-1)-5/9216*(-d*(c^2*x^2-1))^(1/2)*(-I
*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(I+12*arccos(c*x))*cos(5*arccos(c*x))*d
^2/c^3/(c^2*x^2-1)-1/9216*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+
1)^(1/2)-I)*(13*I+12*arccos(c*x))*sin(5*arccos(c*x))*d^2/c^3/(c^2*x^2-1...
```

Fricas [F]

$$\int x^2 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \int \left(-c^2dx^2+d\right)^{\frac{5}{2}} (b\arccos(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
integral((a*c^4*d^2*x^6 - 2*a*c^2*d^2*x^4 + a*d^2*x^2 + (b*c^4*d^2*x^6 - 2
*b*c^2*d^2*x^4 + b*d^2*x^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \text{Timed out}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \int \left(-c^2dx^2+d\right)^{\frac{5}{2}} (b\arccos(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
b*sqrt(d)*integrate((c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2)*sqrt(c*x + 1)*
sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x) + 1/384*(8*(
-c^2*d*x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d) + 10*(-c
^2*d*x^2 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2 + 15*d^(5/
2)*arcsin(c*x)/c^3)*a
```

Giac [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.92

$$\int x^2 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \frac{1}{8}\sqrt{-c^2dx^2+d}ac^4d^2x^7 - \frac{17}{48}\sqrt{-c^2dx^2+d}ac^2d^2x^5 + \frac{59}{192}\sqrt{-c^2dx^2+d}ad^2x^3 - \frac{5\sqrt{-c^2dx^2+d}ad^2x}{128\,c^2} - \frac{5\,ad^3\log\left(\left|-c\sqrt{-d}x+\sqrt{c^2x^2-1}\sqrt{-d}\right|\right)}{128\,c^3\sqrt{-d}} + \frac{1152\,bc^7d^{\frac{5}{2}}x^8 + 9216\sqrt{-c^2x^2+1}bc^6d^{\frac{5}{2}}x^7\arccos(cx) - 4352\,bc^5d^{\frac{5}{2}}x^6 - 26112\sqrt{-c^2x^2+1}bc^4d^{\frac{5}{2}}x^5\arccos}{}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/8*sqrt(-c^2*d*x^2 + d)*a*c^4*d^2*x^7 - 17/48*sqrt(-c^2*d*x^2 + d)*a*c^2*
d^2*x^5 + 59/192*sqrt(-c^2*d*x^2 + d)*a*d^2*x^3 - 5/128*sqrt(-c^2*d*x^2 +
d)*a*d^2*x/c^2 - 5/128*a*d^3*log(abs(-c*sqrt(-d)*x + sqrt(c^2*x^2 - 1)*sqr
t(-d)))/(c^3*sqrt(-d)) + 1/73728*(1152*b*c^7*d^(5/2)*x^8 + 9216*sqrt(-c^2*
x^2 + 1)*b*c^6*d^(5/2)*x^7*arccos(c*x) - 4352*b*c^5*d^(5/2)*x^6 - 26112*sq
rt(-c^2*x^2 + 1)*b*c^4*d^(5/2)*x^5*arccos(c*x) + 5664*b*c^3*d^(5/2)*x^4 +
22656*sqrt(-c^2*x^2 + 1)*b*c^2*d^(5/2)*x^3*arccos(c*x) - 1440*b*c*d^(5/2)*
x^2 - 2880*sqrt(-c^2*x^2 + 1)*b*d^(5/2)*x*arccos(c*x) - 1440*b*d^(5/2)*arc
cos(c*x)^2/c - 359*b*d^(5/2)/c)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b\arccos(cx))\, dx = \int x^2 (a + b\,\mathrm{acos}(c\,x))\, \left(d - c^2\,d\,x^2\right)^{5/2} dx$$

input
```
int(x^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b\arccos(cx))\, dx = \frac{\sqrt{d}\, d^2 \big(15 asin(cx)\, a + 48\sqrt{-c^2x^2+1}\, a\, c^7 x^7 - 136\sqrt{-c^2x^2+1}\, a\, c^5 x^5 + 118\sqrt{-c^2x^2+}$$

input
```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d**2*(15*asin(c*x)*a + 48*sqrt( - c**2*x**2 + 1)*a*c**7*x**7 - 13
6*sqrt( - c**2*x**2 + 1)*a*c**5*x**5 + 118*sqrt( - c**2*x**2 + 1)*a*c**3*x
**3 - 15*sqrt( - c**2*x**2 + 1)*a*c*x + 384*int(sqrt( - c**2*x**2 + 1)*aco
s(c*x)*x**6,x)*b*c**7 - 768*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**4,x)*b
*c**5 + 384*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**2,x)*b*c**3))/(384*c**
3)
```

3.88 $\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 262

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=-\frac{5bcd^2x^2\sqrt{d-c^2dx^2}}{32\sqrt{1-c^2x^2}}+\frac{5bd^2(1-c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}{96c}+\frac{bd^2(1-c^2x^2)^{5/2}\sqrt{d-c^2dx^2}}{36c}+\frac{5}{16}d^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{5}{24}dx\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos$$

output

```
-5/32*b*c*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+5/96*b*d^2*(-c^2
*x^2+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)/c+1/36*b*d^2*(-c^2*x^2+1)^(5/2)*(-c^2*d
*x^2+d)^(1/2)/c+5/16*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))+5/24*d*x
*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))+1/6*x*(-c^2*d*x^2+d)^(5/2)*(a+b*ar
ccos(c*x))+5/32*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/b/c/(-c^2*x^2
+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.02

$$\int \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \frac{d^2\left(-360b\sqrt{d-c^2dx^2}\arccos(cx)^2-720a\sqrt{d}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+\sqrt{d-c\cdots}\right.}{\cdots}$$

input
```
Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*(-360*b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 720*a*Sqrt[d]*Sqrt[1 - c^
2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d
 - c^2*d*x^2]*(1584*a*c*x*Sqrt[1 - c^2*x^2] - 1248*a*c^3*x^3*Sqrt[1 - c^2*
x^2] + 384*a*c^5*x^5*Sqrt[1 - c^2*x^2] + 270*b*Cos[2*ArcCos[c*x]] - 27*b*C
os[4*ArcCos[c*x]] + 2*b*Cos[6*ArcCos[c*x]]) + 12*b*Sqrt[d - c^2*d*x^2]*Arc
Cos[c*x]*(45*Sin[2*ArcCos[c*x]] - 9*Sin[4*ArcCos[c*x]] + Sin[6*ArcCos[c*x]
])))/(2304*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5159, 241, 5159, 244, 2009, 5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{5}{6}d\int \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)^2dx}{6\sqrt{1-c^2x^2}}+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))$$

$$\Big\downarrow \text{241}$$

$$\frac{5}{6}d\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))-\frac{bd^2\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}$$

$\downarrow$ 5159

$$\frac{5}{6}d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))-\frac{bd^2\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}$$

$\downarrow$ 244

$$\frac{5}{6}d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int \left(x-c^2x^3\right)dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))-\frac{bd^2\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}$$

$\downarrow$ 2009

$$\frac{5}{6}d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c^2dx}}{4\sqrt{1-c^2x^2}}\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))-\frac{bd^2\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}$$

$\downarrow$ 5157

$$\frac{5}{6}d\left(\frac{3}{4}d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int xdx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)+\frac{1}{4}x(d-\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))-\frac{bd^2\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}$$

$\downarrow$ 15

$$\frac{5}{6}d\left(\frac{3}{4}d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}\right)+\frac{1}{4}x(d-c^2dx\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))-\frac{bd^2\left(1-c^2x^2\right)^{5/2}\sqrt{d-c^2dx^2}}{36c}$$

$$\Big\downarrow \text{5153}$$

$$\frac{1}{6}x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))+$$
$$\frac{5}{6}d\left(\frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))+\frac{3}{4}d\left(\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx}{4bc\sqrt{1-c^2x^2}}\right.\right.$$
$$\frac{bd^2(1-c^2x^2)^{5/2}\sqrt{d-c^2dx^2}}{36c}$$

input
```
Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/36*(b*d^2*(1 - c^2*x^2)^(5/2)*Sqrt[d - c^2*d*x^2])/c + (x*(d - c^2*d*x^
2)^(5/2)*(a + b*ArcCos[c*x]))/6 + (5*d*((b*c*d*Sqrt[d - c^2*d*x^2]*(x^2/2
- (c^2*x^4)/4))/(4*Sqrt[1 - c^2*x^2]) + (x*(d - c^2*d*x^2)^(3/2)*(a + b*Ar
cCos[c*x]))/4 + (3*d*((b*c*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[1 - c^2*x^2])
+ (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2])))/4))/6
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 689, normalized size of antiderivative = 2.63

method	result
default	$\frac{ax(-c^2dx^2+d)^{\frac{5}{2}}}{6}+\frac{5adx(-c^2dx^2+d)^{\frac{3}{2}}}{24}+\frac{5a\,d^2x\sqrt{-c^2dx^2+d}}{16}+\frac{5a\,d^3\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{16\sqrt{c^2d}}+b\left(\frac{5\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2}}{32c(c^2}\right.$
parts	$\frac{ax(-c^2dx^2+d)^{\frac{5}{2}}}{6}+\frac{5adx(-c^2dx^2+d)^{\frac{3}{2}}}{24}+\frac{5a\,d^2x\sqrt{-c^2dx^2+d}}{16}+\frac{5a\,d^3\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{16\sqrt{c^2d}}+b\left(\frac{5\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2}}{32c(c^2}\right.$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*x*(-c^2*d*x^2+d)^(5/2)+5/24*a*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*d^2*x*
(-c^2*d*x^2+d)^(1/2)+5/16*a*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2
*d*x^2+d)^(1/2))+b*(5/32*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/(c^2*
x^2-1)*arccos(c*x)^2*d^2+1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^7-64*c^5*
x^5+32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+38*c^3*x^3-48*I*(-c^2*x^2+1)^(1/2)*x^4
*c^4-6*c*x+18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(I+6*arcc
os(c*x))*d^2/(c^2*x^2-1)/c+15/256*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1
)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))*d
^2/(c^2*x^2-1)/c+5/4608*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+
c^2*x^2-1)*(5*I+24*arccos(c*x))*cos(5*arccos(c*x))*d^2/(c^2*x^2-1)/c+1/460
8*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(29*I+96*arc
cos(c*x))*sin(5*arccos(c*x))*d^2/(c^2*x^2-1)/c-9/512*(-d*(c^2*x^2-1))^(1/2
)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(3*I+8*arccos(c*x))*cos(3*arccos(c
*x))*d^2/(c^2*x^2-1)/c-3/512*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x
^2+1)^(1/2)-I)*(11*I+16*arccos(c*x))*sin(3*arccos(c*x))*d^2/(c^2*x^2-1)/c)
```

Fricas [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\arccos\left(cx\right)+a)\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
b*sqrt(d)*integrate((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x) + 1/48*(8*(-c^2*
d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(-c^2*d*x^2 +
d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx = \frac{\sqrt{d}\,d^2\left(15asin(cx)\,a+8\sqrt{-c^2x^2+1}\,a\,c^5x^5-26\sqrt{-c^2x^2+1}\,a\,c^3x^3+33\sqrt{-c^2x^2+1}\,ac\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d**2*(15*asin(c*x)*a + 8*sqrt( - c**2*x**2 + 1)*a*c**5*x**5 - 26*
sqrt( - c**2*x**2 + 1)*a*c**3*x**3 + 33*sqrt( - c**2*x**2 + 1)*a*c*x + 48*
int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**4,x)*b*c**5 - 96*int(sqrt( - c**2*
x**2 + 1)*acos(c*x)*x**2,x)*b*c**3 + 48*int(sqrt( - c**2*x**2 + 1)*acos(c*
x),x)*b*c))/(48*c)
```

3.89 $\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 306

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{x^2}\,dx = -\frac{bc^3d^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}+\frac{bc^5d^2x^4\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$
$$-\frac{5}{16}bcd^2\left(1-c^2x^2\right)^{3/2}\sqrt{d-c^2dx^2}-\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$-\frac{5}{4}c^2dx\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{x}-\frac{15cd^2\sqrt{d-c^2dx^2}(a+b\,\text{arcco}}{16b\sqrt{1-c^2x^2}}$$

output
```
-1/16*b*c^3*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/4*b*c^5*d^2*
x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-5/16*b*c*d^2*(-c^2*x^2+1)^(3/2
)*(-c^2*d*x^2+d)^(1/2)-15/8*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x
))-5/4*c^2*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))-(-c^2*d*x^2+d)^(5/2)
*(a+b*arccos(c*x))/x-15/16*c*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/
b/(-c^2*x^2+1)^(1/2)+b*c*d^2*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.84

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^2}\,dx = \frac{d^2\Big(120bcx\sqrt{d-c^2dx^2}\arccos(cx)^2+240ac\sqrt{d}x\sqrt{1-c^2x^2}\arctan$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
(d^2*(120*b*c*x*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 + 240*a*c*Sqrt[d]*x*Sqrt
[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] +
 Sqrt[d - c^2*d*x^2]*(-32*b*c*x*Cos[2*ArcCos[c*x]] + b*c*x*Cos[4*ArcCos[c*
x]] + 16*(a*Sqrt[1 - c^2*x^2]*(-8 - 9*c^2*x^2 + 2*c^4*x^4) - 8*b*c*x*Log[c
*x])) + 4*b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(-32*Sqrt[1 - c^2*x^2] - 16*c*
x*Sin[2*ArcCos[c*x]] + c*x*Sin[4*ArcCos[c*x]])))/(128*x*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5201, 243, 49, 2009, 5159, 244, 2009, 5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^2}dx$$

$$\Big\downarrow \text{5201}$$

$$-5c^2d\int (d-c^2dx^2)^{3/2}(a+b\arccos(cx))dx - \frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2}{x}dx}{\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{243}$$

$$-5c^2d\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2}{x^2}dx^2}{2\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{x}$$

$\downarrow$ 49

$$-5c^2d\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(x^2c^4-2c^2+\frac{1}{x^2}\right)dx^2}{2\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{x}$$

$\downarrow$ 2009

$$-5c^2d\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{x}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{1-c^2x^2}}$$

$\downarrow$ 5159

$$-5c^2d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{ar}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{x}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{1-c^2x^2}}$$

$\downarrow$ 244

$$-5c^2d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{bcd\sqrt{d-c^2dx^2}\int \left(x-c^2x^3\right)dx}{4\sqrt{1-c^2x^2}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\text{arcc}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{x}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{1-c^2x^2}}$$

$\downarrow$ 2009

$$-5c^2d\left(\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\sqrt{d-c}}{4\sqrt{1-c^2x^2}}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{x}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{1-c^2x^2}}$$

$\downarrow$ 5157

$$-5c^2d\left(\frac{3}{4}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int xdx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)+\frac{1}{4}x(d\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{x}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 15

$$-5c^2d\left(\frac{3}{4}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}\right)+\frac{1}{4}x(d-c\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{x}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 5153

$$-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{x}-$$
$$5c^2d\left(\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}d\left(\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(c\right.\right.}{4bc\sqrt{1-c^2x^2}}$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^4}{2}-2c^2x^2+\log\left(x^2\right)\right)}{2\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
-(((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x) - 5*c^2*d*((b*c*d*Sqrt[d
- c^2*d*x^2]*(x^2/2 - (c^2*x^4)/4))/(4*Sqrt[1 - c^2*x^2]) + (x*(d - c^2*d*
x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 + (3*d*((b*c*x^2*Sqrt[d - c^2*d*x^2])/(4
*Sqrt[1 - c^2*x^2]) + (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/2 - (Sqr
t[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2])))/4) - (
b*c*d^2*Sqrt[d - c^2*d*x^2]*(-2*c^2*x^2 + (c^4*x^4)/2 + Log[x^2]))/(2*Sqrt
[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{dx}-ac^2x(-c^2dx^2+d)^{\frac{5}{2}}-\frac{5ac^2dx(-c^2dx^2+d)^{\frac{3}{2}}}{4}-\frac{15ac^2d^2x\sqrt{-c^2dx^2+d}}{8}-\frac{15ac^2d^3\arctan\left(\frac{}{\sqrt{}}\right.}{8\sqrt{c^2d}}$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{dx}-ac^2x(-c^2dx^2+d)^{\frac{5}{2}}-\frac{5ac^2dx(-c^2dx^2+d)^{\frac{3}{2}}}{4}-\frac{15ac^2d^2x\sqrt{-c^2dx^2+d}}{8}-\frac{15ac^2d^3\arctan\left(\frac{}{\sqrt{}}\right.}{8\sqrt{c^2d}}$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/x*(-c^2*d*x^2+d)^(7/2)-a*c^2*x*(-c^2*d*x^2+d)^(5/2)-5/4*a*c^2*d*x*(-c
^2*d*x^2+d)^(3/2)-15/8*a*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)-15/8*a*c^2*d^3/(c^
2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/128*b*(-d*(c^2*x
^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/x/(c^2*x^2-1)*(32*(-c^2*x^2+1)^(1/2)*arcco
s(c*x)*c^4*x^4+8*c^5*x^5-144*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^2-72*c^3
*x^3+120*arccos(c*x)^2*c*x+128*I*arccos(c*x)*x*c-128*ln(1+(c*x+I*(-c^2*x^2
+1)^(1/2))^2)*x*c-128*arccos(c*x)*(-c^2*x^2+1)^(1/2)+33*c*x)*d^2
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^2}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^2,x, algorithm="fricas"
)
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^2}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^2,x, algorithm="maxima"
)
```

output
```
b*sqrt(d)*integrate((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x^2, x) - 1/8*(10*(-
c^2*d*x^2 + d)^(3/2)*c^2*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^2*d^2*x + 15*c*d^
(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)/x)*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^2,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(d - c^2 d x^2)^{5/2} (a + b \arccos(cx))}{x^2} \, dx = \frac{\sqrt{d}\, d^2 \Big(4 a cos(cx)^2 \, bcx - 15 a sin(cx) \, acx + 2\sqrt{-c^2 x^2 + 1} \, a \, c^4 x^4 -$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))/x^2,x)
```

output
```
(sqrt(d)*d**2*(4*acos(c*x)**2*b*c*x - 15*asin(c*x)*a*c*x + 2*sqrt( - c**2*
x**2 + 1)*a*c**4*x**4 - 9*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 8*sqrt( - c
**2*x**2 + 1)*a + 8*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*x**2),x)*b*x + 8
*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**2,x)*b*c**4*x - 16*int(sqrt( - c*
*2*x**2 + 1)*acos(c*x),x)*b*c**2*x))/(8*x)
```

3.90 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 277

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^4}\,dx = -\frac{bcd^2\sqrt{d-c^2dx^2}}{6x^2\sqrt{1-c^2x^2}} - \frac{bc^5d^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}} + \frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx)) + \frac{5c^2d(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{3x} - \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{3x^3} + \frac{5c^3d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4b\sqrt{1-c^2x^2}} - \frac{7bc^3d^2\sqrt{d-c^2dx^2}\log(x)}{3\sqrt{1-c^2x^2}}$$

output
```
-1/6*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/4*b*c^5*d^2*x^2
*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+5/2*c^4*d^2*x*(-c^2*d*x^2+d)^(1/2
)*(a+b*arccos(c*x))+5/3*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x-1/3
*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^3+5/4*c^3*d^2*(-c^2*d*x^2+d)^(1/
2)*(a+b*arccos(c*x))^2/b/(-c^2*x^2+1)^(1/2)-7/3*b*c^3*d^2*(-c^2*d*x^2+d)^(
1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.87

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^4}\,dx = \frac{1}{24}d^2\left(\frac{4b\sqrt{d-c^2dx^2}(-2+14c^2x^2+3c^4x^4)\arccos(cx)}{x^3} - \frac{30bc^3\sqrt{d-c^2dx^2}\arccos(cx)^2}{\sqrt{1-c^2x^2}} - 60ac^3\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right) + \frac{\sqrt{d-c^2dx^2}\left(4a\sqrt{1-c^2x^2}(-2+14c^2x^2+3c^4x^4)+bcx(4-3c^2x^2+6c^4x^4)+56bc^3x^3\log(cx)\right)}{x^3\sqrt{1-c^2x^2}}\right)$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
(d^2*((4*b*Sqrt[d - c^2*d*x^2]*(-2 + 14*c^2*x^2 + 3*c^4*x^4)*ArcCos[c*x])/
x^3 - (30*b*c^3*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2)/Sqrt[1 - c^2*x^2] - 60*
a*c^3*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] +
 (Sqrt[d - c^2*d*x^2]*(4*a*Sqrt[1 - c^2*x^2]*(-2 + 14*c^2*x^2 + 3*c^4*x^4)
 + b*c*x*(4 - 3*c^2*x^2 + 6*c^4*x^4) + 56*b*c^3*x^3*Log[c*x]))/(x^3*Sqrt[1
 - c^2*x^2])))/24
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5201, 243, 49, 2009, 5201, 244, 2009, 5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^4}\,dx$$

$$\Big\downarrow \text{ 5201}$$

$$-\frac{5}{3}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x^2}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2}{x^3}dx}{3\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{5}{3}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x^2}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2}{x^4}dx^2}{6\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{5}{3}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x^2}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(c^4-\frac{2c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{6\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{3x^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{5}{3}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{3x^3}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 5201}$$

$$-\frac{5}{3}c^2d\left(-3c^2d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int \frac{1-c^2x^2}{x}dx}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(}{x}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{3x^3}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 244}$$

$$-\frac{5}{3}c^2d\left(-3c^2d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))dx-\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{1}{x}-c^2x\right)dx}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\,\mathrm{arc}}{x}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{3x^3}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{5}{3}c^2d\left(-3c^2d\int\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x}-\frac{bcd\sqrt{d-c^2dx^2}\left(\log(x\right.}{\sqrt{1-c^2x^2}}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{3x^3}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 5157

$$-\frac{5}{3}c^2d\left(-3c^2d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int xdx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)-\frac{(}{}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{3x^3}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 15

$$-\frac{5}{3}c^2d\left(-3c^2d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}\right)-\frac{(d-}{}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{3x^3}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 5153

$$-\frac{5}{3}c^2d\left(-3c^2d\left(\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4bc\sqrt{1-c^2x^2}}+\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}\right)-\frac{(d-}{}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{3x^3}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4x^2-2c^2\log\left(x^2\right)-\frac{1}{x^2}\right)}{6\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
-1/3*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^3 - (5*c^2*d*(-(((d - c
^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x) - 3*c^2*d*((b*c*x^2*Sqrt[d - c^2*d
*x^2])/(4*Sqrt[1 - c^2*x^2]) + (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))
/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c*Sqrt[1 - c^2*x^2])
) - (b*c*d*Sqrt[d - c^2*d*x^2]*(-1/2*(c^2*x^2) + Log[x]))/Sqrt[1 - c^2*x^2
]))/3 - (b*c*d^2*Sqrt[d - c^2*d*x^2]*(-x^(-2) + c^4*x^2 - 2*c^2*Log[x^2]))
/(6*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.24

method	result
default	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{3dx^3}+\frac{4ac^2(-c^2dx^2+d)^{\frac{7}{2}}}{3dx}+\frac{4ac^4x(-c^2dx^2+d)^{\frac{5}{2}}}{3}+\frac{5ac^4dx(-c^2dx^2+d)^{\frac{3}{2}}}{3}+\frac{5ac^4d^2x\sqrt{-c^2dx^2+d}}{2}+$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{3dx^3}+\frac{4ac^2(-c^2dx^2+d)^{\frac{7}{2}}}{3dx}+\frac{4ac^4x(-c^2dx^2+d)^{\frac{5}{2}}}{3}+\frac{5ac^4dx(-c^2dx^2+d)^{\frac{3}{2}}}{3}+\frac{5ac^4d^2x\sqrt{-c^2dx^2+d}}{2}+$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/d/x^3*(-c^2*d*x^2+d)^(7/2)+4/3*a*c^2/d/x*(-c^2*d*x^2+d)^(7/2)+4/3*a
*c^4*x*(-c^2*d*x^2+d)^(5/2)+5/3*a*c^4*d*x*(-c^2*d*x^2+d)^(3/2)+5/2*a*c^4*d
^2*x*(-c^2*d*x^2+d)^(1/2)+5/2*a*c^4*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)
*x/(-c^2*d*x^2+d)^(1/2))-1/24*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*
(12*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^4*x^4+6*c^5*x^5-30*arccos(c*x)^2*c^3*
x^3-56*I*arccos(c*x)*x^3*c^3+56*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*x^3*c^3
+56*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^2-3*c^3*x^3-8*arccos(c*x)*(-c^2*x
^2+1)^(1/2)+4*c*x)*d^2/(c^2*x^2-1)/x^3
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^4}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^4,x, algorithm="fricas"
)
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^4}\,dx=\int \frac{(-d(cx-1)(cx+1))^{\frac{5}{2}}(a+b\operatorname{acos}(cx))}{x^4}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))/x**4,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))/x**4, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^4}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^4,x, algorithm="maxima"
)
```

output
```
b*sqrt(d)*integrate((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x^4, x) + 1/6*(10*(-
c^2*d*x^2 + d)^(3/2)*c^4*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^4*d^2*x + 15*c^3*
d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)*c^2/x - 2*(-c^2*d*x^2 + d)^
(7/2)/(d*x^3))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^4}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^4,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}{x^4}\,dx = \frac{\sqrt{d}\,d^2\Big(-6acos(cx)^2\,b\,c^3x^3+15asin(cx)\,a\,c^3x^3+3\sqrt{-c^2x^2+1}\,a$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))/x^4,x)
```

output
```
(sqrt(d)*d**2*( - 6*acos(c*x)**2*b*c**3*x**3 + 15*asin(c*x)*a*c**3*x**3 +
3*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 + 14*sqrt( - c**2*x**2 + 1)*a*c**2*x*
*2 - 2*sqrt( - c**2*x**2 + 1)*a - 12*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)
*x**2),x)*b*c**2*x**3 + 6*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**4,x)*b
*x**3 + 6*int(sqrt( - c**2*x**2 + 1)*acos(c*x),x)*b*c**4*x**3))/(6*x**3)
```

3.91 $\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{x^6}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 277

$$\begin{aligned}\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{x^6}\,dx &= -\frac{bcd^2\sqrt{d-c^2dx^2}}{20x^4\sqrt{1-c^2x^2}} \\ &+ \frac{11bc^3d^2\sqrt{d-c^2dx^2}}{30x^2\sqrt{1-c^2x^2}} - \frac{c^4d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x} \\ &+ \frac{c^2d(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{3x^3} - \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{5x^5} \\ &- \frac{c^5d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2b\sqrt{1-c^2x^2}} + \frac{23bc^5d^2\sqrt{d-c^2dx^2}\log(x)}{15\sqrt{1-c^2x^2}}\end{aligned}$$

output

```
-1/20*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)+11/30*b*c^3*d^2*
(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-c^4*d^2*(-c^2*d*x^2+d)^(1/2)*(
a+b*arccos(c*x))/x+1/3*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^3-1/
5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^5-1/2*c^5*d^2*(-c^2*d*x^2+d)^(1
/2)*(a+b*arccos(c*x))^2/b/(-c^2*x^2+1)^(1/2)+23/15*b*c^5*d^2*(-c^2*d*x^2+d
)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.87

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^6}\,dx = -\frac{bd^2\sqrt{d-c^2dx^2}(3-11c^2x^2+23c^4x^4)\arccos(cx)}{15x^5} + \frac{bc^5d^2\sqrt{d-c^2dx^2}\arccos(cx)^2}{2\sqrt{1-c^2x^2}} + ac^5d^{5/2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right) - \frac{d^2\sqrt{d-c^2dx^2}\left(bcx(-3+22c^2x^2)+4a\sqrt{1-c^2x^2}(3-11c^2x^2+23c^4x^4)+92bc^5x^5\log(cx)\right)}{60x^5\sqrt{1-c^2x^2}}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^6,x]
```

output
```
-1/15*(b*d^2*Sqrt[d - c^2*d*x^2]*(3 - 11*c^2*x^2 + 23*c^4*x^4)*ArcCos[c*x]
)/x^5 + (b*c^5*d^2*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2)/(2*Sqrt[1 - c^2*x^2]
) + a*c^5*d^(5/2)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2)
)] - (d^2*Sqrt[d - c^2*d*x^2]*(b*c*x*(-3 + 22*c^2*x^2) + 4*a*Sqrt[1 - c^2*
x^2]*(3 - 11*c^2*x^2 + 23*c^4*x^4) + 92*b*c^5*x^5*Log[c*x]))/(60*x^5*Sqrt[
1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5201, 243, 49, 2009, 5201, 244, 2009, 5197, 14, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^6}\,dx$$

$$\Big\downarrow \text{ 5201}$$

$$-c^2 d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x^4}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2}{x^5}dx}{5\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5x^5}$$

$\big\downarrow$ 243

$$-c^2 d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x^4}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2}{x^6}dx^2}{10\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5x^5}$$

$\big\downarrow$ 49

$$-c^2 d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x^4}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(\frac{c^4}{x^2}-\frac{2c^2}{x^4}+\frac{1}{x^6}\right)dx^2}{10\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5x^5}$$

$\big\downarrow$ 2009

$$-c^2 d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x^4}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5x^5}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 5201

$$-c^2 d\left(c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}dx-\frac{bcd\sqrt{d-c^2dx^2}\int \frac{1-c^2x^2}{x^3}dx}{3\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(c\cdots}{3x^3}\cdots\right.$$
$$-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5x^5}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 244

$$-c^2 d\left(c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}dx-\frac{bcd\sqrt{d-c^2dx^2}\int \left(\frac{1}{x^3}-\frac{c^2}{x}\right)dx}{3\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\,\mathrm{arc}\cdots}{3x^3}\cdots\right.$$
$$-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5x^5}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 2009

$$-c^2d\left(c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3x^3}-\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-\log\right.}{3\sqrt{1-c^2x^2}}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5x^5}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 5197

$$-c^2d\left(c^2(-d)\left(-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}-\frac{bc\sqrt{d-c^2dx^2}\int\frac{1}{x}dx}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}\right)-\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5x^5}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 14

$$-c^2d\left(c^2(-d)\left(-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}-\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}\right)-\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5x^5}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 5153

$$-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{5x^5}-$$
$$c^2d\left(c^2(-d)\left(\frac{c\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2b\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}-\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}\right)-\frac{(d-}{}\right.$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^4\log\left(x^2\right)+\frac{2c^2}{x^2}-\frac{1}{2x^4}\right)}{10\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^6,x]
```

output
```
-1/5*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^5 - c^2*d*(-1/3*((d - c
^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x^3 - (b*c*d*Sqrt[d - c^2*d*x^2]*(-1/
2*1/x^2 - c^2*Log[x]))/(3*Sqrt[1 - c^2*x^2]) - c^2*d*(-((Sqrt[d - c^2*d*x^
2]*(a + b*ArcCos[c*x]))/x) + (c*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)
/(2*b*Sqrt[1 - c^2*x^2]) - (b*c*Sqrt[d - c^2*d*x^2]*Log[x])/Sqrt[1 - c^2*x
^2])) - (b*c*d^2*Sqrt[d - c^2*d*x^2]*(-1/2*1/x^4 + (2*c^2)/x^2 + c^4*Log[x
^2]))/(10*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5197
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^2
]/Sqrt[1 - c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x
] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 2)*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 2615, normalized size of antiderivative = 9.44

method	result	size
default	Expression too large to display	2615
parts	Expression too large to display	2615

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
9/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*
c^2*x^2+9)/x^5/(c^2*x^2-1)*arccos(c*x)-175/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2/
(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/(c^2*x^2-1)*c^5*(-c^2*
x^2+1)^(1/2)+23/15*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)
*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*d^2*c^5-1/2*b*(-d*(c^2*x^2-1))^(1/2)*(
-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^2*d^2*c^5+69/5*I*b*(-d*(c^2*x^2-
1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/(c^2*x^2
-1)*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^5+759/20*I*b*(-d*(c^2*x^2-1))^(1/2)*d
^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^3/(c^2*x^2-1)*(-c
^2*x^2+1)*c^8-69/20*I*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x
^6+325*c^4*x^4-75*c^2*x^2+9)*x/(c^2*x^2-1)*(-c^2*x^2+1)*c^6+5819/30*I*b*(-
d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+
9)*x^7/(c^2*x^2-1)*(-c^2*x^2+1)*c^12-7153/60*I*b*(-d*(c^2*x^2-1))^(1/2)*d^
2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^5/(c^2*x^2-1)*(-c^
2*x^2+1)*c^10-1/5*a/d/x^5*(-c^2*d*x^2+d)^(7/2)-8/15*a*c^6*x*(-c^2*d*x^2+d)
^(5/2)+69/20*I*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*
c^4*x^4-75*c^2*x^2+9)*x/(c^2*x^2-1)*c^6-207/5*I*b*(-d*(c^2*x^2-1))^(1/2)*d
^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^3/(c^2*x^2-1)*c^8
-46/15*I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*
x)*d^2*c^5+5819/30*I*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6...
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^6}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{x^6}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^6,x, algorithm="fricas"
)
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/x^6, x)
```

Sympy [F]

$$\int \frac{(d - c^2dx^2)^{5/2}(a + b\arccos(cx))}{x^6}\,dx = \int \frac{(-d(cx-1)(cx+1))^{\frac{5}{2}}(a + b\operatorname{acos}(cx))}{x^6}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))/x**6,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))/x**6, x)
```

Maxima [F]

$$\int \frac{(d - c^2dx^2)^{5/2}(a + b\arccos(cx))}{x^6}\,dx = \int \frac{(-c^2dx^2 + d)^{\frac{5}{2}}(b\arccos(cx) + a)}{x^6}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^6,x, algorithm="maxima"
)
```

output
```
b*sqrt(d)*integrate((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x^6, x) - 1/15*(10*(
-c^2*d*x^2 + d)^(3/2)*c^6*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^6*d^2*x + 15*c^5
*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)*c^4/x - 2*(-c^2*d*x^2 + d)
^(7/2)*c^2/(d*x^3) + 3*(-c^2*d*x^2 + d)^(7/2)/(d*x^5))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2dx^2)^{5/2}(a + b\arccos(cx))}{x^6}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^6,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^6}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^{5/2}}{x^6}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^6,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^6, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^6}\,dx = \frac{\sqrt{d}\,d^2\Big(15acos(cx)^2\,b\,c^5x^5-30asin(cx)\,a\,c^5x^5-46\sqrt{-c^2x^2+1}\,a$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))/x^6,x)
```

output
```
(sqrt(d)*d**2*(15*acos(c*x)**2*b*c**5*x**5 - 30*asin(c*x)*a*c**5*x**5 - 46
*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 + 22*sqrt( - c**2*x**2 + 1)*a*c**2*x**
2 - 6*sqrt( - c**2*x**2 + 1)*a + 30*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*
x**2),x)*b*c**4*x**5 + 30*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**6,x)*b
*x**5 - 60*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**4,x)*b*c**2*x**5))/(3
0*x**5)
```

3.92 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^8}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 203

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^8}\,dx = -\frac{bcd^2\sqrt{d-c^2dx^2}}{42x^6\sqrt{1-c^2x^2}}+\frac{3bc^3d^2\sqrt{d-c^2dx^2}}{28x^4\sqrt{1-c^2x^2}}-\frac{3bc^5d^2\sqrt{d-c^2dx^2}}{14x^2\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{7dx^7}-\frac{bc^7d^2\sqrt{d-c^2dx^2}\log(x)}{7\sqrt{1-c^2x^2}}$$

output
```
-1/42*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^6/(-c^2*x^2+1)^(1/2)+3/28*b*c^3*d^2*(
-c^2*d*x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)-3/14*b*c^5*d^2*(-c^2*d*x^2+d)^(
1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccos(c*x))/d/x
^7-1/7*b*c^7*d^2*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.94

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^8}\,dx = \frac{d^2\sqrt{d-c^2dx^2}\left(10bcx-45bc^3x^3+90bc^5x^5-147bc^7x^7-60a\sqrt{1\cdots}\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^8,x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(10*b*c*x - 45*b*c^3*x^3 + 90*b*c^5*x^5 - 147*b*c
^7*x^7 - 60*a*Sqrt[1 - c^2*x^2] + 180*a*c^2*x^2*Sqrt[1 - c^2*x^2] - 180*a*
c^4*x^4*Sqrt[1 - c^2*x^2] + 60*a*c^6*x^6*Sqrt[1 - c^2*x^2] - 60*b*(1 - c^2
*x^2)^(7/2)*ArcCos[c*x] + 60*b*c^7*x^7*Log[x]))/(420*x^7*Sqrt[1 - c^2*x^2]
)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.53, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5187, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^8}dx$$
$$\Big\downarrow \text{5187}$$
$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^3}{x^7}dx}{7\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{7dx^7}$$
$$\Big\downarrow \text{243}$$
$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^3}{x^8}dx^2}{14\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{7dx^7}$$
$$\Big\downarrow \text{49}$$
$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(-\frac{c^6}{x^2}+\frac{3c^4}{x^4}-\frac{3c^2}{x^6}+\frac{1}{x^8}\right)dx^2}{14\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{7dx^7}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{7dx^7}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(c^6(-\log(x^2))-\frac{3c^4}{x^2}+\frac{3c^2}{2x^4}-\frac{1}{3x^6}\right)}{14\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^8,x]
```

output
```
-1/7*((d - c^2*d*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(d*x^7) - (b*c*d^2*Sqrt[d
 - c^2*d*x^2]*(-1/3*1/x^6 + (3*c^2)/(2*x^4) - (3*c^4)/x^2 - c^6*Log[x^2]))
/(14*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 4031, normalized size of antiderivative = 19.86

method	result	size
default	Expression too large to display	4031
parts	Expression too large to display	4031

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
17/28*I*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-
35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^5/(c^2*x^2-1)*(-c^2*x^2+1)*c^12-5/28*
I*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6
*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^3/(c^2*x^2-1)*(-c^2*x^2+1)*c^10+1/42*I*b*(-
d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+2
1*c^4*x^4-7*c^2*x^2+1)*x/(c^2*x^2-1)*(-c^2*x^2+1)*c^8-1/7*I*b*(-d*(c^2*x^2
-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-
7*c^2*x^2+1)/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^7-3/14*I*b*(-d*(
c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c
^4*x^4-7*c^2*x^2+1)*x^11/(c^2*x^2-1)*(-c^2*x^2+1)*c^18+3/4*I*b*(-d*(c^2*x^
2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4
-7*c^2*x^2+1)*x^9/(c^2*x^2-1)*(-c^2*x^2+1)*c^16-83/84*I*b*(-d*(c^2*x^2-1))
^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^
2*x^2+1)*x^7/(c^2*x^2-1)*(-c^2*x^2+1)*c^14+1/7*b*(-d*(c^2*x^2-1))^(1/2)*d^
2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/
x^7/(c^2*x^2-1)*arccos(c*x)-165/7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^1
2-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x/(c^2*x^2-1)
*arccos(c*x)*c^6-41/28*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x
^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x^2/(c^2*x^2-1)*c^5*(-c^
2*x^2+1)^(1/2)+55/7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x...
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 658, normalized size of antiderivative = 3.24

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^8}\,dx = \left[\frac{6\left(bc^9d^2x^9-bc^7d^2x^7\right)\sqrt{d}\log\left(\frac{c^2dx^6+c^2dx^2-dx^4-\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}}{c^2x^4-x^2}\right.}{}\right.$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^8,x, algorithm="fricas"
)
```

output
```
[1/84*(6*(b*c^9*d^2*x^9 - b*c^7*d^2*x^7)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^
2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)
/(c^2*x^4 - x^2)) - (18*b*c^5*d^2*x^5 - (18*b*c^5 - 9*b*c^3 + 2*b*c)*d^2*x
^7 - 9*b*c^3*d^2*x^3 + 2*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1
) + 12*(a*c^8*d^2*x^8 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^
2 + a*d^2 + (b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 - 4*b*c^2*d
^2*x^2 + b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7), 1/84*(
12*(b*c^9*d^2*x^9 - b*c^7*d^2*x^7)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sq
rt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) - (
18*b*c^5*d^2*x^5 - (18*b*c^5 - 9*b*c^3 + 2*b*c)*d^2*x^7 - 9*b*c^3*d^2*x^3
+ 2*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 12*(a*c^8*d^2*x^8
 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2 + (b*c^8*d^
2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*arcco
s(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^8}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))/x**8,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.01

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^8}\,dx =$$
$$-\frac{\left(6\,(-1)^{-2\,c^2dx^2+2\,d}\,c^6d^{\frac{7}{2}}\log\left(-2\,c^2d+\frac{2\,d}{x^2}\right)+6\,c^6d^{\frac{7}{2}}\log\left(x^2-\frac{1}{c^2}\right)-\frac{11\,\sqrt{c^4dx^4-2\,c^2dx^2+d}c^4d^3}{x^2}+\frac{7\,\sqrt{c^4dx^4-2\,c^2dx^2}}{x^4}\right.}{84\,d}$$
$$-\frac{(-c^2dx^2+d)^{\frac{7}{2}}b\arccos(cx)}{7\,dx^7}-\frac{(-c^2dx^2+d)^{\frac{7}{2}}a}{7\,dx^7}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^8,x, algorithm="maxima"
)
```

output
```
-1/84*(6*(-1)^(-2*c^2*d*x^2 + 2*d)*c^6*d^(7/2)*log(-2*c^2*d + 2*d/x^2) + 6
*c^6*d^(7/2)*log(x^2 - 1/c^2) - 11*sqrt(c^4*d*x^4 - 2*c^2*d*x^2 + d)*c^4*d
^3/x^2 + 7*sqrt(c^4*d*x^4 - 2*c^2*d*x^2 + d)*c^2*d^3/x^4 - 2*sqrt(c^4*d*x^
4 - 2*c^2*d*x^2 + d)*d^3/x^6)*b*c/d - 1/7*(-c^2*d*x^2 + d)^(7/2)*b*arccos(
c*x)/(d*x^7) - 1/7*(-c^2*d*x^2 + d)^(7/2)*a/(d*x^7)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}{x^8}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^8,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}{x^8}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^8}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^8,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^8, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^8}\,dx = \frac{\sqrt{d}\,d^2\left(\sqrt{-c^2x^2+1}\,a\,c^6x^6-3\sqrt{-c^2x^2+1}\,a\,c^4x^4+3\sqrt{-c^2x^2+1}\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))/x^8,x)
```

output
```
(sqrt(d)*d**2*(sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - 3*sqrt( - c**2*x**2 +
1)*a*c**4*x**4 + 3*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x**2
+ 1)*a + 7*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**8,x)*b*x**7 - 14*int(
(sqrt( - c**2*x**2 + 1)*acos(c*x))/x**6,x)*b*c**2*x**7 + 7*int((sqrt( - c*
*2*x**2 + 1)*acos(c*x))/x**4,x)*b*c**4*x**7))/(7*x**7)
```

3.93 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{10}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 282

$$\begin{aligned}\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{10}}\,dx = {} & -\frac{bc^3d^2\sqrt{d-c^2dx^2}}{189x^6\sqrt{1-c^2x^2}}+\frac{bc^5d^2\sqrt{d-c^2dx^2}}{42x^4\sqrt{1-c^2x^2}}-\frac{bc^7d^2\sqrt{d-c^2dx^2}}{21x^2\sqrt{1-c^2x^2}} \\ & -\frac{bcd^2(1-c^2x^2)^{7/2}\sqrt{d-c^2dx^2}}{72x^8}-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{9dx^9} \\ & -\frac{2c^2(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{63dx^7}-\frac{2bc^9d^2\sqrt{d-c^2dx^2}\log(x)}{63\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-1/189*b*c^3*d^2*(-c^2*d*x^2+d)^(1/2)/x^6/(-c^2*x^2+1)^(1/2)+1/42*b*c^5*d^
2*(-c^2*d*x^2+d)^(1/2)/x^4/(-c^2*x^2+1)^(1/2)-1/21*b*c^7*d^2*(-c^2*d*x^2+d
)^(1/2)/x^2/(-c^2*x^2+1)^(1/2)-1/72*b*c*d^2*(-c^2*x^2+1)^(7/2)*(-c^2*d*x^2
+d)^(1/2)/x^8-1/9*(-c^2*d*x^2+d)^(7/2)*(a+b*arccos(c*x))/d/x^9-2/63*c^2*(-
c^2*d*x^2+d)^(7/2)*(a+b*arccos(c*x))/d/x^7-2/63*b*c^9*d^2*(-c^2*d*x^2+d)^(
1/2)*ln(x)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.82

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{10}}\,dx = \frac{d^2\sqrt{d-c^2dx^2}\Big(735bcx-2660bc^3x^3+3150bc^5x^5-420bc^7x^7-45}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^10,x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(735*b*c*x - 2660*b*c^3*x^3 + 3150*b*c^5*x^5 - 42
0*b*c^7*x^7 - 4566*b*c^9*x^9 - 5880*a*Sqrt[1 - c^2*x^2] + 15960*a*c^2*x^2*
Sqrt[1 - c^2*x^2] - 12600*a*c^4*x^4*Sqrt[1 - c^2*x^2] + 840*a*c^6*x^6*Sqrt
[1 - c^2*x^2] + 1680*a*c^8*x^8*Sqrt[1 - c^2*x^2] - 840*b*(1 - c^2*x^2)^(7/
2)*(7 + 2*c^2*x^2)*ArcCos[c*x] + 1680*b*c^9*x^9*Log[x]))/(52920*x^9*Sqrt[1
 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.60, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5195, 27, 354, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{10}}dx$$

$$\Big\downarrow \text{5195}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d^2(1-c^2x^2)^3(2c^2x^2+7)}{63x^9}dx}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{9dx^9}-\frac{2c^2(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{63dx^7}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^3(2c^2x^2+7)}{x^9}dx}{63\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{9dx^9}-\frac{2c^2(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{63dx^7}$$

$$\Big\downarrow \text{354}$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^3(2c^2x^2+7)}{x^{10}}dx^2}{126\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{9dx^9}-\frac{2c^2(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{63dx^7}$$

$$\Big\downarrow \text{87}$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\left(2c^2\int \frac{(1-c^2x^2)^3}{x^8}dx^2-\frac{7(1-c^2x^2)^4}{4x^8}\right)}{126\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{9dx^9}-\frac{2c^2(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{63dx^7}$$

$$\Big\downarrow \text{49}$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\left(2c^2\int \left(-\frac{c^6}{x^2}+\frac{3c^4}{x^4}-\frac{3c^2}{x^6}+\frac{1}{x^8}\right)dx^2-\frac{7(1-c^2x^2)^4}{4x^8}\right)}{126\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{9dx^9}-\frac{2c^2(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{63dx^7}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{9dx^9}-\frac{2c^2(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{63dx^7}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(2c^2\left(c^6\left(-\log\left(x^2\right)\right)-\frac{3c^4}{x^2}+\frac{3c^2}{2x^4}-\frac{1}{3x^6}\right)-\frac{7(1-c^2x^2)^4}{4x^8}\right)}{126\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^10,x]
```

output
```
-1/9*((d - c^2*d*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(d*x^9) - (2*c^2*(d - c^2
*d*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(63*d*x^7) - (b*c*d^2*Sqrt[d - c^2*d*x^
2]*((-7*(1 - c^2*x^2)^4)/(4*x^8) + 2*c^2*(-1/3*1/x^6 + (3*c^2)/(2*x^4) - (
3*c^4)/x^2 - c^6*Log[x^2])))/(126*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 5324, normalized size of antiderivative = 18.88

method	result	size
default	Expression too large to display	5324
parts	Expression too large to display	5324

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^10,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 748, normalized size of antiderivative = 2.65

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\arccos(cx)\right)}{x^{10}}\,dx = \Bigg[\frac{24\left(bc^{11}d^2x^{11}-bc^9d^2x^9\right)\sqrt{d}\log\left(\frac{c^2dx^6+c^2dx^2-dx^4-\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2}}{c^2x^4-x^2}\right.}{}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^10,x, algorithm="fricas
")
```

output
```
[1/1512*(24*(b*c^11*d^2*x^11 - b*c^9*d^2*x^9)*sqrt(d)*log((c^2*d*x^6 + c^2
*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d)
 - d)/(c^2*x^4 - x^2)) + (12*b*c^7*d^2*x^7 - 90*b*c^5*d^2*x^5 - (12*b*c^7
- 90*b*c^5 + 76*b*c^3 - 21*b*c)*d^2*x^9 + 76*b*c^3*d^2*x^3 - 21*b*c*d^2*x)
*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 24*(2*a*c^10*d^2*x^10 - a*c^8*d
^2*x^8 - 16*a*c^6*d^2*x^6 + 34*a*c^4*d^2*x^4 - 26*a*c^2*d^2*x^2 + 7*a*d^2
+ (2*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 16*b*c^6*d^2*x^6 + 34*b*c^4*d^2*x^4
 - 26*b*c^2*d^2*x^2 + 7*b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^1
1 - x^9), 1/1512*(48*(b*c^11*d^2*x^11 - b*c^9*d^2*x^9)*sqrt(-d)*arctan(sqr
t(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)/(c^2*d*x^4 + (c^2
- 1)*d*x^2 - d)) + (12*b*c^7*d^2*x^7 - 90*b*c^5*d^2*x^5 - (12*b*c^7 - 90*b
*c^5 + 76*b*c^3 - 21*b*c)*d^2*x^9 + 76*b*c^3*d^2*x^3 - 21*b*c*d^2*x)*sqrt(
-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 24*(2*a*c^10*d^2*x^10 - a*c^8*d^2*x^8
 - 16*a*c^6*d^2*x^6 + 34*a*c^4*d^2*x^4 - 26*a*c^2*d^2*x^2 + 7*a*d^2 + (2*b
*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 16*b*c^6*d^2*x^6 + 34*b*c^4*d^2*x^4 - 26*
b*c^2*d^2*x^2 + 7*b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^
9)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{10}}\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))/x**10,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.57

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{10}}\,dx = \frac{1}{1512}\left(48\,c^8d^{\frac{5}{2}}\log(x) - \frac{12\,c^6d^{\frac{5}{2}}x^6 - 90\,c^4d^{\frac{5}{2}}x^4 + 76\,c^2d^{\frac{5}{2}}x^2 - 21}{x^8}\right.$$
$$- \frac{1}{63}b\left(\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}c^2}{dx^7} + \frac{7\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{dx^9}\right)\arccos(cx)$$
$$- \frac{1}{63}a\left(\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}c^2}{dx^7} + \frac{7\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{dx^9}\right)$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^10,x, algorithm="maxima
")
```

output
```
1/1512*(48*c^8*d^(5/2)*log(x) - (12*c^6*d^(5/2)*x^6 - 90*c^4*d^(5/2)*x^4 +
 76*c^2*d^(5/2)*x^2 - 21*d^(5/2))/x^8)*b*c - 1/63*b*(2*(-c^2*d*x^2 + d)^(7
/2)*c^2/(d*x^7) + 7*(-c^2*d*x^2 + d)^(7/2)/(d*x^9))*arccos(c*x) - 1/63*a*(
2*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^7) + 7*(-c^2*d*x^2 + d)^(7/2)/(d*x^9))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{10}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^10,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{10}}\,dx = \int \frac{(a+b\operatorname{acos}(c\,x))\,(d-c^2\,d\,x^2)^{5/2}}{x^{10}}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^10,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^10, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{10}}\,dx = \frac{\sqrt{d}\,d^2\Big(2\sqrt{-c^2x^2+1}\,a\,c^8x^8+\sqrt{-c^2x^2+1}\,a\,c^6x^6-15\sqrt{-c^2x^2+}}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))/x^10,x)
```

output
```
(sqrt(d)*d**2*(2*sqrt( - c**2*x**2 + 1)*a*c**8*x**8 + sqrt( - c**2*x**2 +
1)*a*c**6*x**6 - 15*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 + 19*sqrt( - c**2*x
**2 + 1)*a*c**2*x**2 - 7*sqrt( - c**2*x**2 + 1)*a + 63*int((sqrt( - c**2*x
**2 + 1)*acos(c*x))/x**10,x)*b*x**9 - 126*int((sqrt( - c**2*x**2 + 1)*acos
(c*x))/x**8,x)*b*c**2*x**9 + 63*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**
6,x)*b*c**4*x**9))/(63*x**9)
```

3.94 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{12}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 361

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{12}}\,dx = -\frac{bcd^2\sqrt{d-c^2dx^2}}{110x^{10}\sqrt{1-c^2x^2}} + \frac{23bc^3d^2\sqrt{d-c^2dx^2}}{792x^8\sqrt{1-c^2x^2}} - \frac{113bc^5d^2\sqrt{d-c^2dx^2}}{4158x^6\sqrt{1-c^2x^2}} + \frac{bc^7d^2\sqrt{d-c^2dx^2}}{924x^4\sqrt{1-c^2x^2}} + \frac{2bc^9d^2\sqrt{d-c^2dx^2}}{693x^2\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{11dx^{11}} - \frac{4c^2(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{99dx^9} - \frac{8c^4(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{693dx^7} - \frac{8bc^{11}d^2\sqrt{d-c^2dx^2}\log(x)}{693\sqrt{1-c^2x^2}}$$

output
```
-1/110*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^10/(-c^2*x^2+1)^(1/2)+23/792*b*c^3*d
^2*(-c^2*d*x^2+d)^(1/2)/x^8/(-c^2*x^2+1)^(1/2)-113/4158*b*c^5*d^2*(-c^2*d*
x^2+d)^(1/2)/x^6/(-c^2*x^2+1)^(1/2)+1/924*b*c^7*d^2*(-c^2*d*x^2+d)^(1/2)/x
^4/(-c^2*x^2+1)^(1/2)+2/693*b*c^9*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(-c^2*x^2+1
)^(1/2)-1/11*(-c^2*d*x^2+d)^(7/2)*(a+b*arccos(c*x))/d/x^11-4/99*c^2*(-c^2*
d*x^2+d)^(7/2)*(a+b*arccos(c*x))/d/x^9-8/693*c^4*(-c^2*d*x^2+d)^(7/2)*(a+b
*arccos(c*x))/d/x^7-8/693*b*c^11*d^2*(-c^2*d*x^2+d)^(1/2)*ln(x)/(-c^2*x^2+
1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.75

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{12}}\,dx = \frac{d^2\sqrt{d-c^2dx^2}\left(15876bcx-50715bc^3x^3+47460bc^5x^5-1890bc^7x\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^12,x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(15876*b*c*x - 50715*b*c^3*x^3 + 47460*b*c^5*x^5
- 1890*b*c^7*x^7 - 5040*b*c^9*x^9 - 59048*b*c^11*x^11 - 158760*a*Sqrt[1 -
c^2*x^2] + 405720*a*c^2*x^2*Sqrt[1 - c^2*x^2] - 284760*a*c^4*x^4*Sqrt[1 -
c^2*x^2] + 7560*a*c^6*x^6*Sqrt[1 - c^2*x^2] + 10080*a*c^8*x^8*Sqrt[1 - c^2
*x^2] + 20160*a*c^10*x^10*Sqrt[1 - c^2*x^2] - 2520*b*(1 - c^2*x^2)^(7/2)*(
63 + 28*c^2*x^2 + 8*c^4*x^4)*ArcCos[c*x] + 20160*b*c^11*x^11*Log[x]))/(174
6360*x^11*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.55, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5195, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{12}}dx$$

$$\Big\downarrow \; 5195$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d^2(1-c^2x^2)^3(8c^4x^4+28c^2x^2+63)}{693x^{11}}dx}{\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{11dx^{11}} - \frac{4c^2(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{99dx^9} - \frac{8c^4(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{693dx^7}$$

$$\Big\downarrow \; 27$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^3(8c^4x^4+28c^2x^2+63)}{x^{11}}dx}{693\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{11dx^{11}}-\frac{4c^2(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{99dx^9}-\frac{8c^4(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{693dx^7}$$

$$\Big\downarrow \text{ 1578}$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^3(8c^4x^4+28c^2x^2+63)}{x^{12}}dx^2}{1386\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{11dx^{11}}-\frac{4c^2(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{99dx^9}-\frac{8c^4(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{693dx^7}$$

$$\Big\downarrow \text{ 1195}$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(-\frac{8c^{10}}{x^2}-\frac{4c^8}{x^4}-\frac{3c^6}{x^6}+\frac{113c^4}{x^8}-\frac{161c^2}{x^{10}}+\frac{63}{x^{12}}\right)dx^2}{1386\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{11dx^{11}}-\frac{4c^2(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{99dx^9}-\frac{8c^4(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{693dx^7}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{11dx^{11}}-\frac{4c^2(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{99dx^9}-\frac{8c^4(d-c^2dx^2)^{7/2}(a+b\arccos(cx))}{693dx^7}-\frac{bcd^2\sqrt{d-c^2dx^2}\left(-8c^{10}\log(x^2)+\frac{4c^8}{x^2}+\frac{3c^6}{2x^4}-\frac{113c^4}{3x^6}+\frac{161c^2}{4x^8}-\frac{63}{5x^{10}}\right)}{1386\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^12,x]
```

output
```
-1/11*((d - c^2*d*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(d*x^11) - (4*c^2*(d - c
^2*d*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(99*d*x^9) - (8*c^4*(d - c^2*d*x^2)^(
7/2)*(a + b*ArcCos[c*x]))/(693*d*x^7) - (b*c*d^2*Sqrt[d - c^2*d*x^2]*(-63/
(5*x^10) + (161*c^2)/(4*x^8) - (113*c^4)/(3*x^6) + (3*c^6)/(2*x^4) + (4*c^
8)/x^2 - 8*c^10*Log[x^2]))/(1386*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 6761, normalized size of antiderivative = 18.73

method	result	size
default	Expression too large to display	6761
parts	Expression too large to display	6761

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^12,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 832, normalized size of antiderivative = 2.30

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{12}}\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^12,x, algorithm="fricas
")
```

output
```
[1/83160*(480*(b*c^13*d^2*x^13 - b*c^11*d^2*x^11)*sqrt(d)*log((c^2*d*x^6 +
 c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqr
t(d) - d)/(c^2*x^4 - x^2)) + (240*b*c^9*d^2*x^9 + 90*b*c^7*d^2*x^7 - (240*
b*c^9 + 90*b*c^7 - 2260*b*c^5 + 2415*b*c^3 - 756*b*c)*d^2*x^11 - 2260*b*c^
5*d^2*x^5 + 2415*b*c^3*d^2*x^3 - 756*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(
-c^2*x^2 + 1) + 120*(8*a*c^12*d^2*x^12 - 4*a*c^10*d^2*x^10 - a*c^8*d^2*x^8
 - 116*a*c^6*d^2*x^6 + 274*a*c^4*d^2*x^4 - 224*a*c^2*d^2*x^2 + 63*a*d^2 +
(8*b*c^12*d^2*x^12 - 4*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 116*b*c^6*d^2*x^6
 + 274*b*c^4*d^2*x^4 - 224*b*c^2*d^2*x^2 + 63*b*d^2)*arccos(c*x))*sqrt(-c^
2*d*x^2 + d))/(c^2*x^13 - x^11), 1/83160*(960*(b*c^13*d^2*x^13 - b*c^11*d^
2*x^11)*sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*
sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) + (240*b*c^9*d^2*x^9 + 90*b*c^
7*d^2*x^7 - (240*b*c^9 + 90*b*c^7 - 2260*b*c^5 + 2415*b*c^3 - 756*b*c)*d^2
*x^11 - 2260*b*c^5*d^2*x^5 + 2415*b*c^3*d^2*x^3 - 756*b*c*d^2*x)*sqrt(-c^2
*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 120*(8*a*c^12*d^2*x^12 - 4*a*c^10*d^2*x^1
0 - a*c^8*d^2*x^8 - 116*a*c^6*d^2*x^6 + 274*a*c^4*d^2*x^4 - 224*a*c^2*d^2*
x^2 + 63*a*d^2 + (8*b*c^12*d^2*x^12 - 4*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 -
116*b*c^6*d^2*x^6 + 274*b*c^4*d^2*x^4 - 224*b*c^2*d^2*x^2 + 63*b*d^2)*arcc
os(c*x))*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{12}}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))/x**12,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.61

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{12}}\,dx = \frac{1}{83160}\left(960\,c^{10}d^{\frac{5}{2}}\log\left(x\right) - \frac{240\,c^8d^{\frac{5}{2}}x^8 + 90\,c^6d^{\frac{5}{2}}x^6 - 2260\,c^4d^{\frac{5}{2}}x}{x^{10}}\right.$$
$$- \frac{1}{693}\,b\left(\frac{8\left(-c^2dx^2+d\right)^{\frac{7}{2}}c^4}{dx^7} + \frac{28\left(-c^2dx^2+d\right)^{\frac{7}{2}}c^2}{dx^9} + \frac{63\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{dx^{11}}\right)\arccos\left(cx\right)$$
$$- \frac{1}{693}\,a\left(\frac{8\left(-c^2dx^2+d\right)^{\frac{7}{2}}c^4}{dx^7} + \frac{28\left(-c^2dx^2+d\right)^{\frac{7}{2}}c^2}{dx^9} + \frac{63\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{dx^{11}}\right)$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^12,x, algorithm="maxima
")
```

output
```
1/83160*(960*c^10*d^(5/2)*log(x) - (240*c^8*d^(5/2)*x^8 + 90*c^6*d^(5/2)*x
^6 - 2260*c^4*d^(5/2)*x^4 + 2415*c^2*d^(5/2)*x^2 - 756*d^(5/2))/x^10)*b*c
- 1/693*b*(8*(-c^2*d*x^2 + d)^(7/2)*c^4/(d*x^7) + 28*(-c^2*d*x^2 + d)^(7/2
)*c^2/(d*x^9) + 63*(-c^2*d*x^2 + d)^(7/2)/(d*x^11))*arccos(c*x) - 1/693*a*
(8*(-c^2*d*x^2 + d)^(7/2)*c^4/(d*x^7) + 28*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x
^9) + 63*(-c^2*d*x^2 + d)^(7/2)/(d*x^11))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{12}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^12,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{12}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^{12}}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^12,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^12, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^{12}}\,dx = \frac{\sqrt{d}\,d^2\Big(8\sqrt{-c^2x^2+1}\,a\,c^{10}x^{10}+4\sqrt{-c^2x^2+1}\,a\,c^8x^8+3\sqrt{-c^2x^2}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))/x^12,x)
```

output
```
(sqrt(d)*d**2*(8*sqrt( - c**2*x**2 + 1)*a*c**10*x**10 + 4*sqrt( - c**2*x**
2 + 1)*a*c**8*x**8 + 3*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - 113*sqrt( - c*
*2*x**2 + 1)*a*c**4*x**4 + 161*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 63*sqr
t( - c**2*x**2 + 1)*a + 693*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**12,x
)*b*x**11 - 1386*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**10,x)*b*c**2*x*
*11 + 693*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**8,x)*b*c**4*x**11))/(6
93*x**11)
```

3.95 $\int x^5(d-c^2dx^2)^{5/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 354

$$\int x^5\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\frac{8bd^2x\sqrt{d-c^2dx^2}}{693c^5\sqrt{1-c^2x^2}}+\frac{4bd^2x^3\sqrt{d-c^2dx^2}}{2079c^3\sqrt{1-c^2x^2}}+\frac{bd^2x^5\sqrt{d-c^2dx^2}}{1155c\sqrt{1-c^2x^2}}-\frac{113bcd^2x^7\sqrt{d-c^2dx^2}}{4851\sqrt{1-c^2x^2}}+\frac{23bc^3d^2x^9\sqrt{d-c^2dx^2}}{891\sqrt{1-c^2x^2}}-\frac{bc^5d^2x^{11}\sqrt{d-c^2dx^2}}{121\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^6d}+\frac{2\left(d-c^2dx^2\right)^{9/2}(a+b\arccos(cx))}{9c^6d^2}-\frac{\left(d-c^2dx^2\right)^{11/2}(a+b\arccos(cx))}{11c^6d^3}$$

output
```
8/693*b*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^5/(-c^2*x^2+1)^(1/2)+4/2079*b*d^2*x^3
*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/1155*b*d^2*x^5*(-c^2*d*x^2+
d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-113/4851*b*c*d^2*x^7*(-c^2*d*x^2+d)^(1/2)/(-
c^2*x^2+1)^(1/2)+23/891*b*c^3*d^2*x^9*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1
/2)-1/121*b*c^5*d^2*x^11*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/7*(-c^2
*d*x^2+d)^(7/2)*(a+b*arccos(c*x))/c^6/d+2/9*(-c^2*d*x^2+d)^(9/2)*(a+b*arcc
os(c*x))/c^6/d^2-1/11*(-c^2*d*x^2+d)^(11/2)*(a+b*arccos(c*x))/c^6/d^3
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.47

$$\int x^5 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos (c x)) \, dx = \frac{d^2 \sqrt{d-c^2 d x^2} \left(3465 a \left(-1+c^2 x^2\right)^4 \left(8+28 c^2 x^2+63 c^4 x^4\right)+b c x \sqrt{1-c^2 x^2} (27720+462$$

input
```
Integrate[x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(3465*a*(-1 + c^2*x^2)^4*(8 + 28*c^2*x^2 + 63*c^4
*x^4) + b*c*x*Sqrt[1 - c^2*x^2]*(27720 + 4620*c^2*x^2 + 2079*c^4*x^4 - 559
35*c^6*x^6 + 61985*c^8*x^8 - 19845*c^10*x^10) + 3465*b*(-1 + c^2*x^2)^4*(8
 + 28*c^2*x^2 + 63*c^4*x^4)*ArcCos[c*x]))/(2401245*c^6*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5195, 27, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos (c x)) \, dx$$

$$\Big\downarrow \text{5195}$$

$$\frac{b c \sqrt{d-c^2 d x^2} \int -\frac{d^2 \left(1-c^2 x^2\right)^3 \left(63 c^4 x^4+28 c^2 x^2+8\right)}{693 c^6}dx}{\sqrt{1-c^2 x^2}}-\frac{\left(d-c^2 d x^2\right)^{11/2} (a+b \arccos (c x))}{11 c^6 d^3}+\frac{2 \left(d-c^2 d x^2\right)^{9/2} (a+b \arccos (c x))}{9 c^6 d^2}-\frac{\left(d-c^2 d x^2\right)^{7/2} (a+b \arccos (c x))}{7 c^6 d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bd^2\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)^3\left(63c^4x^4+28c^2x^2+8\right)dx}{693c^5\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{11/2}(a+b\arccos(cx))}{11c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{9/2}(a+b\arccos(cx))}{9c^6d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^6d}$$

$$\Big\downarrow \text{ 1467}$$

$$-\frac{bd^2\sqrt{d-c^2dx^2}\int \left(-63c^{10}x^{10}+161c^8x^8-113c^6x^6+3c^4x^4+4c^2x^2+8\right)dx}{693c^5\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{11/2}(a+b\arccos(cx))}{11c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{9/2}(a+b\arccos(cx))}{9c^6d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^6d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(d-c^2dx^2\right)^{11/2}(a+b\arccos(cx))}{11c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{9/2}(a+b\arccos(cx))}{9c^6d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^6d}-\frac{bd^2\left(-\frac{63}{11}c^{10}x^{11}+\frac{161c^8x^9}{9}-\frac{113c^6x^7}{7}+\frac{3c^4x^5}{5}+\frac{4c^2x^3}{3}+8x\right)\sqrt{d-c^2dx^2}}{693c^5\sqrt{1-c^2x^2}}$$

input
```
Int[x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/693*(b*d^2*Sqrt[d - c^2*d*x^2]*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (
113*c^6*x^7)/7 + (161*c^8*x^9)/9 - (63*c^10*x^11)/11))/(c^5*Sqrt[1 - c^2*x
^2]) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(7*c^6*d) + (2*(d - c^2
*d*x^2)^(9/2)*(a + b*ArcCos[c*x]))/(9*c^6*d^2) - ((d - c^2*d*x^2)^(11/2)*(
a + b*ArcCos[c*x]))/(11*c^6*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.72

method	result
orering	$\frac{\left(83349 x^{12} c^{12}-299047 c^{10} x^{10}+363737 c^{8} x^{8}-140481 c^{6} x^{6}-7854 c^{4} x^{4}-53592 c^{2} x^{2}+33264\right) \left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}} \left(a +b \arccos \left(c x \right)\right)}{480249 c^{6} \left(c x -1\right)^{2} \left(c x +1\right)^{2} \left(c^{2} x^{2}-1\right)}-\frac{(198}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/480249*(83349*c^12*x^12-299047*c^10*x^10+363737*c^8*x^8-140481*c^6*x^6-7
854*c^4*x^4-53592*c^2*x^2+33264)/c^6/(c*x-1)^2/(c*x+1)^2/(c^2*x^2-1)*(-c^2
*d*x^2+d)^(5/2)*(a+b*arccos(c*x))-1/2401245/x^4*(19845*c^10*x^10-61985*c^8
*x^8+55935*c^6*x^6-2079*c^4*x^4-4620*c^2*x^2-27720)/c^6/(c*x-1)^2/(c*x+1)^
2*(5*x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))-5*x^6*(-c^2*d*x^2+d)^(3/2)
*(a+b*arccos(c*x))*d*c^2-x^5*(-c^2*d*x^2+d)^(5/2)*b*c/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.82

$$\int x^5 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx =$$

$$-\frac{(19845\,bc^{11}d^2x^{11} - 61985\,bc^9d^2x^9 + 55935\,bc^7d^2x^7 - 2079\,bc^5d^2x^5 - 4620\,bc^3d^2x^3 - 27720\,bcd^2x)\sqrt{-c^2d}}{}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
-1/2401245*((19845*b*c^11*d^2*x^11 - 61985*b*c^9*d^2*x^9 + 55935*b*c^7*d^2
*x^7 - 2079*b*c^5*d^2*x^5 - 4620*b*c^3*d^2*x^3 - 27720*b*c*d^2*x)*sqrt(-c^
2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 3465*(63*a*c^12*d^2*x^12 - 224*a*c^10*d^
2*x^10 + 274*a*c^8*d^2*x^8 - 116*a*c^6*d^2*x^6 - a*c^4*d^2*x^4 - 4*a*c^2*d
^2*x^2 + 8*a*d^2 + (63*b*c^12*d^2*x^12 - 224*b*c^10*d^2*x^10 + 274*b*c^8*d
^2*x^8 - 116*b*c^6*d^2*x^6 - b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + 8*b*d^2)*ar
ccos(c*x))*sqrt(-c^2*d*x^2 + d))/(c^8*x^2 - c^6)
```

Sympy [F(-1)]

Timed out.

$$\int x^5 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \text{Timed out}$$

input
```
integrate(x**5*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.62

$$\int x^5 \left(d - c^2 d x^2\right)^{5/2} (a + b \arccos(cx))\, dx =$$
$$-\frac{1}{693}\left(\frac{63\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^4}{c^2d} + \frac{28\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^4d} + \frac{8\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^6d}\right) b \arccos(cx)$$
$$-\frac{1}{693}\left(\frac{63\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^4}{c^2d} + \frac{28\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^4d} + \frac{8\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^6d}\right) a$$
$$+\frac{\left(19845\,c^{10}d^{\frac{5}{2}}x^{11} - 61985\,c^8d^{\frac{5}{2}}x^9 + 55935\,c^6d^{\frac{5}{2}}x^7 - 2079\,c^4d^{\frac{5}{2}}x^5 - 4620\,c^2d^{\frac{5}{2}}x^3 - 27720\,d^{\frac{5}{2}}x\right)b}{2401245\,c^5}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
-1/693*(63*(-c^2*d*x^2 + d)^(7/2)*x^4/(c^2*d) + 28*(-c^2*d*x^2 + d)^(7/2)*
x^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(7/2)/(c^6*d))*b*arccos(c*x) - 1/693*(63*
(-c^2*d*x^2 + d)^(7/2)*x^4/(c^2*d) + 28*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^4*d)
 + 8*(-c^2*d*x^2 + d)^(7/2)/(c^6*d))*a + 1/2401245*(19845*c^10*d^(5/2)*x^1
1 - 61985*c^8*d^(5/2)*x^9 + 55935*c^6*d^(5/2)*x^7 - 2079*c^4*d^(5/2)*x^5 -
 4620*c^2*d^(5/2)*x^3 - 27720*d^(5/2)*x)*b/c^5
```

Giac [F(-2)]

Exception generated.

$$\int x^5 \left(d - c^2 d x^2\right)^{5/2} (a + b \arccos(cx))\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \int x^5\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int(x^5*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^5*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^5 \left(d-c^2dx^2\right)^{5/2} (a +b\arccos(cx))\,dx = \frac{\sqrt{d}\,d^2\left(63\sqrt{-c^2x^2+1}\,a\,c^{10}x^{10}-161\sqrt{-c^2x^2+1}\,a\,c^8x^8+113\sqrt{-c^2x^2+1}\,a\,c^6x^6-3\sqrt{}\right.}{}$$

input
```
int(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d**2*(63*sqrt( - c**2*x**2 + 1)*a*c**10*x**10 - 161*sqrt( - c**2*
x**2 + 1)*a*c**8*x**8 + 113*sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - 3*sqrt( -
 c**2*x**2 + 1)*a*c**4*x**4 - 4*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 8*sqr
t( - c**2*x**2 + 1)*a + 693*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**9,x)*b
*c**10 - 1386*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**7,x)*b*c**8 + 693*in
t(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**5,x)*b*c**6))/(693*c**6)
```

3.96 $\int x^3(d-c^2dx^2)^{5/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 278

$$\int x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\frac{2bd^2x\sqrt{d-c^2dx^2}}{63c^3\sqrt{1-c^2x^2}}+\frac{bd^2x^3\sqrt{d-c^2dx^2}}{189c\sqrt{1-c^2x^2}}-\frac{bcd^2x^5\sqrt{d-c^2dx^2}}{21\sqrt{1-c^2x^2}}+\frac{19bc^3d^2x^7\sqrt{d-c^2dx^2}}{441\sqrt{1-c^2x^2}}-\frac{bc^5d^2x^9\sqrt{d-c^2dx^2}}{81\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^4d}+\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\arccos(cx))}{9c^4d^2}$$

output
```
2/63*b*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2*x^2+1)^(1/2)+1/189*b*d^2*x^3*(
-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/21*b*c*d^2*x^5*(-c^2*d*x^2+d)^(
1/2)/(-c^2*x^2+1)^(1/2)+19/441*b*c^3*d^2*x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^
2+1)^(1/2)-1/81*b*c^5*d^2*x^9*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/7*
(-c^2*d*x^2+d)^(7/2)*(a+b*arccos(c*x))/c^4/d+1/9*(-c^2*d*x^2+d)^(9/2)*(a+b
*arccos(c*x))/c^4/d^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.51

$$\int x^3 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \frac{d^2\sqrt{d-c^2dx^2}\Big(63a(-1+c^2x^2)^4\left(2+7c^2x^2\right)+bcx\sqrt{1-c^2x^2}(126+21c^2x^2-189c^4x^4}{3969c^4\left(-1+c^2x^2\right)}$$

input
```
Integrate[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(63*a*(-1 + c^2*x^2)^4*(2 + 7*c^2*x^2) + b*c*x*Sq
rt[1 - c^2*x^2]*(126 + 21*c^2*x^2 - 189*c^4*x^4 + 171*c^6*x^6 - 49*c^8*x^8
) + 63*b*(-1 + c^2*x^2)^4*(2 + 7*c^2*x^2)*ArcCos[c*x]))/(3969*c^4*(-1 + c^
2*x^2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5195, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5195}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{d^2\left(1-c^2x^2\right)^3\left(7c^2x^2+2\right)}{63c^4}dx}{\sqrt{1-c^2x^2}}+\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\arccos(cx))}{9c^4d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^4d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bd^2\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)^3\left(7c^2x^2+2\right)dx}{63c^3\sqrt{1-c^2x^2}}+\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\arccos(cx))}{9c^4d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^4d}$$

$$\Big\downarrow \text{ 290}$$

$$-\frac{bd^2\sqrt{d-c^2dx^2}\int \left(-7c^8x^8+19c^6x^6-15c^4x^4+c^2x^2+2\right)dx}{63c^3\sqrt{1-c^2x^2}}+\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\arccos(cx))}{9c^4d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^4d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(d-c^2dx^2\right)^{9/2}(a+b\arccos(cx))}{9c^4d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^4d}-\frac{bd^2\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}+2x\right)\sqrt{d-c^2dx^2}}{63c^3\sqrt{1-c^2x^2}}$$

input
```
Int[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/63*(b*d^2*Sqrt[d - c^2*d*x^2]*(2*x + (c^2*x^3)/3 - 3*c^4*x^5 + (19*c^6*
x^7)/7 - (7*c^8*x^9)/9))/(c^3*Sqrt[1 - c^2*x^2]) - ((d - c^2*d*x^2)^(7/2)*
(a + b*ArcCos[c*x]))/(7*c^4*d) + ((d - c^2*d*x^2)^(9/2)*(a + b*ArcCos[c*x]
))/(9*c^4*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.86

method	result
orering	$\frac{(833c^{10}x^{10}-3153c^8x^8+4167c^6x^6-1743c^4x^4-1008c^2x^2+504)\left(-c^2dx^2+d\right)^{\frac{5}{2}}(a+b\arccos(cx))}{3969c^4(cx-1)^2(cx+1)^2(c^2x^2-1)} - \frac{(49c^8x^8-171c^6x^6+189c^4x^4-2}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3969*(833*c^10*x^10-3153*c^8*x^8+4167*c^6*x^6-1743*c^4*x^4-1008*c^2*x^2+
504)/c^4/(c*x-1)^2/(c*x+1)^2/(c^2*x^2-1)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(
c*x))-1/3969/x^2*(49*c^8*x^8-171*c^6*x^6+189*c^4*x^4-21*c^2*x^2-126)/c^4/(
c*x-1)^2/(c*x+1)^2*(3*x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))-5*x^4*(-c
^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))*d*c^2-x^3*(-c^2*d*x^2+d)^(5/2)*b*c/(-c
^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.92

$$\int x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx =$$

$$-\frac{(49\,bc^9d^2x^9-171\,bc^7d^2x^7+189\,bc^5d^2x^5-21\,bc^3d^2x^3-126\,bcd^2x)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}-63\,(7\,ac^{10}}{}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
-1/3969*((49*b*c^9*d^2*x^9 - 171*b*c^7*d^2*x^7 + 189*b*c^5*d^2*x^5 - 21*b*
c^3*d^2*x^3 - 126*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 63*
(7*a*c^10*d^2*x^10 - 26*a*c^8*d^2*x^8 + 34*a*c^6*d^2*x^6 - 16*a*c^4*d^2*x^
4 - a*c^2*d^2*x^2 + 2*a*d^2 + (7*b*c^10*d^2*x^10 - 26*b*c^8*d^2*x^8 + 34*b
*c^6*d^2*x^6 - 16*b*c^4*d^2*x^4 - b*c^2*d^2*x^2 + 2*b*d^2)*arccos(c*x))*sq
rt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 (d-c^2dx^2)^{5/2} (a+b\arccos(cx))\,dx = \text{Timed out}$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.58

$$\int x^3 (d-c^2dx^2)^{5/2} (a+b\arccos(cx))\,dx = -\frac{1}{63}\left(\frac{7\,(-c^2dx^2+d)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{7}{2}}}{c^4d}\right)b\arccos(cx) - \frac{1}{63}\left(\frac{7\,(-c^2dx^2+d)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{7}{2}}}{c^4d}\right)a + \frac{\left(49\,c^8d^{\frac{5}{2}}x^9-171\,c^6d^{\frac{5}{2}}x^7+189\,c^4d^{\frac{5}{2}}x^5-21\,c^2d^{\frac{5}{2}}x^3-126\,d^{\frac{5}{2}}x\right)b}{3969\,c^3}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
-1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^
4*d))*b*arccos(c*x) - 1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2
*d*x^2 + d)^(7/2)/(c^4*d))*a + 1/3969*(49*c^8*d^(5/2)*x^9 - 171*c^6*d^(5/2
)*x^7 + 189*c^4*d^(5/2)*x^5 - 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3 (d - c^2 dx^2)^{5/2} (a + b \arccos(cx)) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d - c^2 dx^2)^{5/2} (a + b \arccos(cx)) \, dx = \int x^3 (a + b \operatorname{acos}(c x)) \left(d - c^2 d x^2\right)^{5/2} dx$$

input
```
int(x^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^3 (d-c^2dx^2)^{5/2} (a+b\arccos(cx))\,dx = \frac{\sqrt{d}\,d^2\left(7\sqrt{-c^2x^2+1}\,a\,c^8x^8-19\sqrt{-c^2x^2+1}\,a\,c^6x^6+15\sqrt{-c^2x^2+1}\,a\,c^4x^4-\sqrt{-c^2x^2}\right.}{}$$

input
```
int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d**2*(7*sqrt( - c**2*x**2 + 1)*a*c**8*x**8 - 19*sqrt( - c**2*x**2
 + 1)*a*c**6*x**6 + 15*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 - sqrt( - c**2*x
**2 + 1)*a*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a + 63*int(sqrt( - c**2*x*
*2 + 1)*acos(c*x)*x**7,x)*b*c**8 - 126*int(sqrt( - c**2*x**2 + 1)*acos(c*x
)*x**5,x)*b*c**6 + 63*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**3,x)*b*c**4)
)/(63*c**4)
```

3.97 $\int x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 202

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\frac{bd^2x\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}-\frac{bcd^2x^3\sqrt{d-c^2dx^2}}{7\sqrt{1-c^2x^2}}+\frac{3bc^3d^2x^5\sqrt{d-c^2dx^2}}{35\sqrt{1-c^2x^2}}-\frac{bc^5d^2x^7\sqrt{d-c^2dx^2}}{49\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^2d}$$

output
```
1/7*b*d^2*x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/7*b*c*d^2*x^3*(-c^
2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+3/35*b*c^3*d^2*x^5*(-c^2*d*x^2+d)^(1/2
)/(-c^2*x^2+1)^(1/2)-1/49*b*c^5*d^2*x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^
(1/2)-1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccos(c*x))/c^2/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.56

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\frac{d^2\sqrt{d-c^2dx^2}\Big(35a(-1+c^2x^2)^4+bcx\sqrt{1-c^2x^2}(35-35c^2x^2+21c^4x^4-5c^6x^6)+35b}{245c^2\left(-1+c^2x^2\right)}$$

input
```
Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(35*a*(-1 + c^2*x^2)^4 + b*c*x*Sqrt[1 - c^2*x^2]*
(35 - 35*c^2*x^2 + 21*c^4*x^4 - 5*c^6*x^6) + 35*b*(-1 + c^2*x^2)^4*ArcCos[
c*x]))/(245*c^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.51, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5183, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5183}$$

$$-\frac{bd^2\sqrt{d-c^2dx^2}\int\left(1-c^2x^2\right)^3dx}{7c\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^2d}$$

$$\Big\downarrow \text{ 210}$$

$$-\frac{bd^2\sqrt{d-c^2dx^2}\int\left(-c^6x^6+3c^4x^4-3c^2x^2+1\right)dx}{7c\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^2d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))}{7c^2d}-\frac{bd^2\left(-\frac{1}{7}c^6x^7+\frac{3c^4x^5}{5}-c^2x^3+x\right)\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}$$

input
```
Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/7*(b*d^2*Sqrt[d - c^2*d*x^2]*(x - c^2*x^3 + (3*c^4*x^5)/5 - (c^6*x^7)/7
))/(c*Sqrt[1 - c^2*x^2]) - ((d - c^2*d*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(7*
c^2*d)
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.06

method	result
orering	$\frac{(65c^8x^8-271c^6x^6+441c^4x^4-385c^2x^2+70)(-c^2dx^2+d)^{\frac{5}{2}}(a+b\arccos(cx))}{245c^2(cx-1)^2(cx+1)^2(c^2x^2-1)} - \frac{(5c^6x^6-21c^4x^4+35c^2x^2-35)\left((-c^2dx^2+d)^{\frac{5}{2}}(a\cdots\right.}{}$
default	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{7c^2d} + b\left(\frac{\sqrt{-d(c^2x^2-1)}\left(64c^8x^8-144c^6x^6+64i\sqrt{-c^2x^2+1}\,x^7c^7+104c^4x^4-112i\sqrt{-c^2x^2+1}\,x^5c^5-25c^2x^2+56i\cdots\right.}{6272c^2(c^2x^2-1)}\right.$
parts	$-\frac{a(-c^2dx^2+d)^{\frac{7}{2}}}{7c^2d} + b\left(\frac{\sqrt{-d(c^2x^2-1)}\left(64c^8x^8-144c^6x^6+64i\sqrt{-c^2x^2+1}\,x^7c^7+104c^4x^4-112i\sqrt{-c^2x^2+1}\,x^5c^5-25c^2x^2+56i\cdots\right.}{6272c^2(c^2x^2-1)}\right.$

input
```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/245*(65*c^8*x^8-271*c^6*x^6+441*c^4*x^4-385*c^2*x^2+70)/c^2/(c*x-1)^2/(c
*x+1)^2/(c^2*x^2-1)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))-1/245*(5*c^6*x^
6-21*c^4*x^4+35*c^2*x^2-35)/c^2/(c*x-1)^2/(c*x+1)^2*((-c^2*d*x^2+d)^(5/2)*
(a+b*arccos(c*x))-5*x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))*d*c^2-x*(-c
^2*d*x^2+d)^(5/2)*b*c/(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.06

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx =$$
$$-\frac{(5\,bc^7d^2x^7 - 21\,bc^5d^2x^5 + 35\,bc^3d^2x^3 - 35\,bcd^2x)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1} - 35\,(ac^8d^2x^8 - 4\,ac^6d^2x^6 + \cdots}{245\,(c\cdots}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-1/245*((5*b*c^7*d^2*x^7 - 21*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3 - 35*b*c*d^
2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 35*(a*c^8*d^2*x^8 - 4*a*c^6
*d^2*x^6 + 6*a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2 + (b*c^8*d^2*x^8 - 4*
b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*arccos(c*x))*sq
rt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx = \text{Timed out}$$

input
```
integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.49

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx = -\frac{\left(-c^2dx^2+d\right)^{\frac{7}{2}}b\arccos\left(cx\right)}{7\,c^2d} - \frac{\left(-c^2dx^2+d\right)^{\frac{7}{2}}a}{7\,c^2d} + \frac{\left(5\,c^6d^{\frac{7}{2}}x^7 - 21\,c^4d^{\frac{7}{2}}x^5 + 35\,c^2d^{\frac{7}{2}}x^3 - 35\,d^{\frac{7}{2}}x\right)b}{245\,cd}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-1/7*(-c^2*d*x^2 + d)^(7/2)*b*arccos(c*x)/(c^2*d) - 1/7*(-c^2*d*x^2 + d)^(
7/2)*a/(c^2*d) + 1/245*(5*c^6*d^(7/2)*x^7 - 21*c^4*d^(7/2)*x^5 + 35*c^2*d^
(7/2)*x^3 - 35*d^(7/2)*x)*b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\int x\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int(x*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))\,dx = \frac{\sqrt{d}\,d^2\left(\sqrt{-c^2x^2+1}\,a\,c^6x^6-3\sqrt{-c^2x^2+1}\,a\,c^4x^4+3\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\right.}{}$$

input
```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d)*d**2*(sqrt( - c**2*x**2 + 1)*a*c**6*x**6 - 3*sqrt( - c**2*x**2 +
1)*a*c**4*x**4 + 3*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x**2
+ 1)*a + 7*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**5,x)*b*c**6 - 14*int(sq
rt( - c**2*x**2 + 1)*acos(c*x)*x**3,x)*b*c**4 + 7*int(sqrt( - c**2*x**2 +
1)*acos(c*x)*x,x)*b*c**2))/(7*c**2)
```

3.98 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 361

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x}\,dx = -\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{1-c^2x^2}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{1-c^2x^2}} - \frac{bc^5d^2x^5\sqrt{d-c^2dx^2}}{25\sqrt{1-c^2x^2}} + d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx)) + \frac{1}{3}d(d-c^2dx^2)^{3/2}(a+b\arccos(cx)) + \frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\arccos(cx)) - \frac{2d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} + \frac{ibd^2\sqrt{d-c^2dx}{}$$

output

```
-23/15*b*c*d^2*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+11/45*b*c^3*d^2*x
^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/25*b*c^5*d^2*x^5*(-c^2*d*x^2+
d)^(1/2)/(-c^2*x^2+1)^(1/2)+d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))+1/3
*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))+1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*ar
ccos(c*x))-2*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*arctanh(c*x+I*(-c^
2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+I*b*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,
-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-I*b*d^2*(-c^2*d*x^2+d)^(1/2)
*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.09

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x}\,dx = \frac{1}{15}ad^2\sqrt{d-c^2dx^2}(23-11c^2x^2+3c^4x^4) - \frac{bd^2\sqrt{d-c^2dx^2}\left(-9cx-12(1-c^2x^2)^{3/2}\arccos(cx)+\cos(3\arccos(cx))\right)}{18\sqrt{1-c^2x^2}} + ad^{5/2}\log(x) - ad^{5/2}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right) + \frac{bd^2\sqrt{d-c^2dx^2}(cx+\sqrt{1-c^2x^2}\arccos(cx)-\arccos(cx)\,\text{lo}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x,x]
```

output
```
(a*d^2*Sqrt[d - c^2*d*x^2]*(23 - 11*c^2*x^2 + 3*c^4*x^4))/15 - (b*d^2*Sqrt
[d - c^2*d*x^2]*(-9*c*x - 12*(1 - c^2*x^2)^(3/2)*ArcCos[c*x] + Cos[3*ArcCo
s[c*x]]))/(18*Sqrt[1 - c^2*x^2]) + a*d^(5/2)*Log[x] - a*d^(5/2)*Log[d + Sq
rt[d]*Sqrt[d - c^2*d*x^2]] + (b*d^2*Sqrt[d - c^2*d*x^2]*(c*x + Sqrt[1 - c^
2*x^2]*ArcCos[c*x] - ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] + ArcCos[c*x
]*Log[1 + I*E^(I*ArcCos[c*x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I*
PolyLog[2, I*E^(I*ArcCos[c*x])]))/Sqrt[1 - c^2*x^2] + (b*d^2*Sqrt[d - c^2*
d*x^2]*(16*c*x*(-30 - 5*c^2*x^2 + 9*c^4*x^4) + 15*ArcCos[c*x]*(-30*Sqrt[1
- c^2*x^2] + 5*Sin[3*ArcCos[c*x]] + 3*Sin[5*ArcCos[c*x]])))/(3600*Sqrt[1 -
 c^2*x^2])
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.89, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5203, 210, 2009, 5203, 2009, 5199, 24, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x}dx$$

$$\Big\downarrow \text{ 5203}$$

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)^2dx}{5\sqrt{1-c^2x^2}}+\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))$$

$\big\downarrow$ 210

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x}dx+\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(c^4x^4-2c^2x^2+1\right)dx}{5\sqrt{1-c^2x^2}}+\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))$$

$\big\downarrow$ 2009

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x}dx+\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 5203

$$d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)dx}{3\sqrt{1-c^2x^2}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\right)+\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 2009

$$d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}}\right)+\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 5199

$$d\left(d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int 1dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)+\frac{1}{3}\left(d-c^2dx^2\right)^{3/}\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 24

$$d\left(d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}\right)+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 5219

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}\right)+\frac{1}{3}\left(d-c^2\right.\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 3042

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bc}{}\right.\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 4669

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(-b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(\right.\right.}{\sqrt{1-c^2x^2}}\right.\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 2715

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de\right.}{\sqrt{1-c^2x^2}}\right.\right.$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 2838

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\operatorname{PolyLog}\left(2,i\right.\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))+\frac{bcd^2\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x,x]
```

output
```
(b*c*d^2*Sqrt[d - c^2*d*x^2]*(x - (2*c^2*x^3)/3 + (c^4*x^5)/5))/(5*Sqrt[1
- c^2*x^2]) + ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/5 + d*((b*c*d*Sq
rt[d - c^2*d*x^2]*(x - (c^2*x^3)/3))/(3*Sqrt[1 - c^2*x^2]) + ((d - c^2*d*x
^2)^(3/2)*(a + b*ArcCos[c*x]))/3 + d*((b*c*x*Sqrt[d - c^2*d*x^2])/Sqrt[1 -
 c^2*x^2] + Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]) - (Sqrt[d - c^2*d*x^2]
*((-2*I)*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, (-
I)*E^(I*ArcCos[c*x])] - I*b*PolyLog[2, I*E^(I*ArcCos[c*x])]))/Sqrt[1 - c^2
*x^2]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.85

method	result
default	$\frac{(-c^2 d x^2+d)^{\frac{5}{2}} a}{5}+\frac{a d(-c^2 d x^2+d)^{\frac{3}{2}}}{3}-a\, d^{\frac{5}{2}} \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\, x^2+d}}{x}\right)+a\, d^2\sqrt{-c^2 d\, x^2+d}-\frac{ib\sqrt{-d(c^2x^2-1)}}{}$
parts	$\frac{(-c^2 d x^2+d)^{\frac{5}{2}} a}{5}+\frac{a d(-c^2 d x^2+d)^{\frac{3}{2}}}{3}-a\, d^{\frac{5}{2}} \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\, x^2+d}}{x}\right)+a\, d^2\sqrt{-c^2 d\, x^2+d}-\frac{ib\sqrt{-d(c^2x^2-1)}}{}$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/5*(-c^2*d*x^2+d)^(5/2)*a+1/3*a*d*(-c^2*d*x^2+d)^(3/2)-a*d^(5/2)*ln((2*d+
2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+a*d^2*(-c^2*d*x^2+d)^(1/2)-I*b*(-d*(c^2
*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*d^2*dilog(1-I*(c*x+I*(-c^2*x
^2+1)^(1/2)))-23/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)*arccos(c*x)+1
/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)*arccos(c*x)*x^6*c^6-14/15*b*(-
d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)*arccos(c*x)*x^4*c^4+34/15*b*(-d*(c^2*
x^2-1))^(1/2)*d^2/(c^2*x^2-1)*arccos(c*x)*x^2*c^2-b*(-d*(c^2*x^2-1))^(1/2)
*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*d^2*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)
^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*d^2*arcco
s(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2
*x^2+1)^(1/2)/(c^2*x^2-1)*d^2*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-1/25*b
*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^5*c^5+11/45*b
*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x^3*c^3-23/15*b
*(-d*(c^2*x^2-1))^(1/2)*d^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x*c
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}{x}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x}\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x,x, algorithm="maxima")
```

output
```
b*sqrt(d)*integrate((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x, x) - 1/15*(15*d^(
5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - 3*(-c^2*d*x
^2 + d)^(5/2) - 5*(-c^2*d*x^2 + d)^(3/2)*d - 15*sqrt(-c^2*d*x^2 + d)*d^2)*
a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^{5/2}}{x}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x}\,dx = \frac{\sqrt{d}\,d^2\Big(3\sqrt{-c^2x^2+1}\,a\,c^4x^4 - 11\sqrt{-c^2x^2+1}\,a\,c^2x^2 + 23\sqrt{-c^2x^2}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))/x,x)
```

output
```
(sqrt(d)*d**2*(3*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 - 11*sqrt( - c**2*x**2
 + 1)*a*c**2*x**2 + 23*sqrt( - c**2*x**2 + 1)*a + 15*int((sqrt( - c**2*x**
2 + 1)*acos(c*x))/x,x)*b + 15*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**3,x)
*b*c**4 - 30*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x,x)*b*c**2 + 15*log(tan
(asin(c*x)/2))*a - 23*a))/15
```

3.99 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 386

$$\begin{aligned}
\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^3}\,dx &= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{1-c^2x^2}} \\
&+ \frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}} - \frac{bc^5d^2x^3\sqrt{d-c^2dx^2}}{9\sqrt{1-c^2x^2}} - \frac{5}{2}c^2d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx)) \\
&- \frac{5}{6}c^2d\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx)) - \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{2x^2} \\
&+ \frac{5c^2d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&- \frac{5ibc^2d^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{2\sqrt{1-c^2x^2}} \\
&+ \frac{5ibc^2d^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2\sqrt{1-c^2x^2}}
\end{aligned}$$

output
```
-1/2*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x/(-c^2*x^2+1)^(1/2)+7/3*b*c^3*d^2*x*(-c
^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/9*b*c^5*d^2*x^3*(-c^2*d*x^2+d)^(1/2
)/(-c^2*x^2+1)^(1/2)-5/2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))-5/
6*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))-1/2*(-c^2*d*x^2+d)^(5/2)*(a
+b*arccos(c*x))/x^2+5*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*arcta
nh(c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-5/2*I*b*c^2*d^2*(-c^2*d*x^
2+d)^(1/2)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+5/2*I*b
*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^
2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.20

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^3}\,dx =$$

$$-\frac{9ad^3+33ac^2d^3x^2-48ac^4d^3x^4+6ac^6d^3x^6-9bcd^3x\sqrt{1-c^2x^2}+42bc^3d^3x^3\sqrt{1-c^2x^2}-2bc^5d^3x^5\sqrt{1-c}}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
-1/18*(9*a*d^3 + 33*a*c^2*d^3*x^2 - 48*a*c^4*d^3*x^4 + 6*a*c^6*d^3*x^6 - 9
*b*c*d^3*x*Sqrt[1 - c^2*x^2] + 42*b*c^3*d^3*x^3*Sqrt[1 - c^2*x^2] - 2*b*c^
5*d^3*x^5*Sqrt[1 - c^2*x^2] + 9*b*d^3*ArcCos[c*x] + 33*b*c^2*d^3*x^2*ArcCo
s[c*x] - 48*b*c^4*d^3*x^4*ArcCos[c*x] + 6*b*c^6*d^3*x^6*ArcCos[c*x] - 45*b
*c^2*d^3*x^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] +
45*b*c^2*d^3*x^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])
] + 45*a*c^2*d^(5/2)*x^2*Sqrt[d - c^2*d*x^2]*Log[x] - 45*a*c^2*d^(5/2)*x^2
*Sqrt[d - c^2*d*x^2]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (45*I)*b*c^2*d
^3*x^2*Sqrt[1 - c^2*x^2]*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + (45*I)*b*c^2
*d^3*x^2*Sqrt[1 - c^2*x^2]*PolyLog[2, I*E^(I*ArcCos[c*x])])/(x^2*Sqrt[d -
c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.86, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5201, 244, 2009, 5203, 2009, 5199, 24, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{x^3}dx$$

$$\Big\downarrow \text{5201}$$

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2}{x^2}dx}{2\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}$$

$$\Big\downarrow \text{244}$$

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(x^2c^4-2c^2+\frac{1}{x^2}\right)dx}{2\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5203}$$

$$-\frac{5}{2}c^2d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}dx+\frac{bcd\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)dx}{3\sqrt{1-c^2x^2}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5}{2}c^2d\left(d\int\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))+\frac{bcd\left(x-\frac{c^2x^3}{3}\right)\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$\downarrow$ 5199

$$-\frac{5}{2}c^2d\left(d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int 1dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)+\frac{1}{3}(d-c^2\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$\downarrow$ 24

$$-\frac{5}{2}c^2d\left(d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}\right)+\frac{1}{3}\left(d-c^2dx^2\right)^3\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$\downarrow$ 5219

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{cx}d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}\right)+\frac{1}{3}(\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$\downarrow$ 3042

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int(a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right.\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$\downarrow$ 4669

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(-b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\,\mathrm{ar}\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$\downarrow$ 2715

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(\right.\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

$\downarrow$ 2838

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\mathrm{PolyLo}\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d^2*Sqrt[d - c^2*d*x^2]*(-x^(-1) - 2*c^2*x + (c^4*x^3)/3))/Sqrt[
1 - c^2*x^2] - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(2*x^2) - (5*c^
2*d*((b*c*d*Sqrt[d - c^2*d*x^2]*(x - (c^2*x^3)/3))/(3*Sqrt[1 - c^2*x^2]) +
 ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/3 + d*((b*c*x*Sqrt[d - c^2*d*
x^2])/Sqrt[1 - c^2*x^2] + Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]) - (Sqrt[
d - c^2*d*x^2]*((-2*I)*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcCos[c*x])] + I*b
*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*b*PolyLog[2, I*E^(I*ArcCos[c*x])])
)/Sqrt[1 - c^2*x^2])))/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.74

method	result
default	$a\left(-\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{2 d\,x^{2}}-\frac{5 c^{2}\left(\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{5}+d\left(\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{-c^{2} d\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2 d+2\sqrt{d}\,\sqrt{-c^{2} d\,x^{2}+d}}{x}\right)\right)\right)\right)}{2}\right)+$
parts	$a\left(-\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{7}{2}}}{2 d\,x^{2}}-\frac{5 c^{2}\left(\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}}}{5}+d\left(\frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}}}{3}+d\left(\sqrt{-c^{2} d\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2 d+2\sqrt{d}\,\sqrt{-c^{2} d\,x^{2}+d}}{x}\right)\right)\right)\right)}{2}\right)+$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/d/x^2*(-c^2*d*x^2+d)^(7/2)-5/2*c^2*(1/5*(-c^2*d*x^2+d)^(5/2)+d*(1/
3*(-c^2*d*x^2+d)^(3/2)+d*((-c^2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(
-c^2*d*x^2+d)^(1/2))/x)))))+b*(1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^
2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-3*I*(-c^2*x^2+1)^(1/2)*c*x+1)*(I+3*ar
ccos(c*x))*c^2*d^2/(c^2*x^2-1)-9/8*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^
(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)+I)*c^2*d^2/(c^2*x^2-1)-9/8*(-d*(c^2*x^2-
1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)-I)*c^2*d^2/(c
^2*x^2-1)+1/72*(-d*(c^2*x^2-1))^(1/2)*(-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+4*c
^4*x^4+3*I*(-c^2*x^2+1)^(1/2)*c*x-5*c^2*x^2+1)*(-I+3*arccos(c*x))*c^2*d^2/
(c^2*x^2-1)-1/2*d^2*(c^2*x^2*arccos(c*x)+c*x*(-c^2*x^2+1)^(1/2)-arccos(c*x
))*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/x^2+5/2*(-d*(c^2*x^2-1))^(1/2)*(-c^2
*x^2+1)^(1/2)/(c^2*x^2-1)*(arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-
arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-I*dilog(1+I*(c*x+I*(-c^2*x^
2+1)^(1/2)))+I*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2))))*c^2*d^2)
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^3}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^3,x, algorithm="fricas"
)
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^3}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^3,x, algorithm="maxima"
)
```

output
```
b*sqrt(d)*integrate((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x^3, x) + 1/6*(15*c^
2*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - 3*(-c^
2*d*x^2 + d)^(5/2)*c^2 - 5*(-c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqrt(-c^2*d*x
^2 + d)*c^2*d^2 - 3*(-c^2*d*x^2 + d)^(7/2)/(d*x^2))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^{5/2}}{x^3}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^3,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^3}\,dx = \frac{\sqrt{d}\,d^2\Big(8\sqrt{-c^2x^2+1}\,a\,c^4x^4-56\sqrt{-c^2x^2+1}\,a\,c^2x^2-12\sqrt{-c^2x^2}}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))/x^3,x)
```

output
```
(sqrt(d)*d**2*(8*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 - 56*sqrt( - c**2*x**2
 + 1)*a*c**2*x**2 - 12*sqrt( - c**2*x**2 + 1)*a + 24*int((sqrt( - c**2*x**
2 + 1)*acos(c*x))/x**3,x)*b*x**2 - 48*int((sqrt( - c**2*x**2 + 1)*acos(c*x
))/x,x)*b*c**2*x**2 + 24*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x,x)*b*c**4*
x**2 - 60*log(tan(asin(c*x)/2))*a*c**2*x**2 + 65*a*c**2*x**2))/(24*x**2)
```

3.100 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^5}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 389

$$\begin{aligned}
\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^5}\,dx &= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{1-c^2x^2}} \\
&+\frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{1-c^2x^2}}-\frac{bc^5d^2x\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}+\frac{15}{8}c^4d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx)) \\
&+\frac{5c^2d(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{8x^2}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{4x^4} \\
&-\frac{15c^4d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{4\sqrt{1-c^2x^2}} \\
&+\frac{15ibc^4d^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{8\sqrt{1-c^2x^2}} \\
&-\frac{15ibc^4d^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{8\sqrt{1-c^2x^2}}
\end{aligned}$$

output
```
-1/12*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^3/(-c^2*x^2+1)^(1/2)+9/8*b*c^3*d^2*(-
c^2*d*x^2+d)^(1/2)/x/(-c^2*x^2+1)^(1/2)-b*c^5*d^2*x*(-c^2*d*x^2+d)^(1/2)/(
-c^2*x^2+1)^(1/2)+15/8*c^4*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))+5/8*
c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/x^2-1/4*(-c^2*d*x^2+d)^(5/2)*
(a+b*arccos(c*x))/x^4-15/4*c^4*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*
arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+15/8*I*b*c^4*d^2*(-c^
2*d*x^2+d)^(1/2)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-1
5/8*I*b*c^4*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/(
-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.58

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^5}\,dx =$$

$$-\frac{-2bcd^3x+29bc^3d^3x^3-51bc^5d^3x^5+24bc^7d^3x^7+6ad^3\sqrt{1-c^2x^2}-33ac^2d^3x^2\sqrt{1-c^2x^2}+3ac^4d^3x^4\sqrt{1-}}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^5,x]
```

output
```
-1/24*(-2*b*c*d^3*x + 29*b*c^3*d^3*x^3 - 51*b*c^5*d^3*x^5 + 24*b*c^7*d^3*x
^7 + 6*a*d^3*Sqrt[1 - c^2*x^2] - 33*a*c^2*d^3*x^2*Sqrt[1 - c^2*x^2] + 3*a*
c^4*d^3*x^4*Sqrt[1 - c^2*x^2] + 24*a*c^6*d^3*x^6*Sqrt[1 - c^2*x^2] + 6*b*d
^3*Sqrt[1 - c^2*x^2]*ArcCos[c*x] - 33*b*c^2*d^3*x^2*Sqrt[1 - c^2*x^2]*ArcC
os[c*x] + 3*b*c^4*d^3*x^4*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + 24*b*c^6*d^3*x^6
*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + 45*b*c^4*d^3*x^4*ArcCos[c*x]*Log[1 - I*E^
(I*ArcCos[c*x])] - 45*b*c^6*d^3*x^6*ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x]
)] - 45*b*c^4*d^3*x^4*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] + 45*b*c^6*
d^3*x^6*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] - 45*a*c^4*d^(5/2)*x^4*Sq
rt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*Log[x] + 45*a*c^4*d^(5/2)*x^4*Sqrt[1 -
 c^2*x^2]*Sqrt[d - c^2*d*x^2]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (45*I
)*b*c^4*d^3*x^4*(-1 + c^2*x^2)*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + (45*I)
*b*c^4*d^3*x^4*(-1 + c^2*x^2)*PolyLog[2, I*E^(I*ArcCos[c*x])])/(x^4*Sqrt[1
 - c^2*x^2]*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.87, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {5201, 244, 2009, 5201, 244, 2009, 5199, 24, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^5}dx$$

$$\Big\downarrow \text{5201}$$

$$-\frac{5}{4}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^3}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2}{x^4}dx}{4\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{4x^4}$$

$$\Big\downarrow \text{244}$$

$$-\frac{5}{4}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^3}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\int \left(c^4-\frac{2c^2}{x^2}+\frac{1}{x^4}\right)dx}{4\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5}{4}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{x^3}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5201}$$

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\int \frac{1-c^2x^2}{x^2}dx}{2\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos\right.$$
$$\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{244}$$

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\int\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\int\left(\frac{1}{x^2}-c^2\right)dx}{2\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$\downarrow$ 2009

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\int\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}-\frac{bcd\left(c^2(-x)-\frac{1}{x}\right)\sqrt{d-c^2dx^2}}{2\sqrt{1-c^2x^2}}\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$\downarrow$ 5199

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int 1dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))\right)-\frac{(d-\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$\downarrow$ 24

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}\right)-\frac{(d-c^2d\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$\downarrow$ 5219

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{cx}d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}\right)\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$\downarrow$ 3042

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int(a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(-b\int\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\right.\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{5}{4}c^2d\left(-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\text{Po}\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{4x^4}-\frac{bcd^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x^5,x]
```

output
```
-1/4*(b*c*d^2*(-1/3*1/x^3 + (2*c^2)/x + c^4*x)*Sqrt[d - c^2*d*x^2])/Sqrt[1
 - c^2*x^2] - ((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(4*x^4) - (5*c^2
*d*(-1/2*(b*c*d*(-x^(-1) - c^2*x)*Sqrt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2] -
 ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(2*x^2) - (3*c^2*d*((b*c*x*Sq
rt[d - c^2*d*x^2])/Sqrt[1 - c^2*x^2] + Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c
*x]) - (Sqrt[d - c^2*d*x^2]*((-2*I)*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcCos
[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*b*PolyLog[2, I*E^(I*A
rcCos[c*x])]))/Sqrt[1 - c^2*x^2]))/2))/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.40

method	result
default	$-\frac{a\left(-c^{2} d \,x^{2}+d \right)^{\frac{7}{2}}}{4 d \,x^{4}}+\frac{3 a \,c^{2} \left(-c^{2} d \,x^{2}+d \right)^{\frac{7}{2}}}{8 d \,x^{2}}+\frac{3 a \,c^{4} \left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{8}+\frac{5 a \,c^{4} d \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}{8}-\frac{15 a \,c^{4} d^{\frac{5}{2}} \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{-c^{2} d \,x^{2}}}{x}\right.}{8}$
parts	$-\frac{a\left(-c^{2} d \,x^{2}+d \right)^{\frac{7}{2}}}{4 d \,x^{4}}+\frac{3 a \,c^{2} \left(-c^{2} d \,x^{2}+d \right)^{\frac{7}{2}}}{8 d \,x^{2}}+\frac{3 a \,c^{4} \left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}}}{8}+\frac{5 a \,c^{4} d \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}{8}-\frac{15 a \,c^{4} d^{\frac{5}{2}} \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{-c^{2} d \,x^{2}}}{x}\right.}{8}$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/d/x^4*(-c^2*d*x^2+d)^(7/2)+3/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(7/2)+3/8
*a*c^4*(-c^2*d*x^2+d)^(5/2)+5/8*a*c^4*d*(-c^2*d*x^2+d)^(3/2)-15/8*a*c^4*d^
(5/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+15/8*a*c^4*d^2*(-c^2*d*x^
2+d)^(1/2)+b*(1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2
-1)*(arccos(c*x)+I)*c^4*d^2/(c^2*x^2-1)+1/2*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c
^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)-I)*c^4*d^2/(c^2*x^2-1)+1/24*d^
2*(27*c^4*x^4*arccos(c*x)+27*c^3*x^3*(-c^2*x^2+1)^(1/2)-33*c^2*x^2*arccos(
c*x)-2*c*x*(-c^2*x^2+1)^(1/2)+6*arccos(c*x))*(-d*(c^2*x^2-1))^(1/2)/(c^2*x
^2-1)/x^4-15/8*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*(arcc
os(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-arccos(c*x)*ln(1-I*(c*x+I*(-c^2
*x^2+1)^(1/2)))-I*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+I*dilog(1-I*(c*x+I
*(-c^2*x^2+1)^(1/2))))*c^4*d^2)
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}{x^5}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{x^5}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^5,x, algorithm="fricas"
)
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/x^5, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^5}\,dx=\int \frac{(-d(cx-1)(cx+1))^{\frac{5}{2}}(a+b\operatorname{acos}(cx))}{x^5}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))/x**5,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))/x**5, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^5}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{x^5}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^5,x, algorithm="maxima"
)
```

output
```
b*sqrt(d)*integrate((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x^5, x) - 1/8*(15*c^
4*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - 3*(-c^
2*d*x^2 + d)^(5/2)*c^4 - 5*(-c^2*d*x^2 + d)^(3/2)*c^4*d - 15*sqrt(-c^2*d*x
^2 + d)*c^4*d^2 - 3*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^2) + 2*(-c^2*d*x^2 + d
)^(7/2)/(d*x^4))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^5}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^5}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^5,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/x^5, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{x^5}\,dx = \frac{\sqrt{d}\,d^2\Big(8\sqrt{-c^2x^2+1}\,a\,c^4x^4+9\sqrt{-c^2x^2+1}\,a\,c^2x^2-2\sqrt{-c^2x^2+}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))/x^5,x)
```

output
```
(sqrt(d)*d**2*(8*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 + 9*sqrt( - c**2*x**2
+ 1)*a*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a + 8*int((sqrt( - c**2*x**2 +
 1)*acos(c*x))/x**5,x)*b*x**4 - 16*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/
x**3,x)*b*c**2*x**4 + 8*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x,x)*b*c**4
*x**4 + 15*log(tan(asin(c*x)/2))*a*c**4*x**4 - 10*a*c**4*x**4))/(8*x**4)
```

3.101 $\int \sqrt{1-x^2} \arccos(x) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \sqrt{1-x^2} \arccos(x) \, dx = -\frac{x^2}{4} + \frac{1}{2} x \sqrt{1-x^2} \arccos(x) + \frac{\arccos(x)^2}{4}$$

output
```
-1/4*x^2+1/2*x*(-x^2+1)^(1/2)*arccos(x)+1/4*arccos(x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2} \arccos(x) \, dx = \frac{1}{4} \left(x^2 + 2x\sqrt{1-x^2} \arccos(x) - \arccos(x)^2\right)$$

input
```
Integrate[Sqrt[1 - x^2]*ArcCos[x],x]
```

output
```
(x^2 + 2*x*Sqrt[1 - x^2]*ArcCos[x] - ArcCos[x]^2)/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x^2}\arccos(x)\,dx$$

$$\Big\downarrow \text{5157}$$

$$\frac{1}{2}\int \frac{\arccos(x)}{\sqrt{1-x^2}}dx+\frac{\int xdx}{2}+\frac{1}{2}x\sqrt{1-x^2}\arccos(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}\int \frac{\arccos(x)}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x\arccos(x)+\frac{x^2}{4}$$

$$\Big\downarrow \text{5153}$$

$$\frac{1}{2}\sqrt{1-x^2}x\arccos(x)-\frac{\arccos(x)^2}{4}+\frac{x^2}{4}$$

input
```
Int[Sqrt[1 - x^2]*ArcCos[x],x]
```

output
```
x^2/4 + (x*Sqrt[1 - x^2]*ArcCos[x])/2 - ArcCos[x]^2/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\arccos(x)\left(-x\sqrt{-x^{2}+1}+\arccos(x)\right)}{2}+\frac{\arccos(x)^{2}}{4}+\frac{x^{2}}{4}-\frac{1}{4}$	33

input
```
int((-x^2+1)^(1/2)*arccos(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccos(x)*(-x*(-x^2+1)^(1/2)+arccos(x))+1/4*arccos(x)^2+1/4*x^2-1/4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \sqrt{1-x^2}\arccos(x)\,dx=\frac{1}{2}\sqrt{-x^2+1}x\arccos{(x)}+\frac{1}{4}x^2-\frac{1}{4}\arccos{(x)}^2$$

input
```
integrate((-x^2+1)^(1/2)*arccos(x),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{x^2}{4} + \left(\frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2}\right)\operatorname{acos}(x) + \frac{\operatorname{asin}^2(x)}{4}$$

input
```
integrate((-x**2+1)**(1/2)*acos(x),x)
```

output
```
x**2/4 + (x*sqrt(1 - x**2)/2 + asin(x)/2)*acos(x) + asin(x)**2/4
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{4}x^2 + \frac{1}{2}\left(\sqrt{-x^2+1}x + \arcsin(x)\right)\arccos(x) + \frac{1}{4}\arcsin(x)^2$$

input
```
integrate((-x^2+1)^(1/2)*arccos(x),x, algorithm="maxima")
```

output
```
1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arccos(x) + 1/4*arcsin(x)^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{2}\sqrt{-x^2+1}x\arccos(x) + \frac{1}{4}x^2 - \frac{1}{4}\arccos(x)^2 - \frac{1}{8}$$

input
```
integrate((-x^2+1)^(1/2)*arccos(x),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2 - 1/8
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \int \mathrm{acos}(x)\,\sqrt{1-x^2}\,dx$$

input
```
int(acos(x)*(1 - x^2)^(1/2),x)
```

output
```
int(acos(x)*(1 - x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \int \sqrt{-x^2+1}\,acos(x)\,dx$$

input
```
int((-x^2+1)^(1/2)*acos(x),x)
```

output
```
int(sqrt( - x**2 + 1)*acos(x),x)
```

3.102 $\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))\,dx = -\frac{1}{4}bc\sqrt{\pi}x^2 + \frac{1}{2}x\sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx)) + \frac{\sqrt{\pi}(a + b\arccos(cx))^2}{4bc}$$

output
```
-1/4*b*c*Pi^(1/2)*x^2+1/2*x*(-Pi*c^2*x^2+Pi)^(1/2)*(a+b*arccos(c*x))+1/4*P
i^(1/2)*(a+b*arccos(c*x))^2/b/c
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))\,dx = \frac{\sqrt{\pi}\left(4acx\sqrt{1 - c^2x^2} - 2b\arccos(cx)^2 + 4a\arcsin(cx) + b\cos(2\arccos(cx)) + 2b\arccos(cx)\sin(2\arccos(\right.}{8c}$$

input
```
Integrate[Sqrt[Pi - c^2*Pi*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(Sqrt[Pi]*(4*a*c*x*Sqrt[1 - c^2*x^2] - 2*b*ArcCos[c*x]^2 + 4*a*ArcSin[c*x]
 + b*Cos[2*ArcCos[c*x]] + 2*b*ArcCos[c*x]*Sin[2*ArcCos[c*x]]))/(8*c)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\pi - \pi c^2 x^2}(a + b \arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5157}$$

$$\frac{1}{2}\sqrt{\pi}\int \frac{a + b\arccos(cx)}{\sqrt{1-c^2x^2}}dx + \frac{1}{2}\sqrt{\pi}bc\int xdx + \frac{1}{2}x\sqrt{\pi - \pi c^2x^2}(a + b\arccos(cx))$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{2}\sqrt{\pi}\int \frac{a + b\arccos(cx)}{\sqrt{1-c^2x^2}}dx + \frac{1}{2}x\sqrt{\pi - \pi c^2x^2}(a + b\arccos(cx)) + \frac{1}{4}\sqrt{\pi}bcx^2$$

$$\Big\downarrow \text{ 5153}$$

$$\frac{1}{2}x\sqrt{\pi - \pi c^2x^2}(a + b\arccos(cx)) - \frac{\sqrt{\pi}(a + b\arccos(cx))^2}{4bc} + \frac{1}{4}\sqrt{\pi}bcx^2$$

input
```
Int[Sqrt[Pi - c^2*Pi*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*Sqrt[Pi]*x^2)/4 + (x*Sqrt[Pi - c^2*Pi*x^2]*(a + b*ArcCos[c*x]))/2 - (
Sqrt[Pi]*(a + b*ArcCos[c*x])^2)/(4*b*c)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.44

method	result	size
default	$\frac{ax\sqrt{-\pi\,c^{2}x^{2}+\pi}}{2}+\frac{a\pi\arctan\left(\frac{\sqrt{\pi\,c^{2}}\,x}{\sqrt{-\pi\,c^{2}x^{2}+\pi}}\right)}{2\sqrt{\pi\,c^{2}}}-\frac{b\sqrt{\pi}\,\left(-2\sqrt{-c^{2}x^{2}+1}\,\arccos\left(cx\right)xc-c^{2}x^{2}+\arccos\left(cx\right)^{2}+1\right)}{4c}$	98
parts	$\frac{ax\sqrt{-\pi\,c^{2}x^{2}+\pi}}{2}+\frac{a\pi\arctan\left(\frac{\sqrt{\pi\,c^{2}}\,x}{\sqrt{-\pi\,c^{2}x^{2}+\pi}}\right)}{2\sqrt{\pi\,c^{2}}}-\frac{b\sqrt{\pi}\,\left(-2\sqrt{-c^{2}x^{2}+1}\,\arccos\left(cx\right)xc-c^{2}x^{2}+\arccos\left(cx\right)^{2}+1\right)}{4c}$	98

input
```
int((-Pi*c^2*x^2+Pi)^(1/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x*(-Pi*c^2*x^2+Pi)^(1/2)+1/2*a*Pi/(Pi*c^2)^(1/2)*arctan((Pi*c^2)^(1/
2)*x/(-Pi*c^2*x^2+Pi)^(1/2))-1/4*b*Pi^(1/2)*(-2*(-c^2*x^2+1)^(1/2)*arccos(
c*x)*x*c-c^2*x^2+arccos(c*x)^2+1)/c
```

Fricas [F]

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))\,dx = \int \sqrt{\pi - \pi c^2x^2}(b\arccos(cx) + a)\,dx$$

input
```
integrate((-pi*c^2*x^2+pi)^(1/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(pi - pi*c^2*x^2)*(b*arccos(c*x) + a), x)
```

Sympy [A] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.43

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))\,dx$$
$$= \begin{cases} \frac{\sqrt{\pi}a\left(\frac{cx\sqrt{-c^2x^2+1}}{2} + \frac{\operatorname{asin}(cx)}{2}\right) + \sqrt{\pi}b\left(\frac{c^2x^2}{4} + \left(\frac{cx\sqrt{-c^2x^2+1}}{2} + \frac{\operatorname{asin}(cx)}{2}\right)\operatorname{acos}(cx) + \frac{\operatorname{asin}^2(cx)}{4}\right)}{c} & \text{for } c \neq 0 \\ \sqrt{\pi}x\left(a + \frac{\pi b}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((-pi*c**2*x**2+pi)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
Piecewise(((sqrt(pi)*a*(c*x*sqrt(-c**2*x**2 + 1)/2 + asin(c*x)/2) + sqrt(p
i)*b*(c**2*x**2/4 + (c*x*sqrt(-c**2*x**2 + 1)/2 + asin(c*x)/2)*acos(c*x) +
 asin(c*x)**2/4))/c, Ne(c, 0)), (sqrt(pi)*x*(a + pi*b/2), True))
```

Maxima [F]

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))\,dx = \int \sqrt{\pi - \pi c^2x^2}(b\arccos(cx) + a)\,dx$$

input
```
integrate((-pi*c^2*x^2+pi)^(1/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
sqrt(pi)*b*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sq
rt(-c*x + 1), c*x), x) + 1/2*(sqrt(pi - pi*c^2*x^2)*x + sqrt(pi)*arcsin(c*
x)/c)*a
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-pi*c^2*x^2+pi)^(1/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))\,dx = \int (a + b\,\mathrm{acos}(c\,x))\,\sqrt{\Pi - \Pi\,c^2\,x^2}\,dx$$

input
```
int((a + b*acos(c*x))*(Pi - Pi*c^2*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))*(Pi - Pi*c^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\pi - c^2\pi x^2}(a + b\arccos(cx))\,dx$$
$$= \frac{\sqrt{\pi}\left(asin(cx)\,a + \sqrt{-c^2x^2+1}\,acx + 2\left(\int \sqrt{-c^2x^2+1}\,acos(cx)\,dx\right)bc\right)}{2c}$$

input
```
int((-Pi*c^2*x^2+Pi)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(pi)*(asin(c*x)*a + sqrt( - c**2*x**2 + 1)*a*c*x + 2*int(sqrt( - c**2
*x**2 + 1)*acos(c*x),x)*b*c))/(2*c)
```

3.103 $\int \frac{x^4 \arccos(ax)}{\sqrt{1-a^2x^2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{x^4 \arccos(ax)}{\sqrt{1-a^2x^2}} \, dx = \frac{3x^2}{16a^3} + \frac{x^4}{16a} - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2} + \frac{3\arccos(ax)^2}{16a^5}$$

output
```
3/16*x^2/a^3+1/16*x^4/a-3/8*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^4-1/4*x^3*(
-a^2*x^2+1)^(1/2)*arccos(a*x)/a^2+3/16*arccos(a*x)^2/a^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.73

$$\int \frac{x^4 \arccos(ax)}{\sqrt{1-a^2x^2}} \, dx = -\frac{a^2x^2(3+a^2x^2) + 2ax\sqrt{1-a^2x^2}(3+2a^2x^2)\arccos(ax) + 3\arccos(ax)^2}{16a^5}$$

input
```
Integrate[(x^4*ArcCos[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/16*(a^2*x^2*(3 + a^2*x^2) + 2*a*x*Sqrt[1 - a^2*x^2]*(3 + 2*a^2*x^2)*Arc
Cos[a*x] + 3*ArcCos[a*x]^2)/a^5
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5211, 15, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arccos(ax)}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{5211}$$

$$\frac{3\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\int x^3dx}{4a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{3\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}$$

$$\Big\downarrow \text{5211}$$

$$\frac{3\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int xdx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}$$

$$\Big\downarrow \text{15}$$

$$\frac{3\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}$$

$$\Big\downarrow \text{5153}$$

$$-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}+\frac{3\left(-\frac{\arccos(ax)^2}{4a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}-\frac{x^4}{16a}$$

input
```
Int[(x^4*ArcCos[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/16*x^4/a - (x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(4*a^2) + (3*(-1/4*x^2/a
 - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a^2) - ArcCos[a*x]^2/(4*a^3)))/(4*
a^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{16\sqrt{-a^{2}x^{2}+1}\, \arccos\left(a x\right) a^{3} x^{3}+4 a^{4} x^{4}+24 \arccos\left(a x\right) \sqrt{-a^{2}x^{2}+1}\, a x+12 a^{2} x^{2}+12 \arccos\left(a x\right)^{2}+9}{64 a^{5}}$	76

input
```
int(x^4*arccos(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64*(16*(-a^2*x^2+1)^(1/2)*arccos(a*x)*a^3*x^3+4*a^4*x^4+24*arccos(a*x)*
(-a^2*x^2+1)^(1/2)*a*x+12*a^2*x^2+12*arccos(a*x)^2+9)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.68

$$\int \frac{x^4 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{a^4x^4 + 3\,a^2x^2 + 2\,(2\,a^3x^3 + 3\,ax)\sqrt{-a^2x^2+1}\arccos{(ax)} + 3\,\arccos{(ax)}^2}{16\,a^5}$$

input
```
integrate(x^4*arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/16*(a^4*x^4 + 3*a^2*x^2 + 2*(2*a^3*x^3 + 3*a*x)*sqrt(-a^2*x^2 + 1)*arcc
os(a*x) + 3*arccos(a*x)^2)/a^5
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{x^4 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} -\frac{x^4}{16a} - \frac{x^3\sqrt{-a^2x^2+1}\,\operatorname{acos}{(ax)}}{4a^2} - \frac{3x^2}{16a^3} - \frac{3x\sqrt{-a^2x^2+1}\,\operatorname{acos}{(ax)}}{8a^4} - \frac{3\operatorname{acos}^2{(ax)}}{16a^5} & \text{for}\ a \neq 0 \\ \frac{\pi x^5}{10} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*acos(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x**4/(16*a) - x**3*sqrt(-a**2*x**2 + 1)*acos(a*x)/(4*a**2) - 3
*x**2/(16*a**3) - 3*x*sqrt(-a**2*x**2 + 1)*acos(a*x)/(8*a**4) - 3*acos(a*x
)**2/(16*a**5), Ne(a, 0)), (pi*x**5/10, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{x^4 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{1}{16}\left(\frac{x^4}{a^2}+\frac{3\,x^2}{a^4}-\frac{3\,\arcsin{(ax)}^2}{a^6}\right)a - \frac{1}{8}\left(\frac{2\,\sqrt{-a^2x^2+1}x^3}{a^2}+\frac{3\,\sqrt{-a^2x^2+1}x}{a^4}-\frac{3\,\arcsin{(ax)}}{a^5}\right)\arccos{(ax)}$$

input
```
integrate(x^4*arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(x^4/a^2 + 3*x^2/a^4 - 3*arcsin(a*x)^2/a^6)*a - 1/8*(2*sqrt(-a^2*x^2
 + 1)*x^3/a^2 + 3*sqrt(-a^2*x^2 + 1)*x/a^4 - 3*arcsin(a*x)/a^5)*arccos(a*x
)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

$$\int \frac{x^4 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^4}{16\,a}-\frac{\sqrt{-a^2x^2+1}x^3\arccos{(ax)}}{4\,a^2}-\frac{3\,x^2}{16\,a^3}-\frac{3\,\sqrt{-a^2x^2+1}x\arccos{(ax)}}{8\,a^4}-\frac{3\,\arccos{(ax)}^2}{16\,a^5}+\frac{15}{128\,a^5}$$

input
```
integrate(x^4*arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/16*x^4/a - 1/4*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)/a^2 - 3/16*x^2/a^3 -
3/8*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)/a^4 - 3/16*arccos(a*x)^2/a^5 + 15/128
/a^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4\,\mathrm{acos}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^4*acos(a*x))/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^4*acos(a*x))/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)\,x^4}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^4*acos(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acos(a*x)*x**4)/sqrt( - a**2*x**2 + 1),x)
```

3.104 $\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}\, dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}\, dx = \frac{2x}{3a^3} + \frac{x^3}{9a} - \frac{2\sqrt{1-a^2x^2}\arccos(ax)}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}$$

output
```
2/3*x/a^3+1/9*x^3/a-2/3*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^4-1/3*x^2*(-a^2*x
^2+1)^(1/2)*arccos(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}\, dx = -\frac{ax(6+a^2x^2) + 3\sqrt{1-a^2x^2}(2+a^2x^2)\arccos(ax)}{9a^4}$$

input
```
Integrate[(x^3*ArcCos[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/9*(a*x*(6 + a^2*x^2) + 3*Sqrt[1 - a^2*x^2]*(2 + a^2*x^2)*ArcCos[a*x])/a
^4
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5211, 15, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}} \, dx$$

$$\Big\downarrow \text{5211}$$

$$\frac{2 \int \frac{x \arccos(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2} - \frac{\int x^2dx}{3a} - \frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{2 \int \frac{x \arccos(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2} - \frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2} - \frac{x^3}{9a}$$

$$\Big\downarrow \text{5183}$$

$$\frac{2\left(-\frac{\int 1dx}{a} - \frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}\right)}{3a^2} - \frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2} - \frac{x^3}{9a}$$

$$\Big\downarrow \text{24}$$

$$-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2} + \frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2} - \frac{x}{a}\right)}{3a^2} - \frac{x^3}{9a}$$

input
```
Int[(x^3*ArcCos[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/9*x^3/a - (x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(3*a^2) + (2*(-(x/a) - (S
qrt[1 - a^2*x^2]*ArcCos[a*x])/a^2))/(3*a^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.33

method	result	size
default	$-\frac{\sqrt{-a^{2} x^{2}+1}\, \left(3 a^{4} x^{4} \arccos \left(a x \right)+3 a^{2} x^{2} \arccos \left(a x \right)-a^{3} x^{3} \sqrt{-a^{2} x^{2}+1}-6 \arccos \left(a x \right)-6 \sqrt{-a^{2} x^{2}+1}\, a x \right)}{9 a^{4} \left(a^{2} x^{2}-1\right)}$	96
orering	$\frac{\left(5 a^{4} x^{4}+12 a^{2} x^{2}-24\right) \arccos \left(a x \right)}{9 a^{4} \sqrt{-a^{2} x^{2}+1}}-\frac{\left(a^{2} x^{2}+6\right) \left(a x -1\right) \left(a x +1\right) \left(\frac{3 x^{2} \arccos \left(a x \right)}{\sqrt{-a^{2} x^{2}+1}}-\frac{x^{3} a}{-a^{2} x^{2}+1}+\frac{x^{4} \arccos \left(a x \right) a^{2}}{\left(-a^{2} x^{2}+1\right)^{\frac{3}{2}}}\right)}{9 x^{2} a^{4}}$	131

input
```
int(x^3*arccos(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9/a^4*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)*(3*a^4*x^4*arccos(a*x)+3*a^2*x^2*a
rccos(a*x)-a^3*x^3*(-a^2*x^2+1)^(1/2)-6*arccos(a*x)-6*(-a^2*x^2+1)^(1/2)*a
*x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.61

$$\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{a^3x^3 + 3\,(a^2x^2+2)\sqrt{-a^2x^2+1}\arccos{(ax)} + 6\,ax}{9\,a^4}$$

input
```
integrate(x^3*arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*(a^3*x^3 + 3*(a^2*x^2 + 2)*sqrt(-a^2*x^2 + 1)*arccos(a*x) + 6*a*x)/a^
4
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} -\frac{x^3}{9a} - \frac{x^2\sqrt{-a^2x^2+1}\operatorname{acos}{(ax)}}{3a^2} - \frac{2x}{3a^3} - \frac{2\sqrt{-a^2x^2+1}\operatorname{acos}{(ax)}}{3a^4} & \text{for}\ a \neq 0 \\ \frac{\pi x^4}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acos(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x**3/(9*a) - x**2*sqrt(-a**2*x**2 + 1)*acos(a*x)/(3*a**2) - 2*
x/(3*a**3) - 2*sqrt(-a**2*x**2 + 1)*acos(a*x)/(3*a**4), Ne(a, 0)), (pi*x**
4/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{1}{9}\,a\left(\frac{x^3}{a^2}+\frac{6\,x}{a^4}\right) - \frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2}+\frac{2\,\sqrt{-a^2x^2+1}}{a^4}\right)\arccos\left(ax\right)$$

input
```
integrate(x^3*arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/9*a*(x^3/a^2 + 6*x/a^4) - 1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2
*x^2 + 1)/a^4)*arccos(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3\,\mathrm{acos}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^3*acos(a*x))/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^3*acos(a*x))/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)\,x^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^3*acos(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acos(a*x)*x**3)/sqrt( - a**2*x**2 + 1),x)
```

3.105 $\int \frac{x^2 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{x^2 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{x^2}{4a} - \frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2} + \frac{\arccos(ax)^2}{4a^3}$$

output
```
1/4*x^2/a-1/2*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^2+1/4*arccos(a*x)^2/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{x^2 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{a^2x^2 + 2ax\sqrt{1-a^2x^2}\arccos(ax) + \arccos(ax)^2}{4a^3}$$

input
```
Integrate[(x^2*ArcCos[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/4*(a^2*x^2 + 2*a*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x] + ArcCos[a*x]^2)/a^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arccos(ax)}{\sqrt{1-a^2x^2}} \, dx$$

$$\Big\downarrow \; 5211$$

$$\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2} - \frac{\int xdx}{2a} - \frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}$$

$$\Big\downarrow \; 15$$

$$\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2} - \frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2} - \frac{x^2}{4a}$$

$$\Big\downarrow \; 5153$$

$$-\frac{\arccos(ax)^2}{4a^3} - \frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2} - \frac{x^2}{4a}$$

input
```
Int[(x^2*ArcCos[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/4*x^2/a - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a^2) - ArcCos[a*x]^2/(4*
a^3)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{2 \arccos(ax)\sqrt{-a^2x^2+1}\, ax+a^2x^2+\arccos(ax)^2-1}{4a^3}$	41

input
```
int(x^2*arccos(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*arccos(a*x)*(-a^2*x^2+1)^(1/2)*a*x+a^2*x^2+arccos(a*x)^2-1)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{x^2 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{a^2x^2 + 2\sqrt{-a^2x^2+1}ax \arccos{(ax)} + \arccos{(ax)}^2}{4\,a^3}$$

input
```
integrate(x^2*arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(a^2*x^2 + 2*sqrt(-a^2*x^2 + 1)*a*x*arccos(a*x) + arccos(a*x)^2)/a^3
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{x^2 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} -\frac{x^2}{4a} - \frac{x\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{2a^2} - \frac{\operatorname{acos}^2(ax)}{4a^3} & \text{for } a \neq 0 \\ \frac{\pi x^3}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*acos(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x**2/(4*a) - x*sqrt(-a**2*x**2 + 1)*acos(a*x)/(2*a**2) - acos(
a*x)**2/(4*a**3), Ne(a, 0)), (pi*x**3/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{x^2 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{1}{4}a\left(\frac{x^2}{a^2} - \frac{\arcsin(ax)^2}{a^4}\right) - \frac{1}{2}\left(\frac{\sqrt{-a^2x^2+1}x}{a^2} - \frac{\arcsin(ax)}{a^3}\right)\arccos(ax)$$

input
```
integrate(x^2*arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*a*(x^2/a^2 - arcsin(a*x)^2/a^4) - 1/2*(sqrt(-a^2*x^2 + 1)*x/a^2 - arc
sin(a*x)/a^3)*arccos(a*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{x^2 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^2}{4\,a} - \frac{\sqrt{-a^2x^2+1}x \arccos{(ax)}}{2\,a^2} - \frac{\arccos{(ax)}^2}{4\,a^3} + \frac{1}{8\,a^3}$$

input
```
integrate(x^2*arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/4*x^2/a - 1/2*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)/a^2 - 1/4*arccos(a*x)^2/
a^3 + 1/8/a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2 \operatorname{acos}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^2*acos(a*x))/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^2*acos(a*x))/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2 \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acos}(ax)\,x^2}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^2*acos(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acos(a*x)*x**2)/sqrt( - a**2*x**2 + 1),x)
```

3.106 $\int \frac{x \arccos(ax)}{\sqrt{1-a^2x^2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{x \arccos(ax)}{\sqrt{1-a^2x^2}} \, dx = \frac{x}{a} - \frac{\sqrt{1-a^2x^2} \arccos(ax)}{a^2}$$

output
```
x/a-(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{x \arccos(ax)}{\sqrt{1-a^2x^2}} \, dx = -\frac{x}{a} - \frac{\sqrt{1-a^2x^2} \arccos(ax)}{a^2}$$

input
```
Integrate[(x*ArcCos[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-(x/a) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arccos(ax)}{\sqrt{1-a^2x^2}} \, dx$$

$$\Big\downarrow \text{5183}$$

$$-\frac{\int 1dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}-\frac{x}{a}$$

input
```
Int[(x*ArcCos[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
-(x/a) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(27) = 54.

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.17

method	result	size
default	$-\frac{\sqrt{-a^2x^2+1}\left(a^2x^2\arccos(ax)-\arccos(ax)-\sqrt{-a^2x^2+1}\,ax\right)}{a^2(a^2x^2-1)}$	63
orering	$\frac{(a^2x^2-2)\arccos(ax)}{a^2\sqrt{-a^2x^2+1}}-\frac{(ax-1)(ax+1)\left(\frac{\arccos(ax)}{\sqrt{-a^2x^2+1}}-\frac{xa}{-a^2x^2+1}+\frac{x^2\arccos(ax)a^2}{(-a^2x^2+1)^{\frac{3}{2}}}\right)}{a^2}$	103

input
```
int(x*arccos(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)*(a^2*x^2*arccos(a*x)-arccos(a*x)-(-a
^2*x^2+1)^(1/2)*a*x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx=-\frac{ax+\sqrt{-a^2x^2+1}\arccos{(ax)}}{a^2}$$

input
```
integrate(x*arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(a*x + sqrt(-a^2*x^2 + 1)*arccos(a*x))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} -\frac{x}{a} - \frac{\sqrt{-a^2x^2+1}\,\mathrm{acos}\,(ax)}{a^2} & \text{for } a \neq 0 \\ \frac{\pi x^2}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acos(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x/a - sqrt(-a**2*x**2 + 1)*acos(a*x)/a**2, Ne(a, 0)), (pi*x**2
/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{x \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x}{a} - \frac{\sqrt{-a^2x^2+1}\arccos(ax)}{a^2}$$

input
```
integrate(x*arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-x/a - sqrt(-a^2*x^2 + 1)*arccos(a*x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{x \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x}{a} - \frac{\sqrt{-a^2x^2+1}\arccos(ax)}{a^2}$$

input
```
integrate(x*arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-x/a - sqrt(-a^2*x^2 + 1)*arccos(a*x)/a^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\mathrm{acos}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x*acos(a*x))/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x*acos(a*x))/(1 - a^2*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{x \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{-\sqrt{-a^2x^2+1}\,acos(ax)-ax}{a^2}$$

input
```
int(x*acos(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
( - (sqrt( - a**2*x**2 + 1)*acos(a*x) + a*x))/a**2
```

3.107 $\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\arccos(ax)^2}{2a}$$

output
```
1/2*arccos(a*x)^2/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos(ax)^2}{2a}$$

input
```
Integrate[ArcCos[a*x]/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/2*ArcCos[a*x]^2/a
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx$$
$$\Big\downarrow \; 5153$$
$$-\frac{\arccos(ax)^2}{2a}$$

input
```
Int[ArcCos[a*x]/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/2*ArcCos[a*x]^2/a
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{\arccos(ax)^2}{2a}$	12
default	$-\frac{\arccos(ax)^2}{2a}$	12

input
```
int(arccos(a*x)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccos(a*x)^2/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos{(ax)}^2}{2\,a}$$

input
```
integrate(arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*arccos(a*x)^2/a
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} -\frac{\operatorname{acos}^2{(ax)}}{2a} & \text{for}\ a \neq 0 \\ \frac{\pi x}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(acos(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-acos(a*x)**2/(2*a), Ne(a, 0)), (pi*x/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos{(ax)}^2}{2\,a}$$

input
```
integrate(arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arccos(a*x)^2/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos{(ax)}^2}{2\,a}$$

input
```
integrate(arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*arccos(a*x)^2/a
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{\mathrm{acos}(a\,x)^2}{2\,a}$$

input
```
int(acos(a*x)/(1 - a^2*x^2)^(1/2),x)
```

output
```
-acos(a*x)^2/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{acos(ax)^2}{2a}$$

input
```
int(acos(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
( - acos(a*x)**2)/(2*a)
```

3.108 $\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}\,dx = -2\arccos(ax)\text{arctanh}\left(e^{i\arccos(ax)}\right) + i\,\text{PolyLog}\left(2,-e^{i\arccos(ax)}\right) - i\,\text{PolyLog}\left(2,e^{i\arccos(ax)}\right)$$

output
```
-2*arccos(a*x)*arctanh(a*x+I*(-a^2*x^2+1)^(1/2))+I*polylog(2,-a*x-I*(-a^2*
x^2+1)^(1/2))-I*polylog(2,a*x+I*(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}\,dx = i\left(2\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right) - \text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) + \text{PolyLog}\left(2,ie^{i\arccos(ax)}\right)\right)$$

input
```
Integrate[ArcCos[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
I*(2*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])] - PolyLog[2, (-I)*E^(I*ArcCos[a
*x])] + PolyLog[2, I*E^(I*ArcCos[a*x])])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}dx$$
$$\Big\downarrow \text{5219}$$
$$-\int \frac{\arccos(ax)}{ax}d\arccos(ax)$$
$$\Big\downarrow \text{3042}$$
$$-\int \arccos(ax)\csc\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)$$
$$\Big\downarrow \text{4669}$$
$$\int \log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)-\int \log\left(1+ie^{i\arccos(ax)}\right)d\arccos(ax)+2i\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right)$$
$$\Big\downarrow \text{2715}$$
$$-i\int e^{-i\arccos(ax)}\log\left(1-ie^{i\arccos(ax)}\right)de^{i\arccos(ax)}+i\int e^{-i\arccos(ax)}\log\left(1+ie^{i\arccos(ax)}\right)de^{i\arccos(ax)}+2i\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right)$$
$$\Big\downarrow \text{2838}$$
$$2i\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right)-i\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)+i\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right)$$

input
```
Int[ArcCos[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
(2*I)*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])] - I*PolyLog[2, (-I)*E^(I*ArcCo
s[a*x])] + I*PolyLog[2, I*E^(I*ArcCos[a*x])]
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.29

method	result
default	$\arccos(ax)\ln\left(1+i\left(ax+i\sqrt{-a^2x^2+1}\right)\right)-\arccos(ax)\ln\left(1-i\left(ax+i\sqrt{-a^2x^2+1}\right)\right)-i\,\mathrm{dilo}$

input
```
int(arccos(a*x)/x/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arccos(a*x)*ln(1+I*(a*x+I*(-a^2*x^2+1)^(1/2)))-arccos(a*x)*ln(1-I*(a*x+I*(
-a^2*x^2+1)^(1/2)))-I*dilog(1+I*(a*x+I*(-a^2*x^2+1)^(1/2)))+I*dilog(1-I*(a
*x+I*(-a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos{(ax)}}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccos(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccos(a*x)/(a^2*x^3 - x), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acos}{(ax)}}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acos(a*x)/x/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acos(a*x)/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos{(ax)}}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccos(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Giac [F]

$$\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos(ax)}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccos(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\mathrm{acos}(a\,x)}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acos(a*x)/(x*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acos(a*x)/(x*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)}{\sqrt{-a^2x^2+1}\,x}dx$$

input
```
int(acos(a*x)/x/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acos(a*x)/(sqrt( - a**2*x**2 + 1)*x),x)
```

3.109 $\int \frac{\arccos(ax)}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{\arccos(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\arccos(ax)}{x} + a\log(x)$$

output
```
-(-a^2*x^2+1)^(1/2)*arccos(a*x)/x+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\arccos(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\arccos(ax)}{x} - a\log(x)$$

input
```
Integrate[ArcCos[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcCos[a*x])/x) - a*Log[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5187, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)}{x^2\sqrt{1-a^2x^2}}dx$$
$$\Big\downarrow \text{5187}$$
$$-a\int \frac{1}{x}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{x}$$
$$\Big\downarrow \text{14}$$
$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{x}-a\log(x)$$

input
```
Int[ArcCos[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcCos[a*x])/x) - a*Log[x]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
default	$-\frac{\ln(ax)ax+\arccos(ax)\sqrt{-a^2x^2+1}}{x}$	31

input
```
int(arccos(a*x)/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(ln(a*x)*a*x+arccos(a*x)*(-a^2*x^2+1)^(1/2))/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = -\frac{ax\log(x)+\sqrt{-a^2x^2+1}\arccos(ax)}{x}$$

input
```
integrate(arccos(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(a*x*log(x) + sqrt(-a^2*x^2 + 1)*arccos(a*x))/x
```

Sympy [F]

$$\int \frac{\arccos(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acos}(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acos(a*x)/x**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acos(a*x)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\arccos(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = -a\log(x) - \frac{\sqrt{-a^2x^2+1}\arccos(ax)}{x}$$

input
```
integrate(arccos(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-a*log(x) - sqrt(-a^2*x^2 + 1)*arccos(a*x)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(26) = 52.

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int \frac{\arccos(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = \frac{1}{2}\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|} - \frac{\sqrt{-a^2x^2+1}|a|+a}{x|a|}\right)\arccos(ax) - a\log(|x|)$$

input
```
integrate(arccos(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*(a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - (sqrt(-a^2*x^2 + 1)*
abs(a) + a)/(x*abs(a)))*arccos(a*x) - a*log(abs(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\mathrm{acos}(a\,x)}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acos(a*x)/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acos(a*x)/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)}{\sqrt{-a^2x^2+1}\,x^2}dx$$

input
```
int(acos(a*x)/x^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acos(a*x)/(sqrt( - a**2*x**2 + 1)*x**2),x)
```

3.110 $\int \frac{\arccos(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{\arccos(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = -\frac{a}{2x} - \frac{\sqrt{1-a^2x^2}\arccos(ax)}{2x^2} - a^2\arccos(ax)\operatorname{arctanh}\left(e^{i\arccos(ax)}\right) + \frac{1}{2}ia^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right) - \frac{1}{2}ia^2\operatorname{PolyLog}\left(2,e^{i\arccos(ax)}\right)$$

output
```
-1/2*a/x-1/2*(-a^2*x^2+1)^(1/2)*arccos(a*x)/x^2-a^2*arccos(a*x)*arctanh(a*
x+I*(-a^2*x^2+1)^(1/2))+1/2*I*a^2*polylog(2,-a*x-I*(-a^2*x^2+1)^(1/2))-1/2
*I*a^2*polylog(2,a*x+I*(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

$$\int \frac{\arccos(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \frac{1}{2}a^2\left(-\frac{-1+\frac{\sqrt{1-a^2x^2}\arccos(ax)}{ax}}{ax} - \arccos(ax)\left(\log\left(1-ie^{i\arccos(ax)}\right)-\log\left(1+ie^{i\arccos(ax)}\right)\right) - i\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) + i\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)\right)$$

input
```
Integrate[ArcCos[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
(a^2*(-((-1 + (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(a*x))/(a*x)) - ArcCos[a*x]*
(Log[1 - I*E^(I*ArcCos[a*x])] - Log[1 + I*E^(I*ArcCos[a*x])]) - I*PolyLog[
2, (-I)*E^(I*ArcCos[a*x])] + I*PolyLog[2, I*E^(I*ArcCos[a*x])]))/2
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5205, 15, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{5205}$$

$$\frac{1}{2}a^2\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{1}{2}a\int \frac{1}{x^2}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{2x^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}a^2\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{2x^2}+\frac{a}{2x}$$

$$\Big\downarrow \text{5219}$$

$$-\frac{1}{2}a^2\int \frac{\arccos(ax)}{ax}d\arccos(ax)-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{2x^2}+\frac{a}{2x}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}a^2\int \arccos(ax)\csc\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{2x^2}+\frac{a}{2x}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{1}{2}a^2\left(-\int \log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)+\int \log\left(1+ie^{i\arccos(ax)}\right)d\arccos(ax)-2i\arccos(ax)\arctan\left(e\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\arccos(ax)}{2x^2}+\frac{a}{2x}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{1}{2}a^2\left(i\int e^{-i\arccos(ax)}\log\left(1-ie^{i\arccos(ax)}\right)de^{i\arccos(ax)}-i\int e^{-i\arccos(ax)}\log\left(1+ie^{i\arccos(ax)}\right)de^{i\arccos(ax)}-\right.$$
$$\frac{\sqrt{1-a^2x^2}\arccos(ax)}{2x^2}+\frac{a}{2x}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{1}{2}a^2\left(-2i\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right)+i\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-i\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right)\right)-$$
$$\frac{\sqrt{1-a^2x^2}\arccos(ax)}{2x^2}+\frac{a}{2x}$$

input
```
Int[ArcCos[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
a/(2*x) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*x^2) - (a^2*((-2*I)*ArcCos[a*
x]*ArcTan[E^(I*ArcCos[a*x])] + I*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - I*Po
lyLog[2, I*E^(I*ArcCos[a*x])]))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.90

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\left(a^2x^2\arccos(ax)+\sqrt{-a^2x^2+1}\,ax-\arccos(ax)\right)}{2(a^2x^2-1)x^2}+\frac{a^2\left(\arccos(ax)\ln\left(1+i\left(ax+i\sqrt{-a^2x^2+1}\right)\right)-\arccos(ax)\ln\left(1-\right.\right.}{}$

input
```
int(arccos(a*x)/x^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-a^2*x^2+1)^(1/2)*(a^2*x^2*arccos(a*x)+(-a^2*x^2+1)^(1/2)*a*x-arccos
(a*x))/(a^2*x^2-1)/x^2+1/2*a^2*(arccos(a*x)*ln(1+I*(a*x+I*(-a^2*x^2+1)^(1/
2)))-arccos(a*x)*ln(1-I*(a*x+I*(-a^2*x^2+1)^(1/2)))-I*dilog(1+I*(a*x+I*(-a
^2*x^2+1)^(1/2)))+I*dilog(1-I*(a*x+I*(-a^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{\arccos(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos(ax)}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccos(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccos(a*x)/(a^2*x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acos}(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acos(a*x)/x**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acos(a*x)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos(ax)}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccos(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{\arccos(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos{(ax)}}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccos(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\mathrm{acos}(a\,x)}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acos(a*x)/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acos(a*x)/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)}{\sqrt{-a^2x^2+1}\,x^3}dx$$

input
```
int(acos(a*x)/x^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acos(a*x)/(sqrt( - a**2*x**2 + 1)*x**3),x)
```

3.111 $\int \frac{x^5(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 224

$$\int \frac{x^5(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{8bx\sqrt{1-c^2x^2}}{15c^5\sqrt{d-c^2dx^2}}+\frac{4bx^3\sqrt{1-c^2x^2}}{45c^3\sqrt{d-c^2dx^2}}+\frac{bx^5\sqrt{1-c^2x^2}}{25c\sqrt{d-c^2dx^2}}-\frac{8\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{15c^6d}-\frac{4x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{15c^4d}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{5c^2d}$$

output
```
8/15*b*x*(-c^2*x^2+1)^(1/2)/c^5/(-c^2*d*x^2+d)^(1/2)+4/45*b*x^3*(-c^2*x^2+
1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)+1/25*b*x^5*(-c^2*x^2+1)^(1/2)/c/(-c^2*d*
x^2+d)^(1/2)-8/15*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c^6/d-4/15*x^2*(-
c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c^4/d-1/5*x^4*(-c^2*d*x^2+d)^(1/2)*(a
+b*arccos(c*x))/c^2/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.59

$$\int \frac{x^5(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{\sqrt{d-c^2dx^2}\left(bcx\sqrt{1-c^2x^2}(120+20c^2x^2+9c^4x^4)-15a(-8+4c^2x^2+c^4x^4+3c^6x^6)-15b(-8+4c^2x^2}{225c^6d\,(-1+c^2x^2)}$$

input
```
Integrate[(x^5*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(b*c*x*Sqrt[1 - c^2*x^2]*(120 + 20*c^2*x^2 + 9*c^4*x^
4) - 15*a*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c^6*x^6) - 15*b*(-8 + 4*c^2*x^2 +
c^4*x^4 + 3*c^6*x^6)*ArcCos[c*x]))/(225*c^6*d*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5211, 15, 5211, 15, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$$
$$\Big\downarrow \text{ 5211}$$
$$\frac{4\int \frac{x^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{5c^2}-\frac{b\sqrt{1-c^2x^2}\int x^4dx}{5c\sqrt{d-c^2dx^2}}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{5c^2d}$$
$$\Big\downarrow \text{ 15}$$
$$\frac{4\int \frac{x^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{5c^2}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{5c^2d}-\frac{bx^5\sqrt{1-c^2x^2}}{25c\sqrt{d-c^2dx^2}}$$
$$\Big\downarrow \text{ 5211}$$

$$\frac{4\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{b\sqrt{1-c^2x^2}\int x^2dx}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3c^2d}\right)}{5c^2}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{5c^2d}-\frac{bx^5\sqrt{1-c^2x^2}}{25c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{4\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3c^2d}-\frac{bx^3\sqrt{1-c^2x^2}}{9c\sqrt{d-c^2dx^2}}\right)}{5c^2}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{5c^2d}-\frac{bx^5\sqrt{1-c^2x^2}}{25c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 5183}$$

$$\frac{4\left(\frac{2\left(-\frac{b\sqrt{1-c^2x^2}\int 1dx}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^2d}\right)}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3c^2d}-\frac{bx^3\sqrt{1-c^2x^2}}{9c\sqrt{d-c^2dx^2}}\right)}{5c^2}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{5c^2d}-\frac{bx^5\sqrt{1-c^2x^2}}{25c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{5c^2d}+\frac{4\left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3c^2d}+\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^2d}-\frac{bx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}}\right)}{3c^2}-\frac{bx^3\sqrt{1-c^2x^2}}{9c\sqrt{d-c^2dx^2}}\right)}{5c^2}-\frac{bx^5\sqrt{1-c^2x^2}}{25c\sqrt{d-c^2dx^2}}$$

input

```
Int[(x^5*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output

```
-1/25*(b*x^5*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2]) - (x^4*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCos[c*x]))/(5*c^2*d) + (4*(-1/9*(b*x^3*Sqrt[1 - c^2*x^2
])/(c*Sqrt[d - c^2*d*x^2]) - (x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))
/(3*c^2*d) + (2*(-((b*x*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2])) - (Sqr
t[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(c^2*d)))/(3*c^2)))/(5*c^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.79

method	result
orering	$\frac{\left(81 c^{6} x^{6}+50 c^{4} x^{4}+440 c^{2} x^{2}-720\right) \left(a +b \arccos \left(c x \right)\right)}{225 c^{6} \sqrt{-c^{2} d \,x^{2}+d}}-\frac{\left(9 c^{4} x^{4}+20 c^{2} x^{2}+120\right) \left(c x -1\right) \left(c x +1\right) \left(\frac{5 x^{4} \left(a +b \arccos \left(c x \right)\right)}{\sqrt{-c^{2} d \,x^{2}+d}}-\frac{x^{5} b c}{\sqrt{-c^{2} x^{2}+1}\, \sqrt{-c^{2}}}\right.}{225 x^{4} c^{6}}$
default	$a \left(-\frac{x^{4} \sqrt{-c^{2} d \,x^{2}+d}}{5 c^{2} d}+\frac{-\frac{4 x^{2} \sqrt{-c^{2} d \,x^{2}+d}}{15 c^{2} d}-\frac{8 \sqrt{-c^{2} d \,x^{2}+d}}{15 d \,c^{4}}}{c^{2}}\right)+b \left(\frac{5 \sqrt{-d \left(c^{2} x^{2}-1\right)}\, \left(2 c^{2} x^{2}-1+2 i \sqrt{-c^{2} x^{2}+1}\, x c \right) \left(i+3 \arccos \left(\right.\right.}{576 c^{6} d \left(c^{2} x^{2}-1\right)}\right.$
parts	$a \left(-\frac{x^{4} \sqrt{-c^{2} d \,x^{2}+d}}{5 c^{2} d}+\frac{-\frac{4 x^{2} \sqrt{-c^{2} d \,x^{2}+d}}{15 c^{2} d}-\frac{8 \sqrt{-c^{2} d \,x^{2}+d}}{15 d \,c^{4}}}{c^{2}}\right)+b \left(\frac{5 \sqrt{-d \left(c^{2} x^{2}-1\right)}\, \left(2 c^{2} x^{2}-1+2 i \sqrt{-c^{2} x^{2}+1}\, x c \right) \left(i+3 \arccos \left(\right.\right.}{576 c^{6} d \left(c^{2} x^{2}-1\right)}\right.$

input
```
int(x^5*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/225*(81*c^6*x^6+50*c^4*x^4+440*c^2*x^2-720)/c^6*(a+b*arccos(c*x))/(-c^2*
d*x^2+d)^(1/2)-1/225/x^4*(9*c^4*x^4+20*c^2*x^2+120)/c^6*(c*x-1)*(c*x+1)*(5
*x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2)-x^5*b*c/(-c^2*x^2+1)^(1/2)/(-c
^2*d*x^2+d)^(1/2)+x^6*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2)*d*c^2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

$$\int \frac{x^5(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{(9\,bc^5x^5+20\,bc^3x^3+120\,bcx)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}-15\,(3\,ac^6x^6+ac^4x^4+4\,ac^2x^2+(3\,bc^6x^6+bc^4x^4+4\,bc^2x^2-8\,b)\arccos(cx)-8\,a)\sqrt{-c^2dx^2+d}}{225\,(c^8dx^2-c^6d)}$$

input
```
integrate(x^5*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
1/225*((9*b*c^5*x^5 + 20*b*c^3*x^3 + 120*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(
-c^2*x^2 + 1) - 15*(3*a*c^6*x^6 + a*c^4*x^4 + 4*a*c^2*x^2 + (3*b*c^6*x^6 +
 b*c^4*x^4 + 4*b*c^2*x^2 - 8*b)*arccos(c*x) - 8*a)*sqrt(-c^2*d*x^2 + d))/(
c^8*d*x^2 - c^6*d)
```

Sympy [F]

$$\int \frac{x^5(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^5(a+b\,\mathrm{acos}\,(cx))}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate(x**5*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**5*(a + b*acos(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.80

$$\int \frac{x^5(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = -\frac{1}{15}\left(\frac{3\sqrt{-c^2dx^2+d}x^4}{c^2d}+\frac{4\sqrt{-c^2dx^2+d}x^2}{c^4d}+\frac{8\sqrt{-c^2dx^2+d}}{c^6d}\right)b\arccos(cx) - \frac{1}{15}\left(\frac{3\sqrt{-c^2dx^2+d}x^4}{c^2d}+\frac{4\sqrt{-c^2dx^2+d}x^2}{c^4d}+\frac{8\sqrt{-c^2dx^2+d}}{c^6d}\right)a - \frac{(9\,c^4x^5+20\,c^2x^3+120\,x)b}{225\,c^5\sqrt{d}}$$

input
```
integrate(x^5*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
-1/15*(3*sqrt(-c^2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^
4*d) + 8*sqrt(-c^2*d*x^2 + d)/(c^6*d))*b*arccos(c*x) - 1/15*(3*sqrt(-c^2*d
*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-c^2*d
*x^2 + d)/(c^6*d))*a - 1/225*(9*c^4*x^5 + 20*c^2*x^3 + 120*x)*b/(c^5*sqrt(
d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x^5\,(a+b\,\mathrm{acos}(c\,x))}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x^5*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^5(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$$
$$=\frac{-3\sqrt{-c^2x^2+1}\,a\,c^4x^4-4\sqrt{-c^2x^2+1}\,a\,c^2x^2-8\sqrt{-c^2x^2+1}\,a+15\left(\int \frac{acos(cx)\,x^5}{\sqrt{-c^2x^2+1}}dx\right)b\,c^6}{15\sqrt{d}\,c^6}$$

input
```
int(x^5*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - 3*sqrt( - c**2*x**2 + 1)*a*c**4*x**4 - 4*sqrt( - c**2*x**2 + 1)*a*c**2
*x**2 - 8*sqrt( - c**2*x**2 + 1)*a + 15*int((acos(c*x)*x**5)/sqrt( - c**2*
x**2 + 1),x)*b*c**6)/(15*sqrt(d)*c**6)
```

3.112 $\int \frac{x^4(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 200

$$\int \frac{x^4(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{3bx^2\sqrt{1-c^2x^2}}{16c^3\sqrt{d-c^2dx^2}}+\frac{bx^4\sqrt{1-c^2x^2}}{16c\sqrt{d-c^2dx^2}} - \frac{3x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{8c^4d} - \frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4c^2d} + \frac{3\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{16bc^5\sqrt{d-c^2dx^2}}$$

output
```
3/16*b*x^2*(-c^2*x^2+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)+1/16*b*x^4*(-c^2*x^
2+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-3/8*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c
*x))/c^4/d-1/4*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c^2/d+3/16*(-c^2
*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/b/c^5/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.80

$$\int \frac{x^4(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx =$$

$$-\frac{\frac{16acx(3+2c^2x^2)\sqrt{d-c^2dx^2}}{d}+\frac{48a\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{\sqrt{d}}+\frac{b\sqrt{1-c^2x^2}(16\cos(2\arccos(cx))+\cos(4\arccos(cx))+4\arccos(cx)(6\arccos(}{\sqrt{d-c^2dx^2}}}{128c^5}$$

input

```
Integrate[(x^4*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output

```
-1/128*((16*a*c*x*(3 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2])/d + (48*a*ArcTan[(c
*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d] + (b*Sqrt[1 - c
^2*x^2]*(16*Cos[2*ArcCos[c*x]] + Cos[4*ArcCos[c*x]] + 4*ArcCos[c*x]*(6*Arc
Cos[c*x] + 8*Sin[2*ArcCos[c*x]] + Sin[4*ArcCos[c*x]])))/Sqrt[d - c^2*d*x^2
])/c^5
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5211, 15, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \text{5211}$$

$$\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{4c^2}-\frac{b\sqrt{1-c^2x^2}\int x^3dx}{4c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4c^2d}$$

$$\Big\downarrow \text{15}$$

$$\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{4c^2}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4c^2d}-\frac{bx^4\sqrt{1-c^2x^2}}{16c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5211}$$

$$\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{b\sqrt{1-c^2x^2}\int xdx}{2c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2c^2d}\right)}{4c^2}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4c^2d}-\frac{bx^4\sqrt{1-c^2x^2}}{16c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2c^2d}-\frac{bx^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}}\right)}{4c^2}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4c^2d}-\frac{bx^4\sqrt{1-c^2x^2}}{16c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5153}$$

$$-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4c^2d}+\frac{3\left(-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2c^2d}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{bx^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}}\right)}{4c^2}-\frac{bx^4\sqrt{1-c^2x^2}}{16c\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^4*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/16*(b*x^4*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2]) - (x^3*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCos[c*x]))/(4*c^2*d) + (3*(-1/4*(b*x^2*Sqrt[1 - c^2*x^2
])/(c*Sqrt[d - c^2*d*x^2]) - (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(
2*c^2*d) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c^3*Sqrt[d - c^2
*d*x^2])))/(4*c^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(174) = 348$.

Time = 0.51 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.88

method	result
default	$-\frac{a\,x^{3}\sqrt{-c^{2}d\,x^{2}+d}}{4c^{2}d}-\frac{3ax\sqrt{-c^{2}d\,x^{2}+d}}{8c^{4}d}+\frac{3a\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{8c^{4}\sqrt{c^{2}d}}+b\left(\frac{3\sqrt{-d(c^{2}x^{2}-1)}\,\sqrt{-c^{2}x^{2}+1}\,\arccos(cx)^{2}}{16c^{5}d(c^{2}x^{2}-1)}+\frac{\sqrt{-d}\right.$
parts	$-\frac{a\,x^{3}\sqrt{-c^{2}d\,x^{2}+d}}{4c^{2}d}-\frac{3ax\sqrt{-c^{2}d\,x^{2}+d}}{8c^{4}d}+\frac{3a\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{8c^{4}\sqrt{c^{2}d}}+b\left(\frac{3\sqrt{-d(c^{2}x^{2}-1)}\,\sqrt{-c^{2}x^{2}+1}\,\arccos(cx)^{2}}{16c^{5}d(c^{2}x^{2}-1)}+\frac{\sqrt{-d}\right.$

input
```
int(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*x^3/c^2/d*(-c^2*d*x^2+d)^(1/2)-3/8*a/c^4*x/d*(-c^2*d*x^2+d)^(1/2)+3
/8*a/c^4/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(3/1
6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^5/d/(c^2*x^2-1)*arccos(c*x)^
2+1/8*(-d*(c^2*x^2-1))^(1/2)/c^4/d/(c^2*x^2-1)*arccos(c*x)*x+1/16/c^5/(-d*
(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)-1/64*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c
^2*x^2-1)*arccos(c*x)*cos(5*arccos(c*x))+1/256*(-d*(c^2*x^2-1))^(1/2)/c^5/
d/(c^2*x^2-1)*sin(5*arccos(c*x))-7/64*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^
2-1)*arccos(c*x)*cos(3*arccos(c*x))+15/256*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c
^2*x^2-1)*sin(3*arccos(c*x)))
```

Fricas [F]

$$\int \frac{x^4(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\arccos(cx)+a)x^4}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(-(b*x^4*arccos(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^2*d*x^2 - d)
, x)
```

Sympy [F]

$$\int \frac{x^4(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^4(a+b\,\mathrm{acos}(cx))}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate(x**4*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**4*(a + b*acos(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^4(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\arccos{(cx)}+a)x^4}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
-1/8*a*(2*sqrt(-c^2*d*x^2 + d)*x^3/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*x/(c^4
*d) - 3*arcsin(c*x)/(c^5*sqrt(d))) + b*integrate(x^4*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x)/(sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.68

$$\int \frac{x^4(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx=-\frac{8\,bc^3x^4+32\,\sqrt{-c^2x^2+1}bc^2x^3\arccos{(cx)}+32\,\sqrt{-c^2x^2+1}ac^2x^3+24\,bcx^2+48\,\sqrt{-c^2x^2+1}bx\arccos}{128\,c^4\sqrt{d}}$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
-1/128*(8*b*c^3*x^4 + 32*sqrt(-c^2*x^2 + 1)*b*c^2*x^3*arccos(c*x) + 32*sqr
t(-c^2*x^2 + 1)*a*c^2*x^3 + 24*b*c*x^2 + 48*sqrt(-c^2*x^2 + 1)*b*x*arccos(
c*x) + 48*sqrt(-c^2*x^2 + 1)*a*x + 24*b*arccos(c*x)^2/c + 48*a*arccos(c*x)
/c - 15*b/c)/(c^4*sqrt(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^4\,(a+b\,\mathrm{acos}(c\,x))}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x^4*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^4*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{3asin(cx)\,a-2\sqrt{-c^2x^2+1}\,a\,c^3x^3-3\sqrt{-c^2x^2+1}\,acx+8\left(\int \frac{acos(cx)\,x^4}{\sqrt{-c^2x^2+1}}dx\right)b\,c^5}{8\sqrt{d}\,c^5}$$

input
```
int(x^4*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(3*asin(c*x)*a - 2*sqrt( - c**2*x**2 + 1)*a*c**3*x**3 - 3*sqrt( - c**2*x**
2 + 1)*a*c*x + 8*int((acos(c*x)*x**4)/sqrt( - c**2*x**2 + 1),x)*b*c**5)/(8
*sqrt(d)*c**5)
```

3.113 $\int \frac{x^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 148

$$\int \frac{x^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{2bx\sqrt{1-c^2x^2}}{3c^3\sqrt{d-c^2dx^2}}+\frac{bx^3\sqrt{1-c^2x^2}}{9c\sqrt{d-c^2dx^2}} - \frac{2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3c^4d} - \frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3c^2d}$$

output
```
2/3*b*x*(-c^2*x^2+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)+1/9*b*x^3*(-c^2*x^2+1)
^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-2/3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c
^4/d-1/3*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c^2/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.72

$$\int \frac{x^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{d-c^2dx^2}\left(bcx\sqrt{1-c^2x^2}(6+c^2x^2)-3a(-2+c^2x^2+c^4x^4)-3b(-2+c^2x^2+c^4x^4)\arccos(cx)\right)}{9c^4d\,(-1+c^2x^2)}$$

input
```
Integrate[(x^3*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(b*c*x*Sqrt[1 - c^2*x^2]*(6 + c^2*x^2) - 3*a*(-2 + c^
2*x^2 + c^4*x^4) - 3*b*(-2 + c^2*x^2 + c^4*x^4)*ArcCos[c*x]))/(9*c^4*d*(-1
 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5211, 15, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{5211}$$

$$\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{b\sqrt{1-c^2x^2}\int x^2dx}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3c^2d}$$

$$\Big\downarrow \text{15}$$

$$\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3c^2d}-\frac{bx^3\sqrt{1-c^2x^2}}{9c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5183}$$

$$\frac{2\left(-\frac{b\sqrt{1-c^2x^2}\int 1dx}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^2d}\right)}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3c^2d}-\frac{bx^3\sqrt{1-c^2x^2}}{9c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{24}$$

$$-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3c^2d}+\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^2d}-\frac{bx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}}\right)}{3c^2}-\frac{bx^3\sqrt{1-c^2x^2}}{9c\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^3*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/9*(b*x^3*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2]) - (x^2*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCos[c*x]))/(3*c^2*d) + (2*(-((b*x*Sqrt[1 - c^2*x^2])/(c*
Sqrt[d - c^2*d*x^2])) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(c^2*d))
)/(3*c^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.09

method	result
orering	$\frac{(5c^4x^4+12c^2x^2-24)(a+b\arccos(cx))}{9c^4\sqrt{-c^2d\,x^2+d}}-\frac{(c^2x^2+6)(cx-1)(cx+1)\left(\frac{3x^2(a+b\arccos(cx))}{\sqrt{-c^2d\,x^2+d}}-\frac{x^3bc}{\sqrt{-c^2x^2+1}\,\sqrt{-c^2d\,x^2+d}}+\frac{x^4(a+b\arccos(cx}{(-c^2d\,x^2+d)}\right.}{9x^2c^4}$
default	$a\left(-\frac{x^2\sqrt{-c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{-c^2d\,x^2+d}}{3d\,c^4}\right)+b\left(\frac{\sqrt{-d(c^2x^2-1)}\left(2c^2x^2-1+2i\sqrt{-c^2x^2+1}\,xc\right)(i+3\arccos(cx))}{144c^4d(c^2x^2-1)}-\frac{3\sqrt{-d(c^2x^2}{}\right.$
parts	$a\left(-\frac{x^2\sqrt{-c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{-c^2d\,x^2+d}}{3d\,c^4}\right)+b\left(\frac{\sqrt{-d(c^2x^2-1)}\left(2c^2x^2-1+2i\sqrt{-c^2x^2+1}\,xc\right)(i+3\arccos(cx))}{144c^4d(c^2x^2-1)}-\frac{3\sqrt{-d(c^2x^2}{}\right.$

input
```
int(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(5*c^4*x^4+12*c^2*x^2-24)/c^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2)-1
/9/x^2*(c^2*x^2+6)/c^4*(c*x-1)*(c*x+1)*(3*x^2*(a+b*arccos(c*x))/(-c^2*d*x^
2+d)^(1/2)-x^3*b*c/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+x^4*(a+b*arccos
(c*x))/(-c^2*d*x^2+d)^(3/2)*d*c^2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int \frac{x^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{(bc^3x^3+6\,bcx)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}-3\left(ac^4x^4+ac^2x^2+(bc^4x^4+bc^2x^2-2\,b)\arccos(cx)-2\,a\right)\surd}{9\left(c^6dx^2-c^4d\right)}$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
1/9*((b*c^3*x^3 + 6*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 3*(a*
c^4*x^4 + a*c^2*x^2 + (b*c^4*x^4 + b*c^2*x^2 - 2*b)*arccos(c*x) - 2*a)*sqr
t(-c^2*d*x^2 + d))/(c^6*d*x^2 - c^4*d)
```

Sympy [F]

$$\int \frac{x^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^3(a+b\,\mathrm{acos}\,(cx))}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate(x**3*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*acos(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

$$\int \frac{x^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = -\frac{1}{3}\,b\left(\frac{\sqrt{-c^2dx^2+d}x^2}{c^2d}+\frac{2\,\sqrt{-c^2dx^2+d}}{c^4d}\right)\arccos\,(cx) - \frac{1}{3}\,a\left(\frac{\sqrt{-c^2dx^2+d}x^2}{c^2d}+\frac{2\,\sqrt{-c^2dx^2+d}}{c^4d}\right) - \frac{(c^2x^3+6\,x)b}{9\,c^3\sqrt{d}}$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
-1/3*b*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d))
*arccos(c*x) - 1/3*a*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2
 + d)/(c^4*d)) - 1/9*(c^2*x^3 + 6*x)*b/(c^3*sqrt(d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^3\,(a+b\,\mathrm{acos}(c\,x))}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x^3*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\,a\,c^2x^2-2\sqrt{-c^2x^2+1}\,a+3\left(\int \frac{acos(cx)x^3}{\sqrt{-c^2x^2+1}}dx\right)b\,c^4}{3\sqrt{d}\,c^4}$$

input
```
int(x^3*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a + 3*in
t((acos(c*x)*x**3)/sqrt( - c**2*x**2 + 1),x)*b*c**4)/(3*sqrt(d)*c**4)
```

3.114 $\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{bx^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}} - \frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2c^2d} + \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4bc^3\sqrt{d-c^2dx^2}}$$

output
```
1/4*b*x^2*(-c^2*x^2+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-1/2*x*(-c^2*d*x^2+d)^(
1/2)*(a+b*arccos(c*x))/c^2/d+1/4*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/b/
c^3/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.06

$$\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = -\frac{\frac{4acx\sqrt{d-c^2dx^2}}{d} + \frac{4a\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{\sqrt{d}} + \frac{b\sqrt{1-c^2x^2}(\cos(2\arccos(cx))+2\arccos(cx)(\arccos(cx)+\sin(2\arccos(cx))))}{\sqrt{d-c^2dx^2}}}{8c^3}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/8*((4*a*c*x*Sqrt[d - c^2*d*x^2])/d + (4*a*ArcTan[(c*x*Sqrt[d - c^2*d*x^
2])/(Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d] + (b*Sqrt[1 - c^2*x^2]*(Cos[2*ArcCo
s[c*x]] + 2*ArcCos[c*x]*(ArcCos[c*x] + Sin[2*ArcCos[c*x]])))/Sqrt[d - c^2*
d*x^2])/c^3
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \text{5211}$$

$$\frac{\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{b\sqrt{1-c^2x^2}\int xdx}{2c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2c^2d}$$

$$\Big\downarrow \text{15}$$

$$\frac{\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2c^2d}-\frac{bx^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5153}$$

$$-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2c^2d}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{bx^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/4*(b*x^2*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2]) - (x*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCos[c*x]))/(2*c^2*d) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*
x])^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(108) = 216$.

Time = 0.45 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.16

method	result
default	$-\frac{ax\sqrt{-c^2d\,x^2+d}}{2c^2d}+\frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^2\sqrt{c^2d}}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2}{4c^3d(c^2x^2-1)}+\frac{\sqrt{-d(c^2x^2-1)}\,\arccos(cx)x}{8c^2d(c^2x^2-1)}+\right.$
parts	$-\frac{ax\sqrt{-c^2d\,x^2+d}}{2c^2d}+\frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^2\sqrt{c^2d}}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2}{4c^3d(c^2x^2-1)}+\frac{\sqrt{-d(c^2x^2-1)}\,\arccos(cx)x}{8c^2d(c^2x^2-1)}+\right.$

input
```
int(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a/c^2/(c^2*d)^(1/2)*arctan((c^2*d)
^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(1/4*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^
(1/2)/c^3/d/(c^2*x^2-1)*arccos(c*x)^2+1/8*(-d*(c^2*x^2-1))^(1/2)/c^2/d/(c^
2*x^2-1)*arccos(c*x)*x+1/16/c^3/(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)-
1/8*(-d*(c^2*x^2-1))^(1/2)/c^3/d/(c^2*x^2-1)*arccos(c*x)*cos(3*arccos(c*x)
)+1/16*(-d*(c^2*x^2-1))^(1/2)/c^3/d/(c^2*x^2-1)*sin(3*arccos(c*x)))
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\arccos(cx)+a)x^2}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*x^2*arccos(c*x) + a*x^2)/(c^2*d*x^2 - d)
, x)
```

Sympy [F]

$$\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x^2(a+b\,\mathrm{acos}\,(cx))}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate(x**2*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*acos(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\arccos(cx)+a)x^2}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
-1/2*a*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d))) + b*in
tegrate(x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(sqrt(c*x + 1)*sqrt
(-c*x + 1)), x)/sqrt(d)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.65

$$\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = -\frac{2\,bcx^2+4\,\sqrt{-c^2x^2+1}bx\arccos(cx)+4\,\sqrt{-c^2x^2+1}ax+\frac{2\,b\arccos(cx)^2}{c}+\frac{4\,a\arccos(cx)}{c}-\frac{b}{c}}{8\,c^2\sqrt{d}}$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(2*b*c*x^2 + 4*sqrt(-c^2*x^2 + 1)*b*x*arccos(c*x) + 4*sqrt(-c^2*x^2 +
 1)*a*x + 2*b*arccos(c*x)^2/c + 4*a*arccos(c*x)/c - b/c)/(c^2*sqrt(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^2\,(a+b\,\mathrm{acos}(c\,x))}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x^2*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{asin(cx)\,a-\sqrt{-c^2x^2+1}\,acx+2\left(\int \frac{acos(cx)x^2}{\sqrt{-c^2x^2+1}}dx\right)b\,c^3}{2\sqrt{d}\,c^3}$$

input
```
int(x^2*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(asin(c*x)*a - sqrt( - c**2*x**2 + 1)*a*c*x + 2*int((acos(c*x)*x**2)/sqrt(
 - c**2*x**2 + 1),x)*b*c**3)/(2*sqrt(d)*c**3)
```

3.115 $\int \frac{x(a+b \arccos(cx))}{\sqrt{d-c^2dx^2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 67

$$\int \frac{x(a+b \arccos(cx))}{\sqrt{d-c^2dx^2}} \, dx = \frac{bx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b \arccos(cx))}{c^2d}$$

output
```
b*x*(-c^2*x^2+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-(-c^2*d*x^2+d)^(1/2)*(a+b*ar
ccos(c*x))/c^2/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int \frac{x(a+b \arccos(cx))}{\sqrt{d-c^2dx^2}} \, dx = \frac{\sqrt{d-c^2dx^2}\left(a-ac^2x^2+bcx\sqrt{1-c^2x^2}+(b-bc^2x^2) \arccos(cx)\right)}{c^2d\,(-1+c^2x^2)}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(a - a*c^2*x^2 + b*c*x*Sqrt[1 - c^2*x^2] + (b - b*c^2
*x^2)*ArcCos[c*x]))/(c^2*d*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \; 5183$$

$$-\frac{b\sqrt{1-c^2x^2}\int 1dx}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^2d}$$

$$\Big\downarrow \; 24$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^2d}-\frac{bx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}}$$

input
```
Int[(x*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
-((b*x*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2])) - (Sqrt[d - c^2*d*x^2]*
(a + b*ArcCos[c*x]))/(c^2*d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(61) = 122$.

Time = 0.56 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.99

method	result
orering	$\frac{(c^2x^2-2)(a+b\arccos(cx))}{c^2\sqrt{-c^2dx^2+d}}-\frac{(cx-1)(cx+1)\left(\frac{a+b\arccos(cx)}{\sqrt{-c^2dx^2+d}}-\frac{xbc}{\sqrt{-c^2x^2+1}\sqrt{-c^2dx^2+d}}+\frac{x^2(a+b\arccos(cx))dc^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\right)}{c^2}$
default	$-\frac{a\sqrt{-c^2dx^2+d}}{c^2d}+b\Bigg(-\frac{\sqrt{-d(c^2x^2-1)}\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)(\arccos(cx)+i)}{2c^2d(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\left(-i\sqrt{-c^2x^2+1}\,xc+c^2x^2\right.}{2c^2d(c^2x^2-1)}$
parts	$-\frac{a\sqrt{-c^2dx^2+d}}{c^2d}+b\Bigg(-\frac{\sqrt{-d(c^2x^2-1)}\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)(\arccos(cx)+i)}{2c^2d(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\left(-i\sqrt{-c^2x^2+1}\,xc+c^2x^2\right.}{2c^2d(c^2x^2-1)}$

input
```
int(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c^2*x^2-2)/c^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2)-1/c^2*(c*x-1)*(c*x+
1)*((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2)-x*b*c/(-c^2*x^2+1)^(1/2)/(-c^2*
d*x^2+d)^(1/2)+x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2)*d*c^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

$$\int \frac{x(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}bcx-(ac^2x^2+(bc^2x^2-b)\arccos(cx)-a)\sqrt{-c^2dx^2+d}}{c^4dx^2-c^2d}$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*b*c*x - (a*c^2*x^2 + (b*c^2*x^2 -
 b)*arccos(c*x) - a)*sqrt(-c^2*d*x^2 + d))/(c^4*d*x^2 - c^2*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = -\frac{bx}{c\sqrt{d}} - \frac{\sqrt{-c^2dx^2+d}b\arccos{(cx)}}{c^2d} - \frac{\sqrt{-c^2dx^2+d}a}{c^2d}$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
-b*x/(c*sqrt(d)) - sqrt(-c^2*d*x^2 + d)*b*arccos(c*x)/(c^2*d) - sqrt(-c^2*
d*x^2 + d)*a/(c^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x\,(a+b\,\mathrm{acos}(c\,x))}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

$$\int \frac{x(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx=-\frac{\sqrt{d}\,\left(\sqrt{-c^2x^2+1}\,acos(cx)\,b+\sqrt{-c^2x^2+1}\,a+bcx\right)}{c^2d}$$

input
```
int(x*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - sqrt(d)*(sqrt( - c**2*x**2 + 1)*acos(c*x)*b + sqrt( - c**2*x**2 + 1)*a
 + b*c*x))/(c**2*d)
```

3.116 $\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2bc\sqrt{d-c^2dx^2}}$$

output
```
1/2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/b/c/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx = -\frac{\sqrt{1-c^2x^2}\arccos(cx)(2a+b\arccos(cx))}{2c\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/2*(Sqrt[1 - c^2*x^2]*ArcCos[c*x]*(2*a + b*ArcCos[c*x]))/(c*Sqrt[d - c^2
*d*x^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{5153\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \text{5153}$$

$$-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2bc\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/2*(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(b*c*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

method	result	size
default	$\frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2}{2cd(c^2x^2-1)}$	86
parts	$\frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2}{2cd(c^2x^2-1)}$	86

input
```
int((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/2*b*(-d*(c^
2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/(c^2*x^2-1)*arccos(c*x)^2
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{b\arccos(cx)+a}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{a+b\,\mathrm{acos}\,(cx)}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx = \frac{b\arccos(cx)\arcsin(cx)}{c\sqrt{d}} + \frac{b\arcsin(cx)^2}{2\,c\sqrt{d}} + \frac{a\arcsin(cx)}{c\sqrt{d}}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
b*arccos(c*x)*arcsin(c*x)/(c*sqrt(d)) + 1/2*b*arcsin(c*x)^2/(c*sqrt(d)) +
a*arcsin(c*x)/(c*sqrt(d))
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx = -\frac{b\arccos(cx)^2+2\,a\arccos(cx)}{2\,c\sqrt{d}}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(b*arccos(c*x)^2 + 2*a*arccos(c*x))/(c*sqrt(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.55

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{d}\,\left(-acos(cx)^2\,b+2asin(cx)\,a\right)}{2cd}$$

input
```
int((a+b*acos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(sqrt(d)*( - acos(c*x)**2*b + 2*asin(c*x)*a))/(2*c*d)
```

3.117 $\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 145

$$\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}\,dx = -\frac{2\sqrt{1-c^2x^2}(a+b\arccos(cx))\text{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}} + \frac{ib\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{ib\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}}$$

output
```
-2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/
(-c^2*d*x^2+d)^(1/2)+I*b*(-c^2*x^2+1)^(1/2)*polylog(2,-c*x-I*(-c^2*x^2+1)^
(1/2))/(-c^2*d*x^2+d)^(1/2)-I*b*(-c^2*x^2+1)^(1/2)*polylog(2,c*x+I*(-c^2*x
^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90

$$\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}\,dx = \frac{a\log(x)}{\sqrt{d}} - \frac{a\log\left(d+\sqrt{d}\sqrt{-d(-1+c^2x^2)}\right)}{\sqrt{d}} + \frac{ib\sqrt{1-c^2x^2}\left(2\arccos(cx)\arctan\left(e^{i\arccos(cx)}\right) - \text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right) + \text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\right)}{\sqrt{d(1-c^2x^2)}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(a*Log[x])/Sqrt[d] - (a*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/Sqrt[d
] + (I*b*Sqrt[1 - c^2*x^2]*(2*ArcCos[c*x]*ArcTan[E^(I*ArcCos[c*x])] - Poly
Log[2, (-I)*E^(I*ArcCos[c*x])] + PolyLog[2, I*E^(I*ArcCos[c*x])]))/Sqrt[d*
(1 - c^2*x^2)]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \text{5219}$$

$$-\frac{\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\sqrt{1-c^2x^2}\left(-b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(e^{i\arccos(\right.\right.}{\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 2715

$$-\frac{\sqrt{1-c^2x^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(\right.}{\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 2838

$$-\frac{\sqrt{1-c^2x^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,ie^{i\arccos(\right.\right.}{\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(x*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-((Sqrt[1 - c^2*x^2]*((-2*I)*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcCos[c*x])]
 + I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*b*PolyLog[2, I*E^(I*ArcCos[c
*x])]))/Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.35

method	result
default	$-\frac{a \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)}{\sqrt{d}}-\frac{b\sqrt{-c^2x^2+1}\,\sqrt{-d(c^2x^2-1)}\,\left(\arccos(cx)\ln\left(1+i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)-\arccos(cx)\ln\left(1-i\left(cx+i\right.\right.\right.}{d(c^2x^2-1)}$
parts	$-\frac{a \ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)}{\sqrt{d}}-\frac{b\sqrt{-c^2x^2+1}\,\sqrt{-d(c^2x^2-1)}\,\left(\arccos(cx)\ln\left(1+i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)-\arccos(cx)\ln\left(1-i\left(cx+i\right.\right.\right.}{d(c^2x^2-1)}$

input
```
int((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a/d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-b*(-c^2*x^2+1)^(1/2)
*(-d*(c^2*x^2-1))^(1/2)*(arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-ar
ccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-I*dilog(1+I*(c*x+I*(-c^2*x^2+
1)^(1/2)))+I*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2))))/d/(c^2*x^2-1)
```

Fricas [F]

$$\int \frac{a + b\arccos(cx)}{x\sqrt{d - c^2dx^2}}\,dx = \int \frac{b\arccos(cx) + a}{\sqrt{-c^2dx^2 + d}x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^2*d*x^3 - d*x), x)
```

Sympy [F]

$$\int \frac{a + b\arccos(cx)}{x\sqrt{d - c^2dx^2}}\,dx = \int \frac{a + b\,\mathrm{acos}\,(cx)}{x\sqrt{-d\,(cx - 1)\,(cx + 1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))/x/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [F]

$$\int \frac{a + b\arccos(cx)}{x\sqrt{d - c^2dx^2}}\,dx = \int \frac{b\arccos(cx) + a}{\sqrt{-c^2dx^2 + d}x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
b*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(sqrt(c*x + 1)*sqrt
(-c*x + 1)*x), x)/sqrt(d) - a*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) +
2*d/abs(x))/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))/(x*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))/(x*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,x}dx\right)b+\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a}{\sqrt{d}}$$

input
```
int((a+b*acos(c*x))/x/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*x),x)*b + log(tan(asin(c*x)/2))*a)/
sqrt(d)
```

3.118 $\int \frac{a+b\arccos(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{a+b\arccos(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx = -\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{dx}+\frac{bc\sqrt{1-c^2x^2}\log(x)}{\sqrt{d-c^2dx^2}}$$

output
```
-(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/d/x+b*c*(-c^2*x^2+1)^(1/2)*ln(x)/(
-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \frac{a+b\arccos(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx = -\frac{\sqrt{d-c^2dx^2}\left(a\sqrt{1-c^2x^2}+b\sqrt{1-c^2x^2}\arccos(cx)+bcx\log(x)\right)}{dx\sqrt{1-c^2x^2}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*(a*Sqrt[1 - c^2*x^2] + b*Sqrt[1 - c^2*x^2]*ArcCos[c
*x] + b*c*x*Log[x]))/(d*x*Sqrt[1 - c^2*x^2]))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {5187, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^2\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{5187}$$

$$-\frac{bc\sqrt{1-c^2x^2}\int \frac{1}{x}dx}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{dx}$$

$$\Big\downarrow \text{14}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{dx}-\frac{bc\sqrt{1-c^2x^2}\log(x)}{\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(d*x)) - (b*c*Sqrt[1 - c^2*x^2
]*Log[x])/Sqrt[d - c^2*d*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.09

method	result
default	$-\frac{a\sqrt{-c^2 d\,x^2+d}}{dx}+b\left(-\frac{2i\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)c}{d(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\,\left(-i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\arccos(cx)}{dx(c^2x^2-1)}\right.$
parts	$-\frac{a\sqrt{-c^2 d\,x^2+d}}{dx}+b\left(-\frac{2i\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)c}{d(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\,\left(-i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\arccos(cx)}{dx(c^2x^2-1)}\right.$

input
```
int((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a/d/x*(-c^2*d*x^2+d)^(1/2)+b*(-2*I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1
/2)/d/(c^2*x^2-1)*arccos(c*x)*c-(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1
/2)*x*c+c^2*x^2-1)*arccos(c*x)/d/x/(c^2*x^2-1)+(-d*(c^2*x^2-1))^(1/2)*(-c^
2*x^2+1)^(1/2)/d/(c^2*x^2-1)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*c)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.30

$$\int \frac{a+b\arccos(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx = \left[\frac{bc\sqrt{d}x\log\left(\frac{c^2dx^6+c^2dx^2-dx^4+\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}(x^4-1)\sqrt{d}-d}{c^2x^4-x^2}\right)-2\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)}{2\,dx},\right.$$
$$\left.-\frac{bc\sqrt{-d}x\arctan\left(\frac{\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}(x^2-1)\sqrt{-d}}{c^2dx^4+(c^2-1)dx^2-d}\right)+\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)}{dx}\right]$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/2*(b*c*sqrt(d)*x*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 +
 d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^2*x^4 - x^2)) - 2*sqrt(-c
^2*d*x^2 + d)*(b*arccos(c*x) + a))/(d*x), -(b*c*sqrt(-d)*x*arctan(sqrt(-c^
2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)/(c^2*d*x^4 + (c^2 - 1)*
d*x^2 - d)) + sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a))/(d*x)]
```

Sympy [F]

$$\int \frac{a + b \arccos(cx)}{x^2\sqrt{d - c^2dx^2}}\,dx = \int \frac{a + b\,\mathrm{acos}\,(cx)}{x^2\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))/x**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.58

$$\int \frac{a + b \arccos(cx)}{x^2\sqrt{d - c^2dx^2}}\,dx = \frac{\left((-1)^{-2\,c^2dx^2+2\,d}\,\sqrt{d}\log\left(-2\,c^2d + \frac{2\,d}{x^2}\right) + \sqrt{d}\log\left(x^2 - \frac{1}{c^2}\right)\right)bc}{2\,d} - \frac{\sqrt{-c^2dx^2+d}b\arccos\left(cx\right)}{dx} - \frac{\sqrt{-c^2dx^2+d}a}{dx}$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
1/2*((-1)^(-2*c^2*d*x^2 + 2*d)*sqrt(d)*log(-2*c^2*d + 2*d/x^2) + sqrt(d)*l
og(x^2 - 1/c^2))*b*c/d - sqrt(-c^2*d*x^2 + d)*b*arccos(c*x)/(d*x) - sqrt(-
c^2*d*x^2 + d)*a/(d*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x^2\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))/(x^2*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))/(x^2*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\,a+\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,x^2}dx\right)bx}{\sqrt{d}\,x}$$

input
```
int((a+b*acos(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*a + int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*x**2)
,x)*b*x)/(sqrt(d)*x)
```

3.119 $\int \frac{a+b\arccos(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 229

$$\int \frac{a+b\arccos(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx = -\frac{bc\sqrt{1-c^2x^2}}{2x\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2dx^2} - \frac{c^2\sqrt{1-c^2x^2}(a+b\arccos(cx))\text{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}} + \frac{ibc^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{2\sqrt{d-c^2dx^2}} - \frac{ibc^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2\sqrt{d-c^2dx^2}}$$

output
```
-1/2*b*c*(-c^2*x^2+1)^(1/2)/x/(-c^2*d*x^2+d)^(1/2)-1/2*(-c^2*d*x^2+d)^(1/2
)*(a+b*arccos(c*x))/d/x^2-c^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*arctanh
(c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)+1/2*I*b*c^2*(-c^2*x^2+1)^(
1/2)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)-1/2*I*b*c^2
*(-c^2*x^2+1)^(1/2)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/
2)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.07

$$\int \frac{a+b\arccos(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx =$$

$$-\frac{\frac{2a\sqrt{d-c^2dx^2}}{x^2}-2ac^2\sqrt{d}\log(x)+2ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)+\frac{2bd\sqrt{1-c^2x^2}\left(-cx+\sqrt{1-c^2x^2}\arccos(cx)+c^2x^2\right.}{}}{4}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-1/4*((2*a*Sqrt[d - c^2*d*x^2])/x^2 - 2*a*c^2*Sqrt[d]*Log[x] + 2*a*c^2*Sqr
t[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (2*b*d*Sqrt[1 - c^2*x^2]*(-(c*
x) + Sqrt[1 - c^2*x^2]*ArcCos[c*x] + c^2*x^2*ArcCos[c*x]*Log[1 - I*E^(I*Ar
cCos[c*x])] - c^2*x^2*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] + I*c^2*x^2
*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*c^2*x^2*PolyLog[2, I*E^(I*ArcCos[c
*x])]))/(x^2*Sqrt[d - c^2*d*x^2]))/d
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.76, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5205, 15, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \; 5205$$

$$\frac{1}{2}c^2\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}dx-\frac{bc\sqrt{1-c^2x^2}\int \frac{1}{x^2}dx}{2\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2dx^2}$$

$$\Big\downarrow \; 15$$

$$\frac{1}{2}c^2\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}dx-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2dx^2}+\frac{bc\sqrt{1-c^2x^2}}{2x\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5219}$$

$$-\frac{c^2\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)}{2\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2dx^2}+\frac{bc\sqrt{1-c^2x^2}}{2x\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{c^2\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{2\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2dx^2}+\frac{bc\sqrt{1-c^2x^2}}{2x\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{c^2\sqrt{1-c^2x^2}\left(-b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(e^{i\arccos}\right.\right.}{2\sqrt{d-c^2dx^2}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2dx^2}+\frac{bc\sqrt{1-c^2x^2}}{2x\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{c^2\sqrt{1-c^2x^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos}\right.}{2\sqrt{d-c^2dx^2}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2dx^2}+\frac{bc\sqrt{1-c^2x^2}}{2x\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{c^2\sqrt{1-c^2x^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,ie^{i\arccos}\right.\right.}{2\sqrt{d-c^2dx^2}}$$
$$\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2dx^2}+\frac{bc\sqrt{1-c^2x^2}}{2x\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(b*c*Sqrt[1 - c^2*x^2])/(2*x*Sqrt[d - c^2*d*x^2]) - (Sqrt[d - c^2*d*x^2]*(
a + b*ArcCos[c*x]))/(2*d*x^2) - (c^2*Sqrt[1 - c^2*x^2]*((-2*I)*(a + b*ArcC
os[c*x])*ArcTan[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])
] - I*b*PolyLog[2, I*E^(I*ArcCos[c*x])]))/(2*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.28

method	result
default	$-\frac{a\sqrt{-c^2 d\,x^2+d}}{2d\,x^2}-\frac{a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)}{2\sqrt{d}}+b\left(-\frac{\left(c^2x^2\arccos(cx)+cx\sqrt{-c^2x^2+1}-\arccos(cx)\right)\sqrt{-d(c^2x^2-1)}}{2x^2d(c^2x^2-1)}-\right.$
parts	$-\frac{a\sqrt{-c^2 d\,x^2+d}}{2d\,x^2}-\frac{a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)}{2\sqrt{d}}+b\left(-\frac{\left(c^2x^2\arccos(cx)+cx\sqrt{-c^2x^2+1}-\arccos(cx)\right)\sqrt{-d(c^2x^2-1)}}{2x^2d(c^2x^2-1)}-\right.$

input
```
int((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/x^2*(-c^2*d*x^2+d)^(1/2)-1/2*a*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(-c^
2*d*x^2+d)^(1/2))/x)+b*(-1/2*(c^2*x^2*arccos(c*x)+c*x*(-c^2*x^2+1)^(1/2)-a
rccos(c*x))*(-d*(c^2*x^2-1))^(1/2)/x^2/d/(c^2*x^2-1)-1/2*(-c^2*x^2+1)^(1/2
)*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*(arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^
2+1)^(1/2)))-arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-I*dilog(1+I*(c
*x+I*(-c^2*x^2+1)^(1/2)))+I*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2))))*c^2)
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx=\int \frac{b\arccos{(cx)}+a}{\sqrt{-c^2dx^2+dx^3}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^2*d*x^5 - d*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx=\int \frac{a+b\,\mathrm{acos}\,(cx)}{x^3\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))/x**3/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))/(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx=\int \frac{b\arccos\,(cx)+a}{\sqrt{-c^2dx^2+d}x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
-1/2*(c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d)
+ sqrt(-c^2*d*x^2 + d)/(d*x^2))*a + b*integrate(arctan2(sqrt(c*x + 1)*sqrt
(-c*x + 1), c*x)/(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^3), x)/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{x^3\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))/(x^3*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))/(x^3*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\,a+2\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,x^3}dx\right)b\,x^2+\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a\,c^2x^2}{2\sqrt{d}\,x^2}$$

input
```
int((a+b*acos(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*a + 2*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*x**
3),x)*b*x**2 + log(tan(asin(c*x)/2))*a*c**2*x**2)/(2*sqrt(d)*x**2)
```

3.120 $\int \frac{a+b\arccos(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{a+b\arccos(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx=-\frac{bc\sqrt{1-c^2x^2}}{6x^2\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3dx^3}-\frac{2c^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3dx}+\frac{2bc^3\sqrt{1-c^2x^2}\log(x)}{3\sqrt{d-c^2dx^2}}$$

output
```
-1/6*b*c*(-c^2*x^2+1)^(1/2)/x^2/(-c^2*d*x^2+d)^(1/2)-1/3*(-c^2*d*x^2+d)^(1
/2)*(a+b*arccos(c*x))/d/x^3-2/3*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))
/d/x+2/3*b*c^3*(-c^2*x^2+1)^(1/2)*ln(x)/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{a+b\arccos(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx$$
$$=\frac{\sqrt{d-c^2dx^2}\left(bcx+6bc^3x^3-2a\sqrt{1-c^2x^2}-4ac^2x^2\sqrt{1-c^2x^2}-2b\sqrt{1-c^2x^2}(1+2c^2x^2)\arccos(cx)-4\right.}{6dx^3\sqrt{1-c^2x^2}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(b*c*x + 6*b*c^3*x^3 - 2*a*Sqrt[1 - c^2*x^2] - 4*a*c^
2*x^2*Sqrt[1 - c^2*x^2] - 2*b*Sqrt[1 - c^2*x^2]*(1 + 2*c^2*x^2)*ArcCos[c*x
] - 4*b*c^3*x^3*Log[x]))/(6*d*x^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5205, 15, 5187, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^4\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{5205}$$

$$\frac{2}{3}c^2\int \frac{a+b\arccos(cx)}{x^2\sqrt{d-c^2dx^2}}dx-\frac{bc\sqrt{1-c^2x^2}\int \frac{1}{x^3}dx}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3dx^3}$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{3}c^2\int \frac{a+b\arccos(cx)}{x^2\sqrt{d-c^2dx^2}}dx-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3dx^3}+\frac{bc\sqrt{1-c^2x^2}}{6x^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5187}$$

$$\frac{2}{3}c^2\left(-\frac{bc\sqrt{1-c^2x^2}\int \frac{1}{x}dx}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{dx}\right)-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3dx^3}+\frac{bc\sqrt{1-c^2x^2}}{6x^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{14}$$

$$\frac{2}{3}c^2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{dx}-\frac{bc\sqrt{1-c^2x^2}\log(x)}{\sqrt{d-c^2dx^2}}\right)-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3dx^3}+\frac{bc\sqrt{1-c^2x^2}}{6x^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(b*c*Sqrt[1 - c^2*x^2])/(6*x^2*Sqrt[d - c^2*d*x^2]) - (Sqrt[d - c^2*d*x^2]
*(a + b*ArcCos[c*x]))/(3*d*x^3) + (2*c^2*(-((Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCos[c*x]))/(d*x)) - (b*c*Sqrt[1 - c^2*x^2]*Log[x])/Sqrt[d - c^2*d*x^2]))/
3
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 851, normalized size of antiderivative = 5.79

method	result
default	$a\left(-\frac{\sqrt{-c^2d\,x^2+d}}{3d\,x^3}-\frac{2c^2\sqrt{-c^2d\,x^2+d}}{3dx}\right)+\frac{2ib\sqrt{-d(c^2x^2-1)}\,x^2\arccos(cx)\sqrt{-c^2x^2+1}\,c^5}{(3c^4x^4-2c^2x^2-1)d}-\frac{2ib\sqrt{-d(c^2x^2-1)}\,x^5c^8}{3(3c^4x^4-2c^2x^2-1)d}-\frac{4ib\sqrt{}}{}$
parts	$a\left(-\frac{\sqrt{-c^2d\,x^2+d}}{3d\,x^3}-\frac{2c^2\sqrt{-c^2d\,x^2+d}}{3dx}\right)+\frac{2ib\sqrt{-d(c^2x^2-1)}\,x^2\arccos(cx)\sqrt{-c^2x^2+1}\,c^5}{(3c^4x^4-2c^2x^2-1)d}-\frac{2ib\sqrt{-d(c^2x^2-1)}\,x^5c^8}{3(3c^4x^4-2c^2x^2-1)d}-\frac{4ib\sqrt{}}{}$

input
```
int((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3/d/x^3*(-c^2*d*x^2+d)^(1/2)-2/3*c^2/d/x*(-c^2*d*x^2+d)^(1/2))+2*I*b
*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x^2*arccos(c*x)*(-c^2*x^
2+1)^(1/2)*c^5-2/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x^
5*c^8-4/3*I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)*arcc
os(c*x)*c^3-2/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x^3*(
-c^2*x^2+1)*c^6-2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x^3*a
rccos(c*x)*c^6+2/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*ar
ccos(c*x)*(-c^2*x^2+1)^(1/2)*c^3-1/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4
-2*c^2*x^2-1)/d*x*(-c^2*x^2+1)*c^4+1/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x
^4-2*c^2*x^2-1)/d*x^3*c^6+1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^
2-1)/d*x*arccos(c*x)*c^4+1/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x
^2-1)/d*x*c^4-1/2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*c^3*(
-c^2*x^2+1)^(1/2)+4/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d/x
*arccos(c*x)*c^2-1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d/x^
2*(-c^2*x^2+1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1
)/d/x^3*arccos(c*x)+2/3*b*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d/(c^2
*x^2-1)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*c^3
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.94

$$\int \frac{a + b \arccos(cx)}{x^4\sqrt{d - c^2dx^2}}\,dx$$
$$= \left[\frac{2\,(bc^5x^5 - bc^3x^3)\sqrt{d}\log\left(\frac{c^2dx^6 + c^2dx^2 - dx^4 + \sqrt{-c^2dx^2 + d}\sqrt{-c^2x^2 + 1}\,(x^4 - 1)\sqrt{d} - d}{c^2x^4 - x^2}\right) + \sqrt{-c^2dx^2 + d}(bcx^3 - bcx)\sqrt{-}}{6\,(c^2dx^5 - dx^3)}\right.$$
$$-\frac{4\,(bc^5x^5 - bc^3x^3)\sqrt{-d}\arctan\left(\frac{\sqrt{-c^2dx^2 + d}\sqrt{-c^2x^2 + 1}\,(x^2 - 1)\sqrt{-d}}{c^2dx^4 + (c^2 - 1)dx^2 - d}\right) - \sqrt{-c^2dx^2 + d}(bcx^3 - bcx)\sqrt{-c^2x^2 + 1}}{6\,(c^2dx^5 - dx^3)}$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/6*(2*(b*c^5*x^5 - b*c^3*x^3)*sqrt(d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4
 + sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^4 - 1)*sqrt(d) - d)/(c^2*x^4
 - x^2)) + sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(-c^2*x^2 + 1) - 2*(
2*a*c^4*x^4 - a*c^2*x^2 + (2*b*c^4*x^4 - b*c^2*x^2 - b)*arccos(c*x) - a)*s
qrt(-c^2*d*x^2 + d))/(c^2*d*x^5 - d*x^3), -1/6*(4*(b*c^5*x^5 - b*c^3*x^3)*
sqrt(-d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*(x^2 - 1)*sqrt(-d)
/(c^2*d*x^4 + (c^2 - 1)*d*x^2 - d)) - sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*
x)*sqrt(-c^2*x^2 + 1) + 2*(2*a*c^4*x^4 - a*c^2*x^2 + (2*b*c^4*x^4 - b*c^2*
x^2 - b)*arccos(c*x) - a)*sqrt(-c^2*d*x^2 + d))/(c^2*d*x^5 - d*x^3)]
```

Sympy [F]

$$\int \frac{a + b \arccos(cx)}{x^4\sqrt{d - c^2dx^2}}\,dx = \int \frac{a + b\,\mathrm{acos}\,(cx)}{x^4\sqrt{-d\,(cx - 1)\,(cx + 1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))/x**4/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))/(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int \frac{a+b\arccos(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx = -\frac{1}{6}\left(\frac{4\,c^2\log\left(x\right)}{\sqrt{d}}-\frac{1}{\sqrt{d}x^2}\right)bc - \frac{1}{3}\,b\left(\frac{2\,\sqrt{-c^2dx^2+d}c^2}{dx}+\frac{\sqrt{-c^2dx^2+d}}{dx^3}\right)\arccos\left(cx\right) - \frac{1}{3}\,a\left(\frac{2\,\sqrt{-c^2dx^2+d}c^2}{dx}+\frac{\sqrt{-c^2dx^2+d}}{dx^3}\right)$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
-1/6*(4*c^2*log(x)/sqrt(d) - 1/(sqrt(d)*x^2))*b*c - 1/3*b*(2*sqrt(-c^2*d*x
^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3))*arccos(c*x) - 1/3*a*(2*s
qrt(-c^2*d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{x^4\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))/(x^4*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))/(x^4*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx = \frac{-2\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a+3\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,x^4}dx\right)b\,x^3}{3\sqrt{d}\,x^3}$$

input
```
int((a+b*acos(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - 2*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a + 3*in
t(acos(c*x)/(sqrt( - c**2*x**2 + 1)*x**4),x)*b*x**3)/(3*sqrt(d)*x**3)
```

3.121 $\int \frac{x^5(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 221

$$\begin{aligned}\int \frac{x^5(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^{3/2}}\,dx &= -\frac{5bx\sqrt{d-c^2dx^2}}{3c^5d^2\sqrt{1-c^2x^2}}-\frac{bx^3\sqrt{d-c^2dx^2}}{9c^3d^2\sqrt{1-c^2x^2}}\\ &\quad+\frac{a+b\arccos(cx)}{c^6d\sqrt{d-c^2dx^2}}+\frac{2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^6d^2}\\ &\quad-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{3c^6d^3}-\frac{b\sqrt{d-c^2dx^2}\operatorname{arctanh}(cx)}{c^6d^2\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-5/3*b*x*(-c^2*d*x^2+d)^(1/2)/c^5/d^2/(-c^2*x^2+1)^(1/2)-1/9*b*x^3*(-c^2*d
*x^2+d)^(1/2)/c^3/d^2/(-c^2*x^2+1)^(1/2)+(a+b*arccos(c*x))/c^6/d/(-c^2*d*x
^2+d)^(1/2)+2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c^6/d^2-1/3*(-c^2*d*x
^2+d)^(3/2)*(a+b*arccos(c*x))/c^6/d^3-b*(-c^2*d*x^2+d)^(1/2)*arctanh(c*x)/
c^6/d^2/(-c^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.47 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{\sqrt{d-c^2dx^2}\left(c\left(-bcx\sqrt{1-c^2x^2}(15+c^2x^2)+3a(-8+4c^2x^2+c^4x^4)+3b(-8\right.\right.}{9c^7d^2}$$

input
```
Integrate[(x^5*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(c*(-(b*c*x*Sqrt[1 - c^2*x^2]*(15 + c^2*x^2)) + 3*a*(
-8 + 4*c^2*x^2 + c^4*x^4) + 3*b*(-8 + 4*c^2*x^2 + c^4*x^4)*ArcCos[c*x]) -
(9*I)*b*Sqrt[-c^2]*Sqrt[1 - c^2*x^2]*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], 1]
))/(9*c^7*d^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5195, 27, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5195}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \frac{-c^4x^4-4c^2x^2+8}{3c^6d^2(1-c^2x^2)}dx}{\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{3c^6d^3} + \frac{2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^6d^2} + \frac{a+b\arccos(cx)}{c^6d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\sqrt{d-c^2dx^2}\int\frac{-c^4x^4-4c^2x^2+8}{1-c^2x^2}dx}{3c^5d^2\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{3c^6d^3}+\frac{2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^6d^2}+\frac{a+b\arccos(cx)}{c^6d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{1467}$$

$$\frac{b\sqrt{d-c^2dx^2}\int\left(c^2x^2+\frac{3}{1-c^2x^2}+5\right)dx}{3c^5d^2\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{3c^6d^3}+\frac{2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^6d^2}+\frac{a+b\arccos(cx)}{c^6d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{3c^6d^3}+\frac{2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^6d^2}+\frac{a+b\arccos(cx)}{c^6d\sqrt{d-c^2dx^2}}+\frac{b\left(\frac{3\text{arctanh}(cx)}{c}+\frac{c^2x^3}{3}+5x\right)\sqrt{d-c^2dx^2}}{3c^5d^2\sqrt{1-c^2x^2}}$$

input
```
Int[(x^5*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(a + b*ArcCos[c*x])/(c^6*d*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCos[c*x]))/(c^6*d^2) - ((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])
)/(3*c^6*d^3) + (b*Sqrt[d - c^2*d*x^2]*(5*x + (c^2*x^3)/3 + (3*ArcTanh[c*x
])/c))/(3*c^5*d^2*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.92

method	result
default	$a\left(-\frac{x^4}{3c^2d\sqrt{-c^2d\,x^2+d}}+\frac{-\frac{4x^2}{3c^2d\sqrt{-c^2d\,x^2+d}}+\frac{8}{3d\,c^4\sqrt{-c^2d\,x^2+d}}}{c^2}\right)-\frac{31b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,x}{18d^2c^5(c^2x^2-1)}+\frac{5b\sqrt{-d(c^2x^2-1)}\,\mathrm{a}}{3d^2c^4(c^2x^2}$
parts	$a\left(-\frac{x^4}{3c^2d\sqrt{-c^2d\,x^2+d}}+\frac{-\frac{4x^2}{3c^2d\sqrt{-c^2d\,x^2+d}}+\frac{8}{3d\,c^4\sqrt{-c^2d\,x^2+d}}}{c^2}\right)-\frac{31b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,x}{18d^2c^5(c^2x^2-1)}+\frac{5b\sqrt{-d(c^2x^2-1)}\,\mathrm{a}}{3d^2c^4(c^2x^2}$

input
```
int(x^5*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3*x^4/c^2/d/(-c^2*d*x^2+d)^(1/2)+4/3/c^2*(-x^2/c^2/d/(-c^2*d*x^2+d)^
(1/2)+2/d/c^4/(-c^2*d*x^2+d)^(1/2)))-31/18*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^
5/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x+5/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c
^2*x^2-1)*arccos(c*x)*x^2-65/24*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-
1)*arccos(c*x)+1/24*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*arccos(c*
x)*cos(4*arccos(c*x))-b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^6/
(c^2*x^2-1)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*
x^2+1)^(1/2)/d^2/c^6/(c^2*x^2-1)*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)-1/72*b*(-d
*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*sin(4*arccos(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.00

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \left[\frac{9\left(bc^2x^2-b\right)\sqrt{d}\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2-4\,(c^3x^3+cx)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}\sqrt{d}-d}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right)}{\quad}\right.$$

input
```
integrate(x^5*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
[1/36*(9*(b*c^2*x^2 - b)*sqrt(d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x
^2 - 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*sqrt(d) - d
)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) - 4*(b*c^3*x^3 + 15*b*c*x)*sqrt(-
c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 12*(a*c^4*x^4 + 4*a*c^2*x^2 + (b*c^4*x
^4 + 4*b*c^2*x^2 - 8*b)*arccos(c*x) - 8*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d^2*
x^2 - c^6*d^2), 1/18*(9*(b*c^2*x^2 - b)*sqrt(-d)*arctan(2*sqrt(-c^2*d*x^2
+ d)*sqrt(-c^2*x^2 + 1)*c*sqrt(-d)*x/(c^4*d*x^4 - d)) - 2*(b*c^3*x^3 + 15*
b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) + 6*(a*c^4*x^4 + 4*a*c^2*x^
2 + (b*c^4*x^4 + 4*b*c^2*x^2 - 8*b)*arccos(c*x) - 8*a)*sqrt(-c^2*d*x^2 + d
))/(c^8*d^2*x^2 - c^6*d^2)]
```

Sympy [F]

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^5(a+b\operatorname{acos}(cx))}{\left(-d\,(cx-1)\,(cx+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**5*(a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)x^5}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
-1/3*a*(x^4/(sqrt(-c^2*d*x^2 + d)*c^2*d) + 4*x^2/(sqrt(-c^2*d*x^2 + d)*c^4
*d) - 8/(sqrt(-c^2*d*x^2 + d)*c^6*d)) + 1/3*(3*sqrt(c*x + 1)*sqrt(-c*x + 1
)*c^6*d^2*integrate(1/3*(c^4*x^6 + 4*c^2*x^4 - 8*x^2)/(c^7*d^2*x^4 - c^5*d
^2*x^2 + (c^5*d^2*x^2 - c^3*d^2)*e^(log(c*x + 1) + log(-c*x + 1))), x) - (
c^4*x^4 + 4*c^2*x^2 - 8)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*b/(sq
rt(c*x + 1)*sqrt(-c*x + 1)*c^6*d^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^5\,(a+b\,\mathrm{acos}(c\,x))}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x^5*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^5*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-3\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)x^5}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)b\,c^6-a\,c^4x^4-4a\,c^2x^2+8a}{3\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^6d}$$

input
```
int(x^5*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**5)/(sqrt( - c**2*x**2 + 1)*
c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b*c**6 - a*c**4*x**4 - 4*a*c**2*x**
2 + 8*a)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*c**6*d)
```

3.122 $\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 214

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = -\frac{bx^2\sqrt{1-c^2x^2}}{4c^3d\sqrt{d-c^2dx^2}} + \frac{x^3(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}} + \frac{3x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2c^4d^2} - \frac{3\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4bc^5d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{1-c^2x^2}\log(1-c^2x^2)}{2c^5d\sqrt{d-c^2dx^2}}$$

output
```
-1/4*b*x^2*(-c^2*x^2+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)+x^3*(a+b*arccos(c
*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)+3/2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x
))/c^4/d^2-3/4*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/b/c^5/d/(-c^2*d*x^2+
d)^(1/2)+1/2*b*(-c^2*x^2+1)^(1/2)*ln(-c^2*x^2+1)/c^5/d/(-c^2*d*x^2+d)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.93

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \sqrt{-d\left(-1+c^2x^2\right)}\left(\frac{ax}{2c^4d^2}-\frac{ax}{c^4d^2\left(-1+c^2x^2\right)}\right) + \frac{3a\arctan\left(\frac{cx\sqrt{-d(-1+c^2x^2)}}{\sqrt{d}(-1+c^2x^2)}\right)}{2c^5d^{3/2}} - \frac{b\left(-8cx\arccos(cx)-\sqrt{1-c^2x^2}\left(6\arccos(cx)^2+\cos(2\arccos(cx))-8\log\left(\sqrt{1-c^2x^2}\right)+2\arccos(cx)\operatorname{si}\right.\right.}{8c^5d\sqrt{d\left(1-c^2x^2\right)}}$$

input
```
Integrate[(x^4*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
Sqrt[-(d*(-1 + c^2*x^2))]*((a*x)/(2*c^4*d^2) - (a*x)/(c^4*d^2*(-1 + c^2*x^
2))) + (3*a*ArcTan[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2)
)])/(2*c^5*d^(3/2)) - (b*(-8*c*x*ArcCos[c*x] - Sqrt[1 - c^2*x^2]*(6*ArcCos
[c*x]^2 + Cos[2*ArcCos[c*x]] - 8*Log[Sqrt[1 - c^2*x^2]] + 2*ArcCos[c*x]*Si
n[2*ArcCos[c*x]])))/(8*c^5*d*Sqrt[d*(1 - c^2*x^2)])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5207, 243, 49, 2009, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5207}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{1-c^2x^2}\int \frac{x^3}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{1-c^2x^2}\int \frac{x^2}{1-c^2x^2}dx^2}{2cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{1-c^2x^2}\int \left(-\frac{1}{c^2}-\frac{1}{c^2(c^2x^2-1)}\right)dx^2}{2cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x^3(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{2cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5211}$$

$$-\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{b\sqrt{1-c^2x^2}\int xdx}{2c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2c^2d}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{2cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2c^2d}-\frac{bx^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{2cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{x^3(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2c^2d}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{bx^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}}\right)}{c^2d}+\frac{b\sqrt{1-c^2x^2}\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{2cd\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^4*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x^3*(a + b*ArcCos[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) - (3*(-1/4*(b*x^2*Sq
rt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2]) - (x*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCos[c*x]))/(2*c^2*d) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(4*b*c^
3*Sqrt[d - c^2*d*x^2])))/(c^2*d) + (b*Sqrt[1 - c^2*x^2]*(-(x^2/c^2) - Log[
1 - c^2*x^2]/c^4))/(2*c*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.02

method	result
default	$-\frac{a\,x^{3}}{2c^{2}d\sqrt{-c^{2}d\,x^{2}+d}}+\frac{3ax}{2c^{4}d\sqrt{-c^{2}d\,x^{2}+d}}-\frac{3a\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{2c^{4}d\sqrt{c^{2}d}}-\frac{3b\sqrt{-d(c^{2}x^{2}-1)}\,\sqrt{-c^{2}x^{2}+1}\,\arccos(cx)^{2}}{4d^{2}c^{5}(c^{2}x^{2}-1)}-\frac{ib\sqrt{-}}{}$
parts	$-\frac{a\,x^{3}}{2c^{2}d\sqrt{-c^{2}d\,x^{2}+d}}+\frac{3ax}{2c^{4}d\sqrt{-c^{2}d\,x^{2}+d}}-\frac{3a\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{2c^{4}d\sqrt{c^{2}d}}-\frac{3b\sqrt{-d(c^{2}x^{2}-1)}\,\sqrt{-c^{2}x^{2}+1}\,\arccos(cx)^{2}}{4d^{2}c^{5}(c^{2}x^{2}-1)}-\frac{ib\sqrt{-}}{}$

input
```
int(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*x^3/c^2/d/(-c^2*d*x^2+d)^(1/2)+3/2*a/c^4*x/d/(-c^2*d*x^2+d)^(1/2)-3
/2*a/c^4/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-3/4*
b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arccos(c*x
)^2-I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arcc
os(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*ln
((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)-9/8*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2
*x^2-1)*arccos(c*x)*x+1/16*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*(-
c^2*x^2+1)^(1/2)+1/8*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*arccos(c
*x)*cos(3*arccos(c*x))-1/16*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*s
in(3*arccos(c*x))
```

Fricas [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)x^4}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral((b*x^4*arccos(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2
*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x^4(a+b\operatorname{acos}(cx))}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**4*(a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)x^4}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
-1/2*a*(x^3/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 3*x/(sqrt(-c^2*d*x^2 + d)*c^4*d
) + 3*arcsin(c*x)/(c^5*d^(3/2))) - b*integrate(x^4*arctan2(sqrt(c*x + 1)*s
qrt(-c*x + 1), c*x)/((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqr
t(d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 969 vs. 2(190) = 380.

Time = 0.87 (sec) , antiderivative size = 969, normalized size of antiderivative = 4.53

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
1/8*c*(8*b*arccos(c*x)/(sqrt(-c^2*x^2 + 1)*c^5*d^(3/2)/x + (-c^2*x^2 + 1)^
(3/2)*c^3*d^(3/2)/x^3) + 6*sqrt(-c^2*x^2 + 1)*b*arccos(c*x)^2/((sqrt(-c^2*
x^2 + 1)*c^5*d^(3/2)/x + (-c^2*x^2 + 1)^(3/2)*c^3*d^(3/2)/x^3)*c*x) + 8*a/
(sqrt(-c^2*x^2 + 1)*c^5*d^(3/2)/x + (-c^2*x^2 + 1)^(3/2)*c^3*d^(3/2)/x^3)
+ 12*sqrt(-c^2*x^2 + 1)*a*arccos(c*x)/((sqrt(-c^2*x^2 + 1)*c^5*d^(3/2)/x +
 (-c^2*x^2 + 1)^(3/2)*c^3*d^(3/2)/x^3)*c*x) - 8*sqrt(-c^2*x^2 + 1)*b*log(2
)/((sqrt(-c^2*x^2 + 1)*c^5*d^(3/2)/x + (-c^2*x^2 + 1)^(3/2)*c^3*d^(3/2)/x^
3)*c*x) - 4*sqrt(-c^2*x^2 + 1)*b*log(abs(-c^2*x^2 + 1))/((sqrt(-c^2*x^2 +
1)*c^5*d^(3/2)/x + (-c^2*x^2 + 1)^(3/2)*c^3*d^(3/2)/x^3)*c*x) + sqrt(-c^2*
x^2 + 1)*b/((sqrt(-c^2*x^2 + 1)*c^5*d^(3/2)/x + (-c^2*x^2 + 1)^(3/2)*c^3*d
^(3/2)/x^3)*c*x) - 12*(c^2*x^2 - 1)*b*arccos(c*x)/((sqrt(-c^2*x^2 + 1)*c^5
*d^(3/2)/x + (-c^2*x^2 + 1)^(3/2)*c^3*d^(3/2)/x^3)*c^2*x^2) + 6*(-c^2*x^2
+ 1)^(3/2)*b*arccos(c*x)^2/((sqrt(-c^2*x^2 + 1)*c^5*d^(3/2)/x + (-c^2*x^2
+ 1)^(3/2)*c^3*d^(3/2)/x^3)*c^3*x^3) - 12*(c^2*x^2 - 1)*a/((sqrt(-c^2*x^2
+ 1)*c^5*d^(3/2)/x + (-c^2*x^2 + 1)^(3/2)*c^3*d^(3/2)/x^3)*c^2*x^2) + 12*(
-c^2*x^2 + 1)^(3/2)*a*arccos(c*x)/((sqrt(-c^2*x^2 + 1)*c^5*d^(3/2)/x + (-c
^2*x^2 + 1)^(3/2)*c^3*d^(3/2)/x^3)*c^3*x^3) - 8*(-c^2*x^2 + 1)^(3/2)*b*log
(2)/((sqrt(-c^2*x^2 + 1)*c^5*d^(3/2)/x + (-c^2*x^2 + 1)^(3/2)*c^3*d^(3/2)/
x^3)*c^3*x^3) - 4*(-c^2*x^2 + 1)^(3/2)*b*log(abs(-c^2*x^2 + 1))/((sqrt(-c^
2*x^2 + 1)*c^5*d^(3/2)/x + (-c^2*x^2 + 1)^(3/2)*c^3*d^(3/2)/x^3)*c^3*x^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x^4\,(a+b\,\mathrm{acos}(c\,x))}{{(d-c^2\,d\,x^2)}^{3/2}}\,dx$$

input
```
int((x^4*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\,acos(cx)^2\,b-2acos(cx)\,b\,c^3x^3+2acos(cx)\,bcx-6\sqrt{-c^2x^2+1}}{}$$

input

```
int(x^4*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output

```
(3*sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b - 2*acos(c*x)*b*c**3*x**3 + 2*aco
s(c*x)*b*c*x - 6*sqrt( - c**2*x**2 + 1)*asin(c*x)*a - 4*sqrt( - c**2*x**2
+ 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 +
 1)),x)*b*c + sqrt( - c**2*x**2 + 1)*b*c**2*x**2 - sqrt( - c**2*x**2 + 1)*
b - 2*a*c**3*x**3 + 6*a*c*x)/(4*sqrt(d)*sqrt( - c**2*x**2 + 1)*c**5*d)
```

3.123 $\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 142

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = -\frac{bx\sqrt{d-c^2dx^2}}{c^3d^2\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{c^4d\sqrt{d-c^2dx^2}} + \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^4d^2}-\frac{b\sqrt{d-c^2dx^2}\text{arctanh}(cx)}{c^4d^2\sqrt{1-c^2x^2}}$$

output

```
-b*x*(-c^2*d*x^2+d)^(1/2)/c^3/d^2/(-c^2*x^2+1)^(1/2)+(a+b*arccos(c*x))/c^4
/d/(-c^2*d*x^2+d)^(1/2)+(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c^4/d^2-b*(
-c^2*d*x^2+d)^(1/2)*arctanh(c*x)/c^4/d^2/(-c^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{\sqrt{d-c^2dx^2}\left(\sqrt{-c^2}\left(-2a+ac^2x^2-bcx\sqrt{1-c^2x^2}+b(-2+c^2x^2)\arccos(cx)\right)\right.}{c^4\sqrt{-c^2}d^2\left(-1+c^2x^2\right)}$$

input

```
Integrate[(x^3*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(Sqrt[-c^2]*(-2*a + a*c^2*x^2 - b*c*x*Sqrt[1 - c^2*x^
2] + b*(-2 + c^2*x^2)*ArcCos[c*x]) + I*b*c*Sqrt[1 - c^2*x^2]*EllipticF[I*A
rcSinh[Sqrt[-c^2]*x], 1]))/(c^4*Sqrt[-c^2]*d^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5195, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5195}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int \frac{2-c^2x^2}{c^4d^2(1-c^2x^2)}dx}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^4d^2}+\frac{a+b\arccos(cx)}{c^4d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\sqrt{d-c^2dx^2}\int \frac{2-c^2x^2}{1-c^2x^2}dx}{c^3d^2\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^4d^2}+\frac{a+b\arccos(cx)}{c^4d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{b\sqrt{d-c^2dx^2}\left(\int \frac{1}{1-c^2x^2}dx+x\right)}{c^3d^2\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^4d^2}+\frac{a+b\arccos(cx)}{c^4d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^4d^2}+\frac{a+b\arccos(cx)}{c^4d\sqrt{d-c^2dx^2}}+\frac{b\left(\frac{\text{arctanh}(cx)}{c}+x\right)\sqrt{d-c^2dx^2}}{c^3d^2\sqrt{1-c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(a + b*ArcCos[c*x])/(c^4*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCos[c*x]))/(c^4*d^2) + (b*Sqrt[d - c^2*d*x^2]*(x + ArcTanh[c*x]/c))
/(c^3*d^2*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.17

method	result
default	$a\left(-\frac{x^2}{c^2d\sqrt{-c^2d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{-c^2d\,x^2+d}}\right)+\frac{b\sqrt{-d(c^2x^2-1)}\,\arccos(cx)x^2}{d^2c^2(c^2x^2-1)}-\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,x}{d^2c^3(c^2x^2-1)}-\frac{2b\sqrt{-d(c^2}{d^2c}$
parts	$a\left(-\frac{x^2}{c^2d\sqrt{-c^2d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{-c^2d\,x^2+d}}\right)+\frac{b\sqrt{-d(c^2x^2-1)}\,\arccos(cx)x^2}{d^2c^2(c^2x^2-1)}-\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,x}{d^2c^3(c^2x^2-1)}-\frac{2b\sqrt{-d(c^2}{d^2c}$

input
```
int(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2/d/c^4/(-c^2*d*x^2+d)^(1/2))+b*(-d*(c^
2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arccos(c*x)*x^2-b*(-d*(c^2*x^2-1))^(1/
2)/d^2/c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*x-2*b*(-d*(c^2*x^2-1))^(1/2)/d^2
/c^4/(c^2*x^2-1)*arccos(c*x)-b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d
^2/c^4/(c^2*x^2-1)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+b*(-d*(c^2*x^2-1))^(1/2)
*(-c^2*x^2+1)^(1/2)/d^2/c^4/(c^2*x^2-1)*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.70

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \left[-\frac{4\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}bcx-(bc^2x^2-b)\sqrt{d}\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2}{c^6}\right.}{}\right.$$

$$-\frac{2\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}bcx-(bc^2x^2-b)\sqrt{-d}\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}c\sqrt{-d}x}{c^4dx^4-d}\right)-2\left(ac^2x^2+(bc^2x\right.}{2\left(c^6d^2x^2-c^4d^2\right)}$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
[-1/4*(4*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*b*c*x - (b*c^2*x^2 - b)*s
qrt(d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x)*sqr
t(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*sqrt(d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3
*c^2*x^2 - 1)) - 4*(a*c^2*x^2 + (b*c^2*x^2 - 2*b)*arccos(c*x) - 2*a)*sqrt(
-c^2*d*x^2 + d))/(c^6*d^2*x^2 - c^4*d^2), -1/2*(2*sqrt(-c^2*d*x^2 + d)*sqr
t(-c^2*x^2 + 1)*b*c*x - (b*c^2*x^2 - b)*sqrt(-d)*arctan(2*sqrt(-c^2*d*x^2
+ d)*sqrt(-c^2*x^2 + 1)*c*sqrt(-d)*x/(c^4*d*x^4 - d)) - 2*(a*c^2*x^2 + (b*
c^2*x^2 - 2*b)*arccos(c*x) - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*d^2*x^2 - c^4
*d^2)]
```

Sympy [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^3(a+b\operatorname{acos}(cx))}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**3*(a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{1}{2}bc\left(\frac{2x}{c^4d^{\frac{3}{2}}}+\frac{\log(cx+1)}{c^5d^{\frac{3}{2}}}-\frac{\log(cx-1)}{c^5d^{\frac{3}{2}}}\right) - b\left(\frac{x^2}{\sqrt{-c^2dx^2+d}c^2d}-\frac{2}{\sqrt{-c^2dx^2+d}c^4d}\right)\arccos(cx) - a\left(\frac{x^2}{\sqrt{-c^2dx^2+d}c^2d}-\frac{2}{\sqrt{-c^2dx^2+d}c^4d}\right)$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
1/2*b*c*(2*x/(c^4*d^(3/2)) + log(c*x + 1)/(c^5*d^(3/2)) - log(c*x - 1)/(c^
5*d^(3/2))) - b*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 2/(sqrt(-c^2*d*x^2 + d
)*c^4*d))*arccos(c*x) - a*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 2/(sqrt(-c^2
*d*x^2 + d)*c^4*d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^3\,(a+b\,\mathrm{acos}(c\,x))}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x^3*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3(a + b \arccos(cx))}{(d - c^2dx^2)^{3/2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)x^3}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right) b\,c^4 - a\,c^2x^2 + 2a}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^4d}$$

input
```
int(x^3*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**3)/(sqrt( - c**2*x**2 + 1)*c*
*2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b*c**4 - a*c**2*x**2 + 2*a)/(sqrt(d)*
sqrt( - c**2*x**2 + 1)*c**4*d)
```

3.124 $\int \frac{x^2(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 135

$$\int \frac{x^2(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^{3/2}}\,dx=\frac{x(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2bc^3d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\log\left(1-c^2x^2\right)}{2c^3d\sqrt{d-c^2dx^2}}$$

output
```
x*(a+b*arccos(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)-1/2*(-c^2*x^2+1)^(1/2)*(a+b
*arccos(c*x))^2/b/c^3/d/(-c^2*d*x^2+d)^(1/2)+1/2*b*(-c^2*x^2+1)^(1/2)*ln(-
c^2*x^2+1)/c^3/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.19

$$\int \frac{x^2(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^{3/2}}\,dx=-\frac{ax\sqrt{-d\left(-1+c^2x^2\right)}}{c^2d^2\left(-1+c^2x^2\right)}+\frac{a\arctan\left(\frac{cx\sqrt{-d(-1+c^2x^2)}}{\sqrt{d}(-1+c^2x^2)}\right)}{c^3d^{3/2}}-\frac{b\left(-2cx\arccos(cx)-\sqrt{1-c^2x^2}\left(\arccos(cx)^2-2\log\left(\sqrt{1-c^2x^2}\right)\right)\right)}{2c^3d\sqrt{d\left(1-c^2x^2\right)}}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
-((a*x*Sqrt[-(d*(-1 + c^2*x^2))])/(c^2*d^2*(-1 + c^2*x^2))) + (a*ArcTan[(c
*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/(c^3*d^(3/2)) - (
b*(-2*c*x*ArcCos[c*x] - Sqrt[1 - c^2*x^2]*(ArcCos[c*x]^2 - 2*Log[Sqrt[1 -
c^2*x^2]])))/(2*c^3*d*Sqrt[d*(1 - c^2*x^2)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5207, 240, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5207}$$

$$-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{1-c^2x^2}\int \frac{x}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{240}$$

$$-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{1-c^2x^2}\log\left(1-c^2x^2\right)}{2c^3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{x(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2bc^3d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{1-c^2x^2}\log\left(1-c^2x^2\right)}{2c^3d\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCos[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[1 - c^2*x^2]*(
a + b*ArcCos[c*x])^2)/(2*b*c^3*d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^
2]*Log[1 - c^2*x^2])/(2*c^3*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.03

method	result
default	$\frac{ax}{c^2 d\sqrt{-c^2 d\,x^2+d}}-\frac{a\arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d\,x^2+d}}\right)}{c^2 d\sqrt{c^2 d}}-\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2}{2d^2c^3(c^2x^2-1)}-\frac{ib\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\text{arcco}}{d^2c^3(c^2x^2-1)}$
parts	$\frac{ax}{c^2 d\sqrt{-c^2 d\,x^2+d}}-\frac{a\arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d\,x^2+d}}\right)}{c^2 d\sqrt{c^2 d}}-\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2}{2d^2c^3(c^2x^2-1)}-\frac{ib\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\text{arcco}}{d^2c^3(c^2x^2-1)}$

input
```
int(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-a/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*
x/(-c^2*d*x^2+d)^(1/2))-1/2*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^
2/c^3/(c^2*x^2-1)*arccos(c*x)^2-I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1
/2)/d^2/c^3/(c^2*x^2-1)*arccos(c*x)-b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*
x^2-1)*arccos(c*x)*x+b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^3/(
c^2*x^2-1)*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\arccos{(cx)}+a)x^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*x^2*arccos(c*x) + a*x^2)/(c^4*d^2*x^4 - 2
*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x^2(a+b\operatorname{acos}{(cx)})}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)x^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
a*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d) - arcsin(c*x)/(c^3*d^(3/2))) - b*integra
te(x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/((c^2*d*x^2 - d)*sqrt(c*
x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.82

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{1}{2}c\left(\frac{2\,bx\arccos(cx)}{\sqrt{-c^2x^2+1}c^3d^{\frac{3}{2}}}+\frac{b\arccos(cx)^2}{c^4d^{\frac{3}{2}}}+\frac{2\,ax}{\sqrt{-c^2x^2+1}c^3d^{\frac{3}{2}}}+\frac{2\,a\arccos(cx)}{c^4d^{\frac{3}{2}}}\right.$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
1/2*c*(2*b*x*arccos(c*x)/(sqrt(-c^2*x^2 + 1)*c^3*d^(3/2)) + b*arccos(c*x)^
2/(c^4*d^(3/2)) + 2*a*x/(sqrt(-c^2*x^2 + 1)*c^3*d^(3/2)) + 2*a*arccos(c*x)
/(c^4*d^(3/2)) - 2*b*log(2)/(c^4*d^(3/2)) - b*log(abs(-c^2*x^2 + 1))/(c^4*
d^(3/2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^2\,(a+b\,\mathrm{acos}(c\,x))}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{\sqrt{-c^2x^2+1}\,acos(cx)^2\,b-2\sqrt{-c^2x^2+1}\,asin(cx)\,a-2\sqrt{-c^2x^2+1}\left(\int \frac{}{\sqrt{-c^2x^2}}\right.}{2\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^3d}$$

input
```
int(x^2*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b - 2*sqrt( - c**2*x**2 + 1)*asin(c*x
)*a - 2*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*
x**2 - sqrt( - c**2*x**2 + 1)),x)*b*c + 2*a*c*x)/(2*sqrt(d)*sqrt( - c**2*x
**2 + 1)*c**3*d)
```

3.125 $\int \frac{x(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{x(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^{3/2}}\,dx=\frac{a+b\arccos(cx)}{c^2d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

output
```
(a+b*arccos(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)-b*(-c^2*x^2+1)^(1/2)*arctanh(
c*x)/c^2/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.34

$$\int \frac{x(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\sqrt{d-c^2dx^2}\left(-\sqrt{-c^2}(a+b\arccos(cx))+ibc\sqrt{1-c^2x^2}\operatorname{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right),1\right)\right)}{\left(-c^2\right)^{3/2}d^2\left(-1+c^2x^2\right)}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*(-(Sqrt[-c^2]*(a + b*ArcCos[c*x])) + I*b*c*Sqrt[1 -
 c^2*x^2]*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], 1]))/((-c^2)^(3/2)*d^2*(-1 +
c^2*x^2)))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5183, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5183}$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{1}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{a+b\arccos(cx)}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

input
```
Int[(x*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(a + b*ArcCos[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[1 - c^2*x^2]*Arc
Tanh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.66

method	result
default	$\frac{a}{c^{2} d \sqrt{-c^{2} d \,x^{2}+d}}+b \left(-\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \arccos \left(c x \right)}{d^{2} c^{2} \left(c^{2} x^{2}-1\right)}+\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \sqrt{-c^{2} x^{2}+1}\, \ln \left(i \sqrt{-c^{2} x^{2}+1}+c x -1\right)}{d^{2} c^{2} \left(c^{2} x^{2}-1\right)}-\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}}{}\right.$
parts	$\frac{a}{c^{2} d \sqrt{-c^{2} d \,x^{2}+d}}+b \left(-\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \arccos \left(c x \right)}{d^{2} c^{2} \left(c^{2} x^{2}-1\right)}+\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \sqrt{-c^{2} x^{2}+1}\, \ln \left(i \sqrt{-c^{2} x^{2}+1}+c x -1\right)}{d^{2} c^{2} \left(c^{2} x^{2}-1\right)}-\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}}{}\right.$

input
```
int(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a/c^2/d/(-c^2*d*x^2+d)^(1/2)+b*(-(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1
)*arccos(c*x)+(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^2/(c^2*x^2-1
)*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)-(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)
/d^2/c^2/(c^2*x^2-1)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.82

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \left[\frac{(bc^2x^2-b)\sqrt{d}\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2-4\,(c^3x^3+cx)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}\sqrt{d}-d}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right)-}{4\,(c^4d^2x^2-c^2d^2)}\right.$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((b*c^2*x^2 - b)*sqrt(d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2
- 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*sqrt(d) - d)/(
c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) - 4*sqrt(-c^2*d*x^2 + d)*(b*arccos(c
*x) + a))/(c^4*d^2*x^2 - c^2*d^2), 1/2*((b*c^2*x^2 - b)*sqrt(-d)*arctan(2*
sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*c*sqrt(-d)*x/(c^4*d*x^4 - d)) - 2*
sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a))/(c^4*d^2*x^2 - c^2*d^2)]
```

Sympy [F]

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x(a+b\operatorname{acos}(cx))}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x*(a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)x}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
-(sqrt(c*x + 1)*sqrt(-c*x + 1)*c^3*d^2*integrate(x^2/(c^4*d^2*x^4 - c^2*d^
2*x^2 + (c^2*d^2*x^2 - d^2)*e^(log(c*x + 1) + log(-c*x + 1))), x) - arctan
2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*b/(sqrt(c*x + 1)*sqrt(-c*x + 1)*c^2*
d^(3/2)) + a/(sqrt(-c^2*d*x^2 + d)*c^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b\arccos(cx))}{(d - c^2dx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\arccos(cx))}{(d - c^2dx^2)^{3/2}}\,dx = \int \frac{x\,(a + b\,\mathrm{acos}(c\,x))}{(d - c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)x}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)b\,c^2+a}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^2d}$$

input
```
int(x*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x)/(sqrt( - c**2*x**2 + 1)*c**2*
x**2 - sqrt( - c**2*x**2 + 1)),x)*b*c**2 + a)/(sqrt(d)*sqrt( - c**2*x**2 +
 1)*c**2*d)
```

3.126 $\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx=\frac{x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\log(1-c^2x^2)}{2cd\sqrt{d-c^2dx^2}}$$

output
```
x*(a+b*arccos(c*x))/d/(-c^2*d*x^2+d)^(1/2)+1/2*b*(-c^2*x^2+1)^(1/2)*ln(-c^
2*x^2+1)/c/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx=\frac{\sqrt{d-c^2dx^2}\left(-2acx-2bcx\arccos(cx)+b\sqrt{1-c^2x^2}\log(-1+c^2x^2)\right)}{2cd^2(-1+c^2x^2)}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(-2*a*c*x - 2*b*c*x*ArcCos[c*x] + b*Sqrt[1 - c^2*x^2]
*Log[-1 + c^2*x^2]))/(2*c*d^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5161, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}dx$$
$$\Big\downarrow \text{5161}$$
$$\frac{bc\sqrt{1-c^2x^2}\int \frac{x}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}$$
$$\Big\downarrow \text{240}$$
$$\frac{x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{1-c^2x^2}\log(1-c^2x^2)}{2cd\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCos[c*x]))/(d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*Log
[1 - c^2*x^2])/(2*c*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.21

method	result
default	$\frac{ax}{d\sqrt{-c^2dx^2+d}} - \frac{ib\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)}{c\,d^2(c^2x^2-1)} - \frac{b\sqrt{-d(c^2x^2-1)}\,\arccos(cx)x}{d^2(c^2x^2-1)} + \frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\ln\Big(\big(}{c\,d^2(c^2x^2-}$
parts	$\frac{ax}{d\sqrt{-c^2dx^2+d}} - \frac{ib\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)}{c\,d^2(c^2x^2-1)} - \frac{b\sqrt{-d(c^2x^2-1)}\,\arccos(cx)x}{d^2(c^2x^2-1)} + \frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\ln\Big(\big(}{c\,d^2(c^2x^2-}$

input
```
int((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a/d*x/(-c^2*d*x^2+d)^(1/2)-I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c
/d^2/(c^2*x^2-1)*arccos(c*x)-b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d^2/(c^2
*x^2-1)*x+b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d^2/(c^2*x^2-1)*ln
((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{b\arccos(cx)+a}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^4*d^2*x^4 - 2*c^2*d^2
*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{a+b\operatorname{acos}(cx)}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx = \frac{bx\arccos(cx)}{\sqrt{-c^2dx^2+d}d} + \frac{ax}{\sqrt{-c^2dx^2+d}d} + \frac{b\log\left(x^2-\frac{1}{c^2}\right)}{2\,cd^{\frac{3}{2}}}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
b*x*arccos(c*x)/(sqrt(-c^2*d*x^2 + d)*d) + a*x/(sqrt(-c^2*d*x^2 + d)*d) +
1/2*b*log(x^2 - 1/c^2)/(c*d^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)b+ax}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,d}$$

input
```
int((a+b*acos(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x**2
 - sqrt( - c**2*x**2 + 1)),x)*b + a*x)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*d)
```

3.127 $\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 220

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = \frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}} - \frac{2\sqrt{1-c^2x^2}(a+b\arccos(cx))\text{arctanh}\left(e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}} - \frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{ib\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}} - \frac{ib\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}}$$

output
```
(a+b*arccos(c*x))/d/(-c^2*d*x^2+d)^(1/2)-2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(
c*x))*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-b*(-c^2*x^2
+1)^(1/2)*arctanh(c*x)/d/(-c^2*d*x^2+d)^(1/2)+I*b*(-c^2*x^2+1)^(1/2)*polyl
og(2,-c*x-I*(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-I*b*(-c^2*x^2+1)^(1
/2)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.27

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = \frac{\frac{ad}{\sqrt{d-c^2dx^2}} + a\sqrt{d}\log(x) - a\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right) + \frac{bd\left(\arccos(cx)-\sqrt{1-c^2x^2}\,\mathrm{arcc}\right.}{}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
((a*d)/Sqrt[d - c^2*d*x^2] + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d + Sqrt[d]*
Sqrt[d - c^2*d*x^2]] + (b*d*(ArcCos[c*x] - Sqrt[1 - c^2*x^2]*ArcCos[c*x]*L
og[1 - I*E^(I*ArcCos[c*x])] + Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 + I*E^(I
*ArcCos[c*x])] + Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/2]] - Sqrt[1 - c^2*
x^2]*Log[Sin[ArcCos[c*x]/2]] - I*Sqrt[1 - c^2*x^2]*PolyLog[2, (-I)*E^(I*Ar
cCos[c*x])] + I*Sqrt[1 - c^2*x^2]*PolyLog[2, I*E^(I*ArcCos[c*x])]))/Sqrt[d
 - c^2*d*x^2])/d^2
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5209, 219, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{5209}$$

$$\frac{\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}dx}{d} + \frac{bc\sqrt{1-c^2x^2}\int \frac{1}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}} + \frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}dx}{d} + \frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{1-c^2x^2}\,\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 5219$$

$$-\frac{\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 4669$$

$$-\frac{\sqrt{1-c^2x^2}\left(-b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(e^{i\arccos(}\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2715$$

$$-\frac{\sqrt{1-c^2x^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(c}\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2838$$

$$-\frac{\sqrt{1-c^2x^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,ie^{i\arccos(}\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(x*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
(a + b*ArcCos[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[1 - c^2*x^2]*ArcTanh
[c*x])/(d*Sqrt[d - c^2*d*x^2]) - (Sqrt[1 - c^2*x^2]*((-2*I)*(a + b*ArcCos[
c*x])*ArcTan[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] -
 I*b*PolyLog[2, I*E^(I*ArcCos[c*x])]))/(d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.35

method	result
default	$\frac{a}{d\sqrt{-c^2 d\,x^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}}+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\arccos(cx)}{d^2(c^2x^2-1)}+\frac{i\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\left(i\arccos(cx)\right.}{}\right.$
parts	$\frac{a}{d\sqrt{-c^2 d\,x^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}}+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\arccos(cx)}{d^2(c^2x^2-1)}+\frac{i\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\left(i\arccos(cx)\right.}{}\right.$

input
```
int((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a/d/(-c^2*d*x^2+d)^(1/2)-a/d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))
/x)+b*(-(-d*(c^2*x^2-1))^(1/2)/d^2/(c^2*x^2-1)*arccos(c*x)+I*(-d*(c^2*x^2-
1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/(c^2*x^2-1)*(I*arccos(c*x)*ln(1+I*(c*x+I*
(-c^2*x^2+1)^(1/2)))-I*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-I*ln
(I*(-c^2*x^2+1)^(1/2)+c*x-1)+I*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+dilog(1+I*(c
*x+I*(-c^2*x^2+1)^(1/2)))-dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x\,(d-c^2dx^2)^{3/2}}\,dx=\int \frac{b\arccos{(cx)}+a}{(-c^2dx^2+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^4*d^2*x^5 - 2*c^2*d^2
*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\operatorname{acos}(cx)}{x\left(-d(cx-1)(cx+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/x/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))/(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\arccos(cx)+a}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
-a*(log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2) - 1/(s
qrt(-c^2*d*x^2 + d)*d)) - b*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1)
, c*x)/((c^2*d*x^3 - d*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(x*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))/(x*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^3-\sqrt{-c^2x^2+1}\,x}dx\right)b+\sqrt{-c^2x^2+1}\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,d}$$

input
```
int((a+b*acos(c*x))/x/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x**3
 - sqrt( - c**2*x**2 + 1)*x),x)*b + sqrt( - c**2*x**2 + 1)*log(tan(asin(c*
x)/2))*a - sqrt( - c**2*x**2 + 1)*a + a)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*d
)
```

3.128 $\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 150

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^{3/2}}\,dx = -\frac{a+b\arccos(cx)}{dx\sqrt{d-c^2dx^2}}+\frac{2c^2x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{d-c^2dx^2}\log(x)}{d^2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\log(1-c^2x^2)}{2d^2\sqrt{1-c^2x^2}}$$

output
```
-(a+b*arccos(c*x))/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2*x*(a+b*arccos(c*x))/d/(-
c^2*d*x^2+d)^(1/2)+b*c*(-c^2*d*x^2+d)^(1/2)*ln(x)/d^2/(-c^2*x^2+1)^(1/2)+1
/2*b*c*(-c^2*d*x^2+d)^(1/2)*ln(-c^2*x^2+1)/d^2/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.14

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^{3/2}}\,dx = -\frac{\sqrt{d-c^2dx^2}\left(2a\sqrt{1-c^2x^2}-4ac^2x^2\sqrt{1-c^2x^2}-2b\sqrt{1-c^2x^2}(-1+2c^2x^2)\arccos(cx)+bcx(-1+c^2x^2)\right.}{2d^2x(1-c^2x^2)^{3/2}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-1/2*(Sqrt[d - c^2*d*x^2]*(2*a*Sqrt[1 - c^2*x^2] - 4*a*c^2*x^2*Sqrt[1 - c^
2*x^2] - 2*b*Sqrt[1 - c^2*x^2]*(-1 + 2*c^2*x^2)*ArcCos[c*x] + b*c*x*(-1 +
c^2*x^2)*Log[1 - 1/(c^2*x^2)] + 2*b*c*x*Log[1 - c^2*x^2] - 2*b*c^3*x^3*Log
[1 - c^2*x^2]))/(d^2*x*(1 - c^2*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5195, 25, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{5195}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{1-2c^2x^2}{d^2x(1-c^2x^2)}dx}{\sqrt{1-c^2x^2}}+\frac{2c^2x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \frac{1-2c^2x^2}{d^2x(1-c^2x^2)}dx}{\sqrt{1-c^2x^2}}+\frac{2c^2x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \frac{1-2c^2x^2}{x(1-c^2x^2)}dx}{d^2\sqrt{1-c^2x^2}}+\frac{2c^2x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{354}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \frac{1-2c^2x^2}{x^2(1-c^2x^2)}dx^2}{2d^2\sqrt{1-c^2x^2}}+\frac{2c^2x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{86}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \left(\frac{c^2}{c^2x^2-1}+\frac{1}{x^2}\right)dx^2}{2d^2\sqrt{1-c^2x^2}}+\frac{2c^2x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2c^2x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{dx\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{d-c^2dx^2}\left(\log\left(1-c^2x^2\right)+\log\left(x^2\right)\right)}{2d^2\sqrt{1-c^2x^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-((a + b*ArcCos[c*x])/(d*x*Sqrt[d - c^2*d*x^2])) + (2*c^2*x*(a + b*ArcCos[
c*x]))/(d*Sqrt[d - c^2*d*x^2]) - (b*c*Sqrt[d - c^2*d*x^2]*(Log[x^2] + Log[
1 - c^2*x^2]))/(2*d^2*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.53

method	result
default	$a\left(-\frac{1}{dx\sqrt{-c^2dx^2+d}}+\frac{2c^2x}{d\sqrt{-c^2dx^2+d}}\right)+b\left(-\frac{4i\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)c}{d^2(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\,\left(-2i\sqrt{-c^2x^2+\right.}{d^2x(c^2x}\right.$
parts	$a\left(-\frac{1}{dx\sqrt{-c^2dx^2+d}}+\frac{2c^2x}{d\sqrt{-c^2dx^2+d}}\right)+b\left(-\frac{4i\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)c}{d^2(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\,\left(-2i\sqrt{-c^2x^2+\right.}{d^2x(c^2x}\right.$

input
```
int((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2/d*x/(-c^2*d*x^2+d)^(1/2))+b*(-4*I*(-d
*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/(c^2*x^2-1)*arccos(c*x)*c-(-d*(
c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*arccos(c*x)/d^
2/x/(c^2*x^2-1)+(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/(c^2*x^2-1)*
ln((c*x+I*(-c^2*x^2+1)^(1/2))^4-1)*c)
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^4*d^2*x^6 - 2*c^2*d^2
*x^4 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\operatorname{acos}\left(cx\right)}{x^2\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/x**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))/(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.86

$$\int \frac{a+b\arccos(cx)}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=-\frac{1}{2}\,bc\left(\frac{\log\left(cx+1\right)}{d^{\frac{3}{2}}}+\frac{\log\left(cx-1\right)}{d^{\frac{3}{2}}}+\frac{2\,\log\left(x\right)}{d^{\frac{3}{2}}}\right)+\left(\frac{2\,c^2x}{\sqrt{-c^2dx^2+d}d}-\frac{1}{\sqrt{-c^2dx^2+d}dx}\right)b\arccos\left(cx\right)+\left(\frac{2\,c^2x}{\sqrt{-c^2dx^2+d}d}-\frac{1}{\sqrt{-c^2dx^2+d}dx}\right)a$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
-1/2*b*c*(log(c*x + 1)/d^(3/2) + log(c*x - 1)/d^(3/2) + 2*log(x)/d^(3/2))
+ (2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x^2 + d)*d*x))*b*arcc
os(c*x) + (2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x^2 + d)*d*x)
)*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x^2\,(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(x^2*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))/(x^2*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^{3/2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^4-\sqrt{-c^2x^2+1}\,x^2}dx\right)bx+2a\,c^2x^2-a}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,dx}$$

input
```
int((a+b*acos(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x**4
 - sqrt( - c**2*x**2 + 1)*x**2),x)*b*x + 2*a*c**2*x**2 - a)/(sqrt(d)*sqrt(
 - c**2*x**2 + 1)*d*x)
```

3.129 $\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 316

$$\begin{aligned}\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^{3/2}}\,dx =& -\frac{bc\sqrt{1-c^2x^2}}{2dx\sqrt{d-c^2dx^2}}+\frac{3c^2(a+b\arccos(cx))}{2d\sqrt{d-c^2dx^2}} \\ &-\frac{a+b\arccos(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{3c^2\sqrt{1-c^2x^2}(a+b\arccos(cx))\text{arctanh}\left(e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}} \\ &-\frac{bc^2\sqrt{1-c^2x^2}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{3ibc^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{2d\sqrt{d-c^2dx^2}} \\ &-\frac{3ibc^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2d\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
-1/2*b*c*(-c^2*x^2+1)^(1/2)/d/x/(-c^2*d*x^2+d)^(1/2)+3/2*c^2*(a+b*arccos(c
*x))/d/(-c^2*d*x^2+d)^(1/2)-1/2*(a+b*arccos(c*x))/d/x^2/(-c^2*d*x^2+d)^(1/
2)-3*c^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*arctanh(c*x+I*(-c^2*x^2+1)^(
1/2))/d/(-c^2*d*x^2+d)^(1/2)-b*c^2*(-c^2*x^2+1)^(1/2)*arctanh(c*x)/d/(-c^2
*d*x^2+d)^(1/2)+3/2*I*b*c^2*(-c^2*x^2+1)^(1/2)*polylog(2,-c*x-I*(-c^2*x^2+
1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-3/2*I*b*c^2*(-c^2*x^2+1)^(1/2)*polylog(2,
c*x+I*(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 4.53 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.25

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx =$$

$$-\frac{\frac{2a(-1+3c^2x^2)\sqrt{d-c^2dx^2}}{x^2(-1+c^2x^2)} - 6ac^2\sqrt{d}\log(x) + 6ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right) + \frac{bc^2d\sqrt{1-c^2x^2}\left(-2-2\arccos(cx)\cot\left(\frac{1}{2}\right.\right.}{}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-1/4*((2*a*(-1 + 3*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(x^2*(-1 + c^2*x^2)) - 6*
a*c^2*Sqrt[d]*Log[x] + 6*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]
] + (b*c^2*d*Sqrt[1 - c^2*x^2]*(-2 - 2*ArcCos[c*x]*Cot[ArcCos[c*x]/2] + 6*
ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] - 6*ArcCos[c*x]*Log[1 + I*E^(I*Ar
cCos[c*x])] - 4*Log[Cos[ArcCos[c*x]/2]] + 4*Log[Sin[ArcCos[c*x]/2]] + (6*I
)*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - (6*I)*PolyLog[2, I*E^(I*ArcCos[c*x]
)] + ArcCos[c*x]/(Cos[ArcCos[c*x]/2] - Sin[ArcCos[c*x]/2])^2 - (2*Sin[ArcC
os[c*x]/2])/(Cos[ArcCos[c*x]/2] - Sin[ArcCos[c*x]/2]) - ArcCos[c*x]/(Cos[A
rcCos[c*x]/2] + Sin[ArcCos[c*x]/2])^2 + (2*Sin[ArcCos[c*x]/2])/(Cos[ArcCos
[c*x]/2] + Sin[ArcCos[c*x]/2]) - 2*ArcCos[c*x]*Tan[ArcCos[c*x]/2]))/Sqrt[d
 - c^2*d*x^2])/d^2
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.81, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {5205, 264, 219, 5209, 219, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 5205$$

$$\frac{3}{2}c^2\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{bc\sqrt{1-c^2x^2}\int \frac{1}{x^2(1-c^2x^2)}dx}{2d\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{3}{2}c^2\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{bc\sqrt{1-c^2x^2}\left(c^2\int \frac{1}{1-c^2x^2}dx-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{2}c^2\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{a+b\arccos(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{1-c^2x^2}\left(\text{carctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5209}$$

$$\frac{3}{2}c^2\left(\frac{\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{1-c^2x^2}\int \frac{1}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}\right)-\frac{a+b\arccos(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{1-c^2x^2}\left(\text{carctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{2}c^2\left(\frac{\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}\right)-\frac{a+b\arccos(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{1-c^2x^2}\left(\text{carctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5219}$$

$$\frac{3}{2}c^2\left(-\frac{\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}\right)-\frac{a+b\arccos(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{1-c^2x^2}\left(\text{carctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}c^2\left(-\frac{\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\text{arctan}}{d\sqrt{d-c^2dx^2}}\right.$$
$$\frac{a+b\arccos(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{1-c^2x^2}\left(\text{carctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4669}$$

$$\frac{3}{2}c^2\left(-\frac{\sqrt{1-c^2x^2}\left(-b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(e^{i\right.}\right.}{d\sqrt{d-c^2dx^2}}\right.$$

$$\frac{a+b\arccos(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{1-c^2x^2}\left(c\operatorname{arctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2715

$$\frac{3}{2}c^2\left(-\frac{\sqrt{1-c^2x^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\,a}\right.}{d\sqrt{d-c^2dx^2}}\right.$$

$$\frac{a+b\arccos(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{1-c^2x^2}\left(c\operatorname{arctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2838

$$\frac{3}{2}c^2\left(-\frac{\sqrt{1-c^2x^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\operatorname{PolyLog}\left(2,ie^{i\right.}\right.}{d\sqrt{d-c^2dx^2}}\right.$$

$$\frac{a+b\arccos(cx)}{2dx^2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{1-c^2x^2}\left(c\operatorname{arctanh}(cx)-\frac{1}{x}\right)}{2d\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-1/2*(a + b*ArcCos[c*x])/(d*x^2*Sqrt[d - c^2*d*x^2]) - (b*c*Sqrt[1 - c^2*x
^2]*(-x^(-1) + c*ArcTanh[c*x]))/(2*d*Sqrt[d - c^2*d*x^2]) + (3*c^2*((a + b
*ArcCos[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[1 - c^2*x^2]*ArcTanh[c*x])
/(d*Sqrt[d - c^2*d*x^2]) - (Sqrt[1 - c^2*x^2]*((-2*I)*(a + b*ArcCos[c*x])*
ArcTan[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*b*P
olyLog[2, I*E^(I*ArcCos[c*x])]))/(d*Sqrt[d - c^2*d*x^2])))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.14

method	result
default	$a\left(-\frac{1}{2d\,x^{2}\sqrt{-c^{2}d\,x^{2}+d}}+\frac{3c^{2}\left(\frac{1}{d\sqrt{-c^{2}d\,x^{2}+d}}-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^{2}d\,x^{2}+d}}{x}\right)}{d^{\frac{3}{2}}}\right)}{2}\right)+b\left(-\frac{\sqrt{-d(c^{2}x^{2}-1)}\,\left(3c^{2}x^{2}\arccos(cx)+cx\right.}{2d^{2}(c^{2}x^{2}-1)x}\right.$
parts	$a\left(-\frac{1}{2d\,x^{2}\sqrt{-c^{2}d\,x^{2}+d}}+\frac{3c^{2}\left(\frac{1}{d\sqrt{-c^{2}d\,x^{2}+d}}-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^{2}d\,x^{2}+d}}{x}\right)}{d^{\frac{3}{2}}}\right)}{2}\right)+b\left(-\frac{\sqrt{-d(c^{2}x^{2}-1)}\,\left(3c^{2}x^{2}\arccos(cx)+cx\right.}{2d^{2}(c^{2}x^{2}-1)x}\right.$

input
```
int((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/d/x^2/(-c^2*d*x^2+d)^(1/2)+3/2*c^2*(1/d/(-c^2*d*x^2+d)^(1/2)-1/d^(
3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)))+b*(-1/2*(-d*(c^2*x^2-1))
^(1/2)*(3*c^2*x^2*arccos(c*x)+c*x*(-c^2*x^2+1)^(1/2)-arccos(c*x))/d^2/(c^2
*x^2-1)/x^2-1/2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/d^2*
(3*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-3*arccos(c*x)*ln(1-I*(c*
x+I*(-c^2*x^2+1)^(1/2)))-2*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)+2*ln(1+c*x+I*(-c
^2*x^2+1)^(1/2))-3*I*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+3*I*dilog(1-I*(
c*x+I*(-c^2*x^2+1)^(1/2))))*c^2)
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^{3/2}}\,dx=\int \frac{b\arccos(cx)+a}{(-c^2dx^2+d)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^4*d^2*x^7 - 2*c^2*d^2
*x^5 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\,(cx)}{x^3\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/x**3/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))/(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^{3/2}}\,dx = \int \frac{b\arccos(cx)+a}{(-c^2dx^2+d)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
-1/2*(3*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2
) - 3*c^2/(sqrt(-c^2*d*x^2 + d)*d) + 1/(sqrt(-c^2*d*x^2 + d)*d*x^2))*a - b
*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/((c^2*d*x^5 - d*x^3)
*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^3(d-c^2dx^2)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x^3\,(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(x^3*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))/(x^3*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^3\,(d-c^2dx^2)^{3/2}}\,dx = \frac{-8\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^5-\sqrt{-c^2x^2+1}\,x^3}dx\right)b\,x^2+12\sqrt{-c^2x^2+1}\log\left(\tan\left(\frac{a}{}\right.\right.}{8\sqrt{d}\,\sqrt{-c^2x^2+1}\,d\,x^2}$$

input
```
int((a+b*acos(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 8*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x*
*5 - sqrt( - c**2*x**2 + 1)*x**3),x)*b*x**2 + 12*sqrt( - c**2*x**2 + 1)*lo
g(tan(asin(c*x)/2))*a*c**2*x**2 - 9*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 + 1
2*a*c**2*x**2 - 4*a)/(8*sqrt(d)*sqrt( - c**2*x**2 + 1)*d*x**2)
```

3.130 $\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 238

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)^{3/2}}\,dx = -\frac{bc\sqrt{d-c^2dx^2}}{6d^2x^2\sqrt{1-c^2x^2}} - \frac{a+b\arccos(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2(a+b\arccos(cx))}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x(a+b\arccos(cx))}{3d\sqrt{d-c^2dx^2}} + \frac{5bc^3\sqrt{d-c^2dx^2}\log(x)}{3d^2\sqrt{1-c^2x^2}} + \frac{bc^3\sqrt{d-c^2dx^2}\log(1-c^2x^2)}{2d^2\sqrt{1-c^2x^2}}$$

output

```
-1/6*b*c*(-c^2*d*x^2+d)^(1/2)/d^2/x^2/(-c^2*x^2+1)^(1/2)-1/3*(a+b*arccos(c
*x))/d/x^3/(-c^2*d*x^2+d)^(1/2)-4/3*c^2*(a+b*arccos(c*x))/d/x/(-c^2*d*x^2+
d)^(1/2)+8/3*c^4*x*(a+b*arccos(c*x))/d/(-c^2*d*x^2+d)^(1/2)+5/3*b*c^3*(-c^
2*d*x^2+d)^(1/2)*ln(x)/d^2/(-c^2*x^2+1)^(1/2)+1/2*b*c^3*(-c^2*d*x^2+d)^(1/
2)*ln(-c^2*x^2+1)/d^2/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx =$$
$$-\frac{\sqrt{d-c^2dx^2}\left(-bcx+bc^3x^3+2a\sqrt{1-c^2x^2}+8ac^2x^2\sqrt{1-c^2x^2}-16ac^4x^4\sqrt{1-c^2x^2}-2b\sqrt{1-c^2x^2}(-1\right.}{6d^2x^3}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-1/6*(Sqrt[d - c^2*d*x^2]*(-(b*c*x) + b*c^3*x^3 + 2*a*Sqrt[1 - c^2*x^2] +
8*a*c^2*x^2*Sqrt[1 - c^2*x^2] - 16*a*c^4*x^4*Sqrt[1 - c^2*x^2] - 2*b*Sqrt[
1 - c^2*x^2]*(-1 - 4*c^2*x^2 + 8*c^4*x^4)*ArcCos[c*x] + 5*b*c^3*x^3*(-1 +
c^2*x^2)*Log[1 - 1/(c^2*x^2)] + 8*b*c^3*x^3*Log[1 - c^2*x^2] - 8*b*c^5*x^5
*Log[1 - c^2*x^2]))/(d^2*x^3*(1 - c^2*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5195, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \ 5195$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{-8c^4x^4+4c^2x^2+1}{3d^2x^3(1-c^2x^2)}dx}{\sqrt{1-c^2x^2}}-\frac{4c^2(a+b\arccos(cx))}{3dx\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{3dx^3\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\arccos(cx))}{3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \frac{-8c^4x^4+4c^2x^2+1}{x^3(1-c^2x^2)}dx}{3d^2\sqrt{1-c^2x^2}}-\frac{4c^2(a+b\arccos(cx))}{3dx\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{3dx^3\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\arccos(cx))}{3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{1578}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \frac{-8c^4x^4+4c^2x^2+1}{x^4(1-c^2x^2)}dx^2}{6d^2\sqrt{1-c^2x^2}}-\frac{4c^2(a+b\arccos(cx))}{3dx\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{3dx^3\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\arccos(cx))}{3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{1195}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \left(\frac{3c^4}{c^2x^2-1}+\frac{5c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{6d^2\sqrt{1-c^2x^2}}-\frac{4c^2(a+b\arccos(cx))}{3dx\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{3dx^3\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\arccos(cx))}{3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4c^2(a+b\arccos(cx))}{3dx\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{3dx^3\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\arccos(cx))}{3d\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{d-c^2dx^2}\left(5c^2\log\left(x^2\right)+3c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)}{6d^2\sqrt{1-c^2x^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-1/3*(a + b*ArcCos[c*x])/(d*x^3*Sqrt[d - c^2*d*x^2]) - (4*c^2*(a + b*ArcCo
s[c*x]))/(3*d*x*Sqrt[d - c^2*d*x^2]) + (8*c^4*x*(a + b*ArcCos[c*x]))/(3*d*
Sqrt[d - c^2*d*x^2]) - (b*c*Sqrt[d - c^2*d*x^2]*(-x^(-2) + 5*c^2*Log[x^2]
+ 3*c^2*Log[1 - c^2*x^2]))/(6*d^2*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 1049, normalized size of antiderivative = 4.41

method	result	size
default	Expression too large to display	1049
parts	Expression too large to display	1049

input
```
int((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3/d/x^3/(-c^2*d*x^2+d)^(1/2)+4/3*c^2*(-1/d/x/(-c^2*d*x^2+d)^(1/2)+2*
c^2/d*x/(-c^2*d*x^2+d)^(1/2)))+32/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-
7*c^2*x^2-1)/d^2*x^5*(-c^2*x^2+1)*c^8-16/3*I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^
2*x^2+1)^(1/2)/d^2/(c^2*x^2-1)*arccos(c*x)*c^3+4*I*b*(-d*(c^2*x^2-1))^(1/2
)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^3*c^6+4/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^
4*x^4-7*c^2*x^2-1)/d^2*x*c^4-4/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c
^2*x^2-1)/d^2*x*(-c^2*x^2+1)*c^4+32/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^
4-7*c^2*x^2-1)/d^2*x^7*c^10-64/3*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2
*x^2-1)/d^2*x^3*arccos(c*x)*c^6-16/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4
-7*c^2*x^2-1)/d^2*x^3*(-c^2*x^2+1)*c^6-16*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^
4*x^4-7*c^2*x^2-1)/d^2*x^5*c^8+8/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7
*c^2*x^2-1)/d^2*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^3+8*b*(-d*(c^2*x^2-1))^(1
/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x*arccos(c*x)*c^4+64/3*I*b*(-d*(c^2*x^2-1)
)^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^2*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^5
-4/3*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*c^3*(-c^2*x^2+1)
^(1/2)+4*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2/x*arccos(c*x
)*c^2-1/6*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2/x^2*(-c^2*x
^2+1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2/x^3
*arccos(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/(c^2*x^2-1)*l
n((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)*c^3+5/3*b*(-d*(c^2*x^2-1))^(1/2)*(-c^...
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^4*d^2*x^8 - 2*c^2*d^2
*x^6 + d^2*x^4), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\,(cx)}{x^4\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/x**4/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))/(x**4*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
1/3*(8*c^4*x/(sqrt(-c^2*d*x^2 + d)*d) - 4*c^2/(sqrt(-c^2*d*x^2 + d)*d*x) -
 1/(sqrt(-c^2*d*x^2 + d)*d*x^3))*a - b*integrate(arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x)/((c^2*d*x^6 - d*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/s
qrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{x^4\,\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(x^4*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))/(x^4*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx=\frac{-3\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^6-\sqrt{-c^2x^2+1}\,x^4}dx\right)b\,x^3+8a\,c^4x^4-4a\,c^2x^2-a}{3\sqrt{d}\,\sqrt{-c^2x^2+1}\,d\,x^3}$$

input
```
int((a+b*acos(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x*
*6 - sqrt( - c**2*x**2 + 1)*x**4),x)*b*x**3 + 8*a*c**4*x**4 - 4*a*c**2*x**
2 - a)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*d*x**3)
```

3.131 $\int \frac{x^6(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 293

$$\begin{aligned}\int \frac{x^6(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx =& -\frac{b}{6c^7d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{1-c^2x^2}}{4c^5d^2\sqrt{d-c^2dx^2}} \\ &+ \frac{x^5(a+b\arccos(cx))}{3c^2d(d-c^2dx^2)^{3/2}} - \frac{5x^3(a+b\arccos(cx))}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{5x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2c^6d^3} \\ &+ \frac{5\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4bc^7d^2\sqrt{d-c^2dx^2}} - \frac{7b\sqrt{1-c^2x^2}\log(1-c^2x^2)}{6c^7d^2\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
-1/6*b/c^7/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/4*b*x^2*(-c^2*x^2
+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*x^5*(a+b*arccos(c*x))/c^2/d/(-c
^2*d*x^2+d)^(3/2)-5/3*x^3*(a+b*arccos(c*x))/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-5
/2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c^6/d^3+5/4*(-c^2*x^2+1)^(1/2)
*(a+b*arccos(c*x))^2/b/c^7/d^2/(-c^2*d*x^2+d)^(1/2)-7/6*b*(-c^2*x^2+1)^(1/
2)*ln(-c^2*x^2+1)/c^7/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.86

$$\int \frac{x^6(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{4bc\sqrt{d}x(15-20c^2x^2+3c^4x^4)\arccos(cx)+30b\sqrt{d}(1-c^2x^2)^{3/2}\arccos(cx)^2 -}{}$$

input
```
Integrate[(x^6*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(4*b*c*Sqrt[d]*x*(15 - 20*c^2*x^2 + 3*c^4*x^4)*ArcCos[c*x] + 30*b*Sqrt[d]*
(1 - c^2*x^2)^(3/2)*ArcCos[c*x]^2 - 60*a*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2
]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d]*(b*
Sqrt[1 - c^2*x^2]*(-7 + 9*c^2*x^2 - 6*c^4*x^4) + 4*a*c*x*(15 - 20*c^2*x^2
+ 3*c^4*x^4) - 28*b*(1 - c^2*x^2)^(3/2)*Log[1 - c^2*x^2]))/(24*c^7*d^(5/2)
*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {5207, 243, 49, 2009, 5207, 243, 49, 2009, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5207}$$

$$-\frac{5\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}dx}{3c^2d}+\frac{b\sqrt{1-c^2x^2}\int \frac{x^5}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}+\frac{x^5(a+b\arccos(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{5\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}dx}{3c^2d}+\frac{b\sqrt{1-c^2x^2}\int \frac{x^4}{(1-c^2x^2)^2}dx^2}{6cd^2\sqrt{d-c^2dx^2}}+\frac{x^5(a+b\arccos(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{5\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}dx}{3c^2d}+\frac{b\sqrt{1-c^2x^2}\int \left(\frac{1}{c^4}+\frac{2}{c^4(c^2x^2-1)}+\frac{1}{c^4(c^2x^2-1)^2}\right)dx^2}{6cd^2\sqrt{d-c^2dx^2}}+\frac{x^5(a+b\arccos(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}dx}{3c^2d}+\frac{x^5(a+b\arccos(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{1-c^2x^2}\left(\frac{x^2}{c^4}+\frac{1}{c^6(1-c^2x^2)}+\frac{2\log(1-c^2x^2)}{c^6}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5207}$$

$$-\frac{5\left(-\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{1-c^2x^2}\int \frac{x^3}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^5(a+b\arccos(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{1-c^2x^2}\left(\frac{x^2}{c^4}+\frac{1}{c^6(1-c^2x^2)}+\frac{2\log(1-c^2x^2)}{c^6}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{5\left(-\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{1-c^2x^2}\int \frac{x^2}{1-c^2x^2}dx^2}{2cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^5(a+b\arccos(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{1-c^2x^2}\left(\frac{x^2}{c^4}+\frac{1}{c^6(1-c^2x^2)}+\frac{2\log(1-c^2x^2)}{c^6}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{5\left(-\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{1-c^2x^2}\int \left(-\frac{1}{c^2}-\frac{1}{c^2\left(c^2x^2-1\right)}\right)dx^2}{2cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^5(a+b\arccos(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{1-c^2x^2}\left(\frac{x^2}{c^4}+\frac{1}{c^6(1-c^2x^2)}+\frac{2\log(1-c^2x^2)}{c^6}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5\left(-\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x^3(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{2cd\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^5(a+b\arccos(cx))}{3c^2d(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{1-c^2x^2}\left(\frac{x^2}{c^4}+\frac{1}{c^6(1-c^2x^2)}+\frac{2\log(1-c^2x^2)}{c^6}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5211}$$

$$-\frac{5\left(-\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{b\sqrt{1-c^2x^2}\int xdx}{2c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2c^2d}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{2cd\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^5(a+b\arccos(cx))}{3c^2d(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{1-c^2x^2}\left(\frac{x^2}{c^4}+\frac{1}{c^6(1-c^2x^2)}+\frac{2\log(1-c^2x^2)}{c^6}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{15}$$

$$-\frac{5\left(-\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2c^2d}-\frac{bx^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{2cd\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^5(a+b\arccos(cx))}{3c^2d(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{1-c^2x^2}\left(\frac{x^2}{c^4}+\frac{1}{c^6(1-c^2x^2)}+\frac{2\log(1-c^2x^2)}{c^6}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{x^5(a+b\arccos(cx))}{3c^2d(d-c^2dx^2)^{3/2}}-\frac{5\left(\frac{x^3(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2c^2d}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{bx^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}}\right)}{c^2d}+\frac{b\sqrt{1-c^2x^2}\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{2cd\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{b\sqrt{1-c^2x^2}\left(\frac{x^2}{c^4}+\frac{1}{c^6(1-c^2x^2)}+\frac{2\log(1-c^2x^2)}{c^6}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^6*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x^5*(a + b*ArcCos[c*x]))/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (b*Sqrt[1 - c^
2*x^2]*(x^2/c^4 + 1/(c^6*(1 - c^2*x^2)) + (2*Log[1 - c^2*x^2])/c^6))/(6*c*
d^2*Sqrt[d - c^2*d*x^2]) - (5*((x^3*(a + b*ArcCos[c*x]))/(c^2*d*Sqrt[d - c
^2*d*x^2]) - (3*(-1/4*(b*x^2*Sqrt[1 - c^2*x^2])/(c*Sqrt[d - c^2*d*x^2]) -
(x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(2*c^2*d) - (Sqrt[1 - c^2*x^2]
*(a + b*ArcCos[c*x])^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])))/(c^2*d) + (b*Sqrt[
1 - c^2*x^2]*(-(x^2/c^2) - Log[1 - c^2*x^2]/c^4))/(2*c*d*Sqrt[d - c^2*d*x^
2])))/(3*c^2*d)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.46

method	result
default	$-\frac{a\,x^5}{2c^2d(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a\,x^3}{6c^4d(-c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{5ax}{2c^6d^2\sqrt{-c^2d\,x^2+d}}+\frac{5a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^6d^2\sqrt{c^2d}}-\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2-}}{}$
parts	$-\frac{a\,x^5}{2c^2d(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a\,x^3}{6c^4d(-c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{5ax}{2c^6d^2\sqrt{-c^2d\,x^2+d}}+\frac{5a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^6d^2\sqrt{c^2d}}-\frac{b\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2-}}{}$

input
```
int(x^6*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*x^5/c^2/d/(-c^2*d*x^2+d)^(3/2)+5/6*a/c^4*x^3/d/(-c^2*d*x^2+d)^(3/2)
-5/2*a/c^6/d^2*x/(-c^2*d*x^2+d)^(1/2)+5/2*a/c^6/d^2/(c^2*d)^(1/2)*arctan((
c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/24*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x
^2+1)^(1/2)*(-12*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^5*c^5-6*c^6*x^6-30*arcco
s(c*x)^2*x^4*c^4+112*I*arccos(c*x)*x^2*c^2+56*ln((c*x+I*(-c^2*x^2+1)^(1/2)
)^2-1)*x^4*c^4+80*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^3*c^3+15*c^4*x^4+60*arc
cos(c*x)^2*x^2*c^2-56*I*arccos(c*x)*x^4*c^4-112*ln((c*x+I*(-c^2*x^2+1)^(1/
2))^2-1)*x^2*c^2-60*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c-16*c^2*x^2-30*arcco
s(c*x)^2-56*I*arccos(c*x)+56*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)+7)/d^3/(c^
6*x^6-3*c^4*x^4+3*c^2*x^2-1)/c^7
```

Fricas [F]

$$\int \frac{x^6(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\arccos{(cx)}+a)x^6}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(-(b*x^6*arccos(c*x) + a*x^6)*sqrt(-c^2*d*x^2 + d)/(c^6*d^3*x^6 -
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^6(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^6(a+b\,\mathrm{acos}\,(cx))}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**6*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**6*(a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^6(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)x^6}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/6*a*(3*x^5/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 5*x*(3*x^2/((-c^2*d*x^2 + d
)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))/c^2 + 5*x/(sqrt(-c^2*d*
x^2 + d)*c^6*d^2) - 15*arcsin(c*x)/(c^7*d^(5/2))) + b*integrate(x^6*arctan
2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*
sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1418 vs. $2(257) = 514$.

Time = 1.45 (sec) , antiderivative size = 1418, normalized size of antiderivative = 4.84

$$\int \frac{x^6(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate(x^6*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
1/24*c*(8*b*arccos(c*x)/((-c^2*x^2 + 1)^(3/2)*c^5*d^(5/2)/x^3 + (c^2*x^2 -
 1)^2*sqrt(-c^2*x^2 + 1)*c^3*d^(5/2)/x^5) + 8*a/((-c^2*x^2 + 1)^(3/2)*c^5*
d^(5/2)/x^3 + (c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*c^3*d^(5/2)/x^5) + 4*sqrt
(-c^2*x^2 + 1)*b/(((-c^2*x^2 + 1)^(3/2)*c^5*d^(5/2)/x^3 + (c^2*x^2 - 1)^2*
sqrt(-c^2*x^2 + 1)*c^3*d^(5/2)/x^5)*c*x) + 40*(c^2*x^2 - 1)*b*arccos(c*x)/
(((-c^2*x^2 + 1)^(3/2)*c^5*d^(5/2)/x^3 + (c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1
)*c^3*d^(5/2)/x^5)*c^2*x^2) - 30*(-c^2*x^2 + 1)^(3/2)*b*arccos(c*x)^2/(((-
c^2*x^2 + 1)^(3/2)*c^5*d^(5/2)/x^3 + (c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*c^
3*d^(5/2)/x^5)*c^3*x^3) + 40*(c^2*x^2 - 1)*a/(((-c^2*x^2 + 1)^(3/2)*c^5*d^
(5/2)/x^3 + (c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*c^3*d^(5/2)/x^5)*c^2*x^2) -
 60*(-c^2*x^2 + 1)^(3/2)*a*arccos(c*x)/(((-c^2*x^2 + 1)^(3/2)*c^5*d^(5/2)/
x^3 + (c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*c^3*d^(5/2)/x^5)*c^3*x^3) + 56*(-
c^2*x^2 + 1)^(3/2)*b*log(2)/(((-c^2*x^2 + 1)^(3/2)*c^5*d^(5/2)/x^3 + (c^2*
x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*c^3*d^(5/2)/x^5)*c^3*x^3) + 28*(-c^2*x^2 + 1
)^(3/2)*b*log(abs(-c^2*x^2 + 1))/(((-c^2*x^2 + 1)^(3/2)*c^5*d^(5/2)/x^3 +
(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*c^3*d^(5/2)/x^5)*c^3*x^3) + 5*(-c^2*x^2
 + 1)^(3/2)*b/(((-c^2*x^2 + 1)^(3/2)*c^5*d^(5/2)/x^3 + (c^2*x^2 - 1)^2*sqr
t(-c^2*x^2 + 1)*c^3*d^(5/2)/x^5)*c^3*x^3) - 60*(c^2*x^2 - 1)^2*b*arccos(c*
x)/(((-c^2*x^2 + 1)^(3/2)*c^5*d^(5/2)/x^3 + (c^2*x^2 - 1)^2*sqrt(-c^2*x^2
+ 1)*c^3*d^(5/2)/x^5)*c^4*x^4) - 30*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^6\,(a+b\,\mathrm{acos}(c\,x))}{{(d-c^2\,d\,x^2)}^{5/2}}\,dx$$

input
```
int((x^6*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^6*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^6(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{15\sqrt{-c^2x^2+1}\,asin(cx)\,a\,c^2x^2-15\sqrt{-c^2x^2+1}\,asin(cx)\,a+6\sqrt{-c^2x^2+1}\left(\right.}{}$$

input
```
int(x^6*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(15*sqrt( - c**2*x**2 + 1)*asin(c*x)*a*c**2*x**2 - 15*sqrt( - c**2*x**2 +
1)*asin(c*x)*a + 6*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**6)/(sqrt( - c*
*2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2
*x**2 + 1)),x)*b*c**9*x**2 - 6*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**6)
/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 +
sqrt( - c**2*x**2 + 1)),x)*b*c**7 + 3*a*c**5*x**5 - 20*a*c**3*x**3 + 15*a*
c*x)/(6*sqrt(d)*sqrt( - c**2*x**2 + 1)*c**7*d**2*(c**2*x**2 - 1))
```

3.132 $\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 219

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = -\frac{bx\sqrt{d-c^2dx^2}}{6c^5d^3(1-c^2x^2)^{3/2}} + \frac{bx\sqrt{d-c^2dx^2}}{c^5d^3\sqrt{1-c^2x^2}} + \frac{a+b\arccos(cx)}{3c^6d(d-c^2dx^2)^{3/2}} - \frac{2(a+b\arccos(cx))}{c^6d^2\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^6d^3} + \frac{11b\sqrt{d-c^2dx^2}\operatorname{arctanh}(cx)}{6c^6d^3\sqrt{1-c^2x^2}}$$

output

```
-1/6*b*x*(-c^2*d*x^2+d)^(1/2)/c^5/d^3/(-c^2*x^2+1)^(3/2)+b*x*(-c^2*d*x^2+d
)^(1/2)/c^5/d^3/(-c^2*x^2+1)^(1/2)+1/3*(a+b*arccos(c*x))/c^6/d/(-c^2*d*x^2
+d)^(3/2)-2*(a+b*arccos(c*x))/c^6/d^2/(-c^2*d*x^2+d)^(1/2)-(-c^2*d*x^2+d)^
(1/2)*(a+b*arccos(c*x))/c^6/d^3+11/6*b*(-c^2*d*x^2+d)^(1/2)*arctanh(c*x)/c
^6/d^3/(-c^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.78

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d-c^2dx^2}\left(-\sqrt{-c^2}\left(bcx(5-6c^2x^2)\sqrt{1-c^2x^2}+2a(8-12c^2x^2+3c^4x^4)+\right.\right.}{6c^6\sqrt{-}}$$

input
```
Integrate[(x^5*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(-(Sqrt[-c^2]*(b*c*x*(5 - 6*c^2*x^2)*Sqrt[1 - c^2*x^2
] + 2*a*(8 - 12*c^2*x^2 + 3*c^4*x^4) + 2*b*(8 - 12*c^2*x^2 + 3*c^4*x^4)*Ar
cCos[c*x])) + (11*I)*b*c*(1 - c^2*x^2)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c^2
]*x], 1]))/(6*c^6*Sqrt[-c^2]*d^3*(-1 + c^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5195, 27, 1471, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 5195$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{3c^4x^4-12c^2x^2+8}{3c^6d^3(1-c^2x^2)^2}dx}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^6d^3}-\frac{2(a+b\arccos(cx))}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3c^6d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{b\sqrt{d-c^2dx^2}\int \frac{3c^4x^4-12c^2x^2+8}{(1-c^2x^2)^2}dx}{3c^5d^3\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^6d^3}-\frac{2(a+b\arccos(cx))}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3c^6d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{1471}$$

$$-\frac{b\sqrt{d-c^2dx^2}\left(-\frac{1}{2}\int -\frac{17-6c^2x^2}{1-c^2x^2}dx-\frac{x}{2(1-c^2x^2)}\right)}{3c^5d^3\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^6d^3}-\frac{2(a+b\arccos(cx))}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3c^6d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{b\sqrt{d-c^2dx^2}\left(\frac{1}{2}\int \frac{17-6c^2x^2}{1-c^2x^2}dx-\frac{x}{2(1-c^2x^2)}\right)}{3c^5d^3\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^6d^3}-\frac{2(a+b\arccos(cx))}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3c^6d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{299}$$

$$-\frac{b\sqrt{d-c^2dx^2}\left(\frac{1}{2}\left(11\int \frac{1}{1-c^2x^2}dx+6x\right)-\frac{x}{2(1-c^2x^2)}\right)}{3c^5d^3\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^6d^3}-\frac{2(a+b\arccos(cx))}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3c^6d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^6d^3}-\frac{2(a+b\arccos(cx))}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3c^6d\left(d-c^2dx^2\right)^{3/2}}-\frac{b\left(\frac{1}{2}\left(\frac{11\text{arctanh}(cx)}{c}+6x\right)-\frac{x}{2(1-c^2x^2)}\right)\sqrt{d-c^2dx^2}}{3c^5d^3\sqrt{1-c^2x^2}}$$

input
```
Int[(x^5*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(a + b*ArcCos[c*x])/(3*c^6*d*(d - c^2*d*x^2)^(3/2)) - (2*(a + b*ArcCos[c*x
]))/(c^6*d^2*Sqrt[d - c^2*d*x^2]) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x
]))/(c^6*d^3) - (b*Sqrt[d - c^2*d*x^2]*(-1/2*x/(1 - c^2*x^2) + (6*x + (11*
ArcTanh[c*x])/c)/2))/(3*c^5*d^3*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.89

method	result
default	$a\left(-\frac{x^4}{c^2d(-c^2dx^2+d)^{\frac{3}{2}}}+\frac{\frac{4x^2}{c^2d\left(-c^2dx^2+d\right)^{\frac{3}{2}}}-\frac{8}{3dc^4\left(-c^2dx^2+d\right)^{\frac{3}{2}}}}{c^2}\right)+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)(\mathrm{arc}}{2d^3c^6(c^2x^2-1)}\right.$
parts	$a\left(-\frac{x^4}{c^2d(-c^2dx^2+d)^{\frac{3}{2}}}+\frac{\frac{4x^2}{c^2d\left(-c^2dx^2+d\right)^{\frac{3}{2}}}-\frac{8}{3dc^4\left(-c^2dx^2+d\right)^{\frac{3}{2}}}}{c^2}\right)+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)(\mathrm{arc}}{2d^3c^6(c^2x^2-1)}\right.$

input
```
int(x^5*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-x^4/c^2/d/(-c^2*d*x^2+d)^(3/2)+4/c^2*(x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2
/3/d/c^4/(-c^2*d*x^2+d)^(3/2)))+b*(-1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^
2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)+I)/d^3/c^6/(c^2*x^2-1)-1/2*(-d*(c^2
*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)-I)/d^3/c
^6/(c^2*x^2-1)+1/6*(-d*(c^2*x^2-1))^(1/2)*(12*c^2*x^2*arccos(c*x)+c*x*(-c^
2*x^2+1)^(1/2)-10*arccos(c*x))/c^6/(c^2*x^2-1)^2/d^3-11/6*(-d*(c^2*x^2-1))
^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^6/(c^2*x^2-1)*ln(I*(-c^2*x^2+1)^(1/2)+c*x-
1)+11/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^6/(c^2*x^2-1)*ln(1
+c*x+I*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.20

$$\int \frac{x^5(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \left[\frac{11\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{d}\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2+4\,\left(c^3x^3+cx\right)\sqrt{-c^2dx^2+d}\sqrt{-c}}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right.}{}\right.$$

$$-\frac{11\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{-d}\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}c\sqrt{-d}x}{c^4dx^4-d}\right)-2\left(6\,bc^3x^3-5\,bcx\right)\sqrt{-c^2dx^2+d}\sqrt{-c^2}}{12\left(c^{10}d^3x^4-2\,c^8d^3x^2\right.}$$

input
```
integrate(x^5*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
[1/24*(11*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(d)*log(-(c^6*d*x^6 + 5*c^4*d*
x^4 - 5*c^2*d*x^2 + 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 +
 1)*sqrt(d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) + 4*(6*b*c^3*x^3 -
 5*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - 8*(3*a*c^4*x^4 - 12*a*
c^2*x^2 + (3*b*c^4*x^4 - 12*b*c^2*x^2 + 8*b)*arccos(c*x) + 8*a)*sqrt(-c^2*
d*x^2 + d))/(c^10*d^3*x^4 - 2*c^8*d^3*x^2 + c^6*d^3), -1/12*(11*(b*c^4*x^4
 - 2*b*c^2*x^2 + b)*sqrt(-d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 +
 1)*c*sqrt(-d)*x/(c^4*d*x^4 - d)) - 2*(6*b*c^3*x^3 - 5*b*c*x)*sqrt(-c^2*d*
x^2 + d)*sqrt(-c^2*x^2 + 1) + 4*(3*a*c^4*x^4 - 12*a*c^2*x^2 + (3*b*c^4*x^4
 - 12*b*c^2*x^2 + 8*b)*arccos(c*x) + 8*a)*sqrt(-c^2*d*x^2 + d))/(c^10*d^3*
x^4 - 2*c^8*d^3*x^2 + c^6*d^3)]
```

Sympy [F]

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^5(a+b\,\mathrm{acos}\,(cx))}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**5*(a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)x^5}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/3*a*(3*x^4/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 12*x^2/((-c^2*d*x^2 + d)^(3
/2)*c^4*d) + 8/((-c^2*d*x^2 + d)^(3/2)*c^6*d)) - 1/3*(3*(c^8*d^3*x^2 - c^6
*d^3)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(d)*integrate(1/3*(3*c^4*x^6 - 12*c
^2*x^4 + 8*x^2)/(c^9*d^3*x^6 - 2*c^7*d^3*x^4 + c^5*d^3*x^2 + (c^7*d^3*x^4
- 2*c^5*d^3*x^2 + c^3*d^3)*e^(log(c*x + 1) + log(-c*x + 1))), x) - (3*c^4*
x^4 - 12*c^2*x^2 + 8)*sqrt(d)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*
b/((c^8*d^3*x^2 - c^6*d^3)*sqrt(c*x + 1)*sqrt(-c*x + 1))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^5\,(a+b\,\mathrm{acos}(c\,x))}{{(d-c^2\,d\,x^2)}^{5/2}}\,dx$$

input
```
int((x^5*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^5*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^5(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)x^5}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)b\,c^8x^2-3\sqrt{-c^2x^2}}{3\sqrt{d}\,\sqrt{-c^2x^2+}}$$

input
```
int(x^5*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**5)/(sqrt( - c**2*x**2 + 1)*c**
4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b
*c**8*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**5)/(sqrt( - c**2*x
**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**
2 + 1)),x)*b*c**6 + 3*a*c**4*x**4 - 12*a*c**2*x**2 + 8*a)/(3*sqrt(d)*sqrt(
 - c**2*x**2 + 1)*c**6*d**2*(c**2*x**2 - 1))
```

3.133 $\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 212

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = -\frac{b}{6c^5d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}} -\frac{x(a+b\arccos(cx))}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2bc^5d^2\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\log(1-c^2x^2)}{3c^5d^2\sqrt{d-c^2dx^2}}$$

output
```
-1/6*b/c^5/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x^3*(a+b*arccos
(c*x))/c^2/d/(-c^2*d*x^2+d)^(3/2)-x*(a+b*arccos(c*x))/c^4/d^2/(-c^2*d*x^2+
d)^(1/2)+1/2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/b/c^5/d^2/(-c^2*d*x^2+
d)^(1/2)-2/3*b*(-c^2*x^2+1)^(1/2)*ln(-c^2*x^2+1)/c^5/d^2/(-c^2*d*x^2+d)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.03

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{-2bc\sqrt{d}x(-3+4c^2x^2)\arccos(cx)+3b\sqrt{d}(1-c^2x^2)^{3/2}\arccos(cx)^2-6a(-1}{}$$

input
```
Integrate[(x^4*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(-2*b*c*Sqrt[d]*x*(-3 + 4*c^2*x^2)*ArcCos[c*x] + 3*b*Sqrt[d]*(1 - c^2*x^2)
^(3/2)*ArcCos[c*x]^2 - 6*a*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*
Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - Sqrt[d]*(-6*a*c*x + 8*a*c
^3*x^3 + b*Sqrt[1 - c^2*x^2] + 4*b*(1 - c^2*x^2)^(3/2)*Log[1 - c^2*x^2]))/
(6*c^5*d^(5/2)*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5207, 243, 49, 2009, 5207, 240, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5207}$$

$$-\frac{\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}dx}{c^2d}+\frac{b\sqrt{1-c^2x^2}\int \frac{x^3}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}dx}{c^2d}+\frac{b\sqrt{1-c^2x^2}\int \frac{x^2}{(1-c^2x^2)^2}dx^2}{6cd^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}dx}{c^2d}+\frac{b\sqrt{1-c^2x^2}\int \left(\frac{1}{c^2(c^2x^2-1)}+\frac{1}{c^2(c^2x^2-1)^2}\right)dx^2}{6cd^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}dx}{c^2d}+\frac{x^3(a+b\arccos(cx))}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{c^4(1-c^2x^2)}+\frac{\log(1-c^2x^2)}{c^4}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 5207$$

$$-\frac{-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{b\sqrt{1-c^2x^2}\int \frac{x}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\arccos(cx))}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{c^4(1-c^2x^2)}+\frac{\log(1-c^2x^2)}{c^4}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 240$$

$$-\frac{-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{1-c^2x^2}\log(1-c^2x^2)}{2c^3d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\arccos(cx))}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{c^4(1-c^2x^2)}+\frac{\log(1-c^2x^2)}{c^4}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 5153$$

$$\frac{x^3(a+b\arccos(cx))}{3c^2d\left(d-c^2dx^2\right)^{3/2}}-\frac{\frac{x(a+b\arccos(cx))}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2bc^3d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{1-c^2x^2}\log(1-c^2x^2)}{2c^3d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{c^4(1-c^2x^2)}+\frac{\log(1-c^2x^2)}{c^4}\right)}{6cd^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^4*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcCos[c*x]))/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (b*Sqrt[1 - c^
2*x^2]*(1/(c^4*(1 - c^2*x^2)) + Log[1 - c^2*x^2]/c^4))/(6*c*d^2*Sqrt[d - c
^2*d*x^2]) - ((x*(a + b*ArcCos[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[
1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(2*b*c^3*d*Sqrt[d - c^2*d*x^2]) - (b*S
qrt[1 - c^2*x^2]*Log[1 - c^2*x^2])/(2*c^3*d*Sqrt[d - c^2*d*x^2]))/(c^2*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 1511, normalized size of antiderivative = 7.13

method	result	size
default	Expression too large to display	1511
parts	Expression too large to display	1511

input

```
int(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/3*a*x^3/c^2/d/(-c^2*d*x^2+d)^(3/2)-a/c^4/d^2*x/(-c^2*d*x^2+d)^(1/2)+a/c^
4/d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-32*I*b*(-
d*(c^2*x^2-1))^(1/2)/d^3/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)
*c*arccos(c*x)*(-c^2*x^2+1)^(1/2)*x^6-20/3*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/
(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2*x^3-2*I*b*(-d*(c^2*x
^2-1))^(1/2)/d^3/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4*(-c
^2*x^2+1)*x+32*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(24*c^8*x^8-87*c^6*x^6+118*c^4
*x^4-71*c^2*x^2+16)*c^2*arccos(c*x)*x^7+1/2*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2
*x^2+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arccos(c*x)^2-4/3*b*(-d*(c^2*x^2-1))^(1/
2)*(-c^2*x^2+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-
1)+4*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^
2*x^2+16)/c*(-c^2*x^2+1)^(1/2)*x^4+181/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(24*
c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2*arccos(c*x)*x^3-13/2*b*(
-d*(c^2*x^2-1))^(1/2)/d^3/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16
)/c^3*x^2*(-c^2*x^2+1)^(1/2)-16*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(24*c^8*x^8-8
7*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4*arccos(c*x)*x+64/3*I*b*(-d*(c^2*x
^2-1))^(1/2)/d^3/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5*arc
cos(c*x)*(-c^2*x^2+1)^(1/2)-220/3*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(24*c^8*x
^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^3*arccos(c*x)*(-c^2*x^2+1)^(1/2
)*x^2+8/3*I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^5/(c^2*x^...
```

Fricas [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos{(cx)}+a)x^4}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(-(b*x^4*arccos(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^6*d^3*x^6 -
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^4(a+b\operatorname{acos}{(cx)})}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**4*(a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos{(cx)}+a)x^4}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
1/3*(x*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c
^4*d)) - x/(sqrt(-c^2*d*x^2 + d)*c^4*d^2) + 3*arcsin(c*x)/(c^5*d^(5/2)))*a
 + b*integrate(x^4*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/((c^4*d^2*x^
4 - 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{1}{6}c\left(\frac{2\,bx^3\arccos{(cx)}}{(-c^2x^2+1)^{\frac{3}{2}}c^3d^{\frac{5}{2}}} + \frac{2\,ax^3}{(-c^2x^2+1)^{\frac{3}{2}}c^3d^{\frac{5}{2}}} - \frac{bx^2}{(c^2x^2-1)c^4d^{\frac{5}{2}}} + \frac{6\sqrt{-c^2x^2 \cdots}}{(c^2x^2 \cdots}\right.$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
1/6*c*(2*b*x^3*arccos(c*x)/((-c^2*x^2 + 1)^(3/2)*c^3*d^(5/2)) + 2*a*x^3/((
-c^2*x^2 + 1)^(3/2)*c^3*d^(5/2)) - b*x^2/((c^2*x^2 - 1)*c^4*d^(5/2)) + 6*s
qrt(-c^2*x^2 + 1)*b*x*arccos(c*x)/((c^2*x^2 - 1)*c^5*d^(5/2)) - 3*b*arccos
(c*x)^2/(c^6*d^(5/2)) + 6*sqrt(-c^2*x^2 + 1)*a*x/((c^2*x^2 - 1)*c^5*d^(5/2
)) - 6*a*arccos(c*x)/(c^6*d^(5/2)) + 8*b*log(2)/(c^6*d^(5/2)) + 4*b*log(ab
s(-c^2*x^2 + 1))/(c^6*d^(5/2)) + b/(c^6*d^(5/2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^4\,(a+b\,\mathrm{acos}(c\,x))}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((x^4*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^4(a + b\arccos(cx))}{(d - c^2dx^2)^{5/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\,asin(cx)\,a\,c^2x^2 - 3\sqrt{-c^2x^2+1}\,asin(cx)\,a + 3\sqrt{-c^2x^2+1}\left(\int \frac{}{\sqrt{}}\right.}{}$$

input
```
int(x^4*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*asin(c*x)*a*c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)
*asin(c*x)*a + 3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**4)/(sqrt( - c**2
*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x
**2 + 1)),x)*b*c**7*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**4)/(
sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sq
rt( - c**2*x**2 + 1)),x)*b*c**5 - 4*a*c**3*x**3 + 3*a*c*x)/(3*sqrt(d)*sqrt
( - c**2*x**2 + 1)*c**5*d**2*(c**2*x**2 - 1))
```

3.134 $\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 150

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = -\frac{bx\sqrt{d-c^2dx^2}}{6c^3d^3(1-c^2x^2)^{3/2}} + \frac{a+b\arccos(cx)}{3c^4d(d-c^2dx^2)^{3/2}} - \frac{a+b\arccos(cx)}{c^4d^2\sqrt{d-c^2dx^2}} + \frac{5b\sqrt{d-c^2dx^2}\operatorname{arctanh}(cx)}{6c^4d^3\sqrt{1-c^2x^2}}$$

output
```
-1/6*b*x*(-c^2*d*x^2+d)^(1/2)/c^3/d^3/(-c^2*x^2+1)^(3/2)+1/3*(a+b*arccos(c
*x))/c^4/d/(-c^2*d*x^2+d)^(3/2)-(a+b*arccos(c*x))/c^4/d^2/(-c^2*d*x^2+d)^(
1/2)+5/6*b*(-c^2*d*x^2+d)^(1/2)*arctanh(c*x)/c^4/d^3/(-c^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d-c^2dx^2}\left(\sqrt{-c^2}\left(-4a+6ac^2x^2+bcx\sqrt{1-c^2x^2}+2b(-2+3c^2x^2)\arccos(\right.\right.}{6c^4\sqrt{-c^2}d^3(-1+c^2x}$$

input
```
Integrate[(x^3*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(Sqrt[-c^2]*(-4*a + 6*a*c^2*x^2 + b*c*x*Sqrt[1 - c^2*
x^2] + 2*b*(-2 + 3*c^2*x^2)*ArcCos[c*x]) + (5*I)*b*c*(1 - c^2*x^2)^(3/2)*E
llipticF[I*ArcSinh[Sqrt[-c^2]*x], 1]))/(6*c^4*Sqrt[-c^2]*d^3*(-1 + c^2*x^2
)^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5195, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5195}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{2-3c^2x^2}{3c^4d^3(1-c^2x^2)^2}dx}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3c^4d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\sqrt{d-c^2dx^2}\int \frac{2-3c^2x^2}{(1-c^2x^2)^2}dx}{3c^3d^3\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3c^4d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{298}$$

$$-\frac{b\sqrt{d-c^2dx^2}\left(\frac{5}{2}\int \frac{1}{1-c^2x^2}dx-\frac{x}{2(1-c^2x^2)}\right)}{3c^3d^3\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3c^4d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a+b\arccos(cx)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3c^4d\left(d-c^2dx^2\right)^{3/2}}-\frac{b\left(\frac{5\text{arctanh}(cx)}{2c}-\frac{x}{2(1-c^2x^2)}\right)\sqrt{d-c^2dx^2}}{3c^3d^3\sqrt{1-c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(a + b*ArcCos[c*x])/(3*c^4*d*(d - c^2*d*x^2)^(3/2)) - (a + b*ArcCos[c*x])/
(c^4*d^2*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[d - c^2*d*x^2]*(-1/2*x/(1 - c^2*x^
2) + (5*ArcTanh[c*x])/(2*c)))/(3*c^3*d^3*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.67

method	result
default	$a\left(\frac{x^{2}}{c^{2} d \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}-\frac{2}{3 d \,c^{4} \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}\right)+b\left(\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \left(6 c^{2} x^{2} \arccos \left(c x \right)+c x \sqrt{-c^{2} x^{2}+1}-4 \arccos \left(c x \right)\right)}{6 \left(c^{2} x^{2}-1\right)^{2} d^{3} c^{4}}-\frac{5\sqrt{}}{}\right.$
parts	$a\left(\frac{x^{2}}{c^{2} d \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}-\frac{2}{3 d \,c^{4} \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}\right)+b\left(\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \left(6 c^{2} x^{2} \arccos \left(c x \right)+c x \sqrt{-c^{2} x^{2}+1}-4 \arccos \left(c x \right)\right)}{6 \left(c^{2} x^{2}-1\right)^{2} d^{3} c^{4}}-\frac{5\sqrt{}}{}\right.$

input
```
int(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(-c^2*d*x^2+d)^(3/2))+b*(1/6*(
-d*(c^2*x^2-1))^(1/2)*(6*c^2*x^2*arccos(c*x)+c*x*(-c^2*x^2+1)^(1/2)-4*arcc
os(c*x))/(c^2*x^2-1)^2/d^3/c^4-5/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/
2)/d^3/c^4/(c^2*x^2-1)*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)+5/6*(-d*(c^2*x^2-1))
^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2
)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.81

$$\int \frac{x^3(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \left[\frac{4\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}bcx+5\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{d}\log\left(-\frac{c^6dx^6+5\,c^4}{}\right.}{}\right.$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
[1/24*(4*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*b*c*x + 5*(b*c^4*x^4 - 2*
b*c^2*x^2 + b)*sqrt(d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^
3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*sqrt(d) - d)/(c^6*x^6
 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) + 8*(3*a*c^2*x^2 + (3*b*c^2*x^2 - 2*b)*arcc
os(c*x) - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^
3), 1/12*(2*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*b*c*x - 5*(b*c^4*x^4 -
 2*b*c^2*x^2 + b)*sqrt(-d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1
)*c*sqrt(-d)*x/(c^4*d*x^4 - d)) + 4*(3*a*c^2*x^2 + (3*b*c^2*x^2 - 2*b)*arc
cos(c*x) - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d
^3)]
```

Sympy [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^3(a+b\operatorname{acos}(cx))}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**3*(a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = & -\frac{1}{12}\,bc\left(\frac{2\,x}{c^6d^{\frac{5}{2}}x^2-c^4d^{\frac{5}{2}}}+\frac{5\,\log(cx+1)}{c^5d^{\frac{5}{2}}}-\frac{5\,\log(cx-1)}{c^5d^{\frac{5}{2}}}\right)\\ & +\frac{1}{3}\,b\left(\frac{3\,x^2}{(-c^2dx^2+d)^{\frac{3}{2}}c^2d}-\frac{2}{(-c^2dx^2+d)^{\frac{3}{2}}c^4d}\right)\arccos(cx)\\ & +\frac{1}{3}\,a\left(\frac{3\,x^2}{(-c^2dx^2+d)^{\frac{3}{2}}c^2d}-\frac{2}{(-c^2dx^2+d)^{\frac{3}{2}}c^4d}\right)\end{aligned}$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/12*b*c*(2*x/(c^6*d^(5/2)*x^2 - c^4*d^(5/2)) + 5*log(c*x + 1)/(c^5*d^(5/
2)) - 5*log(c*x - 1)/(c^5*d^(5/2))) + 1/3*b*(3*x^2/((-c^2*d*x^2 + d)^(3/2)
*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))*arccos(c*x) + 1/3*a*(3*x^2/((-
c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^3\,(a+b\,\mathrm{acos}(c\,x))}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)x^3}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)b\,c^6x^2-3\sqrt{-c^2x^2}}{3\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^4d^2}$$

input
```
int(x^3*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**3)/(sqrt( - c**2*x**2 + 1)*c**
4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b
*c**6*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**3)/(sqrt( - c**2*x
**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**
2 + 1)),x)*b*c**4 - 3*a*c**2*x**2 + 2*a)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)
*c**4*d**2*(c**2*x**2 - 1))
```

3.135 $\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 125

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = -\frac{b}{6c^3d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{x^3(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}} - \frac{b\sqrt{1-c^2x^2}\log\left(1-c^2x^2\right)}{6c^3d^2\sqrt{d-c^2dx^2}}$$

output
```
-1/6*b/c^3/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x^3*(a+b*arccos
(c*x))/d/(-c^2*d*x^2+d)^(3/2)-1/6*b*(-c^2*x^2+1)^(1/2)*ln(-c^2*x^2+1)/c^3/
d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.81

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d-c^2dx^2}\left(2ac^3x^3+b\sqrt{1-c^2x^2}+2bc^3x^3\arccos(cx)+b(1-c^2x^2)^{3/2}\log(-\right.}{6c^3d^3\left(-1+c^2x^2\right)^2}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(2*a*c^3*x^3 + b*Sqrt[1 - c^2*x^2] + 2*b*c^3*x^3*ArcC
os[c*x] + b*(1 - c^2*x^2)^(3/2)*Log[-1 + c^2*x^2]))/(6*c^3*d^3*(-1 + c^2*x
^2)^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5187, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{5187}$$

$$\frac{bc\sqrt{1-c^2x^2}\int \frac{x^3}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))}{3d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{bc\sqrt{1-c^2x^2}\int \frac{x^2}{(1-c^2x^2)^2}dx^2}{6d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))}{3d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{bc\sqrt{1-c^2x^2}\int \left(\frac{1}{c^2(c^2x^2-1)}+\frac{1}{c^2(c^2x^2-1)^2}\right)dx^2}{6d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))}{3d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3(a+b\arccos(cx))}{3d\,(d-c^2dx^2)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}\left(\frac{1}{c^4(1-c^2x^2)}+\frac{\log(1-c^2x^2)}{c^4}\right)}{6d^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcCos[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (b*c*Sqrt[1 - c^2*
x^2]*(1/(c^4*(1 - c^2*x^2)) + Log[1 - c^2*x^2]/c^4))/(6*d^2*Sqrt[d - c^2*d
*x^2])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 1243, normalized size of antiderivative = 9.94

method	result	size
default	Expression too large to display	1243
parts	Expression too large to display	1243

input
```
int(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*x/c^2/d/(-c^2*d*x^2+d)^(3/2)-1/2/c^2*(1/3/d*x/(-c^2*d*x^2+d)^(3/2)+
2/3/d^2*x/(-c^2*d*x^2+d)^(1/2)))+2/3*I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+
1)^(1/2)/d^3/c^3/(c^2*x^2-1)*arccos(c*x)-1/6*I*b*(-d*(c^2*x^2-1))^(1/2)/d^
3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2*(-c^2*x^2+1)*x^5+b*(-d*
(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4*ar
ccos(c*x)*x^7-I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x
^4-5*c^2*x^2+1)*c^3*arccos(c*x)*(-c^2*x^2+1)^(1/2)*x^6+1/3*I*b*(-d*(c^2*x^
2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3*arccos(c*
x)*(-c^2*x^2+1)^(1/2)-1/6*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*
x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4*x^7-b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^
8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2*arccos(c*x)*x^5+1/2*b*(-d*(c^2*x^2
-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c*(-c^2*x^2+1)
^(1/2)*x^4+1/6*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*
x^4-5*c^2*x^2+1)*(-c^2*x^2+1)*x^3+2*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*
x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c*arccos(c*x)*(-c^2*x^2+1)^(1/2)*x^4
-1/6*I*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*
x^2+1)*x^3+1/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^
4-5*c^2*x^2+1)*arccos(c*x)*x^3-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8
-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c*(-c^2*x^2+1)^(1/2)*x^2-4/3*I*b*(-d*(c
^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c*arc...
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos{(cx)}+a)x^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*x^2*arccos(c*x) + a*x^2)/(c^6*d^3*x^6 -
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^2(a+b\operatorname{acos}(cx))}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**2*(a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{x^2(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \\ &-\frac{1}{6}bc\left(\frac{1}{c^6d^{\frac{5}{2}}x^2-c^4d^{\frac{5}{2}}}-\frac{\log(cx+1)}{c^4d^{\frac{5}{2}}}-\frac{\log(cx-1)}{c^4d^{\frac{5}{2}}}\right) \\ &-\frac{1}{3}b\left(\frac{x}{\sqrt{-c^2dx^2+d}c^2d^2}-\frac{x}{(-c^2dx^2+d)^{\frac{3}{2}}c^2d}\right)\arccos(cx) \\ &-\frac{1}{3}a\left(\frac{x}{\sqrt{-c^2dx^2+d}c^2d^2}-\frac{x}{(-c^2dx^2+d)^{\frac{3}{2}}c^2d}\right)\end{aligned}$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/6*b*c*(1/(c^6*d^(5/2)*x^2 - c^4*d^(5/2)) - log(c*x + 1)/(c^4*d^(5/2)) -
 log(c*x - 1)/(c^4*d^(5/2))) - 1/3*b*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d^2) - x
/((-c^2*d*x^2 + d)^(3/2)*c^2*d))*arccos(c*x) - 1/3*a*(x/(sqrt(-c^2*d*x^2 +
 d)*c^2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))
```

Giac [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{x^2(a + b \arccos(cx))}{(d - c^2dx^2)^{5/2}} dx = \frac{1}{6}c\left(\frac{2 bx^3 \arccos(cx)}{(-c^2x^2 + 1)^{\frac{3}{2}}cd^{\frac{5}{2}}} + \frac{2 ax^3}{(-c^2x^2 + 1)^{\frac{3}{2}}cd^{\frac{5}{2}}} - \frac{bx^2}{(c^2x^2 - 1)c^2d^{\frac{5}{2}}} + \frac{2 b \log(2)}{c^4d^{\frac{5}{2}}} +$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
1/6*c*(2*b*x^3*arccos(c*x)/((-c^2*x^2 + 1)^(3/2)*c*d^(5/2)) + 2*a*x^3/((-c
^2*x^2 + 1)^(3/2)*c*d^(5/2)) - b*x^2/((c^2*x^2 - 1)*c^2*d^(5/2)) + 2*b*log
(2)/(c^4*d^(5/2)) + b*log(abs(-c^2*x^2 + 1))/(c^4*d^(5/2)) + b/(c^4*d^(5/2
)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b \arccos(cx))}{(d - c^2dx^2)^{5/2}} dx = \int \frac{x^2 (a + b \operatorname{acos}(c x))}{(d - c^2 d x^2)^{5/2}} dx$$

input
```
int((x^2*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2(a + b \arccos(cx))}{(d - c^2dx^2)^{5/2}} dx = \frac{3\sqrt{-c^2x^2 + 1} \left(\int \frac{acos(cx)x^2}{\sqrt{-c^2x^2+1}\, c^4x^4 - 2\sqrt{-c^2x^2+1}\, c^2x^2 + \sqrt{-c^2x^2+1}} dx\right) b c^2x^2 - 3\sqrt{-c^2x^2}}{3\sqrt{d}\, \sqrt{-c^2x^2 + 1}\, d^2 (c^2x^2}$$

input
```
int(x^2*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**2)/(sqrt( - c**2*x**2 + 1)*c**
4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b
*c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**2)/(sqrt( - c**2*x
**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**
2 + 1)),x)*b - a*x**3)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*d**2*(c**2*x**2 -
 1))
```

3.136 $\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = -\frac{bx}{6cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{a+b\arccos(cx)}{3c^2d\,(d-c^2dx^2)^{3/2}} - \frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{6c^2d^2\sqrt{d-c^2dx^2}}$$

output
```
-1/6*b*x/c/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arccos(c*x
))/c^2/d/(-c^2*d*x^2+d)^(3/2)-1/6*b*(-c^2*x^2+1)^(1/2)*arctanh(c*x)/c^2/d^
2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = -\frac{\sqrt{d-c^2dx^2}\left(\sqrt{-c^2}\left(2a+bcx\sqrt{1-c^2x^2}+2b\arccos(cx)\right)-ibc(1-c^2x^2)^{3/2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\right.\right.}{6\,(-c^2)^{3/2}\,d^3\,(-1+c^2x^2)^2}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
-1/6*(Sqrt[d - c^2*d*x^2]*(Sqrt[-c^2]*(2*a + b*c*x*Sqrt[1 - c^2*x^2] + 2*b
*ArcCos[c*x]) - I*b*c*(1 - c^2*x^2)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x
], 1]))/((-c^2)^(3/2)*d^3*(-1 + c^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5183, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5183}$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{1}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{215}$$

$$\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}\int \frac{1}{1-c^2x^2}dx+\frac{x}{2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{a+b\arccos(cx)}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{b\sqrt{1-c^2x^2}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(x*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(a + b*ArcCos[c*x])/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (b*Sqrt[1 - c^2*x^2]
*(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/(3*c*d^2*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.87

method	result
default	$\frac{a}{3c^2 d(-c^2 d x^2+d)^{\frac{3}{2}}}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\left(cx\sqrt{-c^2x^2+1}+2\arccos(cx)\right)}{6d^3(c^4x^4-2c^2x^2+1)c^2}-\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)}{6d^3c^2(c^2x^2-1)}\right.$
parts	$\frac{a}{3c^2 d(-c^2 d x^2+d)^{\frac{3}{2}}}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\left(cx\sqrt{-c^2x^2+1}+2\arccos(cx)\right)}{6d^3(c^4x^4-2c^2x^2+1)c^2}-\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)}{6d^3c^2(c^2x^2-1)}\right.$

input
```
int(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a/c^2/d/(-c^2*d*x^2+d)^(3/2)+b*(1/6*(-d*(c^2*x^2-1))^(1/2)*(c*x*(-c^2*
x^2+1)^(1/2)+2*arccos(c*x))/d^3/(c^4*x^4-2*c^2*x^2+1)/c^2-1/6*(-d*(c^2*x^2
-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*ln(1+c*x+I*(-c^2*x^2+1)^
(1/2))+1/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*l
n(I*(-c^2*x^2+1)^(1/2)+c*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.13

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \left[\frac{4\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}bcx+(bc^4x^4-2\,bc^2x^2+b)\sqrt{d}\log\left(-\frac{c^6dx^6+5\,c^4dx^4}{}\right.}{24\,(c^6d^3x^4-2}\right.$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(4*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*b*c*x + (b*c^4*x^4 - 2*b*
c^2*x^2 + b)*sqrt(d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*
x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*sqrt(d) - d)/(c^6*x^6 -
 3*c^4*x^4 + 3*c^2*x^2 - 1)) + 8*sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a))
/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3), 1/12*(2*sqrt(-c^2*d*x^2 + d)*sqr
t(-c^2*x^2 + 1)*b*c*x + (b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-d)*arctan(2*sq
rt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1)*c*sqrt(-d)*x/(c^4*d*x^4 - d)) + 4*sq
rt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a))/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2
*d^3)]
```

Sympy [F]

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x(a+b\operatorname{acos}(cx))}{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x*(a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)x}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
b*integrate(x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/((c^4*d^2*x^4 - 2
*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d) + 1/3*a/((-c
^2*d*x^2 + d)^(3/2)*c^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x\,(a+b\,\mathrm{acos}(c\,x))}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((x*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)x}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)b\,c^4x^2 - 3\sqrt{-c^2x^2+}}{3\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^2d^2\,(c^2x^2}$$

input
```
int(x*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x)/(sqrt( - c**2*x**2 + 1)*c**4*x
**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b*c*
*4*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x)/(sqrt( - c**2*x**2 +
1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)
),x)*b*c**2 - a)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*c**2*d**2*(c**2*x**2 -
1))
```

3.137 $\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 154

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}\,dx = -\frac{b}{6cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{x(a+b\arccos(cx))}{3d\,(d-c^2dx^2)^{3/2}} + \frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{1-c^2x^2}\log(1-c^2x^2)}{3cd^2\sqrt{d-c^2dx^2}}$$

output
```
-1/6*b/c/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x*(a+b*arccos(c*x
))/d/(-c^2*d*x^2+d)^(3/2)+2/3*x*(a+b*arccos(c*x))/d^2/(-c^2*d*x^2+d)^(1/2)
+1/3*b*(-c^2*x^2+1)^(1/2)*ln(-c^2*x^2+1)/c/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.73

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d-c^2dx^2}\left(6acx-4ac^3x^3+b\sqrt{1-c^2x^2}+b(6cx-4c^3x^3)\arccos(cx)-2b(1-\right.}{6cd^3\left(-1+c^2x^2\right)^2}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(6*a*c*x - 4*a*c^3*x^3 + b*Sqrt[1 - c^2*x^2] + b*(6*c
*x - 4*c^3*x^3)*ArcCos[c*x] - 2*b*(1 - c^2*x^2)^(3/2)*Log[-1 + c^2*x^2]))/
(6*c*d^3*(-1 + c^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5163, 241, 5161, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{5163}$$

$$\frac{2\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{bc\sqrt{1-c^2x^2}\int \frac{x}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{2\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{x(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{b}{6cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5161}$$

$$\frac{2\left(\frac{bc\sqrt{1-c^2x^2}\int \frac{x}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{b}{6cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{240}$$

$$\frac{x(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{2\left(\frac{x(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{1-c^2x^2}\log\left(1-c^2x^2\right)}{2cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{b}{6cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d - c^2*d*x^2)^(5/2),x]
```

output
```
b/(6*c*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]) + (x*(a + b*ArcCos[c*x])
)/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*((x*(a + b*ArcCos[c*x]))/(d*Sqrt[d - c^
2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*Log[1 - c^2*x^2])/(2*c*d*Sqrt[d - c^2*d*x
^2])))/(3*d)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.06

method	result
default	$a\left(\frac{x}{3d(-c^2dx^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{-c^2dx^2+d}}\right)-\frac{b\sqrt{-d(c^2x^2-1)}\left(2c^3x^3-3cx+2i\sqrt{-c^2x^2+1}\,x^2c^2-2i\sqrt{-c^2x^2+1}\right)\left(8i\ln\left(\left(cx+i\right.\right.\right.}{}$
parts	$a\left(\frac{x}{3d(-c^2dx^2+d)^{\frac{3}{2}}}+\frac{2x}{3d^2\sqrt{-c^2dx^2+d}}\right)-\frac{b\sqrt{-d(c^2x^2-1)}\left(2c^3x^3-3cx+2i\sqrt{-c^2x^2+1}\,x^2c^2-2i\sqrt{-c^2x^2+1}\right)\left(8i\ln\left(\left(cx+i\right.\right.\right.}{}$

input
```
int((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3/d*x/(-c^2*d*x^2+d)^(3/2)+2/3/d^2*x/(-c^2*d*x^2+d)^(1/2))-1/6*b*(-d*
(c^2*x^2-1))^(1/2)*(2*c^3*x^3-3*c*x+2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-2*I*(-c
^2*x^2+1)^(1/2))*(8*I*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)*x^6*c^6+8*ln((c*x
+I*(-c^2*x^2+1)^(1/2))^2-1)*(-c^2*x^2+1)^(1/2)*x^5*c^5-24*I*ln((c*x+I*(-c^
2*x^2+1)^(1/2))^2-1)*x^4*c^4+2*I*x^4*c^4-20*ln((c*x+I*(-c^2*x^2+1)^(1/2))^
2-1)*(-c^2*x^2+1)^(1/2)*x^3*c^3+2*c^3*x^3*(-c^2*x^2+1)^(1/2)+24*I*ln((c*x+
I*(-c^2*x^2+1)^(1/2))^2-1)*x^2*c^2+6*c^2*x^2*arccos(c*x)-4*I*c^2*x^2+12*ln
((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)*(-c^2*x^2+1)^(1/2)*x*c-3*c*x*(-c^2*x^2+1)
^(1/2)-8*I*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)-8*arccos(c*x)+2*I)/d^3/(3*c^
6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^6*d^3*x^6 - 3*c^4*d^
3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\,(cx)}{\left(-d\,(cx-1)\,(cx+1)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\int \frac{a+b\arccos(cx)}{\left(d-c^2dx^2\right)^{5/2}}\,dx = -\frac{1}{6}\,bc\left(\frac{1}{c^4d^{\frac{5}{2}}x^2-c^2d^{\frac{5}{2}}}+\frac{2\,\log\left(cx+1\right)}{c^2d^{\frac{5}{2}}}+\frac{2\,\log\left(cx-1\right)}{c^2d^{\frac{5}{2}}}\right) + \frac{1}{3}\,b\left(\frac{2\,x}{\sqrt{-c^2dx^2+d}d^2}+\frac{x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}d}\right)\arccos\left(cx\right) + \frac{1}{3}\,a\left(\frac{2\,x}{\sqrt{-c^2dx^2+d}d^2}+\frac{x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}d}\right)$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*b*c*(1/(c^4*d^(5/2)*x^2 - c^2*d^(5/2)) + 2*log(c*x + 1)/(c^2*d^(5/2))
 + 2*log(c*x - 1)/(c^2*d^(5/2))) + 1/3*b*(2*x/(sqrt(-c^2*d*x^2 + d)*d^2) +
 x/((-c^2*d*x^2 + d)^(3/2)*d))*arccos(c*x) + 1/3*a*(2*x/(sqrt(-c^2*d*x^2 +
 d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*acos(c*x))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d-c^2dx^2)^{5/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)b\,c^2x^2-3\sqrt{-c^2x^2+1}}{3\sqrt{d}\,\sqrt{-c^2x^2+1}\,d^2\,(c^2x^2}$$

input
```
int((a+b*acos(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**4
- 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b*c**2*x
**2 - 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*
x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b +
 2*a*c**2*x**3 - 3*a*x)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*d**2*(c**2*x**2
- 1))
```

3.138 $\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 291

$$\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^{5/2}}\,dx = -\frac{bcx}{6d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3d(d-c^2dx^2)^{3/2}} + \frac{a+b\arccos(cx)}{d^2\sqrt{d-c^2dx^2}}-\frac{2\sqrt{1-c^2x^2}(a+b\arccos(cx))\text{arctanh}\left(e^{i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}} - \frac{7b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{6d^2\sqrt{d-c^2dx^2}}+\frac{ib\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}} - \frac{ib\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}$$

output
```
-1/6*b*c*x/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arccos(c*x
))/d/(-c^2*d*x^2+d)^(3/2)+(a+b*arccos(c*x))/d^2/(-c^2*d*x^2+d)^(1/2)-2*(-c
^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/d^2/(-
c^2*d*x^2+d)^(1/2)-7/6*b*(-c^2*x^2+1)^(1/2)*arctanh(c*x)/d^2/(-c^2*d*x^2+d
)^(1/2)+I*b*(-c^2*x^2+1)^(1/2)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/d^2/(-
c^2*d*x^2+d)^(1/2)-I*b*(-c^2*x^2+1)^(1/2)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/
2))/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 3.77 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.21

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx = -\frac{a(-4+3c^2x^2)\sqrt{d-c^2dx^2}}{3d^3\left(-1+c^2x^2\right)^2} + \frac{a\log(x)}{d^{5/2}} - \frac{a\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)}{d^{5/2}} - \frac{b(1-c^2x^2)^{3/2}\left(-14\arccos(cx)\cot\left(\frac{1}{2}\arccos(cx)\right)-\csc^2\left(\frac{1}{2}\arccos(cx)\right)-\frac{1}{2}\sqrt{1-c^2x^2}\arccos(cx)\csc^4\left(\frac{1}{2}\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
-1/3*(a*(-4 + 3*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(d^3*(-1 + c^2*x^2)^2) + (a*
Log[x])/d^(5/2) - (a*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/d^(5/2) - (b*(1
 - c^2*x^2)^(3/2)*(-14*ArcCos[c*x]*Cot[ArcCos[c*x]/2] - Csc[ArcCos[c*x]/2]
^2 - (Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Csc[ArcCos[c*x]/2]^4)/2 + 24*ArcCos[c*
x]*Log[1 - I*E^(I*ArcCos[c*x])] - 24*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x
])] - 28*Log[Cos[ArcCos[c*x]/2]] + 28*Log[Sin[ArcCos[c*x]/2]] + (24*I)*Pol
yLog[2, (-I)*E^(I*ArcCos[c*x])] - (24*I)*PolyLog[2, I*E^(I*ArcCos[c*x])] +
 Sec[ArcCos[c*x]/2]^2 - (8*ArcCos[c*x]*Sin[ArcCos[c*x]/2]^4)/(1 - c^2*x^2)
^(3/2) - 14*ArcCos[c*x]*Tan[ArcCos[c*x]/2]))/(24*d*(d - c^2*d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5209, 215, 219, 5209, 219, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 5209$$

$$\frac{\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{bc\sqrt{1-c^2x^2}\int \frac{1}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 215

$$\frac{\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{bc\sqrt{1-c^2x^2}\left(\frac{1}{2}\int \frac{1}{1-c^2x^2}dx+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 219

$$\frac{\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 5209

$$\frac{\frac{\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{1-c^2x^2}\int \frac{1}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 219

$$\frac{\frac{\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 5219

$$\frac{-\frac{\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 3042

$$\frac{-\frac{\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 4669$$

$$\frac{-\frac{\sqrt{1-c^2x^2}\left(-b\int\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))\right)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2715$$

$$\frac{-\frac{\sqrt{1-c^2x^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2i\arctan\left(e^{i\arccos(cx)}\right)(a-\right.}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2838$$

$$\frac{-\frac{\sqrt{1-c^2x^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\right)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-}}{}}{d}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(x*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
(a + b*ArcCos[c*x])/(3*d*(d - c^2*d*x^2)^(3/2)) + (b*c*Sqrt[1 - c^2*x^2]*(
x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/(3*d^2*Sqrt[d - c^2*d*x^2]) + (
(a + b*ArcCos[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[1 - c^2*x^2]*ArcTanh
[c*x])/(d*Sqrt[d - c^2*d*x^2]) - (Sqrt[1 - c^2*x^2]*((-2*I)*(a + b*ArcCos[
c*x])*ArcTan[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] -
 I*b*PolyLog[2, I*E^(I*ArcCos[c*x])]))/(d*Sqrt[d - c^2*d*x^2]))/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.20

method	result
default	$\frac{a}{3d(-c^2dx^2+d)^{\frac{3}{2}}}+\frac{a}{d^2\sqrt{-c^2dx^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{5}{2}}}+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\left(6c^2x^2\arccos(cx)-cx\sqrt{-c^2x^2+1}\right.}{6(c^2x^2-1)^2d^3}\right.$
parts	$\frac{a}{3d(-c^2dx^2+d)^{\frac{3}{2}}}+\frac{a}{d^2\sqrt{-c^2dx^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{5}{2}}}+b\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\left(6c^2x^2\arccos(cx)-cx\sqrt{-c^2x^2+1}\right.}{6(c^2x^2-1)^2d^3}\right.$

input
```
int((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a/d/(-c^2*d*x^2+d)^(3/2)+a/d^2/(-c^2*d*x^2+d)^(1/2)-a/d^(5/2)*ln((2*d+
2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+b*(-1/6*(-d*(c^2*x^2-1))^(1/2)*(6*c^2*x
^2*arccos(c*x)-c*x*(-c^2*x^2+1)^(1/2)-8*arccos(c*x))/(c^2*x^2-1)^2/d^3+1/6
*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(6*I*arccos(c*x)*ln(1+I*(c*x+
I*(-c^2*x^2+1)^(1/2)))-6*I*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-
7*I*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)+7*I*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+6*di
log(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-6*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2))
))/d^3/(c^2*x^2-1))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^6*d^3*x^7 - 3*c^4*d^
3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\left(cx\right)}{x\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/x/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))/(x*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a*(3*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(5/2)
- 3/(sqrt(-c^2*d*x^2 + d)*d^2) - 1/((-c^2*d*x^2 + d)^(3/2)*d)) + b*integra
te(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/((c^4*d^2*x^5 - 2*c^2*d^2*x^
3 + d^2*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x\,(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(x*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*acos(c*x))/(x*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^{5/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^5-2\sqrt{-c^2x^2+1}\,c^2x^3+\sqrt{-c^2x^2+1}\,x}dx\right)b\,c^2x^2-3\sqrt{-c^2x^2+1}}{}$$

input
```
int((a+b*acos(c*x))/x/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**5
- 2*sqrt( - c**2*x**2 + 1)*c**2*x**3 + sqrt( - c**2*x**2 + 1)*x),x)*b*c**2
*x**2 - 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**
4*x**5 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**3 + sqrt( - c**2*x**2 + 1)*x),x)
*b + 3*sqrt( - c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a*c**2*x**2 - 3*sqrt(
- c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a - 4*sqrt( - c**2*x**2 + 1)*a*c**2
*x**2 + 4*sqrt( - c**2*x**2 + 1)*a + 3*a*c**2*x**2 - 4*a)/(3*sqrt(d)*sqrt(
 - c**2*x**2 + 1)*d**2*(c**2*x**2 - 1))
```

3.139 $\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 224

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^{5/2}}\,dx = -\frac{bc\sqrt{d-c^2dx^2}}{6d^3(1-c^2x^2)^{3/2}} - \frac{a+b\arccos(cx)}{dx(d-c^2dx^2)^{3/2}} + \frac{4c^2x(a+b\arccos(cx))}{3d(d-c^2dx^2)^{3/2}} + \frac{8c^2x(a+b\arccos(cx))}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{d-c^2dx^2}\log(x)}{d^3\sqrt{1-c^2x^2}} + \frac{5bc\sqrt{d-c^2dx^2}\log(1-c^2x^2)}{6d^3\sqrt{1-c^2x^2}}$$

output

```
-1/6*b*c*(-c^2*d*x^2+d)^(1/2)/d^3/(-c^2*x^2+1)^(3/2)-(a+b*arccos(c*x))/d/x
/(-c^2*d*x^2+d)^(3/2)+4/3*c^2*x*(a+b*arccos(c*x))/d/(-c^2*d*x^2+d)^(3/2)+8
/3*c^2*x*(a+b*arccos(c*x))/d^2/(-c^2*d*x^2+d)^(1/2)+b*c*(-c^2*d*x^2+d)^(1/
2)*ln(x)/d^3/(-c^2*x^2+1)^(1/2)+5/6*b*c*(-c^2*d*x^2+d)^(1/2)*ln(-c^2*x^2+1
)/d^3/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.06

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^{5/2}}\,dx = \frac{\sqrt{d-c^2dx^2}\Big(bcx-bc^3x^3-6a\sqrt{1-c^2x^2}+24ac^2x^2\sqrt{1-c^2x^2}-16ac^4x^4\sqrt{1-c^2x^2}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(b*c*x - b*c^3*x^3 - 6*a*Sqrt[1 - c^2*x^2] + 24*a*c^2
*x^2*Sqrt[1 - c^2*x^2] - 16*a*c^4*x^4*Sqrt[1 - c^2*x^2] - 2*b*Sqrt[1 - c^2
*x^2]*(3 - 12*c^2*x^2 + 8*c^4*x^4)*ArcCos[c*x] + 3*b*c*x*(-1 + c^2*x^2)^2*
Log[1 - 1/(c^2*x^2)] - 8*b*c*x*Log[1 - c^2*x^2] + 16*b*c^3*x^3*Log[1 - c^2
*x^2] - 8*b*c^5*x^5*Log[1 - c^2*x^2]))/(6*d^3*x*(1 - c^2*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5195, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 5195}$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{8c^4x^4-12c^2x^2+3}{3d^3x(1-c^2x^2)^2}dx}{\sqrt{1-c^2x^2}}+\frac{8c^2x(a+b\arccos(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x(a+b\arccos(cx))}{3d(d-c^2dx^2)^{3/2}}-\frac{a+b\arccos(cx)}{dx(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \frac{8c^4x^4-12c^2x^2+3}{x(1-c^2x^2)^2}dx}{3d^3\sqrt{1-c^2x^2}}+\frac{8c^2x(a+b\arccos(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\arccos(cx)}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1578}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \frac{8c^4x^4-12c^2x^2+3}{x^2(1-c^2x^2)^2}dx^2}{6d^3\sqrt{1-c^2x^2}}+\frac{8c^2x(a+b\arccos(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\arccos(cx)}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1195}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \left(\frac{5c^2}{c^2x^2-1}-\frac{c^2}{(c^2x^2-1)^2}+\frac{3}{x^2}\right)dx^2}{6d^3\sqrt{1-c^2x^2}}+\frac{8c^2x(a+b\arccos(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\arccos(cx)}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8c^2x(a+b\arccos(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\arccos(cx)}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{c^2x^2-1}+5\log\left(1-c^2x^2\right)+3\log\left(x^2\right)\right)}{6d^3\sqrt{1-c^2x^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
-((a + b*ArcCos[c*x])/(d*x*(d - c^2*d*x^2)^(3/2))) + (4*c^2*x*(a + b*ArcCo
s[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (8*c^2*x*(a + b*ArcCos[c*x]))/(3*d^
2*Sqrt[d - c^2*d*x^2]) - (b*c*Sqrt[d - c^2*d*x^2]*((-1 + c^2*x^2)^(-1) + 3
*Log[x^2] + 5*Log[1 - c^2*x^2]))/(6*d^3*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 1347, normalized size of antiderivative = 6.01

method	result	size
default	Expression too large to display	1347
parts	Expression too large to display	1347

input
```
int((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/d/x/(-c^2*d*x^2+d)^(3/2)+4*c^2*(1/3/d*x/(-c^2*d*x^2+d)^(3/2)+2/3/d^2
*x/(-c^2*d*x^2+d)^(1/2)))+56*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^
4+26*c^2*x^2-9)/d^3*x^3*arccos(c*x)*c^4+4/3*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^
6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^2*(-c^2*x^2+1)^(1/2)*c^3-44*b*(-d*(c^
2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x*arccos(c*x)*c^2+
b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/(c^2*x^2-1)*ln(1+(c*x+I*(-
c^2*x^2+1)^(1/2))^2)*c+5/3*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3
/(c^2*x^2-1)*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)*c-80/3*I*b*(-d*(c^2*x^2-1)
)^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^5*(-c^2*x^2+1)*c^6-4*I*b
*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x*(-c^2*x^
2+1)*c^2-3/2*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/
d^3*(-c^2*x^2+1)^(1/2)*c+9*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+
26*c^2*x^2-9)/d^3/x*arccos(c*x)-16/3*I*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+
1)^(1/2)/d^3/(c^2*x^2-1)*arccos(c*x)*c-112/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(8
*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^7*c^8-64/3*b*(-d*(c^2*x^2-1))^(1/2
)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^5*arccos(c*x)*c^6+140/3*I*b*(-
d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^5*c^6+24*I*
b*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*arccos(c*
x)*(-c^2*x^2+1)^(1/2)*c-136/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4
*x^4+26*c^2*x^2-9)/d^3*x^2*arccos(c*x)*(-c^2*x^2+1)^(1/2)*c^3-24*I*b*(-...
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^6*d^3*x^8 - 3*c^4*d^
3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^{5/2}}\,dx=\int \frac{a+b\operatorname{acos}(cx)}{x^2\left(-d(cx-1)(cx+1)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/x**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))/(x**2*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^{5/2}}\,dx=\int \frac{b\arccos(cx)+a}{(-c^2dx^2+d)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
1/3*a*(8*c^2*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 4*c^2*x/((-c^2*d*x^2 + d)^(3/2
)*d) - 3/((-c^2*d*x^2 + d)^(3/2)*d*x)) + b*integrate(arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x)/((c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2)*sqrt(c*x +
1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^2(d-c^2dx^2)^{5/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{x^2\,\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(x^2*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*acos(c*x))/(x^2*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx=\frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^6-2\sqrt{-c^2x^2+1}\,c^2x^4+\sqrt{-c^2x^2+1}\,x^2}dx\right)b\,c^2x^3-3\sqrt{-c^2x^2+}}{3\sqrt{d}\,\sqrt{-c^2x^2+1}}$$

input
```
int((a+b*acos(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**6
- 2*sqrt( - c**2*x**2 + 1)*c**2*x**4 + sqrt( - c**2*x**2 + 1)*x**2),x)*b*c
**2*x**3 - 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*
c**4*x**6 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**4 + sqrt( - c**2*x**2 + 1)*x*
*2),x)*b*x + 8*a*c**4*x**4 - 12*a*c**2*x**2 + 3*a)/(3*sqrt(d)*sqrt( - c**2
*x**2 + 1)*d**2*x*(c**2*x**2 - 1))
```

3.140 $\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 433

$$
\begin{aligned}
\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx &= \frac{bc}{4d^2x\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} \\
&- \frac{5bc^3x}{12d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} - \frac{3bc\sqrt{1-c^2x^2}}{4d^2x\sqrt{d-c^2dx^2}} \\
&+ \frac{5c^2(a+b\arccos(cx))}{6d\left(d-c^2dx^2\right)^{3/2}} - \frac{a+b\arccos(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}} + \frac{5c^2(a+b\arccos(cx))}{2d^2\sqrt{d-c^2dx^2}} \\
&- \frac{5c^2\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}} \\
&- \frac{13bc^2\sqrt{1-c^2x^2}\operatorname{arctanh}(cx)}{6d^2\sqrt{d-c^2dx^2}} + \frac{5ibc^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{2d^2\sqrt{d-c^2dx^2}} \\
&- \frac{5ibc^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2d^2\sqrt{d-c^2dx^2}}
\end{aligned}
$$

output
```
1/4*b*c/d^2/x/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-5/12*b*c^3*x/d^2/(-c
^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-3/4*b*c*(-c^2*x^2+1)^(1/2)/d^2/x/(-c^
2*d*x^2+d)^(1/2)+5/6*c^2*(a+b*arccos(c*x))/d/(-c^2*d*x^2+d)^(3/2)-1/2*(a+b
*arccos(c*x))/d/x^2/(-c^2*d*x^2+d)^(3/2)+5/2*c^2*(a+b*arccos(c*x))/d^2/(-c
^2*d*x^2+d)^(1/2)-5*c^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*arctanh(c*x+I
*(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)-13/6*b*c^2*(-c^2*x^2+1)^(1/2
)*arctanh(c*x)/d^2/(-c^2*d*x^2+d)^(1/2)+5/2*I*b*c^2*(-c^2*x^2+1)^(1/2)*pol
ylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)-5/2*I*b*c^2*(-c
^2*x^2+1)^(1/2)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/
2)
```

Mathematica [A] (verified)

Time = 3.31 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.12

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{-\frac{a\sqrt{d-c^2dx^2}\left(3-20c^2x^2+15c^4x^4\right)}{x^2\left(-1+c^2x^2\right)^2}+15ac^2\sqrt{d}\log(x)-15ac^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
(-((a*Sqrt[d - c^2*d*x^2]*(3 - 20*c^2*x^2 + 15*c^4*x^4))/(x^2*(-1 + c^2*x^
2)^2)) + 15*a*c^2*Sqrt[d]*Log[x] - 15*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d
 - c^2*d*x^2]] + (b*d^2*Sqrt[1 - c^2*x^2]*(3*c*x - 2*c^3*x^3 - (3*ArcCos[c
*x])/Sqrt[1 - c^2*x^2] + (20*c^2*x^2*ArcCos[c*x])/Sqrt[1 - c^2*x^2] - (15*
c^4*x^4*ArcCos[c*x])/Sqrt[1 - c^2*x^2] - 15*c^2*x^2*ArcCos[c*x]*Log[1 - I*
E^(I*ArcCos[c*x])] + 15*c^4*x^4*ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] +
 15*c^2*x^2*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] - 15*c^4*x^4*ArcCos[c
*x]*Log[1 + I*E^(I*ArcCos[c*x])] + 13*c^2*x^2*Log[Cos[ArcCos[c*x]/2]] - 13
*c^4*x^4*Log[Cos[ArcCos[c*x]/2]] - 13*c^2*x^2*Log[Sin[ArcCos[c*x]/2]] + 13
*c^4*x^4*Log[Sin[ArcCos[c*x]/2]] + (15*I)*c^2*x^2*(-1 + c^2*x^2)*PolyLog[2
, (-I)*E^(I*ArcCos[c*x])] - (15*I)*c^2*x^2*(-1 + c^2*x^2)*PolyLog[2, I*E^(
I*ArcCos[c*x])]))/(x^2*(d - c^2*d*x^2)^(3/2)))/(6*d^3)
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.88, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {5205, 253, 264, 219, 5209, 215, 219, 5209, 219, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{5205}$$

$$\frac{5}{2}c^2\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{bc\sqrt{1-c^2x^2}\int \frac{1}{x^2\left(1-c^2x^2\right)^2}dx}{2d^2\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$\frac{5}{2}c^2\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{bc\sqrt{1-c^2x^2}\left(\frac{3}{2}\int \frac{1}{x^2\left(1-c^2x^2\right)}dx+\frac{1}{2x\left(1-c^2x^2\right)}\right)}{2d^2\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{5}{2}c^2\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{bc\sqrt{1-c^2x^2}\left(\frac{3}{2}\left(c^2\int \frac{1}{1-c^2x^2}dx-\frac{1}{x}\right)+\frac{1}{2x\left(1-c^2x^2\right)}\right)}{2d^2\sqrt{d-c^2dx^2}}-\frac{a+b\arccos(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{2}c^2\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{a+b\arccos(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{1-c^2x^2}\left(\frac{3}{2}\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x\left(1-c^2x^2\right)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5209}$$

$$\frac{5}{2}c^2\left(\frac{\int \frac{a+b\arccos(cx)}{x\left(d-c^2dx^2\right)^{3/2}}dx}{d}+\frac{bc\sqrt{1-c^2x^2}\int \frac{1}{\left(1-c^2x^2\right)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}\right)-\frac{a+b\arccos(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{1-c^2x^2}\left(\frac{3}{2}\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x\left(1-c^2x^2\right)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{215}$$

$$\frac{5}{2}c^2\left(\frac{\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{bc\sqrt{1-c^2x^2}\left(\frac{1}{2}\int\frac{1}{1-c^2x^2}dx+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}\right)-\frac{a+b\arccos(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{1-c^2x^2}\left(\frac{3}{2}\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{2}c^2\left(\frac{\int \frac{a+b\arccos(cx)}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}\right)-\frac{a+b\arccos(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{1-c^2x^2}\left(\frac{3}{2}\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5209}$$

$$\frac{5}{2}c^2\left(\frac{\frac{\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{1-c^2x^2}\int\frac{1}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}\left(\frac{\text{arctanh}(cx)}{2c}+\cdots\right.}{3d^2\sqrt{d-c^2dx^2}}\cdots\right.$$
$$-\frac{a+b\arccos(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{1-c^2x^2}\left(\frac{3}{2}\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{2}c^2\left(\frac{\frac{\int \frac{a+b\arccos(cx)}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}\left(\frac{\text{arctanh}(cx)}{2c}+\cdots\right.}{3d^2\sqrt{d-c^2dx^2}}\cdots\right.$$
$$-\frac{a+b\arccos(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{1-c^2x^2}\left(\frac{3}{2}\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5219}$$

$$\frac{5}{2}c^2\left(\frac{-\frac{\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{cx}d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\arccos(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}\cdots}{3\cdots}\cdots\right.$$
$$-\frac{a+b\arccos(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{1-c^2x^2}\left(\frac{3}{2}\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 3042

$$\frac{5}{2}c^2\left(\frac{-\frac{\sqrt{1-c^2x^2}\int(a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{1-c^2x^2}\text{arctanh}(cx)}{d\sqrt{d-c^2dx^2}}}{d}+\frac{a+b\arccos(cx}{3d\left(d-c^2dx^2\right)^{3/}}\right.$$
$$\frac{a+b\arccos(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{1-c^2x^2}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 4669

$$\frac{5}{2}c^2\left(\frac{-\frac{\sqrt{1-c^2x^2}\left(-b\int\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))\right)}{d\sqrt{d-c^2dx^2}}+\frac{a}{}}{d}\right.$$
$$\frac{a+b\arccos(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{1-c^2x^2}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2715

$$\frac{5}{2}c^2\left(\frac{-\frac{\sqrt{1-c^2x^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2i\arctan\left(e^{i\arccos(}\right.\right.}{d\sqrt{d-c^2dx^2}}}{d}\right.$$
$$\frac{a+b\arccos(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{1-c^2x^2}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2838

$$\frac{5}{2}c^2\left(\frac{-\frac{\sqrt{1-c^2x^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\right)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\arccos(cx)}{d\sqrt{d-c^2dx^2}}+}{d}\right.$$
$$\frac{a+b\arccos(cx)}{2dx^2\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{1-c^2x^2}\left(\frac{3}{2}\left(c\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)}{2d^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
-1/2*(a + b*ArcCos[c*x])/(d*x^2*(d - c^2*d*x^2)^(3/2)) - (b*c*Sqrt[1 - c^2
*x^2]*(1/(2*x*(1 - c^2*x^2)) + (3*(-x^(-1) + c*ArcTanh[c*x]))/2))/(2*d^2*S
qrt[d - c^2*d*x^2]) + (5*c^2*((a + b*ArcCos[c*x])/(3*d*(d - c^2*d*x^2)^(3/
2)) + (b*c*Sqrt[1 - c^2*x^2]*(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/(
3*d^2*Sqrt[d - c^2*d*x^2]) + ((a + b*ArcCos[c*x])/(d*Sqrt[d - c^2*d*x^2])
+ (b*Sqrt[1 - c^2*x^2]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) - (Sqrt[1 - c
^2*x^2]*((-2*I)*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcCos[c*x])] + I*b*PolyLo
g[2, (-I)*E^(I*ArcCos[c*x])] - I*b*PolyLog[2, I*E^(I*ArcCos[c*x])]))/(d*Sq
rt[d - c^2*d*x^2]))/d))/2
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.99

method	result
default	$-\frac{a}{2d\,x^2(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a\,c^2}{6d(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a\,c^2}{2d^2\sqrt{-c^2d\,x^2+d}}-\frac{5a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{2d^{\frac{5}{2}}}+b\Bigg(-\frac{\sqrt{-d(c^2x^2-1)}\,\big(}{}$
parts	$-\frac{a}{2d\,x^2(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a\,c^2}{6d(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a\,c^2}{2d^2\sqrt{-c^2d\,x^2+d}}-\frac{5a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{2d^{\frac{5}{2}}}+b\Bigg(-\frac{\sqrt{-d(c^2x^2-1)}\,\big(}{}$

input
```
int((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/x^2/(-c^2*d*x^2+d)^(3/2)+5/6*a*c^2/d/(-c^2*d*x^2+d)^(3/2)+5/2*a*c
^2/d^2/(-c^2*d*x^2+d)^(1/2)-5/2*a*c^2/d^(5/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^
2+d)^(1/2))/x)+b*(-1/6*(-d*(c^2*x^2-1))^(1/2)*(15*c^4*x^4*arccos(c*x)+2*c^
3*x^3*(-c^2*x^2+1)^(1/2)-20*c^2*x^2*arccos(c*x)-3*c*x*(-c^2*x^2+1)^(1/2)+3
*arccos(c*x))/d^3/(c^4*x^4-2*c^2*x^2+1)/x^2+1/6*I*(-c^2*x^2+1)^(1/2)*(-d*(
c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)*(15*I*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^
2+1)^(1/2)))-15*I*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-13*I*ln(I
*(-c^2*x^2+1)^(1/2)+c*x-1)+13*I*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+15*dilog(1+
I*(c*x+I*(-c^2*x^2+1)^(1/2)))-15*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2))))*c^
2)
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^6*d^3*x^9 - 3*c^4*d^
3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

Sympy [F]

$$\int \frac{a + b\arccos(cx)}{x^3 (d - c^2dx^2)^{5/2}}\,dx = \int \frac{a + b\,\mathrm{acos}\,(cx)}{x^3 \left(-d\,(cx - 1)\,(cx + 1)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/x**3/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))/(x**3*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{a + b\arccos(cx)}{x^3 (d - c^2dx^2)^{5/2}}\,dx = \int \frac{b\arccos(cx) + a}{(-c^2dx^2 + d)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/6*a*(15*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(
5/2) - 15*c^2/(sqrt(-c^2*d*x^2 + d)*d^2) - 5*c^2/((-c^2*d*x^2 + d)^(3/2)*d
) + 3/((-c^2*d*x^2 + d)^(3/2)*d*x^2)) + b*integrate(arctan2(sqrt(c*x + 1)*
sqrt(-c*x + 1), c*x)/((c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3)*sqrt(c*x + 1
)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\arccos(cx)}{x^3 (d - c^2dx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x^3\,\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(x^3*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*acos(c*x))/(x^3*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{24\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^7-2\sqrt{-c^2x^2+1}\,c^2x^5+\sqrt{-c^2x^2+1}\,x^3}dx\right)b\,c^2x^4-24\sqrt{-c^2x^2}}{}$$

input
```
int((a+b*acos(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(24*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**7
 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**5 + sqrt( - c**2*x**2 + 1)*x**3),x)*b*
c**2*x**4 - 24*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1
)*c**4*x**7 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**5 + sqrt( - c**2*x**2 + 1)*
x**3),x)*b*x**2 + 60*sqrt( - c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a*c**4*x
**4 - 60*sqrt( - c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a*c**2*x**2 - 65*sqr
t( - c**2*x**2 + 1)*a*c**4*x**4 + 65*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 +
60*a*c**4*x**4 - 80*a*c**2*x**2 + 12*a)/(24*sqrt(d)*sqrt( - c**2*x**2 + 1)
*d**2*x**2*(c**2*x**2 - 1))
```

3.141 $\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 310

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)^{5/2}}\,dx = -\frac{bc^3\sqrt{d-c^2dx^2}}{6d^3(1-c^2x^2)^{3/2}} - \frac{bc\sqrt{d-c^2dx^2}}{6d^3x^2\sqrt{1-c^2x^2}} - \frac{a+b\arccos(cx)}{3dx^3(d-c^2dx^2)^{3/2}} - \frac{2c^2(a+b\arccos(cx))}{dx(d-c^2dx^2)^{3/2}} + \frac{8c^4x(a+b\arccos(cx))}{3d(d-c^2dx^2)^{3/2}} + \frac{16c^4x(a+b\arccos(cx))}{3d^2\sqrt{d-c^2dx^2}} + \frac{8bc^3\sqrt{d-c^2dx^2}\log(x)}{3d^3\sqrt{1-c^2x^2}} + \frac{4bc^3\sqrt{d-c^2dx^2}\log(1-c^2x^2)}{3d^3\sqrt{1-c^2x^2}}$$

output
```
-1/6*b*c^3*(-c^2*d*x^2+d)^(1/2)/d^3/(-c^2*x^2+1)^(3/2)-1/6*b*c*(-c^2*d*x^2
+d)^(1/2)/d^3/x^2/(-c^2*x^2+1)^(1/2)-1/3*(a+b*arccos(c*x))/d/x^3/(-c^2*d*x
^2+d)^(3/2)-2*c^2*(a+b*arccos(c*x))/d/x/(-c^2*d*x^2+d)^(3/2)+8/3*c^4*x*(a+
b*arccos(c*x))/d/(-c^2*d*x^2+d)^(3/2)+16/3*c^4*x*(a+b*arccos(c*x))/d^2/(-c
^2*d*x^2+d)^(1/2)+8/3*b*c^3*(-c^2*d*x^2+d)^(1/2)*ln(x)/d^3/(-c^2*x^2+1)^(1
/2)+4/3*b*c^3*(-c^2*d*x^2+d)^(1/2)*ln(-c^2*x^2+1)/d^3/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.89

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)^{5/2}}\,dx = -\frac{\sqrt{d-c^2dx^2}\left(-bcx+bc^3x^3+2a\sqrt{1-c^2x^2}+12ac^2x^2\sqrt{1-c^2x^2}-48ac^4x^4\sqrt{1-c^2x^2}+32ac^6x^6\sqrt{1-c^2x^2}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
-1/6*(Sqrt[d - c^2*d*x^2]*(-(b*c*x) + b*c^3*x^3 + 2*a*Sqrt[1 - c^2*x^2] +
12*a*c^2*x^2*Sqrt[1 - c^2*x^2] - 48*a*c^4*x^4*Sqrt[1 - c^2*x^2] + 32*a*c^6
*x^6*Sqrt[1 - c^2*x^2] + 2*b*Sqrt[1 - c^2*x^2]*(1 + 6*c^2*x^2 - 24*c^4*x^4
 + 16*c^6*x^6)*ArcCos[c*x] - 8*b*c^3*x^3*(-1 + c^2*x^2)^2*Log[1 - 1/(c^2*x
^2)] + 16*b*c^3*x^3*Log[1 - c^2*x^2] - 32*b*c^5*x^5*Log[1 - c^2*x^2] + 16*
b*c^7*x^7*Log[1 - c^2*x^2]))/(d^3*x^3*(1 - c^2*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5195, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 5195$$

$$\frac{bc\sqrt{d-c^2dx^2}\int -\frac{16c^6x^6-24c^4x^4+6c^2x^2+1}{3d^3x^3(1-c^2x^2)^2}dx}{\sqrt{1-c^2x^2}}-\frac{2c^2(a+b\arccos(cx))}{dx(d-c^2dx^2)^{3/2}}-\frac{a+b\arccos(cx)}{3dx^3(d-c^2dx^2)^{3/2}}+\frac{16c^4x(a+b\arccos(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\arccos(cx))}{3d(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \frac{16c^6x^6-24c^4x^4+6c^2x^2+1}{x^3(1-c^2x^2)^2}dx}{3d^3\sqrt{1-c^2x^2}}-\frac{2c^2(a+b\arccos(cx))}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\arccos(cx)}{3dx^3\left(d-c^2dx^2\right)^{3/2}}+\frac{16c^4x(a+b\arccos(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2331}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \frac{16c^6x^6-24c^4x^4+6c^2x^2+1}{x^4(1-c^2x^2)^2}dx^2}{6d^3\sqrt{1-c^2x^2}}-\frac{2c^2(a+b\arccos(cx))}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\arccos(cx)}{3dx^3\left(d-c^2dx^2\right)^{3/2}}+\frac{16c^4x(a+b\arccos(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2123}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\int \left(\frac{8c^4}{c^2x^2-1}-\frac{c^4}{(c^2x^2-1)^2}+\frac{8c^2}{x^2}+\frac{1}{x^4}\right)dx^2}{6d^3\sqrt{1-c^2x^2}}-\frac{2c^2(a+b\arccos(cx))}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\arccos(cx)}{3dx^3\left(d-c^2dx^2\right)^{3/2}}+\frac{16c^4x(a+b\arccos(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2c^2(a+b\arccos(cx))}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\arccos(cx)}{3dx^3\left(d-c^2dx^2\right)^{3/2}}+\frac{16c^4x(a+b\arccos(cx))}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{d-c^2dx^2}\left(-\frac{c^2}{1-c^2x^2}+8c^2\log\left(x^2\right)+8c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)}{6d^3\sqrt{1-c^2x^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
-1/3*(a + b*ArcCos[c*x])/(d*x^3*(d - c^2*d*x^2)^(3/2)) - (2*c^2*(a + b*Arc
Cos[c*x]))/(d*x*(d - c^2*d*x^2)^(3/2)) + (8*c^4*x*(a + b*ArcCos[c*x]))/(3*
d*(d - c^2*d*x^2)^(3/2)) + (16*c^4*x*(a + b*ArcCos[c*x]))/(3*d^2*Sqrt[d -
c^2*d*x^2]) - (b*c*Sqrt[d - c^2*d*x^2]*(-x^(-2) - c^2/(1 - c^2*x^2) + 8*c^
2*Log[x^2] + 8*c^2*Log[1 - c^2*x^2]))/(6*d^3*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 1878, normalized size of antiderivative = 6.06

method	result	size
default	Expression too large to display	1878
parts	Expression too large to display	1878

input
```
int((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)
/d^3*x^2*c^5*(-c^2*x^2+1)^(1/2)+12*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-36
*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)/d^3*x*arccos(c*x)*c^4+6*b*(-d*(c^2*x^2-1
))^(1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)/d^3/x*arccos(c*x)
*c^2-1/6*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2
*x^2-1)/d^3/x^2*(-c^2*x^2+1)^(1/2)*c+128/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(12*
c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)/d^3*x^11*c^14-448/3*I*b*(-d*(c
^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)/d^3*x^9*c
^12+560/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*
c^2*x^2-1)/d^3*x^7*c^10+1/3*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^
6+35*c^4*x^4-10*c^2*x^2-1)/d^3/x^3*arccos(c*x)-2*b*(-d*(c^2*x^2-1))^(1/2)/
(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)/d^3*(-c^2*x^2+1)^(1/2)*c^3
+64*I*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^
2-1)/d^3*x^6*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^9-128*I*b*(-d*(c^2*x^2-1))^(
1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)/d^3*x^4*(-c^2*x^2+1)^
(1/2)*arccos(c*x)*c^7+176/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*
x^6+35*c^4*x^4-10*c^2*x^2-1)/d^3*x^2*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^5-40
/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2
-1)/d^3*x^3*(-c^2*x^2+1)*c^6+128/3*I*b*(-d*(c^2*x^2-1))^(1/2)/(12*c^8*x^8-
36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)/d^3*x^9*(-c^2*x^2+1)*c^12-320/3*I*b...
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^6*d^3*x^10 - 3*c^4*d
^3*x^8 + 3*c^2*d^3*x^6 - d^3*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acos(c*x))/x**4/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.82

$$\int \frac{a+b\arccos(cx)}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx=-\frac{1}{6}\,bc\left(\frac{8\,c^2\log\left(cx+1\right)}{d^{\frac{5}{2}}}+\frac{8\,c^2\log\left(cx-1\right)}{d^{\frac{5}{2}}}+\frac{16\,c^2\log\left(x\right)}{d^{\frac{5}{2}}}+\frac{1}{c^2d^{\frac{5}{2}}x^4-d^{\frac{5}{2}}x^2}\right)+\frac{1}{3}\left(\frac{16\,c^4x}{\sqrt{-c^2dx^2+d}\,d^2}+\frac{8\,c^4x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}d}-\frac{6\,c^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx}-\frac{1}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx^3}\right)b\arccos\left(cx\right)+\frac{1}{3}\left(\frac{16\,c^4x}{\sqrt{-c^2dx^2+d}\,d^2}+\frac{8\,c^4x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}d}-\frac{6\,c^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx}-\frac{1}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx^3}\right)a$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/6*b*c*(8*c^2*log(c*x + 1)/d^(5/2) + 8*c^2*log(c*x - 1)/d^(5/2) + 16*c^2
*log(x)/d^(5/2) + 1/(c^2*d^(5/2)*x^4 - d^(5/2)*x^2)) + 1/3*(16*c^4*x/(sqrt
(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x^2 + d)^(3/2)*d) - 6*c^2/((-c^2*
d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)^(3/2)*d*x^3))*b*arccos(c*x) +
1/3*(16*c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x^2 + d)^(3/2)
*d) - 6*c^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)^(3/2)*d*x^3
))*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x^4\,(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(x^4*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*acos(c*x))/(x^4*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^4(d-c^2dx^2)^{5/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^8-2\sqrt{-c^2x^2+1}\,c^2x^6+\sqrt{-c^2x^2+1}\,x^4}dx\right)b\,c^2x^5-3\sqrt{-c^2x^2+}}{3\sqrt{d}\,\sqrt{-c^2x}}$$

input
```
int((a+b*acos(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x)
```

output

```
(3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**8
- 2*sqrt( - c**2*x**2 + 1)*c**2*x**6 + sqrt( - c**2*x**2 + 1)*x**4),x)*b*c
**2*x**5 - 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*
c**4*x**8 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**6 + sqrt( - c**2*x**2 + 1)*x*
*4),x)*b*x**3 + 16*a*c**6*x**6 - 24*a*c**4*x**4 + 6*a*c**2*x**2 + a)/(3*sq
rt(d)*sqrt( - c**2*x**2 + 1)*d**2*x**3*(c**2*x**2 - 1))
```

3.142 $\int \frac{\arccos(ax)}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 210

$$\int \frac{\arccos(ax)}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{1}{20ac^3(1-a^2x^2)^{3/2}\sqrt{c-a^2cx^2}} - \frac{2}{15ac^3\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}} + \frac{x\arccos(ax)}{5c(c-a^2cx^2)^{5/2}} + \frac{4x\arccos(ax)}{15c^2(c-a^2cx^2)^{3/2}} + \frac{8x\arccos(ax)}{15c^3\sqrt{c-a^2cx^2}} + \frac{4\sqrt{1-a^2x^2}\log(1-a^2x^2)}{15ac^3\sqrt{c-a^2cx^2}}$$

output
```
-1/20/a/c^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2)-2/15/a/c^3/(-a^2*x^2+1
)^(1/2)/(-a^2*c*x^2+c)^(1/2)+1/5*x*arccos(a*x)/c/(-a^2*c*x^2+c)^(5/2)+4/15
*x*arccos(a*x)/c^2/(-a^2*c*x^2+c)^(3/2)+8/15*x*arccos(a*x)/c^3/(-a^2*c*x^2
+c)^(1/2)+4/15*(-a^2*x^2+1)^(1/2)*ln(-a^2*x^2+1)/a/c^3/(-a^2*c*x^2+c)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.53

$$\int \frac{\arccos(ax)}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\sqrt{c-a^2cx^2}\left(4ax(15-20a^2x^2+8a^4x^4)\arccos(ax)+\sqrt{1-a^2x^2}\left(11-8a^2x^2-16(-1+a^2x^2)^2\log\left(-1+\right.\right.\right.}{60ac^4\left(-1+a^2x^2\right)^3}$$

input
```
Integrate[ArcCos[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

output
```
-1/60*(Sqrt[c - a^2*c*x^2]*(4*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*ArcCos[a*x
] + Sqrt[1 - a^2*x^2]*(11 - 8*a^2*x^2 - 16*(-1 + a^2*x^2)^2*Log[-1 + a^2*x
^2])))/(a*c^4*(-1 + a^2*x^2)^3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5163, 241, 5163, 241, 5161, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{5163}$$

$$\frac{4\int \frac{\arccos(ax)}{(c-a^2cx^2)^{5/2}}dx}{5c}+\frac{a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{4\int \frac{\arccos(ax)}{(c-a^2cx^2)^{5/2}}dx}{5c}+\frac{x\arccos(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}+\frac{1}{20ac^3\left(1-a^2x^2\right)^{3/2}\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{5163}$$

$$\frac{4\left(\frac{2\int \frac{\arccos(ax)}{\left(c-a^2cx^2\right)^{3/2}}dx}{3c}+\frac{a\sqrt{1-a^2x^2}\int \frac{x}{\left(1-a^2x^2\right)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{x\arccos(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}+\frac{1}{20ac^3\left(1-a^2x^2\right)^{3/2}\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \ 241$$

$$\frac{4\left(\frac{2\int \frac{\arccos(ax)}{\left(c-a^2cx^2\right)^{3/2}}dx}{3c}+\frac{x\arccos(ax)}{3c(c-a^2cx^2)^{3/2}}+\frac{1}{6ac^2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}\right)}{5c}+\frac{x\arccos(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}+\frac{1}{20ac^3\left(1-a^2x^2\right)^{3/2}\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \ 5161$$

$$\frac{4\left(\frac{2\left(\frac{a\sqrt{1-a^2x^2}\int \frac{x}{1-a^2x^2}dx}{c\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)}{c\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)}{3c(c-a^2cx^2)^{3/2}}+\frac{1}{6ac^2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}\right)}{5c}+\frac{x\arccos(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}+\frac{1}{20ac^3\left(1-a^2x^2\right)^{3/2}\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \ 240$$

$$\frac{4\left(\frac{x\arccos(ax)}{3c(c-a^2cx^2)^{3/2}}+\frac{2\left(\frac{x\arccos(ax)}{c\sqrt{c-a^2cx^2}}-\frac{\sqrt{1-a^2x^2}\log\left(1-a^2x^2\right)}{2ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{1}{6ac^2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}\right)}{5c}+\frac{x\arccos(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}+\frac{1}{20ac^3\left(1-a^2x^2\right)^{3/2}\sqrt{c-a^2cx^2}}$$

input
```
Int[ArcCos[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

output
```
1/(20*a*c^3*(1 - a^2*x^2)^(3/2)*Sqrt[c - a^2*c*x^2]) + (x*ArcCos[a*x])/(5*
c*(c - a^2*c*x^2)^(5/2)) + (4*(1/(6*a*c^2*Sqrt[1 - a^2*x^2]*Sqrt[c - a^2*c
*x^2]) + (x*ArcCos[a*x])/(3*c*(c - a^2*c*x^2)^(3/2)) + (2*((x*ArcCos[a*x])
/(c*Sqrt[c - a^2*c*x^2]) - (Sqrt[1 - a^2*x^2]*Log[1 - a^2*x^2])/(2*a*c*Sqr
t[c - a^2*c*x^2])))/(3*c)))/(5*c)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.95

method	result
default	$-\frac{16 i \sqrt{-c\left(a^{2} x^{2}-1\right)}\, \sqrt{-a^{2} x^{2}+1}\, \arccos\left(a x\right)}{15 c^{4} a \left(a^{2} x^{2}-1\right)}-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(-8 i \sqrt{-a^{2} x^{2}+1}\, a^{4} x^{4}+8 a^{5} x^{5}+16 i \sqrt{-a^{2} x^{2}+1}\, a^{2} x^{2}-20 a^{3} x^{3}-8 i\right.}{}$

input
```
int(arccos(a*x)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-16/15*I*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/c^4/a/(a^2*x^2-1)*arcco
s(a*x)-1/60*(-c*(a^2*x^2-1))^(1/2)*(-8*I*(-a^2*x^2+1)^(1/2)*a^4*x^4+8*a^5*
x^5+16*I*(-a^2*x^2+1)^(1/2)*a^2*x^2-20*a^3*x^3-8*I*(-a^2*x^2+1)^(1/2)+15*a
*x)*(64*I*a^8*x^8-64*(-a^2*x^2+1)^(1/2)*a^7*x^7-280*I*a^6*x^6+248*(-a^2*x^
2+1)^(1/2)*a^5*x^5+160*a^4*x^4*arccos(a*x)+456*I*a^4*x^4-340*a^3*x^3*(-a^2
*x^2+1)^(1/2)-380*a^2*x^2*arccos(a*x)-328*I*x^2*a^2+165*(-a^2*x^2+1)^(1/2)
*a*x+256*arccos(a*x)+88*I)/c^4/(40*a^10*x^10-215*a^8*x^8+469*a^6*x^6-517*a
^4*x^4+287*a^2*x^2-64)/a+8/15*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/c^
4/a/(a^2*x^2-1)*ln((a*x+I*(-a^2*x^2+1)^(1/2))^2-1)
```

Fricas [F]

$$\int \frac{\arccos(ax)}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\arccos(ax)}{(-a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(arccos(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*arccos(a*x)/(a^8*c^4*x^8 - 4*a^6*c^4*x^6 + 6
*a^4*c^4*x^4 - 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{acos}(ax)}{(-c(ax-1)(ax+1))^{\frac{7}{2}}}\,dx$$

input
```
integrate(acos(a*x)/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Integral(acos(a*x)/(-c*(a*x - 1)*(a*x + 1))**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

$$\int \frac{\arccos(ax)}{(c-a^2cx^2)^{7/2}}\,dx = \frac{1}{60}\,a\left(\frac{3}{\left(a^6c^{\frac{5}{2}}x^4-2\,a^4c^{\frac{5}{2}}x^2+a^2c^{\frac{5}{2}}\right)c}-\frac{8}{\left(a^4c^{\frac{3}{2}}x^2-a^2c^{\frac{3}{2}}\right)c^2}+\frac{16\,\log\left(x^2-\frac{1}{a^2}\right)}{a^2c^{\frac{7}{2}}}\right) + \frac{1}{15}\left(\frac{8\,x}{\sqrt{-a^2cx^2+c}c^3}+\frac{4\,x}{(-a^2cx^2+c)^{\frac{3}{2}}c^2}+\frac{3\,x}{(-a^2cx^2+c)^{\frac{5}{2}}c}\right)\arccos\left(ax\right)$$

input
```
integrate(arccos(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
1/60*a*(3/((a^6*c^(5/2)*x^4 - 2*a^4*c^(5/2)*x^2 + a^2*c^(5/2))*c) - 8/((a^
4*c^(3/2)*x^2 - a^2*c^(3/2))*c^2) + 16*log(x^2 - 1/a^2)/(a^2*c^(7/2))) + 1
/15*(8*x/(sqrt(-a^2*c*x^2 + c)*c^3) + 4*x/((-a^2*c*x^2 + c)^(3/2)*c^2) + 3
*x/((-a^2*c*x^2 + c)^(5/2)*c))*arccos(a*x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.58

$$\int \frac{\arccos(ax)}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\sqrt{-a^2cx^2+c}\left(4\left(\frac{2\,a^4x^2}{c}-\frac{5\,a^2}{c}\right)x^2+\frac{15}{c}\right)x\arccos\left(ax\right)}{15\left(a^2cx^2-c\right)^3} - \frac{\frac{16\,\log\left(\left|a^2x^2-1\right|\right)}{a}-\frac{24\,a^4x^4-56\,a^2x^2+35}{(a^2x^2-1)^2a}}{60\,c^{\frac{7}{2}}}$$

input
```
integrate(arccos(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
-1/15*sqrt(-a^2*c*x^2 + c)*(4*(2*a^4*x^2/c - 5*a^2/c)*x^2 + 15/c)*x*arccos
(a*x)/(a^2*c*x^2 - c)^3 - 1/60*(16*log(abs(a^2*x^2 - 1))/a - (24*a^4*x^4 -
 56*a^2*x^2 + 35)/((a^2*x^2 - 1)^2*a))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\mathrm{acos}(a\,x)}{(c-a^2\,c\,x^2)^{7/2}}\,dx$$

input
```
int(acos(a*x)/(c - a^2*c*x^2)^(7/2),x)
```

output
```
int(acos(a*x)/(c - a^2*c*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\int \frac{acos(ax)}{\sqrt{-a^2x^2+1}\,a^6x^6-3\sqrt{-a^2x^2+1}\,a^4x^4+3\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^3}$$

input
```
int(acos(a*x)/(-a^2*c*x^2+c)^(7/2),x)
```

output
```
( - int(acos(a*x)/(sqrt( - a**2*x**2 + 1)*a**6*x**6 - 3*sqrt( - a**2*x**2
+ 1)*a**4*x**4 + 3*sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*x**2 +
1)),x))/(sqrt(c)*c**3)
```

3.143 $\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 79

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx = \frac{2(fx)^{5/2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)}{5f} - \frac{4bc(fx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2}$$

output
```
2/5*(f*x)^(5/2)*(a+b*arccos(c*x))*hypergeom([1/2, 5/4],[9/4],c^2*x^2)/f-4/
35*b*c*(f*x)^(7/2)*hypergeom([1, 7/4, 7/4],[9/4, 11/4],c^2*x^2)/f^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.96

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx = \frac{f\sqrt{fx}\left(-8\,\text{Gamma}\left(\frac{5}{4}\right)\text{Gamma}\left(\frac{7}{4}\right)\left(3a-3ac^2x^2+2bcx\sqrt{1-c^2x^2}+3\right.\right.}{}$$

input
```
Integrate[((f*x)^(3/2)*(a + b*ArcCos[c*x]))/Sqrt[1 - c^2*x^2],x]
```

output
```
(f*Sqrt[f*x]*(-8*Gamma[5/4]*Gamma[7/4]*(3*a - 3*a*c^2*x^2 + 2*b*c*x*Sqrt[1
 - c^2*x^2] + 3*b*ArcCos[c*x] - 3*b*c^2*x^2*ArcCos[c*x] + (3*I)*a*Sqrt[-c^
(-1)]*c*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[-c^(-1)]/Sq
rt[x]], -1] + 3*b*(-1 + c^2*x^2)*ArcCos[c*x]*Hypergeometric2F1[3/4, 1, 5/4
, c^2*x^2]) + 3*b*c*Pi*x*Sqrt[2 - 2*c^2*x^2]*HypergeometricPFQ[{3/4, 3/4,
1}, {5/4, 7/4}, c^2*x^2]))/(36*c^2*Sqrt[1 - c^2*x^2]*Gamma[5/4]*Gamma[7/4]
)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx$$
$$\Big\downarrow \; 5221$$
$$\frac{4bc(fx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2}+\frac{2(fx)^{5/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)(a+b\arccos(cx))}{5f}$$

input
```
Int[((f*x)^(3/2)*(a + b*ArcCos[c*x]))/Sqrt[1 - c^2*x^2],x]
```

output
```
(2*(f*x)^(5/2)*(a + b*ArcCos[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x^
2])/(5*f) + (4*b*c*(f*x)^(7/2)*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4
}, c^2*x^2])/(35*f^2)
```

Defintions of rubi rules used

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(fx)^{\frac{3}{2}} (a+b\arccos(cx))}{\sqrt{-c^2x^2+1}}dx$$

input
```
int((f*x)^(3/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x)
```

output
```
int((f*x)^(3/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(fx)^{\frac{3}{2}} (b\arccos(cx)+a)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((f*x)^(3/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*(b*f*x*arccos(c*x) + a*f*x)*sqrt(f*x)/(c^2*x^
2 - 1), x)
```

Sympy [F]

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(fx)^{\frac{3}{2}}\,(a+b\,\mathrm{acos}\,(cx))}{\sqrt{-\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate((f*x)**(3/2)*(a+b*acos(c*x))/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral((f*x)**(3/2)*(a + b*acos(c*x))/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(fx)^{\frac{3}{2}}\,(b\arccos\,(cx)+a)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((f*x)^(3/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((f*x)^(3/2)*(b*arccos(c*x) + a)/sqrt(-c^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(fx)^{\frac{3}{2}}\,(b\arccos\,(cx)+a)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((f*x)^(3/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((f*x)^(3/2)*(b*arccos(c*x) + a)/sqrt(-c^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,(f\,x)^{3/2}}{\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(((a + b*acos(c*x))*(f*x)^(3/2))/(1 - c^2*x^2)^(1/2),x)
```

output
```
int(((a + b*acos(c*x))*(f*x)^(3/2))/(1 - c^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx=\frac{\sqrt{f}\,f\left(-6\sqrt{x}\,\sqrt{-c^2x^2+1}\,acos(cx)\,b-6\sqrt{x}\,\sqrt{-c^2x^2+1}\,a-4\sqrt{x}\,bcx-\right.}{9c^2}$$

input
```
int((f*x)^(3/2)*(a+b*acos(c*x))/(-c^2*x^2+1)^(1/2),x)
```

output
```
(sqrt(f)*f*( - 6*sqrt(x)*sqrt( - c**2*x**2 + 1)*acos(c*x)*b - 6*sqrt(x)*sq
rt( - c**2*x**2 + 1)*a - 4*sqrt(x)*b*c*x - 3*int((sqrt(x)*sqrt( - c**2*x**
2 + 1)*acos(c*x))/(c**2*x**3 - x),x)*b - 3*int((sqrt(x)*sqrt( - c**2*x**2
+ 1))/(c**2*x**3 - x),x)*a))/(9*c**2)
```

3.144 $\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 137

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{2(fx)^{5/2}\sqrt{1-c^2x^2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)}{5f\sqrt{d-c^2dx^2}} - \frac{4bc(fx)^{7/2}\sqrt{1-c^2x^2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2\sqrt{d-c^2dx^2}}$$

output
```
2/5*(f*x)^(5/2)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*hypergeom([1/2, 5/4],
[9/4],c^2*x^2)/f/(-c^2*d*x^2+d)^(1/2)-4/35*b*c*(f*x)^(7/2)*(-c^2*x^2+1)^(1
/2)*hypergeom([1, 7/4, 7/4],[9/4, 11/4],c^2*x^2)/f^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.18 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.72

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{f\sqrt{fx}\Big(-8\,\text{Gamma}\left(\frac{5}{4}\right)\text{Gamma}\left(\frac{7}{4}\right)\left(3a-3ac^2x^2+2bcx\sqrt{1-c^2x^2}+3\right.}{}$$

input
```
Integrate[((f*x)^(3/2)*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(f*Sqrt[f*x]*(-8*Gamma[5/4]*Gamma[7/4]*(3*a - 3*a*c^2*x^2 + 2*b*c*x*Sqrt[1
 - c^2*x^2] + 3*b*ArcCos[c*x] - 3*b*c^2*x^2*ArcCos[c*x] + (3*I)*a*Sqrt[-c^
(-1)]*c*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[-c^(-1)]/Sq
rt[x]], -1] + 3*b*(-1 + c^2*x^2)*ArcCos[c*x]*Hypergeometric2F1[3/4, 1, 5/4
, c^2*x^2]) + 3*b*c*Pi*x*Sqrt[2 - 2*c^2*x^2]*HypergeometricPFQ[{3/4, 3/4,
1}, {5/4, 7/4}, c^2*x^2]))/(36*c^2*Sqrt[d - c^2*d*x^2]*Gamma[5/4]*Gamma[7/
4])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = $\{5221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \text{5221}$$

$$\frac{4bc\sqrt{1-c^2x^2}(fx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2\sqrt{d-c^2dx^2}}+\frac{2\sqrt{1-c^2x^2}(fx)^{5/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)(a+b\arccos(cx))}{5f\sqrt{d-c^2dx^2}}$$

input
```
Int[((f*x)^(3/2)*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(2*(f*x)^(5/2)*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Hypergeometric2F1[1/2
, 5/4, 9/4, c^2*x^2])/(5*f*Sqrt[d - c^2*d*x^2]) + (4*b*c*(f*x)^(7/2)*Sqrt[
1 - c^2*x^2]*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^2])/(35*f
^2*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(fx)^{\frac{3}{2}} (a+b\arccos(cx))}{\sqrt{-c^2 d\, x^2+d}}dx$$

input
```
int((f*x)^(3/2)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int((f*x)^(3/2)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(fx)^{\frac{3}{2}}\,(b\arccos(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((f*x)^(3/2)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*f*x*arccos(c*x) + a*f*x)*sqrt(f*x)/(c^2*
d*x^2 - d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x)**(3/2)*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(fx)^{\frac{3}{2}}\left(b\arccos\left(cx\right)+a\right)}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((f*x)^(3/2)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((f*x)^(3/2)*(b*arccos(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

Giac [F]

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(fx)^{\frac{3}{2}}\left(b\arccos\left(cx\right)+a\right)}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((f*x)^(3/2)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((f*x)^(3/2)*(b*arccos(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(f\,x)^{3/2}}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int(((a + b*acos(c*x))*(f*x)^(3/2))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(((a + b*acos(c*x))*(f*x)^(3/2))/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(fx)^{3/2}(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{f}\,\sqrt{d}\,f\left(-6\sqrt{x}\,\sqrt{-c^2x^2+1}\,acos(cx)\,b-6\sqrt{x}\,\sqrt{-c^2x^2+1}\,a-4\sqrt{x}\,bc\right.}{9c^2d}$$

input
```
int((f*x)^(3/2)*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(sqrt(f)*sqrt(d)*f*( - 6*sqrt(x)*sqrt( - c**2*x**2 + 1)*acos(c*x)*b - 6*sq
rt(x)*sqrt( - c**2*x**2 + 1)*a - 4*sqrt(x)*b*c*x - 3*int((sqrt(x)*sqrt( -
c**2*x**2 + 1)*acos(c*x))/(c**2*x**3 - x),x)*b - 3*int((sqrt(x)*sqrt( - c*
*2*x**2 + 1))/(c**2*x**3 - x),x)*a))/(9*c**2*d)
```

3.145 $\int x^m (d-c^2dx^2)^3 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 315

$$\begin{aligned}
&\int x^m (d-c^2dx^2)^3 (a+b\arccos(cx))\,dx \\
&= -\frac{bcd^3(2271+1329m+284m^2+27m^3+m^4)\,x^{2+m}\sqrt{1-c^2x^2}}{(3+m)^2(5+m)^2(7+m)^2} \\
&\quad+\frac{bc^3d^3(9+m)(13+2m)x^{4+m}\sqrt{1-c^2x^2}}{(5+m)^2(7+m)^2}-\frac{bc^5d^3x^{6+m}\sqrt{1-c^2x^2}}{(7+m)^2} \\
&\quad+\frac{d^3x^{1+m}(a+b\arccos(cx))}{1+m}-\frac{3c^2d^3x^{3+m}(a+b\arccos(cx))}{3+m} \\
&\quad+\frac{3c^4d^3x^{5+m}(a+b\arccos(cx))}{5+m}-\frac{c^6d^3x^{7+m}(a+b\arccos(cx))}{7+m} \\
&\quad-\frac{3bcd^3(2161+1813m+455m^2+35m^3)\,x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{(1+m)(2+m)(3+m)^2(5+m)^2(7+m)^2}
\end{aligned}$$

output

```
-b*c*d^3*(m^4+27*m^3+284*m^2+1329*m+2271)*x^(2+m)*(-c^2*x^2+1)^(1/2)/(3+m)
^2/(5+m)^2/(7+m)^2+b*c^3*d^3*(9+m)*(13+2*m)*x^(4+m)*(-c^2*x^2+1)^(1/2)/(5+
m)^2/(7+m)^2-b*c^5*d^3*x^(6+m)*(-c^2*x^2+1)^(1/2)/(7+m)^2+d^3*x^(1+m)*(a+b
*arccos(c*x))/(1+m)-3*c^2*d^3*x^(3+m)*(a+b*arccos(c*x))/(3+m)+3*c^4*d^3*x^
(5+m)*(a+b*arccos(c*x))/(5+m)-c^6*d^3*x^(7+m)*(a+b*arccos(c*x))/(7+m)-3*b*
c*d^3*(35*m^3+455*m^2+1813*m+2161)*x^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2
*m],c^2*x^2)/(1+m)/(2+m)/(3+m)^2/(5+m)^2/(7+m)^2
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.88

$$\int x^m (d-c^2dx^2)^3 (a+b\arccos(cx))\,dx$$
$$= d^3x^{1+m}\left(\frac{a}{1+m}-\frac{3ac^2x^2}{3+m}+\frac{3ac^4x^4}{5+m}-\frac{ac^6x^6}{7+m}\right.$$
$$+\frac{b\left((2+m)\arccos(cx)+cx\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{m}{2},2+\frac{m}{2},c^2x^2\right)\right)}{(1+m)(2+m)}$$
$$-\frac{3bc^2x^2\left((4+m)\arccos(cx)+cx\,\text{Hypergeometric2F1}\left(\frac{1}{2},2+\frac{m}{2},3+\frac{m}{2},c^2x^2\right)\right)}{(3+m)(4+m)}$$
$$+\frac{3bc^4x^4\left((6+m)\arccos(cx)+cx\,\text{Hypergeometric2F1}\left(\frac{1}{2},3+\frac{m}{2},4+\frac{m}{2},c^2x^2\right)\right)}{(5+m)(6+m)}$$
$$\left.-\frac{bc^6x^6\left((8+m)\arccos(cx)+cx\,\text{Hypergeometric2F1}\left(\frac{1}{2},4+\frac{m}{2},5+\frac{m}{2},c^2x^2\right)\right)}{(7+m)(8+m)}\right)$$

input
```
Integrate[x^m*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
d^3*x^(1 + m)*(a/(1 + m) - (3*a*c^2*x^2)/(3 + m) + (3*a*c^4*x^4)/(5 + m) -
 (a*c^6*x^6)/(7 + m) + (b*((2 + m)*ArcCos[c*x] + c*x*Hypergeometric2F1[1/2
, 1 + m/2, 2 + m/2, c^2*x^2]))/((1 + m)*(2 + m)) - (3*b*c^2*x^2*((4 + m)*A
rcCos[c*x] + c*x*Hypergeometric2F1[1/2, 2 + m/2, 3 + m/2, c^2*x^2]))/((3 +
 m)*(4 + m)) + (3*b*c^4*x^4*((6 + m)*ArcCos[c*x] + c*x*Hypergeometric2F1[1
/2, 3 + m/2, 4 + m/2, c^2*x^2]))/((5 + m)*(6 + m)) - (b*c^6*x^6*((8 + m)*A
rcCos[c*x] + c*x*Hypergeometric2F1[1/2, 4 + m/2, 5 + m/2, c^2*x^2]))/((7 +
 m)*(8 + m)))
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5193, 27, 2340, 25, 1590, 25, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \; 5193$$

$$bc\int \frac{d^3x^{m+1}\left(-\frac{c^6x^6}{m+7}+\frac{3c^4x^4}{m+5}-\frac{3c^2x^2}{m+3}+\frac{1}{m+1}\right)}{\sqrt{1-c^2x^2}}dx-\frac{c^6d^3x^{m+7}(a+b\arccos(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{3c^2d^3x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \; 27$$

$$bcd^3\int \frac{x^{m+1}\left(-\frac{c^6x^6}{m+7}+\frac{3c^4x^4}{m+5}-\frac{3c^2x^2}{m+3}+\frac{1}{m+1}\right)}{\sqrt{1-c^2x^2}}dx-\frac{c^6d^3x^{m+7}(a+b\arccos(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{3c^2d^3x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \; 2340$$

$$bcd^3\left(\frac{c^4\sqrt{1-c^2x^2}x^{m+6}}{(m+7)^2}-\frac{\int -\frac{x^{m+1}\left(\frac{(m+9)(2m+13)x^4c^6}{(m+5)(m+7)}-\frac{3(m+7)x^2c^4}{m+3}+\frac{(m+7)c^2}{m+1}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+7)}\right)-\frac{c^6d^3x^{m+7}(a+b\arccos(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{3c^2d^3x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \; 25$$

$$bcd^3\left(\frac{\int \frac{x^{m+1}\left(\frac{(m+9)(2m+13)x^4c^6}{(m+5)(m+7)}-\frac{3(m+7)x^2c^4}{m+3}+\frac{(m+7)c^2}{m+1}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+7)}+\frac{c^4\sqrt{1-c^2x^2}x^{m+6}}{(m+7)^2}\right)-\frac{c^6d^3x^{m+7}(a+b\arccos(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{3c^2d^3x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \; 1590$$

$$bcd^3\left(\frac{-\frac{\int -\frac{c^4x^{m+1}\left(\frac{(m+5)(m+7)}{m+1}-\frac{c^2\left(m^4+27m^3+284m^2+1329m+2271\right)x^2}{(m+3)(m+5)(m+7)}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+5)}-\frac{c^4(m+9)(2m+13)\sqrt{1-c^2x^2}x^{m+4}}{(m+5)^2(m+7)}}{c^2(m+7)}+\frac{c^4\sqrt{1-c^2x^2}x^{m+6}}{(m+7)^2}\right)+$$

$$\frac{c^6d^3x^{m+7}(a+b\arccos(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{3c^2d^3x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \; 25$$

$$bcd^3\left(\frac{\frac{\int \frac{c^4x^{m+1}\left(\frac{(m+5)(m+7)}{m+1}-\frac{c^2\left(m^4+27m^3+284m^2+1329m+2271\right)x^2}{(m+3)(m+5)(m+7)}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+5)}-\frac{c^4(m+9)(2m+13)\sqrt{1-c^2x^2}x^{m+4}}{(m+5)^2(m+7)}}{c^2(m+7)}+\frac{c^4\sqrt{1-c^2x^2}x^{m+6}}{(m+7)^2}\right)+$$

$$\frac{c^6d^3x^{m+7}(a+b\arccos(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{3c^2d^3x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \; 27$$

$$bcd^3\left(\frac{\frac{c^2\int \frac{x^{m+1}\left(\frac{(m+5)(m+7)}{m+1}-\frac{c^2\left(m^4+27m^3+284m^2+1329m+2271\right)x^2}{(m+3)(m+5)(m+7)}\right)}{\sqrt{1-c^2x^2}}dx}{m+5}-\frac{c^4(m+9)(2m+13)\sqrt{1-c^2x^2}x^{m+4}}{(m+5)^2(m+7)}}{c^2(m+7)}+\frac{c^4\sqrt{1-c^2x^2}x^{m+6}}{(m+7)^2}\right)+$$

$$\frac{c^6d^3x^{m+7}(a+b\arccos(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{3c^2d^3x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \; 363$$

$$bcd^3\left(\frac{\frac{c^2\left(\frac{3\left(35m^3+455m^2+1813m+2161\right)\int\frac{x^{m+1}}{\sqrt{1-c^2x^2}}dx}{(m+1)(m+3)^2(m+5)(m+7)}+\frac{\left(m^4+27m^3+284m^2+1329m+2271\right)\sqrt{1-c^2x^2}x^{m+2}}{(m+3)^2(m+5)(m+7)}\right)}{m+5}-\frac{c^4(m+9)(2m+13)\sqrt{1-c^2x^2}x^{m+}}{(m+5)^2(m+7)}}{c^2(m+7)}\right.$$

$$\frac{c^6d^3x^{m+7}(a+b\arccos(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{3c^2d^3x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \text{278}$$

$$-\frac{c^6d^3x^{m+7}(a+b\arccos(cx))}{m+7}+\frac{3c^4d^3x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{3c^2d^3x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^3x^{m+1}(a+b\arccos(cx))}{m+1}+$$

$$bcd^3\left(\frac{\frac{c^2\left(\frac{3\left(35m^3+455m^2+1813m+2161\right)x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{(m+1)(m+2)(m+3)^2(m+5)(m+7)}+\frac{\left(m^4+27m^3+284m^2+1329m+2271\right)\sqrt{1-c^2x^2}x^{m+2}}{(m+3)^2(m+5)(m+7)}\right)}{m+5}-c^4(}{c^2(m+7)}\right.$$

input
```
Int[x^m*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
(d^3*x^(1 + m)*(a + b*ArcCos[c*x]))/(1 + m) - (3*c^2*d^3*x^(3 + m)*(a + b*
ArcCos[c*x]))/(3 + m) + (3*c^4*d^3*x^(5 + m)*(a + b*ArcCos[c*x]))/(5 + m)
- (c^6*d^3*x^(7 + m)*(a + b*ArcCos[c*x]))/(7 + m) + b*c*d^3*((c^4*x^(6 + m
)*Sqrt[1 - c^2*x^2])/(7 + m)^2 + (-((c^4*(9 + m)*(13 + 2*m)*x^(4 + m)*Sqrt
[1 - c^2*x^2])/((5 + m)^2*(7 + m))) + (c^2*(((2271 + 1329*m + 284*m^2 + 27
*m^3 + m^4)*x^(2 + m)*Sqrt[1 - c^2*x^2])/((3 + m)^2*(5 + m)*(7 + m)) + (3*
(2161 + 1813*m + 455*m^2 + 35*m^3)*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m
)/2, (4 + m)/2, c^2*x^2])/((1 + m)*(2 + m)*(3 + m)^2*(5 + m)*(7 + m))))/(5
 + m))/(c^2*(7 + m)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [F]

$$\int x^{m} \left(-c^{2} d\,x^{2}+d \right)^{3} \left(a +b \arccos \left(c x \right)\right)d x$$

input
```
int(x^m*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x)
```

output
```
int(x^m*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x)
```

Fricas [F]

$$\int x^m \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx=\int -\left(c^{2} d x^{2} - d\right)^{3} {\left(b \arccos\left(c x\right) + a\right)} x^{m} \,d x$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*
c^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arccos(c*x))*x^m,
 x)
```

Sympy [F]

$$\int x^m \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx = -d^3\left(\int \left(-ax^m\right)\,dx + \int \left(-bx^m \operatorname{acos}(cx)\right)\,dx + \int 3ac^2x^2x^m\,dx + \int \left(-3ac^4x^4x^m\right)\,dx + \int ac^6x^6x^m\,dx + \int 3bc^2x^2x^m \operatorname{acos}(cx)\,dx + \int \left(-3bc^4x^4x^m \operatorname{acos}(cx)\right)\,dx + \int bc^6x^6x^m \operatorname{acos}(cx)\,dx\right)$$

input
```
integrate(x**m*(-c**2*d*x**2+d)**3*(a+b*acos(c*x)),x)
```

output
```
-d**3*(Integral(-a*x**m, x) + Integral(-b*x**m*acos(c*x), x) + Integral(3*
a*c**2*x**2*x**m, x) + Integral(-3*a*c**4*x**4*x**m, x) + Integral(a*c**6*
x**6*x**m, x) + Integral(3*b*c**2*x**2*x**m*acos(c*x), x) + Integral(-3*b*
c**4*x**4*x**m*acos(c*x), x) + Integral(b*c**6*x**6*x**m*acos(c*x), x))
```

Maxima [F]

$$\int x^m \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))\,dx = \int -\left(c^2dx^2-d\right)^3 (b\arccos(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-a*c^6*d^3*x^(m + 7)/(m + 7) + 3*a*c^4*d^3*x^(m + 5)/(m + 5) - 3*a*c^2*d^3
*x^(m + 3)/(m + 3) + a*d^3*x^(m + 1)/(m + 1) - (((b*c^6*d^3*m^3 + 9*b*c^6*
d^3*m^2 + 23*b*c^6*d^3*m + 15*b*c^6*d^3)*x^7 - 3*(b*c^4*d^3*m^3 + 11*b*c^4
*d^3*m^2 + 31*b*c^4*d^3*m + 21*b*c^4*d^3)*x^5 + 3*(b*c^2*d^3*m^3 + 13*b*c^
2*d^3*m^2 + 47*b*c^2*d^3*m + 35*b*c^2*d^3)*x^3 - (b*d^3*m^3 + 15*b*d^3*m^2
 + 71*b*d^3*m + 105*b*d^3)*x)*x^m*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*
x) - (m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*integrate(-((b*c^7*d^3*m^3 + 9*
b*c^7*d^3*m^2 + 23*b*c^7*d^3*m + 15*b*c^7*d^3)*x^7 - 3*(b*c^5*d^3*m^3 + 11
*b*c^5*d^3*m^2 + 31*b*c^5*d^3*m + 21*b*c^5*d^3)*x^5 + 3*(b*c^3*d^3*m^3 + 1
3*b*c^3*d^3*m^2 + 47*b*c^3*d^3*m + 35*b*c^3*d^3)*x^3 - (b*c*d^3*m^3 + 15*b
*c*d^3*m^2 + 71*b*c*d^3*m + 105*b*c*d^3)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x
^m/(m^4 + 16*m^3 - (c^2*m^4 + 16*c^2*m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^
2)*x^2 + 86*m^2 + 176*m + 105), x))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Giac [F]

$$\int x^m (d-c^2dx^2)^3 (a+b\arccos(cx))\,dx = \int -(c^2dx^2-d)^3(b\arccos(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(-(c^2*d*x^2 - d)^3*(b*arccos(c*x) + a)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m (d-c^2dx^2)^3 (a+b\arccos(cx))\,dx = \int x^m (a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

input
```
int(x^m*(a + b*acos(c*x))*(d - c^2*d*x^2)^3,x)
```

output
```
int(x^m*(a + b*acos(c*x))*(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int x^m \left(d-c^2 d x^2\right)^3 (a+b \arccos(cx))\,dx = \text{Too large to display}$$

input
```
int(x^m*(-c^2*d*x^2+d)^3*(a+b*acos(c*x)),x)
```

output
```
(d**3*( - x**m*a*c**6*m**3*x**7 - 9*x**m*a*c**6*m**2*x**7 - 23*x**m*a*c**6
*m*x**7 - 15*x**m*a*c**6*x**7 + 3*x**m*a*c**4*m**3*x**5 + 33*x**m*a*c**4*m
**2*x**5 + 93*x**m*a*c**4*m*x**5 + 63*x**m*a*c**4*x**5 - 3*x**m*a*c**2*m**
3*x**3 - 39*x**m*a*c**2*m**2*x**3 - 141*x**m*a*c**2*m*x**3 - 105*x**m*a*c*
*2*x**3 + x**m*a*m**3*x + 15*x**m*a*m**2*x + 71*x**m*a*m*x + 105*x**m*a*x
- int(x**m*acos(c*x)*x**6,x)*b*c**6*m**4 - 16*int(x**m*acos(c*x)*x**6,x)*b
*c**6*m**3 - 86*int(x**m*acos(c*x)*x**6,x)*b*c**6*m**2 - 176*int(x**m*acos
(c*x)*x**6,x)*b*c**6*m - 105*int(x**m*acos(c*x)*x**6,x)*b*c**6 + 3*int(x**
m*acos(c*x)*x**4,x)*b*c**4*m**4 + 48*int(x**m*acos(c*x)*x**4,x)*b*c**4*m**
3 + 258*int(x**m*acos(c*x)*x**4,x)*b*c**4*m**2 + 528*int(x**m*acos(c*x)*x*
*4,x)*b*c**4*m + 315*int(x**m*acos(c*x)*x**4,x)*b*c**4 - 3*int(x**m*acos(c
*x)*x**2,x)*b*c**2*m**4 - 48*int(x**m*acos(c*x)*x**2,x)*b*c**2*m**3 - 258*
int(x**m*acos(c*x)*x**2,x)*b*c**2*m**2 - 528*int(x**m*acos(c*x)*x**2,x)*b*
c**2*m - 315*int(x**m*acos(c*x)*x**2,x)*b*c**2 + int(x**m*acos(c*x),x)*b*m
**4 + 16*int(x**m*acos(c*x),x)*b*m**3 + 86*int(x**m*acos(c*x),x)*b*m**2 +
176*int(x**m*acos(c*x),x)*b*m + 105*int(x**m*acos(c*x),x)*b))/(m**4 + 16*m
**3 + 86*m**2 + 176*m + 105)
```

3.146 $\int x^m (d-c^2dx^2)^2 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 217

$$\begin{aligned}\int x^m (d-c^2dx^2)^2 (a+b\arccos(cx))\,dx &= -\frac{bcd^2(38+13m+m^2)\,x^{2+m}\sqrt{1-c^2x^2}}{(3+m)^2(5+m)^2} \\ &\quad+\frac{bc^3d^2x^{4+m}\sqrt{1-c^2x^2}}{(5+m)^2}+\frac{d^2x^{1+m}(a+b\arccos(cx))}{1+m} \\ &\quad-\frac{2c^2d^2x^{3+m}(a+b\arccos(cx))}{3+m}+\frac{c^4d^2x^{5+m}(a+b\arccos(cx))}{5+m} \\ &\quad-\frac{bcd^2(149+100m+15m^2)\,x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{(1+m)(2+m)(3+m)^2(5+m)^2}\end{aligned}$$

output
```
-b*c*d^2*(m^2+13*m+38)*x^(2+m)*(-c^2*x^2+1)^(1/2)/(3+m)^2/(5+m)^2+b*c^3*d^
2*x^(4+m)*(-c^2*x^2+1)^(1/2)/(5+m)^2+d^2*x^(1+m)*(a+b*arccos(c*x))/(1+m)-2
*c^2*d^2*x^(3+m)*(a+b*arccos(c*x))/(3+m)+c^4*d^2*x^(5+m)*(a+b*arccos(c*x))
/(5+m)-b*c*d^2*(15*m^2+100*m+149)*x^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*
m],c^2*x^2)/(1+m)/(2+m)/(3+m)^2/(5+m)^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.94

$$\int x^m (d - c^2 d x^2)^2 (a + b \arccos(cx)) \, dx$$
$$= d^2 x^{1+m} \left(\frac{a}{1+m} - \frac{2ac^2x^2}{3+m} + \frac{ac^4x^4}{5+m} + \frac{b\left((2+m)\arccos(cx) + cx \text{ Hypergeometric2F1}\left(\frac{1}{2}, 1+\frac{m}{2}, 2+\frac{m}{2}, c^2x^2\right)\right)}{(1+m)(2+m)} - \frac{2bc^2x^2\left((4+m)\arccos(cx) + cx \text{ Hypergeometric2F1}\left(\frac{1}{2}, 2+\frac{m}{2}, 3+\frac{m}{2}, c^2x^2\right)\right)}{(3+m)(4+m)} + \frac{bc^4x^4\left((6+m)\arccos(cx) + cx \text{ Hypergeometric2F1}\left(\frac{1}{2}, 3+\frac{m}{2}, 4+\frac{m}{2}, c^2x^2\right)\right)}{(5+m)(6+m)} \right)$$

input
```
Integrate[x^m*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
d^2*x^(1 + m)*(a/(1 + m) - (2*a*c^2*x^2)/(3 + m) + (a*c^4*x^4)/(5 + m) + (
b*((2 + m)*ArcCos[c*x] + c*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, c^2*
x^2]))/((1 + m)*(2 + m)) - (2*b*c^2*x^2*((4 + m)*ArcCos[c*x] + c*x*Hyperge
ometric2F1[1/2, 2 + m/2, 3 + m/2, c^2*x^2]))/((3 + m)*(4 + m)) + (b*c^4*x^
4*((6 + m)*ArcCos[c*x] + c*x*Hypergeometric2F1[1/2, 3 + m/2, 4 + m/2, c^2*
x^2]))/((5 + m)*(6 + m)))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5193, 27, 1590, 25, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (d - c^2 d x^2)^2 (a + b \arccos(cx)) \, dx$$
$$\Big\downarrow \; 5193$$

$$bc\int \frac{d^2x^{m+1}\left(\frac{c^4x^4}{m+5}-\frac{2c^2x^2}{m+3}+\frac{1}{m+1}\right)}{\sqrt{1-c^2x^2}}dx+\frac{c^4d^2x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{2c^2d^2x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^2x^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \text{27}$$

$$bcd^2\int \frac{x^{m+1}\left(\frac{c^4x^4}{m+5}-\frac{2c^2x^2}{m+3}+\frac{1}{m+1}\right)}{\sqrt{1-c^2x^2}}dx+\frac{c^4d^2x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{2c^2d^2x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^2x^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \text{1590}$$

$$bcd^2\left(-\frac{\int -\frac{c^2x^{m+1}\left(\frac{m+5}{m+1}-\frac{c^2\left(m^2+13m+38\right)x^2}{(m+3)(m+5)}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+5)}-\frac{c^2\sqrt{1-c^2x^2}x^{m+4}}{(m+5)^2}\right)+\frac{c^4d^2x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{2c^2d^2x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^2x^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \text{25}$$

$$bcd^2\left(\frac{\int \frac{c^2x^{m+1}\left(\frac{m+5}{m+1}-\frac{c^2\left(m^2+13m+38\right)x^2}{(m+3)(m+5)}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+5)}-\frac{c^2\sqrt{1-c^2x^2}x^{m+4}}{(m+5)^2}\right)+\frac{c^4d^2x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{2c^2d^2x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^2x^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \text{27}$$

$$bcd^2\left(\frac{\int \frac{x^{m+1}\left(\frac{m+5}{m+1}-\frac{c^2\left(m^2+13m+38\right)x^2}{(m+3)(m+5)}\right)}{\sqrt{1-c^2x^2}}dx}{m+5}-\frac{c^2\sqrt{1-c^2x^2}x^{m+4}}{(m+5)^2}\right)+\frac{c^4d^2x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{2c^2d^2x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^2x^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \text{363}$$

$$bcd^2\left(\frac{\frac{(15m^2+100m+149)\int \frac{x^{m+1}}{\sqrt{1-c^2x^2}}dx}{(m+1)(m+3)^2(m+5)}+\frac{(m^2+13m+38)\sqrt{1-c^2x^2}x^{m+2}}{(m+3)^2(m+5)}}{m+5}-\frac{c^2\sqrt{1-c^2x^2}x^{m+4}}{(m+5)^2}\right)+$$
$$\frac{c^4d^2x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{2c^2d^2x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^2x^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \text{278}$$

$$\frac{c^4d^2x^{m+5}(a+b\arccos(cx))}{m+5}-\frac{2c^2d^2x^{m+3}(a+b\arccos(cx))}{m+3}+\frac{d^2x^{m+1}(a+b\arccos(cx))}{m+1}+$$
$$bcd^2\left(\frac{\frac{(15m^2+100m+149)x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{(m+1)(m+2)(m+3)^2(m+5)}+\frac{(m^2+13m+38)\sqrt{1-c^2x^2}x^{m+2}}{(m+3)^2(m+5)}}{m+5}-\frac{c^2\sqrt{1-c^2x^2}x^{m+4}}{(m+5)^2}\right)$$

input
```
Int[x^m*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*x^(1 + m)*(a + b*ArcCos[c*x]))/(1 + m) - (2*c^2*d^2*x^(3 + m)*(a + b*
ArcCos[c*x]))/(3 + m) + (c^4*d^2*x^(5 + m)*(a + b*ArcCos[c*x]))/(5 + m) +
b*c*d^2*(-((c^2*x^(4 + m)*Sqrt[1 - c^2*x^2])/(5 + m)^2) + (((38 + 13*m + m
^2)*x^(2 + m)*Sqrt[1 - c^2*x^2])/((3 + m)^2*(5 + m)) + ((149 + 100*m + 15*
m^2)*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/((1
+ m)*(2 + m)*(3 + m)^2*(5 + m)))/(5 + m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [F]

$$\int x^m (-c^2 d x^2 + d)^2 (a + b \arccos(cx)) \, dx$$

input
```
int(x^m*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x)
```

output
```
int(x^m*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x)
```

Fricas [F]

$$\int x^m \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx = \int \left(c^2dx^2-d\right)^2 (b\arccos(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccos(c*x))*x^m, x)
```

Sympy [F]

$$\int x^m \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx = d^2\left(\int ax^m\,dx + \int bx^m \operatorname{acos}(cx)\,dx + \int \left(-2ac^2x^2x^m\right)dx + \int ac^4x^4x^m\,dx + \int \left(-2bc^2x^2x^m \operatorname{acos}(cx)\right)dx + \int bc^4x^4x^m \operatorname{acos}(cx)\,dx\right)$$

input
```
integrate(x**m*(-c**2*d*x**2+d)**2*(a+b*acos(c*x)),x)
```

output
```
d**2*(Integral(a*x**m, x) + Integral(b*x**m*acos(c*x), x) + Integral(-2*a*
c**2*x**2*x**m, x) + Integral(a*c**4*x**4*x**m, x) + Integral(-2*b*c**2*x*
*2*x**m*acos(c*x), x) + Integral(b*c**4*x**4*x**m*acos(c*x), x))
```

Maxima [F]

$$\int x^m \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx=\int \left(c^2dx^2-d\right)^2(b\arccos\left(cx\right)+a)x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
a*c^4*d^2*x^(m + 5)/(m + 5) - 2*a*c^2*d^2*x^(m + 3)/(m + 3) + a*d^2*x^(m +
 1)/(m + 1) + (((b*c^4*d^2*m^2 + 4*b*c^4*d^2*m + 3*b*c^4*d^2)*x^5 - 2*(b*c
^2*d^2*m^2 + 6*b*c^2*d^2*m + 5*b*c^2*d^2)*x^3 + (b*d^2*m^2 + 8*b*d^2*m + 1
5*b*d^2)*x)*x^m*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - (m^3 + 9*m^2
+ 23*m + 15)*integrate(-((b*c^5*d^2*m^2 + 4*b*c^5*d^2*m + 3*b*c^5*d^2)*x^5
 - 2*(b*c^3*d^2*m^2 + 6*b*c^3*d^2*m + 5*b*c^3*d^2)*x^3 + (b*c*d^2*m^2 + 8*
b*c*d^2*m + 15*b*c*d^2)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m/(m^3 - (c^2*m^
3 + 9*c^2*m^2 + 23*c^2*m + 15*c^2)*x^2 + 9*m^2 + 23*m + 15), x))/(m^3 + 9*
m^2 + 23*m + 15)
```

Giac [F]

$$\int x^m \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))\,dx=\int \left(c^2dx^2-d\right)^2(b\arccos\left(cx\right)+a)x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 - d)^2*(b*arccos(c*x) + a)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m (d-c^2dx^2)^2 (a+b\arccos(cx))\,dx = \int x^m (a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2\,dx$$

input
```
int(x^m*(a + b*acos(c*x))*(d - c^2*d*x^2)^2,x)
```

output
```
int(x^m*(a + b*acos(c*x))*(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int x^m (d-c^2dx^2)^2 (a+b\arccos(cx))\,dx = \frac{d^2\big(x^m a\,c^4m^2x^5+4x^m a\,c^4m\,x^5+3x^m a\,c^4x^5-2x^m a\,c^2m^2x^3-12x^m a\,c^2m\,x^3-10x^m a\,c^2x^3+x^m a\,m^2x-}{}$$

input
```
int(x^m*(-c^2*d*x^2+d)^2*(a+b*acos(c*x)),x)
```

output
```
(d**2*(x**m*a*c**4*m**2*x**5 + 4*x**m*a*c**4*m*x**5 + 3*x**m*a*c**4*x**5 -
 2*x**m*a*c**2*m**2*x**3 - 12*x**m*a*c**2*m*x**3 - 10*x**m*a*c**2*x**3 + x
**m*a*m**2*x + 8*x**m*a*m*x + 15*x**m*a*x + int(x**m*acos(c*x)*x**4,x)*b*c
**4*m**3 + 9*int(x**m*acos(c*x)*x**4,x)*b*c**4*m**2 + 23*int(x**m*acos(c*x
)*x**4,x)*b*c**4*m + 15*int(x**m*acos(c*x)*x**4,x)*b*c**4 - 2*int(x**m*aco
s(c*x)*x**2,x)*b*c**2*m**3 - 18*int(x**m*acos(c*x)*x**2,x)*b*c**2*m**2 - 4
6*int(x**m*acos(c*x)*x**2,x)*b*c**2*m - 30*int(x**m*acos(c*x)*x**2,x)*b*c*
*2 + int(x**m*acos(c*x),x)*b*m**3 + 9*int(x**m*acos(c*x),x)*b*m**2 + 23*in
t(x**m*acos(c*x),x)*b*m + 15*int(x**m*acos(c*x),x)*b))/(m**3 + 9*m**2 + 23
*m + 15)
```

3.147 $\int x^m(d - c^2dx^2)(a + b\arccos(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int x^m(d - c^2dx^2)(a + b\arccos(cx))\,dx = -\frac{bcdx^{2+m}\sqrt{1-c^2x^2}}{(3+m)^2} + \frac{dx^{1+m}(a+b\arccos(cx))}{1+m} - \frac{c^2dx^{3+m}(a+b\arccos(cx))}{3+m} - \frac{bcd(7+3m)x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{(1+m)(2+m)(3+m)^2}$$

output

```
-b*c*d*x^(2+m)*(-c^2*x^2+1)^(1/2)/(3+m)^2+d*x^(1+m)*(a+b*arccos(c*x))/(1+m
)-c^2*d*x^(3+m)*(a+b*arccos(c*x))/(3+m)-b*c*d*(7+3*m)*x^(2+m)*hypergeom([1
/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/(1+m)/(2+m)/(3+m)^2
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

$$\int x^m(d-c^2dx^2)(a+b\arccos(cx))\,dx$$
$$=dx^{1+m}\left(\frac{bcx\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{m}{2},2+\frac{m}{2},c^2x^2\right)}{2+3m+m^2}-\frac{\frac{(-3+c^2x^2+m(-1+c^2x^2))(a+b\arccos(cx))}{1+m}+\frac{bc^3x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},2+\frac{m}{2},3+\frac{m}{2},c^2x^2\right)}{4+m}}{3+m}\right)$$

input
```
Integrate[x^m*(d - c^2*d*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
d*x^(1 + m)*((b*c*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, c^2*x^2])/(2
+ 3*m + m^2) - (((-3 + c^2*x^2 + m*(-1 + c^2*x^2))*(a + b*ArcCos[c*x]))/(1
 + m) + (b*c^3*x^3*Hypergeometric2F1[1/2, 2 + m/2, 3 + m/2, c^2*x^2])/(4 +
 m))/(3 + m))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5193, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m(d-c^2dx^2)(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5193}$$

$$bc\int \frac{dx^{m+1}\left(\frac{1}{m+1}-\frac{c^2x^2}{m+3}\right)}{\sqrt{1-c^2x^2}}dx-\frac{c^2dx^{m+3}(a+b\arccos(cx))}{m+3}+\frac{dx^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \text{27}$$

$$bcd\int \frac{x^{m+1}\left(\frac{1}{m+1}-\frac{c^2x^2}{m+3}\right)}{\sqrt{1-c^2x^2}}dx-\frac{c^2dx^{m+3}(a+b\arccos(cx))}{m+3}+\frac{dx^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \text{363}$$

$$bcd\left(\frac{(3m+7)\int \frac{x^{m+1}}{\sqrt{1-c^2x^2}}dx}{(m+1)(m+3)^2}+\frac{\sqrt{1-c^2x^2}x^{m+2}}{(m+3)^2}\right)-\frac{c^2dx^{m+3}(a+b\arccos(cx))}{m+3}+\frac{dx^{m+1}(a+b\arccos(cx))}{m+1}$$

$$\Big\downarrow \text{278}$$

$$-\frac{c^2dx^{m+3}(a+b\arccos(cx))}{m+3}+\frac{dx^{m+1}(a+b\arccos(cx))}{m+1}+bcd\left(\frac{(3m+7)x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{(m+1)(m+2)(m+3)^2}+\frac{\sqrt{1-c^2x^2}x^{m+2}}{(m+3)^2}\right)$$

input
```
Int[x^m*(d - c^2*d*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d*x^(1 + m)*(a + b*ArcCos[c*x]))/(1 + m) - (c^2*d*x^(3 + m)*(a + b*ArcCos
[c*x]))/(3 + m) + b*c*d*((x^(2 + m)*Sqrt[1 - c^2*x^2])/(3 + m)^2 + ((7 + 3
*m)*x^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/((1 +
 m)*(2 + m)*(3 + m)^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

Maple [F]

$$\int x^{m} \left(-c^{2} d\, x^{2}+d \right) \left(a +b \arccos \left(c x \right)\right)d x$$

input
```
int(x^m*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x)
```

output
```
int(x^m*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x)
```

Fricas [F]

$$\int x^m \left(d-c^2 d x^2\right) (a+b \arccos (c x)) \, dx=\int -{\left(c^{2} d x^{2} - d\right)} {\left(b \arccos \left(c x\right) + a\right)} x^{m} \,d x$$

input
```
integrate(x^m*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccos(c*x))*x^m, x)
```

Sympy [F]

$$\int x^m(d-c^2dx^2)(a+b\arccos(cx))\,dx = -d\left(\int(-ax^m)\,dx + \int(-bx^m\,\mathrm{acos}\,(cx))\,dx + \int ac^2x^2x^m\,dx + \int bc^2x^2x^m\,\mathrm{acos}\,(cx)\,dx\right)$$

input
```
integrate(x**m*(-c**2*d*x**2+d)*(a+b*acos(c*x)),x)
```

output
```
-d*(Integral(-a*x**m, x) + Integral(-b*x**m*acos(c*x), x) + Integral(a*c**
2*x**2*x**m, x) + Integral(b*c**2*x**2*x**m*acos(c*x), x))
```

Maxima [F]

$$\int x^m(d-c^2dx^2)(a+b\arccos(cx))\,dx = \int -(c^2dx^2-d)(b\arccos(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-a*c^2*d*x^(m + 3)/(m + 3) + a*d*x^(m + 1)/(m + 1) - (((b*c^2*d*m + b*c^2*
d)*x^3 - (b*d*m + 3*b*d)*x)*x^m*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)
 - (m^2 + 4*m + 3)*integrate(((b*c^3*d*m + b*c^3*d)*x^3 - (b*c*d*m + 3*b*c
*d)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m/((c^2*m^2 + 4*c^2*m + 3*c^2)*x^2 -
 m^2 - 4*m - 3), x))/(m^2 + 4*m + 3)
```

Giac [F]

$$\int x^m(d-c^2dx^2)(a+b\arccos(cx))\,dx = \int -(c^2dx^2-d)(b\arccos(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(-(c^2*d*x^2 - d)*(b*arccos(c*x) + a)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m(d-c^2dx^2)(a+b\arccos(cx))\,dx=\int x^m\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)\,dx$$

input
```
int(x^m*(a + b*acos(c*x))*(d - c^2*d*x^2),x)
```

output
```
int(x^m*(a + b*acos(c*x))*(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int x^m(d-c^2dx^2)(a+b\arccos(cx))\,dx$$
$$=\frac{d\left(-x^m a\,c^2m\,x^3-x^m a\,c^2x^3+x^m amx+3x^m ax-\left(\int x^m acos(cx)\,x^2dx\right)b\,c^2m^2-4\left(\int x^m acos(cx)\,x^2dx\right)\right.}{m}$$

input
```
int(x^m*(-c^2*d*x^2+d)*(a+b*acos(c*x)),x)
```

output
```
(d*( - x**m*a*c**2*m*x**3 - x**m*a*c**2*x**3 + x**m*a*m*x + 3*x**m*a*x - i
nt(x**m*acos(c*x)*x**2,x)*b*c**2*m**2 - 4*int(x**m*acos(c*x)*x**2,x)*b*c**
2*m - 3*int(x**m*acos(c*x)*x**2,x)*b*c**2 + int(x**m*acos(c*x),x)*b*m**2 +
 4*int(x**m*acos(c*x),x)*b*m + 3*int(x**m*acos(c*x),x)*b))/(m**2 + 4*m + 3
)
```

3.148 $\int \frac{x^m(a+b\arccos(cx))}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{x^m(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \text{Int}\left(\frac{x^m(a+b\arccos(cx))}{d-c^2dx^2},x\right)$$

output
```
Defer(Int)(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x)
```

Mathematica [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \int \frac{x^m(a+b\arccos(cx))}{d-c^2dx^2}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2),x]
```

output
```
Integrate[(x^m*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arccos(cx))}{d-c^2dx^2}\,dx$$

$$\Big\downarrow \; 5235$$

$$\int \frac{x^m(a+b\arccos(cx))}{d-c^2dx^2}dx$$

input
```
Int[(x^m*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arccos(cx))}{-c^2d\,x^2+d}dx$$

input
```
int(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x)
```

output
```
int(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x^m(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos{(cx)}+a)x^m}{c^2dx^2-d}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b*arccos(c*x) + a)*x^m/(c^2*d*x^2 - d), x)
```

Sympy [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{x^m(a+b\arccos(cx))}{d-c^2dx^2}\,dx = -\frac{\int \frac{ax^m}{c^2x^2-1}\,dx + \int \frac{bx^m\operatorname{acos}{(cx)}}{c^2x^2-1}\,dx}{d}$$

input
```
integrate(x**m*(a+b*acos(c*x))/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a*x**m/(c**2*x**2 - 1), x) + Integral(b*x**m*acos(c*x)/(c**2*x*
*2 - 1), x))/d
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{x^m(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos{(cx)}+a)x^m}{c^2dx^2-d}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-integrate((b*arccos(c*x) + a)*x^m/(c^2*d*x^2 - d), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \int \frac{x^m\,(a+b\,\mathrm{acos}(c\,x))}{d-c^2\,d\,x^2}\,dx$$

input
```
int((x^m*(a + b*acos(c*x)))/(d - c^2*d*x^2),x)
```

output
```
int((x^m*(a + b*acos(c*x)))/(d - c^2*d*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{x^m(a+b\arccos(cx))}{d-c^2dx^2}\,dx = \frac{-\left(\int \frac{x^m}{c^2x^2-1}dx\right)a - \left(\int \frac{x^m acos(cx)}{c^2x^2-1}dx\right)b}{d}$$

input
```
int(x^m*(a+b*acos(c*x))/(-c^2*d*x^2+d),x)
```

output
```
( - (int(x**m/(c**2*x**2 - 1),x)*a + int((x**m*acos(c*x))/(c**2*x**2 - 1),
x)*b))/d
```

3.149 $\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = \frac{x^{1+m}(a+b\arccos(cx))}{2d^2(1-c^2x^2)} - \frac{bcx^{2+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{2d^2(2+m)} + \frac{(1-m)\text{Int}\left(\frac{x^m(a+b\arccos(cx))}{d-c^2dx^2},x\right)}{2d}$$

output
```
1/2*x^(1+m)*(a+b*arccos(c*x))/d^2/(-c^2*x^2+1)-1/2*b*c*x^(2+m)*hypergeom([
3/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/d^2/(2+m)+1/2*(1-m)*Defer(Int)(x^m*(a+b*a
rccos(c*x))/(-c^2*d*x^2+d),x)/d
```

Mathematica [N/A]

Not integrable

Time = 8.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx=\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
Integrate[(x^m*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx$$

$$\Big\downarrow \text{5209}$$

$$\frac{(1-m)\int \frac{x^m(a+b\arccos(cx))}{d(1-c^2x^2)}dx}{2d}+\frac{bc\int \frac{x^{m+1}}{(1-c^2x^2)^{3/2}}dx}{2d^2}+\frac{x^{m+1}(a+b\arccos(cx))}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(1-m)\int \frac{x^m(a+b\arccos(cx))}{1-c^2x^2}dx}{2d^2}+\frac{bc\int \frac{x^{m+1}}{(1-c^2x^2)^{3/2}}dx}{2d^2}+\frac{x^{m+1}(a+b\arccos(cx))}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{278}$$

$$\frac{(1-m)\int \frac{x^m(a+b\arccos(cx))}{1-c^2x^2}dx}{2d^2}+\frac{x^{m+1}(a+b\arccos(cx))}{2d^2(1-c^2x^2)}+\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{2d^2(m+2)}$$

$$\Big\downarrow \; 5235$$

$$\frac{(1-m)\int \frac{x^m(a+b\arccos(cx))}{1-c^2x^2}dx}{2d^2}+\frac{x^{m+1}(a+b\arccos(cx))}{2d^2(1-c^2x^2)}+\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{2d^2(m+2)}$$

input
```
Int[(x^m*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arccos(cx))}{(-c^2d\,x^2+d)^2}dx$$

input
```
int(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x)
```

output
```
int(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{x^m(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^2}\,dx=\int \frac{(b\arccos{(cx)}+a)x^m}{\left(c^2dx^2-d\right)^2}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)*x^m/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 15.71 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \frac{x^m(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^2}\,dx=\frac{\int \frac{ax^m}{c^4x^4-2c^2x^2+1}\,dx+\int \frac{bx^m\operatorname{acos}{(cx)}}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**m*(a+b*acos(c*x))/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a*x**m/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**m*acos(
c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{x^m(a+b\arccos(cx))}{\left(d-c^2dx^2\right)^2}\,dx=\int \frac{(b\arccos{(cx)}+a)x^m}{\left(c^2dx^2-d\right)^2}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)*x^m/(c^2*d*x^2 - d)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = \int \frac{x^m\,(a+b\,\mathrm{acos}(c\,x))}{{(d-c^2\,d\,x^2)}^2}\,dx$$

input
```
int((x^m*(a + b*acos(c*x)))/(d - c^2*d*x^2)^2,x)
```

output
```
int((x^m*(a + b*acos(c*x)))/(d - c^2*d*x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^2}\,dx = \frac{\left(\int \frac{x^m}{c^4x^4-2c^2x^2+1}dx\right)a+\left(\int \frac{x^m\,acos(cx)}{c^4x^4-2c^2x^2+1}dx\right)b}{d^2}$$

input
```
int(x^m*(a+b*acos(c*x))/(-c^2*d*x^2+d)^2,x)
```

output
```
(int(x**m/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a + int((x**m*acos(c*x))/(c**4*
x**4 - 2*c**2*x**2 + 1),x)*b)/d**2
```

3.150 $\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \frac{x^{1+m}(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}+\frac{(3-m)x^{1+m}(a+b\arccos(cx))}{8d^3(1-c^2x^2)} - \frac{bc(3-m)x^{2+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{8d^3(2+m)} - \frac{bcx^{2+m}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{4d^3(2+m)} + \frac{(1-m)(3-m)\text{Int}\left(\frac{x^m(a+b\arccos(cx))}{d-c^2dx^2},x\right)}{8d^2}$$

output
```
1/4*x^(1+m)*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)^2+1/8*(3-m)*x^(1+m)*(a+b*ar
ccos(c*x))/d^3/(-c^2*x^2+1)-1/8*b*c*(3-m)*x^(2+m)*hypergeom([3/2, 1+1/2*m]
,[2+1/2*m],c^2*x^2)/d^3/(2+m)-1/4*b*c*x^(2+m)*hypergeom([5/2, 1+1/2*m],[2+
1/2*m],c^2*x^2)/d^3/(2+m)+1/8*(1-m)*(3-m)*Defer(Int)(x^m*(a+b*arccos(c*x))
/(-c^2*d*x^2+d),x)/d^2
```

Mathematica [N/A]

Not integrable

Time = 11.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx=\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
Integrate[(x^m*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx$$

$$\Big\downarrow \text{5209}$$

$$\frac{(3-m)\int \frac{x^m(a+b\arccos(cx))}{d^2(1-c^2x^2)^2}dx}{4d}+\frac{bc\int \frac{x^{m+1}}{(1-c^2x^2)^{5/2}}dx}{4d^3}+\frac{x^{m+1}(a+b\arccos(cx))}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(3-m)\int \frac{x^m(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{4d^3}+\frac{bc\int \frac{x^{m+1}}{(1-c^2x^2)^{5/2}}dx}{4d^3}+\frac{x^{m+1}(a+b\arccos(cx))}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{278}$$

$$\frac{(3-m)\int \frac{x^m(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{4d^3}+\frac{x^{m+1}(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}+\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{4d^3(m+2)}$$

$\Big\downarrow$ 5209

$$\frac{(3-m)\left(\frac{1}{2}(1-m)\int \frac{x^m(a+b\arccos(cx))}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{x^{m+1}}{(1-c^2x^2)^{3/2}}dx+\frac{x^{m+1}(a+b\arccos(cx))}{2(1-c^2x^2)}\right)}{4d^3}+\frac{x^{m+1}(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}+\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{4d^3(m+2)}$$

$\Big\downarrow$ 278

$$\frac{(3-m)\left(\frac{1}{2}(1-m)\int \frac{x^m(a+b\arccos(cx))}{1-c^2x^2}dx+\frac{x^{m+1}(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{2(m+2)}\right)}{4d^3}+\frac{x^{m+1}(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}+\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{4d^3(m+2)}$$

$\Big\downarrow$ 5235

$$\frac{(3-m)\left(\frac{1}{2}(1-m)\int \frac{x^m(a+b\arccos(cx))}{1-c^2x^2}dx+\frac{x^{m+1}(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{2(m+2)}\right)}{4d^3}+\frac{x^{m+1}(a+b\arccos(cx))}{4d^3(1-c^2x^2)^2}+\frac{bcx^{m+2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{4d^3(m+2)}$$

input
```
Int[(x^m*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arccos(cx))}{(-c^2d\,x^2+d)^3}dx$$

input
```
int(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x)
```

output
```
int(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.20

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)x^m}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b*arccos(c*x) + a)*x^m/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3
*x^2 - d^3), x)
```

Sympy [N/A]

Not integrable

Time = 106.72 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.92

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = -\frac{\int \frac{ax^m}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{bx^m\operatorname{acos}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

input
```
integrate(x**m*(a+b*acos(c*x))/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a*x**m/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integr
al(b*x**m*acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx=\int -\frac{(b\arccos{(cx)}+a)x^m}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-integrate((b*arccos(c*x) + a)*x^m/(c^2*d*x^2 - d)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \int \frac{x^m\,(a+b\,\mathrm{acos}(c\,x))}{(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((x^m*(a + b*acos(c*x)))/(d - c^2*d*x^2)^3,x)
```

output
```
int((x^m*(a + b*acos(c*x)))/(d - c^2*d*x^2)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.24

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^3}\,dx = \frac{-\left(\int \frac{x^m}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)a-\left(\int \frac{x^m\,acos(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b}{d^3}$$

input
```
int(x^m*(a+b*acos(c*x))/(-c^2*d*x^2+d)^3,x)
```

output
```
( - (int(x**m/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*a + int((x**m
*acos(c*x))/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b))/d**3
```

3.151 $\int x^m(d-c^2dx^2)^{5/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 635

$$\begin{aligned}\int x^m\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx &= -\frac{15bcd^2x^{2+m}\sqrt{d-c^2dx^2}}{(2+m)^2(4+m)(6+m)\sqrt{1-c^2x^2}}\\
&-\frac{5bcd^2x^{2+m}\sqrt{d-c^2dx^2}}{(6+m)\left(8+6m+m^2\right)\sqrt{1-c^2x^2}}-\frac{bcd^2x^{2+m}\sqrt{d-c^2dx^2}}{\left(12+8m+m^2\right)\sqrt{1-c^2x^2}}\\
&+\frac{5bc^3d^2x^{4+m}\sqrt{d-c^2dx^2}}{(4+m)^2(6+m)\sqrt{1-c^2x^2}}+\frac{2bc^3d^2x^{4+m}\sqrt{d-c^2dx^2}}{(4+m)(6+m)\sqrt{1-c^2x^2}}\\
&-\frac{bc^5d^2x^{6+m}\sqrt{d-c^2dx^2}}{(6+m)^2\sqrt{1-c^2x^2}}+\frac{15d^2x^{1+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(6+m)\left(8+6m+m^2\right)}\\
&+\frac{5dx^{1+m}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{(4+m)(6+m)}+\frac{x^{1+m}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{6+m}\\
&+\frac{15d^2x^{1+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(6+m)\left(8+14m+7m^2+m^3\right)\sqrt{1-c^2x^2}}\\
&-\frac{15bcd^2x^{2+m}\sqrt{d-c^2dx^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{(1+m)(2+m)^2(4+m)(6+m)\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-15*b*c*d^2*x^(2+m)*(-c^2*d*x^2+d)^(1/2)/(2+m)^2/(4+m)/(6+m)/(-c^2*x^2+1)^
(1/2)-5*b*c*d^2*x^(2+m)*(-c^2*d*x^2+d)^(1/2)/(6+m)/(m^2+6*m+8)/(-c^2*x^2+1
)^(1/2)-b*c*d^2*x^(2+m)*(-c^2*d*x^2+d)^(1/2)/(m^2+8*m+12)/(-c^2*x^2+1)^(1/
2)+5*b*c^3*d^2*x^(4+m)*(-c^2*d*x^2+d)^(1/2)/(4+m)^2/(6+m)/(-c^2*x^2+1)^(1/
2)+2*b*c^3*d^2*x^(4+m)*(-c^2*d*x^2+d)^(1/2)/(4+m)/(6+m)/(-c^2*x^2+1)^(1/2)
-b*c^5*d^2*x^(6+m)*(-c^2*d*x^2+d)^(1/2)/(6+m)^2/(-c^2*x^2+1)^(1/2)+15*d^2*
x^(1+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(6+m)/(m^2+6*m+8)+5*d*x^(1+
m)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/(4+m)/(6+m)+x^(1+m)*(-c^2*d*x^2+
d)^(5/2)*(a+b*arccos(c*x))/(6+m)+15*d^2*x^(1+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*
arccos(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)/(6+m)/(m^3+7*
m^2+14*m+8)/(-c^2*x^2+1)^(1/2)-15*b*c*d^2*x^(2+m)*(-c^2*d*x^2+d)^(1/2)*hyp
ergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)/(1+m)/(2+m)^2/(
4+m)/(6+m)/(-c^2*x^2+1)^(1/2)
```

Mathematica [F]

$$\int x^m \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\int x^m \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx$$

input
```
Integrate[x^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
Integrate[x^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]), x]
```

Rubi [A] (warning: unable to verify)

Time = 1.32 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.71, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {5203, 244, 2009, 5203, 244, 2009, 5199, 15, 5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5203}$$

$$\frac{5d\int x^m\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx}{m+6}+\frac{bcd^2\sqrt{d-c^2dx^2}\int x^{m+1}\left(1-c^2x^2\right)^2dx}{(m+6)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{m+6}$$

$$\Big\downarrow \; 244$$

$$\frac{5d\int x^m\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx}{m+6}+\frac{bcd^2\sqrt{d-c^2dx^2}\int\left(x^{m+1}-2c^2x^{m+3}+c^4x^{m+5}\right)dx}{(m+6)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{m+6}$$

$$\Big\downarrow \; 2009$$

$$\frac{5d\int x^m\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))dx}{m+6}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{m+6}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}}{m+6}-\frac{2c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+6)\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 5203$$

$$\frac{5d\left(\frac{3d\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx}{m+4}+\frac{bcd\sqrt{d-c^2dx^2}\int x^{m+1}\left(1-c^2x^2\right)dx}{(m+4)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{m+4}\right)}{m+6}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{m+6}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}}{m+6}-\frac{2c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+6)\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 244$$

$$\frac{5d\left(\frac{3d\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx}{m+4}+\frac{bcd\sqrt{d-c^2dx^2}\int\left(x^{m+1}-c^2x^{m+3}\right)dx}{(m+4)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{m+4}\right)}{m+6}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{m+6}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}}{m+6}-\frac{2c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+6)\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{5d\left(\frac{3d\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx}{m+4}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{m+4}+\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{x^{m+2}}{m+2}-\frac{c^2x^{m+4}}{m+4}\right)}{(m+4)\sqrt{1-c^2x^2}}\right)}{m+6}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{m+6}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}}{m+6}-\frac{2c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+6)\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5199

$$5d\left(\frac{3d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^m(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{(m+2)\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int x^{m+1}dx}{(m+2)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{m+2}\right)}{m+4}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{m+4}\right.$$

$$\frac{}{m+6}$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{m+6}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}}{m+6}-\frac{2c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+6)\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 15

$$5d\left(\frac{3d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^m(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{(m+2)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{m+2}+\frac{bcx^{m+2}\sqrt{d-c^2dx^2}}{(m+2)^2\sqrt{1-c^2x^2}}\right)}{m+4}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{m+4}+\right.$$

$$\frac{}{m+6}$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{m+6}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}}{m+6}-\frac{2c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+6)\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5221

$$5d\left(\frac{3d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{bcx^{m+2}{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{m^2+3m+2}+\frac{x^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\arccos(cx))}{m+1}\right)}{(m+2)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\sqrt{d-c^2d}}{}\right.}{m+4}\right.$$

$$\frac{}{m+6}$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{m+6}+\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}}{m+6}-\frac{2c^2x^{m+4}}{m+4}+\frac{x^{m+2}}{m+2}\right)}{(m+6)\sqrt{1-c^2x^2}}$$

input
```
Int[x^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d^2*Sqrt[d - c^2*d*x^2]*(x^(2 + m)/(2 + m) - (2*c^2*x^(4 + m))/(4 + m
) + (c^4*x^(6 + m))/(6 + m)))/((6 + m)*Sqrt[1 - c^2*x^2]) + (x^(1 + m)*(d
- c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(6 + m) + (5*d*((b*c*d*Sqrt[d - c^
2*d*x^2]*(x^(2 + m)/(2 + m) - (c^2*x^(4 + m))/(4 + m)))/((4 + m)*Sqrt[1 -
c^2*x^2]) + (x^(1 + m)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(4 + m)
+ (3*d*((b*c*x^(2 + m)*Sqrt[d - c^2*d*x^2])/((2 + m)^2*Sqrt[1 - c^2*x^2])
+ (x^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(2 + m) + (Sqrt[d -
c^2*d*x^2]*((x^(1 + m)*(a + b*ArcCos[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, c^2*x^2])/(1 + m) + (b*c*x^(2 + m)*HypergeometricPFQ[{1, 1 +
 m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(2 + 3*m + m^2)))/((2 + m)
*Sqrt[1 - c^2*x^2])))/(4 + m)))/(6 + m)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int x^m \left(-c^2 d\,x^2+d\right)^{\frac{5}{2}} \left(a+b\arccos\left(cx\right)\right) dx$$

input
```
int(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x)
```

output
```
int(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x)
```

Fricas [F]

$$\int x^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\arccos(cx)\right) dx = \int \left(-c^2dx^2+d\right)^{\frac{5}{2}} \left(b\arccos\left(cx\right)+a\right)x^m\, dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)*x^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m (d - c^2 dx^2)^{5/2} (a + b \arccos(cx)) \, dx = \text{Timed out}$$

input
```
integrate(x**m*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^m (d - c^2 dx^2)^{5/2} (a + b \arccos(cx)) \, dx = \int (-c^2 dx^2 + d)^{\frac{5}{2}} (b \arccos(cx) + a) x^m \, dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccos(c*x) + a)*x^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^m (d - c^2 dx^2)^{5/2} (a + b \arccos(cx)) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx=\int x^m\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int(x^m*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^m*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\begin{aligned}&\int x^m \left(d-c^2dx^2\right)^{5/2} (a\\&+b\arccos(cx))\,dx=\sqrt{d}\,d^2\Bigg(\left(\left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)\,x^4dx\right)b\,c^4\\&-2\left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)\,x^2dx\right)b\,c^2\\&+\left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)\,dx\right)b+\left(\int x^m\sqrt{-c^2x^2+1}\,x^4dx\right)a\,c^4\\&-2\left(\int x^m\sqrt{-c^2x^2+1}\,x^2dx\right)a\,c^2+\left(\int x^m\sqrt{-c^2x^2+1}dx\right)a\right)\end{aligned}$$

input
```
int(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x)),x)
```

output
```
sqrt(d)*d**2*(int(x**m*sqrt( - c**2*x**2 + 1)*acos(c*x)*x**4,x)*b*c**4 - 2
*int(x**m*sqrt( - c**2*x**2 + 1)*acos(c*x)*x**2,x)*b*c**2 + int(x**m*sqrt(
 - c**2*x**2 + 1)*acos(c*x),x)*b + int(x**m*sqrt( - c**2*x**2 + 1)*x**4,x)
*a*c**4 - 2*int(x**m*sqrt( - c**2*x**2 + 1)*x**2,x)*a*c**2 + int(x**m*sqrt
( - c**2*x**2 + 1),x)*a)
```

3.152 $\int x^m(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 399

$$\begin{aligned}\int x^m\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx=&-\frac{3bcdx^{2+m}\sqrt{d-c^2dx^2}}{(2+m)^2(4+m)\sqrt{1-c^2x^2}}-\frac{bcdx^{2+m}\sqrt{d-c^2dx^2}}{(8+6m+m^2)\sqrt{1-c^2x^2}}\\&+\frac{bc^3dx^{4+m}\sqrt{d-c^2dx^2}}{(4+m)^2\sqrt{1-c^2x^2}}+\frac{3dx^{1+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{8+6m+m^2}\\&+\frac{x^{1+m}(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{4+m}\\&+\frac{3dx^{1+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(8+14m+7m^2+m^3)\sqrt{1-c^2x^2}}\\&-\frac{3bcdx^{2+m}\sqrt{d-c^2dx^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{(1+m)(2+m)^2(4+m)\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-3*b*c*d*x^(2+m)*(-c^2*d*x^2+d)^(1/2)/(2+m)^2/(4+m)/(-c^2*x^2+1)^(1/2)-b*c
*d*x^(2+m)*(-c^2*d*x^2+d)^(1/2)/(m^2+6*m+8)/(-c^2*x^2+1)^(1/2)+b*c^3*d*x^(
4+m)*(-c^2*d*x^2+d)^(1/2)/(4+m)^2/(-c^2*x^2+1)^(1/2)+3*d*x^(1+m)*(-c^2*d*x
^2+d)^(1/2)*(a+b*arccos(c*x))/(m^2+6*m+8)+x^(1+m)*(-c^2*d*x^2+d)^(3/2)*(a+
b*arccos(c*x))/(4+m)+3*d*x^(1+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*hy
pergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)/(m^3+7*m^2+14*m+8)/(-c^2*x^2
+1)^(1/2)-3*b*c*d*x^(2+m)*(-c^2*d*x^2+d)^(1/2)*hypergeom([1, 1+1/2*m, 1+1/
2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)/(1+m)/(2+m)^2/(4+m)/(-c^2*x^2+1)^(1/2)
```

Mathematica [F]

$$\int x^m \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos(cx)) \, dx = \int x^m \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos(cx)) \, dx$$

input
```
Integrate[x^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
Integrate[x^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]), x]
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5203, 244, 2009, 5199, 15, 5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos(cx)) \, dx$$

$$\Big\downarrow \text{ 5203}$$

$$\frac{3d \int x^m \sqrt{d-c^2 d x^2} (a+b \arccos(cx)) dx}{m+4} + \frac{b c d \sqrt{d-c^2 d x^2} \int x^{m+1} \left(1-c^2 x^2\right) dx}{(m+4) \sqrt{1-c^2 x^2}} + \frac{x^{m+1} \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos(cx))}{m+4}$$

$$\Big\downarrow \text{244}$$

$$\frac{3d\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx}{m+4}+\frac{bcd\sqrt{d-c^2dx^2}\int\left(x^{m+1}-c^2x^{m+3}\right)dx}{(m+4)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{m+4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3d\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))dx}{m+4}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{m+4}+\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{x^{m+2}}{m+2}-\frac{c^2x^{m+4}}{m+4}\right)}{(m+4)\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5199}$$

$$\frac{3d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^m(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{(m+2)\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int x^{m+1}dx}{(m+2)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{m+2}\right)}{m+4}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{m+4}+\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{x^{m+2}}{m+2}-\frac{c^2x^{m+4}}{m+4}\right)}{(m+4)\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{3d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^m(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{(m+2)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{m+2}+\frac{bcx^{m+2}\sqrt{d-c^2dx^2}}{(m+2)^2\sqrt{1-c^2x^2}}\right)}{m+4}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{m+4}+\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{x^{m+2}}{m+2}-\frac{c^2x^{m+4}}{m+4}\right)}{(m+4)\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5221}$$

$$\frac{3d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{bcx^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{m^2+3m+2}+\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\arccos(cx))}{m+1}\right)}{(m+2)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\sqrt{d-c^2d}}{}\right.}{m+4}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))}{m+4}+\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{x^{m+2}}{m+2}-\frac{c^2x^{m+4}}{m+4}\right)}{(m+4)\sqrt{1-c^2x^2}}$$

input
```
Int[x^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*d*Sqrt[d - c^2*d*x^2]*(x^(2 + m)/(2 + m) - (c^2*x^(4 + m))/(4 + m)))/
((4 + m)*Sqrt[1 - c^2*x^2]) + (x^(1 + m)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcC
os[c*x]))/(4 + m) + (3*d*((b*c*x^(2 + m)*Sqrt[d - c^2*d*x^2])/((2 + m)^2*S
qrt[1 - c^2*x^2]) + (x^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(2
 + m) + (Sqrt[d - c^2*d*x^2]*((x^(1 + m)*(a + b*ArcCos[c*x])*Hypergeometri
c2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(1 + m) + (b*c*x^(2 + m)*Hyperge
ometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(2 + 3*m
 + m^2)))/((2 + m)*Sqrt[1 - c^2*x^2])))/(4 + m)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int x^{m} \left(-c^{2} d\,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \arccos \left(c x \right)\right)d x$$

input
```
int(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x)
```

output
```
int(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x)
```

Fricas [F]

$$\int x^m \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \int \left(-c^{2} d x^{2} + d\right)^{\frac{3}{2}} {\left(b \arccos \left(c x\right) + a\right)} x^{m}\,{d x}$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccos(c*x))*sqrt(-c^2*
d*x^2 + d)*x^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m (d - c^2dx^2)^{3/2} (a + b\arccos(cx))\, dx = \text{Timed out}$$

input
```
integrate(x**m*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^m (d - c^2dx^2)^{3/2} (a + b\arccos(cx))\, dx = \int (-c^2dx^2 + d)^{\frac{3}{2}} (b\arccos(cx) + a)x^m\, dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccos(c*x) + a)*x^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^m (d - c^2dx^2)^{3/2} (a + b\arccos(cx))\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^m (d-c^2dx^2)^{3/2} (a+b\arccos(cx))\,dx = \int x^m\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int(x^m*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^m*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\begin{aligned}&\int x^m (d-c^2dx^2)^{3/2} (a\\&+b\arccos(cx))\,dx = \sqrt{d}\,d\left(-\left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)\,x^2dx\right)b\,c^2\right.\\&+\left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)\,dx\right)b\\&\left.-\left(\int x^m\sqrt{-c^2x^2+1}\,x^2dx\right)a\,c^2+\left(\int x^m\sqrt{-c^2x^2+1}dx\right)a\right)\end{aligned}$$

input
```
int(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x)),x)
```

output
```
sqrt(d)*d*( - int(x**m*sqrt( - c**2*x**2 + 1)*acos(c*x)*x**2,x)*b*c**2 + i
nt(x**m*sqrt( - c**2*x**2 + 1)*acos(c*x),x)*b - int(x**m*sqrt( - c**2*x**2
 + 1)*x**2,x)*a*c**2 + int(x**m*sqrt( - c**2*x**2 + 1),x)*a)
```

3.153 $\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 245

$$\begin{aligned}\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx &= -\frac{bcx^{2+m}\sqrt{d-c^2dx^2}}{(2+m)^2\sqrt{1-c^2x^2}}+\frac{x^{1+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2+m}\\ &\quad+\frac{x^{1+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(2+3m+m^2)\sqrt{1-c^2x^2}}\\ &\quad-\frac{bcx^{2+m}\sqrt{d-c^2dx^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{(1+m)(2+m)^2\sqrt{1-c^2x^2}}\end{aligned}$$

output

```
-b*c*x^(2+m)*(-c^2*d*x^2+d)^(1/2)/(2+m)^2/(-c^2*x^2+1)^(1/2)+x^(1+m)*(-c^2
*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(2+m)+x^(1+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*
arccos(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)/(m^2+3*m+2)/(
-c^2*x^2+1)^(1/2)-b*c*x^(2+m)*(-c^2*d*x^2+d)^(1/2)*hypergeom([1, 1+1/2*m,
1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)/(1+m)/(2+m)^2/(-c^2*x^2+1)^(1/2)
```

Mathematica [F]

$$\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$

input
```
Integrate[x^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
Integrate[x^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]), x]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.88, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5199, 15, 5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5199}$$

$$\frac{\sqrt{d-c^2dx^2}\int\frac{x^m(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{(m+2)\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int x^{m+1}dx}{(m+2)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{m+2}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{d-c^2dx^2}\int\frac{x^m(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{(m+2)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{m+2}+\frac{bcx^{m+2}\sqrt{d-c^2dx^2}}{(m+2)^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5221}$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{bcx^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{m^2+3m+2}+\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\arccos(cx))}{m+1}\right)}{(m+2)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{m+2}+\frac{bcx^{m+2}\sqrt{d-c^2dx^2}}{(m+2)^2\sqrt{1-c^2x^2}}$$

input
```
Int[x^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*x^(2 + m)*Sqrt[d - c^2*d*x^2])/((2 + m)^2*Sqrt[1 - c^2*x^2]) + (x^(1
+ m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(2 + m) + (Sqrt[d - c^2*d*x^
2]*((x^(1 + m)*(a + b*ArcCos[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, c^2*x^2])/(1 + m) + (b*c*x^(2 + m)*HypergeometricPFQ[{1, 1 + m/2, 1
+ m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(2 + 3*m + m^2)))/((2 + m)*Sqrt[1
- c^2*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int x^{m}\sqrt{-c^{2}d\,x^{2}+d}\,(a+b\arccos(cx))\,dx$$

input
```
int(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x)
```

output
```
int(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x)
```

Fricas [F]

$$\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)*x^m, x)
```

Sympy [F]

$$\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int x^m\sqrt{-d\left(cx-1\right)\left(cx+1\right)}(a+b\,\mathrm{acos}\left(cx\right))\,dx$$

input
```
integrate(x**m*(-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
Integral(x**m*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int\sqrt{-c^2dx^2+d}(b\arccos\left(cx\right)+a)x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)*x^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int x^m\,(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^m*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^m*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^m \sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \sqrt{d}\left(\left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)\,dx\right)b + \left(\int x^m\sqrt{-c^2x^2+1}dx\right)a\right)$$

input
```
int(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
sqrt(d)*(int(x**m*sqrt( - c**2*x**2 + 1)*acos(c*x),x)*b + int(x**m*sqrt( -
 c**2*x**2 + 1),x)*a)
```

3.154 $\int \frac{x^m(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 163

$$\int \frac{x^m(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{x^{1+m}\sqrt{1-c^2x^2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)\sqrt{d-c^2dx^2}} - \frac{bcx^{2+m}\sqrt{1-c^2x^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{(2+3m+m^2)\sqrt{d-c^2dx^2}}$$

output
```
x^(1+m)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*hypergeom([1/2, 1/2+1/2*m],[3
/2+1/2*m],c^2*x^2)/(1+m)/(-c^2*d*x^2+d)^(1/2)-b*c*x^(2+m)*(-c^2*x^2+1)^(1/
2)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)/(m^2+3*m+
2)/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.61 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{2^{-2-m}x^{1+m}\sqrt{1-c^2x^2}\left(2^{2+m}\left(a\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)+b\sqrt{1-c^2x^2}\arccos(cx)\,\text{Hyperg}\right.\right.}{(1+m)\sqrt{}}$$

input
```
Integrate[(x^m*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(2^(-2 - m)*x^(1 + m)*Sqrt[1 - c^2*x^2]*(2^(2 + m)*(a*Hypergeometric2F1[1/
2, (1 + m)/2, (3 + m)/2, c^2*x^2] + b*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Hyperg
eometric2F1[1, (2 + m)/2, (3 + m)/2, c^2*x^2]) + b*c*(1 + m)*Sqrt[Pi]*x*Ga
mma[1 + m]*HypergeometricPFQRegularized[{1, (2 + m)/2, (2 + m)/2}, {(3 + m
)/2, (4 + m)/2}, c^2*x^2]))/((1 + m)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \; 5221$$

$$\frac{bc\sqrt{1-c^2x^2}x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{(m^2+3m+2)\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\arccos(cx))}{(m+1)\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^m*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(x^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)*Sqrt[d - c^2*d*x^2]) + (b*c*x^(2 +
m)*Sqrt[1 - c^2*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2,
2 + m/2}, c^2*x^2])/((2 + 3*m + m^2)*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{x^m(a+b\arccos(cx))}{\sqrt{-c^2d\,x^2+d}}dx$$

input
```
int(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^m(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\arccos(cx)+a)x^m}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)*x^m/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^m(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x^m(a+b\,\mathrm{acos}\,(cx))}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate(x**m*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**m*(a + b*acos(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^m(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\arccos\,(cx)+a)x^m}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((b*arccos(c*x) + a)*x^m/sqrt(-c^2*d*x^2 + d), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^m\,(a+b\,\mathrm{acos}(c\,x))}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x^m*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^m*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}}dx\right)a+\left(\int \frac{x^m acos(cx)}{\sqrt{-c^2x^2+1}}dx\right)b}{\sqrt{d}}$$

input
```
int(x^m*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(int(x**m/sqrt( - c**2*x**2 + 1),x)*a + int((x**m*acos(c*x))/sqrt( - c**2*
x**2 + 1),x)*b)/sqrt(d)
```

3.155 $\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 272

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{x^{1+m}(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}} - \frac{mx^{1+m}\sqrt{1-c^2x^2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{d(1+m)\sqrt{d-c^2dx^2}} - \frac{bcx^{2+m}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{d(2+m)\sqrt{d-c^2dx^2}} + \frac{bcmx^{2+m}\sqrt{1-c^2x^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{d(2+3m+m^2)\sqrt{d-c^2dx^2}}$$

output
```
x^(1+m)*(a+b*arccos(c*x))/d/(-c^2*d*x^2+d)^(1/2)-m*x^(1+m)*(-c^2*x^2+1)^(1
/2)*(a+b*arccos(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)/d/(1
+m)/(-c^2*d*x^2+d)^(1/2)-b*c*x^(2+m)*(-c^2*x^2+1)^(1/2)*hypergeom([1, 1+1/
2*m],[2+1/2*m],c^2*x^2)/d/(2+m)/(-c^2*d*x^2+d)^(1/2)+b*c*m*x^(2+m)*(-c^2*x
^2+1)^(1/2)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)/
d/(m^2+3*m+2)/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [F]

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5209, 278, 5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5209}$$

$$-\frac{m\int \frac{x^m(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{1-c^2x^2}\int \frac{x^{m+1}}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x^{m+1}(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{m\int \frac{x^m(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{d}+\frac{x^{m+1}(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{1-c^2x^2}x^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{d(m+2)\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5221}$$

$$-\frac{m\left(\frac{bc\sqrt{1-c^2x^2}x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{(m^2+3m+2)\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\arccos(cx))}{(m+1)\sqrt{d-c^2dx^2}}\right)}{d}$$

$$\frac{x^{m+1}(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{1-c^2x^2}x^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{d(m+2)\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^m*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x^(1 + m)*(a + b*ArcCos[c*x]))/(d*Sqrt[d - c^2*d*x^2]) + (b*c*x^(2 + m)*S
qrt[1 - c^2*x^2]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(d*(
2 + m)*Sqrt[d - c^2*d*x^2]) - (m*((x^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcC
os[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)*S
qrt[d - c^2*d*x^2]) + (b*c*x^(2 + m)*Sqrt[1 - c^2*x^2]*HypergeometricPFQ[{
1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/((2 + 3*m + m^2)*Sqr
t[d - c^2*d*x^2])))/d
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
 + b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{x^{m}(a+b\arccos(cx))}{(-c^{2}d\,x^{2}+d)^{\frac{3}{2}}}dx$$

input
```
int(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
int(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)x^m}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)*x^m/(c^4*d^2*x^4 - 2*c^2
*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^m(a+b\operatorname{acos}(cx))}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**m*(a + b*acos(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)x^m}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((b*arccos(c*x) + a)*x^m/(-c^2*d*x^2 + d)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^m\,(a+b\,\mathrm{acos}(c\,x))}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x^m*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^m*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)a-\left(\int \frac{x^m\,acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)b}{\sqrt{d}\,d}$$

input
```
int(x^m*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - (int(x**m/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),
x)*a + int((x**m*acos(c*x))/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c*
*2*x**2 + 1)),x)*b))/(sqrt(d)*d)
```

3.156 $\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 408

$$\begin{aligned}\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx &= \frac{x^{1+m}(a+b\arccos(cx))}{3d(d-c^2dx^2)^{3/2}}+\frac{(2-m)x^{1+m}(a+b\arccos(cx))}{3d^2\sqrt{d-c^2dx^2}}\\ &-\frac{(2-m)mx^{1+m}\sqrt{1-c^2x^2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{3d^2(1+m)\sqrt{d-c^2dx^2}}\\ &-\frac{bc(2-m)x^{2+m}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{3d^2(2+m)\sqrt{d-c^2dx^2}}\\ &-\frac{bcx^{2+m}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(2,\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{3d^2(2+m)\sqrt{d-c^2dx^2}}\\ &+\frac{bc(2-m)mx^{2+m}\sqrt{1-c^2x^2}\,{}_3F_2\left(1,1+\frac{m}{2},1+\frac{m}{2};\frac{3}{2}+\frac{m}{2},2+\frac{m}{2};c^2x^2\right)}{3d^2(2+3m+m^2)\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
1/3*x^(1+m)*(a+b*arccos(c*x))/d/(-c^2*d*x^2+d)^(3/2)+1/3*(2-m)*x^(1+m)*(a+
b*arccos(c*x))/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*(2-m)*m*x^(1+m)*(-c^2*x^2+1)^(
1/2)*(a+b*arccos(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)/d^2
/(1+m)/(-c^2*d*x^2+d)^(1/2)-1/3*b*c*(2-m)*x^(2+m)*(-c^2*x^2+1)^(1/2)*hyper
geom([1, 1+1/2*m],[2+1/2*m],c^2*x^2)/d^2/(2+m)/(-c^2*d*x^2+d)^(1/2)-1/3*b*
c*x^(2+m)*(-c^2*x^2+1)^(1/2)*hypergeom([2, 1+1/2*m],[2+1/2*m],c^2*x^2)/d^2
/(2+m)/(-c^2*d*x^2+d)^(1/2)+1/3*b*c*(2-m)*m*x^(2+m)*(-c^2*x^2+1)^(1/2)*hyp
ergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],c^2*x^2)/d^2/(m^2+3*m+2)
/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [F]

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
Integrate[(x^m*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(5/2), x]
```

Rubi [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5209, 278, 5209, 278, 5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5209}$$

$$\frac{(2-m)\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{bc\sqrt{1-c^2x^2}\int \frac{x^{m+1}}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^{m+1}(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{(2-m)\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{x^{m+1}(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}x^{m+2}\,\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3d^2(m+2)\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5209}$$

$$\frac{(2-m)\left(-\frac{m\int \frac{x^m(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{d}+\frac{bc\sqrt{1-c^2x^2}\int \frac{x^{m+1}}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x^{m+1}(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x^{m+1}(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}x^{m+2}\,\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3d^2(m+2)\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{(2-m)\left(-\frac{m\int \frac{x^m(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}dx}{d}+\frac{x^{m+1}(a+b\arccos(cx))}{d\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{1-c^2x^2}x^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{d(m+2)\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x^{m+1}(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}x^{m+2}\,\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3d^2(m+2)\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5221}$$

$$\frac{(2-m)\left(-\frac{m\left(\frac{bc\sqrt{1-c^2x^2}x^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{\left(m^2+3m+2\right)\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}x^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)(a+b\arccos(cx))}{(m+1)\sqrt{d-c^2dx^2}}\right)}{d}}{3d}+\frac{x^{m+1}(a+b\arccos(cx))}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{bc\sqrt{1-c^2x^2}x^{m+2}\,\text{Hypergeometric2F1}\left(2,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{3d^2(m+2)\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^m*(a + b*ArcCos[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x^(1 + m)*(a + b*ArcCos[c*x]))/(3*d*(d - c^2*d*x^2)^(3/2)) + (b*c*x^(2 +
m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[2, (2 + m)/2, (4 + m)/2, c^2*x^2])/
(3*d^2*(2 + m)*Sqrt[d - c^2*d*x^2]) + ((2 - m)*((x^(1 + m)*(a + b*ArcCos[c
*x]))/(d*Sqrt[d - c^2*d*x^2]) + (b*c*x^(2 + m)*Sqrt[1 - c^2*x^2]*Hypergeom
etric2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(d*(2 + m)*Sqrt[d - c^2*d*x^2]
) - (m*((x^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])*Hypergeometric2F1
[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)*Sqrt[d - c^2*d*x^2]) + (b*c
*x^(2 + m)*Sqrt[1 - c^2*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2
 + m/2, 2 + m/2}, c^2*x^2])/((2 + 3*m + m^2)*Sqrt[d - c^2*d*x^2])))/d))/(3
*d)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{x^m(a+b\arccos(cx))}{(-c^2d\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
int(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)x^m}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)*x^m/(c^6*d^3*x^6 - 3*c^
4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)x^m}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate((b*arccos(c*x) + a)*x^m/(-c^2*d*x^2 + d)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^m\,(a+b\,\mathrm{acos}(c\,x))}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((x^m*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^m*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^m(a+b\arccos(cx))}{(d-c^2dx^2)^{5/2}}\,dx = \frac{\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)a+\left(\int \frac{x^m\,acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2}\right.}{\sqrt{d}\,d^2}$$

input
```
int(x^m*(a+b*acos(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(int(x**m/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**
2*x**2 + sqrt( - c**2*x**2 + 1)),x)*a + int((x**m*acos(c*x))/(sqrt( - c**2
*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x
**2 + 1)),x)*b)/(sqrt(d)*d**2)
```

3.157 $\int \frac{x^m \arccos(ax)}{\sqrt{1-a^2x^2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{x^m \arccos(ax)}{\sqrt{1-a^2x^2}} \, dx = \frac{x^{1+m} \arccos(ax) \,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2x^2\right)}{1+m} - \frac{ax^{2+m} \,{}_3F_2\left(1, 1+\frac{m}{2}, 1+\frac{m}{2}; \frac{3}{2}+\frac{m}{2}, 2+\frac{m}{2}; a^2x^2\right)}{2+3m+m^2}$$

output
```
x^(1+m)*arccos(a*x)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/(1+m)-
a*x^(2+m)*hypergeom([1, 1+1/2*m, 1+1/2*m],[2+1/2*m, 3/2+1/2*m],a^2*x^2)/(m
^2+3*m+2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.92 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int \frac{x^m \arccos(ax)}{\sqrt{1-a^2x^2}} \, dx = \frac{1}{4} x^{1+m} \left(\frac{4\sqrt{1-a^2x^2} \arccos(ax) \,\text{Hypergeometric2F1}\left(1, \frac{2+m}{2}, \frac{3+m}{2}, a^2x^2\right)}{1+m} + 2^{-m} a \sqrt{\pi} x \,\text{Gamma}(1+m) \,{}_3\tilde{F}_2\left(1, \frac{2+m}{2}, \frac{2+m}{2}; \frac{3+m}{2}, \frac{4+m}{2}; a^2x^2\right) \right)$$

input
```
Integrate[(x^m*ArcCos[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
(x^(1 + m)*((4*Sqrt[1 - a^2*x^2]*ArcCos[a*x]*Hypergeometric2F1[1, (2 + m)/
2, (3 + m)/2, a^2*x^2])/(1 + m) + (a*Sqrt[Pi]*x*Gamma[1 + m]*Hypergeometri
cPFQRegularized[{1, (2 + m)/2, (2 + m)/2}, {(3 + m)/2, (4 + m)/2}, a^2*x^2
])/2^m))/4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arccos(ax)}{\sqrt{1-a^2x^2}}dx$$
$$\Big\downarrow \text{5221}$$
$$\frac{ax^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;a^2x^2\right)}{m^2+3m+2}+\frac{x^{m+1}\arccos(ax)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},a^2x^2\right)}{m+1}$$

input
```
Int[(x^m*ArcCos[a*x])/Sqrt[1 - a^2*x^2],x]
```

output
```
(x^(1 + m)*ArcCos[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^
2])/(1 + m) + (a*x^(2 + m)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 +
 m/2, 2 + m/2}, a^2*x^2])/(2 + 3*m + m^2)
```

Defintions of rubi rules used

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{x^m \arccos(ax)}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^m*arccos(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(x^m*arccos(a*x)/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^m \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m \arccos(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^m*arccos(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^m \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m \operatorname{acos}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x**m*acos(a*x)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**m*acos(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^m \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m \arccos(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arccos(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{x^m \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m \arccos(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arccos(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*arccos(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m\,\mathrm{acos}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^m*acos(a*x))/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^m*acos(a*x))/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m \arccos(ax)}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m acos(ax)}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^m*acos(a*x)/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((x**m*acos(a*x))/sqrt( - a**2*x**2 + 1),x)
```

3.158 $\int x^4(d-c^2dx^2)(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 290

$$\begin{aligned}\int x^4(d-c^2dx^2)(a+b\arccos(cx))^2\,dx &= -\frac{304b^2dx}{3675c^4}-\frac{152b^2dx^3}{11025c^2}-\frac{38b^2dx^5}{6125}+\frac{2}{343}b^2c^2dx^7\\ &+\frac{32bd\sqrt{1-c^2x^2}(a+b\arccos(cx))}{525c^5}+\frac{16bdx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{525c^3}\\ &+\frac{4bdx^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{175c}+\frac{2bd(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{21c^5}\\ &-\frac{4bd(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{35c^5}+\frac{2bd(1-c^2x^2)^{7/2}(a+b\arccos(cx))}{49c^5}\\ &+\frac{2}{35}dx^5(a+b\arccos(cx))^2+\frac{1}{7}dx^5(1-c^2x^2)(a+b\arccos(cx))^2\end{aligned}$$

output
```
-304/3675*b^2*d*x/c^4-152/11025*b^2*d*x^3/c^2-38/6125*b^2*d*x^5+2/343*b^2*
c^2*d*x^7+32/525*b*d*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^5+16/525*b*d*x
^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3+4/175*b*d*x^4*(-c^2*x^2+1)^(1/
2)*(a+b*arccos(c*x))/c+2/21*b*d*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/c^5-4
/35*b*d*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))/c^5+2/49*b*d*(-c^2*x^2+1)^(7/
2)*(a+b*arccos(c*x))/c^5+2/35*d*x^5*(a+b*arccos(c*x))^2+1/7*d*x^5*(-c^2*x^
2+1)*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.70

$$\int x^4(d-c^2dx^2)(a+b\arccos(cx))^2\,dx =$$
$$-\frac{d\left(11025a^2c^5x^5(-7+5c^2x^2)-210ab\sqrt{1-c^2x^2}(-152-76c^2x^2-57c^4x^4+75c^6x^6)+b^2(31920cx+5\right.}{}$$

input
```
Integrate[x^4*(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/385875*(d*(11025*a^2*c^5*x^5*(-7 + 5*c^2*x^2) - 210*a*b*Sqrt[1 - c^2*x^
2]*(-152 - 76*c^2*x^2 - 57*c^4*x^4 + 75*c^6*x^6) + b^2*(31920*c*x + 5320*c
^3*x^3 + 2394*c^5*x^5 - 2250*c^7*x^7) - 210*b*(-105*a*c^5*x^5*(-7 + 5*c^2*
x^2) + b*Sqrt[1 - c^2*x^2]*(-152 - 76*c^2*x^2 - 57*c^4*x^4 + 75*c^6*x^6))*
ArcCos[c*x] + 11025*b^2*c^5*x^5*(-7 + 5*c^2*x^2)*ArcCos[c*x]^2))/c^5
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5203, 5139, 5195, 27, 2009, 5211, 15, 5211, 15, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d-c^2dx^2)(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{2}{7}bcd\int x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{2}{7}d\int x^4(a+b\arccos(cx))^2dx+\frac{1}{7}dx^5(1-c^2x^2)(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5139}$$

$$\frac{2}{7}d\left(\frac{2}{5}bc\int \frac{x^5(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))^2\right)+\frac{2}{7}bcd\int x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{7}dx^5\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$\Big\downarrow$ 5195

$$\frac{2}{7}d\left(\frac{2}{5}bc\int \frac{x^5(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))^2\right)+\frac{2}{7}bcd\left(bc\int -\frac{-15c^6x^6+3c^4x^4+4c^2x^2+8}{105c^6}dx-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}+\frac{2\left(1-c^2x^2\right)^{5/2}(a+b\arccos\right.$$
$$\frac{1}{7}dx^5\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$\Big\downarrow$ 27

$$\frac{2}{7}d\left(\frac{2}{5}bc\int \frac{x^5(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))^2\right)+\frac{2}{7}bcd\left(-\frac{b\int\left(-15c^6x^6+3c^4x^4+4c^2x^2+8\right)dx}{105c^5}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}+\frac{2\left(1-c^2x^2\right)^{5/2}(a+b\arcco\right.$$
$$\frac{1}{7}dx^5\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$\Big\downarrow$ 2009

$$\frac{2}{7}d\left(\frac{2}{5}bc\int \frac{x^5(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))^2\right)+\frac{1}{7}dx^5\left(1-c^2x^2\right)(a+b\arccos(cx))^2+$$
$$\frac{2}{7}bcd\left(-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}+\frac{2\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^6}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^6}\right.$$

$\Big\downarrow$ 5211

$$\frac{2}{7}d\left(\frac{2}{5}bc\left(\frac{4\int \frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{5c^2}-\frac{b\int x^4dx}{5c}-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{5c^2}\right)+\frac{1}{5}x^5(a+b\arccos(cx))^2\right)+$$
$$\frac{1}{7}dx^5\left(1-c^2x^2\right)(a+b\arccos(cx))^2+$$
$$\frac{2}{7}bcd\left(-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}+\frac{2\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^6}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^6}\right.$$

$\Big\downarrow$ 15

$$\frac{2}{7}d\left(\frac{2}{5}bc\left(\frac{4\int \frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{5c^2}-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{5c^2}-\frac{bx^5}{25c}\right)+\frac{1}{5}x^5(a+b\arccos(cx))^2\right)+$$
$$\frac{1}{7}dx^5\left(1-c^2x^2\right)(a+b\arccos(cx))^2+$$
$$\frac{2}{7}bcd\left(-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}+\frac{2\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^6}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^6}\right.$$

$$\Big\downarrow \text{5211}$$

$$\frac{2}{7}d\left(\frac{2}{5}bc\left(\frac{4\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{b\int x^2dx}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}\right)}{5c^2}-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{5c^2}-\frac{bx^5}{25c}\right)\right.$$
$$\frac{1}{7}dx^5\left(1-c^2x^2\right)(a+b\arccos(cx))^2+$$
$$\frac{2}{7}bcd\left(-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}+\frac{2\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^6}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^6}\right.$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{7}d\left(\frac{2}{5}bc\left(\frac{4\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{5c^2}-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{5c^2}-\frac{bx^5}{25c}\right)+\right.$$
$$\frac{1}{7}dx^5\left(1-c^2x^2\right)(a+b\arccos(cx))^2+$$
$$\frac{2}{7}bcd\left(-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}+\frac{2\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^6}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^6}\right.$$

$$\Big\downarrow \text{5183}$$

$$\frac{2}{7}d\left(\frac{2}{5}bc\left(\frac{4\left(\frac{2\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{5c^2}-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos(cx)}{5c^2}\right.\right.$$

$$\frac{1}{7}dx^5\left(1-c^2x^2\right)(a+b\arccos(cx))^2+$$

$$\frac{2}{7}bcd\left(-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}+\frac{2\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^6}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^6}\right.$$

$$\downarrow 24$$

$$\frac{1}{7}dx^5\left(1-c^2x^2\right)(a+b\arccos(cx))^2+$$

$$\frac{2}{7}d\left(\frac{2}{5}bc\left(-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{5c^2}+\frac{4\left(-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}+\frac{2\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)}{3c^2}-\frac{bx^3}{9c}\right)}{5c^2}\right.\right.$$

$$\frac{2}{7}bcd\left(-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}+\frac{2\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^6}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^6}\right.$$

input
```
Int[x^4*(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*x^5*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/7 + (2*b*c*d*(-1/105*(b*(8*x +
 (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (15*c^6*x^7)/7))/c^5 - ((1 - c^2*x^2)^(3/
2)*(a + b*ArcCos[c*x]))/(3*c^6) + (2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x
]))/(5*c^6) - ((1 - c^2*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(7*c^6)))/7 + (2*d
*((x^5*(a + b*ArcCos[c*x])^2)/5 + (2*b*c*(-1/25*(b*x^5)/c - (x^4*Sqrt[1 -
c^2*x^2]*(a + b*ArcCos[c*x]))/(5*c^2) + (4*(-1/9*(b*x^3)/c - (x^2*Sqrt[1 -
 c^2*x^2]*(a + b*ArcCos[c*x]))/(3*c^2) + (2*(-((b*x)/c) - (Sqrt[1 - c^2*x^
2]*(a + b*ArcCos[c*x]))/c^2))/(3*c^2)))/(5*c^2)))/5))/7
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.95

method	result
parts	$-d\,a^2\left(\frac{1}{7}c^2x^7-\frac{1}{5}x^5\right)-\frac{d\,b^2\left(-\frac{\arccos(cx)^2c^5x^5}{5}+\frac{2\arccos(cx)\left(3c^4x^4+4c^2x^2+8\right)\sqrt{-c^2x^2+1}}{75}+\frac{38c^5x^5}{6125}+\frac{152c^3x^3}{11025}+\frac{3}{}\right.}{c^5}$
derivativedivides	$\frac{-d\,a^2\left(\frac{1}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-d\,b^2\left(-\frac{\arccos(cx)^2c^5x^5}{5}+\frac{2\arccos(cx)\left(3c^4x^4+4c^2x^2+8\right)\sqrt{-c^2x^2+1}}{75}+\frac{38c^5x^5}{6125}+\frac{152c^3x^3}{11025}+\frac{304cx}{3675}+\frac{5}{}\right.}{}$
default	$\frac{-d\,a^2\left(\frac{1}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-d\,b^2\left(-\frac{\arccos(cx)^2c^5x^5}{5}+\frac{2\arccos(cx)\left(3c^4x^4+4c^2x^2+8\right)\sqrt{-c^2x^2+1}}{75}+\frac{38c^5x^5}{6125}+\frac{152c^3x^3}{11025}+\frac{304cx}{3675}+\frac{5}{}\right.}{}$
orering	$\frac{(142875c^{10}x^{10}-346302c^8x^8+107235c^6x^6-505400c^4x^4+872480c^2x^2-383040)\left(-c^2d\,x^2+d\right)(a+b\arccos(cx))^2}{385875x\,c^6(c^2x^2-1)^2}-\frac{(2}{}$

input
```
int(x^4*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-d*a^2*(1/7*c^2*x^7-1/5*x^5)-d*b^2/c^5*(-1/5*arccos(c*x)^2*c^5*x^5+2/75*ar
ccos(c*x)*(3*c^4*x^4+4*c^2*x^2+8)*(-c^2*x^2+1)^(1/2)+38/6125*c^5*x^5+152/1
1025*c^3*x^3+304/3675*c*x+1/7*arccos(c*x)^2*c^7*x^7-2/245*arccos(c*x)*(5*c
^6*x^6+6*c^4*x^4+8*c^2*x^2+16)*(-c^2*x^2+1)^(1/2)-2/343*c^7*x^7)-2*d*a*b/c
^5*(1/7*arccos(c*x)*c^7*x^7-1/5*arccos(c*x)*c^5*x^5+19/1225*c^4*x^4*(-c^2*
x^2+1)^(1/2)+76/3675*c^2*x^2*(-c^2*x^2+1)^(1/2)+152/3675*(-c^2*x^2+1)^(1/2
)-1/49*c^6*x^6*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.79

$$\int x^4(d-c^2dx^2)(a+b\arccos(cx))^2\,dx = -\frac{1125\,(49\,a^2-2\,b^2)c^7dx^7-63\,(1225\,a^2-38\,b^2)c^5dx^5+5320\,b^2c^3dx^3+31920\,b^2cdx+11025\,(5\,b^2c^7dx^7}{}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
-1/385875*(1125*(49*a^2 - 2*b^2)*c^7*d*x^7 - 63*(1225*a^2 - 38*b^2)*c^5*d*
x^5 + 5320*b^2*c^3*d*x^3 + 31920*b^2*c*d*x + 11025*(5*b^2*c^7*d*x^7 - 7*b^
2*c^5*d*x^5)*arccos(c*x)^2 + 22050*(5*a*b*c^7*d*x^7 - 7*a*b*c^5*d*x^5)*arc
cos(c*x) - 210*(75*a*b*c^6*d*x^6 - 57*a*b*c^4*d*x^4 - 76*a*b*c^2*d*x^2 - 1
52*a*b*d + (75*b^2*c^6*d*x^6 - 57*b^2*c^4*d*x^4 - 76*b^2*c^2*d*x^2 - 152*b
^2*d)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.36

$$\int x^4(d-c^2dx^2)(a+b\arccos(cx))^2\,dx = \begin{cases} -\frac{a^2c^2dx^7}{7}+\frac{a^2dx^5}{5}-\frac{2abc^2dx^7\operatorname{acos}(cx)}{7}+\frac{2abcdx^6\sqrt{-c^2x^2+1}}{49}+\frac{2abdx^5\operatorname{acos}(cx)}{5}-\frac{38abdx^4\sqrt{-c^2x^2+1}}{1225c}-\frac{152abdx^2\sqrt{-c^2x^2+1}}{3675c^3} \\ \frac{dx^5\left(a+\frac{\pi b}{2}\right)^2}{5} \end{cases}$$

input
```
integrate(x**4*(-c**2*d*x**2+d)*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((-a**2*c**2*d*x**7/7 + a**2*d*x**5/5 - 2*a*b*c**2*d*x**7*acos(c*
x)/7 + 2*a*b*c*d*x**6*sqrt(-c**2*x**2 + 1)/49 + 2*a*b*d*x**5*acos(c*x)/5 -
 38*a*b*d*x**4*sqrt(-c**2*x**2 + 1)/(1225*c) - 152*a*b*d*x**2*sqrt(-c**2*x
**2 + 1)/(3675*c**3) - 304*a*b*d*sqrt(-c**2*x**2 + 1)/(3675*c**5) - b**2*c
**2*d*x**7*acos(c*x)**2/7 + 2*b**2*c**2*d*x**7/343 + 2*b**2*c*d*x**6*sqrt(
-c**2*x**2 + 1)*acos(c*x)/49 + b**2*d*x**5*acos(c*x)**2/5 - 38*b**2*d*x**5
/6125 - 38*b**2*d*x**4*sqrt(-c**2*x**2 + 1)*acos(c*x)/(1225*c) - 152*b**2*
d*x**3/(11025*c**2) - 152*b**2*d*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(3675
*c**3) - 304*b**2*d*x/(3675*c**4) - 304*b**2*d*sqrt(-c**2*x**2 + 1)*acos(c
*x)/(3675*c**5), Ne(c, 0)), (d*x**5*(a + pi*b/2)**2/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.56

$$\int x^4(d-c^2dx^2)\,(a+b\arccos(cx))^2\,dx$$
$$= -\frac{1}{7}\,b^2c^2dx^7\arccos(cx)^2 - \frac{1}{7}\,a^2c^2dx^7 + \frac{1}{5}\,b^2dx^5\arccos(cx)^2 + \frac{1}{5}\,a^2dx^5$$
$$-\frac{2}{245}\left(35\,x^7\arccos(cx) - \left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\sqrt{-c^2x^2+1}}{c^8}\right.\right.$$
$$+\frac{2}{25725}\left(105\left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\sqrt{-c^2x^2+1}}{c^8}\right)c\arccos(cx\right.$$
$$+\frac{2}{75}\left(15\,x^5\arccos(cx) - \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)abd$$
$$-\frac{2}{1125}\left(15\left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\arccos(cx) + \frac{9\,c^4x^5+20\,c^2x^3+}{c^4}\right.$$

input
```
integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-1/7*b^2*c^2*d*x^7*arccos(c*x)^2 - 1/7*a^2*c^2*d*x^7 + 1/5*b^2*d*x^5*arcco
s(c*x)^2 + 1/5*a^2*d*x^5 - 2/245*(35*x^7*arccos(c*x) - (5*sqrt(-c^2*x^2 +
1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 +
 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d + 2/25725*(105*(5*sqrt(-c^2*x^2 +
 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6
+ 16*sqrt(-c^2*x^2 + 1)/c^8)*c*arccos(c*x) + (75*c^6*x^7 + 126*c^4*x^5 + 2
80*c^2*x^3 + 1680*x)/c^6)*b^2*c^2*d + 2/75*(15*x^5*arccos(c*x) - (3*sqrt(-
c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)
/c^6)*c)*a*b*d - 2/1125*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^
2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c*arccos(c*x) + (9*c^4*x^5 + 20
*c^2*x^3 + 120*x)/c^4)*b^2*d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.10

$$\begin{aligned}\int x^4(d-c^2dx^2)\,(a+b\arccos(cx))^2\,dx = &-\frac{1}{7}\,b^2c^2dx^7\arccos(cx)^2-\frac{2}{7}\,abc^2dx^7\arccos(cx)\\&-\frac{1}{7}\,a^2c^2dx^7+\frac{2}{343}\,b^2c^2dx^7\\&+\frac{2}{49}\,\sqrt{-c^2x^2+1}b^2cdx^6\arccos(cx)\\&+\frac{2}{49}\,\sqrt{-c^2x^2+1}abcdx^6\\&+\frac{1}{5}\,b^2dx^5\arccos(cx)^2+\frac{2}{5}\,abdx^5\arccos(cx)\\&+\frac{1}{5}\,a^2dx^5-\frac{38}{6125}\,b^2dx^5\\&-\frac{38\,\sqrt{-c^2x^2+1}b^2dx^4\arccos(cx)}{1225\,c}\\&-\frac{38\,\sqrt{-c^2x^2+1}abdx^4}{1225\,c}-\frac{152\,b^2dx^3}{11025\,c^2}\\&-\frac{152\,\sqrt{-c^2x^2+1}b^2dx^2\arccos(cx)}{3675\,c^3}\\&-\frac{152\,\sqrt{-c^2x^2+1}abdx^2}{3675\,c^3}-\frac{304\,b^2dx}{3675\,c^4}\\&-\frac{304\,\sqrt{-c^2x^2+1}b^2d\arccos(cx)}{3675\,c^5}\\&-\frac{304\,\sqrt{-c^2x^2+1}abd}{3675\,c^5}\end{aligned}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-1/7*b^2*c^2*d*x^7*arccos(c*x)^2 - 2/7*a*b*c^2*d*x^7*arccos(c*x) - 1/7*a^2
*c^2*d*x^7 + 2/343*b^2*c^2*d*x^7 + 2/49*sqrt(-c^2*x^2 + 1)*b^2*c*d*x^6*arc
cos(c*x) + 2/49*sqrt(-c^2*x^2 + 1)*a*b*c*d*x^6 + 1/5*b^2*d*x^5*arccos(c*x)
^2 + 2/5*a*b*d*x^5*arccos(c*x) + 1/5*a^2*d*x^5 - 38/6125*b^2*d*x^5 - 38/12
25*sqrt(-c^2*x^2 + 1)*b^2*d*x^4*arccos(c*x)/c - 38/1225*sqrt(-c^2*x^2 + 1)
*a*b*d*x^4/c - 152/11025*b^2*d*x^3/c^2 - 152/3675*sqrt(-c^2*x^2 + 1)*b^2*d
*x^2*arccos(c*x)/c^3 - 152/3675*sqrt(-c^2*x^2 + 1)*a*b*d*x^2/c^3 - 304/367
5*b^2*d*x/c^4 - 304/3675*sqrt(-c^2*x^2 + 1)*b^2*d*arccos(c*x)/c^5 - 304/36
75*sqrt(-c^2*x^2 + 1)*a*b*d/c^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d-c^2dx^2)(a+b\arccos(cx))^2\,dx=\int x^4\,(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)\,dx$$

input
```
int(x^4*(a + b*acos(c*x))^2*(d - c^2*d*x^2),x)
```

output
```
int(x^4*(a + b*acos(c*x))^2*(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int x^4(d-c^2dx^2)(a+b\arccos(cx))^2\,dx$$
$$=\frac{d\left(-1050acos(cx)\,ab\,c^7x^7+1470acos(cx)\,ab\,c^5x^5+150\sqrt{-c^2x^2+1}\,ab\,c^6x^6-114\sqrt{-c^2x^2+1}\,ab\,c^4x^4-\right.}{}$$

input
```
int(x^4*(-c^2*d*x^2+d)*(a+b*acos(c*x))^2,x)
```

output
```
(d*( - 1050*acos(c*x)*a*b*c**7*x**7 + 1470*acos(c*x)*a*b*c**5*x**5 + 150*s
qrt( - c**2*x**2 + 1)*a*b*c**6*x**6 - 114*sqrt( - c**2*x**2 + 1)*a*b*c**4*
x**4 - 152*sqrt( - c**2*x**2 + 1)*a*b*c**2*x**2 - 304*sqrt( - c**2*x**2 +
1)*a*b - 3675*int(acos(c*x)**2*x**6,x)*b**2*c**7 + 3675*int(acos(c*x)**2*x
**4,x)*b**2*c**5 - 525*a**2*c**7*x**7 + 735*a**2*c**5*x**5))/(3675*c**5)
```

3.159 $\int x^3(d-c^2dx^2)(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 202

$$\begin{aligned}\int x^3(d-c^2dx^2)(a+b\arccos(cx))^2\,dx = &-\frac{b^2dx^2}{24c^2}-\frac{1}{72}b^2dx^4+\frac{1}{108}b^2c^2dx^6\\&+\frac{bdx\sqrt{1-c^2x^2}(a+b\arccos(cx))}{12c^3}\\&+\frac{bdx^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{18c}\\&-\frac{1}{18}bcdx^5\sqrt{1-c^2x^2}(a+b\arccos(cx))\\&-\frac{d(a+b\arccos(cx))^2}{24c^4}\\&+\frac{1}{12}dx^4(a+b\arccos(cx))^2\\&+\frac{1}{6}dx^4(1-c^2x^2)(a+b\arccos(cx))^2\end{aligned}$$

output
```
-1/24*b^2*d*x^2/c^2-1/72*b^2*d*x^4+1/108*b^2*c^2*d*x^6+1/12*b*d*x*(-c^2*x^
2+1)^(1/2)*(a+b*arccos(c*x))/c^3+1/18*b*d*x^3*(-c^2*x^2+1)^(1/2)*(a+b*arcc
os(c*x))/c-1/18*b*c*d*x^5*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))-1/24*d*(a+b
*arccos(c*x))^2/c^4+1/12*d*x^4*(a+b*arccos(c*x))^2+1/6*d*x^4*(-c^2*x^2+1)*
(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.22 (sec), antiderivative size = 195, normalized size of antiderivative = 0.97

$$\int x^3(d-c^2dx^2)(a+b\arccos(cx))^2\,dx = -\frac{d\left(cx\left(18a^2c^3x^3(-3+2c^2x^2)-6ab\sqrt{1-c^2x^2}(-3-2c^2x^2+2c^4x^4)+b^2(9cx+3c^3x^3-2c^5x^5)\right)+6bcx\right.}{}$$

input
```
Integrate[x^3*(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/216*(d*(c*x*(18*a^2*c^3*x^3*(-3 + 2*c^2*x^2) - 6*a*b*Sqrt[1 - c^2*x^2]*
(-3 - 2*c^2*x^2 + 2*c^4*x^4) + b^2*(9*c*x + 3*c^3*x^3 - 2*c^5*x^5)) + 6*b*
c*x*(6*a*c^3*x^3*(-3 + 2*c^2*x^2) + b*Sqrt[1 - c^2*x^2]*(3 + 2*c^2*x^2 - 2
*c^4*x^4))*ArcCos[c*x] + 9*b^2*(1 - 6*c^4*x^4 + 4*c^6*x^6)*ArcCos[c*x]^2 -
 18*a*b*ArcSin[c*x]))/c^4
```

Rubi [A] (verified)

Time = 1.39 (sec), antiderivative size = 334, normalized size of antiderivative = 1.65, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5203, 5139, 5199, 15, 5211, 15, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d-c^2dx^2)(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{1}{3}bcd\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{3}d\int x^3(a+b\arccos(cx))^2dx+\frac{1}{6}dx^4(1-c^2x^2)(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5139}$$

$$\frac{1}{3}d\left(\frac{1}{2}bc\int \frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))^2\right)+\frac{1}{3}bcd\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{6}dx^4\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$\Big\downarrow$ 5199

$$\frac{1}{3}d\left(\frac{1}{2}bc\int \frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))^2\right)+\frac{1}{3}bcd\left(\frac{1}{6}\int \frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{6}bc\int x^5dx+\frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)+\frac{1}{6}dx^4\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$\Big\downarrow$ 15

$$\frac{1}{3}d\left(\frac{1}{2}bc\int \frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))^2\right)+\frac{1}{3}bcd\left(\frac{1}{6}\int \frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{36}bcx^6\right)+\frac{1}{6}dx^4\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$\Big\downarrow$ 5211

$$\frac{1}{3}d\left(\frac{1}{2}bc\left(\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{b\int x^3dx}{4c}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}\right)+\frac{1}{4}x^4(a+b\arccos(cx))^2\right)+$$
$$\frac{1}{3}bcd\Bigg(\frac{1}{6}\left(\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{b\int x^3dx}{4c}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}\right)+\frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arccos($$
$$\frac{1}{6}dx^4\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$\Big\downarrow$ 15

$$\frac{1}{3}d\left(\frac{1}{2}bc\left(\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)+\frac{1}{4}x^4(a+b\arccos(cx))^2\right)+$$
$$\frac{1}{3}bcd\Bigg(\frac{1}{6}\left(\frac{3\int \frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)+\frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))$$
$$\frac{1}{6}dx^4\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$\Big\downarrow$ 5211

$$\frac{1}{3}d\left(\frac{1}{2}bc\left(\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int xdx}{2c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)+\frac{1}{4}\right.$$
$$\frac{1}{3}bcd\left(\frac{1}{6}\left(\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int xdx}{2c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)+\frac{1}{6}\right.$$
$$\frac{1}{6}dx^4\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{3}d\left(\frac{1}{2}bc\left(\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)+\frac{1}{4}x^4(\right.$$
$$\frac{1}{3}bcd\left(\frac{1}{6}\left(\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)+\frac{1}{6}x^5\right.$$
$$\frac{1}{6}dx^4\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{ 5153}$$

$$\frac{1}{6}dx^4\left(1-c^2x^2\right)(a+b\arccos(cx))^2+$$
$$\frac{1}{3}d\left(\frac{1}{2}bc\left(-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}+\frac{3\left(-\frac{(a+b\arccos(cx))^2}{4bc^3}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}-\frac{bx^4}{16c}\right)+\frac{1}{4}\right.$$
$$\frac{1}{3}bcd\left(\frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{6}\left(-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}+\frac{3\left(-\frac{(a+b\arccos(cx))^2}{4bc^3}-\frac{x\sqrt{1-c^2x}}{}\right.}{4c^2}\right.\right.$$

input
```
Int[x^3*(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*x^4*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/6 + (b*c*d*((b*c*x^6)/36 + (x^
5*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/6 + (-1/16*(b*x^4)/c - (x^3*Sqrt[
1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(4*c^2) + (3*(-1/4*(b*x^2)/c - (x*Sqrt[1
 - c^2*x^2]*(a + b*ArcCos[c*x]))/(2*c^2) - (a + b*ArcCos[c*x])^2/(4*b*c^3)
))/(4*c^2))/6))/3 + (d*((x^4*(a + b*ArcCos[c*x])^2)/4 + (b*c*(-1/16*(b*x^4
)/c - (x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(4*c^2) + (3*(-1/4*(b*x^
2)/c - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(2*c^2) - (a + b*ArcCos[c
*x])^2/(4*b*c^3)))/(4*c^2)))/2))/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.63

method	result
parts	$-d\,a^2\left(\frac{1}{6}c^2x^6-\frac{1}{4}x^4\right)-\frac{d\,b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-1\right)^2}{4}-\frac{\arccos(cx)\left(2c^3x^3\sqrt{-c^2x^2+1}-5cx\sqrt{-c^2x^2+1}+3\arccos(cx)\right)}{16}+\right.}{}$
derivativedivides	$\frac{-d\,a^2\left(\frac{1}{6}c^6x^6-\frac{1}{4}c^4x^4\right)-d\,b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-1\right)^2}{4}-\frac{\arccos(cx)\left(2c^3x^3\sqrt{-c^2x^2+1}-5cx\sqrt{-c^2x^2+1}+3\arccos(cx)\right)}{16}+\frac{\arccos(}{24}\right.}{}$
default	$\frac{-d\,a^2\left(\frac{1}{6}c^6x^6-\frac{1}{4}c^4x^4\right)-d\,b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-1\right)^2}{4}-\frac{\arccos(cx)\left(2c^3x^3\sqrt{-c^2x^2+1}-5cx\sqrt{-c^2x^2+1}+3\arccos(cx)\right)}{16}+\frac{\arccos(}{24}\right.}{}$
orering	$\frac{\left(182c^8x^8-473c^6x^6+42c^4x^4+369c^2x^2-180\right)\left(-c^2d\,x^2+d\right)\left(a+b\arccos(cx)\right)^2}{432c^4\left(c^2x^2-1\right)^2}-\frac{\left(10c^6x^6-21c^4x^4-23c^2x^2+24\right)\left(3x^2\right.}{}$

input
```
int(x^3*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-d*a^2*(1/6*c^2*x^6-1/4*x^4)-d*b^2/c^4*(1/4*arccos(c*x)^2*(c^2*x^2-1)^2-1/
16*arccos(c*x)*(2*c^3*x^3*(-c^2*x^2+1)^(1/2)-5*c*x*(-c^2*x^2+1)^(1/2)+3*ar
ccos(c*x))+1/24*arccos(c*x)^2-1/128*(2*c^2*x^2-5)^2+1/6*arccos(c*x)^2*(c^2
*x^2-1)^3+1/144*arccos(c*x)*(-8*c^5*x^5*(-c^2*x^2+1)^(1/2)+26*c^3*x^3*(-c^
2*x^2+1)^(1/2)-33*c*x*(-c^2*x^2+1)^(1/2)+15*arccos(c*x))-1/108*c^6*x^6+13/
288*c^4*x^4-11/96*c^2*x^2)-2*d*a*b/c^4*(1/6*arccos(c*x)*c^6*x^6-1/4*c^4*x^
4*arccos(c*x)+1/36*c^3*x^3*(-c^2*x^2+1)^(1/2)+1/24*c*x*(-c^2*x^2+1)^(1/2)-
1/24*arcsin(c*x)-1/36*c^5*x^5*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.04

$$\int x^3(d-c^2dx^2)(a+b\arccos(cx))^2\,dx =$$
$$-\frac{2(18a^2-b^2)c^6dx^6-3(18a^2-b^2)c^4dx^4+9b^2c^2dx^2+9(4b^2c^6dx^6-6b^2c^4dx^4+b^2d)\arccos(cx)^2+}{}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
-1/216*(2*(18*a^2 - b^2)*c^6*d*x^6 - 3*(18*a^2 - b^2)*c^4*d*x^4 + 9*b^2*c^
2*d*x^2 + 9*(4*b^2*c^6*d*x^6 - 6*b^2*c^4*d*x^4 + b^2*d)*arccos(c*x)^2 + 18
*(4*a*b*c^6*d*x^6 - 6*a*b*c^4*d*x^4 + a*b*d)*arccos(c*x) - 6*(2*a*b*c^5*d*
x^5 - 2*a*b*c^3*d*x^3 - 3*a*b*c*d*x + (2*b^2*c^5*d*x^5 - 2*b^2*c^3*d*x^3 -
 3*b^2*c*d*x)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.67

$$\int x^3(d-c^2dx^2)(a+b\arccos(cx))^2\,dx$$
$$=\begin{cases} -\frac{a^2c^2dx^6}{6}+\frac{a^2dx^4}{4}-\frac{abc^2dx^6\operatorname{acos}(cx)}{3}+\frac{abcdx^5\sqrt{-c^2x^2+1}}{18}+\frac{abdx^4\operatorname{acos}(cx)}{2}-\frac{abdx^3\sqrt{-c^2x^2+1}}{18c}-\frac{abdx\sqrt{-c^2x^2+1}}{12c^3}-\frac{abd\,\mathrm{a}}{1} \\ \frac{dx^4\left(a+\frac{\pi b}{2}\right)^2}{4} \end{cases}$$

input
```
integrate(x**3*(-c**2*d*x**2+d)*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((-a**2*c**2*d*x**6/6 + a**2*d*x**4/4 - a*b*c**2*d*x**6*acos(c*x)
/3 + a*b*c*d*x**5*sqrt(-c**2*x**2 + 1)/18 + a*b*d*x**4*acos(c*x)/2 - a*b*d
*x**3*sqrt(-c**2*x**2 + 1)/(18*c) - a*b*d*x*sqrt(-c**2*x**2 + 1)/(12*c**3)
 - a*b*d*acos(c*x)/(12*c**4) - b**2*c**2*d*x**6*acos(c*x)**2/6 + b**2*c**2
*d*x**6/108 + b**2*c*d*x**5*sqrt(-c**2*x**2 + 1)*acos(c*x)/18 + b**2*d*x**
4*acos(c*x)**2/4 - b**2*d*x**4/72 - b**2*d*x**3*sqrt(-c**2*x**2 + 1)*acos(
c*x)/(18*c) - b**2*d*x**2/(24*c**2) - b**2*d*x*sqrt(-c**2*x**2 + 1)*acos(c
*x)/(12*c**3) - b**2*d*acos(c*x)**2/(24*c**4), Ne(c, 0)), (d*x**4*(a + pi*
b/2)**2/4, True))
```

Maxima [F]

$$\int x^3 (d - c^2 dx^2) (a + b \arccos(cx))^2 \, dx = \int -(c^2 dx^2 - d)(b \arccos(cx) + a)^2 x^3 \, dx$$

input
```
integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-1/6*a^2*c^2*d*x^6 + 1/4*a^2*d*x^4 - 1/144*(48*x^6*arccos(c*x) - (8*sqrt(-
c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 +
1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*a*b*c^2*d + 1/16*(8*x^4*arccos(c*x) - (2
*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c
^5)*c)*a*b*d - 1/12*(2*b^2*c^2*d*x^6 - 3*b^2*d*x^4)*arctan2(sqrt(c*x + 1)*
sqrt(-c*x + 1), c*x)^2 + integrate(1/6*(2*b^2*c^3*d*x^6 - 3*b^2*c*d*x^4)*s
qrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^
2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.47

$$\int x^3(d-c^2dx^2)(a+b\arccos(cx))^2\,dx = -\frac{1}{6}b^2c^2dx^6\arccos(cx)^2-\frac{1}{3}abc^2dx^6\arccos(cx) - \frac{1}{6}a^2c^2dx^6+\frac{1}{108}b^2c^2dx^6 + \frac{1}{18}\sqrt{-c^2x^2+1}b^2cdx^5\arccos(cx) + \frac{1}{18}\sqrt{-c^2x^2+1}abcdx^5 + \frac{1}{4}b^2dx^4\arccos(cx)^2 + \frac{1}{2}abdx^4\arccos(cx)+\frac{1}{4}a^2dx^4 - \frac{1}{72}b^2dx^4-\frac{\sqrt{-c^2x^2+1}b^2dx^3\arccos(cx)}{18\,c} - \frac{\sqrt{-c^2x^2+1}abdx^3}{18\,c}-\frac{b^2dx^2}{24\,c^2} - \frac{\sqrt{-c^2x^2+1}b^2dx\arccos(cx)}{12\,c^3} - \frac{\sqrt{-c^2x^2+1}abdx}{12\,c^3}-\frac{b^2d\arccos(cx)^2}{24\,c^4} - \frac{abd\arccos(cx)}{12\,c^4}+\frac{5\,b^2d}{216\,c^4}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-1/6*b^2*c^2*d*x^6*arccos(c*x)^2 - 1/3*a*b*c^2*d*x^6*arccos(c*x) - 1/6*a^2
*c^2*d*x^6 + 1/108*b^2*c^2*d*x^6 + 1/18*sqrt(-c^2*x^2 + 1)*b^2*c*d*x^5*arc
cos(c*x) + 1/18*sqrt(-c^2*x^2 + 1)*a*b*c*d*x^5 + 1/4*b^2*d*x^4*arccos(c*x)
^2 + 1/2*a*b*d*x^4*arccos(c*x) + 1/4*a^2*d*x^4 - 1/72*b^2*d*x^4 - 1/18*sqr
t(-c^2*x^2 + 1)*b^2*d*x^3*arccos(c*x)/c - 1/18*sqrt(-c^2*x^2 + 1)*a*b*d*x^
3/c - 1/24*b^2*d*x^2/c^2 - 1/12*sqrt(-c^2*x^2 + 1)*b^2*d*x*arccos(c*x)/c^3
 - 1/12*sqrt(-c^2*x^2 + 1)*a*b*d*x/c^3 - 1/24*b^2*d*arccos(c*x)^2/c^4 - 1/
12*a*b*d*arccos(c*x)/c^4 + 5/216*b^2*d/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d - c^2 d x^2) (a + b \arccos(cx))^2 \, dx = \int x^3 (a + b \operatorname{acos}(c\,x))^2 (d - c^2\,d\,x^2) \, dx$$

input
```
int(x^3*(a + b*acos(c*x))^2*(d - c^2*d*x^2),x)
```

output
```
int(x^3*(a + b*acos(c*x))^2*(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int x^3 (d - c^2 d x^2) (a + b \arccos(cx))^2 \, dx = \frac{d(-12 acos(cx)\, ab\, c^6 x^6 + 18 acos(cx)\, ab\, c^4 x^4 + 3 asin(cx)\, ab + 2\sqrt{-c^2x^2+1}\, ab\, c^5 x^5 - 2\sqrt{-c^2x^2+1}\, ab\, c^3}{36}$$

input
```
int(x^3*(-c^2*d*x^2+d)*(a+b*acos(c*x))^2,x)
```

output
```
(d*( - 12*acos(c*x)*a*b*c**6*x**6 + 18*acos(c*x)*a*b*c**4*x**4 + 3*asin(c*
x)*a*b + 2*sqrt( - c**2*x**2 + 1)*a*b*c**5*x**5 - 2*sqrt( - c**2*x**2 + 1)
*a*b*c**3*x**3 - 3*sqrt( - c**2*x**2 + 1)*a*b*c*x - 36*int(acos(c*x)**2*x*
*5,x)*b**2*c**6 + 36*int(acos(c*x)**2*x**3,x)*b**2*c**4 - 6*a**2*c**6*x**6
 + 9*a**2*c**4*x**4))/(36*c**4)
```

3.160 $\int x^2(d-c^2dx^2)(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 211

$$\begin{aligned}\int x^2(d-c^2dx^2)(a+b\arccos(cx))^2\,dx = &-\frac{52b^2dx}{225c^2}-\frac{26}{675}b^2dx^3+\frac{2}{125}b^2c^2dx^5\\ &+\frac{8bd\sqrt{1-c^2x^2}(a+b\arccos(cx))}{45c^3}\\ &+\frac{4bdx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{45c}\\ &+\frac{2bd(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{15c^3}\\ &-\frac{2bd(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{25c^3}\\ &+\frac{2}{15}dx^3(a+b\arccos(cx))^2\\ &+\frac{1}{5}dx^3(1-c^2x^2)(a+b\arccos(cx))^2\end{aligned}$$

output
```
-52/225*b^2*d*x/c^2-26/675*b^2*d*x^3+2/125*b^2*c^2*d*x^5+8/45*b*d*(-c^2*x^
2+1)^(1/2)*(a+b*arccos(c*x))/c^3+4/45*b*d*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arcc
os(c*x))/c+2/15*b*d*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/c^3-2/25*b*d*(-c^
2*x^2+1)^(5/2)*(a+b*arccos(c*x))/c^3+2/15*d*x^3*(a+b*arccos(c*x))^2+1/5*d*
x^3*(-c^2*x^2+1)*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.85

$$\int x^2 (d - c^2 dx^2) (a + b \arccos(cx))^2 dx =$$
$$-\frac{d(225a^2c^3x^3(-5 + 3c^2x^2) - 30ab\sqrt{1 - c^2x^2}(-26 - 13c^2x^2 + 9c^4x^4) + b^2(780cx + 130c^3x^3 - 54c^5x^5)}{}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/3375*(d*(225*a^2*c^3*x^3*(-5 + 3*c^2*x^2) - 30*a*b*Sqrt[1 - c^2*x^2]*(-
26 - 13*c^2*x^2 + 9*c^4*x^4) + b^2*(780*c*x + 130*c^3*x^3 - 54*c^5*x^5) -
30*b*(b*Sqrt[1 - c^2*x^2]*(-26 - 13*c^2*x^2 + 9*c^4*x^4) + a*(75*c^3*x^3 -
 45*c^5*x^5))*ArcCos[c*x] + 225*b^2*c^3*x^3*(-5 + 3*c^2*x^2)*ArcCos[c*x]^2
))/c^3
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5203, 5139, 5195, 27, 2009, 5211, 15, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d - c^2 dx^2) (a + b \arccos(cx))^2 dx$$

$$\Big\downarrow \text{ 5203}$$

$$\frac{2}{5} bcd \int x^3 \sqrt{1 - c^2x^2} (a + b \arccos(cx)) dx + \frac{2}{5} d \int x^2 (a + b \arccos(cx))^2 dx + \frac{1}{5} dx^3 (1 - c^2x^2) (a + b \arccos(cx))^2$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{2}{5}d\left(\frac{2}{3}bc\int \frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)+\frac{2}{5}bcd\int x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{5}dx^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5195}$$

$$\frac{2}{5}d\left(\frac{2}{3}bc\int \frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)+\frac{2}{5}bcd\left(bc\int -\frac{-3c^4x^4+c^2x^2+2}{15c^4}dx+\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^4}\right)+\frac{1}{5}dx^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{5}d\left(\frac{2}{3}bc\int \frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)+\frac{2}{5}bcd\left(-\frac{b\int\left(-3c^4x^4+c^2x^2+2\right)dx}{15c^3}+\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^4}\right)+\frac{1}{5}dx^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}d\left(\frac{2}{3}bc\int \frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)+\frac{1}{5}dx^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{5}bcd\left(\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^4}-\frac{b\left(-\frac{3}{5}c^4x^5+\frac{c^2x^3}{3}+2x\right)}{15c^3}\right)$$

$$\Big\downarrow \text{5211}$$

$$\frac{2}{5}d\left(\frac{2}{3}bc\left(\frac{2\int\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{b\int x^2dx}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)+\frac{1}{5}dx^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{5}bcd\left(\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^4}-\frac{b\left(-\frac{3}{5}c^4x^5+\frac{c^2x^3}{3}+2x\right)}{15c^3}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{5}d\left(\frac{2}{3}bc\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)+$$
$$\frac{1}{5}dx^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2+$$
$$\frac{2}{5}bcd\left(\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^4}-\frac{b\left(-\frac{3}{5}c^4x^5+\frac{c^2x^3}{3}+2x\right)}{15c^3}\right)$$

$$\Big\downarrow \; 5183$$

$$\frac{2}{5}d\left(\frac{2}{3}bc\left(\frac{2\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)+\frac{1}{3}x^3(a+b\arccos(cx\right.$$
$$\frac{1}{5}dx^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2+$$
$$\frac{2}{5}bcd\left(\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^4}-\frac{b\left(-\frac{3}{5}c^4x^5+\frac{c^2x^3}{3}+2x\right)}{15c^3}\right)$$

$$\Big\downarrow \; 24$$

$$\frac{1}{5}dx^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2+$$
$$\frac{2}{5}d\left(\frac{2}{3}bc\left(-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}+\frac{2\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)}{3c^2}-\frac{bx^3}{9c}\right)+\frac{1}{3}x^3(a+b\arccos(cx)\right.$$
$$\frac{2}{5}bcd\left(\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^4}-\frac{b\left(-\frac{3}{5}c^4x^5+\frac{c^2x^3}{3}+2x\right)}{15c^3}\right)$$

input
```
Int[x^2*(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*x^3*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/5 + (2*b*c*d*(-1/15*(b*(2*x +
(c^2*x^3)/3 - (3*c^4*x^5)/5))/c^3 - ((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x
]))/(3*c^4) + ((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^4)))/5 + (2*d
*((x^3*(a + b*ArcCos[c*x])^2)/3 + (2*b*c*(-1/9*(b*x^3)/c - (x^2*Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x]))/(3*c^2) + (2*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]
*(a + b*ArcCos[c*x]))/c^2))/(3*c^2))))/3))/5
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.32

method	result
parts	$-d\,a^2\left(\frac{1}{5}c^2x^5-\frac{1}{3}x^3\right)-\frac{d\,b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-3\right)cx}{3}+\frac{4cx}{15}+\frac{4\arccos(cx)\sqrt{-c^2x^2+1}}{15}-\frac{2\arccos(cx)\left(c^2x^2-1\right)\sqrt{-c^2x}}{45}\right.}{}$
derivativedivides	$\frac{-d\,a^2\left(\frac{1}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-d\,b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-3\right)cx}{3}+\frac{4cx}{15}+\frac{4\arccos(cx)\sqrt{-c^2x^2+1}}{15}-\frac{2\arccos(cx)\left(c^2x^2-1\right)\sqrt{-c^2x^2+1}}{45}-\right.}{}$
default	$\frac{-d\,a^2\left(\frac{1}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-d\,b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-3\right)cx}{3}+\frac{4cx}{15}+\frac{4\arccos(cx)\sqrt{-c^2x^2+1}}{15}-\frac{2\arccos(cx)\left(c^2x^2-1\right)\sqrt{-c^2x^2+1}}{45}-\right.}{}$
orering	$\frac{(1647c^8x^8-4862c^6x^6-4033c^4x^4+7800c^2x^2-3120)\left(-c^2d\,x^2+d\right)(a+b\arccos(cx))^2}{3375x\,c^4(c^2x^2-1)^2}-\frac{(324c^6x^6-893c^4x^4-2665c^2x}{}$

input
```
int(x^2*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-d*a^2*(1/5*c^2*x^5-1/3*x^3)-d*b^2/c^3*(1/3*arccos(c*x)^2*(c^2*x^2-3)*c*x+
4/15*c*x+4/15*arccos(c*x)*(-c^2*x^2+1)^(1/2)-2/45*arccos(c*x)*(c^2*x^2-1)*
(-c^2*x^2+1)^(1/2)-2/135*(c^2*x^2-3)*c*x+1/15*arccos(c*x)^2*(3*c^4*x^4-10*
c^2*x^2+15)*c*x-2/25*arccos(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)^(1/2)-2/375*(3
*c^4*x^4-10*c^2*x^2+15)*c*x)-2*d*a*b/c^3*(1/5*arccos(c*x)*c^5*x^5-1/3*c^3*
x^3*arccos(c*x)+13/225*c^2*x^2*(-c^2*x^2+1)^(1/2)+26/225*(-c^2*x^2+1)^(1/2
)-1/25*c^4*x^4*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.92

$$\int x^2(d-c^2dx^2)(a+b\arccos(cx))^2\,dx =$$
$$-\frac{27\,(25\,a^2-2\,b^2)c^5dx^5-5\,(225\,a^2-26\,b^2)c^3dx^3+780\,b^2cdx+225\,(3\,b^2c^5dx^5-5\,b^2c^3dx^3)\arccos{(cx)}^2}{}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
-1/3375*(27*(25*a^2 - 2*b^2)*c^5*d*x^5 - 5*(225*a^2 - 26*b^2)*c^3*d*x^3 +
780*b^2*c*d*x + 225*(3*b^2*c^5*d*x^5 - 5*b^2*c^3*d*x^3)*arccos(c*x)^2 + 45
0*(3*a*b*c^5*d*x^5 - 5*a*b*c^3*d*x^3)*arccos(c*x) - 30*(9*a*b*c^4*d*x^4 -
13*a*b*c^2*d*x^2 - 26*a*b*d + (9*b^2*c^4*d*x^4 - 13*b^2*c^2*d*x^2 - 26*b^2
*d)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.51

$$\int x^2(d-c^2dx^2)(a+b\arccos(cx))^2\,dx$$
$$=\begin{cases}-\frac{a^2c^2dx^5}{5}+\frac{a^2dx^3}{3}-\frac{2abc^2dx^5\operatorname{acos}(cx)}{5}+\frac{2abcdx^4\sqrt{-c^2x^2+1}}{25}+\frac{2abdx^3\operatorname{acos}(cx)}{3}-\frac{26abdx^2\sqrt{-c^2x^2+1}}{225c}-\frac{52abd\sqrt{-c^2x^2+1}}{225c^3}-\\ \frac{dx^3\left(a+\frac{\pi b}{2}\right)^2}{3}\end{cases}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((-a**2*c**2*d*x**5/5 + a**2*d*x**3/3 - 2*a*b*c**2*d*x**5*acos(c*
x)/5 + 2*a*b*c*d*x**4*sqrt(-c**2*x**2 + 1)/25 + 2*a*b*d*x**3*acos(c*x)/3 -
 26*a*b*d*x**2*sqrt(-c**2*x**2 + 1)/(225*c) - 52*a*b*d*sqrt(-c**2*x**2 + 1
)/(225*c**3) - b**2*c**2*d*x**5*acos(c*x)**2/5 + 2*b**2*c**2*d*x**5/125 +
2*b**2*c*d*x**4*sqrt(-c**2*x**2 + 1)*acos(c*x)/25 + b**2*d*x**3*acos(c*x)*
*2/3 - 26*b**2*d*x**3/675 - 26*b**2*d*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/
(225*c) - 52*b**2*d*x/(225*c**2) - 52*b**2*d*sqrt(-c**2*x**2 + 1)*acos(c*x
)/(225*c**3), Ne(c, 0)), (d*x**3*(a + pi*b/2)**2/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.68

$$
\begin{aligned}
&\int x^2\left(d-c^2dx^2\right)(a+b\arccos(cx))^2\,dx \\
&= -\frac{1}{5}\,b^2c^2dx^5\arccos\left(cx\right)^2 - \frac{1}{5}\,a^2c^2dx^5 + \frac{1}{3}\,b^2dx^3\arccos\left(cx\right)^2 \\
&\quad - \frac{2}{75}\left(15\,x^5\arccos\left(cx\right) - \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)abc^2d \\
&\quad + \frac{2}{1125}\left(15\left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\arccos\left(cx\right) + \frac{9\,c^4x^5 + 20\,c^2x^3 +}{c^4}\right. \\
&\quad + \frac{1}{3}\,a^2dx^3 + \frac{2}{9}\left(3\,x^3\arccos\left(cx\right) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)abd \\
&\quad - \frac{2}{27}\left(3\,c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\arccos\left(cx\right) + \frac{c^2x^3+6\,x}{c^2}\right)b^2d
\end{aligned}
$$

input
```
integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-1/5*b^2*c^2*d*x^5*arccos(c*x)^2 - 1/5*a^2*c^2*d*x^5 + 1/3*b^2*d*x^3*arcco
s(c*x)^2 - 2/75*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sq
rt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d + 2/1125
*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt
(-c^2*x^2 + 1)/c^6)*c*arccos(c*x) + (9*c^4*x^5 + 20*c^2*x^3 + 120*x)/c^4)*
b^2*c^2*d + 1/3*a^2*d*x^3 + 2/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)
*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*d - 2/27*(3*c*(sqrt(-c^2*x^2 + 1
)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x) + (c^2*x^3 + 6*x)/c^2)*b
^2*d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.21

$$\int x^2(d-c^2dx^2)(a+b\arccos(cx))^2\,dx = -\frac{1}{5}\,b^2c^2dx^5\arccos(cx)^2 - \frac{2}{5}\,abc^2dx^5\arccos(cx) - \frac{1}{5}\,a^2c^2dx^5 + \frac{2}{125}\,b^2c^2dx^5 + \frac{2}{25}\,\sqrt{-c^2x^2+1}b^2cdx^4\arccos(cx) + \frac{2}{25}\,\sqrt{-c^2x^2+1}abcdx^4 + \frac{1}{3}\,b^2dx^3\arccos(cx)^2 + \frac{2}{3}\,abdx^3\arccos(cx) + \frac{1}{3}\,a^2dx^3 - \frac{26}{675}\,b^2dx^3 - \frac{26\,\sqrt{-c^2x^2+1}b^2dx^2\arccos(cx)}{225\,c} - \frac{26\,\sqrt{-c^2x^2+1}abdx^2}{225\,c} - \frac{52\,b^2dx}{225\,c^2} - \frac{52\,\sqrt{-c^2x^2+1}b^2d\arccos(cx)}{225\,c^3} - \frac{52\,\sqrt{-c^2x^2+1}abd}{225\,c^3}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-1/5*b^2*c^2*d*x^5*arccos(c*x)^2 - 2/5*a*b*c^2*d*x^5*arccos(c*x) - 1/5*a^2
*c^2*d*x^5 + 2/125*b^2*c^2*d*x^5 + 2/25*sqrt(-c^2*x^2 + 1)*b^2*c*d*x^4*arc
cos(c*x) + 2/25*sqrt(-c^2*x^2 + 1)*a*b*c*d*x^4 + 1/3*b^2*d*x^3*arccos(c*x)
^2 + 2/3*a*b*d*x^3*arccos(c*x) + 1/3*a^2*d*x^3 - 26/675*b^2*d*x^3 - 26/225
*sqrt(-c^2*x^2 + 1)*b^2*d*x^2*arccos(c*x)/c - 26/225*sqrt(-c^2*x^2 + 1)*a*
b*d*x^2/c - 52/225*b^2*d*x/c^2 - 52/225*sqrt(-c^2*x^2 + 1)*b^2*d*arccos(c*
x)/c^3 - 52/225*sqrt(-c^2*x^2 + 1)*a*b*d/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(d-c^2dx^2\right) (a+b\arccos(cx))^2\,dx = \int x^2\,(a+b\,\mathrm{acos}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)\,dx$$

input
```
int(x^2*(a + b*acos(c*x))^2*(d - c^2*d*x^2),x)
```

output
```
int(x^2*(a + b*acos(c*x))^2*(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int x^2 \left(d-c^2dx^2\right) (a+b\arccos(cx))^2\,dx = \frac{d\left(-90acos(cx)\,ab\,c^5x^5+150acos(cx)\,ab\,c^3x^3+18\sqrt{-c^2x^2+1}\,ab\,c^4x^4-26\sqrt{-c^2x^2+1}\,ab\,c^2x^2-52\sqrt{-\right.}{225c^3}$$

input
```
int(x^2*(-c^2*d*x^2+d)*(a+b*acos(c*x))^2,x)
```

output
```
(d*( - 90*acos(c*x)*a*b*c**5*x**5 + 150*acos(c*x)*a*b*c**3*x**3 + 18*sqrt(
 - c**2*x**2 + 1)*a*b*c**4*x**4 - 26*sqrt( - c**2*x**2 + 1)*a*b*c**2*x**2
- 52*sqrt( - c**2*x**2 + 1)*a*b - 225*int(acos(c*x)**2*x**4,x)*b**2*c**5 +
 225*int(acos(c*x)**2*x**2,x)*b**2*c**3 - 45*a**2*c**5*x**5 + 75*a**2*c**3
*x**3))/(225*c**3)
```

3.161 $\int x(d-c^2dx^2)(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int x(d-c^2dx^2)(a+b\arccos(cx))^2\,dx = -\frac{3}{32}b^2dx^2+\frac{b^2d(1-c^2x^2)^2}{32c^2} + \frac{3bdx\sqrt{1-c^2x^2}(a+b\arccos(cx))}{16c} + \frac{bdx(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{8c} + \frac{3d(a+b\arccos(cx))^2}{32c^2} - \frac{d(1-c^2x^2)^2(a+b\arccos(cx))^2}{4c^2}$$

output
```
-3/32*b^2*d*x^2+1/32*b^2*d*(-c^2*x^2+1)^2/c^2+3/16*b*d*x*(-c^2*x^2+1)^(1/2
)*(a+b*arccos(c*x))/c+1/8*b*d*x*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/c+3/3
2*d*(a+b*arccos(c*x))^2/c^2-1/4*d*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2/c^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.07

$$\int x(d-c^2dx^2)(a+b\arccos(cx))^2\,dx$$
$$=\frac{d\left(cx\left(b^2cx(-5+c^2x^2)-8a^2cx(-2+c^2x^2)+2ab\sqrt{1-c^2x^2}(-5+2c^2x^2)\right)+2bcx\left(-8acx(-2+c^2x^2)+\right.\right.}{32c^2}$$

input
```
Integrate[x*(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*(c*x*(b^2*c*x*(-5 + c^2*x^2) - 8*a^2*c*x*(-2 + c^2*x^2) + 2*a*b*Sqrt[1
- c^2*x^2]*(-5 + 2*c^2*x^2)) + 2*b*c*x*(-8*a*c*x*(-2 + c^2*x^2) + b*Sqrt[1
 - c^2*x^2]*(-5 + 2*c^2*x^2))*ArcCos[c*x] + b^2*(-5 + 16*c^2*x^2 - 8*c^4*x
^4)*ArcCos[c*x]^2 + 10*a*b*ArcSin[c*x]))/(32*c^2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5183, 5159, 244, 2009, 5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d-c^2dx^2)(a+b\arccos(cx))^2\,dx$$
$$\Big\downarrow \text{5183}$$
$$-\frac{bd\int(1-c^2x^2)^{3/2}(a+b\arccos(cx))dx}{2c}-\frac{d(1-c^2x^2)^2(a+b\arccos(cx))^2}{4c^2}$$
$$\Big\downarrow \text{5159}$$
$$-\frac{bd\left(\frac{3}{4}\int\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int x(1-c^2x^2)\,dx+\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arccos(cx))\right)}{2c}-\frac{d(1-c^2x^2)^2(a+b\arccos(cx))^2}{4c^2}$$

$$\Big\downarrow \; 244$$

$$-\frac{bd\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int \left(x-c^2x^3\right)dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)}{2c}-\frac{d\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{4c^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{bd\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\right)}{2c}-\frac{d\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{4c^2}$$

$$\Big\downarrow \; 5157$$

$$-\frac{bd\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{2c}\frac{d\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{4c^2}$$

$$\Big\downarrow \; 15$$

$$-\frac{bd\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{4}bcx^2\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{4}}{2c}\frac{d\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{4c^2}$$

$$\Big\downarrow \; 5153$$

$$-\frac{d\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{4c^2}-$$

$$\frac{bd\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)+\frac{1}{4}bc\left(\frac{x^2}{2}\right.}{2c}$$

input
```
Int[x*(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/4*(d*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/c^2 - (b*d*((b*c*(x^2/2 - (
c^2*x^4)/4))/4 + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 + (3*((b*c*
x^2)/4 + (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/2 - (a + b*ArcCos[c*x])
^2/(4*b*c)))/4))/(2*c)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{-\frac{d\,a^2\left(c^2x^2-1\right)^2}{4}-d\,b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-1\right)^2}{4}-\frac{\arccos(cx)\left(2c^3x^3\sqrt{-c^2x^2+1}-5cx\sqrt{-c^2x^2+1}+3\arccos(cx)\right)}{16}+\frac{3\arccos(cx)^2}{32}}{c^2}$
default	$\frac{-\frac{d\,a^2\left(c^2x^2-1\right)^2}{4}-d\,b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-1\right)^2}{4}-\frac{\arccos(cx)\left(2c^3x^3\sqrt{-c^2x^2+1}-5cx\sqrt{-c^2x^2+1}+3\arccos(cx)\right)}{16}+\frac{3\arccos(cx)^2}{32}}{c^2}$
parts	$-\frac{d\,a^2\left(c^2x^2-1\right)^2}{4c^2}-\frac{d\,b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-1\right)^2}{4}-\frac{\arccos(cx)\left(2c^3x^3\sqrt{-c^2x^2+1}-5cx\sqrt{-c^2x^2+1}+3\arccos(cx)\right)}{16}+\frac{3\arccos(c}{32}}{c^2}$
orering	$\frac{\left(37c^6x^6-144c^4x^4+113c^2x^2-30\right)\left(-c^2d\,x^2+d\right)\left(a+b\arccos(cx)\right)^2}{64c^2\left(c^2x^2-1\right)^2}-\frac{\left(9c^4x^4-41c^2x^2+20\right)\left(\left(-c^2d\,x^2+d\right)\left(a+b\arccos\right.\right.}{}$

input
```
int(x*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/4*d*a^2*(c^2*x^2-1)^2-d*b^2*(1/4*arccos(c*x)^2*(c^2*x^2-1)^2-1/1
6*arccos(c*x)*(2*c^3*x^3*(-c^2*x^2+1)^(1/2)-5*c*x*(-c^2*x^2+1)^(1/2)+3*arc
cos(c*x))+3/32*arccos(c*x)^2-1/128*(2*c^2*x^2-5)^2)-2*d*a*b*(1/4*c^4*x^4*a
rccos(c*x)-1/2*c^2*x^2*arccos(c*x)+1/4*arccos(c*x)+3/32*arcsin(c*x)-1/16*c
^3*x^3*(-c^2*x^2+1)^(1/2)+5/32*c*x*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.20

$$\int x(d-c^2dx^2)(a+b\arccos(cx))^2\,dx =$$
$$-\frac{(8a^2-b^2)c^4dx^4-(16a^2-5b^2)c^2dx^2+(8b^2c^4dx^4-16b^2c^2dx^2+5b^2d)\arccos(cx)^2+2(8abc^4dx^4-$$

input
```
integrate(x*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
-1/32*((8*a^2 - b^2)*c^4*d*x^4 - (16*a^2 - 5*b^2)*c^2*d*x^2 + (8*b^2*c^4*d
*x^4 - 16*b^2*c^2*d*x^2 + 5*b^2*d)*arccos(c*x)^2 + 2*(8*a*b*c^4*d*x^4 - 16
*a*b*c^2*d*x^2 + 5*a*b*d)*arccos(c*x) - 2*(2*a*b*c^3*d*x^3 - 5*a*b*c*d*x +
 (2*b^2*c^3*d*x^3 - 5*b^2*c*d*x)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(136) = 272$.

Time = 0.37 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.86

$$\int x(d-c^2dx^2)(a+b\arccos(cx))^2\,dx$$
$$=\begin{cases}-\frac{a^2c^2dx^4}{4}+\frac{a^2dx^2}{2}-\frac{abc^2dx^4\operatorname{acos}(cx)}{2}+\frac{abcdx^3\sqrt{-c^2x^2+1}}{8}+abdx^2\operatorname{acos}(cx)-\frac{5abdx\sqrt{-c^2x^2+1}}{16c}-\frac{5abd\operatorname{acos}(cx)}{16c^2}-\frac{b^2}{}\\ \frac{dx^2\left(a+\frac{\pi b}{2}\right)^2}{2}\end{cases}$$

input
```
integrate(x*(-c**2*d*x**2+d)*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((-a**2*c**2*d*x**4/4 + a**2*d*x**2/2 - a*b*c**2*d*x**4*acos(c*x)
/2 + a*b*c*d*x**3*sqrt(-c**2*x**2 + 1)/8 + a*b*d*x**2*acos(c*x) - 5*a*b*d*
x*sqrt(-c**2*x**2 + 1)/(16*c) - 5*a*b*d*acos(c*x)/(16*c**2) - b**2*c**2*d*
x**4*acos(c*x)**2/4 + b**2*c**2*d*x**4/32 + b**2*c*d*x**3*sqrt(-c**2*x**2
+ 1)*acos(c*x)/8 + b**2*d*x**2*acos(c*x)**2/2 - 5*b**2*d*x**2/32 - 5*b**2*
d*x*sqrt(-c**2*x**2 + 1)*acos(c*x)/(16*c) - 5*b**2*d*acos(c*x)**2/(32*c**2
), Ne(c, 0)), (d*x**2*(a + pi*b/2)**2/2, True))
```

Maxima [F]

$$\int x(d-c^2dx^2)\,(a+b\arccos(cx))^2\,dx = \int -\left(c^2dx^2-d\right)\left(b\arccos\left(cx\right)+a\right)^2x\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-1/4*a^2*c^2*d*x^4 - 1/16*(8*x^4*arccos(c*x) - (2*sqrt(-c^2*x^2 + 1)*x^3/c
^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*a*b*c^2*d + 1/2*a^
2*d*x^2 + 1/2*(2*x^2*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*
x)/c^3))*a*b*d - 1/4*(b^2*c^2*d*x^4 - 2*b^2*d*x^2)*arctan2(sqrt(c*x + 1)*s
qrt(-c*x + 1), c*x)^2 + integrate(1/2*(b^2*c^3*d*x^4 - 2*b^2*c*d*x^2)*sqrt
(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^2*x
^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.58

$$\int x(d-c^2dx^2)\,(a+b\arccos(cx))^2\,dx = -\frac{1}{4}\,b^2c^2dx^4\arccos\left(cx\right)^2-\frac{1}{2}\,abc^2dx^4\arccos\left(cx\right) -\frac{1}{4}\,a^2c^2dx^4+\frac{1}{32}\,b^2c^2dx^4 +\frac{1}{8}\,\sqrt{-c^2x^2+1}b^2cdx^3\arccos\left(cx\right) +\frac{1}{8}\,\sqrt{-c^2x^2+1}abcdx^3+\frac{1}{2}\,b^2dx^2\arccos\left(cx\right)^2 +abdx^2\arccos\left(cx\right)+\frac{1}{2}\,a^2dx^2-\frac{5}{32}\,b^2dx^2 -\frac{5\,\sqrt{-c^2x^2+1}b^2dx\arccos\left(cx\right)}{16\,c} -\frac{5\,\sqrt{-c^2x^2+1}abdx}{16\,c}-\frac{5\,b^2d\arccos\left(cx\right)^2}{32\,c^2} -\frac{5\,abd\arccos\left(cx\right)}{16\,c^2}+\frac{17\,b^2d}{256\,c^2}$$

input
```
integrate(x*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-1/4*b^2*c^2*d*x^4*arccos(c*x)^2 - 1/2*a*b*c^2*d*x^4*arccos(c*x) - 1/4*a^2
*c^2*d*x^4 + 1/32*b^2*c^2*d*x^4 + 1/8*sqrt(-c^2*x^2 + 1)*b^2*c*d*x^3*arcco
s(c*x) + 1/8*sqrt(-c^2*x^2 + 1)*a*b*c*d*x^3 + 1/2*b^2*d*x^2*arccos(c*x)^2
+ a*b*d*x^2*arccos(c*x) + 1/2*a^2*d*x^2 - 5/32*b^2*d*x^2 - 5/16*sqrt(-c^2*
x^2 + 1)*b^2*d*x*arccos(c*x)/c - 5/16*sqrt(-c^2*x^2 + 1)*a*b*d*x/c - 5/32*
b^2*d*arccos(c*x)^2/c^2 - 5/16*a*b*d*arccos(c*x)/c^2 + 17/256*b^2*d/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)\,(a+b\arccos(cx))^2\,dx = \int x\,{\left(a+b\,\mathrm{acos}(c\,x)\right)}^2\,\left(d-c^2\,d\,x^2\right)\,dx$$

input
```
int(x*(a + b*acos(c*x))^2*(d - c^2*d*x^2),x)
```

output
```
int(x*(a + b*acos(c*x))^2*(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int x(d-c^2dx^2)\,(a+b\arccos(cx))^2\,dx$$
$$=\frac{d\left(8acos(cx)^2\,b^2c^2x^2-4acos(cx)^2\,b^2-8\sqrt{-c^2x^2+1}\,acos(cx)\,b^2cx-8acos(cx)\,ab\,c^4x^4+16acos(cx)\,ab\,c\right.}{}$$

input
```
int(x*(-c^2*d*x^2+d)*(a+b*acos(c*x))^2,x)
```

output
```
(d*(8*acos(c*x)**2*b**2*c**2*x**2 - 4*acos(c*x)**2*b**2 - 8*sqrt( - c**2*x
**2 + 1)*acos(c*x)*b**2*c*x - 8*acos(c*x)*a*b*c**4*x**4 + 16*acos(c*x)*a*b
*c**2*x**2 + 5*asin(c*x)*a*b + 2*sqrt( - c**2*x**2 + 1)*a*b*c**3*x**3 - 5*
sqrt( - c**2*x**2 + 1)*a*b*c*x - 16*int(acos(c*x)**2*x**3,x)*b**2*c**4 - 4
*a**2*c**4*x**4 + 8*a**2*c**2*x**2 - 4*b**2*c**2*x**2))/(16*c**2)
```

3.162 $\int (d-c^2dx^2) (a+b\arccos(cx))^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\begin{aligned}\int (d-c^2dx^2) (a+b\arccos(cx))^2 \, dx = & -\frac{14}{9}b^2dx+\frac{2}{27}b^2c^2dx^3 \\ & +\frac{4bd\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c} \\ & +\frac{2bd(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{9c} \\ & +\frac{2}{3}dx(a+b\arccos(cx))^2 \\ & +\frac{1}{3}dx(1-c^2x^2)(a+b\arccos(cx))^2\end{aligned}$$

output
```
-14/9*b^2*d*x+2/27*b^2*c^2*d*x^3+4/3*b*d*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*
x))/c+2/9*b*d*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/c+2/3*d*x*(a+b*arccos(c
*x))^2+1/3*d*x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.08

$$\int \left(d-c^2dx^2\right)(a+b\arccos(cx))^2\,dx$$
$$=\frac{d\left(2b^2cx(-21+c^2x^2)+6ab\sqrt{1-c^2x^2}(-7+c^2x^2)-9a^2cx(-3+c^2x^2)+6b\left(b\sqrt{1-c^2x^2}(-7+c^2x^2)+\right.\right.}{27c}$$

input
```
Integrate[(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*(2*b^2*c*x*(-21 + c^2*x^2) + 6*a*b*Sqrt[1 - c^2*x^2]*(-7 + c^2*x^2) - 9
*a^2*c*x*(-3 + c^2*x^2) + 6*b*(b*Sqrt[1 - c^2*x^2]*(-7 + c^2*x^2) + a*(9*c
*x - 3*c^3*x^3))*ArcCos[c*x] - 9*b^2*c*x*(-3 + c^2*x^2)*ArcCos[c*x]^2))/(2
7*c)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5159, 5131, 5183, 24, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{ 5159}$$

$$\frac{2}{3}bcd\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{2}{3}d\int (a+b\arccos(cx))^2dx+\frac{1}{3}dx\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{ 5131}$$

$$\frac{2}{3}d\left(2bc\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bcd\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{3}dx\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 5183$$

$$\frac{2}{3}d\left(2bc\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bcd\left(-\frac{b\int \left(1-c^2x^2\right)dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}dx\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 24$$

$$\frac{2}{3}bcd\left(-\frac{b\int \left(1-c^2x^2\right)dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}dx\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}d\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}dx\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}d\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bcd\left(-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^2}-\frac{b\left(x-\frac{c^2x^3}{3}\right)}{3c}\right)$$

input
```
Int[(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*x*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/3 + (2*b*c*d*(-1/3*(b*(x - (c^2*
x^3)/3))/c - ((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(3*c^2)))/3 + (2*d*
(x*(a + b*ArcCos[c*x])^2 + 2*b*c*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*A
rcCos[c*x]))/c^2)))/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{-d\,a^2\left(\frac{1}{3}c^3x^3-cx\right)-d\,b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-3\right)cx}{3}+\frac{4cx}{3}+\frac{4\arccos(cx)\sqrt{-c^2x^2+1}}{3}-\frac{2\arccos(cx)\left(c^2x^2-1\right)\sqrt{-c^2x^2+1}}{9}-\frac{2\left(c^2\right.}{}\right.}{c}$
default	$\frac{-d\,a^2\left(\frac{1}{3}c^3x^3-cx\right)-d\,b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-3\right)cx}{3}+\frac{4cx}{3}+\frac{4\arccos(cx)\sqrt{-c^2x^2+1}}{3}-\frac{2\arccos(cx)\left(c^2x^2-1\right)\sqrt{-c^2x^2+1}}{9}-\frac{2\left(c^2\right.}{}\right.}{c}$
parts	$-d\,a^2\left(\frac{1}{3}c^2x^3-x\right)-\frac{d\,b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-3\right)cx}{3}+\frac{4cx}{3}+\frac{4\arccos(cx)\sqrt{-c^2x^2+1}}{3}-\frac{2\arccos(cx)\left(c^2x^2-1\right)\sqrt{-c^2x^2+}}{9}\right.}{c}$
orering	$\frac{x(19c^4x^4-166c^2x^2+27)\left(-c^2d\,x^2+d\right)(a+b\arccos(cx))^2}{27(c^2x^2-1)^2}-\frac{(2c^4x^4-29c^2x^2+7)\left(-2c^2dx(a+b\arccos(cx))^2-\frac{2\left(-c^2d\right.}{}\right.}{9c^2(c^2x^2-1)}$

input
```
int((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-d*a^2*(1/3*c^3*x^3-c*x)-d*b^2*(1/3*arccos(c*x)^2*(c^2*x^2-3)*c*x+4/3
*c*x+4/3*arccos(c*x)*(-c^2*x^2+1)^(1/2)-2/9*arccos(c*x)*(c^2*x^2-1)*(-c^2*
x^2+1)^(1/2)-2/27*(c^2*x^2-3)*c*x)-2*d*a*b*(1/3*c^3*x^3*arccos(c*x)-c*x*ar
ccos(c*x)-1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)+7/9*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.14

$$\int \left(d-c^2dx^2\right)(a+b\arccos(cx))^2\,dx = -\frac{(9\,a^2-2\,b^2)c^3dx^3-3\,(9\,a^2-14\,b^2)cdx+9\,(b^2c^3dx^3-3\,b^2cdx)\arccos{(cx)}^2+18\,(abc^3dx^3-3\,abcdx)\cdots}{27\,c}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
-1/27*((9*a^2 - 2*b^2)*c^3*d*x^3 - 3*(9*a^2 - 14*b^2)*c*d*x + 9*(b^2*c^3*d
*x^3 - 3*b^2*c*d*x)*arccos(c*x)^2 + 18*(a*b*c^3*d*x^3 - 3*a*b*c*d*x)*arcco
s(c*x) - 6*(a*b*c^2*d*x^2 - 7*a*b*d + (b^2*c^2*d*x^2 - 7*b^2*d)*arccos(c*x
))*sqrt(-c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.80

$$\int (d - c^2dx^2) (a + b\arccos(cx))^2 \, dx$$
$$= \begin{cases} -\frac{a^2c^2dx^3}{3} + a^2dx - \frac{2abc^2dx^3\operatorname{acos}(cx)}{3} + \frac{2abcdx^2\sqrt{-c^2x^2+1}}{9} + 2abdx\operatorname{acos}(cx) - \frac{14abd\sqrt{-c^2x^2+1}}{9c} - \frac{b^2c^2dx^3\operatorname{acos}^2(cx)}{3} \\ dx\left(a + \frac{\pi b}{2}\right)^2 \end{cases}$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((-a**2*c**2*d*x**3/3 + a**2*d*x - 2*a*b*c**2*d*x**3*acos(c*x)/3
+ 2*a*b*c*d*x**2*sqrt(-c**2*x**2 + 1)/9 + 2*a*b*d*x*acos(c*x) - 14*a*b*d*s
qrt(-c**2*x**2 + 1)/(9*c) - b**2*c**2*d*x**3*acos(c*x)**2/3 + 2*b**2*c**2*
d*x**3/27 + 2*b**2*c*d*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/9 + b**2*d*x*ac
os(c*x)**2 - 14*b**2*d*x/9 - 14*b**2*d*sqrt(-c**2*x**2 + 1)*acos(c*x)/(9*c
), Ne(c, 0)), (d*x*(a + pi*b/2)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(111) = 222$.

Time = 0.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.83

$$\int (d - c^2dx^2) (a + b\arccos(cx))^2 \, dx$$
$$= -\frac{1}{3}b^2c^2dx^3\arccos(cx)^2 - \frac{1}{3}a^2c^2dx^3$$
$$- \frac{2}{9}\left(3x^3\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)abc^2d$$
$$+ \frac{2}{27}\left(3c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\arccos(cx) + \frac{c^2x^3+6x}{c^2}\right)b^2c^2d$$
$$+ b^2dx\arccos(cx)^2 - 2b^2d\left(x + \frac{\sqrt{-c^2x^2+1}\arccos(cx)}{c}\right)$$
$$+ a^2dx + \frac{2\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)abd}{c}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-1/3*b^2*c^2*d*x^3*arccos(c*x)^2 - 1/3*a^2*c^2*d*x^3 - 2/9*(3*x^3*arccos(c
*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*c^2*d
 + 2/27*(3*c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arcco
s(c*x) + (c^2*x^3 + 6*x)/c^2)*b^2*c^2*d + b^2*d*x*arccos(c*x)^2 - 2*b^2*d*
(x + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + a^2*d*x + 2*(c*x*arccos(c*x) - sq
rt(-c^2*x^2 + 1))*a*b*d/c
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.45

$$\int (d-c^2dx^2)(a+b\arccos(cx))^2\,dx = -\frac{1}{3}b^2c^2dx^3\arccos(cx)^2-\frac{2}{3}abc^2dx^3\arccos(cx) -\frac{1}{3}a^2c^2dx^3+\frac{2}{27}b^2c^2dx^3 +\frac{2}{9}\sqrt{-c^2x^2+1}b^2cdx^2\arccos(cx) +\frac{2}{9}\sqrt{-c^2x^2+1}abcdx^2+b^2dx\arccos(cx)^2 +2abdx\arccos(cx)+a^2dx-\frac{14}{9}b^2dx -\frac{14\sqrt{-c^2x^2+1}b^2d\arccos(cx)}{9c} -\frac{14\sqrt{-c^2x^2+1}abd}{9c}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-1/3*b^2*c^2*d*x^3*arccos(c*x)^2 - 2/3*a*b*c^2*d*x^3*arccos(c*x) - 1/3*a^2
*c^2*d*x^3 + 2/27*b^2*c^2*d*x^3 + 2/9*sqrt(-c^2*x^2 + 1)*b^2*c*d*x^2*arcco
s(c*x) + 2/9*sqrt(-c^2*x^2 + 1)*a*b*c*d*x^2 + b^2*d*x*arccos(c*x)^2 + 2*a*
b*d*x*arccos(c*x) + a^2*d*x - 14/9*b^2*d*x - 14/9*sqrt(-c^2*x^2 + 1)*b^2*d
*arccos(c*x)/c - 14/9*sqrt(-c^2*x^2 + 1)*a*b*d/c
```

Mupad [F(-1)]

Timed out.

$$\int (d - c^2dx^2) (a + b \arccos(cx))^2 dx = \int (a + b \operatorname{acos}(c x))^2 (d - c^2 d x^2) dx$$

input
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2),x)
```

output
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int (d - c^2dx^2) (a + b \arccos(cx))^2 dx = \frac{d(9acos(cx)^2 b^2cx - 18\sqrt{-c^2x^2 + 1}\, acos(cx)\, b^2 - 6acos(cx)\, ab\, c^3x^3 + 18acos(cx)\, abcx + 2\sqrt{-c^2x^2 + 1}\, a}{9c}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acos(c*x))^2,x)
```

output
```
(d*(9*acos(c*x)**2*b**2*c*x - 18*sqrt( - c**2*x**2 + 1)*acos(c*x)*b**2 - 6
*acos(c*x)*a*b*c**3*x**3 + 18*acos(c*x)*a*b*c*x + 2*sqrt( - c**2*x**2 + 1)
*a*b*c**2*x**2 - 14*sqrt( - c**2*x**2 + 1)*a*b - 9*int(acos(c*x)**2*x**2,x
)*b**2*c**3 - 3*a**2*c**3*x**3 + 9*a**2*c*x - 18*b**2*c*x))/(9*c)
```

3.163 $\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x} dx$

Optimal result

Integrand size = 25, antiderivative size = 178

$$\begin{aligned}\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x} dx &= \frac{1}{4}b^2c^2dx^2 - \frac{1}{2}bcdx\sqrt{1-c^2x^2}(a+b\arccos(cx)) \\ &- \frac{1}{4}d(a+b\arccos(cx))^2 \\ &+ \frac{1}{2}d(1-c^2x^2)(a+b\arccos(cx))^2 \\ &- \frac{id(a+b\arccos(cx))^3}{3b} \\ &+ d(a+b\arccos(cx))^2\log\left(1-e^{2i\arccos(cx)}\right) \\ &- ibd(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right) \\ &+ \frac{1}{2}b^2d\operatorname{PolyLog}\left(3,e^{2i\arccos(cx)}\right)\end{aligned}$$

output
```
1/4*b^2*c^2*d*x^2-1/2*b*c*d*x*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))-1/4*d*(
a+b*arccos(c*x))^2+1/2*d*(-c^2*x^2+1)*(a+b*arccos(c*x))^2-1/3*I*d*(a+b*arc
cos(c*x))^3/b+d*(a+b*arccos(c*x))^2*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-I*b
*d*(a+b*arccos(c*x))*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/2*b^2*d*pol
ylog(3,(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.38

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x}\,dx = -\frac{1}{24}d\Bigg(12a^2c^2x^2 - 12abcx\sqrt{1-c^2x^2} + 24abc^2x^2\arccos(cx) + 24iab\arccos(cx)^2 + 8ib^2\arccos(cx)^3 + 24ab\arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right) - 3b^2\cos(2\arccos(cx)) + 6b^2\arccos(cx)^2\cos(2\arccos(cx)) - 48ab\arccos(cx)\log\left(1+e^{2i\arccos(cx)}\right) - 24b^2\arccos(cx)^2\log\left(1+e^{2i\arccos(cx)}\right) - 24a^2\log(x) + 24ib(a + b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right) - 12b^2\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right) - 6b^2\arccos(cx)\sin(2\arccos(cx))\Bigg)$$

input
```
Integrate[((d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
-1/24*(d*(12*a^2*c^2*x^2 - 12*a*b*c*x*Sqrt[1 - c^2*x^2] + 24*a*b*c^2*x^2*A
rcCos[c*x] + (24*I)*a*b*ArcCos[c*x]^2 + (8*I)*b^2*ArcCos[c*x]^3 + 24*a*b*A
rcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])] - 3*b^2*Cos[2*ArcCos[c*x]] + 6*b^2*A
rcCos[c*x]^2*Cos[2*ArcCos[c*x]] - 48*a*b*ArcCos[c*x]*Log[1 + E^((2*I)*ArcC
os[c*x])] - 24*b^2*ArcCos[c*x]^2*Log[1 + E^((2*I)*ArcCos[c*x])] - 24*a^2*L
og[x] + (24*I)*b*(a + b*ArcCos[c*x])*PolyLog[2, -E^((2*I)*ArcCos[c*x])] -
12*b^2*PolyLog[3, -E^((2*I)*ArcCos[c*x])] - 6*b^2*ArcCos[c*x]*Sin[2*ArcCos
[c*x]]))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5203, 5137, 3042, 4202, 2620, 3011, 2720, 5157, 15, 5153, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x}dx$$

$$\Big\downarrow \text{5203}$$

$$bcd\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+d\int \frac{(a+b\arccos(cx))^2}{x}dx+\frac{1}{2}d(1-c^2x^2)(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5137}$$

$$bcd\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-d\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{cx}d\arccos(cx)+\frac{1}{2}d(1-c^2x^2)(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{3042}$$

$$bcd\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-d\int (a+b\arccos(cx))^2\tan(\arccos(cx))d\arccos(cx)+\frac{1}{2}d(1-c^2x^2)(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{4202}$$

$$bcd\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))^2}{1+e^{2i\arccos(cx)}}d\arccos(cx)\right)+\frac{1}{2}d(1-c^2x^2)(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{2620}$$

$$bcd\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\left(ib\int (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)\right.\right.$$
$$\frac{1}{2}d(1-c^2x^2)(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{3011}$$

$$bcd\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-$$
$$d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int \text{PolyLog}\left(2,-e^{2i\arccos\right.\right.\right.\right.$$
$$\frac{1}{2}d(1-c^2x^2)\,(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{2720}$$

$$bcd\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-$$
$$d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\,\text{PolyLog}\right.\right.\right.$$
$$\frac{1}{2}d(1-c^2x^2)\,(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5157}$$

$$bcd\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)-$$
$$d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\,\text{PolyLog}\right.\right.\right.$$
$$\frac{1}{2}d(1-c^2x^2)\,(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{15}$$

$$bcd\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{4}bcx^2\right)-$$
$$d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\,\text{PolyLog}\right.\right.\right.$$
$$\frac{1}{2}d(1-c^2x^2)\,(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5153}$$

$$-d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\,\text{PolyLo}\right.\right.\right.$$
$$\frac{1}{2}d(1-c^2x^2)\,(a+b\arccos(cx))^2+$$
$$bcd\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}d(1-c^2x^2)\left(a+b\arccos(cx)\right)^2+$$
$$bcd\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)-$$
$$d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\left(ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right.\right.\right.\right.$$

input
```
Int[((d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
(d*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/2 + b*c*d*((b*c*x^2)/4 + (x*Sqrt[1
 - c^2*x^2]*(a + b*ArcCos[c*x]))/2 - (a + b*ArcCos[c*x])^2/(4*b*c)) - d*((
(I/3)*(a + b*ArcCos[c*x])^3)/b - (2*I)*((-1/2*I)*(a + b*ArcCos[c*x])^2*Log
[1 + E^((2*I)*ArcCos[c*x])] + I*b*((I/2)*(a + b*ArcCos[c*x])*PolyLog[2, -E
^((2*I)*ArcCos[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/4)))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.84

method	result
parts	$-d\,a^2\left(\frac{c^2x^2}{2}-\ln\left(x\right)\right)-idab\,\mathrm{polylog}\left(2,-\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)+\frac{cdx\,b^2\arccos(cx)\sqrt{-c^2}}{2}$
derivativedivides	$-d\,a^2\left(\frac{c^2x^2}{2}-\ln\left(cx\right)\right)-idab\,\mathrm{polylog}\left(2,-\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)+\frac{cdx\,b^2\arccos(cx)\sqrt{-c}}{2}$
default	$-d\,a^2\left(\frac{c^2x^2}{2}-\ln\left(cx\right)\right)-idab\,\mathrm{polylog}\left(2,-\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)+\frac{cdx\,b^2\arccos(cx)\sqrt{-c}}{2}$

input
```
int((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
-d*a^2*(1/2*c^2*x^2-ln(x))-I*d*a*b*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2
)+1/2*c*d*x*b^2*arccos(c*x)*(-c^2*x^2+1)^(1/2)-1/2*d*b^2*arccos(c*x)^2*c^2
*x^2+1/4*b^2*c^2*d*x^2+1/4*d*b^2*arccos(c*x)^2-1/8*d*b^2+d*b^2*arccos(c*x)
^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1/3*I*d*b^2*arccos(c*x)^3+1/2*d*b^2*
polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-I*d*a*b*arccos(c*x)^2+1/2*a*b*c*d
*x*(-c^2*x^2+1)^(1/2)-arccos(c*x)*a*b*c^2*d*x^2+1/2*arccos(c*x)*a*b*d+2*d*
a*b*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-I*d*b^2*arccos(c*x)*pol
ylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x}\,dx=\int -\frac{(c^2dx^2-d)(b\arccos(cx)+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccos(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccos(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x}\,dx=-d\left(\int\left(-\frac{a^2}{x}\right)dx+\int a^2c^2x\,dx+\int\left(-\frac{b^2\operatorname{acos}^2(cx)}{x}\right)dx+\int\left(-\frac{2ab\operatorname{acos}(cx)}{x}\right)dx+\int b^2c^2x\operatorname{acos}^2(cx)\,dx+\int 2abc^2x\operatorname{acos}(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acos(c*x))**2/x,x)
```

output
```
-d*(Integral(-a**2/x, x) + Integral(a**2*c**2*x, x) + Integral(-b**2*acos(
c*x)**2/x, x) + Integral(-2*a*b*acos(c*x)/x, x) + Integral(b**2*c**2*x*aco
s(c*x)**2, x) + Integral(2*a*b*c**2*x*acos(c*x), x))
```

Maxima [F]

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))^2}{x} dx = \int -\frac{(c^2dx^2 - d)(b\arccos(cx) + a)^2}{x} dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x,x, algorithm="maxima")
```

output
```
-1/2*a^2*c^2*d*x^2 + a^2*d*log(x) - integrate(((b^2*c^2*d*x^2 - b^2*d)*arc
tan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arct
an2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))^2}{x} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))^2}{x} dx = \int \frac{(a + b\,\mathrm{acos}(c\,x))^2\,(d - c^2\,d\,x^2)}{x} dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2))/x,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2))/x, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x}\,dx$$

$$= \frac{d\Big(-2acos(cx)^2 b^2c^2x^2 + acos(cx)^2 b^2 + 2\sqrt{-c^2x^2+1}\,acos(cx)\,b^2cx - 4acos(cx)\,ab\,c^2x^2 - 2asin(cx)\,ab -}{4}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acos(c*x))^2/x,x)
```

output
```
(d*( - 2*acos(c*x)**2*b**2*c**2*x**2 + acos(c*x)**2*b**2 + 2*sqrt( - c**2*
x**2 + 1)*acos(c*x)*b**2*c*x - 4*acos(c*x)*a*b*c**2*x**2 - 2*asin(c*x)*a*b
 + 2*sqrt( - c**2*x**2 + 1)*a*b*c*x + 8*int(acos(c*x)/x,x)*a*b + 4*int(aco
s(c*x)**2/x,x)*b**2 + 4*log(x)*a**2 - 2*a**2*c**2*x**2 + b**2*c**2*x**2))/
4
```

3.164 $\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 149

$$\begin{aligned}\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^2}\,dx = {} & 2b^2c^2dx-2bcd\sqrt{1-c^2x^2}(a+b\arccos(cx)) \\ & -2c^2dx(a+b\arccos(cx))^2 \\ & -\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{x} \\ & -4bcd(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right) \\ & +2ib^2cd\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right) \\ & -2ib^2cd\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)\end{aligned}$$

output
```
2*b^2*c^2*d*x-2*b*c*d*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))-2*c^2*d*x*(a+b*
arccos(c*x))^2-d*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/x-4*b*c*d*(a+b*arccos(c*
x))*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))+2*I*b^2*c*d*polylog(2,-c*x-I*(-c^2*x
^2+1)^(1/2))-2*I*b^2*c*d*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.44

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^2}\,dx =$$
$$-\frac{d\left(a^2+a^2c^2x^2+2abcx\left(-\sqrt{1-c^2x^2}+cx\arccos(cx)\right)+b^2cx\left(-2cx-2\sqrt{1-c^2x^2}\arccos(cx)+cx\,\mathrm{arc}\right.\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2)/x^2,x]
```

output
```
-((d*(a^2 + a^2*c^2*x^2 + 2*a*b*c*x*(-Sqrt[1 - c^2*x^2] + c*x*ArcCos[c*x])
 + b^2*c*x*(-2*c*x - 2*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + c*x*ArcCos[c*x]^2)
+ 2*a*b*(ArcCos[c*x] - c*x*ArcTanh[Sqrt[1 - c^2*x^2]]) + b^2*(ArcCos[c*x]^
2 - 2*c*x*(ArcCos[c*x]*(Log[1 - I*E^(I*ArcCos[c*x])] - Log[1 + I*E^(I*ArcC
os[c*x])]) + I*(PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - PolyLog[2, I*E^(I*Arc
Cos[c*x])])))))/x)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5201, 5131, 5183, 24, 5199, 24, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^2}dx$$

$$\Big\downarrow \text{5201}$$

$$-2bcd\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx-2c^2d\int (a+b\arccos(cx))^2dx-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{5131}$$

$$-2c^2d\left(2bc\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^2\right)-2bcd\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{5183}$$

$$-2c^2d\left(2bc\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)+x(a+b\arccos(cx))^2\right)-2bcd\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{24}$$

$$-2bcd\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx-2c^2d\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2\right)-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{5199}$$

$$-2bcd\left(\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+bc\int 1dx+\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)-2c^2d\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2\right)-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{24}$$

$$-2bcd\left(\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+\sqrt{1-c^2x^2}(a+b\arccos(cx))+bcx\right)-2c^2d\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2\right)-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{5219}$$

$$-2bcd\left(-\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)+\sqrt{1-c^2x^2}(a+b\arccos(cx))+bcx\right)-2c^2d\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2\right)-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{3042}$$

$$-2bcd\left(-\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)+\sqrt{1-c^2x^2}(a+b\arccos(cx))+bcx\right)-2c^2d\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2\right)-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{4669}$$

$$-2bcd\left(b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)-b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)+2i\arctan\left(e^{i\arccos(cx)}\right)\right.$$
$$2c^2d\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2\right)-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{2715}$$

$$-2bcd\left(-ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}+ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(cx}\right.$$
$$2c^2d\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2\right)-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{2838}$$

$$-2bcd\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\arccos(cx))-ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)\right.$$
$$2c^2d\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2\right)-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{x}$$

input
```
Int[((d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2)/x^2,x]
```

output
```
-((d*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/x) - 2*c^2*d*(x*(a + b*ArcCos[c*
x])^2 + 2*b*c*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c^2))
- 2*b*c*d*(b*c*x + Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]) + (2*I)*(a + b*Ar
cCos[c*x])*ArcTan[E^(I*ArcCos[c*x])] - I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x
])] + I*b*PolyLog[2, I*E^(I*ArcCos[c*x])])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.81

method	result
derivativedivides	$c\left(-d\,a^2\left(cx+\frac{1}{cx}\right)+2d\,b^2\arccos\left(cx\right)\sqrt{-c^2x^2+1}-d\,b^2\arccos\left(cx\right)^2cx+2d\,b^2cx-\frac{d}{}\right.$
default	$c\left(-d\,a^2\left(cx+\frac{1}{cx}\right)+2d\,b^2\arccos\left(cx\right)\sqrt{-c^2x^2+1}-d\,b^2\arccos\left(cx\right)^2cx+2d\,b^2cx-\frac{d}{}\right.$
parts	$-d\,a^2\left(c^2x+\frac{1}{x}\right)+2cd\,b^2\arccos\left(cx\right)\sqrt{-c^2x^2+1}-d\,b^2\arccos\left(cx\right)^2c^2x+2b^2c^2dx-\frac{c}{}$

input
```
int((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
c*(-d*a^2*(c*x+1/c/x)+2*d*b^2*arccos(c*x)*(-c^2*x^2+1)^(1/2)-d*b^2*arccos(
c*x)^2*c*x+2*d*b^2*c*x-d*b^2*arccos(c*x)^2/c/x-2*d*b^2*arccos(c*x)*ln(1+I*
(c*x+I*(-c^2*x^2+1)^(1/2)))+2*d*b^2*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)
^(1/2)))+2*I*d*b^2*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*I*d*b^2*dilog(1
-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*d*a*b*(c*x*arccos(c*x)+arccos(c*x)/c/x-ar
ctanh(1/(-c^2*x^2+1)^(1/2))-(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int\frac{(d-c^2dx^2)\,(a+b\arccos(cx))^2}{x^2}\,dx=\int-\frac{(c^2dx^2-d)(b\arccos\left(cx\right)+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccos(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccos(c*x))/x^2, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^2}\,dx = -d\left(\int a^2c^2\,dx + \int\left(-\frac{a^2}{x^2}\right)dx + \int b^2c^2\operatorname{acos}^2(cx)\,dx + \int\left(-\frac{b^2\operatorname{acos}^2(cx)}{x^2}\right)dx + \int 2abc^2\operatorname{acos}(cx)\,dx + \int\left(-\frac{2ab\operatorname{acos}(cx)}{x^2}\right)dx\right)$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acos(c*x))**2/x**2,x)
```

output
```
-d*(Integral(a**2*c**2, x) + Integral(-a**2/x**2, x) + Integral(b**2*c**2*
acos(c*x)**2, x) + Integral(-b**2*acos(c*x)**2/x**2, x) + Integral(2*a*b*c
**2*acos(c*x), x) + Integral(-2*a*b*acos(c*x)/x**2, x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^2}\,dx = \int -\frac{(c^2dx^2-d)(b\arccos(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
-b^2*c^2*d*x*arccos(c*x)^2 + 2*b^2*c^2*d*(x + sqrt(-c^2*x^2 + 1)*arccos(c*
x)/c) - a^2*c^2*d*x - 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a*b*c*d + 2
*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*a*b*d + (
2*c*x*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c
*x + 1), c*x)/(c^2*x^3 - x), x) - arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*
x)^2)*b^2*d/x - a^2*d/x
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2))/x^2,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2))/x^2, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^2}\,dx$$
$$= \frac{d\Big(-acos(cx)^2\,b^2c^2x^2+2\sqrt{-c^2x^2+1}\,acos(cx)\,b^2cx-2acos(cx)\,ab\,c^2x^2-2acos(cx)\,ab+2\sqrt{-c^2x^2+1}\,a}{x}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acos(c*x))^2/x^2,x)
```

output
```
(d*( - acos(c*x)**2*b**2*c**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*b*
*2*c*x - 2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b + 2*sqrt( - c**2*x**2
 + 1)*a*b*c*x + int(acos(c*x)**2/x**2,x)*b**2*x - 2*log(tan(asin(c*x)/2))*
a*b*c*x - a**2*c**2*x**2 - a**2 + 2*b**2*c**2*x**2))/x
```

3.165 $\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^3}\,dx = -\frac{bcd\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x} - \frac{1}{2}c^2d(a+b\arccos(cx))^2 - \frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{2x^2} + \frac{ic^2d(a+b\arccos(cx))^3}{3b} - c^2d(a+b\arccos(cx))^2\log\left(1-e^{2i\arccos(cx)}\right) + b^2c^2d\log(x) + ibc^2d(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right) - \frac{1}{2}b^2c^2d\operatorname{PolyLog}\left(3,e^{2i\arccos(cx)}\right)$$

output
```
-b*c*d*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/x-1/2*c^2*d*(a+b*arccos(c*x))^
2-1/2*d*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/x^2+1/3*I*c^2*d*(a+b*arccos(c*x))
^3/b-c^2*d*(a+b*arccos(c*x))^2*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+b^2*c^2*
d*ln(x)+I*b*c^2*d*(a+b*arccos(c*x))*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2
)-1/2*b^2*c^2*d*polylog(3,(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.34

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^3}\,dx$$
$$= \frac{d\left(-3a^2+6abcx\sqrt{1-c^2x^2}-6ab\arccos(cx)+6b^2cx\sqrt{1-c^2x^2}\arccos(cx)-3b^2\arccos(cx)^2+6iabc^2x^2\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2)/x^3,x]
```

output
```
(d*(-3*a^2 + 6*a*b*c*x*Sqrt[1 - c^2*x^2] - 6*a*b*ArcCos[c*x] + 6*b^2*c*x*S
qrt[1 - c^2*x^2]*ArcCos[c*x] - 3*b^2*ArcCos[c*x]^2 + (6*I)*a*b*c^2*x^2*Arc
Cos[c*x]^2 + (2*I)*b^2*c^2*x^2*ArcCos[c*x]^3 - 12*a*b*c^2*x^2*ArcCos[c*x]*
Log[1 + E^((2*I)*ArcCos[c*x])] - 6*b^2*c^2*x^2*ArcCos[c*x]^2*Log[1 + E^((2
*I)*ArcCos[c*x])] - 6*a^2*c^2*x^2*Log[x] + 6*b^2*c^2*x^2*Log[c*x] + (6*I)*
b*c^2*x^2*(a + b*ArcCos[c*x])*PolyLog[2, -E^((2*I)*ArcCos[c*x])] - 3*b^2*c
^2*x^2*PolyLog[3, -E^((2*I)*ArcCos[c*x])]))/(6*x^2)
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5201, 5137, 3042, 4202, 2620, 3011, 2720, 5197, 14, 5153, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^3}\,dx$$

$$\Big\downarrow \text{5201}$$

$$-bcd\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x^2}dx+c^2(-d)\int \frac{(a+b\arccos(cx))^2}{x}dx-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{5137}$$

$$c^2d\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{cx}d\arccos(cx)-bcd\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x^2}dx-\frac{d\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$-bcd\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x^2}dx+c^2d\int (a+b\arccos(cx))^2\tan(\arccos(cx))d\arccos(cx)-\frac{d\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{4202}$$

$$-bcd\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x^2}dx+c^2d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))^2}{1+e^{2i\arccos(cx)}}d\arccos(cx)\right)-\frac{d\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{2620}$$

$$-bcd\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x^2}dx+c^2d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\left(ib\int (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right.\right.\right.$$
$$\frac{d\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{3011}$$

$$c^2d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int \text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right.\right.\right.\right.$$
$$bcd\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x^2}dx-\frac{d\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{2720}$$

$$c^2d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\,\text{PolyLo}\right.\right.\right.$$
$$bcd\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x^2}dx-\frac{d\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{5197}$$

$$c^2d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\left(ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\operatorname{PolyLo}\right.\right.\right.$$
$$bcd\left(c^2\left(-\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx\right)-bc\int\frac{1}{x}dx-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}\right)-$$
$$\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{2x^2}$$

$\downarrow$ 14

$$c^2d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\left(ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\operatorname{PolyLo}\right.\right.\right.$$
$$bcd\left(c^2\left(-\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx\right)-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}-bc\log(x)\right)-$$
$$\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{2x^2}$$

$\downarrow$ 5153

$$c^2d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\left(ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\operatorname{PolyLo}\right.\right.\right.$$
$$\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}+\frac{c(a+b\arccos(cx))^2}{2b}-bc\log(x)\right)$$

$\downarrow$ 7143

$$c^2d\left(\frac{i(a+b\arccos(cx))^3}{3b}-2i\left(ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\operatorname{PolyLog}\left(3,-e^{2i\arccos(}\right.\right.\right.\right.$$
$$\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}+\frac{c(a+b\arccos(cx))^2}{2b}-bc\log(x)\right)$$

input
```
Int[((d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2)/x^3,x]
```

output
```
-1/2*(d*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/x^2 - b*c*d*(-((Sqrt[1 - c^2*
x^2]*(a + b*ArcCos[c*x]))/x) + (c*(a + b*ArcCos[c*x])^2)/(2*b) - b*c*Log[x
]) + c^2*d*(((I/3)*(a + b*ArcCos[c*x])^3)/b - (2*I)*((-1/2*I)*(a + b*ArcCo
s[c*x])^2*Log[1 + E^((2*I)*ArcCos[c*x])] + I*b*((I/2)*(a + b*ArcCos[c*x])*
PolyLog[2, -E^((2*I)*ArcCos[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcCos[c*x])]
)/4)))
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5197
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^2
]/Sqrt[1 - c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x
] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 2)*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.72

method	result
derivativedivides	$c^{2}\left(-d\,a^{2}\left(\frac{1}{2c^{2}x^{2}}+\ln\left(cx\right)\right)-d\,b^{2}\left(-\frac{i\arccos(cx)^{3}}{3}+\frac{\arccos(cx)\left(-2ic^{2}x^{2}-2cx\sqrt{-c^{2}x^{2}+1}+\arccos(cx)\right)}{2c^{2}x^{2}}\right.\right.$
default	$c^{2}\left(-d\,a^{2}\left(\frac{1}{2c^{2}x^{2}}+\ln\left(cx\right)\right)-d\,b^{2}\left(-\frac{i\arccos(cx)^{3}}{3}+\frac{\arccos(cx)\left(-2ic^{2}x^{2}-2cx\sqrt{-c^{2}x^{2}+1}+\arccos(cx)\right)}{2c^{2}x^{2}}\right.\right.$
parts	$-d\,a^{2}c^{2}\ln\left(x\right)-\frac{d\,a^{2}}{2x^{2}}-d\,b^{2}c^{2}\left(-\frac{i\arccos(cx)^{3}}{3}+\frac{\arccos(cx)\left(-2ic^{2}x^{2}-2cx\sqrt{-c^{2}x^{2}+1}+\arccos(cx)\right)}{2c^{2}x^{2}}+\right.$

input
```
int((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-d*a^2*(1/2/c^2/x^2+ln(c*x))-d*b^2*(-1/3*I*arccos(c*x)^3+1/2*arccos(c
*x)*(-2*I*c^2*x^2-2*c*x*(-c^2*x^2+1)^(1/2)+arccos(c*x))/c^2/x^2+arccos(c*x
)^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-I*arccos(c*x)*polylog(2,-(c*x+I*(-c
^2*x^2+1)^(1/2))^2)+1/2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-ln(1+(c*x
+I*(-c^2*x^2+1)^(1/2))^2)+2*ln(c*x+I*(-c^2*x^2+1)^(1/2)))-2*d*a*b*(-1/2*I*
arccos(c*x)^2+1/2*(-I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)+arccos(c*x))/c^2/x^2+
arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1/2*I*polylog(2,-(c*x+I*(-c
^2*x^2+1)^(1/2))^2)))
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)\,(a+b\arccos(cx))^2}{x^3}\,dx=\int -\frac{(c^2dx^2-d)(b\arccos{(cx)}+a)^2}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccos(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccos(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)\,(a+b\arccos(cx))^2}{x^3}\,dx = -d\left(\int\left(-\frac{a^2}{x^3}\right)dx + \int\frac{a^2c^2}{x}\,dx + \int\left(-\frac{b^2\operatorname{acos}^2(cx)}{x^3}\right)dx + \int\left(-\frac{2ab\operatorname{acos}(cx)}{x^3}\right)dx + \int\frac{b^2c^2\operatorname{acos}^2(cx)}{x}\,dx + \int\frac{2abc^2\operatorname{acos}(cx)}{x}\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acos(c*x))**2/x**3,x)
```

output
```
-d*(Integral(-a**2/x**3, x) + Integral(a**2*c**2/x, x) + Integral(-b**2*ac
os(c*x)**2/x**3, x) + Integral(-2*a*b*acos(c*x)/x**3, x) + Integral(b**2*c
**2*acos(c*x)**2/x, x) + Integral(2*a*b*c**2*acos(c*x)/x, x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)\,(a+b\arccos(cx))^2}{x^3}\,dx = \int -\frac{(c^2dx^2-d)(b\arccos(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-a^2*c^2*d*log(x) + a*b*d*(sqrt(-c^2*x^2 + 1)*c/x - arccos(c*x)/x^2) - 1/2
*a^2*d/x^2 - integrate((2*a*b*c^2*d*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x) + (b^2*c^2*d*x^2 - b^2*d)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c
*x)^2)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)\,(a+b\arccos(cx))^2}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)\,(a+b\arccos(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)}{x^3}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2))/x^3,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)\,(a+b\arccos(cx))^2}{x^3}\,dx$$
$$= \frac{d\left(-2acos(cx)\,ab + 2\sqrt{-c^2x^2+1}\,abcx - 4\left(\int \frac{acos(cx)}{x}dx\right)ab\,c^2x^2 + 2\left(\int \frac{acos(cx)^2}{x^3}dx\right)b^2x^2 - 2\left(\int \frac{acos(cx)^2}{x}\right.\right.}{2x^2}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acos(c*x))^2/x^3,x)
```

output
```
(d*( - 2*acos(c*x)*a*b + 2*sqrt( - c**2*x**2 + 1)*a*b*c*x - 4*int(acos(c*x
)/x,x)*a*b*c**2*x**2 + 2*int(acos(c*x)**2/x**3,x)*b**2*x**2 - 2*int(acos(c
*x)**2/x,x)*b**2*c**2*x**2 - 2*log(x)*a**2*c**2*x**2 - a**2))/(2*x**2)
```

3.166 $\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 176

$$\begin{aligned}\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^4}\,dx = & -\frac{b^2c^2d}{3x}-\frac{bcd\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3x^2} \\ & +\frac{2c^2d(a+b\arccos(cx))^2}{3x} \\ & -\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{3x^3} \\ & +\frac{10}{3}bc^3d(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right) \\ & -\frac{5}{3}ib^2c^3d\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right) \\ & +\frac{5}{3}ib^2c^3d\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\end{aligned}$$

output

```
-1/3*b^2*c^2*d/x-1/3*b*c*d*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/x^2+2/3*c^
2*d*(a+b*arccos(c*x))^2/x-1/3*d*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/x^3+10/3*
b*c^3*d*(a+b*arccos(c*x))*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))-5/3*I*b^2*c^3*
d*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+5/3*I*b^2*c^3*d*polylog(2,c*x+I*(-c
^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.56

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^4}\,dx =$$

$$-\frac{d\left(a^2-3a^2c^2x^2+b^2c^2x^2-abcx\sqrt{1-c^2x^2}+2ab\arccos(cx)-6abc^2x^2\arccos(cx)-b^2cx\sqrt{1-c^2x^2}\,\text{ar}\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2)/x^4,x]
```

output
```
-1/3*(d*(a^2 - 3*a^2*c^2*x^2 + b^2*c^2*x^2 - a*b*c*x*Sqrt[1 - c^2*x^2] + 2
*a*b*ArcCos[c*x] - 6*a*b*c^2*x^2*ArcCos[c*x] - b^2*c*x*Sqrt[1 - c^2*x^2]*A
rcCos[c*x] + b^2*ArcCos[c*x]^2 - 3*b^2*c^2*x^2*ArcCos[c*x]^2 + 5*a*b*c^3*x
^3*ArcTanh[Sqrt[1 - c^2*x^2]] + 5*b^2*c^3*x^3*ArcCos[c*x]*Log[1 - I*E^(I*A
rcCos[c*x])] - 5*b^2*c^3*x^3*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] + (5
*I)*b^2*c^3*x^3*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - (5*I)*b^2*c^3*x^3*Pol
yLog[2, I*E^(I*ArcCos[c*x])]))/x^3
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.38, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5201, 5139, 5197, 15, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^4}\,dx$$

$$\Big\downarrow \text{ 5201}$$

$$-\frac{2}{3}c^2d\int \frac{(a+b\arccos(cx))^2}{x^2}dx-\frac{2}{3}bcd\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x^3}dx-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 5139}$$

$$-\frac{2}{3}c^2d\left(-2bc\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{(a+b\arccos(cx))^2}{x}\right)-\frac{2}{3}bcd\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x^3}dx-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 5197$$

$$-\frac{2}{3}c^2d\left(-2bc\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{(a+b\arccos(cx))^2}{x}\right)-\frac{2}{3}bcd\left(-\frac{1}{2}c^2\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{1}{2}bc\int \frac{1}{x^2}dx-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}\right)-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 15$$

$$-\frac{2}{3}c^2d\left(-2bc\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{(a+b\arccos(cx))^2}{x}\right)-\frac{2}{3}bcd\left(-\frac{1}{2}c^2\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}+\frac{bc}{2x}\right)-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 5219$$

$$-\frac{2}{3}bcd\left(\frac{1}{2}c^2\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}+\frac{bc}{2x}\right)-\frac{2}{3}c^2d\left(2bc\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)-\frac{(a+b\arccos(cx))^2}{x}\right)-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2}{3}bcd\left(\frac{1}{2}c^2\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}+\frac{bc}{2x}\right)-\frac{2}{3}c^2d\left(2bc\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)-\frac{(a+b\arccos(cx))^2}{x}\right)-\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 4669$$

$$-\frac{2}{3}bcd\left(\frac{1}{2}c^2\left(-b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(e^{i\,\mathrm{ar}}\right.\right.\right.$$
$$\frac{2}{3}c^2d\left(-\frac{(a+b\arccos(cx))^2}{x}+2bc\left(-b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos\right.\right.$$
$$\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2715$$

$$-\frac{2}{3}bcd\left(\frac{1}{2}c^2\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\,\mathrm{arcc}}\right.\right.$$
$$\frac{2}{3}c^2d\left(-\frac{(a+b\arccos(cx))^2}{x}+2bc\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1\right.\right.\right.$$
$$\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2838$$

$$-\frac{2}{3}bcd\left(\frac{1}{2}c^2\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\mathrm{PolyLog}\left(2,ie^{i\,\mathrm{arcc}}\right.\right.\right.$$
$$\frac{2}{3}c^2d\left(-\frac{(a+b\arccos(cx))^2}{x}+2bc\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-\right.\right.$$
$$\frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{3x^3}$$

input
```
Int[((d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2)/x^4,x]
```

output
```
-1/3*(d*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/x^3 - (2*c^2*d*(-((a + b*ArcC
os[c*x])^2/x) + 2*b*c*((-2*I)*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcCos[c*x])
] + I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*b*PolyLog[2, I*E^(I*ArcCos[
c*x])])))/3 - (2*b*c*d*((b*c)/(2*x) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x
]))/(2*x^2) + (c^2*((-2*I)*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcCos[c*x])] +
 I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*b*PolyLog[2, I*E^(I*ArcCos[c*x
])]))/2))/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5197
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^2
]/Sqrt[1 - c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x
] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 2)*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.57

method	result
parts	$-d\,a^{2}\left(-\frac{c^{2}}{x}+\frac{1}{3x^{3}}\right)-d\,b^{2}c^{3}\left(-\frac{3\arccos\left(cx\right)^{2}x^{2}c^{2}+\sqrt{-c^{2}x^{2}+1}\,\arccos\left(cx\right)xc-\arccos\left(cx\right)^{2}-c^{2}x^{2}}{3c^{3}x^{3}}-\frac{5\,\mathrm{arc}}{}\right.$
derivativedivides	$c^{3}\left(-d\,a^{2}\left(\frac{1}{3c^{3}x^{3}}-\frac{1}{cx}\right)-d\,b^{2}\left(-\frac{3\arccos\left(cx\right)^{2}x^{2}c^{2}+\sqrt{-c^{2}x^{2}+1}\,\arccos\left(cx\right)xc-\arccos\left(cx\right)^{2}-c^{2}x^{2}}{3c^{3}x^{3}}-\frac{5\,\mathrm{ar}}{}\right.\right.$
default	$c^{3}\left(-d\,a^{2}\left(\frac{1}{3c^{3}x^{3}}-\frac{1}{cx}\right)-d\,b^{2}\left(-\frac{3\arccos\left(cx\right)^{2}x^{2}c^{2}+\sqrt{-c^{2}x^{2}+1}\,\arccos\left(cx\right)xc-\arccos\left(cx\right)^{2}-c^{2}x^{2}}{3c^{3}x^{3}}-\frac{5\,\mathrm{ar}}{}\right.\right.$

input
```
int((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-d*a^2*(-c^2/x+1/3/x^3)-d*b^2*c^3*(-1/3*(3*arccos(c*x)^2*x^2*c^2+(-c^2*x^2
+1)^(1/2)*arccos(c*x)*x*c-arccos(c*x)^2-c^2*x^2)/c^3/x^3-5/3*arccos(c*x)*l
n(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+5/3*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+
1)^(1/2)))+5/3*I*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-5/3*I*dilog(1-I*(c*
x+I*(-c^2*x^2+1)^(1/2))))-2*d*a*b*c^3*(1/3*arccos(c*x)/c^3/x^3-arccos(c*x)
/c/x-1/6/c^2/x^2*(-c^2*x^2+1)^(1/2)+5/6*arctanh(1/(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^4}\,dx = \int -\frac{(c^2dx^2-d)(b\arccos{(cx)}+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccos(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccos(c*x))/x^4, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^4}\,dx = -d\left(\int \left(-\frac{a^2}{x^4}\right)dx + \int \frac{a^2c^2}{x^2}\,dx + \int \left(-\frac{b^2\operatorname{acos}^2{(cx)}}{x^4}\right)dx + \int \left(-\frac{2ab\operatorname{acos}{(cx)}}{x^4}\right)dx + \int \frac{b^2c^2\operatorname{acos}^2{(cx)}}{x^2}\,dx + \int \frac{2abc^2\operatorname{acos}{(cx)}}{x^2}\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)*(a+b*acos(c*x))**2/x**4,x)
```

output
```
-d*(Integral(-a**2/x**4, x) + Integral(a**2*c**2/x**2, x) + Integral(-b**2
*acos(c*x)**2/x**4, x) + Integral(-2*a*b*acos(c*x)/x**4, x) + Integral(b**
2*c**2*acos(c*x)**2/x**2, x) + Integral(2*a*b*c**2*acos(c*x)/x**2, x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^4}\,dx=\int -\frac{(c^2dx^2-d)(b\arccos(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
-2*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*a*b*c^2
*d + 1/3*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2
 + 1)/x^2)*c - 2*arccos(c*x)/x^3)*a*b*d + a^2*c^2*d/x - 1/3*a^2*d/x^3 - 1/
3*(3*x^3*integrate(2/3*(3*b^2*c^3*d*x^2 - b^2*c*d)*sqrt(c*x + 1)*sqrt(-c*x
 + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^2*x^5 - x^3), x) - (3*
b^2*c^2*d*x^2 - b^2*d)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2)/x^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)(a+b\arccos(cx))^2}{x^4}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)*(a+b*arccos(c*x))^2/x^4,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))^2}{x^4}\,dx = \int \frac{(a + b\,\mathrm{acos}(c\,x))^2\,(d - c^2\,d\,x^2)}{x^4}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2))/x^4,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2))/x^4, x)
```

Reduce [F]

$$\int \frac{(d - c^2dx^2)(a + b\arccos(cx))^2}{x^4}\,dx$$
$$= \frac{d\left(6acos(cx)\,ab\,c^2x^2 - 2acos(cx)\,ab + \sqrt{-c^2x^2+1}\,abcx + 3\left(\int \frac{acos(cx)^2}{x^4}dx\right)b^2x^3 - 3\left(\int \frac{acos(cx)^2}{x^2}dx\right)b^2c^2x\right.}{3x^3}$$

input
```
int((-c^2*d*x^2+d)*(a+b*acos(c*x))^2/x^4,x)
```

output
```
(d*(6*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a
*b*c*x + 3*int(acos(c*x)**2/x**4,x)*b**2*x**3 - 3*int(acos(c*x)**2/x**2,x)
*b**2*c**2*x**3 + 5*log(tan(asin(c*x)/2))*a*b*c**3*x**3 + 3*a**2*c**2*x**2
 - a**2))/(3*x**3)
```

3.167 $\int x^4(d-c^2dx^2)^2(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 395

$$\int x^4\left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2\,dx = -\frac{4208b^2d^2x}{99225c^4}-\frac{2104b^2d^2x^3}{297675c^2}-\frac{526b^2d^2x^5}{165375} + \frac{212b^2c^2d^2x^7}{27783}-\frac{2}{729}b^2c^4d^2x^9+\frac{128bd^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4725c^5} + \frac{64bd^2x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4725c^3}+\frac{16bd^2x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{1575c} + \frac{8bd^2\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{189c^5}-\frac{2bd^2\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{315c^5} - \frac{20bd^2\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{441c^5}+\frac{2bd^2\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{81c^5} + \frac{8}{315}d^2x^5(a+b\arccos(cx))^2+\frac{4}{63}d^2x^5\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{1}{9}d^2x^5\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

output
```
-4208/99225*b^2*d^2*x/c^4-2104/297675*b^2*d^2*x^3/c^2-526/165375*b^2*d^2*x
^5+212/27783*b^2*c^2*d^2*x^7-2/729*b^2*c^4*d^2*x^9+128/4725*b*d^2*(-c^2*x^
2+1)^(1/2)*(a+b*arccos(c*x))/c^5+64/4725*b*d^2*x^2*(-c^2*x^2+1)^(1/2)*(a+b
*arccos(c*x))/c^3+16/1575*b*d^2*x^4*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c
+8/189*b*d^2*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/c^5-2/315*b*d^2*(-c^2*x^
2+1)^(5/2)*(a+b*arccos(c*x))/c^5-20/441*b*d^2*(-c^2*x^2+1)^(7/2)*(a+b*arcc
os(c*x))/c^5+2/81*b*d^2*(-c^2*x^2+1)^(9/2)*(a+b*arccos(c*x))/c^5+8/315*d^2
*x^5*(a+b*arccos(c*x))^2+4/63*d^2*x^5*(-c^2*x^2+1)*(a+b*arccos(c*x))^2+1/9
*d^2*x^5*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.64

$$\int x^4 \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2\,dx$$
$$=\frac{d^2\left(99225a^2c^5x^5(63-90c^2x^2+35c^4x^4)-630ab\sqrt{1-c^2x^2}(2104+1052c^2x^2+789c^4x^4-2650c^6x^6+12\right.}{}$$

input
```
Integrate[x^4*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*(99225*a^2*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4) - 630*a*b*Sqrt[1 -
c^2*x^2]*(2104 + 1052*c^2*x^2 + 789*c^4*x^4 - 2650*c^6*x^6 + 1225*c^8*x^8)
 - 2*b^2*c*x*(662760 + 110460*c^2*x^2 + 49707*c^4*x^4 - 119250*c^6*x^6 + 4
2875*c^8*x^8) - 630*b*(-315*a*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4) + b*S
qrt[1 - c^2*x^2]*(2104 + 1052*c^2*x^2 + 789*c^4*x^4 - 2650*c^6*x^6 + 1225*
c^8*x^8))*ArcCos[c*x] + 99225*b^2*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4)*A
rcCos[c*x]^2))/(31255875*c^5)
```

Rubi [A] (verified)

Time = 2.39 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.31, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.630, Rules used = {5203, 27, 5195, 27, 1467, 2009, 5203, 5139, 5195, 27, 2009, 5211, 15, 5211, 15, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(d-c^2 d x^2\right)^2 (a+b \arccos (c x))^2 \, dx$$

$$\Big\downarrow \text{ 5203}$$

$$\frac{2}{9} b c d^2 \int x^5 \left(1-c^2 x^2\right)^{3/2} (a+b \arccos (c x))dx+\frac{4}{9} d \int d x^4 \left(1-c^2 x^2\right) (a+b \arccos (c x))^2dx+\frac{1}{9} d^2 x^5 \left(1-c^2 x^2\right)^2 (a+b \arccos (c x))^2$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{9} b c d^2 \int x^5 \left(1-c^2 x^2\right)^{3/2} (a+b \arccos (c x))dx+\frac{4}{9} d^2 \int x^4 \left(1-c^2 x^2\right) (a+b \arccos (c x))^2dx+\frac{1}{9} d^2 x^5 \left(1-c^2 x^2\right)^2 (a+b \arccos (c x))^2$$

$$\Big\downarrow \text{ 5195}$$

$$\frac{4}{9} d^2 \int x^4 \left(1-c^2 x^2\right) (a+b \arccos (c x))^2dx+\frac{2}{9} b c d^2 \left(b c \int -\frac{\left(1-c^2 x^2\right)^2 \left(35 c^4 x^4+20 c^2 x^2+8\right)}{315 c^6}dx-\frac{\left(1-c^2 x^2\right)^{9/2} (a+b \arccos (c x))}{9 c^6}+\frac{2 \left(1-c^2 x^2\right)^{7/2} (a+b\right.$$
$$\frac{1}{9} d^2 x^5 \left(1-c^2 x^2\right)^2 (a+b \arccos (c x))^2$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4}{9} d^2 \int x^4 \left(1-c^2 x^2\right) (a+b \arccos (c x))^2dx+\frac{2}{9} b c d^2 \left(-\frac{b \int \left(1-c^2 x^2\right)^2 \left(35 c^4 x^4+20 c^2 x^2+8\right) dx}{315 c^5}-\frac{\left(1-c^2 x^2\right)^{9/2} (a+b \arccos (c x))}{9 c^6}+\frac{2 \left(1-c^2 x^2\right)^{7/2} (a+b \text{ a}\right.$$
$$\frac{1}{9} d^2 x^5 \left(1-c^2 x^2\right)^2 (a+b \arccos (c x))^2$$

$$\Big\downarrow \text{ 1467}$$

$$\frac{4}{9}d^2\int x^4\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+$$
$$\frac{2}{9}bcd^2\left(-\frac{b\int\left(35c^8x^8-50c^6x^6+3c^4x^4+4c^2x^2+8\right)dx}{315c^5}-\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}+\frac{2\left(1-c^2x^2\right)^{7/2}(a}{7c^6}\right.$$
$$\frac{1}{9}d^2x^5\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4}{9}d^2\int x^4\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{1}{9}d^2x^5\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{2}{9}bcd^2\left(-\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}+\frac{2\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx)}{5c^6}\right.$$

$$\Big\downarrow \text{ 5203}$$

$$\frac{4}{9}d^2\left(\frac{2}{7}bc\int x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{2}{7}\int x^4(a+b\arccos(cx))^2dx+\frac{1}{7}x^5\left(1-c^2x^2\right)(a+b\arccos(cx)\right.$$
$$\frac{1}{9}d^2x^5\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{2}{9}bcd^2\left(-\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}+\frac{2\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx)}{5c^6}\right.$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{2}{5}bc\int\frac{x^5(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))^2\right)+\frac{2}{7}bc\int x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\right.$$
$$\frac{1}{9}d^2x^5\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{2}{9}bcd^2\left(-\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}+\frac{2\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx)}{5c^6}\right.$$

$$\Big\downarrow \text{ 5195}$$

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{2}{5}bc\int\frac{x^5(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))^2\right)+\frac{2}{7}bc\left(bc\int-\frac{-15c^6x^6+3c^4x^4+4c^2x^2+8}{105c^6}\right.\right.$$
$$\frac{1}{9}d^2x^5\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{2}{9}bcd^2\left(-\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}+\frac{2\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx)}{5c^6}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{2}{5}bc\int\frac{x^5(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))^2\right)+\frac{2}{7}bc\left(-\frac{b\int\left(-15c^6x^6+3c^4x^4+4c^2x^2+8\right)}{105c^5}\right.\right.$$

$$\frac{1}{9}d^2x^5\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$

$$\frac{2}{9}bcd^2\left(-\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}+\frac{2\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx)}{5c^6}\right.$$

$\big\downarrow$ 2009

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{2}{5}bc\int\frac{x^5(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))^2\right)+\frac{1}{7}x^5\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{7}bc\left(\right.\right.$$

$$\frac{1}{9}d^2x^5\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$

$$\frac{2}{9}bcd^2\left(-\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}+\frac{2\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx)}{5c^6}\right.$$

$\big\downarrow$ 5211

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{2}{5}bc\left(\frac{4\int\frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{5c^2}-\frac{b\int x^4dx}{5c}-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{5c^2}\right)+\frac{1}{5}x^5(a+b\arccos(cx))^2\right.\right.$$

$$\frac{1}{9}d^2x^5\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$

$$\frac{2}{9}bcd^2\left(-\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}+\frac{2\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx)}{5c^6}\right.$$

$\big\downarrow$ 15

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{2}{5}bc\left(\frac{4\int\frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{5c^2}-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{5c^2}-\frac{bx^5}{25c}\right)+\frac{1}{5}x^5(a+b\arccos(cx))^2\right)+\frac{1}{7}\right.$$

$$\frac{1}{9}d^2x^5\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$

$$\frac{2}{9}bcd^2\left(-\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}+\frac{2\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx)}{5c^6}\right.$$

$\big\downarrow$ 5211

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{2}{5}bc\left(\frac{4\left(\frac{2\int\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{b\int x^2dx}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}\right)}{5c^2}-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{5c^2}-\right.\right.\right.$$

$$\frac{1}{9}d^2x^5\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$

$$\frac{2}{9}bcd^2\left(-\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}+\frac{2\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx)}{5c^6}\right.$$

$$\Big\downarrow 15$$

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{2}{5}bc\left(\frac{4\left(\frac{2\int\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{5c^2}-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{5c^2}-\frac{bx^5}{25c}\right.\right.\right.$$

$$\frac{1}{9}d^2x^5\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$

$$\frac{2}{9}bcd^2\left(-\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}+\frac{2\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx)}{5c^6}\right.$$

$$\Big\downarrow 5183$$

$$\frac{4}{9}d^2\left(\frac{2}{7}\left(\frac{2}{5}bc\left(\frac{4\left(\frac{2\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{5c^2}-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos}{5c^2}\right.\right.\right.$$

$$\frac{1}{9}d^2x^5\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$

$$\frac{2}{9}bcd^2\left(-\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}+\frac{2\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx)}{5c^6}\right.$$

$$\Big\downarrow 24$$

$$\frac{1}{9}d^2x^5\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$

$$\frac{4}{9}d^2\left(\frac{1}{7}x^5\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{7}\left(\frac{2}{5}bc\left(-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{5c^2}+\frac{4\left(-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos}{3c^2}\right.}{}\right.\right.\right.$$

$$\frac{2}{9}bcd^2\left(-\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}+\frac{2\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^6}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx)}{5c^6}\right.$$

input

```
Int[x^4*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output

```
(d^2*x^5*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/9 + (2*b*c*d^2*(-1/315*(b*
(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (50*c^6*x^7)/7 + (35*c^8*x^9)/9))/c
^5 - ((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^6) + (2*(1 - c^2*x^2)^
(7/2)*(a + b*ArcCos[c*x]))/(7*c^6) - ((1 - c^2*x^2)^(9/2)*(a + b*ArcCos[c*
x]))/(9*c^6)))/9 + (4*d^2*((x^5*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/7 + (
2*b*c*(-1/105*(b*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (15*c^6*x^7)/7))/c
^5 - ((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(3*c^6) + (2*(1 - c^2*x^2)^
(5/2)*(a + b*ArcCos[c*x]))/(5*c^6) - ((1 - c^2*x^2)^(7/2)*(a + b*ArcCos[c*
x]))/(7*c^6)))/7 + (2*((x^5*(a + b*ArcCos[c*x])^2)/5 + (2*b*c*(-1/25*(b*x^
5)/c - (x^4*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(5*c^2) + (4*(-1/9*(b*x
^3)/c - (x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(3*c^2) + (2*(-((b*x)/
c) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c^2))/(3*c^2)))/(5*c^2)))/5))
/7))/9
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.34

method	result
parts	$d^2a^2\left(\frac{1}{9}c^4x^9-\frac{2}{7}c^2x^7+\frac{1}{5}x^5\right)+\frac{d^2b^2\left(\frac{\arccos(cx)^2\left(3c^4x^4-10c^2x^2+15\right)cx}{15}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^2\sqrt{-c^2x^2+1}}{525}-\right.}{}$
derivativedivides	$\frac{d^2a^2\left(\frac{1}{9}c^9x^9-\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b^2\left(\frac{\arccos(cx)^2\left(3c^4x^4-10c^2x^2+15\right)cx}{15}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^2\sqrt{-c^2x^2+1}}{525}-\frac{2\left(3c^4x^4-10\right.}{78}\right.}{}$
default	$\frac{d^2a^2\left(\frac{1}{9}c^9x^9-\frac{2}{7}c^7x^7+\frac{1}{5}c^5x^5\right)+d^2b^2\left(\frac{\arccos(cx)^2\left(3c^4x^4-10c^2x^2+15\right)cx}{15}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^2\sqrt{-c^2x^2+1}}{525}-\frac{2\left(3c^4x^4-10\right.}{78}\right.}{}$
orering	$\frac{\left(9303875x^{12}c^{12}-34087625c^{10}x^{10}+40400953c^8x^8-8418363c^6x^6+38661000c^4x^4-46835040c^2x^2+15906240\right)\left(-c^2d\,x\right.}{31255875x\,c^6(cx-1)(cx+1)\left(c^2x^2-1\right)^2}$

input
```
int(x^4*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(1/9*c^4*x^9-2/7*c^2*x^7+1/5*x^5)+d^2*b^2/c^5*(1/15*arccos(c*x)^2*
(3*c^4*x^4-10*c^2*x^2+15)*c*x-2/525*arccos(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)
^(1/2)-2/7875*(3*c^4*x^4-10*c^2*x^2+15)*c*x+8/945*arccos(c*x)*(c^2*x^2-1)*
(-c^2*x^2+1)^(1/2)+8/2835*(c^2*x^2-3)*c*x-16/315*c*x-16/315*arccos(c*x)*(-
c^2*x^2+1)^(1/2)+2/35*arccos(c*x)^2*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c
*x-20/441*arccos(c*x)*(c^2*x^2-1)^3*(-c^2*x^2+1)^(1/2)-4/3087*(5*c^6*x^6-2
1*c^4*x^4+35*c^2*x^2-35)*c*x+1/315*arccos(c*x)^2*(35*c^8*x^8-180*c^6*x^6+3
78*c^4*x^4-420*c^2*x^2+315)*c*x-2/81*arccos(c*x)*(c^2*x^2-1)^4*(-c^2*x^2+1
)^(1/2)-2/25515*(35*c^8*x^8-180*c^6*x^6+378*c^4*x^4-420*c^2*x^2+315)*c*x)+
2*d^2*a*b/c^5*(1/9*arccos(c*x)*c^9*x^9-2/7*arccos(c*x)*c^7*x^7+1/5*arccos(
c*x)*c^5*x^5-263/33075*c^4*x^4*(-c^2*x^2+1)^(1/2)-1052/99225*c^2*x^2*(-c^2
*x^2+1)^(1/2)-2104/99225*(-c^2*x^2+1)^(1/2)+106/3969*c^6*x^6*(-c^2*x^2+1)^
(1/2)-1/81*c^8*x^8*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.85

$$\int x^4 (d-c^2dx^2)^2 (a+b\arccos(cx))^2\,dx = \frac{42875\,(81\,a^2-2\,b^2)c^9d^2x^9-2250\,(3969\,a^2-106\,b^2)c^7d^2x^7+189\,(33075\,a^2-526\,b^2)c^5d^2x^5-220920\,b^2 \cdots}{\cdots}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
1/31255875*(42875*(81*a^2 - 2*b^2)*c^9*d^2*x^9 - 2250*(3969*a^2 - 106*b^2)
*c^7*d^2*x^7 + 189*(33075*a^2 - 526*b^2)*c^5*d^2*x^5 - 220920*b^2*c^3*d^2*
x^3 - 1325520*b^2*c*d^2*x + 99225*(35*b^2*c^9*d^2*x^9 - 90*b^2*c^7*d^2*x^7
 + 63*b^2*c^5*d^2*x^5)*arccos(c*x)^2 + 198450*(35*a*b*c^9*d^2*x^9 - 90*a*b
*c^7*d^2*x^7 + 63*a*b*c^5*d^2*x^5)*arccos(c*x) - 630*(1225*a*b*c^8*d^2*x^8
 - 2650*a*b*c^6*d^2*x^6 + 789*a*b*c^4*d^2*x^4 + 1052*a*b*c^2*d^2*x^2 + 210
4*a*b*d^2 + (1225*b^2*c^8*d^2*x^8 - 2650*b^2*c^6*d^2*x^6 + 789*b^2*c^4*d^2
*x^4 + 1052*b^2*c^2*d^2*x^2 + 2104*b^2*d^2)*arccos(c*x))*sqrt(-c^2*x^2 + 1
))/c^5
```

Sympy [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.44

$$\int x^4\left(d-c^2dx^2\right)^2\left(a+b\arccos(cx)\right)^2dx$$
$$=\begin{cases}\frac{a^2c^4d^2x^9}{9}-\frac{2a^2c^2d^2x^7}{7}+\frac{a^2d^2x^5}{5}+\frac{2abc^4d^2x^9\operatorname{acos}(cx)}{9}-\frac{2abc^3d^2x^8\sqrt{-c^2x^2+1}}{81}-\frac{4abc^2d^2x^7\operatorname{acos}(cx)}{7}+\frac{212abcd^2x^6\sqrt{-c^2x^2+1}}{3969}\\ \frac{d^2x^5\left(a+\frac{\pi b}{2}\right)^2}{5}\end{cases}$$

input
```
integrate(x**4*(-c**2*d*x**2+d)**2*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*c**4*d**2*x**9/9 - 2*a**2*c**2*d**2*x**7/7 + a**2*d**2*x**
5/5 + 2*a*b*c**4*d**2*x**9*acos(c*x)/9 - 2*a*b*c**3*d**2*x**8*sqrt(-c**2*x
**2 + 1)/81 - 4*a*b*c**2*d**2*x**7*acos(c*x)/7 + 212*a*b*c*d**2*x**6*sqrt(
-c**2*x**2 + 1)/3969 + 2*a*b*d**2*x**5*acos(c*x)/5 - 526*a*b*d**2*x**4*sqr
t(-c**2*x**2 + 1)/(33075*c) - 2104*a*b*d**2*x**2*sqrt(-c**2*x**2 + 1)/(992
25*c**3) - 4208*a*b*d**2*sqrt(-c**2*x**2 + 1)/(99225*c**5) + b**2*c**4*d**
2*x**9*acos(c*x)**2/9 - 2*b**2*c**4*d**2*x**9/729 - 2*b**2*c**3*d**2*x**8*
sqrt(-c**2*x**2 + 1)*acos(c*x)/81 - 2*b**2*c**2*d**2*x**7*acos(c*x)**2/7 +
 212*b**2*c**2*d**2*x**7/27783 + 212*b**2*c*d**2*x**6*sqrt(-c**2*x**2 + 1)
*acos(c*x)/3969 + b**2*d**2*x**5*acos(c*x)**2/5 - 526*b**2*d**2*x**5/16537
5 - 526*b**2*d**2*x**4*sqrt(-c**2*x**2 + 1)*acos(c*x)/(33075*c) - 2104*b**
2*d**2*x**3/(297675*c**2) - 2104*b**2*d**2*x**2*sqrt(-c**2*x**2 + 1)*acos(
c*x)/(99225*c**3) - 4208*b**2*d**2*x/(99225*c**4) - 4208*b**2*d**2*sqrt(-c
**2*x**2 + 1)*acos(c*x)/(99225*c**5), Ne(c, 0)), (d**2*x**5*(a + pi*b/2)**
2/5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(349) = 698$.

Time = 0.16 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.98

$$\int x^4\left(d-c^2dx^2\right)^2\left(a+b\arccos(cx)\right)^2dx=\text{Too large to display}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/9*b^2*c^4*d^2*x^9*arccos(c*x)^2 + 1/9*a^2*c^4*d^2*x^9 - 2/7*b^2*c^2*d^2*
x^7*arccos(c*x)^2 - 2/7*a^2*c^2*d^2*x^7 + 1/5*b^2*d^2*x^5*arccos(c*x)^2 +
2/2835*(315*x^9*arccos(c*x) - (35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^
2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)
*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/c^10)*c)*a*b*c^4*d^2 - 2/893025*(315*(35
*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2
*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)
/c^10)*c*arccos(c*x) + (1225*c^8*x^9 + 1800*c^6*x^7 + 3024*c^4*x^5 + 6720*
c^2*x^3 + 40320*x)/c^8)*b^2*c^4*d^2 + 1/5*a^2*d^2*x^5 - 4/245*(35*x^7*arcc
os(c*x) - (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8
*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d^2 +
4/25725*(105*(5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4
+ 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c*arccos(c*x)
+ (75*c^6*x^7 + 126*c^4*x^5 + 280*c^2*x^3 + 1680*x)/c^6)*b^2*c^2*d^2 + 2/7
5*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 +
1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a*b*d^2 - 2/1125*(15*(3*sqrt(-c^
2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c
^6)*c*arccos(c*x) + (9*c^4*x^5 + 20*c^2*x^3 + 120*x)/c^4)*b^2*d^2
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.20

$$\int x^4 (d-c^2dx^2)^2 (a+b\arccos(cx))^2\,dx = \frac{1}{9}b^2c^4d^2x^9\arccos(cx)^2 + \frac{2}{9}abc^4d^2x^9\arccos(cx) + \frac{1}{9}a^2c^4d^2x^9 - \frac{2}{729}b^2c^4d^2x^9 - \frac{2}{81}\sqrt{-c^2x^2+1}b^2c^3d^2x^8\arccos(cx) - \frac{2}{81}\sqrt{-c^2x^2+1}abc^3d^2x^8 - \frac{2}{7}b^2c^2d^2x^7\arccos(cx)^2 - \frac{4}{7}abc^2d^2x^7\arccos(cx) - \frac{2}{7}a^2c^2d^2x^7 + \frac{212}{27783}b^2c^2d^2x^7 + \frac{212}{3969}\sqrt{-c^2x^2+1}b^2cd^2x^6\arccos(cx) + \frac{212}{3969}\sqrt{-c^2x^2+1}abcd^2x^6 + \frac{1}{5}b^2d^2x^5\arccos(cx)^2 + \frac{2}{5}abd^2x^5\arccos(cx) + \frac{1}{5}a^2d^2x^5 - \frac{526}{165375}b^2d^2x^5 - \frac{526\sqrt{-c^2x^2+1}b^2d^2x^4\arccos(cx)}{33075\,c} - \frac{526\sqrt{-c^2x^2+1}abd^2x^4}{33075\,c} - \frac{2104\,b^2d^2x^3}{297675\,c^2} - \frac{2104\sqrt{-c^2x^2+1}b^2d^2x^2\arccos(cx)}{99225\,c^3} - \frac{2104\sqrt{-c^2x^2+1}abd^2x^2}{99225\,c^3} - \frac{4208\,b^2d^2x}{99225\,c^4} - \frac{4208\sqrt{-c^2x^2+1}b^2d^2\arccos(cx)}{99225\,c^5} - \frac{4208\sqrt{-c^2x^2+1}abd^2}{99225\,c^5}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
1/9*b^2*c^4*d^2*x^9*arccos(c*x)^2 + 2/9*a*b*c^4*d^2*x^9*arccos(c*x) + 1/9*
a^2*c^4*d^2*x^9 - 2/729*b^2*c^4*d^2*x^9 - 2/81*sqrt(-c^2*x^2 + 1)*b^2*c^3*
d^2*x^8*arccos(c*x) - 2/81*sqrt(-c^2*x^2 + 1)*a*b*c^3*d^2*x^8 - 2/7*b^2*c^
2*d^2*x^7*arccos(c*x)^2 - 4/7*a*b*c^2*d^2*x^7*arccos(c*x) - 2/7*a^2*c^2*d^
2*x^7 + 212/27783*b^2*c^2*d^2*x^7 + 212/3969*sqrt(-c^2*x^2 + 1)*b^2*c*d^2*
x^6*arccos(c*x) + 212/3969*sqrt(-c^2*x^2 + 1)*a*b*c*d^2*x^6 + 1/5*b^2*d^2*
x^5*arccos(c*x)^2 + 2/5*a*b*d^2*x^5*arccos(c*x) + 1/5*a^2*d^2*x^5 - 526/16
5375*b^2*d^2*x^5 - 526/33075*sqrt(-c^2*x^2 + 1)*b^2*d^2*x^4*arccos(c*x)/c
- 526/33075*sqrt(-c^2*x^2 + 1)*a*b*d^2*x^4/c - 2104/297675*b^2*d^2*x^3/c^2
 - 2104/99225*sqrt(-c^2*x^2 + 1)*b^2*d^2*x^2*arccos(c*x)/c^3 - 2104/99225*
sqrt(-c^2*x^2 + 1)*a*b*d^2*x^2/c^3 - 4208/99225*b^2*d^2*x/c^4 - 4208/99225
*sqrt(-c^2*x^2 + 1)*b^2*d^2*arccos(c*x)/c^5 - 4208/99225*sqrt(-c^2*x^2 + 1
)*a*b*d^2/c^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d-c^2dx^2)^2 (a+b\arccos(cx))^2\,dx=\int x^4\,(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^2\,dx$$

input
```
int(x^4*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^2,x)
```

output
```
int(x^4*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int x^4 (d-c^2dx^2)^2 (a+b\arccos(cx))^2\,dx$$
$$=\frac{d^2\left(22050\,acos(cx)\,ab\,c^9x^9-56700\,acos(cx)\,ab\,c^7x^7+39690\,acos(cx)\,ab\,c^5x^5-2450\sqrt{-c^2x^2+1}\,ab\,c^8x^8+\right.}{}$$

input
```
int(x^4*(-c^2*d*x^2+d)^2*(a+b*acos(c*x))^2,x)
```

output
```
(d**2*(22050*acos(c*x)*a*b*c**9*x**9 - 56700*acos(c*x)*a*b*c**7*x**7 + 396
90*acos(c*x)*a*b*c**5*x**5 - 2450*sqrt( - c**2*x**2 + 1)*a*b*c**8*x**8 + 5
300*sqrt( - c**2*x**2 + 1)*a*b*c**6*x**6 - 1578*sqrt( - c**2*x**2 + 1)*a*b
*c**4*x**4 - 2104*sqrt( - c**2*x**2 + 1)*a*b*c**2*x**2 - 4208*sqrt( - c**2
*x**2 + 1)*a*b + 99225*int(acos(c*x)**2*x**8,x)*b**2*c**9 - 198450*int(aco
s(c*x)**2*x**6,x)*b**2*c**7 + 99225*int(acos(c*x)**2*x**4,x)*b**2*c**5 + 1
1025*a**2*c**9*x**9 - 28350*a**2*c**7*x**7 + 19845*a**2*c**5*x**5))/(99225
*c**5)
```

3.168 $\int x^3(d-c^2dx^2)^2 (a+b\arccos(cx))^2 \, dx$

Optimal result

Integrand size = 27, antiderivative size = 302

$$\begin{aligned}\int x^3(d-c^2dx^2)^2 (a+b\arccos(cx))^2 \, dx &= -\frac{73b^2d^2x^2}{3072c^2} - \frac{73b^2d^2x^4}{9216} \\ &+ \frac{43b^2c^2d^2x^6}{3456} - \frac{1}{256}b^2c^4d^2x^8 \\ &+ \frac{73bd^2x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{1536c^3} \\ &+ \frac{73bd^2x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2304c} \\ &- \frac{25}{576}bcd^2x^5\sqrt{1-c^2x^2}(a+b\arccos(cx)) \\ &- \frac{1}{32}bcd^2x^5(1-c^2x^2)^{3/2}(a+b\arccos(cx)) \\ &- \frac{73d^2(a+b\arccos(cx))^2}{3072c^4} \\ &+ \frac{1}{24}d^2x^4(a+b\arccos(cx))^2 \\ &+ \frac{1}{12}d^2x^4(1-c^2x^2)(a+b\arccos(cx))^2 \\ &+ \frac{1}{8}d^2x^4(1-c^2x^2)^2(a+b\arccos(cx))^2\end{aligned}$$

output
```
-73/3072*b^2*d^2*x^2/c^2-73/9216*b^2*d^2*x^4+43/3456*b^2*c^2*d^2*x^6-1/256
*b^2*c^4*d^2*x^8+73/1536*b*d^2*x*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3+
73/2304*b*d^2*x^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c-25/576*b*c*d^2*x^
5*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))-1/32*b*c*d^2*x^5*(-c^2*x^2+1)^(3/2)
*(a+b*arccos(c*x))-73/3072*d^2*(a+b*arccos(c*x))^2/c^4+1/24*d^2*x^4*(a+b*a
rccos(c*x))^2+1/12*d^2*x^4*(-c^2*x^2+1)*(a+b*arccos(c*x))^2+1/8*d^2*x^4*(-
c^2*x^2+1)^2*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.81

$$\int x^3 \left(d - c^2 d x^2\right)^2 (a + b \arccos(cx))^2 \, dx$$
$$= \frac{d^2 \left(-cx \left(-1152 a^2 c^3 x^3 (6 - 8c^2x^2 + 3c^4x^4) + b^2 cx(657 + 219c^2x^2 - 344c^4x^4 + 108c^6x^6) + 6ab\sqrt{1-c^2x^2}(\right.\right.}{}$$

input
```
Integrate[x^3*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*(-(c*x*(-1152*a^2*c^3*x^3*(6 - 8*c^2*x^2 + 3*c^4*x^4) + b^2*c*x*(657
+ 219*c^2*x^2 - 344*c^4*x^4 + 108*c^6*x^6) + 6*a*b*Sqrt[1 - c^2*x^2]*(219
+ 146*c^2*x^2 - 344*c^4*x^4 + 144*c^6*x^6))) + 6*b*c*x*(384*a*c^3*x^3*(6 -
 8*c^2*x^2 + 3*c^4*x^4) - b*Sqrt[1 - c^2*x^2]*(219 + 146*c^2*x^2 - 344*c^4
*x^4 + 144*c^6*x^6))*ArcCos[c*x] + 9*b^2*(-73 + 768*c^4*x^4 - 1024*c^6*x^6
 + 384*c^8*x^8)*ArcCos[c*x]^2 + 1314*a*b*ArcSin[c*x]))/(27648*c^4)
```

Rubi [A] (verified)

Time = 2.42 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.97, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {5203, 27, 5203, 244, 2009, 5139, 5199, 15, 5211, 15, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2\,dx$$

$\Big\downarrow$ 5203

$$\frac{1}{4}bcd^2\int x^4\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{2}d\int dx^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{1}{8}d^2x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\Big\downarrow$ 27

$$\frac{1}{4}bcd^2\int x^4\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{2}d^2\int x^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{1}{8}d^2x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\Big\downarrow$ 5203

$$\frac{1}{4}bcd^2\left(\frac{3}{8}\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{8}bc\int x^5\left(1-c^2x^2\right)dx+\frac{1}{8}x^5\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)$$
$$\frac{1}{2}d^2\left(\frac{1}{3}bc\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{3}\int x^3(a+b\arccos(cx))^2dx+\frac{1}{6}x^4\left(1-c^2x^2\right)(a+b\arccos(cx)\right.$$
$$\frac{1}{8}d^2x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\Big\downarrow$ 244

$$\frac{1}{2}d^2\left(\frac{1}{3}bc\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{3}\int x^3(a+b\arccos(cx))^2dx+\frac{1}{6}x^4\left(1-c^2x^2\right)(a+b\arccos(cx)\right.$$
$$\frac{1}{4}bcd^2\left(\frac{3}{8}\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{8}bc\int\left(x^5-c^2x^7\right)dx+\frac{1}{8}x^5\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)+$$
$$\frac{1}{8}d^2x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\Big\downarrow$ 2009

$$\frac{1}{2}d^2\left(\frac{1}{3}bc\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{3}\int x^3(a+b\arccos(cx))^2dx+\frac{1}{6}x^4\left(1-c^2x^2\right)(a+b\arccos(cx)\right.$$
$$\frac{1}{4}bcd^2\left(\frac{3}{8}\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{8}x^5\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{8}bc\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\right)+$$
$$\frac{1}{8}d^2x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\Big\downarrow$ 5139

$$\frac{1}{2}d^2\left(\frac{1}{3}\left(\frac{1}{2}bc\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))^2\right)+\frac{1}{3}bc\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\right.$$
$$\frac{1}{4}bcd^2\left(\frac{3}{8}\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{8}x^5\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{8}bc\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\right)+$$
$$\frac{1}{8}d^2x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\downarrow$ 5199

$$\frac{1}{2}d^2\left(\frac{1}{3}\left(\frac{1}{2}bc\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))^2\right)+\frac{1}{3}bc\left(\frac{1}{6}\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{6}bc\int\right.\right.$$
$$\frac{1}{4}bcd^2\left(\frac{3}{8}\left(\frac{1}{6}\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{6}bc\int x^5dx+\frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)+\frac{1}{8}x^5\left(1-c^2x^2\right)^{3/2}\right.$$
$$\frac{1}{8}d^2x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\downarrow$ 15

$$\frac{1}{2}d^2\left(\frac{1}{3}\left(\frac{1}{2}bc\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))^2\right)+\frac{1}{3}bc\left(\frac{1}{6}\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{6}x^5\sqrt{\right.\right.$$
$$\frac{1}{4}bcd^2\left(\frac{3}{8}\left(\frac{1}{6}\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{36}bcx^6\right)+\frac{1}{8}x^5\left(1-c^2x^2\right)^{3/2}(a+\right.$$
$$\frac{1}{8}d^2x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\downarrow$ 5211

$$\frac{1}{4}bcd^2\left(\frac{3}{8}\left(\frac{1}{6}\left(\frac{3\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{b\int x^3dx}{4c}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}\right)+\frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\,\mathrm{arc}\right.\right.$$
$$\frac{1}{2}d^2\left(\frac{1}{3}\left(\frac{1}{2}bc\left(\frac{3\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{b\int x^3dx}{4c}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}\right)+\frac{1}{4}x^4(a+b\arccos(cx))^2\right)\right.$$
$$\frac{1}{8}d^2x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\downarrow$ 15

$$\frac{1}{2}d^2\left(\frac{1}{3}\left(\frac{1}{2}bc\left(\frac{3\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)+\frac{1}{4}x^4(a+b\arccos(cx))^2\right)+\frac{1}{3}\right.$$
$$\frac{1}{4}bcd^2\left(\frac{3}{8}\left(\frac{1}{6}\left(\frac{3\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)+\frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arccos(\right.\right.\right.$$
$$\frac{1}{8}d^2x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\downarrow$ 5211

$$\frac{1}{2}d^2\left(\frac{1}{3}\left(\frac{1}{2}bc\left(\frac{3\left(\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int xdx}{2c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)\right.\right.$$
$$\frac{1}{4}bcd^2\left(\frac{3}{8}\left(\frac{1}{6}\left(\frac{3\left(\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int xdx}{2c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)\right.\right.$$
$$\frac{1}{8}d^2x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\downarrow$ 15

$$\frac{1}{2}d^2\left(\frac{1}{3}\left(\frac{1}{2}bc\left(\frac{3\left(\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)+\right.\right.$$
$$\frac{1}{4}bcd^2\left(\frac{3}{8}\left(\frac{1}{6}\left(\frac{3\left(\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)+\right.\right.$$
$$\frac{1}{8}d^2x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\downarrow$ 5153

$$\frac{1}{8}d^2x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{1}{2}d^2\left(\frac{1}{6}x^4\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{1}{3}\left(\frac{1}{2}bc\left(-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}+\frac{3\left(-\frac{(a+b\arccos(cx))^2}{4bc^3}-\frac{x\sqrt{}}{}\right.}{}\right.\right.\right.$$
$$\frac{1}{4}bcd^2\left(\frac{1}{8}x^5\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{8}\left(\frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{6}\left(-\frac{x^3\sqrt{1-c^2x^2}(a+b\,\mathrm{a}}{4c^2}\right.\right.\right.$$

input
```
Int[x^3*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*x^4*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/8 + (b*c*d^2*((b*c*(x^6/6
- (c^2*x^8)/8))/8 + (x^5*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/8 + (3*(
(b*c*x^6)/36 + (x^5*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/6 + (-1/16*(b*x
^4)/c - (x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(4*c^2) + (3*(-1/4*(b*
x^2)/c - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(2*c^2) - (a + b*ArcCos
[c*x])^2/(4*b*c^3)))/(4*c^2))/6))/8))/4 + (d^2*((x^4*(1 - c^2*x^2)*(a + b*
ArcCos[c*x])^2)/6 + (b*c*((b*c*x^6)/36 + (x^5*Sqrt[1 - c^2*x^2]*(a + b*Arc
Cos[c*x]))/6 + (-1/16*(b*x^4)/c - (x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x
]))/(4*c^2) + (3*(-1/4*(b*x^2)/c - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]
))/(2*c^2) - (a + b*ArcCos[c*x])^2/(4*b*c^3)))/(4*c^2))/6))/3 + ((x^4*(a +
 b*ArcCos[c*x])^2)/4 + (b*c*(-1/16*(b*x^4)/c - (x^3*Sqrt[1 - c^2*x^2]*(a +
 b*ArcCos[c*x]))/(4*c^2) + (3*(-1/4*(b*x^2)/c - (x*Sqrt[1 - c^2*x^2]*(a +
b*ArcCos[c*x]))/(2*c^2) - (a + b*ArcCos[c*x])^2/(4*b*c^3)))/(4*c^2)))/2)/3
))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.45

method	result
parts	$d^2a^2\left(\frac{1}{8}c^4x^8-\frac{1}{3}c^2x^6+\frac{1}{4}x^4\right)+\frac{d^2b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-1\right)^3}{6}+\frac{\arccos(cx)\left(-8c^5x^5\sqrt{-c^2x^2+1}+26c^3x^3\sqrt{-c^2x^2+\right.}{144}$
derivativedivides	$\frac{d^2a^2\left(\frac{1}{8}c^8x^8-\frac{1}{3}c^6x^6+\frac{1}{4}c^4x^4\right)+d^2b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-1\right)^3}{6}+\frac{\arccos(cx)\left(-8c^5x^5\sqrt{-c^2x^2+1}+26c^3x^3\sqrt{-c^2x^2+1}-33cx\sqrt{-\right.}{144}\right.}{}$
default	$\frac{d^2a^2\left(\frac{1}{8}c^8x^8-\frac{1}{3}c^6x^6+\frac{1}{4}c^4x^4\right)+d^2b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-1\right)^3}{6}+\frac{\arccos(cx)\left(-8c^5x^5\sqrt{-c^2x^2+1}+26c^3x^3\sqrt{-c^2x^2+1}-33cx\sqrt{-\right.}{144}\right.}{}$
orering	$\frac{(18252c^{10}x^{10}-69716c^8x^8+87751c^6x^6-492c^4x^4-36135c^2x^2+13140)\left(-c^2d\,x^2+d\right)^2(a+b\arccos(cx))^2}{55296c^4(cx-1)(cx+1)(c^2x^2-1)^2}-\frac{(2268c^8x^8}{}$

input
```
int(x^3*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(1/8*c^4*x^8-1/3*c^2*x^6+1/4*x^4)+d^2*b^2/c^4*(1/6*arccos(c*x)^2*(
c^2*x^2-1)^3+1/144*arccos(c*x)*(-8*c^5*x^5*(-c^2*x^2+1)^(1/2)+26*c^3*x^3*(
-c^2*x^2+1)^(1/2)-33*c*x*(-c^2*x^2+1)^(1/2)+15*arccos(c*x))-55/3072*arccos
(c*x)^2-1/108*c^6*x^6+13/288*c^4*x^4-247/3072*c^2*x^2+1/8*arccos(c*x)^2*(c
^2*x^2-1)^4-1/1536*arccos(c*x)*(48*c^7*x^7*(-c^2*x^2+1)^(1/2)-200*c^5*x^5*
(-c^2*x^2+1)^(1/2)+326*c^3*x^3*(-c^2*x^2+1)^(1/2)-279*c*x*(-c^2*x^2+1)^(1/
2)+105*arccos(c*x))-1/256*(c^2*x^2-1)^4+7/1152*(c^2*x^2-1)^3-35/3072*(c^2*
x^2-1)^2-35/1024)+2*d^2*a*b/c^4*(1/8*arccos(c*x)*c^8*x^8-1/3*arccos(c*x)*c
^6*x^6+1/4*c^4*x^4*arccos(c*x)-73/4608*c^3*x^3*(-c^2*x^2+1)^(1/2)-73/3072*
c*x*(-c^2*x^2+1)^(1/2)+73/3072*arcsin(c*x)+43/1152*c^5*x^5*(-c^2*x^2+1)^(1
/2)-1/64*c^7*x^7*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.06

$$\int x^3(d-c^2dx^2)^2(a+b\arccos(cx))^2\,dx = \frac{108\,(32\,a^2-b^2)c^8d^2x^8-8\,(1152\,a^2-43\,b^2)c^6d^2x^6+3\,(2304\,a^2-73\,b^2)c^4d^2x^4-657\,b^2c^2d^2x^2+9\,(384\,b^2\cdots}{\cdots}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
1/27648*(108*(32*a^2 - b^2)*c^8*d^2*x^8 - 8*(1152*a^2 - 43*b^2)*c^6*d^2*x^
6 + 3*(2304*a^2 - 73*b^2)*c^4*d^2*x^4 - 657*b^2*c^2*d^2*x^2 + 9*(384*b^2*c
^8*d^2*x^8 - 1024*b^2*c^6*d^2*x^6 + 768*b^2*c^4*d^2*x^4 - 73*b^2*d^2)*arcc
os(c*x)^2 + 18*(384*a*b*c^8*d^2*x^8 - 1024*a*b*c^6*d^2*x^6 + 768*a*b*c^4*d
^2*x^4 - 73*a*b*d^2)*arccos(c*x) - 6*(144*a*b*c^7*d^2*x^7 - 344*a*b*c^5*d^
2*x^5 + 146*a*b*c^3*d^2*x^3 + 219*a*b*c*d^2*x + (144*b^2*c^7*d^2*x^7 - 344
*b^2*c^5*d^2*x^5 + 146*b^2*c^3*d^2*x^3 + 219*b^2*c*d^2*x)*arccos(c*x))*sqr
t(-c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.72

$$\int x^3 (d - c^2 d x^2)^2 (a + b \arccos(cx))^2 \, dx$$
$$= \begin{cases} \frac{a^2c^4d^2x^8}{8} - \frac{a^2c^2d^2x^6}{3} + \frac{a^2d^2x^4}{4} + \frac{abc^4d^2x^8 \operatorname{acos}(cx)}{4} - \frac{abc^3d^2x^7\sqrt{-c^2x^2+1}}{32} - \frac{2abc^2d^2x^6 \operatorname{acos}(cx)}{3} + \frac{43abcd^2x^5\sqrt{-c^2x^2+1}}{576} + \\ \frac{d^2x^4\left(a+\frac{\pi b}{2}\right)^2}{4} \end{cases}$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**2*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*c**4*d**2*x**8/8 - a**2*c**2*d**2*x**6/3 + a**2*d**2*x**4/
4 + a*b*c**4*d**2*x**8*acos(c*x)/4 - a*b*c**3*d**2*x**7*sqrt(-c**2*x**2 +
1)/32 - 2*a*b*c**2*d**2*x**6*acos(c*x)/3 + 43*a*b*c*d**2*x**5*sqrt(-c**2*x
**2 + 1)/576 + a*b*d**2*x**4*acos(c*x)/2 - 73*a*b*d**2*x**3*sqrt(-c**2*x**
2 + 1)/(2304*c) - 73*a*b*d**2*x*sqrt(-c**2*x**2 + 1)/(1536*c**3) - 73*a*b*
d**2*acos(c*x)/(1536*c**4) + b**2*c**4*d**2*x**8*acos(c*x)**2/8 - b**2*c**
4*d**2*x**8/256 - b**2*c**3*d**2*x**7*sqrt(-c**2*x**2 + 1)*acos(c*x)/32 -
b**2*c**2*d**2*x**6*acos(c*x)**2/3 + 43*b**2*c**2*d**2*x**6/3456 + 43*b**2
*c*d**2*x**5*sqrt(-c**2*x**2 + 1)*acos(c*x)/576 + b**2*d**2*x**4*acos(c*x)
**2/4 - 73*b**2*d**2*x**4/9216 - 73*b**2*d**2*x**3*sqrt(-c**2*x**2 + 1)*ac
os(c*x)/(2304*c) - 73*b**2*d**2*x**2/(3072*c**2) - 73*b**2*d**2*x*sqrt(-c*
*2*x**2 + 1)*acos(c*x)/(1536*c**3) - 73*b**2*d**2*acos(c*x)**2/(3072*c**4)
, Ne(c, 0)), (d**2*x**4*(a + pi*b/2)**2/4, True))
```

Maxima [F]

$$\int x^3 (d - c^2 d x^2)^2 (a + b \arccos(cx))^2 \, dx = \int (c^2 d x^2 - d)^2 (b \arccos(cx) + a)^2 x^3 \, dx$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/8*a^2*c^4*d^2*x^8 - 1/3*a^2*c^2*d^2*x^6 + 1/1536*(384*x^8*arccos(c*x) -
(48*sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(-
c^2*x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9)
*c)*a*b*c^4*d^2 + 1/4*a^2*d^2*x^4 - 1/72*(48*x^6*arccos(c*x) - (8*sqrt(-c^
2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)
*x/c^6 - 15*arcsin(c*x)/c^7)*c)*a*b*c^2*d^2 + 1/16*(8*x^4*arccos(c*x) - (2
*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c
^5)*c)*a*b*d^2 + 1/24*(3*b^2*c^4*d^2*x^8 - 8*b^2*c^2*d^2*x^6 + 6*b^2*d^2*x
^4)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 - integrate(1/12*(3*b^2*c
^5*d^2*x^8 - 8*b^2*c^3*d^2*x^6 + 6*b^2*c*d^2*x^4)*sqrt(c*x + 1)*sqrt(-c*x
+ 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.50

$$\begin{aligned}
&\int x^3\left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2\,dx \\
&= \frac{1}{8}\,b^2c^4d^2x^8\arccos(cx)^2 + \frac{1}{4}\,abc^4d^2x^8\arccos(cx) + \frac{1}{8}\,a^2c^4d^2x^8 - \frac{1}{256}\,b^2c^4d^2x^8 \\
&- \frac{1}{32}\,\sqrt{-c^2x^2+1}b^2c^3d^2x^7\arccos(cx) - \frac{1}{32}\,\sqrt{-c^2x^2+1}abc^3d^2x^7 \\
&- \frac{1}{3}\,b^2c^2d^2x^6\arccos(cx)^2 - \frac{2}{3}\,abc^2d^2x^6\arccos(cx) - \frac{1}{3}\,a^2c^2d^2x^6 + \frac{43}{3456}\,b^2c^2d^2x^6 \\
&+ \frac{43}{576}\,\sqrt{-c^2x^2+1}b^2cd^2x^5\arccos(cx) + \frac{43}{576}\,\sqrt{-c^2x^2+1}abcd^2x^5 \\
&+ \frac{1}{4}\,b^2d^2x^4\arccos(cx)^2 + \frac{1}{2}\,abd^2x^4\arccos(cx) + \frac{1}{4}\,a^2d^2x^4 - \frac{73}{9216}\,b^2d^2x^4 \\
&- \frac{73\,\sqrt{-c^2x^2+1}b^2d^2x^3\arccos(cx)}{2304\,c} - \frac{73\,\sqrt{-c^2x^2+1}abd^2x^3}{2304\,c} \\
&- \frac{73\,b^2d^2x^2}{3072\,c^2} - \frac{73\,\sqrt{-c^2x^2+1}b^2d^2x\arccos(cx)}{1536\,c^3} - \frac{73\,\sqrt{-c^2x^2+1}abd^2x}{1536\,c^3} \\
&- \frac{73\,b^2d^2\arccos(cx)^2}{3072\,c^4} - \frac{73\,abd^2\arccos(cx)}{1536\,c^4} + \frac{10645\,b^2d^2}{884736\,c^4}
\end{aligned}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
1/8*b^2*c^4*d^2*x^8*arccos(c*x)^2 + 1/4*a*b*c^4*d^2*x^8*arccos(c*x) + 1/8*
a^2*c^4*d^2*x^8 - 1/256*b^2*c^4*d^2*x^8 - 1/32*sqrt(-c^2*x^2 + 1)*b^2*c^3*
d^2*x^7*arccos(c*x) - 1/32*sqrt(-c^2*x^2 + 1)*a*b*c^3*d^2*x^7 - 1/3*b^2*c^
2*d^2*x^6*arccos(c*x)^2 - 2/3*a*b*c^2*d^2*x^6*arccos(c*x) - 1/3*a^2*c^2*d^
2*x^6 + 43/3456*b^2*c^2*d^2*x^6 + 43/576*sqrt(-c^2*x^2 + 1)*b^2*c*d^2*x^5*
arccos(c*x) + 43/576*sqrt(-c^2*x^2 + 1)*a*b*c*d^2*x^5 + 1/4*b^2*d^2*x^4*ar
ccos(c*x)^2 + 1/2*a*b*d^2*x^4*arccos(c*x) + 1/4*a^2*d^2*x^4 - 73/9216*b^2*
d^2*x^4 - 73/2304*sqrt(-c^2*x^2 + 1)*b^2*d^2*x^3*arccos(c*x)/c - 73/2304*s
qrt(-c^2*x^2 + 1)*a*b*d^2*x^3/c - 73/3072*b^2*d^2*x^2/c^2 - 73/1536*sqrt(-
c^2*x^2 + 1)*b^2*d^2*x*arccos(c*x)/c^3 - 73/1536*sqrt(-c^2*x^2 + 1)*a*b*d^
2*x/c^3 - 73/3072*b^2*d^2*arccos(c*x)^2/c^4 - 73/1536*a*b*d^2*arccos(c*x)/
c^4 + 10645/884736*b^2*d^2/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d-c^2dx^2)^2 (a+b\arccos(cx))^2\,dx = \int x^3\,(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^2\,dx$$

input
```
int(x^3*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^2,x)
```

output
```
int(x^3*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int x^3 (d-c^2dx^2)^2 (a+b\arccos(cx))^2\,dx$$
$$= \frac{d^2\left(1152acos(cx)\,ab\,c^8x^8 - 3072acos(cx)\,ab\,c^6x^6 + 2304acos(cx)\,ab\,c^4x^4 + 219asin(cx)\,ab - 144\sqrt{-c^2x^2}\right.}{}$$

input
```
int(x^3*(-c^2*d*x^2+d)^2*(a+b*acos(c*x))^2,x)
```

output

```
(d**2*(1152*acos(c*x)*a*b*c**8*x**8 - 3072*acos(c*x)*a*b*c**6*x**6 + 2304*
acos(c*x)*a*b*c**4*x**4 + 219*asin(c*x)*a*b - 144*sqrt( - c**2*x**2 + 1)*a
*b*c**7*x**7 + 344*sqrt( - c**2*x**2 + 1)*a*b*c**5*x**5 - 146*sqrt( - c**2
*x**2 + 1)*a*b*c**3*x**3 - 219*sqrt( - c**2*x**2 + 1)*a*b*c*x + 4608*int(a
cos(c*x)**2*x**7,x)*b**2*c**8 - 9216*int(acos(c*x)**2*x**5,x)*b**2*c**6 +
4608*int(acos(c*x)**2*x**3,x)*b**2*c**4 + 576*a**2*c**8*x**8 - 1536*a**2*c
**6*x**6 + 1152*a**2*c**4*x**4))/(4608*c**4)
```

3.169 $\int x^2(d-c^2dx^2)^2 (a+b\arccos(cx))^2 dx$

Optimal result

Integrand size = 27, antiderivative size = 310

$$\begin{aligned}\int x^2(d-c^2dx^2)^2 (a+b\arccos(cx))^2 dx &= -\frac{1636b^2d^2x}{11025c^2}-\frac{818b^2d^2x^3}{33075} \\ &+\frac{136b^2c^2d^2x^5}{6125}-\frac{2}{343}b^2c^4d^2x^7+\frac{32bd^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{315c^3} \\ &+\frac{16bd^2x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{315c}+\frac{8bd^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{105c^3} \\ &+\frac{2bd^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{175c^3}-\frac{2bd^2(1-c^2x^2)^{7/2}(a+b\arccos(cx))}{49c^3} \\ &+\frac{8}{105}d^2x^3(a+b\arccos(cx))^2+\frac{4}{35}d^2x^3(1-c^2x^2)(a+b\arccos(cx))^2+\frac{1}{7}d^2x^3(1-c^2x^2)^2(a+b\arccos(cx))^2\end{aligned}$$

output
```
-1636/11025*b^2*d^2*x/c^2-818/33075*b^2*d^2*x^3+136/6125*b^2*c^2*d^2*x^5-2
/343*b^2*c^4*d^2*x^7+32/315*b*d^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3
+16/315*b*d^2*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+8/105*b*d^2*(-c^2
*x^2+1)^(3/2)*(a+b*arccos(c*x))/c^3+2/175*b*d^2*(-c^2*x^2+1)^(5/2)*(a+b*ar
ccos(c*x))/c^3-2/49*b*d^2*(-c^2*x^2+1)^(7/2)*(a+b*arccos(c*x))/c^3+8/105*d
^2*x^3*(a+b*arccos(c*x))^2+4/35*d^2*x^3*(-c^2*x^2+1)*(a+b*arccos(c*x))^2+1
/7*d^2*x^3*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.74

$$\int x^2 (d - c^2 dx^2)^2 (a + b \arccos(cx))^2 \, dx$$
$$= \frac{d^2 (11025a^2c^3x^3(35 - 42c^2x^2 + 15c^4x^4) - 210ab\sqrt{1 - c^2x^2}(818 + 409c^2x^2 - 612c^4x^4 + 225c^6x^6) - 2b^2cx}{}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*(11025*a^2*c^3*x^3*(35 - 42*c^2*x^2 + 15*c^4*x^4) - 210*a*b*Sqrt[1 -
c^2*x^2]*(818 + 409*c^2*x^2 - 612*c^4*x^4 + 225*c^6*x^6) - 2*b^2*c*x*(8589
0 + 14315*c^2*x^2 - 12852*c^4*x^4 + 3375*c^6*x^6) - 210*b*(-105*a*c^3*x^3*
(35 - 42*c^2*x^2 + 15*c^4*x^4) + b*Sqrt[1 - c^2*x^2]*(818 + 409*c^2*x^2 -
612*c^4*x^4 + 225*c^6*x^6))*ArcCos[c*x] + 11025*b^2*c^3*x^3*(35 - 42*c^2*x
^2 + 15*c^4*x^4)*ArcCos[c*x]^2))/(1157625*c^3)
```

Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.25, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5203, 27, 5195, 27, 290, 2009, 5203, 5139, 5195, 27, 2009, 5211, 15, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d - c^2 dx^2)^2 (a + b \arccos(cx))^2 \, dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{2}{7} bcd^2 \int x^3 (1 - c^2x^2)^{3/2} (a + b \arccos(cx)) dx + \frac{4}{7} d \int dx^2 (1 - c^2x^2) (a + b \arccos(cx))^2 dx + \frac{1}{7} d^2 x^3 (1 - c^2x^2)^2 (a + b \arccos(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{7}d^2\int x^2\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{2}{7}bcd^2\int x^3\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{7}d^2x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\Big\downarrow$ 5195

$$\frac{4}{7}d^2\int x^2\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{2}{7}bcd^2\left(bc\int -\frac{\left(1-c^2x^2\right)^2\left(5c^2x^2+2\right)}{35c^4}dx+\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}\right)+\frac{1}{7}d^2x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\Big\downarrow$ 27

$$\frac{4}{7}d^2\int x^2\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{2}{7}bcd^2\left(-\frac{b\int\left(1-c^2x^2\right)^2\left(5c^2x^2+2\right)dx}{35c^3}+\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}\right)+\frac{1}{7}d^2x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\Big\downarrow$ 290

$$\frac{4}{7}d^2\int x^2\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{2}{7}bcd^2\left(-\frac{b\int\left(5c^6x^6-8c^4x^4+c^2x^2+2\right)dx}{35c^3}+\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}\right)+\frac{1}{7}d^2x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$\Big\downarrow$ 2009

$$\frac{4}{7}d^2\int x^2\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{1}{7}d^2x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{2}{7}bcd^2\left(\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{b\left(\frac{5c^6x^7}{7}-\frac{8c^4x^5}{5}+\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$\Big\downarrow$ 5203

$$\frac{4}{7}d^2\left(\frac{2}{5}bc\int x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{2}{5}\int x^2(a+b\arccos(cx))^2dx+\frac{1}{5}x^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2\right)+\frac{1}{7}d^2x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{2}{7}bcd^2\left(\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{b\left(\frac{5c^6x^7}{7}-\frac{8c^4x^5}{5}+\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$$\Big\downarrow \text{5139}$$

$$\frac{4}{7}d^2\left(\frac{2}{5}\left(\frac{2}{3}bc\int\frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)+\frac{2}{5}bc\int x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\right.$$
$$\frac{1}{7}d^2x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{2}{7}bcd^2\left(\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{b\left(\frac{5c^6x^7}{7}-\frac{8c^4x^5}{5}+\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$$\Big\downarrow \text{5195}$$

$$\frac{4}{7}d^2\left(\frac{2}{5}\left(\frac{2}{3}bc\int\frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)+\frac{2}{5}bc\left(bc\int-\frac{-3c^4x^4+c^2x^2+2}{15c^4}dx+\frac{(1-\right.\right.$$
$$\frac{1}{7}d^2x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{2}{7}bcd^2\left(\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{b\left(\frac{5c^6x^7}{7}-\frac{8c^4x^5}{5}+\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{7}d^2\left(\frac{2}{5}\left(\frac{2}{3}bc\int\frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)+\frac{2}{5}bc\left(-\frac{b\int\left(-3c^4x^4+c^2x^2+2\right)dx}{15c^3}+\frac{(1-\right.\right.$$
$$\frac{1}{7}d^2x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{2}{7}bcd^2\left(\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{b\left(\frac{5c^6x^7}{7}-\frac{8c^4x^5}{5}+\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{7}d^2\left(\frac{2}{5}\left(\frac{2}{3}bc\int\frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)+\frac{1}{5}x^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{5}bc\left(\right.\right.$$
$$\frac{1}{7}d^2x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{2}{7}bcd^2\left(\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{b\left(\frac{5c^6x^7}{7}-\frac{8c^4x^5}{5}+\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$$\Big\downarrow \text{5211}$$

$$\frac{4}{7}d^2\left(\frac{2}{5}\left(\frac{2}{3}bc\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{b\int x^2dx}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)\right.$$
$$\frac{1}{7}d^2x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{2}{7}bcd^2\left(\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{b\left(\frac{5c^6x^7}{7}-\frac{8c^4x^5}{5}+\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$\Big\downarrow$ 15

$$\frac{4}{7}d^2\left(\frac{2}{5}\left(\frac{2}{3}bc\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)+\frac{1}{5}\right.$$
$$\frac{1}{7}d^2x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{2}{7}bcd^2\left(\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{b\left(\frac{5c^6x^7}{7}-\frac{8c^4x^5}{5}+\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$\Big\downarrow$ 5183

$$\frac{4}{7}d^2\left(\frac{2}{5}\left(\frac{2}{3}bc\left(\frac{2\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)+\frac{1}{3}x^3(a+b\arccos\right.\right.\right.$$
$$\frac{1}{7}d^2x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{2}{7}bcd^2\left(\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{b\left(\frac{5c^6x^7}{7}-\frac{8c^4x^5}{5}+\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

$\Big\downarrow$ 24

$$\frac{1}{7}d^2x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{4}{7}d^2\left(\frac{1}{5}x^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{5}\left(\frac{2}{3}bc\left(-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}+\frac{2\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx}{c^2}\right.}{3c^2}\right.\right.\right.$$
$$\frac{2}{7}bcd^2\left(\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^4}-\frac{b\left(\frac{5c^6x^7}{7}-\frac{8c^4x^5}{5}+\frac{c^2x^3}{3}+2x\right)}{35c^3}\right)$$

input
```
Int[x^2*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*x^3*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/7 + (2*b*c*d^2*(-1/35*(b*(
2*x + (c^2*x^3)/3 - (8*c^4*x^5)/5 + (5*c^6*x^7)/7))/c^3 - ((1 - c^2*x^2)^(
5/2)*(a + b*ArcCos[c*x]))/(5*c^4) + ((1 - c^2*x^2)^(7/2)*(a + b*ArcCos[c*x
]))/(7*c^4)))/7 + (4*d^2*((x^3*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/5 + (2
*b*c*(-1/15*(b*(2*x + (c^2*x^3)/3 - (3*c^4*x^5)/5))/c^3 - ((1 - c^2*x^2)^(
3/2)*(a + b*ArcCos[c*x]))/(3*c^4) + ((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x
]))/(5*c^4)))/5 + (2*((x^3*(a + b*ArcCos[c*x])^2)/3 + (2*b*c*(-1/9*(b*x^3)
/c - (x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(3*c^2) + (2*(-((b*x)/c)
- (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c^2))/(3*c^2)))/3))/5))/7
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.29

method	result
parts	$d^2a^2\left(\frac{1}{7}c^4x^7-\frac{2}{5}c^2x^5+\frac{1}{3}x^3\right)+\frac{d^2b^2\left(\frac{\arccos(cx)^2\left(3c^4x^4-10c^2x^2+15\right)cx}{15}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^2\sqrt{-c^2x^2+1}}{175}-\right.}{}$
derivativedivides	$\frac{d^2a^2\left(\frac{1}{7}c^7x^7-\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b^2\left(\frac{\arccos(cx)^2\left(3c^4x^4-10c^2x^2+15\right)cx}{15}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^2\sqrt{-c^2x^2+1}}{175}-\frac{2\left(3c^4x^4-10\right.}{26}\right.}{}$
default	$\frac{d^2a^2\left(\frac{1}{7}c^7x^7-\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b^2\left(\frac{\arccos(cx)^2\left(3c^4x^4-10c^2x^2+15\right)cx}{15}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^2\sqrt{-c^2x^2+1}}{175}-\frac{2\left(3c^4x^4-10\right.}{26}\right.}{}$
orering	$\frac{\left(428625c^{10}x^{10}-1739907c^8x^8+2486259c^6x^6+2357383c^4x^4-2404920c^2x^2+687120\right)\left(-c^2d\,x^2+d\right)^2\left(a+b\arccos(cx)\right)^2}{1157625x\,c^4(cx-1)(cx+1)\left(c^2x^2-1\right)^2}$

input
```
int(x^2*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(1/7*c^4*x^7-2/5*c^2*x^5+1/3*x^3)+d^2*b^2/c^3*(1/15*arccos(c*x)^2*
(3*c^4*x^4-10*c^2*x^2+15)*c*x-2/175*arccos(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)
^(1/2)-2/2625*(3*c^4*x^4-10*c^2*x^2+15)*c*x+8/315*arccos(c*x)*(c^2*x^2-1)*
(-c^2*x^2+1)^(1/2)+8/945*(c^2*x^2-3)*c*x-16/105*c*x-16/105*arccos(c*x)*(-c
^2*x^2+1)^(1/2)+1/35*arccos(c*x)^2*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c*
x-2/49*arccos(c*x)*(c^2*x^2-1)^3*(-c^2*x^2+1)^(1/2)-2/1715*(5*c^6*x^6-21*c
^4*x^4+35*c^2*x^2-35)*c*x)+2*d^2*a*b/c^3*(1/7*arccos(c*x)*c^7*x^7-2/5*arcc
os(c*x)*c^5*x^5+1/3*c^3*x^3*arccos(c*x)-409/11025*c^2*x^2*(-c^2*x^2+1)^(1/
2)-818/11025*(-c^2*x^2+1)^(1/2)+68/1225*c^4*x^4*(-c^2*x^2+1)^(1/2)-1/49*c^
6*x^6*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.95

$$\int x^2\left(d-c^2dx^2\right)^2\left(a+b\arccos(cx)\right)^2dx$$

$$=\frac{3375\left(49a^2-2b^2\right)c^7d^2x^7-378\left(1225a^2-68b^2\right)c^5d^2x^5+35\left(11025a^2-818b^2\right)c^3d^2x^3-171780b^2cd^2x}{}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
1/1157625*(3375*(49*a^2 - 2*b^2)*c^7*d^2*x^7 - 378*(1225*a^2 - 68*b^2)*c^5
*d^2*x^5 + 35*(11025*a^2 - 818*b^2)*c^3*d^2*x^3 - 171780*b^2*c*d^2*x + 110
25*(15*b^2*c^7*d^2*x^7 - 42*b^2*c^5*d^2*x^5 + 35*b^2*c^3*d^2*x^3)*arccos(c
*x)^2 + 22050*(15*a*b*c^7*d^2*x^7 - 42*a*b*c^5*d^2*x^5 + 35*a*b*c^3*d^2*x^
3)*arccos(c*x) - 210*(225*a*b*c^6*d^2*x^6 - 612*a*b*c^4*d^2*x^4 + 409*a*b*
c^2*d^2*x^2 + 818*a*b*d^2 + (225*b^2*c^6*d^2*x^6 - 612*b^2*c^4*d^2*x^4 + 4
09*b^2*c^2*d^2*x^2 + 818*b^2*d^2)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.57

$$\int x^2 (d - c^2 d x^2)^2 (a + b \arccos(cx))^2 \, dx$$

$$= \begin{cases} \frac{a^2c^4d^2x^7}{7} - \frac{2a^2c^2d^2x^5}{5} + \frac{a^2d^2x^3}{3} + \frac{2abc^4d^2x^7 \operatorname{acos}(cx)}{7} - \frac{2abc^3d^2x^6\sqrt{-c^2x^2+1}}{49} - \frac{4abc^2d^2x^5 \operatorname{acos}(cx)}{5} + \frac{136abcd^2x^4\sqrt{-c^2x^2+1}}{1225} \\ \frac{d^2x^3\left(a+\frac{\pi b}{2}\right)^2}{3} \end{cases}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**2*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*c**4*d**2*x**7/7 - 2*a**2*c**2*d**2*x**5/5 + a**2*d**2*x**
3/3 + 2*a*b*c**4*d**2*x**7*acos(c*x)/7 - 2*a*b*c**3*d**2*x**6*sqrt(-c**2*x
**2 + 1)/49 - 4*a*b*c**2*d**2*x**5*acos(c*x)/5 + 136*a*b*c*d**2*x**4*sqrt(
-c**2*x**2 + 1)/1225 + 2*a*b*d**2*x**3*acos(c*x)/3 - 818*a*b*d**2*x**2*sqr
t(-c**2*x**2 + 1)/(11025*c) - 1636*a*b*d**2*sqrt(-c**2*x**2 + 1)/(11025*c*
*3) + b**2*c**4*d**2*x**7*acos(c*x)**2/7 - 2*b**2*c**4*d**2*x**7/343 - 2*b
**2*c**3*d**2*x**6*sqrt(-c**2*x**2 + 1)*acos(c*x)/49 - 2*b**2*c**2*d**2*x*
*5*acos(c*x)**2/5 + 136*b**2*c**2*d**2*x**5/6125 + 136*b**2*c*d**2*x**4*sq
rt(-c**2*x**2 + 1)*acos(c*x)/1225 + b**2*d**2*x**3*acos(c*x)**2/3 - 818*b*
*2*d**2*x**3/33075 - 818*b**2*d**2*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(11
025*c) - 1636*b**2*d**2*x/(11025*c**2) - 1636*b**2*d**2*sqrt(-c**2*x**2 +
1)*acos(c*x)/(11025*c**3), Ne(c, 0)), (d**2*x**3*(a + pi*b/2)**2/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(274) = 548$.

Time = 0.14 (sec) , antiderivative size = 634, normalized size of antiderivative = 2.05

$$\int x^2(d-c^2dx^2)^2(a+b\arccos(cx))^2\,dx$$
$$= \frac{1}{7}b^2c^4d^2x^7\arccos(cx)^2 + \frac{1}{7}a^2c^4d^2x^7 - \frac{2}{5}b^2c^2d^2x^5\arccos(cx)^2 - \frac{2}{5}a^2c^2d^2x^5$$
$$+ \frac{2}{245}\left(35\,x^7\arccos(cx) - \left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\sqrt{-c^2x^2+1}}{c^8}\right.\right.$$
$$- \frac{2}{25725}\left(105\left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\sqrt{-c^2x^2+1}}{c^8}\right)c\arccos(cx\right.$$
$$+ \frac{1}{3}b^2d^2x^3\arccos(cx)^2$$
$$- \frac{4}{75}\left(15\,x^5\arccos(cx) - \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)abc^2d^2$$
$$+ \frac{4}{1125}\left(15\left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\arccos(cx) + \frac{9\,c^4x^5 + 20\,c^2x^3 +}{c^4}\right.$$
$$+ \frac{1}{3}a^2d^2x^3 + \frac{2}{9}\left(3\,x^3\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)abd^2$$
$$- \frac{2}{27}\left(3\,c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\arccos(cx) + \frac{c^2x^3+6\,x}{c^2}\right)b^2d^2$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/7*b^2*c^4*d^2*x^7*arccos(c*x)^2 + 1/7*a^2*c^4*d^2*x^7 - 2/5*b^2*c^2*d^2*
x^5*arccos(c*x)^2 - 2/5*a^2*c^2*d^2*x^5 + 2/245*(35*x^7*arccos(c*x) - (5*s
qrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2
 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*a*b*c^4*d^2 - 2/25725*(105*(
5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*
x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c*arccos(c*x) + (75*c^6*x^7
+ 126*c^4*x^5 + 280*c^2*x^3 + 1680*x)/c^6)*b^2*c^4*d^2 + 1/3*b^2*d^2*x^3*a
rccos(c*x)^2 - 4/75*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 +
4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a*b*c^2*d^2 +
4/1125*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 +
8*sqrt(-c^2*x^2 + 1)/c^6)*c*arccos(c*x) + (9*c^4*x^5 + 20*c^2*x^3 + 120*x)
/c^4)*b^2*c^2*d^2 + 1/3*a^2*d^2*x^3 + 2/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^
2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*d^2 - 2/27*(3*c*(sqrt(
-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x) + (c^2*x^3 +
 6*x)/c^2)*b^2*d^2
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.31

$$\begin{aligned}
&\int x^2\left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2\,dx \\
&= \frac{1}{7}\,b^2c^4d^2x^7\arccos\left(cx\right)^2 + \frac{2}{7}\,abc^4d^2x^7\arccos\left(cx\right) + \frac{1}{7}\,a^2c^4d^2x^7 - \frac{2}{343}\,b^2c^4d^2x^7 \\
&\quad - \frac{2}{49}\,\sqrt{-c^2x^2+1}b^2c^3d^2x^6\arccos\left(cx\right) - \frac{2}{49}\,\sqrt{-c^2x^2+1}abc^3d^2x^6 \\
&\quad - \frac{2}{5}\,b^2c^2d^2x^5\arccos\left(cx\right)^2 - \frac{4}{5}\,abc^2d^2x^5\arccos\left(cx\right) - \frac{2}{5}\,a^2c^2d^2x^5 + \frac{136}{6125}\,b^2c^2d^2x^5 \\
&\quad + \frac{136}{1225}\,\sqrt{-c^2x^2+1}b^2cd^2x^4\arccos\left(cx\right) + \frac{136}{1225}\,\sqrt{-c^2x^2+1}abcd^2x^4 \\
&\quad + \frac{1}{3}\,b^2d^2x^3\arccos\left(cx\right)^2 + \frac{2}{3}\,abd^2x^3\arccos\left(cx\right) + \frac{1}{3}\,a^2d^2x^3 - \frac{818}{33075}\,b^2d^2x^3 \\
&\quad - \frac{818\,\sqrt{-c^2x^2+1}b^2d^2x^2\arccos\left(cx\right)}{11025\,c} - \frac{818\,\sqrt{-c^2x^2+1}abd^2x^2}{11025\,c} \\
&\quad - \frac{1636\,b^2d^2x}{11025\,c^2} - \frac{1636\,\sqrt{-c^2x^2+1}b^2d^2\arccos\left(cx\right)}{11025\,c^3} - \frac{1636\,\sqrt{-c^2x^2+1}abd^2}{11025\,c^3}
\end{aligned}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
1/7*b^2*c^4*d^2*x^7*arccos(c*x)^2 + 2/7*a*b*c^4*d^2*x^7*arccos(c*x) + 1/7*
a^2*c^4*d^2*x^7 - 2/343*b^2*c^4*d^2*x^7 - 2/49*sqrt(-c^2*x^2 + 1)*b^2*c^3*
d^2*x^6*arccos(c*x) - 2/49*sqrt(-c^2*x^2 + 1)*a*b*c^3*d^2*x^6 - 2/5*b^2*c^
2*d^2*x^5*arccos(c*x)^2 - 4/5*a*b*c^2*d^2*x^5*arccos(c*x) - 2/5*a^2*c^2*d^
2*x^5 + 136/6125*b^2*c^2*d^2*x^5 + 136/1225*sqrt(-c^2*x^2 + 1)*b^2*c*d^2*x
^4*arccos(c*x) + 136/1225*sqrt(-c^2*x^2 + 1)*a*b*c*d^2*x^4 + 1/3*b^2*d^2*x
^3*arccos(c*x)^2 + 2/3*a*b*d^2*x^3*arccos(c*x) + 1/3*a^2*d^2*x^3 - 818/330
75*b^2*d^2*x^3 - 818/11025*sqrt(-c^2*x^2 + 1)*b^2*d^2*x^2*arccos(c*x)/c -
818/11025*sqrt(-c^2*x^2 + 1)*a*b*d^2*x^2/c - 1636/11025*b^2*d^2*x/c^2 - 16
36/11025*sqrt(-c^2*x^2 + 1)*b^2*d^2*arccos(c*x)/c^3 - 1636/11025*sqrt(-c^2
*x^2 + 1)*a*b*d^2/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d-c^2dx^2)^2 (a+b\arccos(cx))^2\,dx=\int x^2\,(a+b\,\mathrm{acos}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^2\,dx$$

input
```
int(x^2*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^2,x)
```

output
```
int(x^2*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int x^2 (d-c^2dx^2)^2 (a+b\arccos(cx))^2\,dx$$
$$=\frac{d^2\left(3150acos(cx)\,ab\,c^7x^7-8820acos(cx)\,ab\,c^5x^5+7350acos(cx)\,ab\,c^3x^3-450\sqrt{-c^2x^2+1}\,ab\,c^6x^6+122\right.}{}$$

input
```
int(x^2*(-c^2*d*x^2+d)^2*(a+b*acos(c*x))^2,x)
```

output
```
(d**2*(3150*acos(c*x)*a*b*c**7*x**7 - 8820*acos(c*x)*a*b*c**5*x**5 + 7350*
acos(c*x)*a*b*c**3*x**3 - 450*sqrt( - c**2*x**2 + 1)*a*b*c**6*x**6 + 1224*
sqrt( - c**2*x**2 + 1)*a*b*c**4*x**4 - 818*sqrt( - c**2*x**2 + 1)*a*b*c**2
*x**2 - 1636*sqrt( - c**2*x**2 + 1)*a*b + 11025*int(acos(c*x)**2*x**6,x)*b
**2*c**7 - 22050*int(acos(c*x)**2*x**4,x)*b**2*c**5 + 11025*int(acos(c*x)*
*2*x**2,x)*b**2*c**3 + 1575*a**2*c**7*x**7 - 4410*a**2*c**5*x**5 + 3675*a*
*2*c**3*x**3))/(11025*c**3)
```

3.170 $\int x(d-c^2dx^2)^2(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 218

$$\begin{aligned}\int x(d-c^2dx^2)^2(a+b\arccos(cx))^2\,dx = &-\frac{5}{96}b^2d^2x^2+\frac{5b^2d^2(1-c^2x^2)^2}{288c^2}\\&+\frac{b^2d^2(1-c^2x^2)^3}{108c^2}\\&+\frac{5bd^2x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{48c}\\&+\frac{5bd^2x(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{72c}\\&+\frac{bd^2x(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{18c}\\&+\frac{5d^2(a+b\arccos(cx))^2}{96c^2}\\&-\frac{d^2(1-c^2x^2)^3(a+b\arccos(cx))^2}{6c^2}\end{aligned}$$

output

```
-5/96*b^2*d^2*x^2+5/288*b^2*d^2*(-c^2*x^2+1)^2/c^2+1/108*b^2*d^2*(-c^2*x^2
+1)^3/c^2+5/48*b*d^2*x*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+5/72*b*d^2*x
*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/c+1/18*b*d^2*x*(-c^2*x^2+1)^(5/2)*(a
+b*arccos(c*x))/c+5/96*d^2*(a+b*arccos(c*x))^2/c^2-1/6*d^2*(-c^2*x^2+1)^3*
(a+b*arccos(c*x))^2/c^2
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.96

$$\int x\left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2\,dx$$
$$=\frac{d^2\left(cx\left(b^2cx(-99+39c^2x^2-8c^4x^4)+144a^2cx(3-3c^2x^2+c^4x^4)-6ab\sqrt{1-c^2x^2}(33-26c^2x^2+8c^4x^4)\right.\right.}{}$$

input
```
Integrate[x*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*(c*x*(b^2*c*x*(-99 + 39*c^2*x^2 - 8*c^4*x^4) + 144*a^2*c*x*(3 - 3*c^2
*x^2 + c^4*x^4) - 6*a*b*Sqrt[1 - c^2*x^2]*(33 - 26*c^2*x^2 + 8*c^4*x^4)) +
 6*b*c*x*(b*Sqrt[1 - c^2*x^2]*(-33 + 26*c^2*x^2 - 8*c^4*x^4) + 48*a*c*x*(3
 - 3*c^2*x^2 + c^4*x^4))*ArcCos[c*x] + 9*b^2*(-11 + 48*c^2*x^2 - 48*c^4*x^
4 + 16*c^6*x^6)*ArcCos[c*x]^2 + 198*a*b*ArcSin[c*x]))/(864*c^2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.360$, Rules used = {5183, 5159, 241, 5159, 244, 2009, 5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5183}$$

$$-\frac{bd^2\int\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx}{3c}-\frac{d^2\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{6c^2}$$

$$\Big\downarrow \text{5159}$$

$$-\frac{bd^2\left(\frac{5}{6}\int \left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{6}bc\int x\left(1-c^2x^2\right)^2dx+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))\right)}{3c}-\frac{d^2\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{6c^2}$$

$\downarrow$ 241

$$-\frac{bd^2\left(\frac{5}{6}\int \left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))-\frac{b\left(1-c^2x^2\right)^3}{36c}\right)}{3c}-\frac{d^2\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{6c^2}$$

$\downarrow$ 5159

$$-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int x\left(1-c^2x^2\right)dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)+\frac{1}{6}x\right.}{3c}$$
$$\frac{d^2\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{6c^2}$$

$\downarrow$ 244

$$-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int \left(x-c^2x^3\right)dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)+\frac{1}{6}x(\right.}{3c}$$
$$\frac{d^2\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{6c^2}$$

$\downarrow$ 2009

$$-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\right)+\frac{1}{6}x(1-\right.}{3c}$$
$$\frac{d^2\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{6c^2}$$

$\downarrow$ 5157

$$-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(\right.\right.}{3c}$$
$$\frac{d^2\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{6c^2}$$

$$\downarrow 15$$

$$-\frac{bd^2\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{4}bcx^2\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx)\right.}{3c}$$

$$\frac{d^2\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{6c^2}$$

$$\downarrow 5153$$

$$-\frac{d^2\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{6c^2}-$$

$$\frac{bd^2\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(\right.\right.\right.}{3c}$$

input
```
Int[x*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/6*(d^2*(1 - c^2*x^2)^3*(a + b*ArcCos[c*x])^2)/c^2 - (b*d^2*(-1/36*(b*(1
 - c^2*x^2)^3)/c + (x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/6 + (5*((b*
c*(x^2/2 - (c^2*x^4)/4))/4 + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4
 + (3*((b*c*x^2)/4 + (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/2 - (a + b*
ArcCos[c*x])^2/(4*b*c)))/4))/6))/(3*c)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{\frac{d^2a^2(c^2x^2-1)^3}{6}+d^2b^2\left(\frac{\arccos(cx)^2(c^2x^2-1)^3}{6}+\frac{\arccos(cx)\left(-8c^5x^5\sqrt{-c^2x^2+1}+26c^3x^3\sqrt{-c^2x^2+1}-33cx\sqrt{-c^2x^2+1}+15\,\mathrm{arcc}\right.}{144}\right.}{}$
default	$\frac{\frac{d^2a^2(c^2x^2-1)^3}{6}+d^2b^2\left(\frac{\arccos(cx)^2(c^2x^2-1)^3}{6}+\frac{\arccos(cx)\left(-8c^5x^5\sqrt{-c^2x^2+1}+26c^3x^3\sqrt{-c^2x^2+1}-33cx\sqrt{-c^2x^2+1}+15\,\mathrm{arcc}\right.}{144}\right.}{}$
parts	$\frac{d^2a^2(c^2x^2-1)^3}{6c^2}+\frac{d^2b^2\left(\frac{\arccos(cx)^2(c^2x^2-1)^3}{6}+\frac{\arccos(cx)\left(-8c^5x^5\sqrt{-c^2x^2+1}+26c^3x^3\sqrt{-c^2x^2+1}-33cx\sqrt{-c^2x^2+1}+15\right.}{144}\right.}{c^2}$
orering	$\frac{(728c^8x^8-3251c^6x^6+6466c^4x^4-3177c^2x^2+594)(-c^2d\,x^2+d)^2(a+b\arccos(cx))^2}{1728c^2(cx-1)(cx+1)(c^2x^2-1)^2}-\frac{(120c^6x^6-571c^4x^4+1323c^2x^2}{}$

input
```
int(x*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/6*d^2*a^2*(c^2*x^2-1)^3+d^2*b^2*(1/6*arccos(c*x)^2*(c^2*x^2-1)^3+
1/144*arccos(c*x)*(-8*c^5*x^5*(-c^2*x^2+1)^(1/2)+26*c^3*x^3*(-c^2*x^2+1)^(
1/2)-33*c*x*(-c^2*x^2+1)^(1/2)+15*arccos(c*x))-5/96*arccos(c*x)^2-1/108*c^
6*x^6+13/288*c^4*x^4-11/96*c^2*x^2)+2*d^2*a*b*(1/6*arccos(c*x)*c^6*x^6-1/2
*c^4*x^4*arccos(c*x)+1/2*c^2*x^2*arccos(c*x)-1/6*arccos(c*x)-1/36*c^5*x^5*
(-c^2*x^2+1)^(1/2)+13/144*c^3*x^3*(-c^2*x^2+1)^(1/2)-11/96*c*x*(-c^2*x^2+1
)^(1/2)-5/96*arcsin(c*x)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.28

$$\int x(d-c^2dx^2)^2(a+b\arccos(cx))^2\,dx$$
$$=\frac{8\,(18\,a^2-b^2)c^6d^2x^6-3\,(144\,a^2-13\,b^2)c^4d^2x^4+9\,(48\,a^2-11\,b^2)c^2d^2x^2+9\,(16\,b^2c^6d^2x^6-48\,b^2c^4d^2x^4}{}$$

input
```
integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
1/864*(8*(18*a^2 - b^2)*c^6*d^2*x^6 - 3*(144*a^2 - 13*b^2)*c^4*d^2*x^4 + 9
*(48*a^2 - 11*b^2)*c^2*d^2*x^2 + 9*(16*b^2*c^6*d^2*x^6 - 48*b^2*c^4*d^2*x^
4 + 48*b^2*c^2*d^2*x^2 - 11*b^2*d^2)*arccos(c*x)^2 + 18*(16*a*b*c^6*d^2*x^
6 - 48*a*b*c^4*d^2*x^4 + 48*a*b*c^2*d^2*x^2 - 11*a*b*d^2)*arccos(c*x) - 6*
(8*a*b*c^5*d^2*x^5 - 26*a*b*c^3*d^2*x^3 + 33*a*b*c*d^2*x + (8*b^2*c^5*d^2*
x^5 - 26*b^2*c^3*d^2*x^3 + 33*b^2*c*d^2*x)*arccos(c*x))*sqrt(-c^2*x^2 + 1)
)/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(202) = 404$.

Time = 0.68 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.00

$$\int x(d-c^2dx^2)^2 (a+b\arccos(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2c^4d^2x^6}{6} - \frac{a^2c^2d^2x^4}{2} + \frac{a^2d^2x^2}{2} + \frac{abc^4d^2x^6\operatorname{acos}(cx)}{3} - \frac{abc^3d^2x^5\sqrt{-c^2x^2+1}}{18} - abc^2d^2x^4\operatorname{acos}(cx) + \frac{13abcd^2x^3\sqrt{-c^2x^2+1}}{72} \\ \frac{d^2x^2\left(a+\frac{\pi b}{2}\right)^2}{2} \end{cases}$$

input
```
integrate(x*(-c**2*d*x**2+d)**2*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*c**4*d**2*x**6/6 - a**2*c**2*d**2*x**4/2 + a**2*d**2*x**2/
2 + a*b*c**4*d**2*x**6*acos(c*x)/3 - a*b*c**3*d**2*x**5*sqrt(-c**2*x**2 +
1)/18 - a*b*c**2*d**2*x**4*acos(c*x) + 13*a*b*c*d**2*x**3*sqrt(-c**2*x**2
+ 1)/72 + a*b*d**2*x**2*acos(c*x) - 11*a*b*d**2*x*sqrt(-c**2*x**2 + 1)/(48
*c) - 11*a*b*d**2*acos(c*x)/(48*c**2) + b**2*c**4*d**2*x**6*acos(c*x)**2/6
 - b**2*c**4*d**2*x**6/108 - b**2*c**3*d**2*x**5*sqrt(-c**2*x**2 + 1)*acos
(c*x)/18 - b**2*c**2*d**2*x**4*acos(c*x)**2/2 + 13*b**2*c**2*d**2*x**4/288
 + 13*b**2*c*d**2*x**3*sqrt(-c**2*x**2 + 1)*acos(c*x)/72 + b**2*d**2*x**2*
acos(c*x)**2/2 - 11*b**2*d**2*x**2/96 - 11*b**2*d**2*x*sqrt(-c**2*x**2 + 1
)*acos(c*x)/(48*c) - 11*b**2*d**2*acos(c*x)**2/(96*c**2), Ne(c, 0)), (d**2
*x**2*(a + pi*b/2)**2/2, True))
```

Maxima [F]

$$\int x\left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2\,dx=\int \left(c^2dx^2-d\right)^2(b\arccos{(cx)}+a)^2x\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/6*a^2*c^4*d^2*x^6 - 1/2*a^2*c^2*d^2*x^4 + 1/144*(48*x^6*arccos(c*x) - (8
*sqrt(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2
*x^2 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*a*b*c^4*d^2 - 1/8*(8*x^4*arccos(c
*x) - (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsi
n(c*x)/c^5)*c)*a*b*c^2*d^2 + 1/2*a^2*d^2*x^2 + 1/2*(2*x^2*arccos(c*x) - c*
(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*x)/c^3))*a*b*d^2 + 1/6*(b^2*c^4*d^2*x
^6 - 3*b^2*c^2*d^2*x^4 + 3*b^2*d^2*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x)^2 - integrate(1/3*(b^2*c^5*d^2*x^6 - 3*b^2*c^3*d^2*x^4 + 3*b^2*c*
d^2*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1)
, c*x)/(c^2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.75

$$\begin{aligned}&\int x\left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2\,dx\\&=\frac{1}{6}\,b^2c^4d^2x^6\arccos{(cx)}^2+\frac{1}{3}\,abc^4d^2x^6\arccos{(cx)}+\frac{1}{6}\,a^2c^4d^2x^6-\frac{1}{108}\,b^2c^4d^2x^6\\&\quad-\frac{1}{18}\,\sqrt{-c^2x^2+1}b^2c^3d^2x^5\arccos{(cx)}-\frac{1}{18}\,\sqrt{-c^2x^2+1}abc^3d^2x^5\\&\quad-\frac{1}{2}\,b^2c^2d^2x^4\arccos{(cx)}^2-abc^2d^2x^4\arccos{(cx)}-\frac{1}{2}\,a^2c^2d^2x^4+\frac{13}{288}\,b^2c^2d^2x^4\\&\quad+\frac{13}{72}\,\sqrt{-c^2x^2+1}b^2cd^2x^3\arccos{(cx)}+\frac{13}{72}\,\sqrt{-c^2x^2+1}abcd^2x^3+\frac{1}{2}\,b^2d^2x^2\arccos{(cx)}^2\\&\quad+abd^2x^2\arccos{(cx)}+\frac{1}{2}\,a^2d^2x^2-\frac{11}{96}\,b^2d^2x^2-\frac{11\,\sqrt{-c^2x^2+1}b^2d^2x\arccos{(cx)}}{48\,c}\\&\quad-\frac{11\,\sqrt{-c^2x^2+1}abd^2x}{48\,c}-\frac{11\,b^2d^2\arccos{(cx)}^2}{96\,c^2}-\frac{11\,abd^2\arccos{(cx)}}{48\,c^2}+\frac{299\,b^2d^2}{6912\,c^2}\end{aligned}$$

input
```
integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
1/6*b^2*c^4*d^2*x^6*arccos(c*x)^2 + 1/3*a*b*c^4*d^2*x^6*arccos(c*x) + 1/6*
a^2*c^4*d^2*x^6 - 1/108*b^2*c^4*d^2*x^6 - 1/18*sqrt(-c^2*x^2 + 1)*b^2*c^3*
d^2*x^5*arccos(c*x) - 1/18*sqrt(-c^2*x^2 + 1)*a*b*c^3*d^2*x^5 - 1/2*b^2*c^
2*d^2*x^4*arccos(c*x)^2 - a*b*c^2*d^2*x^4*arccos(c*x) - 1/2*a^2*c^2*d^2*x^
4 + 13/288*b^2*c^2*d^2*x^4 + 13/72*sqrt(-c^2*x^2 + 1)*b^2*c*d^2*x^3*arccos
(c*x) + 13/72*sqrt(-c^2*x^2 + 1)*a*b*c*d^2*x^3 + 1/2*b^2*d^2*x^2*arccos(c*
x)^2 + a*b*d^2*x^2*arccos(c*x) + 1/2*a^2*d^2*x^2 - 11/96*b^2*d^2*x^2 - 11/
48*sqrt(-c^2*x^2 + 1)*b^2*d^2*x*arccos(c*x)/c - 11/48*sqrt(-c^2*x^2 + 1)*a
*b*d^2*x/c - 11/96*b^2*d^2*arccos(c*x)^2/c^2 - 11/48*a*b*d^2*arccos(c*x)/c
^2 + 299/6912*b^2*d^2/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)^2 (a+b\arccos(cx))^2 \,dx = \int x\,(a+b\,\mathrm{acos}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^2 \,dx$$

input
```
int(x*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^2,x)
```

output
```
int(x*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int x(d-c^2dx^2)^2 (a+b\arccos(cx))^2 \,dx$$
$$= \frac{d^2\left(72acos(cx)^2 b^2c^2x^2 - 36acos(cx)^2 b^2 - 72\sqrt{-c^2x^2+1}\,acos(cx)\,b^2cx + 48acos(cx)\,ab\,c^6x^6 - 144acos(\right.}{}$$

input
```
int(x*(-c^2*d*x^2+d)^2*(a+b*acos(c*x))^2,x)
```

output
```
(d**2*(72*acos(c*x)**2*b**2*c**2*x**2 - 36*acos(c*x)**2*b**2 - 72*sqrt( -
c**2*x**2 + 1)*acos(c*x)*b**2*c*x + 48*acos(c*x)*a*b*c**6*x**6 - 144*acos(
c*x)*a*b*c**4*x**4 + 144*acos(c*x)*a*b*c**2*x**2 + 33*asin(c*x)*a*b - 8*sq
rt( - c**2*x**2 + 1)*a*b*c**5*x**5 + 26*sqrt( - c**2*x**2 + 1)*a*b*c**3*x*
*3 - 33*sqrt( - c**2*x**2 + 1)*a*b*c*x + 144*int(acos(c*x)**2*x**5,x)*b**2
*c**6 - 288*int(acos(c*x)**2*x**3,x)*b**2*c**4 + 24*a**2*c**6*x**6 - 72*a*
*2*c**4*x**4 + 72*a**2*c**2*x**2 - 36*b**2*c**2*x**2))/(144*c**2)
```

3.171 $\int (d-c^2dx^2)^2 (a+b\arccos(cx))^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 219

$$\begin{aligned}\int (d-c^2dx^2)^2 (a+b\arccos(cx))^2 \, dx = & -\frac{298}{225}b^2d^2x+\frac{76}{675}b^2c^2d^2x^3-\frac{2}{125}b^2c^4d^2x^5 \\ & +\frac{16bd^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{15c} \\ & +\frac{8bd^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{45c} \\ & +\frac{2bd^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{25c} \\ & +\frac{8}{15}d^2x(a+b\arccos(cx))^2 \\ & +\frac{4}{15}d^2x(1-c^2x^2)(a+b\arccos(cx))^2 \\ & +\frac{1}{5}d^2x(1-c^2x^2)^2(a+b\arccos(cx))^2\end{aligned}$$

output
```
-298/225*b^2*d^2*x+76/675*b^2*c^2*d^2*x^3-2/125*b^2*c^4*d^2*x^5+16/15*b*d^
2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+8/45*b*d^2*(-c^2*x^2+1)^(3/2)*(a+
b*arccos(c*x))/c+2/25*b*d^2*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))/c+8/15*d^
2*x*(a+b*arccos(c*x))^2+4/15*d^2*x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2+1/5*d^
2*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.88

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2 \, dx$$
$$= \frac{d^2\left(225a^2cx(15-10c^2x^2+3c^4x^4)-30ab\sqrt{1-c^2x^2}(149-38c^2x^2+9c^4x^4)-2b^2cx(2235-190c^2x^2+2\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*(225*a^2*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) - 30*a*b*Sqrt[1 - c^2*x^2]
*(149 - 38*c^2*x^2 + 9*c^4*x^4) - 2*b^2*c*x*(2235 - 190*c^2*x^2 + 27*c^4*x
^4) - 30*b*(-15*a*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) + b*Sqrt[1 - c^2*x^2]*
(149 - 38*c^2*x^2 + 9*c^4*x^4))*ArcCos[c*x] + 225*b^2*c*x*(15 - 10*c^2*x^2
 + 3*c^4*x^4)*ArcCos[c*x]^2))/(3375*c)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5159, 27, 5159, 5131, 5183, 24, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2 \, dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{2}{5}bcd^2\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{4}{5}d\int d\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{5}bcd^2\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{4}{5}d^2\int \left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5159}$$

$$\frac{2}{5}bcd^2\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{4}{5}d^2\left(\frac{2}{3}bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{2}{3}\int (a+b\arccos(cx))^2dx+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2\right)+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5131}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(2bc\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{3}x(\right.$$
$$\frac{2}{5}bcd^2\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5183}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}\left(2bc\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bc\left(-\frac{b\int \left(1-c^2x^2\right)dx}{3c}-\right.\right.$$
$$\frac{2}{5}bcd^2\left(-\frac{b\int \left(1-c^2x^2\right)^2dx}{5c}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^2}\right)+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{24}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}bc\left(-\frac{b\int \left(1-c^2x^2\right)dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2b\right.\right.$$
$$\frac{2}{5}bcd^2\left(-\frac{b\int \left(1-c^2x^2\right)^2dx}{5c}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^2}\right)+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{210}$$

$$\frac{4}{5}d^2\left(\frac{2}{3}bc\left(-\frac{b\int \left(1-c^2x^2\right)dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2b\right.\right.$$
$$\frac{2}{5}bcd^2\left(-\frac{b\int \left(c^4x^4-2c^2x^2+1\right)dx}{5c}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{5c^2}\right)+\frac{1}{5}d^2x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}d^2x(1-c^2x^2)^2(a+b\arccos(cx))^2+$$
$$\frac{4}{5}d^2\left(\frac{1}{3}x(1-c^2x^2)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2\right.\right.$$
$$\frac{2}{5}bcd^2\left(-\frac{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{5c^2}-\frac{b\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)}{5c}\right)$$

input
```
Int[(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*x*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/5 + (2*b*c*d^2*(-1/5*(b*(x -
 (2*c^2*x^3)/3 + (c^4*x^5)/5))/c - ((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]
))/(5*c^2)))/5 + (4*d^2*((x*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/3 + (2*b*
c*(-1/3*(b*(x - (c^2*x^3)/3))/c - ((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])
)/(3*c^2)))/3 + (2*(x*(a + b*ArcCos[c*x])^2 + 2*b*c*(-((b*x)/c) - (Sqrt[1
- c^2*x^2]*(a + b*ArcCos[c*x]))/c^2)))/3))/5
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.26

method	result
derivativedivides	$d^2a^2\left(\frac{1}{5}c^5x^5-\frac{2}{3}c^3x^3+cx\right)+d^2b^2\left(\frac{\arccos(cx)^2\left(3c^4x^4-10c^2x^2+15\right)cx}{15}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^2\sqrt{-c^2x^2+1}}{25}-\frac{2\left(3c^4x^4-10c^2x^2\right.}{375}\right.$
default	$d^2a^2\left(\frac{1}{5}c^5x^5-\frac{2}{3}c^3x^3+cx\right)+d^2b^2\left(\frac{\arccos(cx)^2\left(3c^4x^4-10c^2x^2+15\right)cx}{15}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^2\sqrt{-c^2x^2+1}}{25}-\frac{2\left(3c^4x^4-10c^2x^2\right.}{375}\right.$
parts	$d^2a^2\left(\frac{1}{5}c^4x^5-\frac{2}{3}c^2x^3+x\right)+d^2b^2\left(\frac{\arccos(cx)^2\left(3c^4x^4-10c^2x^2+15\right)cx}{15}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^2\sqrt{-c^2x^2+1}}{25}-\frac{2\left(3c\right.}{}\right.$
orering	$\frac{x\left(1647c^6x^6-8677c^4x^4+51845c^2x^2-3375\right)\left(-c^2d\,x^2+d\right)^2\left(a+b\arccos(cx)\right)^2}{3375(cx-1)(cx+1)\left(c^2x^2-1\right)^2}-\frac{\left(324c^6x^6-2035c^4x^4+18450c^2x^2-22\right.}{}$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(d^2*a^2*(1/5*c^5*x^5-2/3*c^3*x^3+c*x)+d^2*b^2*(1/15*arccos(c*x)^2*(3*
c^4*x^4-10*c^2*x^2+15)*c*x-2/25*arccos(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)^(1/
2)-2/375*(3*c^4*x^4-10*c^2*x^2+15)*c*x+8/45*arccos(c*x)*(c^2*x^2-1)*(-c^2*
x^2+1)^(1/2)+8/135*(c^2*x^2-3)*c*x-16/15*c*x-16/15*arccos(c*x)*(-c^2*x^2+1
)^(1/2))+2*d^2*a*b*(1/5*arccos(c*x)*c^5*x^5-2/3*c^3*x^3*arccos(c*x)+c*x*ar
ccos(c*x)-149/225*(-c^2*x^2+1)^(1/2)+38/225*c^2*x^2*(-c^2*x^2+1)^(1/2)-1/2
5*c^4*x^4*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.13

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2\,dx$$
$$= \frac{27\,(25\,a^2-2\,b^2)c^5d^2x^5-10\,(225\,a^2-38\,b^2)c^3d^2x^3+15\,(225\,a^2-298\,b^2)cd^2x+225\,(3\,b^2c^5d^2x^5-10\,b^2c}{}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
1/3375*(27*(25*a^2 - 2*b^2)*c^5*d^2*x^5 - 10*(225*a^2 - 38*b^2)*c^3*d^2*x^
3 + 15*(225*a^2 - 298*b^2)*c*d^2*x + 225*(3*b^2*c^5*d^2*x^5 - 10*b^2*c^3*d
^2*x^3 + 15*b^2*c*d^2*x)*arccos(c*x)^2 + 450*(3*a*b*c^5*d^2*x^5 - 10*a*b*c
^3*d^2*x^3 + 15*a*b*c*d^2*x)*arccos(c*x) - 30*(9*a*b*c^4*d^2*x^4 - 38*a*b*
c^2*d^2*x^2 + 149*a*b*d^2 + (9*b^2*c^4*d^2*x^4 - 38*b^2*c^2*d^2*x^2 + 149*
b^2*d^2)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.80

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2c^4d^2x^5}{5} - \frac{2a^2c^2d^2x^3}{3} + a^2d^2x + \frac{2abc^4d^2x^5\operatorname{acos}(cx)}{5} - \frac{2abc^3d^2x^4\sqrt{-c^2x^2+1}}{25} - \frac{4abc^2d^2x^3\operatorname{acos}(cx)}{3} + \frac{76abcd^2x^2\sqrt{-c^2x^2+1}}{225} \\ d^2x\left(a+\frac{\pi b}{2}\right)^2 \end{cases}$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*c**4*d**2*x**5/5 - 2*a**2*c**2*d**2*x**3/3 + a**2*d**2*x +
 2*a*b*c**4*d**2*x**5*acos(c*x)/5 - 2*a*b*c**3*d**2*x**4*sqrt(-c**2*x**2 +
 1)/25 - 4*a*b*c**2*d**2*x**3*acos(c*x)/3 + 76*a*b*c*d**2*x**2*sqrt(-c**2*
x**2 + 1)/225 + 2*a*b*d**2*x*acos(c*x) - 298*a*b*d**2*sqrt(-c**2*x**2 + 1)
/(225*c) + b**2*c**4*d**2*x**5*acos(c*x)**2/5 - 2*b**2*c**4*d**2*x**5/125
- 2*b**2*c**3*d**2*x**4*sqrt(-c**2*x**2 + 1)*acos(c*x)/25 - 2*b**2*c**2*d*
*2*x**3*acos(c*x)**2/3 + 76*b**2*c**2*d**2*x**3/675 + 76*b**2*c*d**2*x**2*
sqrt(-c**2*x**2 + 1)*acos(c*x)/225 + b**2*d**2*x*acos(c*x)**2 - 298*b**2*d
**2*x/225 - 298*b**2*d**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(225*c), Ne(c, 0)
), (d**2*x*(a + pi*b/2)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(193) = 386$.

Time = 0.14 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.13

$$\int \left(d - c^2dx^2\right)^2 (a + b\arccos(cx))^2 \, dx$$
$$= \frac{1}{5}\,b^2c^4d^2x^5\arccos\left(cx\right)^2 + \frac{1}{5}\,a^2c^4d^2x^5 - \frac{2}{3}\,b^2c^2d^2x^3\arccos\left(cx\right)^2$$
$$+ \frac{2}{75}\left(15\,x^5\arccos\left(cx\right) - \left(\frac{3\,\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\,\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{-c^2x^2+1}}{c^6}\right)c\right)abc^4d^2$$
$$- \frac{2}{1125}\left(15\left(\frac{3\,\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\,\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{-c^2x^2+1}}{c^6}\right)c\arccos\left(cx\right) + \frac{9\,c^4x^5 + 20\,c^2x^3 +}{c^4}\right.$$
$$- \frac{2}{3}\,a^2c^2d^2x^3 - \frac{4}{9}\left(3\,x^3\arccos\left(cx\right) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\,\sqrt{-c^2x^2+1}}{c^4}\right)\right)abc^2d^2$$
$$+ \frac{4}{27}\left(3\,c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\,\sqrt{-c^2x^2+1}}{c^4}\right)\arccos\left(cx\right) + \frac{c^2x^3 + 6\,x}{c^2}\right)b^2c^2d^2$$
$$+ b^2d^2x\arccos\left(cx\right)^2 - 2\,b^2d^2\left(x + \frac{\sqrt{-c^2x^2+1}\arccos\left(cx\right)}{c}\right)$$
$$+ a^2d^2x + \frac{2\left(cx\arccos\left(cx\right) - \sqrt{-c^2x^2+1}\right)abd^2}{c}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*b^2*c^4*d^2*x^5*arccos(c*x)^2 + 1/5*a^2*c^4*d^2*x^5 - 2/3*b^2*c^2*d^2*
x^3*arccos(c*x)^2 + 2/75*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c
^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d
^2 - 2/1125*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c
^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c*arccos(c*x) + (9*c^4*x^5 + 20*c^2*x^3 + 1
20*x)/c^4)*b^2*c^4*d^2 - 2/3*a^2*c^2*d^2*x^3 - 4/9*(3*x^3*arccos(c*x) - c*
(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*c^2*d^2 + 4/2
7*(3*c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x)
 + (c^2*x^3 + 6*x)/c^2)*b^2*c^2*d^2 + b^2*d^2*x*arccos(c*x)^2 - 2*b^2*d^2*
(x + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + a^2*d^2*x + 2*(c*x*arccos(c*x) -
sqrt(-c^2*x^2 + 1))*a*b*d^2/c
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.50

$$\begin{aligned}\int \left(d-c^2dx^2\right)^2(a+b\arccos(cx))^2\,dx &= \frac{1}{5}\,b^2c^4d^2x^5\arccos(cx)^2+\frac{2}{5}\,abc^4d^2x^5\arccos(cx)\\&\quad+\frac{1}{5}\,a^2c^4d^2x^5-\frac{2}{125}\,b^2c^4d^2x^5\\&\quad-\frac{2}{25}\,\sqrt{-c^2x^2+1}b^2c^3d^2x^4\arccos(cx)\\&\quad-\frac{2}{25}\,\sqrt{-c^2x^2+1}abc^3d^2x^4\\&\quad-\frac{2}{3}\,b^2c^2d^2x^3\arccos(cx)^2\\&\quad-\frac{4}{3}\,abc^2d^2x^3\arccos(cx)\\&\quad-\frac{2}{3}\,a^2c^2d^2x^3+\frac{76}{675}\,b^2c^2d^2x^3\\&\quad+\frac{76}{225}\,\sqrt{-c^2x^2+1}b^2cd^2x^2\arccos(cx)\\&\quad+\frac{76}{225}\,\sqrt{-c^2x^2+1}abcd^2x^2+b^2d^2x\arccos(cx)^2\\&\quad+2\,abd^2x\arccos(cx)+a^2d^2x-\frac{298}{225}\,b^2d^2x\\&\quad-\frac{298\,\sqrt{-c^2x^2+1}b^2d^2\arccos(cx)}{225\,c}\\&\quad-\frac{298\,\sqrt{-c^2x^2+1}abd^2}{225\,c}\end{aligned}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
1/5*b^2*c^4*d^2*x^5*arccos(c*x)^2 + 2/5*a*b*c^4*d^2*x^5*arccos(c*x) + 1/5*
a^2*c^4*d^2*x^5 - 2/125*b^2*c^4*d^2*x^5 - 2/25*sqrt(-c^2*x^2 + 1)*b^2*c^3*
d^2*x^4*arccos(c*x) - 2/25*sqrt(-c^2*x^2 + 1)*a*b*c^3*d^2*x^4 - 2/3*b^2*c^
2*d^2*x^3*arccos(c*x)^2 - 4/3*a*b*c^2*d^2*x^3*arccos(c*x) - 2/3*a^2*c^2*d^
2*x^3 + 76/675*b^2*c^2*d^2*x^3 + 76/225*sqrt(-c^2*x^2 + 1)*b^2*c*d^2*x^2*a
rccos(c*x) + 76/225*sqrt(-c^2*x^2 + 1)*a*b*c*d^2*x^2 + b^2*d^2*x*arccos(c*
x)^2 + 2*a*b*d^2*x*arccos(c*x) + a^2*d^2*x - 298/225*b^2*d^2*x - 298/225*s
qrt(-c^2*x^2 + 1)*b^2*d^2*arccos(c*x)/c - 298/225*sqrt(-c^2*x^2 + 1)*a*b*d
^2/c
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^2\,dx$$

input
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2\,dx$$
$$=\frac{d^2\left(225acos(cx)^2\,b^2cx-450\sqrt{-c^2x^2+1}\,acos(cx)\,b^2+90acos(cx)\,ab\,c^5x^5-300acos(cx)\,ab\,c^3x^3+450ac$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acos(c*x))^2,x)
```

output
```
(d**2*(225*acos(c*x)**2*b**2*c*x - 450*sqrt( - c**2*x**2 + 1)*acos(c*x)*b*
*2 + 90*acos(c*x)*a*b*c**5*x**5 - 300*acos(c*x)*a*b*c**3*x**3 + 450*acos(c
*x)*a*b*c*x - 18*sqrt( - c**2*x**2 + 1)*a*b*c**4*x**4 + 76*sqrt( - c**2*x*
*2 + 1)*a*b*c**2*x**2 - 298*sqrt( - c**2*x**2 + 1)*a*b + 225*int(acos(c*x)
**2*x**4,x)*b**2*c**5 - 450*int(acos(c*x)**2*x**2,x)*b**2*c**3 + 45*a**2*c
**5*x**5 - 150*a**2*c**3*x**3 + 225*a**2*c*x - 450*b**2*c*x))/(225*c)
```

3.172 $\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))^2}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 277

$$\begin{aligned}\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))^2}{x}\,dx &= \frac{11}{32}b^2c^2d^2x^2-\frac{1}{32}b^2d^2(1-c^2x^2)^2 \\ &\quad-\frac{11}{16}bcd^2x\sqrt{1-c^2x^2}(a+b\arccos(cx)) \\ &\quad-\frac{1}{8}bcd^2x(1-c^2x^2)^{3/2}(a+b\arccos(cx)) \\ &\quad-\frac{11}{32}d^2(a+b\arccos(cx))^2 \\ &\quad+\frac{1}{2}d^2(1-c^2x^2)(a+b\arccos(cx))^2 \\ &\quad+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\arccos(cx))^2 \\ &\quad-\frac{id^2(a+b\arccos(cx))^3}{3b} \\ &\quad+d^2(a+b\arccos(cx))^2\log\left(1-e^{2i\arccos(cx)}\right) \\ &\quad-ibd^2(a \\ &\quad+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right) \\ &\quad+\frac{1}{2}b^2d^2\operatorname{PolyLog}\left(3,e^{2i\arccos(cx)}\right)\end{aligned}$$

output
```
11/32*b^2*c^2*d^2*x^2-1/32*b^2*d^2*(-c^2*x^2+1)^2-11/16*b*c*d^2*x*(-c^2*x^
2+1)^(1/2)*(a+b*arccos(c*x))-1/8*b*c*d^2*x*(-c^2*x^2+1)^(3/2)*(a+b*arccos(
c*x))-11/32*d^2*(a+b*arccos(c*x))^2+1/2*d^2*(-c^2*x^2+1)*(a+b*arccos(c*x))
^2+1/4*d^2*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2-1/3*I*d^2*(a+b*arccos(c*x))^
3/b+d^2*(a+b*arccos(c*x))^2*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-I*b*d^2*(a+
b*arccos(c*x))*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/2*b^2*d^2*polylog
(3,(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.25

$$\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))^2}{x}\,dx = \frac{1}{768}d^2\Big(-768a^2c^2x^2+192a^2c^4x^4 + 624abcx\sqrt{1-c^2x^2}-96abc^3x^3\sqrt{1-c^2x^2} - 1536abc^2x^2\arccos(cx)+384abc^4x^4\arccos(cx) - 768iab\arccos(cx)^2-256ib^2\arccos(cx)^3 - 1248ab\arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right) + 144b^2\cos(2\arccos(cx)) - 288b^2\arccos(cx)^2\cos(2\arccos(cx)) - 3b^2\cos(4\arccos(cx)) + 24b^2\arccos(cx)^2\cos(4\arccos(cx)) + 1536ab\arccos(cx)\log\left(1+e^{2i\arccos(cx)}\right) + 768b^2\arccos(cx)^2\log\left(1+e^{2i\arccos(cx)}\right) + 768a^2\log(cx)-768ib(a + b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right) + 384b^2\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right) + 288b^2\arccos(cx)\sin(2\arccos(cx)) - 12b^2\arccos(cx)\sin(4\arccos(cx))\Big)$$

input
```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
(d^2*(-768*a^2*c^2*x^2 + 192*a^2*c^4*x^4 + 624*a*b*c*x*Sqrt[1 - c^2*x^2] -
 96*a*b*c^3*x^3*Sqrt[1 - c^2*x^2] - 1536*a*b*c^2*x^2*ArcCos[c*x] + 384*a*b
*c^4*x^4*ArcCos[c*x] - (768*I)*a*b*ArcCos[c*x]^2 - (256*I)*b^2*ArcCos[c*x]
^3 - 1248*a*b*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])] + 144*b^2*Cos[2*ArcCo
s[c*x]] - 288*b^2*ArcCos[c*x]^2*Cos[2*ArcCos[c*x]] - 3*b^2*Cos[4*ArcCos[c*
x]] + 24*b^2*ArcCos[c*x]^2*Cos[4*ArcCos[c*x]] + 1536*a*b*ArcCos[c*x]*Log[1
 + E^((2*I)*ArcCos[c*x])] + 768*b^2*ArcCos[c*x]^2*Log[1 + E^((2*I)*ArcCos[
c*x])] + 768*a^2*Log[c*x] - (768*I)*b*(a + b*ArcCos[c*x])*PolyLog[2, -E^((
2*I)*ArcCos[c*x])] + 384*b^2*PolyLog[3, -E^((2*I)*ArcCos[c*x])] + 288*b^2*
ArcCos[c*x]*Sin[2*ArcCos[c*x]] - 12*b^2*ArcCos[c*x]*Sin[4*ArcCos[c*x]]))/7
68
```

Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.25, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.704$, Rules used = {5203, 27, 5159, 244, 2009, 5157, 15, 5153, 5203, 5137, 3042, 4202, 2620, 3011, 2720, 5157, 15, 5153, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x}dx$$

$$\Big\downarrow \text{ 5203}$$

$$\frac{1}{2}bcd^2\int (1-c^2x^2)^{3/2}(a+b\arccos(cx))dx+d\int \frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{x}dx+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}bcd^2\int (1-c^2x^2)^{3/2}(a+b\arccos(cx))dx+d^2\int \frac{(1-c^2x^2)(a+b\arccos(cx))^2}{x}dx+\frac{1}{4}d^2(1-c^2x^2)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{ 5159}$$

$$\frac{1}{2}bcd^2\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int x\left(1-c^2x^2\right)dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)+ d^2\int \frac{\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{x}dx+\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \textbf{244}$$

$$d^2\int \frac{\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{x}dx+ \frac{1}{2}bcd^2\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int \left(x-c^2x^3\right)dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)+ \frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \textbf{2009}$$

$$d^2\int \frac{\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{x}dx+ \frac{1}{2}bcd^2\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\right)+ \frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \textbf{5157}$$

$$d^2\int \frac{\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{x}dx+ \frac{1}{2}bcd^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\text{a} \right. \frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \textbf{15}$$

$$d^2\int \frac{\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{x}dx+ \frac{1}{2}bcd^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{4}bcx^2\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos \right. \frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2$$

$$\Big\downarrow \textbf{5153}$$

$$d^2\int \frac{\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{x}dx+\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+ \frac{1}{2}bcd^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right.\right.$$

$$\Big\downarrow \textbf{5203}$$

$$d^2\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\int \frac{(a+b\arccos(cx))^2}{x}dx+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))^2\right)+$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right.\right.$$

$\downarrow$ 5137

$$d^2\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{cx}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\,\text{ar}\right.$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right.\right.$$

$\downarrow$ 3042

$$d^2\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-\int (a+b\arccos(cx))^2\tan(\arccos(cx))d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+\right.$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right.\right.$$

$\downarrow$ 4202

$$d^2\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))^2}{1+e^{2i\arccos(cx)}}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+\right.$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right.\right.$$

$\downarrow$ 2620

$$d^2\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\left(ib\int (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\right.\right.$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right.\right.$$

$\downarrow$ 3011

$$d^2\left(bc\int\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\left(ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\operatorname{Poly}\right.\right.\right.$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right.\right.$$

$\downarrow$ 2720

$$d^2\left(bc\int\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\left(ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i}\right.\right.\right.$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right.\right.$$

$\downarrow$ 5157

$$d^2\left(bc\left(\frac{1}{2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)+2i\left(ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i}\right.\right.\right.\right.$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right.\right.$$

$\downarrow$ 15

$$d^2\left(bc\left(\frac{1}{2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{4}bcx^2\right)+2i\left(ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos}\right.\right.\right.\right.$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right.\right.$$

$\downarrow$ 5153

$$d^2\left(2i\left(ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)de^{2i}\right.\right.\right.$$
$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+$$
$$\frac{1}{2}bcd^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right.\right.$$

$\downarrow$ 7143

$$d^2\left(\frac{1}{2}(1-c^2x^2)(a+b\arccos(cx))^2+bc\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)+2i\right.$$
$$\frac{1}{4}d^2(1-c^2x^2)^2(a+b\arccos(cx))^2+$$
$$\frac{1}{2}bcd^2\left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right.\right.$$

input
```
Int[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
(d^2*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/4 + (b*c*d^2*((b*c*(x^2/2 - (c
^2*x^4)/4))/4 + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 + (3*((b*c*x
^2)/4 + (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/2 - (a + b*ArcCos[c*x])^
2/(4*b*c)))/4))/2 + d^2*(((1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/2 - ((I/3)*
(a + b*ArcCos[c*x])^3)/b + b*c*((b*c*x^2)/4 + (x*Sqrt[1 - c^2*x^2]*(a + b*
ArcCos[c*x]))/2 - (a + b*ArcCos[c*x])^2/(4*b*c)) + (2*I)*((-1/2*I)*(a + b*
ArcCos[c*x])^2*Log[1 + E^((2*I)*ArcCos[c*x])] + I*b*((I/2)*(a + b*ArcCos[c
*x])*PolyLog[2, -E^((2*I)*ArcCos[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcCos[c
*x])])/4)))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.53

method	result
derivativedivides	$d^2a^2\left(\frac{c^4x^4}{4}-c^2x^2+\ln(cx)\right)+d^2b^2\left(-\frac{i\arccos(cx)^3}{3}-\frac{3\left(2\arccos(cx)^2-1+2i\arccos(cx)\right)\left(2c^2x^2-1+\right.}{32}\right.$
default	$d^2a^2\left(\frac{c^4x^4}{4}-c^2x^2+\ln(cx)\right)+d^2b^2\left(-\frac{i\arccos(cx)^3}{3}-\frac{3\left(2\arccos(cx)^2-1+2i\arccos(cx)\right)\left(2c^2x^2-1+\right.}{32}\right.$
parts	$d^2a^2\left(\frac{c^4x^4}{4}-c^2x^2+\ln(x)\right)-id^2ab\arccos(cx)^2+\frac{3d^2b^2\sqrt{-c^2x^2+1}\,\arccos(cx)xc}{4}-\frac{3d^2b^2\arccos}{4}$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(1/4*c^4*x^4-c^2*x^2+ln(c*x))+d^2*b^2*(-1/3*I*arccos(c*x)^3-3/32*(
2*arccos(c*x)^2-1+2*I*arccos(c*x))*(2*c^2*x^2-1+2*I*(-c^2*x^2+1)^(1/2)*c*x
)-3/32*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*(2*arccos(c*x)^2-1-2*I*ar
ccos(c*x))+arccos(c*x)^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-I*arccos(c*x)*
polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/2*polylog(3,-(c*x+I*(-c^2*x^2+1
)^(1/2))^2)+1/256*(8*arccos(c*x)^2-1)*cos(4*arccos(c*x))-1/64*arccos(c*x)*
sin(4*arccos(c*x)))-I*d^2*a*b*arccos(c*x)^2+3/4*d^2*a*b*(-c^2*x^2+1)^(1/2)
*c*x-3/2*d^2*a*b*arccos(c*x)*c^2*x^2+3/4*d^2*a*b*arccos(c*x)+2*d^2*a*b*arc
cos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-I*d^2*a*b*polylog(2,-(c*x+I*(-
c^2*x^2+1)^(1/2))^2)+1/16*d^2*a*b*arccos(c*x)*cos(4*arccos(c*x))-1/64*d^2*
a*b*sin(4*arccos(c*x))
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^2\,(a+b\arccos(cx))^2}{x}\,dx=\int \frac{(c^2dx^2-d)^2(b\arccos(cx)+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arccos(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x} dx = d^2\left(\int \frac{a^2}{x} dx + \int \left(-2a^2c^2x\right) dx + \int a^2c^4x^3 dx + \int \frac{b^2 \operatorname{acos}^2(cx)}{x} dx + \int \frac{2ab \operatorname{acos}(cx)}{x} dx + \int \left(-2b^2c^2x \operatorname{acos}^2(cx)\right) dx + \int b^2c^4x^3 \operatorname{acos}^2(cx) dx + \int \left(-4abc^2x \operatorname{acos}(cx)\right) dx + \int 2abc^4x^3 \operatorname{acos}(cx) dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acos(c*x))**2/x,x)
```

output
```
d**2*(Integral(a**2/x, x) + Integral(-2*a**2*c**2*x, x) + Integral(a**2*c*
*4*x**3, x) + Integral(b**2*acos(c*x)**2/x, x) + Integral(2*a*b*acos(c*x)/
x, x) + Integral(-2*b**2*c**2*x*acos(c*x)**2, x) + Integral(b**2*c**4*x**3
*acos(c*x)**2, x) + Integral(-4*a*b*c**2*x*acos(c*x), x) + Integral(2*a*b*
c**4*x**3*acos(c*x), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))^2}{x}\,dx=\int \frac{(c^2dx^2-d)^2(b\arccos(cx)+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x,x, algorithm="maxima")
```

output
```
1/4*a^2*c^4*d^2*x^4 - a^2*c^2*d^2*x^2 + a^2*d^2*log(x) + integrate(((b^2*c
^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x
+ 1), c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arctan2(s
qrt(c*x + 1)*sqrt(-c*x + 1), c*x))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))^2}{x}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))^2}{x}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^2}{x}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^2)/x,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^2)/x, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x}\,dx$$
$$= \frac{d^2\left(-16acos(cx)^2 b^2c^2x^2+8acos(cx)^2 b^2+16\sqrt{-c^2x^2+1}\,acos(cx)\,b^2cx+8acos(cx)\,ab\,c^4x^4-32acos(cx\right.}{}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acos(c*x))^2/x,x)
```

output
```
(d**2*( - 16*acos(c*x)**2*b**2*c**2*x**2 + 8*acos(c*x)**2*b**2 + 16*sqrt(
- c**2*x**2 + 1)*acos(c*x)*b**2*c*x + 8*acos(c*x)*a*b*c**4*x**4 - 32*acos(
c*x)*a*b*c**2*x**2 - 13*asin(c*x)*a*b - 2*sqrt( - c**2*x**2 + 1)*a*b*c**3*
x**3 + 13*sqrt( - c**2*x**2 + 1)*a*b*c*x + 32*int(acos(c*x)/x,x)*a*b + 16*
int(acos(c*x)**2/x,x)*b**2 + 16*int(acos(c*x)**2*x**3,x)*b**2*c**4 + 16*lo
g(x)*a**2 + 4*a**2*c**4*x**4 - 16*a**2*c**2*x**2 + 8*b**2*c**2*x**2))/16
```

3.173 $\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 249

$$\begin{aligned}\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))^2}{x^2}\,dx &= \frac{32}{9}b^2c^2d^2x-\frac{2}{27}b^2c^4d^2x^3 \\ &\quad -\frac{10}{3}bcd^2\sqrt{1-c^2x^2}(a+b\arccos(cx)) \\ &\quad -\frac{2}{9}bcd^2\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx)) \\ &\quad -\frac{8}{3}c^2d^2x(a+b\arccos(cx))^2 \\ &\quad -\frac{4}{3}c^2d^2x\left(1-c^2x^2\right)(a+b\arccos(cx))^2 \\ &\quad -\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{x} \\ &\quad -4bcd^2(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right) \\ &\quad +2ib^2cd^2\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right) \\ &\quad -2ib^2cd^2\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)\end{aligned}$$

output
```
32/9*b^2*c^2*d^2*x-2/27*b^2*c^4*d^2*x^3-10/3*b*c*d^2*(-c^2*x^2+1)^(1/2)*(a
+b*arccos(c*x))-2/9*b*c*d^2*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))-8/3*c^2*d
^2*x*(a+b*arccos(c*x))^2-4/3*c^2*d^2*x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2-d^
2*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2/x-4*b*c*d^2*(a+b*arccos(c*x))*arctanh
(c*x+I*(-c^2*x^2+1)^(1/2))+2*I*b^2*c*d^2*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/
2))-2*I*b^2*c*d^2*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.40

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^2}\,dx = \frac{1}{108}d^2\left(-\frac{108a^2}{x}-216a^2c^2x+378b^2c^2x+36a^2c^4x^3+384abc\sqrt{1-c^2x^2}-24abc^3x^2\sqrt{1-c^2x^2}-\frac{216ab\arccos(cx)}{x}-432abc^2x\arccos(cx)+72abc^4x^3\arccos(cx)+378b^2c\sqrt{1-c^2x^2}\arccos(cx)-\frac{108b^2\arccos(cx)^2}{x}-189b^2c^2x\arccos(cx)^2+216abc\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)-2b^2c\cos(3\arccos(cx))+9b^2c\arccos(cx)^2\cos(3\arccos(cx))+216b^2c\arccos(cx)\log\left(1-ie^{i\arccos(cx)}\right)-216b^2c\arccos(cx)\log\left(1+ie^{i\arccos(cx)}\right)+216ib^2c\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-216ib^2c\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)-6b^2c\arccos(cx)\sin(3\arccos(cx))\right)$$

input
```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2)/x^2,x]
```

output
```
(d^2*((-108*a^2)/x - 216*a^2*c^2*x + 378*b^2*c^2*x + 36*a^2*c^4*x^3 + 384*
a*b*c*Sqrt[1 - c^2*x^2] - 24*a*b*c^3*x^2*Sqrt[1 - c^2*x^2] - (216*a*b*ArcC
os[c*x])/x - 432*a*b*c^2*x*ArcCos[c*x] + 72*a*b*c^4*x^3*ArcCos[c*x] + 378*
b^2*c*Sqrt[1 - c^2*x^2]*ArcCos[c*x] - (108*b^2*ArcCos[c*x]^2)/x - 189*b^2*
c^2*x*ArcCos[c*x]^2 + 216*a*b*c*ArcTanh[Sqrt[1 - c^2*x^2]] - 2*b^2*c*Cos[3
*ArcCos[c*x]] + 9*b^2*c*ArcCos[c*x]^2*Cos[3*ArcCos[c*x]] + 216*b^2*c*ArcCo
s[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] - 216*b^2*c*ArcCos[c*x]*Log[1 + I*E^(I
*ArcCos[c*x])] + (216*I)*b^2*c*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - (216*I
)*b^2*c*PolyLog[2, I*E^(I*ArcCos[c*x])] - 6*b^2*c*ArcCos[c*x]*Sin[3*ArcCos
[c*x]]))/108
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.29, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {5201, 27, 5159, 5131, 5183, 24, 2009, 5203, 2009, 5199, 24, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^2} \, dx$$

$$\Big\downarrow \text{ 5201}$$

$$-2bcd^2\int \frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{x}dx-4c^2d\int d(1-c^2x^2)(a+b\arccos(cx))^2dx-\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-2bcd^2\int \frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{x}dx-4c^2d^2\int (1-c^2x^2)(a+b\arccos(cx))^2dx-\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{ 5159}$$

$$-2bcd^2\int \frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-$$
$$4c^2d^2\left(\frac{2}{3}bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{2}{3}\int (a+b\arccos(cx))^2dx+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2\right)$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{5131}$$

$$-4c^2d^2\left(\frac{2}{3}\left(2bc\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{3}\right.$$
$$2bcd^2\int \frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{5183}$$

$$-4c^2d^2\left(\frac{2}{3}\left(2bc\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bc\left(-\frac{b\int \left(1-c^2x^2\right)dx}{3c}\right.\right.$$
$$2bcd^2\int \frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{24}$$

$$-4c^2d^2\left(\frac{2}{3}bc\left(-\frac{b\int \left(1-c^2x^2\right)dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\right.$$
$$2bcd^2\int \frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{2009}$$

$$-2bcd^2\int \frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}-$$
$$4c^2d^2\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))\right.\right.$$

$$\Big\downarrow \text{5203}$$

$$-2bcd^2\left(\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx+\frac{1}{3}bc\int (1-c^2x^2)\,dx+\frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arccos(cx))\right)-\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{x}-4c^2d^2\left(\frac{1}{3}x(1-c^2x^2)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))\right.\right.$$

$\downarrow$ 2009

$$-2bcd^2\left(\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx+\frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\frac{1}{3}bc\left(x-\frac{c^2x^3}{3}\right)\right)-\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{x}-4c^2d^2\left(\frac{1}{3}x(1-c^2x^2)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))\right.\right.$$

$\downarrow$ 5199

$$-2bcd^2\left(\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+bc\int 1dx+\frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\arccos(cx))+\right.$$
$$\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{x}-4c^2d^2\left(\frac{1}{3}x(1-c^2x^2)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))\right.\right.$$

$\downarrow$ 24

$$-2bcd^2\left(\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+\frac{1}{3}(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{3}bc\left(x-\right.\right.$$
$$\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{x}-4c^2d^2\left(\frac{1}{3}x(1-c^2x^2)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))\right.\right.$$

$\downarrow$ 5219

$$-2bcd^2\left(-\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\arccos(cx))+\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}-$$
$$4c^2d^2\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))\right.\right.$$

$\downarrow$ 3042

$$-2bcd^2\left(-\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\sqrt{1-c}\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}-$$
$$4c^2d^2\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))\right.\right.$$

$\downarrow$ 4669

$$-2bcd^2\left(b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)-b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)+2i\arctan\left(e^{i\arccos(cx)}\right)\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}-$$
$$4c^2d^2\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))\right.\right.$$

$\downarrow$ 2715

$$-2bcd^2\left(-ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}+ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(c}\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}-$$
$$4c^2d^2\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))\right.\right.$$

$\downarrow$ 2838

$$-2bcd^2\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\arccos\right.$$

$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}-$$

$$4c^2d^2\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))\right.\right.$$

input
```
Int[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2)/x^2,x]
```

output
```
-((d^2*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/x) - 4*c^2*d^2*((x*(1 - c^2*
x^2)*(a + b*ArcCos[c*x])^2)/3 + (2*b*c*(-1/3*(b*(x - (c^2*x^3)/3))/c - ((1
 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(3*c^2)))/3 + (2*(x*(a + b*ArcCos[c
*x])^2 + 2*b*c*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c^2))
)/3) - 2*b*c*d^2*(b*c*x + (b*c*(x - (c^2*x^3)/3))/3 + Sqrt[1 - c^2*x^2]*(a
 + b*ArcCos[c*x]) + ((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/3 + (2*I)*(a
 + b*ArcCos[c*x])*ArcTan[E^(I*ArcCos[c*x])] - I*b*PolyLog[2, (-I)*E^(I*Arc
Cos[c*x])] + I*b*PolyLog[2, I*E^(I*ArcCos[c*x])])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.55

method	result
derivativedivides	$c\left(d^2a^2\left(\frac{c^3x^3}{3}-2cx-\frac{1}{cx}\right)+\frac{7d^2b^2\arccos(cx)\sqrt{-c^2x^2+1}}{2}-\frac{7d^2b^2\arccos(cx)^2cx}{4}+\frac{7d^2b^2cx}{2}-\frac{d^2b^2\,\mathrm{arc}}{c}\right.$
default	$c\left(d^2a^2\left(\frac{c^3x^3}{3}-2cx-\frac{1}{cx}\right)+\frac{7d^2b^2\arccos(cx)\sqrt{-c^2x^2+1}}{2}-\frac{7d^2b^2\arccos(cx)^2cx}{4}+\frac{7d^2b^2cx}{2}-\frac{d^2b^2\,\mathrm{arc}}{c}\right.$
parts	$d^2a^2\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)+\frac{7d^2b^2c\sqrt{-c^2x^2+1}\,\arccos(cx)}{2}-\frac{7d^2b^2c^2\arccos(cx)^2x}{4}+\frac{7b^2c^2d^2x}{2}-\frac{d^2b^2\,\mathrm{arc}}{}$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
c*(d^2*a^2*(1/3*c^3*x^3-2*c*x-1/c/x)+7/2*d^2*b^2*arccos(c*x)*(-c^2*x^2+1)^
(1/2)-7/4*d^2*b^2*arccos(c*x)^2*c*x+7/2*d^2*b^2*c*x-d^2*b^2*arccos(c*x)^2/
c/x-2*d^2*b^2*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*d^2*b^2*arc
cos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*I*d^2*b^2*dilog(1+I*(c*x+I*(
-c^2*x^2+1)^(1/2)))-2*I*d^2*b^2*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+1/12
*d^2*b^2*arccos(c*x)^2*cos(3*arccos(c*x))-1/54*d^2*b^2*cos(3*arccos(c*x))-
1/18*d^2*b^2*arccos(c*x)*sin(3*arccos(c*x))+2*d^2*a*b*(1/3*c^3*x^3*arccos(
c*x)-2*c*x*arccos(c*x)-arccos(c*x)/c/x-1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)+16/9
*(-c^2*x^2+1)^(1/2)+arctanh(1/(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^2\,(a+b\arccos(cx))^2}{x^2}\,dx=\int \frac{(c^2dx^2-d)^2(b\arccos{(cx)}+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arccos(c*x))/x^2, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^2}\,dx = d^2\left(\int \left(-2a^2c^2\right)\,dx + \int \frac{a^2}{x^2}\,dx + \int a^2c^4x^2\,dx + \int \left(-2b^2c^2\operatorname{acos}^2(cx)\right)\,dx + \int \frac{b^2\operatorname{acos}^2(cx)}{x^2}\,dx + \int \left(-4abc^2\operatorname{acos}(cx)\right)\,dx + \int \frac{2ab\operatorname{acos}(cx)}{x^2}\,dx + \int b^2c^4x^2\operatorname{acos}^2(cx)\,dx + \int 2abc^4x^2\operatorname{acos}(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acos(c*x))**2/x**2,x)
```

output
```
d**2*(Integral(-2*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(a**2*c
**4*x**2, x) + Integral(-2*b**2*c**2*acos(c*x)**2, x) + Integral(b**2*acos
(c*x)**2/x**2, x) + Integral(-4*a*b*c**2*acos(c*x), x) + Integral(2*a*b*ac
os(c*x)/x**2, x) + Integral(b**2*c**4*x**2*acos(c*x)**2, x) + Integral(2*a
*b*c**4*x**2*acos(c*x), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{(c^2dx^2-d)^2(b\arccos(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
1/3*a^2*c^4*d^2*x^3 + 2/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c
^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*c^4*d^2 - 2*b^2*c^2*d^2*x*arccos(c*x)^
2 + 4*b^2*c^2*d^2*(x + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) - 2*a^2*c^2*d^2*x
 - 4*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a*b*c*d^2 + 2*(c*log(2*sqrt(-c
^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*a*b*d^2 - a^2*d^2/x + 1/3*
((b^2*c^4*d^2*x^4 - 3*b^2*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^
2 - 3*x*integrate(2/3*(b^2*c^5*d^2*x^4 - 3*b^2*c*d^2)*sqrt(c*x + 1)*sqrt(-
c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^2*x^3 - x), x))/x
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^2}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^2)/x^2,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^2)/x^2, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^2}\,dx$$
$$= \frac{d^2\Big(-18acos(cx)^2 b^2c^2x^2+36\sqrt{-c^2x^2+1}\,acos(cx)\,b^2cx+6acos(cx)\,ab\,c^4x^4-36acos(cx)\,ab\,c^2x^2-18a$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acos(c*x))^2/x^2,x)
```

output
```
(d**2*( - 18*acos(c*x)**2*b**2*c**2*x**2 + 36*sqrt( - c**2*x**2 + 1)*acos(
c*x)*b**2*c*x + 6*acos(c*x)*a*b*c**4*x**4 - 36*acos(c*x)*a*b*c**2*x**2 - 1
8*acos(c*x)*a*b - 2*sqrt( - c**2*x**2 + 1)*a*b*c**3*x**3 + 32*sqrt( - c**2
*x**2 + 1)*a*b*c*x + 9*int(acos(c*x)**2/x**2,x)*b**2*x + 9*int(acos(c*x)**
2*x**2,x)*b**2*c**4*x - 18*log(tan(asin(c*x)/2))*a*b*c*x + 3*a**2*c**4*x**
4 - 18*a**2*c**2*x**2 - 9*a**2 + 36*b**2*c**2*x**2))/(9*x)
```

3.174 $\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 287

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^3}\,dx = -\frac{1}{4}b^2c^4d^2x^2 - \frac{1}{2}bc^3d^2x\sqrt{1-c^2x^2}(a+b\arccos(cx)) - \frac{bcd^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{x} - \frac{1}{4}c^2d^2(a+b\arccos(cx))^2 - c^2d^2(1-c^2x^2)(a+b\arccos(cx))^2 - \frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{2x^2} + \frac{2ic^2d^2(a+b\arccos(cx))^3}{3b} - 2c^2d^2(a+b\arccos(cx))^2\log\left(1-e^{2i\arccos(cx)}\right) + b^2c^2d^2\log(x) + 2ibc^2d^2(a+b\arccos(cx))\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right) - b^2c^2d^2\,\text{PolyLog}\left(3,e^{2i\arccos(cx)}\right)$$

output
```
-1/4*b^2*c^4*d^2*x^2-1/2*b*c^3*d^2*x*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))-
b*c*d^2*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/x-1/4*c^2*d^2*(a+b*arccos(c*x
))^2-c^2*d^2*(-c^2*x^2+1)*(a+b*arccos(c*x))^2-1/2*d^2*(-c^2*x^2+1)^2*(a+b*
arccos(c*x))^2/x^2+2/3*I*c^2*d^2*(a+b*arccos(c*x))^3/b-2*c^2*d^2*(a+b*arcc
os(c*x))^2*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+b^2*c^2*d^2*ln(x)+2*I*b*c^2*
d^2*(a+b*arccos(c*x))*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)-b^2*c^2*d^2*
polylog(3,(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.41

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^3}\,dx$$
$$= \frac{d^2\Big(-12a^2+12a^2c^4x^4+24abcx\sqrt{1-c^2x^2}-12abc^3x^3\sqrt{1-c^2x^2}-24ab\arccos(cx)+24abc^4x^4\arccos(c}{}$$

input
```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2)/x^3,x]
```

output
```
(d^2*(-12*a^2 + 12*a^2*c^4*x^4 + 24*a*b*c*x*Sqrt[1 - c^2*x^2] - 12*a*b*c^3
*x^3*Sqrt[1 - c^2*x^2] - 24*a*b*ArcCos[c*x] + 24*a*b*c^4*x^4*ArcCos[c*x] +
 24*b^2*c*x*Sqrt[1 - c^2*x^2]*ArcCos[c*x] - 12*b^2*ArcCos[c*x]^2 + (48*I)*
a*b*c^2*x^2*ArcCos[c*x]^2 + (16*I)*b^2*c^2*x^2*ArcCos[c*x]^3 + 24*a*b*c^2*
x^2*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])] - 3*b^2*c^2*x^2*Cos[2*ArcCos[c*
x]] + 6*b^2*c^2*x^2*ArcCos[c*x]^2*Cos[2*ArcCos[c*x]] - 96*a*b*c^2*x^2*ArcC
os[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] - 48*b^2*c^2*x^2*ArcCos[c*x]^2*Log[
1 + E^((2*I)*ArcCos[c*x])] - 48*a^2*c^2*x^2*Log[x] + 24*b^2*c^2*x^2*Log[c*
x] + (48*I)*b*c^2*x^2*(a + b*ArcCos[c*x])*PolyLog[2, -E^((2*I)*ArcCos[c*x]
)] - 24*b^2*c^2*x^2*PolyLog[3, -E^((2*I)*ArcCos[c*x])] - 6*b^2*c^2*x^2*Arc
Cos[c*x]*Sin[2*ArcCos[c*x]]))/(24*x^2)
```

Rubi [A] (verified)

Time = 2.35 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.21, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.704, Rules used = {5201, 27, 5201, 244, 2009, 5157, 15, 5153, 5203, 5137, 3042, 4202, 2620, 3011, 2720, 5157, 15, 5153, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^3}dx$$

$$\Big\downarrow \text{5201}$$

$$-bcd^2\int \frac{(1-c^2x^2)^{3/2} (a+b\arccos(cx))}{x^2}dx-2c^2d\int \frac{d(1-c^2x^2) (a+b\arccos(cx))^2}{x}dx-\frac{d^2(1-c^2x^2)^2 (a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$-bcd^2\int \frac{(1-c^2x^2)^{3/2} (a+b\arccos(cx))}{x^2}dx-2c^2d^2\int \frac{(1-c^2x^2) (a+b\arccos(cx))^2}{x}dx-\frac{d^2(1-c^2x^2)^2 (a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{5201}$$

$$-bcd^2\left(-3c^2\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-bc\int \frac{1-c^2x^2}{x}dx-\frac{(1-c^2x^2)^{3/2} (a+b\arccos(cx))}{x}\right)-2c^2d^2\int \frac{(1-c^2x^2) (a+b\arccos(cx))^2}{x}dx-\frac{d^2(1-c^2x^2)^2 (a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{244}$$

$$-bcd^2\left(-3c^2\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-bc\int \left(\frac{1}{x}-c^2x\right)dx-\frac{(1-c^2x^2)^{3/2} (a+b\arccos(cx))}{x}\right)-2c^2d^2\int \frac{(1-c^2x^2) (a+b\arccos(cx))^2}{x}dx-\frac{d^2(1-c^2x^2)^2 (a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$-2c^2d^2\int \frac{(1-c^2x^2)(a+b\arccos(cx))^2}{x}dx-$$
$$bcd^2\left(-3c^2\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{x}-bc\left(\log(x)-\frac{c^2x^2}{2}\right)\right)-$$
$$\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{2x^2}$$

$\downarrow$ 5157

$$-2c^2d^2\int \frac{(1-c^2x^2)(a+b\arccos(cx))^2}{x}dx-$$
$$bcd^2\left(-3c^2\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)-\frac{(1-c^2x^2)^{3/2}(a+b\text{ar}}{x}\right.$$
$$\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{2x^2}$$

$\downarrow$ 15

$$-2c^2d^2\int \frac{(1-c^2x^2)(a+b\arccos(cx))^2}{x}dx-$$
$$bcd^2\left(-3c^2\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{4}bcx^2\right)-\frac{(1-c^2x^2)^{3/2}(a+b\text{arccos}}{x}\right.$$
$$\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{2x^2}$$

$\downarrow$ 5153

$$-2c^2d^2\int \frac{(1-c^2x^2)(a+b\arccos(cx))^2}{x}dx-\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^2\left(-3c^2\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)-\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{x}\right.$$

$\downarrow$ 5203

$$-2c^2d^2\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\int \frac{(a+b\arccos(cx))^2}{x}dx+\frac{1}{2}(1-c^2x^2)(a+b\arccos(cx))^2\right)-$$
$$\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^2\left(-3c^2\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)-\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{x}\right.$$

$\downarrow$ 5137

$$-2c^2d^2\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{cx}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^2\left(-3c^2\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}\right.$$

$\downarrow$ 3042

$$-2c^2d^2\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-\int (a+b\arccos(cx))^2\tan(\arccos(cx))d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^2\left(-3c^2\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}\right.$$

$\downarrow$ 4202

$$-2c^2d^2\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))^2}{1+e^{2i\arccos(cx)}}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^2\left(-3c^2\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}\right.$$

$\downarrow$ 2620

$$-2c^2d^2\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\left(ib\int (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\right.\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^2\left(-3c^2\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}\right.$$

$\downarrow$ 3011

$$-2c^2d^2\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\right.\right.\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^2\left(-3c^2\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}\right.$$

$$\Big\downarrow \text{2720}$$

$$-2c^2d^2\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int\right.\right.\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^2\left(-3c^2\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}\right.$$

$$\Big\downarrow \text{5157}$$

$$-2c^2d^2\left(bc\left(\frac{1}{2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)+2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,\right.\right.\right.\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^2\left(-3c^2\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}\right.$$

$$\Big\downarrow \text{15}$$

$$-2c^2d^2\left(bc\left(\frac{1}{2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{4}bcx^2\right)+2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i}\right.\right.\right.\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^2\left(-3c^2\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}\right.$$

$$\Big\downarrow \text{5153}$$

$$-2c^2d^2\left(2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\right.\right.\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^2\left(-3c^2\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}\right.$$

$\downarrow$ 7143

$$-2c^2d^2\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))^2+bc\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^2\left(-3c^2\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}\right.$$

input
```
Int[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2)/x^3,x]
```

output
```
-1/2*(d^2*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/x^2 - b*c*d^2*(-(((1 - c^
2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/x) - 3*c^2*((b*c*x^2)/4 + (x*Sqrt[1 - c^
2*x^2]*(a + b*ArcCos[c*x]))/2 - (a + b*ArcCos[c*x])^2/(4*b*c)) - b*c*(-1/2
*(c^2*x^2) + Log[x])) - 2*c^2*d^2*(((1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/2
 - ((I/3)*(a + b*ArcCos[c*x])^3)/b + b*c*((b*c*x^2)/4 + (x*Sqrt[1 - c^2*x^
2]*(a + b*ArcCos[c*x]))/2 - (a + b*ArcCos[c*x])^2/(4*b*c)) + (2*I)*((-1/2*
I)*(a + b*ArcCos[c*x])^2*Log[1 + E^((2*I)*ArcCos[c*x])] + I*b*((I/2)*(a +
b*ArcCos[c*x])*PolyLog[2, -E^((2*I)*ArcCos[c*x])] - (b*PolyLog[3, -E^((2*I
)*ArcCos[c*x])])/4)))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.75

method	result
derivativedivides	$c^2\left(d^2a^2\left(\frac{c^2x^2}{2}-\frac{1}{2c^2x^2}-2\ln(cx)\right)+d^2b^2\left(\frac{2i\arccos(cx)^3}{3}+\frac{\left(2\arccos(cx)^2-1+2i\arccos(cx)\right)\left(2c^2x\right.}{16}\right.\right.$
default	$c^2\left(d^2a^2\left(\frac{c^2x^2}{2}-\frac{1}{2c^2x^2}-2\ln(cx)\right)+d^2b^2\left(\frac{2i\arccos(cx)^3}{3}+\frac{\left(2\arccos(cx)^2-1+2i\arccos(cx)\right)\left(2c^2x\right.}{16}\right.\right.$
parts	$d^2a^2\left(\frac{c^4x^2}{2}-2c^2\ln(x)-\frac{1}{2x^2}\right)+d^2b^2c^2\left(\frac{2i\arccos(cx)^3}{3}+\frac{\left(2\arccos(cx)^2-1+2i\arccos(cx)\right)\left(2c^2x^2-\right.}{16}\right.$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(d^2*a^2*(1/2*c^2*x^2-1/2/c^2/x^2-2*ln(c*x))+d^2*b^2*(2/3*I*arccos(c*x
)^3+1/16*(2*arccos(c*x)^2-1+2*I*arccos(c*x))*(2*c^2*x^2-1+2*I*(-c^2*x^2+1)
^(1/2)*c*x)+1/16*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*(2*arccos(c*x)^
2-1-2*I*arccos(c*x))-1/2*arccos(c*x)*(-2*I*c^2*x^2-2*c*x*(-c^2*x^2+1)^(1/2
)+arccos(c*x))/c^2/x^2+ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-2*ln(c*x+I*(-c^2
*x^2+1)^(1/2))-2*arccos(c*x)^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+2*I*arcc
os(c*x)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-polylog(3,-(c*x+I*(-c^2*x
^2+1)^(1/2))^2))+2*I*d^2*a*b*arccos(c*x)^2-1/2*d^2*a*b*(-c^2*x^2+1)^(1/2)*
c*x+d^2*a*b*arccos(c*x)*c^2*x^2-1/2*d^2*a*b*arccos(c*x)+I*d^2*a*b+d^2*a*b/
c/x*(-c^2*x^2+1)^(1/2)-d^2*a*b*arccos(c*x)/c^2/x^2-4*d^2*a*b*arccos(c*x)*l
n(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+2*I*d^2*a*b*polylog(2,-(c*x+I*(-c^2*x^2+
1)^(1/2))^2))
```

Fricas [F]

$$\int\frac{(d-c^2dx^2)^2\,(a+b\arccos(cx))^2}{x^3}\,dx=\int\frac{(c^2dx^2-d)^2(b\arccos(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arccos(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^3}\,dx = d^2\left(\int \frac{a^2}{x^3}\,dx + \int \left(-\frac{2a^2c^2}{x}\right)dx + \int a^2c^4x\,dx + \int \frac{b^2\operatorname{acos}^2(cx)}{x^3}\,dx + \int \frac{2ab\operatorname{acos}(cx)}{x^3}\,dx + \int \left(-\frac{2b^2c^2\operatorname{acos}^2(cx)}{x}\right)dx + \int b^2c^4x\operatorname{acos}^2(cx)\,dx + \int \left(-\frac{4abc^2\operatorname{acos}(cx)}{x}\right)dx + \int 2abc^4x\operatorname{acos}(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acos(c*x))**2/x**3,x)
```

output
```
d**2*(Integral(a**2/x**3, x) + Integral(-2*a**2*c**2/x, x) + Integral(a**2
*c**4*x, x) + Integral(b**2*acos(c*x)**2/x**3, x) + Integral(2*a*b*acos(c*
x)/x**3, x) + Integral(-2*b**2*c**2*acos(c*x)**2/x, x) + Integral(b**2*c**
4*x*acos(c*x)**2, x) + Integral(-4*a*b*c**2*acos(c*x)/x, x) + Integral(2*a
*b*c**4*x*acos(c*x), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^3} dx = \int \frac{(c^2dx^2-d)^2(b\arccos(cx)+a)^2}{x^3} dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
1/2*a^2*c^4*d^2*x^2 - 2*a^2*c^2*d^2*log(x) + a*b*d^2*(sqrt(-c^2*x^2 + 1)*c
/x - arccos(c*x)/x^2) - 1/2*a^2*d^2/x^2 + integrate(((b^2*c^4*d^2*x^4 - 2*
b^2*c^2*d^2*x^2 + b^2*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 +
2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x))/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^3} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^3} dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^2}{x^3} dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^2)/x^3,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^3}\,dx$$
$$= \frac{d^2\left(2acos(cx)^2 b^2c^4x^4 - acos(cx)^2 b^2c^2x^2 - 2\sqrt{-c^2x^2+1}\,acos(cx)\,b^2c^3x^3 + 4acos(cx)\,ab\,c^4x^4 - 4acos(cx\right.}{}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acos(c*x))^2/x^3,x)
```

output
```
(d**2*(2*acos(c*x)**2*b**2*c**4*x**4 - acos(c*x)**2*b**2*c**2*x**2 - 2*sqr
t( - c**2*x**2 + 1)*acos(c*x)*b**2*c**3*x**3 + 4*acos(c*x)*a*b*c**4*x**4 -
 4*acos(c*x)*a*b + 2*asin(c*x)*a*b*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a*
b*c**3*x**3 + 4*sqrt( - c**2*x**2 + 1)*a*b*c*x - 16*int(acos(c*x)/x,x)*a*b
*c**2*x**2 + 4*int(acos(c*x)**2/x**3,x)*b**2*x**2 - 8*int(acos(c*x)**2/x,x
)*b**2*c**2*x**2 - 8*log(x)*a**2*c**2*x**2 + 2*a**2*c**4*x**4 - 2*a**2 - b
**2*c**4*x**4))/(4*x**2)
```

3.175 $\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 268

$$\begin{aligned}\int \frac{(d-c^2dx^2)^2(a+b\arccos(cx))^2}{x^4}\,dx = & -\frac{b^2c^2d^2}{3x}-2b^2c^4d^2x \\ & +\frac{5}{3}bc^3d^2\sqrt{1-c^2x^2}(a+b\arccos(cx)) \\ & -\frac{bcd^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3x^2} \\ & +\frac{8}{3}c^4d^2x(a+b\arccos(cx))^2 \\ & +\frac{4c^2d^2(1-c^2x^2)(a+b\arccos(cx))^2}{3x} \\ & -\frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{3x^3} \\ & +\frac{22}{3}bc^3d^2(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right) \\ & -\frac{11}{3}ib^2c^3d^2\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right) \\ & +\frac{11}{3}ib^2c^3d^2\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)\end{aligned}$$

output
```
-1/3*b^2*c^2*d^2/x-2*b^2*c^4*d^2*x+5/3*b*c^3*d^2*(-c^2*x^2+1)^(1/2)*(a+b*a
rccos(c*x))-1/3*b*c*d^2*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/x^2+8/3*c^4*d
^2*x*(a+b*arccos(c*x))^2+4/3*c^2*d^2*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/x-1/
3*d^2*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2/x^3+22/3*b*c^3*d^2*(a+b*arccos(c*
x))*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))-11/3*I*b^2*c^3*d^2*polylog(2,-c*x-I*
(-c^2*x^2+1)^(1/2))+11/3*I*b^2*c^3*d^2*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.43

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^4}\,dx$$
$$= \frac{d^2\left(-a^2+6a^2c^2x^2-b^2c^2x^2+3a^2c^4x^4-6b^2c^4x^4+abcx\sqrt{1-c^2x^2}-6abc^3x^3\sqrt{1-c^2x^2}-2ab\arccos(cx)\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2)/x^4,x]
```

output
```
(d^2*(-a^2 + 6*a^2*c^2*x^2 - b^2*c^2*x^2 + 3*a^2*c^4*x^4 - 6*b^2*c^4*x^4 +
 a*b*c*x*Sqrt[1 - c^2*x^2] - 6*a*b*c^3*x^3*Sqrt[1 - c^2*x^2] - 2*a*b*ArcCo
s[c*x] + 12*a*b*c^2*x^2*ArcCos[c*x] + 6*a*b*c^4*x^4*ArcCos[c*x] + b^2*c*x*
Sqrt[1 - c^2*x^2]*ArcCos[c*x] - 6*b^2*c^3*x^3*Sqrt[1 - c^2*x^2]*ArcCos[c*x
] - b^2*ArcCos[c*x]^2 + 6*b^2*c^2*x^2*ArcCos[c*x]^2 + 3*b^2*c^4*x^4*ArcCos
[c*x]^2 - 11*a*b*c^3*x^3*ArcTanh[Sqrt[1 - c^2*x^2]] - 11*b^2*c^3*x^3*ArcCo
s[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] + 11*b^2*c^3*x^3*ArcCos[c*x]*Log[1 + I
*E^(I*ArcCos[c*x])] - (11*I)*b^2*c^3*x^3*PolyLog[2, (-I)*E^(I*ArcCos[c*x])
] + (11*I)*b^2*c^3*x^3*PolyLog[2, I*E^(I*ArcCos[c*x])]))/(3*x^3)
```

Rubi [A] (verified)

Time = 2.21 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.42, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5201, 27, 5201, 244, 2009, 5131, 5183, 24, 5199, 24, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2}{x^4}dx$$

$$\Big\downarrow \; 5201$$

$$-\frac{2}{3}bcd^2\int \frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x^3}dx-\frac{4}{3}c^2d\int \frac{d\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{x^2}dx-\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{4}{3}c^2d^2\int \frac{\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{x^2}dx-\frac{2}{3}bcd^2\int \frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x^3}dx-\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 5201$$

$$-\frac{2}{3}bcd^2\left(-\frac{3}{2}c^2\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx-\frac{1}{2}bc\int \frac{1-c^2x^2}{x^2}dx-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}\right)-\frac{4}{3}c^2d^2\left(-2bc\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx-2c^2\int (a+b\arccos(cx))^2dx-\frac{\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{x}\right)-\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 244$$

$$-\frac{2}{3}bcd^2\left(-\frac{3}{2}c^2\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx-\frac{1}{2}bc\int \left(\frac{1}{x^2}-c^2\right)dx-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}\right)-\frac{4}{3}c^2d^2\left(-2bc\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx-2c^2\int (a+b\arccos(cx))^2dx-\frac{\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{x}\right)-\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2}{3}bcd^2\left(-\frac{3}{2}c^2\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(c^2(-x)-\frac{1}{x}\right)\right)-$$
$$\frac{4}{3}c^2d^2\left(-2bc\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx-2c^2\int (a+b\arccos(cx))^2dx-\frac{\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{x}\right)$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 5131

$$-\frac{4}{3}c^2d^2\left(-2c^2\left(2bc\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^2\right)-2bc\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx-\right.$$
$$\frac{2}{3}bcd^2\left(-\frac{3}{2}c^2\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(c^2(-x)-\frac{1}{x}\right)\right)-$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 5183

$$-\frac{4}{3}c^2d^2\left(-2c^2\left(2bc\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)+x(a+b\arccos(cx))^2\right)-2bc\int \frac{\sqrt{1-c^2x^2}(a\right.$$
$$\frac{2}{3}bcd^2\left(-\frac{3}{2}c^2\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(c^2(-x)-\frac{1}{x}\right)\right)-$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 24

$$-\frac{4}{3}c^2d^2\left(-2bc\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx-2c^2\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\,\mathrm{a}\right.\right.$$
$$\frac{2}{3}bcd^2\left(-\frac{3}{2}c^2\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(c^2(-x)-\frac{1}{x}\right)\right)-$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 5199

$$-\frac{4}{3}c^2d^2\left(-2bc\left(\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+bc\int 1dx+\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)-2c^2\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+}{c}\right.\right.\right.$$
$$\frac{2}{3}bcd^2\left(-\frac{3}{2}c^2\left(\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+bc\int 1dx+\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(}{2x^2}\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{24}$$

$$-\frac{4}{3}c^2d^2\left(-2bc\left(\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+\sqrt{1-c^2x^2}(a+b\arccos(cx))+bcx\right)-2c^2\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\,\mathrm{ar}}{c^2}\right.\right.\right.$$
$$\frac{2}{3}bcd^2\left(-\frac{3}{2}c^2\left(\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+\sqrt{1-c^2x^2}(a+b\arccos(cx))+bcx\right)-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{2x^2}\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{5219}$$

$$-\frac{4}{3}c^2d^2\left(-2bc\left(-\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)+\sqrt{1-c^2x^2}(a+b\arccos(cx))+bcx\right)-2c^2\left(2bc\left(-\frac{\sqrt{1-c^2}}{}\right.\right.\right.$$
$$\frac{2}{3}bcd^2\left(-\frac{3}{2}c^2\left(-\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)+\sqrt{1-c^2x^2}(a+b\arccos(cx))+bcx\right)-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\,\mathrm{a}}{2x^2}\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4}{3}c^2d^2\left(-2bc\left(-\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)+\sqrt{1-c^2x^2}(a+b\arccos(cx))+bcx\right)\right.$$
$$\frac{2}{3}bcd^2\left(-\frac{3}{2}c^2\left(-\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)+\sqrt{1-c^2x^2}(a+b\arccos(cx))+bcx\right)\right.$$
$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{4}{3}c^2d^2\Bigg(-2bc\Bigg(b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)-b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)+2i\arctan\left(e^{i\,\mathrm{ar}}\right.$$

$$\frac{2}{3}bcd^2\Bigg(-\frac{3}{2}c^2\Bigg(b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)-b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)+2i\arctan\left(e^{i\,\mathrm{arc}}\right.$$

$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 2715

$$-\frac{4}{3}c^2d^2\Bigg(-2bc\Bigg(-ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}+ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de$$

$$\frac{2}{3}bcd^2\Bigg(-\frac{3}{2}c^2\Bigg(-ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}+ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i}$$

$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 2838

$$-\frac{4}{3}c^2d^2\Bigg(-2bc\Bigg(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\arccos(cx))-ib\,\mathrm{PolyLog}\left(2,-ie^{i\,\mathrm{ar}}\right.$$

$$\frac{2}{3}bcd^2\Bigg(-\frac{3}{2}c^2\Bigg(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\arccos(cx))-ib\,\mathrm{PolyLog}\left(2,-ie^{i\,\mathrm{arc}}\right.$$

$$\frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{3x^3}$$

input
```
Int[((d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2)/x^4,x]
```

output
```
-1/3*(d^2*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/x^3 - (4*c^2*d^2*(-(((1 -
 c^2*x^2)*(a + b*ArcCos[c*x])^2)/x) - 2*c^2*(x*(a + b*ArcCos[c*x])^2 + 2*b
*c*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c^2)) - 2*b*c*(b*
c*x + Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]) + (2*I)*(a + b*ArcCos[c*x])*Ar
cTan[E^(I*ArcCos[c*x])] - I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I*b*Pol
yLog[2, I*E^(I*ArcCos[c*x])])))/3 - (2*b*c*d^2*(-1/2*(b*c*(-x^(-1) - c^2*x
)) - ((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(2*x^2) - (3*c^2*(b*c*x + S
qrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]) + (2*I)*(a + b*ArcCos[c*x])*ArcTan[E^
(I*ArcCos[c*x])] - I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I*b*PolyLog[2,
 I*E^(I*ArcCos[c*x])]))/2))/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.48

method	result
derivativedivides	$c^3\left(d^2a^2\left(cx-\frac{1}{3c^3x^3}+\frac{2}{cx}\right)-2d^2b^2\arccos(cx)\sqrt{-c^2x^2+1}+d^2b^2\arccos(cx)^2cx-2d\right.$
default	$c^3\left(d^2a^2\left(cx-\frac{1}{3c^3x^3}+\frac{2}{cx}\right)-2d^2b^2\arccos(cx)\sqrt{-c^2x^2+1}+d^2b^2\arccos(cx)^2cx-2d\right.$
parts	$d^2a^2\left(c^4x+\frac{2c^2}{x}-\frac{1}{3x^3}\right)-2d^2b^2c^3\sqrt{-c^2x^2+1}\,\arccos(cx)+d^2b^2c^4\arccos(cx)^2x-2b$

input
```
int((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(d^2*a^2*(c*x-1/3/c^3/x^3+2/c/x)-2*d^2*b^2*arccos(c*x)*(-c^2*x^2+1)^(1
/2)+d^2*b^2*arccos(c*x)^2*c*x-2*d^2*b^2*c*x+2*d^2*b^2*arccos(c*x)^2/c/x+1/
3*d^2*b^2/c^2/x^2*arccos(c*x)*(-c^2*x^2+1)^(1/2)-1/3*d^2*b^2/c^3/x^3*arcco
s(c*x)^2-1/3*d^2*b^2/c/x+11/3*d^2*b^2*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+
1)^(1/2)))-11/3*d^2*b^2*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-11/
3*I*d^2*b^2*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+11/3*I*d^2*b^2*dilog(1-I
*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*d^2*a*b*(c*x*arccos(c*x)-1/3*arccos(c*x)/c^
3/x^3+2*arccos(c*x)/c/x+1/6/c^2/x^2*(-c^2*x^2+1)^(1/2)-11/6*arctanh(1/(-c^
2*x^2+1)^(1/2))-(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int\frac{(d-c^2dx^2)^2(a+b\arccos(cx))^2}{x^4}\,dx=\int\frac{(c^2dx^2-d)^2(b\arccos(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arccos(c*x))/x^4, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^4}\,dx = d^2\left(\int a^2c^4\,dx + \int \frac{a^2}{x^4}\,dx + \int \left(-\frac{2a^2c^2}{x^2}\right)dx + \int b^2c^4\operatorname{acos}^2(cx)\,dx + \int \frac{b^2\operatorname{acos}^2(cx)}{x^4}\,dx + \int 2abc^4\operatorname{acos}(cx)\,dx + \int \frac{2ab\operatorname{acos}(cx)}{x^4}\,dx + \int \left(-\frac{2b^2c^2\operatorname{acos}^2(cx)}{x^2}\right)dx + \int \left(-\frac{4abc^2\operatorname{acos}(cx)}{x^2}\right)dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2*(a+b*acos(c*x))**2/x**4,x)
```

output
```
d**2*(Integral(a**2*c**4, x) + Integral(a**2/x**4, x) + Integral(-2*a**2*c
**2/x**2, x) + Integral(b**2*c**4*acos(c*x)**2, x) + Integral(b**2*acos(c*
x)**2/x**4, x) + Integral(2*a*b*c**4*acos(c*x), x) + Integral(2*a*b*acos(c
*x)/x**4, x) + Integral(-2*b**2*c**2*acos(c*x)**2/x**2, x) + Integral(-4*a
*b*c**2*acos(c*x)/x**2, x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^4}\,dx = \int \frac{(c^2dx^2-d)^2(b\arccos(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
b^2*c^4*d^2*x*arccos(c*x)^2 - 2*b^2*c^4*d^2*(x + sqrt(-c^2*x^2 + 1)*arccos
(c*x)/c) + a^2*c^4*d^2*x + 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a*b*c^
3*d^2 - 4*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*
a*b*c^2*d^2 + 1/3*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt
(-c^2*x^2 + 1)/x^2)*c - 2*arccos(c*x)/x^3)*a*b*d^2 + 2*a^2*c^2*d^2/x - 1/3
*a^2*d^2/x^3 - 1/3*(3*x^3*integrate(2/3*(6*b^2*c^3*d^2*x^2 - b^2*c*d^2)*sq
rt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^2
*x^5 - x^3), x) - (6*b^2*c^2*d^2*x^2 - b^2*d^2)*arctan2(sqrt(c*x + 1)*sqrt
(-c*x + 1), c*x)^2)/x^3
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2\,(a+b\arccos(cx))^2}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2/x^4,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2\,(a+b\arccos(cx))^2}{x^4}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^2}{x^4}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^2)/x^4,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^2)/x^4, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^2 (a+b\arccos(cx))^2}{x^4} dx$$
$$= \frac{d^2 \Big(3acos(cx)^2 b^2c^4x^4 - 6\sqrt{-c^2x^2+1}\, acos(cx)\, b^2c^3x^3 + 6acos(cx)\, ab\, c^4x^4 + 12acos(cx)\, ab\, c^2x^2 - 2acos(}{}$$

input
```
int((-c^2*d*x^2+d)^2*(a+b*acos(c*x))^2/x^4,x)
```

output
```
(d**2*(3*acos(c*x)**2*b**2*c**4*x**4 - 6*sqrt( - c**2*x**2 + 1)*acos(c*x)*
b**2*c**3*x**3 + 6*acos(c*x)*a*b*c**4*x**4 + 12*acos(c*x)*a*b*c**2*x**2 -
2*acos(c*x)*a*b - 6*sqrt( - c**2*x**2 + 1)*a*b*c**3*x**3 + sqrt( - c**2*x*
*2 + 1)*a*b*c*x + 3*int(acos(c*x)**2/x**4,x)*b**2*x**3 - 6*int(acos(c*x)**
2/x**2,x)*b**2*c**2*x**3 + 11*log(tan(asin(c*x)/2))*a*b*c**3*x**3 + 3*a**2
*c**4*x**4 + 6*a**2*c**2*x**2 - a**2 - 6*b**2*c**4*x**4))/(3*x**3)
```

3.176 $\int x^4(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 476

$$\begin{aligned}\int x^4(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx &= -\frac{100976b^2d^3x}{4002075c^4}-\frac{50488b^2d^3x^3}{12006225c^2}-\frac{12622b^2d^3x^5}{6670125} \\ &+\frac{9410b^2c^2d^3x^7}{1120581}-\frac{182b^2c^4d^3x^9}{29403}+\frac{2b^2c^6d^3x^{11}}{1331}+\frac{256bd^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{17325c^5} \\ &+\frac{128bd^3x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{17325c^3}+\frac{32bd^3x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{5775c} \\ &+\frac{16bd^3(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{693c^5}-\frac{4bd^3(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{1155c^5} \\ &+\frac{2bd^3(1-c^2x^2)^{7/2}(a+b\arccos(cx))}{1617c^5}-\frac{8bd^3(1-c^2x^2)^{9/2}(a+b\arccos(cx))}{297c^5} \\ &+\frac{2bd^3(1-c^2x^2)^{11/2}(a+b\arccos(cx))}{121c^5}+\frac{16d^3x^5(a+b\arccos(cx))^2}{1155} \\ &+\frac{8}{231}d^3x^5(1-c^2x^2)(a+b\arccos(cx))^2+\frac{2}{33}d^3x^5(1-c^2x^2)^2(a+b\arccos(cx))^2+\frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\end{aligned}$$

output
```
-100976/4002075*b^2*d^3*x/c^4-50488/12006225*b^2*d^3*x^3/c^2-12622/6670125
*b^2*d^3*x^5+9410/1120581*b^2*c^2*d^3*x^7-182/29403*b^2*c^4*d^3*x^9+2/1331
*b^2*c^6*d^3*x^11+256/17325*b*d^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^5
+128/17325*b*d^3*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3+32/5775*b*d^
3*x^4*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+16/693*b*d^3*(-c^2*x^2+1)^(3/
2)*(a+b*arccos(c*x))/c^5-4/1155*b*d^3*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))
/c^5+2/1617*b*d^3*(-c^2*x^2+1)^(7/2)*(a+b*arccos(c*x))/c^5-8/297*b*d^3*(-c
^2*x^2+1)^(9/2)*(a+b*arccos(c*x))/c^5+2/121*b*d^3*(-c^2*x^2+1)^(11/2)*(a+b
*arccos(c*x))/c^5+16/1155*d^3*x^5*(a+b*arccos(c*x))^2+8/231*d^3*x^5*(-c^2*
x^2+1)*(a+b*arccos(c*x))^2+2/33*d^3*x^5*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2
+1/11*d^3*x^5*(-c^2*x^2+1)^3*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.63

$$\int x^4(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx =$$
$$-\frac{d^3\left(12006225a^2c^5x^5(-231+495c^2x^2-385c^4x^4+105c^6x^6)-6930ab\sqrt{1-c^2x^2}(-50488-25244c^2x^2\right.}{}$$

input
```
Integrate[x^4*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/13867189875*(d^3*(12006225*a^2*c^5*x^5*(-231 + 495*c^2*x^2 - 385*c^4*x^
4 + 105*c^6*x^6) - 6930*a*b*Sqrt[1 - c^2*x^2]*(-50488 - 25244*c^2*x^2 - 18
933*c^4*x^4 + 117625*c^6*x^6 - 111475*c^8*x^8 + 33075*c^10*x^10) + b^2*(34
9881840*c*x + 58313640*c^3*x^3 + 26241138*c^5*x^5 - 116448750*c^7*x^7 + 85
835750*c^9*x^9 - 20837250*c^11*x^11) - 6930*b*(-3465*a*c^5*x^5*(-231 + 495
*c^2*x^2 - 385*c^4*x^4 + 105*c^6*x^6) + b*Sqrt[1 - c^2*x^2]*(-50488 - 2524
4*c^2*x^2 - 18933*c^4*x^4 + 117625*c^6*x^6 - 111475*c^8*x^8 + 33075*c^10*x
^10))*ArcCos[c*x] + 12006225*b^2*c^5*x^5*(-231 + 495*c^2*x^2 - 385*c^4*x^4
 + 105*c^6*x^6)*ArcCos[c*x]^2))/c^5
```

Rubi [A] (verified)

Time = 3.53 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.49, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.815, Rules used = {5203, 27, 5195, 27, 1467, 2009, 5203, 5195, 27, 1467, 2009, 5203, 5139, 5195, 27, 2009, 5211, 15, 5211, 15, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(d-c^2 d x^2\right)^3 (a+b \arccos (c x))^2 \, dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{2}{11} b c d^3 \int x^5 \left(1-c^2 x^2\right)^{5/2} (a+b \arccos (c x))dx+\frac{6}{11} d \int d^2 x^4 \left(1-c^2 x^2\right)^2 (a+b \arccos (c x))^2dx+\frac{1}{11} d^3 x^5 \left(1-c^2 x^2\right)^3 (a+b \arccos (c x))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{11} b c d^3 \int x^5 \left(1-c^2 x^2\right)^{5/2} (a+b \arccos (c x))dx+\frac{6}{11} d^3 \int x^4 \left(1-c^2 x^2\right)^2 (a+b \arccos (c x))^2dx+\frac{1}{11} d^3 x^5 \left(1-c^2 x^2\right)^3 (a+b \arccos (c x))^2$$

$$\Big\downarrow \text{5195}$$

$$\frac{6}{11} d^3 \int x^4 \left(1-c^2 x^2\right)^2 (a+b \arccos (c x))^2dx+\frac{2}{11} b c d^3 \left(b c \int -\frac{\left(1-c^2 x^2\right)^3 \left(63 c^4 x^4+28 c^2 x^2+8\right)}{693 c^6}dx-\frac{\left(1-c^2 x^2\right)^{11/2} (a+b \arccos (c x))}{11 c^6}+\frac{2 \left(1-c^2 x^2\right)^{9/2} (a+\right.$$
$$\frac{1}{11} d^3 x^5 \left(1-c^2 x^2\right)^3 (a+b \arccos (c x))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{11} d^3 \int x^4 \left(1-c^2 x^2\right)^2 (a+b \arccos (c x))^2dx+\frac{2}{11} b c d^3 \left(-\frac{b \int \left(1-c^2 x^2\right)^3 \left(63 c^4 x^4+28 c^2 x^2+8\right)dx}{693 c^5}-\frac{\left(1-c^2 x^2\right)^{11/2} (a+b \arccos (c x))}{11 c^6}+\frac{2 \left(1-c^2 x^2\right)^{9/2} (a+b\right.$$
$$\frac{1}{11} d^3 x^5 \left(1-c^2 x^2\right)^3 (a+b \arccos (c x))^2$$

$$\Big\downarrow \text{1467}$$

$$\frac{6}{11}d^3\int x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2dx+$$
$$\frac{2}{11}bcd^3\left(-\frac{b\int\left(-63c^{10}x^{10}+161c^8x^8-113c^6x^6+3c^4x^4+4c^2x^2+8\right)dx}{693c^5}-\frac{\left(1-c^2x^2\right)^{11/2}(a+b\arccos(cx))}{11c^6}+\right.$$
$$\frac{1}{11}d^3x^5\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{6}{11}d^3\int x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2dx+\frac{1}{11}d^3x^5\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{11}bcd^3\left(-\frac{\left(1-c^2x^2\right)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(c\right.$$

$$\Big\downarrow \text{5203}$$

$$\frac{6}{11}d^3\left(\frac{2}{9}bc\int x^5\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{4}{9}\int x^4\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{1}{9}x^5\left(1-c^2x^2\right)^2\right.$$
$$\frac{1}{11}d^3x^5\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{11}bcd^3\left(-\frac{\left(1-c^2x^2\right)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(c\right.$$

$$\Big\downarrow \text{5195}$$

$$\frac{6}{11}d^3\left(\frac{4}{9}\int x^4\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{2}{9}bc\left(bc\int-\frac{\left(1-c^2x^2\right)^2\left(35c^4x^4+20c^2x^2+8\right)}{315c^6}dx-\frac{(1-c^2x^2\right.\right.$$
$$\frac{1}{11}d^3x^5\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{11}bcd^3\left(-\frac{\left(1-c^2x^2\right)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(c\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{11}d^3\left(\frac{4}{9}\int x^4\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{2}{9}bc\left(-\frac{b\int\left(1-c^2x^2\right)^2\left(35c^4x^4+20c^2x^2+8\right)dx}{315c^5}-\frac{(1-c^2x^2)}{}\right.\right.$$
$$\frac{1}{11}d^3x^5\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{11}bcd^3\left(-\frac{\left(1-c^2x^2\right)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(c\right.$$

$$\Big\downarrow \text{1467}$$

$$\frac{6}{11}d^3\left(\frac{4}{9}\int x^4(1-c^2x^2)(a+b\arccos(cx))^2dx+\frac{2}{9}bc\left(-\frac{b\int(35c^8x^8-50c^6x^6+3c^4x^4+4c^2x^2+8)\,dx}{315c^5}-\frac{(1-c\right.\right.$$
$$\frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{11}bcd^3\left(-\frac{(1-c^2x^2)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2(1-c^2x^2)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{(1-c^2x^2)^{7/2}(a+b\arccos(c}{7c^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{6}{11}d^3\left(\frac{4}{9}\int x^4(1-c^2x^2)(a+b\arccos(cx))^2dx+\frac{1}{9}x^5(1-c^2x^2)^2(a+b\arccos(cx))^2+\frac{2}{9}bc\left(-\frac{(1-c^2x^2)^{9/2}(a}{9c}\right.\right.$$
$$\frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{11}bcd^3\left(-\frac{(1-c^2x^2)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2(1-c^2x^2)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{(1-c^2x^2)^{7/2}(a+b\arccos(c}{7c^6}\right.$$

$$\Big\downarrow \text{5203}$$

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}bc\int x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{2}{7}\int x^4(a+b\arccos(cx))^2dx+\frac{1}{7}x^5(1-c^2x^2)(a+b\arcco\right.\right.$$
$$\frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{11}bcd^3\left(-\frac{(1-c^2x^2)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2(1-c^2x^2)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{(1-c^2x^2)^{7/2}(a+b\arccos(c}{7c^6}\right.$$

$$\Big\downarrow \text{5139}$$

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{2}{5}bc\int\frac{x^5(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))^2\right)+\frac{2}{7}bc\int x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))\right.\right.$$
$$\frac{1}{11}d^3x^5(1-c^2x^2)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{11}bcd^3\left(-\frac{(1-c^2x^2)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2(1-c^2x^2)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{(1-c^2x^2)^{7/2}(a+b\arccos(c}{7c^6}\right.$$

$$\Big\downarrow \text{5195}$$

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{2}{5}bc\int\frac{x^5(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))^2\right)+\frac{2}{7}bc\left(bc\int-\frac{-15c^6x^6+3c^4x^4+4c^2x\right.\right.\right.$$
$$\frac{1}{11}d^3x^5\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{11}bcd^3\left(-\frac{\left(1-c^2x^2\right)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(c\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{2}{5}bc\int\frac{x^5(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))^2\right)+\frac{2}{7}bc\left(-\frac{b\int\left(-15c^6x^6+3c^4x^4+4c^2x^2\right.}{105c^5}\right.\right.\right.$$
$$\frac{1}{11}d^3x^5\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{11}bcd^3\left(-\frac{\left(1-c^2x^2\right)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(c\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{2}{5}bc\int\frac{x^5(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))^2\right)+\frac{1}{7}x^5\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\right.\right.$$
$$\frac{1}{11}d^3x^5\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{11}bcd^3\left(-\frac{\left(1-c^2x^2\right)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(c\right.$$

$$\Big\downarrow \text{5211}$$

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{2}{5}bc\left(\frac{4\int\frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{5c^2}-\frac{b\int x^4dx}{5c}-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{5c^2}\right)+\frac{1}{5}x^5(a+b\arccos(c\right.\right.\right.$$
$$\frac{1}{11}d^3x^5\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{11}bcd^3\left(-\frac{\left(1-c^2x^2\right)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(c\right.$$

$$\Big\downarrow \text{15}$$

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{2}{5}bc\left(\frac{4\int\frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{5c^2}-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{5c^2}-\frac{bx^5}{25c}\right)+\frac{1}{5}x^5(a+b\arccos(cx))^2\right.\right.\right.$$

$$\frac{1}{11}d^3x^5\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$

$$\frac{2}{11}bcd^3\left(-\frac{\left(1-c^2x^2\right)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(c\right.}{7c^6}$$

$$\Big\downarrow \text{5211}$$

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{2}{5}bc\left(\frac{4\left(\frac{2\int\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{b\int x^2dx}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}\right)}{5c^2}-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos(cx)}{5c^2}\right.\right.\right.\right.$$

$$\frac{1}{11}d^3x^5\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$

$$\frac{2}{11}bcd^3\left(-\frac{\left(1-c^2x^2\right)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(c\right.}{7c^6}$$

$$\Big\downarrow \text{15}$$

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{2}{5}bc\left(\frac{4\left(\frac{2\int\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{5c^2}-\frac{x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{5c^2}-\right.\right.\right.\right.$$

$$\frac{1}{11}d^3x^5\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$

$$\frac{2}{11}bcd^3\left(-\frac{\left(1-c^2x^2\right)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(c\right.}{7c^6}$$

$$\Big\downarrow \text{5183}$$

$$\frac{6}{11}d^3\left(\frac{4}{9}\left(\frac{2}{7}\left(\frac{2}{5}bc\left(\frac{4\left(\frac{2\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{5c^2}-\frac{x^4\sqrt{1-c^2x^2}(a+b\,a\right.}{5c^2}\right.\right.\right.\right.$$

$$\frac{1}{11}d^3x^5\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$

$$\frac{2}{11}bcd^3\left(-\frac{\left(1-c^2x^2\right)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(c\right.}{7c^6}\right.$$

$$\Big\downarrow \textbf{24}$$

$$\frac{1}{11}d^3x^5\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$

$$\frac{6}{11}d^3\left(\frac{1}{9}x^5\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{4}{9}\left(\frac{1}{7}x^5\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{7}\left(\frac{2}{5}bc\left(-\frac{x^4\sqrt{1-c^2x^2}(a\right.}{5}\right.\right.\right.\right.$$

$$\frac{2}{11}bcd^3\left(-\frac{\left(1-c^2x^2\right)^{11/2}(a+b\arccos(cx))}{11c^6}+\frac{2\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^6}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(c\right.}{7c^6}\right.$$

input
```
Int[x^4*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^3*x^5*(1 - c^2*x^2)^3*(a + b*ArcCos[c*x])^2)/11 + (2*b*c*d^3*(-1/693*(b
*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (113*c^6*x^7)/7 + (161*c^8*x^9)/9
- (63*c^10*x^11)/11))/c^5 - ((1 - c^2*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(7*c
^6) + (2*(1 - c^2*x^2)^(9/2)*(a + b*ArcCos[c*x]))/(9*c^6) - ((1 - c^2*x^2)
^(11/2)*(a + b*ArcCos[c*x]))/(11*c^6)))/11 + (6*d^3*((x^5*(1 - c^2*x^2)^2*
(a + b*ArcCos[c*x])^2)/9 + (2*b*c*(-1/315*(b*(8*x + (4*c^2*x^3)/3 + (3*c^4
*x^5)/5 - (50*c^6*x^7)/7 + (35*c^8*x^9)/9))/c^5 - ((1 - c^2*x^2)^(5/2)*(a
+ b*ArcCos[c*x]))/(5*c^6) + (2*(1 - c^2*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(7
*c^6) - ((1 - c^2*x^2)^(9/2)*(a + b*ArcCos[c*x]))/(9*c^6)))/9 + (4*((x^5*(
1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/7 + (2*b*c*(-1/105*(b*(8*x + (4*c^2*x^
3)/3 + (3*c^4*x^5)/5 - (15*c^6*x^7)/7))/c^5 - ((1 - c^2*x^2)^(3/2)*(a + b*
ArcCos[c*x]))/(3*c^6) + (2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^6
) - ((1 - c^2*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(7*c^6)))/7 + (2*((x^5*(a +
b*ArcCos[c*x])^2)/5 + (2*b*c*(-1/25*(b*x^5)/c - (x^4*Sqrt[1 - c^2*x^2]*(a
+ b*ArcCos[c*x]))/(5*c^2) + (4*(-1/9*(b*x^3)/c - (x^2*Sqrt[1 - c^2*x^2]*(a
 + b*ArcCos[c*x]))/(3*c^2) + (2*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*Ar
cCos[c*x]))/c^2))/(3*c^2)))/(5*c^2)))/5))/7))/9))/11
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 618, normalized size of antiderivative = 1.30

method	result
orering	$\frac{(3448564875c^{14}x^{14}-16454567500x^{12}c^{12}+29885660250c^{10}x^{10}-23335495700c^{8}x^{8}+3719665587c^{6}x^{6}-16269505560c^{4}}{13867189875x\,c^{6}(cx-1)^{2}(cx+1)^{2}(c^{2}x^{2}-1)^{2}}$
parts	$-d^{3}a^{2}\left(\frac{1}{11}c^{6}x^{11}-\frac{1}{3}c^{4}x^{9}+\frac{3}{7}c^{2}x^{7}-\frac{1}{5}x^{5}\right)-\frac{d^{3}b^{2}\left(-\frac{8\arccos(cx)\left(c^{2}x^{2}-1\right)^{4}\sqrt{-c^{2}x^{2}+1}}{297}+\frac{2\arccos(cx)^{2}\left(35c^{8}\right.}{}\right.}{}$
derivativedivides	$\frac{-d^{3}a^{2}\left(\frac{1}{11}c^{11}x^{11}-\frac{1}{3}c^{9}x^{9}+\frac{3}{7}c^{7}x^{7}-\frac{1}{5}c^{5}x^{5}\right)-d^{3}b^{2}\left(-\frac{8\arccos(cx)\left(c^{2}x^{2}-1\right)^{4}\sqrt{-c^{2}x^{2}+1}}{297}+\frac{2\arccos(cx)^{2}\left(35c^{8}x^{8}-180c^{6}x^{6}+\right.}{315}\right.}{}$
default	$\frac{-d^{3}a^{2}\left(\frac{1}{11}c^{11}x^{11}-\frac{1}{3}c^{9}x^{9}+\frac{3}{7}c^{7}x^{7}-\frac{1}{5}c^{5}x^{5}\right)-d^{3}b^{2}\left(-\frac{8\arccos(cx)\left(c^{2}x^{2}-1\right)^{4}\sqrt{-c^{2}x^{2}+1}}{297}+\frac{2\arccos(cx)^{2}\left(35c^{8}x^{8}-180c^{6}x^{6}+\right.}{315}\right.}{}$

input
```
int(x^4*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/13867189875*(3448564875*c^14*x^14-16454567500*c^12*x^12+29885660250*c^10
*x^10-23335495700*c^8*x^8+3719665587*c^6*x^6-16269505560*c^4*x^4+151615464
00*c^2*x^2-4198582080)/x/c^6/(c*x-1)^2/(c*x+1)^2/(c^2*x^2-1)^2*(-c^2*d*x^2
+d)^3*(a+b*arccos(c*x))^2-1/13867189875*(312558750*c^12*x^12-1399654375*c^
10*x^10+2243437625*c^8*x^8-1188259281*c^6*x^6-470882643*c^4*x^4-3178093380
*c^2*x^2+1574468280)/x^4/c^6/(c*x-1)^2/(c*x+1)^2/(c^2*x^2-1)*(4*x^3*(-c^2*
d*x^2+d)^3*(a+b*arccos(c*x))^2-6*x^5*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2*
d*c^2-2*x^4*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))*b*c/(-c^2*x^2+1)^(1/2))+1/1
3867189875/x^3*(10418625*c^10*x^10-42917875*c^8*x^8+58224375*c^6*x^6-13120
569*c^4*x^4-29156820*c^2*x^2-174940920)/c^6/(c*x-1)^2/(c*x+1)^2*(12*x^2*(-
c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2-54*x^4*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x
))^2*d*c^2-16*x^3*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))*b*c/(-c^2*x^2+1)^(1/2
)+24*x^6*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2*d^2*c^4+24*x^5*(-c^2*d*x^2+d)^
2*(a+b*arccos(c*x))*d*c^3*b/(-c^2*x^2+1)^(1/2)+2*x^4*(-c^2*d*x^2+d)^3*b^2*
c^2/(-c^2*x^2+1)-2*x^5*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))*b*c^3/(-c^2*x^2+
1)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.87

$$\int x^4 (d - c^2 d x^2)^3 (a + b \arccos(cx))^2 \, dx =$$

$$-\frac{10418625 (121 a^2 - 2 b^2) c^{11} d^3 x^{11} - 471625 (9801 a^2 - 182 b^2) c^9 d^3 x^9 + 12375 (480249 a^2 - 9410 b^2) c^7 d}{}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
-1/13867189875*(10418625*(121*a^2 - 2*b^2)*c^11*d^3*x^11 - 471625*(9801*a^
2 - 182*b^2)*c^9*d^3*x^9 + 12375*(480249*a^2 - 9410*b^2)*c^7*d^3*x^7 - 207
9*(1334025*a^2 - 12622*b^2)*c^5*d^3*x^5 + 58313640*b^2*c^3*d^3*x^3 + 34988
1840*b^2*c*d^3*x + 12006225*(105*b^2*c^11*d^3*x^11 - 385*b^2*c^9*d^3*x^9 +
 495*b^2*c^7*d^3*x^7 - 231*b^2*c^5*d^3*x^5)*arccos(c*x)^2 + 24012450*(105*
a*b*c^11*d^3*x^11 - 385*a*b*c^9*d^3*x^9 + 495*a*b*c^7*d^3*x^7 - 231*a*b*c^
5*d^3*x^5)*arccos(c*x) - 6930*(33075*a*b*c^10*d^3*x^10 - 111475*a*b*c^8*d^
3*x^8 + 117625*a*b*c^6*d^3*x^6 - 18933*a*b*c^4*d^3*x^4 - 25244*a*b*c^2*d^3
*x^2 - 50488*a*b*d^3 + (33075*b^2*c^10*d^3*x^10 - 111475*b^2*c^8*d^3*x^8 +
 117625*b^2*c^6*d^3*x^6 - 18933*b^2*c^4*d^3*x^4 - 25244*b^2*c^2*d^3*x^2 -
50488*b^2*d^3)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.49

$$\int x^4 (d - c^2 d x^2)^3 (a + b \arccos(cx))^2 \, dx = \text{Too large to display}$$

input
```
integrate(x**4*(-c**2*d*x**2+d)**3*(a+b*acos(c*x))**2,x)
```

output

```
Piecewise((-a**2*c**6*d**3*x**11/11 + a**2*c**4*d**3*x**9/3 - 3*a**2*c**2*
d**3*x**7/7 + a**2*d**3*x**5/5 - 2*a*b*c**6*d**3*x**11*acos(c*x)/11 + 2*a*
b*c**5*d**3*x**10*sqrt(-c**2*x**2 + 1)/121 + 2*a*b*c**4*d**3*x**9*acos(c*x
)/3 - 182*a*b*c**3*d**3*x**8*sqrt(-c**2*x**2 + 1)/3267 - 6*a*b*c**2*d**3*x
**7*acos(c*x)/7 + 9410*a*b*c*d**3*x**6*sqrt(-c**2*x**2 + 1)/160083 + 2*a*b
*d**3*x**5*acos(c*x)/5 - 12622*a*b*d**3*x**4*sqrt(-c**2*x**2 + 1)/(1334025
*c) - 50488*a*b*d**3*x**2*sqrt(-c**2*x**2 + 1)/(4002075*c**3) - 100976*a*b
*d**3*sqrt(-c**2*x**2 + 1)/(4002075*c**5) - b**2*c**6*d**3*x**11*acos(c*x)
**2/11 + 2*b**2*c**6*d**3*x**11/1331 + 2*b**2*c**5*d**3*x**10*sqrt(-c**2*x
**2 + 1)*acos(c*x)/121 + b**2*c**4*d**3*x**9*acos(c*x)**2/3 - 182*b**2*c**
4*d**3*x**9/29403 - 182*b**2*c**3*d**3*x**8*sqrt(-c**2*x**2 + 1)*acos(c*x)
/3267 - 3*b**2*c**2*d**3*x**7*acos(c*x)**2/7 + 9410*b**2*c**2*d**3*x**7/11
20581 + 9410*b**2*c*d**3*x**6*sqrt(-c**2*x**2 + 1)*acos(c*x)/160083 + b**2
*d**3*x**5*acos(c*x)**2/5 - 12622*b**2*d**3*x**5/6670125 - 12622*b**2*d**3
*x**4*sqrt(-c**2*x**2 + 1)*acos(c*x)/(1334025*c) - 50488*b**2*d**3*x**3/(1
2006225*c**2) - 50488*b**2*d**3*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(40020
75*c**3) - 100976*b**2*d**3*x/(4002075*c**4) - 100976*b**2*d**3*sqrt(-c**2
*x**2 + 1)*acos(c*x)/(4002075*c**5), Ne(c, 0)), (d**3*x**5*(a + pi*b/2)**2
/5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. $2(421) = 842$.

Time = 0.21 (sec) , antiderivative size = 1141, normalized size of antiderivative = 2.40

$$\int x^4 \left(d - c^2 d x^2\right)^3 (a + b \arccos(cx))^2 \, dx = \text{Too large to display}$$

input

```
integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output

```
-1/11*b^2*c^6*d^3*x^11*arccos(c*x)^2 - 1/11*a^2*c^6*d^3*x^11 + 1/3*b^2*c^4
*d^3*x^9*arccos(c*x)^2 + 1/3*a^2*c^4*d^3*x^9 - 3/7*b^2*c^2*d^3*x^7*arccos(
c*x)^2 - 3/7*a^2*c^2*d^3*x^7 - 2/7623*(693*x^11*arccos(c*x) - (63*sqrt(-c^
2*x^2 + 1)*x^10/c^2 + 70*sqrt(-c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(-c^2*x^2 + 1
)*x^6/c^6 + 96*sqrt(-c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(-c^2*x^2 + 1)*x^2/c^1
0 + 256*sqrt(-c^2*x^2 + 1)/c^12)*c)*a*b*c^6*d^3 + 2/26413695*(3465*(63*sqr
t(-c^2*x^2 + 1)*x^10/c^2 + 70*sqrt(-c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(-c^2*x^
2 + 1)*x^6/c^6 + 96*sqrt(-c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(-c^2*x^2 + 1)*x^
2/c^10 + 256*sqrt(-c^2*x^2 + 1)/c^12)*c*arccos(c*x) + (19845*c^10*x^11 + 2
6950*c^8*x^9 + 39600*c^6*x^7 + 66528*c^4*x^5 + 147840*c^2*x^3 + 887040*x)/
c^10)*b^2*c^6*d^3 + 1/5*b^2*d^3*x^5*arccos(c*x)^2 + 2/945*(315*x^9*arccos(
c*x) - (35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48
*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^
2*x^2 + 1)/c^10)*c)*a*b*c^4*d^3 - 2/297675*(315*(35*sqrt(-c^2*x^2 + 1)*x^8
/c^2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*
sqrt(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/c^10)*c*arccos(c*x) +
(1225*c^8*x^9 + 1800*c^6*x^7 + 3024*c^4*x^5 + 6720*c^2*x^3 + 40320*x)/c^8)
*b^2*c^4*d^3 + 1/5*a^2*d^3*x^5 - 6/245*(35*x^7*arccos(c*x) - (5*sqrt(-c^2*
x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2
/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*a*b*c^2*d^3 + 2/8575*(105*(5*sqrt(...
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.25

$$\int x^4(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx = -\frac{1}{11}b^2c^6d^3x^{11}\arccos(cx)^2 - \frac{2}{11}abc^6d^3x^{11}\arccos(cx) - \frac{1}{11}a^2c^6d^3x^{11} + \frac{2}{1331}b^2c^6d^3x^{11} + \frac{2}{121}\sqrt{-c^2x^2+1}b^2c^5d^3x^{10}\arccos(cx) + \frac{2}{121}\sqrt{-c^2x^2+1}abc^5d^3x^{10} + \frac{1}{3}b^2c^4d^3x^9\arccos(cx)^2 + \frac{2}{3}abc^4d^3x^9\arccos(cx) + \frac{1}{3}a^2c^4d^3x^9 - \frac{182}{29403}b^2c^4d^3x^9 - \frac{182}{3267}\sqrt{-c^2x^2+1}b^2c^3d^3x^8\arccos(cx) - \frac{182}{3267}\sqrt{-c^2x^2+1}abc^3d^3x^8 - \frac{3}{7}b^2c^2d^3x^7\arccos(cx)^2 - \frac{6}{7}abc^2d^3x^7\arccos(cx) - \frac{3}{7}a^2c^2d^3x^7 + \frac{9410}{1120581}b^2c^2d^3x^7 + \frac{9410}{160083}\sqrt{-c^2x^2+1}b^2cd^3x^6\arccos(cx) + \frac{9410}{160083}\sqrt{-c^2x^2+1}abcd^3x^6 + \frac{1}{5}b^2d^3x^5\arccos(cx)^2 + \frac{2}{5}abd^3x^5\arccos(cx) + \frac{1}{5}a^2d^3x^5 - \frac{12622}{6670125}b^2d^3x^5 - \frac{12622\sqrt{-c^2x^2+1}b^2d^3x^4\arccos(cx)}{1334025\,c} - \frac{12622\sqrt{-c^2x^2+1}abd^3x^4}{1334025\,c} - \frac{50488\,b^2d^3x^3}{12006225\,c^2} - \frac{50488\sqrt{-c^2x^2+1}b^2d^3x^2\arccos(cx)}{4002075\,c^3} - \frac{50488\sqrt{-c^2x^2+1}abd^3x^2}{4002075\,c^3} - \frac{100976\,b^2d^3x}{4002075\,c^4} - \frac{100976\sqrt{-c^2x^2+1}b^2d^3\arccos(cx)}{4002075\,c^5} - \frac{100976\sqrt{-c^2x^2+1}abd^3}{4002075\,c^5}$$

input
```
integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-1/11*b^2*c^6*d^3*x^11*arccos(c*x)^2 - 2/11*a*b*c^6*d^3*x^11*arccos(c*x) -
 1/11*a^2*c^6*d^3*x^11 + 2/1331*b^2*c^6*d^3*x^11 + 2/121*sqrt(-c^2*x^2 + 1
)*b^2*c^5*d^3*x^10*arccos(c*x) + 2/121*sqrt(-c^2*x^2 + 1)*a*b*c^5*d^3*x^10
 + 1/3*b^2*c^4*d^3*x^9*arccos(c*x)^2 + 2/3*a*b*c^4*d^3*x^9*arccos(c*x) + 1
/3*a^2*c^4*d^3*x^9 - 182/29403*b^2*c^4*d^3*x^9 - 182/3267*sqrt(-c^2*x^2 +
1)*b^2*c^3*d^3*x^8*arccos(c*x) - 182/3267*sqrt(-c^2*x^2 + 1)*a*b*c^3*d^3*x
^8 - 3/7*b^2*c^2*d^3*x^7*arccos(c*x)^2 - 6/7*a*b*c^2*d^3*x^7*arccos(c*x) -
 3/7*a^2*c^2*d^3*x^7 + 9410/1120581*b^2*c^2*d^3*x^7 + 9410/160083*sqrt(-c^
2*x^2 + 1)*b^2*c*d^3*x^6*arccos(c*x) + 9410/160083*sqrt(-c^2*x^2 + 1)*a*b*
c*d^3*x^6 + 1/5*b^2*d^3*x^5*arccos(c*x)^2 + 2/5*a*b*d^3*x^5*arccos(c*x) +
1/5*a^2*d^3*x^5 - 12622/6670125*b^2*d^3*x^5 - 12622/1334025*sqrt(-c^2*x^2
+ 1)*b^2*d^3*x^4*arccos(c*x)/c - 12622/1334025*sqrt(-c^2*x^2 + 1)*a*b*d^3*
x^4/c - 50488/12006225*b^2*d^3*x^3/c^2 - 50488/4002075*sqrt(-c^2*x^2 + 1)*
b^2*d^3*x^2*arccos(c*x)/c^3 - 50488/4002075*sqrt(-c^2*x^2 + 1)*a*b*d^3*x^2
/c^3 - 100976/4002075*b^2*d^3*x/c^4 - 100976/4002075*sqrt(-c^2*x^2 + 1)*b^
2*d^3*arccos(c*x)/c^5 - 100976/4002075*sqrt(-c^2*x^2 + 1)*a*b*d^3/c^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx=\int x^4\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

input
```
int(x^4*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^3,x)
```

output
```
int(x^4*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int x^4(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx$$
$$=\frac{d^3(-727650acos(cx)\,ab\,c^{11}x^{11}+2668050acos(cx)\,ab\,c^9x^9-3430350acos(cx)\,ab\,c^7x^7+1600830acos(cx)}{}$$

input
```
int(x^4*(-c^2*d*x^2+d)^3*(a+b*acos(c*x))^2,x)
```

output
```
(d**3*( - 727650*acos(c*x)*a*b*c**11*x**11 + 2668050*acos(c*x)*a*b*c**9*x*
*9 - 3430350*acos(c*x)*a*b*c**7*x**7 + 1600830*acos(c*x)*a*b*c**5*x**5 + 6
6150*sqrt( - c**2*x**2 + 1)*a*b*c**10*x**10 - 222950*sqrt( - c**2*x**2 + 1
)*a*b*c**8*x**8 + 235250*sqrt( - c**2*x**2 + 1)*a*b*c**6*x**6 - 37866*sqrt
( - c**2*x**2 + 1)*a*b*c**4*x**4 - 50488*sqrt( - c**2*x**2 + 1)*a*b*c**2*x
**2 - 100976*sqrt( - c**2*x**2 + 1)*a*b - 4002075*int(acos(c*x)**2*x**10,x
)*b**2*c**11 + 12006225*int(acos(c*x)**2*x**8,x)*b**2*c**9 - 12006225*int(
acos(c*x)**2*x**6,x)*b**2*c**7 + 4002075*int(acos(c*x)**2*x**4,x)*b**2*c**
5 - 363825*a**2*c**11*x**11 + 1334025*a**2*c**9*x**9 - 1715175*a**2*c**7*x
**7 + 800415*a**2*c**5*x**5))/(4002075*c**5)
```

3.177 $\int x^3(d-c^2dx^2)^3\,(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 384

$$\begin{aligned}\int x^3\left(d-c^2dx^2\right)^3(a+b\arccos(cx))^2\,dx=&-\frac{79b^2d^3x^2}{5120c^2}-\frac{79b^2d^3x^4}{15360}+\frac{401b^2c^2d^3x^6}{28800}\\&-\frac{57b^2c^4d^3x^8}{6400}+\frac{1}{500}b^2c^6d^3x^{10}\\&+\frac{79bd^3x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2560c^3}\\&+\frac{79bd^3x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3840c}\\&-\frac{31}{960}bcd^3x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))\\&-\frac{1}{32}bcd^3x^5\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\\&-\frac{1}{50}bcd^3x^5\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))\\&-\frac{79d^3(a+b\arccos(cx))^2}{5120c^4}\\&+\frac{1}{40}d^3x^4(a+b\arccos(cx))^2\\&+\frac{1}{20}d^3x^4\left(1-c^2x^2\right)(a+b\arccos(cx))^2\\&+\frac{3}{40}d^3x^4\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2\\&+\frac{1}{10}d^3x^4\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2\end{aligned}$$

output
```
-79/5120*b^2*d^3*x^2/c^2-79/15360*b^2*d^3*x^4+401/28800*b^2*c^2*d^3*x^6-57
/6400*b^2*c^4*d^3*x^8+1/500*b^2*c^6*d^3*x^10+79/2560*b*d^3*x*(-c^2*x^2+1)^
(1/2)*(a+b*arccos(c*x))/c^3+79/3840*b*d^3*x^3*(-c^2*x^2+1)^(1/2)*(a+b*arcc
os(c*x))/c-31/960*b*c*d^3*x^5*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))-1/32*b*
c*d^3*x^5*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))-1/50*b*c*d^3*x^5*(-c^2*x^2+
1)^(5/2)*(a+b*arccos(c*x))-79/5120*d^3*(a+b*arccos(c*x))^2/c^4+1/40*d^3*x^
4*(a+b*arccos(c*x))^2+1/20*d^3*x^4*(-c^2*x^2+1)*(a+b*arccos(c*x))^2+3/40*d
^3*x^4*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2+1/10*d^3*x^4*(-c^2*x^2+1)^3*(a+b
*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.76

$$\int x^3(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx =$$
$$-\frac{d^3\left(cx\left(28800a^2c^3x^3(-10+20c^2x^2-15c^4x^4+4c^6x^6)-30ab\sqrt{1-c^2x^2}(-1185-790c^2x^2+3208c^4x^4\right.\right.}{}$$

input
```
Integrate[x^3*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/1152000*(d^3*(c*x*(28800*a^2*c^3*x^3*(-10 + 20*c^2*x^2 - 15*c^4*x^4 + 4
*c^6*x^6) - 30*a*b*Sqrt[1 - c^2*x^2]*(-1185 - 790*c^2*x^2 + 3208*c^4*x^4 -
 2736*c^6*x^6 + 768*c^8*x^8) + b^2*(17775*c*x + 5925*c^3*x^3 - 16040*c^5*x
^5 + 10260*c^7*x^7 - 2304*c^9*x^9)) + 30*b*c*x*(1920*a*c^3*x^3*(-10 + 20*c
^2*x^2 - 15*c^4*x^4 + 4*c^6*x^6) + b*Sqrt[1 - c^2*x^2]*(1185 + 790*c^2*x^2
 - 3208*c^4*x^4 + 2736*c^6*x^6 - 768*c^8*x^8))*ArcCos[c*x] + 225*b^2*(79 -
 1280*c^4*x^4 + 2560*c^6*x^6 - 1920*c^8*x^8 + 512*c^10*x^10)*ArcCos[c*x]^2
 - 35550*a*b*ArcSin[c*x]))/c^4
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 916 vs. 2(384) = 768.

Time = 3.73 (sec) , antiderivative size = 916, normalized size of antiderivative = 2.39, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {5203, 27, 5203, 243, 49, 2009, 5203, 244, 2009, 5139, 5199, 15, 5211, 15, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d-c^2 d x^2\right)^3 (a+b \arccos(c x))^2 \, dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{1}{5} b c d^3 \int x^4 \left(1-c^2 x^2\right)^{5/2} (a+b \arccos(c x)) dx+\frac{3}{5} d \int d^2 x^3 \left(1-c^2 x^2\right)^2 (a+b \arccos(c x))^2 dx+\frac{1}{10} d^3 x^4 \left(1-c^2 x^2\right)^3 (a+b \arccos(c x))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5} b c d^3 \int x^4 \left(1-c^2 x^2\right)^{5/2} (a+b \arccos(c x)) dx+\frac{3}{5} d^3 \int x^3 \left(1-c^2 x^2\right)^2 (a+b \arccos(c x))^2 dx+\frac{1}{10} d^3 x^4 \left(1-c^2 x^2\right)^3 (a+b \arccos(c x))^2$$

$$\Big\downarrow \text{5203}$$

$$\frac{1}{5} b c d^3 \left(\frac{1}{2} \int x^4 \left(1-c^2 x^2\right)^{3/2} (a+b \arccos(c x)) dx+\frac{1}{10} b c \int x^5 \left(1-c^2 x^2\right)^2 dx+\frac{1}{10} x^5 \left(1-c^2 x^2\right)^{5/2} (a+b \arccos(\right.$$
$$\frac{3}{5} d^3 \left(\frac{1}{4} b c \int x^4 \left(1-c^2 x^2\right)^{3/2} (a+b \arccos(c x)) dx+\frac{1}{2} \int x^3 \left(1-c^2 x^2\right) (a+b \arccos(c x))^2 dx+\frac{1}{8} x^4 \left(1-c^2 x^2\right)^2 (\right.$$
$$\frac{1}{10} d^3 x^4 \left(1-c^2 x^2\right)^3 (a+b \arccos(c x))^2$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{5} b c d^3 \left(\frac{1}{2} \int x^4 \left(1-c^2 x^2\right)^{3/2} (a+b \arccos(c x)) dx+\frac{1}{20} b c \int x^4 \left(1-c^2 x^2\right)^2 dx^2+\frac{1}{10} x^5 \left(1-c^2 x^2\right)^{5/2} (a+b \arccos\right.$$
$$\frac{3}{5} d^3 \left(\frac{1}{4} b c \int x^4 \left(1-c^2 x^2\right)^{3/2} (a+b \arccos(c x)) dx+\frac{1}{2} \int x^3 \left(1-c^2 x^2\right) (a+b \arccos(c x))^2 dx+\frac{1}{8} x^4 \left(1-c^2 x^2\right)^2 (\right.$$
$$\frac{1}{10} d^3 x^4 \left(1-c^2 x^2\right)^3 (a+b \arccos(c x))^2$$

$\downarrow$ 49

$$\frac{3}{5}d^3\left(\frac{1}{4}bc\int x^4\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{2}\int x^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{1}{8}x^4\left(1-c^2x^2\right)^2(a\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\int x^4\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{20}bc\int\left(c^4x^8-2c^2x^6+x^4\right)dx^2+\frac{1}{10}x^5\left(1-c^2x^2\right)^{5/2}(a+b\right.$$
$$\frac{1}{10}d^3x^4\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$\downarrow$ 2009

$$\frac{3}{5}d^3\left(\frac{1}{4}bc\int x^4\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{2}\int x^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{1}{8}x^4\left(1-c^2x^2\right)^2(a\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\int x^4\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{10}x^5\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{1}{20}bc\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}\right.\right.$$
$$\frac{1}{10}d^3x^4\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$\downarrow$ 5203

$$\frac{3}{5}d^3\left(\frac{1}{4}bc\left(\frac{3}{8}\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{8}bc\int x^5\left(1-c^2x^2\right)dx+\frac{1}{8}x^5\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx\right.\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\left(\frac{3}{8}\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{8}bc\int x^5\left(1-c^2x^2\right)dx+\frac{1}{8}x^5\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx\right.\right.$$
$$\frac{1}{10}d^3x^4\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$\downarrow$ 244

$$\frac{3}{5}d^3\left(\frac{1}{2}\left(\frac{1}{3}bc\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{3}\int x^3(a+b\arccos(cx))^2dx+\frac{1}{6}x^4\left(1-c^2x^2\right)(a+b\arccos(\right.\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\left(\frac{3}{8}\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{8}bc\int\left(x^5-c^2x^7\right)dx+\frac{1}{8}x^5\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx)\right.\right.$$
$$\frac{1}{10}d^3x^4\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$\downarrow$ 2009

$$\frac{3}{5}d^3\left(\frac{1}{2}\left(\frac{1}{3}bc\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{3}\int x^3(a+b\arccos(cx))^2dx+\frac{1}{6}x^4\left(1-c^2x^2\right)(a+b\arccos(\right.\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\left(\frac{3}{8}\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{8}x^5\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{8}bc\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\right)\right.+$$
$$\frac{1}{10}d^3x^4\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$\downarrow$ 5139

$$\frac{3}{5}d^3\left(\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}bc\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))^2\right)+\frac{1}{3}bc\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))d\right.\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\left(\frac{3}{8}\int x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{8}x^5\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{8}bc\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)\right)+\right.$$
$$\frac{1}{10}d^3x^4\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \; 5199$$

$$\frac{3}{5}d^3\left(\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}bc\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))^2\right)+\frac{1}{3}bc\left(\frac{1}{6}\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{6}b\right.\right.\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\left(\frac{3}{8}\left(\frac{1}{6}\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{6}bc\int x^5dx+\frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)+\frac{1}{8}x^5\left(1-c^2x^2\right)\right.\right.$$
$$\frac{1}{10}d^3x^4\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \; 15$$

$$\frac{3}{5}d^3\left(\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}bc\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))^2\right)+\frac{1}{3}bc\left(\frac{1}{6}\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{6}x\right.\right.\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\left(\frac{3}{8}\left(\frac{1}{6}\int\frac{x^4(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{36}bcx^6\right)+\frac{1}{8}x^5\left(1-c^2x^2\right)^{3/2}(\right.\right.$$
$$\frac{1}{10}d^3x^4\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \; 5211$$

$$\frac{1}{10}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2x^4+$$
$$\frac{1}{5}bcd^3\left(\frac{1}{10}\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))x^5+\frac{1}{20}bc\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)+\frac{1}{2}\left(\frac{1}{8}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(c\right.\right.$$
$$\frac{3}{5}d^3\left(\frac{1}{8}\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2x^4+\frac{1}{4}bc\left(\frac{1}{8}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))x^5+\frac{1}{8}bc\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)+\frac{3}{8}\right.\right.$$

$$\Big\downarrow \; 15$$

$$\frac{3}{5}d^3\left(\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}bc\left(\frac{3\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)+\frac{1}{4}x^4(a+b\arccos(cx))^2\right)\right.\right.$$
$$\frac{1}{5}bcd^3\left(\frac{1}{2}\left(\frac{3}{8}\left(\frac{1}{6}\left(\frac{3\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{4c^2}-\frac{bx^4}{16c}\right)+\frac{1}{6}x^5\sqrt{1-c^2x^2}(a+b\arccos\right.\right.\right.$$
$$\frac{1}{10}d^3x^4\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5211}$$

$$\frac{1}{10}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2x^4+$$

$$\frac{1}{5}bcd^3\left(\frac{1}{10}\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))x^5+\frac{1}{20}bc\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)+\frac{1}{2}\left(\frac{1}{8}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(c\right.\right.$$

$$\frac{3}{5}d^3\left(\frac{1}{8}\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2x^4+\frac{1}{4}bc\left(\frac{1}{8}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))x^5+\frac{1}{8}bc\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)+\frac{3}{8}\right.\right.$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{10}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2x^4+$$

$$\frac{1}{5}bcd^3\left(\frac{1}{10}\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))x^5+\frac{1}{20}bc\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)+\frac{1}{2}\left(\frac{1}{8}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(c\right.\right.$$

$$\frac{3}{5}d^3\left(\frac{1}{8}\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2x^4+\frac{1}{4}bc\left(\frac{1}{8}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))x^5+\frac{1}{8}bc\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)+\frac{3}{8}\right.\right.$$

$$\Big\downarrow \text{5153}$$

$$\frac{1}{10}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2x^4+$$

$$\frac{1}{5}bcd^3\left(\frac{1}{10}\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))x^5+\frac{1}{20}bc\left(\frac{c^4x^{10}}{5}-\frac{c^2x^8}{2}+\frac{x^6}{3}\right)+\frac{1}{2}\left(\frac{1}{8}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(c\right.\right.$$

$$\frac{3}{5}d^3\left(\frac{1}{8}\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2x^4+\frac{1}{4}bc\left(\frac{1}{8}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))x^5+\frac{1}{8}bc\left(\frac{x^6}{6}-\frac{c^2x^8}{8}\right)+\frac{3}{8}\right.\right.$$

input
```
Int[x^3*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^3*x^4*(1 - c^2*x^2)^3*(a + b*ArcCos[c*x])^2)/10 + (b*c*d^3*((b*c*(x^6/3
 - (c^2*x^8)/2 + (c^4*x^10)/5))/20 + (x^5*(1 - c^2*x^2)^(5/2)*(a + b*ArcCo
s[c*x]))/10 + ((b*c*(x^6/6 - (c^2*x^8)/8))/8 + (x^5*(1 - c^2*x^2)^(3/2)*(a
 + b*ArcCos[c*x]))/8 + (3*((b*c*x^6)/36 + (x^5*Sqrt[1 - c^2*x^2]*(a + b*Ar
cCos[c*x]))/6 + (-1/16*(b*x^4)/c - (x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*
x]))/(4*c^2) + (3*(-1/4*(b*x^2)/c - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x
]))/(2*c^2) - (a + b*ArcCos[c*x])^2/(4*b*c^3)))/(4*c^2))/6))/8)/2))/5 + (3
*d^3*((x^4*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/8 + (b*c*((b*c*(x^6/6 -
(c^2*x^8)/8))/8 + (x^5*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/8 + (3*((b
*c*x^6)/36 + (x^5*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/6 + (-1/16*(b*x^4
)/c - (x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(4*c^2) + (3*(-1/4*(b*x^
2)/c - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(2*c^2) - (a + b*ArcCos[c
*x])^2/(4*b*c^3)))/(4*c^2))/6))/8))/4 + ((x^4*(1 - c^2*x^2)*(a + b*ArcCos[
c*x])^2)/6 + (b*c*((b*c*x^6)/36 + (x^5*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x
]))/6 + (-1/16*(b*x^4)/c - (x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(4*
c^2) + (3*(-1/4*(b*x^2)/c - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(2*c
^2) - (a + b*ArcCos[c*x])^2/(4*b*c^3)))/(4*c^2))/6))/3 + ((x^4*(a + b*ArcC
os[c*x])^2)/4 + (b*c*(-1/16*(b*x^4)/c - (x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcC
os[c*x]))/(4*c^2) + (3*(-1/4*(b*x^2)/c - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCo
s[c*x]))/(2*c^2) - (a + b*ArcCos[c*x])^2/(4*b*c^3)))/(4*c^2)))/2)/3)/2)...
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.35

method	result
parts	$-d^3a^2\left(\frac{1}{10}c^6x^{10}-\frac{3}{8}c^4x^8+\frac{1}{2}c^2x^6-\frac{1}{4}x^4\right)-\frac{d^3b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-1\right)^4}{8}-\frac{\arccos(cx)\left(48c^7x^7\sqrt{-c^2x^2+1}-\right.}{}\right.}{}$
derivativedivides	$\frac{-d^3a^2\left(\frac{1}{10}c^{10}x^{10}-\frac{3}{8}c^8x^8+\frac{1}{2}c^6x^6-\frac{1}{4}c^4x^4\right)-d^3b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-1\right)^4}{8}-\frac{\arccos(cx)\left(48c^7x^7\sqrt{-c^2x^2+1}-200c^5x^5\sqrt{-c^2x\right.}{}\right.}{}$
default	$\frac{-d^3a^2\left(\frac{1}{10}c^{10}x^{10}-\frac{3}{8}c^8x^8+\frac{1}{2}c^6x^6-\frac{1}{4}c^4x^4\right)-d^3b^2\left(\frac{\arccos(cx)^2\left(c^2x^2-1\right)^4}{8}-\frac{\arccos(cx)\left(48c^7x^7\sqrt{-c^2x^2+1}-200c^5x^5\sqrt{-c^2x\right.}{}\right.}{}$
orering	$\frac{\left(208128x^{12}c^{12}-1019388c^{10}x^{10}+1928796c^8x^8-1587835c^6x^6-38650c^4x^4+408825c^2x^2-118500\right)\left(-c^2d\,x^2+d\right)^3(a+b}{768000c^4(cx-1)^2(cx+1)^2(c^2x^2-1)^2}$

input
```
int(x^3*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-d^3*a^2*(1/10*c^6*x^10-3/8*c^4*x^8+1/2*c^2*x^6-1/4*x^4)-d^3*b^2/c^4*(1/8*
arccos(c*x)^2*(c^2*x^2-1)^4-1/1536*arccos(c*x)*(48*c^7*x^7*(-c^2*x^2+1)^(1
/2)-200*c^5*x^5*(-c^2*x^2+1)^(1/2)+326*c^3*x^3*(-c^2*x^2+1)^(1/2)-279*c*x*
(-c^2*x^2+1)^(1/2)+105*arccos(c*x))+49/5120*arccos(c*x)^2-7/6400*(c^2*x^2-
1)^4+49/28800*(c^2*x^2-1)^3-49/15360*(c^2*x^2-1)^2+49/5120*c^2*x^2-49/5120
+1/10*arccos(c*x)^2*(c^2*x^2-1)^5+1/6400*arccos(c*x)*(-128*c^9*x^9*(-c^2*x
^2+1)^(1/2)+656*c^7*x^7*(-c^2*x^2+1)^(1/2)-1368*c^5*x^5*(-c^2*x^2+1)^(1/2)
+1490*c^3*x^3*(-c^2*x^2+1)^(1/2)-965*c*x*(-c^2*x^2+1)^(1/2)+315*arccos(c*x
))-1/500*(c^2*x^2-1)^5)-2*d^3*a*b/c^4*(1/10*arccos(c*x)*c^10*x^10-3/8*arcc
os(c*x)*c^8*x^8+1/2*arccos(c*x)*c^6*x^6-1/4*c^4*x^4*arccos(c*x)+79/7680*c^
3*x^3*(-c^2*x^2+1)^(1/2)+79/5120*c*x*(-c^2*x^2+1)^(1/2)-79/5120*arcsin(c*x
)-401/9600*c^5*x^5*(-c^2*x^2+1)^(1/2)+57/1600*c^7*x^7*(-c^2*x^2+1)^(1/2)-1
/100*c^9*x^9*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.03

$$\int x^3\left(d-c^2dx^2\right)^3(a+b\arccos(cx))^2\,dx =$$
$$-\frac{2304\,(50\,a^2-b^2)c^{10}d^3x^{10}-540\,(800\,a^2-19\,b^2)c^8d^3x^8+40\,(14400\,a^2-401\,b^2)c^6d^3x^6-75\,(3840\,a^2-$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
-1/1152000*(2304*(50*a^2 - b^2)*c^10*d^3*x^10 - 540*(800*a^2 - 19*b^2)*c^8
*d^3*x^8 + 40*(14400*a^2 - 401*b^2)*c^6*d^3*x^6 - 75*(3840*a^2 - 79*b^2)*c
^4*d^3*x^4 + 17775*b^2*c^2*d^3*x^2 + 225*(512*b^2*c^10*d^3*x^10 - 1920*b^2
*c^8*d^3*x^8 + 2560*b^2*c^6*d^3*x^6 - 1280*b^2*c^4*d^3*x^4 + 79*b^2*d^3)*a
rccos(c*x)^2 + 450*(512*a*b*c^10*d^3*x^10 - 1920*a*b*c^8*d^3*x^8 + 2560*a*
b*c^6*d^3*x^6 - 1280*a*b*c^4*d^3*x^4 + 79*a*b*d^3)*arccos(c*x) - 30*(768*a
*b*c^9*d^3*x^9 - 2736*a*b*c^7*d^3*x^7 + 3208*a*b*c^5*d^3*x^5 - 790*a*b*c^3
*d^3*x^3 - 1185*a*b*c*d^3*x + (768*b^2*c^9*d^3*x^9 - 2736*b^2*c^7*d^3*x^7
+ 3208*b^2*c^5*d^3*x^5 - 790*b^2*c^3*d^3*x^3 - 1185*b^2*c*d^3*x)*arccos(c*
x))*sqrt(-c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.72

$$\int x^3 (d - c^2 d x^2)^3 (a + b \arccos(cx))^2 \, dx = \text{Too large to display}$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**3*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((-a**2*c**6*d**3*x**10/10 + 3*a**2*c**4*d**3*x**8/8 - a**2*c**2*
d**3*x**6/2 + a**2*d**3*x**4/4 - a*b*c**6*d**3*x**10*acos(c*x)/5 + a*b*c**
5*d**3*x**9*sqrt(-c**2*x**2 + 1)/50 + 3*a*b*c**4*d**3*x**8*acos(c*x)/4 - 5
7*a*b*c**3*d**3*x**7*sqrt(-c**2*x**2 + 1)/800 - a*b*c**2*d**3*x**6*acos(c*
x) + 401*a*b*c*d**3*x**5*sqrt(-c**2*x**2 + 1)/4800 + a*b*d**3*x**4*acos(c*
x)/2 - 79*a*b*d**3*x**3*sqrt(-c**2*x**2 + 1)/(3840*c) - 79*a*b*d**3*x*sqrt
(-c**2*x**2 + 1)/(2560*c**3) - 79*a*b*d**3*acos(c*x)/(2560*c**4) - b**2*c*
*6*d**3*x**10*acos(c*x)**2/10 + b**2*c**6*d**3*x**10/500 + b**2*c**5*d**3*
x**9*sqrt(-c**2*x**2 + 1)*acos(c*x)/50 + 3*b**2*c**4*d**3*x**8*acos(c*x)**
2/8 - 57*b**2*c**4*d**3*x**8/6400 - 57*b**2*c**3*d**3*x**7*sqrt(-c**2*x**2
 + 1)*acos(c*x)/800 - b**2*c**2*d**3*x**6*acos(c*x)**2/2 + 401*b**2*c**2*d
**3*x**6/28800 + 401*b**2*c*d**3*x**5*sqrt(-c**2*x**2 + 1)*acos(c*x)/4800
+ b**2*d**3*x**4*acos(c*x)**2/4 - 79*b**2*d**3*x**4/15360 - 79*b**2*d**3*x
**3*sqrt(-c**2*x**2 + 1)*acos(c*x)/(3840*c) - 79*b**2*d**3*x**2/(5120*c**2
) - 79*b**2*d**3*x*sqrt(-c**2*x**2 + 1)*acos(c*x)/(2560*c**3) - 79*b**2*d*
*3*acos(c*x)**2/(5120*c**4), Ne(c, 0)), (d**3*x**4*(a + pi*b/2)**2/4, True
))
```

Maxima [F]

$$\int x^3 (d - c^2 d x^2)^3 (a + b \arccos(cx))^2 \, dx = \int -(c^2 d x^2 - d)^3 (b \arccos(cx) + a)^2 x^3 \, dx$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output

```
-1/10*a^2*c^6*d^3*x^10 + 3/8*a^2*c^4*d^3*x^8 - 1/2*a^2*c^2*d^3*x^6 - 1/640
0*(1280*x^10*arccos(c*x) - (128*sqrt(-c^2*x^2 + 1)*x^9/c^2 + 144*sqrt(-c^2
*x^2 + 1)*x^7/c^4 + 168*sqrt(-c^2*x^2 + 1)*x^5/c^6 + 210*sqrt(-c^2*x^2 + 1
)*x^3/c^8 + 315*sqrt(-c^2*x^2 + 1)*x/c^10 - 315*arcsin(c*x)/c^11)*c)*a*b*c
^6*d^3 + 1/512*(384*x^8*arccos(c*x) - (48*sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*
sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(-c^2*x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2
*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9)*c)*a*b*c^4*d^3 + 1/4*a^2*d^3*x^4 -
1/48*(48*x^6*arccos(c*x) - (8*sqrt(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^
2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*a*b*
c^2*d^3 + 1/16*(8*x^4*arccos(c*x) - (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt
(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*a*b*d^3 - 1/40*(4*b^2*c^6*d^3
*x^10 - 15*b^2*c^4*d^3*x^8 + 20*b^2*c^2*d^3*x^6 - 10*b^2*d^3*x^4)*arctan2(
sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + integrate(1/20*(4*b^2*c^7*d^3*x^10
- 15*b^2*c^5*d^3*x^8 + 20*b^2*c^3*d^3*x^6 - 10*b^2*c*d^3*x^4)*sqrt(c*x + 1
)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^2*x^2 - 1),
 x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 572, normalized size of antiderivative = 1.49

$$\int x^3 \left(d-c^2 d x^2\right)^3 (a+b \arccos(cx))^2 \, dx = -\frac{1}{10} b^2 c^6 d^3 x^{10} \arccos\left(cx\right)^2 - \frac{1}{5} a b c^6 d^3 x^{10} \arccos\left(cx\right) - \frac{1}{10} a^2 c^6 d^3 x^{10} + \frac{1}{500} b^2 c^6 d^3 x^{10} + \frac{1}{50} \sqrt{-c^2 x^2 + 1} b^2 c^5 d^3 x^9 \arccos\left(cx\right) + \frac{1}{50} \sqrt{-c^2 x^2 + 1} a b c^5 d^3 x^9 + \frac{3}{8} b^2 c^4 d^3 x^8 \arccos\left(cx\right)^2 + \frac{3}{4} a b c^4 d^3 x^8 \arccos\left(cx\right) + \frac{3}{8} a^2 c^4 d^3 x^8 - \frac{57}{6400} b^2 c^4 d^3 x^8 - \frac{57}{800} \sqrt{-c^2 x^2 + 1} b^2 c^3 d^3 x^7 \arccos\left(cx\right) - \frac{57}{800} \sqrt{-c^2 x^2 + 1} a b c^3 d^3 x^7 - \frac{1}{2} b^2 c^2 d^3 x^6 \arccos\left(cx\right)^2 - a b c^2 d^3 x^6 \arccos\left(cx\right) - \frac{1}{2} a^2 c^2 d^3 x^6 + \frac{401}{28800} b^2 c^2 d^3 x^6 + \frac{401}{4800} \sqrt{-c^2 x^2 + 1} b^2 c d^3 x^5 \arccos\left(cx\right) + \frac{401}{4800} \sqrt{-c^2 x^2 + 1} a b c d^3 x^5 + \frac{1}{4} b^2 d^3 x^4 \arccos\left(cx\right)^2 + \frac{1}{2} a b d^3 x^4 \arccos\left(cx\right) + \frac{1}{4} a^2 d^3 x^4 - \frac{79}{15360} b^2 d^3 x^4 - \frac{79 \sqrt{-c^2 x^2 + 1} b^2 d^3 x^3 \arccos\left(cx\right)}{3840 \, c} - \frac{79 \sqrt{-c^2 x^2 + 1} a b d^3 x^3}{3840 \, c} - \frac{79 \, b^2 d^3 x^2}{5120 \, c^2} - \frac{79 \sqrt{-c^2 x^2 + 1} b^2 d^3 x \arccos\left(cx\right)}{2560 \, c^3} - \frac{79 \sqrt{-c^2 x^2 + 1} a b d^3 x}{2560 \, c^3} - \frac{79 \, b^2 d^3 \arccos\left(cx\right)^2}{5120 \, c^4} - \frac{79 \, a b d^3 \arccos\left(cx\right)}{2560 \, c^4} + \frac{266731 \, b^2 d^3}{36864000 \, c^4}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-1/10*b^2*c^6*d^3*x^10*arccos(c*x)^2 - 1/5*a*b*c^6*d^3*x^10*arccos(c*x) -
1/10*a^2*c^6*d^3*x^10 + 1/500*b^2*c^6*d^3*x^10 + 1/50*sqrt(-c^2*x^2 + 1)*b
^2*c^5*d^3*x^9*arccos(c*x) + 1/50*sqrt(-c^2*x^2 + 1)*a*b*c^5*d^3*x^9 + 3/8
*b^2*c^4*d^3*x^8*arccos(c*x)^2 + 3/4*a*b*c^4*d^3*x^8*arccos(c*x) + 3/8*a^2
*c^4*d^3*x^8 - 57/6400*b^2*c^4*d^3*x^8 - 57/800*sqrt(-c^2*x^2 + 1)*b^2*c^3
*d^3*x^7*arccos(c*x) - 57/800*sqrt(-c^2*x^2 + 1)*a*b*c^3*d^3*x^7 - 1/2*b^2
*c^2*d^3*x^6*arccos(c*x)^2 - a*b*c^2*d^3*x^6*arccos(c*x) - 1/2*a^2*c^2*d^3
*x^6 + 401/28800*b^2*c^2*d^3*x^6 + 401/4800*sqrt(-c^2*x^2 + 1)*b^2*c*d^3*x
^5*arccos(c*x) + 401/4800*sqrt(-c^2*x^2 + 1)*a*b*c*d^3*x^5 + 1/4*b^2*d^3*x
^4*arccos(c*x)^2 + 1/2*a*b*d^3*x^4*arccos(c*x) + 1/4*a^2*d^3*x^4 - 79/1536
0*b^2*d^3*x^4 - 79/3840*sqrt(-c^2*x^2 + 1)*b^2*d^3*x^3*arccos(c*x)/c - 79/
3840*sqrt(-c^2*x^2 + 1)*a*b*d^3*x^3/c - 79/5120*b^2*d^3*x^2/c^2 - 79/2560*
sqrt(-c^2*x^2 + 1)*b^2*d^3*x*arccos(c*x)/c^3 - 79/2560*sqrt(-c^2*x^2 + 1)*
a*b*d^3*x/c^3 - 79/5120*b^2*d^3*arccos(c*x)^2/c^4 - 79/2560*a*b*d^3*arccos
(c*x)/c^4 + 266731/36864000*b^2*d^3/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d-c^2dx^2)^3 (a+b\arccos(cx))^2\,dx = \int x^3\,(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^3\,dx$$

input
```
int(x^3*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^3,x)
```

output
```
int(x^3*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int x^3\left(d-c^2dx^2\right)^3(a+b\arccos(cx))^2\,dx$$
$$=\frac{d^3\left(-7680acos(cx)\,ab\,c^{10}x^{10}+28800acos(cx)\,ab\,c^8x^8-38400acos(cx)\,ab\,c^6x^6+19200acos(cx)\,ab\,c^4x^4+\right.}{}$$

input
```
int(x^3*(-c^2*d*x^2+d)^3*(a+b*acos(c*x))^2,x)
```

output
```
(d**3*( - 7680*acos(c*x)*a*b*c**10*x**10 + 28800*acos(c*x)*a*b*c**8*x**8 -
 38400*acos(c*x)*a*b*c**6*x**6 + 19200*acos(c*x)*a*b*c**4*x**4 + 1185*asin
(c*x)*a*b + 768*sqrt( - c**2*x**2 + 1)*a*b*c**9*x**9 - 2736*sqrt( - c**2*x
**2 + 1)*a*b*c**7*x**7 + 3208*sqrt( - c**2*x**2 + 1)*a*b*c**5*x**5 - 790*s
qrt( - c**2*x**2 + 1)*a*b*c**3*x**3 - 1185*sqrt( - c**2*x**2 + 1)*a*b*c*x
- 38400*int(acos(c*x)**2*x**9,x)*b**2*c**10 + 115200*int(acos(c*x)**2*x**7
,x)*b**2*c**8 - 115200*int(acos(c*x)**2*x**5,x)*b**2*c**6 + 38400*int(acos
(c*x)**2*x**3,x)*b**2*c**4 - 3840*a**2*c**10*x**10 + 14400*a**2*c**8*x**8
- 19200*a**2*c**6*x**6 + 9600*a**2*c**4*x**4))/(38400*c**4)
```

3.178 $\int x^2(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 391

$$\int x^2(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx$$
$$= -\frac{10516b^2d^3x}{99225c^2} - \frac{5258b^2d^3x^3}{297675} + \frac{4198b^2c^2d^3x^5}{165375} - \frac{374b^2c^4d^3x^7}{27783} + \frac{2}{729}b^2c^6d^3x^9$$
$$+ \frac{64bd^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{945c^3} + \frac{32bd^3x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{945c}$$
$$+ \frac{16bd^3(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{315c^3} + \frac{4bd^3(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{525c^3}$$
$$+ \frac{2bd^3(1-c^2x^2)^{7/2}(a+b\arccos(cx))}{441c^3} - \frac{2bd^3(1-c^2x^2)^{9/2}(a+b\arccos(cx))}{81c^3}$$
$$+ \frac{16}{315}d^3x^3(a+b\arccos(cx))^2 + \frac{8}{105}d^3x^3(1-c^2x^2)(a+b\arccos(cx))^2 + \frac{2}{21}d^3x^3(1-c^2x^2)^2(a+b\arccos(cx$$

output
```
-10516/99225*b^2*d^3*x/c^2-5258/297675*b^2*d^3*x^3+4198/165375*b^2*c^2*d^3
*x^5-374/27783*b^2*c^4*d^3*x^7+2/729*b^2*c^6*d^3*x^9+64/945*b*d^3*(-c^2*x^
2+1)^(1/2)*(a+b*arccos(c*x))/c^3+32/945*b*d^3*x^2*(-c^2*x^2+1)^(1/2)*(a+b*
arccos(c*x))/c+16/315*b*d^3*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/c^3+4/525
*b*d^3*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))/c^3+2/441*b*d^3*(-c^2*x^2+1)^(
7/2)*(a+b*arccos(c*x))/c^3-2/81*b*d^3*(-c^2*x^2+1)^(9/2)*(a+b*arccos(c*x))
/c^3+16/315*d^3*x^3*(a+b*arccos(c*x))^2+8/105*d^3*x^3*(-c^2*x^2+1)*(a+b*ar
ccos(c*x))^2+2/21*d^3*x^3*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2+1/9*d^3*x^3*(
-c^2*x^2+1)^3*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.71

$$\int x^2 \left(d-c^2 d x^2\right)^3 (a+b \arccos(cx))^2 \, dx =$$
$$-\frac{d^3 \left(99225 a^2 c^3 x^3 (-105+189 c^2 x^2-135 c^4 x^4+35 c^6 x^6)-630 a b \sqrt{1-c^2 x^2} (-5258-2629 c^2 x^2+6297 c\right.}{}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/31255875*(d^3*(99225*a^2*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^4 + 35
*c^6*x^6) - 630*a*b*Sqrt[1 - c^2*x^2]*(-5258 - 2629*c^2*x^2 + 6297*c^4*x^4
 - 4675*c^6*x^6 + 1225*c^8*x^8) + b^2*(3312540*c*x + 552090*c^3*x^3 - 7934
22*c^5*x^5 + 420750*c^7*x^7 - 85750*c^9*x^9) - 630*b*(-315*a*c^3*x^3*(-105
 + 189*c^2*x^2 - 135*c^4*x^4 + 35*c^6*x^6) + b*Sqrt[1 - c^2*x^2]*(-5258 -
2629*c^2*x^2 + 6297*c^4*x^4 - 4675*c^6*x^6 + 1225*c^8*x^8))*ArcCos[c*x] +
99225*b^2*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^4 + 35*c^6*x^6)*ArcCos[c
*x]^2))/c^3
```

Rubi [A] (verified)

Time = 2.70 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.38, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.741$, Rules used = {5203, 27, 5195, 27, 290, 2009, 5203, 5195, 27, 290, 2009, 5203, 5139, 5195, 27, 2009, 5211, 15, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d-c^2 d x^2\right)^3 (a+b \arccos(cx))^2 \, dx$$

$$\Big\downarrow \text{ 5203}$$

$$\frac{2}{9} b c d^3 \int x^3 \left(1-c^2 x^2\right)^{5/2} (a+b \arccos(cx))dx+\frac{2}{3} d \int d^2 x^2 \left(1-c^2 x^2\right)^2 (a+b \arccos(cx))^2 dx+\frac{1}{9} d^3 x^3 \left(1-c^2 x^2\right)^3 (a+b \arccos(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}d^3\int x^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2dx+\frac{2}{9}bcd^3\int x^3\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx+\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5195}$$

$$\frac{2}{3}d^3\int x^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2dx+\frac{2}{9}bcd^3\left(bc\int -\frac{\left(1-c^2x^2\right)^3\left(7c^2x^2+2\right)}{63c^4}dx+\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}\right.$$
$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}d^3\int x^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2dx+\frac{2}{9}bcd^3\left(-\frac{b\int\left(1-c^2x^2\right)^3\left(7c^2x^2+2\right)dx}{63c^3}+\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}\right)$$
$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{290}$$

$$\frac{2}{3}d^3\int x^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2dx+\frac{2}{9}bcd^3\left(-\frac{b\int\left(-7c^8x^8+19c^6x^6-15c^4x^4+c^2x^2+2\right)dx}{63c^3}+\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}(a\right.}{7c^4}$$
$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}d^3\int x^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2dx+\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+\frac{2}{9}bcd^3\left(\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{b\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$$\Big\downarrow \text{5203}$$

$$\frac{2}{3}d^3\left(\frac{4}{7}\int x^2\left(1-c^2x^2\right)\left(a+b\arccos(cx)\right)^2dx+\frac{2}{7}bc\int x^3\left(1-c^2x^2\right)^{3/2}\left(a+b\arccos(cx)\right)dx+\frac{1}{7}x^3\left(1-c^2x^2\right)^2\left(a\right.\right.$$
$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3\left(a+b\arccos(cx)\right)^2+$$
$$\frac{2}{9}bcd^3\left(\frac{\left(1-c^2x^2\right)^{9/2}\left(a+b\arccos(cx)\right)}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}\left(a+b\arccos(cx)\right)}{7c^4}-\frac{b\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$$\Big\downarrow \text{5195}$$

$$\frac{2}{3}d^3\left(\frac{4}{7}\int x^2\left(1-c^2x^2\right)\left(a+b\arccos(cx)\right)^2dx+\frac{2}{7}bc\left(bc\int -\frac{\left(1-c^2x^2\right)^2\left(5c^2x^2+2\right)}{35c^4}dx+\frac{\left(1-c^2x^2\right)^{7/2}\left(a+b\,\mathrm{a}\right.}{7c^4}\right.\right.$$
$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3\left(a+b\arccos(cx)\right)^2+$$
$$\frac{2}{9}bcd^3\left(\frac{\left(1-c^2x^2\right)^{9/2}\left(a+b\arccos(cx)\right)}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}\left(a+b\arccos(cx)\right)}{7c^4}-\frac{b\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}d^3\left(\frac{4}{7}\int x^2\left(1-c^2x^2\right)\left(a+b\arccos(cx)\right)^2dx+\frac{2}{7}bc\left(-\frac{b\int\left(1-c^2x^2\right)^2\left(5c^2x^2+2\right)dx}{35c^3}+\frac{\left(1-c^2x^2\right)^{7/2}\left(a+b\,\mathrm{arc}\right.}{7c^4}\right.\right.$$
$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3\left(a+b\arccos(cx)\right)^2+$$
$$\frac{2}{9}bcd^3\left(\frac{\left(1-c^2x^2\right)^{9/2}\left(a+b\arccos(cx)\right)}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}\left(a+b\arccos(cx)\right)}{7c^4}-\frac{b\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$$\Big\downarrow \text{290}$$

$$\frac{2}{3}d^3\left(\frac{4}{7}\int x^2\left(1-c^2x^2\right)\left(a+b\arccos(cx)\right)^2dx+\frac{2}{7}bc\left(-\frac{b\int\left(5c^6x^6-8c^4x^4+c^2x^2+2\right)dx}{35c^3}+\frac{\left(1-c^2x^2\right)^{7/2}\left(a+\right.}{7c^4}\right.\right.$$
$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3\left(a+b\arccos(cx)\right)^2+$$
$$\frac{2}{9}bcd^3\left(\frac{\left(1-c^2x^2\right)^{9/2}\left(a+b\arccos(cx)\right)}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}\left(a+b\arccos(cx)\right)}{7c^4}-\frac{b\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}d^3\left(\frac{4}{7}\int x^2\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{1}{7}x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{2}{7}bc\left(\frac{\left(1-c^2x^2\right)^{7/2}(a+b}{7c^4}\right.\right.$$

$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$

$$\frac{2}{9}bcd^3\left(\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{b\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 5203

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(\frac{2}{5}bc\int x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{2}{5}\int x^2(a+b\arccos(cx))^2dx+\frac{1}{5}x^3\left(1-c^2x^2\right)(a+b\arccos\right.\right.$$

$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$

$$\frac{2}{9}bcd^3\left(\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{b\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 5139

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(\frac{2}{5}\left(\frac{2}{3}bc\int\frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)+\frac{2}{5}bc\int x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))d\right.\right.$$

$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$

$$\frac{2}{9}bcd^3\left(\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{b\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 5195

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(\frac{2}{5}\left(\frac{2}{3}bc\int\frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)+\frac{2}{5}bc\left(bc\int-\frac{-3c^4x^4+c^2x^2+2}{15c^4}dx+\right.\right.\right.$$

$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$

$$\frac{2}{9}bcd^3\left(\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{b\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\downarrow$ 27

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(\frac{2}{5}\left(\frac{2}{3}bc\int\frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)+\frac{2}{5}bc\left(-\frac{b\int\left(-3c^4x^4+c^2x^2+2\right)dx}{15c^3}+\right.\right.\right.$$
$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{9}bcd^3\left(\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{b\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\Big\downarrow$ 2009

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(\frac{2}{5}\left(\frac{2}{3}bc\int\frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)+\frac{1}{5}x^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{5}\right.\right.$$
$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{9}bcd^3\left(\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{b\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\Big\downarrow$ 5211

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(\frac{2}{5}\left(\frac{2}{3}bc\left(\frac{2\int\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{b\int x^2dx}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x^3(a+b\arccos(cx)\right.\right.\right.$$
$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{9}bcd^3\left(\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{b\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\Big\downarrow$ 15

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(\frac{2}{5}\left(\frac{2}{3}bc\left(\frac{2\int\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)+\frac{1}{3}x^3(a+b\arccos(cx))^2\right)\right.\right.$$
$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{2}{9}bcd^3\left(\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{b\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$\Big\downarrow$ 5183

$$\frac{2}{3}d^3\left(\frac{4}{7}\left(\frac{2}{5}\left(\frac{2}{3}bc\left(\frac{2\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)+\frac{1}{3}x^3(a+b\right.\right.\right.$$

$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$

$$\frac{2}{9}bcd^3\left(\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{b\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{9}d^3x^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$

$$\frac{2}{3}d^3\left(\frac{1}{7}x^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{4}{7}\left(\frac{1}{5}x^3\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{5}\left(\frac{2}{3}bc\left(-\frac{x^2\sqrt{1-c^2x^2}(a\right.}{3c^2}\right.\right.\right.$$

$$\frac{2}{9}bcd^3\left(\frac{\left(1-c^2x^2\right)^{9/2}(a+b\arccos(cx))}{9c^4}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^4}-\frac{b\left(-\frac{7}{9}c^8x^9+\frac{19c^6x^7}{7}-3c^4x^5+\frac{c^2x^3}{3}\right.}{63c^3}\right.$$

input
```
Int[x^2*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^3*x^3*(1 - c^2*x^2)^3*(a + b*ArcCos[c*x])^2)/9 + (2*b*c*d^3*(-1/63*(b*(
2*x + (c^2*x^3)/3 - 3*c^4*x^5 + (19*c^6*x^7)/7 - (7*c^8*x^9)/9))/c^3 - ((1
 - c^2*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(7*c^4) + ((1 - c^2*x^2)^(9/2)*(a +
 b*ArcCos[c*x]))/(9*c^4)))/9 + (2*d^3*((x^3*(1 - c^2*x^2)^2*(a + b*ArcCos[
c*x])^2)/7 + (2*b*c*(-1/35*(b*(2*x + (c^2*x^3)/3 - (8*c^4*x^5)/5 + (5*c^6*
x^7)/7))/c^3 - ((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^4) + ((1 - c
^2*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(7*c^4)))/7 + (4*((x^3*(1 - c^2*x^2)*(a
 + b*ArcCos[c*x])^2)/5 + (2*b*c*(-1/15*(b*(2*x + (c^2*x^3)/3 - (3*c^4*x^5)
/5))/c^3 - ((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(3*c^4) + ((1 - c^2*x
^2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^4)))/5 + (2*((x^3*(a + b*ArcCos[c*x])^
2)/3 + (2*b*c*(-1/9*(b*x^3)/c - (x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])
)/(3*c^2) + (2*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c^2))
/(3*c^2)))/3))/5))/7))/3
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.34

method	result
parts	$-d^3a^2\left(\frac{1}{9}c^6x^9-\frac{3}{7}c^4x^7+\frac{3}{5}c^2x^5-\frac{1}{3}x^3\right)-\frac{d^3b^2\left(\frac{\arccos(cx)^2\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)cx}{35}-\frac{2\arccos(cx)\left(\cdots\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{-d^3a^2\left(\frac{1}{9}c^9x^9-\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-d^3b^2\left(\frac{\arccos(cx)^2\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)cx}{35}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^3\sqrt{-c^2\cdots}}{441}\right.}{\cdots}$
default	$\frac{-d^3a^2\left(\frac{1}{9}c^9x^9-\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-d^3b^2\left(\frac{\arccos(cx)^2\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)cx}{35}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^3\sqrt{-c^2\cdots}}{441}\right.}{\cdots}$
orering	$\frac{\left(9303875x^{12}c^{12}-47172500c^{10}x^{10}+95052594c^8x^8-88615068c^6x^6-86474829c^4x^4+59625720c^2x^2-13250160\right)\left(-c^2d\cdots\right.}{31255875x\,c^4(cx-1)^2(cx+1)^2(c^2x^2-1)^2}$

input
```
int(x^2*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-d^3*a^2*(1/9*c^6*x^9-3/7*c^4*x^7+3/5*c^2*x^5-1/3*x^3)-d^3*b^2/c^3*(1/35*a
rccos(c*x)^2*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c*x-2/441*arccos(c*x)*(c
^2*x^2-1)^3*(-c^2*x^2+1)^(1/2)-2/15435*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35
)*c*x+4/525*arccos(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)^(1/2)+4/7875*(3*c^4*x^4
-10*c^2*x^2+15)*c*x-16/945*arccos(c*x)*(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)-16/2
835*(c^2*x^2-3)*c*x+32/315*c*x+32/315*arccos(c*x)*(-c^2*x^2+1)^(1/2)+1/315
*arccos(c*x)^2*(35*c^8*x^8-180*c^6*x^6+378*c^4*x^4-420*c^2*x^2+315)*c*x-2/
81*arccos(c*x)*(c^2*x^2-1)^4*(-c^2*x^2+1)^(1/2)-2/25515*(35*c^8*x^8-180*c^
6*x^6+378*c^4*x^4-420*c^2*x^2+315)*c*x)-2*d^3*a*b/c^3*(1/9*arccos(c*x)*c^9
*x^9-3/7*arccos(c*x)*c^7*x^7+3/5*arccos(c*x)*c^5*x^5-1/3*c^3*x^3*arccos(c*
x)+2629/99225*c^2*x^2*(-c^2*x^2+1)^(1/2)+5258/99225*(-c^2*x^2+1)^(1/2)-209
9/33075*c^4*x^4*(-c^2*x^2+1)^(1/2)+187/3969*c^6*x^6*(-c^2*x^2+1)^(1/2)-1/8
1*c^8*x^8*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.95

$$\int x^2(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx =$$
$$-\frac{42875\,(81\,a^2-2\,b^2)c^9d^3x^9-1125\,(11907\,a^2-374\,b^2)c^7d^3x^7+189\,(99225\,a^2-4198\,b^2)c^5d^3x^5-105\,(\ldots}{\ldots}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
-1/31255875*(42875*(81*a^2 - 2*b^2)*c^9*d^3*x^9 - 1125*(11907*a^2 - 374*b^
2)*c^7*d^3*x^7 + 189*(99225*a^2 - 4198*b^2)*c^5*d^3*x^5 - 105*(99225*a^2 -
 5258*b^2)*c^3*d^3*x^3 + 3312540*b^2*c*d^3*x + 99225*(35*b^2*c^9*d^3*x^9 -
 135*b^2*c^7*d^3*x^7 + 189*b^2*c^5*d^3*x^5 - 105*b^2*c^3*d^3*x^3)*arccos(c
*x)^2 + 198450*(35*a*b*c^9*d^3*x^9 - 135*a*b*c^7*d^3*x^7 + 189*a*b*c^5*d^3
*x^5 - 105*a*b*c^3*d^3*x^3)*arccos(c*x) - 630*(1225*a*b*c^8*d^3*x^8 - 4675
*a*b*c^6*d^3*x^6 + 6297*a*b*c^4*d^3*x^4 - 2629*a*b*c^2*d^3*x^2 - 5258*a*b*
d^3 + (1225*b^2*c^8*d^3*x^8 - 4675*b^2*c^6*d^3*x^6 + 6297*b^2*c^4*d^3*x^4
- 2629*b^2*c^2*d^3*x^2 - 5258*b^2*d^3)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^
3
```

Sympy [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.61

$$\int x^2(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx$$
$$=\begin{cases}-\frac{a^2c^6d^3x^9}{9}+\frac{3a^2c^4d^3x^7}{7}-\frac{3a^2c^2d^3x^5}{5}+\frac{a^2d^3x^3}{3}-\frac{2abc^6d^3x^9\operatorname{acos}(cx)}{9}+\frac{2abc^5d^3x^8\sqrt{-c^2x^2+1}}{81}+\frac{6abc^4d^3x^7\operatorname{acos}(cx)}{7}-374ab \\ \frac{d^3x^3\left(a+\frac{\pi b}{2}\right)^2}{3}\end{cases}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**3*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((-a**2*c**6*d**3*x**9/9 + 3*a**2*c**4*d**3*x**7/7 - 3*a**2*c**2*
d**3*x**5/5 + a**2*d**3*x**3/3 - 2*a*b*c**6*d**3*x**9*acos(c*x)/9 + 2*a*b*
c**5*d**3*x**8*sqrt(-c**2*x**2 + 1)/81 + 6*a*b*c**4*d**3*x**7*acos(c*x)/7
- 374*a*b*c**3*d**3*x**6*sqrt(-c**2*x**2 + 1)/3969 - 6*a*b*c**2*d**3*x**5*
acos(c*x)/5 + 4198*a*b*c*d**3*x**4*sqrt(-c**2*x**2 + 1)/33075 + 2*a*b*d**3
*x**3*acos(c*x)/3 - 5258*a*b*d**3*x**2*sqrt(-c**2*x**2 + 1)/(99225*c) - 10
516*a*b*d**3*sqrt(-c**2*x**2 + 1)/(99225*c**3) - b**2*c**6*d**3*x**9*acos(
c*x)**2/9 + 2*b**2*c**6*d**3*x**9/729 + 2*b**2*c**5*d**3*x**8*sqrt(-c**2*x
**2 + 1)*acos(c*x)/81 + 3*b**2*c**4*d**3*x**7*acos(c*x)**2/7 - 374*b**2*c*
*4*d**3*x**7/27783 - 374*b**2*c**3*d**3*x**6*sqrt(-c**2*x**2 + 1)*acos(c*x
)/3969 - 3*b**2*c**2*d**3*x**5*acos(c*x)**2/5 + 4198*b**2*c**2*d**3*x**5/1
65375 + 4198*b**2*c*d**3*x**4*sqrt(-c**2*x**2 + 1)*acos(c*x)/33075 + b**2*
d**3*x**3*acos(c*x)**2/3 - 5258*b**2*d**3*x**3/297675 - 5258*b**2*d**3*x**
2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(99225*c) - 10516*b**2*d**3*x/(99225*c**2
) - 10516*b**2*d**3*sqrt(-c**2*x**2 + 1)*acos(c*x)/(99225*c**3), Ne(c, 0))
, (d**3*x**3*(a + pi*b/2)**2/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 946 vs. 2(346) = 692.

Time = 0.17 (sec) , antiderivative size = 946, normalized size of antiderivative = 2.42

$$\int x^2(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-1/9*b^2*c^6*d^3*x^9*arccos(c*x)^2 - 1/9*a^2*c^6*d^3*x^9 + 3/7*b^2*c^4*d^3
*x^7*arccos(c*x)^2 + 3/7*a^2*c^4*d^3*x^7 - 3/5*b^2*c^2*d^3*x^5*arccos(c*x)
^2 - 2/2835*(315*x^9*arccos(c*x) - (35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqr
t(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2
 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/c^10)*c)*a*b*c^6*d^3 + 2/893025*(31
5*(35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt
(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2
 + 1)/c^10)*c*arccos(c*x) + (1225*c^8*x^9 + 1800*c^6*x^7 + 3024*c^4*x^5 +
6720*c^2*x^3 + 40320*x)/c^8)*b^2*c^6*d^3 - 3/5*a^2*c^2*d^3*x^5 + 6/245*(35
*x^7*arccos(c*x) - (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^
4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*a*b*c
^4*d^3 - 2/8575*(105*(5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*
x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c*arcc
os(c*x) + (75*c^6*x^7 + 126*c^4*x^5 + 280*c^2*x^3 + 1680*x)/c^6)*b^2*c^4*d
^3 + 1/3*b^2*d^3*x^3*arccos(c*x)^2 - 2/25*(15*x^5*arccos(c*x) - (3*sqrt(-c
^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/
c^6)*c)*a*b*c^2*d^3 + 2/375*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^
2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c*arccos(c*x) + (9*c^4*x^5
+ 20*c^2*x^3 + 120*x)/c^4)*b^2*c^2*d^3 + 1/3*a^2*d^3*x^3 + 2/9*(3*x^3*arcc
os(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*...
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.35

$$\int x^2(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx = -\frac{1}{9}b^2c^6d^3x^9\arccos(cx)^2 - \frac{2}{9}abc^6d^3x^9\arccos(cx) - \frac{1}{9}a^2c^6d^3x^9 + \frac{2}{729}b^2c^6d^3x^9 + \frac{2}{81}\sqrt{-c^2x^2+1}b^2c^5d^3x^8\arccos(cx) + \frac{2}{81}\sqrt{-c^2x^2+1}abc^5d^3x^8 + \frac{3}{7}b^2c^4d^3x^7\arccos(cx)^2 + \frac{6}{7}abc^4d^3x^7\arccos(cx) + \frac{3}{7}a^2c^4d^3x^7 - \frac{374}{27783}b^2c^4d^3x^7 - \frac{374}{3969}\sqrt{-c^2x^2+1}b^2c^3d^3x^6\arccos(cx) - \frac{374}{3969}\sqrt{-c^2x^2+1}abc^3d^3x^6 - \frac{3}{5}b^2c^2d^3x^5\arccos(cx)^2 - \frac{6}{5}abc^2d^3x^5\arccos(cx) - \frac{3}{5}a^2c^2d^3x^5 + \frac{4198}{165375}b^2c^2d^3x^5 + \frac{4198}{33075}\sqrt{-c^2x^2+1}b^2cd^3x^4\arccos(cx) + \frac{4198}{33075}\sqrt{-c^2x^2+1}abcd^3x^4 + \frac{1}{3}b^2d^3x^3\arccos(cx)^2 + \frac{2}{3}abd^3x^3\arccos(cx) + \frac{1}{3}a^2d^3x^3 - \frac{5258}{297675}b^2d^3x^3 - \frac{5258\sqrt{-c^2x^2+1}b^2d^3x^2\arccos(cx)}{99225\,c} - \frac{5258\sqrt{-c^2x^2+1}abd^3x^2}{99225\,c} - \frac{10516\,b^2d^3x}{99225\,c^2} - \frac{10516\sqrt{-c^2x^2+1}b^2d^3\arccos(cx)}{99225\,c^3} - \frac{10516\sqrt{-c^2x^2+1}abd^3}{99225\,c^3}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-1/9*b^2*c^6*d^3*x^9*arccos(c*x)^2 - 2/9*a*b*c^6*d^3*x^9*arccos(c*x) - 1/9
*a^2*c^6*d^3*x^9 + 2/729*b^2*c^6*d^3*x^9 + 2/81*sqrt(-c^2*x^2 + 1)*b^2*c^5
*d^3*x^8*arccos(c*x) + 2/81*sqrt(-c^2*x^2 + 1)*a*b*c^5*d^3*x^8 + 3/7*b^2*c
^4*d^3*x^7*arccos(c*x)^2 + 6/7*a*b*c^4*d^3*x^7*arccos(c*x) + 3/7*a^2*c^4*d
^3*x^7 - 374/27783*b^2*c^4*d^3*x^7 - 374/3969*sqrt(-c^2*x^2 + 1)*b^2*c^3*d
^3*x^6*arccos(c*x) - 374/3969*sqrt(-c^2*x^2 + 1)*a*b*c^3*d^3*x^6 - 3/5*b^2
*c^2*d^3*x^5*arccos(c*x)^2 - 6/5*a*b*c^2*d^3*x^5*arccos(c*x) - 3/5*a^2*c^2
*d^3*x^5 + 4198/165375*b^2*c^2*d^3*x^5 + 4198/33075*sqrt(-c^2*x^2 + 1)*b^2
*c*d^3*x^4*arccos(c*x) + 4198/33075*sqrt(-c^2*x^2 + 1)*a*b*c*d^3*x^4 + 1/3
*b^2*d^3*x^3*arccos(c*x)^2 + 2/3*a*b*d^3*x^3*arccos(c*x) + 1/3*a^2*d^3*x^3
 - 5258/297675*b^2*d^3*x^3 - 5258/99225*sqrt(-c^2*x^2 + 1)*b^2*d^3*x^2*arc
cos(c*x)/c - 5258/99225*sqrt(-c^2*x^2 + 1)*a*b*d^3*x^2/c - 10516/99225*b^2
*d^3*x/c^2 - 10516/99225*sqrt(-c^2*x^2 + 1)*b^2*d^3*arccos(c*x)/c^3 - 1051
6/99225*sqrt(-c^2*x^2 + 1)*a*b*d^3/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(d-c^2dx^2\right)^3(a+b\arccos(cx))^2\,dx=\int x^2\,\left(a+b\,\mathrm{acos}(c\,x)\right)^2\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

input
```
int(x^2*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^3,x)
```

output
```
int(x^2*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int x^2\left(d-c^2dx^2\right)^3(a+b\arccos(cx))^2\,dx$$
$$=\frac{d^3\left(-22050acos(cx)\,ab\,c^9x^9+85050acos(cx)\,ab\,c^7x^7-119070acos(cx)\,ab\,c^5x^5+66150acos(cx)\,ab\,c^3x^3\right.}{}$$

input
```
int(x^2*(-c^2*d*x^2+d)^3*(a+b*acos(c*x))^2,x)
```

output
```
(d**3*( - 22050*acos(c*x)*a*b*c**9*x**9 + 85050*acos(c*x)*a*b*c**7*x**7 -
119070*acos(c*x)*a*b*c**5*x**5 + 66150*acos(c*x)*a*b*c**3*x**3 + 2450*sqrt
( - c**2*x**2 + 1)*a*b*c**8*x**8 - 9350*sqrt( - c**2*x**2 + 1)*a*b*c**6*x*
*6 + 12594*sqrt( - c**2*x**2 + 1)*a*b*c**4*x**4 - 5258*sqrt( - c**2*x**2 +
 1)*a*b*c**2*x**2 - 10516*sqrt( - c**2*x**2 + 1)*a*b - 99225*int(acos(c*x)
**2*x**8,x)*b**2*c**9 + 297675*int(acos(c*x)**2*x**6,x)*b**2*c**7 - 297675
*int(acos(c*x)**2*x**4,x)*b**2*c**5 + 99225*int(acos(c*x)**2*x**2,x)*b**2*
c**3 - 11025*a**2*c**9*x**9 + 42525*a**2*c**7*x**7 - 59535*a**2*c**5*x**5
+ 33075*a**2*c**3*x**3))/(99225*c**3)
```

3.179 $\int x(d-c^2dx^2)^3 (a+b\arccos(cx))^2 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 277

$$\begin{aligned}\int x(d-c^2dx^2)^3 (a+b\arccos(cx))^2 \, dx = &-\frac{35b^2d^3x^2}{1024}+\frac{35b^2d^3(1-c^2x^2)^2}{3072c^2} \\ &+\frac{7b^2d^3(1-c^2x^2)^3}{1152c^2}+\frac{b^2d^3(1-c^2x^2)^4}{256c^2} \\ &+\frac{35bd^3x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{512c} \\ &+\frac{35bd^3x(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{768c} \\ &+\frac{7bd^3x(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{192c} \\ &+\frac{bd^3x(1-c^2x^2)^{7/2}(a+b\arccos(cx))}{32c} \\ &+\frac{35d^3(a+b\arccos(cx))^2}{1024c^2} \\ &-\frac{d^3(1-c^2x^2)^4(a+b\arccos(cx))^2}{8c^2}\end{aligned}$$

output
```
-35/1024*b^2*d^3*x^2+35/3072*b^2*d^3*(-c^2*x^2+1)^2/c^2+7/1152*b^2*d^3*(-c
^2*x^2+1)^3/c^2+1/256*b^2*d^3*(-c^2*x^2+1)^4/c^2+35/512*b*d^3*x*(-c^2*x^2+
1)^(1/2)*(a+b*arccos(c*x))/c+35/768*b*d^3*x*(-c^2*x^2+1)^(3/2)*(a+b*arccos
(c*x))/c+7/192*b*d^3*x*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))/c+1/32*b*d^3*x
*(-c^2*x^2+1)^(7/2)*(a+b*arccos(c*x))/c+35/1024*d^3*(a+b*arccos(c*x))^2/c^
2-1/8*d^3*(-c^2*x^2+1)^4*(a+b*arccos(c*x))^2/c^2
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.93

$$\int x(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx$$
$$=\frac{d^3\left(cx\left(-1152a^2cx(-4+6c^2x^2-4c^4x^4+c^6x^6)+b^2cx(-837+489c^2x^2-200c^4x^4+36c^6x^6)+6ab\sqrt{1-\right.\right.}{}$$

input
```
Integrate[x*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^3*(c*x*(-1152*a^2*c*x*(-4 + 6*c^2*x^2 - 4*c^4*x^4 + c^6*x^6) + b^2*c*x*
(-837 + 489*c^2*x^2 - 200*c^4*x^4 + 36*c^6*x^6) + 6*a*b*Sqrt[1 - c^2*x^2]*
(-279 + 326*c^2*x^2 - 200*c^4*x^4 + 48*c^6*x^6)) + 6*b*c*x*(-384*a*c*x*(-4
 + 6*c^2*x^2 - 4*c^4*x^4 + c^6*x^6) + b*Sqrt[1 - c^2*x^2]*(-279 + 326*c^2*
x^2 - 200*c^4*x^4 + 48*c^6*x^6))*ArcCos[c*x] - 9*b^2*(93 - 512*c^2*x^2 + 7
68*c^4*x^4 - 512*c^6*x^6 + 128*c^8*x^8)*ArcCos[c*x]^2 + 1674*a*b*ArcSin[c*
x]))/(9216*c^2)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5183, 5159, 241, 5159, 241, 5159, 244, 2009, 5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d-c^2dx^2\right)^3(a+b\arccos(cx))^2dx$$

$$\Big\downarrow \ 5183$$

$$-\frac{bd^3\int\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))dx}{4c}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))^2}{8c^2}$$

$$\Big\downarrow \ 5159$$

$$-\frac{bd^3\left(\frac{7}{8}\int\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx+\frac{1}{8}bc\int x\left(1-c^2x^2\right)^3dx+\frac{1}{8}x\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))\right)}{4c}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))^2}{8c^2}$$

$$\Big\downarrow \ 241$$

$$-\frac{bd^3\left(\frac{7}{8}\int\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx+\frac{1}{8}x\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))-\frac{b\left(1-c^2x^2\right)^4}{64c}\right)}{4c}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))^2}{8c^2}$$

$$\Big\downarrow \ 5159$$

$$-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\int\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{6}bc\int x\left(1-c^2x^2\right)^2dx+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))\right)\cdots\right.}{4c}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))^2}{8c^2}$$

$$\Big\downarrow \ 241$$

$$-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\int\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))-\frac{b\left(1-c^2x^2\right)^3}{36c}\right)+\frac{1}{8}x\left(1-c\cdots\right.\right.}{4c}-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))^2}{8c^2}$$

$$\Big\downarrow \ 5159$$

$$-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int x\left(1-c^2x^2\right)dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)+\right.\right.}{4c}$$
$$\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))^2}{8c^2}$$

$\downarrow$ 244

$$-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int \left(x-c^2x^3\right)dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)+\right.\right.}{4c}$$
$$\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))^2}{8c^2}$$

$\downarrow$ 2009

$$-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\right)+\frac{1}{6}x(\right.\right.}{4c}$$
$$\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))^2}{8c^2}$$

$\downarrow$ 5157

$$-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\,\mathrm{arcc}\right.\right.\right.}{}$$
$$\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))^2}{8c^2}$$

$\downarrow$ 15

$$-\frac{bd^3\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{4}bcx^2\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(\right.\right.\right.}{}$$
$$\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))^2}{8c^2}$$

$\downarrow$ 5153

$$-\frac{d^3\left(1-c^2x^2\right)^4(a+b\arccos(cx))^2}{8c^2}-$$
$$\frac{bd^3\left(\frac{1}{8}x\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))+\frac{7}{8}\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\,\mathrm{arc}\right.\right.\right.}{}$$

input
```
Int[x*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/8*(d^3*(1 - c^2*x^2)^4*(a + b*ArcCos[c*x])^2)/c^2 - (b*d^3*(-1/64*(b*(1
 - c^2*x^2)^4)/c + (x*(1 - c^2*x^2)^(7/2)*(a + b*ArcCos[c*x]))/8 + (7*(-1/
36*(b*(1 - c^2*x^2)^3)/c + (x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/6 +
 (5*((b*c*(x^2/2 - (c^2*x^4)/4))/4 + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[
c*x]))/4 + (3*((b*c*x^2)/4 + (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/2 -
 (a + b*ArcCos[c*x])^2/(4*b*c)))/4))/6))/8))/(4*c)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{-\frac{d^3a^2(c^2x^2-1)^4}{8}-d^3b^2\left(\frac{\arccos(cx)^2(c^2x^2-1)^4}{8}-\frac{\arccos(cx)\left(48c^7x^7\sqrt{-c^2x^2+1}-200c^5x^5\sqrt{-c^2x^2+1}+326c^3x^3\sqrt{-c^2x^2+1}-\right.}{1536}\right.}{}$
default	$\frac{-\frac{d^3a^2(c^2x^2-1)^4}{8}-d^3b^2\left(\frac{\arccos(cx)^2(c^2x^2-1)^4}{8}-\frac{\arccos(cx)\left(48c^7x^7\sqrt{-c^2x^2+1}-200c^5x^5\sqrt{-c^2x^2+1}+326c^3x^3\sqrt{-c^2x^2+1}-\right.}{1536}\right.}{}$
parts	$-\frac{d^3a^2(c^2x^2-1)^4}{8c^2}-\frac{d^3b^2\left(\frac{\arccos(cx)^2(c^2x^2-1)^4}{8}-\frac{\arccos(cx)\left(48c^7x^7\sqrt{-c^2x^2+1}-200c^5x^5\sqrt{-c^2x^2+1}+326c^3x^3\sqrt{-c^2x^2\right.}{1536}\right.}{}$
orering	$\frac{(6084c^{10}x^{10}-32348c^8x^8+72453c^6x^6-97420c^4x^4+34749c^2x^2-5022)(-c^2d\,x^2+d)^3(a+b\arccos(cx))^2}{18432c^2(cx-1)^2(cx+1)^2(c^2x^2-1)^2}-\frac{(756c^8x^8-}{}$

input
```
int(x*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/8*d^3*a^2*(c^2*x^2-1)^4-d^3*b^2*(1/8*arccos(c*x)^2*(c^2*x^2-1)^4
-1/1536*arccos(c*x)*(48*c^7*x^7*(-c^2*x^2+1)^(1/2)-200*c^5*x^5*(-c^2*x^2+1
)^(1/2)+326*c^3*x^3*(-c^2*x^2+1)^(1/2)-279*c*x*(-c^2*x^2+1)^(1/2)+105*arcc
os(c*x))+35/1024*arccos(c*x)^2-1/256*(c^2*x^2-1)^4+7/1152*(c^2*x^2-1)^3-35
/3072*(c^2*x^2-1)^2+35/1024*c^2*x^2-35/1024)-2*d^3*a*b*(1/8*arccos(c*x)*c^
8*x^8-1/2*arccos(c*x)*c^6*x^6+3/4*c^4*x^4*arccos(c*x)-1/2*c^2*x^2*arccos(c
*x)+1/8*arccos(c*x)+35/1024*arcsin(c*x)-1/64*c^7*x^7*(-c^2*x^2+1)^(1/2)+25
/384*c^5*x^5*(-c^2*x^2+1)^(1/2)-163/1536*c^3*x^3*(-c^2*x^2+1)^(1/2)+93/102
4*c*x*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.28

$$\int x\left(d-c^2dx^2\right)^3(a+b\arccos(cx))^2\,dx =$$

$$-\frac{36\,(32\,a^2-b^2)c^8d^3x^8-8\,(576\,a^2-25\,b^2)c^6d^3x^6+3\,(2304\,a^2-163\,b^2)c^4d^3x^4-9\,(512\,a^2-93\,b^2)c^2d^3\cdots}{}$$

input
```
integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
-1/9216*(36*(32*a^2 - b^2)*c^8*d^3*x^8 - 8*(576*a^2 - 25*b^2)*c^6*d^3*x^6
+ 3*(2304*a^2 - 163*b^2)*c^4*d^3*x^4 - 9*(512*a^2 - 93*b^2)*c^2*d^3*x^2 +
9*(128*b^2*c^8*d^3*x^8 - 512*b^2*c^6*d^3*x^6 + 768*b^2*c^4*d^3*x^4 - 512*b
^2*c^2*d^3*x^2 + 93*b^2*d^3)*arccos(c*x)^2 + 18*(128*a*b*c^8*d^3*x^8 - 512
*a*b*c^6*d^3*x^6 + 768*a*b*c^4*d^3*x^4 - 512*a*b*c^2*d^3*x^2 + 93*a*b*d^3)
*arccos(c*x) - 6*(48*a*b*c^7*d^3*x^7 - 200*a*b*c^5*d^3*x^5 + 326*a*b*c^3*d
^3*x^3 - 279*a*b*c*d^3*x + (48*b^2*c^7*d^3*x^7 - 200*b^2*c^5*d^3*x^5 + 326
*b^2*c^3*d^3*x^3 - 279*b^2*c*d^3*x)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(258) = 516.

Time = 1.23 (sec) , antiderivative size = 578, normalized size of antiderivative = 2.09

$$\int x(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx$$
$$=\begin{cases}-\frac{a^2c^6d^3x^8}{8}+\frac{a^2c^4d^3x^6}{2}-\frac{3a^2c^2d^3x^4}{4}+\frac{a^2d^3x^2}{2}-\frac{abc^6d^3x^8\operatorname{acos}(cx)}{4}+\frac{abc^5d^3x^7\sqrt{-c^2x^2+1}}{32}+abc^4d^3x^6\operatorname{acos}(cx)-\frac{25ab}{} \\ \frac{d^3x^2\left(a+\frac{\pi b}{2}\right)^2}{2}\end{cases}$$

input
```
integrate(x*(-c**2*d*x**2+d)**3*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((-a**2*c**6*d**3*x**8/8 + a**2*c**4*d**3*x**6/2 - 3*a**2*c**2*d*
*3*x**4/4 + a**2*d**3*x**2/2 - a*b*c**6*d**3*x**8*acos(c*x)/4 + a*b*c**5*d
**3*x**7*sqrt(-c**2*x**2 + 1)/32 + a*b*c**4*d**3*x**6*acos(c*x) - 25*a*b*c
**3*d**3*x**5*sqrt(-c**2*x**2 + 1)/192 - 3*a*b*c**2*d**3*x**4*acos(c*x)/2
+ 163*a*b*c*d**3*x**3*sqrt(-c**2*x**2 + 1)/768 + a*b*d**3*x**2*acos(c*x) -
 93*a*b*d**3*x*sqrt(-c**2*x**2 + 1)/(512*c) - 93*a*b*d**3*acos(c*x)/(512*c
**2) - b**2*c**6*d**3*x**8*acos(c*x)**2/8 + b**2*c**6*d**3*x**8/256 + b**2
*c**5*d**3*x**7*sqrt(-c**2*x**2 + 1)*acos(c*x)/32 + b**2*c**4*d**3*x**6*ac
os(c*x)**2/2 - 25*b**2*c**4*d**3*x**6/1152 - 25*b**2*c**3*d**3*x**5*sqrt(-
c**2*x**2 + 1)*acos(c*x)/192 - 3*b**2*c**2*d**3*x**4*acos(c*x)**2/4 + 163*
b**2*c**2*d**3*x**4/3072 + 163*b**2*c*d**3*x**3*sqrt(-c**2*x**2 + 1)*acos(
c*x)/768 + b**2*d**3*x**2*acos(c*x)**2/2 - 93*b**2*d**3*x**2/1024 - 93*b**
2*d**3*x*sqrt(-c**2*x**2 + 1)*acos(c*x)/(512*c) - 93*b**2*d**3*acos(c*x)**
2/(1024*c**2), Ne(c, 0)), (d**3*x**2*(a + pi*b/2)**2/2, True))
```

Maxima [F]

$$\int x(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx=\int -\left(c^2dx^2-d\right)^3\left(b\arccos(cx)+a\right)^2x\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-1/8*a^2*c^6*d^3*x^8 + 1/2*a^2*c^4*d^3*x^6 - 1/1536*(384*x^8*arccos(c*x) -
 (48*sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(
-c^2*x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9
)*c)*a*b*c^6*d^3 - 3/4*a^2*c^2*d^3*x^4 + 1/48*(48*x^6*arccos(c*x) - (8*sqr
t(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2
 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*a*b*c^4*d^3 - 3/16*(8*x^4*arccos(c*x)
 - (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c
*x)/c^5)*c)*a*b*c^2*d^3 + 1/2*a^2*d^3*x^2 + 1/2*(2*x^2*arccos(c*x) - c*(sq
rt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*x)/c^3))*a*b*d^3 - 1/8*(b^2*c^6*d^3*x^8
- 4*b^2*c^4*d^3*x^6 + 6*b^2*c^2*d^3*x^4 - 4*b^2*d^3*x^2)*arctan2(sqrt(c*x
+ 1)*sqrt(-c*x + 1), c*x)^2 + integrate(1/4*(b^2*c^7*d^3*x^8 - 4*b^2*c^5*d
^3*x^6 + 6*b^2*c^3*d^3*x^4 - 4*b^2*c*d^3*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)
*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^2*x^2 - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(245) = 490$.

Time = 0.17 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.81

$$\int x(d-c^2dx^2)^3(a+b\arccos(cx))^2\,dx = -\frac{1}{8}b^2c^6d^3x^8\arccos(cx)^2 - \frac{1}{4}abc^6d^3x^8\arccos(cx) - \frac{1}{8}a^2c^6d^3x^8 + \frac{1}{256}b^2c^6d^3x^8 + \frac{1}{32}\sqrt{-c^2x^2+1}b^2c^5d^3x^7\arccos(cx) + \frac{1}{32}\sqrt{-c^2x^2+1}abc^5d^3x^7 + \frac{1}{2}b^2c^4d^3x^6\arccos(cx)^2 + abc^4d^3x^6\arccos(cx) + \frac{1}{2}a^2c^4d^3x^6 - \frac{25}{1152}b^2c^4d^3x^6 - \frac{25}{192}\sqrt{-c^2x^2+1}b^2c^3d^3x^5\arccos(cx) - \frac{25}{192}\sqrt{-c^2x^2+1}abc^3d^3x^5 - \frac{3}{4}b^2c^2d^3x^4\arccos(cx)^2 - \frac{3}{2}abc^2d^3x^4\arccos(cx) - \frac{3}{4}a^2c^2d^3x^4 + \frac{163}{3072}b^2c^2d^3x^4 + \frac{163}{768}\sqrt{-c^2x^2+1}b^2cd^3x^3\arccos(cx) + \frac{163}{768}\sqrt{-c^2x^2+1}abcd^3x^3 + \frac{1}{2}b^2d^3x^2\arccos(cx)^2 + abd^3x^2\arccos(cx) + \frac{1}{2}a^2d^3x^2 - \frac{93}{1024}b^2d^3x^2 - \frac{93\sqrt{-c^2x^2+1}b^2d^3x\arccos(cx)}{512\,c} - \frac{93\sqrt{-c^2x^2+1}abd^3x}{512\,c} - \frac{93\,b^2d^3\arccos(cx)^2}{1024\,c^2} - \frac{93\,abd^3\arccos(cx)}{512\,c^2} + \frac{9209\,b^2d^3}{294912\,c^2}$$

input
```
integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-1/8*b^2*c^6*d^3*x^8*arccos(c*x)^2 - 1/4*a*b*c^6*d^3*x^8*arccos(c*x) - 1/8
*a^2*c^6*d^3*x^8 + 1/256*b^2*c^6*d^3*x^8 + 1/32*sqrt(-c^2*x^2 + 1)*b^2*c^5
*d^3*x^7*arccos(c*x) + 1/32*sqrt(-c^2*x^2 + 1)*a*b*c^5*d^3*x^7 + 1/2*b^2*c
^4*d^3*x^6*arccos(c*x)^2 + a*b*c^4*d^3*x^6*arccos(c*x) + 1/2*a^2*c^4*d^3*x
^6 - 25/1152*b^2*c^4*d^3*x^6 - 25/192*sqrt(-c^2*x^2 + 1)*b^2*c^3*d^3*x^5*a
rccos(c*x) - 25/192*sqrt(-c^2*x^2 + 1)*a*b*c^3*d^3*x^5 - 3/4*b^2*c^2*d^3*x
^4*arccos(c*x)^2 - 3/2*a*b*c^2*d^3*x^4*arccos(c*x) - 3/4*a^2*c^2*d^3*x^4 +
 163/3072*b^2*c^2*d^3*x^4 + 163/768*sqrt(-c^2*x^2 + 1)*b^2*c*d^3*x^3*arcco
s(c*x) + 163/768*sqrt(-c^2*x^2 + 1)*a*b*c*d^3*x^3 + 1/2*b^2*d^3*x^2*arccos
(c*x)^2 + a*b*d^3*x^2*arccos(c*x) + 1/2*a^2*d^3*x^2 - 93/1024*b^2*d^3*x^2
- 93/512*sqrt(-c^2*x^2 + 1)*b^2*d^3*x*arccos(c*x)/c - 93/512*sqrt(-c^2*x^2
 + 1)*a*b*d^3*x/c - 93/1024*b^2*d^3*arccos(c*x)^2/c^2 - 93/512*a*b*d^3*arc
cos(c*x)/c^2 + 9209/294912*b^2*d^3/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x\left(d-c^2dx^2\right)^3(a+b\arccos(cx))^2\,dx=\int x\,\left(a+b\,\mathrm{acos}(c\,x)\right)^2\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

input
```
int(x*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^3,x)
```

output
```
int(x*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int x\left(d-c^2dx^2\right)^3(a+b\arccos(cx))^2\,dx$$
$$=\frac{d^3\left(768acos(cx)^2b^2c^2x^2-384acos(cx)^2b^2-768\sqrt{-c^2x^2+1}\,acos(cx)\,b^2cx-384acos(cx)\,ab\,c^8x^8+1536\right.}{}$$

input
```
int(x*(-c^2*d*x^2+d)^3*(a+b*acos(c*x))^2,x)
```

output
```
(d**3*(768*acos(c*x)**2*b**2*c**2*x**2 - 384*acos(c*x)**2*b**2 - 768*sqrt(
 - c**2*x**2 + 1)*acos(c*x)*b**2*c*x - 384*acos(c*x)*a*b*c**8*x**8 + 1536*
acos(c*x)*a*b*c**6*x**6 - 2304*acos(c*x)*a*b*c**4*x**4 + 1536*acos(c*x)*a*
b*c**2*x**2 + 279*asin(c*x)*a*b + 48*sqrt( - c**2*x**2 + 1)*a*b*c**7*x**7
- 200*sqrt( - c**2*x**2 + 1)*a*b*c**5*x**5 + 326*sqrt( - c**2*x**2 + 1)*a*
b*c**3*x**3 - 279*sqrt( - c**2*x**2 + 1)*a*b*c*x - 1536*int(acos(c*x)**2*x
**7,x)*b**2*c**8 + 4608*int(acos(c*x)**2*x**5,x)*b**2*c**6 - 4608*int(acos
(c*x)**2*x**3,x)*b**2*c**4 - 192*a**2*c**8*x**8 + 768*a**2*c**6*x**6 - 115
2*a**2*c**4*x**4 + 768*a**2*c**2*x**2 - 384*b**2*c**2*x**2))/(1536*c**2)
```

3.180 $\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 298

$$
\begin{aligned}
&\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2 \, dx \\
&= -\frac{4322b^2d^3x}{3675} + \frac{1514b^2c^2d^3x^3}{11025} - \frac{234b^2c^4d^3x^5}{6125} + \frac{2}{343}b^2c^6d^3x^7 \\
&\quad + \frac{32bd^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{35c} + \frac{16bd^3\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{105c} \\
&\quad + \frac{12bd^3\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{175c} + \frac{2bd^3\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{49c} \\
&\quad + \frac{16}{35}d^3x(a+b\arccos(cx))^2 + \frac{8}{35}d^3x\left(1-c^2x^2\right)(a+b\arccos(cx))^2 + \frac{6}{35}d^3x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2 + \frac{1}{7}
\end{aligned}
$$

output

```
-4322/3675*b^2*d^3*x+1514/11025*b^2*c^2*d^3*x^3-234/6125*b^2*c^4*d^3*x^5+2
/343*b^2*c^6*d^3*x^7+32/35*b*d^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+16
/105*b*d^3*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/c+12/175*b*d^3*(-c^2*x^2+1
)^(5/2)*(a+b*arccos(c*x))/c+2/49*b*d^3*(-c^2*x^2+1)^(7/2)*(a+b*arccos(c*x)
)/c+16/35*d^3*x*(a+b*arccos(c*x))^2+8/35*d^3*x*(-c^2*x^2+1)*(a+b*arccos(c*
x))^2+6/35*d^3*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2+1/7*d^3*x*(-c^2*x^2+1)
^3*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.81

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx$$
$$=\frac{d^3\left(-11025a^2cx(-35+35c^2x^2-21c^4x^4+5c^6x^6)+210ab\sqrt{1-c^2x^2}(-2161+757c^2x^2-351c^4x^4+75c\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^3*(-11025*a^2*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) + 210*a*b
*Sqrt[1 - c^2*x^2]*(-2161 + 757*c^2*x^2 - 351*c^4*x^4 + 75*c^6*x^6) + 2*b^
2*c*x*(-226905 + 26495*c^2*x^2 - 7371*c^4*x^4 + 1125*c^6*x^6) + 210*b*(-10
5*a*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) + b*Sqrt[1 - c^2*x^2]*
(-2161 + 757*c^2*x^2 - 351*c^4*x^4 + 75*c^6*x^6))*ArcCos[c*x] - 11025*b^2*
c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6)*ArcCos[c*x]^2))/(385875*c)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5159, 27, 5159, 5159, 5131, 5183, 24, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{ 5159}$$

$$\frac{2}{7}bcd^3\int x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx+\frac{6}{7}d\int d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2dx+\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{7}bcd^3\int x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx+\frac{6}{7}d^3\int \left(1-c^2x^2\right)^2(a+b\arccos(cx))^2dx+\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5159}$$

$$\frac{2}{7}bcd^3\int x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx+\frac{6}{7}d^3\left(\frac{2}{5}bc\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{4}{5}\int \left(1-c^2x^2\right)(a+b\arccos(cx))^2dx+\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\right.$$
$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5159}$$

$$\frac{2}{7}bcd^3\int x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx+\frac{6}{7}d^3\left(\frac{2}{5}bc\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{4}{5}\left(\frac{2}{3}bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{2}{3}\int (a+b\arccos\right.\right.$$
$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5131}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(2bc\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{3}\right.\right.$$
$$\frac{2}{7}bcd^3\int x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx+\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5183}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(2bc\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bc\left(-\frac{b\int \left(1-c^2x^2\right)d}{3c}\right.\right.\right.$$
$$\frac{2}{7}bcd^3\left(-\frac{b\int \left(1-c^2x^2\right)^3dx}{7c}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^2}\right)+\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{24}$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}bc\left(-\frac{b\int \left(1-c^2x^2\right)dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\right.\right.$$
$$\frac{2}{7}bcd^3\left(-\frac{b\int \left(1-c^2x^2\right)^3dx}{7c}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^2}\right)+\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \; 210$$

$$\frac{6}{7}d^3\left(\frac{4}{5}\left(\frac{2}{3}bc\left(-\frac{b\int\left(1-c^2x^2\right)dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\right.\right.$$
$$\frac{2}{7}bcd^3\left(-\frac{b\int\left(-c^6x^6+3c^4x^4-3c^2x^2+1\right)dx}{7c}-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^2}\right)+$$
$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{7}d^3x\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{6}{7}d^3\left(\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right.\right.\right.\right.$$
$$\frac{2}{7}bcd^3\left(-\frac{\left(1-c^2x^2\right)^{7/2}(a+b\arccos(cx))}{7c^2}-\frac{b\left(-\frac{1}{7}c^6x^7+\frac{3c^4x^5}{5}-c^2x^3+x\right)}{7c}\right)$$

input
```
Int[(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^3*x*(1 - c^2*x^2)^3*(a + b*ArcCos[c*x])^2)/7 + (2*b*c*d^3*(-1/7*(b*(x -
 c^2*x^3 + (3*c^4*x^5)/5 - (c^6*x^7)/7))/c - ((1 - c^2*x^2)^(7/2)*(a + b*A
rcCos[c*x]))/(7*c^2)))/7 + (6*d^3*((x*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^
2)/5 + (2*b*c*(-1/5*(b*(x - (2*c^2*x^3)/3 + (c^4*x^5)/5))/c - ((1 - c^2*x^
2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^2)))/5 + (4*((x*(1 - c^2*x^2)*(a + b*Ar
cCos[c*x])^2)/3 + (2*b*c*(-1/3*(b*(x - (c^2*x^3)/3))/c - ((1 - c^2*x^2)^(3
/2)*(a + b*ArcCos[c*x]))/(3*c^2)))/3 + (2*(x*(a + b*ArcCos[c*x])^2 + 2*b*c
*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c^2)))/3))/5))/7
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-d^3a^2\left(\frac{1}{7}c^7x^7-\frac{3}{5}c^5x^5+c^3x^3-cx\right)-d^3b^2\left(\frac{\arccos(cx)^2\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)cx}{35}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^3\sqrt{-c^2x^2+1}}{49}\right.}{}$
default	$\frac{-d^3a^2\left(\frac{1}{7}c^7x^7-\frac{3}{5}c^5x^5+c^3x^3-cx\right)-d^3b^2\left(\frac{\arccos(cx)^2\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)cx}{35}-\frac{2\arccos(cx)\left(c^2x^2-1\right)^3\sqrt{-c^2x^2+1}}{49}\right.}{}$
parts	$-d^3a^2\left(\frac{1}{7}c^6x^7-\frac{3}{5}c^4x^5+c^2x^3-x\right)-\frac{d^3b^2\left(\frac{\arccos(cx)^2\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)cx}{35}-\frac{2\arccos(cx)\left(c^2x^2-\right.}{49}\right.}{}$
orering	$\frac{x\left(47625c^8x^8-271212c^6x^6+741678c^4x^4-3539900c^2x^2+128625\right)\left(-c^2d\,x^2+d\right)^3\left(a+b\arccos(cx)\right)^2}{128625(cx-1)^2(cx+1)^2\left(c^2x^2-1\right)^2}-\frac{\left(20250c^8x^8-1\right.}{}$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-d^3*a^2*(1/7*c^7*x^7-3/5*c^5*x^5+c^3*x^3-c*x)-d^3*b^2*(1/35*arccos(c
*x)^2*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c*x-2/49*arccos(c*x)*(c^2*x^2-1
)^3*(-c^2*x^2+1)^(1/2)-2/1715*(5*c^6*x^6-21*c^4*x^4+35*c^2*x^2-35)*c*x+12/
175*arccos(c*x)*(c^2*x^2-1)^2*(-c^2*x^2+1)^(1/2)+4/875*(3*c^4*x^4-10*c^2*x
^2+15)*c*x-16/105*arccos(c*x)*(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)-16/315*(c^2*x
^2-3)*c*x+32/35*c*x+32/35*arccos(c*x)*(-c^2*x^2+1)^(1/2))-2*d^3*a*b*(1/7*a
rccos(c*x)*c^7*x^7-3/5*arccos(c*x)*c^5*x^5+c^3*x^3*arccos(c*x)-c*x*arccos(
c*x)+2161/3675*(-c^2*x^2+1)^(1/2)-757/3675*c^2*x^2*(-c^2*x^2+1)^(1/2)+117/
1225*c^4*x^4*(-c^2*x^2+1)^(1/2)-1/49*c^6*x^6*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.08

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx =$$
$$-\frac{1125\,(49\,a^2-2\,b^2)c^7d^3x^7-189\,(1225\,a^2-78\,b^2)c^5d^3x^5+35\,(11025\,a^2-1514\,b^2)c^3d^3x^3-105\,(3675\,a}{}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
-1/385875*(1125*(49*a^2 - 2*b^2)*c^7*d^3*x^7 - 189*(1225*a^2 - 78*b^2)*c^5
*d^3*x^5 + 35*(11025*a^2 - 1514*b^2)*c^3*d^3*x^3 - 105*(3675*a^2 - 4322*b^
2)*c*d^3*x + 11025*(5*b^2*c^7*d^3*x^7 - 21*b^2*c^5*d^3*x^5 + 35*b^2*c^3*d^
3*x^3 - 35*b^2*c*d^3*x)*arccos(c*x)^2 + 22050*(5*a*b*c^7*d^3*x^7 - 21*a*b*
c^5*d^3*x^5 + 35*a*b*c^3*d^3*x^3 - 35*a*b*c*d^3*x)*arccos(c*x) - 210*(75*a
*b*c^6*d^3*x^6 - 351*a*b*c^4*d^3*x^4 + 757*a*b*c^2*d^3*x^2 - 2161*a*b*d^3
+ (75*b^2*c^6*d^3*x^6 - 351*b^2*c^4*d^3*x^4 + 757*b^2*c^2*d^3*x^2 - 2161*b
^2*d^3)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.78

$$\int \left(d-c^2dx^2\right)^3(a+b\arccos(cx))^2\,dx$$

$$= \begin{cases} -\frac{a^2c^6d^3x^7}{7} + \frac{3a^2c^4d^3x^5}{5} - a^2c^2d^3x^3 + a^2d^3x - \frac{2abc^6d^3x^7\operatorname{acos}(cx)}{7} + \frac{2abc^5d^3x^6\sqrt{-c^2x^2+1}}{49} + \frac{6abc^4d^3x^5\operatorname{acos}(cx)}{5} - 234a \\ d^3x\left(a+\frac{\pi b}{2}\right)^2 \end{cases}$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((-a**2*c**6*d**3*x**7/7 + 3*a**2*c**4*d**3*x**5/5 - a**2*c**2*d*
*3*x**3 + a**2*d**3*x - 2*a*b*c**6*d**3*x**7*acos(c*x)/7 + 2*a*b*c**5*d**3
*x**6*sqrt(-c**2*x**2 + 1)/49 + 6*a*b*c**4*d**3*x**5*acos(c*x)/5 - 234*a*b
*c**3*d**3*x**4*sqrt(-c**2*x**2 + 1)/1225 - 2*a*b*c**2*d**3*x**3*acos(c*x)
 + 1514*a*b*c*d**3*x**2*sqrt(-c**2*x**2 + 1)/3675 + 2*a*b*d**3*x*acos(c*x)
 - 4322*a*b*d**3*sqrt(-c**2*x**2 + 1)/(3675*c) - b**2*c**6*d**3*x**7*acos(
c*x)**2/7 + 2*b**2*c**6*d**3*x**7/343 + 2*b**2*c**5*d**3*x**6*sqrt(-c**2*x
**2 + 1)*acos(c*x)/49 + 3*b**2*c**4*d**3*x**5*acos(c*x)**2/5 - 234*b**2*c*
*4*d**3*x**5/6125 - 234*b**2*c**3*d**3*x**4*sqrt(-c**2*x**2 + 1)*acos(c*x)
/1225 - b**2*c**2*d**3*x**3*acos(c*x)**2 + 1514*b**2*c**2*d**3*x**3/11025
+ 1514*b**2*c*d**3*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/3675 + b**2*d**3*x*
acos(c*x)**2 - 4322*b**2*d**3*x/3675 - 4322*b**2*d**3*sqrt(-c**2*x**2 + 1)
*acos(c*x)/(3675*c), Ne(c, 0)), (d**3*x*(a + pi*b/2)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(263) = 526$.

Time = 0.15 (sec) , antiderivative size = 730, normalized size of antiderivative = 2.45

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-1/7*b^2*c^6*d^3*x^7*arccos(c*x)^2 - 1/7*a^2*c^6*d^3*x^7 + 3/5*b^2*c^4*d^3
*x^5*arccos(c*x)^2 + 3/5*a^2*c^4*d^3*x^5 - 2/245*(35*x^7*arccos(c*x) - (5*
sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^
2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*a*b*c^6*d^3 + 2/25725*(105*
(5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2
*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c*arccos(c*x) + (75*c^6*x^7
 + 126*c^4*x^5 + 280*c^2*x^3 + 1680*x)/c^6)*b^2*c^6*d^3 - b^2*c^2*d^3*x^3*
arccos(c*x)^2 + 2/25*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 +
 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a*b*c^4*d^3 -
 2/375*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 +
8*sqrt(-c^2*x^2 + 1)/c^6)*c*arccos(c*x) + (9*c^4*x^5 + 20*c^2*x^3 + 120*x)
/c^4)*b^2*c^4*d^3 - a^2*c^2*d^3*x^3 - 2/3*(3*x^3*arccos(c*x) - c*(sqrt(-c^
2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*c^2*d^3 + 2/9*(3*c*(sq
rt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x) + (c^2*x^
3 + 6*x)/c^2)*b^2*c^2*d^3 + b^2*d^3*x*arccos(c*x)^2 - 2*b^2*d^3*(x + sqrt(
-c^2*x^2 + 1)*arccos(c*x)/c) + a^2*d^3*x + 2*(c*x*arccos(c*x) - sqrt(-c^2*
x^2 + 1))*a*b*d^3/c
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.51

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx = -\frac{1}{7}\,b^2c^6d^3x^7\arccos\left(cx\right)^2-\frac{2}{7}\,abc^6d^3x^7\arccos\left(cx\right) -\frac{1}{7}\,a^2c^6d^3x^7+\frac{2}{343}\,b^2c^6d^3x^7 +\frac{2}{49}\,\sqrt{-c^2x^2+1}b^2c^5d^3x^6\arccos\left(cx\right) +\frac{2}{49}\,\sqrt{-c^2x^2+1}abc^5d^3x^6 +\frac{3}{5}\,b^2c^4d^3x^5\arccos\left(cx\right)^2 +\frac{6}{5}\,abc^4d^3x^5\arccos\left(cx\right) +\frac{3}{5}\,a^2c^4d^3x^5-\frac{234}{6125}\,b^2c^4d^3x^5 -\frac{234}{1225}\,\sqrt{-c^2x^2+1}b^2c^3d^3x^4\arccos\left(cx\right) -\frac{234}{1225}\,\sqrt{-c^2x^2+1}abc^3d^3x^4 -b^2c^2d^3x^3\arccos\left(cx\right)^2-2\,abc^2d^3x^3\arccos\left(cx\right) -a^2c^2d^3x^3+\frac{1514}{11025}\,b^2c^2d^3x^3 +\frac{1514}{3675}\,\sqrt{-c^2x^2+1}b^2cd^3x^2\arccos\left(cx\right) +\frac{1514}{3675}\,\sqrt{-c^2x^2+1}abcd^3x^2+b^2d^3x\arccos\left(cx\right)^2 +2\,abd^3x\arccos\left(cx\right)+a^2d^3x-\frac{4322}{3675}\,b^2d^3x -\frac{4322\,\sqrt{-c^2x^2+1}b^2d^3\arccos\left(cx\right)}{3675\,c} -\frac{4322\,\sqrt{-c^2x^2+1}abd^3}{3675\,c}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-1/7*b^2*c^6*d^3*x^7*arccos(c*x)^2 - 2/7*a*b*c^6*d^3*x^7*arccos(c*x) - 1/7
*a^2*c^6*d^3*x^7 + 2/343*b^2*c^6*d^3*x^7 + 2/49*sqrt(-c^2*x^2 + 1)*b^2*c^5
*d^3*x^6*arccos(c*x) + 2/49*sqrt(-c^2*x^2 + 1)*a*b*c^5*d^3*x^6 + 3/5*b^2*c
^4*d^3*x^5*arccos(c*x)^2 + 6/5*a*b*c^4*d^3*x^5*arccos(c*x) + 3/5*a^2*c^4*d
^3*x^5 - 234/6125*b^2*c^4*d^3*x^5 - 234/1225*sqrt(-c^2*x^2 + 1)*b^2*c^3*d^
3*x^4*arccos(c*x) - 234/1225*sqrt(-c^2*x^2 + 1)*a*b*c^3*d^3*x^4 - b^2*c^2*
d^3*x^3*arccos(c*x)^2 - 2*a*b*c^2*d^3*x^3*arccos(c*x) - a^2*c^2*d^3*x^3 +
1514/11025*b^2*c^2*d^3*x^3 + 1514/3675*sqrt(-c^2*x^2 + 1)*b^2*c*d^3*x^2*ar
ccos(c*x) + 1514/3675*sqrt(-c^2*x^2 + 1)*a*b*c*d^3*x^2 + b^2*d^3*x*arccos(
c*x)^2 + 2*a*b*d^3*x*arccos(c*x) + a^2*d^3*x - 4322/3675*b^2*d^3*x - 4322/
3675*sqrt(-c^2*x^2 + 1)*b^2*d^3*arccos(c*x)/c - 4322/3675*sqrt(-c^2*x^2 +
1)*a*b*d^3/c
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx = \int (a+b\,\mathrm{acos}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

input
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^3,x)
```

output
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx$$
$$= \frac{d^3\left(3675 acos(cx)^2 b^2cx - 7350\sqrt{-c^2x^2+1}\, acos(cx)\, b^2 - 1050 acos(cx)\, ab\, c^7x^7 + 4410 acos(cx)\, ab\, c^5x^5 - \right.}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acos(c*x))^2,x)
```

output
```
(d**3*(3675*acos(c*x)**2*b**2*c*x - 7350*sqrt( - c**2*x**2 + 1)*acos(c*x)*
b**2 - 1050*acos(c*x)*a*b*c**7*x**7 + 4410*acos(c*x)*a*b*c**5*x**5 - 7350*
acos(c*x)*a*b*c**3*x**3 + 7350*acos(c*x)*a*b*c*x + 150*sqrt( - c**2*x**2 +
 1)*a*b*c**6*x**6 - 702*sqrt( - c**2*x**2 + 1)*a*b*c**4*x**4 + 1514*sqrt(
- c**2*x**2 + 1)*a*b*c**2*x**2 - 4322*sqrt( - c**2*x**2 + 1)*a*b - 3675*in
t(acos(c*x)**2*x**6,x)*b**2*c**7 + 11025*int(acos(c*x)**2*x**4,x)*b**2*c**
5 - 11025*int(acos(c*x)**2*x**2,x)*b**2*c**3 - 525*a**2*c**7*x**7 + 2205*a
**2*c**5*x**5 - 3675*a**2*c**3*x**3 + 3675*a**2*c*x - 7350*b**2*c*x))/(367
5*c)
```

3.181 $\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))^2}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 360

$$\begin{aligned}\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))^2}{x}\,dx &= \frac{19}{48}b^2c^2d^3x^2-\frac{7}{144}b^2d^3(1-c^2x^2)^2\\
&-\frac{1}{108}b^2d^3(1-c^2x^2)^3\\
&-\frac{19}{24}bcd^3x\sqrt{1-c^2x^2}(a+b\arccos(cx))\\
&-\frac{7}{36}bcd^3x(1-c^2x^2)^{3/2}(a+b\arccos(cx))\\
&-\frac{1}{18}bcd^3x(1-c^2x^2)^{5/2}(a+b\arccos(cx))\\
&-\frac{19}{48}d^3(a+b\arccos(cx))^2\\
&+\frac{1}{2}d^3(1-c^2x^2)(a+b\arccos(cx))^2\\
&+\frac{1}{4}d^3(1-c^2x^2)^2(a+b\arccos(cx))^2\\
&+\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))^2\\
&-\frac{id^3(a+b\arccos(cx))^3}{3b}\\
&+d^3(a+b\arccos(cx))^2\log\left(1-e^{2i\arccos(cx)}\right)\\
&-ibd^3(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\\
&+\frac{1}{2}b^2d^3\operatorname{PolyLog}\left(3,e^{2i\arccos(cx)}\right)\end{aligned}$$

output
```
19/48*b^2*c^2*d^3*x^2-7/144*b^2*d^3*(-c^2*x^2+1)^2-1/108*b^2*d^3*(-c^2*x^2
+1)^3-19/24*b*c*d^3*x*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))-7/36*b*c*d^3*x*
(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))-1/18*b*c*d^3*x*(-c^2*x^2+1)^(5/2)*(a+
b*arccos(c*x))-19/48*d^3*(a+b*arccos(c*x))^2+1/2*d^3*(-c^2*x^2+1)*(a+b*arc
cos(c*x))^2+1/4*d^3*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2+1/6*d^3*(-c^2*x^2+1
)^3*(a+b*arccos(c*x))^2-1/3*I*d^3*(a+b*arccos(c*x))^3/b+d^3*(a+b*arccos(c*
x))^2*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-I*b*d^3*(a+b*arccos(c*x))*polylog
(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/2*b^2*d^3*polylog(3,(c*x+I*(-c^2*x^2+1)
^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.22

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x}\,dx =$$
$$-\frac{d^3\Big(5184a^2c^2x^2 - 2592a^2c^4x^4 + 576a^2c^6x^6 - 3600abcx\sqrt{1-c^2x^2} + 1056abc^3x^3\sqrt{1-c^2x^2} - 192abc^5x$$

input
```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
-1/3456*(d^3*(5184*a^2*c^2*x^2 - 2592*a^2*c^4*x^4 + 576*a^2*c^6*x^6 - 3600
*a*b*c*x*Sqrt[1 - c^2*x^2] + 1056*a*b*c^3*x^3*Sqrt[1 - c^2*x^2] - 192*a*b*
c^5*x^5*Sqrt[1 - c^2*x^2] + 10368*a*b*c^2*x^2*ArcCos[c*x] - 5184*a*b*c^4*x
^4*ArcCos[c*x] + 1152*a*b*c^6*x^6*ArcCos[c*x] + (3456*I)*a*b*ArcCos[c*x]^2
 + (1152*I)*b^2*ArcCos[c*x]^3 + 7200*a*b*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x
^2])] - 783*b^2*Cos[2*ArcCos[c*x]] + 1566*b^2*ArcCos[c*x]^2*Cos[2*ArcCos[c
*x]] + 27*b^2*Cos[4*ArcCos[c*x]] - 216*b^2*ArcCos[c*x]^2*Cos[4*ArcCos[c*x]
] - b^2*Cos[6*ArcCos[c*x]] + 18*b^2*ArcCos[c*x]^2*Cos[6*ArcCos[c*x]] - 691
2*a*b*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] - 3456*b^2*ArcCos[c*x]^2*
Log[1 + E^((2*I)*ArcCos[c*x])] - 3456*a^2*Log[c*x] + (3456*I)*b*(a + b*Arc
Cos[c*x])*PolyLog[2, -E^((2*I)*ArcCos[c*x])] - 1728*b^2*PolyLog[3, -E^((2*
I)*ArcCos[c*x])] - 1566*b^2*ArcCos[c*x]*Sin[2*ArcCos[c*x]] + 108*b^2*ArcCo
s[c*x]*Sin[4*ArcCos[c*x]] - 6*b^2*ArcCos[c*x]*Sin[6*ArcCos[c*x]]))
```

Rubi [A] (verified)

Time = 4.26 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.51, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.037$, Rules used = {5203, 27, 5159, 241, 5159, 244, 2009, 5157, 15, 5153, 5203, 5159, 244, 2009, 5157, 15, 5153, 5203, 5137, 3042, 4202, 2620, 3011, 2720, 5157, 15, 5153, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x}dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{1}{3}bcd^3\int (1-c^2x^2)^{5/2}(a+b\arccos(cx))dx+d\int \frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{x}dx+\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bcd^3\int (1-c^2x^2)^{5/2}(a+b\arccos(cx))dx+d^3\int \frac{(1-c^2x^2)^2(a+b\arccos(cx))^2}{x}dx+\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5159}$$

$$\frac{1}{3}bcd^3\left(\frac{5}{6}\int (1-c^2x^2)^{3/2}(a+b\arccos(cx))dx+\frac{1}{6}bc\int x(1-c^2x^2)^2dx+\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arccos(cx))\right)+d^3\int \frac{(1-c^2x^2)^2(a+b\arccos(cx))^2}{x}dx+\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{241}$$

$$\frac{1}{3}bcd^3\left(\frac{5}{6}\int (1-c^2x^2)^{3/2}(a+b\arccos(cx))dx+\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arccos(cx))-\frac{b(1-c^2x^2)^3}{36c}\right)+d^3\int \frac{(1-c^2x^2)^2(a+b\arccos(cx))^2}{x}dx+\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5159}$$

$$\frac{1}{3}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int x\left(1-c^2x^2\right)dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)\right.$$
$$d^3\int \frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}dx+\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$\Big\downarrow$ 244

$$d^3\int \frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}dx+$$
$$\frac{1}{3}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int \left(x-c^2x^3\right)dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)+\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$\Big\downarrow$ 2009

$$d^3\int \frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}dx+$$
$$\frac{1}{3}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\right)+\frac{1}{6}\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$\Big\downarrow$ 5157

$$d^3\int \frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}dx+$$
$$\frac{1}{3}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+\right.\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$\Big\downarrow$ 15

$$d^3\int \frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}dx+$$
$$\frac{1}{3}bcd^3\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{4}bcx^2\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\,\mathrm{ar}\right.\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2$$

$\Big\downarrow$ 5153

$$d^3\int \frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}dx+\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\text{ar}\right.\right.\right.$$

$\big\downarrow$ 5203

$$d^3\left(\frac{1}{2}bc\int\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\int\frac{\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{x}dx+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\,\text{arcco}\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\text{ar}\right.\right.\right.$$

$\big\downarrow$ 5159

$$d^3\left(\frac{1}{2}bc\left(\frac{3}{4}\int\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int x\left(1-c^2x^2\right)dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)+\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\text{ar}\right.\right.\right.$$

$\big\downarrow$ 244

$$d^3\left(\int\frac{\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{x}dx+\frac{1}{2}bc\left(\frac{3}{4}\int\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int\left(x-c^2x^3\right)dx+\frac{1}{4}\right.\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\text{ar}\right.\right.\right.$$

$\big\downarrow$ 2009

$$d^3\left(\int\frac{\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{x}dx+\frac{1}{2}bc\left(\frac{3}{4}\int\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\,\text{ar}\right.\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\text{ar}\right.\right.\right.$$

$\big\downarrow$ 5157

$$d^3\left(\int \frac{(1-c^2x^2)(a+b\arccos(cx))^2}{x}dx+\frac{1}{2}bc\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\right.\right.\right.$$
$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\mathrm{ar}\right.\right.\right.$$

$\Big\downarrow$ 15

$$d^3\left(\int \frac{(1-c^2x^2)(a+b\arccos(cx))^2}{x}dx+\frac{1}{2}bc\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\right.\right.\right.$$
$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\mathrm{ar}\right.\right.\right.$$

$\Big\downarrow$ 5153

$$d^3\left(\int \frac{(1-c^2x^2)(a+b\arccos(cx))^2}{x}dx+\frac{1}{4}(1-c^2x^2)^2(a+b\arccos(cx))^2+\frac{1}{2}bc\left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\,\mathrm{arcco}\right.\right.$$
$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\mathrm{ar}\right.\right.\right.$$

$\Big\downarrow$ 5203

$$d^3\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\int \frac{(a+b\arccos(cx))^2}{x}dx+\frac{1}{4}(1-c^2x^2)^2(a+b\arccos(cx))^2+\frac{1}{2}(1-\right.$$
$$\frac{1}{6}d^3(1-c^2x^2)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\mathrm{ar}\right.\right.\right.$$

$\Big\downarrow$ 5137

$$d^3\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{cx}d\arccos(cx)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\,\mathrm{a}$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\mathrm{ar}\right.\right.\right.$$

$\downarrow$ 3042

$$d^3\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-\int (a+b\arccos(cx))^2\tan(\arccos(cx))d\arccos(cx)+\frac{1}{4}\left(1-c^2x^2\right)^2(a\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\mathrm{ar}\right.\right.\right.$$

$\downarrow$ 4202

$$d^3\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))^2}{1+e^{2i\arccos(cx)}}d\arccos(cx)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\mathrm{ar}\right.\right.\right.$$

$\downarrow$ 2620

$$d^3\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\left(ib\int (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\right.\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\mathrm{ar}\right.\right.\right.$$

$\downarrow$ 3011

$$d^3\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\left(ib\left(\frac{1}{2}i\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int \mathrm{Poly}\right.\right.\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\mathrm{ar}\right.\right.\right.$$

$$\downarrow \text{2720}$$

$$d^3\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i a}\right.\right.\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\text{ar}\right.\right.\right.$$

$$\downarrow \text{5157}$$

$$d^3\left(bc\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)+2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i}\right.\right.\right.\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\text{ar}\right.\right.\right.$$

$$\downarrow \text{15}$$

$$d^3\left(bc\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{4}bcx^2\right)+2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos}\right.\right.\right.\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\text{ar}\right.\right.\right.$$

$$\downarrow \text{5153}$$

$$d^3\left(2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)de^{2i}\right.\right.\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\text{ar}\right.\right.\right.$$

$$\downarrow \text{7143}$$

$$d^3\left(\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))^2+bc\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\right.\right.$$
$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2+$$
$$\frac{1}{3}bcd^3\left(\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\mathrm{ar}\right.\right.\right.$$

input
```
Int[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
(d^3*(1 - c^2*x^2)^3*(a + b*ArcCos[c*x])^2)/6 + (b*c*d^3*(-1/36*(b*(1 - c^
2*x^2)^3)/c + (x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/6 + (5*((b*c*(x^
2/2 - (c^2*x^4)/4))/4 + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 + (3
*((b*c*x^2)/4 + (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/2 - (a + b*ArcCo
s[c*x])^2/(4*b*c)))/4))/6))/3 + d^3*(((1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)
/2 + ((1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/4 - ((I/3)*(a + b*ArcCos[c*x]
)^3)/b + b*c*((b*c*x^2)/4 + (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/2 -
(a + b*ArcCos[c*x])^2/(4*b*c)) + (b*c*((b*c*(x^2/2 - (c^2*x^4)/4))/4 + (x*
(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 + (3*((b*c*x^2)/4 + (x*Sqrt[1 -
 c^2*x^2]*(a + b*ArcCos[c*x]))/2 - (a + b*ArcCos[c*x])^2/(4*b*c)))/4))/2 +
 (2*I)*((-1/2*I)*(a + b*ArcCos[c*x])^2*Log[1 + E^((2*I)*ArcCos[c*x])] + I*
b*((I/2)*(a + b*ArcCos[c*x])*PolyLog[2, -E^((2*I)*ArcCos[c*x])] - (b*PolyL
og[3, -E^((2*I)*ArcCos[c*x])])/4)))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.39

method	result
parts	$-d^3a^2\left(\frac{c^6x^6}{6}-\frac{3c^4x^4}{4}+\frac{3c^2x^2}{2}-\ln(x)\right)-d^3b^2\left(\frac{i\arccos(cx)^3}{3}+\frac{29\left(2\arccos(cx)^2-1+2i\arccos(cx)\right)}{256}\right.$
derivativedivides	$-d^3a^2\left(\frac{c^6x^6}{6}-\frac{3c^4x^4}{4}+\frac{3c^2x^2}{2}-\ln(cx)\right)-d^3b^2\left(\frac{i\arccos(cx)^3}{3}+\frac{29\left(2\arccos(cx)^2-1+2i\arccos(cx)\right.}{256}\right.$
default	$-d^3a^2\left(\frac{c^6x^6}{6}-\frac{3c^4x^4}{4}+\frac{3c^2x^2}{2}-\ln(cx)\right)-d^3b^2\left(\frac{i\arccos(cx)^3}{3}+\frac{29\left(2\arccos(cx)^2-1+2i\arccos(cx)\right.}{256}\right.$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
-d^3*a^2*(1/6*c^6*x^6-3/4*c^4*x^4+3/2*c^2*x^2-ln(x))-d^3*b^2*(1/3*I*arccos
(c*x)^3+29/256*(2*arccos(c*x)^2-1+2*I*arccos(c*x))*(2*c^2*x^2-1+2*I*(-c^2*
x^2+1)^(1/2)*c*x)+29/256*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*(2*arcc
os(c*x)^2-1-2*I*arccos(c*x))-arccos(c*x)^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))
^2)+I*arccos(c*x)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1/2*polylog(3,-
(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/3456*(18*arccos(c*x)^2-1)*cos(6*arccos(c*x
))-1/576*arccos(c*x)*sin(6*arccos(c*x))-1/128*(8*arccos(c*x)^2-1)*cos(4*ar
ccos(c*x))+1/32*arccos(c*x)*sin(4*arccos(c*x)))-I*d^3*a*b*arccos(c*x)^2+29
/32*d^3*a*b*(-c^2*x^2+1)^(1/2)*x*c-29/16*d^3*a*b*arccos(c*x)*x^2*c^2+29/32
*d^3*a*b*arccos(c*x)+2*d^3*a*b*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))
^2)-I*d^3*a*b*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1/96*d^3*a*b*arccos
(c*x)*cos(6*arccos(c*x))+1/576*d^3*a*b*sin(6*arccos(c*x))+1/8*d^3*a*b*arcc
os(c*x)*cos(4*arccos(c*x))-1/32*d^3*a*b*sin(4*arccos(c*x))
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^3\,(a+b\arccos(cx))^2}{x}\,dx=\int -\frac{(c^2dx^2-d)^3(b\arccos(cx)+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral(-(a^2*c^6*d^3*x^6 - 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 - a^2*d
^3 + (b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 - b^2*d^3)*a
rccos(c*x)^2 + 2*(a*b*c^6*d^3*x^6 - 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2
- a*b*d^3)*arccos(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x}\,dx$$
$$= -d^3\left(\int \left(-\frac{a^2}{x}\right)\,dx + \int 3a^2c^2x\,dx + \int \left(-3a^2c^4x^3\right)\,dx + \int a^2c^6x^5\,dx + \int \left(-\frac{b^2\operatorname{acos}^2(cx)}{x}\right)\,dx + \int \left(-\frac{2ab\operatorname{acos}(cx)}{x}\right)\,dx + \int 3b^2c^2x\operatorname{acos}^2(cx)\,dx + \int \left(-3b^2c^4x^3\operatorname{acos}^2(cx)\right)\,dx + \int b^2c^6x^5\operatorname{acos}^2(cx)\,dx + \int 6abc^2x\operatorname{acos}(cx)\,dx + \int \left(-6abc^4x^3\operatorname{acos}(cx)\right)\,dx + \int 2abc^6x^5\operatorname{acos}(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acos(c*x))**2/x,x)
```

output
```
-d**3*(Integral(-a**2/x, x) + Integral(3*a**2*c**2*x, x) + Integral(-3*a**
2*c**4*x**3, x) + Integral(a**2*c**6*x**5, x) + Integral(-b**2*acos(c*x)**
2/x, x) + Integral(-2*a*b*acos(c*x)/x, x) + Integral(3*b**2*c**2*x*acos(c*
x)**2, x) + Integral(-3*b**2*c**4*x**3*acos(c*x)**2, x) + Integral(b**2*c*
*6*x**5*acos(c*x)**2, x) + Integral(6*a*b*c**2*x*acos(c*x), x) + Integral(
-6*a*b*c**4*x**3*acos(c*x), x) + Integral(2*a*b*c**6*x**5*acos(c*x), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x}\,dx = \int -\frac{(c^2dx^2-d)^3(b\arccos(cx)+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x,x, algorithm="maxima")
```

output
```
-1/6*a^2*c^6*d^3*x^6 + 3/4*a^2*c^4*d^3*x^4 - 3/2*a^2*c^2*d^3*x^2 + a^2*d^3
*log(x) - integrate(((b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*
x^2 - b^2*d^3)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*(a*b*c^6*d
^3*x^6 - 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2 - a*b*d^3)*arctan2(sqrt(c*x
 + 1)*sqrt(-c*x + 1), c*x))/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^3}{x}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^3)/x,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^3)/x, x)
```

Reduce [F]

$$\int \frac{(d - c^2 d x^2)^3 (a + b \arccos(cx))^2}{x} \, dx$$
$$= \frac{d^3 \left(-108 acos(cx)^2 b^2 c^2 x^2 + 54 acos(cx)^2 b^2 + 108\sqrt{-c^2 x^2 + 1}\, acos(cx)\, b^2 c x - 24 acos(cx)\, ab\, c^6 x^6 + 108 a\right.}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acos(c*x))^2/x,x)
```

output
```
(d**3*( - 108*acos(c*x)**2*b**2*c**2*x**2 + 54*acos(c*x)**2*b**2 + 108*sqr
t( - c**2*x**2 + 1)*acos(c*x)*b**2*c*x - 24*acos(c*x)*a*b*c**6*x**6 + 108*
acos(c*x)*a*b*c**4*x**4 - 216*acos(c*x)*a*b*c**2*x**2 - 75*asin(c*x)*a*b +
 4*sqrt( - c**2*x**2 + 1)*a*b*c**5*x**5 - 22*sqrt( - c**2*x**2 + 1)*a*b*c*
*3*x**3 + 75*sqrt( - c**2*x**2 + 1)*a*b*c*x + 144*int(acos(c*x)/x,x)*a*b +
 72*int(acos(c*x)**2/x,x)*b**2 - 72*int(acos(c*x)**2*x**5,x)*b**2*c**6 + 2
16*int(acos(c*x)**2*x**3,x)*b**2*c**4 + 72*log(x)*a**2 - 12*a**2*c**6*x**6
 + 54*a**2*c**4*x**4 - 108*a**2*c**2*x**2 + 54*b**2*c**2*x**2))/72
```

3.182 $\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 329

$$\begin{aligned}\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))^2}{x^2}\,dx &= \frac{122}{25}b^2c^2d^3x-\frac{14}{75}b^2c^4d^3x^3+\frac{2}{125}b^2c^6d^3x^5\\
&\quad-\frac{22}{5}bcd^3\sqrt{1-c^2x^2}(a+b\arccos(cx))\\
&\quad-\frac{2}{5}bcd^3\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\\
&\quad-\frac{2}{25}bcd^3\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))\\
&\quad-\frac{16}{5}c^2d^3x(a+b\arccos(cx))^2\\
&\quad-\frac{8}{5}c^2d^3x\left(1-c^2x^2\right)(a+b\arccos(cx))^2\\
&\quad-\frac{6}{5}c^2d^3x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2\\
&\quad-\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{x}\\
&\quad-4bcd^3(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)\\
&\quad+2ib^2cd^3\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)\\
&\quad-2ib^2cd^3\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)\end{aligned}$$

output
```
122/25*b^2*c^2*d^3*x-14/75*b^2*c^4*d^3*x^3+2/125*b^2*c^6*d^3*x^5-22/5*b*c*
d^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))-2/5*b*c*d^3*(-c^2*x^2+1)^(3/2)*(a
+b*arccos(c*x))-2/25*b*c*d^3*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))-16/5*c^2
*d^3*x*(a+b*arccos(c*x))^2-8/5*c^2*d^3*x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2-
6/5*c^2*d^3*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2-d^3*(-c^2*x^2+1)^3*(a+b*a
rccos(c*x))^2/x-4*b*c*d^3*(a+b*arccos(c*x))*arctanh(c*x+I*(-c^2*x^2+1)^(1/
2))+2*I*b^2*c*d^3*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-2*I*b^2*c*d^3*polyl
og(2,c*x+I*(-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.40

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^2}\,dx =$$
$$-\frac{d^3\left(6000a^2+18000a^2c^2x^2-28500b^2c^2x^2-6000a^2c^4x^4+1200a^2c^6x^6-29280abcx\sqrt{1-c^2x^2}+3360a\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2)/x^2,x]
```

output
```
-1/6000*(d^3*(6000*a^2 + 18000*a^2*c^2*x^2 - 28500*b^2*c^2*x^2 - 6000*a^2*
c^4*x^4 + 1200*a^2*c^6*x^6 - 29280*a*b*c*x*Sqrt[1 - c^2*x^2] + 3360*a*b*c^
3*x^3*Sqrt[1 - c^2*x^2] - 480*a*b*c^5*x^5*Sqrt[1 - c^2*x^2] + 12000*a*b*Ar
cCos[c*x] + 36000*a*b*c^2*x^2*ArcCos[c*x] - 12000*a*b*c^4*x^4*ArcCos[c*x]
+ 2400*a*b*c^6*x^6*ArcCos[c*x] - 28500*b^2*c*x*Sqrt[1 - c^2*x^2]*ArcCos[c*
x] + 6000*b^2*ArcCos[c*x]^2 + 14250*b^2*c^2*x^2*ArcCos[c*x]^2 - 12000*a*b*
c*x*ArcTanh[Sqrt[1 - c^2*x^2]] + 250*b^2*c*x*Cos[3*ArcCos[c*x]] - 1125*b^2
*c*x*ArcCos[c*x]^2*Cos[3*ArcCos[c*x]] - 6*b^2*c*x*Cos[5*ArcCos[c*x]] + 75*
b^2*c*x*ArcCos[c*x]^2*Cos[5*ArcCos[c*x]] - 12000*b^2*c*x*ArcCos[c*x]*Log[1
 - I*E^(I*ArcCos[c*x])] + 12000*b^2*c*x*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[
c*x])] - (12000*I)*b^2*c*x*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + (12000*I)*
b^2*c*x*PolyLog[2, I*E^(I*ArcCos[c*x])] + 750*b^2*c*x*ArcCos[c*x]*Sin[3*Ar
cCos[c*x]] - 30*b^2*c*x*ArcCos[c*x]*Sin[5*ArcCos[c*x]]))/x
```

Rubi [A] (verified)

Time = 3.35 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.44, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {5201, 27, 5159, 5159, 5131, 5183, 24, 210, 2009, 5203, 210, 2009, 5203, 2009, 5199, 24, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))^2}{x^2}dx$$

$$\Big\downarrow \text{5201}$$

$$-2bcd^3\int \frac{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{x}dx-6c^2d\int d^2(1-c^2x^2)^2(a+b\arccos(cx))^2dx-\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{27}$$

$$-2bcd^3\int \frac{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{x}dx-6c^2d^3\int (1-c^2x^2)^2(a+b\arccos(cx))^2dx-\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{5159}$$

$$-2bcd^3\int \frac{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{x}dx-6c^2d^3\left(\frac{2}{5}bc\int x(1-c^2x^2)^{3/2}(a+b\arccos(cx))dx+\frac{4}{5}\int (1-c^2x^2)(a+b\arccos(cx))^2dx+\frac{1}{5}x(1-c^2x^2)^2(a+\right.$$
$$\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{5159}$$

$$-2bcd^3\int \frac{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{x}dx-6c^2d^3\left(\frac{2}{5}bc\int x(1-c^2x^2)^{3/2}(a+b\arccos(cx))dx+\frac{4}{5}\left(\frac{2}{3}bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{2}{3}\int (a+b\,\text{arc}\right.\right.$$
$$\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{x}$$

$\Big\downarrow$ 5131

$$-6c^2d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(2bc\int\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))dx\right.\right.$$
$$2bcd^3\int\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}dx-\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{x}$$

$\Big\downarrow$ 5183

$$-6c^2d^3\left(\frac{4}{5}\left(\frac{2}{3}\left(2bc\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bc\left(-\frac{b\int(1-c^2x^2}{3c}\right.\right.\right.$$
$$2bcd^3\int\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}dx-\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{x}$$

$\Big\downarrow$ 24

$$-6c^2d^3\left(\frac{4}{5}\left(\frac{2}{3}bc\left(-\frac{b\int\left(1-c^2x^2\right)dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\right.\right.$$
$$2bcd^3\int\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}dx-\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{x}$$

$\Big\downarrow$ 210

$$-2bcd^3\int\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}dx-$$
$$6c^2d^3\left(\frac{4}{5}\left(\frac{2}{3}bc\left(-\frac{b\int\left(1-c^2x^2\right)dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{x}$$

$\Big\downarrow$ 2009

$$-2bcd^3\int\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}dx-\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{x}-$$
$$6c^2d^3\left(\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos}{c^2}\right.\right.\right.\right.$$

$\Big\downarrow$ 5203

$$-2bcd^3\left(\int \frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{x}dx+\frac{1}{5}bc\int (1-c^2x^2)^2dx+\frac{1}{5}(1-c^2x^2)^{5/2}(a+b\arccos(cx))\right)-\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{x}-$$
$$6c^2d^3\left(\frac{1}{5}x(1-c^2x^2)^2(a+b\arccos(cx))^2+\frac{4}{5}\left(\frac{1}{3}x(1-c^2x^2)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\,\text{a}}{c^2}\right.\right.\right.\right.$$

$\downarrow$ 210

$$-2bcd^3\left(\int \frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{x}dx+\frac{1}{5}bc\int (c^4x^4-2c^2x^2+1)dx+\frac{1}{5}(1-c^2x^2)^{5/2}(a+b\arccos(c\right.$$
$$\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{x}-$$
$$6c^2d^3\left(\frac{1}{5}x(1-c^2x^2)^2(a+b\arccos(cx))^2+\frac{4}{5}\left(\frac{1}{3}x(1-c^2x^2)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\,\text{a}}{c^2}\right.\right.\right.\right.$$

$\downarrow$ 2009

$$-2bcd^3\left(\int \frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{x}dx+\frac{1}{5}(1-c^2x^2)^{5/2}(a+b\arccos(cx))+\frac{1}{5}bc\left(\frac{c^4x^5}{5}-\frac{2c^2x^3}{3}+x\right)\right)$$
$$\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{x}-$$
$$6c^2d^3\left(\frac{1}{5}x(1-c^2x^2)^2(a+b\arccos(cx))^2+\frac{4}{5}\left(\frac{1}{3}x(1-c^2x^2)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\,\text{a}}{c^2}\right.\right.\right.\right.$$

$\downarrow$ 5203

$$-2bcd^3\left(\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx+\frac{1}{3}bc\int (1-c^2x^2)dx+\frac{1}{5}(1-c^2x^2)^{5/2}(a+b\arccos(cx))+\frac{1}{3}(1-\right.$$
$$\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{x}-$$
$$6c^2d^3\left(\frac{1}{5}x(1-c^2x^2)^2(a+b\arccos(cx))^2+\frac{4}{5}\left(\frac{1}{3}x(1-c^2x^2)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\,\text{a}}{c^2}\right.\right.\right.\right.$$

$\downarrow$ 2009

$$-2bcd^3\left(\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx+\frac{1}{5}\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos($$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{x}-$$
$$6c^2d^3\left(\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\,a}{c^2}\right.\right.\right.\right.$$

$\big\downarrow$ 5199

$$-2bcd^3\left(\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+bc\int 1dx+\frac{1}{5}\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(c$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{x}-$$
$$6c^2d^3\left(\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\,a}{c^2}\right.\right.\right.\right.$$

$\big\downarrow$ 24

$$-2bcd^3\left(\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+\frac{1}{5}\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\sqrt{1-}\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{x}-$$
$$6c^2d^3\left(\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\,a}{c^2}\right.\right.\right.\right.$$

$\big\downarrow$ 5219

$$-2bcd^3\left(-\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)+\frac{1}{5}\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(c\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{x}-$$
$$6c^2d^3\left(\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\,a}{c^2}\right.\right.\right.\right.$$

$\big\downarrow$ 3042

$$-2bcd^3\left(-\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)+\frac{1}{5}\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{1}{3}(1-c\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{x}-$$
$$6c^2d^3\left(\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\,a}{c^2}\right.\right.\right.\right.$$

$$\Big\downarrow \; 4669$$

$$-2bcd^3\left(b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)-b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)+2i\arctan\left(e^{i\arccos(cx)}\right)\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{x}-$$
$$6c^2d^3\left(\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\,a}{c^2}\right.\right.\right.\right.$$

$$\Big\downarrow \; 2715$$

$$-2bcd^3\left(-ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}+ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(c}\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{x}-$$
$$6c^2d^3\left(\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\,a}{c^2}\right.\right.\right.\right.$$

$$\Big\downarrow \; 2838$$

$$-2bcd^3\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{5}\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\,a\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{x}-$$
$$6c^2d^3\left(\frac{1}{5}x\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{4}{5}\left(\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx))^2+\frac{2}{3}\left(2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\,a}{c^2}\right.\right.\right.\right.$$

input
```
Int[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2)/x^2,x]
```

output
```
-((d^3*(1 - c^2*x^2)^3*(a + b*ArcCos[c*x])^2)/x) - 6*c^2*d^3*((x*(1 - c^2*
x^2)^2*(a + b*ArcCos[c*x])^2)/5 + (2*b*c*(-1/5*(b*(x - (2*c^2*x^3)/3 + (c^
4*x^5)/5))/c - ((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^2)))/5 + (4*
((x*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/3 + (2*b*c*(-1/3*(b*(x - (c^2*x^3
)/3))/c - ((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(3*c^2)))/3 + (2*(x*(a
 + b*ArcCos[c*x])^2 + 2*b*c*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos
[c*x]))/c^2)))/3))/5) - 2*b*c*d^3*(b*c*x + (b*c*(x - (c^2*x^3)/3))/3 + (b*
c*(x - (2*c^2*x^3)/3 + (c^4*x^5)/5))/5 + Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c
*x]) + ((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/3 + ((1 - c^2*x^2)^(5/2)*
(a + b*ArcCos[c*x]))/5 + (2*I)*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcCos[c*x]
)] - I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I*b*PolyLog[2, I*E^(I*ArcCos
[c*x])])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.32

method	result
parts	$-d^3a^2\left(\frac{c^6x^5}{5}-c^4x^3+3c^2x+\frac{1}{x}\right)-d^3b^2c\left(\frac{19\left(\arccos(cx)^2-2+2i\arccos(cx)\right)\left(cx+i\sqrt{-c^2x^2+1}\right)}{16}+\right.$
derivativedivides	$c\left(-d^3a^2\left(\frac{c^5x^5}{5}-c^3x^3+3cx+\frac{1}{cx}\right)-d^3b^2\left(\frac{19\left(\arccos(cx)^2-2+2i\arccos(cx)\right)\left(cx+i\sqrt{-c^2x^2+1}\right)}{16}+\right.\right.$
default	$c\left(-d^3a^2\left(\frac{c^5x^5}{5}-c^3x^3+3cx+\frac{1}{cx}\right)-d^3b^2\left(\frac{19\left(\arccos(cx)^2-2+2i\arccos(cx)\right)\left(cx+i\sqrt{-c^2x^2+1}\right)}{16}+\right.\right.$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-d^3*a^2*(1/5*c^6*x^5-c^4*x^3+3*c^2*x+1/x)-d^3*b^2*c*(19/16*(arccos(c*x)^2
-2+2*I*arccos(c*x))*(c*x+I*(-c^2*x^2+1)^(1/2))+19/16*(-I*(-c^2*x^2+1)^(1/2
)+c*x)*(arccos(c*x)^2-2-2*I*arccos(c*x))+arccos(c*x)^2/c/x+2*arccos(c*x)*l
n(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)
^(1/2)))-2*I*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*I*dilog(1-I*(c*x+I*(-
c^2*x^2+1)^(1/2)))+1/2000*(25*arccos(c*x)^2-2)*cos(5*arccos(c*x))-1/200*ar
ccos(c*x)*sin(5*arccos(c*x))-1/48*(9*arccos(c*x)^2-2)*cos(3*arccos(c*x))+1
/8*arccos(c*x)*sin(3*arccos(c*x)))-2*d^3*a*b*c*(1/5*arccos(c*x)*c^5*x^5-c^
3*x^3*arccos(c*x)+3*c*x*arccos(c*x)+arccos(c*x)/c/x-1/25*c^4*x^4*(-c^2*x^2
+1)^(1/2)+7/25*c^2*x^2*(-c^2*x^2+1)^(1/2)-61/25*(-c^2*x^2+1)^(1/2)-arctanh
(1/(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^2}\,dx = \int -\frac{(c^2dx^2-d)^3(b\arccos(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*c^6*d^3*x^6 - 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 - a^2*d
^3 + (b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 - b^2*d^3)*a
rccos(c*x)^2 + 2*(a*b*c^6*d^3*x^6 - 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2
- a*b*d^3)*arccos(c*x))/x^2, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^2}\,dx = -d^3\left(\int 3a^2c^2\,dx + \int \left(-\frac{a^2}{x^2}\right)\,dx + \int \left(-3a^2c^4x^2\right)\,dx + \int a^2c^6x^4\,dx + \int 3b^2c^2\operatorname{acos}^2(cx)\,dx + \int \left(-\frac{b^2\operatorname{acos}^2(cx)}{x^2}\right)\,dx + \int 6abc^2\operatorname{acos}(cx)\,dx + \int \left(-\frac{2ab\operatorname{acos}(cx)}{x^2}\right)\,dx + \int \left(-3b^2c^4x^2\operatorname{acos}^2(cx)\right)\,dx + \int b^2c^6x^4\operatorname{acos}^2(cx)\,dx + \int \left(-6abc^4x^2\operatorname{acos}(cx)\right)\,dx + \int 2abc^6x^4\operatorname{acos}(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acos(c*x))**2/x**2,x)
```

output
```
-d**3*(Integral(3*a**2*c**2, x) + Integral(-a**2/x**2, x) + Integral(-3*a*
*2*c**4*x**2, x) + Integral(a**2*c**6*x**4, x) + Integral(3*b**2*c**2*acos
(c*x)**2, x) + Integral(-b**2*acos(c*x)**2/x**2, x) + Integral(6*a*b*c**2*
acos(c*x), x) + Integral(-2*a*b*acos(c*x)/x**2, x) + Integral(-3*b**2*c**4
*x**2*acos(c*x)**2, x) + Integral(b**2*c**6*x**4*acos(c*x)**2, x) + Integr
al(-6*a*b*c**4*x**2*acos(c*x), x) + Integral(2*a*b*c**6*x**4*acos(c*x), x)
)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^2}\,dx = \int -\frac{(c^2dx^2-d)^3(b\arccos(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
-1/5*a^2*c^6*d^3*x^5 - 2/75*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^
4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a*b*c^
6*d^3 + a^2*c^4*d^3*x^3 + 2/3*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x
^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*c^4*d^3 - 3*b^2*c^2*d^3*x*arccos(c
*x)^2 + 6*b^2*c^2*d^3*(x + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) - 3*a^2*c^2*d
^3*x - 6*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a*b*c*d^3 + 2*(c*log(2*sqr
t(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*a*b*d^3 - a^2*d^3/x -
1/5*((b^2*c^6*d^3*x^6 - 5*b^2*c^4*d^3*x^4 + 5*b^2*d^3)*arctan2(sqrt(c*x +
1)*sqrt(-c*x + 1), c*x)^2 - 5*x*integrate(2/5*(b^2*c^7*d^3*x^6 - 5*b^2*c^5
*d^3*x^4 + 5*b^2*c*d^3)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x)/(c^2*x^3 - x), x))/x
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^3}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^3)/x^2,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^3)/x^2, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^2}\,dx$$
$$= \frac{d^3\left(-75acos(cx)^2 b^2c^2x^2+150\sqrt{-c^2x^2+1}\,acos(cx)\,b^2cx-10acos(cx)\,ab\,c^6x^6+50acos(cx)\,ab\,c^4x^4-15\right.}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acos(c*x))^2/x^2,x)
```

output
```
(d**3*( - 75*acos(c*x)**2*b**2*c**2*x**2 + 150*sqrt( - c**2*x**2 + 1)*acos
(c*x)*b**2*c*x - 10*acos(c*x)*a*b*c**6*x**6 + 50*acos(c*x)*a*b*c**4*x**4 -
 150*acos(c*x)*a*b*c**2*x**2 - 50*acos(c*x)*a*b + 2*sqrt( - c**2*x**2 + 1)
*a*b*c**5*x**5 - 14*sqrt( - c**2*x**2 + 1)*a*b*c**3*x**3 + 122*sqrt( - c**
2*x**2 + 1)*a*b*c*x + 25*int(acos(c*x)**2/x**2,x)*b**2*x - 25*int(acos(c*x
)**2*x**4,x)*b**2*c**6*x + 75*int(acos(c*x)**2*x**2,x)*b**2*c**4*x - 50*lo
g(tan(asin(c*x)/2))*a*b*c*x - 5*a**2*c**6*x**6 + 25*a**2*c**4*x**4 - 75*a*
*2*c**2*x**2 - 25*a**2 + 150*b**2*c**2*x**2))/(25*x)
```

3.183 $\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 396

$$\begin{aligned}\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))^2}{x^3}\,dx &= -\frac{35}{32}b^2c^4d^3x^2+\frac{1}{4}b^2c^6d^3x^4-\frac{7}{32}b^2c^2d^3(1-c^2x^2)^2 \\ &+\frac{3}{16}bc^3d^3x\sqrt{1-c^2x^2}(a+b\arccos(cx)) \\ &-\frac{7}{8}bc^3d^3x(1-c^2x^2)^{3/2}(a+b\arccos(cx))-\frac{bcd^3(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{x} \\ &+\frac{3}{32}c^2d^3(a+b\arccos(cx))^2-\frac{3}{2}c^2d^3(1-c^2x^2)(a+b\arccos(cx))^2 \\ &-\frac{3}{4}c^2d^3(1-c^2x^2)^2(a+b\arccos(cx))^2-\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{2x^2} \\ &+\frac{ic^2d^3(a+b\arccos(cx))^3}{b}-3c^2d^3(a+b\arccos(cx))^2\log\left(1-e^{2i\arccos(cx)}\right) \\ &+b^2c^2d^3\log(x)+3ibc^2d^3(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right) \\ &-\frac{3}{2}b^2c^2d^3\operatorname{PolyLog}\left(3,e^{2i\arccos(cx)}\right)\end{aligned}$$

output
```
-35/32*b^2*c^4*d^3*x^2+1/4*b^2*c^6*d^3*x^4-7/32*b^2*c^2*d^3*(-c^2*x^2+1)^2
+3/16*b*c^3*d^3*x*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))-7/8*b*c^3*d^3*x*(-c
^2*x^2+1)^(3/2)*(a+b*arccos(c*x))-b*c*d^3*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c
*x))/x+3/32*c^2*d^3*(a+b*arccos(c*x))^2-3/2*c^2*d^3*(-c^2*x^2+1)*(a+b*arcc
os(c*x))^2-3/4*c^2*d^3*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2-1/2*d^3*(-c^2*x^
2+1)^3*(a+b*arccos(c*x))^2/x^2+I*c^2*d^3*(a+b*arccos(c*x))^3/b-3*c^2*d^3*(
a+b*arccos(c*x))^2*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+b^2*c^2*d^3*ln(x)+3*
I*b*c^2*d^3*(a+b*arccos(c*x))*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)-3/2*
b^2*c^2*d^3*polylog(3,(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.31

$$\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))^2}{x^3}\,dx = \frac{d^3\left(-128a^2+384a^2c^4x^4-64a^2c^6x^6+256abcx\sqrt{1-c^2x^2}-336abc^3x^3\sqrt{1-c^2x^2}+32abc^5x^5\sqrt{1-c^2x^2}\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2)/x^3,x]
```

output
```
(d^3*(-128*a^2 + 384*a^2*c^4*x^4 - 64*a^2*c^6*x^6 + 256*a*b*c*x*Sqrt[1 - c
^2*x^2] - 336*a*b*c^3*x^3*Sqrt[1 - c^2*x^2] + 32*a*b*c^5*x^5*Sqrt[1 - c^2*
x^2] - 256*a*b*ArcCos[c*x] + 768*a*b*c^4*x^4*ArcCos[c*x] - 128*a*b*c^6*x^6
*ArcCos[c*x] + 256*b^2*c*x*Sqrt[1 - c^2*x^2]*ArcCos[c*x] - 128*b^2*ArcCos[
c*x]^2 + (768*I)*a*b*c^2*x^2*ArcCos[c*x]^2 + (256*I)*b^2*c^2*x^2*ArcCos[c*
x]^3 + 672*a*b*c^2*x^2*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])] - 80*b^2*c^2
*x^2*Cos[2*ArcCos[c*x]] + 160*b^2*c^2*x^2*ArcCos[c*x]^2*Cos[2*ArcCos[c*x]]
 + b^2*c^2*x^2*Cos[4*ArcCos[c*x]] - 8*b^2*c^2*x^2*ArcCos[c*x]^2*Cos[4*ArcC
os[c*x]] - 1536*a*b*c^2*x^2*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] - 7
68*b^2*c^2*x^2*ArcCos[c*x]^2*Log[1 + E^((2*I)*ArcCos[c*x])] - 768*a^2*c^2*
x^2*Log[x] + 256*b^2*c^2*x^2*Log[c*x] + (768*I)*b*c^2*x^2*(a + b*ArcCos[c*
x])*PolyLog[2, -E^((2*I)*ArcCos[c*x])] - 384*b^2*c^2*x^2*PolyLog[3, -E^((2
*I)*ArcCos[c*x])] - 160*b^2*c^2*x^2*ArcCos[c*x]*Sin[2*ArcCos[c*x]] + 4*b^2
*c^2*x^2*ArcCos[c*x]*Sin[4*ArcCos[c*x]]))/(256*x^2)
```

Rubi [A] (verified)

Time = 3.97 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.41, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.111, Rules used = {5201, 27, 5201, 243, 49, 2009, 5159, 244, 2009, 5157, 15, 5153, 5203, 5159, 244, 2009, 5157, 15, 5153, 5203, 5137, 3042, 4202, 2620, 3011, 2720, 5157, 15, 5153, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2}{x^3}dx$$

$$\Big\downarrow \text{5201}$$

$$-bcd^3\int \frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x^2}dx-3c^2d\int \frac{d^2\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}dx-\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$-bcd^3\int \frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x^2}dx-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}dx-\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{5201}$$

$$-bcd^3\left(-5c^2\int \left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx-bc\int \frac{\left(1-c^2x^2\right)^2}{x}dx-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}\right)-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}dx-\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$-bcd^3\left(-5c^2\int \left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx-\frac{1}{2}bc\int \frac{\left(1-c^2x^2\right)^2}{x^2}dx^2-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}\right)-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}dx-\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{49}$$

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))^2}{x}dx-$$
$$bcd^3\left(-5c^2\int \left(1-c^2x^2\right)^{3/2} (a+b\arccos(cx))dx-\frac{1}{2}bc\int \left(x^2c^4-2c^2+\frac{1}{x^2}\right)dx^2-\frac{\left(1-c^2x^2\right)^{5/2} (a+b\arccos(\right.}{x}$$
$$\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))^2}{2x^2}$$

$\Big\downarrow$ 2009

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))^2}{x}dx-$$
$$bcd^3\left(-5c^2\int \left(1-c^2x^2\right)^{3/2} (a+b\arccos(cx))dx-\frac{\left(1-c^2x^2\right)^{5/2} (a+b\arccos(cx))}{x}-\frac{1}{2}bc\left(\frac{c^4x^4}{2}-2c^2x^2+\log\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))^2}{2x^2}$$

$\Big\downarrow$ 5159

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))^2}{x}dx-$$
$$bcd^3\left(-5c^2\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int x\left(1-c^2x^2\right)dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2} (a+b\arccos(cx))\right)\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))^2}{2x^2}$$

$\Big\downarrow$ 244

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))^2}{x}dx-$$
$$bcd^3\left(-5c^2\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int \left(x-c^2x^3\right)dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2} (a+b\arccos(cx))\right)\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))^2}{2x^2}$$

$\Big\downarrow$ 2009

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))^2}{x}dx-$$
$$bcd^3\left(-5c^2\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2} (a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{x^2}{2}-\frac{c^2x^4}{4}\right)\right)-\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))^2}{2x^2}$$

$\Big\downarrow$ 5157

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))^2}{x}dx-$$
$$bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a$$
$$\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))^2}{2x^2}$$

$\downarrow$ 15

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))^2}{x}dx-$$
$$bcd^3\left(-5c^2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{4}bcx^2\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b$$
$$\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))^2}{2x^2}$$

$\downarrow$ 5153

$$-3c^2d^3\int \frac{\left(1-c^2x^2\right)^2 (a+b\arccos(cx))^2}{x}dx-\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^3\left(-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arc$$

$\downarrow$ 5203

$$-3c^2d^3\left(\frac{1}{2}bc\int \left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\int \frac{\left(1-c^2x^2\right)(a+b\arccos(cx))^2}{x}dx+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b$$
$$\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^3\left(-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arc$$

$\downarrow$ 5159

$$-3c^2d^3\left(\frac{1}{2}bc\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int x\left(1-c^2x^2\right)dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx)\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3 (a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^3\left(-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arc$$

$\downarrow$ 244

$$-3c^2d^3\left(\int \frac{(1-c^2x^2)(a+b\arccos(cx))^2}{x}dx+\frac{1}{2}bc\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int (x-c^2x^3)\,dx\right.\right.$$
$$\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^3\left(-\frac{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\mathrm{arc}\right.\right.\right.$$

$\downarrow$ 2009

$$-3c^2d^3\left(\int \frac{(1-c^2x^2)(a+b\arccos(cx))^2}{x}dx+\frac{1}{2}bc\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}x(1-c^2x^2)^{3/2}(a+\right.\right.$$
$$\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^3\left(-\frac{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\mathrm{arc}\right.\right.\right.$$

$\downarrow$ 5157

$$-3c^2d^3\left(\int \frac{(1-c^2x^2)(a+b\arccos(cx))^2}{x}dx+\frac{1}{2}bc\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(\right.\right.\right.$$
$$\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^3\left(-\frac{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\mathrm{arc}\right.\right.\right.$$

$\downarrow$ 15

$$-3c^2d^3\left(\int \frac{(1-c^2x^2)(a+b\arccos(cx))^2}{x}dx+\frac{1}{2}bc\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(c\right.\right.\right.$$
$$\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^3\left(-\frac{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\mathrm{arc}\right.\right.\right.$$

$\downarrow$ 5153

$$-3c^2d^3\left(\int \frac{(1-c^2x^2)\,(a+b\arccos(cx))^2}{x}dx+\frac{1}{4}(1-c^2x^2)^2\,(a+b\arccos(cx))^2+\frac{1}{2}bc\left(\frac{1}{4}x(1-c^2x^2)^{3/2}\,(a+b\right.\right.$$

$$\frac{d^3(1-c^2x^2)^3\,(a+b\arccos(cx))^2}{2x^2}-$$

$$bcd^3\left(-\frac{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arc\right.\right.\right.$$

$$\Big\downarrow \text{ 5203}$$

$$-3c^2d^3\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\int \frac{(a+b\arccos(cx))^2}{x}dx+\frac{1}{4}(1-c^2x^2)^2\,(a+b\arccos(cx))^2+\frac{1}{2}\right.$$

$$\frac{d^3(1-c^2x^2)^3\,(a+b\arccos(cx))^2}{2x^2}-$$

$$bcd^3\left(-\frac{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arc\right.\right.\right.$$

$$\Big\downarrow \text{ 5137}$$

$$-3c^2d^3\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{cx}d\arccos(cx)+\frac{1}{4}(1-c^2x^2)^2\,(a\right.$$

$$\frac{d^3(1-c^2x^2)^3\,(a+b\arccos(cx))^2}{2x^2}-$$

$$bcd^3\left(-\frac{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arc\right.\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$-3c^2d^3\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx-\int (a+b\arccos(cx))^2\tan(\arccos(cx))d\arccos(cx)+\frac{1}{4}(1-c^2x^2\right.$$

$$\frac{d^3(1-c^2x^2)^3\,(a+b\arccos(cx))^2}{2x^2}-$$

$$bcd^3\left(-\frac{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arc\right.\right.\right.$$

$$\Big\downarrow \text{ 4202}$$

$$-3c^2d^3\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))^2}{1+e^{2i\arccos(cx)}}d\arccos(cx)+\frac{1}{4}(1-c^2x^2)^2\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^3\left(-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arc\right.\right.\right.$$

$\downarrow$ 2620

$$-3c^2d^3\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\left(ib\int (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^3\left(-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arc\right.\right.\right.$$

$\downarrow$ 3011

$$-3c^2d^3\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\right.\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^3\left(-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arc\right.\right.\right.$$

$\downarrow$ 2720

$$-3c^2d^3\left(bc\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int\right.\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{2x^2}-$$
$$bcd^3\left(-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arc\right.\right.\right.$$

$\downarrow$ 5157

$$-3c^2d^3\left(bc\left(\frac{1}{2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)+2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,\right.\right.\right.\right.$$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{2x^2}-$$

$$bcd^3\left(-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\text{arc}\right.\right.\right.$$

$\Big\downarrow$ 15

$$-3c^2d^3\left(bc\left(\frac{1}{2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{4}bcx^2\right)+2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i}\right.\right.\right.\right.$$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{2x^2}-$$

$$bcd^3\left(-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\text{arc}\right.\right.\right.$$

$\Big\downarrow$ 5153

$$-3c^2d^3\left(2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\right.\right.\right.$$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{2x^2}-$$

$$bcd^3\left(-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\text{arc}\right.\right.\right.$$

$\Big\downarrow$ 7143

$$-3c^2d^3\left(\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))^2+bc\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(ca\right.\right.$$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{2x^2}-$$

$$bcd^3\left(-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{x}-5c^2\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\,\text{arc}\right.\right.\right.$$

input
```
Int[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2)/x^3,x]
```

output

```
-1/2*(d^3*(1 - c^2*x^2)^3*(a + b*ArcCos[c*x])^2)/x^2 - b*c*d^3*(-(((1 - c^
2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/x) - 5*c^2*((b*c*(x^2/2 - (c^2*x^4)/4))/
4 + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 + (3*((b*c*x^2)/4 + (x*S
qrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/2 - (a + b*ArcCos[c*x])^2/(4*b*c)))/
4) - (b*c*(-2*c^2*x^2 + (c^4*x^4)/2 + Log[x^2]))/2) - 3*c^2*d^3*(((1 - c^2
*x^2)*(a + b*ArcCos[c*x])^2)/2 + ((1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/4
 - ((I/3)*(a + b*ArcCos[c*x])^3)/b + b*c*((b*c*x^2)/4 + (x*Sqrt[1 - c^2*x^
2]*(a + b*ArcCos[c*x]))/2 - (a + b*ArcCos[c*x])^2/(4*b*c)) + (b*c*((b*c*(x
^2/2 - (c^2*x^4)/4))/4 + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 + (
3*((b*c*x^2)/4 + (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/2 - (a + b*ArcC
os[c*x])^2/(4*b*c)))/4))/2 + (2*I)*((-1/2*I)*(a + b*ArcCos[c*x])^2*Log[1 +
 E^((2*I)*ArcCos[c*x])] + I*b*((I/2)*(a + b*ArcCos[c*x])*PolyLog[2, -E^((2
*I)*ArcCos[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/4)))
```

Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 244

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.46

method	result
parts	$-d^3a^2\left(\frac{c^6x^4}{4}-\frac{3c^4x^2}{2}+3c^2\ln\left(x\right)+\frac{1}{2x^2}\right)-d^3b^2c^2\left(-i\arccos\left(cx\right)^3-\frac{5\left(2\arccos(cx)^2-1+2i\right.}{}\right.$
derivativedivides	$c^2\left(-d^3a^2\left(\frac{c^4x^4}{4}-\frac{3c^2x^2}{2}+\frac{1}{2c^2x^2}+3\ln\left(cx\right)\right)-d^3b^2\left(-i\arccos\left(cx\right)^3-\frac{5\left(2\arccos(cx)^2-1+\right.}{}\right.\right.$
default	$c^2\left(-d^3a^2\left(\frac{c^4x^4}{4}-\frac{3c^2x^2}{2}+\frac{1}{2c^2x^2}+3\ln\left(cx\right)\right)-d^3b^2\left(-i\arccos\left(cx\right)^3-\frac{5\left(2\arccos(cx)^2-1+\right.}{}\right.\right.$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-d^3*a^2*(1/4*c^6*x^4-3/2*c^4*x^2+3*c^2*ln(x)+1/2/x^2)-d^3*b^2*c^2*(-I*arc
cos(c*x)^3-5/32*(2*arccos(c*x)^2-1+2*I*arccos(c*x))*(2*c^2*x^2-1+2*I*(-c^2
*x^2+1)^(1/2)*c*x)-5/32*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*(2*arcco
s(c*x)^2-1-2*I*arccos(c*x))+1/2*arccos(c*x)*(-2*I*c^2*x^2-2*c*x*(-c^2*x^2+
1)^(1/2)+arccos(c*x))/c^2/x^2-ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+2*ln(c*x+
I*(-c^2*x^2+1)^(1/2))+3*arccos(c*x)^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-3
*I*arccos(c*x)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3/2*polylog(3,-(c*
x+I*(-c^2*x^2+1)^(1/2))^2)+1/256*(8*arccos(c*x)^2-1)*cos(4*arccos(c*x))-1/
64*arccos(c*x)*sin(4*arccos(c*x)))-2*d^3*a*b*c^2*(-3/2*I*arccos(c*x)^2-5/3
2*(I+2*arccos(c*x))*(2*c^2*x^2-1+2*I*(-c^2*x^2+1)^(1/2)*c*x)-5/32*(-2*I*(-
c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*(-I+2*arccos(c*x))+1/2*(-I*c^2*x^2-c*x*(
-c^2*x^2+1)^(1/2)+arccos(c*x))/c^2/x^2+3*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2
+1)^(1/2))^2)-3/2*I*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/32*arccos(c
*x)*cos(4*arccos(c*x))-1/128*sin(4*arccos(c*x)))
```

Fricas [F]

$$\int \frac{(d - c^2dx^2)^3 (a + b\arccos(cx))^2}{x^3} \, dx = \int -\frac{(c^2dx^2 - d)^3(b\arccos(cx) + a)^2}{x^3} \, dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral(-(a^2*c^6*d^3*x^6 - 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 - a^2*d
^3 + (b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 - b^2*d^3)*a
rccos(c*x)^2 + 2*(a*b*c^6*d^3*x^6 - 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2
- a*b*d^3)*arccos(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d - c^2dx^2)^3 (a + b\arccos(cx))^2}{x^3} \, dx = -d^3\left(\int \left(-\frac{a^2}{x^3}\right) dx + \int \frac{3a^2c^2}{x} \, dx + \int \left(-3a^2c^4x\right) dx + \int a^2c^6x^3 \, dx + \int \left(-\frac{b^2 \operatorname{acos}^2(cx)}{x^3}\right) dx + \int \left(-\frac{2ab \operatorname{acos}(cx)}{x^3}\right) dx + \int \frac{3b^2c^2 \operatorname{acos}^2(cx)}{x} \, dx + \int \left(-3b^2c^4x \operatorname{acos}^2(cx)\right) dx + \int b^2c^6x^3 \operatorname{acos}^2(cx) \, dx + \int \frac{6abc^2 \operatorname{acos}(cx)}{x} \, dx + \int \left(-6abc^4x \operatorname{acos}(cx)\right) dx + \int 2abc^6x^3 \operatorname{acos}(cx) \, dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acos(c*x))**2/x**3,x)
```

output
```
-d**3*(Integral(-a**2/x**3, x) + Integral(3*a**2*c**2/x, x) + Integral(-3*
a**2*c**4*x, x) + Integral(a**2*c**6*x**3, x) + Integral(-b**2*acos(c*x)**
2/x**3, x) + Integral(-2*a*b*acos(c*x)/x**3, x) + Integral(3*b**2*c**2*aco
s(c*x)**2/x, x) + Integral(-3*b**2*c**4*x*acos(c*x)**2, x) + Integral(b**2
*c**6*x**3*acos(c*x)**2, x) + Integral(6*a*b*c**2*acos(c*x)/x, x) + Integr
al(-6*a*b*c**4*x*acos(c*x), x) + Integral(2*a*b*c**6*x**3*acos(c*x), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^3}\,dx = \int -\frac{(c^2dx^2-d)^3(b\arccos(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-1/4*a^2*c^6*d^3*x^4 + 3/2*a^2*c^4*d^3*x^2 - 3*a^2*c^2*d^3*log(x) + a*b*d^
3*(sqrt(-c^2*x^2 + 1)*c/x - arccos(c*x)/x^2) - 1/2*a^2*d^3/x^2 - integrate
(((b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 - b^2*d^3)*arct
an2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*(a*b*c^6*d^3*x^6 - 3*a*b*c^4*
d^3*x^4 + 3*a*b*c^2*d^3*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/x
^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^3}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^3}{x^3}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^3)/x^3,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^3)/x^3, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^3}\,dx = \frac{d^3\Big(24acos(cx)^2\,b^2c^4x^4-12acos(cx)^2\,b^2c^2x^2-24\sqrt{-c^2x^2+1}\,acos(cx)\,b^2c^3x^3-8acos(cx)\,ab\,c^6x^6+48a$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acos(c*x))^2/x^3,x)
```

output
```
(d**3*(24*acos(c*x)**2*b**2*c**4*x**4 - 12*acos(c*x)**2*b**2*c**2*x**2 - 2
4*sqrt( - c**2*x**2 + 1)*acos(c*x)*b**2*c**3*x**3 - 8*acos(c*x)*a*b*c**6*x
**6 + 48*acos(c*x)*a*b*c**4*x**4 - 16*acos(c*x)*a*b + 21*asin(c*x)*a*b*c**
2*x**2 + 2*sqrt( - c**2*x**2 + 1)*a*b*c**5*x**5 - 21*sqrt( - c**2*x**2 + 1
)*a*b*c**3*x**3 + 16*sqrt( - c**2*x**2 + 1)*a*b*c*x - 96*int(acos(c*x)/x,x
)*a*b*c**2*x**2 + 16*int(acos(c*x)**2/x**3,x)*b**2*x**2 - 48*int(acos(c*x)
**2/x,x)*b**2*c**2*x**2 - 16*int(acos(c*x)**2*x**3,x)*b**2*c**6*x**2 - 48*
log(x)*a**2*c**2*x**2 - 4*a**2*c**6*x**6 + 24*a**2*c**4*x**4 - 8*a**2 - 12
*b**2*c**4*x**4))/(16*x**2)
```

3.184 $\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 348

$$\begin{aligned}\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))^2}{x^4}\,dx = & -\frac{b^2c^2d^3}{3x}-\frac{50}{9}b^2c^4d^3x+\frac{2}{27}b^2c^6d^3x^3 \\ & +5bc^3d^3\sqrt{1-c^2x^2}(a+b\arccos(cx)) \\ & -\frac{1}{9}bc^3d^3\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx)) \\ & -\frac{bcd^3\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{3x^2} \\ & +\frac{16}{3}c^4d^3x(a+b\arccos(cx))^2 \\ & +\frac{8}{3}c^4d^3x\left(1-c^2x^2\right)(a+b\arccos(cx))^2 \\ & +\frac{2c^2d^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x} \\ & -\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3} \\ & +\frac{34}{3}bc^3d^3(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right) \\ & -\frac{17}{3}ib^2c^3d^3\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right) \\ & +\frac{17}{3}ib^2c^3d^3\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)\end{aligned}$$

output
```
-1/3*b^2*c^2*d^3/x-50/9*b^2*c^4*d^3*x+2/27*b^2*c^6*d^3*x^3+5*b*c^3*d^3*(-c
^2*x^2+1)^(1/2)*(a+b*arccos(c*x))-1/9*b*c^3*d^3*(-c^2*x^2+1)^(3/2)*(a+b*ar
ccos(c*x))-1/3*b*c*d^3*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))/x^2+16/3*c^4*d
^3*x*(a+b*arccos(c*x))^2+8/3*c^4*d^3*x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2+2*
c^2*d^3*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2/x-1/3*d^3*(-c^2*x^2+1)^3*(a+b*a
rccos(c*x))^2/x^3+34/3*b*c^3*d^3*(a+b*arccos(c*x))*arctanh(c*x+I*(-c^2*x^2
+1)^(1/2))-17/3*I*b^2*c^3*d^3*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+17/3*I*
b^2*c^3*d^3*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.41

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^4}\,dx = -\frac{d^3\left(9a^2-81a^2c^2x^2+9b^2c^2x^2-81a^2c^4x^4+150b^2c^4x^4+9a^2c^6x^6-2b^2c^6x^6-9abcx\sqrt{1-c^2x^2}+150a\right.}{}$$

input
```
Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2)/x^4,x]
```

output
```
-1/27*(d^3*(9*a^2 - 81*a^2*c^2*x^2 + 9*b^2*c^2*x^2 - 81*a^2*c^4*x^4 + 150*
b^2*c^4*x^4 + 9*a^2*c^6*x^6 - 2*b^2*c^6*x^6 - 9*a*b*c*x*Sqrt[1 - c^2*x^2]
+ 150*a*b*c^3*x^3*Sqrt[1 - c^2*x^2] - 6*a*b*c^5*x^5*Sqrt[1 - c^2*x^2] + 18
*a*b*ArcCos[c*x] - 162*a*b*c^2*x^2*ArcCos[c*x] - 162*a*b*c^4*x^4*ArcCos[c*
x] + 18*a*b*c^6*x^6*ArcCos[c*x] - 9*b^2*c*x*Sqrt[1 - c^2*x^2]*ArcCos[c*x]
+ 150*b^2*c^3*x^3*Sqrt[1 - c^2*x^2]*ArcCos[c*x] - 6*b^2*c^5*x^5*Sqrt[1 - c
^2*x^2]*ArcCos[c*x] + 9*b^2*ArcCos[c*x]^2 - 81*b^2*c^2*x^2*ArcCos[c*x]^2 -
 81*b^2*c^4*x^4*ArcCos[c*x]^2 + 9*b^2*c^6*x^6*ArcCos[c*x]^2 + 153*a*b*c^3*
x^3*ArcTanh[Sqrt[1 - c^2*x^2]] + 153*b^2*c^3*x^3*ArcCos[c*x]*Log[1 - I*E^(
I*ArcCos[c*x])] - 153*b^2*c^3*x^3*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])]
 + (153*I)*b^2*c^3*x^3*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - (153*I)*b^2*c^
3*x^3*PolyLog[2, I*E^(I*ArcCos[c*x])]))/x^3
```

Rubi [A] (verified)

Time = 3.40 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.62, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.704$, Rules used = {5201, 27, 5201, 244, 2009, 5159, 5131, 5183, 24, 2009, 5203, 2009, 5199, 24, 5219, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^4}dx$$

$$\Big\downarrow \text{ 5201}$$

$$-\frac{2}{3}bcd^3\int \frac{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{x^3}dx-2c^2d\int \frac{d^2(1-c^2x^2)^2(a+b\arccos(cx))^2}{x^2}dx-\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 27}$$

$$-2c^2d^3\int \frac{(1-c^2x^2)^2(a+b\arccos(cx))^2}{x^2}dx-\frac{2}{3}bcd^3\int \frac{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{x^3}dx-\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 5201}$$

$$-\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\int \frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{1}{2}bc\int \frac{(1-c^2x^2)^2}{x^2}dx-\frac{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{2x^2}\right.$$
$$2c^2d^3\left(-4c^2\int (1-c^2x^2)(a+b\arccos(cx))^2dx-2bc\int \frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{(1-c^2x^2)^2(a+\right.}{x}$$
$$\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 244}$$

$$-2c^2d^3\left(-4c^2\int\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx-2bc\int\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{\left(1-c^2x^2\right)^2(a\cdots}{\cdots}\right.$$
$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\int\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{1}{2}bc\int\left(x^2c^4-2c^2+\frac{1}{x^2}\right)dx-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos\cdots}{2x^2}\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 2009

$$-2c^2d^3\left(-4c^2\int\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx-2bc\int\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{\left(1-c^2x^2\right)^2(a\cdots}{\cdots}\right.$$
$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\int\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\cdots\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 5159

$$-2c^2d^3\left(-4c^2\left(\frac{2}{3}bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{2}{3}\int(a+b\arccos(cx))^2dx+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos\cdots\right.\right.$$
$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\int\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\cdots\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 5131

$$-2c^2d^3\left(-4c^2\left(\frac{2}{3}\left(2bc\int\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bc\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))\cdots\right.\right.$$
$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\int\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\cdots\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 5183

$$-2c^2d^3\left(-4c^2\left(\frac{2}{3}\left(2bc\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)+x(a+b\arccos(cx))^2\right)+\frac{2}{3}bc\left(-\frac{b\int (1-c\right.\right.\right.$$
$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\int \frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 24

$$-2c^2d^3\left(-4c^2\left(\frac{2}{3}bc\left(-\frac{b\int \left(1-c^2x^2\right)dx}{3c}-\frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{3c^2}\right)+\frac{1}{3}x\left(1-c^2x^2\right)(a+b\arccos(cx)\right.\right.$$
$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\int \frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 2009

$$-2c^2d^3\left(-2bc\int \frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))^2}{x}-4c^2\left(\frac{1}{3}x\left(1-c^2x^2\right)(a\right.\right.$$
$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\int \frac{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{x}dx-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(\frac{c^4x^3}{3}-2c^2x-\frac{1}{x}\right)\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 5203

$$-2c^2d^3\left(-2bc\left(\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx+\frac{1}{3}bc\int \left(1-c^2x^2\right)dx+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)\right.$$
$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\left(\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx+\frac{1}{3}bc\int \left(1-c^2x^2\right)dx+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)-\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 2009

$$-2c^2d^3\left(-2bc\left(\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{3}bc\left(x-\frac{c^2x^3}{3}\right)\right)-\frac{(1}{}\right.$$
$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\left(\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}dx+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{3}bc\left(x-\frac{c^2x^3}{3}\right)\right)-\frac{(1}{}\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 5199

$$-2c^2d^3\left(-2bc\left(\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+bc\int 1dx+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\arccos\right.\right.$$
$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\left(\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+bc\int 1dx+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\arccos(\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 24

$$-2c^2d^3\left(-2bc\left(\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{3}bc\right.\right.$$
$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\left(\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{3}bc\left(\right.\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 5219

$$-2c^2d^3\left(-2bc\left(-\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\arccos\right.\right.$$
$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\left(-\int \frac{a+b\arccos(cx)}{cx}d\arccos(cx)+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\arccos(\right.\right.$$
$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 3042

$$-2c^2d^3\left(-2bc\left(-\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\right.\right.$$

$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\left(-\int (a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\sqrt{}\right.\right.$$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 4669$$

$$-2c^2d^3\left(-2bc\left(b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)-b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)+2i\arctan\left(e^{i\arccos(cx)}\right)\right.\right.$$

$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\left(b\int \log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)-b\int \log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)+2i\arctan\left(e^{i\arccos(cx)}\right)\right.\right.$$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2715$$

$$-2c^2d^3\left(-2bc\left(-ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}+ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right.\right.$$

$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\left(-ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}+ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right.\right.$$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2838$$

$$-2c^2d^3\left(-2bc\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\right.\right.$$

$$\frac{2}{3}bcd^3\left(-\frac{5}{2}c^2\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\right.\right.$$

$$\frac{d^3\left(1-c^2x^2\right)^3(a+b\arccos(cx))^2}{3x^3}$$

input
```
Int[((d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2)/x^4,x]
```

output
```
-1/3*(d^3*(1 - c^2*x^2)^3*(a + b*ArcCos[c*x])^2)/x^3 - 2*c^2*d^3*(-(((1 -
c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/x) - 4*c^2*((x*(1 - c^2*x^2)*(a + b*ArcC
os[c*x])^2)/3 + (2*b*c*(-1/3*(b*(x - (c^2*x^3)/3))/c - ((1 - c^2*x^2)^(3/2
)*(a + b*ArcCos[c*x]))/(3*c^2)))/3 + (2*(x*(a + b*ArcCos[c*x])^2 + 2*b*c*(
-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c^2)))/3) - 2*b*c*(b*
c*x + (b*c*(x - (c^2*x^3)/3))/3 + Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]) +
((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/3 + (2*I)*(a + b*ArcCos[c*x])*Ar
cTan[E^(I*ArcCos[c*x])] - I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I*b*Pol
yLog[2, I*E^(I*ArcCos[c*x])])) - (2*b*c*d^3*(-1/2*(b*c*(-x^(-1) - 2*c^2*x
+ (c^4*x^3)/3)) - ((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(2*x^2) - (5*c
^2*(b*c*x + (b*c*(x - (c^2*x^3)/3))/3 + Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*
x]) + ((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/3 + (2*I)*(a + b*ArcCos[c*
x])*ArcTan[E^(I*ArcCos[c*x])] - I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I
*b*PolyLog[2, I*E^(I*ArcCos[c*x])]))/2))/3
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.45

method	result
derivativedivides	$c^3\left(-d^3a^2\left(\frac{c^3x^3}{3}-3cx+\frac{1}{3c^3x^3}-\frac{3}{cx}\right)-\frac{d^3b^2}{3cx}-\frac{50d^3b^2cx}{9}+\frac{2d^3b^2c^3x^3}{27}+\frac{2d^3b^2\arccos(cx)\sqrt{-c^2x^2}}{9}\right.$
default	$c^3\left(-d^3a^2\left(\frac{c^3x^3}{3}-3cx+\frac{1}{3c^3x^3}-\frac{3}{cx}\right)-\frac{d^3b^2}{3cx}-\frac{50d^3b^2cx}{9}+\frac{2d^3b^2c^3x^3}{27}+\frac{2d^3b^2\arccos(cx)\sqrt{-c^2x^2}}{9}\right.$
parts	$-d^3a^2\left(\frac{c^6x^3}{3}-3c^4x-\frac{3c^2}{x}+\frac{1}{3x^3}\right)+\frac{d^3b^2c\sqrt{-c^2x^2+1}\arccos(cx)}{3x^2}+\frac{2b^2c^6d^3x^3}{27}-\frac{50b^2c^4d^3x}{9}-\frac{b^2c^2d}{3x}$

input
```
int((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-d^3*a^2*(1/3*c^3*x^3-3*c*x+1/3/c^3/x^3-3/c/x)-1/3*d^3*b^2/c/x-50/9*d
^3*b^2*c*x+2/27*d^3*b^2*c^3*x^3+2/9*d^3*b^2*arccos(c*x)*(-c^2*x^2+1)^(1/2)
*c^2*x^2+17/3*I*d^3*b^2*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-17/3*d^3*b^2
*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+3*d^3*b^2/c/x*arccos(c*x)^
2-1/3*d^3*b^2/c^3/x^3*arccos(c*x)^2-1/3*d^3*b^2*arccos(c*x)^2*c^3*x^3+3*d^
3*b^2*arccos(c*x)^2*c*x+17/3*d^3*b^2*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1
)^(1/2)))-17/3*I*d^3*b^2*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+1/3*d^3*b^2
/c^2/x^2*arccos(c*x)*(-c^2*x^2+1)^(1/2)-50/9*d^3*b^2*arccos(c*x)*(-c^2*x^2
+1)^(1/2)-2*d^3*a*b*(1/3*c^3*x^3*arccos(c*x)-3*c*x*arccos(c*x)+1/3*arccos(
c*x)/c^3/x^3-3*arccos(c*x)/c/x-1/6/c^2/x^2*(-c^2*x^2+1)^(1/2)+17/6*arctanh
(1/(-c^2*x^2+1)^(1/2))-1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)+25/9*(-c^2*x^2+1)^(1
/2)))
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))^2}{x^4}\,dx=\int -\frac{(c^2dx^2-d)^3(b\arccos(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
integral(-(a^2*c^6*d^3*x^6 - 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 - a^2*d
^3 + (b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 - b^2*d^3)*a
rccos(c*x)^2 + 2*(a*b*c^6*d^3*x^6 - 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2
- a*b*d^3)*arccos(c*x))/x^4, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^3(a+b\arccos(cx))^2}{x^4}\,dx = -d^3\left(\int\left(-3a^2c^4\right)dx+\int\left(-\frac{a^2}{x^4}\right)dx+\int\frac{3a^2c^2}{x^2}\,dx+\int a^2c^6x^2\,dx+\int\left(-3b^2c^4\operatorname{acos}^2(cx)\right)dx+\int\left(-\frac{b^2\operatorname{acos}^2(cx)}{x^4}\right)dx+\int\left(-6abc^4\operatorname{acos}(cx)\right)dx+\int\left(-\frac{2ab\operatorname{acos}(cx)}{x^4}\right)dx+\int\frac{3b^2c^2\operatorname{acos}^2(cx)}{x^2}\,dx+\int b^2c^6x^2\operatorname{acos}^2(cx)\,dx+\int\frac{6abc^2\operatorname{acos}(cx)}{x^2}\,dx+\int 2abc^6x^2\operatorname{acos}(cx)\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**3*(a+b*acos(c*x))**2/x**4,x)
```

output
```
-d**3*(Integral(-3*a**2*c**4, x) + Integral(-a**2/x**4, x) + Integral(3*a*
*2*c**2/x**2, x) + Integral(a**2*c**6*x**2, x) + Integral(-3*b**2*c**4*aco
s(c*x)**2, x) + Integral(-b**2*acos(c*x)**2/x**4, x) + Integral(-6*a*b*c**
4*acos(c*x), x) + Integral(-2*a*b*acos(c*x)/x**4, x) + Integral(3*b**2*c**
2*acos(c*x)**2/x**2, x) + Integral(b**2*c**6*x**2*acos(c*x)**2, x) + Integ
ral(6*a*b*c**2*acos(c*x)/x**2, x) + Integral(2*a*b*c**6*x**2*acos(c*x), x)
)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^4}\,dx = \int -\frac{(c^2dx^2-d)^3(b\arccos(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*a^2*c^6*d^3*x^3 - 2/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/
c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*c^6*d^3 + 3*b^2*c^4*d^3*x*arccos(c*x)
^2 - 6*b^2*c^4*d^3*(x + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + 3*a^2*c^4*d^3*
x + 6*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a*b*c^3*d^3 - 6*(c*log(2*sqrt
(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*a*b*c^2*d^3 + 1/3*((c^2
*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)/x^2)*c -
 2*arccos(c*x)/x^3)*a*b*d^3 + 3*a^2*c^2*d^3/x - 1/3*a^2*d^3/x^3 + 1/3*(3*x
^3*integrate(2/3*(b^2*c^7*d^3*x^6 - 9*b^2*c^3*d^3*x^2 + b^2*c*d^3)*sqrt(c*
x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^2*x^5
- x^3), x) - (b^2*c^6*d^3*x^6 - 9*b^2*c^2*d^3*x^2 + b^2*d^3)*arctan2(sqrt(
c*x + 1)*sqrt(-c*x + 1), c*x)^2)/x^3
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2/x^4,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^4} \, dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2 \,(d-c^2\,d\,x^2)^3}{x^4} \, dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^3)/x^4,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^3)/x^4, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^3 (a+b\arccos(cx))^2}{x^4} \, dx$$
$$= \frac{d^3\left(27acos(cx)^2 b^2c^4x^4 - 54\sqrt{-c^2x^2+1}\,acos(cx)\,b^2c^3x^3 - 6acos(cx)\,ab\,c^6x^6 + 54acos(cx)\,ab\,c^4x^4 + 54a\right.}{}$$

input
```
int((-c^2*d*x^2+d)^3*(a+b*acos(c*x))^2/x^4,x)
```

output
```
(d**3*(27*acos(c*x)**2*b**2*c**4*x**4 - 54*sqrt( - c**2*x**2 + 1)*acos(c*x
)*b**2*c**3*x**3 - 6*acos(c*x)*a*b*c**6*x**6 + 54*acos(c*x)*a*b*c**4*x**4
+ 54*acos(c*x)*a*b*c**2*x**2 - 6*acos(c*x)*a*b + 2*sqrt( - c**2*x**2 + 1)*
a*b*c**5*x**5 - 50*sqrt( - c**2*x**2 + 1)*a*b*c**3*x**3 + 3*sqrt( - c**2*x
**2 + 1)*a*b*c*x + 9*int(acos(c*x)**2/x**4,x)*b**2*x**3 - 27*int(acos(c*x)
**2/x**2,x)*b**2*c**2*x**3 - 9*int(acos(c*x)**2*x**2,x)*b**2*c**6*x**3 + 5
1*log(tan(asin(c*x)/2))*a*b*c**3*x**3 - 3*a**2*c**6*x**6 + 27*a**2*c**4*x*
*4 + 27*a**2*c**2*x**2 - 3*a**2 - 54*b**2*c**4*x**4))/(9*x**3)
```

3.185 $\int \frac{x^4(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 297

$$\int \frac{x^4(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \frac{22b^2x}{9c^4d}+\frac{2b^2x^3}{27c^2d}-\frac{22b\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^5d} - \frac{2bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^3d} - \frac{x(a+b\arccos(cx))^2}{c^4d}-\frac{x^3(a+b\arccos(cx))^2}{3c^2d} - \frac{2i(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)}{c^5d} + \frac{2ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c^5d} - \frac{2ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c^5d} - \frac{2b^2\operatorname{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)}{c^5d} + \frac{2b^2\operatorname{PolyLog}\left(3,ie^{i\arccos(cx)}\right)}{c^5d}$$

output
```
22/9*b^2*x/c^4/d+2/27*b^2*x^3/c^2/d-22/9*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(
c*x))/c^5/d-2/9*b*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3/d-x*(a+b*ar
ccos(c*x))^2/c^4/d-1/3*x^3*(a+b*arccos(c*x))^2/c^2/d-2*I*(a+b*arccos(c*x))
^2*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c^5/d+2*I*b*(a+b*arccos(c*x))*polylog(
2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^5/d-2*I*b*(a+b*arccos(c*x))*polylog(2,I
*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^5/d-2*b^2*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^
(1/2)))/c^5/d+2*b^2*polylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^5/d
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.31

$$\int \frac{x^4(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$$
$$= \frac{-108a^2cx+270b^2cx-36a^2c^3x^3+264ab\sqrt{1-c^2x^2}+24abc^2x^2\sqrt{1-c^2x^2}-216abcx\arccos(cx)-72ab}{}$$

input
```
Integrate[(x^4*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2),x]
```

output
```
(-108*a^2*c*x + 270*b^2*c*x - 36*a^2*c^3*x^3 + 264*a*b*Sqrt[1 - c^2*x^2] +
 24*a*b*c^2*x^2*Sqrt[1 - c^2*x^2] - 216*a*b*c*x*ArcCos[c*x] - 72*a*b*c^3*x
^3*ArcCos[c*x] + 270*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x] - 135*b^2*c*x*ArcCo
s[c*x]^2 + 2*b^2*Cos[3*ArcCos[c*x]] - 9*b^2*ArcCos[c*x]^2*Cos[3*ArcCos[c*x
]] - 216*a*b*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] - 108*b^2*ArcCos[c*x]^
2*Log[1 - E^(I*ArcCos[c*x])] + 216*a*b*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x
])] + 108*b^2*ArcCos[c*x]^2*Log[1 + E^(I*ArcCos[c*x])] - 54*a^2*Log[1 - c*
x] + 54*a^2*Log[1 + c*x] - (216*I)*b*(a + b*ArcCos[c*x])*PolyLog[2, -E^(I*
ArcCos[c*x])] + (216*I)*b*(a + b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])
] + 216*b^2*PolyLog[3, -E^(I*ArcCos[c*x])] - 216*b^2*PolyLog[3, E^(I*ArcCo
s[c*x])] + 6*b^2*ArcCos[c*x]*Sin[3*ArcCos[c*x]])/(108*c^5*d)
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5211, 27, 5211, 15, 5165, 3042, 4671, 3011, 2720, 5183, 24, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arccos(cx))^2}{d-c^2dx^2}dx$$

$$\Big\downarrow \text{5211}$$

$$\frac{\int \frac{x^2(a+b\arccos(cx))^2}{d(1-c^2x^2)}dx}{c^2}-\frac{2b\int \frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3cd}-\frac{x^3(a+b\arccos(cx))^2}{3c^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^2(a+b\arccos(cx))^2}{1-c^2x^2}dx}{c^2d}-\frac{2b\int \frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3cd}-\frac{x^3(a+b\arccos(cx))^2}{3c^2d}$$

$$\Big\downarrow \text{5211}$$

$$-\frac{2b\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{b\int x^2dx}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}\right)}{3cd}+\frac{-\frac{2b\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{c}+\frac{\int \frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\arccos(cx))^2}{c^2}}{c^2d}-\frac{x^3(a+b\arccos(cx))^2}{3c^2d}$$

$$\Big\downarrow \text{15}$$

$$\frac{-\frac{2b\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{c}+\frac{\int \frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\arccos(cx))^2}{c^2}}{c^2d}-\frac{2b\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}-\frac{x^3(a+b\arccos(cx))^2}{3c^2d}$$

$$\Big\downarrow \text{5165}$$

$$-\frac{2b\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}+\frac{-\frac{2b\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{c}-\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^3}-\frac{x(a+b\arccos(cx))^2}{c^2}}{c^2d}-\frac{x^3(a+b\arccos(cx))^2}{3c^2d}$$

$\downarrow$ 3042

$$-\frac{2b\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}+\frac{-\frac{\int (a+b\arccos(cx))^2\csc(\arccos(cx))d\arccos(cx)}{c^3}-\frac{2b\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{c}-\frac{x(a+b\arccos(cx))^2}{c^2}}{c^2d}-\frac{x^3(a+b\arccos(cx))^2}{3c^2d}$$

$\downarrow$ 4671

$$-\frac{\frac{-2b\int (a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\cdots}{c^3}}{c^2d}$$

$$\frac{2b\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}-\frac{x^3(a+b\arccos(cx))^2}{3c^2d}$$

$\downarrow$ 3011

$$-\frac{\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a+b\arccos(cx))-i\cdots\right.}{c^3}}{c^2d}$$

$$\frac{2b\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}-\frac{x^3(a+b\arccos(cx))^2}{3c^2d}$$

$\downarrow$ 2720

$$-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a+b\cdots\right.}{c^3}$$

$$\frac{2b\left(\frac{2\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}-\frac{x^3(a+b\arccos(cx))^2}{3c^2d}$$

$\downarrow$ 5183

$$-\frac{\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a+b\ldots\right.}{c^3}}{\ldots}$$

$$\frac{2b\left(\frac{2\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}-\frac{x^3(a+b\arccos(cx))^2}{3c^2d}$$

$\big\downarrow$ 24

$$-\frac{\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a+b\ldots\right.}{c^3}}{\ldots}$$

$$\frac{x^3(a+b\arccos(cx))^2}{3c^2d}-\frac{2b\left(-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}+\frac{2\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}$$

$\big\downarrow$ 7143

$$-\frac{\frac{-2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)\right)-2b\left(i\operatorname{PolyLog}\left(2,\ldots\right.\right.}{c^3}}{c^2d}$$

$$\frac{x^3(a+b\arccos(cx))^2}{3c^2d}-\frac{2b\left(-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^2}+\frac{2\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)}{3c^2}-\frac{bx^3}{9c}\right)}{3cd}$$

input
```
Int[(x^4*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2),x]
```

output
```
-1/3*(x^3*(a + b*ArcCos[c*x])^2)/(c^2*d) - (2*b*(-1/9*(b*x^3)/c - (x^2*Sqr
t[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(3*c^2) + (2*(-((b*x)/c) - (Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x]))/c^2))/(3*c^2)))/(3*c*d) + (-((x*(a + b*ArcCos
[c*x])^2)/c^2) - (2*b*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])
)/c^2))/c - (-2*(a + b*ArcCos[c*x])^2*ArcTanh[E^(I*ArcCos[c*x])] + 2*b*(I*
(a + b*ArcCos[c*x])*PolyLog[2, -E^(I*ArcCos[c*x])] - b*PolyLog[3, -E^(I*Ar
cCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] -
b*PolyLog[3, E^(I*ArcCos[c*x])]))/c^3)/(c^2*d)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.79

method	result
derivativedivides	$-\frac{a^{2}\left(\frac{c^{3}x^{3}}{3}+cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}+\frac{5b^{2}\arccos(cx)\sqrt{-c^{2}x^{2}+1}}{2d}-\frac{5b^{2}\arccos(cx)^{2}cx}{4d}+\frac{5b^{2}cx}{2d}-\frac{b^{2}\arccos(cx)^{2}\ln\left(1-cx-i\sqrt{-\right.}}{d}$
default	$-\frac{a^{2}\left(\frac{c^{3}x^{3}}{3}+cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}+\frac{5b^{2}\arccos(cx)\sqrt{-c^{2}x^{2}+1}}{2d}-\frac{5b^{2}\arccos(cx)^{2}cx}{4d}+\frac{5b^{2}cx}{2d}-\frac{b^{2}\arccos(cx)^{2}\ln\left(1-cx-i\sqrt{-\right.}}{d}$
parts	$-\frac{a^{2}\left(\frac{\frac{1}{3}c^{2}x^{3}+x}{c^{4}}+\frac{\ln(cx-1)}{2c^{5}}-\frac{\ln(cx+1)}{2c^{5}}\right)}{d}-\frac{b^{2}\left(\frac{5\left(\arccos(cx)^{2}-2+2i\arccos(cx)\right)\left(cx+i\sqrt{-c^{2}x^{2}+1}\right)}{8}+\frac{5\left(-i\sqrt{-c^{2}x^{2}+1}+cx\right)\left(\right.}{}\right.}{}$

input
```
int(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(-a^2/d*(1/3*c^3*x^3+c*x+1/2*ln(c*x-1)-1/2*ln(c*x+1))+5/2*b^2/d*arcc
os(c*x)*(-c^2*x^2+1)^(1/2)-5/4*b^2/d*arccos(c*x)^2*c*x+5/2*b^2/d*c*x-b^2/d
*arccos(c*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-2*I*a*b/d*polylog(2,-c*x-I*(
-c^2*x^2+1)^(1/2))-2*b^2/d*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))+b^2/d*arcco
s(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-2*I*b^2/d*arccos(c*x)*polylog(2,-c
*x-I*(-c^2*x^2+1)^(1/2))+2*b^2/d*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))-1/12
*b^2/d*arccos(c*x)^2*cos(3*arccos(c*x))+1/54*b^2/d*cos(3*arccos(c*x))+1/18
*b^2/d*arccos(c*x)*sin(3*arccos(c*x))+5/2*a*b/d*(-c^2*x^2+1)^(1/2)-5/2*a*b
/d*arccos(c*x)*c*x-2*a*b/d*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+2*a*
b/d*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+2*I*b^2/d*arccos(c*x)*polyl
og(2,c*x+I*(-c^2*x^2+1)^(1/2))+2*I*a*b/d*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2
))-1/6*a*b/d*arccos(c*x)*cos(3*arccos(c*x))+1/18*a*b/d*sin(3*arccos(c*x)))
```

Fricas [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos(cx)+a)^2x^4}{c^2dx^2-d}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b^2*x^4*arccos(c*x)^2 + 2*a*b*x^4*arccos(c*x) + a^2*x^4)/(c^2*d
*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = -\frac{\int \frac{a^2x^4}{c^2x^2-1}\,dx + \int \frac{b^2x^4\operatorname{acos}^2(cx)}{c^2x^2-1}\,dx + \int \frac{2abx^4\operatorname{acos}(cx)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate(x**4*(a+b*acos(c*x))**2/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a**2*x**4/(c**2*x**2 - 1), x) + Integral(b**2*x**4*acos(c*x)**2
/(c**2*x**2 - 1), x) + Integral(2*a*b*x**4*acos(c*x)/(c**2*x**2 - 1), x))/
d
```

Maxima [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos{(cx)}+a)^2x^4}{c^2dx^2-d}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/6*a^2*(2*(c^2*x^3 + 3*x)/(c^4*d) - 3*log(c*x + 1)/(c^5*d) + 3*log(c*x -
 1)/(c^5*d)) - 1/6*(6*c^5*d*integrate(1/3*(6*a*b*c^4*x^4*arctan2(sqrt(c*x
+ 1)*sqrt(-c*x + 1), c*x) - (2*b^2*c^3*x^3 + 6*b^2*c*x - 3*b^2*log(c*x + 1
) + 3*b^2*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1
)*sqrt(-c*x + 1), c*x))/(c^6*d*x^2 - c^4*d), x) + (2*b^2*c^3*x^3 + 6*b^2*c
*x - 3*b^2*log(c*x + 1) + 3*b^2*log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(
-c*x + 1), c*x)^2)/(c^5*d)
```

Giac [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos{(cx)}+a)^2x^4}{c^2dx^2-d}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate(-(b*arccos(c*x) + a)^2*x^4/(c^2*d*x^2 - d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx=\int \frac{x^4\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}{d-c^2\,d\,x^2}\,dx$$

input
```
int((x^4*(a + b*acos(c*x))^2)/(d - c^2*d*x^2),x)
```

output
```
int((x^4*(a + b*acos(c*x))^2)/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \frac{-12\left(\int \frac{acos(cx)x^4}{c^2x^2-1}dx\right)ab\,c^5-6\left(\int \frac{acos(cx)^2x^4}{c^2x^2-1}dx\right)b^2c^5-3\log(c^2x-c)\,a^2+3\log(c^2x+c)\,a^2-2a^2c^3x^3-6}{6c^5d}$$

input
```
int(x^4*(a+b*acos(c*x))^2/(-c^2*d*x^2+d),x)
```

output
```
( - 12*int((acos(c*x)*x**4)/(c**2*x**2 - 1),x)*a*b*c**5 - 6*int((acos(c*x)
**2*x**4)/(c**2*x**2 - 1),x)*b**2*c**5 - 3*log(c**2*x - c)*a**2 + 3*log(c*
*2*x + c)*a**2 - 2*a**2*c**3*x**3 - 6*a**2*c*x)/(6*c**5*d)
```

3.186 $\int \frac{x^3(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 210

$$\int \frac{x^3(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \frac{b^2x^2}{4c^2d}-\frac{bx\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^3d}+\frac{(a+b\arccos(cx))^2}{4c^4d}-\frac{x^2(a+b\arccos(cx))^2}{2c^2d}+\frac{i(a+b\arccos(cx))^3}{3bc^4d}-\frac{(a+b\arccos(cx))^2\log\left(1+e^{2i\arccos(cx)}\right)}{c^4d}+\frac{ib(a+b\arccos(cx))\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c^4d}-\frac{b^2\,\mathrm{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)}{2c^4d}$$

output
```
1/4*b^2*x^2/c^2/d-1/2*b*x*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3/d+1/4*(
a+b*arccos(c*x))^2/c^4/d-1/2*x^2*(a+b*arccos(c*x))^2/c^2/d+1/3*I*(a+b*arcc
os(c*x))^3/b/c^4/d-(a+b*arccos(c*x))^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/
c^4/d+I*b*(a+b*arccos(c*x))*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c^4/d
-1/2*b^2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c^4/d
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.56

$$\int \frac{x^3(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx =$$
$$-\frac{-ib^2\pi^3+12a^2c^2x^2-12abcx\sqrt{1-c^2x^2}+24abc^2x^2\arccos(cx)-24iab\arccos(cx)^2+8ib^2\arccos(cx)^3}{}$$

input
```
Integrate[(x^3*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2),x]
```

output
```
-1/24*((-I)*b^2*Pi^3 + 12*a^2*c^2*x^2 - 12*a*b*c*x*Sqrt[1 - c^2*x^2] + 24*
a*b*c^2*x^2*ArcCos[c*x] - (24*I)*a*b*ArcCos[c*x]^2 + (8*I)*b^2*ArcCos[c*x]
^3 + 24*a*b*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])] - 3*b^2*Cos[2*ArcCos[c*
x]] + 6*b^2*ArcCos[c*x]^2*Cos[2*ArcCos[c*x]] + 48*a*b*ArcCos[c*x]*Log[1 -
E^(I*ArcCos[c*x])] + 48*a*b*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] + 24*b^
2*ArcCos[c*x]^2*Log[1 - E^((-2*I)*ArcCos[c*x])] + 12*a^2*Log[1 - c^2*x^2]
- (48*I)*a*b*PolyLog[2, -E^(I*ArcCos[c*x])] - (48*I)*a*b*PolyLog[2, E^(I*A
rcCos[c*x])] + (24*I)*b^2*ArcCos[c*x]*PolyLog[2, E^((-2*I)*ArcCos[c*x])] +
 12*b^2*PolyLog[3, E^((-2*I)*ArcCos[c*x])] - 6*b^2*ArcCos[c*x]*Sin[2*ArcCo
s[c*x]])/(c^4*d)
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {5211, 27, 5181, 3042, 25, 4200, 25, 2620, 3011, 2720, 5211, 15, 5153, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$$
$$\Big\downarrow \; 5211$$
$$-\frac{b\int \frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}+\frac{\int \frac{x(a+b\arccos(cx))^2}{d(1-c^2x^2)}dx}{c^2}-\frac{x^2(a+b\arccos(cx))^2}{2c^2d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}+\frac{\int \frac{x(a+b\arccos(cx))^2}{1-c^2x^2}dx}{c^2d}-\frac{x^2(a+b\arccos(cx))^2}{2c^2d}$$

$$\Big\downarrow \text{5181}$$

$$-\frac{b\int \frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}-\frac{\int \frac{cx(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^4d}-\frac{x^2(a+b\arccos(cx))^2}{2c^2d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int -(a+b\arccos(cx))^2\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c^4d}-\frac{b\int \frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}-\frac{x^2(a+b\arccos(cx))^2}{2c^2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (a+b\arccos(cx))^2\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c^4d}-\frac{b\int \frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}-\frac{x^2(a+b\arccos(cx))^2}{2c^2d}$$

$$\Big\downarrow \text{4200}$$

$$-\frac{2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))^2}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^3}{3b}}{c^4d}-\frac{b\int \frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}-\frac{x^2(a+b\arccos(cx))^2}{2c^2d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))^2}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^3}{3b}}{c^4d}-\frac{b\int \frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}-\frac{x^2(a+b\arccos(cx))^2}{2c^2d}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\int (a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-}{c^4d}$$

$$\frac{b\int \frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}-\frac{x^2(a+b\arccos(cx))^2}{2c^2d}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\text{Po}\right.\right.}{c^4d}$$
$$\frac{b\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}-\frac{x^2(a+b\arccos(cx))^2}{2c^2d}$$

$\big\downarrow$ 2720

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2}\right.\right.}{c^4d}$$
$$\frac{b\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}-\frac{x^2(a+b\arccos(cx))^2}{2c^2d}$$

$\big\downarrow$ 5211

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2}\right.\right.}{c^4d}$$
$$\frac{b\left(\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int xdx}{2c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}\right)}{cd}-\frac{x^2(a+b\arccos(cx))^2}{2c^2d}$$

$\big\downarrow$ 15

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2}\right.\right.}{c^4d}$$
$$\frac{b\left(\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{cd}-\frac{x^2(a+b\arccos(cx))^2}{2c^2d}$$

$\big\downarrow$ 5153

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2}\right.\right.}{c^4d}$$
$$\frac{x^2(a+b\arccos(cx))^2}{2c^2d}-\frac{b\left(-\frac{(a+b\arccos(cx))^2}{4bc^3}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{cd}$$

$\big\downarrow$ 7143

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\text{PolyL}\right.\right.}{c^4d}$$
$$\frac{x^2(a+b\arccos(cx))^2}{2c^2d}-\frac{b\left(-\frac{(a+b\arccos(cx))^2}{4bc^3}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{bx^2}{4c}\right)}{cd}$$

input
```
Int[(x^3*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2),x]
```

output
```
-1/2*(x^2*(a + b*ArcCos[c*x])^2)/(c^2*d) - (b*(-1/4*(b*x^2)/c - (x*Sqrt[1
- c^2*x^2]*(a + b*ArcCos[c*x]))/(2*c^2) - (a + b*ArcCos[c*x])^2/(4*b*c^3))
)/(c*d) - (((-1/3*I)*(a + b*ArcCos[c*x])^3)/b - (2*I)*((I/2)*(a + b*ArcCos
[c*x])^2*Log[1 - E^((2*I)*ArcCos[c*x])] - I*b*((I/2)*(a + b*ArcCos[c*x])*P
olyLog[2, E^((2*I)*ArcCos[c*x])] - (b*PolyLog[3, E^((2*I)*ArcCos[c*x])])/4
)))/(c^4*d)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{-\frac{a^2\left(\frac{c^2x^2}{2}+\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b^2\left(-\frac{i\arccos(cx)^3}{3}+\arccos(cx)^2\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)-2i\arccos(cx)\operatorname{polylog}\left(2,-cx-i\sqrt{\right.}\right.}{}}{}$
default	$\frac{-\frac{a^2\left(\frac{c^2x^2}{2}+\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b^2\left(-\frac{i\arccos(cx)^3}{3}+\arccos(cx)^2\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)-2i\arccos(cx)\operatorname{polylog}\left(2,-cx-i\sqrt{\right.}\right.}{}}{}$
parts	$-\frac{a^2x^2}{2d\,c^2}-\frac{a^2\ln(c^2x^2-1)}{2d\,c^4}-\frac{b^2\left(-\frac{i\arccos(cx)^3}{3}+\arccos(cx)^2\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)-2i\arccos(cx)\operatorname{polylog}\left(2,-cx\right.\right.}{}$

input
```
int(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(-a^2/d*(1/2*c^2*x^2+1/2*ln(c*x-1)+1/2*ln(c*x+1))-b^2/d*(-1/3*I*arcc
os(c*x)^3+arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-2*I*arccos(c*x)*pol
ylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+2*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))+a
rccos(c*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-2*I*arccos(c*x)*polylog(2,c*x+
I*(-c^2*x^2+1)^(1/2))+2*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))+1/8*(2*arccos(
c*x)^2-1)*cos(2*arccos(c*x))-1/4*arccos(c*x)*sin(2*arccos(c*x)))-2*a*b/d*(
-1/2*I*arccos(c*x)^2+arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-I*polylog(
2,-c*x-I*(-c^2*x^2+1)^(1/2))+arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-I*
polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+1/4*arccos(c*x)*cos(2*arccos(c*x))-1/8
*sin(2*arccos(c*x))))
```

Fricas [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos(cx)+a)^2x^3}{c^2dx^2-d}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b^2*x^3*arccos(c*x)^2 + 2*a*b*x^3*arccos(c*x) + a^2*x^3)/(c^2*d
*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = -\frac{\int \frac{a^2x^3}{c^2x^2-1}\,dx + \int \frac{b^2x^3\operatorname{acos}^2(cx)}{c^2x^2-1}\,dx + \int \frac{2abx^3\operatorname{acos}(cx)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate(x**3*(a+b*acos(c*x))**2/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a**2*x**3/(c**2*x**2 - 1), x) + Integral(b**2*x**3*acos(c*x)**2
/(c**2*x**2 - 1), x) + Integral(2*a*b*x**3*acos(c*x)/(c**2*x**2 - 1), x))/
d
```

Maxima [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos(cx)+a)^2x^3}{c^2dx^2-d}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a^2*(x^2/(c^2*d) + log(c^2*x^2 - 1)/(c^4*d)) + 1/2*(2*c^4*d*integrate
(-(2*a*b*c^3*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - (b^2*c^2*x^2
 + b^2*log(c*x + 1) + b^2*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)*arct
an2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/(c^5*d*x^2 - c^3*d), x) - (b^2*c^2
*x^2 + b^2*log(c*x + 1) + b^2*log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c
*x + 1), c*x)^2)/(c^4*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\arccos(cx))^2}{d - c^2dx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\arccos(cx))^2}{d - c^2dx^2}\,dx = \int \frac{x^3\,(a + b\,\mathrm{acos}(c\,x))^2}{d - c^2\,d\,x^2}\,dx$$

input
```
int((x^3*(a + b*acos(c*x))^2)/(d - c^2*d*x^2),x)
```

output
```
int((x^3*(a + b*acos(c*x))^2)/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$$
$$= \frac{-2acos(cx)^2 b^2c^2x^2 + acos(cx)^2 b^2 + 2\sqrt{-c^2x^2+1}\,acos(cx)\,b^2cx - 4acos(cx)\,ab\,c^2x^2 + 4acos(cx)\,ab + 2}{}$$

input
```
int(x^3*(a+b*acos(c*x))^2/(-c^2*d*x^2+d),x)
```

output
```
( - 2*acos(c*x)**2*b**2*c**2*x**2 + acos(c*x)**2*b**2 + 2*sqrt( - c**2*x**
2 + 1)*acos(c*x)*b**2*c*x - 4*acos(c*x)*a*b*c**2*x**2 + 4*acos(c*x)*a*b +
2*asin(c*x)*a*b + 2*sqrt( - c**2*x**2 + 1)*a*b*c*x - 8*int((acos(c*x)*x)/(
c**2*x**2 - 1),x)*a*b*c**2 - 4*int((acos(c*x)**2*x)/(c**2*x**2 - 1),x)*b**
2*c**2 - 2*log(c**2*x - c)*a**2 - 2*log(c**2*x + c)*a**2 - 2*a**2*c**2*x**
2 + b**2*c**2*x**2 - b**2)/(4*c**4*d)
```

3.187 $\int \frac{x^2(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 218

$$\begin{aligned}\int \frac{x^2(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = {} & \frac{2b^2x}{c^2d}-\frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^3d}\\ & -\frac{x(a+b\arccos(cx))^2}{c^2d}\\ & -\frac{2i(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)}{c^3d}\\ & +\frac{2ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c^3d}\\ & -\frac{2ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c^3d}\\ & -\frac{2b^2\operatorname{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)}{c^3d}\\ & +\frac{2b^2\operatorname{PolyLog}\left(3,ie^{i\arccos(cx)}\right)}{c^3d}\end{aligned}$$

output
```
2*b^2*x/c^2/d-2*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3/d-x*(a+b*arccos
(c*x))^2/c^2/d-2*I*(a+b*arccos(c*x))^2*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c^
3/d+2*I*b*(a+b*arccos(c*x))*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^3/d
-2*I*b*(a+b*arccos(c*x))*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^3/d-2*b
^2*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^3/d+2*b^2*polylog(3,I*(c*x+I
*(-c^2*x^2+1)^(1/2)))/c^3/d
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.34

$$\int \frac{x^2(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$$
$$= \frac{-2a^2cx+4b^2cx+4ab\sqrt{1-c^2x^2}-4abcx\arccos(cx)+4b^2\sqrt{1-c^2x^2}\arccos(cx)-2b^2cx\arccos(cx)^2-4}{}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2),x]
```

output
```
(-2*a^2*c*x + 4*b^2*c*x + 4*a*b*Sqrt[1 - c^2*x^2] - 4*a*b*c*x*ArcCos[c*x]
+ 4*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x] - 2*b^2*c*x*ArcCos[c*x]^2 - 4*a*b*Ar
cCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] - 2*b^2*ArcCos[c*x]^2*Log[1 - E^(I*Ar
cCos[c*x])] + 4*a*b*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] + 2*b^2*ArcCos[
c*x]^2*Log[1 + E^(I*ArcCos[c*x])] - a^2*Log[1 - c*x] + a^2*Log[1 + c*x] -
(4*I)*b*(a + b*ArcCos[c*x])*PolyLog[2, -E^(I*ArcCos[c*x])] + (4*I)*b*(a +
b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] + 4*b^2*PolyLog[3, -E^(I*ArcC
os[c*x])] - 4*b^2*PolyLog[3, E^(I*ArcCos[c*x])])/(2*c^3*d)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5211, 27, 5165, 3042, 4671, 3011, 2720, 5183, 24, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$$
$$\Big\downarrow \; 5211$$
$$-\frac{2b\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}+\frac{\int \frac{(a+b\arccos(cx))^2}{d(1-c^2x^2)}dx}{c^2}-\frac{x(a+b\arccos(cx))^2}{c^2d}$$
$$\Big\downarrow \; 27$$

$$-\frac{2b\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}+\frac{\int \frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx}{c^2d}-\frac{x(a+b\arccos(cx))^2}{c^2d}$$

$$\Big\downarrow \text{5165}$$

$$-\frac{2b\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}-\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^3d}-\frac{x(a+b\arccos(cx))^2}{c^2d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int (a+b\arccos(cx))^2\csc(\arccos(cx))d\arccos(cx)}{c^3d}-\frac{2b\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}-\frac{x(a+b\arccos(cx))^2}{c^2d}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{-2b\int (a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arccos}{c^3d}$$
$$\frac{2b\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}-\frac{x(a+b\arccos(cx))^2}{c^2d}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\,\text{PolyLog}\right.}{}$$
$$\frac{2b\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}-\frac{x(a+b\arccos(cx))^2}{c^2d}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b}{}$$
$$\frac{2b\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{cd}-\frac{x(a+b\arccos(cx))^2}{c^2d}$$

$$\Big\downarrow \text{5183}$$

$$-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b}{}$$
$$\frac{2b\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)}{cd}-\frac{x(a+b\arccos(cx))^2}{c^2d}$$

$$\Big\downarrow \text{24}$$

$$-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b}{}$$

$$\frac{2b\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)}{cd}-\frac{x(a+b\arccos(cx))^2}{c^2d}$$

$$\Big\downarrow \ 7143$$

$$-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\,\text{PolyLog}\left(3,\right.\right.}{c^3d}$$

$$\frac{2b\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)}{cd}-\frac{x(a+b\arccos(cx))^2}{c^2d}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2),x]
```

output
```
-((x*(a + b*ArcCos[c*x])^2)/(c^2*d)) - (2*b*(-((b*x)/c) - (Sqrt[1 - c^2*x^
2]*(a + b*ArcCos[c*x]))/c^2))/(c*d) - (-2*(a + b*ArcCos[c*x])^2*ArcTanh[E^
(I*ArcCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, -E^(I*ArcCos[c*x]
)] - b*PolyLog[3, -E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLo
g[2, E^(I*ArcCos[c*x])] - b*PolyLog[3, E^(I*ArcCos[c*x])]))/(c^3*d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.01

method	result
derivativedivides	$\frac{-\frac{a^2\left(cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}+\frac{2b^2\arccos(cx)\sqrt{-c^2x^2+1}}{d}+\frac{b^2\arccos(cx)^2\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)}{d}-\frac{b^2\arccos(cx)^2\ln\left(1-cx-\right.}{d}}{}$
default	$\frac{-\frac{a^2\left(cx+\frac{\ln(cx-1)}{2}-\frac{\ln(cx+1)}{2}\right)}{d}+\frac{2b^2\arccos(cx)\sqrt{-c^2x^2+1}}{d}+\frac{b^2\arccos(cx)^2\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)}{d}-\frac{b^2\arccos(cx)^2\ln\left(1-cx-\right.}{d}}{}$
parts	$-\frac{a^2\left(\frac{x}{c^2}+\frac{\ln(cx-1)}{2c^3}-\frac{\ln(cx+1)}{2c^3}\right)}{d}+\frac{2b^2\sqrt{-c^2x^2+1}\,\arccos(cx)}{d\,c^3}-\frac{b^2\arccos(cx)^2x}{d\,c^2}-\frac{b^2\arccos(cx)^2\ln\left(1-cx-i\sqrt{-\right.}{d\,c^3}$

input
```
int(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-a^2/d*(c*x+1/2*ln(c*x-1)-1/2*ln(c*x+1))+2*b^2/d*arccos(c*x)*(-c^2*
x^2+1)^(1/2)+b^2/d*arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-b^2/d*arcc
os(c*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-b^2/d*arccos(c*x)^2*c*x-2*I*b^2/d
*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-2*I*a*b/d*polylog(2,-c*x
-I*(-c^2*x^2+1)^(1/2))+2*b^2/d*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))-2*b^2/
d*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))+2*b^2/d*c*x+2*a*b/d*(-c^2*x^2+1)^(1/
2)+2*a*b/d*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-2*a*b/d*arccos(c*x)*
ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-2*a*b/d*arccos(c*x)*c*x+2*I*b^2/d*arccos(c*
x)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+2*I*a*b/d*polylog(2,c*x+I*(-c^2*x^2
+1)^(1/2)))
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int -\frac{\left(b\arccos\left(cx\right)+a\right)^2x^2}{c^2dx^2-d}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b^2*x^2*arccos(c*x)^2 + 2*a*b*x^2*arccos(c*x) + a^2*x^2)/(c^2*d
*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = -\frac{\int \frac{a^2x^2}{c^2x^2-1}\,dx+\int \frac{b^2x^2\operatorname{acos}^2\left(cx\right)}{c^2x^2-1}\,dx+\int \frac{2abx^2\operatorname{acos}\left(cx\right)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate(x**2*(a+b*acos(c*x))**2/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a**2*x**2/(c**2*x**2 - 1), x) + Integral(b**2*x**2*acos(c*x)**2
/(c**2*x**2 - 1), x) + Integral(2*a*b*x**2*acos(c*x)/(c**2*x**2 - 1), x))/
d
```

Maxima [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int -\frac{\left(b\arccos\left(cx\right)+a\right)^2x^2}{c^2dx^2-d}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a^2*(2*x/(c^2*d) - log(c*x + 1)/(c^3*d) + log(c*x - 1)/(c^3*d)) - 1/2
*(2*c^3*d*integrate((2*a*b*c^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c
*x) - (2*b^2*c*x - b^2*log(c*x + 1) + b^2*log(-c*x + 1))*sqrt(c*x + 1)*sqr
t(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/(c^4*d*x^2 - c^2*d
), x) + (2*b^2*c*x - b^2*log(c*x + 1) + b^2*log(-c*x + 1))*arctan2(sqrt(c*
x + 1)*sqrt(-c*x + 1), c*x)^2)/(c^3*d)
```

Giac [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx=\int -\frac{(b\arccos{(cx)}+a)^2x^2}{c^2dx^2-d}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
integrate(-(b*arccos(c*x) + a)^2*x^2/(c^2*d*x^2 - d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{d-c^2\,d\,x^2}\,dx$$

input
```
int((x^2*(a + b*acos(c*x))^2)/(d - c^2*d*x^2),x)
```

output
```
int((x^2*(a + b*acos(c*x))^2)/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$$
$$=\frac{-2acos(cx)^2\,b^2cx+4\sqrt{-c^2x^2+1}\,acos(cx)\,b^2-4acos(cx)\,abcx+4\sqrt{-c^2x^2+1}\,ab-4\left(\int\frac{acos(cx)}{c^2x^2-1}dx\right)abc}{2c^3d}$$

input
```
int(x^2*(a+b*acos(c*x))^2/(-c^2*d*x^2+d),x)
```

output
```
( - 2*acos(c*x)**2*b**2*c*x + 4*sqrt( - c**2*x**2 + 1)*acos(c*x)*b**2 - 4*
acos(c*x)*a*b*c*x + 4*sqrt( - c**2*x**2 + 1)*a*b - 4*int(acos(c*x)/(c**2*x
**2 - 1),x)*a*b*c - 2*int(acos(c*x)**2/(c**2*x**2 - 1),x)*b**2*c - log(c**
2*x - c)*a**2 + log(c**2*x + c)*a**2 - 2*a**2*c*x + 4*b**2*c*x)/(2*c**3*d)
```

3.188 $\int \frac{x(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{x(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \frac{i(a+b\arccos(cx))^3}{3bc^2d} - \frac{(a+b\arccos(cx))^2\log\left(1+e^{2i\arccos(cx)}\right)}{c^2d} + \frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c^2d} - \frac{b^2\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)}{2c^2d}$$

output
```
1/3*I*(a+b*arccos(c*x))^3/b/c^2/d-(a+b*arccos(c*x))^2*ln(1+(c*x+I*(-c^2*x^
2+1)^(1/2))^2)/c^2/d+I*b*(a+b*arccos(c*x))*polylog(2,-(c*x+I*(-c^2*x^2+1)^
(1/2))^2)/c^2/d-1/2*b^2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c^2/d
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.79

$$\int \frac{x(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$$
$$= \frac{i\left(b^2\pi^3+24ab\arccos(cx)^2-8b^2\arccos(cx)^3+48iab\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)+48iab\arccos(cx)\,\mathrm{lo}\right.}{}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2),x]
```

output
```
((I/24)*(b^2*Pi^3 + 24*a*b*ArcCos[c*x]^2 - 8*b^2*ArcCos[c*x]^3 + (48*I)*a*
b*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] + (48*I)*a*b*ArcCos[c*x]*Log[1 +
E^(I*ArcCos[c*x])] + (24*I)*b^2*ArcCos[c*x]^2*Log[1 - E^((-2*I)*ArcCos[c*x
])] + (12*I)*a^2*Log[1 - c^2*x^2] + 48*a*b*PolyLog[2, -E^(I*ArcCos[c*x])]
+ 48*a*b*PolyLog[2, E^(I*ArcCos[c*x])] - 24*b^2*ArcCos[c*x]*PolyLog[2, E^(
(-2*I)*ArcCos[c*x])] + (12*I)*b^2*PolyLog[3, E^((-2*I)*ArcCos[c*x])]))/(c^
2*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5181, 3042, 25, 4200, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$$
$$\Big\downarrow \ 5181$$
$$-\frac{\int \frac{cx(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^2d}$$
$$\Big\downarrow \ 3042$$
$$-\frac{\int -(a+b\arccos(cx))^2\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c^2d}$$

$$\Big\downarrow \; 25$$

$$\frac{\int (a+b\arccos(cx))^2\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c^2d}$$

$$\Big\downarrow \; 4200$$

$$-\frac{2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))^2}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^3}{3b}}{c^2d}$$

$$\Big\downarrow \; 25$$

$$-\frac{-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))^2}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^3}{3b}}{c^2d}$$

$$\Big\downarrow \; 2620$$

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\int (a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-}{c^2d}$$

$$\Big\downarrow \; 3011$$

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int \operatorname{Po}\right.\right.}{c^2d}$$

$$\Big\downarrow \; 2720$$

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2}\right.\right.}{c^2d}$$

$$\Big\downarrow \; 7143$$

$$-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\operatorname{PolyL}\right.\right.}{c^2d}$$

input
```
Int[(x*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2),x]
```

output
```
-((((-1/3*I)*(a + b*ArcCos[c*x])^3)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])^2
*Log[1 - E^((2*I)*ArcCos[c*x])] - I*b*((I/2)*(a + b*ArcCos[c*x])*PolyLog[2
, E^((2*I)*ArcCos[c*x])] - (b*PolyLog[3, E^((2*I)*ArcCos[c*x])])/4)))/(c^2
*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(144) = 288$.

Time = 0.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.79

method	result
parts	$-\frac{a^2 \ln(c^2x^2-1)}{2d\,c^2} - \frac{b^2\left(-\frac{i\arccos(cx)^3}{3}+\arccos(cx)^2\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)-2i\arccos(cx)\,\mathrm{polylog}\left(2,-cx-i\sqrt{-c^2x^2\ldots}\right.\right.}{}$
derivativedivides	$\frac{-\frac{a^2\left(\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b^2\left(-\frac{i\arccos(cx)^3}{3}+\arccos(cx)^2\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)-2i\arccos(cx)\,\mathrm{polylog}\left(2,-cx-i\sqrt{-c^2x^2+\ldots}\right.\right.}{}}{}$
default	$\frac{-\frac{a^2\left(\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b^2\left(-\frac{i\arccos(cx)^3}{3}+\arccos(cx)^2\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)-2i\arccos(cx)\,\mathrm{polylog}\left(2,-cx-i\sqrt{-c^2x^2+\ldots}\right.\right.}{}}{}$

input
```
int(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/d/c^2*ln(c^2*x^2-1)-b^2/d/c^2*(-1/3*I*arccos(c*x)^3+arccos(c*x)^2
*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-2*I*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2
+1)^(1/2))+2*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))+arccos(c*x)^2*ln(1-c*x-I
*(-c^2*x^2+1)^(1/2))-2*I*arccos(c*x)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+2
*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2)))-2*a*b/d/c^2*(-1/2*I*arccos(c*x)^2+ar
ccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+
1)^(1/2))-I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-I*polylog(2,c*x+I*(-c^2*x
^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{x(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int -\frac{\left(b\arccos\left(cx\right)+a\right)^2x}{c^2dx^2-d}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b^2*x*arccos(c*x)^2 + 2*a*b*x*arccos(c*x) + a^2*x)/(c^2*d*x^2 -
 d), x)
```

Sympy [F]

$$\int \frac{x(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = -\frac{\int \frac{a^2x}{c^2x^2-1}\,dx + \int \frac{b^2x\operatorname{acos}^2\left(cx\right)}{c^2x^2-1}\,dx + \int \frac{2abx\operatorname{acos}\left(cx\right)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate(x*(a+b*acos(c*x))**2/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a**2*x/(c**2*x**2 - 1), x) + Integral(b**2*x*acos(c*x)**2/(c**2
*x**2 - 1), x) + Integral(2*a*b*x*acos(c*x)/(c**2*x**2 - 1), x))/d
```

Maxima [F]

$$\int \frac{x(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int -\frac{\left(b\arccos\left(cx\right)+a\right)^2x}{c^2dx^2-d}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a^2*log(c^2*d*x^2 - d)/(c^2*d) + 1/2*(2*c^2*d*integrate(-(2*a*b*c*x*a
rctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - (b^2*log(c*x + 1) + b^2*log(-c
*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1)
, c*x))/(c^3*d*x^2 - c*d), x) - (b^2*log(c*x + 1) + b^2*log(-c*x + 1))*arc
tan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2)/(c^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int \frac{x\,(a+b\,\mathrm{acos}(c\,x))^2}{d-c^2\,d\,x^2}\,dx$$

input
```
int((x*(a + b*acos(c*x))^2)/(d - c^2*d*x^2),x)
```

output
```
int((x*(a + b*acos(c*x))^2)/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{x(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \frac{-4\left(\int \frac{acos(cx)x}{c^2x^2-1}dx\right)ab\,c^2-2\left(\int \frac{acos(cx)^2x}{c^2x^2-1}dx\right)b^2c^2-\log(c^2x-c)\,a^2-\log(c^2x+c)\,a^2}{2c^2d}$$

input
```
int(x*(a+b*acos(c*x))^2/(-c^2*d*x^2+d),x)
```

output
```
( - 4*int((acos(c*x)*x)/(c**2*x**2 - 1),x)*a*b*c**2 - 2*int((acos(c*x)**2*
x)/(c**2*x**2 - 1),x)*b**2*c**2 - log(c**2*x - c)*a**2 - log(c**2*x + c)*a
**2)/(2*c**2*d)
```

3.189 $\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = -\frac{2i(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)}{cd} + \frac{2ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{cd} - \frac{2ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{cd} - \frac{2b^2\operatorname{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)}{cd} + \frac{2b^2\operatorname{PolyLog}\left(3,ie^{i\arccos(cx)}\right)}{cd}$$

output
```
-2*I*(a+b*arccos(c*x))^2*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c/d+2*I*b*(a+b*a
rccos(c*x))*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/d-2*I*b*(a+b*arccos
(c*x))*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/d-2*b^2*polylog(3,-I*(c*x
+I*(-c^2*x^2+1)^(1/2)))/c/d+2*b^2*polylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/
c/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.38

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \frac{-4ab\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)-2b^2\arccos(cx)^2\log\left(1-e^{i\arccos(cx)}\right)+4ab\arccos(cx)\log\left(1+e^{i\arccos(cx)}\right)}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2),x]
```

output
```
(-4*a*b*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] - 2*b^2*ArcCos[c*x]^2*Log[1
 - E^(I*ArcCos[c*x])] + 4*a*b*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] + 2*b
^2*ArcCos[c*x]^2*Log[1 + E^(I*ArcCos[c*x])] - a^2*Log[1 - c*x] + a^2*Log[1
 + c*x] - (4*I)*b*(a + b*ArcCos[c*x])*PolyLog[2, -E^(I*ArcCos[c*x])] + (4*
I)*b*(a + b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] + 4*b^2*PolyLog[3,
-E^(I*ArcCos[c*x])] - 4*b^2*PolyLog[3, E^(I*ArcCos[c*x])])/(2*c*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5165, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$$

$$\Big\downarrow \text{ 5165}$$

$$-\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\,d\arccos(cx)}{cd}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int (a+b\arccos(cx))^2\csc(\arccos(cx))\,d\arccos(cx)}{cd}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{-2b\int(a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int(a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arccos}{cd}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\,\text{PolyLog}\right.}{}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b}{}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\,\text{PolyLog}\left(3,\right.\right.}{cd}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2),x]
```

output
```
-((-2*(a + b*ArcCos[c*x])^2*ArcTanh[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*Arc
Cos[c*x])*PolyLog[2, -E^(I*ArcCos[c*x])] - b*PolyLog[3, -E^(I*ArcCos[c*x])
]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] - b*PolyLog[
3, E^(I*ArcCos[c*x])]))/(c*d))
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.01

method	result
derivativedivides	$\frac{a^2 \operatorname{arctanh}(cx)}{d} - \frac{b^2 \left(\arccos(cx)^2 \ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)-2i\arccos(cx)\operatorname{polylog}\left(2,cx+i\sqrt{-c^2x^2+1}\right)+2\operatorname{polylog}\left(3,cx+i\sqrt{-c^2x^2+1}\right.\right.}{}$
default	$\frac{a^2 \operatorname{arctanh}(cx)}{d} - \frac{b^2 \left(\arccos(cx)^2 \ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)-2i\arccos(cx)\operatorname{polylog}\left(2,cx+i\sqrt{-c^2x^2+1}\right)+2\operatorname{polylog}\left(3,cx+i\sqrt{-c^2x^2+1}\right.\right.}{}$
parts	$-\frac{a^2 \ln(cx-1)}{2dc} + \frac{a^2 \ln(cx+1)}{2dc} - \frac{b^2 \left(\arccos(cx)^2 \ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)-2i\arccos(cx)\operatorname{polylog}\left(2,cx+i\sqrt{-c^2x^2+\right.\right.}{}$

input
```
int((a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2/d*arctanh(c*x)-b^2/d*(arccos(c*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2)
)-2*I*arccos(c*x)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+2*polylog(3,c*x+I*(-
c^2*x^2+1)^(1/2))-arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+2*I*arccos(
c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-2*polylog(3,-c*x-I*(-c^2*x^2+1)^
(1/2)))-2*a*b/d*(-arctanh(c*x)*arccos(c*x)-I*arctanh(c*x)*(ln(1-I*(c*x+1)/
(-c^2*x^2+1)^(1/2))-ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2)))+I*dilog(1+I*(c*x+1
)/(-c^2*x^2+1)^(1/2))-I*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int -\frac{(b\arccos(cx)+a)^2}{c^2dx^2-d}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^2*d*x^2 - d), x
)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = -\frac{\int \frac{a^2}{c^2x^2-1}\,dx + \int \frac{b^2\operatorname{acos}^2(cx)}{c^2x^2-1}\,dx + \int \frac{2ab\operatorname{acos}(cx)}{c^2x^2-1}\,dx}{d}$$

input
```
integrate((a+b*acos(c*x))**2/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a**2/(c**2*x**2 - 1), x) + Integral(b**2*acos(c*x)**2/(c**2*x**
2 - 1), x) + Integral(2*a*b*acos(c*x)/(c**2*x**2 - 1), x))/d
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int -\frac{\left(b\arccos\left(cx\right)+a\right)^2}{c^2dx^2-d}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a^2*(log(c*x + 1)/(c*d) - log(c*x - 1)/(c*d)) + 1/2*((b^2*log(c*x + 1)
 - b^2*log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 - 2*c*d
*integrate(((b^2*log(c*x + 1) - b^2*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x
 + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 2*a*b*arctan2(sqrt(c*x
+ 1)*sqrt(-c*x + 1), c*x))/(c^2*d*x^2 - d), x))/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int \frac{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}{d-c^2\,d\,x^2}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2),x)
```

output
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \frac{-4\left(\int \frac{acos(cx)}{c^2x^2-1}dx\right)abc-2\left(\int \frac{acos(cx)^2}{c^2x^2-1}dx\right)b^2c-\log(c^2x-c)\,a^2+\log(c^2x+c)\,a^2}{2cd}$$

input
```
int((a+b*acos(c*x))^2/(-c^2*d*x^2+d),x)
```

output
```
( - 4*int(acos(c*x)/(c**2*x**2 - 1),x)*a*b*c - 2*int(acos(c*x)**2/(c**2*x*
*2 - 1),x)*b**2*c - log(c**2*x - c)*a**2 + log(c**2*x + c)*a**2)/(2*c*d)
```

3.190 $\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 131

$$\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)}\,dx = -\frac{2(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)}{d} + \frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{d} - \frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{d} - \frac{b^2\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)}{2d} + \frac{b^2\operatorname{PolyLog}\left(3,e^{2i\arccos(cx)}\right)}{2d}$$

output
```
-2*(a+b*arccos(c*x))^2*arctanh((c*x+I*(-c^2*x^2+1)^(1/2))^2)/d+I*b*(a+b*ar
ccos(c*x))*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d-I*b*(a+b*arccos(c*x)
)*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d-1/2*b^2*polylog(3,-(c*x+I*(-c^
2*x^2+1)^(1/2))^2)/d+1/2*b^2*polylog(3,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 313 vs. 2(131) = 262.

Time = 0.49 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.39

$$\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)}\,dx = -\frac{-ib^2\pi^3+16ib^2\arccos(cx)^3+48ab\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)+48ab\arccos(cx)\log\left(1+e^{i\arccos(cx)}\right)}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x*(d - c^2*d*x^2)),x]
```

output
```
-1/24*((-I)*b^2*Pi^3 + (16*I)*b^2*ArcCos[c*x]^3 + 48*a*b*ArcCos[c*x]*Log[1
 - E^(I*ArcCos[c*x])] + 48*a*b*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] + 24
*b^2*ArcCos[c*x]^2*Log[1 - E^((-2*I)*ArcCos[c*x])] - 48*a*b*ArcCos[c*x]*Lo
g[1 + E^((2*I)*ArcCos[c*x])] - 24*b^2*ArcCos[c*x]^2*Log[1 + E^((2*I)*ArcCo
s[c*x])] - 24*a^2*Log[c*x] + 12*a^2*Log[1 - c^2*x^2] - (48*I)*a*b*PolyLog[
2, -E^(I*ArcCos[c*x])] - (48*I)*a*b*PolyLog[2, E^(I*ArcCos[c*x])] + (24*I)
*b^2*ArcCos[c*x]*PolyLog[2, E^((-2*I)*ArcCos[c*x])] + (24*I)*a*b*PolyLog[2
, -E^((2*I)*ArcCos[c*x])] + (24*I)*b^2*ArcCos[c*x]*PolyLog[2, -E^((2*I)*Ar
cCos[c*x])] + 12*b^2*PolyLog[3, E^((-2*I)*ArcCos[c*x])] - 12*b^2*PolyLog[3
, -E^((2*I)*ArcCos[c*x])])/d
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5185, 4919, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)}\,dx$$

$$\Big\downarrow \text{ 5185}$$

$$-\frac{\int \frac{(a+b\arccos(cx))^2}{cx\sqrt{1-c^2x^2}}d\arccos(cx)}{d}$$

$\Big\downarrow$ 4919

$$-\frac{2\int (a+b\arccos(cx))^2\csc(2\arccos(cx))d\arccos(cx)}{d}$$

$\Big\downarrow$ 3042

$$-\frac{2\int (a+b\arccos(cx))^2\csc(2\arccos(cx))d\arccos(cx)}{d}$$

$\Big\downarrow$ 4671

$$-\frac{2\left(-b\int (a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+b\int (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)d\arcc\right.}{d}$$

$\Big\downarrow$ 3011

$$-\frac{2\left(b\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int \operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-b\left(\frac{1}{2}i\operatorname{Po}\right.\right.}{}$$

$\Big\downarrow$ 2720

$$-\frac{2\left(b\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx}\right.\right.}{}$$

$\Big\downarrow$ 7143

$$-\frac{2\left(-\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2+b\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\operatorname{PolyLog}\right.\right.}{d}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x*(d - c^2*d*x^2)),x]
```

output
```
(-2*(-((a + b*ArcCos[c*x])^2*ArcTanh[E^((2*I)*ArcCos[c*x])]) + b*((I/2)*(a
 + b*ArcCos[c*x])*PolyLog[2, -E^((2*I)*ArcCos[c*x])] - (b*PolyLog[3, -E^((
2*I)*ArcCos[c*x])])/4) - b*((I/2)*(a + b*ArcCos[c*x])*PolyLog[2, E^((2*I)*
ArcCos[c*x])] - (b*PolyLog[3, E^((2*I)*ArcCos[c*x])])/4)))/d
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(180) = 360$.

Time = 0.42 (sec) , antiderivative size = 460, normalized size of antiderivative = 3.51

method	result
parts	$-\frac{a^2\left(-\ln(x)+\frac{\ln(cx-1)}{2}+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b^2\left(-\arccos(cx)^2\ln\left(1+\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)+i\arccos(cx)\,\mathrm{polylog}\left(2,-\left(c\right.\right.\right.}{}$
derivativedivides	$-\frac{a^2\left(\frac{\ln(cx-1)}{2}-\ln(cx)+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b^2\left(-\arccos(cx)^2\ln\left(1+\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)+i\arccos(cx)\,\mathrm{polylog}\left(2,-\left(cx\right.\right.\right.}{}$
default	$-\frac{a^2\left(\frac{\ln(cx-1)}{2}-\ln(cx)+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b^2\left(-\arccos(cx)^2\ln\left(1+\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)+i\arccos(cx)\,\mathrm{polylog}\left(2,-\left(cx\right.\right.\right.}{}$

input
```
int((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-a^2/d*(-ln(x)+1/2*ln(c*x-1)+1/2*ln(c*x+1))-b^2/d*(-arccos(c*x)^2*ln(1+(c*
x+I*(-c^2*x^2+1)^(1/2))^2)+I*arccos(c*x)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1
/2))^2)-1/2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+arccos(c*x)^2*ln(1+c*
x+I*(-c^2*x^2+1)^(1/2))-2*I*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2
))+2*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))+arccos(c*x)^2*ln(1-c*x-I*(-c^2*x
^2+1)^(1/2))-2*I*arccos(c*x)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+2*polylog
(3,c*x+I*(-c^2*x^2+1)^(1/2)))-2*a*b/d*(arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)
^(1/2))-I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-arccos(c*x)*ln(1+(c*x+I*(-c
^2*x^2+1)^(1/2))^2)+1/2*I*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+arccos(
c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-I*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)}\,dx = \int -\frac{\left(b\arccos\left(cx\right)+a\right)^2}{(c^2dx^2-d)x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^2*d*x^3 - d*x),
 x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)}\,dx = -\frac{\int \frac{a^2}{c^2x^3-x}\,dx+\int \frac{b^2\operatorname{acos}^2\left(cx\right)}{c^2x^3-x}\,dx+\int \frac{2ab\operatorname{acos}\left(cx\right)}{c^2x^3-x}\,dx}{d}$$

input
```
integrate((a+b*acos(c*x))**2/x/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a**2/(c**2*x**3 - x), x) + Integral(b**2*acos(c*x)**2/(c**2*x**
3 - x), x) + Integral(2*a*b*acos(c*x)/(c**2*x**3 - x), x))/d
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)}\,dx = \int -\frac{\left(b\arccos\left(cx\right)+a\right)^2}{(c^2dx^2-d)x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a^2*(log(c*x + 1)/d + log(c*x - 1)/d - 2*log(x)/d) - integrate((b^2*a
rctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arctan2(sqrt(c*x + 1)*
sqrt(-c*x + 1), c*x))/(c^2*d*x^3 - d*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x\,(d-c^2dx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x\,(d-c^2dx^2)}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x\,(d-c^2\,d\,x^2)}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x*(d - c^2*d*x^2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x*(d - c^2*d*x^2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\,(d-c^2dx^2)}\,dx = \frac{-4\left(\int \frac{acos(cx)}{c^2x^3-x}dx\right)ab-2\left(\int \frac{acos(cx)^2}{c^2x^3-x}dx\right)b^2-\log(c^2x-c)\,a^2-\log(c^2x+c)\,a^2+2\log(x)\,a^2}{2d}$$

input
```
int((a+b*acos(c*x))^2/x/(-c^2*d*x^2+d),x)
```

output
```
( - 4*int(acos(c*x)/(c**2*x**3 - x),x)*a*b - 2*int(acos(c*x)**2/(c**2*x**3
 - x),x)*b**2 - log(c**2*x - c)*a**2 - log(c**2*x + c)*a**2 + 2*log(x)*a**
2)/(2*d)
```

3.191 $\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 238

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)}\,dx = &-\frac{(a+b\arccos(cx))^2}{dx} \\ &-\frac{2ic(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)}{d} \\ &-\frac{4bc(a+b\arccos(cx))\text{arctanh}\left(e^{i\arccos(cx)}\right)}{d} \\ &+\frac{2ib^2c\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{d} \\ &+\frac{2ibc(a+b\arccos(cx))\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{d} \\ &-\frac{2ibc(a+b\arccos(cx))\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{d} \\ &-\frac{2ib^2c\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{d} \\ &-\frac{2b^2c\,\text{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)}{d} \\ &+\frac{2b^2c\,\text{PolyLog}\left(3,ie^{i\arccos(cx)}\right)}{d}\end{aligned}$$

output
```
-(a+b*arccos(c*x))^2/d/x-2*I*c*(a+b*arccos(c*x))^2*arctan(c*x+I*(-c^2*x^2+
1)^(1/2))/d-4*b*c*(a+b*arccos(c*x))*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/d+2*
I*b^2*c*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/d+2*I*b*c*(a+b*arccos(c*x))*p
olylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d-2*I*b*c*(a+b*arccos(c*x))*polylo
g(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d-2*I*b^2*c*polylog(2,c*x+I*(-c^2*x^2+1)
^(1/2))/d-2*b^2*c*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d+2*b^2*c*polyl
og(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.72

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)}\,dx = -\frac{a^2}{dx}-\frac{a^2c\log(1-cx)}{2d}+\frac{a^2c\log(1+cx)}{2d}$$
$$-\frac{2abc\left(\frac{\arccos(cx)}{cx}+\log(cx)-\log\left(1+\sqrt{1-c^2x^2}\right)+\frac{1}{2}\left(\frac{1}{2}i\arccos(cx)^2-2\arccos(cx)\log\left(1+e^{i\arccos(cx)}\right)\right.\right.}{}$$
$$-\frac{b^2c\left(\frac{\arccos(cx)^2}{cx}+\arccos(cx)^2\left(\log\left(1-e^{i\arccos(cx)}\right)-\log\left(1+e^{i\arccos(cx)}\right)\right)-2\left(\arccos(cx)\left(\log\left(1-ie^{i a}\right.\right.\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^2*(d - c^2*d*x^2)),x]
```

output
```
-(a^2/(d*x)) - (a^2*c*Log[1 - c*x])/(2*d) + (a^2*c*Log[1 + c*x])/(2*d) - (
2*a*b*c*(ArcCos[c*x]/(c*x) + Log[c*x] - Log[1 + Sqrt[1 - c^2*x^2]] + ((I/2
)*ArcCos[c*x]^2 - 2*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] + (2*I)*PolyLog
[2, -E^(I*ArcCos[c*x])])/2 + (2*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] - (
2*I)*(ArcCos[c*x]^2/4 + PolyLog[2, E^(I*ArcCos[c*x])]))/2))/d - (b^2*c*(Ar
cCos[c*x]^2/(c*x) + ArcCos[c*x]^2*(Log[1 - E^(I*ArcCos[c*x])] - Log[1 + E^
(I*ArcCos[c*x])]) - 2*(ArcCos[c*x]*(Log[1 - I*E^(I*ArcCos[c*x])] - Log[1 +
 I*E^(I*ArcCos[c*x])]) + I*(PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - PolyLog[2
, I*E^(I*ArcCos[c*x])])) + (2*I)*ArcCos[c*x]*(PolyLog[2, -E^(I*ArcCos[c*x]
)] - PolyLog[2, E^(I*ArcCos[c*x])]) + 2*(-PolyLog[3, -E^(I*ArcCos[c*x])] +
 PolyLog[3, E^(I*ArcCos[c*x])])))/d
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {5205, 27, 5165, 3042, 4671, 3011, 2720, 5219, 3042, 4669, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)}dx$$

$$\Big\downarrow \text{5205}$$

$$c^2\int \frac{(a+b\arccos(cx))^2}{d(1-c^2x^2)}dx-\frac{2bc\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{d}-\frac{(a+b\arccos(cx))^2}{dx}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\int \frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx}{d}-\frac{2bc\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{d}-\frac{(a+b\arccos(cx))^2}{dx}$$

$$\Big\downarrow \text{5165}$$

$$-\frac{2bc\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{d}-\frac{c\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{d}-\frac{(a+b\arccos(cx))^2}{dx}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2bc\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{d}-\frac{c\int (a+b\arccos(cx))^2\csc(\arccos(cx))d\arccos(cx)}{d}-\frac{(a+b\arccos(cx))^2}{dx}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{c\left(-2b\int (a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arcc\right.}{d}$$
$$\frac{2bc\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{d}-\frac{(a+b\arccos(cx))^2}{dx}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{c\left(2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\,\text{PolyLo\right.}{}$$

$$\frac{2bc\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{d}-\frac{(a+b\arccos(cx))^2}{dx}$$

$\downarrow$ 2720

$$-\frac{c\left(2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2\right.}{}$$

$$\frac{2bc\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx}{d}-\frac{(a+b\arccos(cx))^2}{dx}$$

$\downarrow$ 5219

$$-\frac{c\left(2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2\right.}{}$$

$$\frac{2bc\int\frac{a+b\arccos(cx)}{cx}d\arccos(cx)}{d}-\frac{(a+b\arccos(cx))^2}{dx}$$

$\downarrow$ 3042

$$-\frac{c\left(2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2\right.}{}$$

$$\frac{2bc\int(a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{d}-\frac{(a+b\arccos(cx))^2}{dx}$$

$\downarrow$ 4669

$$\frac{2bc\left(-b\int\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(e^{i\arccos(cx)}\right)(a+\right.}{d}$$

$$\frac{c\left(2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\right.}{}$$

$$\frac{(a+b\arccos(cx))^2}{dx}$$

$\downarrow$ 2715

$$\frac{2bc\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2i\,\text{ar}\right.}{d}$$

$$\frac{c\left(2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\right.}{}$$

$$\frac{(a+b\arccos(cx))^2}{dx}$$

$\Big\downarrow$ 2838

$$-\frac{c\left(2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2\right.}{}$$

$$\frac{2bc\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\right)}{d}-\frac{(a+b\arccos(cx))^2}{dx}$$

$\Big\downarrow$ 7143

$$\frac{2bc\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\right)}{d}-$$

$$\frac{c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\,\text{PolyLog}\left(3,\right.\right.\right.}{d}$$

$$\frac{(a+b\arccos(cx))^2}{dx}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^2*(d - c^2*d*x^2)),x]
```

output
```
-((a + b*ArcCos[c*x])^2/(d*x)) + (2*b*c*((-2*I)*(a + b*ArcCos[c*x])*ArcTan
[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*b*PolyLog
[2, I*E^(I*ArcCos[c*x])]))/d - (c*(-2*(a + b*ArcCos[c*x])^2*ArcTanh[E^(I*A
rcCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, -E^(I*ArcCos[c*x])] -
 b*PolyLog[3, -E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2,
 E^(I*ArcCos[c*x])] - b*PolyLog[3, E^(I*ArcCos[c*x])])))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.90

method	result
parts	$-\frac{a^2\left(\frac{c\ln(cx-1)}{2}+\frac{1}{x}-\frac{c\ln(cx+1)}{2}\right)}{d}-\frac{b^2c\left(\frac{\arccos(cx)^2}{cx}+\arccos(cx)^2\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)-\arccos(cx)^2\ln\left(1+cx+i\right.\right.}{}$
derivativedivides	$c\left(-\frac{a^2\left(\frac{\ln(cx-1)}{2}+\frac{1}{cx}-\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b^2\left(\frac{\arccos(cx)^2}{cx}+\arccos(cx)^2\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)-\arccos(cx)^2\ln\left(1+cx+\right.\right.}{}\right.$
default	$c\left(-\frac{a^2\left(\frac{\ln(cx-1)}{2}+\frac{1}{cx}-\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b^2\left(\frac{\arccos(cx)^2}{cx}+\arccos(cx)^2\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)-\arccos(cx)^2\ln\left(1+cx+\right.\right.}{}\right.$

input
```
int((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-a^2/d*(1/2*c*ln(c*x-1)+1/x-1/2*c*ln(c*x+1))-b^2/d*c*(arccos(c*x)^2/c/x+ar
ccos(c*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-arccos(c*x)^2*ln(1+c*x+I*(-c^2*
x^2+1)^(1/2))+2*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*arccos(c*
x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2)
)-2*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))-2*I*arccos(c*x)*polylog(2,c*x+I*(
-c^2*x^2+1)^(1/2))+2*I*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-2*
I*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*I*dilog(1-I*(c*x+I*(-c^2*x^2+1)^
(1/2))))-2*a*b/d*c*(arccos(c*x)/c/x+I*dilog(1+c*x+I*(-c^2*x^2+1)^(1/2))+2*
I*arctan(c*x+I*(-c^2*x^2+1)^(1/2))+I*dilog(c*x+I*(-c^2*x^2+1)^(1/2))-arcco
s(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\,(d-c^2dx^2)}\,dx = \int -\frac{\left(b\arccos\left(cx\right)+a\right)^2}{(c^2dx^2-d)x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^2*d*x^4 - d*x^2
), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\,(d-c^2dx^2)}\,dx = -\frac{\int \frac{a^2}{c^2x^4-x^2}\,dx + \int \frac{b^2\operatorname{acos}^2\left(cx\right)}{c^2x^4-x^2}\,dx + \int \frac{2ab\operatorname{acos}\left(cx\right)}{c^2x^4-x^2}\,dx}{d}$$

input
```
integrate((a+b*acos(c*x))**2/x**2/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a**2/(c**2*x**4 - x**2), x) + Integral(b**2*acos(c*x)**2/(c**2*
x**4 - x**2), x) + Integral(2*a*b*acos(c*x)/(c**2*x**4 - x**2), x))/d
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)}\,dx = \int -\frac{(b\arccos(cx)+a)^2}{(c^2dx^2-d)x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a^2*(c*log(c*x + 1)/d - c*log(c*x - 1)/d - 2/(d*x)) + 1/2*((b^2*c*x*lo
g(c*x + 1) - b^2*c*x*log(-c*x + 1) - 2*b^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*
x + 1), c*x)^2 - 2*d*x*integrate(((b^2*c^2*x^2*log(c*x + 1) - b^2*c^2*x^2*
log(-c*x + 1) - 2*b^2*c*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x +
 1)*sqrt(-c*x + 1), c*x) + 2*a*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x
))/(c^2*d*x^4 - d*x^2), x))/(d*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x^2\,(d-c^2\,d\,x^2)}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^2*(d - c^2*d*x^2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x^2*(d - c^2*d*x^2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)}\,dx = \frac{-4\left(\int \frac{acos(cx)}{c^2x^4-x^2}dx\right)abx-2\left(\int \frac{acos(cx)^2}{c^2x^4-x^2}dx\right)b^2x-\log(c^2x-c)\,a^2cx+\log(c^2x+c)\,a^2cx-2a^2}{2dx}$$

input
```
int((a+b*acos(c*x))^2/x^2/(-c^2*d*x^2+d),x)
```

output
```
( - 4*int(acos(c*x)/(c**2*x**4 - x**2),x)*a*b*x - 2*int(acos(c*x)**2/(c**2
*x**4 - x**2),x)*b**2*x - log(c**2*x - c)*a**2*c*x + log(c**2*x + c)*a**2*
c*x - 2*a**2)/(2*d*x)
```

3.192 $\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 210

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)}\,dx = -\frac{bc\sqrt{1-c^2x^2}(a+b\arccos(cx))}{dx} - \frac{(a+b\arccos(cx))^2}{2dx^2} - \frac{2c^2(a+b\arccos(cx))^2\text{arctanh}\left(e^{2i\arccos(cx)}\right)}{d} + \frac{b^2c^2\log(x)}{d} + \frac{ibc^2(a+b\arccos(cx))\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{d} - \frac{ibc^2(a+b\arccos(cx))\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{d} - \frac{b^2c^2\,\text{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)}{2d} + \frac{b^2c^2\,\text{PolyLog}\left(3,e^{2i\arccos(cx)}\right)}{2d}$$

output
```
-b*c*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/d/x-1/2*(a+b*arccos(c*x))^2/d/x^
2-2*c^2*(a+b*arccos(c*x))^2*arctanh((c*x+I*(-c^2*x^2+1)^(1/2))^2)/d+b^2*c^
2*ln(x)/d+I*b*c^2*(a+b*arccos(c*x))*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^
2)/d-I*b*c^2*(a+b*arccos(c*x))*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d-1
/2*b^2*c^2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d+1/2*b^2*c^2*polylog(
3,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 453 vs. 2(210) = 420.

Time = 0.74 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.16

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)}\,dx = -\frac{-\frac{1}{12}ib^2c^2\pi^3+\frac{a^2}{x^2}-\frac{2abc\sqrt{1-c^2x^2}}{x}+\frac{2ab\arccos(cx)}{x^2}-\frac{2b^2c\sqrt{1-c^2x^2}\arccos(cx)}{x}+\frac{b^2\arccos(cx)^2}{x^2}+\frac{4}{3}ib^2c^2\arccos(cx)^3 -}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^3*(d - c^2*d*x^2)),x]
```

output
```
-1/2*((-1/12*I)*b^2*c^2*Pi^3 + a^2/x^2 - (2*a*b*c*Sqrt[1 - c^2*x^2])/x + (
2*a*b*ArcCos[c*x])/x^2 - (2*b^2*c*Sqrt[1 - c^2*x^2]*ArcCos[c*x])/x + (b^2*
ArcCos[c*x]^2)/x^2 + ((4*I)/3)*b^2*c^2*ArcCos[c*x]^3 + 4*a*b*c^2*ArcCos[c*
x]*Log[1 - E^(I*ArcCos[c*x])] + 4*a*b*c^2*ArcCos[c*x]*Log[1 + E^(I*ArcCos[
c*x])] + 2*b^2*c^2*ArcCos[c*x]^2*Log[1 - E^((-2*I)*ArcCos[c*x])] - 4*a*b*c
^2*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] - 2*b^2*c^2*ArcCos[c*x]^2*Lo
g[1 + E^((2*I)*ArcCos[c*x])] - 2*a^2*c^2*Log[x] - 2*b^2*c^2*Log[c*x] + a^2
*c^2*Log[1 - c^2*x^2] - (4*I)*a*b*c^2*PolyLog[2, -E^(I*ArcCos[c*x])] - (4*
I)*a*b*c^2*PolyLog[2, E^(I*ArcCos[c*x])] + (2*I)*b^2*c^2*ArcCos[c*x]*PolyL
og[2, E^((-2*I)*ArcCos[c*x])] + (2*I)*a*b*c^2*PolyLog[2, -E^((2*I)*ArcCos[
c*x])] + (2*I)*b^2*c^2*ArcCos[c*x]*PolyLog[2, -E^((2*I)*ArcCos[c*x])] + b^
2*c^2*PolyLog[3, E^((-2*I)*ArcCos[c*x])] - b^2*c^2*PolyLog[3, -E^((2*I)*Ar
cCos[c*x])])/d
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {5205, 27, 5185, 4919, 3042, 4671, 3011, 2720, 5187, 14, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)}dx$$

$$\Big\downarrow \text{5205}$$

$$c^2\int \frac{(a+b\arccos(cx))^2}{dx\left(1-c^2x^2\right)}dx-\frac{bc\int \frac{a+b\arccos(cx)}{x^2\sqrt{1-c^2x^2}}dx}{d}-\frac{(a+b\arccos(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}dx}{d}-\frac{bc\int \frac{a+b\arccos(cx)}{x^2\sqrt{1-c^2x^2}}dx}{d}-\frac{(a+b\arccos(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{5185}$$

$$-\frac{c^2\int \frac{(a+b\arccos(cx))^2}{cx\sqrt{1-c^2x^2}}d\arccos(cx)}{d}-\frac{bc\int \frac{a+b\arccos(cx)}{x^2\sqrt{1-c^2x^2}}dx}{d}-\frac{(a+b\arccos(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{4919}$$

$$-\frac{bc\int \frac{a+b\arccos(cx)}{x^2\sqrt{1-c^2x^2}}dx}{d}-\frac{2c^2\int (a+b\arccos(cx))^2\csc(2\arccos(cx))d\arccos(cx)}{d}-\frac{(a+b\arccos(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{bc\int \frac{a+b\arccos(cx)}{x^2\sqrt{1-c^2x^2}}dx}{d}-\frac{2c^2\int (a+b\arccos(cx))^2\csc(2\arccos(cx))d\arccos(cx)}{d}-\frac{(a+b\arccos(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{2c^2\left(-b\int (a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+b\int (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)d\text{arc}\right.}{d}$$
$$\frac{bc\int \frac{a+b\arccos(cx)}{x^2\sqrt{1-c^2x^2}}dx}{d}-\frac{(a+b\arccos(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{2c^2\left(b\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int \text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-b\left(\frac{1}{2}i\,\text{I}\right.\right.}{}$$
$$\frac{bc\int \frac{a+b\arccos(cx)}{x^2\sqrt{1-c^2x^2}}dx}{d}-\frac{(a+b\arccos(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{2c^2\left(b\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)de^{2i\arccos}\right.\right.}{}$$

$$\frac{bc\int\frac{a+b\arccos(cx)}{x^2\sqrt{1-c^2x^2}}dx}{d}-\frac{(a+b\arccos(cx))^2}{2dx^2}$$

$$\Big\downarrow \ 5187$$

$$-\frac{2c^2\left(b\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)de^{2i\arccos}\right.\right.}{}$$

$$\frac{bc\left(-bc\int\frac{1}{x}dx-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}\right)}{d}-\frac{(a+b\arccos(cx))^2}{2dx^2}$$

$$\Big\downarrow \ 14$$

$$-\frac{2c^2\left(b\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)de^{2i\arccos}\right.\right.}{}$$

$$\frac{bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}-bc\log(x)\right)}{d}-\frac{(a+b\arccos(cx))^2}{2dx^2}$$

$$\Big\downarrow \ 7143$$

$$-\frac{2c^2\left(-\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2+b\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\operatorname{PolyL}\right.\right.}{d}$$

$$\frac{bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x}-bc\log(x)\right)}{d}-\frac{(a+b\arccos(cx))^2}{2dx^2}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^3*(d - c^2*d*x^2)),x]
```

output
```
-1/2*(a + b*ArcCos[c*x])^2/(d*x^2) - (b*c*(-((Sqrt[1 - c^2*x^2]*(a + b*Arc
Cos[c*x]))/x) - b*c*Log[x]))/d - (2*c^2*(-((a + b*ArcCos[c*x])^2*ArcTanh[E
^((2*I)*ArcCos[c*x])]) + b*((I/2)*(a + b*ArcCos[c*x])*PolyLog[2, -E^((2*I)
*ArcCos[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/4) - b*((I/2)*(a +
 b*ArcCos[c*x])*PolyLog[2, E^((2*I)*ArcCos[c*x])] - (b*PolyLog[3, E^((2*I)
*ArcCos[c*x])])/4)))/d
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(255) = 510$.

Time = 0.56 (sec) , antiderivative size = 602, normalized size of antiderivative = 2.87

method	result
derivativedivides	$c^2\left(-\frac{a^2\left(\frac{\ln(cx-1)}{2}+\frac{1}{2c^2x^2}-\ln(cx)+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b^2\left(\frac{\arccos(cx)\left(-2ic^2x^2-2cx\sqrt{-c^2x^2+1}+\arccos(cx)\right)}{2c^2x^2}-\ln\left(1+\left(cx+\right.\right.\right.}{}\right.$
default	$c^2\left(-\frac{a^2\left(\frac{\ln(cx-1)}{2}+\frac{1}{2c^2x^2}-\ln(cx)+\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b^2\left(\frac{\arccos(cx)\left(-2ic^2x^2-2cx\sqrt{-c^2x^2+1}+\arccos(cx)\right)}{2c^2x^2}-\ln\left(1+\left(cx+\right.\right.\right.}{}\right.$
parts	$-\frac{a^2\left(\frac{1}{2x^2}-c^2\ln(x)+\frac{c^2\ln(cx-1)}{2}+\frac{c^2\ln(cx+1)}{2}\right)}{d}-\frac{b^2c^2\left(\frac{\arccos(cx)\left(-2ic^2x^2-2cx\sqrt{-c^2x^2+1}+\arccos(cx)\right)}{2c^2x^2}-\ln\left(1+\left(cx\right.\right.\right.}{}$

input
```
int((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
c^2*(-a^2/d*(1/2*ln(c*x-1)+1/2/c^2/x^2-ln(c*x)+1/2*ln(c*x+1))-b^2/d*(1/2*a
rccos(c*x)*(-2*I*c^2*x^2-2*c*x*(-c^2*x^2+1)^(1/2)+arccos(c*x))/c^2/x^2-ln(
1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+2*ln(c*x+I*(-c^2*x^2+1)^(1/2))-arccos(c*x)
^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+I*arccos(c*x)*polylog(2,-(c*x+I*(-c^
2*x^2+1)^(1/2))^2)-1/2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+arccos(c*x
)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-2*I*arccos(c*x)*polylog(2,-c*x-I*(-c^2*
x^2+1)^(1/2))+2*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))+arccos(c*x)^2*ln(1-c*
x-I*(-c^2*x^2+1)^(1/2))-2*I*arccos(c*x)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)
)+2*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2)))-2*a*b/d*(1/2*(-I*c^2*x^2-c*x*(-c^
2*x^2+1)^(1/2)+arccos(c*x))/c^2/x^2+arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1
/2))-I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-arccos(c*x)*ln(1+(c*x+I*(-c^2*
x^2+1)^(1/2))^2)+1/2*I*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+arccos(c*x
)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-I*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)}\,dx=\int -\frac{\left(b\arccos\left(cx\right)+a\right)^2}{(c^2dx^2-d)x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^2*d*x^5 - d*x^3
), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)}\,dx=-\frac{\int \frac{a^2}{c^2x^5-x^3}\,dx+\int \frac{b^2\operatorname{acos}^2(cx)}{c^2x^5-x^3}\,dx+\int \frac{2ab\operatorname{acos}(cx)}{c^2x^5-x^3}\,dx}{d}$$

input
```
integrate((a+b*acos(c*x))**2/x**3/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a**2/(c**2*x**5 - x**3), x) + Integral(b**2*acos(c*x)**2/(c**2*
x**5 - x**3), x) + Integral(2*a*b*acos(c*x)/(c**2*x**5 - x**3), x))/d
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)}\,dx=\int -\frac{(b\arccos(cx)+a)^2}{(c^2dx^2-d)x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*(c^2*log(c*x + 1)/d + c^2*log(c*x - 1)/d - 2*c^2*log(x)/d + 1/(d*x^2)
)*a^2 - integrate((b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*
b*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/(c^2*d*x^5 - d*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)}\,dx = \int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{x^3\,\left(d-c^2\,d\,x^2\right)}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^3*(d - c^2*d*x^2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x^3*(d - c^2*d*x^2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)}\,dx$$
$$= \frac{-4\left(\int \frac{acos(cx)}{c^2x^5-x^3}dx\right)ab\,x^2-2\left(\int \frac{acos(cx)^2}{c^2x^5-x^3}dx\right)b^2x^2-\log(c^2x-c)\,a^2c^2x^2-\log(c^2x+c)\,a^2c^2x^2+2\log(x)\,a^2\cdots}{2d\,x^2}$$

input
```
int((a+b*acos(c*x))^2/x^3/(-c^2*d*x^2+d),x)
```

output
```
( - 4*int(acos(c*x)/(c**2*x**5 - x**3),x)*a*b*x**2 - 2*int(acos(c*x)**2/(c
**2*x**5 - x**3),x)*b**2*x**2 - log(c**2*x - c)*a**2*c**2*x**2 - log(c**2*
x + c)*a**2*c**2*x**2 + 2*log(x)*a**2*c**2*x**2 - a**2)/(2*d*x**2)
```

3.193 $\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 333

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)}\,dx = & -\frac{b^2c^2}{3dx}-\frac{bc\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3dx^2}\\ & -\frac{(a+b\arccos(cx))^2}{3dx^3}-\frac{c^2(a+b\arccos(cx))^2}{dx}\\ & -\frac{2ic^3(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)}{d}\\ & -\frac{14bc^3(a+b\arccos(cx))\text{arctanh}\left(e^{i\arccos(cx)}\right)}{3d}\\ & +\frac{7ib^2c^3\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{3d}\\ & +\frac{2ibc^3(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{d}\\ & -\frac{2ibc^3(a+b\arccos(cx))\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{d}\\ & -\frac{7ib^2c^3\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{3d}\\ & -\frac{2b^2c^3\operatorname{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)}{d}\\ & +\frac{2b^2c^3\operatorname{PolyLog}\left(3,ie^{i\arccos(cx)}\right)}{d}\end{aligned}$$

output
```
-1/3*b^2*c^2/d/x-1/3*b*c*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/d/x^2-1/3*(a
+b*arccos(c*x))^2/d/x^3-c^2*(a+b*arccos(c*x))^2/d/x-2*I*c^3*(a+b*arccos(c*
x))^2*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/d-14/3*b*c^3*(a+b*arccos(c*x))*arct
anh(c*x+I*(-c^2*x^2+1)^(1/2))/d+7/3*I*b^2*c^3*polylog(2,-c*x-I*(-c^2*x^2+1
)^(1/2))/d+2*I*b*c^3*(a+b*arccos(c*x))*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1
/2)))/d-2*I*b*c^3*(a+b*arccos(c*x))*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2))
)/d-7/3*I*b^2*c^3*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/d-2*b^2*c^3*polylog(
3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d+2*b^2*c^3*polylog(3,I*(c*x+I*(-c^2*x^2+
1)^(1/2)))/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 739 vs. 2(333) = 666.

Time = 7.32 (sec) , antiderivative size = 739, normalized size of antiderivative = 2.22

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^4*(d - c^2*d*x^2)),x]
```

output
```
-1/3*a^2/(d*x^3) - (a^2*c^2)/(d*x) - (a^2*c^3*Log[1 - c*x])/(2*d) + (a^2*c
^3*Log[1 + c*x])/(2*d) - (2*a*b*(-1/6*(c*Sqrt[1 - c^2*x^2])/x^2 + ArcCos[c
*x]/(3*x^3) + (c^3*Log[x])/6 - (c^3*Log[1 + Sqrt[1 - c^2*x^2]])/6 - c^2*(-
(ArcCos[c*x]/x) - c*Log[x] + c*Log[1 + Sqrt[1 - c^2*x^2]]) - (c^4*(((-1/2*
I)*ArcCos[c*x]^2)/c + (2*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])])/c - ((2*I
)*PolyLog[2, -E^(I*ArcCos[c*x])])/c))/2 - (I/4)*c^3*(ArcCos[c*x]*(ArcCos[c
*x] + (4*I)*Log[1 - E^(I*ArcCos[c*x])]) + 4*PolyLog[2, E^(I*ArcCos[c*x])])
))/d - (b^2*c^3*(4 - ((-2 + ArcCos[c*x])*ArcCos[c*x])/(-1 + Sqrt[1 - c^2*x
^2]) + 14*ArcCos[c*x]^2 + 12*ArcCos[c*x]^2*(Log[1 - E^(I*ArcCos[c*x])] - L
og[1 + E^(I*ArcCos[c*x])]) - 28*(ArcCos[c*x]*(Log[1 - I*E^(I*ArcCos[c*x])]
 - Log[1 + I*E^(I*ArcCos[c*x])]) + I*(PolyLog[2, (-I)*E^(I*ArcCos[c*x])] -
 PolyLog[2, I*E^(I*ArcCos[c*x])])) + (24*I)*ArcCos[c*x]*(PolyLog[2, -E^(I*
ArcCos[c*x])] - PolyLog[2, E^(I*ArcCos[c*x])]) + 24*(-PolyLog[3, -E^(I*Arc
Cos[c*x])] + PolyLog[3, E^(I*ArcCos[c*x])]) + (2*ArcCos[c*x]^2*Sin[ArcCos[
c*x]/2])/(Cos[ArcCos[c*x]/2] - Sin[ArcCos[c*x]/2])^3 + (2*(2 + 7*ArcCos[c*
x]^2)*Sin[ArcCos[c*x]/2])/(Cos[ArcCos[c*x]/2] - Sin[ArcCos[c*x]/2]) - (2*A
rcCos[c*x]^2*Sin[ArcCos[c*x]/2])/(Cos[ArcCos[c*x]/2] + Sin[ArcCos[c*x]/2])
^3 + (ArcCos[c*x]*(2 + ArcCos[c*x]))/(Cos[ArcCos[c*x]/2] + Sin[ArcCos[c*x]
/2])^2 - (2*(2 + 7*ArcCos[c*x]^2)*Sin[ArcCos[c*x]/2])/(Cos[ArcCos[c*x]/2]
+ Sin[ArcCos[c*x]/2])))/(12*d)
```

Rubi [A] (verified)

Time = 2.59 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5205, 27, 5205, 15, 5165, 3042, 4671, 3011, 2720, 5219, 3042, 4669, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)}dx$$

$$\Big\downarrow \; 5205$$

$$c^2\int \frac{(a+b\arccos(cx))^2}{dx^2(1-c^2x^2)}dx-\frac{2bc\int \frac{a+b\arccos(cx)}{x^3\sqrt{1-c^2x^2}}dx}{3d}-\frac{(a+b\arccos(cx))^2}{3dx^3}$$

$$\Big\downarrow \; 27$$

$$\frac{c^2\int \frac{(a+b\arccos(cx))^2}{x^2(1-c^2x^2)}dx}{d}-\frac{2bc\int \frac{a+b\arccos(cx)}{x^3\sqrt{1-c^2x^2}}dx}{3d}-\frac{(a+b\arccos(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{5205}$$

$$\frac{c^2\left(c^2\int \frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx-2bc\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{(a+b\arccos(cx))^2}{x}\right)}{d}-\frac{2bc\left(\frac{1}{2}c^2\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{1}{2}bc\int \frac{1}{x^2}dx-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}\right)}{3d}-\frac{(a+b\arccos(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{15}$$

$$\frac{c^2\left(c^2\int \frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx-2bc\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{(a+b\arccos(cx))^2}{x}\right)}{d}-\frac{2bc\left(\frac{1}{2}c^2\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}+\frac{bc}{2x}\right)}{3d}-\frac{(a+b\arccos(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{5165}$$

$$\frac{c^2\left(-2bc\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-c\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)-\frac{(a+b\arccos(cx))^2}{x}\right)}{d}-\frac{2bc\left(\frac{1}{2}c^2\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}+\frac{bc}{2x}\right)}{3d}-\frac{(a+b\arccos(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2bc\left(\frac{1}{2}c^2\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}+\frac{bc}{2x}\right)}{3d}+\frac{c^2\left(-2bc\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-c\int (a+b\arccos(cx))^2\csc(\arccos(cx))d\arccos(cx)-\frac{(a+b\arccos(cx))^2}{x}\right)}{d}-\frac{(a+b\arccos(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{4671}$$

$$\frac{c^2\left(-c\left(-2b\int (a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\text{ a}\right.\right.}{d}$$

$$\frac{2bc\left(\frac{1}{2}c^2\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}+\frac{bc}{2x}\right)}{3d}-\frac{(a+b\arccos(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{3011}$$

$$\frac{c^2\left(-c\left(2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\,\text{Pol}\right.\right.\right.}{}$$

$$\frac{2bc\left(\frac{1}{2}c^2\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}+\frac{bc}{2x}\right)}{3d}-\frac{(a+b\arccos(cx))^2}{3dx^3}$$

$\downarrow$ 2720

$$\frac{c^2\left(-c\left(2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)\right.\right.}{}$$

$$\frac{2bc\left(\frac{1}{2}c^2\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}+\frac{bc}{2x}\right)}{3d}-\frac{(a+b\arccos(cx))^2}{3dx^3}$$

$\downarrow$ 5219

$$\frac{c^2\left(-c\left(2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)\right.\right.}{}$$

$$\frac{2bc\left(-\frac{1}{2}c^2\int\frac{a+b\arccos(cx)}{cx}d\arccos(cx)-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}+\frac{bc}{2x}\right)}{3d}-\frac{(a+b\arccos(cx))^2}{3dx^3}$$

$\downarrow$ 3042

$$\frac{c^2\left(-c\left(2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)\right.\right.}{}$$

$$\frac{2bc\left(-\frac{1}{2}c^2\int(a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2x^2}+\frac{bc}{2x}\right)}{3d}-\frac{(a+b\arccos(cx))^2}{3dx^3}$$

$\downarrow$ 4669

$$\frac{c^2\left(2bc\left(-b\int\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(e^{i\arccos(cx)}\right)\left(\right.\right.\right.}{}$$

$$\frac{2bc\left(-\frac{1}{2}c^2\left(-b\int\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(e^{i\arccos(cx)}\right.\right.\right.}{3d}$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3}$$

$\downarrow$ 2715

$$\frac{c^2\left(2bc\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\right.}{}$$

$$\frac{2bc\left(-\frac{1}{2}c^2\left(ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right.}{3d}$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3}$$

$$\Big\downarrow \; 2838$$

$$\frac{c^2\left(-c\left(2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)\right.\right.}{}$$

$$\frac{2bc\left(-\frac{1}{2}c^2\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right.\right.\right.}{3d}$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3}$$

$$\Big\downarrow \; 7143$$

$$\frac{c^2\left(2bc\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\right)\right.}{}$$

$$\frac{2bc\left(-\frac{1}{2}c^2\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-ib\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right.\right.\right.}{3d}$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^4*(d - c^2*d*x^2)),x]
```

output
```
-1/3*(a + b*ArcCos[c*x])^2/(d*x^3) - (2*b*c*((b*c)/(2*x) - (Sqrt[1 - c^2*x
^2]*(a + b*ArcCos[c*x]))/(2*x^2) - (c^2*((-2*I)*(a + b*ArcCos[c*x])*ArcTan
[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*b*PolyLog
[2, I*E^(I*ArcCos[c*x])]))/2))/(3*d) + (c^2*(-((a + b*ArcCos[c*x])^2/x) +
2*b*c*((-2*I)*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcCos[c*x])] + I*b*PolyLog[
2, (-I)*E^(I*ArcCos[c*x])] - I*b*PolyLog[2, I*E^(I*ArcCos[c*x])]) - c*(-2*
(a + b*ArcCos[c*x])^2*ArcTanh[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*
x])*PolyLog[2, -E^(I*ArcCos[c*x])] - b*PolyLog[3, -E^(I*ArcCos[c*x])]) - 2
*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] - b*PolyLog[3, E^(
I*ArcCos[c*x])]))))/d
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.68

method	result
parts	$-\frac{a^2\left(\frac{1}{3x^3}+\frac{c^2}{x}+\frac{c^3\ln(cx-1)}{2}-\frac{c^3\ln(cx+1)}{2}\right)}{d}-\frac{b^2c^3\left(\frac{3\arccos(cx)^2x^2c^2-\sqrt{-c^2x^2+1}\,\arccos(cx)xc+\arccos(cx)^2+c^2x^2}{3c^3x^3}+\frac{7\,\mathrm{a}}{}\right.}{}$
derivativedivides	$c^3\left(-\frac{a^2\left(\frac{\ln(cx-1)}{2}+\frac{1}{3c^3x^3}+\frac{1}{cx}-\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b^2\left(\frac{3\arccos(cx)^2x^2c^2-\sqrt{-c^2x^2+1}\,\arccos(cx)xc+\arccos(cx)^2+c^2x^2}{3c^3x^3}+\frac{7\,\mathrm{a}}{}\right.}{}\right.$
default	$c^3\left(-\frac{a^2\left(\frac{\ln(cx-1)}{2}+\frac{1}{3c^3x^3}+\frac{1}{cx}-\frac{\ln(cx+1)}{2}\right)}{d}-\frac{b^2\left(\frac{3\arccos(cx)^2x^2c^2-\sqrt{-c^2x^2+1}\,\arccos(cx)xc+\arccos(cx)^2+c^2x^2}{3c^3x^3}+\frac{7\,\mathrm{a}}{}\right.}{}\right.$

input
```
int((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-a^2/d*(1/3/x^3+c^2/x+1/2*c^3*ln(c*x-1)-1/2*c^3*ln(c*x+1))-b^2/d*c^3*(1/3*
(3*arccos(c*x)^2*x^2*c^2-(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c+arccos(c*x)^2+
c^2*x^2)/c^3/x^3+7/3*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-7/3*ar
ccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-7/3*I*dilog(1+I*(c*x+I*(-c^2*
x^2+1)^(1/2)))+7/3*I*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-arccos(c*x)^2*l
n(1+c*x+I*(-c^2*x^2+1)^(1/2))+2*I*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1
)^(1/2))-2*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))+arccos(c*x)^2*ln(1-c*x-I*(
-c^2*x^2+1)^(1/2))-2*I*arccos(c*x)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+2*p
olylog(3,c*x+I*(-c^2*x^2+1)^(1/2)))-1/3*I*a*b/d/x^3*(6*I*arccos(c*x)*ln(1+
c*x+I*(-c^2*x^2+1)^(1/2))*c^3*x^3-6*I*arccos(c*x)*c^2*x^2+14*arctan(c*x+I*
(-c^2*x^2+1)^(1/2))*c^3*x^3+6*dilog(1+c*x+I*(-c^2*x^2+1)^(1/2))*c^3*x^3+6*
dilog(c*x+I*(-c^2*x^2+1)^(1/2))*c^3*x^3+I*(-c^2*x^2+1)^(1/2)*c*x-2*I*arcco
s(c*x))
```

Fricas [F]

$$\int \frac{(a + b\arccos(cx))^2}{x^4(d - c^2dx^2)}\,dx = \int -\frac{(b\arccos(cx) + a)^2}{(c^2dx^2 - d)x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^2*d*x^6 - d*x^4
), x)
```

Sympy [F]

$$\int \frac{(a + b\arccos(cx))^2}{x^4(d - c^2dx^2)}\,dx = -\frac{\int \frac{a^2}{c^2x^6 - x^4}\,dx + \int \frac{b^2\operatorname{acos}^2(cx)}{c^2x^6 - x^4}\,dx + \int \frac{2ab\operatorname{acos}(cx)}{c^2x^6 - x^4}\,dx}{d}$$

input
```
integrate((a+b*acos(c*x))**2/x**4/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a**2/(c**2*x**6 - x**4), x) + Integral(b**2*acos(c*x)**2/(c**2*
x**6 - x**4), x) + Integral(2*a*b*acos(c*x)/(c**2*x**6 - x**4), x))/d
```

Maxima [F]

$$\int \frac{(a + b\arccos(cx))^2}{x^4(d - c^2dx^2)}\,dx = \int -\frac{(b\arccos(cx) + a)^2}{(c^2dx^2 - d)x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
1/6*(3*c^3*log(c*x + 1)/d - 3*c^3*log(c*x - 1)/d - 2*(3*c^2*x^2 + 1)/(d*x^
3))*a^2 - 1/6*(6*d*x^3*integrate(1/3*((3*b^2*c^4*x^4*log(c*x + 1) - 3*b^2*
c^4*x^4*log(-c*x + 1) - 6*b^2*c^3*x^3 - 2*b^2*c*x)*sqrt(c*x + 1)*sqrt(-c*x
 + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 6*a*b*arctan2(sqrt(c*x
+ 1)*sqrt(-c*x + 1), c*x))/(c^2*d*x^6 - d*x^4), x) - (3*b^2*c^3*x^3*log(c*
x + 1) - 3*b^2*c^3*x^3*log(-c*x + 1) - 6*b^2*c^2*x^2 - 2*b^2)*arctan2(sqrt
(c*x + 1)*sqrt(-c*x + 1), c*x)^2)/(d*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b \arccos(cx))^2}{x^4 (d - c^2 d x^2)} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arccos(cx))^2}{x^4 (d - c^2 d x^2)} dx = \int \frac{(a + b \operatorname{acos}(c x))^2}{x^4 (d - c^2 d x^2)} dx$$

input
```
int((a + b*acos(c*x))^2/(x^4*(d - c^2*d*x^2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x^4*(d - c^2*d*x^2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4\,(d-c^2dx^2)}\,dx$$
$$= \frac{-12\left(\int \frac{acos(cx)}{c^2x^6-x^4}dx\right)ab\,x^3-6\left(\int \frac{acos(cx)^2}{c^2x^6-x^4}dx\right)b^2x^3-3\log(c^2x-c)\,a^2c^3x^3+3\log(c^2x+c)\,a^2c^3x^3-6a^2c^2 \cdots}{6d\,x^3}$$

input
```
int((a+b*acos(c*x))^2/x^4/(-c^2*d*x^2+d),x)
```

output
```
( - 12*int(acos(c*x)/(c**2*x**6 - x**4),x)*a*b*x**3 - 6*int(acos(c*x)**2/(
c**2*x**6 - x**4),x)*b**2*x**3 - 3*log(c**2*x - c)*a**2*c**3*x**3 + 3*log(
c**2*x + c)*a**2*c**3*x**3 - 6*a**2*c**2*x**2 - 2*a**2)/(6*d*x**3)
```

3.194 $\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 300

$$\begin{aligned}\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx =& -\frac{2b^2x}{c^4d^2}-\frac{b(a+b\arccos(cx))}{c^5d^2\sqrt{1-c^2x^2}}\\ &+\frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^5d^2}\\ &+\frac{3x(a+b\arccos(cx))^2}{2c^4d^2}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}\\ &+\frac{3i(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)}{c^5d^2}\\ &+\frac{b^2\text{arctanh}(cx)}{c^5d^2}\\ &-\frac{3ib(a+b\arccos(cx))\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c^5d^2}\\ &+\frac{3ib(a+b\arccos(cx))\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c^5d^2}\\ &+\frac{3b^2\,\text{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)}{c^5d^2}\\ &-\frac{3b^2\,\text{PolyLog}\left(3,ie^{i\arccos(cx)}\right)}{c^5d^2}\end{aligned}$$

output
```
-2*b^2*x/c^4/d^2-b*(a+b*arccos(c*x))/c^5/d^2/(-c^2*x^2+1)^(1/2)+2*b*(-c^2*
x^2+1)^(1/2)*(a+b*arccos(c*x))/c^5/d^2+3/2*x*(a+b*arccos(c*x))^2/c^4/d^2+1
/2*x^3*(a+b*arccos(c*x))^2/c^2/d^2/(-c^2*x^2+1)+3*I*(a+b*arccos(c*x))^2*ar
ctan(c*x+I*(-c^2*x^2+1)^(1/2))/c^5/d^2+b^2*arctanh(c*x)/c^5/d^2-3*I*b*(a+b
*arccos(c*x))*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^5/d^2+3*I*b*(a+b*
arccos(c*x))*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^5/d^2+3*b^2*polylog
(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^5/d^2-3*b^2*polylog(3,I*(c*x+I*(-c^2*x
^2+1)^(1/2)))/c^5/d^2
```

Mathematica [A] (verified)

Time = 4.67 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.71

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$$
$$= \frac{\frac{8a^2x}{c^4}+\frac{4a^2x}{c^4-c^6x^2}+\frac{6a^2\log(1-cx)}{c^5}-\frac{6a^2\log(1+cx)}{c^5}+\frac{8ab\left(\sqrt{1-c^2x^2}-2c^2x^2\sqrt{1-c^2x^2}-3cx\arccos(cx)+2c^3x^3\arccos(cx)-3\arccos(cx)\,\mathrm{l}\right.}{}}{}$$

input
```
Integrate[(x^4*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^2,x]
```

output
```
((8*a^2*x)/c^4 + (4*a^2*x)/(c^4 - c^6*x^2) + (6*a^2*Log[1 - c*x])/c^5 - (6
*a^2*Log[1 + c*x])/c^5 + (8*a*b*(Sqrt[1 - c^2*x^2] - 2*c^2*x^2*Sqrt[1 - c^
2*x^2] - 3*c*x*ArcCos[c*x] + 2*c^3*x^3*ArcCos[c*x] - 3*ArcCos[c*x]*Log[1 -
 E^(I*ArcCos[c*x])] + 3*c^2*x^2*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] + 3
*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] - 3*c^2*x^2*ArcCos[c*x]*Log[1 + E^
(I*ArcCos[c*x])] + (3*I)*(-1 + c^2*x^2)*PolyLog[2, -E^(I*ArcCos[c*x])] - (
3*I)*(-1 + c^2*x^2)*PolyLog[2, E^(I*ArcCos[c*x])]))/(c^5*(-1 + c^2*x^2)) +
 (b^2*(-16*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + 8*c*x*(-2 + ArcCos[c*x]^2) + 4*
ArcCos[c*x]*Cot[ArcCos[c*x]/2] + ArcCos[c*x]^2*Csc[ArcCos[c*x]/2]^2 - 8*Lo
g[Tan[ArcCos[c*x]/2]] + 12*(ArcCos[c*x]^2*(Log[1 - E^(I*ArcCos[c*x])] - Lo
g[1 + E^(I*ArcCos[c*x])]) + (2*I)*ArcCos[c*x]*(PolyLog[2, -E^(I*ArcCos[c*x
])] - PolyLog[2, E^(I*ArcCos[c*x])]) + 2*(-PolyLog[3, -E^(I*ArcCos[c*x])]
+ PolyLog[3, E^(I*ArcCos[c*x])])) - ArcCos[c*x]^2*Sec[ArcCos[c*x]/2]^2 + 4
*ArcCos[c*x]*Tan[ArcCos[c*x]/2]))/c^5)/(8*d^2)
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5207, 27, 5195, 27, 299, 219, 5211, 5165, 3042, 4671, 3011, 2720, 5183, 24, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$$

$$\Big\downarrow \text{5207}$$

$$\frac{b\int \frac{x^3(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}-\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{d(1-c^2x^2)}dx}{2c^2d}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{1-c^2x^2}dx}{2c^2d^2}+\frac{b\int \frac{x^3(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{5195}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{1-c^2x^2}dx}{2c^2d^2}+\frac{b\left(bc\int \frac{2-c^2x^2}{c^4(1-c^2x^2)}dx+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^4}+\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}\right)}{cd^2}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{1-c^2x^2}dx}{2c^2d^2}+\frac{b\left(\frac{b\int \frac{2-c^2x^2}{1-c^2x^2}dx}{c^3}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^4}+\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}\right)}{cd^2}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{299}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{1-c^2x^2}dx}{2c^2d^2}+\frac{b\left(\frac{b\left(\int \frac{1}{1-c^2x^2}dx+x\right)}{c^3}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^4}+\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}\right)}{cd^2}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \ 219$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{1-c^2x^2}dx}{2c^2d^2}+\frac{b\left(\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^4}+\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{b\left(\frac{\operatorname{arctanh}(cx)}{c}+x\right)}{c^3}\right)}{cd^2}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \ 5211$$

$$-\frac{3\left(-\frac{2b\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{c}+\frac{\int \frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\arccos(cx))^2}{c^2}\right)}{2c^2d^2}+\frac{b\left(\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^4}+\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{b\left(\frac{\operatorname{arctanh}(cx)}{c}+x\right)}{c^3}\right)}{cd^2}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \ 5165$$

$$-\frac{3\left(-\frac{2b\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{c}-\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^3}-\frac{x(a+b\arccos(cx))^2}{c^2}\right)}{2c^2d^2}+\frac{b\left(\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^4}+\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{b\left(\frac{\operatorname{arctanh}(cx)}{c}+x\right)}{c^3}\right)}{cd^2}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \ 3042$$

$$-\frac{3\left(-\frac{\int (a+b\arccos(cx))^2\csc(\arccos(cx))d\arccos(cx)}{c^3}-\frac{2b\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{c}-\frac{x(a+b\arccos(cx))^2}{c^2}\right)}{2c^2d^2}+\frac{b\left(\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^4}+\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{b\left(\frac{\operatorname{arctanh}(cx)}{c}+x\right)}{c^3}\right)}{cd^2}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \ 4671$$

$$-\frac{3\left(-\frac{-2b\int(a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int(a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right.}{c^3}\right.}{2c^2d^2}$$

$$\frac{b\left(\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^4}+\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{b\left(\frac{\text{arctanh}(cx)}{c}+x\right)}{c^3}\right)}{cd^2}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$\big\downarrow$ 3011

$$-\frac{3\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a+b\arccos(\right.}{c^3}\right.}{2}$$

$$\frac{b\left(\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^4}+\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{b\left(\frac{\text{arctanh}(cx)}{c}+x\right)}{c^3}\right)}{cd^2}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$\big\downarrow$ 2720

$$-\frac{3\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right.}{c^3}\right.}{}$$

$$\frac{b\left(\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^4}+\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{b\left(\frac{\text{arctanh}(cx)}{c}+x\right)}{c^3}\right)}{cd^2}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$\big\downarrow$ 5183

$$-\frac{3\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right.}{c^3}\right.}{}$$

$$\frac{b\left(\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^4}+\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{b\left(\frac{\text{arctanh}(cx)}{c}+x\right)}{c^3}\right)}{cd^2}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$\big\downarrow$ 24

$$-\frac{3\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right.}{c^3}}{}$$

$$\frac{b\left(\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^4}+\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{b\left(\frac{\text{arctanh}(cx)}{c}+x\right)}{c^3}\right)}{cd^2}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2\left(1-c^2x^2\right)}$$

$\downarrow$ 7143

$$-\frac{3\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\,\text{PolyLog}\left(3,-e^{i\arccos(cx)}\right)\right)-2b\left(i\,\text{PolyI}\right.}{c^3}}{2c^2d^2}$$

$$\frac{b\left(\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^4}+\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{b\left(\frac{\text{arctanh}(cx)}{c}+x\right)}{c^3}\right)}{cd^2}+\frac{x^3(a+b\arccos(cx))^2}{2c^2d^2\left(1-c^2x^2\right)}$$

input
```
Int[(x^4*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^2,x]
```

output
```
(x^3*(a + b*ArcCos[c*x])^2)/(2*c^2*d^2*(1 - c^2*x^2)) + (b*((a + b*ArcCos[
c*x])/(c^4*Sqrt[1 - c^2*x^2]) + (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c^
4 + (b*(x + ArcTanh[c*x]/c))/c^3))/(c*d^2) - (3*(-((x*(a + b*ArcCos[c*x])^
2)/c^2) - (2*b*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c^2))
/c - (-2*(a + b*ArcCos[c*x])^2*ArcTanh[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*
ArcCos[c*x])*PolyLog[2, -E^(I*ArcCos[c*x])] - b*PolyLog[3, -E^(I*ArcCos[c*
x])]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] - b*PolyL
og[3, E^(I*ArcCos[c*x])]))/c^3))/(2*c^2*d^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.00

method	result
derivativedivides	$\frac{\frac{a^2\left(cx-\frac{1}{4(cx-1)}+\frac{3\ln(cx-1)}{4}-\frac{1}{4(cx+1)}-\frac{3\ln(cx+1)}{4}\right)}{d^2}-\frac{2b^2\arccos(cx)\sqrt{-c^2x^2+1}}{d^2}+\frac{b^2\arccos(cx)^2cx}{d^2}-\frac{2b^2cx}{d^2}-\frac{b^2\arccos(cx)^2c\cdots}{2d^2(c^2x^2-1)}\cdots}{\cdots}$
default	$\frac{\frac{a^2\left(cx-\frac{1}{4(cx-1)}+\frac{3\ln(cx-1)}{4}-\frac{1}{4(cx+1)}-\frac{3\ln(cx+1)}{4}\right)}{d^2}-\frac{2b^2\arccos(cx)\sqrt{-c^2x^2+1}}{d^2}+\frac{b^2\arccos(cx)^2cx}{d^2}-\frac{2b^2cx}{d^2}-\frac{b^2\arccos(cx)^2c\cdots}{2d^2(c^2x^2-1)}\cdots}{\cdots}$
parts	$\frac{a^2\left(\frac{x}{c^4}-\frac{1}{4c^5(cx-1)}+\frac{3\ln(cx-1)}{4c^5}-\frac{1}{4c^5(cx+1)}-\frac{3\ln(cx+1)}{4c^5}\right)}{d^2}+\frac{b^2\arccos(cx)^2x}{d^2c^4}-\frac{2b^2\arccos(cx)\sqrt{-c^2x^2+1}}{d^2c^5}-\frac{2b^2x}{c^4d^2}\cdots$

input
```
int(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(a^2/d^2*(c*x-1/4/(c*x-1)+3/4*ln(c*x-1)-1/4/(c*x+1)-3/4*ln(c*x+1))-2
*b^2/d^2*arccos(c*x)*(-c^2*x^2+1)^(1/2)+b^2/d^2*arccos(c*x)^2*c*x-2*b^2/d^
2*c*x-1/2*b^2/d^2/(c^2*x^2-1)*arccos(c*x)^2*c*x-b^2/d^2/(c^2*x^2-1)*arccos
(c*x)*(-c^2*x^2+1)^(1/2)+2*b^2/d^2*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))-3/2*b
^2/d^2*arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-3*I*b^2/d^2*arccos(c*x
)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))-3*b^2/d^2*polylog(3,-c*x-I*(-c^2*x^2
+1)^(1/2))+3/2*b^2/d^2*arccos(c*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+3*I*a*
b/d^2*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+3*b^2/d^2*polylog(3,c*x+I*(-c^2
*x^2+1)^(1/2))-2*a*b/d^2*(-c^2*x^2+1)^(1/2)+2*a*b/d^2*arccos(c*x)*c*x-a*b/
d^2/(c^2*x^2-1)*arccos(c*x)*c*x-a*b/d^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)+3*a
*b/d^2*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-3*I*a*b/d^2*polylog(2,c*
x+I*(-c^2*x^2+1)^(1/2))-3*a*b/d^2*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2
))+3*I*b^2/d^2*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx=\int \frac{(b\arccos(cx)+a)^2x^4}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^4*arccos(c*x)^2 + 2*a*b*x^4*arccos(c*x) + a^2*x^4)/(c^4*d^
2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \frac{\int \frac{a^2x^4}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{b^2x^4\operatorname{acos}^2(cx)}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{2abx^4\operatorname{acos}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**4*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2*x**4/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b**2*x**4
*acos(c*x)**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**4*acos
(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)^2x^4}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/4*a^2*(2*x/(c^6*d^2*x^2 - c^4*d^2) - 4*x/(c^4*d^2) + 3*log(c*x + 1)/(c^
5*d^2) - 3*log(c*x - 1)/(c^5*d^2)) + 1/4*((4*b^2*c^3*x^3 - 6*b^2*c*x - 3*(
b^2*c^2*x^2 - b^2)*log(c*x + 1) + 3*(b^2*c^2*x^2 - b^2)*log(-c*x + 1))*arc
tan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 4*(c^7*d^2*x^2 - c^5*d^2)*inte
grate(1/2*(4*a*b*c^4*x^4*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - (4*b
^2*c^3*x^3 - 6*b^2*c*x - 3*(b^2*c^2*x^2 - b^2)*log(c*x + 1) + 3*(b^2*c^2*x
^2 - b^2)*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1
)*sqrt(-c*x + 1), c*x))/(c^8*d^2*x^4 - 2*c^6*d^2*x^2 + c^4*d^2), x))/(c^7*
d^2*x^2 - c^5*d^2)
```

Giac [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)^2x^4}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2*x^4/(c^2*d*x^2 - d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \int \frac{x^4\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((x^4*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^2,x)
```

output
```
int((x^4*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \frac{8\left(\int \frac{acos(cx)x^4}{c^4x^4-2c^2x^2+1}dx\right)ab\,c^7x^2 - 8\left(\int \frac{acos(cx)x^4}{c^4x^4-2c^2x^2+1}dx\right)ab\,c^5 + 4\left(\int \frac{acos(cx)^2x^4}{c^4x^4-2c^2x^2+1}dx\right)b^2c^7x^2 - 4\left(\int \frac{acos(cx)^2x^4}{c^4x^4-2c^2x^2+}\right.}{4c^5}$$

input
```
int(x^4*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^2,x)
```

output
```
(8*int((acos(c*x)*x**4)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**7*x**2 - 8
*int((acos(c*x)*x**4)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**5 + 4*int((a
cos(c*x)**2*x**4)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2*c**7*x**2 - 4*int(
(acos(c*x)**2*x**4)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2*c**5 + 3*log(c**
2*x - c)*a**2*c**2*x**2 - 3*log(c**2*x - c)*a**2 - 3*log(c**2*x + c)*a**2*
c**2*x**2 + 3*log(c**2*x + c)*a**2 + 4*a**2*c**3*x**3 - 6*a**2*c*x)/(4*c**
5*d**2*(c**2*x**2 - 1))
```

3.195 $\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 227

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = -\frac{bx(a+b\arccos(cx))}{c^3d^2\sqrt{1-c^2x^2}}+\frac{(a+b\arccos(cx))^2}{2c^4d^2}+\frac{x^2(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}-\frac{i(a+b\arccos(cx))^3}{3bc^4d^2}+\frac{(a+b\arccos(cx))^2\log\left(1+e^{2i\arccos(cx)}\right)}{c^4d^2}-\frac{b^2\log(1-c^2x^2)}{2c^4d^2}-\frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c^4d^2}+\frac{b^2\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)}{2c^4d^2}$$

output
```
-b*x*(a+b*arccos(c*x))/c^3/d^2/(-c^2*x^2+1)^(1/2)+1/2*(a+b*arccos(c*x))^2/
c^4/d^2+1/2*x^2*(a+b*arccos(c*x))^2/c^2/d^2/(-c^2*x^2+1)-1/3*I*(a+b*arccos
(c*x))^3/b/c^4/d^2+(a+b*arccos(c*x))^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/
c^4/d^2-1/2*b^2*ln(-c^2*x^2+1)/c^4/d^2-I*b*(a+b*arccos(c*x))*polylog(2,-(c
*x+I*(-c^2*x^2+1)^(1/2))^2)/c^4/d^2+1/2*b^2*polylog(3,-(c*x+I*(-c^2*x^2+1)
^(1/2))^2)/c^4/d^2
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.52

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$$
$$=\frac{\frac{a^2}{1-c^2x^2}-\frac{ab\left(\sqrt{1-c^2x^2}-\arccos(cx)\right)}{1+cx}-\frac{ab\left(\sqrt{1-c^2x^2}+\arccos(cx)\right)}{-1+cx}+a^2\log\left(1-c^2x^2\right)-iab\left(\arccos(cx)\left(\arccos(cx)+\right.\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^2,x]
```

output
```
(a^2/(1 - c^2*x^2) - (a*b*(Sqrt[1 - c^2*x^2] - ArcCos[c*x]))/(1 + c*x) - (
a*b*(Sqrt[1 - c^2*x^2] + ArcCos[c*x]))/(-1 + c*x) + a^2*Log[1 - c^2*x^2] -
 I*a*b*(ArcCos[c*x]*(ArcCos[c*x] + (4*I)*Log[1 + E^(I*ArcCos[c*x])]) + 4*P
olyLog[2, -E^(I*ArcCos[c*x])]) - I*a*b*(ArcCos[c*x]*(ArcCos[c*x] + (4*I)*L
og[1 - E^(I*ArcCos[c*x])]) + 4*PolyLog[2, E^(I*ArcCos[c*x])]) + 2*b^2*((-1
/24*I)*Pi^3 + (c*x*ArcCos[c*x])/Sqrt[1 - c^2*x^2] + ArcCos[c*x]^2/(2 - 2*c
^2*x^2) + (I/3)*ArcCos[c*x]^3 + ArcCos[c*x]^2*Log[1 - E^((-2*I)*ArcCos[c*x
])] - Log[1 - c^2*x^2]/2 + I*ArcCos[c*x]*PolyLog[2, E^((-2*I)*ArcCos[c*x])
] + PolyLog[3, E^((-2*I)*ArcCos[c*x])]/2))/(2*c^4*d^2)
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {5207, 27, 5181, 3042, 25, 4200, 25, 2620, 3011, 2720, 5207, 240, 5153, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$$
$$\Big\downarrow \ 5207$$
$$\frac{b\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}-\frac{\int \frac{x(a+b\arccos(cx))^2}{d(1-c^2x^2)}dx}{c^2d}+\frac{x^2(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}-\frac{\int \frac{x(a+b\arccos(cx))^2}{1-c^2x^2}dx}{c^2d^2}+\frac{x^2(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{5181}$$

$$\frac{b\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}+\frac{\int \frac{cx(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^4d^2}+\frac{x^2(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -(a+b\arccos(cx))^2\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c^4d^2}+\frac{b\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}+\frac{x^2(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int (a+b\arccos(cx))^2\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c^4d^2}+\frac{b\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}+\frac{x^2(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{4200}$$

$$\frac{2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))^2}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^3}{3b}}{c^4d^2}+\frac{b\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}+\frac{x^2(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))^2}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^3}{3b}}{c^4d^2}+\frac{b\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}+\frac{x^2(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{2620}$$

$$\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\int (a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a}{}}{c^4d^2}$$
$$\frac{b\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}+\frac{x^2(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{3011}$$

$$\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\text{Poly}}{c^4d^2}$$

$$\frac{b\int\frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}+\frac{x^2(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{2720}$$

$$\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\,\text{ar}}}{c^4d^2}$$

$$\frac{b\int\frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}+\frac{x^2(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{5207}$$

$$\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\,\text{ar}}}{c^4d^2}$$

$$\frac{b\left(-\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{c^2}+\frac{b\int\frac{x}{1-c^2x^2}dx}{c}+\frac{x(a+b\arccos(cx))}{c^2\sqrt{1-c^2x^2}}\right)}{cd^2}+\frac{x^2(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{240}$$

$$\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\,\text{ar}}}{c^4d^2}$$

$$\frac{b\left(-\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{c^2}+\frac{x(a+b\arccos(cx))}{c^2\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c^3}\right)}{cd^2}+\frac{x^2(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{5153}$$

$$\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\,\text{ar}}}{c^4d^2}$$

$$\frac{x^2(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{(a+b\arccos(cx))^2}{2bc^3}+\frac{x(a+b\arccos(cx))}{c^2\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c^3}\right)}{cd^2}$$

$$\Big\downarrow \text{7143}$$

$$\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2-ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\text{PolyLog}\right.\right.}{c^4d^2}$$

$$\frac{x^2(a+b\arccos(cx))^2}{2c^2d^2\left(1-c^2x^2\right)}+\frac{b\left(\frac{(a+b\arccos(cx))^2}{2bc^3}+\frac{x(a+b\arccos(cx))}{c^2\sqrt{1-c^2x^2}}-\frac{b\log\left(1-c^2x^2\right)}{2c^3}\right)}{cd^2}$$

input
```
Int[(x^3*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^2,x]
```

output
```
(x^2*(a + b*ArcCos[c*x])^2)/(2*c^2*d^2*(1 - c^2*x^2)) + (b*((x*(a + b*ArcC
os[c*x]))/(c^2*Sqrt[1 - c^2*x^2]) + (a + b*ArcCos[c*x])^2/(2*b*c^3) - (b*L
og[1 - c^2*x^2])/(2*c^3)))/(c*d^2) + (((-1/3*I)*(a + b*ArcCos[c*x])^3)/b -
 (2*I)*((I/2)*(a + b*ArcCos[c*x])^2*Log[1 - E^((2*I)*ArcCos[c*x])] - I*b*(
(I/2)*(a + b*ArcCos[c*x])*PolyLog[2, E^((2*I)*ArcCos[c*x])] - (b*PolyLog[3
, E^((2*I)*ArcCos[c*x])])/4)))/(c^4*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(246) = 492$.

Time = 0.46 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.23

method	result
derivativedivides	$\frac{a^2\left(-\frac{1}{4(cx-1)}+\frac{\ln(cx-1)}{2}+\frac{1}{4cx+4}+\frac{\ln(cx+1)}{2}\right)}{d^2}+\frac{b^2\left(-\frac{i\arccos(cx)^3}{3}-\frac{\left(2ic^2x^2+2cx\sqrt{-c^2x^2+1}-2i+\arccos(cx)\right)\arccos(cx)}{2\left(c^2x^2-1\right)}+\text{arc}\right.}{}$
default	$\frac{a^2\left(-\frac{1}{4(cx-1)}+\frac{\ln(cx-1)}{2}+\frac{1}{4cx+4}+\frac{\ln(cx+1)}{2}\right)}{d^2}+\frac{b^2\left(-\frac{i\arccos(cx)^3}{3}-\frac{\left(2ic^2x^2+2cx\sqrt{-c^2x^2+1}-2i+\arccos(cx)\right)\arccos(cx)}{2\left(c^2x^2-1\right)}+\text{arc}\right.}{}$
parts	$\frac{a^2\left(-\frac{1}{4c^4(cx-1)}+\frac{\ln(cx-1)}{2c^4}+\frac{1}{4c^4(cx+1)}+\frac{\ln(cx+1)}{2c^4}\right)}{d^2}+\frac{b^2\left(-\frac{i\arccos(cx)^3}{3}-\frac{\left(2ic^2x^2+2cx\sqrt{-c^2x^2+1}-2i+\arccos(cx)\right)\text{arccc}}{2\left(c^2x^2-1\right)}\right.}{}$

input
```
int(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a^2/d^2*(-1/4/(c*x-1)+1/2*ln(c*x-1)+1/4/(c*x+1)+1/2*ln(c*x+1))+b^2/
d^2*(-1/3*I*arccos(c*x)^3-1/2*(2*I*c^2*x^2+2*c*x*(-c^2*x^2+1)^(1/2)-2*I+ar
ccos(c*x))*arccos(c*x)/(c^2*x^2-1)+arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(
1/2))-2*I*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+2*polylog(3,-c*
x-I*(-c^2*x^2+1)^(1/2))+arccos(c*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-2*I*a
rccos(c*x)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+2*polylog(3,c*x+I*(-c^2*x^2
+1)^(1/2))+2*ln(c*x+I*(-c^2*x^2+1)^(1/2))-ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-l
n(I*(-c^2*x^2+1)^(1/2)+c*x-1))+2*a*b/d^2*(-1/2*I*arccos(c*x)^2-1/2*(I*c^2*
x^2+c*x*(-c^2*x^2+1)^(1/2)+arccos(c*x)-I)/(c^2*x^2-1)+arccos(c*x)*ln(1+c*x
+I*(-c^2*x^2+1)^(1/2))+arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-I*polylo
g(2,-c*x-I*(-c^2*x^2+1)^(1/2))-I*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx=\int \frac{(b\arccos(cx)+a)^2x^3}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^3*arccos(c*x)^2 + 2*a*b*x^3*arccos(c*x) + a^2*x^3)/(c^4*d^
2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx=\frac{\int \frac{a^2x^3}{c^4x^4-2c^2x^2+1}\,dx+\int \frac{b^2x^3\operatorname{acos}^2(cx)}{c^4x^4-2c^2x^2+1}\,dx+\int \frac{2abx^3\operatorname{acos}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**3*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2*x**3/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b**2*x**3
*acos(c*x)**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**3*acos
(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \int \frac{(b\arccos{(cx)}+a)^2x^3}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*(1/(c^6*d^2*x^2 - c^4*d^2) - log(c^2*x^2 - 1)/(c^4*d^2)) - 1/2*((
b^2 - (b^2*c^2*x^2 - b^2)*log(c*x + 1) - (b^2*c^2*x^2 - b^2)*log(-c*x + 1)
)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*(c^6*d^2*x^2 - c^4*d^2)
*integrate(-(2*a*b*c^3*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + (b
^2 - (b^2*c^2*x^2 - b^2)*log(c*x + 1) - (b^2*c^2*x^2 - b^2)*log(-c*x + 1))
*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/
(c^7*d^2*x^4 - 2*c^5*d^2*x^2 + c^3*d^2), x))/(c^6*d^2*x^2 - c^4*d^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \int \frac{x^3\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((x^3*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^2,x)
```

output
```
int((x^3*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \frac{4\left(\int \frac{acos(cx)x^3}{c^4x^4-2c^2x^2+1}dx\right)ab\,c^6x^2 - 4\left(\int \frac{acos(cx)x^3}{c^4x^4-2c^2x^2+1}dx\right)ab\,c^4 + 2\left(\int \frac{acos(cx)^2x^3}{c^4x^4-2c^2x^2+1}dx\right)b^2c^6x^2 - 2\left(\int \frac{acos(cx)^2x^3}{c^4x^4-2c^2x^2+}\right.}{2c^4d^2\left(c^2x^2\right.}$$

input
```
int(x^3*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^2,x)
```

output
```
(4*int((acos(c*x)*x**3)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**6*x**2 - 4
*int((acos(c*x)*x**3)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**4 + 2*int((a
cos(c*x)**2*x**3)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2*c**6*x**2 - 2*int(
(acos(c*x)**2*x**3)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2*c**4 + log(c**2*
x - c)*a**2*c**2*x**2 - log(c**2*x - c)*a**2 + log(c**2*x + c)*a**2*c**2*x
**2 - log(c**2*x + c)*a**2 - a**2*c**2*x**2)/(2*c**4*d**2*(c**2*x**2 - 1))
```

3.196 $\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 233

$$\begin{aligned}\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx =& -\frac{b(a+b\arccos(cx))}{c^3d^2\sqrt{1-c^2x^2}}+\frac{x(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)} \\ &+\frac{i(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)}{c^3d^2}+\frac{b^2\text{arctanh}(cx)}{c^3d^2} \\ &-\frac{ib(a+b\arccos(cx))\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c^3d^2} \\ &+\frac{ib(a+b\arccos(cx))\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c^3d^2} \\ &+\frac{b^2\,\text{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)}{c^3d^2} \\ &-\frac{b^2\,\text{PolyLog}\left(3,ie^{i\arccos(cx)}\right)}{c^3d^2}\end{aligned}$$

output
```
-b*(a+b*arccos(c*x))/c^3/d^2/(-c^2*x^2+1)^(1/2)+1/2*x*(a+b*arccos(c*x))^2/
c^2/d^2/(-c^2*x^2+1)+I*(a+b*arccos(c*x))^2*arctan(c*x+I*(-c^2*x^2+1)^(1/2)
)/c^3/d^2+b^2*arctanh(c*x)/c^3/d^2-I*b*(a+b*arccos(c*x))*polylog(2,-I*(c*x
+I*(-c^2*x^2+1)^(1/2)))/c^3/d^2+I*b*(a+b*arccos(c*x))*polylog(2,I*(c*x+I*(
-c^2*x^2+1)^(1/2)))/c^3/d^2+b^2*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c
^3/d^2-b^2*polylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^3/d^2
```

Mathematica [A] (verified)

Time = 3.97 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.75

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = -\frac{\frac{4ab\sqrt{1-c^2x^2}}{-1+cx}-\frac{4ab\sqrt{1-c^2x^2}}{1+cx}+\frac{4a^2cx}{-1+c^2x^2}+\frac{4ab\arccos(cx)}{-1+cx}+\frac{4ab\arccos(cx)}{1+cx}-4b^2\arccos(cx)\cot\left(\frac{1}{2}\arccos(cx)\right)-b^2$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^2,x]
```

output
```
-1/8*((4*a*b*Sqrt[1 - c^2*x^2])/(-1 + c*x) - (4*a*b*Sqrt[1 - c^2*x^2])/(1
+ c*x) + (4*a^2*c*x)/(-1 + c^2*x^2) + (4*a*b*ArcCos[c*x])/(-1 + c*x) + (4*
a*b*ArcCos[c*x])/(1 + c*x) - 4*b^2*ArcCos[c*x]*Cot[ArcCos[c*x]/2] - b^2*Ar
cCos[c*x]^2*Csc[ArcCos[c*x]/2]^2 - 8*a*b*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c
*x])] - 4*b^2*ArcCos[c*x]^2*Log[1 - E^(I*ArcCos[c*x])] + 8*a*b*ArcCos[c*x]
*Log[1 + E^(I*ArcCos[c*x])] + 4*b^2*ArcCos[c*x]^2*Log[1 + E^(I*ArcCos[c*x]
)] - 2*a^2*Log[1 - c*x] + 2*a^2*Log[1 + c*x] + 8*b^2*Log[Tan[ArcCos[c*x]/2
]] - (8*I)*b*(a + b*ArcCos[c*x])*PolyLog[2, -E^(I*ArcCos[c*x])] + (8*I)*b*
(a + b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] + 8*b^2*PolyLog[3, -E^(I
*ArcCos[c*x])] - 8*b^2*PolyLog[3, E^(I*ArcCos[c*x])] + b^2*ArcCos[c*x]^2*S
ec[ArcCos[c*x]/2]^2 - 4*b^2*ArcCos[c*x]*Tan[ArcCos[c*x]/2])/(c^3*d^2)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {5207, 27, 5165, 3042, 4671, 3011, 2720, 5183, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$$

$$\Big\downarrow \text{ 5207}$$

$$\frac{b\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}-\frac{\int \frac{(a+b\arccos(cx))^2}{d(1-c^2x^2)}dx}{2c^2d}+\frac{x(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{b\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}-\frac{\int \frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx}{2c^2d^2}+\frac{x(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \; 5165$$

$$\frac{b\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}+\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c^3d^2}+\frac{x(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (a+b\arccos(cx))^2\csc(\arccos(cx))d\arccos(cx)}{2c^3d^2}+\frac{b\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}+\frac{x(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \; 4671$$

$$\frac{-2b\int (a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arccos(c}{2c^3d^2}$$

$$\frac{b\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}+\frac{x(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \; 3011$$

$$\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\operatorname{PolyLog}\left(2\right.\right.}{}$$

$$\frac{b\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}+\frac{x(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \; 2720$$

$$\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\right.}{}$$

$$\frac{b\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{cd^2}+\frac{x(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \; 5183$$

$$\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b(i}{}$$
$$\frac{b\left(\frac{b\int\frac{1}{1-c^2x^2}dx}{c}+\frac{a+b\arccos(cx)}{c^2\sqrt{1-c^2x^2}}\right)}{cd^2}+\frac{x(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$\downarrow$ 219

$$\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b(i}{}$$
$$\frac{b\left(\frac{a+b\arccos(cx)}{c^2\sqrt{1-c^2x^2}}+\frac{b\text{arctanh}(cx)}{c^2}\right)}{cd^2}+\frac{x(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$\downarrow$ 7143

$$\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\,\text{PolyLog}\left(3,-\right.\right.}{2c^3d^2}$$
$$\frac{b\left(\frac{a+b\arccos(cx)}{c^2\sqrt{1-c^2x^2}}+\frac{b\text{arctanh}(cx)}{c^2}\right)}{cd^2}+\frac{x(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^2,x]
```

output
```
(x*(a + b*ArcCos[c*x])^2)/(2*c^2*d^2*(1 - c^2*x^2)) + (b*((a + b*ArcCos[c*
x])/(c^2*Sqrt[1 - c^2*x^2]) + (b*ArcTanh[c*x])/c^2))/(c*d^2) + (-2*(a + b*
ArcCos[c*x])^2*ArcTanh[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*Pol
yLog[2, -E^(I*ArcCos[c*x])] - b*PolyLog[3, -E^(I*ArcCos[c*x])]) - 2*b*(I*(
a + b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] - b*PolyLog[3, E^(I*ArcCo
s[c*x])]))/(2*c^3*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{a^2\left(-\frac{1}{4(cx-1)}+\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}-\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b^2\left(-\frac{\arccos(cx)\left(cx\arccos(cx)+2\sqrt{-c^2x^2+1}\right)}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)^2\ln\left(1+cx+i\sqrt{-c^2}\right.}{2}\right.}{}$
default	$\frac{a^2\left(-\frac{1}{4(cx-1)}+\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}-\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b^2\left(-\frac{\arccos(cx)\left(cx\arccos(cx)+2\sqrt{-c^2x^2+1}\right)}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)^2\ln\left(1+cx+i\sqrt{-c^2}\right.}{2}\right.}{}$
parts	$\frac{a^2\left(-\frac{1}{4c^3(cx-1)}+\frac{\ln(cx-1)}{4c^3}-\frac{1}{4c^3(cx+1)}-\frac{\ln(cx+1)}{4c^3}\right)}{d^2}+\frac{b^2\left(-\frac{\arccos(cx)\left(cx\arccos(cx)+2\sqrt{-c^2x^2+1}\right)}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)^2\ln\left(1+c\right.}{2}\right.}{}$

input
```
int(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a^2/d^2*(-1/4/(c*x-1)+1/4*ln(c*x-1)-1/4/(c*x+1)-1/4*ln(c*x+1))+b^2/
d^2*(-1/2/(c^2*x^2-1)*arccos(c*x)*(c*x*arccos(c*x)+2*(-c^2*x^2+1)^(1/2))-1
/2*arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+I*arccos(c*x)*polylog(2,-c
*x-I*(-c^2*x^2+1)^(1/2))-polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))+1/2*arccos(c
*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-I*arccos(c*x)*polylog(2,c*x+I*(-c^2*x
^2+1)^(1/2))+polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))+2*arctanh(c*x+I*(-c^2*x^2
+1)^(1/2)))+2*a*b/d^2*(-1/2*(c*x*arccos(c*x)+(-c^2*x^2+1)^(1/2))/(c^2*x^2-
1)+1/2*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-1/2*I*polylog(2,c*x+I*(-
c^2*x^2+1)^(1/2))-1/2*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+1/2*I*pol
ylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)^2x^2}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^2*arccos(c*x)^2 + 2*a*b*x^2*arccos(c*x) + a^2*x^2)/(c^4*d^
2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \frac{\int \frac{a^2x^2}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{b^2x^2\operatorname{acos}^2(cx)}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{2abx^2\operatorname{acos}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**2*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2*x**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b**2*x**2
*acos(c*x)**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**2*acos
(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx=\int \frac{(b\arccos(cx)+a)^2x^2}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/4*a^2*(2*x/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - log(c*x -
 1)/(c^3*d^2)) - 1/4*((2*b^2*c*x + (b^2*c^2*x^2 - b^2)*log(c*x + 1) - (b^2
*c^2*x^2 - b^2)*log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^
2 - 4*(c^5*d^2*x^2 - c^3*d^2)*integrate(1/2*(4*a*b*c^2*x^2*arctan2(sqrt(c*
x + 1)*sqrt(-c*x + 1), c*x) + (2*b^2*c*x + (b^2*c^2*x^2 - b^2)*log(c*x + 1
) - (b^2*c^2*x^2 - b^2)*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan
2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/(c^6*d^2*x^4 - 2*c^4*d^2*x^2 + c^2*d
^2), x))/(c^5*d^2*x^2 - c^3*d^2)
```

Giac [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx=\int \frac{(b\arccos(cx)+a)^2x^2}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2*x^2/(c^2*d*x^2 - d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \int \frac{x^2\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((x^2*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^2,x)
```

output
```
int((x^2*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$$
$$= \frac{8\left(\int \frac{acos(cx)x^2}{c^4x^4-2c^2x^2+1}dx\right)ab\,c^5x^2 - 8\left(\int \frac{acos(cx)x^2}{c^4x^4-2c^2x^2+1}dx\right)ab\,c^3 + 4\left(\int \frac{acos(cx)^2x^2}{c^4x^4-2c^2x^2+1}dx\right)b^2c^5x^2 - 4\left(\int \frac{acos(cx)^2x^2}{c^4x^4-2c^2x^2+}\right.}{4c^3d^2\left(c^2x^2\right.}$$

input
```
int(x^2*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^2,x)
```

output
```
(8*int((acos(c*x)*x**2)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**5*x**2 - 8
*int((acos(c*x)*x**2)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**3 + 4*int((a
cos(c*x)**2*x**2)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2*c**5*x**2 - 4*int(
(acos(c*x)**2*x**2)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2*c**3 + log(c**2*
x - c)*a**2*c**2*x**2 - log(c**2*x - c)*a**2 - log(c**2*x + c)*a**2*c**2*x
**2 + log(c**2*x + c)*a**2 - 2*a**2*c*x)/(4*c**3*d**2*(c**2*x**2 - 1))
```

3.197 $\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = -\frac{bx(a+b\arccos(cx))}{cd^2\sqrt{1-c^2x^2}} + \frac{(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)} - \frac{b^2\log(1-c^2x^2)}{2c^2d^2}$$

output
```
-b*x*(a+b*arccos(c*x))/c/d^2/(-c^2*x^2+1)^(1/2)+1/2*(a+b*arccos(c*x))^2/c^
2/d^2/(-c^2*x^2+1)-1/2*b^2*ln(-c^2*x^2+1)/c^2/d^2
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.19

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = -\frac{a\left(a+2bcx\sqrt{1-c^2x^2}\right)+2b\left(a+bcx\sqrt{1-c^2x^2}\right)\arccos(cx)+b^2\arccos(cx)^2+b^2(-1+c^2x^2)\log(1-}{2c^2d^2(-1+c^2x^2)}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^2,x]
```

output
```
-1/2*(a*(a + 2*b*c*x*Sqrt[1 - c^2*x^2]) + 2*b*(a + b*c*x*Sqrt[1 - c^2*x^2]
)*ArcCos[c*x] + b^2*ArcCos[c*x]^2 + b^2*(-1 + c^2*x^2)*Log[1 - c^2*x^2])/(
c^2*d^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5183, 5161, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}dx$$

$$\Big\downarrow \text{ 5183}$$

$$\frac{b\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{3/2}}dx}{cd^2}+\frac{(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{ 5161}$$

$$\frac{b\left(bc\int \frac{x}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)}{cd^2}+\frac{(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{(a+b\arccos(cx))^2}{2c^2d^2(1-c^2x^2)}+\frac{b\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)}{cd^2}$$

input
```
Int[(x*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^2,x]
```

output
```
(a + b*ArcCos[c*x])^2/(2*c^2*d^2*(1 - c^2*x^2)) + (b*((x*(a + b*ArcCos[c*x
]))/Sqrt[1 - c^2*x^2] - (b*Log[1 - c^2*x^2])/(2*c)))/(c*d^2)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(83) = 166$.

Time = 0.26 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.94

method	result
derivativedivides	$\frac{-\frac{a^2}{2d^2\left(c^2x^2-1\right)}+\frac{b^2\left(-\frac{\arccos(cx)^2}{2\left(c^2x^2-1\right)}-\frac{cx\arccos(cx)\sqrt{-c^2x^2+1}}{c^2x^2-1}-\frac{\ln\left(-c^2x^2+1\right)}{2}\right)}{d^2}+\frac{2ab\left(-\frac{\arccos(cx)}{2\left(c^2x^2-1\right)}-\frac{\sqrt{-(cx-1)^2-2cx+2}}{4(cx-1)}-\cdots\right.}{d^2}}{c^2}$
default	$\frac{-\frac{a^2}{2d^2\left(c^2x^2-1\right)}+\frac{b^2\left(-\frac{\arccos(cx)^2}{2\left(c^2x^2-1\right)}-\frac{cx\arccos(cx)\sqrt{-c^2x^2+1}}{c^2x^2-1}-\frac{\ln\left(-c^2x^2+1\right)}{2}\right)}{d^2}+\frac{2ab\left(-\frac{\arccos(cx)}{2\left(c^2x^2-1\right)}-\frac{\sqrt{-(cx-1)^2-2cx+2}}{4(cx-1)}-\cdots\right.}{d^2}}{c^2}$
parts	$-\frac{a^2}{2d^2c^2(c^2x^2-1)}+\frac{b^2\left(-\frac{\arccos(cx)^2}{2\left(c^2x^2-1\right)}-\frac{cx\arccos(cx)\sqrt{-c^2x^2+1}}{c^2x^2-1}-\frac{\ln\left(-c^2x^2+1\right)}{2}\right)}{d^2c^2}+\frac{2ab\left(-\frac{\arccos(cx)}{2\left(c^2x^2-1\right)}-\frac{\sqrt{-(cx-1)^2\cdots}}{4(cx-\cdots}\right.}{d^2c\cdots}$

input
```
int(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/2*a^2/d^2/(c^2*x^2-1)+b^2/d^2*(-1/2*arccos(c*x)^2/(c^2*x^2-1)-c*
x/(c^2*x^2-1)*arccos(c*x)*(-c^2*x^2+1)^(1/2)-1/2*ln(-c^2*x^2+1))+2*a*b/d^2
*(-1/2/(c^2*x^2-1)*arccos(c*x)-1/4/(c*x-1)*(-(c*x-1)^2-2*c*x+2)^(1/2)-1/4/
(c*x+1)*(-(c*x+1)^2+2*c*x+2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.70

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = -\frac{2\,abc^2x^2\arccos{(cx)}+b^2\arccos{(cx)}^2+a^2-2\,(abc^2x^2-ab)\arctan\left(\frac{\sqrt{-c^2x^2+1}cx}{c^2x^2-1}\right)+(b^2c^2x^2-b^2)\log\left(\right.}{2\,(c^4d^2x^2-c^2d^2)}$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*c^2*x^2*arccos(c*x) + b^2*arccos(c*x)^2 + a^2 - 2*(a*b*c^2*x^2
 - a*b)*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + (b^2*c^2*x^2 - b^2)
*log(c^2*x^2 - 1) + 2*(b^2*c*x*arccos(c*x) + a*b*c*x)*sqrt(-c^2*x^2 + 1))/
(c^4*d^2*x^2 - c^2*d^2)
```

Sympy [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \frac{\int \frac{a^2x}{c^4x^4-2c^2x^2+1}\,dx+\int \frac{b^2x\operatorname{acos}^2{(cx)}}{c^4x^4-2c^2x^2+1}\,dx+\int \frac{2abx\operatorname{acos}{(cx)}}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2*x/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b**2*x*acos(
c*x)**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(2*a*b*x*acos(c*x)/(c*
*4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(82) = 164$.

Time = 0.14 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.28

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$$
$$= -\frac{1}{2}\left(\left(\frac{\sqrt{-c^2x^2+1}c^2d^2}{c^7d^4x+c^6d^4}+\frac{\sqrt{-c^2x^2+1}c^2d^2}{c^7d^4x-c^6d^4}\right)c^2+\frac{2\arccos(cx)}{c^4d^2x^2-c^2d^2}\right)ab$$
$$-\frac{1}{2}\left(c^3\left(\frac{\log(cx+1)}{c^5d^2}+\frac{\log(cx-1)}{c^5d^2}\right)+\left(\frac{\sqrt{-c^2x^2+1}c^2d^2}{c^7d^4x+c^6d^4}+\frac{\sqrt{-c^2x^2+1}c^2d^2}{c^7d^4x-c^6d^4}\right)c^2\arccos(cx)\right)b^2$$
$$-\frac{b^2\arccos(cx)^2}{2(c^4d^2x^2-c^2d^2)}-\frac{a^2}{2(c^4d^2x^2-c^2d^2)}$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*((sqrt(-c^2*x^2 + 1)*c^2*d^2/(c^7*d^4*x + c^6*d^4) + sqrt(-c^2*x^2 +
1)*c^2*d^2/(c^7*d^4*x - c^6*d^4))*c^2 + 2*arccos(c*x)/(c^4*d^2*x^2 - c^2*d
^2))*a*b - 1/2*(c^3*(log(c*x + 1)/(c^5*d^2) + log(c*x - 1)/(c^5*d^2)) + (s
qrt(-c^2*x^2 + 1)*c^2*d^2/(c^7*d^4*x + c^6*d^4) + sqrt(-c^2*x^2 + 1)*c^2*d
^2/(c^7*d^4*x - c^6*d^4))*c^2*arccos(c*x))*b^2 - 1/2*b^2*arccos(c*x)^2/(c^
4*d^2*x^2 - c^2*d^2) - 1/2*a^2/(c^4*d^2*x^2 - c^2*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(82) = 164$.

Time = 0.19 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.54

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = -\frac{b^2x^2\arccos(cx)^2}{2(c^2x^2-1)d^2}-\frac{abx^2\arccos(cx)}{(c^2x^2-1)d^2}$$
$$-\frac{a^2x^2}{2(c^2x^2-1)d^2}-\frac{\sqrt{-c^2x^2+1}b^2x\arccos(cx)}{(c^2x^2-1)cd^2}$$
$$+\frac{b^2\arccos(cx)^2}{2c^2d^2}-\frac{\sqrt{-c^2x^2+1}abx}{(c^2x^2-1)cd^2}+\frac{ab\arccos(cx)}{c^2d^2}$$
$$-\frac{b^2\log(2)}{c^2d^2}-\frac{b^2\log(|-c^2x^2+1|)}{2c^2d^2}+\frac{a^2}{2c^2d^2}$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/2*b^2*x^2*arccos(c*x)^2/((c^2*x^2 - 1)*d^2) - a*b*x^2*arccos(c*x)/((c^2
*x^2 - 1)*d^2) - 1/2*a^2*x^2/((c^2*x^2 - 1)*d^2) - sqrt(-c^2*x^2 + 1)*b^2*
x*arccos(c*x)/((c^2*x^2 - 1)*c*d^2) + 1/2*b^2*arccos(c*x)^2/(c^2*d^2) - sq
rt(-c^2*x^2 + 1)*a*b*x/((c^2*x^2 - 1)*c*d^2) + a*b*arccos(c*x)/(c^2*d^2) -
 b^2*log(2)/(c^2*d^2) - 1/2*b^2*log(abs(-c^2*x^2 + 1))/(c^2*d^2) + 1/2*a^2
/(c^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx=\int \frac{x\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((x*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^2,x)
```

output
```
int((x*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \frac{4\left(\int \frac{acos(cx)x}{c^4x^4-2c^2x^2+1}dx\right)ab\,c^2x^2-4\left(\int \frac{acos(cx)x}{c^4x^4-2c^2x^2+1}dx\right)ab+2\left(\int \frac{acos(cx)^2x}{c^4x^4-2c^2x^2+1}dx\right)b^2c^2x^2-2\left(\int \frac{acos(cx)^2x}{c^4x^4-2c^2x^2+1}d\right.}{2d^2\left(c^2x^2-1\right)}$$

input
```
int(x*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^2,x)
```

output
```
(4*int((acos(c*x)*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**2*x**2 - 4*in
t((acos(c*x)*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b + 2*int((acos(c*x)**2
*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2*c**2*x**2 - 2*int((acos(c*x)**2*
x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*b**2 - a**2*x**2)/(2*d**2*(c**2*x**2 -
 1))
```

3.198 $\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 230

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx &= -\frac{b(a+b\arccos(cx))}{cd^2\sqrt{1-c^2x^2}}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}\\ &\quad-\frac{i(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)}{cd^2}+\frac{b^2\operatorname{arctanh}(cx)}{cd^2}\\ &\quad+\frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{cd^2}\\ &\quad-\frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{cd^2}\\ &\quad-\frac{b^2\operatorname{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)}{cd^2}+\frac{b^2\operatorname{PolyLog}\left(3,ie^{i\arccos(cx)}\right)}{cd^2}\end{aligned}$$

output
```
-b*(a+b*arccos(c*x))/c/d^2/(-c^2*x^2+1)^(1/2)+1/2*x*(a+b*arccos(c*x))^2/d^
2/(-c^2*x^2+1)-I*(a+b*arccos(c*x))^2*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c/d^
2+b^2*arctanh(c*x)/c/d^2+I*b*(a+b*arccos(c*x))*polylog(2,-I*(c*x+I*(-c^2*x
^2+1)^(1/2)))/c/d^2-I*b*(a+b*arccos(c*x))*polylog(2,I*(c*x+I*(-c^2*x^2+1)^
(1/2)))/c/d^2-b^2*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/d^2+b^2*polyl
og(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/d^2
```

Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.74

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$$
$$= \frac{-\frac{4a^2x}{-1+c^2x^2}-\frac{2a^2\log(1-cx)}{c}+\frac{2a^2\log(1+cx)}{c}+\frac{4ab\left(\frac{\sqrt{1-c^2x^2}}{1-cx}+\frac{\sqrt{1-c^2x^2}}{1+cx}+\frac{\arccos(cx)}{1-cx}-\frac{\arccos(cx)}{1+cx}-2\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)+2\,\mathrm{a}\right.}{}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2)^2,x]
```

output
```
((-4*a^2*x)/(-1 + c^2*x^2) - (2*a^2*Log[1 - c*x])/c + (2*a^2*Log[1 + c*x])
/c + (4*a*b*(Sqrt[1 - c^2*x^2]/(1 - c*x) + Sqrt[1 - c^2*x^2]/(1 + c*x) + A
rcCos[c*x]/(1 - c*x) - ArcCos[c*x]/(1 + c*x) - 2*ArcCos[c*x]*Log[1 - E^(I*
ArcCos[c*x])] + 2*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] - (2*I)*PolyLog[2
, -E^(I*ArcCos[c*x])] + (2*I)*PolyLog[2, E^(I*ArcCos[c*x])]))/c + (b^2*(4*
ArcCos[c*x]*Cot[ArcCos[c*x]/2] + ArcCos[c*x]^2*Csc[ArcCos[c*x]/2]^2 - 4*Ar
cCos[c*x]^2*(Log[1 - E^(I*ArcCos[c*x])] - Log[1 + E^(I*ArcCos[c*x])]) - 8*
Log[Tan[ArcCos[c*x]/2]] - (8*I)*ArcCos[c*x]*(PolyLog[2, -E^(I*ArcCos[c*x])
] - PolyLog[2, E^(I*ArcCos[c*x])]) + 8*(PolyLog[3, -E^(I*ArcCos[c*x])] - P
olyLog[3, E^(I*ArcCos[c*x])]) - ArcCos[c*x]^2*Sec[ArcCos[c*x]/2]^2 + 4*Arc
Cos[c*x]*Tan[ArcCos[c*x]/2]))/c)/(8*d^2)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.86, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5163, 27, 5165, 3042, 4671, 3011, 2720, 5183, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$$
$$\Big\downarrow \; 5163$$

$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}+\frac{\int \frac{(a+b\arccos(cx))^2}{d(1-c^2x^2)}dx}{2d}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}+\frac{\int \frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx}{2d^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \; 5165$$

$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}-\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2cd^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}-\frac{\int (a+b\arccos(cx))^2\csc(\arccos(cx))d\arccos(cx)}{2cd^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \; 4671$$

$$-\frac{-2b\int (a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arccos}{2cd^2}$$
$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \; 3011$$

$$-\frac{2b\big(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\big)-2b\big(i\,\text{PolyLog}}{}$$
$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \; 2720$$

$$-\frac{2b\big(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\big)-2b}{}$$
$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \; 5183$$

$$-\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b}{}$$

$$\frac{bc\left(\frac{b\int\frac{1}{1-c^2x^2}dx}{c}+\frac{a+b\arccos(cx)}{c^2\sqrt{1-c^2x^2}}\right)}{d^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$\downarrow$ 219

$$-\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b}{}$$

$$\frac{bc\left(\frac{a+b\arccos(cx)}{c^2\sqrt{1-c^2x^2}}+\frac{b\operatorname{arctanh}(cx)}{c^2}\right)}{d^2}+\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$\downarrow$ 7143

$$\frac{bc\left(\frac{a+b\arccos(cx)}{c^2\sqrt{1-c^2x^2}}+\frac{b\operatorname{arctanh}(cx)}{c^2}\right)}{d^2}-$$

$$\frac{-2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\operatorname{PolyLog}(3,-\right.}{2cd^2}$$

$$\frac{x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2)^2,x]
```

output
```
(x*(a + b*ArcCos[c*x])^2)/(2*d^2*(1 - c^2*x^2)) + (b*c*((a + b*ArcCos[c*x]
)/(c^2*Sqrt[1 - c^2*x^2]) + (b*ArcTanh[c*x])/c^2))/d^2 - (-2*(a + b*ArcCos
[c*x])^2*ArcTanh[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2
, -E^(I*ArcCos[c*x])] - b*PolyLog[3, -E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*
ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] - b*PolyLog[3, E^(I*ArcCos[c*x]
)]))/(2*c*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.84

method	result
derivativedivides	$\frac{a^2\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b^2\left(-\frac{\arccos(cx)\left(cx\arccos(cx)+2\sqrt{-c^2x^2+1}\right)}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)^2\ln\left(1-cx-i\sqrt{-c^2\right.}}{2}\right.}{}$
default	$\frac{a^2\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{4}-\frac{1}{4(cx+1)}+\frac{\ln(cx+1)}{4}\right)}{d^2}+\frac{b^2\left(-\frac{\arccos(cx)\left(cx\arccos(cx)+2\sqrt{-c^2x^2+1}\right)}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)^2\ln\left(1-cx-i\sqrt{-c^2\right.}}{2}\right.}{}$
parts	$\frac{a^2\left(-\frac{1}{4c(cx-1)}-\frac{\ln(cx-1)}{4c}-\frac{1}{4c(cx+1)}+\frac{\ln(cx+1)}{4c}\right)}{d^2}+\frac{b^2\left(-\frac{\arccos(cx)\left(cx\arccos(cx)+2\sqrt{-c^2x^2+1}\right)}{2\left(c^2x^2-1\right)}-\frac{\arccos(cx)^2\ln\left(1-cx-\right.}{2}\right.}{}$

input
```
int((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2/d^2*(-1/4/(c*x-1)-1/4*ln(c*x-1)-1/4/(c*x+1)+1/4*ln(c*x+1))+b^2/d^
2*(-1/2/(c^2*x^2-1)*arccos(c*x)*(c*x*arccos(c*x)+2*(-c^2*x^2+1)^(1/2))-1/2
*arccos(c*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+I*arccos(c*x)*polylog(2,c*x+
I*(-c^2*x^2+1)^(1/2))-polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))+1/2*arccos(c*x)^
2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-I*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+
1)^(1/2))+polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))+2*arctanh(c*x+I*(-c^2*x^2+1
)^(1/2)))+2*a*b/d^2*(-1/2*(c*x*arccos(c*x)+(-c^2*x^2+1)^(1/2))/(c^2*x^2-1)
-1/2*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+1/2*I*polylog(2,c*x+I*(-c^
2*x^2+1)^(1/2))+1/2*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-1/2*I*polyl
og(2,-c*x-I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a + b\arccos(cx))^2}{(d - c^2dx^2)^2}\,dx = \int \frac{(b\arccos(cx) + a)^2}{(c^2dx^2 - d)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^4*d^2*x^4 - 2*c^
2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(a + b\arccos(cx))^2}{(d - c^2dx^2)^2}\,dx = \frac{\int \frac{a^2}{c^4x^4 - 2c^2x^2 + 1}\,dx + \int \frac{b^2\operatorname{acos}^2(cx)}{c^4x^4 - 2c^2x^2 + 1}\,dx + \int \frac{2ab\operatorname{acos}(cx)}{c^4x^4 - 2c^2x^2 + 1}\,dx}{d^2}$$

input
```
integrate((a+b*acos(c*x))**2/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b**2*acos(c*x)
**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(2*a*b*acos(c*x)/(c**4*x**
4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/4*a^2*(2*x/(c^2*d^2*x^2 - d^2) - log(c*x + 1)/(c*d^2) + log(c*x - 1)/(c
*d^2)) - 1/4*((2*b^2*c*x - (b^2*c^2*x^2 - b^2)*log(c*x + 1) + (b^2*c^2*x^2
 - b^2)*log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 4*(c
^3*d^2*x^2 - c*d^2)*integrate(-1/2*((2*b^2*c*x - (b^2*c^2*x^2 - b^2)*log(c
*x + 1) + (b^2*c^2*x^2 - b^2)*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)*
arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 4*a*b*arctan2(sqrt(c*x + 1)*s
qrt(-c*x + 1), c*x))/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x))/(c^3*d^2*x^2
 - c*d^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$$
$$=\frac{8\left(\int \frac{acos(cx)}{c^4x^4-2c^2x^2+1}dx\right)ab\,c^3x^2-8\left(\int \frac{acos(cx)}{c^4x^4-2c^2x^2+1}dx\right)abc+4\left(\int \frac{acos(cx)^2}{c^4x^4-2c^2x^2+1}dx\right)b^2c^3x^2-4\left(\int \frac{acos(cx)^2}{c^4x^4-2c^2x^2+1}\right.}{4c\,d^2\left(c^2x^2-\right.}$$

input
```
int((a+b*acos(c*x))^2/(-c^2*d*x^2+d)^2,x)
```

output
```
(8*int(acos(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c**3*x**2 - 8*int(ac
os(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b*c + 4*int(acos(c*x)**2/(c**4*
x**4 - 2*c**2*x**2 + 1),x)*b**2*c**3*x**2 - 4*int(acos(c*x)**2/(c**4*x**4
- 2*c**2*x**2 + 1),x)*b**2*c - log(c**2*x - c)*a**2*c**2*x**2 + log(c**2*x
 - c)*a**2 + log(c**2*x + c)*a**2*c**2*x**2 - log(c**2*x + c)*a**2 - 2*a**
2*c*x)/(4*c*d**2*(c**2*x**2 - 1))
```

3.199 $\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 211

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^2}\,dx = & -\frac{bcx(a+b\arccos(cx))}{d^2\sqrt{1-c^2x^2}}+\frac{(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)} \\ & -\frac{2(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)}{d^2} \\ & -\frac{b^2\log(1-c^2x^2)}{2d^2} \\ & +\frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{d^2} \\ & -\frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{d^2} \\ & -\frac{b^2\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)}{2d^2}+\frac{b^2\operatorname{PolyLog}\left(3,e^{2i\arccos(cx)}\right)}{2d^2}\end{aligned}$$

output

```
-b*c*x*(a+b*arccos(c*x))/d^2/(-c^2*x^2+1)^(1/2)+1/2*(a+b*arccos(c*x))^2/d^
2/(-c^2*x^2+1)-2*(a+b*arccos(c*x))^2*arctanh((c*x+I*(-c^2*x^2+1)^(1/2))^2)
/d^2-1/2*b^2*ln(-c^2*x^2+1)/d^2+I*b*(a+b*arccos(c*x))*polylog(2,-(c*x+I*(-
c^2*x^2+1)^(1/2))^2)/d^2-I*b*(a+b*arccos(c*x))*polylog(2,(c*x+I*(-c^2*x^2+
1)^(1/2))^2)/d^2-1/2*b^2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^2+1/2*
b^2*polylog(3,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 474 vs. 2(211) = 422.

Time = 1.09 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.25

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^2}\,dx$$
$$= \frac{\frac{1}{12}ib^2\pi^3+\frac{a^2}{1-c^2x^2}+\frac{ab\sqrt{1-c^2x^2}}{1-cx}-\frac{ab\sqrt{1-c^2x^2}}{1+cx}+\frac{ab\arccos(cx)}{1-cx}+\frac{ab\arccos(cx)}{1+cx}+\frac{2b^2cx\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{b^2\arccos(cx)^2}{1-c^2x^2}-\frac{4}{3}ib^2\,\mathrm{a}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x*(d - c^2*d*x^2)^2),x]
```

output
```
((I/12)*b^2*Pi^3 + a^2/(1 - c^2*x^2) + (a*b*Sqrt[1 - c^2*x^2])/(1 - c*x) -
 (a*b*Sqrt[1 - c^2*x^2])/(1 + c*x) + (a*b*ArcCos[c*x])/(1 - c*x) + (a*b*Ar
cCos[c*x])/(1 + c*x) + (2*b^2*c*x*ArcCos[c*x])/Sqrt[1 - c^2*x^2] + (b^2*Ar
cCos[c*x]^2)/(1 - c^2*x^2) - ((4*I)/3)*b^2*ArcCos[c*x]^3 - 4*a*b*ArcCos[c*
x]*Log[1 - E^(I*ArcCos[c*x])] - 4*a*b*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x]
)] - 2*b^2*ArcCos[c*x]^2*Log[1 - E^((-2*I)*ArcCos[c*x])] + 4*a*b*ArcCos[c*
x]*Log[1 + E^((2*I)*ArcCos[c*x])] + 2*b^2*ArcCos[c*x]^2*Log[1 + E^((2*I)*A
rcCos[c*x])] + 2*a^2*Log[c*x] - a^2*Log[1 - c^2*x^2] - b^2*Log[1 - c^2*x^2
] + (4*I)*a*b*PolyLog[2, -E^(I*ArcCos[c*x])] + (4*I)*a*b*PolyLog[2, E^(I*A
rcCos[c*x])] - (2*I)*b^2*ArcCos[c*x]*PolyLog[2, E^((-2*I)*ArcCos[c*x])] -
(2*I)*a*b*PolyLog[2, -E^((2*I)*ArcCos[c*x])] - (2*I)*b^2*ArcCos[c*x]*PolyL
og[2, -E^((2*I)*ArcCos[c*x])] - b^2*PolyLog[3, E^((-2*I)*ArcCos[c*x])] + b
^2*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/(2*d^2)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {5209, 27, 5161, 240, 5185, 4919, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^2}dx$$

$$\Big\downarrow \text{5209}$$

$$\frac{bc\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{3/2}}dx}{d^2}+\frac{\int \frac{(a+b\arccos(cx))^2}{dx(1-c^2x^2)}dx}{d}+\frac{(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{3/2}}dx}{d^2}+\frac{\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}dx}{d^2}+\frac{(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{5161}$$

$$\frac{bc\left(bc\int \frac{x}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)}{d^2}+\frac{\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}dx}{d^2}+\frac{(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}$$

$$\Big\downarrow \text{240}$$

$$\frac{\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}dx}{d^2}+\frac{(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)}{d^2}$$

$$\Big\downarrow \text{5185}$$

$$-\frac{\int \frac{(a+b\arccos(cx))^2}{cx\sqrt{1-c^2x^2}}d\arccos(cx)}{d^2}+\frac{(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)}{d^2}$$

$$\Big\downarrow \text{4919}$$

$$-\frac{2\int (a+b\arccos(cx))^2\csc(2\arccos(cx))d\arccos(cx)}{d^2}+\frac{(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int (a+b\arccos(cx))^2\csc(2\arccos(cx))d\arccos(cx)}{d^2}+\frac{(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)}{d^2}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{2\left(-b\int (a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+b\int (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)\right.}{d^2}$$
$$\frac{(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)}{d^2}$$

$\Big\downarrow$ 3011

$$-\frac{2\left(b\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int \text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-b\left(\frac{1}{2}i\,\text{Po}\right.\right.}{}$$
$$\frac{(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)}{d^2}$$

$\Big\downarrow$ 2720

$$-\frac{2\left(b\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right.\right.}{}$$
$$\frac{(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)}{d^2}$$

$\Big\downarrow$ 7143

$$-\frac{2\left(-\text{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2+b\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\text{PolyLog}\right.\right.}{d^2}$$
$$\frac{(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)}{d^2}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x*(d - c^2*d*x^2)^2),x]
```

output
```
(a + b*ArcCos[c*x])^2/(2*d^2*(1 - c^2*x^2)) + (b*c*((x*(a + b*ArcCos[c*x])
)/Sqrt[1 - c^2*x^2] - (b*Log[1 - c^2*x^2])/(2*c)))/d^2 - (2*(-((a + b*ArcC
os[c*x])^2*ArcTanh[E^((2*I)*ArcCos[c*x])]) + b*((I/2)*(a + b*ArcCos[c*x])*
PolyLog[2, -E^((2*I)*ArcCos[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcCos[c*x])]
)/4) - b*((I/2)*(a + b*ArcCos[c*x])*PolyLog[2, E^((2*I)*ArcCos[c*x])] - (b
*PolyLog[3, E^((2*I)*ArcCos[c*x])])/4)))/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 636 vs. $2(254) = 508$.

Time = 0.59 (sec) , antiderivative size = 637, normalized size of antiderivative = 3.02

method	result
parts	$\frac{a^2\left(\ln(x)-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{2}+\frac{1}{4cx+4}-\frac{\ln(cx+1)}{2}\right)}{d^2}+\frac{b^2\left(-\frac{\left(2ic^2x^2+2cx\sqrt{-c^2x^2+1}-2i+\arccos(cx)\right)\arccos(cx)}{2\left(c^2x^2-1\right)}+2\ln\left(cx\right.\right.}{}$
derivativedivides	$\frac{a^2\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{2}+\ln(cx)+\frac{1}{4cx+4}-\frac{\ln(cx+1)}{2}\right)}{d^2}+\frac{b^2\left(-\frac{\left(2ic^2x^2+2cx\sqrt{-c^2x^2+1}-2i+\arccos(cx)\right)\arccos(cx)}{2\left(c^2x^2-1\right)}+2\ln\left(\right.\right.}{}$
default	$\frac{a^2\left(-\frac{1}{4(cx-1)}-\frac{\ln(cx-1)}{2}+\ln(cx)+\frac{1}{4cx+4}-\frac{\ln(cx+1)}{2}\right)}{d^2}+\frac{b^2\left(-\frac{\left(2ic^2x^2+2cx\sqrt{-c^2x^2+1}-2i+\arccos(cx)\right)\arccos(cx)}{2\left(c^2x^2-1\right)}+2\ln\left(\right.\right.}{}$

input
```
int((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/d^2*(ln(x)-1/4/(c*x-1)-1/2*ln(c*x-1)+1/4/(c*x+1)-1/2*ln(c*x+1))+b^2/d^
2*(-1/2*(2*I*c^2*x^2+2*c*x*(-c^2*x^2+1)^(1/2)-2*I+arccos(c*x))*arccos(c*x)
/(c^2*x^2-1)+2*ln(c*x+I*(-c^2*x^2+1)^(1/2))-ln(1+c*x+I*(-c^2*x^2+1)^(1/2))
-ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)+arccos(c*x)^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/
2))^2)-I*arccos(c*x)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/2*polylog(
3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/
2))+2*I*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-2*polylog(3,-c*x-
I*(-c^2*x^2+1)^(1/2))-arccos(c*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+2*I*arc
cos(c*x)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))-2*polylog(3,c*x+I*(-c^2*x^2+1
)^(1/2)))+2*a*b/d^2*(-1/2*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)+arccos(c*x)-I)
/(c^2*x^2-1)-arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+I*polylog(2,-c*x-I
*(-c^2*x^2+1)^(1/2))+arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1/2*I*
polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+
1)^(1/2))+I*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^2}\,dx=\int \frac{\left(b\arccos\left(cx\right)+a\right)^2}{\left(c^2dx^2-d\right)^2x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^4*d^2*x^5 - 2*c^
2*d^2*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^2}\,dx = \frac{\int \frac{a^2}{c^4x^5-2c^2x^3+x}\,dx + \int \frac{b^2\operatorname{acos}^2(cx)}{c^4x^5-2c^2x^3+x}\,dx + \int \frac{2ab\operatorname{acos}(cx)}{c^4x^5-2c^2x^3+x}\,dx}{d^2}$$

input
```
integrate((a+b*acos(c*x))**2/x/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2/(c**4*x**5 - 2*c**2*x**3 + x), x) + Integral(b**2*acos(c*x)
**2/(c**4*x**5 - 2*c**2*x**3 + x), x) + Integral(2*a*b*acos(c*x)/(c**4*x**
5 - 2*c**2*x**3 + x), x))/d**2
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^2}\,dx = \int \frac{(b\arccos(cx)+a)^2}{\left(c^2dx^2-d\right)^2x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*(1/(c^2*d^2*x^2 - d^2) + log(c*x + 1)/d^2 + log(c*x - 1)/d^2 - 2*
log(x)/d^2) + integrate((b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2
+ 2*a*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/(c^4*d^2*x^5 - 2*c^2*d
^2*x^3 + d^2*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^2}\,dx = \int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{x\,\left(d-c^2\,d\,x^2\right)^2}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x*(d - c^2*d*x^2)^2),x)
```

output
```
int((a + b*acos(c*x))^2/(x*(d - c^2*d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^2}\,dx = \frac{4\left(\int \frac{acos(cx)}{c^4x^5-2c^2x^3+x}dx\right)ab\,c^2x^2-4\left(\int \frac{acos(cx)}{c^4x^5-2c^2x^3+x}dx\right)ab+2\left(\int \frac{acos(cx)^2}{c^4x^5-2c^2x^3+x}dx\right)b^2c^2x^2-2\left(\int \frac{acos(cx)^2}{c^4x^5-2c^2x^3+x}\right.}{}$$

input
```
int((a+b*acos(c*x))^2/x/(-c^2*d*x^2+d)^2,x)
```

output
```
(4*int(acos(c*x)/(c**4*x**5 - 2*c**2*x**3 + x),x)*a*b*c**2*x**2 - 4*int(ac
os(c*x)/(c**4*x**5 - 2*c**2*x**3 + x),x)*a*b + 2*int(acos(c*x)**2/(c**4*x*
*5 - 2*c**2*x**3 + x),x)*b**2*c**2*x**2 - 2*int(acos(c*x)**2/(c**4*x**5 -
2*c**2*x**3 + x),x)*b**2 - log(c**2*x - c)*a**2*c**2*x**2 + log(c**2*x - c
)*a**2 - log(c**2*x + c)*a**2*c**2*x**2 + log(c**2*x + c)*a**2 + 2*log(x)*
a**2*c**2*x**2 - 2*log(x)*a**2 - a**2*c**2*x**2)/(2*d**2*(c**2*x**2 - 1))
```

3.200 $\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 324

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^2}\,dx = &-\frac{bc(a+b\arccos(cx))}{d^2\sqrt{1-c^2x^2}}-\frac{(a+b\arccos(cx))^2}{d^2x(1-c^2x^2)}\\ &+\frac{3c^2x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}\\ &-\frac{3ic(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)}{d^2}\\ &-\frac{4bc(a+b\arccos(cx))\text{arctanh}\left(e^{i\arccos(cx)}\right)}{d^2}\\ &+\frac{b^2c\,\text{arctanh}(cx)}{d^2}+\frac{2ib^2c\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{d^2}\\ &+\frac{3ibc(a+b\arccos(cx))\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{d^2}\\ &-\frac{3ibc(a+b\arccos(cx))\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{d^2}\\ &-\frac{2ib^2c\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{d^2}\\ &-\frac{3b^2c\,\text{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)}{d^2}\\ &+\frac{3b^2c\,\text{PolyLog}\left(3,ie^{i\arccos(cx)}\right)}{d^2}\end{aligned}$$

output
```
-b*c*(a+b*arccos(c*x))/d^2/(-c^2*x^2+1)^(1/2)-(a+b*arccos(c*x))^2/d^2/x/(-
c^2*x^2+1)+3/2*c^2*x*(a+b*arccos(c*x))^2/d^2/(-c^2*x^2+1)-3*I*c*(a+b*arcco
s(c*x))^2*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/d^2-4*b*c*(a+b*arccos(c*x))*arc
tanh(c*x+I*(-c^2*x^2+1)^(1/2))/d^2+b^2*c*arctanh(c*x)/d^2+2*I*b^2*c*polylo
g(2,-c*x-I*(-c^2*x^2+1)^(1/2))/d^2+3*I*b*c*(a+b*arccos(c*x))*polylog(2,-I*
(c*x+I*(-c^2*x^2+1)^(1/2)))/d^2-3*I*b*c*(a+b*arccos(c*x))*polylog(2,I*(c*x
+I*(-c^2*x^2+1)^(1/2)))/d^2-2*I*b^2*c*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/
d^2-3*b^2*c*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^2+3*b^2*c*polylog(3
,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^2
```

Mathematica [A] (verified)

Time = 6.41 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.89

$$\int \frac{(a+b\arccos(cx))^2}{x^2\left(d-c^2dx^2\right)^2}\,dx$$
$$=\frac{-\frac{4a^2}{x}-\frac{2a^2c^2x}{-1+c^2x^2}-3a^2c\log(1-cx)+3a^2c\log(1+cx)+2abc\left(\frac{\sqrt{1-c^2x^2}}{1-cx}+\frac{\sqrt{1-c^2x^2}}{1+cx}-\frac{4\arccos(cx)}{cx}+\frac{\arccos(cx)}{1-cx}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^2*(d - c^2*d*x^2)^2),x]
```

output
```
((-4*a^2)/x - (2*a^2*c^2*x)/(-1 + c^2*x^2) - 3*a^2*c*Log[1 - c*x] + 3*a^2*
c*Log[1 + c*x] + 2*a*b*c*(Sqrt[1 - c^2*x^2]/(1 - c*x) + Sqrt[1 - c^2*x^2]/
(1 + c*x) - (4*ArcCos[c*x])/(c*x) + ArcCos[c*x]/(1 - c*x) - ArcCos[c*x]/(1
 + c*x) - 6*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] + 6*ArcCos[c*x]*Log[1 +
 E^(I*ArcCos[c*x])] - 4*Log[c*x] + 4*Log[1 + Sqrt[1 - c^2*x^2]] - (6*I)*Po
lyLog[2, -E^(I*ArcCos[c*x])] + (6*I)*PolyLog[2, E^(I*ArcCos[c*x])]) + (b^2
*c*(-8*ArcCos[c*x]^2 + 4*ArcCos[c*x]*Cot[ArcCos[c*x]/2] + ArcCos[c*x]^2*Cs
c[ArcCos[c*x]/2]^2 + 16*ArcCos[c*x]*(Log[1 - I*E^(I*ArcCos[c*x])] - Log[1
+ I*E^(I*ArcCos[c*x])]) - 12*ArcCos[c*x]^2*(Log[1 - E^(I*ArcCos[c*x])] - L
og[1 + E^(I*ArcCos[c*x])]) - 8*Log[Tan[ArcCos[c*x]/2]] + (16*I)*(PolyLog[2
, (-I)*E^(I*ArcCos[c*x])] - PolyLog[2, I*E^(I*ArcCos[c*x])]) - (24*I)*ArcC
os[c*x]*(PolyLog[2, -E^(I*ArcCos[c*x])] - PolyLog[2, E^(I*ArcCos[c*x])]) +
 24*(PolyLog[3, -E^(I*ArcCos[c*x])] - PolyLog[3, E^(I*ArcCos[c*x])]) - Arc
Cos[c*x]^2*Sec[ArcCos[c*x]/2]^2 - (8*ArcCos[c*x]^2*Sin[ArcCos[c*x]/2])/(Co
s[ArcCos[c*x]/2] - Sin[ArcCos[c*x]/2]) + (8*ArcCos[c*x]^2*Sin[ArcCos[c*x]/
2])/(Cos[ArcCos[c*x]/2] + Sin[ArcCos[c*x]/2]) + 4*ArcCos[c*x]*Tan[ArcCos[c
*x]/2]))/2)/(4*d^2)
```

Rubi [A] (verified)

Time = 2.94 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.02, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5205, 27, 5163, 5165, 3042, 4671, 3011, 2720, 5183, 219, 5209, 219, 5219, 3042, 4669, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^2}\,dx$$

$$\Big\downarrow \; 5205$$

$$3c^2\int \frac{(a+b\arccos(cx))^2}{d^2(1-c^2x^2)^2}dx-\frac{2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx}{d^2}-\frac{(a+b\arccos(cx))^2}{d^2x(1-c^2x^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{3c^2\int \frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^2}dx}{d^2}-\frac{2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx}{d^2}-\frac{(a+b\arccos(cx))^2}{d^2x(1-c^2x^2)}$$

$$\Big\downarrow \text{5163}$$

$$\frac{3c^2\left(bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx+\frac{1}{2}\int \frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}\right)}{d^2}-\frac{2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx}{d^2}-\frac{(a+b\arccos(cx))^2}{d^2x\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{5165}$$

$$\frac{3c^2\left(bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx-\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}\right)}{d^2}-\frac{2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx}{d^2}-\frac{(a+b\arccos(cx))^2}{d^2x\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx}{d^2}+\frac{3c^2\left(bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx-\frac{\int (a+b\arccos(cx))^2\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}\right)}{d^2}-\frac{(a+b\arccos(cx))^2}{d^2x\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{4671}$$

$$\frac{3c^2\left(-\frac{-2b\int (a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right.}{2c}\right.}{d^2}$$

$$\frac{2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx}{d^2}-\frac{(a+b\arccos(cx))^2}{d^2x\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{3011}$$

$$3c^2\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a+b\arccos(\right.}{2c}\right.$$

$$\frac{2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx}{d^2}-\frac{(a+b\arccos(cx))^2}{d^2x\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{2720}$$

$$3c^2\left(-\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right.}{2c}\right.$$

$$\frac{2bc\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx}{d^2}-\frac{(a+b\arccos(cx))^2}{d^2x(1-c^2x^2)}$$

$\downarrow$ 5183

$$3c^2\left(-\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right.}{2c}\right.$$

$$\frac{2bc\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx}{d^2}-\frac{(a+b\arccos(cx))^2}{d^2x(1-c^2x^2)}$$

$\downarrow$ 219

$$3c^2\left(-\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right.}{2c}\right.$$

$$\frac{2bc\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx}{d^2}-\frac{(a+b\arccos(cx))^2}{d^2x(1-c^2x^2)}$$

$\downarrow$ 5209

$$3c^2\left(-\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right.}{2c}\right.$$

$$\frac{2bc\left(\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+bc\int\frac{1}{1-c^2x^2}dx+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}\right)}{d^2}-\frac{(a+b\arccos(cx))^2}{d^2x(1-c^2x^2)}$$

$\downarrow$ 219

$$3c^2\left(-\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right.}{2c}\right.$$

$$\frac{2bc\left(\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+b\operatorname{arctanh}(cx)\right)}{d^2}-\frac{(a+b\arccos(cx))^2}{d^2x(1-c^2x^2)}$$

$\downarrow$ 5219

$$3c^2\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right.}{2c}\right.$$

$$\frac{2bc\left(-\int\frac{a+b\arccos(cx)}{cx}d\arccos(cx)+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+b\,\text{arctanh}(cx)\right)}{d^2}-\frac{(a+b\arccos(cx))^2}{d^2x\,(1-c^2x^2)}$$

$\downarrow$ 3042

$$3c^2\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right.}{2c}\right.$$

$$\frac{2bc\left(-\int(a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+b\,\text{arctanh}(cx)\right)}{d^2}-$$

$$\frac{(a+b\arccos(cx))^2}{d^2x\,(1-c^2x^2)}$$

$\downarrow$ 4669

$$-\frac{2bc\left(b\int\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)-b\int\log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)+2i\arctan\left(e^{i\arccos(cx)}\right)(a+\right.}{d^2}$$

$$3c^2\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right.}{2c}\right.$$

$$\frac{(a+b\arccos(cx))^2}{d^2x\,(1-c^2x^2)}$$

$\downarrow$ 2715

$$-\frac{2bc\left(-ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}+ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}+\right.}{d^2}$$

$$3c^2\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right.}{2c}\right.$$

$$\frac{(a+b\arccos(cx))^2}{d^2x\,(1-c^2x^2)}$$

$\downarrow$ 2838

$$\frac{3c^2\left(-\frac{2b(i\,\text{PolyLog}(2,-e^{i\arccos(cx)})(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}(2,-e^{i\arccos(cx)})de^{i\arccos(cx)})-2b(i\,\text{PolyLog}(2,e^{i\arccos(cx)})}{2c}\right.}{}$$

$$\frac{2bc\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}-ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)+ib\,\text{PolyLog}\left(2,ie^{i\,\text{a}}\right.\right.}{d^2}$$

$$\frac{(a+b\arccos(cx))^2}{d^2x\,(1-c^2x^2)}$$

$\downarrow$ 7143

$$-\frac{2bc\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}-ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)+ib\,\text{PolyLog}\left(2,ie\right.\right.}{d^2}$$

$$\frac{3c^2\left(bc\left(\frac{a+b\arccos(cx)}{c^2\sqrt{1-c^2x^2}}+\frac{b\text{arctanh}(cx)}{c^2}\right)-\frac{-2\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))^2+2b(i\,\text{PolyLog}(2,-e^{i\arccos(cx)})(a+b\arccos(cx)}{}\right.}{d^2}$$

$$\frac{(a+b\arccos(cx))^2}{d^2x\,(1-c^2x^2)}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^2*(d - c^2*d*x^2)^2),x]
```

output
```
-((a + b*ArcCos[c*x])^2/(d^2*x*(1 - c^2*x^2))) - (2*b*c*((a + b*ArcCos[c*x
])/Sqrt[1 - c^2*x^2] + (2*I)*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcCos[c*x])]
 + b*ArcTanh[c*x] - I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I*b*PolyLog[2
, I*E^(I*ArcCos[c*x])]))/d^2 + (3*c^2*((x*(a + b*ArcCos[c*x])^2)/(2*(1 - c
^2*x^2)) + b*c*((a + b*ArcCos[c*x])/(c^2*Sqrt[1 - c^2*x^2]) + (b*ArcTanh[c
*x])/c^2) - (-2*(a + b*ArcCos[c*x])^2*ArcTanh[E^(I*ArcCos[c*x])] + 2*b*(I*
(a + b*ArcCos[c*x])*PolyLog[2, -E^(I*ArcCos[c*x])] - b*PolyLog[3, -E^(I*Ar
cCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] -
b*PolyLog[3, E^(I*ArcCos[c*x])]))/(2*c)))/d^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 727, normalized size of antiderivative = 2.24

method	result
parts	$\frac{a^2\left(-\frac{1}{x}-\frac{c}{4(cx-1)}-\frac{3c\ln(cx-1)}{4}-\frac{c}{4(cx+1)}+\frac{3c\ln(cx+1)}{4}\right)}{d^2}+\frac{b^2c\left(-\frac{\left(3c^2x^2\arccos(cx)+2cx\sqrt{-c^2x^2+1}-2\arccos(cx)\right)\arccos(}{2cx\left(c^2x^2-1\right)}\right.}{}$
derivativedivides	$c\left(\frac{a^2\left(-\frac{1}{4(cx-1)}-\frac{3\ln(cx-1)}{4}-\frac{1}{cx}-\frac{1}{4(cx+1)}+\frac{3\ln(cx+1)}{4}\right)}{d^2}+\frac{b^2\left(-\frac{\left(3c^2x^2\arccos(cx)+2cx\sqrt{-c^2x^2+1}-2\arccos(cx)\right)\arccos}{2cx\left(c^2x^2-1\right)}\right.}{}\right.$
default	$c\left(\frac{a^2\left(-\frac{1}{4(cx-1)}-\frac{3\ln(cx-1)}{4}-\frac{1}{cx}-\frac{1}{4(cx+1)}+\frac{3\ln(cx+1)}{4}\right)}{d^2}+\frac{b^2\left(-\frac{\left(3c^2x^2\arccos(cx)+2cx\sqrt{-c^2x^2+1}-2\arccos(cx)\right)\arccos}{2cx\left(c^2x^2-1\right)}\right.}{}\right.$

input
```
int((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/d^2*(-1/x-1/4*c/(c*x-1)-3/4*c*ln(c*x-1)-1/4*c/(c*x+1)+3/4*c*ln(c*x+1))
+b^2/d^2*c*(-1/2*(3*c^2*x^2*arccos(c*x)+2*c*x*(-c^2*x^2+1)^(1/2)-2*arccos(
c*x))*arccos(c*x)/c/x/(c^2*x^2-1)+2*I*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2))
)-2*I*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+3*I*arccos(c*x)*polylog(2,c*x+
I*(-c^2*x^2+1)^(1/2))+3/2*arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-3*I
*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-3/2*arccos(c*x)^2*ln(1-c
*x-I*(-c^2*x^2+1)^(1/2))+ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-ln(I*(-c^2*x^2+1)^
(1/2)+c*x-1)+3*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))-3*polylog(3,c*x+I*(-c^
2*x^2+1)^(1/2))-2*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*arccos(
c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2))))-I*a*b/d^2/(c^2*x^2-1)/x*(3*I*arcc
os(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*c^3*x^3-3*I*arccos(c*x)*ln(1+c*x+I*
(-c^2*x^2+1)^(1/2))*c*x-3*I*arccos(c*x)*c^2*x^2+3*dilog(1+c*x+I*(-c^2*x^2+
1)^(1/2))*c^3*x^3+4*arctan(c*x+I*(-c^2*x^2+1)^(1/2))*c^3*x^3+3*dilog(c*x+I
*(-c^2*x^2+1)^(1/2))*c^3*x^3-I*(-c^2*x^2+1)^(1/2)*x*c+2*I*arccos(c*x)-3*di
log(1+c*x+I*(-c^2*x^2+1)^(1/2))*c*x-4*arctan(c*x+I*(-c^2*x^2+1)^(1/2))*c*x
-3*dilog(c*x+I*(-c^2*x^2+1)^(1/2))*c*x)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\left(d-c^2dx^2\right)^2}\,dx=\int \frac{\left(b\arccos\left(cx\right)+a\right)^2}{\left(c^2dx^2-d\right)^2x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^4*d^2*x^6 - 2*c^
2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\left(d-c^2dx^2\right)^2}\,dx=\frac{\int \frac{a^2}{c^4x^6-2c^2x^4+x^2}\,dx+\int \frac{b^2\operatorname{acos}^2\left(cx\right)}{c^4x^6-2c^2x^4+x^2}\,dx+\int \frac{2ab\operatorname{acos}\left(cx\right)}{c^4x^6-2c^2x^4+x^2}\,dx}{d^2}$$

input
```
integrate((a+b*acos(c*x))**2/x**2/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2/(c**4*x**6 - 2*c**2*x**4 + x**2), x) + Integral(b**2*acos(c
*x)**2/(c**4*x**6 - 2*c**2*x**4 + x**2), x) + Integral(2*a*b*acos(c*x)/(c*
*4*x**6 - 2*c**2*x**4 + x**2), x))/d**2
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(c^2dx^2-d)^2x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/4*a^2*(2*(3*c^2*x^2 - 2)/(c^2*d^2*x^3 - d^2*x) - 3*c*log(c*x + 1)/d^2 +
 3*c*log(c*x - 1)/d^2) - 1/4*((6*b^2*c^2*x^2 - 4*b^2 - 3*(b^2*c^3*x^3 - b^
2*c*x)*log(c*x + 1) + 3*(b^2*c^3*x^3 - b^2*c*x)*log(-c*x + 1))*arctan2(sqr
t(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 4*(c^2*d^2*x^3 - d^2*x)*integrate(-1/2
*((6*b^2*c^3*x^3 - 4*b^2*c*x - 3*(b^2*c^4*x^4 - b^2*c^2*x^2)*log(c*x + 1)
+ 3*(b^2*c^4*x^4 - b^2*c^2*x^2)*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1
)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 4*a*b*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x))/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x))/(c^2*d
^2*x^3 - d^2*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x^2\,(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^2*(d - c^2*d*x^2)^2),x)
```

output
```
int((a + b*acos(c*x))^2/(x^2*(d - c^2*d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^2}\,dx$$
$$= \frac{8\left(\int \frac{acos(cx)}{c^4x^6-2c^2x^4+x^2}dx\right)ab\,c^2x^3 - 8\left(\int \frac{acos(cx)}{c^4x^6-2c^2x^4+x^2}dx\right)abx + 4\left(\int \frac{acos(cx)^2}{c^4x^6-2c^2x^4+x^2}dx\right)b^2c^2x^3 - 4\left(\int \frac{acos(cx)}{c^4x^6-2c^2x^4}\right.}{}$$

input
```
int((a+b*acos(c*x))^2/x^2/(-c^2*d*x^2+d)^2,x)
```

output
```
(8*int(acos(c*x)/(c**4*x**6 - 2*c**2*x**4 + x**2),x)*a*b*c**2*x**3 - 8*int
(acos(c*x)/(c**4*x**6 - 2*c**2*x**4 + x**2),x)*a*b*x + 4*int(acos(c*x)**2/
(c**4*x**6 - 2*c**2*x**4 + x**2),x)*b**2*c**2*x**3 - 4*int(acos(c*x)**2/(c
**4*x**6 - 2*c**2*x**4 + x**2),x)*b**2*x - 3*log(c**2*x - c)*a**2*c**3*x**
3 + 3*log(c**2*x - c)*a**2*c*x + 3*log(c**2*x + c)*a**2*c**3*x**3 - 3*log(
c**2*x + c)*a**2*c*x - 6*a**2*c**2*x**2 + 4*a**2)/(4*d**2*x*(c**2*x**2 - 1
))
```

3.201 $\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 270

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^2}\,dx = -\frac{bc(a+b\arccos(cx))}{d^2x\sqrt{1-c^2x^2}} + \frac{c^2(a+b\arccos(cx))^2}{d^2(1-c^2x^2)} - \frac{(a+b\arccos(cx))^2}{2d^2x^2(1-c^2x^2)} - \frac{4c^2(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)}{d^2} + \frac{b^2c^2\log(x)}{d^2} - \frac{b^2c^2\log(1-c^2x^2)}{2d^2} + \frac{2ibc^2(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{d^2} - \frac{2ibc^2(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{d^2} - \frac{b^2c^2\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)}{d^2} + \frac{b^2c^2\operatorname{PolyLog}\left(3,e^{2i\arccos(cx)}\right)}{d^2}$$

output
```
-b*c*(a+b*arccos(c*x))/d^2/x/(-c^2*x^2+1)^(1/2)+c^2*(a+b*arccos(c*x))^2/d^
2/(-c^2*x^2+1)-1/2*(a+b*arccos(c*x))^2/d^2/x^2/(-c^2*x^2+1)-4*c^2*(a+b*arc
cos(c*x))^2*arctanh((c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^2+b^2*c^2*ln(x)/d^2-1/
2*b^2*c^2*ln(-c^2*x^2+1)/d^2+2*I*b*c^2*(a+b*arccos(c*x))*polylog(2,-(c*x+I
*(-c^2*x^2+1)^(1/2))^2)/d^2-2*I*b*c^2*(a+b*arccos(c*x))*polylog(2,(c*x+I*(
-c^2*x^2+1)^(1/2))^2)/d^2-b^2*c^2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)
/d^2+b^2*c^2*polylog(3,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^2
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^2}\,dx = \frac{-\frac{a^2}{x^2}+\frac{a^2c^2}{1-c^2x^2}+4a^2c^2\log(x)-2a^2c^2\log(1-c^2x^2)+ab\left(\frac{2c\sqrt{1-c^2x^2}}{x}+\frac{c^2\sqrt{1-c^2x^2}}{1-cx}-\frac{c^2\sqrt{1-c^2x^2}}{1+cx}-\frac{2\arccos(cx)}{x^2}+\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^3*(d - c^2*d*x^2)^2),x]
```

output
```
(-(a^2/x^2) + (a^2*c^2)/(1 - c^2*x^2) + 4*a^2*c^2*Log[x] - 2*a^2*c^2*Log[1
 - c^2*x^2] + a*b*((2*c*Sqrt[1 - c^2*x^2])/x + (c^2*Sqrt[1 - c^2*x^2])/(1
- c*x) - (c^2*Sqrt[1 - c^2*x^2])/(1 + c*x) - (2*ArcCos[c*x])/x^2 + (c^2*Ar
cCos[c*x])/(1 - c*x) + (c^2*ArcCos[c*x])/(1 + c*x) - 8*c^2*ArcCos[c*x]*Log
[1 - E^(I*ArcCos[c*x])] - 8*c^2*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] + 8
*c^2*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] + (8*I)*c^2*PolyLog[2, -E^
(I*ArcCos[c*x])] + (8*I)*c^2*PolyLog[2, E^(I*ArcCos[c*x])] - (4*I)*c^2*Pol
yLog[2, -E^((2*I)*ArcCos[c*x])]) + b^2*c^2*((2*c*x*ArcCos[c*x])/Sqrt[1 - c
^2*x^2] + (2*Sqrt[1 - c^2*x^2]*ArcCos[c*x])/(c*x) - ArcCos[c*x]^2/(c^2*x^2
) + ArcCos[c*x]^2/(1 - c^2*x^2) - 4*ArcCos[c*x]^2*(Log[1 - E^((2*I)*ArcCos
[c*x])] - Log[1 + E^((2*I)*ArcCos[c*x])]) - 2*Log[Sqrt[1 - c^2*x^2]/(c*x)]
 - (4*I)*ArcCos[c*x]*(PolyLog[2, -E^((2*I)*ArcCos[c*x])] - PolyLog[2, E^((
2*I)*ArcCos[c*x])]) + 2*(PolyLog[3, -E^((2*I)*ArcCos[c*x])] - PolyLog[3, E
^((2*I)*ArcCos[c*x])])))/(2*d^2)
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.20, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.630, Rules used = {5205, 27, 5195, 25, 354, 86, 2009, 5209, 5161, 240, 5185, 4919, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^2}dx$$

$$\Big\downarrow \text{5205}$$

$$2c^2\int \frac{(a+b\arccos(cx))^2}{d^2x\left(1-c^2x^2\right)^2}dx-\frac{bc\int \frac{a+b\arccos(cx)}{x^2\left(1-c^2x^2\right)^{3/2}}dx}{d^2}-\frac{(a+b\arccos(cx))^2}{2d^2x^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2c^2\int \frac{(a+b\arccos(cx))^2}{x\left(1-c^2x^2\right)^2}dx}{d^2}-\frac{bc\int \frac{a+b\arccos(cx)}{x^2\left(1-c^2x^2\right)^{3/2}}dx}{d^2}-\frac{(a+b\arccos(cx))^2}{2d^2x^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{5195}$$

$$\frac{2c^2\int \frac{(a+b\arccos(cx))^2}{x\left(1-c^2x^2\right)^2}dx}{d^2}-\frac{bc\left(bc\int -\frac{1-2c^2x^2}{x\left(1-c^2x^2\right)}dx+\frac{2c^2x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}\right)}{d^2}-\frac{(a+b\arccos(cx))^2}{2d^2x^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{2c^2\int \frac{(a+b\arccos(cx))^2}{x\left(1-c^2x^2\right)^2}dx}{d^2}-\frac{bc\left(-bc\int \frac{1-2c^2x^2}{x\left(1-c^2x^2\right)}dx+\frac{2c^2x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}\right)}{d^2}-\frac{(a+b\arccos(cx))^2}{2d^2x^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{354}$$

$$\frac{2c^2\int \frac{(a+b\arccos(cx))^2}{x\left(1-c^2x^2\right)^2}dx}{d^2}-\frac{bc\left(-\frac{1}{2}bc\int \frac{1-2c^2x^2}{x^2\left(1-c^2x^2\right)}dx^2+\frac{2c^2x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}\right)}{d^2}-\frac{(a+b\arccos(cx))^2}{2d^2x^2\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{86}$$

$$\frac{2c^2\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}dx}{d^2}-\frac{bc\left(-\frac{1}{2}bc\int\left(\frac{c^2}{c^2x^2-1}+\frac{1}{x^2}\right)dx^2+\frac{2c^2x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}\right)}{d^2}-\frac{(a+b\arccos(cx))^2}{2d^2x^2(1-c^2x^2)}$$

$\big\downarrow$ 2009

$$\frac{2c^2\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}dx}{d^2}-\frac{(a+b\arccos(cx))^2}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{2c^2x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(\log\left(1-c^2x^2\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$\big\downarrow$ 5209

$$\frac{2c^2\left(bc\int\frac{a+b\arccos(cx)}{(1-c^2x^2)^{3/2}}dx+\int\frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}dx+\frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)}\right)}{d^2}-\frac{(a+b\arccos(cx))^2}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{2c^2x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(\log\left(1-c^2x^2\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$\big\downarrow$ 5161

$$\frac{2c^2\left(bc\left(bc\int\frac{x}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)+\int\frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}dx+\frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)}\right)}{d^2}-\frac{(a+b\arccos(cx))^2}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{2c^2x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(\log\left(1-c^2x^2\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$\big\downarrow$ 240

$$\frac{2c^2\left(\int\frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}dx+\frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)}+bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)\right)}{d^2}-\frac{(a+b\arccos(cx))^2}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{2c^2x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(\log\left(1-c^2x^2\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$\big\downarrow$ 5185

$$\frac{2c^2\left(-\int\frac{(a+b\arccos(cx))^2}{cx\sqrt{1-c^2x^2}}d\arccos(cx)+\frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)}+bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)\right)}{d^2}-\frac{(a+b\arccos(cx))^2}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{2c^2x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(\log\left(1-c^2x^2\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$\big\downarrow$ 4919

$$\frac{2c^2\left(-2\int(a+b\arccos(cx))^2\csc(2\arccos(cx))d\arccos(cx)+\frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)}+bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)\right.}{d^2}$$
$$\frac{(a+b\arccos(cx))^2}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{2c^2x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(\log\left(1-c^2x^2\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$\downarrow$ 3042

$$\frac{2c^2\left(-2\int(a+b\arccos(cx))^2\csc(2\arccos(cx))d\arccos(cx)+\frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)}+bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)\right.}{d^2}$$
$$\frac{(a+b\arccos(cx))^2}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{2c^2x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(\log\left(1-c^2x^2\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$\downarrow$ 4671

$$\frac{2c^2\left(-2\left(-b\int(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+b\int(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)d\right.\right.}{d^2}$$
$$\frac{(a+b\arccos(cx))^2}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{2c^2x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(\log\left(1-c^2x^2\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$\downarrow$ 3011

$$\frac{2c^2\left(-2\left(b\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-b\left(\frac{1}{2}\right.\right.\right.}{}$$
$$\frac{(a+b\arccos(cx))^2}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{2c^2x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(\log\left(1-c^2x^2\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$\downarrow$ 2720

$$\frac{2c^2\left(-2\left(b\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right.\right.\right.}{}$$
$$\frac{(a+b\arccos(cx))^2}{2d^2x^2(1-c^2x^2)}-\frac{bc\left(\frac{2c^2x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(\log\left(1-c^2x^2\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

$\downarrow$ 7143

$$\frac{2c^2\left(-2\left(-\text{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2+b\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\text{Pol}\right.\right.\right.}{}$$

$$\frac{(a+b\arccos(cx))^2}{2d^2x^2\left(1-c^2x^2\right)}-\frac{bc\left(\frac{2c^2x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(\log\left(1-c^2x^2\right)+\log\left(x^2\right)\right)\right)}{d^2}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^3*(d - c^2*d*x^2)^2),x]
```

output
```
-1/2*(a + b*ArcCos[c*x])^2/(d^2*x^2*(1 - c^2*x^2)) - (b*c*(-((a + b*ArcCos
[c*x])/(x*Sqrt[1 - c^2*x^2])) + (2*c^2*x*(a + b*ArcCos[c*x]))/Sqrt[1 - c^2
*x^2] - (b*c*(Log[x^2] + Log[1 - c^2*x^2]))/2))/d^2 + (2*c^2*((a + b*ArcCo
s[c*x])^2/(2*(1 - c^2*x^2)) + b*c*((x*(a + b*ArcCos[c*x]))/Sqrt[1 - c^2*x^
2] - (b*Log[1 - c^2*x^2])/(2*c)) - 2*(-((a + b*ArcCos[c*x])^2*ArcTanh[E^((
2*I)*ArcCos[c*x])]) + b*((I/2)*(a + b*ArcCos[c*x])*PolyLog[2, -E^((2*I)*Ar
cCos[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/4) - b*((I/2)*(a + b*
ArcCos[c*x])*PolyLog[2, E^((2*I)*ArcCos[c*x])] - (b*PolyLog[3, E^((2*I)*Ar
cCos[c*x])])/4))))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(317) = 634$.

Time = 0.71 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.49

method	result
derivativedivides	$c^2\left(\frac{a^2\left(-\frac{1}{4(cx-1)}-\ln(cx-1)-\frac{1}{2c^2x^2}+2\ln(cx)+\frac{1}{4cx+4}-\ln(cx+1)\right)}{d^2}+\frac{b^2\left(-\frac{\arccos(cx)\left(2c^2x^2\arccos(cx)+2cx\sqrt{-c^2x^2}\right.}{2\left(c^2x^2-1\right)c^2x^2}\right.}{}\right.$
default	$c^2\left(\frac{a^2\left(-\frac{1}{4(cx-1)}-\ln(cx-1)-\frac{1}{2c^2x^2}+2\ln(cx)+\frac{1}{4cx+4}-\ln(cx+1)\right)}{d^2}+\frac{b^2\left(-\frac{\arccos(cx)\left(2c^2x^2\arccos(cx)+2cx\sqrt{-c^2x^2}\right.}{2\left(c^2x^2-1\right)c^2x^2}\right.}{}\right.$
parts	$\frac{a^2\left(-\frac{1}{2x^2}+2c^2\ln(x)-\frac{c^2}{4(cx-1)}-c^2\ln(cx-1)+\frac{c^2}{4cx+4}-c^2\ln(cx+1)\right)}{d^2}+\frac{b^2c^2\left(-\frac{\arccos(cx)\left(2c^2x^2\arccos(cx)+2cx\sqrt{-c^2x^2}\right.}{2\left(c^2x^2-1\right)c^2x^2}\right.}{}$

input
```
int((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(a^2/d^2*(-1/4/(c*x-1)-ln(c*x-1)-1/2/c^2/x^2+2*ln(c*x)+1/4/(c*x+1)-ln(
c*x+1))+b^2/d^2*(-1/2/(c^2*x^2-1)/c^2/x^2*arccos(c*x)*(2*c^2*x^2*arccos(c*
x)+2*c*x*(-c^2*x^2+1)^(1/2)-arccos(c*x))+ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2
)-ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)+2*arccos(c
*x)^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-2*I*arccos(c*x)*polylog(2,-(c*x+I
*(-c^2*x^2+1)^(1/2))^2)+polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-2*arccos(
c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+4*I*arccos(c*x)*polylog(2,-c*x-I*(-c
^2*x^2+1)^(1/2))-4*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))-2*arccos(c*x)^2*ln
(1-c*x-I*(-c^2*x^2+1)^(1/2))+4*I*arccos(c*x)*polylog(2,c*x+I*(-c^2*x^2+1)^
(1/2))-4*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2)))+2*a*b/d^2*(-1/2*(2*c^2*x^2*a
rccos(c*x)+c*x*(-c^2*x^2+1)^(1/2)-arccos(c*x))/(c^2*x^2-1)/c^2/x^2+2*arcco
s(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-2*arccos(c*x)*ln(1+c*x+I*(-c^2*x
^2+1)^(1/2))-2*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-I*polylog(2,-(c*
x+I*(-c^2*x^2+1)^(1/2))^2)+2*I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+2*I*po
lylog(2,c*x+I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^2}\,dx=\int \frac{\left(b\arccos\left(cx\right)+a\right)^2}{\left(c^2dx^2-d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^4*d^2*x^7 - 2*c^
2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^2}\,dx=\frac{\int \frac{a^2}{c^4x^7-2c^2x^5+x^3}\,dx+\int \frac{b^2\operatorname{acos}^2\left(cx\right)}{c^4x^7-2c^2x^5+x^3}\,dx+\int \frac{2ab\operatorname{acos}\left(cx\right)}{c^4x^7-2c^2x^5+x^3}\,dx}{d^2}$$

input
```
integrate((a+b*acos(c*x))**2/x**3/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2/(c**4*x**7 - 2*c**2*x**5 + x**3), x) + Integral(b**2*acos(c
*x)**2/(c**4*x**7 - 2*c**2*x**5 + x**3), x) + Integral(2*a*b*acos(c*x)/(c*
*4*x**7 - 2*c**2*x**5 + x**3), x))/d**2
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^2}\,dx = \int \frac{\left(b\arccos\left(cx\right)+a\right)^2}{\left(c^2dx^2-d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*(2*c^2*log(c*x + 1)/d^2 + 2*c^2*log(c*x - 1)/d^2 - 4*c^2*log(x)/d
^2 + (2*c^2*x^2 - 1)/(c^2*d^2*x^4 - d^2*x^2)) + integrate((b^2*arctan2(sqr
t(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x
+ 1), c*x))/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x^3\,(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^3*(d - c^2*d*x^2)^2),x)
```

output
```
int((a + b*acos(c*x))^2/(x^3*(d - c^2*d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^2}\,dx$$
$$= \frac{4\left(\int \frac{acos(cx)}{c^4x^7-2c^2x^5+x^3}dx\right)ab\,c^2x^4 - 4\left(\int \frac{acos(cx)}{c^4x^7-2c^2x^5+x^3}dx\right)ab\,x^2 + 2\left(\int \frac{acos(cx)^2}{c^4x^7-2c^2x^5+x^3}dx\right)b^2c^2x^4 - 2\left(\int \frac{acos(c}{c^4x^7-2c^2}\right.}{}$$

input
```
int((a+b*acos(c*x))^2/x^3/(-c^2*d*x^2+d)^2,x)
```

output
```
(4*int(acos(c*x)/(c**4*x**7 - 2*c**2*x**5 + x**3),x)*a*b*c**2*x**4 - 4*int
(acos(c*x)/(c**4*x**7 - 2*c**2*x**5 + x**3),x)*a*b*x**2 + 2*int(acos(c*x)*
*2/(c**4*x**7 - 2*c**2*x**5 + x**3),x)*b**2*c**2*x**4 - 2*int(acos(c*x)**2
/(c**4*x**7 - 2*c**2*x**5 + x**3),x)*b**2*x**2 - 2*log(c**2*x - c)*a**2*c*
*4*x**4 + 2*log(c**2*x - c)*a**2*c**2*x**2 - 2*log(c**2*x + c)*a**2*c**4*x
**4 + 2*log(c**2*x + c)*a**2*c**2*x**2 + 4*log(x)*a**2*c**4*x**4 - 4*log(x
)*a**2*c**2*x**2 - 2*a**2*c**4*x**4 + a**2)/(2*d**2*x**2*(c**2*x**2 - 1))
```

3.202 $\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 439

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^2}\,dx = -\frac{b^2c^2}{3d^2x} - \frac{2bc^3(a+b\arccos(cx))}{3d^2\sqrt{1-c^2x^2}} - \frac{bc(a+b\arccos(cx))}{3d^2x^2\sqrt{1-c^2x^2}} - \frac{(a+b\arccos(cx))^2}{3d^2x^3(1-c^2x^2)} - \frac{5c^2(a+b\arccos(cx))^2}{3d^2x(1-c^2x^2)} + \frac{5c^4x(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)} - \frac{5ic^3(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)}{d^2} - \frac{26bc^3(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{3d^2} + \frac{b^2c^3\operatorname{arctanh}(cx)}{d^2} + \frac{13ib^2c^3\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{3d^2} + \frac{5ibc^3(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{d^2} - \frac{5ibc^3(a+b\arccos(cx))\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{d^2} - \frac{13ib^2c^3\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{3d^2} - \frac{5b^2c^3\operatorname{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)}{d^2} + \frac{5b^2c^3\operatorname{PolyLog}\left(3,ie^{i\arccos(cx)}\right)}{d^2}$$

output

```
-1/3*b^2*c^2/d^2/x-2/3*b*c^3*(a+b*arccos(c*x))/d^2/(-c^2*x^2+1)^(1/2)-1/3*
b*c*(a+b*arccos(c*x))/d^2/x^2/(-c^2*x^2+1)^(1/2)-1/3*(a+b*arccos(c*x))^2/d
^2/x^3/(-c^2*x^2+1)-5/3*c^2*(a+b*arccos(c*x))^2/d^2/x/(-c^2*x^2+1)+5/2*c^4
*x*(a+b*arccos(c*x))^2/d^2/(-c^2*x^2+1)-5*I*c^3*(a+b*arccos(c*x))^2*arctan
(c*x+I*(-c^2*x^2+1)^(1/2))/d^2-26/3*b*c^3*(a+b*arccos(c*x))*arctanh(c*x+I*
(-c^2*x^2+1)^(1/2))/d^2+b^2*c^3*arctanh(c*x)/d^2+13/3*I*b^2*c^3*polylog(2,
-c*x-I*(-c^2*x^2+1)^(1/2))/d^2+5*I*b*c^3*(a+b*arccos(c*x))*polylog(2,-I*(c
*x+I*(-c^2*x^2+1)^(1/2)))/d^2-5*I*b*c^3*(a+b*arccos(c*x))*polylog(2,I*(c*x
+I*(-c^2*x^2+1)^(1/2)))/d^2-13/3*I*b^2*c^3*polylog(2,c*x+I*(-c^2*x^2+1)^(1
/2))/d^2-5*b^2*c^3*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^2+5*b^2*c^3*
polylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 920 vs. $2(439) = 878$.

Time = 8.03 (sec) , antiderivative size = 920, normalized size of antiderivative = 2.10

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^4*(d - c^2*d*x^2)^2),x]
```

output
```
-1/3*a^2/(d^2*x^3) - (2*a^2*c^2)/(d^2*x) - (a^2*c^4*x)/(2*d^2*(-1 + c^2*x^
2)) - (5*a^2*c^3*Log[1 - c*x])/(4*d^2) + (5*a^2*c^3*Log[1 + c*x])/(4*d^2)
+ (2*a*b*((c*Sqrt[1 - c^2*x^2])/(6*x^2) + (c^4*(Sqrt[1 - c^2*x^2] - ArcCos
[c*x]))/(4*(c + c^2*x)) - ArcCos[c*x]/(3*x^3) + (c^4*(Sqrt[1 - c^2*x^2] +
ArcCos[c*x]))/(4*(c - c^2*x)) - (c^3*Log[x])/6 + (c^3*Log[1 + Sqrt[1 - c^2
*x^2]])/6 + 2*c^2*(-(ArcCos[c*x]/x) - c*Log[x] + c*Log[1 + Sqrt[1 - c^2*x^
2]]) + (5*c^4*(((-1/2*I)*ArcCos[c*x]^2)/c + (2*ArcCos[c*x]*Log[1 + E^(I*Ar
cCos[c*x])])/c - ((2*I)*PolyLog[2, -E^(I*ArcCos[c*x])])/c))/4 + ((5*I)/8)*
c^3*(ArcCos[c*x]*(ArcCos[c*x] + (4*I)*Log[1 - E^(I*ArcCos[c*x])]) + 4*Poly
Log[2, E^(I*ArcCos[c*x])])))/d^2 + (b^2*c^3*(-8 + (2*(-2 + ArcCos[c*x])*Ar
cCos[c*x])/(-1 + Sqrt[1 - c^2*x^2]) - 52*ArcCos[c*x]^2 + 12*ArcCos[c*x]*Co
t[ArcCos[c*x]/2] + 3*ArcCos[c*x]^2*Csc[ArcCos[c*x]/2]^2 - 24*Log[Tan[ArcCo
s[c*x]/2]] + 104*(ArcCos[c*x]*(Log[1 - I*E^(I*ArcCos[c*x])] - Log[1 + I*E^
(I*ArcCos[c*x])]) + I*(PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - PolyLog[2, I*E
^(I*ArcCos[c*x])])) - 60*(ArcCos[c*x]^2*(Log[1 - E^(I*ArcCos[c*x])] - Log[
1 + E^(I*ArcCos[c*x])]) + (2*I)*ArcCos[c*x]*(PolyLog[2, -E^(I*ArcCos[c*x])
] - PolyLog[2, E^(I*ArcCos[c*x])]) + 2*(-PolyLog[3, -E^(I*ArcCos[c*x])] +
PolyLog[3, E^(I*ArcCos[c*x])])) - 3*ArcCos[c*x]^2*Sec[ArcCos[c*x]/2]^2 - (
4*ArcCos[c*x]^2*Sin[ArcCos[c*x]/2])/(Cos[ArcCos[c*x]/2] - Sin[ArcCos[c*x]/
2])^3 - (4*(2 + 13*ArcCos[c*x]^2)*Sin[ArcCos[c*x]/2])/(Cos[ArcCos[c*x]/...
```

Rubi [A] (verified)

Time = 4.58 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.19, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {5205, 27, 5205, 264, 219, 5163, 5165, 3042, 4671, 3011, 2720, 5183, 219, 5209, 219, 5219, 3042, 4669, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^4\left(d-c^2dx^2\right)^2}dx$$

$$\Big\downarrow \text{5205}$$

$$\frac{5}{3}c^2\int \frac{(a+b\arccos(cx))^2}{d^2x^2\left(1-c^2x^2\right)^2}dx-\frac{2bc\int \frac{a+b\arccos(cx)}{x^3(1-c^2x^2)^{3/2}}dx}{3d^2}-\frac{(a+b\arccos(cx))^2}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5c^2\int \frac{(a+b\arccos(cx))^2}{x^2(1-c^2x^2)^2}dx}{3d^2}-\frac{2bc\int \frac{a+b\arccos(cx)}{x^3(1-c^2x^2)^{3/2}}dx}{3d^2}-\frac{(a+b\arccos(cx))^2}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{5205}$$

$$\frac{5c^2\left(3c^2\int \frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^2}dx-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}\right)}{3d^2}-\frac{2bc\left(\frac{3}{2}c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx-\frac{a+b\arccos(cx)}{2x^2\sqrt{1-c^2x^2}}\right)}{3d^2}-\frac{(a+b\arccos(cx))^2}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{264}$$

$$\frac{5c^2\left(3c^2\int \frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^2}dx-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}\right)}{3d^2}-\frac{2bc\left(\frac{3}{2}c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx-\frac{1}{x}\right)-\frac{a+b\arccos(cx)}{2x^2\sqrt{1-c^2x^2}}\right)}{3d^2}-\frac{(a+b\arccos(cx))^2}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2bc\left(\frac{3}{2}c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{a+b\arccos(cx)}{2x^2\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)\right)}{3d^2}+\frac{5c^2\left(3c^2\int \frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^2}dx-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}\right)}{3d^2}-\frac{(a+b\arccos(cx))^2}{3d^2x^3\left(1-c^2x^2\right)}$$

$\Big\downarrow$ 5163

$$-\frac{2bc\left(\frac{3}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{a+b\arccos(cx)}{2x^2\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)\right)}{3d^2}+\frac{5c^2\left(3c^2\left(bc\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx+\frac{1}{2}\int\frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}\right)-2bc\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{(a+b\arccos(cx}{x(1-c^2x^2)}\right.}{3d^2}$$

$$\frac{(a+b\arccos(cx))^2}{3d^2x^3\left(1-c^2x^2\right)}$$

$\Big\downarrow$ 5165

$$-\frac{2bc\left(\frac{3}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{a+b\arccos(cx)}{2x^2\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)\right)}{3d^2}+\frac{5c^2\left(3c^2\left(bc\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx-\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}\right)-2bc\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{(a+b\arc}{x(1-}\right.}{3d^2}$$

$$\frac{(a+b\arccos(cx))^2}{3d^2x^3\left(1-c^2x^2\right)}$$

$\Big\downarrow$ 3042

$$-\frac{2bc\left(\frac{3}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{a+b\arccos(cx)}{2x^2\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)\right)}{3d^2}+\frac{5c^2\left(-2bc\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx+3c^2\left(bc\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx-\frac{\int(a+b\arccos(cx))^2\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx}{2(1-c^2x^2)}\right.\right.}{3d^2}$$

$$\frac{(a+b\arccos(cx))^2}{3d^2x^3\left(1-c^2x^2\right)}$$

$\Big\downarrow$ 4671

$$-\frac{2bc\left(\frac{3}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{a+b\arccos(cx)}{2x^2\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)\right)}{3d^2}+\frac{5c^2\left(3c^2\left(-\frac{-2b\int(a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int(a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\,\text{arc}}\right.}{2c}\right.\right.}{3d^2}$$

$$\frac{(a+b\arccos(cx))^2}{3d^2x^3\left(1-c^2x^2\right)}$$

$\Big\downarrow$ 3011

$$5c^2\left(3c^2\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a+b\,\text{a}\right.}{2c}\right.\right.$$

$$\frac{2bc\left(\frac{3}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{a+b\arccos(cx)}{2x^2\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)\right)}{3d^2}-\frac{(a+b\arccos(cx))^2}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \; 2720$$

$$5c^2\left(3c^2\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\,\text{arcc}}\right.\right.}{2c}\right.\right.$$

$$\frac{2bc\left(\frac{3}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{a+b\arccos(cx)}{2x^2\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)\right)}{3d^2}-\frac{(a+b\arccos(cx))^2}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \; 5183$$

$$5c^2\left(3c^2\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\,\text{arcc}}\right.\right.}{2c}\right.\right.$$

$$\frac{2bc\left(\frac{3}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{a+b\arccos(cx)}{2x^2\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)\right)}{3d^2}-\frac{(a+b\arccos(cx))^2}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \; 219$$

$$5c^2\left(3c^2\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\,\text{arcc}}\right.\right.}{2c}\right.\right.$$

$$\frac{2bc\left(\frac{3}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx-\frac{a+b\arccos(cx)}{2x^2\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)\right)}{3d^2}-\frac{(a+b\arccos(cx))^2}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \; 5209$$

$$5c^2\left(3c^2\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\,\text{arcc}}\right.\right.}{2c}\right.\right.$$

$$\frac{2bc\left(\frac{3}{2}c^2\left(\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+bc\int\frac{1}{1-c^2x^2}dx+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}\right)-\frac{a+b\arccos(cx)}{2x^2\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)\right)}{3d^2}-\frac{(a+b\arccos(cx))^2}{3d^2x^3\left(1-c^2x^2\right)}$$

$$\Big\downarrow \; 219$$

$$5c^2\left(3c^2\left(-\frac{2b(i\,\text{PolyLog}(2,-e^{i\arccos(cx)})(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}(2,-e^{i\arccos(cx)})de^{i\arccos(cx)})-2b(i\,\text{PolyLog}(2,e^{i\arccos}}{2c}\right.\right.$$

$$\frac{2bc\left(\frac{3}{2}c^2\left(\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+b\text{arctanh}(cx)\right)-\frac{a+b\arccos(cx)}{2x^2\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)\right)}{3d^2}-$$

$$\frac{(a+b\arccos(cx))^2}{3d^2x^3(1-c^2x^2)}$$

$$\Big\downarrow \text{5219}$$

$$5c^2\left(3c^2\left(-\frac{2b(i\,\text{PolyLog}(2,-e^{i\arccos(cx)})(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}(2,-e^{i\arccos(cx)})de^{i\arccos(cx)})-2b(i\,\text{PolyLog}(2,e^{i\arccos}}{2c}\right.\right.$$

$$\frac{2bc\left(\frac{3}{2}c^2\left(-\int\frac{a+b\arccos(cx)}{cx}d\arccos(cx)+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+b\text{arctanh}(cx)\right)-\frac{a+b\arccos(cx)}{2x^2\sqrt{1-c^2x^2}}-\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)\right.}{3d^2}$$

$$\frac{(a+b\arccos(cx))^2}{3d^2x^3(1-c^2x^2)}$$

$$\Big\downarrow \text{3042}$$

$$5c^2\left(3c^2\left(-\frac{2b(i\,\text{PolyLog}(2,-e^{i\arccos(cx)})(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}(2,-e^{i\arccos(cx)})de^{i\arccos(cx)})-2b(i\,\text{PolyLog}(2,e^{i\arccos}}{2c}\right.\right.$$

$$\frac{2bc\left(\frac{3}{2}c^2\left(-\int(a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+b\text{arctanh}(cx)\right)-\frac{a+b\arccos(cx)}{2x^2\sqrt{1-c^2x^2}}\right.}{3d^2}$$

$$\frac{(a+b\arccos(cx))^2}{3d^2x^3(1-c^2x^2)}$$

$$\Big\downarrow \text{4669}$$

$$5c^2\left(-2bc\left(b\int\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)-b\int\log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)+2i\arctan\left(e^{i\arccos(cx)}\right)\right.\right.$$

$$\frac{2bc\left(\frac{3}{2}c^2\left(b\int\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)-b\int\log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)+2i\arctan\left(e^{i\arccos(cx)}\right)\right.\right.}{3d^2}$$

$$\frac{(a+b\arccos(cx))^2}{3d^2x^3(1-c^2x^2)}$$

$$\Big\downarrow \text{2715}$$

$$\frac{5c^2\left(-2bc\left(-ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}+ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(c\cdots}\right.\right.}{\cdots}$$

$$\frac{2bc\left(\frac{3}{2}c^2\left(-ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}+ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}\cdots\right.\right.}{\cdots}$$

$$\frac{(a+b\arccos(cx))^2}{3d^2x^3(1-c^2x^2)}$$

$\downarrow$ 2838

$$\frac{5c^2\left(3c^2\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos\cdots}\right.\right.}{2c}\right.\right.}{\cdots}$$

$$\frac{2bc\left(\frac{3}{2}c^2\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}-ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)+ib\,\text{PolyLog}\left(2\cdots\right.\right.\right.}{3d^2}$$

$$\frac{(a+b\arccos(cx))^2}{3d^2x^3(1-c^2x^2)}$$

$\downarrow$ 7143

$$\frac{5c^2\left(-2bc\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}-ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)+ib\,\text{PolyLog}\cdots\right.\right.}{\cdots}$$

$$\frac{2bc\left(\frac{3}{2}c^2\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}-ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)+ib\,\text{PolyLog}\left(2\cdots\right.\right.\right.}{3d^2}$$

$$\frac{(a+b\arccos(cx))^2}{3d^2x^3(1-c^2x^2)}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^4*(d - c^2*d*x^2)^2),x]
```

output
```
-1/3*(a + b*ArcCos[c*x])^2/(d^2*x^3*(1 - c^2*x^2)) - (2*b*c*(-1/2*(a + b*A
rcCos[c*x])/(x^2*Sqrt[1 - c^2*x^2]) - (b*c*(-x^(-1) + c*ArcTanh[c*x]))/2 +
 (3*c^2*((a + b*ArcCos[c*x])/Sqrt[1 - c^2*x^2] + (2*I)*(a + b*ArcCos[c*x])
*ArcTan[E^(I*ArcCos[c*x])] + b*ArcTanh[c*x] - I*b*PolyLog[2, (-I)*E^(I*Arc
Cos[c*x])] + I*b*PolyLog[2, I*E^(I*ArcCos[c*x])]))/2))/(3*d^2) + (5*c^2*(-
((a + b*ArcCos[c*x])^2/(x*(1 - c^2*x^2))) - 2*b*c*((a + b*ArcCos[c*x])/Sqr
t[1 - c^2*x^2] + (2*I)*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcCos[c*x])] + b*A
rcTanh[c*x] - I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I*b*PolyLog[2, I*E^
(I*ArcCos[c*x])]) + 3*c^2*((x*(a + b*ArcCos[c*x])^2)/(2*(1 - c^2*x^2)) + b
*c*((a + b*ArcCos[c*x])/(c^2*Sqrt[1 - c^2*x^2]) + (b*ArcTanh[c*x])/c^2) -
(-2*(a + b*ArcCos[c*x])^2*ArcTanh[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*ArcCo
s[c*x])*PolyLog[2, -E^(I*ArcCos[c*x])] - b*PolyLog[3, -E^(I*ArcCos[c*x])])
 - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] - b*PolyLog[3,
 E^(I*ArcCos[c*x])]))/(2*c))))/(3*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.60

method	result
derivativedivides	$c^3\left(\frac{a^2\left(-\frac{1}{4(cx-1)}-\frac{5\ln(cx-1)}{4}-\frac{1}{3c^3x^3}-\frac{2}{cx}-\frac{1}{4(cx+1)}+\frac{5\ln(cx+1)}{4}\right)}{d^2}+\frac{b^2\left(-\frac{15\arccos(cx)^2x^4c^4+4\sqrt{-c^2x^2+1}\,\arccos(cx)}{}\right.}{}\right.$
default	$c^3\left(\frac{a^2\left(-\frac{1}{4(cx-1)}-\frac{5\ln(cx-1)}{4}-\frac{1}{3c^3x^3}-\frac{2}{cx}-\frac{1}{4(cx+1)}+\frac{5\ln(cx+1)}{4}\right)}{d^2}+\frac{b^2\left(-\frac{15\arccos(cx)^2x^4c^4+4\sqrt{-c^2x^2+1}\,\arccos(cx)}{}\right.}{}\right.$
parts	$\frac{a^2\left(-\frac{1}{3x^3}-\frac{2c^2}{x}-\frac{c^3}{4(cx-1)}-\frac{5c^3\ln(cx-1)}{4}-\frac{c^3}{4(cx+1)}+\frac{5c^3\ln(cx+1)}{4}\right)}{d^2}+\frac{b^2c^3\left(-\frac{15\arccos(cx)^2x^4c^4+4\sqrt{-c^2x^2+1}\,\arccos(cx}{}\right.}{}$

input
```
int((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c^3*(a^2/d^2*(-1/4/(c*x-1)-5/4*ln(c*x-1)-1/3/c^3/x^3-2/c/x-1/4/(c*x+1)+5/4
*ln(c*x+1))+b^2/d^2*(-1/6*(15*arccos(c*x)^2*x^4*c^4+4*(-c^2*x^2+1)^(1/2)*a
rccos(c*x)*x^3*c^3-10*arccos(c*x)^2*x^2*c^2+2*c^4*x^4+2*(-c^2*x^2+1)^(1/2)
*arccos(c*x)*x*c-2*arccos(c*x)^2-2*c^2*x^2)/(c^2*x^2-1)/c^3/x^3+5*I*arccos
(c*x)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+5/2*arccos(c*x)^2*ln(1+c*x+I*(-c
^2*x^2+1)^(1/2))-5*I*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-5/2*
arccos(c*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+13/3*I*dilog(1+I*(c*x+I*(-c^2
*x^2+1)^(1/2)))-13/3*I*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+ln(1+c*x+I*(-
c^2*x^2+1)^(1/2))-ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)+5*polylog(3,-c*x-I*(-c^2*
x^2+1)^(1/2))-13/3*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+13/3*arc
cos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-5*polylog(3,c*x+I*(-c^2*x^2+1)
^(1/2)))+2*a*b/d^2*(-1/6*(15*c^4*x^4*arccos(c*x)+2*c^3*x^3*(-c^2*x^2+1)^(1
/2)-10*c^2*x^2*arccos(c*x)+c*x*(-c^2*x^2+1)^(1/2)-2*arccos(c*x))/c^3/x^3/(
c^2*x^2-1)-5/2*I*dilog(1+c*x+I*(-c^2*x^2+1)^(1/2))-13/3*I*arctan(c*x+I*(-c
^2*x^2+1)^(1/2))-5/2*I*dilog(c*x+I*(-c^2*x^2+1)^(1/2))+5/2*arccos(c*x)*ln(
1+c*x+I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^2}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(c^2dx^2-d)^2x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^4*d^2*x^8 - 2*c^
2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^2}\,dx=\frac{\int \frac{a^2}{c^4x^8-2c^2x^6+x^4}\,dx+\int \frac{b^2\operatorname{acos}^2(cx)}{c^4x^8-2c^2x^6+x^4}\,dx+\int \frac{2ab\operatorname{acos}(cx)}{c^4x^8-2c^2x^6+x^4}\,dx}{d^2}$$

input
```
integrate((a+b*acos(c*x))**2/x**4/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2/(c**4*x**8 - 2*c**2*x**6 + x**4), x) + Integral(b**2*acos(c
*x)**2/(c**4*x**8 - 2*c**2*x**6 + x**4), x) + Integral(2*a*b*acos(c*x)/(c*
*4*x**8 - 2*c**2*x**6 + x**4), x))/d**2
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(c^2dx^2-d)^2x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/12*(15*c^3*log(c*x + 1)/d^2 - 15*c^3*log(c*x - 1)/d^2 - 2*(15*c^4*x^4 -
10*c^2*x^2 - 2)/(c^2*d^2*x^5 - d^2*x^3))*a^2 - 1/12*((30*b^2*c^4*x^4 - 20*
b^2*c^2*x^2 - 4*b^2 - 15*(b^2*c^5*x^5 - b^2*c^3*x^3)*log(c*x + 1) + 15*(b^
2*c^5*x^5 - b^2*c^3*x^3)*log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x)^2 + 12*(c^2*d^2*x^5 - d^2*x^3)*integrate(-1/6*((30*b^2*c^5*x^5 -
20*b^2*c^3*x^3 - 4*b^2*c*x - 15*(b^2*c^6*x^6 - b^2*c^4*x^4)*log(c*x + 1) +
 15*(b^2*c^6*x^6 - b^2*c^4*x^4)*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1
)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 12*a*b*arctan2(sqrt(c*x + 1
)*sqrt(-c*x + 1), c*x))/(c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4), x))/(c^2*
d^2*x^5 - d^2*x^3)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x^4\,(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^4*(d - c^2*d*x^2)^2),x)
```

output
```
int((a + b*acos(c*x))^2/(x^4*(d - c^2*d*x^2)^2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^2}\,dx$$
$$= \frac{24\left(\int \frac{acos(cx)}{c^4x^8-2c^2x^6+x^4}dx\right)ab\,c^2x^5 - 24\left(\int \frac{acos(cx)}{c^4x^8-2c^2x^6+x^4}dx\right)ab\,x^3 + 12\left(\int \frac{acos(cx)^2}{c^4x^8-2c^2x^6+x^4}dx\right)b^2c^2x^5 - 12\left(\int \frac{}{c^4x}\right.}{}$$

input
```
int((a+b*acos(c*x))^2/x^4/(-c^2*d*x^2+d)^2,x)
```

output
```
(24*int(acos(c*x)/(c**4*x**8 - 2*c**2*x**6 + x**4),x)*a*b*c**2*x**5 - 24*i
nt(acos(c*x)/(c**4*x**8 - 2*c**2*x**6 + x**4),x)*a*b*x**3 + 12*int(acos(c*
x)**2/(c**4*x**8 - 2*c**2*x**6 + x**4),x)*b**2*c**2*x**5 - 12*int(acos(c*x
)**2/(c**4*x**8 - 2*c**2*x**6 + x**4),x)*b**2*x**3 - 15*log(c**2*x - c)*a*
*2*c**5*x**5 + 15*log(c**2*x - c)*a**2*c**3*x**3 + 15*log(c**2*x + c)*a**2
*c**5*x**5 - 15*log(c**2*x + c)*a**2*c**3*x**3 - 30*a**2*c**4*x**4 + 20*a*
*2*c**2*x**2 + 4*a**2)/(12*d**2*x**3*(c**2*x**2 - 1))
```

3.203 $\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 343

$$\begin{aligned}\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx &= \frac{b^2x}{12c^4d^3(1-c^2x^2)} - \frac{b(a+b\arccos(cx))}{6c^5d^3(1-c^2x^2)^{3/2}} \\ &\quad + \frac{5b(a+b\arccos(cx))}{4c^5d^3\sqrt{1-c^2x^2}} + \frac{x^3(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2} \\ &\quad - \frac{3x(a+b\arccos(cx))^2}{8c^4d^3(1-c^2x^2)} \\ &\quad - \frac{3i(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)}{4c^5d^3} \\ &\quad - \frac{7b^2\text{arctanh}(cx)}{6c^5d^3} \\ &\quad + \frac{3ib(a+b\arccos(cx))\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{4c^5d^3} \\ &\quad - \frac{3ib(a+b\arccos(cx))\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{4c^5d^3} \\ &\quad - \frac{3b^2\,\text{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)}{4c^5d^3} \\ &\quad + \frac{3b^2\,\text{PolyLog}\left(3,ie^{i\arccos(cx)}\right)}{4c^5d^3}\end{aligned}$$

output
```
1/12*b^2*x/c^4/d^3/(-c^2*x^2+1)-1/6*b*(a+b*arccos(c*x))/c^5/d^3/(-c^2*x^2+
1)^(3/2)+5/4*b*(a+b*arccos(c*x))/c^5/d^3/(-c^2*x^2+1)^(1/2)+1/4*x^3*(a+b*a
rccos(c*x))^2/c^2/d^3/(-c^2*x^2+1)^2-3/8*x*(a+b*arccos(c*x))^2/c^4/d^3/(-c
^2*x^2+1)-3/4*I*(a+b*arccos(c*x))^2*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c^5/d
^3-7/6*b^2*arctanh(c*x)/c^5/d^3+3/4*I*b*(a+b*arccos(c*x))*polylog(2,-I*(c*
x+I*(-c^2*x^2+1)^(1/2)))/c^5/d^3-3/4*I*b*(a+b*arccos(c*x))*polylog(2,I*(c*
x+I*(-c^2*x^2+1)^(1/2)))/c^5/d^3-3/4*b^2*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^
(1/2)))/c^5/d^3+3/4*b^2*polylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^5/d^3
```

Mathematica [A] (verified)

Time = 7.20 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.99

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$$

$$= \frac{a^2x}{4c^4d^3(-1+c^2x^2)^2} + \frac{5a^2x}{8c^4d^3(-1+c^2x^2)} - \frac{3a^2\log(1-cx)}{16c^5d^3} + \frac{3a^2\log(1+cx)}{16c^5d^3}$$

$$- \frac{2ab\left(\frac{(-2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)}{48c^5(-1+cx)^2} - \frac{(2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)}{48c^5(1+cx)^2} + \frac{5\left(\sqrt{1-c^2x^2}-\arccos(cx)\right)}{16c^4(c+c^2x)} + \frac{5\left(\sqrt{1-c^2x^2}+\arccos(cx)\right)}{16c^4(c-c^2x)} - \cdots\right.}{\cdots}$$

$$- \frac{b^2\left(112\arccos(cx)\cot\left(\frac{1}{2}\arccos(cx)\right) + 2(-2+15\arccos(cx)^2)\csc^2\left(\frac{1}{2}\arccos(cx)\right) - 2\sqrt{1-c^2x^2}\,\mathrm{arcc}\cdots\right.}{\cdots}$$

input
```
Integrate[(x^4*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^3,x]
```

output
```
(a^2*x)/(4*c^4*d^3*(-1 + c^2*x^2)^2) + (5*a^2*x)/(8*c^4*d^3*(-1 + c^2*x^2)
) - (3*a^2*Log[1 - c*x])/(16*c^5*d^3) + (3*a^2*Log[1 + c*x])/(16*c^5*d^3)
- (2*a*b*(((-2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x])/(48*c^5*(-1 + c*x
)^2) - ((2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x])/(48*c^5*(1 + c*x)^2)
+ (5*(Sqrt[1 - c^2*x^2] - ArcCos[c*x]))/(16*c^4*(c + c^2*x)) + (5*(Sqrt[1
- c^2*x^2] + ArcCos[c*x]))/(16*c^4*(c - c^2*x)) - (3*(((-1/2*I)*ArcCos[c*x
]^2)/c + (2*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])])/c - ((2*I)*PolyLog[2,
-E^(I*ArcCos[c*x])])/c))/(16*c^4) - (((3*I)/32)*(ArcCos[c*x]*(ArcCos[c*x]
+ (4*I)*Log[1 - E^(I*ArcCos[c*x])]) + 4*PolyLog[2, E^(I*ArcCos[c*x])]))/c^
5))/d^3 - (b^2*(112*ArcCos[c*x]*Cot[ArcCos[c*x]/2] + 2*(-2 + 15*ArcCos[c*x
]^2)*Csc[ArcCos[c*x]/2]^2 - 2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Csc[ArcCos[c*x
]/2]^4 - 3*ArcCos[c*x]^2*Csc[ArcCos[c*x]/2]^4 - 224*Log[Tan[ArcCos[c*x]/2]
] + 72*(ArcCos[c*x]^2*(Log[1 - E^(I*ArcCos[c*x])] - Log[1 + E^(I*ArcCos[c*
x])]) + (2*I)*ArcCos[c*x]*(PolyLog[2, -E^(I*ArcCos[c*x])] - PolyLog[2, E^(
I*ArcCos[c*x])]) + 2*(-PolyLog[3, -E^(I*ArcCos[c*x])] + PolyLog[3, E^(I*Ar
cCos[c*x])])) + 2*(2 - 15*ArcCos[c*x]^2)*Sec[ArcCos[c*x]/2]^2 + 3*ArcCos[c
*x]^2*Sec[ArcCos[c*x]/2]^4 - (32*ArcCos[c*x]*Sin[ArcCos[c*x]/2]^4)/(1 - c^
2*x^2)^(3/2) + 112*ArcCos[c*x]*Tan[ArcCos[c*x]/2]))/(192*c^5*d^3)
```

Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5207, 27, 5195, 27, 298, 219, 5207, 5165, 3042, 4671, 3011, 2720, 5183, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}dx$$

$$\Big\downarrow \text{5207}$$

$$\frac{b\int \frac{x^3(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2cd^3}-\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{d^2(1-c^2x^2)^2}dx}{4c^2d}+\frac{x^3(a+b\arccos(cx))^2}{4c^2d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{(1-c^2x^2)^2}dx}{4c^2d^3}+\frac{b\int \frac{x^3(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2cd^3}+\frac{x^3(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\downarrow$ 5195

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{(1-c^2x^2)^2}dx}{4c^2d^3}+\frac{b\left(bc\int -\frac{2-3c^2x^2}{3c^4(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3c^4(1-c^2x^2)^{3/2}}\right)}{2cd^3}+\frac{x^3(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\downarrow$ 27

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{(1-c^2x^2)^2}dx}{4c^2d^3}+\frac{b\left(-\frac{b\int \frac{2-3c^2x^2}{(1-c^2x^2)^2}dx}{3c^3}-\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3c^4(1-c^2x^2)^{3/2}}\right)}{2cd^3}+\frac{x^3(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\downarrow$ 298

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{(1-c^2x^2)^2}dx}{4c^2d^3}+\frac{b\left(-\frac{b\left(\frac{5}{2}\int \frac{1}{1-c^2x^2}dx-\frac{x}{2(1-c^2x^2)}\right)}{3c^3}-\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3c^4(1-c^2x^2)^{3/2}}\right)}{2cd^3}+\frac{x^3(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\downarrow$ 219

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{(1-c^2x^2)^2}dx}{4c^2d^3}+\frac{b\left(-\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3c^4(1-c^2x^2)^{3/2}}-\frac{b\left(\frac{5\text{arctanh}(cx)}{2c}-\frac{x}{2(1-c^2x^2)}\right)}{3c^3}\right)}{2cd^3}+\frac{x^3(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\downarrow$ 5207

$$-\frac{3\left(\frac{b\int \frac{x(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}dx}{c}-\frac{\int \frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx}{2c^2}+\frac{x(a+b\arccos(cx))^2}{2c^2(1-c^2x^2)}\right)}{4c^2d^3}+\frac{b\left(-\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3c^4(1-c^2x^2)^{3/2}}-\frac{b\left(\frac{5\text{arctanh}(cx)}{2c}-\frac{x}{2\left(1-c^2x^2\right)}\right)}{3c^3}\right)}{2cd^3}+\frac{x^3(a+b\arccos(cx))^2}{4c^2d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{ 5165}$$

$$-\frac{3\left(\frac{b\int \frac{x(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}dx}{c}+\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c^3}+\frac{x(a+b\arccos(cx))^2}{2c^2(1-c^2x^2)}\right)}{4c^2d^3}+\frac{b\left(-\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3c^4(1-c^2x^2)^{3/2}}-\frac{b\left(\frac{5\text{arctanh}(cx)}{2c}-\frac{x}{2\left(1-c^2x^2\right)}\right)}{3c^3}\right)}{2cd^3}+\frac{x^3(a+b\arccos(cx))^2}{4c^2d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3\left(\frac{\int (a+b\arccos(cx))^2\csc(\arccos(cx))d\arccos(cx)}{2c^3}+\frac{b\int \frac{x(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}dx}{c}+\frac{x(a+b\arccos(cx))^2}{2c^2(1-c^2x^2)}\right)}{4c^2d^3}+\frac{b\left(-\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3c^4(1-c^2x^2)^{3/2}}-\frac{b\left(\frac{5\text{arctanh}(cx)}{2c}-\frac{x}{2\left(1-c^2x^2\right)}\right)}{3c^3}\right)}{2cd^3}+\frac{x^3(a+b\arccos(cx))^2}{4c^2d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{ 4671}$$

$$-\frac{3\left(\frac{-2b\int (a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)\left(\cdots\right.}{2c^3}\cdots\right)}{4c^2d^3}+\frac{b\left(-\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3c^4(1-c^2x^2)^{3/2}}-\frac{b\left(\frac{5\text{arctanh}(cx)}{2c}-\frac{x}{2\left(1-c^2x^2\right)}\right)}{3c^3}\right)}{2cd^3}+\frac{x^3(a+b\arccos(cx))^2}{4c^2d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{3\left(\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a+b\arccos(cx)\right.}{2c^3}}{4c}$$

$$\frac{b\left(-\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3c^4(1-c^2x^2)^{3/2}}-\frac{b\left(\frac{5\text{arctanh}(cx)}{2c}-\frac{x}{2(1-c^2x^2)}\right)}{3c^3}\right)}{2cd^3}+\frac{x^3(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\downarrow$ 2720

$$-\frac{3\left(\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a\right.}{2c^3}}{}$$

$$\frac{b\left(-\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3c^4(1-c^2x^2)^{3/2}}-\frac{b\left(\frac{5\text{arctanh}(cx)}{2c}-\frac{x}{2(1-c^2x^2)}\right)}{3c^3}\right)}{2cd^3}+\frac{x^3(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\downarrow$ 5183

$$-\frac{3\left(\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a\right.}{2c^3}}{}$$

$$\frac{b\left(-\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3c^4(1-c^2x^2)^{3/2}}-\frac{b\left(\frac{5\text{arctanh}(cx)}{2c}-\frac{x}{2(1-c^2x^2)}\right)}{3c^3}\right)}{2cd^3}+\frac{x^3(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\downarrow$ 219

$$-\frac{3\left(\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a\right.}{2c^3}}{}$$

$$\frac{b\left(-\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3c^4(1-c^2x^2)^{3/2}}-\frac{b\left(\frac{5\text{arctanh}(cx)}{2c}-\frac{x}{2(1-c^2x^2)}\right)}{3c^3}\right)}{2cd^3}+\frac{x^3(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\downarrow$ 7143

$$-\frac{3\left(\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\,\text{PolyLog}\left(3,-e^{i\arccos(cx)}\right)\right)-2b\left(i\,\text{PolyLo}\right.}{2c^3}\right.}{4c^2d^3}$$

$$\frac{b\left(-\frac{a+b\arccos(cx)}{c^4\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3c^4(1-c^2x^2)^{3/2}}-\frac{b\left(\frac{5\text{arctanh}(cx)}{2c}-\frac{x}{2(1-c^2x^2)}\right)}{3c^3}\right)}{2cd^3}+\frac{x^3(a+b\arccos(cx))^2}{4c^2d^3\left(1-c^2x^2\right)^2}$$

input
```
Int[(x^4*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^3,x]
```

output
```
(x^3*(a + b*ArcCos[c*x])^2)/(4*c^2*d^3*(1 - c^2*x^2)^2) + (b*((a + b*ArcCo
s[c*x])/(3*c^4*(1 - c^2*x^2)^(3/2)) - (a + b*ArcCos[c*x])/(c^4*Sqrt[1 - c^
2*x^2]) - (b*(-1/2*x/(1 - c^2*x^2) + (5*ArcTanh[c*x])/(2*c)))/(3*c^3)))/(2
*c*d^3) - (3*((x*(a + b*ArcCos[c*x])^2)/(2*c^2*(1 - c^2*x^2)) + (b*((a + b
*ArcCos[c*x])/(c^2*Sqrt[1 - c^2*x^2]) + (b*ArcTanh[c*x])/c^2))/c + (-2*(a
+ b*ArcCos[c*x])^2*ArcTanh[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])
*PolyLog[2, -E^(I*ArcCos[c*x])] - b*PolyLog[3, -E^(I*ArcCos[c*x])]) - 2*b*
(I*(a + b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] - b*PolyLog[3, E^(I*A
rcCos[c*x])]))/(2*c^3)))/(4*c^2*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.60

method	result
derivativedivides	$-\frac{a^2\left(-\frac{1}{16(cx-1)^2}-\frac{5}{16(cx-1)}+\frac{3\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}-\frac{5}{16(cx+1)}-\frac{3\ln(cx+1)}{16}\right)}{d^3}-\frac{b^2\left(-\frac{15\arccos(cx)^2c^3x^3+30\sqrt{-c^2x^2+1}\ \text{ar}}{}\right.}{}$
default	$-\frac{a^2\left(-\frac{1}{16(cx-1)^2}-\frac{5}{16(cx-1)}+\frac{3\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}-\frac{5}{16(cx+1)}-\frac{3\ln(cx+1)}{16}\right)}{d^3}-\frac{b^2\left(-\frac{15\arccos(cx)^2c^3x^3+30\sqrt{-c^2x^2+1}\ \text{ar}}{}\right.}{}$
parts	$-\frac{a^2\left(-\frac{1}{16c^5(cx-1)^2}-\frac{5}{16c^5(cx-1)}+\frac{3\ln(cx-1)}{16c^5}+\frac{1}{16c^5(cx+1)^2}-\frac{5}{16c^5(cx+1)}-\frac{3\ln(cx+1)}{16c^5}\right)}{d^3}-\frac{b^2\left(-\frac{15\arccos(cx)^2c^3x^3+30}{}\right.}{}$

input
```
int(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(-a^2/d^3*(-1/16/(c*x-1)^2-5/16/(c*x-1)+3/16*ln(c*x-1)+1/16/(c*x+1)^
2-5/16/(c*x+1)-3/16*ln(c*x+1))-b^2/d^3*(-1/24*(15*arccos(c*x)^2*c^3*x^3+30
*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^2-9*arccos(c*x)^2*c*x-2*c^3*x^3-26*a
rccos(c*x)*(-c^2*x^2+1)^(1/2)+2*c*x)/(c^4*x^4-2*c^2*x^2+1)+7/3*arctanh(c*x
+I*(-c^2*x^2+1)^(1/2))-3/8*arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+3/
4*I*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-3/4*polylog(3,-c*x-I*
(-c^2*x^2+1)^(1/2))+3/8*arccos(c*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-3/4*I
*arccos(c*x)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+3/4*polylog(3,c*x+I*(-c^2
*x^2+1)^(1/2)))-2*a*b/d^3*(-1/24*(15*c^3*x^3*arccos(c*x)+15*c^2*x^2*(-c^2*
x^2+1)^(1/2)-9*c*x*arccos(c*x)-13*(-c^2*x^2+1)^(1/2))/(c^4*x^4-2*c^2*x^2+1
)+3/8*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-3/8*I*polylog(2,c*x+I*(-c
^2*x^2+1)^(1/2))-3/8*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+3/8*I*poly
log(2,-c*x-I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx=\int -\frac{(b\arccos{(cx)}+a)^2x^4}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b^2*x^4*arccos(c*x)^2 + 2*a*b*x^4*arccos(c*x) + a^2*x^4)/(c^6*d
^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$$
$$= -\frac{\int \frac{a^2x^4}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{b^2x^4\operatorname{acos}^2(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{2abx^4\operatorname{acos}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

input
```
integrate(x**4*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a**2*x**4/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Int
egral(b**2*x**4*acos(c*x)**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),
x) + Integral(2*a*b*x**4*acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2
- 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)^2x^4}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/16*a^2*(2*(5*c^2*x^3 - 3*x)/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 3*
log(c*x + 1)/(c^5*d^3) - 3*log(c*x - 1)/(c^5*d^3)) + 1/16*((10*b^2*c^3*x^3
 - 6*b^2*c*x + 3*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(c*x + 1) - 3*(b^2
*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(
-c*x + 1), c*x)^2 - 16*(c^9*d^3*x^4 - 2*c^7*d^3*x^2 + c^5*d^3)*integrate(1
/8*(16*a*b*c^4*x^4*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + (10*b^2*c^
3*x^3 - 6*b^2*c*x + 3*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(c*x + 1) - 3
*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*
x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/(c^10*d^3*x^6 - 3*c^8*d
^3*x^4 + 3*c^6*d^3*x^2 - c^4*d^3), x))/(c^9*d^3*x^4 - 2*c^7*d^3*x^2 + c^5*
d^3)
```

Giac [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx=\int -\frac{(b\arccos{(cx)}+a)^2x^4}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(-(b*arccos(c*x) + a)^2*x^4/(c^2*d*x^2 - d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx=\int \frac{x^4\,{(a+b\,\mathrm{acos}(c\,x))}^2}{{(d-c^2\,d\,x^2)}^3}\,dx$$

input
```
int((x^4*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^3,x)
```

output
```
int((x^4*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$$
$$= \frac{-32\left(\int \frac{acos(cx)x^4}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)ab\,c^9x^4+64\left(\int \frac{acos(cx)x^4}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)ab\,c^7x^2-32\left(\int \frac{acos(cx)x^4}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)}{}$$

input
```
int(x^4*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 32*int((acos(c*x)*x**4)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)
*a*b*c**9*x**4 + 64*int((acos(c*x)*x**4)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2
*x**2 - 1),x)*a*b*c**7*x**2 - 32*int((acos(c*x)*x**4)/(c**6*x**6 - 3*c**4*
x**4 + 3*c**2*x**2 - 1),x)*a*b*c**5 - 16*int((acos(c*x)**2*x**4)/(c**6*x**
6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2*c**9*x**4 + 32*int((acos(c*x)**
2*x**4)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2*c**7*x**2 - 16
*int((acos(c*x)**2*x**4)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*
*2*c**5 - 3*log(c**2*x - c)*a**2*c**4*x**4 + 6*log(c**2*x - c)*a**2*c**2*x
**2 - 3*log(c**2*x - c)*a**2 + 3*log(c**2*x + c)*a**2*c**4*x**4 - 6*log(c*
*2*x + c)*a**2*c**2*x**2 + 3*log(c**2*x + c)*a**2 + 10*a**2*c**3*x**3 - 6*
a**2*c*x)/(16*c**5*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.204 $\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 172

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \frac{b^2}{12c^4d^3(1-c^2x^2)} - \frac{bx^3(a+b\arccos(cx))}{6cd^3(1-c^2x^2)^{3/2}} + \frac{bx(a+b\arccos(cx))}{2c^3d^3\sqrt{1-c^2x^2}} - \frac{(a+b\arccos(cx))^2}{4c^4d^3} + \frac{x^4(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2} + \frac{b^2\log(1-c^2x^2)}{3c^4d^3}$$

output

```
1/12*b^2/c^4/d^3/(-c^2*x^2+1)-1/6*b*x^3*(a+b*arccos(c*x))/c/d^3/(-c^2*x^2+
1)^(3/2)+1/2*b*x*(a+b*arccos(c*x))/c^3/d^3/(-c^2*x^2+1)^(1/2)-1/4*(a+b*arc
cos(c*x))^2/c^4/d^3+1/4*x^4*(a+b*arccos(c*x))^2/d^3/(-c^2*x^2+1)^2+1/3*b^2
*ln(-c^2*x^2+1)/c^4/d^3
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.12

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$$
$$= \frac{-3a^2+b^2+6a^2c^2x^2-b^2c^2x^2-6abcx\sqrt{1-c^2x^2}+8abc^3x^3\sqrt{1-c^2x^2}+2b\big(bcx\sqrt{1-c^2x^2}(-3+4c^2x^2)}{12c^4d^3\,(-1+c}$$

input
```
Integrate[(x^3*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^3,x]
```

output
```
(-3*a^2 + b^2 + 6*a^2*c^2*x^2 - b^2*c^2*x^2 - 6*a*b*c*x*Sqrt[1 - c^2*x^2]
+ 8*a*b*c^3*x^3*Sqrt[1 - c^2*x^2] + 2*b*(b*c*x*Sqrt[1 - c^2*x^2]*(-3 + 4*c
^2*x^2) + a*(-3 + 6*c^2*x^2))*ArcCos[c*x] + 3*b^2*(-1 + 2*c^2*x^2)*ArcCos[
c*x]^2 + 4*b^2*(-1 + c^2*x^2)^2*Log[1 - c^2*x^2])/(12*c^4*d^3*(-1 + c^2*x^
2)^2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5187, 5207, 243, 49, 2009, 5207, 240, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$$
$$\Big\downarrow \text{ 5187}$$
$$\frac{bc\int \frac{x^4(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2d^3}+\frac{x^4(a+b\arccos(cx))^2}{4d^3\,(1-c^2x^2)^2}$$
$$\Big\downarrow \text{ 5207}$$

$$\frac{bc\left(-\frac{\int \frac{x^2(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}dx}{c^2}+\frac{b\int \frac{x^3}{\left(1-c^2x^2\right)^2}dx}{3c}+\frac{x^3(a+b\arccos(cx))}{3c^2(1-c^2x^2)^{3/2}}\right)}{2d^3}+\frac{x^4(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{bc\left(-\frac{\int \frac{x^2(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}dx}{c^2}+\frac{b\int \frac{x^2}{\left(1-c^2x^2\right)^2}dx^2}{6c}+\frac{x^3(a+b\arccos(cx))}{3c^2(1-c^2x^2)^{3/2}}\right)}{2d^3}+\frac{x^4(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{bc\left(-\frac{\int \frac{x^2(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}dx}{c^2}+\frac{b\int \left(\frac{1}{c^2\left(c^2x^2-1\right)}+\frac{1}{c^2\left(c^2x^2-1\right)^2}\right)dx^2}{6c}+\frac{x^3(a+b\arccos(cx))}{3c^2(1-c^2x^2)^{3/2}}\right)}{2d^3}+\frac{x^4(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{bc\left(-\frac{\int \frac{x^2(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}dx}{c^2}+\frac{x^3(a+b\arccos(cx))}{3c^2(1-c^2x^2)^{3/2}}+\frac{b\left(\frac{1}{c^4\left(1-c^2x^2\right)}+\frac{\log\left(1-c^2x^2\right)}{c^4}\right)}{6c}\right)}{2d^3}+\frac{x^4(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{ 5207}$$

$$\frac{bc\left(-\frac{-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{c^2}+\frac{b\int \frac{x}{1-c^2x^2}dx}{c}+\frac{x(a+b\arccos(cx))}{c^2\sqrt{1-c^2x^2}}}{c^2}+\frac{x^3(a+b\arccos(cx))}{3c^2(1-c^2x^2)^{3/2}}+\frac{b\left(\frac{1}{c^4\left(1-c^2x^2\right)}+\frac{\log\left(1-c^2x^2\right)}{c^4}\right)}{6c}\right)}{2d^3}+\frac{x^4(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{bc\left(-\frac{-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{c^2}+\frac{x(a+b\arccos(cx))}{c^2\sqrt{1-c^2x^2}}-\frac{b\log\left(1-c^2x^2\right)}{2c^3}}{c^2}+\frac{x^3(a+b\arccos(cx))}{3c^2(1-c^2x^2)^{3/2}}+\frac{b\left(\frac{1}{c^4\left(1-c^2x^2\right)}+\frac{\log\left(1-c^2x^2\right)}{c^4}\right)}{6c}\right)}{2d^3}+\frac{x^4(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \; 5153$$

$$\frac{x^4(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{x^3(a+b\arccos(cx))}{3c^2(1-c^2x^2)^{3/2}}-\frac{\frac{(a+b\arccos(cx))^2}{2bc^3}+\frac{x(a+b\arccos(cx))}{c^2\sqrt{1-c^2x^2}}-\frac{b\log\left(1-c^2x^2\right)}{2c^3}}{c^2}+\frac{b\left(\frac{1}{c^4\left(1-c^2x^2\right)}+\frac{\log\left(1-c^2x^2\right)}{c^4}\right)}{6c}\right)}{2d^3}$$

input
```
Int[(x^3*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^3,x]
```

output
```
(x^4*(a + b*ArcCos[c*x])^2)/(4*d^3*(1 - c^2*x^2)^2) + (b*c*((x^3*(a + b*Ar
cCos[c*x]))/(3*c^2*(1 - c^2*x^2)^(3/2)) + (b*(1/(c^4*(1 - c^2*x^2)) + Log[
1 - c^2*x^2]/c^4))/(6*c) - ((x*(a + b*ArcCos[c*x]))/(c^2*Sqrt[1 - c^2*x^2]
) + (a + b*ArcCos[c*x])^2/(2*b*c^3) - (b*Log[1 - c^2*x^2])/(2*c^3))/c^2))/
(2*d^3)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.25

method	result
derivativedivides	$-\frac{a^2\left(-\frac{1}{16(cx-1)^2}-\frac{3}{16(cx-1)}-\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}\right)}{d^3}-\frac{b^2\left(\frac{4i\arccos(cx)}{3}-\frac{8i\arccos(cx)c^4x^4+8\sqrt{-c^2x^2+1}\,\arccos(cx)x^3c^3+6}{}\right.}{}$
default	$-\frac{a^2\left(-\frac{1}{16(cx-1)^2}-\frac{3}{16(cx-1)}-\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}\right)}{d^3}-\frac{b^2\left(\frac{4i\arccos(cx)}{3}-\frac{8i\arccos(cx)c^4x^4+8\sqrt{-c^2x^2+1}\,\arccos(cx)x^3c^3+6}{}\right.}{}$
parts	$-\frac{a^2\left(-\frac{1}{16c^4(cx-1)^2}-\frac{3}{16c^4(cx-1)}-\frac{1}{16c^4(cx+1)^2}+\frac{3}{16c^4(cx+1)}\right)}{d^3}-\frac{b^2\left(\frac{4i\arccos(cx)}{3}-\frac{8i\arccos(cx)c^4x^4+8\sqrt{-c^2x^2+1}\,\mathrm{arc}}{}\right.}{}$

input
```
int(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(-a^2/d^3*(-1/16/(c*x-1)^2-3/16/(c*x-1)-1/16/(c*x+1)^2+3/16/(c*x+1))
-b^2/d^3*(4/3*I*arccos(c*x)-1/12*(8*I*arccos(c*x)*c^4*x^4+8*(-c^2*x^2+1)^(
1/2)*arccos(c*x)*x^3*c^3+6*arccos(c*x)^2*x^2*c^2-16*I*arccos(c*x)*c^2*x^2-
6*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c-3*arccos(c*x)^2+8*I*arccos(c*x)-c^2*x
^2+1)/(c^4*x^4-2*c^2*x^2+1)-2/3*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1))-2*a*b/
d^3*(-1/16*arccos(c*x)/(c*x-1)^2-3/16*arccos(c*x)/(c*x-1)-1/16*arccos(c*x)
/(c*x+1)^2+3/16*arccos(c*x)/(c*x+1)-1/48/(c*x-1)^2*(-(c*x-1)^2-2*c*x+2)^(1
/2)-1/6/(c*x-1)*(-(c*x-1)^2-2*c*x+2)^(1/2)+1/48/(c*x+1)^2*(-(c*x+1)^2+2*c*
x+2)^(1/2)-1/6/(c*x+1)*(-(c*x+1)^2+2*c*x+2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.41

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$$
$$=\frac{6\,abc^4x^4\arccos(cx)+(6\,a^2-b^2)c^2x^2+3\,(2\,b^2c^2x^2-b^2)\arccos(cx)^2-3\,a^2+b^2-6\,(abc^4x^4-2\,abc^2x^2}{}$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
1/12*(6*a*b*c^4*x^4*arccos(c*x) + (6*a^2 - b^2)*c^2*x^2 + 3*(2*b^2*c^2*x^2
 - b^2)*arccos(c*x)^2 - 3*a^2 + b^2 - 6*(a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b
)*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + 4*(b^2*c^4*x^4 - 2*b^2*c^
2*x^2 + b^2)*log(c^2*x^2 - 1) + 2*(4*a*b*c^3*x^3 - 3*a*b*c*x + (4*b^2*c^3*
x^3 - 3*b^2*c*x)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/(c^8*d^3*x^4 - 2*c^6*d^3
*x^2 + c^4*d^3)
```

Sympy [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = -\frac{\int \frac{a^2x^3}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{b^2x^3\operatorname{acos}^2(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{2abx^3\operatorname{acos}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

input
```
integrate(x**3*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a**2*x**3/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Int
egral(b**2*x**3*acos(c*x)**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),
x) + Integral(2*a*b*x**3*acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2
- 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)^2x^3}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*(2*c^2*x^2 - 1)*a^2/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 1/4*((2*
b^2*c^2*x^2 - b^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 - 4*(c^8*d
^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)*integrate(1/2*(4*a*b*c^3*x^3*arctan2(sqr
t(c*x + 1)*sqrt(-c*x + 1), c*x) + (2*b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/(c^9*d^3*x^6 - 3*c^
7*d^3*x^4 + 3*c^5*d^3*x^2 - c^3*d^3), x))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c
^4*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(154) = 308$.

Time = 0.21 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.85

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \frac{b^2x^4\arccos{(cx)}^2}{4\,(c^2x^2-1)^2d^3} + \frac{abx^4\arccos{(cx)}}{2\,(c^2x^2-1)^2d^3} + \frac{a^2x^4}{4\,(c^2x^2-1)^2d^3} - \frac{b^2x^3\arccos{(cx)}}{6\,(c^2x^2-1)\sqrt{-c^2x^2+1}cd^3} - \frac{abx^3}{6\,(c^2x^2-1)\sqrt{-c^2x^2+1}cd^3} - \frac{b^2x^2}{12\,(c^2x^2-1)c^2d^3} - \frac{b^2x\arccos{(cx)}}{2\sqrt{-c^2x^2+1}c^3d^3} - \frac{b^2\arccos{(cx)}^2}{4\,c^4d^3} - \frac{abx}{2\sqrt{-c^2x^2+1}c^3d^3} - \frac{ab\arccos{(cx)}}{2\,c^4d^3} + \frac{2\,b^2\log{(2)}}{3\,c^4d^3} + \frac{b^2\log{(|-c^2x^2+1|)}}{3\,c^4d^3} - \frac{a^2}{4\,c^4d^3} + \frac{b^2}{12\,c^4d^3}$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
1/4*b^2*x^4*arccos(c*x)^2/((c^2*x^2 - 1)^2*d^3) + 1/2*a*b*x^4*arccos(c*x)/
((c^2*x^2 - 1)^2*d^3) + 1/4*a^2*x^4/((c^2*x^2 - 1)^2*d^3) - 1/6*b^2*x^3*ar
ccos(c*x)/((c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*c*d^3) - 1/6*a*b*x^3/((c^2*x^2
 - 1)*sqrt(-c^2*x^2 + 1)*c*d^3) - 1/12*b^2*x^2/((c^2*x^2 - 1)*c^2*d^3) - 1
/2*b^2*x*arccos(c*x)/(sqrt(-c^2*x^2 + 1)*c^3*d^3) - 1/4*b^2*arccos(c*x)^2/
(c^4*d^3) - 1/2*a*b*x/(sqrt(-c^2*x^2 + 1)*c^3*d^3) - 1/2*a*b*arccos(c*x)/(
c^4*d^3) + 2/3*b^2*log(2)/(c^4*d^3) + 1/3*b^2*log(abs(-c^2*x^2 + 1))/(c^4*
d^3) - 1/4*a^2/(c^4*d^3) + 1/12*b^2/(c^4*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx=\int \frac{x^3\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((x^3*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^3,x)
```

output
```
int((x^3*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$$
$$=\frac{-8\left(\int \frac{acos(cx)x^3}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)ab\,c^4x^4+16\left(\int \frac{acos(cx)x^3}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)ab\,c^2x^2-8\left(\int \frac{acos(cx)x^3}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)ab}{4d^3\,(c^4x^4}$$

input
```
int(x^3*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 8*int((acos(c*x)*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*
a*b*c**4*x**4 + 16*int((acos(c*x)*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*
x**2 - 1),x)*a*b*c**2*x**2 - 8*int((acos(c*x)*x**3)/(c**6*x**6 - 3*c**4*x*
*4 + 3*c**2*x**2 - 1),x)*a*b - 4*int((acos(c*x)**2*x**3)/(c**6*x**6 - 3*c*
*4*x**4 + 3*c**2*x**2 - 1),x)*b**2*c**4*x**4 + 8*int((acos(c*x)**2*x**3)/(
c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2*c**2*x**2 - 4*int((acos
(c*x)**2*x**3)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2 + a**2*
x**4)/(4*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.205 $\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 341

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \frac{b^2x}{12c^2d^3(1-c^2x^2)} - \frac{b(a+b\arccos(cx))}{6c^3d^3(1-c^2x^2)^{3/2}} + \frac{b(a+b\arccos(cx))}{4c^3d^3\sqrt{1-c^2x^2}} + \frac{x(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2} - \frac{x(a+b\arccos(cx))^2}{8c^2d^3(1-c^2x^2)} + \frac{i(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)}{4c^3d^3} - \frac{b^2\operatorname{arctanh}(cx)}{6c^3d^3} - \frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{4c^3d^3} + \frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{4c^3d^3} + \frac{b^2\operatorname{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)}{4c^3d^3} - \frac{b^2\operatorname{PolyLog}\left(3,ie^{i\arccos(cx)}\right)}{4c^3d^3}$$

output
```
1/12*b^2*x/c^2/d^3/(-c^2*x^2+1)-1/6*b*(a+b*arccos(c*x))/c^3/d^3/(-c^2*x^2+
1)^(3/2)+1/4*b*(a+b*arccos(c*x))/c^3/d^3/(-c^2*x^2+1)^(1/2)+1/4*x*(a+b*arc
cos(c*x))^2/c^2/d^3/(-c^2*x^2+1)^2-1/8*x*(a+b*arccos(c*x))^2/c^2/d^3/(-c^2
*x^2+1)+1/4*I*(a+b*arccos(c*x))^2*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c^3/d^3
-1/6*b^2*arctanh(c*x)/c^3/d^3-1/4*I*b*(a+b*arccos(c*x))*polylog(2,-I*(c*x+
I*(-c^2*x^2+1)^(1/2)))/c^3/d^3+1/4*I*b*(a+b*arccos(c*x))*polylog(2,I*(c*x+
I*(-c^2*x^2+1)^(1/2)))/c^3/d^3+1/4*b^2*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1
/2)))/c^3/d^3-1/4*b^2*polylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^3/d^3
```

Mathematica [A] (verified)

Time = 7.64 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.92

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$$
$$= \frac{a^2x}{4c^2d^3(-1+c^2x^2)^2} + \frac{a^2x}{8c^2d^3(-1+c^2x^2)} + \frac{a^2\log(1-cx)}{16c^3d^3} - \frac{a^2\log(1+cx)}{16c^3d^3}$$
$$- \frac{2ab\left(\frac{(-2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)}{48(-1+cx)^2} - \frac{(2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)}{48(1+cx)^2} + \frac{\sqrt{1-c^2x^2}-\arccos(cx)}{16(1+cx)} + \frac{\sqrt{1-c^2x^2}+\arccos(cx)}{16(1-cx)} + \frac{1}{16}\left(-\right.\right.}{}$$
$$- \frac{b^2\left(16\arccos(cx)\cot\left(\frac{1}{2}\arccos(cx)\right) + 2(-2+3\arccos(cx)^2)\csc^2\left(\frac{1}{2}\arccos(cx)\right) - 2\sqrt{1-c^2x^2}\arccos(\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^3,x]
```

output
```
(a^2*x)/(4*c^2*d^3*(-1 + c^2*x^2)^2) + (a^2*x)/(8*c^2*d^3*(-1 + c^2*x^2))
+ (a^2*Log[1 - c*x])/(16*c^3*d^3) - (a^2*Log[1 + c*x])/(16*c^3*d^3) - (2*a
*b*(((-2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x])/(48*(-1 + c*x)^2) - ((2
 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x])/(48*(1 + c*x)^2) + (Sqrt[1 - c^
2*x^2] - ArcCos[c*x])/(16*(1 + c*x)) + (Sqrt[1 - c^2*x^2] + ArcCos[c*x])/(
16*(1 - c*x)) + ((-1/2*I)*ArcCos[c*x]^2 + 2*ArcCos[c*x]*Log[1 + E^(I*ArcCo
s[c*x])] - (2*I)*PolyLog[2, -E^(I*ArcCos[c*x])])/16 + (-2*ArcCos[c*x]*Log[
1 - E^(I*ArcCos[c*x])] + (2*I)*(ArcCos[c*x]^2/4 + PolyLog[2, E^(I*ArcCos[c
*x])]))/16))/(c^3*d^3) - (b^2*(16*ArcCos[c*x]*Cot[ArcCos[c*x]/2] + 2*(-2 +
 3*ArcCos[c*x]^2)*Csc[ArcCos[c*x]/2]^2 - 2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*C
sc[ArcCos[c*x]/2]^4 - 3*ArcCos[c*x]^2*Csc[ArcCos[c*x]/2]^4 - 24*ArcCos[c*x
]^2*(Log[1 - E^(I*ArcCos[c*x])] - Log[1 + E^(I*ArcCos[c*x])]) - 32*Log[Tan
[ArcCos[c*x]/2]] - (48*I)*ArcCos[c*x]*(PolyLog[2, -E^(I*ArcCos[c*x])] - Po
lyLog[2, E^(I*ArcCos[c*x])]) + 48*(PolyLog[3, -E^(I*ArcCos[c*x])] - PolyLo
g[3, E^(I*ArcCos[c*x])]) + 2*(2 - 3*ArcCos[c*x]^2)*Sec[ArcCos[c*x]/2]^2 +
3*ArcCos[c*x]^2*Sec[ArcCos[c*x]/2]^4 - (32*ArcCos[c*x]*Sin[ArcCos[c*x]/2]^
4)/(1 - c^2*x^2)^(3/2) + 16*ArcCos[c*x]*Tan[ArcCos[c*x]/2]))/(192*c^3*d^3)
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.91, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5207, 27, 5163, 5165, 3042, 4671, 3011, 2720, 5183, 215, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (a+b \arccos (c x))^2}{\left(d-c^2 d x^2\right)^3} \, dx$$

$$\Big\downarrow \text{5207}$$

$$\frac{b \int \frac{x (a+b \arccos (c x))}{\left(1-c^2 x^2\right)^{5/2}}dx}{2 c d^3}-\frac{\int \frac{(a+b \arccos (c x))^2}{d^2 \left(1-c^2 x^2\right)^2}dx}{4 c^2 d}+\frac{x (a+b \arccos (c x))^2}{4 c^2 d^3 \left(1-c^2 x^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b \int \frac{x (a+b \arccos (c x))}{\left(1-c^2 x^2\right)^{5/2}}dx}{2 c d^3}-\frac{\int \frac{(a+b \arccos (c x))^2}{\left(1-c^2 x^2\right)^2}dx}{4 c^2 d^3}+\frac{x (a+b \arccos (c x))^2}{4 c^2 d^3 \left(1-c^2 x^2\right)^2}$$

$\Big\downarrow$ 5163

$$\frac{b\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2cd^3}-\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx+\frac{1}{2}\int \frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}}{4c^2d^3}+\frac{x(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\Big\downarrow$ 5165

$$\frac{b\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2cd^3}-\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx-\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}}{4c^2d^3}+\frac{x(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\Big\downarrow$ 3042

$$\frac{b\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2cd^3}-\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx-\frac{\int (a+b\arccos(cx))^2\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}}{4c^2d^3}+\frac{x(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\Big\downarrow$ 4671

$$-\frac{-\frac{-2b\int (a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a\cdots}{2c}}{4c^2d^3}$$

$$\frac{b\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2cd^3}+\frac{x(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\Big\downarrow$ 3011

$$-\frac{-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a+b\arccos(cx))\cdots\right.}{2c}}{4c\cdots}$$

$$\frac{b\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2cd^3}+\frac{x(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\Big\downarrow$ 2720

$$-\frac{-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a+\right.}{2c}}{\quad}$$

$$\frac{b\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2cd^3}+\frac{x(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\downarrow$ 5183

$$-\frac{-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a+\right.}{2c}}{\quad}$$

$$\frac{b\left(\frac{b\int\frac{1}{(1-c^2x^2)^2}dx}{3c}+\frac{a+b\arccos(cx)}{3c^2(1-c^2x^2)^{3/2}}\right)}{2cd^3}+\frac{x(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\downarrow$ 215

$$-\frac{-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a+\right.}{2c}}{\quad}$$

$$\frac{b\left(\frac{b\left(\frac{1}{2}\int\frac{1}{1-c^2x^2}dx+\frac{x}{2(1-c^2x^2)}\right)}{3c}+\frac{a+b\arccos(cx)}{3c^2(1-c^2x^2)^{3/2}}\right)}{2cd^3}+\frac{x(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\downarrow$ 219

$$-\frac{-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a+\right.}{2c}}{\quad}$$

$$\frac{b\left(\frac{a+b\arccos(cx)}{3c^2(1-c^2x^2)^{3/2}}+\frac{b\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3c}\right)}{2cd^3}+\frac{x(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$\downarrow$ 7143

$$-\frac{bc\left(\frac{a+b\arccos(cx)}{c^2\sqrt{1-c^2x^2}}+\frac{b\operatorname{arctanh}(cx)}{c^2}\right)-\frac{-2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\right.}{4c^2d^3}$$

$$\frac{b\left(\frac{a+b\arccos(cx)}{3c^2(1-c^2x^2)^{3/2}}+\frac{b\left(\frac{\operatorname{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3c}\right)}{2cd^3}+\frac{x(a+b\arccos(cx))^2}{4c^2d^3\left(1-c^2x^2\right)^2}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^3,x]
```

output
```
(x*(a + b*ArcCos[c*x])^2)/(4*c^2*d^3*(1 - c^2*x^2)^2) + (b*((a + b*ArcCos[
c*x])/(3*c^2*(1 - c^2*x^2)^(3/2)) + (b*(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]
/(2*c)))/(3*c)))/(2*c*d^3) - ((x*(a + b*ArcCos[c*x])^2)/(2*(1 - c^2*x^2))
+ b*c*((a + b*ArcCos[c*x])/(c^2*Sqrt[1 - c^2*x^2]) + (b*ArcTanh[c*x])/c^2)
 - (-2*(a + b*ArcCos[c*x])^2*ArcTanh[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*Ar
cCos[c*x])*PolyLog[2, -E^(I*ArcCos[c*x])] - b*PolyLog[3, -E^(I*ArcCos[c*x]
)]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] - b*PolyLog
[3, E^(I*ArcCos[c*x])]))/(2*c))/(4*c^2*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.61

method	result
derivativedivides	$-\frac{\frac{a^2\left(-\frac{1}{16(cx-1)^2}-\frac{1}{16(cx-1)}-\frac{\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}-\frac{1}{16(cx+1)}+\frac{\ln(cx+1)}{16}\right)}{d^3}-\frac{b^2\left(-\frac{3\arccos(cx)^2c^3x^3+6\sqrt{-c^2x^2+1}\ \arccos(\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
default	$-\frac{\frac{a^2\left(-\frac{1}{16(cx-1)^2}-\frac{1}{16(cx-1)}-\frac{\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}-\frac{1}{16(cx+1)}+\frac{\ln(cx+1)}{16}\right)}{d^3}-\frac{b^2\left(-\frac{3\arccos(cx)^2c^3x^3+6\sqrt{-c^2x^2+1}\ \arccos(\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
parts	$-\frac{a^2\left(-\frac{1}{16c^3(cx-1)^2}-\frac{1}{16c^3(cx-1)}-\frac{\ln(cx-1)}{16c^3}+\frac{1}{16c^3(cx+1)^2}-\frac{1}{16c^3(cx+1)}+\frac{\ln(cx+1)}{16c^3}\right)}{d^3}-\frac{b^2\left(-\frac{3\arccos(cx)^2c^3x^3+6\sqrt{-c^2\cdots}}{\cdots}\right.}{\cdots}$

input
```
int(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-a^2/d^3*(-1/16/(c*x-1)^2-1/16/(c*x-1)-1/16*ln(c*x-1)+1/16/(c*x+1)^
2-1/16/(c*x+1)+1/16*ln(c*x+1))-b^2/d^3*(-1/24*(3*arccos(c*x)^2*c^3*x^3+6*(
-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^2+3*arccos(c*x)^2*c*x-2*c^3*x^3-2*arcc
os(c*x)*(-c^2*x^2+1)^(1/2)+2*c*x)/(c^4*x^4-2*c^2*x^2+1)+1/3*arctanh(c*x+I*
(-c^2*x^2+1)^(1/2))+1/8*arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-1/4*I
*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+1/4*polylog(3,-c*x-I*(-c
^2*x^2+1)^(1/2))-1/8*arccos(c*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+1/4*I*ar
ccos(c*x)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))-1/4*polylog(3,c*x+I*(-c^2*x^
2+1)^(1/2)))-2*a*b/d^3*(-1/24*(3*c^3*x^3*arccos(c*x)+3*c^2*x^2*(-c^2*x^2+1
)^(1/2)+3*c*x*arccos(c*x)-(-c^2*x^2+1)^(1/2))/(c^4*x^4-2*c^2*x^2+1)-1/8*ar
ccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+1/8*I*polylog(2,c*x+I*(-c^2*x^2+1
)^(1/2))+1/8*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-1/8*I*polylog(2,-c
*x-I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)^2x^2}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b^2*x^2*arccos(c*x)^2 + 2*a*b*x^2*arccos(c*x) + a^2*x^2)/(c^6*d
^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = -\frac{\int \frac{a^2x^2}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{b^2x^2\operatorname{acos}^2(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{2abx^2\operatorname{acos}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

input
```
integrate(x**2*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a**2*x**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Int
egral(b**2*x**2*acos(c*x)**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),
x) + Integral(2*a*b*x**2*acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2
- 1), x))/d**3
```

Maxima [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos{(cx)}+a)^2x^2}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/16*a^2*(2*(c^2*x^3 + x)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3) - log(c*
x + 1)/(c^3*d^3) + log(c*x - 1)/(c^3*d^3)) + 1/16*((2*b^2*c^3*x^3 + 2*b^2*
c*x - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(c*x + 1) + (b^2*c^4*x^4 - 2*
b^2*c^2*x^2 + b^2)*log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*
x)^2 + 16*(c^7*d^3*x^4 - 2*c^5*d^3*x^2 + c^3*d^3)*integrate(-1/8*(16*a*b*c
^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + (2*b^2*c^3*x^3 + 2*b^2
*c*x - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(c*x + 1) + (b^2*c^4*x^4 - 2
*b^2*c^2*x^2 + b^2)*log(-c*x + 1))*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sq
rt(c*x + 1)*sqrt(-c*x + 1), c*x))/(c^8*d^3*x^6 - 3*c^6*d^3*x^4 + 3*c^4*d^3
*x^2 - c^2*d^3), x))/(c^7*d^3*x^4 - 2*c^5*d^3*x^2 + c^3*d^3)
```

Giac [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos{(cx)}+a)^2x^2}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate(-(b*arccos(c*x) + a)^2*x^2/(c^2*d*x^2 - d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \int \frac{x^2\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((x^2*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^3,x)
```

output
```
int((x^2*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \frac{-32\left(\int \frac{acos(cx)x^2}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)ab\,c^7x^4+64\left(\int \frac{acos(cx)x^2}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)ab\,c^5x^2-32\left(\int \frac{acos(cx)x^2}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)}{}$$

input
```
int(x^2*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 32*int((acos(c*x)*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)
*a*b*c**7*x**4 + 64*int((acos(c*x)*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2
*x**2 - 1),x)*a*b*c**5*x**2 - 32*int((acos(c*x)*x**2)/(c**6*x**6 - 3*c**4*
x**4 + 3*c**2*x**2 - 1),x)*a*b*c**3 - 16*int((acos(c*x)**2*x**2)/(c**6*x**
6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2*c**7*x**4 + 32*int((acos(c*x)**
2*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2*c**5*x**2 - 16
*int((acos(c*x)**2*x**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b*
*2*c**3 + log(c**2*x - c)*a**2*c**4*x**4 - 2*log(c**2*x - c)*a**2*c**2*x**
2 + log(c**2*x - c)*a**2 - log(c**2*x + c)*a**2*c**4*x**4 + 2*log(c**2*x +
 c)*a**2*c**2*x**2 - log(c**2*x + c)*a**2 + 2*a**2*c**3*x**3 + 2*a**2*c*x)
/(16*c**3*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.206 $\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 150

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \frac{b^2}{12c^2d^3(1-c^2x^2)} - \frac{bx(a+b\arccos(cx))}{6cd^3(1-c^2x^2)^{3/2}} - \frac{bx(a+b\arccos(cx))}{3cd^3\sqrt{1-c^2x^2}} + \frac{(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2} - \frac{b^2\log(1-c^2x^2)}{6c^2d^3}$$

output
```
1/12*b^2/c^2/d^3/(-c^2*x^2+1)-1/6*b*x*(a+b*arccos(c*x))/c/d^3/(-c^2*x^2+1)
^(3/2)-1/3*b*x*(a+b*arccos(c*x))/c/d^3/(-c^2*x^2+1)^(1/2)+1/4*(a+b*arccos(
c*x))^2/c^2/d^3/(-c^2*x^2+1)^2-1/6*b^2*ln(-c^2*x^2+1)/c^2/d^3
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.08

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \frac{3a^2+b^2-b^2c^2x^2+6abcx\sqrt{1-c^2x^2}-4abc^3x^3\sqrt{1-c^2x^2}+2b\left(3a+bcx(3-2c^2x^2)\sqrt{1-c^2x^2}\right)\arccos(}{12c^2d^3(-1+c^2x^2)^2}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^3,x]
```

output
```
(3*a^2 + b^2 - b^2*c^2*x^2 + 6*a*b*c*x*Sqrt[1 - c^2*x^2] - 4*a*b*c^3*x^3*S
qrt[1 - c^2*x^2] + 2*b*(3*a + b*c*x*(3 - 2*c^2*x^2)*Sqrt[1 - c^2*x^2])*Arc
Cos[c*x] + 3*b^2*ArcCos[c*x]^2 - 2*b^2*(-1 + c^2*x^2)^2*Log[1 - c^2*x^2])/
(12*c^2*d^3*(-1 + c^2*x^2)^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5183, 5163, 241, 5161, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}dx$$

$$\Big\downarrow \text{5183}$$

$$\frac{b\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{5/2}}dx}{2cd^3}+\frac{(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{5163}$$

$$\frac{b\left(\frac{2}{3}\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{3/2}}dx+\frac{1}{3}bc\int \frac{x}{(1-c^2x^2)^2}dx+\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}\right)}{2cd^3}+\frac{(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{b\left(\frac{2}{3}\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{3/2}}dx+\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{b}{6c(1-c^2x^2)}\right)}{2cd^3}+\frac{(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{5161}$$

$$\frac{b\left(\frac{2}{3}\left(bc\int \frac{x}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)+\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{b}{6c(1-c^2x^2)}\right)}{2cd^3}+\frac{(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{240}$$

$$\frac{(a+b\arccos(cx))^2}{4c^2d^3(1-c^2x^2)^2}+\frac{b\left(\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)+\frac{b}{6c(1-c^2x^2)}\right)}{2cd^3}$$

input
```
Int[(x*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^3,x]
```

output
```
(a + b*ArcCos[c*x])^2/(4*c^2*d^3*(1 - c^2*x^2)^2) + (b*(b/(6*c*(1 - c^2*x^
2)) + (x*(a + b*ArcCos[c*x]))/(3*(1 - c^2*x^2)^(3/2)) + (2*((x*(a + b*ArcC
os[c*x]))/Sqrt[1 - c^2*x^2] - (b*Log[1 - c^2*x^2])/(2*c)))/3))/(2*c*d^3)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.80

method	result
derivativedivides	$\frac{\frac{a^2}{4d^3\left(c^2x^2-1\right)^2}-\frac{b^2\left(-\frac{\arccos(cx)^2}{4\left(c^2x^2-1\right)^2}-\frac{cx\arccos(cx)\sqrt{-c^2x^2+1}}{6\left(c^2x^2-1\right)^2}+\frac{1}{12c^2x^2-12}+\frac{\sqrt{-c^2x^2+1}\arccos(cx)xc}{3c^2x^2-3}+\frac{\ln\left(-c^2x^2+1\right)}{6}\right)}{d^3}-\frac{2ab}{}}{c^2}$
default	$\frac{\frac{a^2}{4d^3\left(c^2x^2-1\right)^2}-\frac{b^2\left(-\frac{\arccos(cx)^2}{4\left(c^2x^2-1\right)^2}-\frac{cx\arccos(cx)\sqrt{-c^2x^2+1}}{6\left(c^2x^2-1\right)^2}+\frac{1}{12c^2x^2-12}+\frac{\sqrt{-c^2x^2+1}\arccos(cx)xc}{3c^2x^2-3}+\frac{\ln\left(-c^2x^2+1\right)}{6}\right)}{d^3}-\frac{2ab}{}}{c^2}$
parts	$\frac{a^2}{4d^3c^2(c^2x^2-1)^2}-\frac{b^2\left(-\frac{\arccos(cx)^2}{4\left(c^2x^2-1\right)^2}-\frac{cx\arccos(cx)\sqrt{-c^2x^2+1}}{6\left(c^2x^2-1\right)^2}+\frac{1}{12c^2x^2-12}+\frac{\sqrt{-c^2x^2+1}\arccos(cx)xc}{3c^2x^2-3}+\frac{\ln\left(-c^2x^2+1\right)}{6}\right)}{d^3c^2}$

input
```
int(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/4*a^2/d^3/(c^2*x^2-1)^2-b^2/d^3*(-1/4/(c^2*x^2-1)^2*arccos(c*x)^2
-1/6*c*x/(c^2*x^2-1)^2*arccos(c*x)*(-c^2*x^2+1)^(1/2)+1/12/(c^2*x^2-1)+1/3
*c*x/(c^2*x^2-1)*arccos(c*x)*(-c^2*x^2+1)^(1/2)+1/6*ln(-c^2*x^2+1))-2*a*b/
d^3*(-1/4/(c^2*x^2-1)^2*arccos(c*x)-1/48/(c*x-1)^2*(-(c*x-1)^2-2*c*x+2)^(1
/2)+1/12/(c*x-1)*(-(c*x-1)^2-2*c*x+2)^(1/2)+1/48/(c*x+1)^2*(-(c*x+1)^2+2*c
*x+2)^(1/2)+1/12/(c*x+1)*(-(c*x+1)^2+2*c*x+2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.57

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx =$$
$$-\frac{b^2c^2x^2 - 3\,b^2\arccos{(cx)}^2 - 3\,a^2 - b^2 + 6\,(abc^4x^4 - 2\,abc^2x^2)\arccos{(cx)} - 6\,(abc^4x^4 - 2\,abc^2x^2 + ab)}{}$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
-1/12*(b^2*c^2*x^2 - 3*b^2*arccos(c*x)^2 - 3*a^2 - b^2 + 6*(a*b*c^4*x^4 -
2*a*b*c^2*x^2)*arccos(c*x) - 6*(a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b)*arctan(
sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + 2*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b
^2)*log(c^2*x^2 - 1) + 2*(2*a*b*c^3*x^3 - 3*a*b*c*x + (2*b^2*c^3*x^3 - 3*b
^2*c*x)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^
2*d^3)
```

Sympy [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$$
$$= -\frac{\int \frac{a^2x}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{b^2x\operatorname{acos}^2{(cx)}}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{2abx\operatorname{acos}{(cx)}}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

input
```
integrate(x*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a**2*x/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integr
al(b**2*x*acos(c*x)**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + I
ntegral(2*a*b*x*acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))
/d**3
```

Maxima [F]

$$\int \frac{x(a + b\arccos(cx))^2}{(d - c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx) + a)^2 x}{(c^2dx^2 - d)^3}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a^2/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3) + 1/4*(b^2*arctan2(sqrt(c*
x + 1)*sqrt(-c*x + 1), c*x)^2 - 4*(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3)*
integrate(1/2*(4*a*b*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + sqrt
(c*x + 1)*sqrt(-c*x + 1)*b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/(
c^7*d^3*x^6 - 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 - c*d^3), x))/(c^6*d^3*x^4 - 2
*c^4*d^3*x^2 + c^2*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(134) = 268$.

Time = 0.21 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.63

$$\begin{aligned}\int \frac{x(a + b\arccos(cx))^2}{(d - c^2dx^2)^3}\,dx &= \frac{b^2c^2x^4\arccos(cx)^2}{4(c^2x^2-1)^2d^3} + \frac{abc^2x^4\arccos(cx)}{2(c^2x^2-1)^2d^3} + \frac{a^2c^2x^4}{4(c^2x^2-1)^2d^3} \\ &\quad - \frac{b^2cx^3\arccos(cx)}{6(c^2x^2-1)\sqrt{-c^2x^2+1}d^3} - \frac{b^2x^2\arccos(cx)^2}{2(c^2x^2-1)d^3} \\ &\quad - \frac{abcx^3}{6(c^2x^2-1)\sqrt{-c^2x^2+1}d^3} - \frac{abx^2\arccos(cx)}{(c^2x^2-1)d^3} \\ &\quad - \frac{a^2x^2}{2(c^2x^2-1)d^3} - \frac{b^2x^2}{12(c^2x^2-1)d^3} + \frac{b^2x\arccos(cx)}{2\sqrt{-c^2x^2+1}cd^3} \\ &\quad + \frac{b^2\arccos(cx)^2}{4c^2d^3} + \frac{abx}{2\sqrt{-c^2x^2+1}cd^3} + \frac{ab\arccos(cx)}{2c^2d^3} \\ &\quad - \frac{b^2\log(2)}{3c^2d^3} - \frac{b^2\log(|-c^2x^2+1|)}{6c^2d^3} + \frac{a^2}{4c^2d^3} + \frac{b^2}{12c^2d^3}\end{aligned}$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
1/4*b^2*c^2*x^4*arccos(c*x)^2/((c^2*x^2 - 1)^2*d^3) + 1/2*a*b*c^2*x^4*arcc
os(c*x)/((c^2*x^2 - 1)^2*d^3) + 1/4*a^2*c^2*x^4/((c^2*x^2 - 1)^2*d^3) - 1/
6*b^2*c*x^3*arccos(c*x)/((c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*d^3) - 1/2*b^2*x
^2*arccos(c*x)^2/((c^2*x^2 - 1)*d^3) - 1/6*a*b*c*x^3/((c^2*x^2 - 1)*sqrt(-
c^2*x^2 + 1)*d^3) - a*b*x^2*arccos(c*x)/((c^2*x^2 - 1)*d^3) - 1/2*a^2*x^2/
((c^2*x^2 - 1)*d^3) - 1/12*b^2*x^2/((c^2*x^2 - 1)*d^3) + 1/2*b^2*x*arccos(
c*x)/(sqrt(-c^2*x^2 + 1)*c*d^3) + 1/4*b^2*arccos(c*x)^2/(c^2*d^3) + 1/2*a*
b*x/(sqrt(-c^2*x^2 + 1)*c*d^3) + 1/2*a*b*arccos(c*x)/(c^2*d^3) - 1/3*b^2*l
og(2)/(c^2*d^3) - 1/6*b^2*log(abs(-c^2*x^2 + 1))/(c^2*d^3) + 1/4*a^2/(c^2*
d^3) + 1/12*b^2/(c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx=\int \frac{x\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((x*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^3,x)
```

output
```
int((x*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \frac{-8\left(\int \frac{acos(cx)x}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)ab\,c^6x^4+16\left(\int \frac{acos(cx)x}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)ab\,c^4x^2-8\left(\int \frac{acos(cx)x}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)ab}{4c^2d^3\,(c^4}$$

input
```
int(x*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 8*int((acos(c*x)*x)/(c**6*x***6 - 3*c**4*x***4 + 3*c**2*x***2 - 1),x)*a*b
*c**6*x***4 + 16*int((acos(c*x)*x)/(c**6*x***6 - 3*c**4*x***4 + 3*c**2*x***2 -
 1),x)*a*b*c**4*x***2 - 8*int((acos(c*x)*x)/(c**6*x***6 - 3*c**4*x***4 + 3*c*
*2*x***2 - 1),x)*a*b*c**2 - 4*int((acos(c*x)**2*x)/(c**6*x***6 - 3*c**4*x***4
 + 3*c**2*x***2 - 1),x)*b**2*c**6*x***4 + 8*int((acos(c*x)**2*x)/(c**6*x***6
- 3*c**4*x***4 + 3*c**2*x***2 - 1),x)*b**2*c**4*x***2 - 4*int((acos(c*x)**2*x
)/(c**6*x***6 - 3*c**4*x***4 + 3*c**2*x***2 - 1),x)*b**2*c**2 + a**2)/(4*c**2
*d**3*(c**4*x***4 - 2*c**2*x***2 + 1))
```

3.207 $\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 332

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx &= \frac{b^2x}{12d^3(1-c^2x^2)}-\frac{b(a+b\arccos(cx))}{6cd^3(1-c^2x^2)^{3/2}}-\frac{3b(a+b\arccos(cx))}{4cd^3\sqrt{1-c^2x^2}}\\
&+\frac{x(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}+\frac{3x(a+b\arccos(cx))^2}{8d^3(1-c^2x^2)}\\
&-\frac{3i(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)}{4cd^3}\\
&+\frac{5b^2\text{arctanh}(cx)}{6cd^3}\\
&+\frac{3ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{4cd^3}\\
&-\frac{3ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{4cd^3}\\
&-\frac{3b^2\operatorname{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)}{4cd^3}\\
&+\frac{3b^2\operatorname{PolyLog}\left(3,ie^{i\arccos(cx)}\right)}{4cd^3}\end{aligned}$$

output
```
1/12*b^2*x/d^3/(-c^2*x^2+1)-1/6*b*(a+b*arccos(c*x))/c/d^3/(-c^2*x^2+1)^(3/
2)-3/4*b*(a+b*arccos(c*x))/c/d^3/(-c^2*x^2+1)^(1/2)+1/4*x*(a+b*arccos(c*x)
)^2/d^3/(-c^2*x^2+1)^2+3/8*x*(a+b*arccos(c*x))^2/d^3/(-c^2*x^2+1)-3/4*I*(a
+b*arccos(c*x))^2*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c/d^3+5/6*b^2*arctanh(c
*x)/c/d^3+3/4*I*b*(a+b*arccos(c*x))*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)
))/c/d^3-3/4*I*b*(a+b*arccos(c*x))*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))
/c/d^3-3/4*b^2*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/d^3+3/4*b^2*poly
log(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/d^3
```

Mathematica [A] (verified)

Time = 7.54 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$$
$$= \frac{a^2x}{4d^3(-1+c^2x^2)^2} - \frac{3a^2x}{8d^3(-1+c^2x^2)} - \frac{3a^2\log(1-cx)}{16cd^3} + \frac{3a^2\log(1+cx)}{16cd^3}$$
$$- \frac{2ab\left(\frac{(-2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)}{48(-1+cx)^2} - \frac{(2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)}{48(1+cx)^2} - \frac{3\left(\sqrt{1-c^2x^2}-\arccos(cx)\right)}{16(1+cx)} - \frac{3\left(\sqrt{1-c^2x^2}+\arccos(cx)\right)}{16(1-cx)} - \right.}{}$$
$$- \frac{b^2\left(-80\arccos(cx)\cot\left(\frac{1}{2}\arccos(cx)\right) - 2(2+9\arccos(cx)^2)\csc^2\left(\frac{1}{2}\arccos(cx)\right) - 2\sqrt{1-c^2x^2}\arccos(\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2)^3,x]
```

output
```
(a^2*x)/(4*d^3*(-1 + c^2*x^2)^2) - (3*a^2*x)/(8*d^3*(-1 + c^2*x^2)) - (3*a
^2*Log[1 - c*x])/(16*c*d^3) + (3*a^2*Log[1 + c*x])/(16*c*d^3) - (2*a*b*(((
-2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x])/(48*(-1 + c*x)^2) - ((2 + c*x
)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x])/(48*(1 + c*x)^2) - (3*(Sqrt[1 - c^2*x
^2] - ArcCos[c*x]))/(16*(1 + c*x)) - (3*(Sqrt[1 - c^2*x^2] + ArcCos[c*x]))
/(16*(1 - c*x)) - (3*((-1/2*I)*ArcCos[c*x]^2 + 2*ArcCos[c*x]*Log[1 + E^(I*
ArcCos[c*x])] - (2*I)*PolyLog[2, -E^(I*ArcCos[c*x])]))/16 + (3*(2*ArcCos[c
*x]*Log[1 - E^(I*ArcCos[c*x])] - (2*I)*(ArcCos[c*x]^2/4 + PolyLog[2, E^(I*
ArcCos[c*x])])))/16))/(c*d^3) - (b^2*(-80*ArcCos[c*x]*Cot[ArcCos[c*x]/2] -
 2*(2 + 9*ArcCos[c*x]^2)*Csc[ArcCos[c*x]/2]^2 - 2*Sqrt[1 - c^2*x^2]*ArcCos
[c*x]*Csc[ArcCos[c*x]/2]^4 - 3*ArcCos[c*x]^2*Csc[ArcCos[c*x]/2]^4 + 160*Lo
g[Tan[ArcCos[c*x]/2]] + 72*(ArcCos[c*x]^2*(Log[1 - E^(I*ArcCos[c*x])] - Lo
g[1 + E^(I*ArcCos[c*x])]) + (2*I)*ArcCos[c*x]*(PolyLog[2, -E^(I*ArcCos[c*x
])] - PolyLog[2, E^(I*ArcCos[c*x])]) + 2*(-PolyLog[3, -E^(I*ArcCos[c*x])]
+ PolyLog[3, E^(I*ArcCos[c*x])])) + 2*(2 + 9*ArcCos[c*x]^2)*Sec[ArcCos[c*x
]/2]^2 + 3*ArcCos[c*x]^2*Sec[ArcCos[c*x]/2]^4 - (32*ArcCos[c*x]*Sin[ArcCos
[c*x]/2]^4)/(1 - c^2*x^2)^(3/2) - 80*ArcCos[c*x]*Tan[ArcCos[c*x]/2]))/(192
*c*d^3)
```

Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.91, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5163, 27, 5163, 5165, 3042, 4671, 3011, 2720, 5183, 215, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}dx$$

$$\Big\downarrow \text{5163}$$

$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2d^3}+\frac{3\int \frac{(a+b\arccos(cx))^2}{d^2(1-c^2x^2)^2}dx}{4d}+\frac{x(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2d^3}+\frac{3\int \frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^2}dx}{4d^3}+\frac{x(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \; 5163$$

$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2d^3}+\frac{3\left(bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx+\frac{1}{2}\int \frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}\right)}{4d^3}+\frac{x(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \; 5165$$

$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2d^3}+\frac{3\left(bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx-\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}\right)}{4d^3}+\frac{x(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2d^3}+\frac{3\left(bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx-\frac{\int (a+b\arccos(cx))^2\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}\right)}{4d^3}+\frac{x(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \; 4671$$

$$\frac{3\left(-\frac{-2b\int (a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(\cdots}{2c}\right.}{4d^3}$$
$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2d^3}+\frac{x(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \; 3011$$

$$\frac{3\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a+b\arccos(cx)\cdots\right.}{2c}\right.}{4\cdots}$$
$$\frac{bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2d^3}+\frac{x(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \; 2720$$

$$3\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a\right.}{2c}\right.$$

$$\frac{bc\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2d^3}+\frac{x(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{5183}$$

$$3\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a\right.}{2c}\right.$$

$$\frac{bc\left(\frac{b\int\frac{1}{(1-c^2x^2)^2}dx}{3c}+\frac{a+b\arccos(cx)}{3c^2(1-c^2x^2)^{3/2}}\right)}{2d^3}+\frac{x(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{215}$$

$$3\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a\right.}{2c}\right.$$

$$\frac{bc\left(\frac{b\left(\frac{1}{2}\int\frac{1}{1-c^2x^2}dx+\frac{x}{2(1-c^2x^2)}\right)}{3c}+\frac{a+b\arccos(cx)}{3c^2(1-c^2x^2)^{3/2}}\right)}{2d^3}+\frac{x(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{219}$$

$$3\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a\right.}{2c}\right.$$

$$\frac{bc\left(\frac{a+b\arccos(cx)}{3c^2(1-c^2x^2)^{3/2}}+\frac{b\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3c}\right)}{2d^3}+\frac{x(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{7143}$$

$$\frac{3\left(bc\left(\frac{a+b\arccos(cx)}{c^2\sqrt{1-c^2x^2}}+\frac{b\text{arctanh}(cx)}{c^2}\right)-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-\right.}{}\right.}{4d^3}$$

$$\frac{bc\left(\frac{a+b\arccos(cx)}{3c^2(1-c^2x^2)^{3/2}}+\frac{b\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)}{3c}\right)}{2d^3}+\frac{x(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2)^3,x]
```

output
```
(x*(a + b*ArcCos[c*x])^2)/(4*d^3*(1 - c^2*x^2)^2) + (b*c*((a + b*ArcCos[c*
x])/(3*c^2*(1 - c^2*x^2)^(3/2)) + (b*(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(
2*c)))/(3*c)))/(2*d^3) + (3*((x*(a + b*ArcCos[c*x])^2)/(2*(1 - c^2*x^2)) +
 b*c*((a + b*ArcCos[c*x])/(c^2*Sqrt[1 - c^2*x^2]) + (b*ArcTanh[c*x])/c^2)
- (-2*(a + b*ArcCos[c*x])^2*ArcTanh[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*Arc
Cos[c*x])*PolyLog[2, -E^(I*ArcCos[c*x])] - b*PolyLog[3, -E^(I*ArcCos[c*x])
]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] - b*PolyLog[
3, E^(I*ArcCos[c*x])]))/(2*c)))/(4*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.65

method	result
derivativedivides	$-\frac{a^2\left(-\frac{1}{16(cx-1)^2}+\frac{3}{16(cx-1)}+\frac{3\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}-\frac{3\ln(cx+1)}{16}\right)}{d^3}-\frac{b^2\left(\frac{9\arccos(cx)^2c^3x^3+18\sqrt{-c^2x^2+1}\ \text{arcco}\ldots}{\ldots}\right.}{\ldots}$
default	$-\frac{a^2\left(-\frac{1}{16(cx-1)^2}+\frac{3}{16(cx-1)}+\frac{3\ln(cx-1)}{16}+\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}-\frac{3\ln(cx+1)}{16}\right)}{d^3}-\frac{b^2\left(\frac{9\arccos(cx)^2c^3x^3+18\sqrt{-c^2x^2+1}\ \text{arcco}\ldots}{\ldots}\right.}{\ldots}$
parts	$-\frac{a^2\left(-\frac{1}{16c(cx-1)^2}+\frac{3}{16c(cx-1)}+\frac{3\ln(cx-1)}{16c}+\frac{1}{16c(cx+1)^2}+\frac{3}{16c(cx+1)}-\frac{3\ln(cx+1)}{16c}\right)}{d^3}-\frac{b^2\left(\frac{9\arccos(cx)^2c^3x^3+18\sqrt{-c^2x^2\ldots}}{\ldots}\right.}{\ldots}$

input
```
int((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-a^2/d^3*(-1/16/(c*x-1)^2+3/16/(c*x-1)+3/16*ln(c*x-1)+1/16/(c*x+1)^2+
3/16/(c*x+1)-3/16*ln(c*x+1))-b^2/d^3*(1/24*(9*arccos(c*x)^2*c^3*x^3+18*(-c
^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^2-15*arccos(c*x)^2*c*x+2*c^3*x^3-22*arcc
os(c*x)*(-c^2*x^2+1)^(1/2)-2*c*x)/(c^4*x^4-2*c^2*x^2+1)-5/3*arctanh(c*x+I*
(-c^2*x^2+1)^(1/2))-3/8*arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+3/4*I
*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-3/4*polylog(3,-c*x-I*(-c
^2*x^2+1)^(1/2))+3/8*arccos(c*x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-3/4*I*ar
ccos(c*x)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+3/4*polylog(3,c*x+I*(-c^2*x^
2+1)^(1/2)))-2*a*b/d^3*(1/24*(9*c^3*x^3*arccos(c*x)+9*c^2*x^2*(-c^2*x^2+1)
^(1/2)-15*c*x*arccos(c*x)-11*(-c^2*x^2+1)^(1/2))/(c^4*x^4-2*c^2*x^2+1)+3/8
*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-3/8*arccos(c*x)*ln(1+c*x+I*(-c
^2*x^2+1)^(1/2))-3/8*I*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+3/8*I*polylog(2
,-c*x-I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)^2}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^6*d^3*x^6 - 3*c
^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acos(c*x))**2/(-c**2*d*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)^2}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/16*a^2*(2*(3*c^2*x^3 - 5*x)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) - 3*log
(c*x + 1)/(c*d^3) + 3*log(c*x - 1)/(c*d^3)) - 1/16*((6*b^2*c^3*x^3 - 10*b^
2*c*x - 3*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(c*x + 1) + 3*(b^2*c^4*x^
4 - 2*b^2*c^2*x^2 + b^2)*log(-c*x + 1))*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x)^2 + 16*(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3)*integrate(-1/8*((6*b
^2*c^3*x^3 - 10*b^2*c*x - 3*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(c*x +
1) + 3*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*log(-c*x + 1))*sqrt(c*x + 1)*sq
rt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - 16*a*b*arctan2(s
qrt(c*x + 1)*sqrt(-c*x + 1), c*x))/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^
3*x^2 - d^3), x))/(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^3,x)
```

output
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \frac{-32\left(\int \frac{acos(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)ab\,c^5x^4+64\left(\int \frac{acos(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)ab\,c^3x^2-32\left(\int \frac{acos(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)}{}$$

input
```
int((a+b*acos(c*x))^2/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 32*int(acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*a*b*c*
*5*x**4 + 64*int(acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*
a*b*c**3*x**2 - 32*int(acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 -
1),x)*a*b*c - 16*int(acos(c*x)**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 -
 1),x)*b**2*c**5*x**4 + 32*int(acos(c*x)**2/(c**6*x**6 - 3*c**4*x**4 + 3*c
**2*x**2 - 1),x)*b**2*c**3*x**2 - 16*int(acos(c*x)**2/(c**6*x**6 - 3*c**4*
x**4 + 3*c**2*x**2 - 1),x)*b**2*c - 3*log(c**2*x - c)*a**2*c**4*x**4 + 6*l
og(c**2*x - c)*a**2*c**2*x**2 - 3*log(c**2*x - c)*a**2 + 3*log(c**2*x + c)
*a**2*c**4*x**4 - 6*log(c**2*x + c)*a**2*c**2*x**2 + 3*log(c**2*x + c)*a**
2 - 6*a**2*c**3*x**3 + 10*a**2*c*x)/(16*c*d**3*(c**4*x**4 - 2*c**2*x**2 +
1))
```

3.208 $\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 296

$$
\begin{aligned}
\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^3}\,dx &= \frac{b^2}{12d^3(1-c^2x^2)} - \frac{bcx(a+b\arccos(cx))}{6d^3(1-c^2x^2)^{3/2}} \\
&\quad - \frac{4bcx(a+b\arccos(cx))}{3d^3\sqrt{1-c^2x^2}} \\
&\quad + \frac{(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2} + \frac{(a+b\arccos(cx))^2}{2d^3(1-c^2x^2)} \\
&\quad - \frac{2(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)}{d^3} \\
&\quad - \frac{2b^2\log(1-c^2x^2)}{3d^3} \\
&\quad + \frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{d^3} \\
&\quad - \frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{d^3} \\
&\quad - \frac{b^2\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)}{2d^3} + \frac{b^2\operatorname{PolyLog}\left(3,e^{2i\arccos(cx)}\right)}{2d^3}
\end{aligned}
$$

output
```
1/12*b^2/d^3/(-c^2*x^2+1)-1/6*b*c*x*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)^(3/
2)-4/3*b*c*x*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)^(1/2)+1/4*(a+b*arccos(c*x)
)^2/d^3/(-c^2*x^2+1)^2+1/2*(a+b*arccos(c*x))^2/d^3/(-c^2*x^2+1)-2*(a+b*arc
cos(c*x))^2*arctanh((c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^3-2/3*b^2*ln(-c^2*x^2+
1)/d^3+I*b*(a+b*arccos(c*x))*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^3-
I*b*(a+b*arccos(c*x))*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^3-1/2*b^2*
polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^3+1/2*b^2*polylog(3,(c*x+I*(-c^
2*x^2+1)^(1/2))^2)/d^3
```

Mathematica [A] (verified)

Time = 3.12 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.97

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^3}\,dx =$$

$$-\frac{-\frac{6a^2}{(-1+c^2x^2)^2}+\frac{12a^2}{-1+c^2x^2}-24a^2\log(cx)+12a^2\log\left(1-c^2x^2\right)+ab\left(\frac{(-2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)}{(-1+cx)^2}+\frac{(2+cx)\sqrt{1-\cdots}}{(\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x*(d - c^2*d*x^2)^3),x]
```

output
```
-1/24*((-6*a^2)/(-1 + c^2*x^2)^2 + (12*a^2)/(-1 + c^2*x^2) - 24*a^2*Log[c*
x] + 12*a^2*Log[1 - c^2*x^2] + a*b*(((-2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcC
os[c*x])/(-1 + c*x)^2 + ((2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x])/(1 +
 c*x)^2 + (15*(Sqrt[1 - c^2*x^2] - ArcCos[c*x]))/(1 + c*x) + (24*I)*ArcCos
[c*x]^2 + (15*(Sqrt[1 - c^2*x^2] + ArcCos[c*x]))/(-1 + c*x) + 48*ArcCos[c*
x]*Log[1 - E^(I*ArcCos[c*x])] - 48*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x
])] - (12*I)*(ArcCos[c*x]*(ArcCos[c*x] + (4*I)*Log[1 + E^(I*ArcCos[c*x])])
 + 4*PolyLog[2, -E^(I*ArcCos[c*x])]) - (12*I)*(ArcCos[c*x]^2 + 4*PolyLog[2
, E^(I*ArcCos[c*x])]) + (24*I)*PolyLog[2, -E^((2*I)*ArcCos[c*x])]) + b^2*(
(-I)*Pi^3 + 2/(-1 + c^2*x^2) - (4*c*x*ArcCos[c*x])/(1 - c^2*x^2)^(3/2) - (
32*c*x*ArcCos[c*x])/Sqrt[1 - c^2*x^2] - (6*ArcCos[c*x]^2)/(-1 + c^2*x^2)^2
 + (12*ArcCos[c*x]^2)/(-1 + c^2*x^2) + (16*I)*ArcCos[c*x]^3 + 24*ArcCos[c*
x]^2*Log[1 - E^((-2*I)*ArcCos[c*x])] - 24*ArcCos[c*x]^2*Log[1 + E^((2*I)*A
rcCos[c*x])] + 16*Log[1 - c^2*x^2] + (24*I)*ArcCos[c*x]*PolyLog[2, E^((-2*
I)*ArcCos[c*x])] + (24*I)*ArcCos[c*x]*PolyLog[2, -E^((2*I)*ArcCos[c*x])] +
 12*PolyLog[3, E^((-2*I)*ArcCos[c*x])] - 12*PolyLog[3, -E^((2*I)*ArcCos[c*
x])]))/d^3
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.14, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.593, Rules used = {5209, 27, 5163, 241, 5161, 240, 5209, 5161, 240, 5185, 4919, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^3}dx$$

$$\Big\downarrow \ 5209$$

$$\frac{bc\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{5/2}}dx}{2d^3}+\frac{\int \frac{(a+b\arccos(cx))^2}{d^2x(1-c^2x^2)^2}dx}{d}+\frac{(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{bc\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{5/2}}dx}{2d^3}+\frac{\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}dx}{d^3}+\frac{(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \ 5163$$

$$\frac{bc\left(\frac{2}{3}\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{3/2}}dx+\frac{1}{3}bc\int \frac{x}{(1-c^2x^2)^2}dx+\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}\right)}{2d^3}+\frac{\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}dx}{d^3}+\frac{(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \ 241$$

$$\frac{bc\left(\frac{2}{3}\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{3/2}}dx+\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{b}{6c(1-c^2x^2)}\right)}{2d^3}+\frac{\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}dx}{d^3}+\frac{(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \ 5161$$

$$\frac{bc\left(\frac{2}{3}\left(bc\int \frac{x}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)+\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{b}{6c(1-c^2x^2)}\right)}{2d^3}+\frac{\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}dx}{d^3}+\frac{(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \ 240$$

$$\frac{\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}dx}{d^3}+\frac{(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)+\frac{b}{6c(1-c^2x^2)}\right)}{2d^3}$$

$$\Big\downarrow \ 5209$$

$$\frac{bc\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{3/2}}dx+\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}dx+\frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)}}{d^3}+\frac{(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)+\frac{b}{6c(1-c^2x^2)}\right)}{2d^3}$$

$$\Big\downarrow \ 5161$$

$$\frac{bc\left(bc\int \frac{x}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)+\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}dx+\frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)}}{d^3}+\frac{(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)+\frac{b}{6c(1-c^2x^2)}\right)}{2d^3}$$

$$\Big\downarrow \ 240$$

$$\frac{\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}dx+\frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)}+bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)}{d^3}+\frac{(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)+\frac{b}{6c(1-c^2x^2)}\right)}{2d^3}$$

$$\Big\downarrow \ 5185$$

$$\frac{-\int \frac{(a+b\arccos(cx))^2}{cx\sqrt{1-c^2x^2}}d\arccos(cx)+\frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)}+bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)}{d^3}+\frac{(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)+\frac{b}{6c(1-c^2x^2)}\right)}{2d^3}$$

$$\Big\downarrow \ 4919$$

$$\frac{-2\int (a+b\arccos(cx))^2\csc(2\arccos(cx))d\arccos(cx)+\frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)}+bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)}{d^3}+\frac{(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)+\frac{b}{6c(1-c^2x^2)}\right)}{2d^3}$$

$\Big\downarrow$ 3042

$$\frac{-2\int (a+b\arccos(cx))^2\csc(2\arccos(cx))d\arccos(cx)+\frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)}+bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)}{d^3}+\frac{(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)+\frac{b}{6c(1-c^2x^2)}\right)}{2d^3}$$

$\Big\downarrow$ 4671

$$\frac{-2\left(-b\int (a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+b\int (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)d\arccos\right.}{d^3}$$
$$\frac{(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)+\frac{b}{6c(1-c^2x^2)}\right)}{2d^3}$$

$\Big\downarrow$ 3011

$$-2\left(b\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int \operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-b\left(\frac{1}{2}i\operatorname{Po}\right.\right.$$
$$\frac{(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)+\frac{b}{6c(1-c^2x^2)}\right)}{2d^3}$$

$\Big\downarrow$ 2720

$$-2\left(b\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)de^{2i\arccos(ca}\right.\right.$$
$$\frac{(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)+\frac{b}{6c(1-c^2x^2)}\right)}{2d^3}$$

$\Big\downarrow$ 7143

$$-2\left(-\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2+b\left(\frac{1}{2}i\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\operatorname{PolyLog}\right.\right.$$
$$\frac{(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}+\frac{bc\left(\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)+\frac{b}{6c(1-c^2x^2)}\right)}{2d^3}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x*(d - c^2*d*x^2)^3),x]
```

output
```
(a + b*ArcCos[c*x])^2/(4*d^3*(1 - c^2*x^2)^2) + (b*c*(b/(6*c*(1 - c^2*x^2)
) + (x*(a + b*ArcCos[c*x]))/(3*(1 - c^2*x^2)^(3/2)) + (2*((x*(a + b*ArcCos
[c*x]))/Sqrt[1 - c^2*x^2] - (b*Log[1 - c^2*x^2])/(2*c)))/3))/(2*d^3) + ((a
 + b*ArcCos[c*x])^2/(2*(1 - c^2*x^2)) + b*c*((x*(a + b*ArcCos[c*x]))/Sqrt[
1 - c^2*x^2] - (b*Log[1 - c^2*x^2])/(2*c)) - 2*(-((a + b*ArcCos[c*x])^2*Ar
cTanh[E^((2*I)*ArcCos[c*x])]) + b*((I/2)*(a + b*ArcCos[c*x])*PolyLog[2, -E
^((2*I)*ArcCos[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/4) - b*((I/
2)*(a + b*ArcCos[c*x])*PolyLog[2, E^((2*I)*ArcCos[c*x])] - (b*PolyLog[3, E
^((2*I)*ArcCos[c*x])])/4)))/d^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(329) = 658$.

Time = 0.71 (sec) , antiderivative size = 789, normalized size of antiderivative = 2.67

method	result
parts	$-\frac{a^2\left(-\ln(x)-\frac{1}{16(cx-1)^2}+\frac{5}{16(cx-1)}+\frac{\ln(cx-1)}{2}-\frac{1}{16(cx+1)^2}-\frac{5}{16(cx+1)}+\frac{\ln(cx+1)}{2}\right)}{d^3}-\frac{b^2\left(\frac{16i\arccos(cx)c^4x^4+16\sqrt{-c^2}}{}\right.}{}$
derivativedivides	$-\frac{a^2\left(-\frac{1}{16(cx-1)^2}+\frac{5}{16(cx-1)}+\frac{\ln(cx-1)}{2}-\ln(cx)-\frac{1}{16(cx+1)^2}-\frac{5}{16(cx+1)}+\frac{\ln(cx+1)}{2}\right)}{d^3}-\frac{b^2\left(\frac{16i\arccos(cx)c^4x^4+16\sqrt{-c^2}}{}\right.}{}$
default	$-\frac{a^2\left(-\frac{1}{16(cx-1)^2}+\frac{5}{16(cx-1)}+\frac{\ln(cx-1)}{2}-\ln(cx)-\frac{1}{16(cx+1)^2}-\frac{5}{16(cx+1)}+\frac{\ln(cx+1)}{2}\right)}{d^3}-\frac{b^2\left(\frac{16i\arccos(cx)c^4x^4+16\sqrt{-c^2}}{}\right.}{}$

input
```
int((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-a^2/d^3*(-ln(x)-1/16/(c*x-1)^2+5/16/(c*x-1)+1/2*ln(c*x-1)-1/16/(c*x+1)^2-
5/16/(c*x+1)+1/2*ln(c*x+1))-b^2/d^3*(1/12*(16*I*arccos(c*x)*c^4*x^4+16*(-c
^2*x^2+1)^(1/2)*arccos(c*x)*x^3*c^3+6*arccos(c*x)^2*x^2*c^2-32*I*arccos(c*
x)*c^2*x^2-18*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c-9*arccos(c*x)^2+16*I*arcc
os(c*x)+c^2*x^2-1)/(c^4*x^4-2*c^2*x^2+1)-8/3*ln(c*x+I*(-c^2*x^2+1)^(1/2))+
4/3*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+4/3*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)-arcc
os(c*x)^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+I*arccos(c*x)*polylog(2,-(c*x
+I*(-c^2*x^2+1)^(1/2))^2)-1/2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+arc
cos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-2*I*arccos(c*x)*polylog(2,-c*x-I
*(-c^2*x^2+1)^(1/2))+2*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))+arccos(c*x)^2*
ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-2*I*arccos(c*x)*polylog(2,c*x+I*(-c^2*x^2+1
)^(1/2))+2*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2)))-2*a*b/d^3*(1/12*(8*I*c^4*x
^4+8*c^3*x^3*(-c^2*x^2+1)^(1/2)+6*c^2*x^2*arccos(c*x)-16*I*c^2*x^2-9*c*x*(
-c^2*x^2+1)^(1/2)-9*arccos(c*x)+8*I)/(c^4*x^4-2*c^2*x^2+1)+arccos(c*x)*ln(
1+c*x+I*(-c^2*x^2+1)^(1/2))-I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-arccos(
c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/2*I*polylog(2,-(c*x+I*(-c^2*x^2+
1)^(1/2))^2)+arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-I*polylog(2,c*x+I*
(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^3}\,dx=\int -\frac{\left(b\arccos\left(cx\right)+a\right)^2}{\left(c^2dx^2-d\right)^3x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^6*d^3*x^7 - 3*c
^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\,(d-c^2dx^2)^3}\,dx = -\frac{\int \frac{a^2}{c^6x^7-3c^4x^5+3c^2x^3-x}\,dx+\int \frac{b^2\operatorname{acos}^2(cx)}{c^6x^7-3c^4x^5+3c^2x^3-x}\,dx+\int \frac{2ab\operatorname{acos}(cx)}{c^6x^7-3c^4x^5+3c^2x^3-x}\,dx}{d^3}$$

input
```
integrate((a+b*acos(c*x))**2/x/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a**2/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x), x) + Integral
(b**2*acos(c*x)**2/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x), x) + Integ
ral(2*a*b*acos(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x), x))/d**3
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\,(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)^2}{(c^2dx^2-d)^3x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*a^2*((2*c^2*x^2 - 3)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) + 2*log(c*x
+ 1)/d^3 + 2*log(c*x - 1)/d^3 - 4*log(x)/d^3) - integrate((b^2*arctan2(sqr
t(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x
+ 1), c*x))/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^3}\,dx = \int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{x\,\left(d-c^2\,d\,x^2\right)^3}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x*(d - c^2*d*x^2)^3),x)
```

output
```
int((a + b*acos(c*x))^2/(x*(d - c^2*d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^3}\,dx$$
$$= \frac{-8\left(\int \frac{acos(cx)}{c^6x^7-3c^4x^5+3c^2x^3-x}dx\right)ab\,c^4x^4+16\left(\int \frac{acos(cx)}{c^6x^7-3c^4x^5+3c^2x^3-x}dx\right)ab\,c^2x^2-8\left(\int \frac{acos(cx)}{c^6x^7-3c^4x^5+3c^2x^3-x}dx\right)ab}{}$$

input
```
int((a+b*acos(c*x))^2/x/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 8*int(acos(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*a*b*c**
4*x**4 + 16*int(acos(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x)*a
*b*c**2*x**2 - 8*int(acos(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x)
,x)*a*b - 4*int(acos(c*x)**2/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x),x
)*b**2*c**4*x**4 + 8*int(acos(c*x)**2/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x*
*3 - x),x)*b**2*c**2*x**2 - 4*int(acos(c*x)**2/(c**6*x**7 - 3*c**4*x**5 +
3*c**2*x**3 - x),x)*b**2 - 2*log(c**2*x - c)*a**2*c**4*x**4 + 4*log(c**2*x
 - c)*a**2*c**2*x**2 - 2*log(c**2*x - c)*a**2 - 2*log(c**2*x + c)*a**2*c**
4*x**4 + 4*log(c**2*x + c)*a**2*c**2*x**2 - 2*log(c**2*x + c)*a**2 + 4*log
(x)*a**2*c**4*x**4 - 8*log(x)*a**2*c**2*x**2 + 4*log(x)*a**2 - a**2*c**4*x
**4 + 2*a**2)/(4*d**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.209 $\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 429

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^3}\,dx = \frac{b^2c^2x}{12d^3(1-c^2x^2)} - \frac{bc(a+b\arccos(cx))}{6d^3(1-c^2x^2)^{3/2}} - \frac{7bc(a+b\arccos(cx))}{4d^3\sqrt{1-c^2x^2}} - \frac{(a+b\arccos(cx))^2}{d^3x(1-c^2x^2)^2} + \frac{5c^2x(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2} + \frac{15c^2x(a+b\arccos(cx))^2}{8d^3(1-c^2x^2)} - \frac{15ic(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)}{4d^3} - \frac{4bc(a+b\arccos(cx))\text{arctanh}\left(e^{i\arccos(cx)}\right)}{d^3} + \frac{11b^2c\text{arctanh}(cx)}{6d^3} + \frac{2ib^2c\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{d^3} + \frac{15ibc(a+b\arccos(cx))\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{4d^3} - \frac{15ibc(a+b\arccos(cx))\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{4d^3} - \frac{2ib^2c\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{d^3} - \frac{15b^2c\,\text{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)}{4d^3} + \frac{15b^2c\,\text{PolyLog}\left(3,ie^{i\arccos(cx)}\right)}{4d^3}$$

output

```
1/12*b^2*c^2*x/d^3/(-c^2*x^2+1)-1/6*b*c*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)
^(3/2)-7/4*b*c*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)^(1/2)-(a+b*arccos(c*x))^
2/d^3/x/(-c^2*x^2+1)^2+5/4*c^2*x*(a+b*arccos(c*x))^2/d^3/(-c^2*x^2+1)^2+15
/8*c^2*x*(a+b*arccos(c*x))^2/d^3/(-c^2*x^2+1)-15/4*I*c*(a+b*arccos(c*x))^2
*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/d^3-4*b*c*(a+b*arccos(c*x))*arctanh(c*x+
I*(-c^2*x^2+1)^(1/2))/d^3+11/6*b^2*c*arctanh(c*x)/d^3+2*I*b^2*c*polylog(2,
-c*x-I*(-c^2*x^2+1)^(1/2))/d^3+15/4*I*b*c*(a+b*arccos(c*x))*polylog(2,-I*(
c*x+I*(-c^2*x^2+1)^(1/2)))/d^3-15/4*I*b*c*(a+b*arccos(c*x))*polylog(2,I*(c
*x+I*(-c^2*x^2+1)^(1/2)))/d^3-2*I*b^2*c*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)
)/d^3-15/4*b^2*c*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^3+15/4*b^2*c*p
olylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 865 vs. 2(429) = 858.

Time = 8.43 (sec) , antiderivative size = 865, normalized size of antiderivative = 2.02

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^2*(d - c^2*d*x^2)^3),x]
```

output
```
-(a^2/(d^3*x)) + (a^2*c^2*x)/(4*d^3*(-1 + c^2*x^2)^2) - (7*a^2*c^2*x)/(8*d
^3*(-1 + c^2*x^2)) - (15*a^2*c*Log[1 - c*x])/(16*d^3) + (15*a^2*c*Log[1 +
c*x])/(16*d^3) - (2*a*b*c*(((-2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x])/
(48*(-1 + c*x)^2) - ((2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x])/(48*(1 +
 c*x)^2) - (7*(Sqrt[1 - c^2*x^2] - ArcCos[c*x]))/(16*(1 + c*x)) + ArcCos[c
*x]/(c*x) - (7*(Sqrt[1 - c^2*x^2] + ArcCos[c*x]))/(16*(1 - c*x)) + Log[c*x
] - Log[1 + Sqrt[1 - c^2*x^2]] - (15*((-1/2*I)*ArcCos[c*x]^2 + 2*ArcCos[c*
x]*Log[1 + E^(I*ArcCos[c*x])] - (2*I)*PolyLog[2, -E^(I*ArcCos[c*x])]))/16
+ (15*(2*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] - (2*I)*(ArcCos[c*x]^2/4 +
 PolyLog[2, E^(I*ArcCos[c*x])])))/16))/d^3 - (b^2*c*(192*ArcCos[c*x]^2 - 1
76*ArcCos[c*x]*Cot[ArcCos[c*x]/2] - 2*(2 + 21*ArcCos[c*x]^2)*Csc[ArcCos[c*
x]/2]^2 - 2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Csc[ArcCos[c*x]/2]^4 - 3*ArcCos[
c*x]^2*Csc[ArcCos[c*x]/2]^4 + 352*Log[Tan[ArcCos[c*x]/2]] - 384*(ArcCos[c*
x]*(Log[1 - I*E^(I*ArcCos[c*x])] - Log[1 + I*E^(I*ArcCos[c*x])]) + I*(Poly
Log[2, (-I)*E^(I*ArcCos[c*x])] - PolyLog[2, I*E^(I*ArcCos[c*x])])) + 360*(
ArcCos[c*x]^2*(Log[1 - E^(I*ArcCos[c*x])] - Log[1 + E^(I*ArcCos[c*x])]) +
(2*I)*ArcCos[c*x]*(PolyLog[2, -E^(I*ArcCos[c*x])] - PolyLog[2, E^(I*ArcCos
[c*x])]) + 2*(-PolyLog[3, -E^(I*ArcCos[c*x])] + PolyLog[3, E^(I*ArcCos[c*x
])])) + 2*(2 + 21*ArcCos[c*x]^2)*Sec[ArcCos[c*x]/2]^2 + 3*ArcCos[c*x]^2*Se
c[ArcCos[c*x]/2]^4 + (192*ArcCos[c*x]^2*Sin[ArcCos[c*x]/2])/(Cos[ArcCos...
```

Rubi [A] (verified)

Time = 4.21 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.15, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.852$, Rules used = {5205, 27, 5163, 5163, 5165, 3042, 4671, 3011, 2720, 5183, 215, 219, 5209, 215, 219, 5209, 219, 5219, 3042, 4669, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^2\left(d-c^2dx^2\right)^3}dx$$

$$\Big\downarrow \text{ 5205}$$

$$5c^2\int \frac{(a+b\arccos(cx))^2}{d^3\left(1-c^2x^2\right)^3}dx-\frac{2bc\int \frac{a+b\arccos(cx)}{x\left(1-c^2x^2\right)^{5/2}}dx}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5c^2\int \frac{(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^3}dx}{d^3}-\frac{2bc\int \frac{a+b\arccos(cx)}{x\left(1-c^2x^2\right)^{5/2}}dx}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{ 5163}$$

$$\frac{5c^2\left(\frac{1}{2}bc\int \frac{x(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{5/2}}dx+\frac{3}{4}\int \frac{(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^2}dx+\frac{x(a+b\arccos(cx))^2}{4\left(1-c^2x^2\right)^2}\right)}{d^3}-\frac{2bc\int \frac{a+b\arccos(cx)}{x\left(1-c^2x^2\right)^{5/2}}dx}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{ 5163}$$

$$\frac{5c^2\left(\frac{1}{2}bc\int \frac{x(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{5/2}}dx+\frac{3}{4}\left(bc\int \frac{x(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}dx+\frac{1}{2}\int \frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))^2}{2\left(1-c^2x^2\right)}\right)+\frac{x(a+b\arccos(cx))^2}{4\left(1-c^2x^2\right)^2}\right)}{d^3}-\frac{2bc\int \frac{a+b\arccos(cx)}{x\left(1-c^2x^2\right)^{5/2}}dx}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{ 5165}$$

$$\frac{5c^2\left(\frac{1}{2}bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx+\frac{3}{4}\left(bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx-\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}\right)+\frac{x(a+b a}{4(1-}\right.}{d^3}$$

$$\frac{2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x(1-c^2x^2)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx}{d^3}+$$

$$\frac{5c^2\left(\frac{1}{2}bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx+\frac{3}{4}\left(bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx-\frac{\int (a+b\arccos(cx))^2\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}\right.\right.}{d^3}$$

$$\frac{(a+b\arccos(cx))^2}{d^3x(1-c^2x^2)^2}$$

$$\Big\downarrow \text{4671}$$

$$\frac{5c^2\left(\frac{3}{4}\left(-\frac{-2b\int (a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos}\right.}{2c}\right.\right.}{d^3}$$

$$\frac{2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x(1-c^2x^2)^2}$$

$$\Big\downarrow \text{3011}$$

$$5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)(a+b\arccos\right.}{2c}\right.\right.$$

$$\frac{2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x(1-c^2x^2)^2}$$

$$\Big\downarrow \text{2720}$$

$$5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos(}\right.\right.}{2c}\right.\right.$$

$$\frac{2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x(1-c^2x^2)^2}$$

$$\Big\downarrow \text{5183}$$

$$5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos}\right.\right.}{2c}\right.\right.$$

$$\frac{2bc\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x\,(1-c^2x^2)^2}$$

$$\Big\downarrow \text{215}$$

$$5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos}\right.\right.}{2c}\right.\right.$$

$$\frac{2bc\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x\,(1-c^2x^2)^2}$$

$$\Big\downarrow \text{219}$$

$$5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos}\right.\right.}{2c}\right.\right.$$

$$\frac{2bc\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x\,(1-c^2x^2)^2}$$

$$\Big\downarrow \text{5209}$$

$$5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos}\right.\right.}{2c}\right.\right.$$

$$\frac{2bc\left(\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx+\frac{1}{3}bc\int\frac{1}{(1-c^2x^2)^2}dx+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}\right)}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x\,(1-c^2x^2)^2}$$

$$\Big\downarrow \text{215}$$

$$5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos}\right.\right.}{2c}\right.\right.$$

$$\frac{2bc\left(\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx+\frac{1}{3}bc\left(\frac{1}{2}\int\frac{1}{1-c^2x^2}dx+\frac{x}{2(1-c^2x^2)}\right)+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}\right)}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x\left(1-c^2x^2\right)^2}$$

$\downarrow$ 219

$$5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos}\right.\right.}{2c}\right.\right.$$

$$\frac{2bc\left(\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}+\frac{1}{3}bc\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)\right)}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x\left(1-c^2x^2\right)^2}$$

$\downarrow$ 5209

$$5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos}\right.\right.}{2c}\right.\right.$$

$$\frac{2bc\left(\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+bc\int\frac{1}{1-c^2x^2}dx+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}+\frac{1}{3}bc\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)\right)}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x\left(1-c^2x^2\right)^2}$$

$\downarrow$ 219

$$5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos}\right.\right.}{2c}\right.\right.$$

$$\frac{2bc\left(\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}+\frac{1}{3}bc\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)+b\,\text{arctanh}(cx)\right)}{d^3}-\frac{(a+b\arccos(cx))^2}{d^3x\left(1-c^2x^2\right)^2}$$

$\downarrow$ 5219

$$5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos}\right.\right.}{2c}\right.\right.$$

$$\frac{2bc\left(-\int\frac{a+b\arccos(cx)}{cx}d\arccos(cx)+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}+\frac{1}{3}bc\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)+b\text{arctanh}(cx)\right)}{d^3}$$

$$\frac{(a+b\arccos(cx))^2}{d^3x\left(1-c^2x^2\right)^2}$$

$\downarrow$ 3042

$$5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos}\right.\right.}{2c}\right.\right.$$

$$\frac{2bc\left(-\int(a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}+\frac{1}{3}bc\left(\frac{\text{arctanh}(cx)}{2c}+\right.\right.}{d^3}$$

$$\frac{(a+b\arccos(cx))^2}{d^3x\left(1-c^2x^2\right)^2}$$

$\downarrow$ 4669

$$-\frac{2bc\left(b\int\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)-b\int\log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)+2i\arctan\left(e^{i\arccos(cx)}\right)(a+\right.}{d^3}$$

$$5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos}\right.\right.}{2c}\right.\right.$$

$$\frac{(a+b\arccos(cx))^2}{d^3x\left(1-c^2x^2\right)^2}$$

$\downarrow$ 2715

$$-\frac{2bc\left(-ib\int e^{-i\arccos(cx)}\log\left(1-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}+ib\int e^{-i\arccos(cx)}\log\left(1+ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}+\right.}{5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos}\right.\right.}{2c}\right.\right.}$$

$$\frac{(a+b\arccos(cx))^2}{d^3x\,(1-c^2x^2)^2}$$

$\downarrow$ 2838

$$\frac{5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e^{i\arccos}\right.\right.}{2c}\right.\right.}{}$$

$$\frac{2bc\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}-ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)+ib\,\text{P}\right.}{d^3}$$

$$\frac{(a+b\arccos(cx))^2}{d^3x\,(1-c^2x^2)^2}$$

$\downarrow$ 7143

$$-\frac{2bc\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}-ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)+ib\right.}{d^3}$$

$$5c^2\left(\frac{3}{4}\left(bc\left(\frac{a+b\arccos(cx)}{c^2\sqrt{1-c^2x^2}}+\frac{b\text{arctanh}(cx)}{c^2}\right)-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos\right.}{}\right.\right.$$

$$\frac{(a+b\arccos(cx))^2}{d^3x\,(1-c^2x^2)^2}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^2*(d - c^2*d*x^2)^3),x]
```

output
```
-((a + b*ArcCos[c*x])^2/(d^3*x*(1 - c^2*x^2)^2)) - (2*b*c*((a + b*ArcCos[c
*x])/(3*(1 - c^2*x^2)^(3/2)) + (a + b*ArcCos[c*x])/Sqrt[1 - c^2*x^2] + (2*
I)*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcCos[c*x])] + b*ArcTanh[c*x] + (b*c*(
x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/3 - I*b*PolyLog[2, (-I)*E^(I*Ar
cCos[c*x])] + I*b*PolyLog[2, I*E^(I*ArcCos[c*x])]))/d^3 + (5*c^2*((x*(a +
b*ArcCos[c*x])^2)/(4*(1 - c^2*x^2)^2) + (b*c*((a + b*ArcCos[c*x])/(3*c^2*(
1 - c^2*x^2)^(3/2)) + (b*(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/(3*c)
))/2 + (3*((x*(a + b*ArcCos[c*x])^2)/(2*(1 - c^2*x^2)) + b*c*((a + b*ArcCo
s[c*x])/(c^2*Sqrt[1 - c^2*x^2]) + (b*ArcTanh[c*x])/c^2) - (-2*(a + b*ArcCo
s[c*x])^2*ArcTanh[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[
2, -E^(I*ArcCos[c*x])] - b*PolyLog[3, -E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b
*ArcCos[c*x])*PolyLog[2, E^(I*ArcCos[c*x])] - b*PolyLog[3, E^(I*ArcCos[c*x
])]))/(2*c)))/4))/d^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 731, normalized size of antiderivative = 1.70

method	result
derivativedivides	$c\left(-\frac{a^2\left(-\frac{1}{16(cx-1)^2}+\frac{7}{16(cx-1)}+\frac{15\ln(cx-1)}{16}+\frac{1}{cx}+\frac{1}{16(cx+1)^2}+\frac{7}{16(cx+1)}-\frac{15\ln(cx+1)}{16}\right)}{d^3}-\frac{b^2\left(\frac{45\arccos(cx)^2x^4c^4+4}{}\right.}{}\right.$
default	$c\left(-\frac{a^2\left(-\frac{1}{16(cx-1)^2}+\frac{7}{16(cx-1)}+\frac{15\ln(cx-1)}{16}+\frac{1}{cx}+\frac{1}{16(cx+1)^2}+\frac{7}{16(cx+1)}-\frac{15\ln(cx+1)}{16}\right)}{d^3}-\frac{b^2\left(\frac{45\arccos(cx)^2x^4c^4+4}{}\right.}{}\right.$
parts	$-\frac{a^2\left(\frac{1}{x}-\frac{c}{16(cx-1)^2}+\frac{7c}{16(cx-1)}+\frac{15c\ln(cx-1)}{16}+\frac{c}{16(cx+1)^2}+\frac{7c}{16(cx+1)}-\frac{15c\ln(cx+1)}{16}\right)}{d^3}-\frac{b^2c\left(\frac{45\arccos(cx)^2x^4c^4+42\sqrt{-}}{}\right.}{}$

input
```
int((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c*(-a^2/d^3*(-1/16/(c*x-1)^2+7/16/(c*x-1)+15/16*ln(c*x-1)+1/c/x+1/16/(c*x+
1)^2+7/16/(c*x+1)-15/16*ln(c*x+1))-b^2/d^3*(1/24*(45*arccos(c*x)^2*x^4*c^4
+42*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^3*c^3-75*arccos(c*x)^2*x^2*c^2+2*c^4*
x^4-46*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c+24*arccos(c*x)^2-2*c^2*x^2)/c/x/
(c^4*x^4-2*c^2*x^2+1)-2*I*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*I*dilog(
1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-15/4*I*arccos(c*x)*polylog(2,c*x+I*(-c^2*x
^2+1)^(1/2))-15/8*arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+15/4*I*arcc
os(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+15/8*arccos(c*x)^2*ln(1-c*x-I
*(-c^2*x^2+1)^(1/2))+2*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*ar
ccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-11/6*ln(1+c*x+I*(-c^2*x^2+1)^
(1/2))+11/6*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)-15/4*polylog(3,-c*x-I*(-c^2*x^2
+1)^(1/2))+15/4*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2)))-2*a*b/d^3*(1/24*(45*c
^4*x^4*arccos(c*x)+21*c^3*x^3*(-c^2*x^2+1)^(1/2)-75*c^2*x^2*arccos(c*x)-23
*c*x*(-c^2*x^2+1)^(1/2)+24*arccos(c*x))/c/x/(c^4*x^4-2*c^2*x^2+1)+15/8*I*d
ilog(1+c*x+I*(-c^2*x^2+1)^(1/2))+2*I*arctan(c*x+I*(-c^2*x^2+1)^(1/2))+15/8
*I*dilog(c*x+I*(-c^2*x^2+1)^(1/2))-15/8*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1
)^(1/2))))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)^2}{(c^2dx^2-d)^3x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^6*d^3*x^8 - 3*c
^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)
```

Sympy [F]

$$\int \frac{(a + b \arccos(cx))^2}{x^2 (d - c^2dx^2)^3} dx = -\frac{\int \frac{a^2}{c^6x^8-3c^4x^6+3c^2x^4-x^2} dx + \int \frac{b^2 \operatorname{acos}^2(cx)}{c^6x^8-3c^4x^6+3c^2x^4-x^2} dx + \int \frac{2ab \operatorname{acos}(cx)}{c^6x^8-3c^4x^6+3c^2x^4-x^2} dx}{d^3}$$

input
```
integrate((a+b*acos(c*x))**2/x**2/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a**2/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2), x) + Integ
ral(b**2*acos(c*x)**2/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2), x) +
 Integral(2*a*b*acos(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2),
x))/d**3
```

Maxima [F]

$$\int \frac{(a + b \arccos(cx))^2}{x^2 (d - c^2dx^2)^3} dx = \int -\frac{(b \arccos(cx) + a)^2}{(c^2dx^2 - d)^3 x^2} dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/16*a^2*(2*(15*c^4*x^4 - 25*c^2*x^2 + 8)/(c^4*d^3*x^5 - 2*c^2*d^3*x^3 +
d^3*x) - 15*c*log(c*x + 1)/d^3 + 15*c*log(c*x - 1)/d^3) - 1/16*((30*b^2*c^
4*x^4 - 50*b^2*c^2*x^2 + 16*b^2 - 15*(b^2*c^5*x^5 - 2*b^2*c^3*x^3 + b^2*c*
x)*log(c*x + 1) + 15*(b^2*c^5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*log(-c*x + 1)
)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 16*(c^4*d^3*x^5 - 2*c^2*d
^3*x^3 + d^3*x)*integrate(-1/8*((30*b^2*c^5*x^5 - 50*b^2*c^3*x^3 + 16*b^2*
c*x - 15*(b^2*c^6*x^6 - 2*b^2*c^4*x^4 + b^2*c^2*x^2)*log(c*x + 1) + 15*(b^
2*c^6*x^6 - 2*b^2*c^4*x^4 + b^2*c^2*x^2)*log(-c*x + 1))*sqrt(c*x + 1)*sqrt
(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - 16*a*b*arctan2(sqr
t(c*x + 1)*sqrt(-c*x + 1), c*x))/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*
x^4 - d^3*x^2), x))/(c^4*d^3*x^5 - 2*c^2*d^3*x^3 + d^3*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^3}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x^2\,(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^2*(d - c^2*d*x^2)^3),x)
```

output
```
int((a + b*acos(c*x))^2/(x^2*(d - c^2*d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^3}\,dx = \frac{-32\left(\int \frac{acos(cx)}{c^6x^8-3c^4x^6+3c^2x^4-x^2}dx\right)ab\,c^4x^5+64\left(\int \frac{acos(cx)}{c^6x^8-3c^4x^6+3c^2x^4-x^2}dx\right)ab\,c^2x^3-32\left(\int \frac{acos(cx)}{c^6x^8-3c^4x^6+3c^2x^4-x^2}d\right.}{}$$

input
```
int((a+b*acos(c*x))^2/x^2/(-c^2*d*x^2+d)^3,x)
```

output

```
( - 32*int(acos(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2),x)*a*b
*c**4*x**5 + 64*int(acos(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**
2),x)*a*b*c**2*x**3 - 32*int(acos(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x
**4 - x**2),x)*a*b*x - 16*int(acos(c*x)**2/(c**6*x**8 - 3*c**4*x**6 + 3*c*
*2*x**4 - x**2),x)*b**2*c**4*x**5 + 32*int(acos(c*x)**2/(c**6*x**8 - 3*c**
4*x**6 + 3*c**2*x**4 - x**2),x)*b**2*c**2*x**3 - 16*int(acos(c*x)**2/(c**6
*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2),x)*b**2*x - 15*log(c**2*x - c)*a
**2*c**5*x**5 + 30*log(c**2*x - c)*a**2*c**3*x**3 - 15*log(c**2*x - c)*a**
2*c*x + 15*log(c**2*x + c)*a**2*c**5*x**5 - 30*log(c**2*x + c)*a**2*c**3*x
**3 + 15*log(c**2*x + c)*a**2*c*x - 30*a**2*c**4*x**4 + 50*a**2*c**2*x**2
- 16*a**2)/(16*d**3*x*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.210 $\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 403

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^3}\,dx &= \frac{b^2c^2}{12d^3(1-c^2x^2)}-\frac{bc(a+b\arccos(cx))}{d^3x(1-c^2x^2)^{3/2}}\\&+\frac{5bc^3x(a+b\arccos(cx))}{6d^3(1-c^2x^2)^{3/2}}\\&-\frac{4bc^3x(a+b\arccos(cx))}{3d^3\sqrt{1-c^2x^2}}+\frac{3c^2(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}\\&-\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2}+\frac{3c^2(a+b\arccos(cx))^2}{2d^3(1-c^2x^2)}\\&-\frac{6c^2(a+b\arccos(cx))^2\text{arctanh}\left(e^{2i\arccos(cx)}\right)}{d^3}\\&+\frac{b^2c^2\log(x)}{d^3}-\frac{7b^2c^2\log(1-c^2x^2)}{6d^3}\\&+\frac{3ibc^2(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{d^3}\\&-\frac{3ibc^2(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{d^3}\\&-\frac{3b^2c^2\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)}{2d^3}\\&+\frac{3b^2c^2\operatorname{PolyLog}\left(3,e^{2i\arccos(cx)}\right)}{2d^3}\end{aligned}$$

output
```
1/12*b^2*c^2/d^3/(-c^2*x^2+1)-b*c*(a+b*arccos(c*x))/d^3/x/(-c^2*x^2+1)^(3/
2)+5/6*b*c^3*x*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)^(3/2)-4/3*b*c^3*x*(a+b*a
rccos(c*x))/d^3/(-c^2*x^2+1)^(1/2)+3/4*c^2*(a+b*arccos(c*x))^2/d^3/(-c^2*x
^2+1)^2-1/2*(a+b*arccos(c*x))^2/d^3/x^2/(-c^2*x^2+1)^2+3/2*c^2*(a+b*arccos
(c*x))^2/d^3/(-c^2*x^2+1)-6*c^2*(a+b*arccos(c*x))^2*arctanh((c*x+I*(-c^2*x
^2+1)^(1/2))^2)/d^3+b^2*c^2*ln(x)/d^3-7/6*b^2*c^2*ln(-c^2*x^2+1)/d^3+3*I*b
*c^2*(a+b*arccos(c*x))*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^3-3*I*b*
c^2*(a+b*arccos(c*x))*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^3-3/2*b^2*
c^2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^3+3/2*b^2*c^2*polylog(3,(c*
x+I*(-c^2*x^2+1)^(1/2))^2)/d^3
```

Mathematica [A] (verified)

Time = 4.01 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.77

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^3}\,dx = -\frac{\frac{12a^2}{x^2}-\frac{6a^2c^2}{(-1+c^2x^2)^2}+\frac{24a^2c^2}{-1+c^2x^2}+\frac{abc^2\left((-2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)\right)}{(-1+cx)^2}+\frac{abc^2\left((2+cx)\sqrt{1-c^2x^2}-3\arccos(cx)\right)}{(1+cx)^2}+\frac{27abc^2\left(\sqrt{1\right.}{}}{}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^3*(d - c^2*d*x^2)^3),x]
```

output
```
-1/24*((12*a^2)/x^2 - (6*a^2*c^2)/(-1 + c^2*x^2)^2 + (24*a^2*c^2)/(-1 + c^
2*x^2) + (a*b*c^2*((-2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x]))/(-1 + c*
x)^2 + (a*b*c^2*((2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x]))/(1 + c*x)^2
 + (27*a*b*c^2*(Sqrt[1 - c^2*x^2] - ArcCos[c*x]))/(1 + c*x) - (24*a*b*(c*x
*Sqrt[1 - c^2*x^2] - ArcCos[c*x]))/x^2 + (27*a*b*c^2*(Sqrt[1 - c^2*x^2] +
ArcCos[c*x]))/(-1 + c*x) - 72*a^2*c^2*Log[x] + 36*a^2*c^2*Log[1 - c^2*x^2]
 - (36*I)*a*b*c^2*(ArcCos[c*x]*(ArcCos[c*x] + (4*I)*Log[1 + E^(I*ArcCos[c*
x])]) + 4*PolyLog[2, -E^(I*ArcCos[c*x])]) - (36*I)*a*b*c^2*(ArcCos[c*x]*(A
rcCos[c*x] + (4*I)*Log[1 - E^(I*ArcCos[c*x])]) + 4*PolyLog[2, E^(I*ArcCos[
c*x])]) + (72*I)*a*b*c^2*(ArcCos[c*x]*(ArcCos[c*x] + (2*I)*Log[1 + E^((2*I
)*ArcCos[c*x])]) + PolyLog[2, -E^((2*I)*ArcCos[c*x])]) + b^2*c^2*((-3*I)*P
i^3 + 2/(-1 + c^2*x^2) - (4*c*x*ArcCos[c*x])/(1 - c^2*x^2)^(3/2) - (56*c*x
*ArcCos[c*x])/Sqrt[1 - c^2*x^2] - (24*Sqrt[1 - c^2*x^2]*ArcCos[c*x])/(c*x)
 + (12*ArcCos[c*x]^2)/(c^2*x^2) - (6*ArcCos[c*x]^2)/(-1 + c^2*x^2)^2 + (24
*ArcCos[c*x]^2)/(-1 + c^2*x^2) + (48*I)*ArcCos[c*x]^3 + 72*ArcCos[c*x]^2*L
og[1 - E^((-2*I)*ArcCos[c*x])] - 72*ArcCos[c*x]^2*Log[1 + E^((2*I)*ArcCos[
c*x])] - 24*Log[c*x] + 28*Log[1 - c^2*x^2] + (72*I)*ArcCos[c*x]*PolyLog[2,
 E^((-2*I)*ArcCos[c*x])] + (72*I)*ArcCos[c*x]*PolyLog[2, -E^((2*I)*ArcCos[
c*x])] + 36*PolyLog[3, E^((-2*I)*ArcCos[c*x])] - 36*PolyLog[3, -E^((2*I)*A
rcCos[c*x])]))/d^3
```

Rubi [A] (verified)

Time = 3.28 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.24, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.815$, Rules used = {5205, 27, 5195, 27, 1578, 1195, 2009, 5209, 5163, 241, 5161, 240, 5209, 5161, 240, 5185, 4919, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^3}\,dx$$

$$\Big\downarrow \text{5205}$$

$$3c^2\int \frac{(a+b\arccos(cx))^2}{d^3x\left(1-c^2x^2\right)^3}dx-\frac{bc\int \frac{a+b\arccos(cx)}{x^2\left(1-c^2x^2\right)^{5/2}}dx}{d^3}-\frac{(a+b\arccos(cx))^2}{2d^3x^2\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3c^2\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^3}dx}{d^3}-\frac{bc\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)^{5/2}}dx}{d^3}-\frac{(a+b\arccos(cx))^2}{2d^3x^2\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{5195}$$

$$\frac{3c^2\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^3}dx}{d^3}-\frac{bc\left(bc\int -\frac{8c^4x^4-12c^2x^2+3}{3x(1-c^2x^2)^2}dx+\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}\right)}{d^3}-\frac{(a+b\arccos(cx))^2}{2d^3x^2\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3c^2\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^3}dx}{d^3}-\frac{bc\left(-\frac{1}{3}bc\int \frac{8c^4x^4-12c^2x^2+3}{x(1-c^2x^2)^2}dx+\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}\right)}{d^3}-\frac{(a+b\arccos(cx))^2}{2d^3x^2\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{1578}$$

$$\frac{3c^2\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^3}dx}{d^3}-\frac{bc\left(-\frac{1}{6}bc\int \frac{8c^4x^4-12c^2x^2+3}{x^2(1-c^2x^2)^2}dx^2+\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}\right)}{d^3}-\frac{(a+b\arccos(cx))^2}{2d^3x^2\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{1195}$$

$$\frac{3c^2\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^3}dx}{d^3}-\frac{bc\left(-\frac{1}{6}bc\int \left(\frac{5c^2}{c^2x^2-1}-\frac{c^2}{(c^2x^2-1)^2}+\frac{3}{x^2}\right)dx^2+\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}\right)}{d^3}-\frac{(a+b\arccos(cx))^2}{2d^3x^2\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3c^2\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^3}dx}{d^3}-\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2}-\frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}-\frac{1}{6}bc\left(\frac{1}{c^2x^2-1}+5\log\left(1-c^2x^2\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$$\Big\downarrow \ 5209$$

$$\frac{3c^2\left(\frac{1}{2}bc\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{5/2}}dx+\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}dx+\frac{(a+b\arccos(cx))^2}{4(1-c^2x^2)^2}\right)}{d^3}-\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2}-\frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}-\frac{1}{6}bc\left(\frac{1}{c^2x^2-1}+5\log\left(1-c^2x^2\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$$\Big\downarrow \ 5163$$

$$\frac{3c^2\left(\frac{1}{2}bc\left(\frac{2}{3}\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{3/2}}dx+\frac{1}{3}bc\int \frac{x}{(1-c^2x^2)^2}dx+\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}\right)+\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}dx+\frac{(a+b\arccos(cx))^2}{4(1-c^2x^2)^2}\right)}{d^3}-\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2}-\frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}-\frac{1}{6}bc\left(\frac{1}{c^2x^2-1}+5\log\left(1-c^2x^2\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$$\Big\downarrow \ 241$$

$$\frac{3c^2\left(\frac{1}{2}bc\left(\frac{2}{3}\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{3/2}}dx+\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{b}{6c(1-c^2x^2)}\right)+\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}dx+\frac{(a+b\arccos(cx))^2}{4(1-c^2x^2)^2}\right)}{d^3}-\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2}-\frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}-\frac{1}{6}bc\left(\frac{1}{c^2x^2-1}+5\log\left(1-c^2x^2\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$$\Big\downarrow \ 5161$$

$$\frac{3c^2\left(\frac{1}{2}bc\left(\frac{2}{3}\left(bc\int \frac{x}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)+\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{b}{6c(1-c^2x^2)}\right)+\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}dx+\frac{(a+b\arccos(cx))^2}{4(1-c^2x^2)^2}\right)}{d^3}-\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2}-\frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}-\frac{1}{6}bc\left(\frac{1}{c^2x^2-1}+5\log\left(1-c^2x^2\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$$\Big\downarrow \ 240$$

$$\frac{3c^2\left(\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}dx + \frac{(a+b\arccos(cx))^2}{4(1-c^2x^2)^2} + \frac{1}{2}bc\left(\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3}\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}} - \frac{b\log(1-c^2x^2)}{2c}\right) + \frac{b}{6c(1-c^2x^2)}\right.\right.}{d^3}$$

$$\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2} - \frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}} - \frac{1}{6}bc\left(\frac{1}{c^2x^2-1} + 5\log(1-c^2x^2) + 3\log(x^2)\right)\right)}{d^3}$$

$$\Big\downarrow \text{ 5209}$$

$$\frac{3c^2\left(bc\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{3/2}}dx + \int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}dx + \frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)} + \frac{(a+b\arccos(cx))^2}{4(1-c^2x^2)^2} + \frac{1}{2}bc\left(\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}} + \frac{2}{3}\left(\frac{x(a+\right.\right.\right.}{d^3}$$

$$\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2} - \frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}} - \frac{1}{6}bc\left(\frac{1}{c^2x^2-1} + 5\log(1-c^2x^2) + 3\log(x^2)\right)\right)}{d^3}$$

$$\Big\downarrow \text{ 5161}$$

$$\frac{3c^2\left(bc\left(bc\int \frac{x}{1-c^2x^2}dx + \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right) + \int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}dx + \frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)} + \frac{(a+b\arccos(cx))^2}{4(1-c^2x^2)^2} + \frac{1}{2}bc\left(\frac{x(a+b\,\mathrm{ar}}{3(1-c^2}\right.\right.}{d^3}$$

$$\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2} - \frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}} - \frac{1}{6}bc\left(\frac{1}{c^2x^2-1} + 5\log(1-c^2x^2) + 3\log(x^2)\right)\right)}{d^3}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{3c^2\left(\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)}dx + \frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)} + \frac{(a+b\arccos(cx))^2}{4(1-c^2x^2)^2} + bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}} - \frac{b\log(1-c^2x^2)}{2c}\right) + \frac{1}{2}bc\left(\frac{x(a+b\,\mathrm{arcco}}{3(1-c^2x^2}\right.\right.}{d^3}$$

$$\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2} - \frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}} + \frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}} - \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}} - \frac{1}{6}bc\left(\frac{1}{c^2x^2-1} + 5\log(1-c^2x^2) + 3\log(x^2)\right)\right)}{d^3}$$

$$\Big\downarrow \text{ 5185}$$

$$\frac{3c^2\left(-\int \frac{(a+b\arccos(cx))^2}{cx\sqrt{1-c^2x^2}}d\arccos(cx)+\frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)}+\frac{(a+b\arccos(cx))^2}{4(1-c^2x^2)^2}+bc\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)+\frac{1}{2}b\right.}{d^3}$$

$$\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2}-$$

$$\frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}-\frac{1}{6}bc\left(\frac{1}{c^2x^2-1}+5\log\left(1-c^2x^2\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$\big\downarrow$ 4919

$$\frac{3c^2\left(-2\int (a+b\arccos(cx))^2\csc(2\arccos(cx))d\arccos(cx)+\frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)}+\frac{(a+b\arccos(cx))^2}{4(1-c^2x^2)^2}+bc\left(\frac{x(a+b\arccos(cx)}{\sqrt{1-c^2x^2}}\right.\right.}{d^3}$$

$$\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2}-$$

$$\frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}-\frac{1}{6}bc\left(\frac{1}{c^2x^2-1}+5\log\left(1-c^2x^2\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$\big\downarrow$ 3042

$$\frac{3c^2\left(-2\int (a+b\arccos(cx))^2\csc(2\arccos(cx))d\arccos(cx)+\frac{(a+b\arccos(cx))^2}{2(1-c^2x^2)}+\frac{(a+b\arccos(cx))^2}{4(1-c^2x^2)^2}+bc\left(\frac{x(a+b\arccos(cx)}{\sqrt{1-c^2x^2}}\right.\right.}{d^3}$$

$$\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2}-$$

$$\frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}-\frac{1}{6}bc\left(\frac{1}{c^2x^2-1}+5\log\left(1-c^2x^2\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$\big\downarrow$ 4671

$$\frac{3c^2\left(-2\left(-b\int (a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+b\int (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)d\right.\right.}{}$$

$$\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2}-$$

$$\frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}-\frac{1}{6}bc\left(\frac{1}{c^2x^2-1}+5\log\left(1-c^2x^2\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$\big\downarrow$ 3011

$$\frac{3c^2\left(-2\left(b\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-b\left(\frac{1}{2}\right.\right.\right.}{}$$

$$\frac{\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2}-}{}$$

$$\frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}-\frac{1}{6}bc\left(\frac{1}{c^2x^2-1}+5\log\left(1-c^2x^2\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$$\Big\downarrow \; 2720$$

$$\frac{3c^2\left(-2\left(b\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)de^{2i\,\text{arc}}\right.\right.\right.}{}$$

$$\frac{\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2}-}{}$$

$$\frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}-\frac{1}{6}bc\left(\frac{1}{c^2x^2-1}+5\log\left(1-c^2x^2\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

$$\Big\downarrow \; 7143$$

$$\frac{3c^2\left(-2\left(-\text{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2+b\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\text{Pol}\right.\right.\right.}{}$$

$$\frac{\frac{(a+b\arccos(cx))^2}{2d^3x^2(1-c^2x^2)^2}-}{}$$

$$\frac{bc\left(\frac{8c^2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{4c^2x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}-\frac{1}{6}bc\left(\frac{1}{c^2x^2-1}+5\log\left(1-c^2x^2\right)+3\log\left(x^2\right)\right)\right)}{d^3}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^3*(d - c^2*d*x^2)^3),x]
```

output
```
-1/2*(a + b*ArcCos[c*x])^2/(d^3*x^2*(1 - c^2*x^2)^2) - (b*c*(-((a + b*ArcC
os[c*x])/(x*(1 - c^2*x^2)^(3/2))) + (4*c^2*x*(a + b*ArcCos[c*x]))/(3*(1 -
c^2*x^2)^(3/2)) + (8*c^2*x*(a + b*ArcCos[c*x]))/(3*Sqrt[1 - c^2*x^2]) - (b
*c*((-1 + c^2*x^2)^(-1) + 3*Log[x^2] + 5*Log[1 - c^2*x^2]))/6))/d^3 + (3*c
^2*((a + b*ArcCos[c*x])^2/(4*(1 - c^2*x^2)^2) + (a + b*ArcCos[c*x])^2/(2*(
1 - c^2*x^2)) + b*c*((x*(a + b*ArcCos[c*x]))/Sqrt[1 - c^2*x^2] - (b*Log[1
- c^2*x^2])/(2*c)) + (b*c*(b/(6*c*(1 - c^2*x^2)) + (x*(a + b*ArcCos[c*x]))
/(3*(1 - c^2*x^2)^(3/2)) + (2*((x*(a + b*ArcCos[c*x]))/Sqrt[1 - c^2*x^2] -
 (b*Log[1 - c^2*x^2])/(2*c)))/3))/2 - 2*(-((a + b*ArcCos[c*x])^2*ArcTanh[E
^((2*I)*ArcCos[c*x])]) + b*((I/2)*(a + b*ArcCos[c*x])*PolyLog[2, -E^((2*I)
*ArcCos[c*x])] - (b*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/4) - b*((I/2)*(a +
 b*ArcCos[c*x])*PolyLog[2, E^((2*I)*ArcCos[c*x])] - (b*PolyLog[3, E^((2*I)
*ArcCos[c*x])])/4))))/d^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 5195
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos
[c*x])   u, x] + Simp[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[Sim
plifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m +
1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0])
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 934 vs. 2(432) = 864.

Time = 0.81 (sec) , antiderivative size = 935, normalized size of antiderivative = 2.32

method	result
derivativedivides	$c^2\left(-\frac{a^2\left(-\frac{1}{16(cx-1)^2}+\frac{9}{16(cx-1)}+\frac{3\ln(cx-1)}{2}+\frac{1}{2c^2x^2}-3\ln(cx)-\frac{1}{16(cx+1)^2}-\frac{9}{16(cx+1)}+\frac{3\ln(cx+1)}{2}\right)}{d^3}-\frac{b^2\left(\frac{16i\arccos\right.}{}}{}\right.$
default	$c^2\left(-\frac{a^2\left(-\frac{1}{16(cx-1)^2}+\frac{9}{16(cx-1)}+\frac{3\ln(cx-1)}{2}+\frac{1}{2c^2x^2}-3\ln(cx)-\frac{1}{16(cx+1)^2}-\frac{9}{16(cx+1)}+\frac{3\ln(cx+1)}{2}\right)}{d^3}-\frac{b^2\left(\frac{16i\arccos\right.}{}}{}\right.$
parts	$-\frac{a^2\left(\frac{1}{2x^2}-3c^2\ln(x)-\frac{c^2}{16(cx-1)^2}+\frac{9c^2}{16(cx-1)}+\frac{3c^2\ln(cx-1)}{2}-\frac{c^2}{16(cx+1)^2}-\frac{9c^2}{16(cx+1)}+\frac{3c^2\ln(cx+1)}{2}\right)}{d^3}-\frac{b^2c^2\left(\frac{16i\arccos\right.}{}}{}$

input
```
int((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-a^2/d^3*(-1/16/(c*x-1)^2+9/16/(c*x-1)+3/2*ln(c*x-1)+1/2/c^2/x^2-3*ln
(c*x)-1/16/(c*x+1)^2-9/16/(c*x+1)+3/2*ln(c*x+1))-b^2/d^3*(1/12/(c^4*x^4-2*
c^2*x^2+1)/c^2/x^2*(16*I*arccos(c*x)*c^6*x^6+16*(-c^2*x^2+1)^(1/2)*arccos(
c*x)*x^5*c^5+18*arccos(c*x)^2*x^4*c^4-32*I*arccos(c*x)*c^4*x^4-6*(-c^2*x^2
+1)^(1/2)*arccos(c*x)*x^3*c^3-27*arccos(c*x)^2*x^2*c^2+16*I*arccos(c*x)*c^
2*x^2+c^4*x^4-12*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c+6*arccos(c*x)^2-c^2*x^
2)-ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-8/3*ln(c*x+I*(-c^2*x^2+1)^(1/2))+7/3
*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+7/3*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)-3*arcco
s(c*x)^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3*I*arccos(c*x)*polylog(2,-(c*
x+I*(-c^2*x^2+1)^(1/2))^2)-3/2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3*
arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-6*I*arccos(c*x)*polylog(2,-c*
x-I*(-c^2*x^2+1)^(1/2))+6*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))+3*arccos(c*
x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-6*I*arccos(c*x)*polylog(2,c*x+I*(-c^2*
x^2+1)^(1/2))+6*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2)))-2*a*b/d^3*(1/12/(c^4*
x^4-2*c^2*x^2+1)/c^2/x^2*(8*I*x^6*c^6+8*c^5*x^5*(-c^2*x^2+1)^(1/2)+18*c^4*
x^4*arccos(c*x)-16*I*c^4*x^4-3*c^3*x^3*(-c^2*x^2+1)^(1/2)-27*c^2*x^2*arcco
s(c*x)+8*I*c^2*x^2-6*c*x*(-c^2*x^2+1)^(1/2)+6*arccos(c*x))+3*arccos(c*x)*l
n(1+c*x+I*(-c^2*x^2+1)^(1/2))-3*I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-3*a
rccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3/2*I*polylog(2,-(c*x+I*(-c^
2*x^2+1)^(1/2))^2)+3*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-3*I*pol...
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)^2}{(c^2dx^2-d)^3x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^6*d^3*x^9 - 3*c
^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^3}\,dx = -\frac{\int \frac{a^2}{c^6x^9-3c^4x^7+3c^2x^5-x^3}\,dx + \int \frac{b^2\operatorname{acos}^2(cx)}{c^6x^9-3c^4x^7+3c^2x^5-x^3}\,dx + \int \frac{2ab\operatorname{acos}(cx)}{c^6x^9-3c^4x^7+3c^2x^5-x^3}\,dx}{d^3}$$

input
```
integrate((a+b*acos(c*x))**2/x**3/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a**2/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3), x) + Integ
ral(b**2*acos(c*x)**2/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3), x) +
 Integral(2*a*b*acos(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3),
x))/d**3
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)^2}{(c^2dx^2-d)^3x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*a^2*((6*c^4*x^4 - 9*c^2*x^2 + 2)/(c^4*d^3*x^6 - 2*c^2*d^3*x^4 + d^3*x
^2) + 6*c^2*log(c*x + 1)/d^3 + 6*c^2*log(c*x - 1)/d^3 - 12*c^2*log(x)/d^3)
 - integrate((b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arc
tan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*
c^2*d^3*x^5 - d^3*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^3}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x^3\,(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^3*(d - c^2*d*x^2)^3),x)
```

output
```
int((a + b*acos(c*x))^2/(x^3*(d - c^2*d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^3}\,dx$$
$$= \frac{-8\left(\int \frac{acos(cx)}{c^6x^9-3c^4x^7+3c^2x^5-x^3}dx\right)ab\,c^4x^6+16\left(\int \frac{acos(cx)}{c^6x^9-3c^4x^7+3c^2x^5-x^3}dx\right)ab\,c^2x^4-8\left(\int \frac{acos(cx)}{c^6x^9-3c^4x^7+3c^2x^5-x^3}dx\right)}{}$$

input
```
int((a+b*acos(c*x))^2/x^3/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 8*int(acos(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3),x)*a*b*
c**4*x**6 + 16*int(acos(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3
),x)*a*b*c**2*x**4 - 8*int(acos(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**
5 - x**3),x)*a*b*x**2 - 4*int(acos(c*x)**2/(c**6*x**9 - 3*c**4*x**7 + 3*c*
*2*x**5 - x**3),x)*b**2*c**4*x**6 + 8*int(acos(c*x)**2/(c**6*x**9 - 3*c**4
*x**7 + 3*c**2*x**5 - x**3),x)*b**2*c**2*x**4 - 4*int(acos(c*x)**2/(c**6*x
**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3),x)*b**2*x**2 - 6*log(c**2*x - c)*a
**2*c**6*x**6 + 12*log(c**2*x - c)*a**2*c**4*x**4 - 6*log(c**2*x - c)*a**2
*c**2*x**2 - 6*log(c**2*x + c)*a**2*c**6*x**6 + 12*log(c**2*x + c)*a**2*c*
*4*x**4 - 6*log(c**2*x + c)*a**2*c**2*x**2 + 12*log(x)*a**2*c**6*x**6 - 24
*log(x)*a**2*c**4*x**4 + 12*log(x)*a**2*c**2*x**2 - 3*a**2*c**6*x**6 + 6*a
**2*c**2*x**2 - 2*a**2)/(4*d**3*x**2*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.211 $\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 572

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^3}\,dx = -\frac{b^2c^2}{2d^3x}+\frac{b^2c^2}{6d^3x(1-c^2x^2)}-\frac{b^2c^4x}{12d^3(1-c^2x^2)}+\frac{bc^3(a+b\arccos(cx))}{6d^3(1-c^2x^2)^{3/2}}-\frac{bc(a+b\arccos(cx))}{3d^3x^2(1-c^2x^2)^{3/2}}-\frac{29bc^3(a+b\arccos(cx))}{12d^3\sqrt{1-c^2x^2}}-\frac{(a+b\arccos(cx))^2}{3d^3x^3(1-c^2x^2)^2}-\frac{7c^2(a+b\arccos(cx))^2}{3d^3x(1-c^2x^2)^2}+\frac{35c^4x(a+b\arccos(cx))^2}{12d^3(1-c^2x^2)^2}+\frac{35c^4x(a+b\arccos(cx))^2}{8d^3(1-c^2x^2)}-\frac{35ic^3(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)}{4d^3}-\frac{38bc^3(a+b\arccos(cx))\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{3d^3}+\frac{17b^2c^3\operatorname{arctanh}(cx)}{6d^3}+\frac{19ib^2c^3\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{3d^3}+\frac{35ibc^3(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{4d^3}-\frac{35ibc^3(a+b\arccos(cx))\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{4d^3}-\frac{19ib^2c^3\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{3d^3}-\frac{35b^2c^3\operatorname{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)}{4d^3}+\frac{35b^2c^3\operatorname{PolyLog}\left(3,ie^{i\arccos(cx)}\right)}{4d^3}$$

output
```
-1/2*b^2*c^2/d^3/x+1/6*b^2*c^2/d^3/x/(-c^2*x^2+1)-1/12*b^2*c^4*x/d^3/(-c^2
*x^2+1)+1/6*b*c^3*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)^(3/2)-1/3*b*c*(a+b*ar
ccos(c*x))/d^3/x^2/(-c^2*x^2+1)^(3/2)-29/12*b*c^3*(a+b*arccos(c*x))/d^3/(-
c^2*x^2+1)^(1/2)-1/3*(a+b*arccos(c*x))^2/d^3/x^3/(-c^2*x^2+1)^2-7/3*c^2*(a
+b*arccos(c*x))^2/d^3/x/(-c^2*x^2+1)^2+35/12*c^4*x*(a+b*arccos(c*x))^2/d^3
/(-c^2*x^2+1)^2+35/8*c^4*x*(a+b*arccos(c*x))^2/d^3/(-c^2*x^2+1)+19/3*I*b^2
*c^3*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/d^3-38/3*b*c^3*(a+b*arccos(c*x))
*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/d^3+17/6*b^2*c^3*arctanh(c*x)/d^3-35/4*
I*c^3*(a+b*arccos(c*x))^2*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/d^3-19/3*I*b^2*
c^3*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/d^3-35/4*I*b*c^3*(a+b*arccos(c*x))
*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^3+35/4*I*b*c^3*(a+b*arccos(c*x)
)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^3-35/4*b^2*c^3*polylog(3,-I*(
c*x+I*(-c^2*x^2+1)^(1/2)))/d^3+35/4*b^2*c^3*polylog(3,I*(c*x+I*(-c^2*x^2+1
)^(1/2)))/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 9.24 (sec) , antiderivative size = 1135, normalized size of antiderivative = 1.98

$$\int \frac{(a+b\arccos(cx))^2}{x^4\left(d-c^2dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^4*(d - c^2*d*x^2)^3),x]
```

output
```
-1/3*a^2/(d^3*x^3) - (3*a^2*c^2)/(d^3*x) + (a^2*c^4*x)/(4*d^3*(-1 + c^2*x^
2)^2) - (11*a^2*c^4*x)/(8*d^3*(-1 + c^2*x^2)) - (35*a^2*c^3*Log[1 - c*x])/
(16*d^3) + (35*a^2*c^3*Log[1 + c*x])/(16*d^3) - (2*a*b*(-1/6*(c*Sqrt[1 - c
^2*x^2])/x^2 + (c^3*((-2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x]))/(48*(-
1 + c*x)^2) - (c^3*((2 + c*x)*Sqrt[1 - c^2*x^2] - 3*ArcCos[c*x]))/(48*(1 +
 c*x)^2) - (11*c^4*(Sqrt[1 - c^2*x^2] - ArcCos[c*x]))/(16*(c + c^2*x)) + A
rcCos[c*x]/(3*x^3) - (11*c^4*(Sqrt[1 - c^2*x^2] + ArcCos[c*x]))/(16*(c - c
^2*x)) + (c^3*Log[x])/6 - (c^3*Log[1 + Sqrt[1 - c^2*x^2]])/6 - 3*c^2*(-(Ar
cCos[c*x]/x) - c*Log[x] + c*Log[1 + Sqrt[1 - c^2*x^2]]) - (35*c^4*(((-1/2*
I)*ArcCos[c*x]^2)/c + (2*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])])/c - ((2*I
)*PolyLog[2, -E^(I*ArcCos[c*x])])/c))/16 - ((35*I)/32)*c^3*(ArcCos[c*x]*(A
rcCos[c*x] + (4*I)*Log[1 - E^(I*ArcCos[c*x])]) + 4*PolyLog[2, E^(I*ArcCos[
c*x])])))/d^3 - (b^2*c^3*(64 - (16*(-2 + ArcCos[c*x])*ArcCos[c*x])/(-1 + S
qrt[1 - c^2*x^2]) + 608*ArcCos[c*x]^2 - 272*ArcCos[c*x]*Cot[ArcCos[c*x]/2]
 - 2*(2 + 33*ArcCos[c*x]^2)*Csc[ArcCos[c*x]/2]^2 - 2*Sqrt[1 - c^2*x^2]*Arc
Cos[c*x]*Csc[ArcCos[c*x]/2]^4 - 3*ArcCos[c*x]^2*Csc[ArcCos[c*x]/2]^4 + 544
*Log[Tan[ArcCos[c*x]/2]] - 1216*(ArcCos[c*x]*(Log[1 - I*E^(I*ArcCos[c*x])]
 - Log[1 + I*E^(I*ArcCos[c*x])]) + I*(PolyLog[2, (-I)*E^(I*ArcCos[c*x])] -
 PolyLog[2, I*E^(I*ArcCos[c*x])])) + 840*(ArcCos[c*x]^2*(Log[1 - E^(I*ArcC
os[c*x])] - Log[1 + E^(I*ArcCos[c*x])]) + (2*I)*ArcCos[c*x]*(PolyLog[2,...
```

Rubi [A] (verified)

Time = 6.35 (sec) , antiderivative size = 773, normalized size of antiderivative = 1.35, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5205, 27, 5205, 253, 264, 219, 5163, 5163, 5165, 3042, 4671, 3011, 2720, 5183, 215, 219, 5209, 215, 219, 5209, 219, 5219, 3042, 4669, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^4\left(d-c^2dx^2\right)^3}dx$$

$$\Big\downarrow \text{ 5205}$$

$$\frac{7}{3}c^2\int \frac{(a+b\arccos(cx))^2}{d^3x^2\left(1-c^2x^2\right)^3}dx-\frac{2bc\int \frac{a+b\arccos(cx)}{x^3\left(1-c^2x^2\right)^{5/2}}dx}{3d^3}-\frac{(a+b\arccos(cx))^2}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{7c^2\int \frac{(a+b\arccos(cx))^2}{x^2(1-c^2x^2)^3}dx}{3d^3}-\frac{2bc\int \frac{a+b\arccos(cx)}{x^3(1-c^2x^2)^{5/2}}dx}{3d^3}-\frac{(a+b\arccos(cx))^2}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{5205}$$

$$\frac{7c^2\left(5c^2\int \frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^3}dx-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}\right)}{3d^3}-\frac{2bc\left(\frac{5}{2}c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}\right)}{3d^3}-\frac{(a+b\arccos(cx))^2}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{7c^2\left(5c^2\int \frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^3}dx-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}\right)}{3d^3}-\frac{2bc\left(\frac{5}{2}c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{1}{2}bc\left(\frac{3}{2}\int \frac{1}{x^2(1-c^2x^2)}dx+\frac{1}{2x(1-c^2x^2)}\right)-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}\right)}{3d^3}-\frac{(a+b\arccos(cx))^2}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{7c^2\left(5c^2\int \frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^3}dx-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}\right)}{3d^3}-\frac{2bc\left(\frac{5}{2}c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{1}{2}bc\left(\frac{3}{2}\left(c^2\int \frac{1}{1-c^2x^2}dx-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}\right)}{3d^3}-\frac{(a+b\arccos(cx))^2}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2bc\left(\frac{5}{2}c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}-\frac{1}{2}bc\left(\frac{3}{2}\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)\right)}{3d^3}+\frac{7c^2\left(5c^2\int \frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^3}dx-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^2}\right)}{3d^3}-\frac{(a+b\arccos(cx))^2}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{5163}$$

$$-\frac{2bc\left(\frac{5}{2}c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}-\frac{1}{2}bc\left(\frac{3}{2}\left(c\,\text{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)\right)}{3d^3}+\frac{7c^2\left(5c^2\left(\frac{1}{2}bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx+\frac{3}{4}\int \frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^2}dx+\frac{x(a+b\arccos(cx))^2}{4(1-c^2x^2)^2}\right)-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{(a+b\arccos(\ldots}{x(1-c^2x^2\ldots}}{3d^3}\ldots\frac{(a+b\arccos(cx))^2}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$\downarrow$ 5163

$$-\frac{2bc\left(\frac{5}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}-\frac{1}{2}bc\left(\frac{3}{2}\left(\text{carctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)\right)}{3d^3}+$$
$$\frac{7c^2\left(5c^2\left(\frac{1}{2}bc\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx+\frac{3}{4}\left(bc\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx+\frac{1}{2}\int\frac{(a+b\arccos(cx))^2}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}\right)+\frac{x(a+b\text{a}}{4(1-}\right.\right.}{3d^3}$$
$$\frac{(a+b\arccos(cx))^2}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$\downarrow$ 5165

$$-\frac{2bc\left(\frac{5}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}-\frac{1}{2}bc\left(\frac{3}{2}\left(\text{carctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)\right)}{3d^3}+$$
$$\frac{7c^2\left(5c^2\left(\frac{1}{2}bc\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx+\frac{3}{4}\left(bc\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx-\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}\right)+\frac{x}{}\right.\right.}{3d^3}$$
$$\frac{(a+b\arccos(cx))^2}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$\downarrow$ 3042

$$-\frac{2bc\left(\frac{5}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}-\frac{1}{2}bc\left(\frac{3}{2}\left(\text{carctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)\right)}{3d^3}+$$
$$\frac{7c^2\left(-2bc\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx+5c^2\left(\frac{1}{2}bc\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx+\frac{3}{4}\left(bc\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx-\frac{\int(a+b\arccos(cx))^2\csc(\arccos(}{2c}\right.\right.\right.}{3d^3}$$
$$\frac{(a+b\arccos(cx))^2}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$\downarrow$ 4671

$$-\frac{2bc\left(\frac{5}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}-\frac{1}{2}bc\left(\frac{3}{2}\left(\text{carctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)\right)}{3d^3}+$$
$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{-2b\int(a+b\arccos(cx))\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+2b\int(a+b\arccos(cx))\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}(e}{2c}\right.\right.\right.}{}$$
$$\frac{(a+b\arccos(cx))^2}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$\downarrow$ 3011

$$7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{2b(i\,\text{PolyLog}(2,-e^{i\arccos(cx)})(a+b\arccos(cx))-ib\int\text{PolyLog}(2,-e^{i\arccos(cx)})\,d\arccos(cx))-2b(i\,\text{PolyLog}(2,e^{i\arccos(cx)})(a\cdots}{2c}\right.\right.\right.$$

$$\frac{2bc\left(\frac{5}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}-\frac{1}{2}bc\left(\frac{3}{2}\left(\text{carctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)\right)}{3d^3}-\frac{(a+b\arccos(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

$\Big\downarrow$ 2720

$$7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{2b(i\,\text{PolyLog}(2,-e^{i\arccos(cx)})(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}(2,-e^{i\arccos(cx)})\,de^{i\arccos(cx)})-2b(i\,\text{PolyLog}(2,e^{i\cdots}}{\cdots}\right.\right.\right.$$

$$\frac{2bc\left(\frac{5}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}-\frac{1}{2}bc\left(\frac{3}{2}\left(\text{carctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)\right)}{3d^3}-\frac{(a+b\arccos(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

$\Big\downarrow$ 5183

$$7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{2b(i\,\text{PolyLog}(2,-e^{i\arccos(cx)})(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}(2,-e^{i\arccos(cx)})\,de^{i\arccos(cx)})-2b(i\,\text{PolyLog}(2,e\cdots}{\cdots}\right.\right.\right.$$

$$\frac{2bc\left(\frac{5}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}-\frac{1}{2}bc\left(\frac{3}{2}\left(\text{carctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)\right)}{3d^3}-\frac{(a+b\arccos(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

$\Big\downarrow$ 215

$$7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{2b(i\,\text{PolyLog}(2,-e^{i\arccos(cx)})(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}(2,-e^{i\arccos(cx)})\,de^{i\arccos(cx)})-2b(i\,\text{PolyLog}(2,e\cdots}{\cdots}\right.\right.\right.$$

$$\frac{2bc\left(\frac{5}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}-\frac{1}{2}bc\left(\frac{3}{2}\left(\text{carctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)\right)}{3d^3}-\frac{(a+b\arccos(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

$\Big\downarrow$ 219

$$7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\operatorname{PolyLog}\left(2,e\right.\right.}{}\right.\right.\right.\right.$$

$$\frac{2bc\left(\frac{5}{2}c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{5/2}}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}-\frac{1}{2}bc\left(\frac{3}{2}\left(c\operatorname{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x(1-c^2x^2)}\right)\right)}{3d^3}-\frac{(a+b\arccos(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{5209}$$

$$7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\operatorname{PolyLog}\left(2,e\right.\right.}{}\right.\right.\right.\right.$$

$$\frac{2bc\left(\frac{5}{2}c^2\left(\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx+\frac{1}{3}bc\int\frac{1}{(1-c^2x^2)^2}dx+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}\right)-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}-\frac{1}{2}bc\left(\frac{3}{2}\left(c\operatorname{arctanh}(cx)-\frac{1}{x}\right)+\frac{1}{2x\ldots}\right.\right.}{3d^3}$$

$$\frac{(a+b\arccos(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{215}$$

$$7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\operatorname{PolyLog}\left(2,e\right.\right.}{}\right.\right.\right.\right.$$

$$\frac{2bc\left(\frac{5}{2}c^2\left(\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx+\frac{1}{3}bc\left(\frac{1}{2}\int\frac{1}{1-c^2x^2}dx+\frac{x}{2(1-c^2x^2)}\right)+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}\right)-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}-\frac{1}{2}bc\left(\frac{3}{2}\left(c\operatorname{arctanh}\right.\right.\right.}{3d^3}$$

$$\frac{(a+b\arccos(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e\right.\right.}{}\right.\right.\right.}{\frac{2bc\left(\frac{5}{2}c^2\left(\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^{3/2}}dx+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}+\frac{1}{3}bc\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)\right)-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)^{3/2}}-\frac{1}{2}bc\left(\frac{3}{2}\left(c\,\text{arctanh}(\right.\right.\right.}{3d^3}}$$

$$\frac{(a+b\arccos(cx))^2}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$\downarrow$ 5209

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e\right.\right.}{}\right.\right.\right.}{\frac{2bc\left(\frac{5}{2}c^2\left(\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+bc\int\frac{1}{1-c^2x^2}dx+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}+\frac{1}{3}bc\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)\right)-\frac{a+}{2x^2}\right.}{3d^3}}$$

$$\frac{(a+b\arccos(cx))^2}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$\downarrow$ 219

$$\frac{7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{2b\left(i\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\,\text{PolyLog}\left(2,e\right.\right.}{}\right.\right.\right.}{\frac{2bc\left(\frac{5}{2}c^2\left(\int\frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}dx+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}+\frac{1}{3}bc\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)+b\,\text{arctanh}(cx)\right)-\frac{a+}{2x^2}\right.}{3d^3}}$$

$$\frac{(a+b\arccos(cx))^2}{3d^3x^3\left(1-c^2x^2\right)^2}$$

$\downarrow$ 5219

$$7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{2b(i\,\text{PolyLog}(2,-e^{i\arccos(cx)})(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}(2,-e^{i\arccos(cx)})de^{i\arccos(cx)})-2b(i\,\text{PolyLog}(2,e\right.\right.\right.$$

$$\frac{2bc\left(\frac{5}{2}c^2\left(-\int\frac{a+b\arccos(cx)}{cx}d\arccos(cx)+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}+\frac{1}{3}bc\left(\frac{\text{arctanh}(cx)}{2c}+\frac{x}{2(1-c^2x^2)}\right)+b\text{arctanh}\right.\right.}{3d^3}$$

$$\frac{(a+b\arccos(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

$\downarrow$ 3042

$$7c^2\left(5c^2\left(\frac{3}{4}\left(-\frac{2b(i\,\text{PolyLog}(2,-e^{i\arccos(cx)})(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}(2,-e^{i\arccos(cx)})de^{i\arccos(cx)})-2b(i\,\text{PolyLog}(2,e\right.\right.\right.$$

$$\frac{2bc\left(\frac{5}{2}c^2\left(-\int(a+b\arccos(cx))\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}+\frac{1}{3}bc\left(\frac{\text{arctanh}(c}{2c}\right.\right.\right.}{3d^3}$$

$$\frac{(a+b\arccos(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

$\downarrow$ 4669

$$7\left(5\left(\frac{x(a+b\arccos(cx))^2}{4(1-c^2x^2)^2}+\frac{1}{2}bc\left(\frac{a+b\arccos(cx)}{3c^2(1-c^2x^2)^{3/2}}+\frac{b\left(\frac{x}{2(1-c^2x^2)}+\frac{\text{arctanh}(cx)}{2c}\right)}{3c}\right)+\frac{3}{4}\left(\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}+bc\left(\frac{a+b\arccos(c}{c^2\sqrt{1-c^2x^2}}\right.\right.\right.\right.$$

$$2b\left(\frac{5}{2}\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}+b\text{arctanh}(cx)+\frac{1}{3}bc\left(\frac{x}{2(1-c^2x^2)}+\right.\right.\right.$$

$$\frac{(a+b\arccos(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

$\downarrow$ 2715

$$\frac{7\left(5\left(\frac{x(a+b\arccos(cx))^2}{4(1-c^2x^2)^2}+\frac{1}{2}bc\left(\frac{a+b\arccos(cx)}{3c^2(1-c^2x^2)^{3/2}}+\frac{b\left(\frac{x}{2(1-c^2x^2)}+\frac{\text{arctanh}(cx)}{2c}\right)}{3c}\right)\right)+\frac{3}{4}\left(\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}+bc\left(\frac{a+b\arccos(c}{c^2\sqrt{1-c^2x^2}}\right.\right.}{2b\left(\frac{5}{2}\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}+b\text{arctanh}(cx)+\frac{1}{3}bc\left(\frac{x}{2(1-c^2x^2)}+\right.\right.\right.}$$

$$\frac{(a+b\arccos(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

$\downarrow$ 2838

$$\frac{7\left(5\left(\frac{x(a+b\arccos(cx))^2}{4(1-c^2x^2)^2}+\frac{1}{2}bc\left(\frac{a+b\arccos(cx)}{3c^2(1-c^2x^2)^{3/2}}+\frac{b\left(\frac{x}{2(1-c^2x^2)}+\frac{\text{arctanh}(cx)}{2c}\right)}{3c}\right)\right)+\frac{3}{4}\left(\frac{x(a+b\arccos(cx))^2}{2(1-c^2x^2)}+bc\left(\frac{a+b\arccos(c}{c^2\sqrt{1-c^2x^2}}\right.\right.}{2b\left(\frac{5}{2}\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}+b\text{arctanh}(cx)+\frac{1}{3}bc\left(\frac{x}{2(1-c^2x^2)}+\right.\right.\right.}$$

$$\frac{(a+b\arccos(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

$\downarrow$ 7143

$$\frac{7c^2\left(-2bc\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}-ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)\right.\right.}{2bc\left(\frac{5}{2}c^2\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{3(1-c^2x^2)^{3/2}}-ib\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-\right.\right.}$$

$$\frac{(a+b\arccos(cx))^2}{3d^3x^3(1-c^2x^2)^2}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^4*(d - c^2*d*x^2)^3),x]
```

output
```
-1/3*(a + b*ArcCos[c*x])^2/(d^3*x^3*(1 - c^2*x^2)^2) - (2*b*c*(-1/2*(a + b
*ArcCos[c*x])/(x^2*(1 - c^2*x^2)^(3/2)) - (b*c*(1/(2*x*(1 - c^2*x^2)) + (3
*(-x^(-1) + c*ArcTanh[c*x]))/2))/2 + (5*c^2*((a + b*ArcCos[c*x])/(3*(1 - c
^2*x^2)^(3/2)) + (a + b*ArcCos[c*x])/Sqrt[1 - c^2*x^2] + (2*I)*(a + b*ArcC
os[c*x])*ArcTan[E^(I*ArcCos[c*x])] + b*ArcTanh[c*x] + (b*c*(x/(2*(1 - c^2*
x^2)) + ArcTanh[c*x]/(2*c)))/3 - I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] +
I*b*PolyLog[2, I*E^(I*ArcCos[c*x])]))/2))/(3*d^3) + (7*c^2*(-((a + b*ArcCo
s[c*x])^2/(x*(1 - c^2*x^2)^2)) - 2*b*c*((a + b*ArcCos[c*x])/(3*(1 - c^2*x^
2)^(3/2)) + (a + b*ArcCos[c*x])/Sqrt[1 - c^2*x^2] + (2*I)*(a + b*ArcCos[c*
x])*ArcTan[E^(I*ArcCos[c*x])] + b*ArcTanh[c*x] + (b*c*(x/(2*(1 - c^2*x^2))
 + ArcTanh[c*x]/(2*c)))/3 - I*b*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I*b*P
olyLog[2, I*E^(I*ArcCos[c*x])]) + 5*c^2*((x*(a + b*ArcCos[c*x])^2)/(4*(1 -
 c^2*x^2)^2) + (b*c*((a + b*ArcCos[c*x])/(3*c^2*(1 - c^2*x^2)^(3/2)) + (b*
(x/(2*(1 - c^2*x^2)) + ArcTanh[c*x]/(2*c)))/(3*c)))/2 + (3*((x*(a + b*ArcC
os[c*x])^2)/(2*(1 - c^2*x^2)) + b*c*((a + b*ArcCos[c*x])/(c^2*Sqrt[1 - c^2
*x^2]) + (b*ArcTanh[c*x])/c^2) - (-2*(a + b*ArcCos[c*x])^2*ArcTanh[E^(I*Ar
cCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, -E^(I*ArcCos[c*x])] -
b*PolyLog[3, -E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2,
E^(I*ArcCos[c*x])] - b*PolyLog[3, E^(I*ArcCos[c*x])]))/(2*c)))/4)))/(3*d^3
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 820, normalized size of antiderivative = 1.43

method	result
derivativedivides	$c^3\left(-\frac{a^2\left(-\frac{1}{16(cx-1)^2}+\frac{11}{16(cx-1)}+\frac{35\ln(cx-1)}{16}+\frac{1}{3c^3x^3}+\frac{3}{cx}+\frac{1}{16(cx+1)^2}+\frac{11}{16(cx+1)}-\frac{35\ln(cx+1)}{16}\right)}{d^3}-\frac{b^2\left(\frac{105\arccos(c\right.}{}\right.$
default	$c^3\left(-\frac{a^2\left(-\frac{1}{16(cx-1)^2}+\frac{11}{16(cx-1)}+\frac{35\ln(cx-1)}{16}+\frac{1}{3c^3x^3}+\frac{3}{cx}+\frac{1}{16(cx+1)^2}+\frac{11}{16(cx+1)}-\frac{35\ln(cx+1)}{16}\right)}{d^3}-\frac{b^2\left(\frac{105\arccos(c\right.}{}\right.$
parts	$-\frac{a^2\left(\frac{1}{3x^3}+\frac{3c^2}{x}-\frac{c^3}{16(cx-1)^2}+\frac{11c^3}{16(cx-1)}+\frac{35c^3\ln(cx-1)}{16}+\frac{c^3}{16(cx+1)^2}+\frac{11c^3}{16(cx+1)}-\frac{35c^3\ln(cx+1)}{16}\right)}{d^3}-\frac{b^2c^3\left(\frac{105\arccos(cx)\right.}{}$

input
```
int((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-a^2/d^3*(-1/16/(c*x-1)^2+11/16/(c*x-1)+35/16*ln(c*x-1)+1/3/c^3/x^3+3
/c/x+1/16/(c*x+1)^2+11/16/(c*x+1)-35/16*ln(c*x+1))-b^2/d^3*(1/24*(105*arcc
os(c*x)^2*c^6*x^6+58*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^5*c^5-175*arccos(c*x
)^2*x^4*c^4+10*c^6*x^6-54*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^3*c^3+56*arccos
(c*x)^2*x^2*c^2-18*c^4*x^4-8*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c+8*arccos(c
*x)^2+8*c^2*x^2)/(c^4*x^4-2*c^2*x^2+1)/c^3/x^3-35/4*I*arccos(c*x)*polylog(
2,c*x+I*(-c^2*x^2+1)^(1/2))-35/8*arccos(c*x)^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/
2))+35/4*I*arccos(c*x)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+35/8*arccos(c*
x)^2*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-19/3*I*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(
1/2)))+19/3*I*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+19/3*arccos(c*x)*ln(1+
I*(c*x+I*(-c^2*x^2+1)^(1/2)))-19/3*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^
(1/2)))-17/6*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+17/6*ln(I*(-c^2*x^2+1)^(1/2)+c
*x-1)-35/4*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))+35/4*polylog(3,c*x+I*(-c^2
*x^2+1)^(1/2)))-2*a*b/d^3*(1/24*(105*arccos(c*x)*c^6*x^6+29*c^5*x^5*(-c^2*
x^2+1)^(1/2)-175*c^4*x^4*arccos(c*x)-27*c^3*x^3*(-c^2*x^2+1)^(1/2)+56*c^2*
x^2*arccos(c*x)-4*c*x*(-c^2*x^2+1)^(1/2)+8*arccos(c*x))/(c^4*x^4-2*c^2*x^2
+1)/c^3/x^3+35/8*I*dilog(1+c*x+I*(-c^2*x^2+1)^(1/2))+19/3*I*arctan(c*x+I*(
-c^2*x^2+1)^(1/2))+35/8*I*dilog(c*x+I*(-c^2*x^2+1)^(1/2))-35/8*arccos(c*x)
*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4\left(d-c^2dx^2\right)^3}\,dx=\int -\frac{\left(b\arccos\left(cx\right)+a\right)^2}{\left(c^2dx^2-d\right)^3x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(c^6*d^3*x^10 - 3*
c^4*d^3*x^8 + 3*c^2*d^3*x^6 - d^3*x^4), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4\left(d-c^2dx^2\right)^3}\,dx$$
$$=-\frac{\int \frac{a^2}{c^6x^{10}-3c^4x^8+3c^2x^6-x^4}\,dx+\int \frac{b^2\operatorname{acos}^2\left(cx\right)}{c^6x^{10}-3c^4x^8+3c^2x^6-x^4}\,dx+\int \frac{2ab\operatorname{acos}\left(cx\right)}{c^6x^{10}-3c^4x^8+3c^2x^6-x^4}\,dx}{d^3}$$

input
```
integrate((a+b*acos(c*x))**2/x**4/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a**2/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4), x) + Inte
gral(b**2*acos(c*x)**2/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4), x)
 + Integral(2*a*b*acos(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4
), x))/d**3
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4\left(d-c^2dx^2\right)^3}\,dx=\int -\frac{\left(b\arccos\left(cx\right)+a\right)^2}{\left(c^2dx^2-d\right)^3x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/48*a^2*(105*c^3*log(c*x + 1)/d^3 - 105*c^3*log(c*x - 1)/d^3 - 2*(105*c^6
*x^6 - 175*c^4*x^4 + 56*c^2*x^2 + 8)/(c^4*d^3*x^7 - 2*c^2*d^3*x^5 + d^3*x^
3)) - 1/48*((210*b^2*c^6*x^6 - 350*b^2*c^4*x^4 + 112*b^2*c^2*x^2 + 16*b^2
- 105*(b^2*c^7*x^7 - 2*b^2*c^5*x^5 + b^2*c^3*x^3)*log(c*x + 1) + 105*(b^2*
c^7*x^7 - 2*b^2*c^5*x^5 + b^2*c^3*x^3)*log(-c*x + 1))*arctan2(sqrt(c*x + 1
)*sqrt(-c*x + 1), c*x)^2 + 48*(c^4*d^3*x^7 - 2*c^2*d^3*x^5 + d^3*x^3)*inte
grate(-1/24*((210*b^2*c^7*x^7 - 350*b^2*c^5*x^5 + 112*b^2*c^3*x^3 + 16*b^2
*c*x - 105*(b^2*c^8*x^8 - 2*b^2*c^6*x^6 + b^2*c^4*x^4)*log(c*x + 1) + 105*
(b^2*c^8*x^8 - 2*b^2*c^6*x^6 + b^2*c^4*x^4)*log(-c*x + 1))*sqrt(c*x + 1)*s
qrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - 48*a*b*arctan2(
sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/(c^6*d^3*x^10 - 3*c^4*d^3*x^8 + 3*c^2*
d^3*x^6 - d^3*x^4), x))/(c^4*d^3*x^7 - 2*c^2*d^3*x^5 + d^3*x^3)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^3}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x^4\,(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^4*(d - c^2*d*x^2)^3),x)
```

output
```
int((a + b*acos(c*x))^2/(x^4*(d - c^2*d*x^2)^3), x)
```

Reduce [F]

$$\int \frac{(a + b \arccos(cx))^2}{x^4 (d - c^2 d x^2)^3} \, dx$$
$$= \frac{-96\left(\int \frac{acos(cx)}{c^6x^{10}-3c^4x^8+3c^2x^6-x^4}dx\right)ab\,c^4x^7 + 192\left(\int \frac{acos(cx)}{c^6x^{10}-3c^4x^8+3c^2x^6-x^4}dx\right)ab\,c^2x^5 - 96\left(\int \frac{acos(cx)}{c^6x^{10}-3c^4x^8+3c^2x^6-}\right.}{}$$

input
```
int((a+b*acos(c*x))^2/x^4/(-c^2*d*x^2+d)^3,x)
```

output
```
( - 96*int(acos(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4),x)*a*
b*c**4*x**7 + 192*int(acos(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 -
x**4),x)*a*b*c**2*x**5 - 96*int(acos(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c*
*2*x**6 - x**4),x)*a*b*x**3 - 48*int(acos(c*x)**2/(c**6*x**10 - 3*c**4*x**
8 + 3*c**2*x**6 - x**4),x)*b**2*c**4*x**7 + 96*int(acos(c*x)**2/(c**6*x**1
0 - 3*c**4*x**8 + 3*c**2*x**6 - x**4),x)*b**2*c**2*x**5 - 48*int(acos(c*x)
**2/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4),x)*b**2*x**3 - 105*log
(c**2*x - c)*a**2*c**7*x**7 + 210*log(c**2*x - c)*a**2*c**5*x**5 - 105*log
(c**2*x - c)*a**2*c**3*x**3 + 105*log(c**2*x + c)*a**2*c**7*x**7 - 210*log
(c**2*x + c)*a**2*c**5*x**5 + 105*log(c**2*x + c)*a**2*c**3*x**3 - 210*a**
2*c**6*x**6 + 350*a**2*c**4*x**4 - 112*a**2*c**2*x**2 - 16*a**2)/(48*d**3*
x**3*(c**4*x**4 - 2*c**2*x**2 + 1))
```

3.212 $\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 374

$$\begin{aligned}\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx &= \frac{52b^2\sqrt{d-c^2dx^2}}{225c^4}+\frac{4abx\sqrt{d-c^2dx^2}}{15c^3\sqrt{1-c^2x^2}}\\ &+\frac{26b^2(1-c^2x^2)\sqrt{d-c^2dx^2}}{675c^4}\\ &-\frac{2b^2(1-c^2x^2)^2\sqrt{d-c^2dx^2}}{125c^4}\\ &+\frac{4b^2x\sqrt{d-c^2dx^2}\arccos(cx)}{15c^3\sqrt{1-c^2x^2}}\\ &+\frac{2bx^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{45c\sqrt{1-c^2x^2}}\\ &-\frac{2bcx^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{25\sqrt{1-c^2x^2}}\\ &-\frac{2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{15c^4}\\ &-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{15c^2}\\ &+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\end{aligned}$$

output
```
52/225*b^2*(-c^2*d*x^2+d)^(1/2)/c^4+4/15*a*b*x*(-c^2*d*x^2+d)^(1/2)/c^3/(-
c^2*x^2+1)^(1/2)+26/675*b^2*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/c^4-2/125*b^
2*(-c^2*x^2+1)^2*(-c^2*d*x^2+d)^(1/2)/c^4+4/15*b^2*x*(-c^2*d*x^2+d)^(1/2)*
arccos(c*x)/c^3/(-c^2*x^2+1)^(1/2)+2/45*b*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*ar
ccos(c*x))/c/(-c^2*x^2+1)^(1/2)-2/25*b*c*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arc
cos(c*x))/(-c^2*x^2+1)^(1/2)-2/15*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2
/c^4-1/15*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/c^2+1/5*x^4*(-c^2*d
*x^2+d)^(1/2)*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.61

$$\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$$
$$=\frac{\sqrt{d-c^2dx^2}\Big(225a^2(-1+c^2x^2)^2\,(2+3c^2x^2)-30abcx\sqrt{1-c^2x^2}(-30-5c^2x^2+9c^4x^4)+2b^2(-428+43}{}$$

input
```
Integrate[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(225*a^2*(-1 + c^2*x^2)^2*(2 + 3*c^2*x^2) - 30*a*b*c*
x*Sqrt[1 - c^2*x^2]*(-30 - 5*c^2*x^2 + 9*c^4*x^4) + 2*b^2*(-428 + 439*c^2*
x^2 + 16*c^4*x^4 - 27*c^6*x^6) + 30*b*(15*a*(-1 + c^2*x^2)^2*(2 + 3*c^2*x^
2) + b*c*x*Sqrt[1 - c^2*x^2]*(30 + 5*c^2*x^2 - 9*c^4*x^4))*ArcCos[c*x] + 2
25*b^2*(-1 + c^2*x^2)^2*(2 + 3*c^2*x^2)*ArcCos[c*x]^2))/(3375*c^4*(-1 + c^
2*x^2))
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {5199, 5139, 243, 53, 2009, 5211, 5139, 243, 53, 2009, 5183, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \ 5199$$

$$\frac{2bc\sqrt{d-c^2dx^2}\int x^4(a+b\arccos(cx))dx}{5\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 5139$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}bc\int \frac{x^5}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))\right)}{5\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 243$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{10}bc\int \frac{x^4}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{5}x^5(a+b\arccos(cx))\right)}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 53$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{10}bc\int \left(\frac{(1-c^2x^2)^{3/2}}{c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{1}{c^4\sqrt{1-c^2x^2}}\right)dx^2+\frac{1}{5}x^5(a+b\arccos(cx))\right)}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \ 5211$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\int x^2(a+b\arccos(cx))dx}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}\right)}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5139}$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\left(\frac{1}{3}bc\int \frac{x^3}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}\right)}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\left(\frac{1}{6}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}\right)}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\left(\frac{1}{6}bc\int \left(\frac{1}{c^2\sqrt{1-c^2x^2}}-\frac{\sqrt{1-c^2x^2}}{c^2}\right)dx^2+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}\right)}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\arccos(cx))+\frac{1}{6}bc\left(\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)\right)}{3c}\right.}{5\sqrt{1-c^2x^2}}$$

$$\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 5183$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{2\left(-\frac{2b\int(a+b\arccos(cx))dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}\right)}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\arccos(cx))+\frac{1}{6}bc\left(\right.\right.}{3c}\right.}{5\sqrt{1-c^2x^2}}$$

$$\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5\sqrt{1-c^2x^2}}+$$

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}+\frac{2\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}-\frac{2b\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c}\right)}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\arccos\right.}{}\right.}{5\sqrt{1-c^2x^2}}$$

input

```
Int[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/5 + (2*b*c*Sqrt[d - c^2*d*
x^2]*((b*c*((-2*Sqrt[1 - c^2*x^2])/c^6 + (4*(1 - c^2*x^2)^(3/2))/(3*c^6) -
 (2*(1 - c^2*x^2)^(5/2))/(5*c^6)))/10 + (x^5*(a + b*ArcCos[c*x]))/5))/(5*S
qrt[1 - c^2*x^2]) + (Sqrt[d - c^2*d*x^2]*(-1/3*(x^2*Sqrt[1 - c^2*x^2]*(a +
 b*ArcCos[c*x])^2)/c^2 - (2*b*((b*c*((-2*Sqrt[1 - c^2*x^2])/c^4 + (2*(1 -
c^2*x^2)^(3/2))/(3*c^4)))/6 + (x^3*(a + b*ArcCos[c*x]))/3))/(3*c) + (2*(-(
(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c^2) - (2*b*(a*x - (b*Sqrt[1 - c
^2*x^2])/c + b*x*ArcCos[c*x]))/c))/(3*c^2)))/(5*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.34

method	result
orering	$\frac{\left(1647c^8x^8-2131c^6x^6-8610c^4x^4+13060c^2x^2-5136\right)\sqrt{-c^2d\,x^2+d}\,(a+b\arccos(cx))^2}{3375(c^2x^2-1)c^6x^2}-\frac{4\left(81c^6x^6-40c^4x^4-878c^2x^2+642\right)\left(3x^2\sqrt{\right.}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3375*(1647*c^8*x^8-2131*c^6*x^6-8610*c^4*x^4+13060*c^2*x^2-5136)/(c^2*x^
2-1)/c^6/x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2-4/3375*(81*c^6*x^6-4
0*c^4*x^4-878*c^2*x^2+642)/c^6/x^4*(3*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos
(c*x))^2-x^4/(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2*d*c^2-2*b*c*x^3*(-c^
2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2))+1/3375*(27*c^4*x^4+
11*c^2*x^2-428)/c^6/x^3*(c*x-1)*(c*x+1)*(6*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arc
cos(c*x))^2-7*x^3/(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2*d*c^2-12*b*c*x^
2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-x^5/(-c^2*d*x^
2+d)^(3/2)*(a+b*arccos(c*x))^2*d^2*c^4+4*x^4/(-c^2*d*x^2+d)^(1/2)*(a+b*arc
cos(c*x))*d*c^3*b/(-c^2*x^2+1)^(1/2)+2*b^2*c^2*x^3*(-c^2*d*x^2+d)^(1/2)/(-
c^2*x^2+1)-2*b*c^3*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)
^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx =$$
$$-\frac{30\left(9\,abc^5x^5-5\,abc^3x^3-30\,abcx+\left(9\,b^2c^5x^5-5\,b^2c^3x^3-30\,b^2cx\right)\arccos\left(cx\right)\right)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2}}{}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="frica
s")
```

output
```
-1/3375*(30*(9*a*b*c^5*x^5 - 5*a*b*c^3*x^3 - 30*a*b*c*x + (9*b^2*c^5*x^5 -
 5*b^2*c^3*x^3 - 30*b^2*c*x)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x
^2 + 1) - (27*(25*a^2 - 2*b^2)*c^6*x^6 - 4*(225*a^2 - 8*b^2)*c^4*x^4 - (22
5*a^2 - 878*b^2)*c^2*x^2 + 225*(3*b^2*c^6*x^6 - 4*b^2*c^4*x^4 - b^2*c^2*x^
2 + 2*b^2)*arccos(c*x)^2 + 450*a^2 - 856*b^2 + 450*(3*a*b*c^6*x^6 - 4*a*b*
c^4*x^4 - a*b*c^2*x^2 + 2*a*b)*arccos(c*x))*sqrt(-c^2*d*x^2 + d))/(c^6*x^2
 - c^4)
```

Sympy [F]

$$\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int x^3\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))^2\;dx$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.83

$$\begin{aligned}&\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx\\&=-\frac{1}{15}\,b^2\left(\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\arccos\left(cx\right)^2\\&\quad-\frac{2}{15}\,ab\left(\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\arccos\left(cx\right)\\&\quad-\frac{1}{15}\,a^2\left(\frac{3\,(-c^2dx^2+d)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}}{c^4d}\right)\\&\quad-\frac{2}{3375}\,b^2\left(\frac{27\,\sqrt{-c^2x^2+1}c^2\sqrt{d}x^4+11\,\sqrt{-c^2x^2+1}\sqrt{d}x^2-\frac{428\,\sqrt{-c^2x^2+1}\sqrt{d}}{c^2}}{c^2}-\frac{15\left(9\,c^4\sqrt{d}x^5-5\,c^2\sqrt{d}x^3\right.}{c^3}\right.\\&\quad+\frac{2\left(9\,c^4\sqrt{d}x^5-5\,c^2\sqrt{d}x^3-30\,\sqrt{d}x\right)ab}{225\,c^3}\end{aligned}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="maxim
a")
```

output
```
-1/15*b^2*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)
/(c^4*d))*arccos(c*x)^2 - 2/15*a*b*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) +
 2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d))*arccos(c*x) - 1/15*a^2*(3*(-c^2*d*x^2 +
 d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d)) - 2/3375*b^2*((2
7*sqrt(-c^2*x^2 + 1)*c^2*sqrt(d)*x^4 + 11*sqrt(-c^2*x^2 + 1)*sqrt(d)*x^2 -
 428*sqrt(-c^2*x^2 + 1)*sqrt(d)/c^2)/c^2 - 15*(9*c^4*sqrt(d)*x^5 - 5*c^2*s
qrt(d)*x^3 - 30*sqrt(d)*x)*arccos(c*x)/c^3) + 2/225*(9*c^4*sqrt(d)*x^5 - 5
*c^2*sqrt(d)*x^3 - 30*sqrt(d)*x)*a*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx = \int x^3\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^3*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^3*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$$
$$=\frac{\sqrt{d}\,\left(3\sqrt{-c^2x^2+1}\,a^2c^4x^4-\sqrt{-c^2x^2+1}\,a^2c^2x^2-2\sqrt{-c^2x^2+1}\,a^2+30\left(\int\sqrt{-c^2x^2+1}\,acos(cx)\,x^3dx\right)a}{15c^4}$$

input
```
int(x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*(3*sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - sqrt( - c**2*x**2 + 1)
*a**2*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a**2 + 30*int(sqrt( - c**2*x**2
 + 1)*acos(c*x)*x**3,x)*a*b*c**4 + 15*int(sqrt( - c**2*x**2 + 1)*acos(c*x)
**2*x**3,x)*b**2*c**4))/(15*c**4)
```

3.213 $\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 303

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\frac{b^2x\sqrt{d-c^2dx^2}}{64c^2}-\frac{1}{32}b^2x^3\sqrt{d-c^2dx^2}-\frac{b^2\sqrt{d-c^2dx^2}\arccos(cx)}{64c^3\sqrt{1-c^2x^2}}+\frac{bx^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{8c\sqrt{1-c^2x^2}}-\frac{bcx^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{8\sqrt{1-c^2x^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{8c^2}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{24bc^3\sqrt{1-c^2x^2}}$$

output
```
1/64*b^2*x*(-c^2*d*x^2+d)^(1/2)/c^2-1/32*b^2*x^3*(-c^2*d*x^2+d)^(1/2)-1/64
*b^2*(-c^2*d*x^2+d)^(1/2)*arccos(c*x)/c^3/(-c^2*x^2+1)^(1/2)+1/8*b*x^2*(-c
^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c/(-c^2*x^2+1)^(1/2)-1/8*b*c*x^4*(-c^2
*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-1/8*x*(-c^2*d*x^2+d)^
(1/2)*(a+b*arccos(c*x))^2/c^2+1/4*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x
))^2+1/24*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^3/b/c^3/(-c^2*x^2+1)^(1/2
)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.73

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx = \frac{96a^2cx(-1+2c^2x^2)\sqrt{d-c^2dx^2}-96a^2\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+\frac{12ab\sqrt{d-c^2dx^2}(-8\arccos(cx)^2+\cos(4\arccos(cx))}{\sqrt{1-c^2x^2}}}{768c}$$

input
```
Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(96*a^2*c*x*(-1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2] - 96*a^2*Sqrt[d]*ArcTan[(
c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (12*a*b*Sqrt[d - c^2*
d*x^2]*(-8*ArcCos[c*x]^2 + Cos[4*ArcCos[c*x]] + 4*ArcCos[c*x]*Sin[4*ArcCos
[c*x]]))/Sqrt[1 - c^2*x^2] - (b^2*Sqrt[d - c^2*d*x^2]*(32*ArcCos[c*x]^3 -
12*ArcCos[c*x]*Cos[4*ArcCos[c*x]] + (3 - 24*ArcCos[c*x]^2)*Sin[4*ArcCos[c*
x]]))/Sqrt[1 - c^2*x^2])/(768*c^3)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {5199, 5139, 262, 262, 223, 5211, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5199}$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int x^3(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5139}$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}bc\int \frac{x^4}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))\right)}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{262}$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}bc\left(\frac{3\int \frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{4}x^4(a+b\arccos(cx))\right)}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{262}$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}bc\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{4}x^4(a+b\arccos(cx))\right)}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{223}$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)\right)}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{ 5211}$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int x(a+b\arccos(cx))dx}{c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)\right)}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{1}{2}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c}+\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)\right)}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{1}{2}bc\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c}+\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{4\sqrt{1-c^2x^2}}$$

$$\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)\right)}{2\sqrt{1-c^2x^2}}+$$

$$\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$\downarrow$ 223

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{4\sqrt{1-c^2x^2}}+$$

$$\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)\right)}{2\sqrt{1-c^2x^2}}+$$

$$\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$\downarrow$ 5153

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c}-\frac{(a+b\arccos(cx))^3}{6bc^3}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{4\sqrt{1-c^2x^2}}+$$

$$\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)\right)}{2\sqrt{1-c^2x^2}}+$$

$$\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

input
```
Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/4 + (b*c*Sqrt[d - c^2*d*x^
2]*((x^4*(a + b*ArcCos[c*x]))/4 + (b*c*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 +
 (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/4))/
(2*Sqrt[1 - c^2*x^2]) + (Sqrt[d - c^2*d*x^2]*(-1/2*(x*Sqrt[1 - c^2*x^2]*(a
 + b*ArcCos[c*x])^2)/c^2 - (a + b*ArcCos[c*x])^3/(6*b*c^3) - (b*((x^2*(a +
 b*ArcCos[c*x]))/2 + (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2
*c^3)))/2))/c))/(4*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.24

method	result
default	$-\frac{a^2 x\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}{4c^2 d}+\frac{a^2 x\sqrt{-c^2 d\,x^2+d}}{8c^2}+\frac{a^2 d\arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d\,x^2+d}}\right)}{8c^2\sqrt{c^2 d}}+b^2\left(\frac{\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{-c^2x^2+1}\,\arccos\left(cx\right)^3}{24c^3\left(c^2x^2-1\right)}+\right.$
parts	$-\frac{a^2 x\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}{4c^2 d}+\frac{a^2 x\sqrt{-c^2 d\,x^2+d}}{8c^2}+\frac{a^2 d\arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d\,x^2+d}}\right)}{8c^2\sqrt{c^2 d}}+b^2\left(\frac{\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{-c^2x^2+1}\,\arccos\left(cx\right)^3}{24c^3\left(c^2x^2-1\right)}+\right.$

input
```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a^2/c^2*x*(-c^2*d*x^2+d)^(1/2)+1
/8*a^2/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^
2*(1/24*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c
*x)^3+1/512*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^
(1/2)*x^4*c^4+4*c*x-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(
4*I*arccos(c*x)+8*arccos(c*x)^2-1)/c^3/(c^2*x^2-1)+1/512*(-d*(c^2*x^2-1))^
(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+1)^(1/2)*x^
2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(-4*I*arccos(c*x)+8*arccos(c*
x)^2-1)/c^3/(c^2*x^2-1))+2*a*b*(1/16*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(
1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2+1/256*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5
-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-8*I*(-c^2*x^2+1)^(1/2)*x^
2*c^2+I*(-c^2*x^2+1)^(1/2))*(I+4*arccos(c*x))/c^3/(c^2*x^2-1)+1/256*(-d*(c
^2*x^2-1))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+
1)^(1/2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(-I+4*arccos(c*x))
/c^3/(c^2*x^2-1))
```

Fricas [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="frica
s")
```

output
```
integral((b^2*x^2*arccos(c*x)^2 + 2*a*b*x^2*arccos(c*x) + a^2*x^2)*sqrt(-c
^2*d*x^2 + d), x)
```

Sympy [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int x^2\sqrt{-d(cx-1)(cx+1)}(a+b\operatorname{acos}(cx))^2\,dx$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="maxim
a")
```

output
```
1/8*a^2*(sqrt(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d) +
 sqrt(d)*arcsin(c*x)/c^3) + sqrt(d)*integrate((b^2*x^2*arctan2(sqrt(c*x +
1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1)
, c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.21

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$$
$$=\frac{1}{4}\sqrt{-c^2dx^2+d}a^2x^3-\frac{\sqrt{-c^2dx^2+d}a^2x}{8c^2}-\frac{a^2d\log\left(\left|-c\sqrt{-d}x+\sqrt{c^2x^2-1}\sqrt{-d}\right|\right)}{8c^3\sqrt{-d}}$$
$$+\frac{24b^2c^3\sqrt{d}x^4\arccos(cx)+48\sqrt{-c^2x^2+1}b^2c^2\sqrt{d}x^3\arccos(cx)^2+24abc^3\sqrt{d}x^4+96\sqrt{-c^2x^2+1}abc^2\cdots}{}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="giac"
)
```

output
```
1/4*sqrt(-c^2*d*x^2 + d)*a^2*x^3 - 1/8*sqrt(-c^2*d*x^2 + d)*a^2*x/c^2 - 1/
8*a^2*d*log(abs(-c*sqrt(-d)*x + sqrt(c^2*x^2 - 1)*sqrt(-d)))/(c^3*sqrt(-d)
) + 1/192*(24*b^2*c^3*sqrt(d)*x^4*arccos(c*x) + 48*sqrt(-c^2*x^2 + 1)*b^2*
c^2*sqrt(d)*x^3*arccos(c*x)^2 + 24*a*b*c^3*sqrt(d)*x^4 + 96*sqrt(-c^2*x^2
+ 1)*a*b*c^2*sqrt(d)*x^3*arccos(c*x) - 6*sqrt(-c^2*x^2 + 1)*b^2*c^2*sqrt(d
)*x^3 - 24*b^2*c*sqrt(d)*x^2*arccos(c*x) - 24*sqrt(-c^2*x^2 + 1)*b^2*sqrt(
d)*x*arccos(c*x)^2 - 24*a*b*c*sqrt(d)*x^2 - 48*sqrt(-c^2*x^2 + 1)*a*b*sqrt
(d)*x*arccos(c*x) - 8*b^2*sqrt(d)*arccos(c*x)^3/c + 3*sqrt(-c^2*x^2 + 1)*b
^2*sqrt(d)*x - 24*a*b*sqrt(d)*arccos(c*x)^2/c + 3*b^2*sqrt(d)*arccos(c*x)/
c + 3*a*b*sqrt(d)/c)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int x^2\,(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^2*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^2*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$$
$$=\frac{\sqrt{d}\left(asin(cx)\,a^2+2\sqrt{-c^2x^2+1}\,a^2c^3x^3-\sqrt{-c^2x^2+1}\,a^2cx+16\left(\int\sqrt{-c^2x^2+1}\,acos(cx)\,x^2dx\right)ab\,c^3+8\right.}{8c^3}$$

input
```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*(asin(c*x)*a**2 + 2*sqrt( - c**2*x**2 + 1)*a**2*c**3*x**3 - sqrt(
 - c**2*x**2 + 1)*a**2*c*x + 16*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**2,
x)*a*b*c**3 + 8*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x**2,x)*b**2*c**3)
)/(8*c**3)
```

3.214 $\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 188

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx = \frac{4b^2\sqrt{d-c^2dx^2}}{9c^2}+\frac{2b^2(1-c^2x^2)\sqrt{d-c^2dx^2}}{27c^2}+\frac{2bx\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3c\sqrt{1-c^2x^2}}-\frac{2bcx^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{9\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{3c^2d}$$

output
```
4/9*b^2*(-c^2*d*x^2+d)^(1/2)/c^2+2/27*b^2*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2
)/c^2+2/3*b*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c/(-c^2*x^2+1)^(1/2)-
2/9*b*c*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-1/3*
(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.92

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$$
$$=\frac{\sqrt{d-c^2dx^2}\left(-6abcx\sqrt{1-c^2x^2}(-3+c^2x^2)+9a^2(-1+c^2x^2)^2-2b^2(7-8c^2x^2+c^4x^4)+6b\left(bcx\sqrt{1-c^2x^2}\right.\right.}{27c^2\left(-1+c^2x^2\right)}$$

input
```
Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(-6*a*b*c*x*Sqrt[1 - c^2*x^2]*(-3 + c^2*x^2) + 9*a^2*
(-1 + c^2*x^2)^2 - 2*b^2*(7 - 8*c^2*x^2 + c^4*x^4) + 6*b*(b*c*x*Sqrt[1 - c
^2*x^2]*(3 - c^2*x^2) + 3*a*(-1 + c^2*x^2)^2)*ArcCos[c*x] + 9*b^2*(-1 + c^
2*x^2)^2*ArcCos[c*x]^2))/(27*c^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.79, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5183, 5155, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{ 5183}$$

$$-\frac{2b\sqrt{d-c^2dx^2}\int\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{3c\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3c^2d}$$

$$\Big\downarrow \text{ 5155}$$

$$-\frac{2b\sqrt{d-c^2dx^2}\left(bc\int\frac{x(3-c^2x^2)}{3\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3c^2d}$$

$$\Big\downarrow \; 27$$

$$-\frac{2b\sqrt{d-c^2dx^2}\left(\frac{1}{3}bc\int \frac{x(3-c^2x^2)}{\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3c^2d}$$

$$\Big\downarrow \; 353$$

$$-\frac{2b\sqrt{d-c^2dx^2}\left(\frac{1}{6}bc\int \frac{3-c^2x^2}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3c^2d}$$

$$\Big\downarrow \; 53$$

$$-\frac{2b\sqrt{d-c^2dx^2}\left(\frac{1}{6}bc\int \left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3c^2d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3c^2d}-\frac{2b\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{6}bc\left(-\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^2}-\frac{4\sqrt{1-c^2x^2}}{c^2}\right)\right)}{3c\sqrt{1-c^2x^2}}$$

input
```
Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/3*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/(c^2*d) - (2*b*Sqrt[d -
 c^2*d*x^2]*((b*c*((-4*Sqrt[1 - c^2*x^2])/c^2 - (2*(1 - c^2*x^2)^(3/2))/(3
*c^2)))/6 + x*(a + b*ArcCos[c*x]) - (c^2*x^3*(a + b*ArcCos[c*x]))/3))/(3*c
*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(164) = 328.

Time = 0.36 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.34

method	result
orering	$\frac{(19c^6x^6-71c^4x^4+48c^2x^2-14)\sqrt{-c^2d\,x^2+d}\,(a+b\arccos(cx))^2}{27(c^2x^2-1)c^4x^2}-\frac{2(3c^4x^4-16c^2x^2+7)\left(\sqrt{-c^2d\,x^2+d}\,(a+b\arccos(cx))^2-\frac{x^2(a+b}{\sqrt{}}\right.}{27c^4x^2}$
default	$-\frac{a^2(-c^2d\,x^2+d)^{\frac{3}{2}}}{3c^2d}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\left(4c^4x^4-5c^2x^2+4i\sqrt{-c^2x^2+1}\,x^3c^3-3i\sqrt{-c^2x^2+1}\,xc+1\right)\left(6i\arccos(cx)+9\arccos(cx\right.}{216c^2(c^2x^2-1)}\right.$
parts	$-\frac{a^2(-c^2d\,x^2+d)^{\frac{3}{2}}}{3c^2d}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\left(4c^4x^4-5c^2x^2+4i\sqrt{-c^2x^2+1}\,x^3c^3-3i\sqrt{-c^2x^2+1}\,xc+1\right)\left(6i\arccos(cx)+9\arccos(cx\right.}{216c^2(c^2x^2-1)}\right.$

input
```
int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/27*(19*c^6*x^6-71*c^4*x^4+48*c^2*x^2-14)/(c^2*x^2-1)/c^4/x^2*(-c^2*d*x^2
+d)^(1/2)*(a+b*arccos(c*x))^2-2/27*(3*c^4*x^4-16*c^2*x^2+7)/c^4/x^2*((-c^2
*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2-x^2/(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c
*x))^2*d*c^2-2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*b*c/(-c^2*x^2+1)^(
1/2))+1/27*(c^2*x^2-7)/c^4/x*(c*x-1)*(c*x+1)*(-3/(-c^2*d*x^2+d)^(1/2)*(a+b
*arccos(c*x))^2*d*c^2*x-4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*b*c/(-c^2
*x^2+1)^(1/2)-x^3/(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2*d^2*c^4+4*x^2/(
-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*d*c^3*b/(-c^2*x^2+1)^(1/2)+2*x*(-c^2
*d*x^2+d)^(1/2)*b^2*c^2/(-c^2*x^2+1)-2*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcco
s(c*x))*b*c^3/(-c^2*x^2+1)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.11

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx =$$
$$-\frac{6\left(abc^3x^3-3\,abcx+(b^2c^3x^3-3\,b^2cx)\arccos{(cx)}\right)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}-\left((9\,a^2-2\,b^2)c^4x^4-2\right.}{}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
-1/27*(6*(a*b*c^3*x^3 - 3*a*b*c*x + (b^2*c^3*x^3 - 3*b^2*c*x)*arccos(c*x))
*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - ((9*a^2 - 2*b^2)*c^4*x^4 - 2*(9
*a^2 - 8*b^2)*c^2*x^2 + 9*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arccos(c*x)^
2 + 9*a^2 - 14*b^2 + 18*(a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b)*arccos(c*x))*s
qrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int x\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))^2\,dx$$

input
```
integrate(x*(-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx &= -\frac{2}{27}\,b^2\left(\frac{\sqrt{-c^2x^2+1}d^{\frac{3}{2}}x^2-\frac{7\sqrt{-c^2x^2+1}d^{\frac{3}{2}}}{c^2}}{d}-\frac{3\left(c^2d^{\frac{3}{2}}x^3-3\,d^{\frac{3}{2}}x\right)\arccos\,(cx)}{cd}\right)\\&\quad-\frac{(-c^2dx^2+d)^{\frac{3}{2}}b^2\arccos\,(cx)^2}{3\,c^2d}-\frac{2\,(-c^2dx^2+d)^{\frac{3}{2}}ab\arccos\,(cx)}{3\,c^2d}\\&\quad+\frac{2\left(c^2d^{\frac{3}{2}}x^3-3\,d^{\frac{3}{2}}x\right)ab}{9\,cd}-\frac{(-c^2dx^2+d)^{\frac{3}{2}}a^2}{3\,c^2d}\end{aligned}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-2/27*b^2*((sqrt(-c^2*x^2 + 1)*d^(3/2)*x^2 - 7*sqrt(-c^2*x^2 + 1)*d^(3/2)/
c^2)/d - 3*(c^2*d^(3/2)*x^3 - 3*d^(3/2)*x)*arccos(c*x)/(c*d)) - 1/3*(-c^2*
d*x^2 + d)^(3/2)*b^2*arccos(c*x)^2/(c^2*d) - 2/3*(-c^2*d*x^2 + d)^(3/2)*a*
b*arccos(c*x)/(c^2*d) + 2/9*(c^2*d^(3/2)*x^3 - 3*d^(3/2)*x)*a*b/(c*d) - 1/
3*(-c^2*d*x^2 + d)^(3/2)*a^2/(c^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx = \int x\,(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$$
$$=\frac{\sqrt{d}\left(\sqrt{-c^2x^2+1}\,a^2c^2x^2-\sqrt{-c^2x^2+1}\,a^2+6\left(\int\sqrt{-c^2x^2+1}\,acos(cx)\,xdx\right)ab\,c^2+3\left(\int\sqrt{-c^2x^2+1}\,aco\right.}{3c^2}$$

input
```
int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*(sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a
**2 + 6*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x,x)*a*b*c**2 + 3*int(sqrt( -
 c**2*x**2 + 1)*acos(c*x)**2*x,x)*b**2*c**2))/(3*c**2)
```

3.215 $\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 192

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx = -\frac{1}{4}b^2x\sqrt{d-c^2dx^2}+\frac{b^2\sqrt{d-c^2dx^2}\arccos(cx)}{4c\sqrt{1-c^2x^2}} - \frac{bcx^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2 + \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}$$

output
```
-1/4*b^2*x*(-c^2*d*x^2+d)^(1/2)+1/4*b^2*(-c^2*d*x^2+d)^(1/2)*arccos(c*x)/c
/(-c^2*x^2+1)^(1/2)-1/2*b*c*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c
^2*x^2+1)^(1/2)+1/2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2+1/6*(-c^2*d
*x^2+d)^(1/2)*(a+b*arccos(c*x))^3/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.14

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx = \frac{1}{2}a^2x\sqrt{d-c^2dx^2} - \frac{a^2\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{2c} - \frac{b^2\sqrt{d-c^2dx^2}(4\arccos(cx)^3-6\arccos(cx)\cos(2\arccos(cx))+(3-6\arccos(cx)^2)\sin(2\arccos(cx)))}{24c\sqrt{1-c^2x^2}} + \frac{ab\sqrt{d-c^2dx^2}(\cos(2\arccos(cx))+2\arccos(cx)(-\arccos(cx)+\sin(2\arccos(cx))))}{4c\sqrt{1-c^2x^2}}$$

input
```
Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(a^2*x*Sqrt[d - c^2*d*x^2])/2 - (a^2*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^
2])/(Sqrt[d]*(-1 + c^2*x^2))])/(2*c) - (b^2*Sqrt[d - c^2*d*x^2]*(4*ArcCos[
c*x]^3 - 6*ArcCos[c*x]*Cos[2*ArcCos[c*x]] + (3 - 6*ArcCos[c*x]^2)*Sin[2*Ar
cCos[c*x]]))/(24*c*Sqrt[1 - c^2*x^2]) + (a*b*Sqrt[d - c^2*d*x^2]*(Cos[2*Ar
cCos[c*x]] + 2*ArcCos[c*x]*(-ArcCos[c*x] + Sin[2*ArcCos[c*x]])))/(4*c*Sqrt
[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5157, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \ 5157$$

$$\frac{bc\sqrt{d-c^2dx^2}\int x(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}} + \frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}} + \frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5139}$$

$$\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{262}$$

$$\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{223}$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5153}$$

$$\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

input
```
Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCos[c*x])^3)/(6*b*c*Sqrt[1 - c^2*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*(
(x^2*(a + b*ArcCos[c*x]))/2 + (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSi
n[c*x]/(2*c^3)))/2))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.00 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.77

method	result
default	$\frac{a^2x\sqrt{-c^2d\,x^2+d}}{2}+\frac{a^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3}{6(c^2x^2-1)c}+\frac{\sqrt{-d(c^2x^2-1)}\,\left(2c^3x^3-2cx\right.}{}\right.$
parts	$\frac{a^2x\sqrt{-c^2d\,x^2+d}}{2}+\frac{a^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3}{6(c^2x^2-1)c}+\frac{\sqrt{-d(c^2x^2-1)}\,\left(2c^3x^3-2cx\right.}{}\right.$

input

```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
1/2*a^2*x*(-c^2*d*x^2+d)^(1/2)+1/2*a^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2
)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(1/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/
2)/(c^2*x^2-1)/c*arccos(c*x)^3+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*
x+2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(2*arccos(c*x)^2-1+
2*I*arccos(c*x))/(c^2*x^2-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2
+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*arccos(c*x)^2-1
-2*I*arccos(c*x))/(c^2*x^2-1)/c)+2*a*b*(1/4*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x
^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^2+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3
*x^3-2*c*x+2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(I+2*arcco
s(c*x))/(c^2*x^2-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)
*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))/(c^2*x^2
-1)/c)
```

Fricas [F]

$$\int\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int\sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)^2\,dx$$

input

```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
), x)
```

Sympy [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int \sqrt{-d\,(cx-1)\,(cx+1)}(a+b\operatorname{acos}{(cx)})^2\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int \sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)^2\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a^2 + sqrt(d)*integra
te((b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arctan2(sqrt(
c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$$
$$=\frac{\sqrt{d}\,\left(asin(cx)\,a^2+\sqrt{-c^2x^2+1}\,a^2cx+4\left(\int\sqrt{-c^2x^2+1}\,acos(cx)\,dx\right)abc+2\left(\int\sqrt{-c^2x^2+1}\,acos(cx)^2\,d\right.}{2c}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*(asin(c*x)*a**2 + sqrt( - c**2*x**2 + 1)*a**2*c*x + 4*int(sqrt( -
 c**2*x**2 + 1)*acos(c*x),x)*a*b*c + 2*int(sqrt( - c**2*x**2 + 1)*acos(c*x
)**2,x)*b**2*c))/(2*c)
```

3.216 $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 378

$$\begin{aligned}
&\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}\,dx \\
&= -2b^2\sqrt{d-c^2dx^2} - \frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} \\
&\quad - \frac{2b^2cx\sqrt{d-c^2dx^2}\arccos(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2 \\
&\quad - \frac{2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\text{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&\quad + \frac{2ib\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&\quad - \frac{2ib\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&\quad - \frac{2b^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} + \frac{2b^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}
\end{aligned}$$

output
```
-2*b^2*(-c^2*d*x^2+d)^(1/2)-2*a*b*c*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1
/2)-2*b^2*c*x*(-c^2*d*x^2+d)^(1/2)*arccos(c*x)/(-c^2*x^2+1)^(1/2)+(-c^2*d*
x^2+d)^(1/2)*(a+b*arccos(c*x))^2-2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^
2*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*I*b*(-c^2*d*x^2+d
)^(1/2)*(a+b*arccos(c*x))*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1
)^(1/2)-2*I*b*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*polylog(2,c*x+I*(-c^2
*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-2*b^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,-c*
x-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*b^2*(-c^2*d*x^2+d)^(1/2)*poly
log(3,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}\,dx$$
$$= a^2\sqrt{d-c^2dx^2}+a^2\sqrt{d}\log(cx)-a^2\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)$$
$$+\frac{2ab\sqrt{d-c^2dx^2}\left(cx+\sqrt{1-c^2x^2}\arccos(cx)-\arccos(cx)\log\left(1-ie^{i\arccos(cx)}\right)+\arccos(cx)\log\left(1+ie\right.\right.}{\sqrt{1-c^2x^2}}$$
$$+\frac{b^2\sqrt{d-c^2dx^2}\left(-2\sqrt{1-c^2x^2}+2cx\arccos(cx)+\sqrt{1-c^2x^2}\arccos(cx)^2-\arccos(cx)^2\log\left(1-ie^{i\arccos}\right.\right.}{}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
a^2*Sqrt[d - c^2*d*x^2] + a^2*Sqrt[d]*Log[c*x] - a^2*Sqrt[d]*Log[d + Sqrt[
d]*Sqrt[d - c^2*d*x^2]] + (2*a*b*Sqrt[d - c^2*d*x^2]*(c*x + Sqrt[1 - c^2*x
^2]*ArcCos[c*x] - ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] + ArcCos[c*x]*L
og[1 + I*E^(I*ArcCos[c*x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I*Pol
yLog[2, I*E^(I*ArcCos[c*x])]))/Sqrt[1 - c^2*x^2] + (b^2*Sqrt[d - c^2*d*x^2
]*(-2*Sqrt[1 - c^2*x^2] + 2*c*x*ArcCos[c*x] + Sqrt[1 - c^2*x^2]*ArcCos[c*x
]^2 - ArcCos[c*x]^2*Log[1 - I*E^(I*ArcCos[c*x])] + ArcCos[c*x]^2*Log[1 + I
*E^(I*ArcCos[c*x])] - (2*I)*ArcCos[c*x]*PolyLog[2, (-I)*E^(I*ArcCos[c*x])]
 + (2*I)*ArcCos[c*x]*PolyLog[2, I*E^(I*ArcCos[c*x])] + 2*PolyLog[3, (-I)*E
^(I*ArcCos[c*x])] - 2*PolyLog[3, I*E^(I*ArcCos[c*x])]))/Sqrt[1 - c^2*x^2]
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.66, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {5199, 2009, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx$$

$$\Big\downarrow \text{5199}$$

$$\frac{2bc\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5219}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\sqrt{d-c^2dx^2}\left(-2b\int(a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+2b\int(a+b\arccos(cx))\log\left(1+ie^{i\arcc}\right.\right.}{\sqrt{1-c^2x^2}}$$

$$\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{\sqrt{1-c^2x^2}}$$

$\downarrow$ 3011

$$-\frac{\sqrt{d-c^2dx^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\arccos(cx)\right)-\right.}{}$$

$$\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{\sqrt{1-c^2x^2}}$$

$\downarrow$ 2720

$$-\frac{\sqrt{d-c^2dx^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)de^{i}\right.\right.}{}$$

$$\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{\sqrt{1-c^2x^2}}$$

$\downarrow$ 7143

$$-\frac{\sqrt{d-c^2dx^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-\right.\right.}{\sqrt{1-}}$$

$$\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{\sqrt{1-c^2x^2}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2 + (2*b*c*Sqrt[d - c^2*d*x^2]*(a*
x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]))/Sqrt[1 - c^2*x^2] - (Sqrt[
d - c^2*d*x^2]*((-2*I)*(a + b*ArcCos[c*x])^2*ArcTan[E^(I*ArcCos[c*x])] + 2
*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - b*PolyLog[3
, (-I)*E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, I*E^(I*
ArcCos[c*x])] - b*PolyLog[3, I*E^(I*ArcCos[c*x])])))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.54

method	result
default	$\frac{\sqrt{-d(c^2x^2-1)}\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\left(\arccos(cx)^2b^2+2\arccos(cx)ab+a^2-2b^2+2i\arccos(cx)b^2+2iab\right)}{2c^2x^2-2}+\frac{\sqrt{-d(c^2x^2-1)}\left(-\right.}{}$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*
x)^2*b^2+2*arccos(c*x)*a*b+a^2-2*b^2+2*I*arccos(c*x)*b^2+2*I*a*b)/(c^2*x^2
-1)+1/2*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arcc
os(c*x)^2*b^2+2*arccos(c*x)*a*b+a^2-2*b^2-2*I*b^2*arccos(c*x)-2*I*a*b)/(c^
2*x^2-1)+I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(I*arccos(c*x)^2*ln(1
+I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b^2-I*arccos(c*x)^2*ln(1-I*(c*x+I*(-c^2*x^2
+1)^(1/2)))*b^2+2*I*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))*a*b-2*I
*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))*a*b+2*arccos(c*x)*polylog(
2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b^2-2*arccos(c*x)*polylog(2,I*(c*x+I*(-c^
2*x^2+1)^(1/2)))*b^2-2*I*polylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b^2+2*I*p
olylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b^2+2*polylog(2,-I*(c*x+I*(-c^2*x^
2+1)^(1/2)))*a*b-2*a^2*arctan(c*x+I*(-c^2*x^2+1)^(1/2))-2*polylog(2,I*(c*x
+I*(-c^2*x^2+1)^(1/2)))*a*b)/(c^2*x^2-1)
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
)/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}\,dx = \int \frac{\sqrt{-d(cx-1)(cx+1)}(a+b\operatorname{acos}(cx))^2}{x}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))**2/x,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x,x, algorithm="maxima"
)
```

output
```
-(sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - sqrt(-
c^2*d*x^2 + d))*a^2 + sqrt(d)*integrate((b^2*arctan2(sqrt(c*x + 1)*sqrt(-c
*x + 1), c*x)^2 + 2*a*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c
*x + 1)*sqrt(-c*x + 1)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}}{x}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2))/x,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2))/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}\,dx = \sqrt{d}\left(\sqrt{-c^2x^2+1}\,a^2 + 2\left(\int \frac{\sqrt{-c^2x^2+1}\,acos(cx)}{x}dx\right)ab + \left(\int \frac{\sqrt{-c^2x^2+1}\,acos(cx)^2}{x}dx\right)b^2 + \log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a^2 - a^2\right)$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))^2/x,x)
```

output
```
sqrt(d)*(sqrt( - c**2*x**2 + 1)*a**2 + 2*int((sqrt( - c**2*x**2 + 1)*acos(
c*x))/x,x)*a*b + int((sqrt( - c**2*x**2 + 1)*acos(c*x)**2)/x,x)*b**2 + log
(tan(asin(c*x)/2))*a**2 - a**2)
```

3.217 $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 227

$$\begin{aligned}\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}\,dx &= -\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x} - \frac{ic\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}} \\ &\quad - \frac{c\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{3b\sqrt{1-c^2x^2}} \\ &\quad + \frac{2bc\sqrt{d-c^2dx^2}(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\ &\quad - \frac{ib^2c\sqrt{d-c^2dx^2}\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x-I*c*(-c^2*d*x^2+d)^(1/2)*(a+b*
arccos(c*x))^2/(-c^2*x^2+1)^(1/2)-1/3*c*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c
*x))^3/b/(-c^2*x^2+1)^(1/2)+2*b*c*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*l
n(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)-I*b^2*c*(-c^2*d*x^2+d
)^(1/2)*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}\,dx$$
$$= -\frac{a^2\sqrt{d-c^2dx^2}}{x} + a^2c\sqrt{d}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)$$
$$- abc\sqrt{d-c^2dx^2}\left(\frac{2\arccos(cx)}{cx} - \frac{\arccos(cx)^2-2\log(cx)}{\sqrt{1-c^2x^2}}\right)$$
$$+ \frac{b^2\sqrt{d-c^2dx^2}\,(\arccos(cx)\left(-3(-icx+\sqrt{1-c^2x^2})\arccos(cx)+cx\arccos(cx)^2-6cx\log\left(1+e^{2i\arccos}\right.\right.}{3x\sqrt{1-c^2x^2}}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/x^2,x]
```

output
```
-((a^2*Sqrt[d - c^2*d*x^2])/x) + a^2*c*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*
x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - a*b*c*Sqrt[d - c^2*d*x^2]*((2*ArcCos[c*x
])/(c*x) - (ArcCos[c*x]^2 - 2*Log[c*x])/Sqrt[1 - c^2*x^2]) + (b^2*Sqrt[d -
 c^2*d*x^2]*(ArcCos[c*x]*(-3*((-I)*c*x + Sqrt[1 - c^2*x^2])*ArcCos[c*x] +
c*x*ArcCos[c*x]^2 - 6*c*x*Log[1 + E^((2*I)*ArcCos[c*x])]) + (3*I)*c*x*Poly
Log[2, -E^((2*I)*ArcCos[c*x])]))/(3*x*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {5197, 5137, 3042, 4202, 2620, 2715, 2838, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}\,dx$$
$$\Big\downarrow \; 5197$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}-\frac{2bc\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{x}dx}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}$$

$\Big\downarrow$ 5137

$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}$$

$\Big\downarrow$ 3042

$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}$$

$\Big\downarrow$ 4202

$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}$$

$\Big\downarrow$ 2620

$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\,\text{arc}\right.\right.}{\sqrt{1-c^2x^2}}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}$$

$\Big\downarrow$ 2715

$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{2}i\log\left(1+e^{2i\arccos(c\right.\right.}{\sqrt{1-c^2x^2}}$$

$$\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right.\right.\right.}{\sqrt{1-c^2x^2}}$$

$$\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{ 5153}$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right.\right.\right.}{\sqrt{1-c^2x^2}}$$

$$\frac{c\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{3b\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/x^2,x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/x) + (c*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCos[c*x])^3)/(3*b*Sqrt[1 - c^2*x^2]) + (2*b*c*Sqrt[d - c^2*d*x^2
]*(((I/2)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcCos[c*x])*L
og[1 + E^((2*I)*ArcCos[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/4))
)/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5197
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^2
]/Sqrt[1 - c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x
] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 2)*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(227) = 454$.

Time = 0.56 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.31

method	result
default	$-\frac{a^2(-c^2dx^2+d)^{\frac{3}{2}}}{dx} - a^2c^2x\sqrt{-c^2dx^2+d} - \frac{a^2c^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{\sqrt{c^2d}} + b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\,\mathrm{arcco}}{3(c^2x^2-1)}\right.$
parts	$-\frac{a^2(-c^2dx^2+d)^{\frac{3}{2}}}{dx} - a^2c^2x\sqrt{-c^2dx^2+d} - \frac{a^2c^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{\sqrt{c^2d}} + b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2+1}\,\mathrm{arcco}}{3(c^2x^2-1)}\right.$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/d/x*(-c^2*d*x^2+d)^(3/2)-a^2*c^2*x*(-c^2*d*x^2+d)^(1/2)-a^2*c^2*d/(c^
2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-1/3*(-d*(c^2
*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^3*c-(-d*(c^2*x^2
-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*arccos(c*x)^2/x/(c^2*x^2-
1)-I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*(2*I*arccos(c*x
)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+2*arccos(c*x)^2+polylog(2,-(c*x+I*(-c
^2*x^2+1)^(1/2))^2))*c)+2*a*b*(-1/2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1
/2)/(c^2*x^2-1)*arccos(c*x)^2*c-2*I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1
/2)/(c^2*x^2-1)*arccos(c*x)*c-(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2
)*x*c+c^2*x^2-1)*arccos(c*x)/x/(c^2*x^2-1)+(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^
2+1)^(1/2)/(c^2*x^2-1)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*c)
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x^2,x, algorithm="frica
s")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}\,dx=\int \frac{\sqrt{-d(cx-1)(cx+1)}(a+b\,\mathrm{acos}(cx))^2}{x^2}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))**2/x**2,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x^2,x, algorithm="maxim
a")
```

output
```
-(c*sqrt(d)*arcsin(c*x) + sqrt(-c^2*d*x^2 + d)/x)*a^2 + sqrt(d)*integrate(
(b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arctan2(sqrt(c*x
 + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}\,dx$$
$$= \frac{\sqrt{d}\left(acos(cx)^3 b^2cx + 3acos(cx)^2 abcx - 3asin(cx)\,a^2cx - 3\sqrt{-c^2x^2+1}\,a^2 + 6\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,x^2}dx\right)abx + \right.}{3x}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))^2/x^2,x)
```

output
```
(sqrt(d)*(acos(c*x)**3*b**2*c*x + 3*acos(c*x)**2*a*b*c*x - 3*asin(c*x)*a**
2*c*x - 3*sqrt( - c**2*x**2 + 1)*a**2 + 6*int(acos(c*x)/(sqrt( - c**2*x**2
 + 1)*x**2),x)*a*b*x + 3*int(acos(c*x)**2/(sqrt( - c**2*x**2 + 1)*x**2),x)
*b**2*x))/(3*x)
```

3.218 $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 398

$$\begin{aligned}
\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^3}\,dx
&= -\frac{bc\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x\sqrt{1-c^2x^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2x^2} \\
&\quad + \frac{c^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&\quad - \frac{b^2c^2\sqrt{d-c^2dx^2}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{1-c^2x^2}} \\
&\quad - \frac{ibc^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&\quad + \frac{ibc^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&\quad + \frac{b^2c^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&\quad - \frac{b^2c^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}
\end{aligned}$$

output
```
-b*c*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x/(-c^2*x^2+1)^(1/2)-1/2*(-c^2
*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x^2+c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcc
os(c*x))^2*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-b^2*c^2*(-
c^2*d*x^2+d)^(1/2)*arctanh((-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-I*b*c^2*
(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2)
)/(-c^2*x^2+1)^(1/2)+I*b*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*polylo
g(2,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+b^2*c^2*(-c^2*d*x^2+d)^(1
/2)*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-b^2*c^2*(-c^2*
d*x^2+d)^(1/2)*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^3}\,dx$$
$$= \frac{a^2d(-1+c^2x^2)-a^2c^2\sqrt{d}x^2\sqrt{d-c^2dx^2}\log(x)+a^2c^2\sqrt{d}x^2\sqrt{d-c^2dx^2}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)+2abc}{}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/x^3,x]
```

output
```
(a^2*d*(-1 + c^2*x^2) - a^2*c^2*Sqrt[d]*x^2*Sqrt[d - c^2*d*x^2]*Log[x] + a
^2*c^2*Sqrt[d]*x^2*Sqrt[d - c^2*d*x^2]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]
] + 2*a*b*d*Sqrt[1 - c^2*x^2]*(c*x - Sqrt[1 - c^2*x^2]*ArcCos[c*x] + c^2*x
^2*ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] - c^2*x^2*ArcCos[c*x]*Log[1 +
I*E^(I*ArcCos[c*x])] + I*c^2*x^2*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*c^
2*x^2*PolyLog[2, I*E^(I*ArcCos[c*x])]) + b^2*d*Sqrt[1 - c^2*x^2]*(ArcCos[c
*x]*(2*c*x - Sqrt[1 - c^2*x^2]*ArcCos[c*x]) - 2*c^2*x^2*(ArcCoth[Sqrt[1 -
c^2*x^2]] + I*ArcCos[c*x]^2*ArcTan[E^(I*ArcCos[c*x])] - I*ArcCos[c*x]*Poly
Log[2, (-I)*E^(I*ArcCos[c*x])] + I*ArcCos[c*x]*PolyLog[2, I*E^(I*ArcCos[c*
x])] + PolyLog[3, (-I)*E^(I*ArcCos[c*x])] - PolyLog[3, I*E^(I*ArcCos[c*x])
])))/(2*x^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.66, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {5197, 5139, 243, 73, 221, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^3}dx$$

$$\Big\downarrow \text{ 5197}$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}-\frac{bc\sqrt{d-c^2dx^2}\int \frac{a+b\arccos(cx)}{x^2}dx}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{ 5139}$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}-\frac{bc\sqrt{d-c^2dx^2}\left(-bc\int \frac{1}{x\sqrt{1-c^2x^2}}dx-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}-\frac{bc\sqrt{d-c^2dx^2}\left(-\frac{1}{2}bc\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}-\frac{bc\sqrt{d-c^2dx^2}\left(\frac{b\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c}-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}-\frac{bc\sqrt{d-c^2dx^2}\left(b\text{carctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{5219}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)}{2\sqrt{1-c^2x^2}}-\frac{bc\sqrt{d-c^2dx^2}\left(b\text{carctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{2\sqrt{1-c^2x^2}}-\frac{bc\sqrt{d-c^2dx^2}\left(b\text{carctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{4669}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\left(-2b\int (a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+ie^{i\arccos(cx)}\right)\right.}{2\sqrt{1-c^2x^2}}$$

$$-\frac{bc\sqrt{d-c^2dx^2}\left(b\text{carctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{3011}$$

$$c^2\sqrt{d-c^2dx^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\arccos(cx)\right)-\right.$$

$$-\frac{bc\sqrt{d-c^2dx^2}\left(b\text{carctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{2720}$$

$$c^2\sqrt{d-c^2dx^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)de^{i\ldots}\right.\right.$$

$$-\frac{bc\sqrt{d-c^2dx^2}\left(b\text{carctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{7143}$$

$$\frac{c^2\sqrt{d-c^2dx^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-\right.\right.}{2\sqrt{1-}}$$

$$\frac{bc\sqrt{d-c^2dx^2}\left(b\operatorname{carctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2x^2}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/x^3,x]
```

output
```
-1/2*(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/x^2 - (b*c*Sqrt[d - c^2*d
*x^2]*(-((a + b*ArcCos[c*x])/x) + b*c*ArcTanh[Sqrt[1 - c^2*x^2]]))/Sqrt[1
- c^2*x^2] + (c^2*Sqrt[d - c^2*d*x^2]*((-2*I)*(a + b*ArcCos[c*x])^2*ArcTan
[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*ArcC
os[c*x])] - b*PolyLog[3, (-I)*E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c
*x])*PolyLog[2, I*E^(I*ArcCos[c*x])] - b*PolyLog[3, I*E^(I*ArcCos[c*x])]))
)/(2*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5197
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^2
]/Sqrt[1 - c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x
] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 2)*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.58

method	result
default	$a^2\left(-\frac{(-c^2 d x^2+d)^{\frac{3}{2}}}{2d x^2}-\frac{c^2\left(\sqrt{-c^2 d x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d x^2+d}}{x}\right)\right)}{2}\right)+b^2\left(-\frac{\left(c^2x^2\arccos(cx)+2cx\sqrt{-c^2x^2+1}-a\right.}{2(c^2x^2}\right.$
parts	$a^2\left(-\frac{(-c^2 d x^2+d)^{\frac{3}{2}}}{2d x^2}-\frac{c^2\left(\sqrt{-c^2 d x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d x^2+d}}{x}\right)\right)}{2}\right)+b^2\left(-\frac{\left(c^2x^2\arccos(cx)+2cx\sqrt{-c^2x^2+1}-a\right.}{2(c^2x^2}\right.$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/d/x^2*(-c^2*d*x^2+d)^(3/2)-1/2*c^2*((-c^2*d*x^2+d)^(1/2)-d^(1/2)
*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)))+b^2*(-1/2*(c^2*x^2*arccos(c*
x)+2*c*x*(-c^2*x^2+1)^(1/2)-arccos(c*x))*arccos(c*x)*(-d*(c^2*x^2-1))^(1/2
)/(c^2*x^2-1)/x^2+I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(I*arccos(c*
x)^2*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-I*arccos(c*x)^2*ln(1+I*(c*x+I*(-c^
2*x^2+1)^(1/2)))+2*arccos(c*x)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*a
rccos(c*x)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*I*polylog(3,I*(c*x+I
*(-c^2*x^2+1)^(1/2)))-2*I*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-4*arcta
n(c*x+I*(-c^2*x^2+1)^(1/2)))*c^2/(2*c^2*x^2-2))+2*a*b*(-1/2*(c^2*x^2*arcco
s(c*x)+c*x*(-c^2*x^2+1)^(1/2)-arccos(c*x))*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2
-1)/x^2+I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(I*arccos(c*x)*ln(1-I*
(c*x+I*(-c^2*x^2+1)^(1/2)))-I*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)
))-dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/
2))))*c^2/(2*c^2*x^2-2))
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^3}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)^2}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x^3,x, algorithm="frica
s")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
)/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^3}\,dx=\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))^2}{x^3}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))**2/x**3,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^3}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)^2}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x^3,x, algorithm="maxim
a")
```

output
```
1/2*(c^2*sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
 sqrt(-c^2*d*x^2 + d)*c^2 - (-c^2*d*x^2 + d)^(3/2)/(d*x^2))*a^2 + sqrt(d)*
integrate((b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arctan
2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x^3,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}}{x^3}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^3,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^3}\,dx = \frac{\sqrt{d}\left(-\sqrt{-c^2x^2+1}\,a^2+4\left(\int \frac{\sqrt{-c^2x^2+1}\,acos(cx)}{x^3}dx\right)ab\,x^2+2\left(\int \frac{\sqrt{-c^2x^2+1}\,acos(cx)^2}{x^3}dx\right)b^2x^2-\log\left(\tan\left(\frac{asin(c}{2}\right.\right.\right.}{2x^2}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))^2/x^3,x)
```

output
```
(sqrt(d)*( - sqrt( - c**2*x**2 + 1)*a**2 + 4*int((sqrt( - c**2*x**2 + 1)*a
cos(c*x))/x**3,x)*a*b*x**2 + 2*int((sqrt( - c**2*x**2 + 1)*acos(c*x)**2)/x
**3,x)*b**2*x**2 - log(tan(asin(c*x)/2))*a**2*c**2*x**2))/(2*x**2)
```

3.219 $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 314

$$\begin{aligned}\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^4}\,dx &= -\frac{b^2c^2\sqrt{d-c^2dx^2}}{3x}-\frac{b^2c^3\sqrt{d-c^2dx^2}\arccos(cx)}{3\sqrt{1-c^2x^2}} \\ &\quad -\frac{bc\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3x^2} \\ &\quad +\frac{ic^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{3dx^3} \\ &\quad -\frac{2bc^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{3\sqrt{1-c^2x^2}} \\ &\quad +\frac{ib^2c^3\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{3\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-1/3*b^2*c^2*(-c^2*d*x^2+d)^(1/2)/x-1/3*b^2*c^3*(-c^2*d*x^2+d)^(1/2)*arcco
s(c*x)/(-c^2*x^2+1)^(1/2)-1/3*b*c*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)*
(a+b*arccos(c*x))/x^2+1/3*I*c^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/(
-c^2*x^2+1)^(1/2)-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/d/x^3-2/3*b
*c^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2)
)^2)/(-c^2*x^2+1)^(1/2)+1/3*I*b^2*c^3*(-c^2*d*x^2+d)^(1/2)*polylog(2,(c*x+
I*(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^4}\,dx$$
$$= \frac{\sqrt{d-c^2dx^2}\Big(abcx-a^2\sqrt{1-c^2x^2}+a^2c^2x^2\sqrt{1-c^2x^2}-b^2c^2x^2\sqrt{1-c^2x^2}+b^2\big(-ic^3x^3-\sqrt{1-c^2x^2}+c^2}{}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/x^4,x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(a*b*c*x - a^2*Sqrt[1 - c^2*x^2] + a^2*c^2*x^2*Sqrt[1
 - c^2*x^2] - b^2*c^2*x^2*Sqrt[1 - c^2*x^2] + b^2*((-I)*c^3*x^3 - Sqrt[1 -
 c^2*x^2] + c^2*x^2*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2 + b*ArcCos[c*x]*(b*c*
x - 2*a*(1 - c^2*x^2)^(3/2) + 2*b*c^3*x^3*Log[1 + E^((2*I)*ArcCos[c*x])])
+ 2*a*b*c^3*x^3*Log[c*x] - I*b^2*c^3*x^3*PolyLog[2, -E^((2*I)*ArcCos[c*x])
]))/(3*x^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.65, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {5187, 5191, 247, 223, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^4}\,dx$$
$$\Big\downarrow \text{5187}$$
$$-\frac{2bc\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x^3}dx}{3\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3dx^3}$$
$$\Big\downarrow \text{5191}$$

$$-\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\left(-\int \frac{a+b\arccos(cx)}{x}dx\right)-\frac{1}{2}bc\int \frac{\sqrt{1-c^2x^2}}{x^2}dx-\frac{(1-c^2x^2)(a+b\arccos(cx))}{2x^2}\right)}{3\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{3dx^3}$$

$\Big\downarrow$ 247

$$-\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\left(-\int \frac{a+b\arccos(cx)}{x}dx\right)-\frac{1}{2}bc\left(c^2\left(-\int \frac{1}{\sqrt{1-c^2x^2}}dx\right)-\frac{\sqrt{1-c^2x^2}}{x}\right)-\frac{(1-c^2x^2)(a+b\arccos(cx))}{2x^2}\right)}{3\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{3dx^3}$$

$\Big\downarrow$ 223

$$-\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\left(-\int \frac{a+b\arccos(cx)}{x}dx\right)-\frac{(1-c^2x^2)(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(-c\arcsin(cx)-\frac{\sqrt{1-c^2x^2}}{x}\right)\right)}{3\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{3dx^3}$$

$\Big\downarrow$ 5137

$$-\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)-\frac{(1-c^2x^2)(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(-c\arcsin(cx)-\frac{\sqrt{1-c^2x^2}}{x}\right.\right.}{3\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{3dx^3}$$

$\Big\downarrow$ 3042

$$-\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)-\frac{(1-c^2x^2)(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(-c\arcsin(cx\right.\right.}{3\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{3dx^3}$$

$\Big\downarrow$ 4202

$$-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{3dx^3}-\frac{2bc\sqrt{d-c^2dx^2}\left(c^2\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)\right)-\frac{(1-c^2x^2)(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(-\right.\right.}{3\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 2620

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3dx^3}-$$
$$\frac{2bc\sqrt{d-c^2dx^2}\Big(c^2\Big(\frac{i(a+b\arccos(cx))^2}{2b}-2i\big(\frac{1}{2}ib\int\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b$$
$$3\sqrt{1-c^2x^2}$$

↓ 2715

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3dx^3}-$$
$$\frac{2bc\sqrt{d-c^2dx^2}\Big(c^2\Big(\frac{i(a+b\arccos(cx))^2}{2b}-2i\big(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{2}i\log\left(1+e^{2i\arccos}\right.$$
$$3\sqrt{1-c^2x^2}$$

↓ 2838

$$-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3dx^3}-$$
$$\frac{2bc\sqrt{d-c^2dx^2}\Big(c^2\Big(\frac{i(a+b\arccos(cx))^2}{2b}-2i\big(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{PolyLog}\left(2,-e^{2i\arccos}\right.$$
$$3\sqrt{1-c^2x^2}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/x^4,x]
```

output
```
-1/3*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/(d*x^3) - (2*b*c*Sqrt[d
 - c^2*d*x^2]*(-1/2*((1 - c^2*x^2)*(a + b*ArcCos[c*x]))/x^2 - (b*c*(-(Sqrt
[1 - c^2*x^2]/x) - c*ArcSin[c*x]))/2 + c^2*(((I/2)*(a + b*ArcCos[c*x])^2)/
b - (2*I)*((-1/2*I)*(a + b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCos[c*x])] - (
b*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/4))))/(3*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5191
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCos[c*x
])/(f*(m + 1))), x] + (Simp[b*c*(d^p/(f*(m + 1)))   Int[(f*x)^(m + 1)*(1 -
c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m + 2)
*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x]), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1909 vs. $2(296) = 592$.

Time = 0.66 (sec) , antiderivative size = 1910, normalized size of antiderivative = 6.08

method	result	size
default	Expression too large to display	1910
parts	Expression too large to display	1910

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-5/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x/(c^2*x^2-1)*arcc
os(c*x)^2*c^2+I*b^2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*c^3/(3*c^2*x
^2-3)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-2*b^2*(-d*(c^2*x^2-1))^(1/2
)*(-c^2*x^2+1)^(1/2)*c^3/(3*c^2*x^2-3)*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1
)^(1/2))^2)+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c^
2*x^2-1)*(-c^2*x^2+1)*c^6+1/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^
2*x^2+1)/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*c^3+2*I*b^2*(-d*(c^2*x^2-1))^(1/2)
*(-c^2*x^2+1)^(1/2)*c^3/(3*c^2*x^2-3)*arccos(c*x)^2-1/3*a^2/d/x^3*(-c^2*d*
x^2+d)^(3/2)-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c
^2*x^2-1)*c^8+5/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(
c^2*x^2-1)*c^6-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c
^2*x^2-1)*c^4+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x/(c^
2*x^2-1)*c^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^3/(c
^2*x^2-1)*arccos(c*x)^2+I*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+
1)*x^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arccos(c*x)^2*c^5-I*b^2*(-d*(c^2*x^2
-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^4/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*arcc
os(c*x)^2*c^7-1/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3
/(c^2*x^2-1)*(-c^2*x^2+1)*arccos(c*x)*c^6+1/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)
/(3*c^4*x^4-3*c^2*x^2+1)*x/(c^2*x^2-1)*(-c^2*x^2+1)*arccos(c*x)*c^4-I*b^2*
(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c^2*x^2-1)*(-c^2*x^...
```

Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^4}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x^4,x, algorithm="frica
s")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
)/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^4}\,dx = \int \frac{\sqrt{-d(cx-1)(cx+1)}(a+b\operatorname{acos}(cx))^2}{x^4}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))**2/x**4,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^4}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x^4,x, algorithm="maxim
a")
```

output
```
-1/3*((-1)^(-2*c^2*d*x^2 + 2*d)*c^2*d^(3/2)*log(-2*c^2*d + 2*d/x^2) + c^2*
d^(3/2)*log(x^2 - 1/c^2) - sqrt(c^4*d*x^4 - 2*c^2*d*x^2 + d)*d/x^2)*a*b*c/
d - 2/3*(-c^2*d*x^2 + d)^(3/2)*a*b*arccos(c*x)/(d*x^3) + 1/3*((c^2*x^2 - 1
)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(d)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1
), c*x)^2 + sqrt(d)*x^3*integrate(2*(c^3*x^2 - c)*arctan2(sqrt(c*x + 1)*sq
rt(-c*x + 1), c*x)/x^3, x))*b^2/x^3 - 1/3*(-c^2*d*x^2 + d)^(3/2)*a^2/(d*x^
3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/x^4,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^4}\,dx=\int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2\sqrt{d-c^2\,d\,x^2}}{x^4}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^4,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^4}\,dx$$
$$=\frac{\sqrt{d}\left(\sqrt{-c^2x^2+1}\,a^2c^2x^2-\sqrt{-c^2x^2+1}\,a^2+6\left(\int \frac{\sqrt{-c^2x^2+1}\,acos(cx)}{x^4}dx\right)ab\,x^3+3\left(\int \frac{\sqrt{-c^2x^2+1}\,acos(cx)^2}{x^4}dx\right)b^{}\right.}{3x^3}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))^2/x^4,x)
```

output
```
(sqrt(d)*(sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a
**2 + 6*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**4,x)*a*b*x**3 + 3*int((s
qrt( - c**2*x**2 + 1)*acos(c*x)**2)/x**4,x)*b**2*x**3))/(3*x**3)
```

3.220 $\int x^3(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 503

$$\begin{aligned}\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx &= \frac{304b^2d\sqrt{d-c^2dx^2}}{3675c^4} \\ &+\frac{4abdx\sqrt{d-c^2dx^2}}{35c^3\sqrt{1-c^2x^2}}+\frac{152b^2d(1-c^2x^2)\sqrt{d-c^2dx^2}}{11025c^4} \\ &+\frac{38b^2d(1-c^2x^2)^2\sqrt{d-c^2dx^2}}{6125c^4}-\frac{2b^2d(1-c^2x^2)^3\sqrt{d-c^2dx^2}}{343c^4} \\ &+\frac{4b^2dx\sqrt{d-c^2dx^2}\arccos(cx)}{35c^3\sqrt{1-c^2x^2}}+\frac{2bdx^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{105c\sqrt{1-c^2x^2}} \\ &-\frac{16bcdx^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{175\sqrt{1-c^2x^2}} \\ &+\frac{2bc^3dx^7\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{49\sqrt{1-c^2x^2}}-\frac{2d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{35c^4} \\ &-\frac{dx^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{35c^2}+\frac{3}{35}dx^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2 \\ &+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\end{aligned}$$

output
```
304/3675*b^2*d*(-c^2*d*x^2+d)^(1/2)/c^4+4/35*a*b*d*x*(-c^2*d*x^2+d)^(1/2)/
c^3/(-c^2*x^2+1)^(1/2)+152/11025*b^2*d*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/c
^4+38/6125*b^2*d*(-c^2*x^2+1)^2*(-c^2*d*x^2+d)^(1/2)/c^4-2/343*b^2*d*(-c^2
*x^2+1)^3*(-c^2*d*x^2+d)^(1/2)/c^4+4/35*b^2*d*x*(-c^2*d*x^2+d)^(1/2)*arcco
s(c*x)/c^3/(-c^2*x^2+1)^(1/2)+2/105*b*d*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arcc
os(c*x))/c/(-c^2*x^2+1)^(1/2)-16/175*b*c*d*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*a
rccos(c*x))/(-c^2*x^2+1)^(1/2)+2/49*b*c^3*d*x^7*(-c^2*d*x^2+d)^(1/2)*(a+b*
arccos(c*x))/(-c^2*x^2+1)^(1/2)-2/35*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*
x))^2/c^4-1/35*d*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/c^2+3/35*d*x
^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2+1/7*x^4*(-c^2*d*x^2+d)^(3/2)*(
a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.50

$$\int x^3 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^2\,dx =$$

$$-\frac{d\sqrt{d-c^2dx^2}\Big(11025a^2(-1+c^2x^2)^3\left(2+5c^2x^2\right)-210abcx\sqrt{1-c^2x^2}(210+35c^2x^2-168c^4x^4+75c^6x^6)}{}$$

input
```
Integrate[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/385875*(d*Sqrt[d - c^2*d*x^2]*(11025*a^2*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^
2) - 210*a*b*c*x*Sqrt[1 - c^2*x^2]*(210 + 35*c^2*x^2 - 168*c^4*x^4 + 75*c^
6*x^6) - 2*b^2*(-18692 + 20371*c^2*x^2 + 499*c^4*x^4 - 3303*c^6*x^6 + 1125
*c^8*x^8) - 210*b*(-105*a*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2) + b*c*x*Sqrt[1
- c^2*x^2]*(210 + 35*c^2*x^2 - 168*c^4*x^4 + 75*c^6*x^6))*ArcCos[c*x] + 11
025*b^2*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2)*ArcCos[c*x]^2))/(c^4*(-1 + c^2*x^
2))
```

Rubi [A] (verified)

Time = 2.72 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.12, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.621, Rules used = {5203, 5193, 27, 354, 86, 2009, 5199, 5139, 243, 53, 2009, 5211, 5139, 243, 53, 2009, 5183, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\int x^4\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{7\sqrt{1-c^2x^2}}+\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5193}$$

$$\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(bc\int \frac{x^5\left(7-5c^2x^2\right)}{35\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))\right)}{7\sqrt{1-c^2x^2}}+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{35}bc\int \frac{x^5\left(7-5c^2x^2\right)}{\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))\right)}{7\sqrt{1-c^2x^2}}+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{354}$$

$$\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{70}bc\int \frac{x^4\left(7-5c^2x^2\right)}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))\right)}{7\sqrt{1-c^2x^2}}+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$\Big\downarrow$ 86

$$\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{70}bc\int\left(\frac{5(1-c^2x^2)^{5/2}}{c^4}-\frac{8(1-c^2x^2)^{3/2}}{c^4}+\frac{\sqrt{1-c^2x^2}}{c^4}+\frac{2}{c^4\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}\right.}{7\sqrt{1-c^2x^2}}$$
$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$\Big\downarrow$ 2009

$$\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{70}bc\left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6}+\frac{16(1-c^2x^2)^{5/2}}{5c^6}-\frac{2(1\right.\right.}{7\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5199

$$\frac{3}{7}d\left(\frac{2bc\sqrt{d-c^2dx^2}\int x^4(a+b\arccos(cx))dx}{5\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos\right.$$
$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{70}bc\left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6}+\frac{16(1-c^2x^2)^{5/2}}{5c^6}-\frac{2(1\right.\right.}{7\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5139

$$\frac{3}{7}d\left(\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}bc\int\frac{x^5}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))\right)}{5\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-\right.$$
$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{70}bc\left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6}+\frac{16(1-c^2x^2)^{5/2}}{5c^6}-\frac{2(1\right.\right.}{7\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 243

$$\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{10}bc\int\frac{x^4}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{5}x^5(a+b\arccos(cx))\right)}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{70}bc\left(-\frac{10\left(1-c^2x^2\right)^{7/2}}{7c^6}+\frac{16\left(1-c^2x^2\right)^{5/2}}{5c^6}-\frac{2(\right.\right.}{7\sqrt{1-c^2x^2}}$$

$\downarrow$ 53

$$\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{10}bc\int\left(\frac{\left(1-c^2x^2\right)^{3/2}}{c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{1}{c^4\sqrt{1-c^2x^2}}\right)dx^2+\frac{1}{5}\right.}{5\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{70}bc\left(-\frac{10\left(1-c^2x^2\right)^{7/2}}{7c^6}+\frac{16\left(1-c^2x^2\right)^{5/2}}{5c^6}-\frac{2(\right.\right.}{7\sqrt{1-c^2x^2}}$$

$\downarrow$ 2009

$$\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\arccos\right.}{}\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{70}bc\left(-\frac{10\left(1-c^2x^2\right)^{7/2}}{7c^6}+\frac{16\left(1-c^2x^2\right)^{5/2}}{5c^6}-\frac{2(\right.\right.}{7\sqrt{1-c^2x^2}}$$

$\downarrow$ 5211

$$\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\int x^2(a+b\arccos(cx))dx}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}\right)}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{70}bc\left(-\frac{10\left(1-c^2x^2\right)^{7/2}}{7c^6}+\frac{16\left(1-c^2x^2\right)^{5/2}}{5c^6}-\frac{2(\right.\right.}{7\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5139

$$\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\left(\frac{1}{3}bc\int\frac{x^3}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}\right)}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{70}bc\left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6}+\frac{16(1-c^2x^2)^{5/2}}{5c^6}-\frac{2(1}{}\right.\right.}{7\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 243

$$\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\left(\frac{1}{6}bc\int\frac{x^2}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}\right)}{5\sqrt{1-c^2x^2}}+\frac{1}{5}\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{70}bc\left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6}+\frac{16(1-c^2x^2)^{5/2}}{5c^6}-\frac{2(1}{}\right.\right.}{7\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 53

$$\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\left(\frac{1}{6}bc\int\left(\frac{1}{c^2\sqrt{1-c^2x^2}}-\frac{\sqrt{1-c^2x^2}}{c^2}\right)dx^2+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos}{3c^2}\right.}{5\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{70}bc\left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6}+\frac{16(1-c^2x^2)^{5/2}}{5c^6}-\frac{2(1}{}\right.\right.}{7\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 2009

$$\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\arccos(cx))+\frac{1}{6}bc\left(\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right.\right.}{3c}\right.}{5\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{70}bc\left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6}+\frac{16(1-c^2x^2)^{5/2}}{5c^6}-\frac{2(1}{}\right.\right.}{7\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5183}$$

$$\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{2\left(-\frac{2b\int(a+b\arccos(cx))dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}\right)}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\arccos(cx))+\right.}{}\right.}{5\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{70}bc\left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6}+\frac{16(1-c^2x^2)^{5/2}}{5c^6}-\frac{2(1}{}\right.\right.}{7\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{70}bc\left(-\frac{10(1-c^2x^2)^{7/2}}{7c^6}+\frac{16(1-c^2x^2)^{5/2}}{5c^6}-\frac{2(1\right.\right.}{7\sqrt{1-c^2x^2}}$$

$$\frac{3}{7}d\left(\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2\right.\right.}{5\sqrt{1-c^2x^2}}\right.$$

input
```
Int[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/7 + (2*b*c*d*Sqrt[d - c^
2*d*x^2]*((b*c*((-4*Sqrt[1 - c^2*x^2])/c^6 - (2*(1 - c^2*x^2)^(3/2))/(3*c^
6) + (16*(1 - c^2*x^2)^(5/2))/(5*c^6) - (10*(1 - c^2*x^2)^(7/2))/(7*c^6)))
/70 + (x^5*(a + b*ArcCos[c*x]))/5 - (c^2*x^7*(a + b*ArcCos[c*x]))/7))/(7*S
qrt[1 - c^2*x^2]) + (3*d*((x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/
5 + (2*b*c*Sqrt[d - c^2*d*x^2]*((b*c*((-2*Sqrt[1 - c^2*x^2])/c^6 + (4*(1 -
 c^2*x^2)^(3/2))/(3*c^6) - (2*(1 - c^2*x^2)^(5/2))/(5*c^6)))/10 + (x^5*(a
+ b*ArcCos[c*x]))/5))/(5*Sqrt[1 - c^2*x^2]) + (Sqrt[d - c^2*d*x^2]*(-1/3*(
x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c^2 - (2*b*((b*c*((-2*Sqrt[1
- c^2*x^2])/c^4 + (2*(1 - c^2*x^2)^(3/2))/(3*c^4)))/6 + (x^3*(a + b*ArcCos
[c*x]))/3))/(3*c) + (2*(-((Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c^2) -
 (2*b*(a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]))/c))/(3*c^2)))/(5*
Sqrt[1 - c^2*x^2])))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.05

method	result
orering	$\frac{\left(47625 c^{10} x^{10}-130566 c^{8} x^{8}+68553 c^{6} x^{6}+279840 c^{4} x^{4}-260420 c^{2} x^{2}+74768\right) \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \arccos \left(c x \right)\right)^{2}}{128625 c^{6} x^{2} \left(c^{2} x^{2}-1\right)^{2}}-\frac{2 \left(10125 c^{8} x^{8}-24\right.}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/128625*(47625*c^10*x^10-130566*c^8*x^8+68553*c^6*x^6+279840*c^4*x^4-2604
20*c^2*x^2+74768)/c^6/x^2/(c^2*x^2-1)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c
*x))^2-2/385875*(10125*c^8*x^8-24174*c^6*x^6-863*c^4*x^4+118868*c^2*x^2-56
076)/c^6/x^4/(c^2*x^2-1)*(3*x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2-3
*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2*d*c^2-2*x^3*(-c^2*d*x^2+d)^(
3/2)*(a+b*arccos(c*x))*b*c/(-c^2*x^2+1)^(1/2))+1/385875*(1125*c^6*x^6-2178
*c^4*x^4-1679*c^2*x^2+18692)/c^6/x^3*(6*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos
(c*x))^2-21*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2*d*c^2-12*x^2*(-c^
2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))*b*c/(-c^2*x^2+1)^(1/2)+3*x^5/(-c^2*d*x^
2+d)^(1/2)*(a+b*arccos(c*x))^2*d^2*c^4+12*b*c^3*d*x^4*(-c^2*d*x^2+d)^(1/2)
*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+2*x^3*(-c^2*d*x^2+d)^(3/2)*b^2*c^2/(
-c^2*x^2+1)-2*x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))*b*c^3/(-c^2*x^2+1
)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.72

$$\int x^3 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^2\,dx = \frac{210\,(75\,abc^7dx^7-168\,abc^5dx^5+35\,abc^3dx^3+210\,abcdx+(75\,b^2c^7dx^7-168\,b^2c^5dx^5}{}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="frica
s")
```

output
```
1/385875*(210*(75*a*b*c^7*d*x^7 - 168*a*b*c^5*d*x^5 + 35*a*b*c^3*d*x^3 + 2
10*a*b*c*d*x + (75*b^2*c^7*d*x^7 - 168*b^2*c^5*d*x^5 + 35*b^2*c^3*d*x^3 +
210*b^2*c*d*x)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - (112
5*(49*a^2 - 2*b^2)*c^8*d*x^8 - 9*(15925*a^2 - 734*b^2)*c^6*d*x^6 + (99225*
a^2 - 998*b^2)*c^4*d*x^4 + (11025*a^2 - 40742*b^2)*c^2*d*x^2 + 11025*(5*b^
2*c^8*d*x^8 - 13*b^2*c^6*d*x^6 + 9*b^2*c^4*d*x^4 + b^2*c^2*d*x^2 - 2*b^2*d
)*arccos(c*x)^2 - 2*(11025*a^2 - 18692*b^2)*d + 22050*(5*a*b*c^8*d*x^8 - 1
3*a*b*c^6*d*x^6 + 9*a*b*c^4*d*x^4 + a*b*c^2*d*x^2 - 2*a*b*d)*arccos(c*x))*
sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F]

$$\int x^3 (d - c^2 d x^2)^{3/2} (a + b \arccos(cx))^2 \, dx = \int x^3 (-d(cx-1)(cx+1))^{\frac{3}{2}} (a + b \operatorname{acos}(cx))^2 \, dx$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.71

$$\int x^3(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx =$$
$$-\frac{1}{35}\left(\frac{5\,(-c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)b^2\arccos(cx)^2$$
$$-\frac{2}{35}\left(\frac{5\,(-c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)ab\arccos(cx)$$
$$-\frac{1}{35}\left(\frac{5\,(-c^2dx^2+d)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\,(-c^2dx^2+d)^{\frac{5}{2}}}{c^4d}\right)a^2$$
$$+\frac{2}{385875}b^2\left(\frac{1125\sqrt{-c^2x^2+1}c^4d^{\frac{3}{2}}x^6-2178\sqrt{-c^2x^2+1}c^2d^{\frac{3}{2}}x^4-1679\sqrt{-c^2x^2+1}d^{\frac{3}{2}}x^2+\frac{18692\sqrt{-c^2x^2+1}}{c^2}\right.$$
$$-\frac{2\left(75\,c^6d^{\frac{3}{2}}x^7-168\,c^4d^{\frac{3}{2}}x^5+35\,c^2d^{\frac{3}{2}}x^3+210\,d^{\frac{3}{2}}x\right)ab}{3675\,c^3}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="maxim
a")
```

output
```
-1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^
4*d))*b^2*arccos(c*x)^2 - 2/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(
-c^2*d*x^2 + d)^(5/2)/(c^4*d))*a*b*arccos(c*x) - 1/35*(5*(-c^2*d*x^2 + d)^
(5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^4*d))*a^2 + 2/385875*b^2*(
(1125*sqrt(-c^2*x^2 + 1)*c^4*d^(3/2)*x^6 - 2178*sqrt(-c^2*x^2 + 1)*c^2*d^(
3/2)*x^4 - 1679*sqrt(-c^2*x^2 + 1)*d^(3/2)*x^2 + 18692*sqrt(-c^2*x^2 + 1)*
d^(3/2)/c^2)/c^2 - 105*(75*c^6*d^(3/2)*x^7 - 168*c^4*d^(3/2)*x^5 + 35*c^2*
d^(3/2)*x^3 + 210*d^(3/2)*x)*arccos(c*x)/c^3) - 2/3675*(75*c^6*d^(3/2)*x^7
 - 168*c^4*d^(3/2)*x^5 + 35*c^2*d^(3/2)*x^3 + 210*d^(3/2)*x)*a*b/c^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\int x^3\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

input
```
int(x^3*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^3*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{\sqrt{d}\,d\left(-5\sqrt{-c^2x^2+1}\,a^2c^6x^6+8\sqrt{-c^2x^2+1}\,a^2c^4x^4-\sqrt{-c^2x^2+1}\,a^2c^2x^2-2\sqrt{-c^2x^2}\right.}{}$$

input
```
int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*d*( - 5*sqrt( - c**2*x**2 + 1)*a**2*c**6*x**6 + 8*sqrt( - c**2*x*
*2 + 1)*a**2*c**4*x**4 - sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - 2*sqrt( -
 c**2*x**2 + 1)*a**2 - 70*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**5,x)*a*b
*c**6 + 70*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**3,x)*a*b*c**4 - 35*int(
sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x**5,x)*b**2*c**6 + 35*int(sqrt( - c**
2*x**2 + 1)*acos(c*x)**2*x**3,x)*b**2*c**4))/(35*c**4)
```

3.221 $\int x^2(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 421

$$\begin{aligned}\int x^2(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx &= -\frac{7b^2dx\sqrt{d-c^2dx^2}}{1152c^2} \\ &-\frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728}+\frac{1}{108}b^2c^2dx^5\sqrt{d-c^2dx^2} \\ &+\frac{7b^2d\sqrt{d-c^2dx^2}\arccos(cx)}{1152c^3\sqrt{1-c^2x^2}}+\frac{bdx^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{16c\sqrt{1-c^2x^2}} \\ &-\frac{7bcdx^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{48\sqrt{1-c^2x^2}}+\frac{bc^3dx^6\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{18\sqrt{1-c^2x^2}} \\ &-\frac{dx\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{16c^2}+\frac{1}{8}dx^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2 \\ &+\frac{1}{6}x^3(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2+\frac{d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{48bc^3\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-7/1152*b^2*d*x*(-c^2*d*x^2+d)^(1/2)/c^2-43/1728*b^2*d*x^3*(-c^2*d*x^2+d)^
(1/2)+1/108*b^2*c^2*d*x^5*(-c^2*d*x^2+d)^(1/2)+7/1152*b^2*d*(-c^2*d*x^2+d)
^(1/2)*arccos(c*x)/c^3/(-c^2*x^2+1)^(1/2)+1/16*b*d*x^2*(-c^2*d*x^2+d)^(1/2
)*(a+b*arccos(c*x))/c/(-c^2*x^2+1)^(1/2)-7/48*b*c*d*x^4*(-c^2*d*x^2+d)^(1/
2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+1/18*b*c^3*d*x^6*(-c^2*d*x^2+d)^(1
/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-1/16*d*x*(-c^2*d*x^2+d)^(1/2)*(a+
b*arccos(c*x))^2/c^2+1/8*d*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2+1/
6*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2+1/48*d*(-c^2*d*x^2+d)^(1/2)
*(a+b*arccos(c*x))^3/b/c^3/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.97

$$\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{-288b^2d\sqrt{d-c^2dx^2}\arccos(cx)^3-864a^2d^{3/2}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+12bd\sqrt{\cdots}}{\cdots}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(-288*b^2*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^3 - 864*a^2*d^(3/2)*Sqrt[1 - c
^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 12*b*
d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(18*b*Cos[2*ArcCos[c*x]] + 9*b*Cos[4*Arc
Cos[c*x]] - 2*b*Cos[6*ArcCos[c*x]] + 36*a*Sin[2*ArcCos[c*x]] + 36*a*Sin[4*
ArcCos[c*x]] - 12*a*Sin[6*ArcCos[c*x]]) - 72*b*d*Sqrt[d - c^2*d*x^2]*ArcCo
s[c*x]^2*(12*a - 3*b*Sin[2*ArcCos[c*x]] - 3*b*Sin[4*ArcCos[c*x]] + b*Sin[6
*ArcCos[c*x]]) + d*Sqrt[d - c^2*d*x^2]*(-864*a^2*c*x*Sqrt[1 - c^2*x^2] + 4
032*a^2*c^3*x^3*Sqrt[1 - c^2*x^2] - 2304*a^2*c^5*x^5*Sqrt[1 - c^2*x^2] + 2
16*a*b*Cos[2*ArcCos[c*x]] + 108*a*b*Cos[4*ArcCos[c*x]] - 24*a*b*Cos[6*ArcC
os[c*x]] - 108*b^2*Sin[2*ArcCos[c*x]] - 27*b^2*Sin[4*ArcCos[c*x]] + 4*b^2*
Sin[6*ArcCos[c*x]]))/(13824*c^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.586$, Rules used = {5203, 5193, 27, 363, 262, 262, 223, 5199, 5139, 262, 262, 223, 5211, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos (c x))^2 \, dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{1}{2} d \int x^2 \sqrt{d-c^2 d x^2} (a+b \arccos (c x))^2 dx+\frac{b c d \sqrt{d-c^2 d x^2} \int x^3 \left(1-c^2 x^2\right) (a+b \arccos (c x)) dx}{3 \sqrt{1-c^2 x^2}}+\frac{1}{6} x^3 \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos (c x))^2$$

$$\Big\downarrow \text{5193}$$

$$\frac{1}{2} d \int x^2 \sqrt{d-c^2 d x^2} (a+b \arccos (c x))^2 dx+\frac{b c d \sqrt{d-c^2 d x^2} \left(b c \int \frac{x^4 \left(3-2 c^2 x^2\right)}{12 \sqrt{1-c^2 x^2}} dx-\frac{1}{6} c^2 x^6 (a+b \arccos (c x))+\frac{1}{4} x^4 (a+b \arccos (c x))\right)}{3 \sqrt{1-c^2 x^2}}+\frac{1}{6} x^3 \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos (c x))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} d \int x^2 \sqrt{d-c^2 d x^2} (a+b \arccos (c x))^2 dx+\frac{b c d \sqrt{d-c^2 d x^2} \left(\frac{1}{12} b c \int \frac{x^4 \left(3-2 c^2 x^2\right)}{\sqrt{1-c^2 x^2}} dx-\frac{1}{6} c^2 x^6 (a+b \arccos (c x))+\frac{1}{4} x^4 (a+b \arccos (c x))\right)}{3 \sqrt{1-c^2 x^2}}+\frac{1}{6} x^3 \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos (c x))^2$$

$$\Big\downarrow \text{363}$$

$$\frac{1}{2} d \int x^2 \sqrt{d-c^2 d x^2} (a+b \arccos (c x))^2 dx+\frac{b c d \sqrt{d-c^2 d x^2} \left(\frac{1}{12} b c \left(\frac{4}{3} \int \frac{x^4}{\sqrt{1-c^2 x^2}} dx+\frac{1}{3} x^5 \sqrt{1-c^2 x^2}\right)-\frac{1}{6} c^2 x^6 (a+b \arccos (c x))+\frac{1}{4} x^4 (a+b \arccos (c x))\right)}{3 \sqrt{1-c^2 x^2}}+\frac{1}{6} x^3 \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos (c x))^2$$

$\downarrow$ 262

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\int\frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{3}x^5\sqrt{1-c^2x^2}\right)-\frac{1}{6}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b}{3\sqrt{1-c^2x^2}}$$
$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$\downarrow$ 262

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{3}x^5\sqrt{1-c^2x^2}\right)-\frac{1}{6}c^2x^6(a+b\arccos(cx)}{3\sqrt{1-c^2x^2}}$$
$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$\downarrow$ 223

$$\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2}}{4c^2}\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$
$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$\downarrow$ 5199

$$\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int x^3(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\arccos\right.$$
$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2}}{4c^2}\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$
$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$\downarrow$ 5139

$$\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}bc\int\frac{x^4}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))\right)}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2}}{4c^2}\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow 262$$

$$\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}bc\left(\frac{3\int\frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{4}x^4(a+b\arccos(cx\right.}{2\sqrt{1-c^2x^2}}\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2}}{4c^2}\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow 262$$

$$\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}bc\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{4}x^4(a\right.}{2\sqrt{1-c^2x^2}}\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2}}{4c^2}\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow 223$$

$$\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\right.\right.}{2\sqrt{1-c^2x^2}}\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2}}{4c^2}\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \; 5211$$

$$\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int x(a+b\arccos(cx))dx}{c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+\right.}{}\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2}}{4c^2}\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \; 5139$$

$$\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{1}{2}bc\int\frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c}+\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d}}{}\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2}}{4c^2}\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \; 262$$

$$\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c}+\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx}{2c^2}\right.}{4\sqrt{1-c^2x^2}}\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2}}{4c^2}\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$\downarrow$ 223

$$\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{4\sqrt{1-c^2x^2}}\right. +$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2}}{4c^2}\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$\downarrow$ 5153

$$\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c}-\frac{(a+b\arccos(cx))^3}{6bc^3}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{4\sqrt{1-c^2x^2}}\right. +$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2}}{4c^2}\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

input
```
Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/6 + (b*c*d*Sqrt[d - c^2*
d*x^2]*((x^4*(a + b*ArcCos[c*x]))/4 - (c^2*x^6*(a + b*ArcCos[c*x]))/6 + (b
*c*((x^5*Sqrt[1 - c^2*x^2])/3 + (4*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*
(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/3))/12))
/(3*Sqrt[1 - c^2*x^2]) + (d*((x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^
2)/4 + (b*c*Sqrt[d - c^2*d*x^2]*((x^4*(a + b*ArcCos[c*x]))/4 + (b*c*(-1/4*
(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[
c*x]/(2*c^3)))/(4*c^2)))/4))/(2*Sqrt[1 - c^2*x^2]) + (Sqrt[d - c^2*d*x^2]*
(-1/2*(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c^2 - (a + b*ArcCos[c*x]
)^3/(6*b*c^3) - (b*((x^2*(a + b*ArcCos[c*x]))/2 + (b*c*(-1/2*(x*Sqrt[1 - c
^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2))/c))/(4*Sqrt[1 - c^2*x^2])))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 1432, normalized size of antiderivative = 3.40

method	result	size
default	Expression too large to display	1432
parts	Expression too large to display	1432

input
```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^2*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/24*a^2/c^2*x*(-c^2*d*x^2+d)^(3/2)+
1/16*a^2/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/16*a^2/c^2*d^2/(c^2*d)^(1/2)*arcta
n((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(1/48*(-d*(c^2*x^2-1))^(1/2)*(
-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^3*d-1/6912*(-d*(c^2*x^2-1))^
(1/2)*(32*c^7*x^7-64*c^5*x^5+32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+38*c^3*x^3-48
*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-6*c*x+18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^
2*x^2+1)^(1/2))*(6*I*arccos(c*x)+18*arccos(c*x)^2-1)*d/c^3/(c^2*x^2-1)+1/1
024*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^
4*c^4+4*c*x-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(4*I*arcc
os(c*x)+8*arccos(c*x)^2-1)*d/c^3/(c^2*x^2-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*
(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*
arccos(c*x)^2-1-2*I*arccos(c*x))*d/c^3/(c^2*x^2-1)-1/6912*(-d*(c^2*x^2-1))
^(1/2)*(-32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+32*c^7*x^7+48*I*(-c^2*x^2+1)^(1/2
)*x^4*c^4-64*c^5*x^5-18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3+I*(-c^2*x^
2+1)^(1/2)-6*c*x)*(-6*I*arccos(c*x)+18*arccos(c*x)^2-1)*d/c^3/(c^2*x^2-1)-
3/1024*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(I+4*ar
ccos(c*x))*cos(3*arccos(c*x))*d/c^3/(c^2*x^2-1)-1/1024*(-d*(c^2*x^2-1))^(1
/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(4*I*arccos(c*x)+16*arccos(c*x)^2
-5)*sin(3*arccos(c*x))*d/c^3/(c^2*x^2-1))+2*a*b*(1/32*(-d*(c^2*x^2-1))^(1/
2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2*d-1/2304*(-d*(c^2*x...
```

Fricas [F]

$$\int x^2 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^2\,dx=\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\arccos(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="frica
s")
```

output
```
integral(-(a^2*c^2*d*x^4 - a^2*d*x^2 + (b^2*c^2*d*x^4 - b^2*d*x^2)*arccos(
c*x)^2 + 2*(a*b*c^2*d*x^4 - a*b*d*x^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d),
x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx=\text{Timed out}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx=\int(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="maxim
a")
```

output
```
1/48*a^2*(2*(-c^2*d*x^2 + d)^(3/2)*x/c^2 - 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^2
*d) + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 + 3*d^(3/2)*arcsin(c*x)/c^3) + sqrt(d
)*integrate(-((b^2*c^2*d*x^4 - b^2*d*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x
+ 1), c*x)^2 + 2*(a*b*c^2*d*x^4 - a*b*d*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c
*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.22

$$\int x^2(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx=-\frac{1}{6}\sqrt{-c^2dx^2+d}a^2c^2dx^5$$
$$+\frac{7}{24}\sqrt{-c^2dx^2+d}a^2dx^3-\frac{\sqrt{-c^2dx^2+d}a^2dx}{16c^2}-\frac{a^2d^2\log\left(\left|-c\sqrt{-d}x+\sqrt{c^2x^2-1}\sqrt{-d}\right|\right)}{16c^3\sqrt{-d}}$$
$$-\frac{192b^2c^5d^{\frac{3}{2}}x^6\arccos(cx)+576\sqrt{-c^2x^2+1}b^2c^4d^{\frac{3}{2}}x^5\arccos(cx)^2+192abc^5d^{\frac{3}{2}}x^6+1152\sqrt{-c^2x^2+1}abc}{}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="giac"
)
```

output
```
-1/6*sqrt(-c^2*d*x^2 + d)*a^2*c^2*d*x^5 + 7/24*sqrt(-c^2*d*x^2 + d)*a^2*d*
x^3 - 1/16*sqrt(-c^2*d*x^2 + d)*a^2*d*x/c^2 - 1/16*a^2*d^2*log(abs(-c*sqrt
(-d)*x + sqrt(c^2*x^2 - 1)*sqrt(-d)))/(c^3*sqrt(-d)) - 1/3456*(192*b^2*c^5
*d^(3/2)*x^6*arccos(c*x) + 576*sqrt(-c^2*x^2 + 1)*b^2*c^4*d^(3/2)*x^5*arcc
os(c*x)^2 + 192*a*b*c^5*d^(3/2)*x^6 + 1152*sqrt(-c^2*x^2 + 1)*a*b*c^4*d^(3
/2)*x^5*arccos(c*x) - 32*sqrt(-c^2*x^2 + 1)*b^2*c^4*d^(3/2)*x^5 - 504*b^2*
c^3*d^(3/2)*x^4*arccos(c*x) - 1008*sqrt(-c^2*x^2 + 1)*b^2*c^2*d^(3/2)*x^3*
arccos(c*x)^2 - 504*a*b*c^3*d^(3/2)*x^4 - 2016*sqrt(-c^2*x^2 + 1)*a*b*c^2*
d^(3/2)*x^3*arccos(c*x) + 86*sqrt(-c^2*x^2 + 1)*b^2*c^2*d^(3/2)*x^3 + 216*
b^2*c*d^(3/2)*x^2*arccos(c*x) + 216*sqrt(-c^2*x^2 + 1)*b^2*d^(3/2)*x*arcco
s(c*x)^2 + 216*a*b*c*d^(3/2)*x^2 + 432*sqrt(-c^2*x^2 + 1)*a*b*d^(3/2)*x*ar
ccos(c*x) + 72*b^2*d^(3/2)*arccos(c*x)^3/c + 21*sqrt(-c^2*x^2 + 1)*b^2*d^(
3/2)*x + 216*a*b*d^(3/2)*arccos(c*x)^2/c + 21*b^2*d^(3/2)*arccos(c*x)/c +
21*a*b*d^(3/2)/c)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\int x^2\,(a+b\,\mathrm{acos}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int(x^2*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^2*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^2 (d - c^2 d x^2)^{3/2} (a + b \arccos(cx))^2 \, dx = \frac{\sqrt{d}\, d \big(3 asin(cx)\, a^2 - 8\sqrt{-c^2x^2+1}\, a^2 c^5 x^5 + 14\sqrt{-c^2x^2+1}\, a^2 c^3 x^3 - 3\sqrt{-c^2x^2+1}\, a^2}{}$$

input
```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*d*(3*asin(c*x)*a**2 - 8*sqrt( - c**2*x**2 + 1)*a**2*c**5*x**5 + 1
4*sqrt( - c**2*x**2 + 1)*a**2*c**3*x**3 - 3*sqrt( - c**2*x**2 + 1)*a**2*c*
x - 96*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**4,x)*a*b*c**5 + 96*int(sqrt
( - c**2*x**2 + 1)*acos(c*x)*x**2,x)*a*b*c**3 - 48*int(sqrt( - c**2*x**2 +
 1)*acos(c*x)**2*x**4,x)*b**2*c**5 + 48*int(sqrt( - c**2*x**2 + 1)*acos(c*
x)**2*x**2,x)*b**2*c**3))/(48*c**3)
```

3.222 $\int x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 279

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{16b^2d\sqrt{d-c^2dx^2}}{75c^2}+\frac{8b^2d(1-c^2x^2)\sqrt{d-c^2dx^2}}{225c^2}+\frac{2b^2d(1-c^2x^2)^2\sqrt{d-c^2dx^2}}{125c^2}+\frac{2bdx\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{5c\sqrt{1-c^2x^2}}-\frac{4bcdx^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{15\sqrt{1-c^2x^2}}+\frac{2bc^3dx^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{25\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{5c^2d}$$

output
```
16/75*b^2*d*(-c^2*d*x^2+d)^(1/2)/c^2+8/225*b^2*d*(-c^2*x^2+1)*(-c^2*d*x^2+
d)^(1/2)/c^2+2/125*b^2*d*(-c^2*x^2+1)^2*(-c^2*d*x^2+d)^(1/2)/c^2+2/5*b*d*x
*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c/(-c^2*x^2+1)^(1/2)-4/15*b*c*d*x^
3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+2/25*b*c^3*d*x
^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-1/5*(-c^2*d*x
^2+d)^(5/2)*(a+b*arccos(c*x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.72

$$\int x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx =$$

$$-\frac{d\sqrt{d-c^2dx^2}\Big(225a^2(-1+c^2x^2)^3-30abcx\sqrt{1-c^2x^2}(15-10c^2x^2+3c^4x^4)+2b^2(149-187c^2x^2+47c^4$$

input
```
Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/1125*(d*Sqrt[d - c^2*d*x^2]*(225*a^2*(-1 + c^2*x^2)^3 - 30*a*b*c*x*Sqrt
[1 - c^2*x^2]*(15 - 10*c^2*x^2 + 3*c^4*x^4) + 2*b^2*(149 - 187*c^2*x^2 + 4
7*c^4*x^4 - 9*c^6*x^6) - 30*b*(-15*a*(-1 + c^2*x^2)^3 + b*c*x*Sqrt[1 - c^2
*x^2]*(15 - 10*c^2*x^2 + 3*c^4*x^4))*ArcCos[c*x] + 225*b^2*(-1 + c^2*x^2)^
3*ArcCos[c*x]^2))/(c^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.68, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5183, 5155, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \; 5183$$

$$-\frac{2bd\sqrt{d-c^2dx^2}\int(1-c^2x^2)^2(a+b\arccos(cx))dx}{5c\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{5c^2d}$$

$$\Big\downarrow \; 5155$$

$$-\frac{2bd\sqrt{d-c^2dx^2}\left(bc\int \frac{x(3c^4x^4-10c^2x^2+15)}{15\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\,\mathrm{arcco}\right.}{5c\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{5c^2d}$$

$\downarrow$ 27

$$-\frac{2bd\sqrt{d-c^2dx^2}\left(\frac{1}{15}bc\int \frac{x(3c^4x^4-10c^2x^2+15)}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\,\mathrm{arc}\right.}{5c\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{5c^2d}$$

$\downarrow$ 1576

$$-\frac{2bd\sqrt{d-c^2dx^2}\left(\frac{1}{30}bc\int \frac{3c^4x^4-10c^2x^2+15}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\,\mathrm{arcc}\right.}{5c\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{5c^2d}$$

$\downarrow$ 1140

$$-\frac{2bd\sqrt{d-c^2dx^2}\left(\frac{1}{30}bc\int \left(3\left(1-c^2x^2\right)^{3/2}+4\sqrt{1-c^2x^2}+\frac{8}{\sqrt{1-c^2x^2}}\right)dx^2+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+\right.}{5c\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{5c^2d}$$

$\downarrow$ 2009

$$-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{5c^2d}-$$

$$\frac{2bd\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6(1-c^2x^2)^{5/2}}{5c^2}\right.\right.}{5c\sqrt{1-c^2x^2}}$$

input

```
Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/5*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/(c^2*d) - (2*b*d*Sqrt[d
 - c^2*d*x^2]*((b*c*((-16*Sqrt[1 - c^2*x^2])/c^2 - (8*(1 - c^2*x^2)^(3/2))
/(3*c^2) - (6*(1 - c^2*x^2)^(5/2))/(5*c^2)))/30 + x*(a + b*ArcCos[c*x]) -
(2*c^2*x^3*(a + b*ArcCos[c*x]))/3 + (c^4*x^5*(a + b*ArcCos[c*x]))/5))/(5*c
*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.67

method	result
orering	$\frac{(549c^8x^8-1982c^6x^6+4355c^4x^4-1420c^2x^2+298)(-c^2dx^2+d)^{\frac{3}{2}}(a+b\arccos(cx))^2}{1125c^4x^2(c^2x^2-1)^2} - \frac{2(54c^6x^6-217c^4x^4+672c^2x^2-149)\left((-c^2\right.}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/1125*(549*c^8*x^8-1982*c^6*x^6+4355*c^4*x^4-1420*c^2*x^2+298)/c^4/x^2/(c
^2*x^2-1)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2-2/1125*(54*c^6*x^6-21
7*c^4*x^4+672*c^2*x^2-149)/c^4/x^2/(c^2*x^2-1)*((-c^2*d*x^2+d)^(3/2)*(a+b*
arccos(c*x))^2-3*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2*d*c^2-2*x*(-
c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))*b*c/(-c^2*x^2+1)^(1/2))+1/1125*(9*c^4
*x^4-38*c^2*x^2+149)/c^4/x*(-9*c^2*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*
x))^2-4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))*b*c/(-c^2*x^2+1)^(1/2)+3*x^
3/(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2*d^2*c^4+12*b*c^3*d*x^2*(-c^2*d*
x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+2*x*(-c^2*d*x^2+d)^(3/2)
*b^2*c^2/(-c^2*x^2+1)-2*x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))*b*c^3/(
-c^2*x^2+1)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.06

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx = \frac{30\,(3\,abc^5dx^5-10\,abc^3dx^3+15\,abcdx+(3\,b^2c^5dx^5-10\,b^2c^3dx^3+15\,b^2cdx)\arccos(c}{}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
1/1125*(30*(3*a*b*c^5*d*x^5 - 10*a*b*c^3*d*x^3 + 15*a*b*c*d*x + (3*b^2*c^5
*d*x^5 - 10*b^2*c^3*d*x^3 + 15*b^2*c*d*x)*arccos(c*x))*sqrt(-c^2*d*x^2 + d
)*sqrt(-c^2*x^2 + 1) - (9*(25*a^2 - 2*b^2)*c^6*d*x^6 - (675*a^2 - 94*b^2)*
c^4*d*x^4 + (675*a^2 - 374*b^2)*c^2*d*x^2 + 225*(b^2*c^6*d*x^6 - 3*b^2*c^4
*d*x^4 + 3*b^2*c^2*d*x^2 - b^2*d)*arccos(c*x)^2 - (225*a^2 - 298*b^2)*d +
450*(a*b*c^6*d*x^6 - 3*a*b*c^4*d*x^4 + 3*a*b*c^2*d*x^2 - a*b*d)*arccos(c*x
))*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F]

$$\int x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx=\int x(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))^2\,dx$$

input
```
integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.85

$$\int x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx=-\frac{(-c^2dx^2+d)^{\frac{5}{2}}b^2\arccos(cx)^2}{5c^2d}+\frac{2}{1125}b^2\left(\frac{9\sqrt{-c^2x^2+1}c^2d^{\frac{5}{2}}x^4-38\sqrt{-c^2x^2+1}d^{\frac{5}{2}}x^2+\frac{149\sqrt{-c^2x^2+1}d^{\frac{5}{2}}}{c^2}}{d}-\frac{15\left(3c^4d^{\frac{5}{2}}x^5-10c^2d^{\frac{5}{2}}x^3+15 \right.}{cd}\right.$$
$$-\frac{2(-c^2dx^2+d)^{\frac{5}{2}}ab\arccos(cx)}{5c^2d}-\frac{(-c^2dx^2+d)^{\frac{5}{2}}a^2}{5c^2d}-\frac{2\left(3c^4d^{\frac{5}{2}}x^5-10c^2d^{\frac{5}{2}}x^3+15d^{\frac{5}{2}}x\right)ab}{75cd}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-1/5*(-c^2*d*x^2 + d)^(5/2)*b^2*arccos(c*x)^2/(c^2*d) + 2/1125*b^2*((9*sqr
t(-c^2*x^2 + 1)*c^2*d^(5/2)*x^4 - 38*sqrt(-c^2*x^2 + 1)*d^(5/2)*x^2 + 149*
sqrt(-c^2*x^2 + 1)*d^(5/2)/c^2)/d - 15*(3*c^4*d^(5/2)*x^5 - 10*c^2*d^(5/2)
*x^3 + 15*d^(5/2)*x)*arccos(c*x)/(c*d)) - 2/5*(-c^2*d*x^2 + d)^(5/2)*a*b*a
rccos(c*x)/(c^2*d) - 1/5*(-c^2*d*x^2 + d)^(5/2)*a^2/(c^2*d) - 2/75*(3*c^4*
d^(5/2)*x^5 - 10*c^2*d^(5/2)*x^3 + 15*d^(5/2)*x)*a*b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx = \int x\,(a+b\,\mathrm{acos}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int(x*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx = \frac{\sqrt{d}\,d\left(-\sqrt{-c^2x^2+1}\,a^2c^4x^4+2\sqrt{-c^2x^2+1}\,a^2c^2x^2-\sqrt{-c^2x^2+1}\,a^2-10\left(\int\sqrt{-c^2x^2\right.\right.}{}$$

input
```
int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*d*( - sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 + 2*sqrt( - c**2*x**2
 + 1)*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a**2 - 10*int(sqrt( - c**2*x
**2 + 1)*acos(c*x)*x**3,x)*a*b*c**4 + 10*int(sqrt( - c**2*x**2 + 1)*acos(c
*x)*x,x)*a*b*c**2 - 5*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x**3,x)*b**2
*c**4 + 5*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x,x)*b**2*c**2))/(5*c**2
)
```

3.223 $\int (d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 305

$$\int (d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\,dx = -\frac{17}{64}b^2dx\sqrt{d-c^2dx^2} + \frac{1}{32}b^2c^2dx^3\sqrt{d-c^2dx^2} + \frac{17b^2d\sqrt{d-c^2dx^2}\arccos(cx)}{64c\sqrt{1-c^2x^2}} - \frac{5bcdx^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{8\sqrt{1-c^2x^2}} + \frac{bc^3dx^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{8\sqrt{1-c^2x^2}} + \frac{3}{8}dx\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2 + \frac{1}{4}x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2 + \frac{d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{8bc\sqrt{1-c^2x^2}}$$

output

```
-17/64*b^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/32*b^2*c^2*d*x^3*(-c^2*d*x^2+d)^(1/2
)+17/64*b^2*d*(-c^2*d*x^2+d)^(1/2)*arccos(c*x)/c/(-c^2*x^2+1)^(1/2)-5/8*b*
c*d*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+1/8*b*c^
3*d*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+3/8*d*x*
(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2+1/4*x*(-c^2*d*x^2+d)^(3/2)*(a+b*a
rccos(c*x))^2+1/8*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^3/b/c/(-c^2*x^2
+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.08

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx = \frac{-32b^2d\sqrt{d-c^2dx^2}\arccos(cx)^3-96a^2d^{3/2}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)-8bd\sqrt{d-\cdots}}{\cdots}$$

input
```
Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(-32*b^2*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^3 - 96*a^2*d^(3/2)*Sqrt[1 - c^2
*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 8*b*d*S
qrt[d - c^2*d*x^2]*ArcCos[c*x]^2*(12*a - 8*b*Sin[2*ArcCos[c*x]] + b*Sin[4*
ArcCos[c*x]]) + d*Sqrt[d - c^2*d*x^2]*(160*a^2*c*x*Sqrt[1 - c^2*x^2] - 64*
a^2*c^3*x^3*Sqrt[1 - c^2*x^2] + 64*a*b*Cos[2*ArcCos[c*x]] - 4*a*b*Cos[4*Ar
cCos[c*x]] - 32*b^2*Sin[2*ArcCos[c*x]] + b^2*Sin[4*ArcCos[c*x]]) - 4*b*d*S
qrt[d - c^2*d*x^2]*ArcCos[c*x]*(-16*b*Cos[2*ArcCos[c*x]] + b*Cos[4*ArcCos[
c*x]] + 4*a*(-8*Sin[2*ArcCos[c*x]] + Sin[4*ArcCos[c*x]])))/(256*c*Sqrt[1 -
 c^2*x^2])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5159, 5157, 5139, 262, 223, 5153, 5183, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{ 5159}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5157}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+$$
$$\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\int x(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos(cx)\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5139}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+$$
$$\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{262}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+$$
$$\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{4}d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$
$$\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5153}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+$$
$$\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5183}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{b\int \left(1-c^2x^2\right)^{3/2}dx}{4c}-\frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{211}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{3}{4}\int \sqrt{1-c^2x^2}dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)}{4c}-\frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{211}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)}{4c}-\frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{223}$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$
$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

input
```
Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/4 + (3*d*((x*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCos[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])
^3)/(6*b*c*Sqrt[1 - c^2*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*((x^2*(a + b*ArcC
os[c*x]))/2 + (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))
/2))/Sqrt[1 - c^2*x^2]))/4 + (b*c*d*Sqrt[d - c^2*d*x^2]*(-1/4*((1 - c^2*x^
2)^2*(a + b*ArcCos[c*x]))/c^2 - (b*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqr
t[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/(4*c)))/(2*Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.00 (sec) , antiderivative size = 987, normalized size of antiderivative = 3.24

method	result
default	$\frac{a^2x(-c^2d\,x^2+d)^{\frac{3}{2}}}{4}+\frac{3a^2dx\sqrt{-c^2d\,x^2+d}}{8}+\frac{3a^2d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{8\sqrt{c^2d}}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3d}{8(c^2x^2-1)c}-\right.$
parts	$\frac{a^2x(-c^2d\,x^2+d)^{\frac{3}{2}}}{4}+\frac{3a^2dx\sqrt{-c^2d\,x^2+d}}{8}+\frac{3a^2d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{8\sqrt{c^2d}}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3d}{8(c^2x^2-1)c}-\right.$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x*(-c^2*d*x^2+d)^(3/2)+3/8*a^2*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a^2*d^
2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(1/8*(-d*
(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^3*d-1/512*
(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^
4+4*c*x-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(4*I*arccos(c
*x)+8*arccos(c*x)^2-1)*d/(c^2*x^2-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*
x^3-2*c*x+2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(2*arccos(c
*x)^2-1+2*I*arccos(c*x))*d/(c^2*x^2-1)/c+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I
*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*arcco
s(c*x)^2-1-2*I*arccos(c*x))*d/(c^2*x^2-1)/c-1/512*(-d*(c^2*x^2-1))^(1/2)*(
-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-1
2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(-4*I*arccos(c*x)+8*arccos(c*x)^2-1)
*d/(c^2*x^2-1)/c)+2*a*b*(3/16*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c
^2*x^2-1)/c*arccos(c*x)^2*d-1/256*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3
*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I
*(-c^2*x^2+1)^(1/2))*(I+4*arccos(c*x))*d/(c^2*x^2-1)/c+1/16*(-d*(c^2*x^2-1
))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2
*c*x)*(-I+2*arccos(c*x))*d/(c^2*x^2-1)/c-3/256*(-d*(c^2*x^2-1))^(1/2)*(-I*
(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(5*I+12*arccos(c*x))*cos(3*arccos(c*x))*
d/(c^2*x^2-1)/c-1/256*(-d*(c^2*x^2-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1...
```

Fricas [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\arccos\left(cx\right)+a)^2\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccos(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\int \left(-d(cx-1)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acos}\left(cx\right)\right)^2\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\arccos\left(cx\right)+a)^2\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*a
rcsin(c*x)/c)*a^2 + sqrt(d)*integrate(-((b^2*c^2*d*x^2 - b^2*d)*arctan2(sq
rt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arctan2(sqr
t(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{\sqrt{d}\,d\left(3asin(cx)\,a^2-2\sqrt{-c^2x^2+1}\,a^2c^3x^3+5\sqrt{-c^2x^2+1}\,a^2cx-16\left(\int\sqrt{-c^2x^2+1}\,a\right.\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*d*(3*asin(c*x)*a**2 - 2*sqrt( - c**2*x**2 + 1)*a**2*c**3*x**3 + 5
*sqrt( - c**2*x**2 + 1)*a**2*c*x - 16*int(sqrt( - c**2*x**2 + 1)*acos(c*x)
*x**2,x)*a*b*c**3 + 16*int(sqrt( - c**2*x**2 + 1)*acos(c*x),x)*a*b*c - 8*i
nt(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x**2,x)*b**2*c**3 + 8*int(sqrt( - c
**2*x**2 + 1)*acos(c*x)**2,x)*b**2*c))/(8*c)
```

3.224 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 545

$$\begin{aligned}\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x}\,dx &= -\frac{22}{9}b^2d\sqrt{d-c^2dx^2}\\
&-\frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}-\frac{2}{27}b^2d(1-c^2x^2)\sqrt{d-c^2dx^2}\\
&-\frac{2b^2cdx\sqrt{d-c^2dx^2}\arccos(cx)}{\sqrt{1-c^2x^2}}-\frac{2bcdx\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}\\
&+\frac{2bc^3dx^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{9\sqrt{1-c^2x^2}}\\
&+d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{1}{3}(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\\
&-\frac{2d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}\\
&+\frac{2ibd\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}\\
&-\frac{2ibd\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}\\
&-\frac{2b^2d\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}\\
&+\frac{2b^2d\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-22/9*b^2*d*(-c^2*d*x^2+d)^(1/2)-2*a*b*c*d*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^
2+1)^(1/2)-2/27*b^2*d*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)-2*b^2*c*d*x*(-c^2*
d*x^2+d)^(1/2)*arccos(c*x)/(-c^2*x^2+1)^(1/2)-2/3*b*c*d*x*(-c^2*d*x^2+d)^(
1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+2/9*b*c^3*d*x^3*(-c^2*d*x^2+d)^(
1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcc
os(c*x))^2+1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2-2*d*(-c^2*d*x^2+d)
^(1/2)*(a+b*arccos(c*x))^2*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^
(1/2)+2*I*b*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*polylog(2,-c*x-I*(-c^
2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-2*I*b*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arcco
s(c*x))*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-2*b^2*d*(-c
^2*d*x^2+d)^(1/2)*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+
2*b^2*d*(-c^2*d*x^2+d)^(1/2)*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2
+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.42 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.07

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x}\,dx = -\frac{1}{3}a^2d(-4+c^2x^2)\sqrt{d-c^2dx^2}$$
$$-\frac{abd\sqrt{d-c^2dx^2}\left(-9cx-12(1-c^2x^2)^{3/2}\arccos(cx)+\cos(3\arccos(cx))\right)}{18\sqrt{1-c^2x^2}}$$
$$-\frac{1}{54}b^2d\sqrt{d-c^2dx^2}\left(28-4c^2x^2+9\arccos(cx)^2(-1+\cos(2\arccos(cx)))+\frac{3\arccos(cx)(-9cx+\cos(3\arccos$$
$$+a^2d^{3/2}\log(cx)-a^2d^{3/2}\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)+\frac{2abd\sqrt{d-c^2dx^2}\left(cx+\sqrt{1-c^2x^2}\arccos(cx)-\arccos(c$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
-1/3*(a^2*d*(-4 + c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (a*b*d*Sqrt[d - c^2*d*x^
2]*(-9*c*x - 12*(1 - c^2*x^2)^(3/2)*ArcCos[c*x] + Cos[3*ArcCos[c*x]]))/(18
*Sqrt[1 - c^2*x^2]) - (b^2*d*Sqrt[d - c^2*d*x^2]*(28 - 4*c^2*x^2 + 9*ArcCo
s[c*x]^2*(-1 + Cos[2*ArcCos[c*x]]) + (3*ArcCos[c*x]*(-9*c*x + Cos[3*ArcCos
[c*x]]))/Sqrt[1 - c^2*x^2]))/54 + a^2*d^(3/2)*Log[c*x] - a^2*d^(3/2)*Log[d
 + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (2*a*b*d*Sqrt[d - c^2*d*x^2]*(c*x + Sqrt
[1 - c^2*x^2]*ArcCos[c*x] - ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] + Arc
Cos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[c*x]
)] + I*PolyLog[2, I*E^(I*ArcCos[c*x])]))/Sqrt[1 - c^2*x^2] + (b^2*d*Sqrt[d
 - c^2*d*x^2]*(-2*Sqrt[1 - c^2*x^2] + 2*c*x*ArcCos[c*x] + Sqrt[1 - c^2*x^2
]*ArcCos[c*x]^2 - ArcCos[c*x]^2*Log[1 - I*E^(I*ArcCos[c*x])] + ArcCos[c*x]
^2*Log[1 + I*E^(I*ArcCos[c*x])] - (2*I)*ArcCos[c*x]*PolyLog[2, (-I)*E^(I*A
rcCos[c*x])] + (2*I)*ArcCos[c*x]*PolyLog[2, I*E^(I*ArcCos[c*x])] + 2*PolyL
og[3, (-I)*E^(I*ArcCos[c*x])] - 2*PolyLog[3, I*E^(I*ArcCos[c*x])]))/Sqrt[1
 - c^2*x^2]
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.72, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {5203, 5155, 27, 353, 53, 2009, 5199, 2009, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)(a+b\arccos(cx))dx}{3\sqrt{1-c^2x^2}}+d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5155}$$

$$d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(bc\int \frac{x(3-c^2x^2)}{3\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3\sqrt{1-c^2x^2}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \textbf{27}$$

$$d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{3}bc\int \frac{x(3-c^2x^2)}{\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3\sqrt{1-c^2x^2}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \textbf{353}$$

$$d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{6}bc\int \frac{3-c^2x^2}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3\sqrt{1-c^2x^2}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \textbf{53}$$

$$d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{6}bc\int \left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3\sqrt{1-c^2x^2}}+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \textbf{2009}$$

$$d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{6}bc\left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2}-\frac{4\sqrt{1-c^2x^2}}{c^2}\right)\right)}{3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \textbf{5199}$$

$$\frac{d\left(\frac{2bc\sqrt{d-c^2dx^2}\int(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\right)}{}$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{6}bc\left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2}-\frac{4\sqrt{1-c^2x^2}}{c^2}\right)\right)}{3\sqrt{1-c^2x^2}}$$

$\downarrow$ 2009

$$d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\arccos(cx)-b\sqrt{\right.}}{\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{6}bc\left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2}-\frac{4\sqrt{1-c^2x^2}}{c^2}\right)\right)}{3\sqrt{1-c^2x^2}}$$

$\downarrow$ 5219

$$d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\arccos\right.}{\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{6}bc\left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2}-\frac{4\sqrt{1-c^2x^2}}{c^2}\right)\right)}{3\sqrt{1-c^2x^2}}$$

$\downarrow$ 3042

$$d\left(-\frac{\sqrt{d-c^2dx^2}\int(a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc}{}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{6}bc\left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2}-\frac{4\sqrt{1-c^2x^2}}{c^2}\right)\right)}{3\sqrt{1-c^2x^2}}$$

$\downarrow$ 4669

$$d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2b\int(a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+2b\int(a+b\arccos(cx))\log\left(1+ie^{i\right.}\right.}{\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{6}bc\left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2}-\frac{4\sqrt{1-c^2x^2}}{c^2}\right)\right)}{3\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 3011

$$d\left(-\frac{\sqrt{d-c^2dx^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\arccos(cx\right.\right.\right.}{}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{6}bc\left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2}-\frac{4\sqrt{1-c^2x^2}}{c^2}\right)\right)}{3\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 2720

$$d\left(-\frac{\sqrt{d-c^2dx^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)\right.\right.}{}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{6}bc\left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2}-\frac{4\sqrt{1-c^2x^2}}{c^2}\right)\right)}{3\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 7143

$$d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx)\right.\right.\right.}{\sqrt{}}\right.$$

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{6}bc\left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2}-\frac{4\sqrt{1-c^2x^2}}{c^2}\right)\right)}{3\sqrt{1-c^2x^2}}$$

input

```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/3 + (2*b*c*d*Sqrt[d - c^2*d*
x^2]*((b*c*((-4*Sqrt[1 - c^2*x^2])/c^2 - (2*(1 - c^2*x^2)^(3/2))/(3*c^2)))
/6 + x*(a + b*ArcCos[c*x]) - (c^2*x^3*(a + b*ArcCos[c*x]))/3))/(3*Sqrt[1 -
 c^2*x^2]) + d*(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2 + (2*b*c*Sqrt[d
- c^2*d*x^2]*(a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]))/Sqrt[1 - c
^2*x^2] - (Sqrt[d - c^2*d*x^2]*((-2*I)*(a + b*ArcCos[c*x])^2*ArcTan[E^(I*A
rcCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*ArcCos[c*x]
)] - b*PolyLog[3, (-I)*E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c*x])*Po
lyLog[2, I*E^(I*ArcCos[c*x])] - b*PolyLog[3, I*E^(I*ArcCos[c*x])])))/Sqrt[
1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 843, normalized size of antiderivative = 1.55

method	result
default	$-\frac{\sqrt{-d(c^2x^2-1)}\left(4c^4x^4-5c^2x^2+4i\sqrt{-c^2x^2+1}\,x^3c^3-3i\sqrt{-c^2x^2+1}\,xc+1\right)\left(6i\arccos(cx)b^2+9\arccos(cx)^2b^2+6iab+18\arccos(cx)\right.}{216(c^2x^2-1)}$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/216*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*
x^3*c^3-3*I*(-c^2*x^2+1)^(1/2)*c*x+1)*(6*I*b^2*arccos(c*x)+9*arccos(c*x)^2
*b^2+6*I*a*b+18*arccos(c*x)*a*b+9*a^2-2*b^2)*d/(c^2*x^2-1)+5/8*(-d*(c^2*x^
2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)^2*b^2+2*arcc
os(c*x)*a*b+a^2-2*b^2+2*I*arccos(c*x)*b^2+2*I*a*b)*d/(c^2*x^2-1)+5/8*(-d*(
c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)^2*b^2
+2*arccos(c*x)*a*b+a^2-2*b^2-2*I*b^2*arccos(c*x)-2*I*a*b)*d/(c^2*x^2-1)-1/
216*(-d*(c^2*x^2-1))^(1/2)*(-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+4*c^4*x^4+3*I*
(-c^2*x^2+1)^(1/2)*c*x-5*c^2*x^2+1)*(-6*I*b^2*arccos(c*x)+9*arccos(c*x)^2*
b^2-6*I*a*b+18*arccos(c*x)*a*b+9*a^2-2*b^2)*d/(c^2*x^2-1)-I*(-d*(c^2*x^2-1
))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*(I*arccos(c*x)^2*ln(1-I*(c*x+I*(-c
^2*x^2+1)^(1/2)))*b^2-I*arccos(c*x)^2*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b
^2+2*I*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))*a*b-2*I*arccos(c*x)*
ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))*a*b+2*arccos(c*x)*polylog(2,I*(c*x+I*(-
c^2*x^2+1)^(1/2)))*b^2-2*arccos(c*x)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2
)))*b^2+2*I*polylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b^2-2*I*polylog(3,-I*(
c*x+I*(-c^2*x^2+1)^(1/2)))*b^2+2*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))*a
*b-2*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))*a*b+2*a^2*arctan(c*x+I*(-c^2
*x^2+1)^(1/2)))*d
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))^2}{x}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x,x, algorithm="fricas"
)
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccos(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x}\,dx = \int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))^2}{x}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))**2/x,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2/x, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x,x, algorithm="maxima"
)
```

output
```
-1/3*(3*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
(-c^2*d*x^2 + d)^(3/2) - 3*sqrt(-c^2*d*x^2 + d)*d)*a^2 - sqrt(d)*integrate
(((b^2*c^2*d*x^2 - b^2*d)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2
*(a*b*c^2*d*x^2 - a*b*d)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(
c*x + 1)*sqrt(-c*x + 1)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^{3/2}}{x}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2))/x,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2))/x, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x}\,dx = \frac{\sqrt{d}\,d\left(-\sqrt{-c^2x^2+1}\,a^2c^2x^2+4\sqrt{-c^2x^2+1}\,a^2+6\left(\int \frac{\sqrt{-c^2x^2+1}\,a}{x}\right.\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))^2/x,x)
```

output
```
(sqrt(d)*d*( - sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 + 4*sqrt( - c**2*x**2
 + 1)*a**2 + 6*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x,x)*a*b + 3*int((sq
rt( - c**2*x**2 + 1)*acos(c*x)**2)/x,x)*b**2 - 6*int(sqrt( - c**2*x**2 + 1
)*acos(c*x)*x,x)*a*b*c**2 - 3*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x,x)
*b**2*c**2 + 3*log(tan(asin(c*x)/2))*a**2 - 4*a**2))/3
```

3.225 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 424

$$\begin{aligned}\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx &= \frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2} \\
&- \frac{5b^2cd\sqrt{d-c^2dx^2}\arccos(cx)}{4\sqrt{1-c^2x^2}} + \frac{3bc^3dx^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2\sqrt{1-c^2x^2}} \\
&+ bcd\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\arccos(cx)) \\
&- \frac{3}{2}c^2dx\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2 - \frac{icd\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}} \\
&- \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x} - \frac{cd\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{2b\sqrt{1-c^2x^2}} \\
&+ \frac{2bcd\sqrt{d-c^2dx^2}(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&- \frac{ib^2cd\sqrt{d-c^2dx^2}\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
1/4*b^2*c^2*d*x*(-c^2*d*x^2+d)^(1/2)-5/4*b^2*c*d*(-c^2*d*x^2+d)^(1/2)*arcc
os(c*x)/(-c^2*x^2+1)^(1/2)+3/2*b*c^3*d*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcco
s(c*x))/(-c^2*x^2+1)^(1/2)+b*c*d*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)*(
a+b*arccos(c*x))-3/2*c^2*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2-I*c*
d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/(-c^2*x^2+1)^(1/2)-(-c^2*d*x^2+
d)^(3/2)*(a+b*arccos(c*x))^2/x-1/2*c*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*
x))^3/b/(-c^2*x^2+1)^(1/2)+2*b*c*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*
ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)-I*b^2*c*d*(-c^2*d*x^
2+d)^(1/2)*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 3.87 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.95

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx = \frac{-12a^2d\sqrt{1-c^2x^2}(2+c^2x^2)\sqrt{d-c^2dx^2}+36a^2cd^{3/2}x\sqrt{1-c^2x}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/x^2,x]
```

output
```
(-12*a^2*d*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2)*Sqrt[d - c^2*d*x^2] + 36*a^2*c*
d^(3/2)*x*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1
+ c^2*x^2))] + 24*a*b*d*Sqrt[d - c^2*d*x^2]*(-2*Sqrt[1 - c^2*x^2]*ArcCos[c
*x] + c*x*ArcCos[c*x]^2 - 2*c*x*Log[c*x]) + (8*I)*b^2*d*Sqrt[d - c^2*d*x^2
]*(I*ArcCos[c*x]*(3*Sqrt[1 - c^2*x^2]*ArcCos[c*x] - c*x*ArcCos[c*x]*(3*I +
 ArcCos[c*x]) + 6*c*x*Log[1 + E^((2*I)*ArcCos[c*x])]) + 3*c*x*PolyLog[2, -
E^((2*I)*ArcCos[c*x])]) + b^2*c*d*x*Sqrt[d - c^2*d*x^2]*(4*ArcCos[c*x]^3 -
 6*ArcCos[c*x]*Cos[2*ArcCos[c*x]] + (3 - 6*ArcCos[c*x]^2)*Sin[2*ArcCos[c*x
]]) - 6*a*b*c*d*x*Sqrt[d - c^2*d*x^2]*(Cos[2*ArcCos[c*x]] + 2*ArcCos[c*x]*
(-ArcCos[c*x] + Sin[2*ArcCos[c*x]])))/(24*x*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.87, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules used = {5201, 5157, 5139, 262, 223, 5153, 5189, 211, 223, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^2}dx$$

$$\Big\downarrow \text{ 5201}$$

$$-3c^2d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{ 5157}$$

$$-3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\int x(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+b\arccos\right.$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{ 5139}$$

$$-3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-\right.$$
$$\frac{2bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{ 262}$$

$$-3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}}{2\sqrt{1-c^2x^2}}\right.$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}$$

$\downarrow$ 223

$$-3c^2d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}\right.$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}$$

$\downarrow$ 5153

$$-\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-$$

$$3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}$$

$\downarrow$ 5189

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(\int\frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\int\sqrt{1-c^2x^2}dx+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}-$$

$$3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}$$

$\downarrow$ 211

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}-$$
$$3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\right.$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \; 223$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}-$$
$$3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\right.$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \; 5137$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\right.\right.}{\sqrt{1-c^2x^2}}$$
$$3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\right.$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\right.\right.}{\sqrt{1-c^2x^2}}$$
$$3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\right.$$
$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \; 4202$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))-\frac{i(a+b\arccos(cx))^2}{2b}+\right.}{\sqrt{1-c^2x^2}}$$

$$3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \; 2620$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(2i\left(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{2}(\right.}{\sqrt{1-c^2x^2}}$$

$$3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \; 2715$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos\right.\right.}{\sqrt{1-c^2x^2}}$$

$$3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \; 2838$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\text{PolyLo}\right.\right.}{\sqrt{1-c^2x^2}}$$

$$3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}+\frac{1}{2}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/x^2,x]
```

output
```
-(((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/x) - 3*c^2*d*((x*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c
*x])^3)/(6*b*c*Sqrt[1 - c^2*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*((x^2*(a + b*
ArcCos[c*x]))/2 + (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^
3)))/2))/Sqrt[1 - c^2*x^2]) - (2*b*c*d*Sqrt[d - c^2*d*x^2]*(((1 - c^2*x^2)
*(a + b*ArcCos[c*x]))/2 - ((I/2)*(a + b*ArcCos[c*x])^2)/b + (b*c*((x*Sqrt[
1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 + (2*I)*((-1/2*I)*(a + b*ArcCos[c*
x])*Log[1 + E^((2*I)*ArcCos[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcCos[c*x])]
)/4)))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5189
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCos[c*x])/(2*p)), x] + (Simp[d
 Int[(d + e*x^2)^(p - 1)*((a + b*ArcCos[c*x])/x), x], x] + Simp[b*c*(d^p/(2
*p))   Int[(1 - c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.11

method	result
default	$-\frac{a^2(-c^2dx^2+d)^{\frac{5}{2}}}{dx}-a^2c^2x(-c^2dx^2+d)^{\frac{3}{2}}-\frac{3a^2c^2dx\sqrt{-c^2dx^2+d}}{2}-\frac{3a^2c^2d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}}-\frac{b^2\sqrt{-d(c^2}{}$
parts	$-\frac{a^2(-c^2dx^2+d)^{\frac{5}{2}}}{dx}-a^2c^2x(-c^2dx^2+d)^{\frac{3}{2}}-\frac{3a^2c^2dx\sqrt{-c^2dx^2+d}}{2}-\frac{3a^2c^2d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}}-\frac{b^2\sqrt{-d(c^2}{}$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/d/x*(-c^2*d*x^2+d)^(5/2)-a^2*c^2*x*(-c^2*d*x^2+d)^(3/2)-3/2*a^2*c^2*d
*x*(-c^2*d*x^2+d)^(1/2)-3/2*a^2*c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)
*x/(-c^2*d*x^2+d)^(1/2))-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)
/(c^2*x^2-1)/x*(-2*arccos(c*x)^2*(-c^2*x^2+1)^(1/2)*x^2*c^2-2*c^3*x^3*arcc
os(c*x)+2*arccos(c*x)^3*c*x+4*I*arccos(c*x)^2*x*c+c^2*x^2*(-c^2*x^2+1)^(1/
2)-8*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*x*c+4*I*polylog(2,-(c*
x+I*(-c^2*x^2+1)^(1/2))^2)*x*c-4*arccos(c*x)^2*(-c^2*x^2+1)^(1/2)+c*x*arcc
os(c*x))*d-1/4*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/x
*(-4*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^2-2*c^3*x^3+6*arccos(c*x)^2*c*x+
8*I*arccos(c*x)*x*c-8*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*x*c-8*arccos(c*x)
*(-c^2*x^2+1)^(1/2)+c*x)*d
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x^2,x, algorithm="frica
s")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccos(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))^2}{x^2}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))**2/x**2,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2/x**2, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x^2,x, algorithm="maxim
a")
```

output
```
-1/2*(3*sqrt(-c^2*d*x^2 + d)*c^2*d*x + 3*c*d^(3/2)*arcsin(c*x) + 2*(-c^2*d
*x^2 + d)^(3/2)/x)*a^2 - sqrt(d)*integrate(((b^2*c^2*d*x^2 - b^2*d)*arctan
2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arctan2
(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^{3/2}}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx=\frac{\sqrt{d}\,d\Big(2acos(cx)^3\,b^2cx+6acos(cx)^2\,abcx-9asin(cx)\,a^2cx-3\sqrt{}$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))^2/x^2,x)
```

output
```
(sqrt(d)*d*(2*acos(c*x)**3*b**2*c*x + 6*acos(c*x)**2*a*b*c*x - 9*asin(c*x)
*a**2*c*x - 3*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - 6*sqrt( - c**2*x**2
+ 1)*a**2 + 12*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*x**2),x)*a*b*x + 6*in
t(acos(c*x)**2/(sqrt( - c**2*x**2 + 1)*x**2),x)*b**2*x - 12*int(sqrt( - c*
*2*x**2 + 1)*acos(c*x),x)*a*b*c**2*x - 6*int(sqrt( - c**2*x**2 + 1)*acos(c
*x)**2,x)*b**2*c**2*x))/(6*x)
```

3.226 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 590

$$\begin{aligned}
\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^3}\,dx &= 2b^2c^2d\sqrt{d-c^2dx^2} \\
&+ \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} + \frac{3b^2c^3dx\sqrt{d-c^2dx^2}\arccos(cx)}{\sqrt{1-c^2x^2}} \\
&- \frac{bcd\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x\sqrt{1-c^2x^2}} - \frac{bc^3dx\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}} \\
&- \frac{3}{2}c^2d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2 - \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{2x^2} \\
&+ \frac{3c^2d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&- \frac{b^2c^2d\sqrt{d-c^2dx^2}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{1-c^2x^2}} \\
&- \frac{3ibc^2d\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&+ \frac{3ibc^2d\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&+ \frac{3b^2c^2d\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&- \frac{3b^2c^2d\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}
\end{aligned}$$

output
```
2*b^2*c^2*d*(-c^2*d*x^2+d)^(1/2)+3*a*b*c^3*d*x*(-c^2*d*x^2+d)^(1/2)/(-c^2*
x^2+1)^(1/2)+3*b^2*c^3*d*x*(-c^2*d*x^2+d)^(1/2)*arccos(c*x)/(-c^2*x^2+1)^(
1/2)-b*c*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x/(-c^2*x^2+1)^(1/2)-b*c
^3*d*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-3/2*c^2*d
*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2-1/2*(-c^2*d*x^2+d)^(3/2)*(a+b*ar
ccos(c*x))^2/x^2+3*c^2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2*arctanh(
c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-b^2*c^2*d*(-c^2*d*x^2+d)^(1/2
)*arctanh((-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-3*I*b*c^2*d*(-c^2*d*x^2+d
)^(1/2)*(a+b*arccos(c*x))*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1
)^(1/2)+3*I*b*c^2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*polylog(2,c*x+I
*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+3*b^2*c^2*d*(-c^2*d*x^2+d)^(1/2)*p
olylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-3*b^2*c^2*d*(-c^2*d
*x^2+d)^(1/2)*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.65 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.35

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^3}\,dx = \frac{d\Big(a^2d(-1+c^2x^2)(1+2c^2x^2)-3a^2c^2\sqrt{d}x^2\sqrt{d-c^2dx^2}\log(x)+$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/x^3,x]
```

output
```
(d*(a^2*d*(-1 + c^2*x^2)*(1 + 2*c^2*x^2) - 3*a^2*c^2*Sqrt[d]*x^2*Sqrt[d -
c^2*d*x^2]*Log[x] + 3*a^2*c^2*Sqrt[d]*x^2*Sqrt[d - c^2*d*x^2]*Log[d + Sqrt
[d]*Sqrt[d - c^2*d*x^2]] - 4*a*b*c^2*d*x^2*Sqrt[1 - c^2*x^2]*(c*x + Sqrt[1
 - c^2*x^2]*ArcCos[c*x] - ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] + ArcCo
s[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[c*x])]
 + I*PolyLog[2, I*E^(I*ArcCos[c*x])]) + 2*a*b*d*Sqrt[1 - c^2*x^2]*(c*x - S
qrt[1 - c^2*x^2]*ArcCos[c*x] + c^2*x^2*ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c
*x])] - c^2*x^2*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] + I*c^2*x^2*PolyL
og[2, (-I)*E^(I*ArcCos[c*x])] - I*c^2*x^2*PolyLog[2, I*E^(I*ArcCos[c*x])])
 + b^2*d*Sqrt[1 - c^2*x^2]*(ArcCos[c*x]*(2*c*x - Sqrt[1 - c^2*x^2]*ArcCos[
c*x]) - 2*c^2*x^2*(ArcCoth[Sqrt[1 - c^2*x^2]] + I*ArcCos[c*x]^2*ArcTan[E^(
I*ArcCos[c*x])] - I*ArcCos[c*x]*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I*Arc
Cos[c*x]*PolyLog[2, I*E^(I*ArcCos[c*x])] + PolyLog[3, (-I)*E^(I*ArcCos[c*x
])] - PolyLog[3, I*E^(I*ArcCos[c*x])])) + 2*b^2*c^2*d*x^2*Sqrt[1 - c^2*x^2
]*(2*Sqrt[1 - c^2*x^2] - 2*c*x*ArcCos[c*x] - Sqrt[1 - c^2*x^2]*ArcCos[c*x]
^2 + ArcCos[c*x]^2*(Log[1 - I*E^(I*ArcCos[c*x])] - Log[1 + I*E^(I*ArcCos[c
*x])]) + (2*I)*ArcCos[c*x]*(PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - PolyLog[2
, I*E^(I*ArcCos[c*x])]) - 2*PolyLog[3, (-I)*E^(I*ArcCos[c*x])] + 2*PolyLog
[3, I*E^(I*ArcCos[c*x])])))/(2*x^2*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.67, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules used = {5201, 5193, 25, 354, 90, 73, 221, 5199, 2009, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^2}{x^3} \, dx$$

$$\Big\downarrow \text{5201}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x^2}dx}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{5193}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\left(bc\int -\frac{c^2x^2+1}{x\sqrt{1-c^2x^2}}dx+c^2(-x)(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\left(-bc\int \frac{c^2x^2+1}{x\sqrt{1-c^2x^2}}dx+c^2(-x)(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{354}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{2}bc\int \frac{c^2x^2+1}{x^2\sqrt{1-c^2x^2}}dx^2+c^2(-x)(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{90}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{2}bc\left(\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2-2\sqrt{1-c^2x^2}\right)+c^2(-x)(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{2}bc\left(-\frac{2\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2}-2\sqrt{1-c^2x^2}\right)+c^2(-x)(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{2x^2}$$

$\Big\downarrow$ 221

$$-\frac{3}{2}c^2d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx-\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-2\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{2x^2}$$

$\Big\downarrow$ 5199

$$-\frac{3}{2}c^2d\left(\frac{2bc\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx\right.$$
$$-\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-2\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{2x^2}$$

$\Big\downarrow$ 2009

$$-\frac{3}{2}c^2d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\arccos(cx\right.}{\sqrt{1-c^2x^2}}\right.$$
$$-\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-2\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{2x^2}$$

$\Big\downarrow$ 5219

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\right.}{\sqrt{1-}}\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-2\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}-$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{2x^2}$$

$\big\downarrow$ 3042

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\int(a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-2\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}-$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{2x^2}$$

$\big\downarrow$ 4669

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2b\int(a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+2b\int(a+b\arccos(cx))\log\left(1\right.\right.}{\sqrt{1-c^2x^2}}\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-2\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}-$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{2x^2}$$

$\big\downarrow$ 3011

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\,\text{arcc}\right.\right.}{}\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-2\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}-$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{2x^2}$$

$\big\downarrow$ 2720

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-ie^{i\arccos}\right.\right.\right.}{}\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-2\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}-$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \; 7143$$

$$-\frac{3}{2}c^2d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos\right.\right.}{}\right.$$

$$\frac{bcd\sqrt{d-c^2dx^2}\left(c^2(-x)(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{1}{2}bc\left(-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-2\sqrt{1-c^2x^2}\right)\right)}{\sqrt{1-c^2x^2}}-$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{2x^2}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/x^3,x]
```

output
```
-1/2*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/x^2 - (b*c*d*Sqrt[d - c
^2*d*x^2]*(-((a + b*ArcCos[c*x])/x) - c^2*x*(a + b*ArcCos[c*x]) - (b*c*(-2
*Sqrt[1 - c^2*x^2] - 2*ArcTanh[Sqrt[1 - c^2*x^2]]))/2))/Sqrt[1 - c^2*x^2]
 - (3*c^2*d*(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2 + (2*b*c*Sqrt[d - c^
2*d*x^2]*(a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]))/Sqrt[1 - c^2*x
^2] - (Sqrt[d - c^2*d*x^2]*((-2*I)*(a + b*ArcCos[c*x])^2*ArcTan[E^(I*ArcCo
s[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] -
 b*PolyLog[3, (-I)*E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLo
g[2, I*E^(I*ArcCos[c*x])] - b*PolyLog[3, I*E^(I*ArcCos[c*x])])))/Sqrt[1 -
c^2*x^2]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 924, normalized size of antiderivative = 1.57

method	result
default	$a^2\left(-\frac{(-c^2dx^2+d)^{\frac{5}{2}}}{2dx^2}-\frac{3c^2\left(\frac{(-c^2dx^2+d)^{\frac{3}{2}}}{3}+d\left(\sqrt{-c^2dx^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2dx^2+d}}{x}\right)\right)\right)}{2}\right)+b^2\left(-\frac{\sqrt{-d(c^2x^2-}}{}\right.$
parts	$a^2\left(-\frac{(-c^2dx^2+d)^{\frac{5}{2}}}{2dx^2}-\frac{3c^2\left(\frac{(-c^2dx^2+d)^{\frac{3}{2}}}{3}+d\left(\sqrt{-c^2dx^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2dx^2+d}}{x}\right)\right)\right)}{2}\right)+b^2\left(-\frac{\sqrt{-d(c^2x^2-}}{}\right.$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/d/x^2*(-c^2*d*x^2+d)^(5/2)-3/2*c^2*(1/3*(-c^2*d*x^2+d)^(3/2)+d*(
(-c^2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x))))
+b^2*(-1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(ar
ccos(c*x)^2-2+2*I*arccos(c*x))*c^2*d/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2
)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)^2-2-2*I*arccos(c*x))*
c^2*d/(c^2*x^2-1)-1/2*d*(c^2*x^2*arccos(c*x)+2*c*x*(-c^2*x^2+1)^(1/2)-arcc
os(c*x))*arccos(c*x)*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/x^2-I*(-d*(c^2*x^2
-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(3*I*arccos(c*x)^2*ln(1+I*(c*x+I*(-c^2*x^2+1
)^(1/2)))-3*I*arccos(c*x)^2*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+6*arccos(c*
x)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-6*arccos(c*x)*polylog(2,I*(c*x
+I*(-c^2*x^2+1)^(1/2)))+6*I*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-6*I*p
olylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))+4*arctan(c*x+I*(-c^2*x^2+1)^(1/2)))
*c^2*d/(2*c^2*x^2-2))+2*a*b*(-1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(
1/2)*c*x+c^2*x^2-1)*(arccos(c*x)+I)*c^2*d/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))
^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)-I)*c^2*d/(c^2*x^
2-1)-1/2*d*(c^2*x^2*arccos(c*x)+c*x*(-c^2*x^2+1)^(1/2)-arccos(c*x))*(-d*(c
^2*x^2-1))^(1/2)/(c^2*x^2-1)/x^2-3*I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(
1/2)*(I*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-I*arccos(c*x)*ln(1-
I*(c*x+I*(-c^2*x^2+1)^(1/2)))+dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-dilog(
1-I*(c*x+I*(-c^2*x^2+1)^(1/2))))*c^2*d/(2*c^2*x^2-2))
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^3}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x^3,x, algorithm="frica
s")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccos(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^3}\,dx = \int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))^2}{x^3}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))**2/x**3,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2/x**3, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^3}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x^3,x, algorithm="maxim
a")
```

output
```
1/2*(3*c^2*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
 - (-c^2*d*x^2 + d)^(3/2)*c^2 - 3*sqrt(-c^2*d*x^2 + d)*c^2*d - (-c^2*d*x^2
 + d)^(5/2)/(d*x^2))*a^2 - sqrt(d)*integrate(((b^2*c^2*d*x^2 - b^2*d)*arct
an2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arcta
n2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x^3, x
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x^3,x, algorithm="giac"
)
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2\left(d-c^2\,d\,x^2\right)^{3/2}}{x^3}\,dx$$

input

```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^3,x)
```

output

```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^3}\,dx=\frac{\sqrt{d}\,d\left(-8\sqrt{-c^2x^2+1}\,a^2c^2x^2-4\sqrt{-c^2x^2+1}\,a^2+16\left(\int \frac{\sqrt{-c^2x^2+}}{x}\right.\right.}{}$$

input

```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))^2/x^3,x)
```

output

```
(sqrt(d)*d*( - 8*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - 4*sqrt( - c**2*x*
*2 + 1)*a**2 + 16*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**3,x)*a*b*x**2
- 16*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x,x)*a*b*c**2*x**2 + 8*int((sq
rt( - c**2*x**2 + 1)*acos(c*x)**2)/x**3,x)*b**2*x**2 - 8*int((sqrt( - c**2
*x**2 + 1)*acos(c*x)**2)/x,x)*b**2*c**2*x**2 - 12*log(tan(asin(c*x)/2))*a*
*2*c**2*x**2 + 9*a**2*c**2*x**2))/(8*x**2)
```

3.227 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 400

$$\begin{aligned}\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^4}\,dx &= -\frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} \\ &- \frac{b^2c^3d\sqrt{d-c^2dx^2}\arccos(cx)}{3\sqrt{1-c^2x^2}} - \frac{bcd\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3x^2} \\ &+ \frac{c^2d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x} + \frac{4ic^3d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3\sqrt{1-c^2x^2}} \\ &- \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{3x^3} + \frac{c^3d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{3b\sqrt{1-c^2x^2}} \\ &- \frac{8bc^3d\sqrt{d-c^2dx^2}(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{3\sqrt{1-c^2x^2}} \\ &+ \frac{4ib^2c^3d\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{3\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-1/3*b^2*c^2*d*(-c^2*d*x^2+d)^(1/2)/x-1/3*b^2*c^3*d*(-c^2*d*x^2+d)^(1/2)*a
rccos(c*x)/(-c^2*x^2+1)^(1/2)-1/3*b*c*d*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^
(1/2)*(a+b*arccos(c*x))/x^2+c^2*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2
/x+4/3*I*c^3*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/(-c^2*x^2+1)^(1/2)
-1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x^3+1/3*c^3*d*(-c^2*d*x^2+d)
^(1/2)*(a+b*arccos(c*x))^3/b/(-c^2*x^2+1)^(1/2)-8/3*b*c^3*d*(-c^2*d*x^2+d)
^(1/2)*(a+b*arccos(c*x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(
1/2)+4/3*I*b^2*c^3*d*(-c^2*d*x^2+d)^(1/2)*polylog(2,(c*x+I*(-c^2*x^2+1)^(1
/2))^2)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.23

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^4}\,dx = \frac{abcdx\sqrt{d-c^2dx^2}-a^2d\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}+4a^2c^2dx^2\sqrt{1-}}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/x^4,x]
```

output
```
(a*b*c*d*x*Sqrt[d - c^2*d*x^2] - a^2*d*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^
2] + 4*a^2*c^2*d*x^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2] - b^2*c^2*d*x^2
*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2] + b*d*Sqrt[d - c^2*d*x^2]*(-3*a*c^3
*x^3 + b*((-4*I)*c^3*x^3 - Sqrt[1 - c^2*x^2] + 4*c^2*x^2*Sqrt[1 - c^2*x^2]
))*ArcCos[c*x]^2 - b^2*c^3*d*x^3*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^3 - 3*a^2
*c^3*d^(3/2)*x^3*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[
d]*(-1 + c^2*x^2))] + b*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(b*c*x + 2*a*Sqr
t[1 - c^2*x^2]*(-1 + 4*c^2*x^2) + 8*b*c^3*x^3*Log[1 + E^((2*I)*ArcCos[c*x]
)]) + 8*a*b*c^3*d*x^3*Sqrt[d - c^2*d*x^2]*Log[c*x] - (4*I)*b^2*c^3*d*x^3*S
qrt[d - c^2*d*x^2]*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/(3*x^3*Sqrt[1 - c^2
*x^2])
```

Rubi [A] (verified)

Time = 3.19 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.621$, Rules used = {5201, 5191, 247, 223, 5137, 3042, 4202, 2620, 2715, 2838, 5197, 5137, 3042, 4202, 2620, 2715, 2838, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^4}dx$$

$$\Big\downarrow \text{ 5201}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}dx-\frac{2bcd\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x^3}dx}{3\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 5191}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(-\int \frac{a+b\arccos(cx)}{x}dx\right)-\frac{1}{2}bc\int \frac{\sqrt{1-c^2x^2}}{x^2}dx-\frac{(1-c^2x^2)(a+b\arccos(cx))}{2x^2}\right)}{3\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 247}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}dx-\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(-\int \frac{a+b\arccos(cx)}{x}dx\right)-\frac{1}{2}bc\left(c^2\left(-\int \frac{1}{\sqrt{1-c^2x^2}}dx\right)-\frac{\sqrt{1-c^2x^2}}{x}\right)-\frac{(1-c^2x^2)(a+b\arccos(cx))}{2x^2}\right)}{3\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(-\int \frac{a+b\arccos(cx)}{x}dx\right)-\frac{(1-c^2x^2)(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(-c\arcsin(cx)-\frac{\sqrt{1-c^2x^2}}{x}\right)\right)}{3\sqrt{1-c^2x^2}}+ c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 5137$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)-\frac{(1-c^2x^2)(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(-c\arcsin(cx)-\frac{\sqrt{1-c^2x^2}}{x}\right.\right.}{3\sqrt{1-c^2x^2}} c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)-\frac{(1-c^2x^2)(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(-c\arcsin(\right.\right.}{3\sqrt{1-c^2x^2}} c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 4202$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)\right)-\frac{(1-c^2x^2)(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\right.}{3\sqrt{1-c^2x^2}} c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2620$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a\right.\right.\right.}{3\sqrt{1-c^2x^2}} c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2715$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{2}i\log\left(1+e^{2i\right.\right.\right.\right.}}{3\sqrt{1-c^2x^2}} c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{2838}$$

$$c^2(-d)\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x^2}dx - \frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right.\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{5197}$$

$$c^2(-d)\left(-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}-\frac{2bc\sqrt{d-c^2dx^2}\int\frac{a+b\arccos(cx)}{x}dx}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx)}{x}\right.$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2i\arccos}\right.\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{5137}$$

$$c^2(-d)\left(-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\int\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)}{\sqrt{1-c^2x^2}}-\frac{\sqrt{d-c^2dx}}{}\right.$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2i\arccos}\right.\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{3042}$$

$$c^2(-d)\left(-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\int(a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)}{\sqrt{1-c^2x^2}}\right.$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2i\arccos}\right.\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{4202}$$

$$c^2(-d)\left(-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\int\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}da\right.}{\sqrt{1-c^2x^2}}\right.$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{PolyLog}\left(2,-e^{2i\arccos}\right.\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2620$$

$$c^2(-d)\left(-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{2}ib\int\log\left(1+e^{2i\arccos(cx)}\right)\right.\right.}{\sqrt{1-c^2}}\right.$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{PolyLog}\left(2,-e^{2i\arccos}\right.\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2715$$

$$c^2(-d)\left(-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e\right.\right.\right.}{\sqrt{}}\right.$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{PolyLog}\left(2,-e^{2i\arccos}\right.\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 2838$$

$$c^2(-d)\left(-\frac{c^2\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(\right.\right.}{\sqrt{1-c^2x^2}}\right.$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{PolyLog}\left(2,-e^{2i\arccos}\right.\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \; 5153$$

$$-\frac{2bcd\sqrt{d-c^2dx^2}\left(c^2\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\right)\right)\right.}{3\sqrt{1-c^2x^2}}$$

$$c^2(-d)\left(\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\right)\right)}{\sqrt{1-c^2x^2}}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{3x^3}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/x^4,x]
```

output
```
-1/3*((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/x^3 - (2*b*c*d*Sqrt[d -
 c^2*d*x^2]*(-1/2*((1 - c^2*x^2)*(a + b*ArcCos[c*x]))/x^2 - (b*c*(-(Sqrt[1
 - c^2*x^2]/x) - c*ArcSin[c*x]))/2 + c^2*(((I/2)*(a + b*ArcCos[c*x])^2)/b
- (2*I)*((-1/2*I)*(a + b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCos[c*x])] - (b*
PolyLog[2, -E^((2*I)*ArcCos[c*x])])/4))))/(3*Sqrt[1 - c^2*x^2]) - c^2*d*(-
((Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/x) + (c*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCos[c*x])^3)/(3*b*Sqrt[1 - c^2*x^2]) + (2*b*c*Sqrt[d - c^2*d*x^2]
*(((I/2)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcCos[c*x])*Lo
g[1 + E^((2*I)*ArcCos[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/4)))
/Sqrt[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5191
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCos[c*x
])/(f*(m + 1))), x] + (Simp[b*c*(d^p/(f*(m + 1)))   Int[(f*x)^(m + 1)*(1 -
c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m + 2)
*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x]), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

rule 5197
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^2
]/Sqrt[1 - c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x
] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 2)*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2116 vs. $2(374) = 748$.

Time = 0.71 (sec) , antiderivative size = 2117, normalized size of antiderivative = 5.29

method	result	size
default	Expression too large to display	2117
parts	Expression too large to display	2117

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
73/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/(c^2*x^2-1)*a
rccos(c*x)^2*c^4-14/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1
)/x/(c^2*x^2-1)*arccos(c*x)^2*c^2+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4
*x^4-9*c^2*x^2+1)*x^3/(c^2*x^2-1)*(-c^2*x^2+1)*c^6-8*b^2*(-d*(c^2*x^2-1))^
(1/2)*(-c^2*x^2+1)^(1/2)*d*c^3/(3*c^2*x^2-3)*arccos(c*x)*ln(1+(c*x+I*(-c^2
*x^2+1)^(1/2))^2)+8*I*b^2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*d*c^3/
(3*c^2*x^2-3)*arccos(c*x)^2+32*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*
c^2*x^2+1)*x^5/(c^2*x^2-1)*arccos(c*x)^2*c^8+3*b^2*(-d*(c^2*x^2-1))^(1/2)*
d/(24*c^4*x^4-9*c^2*x^2+1)/(c^2*x^2-1)*arccos(c*x)*(-c^2*x^2+1)^(1/2)*c^3-
52*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c^2*x^2-1)*a
rccos(c*x)^2*c^6-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)
/x^2/(c^2*x^2-1)*arccos(c*x)*(-c^2*x^2+1)^(1/2)*c-16/3*I*b^2*(-d*(c^2*x^2-
1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c^2*x^2-1)*arccos(c*x)*c^8+8*I*b
^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^4/(c^2*x^2-1)*(-c^2
*x^2+1)^(1/2)*c^7+20/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^
2+1)*x^3/(c^2*x^2-1)*arccos(c*x)*c^6-3*I*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*
c^4*x^4-9*c^2*x^2+1)*x^2/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*c^5-4/3*I*b^2*(-d*
(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/(c^2*x^2-1)*arccos(c*x)*c^
4-4/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/(c^2*x^2-1)*
arccos(c*x)^2*(-c^2*x^2+1)^(1/2)*c^3+2/3*a^2*c^2/d/x*(-c^2*d*x^2+d)^(5/...
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^4}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x^4,x, algorithm="frica
s")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccos(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^4}\,dx = \int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))^2}{x^4}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))**2/x**4,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2/x**4, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^4}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x^4,x, algorithm="maxim
a")
```

output
```
1/3*(3*sqrt(-c^2*d*x^2 + d)*c^4*d*x + 3*c^3*d^(3/2)*arcsin(c*x) + 2*(-c^2*
d*x^2 + d)^(3/2)*c^2/x - (-c^2*d*x^2 + d)^(5/2)/(d*x^3))*a^2 - sqrt(d)*int
egrate(((b^2*c^2*d*x^2 - b^2*d)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)
^2 + 2*(a*b*c^2*d*x^2 - a*b*d)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
*sqrt(c*x + 1)*sqrt(-c*x + 1)/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x^4,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^{3/2}}{x^4}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^4,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^4}\,dx = \frac{\sqrt{d}\,d\Big(-acos(cx)^3\,b^2c^3x^3-3acos(cx)^2\,ab\,c^3x^3+3asin(cx)\,a^2c^3x$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))^2/x^4,x)
```

output
```
(sqrt(d)*d*( - acos(c*x)**3*b**2*c**3*x**3 - 3*acos(c*x)**2*a*b*c**3*x**3
+ 3*asin(c*x)*a**2*c**3*x**3 + 4*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - s
qrt( - c**2*x**2 + 1)*a**2 - 6*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*x**2)
,x)*a*b*c**2*x**3 - 3*int(acos(c*x)**2/(sqrt( - c**2*x**2 + 1)*x**2),x)*b*
*2*c**2*x**3 + 6*int((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**4,x)*a*b*x**3 +
 3*int((sqrt( - c**2*x**2 + 1)*acos(c*x)**2)/x**4,x)*b**2*x**3))/(3*x**3)
```

3.228 $\int x^3(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 651

$$\int x^3(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2\,dx = \frac{160b^2d^2\sqrt{d-c^2dx^2}}{3969c^4} + \frac{4abd^2x\sqrt{d-c^2dx^2}}{63c^3\sqrt{1-c^2x^2}} + \frac{80b^2d^2(1-c^2x^2)\sqrt{d-c^2dx^2}}{11907c^4} + \frac{4b^2d^2(1-c^2x^2)^2\sqrt{d-c^2dx^2}}{1323c^4} + \frac{50b^2d^2(1-c^2x^2)^3\sqrt{d-c^2dx^2}}{27783c^4} - \frac{2b^2d^2(1-c^2x^2)^4\sqrt{d-c^2dx^2}}{729c^4} + \frac{4b^2d^2x\sqrt{d-c^2dx^2}\arccos(cx)}{63c^3\sqrt{1-c^2x^2}} + \frac{2bd^2x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{189c\sqrt{1-c^2x^2}} - \frac{2bcd^2x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{21\sqrt{1-c^2x^2}} + \frac{38bc^3d^2x^7\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{441\sqrt{1-c^2x^2}} - \frac{2bc^5d^2x^9\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{81\sqrt{1-c^2x^2}} - \frac{2d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{63c^4} - \frac{d^2x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{63c^2} + \frac{1}{21}d^2x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2 + \frac{5}{63}dx^4(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2 + \frac{1}{9}x^4(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2$$

output
```
160/3969*b^2*d^2*(-c^2*d*x^2+d)^(1/2)/c^4+4/63*a*b*d^2*x*(-c^2*d*x^2+d)^(1
/2)/c^3/(-c^2*x^2+1)^(1/2)+80/11907*b^2*d^2*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1
/2)/c^4+4/1323*b^2*d^2*(-c^2*x^2+1)^2*(-c^2*d*x^2+d)^(1/2)/c^4+50/27783*b^
2*d^2*(-c^2*x^2+1)^3*(-c^2*d*x^2+d)^(1/2)/c^4-2/729*b^2*d^2*(-c^2*x^2+1)^4
*(-c^2*d*x^2+d)^(1/2)/c^4+4/63*b^2*d^2*x*(-c^2*d*x^2+d)^(1/2)*arccos(c*x)/
c^3/(-c^2*x^2+1)^(1/2)+2/189*b*d^2*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*
x))/c/(-c^2*x^2+1)^(1/2)-2/21*b*c*d^2*x^5*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos
(c*x))/(-c^2*x^2+1)^(1/2)+38/441*b*c^3*d^2*x^7*(-c^2*d*x^2+d)^(1/2)*(a+b*a
rccos(c*x))/(-c^2*x^2+1)^(1/2)-2/81*b*c^5*d^2*x^9*(-c^2*d*x^2+d)^(1/2)*(a+
b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-2/63*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arcco
s(c*x))^2/c^4-1/63*d^2*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/c^2+1/
21*d^2*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2+5/63*d*x^4*(-c^2*d*x^2
+d)^(3/2)*(a+b*arccos(c*x))^2+1/9*x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x
))^2
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.43

$$\int x^3 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))^2\,dx = \frac{d^2\sqrt{d-c^2dx^2}\Big(3969a^2\left(-1+c^2x^2\right)^4\left(2+7c^2x^2\right)-126abcx\sqrt{1-c^2x^2}(-126-21c^2x^2}{}$$

input
```
Integrate[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(3969*a^2*(-1 + c^2*x^2)^4*(2 + 7*c^2*x^2) - 126*
a*b*c*x*Sqrt[1 - c^2*x^2]*(-126 - 21*c^2*x^2 + 189*c^4*x^4 - 171*c^6*x^6 +
 49*c^8*x^8) + 2*b^2*(-6140 + 7039*c^2*x^2 + 106*c^4*x^4 - 2152*c^6*x^6 +
1490*c^8*x^8 - 343*c^10*x^10) + 126*b*(63*a*(-1 + c^2*x^2)^4*(2 + 7*c^2*x^
2) + b*c*x*Sqrt[1 - c^2*x^2]*(126 + 21*c^2*x^2 - 189*c^4*x^4 + 171*c^6*x^6
 - 49*c^8*x^8))*ArcCos[c*x] + 3969*b^2*(-1 + c^2*x^2)^4*(2 + 7*c^2*x^2)*Ar
cCos[c*x]^2))/(250047*c^4*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 3.98 (sec) , antiderivative size = 800, normalized size of antiderivative = 1.23, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.828, Rules used = {5203, 5193, 27, 1578, 1195, 2009, 5203, 5193, 27, 354, 86, 2009, 5199, 5139, 243, 53, 2009, 5211, 5139, 243, 53, 2009, 5183, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos (c x))^2 \, dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{2 b c d^2 \sqrt{d-c^2 d x^2} \int x^4 \left(1-c^2 x^2\right)^2 (a+b \arccos (c x))dx}{9 \sqrt{1-c^2 x^2}}+\frac{5}{9} d \int x^3 \left(d-c^2 d x^2\right)^{3/2} (a+ b \arccos (c x))^2dx+\frac{1}{9} x^4 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos (c x))^2$$

$$\Big\downarrow \text{5193}$$

$$\frac{5}{9} d \int x^3 \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos (c x))^2dx+\frac{2 b c d^2 \sqrt{d-c^2 d x^2} \left(b c \int \frac{x^5 \left(35 c^4 x^4-90 c^2 x^2+63\right)}{315 \sqrt{1-c^2 x^2}}dx+\frac{1}{9} c^4 x^9 (a+b \arccos (c x))-\frac{2}{7} c^2 x^7 (a+b \arccos (c x))+\frac{1}{5} x^5 (a+b \text{a}\right.}{9 \sqrt{1-c^2 x^2}}$$
$$\frac{1}{9} x^4 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos (c x))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{9} d \int x^3 \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos (c x))^2dx+\frac{2 b c d^2 \sqrt{d-c^2 d x^2} \left(\frac{1}{315} b c \int \frac{x^5 \left(35 c^4 x^4-90 c^2 x^2+63\right)}{\sqrt{1-c^2 x^2}}dx+\frac{1}{9} c^4 x^9 (a+b \arccos (c x))-\frac{2}{7} c^2 x^7 (a+b \arccos (c x))+\frac{1}{5} x^5 (a+\right.}{9 \sqrt{1-c^2 x^2}}$$
$$\frac{1}{9} x^4 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos (c x))^2$$

$$\Big\downarrow \text{1578}$$

$$\frac{5}{9} d \int x^3 \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos (c x))^2dx+\frac{2 b c d^2 \sqrt{d-c^2 d x^2} \left(\frac{1}{630} b c \int \frac{x^4 \left(35 c^4 x^4-90 c^2 x^2+63\right)}{\sqrt{1-c^2 x^2}}dx^2+\frac{1}{9} c^4 x^9 (a+b \arccos (c x))-\frac{2}{7} c^2 x^7 (a+b \arccos (c x))+\frac{1}{5} x^5 (a\right.}{9 \sqrt{1-c^2 x^2}}$$
$$\frac{1}{9} x^4 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos (c x))^2$$

$\Big\downarrow$ 1195

$$\frac{5}{9}d\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx+$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{630}bc\int\left(\frac{35\left(1-c^2x^2\right)^{7/2}}{c^4}-\frac{50\left(1-c^2x^2\right)^{5/2}}{c^4}+\frac{3\left(1-c^2x^2\right)^{3/2}}{c^4}+\frac{4\sqrt{1-c^2x^2}}{c^4}+\frac{8}{c^4\sqrt{1-c^2x^2}}\right)dx^2+\frac{1}{9}c^4x^9(a}{9\sqrt{1-c^2x^2}}$$
$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\Big\downarrow$ 2009

$$\frac{5}{9}d\int x^3\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx+\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5203

$$\frac{5}{9}d\left(\frac{2bcd\sqrt{d-c^2dx^2}\int x^4\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{7\sqrt{1-c^2x^2}}+\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{1}{7}x^4(d\right.$$
$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5193

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(bc\int\frac{x^5\left(7-5c^2x^2\right)}{35\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^2x^7(a+b\arccos(cx))\right.}{7\sqrt{1-c^2x^2}}\right.$$
$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 27

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{35}bc\int\frac{x^5(7-5c^2x^2)}{\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^2x^7(a+b\arccos(cx\right.}{7\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\downarrow$ 354

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{70}bc\int\frac{x^4(7-5c^2x^2)}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{7}c^2x^7(a+b\arccos(c\right.}{7\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\downarrow$ 86

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{70}bc\int\left(\frac{5(1-c^2x^2)^{5/2}}{c^4}-\frac{8(1-c^2x^2)^{3/2}}{c^4}+\frac{\sqrt{1-c^2x}}{c^4}\right.\right.}{7}\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\downarrow$ 2009

$$\frac{5}{9}d\left(\frac{3}{7}d\int x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}\right.}{}\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\downarrow$ 5199

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{2bc\sqrt{d-c^2dx^2}\int x^4(a+b\arccos(cx))dx}{5\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\downarrow$ 5139

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}bc\int \frac{x^5}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))\right)}{5\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\downarrow$ 243

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{10}bc\int \frac{x^4}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{5}x^5(a+b\arccos(cx))\right)}{5\sqrt{1-c^2x^2}}+\frac{1}{5}\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\downarrow$ 53

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{10}bc\int \left(\frac{(1-c^2x^2)^{3/2}}{c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{1}{c^4\sqrt{1-c^2x^2}}\right)dx^2\right.}{5\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\downarrow$ 2009

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{1}{5}x^5(a+b\,a\right.}{}\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\downarrow$ 5211

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\int x^2(a+b\arccos(cx))dx}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}\right)}{5\sqrt{1-c^2x^2}}+\frac{1}{5}x^4\sqrt{d-c^2d}\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\downarrow$ 5139

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\left(\frac{1}{3}bc\int\frac{x^3}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}\right)}{5\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\downarrow$ 243

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\left(\frac{1}{6}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}\right)}{5\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\downarrow$ 53

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2b\left(\frac{1}{6}bc\int \left(\frac{1}{c^2\sqrt{1-c^2x^2}}-\frac{\sqrt{1-c^2x^2}}{c^2}\right)dx^2+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c}-\frac{x^2\sqrt{1-c^2x^2}(a+b}{3c^2}\right.}{5\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\downarrow$ 2009

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\arccos(cx))+\frac{1}{6}bc\left(\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2}{}\right.\right.}{3c}\right.}{5\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$\downarrow$ 5183

$$\frac{5}{9}d\left(\frac{3}{7}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{2\left(-\frac{2b\int(a+b\arccos(cx))dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}\right)}{3c^2}-\frac{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^2}-\frac{2b\left(\frac{1}{3}x^3(a+b\arccos\right.}{}\right.}{5\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{1}{9}x^4\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4x^9(a+b\arccos(cx))-\frac{2}{7}c^2x^7(a+b\arccos(cx))+\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2x^4+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{9}c^4(a+b\arccos(cx))x^9-\frac{2}{7}c^2(a+b\arccos(cx))x^7+\frac{1}{5}(a+b\arccos(cx))x^5+\frac{1}{630}bc\left(-\frac{70(1-c^2}{9c}\right.\right.}{9\sqrt{1-c^2x^2}}$$

$$\frac{5}{9}d\left(\frac{1}{7}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2x^4+\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^2(a+b\arccos(cx))x^7+\frac{1}{5}(a+b\arccos(cx))\right.}{7\sqrt{1}}\right.$$

input

```
Int[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(x^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/9 + (2*b*c*d^2*Sqrt[d -
c^2*d*x^2]*((b*c*((-16*Sqrt[1 - c^2*x^2])/c^6 - (8*(1 - c^2*x^2)^(3/2))/(3
*c^6) - (6*(1 - c^2*x^2)^(5/2))/(5*c^6) + (100*(1 - c^2*x^2)^(7/2))/(7*c^6
) - (70*(1 - c^2*x^2)^(9/2))/(9*c^6)))/630 + (x^5*(a + b*ArcCos[c*x]))/5 -
 (2*c^2*x^7*(a + b*ArcCos[c*x]))/7 + (c^4*x^9*(a + b*ArcCos[c*x]))/9))/(9*
Sqrt[1 - c^2*x^2]) + (5*d*((x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^
2)/7 + (2*b*c*d*Sqrt[d - c^2*d*x^2]*((b*c*((-4*Sqrt[1 - c^2*x^2])/c^6 - (2
*(1 - c^2*x^2)^(3/2))/(3*c^6) + (16*(1 - c^2*x^2)^(5/2))/(5*c^6) - (10*(1
- c^2*x^2)^(7/2))/(7*c^6)))/70 + (x^5*(a + b*ArcCos[c*x]))/5 - (c^2*x^7*(a
 + b*ArcCos[c*x]))/7))/(7*Sqrt[1 - c^2*x^2]) + (3*d*((x^4*Sqrt[d - c^2*d*x
^2]*(a + b*ArcCos[c*x])^2)/5 + (2*b*c*Sqrt[d - c^2*d*x^2]*((b*c*((-2*Sqrt[
1 - c^2*x^2])/c^6 + (4*(1 - c^2*x^2)^(3/2))/(3*c^6) - (2*(1 - c^2*x^2)^(5/
2))/(5*c^6)))/10 + (x^5*(a + b*ArcCos[c*x]))/5))/(5*Sqrt[1 - c^2*x^2]) + (
Sqrt[d - c^2*d*x^2]*(-1/3*(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c^
2 - (2*b*((b*c*((-2*Sqrt[1 - c^2*x^2])/c^4 + (2*(1 - c^2*x^2)^(3/2))/(3*c^
4)))/6 + (x^3*(a + b*ArcCos[c*x]))/3))/(3*c) + (2*(-((Sqrt[1 - c^2*x^2]*(a
 + b*ArcCos[c*x])^2)/c^2) - (2*b*(a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcC
os[c*x]))/c))/(3*c^2)))/(5*Sqrt[1 - c^2*x^2])))/7))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 594, normalized size of antiderivative = 0.91

method	result
orering	$\frac{(74431x^{12}c^{12}-287825c^{10}x^{10}+383719c^8x^8-112267c^6x^6-485758c^4x^4+324980c^2x^2-73680)(-c^2d\,x^2+d)^{\frac{5}{2}}(a+b\arccos(cx))^2}{250047c^6x^2(cx-1)(cx+1)(c^2x^2-1)^2} -$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/250047*(74431*c^12*x^12-287825*c^10*x^10+383719*c^8*x^8-112267*c^6*x^6-4
85758*c^4*x^4+324980*c^2*x^2-73680)/c^6/x^2/(c*x-1)/(c*x+1)/(c^2*x^2-1)^2*
(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2-4/83349*(686*c^10*x^10-2503*c^8*x
^8+2822*c^6*x^6+511*c^4*x^4-8786*c^2*x^2+3070)/c^6/x^4/(c*x-1)/(c*x+1)/(c^
2*x^2-1)*(3*x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2-5*x^4*(-c^2*d*x^2
+d)^(3/2)*(a+b*arccos(c*x))^2*d*c^2-2*x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos
(c*x))*b*c/(-c^2*x^2+1)^(1/2))+1/250047*(343*c^8*x^8-1147*c^6*x^6+1005*c^4
*x^4+899*c^2*x^2-6140)/c^6/x^3/(c*x-1)/(c*x+1)*(6*x*(-c^2*d*x^2+d)^(5/2)*(
a+b*arccos(c*x))^2-35*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2*d*c^2-1
2*x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))*b*c/(-c^2*x^2+1)^(1/2)+15*x^5
*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2*d^2*c^4+20*x^4*(-c^2*d*x^2+d)^(3
/2)*(a+b*arccos(c*x))*d*c^3*b/(-c^2*x^2+1)^(1/2)+2*x^3*(-c^2*d*x^2+d)^(5/2
)*b^2*c^2/(-c^2*x^2+1)-2*x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))*b*c^3/
(-c^2*x^2+1)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 487, normalized size of antiderivative = 0.75

$$\int x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2\,dx =$$

$$-\frac{126\,(49\,abc^9d^2x^9-171\,abc^7d^2x^7+189\,abc^5d^2x^5-21\,abc^3d^2x^3-126\,abcd^2x+(49\,b^2c^9d^2x^9-171\,b^2c^7d^2}{}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="frica
s")
```

output
```
-1/250047*(126*(49*a*b*c^9*d^2*x^9 - 171*a*b*c^7*d^2*x^7 + 189*a*b*c^5*d^2
*x^5 - 21*a*b*c^3*d^2*x^3 - 126*a*b*c*d^2*x + (49*b^2*c^9*d^2*x^9 - 171*b^
2*c^7*d^2*x^7 + 189*b^2*c^5*d^2*x^5 - 21*b^2*c^3*d^2*x^3 - 126*b^2*c*d^2*x
)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - (343*(81*a^2 - 2*
b^2)*c^10*d^2*x^10 - 2*(51597*a^2 - 1490*b^2)*c^8*d^2*x^8 + 2*(67473*a^2 -
 2152*b^2)*c^6*d^2*x^6 - 4*(15876*a^2 - 53*b^2)*c^4*d^2*x^4 - (3969*a^2 -
14078*b^2)*c^2*d^2*x^2 + 2*(3969*a^2 - 6140*b^2)*d^2 + 3969*(7*b^2*c^10*d^
2*x^10 - 26*b^2*c^8*d^2*x^8 + 34*b^2*c^6*d^2*x^6 - 16*b^2*c^4*d^2*x^4 - b^
2*c^2*d^2*x^2 + 2*b^2*d^2)*arccos(c*x)^2 + 7938*(7*a*b*c^10*d^2*x^10 - 26*
a*b*c^8*d^2*x^8 + 34*a*b*c^6*d^2*x^6 - 16*a*b*c^4*d^2*x^4 - a*b*c^2*d^2*x^
2 + 2*a*b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 (d - c^2 d x^2)^{5/2} (a + b \arccos(cx))^2 \, dx = \text{Timed out}$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.62

$$\int x^3(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2\,dx =$$

$$-\frac{1}{63}\left(\frac{7(-c^2dx^2+d)^{\frac{7}{2}}x^2}{c^2d}+\frac{2(-c^2dx^2+d)^{\frac{7}{2}}}{c^4d}\right)b^2\arccos(cx)^2$$

$$-\frac{2}{63}\left(\frac{7(-c^2dx^2+d)^{\frac{7}{2}}x^2}{c^2d}+\frac{2(-c^2dx^2+d)^{\frac{7}{2}}}{c^4d}\right)ab\arccos(cx)$$

$$-\frac{1}{63}\left(\frac{7(-c^2dx^2+d)^{\frac{7}{2}}x^2}{c^2d}+\frac{2(-c^2dx^2+d)^{\frac{7}{2}}}{c^4d}\right)a^2$$

$$-\frac{2}{250047}b^2\left(\frac{343\sqrt{-c^2x^2+1}c^6d^{\frac{5}{2}}x^8-1147\sqrt{-c^2x^2+1}c^4d^{\frac{5}{2}}x^6+1005\sqrt{-c^2x^2+1}c^2d^{\frac{5}{2}}x^4+899\sqrt{-c^2x^2}}{c^2}\right.$$

$$+\frac{2\left(49c^8d^{\frac{5}{2}}x^9-171c^6d^{\frac{5}{2}}x^7+189c^4d^{\frac{5}{2}}x^5-21c^2d^{\frac{5}{2}}x^3-126d^{\frac{5}{2}}x\right)ab}{3969c^3}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="maxim
a")
```

output
```
-1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^
4*d))*b^2*arccos(c*x)^2 - 2/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(
-c^2*d*x^2 + d)^(7/2)/(c^4*d))*a*b*arccos(c*x) - 1/63*(7*(-c^2*d*x^2 + d)^
(7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^4*d))*a^2 - 2/250047*b^2*(
(343*sqrt(-c^2*x^2 + 1)*c^6*d^(5/2)*x^8 - 1147*sqrt(-c^2*x^2 + 1)*c^4*d^(5
/2)*x^6 + 1005*sqrt(-c^2*x^2 + 1)*c^2*d^(5/2)*x^4 + 899*sqrt(-c^2*x^2 + 1)
*d^(5/2)*x^2 - 6140*sqrt(-c^2*x^2 + 1)*d^(5/2)/c^2)/c^2 - 63*(49*c^8*d^(5/
2)*x^9 - 171*c^6*d^(5/2)*x^7 + 189*c^4*d^(5/2)*x^5 - 21*c^2*d^(5/2)*x^3 -
126*d^(5/2)*x)*arccos(c*x)/c^3) + 2/3969*(49*c^8*d^(5/2)*x^9 - 171*c^6*d^(
5/2)*x^7 + 189*c^4*d^(5/2)*x^5 - 21*c^2*d^(5/2)*x^3 - 126*d^(5/2)*x)*a*b/c
^3
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos(cx))^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos(cx))^2 \, dx = \int x^3 \left(a+b \operatorname{acos}(c\,x)\right)^2 \left(d-c^2\,d\,x^2\right)^{5/2} \, dx$$

input
```
int(x^3*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^3*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^3 \left(d-c^2 d x^2\right)^{5/2} (a + b \arccos(cx))^2 \, dx = \frac{\sqrt{d}\, d^2 \left(7\sqrt{-c^2x^2+1}\, a^2 c^8 x^8 - 19\sqrt{-c^2x^2+1}\, a^2 c^6 x^6 + 15\sqrt{-c^2x^2+1}\, a^2 c^4 x^4 - \sqrt{-c^2 \cdots}\right.}{\cdots}$$

input
```
int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*d**2*(7*sqrt( - c**2*x**2 + 1)*a**2*c**8*x**8 - 19*sqrt( - c**2*x
**2 + 1)*a**2*c**6*x**6 + 15*sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - sqrt(
 - c**2*x**2 + 1)*a**2*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a**2 + 126*int
(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**7,x)*a*b*c**8 - 252*int(sqrt( - c**2*
x**2 + 1)*acos(c*x)*x**5,x)*a*b*c**6 + 126*int(sqrt( - c**2*x**2 + 1)*acos
(c*x)*x**3,x)*a*b*c**4 + 63*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x**7,x
)*b**2*c**8 - 126*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x**5,x)*b**2*c**
6 + 63*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x**3,x)*b**2*c**4))/(63*c**
4)
```

3.229 $\int x^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 556

$$\begin{aligned}\int x^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2\,dx = &-\frac{359b^2d^2x\sqrt{d-c^2dx^2}}{36864c^2}-\frac{1079b^2d^2x^3\sqrt{d-c^2dx^2}}{55296}\\&+\frac{209b^2c^2d^2x^5\sqrt{d-c^2dx^2}}{13824}-\frac{1}{256}b^2c^4d^2x^7\sqrt{d-c^2dx^2}\\&+\frac{359b^2d^2\sqrt{d-c^2dx^2}\arccos(cx)}{36864c^3\sqrt{1-c^2x^2}}+\frac{5bd^2x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{128c\sqrt{1-c^2x^2}}\\&-\frac{59bcd^2x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{384\sqrt{1-c^2x^2}}\\&+\frac{17bc^3d^2x^6\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{144\sqrt{1-c^2x^2}}\\&-\frac{bc^5d^2x^8\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{32\sqrt{1-c^2x^2}}\\&-\frac{5d^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{128c^2}\\&+\frac{5}{64}d^2x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\\&+\frac{5}{48}dx^3(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2\\&+\frac{1}{8}x^3(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2+\frac{5d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{384bc^3\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-359/36864*b^2*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^2-1079/55296*b^2*d^2*x^3*(-c^2
*d*x^2+d)^(1/2)+209/13824*b^2*c^2*d^2*x^5*(-c^2*d*x^2+d)^(1/2)-1/256*b^2*c
^4*d^2*x^7*(-c^2*d*x^2+d)^(1/2)+359/36864*b^2*d^2*(-c^2*d*x^2+d)^(1/2)*arc
cos(c*x)/c^3/(-c^2*x^2+1)^(1/2)+5/128*b*d^2*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*
arccos(c*x))/c/(-c^2*x^2+1)^(1/2)-59/384*b*c*d^2*x^4*(-c^2*d*x^2+d)^(1/2)*
(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+17/144*b*c^3*d^2*x^6*(-c^2*d*x^2+d)^(
1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-1/32*b*c^5*d^2*x^8*(-c^2*d*x^2+d
)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-5/128*d^2*x*(-c^2*d*x^2+d)^(1
/2)*(a+b*arccos(c*x))^2/c^2+5/64*d^2*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(
c*x))^2+5/48*d*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2+1/8*x^3*(-c^2*
d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2+5/384*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arc
cos(c*x))^3/b/c^3/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 3.02 (sec) , antiderivative size = 486, normalized size of antiderivative = 0.87

$$\int x^2\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2\,dx=\frac{d^2\left(-11520b^2\sqrt{d-c^2dx^2}\arccos(cx)^3-34560a^2\sqrt{d}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+\right.}{}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*(-11520*b^2*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^3 - 34560*a^2*Sqrt[d]*Sqr
t[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))]
+ 24*b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(576*b*Cos[2*ArcCos[c*x]] + 144*b*C
os[4*ArcCos[c*x]] - 64*b*Cos[6*ArcCos[c*x]] + 9*b*Cos[8*ArcCos[c*x]] + 115
2*a*Sin[2*ArcCos[c*x]] + 576*a*Sin[4*ArcCos[c*x]] - 384*a*Sin[6*ArcCos[c*x
]] + 72*a*Sin[8*ArcCos[c*x]]) + 288*b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2*(-
120*a + 48*b*Sin[2*ArcCos[c*x]] + 24*b*Sin[4*ArcCos[c*x]] - 16*b*Sin[6*Arc
Cos[c*x]] + 3*b*Sin[8*ArcCos[c*x]]) + Sqrt[d - c^2*d*x^2]*(-34560*a^2*c*x*
Sqrt[1 - c^2*x^2] + 271872*a^2*c^3*x^3*Sqrt[1 - c^2*x^2] - 313344*a^2*c^5*
x^5*Sqrt[1 - c^2*x^2] + 110592*a^2*c^7*x^7*Sqrt[1 - c^2*x^2] + 13824*a*b*C
os[2*ArcCos[c*x]] + 3456*a*b*Cos[4*ArcCos[c*x]] - 1536*a*b*Cos[6*ArcCos[c*
x]] + 216*a*b*Cos[8*ArcCos[c*x]] - 6912*b^2*Sin[2*ArcCos[c*x]] - 864*b^2*S
in[4*ArcCos[c*x]] + 256*b^2*Sin[6*ArcCos[c*x]] - 27*b^2*Sin[8*ArcCos[c*x]]
)))/(884736*c^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 3.63 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.38, number of steps used = 27, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.931$, Rules used = {5203, 5193, 27, 1590, 25, 27, 363, 262, 262, 223, 5203, 5193, 27, 363, 262, 262, 223, 5199, 5139, 262, 262, 223, 5211, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos (c x))^2 \, dx$$

$$\Big\downarrow \text{ 5203}$$

$$\frac{b c d^2 \sqrt{d-c^2 d x^2} \int x^3 \left(1-c^2 x^2\right)^2 (a+b \arccos (c x))dx}{4 \sqrt{1-c^2 x^2}}+\frac{5}{8} d \int x^2 \left(d-c^2 d x^2\right)^{3/2} (a+b \arccos (c x))^2dx+\frac{1}{8} x^3 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos (c x))^2$$

$$\Big\downarrow \text{ 5193}$$

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx+$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(bc\int\frac{x^4\left(3c^4x^4-8c^2x^2+6\right)}{24\sqrt{1-c^2x^2}}dx+\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos$$
$$4\sqrt{1-c^2x^2}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx+$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{24}bc\int\frac{x^4\left(3c^4x^4-8c^2x^2+6\right)}{\sqrt{1-c^2x^2}}dx+\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\text{ ar}$$
$$4\sqrt{1-c^2x^2}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{1590}$$

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx+$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{24}bc\left(-\frac{\int-\frac{c^2x^4\left(48-43c^2x^2\right)}{\sqrt{1-c^2x^2}}dx}{8c^2}-\frac{3}{8}c^2x^7\sqrt{1-c^2x^2}\right)+\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\text{ arcc}$$
$$4\sqrt{1-c^2x^2}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{25}$$

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx+$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{24}bc\left(\frac{\int\frac{c^2x^4\left(48-43c^2x^2\right)}{\sqrt{1-c^2x^2}}dx}{8c^2}-\frac{3}{8}c^2x^7\sqrt{1-c^2x^2}\right)+\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos($$
$$4\sqrt{1-c^2x^2}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx+$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{24}bc\left(\frac{1}{8}\int\frac{x^4\left(48-43c^2x^2\right)}{\sqrt{1-c^2x^2}}dx-\frac{3}{8}c^2x^7\sqrt{1-c^2x^2}\right)+\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos($$
$$4\sqrt{1-c^2x^2}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{363}$$

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx+$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\int\frac{x^4}{\sqrt{1-c^2x^2}}dx+\frac{43}{6}x^5\sqrt{1-c^2x^2}\right)-\frac{3}{8}c^2x^7\sqrt{1-c^2x^2}\right)+\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}\right.}{4\sqrt{1-c^2x^2}}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\downarrow$ 262

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx+$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3\int\frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{43}{6}x^5\sqrt{1-c^2x^2}\right)-\frac{3}{8}c^2x^7\sqrt{1-c^2x^2}\right)+\frac{1}{8}c^4x^8(a-\right.}{4\sqrt{1-c^2x^2}}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\downarrow$ 262

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx+$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{43}{6}x^5\sqrt{1-c^2x^2}\right)-\frac{3}{8}c^2x^7\sqrt{1-c^2x^2}\right.\right.}{4\sqrt{1-c^2x^2}}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\downarrow$ 223

$$\frac{5}{8}d\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx+$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(}{}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$
$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\downarrow$ 5203

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{bcd\sqrt{d-c^2dx^2}\int x^3\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{3\sqrt{1-c^2x^2}}+\frac{1}{6}x^3(d-\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(}{-}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5193}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{bcd\sqrt{d-c^2dx^2}\left(bc\int\frac{x^4(3-2c^2x^2)}{12\sqrt{1-c^2x^2}}dx-\frac{1}{6}c^2x^6(a+b\arccos(cx))-\right.}{3\sqrt{1-c^2x^2}}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(}{-}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{1}{12}bc\int\frac{x^4(3-2c^2x^2)}{\sqrt{1-c^2x^2}}dx-\frac{1}{6}c^2x^6(a+b\arccos(cx)\right.}{3\sqrt{1-c^2x^2}}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(}{-}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{363}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{1}{12}bc\left(\frac{4}{3}\int\frac{x^4}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^5\sqrt{1-c^2x^2}\right)-\frac{1}{6}c^2x\right.}{3\sqrt{1-c^2x^2}}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(}{-}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\downarrow$ 262

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\int\frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{3}x^5\sqrt{\cdots}\right.\right.}{3\sqrt{1\cdots}}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(\cdots}{\cdots}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\downarrow$ 262

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{bcd\sqrt{d-c^2dx^2}\left(\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2\cdots}}{4c^2}\right.\right.\right.}{\cdots}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(\cdots}{\cdots}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\downarrow$ 223

$$\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{6}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos\cdots\right.}{\cdots}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(\cdots}{\cdots}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5199}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\int x^3(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}(a+b\,\mathrm{a}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(}{}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5139}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}bc\int\frac{x^4}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))\right)}{2\sqrt{1-c^2x^2}}+\frac{1}{4}x^3\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(}{}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{262}$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}bc\left(\frac{3\int\frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{4}x^4(a+b\,\mathrm{arcc}\right.}{2\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(}{}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\downarrow$ 262

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}bc\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{4}\right.}{2\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(}{}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\downarrow$ 223

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right.}{4c^2}\right.\right.}{2\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(}{}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\downarrow$ 5211

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int x(a+b\arccos(cx))dx}{c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{4\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{4}x\right.}{}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(}{}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow 5139$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{1}{2}bc\int\frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c}+\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{4\sqrt{1-c^2x^2}}+\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(}{}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow 262$$

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c}+\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos}{2c^2}\right.}{4\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(}{}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\downarrow$ 223

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\left(\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx)}{2c^2}\right.}{4\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(}{}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\downarrow$ 5153

$$\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c}-\frac{(a+b\arccos(cx))^3}{6bc^3}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}\right.}{4\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{8}c^4x^8(a+b\arccos(cx))-\frac{1}{3}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{24}bc\left(\frac{1}{8}\left(\frac{73}{6}\left(\frac{3(}{}\right.\right.\right.\right.}{4\sqrt{1-c^2x^2}}$$

$$\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

input
```
Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/8 + (b*c*d^2*Sqrt[d - c^
2*d*x^2]*((x^4*(a + b*ArcCos[c*x]))/4 - (c^2*x^6*(a + b*ArcCos[c*x]))/3 +
(c^4*x^8*(a + b*ArcCos[c*x]))/8 + (b*c*((-3*c^2*x^7*Sqrt[1 - c^2*x^2])/8 +
 ((43*x^5*Sqrt[1 - c^2*x^2])/6 + (73*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (
3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/6)/8))
/24))/(4*Sqrt[1 - c^2*x^2]) + (5*d*((x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcC
os[c*x])^2)/6 + (b*c*d*Sqrt[d - c^2*d*x^2]*((x^4*(a + b*ArcCos[c*x]))/4 -
(c^2*x^6*(a + b*ArcCos[c*x]))/6 + (b*c*((x^5*Sqrt[1 - c^2*x^2])/3 + (4*(-1
/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcS
in[c*x]/(2*c^3)))/(4*c^2)))/3))/12))/(3*Sqrt[1 - c^2*x^2]) + (d*((x^3*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/4 + (b*c*Sqrt[d - c^2*d*x^2]*((x^4*
(a + b*ArcCos[c*x]))/4 + (b*c*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2
*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/4))/(2*Sqrt[1
 - c^2*x^2]) + (Sqrt[d - c^2*d*x^2]*(-1/2*(x*Sqrt[1 - c^2*x^2]*(a + b*ArcC
os[c*x])^2)/c^2 - (a + b*ArcCos[c*x])^3/(6*b*c^3) - (b*((x^2*(a + b*ArcCos
[c*x]))/2 + (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2
))/c))/(4*Sqrt[1 - c^2*x^2])))/2))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 2228, normalized size of antiderivative = 4.01

method	result	size
default	Expression too large to display	2228
parts	Expression too large to display	2228

input
```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^2*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/48*a^2/c^2*x*(-c^2*d*x^2+d)^(5/2)+
5/192*a^2/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/128*a^2/c^2*d^2*x*(-c^2*d*x^2+d)^
(1/2)+5/128*a^2/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d
)^(1/2))+b^2*(5/384*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2
-1)*arccos(c*x)^3*d^2+1/65536*(-d*(c^2*x^2-1))^(1/2)*(128*c^9*x^9-320*c^7*
x^7+128*I*(-c^2*x^2+1)^(1/2)*x^8*c^8+272*c^5*x^5-256*I*(-c^2*x^2+1)^(1/2)*
x^6*c^6-88*c^3*x^3+160*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c*x-32*I*(-c^2*x^2+1
)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(8*I*arccos(c*x)+32*arccos(c*x)^2-1)
*d^2/c^3/(c^2*x^2-1)-1/6912*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^7-64*c^5*x^5+
32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+38*c^3*x^3-48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4
-6*c*x+18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(6*I*arccos(c
*x)+18*arccos(c*x)^2-1)*d^2/c^3/(c^2*x^2-1)+1/2048*(-d*(c^2*x^2-1))^(1/2)*
(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-8*I*(-c^2*x^2+1
)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(4*I*arccos(c*x)+8*arccos(c*x)^2-1)*
d^2/c^3/(c^2*x^2-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*
x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*arccos(c*x)^2-1-2*I*arcco
s(c*x))*d^2/c^3/(c^2*x^2-1)-1/6912*(-d*(c^2*x^2-1))^(1/2)*(-32*I*(-c^2*x^2
+1)^(1/2)*x^6*c^6+32*c^7*x^7+48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5-18
*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-6*c*x)*(-6*I
*arccos(c*x)+18*arccos(c*x)^2-1)*d^2/c^3/(c^2*x^2-1)+1/65536*(-d*(c^2*x...
```

Fricas [F]

$$\int x^2 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))^2\,dx = \int \left(-c^2dx^2+d\right)^{\frac{5}{2}} (b\arccos{(cx)}+a)^2x^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="frica
s")
```

output
```
integral((a^2*c^4*d^2*x^6 - 2*a^2*c^2*d^2*x^4 + a^2*d^2*x^2 + (b^2*c^4*d^2
*x^6 - 2*b^2*c^2*d^2*x^4 + b^2*d^2*x^2)*arccos(c*x)^2 + 2*(a*b*c^4*d^2*x^6
 - 2*a*b*c^2*d^2*x^4 + a*b*d^2*x^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(d - c^2 d x^2\right)^{5/2} (a + b \arccos(cx))^2 \, dx = \text{Timed out}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 \left(d - c^2 d x^2\right)^{5/2} (a + b \arccos(cx))^2 \, dx = \int \left(-c^2 d x^2 + d\right)^{\frac{5}{2}} (b \arccos(cx) + a)^2 x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="maxim
a")
```

output
```
1/384*(8*(-c^2*d*x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d
) + 10*(-c^2*d*x^2 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2
+ 15*d^(5/2)*arcsin(c*x)/c^3)*a^2 + sqrt(d)*integrate(((b^2*c^4*d^2*x^6 -
2*b^2*c^2*d^2*x^4 + b^2*d^2*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x
)^2 + 2*(a*b*c^4*d^2*x^6 - 2*a*b*c^2*d^2*x^4 + a*b*d^2*x^2)*arctan2(sqrt(c
*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1), x)
```

Giac [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.20

$$\int x^2 \left(d - c^2 d x^2\right)^{5/2} (a + b \arccos(cx))^2 \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="giac"
)
```

output
```
1/8*sqrt(-c^2*d*x^2 + d)*a^2*c^4*d^2*x^7 - 17/48*sqrt(-c^2*d*x^2 + d)*a^2*
c^2*d^2*x^5 + 59/192*sqrt(-c^2*d*x^2 + d)*a^2*d^2*x^3 - 5/128*sqrt(-c^2*d*
x^2 + d)*a^2*d^2*x/c^2 - 5/128*a^2*d^3*log(abs(-c*sqrt(-d)*x + sqrt(c^2*x^
2 - 1)*sqrt(-d)))/(c^3*sqrt(-d)) + 1/110592*(3456*b^2*c^7*d^(5/2)*x^8*arcc
os(c*x) + 13824*sqrt(-c^2*x^2 + 1)*b^2*c^6*d^(5/2)*x^7*arccos(c*x)^2 + 345
6*a*b*c^7*d^(5/2)*x^8 + 27648*sqrt(-c^2*x^2 + 1)*a*b*c^6*d^(5/2)*x^7*arcco
s(c*x) - 432*sqrt(-c^2*x^2 + 1)*b^2*c^6*d^(5/2)*x^7 - 13056*b^2*c^5*d^(5/2
)*x^6*arccos(c*x) - 39168*sqrt(-c^2*x^2 + 1)*b^2*c^4*d^(5/2)*x^5*arccos(c*
x)^2 - 13056*a*b*c^5*d^(5/2)*x^6 - 78336*sqrt(-c^2*x^2 + 1)*a*b*c^4*d^(5/2
)*x^5*arccos(c*x) + 1672*sqrt(-c^2*x^2 + 1)*b^2*c^4*d^(5/2)*x^5 + 16992*b^
2*c^3*d^(5/2)*x^4*arccos(c*x) + 33984*sqrt(-c^2*x^2 + 1)*b^2*c^2*d^(5/2)*x
^3*arccos(c*x)^2 + 16992*a*b*c^3*d^(5/2)*x^4 + 67968*sqrt(-c^2*x^2 + 1)*a*
b*c^2*d^(5/2)*x^3*arccos(c*x) - 2158*sqrt(-c^2*x^2 + 1)*b^2*c^2*d^(5/2)*x^
3 - 4320*b^2*c*d^(5/2)*x^2*arccos(c*x) - 4320*sqrt(-c^2*x^2 + 1)*b^2*d^(5/
2)*x*arccos(c*x)^2 - 4320*a*b*c*d^(5/2)*x^2 - 8640*sqrt(-c^2*x^2 + 1)*a*b*
d^(5/2)*x*arccos(c*x) - 1440*b^2*d^(5/2)*arccos(c*x)^3/c - 1077*sqrt(-c^2*
x^2 + 1)*b^2*d^(5/2)*x - 4320*a*b*d^(5/2)*arccos(c*x)^2/c - 1077*b^2*d^(5/
2)*arccos(c*x)/c - 1077*a*b*d^(5/2)/c)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2\,dx=\int x^2\,(a+b\,\mathrm{acos}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int(x^2*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^2*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2\,dx = \frac{\sqrt{d}\,d^2\left(15asin(cx)\,a^2+48\sqrt{-c^2x^2+1}\,a^2c^7x^7-136\sqrt{-c^2x^2+1}\,a^2c^5x^5+118\sqrt{-c^2x^2}\right.}{}$$

input
```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*d**2*(15*asin(c*x)*a**2 + 48*sqrt( - c**2*x**2 + 1)*a**2*c**7*x**
7 - 136*sqrt( - c**2*x**2 + 1)*a**2*c**5*x**5 + 118*sqrt( - c**2*x**2 + 1)
*a**2*c**3*x**3 - 15*sqrt( - c**2*x**2 + 1)*a**2*c*x + 768*int(sqrt( - c**
2*x**2 + 1)*acos(c*x)*x**6,x)*a*b*c**7 - 1536*int(sqrt( - c**2*x**2 + 1)*a
cos(c*x)*x**4,x)*a*b*c**5 + 768*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**2,
x)*a*b*c**3 + 384*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x**6,x)*b**2*c**
7 - 768*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x**4,x)*b**2*c**5 + 384*in
t(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x**2,x)*b**2*c**3))/(384*c**3)
```

3.230 $\int x(d-c^2dx^2)^{5/2} (a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 382

$$\begin{aligned}\int x(d-c^2dx^2)^{5/2} (a+b\arccos(cx))^2\,dx &= \frac{32b^2d^2\sqrt{d-c^2dx^2}}{245c^2} \\ &+ \frac{16b^2d^2(1-c^2x^2)\sqrt{d-c^2dx^2}}{735c^2} + \frac{12b^2d^2(1-c^2x^2)^2\sqrt{d-c^2dx^2}}{1225c^2} \\ &+ \frac{2b^2d^2(1-c^2x^2)^3\sqrt{d-c^2dx^2}}{343c^2} + \frac{2bd^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{7c\sqrt{1-c^2x^2}} \\ &- \frac{2bcd^2x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{7\sqrt{1-c^2x^2}} \\ &+ \frac{6bc^3d^2x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{35\sqrt{1-c^2x^2}} \\ &- \frac{2bc^5d^2x^7\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{49\sqrt{1-c^2x^2}} - \frac{(d-c^2dx^2)^{7/2}(a+b\arccos(cx))^2}{7c^2d}\end{aligned}$$

output
```
32/245*b^2*d^2*(-c^2*d*x^2+d)^(1/2)/c^2+16/735*b^2*d^2*(-c^2*x^2+1)*(-c^2*
d*x^2+d)^(1/2)/c^2+12/1225*b^2*d^2*(-c^2*x^2+1)^2*(-c^2*d*x^2+d)^(1/2)/c^2
+2/343*b^2*d^2*(-c^2*x^2+1)^3*(-c^2*d*x^2+d)^(1/2)/c^2+2/7*b*d^2*x*(-c^2*d
*x^2+d)^(1/2)*(a+b*arccos(c*x))/c/(-c^2*x^2+1)^(1/2)-2/7*b*c*d^2*x^3*(-c^2
*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+6/35*b*c^3*d^2*x^5*(-
c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-2/49*b*c^5*d^2*x^7
*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-1/7*(-c^2*d*x^2
+d)^(7/2)*(a+b*arccos(c*x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.59

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2\,dx=\frac{d^2\sqrt{d-c^2dx^2}\Big(3675a^2(-1+c^2x^2)^4-210abcx\sqrt{1-c^2x^2}(-35+35c^2x^2-21c^4x^4+5}{}$$

input
```
Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(3675*a^2*(-1 + c^2*x^2)^4 - 210*a*b*c*x*Sqrt[1 -
 c^2*x^2]*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) - 2*b^2*(2161 - 2918
*c^2*x^2 + 1108*c^4*x^4 - 426*c^6*x^6 + 75*c^8*x^8) + 210*b*(35*a*(-1 + c^
2*x^2)^4 + b*c*x*Sqrt[1 - c^2*x^2]*(35 - 35*c^2*x^2 + 21*c^4*x^4 - 5*c^6*x
^6))*ArcCos[c*x] + 3675*b^2*(-1 + c^2*x^2)^4*ArcCos[c*x]^2))/(25725*c^2*(-
1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.60, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5183, 5155, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \; 5183$$

$$-\frac{2bd^2\sqrt{d-c^2dx^2}\int\left(1-c^2x^2\right)^3(a+b\arccos(cx))dx}{7c\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))^2}{7c^2d}$$

$$\Big\downarrow \; 5155$$

$$-\frac{2bd^2\sqrt{d-c^2dx^2}\left(bc\int\frac{x(-5c^6x^6+21c^4x^4-35c^2x^2+35)}{35\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^6x^7(a+b\arccos(cx))+\frac{3}{5}c^4x^5(a+b\arccos(cx))-c^2x\right.}{7c\sqrt{1-c^2x^2}}$$
$$\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))^2}{7c^2d}$$

$$\Big\downarrow \; 27$$

$$-\frac{2bd^2\sqrt{d-c^2dx^2}\left(\frac{1}{35}bc\int\frac{x(-5c^6x^6+21c^4x^4-35c^2x^2+35)}{\sqrt{1-c^2x^2}}dx-\frac{1}{7}c^6x^7(a+b\arccos(cx))+\frac{3}{5}c^4x^5(a+b\arccos(cx))-c\right.}{7c\sqrt{1-c^2x^2}}$$
$$\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))^2}{7c^2d}$$

$$\Big\downarrow \; 2331$$

$$-\frac{2bd^2\sqrt{d-c^2dx^2}\left(\frac{1}{70}bc\int\frac{-5c^6x^6+21c^4x^4-35c^2x^2+35}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{7}c^6x^7(a+b\arccos(cx))+\frac{3}{5}c^4x^5(a+b\arccos(cx))-c^2x\right.}{7c\sqrt{1-c^2x^2}}$$
$$\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))^2}{7c^2d}$$

$$\Big\downarrow \; 2389$$

$$-\frac{2bd^2\sqrt{d-c^2dx^2}\left(\frac{1}{70}bc\int\left(5\left(1-c^2x^2\right)^{5/2}+6\left(1-c^2x^2\right)^{3/2}+8\sqrt{1-c^2x^2}+\frac{16}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{7}c^6x^7(a+b\arccos\right.}{7c\sqrt{1-c^2x^2}}$$
$$\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))^2}{7c^2d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\left(d-c^2dx^2\right)^{7/2}(a+b\arccos(cx))^2}{7c^2d}-$$
$$\frac{2bd^2\sqrt{d-c^2dx^2}\left(-\frac{1}{7}c^6x^7(a+b\arccos(cx))+\frac{3}{5}c^4x^5(a+b\arccos(cx))-c^2x^3(a+b\arccos(cx))+x(a+b\arccos\right.}{7c\sqrt{1-c^2x^2}}$$

input
```
Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/7*((d - c^2*d*x^2)^(7/2)*(a + b*ArcCos[c*x])^2)/(c^2*d) - (2*b*d^2*Sqrt
[d - c^2*d*x^2]*((b*c*((-32*Sqrt[1 - c^2*x^2])/c^2 - (16*(1 - c^2*x^2)^(3/
2))/(3*c^2) - (12*(1 - c^2*x^2)^(5/2))/(5*c^2) - (10*(1 - c^2*x^2)^(7/2))/
(7*c^2)))/70 + x*(a + b*ArcCos[c*x]) - c^2*x^3*(a + b*ArcCos[c*x]) + (3*c^
4*x^5*(a + b*ArcCos[c*x]))/5 - (c^6*x^7*(a + b*ArcCos[c*x]))/7))/(7*c*Sqrt
[1 - c^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.39

method	result
orering	$\frac{(9525c^{10}x^{10}-41691c^8x^8+76515c^6x^6-124979c^4x^4+26152c^2x^2-4322)\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}(a+b\arccos(cx))^2}{25725c^4x^2(cx-1)(cx+1)(c^2x^2-1)^2} - \frac{2(675c^8x^8-3108c^6x^6}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/25725*(9525*c^10*x^10-41691*c^8*x^8+76515*c^6*x^6-124979*c^4*x^4+26152*c
^2*x^2-4322)/c^4/x^2/(c*x-1)/(c*x+1)/(c^2*x^2-1)^2*(-c^2*d*x^2+d)^(5/2)*(a
+b*arccos(c*x))^2-2/25725*(675*c^8*x^8-3108*c^6*x^6+6352*c^4*x^4-14480*c^2
*x^2+2161)/c^4/x^2/(c*x-1)/(c*x+1)/(c^2*x^2-1)*((-c^2*d*x^2+d)^(5/2)*(a+b*
arccos(c*x))^2-5*x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2*d*c^2-2*x*(-
c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))*b*c/(-c^2*x^2+1)^(1/2))+1/25725*(75*c
^6*x^6-351*c^4*x^4+757*c^2*x^2-2161)/c^4/x/(c*x-1)/(c*x+1)*(-15*c^2*d*x*(-
c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2-4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(
c*x))*b*c/(-c^2*x^2+1)^(1/2)+15*x^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))
^2*d^2*c^4+20*x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))*d*c^3*b/(-c^2*x^2
+1)^(1/2)+2*x*(-c^2*d*x^2+d)^(5/2)*b^2*c^2/(-c^2*x^2+1)-2*x^2*(-c^2*d*x^2+
d)^(5/2)*(a+b*arccos(c*x))*b*c^3/(-c^2*x^2+1)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.06

$$\int x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2\,dx =$$
$$-\frac{210\,(5\,abc^7d^2x^7-21\,abc^5d^2x^5+35\,abc^3d^2x^3-35\,abcd^2x+(5\,b^2c^7d^2x^7-21\,b^2c^5d^2x^5+35\,b^2c^3d^2x^3-35}{}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
-1/25725*(210*(5*a*b*c^7*d^2*x^7 - 21*a*b*c^5*d^2*x^5 + 35*a*b*c^3*d^2*x^3
 - 35*a*b*c*d^2*x + (5*b^2*c^7*d^2*x^7 - 21*b^2*c^5*d^2*x^5 + 35*b^2*c^3*d
^2*x^3 - 35*b^2*c*d^2*x)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 +
 1) - (75*(49*a^2 - 2*b^2)*c^8*d^2*x^8 - 12*(1225*a^2 - 71*b^2)*c^6*d^2*x^
6 + 2*(11025*a^2 - 1108*b^2)*c^4*d^2*x^4 - 4*(3675*a^2 - 1459*b^2)*c^2*d^2
*x^2 + (3675*a^2 - 4322*b^2)*d^2 + 3675*(b^2*c^8*d^2*x^8 - 4*b^2*c^6*d^2*x
^6 + 6*b^2*c^4*d^2*x^4 - 4*b^2*c^2*d^2*x^2 + b^2*d^2)*arccos(c*x)^2 + 7350
*(a*b*c^8*d^2*x^8 - 4*a*b*c^6*d^2*x^6 + 6*a*b*c^4*d^2*x^4 - 4*a*b*c^2*d^2*
x^2 + a*b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

Sympy [F]

$$\int x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2\,dx = \int x(-d(cx-1)\,(cx+1))^{\frac{5}{2}}\,(a+b\,\mathrm{acos}\,(cx))^2\,dx$$

input
```
integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral(x*(-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.74

$$\int x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2\,dx =$$

$$-\frac{(-c^2dx^2+d)^{\frac{7}{2}}b^2\arccos(cx)^2}{7c^2d}-\frac{2(-c^2dx^2+d)^{\frac{7}{2}}ab\arccos(cx)}{7c^2d}$$

$$-\frac{2}{25725}b^2\left(\frac{75\sqrt{-c^2x^2+1}c^4d^{\frac{7}{2}}x^6-351\sqrt{-c^2x^2+1}c^2d^{\frac{7}{2}}x^4+757\sqrt{-c^2x^2+1}d^{\frac{7}{2}}x^2-\frac{2161\sqrt{-c^2x^2+1}d^{\frac{7}{2}}}{c^2}}{d}-\right.$$

$$-\frac{(-c^2dx^2+d)^{\frac{7}{2}}a^2}{7c^2d}+\frac{2\left(5c^6d^{\frac{7}{2}}x^7-21c^4d^{\frac{7}{2}}x^5+35c^2d^{\frac{7}{2}}x^3-35d^{\frac{7}{2}}x\right)ab}{245cd}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-1/7*(-c^2*d*x^2 + d)^(7/2)*b^2*arccos(c*x)^2/(c^2*d) - 2/7*(-c^2*d*x^2 +
d)^(7/2)*a*b*arccos(c*x)/(c^2*d) - 2/25725*b^2*((75*sqrt(-c^2*x^2 + 1)*c^4
*d^(7/2)*x^6 - 351*sqrt(-c^2*x^2 + 1)*c^2*d^(7/2)*x^4 + 757*sqrt(-c^2*x^2
+ 1)*d^(7/2)*x^2 - 2161*sqrt(-c^2*x^2 + 1)*d^(7/2)/c^2)/d - 105*(5*c^6*d^(
7/2)*x^7 - 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2)*x^3 - 35*d^(7/2)*x)*arccos(
c*x)/(c*d)) - 1/7*(-c^2*d*x^2 + d)^(7/2)*a^2/(c^2*d) + 2/245*(5*c^6*d^(7/2
)*x^7 - 21*c^4*d^(7/2)*x^5 + 35*c^2*d^(7/2)*x^3 - 35*d^(7/2)*x)*a*b/(c*d)
```

Giac [F(-2)]

Exception generated.

$$\int x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2\,dx=\int x\,(a+b\,\mathrm{acos}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int(x*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2\,dx=\frac{\sqrt{d}\,d^2\left(\sqrt{-c^2x^2+1}\,a^2c^6x^6-3\sqrt{-c^2x^2+1}\,a^2c^4x^4+3\sqrt{-c^2x^2+1}\,a^2c^2x^2-\sqrt{-c^2x^2+1}\right.}{}$$

input
```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*d**2*(sqrt( - c**2*x**2 + 1)*a**2*c**6*x**6 - 3*sqrt( - c**2*x**2
 + 1)*a**2*c**4*x**4 + 3*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt( - c
**2*x**2 + 1)*a**2 + 14*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**5,x)*a*b*c
**6 - 28*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**3,x)*a*b*c**4 + 14*int(sq
rt( - c**2*x**2 + 1)*acos(c*x)*x,x)*a*b*c**2 + 7*int(sqrt( - c**2*x**2 + 1
)*acos(c*x)**2*x**5,x)*b**2*c**6 - 14*int(sqrt( - c**2*x**2 + 1)*acos(c*x)
**2*x**3,x)*b**2*c**4 + 7*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x,x)*b**
2*c**2))/(7*c**2)
```

3.231 $\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 438

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2\,dx=-\frac{245b^2d^2x\sqrt{d-c^2dx^2}}{1152}$$
$$-\frac{65b^2d^2x(1-c^2x^2)\sqrt{d-c^2dx^2}}{1728}-\frac{1}{108}b^2d^2x\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}$$
$$+\frac{115b^2d^2\sqrt{d-c^2dx^2}\arccos(cx)}{1152c\sqrt{1-c^2x^2}}-\frac{5bcd^2x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{16\sqrt{1-c^2x^2}}$$
$$+\frac{5bd^2(1-c^2x^2)^{3/2}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{48c}$$
$$+\frac{bd^2(1-c^2x^2)^{5/2}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{18c}$$
$$+\frac{5}{16}d^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{5}{24}dx\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arcc$$

output
```
-245/1152*b^2*d^2*x*(-c^2*d*x^2+d)^(1/2)-65/1728*b^2*d^2*x*(-c^2*x^2+1)*(-
c^2*d*x^2+d)^(1/2)-1/108*b^2*d^2*x*(-c^2*x^2+1)^2*(-c^2*d*x^2+d)^(1/2)+115
/1152*b^2*d^2*(-c^2*d*x^2+d)^(1/2)*arccos(c*x)/c/(-c^2*x^2+1)^(1/2)-5/16*b
*c*d^2*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+5/48*
b*d^2*(-c^2*x^2+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c+1/18*b*d
^2*(-c^2*x^2+1)^(5/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/c+5/16*d^2*x*
(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2+5/24*d*x*(-c^2*d*x^2+d)^(3/2)*(a+
b*arccos(c*x))^2+1/6*x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2+5/48*d^2*(
-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^3/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.93

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2\,dx = \frac{d^2\left(-1440b^2\sqrt{d-c^2dx^2}\arccos(cx)^3-4320a^2\sqrt{d}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+12\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*(-1440*b^2*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^3 - 4320*a^2*Sqrt[d]*Sqrt[
1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] +
12*b*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(270*b*Cos[2*ArcCos[c*x]] - 27*b*Cos[
4*ArcCos[c*x]] + 2*b*Cos[6*ArcCos[c*x]] + 540*a*Sin[2*ArcCos[c*x]] - 108*a
*Sin[4*ArcCos[c*x]] + 12*a*Sin[6*ArcCos[c*x]]) + 72*b*Sqrt[d - c^2*d*x^2]*
ArcCos[c*x]^2*(-60*a + 45*b*Sin[2*ArcCos[c*x]] - 9*b*Sin[4*ArcCos[c*x]] +
b*Sin[6*ArcCos[c*x]]) + Sqrt[d - c^2*d*x^2]*(9504*a^2*c*x*Sqrt[1 - c^2*x^2
] - 7488*a^2*c^3*x^3*Sqrt[1 - c^2*x^2] + 2304*a^2*c^5*x^5*Sqrt[1 - c^2*x^2
] + 3240*a*b*Cos[2*ArcCos[c*x]] - 324*a*b*Cos[4*ArcCos[c*x]] + 24*a*b*Cos[
6*ArcCos[c*x]] - 1620*b^2*Sin[2*ArcCos[c*x]] + 81*b^2*Sin[4*ArcCos[c*x]] -
 4*b^2*Sin[6*ArcCos[c*x]])))/(13824*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5159, 5159, 5157, 5139, 262, 223, 5153, 5183, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))^2 \, dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)^2(a+b\arccos(cx))dx}{3\sqrt{1-c^2x^2}}+\frac{5}{6}d\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5159}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)^2(a+b\arccos(cx))dx}{3\sqrt{1-c^2x^2}}+\frac{5}{6}d\left(\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{1}{4}x(d-c^2d\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5157}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)^2(a+b\arccos(cx))dx}{3\sqrt{1-c^2x^2}}+\frac{5}{6}d\left(\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\int x(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c}}{}\right.\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \text{5139}$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)^2(a+b\arccos(cx))dx}{3\sqrt{1-c^2x^2}}+\frac{5}{6}d\left(\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\,\text{arcc}\right.}{\sqrt{1-c^2x^2}}\right.\right.$$
$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\Big\downarrow$ 262

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)^2(a+b\arccos(cx))dx}{3\sqrt{1-c^2x^2}}+$$

$$\frac{5}{6}d\left(\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\Big\downarrow$ 223

$$\frac{5}{6}d\left(\frac{3}{4}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)^2(a+b\arccos(cx))dx}{3\sqrt{1-c^2x^2}}+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\Big\downarrow$ 5153

$$\frac{5}{6}d\left(\frac{bcd\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\text{arcsi}}{2}\right.\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\int x\left(1-c^2x^2\right)^2(a+b\arccos(cx))dx}{3\sqrt{1-c^2x^2}}+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$\Big\downarrow$ 5183

$$\frac{5}{6}d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{b\int\left(1-c^2x^2\right)^{3/2}dx}{4c}-\frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{b\int\left(1-c^2x^2\right)^{5/2}dx}{6c}-\frac{\left(1-c^2x^2\right)^3(a+b\arccos(cx))}{6c^2}\right)}{3\sqrt{1-c^2x^2}}+\frac{1}{6}x\left(d-c^2dx^2\right)^{5/2}(a+$$

$$b\arccos(cx))^2$$

$\Big\downarrow$ 211

$$\frac{5}{6}d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{3}{4}\int\sqrt{1-c^2x^2}dx+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+\right.}{}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{5}{6}\int(1-c^2x^2)^{3/2}dx+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)}{6c}-\frac{(1-c^2x^2)^3(a+b\arccos(cx))}{6c^2}\right)}{3\sqrt{1-c^2x^2}}+$$

$$\frac{1}{6}x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2$$

$\downarrow$ 211

$$\frac{5}{6}d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-}{}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{1-c^2x^2}dx+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)}{6c}-\frac{(1-c^2x^2)^3(a+b\arccos(cx))}{6c^2}\right)}{3\sqrt{1-c^2x^2}}+$$

$$\frac{1}{6}x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2$$

$\downarrow$ 211

$$\frac{5}{6}d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-}{}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)}{6c}-\frac{(1-c^2x^2)^3(a+b\arccos(cx))}{6c^2}\right)}{3\sqrt{1-c^2x^2}}+$$

$$\frac{1}{6}x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2$$

$\downarrow$ 223

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)^3(a+b\arccos(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)^{5/2}\right)}{6c}\right)}{3\sqrt{1-c^2x^2}}+$$

$$\frac{5}{6}d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\right.\right.$$

$$\frac{1}{6}x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2$$

input
```
Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/6 + (b*c*d^2*Sqrt[d - c^2*
d*x^2]*(-1/6*((1 - c^2*x^2)^3*(a + b*ArcCos[c*x]))/c^2 - (b*((x*(1 - c^2*x
^2)^(5/2))/6 + (5*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2
 + ArcSin[c*x]/(2*c)))/4))/6))/(6*c)))/(3*Sqrt[1 - c^2*x^2]) + (5*d*((x*(d
 - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/4 + (3*d*((x*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCos[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^3)/(
6*b*c*Sqrt[1 - c^2*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*((x^2*(a + b*ArcCos[c*
x]))/2 + (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2))/
Sqrt[1 - c^2*x^2]))/4 + (b*c*d*Sqrt[d - c^2*d*x^2]*(-1/4*((1 - c^2*x^2)^2*
(a + b*ArcCos[c*x]))/c^2 - (b*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 -
 c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/(4*c)))/(2*Sqrt[1 - c^2*x^2])))/6
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 1463, normalized size of antiderivative = 3.34

method	result	size
default	Expression too large to display	1463
parts	Expression too large to display	1463

input

```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
1/6*x*(-c^2*d*x^2+d)^(5/2)*a^2+5/24*a^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a^2*
d^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)
*x/(-c^2*d*x^2+d)^(1/2))+b^2*(5/48*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/
2)/(c^2*x^2-1)/c*arccos(c*x)^3*d^2+1/6912*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x
^7-64*c^5*x^5+32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+38*c^3*x^3-48*I*(-c^2*x^2+1)
^(1/2)*x^4*c^4-6*c*x+18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))
*(6*I*arccos(c*x)+18*arccos(c*x)^2-1)*d^2/(c^2*x^2-1)/c-3/1024*(-d*(c^2*x^
2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-8*I
*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(4*I*arccos(c*x)+8*arcco
s(c*x)^2-1)*d^2/(c^2*x^2-1)/c+15/256*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^
2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*arccos(c*x)^2-
1-2*I*arccos(c*x))*d^2/(c^2*x^2-1)/c+1/6912*(-d*(c^2*x^2-1))^(1/2)*(-32*I*
(-c^2*x^2+1)^(1/2)*x^6*c^6+32*c^7*x^7+48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c
^5*x^5-18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-6*c
*x)*(-6*I*arccos(c*x)+18*arccos(c*x)^2-1)*d^2/(c^2*x^2-1)/c-9/1024*(-d*(c^
2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(12*I*arccos(c*x)+16
*arccos(c*x)^2-7)*cos(3*arccos(c*x))*d^2/(c^2*x^2-1)/c-3/1024*(-d*(c^2*x^2
-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(44*I*arccos(c*x)+32*arcco
s(c*x)^2-19)*sin(3*arccos(c*x))*d^2/(c^2*x^2-1)/c)+2*a*b*(5/32*(-d*(c^2*x^
2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/(c^2*x^2-1)*arccos(c*x)^2*d^2+1/2304*(...
```

Fricas [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2\,dx=\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2\,dx=\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt
(-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a^2 + sqrt(d)*integrate
(((b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arctan2(sqrt(c*x + 1)*sq
rt(-c*x + 1), c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*a
rctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1), x
)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2\,dx = \frac{\sqrt{d}\,d^2\left(15asin(cx)\,a^2+8\sqrt{-c^2x^2+1}\,a^2c^5x^5-26\sqrt{-c^2x^2+1}\,a^2c^3x^3+33\sqrt{-c^2x^2+1}\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d)*d**2*(15*asin(c*x)*a**2 + 8*sqrt( - c**2*x**2 + 1)*a**2*c**5*x**5
 - 26*sqrt( - c**2*x**2 + 1)*a**2*c**3*x**3 + 33*sqrt( - c**2*x**2 + 1)*a*
*2*c*x + 96*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**4,x)*a*b*c**5 - 192*in
t(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**2,x)*a*b*c**3 + 96*int(sqrt( - c**2*
x**2 + 1)*acos(c*x),x)*a*b*c + 48*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*
x**4,x)*b**2*c**5 - 96*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x**2,x)*b**
2*c**3 + 48*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2,x)*b**2*c))/(48*c)
```

3.232 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 687

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}\,dx = -\frac{598}{225}b^2d^2\sqrt{d-c^2dx^2}-\frac{2abcd^2x\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}}$$
$$-\frac{74}{675}b^2d^2(1-c^2x^2)\sqrt{d-c^2dx^2}-\frac{2}{125}b^2d^2(1-c^2x^2)^2\sqrt{d-c^2dx^2}$$
$$-\frac{2b^2cd^2x\sqrt{d-c^2dx^2}\arccos(cx)}{\sqrt{1-c^2x^2}}-\frac{16bcd^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{15\sqrt{1-c^2x^2}}$$
$$+\frac{22bc^3d^2x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{45\sqrt{1-c^2x^2}}-\frac{2bc^5d^2x^5\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{25\sqrt{1-c^2x^2}}$$
$$+d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{1}{3}d(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2$$
$$+\frac{1}{5}(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2-\frac{2d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}+\frac{2ibd^2\sqrt{d-c}}{}$$

output
```
-598/225*b^2*d^2*(-c^2*d*x^2+d)^(1/2)-2*a*b*c*d^2*x*(-c^2*d*x^2+d)^(1/2)/(
-c^2*x^2+1)^(1/2)-74/675*b^2*d^2*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)-2/125*b
^2*d^2*(-c^2*x^2+1)^2*(-c^2*d*x^2+d)^(1/2)-2*b^2*c*d^2*x*(-c^2*d*x^2+d)^(1
/2)*arccos(c*x)/(-c^2*x^2+1)^(1/2)-16/15*b*c*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a
+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+22/45*b*c^3*d^2*x^3*(-c^2*d*x^2+d)^(1/2
)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-2/25*b*c^5*d^2*x^5*(-c^2*d*x^2+d)^(
1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*ar
ccos(c*x))^2+1/3*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2+1/5*(-c^2*d*x^
2+d)^(5/2)*(a+b*arccos(c*x))^2-2*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)
)^2*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*I*b*d^2*(-c^2*d
*x^2+d)^(1/2)*(a+b*arccos(c*x))*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2
*x^2+1)^(1/2)-2*I*b*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*polylog(2,c
*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-2*b^2*d^2*(-c^2*d*x^2+d)^(1/2)
*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*b^2*d^2*(-c^2*d
*x^2+d)^(1/2)*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 4.27 (sec) , antiderivative size = 816, normalized size of antiderivative = 1.19

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}\,dx = \frac{d^2\Big(3600a^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(23-11c^2x^2+3c^4x^4)+6000a}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
(d^2*(3600*a^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(23 - 11*c^2*x^2 + 3*
c^4*x^4) + 6000*a*b*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*(1 - c^2*x^2)^(3/2)*Ar
cCos[c*x] - Cos[3*ArcCos[c*x]]) - 2000*b^2*Sqrt[d - c^2*d*x^2]*(-18*(1 - c
^2*x^2)^(3/2)*ArcCos[c*x]^2 - 2*Sqrt[1 - c^2*x^2]*(-13 + Cos[2*ArcCos[c*x]
]) + 3*ArcCos[c*x]*(-9*c*x + Cos[3*ArcCos[c*x]])) + 54000*a^2*Sqrt[d]*Sqrt
[1 - c^2*x^2]*Log[c*x] - 54000*a^2*Sqrt[d]*Sqrt[1 - c^2*x^2]*Log[d + Sqrt[
d]*Sqrt[d - c^2*d*x^2]] + 108000*a*b*Sqrt[d - c^2*d*x^2]*(c*x + Sqrt[1 - c
^2*x^2]*ArcCos[c*x] - ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] + ArcCos[c*
x]*Log[1 + I*E^(I*ArcCos[c*x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I
*PolyLog[2, I*E^(I*ArcCos[c*x])]) - 54000*b^2*Sqrt[d - c^2*d*x^2]*(2*Sqrt[
1 - c^2*x^2] - 2*c*x*ArcCos[c*x] - Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2 + ArcCo
s[c*x]^2*(Log[1 - I*E^(I*ArcCos[c*x])] - Log[1 + I*E^(I*ArcCos[c*x])]) + (
2*I)*ArcCos[c*x]*(PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - PolyLog[2, I*E^(I*A
rcCos[c*x])]) - 2*PolyLog[3, (-I)*E^(I*ArcCos[c*x])] + 2*PolyLog[3, I*E^(I
*ArcCos[c*x])]) - 30*a*b*Sqrt[d - c^2*d*x^2]*(16*c*x*(30 + 5*c^2*x^2 - 9*c
^4*x^4) + 15*ArcCos[c*x]*(30*Sqrt[1 - c^2*x^2] - 5*Sin[3*ArcCos[c*x]] - 3*
Sin[5*ArcCos[c*x]])) + b^2*Sqrt[d - c^2*d*x^2]*(13500*Sqrt[1 - c^2*x^2] +
30*ArcCos[c*x]*(25*Cos[3*ArcCos[c*x]] + 9*(-50*c*x + Cos[5*ArcCos[c*x]]))
- 250*Sin[3*ArcCos[c*x]] - 225*ArcCos[c*x]^2*(30*Sqrt[1 - c^2*x^2] - 5*Sin
[3*ArcCos[c*x]] - 3*Sin[5*ArcCos[c*x]]) - 54*Sin[5*ArcCos[c*x]])))/(540...
```

Rubi [A] (verified)

Time = 3.30 (sec) , antiderivative size = 579, normalized size of antiderivative = 0.84, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.690$, Rules used = {5203, 5155, 27, 1576, 1140, 2009, 5203, 5155, 27, 353, 53, 2009, 5199, 2009, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x}dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)^2(a+b\arccos(cx))dx}{5\sqrt{1-c^2x^2}}+ d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}dx+\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 5155$$

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd^2\sqrt{d-c^2dx^2}\left(bc\int \frac{x\left(3c^4x^4-10c^2x^2+15\right)}{15\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos\right.}{5\sqrt{1-c^2x^2}}$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 27$$

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{15}bc\int \frac{x\left(3c^4x^4-10c^2x^2+15\right)}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arc\right.}{5\sqrt{1-c^2x^2}}$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 1576$$

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{30}bc\int \frac{3c^4x^4-10c^2x^2+15}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arcc\right.}{5\sqrt{1-c^2x^2}}$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 1140$$

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{30}bc\int \left(3\left(1-c^2x^2\right)^{3/2}+4\sqrt{1-c^2x^2}+\frac{8}{\sqrt{1-c^2x^2}}\right)dx^2+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+\right.}{5\sqrt{1-c^2x^2}}$$
$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 2009$$

$$d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}dx+\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6\left(1-c^2x^2\right)^{5/}}{5c^2}\right.\right.}{5\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \ 5203$$

$$d\left(\frac{2bcd\sqrt{d-c^2dx^2}\int\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{3\sqrt{1-c^2x^2}}+d\int\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/}\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6\left(1-c^2x^2\right)^{5/}}{5c^2}\right.\right.}{5\sqrt{1-c^2x^2}}$$

$\downarrow$ 5155

$$d\left(d\int\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(bc\int\frac{x\left(3-c^2x^2\right)}{3\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a\right.}{3\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6\left(1-c^2x^2\right)^{5/}}{5c^2}\right.\right.}{5\sqrt{1-c^2x^2}}$$

$\downarrow$ 27

$$d\left(d\int\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{3}bc\int\frac{x\left(3-c^2x^2\right)}{\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a\right.}{3\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6\left(1-c^2x^2\right)^{5/}}{5c^2}\right.\right.}{5\sqrt{1-c^2x^2}}$$

$\downarrow$ 353

$$d\left(d\int\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{6}bc\int\frac{3-c^2x^2}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a\right.}{3\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6\left(1-c^2x^2\right)^{5/}}{5c^2}\right.\right.}{5\sqrt{1-c^2x^2}}$$

$\downarrow$ 53

$$d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{6}bc\int\left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2x^3(a+b\,\mathrm{a}\right.}{3\sqrt{1-c^2x^2}}\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6(1-c^2x^2)^{5/}}{5c^2}\right.\right.}{5\sqrt{1-c^2x^2}}$$

$\downarrow$ 2009

$$d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2x^3(a+\right.}{}\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6(1-c^2x^2)^{5/}}{5c^2}\right.\right.}{5\sqrt{1-c^2x^2}}$$

$\downarrow$ 5199

$$d\left(d\left(\frac{2bc\sqrt{d-c^2dx^2}\int(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6(1-c^2x^2)^{5/}}{5c^2}\right.\right.}{5\sqrt{1-c^2x^2}}$$

$\downarrow$ 2009

$$d\left(d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\arccos(cx)-\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6(1-c^2x^2)^{5/}}{5c^2}\right.\right.}{5\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 5219

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int \frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\,\mathrm{a}\right.}{\sqrt{1-c}}\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6\left(1-c^2x^2\right)^{5/}}{5c^2}\right.\right.}{5\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 3042

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6\left(1-c^2x^2\right)^{5/}}{5c^2}\right.\right.}{5\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 4669

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2b\int (a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+\right.\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6\left(1-c^2x^2\right)^{5/}}{5c^2}\right.\right.}{5\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 3011

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\arccos(\right.\right.}{}\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6(1-c^2x^2)^{5/}}{5c^2}\right.\right.}{5\sqrt{1-c^2x^2}}$$

$\downarrow$ 2720

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-ie^{i\arccos(c}\right.\right.\right.}{}\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6(1-c^2x^2)^{5/}}{5c^2}\right.\right.}{5\sqrt{1-c^2x^2}}$$

$\downarrow$ 7143

$$d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(\right.\right.}{}\right.\right.$$

$$\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}bc\left(-\frac{6(1-c^2x^2)^{5/}}{5c^2}\right.\right.}{5\sqrt{1-c^2x^2}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/5 + (2*b*c*d^2*Sqrt[d - c^2*
d*x^2]*((b*c*((-16*Sqrt[1 - c^2*x^2])/c^2 - (8*(1 - c^2*x^2)^(3/2))/(3*c^2
) - (6*(1 - c^2*x^2)^(5/2))/(5*c^2)))/30 + x*(a + b*ArcCos[c*x]) - (2*c^2*
x^3*(a + b*ArcCos[c*x]))/3 + (c^4*x^5*(a + b*ArcCos[c*x]))/5))/(5*Sqrt[1 -
 c^2*x^2]) + d*(((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/3 + (2*b*c*d
*Sqrt[d - c^2*d*x^2]*((b*c*((-4*Sqrt[1 - c^2*x^2])/c^2 - (2*(1 - c^2*x^2)^
(3/2))/(3*c^2)))/6 + x*(a + b*ArcCos[c*x]) - (c^2*x^3*(a + b*ArcCos[c*x]))
/3))/(3*Sqrt[1 - c^2*x^2]) + d*(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2
+ (2*b*c*Sqrt[d - c^2*d*x^2]*(a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c
*x]))/Sqrt[1 - c^2*x^2] - (Sqrt[d - c^2*d*x^2]*((-2*I)*(a + b*ArcCos[c*x])
^2*ArcTan[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*
E^(I*ArcCos[c*x])] - b*PolyLog[3, (-I)*E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b
*ArcCos[c*x])*PolyLog[2, I*E^(I*ArcCos[c*x])] - b*PolyLog[3, I*E^(I*ArcCos
[c*x])])))/Sqrt[1 - c^2*x^2]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 1173, normalized size of antiderivative = 1.71

method	result	size
default	Expression too large to display	1173

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/4000*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*I*(-c^2*x^2+1)^(1/
2)*x^5*c^5+13*c^2*x^2-20*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+5*I*(-c^2*x^2+1)^(1/
2)*c*x-1)*(10*I*b^2*arccos(c*x)+25*arccos(c*x)^2*b^2+10*I*a*b+50*arccos(c*
x)*a*b+25*a^2-2*b^2)*d^2/(c^2*x^2-1)-7/864*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x
^4-5*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-3*I*(-c^2*x^2+1)^(1/2)*c*x+1)*
(6*I*b^2*arccos(c*x)+9*arccos(c*x)^2*b^2+6*I*a*b+18*arccos(c*x)*a*b+9*a^2-
2*b^2)*d^2/(c^2*x^2-1)+11/16*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*
c*x+c^2*x^2-1)*(arccos(c*x)^2*b^2+2*arccos(c*x)*a*b+a^2-2*b^2+2*I*arccos(c
*x)*b^2+2*I*a*b)*d^2/(c^2*x^2-1)+11/16*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^
2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)^2*b^2+2*arccos(c*x)*a*b+a^2-2*b^2-2
*I*b^2*arccos(c*x)-2*I*a*b)*d^2/(c^2*x^2-1)-7/864*(-d*(c^2*x^2-1))^(1/2)*(
-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+4*c^4*x^4+3*I*(-c^2*x^2+1)^(1/2)*c*x-5*c^2
*x^2+1)*(-6*I*b^2*arccos(c*x)+9*arccos(c*x)^2*b^2-6*I*a*b+18*arccos(c*x)*a
*b+9*a^2-2*b^2)*d^2/(c^2*x^2-1)+1/4000*(-d*(c^2*x^2-1))^(1/2)*(-16*I*(-c^2
*x^2+1)^(1/2)*x^5*c^5+16*c^6*x^6+20*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-28*c^4*x^
4-5*I*(-c^2*x^2+1)^(1/2)*c*x+13*c^2*x^2-1)*(-10*I*b^2*arccos(c*x)+25*arcco
s(c*x)^2*b^2-10*I*a*b+50*arccos(c*x)*a*b+25*a^2-2*b^2)*d^2/(c^2*x^2-1)-I*(
-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*(I*arccos(c*x)^2*ln(1
-I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b^2-I*arccos(c*x)^2*ln(1+I*(c*x+I*(-c^2*x^2
+1)^(1/2)))*b^2+2*I*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))*a*b-...
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x,x, algorithm="fricas"
)
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))**2/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)^2}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x,x, algorithm="maxima"
)
```

output
```
-1/15*(15*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
- 3*(-c^2*d*x^2 + d)^(5/2) - 5*(-c^2*d*x^2 + d)^(3/2)*d - 15*sqrt(-c^2*d*x
^2 + d)*d^2)*a^2 + sqrt(d)*integrate(((b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2
 + b^2*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*(a*b*c^4*d^2*
x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c
*x))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^{5/2}}{x}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2))/x,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2))/x, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}\,dx = \frac{\sqrt{d}\,d^2\Big(3\sqrt{-c^2x^2+1}\,a^2c^4x^4-11\sqrt{-c^2x^2+1}\,a^2c^2x^2+23\sqrt{-c^2}}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))^2/x,x)
```

output
```
(sqrt(d)*d**2*(3*sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - 11*sqrt( - c**2*x
**2 + 1)*a**2*c**2*x**2 + 23*sqrt( - c**2*x**2 + 1)*a**2 + 30*int((sqrt( -
 c**2*x**2 + 1)*acos(c*x))/x,x)*a*b + 15*int((sqrt( - c**2*x**2 + 1)*acos(
c*x)**2)/x,x)*b**2 + 30*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x**3,x)*a*b*c
**4 - 60*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x,x)*a*b*c**2 + 15*int(sqrt(
 - c**2*x**2 + 1)*acos(c*x)**2*x**3,x)*b**2*c**4 - 30*int(sqrt( - c**2*x**
2 + 1)*acos(c*x)**2*x,x)*b**2*c**2 + 15*log(tan(asin(c*x)/2))*a**2 - 23*a*
*2))/15
```

3.233 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 561

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^2}\,dx = \frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2}$$
$$+\frac{1}{32}b^2c^2d^2x(1-c^2x^2)\sqrt{d-c^2dx^2}-\frac{89b^2cd^2\sqrt{d-c^2dx^2}\arccos(cx)}{64\sqrt{1-c^2x^2}}$$
$$+\frac{15bc^3d^2x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{8\sqrt{1-c^2x^2}}+bcd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$-\frac{1}{8}bcd^2(1-c^2x^2)^{3/2}\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$-\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2-\frac{icd^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}$$
$$-\frac{5}{4}c^2dx(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}-\frac{5cd^2\sqrt{d-c^2dx^2}(a+b\arccc}{8b\sqrt{1-c^2x^2}}$$

output
```
31/64*b^2*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+1/32*b^2*c^2*d^2*x*(-c^2*x^2+1)*(
-c^2*d*x^2+d)^(1/2)-89/64*b^2*c*d^2*(-c^2*d*x^2+d)^(1/2)*arccos(c*x)/(-c^2
*x^2+1)^(1/2)+15/8*b*c^3*d^2*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-
c^2*x^2+1)^(1/2)+b*c*d^2*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcc
os(c*x))-1/8*b*c*d^2*(-c^2*x^2+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c
*x))-15/8*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2-I*c*d^2*(-c^2
*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/(-c^2*x^2+1)^(1/2)-5/4*c^2*d*x*(-c^2*d
*x^2+d)^(3/2)*(a+b*arccos(c*x))^2-(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2
/x-5/8*c*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^3/b/(-c^2*x^2+1)^(1/2)
+2*b*c*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*ln(1-(c*x+I*(-c^2*x^2+1)
^(1/2))^2)/(-c^2*x^2+1)^(1/2)-I*b^2*c*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(2,(
c*x+I*(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.05

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))^2}{x^2}\,dx = \frac{d^2\Big(-256a^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}-288a^2c^2x^2\sqrt{1-c^2x^2}\sqrt{d-c}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/x^2,x]
```

output
```
(d^2*(-256*a^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2] - 288*a^2*c^2*x^2*Sqr
t[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2] + 64*a^2*c^4*x^4*Sqrt[1 - c^2*x^2]*Sqrt
[d - c^2*d*x^2] + 160*b^2*c*x*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^3 + 480*a^2*
c*Sqrt[d]*x*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-
1 + c^2*x^2))] - 128*a*b*c*x*Sqrt[d - c^2*d*x^2]*Cos[2*ArcCos[c*x]] + 4*a*
b*c*x*Sqrt[d - c^2*d*x^2]*Cos[4*ArcCos[c*x]] - 512*a*b*c*x*Sqrt[d - c^2*d*
x^2]*Log[c*x] + (256*I)*b^2*c*x*Sqrt[d - c^2*d*x^2]*PolyLog[2, -E^((2*I)*A
rcCos[c*x])] + 64*b^2*c*x*Sqrt[d - c^2*d*x^2]*Sin[2*ArcCos[c*x]] - b^2*c*x
*Sqrt[d - c^2*d*x^2]*Sin[4*ArcCos[c*x]] + 4*b*Sqrt[d - c^2*d*x^2]*ArcCos[c
*x]*(-128*a*Sqrt[1 - c^2*x^2] - 32*b*c*x*Cos[2*ArcCos[c*x]] + b*c*x*Cos[4*
ArcCos[c*x]] - 128*b*c*x*Log[1 + E^((2*I)*ArcCos[c*x])] - 64*a*c*x*Sin[2*A
rcCos[c*x]] + 4*a*c*x*Sin[4*ArcCos[c*x]]) + 8*b*Sqrt[d - c^2*d*x^2]*ArcCos
[c*x]^2*(60*a*c*x + (32*I)*b*c*x - 32*b*Sqrt[1 - c^2*x^2] - 16*b*c*x*Sin[2
*ArcCos[c*x]] + b*c*x*Sin[4*ArcCos[c*x]])))/(256*x*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 3.30 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.10, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.828, Rules used = {5201, 5159, 5157, 5139, 262, 223, 5153, 5183, 211, 211, 223, 5189, 211, 211, 223, 5189, 211, 223, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^2}dx$$

$$\Big\downarrow \text{5201}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-5c^2d\int (d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2dx-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{5159}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\int x(1-c^2x^2)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx+\frac{1}{4}x(d-c^2\right.$$
$$\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{5157}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\int x(1-c^2x^2)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\int x(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-}}{}\right.\right.$$
$$\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{5139}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-$$

$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\int x(1-c^2x^2)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\,\text{ar}\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}$$

$\downarrow$ 262

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-$$

$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\int x(1-c^2x^2)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)-\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}$$

$\downarrow$ 223

$$-5c^2d\left(\frac{3}{4}d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right.\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}$$

$\downarrow$ 5153

$$-5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\int x(1-c^2x^2)(a+b\arccos(cx))dx}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\right.\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}$$

$\downarrow$ 5183

$$-5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{b\int(1-c^2x^2)^{3/2}dx}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx\right.}{\sqrt{1-c}}\right.\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^2(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{211}$$

$$-5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{3}{4}\int\sqrt{1-c^2x^2}dx+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(\right.}{}\right.\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{211}$$

$$-5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{}}{}\right.\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{223}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\int\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-$$

$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx}}{}\right.\right.$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{5189}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx+\frac{1}{4}bc\int \left(1-c^2x^2\right)^{3/2}dx+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}-$$
$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx}}{}\right.\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x}$$
$$\Big\downarrow 211$$
$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx+\frac{1}{4}bc\left(\frac{3}{4}\int \sqrt{1-c^2x^2}dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+\right.}{\sqrt{1-c^2x^2}}$$
$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx}}{}\right.\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x}$$
$$\Big\downarrow 211$$
$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx+\frac{1}{4}bc\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)+\right.}{\sqrt{1-c^2x^2}}$$
$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx}}{}\right.\right.$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x}$$
$$\Big\downarrow 223$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c\right.\right.\right.}{\sqrt{1-c^2x^2}}$$

$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx}}{}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{5189}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\int\sqrt{1-c^2x^2}dx+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{2}\left(1-c^2x^2\right)(a+\right.}{\sqrt{1-c^2x^2}}$$

$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx}}{}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{211}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))\right.}{\sqrt{1-c^2x^2}}$$

$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx}}{}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \text{223}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\right.\right.}{\sqrt{1-c^2x^2}}$$

$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx}}{}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \; 5137$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{2}\left(1-c^2x^2\right)(a+\right.}{\sqrt{1-c^2}}$$

$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx}}{}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{2}(1\right.}{\sqrt{}}$$

$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx}}{}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \; 4202$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)+\frac{1}{4}\left(1-c^2x^2\right)^2(a+b\arccos(cx))+\frac{1}{2}\left(1-c^2x^2\right)(a}{}$$

$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx}}{}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \textbf{2620}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(2i\left(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}\right.}{}$$

$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx}}{}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \textbf{2715}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos\right.\right.}{}$$

$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx}}{}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x}$$

$$\Big\downarrow \textbf{2838}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{4}\left(1-c^2x^2\right)^2\left(a+b\arccos(cx)\right)+\frac{1}{2}\left(1-c^2x^2\right)\left(a+b\arccos(cx)\right)+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos}\right.\right.\right.}{}$$

$$5c^2d\left(\frac{bcd\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}\right)}{2\sqrt{1-c^2x^2}}+\frac{3}{4}d\left(\frac{bc\sqrt{d-c^2dx}}{}\right.\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\arccos(cx)\right)^2}{x}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/x^2,x]
```

output
```
-(((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/x) - 5*c^2*d*((x*(d - c^2*
d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/4 + (3*d*((x*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCos[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^3)/(6*b*c*S
qrt[1 - c^2*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*((x^2*(a + b*ArcCos[c*x]))/2
+ (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2))/Sqrt[1
- c^2*x^2]))/4 + (b*c*d*Sqrt[d - c^2*d*x^2]*(-1/4*((1 - c^2*x^2)^2*(a + b*
ArcCos[c*x]))/c^2 - (b*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^
2])/2 + ArcSin[c*x]/(2*c)))/4))/(4*c)))/(2*Sqrt[1 - c^2*x^2])) - (2*b*c*d^
2*Sqrt[d - c^2*d*x^2]*(((1 - c^2*x^2)*(a + b*ArcCos[c*x]))/2 + ((1 - c^2*x
^2)^2*(a + b*ArcCos[c*x]))/4 - ((I/2)*(a + b*ArcCos[c*x])^2)/b + (b*c*((x*
Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 + (b*c*((x*(1 - c^2*x^2)^(3/2
))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/4 + (2*I)*((-
1/2*I)*(a + b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCos[c*x])] - (b*PolyLog[2,
-E^((2*I)*ArcCos[c*x])])/4)))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5189
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCos[c*x])/(2*p)), x] + (Simp[d
 Int[(d + e*x^2)^(p - 1)*((a + b*ArcCos[c*x])/x), x], x] + Simp[b*c*(d^p/(2
*p))   Int[(1 - c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.07

method	result
default	$-\frac{a^2(-c^2dx^2+d)^{\frac{7}{2}}}{dx}-a^2c^2x(-c^2dx^2+d)^{\frac{5}{2}}-\frac{5a^2c^2dx(-c^2dx^2+d)^{\frac{3}{2}}}{4}-\frac{15a^2c^2d^2x\sqrt{-c^2dx^2+d}}{8}-\frac{15a^2c^2d^3\arctan}{8\sqrt{}}$
parts	$-\frac{a^2(-c^2dx^2+d)^{\frac{7}{2}}}{dx}-a^2c^2x(-c^2dx^2+d)^{\frac{5}{2}}-\frac{5a^2c^2dx(-c^2dx^2+d)^{\frac{3}{2}}}{4}-\frac{15a^2c^2d^2x\sqrt{-c^2dx^2+d}}{8}-\frac{15a^2c^2d^3\arctan}{8\sqrt{}}$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/d/x*(-c^2*d*x^2+d)^(7/2)-a^2*c^2*x*(-c^2*d*x^2+d)^(5/2)-5/4*a^2*c^2*d
*x*(-c^2*d*x^2+d)^(3/2)-15/8*a^2*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)-15/8*a^2*c
^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/64*I*b
^2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(16*I*arccos(c*x)^2*(-c^2*x^2
+1)^(1/2)*x^4*c^4+8*I*arccos(c*x)*x^5*c^5-2*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-7
2*I*arccos(c*x)^2*(-c^2*x^2+1)^(1/2)*x^2*c^2-72*I*arccos(c*x)*x^3*c^3+40*I
*arccos(c*x)^3*x*c+33*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-128*I*arccos(c*x)*ln(1+
(c*x+I*(-c^2*x^2+1)^(1/2))^2)*x*c-64*I*arccos(c*x)^2*(-c^2*x^2+1)^(1/2)-64
*arccos(c*x)^2*c*x+33*I*arccos(c*x)*x*c-64*polylog(2,-(c*x+I*(-c^2*x^2+1)^
(1/2))^2)*x*c)*d^2/x/(c^2*x^2-1)-1/64*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2
+1)^(1/2)/x/(c^2*x^2-1)*(32*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^4*x^4+8*c^5*x
^5-144*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^2-72*c^3*x^3+120*arccos(c*x)^2
*c*x+128*I*arccos(c*x)*x*c-128*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*x*c-128*
arccos(c*x)*(-c^2*x^2+1)^(1/2)+33*c*x)*d^2
```

Fricas [F]

$$\int \frac{(d - c^2dx^2)^{5/2} (a + b\arccos(cx))^2}{x^2} dx = \int \frac{(-c^2dx^2 + d)^{\frac{5}{2}} (b\arccos(cx) + a)^2}{x^2} dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x^2,x, algorithm="frica
s")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)
```

Sympy [F]

$$\int \frac{(d - c^2dx^2)^{5/2} (a + b\arccos(cx))^2}{x^2} dx = \int \frac{(-d(cx - 1)(cx + 1))^{\frac{5}{2}} (a + b\operatorname{acos}(cx))^2}{x^2} dx$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))**2/x**2,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))**2/x**2, x)
```

Maxima [F]

$$\int \frac{(d - c^2dx^2)^{5/2} (a + b\arccos(cx))^2}{x^2} dx = \int \frac{(-c^2dx^2 + d)^{\frac{5}{2}} (b\arccos(cx) + a)^2}{x^2} dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x^2,x, algorithm="maxim
a")
```

output
```
-1/8*(10*(-c^2*d*x^2 + d)^(3/2)*c^2*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^2*d^2*
x + 15*c*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)/x)*a^2 + sqrt(d)*i
ntegrate(((b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arctan2(sqrt(c*x
 + 1)*sqrt(-c*x + 1), c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*
b*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x
 + 1)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^{5/2}}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^2,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^2}\,dx = \frac{\sqrt{d}\,d^2\left(8acos(cx)^3\,b^2cx+24acos(cx)^2\,abcx-45asin(cx)\,a^2cx+\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))^2/x^2,x)
```

output
```
(sqrt(d)*d**2*(8*acos(c*x)**3*b**2*c*x + 24*acos(c*x)**2*a*b*c*x - 45*asin
(c*x)*a**2*c*x + 6*sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - 27*sqrt( - c**2
*x**2 + 1)*a**2*c**2*x**2 - 24*sqrt( - c**2*x**2 + 1)*a**2 + 48*int(acos(c
*x)/(sqrt( - c**2*x**2 + 1)*x**2),x)*a*b*x + 24*int(acos(c*x)**2/(sqrt( -
c**2*x**2 + 1)*x**2),x)*b**2*x + 48*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x
**2,x)*a*b*c**4*x - 96*int(sqrt( - c**2*x**2 + 1)*acos(c*x),x)*a*b*c**2*x
+ 24*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x**2,x)*b**2*c**4*x - 48*int(
sqrt( - c**2*x**2 + 1)*acos(c*x)**2,x)*b**2*c**2*x))/(24*x)
```

3.234 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 740

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^3}\,dx = \frac{40}{9}b^2c^2d^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{d-c^2dx^2}}{\sqrt{1-c^2x^2}} + \frac{2}{27}b^2c^2d^2(1-c^2x^2)\sqrt{d-c^2dx^2} + \frac{5b^2c^3d^2x\sqrt{d-c^2dx^2}\arccos(cx)}{\sqrt{1-c^2x^2}} - \frac{bcd^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{x\sqrt{1-c^2x^2}} - \frac{bc^3d^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}} - \frac{2bc^5d^2x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{9\sqrt{1-c^2x^2}} - \frac{5}{2}c^2d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2 - \frac{5}{6}c^2d(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2 - \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{2x^2} + \frac{5c^2d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} - \frac{b^2c^2d^2\sqrt{d-c^2dx^2}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{1-c^2x^2}} - \frac{5ibc^2d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} + \frac{5ibc^2d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} + \frac{5b^2c^2d^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} - \frac{5b^2c^2d^2\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}$$

output
```
40/9*b^2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)+5*a*b*c^3*d^2*x*(-c^2*d*x^2+d)^(1/2)
/(-c^2*x^2+1)^(1/2)+2/27*b^2*c^2*d^2*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)+5*b
^2*c^3*d^2*x*(-c^2*d*x^2+d)^(1/2)*arccos(c*x)/(-c^2*x^2+1)^(1/2)-b*c*d^2*(
-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/x/(-c^2*x^2+1)^(1/2)-1/3*b*c^3*d^2*x
*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-2/9*b*c^5*d^2*x
^3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-5/2*c^2*d^2*(
-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2-5/6*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+
b*arccos(c*x))^2-1/2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x^2+5*c^2*d^
2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2*arctanh(c*x+I*(-c^2*x^2+1)^(1/2
))/(-c^2*x^2+1)^(1/2)-b^2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*arctanh((-c^2*x^2+1
)^(1/2))/(-c^2*x^2+1)^(1/2)-5*I*b*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos
(c*x))*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+5*I*b*c^2*d
^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))*polylog(2,c*x+I*(-c^2*x^2+1)^(1/
2))/(-c^2*x^2+1)^(1/2)+5*b^2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)*polylog(3,-c*x-I
*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-5*b^2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)
*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.44 (sec) , antiderivative size = 988, normalized size of antiderivative = 1.34

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^3}\,dx = \frac{-27a^2d^3-99a^2c^2d^3x^2+244b^2c^2d^3x^2+144a^2c^4d^3x^4-248b^2c^4d}{}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/x^3,x]
```

output
```
(-27*a^2*d^3 - 99*a^2*c^2*d^3*x^2 + 244*b^2*c^2*d^3*x^2 + 144*a^2*c^4*d^3*
x^4 - 248*b^2*c^4*d^3*x^4 - 18*a^2*c^6*d^3*x^6 + 4*b^2*c^6*d^3*x^6 + 54*a*
b*c*d^3*x*Sqrt[1 - c^2*x^2] - 252*a*b*c^3*d^3*x^3*Sqrt[1 - c^2*x^2] + 12*a
*b*c^5*d^3*x^5*Sqrt[1 - c^2*x^2] - 54*a*b*d^3*ArcCos[c*x] - 198*a*b*c^2*d^
3*x^2*ArcCos[c*x] + 288*a*b*c^4*d^3*x^4*ArcCos[c*x] - 36*a*b*c^6*d^3*x^6*A
rcCos[c*x] + 54*b^2*c*d^3*x*Sqrt[1 - c^2*x^2]*ArcCos[c*x] - 252*b^2*c^3*d^
3*x^3*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + 12*b^2*c^5*d^3*x^5*Sqrt[1 - c^2*x^2]
*ArcCos[c*x] - 27*b^2*d^3*ArcCos[c*x]^2 - 99*b^2*c^2*d^3*x^2*ArcCos[c*x]^2
 + 144*b^2*c^4*d^3*x^4*ArcCos[c*x]^2 - 18*b^2*c^6*d^3*x^6*ArcCos[c*x]^2 -
54*b^2*c^2*d^3*x^2*Sqrt[1 - c^2*x^2]*ArcCoth[Sqrt[1 - c^2*x^2]] - (54*I)*b
^2*c^2*d^3*x^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2*ArcTan[E^(I*ArcCos[c*x])] +
 270*a*b*c^2*d^3*x^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c
*x])] + 108*b^2*c^2*d^3*x^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2*Log[1 - I*E^(I
*ArcCos[c*x])] - 270*a*b*c^2*d^3*x^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 +
 I*E^(I*ArcCos[c*x])] - 108*b^2*c^2*d^3*x^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^
2*Log[1 + I*E^(I*ArcCos[c*x])] - 135*a^2*c^2*d^(5/2)*x^2*Sqrt[d - c^2*d*x^
2]*Log[x] + 135*a^2*c^2*d^(5/2)*x^2*Sqrt[d - c^2*d*x^2]*Log[d + Sqrt[d]*Sq
rt[d - c^2*d*x^2]] + (270*I)*b*c^2*d^3*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos
[c*x])*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - (270*I)*b*c^2*d^3*x^2*Sqrt[1 -
 c^2*x^2]*(a + b*ArcCos[c*x])*PolyLog[2, I*E^(I*ArcCos[c*x])] - 270*b^2...
```

Rubi [A] (warning: unable to verify)

Time = 3.42 (sec) , antiderivative size = 575, normalized size of antiderivative = 0.78, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.759$, Rules used = {5201, 5193, 27, 1578, 1192, 25, 1467, 2009, 5203, 5155, 27, 353, 53, 2009, 5199, 2009, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x^3}dx$$

$$\Big\downarrow \text{5201}$$

$$-\frac{bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{x^2}dx}{\sqrt{1-c^2x^2}}-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \; 5193$$

$$-\frac{5}{2}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\left(bc\int -\frac{-c^4x^4+6c^2x^2+3}{3x\sqrt{1-c^2x^2}}dx+\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{5}{2}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{1}{3}bc\int \frac{-c^4x^4+6c^2x^2+3}{x\sqrt{1-c^2x^2}}dx+\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \; 1578$$

$$-\frac{5}{2}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{1}{6}bc\int \frac{-c^4x^4+6c^2x^2+3}{x^2\sqrt{1-c^2x^2}}dx^2+\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \; 1192$$

$$-\frac{5}{2}c^2d\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\left(-\frac{b\int -\frac{-c^4x^8-4c^4x^4+8c^4}{1-x^4}d\sqrt{1-c^2x^2}}{3c^3}+\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{b\int \frac{-c^4x^8-4c^4x^4+8c^4}{1-x^4}d\sqrt{1-c^2x^2}}{3c^3}+\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{1467}$$

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{b\int \left(x^4c^4+\frac{3c^4}{1-x^4}+5c^4\right)d\sqrt{1-c^2x^2}}{3c^3}+\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{1-c^2x^2}}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5}{2}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x}dx-\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c\right.}{3c^3}\right.}{\sqrt{1-c^2x^2}}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{5203}$$

$$-\frac{5}{2}c^2d\left(\frac{2bcd\sqrt{d-c^2dx^2}\int \left(1-c^2x^2\right)(a+b\arccos(cx))dx}{3\sqrt{1-c^2x^2}}+d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{1}{3}\left(d-c^2\right.\right.$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c\right.}{3c^3}\right.}{\sqrt{1-c^2x^2}}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \text{5155}$$

$$-\frac{5}{2}c^2d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(bc\int \frac{x(3-c^2x^2)}{3\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+\right.}{3\sqrt{1-c^2x^2}}\right.$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c\right.}{3c^3}\right.}{\sqrt{1-c^2x^2}}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{5}{2}c^2d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{3}bc\int \frac{x(3-c^2x^2)}{\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))\right.}{3\sqrt{1-c^2x^2}}\right.$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c\right.}{3c^3}\right.}{\sqrt{1-c^2x^2}}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \ 353$$

$$-\frac{5}{2}c^2d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{6}bc\int \frac{3-c^2x^2}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))\right.}{3\sqrt{1-c^2x^2}}\right.$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c\right.}{3c^3}\right.}{\sqrt{1-c^2x^2}}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$$\Big\downarrow \ 53$$

$$-\frac{5}{2}c^2d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{6}bc\int \left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2x^3(a\right.}{3\sqrt{1-c^2x^2}}\right.$$
$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c\right.}{3c^3}\right.}{\sqrt{1-c^2x^2}}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$\Big\downarrow$ 2009

$$-\frac{5}{2}c^2d\left(d\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{x}dx+\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2+\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{1}{3}c^2\right.}{}\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c\right.}{3c^3}\right.}{\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$\Big\downarrow$ 5199

$$-\frac{5}{2}c^2d\left(d\left(\frac{2bc\sqrt{d-c^2dx^2}\int(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c\right.}{3c^3}\right.}{\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$\Big\downarrow$ 2009

$$-\frac{5}{2}c^2d\left(d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(ax+bx\arccos(\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c\right.}{3c^3}\right.}{\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$\Big\downarrow$ 5219

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2+\frac{2bc\sqrt{d-c^2dx^2}\left(ax\right.}{\sqrt{}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c\right.}{3c^3}\right.}{\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$\downarrow$ 3042

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\int(a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\sqrt{1-c^2x^2}}+\sqrt{d-c^2dx^2}(a+b\arccos(cx)\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c\right.}{3c^3}\right.}{\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$\downarrow$ 4669

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2b\int(a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+2b\int(a+b\arccos(cx))\log\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c\right.}{3c^3}\right.}{\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$\downarrow$ 3011

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\,\text{a}\right.\right.}{}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c\right.}{3c^3}\right.}{\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$\big\downarrow$ 2720

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-ie^{i\,\text{ar}}\right.\right.\right.}{}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c\right.}{3c^3}\right.}{\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

$\big\downarrow$ 7143

$$-\frac{5}{2}c^2d\left(d\left(-\frac{\sqrt{d-c^2dx^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\,\text{ar}\right.\right.\right.}{}\right.\right.$$

$$\frac{bcd^2\sqrt{d-c^2dx^2}\left(\frac{1}{3}c^4x^3(a+b\arccos(cx))-2c^2x(a+b\arccos(cx))-\frac{a+b\arccos(cx)}{x}-\frac{b\left(-3c^4\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{1}{3}c\right.}{3c^3}\right.}{\sqrt{1-c^2x^2}}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{2x^2}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/x^3,x]
```

output
```
-1/2*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/x^2 - (b*c*d^2*Sqrt[d -
 c^2*d*x^2]*(-((a + b*ArcCos[c*x])/x) - 2*c^2*x*(a + b*ArcCos[c*x]) + (c^4
*x^3*(a + b*ArcCos[c*x]))/3 - (b*(-1/3*(c^4*x^6) - 5*c^4*Sqrt[1 - c^2*x^2]
 - 3*c^4*ArcTanh[Sqrt[1 - c^2*x^2]]))/(3*c^3)))/Sqrt[1 - c^2*x^2] - (5*c^2
*d*(((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/3 + (2*b*c*d*Sqrt[d - c^
2*d*x^2]*((b*c*((-4*Sqrt[1 - c^2*x^2])/c^2 - (2*(1 - c^2*x^2)^(3/2))/(3*c^
2)))/6 + x*(a + b*ArcCos[c*x]) - (c^2*x^3*(a + b*ArcCos[c*x]))/3))/(3*Sqrt
[1 - c^2*x^2]) + d*(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2 + (2*b*c*Sqr
t[d - c^2*d*x^2]*(a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]))/Sqrt[1
 - c^2*x^2] - (Sqrt[d - c^2*d*x^2]*((-2*I)*(a + b*ArcCos[c*x])^2*ArcTan[E^
(I*ArcCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*ArcCos[
c*x])] - b*PolyLog[3, (-I)*E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c*x]
)*PolyLog[2, I*E^(I*ArcCos[c*x])] - b*PolyLog[3, I*E^(I*ArcCos[c*x])])))/S
qrt[1 - c^2*x^2])))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 1365, normalized size of antiderivative = 1.84

method	result	size
default	Expression too large to display	1365
parts	Expression too large to display	1365

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/d/x^2*(-c^2*d*x^2+d)^(7/2)-5/2*c^2*(1/5*(-c^2*d*x^2+d)^(5/2)+d*(
1/3*(-c^2*d*x^2+d)^(3/2)+d*((-c^2*d*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)
*(-c^2*d*x^2+d)^(1/2))/x)))))+b^2*(1/216*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4
-5*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-3*I*(-c^2*x^2+1)^(1/2)*c*x+1)*(6
*I*arccos(c*x)+9*arccos(c*x)^2-2)*d^2*c^2/(c^2*x^2-1)-9/8*(-d*(c^2*x^2-1))
^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)^2-2+2*I*arccos(c*
x))*d^2*c^2/(c^2*x^2-1)-9/8*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*
x*c+c^2*x^2-1)*(arccos(c*x)^2-2-2*I*arccos(c*x))*d^2*c^2/(c^2*x^2-1)+1/216
*(-d*(c^2*x^2-1))^(1/2)*(-4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+4*c^4*x^4+3*I*(-c
^2*x^2+1)^(1/2)*c*x-5*c^2*x^2+1)*(-6*I*arccos(c*x)+9*arccos(c*x)^2-2)*d^2*
c^2/(c^2*x^2-1)-1/2*d^2*(c^2*x^2*arccos(c*x)+2*c*x*(-c^2*x^2+1)^(1/2)-arcc
os(c*x))*arccos(c*x)*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/x^2+I*(-d*(c^2*x^2
-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(5*I*arccos(c*x)^2*ln(1-I*(c*x+I*(-c^2*x^2+1
)^(1/2)))-5*I*arccos(c*x)^2*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+10*arccos(c
*x)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))-10*arccos(c*x)*polylog(2,-I*(c
*x+I*(-c^2*x^2+1)^(1/2)))+10*I*polylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))-10*
I*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-4*arctan(c*x+I*(-c^2*x^2+1)^(1/
2)))*d^2*c^2/(2*c^2*x^2-2))+2*a*b*(1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-
5*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-3*I*(-c^2*x^2+1)^(1/2)*c*x+1)*(I+
3*arccos(c*x))*d^2*c^2/(c^2*x^2-1)-9/8*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*...
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^3}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x^3,x, algorithm="frica
s")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^3}\,dx=\int \frac{(-d(cx-1)(cx+1))^{\frac{5}{2}}(a+b\,\mathrm{acos}\,(cx))^2}{x^3}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))**2/x**3,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))**2/x**3, x)
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^3}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x^3,x, algorithm="maxim
a")
```

output
```
1/6*(15*c^2*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)
) - 3*(-c^2*d*x^2 + d)^(5/2)*c^2 - 5*(-c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqr
t(-c^2*d*x^2 + d)*c^2*d^2 - 3*(-c^2*d*x^2 + d)^(7/2)/(d*x^2))*a^2 + sqrt(d
)*integrate(((b^2*c^4*d^2*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arctan2(sqrt(
c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 +
 a*b*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-
c*x + 1)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x^3,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^{5/2}}{x^3}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^3,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^3}\,dx = \frac{\sqrt{d}\,d^2\left(8\sqrt{-c^2x^2+1}\,a^2c^4x^4-56\sqrt{-c^2x^2+1}\,a^2c^2x^2-12\sqrt{-c^2}\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))^2/x^3,x)
```

output
```
(sqrt(d)*d**2*(8*sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - 56*sqrt( - c**2*x
**2 + 1)*a**2*c**2*x**2 - 12*sqrt( - c**2*x**2 + 1)*a**2 + 48*int((sqrt( -
 c**2*x**2 + 1)*acos(c*x))/x**3,x)*a*b*x**2 - 96*int((sqrt( - c**2*x**2 +
1)*acos(c*x))/x,x)*a*b*c**2*x**2 + 24*int((sqrt( - c**2*x**2 + 1)*acos(c*x
)**2)/x**3,x)*b**2*x**2 - 48*int((sqrt( - c**2*x**2 + 1)*acos(c*x)**2)/x,x
)*b**2*c**2*x**2 + 48*int(sqrt( - c**2*x**2 + 1)*acos(c*x)*x,x)*a*b*c**4*x
**2 + 24*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x,x)*b**2*c**4*x**2 - 60*
log(tan(asin(c*x)/2))*a**2*c**2*x**2 + 65*a**2*c**2*x**2))/(24*x**2)
```

3.235 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 591

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^4}\,dx = -\frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2} - \frac{b^2c^2d^2(1-c^2x^2)\sqrt{d-c^2dx^2}}{3x} + \frac{23b^2c^3d^2\sqrt{d-c^2dx^2}\arccos(cx)}{12\sqrt{1-c^2x^2}} - \frac{5bc^5d^2x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2\sqrt{1-c^2x^2}} - \frac{7}{3}bc^3d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\arccos(cx)) - \frac{bcd^2(1-c^2x^2)^{3/2}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3x^2} + \frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2 + \frac{7ic^3d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3\sqrt{1-c^2x^2}} + \frac{5c^2d(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{3x} - \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{3x^3} + \frac{5c^3d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^3}{6b\sqrt{1-c^2x^2}} - \frac{14bc^3d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{3\sqrt{1-c^2x^2}} + \frac{7ib^2c^3d^2\sqrt{d-c^2dx^2}\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{3\sqrt{1-c^2x^2}}$$

output
```
-7/12*b^2*c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)-1/3*b^2*c^2*d^2*(-c^2*x^2+1)*(-c^
2*d*x^2+d)^(1/2)/x+23/12*b^2*c^3*d^2*(-c^2*d*x^2+d)^(1/2)*arccos(c*x)/(-c^
2*x^2+1)^(1/2)-5/2*b*c^5*d^2*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(-
c^2*x^2+1)^(1/2)-7/3*b*c^3*d^2*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)*(a+
b*arccos(c*x))-1/3*b*c*d^2*(-c^2*x^2+1)^(3/2)*(-c^2*d*x^2+d)^(1/2)*(a+b*ar
ccos(c*x))/x^2+5/2*c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2+7/3*
I*c^3*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/(-c^2*x^2+1)^(1/2)+5/3*
c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/x-1/3*(-c^2*d*x^2+d)^(5/2)*
(a+b*arccos(c*x))^2/x^3+5/6*c^3*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))
^3/b/(-c^2*x^2+1)^(1/2)-14/3*b*c^3*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*
x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)+7/3*I*b^2*c^3*d^
2*(-c^2*d*x^2+d)^(1/2)*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1
)^(1/2)
```

Mathematica [A] (verified)

Time = 3.00 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.15

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^4}\,dx = \frac{d^2\Big(4abcx\sqrt{d-c^2dx^2}-3abc^3x^3\sqrt{d-c^2dx^2}+6abc^5x^5\sqrt{d-c^2d}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/x^4,x]
```

output
```
(d^2*(4*a*b*c*x*Sqrt[d - c^2*d*x^2] - 3*a*b*c^3*x^3*Sqrt[d - c^2*d*x^2] +
6*a*b*c^5*x^5*Sqrt[d - c^2*d*x^2] - 4*a^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d
*x^2] + 28*a^2*c^2*x^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2] - 4*b^2*c^2*x
^2*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2] + 6*a^2*c^4*x^4*Sqrt[1 - c^2*x^2]
*Sqrt[d - c^2*d*x^2] - 3*b^2*c^4*x^4*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]
 - 10*b^2*c^3*x^3*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^3 - 30*a^2*c^3*Sqrt[d]*x
^3*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x
^2))] + 56*a*b*c^3*x^3*Sqrt[d - c^2*d*x^2]*Log[c*x] - (28*I)*b^2*c^3*x^3*S
qrt[d - c^2*d*x^2]*PolyLog[2, -E^((2*I)*ArcCos[c*x])] + b*Sqrt[d - c^2*d*x
^2]*ArcCos[c*x]*(4*b*c*x - 8*a*Sqrt[1 - c^2*x^2] + 56*a*c^2*x^2*Sqrt[1 - c
^2*x^2] + 3*b*c^3*x^3*Cos[2*ArcCos[c*x]] + 56*b*c^3*x^3*Log[1 + E^((2*I)*A
rcCos[c*x])] + 6*a*c^3*x^3*Sin[2*ArcCos[c*x]]) + b*Sqrt[d - c^2*d*x^2]*Arc
Cos[c*x]^2*(-30*a*c^3*x^3 + 4*b*((-7*I)*c^3*x^3 - Sqrt[1 - c^2*x^2] + 7*c^
2*x^2*Sqrt[1 - c^2*x^2]) + 3*b*c^3*x^3*Sin[2*ArcCos[c*x]])))/(12*x^3*Sqrt[
1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 5.10 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.14, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5201, 5191, 247, 211, 223, 5189, 211, 223, 5137, 3042, 4202, 2620, 2715, 2838, 5201, 5157, 5139, 262, 223, 5153, 5189, 211, 223, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x^4}dx$$

$$\Big\downarrow \text{5201}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{x^3}dx}{3\sqrt{1-c^2x^2}}-\frac{5}{3}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{5191}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\int\frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx-\frac{1}{2}bc\int\frac{(1-c^2x^2)^{3/2}}{x^2}dx-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{2x^2}\right)}{3\sqrt{1-c^2x^2}}-\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{247}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\int\frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx-\frac{1}{2}bc\left(-3c^2\int\sqrt{1-c^2x^2}dx-\frac{(1-c^2x^2)^{3/2}}{x}\right)-\frac{(1-c^2x^2)^2(a+b\,\text{a}}{2x^2}}{3\sqrt{1-c^2x^2}}\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{211}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\int\frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx-\frac{1}{2}bc\left(-3c^2\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)-\frac{(1-c^2x^2)^{3/2}}{x}\right.\right.}{3\sqrt{1-c^2x^2}}\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{223}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\int\frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{2x^2}-\frac{1}{2}bc\left(-3c^2\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c}\right.\right.\right.}{3\sqrt{1-c^2x^2}}\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{5189}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\int\frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\int\sqrt{1-c^2x^2}dx+\frac{1}{2}(1-c^2x^2)(a+b\arccos(cx))\right)-\frac{(1-c^2x^2)^2}{}\right.}{3\sqrt{1-c^2x^2}}\frac{5}{3}c^2d\int\frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{x^2}dx-\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{211}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}\int \frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{2}\left(1-c^2x^2\right)\left(a+b\arccos(\right.\right.\right.}{3\sqrt{1-c^2x^2}}$$
$$\frac{5}{3}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 223

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\int \frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right)\right.}{3\sqrt{1-c^2x^2}}$$
$$\frac{5}{3}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 5137

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(-\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(c}{2c}\right.\right.\right.}{3\sqrt{1-c^2}}$$
$$\frac{5}{3}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 3042

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(-\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\right.\right.}{3\sqrt{}}$$
$$\frac{5}{3}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 4202

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))-\frac{i(a+b\arccos}{2b}\right.\right.}{}$$
$$\frac{5}{3}c^2d\int \frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$\downarrow$ 2620

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(2i\left(\frac{1}{2}ib\int\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx\right.\right.\right.}{}$$

$$\frac{5}{3}c^2d\int\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+\right.\right.\right.}{}$$

$$\frac{5}{3}c^2d\int\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x^2}dx-\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{5}{3}c^2d\int\frac{\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{x^2}dx-$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{P}\right.\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{5201}$$

$$-\frac{5}{3}c^2d\left(-3c^2d\int\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx-\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{\left(1-c^2x^2\right)(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-\frac{\left(d-c^2dx^2\right)}{}\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{P}\right.\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{5157}$$

$$-\frac{5}{3}c^2d\left(-3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\int x(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d-c^2dx^2}(a+\right.\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{P}\right.\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$\Big\downarrow$ 5139

$$-\frac{5}{3}c^2d\left(-3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\int\frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\right.\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{P}\right.\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$\Big\downarrow$ 262

$$-\frac{5}{3}c^2d\left(-3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}+\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\mathrm{a}}{\sqrt{1}}}{2\sqrt{1-c^2}}\right.\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{P}\right.\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$\Big\downarrow$ 223

$$-\frac{5}{3}c^2d\left(-3c^2d\left(\frac{\sqrt{d-c^2dx^2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2\sqrt{1-c^2x^2}}+\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-}}{2c}\right.\right.}{\sqrt{1-c^2x^2}}\right.\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{P}\right.\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{5153}$$

$$-\frac{5}{3}c^2d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\int\frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx}{\sqrt{1-c^2x^2}}-3c^2d\left(\frac{bc\sqrt{d-c^2dx^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin}{2c}\right.\right.}{\sqrt{1-c^2x^2}}\right.\right.$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}(1-c^2x^2)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{P}\right.\right.\right.}{}$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{5189}$$

$$-\frac{5}{3}c^2d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\left(\int\frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\int\sqrt{1-c^2x^2}dx+\frac{1}{2}(1-c^2x^2)(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}-3c^2d\left(\frac{b}{}\right.\right.$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}(1-c^2x^2)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{P}\right.\right.\right.}{}$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{211}$$

$$-\frac{5}{3}c^2d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\left(\int\frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{2}(1-c^2x^2)(a+b\arccos\right.}{\sqrt{1-c^2x^2}}\right.$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}(1-c^2x^2)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{P}\right.\right.\right.}{}$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{223}$$

$$-\frac{5}{3}c^2d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\left(\int\frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}(1-c^2x^2)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)\right.}{\sqrt{1-c^2x^2}}\right.$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}(1-c^2x^2)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{P}\right.\right.\right.}{}$$

$$\frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{5137}$$

$$-\frac{5}{3}c^2d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\int\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(}{2c}\right.\right.}{\sqrt{1-c^2x^2}}\right.$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{P}\right.\right.\right.}{}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{3}c^2d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\left(-\int(a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))\right.}{\sqrt{1-c^2x^2}}\right.$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{P}\right.\right.\right.}{}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{4202}$$

$$-\frac{5}{3}c^2d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\left(2i\int\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))-\frac{i(a+b\arccos}{2b}\right.}{\sqrt{1-c^2x^2}}\right.$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{P}\right.\right.\right.}{}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{5}{3}c^2d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\left(2i\left(\frac{1}{2}ib\int\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(c\right.\right.}{\sqrt{1-c^2x^2}}\right.$$
$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{P}\right.\right.\right.}{}$$
$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{5}{3}c^2d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\left(2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a\right.\right.}{\sqrt{1-}}\right.$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\,\mathrm{P}\right.\right.\right.}{}$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2bcd^2\sqrt{d-c^2dx^2}\left(-2c^2\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\right.\right.\right.}{}$$

$$\frac{5}{3}c^2d\left(-\frac{2bcd\sqrt{d-c^2dx^2}\left(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\right.\right.}{\sqrt{1-c^2x^2}}\right.$$

$$\frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{3x^3}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/x^4,x]
```

output
```
-1/3*((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/x^3 - (2*b*c*d^2*Sqrt[d
 - c^2*d*x^2]*(-1/2*((1 - c^2*x^2)^2*(a + b*ArcCos[c*x]))/x^2 - (b*c*(-((1
 - c^2*x^2)^(3/2)/x) - 3*c^2*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c))
))/2 - 2*c^2*(((1 - c^2*x^2)*(a + b*ArcCos[c*x]))/2 - ((I/2)*(a + b*ArcCos
[c*x])^2)/b + (b*c*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 + (2*I
)*((-1/2*I)*(a + b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCos[c*x])] - (b*PolyLo
g[2, -E^((2*I)*ArcCos[c*x])])/4))))/(3*Sqrt[1 - c^2*x^2]) - (5*c^2*d*(-(((
d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/x) - 3*c^2*d*((x*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCos[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])
^3)/(6*b*c*Sqrt[1 - c^2*x^2]) + (b*c*Sqrt[d - c^2*d*x^2]*((x^2*(a + b*ArcC
os[c*x]))/2 + (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))
/2))/Sqrt[1 - c^2*x^2]) - (2*b*c*d*Sqrt[d - c^2*d*x^2]*(((1 - c^2*x^2)*(a
+ b*ArcCos[c*x]))/2 - ((I/2)*(a + b*ArcCos[c*x])^2)/b + (b*c*((x*Sqrt[1 -
c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 + (2*I)*((-1/2*I)*(a + b*ArcCos[c*x])*
Log[1 + E^((2*I)*ArcCos[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/4)
))/Sqrt[1 - c^2*x^2]))/3
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5189
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCos[c*x])/(2*p)), x] + (Simp[d
 Int[(d + e*x^2)^(p - 1)*((a + b*ArcCos[c*x])/x), x], x] + Simp[b*c*(d^p/(2
*p))   Int[(1 - c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5191
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcCos[c*x
])/(f*(m + 1))), x] + (Simp[b*c*(d^p/(f*(m + 1)))   Int[(f*x)^(m + 1)*(1 -
c^2*x^2)^(p - 1/2), x], x] - Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m + 2)
*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x]), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m + 1)/2, 0]
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2512 vs. $2(543) = 1086$.

Time = 0.88 (sec) , antiderivative size = 2513, normalized size of antiderivative = 4.25

method	result	size
default	Expression too large to display	2513
parts	Expression too large to display	2513

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^4/(c^2*x^2-1)*x+5/6*b^2*(-d*(c^2*x^2-
1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^3*d^2*c^3-56/3*b^2*(-
d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c^2*x^2-1)*c^8+71/
3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c^2*x^2-1)
*c^6-16/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(c^2*
x^2-1)*c^4+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x/
(c^2*x^2-1)*c^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+
1)/x^3/(c^2*x^2-1)*arccos(c*x)^2+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^3/(c
^2*x^2-1)*arccos(c*x)*(-c^2*x^2+1)^(1/2)+1/2*b^2*(-d*(c^2*x^2-1))^(1/2)*d^
2*c^6/(c^2*x^2-1)*arccos(c*x)^2*x^3-1/2*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^4
/(c^2*x^2-1)*arccos(c*x)^2*x-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^6/(c^2*x
^2-1)*x^3-7/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/(
c^2*x^2-1)*arccos(c*x)^2*(-c^2*x^2+1)^(1/2)*c^3+5/3*a^2*c^4*d*x*(-c^2*d*x^
2+d)^(3/2)+5/2*a^2*c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/2*a^2*c^4*d^3/(c^2*d)^
(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/3*a^2/d/x^3*(-c^2*d*x
^2+d)^(7/2)+4/3*a^2*c^4*x*(-c^2*d*x^2+d)^(5/2)+4/3*a^2*c^2/d/x*(-c^2*d*x^2
+d)^(7/2)-49/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*
x^3/(c^2*x^2-1)*arccos(c*x)*(-c^2*x^2+1)*c^6+35*I*b^2*(-d*(c^2*x^2-1))^(1/
2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c^2*x^2-1)*arccos(c*x)^2*(-c^2*x^2+1
)^(1/2)*c^5+7/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2...
```

Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))^2}{x^4}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}\,(b\arccos(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x^4,x, algorithm="frica
s")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)
```

Sympy [F]

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x^4}\,dx=\int \frac{\left(-d(cx-1)\,(cx+1)\right)^{\frac{5}{2}}\left(a+b\operatorname{acos}\left(cx\right)\right)^2}{x^4}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))**2/x**4,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))**2/x**4, x)
```

Maxima [F]

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x^4}\,dx=\int \frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}\left(b\arccos\left(cx\right)+a\right)^2}{x^4}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x^4,x, algorithm="maxim
a")
```

output
```
1/6*(10*(-c^2*d*x^2 + d)^(3/2)*c^4*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^4*d^2*x
 + 15*c^3*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)*c^2/x - 2*(-c^2*d
*x^2 + d)^(7/2)/(d*x^3))*a^2 + sqrt(d)*integrate(((b^2*c^4*d^2*x^4 - 2*b^2
*c^2*d^2*x^2 + b^2*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*(
a*b*c^4*d^2*x^4 - 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arctan2(sqrt(c*x + 1)*sqrt(
-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1)/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{x^4}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2/x^4,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d-c^2\,d\,x^2)^{5/2}}{x^4}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^4,x)
```

output
```
int(((a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2}{x^4}\,dx = \frac{\sqrt{d}\,d^2\left(-4acos(cx)^3\,b^2c^3x^3-12acos(cx)^2\,ab\,c^3x^3+15asin(cx)\,a\right.}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))^2/x^4,x)
```

output
```
(sqrt(d)*d**2*( - 4*acos(c*x)**3*b**2*c**3*x**3 - 12*acos(c*x)**2*a*b*c**3
*x**3 + 15*asin(c*x)*a**2*c**3*x**3 + 3*sqrt( - c**2*x**2 + 1)*a**2*c**4*x
**4 + 14*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*
a**2 - 24*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*x**2),x)*a*b*c**2*x**3 - 1
2*int(acos(c*x)**2/(sqrt( - c**2*x**2 + 1)*x**2),x)*b**2*c**2*x**3 + 12*in
t((sqrt( - c**2*x**2 + 1)*acos(c*x))/x**4,x)*a*b*x**3 + 6*int((sqrt( - c**
2*x**2 + 1)*acos(c*x)**2)/x**4,x)*b**2*x**3 + 12*int(sqrt( - c**2*x**2 + 1
)*acos(c*x),x)*a*b*c**4*x**3 + 6*int(sqrt( - c**2*x**2 + 1)*acos(c*x)**2,x
)*b**2*c**4*x**3))/(6*x**3)
```

3.236 $\int \frac{x^5(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 400

$$\begin{aligned}\int \frac{x^5(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx &= \frac{16abx\sqrt{1-c^2x^2}}{15c^5\sqrt{d-c^2dx^2}}+\frac{298b^2(1-c^2x^2)}{225c^6\sqrt{d-c^2dx^2}}-\frac{76b^2(1-c^2x^2)^2}{675c^6\sqrt{d-c^2dx^2}}\\
&+\frac{2b^2(1-c^2x^2)^3}{125c^6\sqrt{d-c^2dx^2}}+\frac{16b^2x\sqrt{1-c^2x^2}\arccos(cx)}{15c^5\sqrt{d-c^2dx^2}}\\
&+\frac{8bx^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{45c^3\sqrt{d-c^2dx^2}}\\
&+\frac{2bx^5\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c\sqrt{d-c^2dx^2}}\\
&-\frac{8\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{15c^6d}\\
&-\frac{4x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{15c^4d}\\
&-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{5c^2d}\end{aligned}$$

output
```
16/15*a*b*x*(-c^2*x^2+1)^(1/2)/c^5/(-c^2*d*x^2+d)^(1/2)+298/225*b^2*(-c^2*
x^2+1)/c^6/(-c^2*d*x^2+d)^(1/2)-76/675*b^2*(-c^2*x^2+1)^2/c^6/(-c^2*d*x^2+
d)^(1/2)+2/125*b^2*(-c^2*x^2+1)^3/c^6/(-c^2*d*x^2+d)^(1/2)+16/15*b^2*x*(-c
^2*x^2+1)^(1/2)*arccos(c*x)/c^5/(-c^2*d*x^2+d)^(1/2)+8/45*b*x^3*(-c^2*x^2+
1)^(1/2)*(a+b*arccos(c*x))/c^3/(-c^2*d*x^2+d)^(1/2)+2/25*b*x^5*(-c^2*x^2+1
)^(1/2)*(a+b*arccos(c*x))/c/(-c^2*d*x^2+d)^(1/2)-8/15*(-c^2*d*x^2+d)^(1/2)
*(a+b*arccos(c*x))^2/c^6/d-4/15*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))
^2/c^4/d-1/5*x^4*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.61

$$\int \frac{x^5(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{d-c^2dx^2}\left(30abcx\sqrt{1-c^2x^2}(120+20c^2x^2+9c^4x^4)-225a^2(-8+4c^2x^2+c^4x^4+3c^6x^6)+2b^2(-2072\right.}{}$$

input
```
Integrate[(x^5*(a + b*ArcCos[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(30*a*b*c*x*Sqrt[1 - c^2*x^2]*(120 + 20*c^2*x^2 + 9*c
^4*x^4) - 225*a^2*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c^6*x^6) + 2*b^2*(-2072 +
1936*c^2*x^2 + 109*c^4*x^4 + 27*c^6*x^6) + 30*b*(b*c*x*Sqrt[1 - c^2*x^2]*(
120 + 20*c^2*x^2 + 9*c^4*x^4) - 15*a*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c^6*x^6
))*ArcCos[c*x] - 225*b^2*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c^6*x^6)*ArcCos[c*x
]^2))/(3375*c^6*d*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {5211, 5139, 243, 53, 2009, 5211, 5139, 243, 53, 2009, 5183, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \text{5211}$$

$$-\frac{2b\sqrt{1-c^2x^2}\int x^4(a+b\arccos(cx))dx}{5c\sqrt{d-c^2dx^2}}+\frac{4\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{5c^2}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{5139}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{5}bc\int \frac{x^5}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}x^5(a+b\arccos(cx))\right)}{5c\sqrt{d-c^2dx^2}}+\frac{4\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{5c^2}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{243}$$

$$\frac{4\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{5c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{10}bc\int \frac{x^4}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{5}x^5(a+b\arccos(cx))\right)}{5c\sqrt{d-c^2dx^2}}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{53}$$

$$\frac{4\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{5c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{10}bc\int \left(\frac{(1-c^2x^2)^{3/2}}{c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{1}{c^4\sqrt{1-c^2x^2}}\right)dx^2+\frac{1}{5}x^5(a+b\arccos(cx))\right)}{5c\sqrt{d-c^2dx^2}}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{5c^2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{5c^2}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{5c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5211}$$

$$\frac{4\left(-\frac{2b\sqrt{1-c^2x^2}\int x^2(a+b\arccos(cx))dx}{3c\sqrt{d-c^2dx^2}}+\frac{2\int\frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}\right)}{5c^2}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{5c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5c\sqrt{d-c^2dx^2}}$$

$\downarrow$ 5139

$$\frac{4\left(\frac{2\int\frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}bc\int\frac{x^3}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}\right)}{5c^2}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{5c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5c\sqrt{d-c^2dx^2}}$$

$\downarrow$ 243

$$\frac{4\left(\frac{2\int\frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{6}bc\int\frac{x^2}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}\right)}{5c^2}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{5c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5c\sqrt{d-c^2dx^2}}$$

$\downarrow$ 53

$$\frac{4\left(\frac{2\int\frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{6}bc\int\left(\frac{1}{c^2\sqrt{1-c^2x^2}}-\frac{\sqrt{1-c^2x^2}}{c^2}\right)dx^2+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}\right)}{5c^2}-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{5c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{4\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a+b\arccos(cx))+\frac{1}{6}bc\left(\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)\right)}{3c\sqrt{d-c^2dx^2}}\right)}{5c^2}$$
$$\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{5c^2d}-$$
$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 5183$$

$$\frac{4\left(\frac{2\left(-\frac{2b\sqrt{1-c^2x^2}\int(a+b\arccos(cx))dx}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}\right)}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a+b\arccos(cx))\right.}{3c\sqrt{\cdots}}\right.}{5c^2}$$
$$\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{5c^2d}-$$
$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{x^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{5c^2d}-$$
$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{5}x^5(a+b\arccos(cx))+\frac{1}{10}bc\left(-\frac{2(1-c^2x^2)^{5/2}}{5c^6}+\frac{4(1-c^2x^2)^{3/2}}{3c^6}-\frac{2\sqrt{1-c^2x^2}}{c^6}\right)\right)}{5c\sqrt{d-c^2dx^2}}+$$
$$\frac{4\left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}+\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a\cdots\right.}{\cdots}\right.}{5c^2}$$

input
```
Int[(x^5*(a + b*ArcCos[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/5*(x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(c^2*d) - (2*b*Sqrt[1
 - c^2*x^2]*((b*c*((-2*Sqrt[1 - c^2*x^2])/c^6 + (4*(1 - c^2*x^2)^(3/2))/(3
*c^6) - (2*(1 - c^2*x^2)^(5/2))/(5*c^6)))/10 + (x^5*(a + b*ArcCos[c*x]))/5
))/(5*c*Sqrt[d - c^2*d*x^2]) + (4*(-1/3*(x^2*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCos[c*x])^2)/(c^2*d) - (2*b*Sqrt[1 - c^2*x^2]*((b*c*((-2*Sqrt[1 - c^2*x^2
])/c^4 + (2*(1 - c^2*x^2)^(3/2))/(3*c^4)))/6 + (x^3*(a + b*ArcCos[c*x]))/3
))/(3*c*Sqrt[d - c^2*d*x^2]) + (2*(-((Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*
x])^2)/(c^2*d)) - (2*b*Sqrt[1 - c^2*x^2]*(a*x - (b*Sqrt[1 - c^2*x^2])/c +
b*x*ArcCos[c*x]))/(c*Sqrt[d - c^2*d*x^2])))/(3*c^2)))/(5*c^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.27

method	result
orering	$\frac{(1647c^8x^8+1684c^6x^6+34306c^4x^4-102032c^2x^2+62160)(a+b\arccos(cx))^2}{3375c^8x^2\sqrt{-c^2d\,x^2+d}} - \frac{2(cx-1)(cx+1)(162c^6x^6+491c^4x^4+7472c^2x^2-10}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^5*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3375*(1647*c^8*x^8+1684*c^6*x^6+34306*c^4*x^4-102032*c^2*x^2+62160)/c^8/
x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2)-2/3375*(c*x-1)*(c*x+1)*(162*c
^6*x^6+491*c^4*x^4+7472*c^2*x^2-10360)/x^6/c^8*(5*x^4*(a+b*arccos(c*x))^2/
(-c^2*d*x^2+d)^(1/2)-2*x^5*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2)*b*c/(-c^
2*x^2+1)^(1/2)+x^6*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2)*d*c^2)+1/3375*
(27*c^4*x^4+136*c^2*x^2+2072)/c^8/x^5*(c*x-1)^2*(c*x+1)^2*(20*x^3*(a+b*arc
cos(c*x))^2/(-c^2*d*x^2+d)^(1/2)-20*x^4*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(
1/2)*b*c/(-c^2*x^2+1)^(1/2)+11*x^5*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2
)*d*c^2+2*x^5*b^2*c^2/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)-4*x^6*(a+b*arccos(
c*x))/(-c^2*d*x^2+d)^(3/2)*b*c^3/(-c^2*x^2+1)^(1/2)*d-2*x^6*(a+b*arccos(c*
x))/(-c^2*d*x^2+d)^(1/2)*b*c^3/(-c^2*x^2+1)^(3/2)+3*x^7*(a+b*arccos(c*x))^
2/(-c^2*d*x^2+d)^(5/2)*d^2*c^4)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.69

$$\int \frac{x^5(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{30\,(9\,abc^5x^5+20\,abc^3x^3+120\,abcx+(9\,b^2c^5x^5+20\,b^2c^3x^3+120\,b^2cx)\arccos{(cx)})\sqrt{-c^2dx^2+d}\sqrt{-c^2x}}{}$$

input
```
integrate(x^5*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
1/3375*(30*(9*a*b*c^5*x^5 + 20*a*b*c^3*x^3 + 120*a*b*c*x + (9*b^2*c^5*x^5
+ 20*b^2*c^3*x^3 + 120*b^2*c*x)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)*sqrt(-c^
2*x^2 + 1) - (27*(25*a^2 - 2*b^2)*c^6*x^6 + (225*a^2 - 218*b^2)*c^4*x^4 +
4*(225*a^2 - 968*b^2)*c^2*x^2 + 225*(3*b^2*c^6*x^6 + b^2*c^4*x^4 + 4*b^2*c
^2*x^2 - 8*b^2)*arccos(c*x)^2 - 1800*a^2 + 4144*b^2 + 450*(3*a*b*c^6*x^6 +
 a*b*c^4*x^4 + 4*a*b*c^2*x^2 - 8*a*b)*arccos(c*x))*sqrt(-c^2*d*x^2 + d))/(
c^8*d*x^2 - c^6*d)
```

Sympy [F]

$$\int \frac{x^5(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^5\,(a+b\,\mathrm{acos}\,(cx))^2}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate(x**5*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**5*(a + b*acos(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.91

$$\int \frac{x^5(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$= -\frac{1}{15}\left(\frac{3\sqrt{-c^2dx^2+d}x^4}{c^2d}+\frac{4\sqrt{-c^2dx^2+d}x^2}{c^4d}+\frac{8\sqrt{-c^2dx^2+d}}{c^6d}\right)b^2\arccos(cx)^2$$
$$-\frac{2}{15}\left(\frac{3\sqrt{-c^2dx^2+d}x^4}{c^2d}+\frac{4\sqrt{-c^2dx^2+d}x^2}{c^4d}+\frac{8\sqrt{-c^2dx^2+d}}{c^6d}\right)ab\arccos(cx)$$
$$-\frac{1}{15}\left(\frac{3\sqrt{-c^2dx^2+d}x^4}{c^2d}+\frac{4\sqrt{-c^2dx^2+d}x^2}{c^4d}+\frac{8\sqrt{-c^2dx^2+d}}{c^6d}\right)a^2$$
$$+\frac{2}{3375}b^2\left(\frac{27\sqrt{-c^2x^2+1}c^2x^4+136\sqrt{-c^2x^2+1}x^2+\frac{2072\sqrt{-c^2x^2+1}}{c^2}}{c^4\sqrt{d}}-\frac{15\left(9c^4x^5+20c^2x^3+120x\right)\mathrm{arc}}{c^5\sqrt{d}}\right.$$
$$-\frac{2\left(9c^4x^5+20c^2x^3+120x\right)ab}{225c^5\sqrt{d}}$$

input
```
integrate(x^5*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/15*(3*sqrt(-c^2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^
4*d) + 8*sqrt(-c^2*d*x^2 + d)/(c^6*d))*b^2*arccos(c*x)^2 - 2/15*(3*sqrt(-c
^2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-c
^2*d*x^2 + d)/(c^6*d))*a*b*arccos(c*x) - 1/15*(3*sqrt(-c^2*d*x^2 + d)*x^4/
(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-c^2*d*x^2 + d)/(c^6
*d))*a^2 + 2/3375*b^2*((27*sqrt(-c^2*x^2 + 1)*c^2*x^4 + 136*sqrt(-c^2*x^2
+ 1)*x^2 + 2072*sqrt(-c^2*x^2 + 1)/c^2)/(c^4*sqrt(d)) - 15*(9*c^4*x^5 + 20
*c^2*x^3 + 120*x)*arccos(c*x)/(c^5*sqrt(d))) - 2/225*(9*c^4*x^5 + 20*c^2*x
^3 + 120*x)*a*b/(c^5*sqrt(d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^5\,(a+b\,\mathrm{acos}(c\,x))^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x^5*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^5(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{-3\sqrt{-c^2x^2+1}\,a^2c^4x^4-4\sqrt{-c^2x^2+1}\,a^2c^2x^2-8\sqrt{-c^2x^2+1}\,a^2+30\left(\int \frac{acos(cx)x^5}{\sqrt{-c^2x^2+1}}dx\right)ab\,c^6+15\left(\int \frac{acos(}{\sqrt{-c}}\right.}{15\sqrt{d}\,c^6}$$

input
```
int(x^5*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output

```
( - 3*sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - 4*sqrt( - c**2*x**2 + 1)*a**
2*c**2*x**2 - 8*sqrt( - c**2*x**2 + 1)*a**2 + 30*int((acos(c*x)*x**5)/sqrt
( - c**2*x**2 + 1),x)*a*b*c**6 + 15*int((acos(c*x)**2*x**5)/sqrt( - c**2*x
**2 + 1),x)*b**2*c**6)/(15*sqrt(d)*c**6)
```

3.237 $\int \frac{x^4(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 337

$$\int \frac{x^4(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{15b^2x(1-c^2x^2)}{64c^4\sqrt{d-c^2dx^2}} + \frac{b^2x^3(1-c^2x^2)}{32c^2\sqrt{d-c^2dx^2}} - \frac{15b^2\sqrt{1-c^2x^2}\arccos(cx)}{64c^5\sqrt{d-c^2dx^2}} + \frac{3bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{8c^3\sqrt{d-c^2dx^2}} + \frac{bx^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{8c\sqrt{d-c^2dx^2}} - \frac{3x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{8c^4d} - \frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4c^2d} + \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{8bc^5\sqrt{d-c^2dx^2}}$$

output
```
15/64*b^2*x*(-c^2*x^2+1)/c^4/(-c^2*d*x^2+d)^(1/2)+1/32*b^2*x^3*(-c^2*x^2+1
)/c^2/(-c^2*d*x^2+d)^(1/2)-15/64*b^2*(-c^2*x^2+1)^(1/2)*arccos(c*x)/c^5/(-
c^2*d*x^2+d)^(1/2)+3/8*b*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3/(-c^
2*d*x^2+d)^(1/2)+1/8*b*x^4*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c/(-c^2*d*
x^2+d)^(1/2)-3/8*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/c^4/d-1/4*x^3*
(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/c^2/d+1/8*(-c^2*x^2+1)^(1/2)*(a+b
*arccos(c*x))^3/b/c^5/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.84

$$\int \frac{x^4(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{32a^2c\sqrt{d}x(-1+c^2x^2)\left(3+2c^2x^2\right)-96a^2\sqrt{d-c^2dx^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)-b^2\sqrt{d}\sqrt{1-c^2x^2}(32\arccos(}{}$$

input
```
Integrate[(x^4*(a + b*ArcCos[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
(32*a^2*c*Sqrt[d]*x*(-1 + c^2*x^2)*(3 + 2*c^2*x^2) - 96*a^2*Sqrt[d - c^2*d
*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - b^2*Sqr
t[d]*Sqrt[1 - c^2*x^2]*(32*ArcCos[c*x]^3 + 4*ArcCos[c*x]*(16*Cos[2*ArcCos[
c*x]] + Cos[4*ArcCos[c*x]]) - 32*Sin[2*ArcCos[c*x]] - Sin[4*ArcCos[c*x]] +
 8*ArcCos[c*x]^2*(8*Sin[2*ArcCos[c*x]] + Sin[4*ArcCos[c*x]])) - 4*a*b*Sqrt
[d]*Sqrt[1 - c^2*x^2]*(16*Cos[2*ArcCos[c*x]] + Cos[4*ArcCos[c*x]] + 4*ArcC
os[c*x]*(6*ArcCos[c*x] + 8*Sin[2*ArcCos[c*x]] + Sin[4*ArcCos[c*x]])))/(256
*c^5*Sqrt[d]*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {5211, 5139, 262, 262, 223, 5211, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{5211}$$

$$\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{4c^2}-\frac{b\sqrt{1-c^2x^2}\int x^3(a+b\arccos(cx))dx}{2c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4c^2d}$$

$$\Big\downarrow \text{5139}$$

$$\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{4c^2}-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{4}bc\int \frac{x^4}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))\right)}{2c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4c^2d}$$

$$\Big\downarrow \text{262}$$

$$\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{4c^2}-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{4}bc\left(\frac{3\int \frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{4}x^4(a+b\arccos(cx))\right)}{2c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4c^2d}$$

$$\Big\downarrow \text{262}$$

$$\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{4c^2}-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{4}bc\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{4}x^4(a+b\arccos(cx))\right)}{2c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4c^2d}$$

$$\Big\downarrow \text{223}$$

$$\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{4c^2}-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)\right)}{2c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4c^2d}$$

$$\Big\downarrow \text{ 5211}$$

$$\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\int x(a+b\arccos(cx))dx}{c\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}\right)}{4c^2}-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)\right)}{2c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4c^2d}$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}\right)}{4c^2}-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)\right)}{2c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4c^2d}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}\right)}{4c^2}-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)\right)}{2c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4c^2d}$$

$\Big\downarrow$ 223

$$\frac{3\left(\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}\right)}{4c^2}-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)\right)}{2c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4c^2d}$$

$\Big\downarrow$ 5153

$$\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}\right)}{4c^2}-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)\right)}{2c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4c^2d}$$

input
```
Int[(x^4*(a + b*ArcCos[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/4*(x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(c^2*d) - (b*Sqrt[1 -
 c^2*x^2]*((x^4*(a + b*ArcCos[c*x]))/4 + (b*c*(-1/4*(x^3*Sqrt[1 - c^2*x^2]
)/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)
))/4))/(2*c*Sqrt[d - c^2*d*x^2]) + (3*(-1/2*(x*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCos[c*x])^2)/(c^2*d) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^3)/(6*b*c
^3*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*((x^2*(a + b*ArcCos[c*x]))/
2 + (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2))/(c*Sq
rt[d - c^2*d*x^2])))/(4*c^2)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(297) = 594$.

Time = 0.51 (sec) , antiderivative size = 722, normalized size of antiderivative = 2.14

method	result
default	$-\frac{a^2x^3\sqrt{-c^2d\,x^2+d}}{4c^2d}-\frac{3a^2x\sqrt{-c^2d\,x^2+d}}{8c^4d}+\frac{3a^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{8c^4\sqrt{c^2d}}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3}{8c^5d(c^2x^2-1)}+\cdots\right.$
parts	$-\frac{a^2x^3\sqrt{-c^2d\,x^2+d}}{4c^2d}-\frac{3a^2x\sqrt{-c^2d\,x^2+d}}{8c^4d}+\frac{3a^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{8c^4\sqrt{c^2d}}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3}{8c^5d(c^2x^2-1)}+\cdots\right.$

input
```
int(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2*x^3/c^2/d*(-c^2*d*x^2+d)^(1/2)-3/8*a^2/c^4*x/d*(-c^2*d*x^2+d)^(1/
2)+3/8*a^2/c^4/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+
b^2*(1/8*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^5/d/(c^2*x^2-1)*arcco
s(c*x)^3+1/16*(-d*(c^2*x^2-1))^(1/2)/c^4/d/(c^2*x^2-1)*(2*arccos(c*x)^2-1)
*x-1/8*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^5/d/(c^2*x^2-1)*arccos(
c*x)-1/512*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*(8*arccos(c*x)^2-1)*co
s(5*arccos(c*x))+1/128*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*arccos(c*x
)*sin(5*arccos(c*x))-1/512*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*(56*ar
ccos(c*x)^2-31)*cos(3*arccos(c*x))+15/128*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^
2*x^2-1)*arccos(c*x)*sin(3*arccos(c*x)))+2*a*b*(3/16*(-d*(c^2*x^2-1))^(1/2
)*(-c^2*x^2+1)^(1/2)/c^5/d/(c^2*x^2-1)*arccos(c*x)^2+1/8*(-d*(c^2*x^2-1))^
(1/2)/c^4/d/(c^2*x^2-1)*arccos(c*x)*x+1/16/c^5/(-d*(c^2*x^2-1))^(1/2)*(-c^
2*x^2+1)^(1/2)-1/64*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*arccos(c*x)*c
os(5*arccos(c*x))+1/256*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*sin(5*arc
cos(c*x))-7/64*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*arccos(c*x)*cos(3*
arccos(c*x))+15/256*(-d*(c^2*x^2-1))^(1/2)/c^5/d/(c^2*x^2-1)*sin(3*arccos(
c*x)))
```

Fricas [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2x^4}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
integral(-(b^2*x^4*arccos(c*x)^2 + 2*a*b*x^4*arccos(c*x) + a^2*x^4)*sqrt(-
c^2*d*x^2 + d)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^4(a+b\operatorname{acos}(cx))^2}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate(x**4*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**4*(a + b*acos(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x^4}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/8*a^2*(2*sqrt(-c^2*d*x^2 + d)*x^3/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*x/(c
^4*d) - 3*arcsin(c*x)/(c^5*sqrt(d))) - sqrt(d)*integrate((b^2*x^4*arctan2(
sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*x^4*arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^2*d*x^2 - d), x)
```

Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.88

$$\int \frac{x^4(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx =$$
$$-\frac{8\,b^2c^3x^4\arccos(cx) + 16\sqrt{-c^2x^2+1}b^2c^2x^3\arccos(cx)^2 + 8\,abc^3x^4 + 32\sqrt{-c^2x^2+1}abc^2x^3\arccos(cx}{}$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
-1/64*(8*b^2*c^3*x^4*arccos(c*x) + 16*sqrt(-c^2*x^2 + 1)*b^2*c^2*x^3*arcco
s(c*x)^2 + 8*a*b*c^3*x^4 + 32*sqrt(-c^2*x^2 + 1)*a*b*c^2*x^3*arccos(c*x) +
 16*sqrt(-c^2*x^2 + 1)*a^2*c^2*x^3 - 2*sqrt(-c^2*x^2 + 1)*b^2*c^2*x^3 + 24
*b^2*c*x^2*arccos(c*x) + 24*sqrt(-c^2*x^2 + 1)*b^2*x*arccos(c*x)^2 + 24*a*
b*c*x^2 + 48*sqrt(-c^2*x^2 + 1)*a*b*x*arccos(c*x) + 8*b^2*arccos(c*x)^3/c
+ 24*sqrt(-c^2*x^2 + 1)*a^2*x - 15*sqrt(-c^2*x^2 + 1)*b^2*x + 24*a*b*arcco
s(c*x)^2/c + 24*a^2*arccos(c*x)/c - 15*b^2*arccos(c*x)/c - 15*a*b/c)/(c^4*
sqrt(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x^4\,(a+b\,\mathrm{acos}(c\,x))^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x^4*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^4*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$=\frac{3asin(cx)\,a^2-2\sqrt{-c^2x^2+1}\,a^2c^3x^3-3\sqrt{-c^2x^2+1}\,a^2cx+16\left(\int\frac{acos(cx)x^4}{\sqrt{-c^2x^2+1}}dx\right)ab\,c^5+8\left(\int\frac{acos(cx)^2x^4}{\sqrt{-c^2x^2+1}}dx\right)}{8\sqrt{d}\,c^5}$$

input
```
int(x^4*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(3*asin(c*x)*a**2 - 2*sqrt( - c**2*x**2 + 1)*a**2*c**3*x**3 - 3*sqrt( - c*
*2*x**2 + 1)*a**2*c*x + 16*int((acos(c*x)*x**4)/sqrt( - c**2*x**2 + 1),x)*
a*b*c**5 + 8*int((acos(c*x)**2*x**4)/sqrt( - c**2*x**2 + 1),x)*b**2*c**5)/
(8*sqrt(d)*c**5)
```

3.238 $\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 277

$$\begin{aligned}\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx &= \frac{4abx\sqrt{1-c^2x^2}}{3c^3\sqrt{d-c^2dx^2}}+\frac{14b^2(1-c^2x^2)}{9c^4\sqrt{d-c^2dx^2}}\\&-\frac{2b^2(1-c^2x^2)^2}{27c^4\sqrt{d-c^2dx^2}}+\frac{4b^2x\sqrt{1-c^2x^2}\arccos(cx)}{3c^3\sqrt{d-c^2dx^2}}\\&+\frac{2bx^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c\sqrt{d-c^2dx^2}}\\&-\frac{2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^4d}\\&-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}\end{aligned}$$

output
```
4/3*a*b*x*(-c^2*x^2+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)+14/9*b^2*(-c^2*x^2+1
)/c^4/(-c^2*d*x^2+d)^(1/2)-2/27*b^2*(-c^2*x^2+1)^2/c^4/(-c^2*d*x^2+d)^(1/2
)+4/3*b^2*x*(-c^2*x^2+1)^(1/2)*arccos(c*x)/c^3/(-c^2*d*x^2+d)^(1/2)+2/9*b*
x^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c/(-c^2*d*x^2+d)^(1/2)-2/3*(-c^2*
d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/c^4/d-1/3*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b
*arccos(c*x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.69

$$\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{d-c^2dx^2}\left(6abcx\sqrt{1-c^2x^2}(6+c^2x^2)-9a^2(-2+c^2x^2+c^4x^4)+2b^2(-20+19c^2x^2+c^4x^4)+6b(bcx\right.}{27c^4d\left(-1+c\right.}$$

input
```
Integrate[(x^3*(a + b*ArcCos[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(6*a*b*c*x*Sqrt[1 - c^2*x^2]*(6 + c^2*x^2) - 9*a^2*(-
2 + c^2*x^2 + c^4*x^4) + 2*b^2*(-20 + 19*c^2*x^2 + c^4*x^4) + 6*b*(b*c*x*S
qrt[1 - c^2*x^2]*(6 + c^2*x^2) - 3*a*(-2 + c^2*x^2 + c^4*x^4))*ArcCos[c*x]
 - 9*b^2*(-2 + c^2*x^2 + c^4*x^4)*ArcCos[c*x]^2))/(27*c^4*d*(-1 + c^2*x^2)
)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {5211, 5139, 243, 53, 2009, 5183, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \text{5211}$$

$$-\frac{2b\sqrt{1-c^2x^2}\int x^2(a+b\arccos(cx))dx}{3c\sqrt{d-c^2dx^2}}+\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}$$

$$\Big\downarrow \text{5139}$$

$$\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}bc\int \frac{x^3}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}$$

$$\Big\downarrow \; 243$$

$$\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{6}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}$$

$$\Big\downarrow \; 53$$

$$\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{6}bc\int \left(\frac{1}{c^2\sqrt{1-c^2x^2}}-\frac{\sqrt{1-c^2x^2}}{c^2}\right)dx^2+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a+b\arccos(cx))+\frac{1}{6}bc\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)\right)}{3c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 5183$$

$$\frac{2\left(-\frac{2b\sqrt{1-c^2x^2}\int (a+b\arccos(cx))dx}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}\right)}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a+b\arccos(cx))+\frac{1}{6}bc\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)\right)}{3c\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}+\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a+b\arccos(cx))+\frac{1}{6}bc\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)\right)}{3c\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^3*(a + b*ArcCos[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/3*(x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(c^2*d) - (2*b*Sqrt[1
 - c^2*x^2]*((b*c*((-2*Sqrt[1 - c^2*x^2])/c^4 + (2*(1 - c^2*x^2)^(3/2))/(3
*c^4)))/6 + (x^3*(a + b*ArcCos[c*x]))/3))/(3*c*Sqrt[d - c^2*d*x^2]) + (2*(
-((Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(c^2*d)) - (2*b*Sqrt[1 - c^2
*x^2]*(a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]))/(c*Sqrt[d - c^2*d
*x^2])))/(3*c^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.73

method	result
orering	$\frac{(19c^6x^6+100c^4x^4-380c^2x^2+240)(a+b\arccos(cx))^2}{27c^6x^2\sqrt{-c^2d\,x^2+d}} - \frac{2(cx-1)(cx+1)(c^4x^4+12c^2x^2-20)\left(\frac{3x^2(a+b\arccos(cx))^2}{\sqrt{-c^2d\,x^2+d}} - \frac{2x^3(a+b\arccos\cdots}{\sqrt{-c^2d\,x^2+d}\,\sqrt{\cdots}}\cdots\right.}{9c^6x^4}$
default	$a^2\left(-\frac{x^2\sqrt{-c^2d\,x^2+d}}{3c^2d} - \frac{2\sqrt{-c^2d\,x^2+d}}{3d\,c^4}\right) + b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\left(2c^2x^2-1+2i\sqrt{-c^2x^2+1}\,xc\right)\left(6i\arccos(cx)+9\arccos(cx)^2-\cdots\right.}{432c^4d(c^2x^2-1)}\cdots\right.$
parts	$a^2\left(-\frac{x^2\sqrt{-c^2d\,x^2+d}}{3c^2d} - \frac{2\sqrt{-c^2d\,x^2+d}}{3d\,c^4}\right) + b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\left(2c^2x^2-1+2i\sqrt{-c^2x^2+1}\,xc\right)\left(6i\arccos(cx)+9\arccos(cx)^2-\cdots\right.}{432c^4d(c^2x^2-1)}\cdots\right.$

input
```
int(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/27*(19*c^6*x^6+100*c^4*x^4-380*c^2*x^2+240)/c^6/x^2*(a+b*arccos(c*x))^2/
(-c^2*d*x^2+d)^(1/2)-2/9*(c*x-1)*(c*x+1)*(c^4*x^4+12*c^2*x^2-20)/c^6/x^4*(
3*x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2)-2*x^3*(a+b*arccos(c*x))/(-c
^2*d*x^2+d)^(1/2)*b*c/(-c^2*x^2+1)^(1/2)+x^4*(a+b*arccos(c*x))^2/(-c^2*d*x
^2+d)^(3/2)*d*c^2)+1/27*(c^2*x^2+20)/c^6/x^3*(c*x-1)^2*(c*x+1)^2*(6*x*(a+b
*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2)-12*x^2*(a+b*arccos(c*x))/(-c^2*d*x^2+
d)^(1/2)*b*c/(-c^2*x^2+1)^(1/2)+7*x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(
3/2)*d*c^2+2*x^3*b^2*c^2/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)-4*x^4*(a+b*arcc
os(c*x))/(-c^2*d*x^2+d)^(3/2)*b*c^3/(-c^2*x^2+1)^(1/2)*d-2*x^4*(a+b*arccos
(c*x))/(-c^2*d*x^2+d)^(1/2)*b*c^3/(-c^2*x^2+1)^(3/2)+3*x^5*(a+b*arccos(c*x
))^2/(-c^2*d*x^2+d)^(5/2)*d^2*c^4)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.76

$$\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{6\left(abc^3x^3+6\,abcx+\left(b^2c^3x^3+6\,b^2cx\right)\arccos\left(cx\right)\right)\sqrt{-c^2dx^2+d}\sqrt{-c^2x^2+1}-\left(\left(9\,a^2-2\,b^2\right)c^4x^4+\left(9\,a^2\right.\right.}{}$$

input

```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output

```
1/27*(6*(a*b*c^3*x^3 + 6*a*b*c*x + (b^2*c^3*x^3 + 6*b^2*c*x)*arccos(c*x))*
sqrt(-c^2*d*x^2 + d)*sqrt(-c^2*x^2 + 1) - ((9*a^2 - 2*b^2)*c^4*x^4 + (9*a^
2 - 38*b^2)*c^2*x^2 + 9*(b^2*c^4*x^4 + b^2*c^2*x^2 - 2*b^2)*arccos(c*x)^2
- 18*a^2 + 40*b^2 + 18*(a*b*c^4*x^4 + a*b*c^2*x^2 - 2*a*b)*arccos(c*x))*sq
rt(-c^2*d*x^2 + d))/(c^6*d*x^2 - c^4*d)
```

Sympy [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^3(a+b\operatorname{acos}(cx))^2}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate(x**3*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*acos(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.91

$$\begin{aligned}
&\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx \\
&= -\frac{1}{3}b^2\left(\frac{\sqrt{-c^2dx^2+d}x^2}{c^2d}+\frac{2\sqrt{-c^2dx^2+d}}{c^4d}\right)\arccos(cx)^2 \\
&\quad -\frac{2}{3}ab\left(\frac{\sqrt{-c^2dx^2+d}x^2}{c^2d}+\frac{2\sqrt{-c^2dx^2+d}}{c^4d}\right)\arccos(cx) \\
&\quad -\frac{1}{3}a^2\left(\frac{\sqrt{-c^2dx^2+d}x^2}{c^2d}+\frac{2\sqrt{-c^2dx^2+d}}{c^4d}\right) \\
&\quad +\frac{2}{27}b^2\left(\frac{\sqrt{-c^2x^2+1}x^2+\frac{20\sqrt{-c^2x^2+1}}{c^2}}{c^2\sqrt{d}}-\frac{3(c^2x^3+6x)\arccos(cx)}{c^3\sqrt{d}}\right) \\
&\quad -\frac{2(c^2x^3+6x)ab}{9c^3\sqrt{d}}
\end{aligned}$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/3*b^2*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d
))*arccos(c*x)^2 - 2/3*a*b*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2
*d*x^2 + d)/(c^4*d))*arccos(c*x) - 1/3*a^2*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*
d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d)) + 2/27*b^2*((sqrt(-c^2*x^2 + 1)*x^2 +
 20*sqrt(-c^2*x^2 + 1)/c^2)/(c^2*sqrt(d)) - 3*(c^2*x^3 + 6*x)*arccos(c*x)/
(c^3*sqrt(d))) - 2/9*(c^2*x^3 + 6*x)*a*b/(c^3*sqrt(d))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x^3\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x^3*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$=\frac{-\sqrt{-c^2x^2+1}\,a^2c^2x^2-2\sqrt{-c^2x^2+1}\,a^2+6\left(\int \frac{acos(cx)x^3}{\sqrt{-c^2x^2+1}}dx\right)ab\,c^4+3\left(\int \frac{acos(cx)^2x^3}{\sqrt{-c^2x^2+1}}dx\right)b^2c^4}{3\sqrt{d}\,c^4}$$

input
```
int(x^3*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a**2
+ 6*int((acos(c*x)*x**3)/sqrt( - c**2*x**2 + 1),x)*a*b*c**4 + 3*int((acos(
c*x)**2*x**3)/sqrt( - c**2*x**2 + 1),x)*b**2*c**4)/(3*sqrt(d)*c**4)
```

3.239 $\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 206

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{b^2x\sqrt{d-c^2dx^2}}{4c^2d} - \frac{b^2\sqrt{1-c^2x^2}\arccos(cx)}{4c^3\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c\sqrt{d-c^2dx^2}} - \frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d} + \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}$$

output

```
1/4*b^2*x*(-c^2*d*x^2+d)^(1/2)/c^2/d-1/4*b^2*(-c^2*x^2+1)^(1/2)*arccos(c*x
)/c^3/(-c^2*d*x^2+d)^(1/2)+1/2*b*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/
c/(-c^2*d*x^2+d)^(1/2)-1/2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/c^2/
d+1/6*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^3/b/c^3/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.01

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{12a^2cdx(-1+c^2x^2)-12a^2\sqrt{d}\sqrt{d-c^2dx^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)-b^2d\sqrt{1-c^2x^2}(4\arccos(cx)^3+6\arccos}{}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
(12*a^2*c*d*x*(-1 + c^2*x^2) - 12*a^2*Sqrt[d]*Sqrt[d - c^2*d*x^2]*ArcTan[(
c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - b^2*d*Sqrt[1 - c^2*x^
2]*(4*ArcCos[c*x]^3 + 6*ArcCos[c*x]*Cos[2*ArcCos[c*x]] + (-3 + 6*ArcCos[c*
x]^2)*Sin[2*ArcCos[c*x]]) - 6*a*b*d*Sqrt[1 - c^2*x^2]*(Cos[2*ArcCos[c*x]]
+ 2*ArcCos[c*x]*(ArcCos[c*x] + Sin[2*ArcCos[c*x]])))/(24*c^3*d*Sqrt[d - c^
2*d*x^2])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {5211, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$\Big\downarrow \text{5211}$$
$$-\frac{b\sqrt{1-c^2x^2}\int x(a+b\arccos(cx))dx}{c\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}$$
$$\Big\downarrow \text{5139}$$

$$-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}bc\int\frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}$$

$\Big\downarrow$ 262

$$-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}$$

$\Big\downarrow$ 223

$$\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}$$

$\Big\downarrow$ 5153

$$-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/2*(x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(c^2*d) - (Sqrt[1 - c^2
*x^2]*(a + b*ArcCos[c*x])^3)/(6*b*c^3*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c
^2*x^2]*((x^2*(a + b*ArcCos[c*x]))/2 + (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^
2 + ArcSin[c*x]/(2*c^3)))/2))/(c*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(180) = 360$.

Time = 0.42 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.51

method	result
default	$-\frac{a^{2} x \sqrt{-c^{2} d \,x^{2}+d}}{2 c^{2} d}+\frac{a^{2} \arctan\left(\frac{\sqrt{c^{2} d}\, x}{\sqrt{-c^{2} d \,x^{2}+d}}\right)}{2 c^{2} \sqrt{c^{2} d}}+b^{2} \Bigl(\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \sqrt{-c^{2} x^{2}+1}\, \arccos\left(c x\right)^{3}}{6 c^{3} d \left(c^{2} x^{2}-1\right)}+\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \bigl(2 \arccos(c}{16 c^{2} d \left(c^{2} x^{2}-1\right)}$
parts	$-\frac{a^{2} x \sqrt{-c^{2} d \,x^{2}+d}}{2 c^{2} d}+\frac{a^{2} \arctan\left(\frac{\sqrt{c^{2} d}\, x}{\sqrt{-c^{2} d \,x^{2}+d}}\right)}{2 c^{2} \sqrt{c^{2} d}}+b^{2} \Bigl(\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \sqrt{-c^{2} x^{2}+1}\, \arccos\left(c x\right)^{3}}{6 c^{3} d \left(c^{2} x^{2}-1\right)}+\frac{\sqrt{-d \left(c^{2} x^{2}-1\right)}\, \bigl(2 \arccos(c}{16 c^{2} d \left(c^{2} x^{2}-1\right)}$

input
```
int(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a^2/c^2/(c^2*d)^(1/2)*arctan((c^
2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(1/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x
^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*arccos(c*x)^3+1/16*(-d*(c^2*x^2-1))^(1/2)/c^
2/d/(c^2*x^2-1)*(2*arccos(c*x)^2-1)*x-1/8*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2
+1)^(1/2)/c^3/d/(c^2*x^2-1)*arccos(c*x)-1/16*(-d*(c^2*x^2-1))^(1/2)/c^3/d/
(c^2*x^2-1)*(2*arccos(c*x)^2-1)*cos(3*arccos(c*x))+1/8*(-d*(c^2*x^2-1))^(1
/2)/c^3/d/(c^2*x^2-1)*arccos(c*x)*sin(3*arccos(c*x)))+2*a*b*(1/4*(-d*(c^2*
x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*arccos(c*x)^2+1/8*(-d*(
c^2*x^2-1))^(1/2)/c^2/d/(c^2*x^2-1)*arccos(c*x)*x+1/16/c^3/(-d*(c^2*x^2-1)
)^(1/2)*(-c^2*x^2+1)^(1/2)-1/8*(-d*(c^2*x^2-1))^(1/2)/c^3/d/(c^2*x^2-1)*ar
ccos(c*x)*cos(3*arccos(c*x))+1/16*(-d*(c^2*x^2-1))^(1/2)/c^3/d/(c^2*x^2-1)
*sin(3*arccos(c*x)))
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\arccos{(cx)}+a)^2x^2}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
integral(-(b^2*x^2*arccos(c*x)^2 + 2*a*b*x^2*arccos(c*x) + a^2*x^2)*sqrt(-
c^2*d*x^2 + d)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x^2(a+b\operatorname{acos}(cx))^2}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate(x**2*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*acos(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\arccos(cx)+a)^2x^2}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/2*a^2*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d))) - sq
rt(d)*integrate((b^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*
a*b*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*
x + 1)/(c^2*d*x^2 - d), x)
```

Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.82

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = -\frac{6\,b^2cx^2\arccos{(cx)}+6\,\sqrt{-c^2x^2+1}b^2x\arccos{(cx)}^2+6\,abcx^2+12\,\sqrt{-c^2x^2+1}abx\arccos{(cx)}+\frac{2\,b^2\,\mathrm{ar}}{}}{12\,c^2\sqrt{}}$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
-1/12*(6*b^2*c*x^2*arccos(c*x) + 6*sqrt(-c^2*x^2 + 1)*b^2*x*arccos(c*x)^2
+ 6*a*b*c*x^2 + 12*sqrt(-c^2*x^2 + 1)*a*b*x*arccos(c*x) + 2*b^2*arccos(c*x
)^3/c + 6*sqrt(-c^2*x^2 + 1)*a^2*x - 3*sqrt(-c^2*x^2 + 1)*b^2*x + 6*a*b*ar
ccos(c*x)^2/c + 6*a^2*arccos(c*x)/c - 3*b^2*arccos(c*x)/c - 3*a*b/c)/(c^2*
sqrt(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^2\,{\left(a+b\,\mathrm{acos}(c\,x)\right)}^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x^2*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$=\frac{asin(cx)\,a^2-\sqrt{-c^2x^2+1}\,a^2cx+4\left(\int \frac{acos(cx)x^2}{\sqrt{-c^2x^2+1}}dx\right)ab\,c^3+2\left(\int \frac{acos(cx)^2x^2}{\sqrt{-c^2x^2+1}}dx\right)b^2c^3}{2\sqrt{d}\,c^3}$$

input
```
int(x^2*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(asin(c*x)*a**2 - sqrt( - c**2*x**2 + 1)*a**2*c*x + 4*int((acos(c*x)*x**2)
/sqrt( - c**2*x**2 + 1),x)*a*b*c**3 + 2*int((acos(c*x)**2*x**2)/sqrt( - c*
*2*x**2 + 1),x)*b**2*c**3)/(2*sqrt(d)*c**3)
```

3.240 $\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 146

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{2abx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}} + \frac{2b^2(1-c^2x^2)}{c^2\sqrt{d-c^2dx^2}} + \frac{2b^2x\sqrt{1-c^2x^2}\arccos(cx)}{c\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}$$

output
```
2*a*b*x*(-c^2*x^2+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)+2*b^2*(-c^2*x^2+1)/c^2/(
-c^2*d*x^2+d)^(1/2)+2*b^2*x*(-c^2*x^2+1)^(1/2)*arccos(c*x)/c/(-c^2*d*x^2+d
)^(1/2)-(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/c^2/d
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{d-c^2dx^2}\left(2abcx\sqrt{1-c^2x^2}+a^2(1-c^2x^2)+2b^2(-1+c^2x^2)+2b\left(a-ac^2x^2+bcx\sqrt{1-c^2x^2}\right)\arccos(\right.}{c^2d\,(-1+c^2x^2)}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(2*a*b*c*x*Sqrt[1 - c^2*x^2] + a^2*(1 - c^2*x^2) + 2*
b^2*(-1 + c^2*x^2) + 2*b*(a - a*c^2*x^2 + b*c*x*Sqrt[1 - c^2*x^2])*ArcCos[
c*x] + b^2*(1 - c^2*x^2)*ArcCos[c*x]^2))/(c^2*d*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.68, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {5183, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{5183}$$

$$-\frac{2b\sqrt{1-c^2x^2}\int (a+b\arccos(cx))dx}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}$$

input
```
Int[(x*(a + b*ArcCos[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(c^2*d)) - (2*b*Sqrt[1 - c^2
*x^2]*(a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]))/(c*Sqrt[d - c^2*d
*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.16

method	result
default	$-\frac{a^2\sqrt{-c^2dx^2+d}}{c^2d}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\left(\arccos(cx)^2-2+2i\arccos(cx)\right)}{2c^2d(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\left(-i\right.}{}\right.$
parts	$-\frac{a^2\sqrt{-c^2dx^2+d}}{c^2d}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\left(\arccos(cx)^2-2+2i\arccos(cx)\right)}{2c^2d(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\left(-i\right.}{}\right.$
orering	$\frac{(c^4x^4-4c^2x^2+2)(a+b\arccos(cx))^2}{c^4x^2\sqrt{-c^2dx^2+d}}+\frac{2(cx-1)(cx+1)\left(\frac{(a+b\arccos(cx))^2}{\sqrt{-c^2dx^2+d}}-\frac{2x(a+b\arccos(cx))bc}{\sqrt{-c^2dx^2+d}\,\sqrt{-c^2x^2+1}}+\frac{x^2(a+b\arccos(cx))^2dc^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\right)}{c^4x^2}+$

input
```
int(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a^2/c^2/d*(-c^2*d*x^2+d)^(1/2)+b^2*(-1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*
x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)^2-2+2*I*arccos(c*x))/c^2/d/(c^2*x
^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(ar
ccos(c*x)^2-2-2*I*arccos(c*x))/c^2/d/(c^2*x^2-1))+2*a*b*(-1/2*(-d*(c^2*x^2
-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)+I)/c^2/d/(c^2
*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(
arccos(c*x)-I)/c^2/d/(c^2*x^2-1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{2\sqrt{-c^2dx^2+d}(b^2cx\arccos(cx)+abcx)\sqrt{-c^2x^2+1}-((a^2-2b^2)c^2x^2+(b^2c^2x^2-b^2)\arccos(cx)^2-a}{c^4dx^2-c^2d}$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
(2*sqrt(-c^2*d*x^2 + d)*(b^2*c*x*arccos(c*x) + a*b*c*x)*sqrt(-c^2*x^2 + 1)
 - ((a^2 - 2*b^2)*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccos(c*x)^2 - a^2 + 2*b^
2 + 2*(a*b*c^2*x^2 - a*b)*arccos(c*x))*sqrt(-c^2*d*x^2 + d))/(c^4*d*x^2 -
c^2*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = -2\,b^2\left(\frac{x\arccos{(cx)}}{c\sqrt{d}} - \frac{\sqrt{-c^2x^2+1}}{c^2\sqrt{d}}\right) - \frac{2\,abx}{c\sqrt{d}} - \frac{\sqrt{-c^2dx^2+d}b^2\arccos{(cx)}^2}{c^2d} - \frac{2\,\sqrt{-c^2dx^2+d}ab\arccos{(cx)}}{c^2d} - \frac{\sqrt{-c^2dx^2+d}a^2}{c^2d}$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
-2*b^2*(x*arccos(c*x)/(c*sqrt(d)) - sqrt(-c^2*x^2 + 1)/(c^2*sqrt(d))) - 2*
a*b*x/(c*sqrt(d)) - sqrt(-c^2*d*x^2 + d)*b^2*arccos(c*x)^2/(c^2*d) - 2*sqr
t(-c^2*d*x^2 + d)*a*b*arccos(c*x)/(c^2*d) - sqrt(-c^2*d*x^2 + d)*a^2/(c^2*
d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.68

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$=\frac{\sqrt{d}\left(-\sqrt{-c^2x^2+1}\,acos(cx)^2\,b^2-2\sqrt{-c^2x^2+1}\,acos(cx)\,ab-2acos(cx)\,b^2cx-\sqrt{-c^2x^2+1}\,a^2+2\sqrt{-c}\right.}{c^2d}$$

input
```
int(x*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(sqrt(d)*( - sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2 - 2*sqrt( - c**2*x**
2 + 1)*acos(c*x)*a*b - 2*acos(c*x)*b**2*c*x - sqrt( - c**2*x**2 + 1)*a**2
+ 2*sqrt( - c**2*x**2 + 1)*b**2 - 2*a*b*c*x))/(c**2*d)
```

3.241 $\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 49

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc\sqrt{d-c^2dx^2}}$$

output
```
1/3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^3/b/c/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = -\frac{\sqrt{1-c^2x^2}\arccos(cx)\left(3a^2+3ab\arccos(cx)+b^2\arccos(cx)^2\right)}{3c\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/3*(Sqrt[1 - c^2*x^2]*ArcCos[c*x]*(3*a^2 + 3*a*b*ArcCos[c*x] + b^2*ArcCo
s[c*x]^2))/(c*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$
$$\Big\downarrow \text{5153}$$
$$-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/Sqrt[d - c^2*d*x^2],x]
```

output
```
-1/3*(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^3)/(b*c*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(43) = 86.

Time = 0.00 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.90

method	result	size
default	$\frac{a^{2} \arctan\left(\frac{\sqrt{c^{2} d}\, x}{\sqrt{-c^{2} d\, x^{2}+d}}\right)}{\sqrt{c^{2} d}}+\frac{b^{2} \sqrt{-d\left(c^{2} x^{2}-1\right)}\, \sqrt{-c^{2} x^{2}+1}\, \arccos\left(c x\right)^{3}}{3 c d\left(c^{2} x^{2}-1\right)}+\frac{a b \sqrt{-d\left(c^{2} x^{2}-1\right)}\, \sqrt{-c^{2} x^{2}+1}\, \arccos\left(c x\right)^{2}}{c d\left(c^{2} x^{2}-1\right)}$	142
parts	$\frac{a^{2} \arctan\left(\frac{\sqrt{c^{2} d}\, x}{\sqrt{-c^{2} d\, x^{2}+d}}\right)}{\sqrt{c^{2} d}}+\frac{b^{2} \sqrt{-d\left(c^{2} x^{2}-1\right)}\, \sqrt{-c^{2} x^{2}+1}\, \arccos\left(c x\right)^{3}}{3 c d\left(c^{2} x^{2}-1\right)}+\frac{a b \sqrt{-d\left(c^{2} x^{2}-1\right)}\, \sqrt{-c^{2} x^{2}+1}\, \arccos\left(c x\right)^{2}}{c d\left(c^{2} x^{2}-1\right)}$	142

input
```
int((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/3*b^2*(-d
*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/(c^2*x^2-1)*arccos(c*x)^3+a*b*(
-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/(c^2*x^2-1)*arccos(c*x)^2
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2)/(c^2*d*x^2 - d), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(a+b\operatorname{acos}(cx))^2}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(43) = 86$.

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.14

$$\int \frac{(a + b \arccos(cx))^2}{\sqrt{d - c^2dx^2}}\,dx = \frac{b^2 \arccos(cx)^2 \arcsin(cx)}{c\sqrt{d}} + \frac{1}{3}\left(\frac{3\arccos(cx)\arcsin(cx)^2}{c\sqrt{d}} + \frac{\arcsin(cx)^3}{c\sqrt{d}}\right)b^2 + \frac{2\,ab\arccos(cx)\arcsin(cx)}{c\sqrt{d}} + \frac{ab\arcsin(cx)^2}{c\sqrt{d}} + \frac{a^2\arcsin(cx)}{c\sqrt{d}}$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
b^2*arccos(c*x)^2*arcsin(c*x)/(c*sqrt(d)) + 1/3*(3*arccos(c*x)*arcsin(c*x)
^2/(c*sqrt(d)) + arcsin(c*x)^3/(c*sqrt(d)))*b^2 + 2*a*b*arccos(c*x)*arcsin
(c*x)/(c*sqrt(d)) + a*b*arcsin(c*x)^2/(c*sqrt(d)) + a^2*arcsin(c*x)/(c*sqr
t(d))
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{(a + b \arccos(cx))^2}{\sqrt{d - c^2dx^2}}\,dx = -\frac{b^2\arccos(cx)^3 + 3\,ab\arccos(cx)^2 + 3\,a^2\arccos(cx)}{3\,c\sqrt{d}}$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
-1/3*(b^2*arccos(c*x)^3 + 3*a*b*arccos(c*x)^2 + 3*a^2*arccos(c*x))/(c*sqrt
(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\sqrt{d}\left(-acos(cx)^3\,b^2-3acos(cx)^2\,ab+3asin(cx)\,a^2\right)}{3cd}$$

input
```
int((a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(sqrt(d)*( - acos(c*x)**3*b**2 - 3*acos(c*x)**2*a*b + 3*asin(c*x)*a**2))/(
3*c*d)
```

3.242 $\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 257

$$\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx = -\frac{2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}} + \frac{2ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{2ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{2b^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}} + \frac{2b^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}}$$

output

```
-2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2*arctanh(c*x+I*(-c^2*x^2+1)^(1/2)
)/(-c^2*d*x^2+d)^(1/2)+2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*polylog(
2,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)-2*I*b*(-c^2*x^2+1)^(1/2)
*(a+b*arccos(c*x))*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2
)-2*b^2*(-c^2*x^2+1)^(1/2)*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*d*x^
2+d)^(1/2)+2*b^2*(-c^2*x^2+1)^(1/2)*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))/(-
c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx = \frac{a^2\log(cx)}{\sqrt{d}} - \frac{a^2\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)}{\sqrt{d}} + \frac{2iab\sqrt{1-c^2x^2}\left(2\arccos(cx)\arctan\left(e^{i\arccos(cx)}\right) - \text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right) + \text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\right)}{\sqrt{d-c^2dx^2}} + \frac{2b^2\sqrt{1-c^2x^2}\left(i\arccos(cx)^2\arctan\left(e^{i\arccos(cx)}\right) - i\arccos(cx)\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right) + i\arccos(cx)\cdots\right)}{\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(a^2*Log[c*x])/Sqrt[d] - (a^2*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d
] + ((2*I)*a*b*Sqrt[1 - c^2*x^2]*(2*ArcCos[c*x]*ArcTan[E^(I*ArcCos[c*x])]
- PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + PolyLog[2, I*E^(I*ArcCos[c*x])]))/S
qrt[d - c^2*d*x^2] + (2*b^2*Sqrt[1 - c^2*x^2]*(I*ArcCos[c*x]^2*ArcTan[E^(I
*ArcCos[c*x])] - I*ArcCos[c*x]*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I*ArcC
os[c*x]*PolyLog[2, I*E^(I*ArcCos[c*x])] + PolyLog[3, (-I)*E^(I*ArcCos[c*x]
)] - PolyLog[3, I*E^(I*ArcCos[c*x])]))/Sqrt[d - c^2*d*x^2]
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.62, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \; 5219$$

$$-\frac{\sqrt{1-c^2x^2}\int \frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{1-c^2x^2}\int (a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\sqrt{1-c^2x^2}\left(-2b\int (a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+ie^{i\arccos(cx)}\right)\right.}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\sqrt{1-c^2x^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\arccos(cx)\right)-\right.}{}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\sqrt{1-c^2x^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right.\right.}{}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{\sqrt{1-c^2x^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\right.\right.}{\sqrt{d-c^2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-((Sqrt[1 - c^2*x^2]*((-2*I)*(a + b*ArcCos[c*x])^2*ArcTan[E^(I*ArcCos[c*x]
)] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - b*Pol
yLog[3, (-I)*E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, I
*E^(I*ArcCos[c*x])] - b*PolyLog[3, I*E^(I*ArcCos[c*x])])))/Sqrt[d - c^2*d*
x^2])
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.53

method	result
default	$-\frac{i\sqrt{-c^2x^2+1}\,\sqrt{-d(c^2x^2-1)}\left(i\arccos(cx)^2\ln\left(1-i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)b^2-i\arccos(cx)^2\ln\left(1+i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)b^2+2i\arccos\right.}{}$

input
```
int((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(I*arccos(c*x)^2*ln(1-I*(c*x+
I*(-c^2*x^2+1)^(1/2)))*b^2-I*arccos(c*x)^2*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2
)))*b^2+2*I*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))*a*b-2*I*arccos(
c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))*a*b+2*arccos(c*x)*polylog(2,I*(c*x
+I*(-c^2*x^2+1)^(1/2)))*b^2-2*arccos(c*x)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)
^(1/2)))*b^2+2*I*polylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b^2-2*I*polylog(3
,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b^2+2*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2
)))*a*b-2*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))*a*b+2*a^2*arctan(c*x+I*
(-c^2*x^2+1)^(1/2)))/d/(c^2*x^2-1)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{\sqrt{-c^2dx^2+d}\,x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2)/(c^2*d*x^3 - d*x), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^2}{x\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{\sqrt{-c^2dx^2+dx}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
-a^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d) - sqr
t(d)*integrate((b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*a
rctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c
^2*d*x^3 - d*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx=\int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}{x\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}\,dx = \frac{2\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,x}dx\right)ab+\left(\int \frac{acos(cx)^2}{\sqrt{-c^2x^2+1}\,x}dx\right)b^2+\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a^2}{\sqrt{d}}$$

input
```
int((a+b*acos(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(2*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*x),x)*a*b + int(acos(c*x)**2/(sqr
t( - c**2*x**2 + 1)*x),x)*b**2 + log(tan(asin(c*x)/2))*a**2)/sqrt(d)
```

3.243 $\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 183

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx = -\frac{ic\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{dx} + \frac{2bc\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{ib^2c\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}}$$

output
```
-I*c*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2)-(-c^2*d*x
^2+d)^(1/2)*(a+b*arccos(c*x))^2/d/x+2*b*c*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c
*x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/(-c^2*d*x^2+d)^(1/2)-I*b^2*c*(-c^2
*x^2+1)^(1/2)*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx = -\frac{\sqrt{1-c^2x^2}\left(b^2\left(-icx+\sqrt{1-c^2x^2}\right)\arccos(cx)^2+2b\arccos(cx)\left(a\sqrt{1-c^2x^2}+bcx\log\left(1+e^{2i\arccos(cx)}\right)\right.\right.}{x\sqrt{d-c^2dx^2}}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-((Sqrt[1 - c^2*x^2]*(b^2*((-I)*c*x + Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2 + 2
*b*ArcCos[c*x]*(a*Sqrt[1 - c^2*x^2] + b*c*x*Log[1 + E^((2*I)*ArcCos[c*x])]
) + a*(a*Sqrt[1 - c^2*x^2] + 2*b*c*x*Log[c*x]) - I*b^2*c*x*PolyLog[2, -E^(
(2*I)*ArcCos[c*x])]))/(x*Sqrt[d - c^2*d*x^2]))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {5187, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \text{5187}$$

$$-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x}dx}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{dx}$$

$$\Big\downarrow \text{5137}$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{dx}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2bc\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)}{\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{dx}$$

$\downarrow$ 4202

$$-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{dx} + \frac{2bc\sqrt{1-c^2x^2}\left(\frac{i(a+b\arccos(cx))^2}{2b} - 2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2620

$$-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{dx} + \frac{2bc\sqrt{1-c^2x^2}\left(\frac{i(a+b\arccos(cx))^2}{2b} - 2i\left(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx) - \frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\operatorname{arcc}\right.\right.}{\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2715

$$-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{dx} + \frac{2bc\sqrt{1-c^2x^2}\left(\frac{i(a+b\arccos(cx))^2}{2b} - 2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)} - \frac{1}{2}i\log\left(1+e^{2i\arccos(cx}\right.\right.\right.}{\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2838

$$-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{dx} + \frac{2bc\sqrt{1-c^2x^2}\left(\frac{i(a+b\arccos(cx))^2}{2b} - 2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx)) - \frac{1}{4}b\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\right.\right.}{\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(d*x)) + (2*b*c*Sqrt[1 - c^2
*x^2]*(((I/2)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcCos[c*x
])*Log[1 + E^((2*I)*ArcCos[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcCos[c*x])])
/4)))/Sqrt[d - c^2*d*x^2]
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(189) = 378.

Time = 0.55 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.11

method	result
default	$-\frac{a^2\sqrt{-c^2 d\,x^2+d}}{dx}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(-i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\arccos(cx)^2}{(c^2x^2-1)xd}-\frac{i\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\left(2i\arccos\right.}{}\right.$
parts	$-\frac{a^2\sqrt{-c^2 d\,x^2+d}}{dx}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(-i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\arccos(cx)^2}{(c^2x^2-1)xd}-\frac{i\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\left(2i\arccos\right.}{}\right.$

input
```
int((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a^2/d/x*(-c^2*d*x^2+d)^(1/2)+b^2*(-(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1
)^(1/2)*x*c+c^2*x^2-1)*arccos(c*x)^2/(c^2*x^2-1)/x/d-I*(-d*(c^2*x^2-1))^(1
/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)*(2*I*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^
2+1)^(1/2))^2)+2*arccos(c*x)^2+polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2))*c
)+2*a*b*(-2*I*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*x^2-1)*arcc
os(c*x)*c-(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*arc
cos(c*x)/(c^2*x^2-1)/x/d+(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*
x^2-1)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*c)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{\sqrt{-c^2dx^2+d}x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2)/(c^2*d*x^4 - d*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx=\int \frac{(a+b\operatorname{acos}(cx))^2}{x^2\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{\sqrt{-c^2dx^2+d}x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
((-1)^(-2*c^2*d*x^2 + 2*d)*sqrt(d)*log(-2*c^2*d + 2*d/x^2) + sqrt(d)*log(x
^2 - 1/c^2))*a*b*c/d + b^2*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1),
 c*x)^2/(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2), x)/sqrt(d) - 2*sqrt(-c^2*d*x^2
 + d)*a*b*arccos(c*x)/(d*x) - sqrt(-c^2*d*x^2 + d)*a^2/(d*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx = \int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{x^2\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^2*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x^2*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d-c^2dx^2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\,a^2+2\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,x^2}dx\right)abx+\left(\int \frac{acos(cx)^2}{\sqrt{-c^2x^2+1}\,x^2}dx\right)b^2x}{\sqrt{d}\,x}$$

input
```
int((a+b*acos(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*a**2 + 2*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*
x**2),x)*a*b*x + int(acos(c*x)**2/(sqrt( - c**2*x**2 + 1)*x**2),x)*b**2*x)
/(sqrt(d)*x)
```

3.244 $\int \frac{(a+b\arccos(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 402

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx =& -\frac{bc\sqrt{1-c^2x^2}(a+b\arccos(cx))}{x\sqrt{d-c^2dx^2}} \\ &-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2dx^2} \\ &-\frac{c^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}} \\ &-\frac{b^2c^2\sqrt{1-c^2x^2}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{d-c^2dx^2}} \\ &+\frac{ibc^2\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}} \\ &-\frac{ibc^2\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}} \\ &-\frac{b^2c^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}} \\ &+\frac{b^2c^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
-b*c*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/x/(-c^2*d*x^2+d)^(1/2)-1/2*(-c^2
*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/d/x^2-c^2*(-c^2*x^2+1)^(1/2)*(a+b*arcc
os(c*x))^2*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)-b^2*c^2*
(-c^2*x^2+1)^(1/2)*arctanh((-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)+I*b*c^
2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2)
)/(-c^2*d*x^2+d)^(1/2)-I*b*c^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*polylo
g(2,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)-b^2*c^2*(-c^2*x^2+1)^(1
/2)*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)+b^2*c^2*(-c^
2*x^2+1)^(1/2)*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\arccos(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx$$

$$= \frac{1}{2}\left(-\frac{a^2\sqrt{d-c^2dx^2}}{dx^2}+\frac{a^2c^2\log(x)}{\sqrt{d}}-\frac{a^2c^2\log\left(d+\sqrt{d}\sqrt{d-c^2dx^2}\right)}{\sqrt{d}}\right.$$

$$-\frac{2ab\sqrt{1-c^2x^2}\left(-cx+\sqrt{1-c^2x^2}\arccos(cx)+c^2x^2\arccos(cx)\log\left(1-ie^{i\arccos(cx)}\right)-c^2x^2\arccos(cx)\right.}{x^2\sqrt{d-c^2dx^2}}$$

$$-\frac{b^2c^2\left(-\frac{2\sqrt{1-c^2x^2}\arccos(cx)}{cx}-\arccos(cx)^2+\frac{\arccos(cx)^2}{c^2x^2}+2\sqrt{1-c^2x^2}\coth^{-1}\left(\sqrt{1-c^2x^2}\right)-2i\sqrt{1-c^2x^2}\,\mathrm{a}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(-((a^2*Sqrt[d - c^2*d*x^2])/(d*x^2)) + (a^2*c^2*Log[x])/Sqrt[d] - (a^2*c^
2*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] - (2*a*b*Sqrt[1 - c^2*x^2]
*(-(c*x) + Sqrt[1 - c^2*x^2]*ArcCos[c*x] + c^2*x^2*ArcCos[c*x]*Log[1 - I*E
^(I*ArcCos[c*x])] - c^2*x^2*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] + I*c
^2*x^2*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*c^2*x^2*PolyLog[2, I*E^(I*Ar
cCos[c*x])]))/(x^2*Sqrt[d - c^2*d*x^2]) - (b^2*c^2*((-2*Sqrt[1 - c^2*x^2]*
ArcCos[c*x])/(c*x) - ArcCos[c*x]^2 + ArcCos[c*x]^2/(c^2*x^2) + 2*Sqrt[1 -
c^2*x^2]*ArcCoth[Sqrt[1 - c^2*x^2]] - (2*I)*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^
2*ArcTan[E^(I*ArcCos[c*x])] + (2*I)*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*PolyLog[
2, (-I)*E^(I*ArcCos[c*x])] - (2*I)*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*PolyLog[2
, I*E^(I*ArcCos[c*x])] - 2*Sqrt[1 - c^2*x^2]*PolyLog[3, (-I)*E^(I*ArcCos[c
*x])] + 2*Sqrt[1 - c^2*x^2]*PolyLog[3, I*E^(I*ArcCos[c*x])]))/Sqrt[d - c^2
*d*x^2])/2
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.66, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {5205, 5139, 243, 73, 221, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^3\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{ 5205}$$

$$\frac{1}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx-\frac{bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x^2}dx}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{1}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx-\frac{bc\sqrt{1-c^2x^2}\left(-bc\int \frac{1}{x\sqrt{1-c^2x^2}}dx-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2dx^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx-\frac{bc\sqrt{1-c^2x^2}\left(-\frac{1}{2}bc\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2dx^2}$$

$\downarrow$ 73

$$\frac{1}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx-\frac{bc\sqrt{1-c^2x^2}\left(\frac{b\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c}-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2dx^2}$$

$\downarrow$ 221

$$\frac{1}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx-\frac{bc\sqrt{1-c^2x^2}\left(b\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2dx^2}$$

$\downarrow$ 5219

$$-\frac{c^2\sqrt{1-c^2x^2}\int \frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)}{2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{1-c^2x^2}\left(b\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2dx^2}$$

$\downarrow$ 3042

$$-\frac{c^2\sqrt{1-c^2x^2}\int (a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{2\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{1-c^2x^2}\left(b\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2dx^2}$$

$\downarrow$ 4669

$$-\frac{c^2\sqrt{1-c^2x^2}\left(-2b\int (a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+ie^{i\text{arc}}\right.\right.}{2\sqrt{d-c^2dx^2}}$$
$$-\frac{bc\sqrt{1-c^2x^2}\left(b\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2dx^2}$$

$\downarrow$ 3011

$$-\frac{c^2\sqrt{1-c^2x^2}\left(2b\left(i\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\arccos(cx)\right)\right.}{}$$

$$\frac{bc\sqrt{1-c^2x^2}\left(bc\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2dx^2}$$

$\downarrow$ 2720

$$-\frac{c^2\sqrt{1-c^2x^2}\left(2b\left(i\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right.\right.}{}$$

$$\frac{bc\sqrt{1-c^2x^2}\left(bc\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2dx^2}$$

$\downarrow$ 7143

$$-\frac{c^2\sqrt{1-c^2x^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-\right.\right.}{2\sqrt{d-}}$$

$$\frac{bc\sqrt{1-c^2x^2}\left(bc\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}\right)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2dx^2}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-1/2*(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(d*x^2) - (b*c*Sqrt[1 - c
^2*x^2]*(-((a + b*ArcCos[c*x])/x) + b*c*ArcTanh[Sqrt[1 - c^2*x^2]]))/Sqrt[
d - c^2*d*x^2] - (c^2*Sqrt[1 - c^2*x^2]*((-2*I)*(a + b*ArcCos[c*x])^2*ArcT
an[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*Ar
cCos[c*x])] - b*PolyLog[3, (-I)*E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos
[c*x])*PolyLog[2, I*E^(I*ArcCos[c*x])] - b*PolyLog[3, I*E^(I*ArcCos[c*x])]
)))/(2*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.55

method	result
default	$-\frac{a^2\sqrt{-c^2d\,x^2+d}}{2d\,x^2}-\frac{a^2c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{2\sqrt{d}}+b^2\left(-\frac{\left(c^2x^2\arccos(cx)+2cx\sqrt{-c^2x^2+1}-\arccos(cx)\right)\arccos(cx)\sqrt{-d}}{2x^2d(c^2x^2-1)}\right.$
parts	$-\frac{a^2\sqrt{-c^2d\,x^2+d}}{2d\,x^2}-\frac{a^2c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{2\sqrt{d}}+b^2\left(-\frac{\left(c^2x^2\arccos(cx)+2cx\sqrt{-c^2x^2+1}-\arccos(cx)\right)\arccos(cx)\sqrt{-d}}{2x^2d(c^2x^2-1)}\right.$

input
```
int((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/d/x^2*(-c^2*d*x^2+d)^(1/2)-1/2*a^2*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*
(-c^2*d*x^2+d)^(1/2))/x)+b^2*(-1/2*(c^2*x^2*arccos(c*x)+2*c*x*(-c^2*x^2+1)
^(1/2)-arccos(c*x))*arccos(c*x)*(-d*(c^2*x^2-1))^(1/2)/x^2/d/(c^2*x^2-1)-1
/2*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(I*arccos(c*x)^2*ln(1-I*(c*
x+I*(-c^2*x^2+1)^(1/2)))-I*arccos(c*x)^2*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2))
)+2*arccos(c*x)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*I*polylog(3,I*(c
*x+I*(-c^2*x^2+1)^(1/2)))-2*arccos(c*x)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(
1/2)))-2*I*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+4*arctan(c*x+I*(-c^2*x
^2+1)^(1/2)))*c^2/d/(c^2*x^2-1))+2*a*b*(-1/2*(c^2*x^2*arccos(c*x)+c*x*(-c^
2*x^2+1)^(1/2)-arccos(c*x))*(-d*(c^2*x^2-1))^(1/2)/x^2/d/(c^2*x^2-1)+1/2*(
-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*(arccos(c*x)*ln(1-I
*(c*x+I*(-c^2*x^2+1)^(1/2)))-arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2))
)+I*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-I*dilog(1-I*(c*x+I*(-c^2*x^2+1)^
(1/2))))*c^2)
```

Fricas [F]

$$\int\frac{(a+b\arccos(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx=\int\frac{(b\arccos(cx)+a)^2}{\sqrt{-c^2dx^2+d}\,x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2)/(c^2*d*x^5 - d*x^3), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx=\int \frac{(a+b\operatorname{acos}(cx))^2}{x^3\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x**3/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{\sqrt{-c^2dx^2+d}x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/2*(c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d)
+ sqrt(-c^2*d*x^2 + d)/(d*x^2))*a^2 - sqrt(d)*integrate((b^2*arctan2(sqrt(
c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^2*d*x^5 - d*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x^3\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^3*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x^3*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3\sqrt{d-c^2dx^2}}\,dx = \frac{-\sqrt{-c^2x^2+1}\,a^2+4\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,x^3}dx\right)ab\,x^2+2\left(\int \frac{acos(cx)^2}{\sqrt{-c^2x^2+1}\,x^3}dx\right)b^2x^2+\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)a^2c^2x^2}{2\sqrt{d}\,x^2}$$

input
```
int((a+b*acos(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - sqrt( - c**2*x**2 + 1)*a**2 + 4*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*
x**3),x)*a*b*x**2 + 2*int(acos(c*x)**2/(sqrt( - c**2*x**2 + 1)*x**3),x)*b*
*2*x**2 + log(tan(asin(c*x)/2))*a**2*c**2*x**2)/(2*sqrt(d)*x**2)
```

3.245 $\int \frac{(a+b\arccos(cx))^2}{x^4\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 319

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x^4\sqrt{d-c^2dx^2}}\,dx = &-\frac{b^2c^2(1-c^2x^2)}{3x\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3x^2\sqrt{d-c^2dx^2}}\\ &-\frac{2ic^3\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3\sqrt{d-c^2dx^2}}\\ &-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3dx^3}\\ &-\frac{2c^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3dx}\\ &+\frac{4bc^3\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{3\sqrt{d-c^2dx^2}}\\ &-\frac{2ib^2c^3\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{3\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
-1/3*b^2*c^2*(-c^2*x^2+1)/x/(-c^2*d*x^2+d)^(1/2)-1/3*b*c*(-c^2*x^2+1)^(1/2
)*(a+b*arccos(c*x))/x^2/(-c^2*d*x^2+d)^(1/2)-2/3*I*c^3*(-c^2*x^2+1)^(1/2)*
(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2)-1/3*(-c^2*d*x^2+d)^(1/2)*(a+b*arc
cos(c*x))^2/d/x^3-2/3*c^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/d/x+4/3
*b*c^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2)
)^2)/(-c^2*d*x^2+d)^(1/2)-2/3*I*b^2*c^3*(-c^2*x^2+1)^(1/2)*polylog(2,(c*x+
I*(-c^2*x^2+1)^(1/2))^2)/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\arccos(cx))^2}{x^4\sqrt{d-c^2dx^2}}\,dx =$$
$$-\frac{\sqrt{1-c^2x^2}\left(-abcx+a^2\sqrt{1-c^2x^2}+2a^2c^2x^2\sqrt{1-c^2x^2}+b^2c^2x^2\sqrt{1-c^2x^2}+b^2\left(-2ic^3x^3+\sqrt{1-c^2x^2}\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-1/3*(Sqrt[1 - c^2*x^2]*(-(a*b*c*x) + a^2*Sqrt[1 - c^2*x^2] + 2*a^2*c^2*x^
2*Sqrt[1 - c^2*x^2] + b^2*c^2*x^2*Sqrt[1 - c^2*x^2] + b^2*((-2*I)*c^3*x^3
+ Sqrt[1 - c^2*x^2] + 2*c^2*x^2*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2 + b*ArcCo
s[c*x]*(-(b*c*x) + 2*a*Sqrt[1 - c^2*x^2]*(1 + 2*c^2*x^2) + 4*b*c^3*x^3*Log
[1 + E^((2*I)*ArcCos[c*x])]) + 4*a*b*c^3*x^3*Log[c*x] - (2*I)*b^2*c^3*x^3*
PolyLog[2, -E^((2*I)*ArcCos[c*x])]))/(x^3*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.80, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {5205, 5139, 242, 5187, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^4\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \text{5205}$$

$$\frac{2}{3}c^2\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d-c^2dx^2}}dx-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x^3}dx}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3dx^3}$$

$$\Big\downarrow \text{5139}$$

$$\frac{2}{3}c^2\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d-c^2dx^2}}dx-\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{1}{2}bc\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx-\frac{a+b\arccos(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3dx^3}$$

$\big\downarrow$ 242

$$\frac{2}{3}c^2\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d-c^2dx^2}}dx-\frac{2bc\sqrt{1-c^2x^2}\left(\frac{bc\sqrt{1-c^2x^2}}{2x}-\frac{a+b\arccos(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3dx^3}$$

$\big\downarrow$ 5187

$$\frac{2}{3}c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x}dx}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{dx}\right)-\frac{2bc\sqrt{1-c^2x^2}\left(\frac{bc\sqrt{1-c^2x^2}}{2x}-\frac{a+b\arccos(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3dx^3}$$

$\big\downarrow$ 5137

$$\frac{2}{3}c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{dx}\right)-\frac{2bc\sqrt{1-c^2x^2}\left(\frac{bc\sqrt{1-c^2x^2}}{2x}-\frac{a+b\arccos(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3dx^3}$$

$\big\downarrow$ 3042

$$\frac{2}{3}c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)}{\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{dx}\right)-\frac{2bc\sqrt{1-c^2x^2}\left(\frac{bc\sqrt{1-c^2x^2}}{2x}-\frac{a+b\arccos(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3dx^3}$$

$\big\downarrow$ 4202

$$\frac{2}{3}c^2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{dx}+\frac{2bc\sqrt{1-c^2x^2}\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(c\right.}{\sqrt{d-c^2dx^2}}\right.$$
$$-\frac{2bc\sqrt{1-c^2x^2}\left(\frac{bc\sqrt{1-c^2x^2}}{2x}-\frac{a+b\arccos(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3dx^3}$$

$\big\downarrow$ 2620

$$\frac{2}{3}c^2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{dx}+\frac{2bc\sqrt{1-c^2x^2}\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{2}ib\int\log\left(1+e^{2i\arccos(cx)}\right)d\arccos\right.\right.}{\sqrt{d-c^2dx^2}}\right.$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(\frac{bc\sqrt{1-c^2x^2}}{2x}-\frac{a+b\arccos(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3dx^3}$$

$\Big\downarrow$ 2715

$$\frac{2}{3}c^2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{dx}+\frac{2bc\sqrt{1-c^2x^2}\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos}\right.\right.\right.}{\sqrt{d-c^2d}}\right.$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(\frac{bc\sqrt{1-c^2x^2}}{2x}-\frac{a+b\arccos(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3dx^3}$$

$\Big\downarrow$ 2838

$$\frac{2}{3}c^2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{dx}+\frac{2bc\sqrt{1-c^2x^2}\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\,\mathrm{ar}\right.\right.}{\sqrt{d-c^2dx^2}}\right.$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(\frac{bc\sqrt{1-c^2x^2}}{2x}-\frac{a+b\arccos(cx)}{2x^2}\right)}{3\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3dx^3}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-1/3*(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(d*x^3) - (2*b*c*Sqrt[1 -
 c^2*x^2]*((b*c*Sqrt[1 - c^2*x^2])/(2*x) - (a + b*ArcCos[c*x])/(2*x^2)))/(
3*Sqrt[d - c^2*d*x^2]) + (2*c^2*(-((Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]
)^2)/(d*x)) + (2*b*c*Sqrt[1 - c^2*x^2]*(((I/2)*(a + b*ArcCos[c*x])^2)/b -
(2*I)*((-1/2*I)*(a + b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCos[c*x])] - (b*Po
lyLog[2, -E^((2*I)*ArcCos[c*x])])/4)))/Sqrt[d - c^2*d*x^2]))/3
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2185 vs. $2(303) = 606$.

Time = 0.65 (sec) , antiderivative size = 2186, normalized size of antiderivative = 6.85

method	result	size
default	Expression too large to display	2186
parts	Expression too large to display	2186

input
```
int((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x^5*c^8-1/3*b^2*
(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x^3*c^6+2/3*b^2*(-d*(c^2*
x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x*c^4+1/3*b^2*(-d*(c^2*x^2-1))^(1/
2)/(3*c^4*x^4-2*c^2*x^2-1)/d/x*c^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x
^4-2*c^2*x^2-1)/d/x^3*arccos(c*x)^2-2/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^
4*x^4-2*c^2*x^2-1)/d*x*(-c^2*x^2+1)*arccos(c*x)*c^4-4/3*I*b^2*(-d*(c^2*x^2
-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x^3*(-c^2*x^2+1)*arccos(c*x)*c^6+2*I*
b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x^2*(-c^2*x^2+1)^(1/2
)*arccos(c*x)^2*c^5+4/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-
1)/d*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^3-2/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/(
3*c^4*x^4-2*c^2*x^2-1)/d*x*(-c^2*x^2+1)*c^4-8/3*I*a*b*(-c^2*x^2+1)^(1/2)*(
-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*arccos(c*x)*c^3-4/3*I*a*b*(-d*(c^2*x^2
-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x^3*(-c^2*x^2+1)*c^6+1/3*I*b^2*(-d*(c
^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*(-c^2*x^2+1)^(1/2)*c^3-2/3*b^2*
(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x^3*(-c^2*x^2+1)*c^6-b^2*
(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*(-c^2*x^2+1)^(1/2)*arccos
(c*x)*c^3-2*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x^3*arcco
s(c*x)^2*c^6+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d*x*ar
ccos(c*x)^2*c^4+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^2-1)/d/x
*arccos(c*x)^2*c^2+2/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-2*c^2*x^...
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{\sqrt{-c^2dx^2+dx^4}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2)/(c^2*d*x^6 - d*x^4), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4\sqrt{d-c^2dx^2}}\,dx=\int \frac{(a+b\operatorname{acos}(cx))^2}{x^4\sqrt{-d(cx-1)(cx+1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x**4/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{\sqrt{-c^2dx^2+d}x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/3*(4*c^2*log(x)/sqrt(d) - 1/(sqrt(d)*x^2))*a*b*c - 2/3*a*b*(2*sqrt(-c^2
*d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3))*arccos(c*x) - 1/3*a^
2*(2*sqrt(-c^2*d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3)) + b^2*
integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2/(sqrt(c*x + 1)*sqrt
(-c*x + 1)*x^4), x)/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^4\sqrt{d-c^2dx^2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^4\sqrt{d-c^2dx^2}}\,dx = \int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}{x^4\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^4*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x^4*(d - c^2*d*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4\sqrt{d-c^2dx^2}}\,dx = \frac{-2\sqrt{-c^2x^2+1}\,a^2c^2x^2-\sqrt{-c^2x^2+1}\,a^2+6\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,x^4}dx\right)ab\,x^3+3\left(\int \frac{acos(cx)^2}{\sqrt{-c^2x^2+1}\,x^4}dx\right)b^2x^3}{3\sqrt{d}\,x^3}$$

input
```
int((a+b*acos(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
( - 2*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a**2
+ 6*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*x**4),x)*a*b*x**3 + 3*int(acos(c
*x)**2/(sqrt( - c**2*x**2 + 1)*x**4),x)*b**2*x**3)/(3*sqrt(d)*x**3)
```

3.246 $\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 549

$$
\begin{aligned}
\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = & -\frac{16abx\sqrt{1-c^2x^2}}{3c^5d\sqrt{d-c^2dx^2}} \\
& - \frac{32b^2(1-c^2x^2)}{9c^6d\sqrt{d-c^2dx^2}} + \frac{2b^2(1-c^2x^2)^2}{27c^6d\sqrt{d-c^2dx^2}} \\
& - \frac{16b^2x\sqrt{1-c^2x^2}\arccos(cx)}{3c^5d\sqrt{d-c^2dx^2}} + \frac{2bx\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^5d\sqrt{d-c^2dx^2}} \\
& - \frac{2bx^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^3d\sqrt{d-c^2dx^2}} + \frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}} \\
& + \frac{8\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^6d^2} + \frac{4x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^4d^2} \\
& + \frac{4ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c^6d\sqrt{d-c^2dx^2}} \\
& - \frac{2ib^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c^6d\sqrt{d-c^2dx^2}} \\
& + \frac{2ib^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c^6d\sqrt{d-c^2dx^2}}
\end{aligned}
$$

output
```
-16/3*a*b*x*(-c^2*x^2+1)^(1/2)/c^5/d/(-c^2*d*x^2+d)^(1/2)-32/9*b^2*(-c^2*x
^2+1)/c^6/d/(-c^2*d*x^2+d)^(1/2)+2/27*b^2*(-c^2*x^2+1)^2/c^6/d/(-c^2*d*x^2
+d)^(1/2)-16/3*b^2*x*(-c^2*x^2+1)^(1/2)*arccos(c*x)/c^5/d/(-c^2*d*x^2+d)^(
1/2)+2*b*x*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^5/d/(-c^2*d*x^2+d)^(1/2)
-2/9*b*x^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3/d/(-c^2*d*x^2+d)^(1/2)
+x^4*(a+b*arccos(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+8/3*(-c^2*d*x^2+d)^(1/
2)*(a+b*arccos(c*x))^2/c^6/d^2+4/3*x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*
x))^2/c^4/d^2+4*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*arctan(c*x+I*(-c^
2*x^2+1)^(1/2))/c^6/d/(-c^2*d*x^2+d)^(1/2)-2*I*b^2*(-c^2*x^2+1)^(1/2)*poly
log(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^6/d/(-c^2*d*x^2+d)^(1/2)+2*I*b^2*(-
c^2*x^2+1)^(1/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^6/d/(-c^2*d*x^2
+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.76

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{576a^2-378b^2-288a^2c^2x^2-72a^2c^4x^4+810ab\arccos(cx)+405b^2\arccos(c\ldots}{}$$

input
```
Integrate[(x^5*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(576*a^2 - 378*b^2 - 288*a^2*c^2*x^2 - 72*a^2*c^4*x^4 + 810*a*b*ArcCos[c*x
] + 405*b^2*ArcCos[c*x]^2 + 376*b^2*Cos[2*ArcCos[c*x]] - 360*a*b*ArcCos[c*
x]*Cos[2*ArcCos[c*x]] - 180*b^2*ArcCos[c*x]^2*Cos[2*ArcCos[c*x]] + 2*b^2*C
os[4*ArcCos[c*x]] - 18*a*b*ArcCos[c*x]*Cos[4*ArcCos[c*x]] - 9*b^2*ArcCos[c
*x]^2*Cos[4*ArcCos[c*x]] - 432*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 - E
^(I*ArcCos[c*x])] + 432*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 + E^(I*Arc
Cos[c*x])] + 432*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/2]] - 432*a*b*S
qrt[1 - c^2*x^2]*Log[Sin[ArcCos[c*x]/2]] - (432*I)*b^2*Sqrt[1 - c^2*x^2]*P
olyLog[2, -E^(I*ArcCos[c*x])] + (432*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, E
^(I*ArcCos[c*x])] + 372*a*b*Sin[2*ArcCos[c*x]] + 372*b^2*ArcCos[c*x]*Sin[2
*ArcCos[c*x]] + 6*a*b*Sin[4*ArcCos[c*x]] + 6*b^2*ArcCos[c*x]*Sin[4*ArcCos[
c*x]])/(216*c^6*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 2.68 (sec) , antiderivative size = 497, normalized size of antiderivative = 0.91, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.621, Rules used = {5207, 5211, 243, 53, 2009, 5139, 243, 53, 2009, 5183, 2009, 5211, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 5207$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{x^4(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}-\frac{4\int \frac{x^3(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 5211$$

$$-\frac{4\left(-\frac{2b\sqrt{1-c^2x^2}\int x^2(a+b\arccos(cx))dx}{3c\sqrt{d-c^2dx^2}}+\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}\right)}{c^2d}+\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{b\int \frac{x^3}{\sqrt{1-c^2x^2}}dx}{3c}-\frac{x^3(a+b\arccos(cx))}{3c^2}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 243$$

$$-\frac{4\left(-\frac{2b\sqrt{1-c^2x^2}\int x^2(a+b\arccos(cx))dx}{3c\sqrt{d-c^2dx^2}}+\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}\right)}{c^2d}+\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{b\int \frac{x^2}{\sqrt{1-c^2x^2}}dx^2}{6c}-\frac{x^3(a+b\arccos(cx))}{3c^2}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 53$$

$$-\frac{4\left(-\frac{2b\sqrt{1-c^2x^2}\int x^2(a+b\arccos(cx))dx}{3c\sqrt{d-c^2dx^2}}+\frac{2\int\frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}\right)}{c^2d}+\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int\frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{b\int\left(\frac{1}{c^2\sqrt{1-c^2x^2}}-\frac{\sqrt{1-c^2x^2}}{c^2}\right)dx^2}{6c}-\frac{x^3(a+b\arccos(cx))}{3c^2}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 2009

$$-\frac{4\left(-\frac{2b\sqrt{1-c^2x^2}\int x^2(a+b\arccos(cx))dx}{3c\sqrt{d-c^2dx^2}}+\frac{2\int\frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}\right)}{c^2d}+\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int\frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^3(a+b\arccos(cx))}{3c^2}-\frac{b\left(\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6c}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 5139

$$-\frac{4\left(\frac{2\int\frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}bc\int\frac{x^3}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}\right)}{c^2d}+\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int\frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^3(a+b\arccos(cx))}{3c^2}-\frac{b\left(\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6c}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 243

$$-\frac{4\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{6}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}\right)}{c^2d}+\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^3(a+b\arccos(cx))}{3c^2}-\frac{b\left(\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6c}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{53}$$

$$-\frac{4\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{6}bc\int \left(\frac{1}{c^2\sqrt{1-c^2x^2}}-\frac{\sqrt{1-c^2x^2}}{c^2}\right)dx^2+\frac{1}{3}x^3(a+b\arccos(cx))\right)}{3c\sqrt{d-c^2dx^2}}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}\right)}{c^2d}+\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^3(a+b\arccos(cx))}{3c^2}-\frac{b\left(\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6c}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^3(a+b\arccos(cx))}{3c^2}-\frac{b\left(\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6c}\right)}{cd\sqrt{d-c^2dx^2}}-\frac{4\left(\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a+b\arccos(cx))+\frac{1}{6}bc\left(\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)\right)}{3c\sqrt{d-c^2dx^2}}\right)}{c^2d}+\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5183}$$

$$-\frac{4\left(\frac{2\left(-\frac{2b\sqrt{1-c^2x^2}\int(a+b\arccos(cx))dx}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}\right)}{3c^2}-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a+b\arccos(cx\right.}{3c}}{c^2d}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int\frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^3(a+b\arccos(cx))}{3c^2}-\frac{b\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6c}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int\frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^3(a+b\arccos(cx))}{3c^2}-\frac{b\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6c}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-$$

$$\frac{4\left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}+\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a\right.}{}\right.}{c^2d}$$

$$\Big\downarrow \; 5211$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\frac{\int\frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c^2}-\frac{b\int\frac{x}{\sqrt{1-c^2x^2}}dx}{c}-\frac{x(a+b\arccos(cx))}{c^2}}{c^2}-\frac{x^3(a+b\arccos(cx))}{3c^2}-\frac{b\left(\frac{2(1-c^2x^2)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6c}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-$$

$$\frac{4\left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}+\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a\right.}{}\right.}{c^2d}$$

$$\Big\downarrow \; 241$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}}{c^2}-\frac{x^3(a+b\arccos(cx))}{3c^2}-\frac{b\left(\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6c}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{4\left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}+\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a \cdots\right.}{\cdots}\cdots\right)}{c^2d}$$

$$\Big\downarrow \; 5165$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}}{c^2}-\frac{x^3(a+b\arccos(cx))}{3c^2}-\frac{b\left(\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}}{c^4}\right)}{6c}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{4\left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}+\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a \cdots\right.}{\cdots}\cdots\right)}{c^2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}}{c^2}-\frac{x^3(a+b\arccos(cx))}{3c^2}-\frac{b\left(\frac{2\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2\sqrt{1}}{}\right.}{6c}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-$$

$$\frac{4\left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}+\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a\right.}{}\right.}{c^2d}$$

$\downarrow$ 4671

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{c^3}-\frac{x(a+b\arccos}{c^2}}{c^2}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-$$

$$\frac{4\left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}+\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a\right.}{}\right.}{c^2d}$$

$\downarrow$ 2715

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos(cx)}\right.}{c^3}}{c^2}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-$$

$$\frac{4\left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}+\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a\right.}{}\right.}{c^2d}$$

$\downarrow$ 2838

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{c^3}}{c^2}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2}}{c^3}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{x^4(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-$$

$$\frac{4\left(-\frac{x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^2d}+\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{3c^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{3}x^3(a\right.}{}\right.}{c^2d}$$

input
```
Int[(x^5*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x^4*(a + b*ArcCos[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) - (4*(-1/3*(x^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(c^2*d) - (2*b*Sqrt[1 - c^2*x^2]*
((b*c*((-2*Sqrt[1 - c^2*x^2])/c^4 + (2*(1 - c^2*x^2)^(3/2))/(3*c^4)))/6 +
(x^3*(a + b*ArcCos[c*x]))/3))/(3*c*Sqrt[d - c^2*d*x^2]) + (2*(-((Sqrt[d -
c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(c^2*d)) - (2*b*Sqrt[1 - c^2*x^2]*(a*x -
 (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]))/(c*Sqrt[d - c^2*d*x^2])))/(3*
c^2)))/(c^2*d) + (2*b*Sqrt[1 - c^2*x^2]*(-1/6*(b*((-2*Sqrt[1 - c^2*x^2])/c
^4 + (2*(1 - c^2*x^2)^(3/2))/(3*c^4)))/c - (x^3*(a + b*ArcCos[c*x]))/(3*c^
2) + ((b*Sqrt[1 - c^2*x^2])/c^3 - (x*(a + b*ArcCos[c*x]))/c^2 - (-2*(a + b
*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(I*ArcCos[c*x
])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/c^3)/c^2))/(c*d*Sqrt[d - c^2*d*x^
2])
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5139

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5165

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(519) = 1038$.

Time = 0.80 (sec) , antiderivative size = 1077, normalized size of antiderivative = 1.96

method	result	size
default	Expression too large to display	1077
parts	Expression too large to display	1077

input
```
int(x^5*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/3*x^4/c^2/d/(-c^2*d*x^2+d)^(1/2)+4/3/c^2*(-x^2/c^2/d/(-c^2*d*x^2+d
)^(1/2)+2/d/c^4/(-c^2*d*x^2+d)^(1/2)))-94/27*b^2*(-d*(c^2*x^2-1))^(1/2)/d^
2/c^4/(c^2*x^2-1)*x^2-65/24*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)
*arccos(c*x)^2+2*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^
2*x^2-1)*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-2*b^2*(-c^2*x^2+1)^(1/
2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*arccos(c*x)*ln(1+c*x+I*(-c^2
*x^2+1)^(1/2))-31/9*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*arccos(
c*x)*(-c^2*x^2+1)^(1/2)*x-1/108*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^
2-1)*cos(4*arccos(c*x))+1/24*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1
)*cos(4*arccos(c*x))*arccos(c*x)^2+377/108*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/
c^6/(c^2*x^2-1)+5/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arccos(
c*x)^2*x^2-2*I*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*
x^2-1)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))+2*I*b^2*(-c^2*x^2+1)^(1/2)*(-d*
(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2)
)-1/36*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*arccos(c*x)*sin(4*ar
ccos(c*x))-65/12*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*arccos(c*x
)-1/36*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*sin(4*arccos(c*x))+1
/12*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^6/(c^2*x^2-1)*arccos(c*x)*cos(4*arcco
s(c*x))-2*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^6/(c^2*x^2-1
)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-31/9*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^...
```

Fricas [F]

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2x^5}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
integral((b^2*x^5*arccos(c*x)^2 + 2*a*b*x^5*arccos(c*x) + a^2*x^5)*sqrt(-c
^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^5(a+b\operatorname{acos}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**5*(a + b*acos(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x^5}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
-1/3*a^2*(x^4/(sqrt(-c^2*d*x^2 + d)*c^2*d) + 4*x^2/(sqrt(-c^2*d*x^2 + d)*c
^4*d) - 8/(sqrt(-c^2*d*x^2 + d)*c^6*d)) + 1/3*((b^2*c^4*x^4 + 4*b^2*c^2*x^
2 - 8*b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(d)*arctan2(sqrt(c*x + 1)*sqrt
(-c*x + 1), c*x)^2 + 3*(c^8*d^2*x^2 - c^6*d^2)*integrate(2/3*(3*sqrt(c*x +
 1)*sqrt(-c*x + 1)*a*b*c^5*sqrt(d)*x^5*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1
), c*x) + (b^2*c^6*x^6 + 3*b^2*c^4*x^4 - 12*b^2*c^2*x^2 + 8*b^2)*sqrt(d)*a
rctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/(c^9*d^2*x^4 - 2*c^7*d^2*x^2 +
c^5*d^2), x))/(c^8*d^2*x^2 - c^6*d^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^5\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}{\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

input
```
int((x^5*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^5*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-6\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)x^5}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)ab\,c^6-3\sqrt{-c^2x^2+1}\left(\int \frac{}{\sqrt{-c^2x}}\right.}{3\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^6d}$$

input
```
int(x^5*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 6*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**5)/(sqrt( - c**2*x**2 + 1)*
c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*a*b*c**6 - 3*sqrt( - c**2*x**2 + 1)
*int((acos(c*x)**2*x**5)/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*
x**2 + 1)),x)*b**2*c**6 - a**2*c**4*x**4 - 4*a**2*c**2*x**2 + 8*a**2)/(3*s
qrt(d)*sqrt( - c**2*x**2 + 1)*c**6*d)
```

3.247 $\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 424

$$\begin{aligned}\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx &= -\frac{b^2x(1-c^2x^2)}{4c^4d\sqrt{d-c^2dx^2}} \\ &+ \frac{b^2\sqrt{1-c^2x^2}\arccos(cx)}{4c^5d\sqrt{d-c^2dx^2}} - \frac{bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^3d\sqrt{d-c^2dx^2}} \\ &+ \frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{i\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^5d\sqrt{d-c^2dx^2}} \\ &+ \frac{3x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^4d^2} - \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{2bc^5d\sqrt{d-c^2dx^2}} \\ &+ \frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c^5d\sqrt{d-c^2dx^2}} \\ &- \frac{ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c^5d\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
-1/4*b^2*x*(-c^2*x^2+1)/c^4/d/(-c^2*d*x^2+d)^(1/2)+1/4*b^2*(-c^2*x^2+1)^(1
/2)*arccos(c*x)/c^5/d/(-c^2*d*x^2+d)^(1/2)-1/2*b*x^2*(-c^2*x^2+1)^(1/2)*(a
+b*arccos(c*x))/c^3/d/(-c^2*d*x^2+d)^(1/2)+x^3*(a+b*arccos(c*x))^2/c^2/d/(
-c^2*d*x^2+d)^(1/2)-I*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/c^5/d/(-c^2*d
*x^2+d)^(1/2)+3/2*x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/c^4/d^2-1/2*(
-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^3/b/c^5/d/(-c^2*d*x^2+d)^(1/2)+2*b*(-c
^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c^5/d
/(-c^2*d*x^2+d)^(1/2)-I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-(c*x+I*(-c^2*x^2
+1)^(1/2))^2)/c^5/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.74

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-4a^2c\sqrt{d}x(-3+c^2x^2)+12a^2\sqrt{d-c^2dx^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+2ab\sqrt{d}(8c}{}$$

input
```
Integrate[(x^4*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(-4*a^2*c*Sqrt[d]*x*(-3 + c^2*x^2) + 12*a^2*Sqrt[d - c^2*d*x^2]*ArcTan[(c*
x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 2*a*b*Sqrt[d]*(8*c*x*Ar
cCos[c*x] + Sqrt[1 - c^2*x^2]*(6*ArcCos[c*x]^2 + Cos[2*ArcCos[c*x]] - 4*Lo
g[1 - c^2*x^2] + 2*ArcCos[c*x]*Sin[2*ArcCos[c*x]])) + b^2*Sqrt[d]*(8*c*x*A
rcCos[c*x]^2 + (8*I)*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcCos[c*x])] +
 Sqrt[1 - c^2*x^2]*(4*ArcCos[c*x]^3 + 2*ArcCos[c*x]*(Cos[2*ArcCos[c*x]] -
8*Log[1 - E^((2*I)*ArcCos[c*x])]) - Sin[2*ArcCos[c*x]] + 2*ArcCos[c*x]^2*(
4*I + Sin[2*ArcCos[c*x]]))))/(8*c^5*d^(3/2)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.93, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.552$, Rules used = {5207, 5211, 262, 223, 5139, 262, 223, 5153, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{5207}$$

$$-\frac{3\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{2b\sqrt{1-c^2x^2}\int \frac{x^3(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5211}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{b\int \frac{x^2}{\sqrt{1-c^2x^2}}dx}{2c}-\frac{x^2(a+b\arccos(cx))}{2c^2}\right)}{cd\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\int x(a+b\arccos(cx))dx}{c\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{b\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2c}-\frac{x^2(a+b\arccos(cx))}{2c^2}\right)}{cd\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\int x(a+b\arccos(cx))dx}{c\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{223}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^2(a+b\arccos(cx))}{2c^2}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\int x(a+b\arccos(cx))dx}{c\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5139}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int\frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^2(a+b\arccos(cx))}{2c^2}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}bc\int\frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int\frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^2(a+b\arccos(cx))}{2c^2}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{223}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int\frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^2(a+b\arccos(cx))}{2c^2}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}-\frac{3\left(\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{2c^2}-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}-\frac{x^2(a+b\arccos(cx))}{2c^2}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 5181$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^4}-\frac{x^2(a+b\arccos(cx))}{2c^2}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{c^4}-\frac{x^2(a+b\arccos(cx))}{2c^2}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c^4}-\frac{x^2(a+b\arccos(cx))}{2c^2}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 4200}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4}-\frac{x^2(a+b\arccos(cx))}{2c^2}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4}-\frac{x^2(a+b\arccos(cx))}{2c^2}-\frac{b\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{2c}\right)}{cd\sqrt{d-c^2dx^2}}-\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}\right)}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4}-\frac{x^2(a+b\arccos}{2c^2}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}\right)}{c^2d}+$$

$$\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}\right)}{c^2d}+$$

$$\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{3\left(-\frac{b\sqrt{1-c^2x^2}\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c\sqrt{d-c^2dx^2}}-\frac{x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2c^2d}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{6bc^3\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4}-\frac{x^2(a+b\arccos(cx))}{2c^2}-\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{x^3(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^4*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x^3*(a + b*ArcCos[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) - (3*(-1/2*(x*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(c^2*d) - (Sqrt[1 - c^2*x^2]*(a + b
*ArcCos[c*x])^3)/(6*b*c^3*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[1 - c^2*x^2]*((x^
2*(a + b*ArcCos[c*x]))/2 + (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c
*x]/(2*c^3)))/2))/(c*Sqrt[d - c^2*d*x^2])))/(c^2*d) + (2*b*Sqrt[1 - c^2*x^
2]*(-1/2*(x^2*(a + b*ArcCos[c*x]))/c^2 - (b*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^
2 + ArcSin[c*x]/(2*c^3)))/(2*c) - (((-1/2*I)*(a + b*ArcCos[c*x])^2)/b - (2
*I)*((I/2)*(a + b*ArcCos[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])] + (b*PolyLog
[2, E^((2*I)*ArcCos[c*x])])/4))/c^4))/(c*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(404) = 808$.

Time = 0.75 (sec) , antiderivative size = 945, normalized size of antiderivative = 2.23

method	result
default	$-\frac{a^2x^3}{2c^2d\sqrt{-c^2d\,x^2+d}}+\frac{3a^2x}{2c^4d\sqrt{-c^2d\,x^2+d}}-\frac{3a^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^4d\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3}{2d^2c^5(c^2x^2-1)}-\right.$
parts	$-\frac{a^2x^3}{2c^2d\sqrt{-c^2d\,x^2+d}}+\frac{3a^2x}{2c^4d\sqrt{-c^2d\,x^2+d}}-\frac{3a^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^4d\sqrt{c^2d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3}{2d^2c^5(c^2x^2-1)}-\right.$

input
```
int(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2*a^2*x^3/c^2/d/(-c^2*d*x^2+d)^(1/2)+3/2*a^2/c^4*x/d/(-c^2*d*x^2+d)^(1/
2)-3/2*a^2/c^4/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2)
)+b^2*(-1/2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*
arccos(c*x)^3-1/32*(-d*(c^2*x^2-1))^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))*(2*ar
ccos(c*x)^2-1+2*I*arccos(c*x))/d^2/c^5/(c^2*x^2-1)-1/32*(-d*(c^2*x^2-1))^(
1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x)*(2*arccos(c*x)^2-1-2*I*arccos(c*x))/d^2/c
^5/(c^2*x^2-1)-(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x)*arccos(c
*x)^2/d^2/c^5/(c^2*x^2-1)-2*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(I
*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+I*arccos(c*x)*ln(1-c*x-I*(-c^2
*x^2+1)^(1/2))+arccos(c*x)^2+polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+polylog(
2,c*x+I*(-c^2*x^2+1)^(1/2)))/d^2/c^5/(c^2*x^2-1)+1/16*(-d*(c^2*x^2-1))^(1/
2)/d^2/c^5/(c^2*x^2-1)*(2*arccos(c*x)^2-1)*cos(3*arccos(c*x))-1/8*(-d*(c^2
*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*arccos(c*x)*sin(3*arccos(c*x)))-3/2*a*b
*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arccos(c*x)
^2-2*I*a*b*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*a
rccos(c*x)+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^5/(c^2*x^
2-1)*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)-9/4*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2
/c^4/(c^2*x^2-1)*arccos(c*x)*x+1/8*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2
*x^2-1)*(-c^2*x^2+1)^(1/2)+1/4*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2
-1)*arccos(c*x)*cos(3*arccos(c*x))-1/8*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c...
```

Fricas [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2x^4}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input

```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output

```
integral((b^2*x^4*arccos(c*x)^2 + 2*a*b*x^4*arccos(c*x) + a^2*x^4)*sqrt(-c
^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x^4(a+b\operatorname{acos}{(cx)})^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**4*(a + b*acos(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2x^4}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
-1/2*a^2*(x^3/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 3*x/(sqrt(-c^2*d*x^2 + d)*c^4
*d) + 3*arcsin(c*x)/(c^5*d^(3/2))) + sqrt(d)*integrate((b^2*x^4*arctan2(sq
rt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*x^4*arctan2(sqrt(c*x + 1)*sqrt(
-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2
 + d^2), x)
```

Giac [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2x^4}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b*arccos(c*x) + a)^2*x^4/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x^4\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x^4*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\frac{2\sqrt{-c^2x^2+1}\,acos(cx)^3\,b^2+6\sqrt{-c^2x^2+1}\,acos(cx)^2\,ab-2acos(cx)^2\,b^2c^3x^3}{}$$

input
```
int(x^4*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
(2*sqrt( - c**2*x**2 + 1)*acos(c*x)**3*b**2 + 6*sqrt( - c**2*x**2 + 1)*aco
s(c*x)**2*a*b - 2*acos(c*x)**2*b**2*c**3*x**3 + 2*acos(c*x)**2*b**2*c*x +
2*sqrt( - c**2*x**2 + 1)*acos(c*x)*b**2*c**2*x**2 - 2*sqrt( - c**2*x**2 +
1)*acos(c*x)*b**2 - 4*acos(c*x)*a*b*c**3*x**3 + 4*acos(c*x)*a*b*c*x - 6*sq
rt( - c**2*x**2 + 1)*asin(c*x)*a**2 - sqrt( - c**2*x**2 + 1)*asin(c*x)*b**
2 - 8*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x*
*2 - sqrt( - c**2*x**2 + 1)),x)*a*b*c - 4*sqrt( - c**2*x**2 + 1)*int(acos(
c*x)**2/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b**
2*c + 2*sqrt( - c**2*x**2 + 1)*a*b*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*a*
b - 2*a**2*c**3*x**3 + 6*a**2*c*x + b**2*c**3*x**3 - b**2*c*x)/(4*sqrt(d)*
sqrt( - c**2*x**2 + 1)*c**5*d)
```

3.248 $\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 412

$$\begin{aligned}\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx &= -\frac{4abx\sqrt{1-c^2x^2}}{c^3d\sqrt{d-c^2dx^2}}-\frac{2b^2(1-c^2x^2)}{c^4d\sqrt{d-c^2dx^2}}\\ &-\frac{4b^2x\sqrt{1-c^2x^2}\arccos(cx)}{c^3d\sqrt{d-c^2dx^2}}+\frac{2bx\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^3d\sqrt{d-c^2dx^2}}\\ &+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^4d^2}\\ &+\frac{4ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c^4d\sqrt{d-c^2dx^2}}\\ &-\frac{2ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c^4d\sqrt{d-c^2dx^2}}\\ &+\frac{2ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c^4d\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
-4*a*b*x*(-c^2*x^2+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)-2*b^2*(-c^2*x^2+1)/
c^4/d/(-c^2*d*x^2+d)^(1/2)-4*b^2*x*(-c^2*x^2+1)^(1/2)*arccos(c*x)/c^3/d/(-
c^2*d*x^2+d)^(1/2)+2*b*x*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3/d/(-c^2*
d*x^2+d)^(1/2)+x^2*(a+b*arccos(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2*(-c^2*
d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/c^4/d^2+4*I*b*(-c^2*x^2+1)^(1/2)*(a+b*a
rccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c^4/d/(-c^2*d*x^2+d)^(1/2)-2*
I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^4/d/(-
c^2*d*x^2+d)^(1/2)+2*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*(c*x+I*(-c^2*x^2
+1)^(1/2)))/c^4/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.82

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{4a^2-2b^2-2a^2c^2x^2+6ab\arccos(cx)+3b^2\arccos(cx)^2+2b^2\cos(2\arccos(c$$

input
```
Integrate[(x^3*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(4*a^2 - 2*b^2 - 2*a^2*c^2*x^2 + 6*a*b*ArcCos[c*x] + 3*b^2*ArcCos[c*x]^2 +
 2*b^2*Cos[2*ArcCos[c*x]] - 2*a*b*ArcCos[c*x]*Cos[2*ArcCos[c*x]] - b^2*Arc
Cos[c*x]^2*Cos[2*ArcCos[c*x]] - 4*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1
- E^(I*ArcCos[c*x])] + 4*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 + E^(I*Ar
cCos[c*x])] + 4*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/2]] - 4*a*b*Sqrt
[1 - c^2*x^2]*Log[Sin[ArcCos[c*x]/2]] - (4*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLo
g[2, -E^(I*ArcCos[c*x])] + (4*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, E^(I*Arc
Cos[c*x])] + 2*a*b*Sin[2*ArcCos[c*x]] + 2*b^2*ArcCos[c*x]*Sin[2*ArcCos[c*x
]])/(2*c^4*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.68, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {5207, 5183, 2009, 5211, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{5207}$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}-\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5183}$$

$$-\frac{2\left(-\frac{2b\sqrt{1-c^2x^2}\int (a+b\arccos(cx))dx}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}\right)}{c^2d}+\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$$\Big\downarrow \text{5211}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c^2}-\frac{b\int \frac{x}{\sqrt{1-c^2x^2}}dx}{c}-\frac{x(a+b\arccos(cx))}{c^2}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$$\Big\downarrow \text{241}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$\Big\downarrow$ 5165

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$\Big\downarrow$ 3042

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$\Big\downarrow$ 4671

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{c^3}-\frac{x}{}\right.}{cd\sqrt{d-c^2dx^2}}$$
$$\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$\Big\downarrow$ 2715

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos}\right.}{c^3}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{c^3}-\frac{x(a+b\arccos(cx)}{c^2}\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}$$

input
```
Int[(x^3*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x^2*(a + b*ArcCos[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) - (2*(-((Sqrt[d -
c^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(c^2*d)) - (2*b*Sqrt[1 - c^2*x^2]*(a*x -
 (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]))/(c*Sqrt[d - c^2*d*x^2])))/(c^
2*d) + (2*b*Sqrt[1 - c^2*x^2]*((b*Sqrt[1 - c^2*x^2])/c^3 - (x*(a + b*ArcCo
s[c*x]))/c^2 - (-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*Po
lyLog[2, -E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/c^3))/(c
*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 658, normalized size of antiderivative = 1.60

method	result
default	$a^2\left(-\frac{x^2}{c^2 d\sqrt{-c^2 d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{-c^2 d\,x^2+d}}\right)+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\left(\arccos(cx)^2-2+2i\arccos(cx\right.}{2d^2c^4(c^2x^2-1)}\right.$
parts	$a^2\left(-\frac{x^2}{c^2 d\sqrt{-c^2 d\,x^2+d}}+\frac{2}{d\,c^4\sqrt{-c^2 d\,x^2+d}}\right)+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\left(\arccos(cx)^2-2+2i\arccos(cx\right.}{2d^2c^4(c^2x^2-1)}\right.$

input
```
int(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2/d/c^4/(-c^2*d*x^2+d)^(1/2))+b^2*(1/
2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)
^2-2+2*I*arccos(c*x))/d^2/c^4/(c^2*x^2-1)+1/2*(-d*(c^2*x^2-1))^(1/2)*(-I*(
-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)^2-2-2*I*arccos(c*x))/d^2/c^4
/(c^2*x^2-1)-(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arccos(c*x)^2-2*I*
(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(I*arccos(c*x)*ln(1-c*x-I*(-c^2*
x^2+1)^(1/2))-I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-polylog(2,-c*x-
I*(-c^2*x^2+1)^(1/2))+polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)))/d^2/c^4/(c^2*x^
2-1))-2*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^3/(c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*
x+2*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arccos(c*x)*x^2-4*a*b*(
-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arccos(c*x)-2*a*b*(-d*(c^2*x^2-1
))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^4/(c^2*x^2-1)*ln(1+c*x+I*(-c^2*x^2+1)^(1
/2))+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^4/(c^2*x^2-1)*l
n(I*(-c^2*x^2+1)^(1/2)+c*x-1)
```

Fricas [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x^3}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
integral((b^2*x^3*arccos(c*x)^2 + 2*a*b*x^3*arccos(c*x) + a^2*x^3)*sqrt(-c
^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^3(a+b\operatorname{acos}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**3*(a + b*acos(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x^3}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
a*b*c*(2*x/(c^4*d^(3/2)) + log(c*x + 1)/(c^5*d^(3/2)) - log(c*x - 1)/(c^5*
d^(3/2))) - 2*a*b*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 2/(sqrt(-c^2*d*x^2 +
 d)*c^4*d))*arccos(c*x) - a^2*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 2/(sqrt(
-c^2*d*x^2 + d)*c^4*d)) + ((c^2*x^2 - 2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt
(d)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + (c^6*d^2*x^2 - c^4*d^2)
*sqrt(d)*integrate(2*(c^2*x^4 - 2*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1
), c*x)/(c^3*d^2*x^2 - c*d^2), x))*b^2/(c^6*d^2*x^2 - c^4*d^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^3\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x^3*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-2\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)x^3}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)ab\,c^4-\sqrt{-c^2x^2+1}\left(\int \frac{}{\sqrt{-c^2x^2}}\right.}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^4d}$$

input
```
int(x^3*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 2*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**3)/(sqrt( - c**2*x**2 + 1)*
c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*a*b*c**4 - sqrt( - c**2*x**2 + 1)*i
nt((acos(c*x)**2*x**3)/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x*
*2 + 1)),x)*b**2*c**4 - a**2*c**2*x**2 + 2*a**2)/(sqrt(d)*sqrt( - c**2*x**
2 + 1)*c**4*d)
```

3.249 $\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 250

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{i\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^3d\sqrt{d-c^2dx^2}} - \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}} + \frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c^3d\sqrt{d-c^2dx^2}} - \frac{ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c^3d\sqrt{d-c^2dx^2}}$$

output
```
x*(a+b*arccos(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)-I*(-c^2*x^2+1)^(1/2)*(a+b
*arccos(c*x))^2/c^3/d/(-c^2*d*x^2+d)^(1/2)-1/3*(-c^2*x^2+1)^(1/2)*(a+b*arc
cos(c*x))^3/b/c^3/d/(-c^2*d*x^2+d)^(1/2)+2*b*(-c^2*x^2+1)^(1/2)*(a+b*arcco
s(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c^3/d/(-c^2*d*x^2+d)^(1/2)-I*b^
2*(-c^2*x^2+1)^(1/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c^3/d/(-c^2*
d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.14

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = -\frac{a^2x\sqrt{-d\left(-1+c^2x^2\right)}}{c^2d^2\left(-1+c^2x^2\right)}+\frac{a^2\arctan\left(\frac{cx\sqrt{-d(-1+c^2x^2)}}{\sqrt{d}(-1+c^2x^2)}\right)}{c^3d^{3/2}}$$
$$-\frac{ab\left(-2cx\arccos(cx)-\sqrt{1-c^2x^2}\left(\arccos(cx)^2-2\log\left(\sqrt{1-c^2x^2}\right)\right)\right)}{c^3d\sqrt{d\left(1-c^2x^2\right)}}$$
$$-\frac{b^2\left(-\arccos(cx)\left(3cx\arccos(cx)+\sqrt{1-c^2x^2}\left(\arccos(cx)(3i+\arccos(cx))-6\log\left(1-e^{2i\arccos(cx)}\right)\right)\right)-\right.}{3c^3d\sqrt{d\left(1-c^2x^2\right)}}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
-((a^2*x*Sqrt[-(d*(-1 + c^2*x^2))])/(c^2*d^2*(-1 + c^2*x^2))) + (a^2*ArcTa
n[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/(c^3*d^(3/2))
 - (a*b*(-2*c*x*ArcCos[c*x] - Sqrt[1 - c^2*x^2]*(ArcCos[c*x]^2 - 2*Log[Sqr
t[1 - c^2*x^2]])))/(c^3*d*Sqrt[d*(1 - c^2*x^2)]) - (b^2*(-(ArcCos[c*x]*(3*
c*x*ArcCos[c*x] + Sqrt[1 - c^2*x^2]*(ArcCos[c*x]*(3*I + ArcCos[c*x]) - 6*L
og[1 - E^((2*I)*ArcCos[c*x])]))) - (3*I)*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((
2*I)*ArcCos[c*x])]))/(3*c^3*d*Sqrt[d*(1 - c^2*x^2)])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.78, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {5207, 5153, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 5207$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}-\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5181}$$

$$-\frac{2b\sqrt{1-c^2x^2}\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\sqrt{1-c^2x^2}\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4200}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+}{}\right.}{c^3d\sqrt{d-c^2dx^2}}$$
$$\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$\big\downarrow$ 2715

$$-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos}\right.\right.}{c^3d\sqrt{d-c^2dx^2}}$$
$$\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

$\big\downarrow$ 2838

$$\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-$$
$$\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c^3d\sqrt{d-c^2dx^2}}$$
$$\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCos[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[1 - c^2*x^2]
*(a + b*ArcCos[c*x])^3)/(3*b*c^3*d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^
2*x^2]*(((-1/2*I)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcCos[c*
x])*Log[1 - E^((2*I)*ArcCos[c*x])] + (b*PolyLog[2, E^((2*I)*ArcCos[c*x])])
/4)))/(c^3*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(250) = 500$.

Time = 0.58 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.20

method	result
default	$\frac{a^2x}{c^2d\sqrt{-c^2dx^2+d}} - \frac{a^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{c^2d\sqrt{c^2d}} + b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3}{3d^2c^3(c^2x^2-1)} - \frac{\sqrt{-d(c^2x^2-1)}\,\left(-i\sqrt{-c^2x^2}\right.}{d^2c^3(c^2x^2}\right.$
parts	$\frac{a^2x}{c^2d\sqrt{-c^2dx^2+d}} - \frac{a^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{c^2d\sqrt{c^2d}} + b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3}{3d^2c^3(c^2x^2-1)} - \frac{\sqrt{-d(c^2x^2-1)}\,\left(-i\sqrt{-c^2x^2}\right.}{d^2c^3(c^2x^2}\right.$

input
```
int(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-a^2/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1
/2)*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-1/3*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^
(1/2)/d^2/c^3/(c^2*x^2-1)*arccos(c*x)^3-(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x
^2+1)^(1/2)+c*x)*arccos(c*x)^2/d^2/c^3/(c^2*x^2-1)-2*I*(-c^2*x^2+1)^(1/2)*
(-d*(c^2*x^2-1))^(1/2)*(I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+I*arc
cos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+arccos(c*x)^2+polylog(2,-c*x-I*(-c
^2*x^2+1)^(1/2))+polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)))/d^2/c^3/(c^2*x^2-1))
-a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccos(
c*x)^2-2*I*a*b*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/c^3/(c^2*x^2-
1)*arccos(c*x)-2*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arccos(c*x
)*x+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*ln
((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
integral((b^2*x^2*arccos(c*x)^2 + 2*a*b*x^2*arccos(c*x) + a^2*x^2)*sqrt(-c
^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^2(a+b\operatorname{acos}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*acos(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
a^2*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d) - arcsin(c*x)/(c^3*d^(3/2))) + sqrt(d)
*integrate((b^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*x
^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1
)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Giac [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((b*arccos(c*x) + a)^2*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x^2\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{\sqrt{-c^2x^2+1}\,acos(cx)^3\,b^2+3\sqrt{-c^2x^2+1}\,acos(cx)^2\,ab-3\sqrt{-c^2x^2+1}\,asin(}{}$$

input
```
int(x^2*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
(sqrt( - c**2*x**2 + 1)*acos(c*x)**3*b**2 + 3*sqrt( - c**2*x**2 + 1)*acos(
c*x)**2*a*b - 3*sqrt( - c**2*x**2 + 1)*asin(c*x)*a**2 - 6*sqrt( - c**2*x**
2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2
 + 1)),x)*a*b*c - 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)**2/(sqrt( - c**2*
x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b**2*c + 3*a**2*c*x)/(3*s
qrt(d)*sqrt( - c**2*x**2 + 1)*c**3*d)
```

3.250 $\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 208

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{4ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}} - \frac{2ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}}$$

output
```
(a+b*arccos(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+4*I*b*(-c^2*x^2+1)^(1/2)*(a
+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c^2/d/(-c^2*d*x^2+d)^(1/2
)-2*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^2/
d/(-c^2*d*x^2+d)^(1/2)+2*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*(c*x+I*(-c^2
*x^2+1)^(1/2)))/c^2/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.16

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{a^2+2ab\arccos(cx)+b^2\arccos(cx)^2-2b^2\sqrt{1-c^2x^2}\arccos(cx)\log\left(1-e^{i\,\mathrm{arc}}\right.}{}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(a^2 + 2*a*b*ArcCos[c*x] + b^2*ArcCos[c*x]^2 - 2*b^2*Sqrt[1 - c^2*x^2]*Arc
Cos[c*x]*Log[1 - E^(I*ArcCos[c*x])] + 2*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*
Log[1 + E^(I*ArcCos[c*x])] + 2*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/2
]] - 2*a*b*Sqrt[1 - c^2*x^2]*Log[Sin[ArcCos[c*x]/2]] - (2*I)*b^2*Sqrt[1 -
c^2*x^2]*PolyLog[2, -E^(I*ArcCos[c*x])] + (2*I)*b^2*Sqrt[1 - c^2*x^2]*Poly
Log[2, E^(I*ArcCos[c*x])])/(c^2*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.62, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5183, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 5183}$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 5165}$$

$$\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int(a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4671}$$

$$\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-b\int\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right.\right.}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx}\right.}{c^2d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx}\right.\right.}{c^2d\sqrt{d-c^2dx^2}}$$

input
```
Int[(x*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(a + b*ArcCos[c*x])^2/(c^2*d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]
*(-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(I
*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])]))/(c^2*d*Sqrt[d - c^2*d
*x^2])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.88

method	result
default	$\frac{a^2}{c^2 d\sqrt{-c^2 d\,x^2+d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\arccos(cx)^2}{d^2c^2(c^2x^2-1)}-\right.$ $2i\sqrt{-c^2x^2+1}\,\sqrt{-d(c^2x^2-1)}\,\left(i\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)-\right.$
parts	$\frac{a^2}{c^2 d\sqrt{-c^2 d\,x^2+d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\arccos(cx)^2}{d^2c^2(c^2x^2-1)}-\right.$ $2i\sqrt{-c^2x^2+1}\,\sqrt{-d(c^2x^2-1)}\,\left(i\arccos(cx)\ln\left(1-cx-i\sqrt{-c^2x^2+1}\right)-\right.$

input
```
int(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+b^2*(-(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x
^2-1)*arccos(c*x)^2-2*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(I*arcco
s(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1
)^(1/2))-polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+polylog(2,c*x+I*(-c^2*x^2+1)
^(1/2)))/d^2/c^2/(c^2*x^2-1))+2*a*b*(-(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*
x^2-1)*arccos(c*x)+(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/c^2/(c^2*
x^2-1)*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)-(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^
(1/2)/d^2/c^2/(c^2*x^2-1)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*x*arccos(c*x)^2 + 2*a*b*x*arccos(c*x) +
 a^2*x)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x(a+b\operatorname{acos}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x*(a + b*acos(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
sqrt(d)*integrate((b^2*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*
a*b*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x
+ 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x) + a^2/(sqrt(-c^2*d*x^2 + d)*c
^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{x\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((x*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-2\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)x}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)ab\,c^2-\sqrt{-c^2x^2+1}\left(\int \frac{a}{\sqrt{-c^2x^2+}}\right.}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,c^2d}$$

input
```
int(x*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 2*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x)/(sqrt( - c**2*x**2 + 1)*c**
2*x**2 - sqrt( - c**2*x**2 + 1)),x)*a*b*c**2 - sqrt( - c**2*x**2 + 1)*int(
(acos(c*x)**2*x)/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1
)),x)*b**2*c**2 + a**2)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*c**2*d)
```

3.251 $\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 195

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}} - \frac{i\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{cd\sqrt{d-c^2dx^2}} + \frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{cd\sqrt{d-c^2dx^2}} - \frac{ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{cd\sqrt{d-c^2dx^2}}$$

output
```
x*(a+b*arccos(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)-I*(-c^2*x^2+1)^(1/2)*(a+b*arc
cos(c*x))^2/c/d/(-c^2*d*x^2+d)^(1/2)+2*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*
x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/d/(-c^2*d*x^2+d)^(1/2)-I*b^2*(-c^
2*x^2+1)^(1/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/d/(-c^2*d*x^2+d)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{b^2\left(cx+i\sqrt{1-c^2x^2}\right)\arccos(cx)^2+2b\arccos(cx)\left(acx-b\sqrt{1-c^2x^2}\log\left(1-e\right.\right.}{c}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(b^2*(c*x + I*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2 + 2*b*ArcCos[c*x]*(a*c*x -
b*Sqrt[1 - c^2*x^2]*Log[1 - E^((2*I)*ArcCos[c*x])]) + a*(a*c*x - b*Sqrt[1
- c^2*x^2]*Log[1 - c^2*x^2]) + I*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)
*ArcCos[c*x])])/(c*d*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.72, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5161}$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5181}$$

$$\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{2b\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 4200$$

$$\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2620$$

$$\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int \log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\cdots\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2715$$

$$\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(\cdots}\right.\right.}{cd\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2838$$

$$\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}$$

input

```
Int[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCos[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]
*(((-1/2*I)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])*Lo
g[1 - E^((2*I)*ArcCos[c*x])] + (b*PolyLog[2, E^((2*I)*ArcCos[c*x])])/4)))/
(c*d*Sqrt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.05

method	result
default	$\frac{a^2 x}{d\sqrt{-c^2 d\,x^2+d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(-i\sqrt{-c^2x^2+1}+cx\right)\arccos(cx)^2}{(c^2x^2-1)c\,d^2}-\frac{2i\sqrt{-c^2x^2+1}\,\sqrt{-d(c^2x^2-1)}\left(i\arccos(cx)\ln\left(1+c\right.\right.}{}\right.$
parts	$\frac{a^2 x}{d\sqrt{-c^2 d\,x^2+d}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(-i\sqrt{-c^2x^2+1}+cx\right)\arccos(cx)^2}{(c^2x^2-1)c\,d^2}-\frac{2i\sqrt{-c^2x^2+1}\,\sqrt{-d(c^2x^2-1)}\left(i\arccos(cx)\ln\left(1+c\right.\right.}{}\right.$

input
```
int((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/d*x/(-c^2*d*x^2+d)^(1/2)+b^2*(-(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)
^(1/2)+c*x)*arccos(c*x)^2/(c^2*x^2-1)/c/d^2-2*I*(-c^2*x^2+1)^(1/2)*(-d*(c^
2*x^2-1))^(1/2)*(I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+I*arccos(c*x
)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+arccos(c*x)^2+polylog(2,-c*x-I*(-c^2*x^2+
1)^(1/2))+polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)))/(c^2*x^2-1)/c/d^2)-2*I*a*b*
(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/c/d^2*arccos(c*x)-2*
a*b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/(c^2*x^2-1)/d^2*x+2*a*b*(-d*(c^2*x^
2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c/d^2*ln((c*x+I*(-c^2*x^2+1)^(1
/2))^2-1)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(a+b\operatorname{acos}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
2*a*b*x*arccos(c*x)/(sqrt(-c^2*d*x^2 + d)*d) - b^2*integrate(arctan2(sqrt(
c*x + 1)*sqrt(-c*x + 1), c*x)^2/((c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(-c*x +
 1)), x)/sqrt(d) + a^2*x/(sqrt(-c^2*d*x^2 + d)*d) + a*b*log(x^2 - 1/c^2)/(
c*d^(3/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \frac{-2\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)ab-\sqrt{-c^2x^2+1}\left(\int \frac{acos(}{\sqrt{-c^2x^2+1}\,c^2x}\right.}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,d}$$

input
```
int((a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 2*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x*
*2 - sqrt( - c**2*x**2 + 1)),x)*a*b - sqrt( - c**2*x**2 + 1)*int(acos(c*x)
**2/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b**2 +
a**2*x)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*d)
```

3.252 $\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 467

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}\,dx &= \frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}\\
&+\frac{4ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}}\\
&-\frac{2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\text{arctanh}\left(e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}}\\
&+\frac{2ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}}\\
&-\frac{2ib^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}}\\
&+\frac{2ib^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}}\\
&-\frac{2ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}}\\
&-\frac{2b^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}}\\
&+\frac{2b^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
(a+b*arccos(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)+4*I*b*(-c^2*x^2+1)^(1/2)*(a+b*a
rccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-2*(-c^
2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/d/(-c
^2*d*x^2+d)^(1/2)+2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*polylog(2,-c*
x-I*(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-2*I*b^2*(-c^2*x^2+1)^(1/2)*
polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d/(-c^2*d*x^2+d)^(1/2)+2*I*b^2*(-
c^2*x^2+1)^(1/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d/(-c^2*d*x^2+d)^
(1/2)-2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*polylog(2,c*x+I*(-c^2*x^2
+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-2*b^2*(-c^2*x^2+1)^(1/2)*polylog(3,-c*x-
I*(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)+2*b^2*(-c^2*x^2+1)^(1/2)*poly
log(3,c*x+I*(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\arccos(cx))^2}{x\,(d-c^2dx^2)^{3/2}}\,dx = -\frac{a^2\sqrt{-d\,(-1+c^2x^2)}}{d^2\,(-1+c^2x^2)}$$
$$+\frac{a^2\log(cx)}{d^{3/2}}-\frac{a^2\log\left(d+\sqrt{d}\sqrt{-d\,(-1+c^2x^2)}\right)}{d^{3/2}}$$
$$-\frac{2ab\big(-\arccos(cx)+\sqrt{1-c^2x^2}\arccos(cx)\log\left(1-ie^{i\arccos(cx)}\right)-\sqrt{1-c^2x^2}\arccos(cx)\log\left(1+ie^{i\arccos(c}\right.}{}$$
$$-\frac{b^2\big(-\arccos(cx)^2+2\sqrt{1-c^2x^2}\arccos(cx)\log\left(1-e^{i\arccos(cx)}\right)+\sqrt{1-c^2x^2}\arccos(cx)^2\log\left(1-ie^{i\arccos}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-((a^2*Sqrt[-(d*(-1 + c^2*x^2))])/(d^2*(-1 + c^2*x^2))) + (a^2*Log[c*x])/d
^(3/2) - (a^2*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/d^(3/2) - (2*a*b
*(-ArcCos[c*x] + Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])
] - Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] - Sqrt[1 -
c^2*x^2]*Log[Cos[ArcCos[c*x]/2]] + Sqrt[1 - c^2*x^2]*Log[Sin[ArcCos[c*x]/2
]] + I*Sqrt[1 - c^2*x^2]*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*Sqrt[1 - c
^2*x^2]*PolyLog[2, I*E^(I*ArcCos[c*x])]))/(d*Sqrt[d*(1 - c^2*x^2)]) - (b^2
*(-ArcCos[c*x]^2 + 2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x
])] + Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2*Log[1 - I*E^(I*ArcCos[c*x])] - Sqrt[
1 - c^2*x^2]*ArcCos[c*x]^2*Log[1 + I*E^(I*ArcCos[c*x])] - 2*Sqrt[1 - c^2*x
^2]*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] + (2*I)*Sqrt[1 - c^2*x^2]*PolyL
og[2, -E^(I*ArcCos[c*x])] + (2*I)*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*PolyLog[2,
 (-I)*E^(I*ArcCos[c*x])] - (2*I)*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*PolyLog[2,
I*E^(I*ArcCos[c*x])] - (2*I)*Sqrt[1 - c^2*x^2]*PolyLog[2, E^(I*ArcCos[c*x]
)] - 2*Sqrt[1 - c^2*x^2]*PolyLog[3, (-I)*E^(I*ArcCos[c*x])] + 2*Sqrt[1 - c
^2*x^2]*PolyLog[3, I*E^(I*ArcCos[c*x])]))/(d*Sqrt[d*(1 - c^2*x^2)])
```

Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.61, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {5209, 5165, 3042, 4671, 2715, 2838, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5209}$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5165}$$

$$-\frac{2b\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{2b\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 4671

$$-\frac{2b\sqrt{1-c^2x^2}\left(-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 2715

$$-\frac{2b\sqrt{1-c^2x^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 2838

$$\frac{\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}-$$
$$\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(c\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 5219

$$-\frac{\sqrt{1-c^2x^2}\int \frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)}{d\sqrt{d-c^2dx^2}}-$$
$$\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(c\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 3042

$$-\frac{\sqrt{1-c^2x^2}\int (a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 4669

$$-\frac{\sqrt{1-c^2x^2}\left(-2b\int (a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+ie^{i\arccos}\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 3011

$$-\frac{\sqrt{1-c^2x^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\arccos(cx)\right)-\right.}{}$$
$$\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2720

$$-\frac{\sqrt{1-c^2x^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)de^{i\,\text{ar}}\right.\right.}{}$$
$$\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

$\downarrow$ 7143

$$-\frac{\sqrt{1-c^2x^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\right.\right.}{d\sqrt{d-}}$$
$$\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
(a + b*ArcCos[c*x])^2/(d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*(-2
*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(I*Arc
Cos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])]))/(d*Sqrt[d - c^2*d*x^2]) -
 (Sqrt[1 - c^2*x^2]*((-2*I)*(a + b*ArcCos[c*x])^2*ArcTan[E^(I*ArcCos[c*x])
] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - b*Poly
Log[3, (-I)*E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, I*
E^(I*ArcCos[c*x])] - b*PolyLog[3, I*E^(I*ArcCos[c*x])])))/(d*Sqrt[d - c^2*
d*x^2])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.37

method	result
default	$\frac{a^2}{d\sqrt{-c^2dx^2+d}}-\frac{a^2\ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{3}{2}}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\arccos(cx)^2}{d^2(c^2x^2-1)}+\frac{\sqrt{-c^2x^2+1}\sqrt{-d(c^2x^2-1)}\left(\arccos(cx\right.}{}\right.$
parts	$\frac{a^2}{d\sqrt{-c^2dx^2+d}}-\frac{a^2\ln\left(\frac{2d+2\sqrt{d}\sqrt{-c^2dx^2+d}}{x}\right)}{d^{\frac{3}{2}}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\arccos(cx)^2}{d^2(c^2x^2-1)}+\frac{\sqrt{-c^2x^2+1}\sqrt{-d(c^2x^2-1)}\left(\arccos(cx\right.}{}\right.$

input
```
int((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2/d/(-c^2*d*x^2+d)^(1/2)-a^2/d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1
/2))/x)+b^2*(-(-d*(c^2*x^2-1))^(1/2)/d^2/(c^2*x^2-1)*arccos(c*x)^2+(-c^2*x
^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(arccos(c*x)^2*ln(1-I*(c*x+I*(-c^2*x^2+
1)^(1/2)))-arccos(c*x)^2*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*I*arccos(c*x
)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*I*arccos(c*x)*polylog(2,-I*(c*
x+I*(-c^2*x^2+1)^(1/2)))-2*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+2*I*
dilog(c*x+I*(-c^2*x^2+1)^(1/2))+2*I*dilog(1+c*x+I*(-c^2*x^2+1)^(1/2))+2*po
lylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(
1/2))))/d^2/(c^2*x^2-1))+2*a*b*(-(-d*(c^2*x^2-1))^(1/2)/d^2/(c^2*x^2-1)*ar
ccos(c*x)-I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(I*arccos(c*x)*ln(1-
I*(c*x+I*(-c^2*x^2+1)^(1/2)))-I*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/
2)))-I*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+I*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)-dil
og(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2))))/
d^2/(c^2*x^2-1))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{(b\arccos\left(cx\right)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(a+b\operatorname{acos}\left(cx\right)\right)^2}{x\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{(b\arccos\left(cx\right)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
-a^2*(log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2) - 1/
(sqrt(-c^2*d*x^2 + d)*d)) + sqrt(d)*integrate((b^2*arctan2(sqrt(c*x + 1)*s
qrt(-c*x + 1), c*x)^2 + 2*a*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*
sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x\,(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = \frac{-2\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^3-\sqrt{-c^2x^2+1}\,x}dx\right)ab-\sqrt{-c^2x^2+1}\left(\int \frac{ac}{\sqrt{-c^2x^2+1}\,c^2}\right.}{\sqrt{d}\,\sqrt{-}}$$

input
```
int((a+b*acos(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 2*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x*
*3 - sqrt( - c**2*x**2 + 1)*x),x)*a*b - sqrt( - c**2*x**2 + 1)*int(acos(c*
x)**2/(sqrt( - c**2*x**2 + 1)*c**2*x**3 - sqrt( - c**2*x**2 + 1)*x),x)*b**
2 + sqrt( - c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a**2 - sqrt( - c**2*x**2
+ 1)*a**2 + a**2)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*d)
```

3.253 $\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 333

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^{3/2}}\,dx = &-\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}\\
&+\frac{2c^2x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2ic\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}\\
&-\frac{4bc\sqrt{1-c^2x^2}(a+b\arccos(cx))\text{arctanh}\left(e^{2i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}}\\
&+\frac{4bc\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}}\\
&-\frac{ib^2c\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}}\\
&-\frac{ib^2c\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
-(a+b*arccos(c*x))^2/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2*x*(a+b*arccos(c*x))^2/
d/(-c^2*d*x^2+d)^(1/2)-2*I*c*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/d/(-c^
2*d*x^2+d)^(1/2)-4*b*c*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*arctanh((c*x+I
*(-c^2*x^2+1)^(1/2))^2)/d/(-c^2*d*x^2+d)^(1/2)+4*b*c*(-c^2*x^2+1)^(1/2)*(a
+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d/(-c^2*d*x^2+d)^(1/2)-
I*b^2*c*(-c^2*x^2+1)^(1/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d/(-c^
2*d*x^2+d)^(1/2)-I*b^2*c*(-c^2*x^2+1)^(1/2)*polylog(2,(c*x+I*(-c^2*x^2+1)^
(1/2))^2)/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.97

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^{3/2}}\,dx = \frac{-a^2+2a^2c^2x^2-2ab\arccos(cx)+4abc^2x^2\arccos(cx)-b^2\arccos(cx)^2+2b^2c^2$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
(-a^2 + 2*a^2*c^2*x^2 - 2*a*b*ArcCos[c*x] + 4*a*b*c^2*x^2*ArcCos[c*x] - b^
2*ArcCos[c*x]^2 + 2*b^2*c^2*x^2*ArcCos[c*x]^2 + (2*I)*b^2*c*x*Sqrt[1 - c^2
*x^2]*ArcCos[c*x]^2 - 2*b^2*c*x*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 - E^((
2*I)*ArcCos[c*x])] - 2*b^2*c*x*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 + E^((2
*I)*ArcCos[c*x])] - 2*a*b*c*x*Sqrt[1 - c^2*x^2]*Log[c*x] - a*b*c*x*Sqrt[1
- c^2*x^2]*Log[1 - c^2*x^2] + I*b^2*c*x*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^((
2*I)*ArcCos[c*x])] + I*b^2*c*x*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcCo
s[c*x])])/(d*x*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.83, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.552$, Rules used = {5205, 5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838, 5185, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^2\left(d-c^2dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 5205}$$

$$2c^2\int \frac{(a+b\arccos(cx))^2}{\left(d-c^2dx^2\right)^{3/2}}dx-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 5161}$$

$$2c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}\right)-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 5181}$$

$$2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}\right)-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}+2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}\right)-\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}+2c^2\left(\frac{2b\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}\right)-\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 4200}$$

$$2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$\downarrow$ 25

$$2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2620

$$-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}+2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\log\left(1-e^{2i\arc\right.\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2715

$$-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}+2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\text{lo}\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2838

$$-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}+2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\,\text{ar}}\right.\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5185}$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{cx\sqrt{1-c^2x^2}}d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+$$
$$2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\,\text{ar}}\right.\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$
$$\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4919}$$

$$\frac{4bc\sqrt{1-c^2x^2}\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+$$
$$2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\,\text{ar}}\right.\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$
$$\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4bc\sqrt{1-c^2x^2}\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+$$
$$2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\,\text{ar}}\right.\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$
$$\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4671}$$

$$\frac{4bc\sqrt{1-c^2x^2}\left(-\frac{1}{2}b\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+\frac{1}{2}b\int\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\left(\text{arctanh}\left(e^{2i\,\text{a}}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\,\text{ar}}\right.\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$
$$\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{4bc\sqrt{1-c^2x^2}\left(\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de\right.}{d\sqrt{d-c^2dx^2}}$$
$$2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\mathrm{PolyLog}\left(2,e^{2i\,\mathrm{ar}}\right.\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$
$$\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \ 2838$$

$$\frac{4bc\sqrt{1-c^2x^2}\left(-\left(\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}ib\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)-\frac{1}{4}ib\,\mathrm{PolyLog}\left(2,e^{2i}\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\mathrm{PolyLog}\left(2,e^{2i\,\mathrm{ar}}\right.\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.$$
$$\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-((a + b*ArcCos[c*x])^2/(d*x*Sqrt[d - c^2*d*x^2])) + (4*b*c*Sqrt[1 - c^2*x
^2]*(-((a + b*ArcCos[c*x])*ArcTanh[E^((2*I)*ArcCos[c*x])]) + (I/4)*b*PolyL
og[2, -E^((2*I)*ArcCos[c*x])] - (I/4)*b*PolyLog[2, E^((2*I)*ArcCos[c*x])])
)/(d*Sqrt[d - c^2*d*x^2]) + 2*c^2*((x*(a + b*ArcCos[c*x])^2)/(d*Sqrt[d - c
^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*(((-1/2*I)*(a + b*ArcCos[c*x])^2)/b -
(2*I)*((I/2)*(a + b*ArcCos[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])] + (b*PolyL
og[2, E^((2*I)*ArcCos[c*x])])/4)))/(c*d*Sqrt[d - c^2*d*x^2]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.55

method	result
default	$a^2\left(-\frac{1}{dx\sqrt{-c^2d\,x^2+d}}+\frac{2c^2x}{d\sqrt{-c^2d\,x^2+d}}\right)+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(-2i\sqrt{-c^2x^2+1}\,xc+2c^2x^2-1\right)\arccos(cx)^2}{d^2x(c^2x^2-1)}-\frac{i\sqrt{-c^2x^2}}{}\right.$
parts	$a^2\left(-\frac{1}{dx\sqrt{-c^2d\,x^2+d}}+\frac{2c^2x}{d\sqrt{-c^2d\,x^2+d}}\right)+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(-2i\sqrt{-c^2x^2+1}\,xc+2c^2x^2-1\right)\arccos(cx)^2}{d^2x(c^2x^2-1)}-\frac{i\sqrt{-c^2x^2}}{}\right.$

input
```
int((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2/d*x/(-c^2*d*x^2+d)^(1/2))+b^2*(-(-d
*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*arccos(c*x)^
2/d^2/x/(c^2*x^2-1)-I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1
)/d^2*(2*I*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+2*I*arccos(c*x)*
ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+2*I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/
2))+4*arccos(c*x)^2+polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+2*polylog(2,c
*x+I*(-c^2*x^2+1)^(1/2))+2*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2)))*c)+2*a*b*
(-4*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/d^2*arccos(c*x
)*c-(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*arcco
s(c*x)/d^2/x/(c^2*x^2-1)+(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^
2-1)/d^2*ln((c*x+I*(-c^2*x^2+1)^(1/2))^4-1)*c)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(b\arccos\left(cx\right)+a\right)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^2}{x^2(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
-a*b*c*(log(c*x + 1)/d^(3/2) + log(c*x - 1)/d^(3/2) + 2*log(x)/d^(3/2)) +
2*(2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x^2 + d)*d*x))*a*b*ar
ccos(c*x) + (2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x^2 + d)*d*
x))*a^2 - b^2*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2/((c^2
*d*x^4 - d*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x^2\,(d-c^2\,d\,x^2)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^2*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x^2*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^{3/2}}\,dx = \frac{-2\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^4-\sqrt{-c^2x^2+1}\,x^2}dx\right)abx-\sqrt{-c^2x^2+1}\left(\int \frac{}{\sqrt{-c^2x^2+}}\right.}{\sqrt{d}\,\sqrt{-c^2x^2+1}\,dx}$$

input
```
int((a+b*acos(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 2*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x**
*4 - sqrt( - c**2*x**2 + 1)*x**2),x)*a*b*x - sqrt( - c**2*x**2 + 1)*int(ac
os(c*x)**2/(sqrt( - c**2*x**2 + 1)*c**2*x**4 - sqrt( - c**2*x**2 + 1)*x**2
),x)*b**2*x + 2*a**2*c**2*x**2 - a**2)/(sqrt(d)*sqrt( - c**2*x**2 + 1)*d*x
)
```

3.254 $\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 634

$$
\begin{aligned}
&\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{bc\sqrt{1-c^2x^2}(a+b\arccos(cx))}{dx\sqrt{d-c^2dx^2}} \\
&+\frac{3c^2(a+b\arccos(cx))^2}{2d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}} \\
&+\frac{4ibc^2\sqrt{1-c^2x^2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}} \\
&-\frac{3c^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}} \\
&-\frac{b^2c^2\sqrt{1-c^2x^2}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d\sqrt{d-c^2dx^2}} \\
&+\frac{3ibc^2\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}} \\
&-\frac{2ib^2c^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}} \\
&+\frac{2ib^2c^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}} \\
&-\frac{3ibc^2\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}} \\
&-\frac{3b^2c^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}} \\
&+\frac{3b^2c^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}}
\end{aligned}
$$

output
```
-b*c*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/d/x/(-c^2*d*x^2+d)^(1/2)+3/2*c^2
*(a+b*arccos(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)-1/2*(a+b*arccos(c*x))^2/d/x^2/
(-c^2*d*x^2+d)^(1/2)+4*I*b*c^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*arctan
(c*x+I*(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-3*c^2*(-c^2*x^2+1)^(1/2)
*(a+b*arccos(c*x))^2*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1
/2)-b^2*c^2*(-c^2*x^2+1)^(1/2)*arctanh((-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d
)^(1/2)+3*I*b*c^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*polylog(2,-c*x-I*(-
c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-2*I*b^2*c^2*(-c^2*x^2+1)^(1/2)*po
lylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d/(-c^2*d*x^2+d)^(1/2)+2*I*b^2*c^2*
(-c^2*x^2+1)^(1/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d/(-c^2*d*x^2+d
)^(1/2)-3*I*b*c^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*polylog(2,c*x+I*(-c
^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)-3*b^2*c^2*(-c^2*x^2+1)^(1/2)*polyl
og(3,-c*x-I*(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)+3*b^2*c^2*(-c^2*x^2
+1)^(1/2)*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))/d/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 9.97 (sec) , antiderivative size = 1163, normalized size of antiderivative = 1.83

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
Sqrt[-(d*(-1 + c^2*x^2))]*(-1/2*a^2/(d^2*x^2) - (a^2*c^2)/(d^2*(-1 + c^2*x
^2))) + (3*a^2*c^2*Log[x])/(2*d^(3/2)) - (3*a^2*c^2*Log[d + Sqrt[d]*Sqrt[-
(d*(-1 + c^2*x^2))]])/(2*d^(3/2)) - (a*b*c^2*Sqrt[1 - c^2*x^2]*(-2 - 2*Arc
Cos[c*x]*Cot[ArcCos[c*x]/2] + 6*ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] -
 6*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] - 4*Log[Cos[ArcCos[c*x]/2]] +
4*Log[Sin[ArcCos[c*x]/2]] + (6*I)*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - (6*
I)*PolyLog[2, I*E^(I*ArcCos[c*x])] + ArcCos[c*x]/(Cos[ArcCos[c*x]/2] - Sin
[ArcCos[c*x]/2])^2 - (2*Sin[ArcCos[c*x]/2])/(Cos[ArcCos[c*x]/2] - Sin[ArcC
os[c*x]/2]) - ArcCos[c*x]/(Cos[ArcCos[c*x]/2] + Sin[ArcCos[c*x]/2])^2 + (2
*Sin[ArcCos[c*x]/2])/(Cos[ArcCos[c*x]/2] + Sin[ArcCos[c*x]/2]) - 2*ArcCos[
c*x]*Tan[ArcCos[c*x]/2]))/(2*d*Sqrt[d*(1 - c^2*x^2)]) - (b^2*c^2*Sqrt[1 -
c^2*x^2]*(-4*ArcCos[c*x] - 2*ArcCos[c*x]^2*Cot[ArcCos[c*x]/2] + 8*ArcCos[c
*x]*Log[1 - E^(I*ArcCos[c*x])] + 6*ArcCos[c*x]^2*Log[1 - I*E^(I*ArcCos[c*x
])] + 6*Pi*ArcCos[c*x]*Log[((-1)^(1/4)*(1 - I*E^(I*ArcCos[c*x])))/(2*E^((I
/2)*ArcCos[c*x]))] - 6*ArcCos[c*x]^2*Log[1 + I*E^(I*ArcCos[c*x])] - 6*ArcC
os[c*x]^2*Log[((1/2 + I/2)*(-I + E^(I*ArcCos[c*x])))/E^((I/2)*ArcCos[c*x])
] + 6*Pi*ArcCos[c*x]*Log[-1/2*((-1)^(1/4)*(-I + E^(I*ArcCos[c*x])))/E^((I/
2)*ArcCos[c*x])] - 8*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] + 6*ArcCos[c*x
]^2*Log[((1 + I) + (1 - I)*E^(I*ArcCos[c*x]))/(2*E^((I/2)*ArcCos[c*x]))] -
 6*Pi*ArcCos[c*x]*Log[-Cos[(Pi + 2*ArcCos[c*x])/4]] - 4*Log[Cos[ArcCos[...
```

Rubi [A] (verified)

Time = 4.35 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.72, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.759$, Rules used = {5205, 5205, 243, 73, 221, 5165, 3042, 4671, 2715, 2838, 5209, 5165, 3042, 4671, 2715, 2838, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{5205}$$

$$\frac{3}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5205}$$

$$\frac{3}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{bc\sqrt{1-c^2x^2}\left(c^2\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx-bc\int \frac{1}{x\sqrt{1-c^2x^2}}dx-\frac{a+b\arccos(cx)}{x}\right)}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\big\downarrow$ 243

$$\frac{3}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{bc\sqrt{1-c^2x^2}\left(c^2\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx-\frac{1}{2}bc\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{a+b\arccos(cx)}{x}\right)}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\big\downarrow$ 73

$$\frac{3}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{bc\sqrt{1-c^2x^2}\left(c^2\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{b\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c}-\frac{a+b\arccos(cx)}{x}\right)}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\big\downarrow$ 221

$$-\frac{bc\sqrt{1-c^2x^2}\left(c^2\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx-\frac{a+b\arccos(cx)}{x}+b\text{carctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{d\sqrt{d-c^2dx^2}}+\frac{3}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\big\downarrow$ 5165

$$-\frac{bc\sqrt{1-c^2x^2}\left(-c\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)-\frac{a+b\arccos(cx)}{x}+b\text{carctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{d\sqrt{d-c^2dx^2}}+\frac{3}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\big\downarrow$ 3042

$$-\frac{bc\sqrt{1-c^2x^2}\left(-c\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)-\frac{a+b\arccos(cx)}{x}+b\text{carctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{d\sqrt{d-c^2dx^2}}+\frac{3}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\big\downarrow$ 4671

$$-\frac{bc\sqrt{1-c^2x^2}\left(-c\left(-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\,\text{a}}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{3}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{bc\sqrt{1-c^2x^2}\left(-c\left(ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\,\text{ar}}\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{3}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{3}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{3/2}}dx-$$

$$\frac{bc\sqrt{1-c^2x^2}\left(-c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\,\text{arcc}}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 5209}$$

$$\frac{3}{2}c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}\right)-$$

$$\frac{bc\sqrt{1-c^2x^2}\left(-c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\,\text{arcc}}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 5165}$$

$$\frac{3}{2}c^2\left(-\frac{2b\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}\right)-$$

$$\frac{bc\sqrt{1-c^2x^2}\left(-c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\,\text{arcc}}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{2}c^2\left(\frac{\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{2b\sqrt{1-c^2x^2}\int(a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}\right)$$

$$\frac{bc\sqrt{1-c^2x^2}\left(-c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos\ldots}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4671}$$

$$\frac{3}{2}c^2\left(-\frac{2b\sqrt{1-c^2x^2}\left(-b\int\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{\ldots}\right.\right.}{d\sqrt{d-c^2dx^2}}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(-c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos\ldots}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{3}{2}c^2\left(-\frac{2b\sqrt{1-c^2x^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\ldots}\right.}{d\sqrt{d-c^2dx^2}}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(-c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos\ldots}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{3}{2}c^2\left(\frac{\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx\ldots}\right.\right.}{d\sqrt{d-c^2dx^2}}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(-c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos\ldots}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5219}$$

$$\frac{3}{2}c^2\left(-\frac{\sqrt{1-c^2x^2}\int\frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\text{P}\right.}{d\sqrt{d-c^2d}}\right.$$
$$\frac{bc\sqrt{1-c^2x^2}\left(-c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\text{arc}}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 3042

$$\frac{3}{2}c^2\left(-\frac{\sqrt{1-c^2x^2}\int(a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)\right.}{}\right.$$
$$\frac{bc\sqrt{1-c^2x^2}\left(-c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\text{arc}}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 4669

$$\frac{3}{2}c^2\left(-\frac{\sqrt{1-c^2x^2}\left(-2b\int(a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+2b\int(a+b\arccos(cx))\log\left(1+ie\right.\right.}{d\sqrt{d-c^2dx^2}}\right.$$
$$\frac{bc\sqrt{1-c^2x^2}\left(-c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\text{arc}}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 3011

$$\frac{3}{2}c^2\left(-\frac{\sqrt{1-c^2x^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\arccos(c\right.\right.}{}\right.$$
$$\frac{bc\sqrt{1-c^2x^2}\left(-c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\text{arc}}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2720

$$\frac{3}{2}c^2\left(-\frac{\sqrt{1-c^2x^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)\right.\right.}{}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(-c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 7143$$

$$\frac{3}{2}c^2\left(-\frac{\sqrt{1-c^2x^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx\right.\right.}{d}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(-c\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-1/2*(a + b*ArcCos[c*x])^2/(d*x^2*Sqrt[d - c^2*d*x^2]) - (b*c*Sqrt[1 - c^2
*x^2]*(-((a + b*ArcCos[c*x])/x) + b*c*ArcTanh[Sqrt[1 - c^2*x^2]] - c*(-2*(
a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(I*ArcCo
s[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])))/(d*Sqrt[d - c^2*d*x^2]) +
(3*c^2*((a + b*ArcCos[c*x])^2/(d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*
x^2]*(-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -
E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])]))/(d*Sqrt[d - c^2*d
*x^2]) - (Sqrt[1 - c^2*x^2]*((-2*I)*(a + b*ArcCos[c*x])^2*ArcTan[E^(I*ArcC
os[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*ArcCos[c*x])]
- b*PolyLog[3, (-I)*E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyL
og[2, I*E^(I*ArcCos[c*x])] - b*PolyLog[3, I*E^(I*ArcCos[c*x])])))/(d*Sqrt[
d - c^2*d*x^2])))/2
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.21

method	result
default	$a^2\left(-\frac{1}{2d\,x^2\sqrt{-c^2d\,x^2+d}}+\frac{3c^2\left(\frac{1}{d\sqrt{-c^2d\,x^2+d}}-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}}\right)}{2}\right)+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\ \arccos(cx)\left(3c^2x^2\right.}{2d^2(}\right.$
parts	$a^2\left(-\frac{1}{2d\,x^2\sqrt{-c^2d\,x^2+d}}+\frac{3c^2\left(\frac{1}{d\sqrt{-c^2d\,x^2+d}}-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}}\right)}{2}\right)+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\ \arccos(cx)\left(3c^2x^2\right.}{2d^2(}\right.$

input
```
int((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/d/x^2/(-c^2*d*x^2+d)^(1/2)+3/2*c^2*(1/d/(-c^2*d*x^2+d)^(1/2)-1/d
^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)))+b^2*(-1/2*(-d*(c^2*x^2
-1))^(1/2)/d^2/(c^2*x^2-1)/x^2*arccos(c*x)*(3*c^2*x^2*arccos(c*x)+2*c*x*(-
c^2*x^2+1)^(1/2)-arccos(c*x))+1/2*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1
/2)*(-3*I*arccos(c*x)^2*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+3*I*arccos(c*x)
^2*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+4*I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2
+1)^(1/2))-6*arccos(c*x)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))-6*I*polyl
og(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))+6*arccos(c*x)*polylog(2,-I*(c*x+I*(-c^2
*x^2+1)^(1/2)))+6*I*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+4*dilog(c*x+I
*(-c^2*x^2+1)^(1/2))+4*dilog(1+c*x+I*(-c^2*x^2+1)^(1/2))-4*arctan(c*x+I*(-
c^2*x^2+1)^(1/2)))*c^2/(c^2*x^2-1)/d^2)+2*a*b*(-1/2*(-d*(c^2*x^2-1))^(1/2)
*(3*c^2*x^2*arccos(c*x)+c*x*(-c^2*x^2+1)^(1/2)-arccos(c*x))/d^2/(c^2*x^2-1
)/x^2+1/2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/(c^2*x^2-1)/d^2*(3*arc
cos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-3*arccos(c*x)*ln(1+I*(c*x+I*(-
c^2*x^2+1)^(1/2)))+2*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)-2*ln(1+c*x+I*(-c^2*x^2
+1)^(1/2))+3*I*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-3*I*dilog(1-I*(c*x+I*
(-c^2*x^2+1)^(1/2))))*c^2)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(a+b\operatorname{acos}\left(cx\right)\right)^2}{x^3\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x**3/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(b\arccos\left(cx\right)+a\right)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
-1/2*(3*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2
) - 3*c^2/(sqrt(-c^2*d*x^2 + d)*d) + 1/(sqrt(-c^2*d*x^2 + d)*d*x^2))*a^2 +
 sqrt(d)*integrate((b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a
*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1
)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{x^3\,\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^3*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x^3*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx=\frac{-16\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^5-\sqrt{-c^2x^2+1}\,x^3}dx\right)ab\,x^2-8\sqrt{-c^2x^2+1}\left(\int \frac{}{\sqrt{-c}}\right.}{}$$

input
```
int((a+b*acos(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 16*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x
**5 - sqrt( - c**2*x**2 + 1)*x**3),x)*a*b*x**2 - 8*sqrt( - c**2*x**2 + 1)*
int(acos(c*x)**2/(sqrt( - c**2*x**2 + 1)*c**2*x**5 - sqrt( - c**2*x**2 + 1
)*x**3),x)*b**2*x**2 + 12*sqrt( - c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a**
2*c**2*x**2 - 9*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 + 12*a**2*c**2*x**2
- 4*a**2)/(8*sqrt(d)*sqrt( - c**2*x**2 + 1)*d*x**2)
```

3.255 $\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 483

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx &= -\frac{b^2c^2(1-c^2x^2)}{3dx\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3dx^2\sqrt{d-c^2dx^2}}\\
&\quad-\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}-\frac{4c^2(a+b\arccos(cx))^2}{3dx\sqrt{d-c^2dx^2}}\\
&\quad+\frac{8c^4x(a+b\arccos(cx))^2}{3d\sqrt{d-c^2dx^2}}-\frac{8ic^3\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3d\sqrt{d-c^2dx^2}}\\
&\quad-\frac{20bc^3\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)}{3d\sqrt{d-c^2dx^2}}\\
&\quad+\frac{16bc^3\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{3d\sqrt{d-c^2dx^2}}\\
&\quad-\frac{ib^2c^3\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{d\sqrt{d-c^2dx^2}}\\
&\quad-\frac{5ib^2c^3\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{3d\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
-1/3*b^2*c^2*(-c^2*x^2+1)/d/x/(-c^2*d*x^2+d)^(1/2)-1/3*b*c*(-c^2*x^2+1)^(1
/2)*(a+b*arccos(c*x))/d/x^2/(-c^2*d*x^2+d)^(1/2)-1/3*(a+b*arccos(c*x))^2/d
/x^3/(-c^2*d*x^2+d)^(1/2)-4/3*c^2*(a+b*arccos(c*x))^2/d/x/(-c^2*d*x^2+d)^(
1/2)+8/3*c^4*x*(a+b*arccos(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)-8/3*I*c^3*(-c^2*
x^2+1)^(1/2)*(a+b*arccos(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)-20/3*b*c^3*(-c^2*x
^2+1)^(1/2)*(a+b*arccos(c*x))*arctanh((c*x+I*(-c^2*x^2+1)^(1/2))^2)/d/(-c^
2*d*x^2+d)^(1/2)+16/3*b*c^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*ln(1+(c*x
+I*(-c^2*x^2+1)^(1/2))^2)/d/(-c^2*d*x^2+d)^(1/2)-I*b^2*c^3*(-c^2*x^2+1)^(1
/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d/(-c^2*d*x^2+d)^(1/2)-5/3*I*
b^2*c^3*(-c^2*x^2+1)^(1/2)*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d/(-c^2
*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\arccos(cx))^2}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx = \frac{-a^2-4a^2c^2x^2-b^2c^2x^2+8a^2c^4x^4+b^2c^4x^4+abcx\sqrt{1-c^2x^2}-2ab\arccos(cx)$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
(-a^2 - 4*a^2*c^2*x^2 - b^2*c^2*x^2 + 8*a^2*c^4*x^4 + b^2*c^4*x^4 + a*b*c*
x*Sqrt[1 - c^2*x^2] - 2*a*b*ArcCos[c*x] - 8*a*b*c^2*x^2*ArcCos[c*x] + 16*a
*b*c^4*x^4*ArcCos[c*x] + b^2*c*x*Sqrt[1 - c^2*x^2]*ArcCos[c*x] - b^2*ArcCo
s[c*x]^2 - 4*b^2*c^2*x^2*ArcCos[c*x]^2 + 8*b^2*c^4*x^4*ArcCos[c*x]^2 + (8*
I)*b^2*c^3*x^3*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2 - 6*b^2*c^3*x^3*Sqrt[1 - c^
2*x^2]*ArcCos[c*x]*Log[1 - E^((2*I)*ArcCos[c*x])] - 10*b^2*c^3*x^3*Sqrt[1
- c^2*x^2]*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] - 10*a*b*c^3*x^3*Sqr
t[1 - c^2*x^2]*Log[c*x] - 3*a*b*c^3*x^3*Sqrt[1 - c^2*x^2]*Log[1 - c^2*x^2]
 + (5*I)*b^2*c^3*x^3*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^((2*I)*ArcCos[c*x])]
+ (3*I)*b^2*c^3*x^3*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcCos[c*x])])/(
3*d*x^3*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 3.25 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.96, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.621, Rules used = {5205, 5205, 242, 5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838, 5185, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{5205}$$

$$\frac{4}{3}c^2\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^{3/2}}dx-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x^3(1-c^2x^2)}dx}{3d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5205}$$

$$\frac{4}{3}c^2\left(2c^2\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}dx-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}\right)-\frac{2bc\sqrt{1-c^2x^2}\left(c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx-\frac{1}{2}bc\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx-\frac{a+b\arccos(cx)}{2x^2}\right)}{3d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{242}$$

$$\frac{4}{3}c^2\left(2c^2\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}dx-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}\right)-\frac{2bc\sqrt{1-c^2x^2}\left(c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx-\frac{a+b\arccos(cx)}{2x^2}+\frac{bc\sqrt{1-c^2x^2}}{2x}\right)}{3d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5161}$$

$$\frac{4}{3}c^2\left(2c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}\right)-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{dx\sqrt{d-c^2dx^2}}\right)-\frac{2bc\sqrt{1-c^2x^2}\left(c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx-\frac{a+b\arccos(cx)}{2x^2}+\frac{bc\sqrt{1-c^2x^2}}{2x}\right)}{3d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5181}$$

$$\frac{4}{3}c^2\left(2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int\frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}\right)-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)}d\right.$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx-\frac{a+b\arccos(cx)}{2x^2}+\frac{bc\sqrt{1-c^2x^2}}{2x}\right)}{3d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 3042

$$-\frac{2bc\sqrt{1-c^2x^2}\left(c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx-\frac{a+b\arccos(cx)}{2x^2}+\frac{bc\sqrt{1-c^2x^2}}{2x}\right)}{3d\sqrt{d-c^2dx^2}}+$$
$$\frac{4}{3}c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}+2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int-((a+b\arccos(cx))\tan(a}{cd\sqrt{d-c^2dx^2}}\right.\right.$$
$$\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 25

$$-\frac{2bc\sqrt{1-c^2x^2}\left(c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx-\frac{a+b\arccos(cx)}{2x^2}+\frac{bc\sqrt{1-c^2x^2}}{2x}\right)}{3d\sqrt{d-c^2dx^2}}+$$
$$\frac{4}{3}c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}+2c^2\left(\frac{2b\sqrt{1-c^2x^2}\int(a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx}{cd\sqrt{d-c^2dx^2}}\right.\right.$$
$$\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 4200

$$\frac{4}{3}c^2\left(2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(2i\int-\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)\right.$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx-\frac{a+b\arccos(cx)}{2x^2}+\frac{bc\sqrt{1-c^2x^2}}{2x}\right)}{3d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 25

$$\frac{4}{3}c^2\left(2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\int\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)\right.$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx-\frac{a+b\arccos(cx)}{2x^2}+\frac{bc\sqrt{1-c^2x^2}}{2x}\right)}{3d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 2620

$$-\frac{2bc\sqrt{1-c^2x^2}\left(c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx-\frac{a+b\arccos(cx)}{2x^2}+\frac{bc\sqrt{1-c^2x^2}}{2x}\right)}{3d\sqrt{d-c^2dx^2}}+$$
$$\frac{4}{3}c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}+2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right.\right.\right.}{}\right.\right.$$
$$\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 2715

$$-\frac{2bc\sqrt{1-c^2x^2}\left(c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx-\frac{a+b\arccos(cx)}{2x^2}+\frac{bc\sqrt{1-c^2x^2}}{2x}\right)}{3d\sqrt{d-c^2dx^2}}+$$
$$\frac{4}{3}c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}+2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right.\right.\right.}{}\right.\right.$$
$$\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 2838

$$\frac{4}{3}c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx}{d\sqrt{d-c^2dx^2}}+2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right.\right.\right.}{}\right.\right.$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx-\frac{a+b\arccos(cx)}{2x^2}+\frac{bc\sqrt{1-c^2x^2}}{2x}\right)}{3d\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 5185

$$\frac{4}{3}c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{cx\sqrt{1-c^2x^2}}d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i a}\right.\right.\right.}{}\right.\right.$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(c^2\left(-\int\frac{a+b\arccos(cx)}{cx\sqrt{1-c^2x^2}}d\arccos(cx)\right)-\frac{a+b\arccos(cx)}{2x^2}+\frac{bc\sqrt{1-c^2x^2}}{2x}\right)}{3d\sqrt{d-c^2dx^2}}-$$
$$\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$\Big\downarrow$ 4919

$$\frac{4}{3}c^2\left(\frac{4bc\sqrt{1-c^2x^2}\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x}}{}\right.\right.$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(-2c^2\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)-\frac{a+b\arccos(cx)}{2x^2}+\frac{bc\sqrt{1-c^2x^2}}{2x}\right)}{3d\sqrt{d-c^2dx^2}}-$$
$$\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}c^2\left(\frac{4bc\sqrt{1-c^2x^2}\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+2c^2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x}}{}\right.\right.$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(-2c^2\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)-\frac{a+b\arccos(cx)}{2x^2}+\frac{bc\sqrt{1-c^2x^2}}{2x}\right)}{3d\sqrt{d-c^2dx^2}}-$$
$$\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{4671}$$

$$\frac{4}{3}c^2\left(\frac{4bc\sqrt{1-c^2x^2}\left(-\frac{1}{2}b\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+\frac{1}{2}b\int\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-(\text{arctan}\right.}{d\sqrt{d-c^2dx^2}}\right.$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(-2c^2\left(-\frac{1}{2}b\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+\frac{1}{2}b\int\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-(\text{arctan}\right.\right.}{3d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{4}{3}c^2\left(\frac{4bc\sqrt{1-c^2x^2}\left(\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(c}\right.\right.}{d\sqrt{d-c^2dx^2}}\right.$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(-2c^2\left(\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(}\right.\right.\right.}{3d\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2bc\sqrt{1-c^2x^2}\left(-2c^2\left(-\left(\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}ib\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)-\frac{1}{4}ib\operatorname{PolyL}\right.}{3d\sqrt{d-c^2dx^2}}$$

$$\frac{4}{3}c^2\left(\frac{4bc\sqrt{1-c^2x^2}\left(-\left(\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}ib\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)-\frac{1}{4}ib\operatorname{PolyLog}\right.}{d\sqrt{d-c^2dx^2}}\right.$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

output
```
-1/3*(a + b*ArcCos[c*x])^2/(d*x^3*Sqrt[d - c^2*d*x^2]) - (2*b*c*Sqrt[1 - c
^2*x^2]*((b*c*Sqrt[1 - c^2*x^2])/(2*x) - (a + b*ArcCos[c*x])/(2*x^2) - 2*c
^2*(-((a + b*ArcCos[c*x])*ArcTanh[E^((2*I)*ArcCos[c*x])]) + (I/4)*b*PolyLo
g[2, -E^((2*I)*ArcCos[c*x])] - (I/4)*b*PolyLog[2, E^((2*I)*ArcCos[c*x])]))
)/(3*d*Sqrt[d - c^2*d*x^2]) + (4*c^2*(-((a + b*ArcCos[c*x])^2/(d*x*Sqrt[d
 - c^2*d*x^2])) + (4*b*c*Sqrt[1 - c^2*x^2]*(-((a + b*ArcCos[c*x])*ArcTanh[E
^((2*I)*ArcCos[c*x])]) + (I/4)*b*PolyLog[2, -E^((2*I)*ArcCos[c*x])] - (I/4
)*b*PolyLog[2, E^((2*I)*ArcCos[c*x])]))/(d*Sqrt[d - c^2*d*x^2]) + 2*c^2*((
x*(a + b*ArcCos[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*
(((-1/2*I)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])*Log
[1 - E^((2*I)*ArcCos[c*x])] + (b*PolyLog[2, E^((2*I)*ArcCos[c*x])])/4))/(
c*d*Sqrt[d - c^2*d*x^2]))))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2849 vs. $2(474) = 948$.

Time = 0.79 (sec) , antiderivative size = 2850, normalized size of antiderivative = 5.90

method	result	size
default	Expression too large to display	2850
parts	Expression too large to display	2850

input
```
int((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-32*I*a*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^5*c^8+64/3*
I*a*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^7*c^10-128/3*a*
b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^3*arccos(c*x)*c^6+8
/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x*c^4+16*a*b*(
-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x*arccos(c*x)*c^4+8*a*b*
(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2/x*arccos(c*x)*c^2-1/3*a
*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2/x^2*(-c^2*x^2+1)^(1/
2)*c+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^2/(c^2*x^2-1)*ln((c
*x+I*(-c^2*x^2+1)^(1/2))^2-1)*c^3+10/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^
2+1)^(1/2)/d^2/(c^2*x^2-1)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*c^3+8*I*a*b*
(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^3*c^6+8*I*b^2*(-d*(c^
2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^3*arccos(c*x)*c^6+8/3*I*b^2*
(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x^2*(-c^2*x^2+1)^(1/2)*
c^5+8/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*x*arccos(
c*x)*c^4+8/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^4*x^4-7*c^2*x^2-1)/d^2*arcc
os(c*x)^2*(-c^2*x^2+1)^(1/2)*c^3-16/3*I*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^
2-1))^(1/2)/d^2/(c^2*x^2-1)*c^3*arccos(c*x)^2-5/3*I*b^2*(-c^2*x^2+1)^(1/2)
*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^2*x^2-1)*c^3*polylog(2,-(c*x+I*(-c^2*x^2+1)
^(1/2))^2)-2*I*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^2*x^2-
1)*c^3*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))-8/3*b^2*(-d*(c^2*x^2-1))^(1/...
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
)/(c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^2}{x^4(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x**4/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(x**4*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
1/3*(8*c^4*x/(sqrt(-c^2*d*x^2 + d)*d) - 4*c^2/(sqrt(-c^2*d*x^2 + d)*d*x) -
 1/(sqrt(-c^2*d*x^2 + d)*d*x^3))*a^2 + sqrt(d)*integrate((b^2*arctan2(sqrt
(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2
*x^4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}{x^4\,{\left(d-c^2\,d\,x^2\right)}^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^4*(d - c^2*d*x^2)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x^4*(d - c^2*d*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx=\frac{-6\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^6-\sqrt{-c^2x^2+1}\,x^4}dx\right)ab\,x^3-3\sqrt{-c^2x^2+1}\left(\int \frac{}{\sqrt{-c^2}}\right.}{3\sqrt{d}\,\sqrt{-c^2x^2+1}\,d\,x^3}$$

input
```
int((a+b*acos(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - 6*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**2*x*
*6 - sqrt( - c**2*x**2 + 1)*x**4),x)*a*b*x**3 - 3*sqrt( - c**2*x**2 + 1)*i
nt(acos(c*x)**2/(sqrt( - c**2*x**2 + 1)*c**2*x**6 - sqrt( - c**2*x**2 + 1)
*x**4),x)*b**2*x**3 + 8*a**2*c**4*x**4 - 4*a**2*c**2*x**2 - a**2)/(3*sqrt(
d)*sqrt( - c**2*x**2 + 1)*d*x**3)
```

3.256 $\int \frac{x^5(a+b\arccos(cx))^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 546

$$
\begin{aligned}
\int \frac{x^5(a+b\arccos(cx))^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx &= \frac{b^2}{3c^6d^2\sqrt{d-c^2dx^2}}+\frac{16abx\sqrt{1-c^2x^2}}{3c^5d^2\sqrt{d-c^2dx^2}} \\
&+\frac{2b^2(1-c^2x^2)}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{16b^2x\sqrt{1-c^2x^2}\arccos(cx)}{3c^5d^2\sqrt{d-c^2dx^2}}-\frac{bx^3(a+b\arccos(cx))}{3c^3d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} \\
&-\frac{11bx\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^5d^2\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}} \\
&-\frac{4x^2(a+b\arccos(cx))^2}{3c^4d^2\sqrt{d-c^2dx^2}}-\frac{8\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{3c^6d^3} \\
&-\frac{22ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{3c^6d^2\sqrt{d-c^2dx^2}} \\
&+\frac{11ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{3c^6d^2\sqrt{d-c^2dx^2}} \\
&-\frac{11ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{3c^6d^2\sqrt{d-c^2dx^2}}
\end{aligned}
$$

output
```
1/3*b^2/c^6/d^2/(-c^2*d*x^2+d)^(1/2)+16/3*a*b*x*(-c^2*x^2+1)^(1/2)/c^5/d^2
/(-c^2*d*x^2+d)^(1/2)+2*b^2*(-c^2*x^2+1)/c^6/d^2/(-c^2*d*x^2+d)^(1/2)+16/3
*b^2*x*(-c^2*x^2+1)^(1/2)*arccos(c*x)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*x
^3*(a+b*arccos(c*x))/c^3/d^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-11/3*
b*x*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*
x^4*(a+b*arccos(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-4/3*x^2*(a+b*arccos(c*x
))^2/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-8/3*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x
))^2/c^6/d^3-22/3*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*arctan(c*x+I*(-
c^2*x^2+1)^(1/2))/c^6/d^2/(-c^2*d*x^2+d)^(1/2)+11/3*I*b^2*(-c^2*x^2+1)^(1/
2)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^6/d^2/(-c^2*d*x^2+d)^(1/2)-1
1/3*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^6/d
^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 532, normalized size of antiderivative = 0.97

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx =$$

$$-\frac{\sqrt{d-c^2dx^2}\Big(64a^2-22b^2-96a^2c^2x^2+24a^2c^4x^4+50ab\arccos(cx)+25b^2\arccos(cx)^2+28b^2\cos(2\arccos$$

input
```
Integrate[(x^5*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
-1/24*(Sqrt[d - c^2*d*x^2]*(64*a^2 - 22*b^2 - 96*a^2*c^2*x^2 + 24*a^2*c^4*
x^4 + 50*a*b*ArcCos[c*x] + 25*b^2*ArcCos[c*x]^2 + 28*b^2*Cos[2*ArcCos[c*x]
] - 72*a*b*ArcCos[c*x]*Cos[2*ArcCos[c*x]] - 36*b^2*ArcCos[c*x]^2*Cos[2*Arc
Cos[c*x]] - 6*b^2*Cos[4*ArcCos[c*x]] + 6*a*b*ArcCos[c*x]*Cos[4*ArcCos[c*x]
] + 3*b^2*ArcCos[c*x]^2*Cos[4*ArcCos[c*x]] - 66*b^2*Sqrt[1 - c^2*x^2]*ArcC
os[c*x]*Log[1 - E^(I*ArcCos[c*x])] + 66*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*
Log[1 + E^(I*ArcCos[c*x])] + 66*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/
2]] - 66*a*b*Sqrt[1 - c^2*x^2]*Log[Sin[ArcCos[c*x]/2]] - (88*I)*b^2*(1 - c
^2*x^2)^(3/2)*PolyLog[2, -E^(I*ArcCos[c*x])] + (88*I)*b^2*(1 - c^2*x^2)^(3
/2)*PolyLog[2, E^(I*ArcCos[c*x])] + 8*a*b*Sin[2*ArcCos[c*x]] + 8*b^2*ArcCo
s[c*x]*Sin[2*ArcCos[c*x]] + 22*b^2*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])]*
Sin[3*ArcCos[c*x]] - 22*b^2*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])]*Sin[3*A
rcCos[c*x]] - 22*a*b*Log[Cos[ArcCos[c*x]/2]]*Sin[3*ArcCos[c*x]] + 22*a*b*L
og[Sin[ArcCos[c*x]/2]]*Sin[3*ArcCos[c*x]] - 6*a*b*Sin[4*ArcCos[c*x]] - 6*b
^2*ArcCos[c*x]*Sin[4*ArcCos[c*x]]))/(c^6*d^3*(-1 + c^2*x^2)^2)
```

Rubi [A] (verified)

Time = 2.45 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.483, Rules used = {5207, 5207, 243, 53, 2009, 5183, 2009, 5211, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5207}$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{x^4(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{4\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}dx}{3c^2d}+\frac{x^4(a+b\arccos(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{5207}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{2c^2}+\frac{b\int \frac{x^3}{\left(1-c^2x^2\right)^{3/2}}dx}{2c}+\frac{x^3(a+b\arccos(cx))}{2c^2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{4\left(\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}-\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^4(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{2c^2}+\frac{b\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}}dx^2}{4c}+\frac{x^3(a+b\arccos(cx))}{2c^2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{4\left(\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}-\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^4(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{2c^2}+\frac{b\int \left(\frac{1}{c^2\left(1-c^2x^2\right)^{3/2}}-\frac{1}{c^2\sqrt{1-c^2x^2}}\right)dx^2}{4c}+\frac{x^3(a+b\arccos(cx))}{2c^2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{4\left(\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}-\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^4(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4\left(\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}-\frac{2\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{2c^2}+\frac{x^3(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{4c}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 5183

$$-\frac{4\left(-\frac{2\left(-\frac{2b\sqrt{1-c^2x^2}\int(a+b\arccos(cx))dx}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}\right)}{c^2d}+\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{2c^2}+\frac{x^3(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{4c}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 2009

$$-\frac{4\left(\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x^2}\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c\sqrt{d-c^2dx^2}}\right)}{c^2d}\right)}{3c^2d}+\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3\int \frac{x^2(a+b\arccos(cx))}{1-c^2x^2}dx}{2c^2}+\frac{x^3(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{4c}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{x^4(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 5211

$$-\frac{4\left(\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c^2}-\frac{b\int \frac{x}{\sqrt{1-c^2x^2}}dx}{c}-\frac{x(a+b\arccos(cx))}{c^2}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1}}{}\right.}{c^2}\right.}{3c^2d}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c^2}-\frac{b\int \frac{x}{\sqrt{1-c^2x^2}}dx}{c}-\frac{x(a+b\arccos(cx))}{c^2}\right)}{2c^2}+\frac{x^3(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{4c}\right)}{3cd^2\sqrt{d-c^2dx^2}}+$$

$$\frac{x^4(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$-\frac{4\left(\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\frac{2b\sqrt{1-c^2x}}{}\right.}{c^2d}\right.}{3c^2d}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3\left(\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}\right)}{2c^2}+\frac{x^3(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{4c}\right)}{3cd^2\sqrt{d-c^2dx^2}}+$$

$$\frac{x^4(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{5165}$$

$$-\frac{4\left(\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{c^2d}-\right.}{}\right.}{3c^2d}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3\left(-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}\right)}{2c^2}+\frac{x^3(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{1-c^2x^2}}{c^4}+\frac{2}{c^4\sqrt{1-c^2x^2}}\right)}{4c}\right)}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{x^4(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{4\left(\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2\left(-\frac{\sqrt{d-c^2dx^2}(a+b}{c^2d}\right.}{}\right.}{3c^2d}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3\left(-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-c^2x^2}}{c^3}\right)}{2c^2}+\frac{x^3(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{2\sqrt{1-c^2x^2}}{c^4}+\right.}{4c}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{x^4(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 4671

$$-\frac{4\left(\frac{2b\sqrt{1-c^2x^2}\left(-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{c^3}-\frac{x(a+b\arccos}{c^2}\right.}{cd\sqrt{d-c^2dx^2}}\right.}{3c^2d}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3\left(-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{c^3}-\frac{x(a+b}{}\right.}{2c^2}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{x^4(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 2715

$$-\frac{4\left(\frac{2b\sqrt{1-c^2x^2}\left(-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos(c}\right.}{c^3}\right.}{cd\sqrt{d-c^2dx^2}}\right.}{\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3\left(-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos}\right.}{c^3}\right.}{2c^2}\right.}{3cd^2\sqrt{d-c^2dx^2}}}$$

$$\frac{x^4(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 2838

$$-\frac{4\left(\frac{2b\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b\sqrt{1-}}{c^3}\right.}{cd\sqrt{d-c^2dx^2}}\right.}{3c^2d}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{c^3}-\frac{x(a+b\arccos(cx))}{c^2}+\frac{b}{}\right.}{2c^2}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{x^4(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

input

```
Int[(x^5*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x^4*(a + b*ArcCos[c*x])^2)/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (2*b*Sqrt[1
- c^2*x^2]*((b*(2/(c^4*Sqrt[1 - c^2*x^2]) + (2*Sqrt[1 - c^2*x^2])/c^4))/(4
*c) + (x^3*(a + b*ArcCos[c*x]))/(2*c^2*(1 - c^2*x^2)) - (3*((b*Sqrt[1 - c^
2*x^2])/c^3 - (x*(a + b*ArcCos[c*x]))/c^2 - (-2*(a + b*ArcCos[c*x])*ArcTan
h[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(I*ArcCos[c*x])] - I*b*PolyLog[2,
 E^(I*ArcCos[c*x])])/c^3))/(2*c^2)))/(3*c*d^2*Sqrt[d - c^2*d*x^2]) - (4*((
x^2*(a + b*ArcCos[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) - (2*(-((Sqrt[d - c
^2*d*x^2]*(a + b*ArcCos[c*x])^2)/(c^2*d)) - (2*b*Sqrt[1 - c^2*x^2]*(a*x -
(b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]))/(c*Sqrt[d - c^2*d*x^2])))/(c^2
*d) + (2*b*Sqrt[1 - c^2*x^2]*((b*Sqrt[1 - c^2*x^2])/c^3 - (x*(a + b*ArcCos
[c*x]))/c^2 - (-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*Pol
yLog[2, -E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/c^3))/(c*
d*Sqrt[d - c^2*d*x^2])))/(3*c^2*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 803, normalized size of antiderivative = 1.47

method	result
default	$a^2\left(-\frac{x^4}{c^2d(-c^2dx^2+d)^{\frac{3}{2}}}+\frac{\frac{4x^2}{c^2d\left(-c^2dx^2+d\right)^{\frac{3}{2}}}-\frac{8}{3dc^4\left(-c^2dx^2+d\right)^{\frac{3}{2}}}}{c^2}\right)+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\left(\right.}{2d^3c^6(c^2x^2-1}\right.$
parts	$a^2\left(-\frac{x^4}{c^2d(-c^2dx^2+d)^{\frac{3}{2}}}+\frac{\frac{4x^2}{c^2d\left(-c^2dx^2+d\right)^{\frac{3}{2}}}-\frac{8}{3dc^4\left(-c^2dx^2+d\right)^{\frac{3}{2}}}}{c^2}\right)+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\left(\right.}{2d^3c^6(c^2x^2-1}\right.$

input
```
int(x^5*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-x^4/c^2/d/(-c^2*d*x^2+d)^(3/2)+4/c^2*(x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)
-2/3/d/c^4/(-c^2*d*x^2+d)^(3/2)))+b^2*(-1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^
2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)^2-2+2*I*arccos(c*x))/d^3/c^6/(c
^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)
*(arccos(c*x)^2-2-2*I*arccos(c*x))/d^3/c^6/(c^2*x^2-1)+1/3*(-d*(c^2*x^2-1)
)^(1/2)*(6*arccos(c*x)^2*x^2*c^2+(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c-c^2*x^
2-5*arccos(c*x)^2+1)/(c^2*x^2-1)^2/d^3/c^6+11/3*I*(-c^2*x^2+1)^(1/2)*(-d*(
c^2*x^2-1))^(1/2)*(I*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))-I*arccos(c
*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+po
lylog(2,c*x+I*(-c^2*x^2+1)^(1/2)))/d^3/c^6/(c^2*x^2-1))+2*a*b*(-1/2*(-d*(c
^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)+I)/d^3/
c^6/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*
x^2-1)*(arccos(c*x)-I)/d^3/c^6/(c^2*x^2-1)+1/6*(-d*(c^2*x^2-1))^(1/2)*(12*
c^2*x^2*arccos(c*x)+c*x*(-c^2*x^2+1)^(1/2)-10*arccos(c*x))/(c^2*x^2-1)^2/d
^3/c^6-11/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^6/(c^2*x^2-1)*
ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)+11/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1
/2)/d^3/c^6/(c^2*x^2-1)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2x^5}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral(-(b^2*x^5*arccos(c*x)^2 + 2*a*b*x^5*arccos(c*x) + a^2*x^5)*sqrt(-
c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^5(a+b\operatorname{acos}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**5*(a + b*acos(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2x^5}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
-1/3*a^2*(3*x^4/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 12*x^2/((-c^2*d*x^2 + d)^
(3/2)*c^4*d) + 8/((-c^2*d*x^2 + d)^(3/2)*c^6*d)) - 1/3*((3*b^2*c^4*x^4 - 1
2*b^2*c^2*x^2 + 8*b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(d)*arctan2(sqrt(c
*x + 1)*sqrt(-c*x + 1), c*x)^2 + 3*(c^10*d^3*x^4 - 2*c^8*d^3*x^2 + c^6*d^3
)*integrate(2/3*(3*sqrt(c*x + 1)*sqrt(-c*x + 1)*a*b*c^5*sqrt(d)*x^5*arctan
2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + (3*b^2*c^6*x^6 - 15*b^2*c^4*x^4 + 2
0*b^2*c^2*x^2 - 8*b^2)*sqrt(d)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
/(c^11*d^3*x^6 - 3*c^9*d^3*x^4 + 3*c^7*d^3*x^2 - c^5*d^3), x))/(c^10*d^3*x
^4 - 2*c^8*d^3*x^2 + c^6*d^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^5\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

input
```
int((x^5*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^5*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^5(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)x^5}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)ab\,c^8x^2-6\sqrt{-c^2x}}{}$$

input
```
int(x^5*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(6*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**5)/(sqrt( - c**2*x**2 + 1)*c**
4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*a
*b*c**8*x**2 - 6*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**5)/(sqrt( - c**2
*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x
**2 + 1)),x)*a*b*c**6 + 3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)**2*x**5)/(
sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sq
rt( - c**2*x**2 + 1)),x)*b**2*c**8*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((ac
os(c*x)**2*x**5)/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 +
 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b**2*c**6 + 3*a**2*c**4*x**4 -
12*a**2*c**2*x**2 + 8*a**2)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*c**6*d**2*(c
**2*x**2 - 1))
```

3.257 $\int \frac{x^4(a+b\arccos(cx))^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 421

$$\begin{aligned}\int \frac{x^4(a+b\arccos(cx))^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx &= \frac{b^2x}{3c^4d^2\sqrt{d-c^2dx^2}}-\frac{b^2\sqrt{1-c^2x^2}\arccos(cx)}{3c^5d^2\sqrt{d-c^2dx^2}}\\ &-\frac{bx^2(a+b\arccos(cx))}{3c^3d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}-\frac{x(a+b\arccos(cx))^2}{c^4d^2\sqrt{d-c^2dx^2}}\\ &+\frac{4i\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^5d^2\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^5d^2\sqrt{d-c^2dx^2}}\\ &-\frac{8b\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{3c^5d^2\sqrt{d-c^2dx^2}}\\ &+\frac{4ib^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{3c^5d^2\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
1/3*b^2*x/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b^2*(-c^2*x^2+1)^(1/2)*arccos(c
*x)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*x^2*(a+b*arccos(c*x))/c^3/d^2/(-c^2
*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x^3*(a+b*arccos(c*x))^2/c^2/d/(-c^2
*d*x^2+d)^(3/2)-x*(a+b*arccos(c*x))^2/c^4/d^2/(-c^2*d*x^2+d)^(1/2)+4/3*I*(
-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*(-c
^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^3/b/c^5/d^2/(-c^2*d*x^2+d)^(1/2)-8/3*b*(
-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c^5
/d^2/(-c^2*d*x^2+d)^(1/2)+4/3*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-(c*x+I*(
-c^2*x^2+1)^(1/2))^2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.83

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{a^2c\sqrt{d}x(-3+4c^2x^2)+3a^2(-1+c^2x^2)\sqrt{d-c^2dx^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+}{}$$

input
```
Integrate[(x^4*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(a^2*c*Sqrt[d]*x*(-3 + 4*c^2*x^2) + 3*a^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^
2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + a*b*Sqrt[d
]*(Sqrt[1 - c^2*x^2] + 2*c*x*ArcCos[c*x] + 8*c*x*(-1 + c^2*x^2)*ArcCos[c*x
] + (1 - c^2*x^2)^(3/2)*(-3*ArcCos[c*x]^2 + 4*Log[1 - c^2*x^2])) + b^2*Sqr
t[d]*(c*x*(1 - c^2*x^2 + (-3 + 4*c^2*x^2)*ArcCos[c*x]^2) + Sqrt[1 - c^2*x^
2]*ArcCos[c*x]*(1 + (-1 + c^2*x^2)*ArcCos[c*x]*(4*I + ArcCos[c*x]) - 8*(-1
 + c^2*x^2)*Log[1 - E^((2*I)*ArcCos[c*x])]) - (4*I)*(1 - c^2*x^2)^(3/2)*Po
lyLog[2, E^((2*I)*ArcCos[c*x])]))/(3*c^5*d^(5/2)*(1 - c^2*x^2)*Sqrt[d - c^
2*d*x^2])
```

Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5207, 5207, 252, 223, 5153, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 5207}$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{x^3(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}dx}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{ 5207}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}+\frac{b\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}}dx}{2c}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{\frac{2b\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}-\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{c^2}\right)}{2c}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{\frac{2b\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}-\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int\frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2c}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{\frac{2b\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}-\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{c^2d}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 5153

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int\frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2c}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{\frac{2b\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 5181

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int\frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^4}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2c}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{-\frac{2b\sqrt{1-c^2x^2}\int\frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 3042

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int-\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{c^4}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2c}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{-\frac{2b\sqrt{1-c^2x^2}\int-\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 25

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c^4}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2c}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{\frac{2b\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 4200$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2c}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{-\frac{2b\sqrt{1-c^2x^2}\left(2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2c}\right)}{3cd^2\sqrt{d-c^2dx^2}}-\frac{-\frac{2b\sqrt{1-c^2x^2}\left(-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3d\sqrt{d-c^2dx^2}}}{c^2d}+\frac{x^3(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 2620$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4}+\frac{x^2(a+b\arccos(}{2c^2(1-c^2x^2}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c^3d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx}{c^2d\sqrt{d-c^2dx^2}}}{c^2d}$$

$$\frac{x^3(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 2715$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4}+\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c^3d\sqrt{d-c^2dx^2}}+}{c^2d}$$

$$\frac{x^3(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 2838$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b(}{}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{x^3(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}-$$

$$\frac{\frac{x(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c^3d\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-}}{}}{c^2d}$$

input
```
Int[(x^4*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcCos[c*x])^2)/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (2*b*Sqrt[1
- c^2*x^2]*((x^2*(a + b*ArcCos[c*x]))/(2*c^2*(1 - c^2*x^2)) + (b*(x/(c^2*S
qrt[1 - c^2*x^2]) - ArcSin[c*x]/c^3))/(2*c) + (((-1/2*I)*(a + b*ArcCos[c*x
])^2)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])]
+ (b*PolyLog[2, E^((2*I)*ArcCos[c*x])])/4))/c^4))/(3*c*d^2*Sqrt[d - c^2*d*
x^2]) - ((x*(a + b*ArcCos[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[1 -
 c^2*x^2]*(a + b*ArcCos[c*x])^3)/(3*b*c^3*d*Sqrt[d - c^2*d*x^2]) - (2*b*Sq
rt[1 - c^2*x^2]*(((-1/2*I)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((I/2)*(a + b*
ArcCos[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])] + (b*PolyLog[2, E^((2*I)*ArcCo
s[c*x])])/4)))/(c^3*d*Sqrt[d - c^2*d*x^2]))/(c^2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4046 vs. 2(395) = 790.

Time = 0.80 (sec) , antiderivative size = 4047, normalized size of antiderivative = 9.61

method	result	size
default	Expression too large to display	4047
parts	Expression too large to display	4047

input
```
int(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arcc
os(c*x)^3-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(24*c^8*x^8-87*c^6*x^6+118*c^
4*x^4-71*c^2*x^2+16)/c^2*(-c^2*x^2+1)*x^3+128/3*I*a*b*(-d*(c^2*x^2-1))^(1/
2)/d^3/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5*arccos(c*x)*(
-c^2*x^2+1)^(1/2)+28/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(24*c^8*x^8-87*c^6
*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2*(-c^2*x^2+1)*x^3+16/3*I*a*b*(-c^2*x^2+
1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/c^5/(c^2*x^2-1)*arccos(c*x)-4*I*a*b*(-
d*(c^2*x^2-1))^(1/2)/d^3/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)
/c^4*(-c^2*x^2+1)*x+8*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(24*c^8*x^8-87*c^6*x^
6+118*c^4*x^4-71*c^2*x^2+16)/c*(-c^2*x^2+1)^(1/2)*x^4+64*a*b*(-d*(c^2*x^2-
1))^(1/2)/d^3/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c^2*arccos
(c*x)*x^7-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^5/(c^2*x
^2-1)*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)-13*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3
/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^3*x^2*(-c^2*x^2+1)^(1
/2)+a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arcc
os(c*x)^2-32*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(24*c^8*x^8-87*c^6*x^6+118*c^4
*x^4-71*c^2*x^2+16)/c^4*arccos(c*x)*x-40/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/d^
3/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2*x^3-16/3*I*a*b*(-d
*(c^2*x^2-1))^(1/2)/d^3/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*
c^2*x^7-16/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(24*c^8*x^8-87*c^6*x^6+11...
```

Fricas [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2x^4}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral(-(b^2*x^4*arccos(c*x)^2 + 2*a*b*x^4*arccos(c*x) + a^2*x^4)*sqrt(-
c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^4(a+b\operatorname{acos}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**4*(a + b*acos(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2x^4}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
1/3*(x*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c
^4*d)) - x/(sqrt(-c^2*d*x^2 + d)*c^4*d^2) + 3*arcsin(c*x)/(c^5*d^(5/2)))*a
^2 - sqrt(d)*integrate((b^2*x^4*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)
^2 + 2*a*b*x^4*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*s
qrt(-c*x + 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Giac [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2x^4}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b*arccos(c*x) + a)^2*x^4/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^4\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((x^4*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{3\sqrt{-c^2x^2+1}\,asin(cx)\,a^2c^2x^2-3\sqrt{-c^2x^2+1}\,asin(cx)\,a^2+6\sqrt{-c^2x^2+1}\,\Big(\int}{}$$

input
```
int(x^4*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(3*sqrt( - c**2*x**2 + 1)*asin(c*x)*a**2*c**2*x**2 - 3*sqrt( - c**2*x**2 +
 1)*asin(c*x)*a**2 + 6*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**4)/(sqrt(
- c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( -
c**2*x**2 + 1)),x)*a*b*c**7*x**2 - 6*sqrt( - c**2*x**2 + 1)*int((acos(c*x)
*x**4)/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x
**2 + sqrt( - c**2*x**2 + 1)),x)*a*b*c**5 + 3*sqrt( - c**2*x**2 + 1)*int((
acos(c*x)**2*x**4)/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2
 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b**2*c**7*x**2 - 3*sqrt( - c*
*2*x**2 + 1)*int((acos(c*x)**2*x**4)/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2
*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b**2*c**5 -
 4*a**2*c**3*x**3 + 3*a**2*c*x)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*c**5*d**
2*(c**2*x**2 - 1))
```

3.258 $\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 332

$$\begin{aligned}\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx &= \frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}}\\
&-\frac{bx(a+b\arccos(cx))}{3c^3d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\arccos(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}\\
&-\frac{2(a+b\arccos(cx))^2}{3c^4d^2\sqrt{d-c^2dx^2}}-\frac{10ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{3c^4d^2\sqrt{d-c^2dx^2}}\\
&+\frac{5ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{3c^4d^2\sqrt{d-c^2dx^2}}\\
&-\frac{5ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{3c^4d^2\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
1/3*b^2/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*x*(a+b*arccos(c*x))/c^3/d^2/(-c
^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x^2*(a+b*arccos(c*x))^2/c^2/d/(-c
^2*d*x^2+d)^(3/2)-2/3*(a+b*arccos(c*x))^2/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-10/
3*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2)
)/c^4/d^2/(-c^2*d*x^2+d)^(1/2)+5/3*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*(
c*x+I*(-c^2*x^2+1)^(1/2)))/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-5/3*I*b^2*(-c^2*x^
2+1)^(1/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^4/d^2/(-c^2*d*x^2+d)^
(1/2)
```

Mathematica [A] (verified)

Time = 2.47 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.89

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{4a^2(-2+3c^2x^2)+ib^2\left(20(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-20(1-c\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(4*a^2*(-2 + 3*c^2*x^2) + I*b^2*(20*(1 - c^2*x^2)^(3/2)*PolyLog[2, -E^(I*A
rcCos[c*x])] - 20*(1 - c^2*x^2)^(3/2)*PolyLog[2, E^(I*ArcCos[c*x])] - I*(4
 - 4*c^2*x^2 + ArcCos[c*x]^2*(-2 + 6*Cos[2*ArcCos[c*x]]) + ArcCos[c*x]*(2*
Sin[2*ArcCos[c*x]] + 5*(Log[1 - E^(I*ArcCos[c*x])] - Log[1 + E^(I*ArcCos[c
*x])])*(3*Sqrt[1 - c^2*x^2] - Sin[3*ArcCos[c*x]])))) - a*b*(ArcCos[c*x]*(4
 - 12*Cos[2*ArcCos[c*x]]) - 2*Sin[2*ArcCos[c*x]] + 5*(Log[Cos[ArcCos[c*x]/
2]] - Log[Sin[ArcCos[c*x]/2]])*(3*Sqrt[1 - c^2*x^2] - Sin[3*ArcCos[c*x]]))
)/(12*c^4*d*(d - c^2*d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {5207, 5183, 5165, 3042, 4671, 2715, 2838, 5207, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 5207}$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{2\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}dx}{3c^2d}+\frac{x^2(a+b\arccos(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{ 5183}$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{2\left(\frac{2b\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{cd\sqrt{d-c^2dx^2}}+\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^2(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\Big\downarrow$ 5165

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{2\left(\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^2(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}-\frac{2\left(\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}+\frac{x^2(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\Big\downarrow$ 4671

$$-\frac{2\left(\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(\cdots}{c^2d\sqrt{d-c^2dx^2}}\right.}{3c^2d}$$
$$\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\Big\downarrow$ 2715

$$-\frac{2\left(\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\cdots}{c^2d\sqrt{d-c^2dx^2}}\right.}{3c^2d}$$
$$\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}+\frac{x^2(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\Big\downarrow$ 2838

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}} - \frac{2\left(\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right)}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}$$
$$\frac{x^2(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 5207$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{2c^2}+\frac{b\int \frac{x}{\left(1-c^2x^2\right)^{3/2}}dx}{2c}+\frac{x(a+b\arccos(cx))}{2c^2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}} - \frac{2\left(\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right)}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}$$
$$\frac{x^2(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 241$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{2c^2}+\frac{x(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b}{2c^3\sqrt{1-c^2x^2}}\right)}{3cd^2\sqrt{d-c^2dx^2}} - \frac{2\left(\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right)}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}$$
$$\frac{x^2(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 5165$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c^3}+\frac{x(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b}{2c^3\sqrt{1-c^2x^2}}\right)}{3cd^2\sqrt{d-c^2dx^2}} - \frac{2\left(\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right)}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}$$
$$\frac{x^2(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{\int(a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2c^3}+\frac{x(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b}{2c^3\sqrt{1-c^2x^2}}\right)}{3cd^2\sqrt{d-c^2dx^2}}-$$

$$\frac{2\left(\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right)}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}$$

$$\frac{x^2(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 4671

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{-b\int\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{2c^3}+\frac{x(a}{2}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{2\left(\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right)}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}$$

$$\frac{x^2(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 2715

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos(c}\right.}{2c^3}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{2\left(\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right)}{c^2d\sqrt{d-c^2dx^2}}\right)}{3c^2d}$$

$$\frac{x^2(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 2838

$$-\frac{2\left(\frac{(a+b\arccos(cx))^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right)}{c^2d\sqrt{d-c^2dx^2}}\right.}{3c^2d}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c^3}+\frac{x(a+b\arccos(cx))}{2c^2(1-c^2x^2)}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\frac{x^2(a+b\arccos(cx))^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}$$

input
```
Int[(x^3*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x^2*(a + b*ArcCos[c*x])^2)/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (2*b*Sqrt[1
- c^2*x^2]*(b/(2*c^3*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcCos[c*x]))/(2*c^2*(
1 - c^2*x^2)) + (-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*P
olyLog[2, -E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/(2*c^3)
))/(3*c*d^2*Sqrt[d - c^2*d*x^2]) - (2*((a + b*ArcCos[c*x])^2/(c^2*d*Sqrt[d
 - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*(-2*(a + b*ArcCos[c*x])*ArcTanh[E^
(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(
I*ArcCos[c*x])]))/(c^2*d*Sqrt[d - c^2*d*x^2])))/(3*c^2*d)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.48

method	result
default	$a^2\left(\frac{x^2}{c^2d(-c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{2}{3d\,c^4(-c^2d\,x^2+d)^{\frac{3}{2}}}\right)+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,\left(3\arccos(cx)^2x^2c^2+\sqrt{-c^2x^2+1}\,\arccos(cx)xc-c^2x^2-\right.}{3(c^2x^2-1)^2d^3c^4}\right.$
parts	$a^2\left(\frac{x^2}{c^2d(-c^2d\,x^2+d)^{\frac{3}{2}}}-\frac{2}{3d\,c^4(-c^2d\,x^2+d)^{\frac{3}{2}}}\right)+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\,\left(3\arccos(cx)^2x^2c^2+\sqrt{-c^2x^2+1}\,\arccos(cx)xc-c^2x^2-\right.}{3(c^2x^2-1)^2d^3c^4}\right.$

input
```
int(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/3/d/c^4/(-c^2*d*x^2+d)^(3/2))+b^2*(1
/3*(-d*(c^2*x^2-1))^(1/2)*(3*arccos(c*x)^2*x^2*c^2+(-c^2*x^2+1)^(1/2)*arcc
os(c*x)*x*c-c^2*x^2-2*arccos(c*x)^2+1)/(c^2*x^2-1)^2/d^3/c^4+5/3*I*(-c^2*x
^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(I*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^
(1/2))-I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-polylog(2,-c*x-I*(-c^2
*x^2+1)^(1/2))+polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)))/d^3/(c^2*x^2-1)/c^4)+2
*a*b*(1/6*(-d*(c^2*x^2-1))^(1/2)*(6*c^2*x^2*arccos(c*x)+c*x*(-c^2*x^2+1)^(
1/2)-4*arccos(c*x))/(c^2*x^2-1)^2/d^3/c^4-5/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2
*x^2+1)^(1/2)/d^3/(c^2*x^2-1)/c^4*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)+5/6*(-d*(
c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/(c^2*x^2-1)/c^4*ln(1+c*x+I*(-c^2*
x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2x^3}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral(-(b^2*x^3*arccos(c*x)^2 + 2*a*b*x^3*arccos(c*x) + a^2*x^3)*sqrt(-
c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^3(a+b\operatorname{acos}{(cx)})^2}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**3*(a + b*acos(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x^3}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
-1/6*a*b*c*(2*x/(c^6*d^(5/2)*x^2 - c^4*d^(5/2)) + 5*log(c*x + 1)/(c^5*d^(5
/2)) - 5*log(c*x - 1)/(c^5*d^(5/2))) + 2/3*a*b*(3*x^2/((-c^2*d*x^2 + d)^(3
/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))*arccos(c*x) + 1/3*a^2*(3*x^
2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d)) + b^2
*integrate(x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2/((c^4*d^2*x^4
- 2*c^2*d^2*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^3\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)x^3}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)ab\,c^6x^2-6\sqrt{-c^2x}$$

input
```
int(x^3*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(6*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**3)/(sqrt( - c**2*x**2 + 1)*c**
4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*a
*b*c**6*x**2 - 6*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**3)/(sqrt( - c**2
*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x
**2 + 1)),x)*a*b*c**4 + 3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)**2*x**3)/(
sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sq
rt( - c**2*x**2 + 1)),x)*b**2*c**6*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((ac
os(c*x)**2*x**3)/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 +
 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b**2*c**4 - 3*a**2*c**2*x**2 +
2*a**2)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*c**4*d**2*(c**2*x**2 - 1))
```

3.259 $\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 332

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{b^2x}{3c^2d^2\sqrt{d-c^2dx^2}} - \frac{b^2\sqrt{1-c^2x^2}\arccos(cx)}{3c^3d^2\sqrt{d-c^2dx^2}} - \frac{bx^2(a+b\arccos(cx))}{3cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{x^3(a+b\arccos(cx))^2}{3d(d-c^2dx^2)^{3/2}} + \frac{i\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^3d^2\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{ib^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{3c^3d^2\sqrt{d-c^2dx^2}}$$

output
```
1/3*b^2*x/c^2/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b^2*(-c^2*x^2+1)^(1/2)*arccos(c
*x)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*x^2*(a+b*arccos(c*x))/c/d^2/(-c^2*x
^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x^3*(a+b*arccos(c*x))^2/d/(-c^2*d*x^2
+d)^(3/2)+1/3*I*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/c^3/d^2/(-c^2*d*x^2
+d)^(1/2)-2/3*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2
+1)^(1/2))^2)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*I*b^2*(-c^2*x^2+1)^(1/2)*po
lylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.92

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{-b^2cx-a^2c^3x^3+b^2c^3x^3-ab\sqrt{1-c^2x^2}-ib^2\left(-ic^3x^3-\sqrt{1-c^2x^2}+c^2x^2\sqrt{\quad}\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(-(b^2*c*x) - a^2*c^3*x^3 + b^2*c^3*x^3 - a*b*Sqrt[1 - c^2*x^2] - I*b^2*((
-I)*c^3*x^3 - Sqrt[1 - c^2*x^2] + c^2*x^2*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2
 - b*ArcCos[c*x]*(2*a*c^3*x^3 + b*Sqrt[1 - c^2*x^2] + 2*b*(1 - c^2*x^2)^(3
/2)*Log[1 - E^((2*I)*ArcCos[c*x])]) - a*b*Sqrt[1 - c^2*x^2]*Log[1 - c^2*x^
2] + a*b*c^2*x^2*Sqrt[1 - c^2*x^2]*Log[1 - c^2*x^2] + I*b^2*(1 - c^2*x^2)^
(3/2)*PolyLog[2, E^((2*I)*ArcCos[c*x])])/(3*c^3*d^2*(-1 + c^2*x^2)*Sqrt[d
- c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.65, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {5187, 5207, 252, 223, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5187}$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x^3(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{5207}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}+\frac{b\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}}dx}{2c}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 252$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{c^2}\right)}{2c}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 223$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c^2}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2c}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 5181$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(\frac{\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^4}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2c}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(\frac{\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{c^4}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2c}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c^4}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2c}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x^3(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 4200

$$\frac{x^3(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{2bc\sqrt{1-c^2x^2}\left(\frac{2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2c}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 25

$$\frac{x^3(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{2bc\sqrt{1-c^2x^2}\left(\frac{-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\frac{b\left(\frac{x}{c^2\sqrt{1-c^2x^2}}-\frac{\arcsin(cx)}{c^3}\right)}{2c}\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2620

$$\frac{x^3(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{2bc\sqrt{1-c^2x^2}\left(\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int \log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4}+\frac{x^2(a+b\arccos}{2c^2(1-c^2x}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2715

$$\frac{x^3(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{2bc\sqrt{1-c^2x^2}\left(\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4}+\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$\downarrow$ 2838

$$\frac{x^3(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}+$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(\frac{-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}}{c^4}+\frac{x^2(a+b\arccos(cx))}{2c^2(1-c^2x^2)}+\right.}{3d^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcCos[c*x])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*b*c*Sqrt[1 -
c^2*x^2]*((x^2*(a + b*ArcCos[c*x]))/(2*c^2*(1 - c^2*x^2)) + (b*(x/(c^2*Sqr
t[1 - c^2*x^2]) - ArcSin[c*x]/c^3))/(2*c) + (((-1/2*I)*(a + b*ArcCos[c*x])
^2)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])] +
(b*PolyLog[2, E^((2*I)*ArcCos[c*x])])/4))/c^4))/(3*d^2*Sqrt[d - c^2*d*x^2]
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3438 vs. 2(314) = 628.

Time = 0.76 (sec) , antiderivative size = 3439, normalized size of antiderivative = 10.36

method	result	size
default	Expression too large to display	3439
parts	Expression too large to display	3439

input
```
int(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*
x^2+1)*x^3-4/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^
4*x^4-5*c^2*x^2+1)/c*(-c^2*x^2+1)^(1/2)*arccos(c*x)^2*x^2-I*b^2*(-d*(c^2*x
^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^3*(-c^2*x^
2+1)^(1/2)*arccos(c*x)^2*x^6-8/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x
^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^2+
4*I*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x
^2+1)*c*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^4-2*I*a*b*(-d*(c^2*x^2-1))^(1/2)/
d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^3*(-c^2*x^2+1)^(1/2)*ar
ccos(c*x)*x^6+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c
^4*x^4-5*c^2*x^2+1)/c^3*(-c^2*x^2+1)^(1/2)-1/3*I*b^2*(-d*(c^2*x^2-1))^(1/2
)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*arccos(c*x)*x^3-1/3*I*b
^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)
/c^3*(-c^2*x^2+1)^(1/2)+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^
6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2*(-c^2*x^2+1)*x^5+1/3*b^2*(-d*(c^2*x^2-1)
)^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^2*(-c^2*x^2+1)*
x+b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2
+1)*c^4*arccos(c*x)^2*x^7-b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^8*x^8-9*c^6*
x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2*arccos(c*x)^2*x^5+1/3*b^2*(-d*(c^2*x^2-1))
^(1/2)/d^3/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3*(-c^2*x^2+1...
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2x^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral(-(b^2*x^2*arccos(c*x)^2 + 2*a*b*x^2*arccos(c*x) + a^2*x^2)*sqrt(-
c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^2(a+b\operatorname{acos}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x**2*(a + b*acos(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2x^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
-1/3*a*b*c*(1/(c^6*d^(5/2)*x^2 - c^4*d^(5/2)) - log(c*x + 1)/(c^4*d^(5/2))
 - log(c*x - 1)/(c^4*d^(5/2))) - 2/3*a*b*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d^2)
 - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))*arccos(c*x) - 1/3*a^2*(x/(sqrt(-c^2*d
*x^2 + d)*c^2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d)) + b^2*integrate(x^2
*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2/((c^4*d^2*x^4 - 2*c^2*d^2*x^
2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2x^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((b*arccos(c*x) + a)^2*x^2/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^2\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((x^2*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)x^2}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)ab\,c^2x^2-6\sqrt{-c^2x}$$

input
```
int(x^2*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(6*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**2)/(sqrt( - c**2*x**2 + 1)*c**
4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*a
*b*c**2*x**2 - 6*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x**2)/(sqrt( - c**2
*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x
**2 + 1)),x)*a*b + 3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)**2*x**2)/(sqrt(
 - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( -
 c**2*x**2 + 1)),x)*b**2*c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((acos(c*
x)**2*x**2)/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c
**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b**2 - a**2*x**3)/(3*sqrt(d)*sqrt( -
 c**2*x**2 + 1)*d**2*(c**2*x**2 - 1))
```

3.260 $\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 294

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}}-\frac{bx(a+b\arccos(cx))}{3cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}+\frac{(a+b\arccos(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{2ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{3c^2d^2\sqrt{d-c^2dx^2}}-\frac{ib^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{3c^2d^2\sqrt{d-c^2dx^2}}+\frac{ib^2\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{3c^2d^2\sqrt{d-c^2dx^2}}$$

output
```
1/3*b^2/c^2/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*x*(a+b*arccos(c*x))/c/d^2/(-c^2
*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arccos(c*x))^2/c^2/d/(-c^2*d*x
^2+d)^(3/2)+2/3*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*arctan(c*x+I*(-c^
2*x^2+1)^(1/2))/c^2/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*I*b^2*(-c^2*x^2+1)^(1/2)*
polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^2/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*
I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^2/d^2/(
-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.37

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx =$$

$$-\frac{4a^2+2b^2+8ab\arccos(cx)+4b^2\arccos(cx)^2-2b^2\cos(2\arccos(cx))-3b^2\sqrt{1-c^2x^2}\arccos(cx)\log\left(1-\right.}{}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
-1/12*(4*a^2 + 2*b^2 + 8*a*b*ArcCos[c*x] + 4*b^2*ArcCos[c*x]^2 - 2*b^2*Cos
[2*ArcCos[c*x]] - 3*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 - E^(I*ArcCos[
c*x])] + 3*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] +
3*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/2]] - 3*a*b*Sqrt[1 - c^2*x^2]*
Log[Sin[ArcCos[c*x]/2]] - (4*I)*b^2*(1 - c^2*x^2)^(3/2)*PolyLog[2, -E^(I*A
rcCos[c*x])] + (4*I)*b^2*(1 - c^2*x^2)^(3/2)*PolyLog[2, E^(I*ArcCos[c*x])]
 + 2*a*b*Sin[2*ArcCos[c*x]] + 2*b^2*ArcCos[c*x]*Sin[2*ArcCos[c*x]] + b^2*A
rcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])]*Sin[3*ArcCos[c*x]] - b^2*ArcCos[c*x]
*Log[1 + E^(I*ArcCos[c*x])]*Sin[3*ArcCos[c*x]] - a*b*Log[Cos[ArcCos[c*x]/2
]]*Sin[3*ArcCos[c*x]] + a*b*Log[Sin[ArcCos[c*x]/2]]*Sin[3*ArcCos[c*x]])/(c
^2*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.64, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5183, 5163, 241, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 5183}$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}+\frac{(a+b\arccos(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{5163}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{x}{(1-c^2x^2)^{3/2}}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{(a+b\arccos(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{(a+b\arccos(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{5165}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{(a+b\arccos(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{(a+b\arccos(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{4671}$$

$$\frac{(a+b\arccos(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{2b\sqrt{1-c^2x^2}\left(-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{2c}+\frac{x}{\cdots}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{(a+b\arccos(cx))^2}{3c^2d\,(d-c^2dx^2)^{3/2}}+\frac{2b\sqrt{1-c^2x^2}\left(-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos\cdots}\right.}{2c}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{(a+b\arccos(cx))^2}{3c^2d(d-c^2dx^2)^{3/2}}+\frac{2b\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))+ib\,\text{PolyLog}(2,-e^{i\arccos(cx)})-ib\,\text{PolyLog}(2,e^{i\arccos(cx)})}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

input
```
Int[(x*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(a + b*ArcCos[c*x])^2/(3*c^2*d*(d - c^2*d*x^2)^(3/2)) + (2*b*Sqrt[1 - c^2*
x^2]*(b/(2*c*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcCos[c*x]))/(2*(1 - c^2*x^2)
) - (-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E
^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/(2*c)))/(3*c*d^2*Sq
rt[d - c^2*d*x^2])
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.55

method	result
default	$\frac{a^2}{3c^2d(-c^2dx^2+d)^{\frac{3}{2}}}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\left(\sqrt{-c^2x^2+1}\,\arccos(cx)xc-c^2x^2+\arccos(cx)^2+1\right)}{3d^3(c^4x^4-2c^2x^2+1)c^2}-\frac{i\sqrt{-c^2x^2+1}\,\sqrt{-d(c^2x^2-1)}\,(i}{}\right.$
parts	$\frac{a^2}{3c^2d(-c^2dx^2+d)^{\frac{3}{2}}}+b^2\left(\frac{\sqrt{-d(c^2x^2-1)}\left(\sqrt{-c^2x^2+1}\,\arccos(cx)xc-c^2x^2+\arccos(cx)^2+1\right)}{3d^3(c^4x^4-2c^2x^2+1)c^2}-\frac{i\sqrt{-c^2x^2+1}\,\sqrt{-d(c^2x^2-1)}\,(i}{}\right.$

input
```
int(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2/c^2/d/(-c^2*d*x^2+d)^(3/2)+b^2*(1/3*(-d*(c^2*x^2-1))^(1/2)*((-c^2*
x^2+1)^(1/2)*arccos(c*x)*x*c-c^2*x^2+arccos(c*x)^2+1)/d^3/(c^4*x^4-2*c^2*x
^2+1)/c^2-1/3*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(I*arccos(c*x)*l
n(1-c*x-I*(-c^2*x^2+1)^(1/2))-I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))
-polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))+polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)))
/d^3/(c^2*x^2-1)/c^2)+2*a*b*(1/6*(-d*(c^2*x^2-1))^(1/2)*(c*x*(-c^2*x^2+1)^
(1/2)+2*arccos(c*x))/d^3/(c^4*x^4-2*c^2*x^2+1)/c^2-1/6*(-d*(c^2*x^2-1))^(1
/2)*(-c^2*x^2+1)^(1/2)/d^3/(c^2*x^2-1)/c^2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+
1/6*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d^3/(c^2*x^2-1)/c^2*ln(I*(-c
^2*x^2+1)^(1/2)+c*x-1))
```

Fricas [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2x}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*x*arccos(c*x)^2 + 2*a*b*x*arccos(c*x)
+ a^2*x)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x(a+b\operatorname{acos}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral(x*(a + b*acos(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
-sqrt(d)*integrate((b^2*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2
*a*b*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x
 + 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x) + 1/3*a^2/((
-c^2*d*x^2 + d)^(3/2)*c^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

input
```
int((x*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)x}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)ab\,c^4x^2-6\sqrt{-c^2x^2}$$

input
```
int(x*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(6*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x)/(sqrt( - c**2*x**2 + 1)*c**4*x
**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*a*b*
c**4*x**2 - 6*sqrt( - c**2*x**2 + 1)*int((acos(c*x)*x)/(sqrt( - c**2*x**2
+ 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 +
1)),x)*a*b*c**2 + 3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)**2*x)/(sqrt( - c
**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**
2*x**2 + 1)),x)*b**2*c**4*x**2 - 3*sqrt( - c**2*x**2 + 1)*int((acos(c*x)**
2*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**
2 + sqrt( - c**2*x**2 + 1)),x)*b**2*c**2 - a**2)/(3*sqrt(d)*sqrt( - c**2*x
**2 + 1)*c**2*d**2*(c**2*x**2 - 1))
```

3.261 $\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 311

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} - \frac{b(a+b\arccos(cx))}{3cd^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{x(a+b\arccos(cx))^2}{3d\,(d-c^2dx^2)^{3/2}} + \frac{2x(a+b\arccos(cx))^2}{3d^2\sqrt{d-c^2dx^2}} - \frac{2i\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3cd^2\sqrt{d-c^2dx^2}} + \frac{4b\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{3cd^2\sqrt{d-c^2dx^2}} - \frac{2ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{3cd^2\sqrt{d-c^2dx^2}}$$

output
```
1/3*b^2*x/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*(a+b*arccos(c*x))/c/d^2/(-c^2*x^2
+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*x*(a+b*arccos(c*x))^2/d/(-c^2*d*x^2+d)^
(3/2)+2/3*x*(a+b*arccos(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)-2/3*I*(-c^2*x^2+1
)^(1/2)*(a+b*arccos(c*x))^2/c/d^2/(-c^2*d*x^2+d)^(1/2)+4/3*b*(-c^2*x^2+1)^
(1/2)*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/d^2/(-c^2*d*x
^2+d)^(1/2)-2/3*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1
/2))^2)/c/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{-3a^2cx-b^2cx+2a^2c^3x^3+b^2c^3x^3-ab\sqrt{1-c^2x^2}+b^2\left(-3cx+2c^3x^3-2i\sqrt{1\right.}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(-3*a^2*c*x - b^2*c*x + 2*a^2*c^3*x^3 + b^2*c^3*x^3 - a*b*Sqrt[1 - c^2*x^2
] + b^2*(-3*c*x + 2*c^3*x^3 - (2*I)*Sqrt[1 - c^2*x^2] + (2*I)*c^2*x^2*Sqrt
[1 - c^2*x^2])*ArcCos[c*x]^2 + b*ArcCos[c*x]*(-6*a*c*x + 4*a*c^3*x^3 - b*S
qrt[1 - c^2*x^2] + 4*b*(1 - c^2*x^2)^(3/2)*Log[1 - E^((2*I)*ArcCos[c*x])])
 + 2*a*b*Sqrt[1 - c^2*x^2]*Log[1 - c^2*x^2] - 2*a*b*c^2*x^2*Sqrt[1 - c^2*x
^2]*Log[1 - c^2*x^2] - (2*I)*b^2*(1 - c^2*x^2)^(3/2)*PolyLog[2, E^((2*I)*A
rcCos[c*x])])/(3*c*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.87, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5163, 5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838, 5183, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5163}$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{5161}$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 5181$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{2b\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 4200$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 2620$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int \log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+b\arccos(cx}{2b}}{cd\sqrt{d-c^2dx^2}}}{3d}\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 2715$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right)}{cd\sqrt{d-c^2dx^2}}}{3d}\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 2838$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \ 5183$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(\frac{b\int \frac{1}{\left(1-c^2x^2\right)^{3/2}}dx}{2c}+\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}+\frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d - c^2*d*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcCos[c*x])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*b*c*Sqrt[1 - c^
2*x^2]*((b*x)/(2*c*Sqrt[1 - c^2*x^2]) + (a + b*ArcCos[c*x])/(2*c^2*(1 - c^
2*x^2))))/(3*d^2*Sqrt[d - c^2*d*x^2]) + (2*((x*(a + b*ArcCos[c*x])^2)/(d*S
qrt[d - c^2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*(((-1/2*I)*(a + b*ArcCos[c*x]
)^2)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])] +
 (b*PolyLog[2, E^((2*I)*ArcCos[c*x])])/4)))/(c*d*Sqrt[d - c^2*d*x^2])))/(3
*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2226 vs. $2(295) = 590$.

Time = 0.62 (sec) , antiderivative size = 2227, normalized size of antiderivative = 7.16

method	result	size
default	Expression too large to display	2227
parts	Expression too large to display	2227

input
```
int((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4*
x^5+13/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4
)*c^2*x^3-4*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^
2-4)*arccos(c*x)^2*x+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*
x^4+11*c^2*x^2-4)*c^6*x^7+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10
*c^4*x^4+11*c^2*x^2-4)*(-c^2*x^2+1)*x+4/3*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*
x^2-1))^(1/2)/d^3/(c^2*x^2-1)/c*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))
+b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c*(-c^
2*x^2+1)^(1/2)*arccos(c*x)*x^2+16/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^
6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2*arccos(c*x)*x^3+4/3*I*b^2*(-d*(c^2*x^2-
1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^6*arccos(c*x)*x^7-14/3
*I*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4*
arccos(c*x)*x^5+8/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x^6-10*c^4*x^4
+11*c^2*x^2-4)/c*(-c^2*x^2+1)^(1/2)*arccos(c*x)^2+I*b^2*(-d*(c^2*x^2-1))^(
1/2)/d^3/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^3*(-c^2*x^2+1)^(1/2)*x^4+4/
3*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)/c*arccos(c
*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-4/3*I*b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x
^2-1))^(1/2)/d^3/(c^2*x^2-1)/c*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-4/3*I*
b^2*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)/c*polylog(2,
c*x+I*(-c^2*x^2+1)^(1/2))-7/3*I*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(3*c^6*x...
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a*b*c*(1/(c^4*d^(5/2)*x^2 - c^2*d^(5/2)) + 2*log(c*x + 1)/(c^2*d^(5/2
)) + 2*log(c*x - 1)/(c^2*d^(5/2))) + 2/3*a*b*(2*x/(sqrt(-c^2*d*x^2 + d)*d^
2) + x/((-c^2*d*x^2 + d)^(3/2)*d))*arccos(c*x) + 1/3*a^2*(2*x/(sqrt(-c^2*d
*x^2 + d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d)) + b^2*integrate(arctan2(sqr
t(c*x + 1)*sqrt(-c*x + 1), c*x)^2/((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqr
t(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*acos(c*x))^2/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)ab\,c^2x^2-6\sqrt{-c^2x^2}}{}$$

input
```
int((a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(6*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**4
- 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*a*b*c**2
*x**2 - 6*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**
4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*a
*b + 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)**2/(sqrt( - c**2*x**2 + 1)*c**
4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*b
**2*c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)**2/(sqrt( - c**2*x*
*2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2
 + 1)),x)*b**2 + 2*a**2*c**2*x**3 - 3*a**2*x)/(3*sqrt(d)*sqrt( - c**2*x**2
 + 1)*d**2*(c**2*x**2 - 1))
```

3.262 $\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 577

$$\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{5/2}}\,dx = \frac{b^2}{3d^2\sqrt{d-c^2dx^2}} - \frac{bcx(a+b\arccos(cx))}{3d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{(a+b\arccos(cx))^2}{3d(d-c^2dx^2)^{3/2}} + \frac{(a+b\arccos(cx))^2}{d^2\sqrt{d-c^2dx^2}} + \frac{14ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}} + \frac{2ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}} - \frac{7ib^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{7ib^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{2ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}} - \frac{2b^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}} + \frac{2b^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}$$

output
```
1/3*b^2/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*c*x*(a+b*arccos(c*x))/d^2/(-c^2*x^2
+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/3*(a+b*arccos(c*x))^2/d/(-c^2*d*x^2+d)^(3
/2)+(a+b*arccos(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+14/3*I*b*(-c^2*x^2+1)^(1/
2)*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(
1/2)-2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2*arctanh(c*x+I*(-c^2*x^2+1)^(
1/2))/d^2/(-c^2*d*x^2+d)^(1/2)+2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*
polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)-7/3*I*b^2*(-
c^2*x^2+1)^(1/2)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^2/(-c^2*d*x^2+
d)^(1/2)+7/3*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2
)))/d^2/(-c^2*d*x^2+d)^(1/2)-2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*po
lylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)-2*b^2*(-c^2*x^2
+1)^(1/2)*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)+2*
b^2*(-c^2*x^2+1)^(1/2)*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2
+d)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 10.19 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.36

$$\int \frac{(a+b\arccos(cx))^2}{x\,(d-c^2dx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
Sqrt[-(d*(-1 + c^2*x^2))]*(a^2/(3*d^3*(-1 + c^2*x^2)^2) - a^2/(d^3*(-1 + c
^2*x^2))) + (a^2*Log[c*x])/d^(5/2) - (a^2*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c
^2*x^2))]])/d^(5/2) - (a*b*(1 - c^2*x^2)^(3/2)*(-14*ArcCos[c*x]*Cot[ArcCos
[c*x]/2] - Csc[ArcCos[c*x]/2]^2 - (Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Csc[ArcCo
s[c*x]/2]^4)/2 + 24*ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] - 24*ArcCos[c
*x]*Log[1 + I*E^(I*ArcCos[c*x])] - 28*Log[Cos[ArcCos[c*x]/2]] + 28*Log[Sin
[ArcCos[c*x]/2]] + (24*I)*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - (24*I)*Poly
Log[2, I*E^(I*ArcCos[c*x])] + Sec[ArcCos[c*x]/2]^2 - (8*ArcCos[c*x]*Sin[Ar
cCos[c*x]/2]^4)/(1 - c^2*x^2)^(3/2) - 14*ArcCos[c*x]*Tan[ArcCos[c*x]/2]))/
(12*d*(d*(1 - c^2*x^2))^(3/2)) - (b^2*(1 - c^2*x^2)^(3/2)*(-4*Cot[ArcCos[c
*x]/2] - 14*ArcCos[c*x]^2*Cot[ArcCos[c*x]/2] - 2*ArcCos[c*x]*Csc[ArcCos[c*
x]/2]^2 - (Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2*Csc[ArcCos[c*x]/2]^4)/2 + 56*Ar
cCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] + 24*ArcCos[c*x]^2*Log[1 - I*E^(I*Arc
Cos[c*x])] - 24*ArcCos[c*x]^2*Log[1 + I*E^(I*ArcCos[c*x])] - 56*ArcCos[c*x
]*Log[1 + E^(I*ArcCos[c*x])] + (56*I)*PolyLog[2, -E^(I*ArcCos[c*x])] + (48
*I)*ArcCos[c*x]*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - (48*I)*ArcCos[c*x]*Po
lyLog[2, I*E^(I*ArcCos[c*x])] - (56*I)*PolyLog[2, E^(I*ArcCos[c*x])] - 48*
PolyLog[3, (-I)*E^(I*ArcCos[c*x])] + 48*PolyLog[3, I*E^(I*ArcCos[c*x])] +
2*ArcCos[c*x]*Sec[ArcCos[c*x]/2]^2 - (8*ArcCos[c*x]^2*Sin[ArcCos[c*x]/2]^4
)/(1 - c^2*x^2)^(3/2) - 4*Tan[ArcCos[c*x]/2] - 14*ArcCos[c*x]^2*Tan[Arc...
```

Rubi [A] (verified)

Time = 4.24 (sec) , antiderivative size = 473, normalized size of antiderivative = 0.82, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.690$, Rules used = {5209, 5163, 241, 5165, 3042, 4671, 2715, 2838, 5209, 5165, 3042, 4671, 2715, 2838, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{5209}$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{3d(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{5163}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(\frac{1}{2}\int\frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int\frac{x}{(1-c^2x^2)^{3/2}}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(\frac{1}{2}\int\frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{5165}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{\int(a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{4671}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-b\int\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{2c}+\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{\int\frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos(cx)}\right.}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{\int\frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\int\frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx}{2(1-c^2x^2)}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 5209}$$

$$\frac{\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}}{d}+$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx}{2(1-c^2x^2)}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 5165}$$

$$\frac{-\frac{2b\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}}{d}+$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx}{2(1-c^2x^2)}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{2b\sqrt{1-c^2x^2}\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}}{d}+$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))+ib\,\text{PolyLog}(2,-e^{i\arccos(cx)})-ib\,\text{PolyLog}(2,e^{i\arccos(cx)})}{2c}+\frac{x(a+b\arccos(cx}{2(1-c^2x^2)}\right.}{3d^2\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 4671

$$-\frac{\frac{2b\sqrt{1-c^2x^2}\left(-b\int \log(1-e^{i\arccos(cx)})d\arccos(cx)+b\int \log(1+e^{i\arccos(cx)})d\arccos(cx)-2\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))\right)}{d\sqrt{d-c^2dx^2}}+\int \frac{(a+}{}}{d}$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))+ib\,\text{PolyLog}(2,-e^{i\arccos(cx)})-ib\,\text{PolyLog}(2,e^{i\arccos(cx)})}{2c}+\frac{x(a+b\arccos(cx}{2(1-c^2x^2)}\right.}{3d^2\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 2715

$$-\frac{\frac{2b\sqrt{1-c^2x^2}\left(ib\int e^{-i\arccos(cx)}\log(1-e^{i\arccos(cx)})de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log(1+e^{i\arccos(cx)})de^{i\arccos(cx)}-2\text{arctanh}(e^{i\arccos(cx)})\right.}{d\sqrt{d-c^2dx^2}}}{d}$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))+ib\,\text{PolyLog}(2,-e^{i\arccos(cx)})-ib\,\text{PolyLog}(2,e^{i\arccos(cx)})}{2c}+\frac{x(a+b\arccos(cx}{2(1-c^2x^2)}\right.}{3d^2\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 2838

$$\frac{\frac{\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))+ib\,\text{PolyLog}(2,-e^{i\arccos(cx)})-ib\,\text{PolyLog}(2,e^{i\arccos(cx)})\right)}{d\sqrt{d-c^2dx^2}}+}{d}$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))+ib\,\text{PolyLog}(2,-e^{i\arccos(cx)})-ib\,\text{PolyLog}(2,e^{i\arccos(cx)})}{2c}+\frac{x(a+b\arccos(cx}{2(1-c^2x^2)}\right.}{3d^2\sqrt{d-c^2dx^2}}$$
$$\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 5219

$$\frac{-\frac{\sqrt{1-c^2x^2}\int\frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyL}\right.}{d\sqrt{d-c^2dx^2}}}{d}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx}{2(1-c^2x^2)}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{-\frac{\sqrt{1-c^2x^2}\int(a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i}\right.\right.}{d\sqrt{d-c^2dx^2}}}{d}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx}{2(1-c^2x^2)}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 4669

$$-\frac{\sqrt{1-c^2x^2}\left(-2b\int(a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+2b\int(a+b\arccos(cx))\log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan\left(e^{i\arccos}\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx}{2(1-c^2x^2)}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 3011

$$-\frac{\sqrt{1-c^2x^2}\left(2b\left(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\arccos(cx)\right)-2b\left(i\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)(a+b\,a\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx}{2(1-c^2x^2)}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 2720

$$-\frac{\sqrt{1-c^2x^2}\left(2b\left(i\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right)-2b\left(i\operatorname{PolyLog}\left(2,ie^{i\arccos\cdots}\right.\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx\cdots}{2(1-c^2x^2)}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 7143

$$-\frac{\sqrt{1-c^2x^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\operatorname{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)\right)-2b\left(i\operatorname{P}\cdots\right.\right.}{d\sqrt{d-c^2dx^2}}$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos(cx\cdots}{2(1-c^2x^2)}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
(a + b*ArcCos[c*x])^2/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*b*c*Sqrt[1 - c^2*x^
2]*(b/(2*c*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcCos[c*x]))/(2*(1 - c^2*x^2))
- (-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(
I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])])/(2*c)))/(3*d^2*Sqrt[d
 - c^2*d*x^2]) + ((a + b*ArcCos[c*x])^2/(d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqr
t[1 - c^2*x^2]*(-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*Po
lyLog[2, -E^(I*ArcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])]))/(d*Sqrt
[d - c^2*d*x^2]) - (Sqrt[1 - c^2*x^2]*((-2*I)*(a + b*ArcCos[c*x])^2*ArcTan
[E^(I*ArcCos[c*x])] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*ArcC
os[c*x])] - b*PolyLog[3, (-I)*E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c
*x])*PolyLog[2, I*E^(I*ArcCos[c*x])] - b*PolyLog[3, I*E^(I*ArcCos[c*x])]))
)/(d*Sqrt[d - c^2*d*x^2]))/d
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 741, normalized size of antiderivative = 1.28

method	result
default	$\frac{a^2}{3d(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{a^2}{d^2\sqrt{-c^2d\,x^2+d}}-\frac{a^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{d^{\frac{5}{2}}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\left(3\arccos(cx)^2x^2c^2-\sqrt{-c^2x^2+}\right.}{3(c^2x^2-1)}\right.$
parts	$\frac{a^2}{3d(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{a^2}{d^2\sqrt{-c^2d\,x^2+d}}-\frac{a^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{d^{\frac{5}{2}}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}\,\left(3\arccos(cx)^2x^2c^2-\sqrt{-c^2x^2+}\right.}{3(c^2x^2-1)}\right.$

input
```
int((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2/d/(-c^2*d*x^2+d)^(3/2)+a^2/d^2/(-c^2*d*x^2+d)^(1/2)-a^2/d^(5/2)*ln
((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+b^2*(-1/3*(-d*(c^2*x^2-1))^(1/2)*
(3*arccos(c*x)^2*x^2*c^2-(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c+c^2*x^2-4*arcc
os(c*x)^2-1)/(c^2*x^2-1)^2/d^3-1/3*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/
2)*(3*arccos(c*x)^2*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-3*arccos(c*x)^2*ln(
1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-6*I*arccos(c*x)*polylog(2,-I*(c*x+I*(-c^2*
x^2+1)^(1/2)))+6*I*arccos(c*x)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))+7*a
rccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-7*I*dilog(c*x+I*(-c^2*x^2+1)^(1/
2))-7*I*dilog(1+c*x+I*(-c^2*x^2+1)^(1/2))+6*polylog(3,-I*(c*x+I*(-c^2*x^2+
1)^(1/2)))-6*polylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2))))/d^3/(c^2*x^2-1))+2*a
*b*(-1/6*(-d*(c^2*x^2-1))^(1/2)*(6*c^2*x^2*arccos(c*x)-c*x*(-c^2*x^2+1)^(1
/2)-8*arccos(c*x))/(c^2*x^2-1)^2/d^3+1/6*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2
-1))^(1/2)*(6*I*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-6*I*arccos(
c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-7*I*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)
+7*I*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+6*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2))
)-6*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2))))/d^3/(c^2*x^2-1))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\,(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\,(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(a+b\operatorname{acos}(cx))^2}{x\,(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(x*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\,(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

output
```
-1/3*a^2*(3*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(5/2
) - 3/(sqrt(-c^2*d*x^2 + d)*d^2) - 1/((-c^2*d*x^2 + d)^(3/2)*d)) - sqrt(d)
*integrate((b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arcta
n2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d
^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}\,dx = \int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{x\,\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^5-2\sqrt{-c^2x^2+1}\,c^2x^3+\sqrt{-c^2x^2+1}\,x}dx\right)ab\,c^2x^2-6\sqrt{-c^2x^2}}{}$$

input
```
int((a+b*acos(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x)
```

output

```
(6*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**5
- 2*sqrt( - c**2*x**2 + 1)*c**2*x**3 + sqrt( - c**2*x**2 + 1)*x),x)*a*b*c*
*2*x**2 - 6*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c
**4*x**5 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**3 + sqrt( - c**2*x**2 + 1)*x),
x)*a*b + 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)**2/(sqrt( - c**2*x**2 + 1)
*c**4*x**5 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**3 + sqrt( - c**2*x**2 + 1)*x
),x)*b**2*c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)**2/(sqrt( - c
**2*x**2 + 1)*c**4*x**5 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**3 + sqrt( - c**
2*x**2 + 1)*x),x)*b**2 + 3*sqrt( - c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a*
*2*c**2*x**2 - 3*sqrt( - c**2*x**2 + 1)*log(tan(asin(c*x)/2))*a**2 - 4*sqr
t( - c**2*x**2 + 1)*a**2*c**2*x**2 + 4*sqrt( - c**2*x**2 + 1)*a**2 + 3*a**
2*c**2*x**2 - 4*a**2)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*d**2*(c**2*x**2 -
1))
```

3.263 $\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 452

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx &= \frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}}-\frac{bc(a+b\arccos(cx))}{3d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}\\
&-\frac{(a+b\arccos(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}+\frac{4c^2x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}\\
&+\frac{8c^2x(a+b\arccos(cx))^2}{3d^2\sqrt{d-c^2dx^2}}-\frac{8ic\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3d^2\sqrt{d-c^2dx^2}}\\
&-\frac{4bc\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}\\
&+\frac{16bc\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}\\
&-\frac{5ib^2c\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}\\
&-\frac{ib^2c\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
1/3*b^2*c^2*x/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b*c*(a+b*arccos(c*x))/d^2/(-c^2
*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-(a+b*arccos(c*x))^2/d/x/(-c^2*d*x^2+d)^
(3/2)+4/3*c^2*x*(a+b*arccos(c*x))^2/d/(-c^2*d*x^2+d)^(3/2)+8/3*c^2*x*(a+b*
arccos(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)-8/3*I*c*(-c^2*x^2+1)^(1/2)*(a+b*ar
ccos(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)-4*b*c*(-c^2*x^2+1)^(1/2)*(a+b*arccos
(c*x))*arctanh((c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^2/(-c^2*d*x^2+d)^(1/2)+16/3
*b*c*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^
2)/d^2/(-c^2*d*x^2+d)^(1/2)-5/3*I*b^2*c*(-c^2*x^2+1)^(1/2)*polylog(2,-(c*x
+I*(-c^2*x^2+1)^(1/2))^2)/d^2/(-c^2*d*x^2+d)^(1/2)-I*b^2*c*(-c^2*x^2+1)^(1
/2)*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^{5/2}}\,dx =$$
$$-\frac{3a^2-12a^2c^2x^2-b^2c^2x^2+8a^2c^4x^4+b^2c^4x^4-abcx\sqrt{1-c^2x^2}+6ab\arccos(cx)-24abc^2x^2\arccos(cx)+}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
-1/3*(3*a^2 - 12*a^2*c^2*x^2 - b^2*c^2*x^2 + 8*a^2*c^4*x^4 + b^2*c^4*x^4 -
 a*b*c*x*Sqrt[1 - c^2*x^2] + 6*a*b*ArcCos[c*x] - 24*a*b*c^2*x^2*ArcCos[c*x
] + 16*a*b*c^4*x^4*ArcCos[c*x] - b^2*c*x*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + 3
*b^2*ArcCos[c*x]^2 - 12*b^2*c^2*x^2*ArcCos[c*x]^2 + 8*b^2*c^4*x^4*ArcCos[c
*x]^2 - (8*I)*b^2*c*x*(1 - c^2*x^2)^(3/2)*ArcCos[c*x]^2 + 10*b^2*c*x*(1 -
c^2*x^2)^(3/2)*ArcCos[c*x]*Log[1 - E^((2*I)*ArcCos[c*x])] + 6*b^2*c*x*(1 -
 c^2*x^2)^(3/2)*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] + 6*a*b*c*x*(1
- c^2*x^2)^(3/2)*Log[c*x] + 5*a*b*c*x*(1 - c^2*x^2)^(3/2)*Log[1 - c^2*x^2]
 - (3*I)*b^2*c*x*(1 - c^2*x^2)^(3/2)*PolyLog[2, -E^((2*I)*ArcCos[c*x])] -
(5*I)*b^2*c*x*(1 - c^2*x^2)^(3/2)*PolyLog[2, E^((2*I)*ArcCos[c*x])])/(d*x*
(d - c^2*d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 3.28 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.01, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.724$, Rules used = {5205, 5163, 5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838, 5183, 208, 5209, 208, 5185, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5205}$$

$$-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+4c^2\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{dx(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{5163}$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{x(a+b\arccos(cx))^2}{3d(d-c^2dx^2)^{3/2}}\right)-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{dx(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{5161}$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\arccos(cx))^2}{3d(d-c^2dx^2)^{3/2}}\right)-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{dx(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{5181}$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int\frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\,\mathrm{a}}{3d\,(d-}\right.$$

$$\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int-\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos}{cd\sqrt{d-c^2dx^2}}\right.}{3d}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{2b\sqrt{1-c^2x^2}\int(a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}\right.}{3d}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{4200}$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(2i\int-\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i}{}\right.}{cd\sqrt{d-c^2dx^2}}\right.}{3d}\right.$$

$$\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\int\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i}{\ldots}\right.}{cd\sqrt{d-c^2dx^2}}\right.}{3d}\right.$$

$$\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$\Big\downarrow$ 2620

$$-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}\ldots\right.\right.}{cd\sqrt{d-c\ldots}}\right.}{3d}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$\Big\downarrow$ 2715

$$-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}\ldots\right.\right.}{c\ldots}\right.}{3d}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$\Big\downarrow$ 2838

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}\right.\right.}{cd\sqrt{d-c^2dx^2}}\right.}{3d}\right.$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{5183}$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{b\int \frac{1}{\left(1-c^2x^2\right)^{3/2}}dx}{2c}+\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)\right.\right.}{}\right.}{3d}\right.$$

$$\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}+\frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos\right.\right.}{cd\sqrt{}}\right.}{3d}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{5209}$$

$$-\frac{2bc\sqrt{1-c^2x^2}\left(\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\int \frac{1}{(1-c^2x^2)^{3/2}}dx+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}\right)}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}+\frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos\right.\right.}{cd\sqrt{}}\right.}{3d}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \; 208$$

$$-\frac{2bc\sqrt{1-c^2x^2}\left(\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}+\frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos\right.\right.}{cd\sqrt{}}\right.}{3d}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \; 5185$$

$$-\frac{2bc\sqrt{1-c^2x^2}\left(-\int \frac{a+b\arccos(cx)}{cx\sqrt{1-c^2x^2}}d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}+\frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos\right.\right.}{cd\sqrt{}}\right.}{3d}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \; 4919$$

$$-\frac{2bc\sqrt{1-c^2x^2}\left(-2\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}+\frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos\right.\right.}{cd\sqrt{}}\right.}{3d}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2bc\sqrt{1-c^2x^2}\left(-2\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)}{d^2\sqrt{d-c^2dx^2}}+$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}+\frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos\right.\right.}{cd\sqrt{}}\right.}{3d}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 4671$$

$$-\frac{2bc\sqrt{1-c^2x^2}\left(-2\left(-\frac{1}{2}b\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+\frac{1}{2}b\int\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-(\arctan\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}+\frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos\right.\right.}{cd\sqrt{}}\right.}{3d}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 2715$$

$$-\frac{2bc\sqrt{1-c^2x^2}\left(-2\left(\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(\right.\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}+\frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos\right.\right.}{cd\sqrt{}}\right.}{3d}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 2838

$$-\frac{2bc\sqrt{1-c^2x^2}\left(-2\left(-\left(\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}ib\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)-\frac{1}{4}ib\operatorname{PolyLog}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}+\frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos\right.\right.}{cd\sqrt{}}\right.}{3d}\right.$$

$$\frac{(a+b\arccos(cx))^2}{dx\left(d-c^2dx^2\right)^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
-((a + b*ArcCos[c*x])^2/(d*x*(d - c^2*d*x^2)^(3/2))) - (2*b*c*Sqrt[1 - c^2
*x^2]*((b*c*x)/(2*Sqrt[1 - c^2*x^2]) + (a + b*ArcCos[c*x])/(2*(1 - c^2*x^2
)) - 2*(-((a + b*ArcCos[c*x])*ArcTanh[E^((2*I)*ArcCos[c*x])]) + (I/4)*b*Po
lyLog[2, -E^((2*I)*ArcCos[c*x])] - (I/4)*b*PolyLog[2, E^((2*I)*ArcCos[c*x]
)])))/(d^2*Sqrt[d - c^2*d*x^2]) + 4*c^2*((x*(a + b*ArcCos[c*x])^2)/(3*d*(d
 - c^2*d*x^2)^(3/2)) + (2*b*c*Sqrt[1 - c^2*x^2]*((b*x)/(2*c*Sqrt[1 - c^2*x
^2]) + (a + b*ArcCos[c*x])/(2*c^2*(1 - c^2*x^2))))/(3*d^2*Sqrt[d - c^2*d*x
^2]) + (2*((x*(a + b*ArcCos[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[1
 - c^2*x^2]*(((-1/2*I)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcC
os[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])] + (b*PolyLog[2, E^((2*I)*ArcCos[c*
x])])/4)))/(c*d*Sqrt[d - c^2*d*x^2])))/(3*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3778 vs. $2(447) = 894$.

Time = 0.76 (sec) , antiderivative size = 3779, normalized size of antiderivative = 8.36

method	result	size
default	Expression too large to display	3779
parts	Expression too large to display	3779

input
```
int((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-272/3*I*a*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^
3*x^2*arccos(c*x)*(-c^2*x^2+1)^(1/2)*c^3+128/3*I*a*b*(-d*(c^2*x^2-1))^(1/2
)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^4*arccos(c*x)*(-c^2*x^2+1)^(1/
2)*c^5-8*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^
3*x*(-c^2*x^2+1)*c^2+32/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4
+26*c^2*x^2-9)/d^3*x^7*(-c^2*x^2+1)*c^8-88/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(8
*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^5*(-c^2*x^2+1)*c^6-44*b^2*(-d*(c^2
*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x*arccos(c*x)^2*c^2
-64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x
^5*arccos(c*x)^2*c^6+56*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+2
6*c^2*x^2-9)/d^3*x^3*arccos(c*x)^2*c^4-3*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^6
*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*arccos(c*x)*(-c^2*x^2+1)^(1/2)*c+3*I*b^2
*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*(-c^2*x^2+
1)^(1/2)*c+80/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^
2-9)/d^3*x^3*(-c^2*x^2+1)*c^4+112*a*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25
*c^4*x^4+26*c^2*x^2-9)/d^3*x^3*arccos(c*x)*c^4-128/3*a*b*(-d*(c^2*x^2-1))^
(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x^5*arccos(c*x)*c^6-88*a*b*(
-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d^3*x*arccos(c*x
)*c^2+8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/d
^3*x^2*(-c^2*x^2+1)^(1/2)*c^3+10/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2...
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx=\int \frac{\left(b\arccos\left(cx\right)+a\right)^2}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx = \int \frac{\left(a+b\operatorname{acos}\left(cx\right)\right)^2}{x^2\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(x**2*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx = \int \frac{\left(b\arccos\left(cx\right)+a\right)^2}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
1/3*a^2*(8*c^2*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 4*c^2*x/((-c^2*d*x^2 + d)^(3
/2)*d) - 3/((-c^2*d*x^2 + d)^(3/2)*d*x)) - sqrt(d)*integrate((b^2*arctan2(
sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arctan2(sqrt(c*x + 1)*sqrt(-c
*x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 +
 3*c^2*d^3*x^4 - d^3*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx = \int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{x^2\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^2*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x^2*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^6-2\sqrt{-c^2x^2+1}\,c^2x^4+\sqrt{-c^2x^2+1}\,x^2}dx\right)ab\,c^2x^3-6\sqrt{-c^2x}$$

input
```
int((a+b*acos(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(6*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**6
- 2*sqrt( - c**2*x**2 + 1)*c**2*x**4 + sqrt( - c**2*x**2 + 1)*x**2),x)*a*b
*c**2*x**3 - 6*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1
)*c**4*x**6 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**4 + sqrt( - c**2*x**2 + 1)*
x**2),x)*a*b*x + 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)**2/(sqrt( - c**2*x
**2 + 1)*c**4*x**6 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**4 + sqrt( - c**2*x**
2 + 1)*x**2),x)*b**2*c**2*x**3 - 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)**2
/(sqrt( - c**2*x**2 + 1)*c**4*x**6 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**4 +
sqrt( - c**2*x**2 + 1)*x**2),x)*b**2*x + 8*a**2*c**4*x**4 - 12*a**2*c**2*x
**2 + 3*a**2)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*d**2*x*(c**2*x**2 - 1))
```

3.264 $\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 752

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^{5/2}}\,dx = \frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} - \frac{bc(a+b\arccos(cx))}{d^2x\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}$$
$$+ \frac{2bc^3x(a+b\arccos(cx))}{3d^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}} + \frac{5c^2(a+b\arccos(cx))^2}{6d(d-c^2dx^2)^{3/2}}$$
$$- \frac{(a+b\arccos(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}} + \frac{5c^2(a+b\arccos(cx))^2}{2d^2\sqrt{d-c^2dx^2}}$$
$$+ \frac{26ibc^2\sqrt{1-c^2x^2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$
$$- \frac{5c^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}$$
$$- \frac{b^2c^2\sqrt{1-c^2x^2}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d^2\sqrt{d-c^2dx^2}}$$
$$+ \frac{5ibc^2\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}$$
$$- \frac{13ib^2c^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$
$$+ \frac{13ib^2c^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}$$
$$- \frac{5ibc^2\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}$$
$$- \frac{5b^2c^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}$$
$$+ \frac{5b^2c^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}$$

output
```
1/3*b^2*c^2/d^2/(-c^2*d*x^2+d)^(1/2)-b*c*(a+b*arccos(c*x))/d^2/x/(-c^2*x^2
+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+2/3*b*c^3*x*(a+b*arccos(c*x))/d^2/(-c^2*x^2
+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+5/6*c^2*(a+b*arccos(c*x))^2/d/(-c^2*d*x^2+d
)^(3/2)-1/2*(a+b*arccos(c*x))^2/d/x^2/(-c^2*d*x^2+d)^(3/2)+5/2*c^2*(a+b*ar
ccos(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+26/3*I*b*c^2*(-c^2*x^2+1)^(1/2)*(a+b
*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)-5*
c^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2*arctanh(c*x+I*(-c^2*x^2+1)^(1/2
))/d^2/(-c^2*d*x^2+d)^(1/2)-b^2*c^2*(-c^2*x^2+1)^(1/2)*arctanh((-c^2*x^2+1
)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)+5*I*b*c^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos
(c*x))*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)-13/3*
I*b^2*c^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d^2/
(-c^2*d*x^2+d)^(1/2)+13/3*I*b^2*c^2*(-c^2*x^2+1)^(1/2)*polylog(2,I*(c*x+I*
(-c^2*x^2+1)^(1/2)))/d^2/(-c^2*d*x^2+d)^(1/2)-5*I*b*c^2*(-c^2*x^2+1)^(1/2)
*(a+b*arccos(c*x))*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^
(1/2)-5*b^2*c^2*(-c^2*x^2+1)^(1/2)*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))/d^
2/(-c^2*d*x^2+d)^(1/2)+5*b^2*c^2*(-c^2*x^2+1)^(1/2)*polylog(3,c*x+I*(-c^2*
x^2+1)^(1/2))/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 9.13 (sec) , antiderivative size = 1292, normalized size of antiderivative = 1.72

$$\int \frac{(a + b\arccos(cx))^2}{x^3\left(d - c^2dx^2\right)^{5/2}}\, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
Sqrt[-(d*(-1 + c^2*x^2))]*(-1/2*a^2/(d^3*x^2) + (a^2*c^2)/(3*d^3*(-1 + c^2
*x^2)^2) - (2*a^2*c^2)/(d^3*(-1 + c^2*x^2))) + (5*a^2*c^2*Log[x])/(2*d^(5/
2)) - (5*a^2*c^2*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/(2*d^(5/2)) +
 (a*b*Sqrt[1 - c^2*x^2]*((-60*I)*PolyLog[2, (-I)*E^(I*ArcCos[c*x])]*Sin[2*
ArcCos[c*x]]^2 + (60*I)*PolyLog[2, I*E^(I*ArcCos[c*x])]*Sin[2*ArcCos[c*x]]
^2 + (22*ArcCos[c*x] + 40*ArcCos[c*x]*Cos[2*ArcCos[c*x]] - 30*ArcCos[c*x]*
Cos[4*ArcCos[c*x]] - 30*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 - I*E^(I*ArcCo
s[c*x])] + 30*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] +
 26*Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/2]] - 26*Sqrt[1 - c^2*x^2]*Log[S
in[ArcCos[c*x]/2]] + 16*Sin[2*ArcCos[c*x]] - 15*ArcCos[c*x]*Log[1 - I*E^(I
*ArcCos[c*x])]*Sin[3*ArcCos[c*x]] + 15*ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c
*x])]*Sin[3*ArcCos[c*x]] + 13*Log[Cos[ArcCos[c*x]/2]]*Sin[3*ArcCos[c*x]] -
 13*Log[Sin[ArcCos[c*x]/2]]*Sin[3*ArcCos[c*x]] - 4*Sin[4*ArcCos[c*x]] + 15
*ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])]*Sin[5*ArcCos[c*x]] - 15*ArcCos[c
*x]*Log[1 + I*E^(I*ArcCos[c*x])]*Sin[5*ArcCos[c*x]] - 13*Log[Cos[ArcCos[c*
x]/2]]*Sin[5*ArcCos[c*x]] + 13*Log[Sin[ArcCos[c*x]/2]]*Sin[5*ArcCos[c*x]])
/Sqrt[1 - c^2*x^2]))/(48*d*x^2*(d*(1 - c^2*x^2))^(3/2)) + (b^2*c^2*((Sqrt[
1 - c^2*x^2]*(6*ArcCos[c*x] + 5*ArcCos[c*x]^3 - 6*ArcCoth[Sqrt[1 - c^2*x^2
]] - 15*ArcCos[c*x]^2*Log[1 - I*E^(I*ArcCos[c*x])] + 15*ArcCos[c*x]^2*Log[
1 + I*E^(I*ArcCos[c*x])] - (30*I)*ArcCos[c*x]*PolyLog[2, (-I)*E^(I*ArcC...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{5205}$$

$$-\frac{bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x^2(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+\frac{5}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{5205}$$

$$-\frac{bc\sqrt{1-c^2x^2}\left(3c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx-bc\int \frac{1}{x(1-c^2x^2)^{3/2}}dx-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{5}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 243$$

$$-\frac{bc\sqrt{1-c^2x^2}\left(3c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx-\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)^{3/2}}dx^2-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{5}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 61$$

$$-\frac{bc\sqrt{1-c^2x^2}\left(3c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx-\frac{1}{2}bc\left(\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2+\frac{2}{\sqrt{1-c^2x^2}}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{5}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 73$$

$$-\frac{bc\sqrt{1-c^2x^2}\left(3c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx-\frac{1}{2}bc\left(\frac{2}{\sqrt{1-c^2x^2}}-\frac{2\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{5}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$-\frac{bc\sqrt{1-c^2x^2}\left(3c^2\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{5}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 5163$$

$$-\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{x}{(1-c^2x^2)^{3/2}}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2}{\sqrt{1-c^2x^2}}-\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{5}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 241$$

$$-\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(\frac{1}{2}\int\frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2}{\sqrt{1-c^2x^2}}-2\text{arctan}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{5}{2}c^2\int\frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 5165

$$-\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2}{\sqrt{1-c^2x^2}}-2\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{5}{2}c^2\int\frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{\int(a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)-\frac{a+b\arccos(cx)}{x(1-c^2x^2)}-\frac{1}{2}bc\left(\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{5}{2}c^2\int\frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 4671

$$-\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-b\int\log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int\log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(c\right.}{2c}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{5}{2}c^2\int\frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 2715

$$-\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}}{2c}\right.\right.}{d^2\sqrt{}}$$

$$\frac{5}{2}c^2\int\frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 2838

$$\frac{5}{2}c^2\int \frac{(a+b\arccos(cx))^2}{x\left(d-c^2dx^2\right)^{5/2}}dx-$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\Big\downarrow$ 5209

$$\frac{5}{2}c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}\right)-$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\Big\downarrow$ 5163

$$\frac{5}{2}c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{x}{(1-c^2x^2)^{3/2}}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b}{3d\,(d}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\Big\downarrow$ 241

$$\frac{5}{2}c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\arccos(cx}{3d\,(d-c^2dx^2)}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\Big\downarrow$ 5165

$$\frac{5}{2}c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{(a+b\arc}{3d\,(d-c}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arcc}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{2}c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{2c}+\frac{x(a+b\arccos(cx))}{2(1-c^2x^2)}+\frac{b}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{\int \frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arcc}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{4671}$$

$$\frac{5}{2}c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(c}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arcc}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{5}{2}c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 2838

$$\frac{5}{2}c^2\left(\frac{\int\frac{(a+b\arccos(cx))^2}{x(d-c^2dx^2)^{3/2}}dx}{d}+\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}}{2c}\right.}{3d^2\sqrt{d-c^2dx^2}}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 5209

$$\frac{5}{2}c^2\left(\frac{\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}}{d}+\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+}{}\right.}{}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 5165

$$\frac{5}{2}c^2\left(\frac{-\frac{2b\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{\int\frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}+\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}}{d}+\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2\text{arctanh}(e^{i\arccos(cx)}\right.}{}}{}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))+ib\,\text{PolyLog}(2,-e^{i\arccos(cx)})-ib\,\text{PolyLog}(2,e^{i\arccos(cx)})}{2c}+\frac{x(a+b\arccos(cx)}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 3042

$$\frac{5}{2}c^2\left(\frac{\frac{\int\frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{2b\sqrt{1-c^2x^2}\int(a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{d\sqrt{d-c^2dx^2}}+\frac{(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}}{d}+\frac{2bc\sqrt{1-c^2x^2}\left(-\frac{-2}{}\right.}{}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))+ib\,\text{PolyLog}(2,-e^{i\arccos(cx)})-ib\,\text{PolyLog}(2,e^{i\arccos(cx)})}{2c}+\frac{x(a+b\arccos(cx)}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 4671

$$\frac{5}{2}c^2\left(\frac{-\frac{2b\sqrt{1-c^2x^2}\left(-b\int\log(1-e^{i\arccos(cx)})d\arccos(cx)+b\int\log(1+e^{i\arccos(cx)})d\arccos(cx)-2\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))\right)}{d\sqrt{d-c^2dx^2}}}{d}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))+ib\,\text{PolyLog}(2,-e^{i\arccos(cx)})-ib\,\text{PolyLog}(2,e^{i\arccos(cx)})}{2c}+\frac{x(a+b\arccos(cx)}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2(d-c^2dx^2)^{3/2}}$$

$\downarrow$ 2715

$$\frac{5}{2}c^2\left(-\frac{\frac{2b\sqrt{1-c^2x^2}\left(ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-2\text{arctanh}\left(e^{i\arccos(cx)}\right.\right.}{d\sqrt{d-c^2dx^2}}}{d}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 2838

$$\frac{5}{2}c^2\left(\frac{\frac{\int\frac{(a+b\arccos(cx))^2}{x\sqrt{d-c^2dx^2}}dx}{d}-\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(c}\right.\right.}{d\sqrt{d-c^2dx^2}}}{d}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 5219

$$\frac{5}{2}c^2\left(\frac{-\frac{\sqrt{1-c^2x^2}\int\frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-i\right.}{d\sqrt{d-c^2dx^2}}}{d}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 3042

$$\frac{5}{2}c^2\left(\frac{-\frac{\sqrt{1-c^2x^2}\int(a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\right.}{d\sqrt{d-c^2dx^2}}}{d}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 4669

$$\frac{5}{2}c^2\left(-\frac{\sqrt{1-c^2x^2}\left(-2b\int(a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+2b\int(a+b\arccos(cx))\log\left(1+ie^{i\arccos(cx)}\right)d\arccos(cx)-2i\arctan(\right.}{d\sqrt{d-c^2dx^2}}\right.$$

$$\frac{bc\sqrt{1-c^2x^2}\left(3c^2\left(-\frac{-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{2c}+\frac{x(a+b\arccos}{2(1-c^2}\right.\right.}{d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{2dx^2\left(d-c^2dx^2\right)^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 905, normalized size of antiderivative = 1.20

method	result
default	$-\frac{a^2}{2d\,x^2(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a^2c^2}{6d(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a^2c^2}{2d^2\sqrt{-c^2d\,x^2+d}}-\frac{5a^2c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{2d^{\frac{5}{2}}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}}{}\right.$
parts	$-\frac{a^2}{2d\,x^2(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a^2c^2}{6d(-c^2d\,x^2+d)^{\frac{3}{2}}}+\frac{5a^2c^2}{2d^2\sqrt{-c^2d\,x^2+d}}-\frac{5a^2c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{2d^{\frac{5}{2}}}+b^2\left(-\frac{\sqrt{-d(c^2x^2-1)}}{}\right.$

input
```
int((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/d/x^2/(-c^2*d*x^2+d)^(3/2)+5/6*a^2*c^2/d/(-c^2*d*x^2+d)^(3/2)+5/2
*a^2*c^2/d^2/(-c^2*d*x^2+d)^(1/2)-5/2*a^2*c^2/d^(5/2)*ln((2*d+2*d^(1/2)*(-
c^2*d*x^2+d)^(1/2))/x)+b^2*(-1/6*(-d*(c^2*x^2-1))^(1/2)*(15*arccos(c*x)^2*
x^4*c^4+4*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^3*c^3+2*c^4*x^4-20*arccos(c*x)^
2*x^2*c^2-6*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c-2*c^2*x^2+3*arccos(c*x)^2)/
d^3/(c^4*x^4-2*c^2*x^2+1)/x^2-1/6*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1
/2)/d^3/(c^2*x^2-1)*(15*I*arccos(c*x)^2*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))
-15*I*arccos(c*x)^2*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-26*I*arccos(c*x)*ln
(1+c*x+I*(-c^2*x^2+1)^(1/2))+30*arccos(c*x)*polylog(2,I*(c*x+I*(-c^2*x^2+1
)^(1/2)))-30*arccos(c*x)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+30*I*pol
ylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))-30*I*polylog(3,-I*(c*x+I*(-c^2*x^2+1)
^(1/2)))-26*dilog(c*x+I*(-c^2*x^2+1)^(1/2))-26*dilog(1+c*x+I*(-c^2*x^2+1)^
(1/2))+12*arctan(c*x+I*(-c^2*x^2+1)^(1/2)))*c^2)+2*a*b*(-1/6*(-d*(c^2*x^2-
1))^(1/2)*(15*c^4*x^4*arccos(c*x)+2*c^3*x^3*(-c^2*x^2+1)^(1/2)-20*c^2*x^2*
arccos(c*x)-3*c*x*(-c^2*x^2+1)^(1/2)+3*arccos(c*x))/d^3/(c^4*x^4-2*c^2*x^2
+1)/x^2-1/6*I*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(15*I*arccos(c*x)*
ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-15*I*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^
2+1)^(1/2)))-13*I*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+13*I*ln(I*(-c^2*x^2+1)^(1
/2)+c*x-1)-15*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+15*dilog(1-I*(c*x+I*(-
c^2*x^2+1)^(1/2))))*c^2/d^3/(c^2*x^2-1))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2)/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}\,(cx))^2}{x^3\,(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x**3/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(x**3*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
-1/6*a^2*(15*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d
^(5/2) - 15*c^2/(sqrt(-c^2*d*x^2 + d)*d^2) - 5*c^2/((-c^2*d*x^2 + d)^(3/2)
*d) + 3/((-c^2*d*x^2 + d)^(3/2)*d*x^2)) - sqrt(d)*integrate((b^2*arctan2(s
qrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arctan2(sqrt(c*x + 1)*sqrt(-c*
x + 1), c*x))*sqrt(c*x + 1)*sqrt(-c*x + 1)/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 +
3*c^2*d^3*x^5 - d^3*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^3(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx = \int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{x^3\,\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^3*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x^3*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{48\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^7-2\sqrt{-c^2x^2+1}\,c^2x^5+\sqrt{-c^2x^2+1}\,x^3}dx\right)ab\,c^2x^4-48\sqrt{-c}}{}$$

input
```
int((a+b*acos(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x)
```

output

```
(48*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**7
 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**5 + sqrt( - c**2*x**2 + 1)*x**3),x)*a*
b*c**2*x**4 - 48*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 +
 1)*c**4*x**7 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**5 + sqrt( - c**2*x**2 + 1
)*x**3),x)*a*b*x**2 + 24*sqrt( - c**2*x**2 + 1)*int(acos(c*x)**2/(sqrt( -
c**2*x**2 + 1)*c**4*x**7 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**5 + sqrt( - c*
*2*x**2 + 1)*x**3),x)*b**2*c**2*x**4 - 24*sqrt( - c**2*x**2 + 1)*int(acos(
c*x)**2/(sqrt( - c**2*x**2 + 1)*c**4*x**7 - 2*sqrt( - c**2*x**2 + 1)*c**2*
x**5 + sqrt( - c**2*x**2 + 1)*x**3),x)*b**2*x**2 + 60*sqrt( - c**2*x**2 +
1)*log(tan(asin(c*x)/2))*a**2*c**4*x**4 - 60*sqrt( - c**2*x**2 + 1)*log(ta
n(asin(c*x)/2))*a**2*c**2*x**2 - 65*sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4
+ 65*sqrt( - c**2*x**2 + 1)*a**2*c**2*x**2 + 60*a**2*c**4*x**4 - 80*a**2*c
**2*x**2 + 12*a**2)/(24*sqrt(d)*sqrt( - c**2*x**2 + 1)*d**2*x**2*(c**2*x**
2 - 1))
```

3.265 $\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 538

$$\begin{aligned}
\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^{5/2}}\,dx &= -\frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}}+\frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} \\
&\quad -\frac{bc(a+b\arccos(cx))}{3d^2x^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}} \\
&\quad -\frac{2c^2(a+b\arccos(cx))^2}{dx(d-c^2dx^2)^{3/2}}+\frac{8c^4x(a+b\arccos(cx))^2}{3d(d-c^2dx^2)^{3/2}} \\
&\quad +\frac{16c^4x(a+b\arccos(cx))^2}{3d^2\sqrt{d-c^2dx^2}}-\frac{16ic^3\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3d^2\sqrt{d-c^2dx^2}} \\
&\quad -\frac{32bc^3\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}} \\
&\quad +\frac{32bc^3\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}} \\
&\quad -\frac{8ib^2c^3\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}} \\
&\quad -\frac{8ib^2c^3\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}
\end{aligned}$$

output
```
-1/3*b^2*c^2/d^2/x/(-c^2*d*x^2+d)^(1/2)+2/3*b^2*c^4*x/d^2/(-c^2*d*x^2+d)^(
1/2)-1/3*b*c*(a+b*arccos(c*x))/d^2/x^2/(-c^2*x^2+1)^(1/2)/(-c^2*d*x^2+d)^(
1/2)-1/3*(a+b*arccos(c*x))^2/d/x^3/(-c^2*d*x^2+d)^(3/2)-2*c^2*(a+b*arccos(
c*x))^2/d/x/(-c^2*d*x^2+d)^(3/2)+8/3*c^4*x*(a+b*arccos(c*x))^2/d/(-c^2*d*x
^2+d)^(3/2)+16/3*c^4*x*(a+b*arccos(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)-16/3*I
*c^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)-32/3*
b*c^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*arctanh((c*x+I*(-c^2*x^2+1)^(1/
2))^2)/d^2/(-c^2*d*x^2+d)^(1/2)+32/3*b*c^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(
c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^2/(-c^2*d*x^2+d)^(1/2)-8/3*I*b^
2*c^3*(-c^2*x^2+1)^(1/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^2/(-c^
2*d*x^2+d)^(1/2)-8/3*I*b^2*c^3*(-c^2*x^2+1)^(1/2)*polylog(2,(c*x+I*(-c^2*x
^2+1)^(1/2))^2)/d^2/(-c^2*d*x^2+d)^(1/2)
```

Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.82

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^{5/2}}\,dx = \frac{-\frac{a^2(1+6c^2x^2-24c^4x^4+16c^6x^6)}{x^3}+\frac{ab\left(-2(1+6c^2x^2-24c^4x^4+16c^6x^6)\arccos(cx)+cx\sqrt{1-c^2x^2}(1+16c^2\cdots\right.}{x^3}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
(-((a^2*(1 + 6*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6))/x^3) + (a*b*(-2*(1 + 6*
c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6)*ArcCos[c*x] + c*x*Sqrt[1 - c^2*x^2]*(1
+ 16*c^2*x^2*(-1 + c^2*x^2)*Log[c*x] + 8*c^2*x^2*(-1 + c^2*x^2)*Log[1 - c^
2*x^2])))/x^3 - b^2*c^3*(1 - c^2*x^2)^(3/2)*(-((c*x)/Sqrt[1 - c^2*x^2]) +
Sqrt[1 - c^2*x^2]/(c*x) - ArcCos[c*x]/(c^2*x^2) + ArcCos[c*x]/(-1 + c^2*x^
2) - (16*I)*ArcCos[c*x]^2 - (c*x*ArcCos[c*x]^2)/(1 - c^2*x^2)^(3/2) - (8*c
*x*ArcCos[c*x]^2)/Sqrt[1 - c^2*x^2] + (Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2)/(c
^3*x^3) + (8*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2)/(c*x) + 16*ArcCos[c*x]*Log[1
 - E^((2*I)*ArcCos[c*x])] + 16*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])]
- (8*I)*PolyLog[2, -E^((2*I)*ArcCos[c*x])] - (8*I)*PolyLog[2, E^((2*I)*Arc
Cos[c*x])]))/(3*d*(d - c^2*d*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 5.00 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.34, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.828, Rules used = {5205, 5205, 245, 208, 5163, 5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838, 5183, 208, 5209, 208, 5185, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{5205}$$

$$-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x^3(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+2c^2\int \frac{(a+b\arccos(cx))^2}{x^2(d-c^2dx^2)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{5205}$$

$$2c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+4c^2\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{dx(d-c^2dx^2)^{3/2}}\right)-\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx-\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)^{3/2}}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{245}$$

$$2c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+4c^2\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{dx(d-c^2dx^2)^{3/2}}\right)-\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx-\frac{1}{2}bc\left(2c^2\int \frac{1}{(1-c^2x^2)^{3/2}}dx-\frac{1}{x\sqrt{1-c^2x^2}}\right)-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{3dx^3(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$2c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+4c^2\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}dx-\frac{(a+b\arccos(cx))^2}{dx\,(d-c^2dx^2)^{3/2}}\right)-\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{3dx^3\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \; 5163$$

$$2c^2\left(4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\int \frac{(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}dx}{3d}+\frac{x(a+b\arccos(cx))^2}{3d\,(d-c^2dx^2)^{3/2}}\right)-\frac{2bc\sqrt{1-c^2x^2}\int}{d^2\sqrt{d-}}\right.$$
$$-\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{3dx^3\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \; 5161$$

$$2c^2\left(4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{d\sqrt{d-c^2dx^2}}+\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a+b\,\mathrm{arcc}}{3d\,(d-c^2}\right.\right.$$
$$-\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{3dx^3\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow \; 5181$$

$$2c^2\left(4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{cd\sqrt{d-c^2dx^2}}\right)}{3d}+\frac{x(a}{3d}\right.\right.$$
$$-\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}-\frac{(a+b\arccos(cx))^2}{3dx^3\,(d-c^2dx^2)^{3/2}}$$

$$\Big\downarrow 3042$$

$$-\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}+$$
$$2c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2}}{}\right.}{}\right.\right.$$
$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow 25$$

$$-\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}+$$
$$2c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{2b\sqrt{1-c^2x^2}\int(a+b\arccos(cx))\tan}{cd\sqrt{d-c}}\right.}{}\right.\right.$$
$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow 4200$$

$$2c^2\left(4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(2i\int-\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos\right.}{cd\sqrt{d-c^2dx^2}}\right.}{3d}\right.\right.$$
$$\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}-$$
$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$$\Big\downarrow 25$$

$$2c^2\left(4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\int\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos\right.}{cd\sqrt{d-c^2dx^2}}\right.}{3d}\right.\right.$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}-$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$$\downarrow 2620$$

$$-\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}+$$

$$2c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2}}{}\right.}{}\right.\right.$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$$\downarrow 2715$$

$$-\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}+$$

$$2c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2}}{}\right.}{}\right.\right.$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$$\downarrow 2838$$

$$2c^2\left(4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(c\right.\right.}{cd\sqrt{d-}}\right.}{3d}\right.\right.$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}-$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 5183

$$2c^2\left(4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{b\int\frac{1}{\left(1-c^2x^2\right)^{3/2}}dx}{2c}+\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\,\mathrm{arcco}}\right.\right.\right.}{}\right.}{}\right.\right.$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}-$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 208

$$2c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx}{d^2\sqrt{d-c^2dx^2}}+4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}+\frac{bx}{2c\sqrt{1-c^2x^2}}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\right.}{}\right.\right.$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\int\frac{a+b\arccos(cx)}{x(1-c^2x^2)^2}dx-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}-$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 5209

$$2c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\left(\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\int \frac{1}{(1-c^2x^2)^{3/2}}dx+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}\right)}{d^2\sqrt{d-c^2dx^2}}+4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}\right.}{3d^2\sqrt{d-c^2dx}}\right.\right.$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\left(\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\int \frac{1}{(1-c^2x^2)^{3/2}}dx+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}\right)-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{}{x\sqrt{1}}\right.\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 208

$$2c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\left(\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)}{d^2\sqrt{d-c^2dx^2}}+4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}+\frac{bx}{2c\sqrt{1-c}}\right.}{3d^2\sqrt{d-c^2dx^2}}\right.\right.$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\left(\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{1}{x\sqrt{1-c^2x^2}}\right)\right)}{3d^2\sqrt{d-c^2dx^2}}-$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 5185

$$2c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\left(-\int \frac{a+b\arccos(cx)}{cx\sqrt{1-c^2x^2}}d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)}{d^2\sqrt{d-c^2dx^2}}+4c^2\left(\frac{2bc\sqrt{1-c^2x^2}\left(\frac{a+b\arccos(}{2c^2(1-c^2x}\right.}{3d^2\sqrt{d-c^2}}\right.\right.$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\left(-\int \frac{a+b\arccos(cx)}{cx\sqrt{1-c^2x^2}}d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)-\frac{a+b\arccos(cx)}{2x^2(1-c^2x^2)}-\frac{1}{2}bc\left(\frac{2c^2x}{\sqrt{1-c^2x^2}}-\frac{}{x\sqrt{}}\right.\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 4919

$$2c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\left(-2\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)}{d^2\sqrt{d-c^2dx^2}}+4c^2\left(\right.\right.$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\left(-2\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)-\frac{a+b\arccos(}{2x^2(1-c^2}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 3042

$$2c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\left(-2\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)}{d^2\sqrt{d-c^2dx^2}}+4c^2\left(\right.\right.$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\left(-2\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}+\frac{bcx}{2\sqrt{1-c^2x^2}}\right)-\frac{a+b\arccos(}{2x^2(1-c^2}\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 4671

$$2c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\left(-2\left(-\frac{1}{2}b\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+\frac{1}{2}b\int\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-(a\right.\right.}{d^2\sqrt{d-c^2dx^2}}\right.$$

$$\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\left(-2\left(-\frac{1}{2}b\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+\frac{1}{2}b\int\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-(\mathrm{arc}\right.\right.\right.}{3d^2\sqrt{d-}}$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$\big\downarrow$ 2715

$$2\left(4\left(\frac{x(a+b\arccos(cx))^2}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{2bc\sqrt{1-c^2x^2}\left(\frac{bx}{2c\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\left(\frac{x(a+b\arccos(cx))^2}{d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{1-c^2x^2}\left(-\frac{i(a+\right.}{}\right)}{}\right.\right.\right.$$

$$\frac{2b\sqrt{1-c^2x^2}\left(2\left(\frac{bcx}{2\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{2(1-c^2x^2)}-2\left(-\left((a+b\arccos(cx))\text{arctanh}\left(e^{2i\arccos(cx)}\right)\right)+\frac{1}{4}ib\int e^{-2i\arccos(cx)}\text{lo}\right.\right.\right.}{}$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

$\downarrow$ 2838

$$-\frac{2bc\sqrt{1-c^2x^2}\left(2c^2\left(-2\left(-\left(\text{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)-\frac{1}{4}ib\,\text{Poly}\right.\right.\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$2c^2\left(-\frac{2bc\sqrt{1-c^2x^2}\left(-2\left(-\left(\text{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)-\frac{1}{4}ib\,\text{Pol}\right.\right.}{d^2\sqrt{d-c^2dx^2}}\right.$$

$$\frac{(a+b\arccos(cx))^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

output
```
-1/3*(a + b*ArcCos[c*x])^2/(d*x^3*(d - c^2*d*x^2)^(3/2)) - (2*b*c*Sqrt[1 -
 c^2*x^2]*(-1/2*(b*c*(-(1/(x*Sqrt[1 - c^2*x^2])) + (2*c^2*x)/Sqrt[1 - c^2*
x^2])) - (a + b*ArcCos[c*x])/(2*x^2*(1 - c^2*x^2)) + 2*c^2*((b*c*x)/(2*Sqr
t[1 - c^2*x^2]) + (a + b*ArcCos[c*x])/(2*(1 - c^2*x^2)) - 2*(-((a + b*ArcC
os[c*x])*ArcTanh[E^((2*I)*ArcCos[c*x])]) + (I/4)*b*PolyLog[2, -E^((2*I)*Ar
cCos[c*x])] - (I/4)*b*PolyLog[2, E^((2*I)*ArcCos[c*x])]))))/(3*d^2*Sqrt[d
- c^2*d*x^2]) + 2*c^2*(-((a + b*ArcCos[c*x])^2/(d*x*(d - c^2*d*x^2)^(3/2))
) - (2*b*c*Sqrt[1 - c^2*x^2]*((b*c*x)/(2*Sqrt[1 - c^2*x^2]) + (a + b*ArcCo
s[c*x])/(2*(1 - c^2*x^2)) - 2*(-((a + b*ArcCos[c*x])*ArcTanh[E^((2*I)*ArcC
os[c*x])]) + (I/4)*b*PolyLog[2, -E^((2*I)*ArcCos[c*x])] - (I/4)*b*PolyLog[
2, E^((2*I)*ArcCos[c*x])])))/(d^2*Sqrt[d - c^2*d*x^2]) + 4*c^2*((x*(a + b*
ArcCos[c*x])^2)/(3*d*(d - c^2*d*x^2)^(3/2)) + (2*b*c*Sqrt[1 - c^2*x^2]*((b
*x)/(2*c*Sqrt[1 - c^2*x^2]) + (a + b*ArcCos[c*x])/(2*c^2*(1 - c^2*x^2))))/
(3*d^2*Sqrt[d - c^2*d*x^2]) + (2*((x*(a + b*ArcCos[c*x])^2)/(d*Sqrt[d - c^
2*d*x^2]) - (2*b*Sqrt[1 - c^2*x^2]*(((-1/2*I)*(a + b*ArcCos[c*x])^2)/b - (
2*I)*((I/2)*(a + b*ArcCos[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])] + (b*PolyLo
g[2, E^((2*I)*ArcCos[c*x])])/4)))/(c*d*Sqrt[d - c^2*d*x^2])))/(3*d)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5229 vs. $2(521) = 1042$.

Time = 0.79 (sec) , antiderivative size = 5230, normalized size of antiderivative = 9.72

method	result	size
default	Expression too large to display	5230
parts	Expression too large to display	5230

input
```
int((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4 (d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2)/(c^6*d^3*x^10 - 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 - d^3*x^4), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^2}{x^4(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x**4/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(x**4*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(-c^2dx^2+d)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
-1/3*a*b*c*(8*c^2*log(c*x + 1)/d^(5/2) + 8*c^2*log(c*x - 1)/d^(5/2) + 16*c
^2*log(x)/d^(5/2) + 1/(c^2*d^(5/2)*x^4 - d^(5/2)*x^2)) + 2/3*(16*c^4*x/(sq
rt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x^2 + d)^(3/2)*d) - 6*c^2/((-c^
2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)^(3/2)*d*x^3))*a*b*arccos(c*x
) + 1/3*(16*c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x^2 + d)^(
3/2)*d) - 6*c^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)^(3/2)*d
*x^3))*a^2 + b^2*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2/((
c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/s
qrt(d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^4\,(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^4\,(d-c^2dx^2)^{5/2}}\,dx = \int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{x^4\,\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^4*(d - c^2*d*x^2)^(5/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x^4*(d - c^2*d*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^4\,(d-c^2dx^2)^{5/2}}\,dx = \frac{6\sqrt{-c^2x^2+1}\left(\int \frac{acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^8-2\sqrt{-c^2x^2+1}\,c^2x^6+\sqrt{-c^2x^2+1}\,x^4}dx\right)ab\,c^2x^5-6\sqrt{-c^2x}}{}$$

input
```
int((a+b*acos(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(6*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1)*c**4*x**8
- 2*sqrt( - c**2*x**2 + 1)*c**2*x**6 + sqrt( - c**2*x**2 + 1)*x**4),x)*a*b
*c**2*x**5 - 6*sqrt( - c**2*x**2 + 1)*int(acos(c*x)/(sqrt( - c**2*x**2 + 1
)*c**4*x**8 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**6 + sqrt( - c**2*x**2 + 1)*
x**4),x)*a*b*x**3 + 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)**2/(sqrt( - c**
2*x**2 + 1)*c**4*x**8 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**6 + sqrt( - c**2*
x**2 + 1)*x**4),x)*b**2*c**2*x**5 - 3*sqrt( - c**2*x**2 + 1)*int(acos(c*x)
**2/(sqrt( - c**2*x**2 + 1)*c**4*x**8 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**6
 + sqrt( - c**2*x**2 + 1)*x**4),x)*b**2*x**3 + 16*a**2*c**6*x**6 - 24*a**2
*c**4*x**4 + 6*a**2*c**2*x**2 + a**2)/(3*sqrt(d)*sqrt( - c**2*x**2 + 1)*d*
*2*x**3*(c**2*x**2 - 1))
```

3.266 $\int \frac{x^4 \arccos(ax)^2}{\sqrt{1-a^2x^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 157

$$\int \frac{x^4 \arccos(ax)^2}{\sqrt{1-a^2x^2}} \, dx = \frac{15x\sqrt{1-a^2x^2}}{64a^4} + \frac{x^3\sqrt{1-a^2x^2}}{32a^2} - \frac{15\arccos(ax)}{64a^5} + \frac{3x^2\arccos(ax)}{8a^3} + \frac{x^4\arccos(ax)}{8a} - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)^2}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2} + \frac{\arccos(ax)^3}{8a^5}$$

output
```
15/64*x*(-a^2*x^2+1)^(1/2)/a^4+1/32*x^3*(-a^2*x^2+1)^(1/2)/a^2-15/64*arcco
s(a*x)/a^5+3/8*x^2*arccos(a*x)/a^3+1/8*x^4*arccos(a*x)/a-3/8*x*(-a^2*x^2+1
)^(1/2)*arccos(a*x)^2/a^4-1/4*x^3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/a^2+1/8
*arccos(a*x)^3/a^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.67

$$\int \frac{x^4 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{-ax\sqrt{1-a^2x^2}(15+2a^2x^2)+8a^2x^2(3+a^2x^2)\arccos(ax)+8ax\sqrt{1-a^2x^2}(3+2a^2x^2)\arccos(ax)^2+}{64a^5}$$

input
```
Integrate[(x^4*ArcCos[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/64*(-(a*x*Sqrt[1 - a^2*x^2]*(15 + 2*a^2*x^2)) + 8*a^2*x^2*(3 + a^2*x^2)
*ArcCos[a*x] + 8*a*x*Sqrt[1 - a^2*x^2]*(3 + 2*a^2*x^2)*ArcCos[a*x]^2 + 8*A
rcCos[a*x]^3 + 15*ArcSin[a*x])/a^5
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.44, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5211, 5139, 262, 262, 223, 5211, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{5211}$$

$$\frac{3\int \frac{x^2\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\int x^3\arccos(ax)dx}{2a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}$$

$$\Big\downarrow \text{5139}$$

$$\frac{3\int \frac{x^2\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\frac{1}{4}a\int \frac{x^4}{\sqrt{1-a^2x^2}}dx+\frac{1}{4}x^4\arccos(ax)}{2a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{3\int \frac{x^2 \arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\frac{1}{4}a\left(\frac{3\int \frac{x^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)}{2a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{3\int \frac{x^2 \arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\frac{1}{4}a\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)}{2a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{3\int \frac{x^2 \arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)}{2a}$$

$$\Big\downarrow \text{5211}$$

$$\frac{3\left(\frac{\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int x\arccos(ax)dx}{a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)}{2a}$$

$$\Big\downarrow \text{5139}$$

$$\frac{3\left(-\frac{\frac{1}{2}a\int \frac{x^2}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)}{a}+\frac{\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)}{2a}$$

$$\Big\downarrow \text{262}$$

$$\frac{3\left(-\frac{\frac{1}{2}a\left(\frac{\int\frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)}{a}+\frac{\int\frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)}{2a}$$

$$\Big\downarrow \textbf{223}$$

$$\frac{3\left(\frac{\int\frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}-\frac{\frac{1}{2}a\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)}{a}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)}{2a}$$

$$\Big\downarrow \textbf{5153}$$

$$-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{4a^2}+\frac{3\left(-\frac{\arccos(ax)^3}{6a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}-\frac{\frac{1}{2}a\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)}{a}\right)}{4a^2}-\frac{\frac{1}{4}a\left(\frac{3\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)}{2a}$$

input
```
Int[(x^4*ArcCos[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/4*(x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a^2 - ((x^4*ArcCos[a*x])/4 + (a
*(-1/4*(x^3*Sqrt[1 - a^2*x^2])/a^2 + (3*(-1/2*(x*Sqrt[1 - a^2*x^2])/a^2 +
ArcSin[a*x]/(2*a^3)))/(4*a^2)))/4)/(2*a) + (3*(-1/2*(x*Sqrt[1 - a^2*x^2]*A
rcCos[a*x]^2)/a^2 - ArcCos[a*x]^3/(6*a^3) - ((x^2*ArcCos[a*x])/2 + (a*(-1/
2*(x*Sqrt[1 - a^2*x^2])/a^2 + ArcSin[a*x]/(2*a^3)))/2)/a))/(4*a^2)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.82

method	result
default	$-\frac{16 \arccos(ax)^2 \sqrt{-a^2x^2+1}\, a^3x^3+8a^4x^4 \arccos(ax)-2a^3x^3\sqrt{-a^2x^2+1}+24 \arccos(ax)^2 \sqrt{-a^2x^2+1}\, ax+24a^2x^2 \arccos(ax)+8 \arccos(ax)^3-15\sqrt{-a^2x^2+1}\, ax-15\arccos(ax)}{64a^5}$

input
```
int(x^4*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64*(16*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)*a^3*x^3+8*a^4*x^4*arccos(a*x)-2
*a^3*x^3*(-a^2*x^2+1)^(1/2)+24*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)*a*x+24*a^2
*x^2*arccos(a*x)+8*arccos(a*x)^3-15*(-a^2*x^2+1)^(1/2)*a*x-15*arccos(a*x))
/a^5
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.54

$$\int \frac{x^4 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\, dx = -\frac{8 \arccos(ax)^3 + (8 a^4x^4 + 24 a^2x^2 - 15) \arccos(ax) - (2 a^3x^3 - 8 (2 a^3x^3 + 3 ax) \arccos(ax)^2 + 15 ax)\sqrt{-a^2x^2+1}}{64 a^5}$$

input
```
integrate(x^4*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/64*(8*arccos(a*x)^3 + (8*a^4*x^4 + 24*a^2*x^2 - 15)*arccos(a*x) - (2*a^
3*x^3 - 8*(2*a^3*x^3 + 3*a*x)*arccos(a*x)^2 + 15*a*x)*sqrt(-a^2*x^2 + 1))/
a^5
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.97

$$\int \frac{x^4 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$
$$= \begin{cases} -\frac{x^4 \operatorname{acos}(ax)}{8a} - \frac{x^3\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{4a^2} + \frac{x^3\sqrt{-a^2x^2+1}}{32a^2} - \frac{3x^2\operatorname{acos}(ax)}{8a^3} - \frac{3x\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{8a^4} + \frac{15x\sqrt{-a^2x^2+1}}{64a^4} - \frac{\operatorname{acos}}{8} \\ \frac{\pi^2x^5}{20} \end{cases}$$

input
```
integrate(x**4*acos(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x**4*acos(a*x)/(8*a) - x**3*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/
(4*a**2) + x**3*sqrt(-a**2*x**2 + 1)/(32*a**2) - 3*x**2*acos(a*x)/(8*a**3)
 - 3*x*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/(8*a**4) + 15*x*sqrt(-a**2*x**2 +
 1)/(64*a**4) - acos(a*x)**3/(8*a**5) + 15*acos(a*x)/(64*a**5), Ne(a, 0)),
 (pi**2*x**5/20, True))
```

Maxima [F]

$$\int \frac{x^4 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4 \arccos(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4*arccos(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85

$$\int \frac{x^4 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^4\arccos(ax)}{8a} - \frac{\sqrt{-a^2x^2+1}x^3\arccos(ax)^2}{4a^2} + \frac{\sqrt{-a^2x^2+1}x^3}{32a^2} - \frac{3x^2\arccos(ax)}{8a^3} - \frac{3\sqrt{-a^2x^2+1}x\arccos(ax)^2}{8a^4} - \frac{\arccos(ax)^3}{8a^5} + \frac{15\sqrt{-a^2x^2+1}x}{64a^4} + \frac{15\arccos(ax)}{64a^5}$$

input
```
integrate(x^4*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/8*x^4*arccos(a*x)/a - 1/4*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)^2/a^2 + 1/
32*sqrt(-a^2*x^2 + 1)*x^3/a^2 - 3/8*x^2*arccos(a*x)/a^3 - 3/8*sqrt(-a^2*x^
2 + 1)*x*arccos(a*x)^2/a^4 - 1/8*arccos(a*x)^3/a^5 + 15/64*sqrt(-a^2*x^2 +
 1)*x/a^4 + 15/64*arccos(a*x)/a^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4\,\mathrm{acos}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^4*acos(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^4*acos(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\, dx = \int \frac{acos(ax)^2 x^4}{\sqrt{-a^2x^2+1}} dx$$

input
```
int(x^4*acos(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acos(a*x)**2*x**4)/sqrt( - a**2*x**2 + 1),x)
```

3.267 $\int \frac{x^3 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{x^3 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{14\sqrt{1-a^2x^2}}{9a^4} - \frac{2(1-a^2x^2)^{3/2}}{27a^4} + \frac{4x\arccos(ax)}{3a^3} + \frac{2x^3\arccos(ax)}{9a} - \frac{2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}$$

output
```
14/9*(-a^2*x^2+1)^(1/2)/a^4-2/27*(-a^2*x^2+1)^(3/2)/a^4+4/3*x*arccos(a*x)/
a^3+2/9*x^3*arccos(a*x)/a-2/3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/a^4-1/3*x^2
*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/a^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.64

$$\int \frac{x^3 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{2\sqrt{1-a^2x^2}(20+a^2x^2) - 6ax(6+a^2x^2)\arccos(ax) - 9\sqrt{1-a^2x^2}(2+a^2x^2)\arccos(ax)^2}{27a^4}$$

input
```
Integrate[(x^3*ArcCos[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
(2*Sqrt[1 - a^2*x^2]*(20 + a^2*x^2) - 6*a*x*(6 + a^2*x^2)*ArcCos[a*x] - 9*
Sqrt[1 - a^2*x^2]*(2 + a^2*x^2)*ArcCos[a*x]^2)/(27*a^4)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5211, 5139, 243, 53, 2009, 5183, 5131, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arccos(ax)^2}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 5211}$$

$$\frac{2\int \frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{2\int x^2\arccos(ax)dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{2\int \frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{2\left(\frac{1}{3}a\int \frac{x^3}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)\right)}{3a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{2\int \frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{2\left(\frac{1}{6}a\int \frac{x^2}{\sqrt{1-a^2x^2}}dx^2+\frac{1}{3}x^3\arccos(ax)\right)}{3a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{2\int \frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{2\left(\frac{1}{6}a\int \left(\frac{1}{a^2\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}}{a^2}\right)dx^2+\frac{1}{3}x^3\arccos(ax)\right)}{3a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\int \frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}-\frac{2\left(\frac{1}{6}a\left(\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)+\frac{1}{3}x^3\arccos(ax)\right)}{3a}$$

$$\Big\downarrow \ 5183$$

$$\frac{2\left(-\frac{2\int \arccos(ax)dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}-\frac{2\left(\frac{1}{6}a\left(\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)+\frac{1}{3}x^3\arccos(ax)\right)}{3a}$$

$$\Big\downarrow \ 5131$$

$$\frac{2\left(-\frac{2\left(a\int \frac{x}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)\right)}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}-\frac{2\left(\frac{1}{6}a\left(\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)+\frac{1}{3}x^3\arccos(ax)\right)}{3a}$$

$$\Big\downarrow \ 241$$

$$-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}-\frac{2\left(x\arccos(ax)-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a}\right)}{3a^2}-\frac{2\left(\frac{1}{6}a\left(\frac{2(1-a^2x^2)^{3/2}}{3a^4}-\frac{2\sqrt{1-a^2x^2}}{a^4}\right)+\frac{1}{3}x^3\arccos(ax)\right)}{3a}$$

input

```
Int[(x^3*ArcCos[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output

```
-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a^2 - (2*((a*((-2*Sqrt[1 - a^2*
x^2])/a^4 + (2*(1 - a^2*x^2)^(3/2))/(3*a^4)))/6 + (x^3*ArcCos[a*x])/3))/(3
*a) + (2*(-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a^2) - (2*(-(Sqrt[1 - a^2*x^
2]/a) + x*ArcCos[a*x]))/a))/(3*a^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.01

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(9a^4x^4\arccos(ax)^2+9a^2x^2\arccos(ax)^2-6\sqrt{-a^2x^2+1}\,\arccos(ax)a^3x^3-2a^4x^4-38a^2x^2-18\arccos(ax)^2-36\arccos\right.}{27a^4(a^2x^2-1)}$
orering	$\frac{(19a^6x^6+100a^4x^4-380a^2x^2+240)\arccos(ax)^2}{27a^6x^2\sqrt{-a^2x^2+1}}-\frac{2(ax-1)(ax+1)(a^4x^4+12a^2x^2-20)\left(\frac{3x^2\arccos(ax)^2}{\sqrt{-a^2x^2+1}}-\frac{2x^3\arccos(ax)a}{-a^2x^2+1}+\frac{x^4\arccos}{(-a^2}\right.}{9x^4a^6}$

input
```
int(x^3*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27/a^4*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)*(9*a^4*x^4*arccos(a*x)^2+9*a^2*x^
2*arccos(a*x)^2-6*(-a^2*x^2+1)^(1/2)*arccos(a*x)*a^3*x^3-2*a^4*x^4-38*a^2*
x^2-18*arccos(a*x)^2-36*arccos(a*x)*(-a^2*x^2+1)^(1/2)*a*x+40)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.52

$$\int \frac{x^3\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{6\,(a^3x^3+6\,ax)\arccos(ax)-\left(2\,a^2x^2-9\,(a^2x^2+2)\arccos(ax)^2+40\right)\sqrt{-a^2x^2+1}}{27\,a^4}$$

input
```
integrate(x^3*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/27*(6*(a^3*x^3 + 6*a*x)*arccos(a*x) - (2*a^2*x^2 - 9*(a^2*x^2 + 2)*arcc
os(a*x)^2 + 40)*sqrt(-a^2*x^2 + 1))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02

$$\int \frac{x^3 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} -\frac{2x^3 \operatorname{acos}(ax)}{9a} - \frac{x^2\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{3a^2} + \frac{2x^2\sqrt{-a^2x^2+1}}{27a^2} - \frac{4x\operatorname{acos}(ax)}{3a^3} - \frac{2\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{3a^4} + \frac{40\sqrt{-a^2x^2+1}}{27a^4} & \text{for } a \\ \frac{\pi^2x^4}{16} & \text{other} \end{cases}$$

input
```
integrate(x**3*acos(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-2*x**3*acos(a*x)/(9*a) - x**2*sqrt(-a**2*x**2 + 1)*acos(a*x)**
2/(3*a**2) + 2*x**2*sqrt(-a**2*x**2 + 1)/(27*a**2) - 4*x*acos(a*x)/(3*a**3
) - 2*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/(3*a**4) + 40*sqrt(-a**2*x**2 + 1)
/(27*a**4), Ne(a, 0)), (pi**2*x**4/16, True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \frac{x^3 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\arccos(ax)^2 + \frac{2\left(\sqrt{-a^2x^2+1}x^2 + \frac{20\sqrt{-a^2x^2+1}}{a^2}\right)}{27\,a^2} - \frac{2\left(a^2x^3+6\,x\right)\arccos(ax)}{9\,a^3}$$

input
```
integrate(x^3*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arccos(a*x)^2
 + 2/27*(sqrt(-a^2*x^2 + 1)*x^2 + 20*sqrt(-a^2*x^2 + 1)/a^2)/a^2 - 2/9*(a^
2*x^3 + 6*x)*arccos(a*x)/a^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3 \operatorname{acos}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^3*acos(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^3*acos(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)^2\,x^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^3*acos(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acos(a*x)**2*x**3)/sqrt( - a**2*x**2 + 1),x)
```

3.268 $\int \frac{x^2 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{x^2 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{x\sqrt{1-a^2x^2}}{4a^2} - \frac{\arccos(ax)}{4a^3} + \frac{x^2\arccos(ax)}{2a} - \frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2} + \frac{\arccos(ax)^3}{6a^3}$$

output
```
1/4*x*(-a^2*x^2+1)^(1/2)/a^2-1/4*arccos(a*x)/a^3+1/2*x^2*arccos(a*x)/a-1/2
*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/a^2+1/6*arccos(a*x)^3/a^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int \frac{x^2 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{-3ax\sqrt{1-a^2x^2} + 6a^2x^2\arccos(ax) + 6ax\sqrt{1-a^2x^2}\arccos(ax)^2 + 2\arccos(ax)^3 + 3\arcsin(ax)}{12a^3}$$

input
```
Integrate[(x^2*ArcCos[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/12*(-3*a*x*Sqrt[1 - a^2*x^2] + 6*a^2*x^2*ArcCos[a*x] + 6*a*x*Sqrt[1 - a
^2*x^2]*ArcCos[a*x]^2 + 2*ArcCos[a*x]^3 + 3*ArcSin[a*x])/a^3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5211, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arccos(ax)^2}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{5211}$$

$$\frac{\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int x\arccos(ax)dx}{a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}$$

$$\Big\downarrow \text{5139}$$

$$-\frac{\frac{1}{2}a\int \frac{x^2}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)}{a}+\frac{\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\frac{1}{2}a\left(\frac{\int \frac{1}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)}{a}+\frac{\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}-\frac{\frac{1}{2}a\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)}{a}$$

$$\Big\downarrow \text{5153}$$

$$-\frac{\arccos(ax)^3}{6a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^2}{2a^2}-\frac{\frac{1}{2}a\left(\frac{\arcsin(ax)}{2a^3}-\frac{x\sqrt{1-a^2x^2}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)}{a}$$

input
```
Int[(x^2*ArcCos[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/2*(x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a^2 - ArcCos[a*x]^3/(6*a^3) - ((x
^2*ArcCos[a*x])/2 + (a*(-1/2*(x*Sqrt[1 - a^2*x^2])/a^2 + ArcSin[a*x]/(2*a^
3)))/2)/a
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{6\arccos(ax)^{2}\sqrt{-a^{2}x^{2}+1}\,ax+6a^{2}x^{2}\arccos(ax)+2\arccos(ax)^{3}-3\sqrt{-a^{2}x^{2}+1}\,ax-3\arccos(ax)}{12a^{3}}$	71

input
```
int(x^2*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(6*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)*a*x+6*a^2*x^2*arccos(a*x)+2*arcc
os(a*x)^3-3*(-a^2*x^2+1)^(1/2)*a*x-3*arccos(a*x))/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

$$\int \frac{x^2\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{2\arccos(ax)^3+3\,(2\,a^2x^2-1)\arccos(ax)+3\,\sqrt{-a^2x^2+1}\,(2\,ax\arccos(ax)^2-ax)}{12\,a^3}$$

input
```
integrate(x^2*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*(2*arccos(a*x)^3 + 3*(2*a^2*x^2 - 1)*arccos(a*x) + 3*sqrt(-a^2*x^2 +
 1)*(2*a*x*arccos(a*x)^2 - a*x))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int \frac{x^2\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} -\frac{x^2\operatorname{acos}(ax)}{2a}-\frac{x\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{2a^2}+\frac{x\sqrt{-a^2x^2+1}}{4a^2}-\frac{\operatorname{acos}^3(ax)}{6a^3}+\frac{\operatorname{acos}(ax)}{4a^3} & \text{for}\ a\neq 0 \\ \frac{\pi^2x^3}{12} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*acos(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x**2*acos(a*x)/(2*a) - x*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/(2*
a**2) + x*sqrt(-a**2*x**2 + 1)/(4*a**2) - acos(a*x)**3/(6*a**3) + acos(a*x
)/(4*a**3), Ne(a, 0)), (pi**2*x**3/12, True))
```

Maxima [F]

$$\int \frac{x^2 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2 \arccos{(ax)}^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arccos(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

$$\int \frac{x^2 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^2 \arccos{(ax)}}{2\,a} - \frac{\sqrt{-a^2x^2+1}x \arccos{(ax)}^2}{2\,a^2} - \frac{\arccos{(ax)}^3}{6\,a^3} + \frac{\sqrt{-a^2x^2+1}x}{4\,a^2} + \frac{\arccos{(ax)}}{4\,a^3}$$

input
```
integrate(x^2*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*x^2*arccos(a*x)/a - 1/2*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)^2/a^2 - 1/6*
arccos(a*x)^3/a^3 + 1/4*sqrt(-a^2*x^2 + 1)*x/a^2 + 1/4*arccos(a*x)/a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2\,\mathrm{acos}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^2*acos(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^2*acos(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2 \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)^2\,x^2}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^2*acos(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acos(a*x)**2*x**2)/sqrt( - a**2*x**2 + 1),x)
```

3.269 $\int \frac{x \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{x \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{2\sqrt{1-a^2x^2}}{a^2} + \frac{2x \arccos(ax)}{a} - \frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}$$

output
```
2*(-a^2*x^2+1)^(1/2)/a^2+2*x*arccos(a*x)/a-(-a^2*x^2+1)^(1/2)*arccos(a*x)^
2/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{x \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{2\sqrt{1-a^2x^2} - 2ax\arccos(ax) - \sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}$$

input
```
Integrate[(x*ArcCos[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
(2*Sqrt[1 - a^2*x^2] - 2*a*x*ArcCos[a*x] - Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2
)/a^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5183, 5131, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arccos(ax)^2}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 5183}$$

$$-\frac{2\int \arccos(ax)dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}$$

$$\Big\downarrow \text{ 5131}$$

$$-\frac{2\left(a\int \frac{x}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)\right)}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}$$

$$\Big\downarrow \text{ 241}$$

$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}-\frac{2\left(x\arccos(ax)-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a}$$

input
```
Int[(x*ArcCos[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a^2) - (2*(-(Sqrt[1 - a^2*x^2]/a) + x*
ArcCos[a*x]))/a
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\left(a^2x^2\arccos(ax)^2-\arccos(ax)^2-2\arccos(ax)\sqrt{-a^2x^2+1}\,ax-2a^2x^2+2\right)}{a^2(a^2x^2-1)}$
orering	$\frac{(a^4x^4-4a^2x^2+2)\arccos(ax)^2}{a^4x^2\sqrt{-a^2x^2+1}}+\frac{2(ax-1)(ax+1)\left(\frac{\arccos(ax)^2}{\sqrt{-a^2x^2+1}}-\frac{2x\arccos(ax)a}{-a^2x^2+1}+\frac{x^2\arccos(ax)^2a^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\right)}{x^2a^4}+\frac{(ax-1)^2(ax+1)^2\left(-\frac{4\,\mathrm{arc}}{-a}\right.}{}$

input
```
int(x*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)*(a^2*x^2*arccos(a*x)^2-arccos(a*x)^2
-2*arccos(a*x)*(-a^2*x^2+1)^(1/2)*a*x-2*a^2*x^2+2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

$$\int \frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx=-\frac{2\,ax\arccos\left(ax\right)+\sqrt{-a^2x^2+1}\left(\arccos\left(ax\right)^2-2\right)}{a^2}$$

input
```
integrate(x*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(2*a*x*arccos(a*x) + sqrt(-a^2*x^2 + 1)*(arccos(a*x)^2 - 2))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{x \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} -\frac{2x\,\mathrm{acos}\,(ax)}{a} - \frac{\sqrt{-a^2x^2+1}\,\mathrm{acos}^2\,(ax)}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^2} & \text{for } a \neq 0 \\ \frac{\pi^2x^2}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acos(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-2*x*acos(a*x)/a - sqrt(-a**2*x**2 + 1)*acos(a*x)**2/a**2 + 2*s
qrt(-a**2*x**2 + 1)/a**2, Ne(a, 0)), (pi**2*x**2/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{x \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{-a^2x^2+1}\arccos{(ax)}^2}{a^2} - \frac{2\left(ax\arccos{(ax)} - \sqrt{-a^2x^2+1}\right)}{a^2}$$

input
```
integrate(x*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-a^2*x^2 + 1)*arccos(a*x)^2/a^2 - 2*(a*x*arccos(a*x) - sqrt(-a^2*x^2
 + 1))/a^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{x \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{-a^2x^2+1}\arccos{(ax)}^2}{a^2} - \frac{2\left(ax\arccos{(ax)} - \sqrt{-a^2x^2+1}\right)}{a^2}$$

input
```
integrate(x*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-a^2*x^2 + 1)*arccos(a*x)^2/a^2 - 2*(a*x*arccos(a*x) - sqrt(-a^2*x^2
 + 1))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\mathrm{acos}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x*acos(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x*acos(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{x \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{-\sqrt{-a^2x^2+1}\,acos(ax)^2 - 2acos(ax)\,ax + 2\sqrt{-a^2x^2+1}}{a^2}$$

input
```
int(x*acos(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
( - sqrt( - a**2*x**2 + 1)*acos(a*x)**2 - 2*acos(a*x)*a*x + 2*sqrt( - a**2
*x**2 + 1))/a**2
```

3.270 $\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 13

$$\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}} dx = \frac{\arccos(ax)^3}{3a}$$

output
```
1/3*arccos(a*x)^3/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}} dx = -\frac{\arccos(ax)^3}{3a}$$

input
```
Integrate[ArcCos[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/3*ArcCos[a*x]^3/a
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$
$$\Big\downarrow \text{ 5153}$$
$$-\frac{\arccos(ax)^3}{3a}$$

input
```
Int[ArcCos[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/3*ArcCos[a*x]^3/a
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{\arccos(ax)^3}{3a}$	12
default	$-\frac{\arccos(ax)^3}{3a}$	12

input
```
int(arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*arccos(a*x)^3/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos{(ax)}^3}{3\,a}$$

input
```
integrate(arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*arccos(a*x)^3/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} -\frac{\operatorname{acos}^3{(ax)}}{3a} & \text{for } a \neq 0 \\ \frac{\pi^2 x}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(acos(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-acos(a*x)**3/(3*a), Ne(a, 0)), (pi**2*x/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos{(ax)}^3}{3\,a}$$

input
```
integrate(arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*arccos(a*x)^3/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos{(ax)}^3}{3\,a}$$

input
```
integrate(arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/3*arccos(a*x)^3/a
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{\mathrm{acos}(a\,x)^3}{3\,a}$$

input
```
int(acos(a*x)^2/(1 - a^2*x^2)^(1/2),x)
```

output
```
-acos(a*x)^3/(3*a)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{acos(ax)^3}{3a}$$

input
```
int(acos(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
( - acos(a*x)**3)/(3*a)
```

3.271 $\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = -2\arccos(ax)^2\operatorname{arctanh}\left(e^{i\arccos(ax)}\right) + 2i\arccos(ax)\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right) - 2i\arccos(ax)\operatorname{PolyLog}\left(2,e^{i\arccos(ax)}\right) - 2\operatorname{PolyLog}\left(3,-e^{i\arccos(ax)}\right) + 2\operatorname{PolyLog}\left(3,e^{i\arccos(ax)}\right)$$

output
```
-2*arccos(a*x)^2*arctanh(a*x+I*(-a^2*x^2+1)^(1/2))+2*I*arccos(a*x)*polylog
(2,-a*x-I*(-a^2*x^2+1)^(1/2))-2*I*arccos(a*x)*polylog(2,a*x+I*(-a^2*x^2+1)
^(1/2))-2*polylog(3,-a*x-I*(-a^2*x^2+1)^(1/2))+2*polylog(3,a*x+I*(-a^2*x^2
+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = -2\left(-i\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right) + i\arccos(ax)\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) - i\arccos(ax)\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right) - \operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right) + \operatorname{PolyLog}\left(3,ie^{i\arccos(ax)}\right)\right)$$

input
```
Integrate[ArcCos[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
-2*((-I)*ArcCos[a*x]^2*ArcTan[E^(I*ArcCos[a*x])] + I*ArcCos[a*x]*PolyLog[2
, (-I)*E^(I*ArcCos[a*x])] - I*ArcCos[a*x]*PolyLog[2, I*E^(I*ArcCos[a*x])]
- PolyLog[3, (-I)*E^(I*ArcCos[a*x])] + PolyLog[3, I*E^(I*ArcCos[a*x])])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{5219}$$

$$-\int \frac{\arccos(ax)^2}{ax}d\arccos(ax)$$

$$\Big\downarrow \text{3042}$$

$$-\int \arccos(ax)^2\csc\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)$$

$$\Big\downarrow \text{4669}$$

$$2\int \arccos(ax)\log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)-2\int \arccos(ax)\log\left(1+ie^{i\arccos(ax)}\right)d\arccos(ax)+2i\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right)$$

$$\Big\downarrow \text{3011}$$

$$-2\left(i\arccos(ax)\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-i\int \text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)d\arccos(ax)\right)+2\left(i\arccos(ax)\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right)-i\int \text{PolyLog}\left(2,ie^{i\arccos(ax)}\right)d\arccos(ax)\right)+2i\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right)$$

$$\Big\downarrow \text{2720}$$

$$-2\left(i\arccos(ax)\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-\int e^{-i\arccos(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)de^{i\arccos(ax)}\right)+2\left(i\arccos(ax)\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)-\int e^{-i\arccos(ax)}\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)de^{i\arccos(ax)}\right)+2i\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right)$$

$$\Big\downarrow \text{7143}$$

$$2i\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right)-2\left(i\arccos(ax)\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)\right)+2\left(i\arccos(ax)\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)-\operatorname{PolyLog}\left(3,ie^{i\arccos(ax)}\right)\right)$$

input
```
Int[ArcCos[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
(2*I)*ArcCos[a*x]^2*ArcTan[E^(I*ArcCos[a*x])] - 2*(I*ArcCos[a*x]*PolyLog[2
, (-I)*E^(I*ArcCos[a*x])] - PolyLog[3, (-I)*E^(I*ArcCos[a*x])]) + 2*(I*Arc
Cos[a*x]*PolyLog[2, I*E^(I*ArcCos[a*x])] - PolyLog[3, I*E^(I*ArcCos[a*x])]
)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\arccos(ax)^2}{x\sqrt{-a^2x^2+1}}dx$$

input
```
int(arccos(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(arccos(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos{(ax)}^2}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccos(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccos(a*x)^2/(a^2*x^3 - x), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acos}^2{(ax)}}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acos(a*x)**2/x/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acos(a*x)**2/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos{(ax)}^2}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccos(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

Giac [F]

$$\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos(ax)^2}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccos(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^2}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acos(a*x)^2/(x*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acos(a*x)^2/(x*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)^2}{\sqrt{-a^2x^2+1}\,x}dx$$

input
```
int(acos(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acos(a*x)**2/(sqrt( - a**2*x**2 + 1)*x),x)
```

3.272 $\int \frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = -ia\arccos(ax)^2 - \frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x} + 2a\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right) - ia\,\text{PolyLog}\left(2,e^{2i\arccos(ax)}\right)$$

output
```
-I*a*arccos(a*x)^2-(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/x+2*a*arccos(a*x)*ln(1
-(a*x+I*(-a^2*x^2+1)^(1/2))^2)-I*a*polylog(2,(a*x+I*(-a^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos(ax)\left(\left(-iax+\sqrt{1-a^2x^2}\right)\arccos(ax)+2ax\log\left(1+e^{2i\arccos(ax)}\right)\right)}{x} + ia\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)$$

input
```
Integrate[ArcCos[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((ArcCos[a*x]*(((-I)*a*x + Sqrt[1 - a^2*x^2])*ArcCos[a*x] + 2*a*x*Log[1 +
 E^((2*I)*ArcCos[a*x])]))/x) + I*a*PolyLog[2, -E^((2*I)*ArcCos[a*x])]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5187, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{5187}$$

$$-2a\int \frac{\arccos(ax)}{x}dx - \frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}$$

$$\Big\downarrow \text{5137}$$

$$2a\int \frac{\sqrt{1-a^2x^2}\arccos(ax)}{ax}d\arccos(ax) - \frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}$$

$$\Big\downarrow \text{3042}$$

$$2a\int \arccos(ax)\tan(\arccos(ax))d\arccos(ax) - \frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}$$

$$\Big\downarrow \text{4202}$$

$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x} + 2a\left(\frac{1}{2}i\arccos(ax)^2 - 2i\int \frac{e^{2i\arccos(ax)}\arccos(ax)}{1+e^{2i\arccos(ax)}}d\arccos(ax)\right)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x} + 2a\left(\frac{1}{2}i\arccos(ax)^2 - 2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\arccos(ax)}\right)d\arccos(ax) - \frac{1}{2}i\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right)\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}+$$
$$2a\left(\frac{1}{2}i\arccos(ax)^2-2i\left(\frac{1}{4}\int e^{-2i\arccos(ax)}\log\left(1+e^{2i\arccos(ax)}\right)de^{2i\arccos(ax)}-\frac{1}{2}i\arccos(ax)\log\left(1+e^{2i\arccos(\right.\right.\right.$$

$$\Big\downarrow \text{2838}$$

$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{x}+$$
$$2a\left(\frac{1}{2}i\arccos(ax)^2-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{1}{2}i\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right)\right)\right)$$

input
```
Int[ArcCos[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/x) + 2*a*((I/2)*ArcCos[a*x]^2 - (2*I)*
((-1/2*I)*ArcCos[a*x]*Log[1 + E^((2*I)*ArcCos[a*x])] - PolyLog[2, -E^((2*I
)*ArcCos[a*x])]/4))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\left(iax+\sqrt{-a^2x^2+1}\right)\arccos(ax)^2}{x}+ia\left(2i\arccos\left(ax\right)\ln\left(1+\left(ax+i\sqrt{-a^2x^2+1}\right)^2\right)+2\arccos\left(ax\right)^2\right.$

input
```
int(arccos(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(I*a*x+(-a^2*x^2+1)^(1/2))/x*arccos(a*x)^2+I*a*(2*I*arccos(a*x)*ln(1+(a*x
+I*(-a^2*x^2+1)^(1/2))^2)+2*arccos(a*x)^2+polylog(2,-(a*x+I*(-a^2*x^2+1)^(
1/2))^2))
```

Fricas [F]

$$\int \frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos{(ax)}^2}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arccos(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccos(a*x)^2/(a^2*x^4 - x^2), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acos}^2{(ax)}}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acos(a*x)**2/x**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acos(a*x)**2/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos{(ax)}^2}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arccos(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^
2 + 2*a*x*integrate(arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)/x, x))/x
```

Giac [F]

$$\int \frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx=\int \frac{\arccos{(ax)}^2}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arccos(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx=\int \frac{\mathrm{acos}(a\,x)^2}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acos(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acos(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx=\int \frac{acos(ax)^2}{\sqrt{-a^2x^2+1}\,x^2}dx$$

input
```
int(acos(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acos(a*x)**2/(sqrt( - a**2*x**2 + 1)*x**2),x)
```

3.273 $\int \frac{\arccos(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\begin{aligned}\int \frac{\arccos(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx &= -\frac{a\arccos(ax)}{x}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}\\ &\quad -a^2\arccos(ax)^2\text{arctanh}\left(e^{i\arccos(ax)}\right)-a^2\text{arctanh}\left(\sqrt{1-a^2x^2}\right)\\ &\quad +ia^2\arccos(ax)\,\text{PolyLog}\left(2,-e^{i\arccos(ax)}\right)\\ &\quad -ia^2\arccos(ax)\,\text{PolyLog}\left(2,e^{i\arccos(ax)}\right)\\ &\quad -a^2\,\text{PolyLog}\left(3,-e^{i\arccos(ax)}\right)+a^2\,\text{PolyLog}\left(3,e^{i\arccos(ax)}\right)\end{aligned}$$

output
```
-a*arccos(a*x)/x-1/2*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/x^2-a^2*arccos(a*x)^
2*arctanh(a*x+I*(-a^2*x^2+1)^(1/2))-a^2*arctanh((-a^2*x^2+1)^(1/2))+I*a^2*
arccos(a*x)*polylog(2,-a*x-I*(-a^2*x^2+1)^(1/2))-I*a^2*arccos(a*x)*polylog
(2,a*x+I*(-a^2*x^2+1)^(1/2))-a^2*polylog(3,-a*x-I*(-a^2*x^2+1)^(1/2))+a^2*
polylog(3,a*x+I*(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.08

$$\int \frac{\arccos(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \frac{a\arccos(ax)}{x} - \frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2} - a^2\coth^{-1}\left(\sqrt{1-a^2x^2}\right) + ia^2\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right) - ia^2\arccos(ax)\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) + ia^2\arccos(ax)\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right) + a^2\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right) - a^2\operatorname{PolyLog}\left(3,ie^{i\arccos(ax)}\right)$$

input
```
Integrate[ArcCos[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
(a*ArcCos[a*x])/x - (Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(2*x^2) - a^2*ArcCot
h[Sqrt[1 - a^2*x^2]] + I*a^2*ArcCos[a*x]^2*ArcTan[E^(I*ArcCos[a*x])] - I*a
^2*ArcCos[a*x]*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] + I*a^2*ArcCos[a*x]*Poly
Log[2, I*E^(I*ArcCos[a*x])] + a^2*PolyLog[3, (-I)*E^(I*ArcCos[a*x])] - a^2
*PolyLog[3, I*E^(I*ArcCos[a*x])]
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5205, 5139, 243, 73, 221, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^2}{x^3\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{ 5205}$$

$$\frac{1}{2}a^2\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}dx - a\int \frac{\arccos(ax)}{x^2}dx - \frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{1}{2}a^2\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}dx-a\left(-a\int \frac{1}{x\sqrt{1-a^2x^2}}dx-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}a^2\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}dx-a\left(-\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}a^2\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}dx-a\left(\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a}-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}a^2\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}dx-a\left(a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 5219}$$

$$-\frac{1}{2}a^2\int \frac{\arccos(ax)^2}{ax}d\arccos(ax)-a\left(a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{2}a^2\int \arccos(ax)^2\csc\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)-a\left(a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 4669}$$

$$-\frac{1}{2}a^2\left(-2\int \arccos(ax)\log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)+2\int \arccos(ax)\log\left(1+ie^{i\arccos(ax)}\right)d\arccos(ax)\right.$$
$$a\left(a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{1}{2}a^2\left(2\left(i\arccos(ax)\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-i\int \text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)d\arccos(ax)\right)-2\left(i\arccos(\right.\right.$$
$$a\left(a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}$$

$$\Big\downarrow \text{ 2720}$$

$$-\frac{1}{2}a^2\left(2\left(i\arccos(ax)\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-\int e^{-i\arccos(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)de^{i\arccos(ax)}\right)-2\right.$$
$$a\left(a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{1}{2}a^2\left(-2i\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right)+2\left(i\arccos(ax)\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arccos}\right.\right.\right.$$
$$a\left(a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)-\frac{\arccos(ax)}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2}$$

input
```
Int[ArcCos[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
-1/2*(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/x^2 - a*(-(ArcCos[a*x]/x) + a*ArcTa
nh[Sqrt[1 - a^2*x^2]]) - (a^2*((-2*I)*ArcCos[a*x]^2*ArcTan[E^(I*ArcCos[a*x
])] + 2*(I*ArcCos[a*x]*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - PolyLog[3, (-I
)*E^(I*ArcCos[a*x])]) - 2*(I*ArcCos[a*x]*PolyLog[2, I*E^(I*ArcCos[a*x])] -
 PolyLog[3, I*E^(I*ArcCos[a*x])])))/2
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.71

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\,\arccos(ax)\left(a^2x^2\arccos(ax)+2\sqrt{-a^2x^2+1}\,ax-\arccos(ax)\right)}{2x^2(a^2x^2-1)}+\frac{ia^2\left(i\arccos(ax)^2\ln\left(1-i\left(ax+i\sqrt{-a^2x^2+1}\right)\right)-i\,a\right.}{}$

input
```
int(arccos(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-a^2*x^2+1)^(1/2)/x^2/(a^2*x^2-1)*arccos(a*x)*(a^2*x^2*arccos(a*x)+2
*(-a^2*x^2+1)^(1/2)*a*x-arccos(a*x))+1/2*I*a^2*(I*arccos(a*x)^2*ln(1-I*(a*
x+I*(-a^2*x^2+1)^(1/2)))-I*arccos(a*x)^2*ln(1+I*(a*x+I*(-a^2*x^2+1)^(1/2))
)+2*arccos(a*x)*polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))-2*arccos(a*x)*poly
log(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))+2*I*polylog(3,I*(a*x+I*(-a^2*x^2+1)^(
1/2)))-2*I*polylog(3,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))+4*arctan(a*x+I*(-a^2*x
^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\arccos(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos(ax)^2}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccos(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccos(a*x)^2/(a^2*x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acos}^2(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acos(a*x)**2/x**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acos(a*x)**2/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos(ax)^2}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccos(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{\arccos(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos(ax)^2}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccos(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^2}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acos(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acos(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)^2}{\sqrt{-a^2x^2+1}\,x^3}dx$$

input
```
int(acos(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acos(a*x)**2/(sqrt( - a**2*x**2 + 1)*x**3),x)
```

3.274 $\int \frac{\arccos(ax)^2}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 42

$$\int \frac{\arccos(ax)^2}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{3a\sqrt{c-a^2cx^2}}$$

output
```
1/3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/a/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^2}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{3a\sqrt{c-a^2cx^2}}$$

input
```
Integrate[ArcCos[a*x]^2/Sqrt[c - a^2*c*x^2],x]
```

output
```
-1/3*(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(a*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^2}{\sqrt{c-a^2cx^2}}\,dx$$

$$\Big\downarrow \ 5153$$

$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{3a\sqrt{c-a^2cx^2}}$$

input
```
Int[ArcCos[a*x]^2/Sqrt[c - a^2*c*x^2],x]
```

output
```
-1/3*(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(a*Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{\sqrt{-c(a^2x^2-1)}\,\sqrt{-a^2x^2+1}\,\arccos(ax)^3}{3c(a^2x^2-1)a}$	52

input
```
int(arccos(a*x)^2/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/c/(a^2*x^2-1)/a*arccos(a*x)^
3
```

Fricas [F]

$$\int \frac{\arccos(ax)^2}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\arccos\left(ax\right)^2}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(arccos(a*x)^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*c*x^2 + c)*arccos(a*x)^2/(a^2*c*x^2 - c), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\operatorname{acos}^2\left(ax\right)}{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(acos(a*x)**2/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(acos(a*x)**2/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{\arccos(ax)^2}{\sqrt{c-a^2cx^2}}\,dx = \frac{\arccos(ax)^2\arcsin(ax)}{a\sqrt{c}} + \frac{\arccos(ax)\arcsin(ax)^2}{a\sqrt{c}} + \frac{\arcsin(ax)^3}{3\,a\sqrt{c}}$$

input
```
integrate(arccos(a*x)^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
arccos(a*x)^2*arcsin(a*x)/(a*sqrt(c)) + arccos(a*x)*arcsin(a*x)^2/(a*sqrt(
c)) + 1/3*arcsin(a*x)^3/(a*sqrt(c))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.33

$$\int \frac{\arccos(ax)^2}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\arccos(ax)^3}{3\,a\sqrt{c}}$$

input
```
integrate(arccos(a*x)^2/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/3*arccos(a*x)^3/(a*sqrt(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^2}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

input
```
int(acos(a*x)^2/(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(acos(a*x)^2/(c - a^2*c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.38

$$\int \frac{\arccos(ax)^2}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\sqrt{c}\,acos(ax)^3}{3ac}$$

input
```
int(acos(a*x)^2/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c)*acos(a*x)**3)/(3*a*c)
```

3.275 $\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 179

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{3/2}}\,dx = \frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}} - \frac{i\sqrt{1-a^2x^2}\arccos(ax)^2}{ac\sqrt{c-a^2cx^2}} + \frac{2\sqrt{1-a^2x^2}\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right)}{ac\sqrt{c-a^2cx^2}} - \frac{i\sqrt{1-a^2x^2}\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)}{ac\sqrt{c-a^2cx^2}}$$

output
```
x*arccos(a*x)^2/c/(-a^2*c*x^2+c)^(1/2)-I*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/
a/c/(-a^2*c*x^2+c)^(1/2)+2*(-a^2*x^2+1)^(1/2)*arccos(a*x)*ln(1+(a*x+I*(-a^
2*x^2+1)^(1/2))^2)/a/c/(-a^2*c*x^2+c)^(1/2)-I*(-a^2*x^2+1)^(1/2)*polylog(2
,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.62

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{3/2}}\,dx =$$
$$-\frac{-\arccos(ax)\left(ax\arccos(ax)+i\sqrt{1-a^2x^2}\left(\arccos(ax)+2i\log\left(1-e^{2i\arccos(ax)}\right)\right)\right)-i\sqrt{1-a^2x^2}\,\text{PolyLo}}{ac\sqrt{c\left(1-a^2x^2\right)}}$$

input
```
Integrate[ArcCos[a*x]^2/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-((-(ArcCos[a*x]*(a*x*ArcCos[a*x] + I*Sqrt[1 - a^2*x^2]*(ArcCos[a*x] + (2*
I)*Log[1 - E^((2*I)*ArcCos[a*x])]))) - I*Sqrt[1 - a^2*x^2]*PolyLog[2, E^((
2*I)*ArcCos[a*x])])/(a*c*Sqrt[c*(1 - a^2*x^2)]))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.69, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5161}$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{1-a^2x^2}dx}{c\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 5181}$$

$$\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\int \frac{ax\arccos(ax)}{\sqrt{1-a^2x^2}}d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\int -\arccos(ax)\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\sqrt{1-a^2x^2}\int \arccos(ax)\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{4200}$$

$$\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(2i\int -\frac{e^{2i\arccos(ax)}\arccos(ax)}{1-e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{2}i\arccos(ax)^2\right)}{ac\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(-2i\int \frac{e^{2i\arccos(ax)}\arccos(ax)}{1-e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{2}i\arccos(ax)^2\right)}{ac\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right)-\frac{1}{2}i\int\log\left(1-e^{2i\arccos(ax)}\right)d\arccos(ax)\right)-\frac{1}{2}i\arccos(ax)^2\right.}{ac\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\arccos(ax)}\log\left(1-e^{2i\arccos(ax)}\right)de^{2i\arccos(ax)}\right)-\frac{1}{2}\right.}{ac\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{4}\text{PolyLog}\left(2,e^{2i\arccos(ax)}\right)+\frac{1}{2}i\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right)\right)-\frac{1}{2}i\arccos(ax)^2\right)}{ac\sqrt{c-a^2cx^2}}$$

input
```
Int[ArcCos[a*x]^2/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(x*ArcCos[a*x]^2)/(c*Sqrt[c - a^2*c*x^2]) - (2*Sqrt[1 - a^2*x^2]*((-1/2*I)
*ArcCos[a*x]^2 - (2*I)*((I/2)*ArcCos[a*x]*Log[1 - E^((2*I)*ArcCos[a*x])] +
 PolyLog[2, E^((2*I)*ArcCos[a*x])]/4)))/(a*c*Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.19

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\left(-i\sqrt{-a^2x^2+1}+ax\right)\arccos(ax)^2}{(a^2x^2-1)a\,c^2}-\frac{2i\sqrt{-a^2x^2+1}\,\sqrt{-c(a^2x^2-1)}\left(i\arccos(ax)\ln\left(1-ax-i\sqrt{-a^2x^2+1}\right)+i\,\mathrm{arc}\right.}{}$

input
```
int(arccos(a*x)^2/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-c*(a^2*x^2-1))^(1/2)*(-I*(-a^2*x^2+1)^(1/2)+a*x)*arccos(a*x)^2/(a^2*x^2
-1)/a/c^2-2*I*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(I*arccos(a*x)*ln(
1-a*x-I*(-a^2*x^2+1)^(1/2))+I*arccos(a*x)*ln(1+a*x+I*(-a^2*x^2+1)^(1/2))+a
rccos(a*x)^2+polylog(2,a*x+I*(-a^2*x^2+1)^(1/2))+polylog(2,-a*x-I*(-a^2*x^
2+1)^(1/2)))/(a^2*x^2-1)/a/c^2
```

Fricas [F]

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{3/2}}\,dx=\int \frac{\arccos\left(ax\right)^2}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*arccos(a*x)^2/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 +
 c^2), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{acos}^2(ax)}{(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(acos(a*x)**2/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(acos(a*x)**2/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\arccos(ax)^2}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)^2/(-a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\arccos(ax)^2}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^2/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^2}{\left(c-a^2\,c\,x^2\right)^{3/2}}\,dx$$

input
```
int(acos(a*x)^2/(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(acos(a*x)^2/(c - a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{3/2}}\,dx = -\frac{\int \frac{acos(ax)^2}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(acos(a*x)^2/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
( - int(acos(a*x)**2/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*x**2
 + 1)),x))/(sqrt(c)*c)
```

3.276 $\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 283

$$\begin{aligned}\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{5/2}}\,dx &= \frac{x}{3c^2\sqrt{c-a^2cx^2}}-\frac{\arccos(ax)}{3ac^2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}} \\ &+\frac{x\arccos(ax)^2}{3c\,(c-a^2cx^2)^{3/2}}+\frac{2x\arccos(ax)^2}{3c^2\sqrt{c-a^2cx^2}}-\frac{2i\sqrt{1-a^2x^2}\arccos(ax)^2}{3ac^2\sqrt{c-a^2cx^2}} \\ &+\frac{4\sqrt{1-a^2x^2}\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right)}{3ac^2\sqrt{c-a^2cx^2}} \\ &-\frac{2i\sqrt{1-a^2x^2}\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)}{3ac^2\sqrt{c-a^2cx^2}}\end{aligned}$$

output
```
1/3*x/c^2/(-a^2*c*x^2+c)^(1/2)-1/3*arccos(a*x)/a/c^2/(-a^2*x^2+1)^(1/2)/(-
a^2*c*x^2+c)^(1/2)+1/3*x*arccos(a*x)^2/c/(-a^2*c*x^2+c)^(3/2)+2/3*x*arccos
(a*x)^2/c^2/(-a^2*c*x^2+c)^(1/2)-2/3*I*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/a/
c^2/(-a^2*c*x^2+c)^(1/2)+4/3*(-a^2*x^2+1)^(1/2)*arccos(a*x)*ln(1+(a*x+I*(-
a^2*x^2+1)^(1/2))^2)/a/c^2/(-a^2*c*x^2+c)^(1/2)-2/3*I*(-a^2*x^2+1)^(1/2)*p
olylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c^2/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.51

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{5/2}}\,dx = \frac{(1-a^2x^2)^{3/2}\left(-\frac{ax(-1+a^2x^2+(-3+2a^2x^2)\arccos(ax)^2)}{(1-a^2x^2)^{3/2}}+\arccos(ax)\left(\frac{1}{1-a^2x^2}+2i\arccos(a\right.\right.}{3ac\,(c-a^2cx^2)^{3/2}}$$

input
```
Integrate[ArcCos[a*x]^2/(c - a^2*c*x^2)^(5/2),x]
```

output
```
((1 - a^2*x^2)^(3/2)*(-((a*x*(-1 + a^2*x^2 + (-3 + 2*a^2*x^2)*ArcCos[a*x]^
2))/(1 - a^2*x^2)^(3/2)) + ArcCos[a*x]*((1 - a^2*x^2)^(-1) + (2*I)*ArcCos[
a*x] - 4*Log[1 - E^((2*I)*ArcCos[a*x])]) + (2*I)*PolyLog[2, E^((2*I)*ArcCo
s[a*x])]))/(3*a*c*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.86, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {5163, 5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838, 5183, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 5163$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{(1-a^2x^2)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{3/2}}dx}{3c}+\frac{x\arccos(ax)^2}{3c\,(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \; 5161$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{(1-a^2x^2)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{1-a^2x^2}dx}{c\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^2}{3c\,(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \; 5181$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x \arccos(ax)}{(1-a^2x^2)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x \arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\int \frac{ax \arccos(ax)}{\sqrt{1-a^2x^2}}d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x \arccos(ax)^2}{3c(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x \arccos(ax)}{(1-a^2x^2)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x \arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\int -\arccos(ax)\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x \arccos(ax)^2}{3c(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x \arccos(ax)}{(1-a^2x^2)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{2\sqrt{1-a^2x^2}\int \arccos(ax)\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}+\frac{x \arccos(ax)^2}{c\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x \arccos(ax)^2}{3c(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 4200}$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x \arccos(ax)}{(1-a^2x^2)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x \arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(2i\int -\frac{e^{2i\arccos(ax)}\arccos(ax)}{1-e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{2}i\arccos(ax)^2\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x \arccos(ax)^2}{3c(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x \arccos(ax)}{(1-a^2x^2)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x \arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(-2i\int \frac{e^{2i\arccos(ax)}\arccos(ax)}{1-e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{2}i\arccos(ax)^2\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x \arccos(ax)^2}{3c(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{\left(1-a^2x^2\right)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right)-\frac{1}{2}i\int\log\left(1-e^{2i\arccos(ax)}\right)d\arccos(ax)\right)-\frac{1}{2}i\arccos(ax)^2\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^2}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{\left(1-a^2x^2\right)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\arccos(ax)}\log\left(1-e^{2i\arccos(ax)}\right)de^{2i\arccos(ax)}\right)-\frac{1}{2}i\arccos(ax)^2\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^2}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{\left(1-a^2x^2\right)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)+\frac{1}{2}i\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right)\right)-\frac{1}{2}i\arccos(ax)^2\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^2}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{5183}$$

$$\frac{2a\sqrt{1-a^2x^2}\left(\frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}dx}{2a}+\frac{\arccos(ax)}{2a^2\left(1-a^2x^2\right)}\right)}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)+\frac{1}{2}i\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right)\right)-\frac{1}{2}i\arccos(ax)^2\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^2}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2a\sqrt{1-a^2x^2}\left(\frac{\arccos(ax)}{2a^2(1-a^2x^2)}+\frac{x}{2a\sqrt{1-a^2x^2}}\right)}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)+\frac{1}{2}i\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right)\right)-\frac{1}{2}i\arccos(ax)^2\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^2}{3c\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Int[ArcCos[a*x]^2/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(x*ArcCos[a*x]^2)/(3*c*(c - a^2*c*x^2)^(3/2)) + (2*a*Sqrt[1 - a^2*x^2]*(x/
(2*a*Sqrt[1 - a^2*x^2]) + ArcCos[a*x]/(2*a^2*(1 - a^2*x^2))))/(3*c^2*Sqrt[
c - a^2*c*x^2]) + (2*((x*ArcCos[a*x]^2)/(c*Sqrt[c - a^2*c*x^2]) - (2*Sqrt[
1 - a^2*x^2]*((-1/2*I)*ArcCos[a*x]^2 - (2*I)*((I/2)*ArcCos[a*x]*Log[1 - E^
((2*I)*ArcCos[a*x])] + PolyLog[2, E^((2*I)*ArcCos[a*x])]/4)))/(a*c*Sqrt[c
- a^2*c*x^2])))/(3*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.45

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\left(-2i\sqrt{-a^2x^2+1}\,a^2x^2+2a^3x^3+2i\sqrt{-a^2x^2+1}-3ax\right)\left(-2i\arccos(ax)a^4x^4+2\sqrt{-a^2x^2+1}\,\arccos(ax)a^3x^3-i\sqrt{-a}\right.}{3c^3(3a}$

input
```
int(arccos(a*x)^2/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-c*(a^2*x^2-1))^(1/2)*(-2*I*(-a^2*x^2+1)^(1/2)*a^2*x^2+2*a^3*x^3+2*I
*(-a^2*x^2+1)^(1/2)-3*a*x)*(-2*I*arccos(a*x)*a^4*x^4+2*(-a^2*x^2+1)^(1/2)*
arccos(a*x)*a^3*x^3-I*(-a^2*x^2+1)^(1/2)*a^3*x^3-a^4*x^4+3*a^2*x^2*arccos(
a*x)^2+4*I*arccos(a*x)*a^2*x^2-3*arccos(a*x)*(-a^2*x^2+1)^(1/2)*a*x+I*(-a^
2*x^2+1)^(1/2)*a*x+3*a^2*x^2-4*arccos(a*x)^2-2*I*arccos(a*x)-2)/c^3/(3*a^6
*x^6-10*a^4*x^4+11*a^2*x^2-4)/a-4/3*I*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^
(1/2)*(I*arccos(a*x)*ln(1-a*x-I*(-a^2*x^2+1)^(1/2))+I*arccos(a*x)*ln(1+a*x
+I*(-a^2*x^2+1)^(1/2))+arccos(a*x)^2+polylog(2,a*x+I*(-a^2*x^2+1)^(1/2))+p
olylog(2,-a*x-I*(-a^2*x^2+1)^(1/2)))/c^3/(a^2*x^2-1)/a
```

Fricas [F]

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\arccos{(ax)}^2}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*c*x^2 + c)*arccos(a*x)^2/(a^6*c^3*x^6 - 3*a^4*c^3*x^4
+ 3*a^2*c^3*x^2 - c^3), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{acos}^2(ax)}{(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(acos(a*x)**2/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(acos(a*x)**2/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\arccos(ax)^2}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)^2/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^2}{(c-a^2\,c\,x^2)^{5/2}}\,dx$$

input
```
int(acos(a*x)^2/(c - a^2*c*x^2)^(5/2),x)
```

output
```
int(acos(a*x)^2/(c - a^2*c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{acos(ax)^2}{\sqrt{-a^2x^2+1}\,a^4x^4-2\sqrt{-a^2x^2+1}\,a^2x^2+\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(acos(a*x)^2/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
int(acos(a*x)**2/(sqrt( - a**2*x**2 + 1)*a**4*x**4 - 2*sqrt( - a**2*x**2 +
 1)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

3.277 $\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 390

$$\begin{aligned}\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{7/2}}\,dx &= \frac{x}{3c^3\sqrt{c-a^2cx^2}}+\frac{x}{30c^3(1-a^2x^2)\sqrt{c-a^2cx^2}}\\
&-\frac{\arccos(ax)}{10ac^3(1-a^2x^2)^{3/2}\sqrt{c-a^2cx^2}}-\frac{4\arccos(ax)}{15ac^3\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}\\
&+\frac{x\arccos(ax)^2}{5c(c-a^2cx^2)^{5/2}}+\frac{4x\arccos(ax)^2}{15c^2(c-a^2cx^2)^{3/2}}+\frac{8x\arccos(ax)^2}{15c^3\sqrt{c-a^2cx^2}}\\
&-\frac{8i\sqrt{1-a^2x^2}\arccos(ax)^2}{15ac^3\sqrt{c-a^2cx^2}}+\frac{16\sqrt{1-a^2x^2}\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right)}{15ac^3\sqrt{c-a^2cx^2}}\\
&-\frac{8i\sqrt{1-a^2x^2}\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)}{15ac^3\sqrt{c-a^2cx^2}}\end{aligned}$$

output
```
1/3*x/c^3/(-a^2*c*x^2+c)^(1/2)+1/30*x/c^3/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2
)-1/10*arccos(a*x)/a/c^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2)-4/15*arcc
os(a*x)/a/c^3/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2)+1/5*x*arccos(a*x)^2/
c/(-a^2*c*x^2+c)^(5/2)+4/15*x*arccos(a*x)^2/c^2/(-a^2*c*x^2+c)^(3/2)+8/15*
x*arccos(a*x)^2/c^3/(-a^2*c*x^2+c)^(1/2)-8/15*I*(-a^2*x^2+1)^(1/2)*arccos(
a*x)^2/a/c^3/(-a^2*c*x^2+c)^(1/2)+16/15*(-a^2*x^2+1)^(1/2)*arccos(a*x)*ln(
1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c^3/(-a^2*c*x^2+c)^(1/2)-8/15*I*(-a^2*x^
2+1)^(1/2)*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c^3/(-a^2*c*x^2+c)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.44

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\sqrt{1-a^2x^2}\left(\frac{ax(11-21a^2x^2+10a^4x^4+2(15-20a^2x^2+8a^4x^4)\arccos(ax)^2)}{(1-a^2x^2)^{5/2}}+\arccos(ax)\left(\frac{11-8a^2x^2}{(-1+a^2x^2)^2}\right.\right.}{30ac^3\sqrt{c-}}$$

input
```
Integrate[ArcCos[a*x]^2/(c - a^2*c*x^2)^(7/2),x]
```

output
```
(Sqrt[1 - a^2*x^2]*((a*x*(11 - 21*a^2*x^2 + 10*a^4*x^4 + 2*(15 - 20*a^2*x^
2 + 8*a^4*x^4)*ArcCos[a*x]^2))/(1 - a^2*x^2)^(5/2) + ArcCos[a*x]*((11 - 8*
a^2*x^2)/(-1 + a^2*x^2)^2 + (16*I)*ArcCos[a*x] - 32*Log[1 - E^((2*I)*ArcCo
s[a*x])]) + (16*I)*PolyLog[2, E^((2*I)*ArcCos[a*x])]))/(30*a*c^3*Sqrt[c -
a^2*c*x^2])
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.99, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {5163, 5163, 5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838, 5183, 208, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{7/2}}\,dx$$

$$\Big\downarrow \; 5163$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{5/2}}dx}{5c}+\frac{x\arccos(ax)^2}{5c\,(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \; 5163$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{\left(1-a^2x^2\right)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\int \frac{\arccos(ax)^2}{\left(c-a^2cx^2\right)^{3/2}}dx}{3c}+\frac{x\arccos(ax)^2}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{x\arccos(ax)^2}{5c\,(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \; 5161$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{\left(1-a^2x^2\right)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{1-a^2x^2}dx}{c\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^2}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{x\arccos(ax)^2}{5c\,(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \; 5181$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{\left(1-a^2x^2\right)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\int \frac{ax\arccos(ax)}{\sqrt{1-a^2x^2}}d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^2}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{x\arccos(ax)^2}{5c\,(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{\left(1-a^2x^2\right)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\int -\arccos(ax)\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^2}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{x\arccos(ax)^2}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{\left(1-a^2x^2\right)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{2\sqrt{1-a^2x^2}\int \arccos(ax)\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^2}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{x\arccos(ax)^2}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 4200$$

$$\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{\left(1-a^2x^2\right)^2}dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(2i\int -\frac{e^{2i\arccos(ax)}\arccos(ax)}{1-e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{2}i\arccos(ax)^2\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^2}{3c(c-a^2cx^2)^{3/}}\right.}{5c}$$

$$\frac{x\arccos(ax)^2}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}} + 4\left(\frac{2a\sqrt{1-a^2x^2}\int\frac{x\arccos(ax)}{\left(1-a^2x^2\right)^2}dx}{3c^2\sqrt{c-a^2cx^2}} + \frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}} - \frac{2\sqrt{1-a^2x^2}\left(-2i\int\frac{e^{2i\arccos(ax)}\arccos(ax)}{1-e^{2i\arccos(ax)}}d\arccos(ax) - \frac{1}{2}i\arccos(ax)^2\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c} + \frac{x\arccos(ax)^2}{3c(c-a^2cx^2)^{3/}}\right)}{5c}$$

$$\frac{x\arccos(ax)^2}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$\Big\downarrow$ 2620

$$\frac{\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}} + 4\left(\frac{2a\sqrt{1-a^2x^2}\int\frac{x\arccos(ax)}{\left(1-a^2x^2\right)^2}dx}{3c^2\sqrt{c-a^2cx^2}} + \frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}} - \frac{2\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right) - \frac{1}{2}i\int\log\left(1-e^{2i\arccos(ax)}\right)d\arccos(ax)\right) - \right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}\right)}{5c}$$

$$\frac{x\arccos(ax)^2}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$\Big\downarrow$ 2715

$$\frac{\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}} + 4\left(\frac{2a\sqrt{1-a^2x^2}\int\frac{x\arccos(ax)}{\left(1-a^2x^2\right)^2}dx}{3c^2\sqrt{c-a^2cx^2}} + \frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}} - \frac{2\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right) - \frac{1}{4}\int e^{-2i\arccos(ax)}\log\left(1-e^{2i\arccos(ax)}\right)\right)\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}\right)}{5c}$$

$$\frac{x\arccos(ax)^2}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$\Big\downarrow$ 2838

$$\frac{\frac{2a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}} + 4\left(\frac{2a\sqrt{1-a^2x^2}\int\frac{x\arccos(ax)}{\left(1-a^2x^2\right)^2}dx}{3c^2\sqrt{c-a^2cx^2}} + \frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}} - \frac{2\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right) + \frac{1}{2}i\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right)\right) - \frac{1}{2}i\arccos(a\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}\right)}{5c}$$

$$\frac{x\arccos(ax)^2}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 5183$$

$$\frac{2a\sqrt{1-a^2x^2}\left(\frac{\int \frac{1}{\left(1-a^2x^2\right)^{5/2}}dx}{4a}+\frac{\arccos(ax)}{4a^2(1-a^2x^2)^2}\right)}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{2a\sqrt{1-a^2x^2}\left(\frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}dx}{2a}+\frac{\arccos(ax)}{2a^2\left(1-a^2x^2\right)}\right)}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{4}\text{PolyLog}\left(2,e^{2i\arccos(ax)}\right)+\frac{1}{2}i\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}\right.}{3c}\right.}{5c}$$

$$\frac{x\arccos(ax)^2}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{2a\sqrt{1-a^2x^2}\left(\frac{\int \frac{1}{\left(1-a^2x^2\right)^{5/2}}dx}{4a}+\frac{\arccos(ax)}{4a^2(1-a^2x^2)^2}\right)}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{2a\sqrt{1-a^2x^2}\left(\frac{\arccos(ax)}{2a^2\left(1-a^2x^2\right)}+\frac{x}{2a\sqrt{1-a^2x^2}}\right)}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{4}\text{PolyLog}\left(2,e^{2i\arccos(ax)}\right)+\frac{1}{2}i\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}\right.}{3c}\right.}{5c}$$

$$\frac{x\arccos(ax)^2}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 209$$

$$\frac{2a\sqrt{1-a^2x^2}\left(\frac{\frac{2}{3}\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}dx+\frac{x}{3\left(1-a^2x^2\right)^{3/2}}}{4a}+\frac{\arccos(ax)}{4a^2(1-a^2x^2)^2}\right)}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{2a\sqrt{1-a^2x^2}\left(\frac{\arccos(ax)}{2a^2\left(1-a^2x^2\right)}+\frac{x}{2a\sqrt{1-a^2x^2}}\right)}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{4}\text{PolyLog}\left(2,e^{2i\arccos(ax)}\right)+\frac{1}{2}i\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}\right.}{3c}\right.}{5c}$$

$$\frac{x\arccos(ax)^2}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{2a\sqrt{1-a^2x^2}\left(\frac{\arccos(ax)}{4a^2(1-a^2x^2)^2}+\frac{\frac{2x}{3\sqrt{1-a^2x^2}}+\frac{x}{3\left(1-a^2x^2\right)^{3/2}}}{4a}\right)}{5c^3\sqrt{c-a^2cx^2}}+$$

$$\frac{4\left(\frac{2a\sqrt{1-a^2x^2}\left(\frac{\arccos(ax)}{2a^2\left(1-a^2x^2\right)}+\frac{x}{2a\sqrt{1-a^2x^2}}\right)}{3c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^2}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)+\frac{1}{2}i\arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}\right.}{3c}\right.}{5c}$$

$$\frac{x\arccos(ax)^2}{5c\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Int[ArcCos[a*x]^2/(c - a^2*c*x^2)^(7/2),x]
```

output
```
(x*ArcCos[a*x]^2)/(5*c*(c - a^2*c*x^2)^(5/2)) + (2*a*Sqrt[1 - a^2*x^2]*((x
/(3*(1 - a^2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 - a^2*x^2]))/(4*a) + ArcCos[a*x
]/(4*a^2*(1 - a^2*x^2)^2)))/(5*c^3*Sqrt[c - a^2*c*x^2]) + (4*((x*ArcCos[a*
x]^2)/(3*c*(c - a^2*c*x^2)^(3/2)) + (2*a*Sqrt[1 - a^2*x^2]*(x/(2*a*Sqrt[1
- a^2*x^2]) + ArcCos[a*x]/(2*a^2*(1 - a^2*x^2))))/(3*c^2*Sqrt[c - a^2*c*x^
2]) + (2*((x*ArcCos[a*x]^2)/(c*Sqrt[c - a^2*c*x^2]) - (2*Sqrt[1 - a^2*x^2]
*((-1/2*I)*ArcCos[a*x]^2 - (2*I)*((I/2)*ArcCos[a*x]*Log[1 - E^((2*I)*ArcCo
s[a*x])] + PolyLog[2, E^((2*I)*ArcCos[a*x])]/4)))/(a*c*Sqrt[c - a^2*c*x^2]
)))/(3*c)))/(5*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\left(-8i\sqrt{-a^2x^2+1}\,a^4x^4+8a^5x^5+16i\sqrt{-a^2x^2+1}\,a^2x^2-20a^3x^3-8i\sqrt{-a^2x^2+1}+15ax\right)\left(456i\arccos(ax)a^4x^4-64a\right.}{}$

input
```
int(arccos(a*x)^2/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*(-c*(a^2*x^2-1))^(1/2)*(-8*I*(-a^2*x^2+1)^(1/2)*a^4*x^4+8*a^5*x^5+16
*I*(-a^2*x^2+1)^(1/2)*a^2*x^2-20*a^3*x^3-8*I*(-a^2*x^2+1)^(1/2)+15*a*x)*(4
56*I*arccos(a*x)*a^4*x^4-64*arccos(a*x)*(-a^2*x^2+1)^(1/2)*a^7*x^7-280*I*a
rccos(a*x)*a^6*x^6+32*a^8*x^8+32*I*(-a^2*x^2+1)^(1/2)*a^7*x^7+248*arccos(a
*x)*(-a^2*x^2+1)^(1/2)*a^5*x^5+64*I*arccos(a*x)*a^8*x^8-142*a^6*x^6+80*a^4
*x^4*arccos(a*x)^2-126*I*(-a^2*x^2+1)^(1/2)*a^5*x^5-340*(-a^2*x^2+1)^(1/2)
*arccos(a*x)*a^3*x^3+156*I*(-a^2*x^2+1)^(1/2)*a^3*x^3+265*a^4*x^4-190*a^2*
x^2*arccos(a*x)^2+88*I*arccos(a*x)+165*arccos(a*x)*(-a^2*x^2+1)^(1/2)*a*x-
62*I*(-a^2*x^2+1)^(1/2)*a*x-235*a^2*x^2+128*arccos(a*x)^2-328*I*arccos(a*x
)*a^2*x^2+80)/c^4/(40*a^10*x^10-215*a^8*x^8+469*a^6*x^6-517*a^4*x^4+287*a^
2*x^2-64)/a-16/15*I*(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(I*arccos(a*
x)*ln(1-a*x-I*(-a^2*x^2+1)^(1/2))+I*arccos(a*x)*ln(1+a*x+I*(-a^2*x^2+1)^(1
/2))+arccos(a*x)^2+polylog(2,a*x+I*(-a^2*x^2+1)^(1/2))+polylog(2,-a*x-I*(-
a^2*x^2+1)^(1/2)))/c^4/(a^2*x^2-1)/a
```

Fricas [F]

$$\int \frac{\arccos(ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\int \frac{\arccos\left(ax\right)^2}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^2/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*arccos(a*x)^2/(a^8*c^4*x^8 - 4*a^6*c^4*x^6 +
 6*a^4*c^4*x^4 - 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\int \frac{\operatorname{acos}^2\left(ax\right)}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(acos(a*x)**2/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Integral(acos(a*x)**2/(-c*(a*x - 1)*(a*x + 1))**(7/2), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx=\int \frac{\arccos\left(ax\right)^2}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^2/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)^2/(-a^2*c*x^2 + c)^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^2/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^2}{(c-a^2\,c\,x^2)^{7/2}}\,dx$$

input
```
int(acos(a*x)^2/(c - a^2*c*x^2)^(7/2),x)
```

output
```
int(acos(a*x)^2/(c - a^2*c*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^2}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\int \frac{\mathrm{acos}(ax)^2}{\sqrt{-a^2x^2+1}\,a^6x^6-3\sqrt{-a^2x^2+1}\,a^4x^4+3\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^3}$$

input
```
int(acos(a*x)^2/(-a^2*c*x^2+c)^(7/2),x)
```

output
```
( - int(acos(a*x)**2/(sqrt( - a**2*x**2 + 1)*a**6*x**6 - 3*sqrt( - a**2*x*
*2 + 1)*a**4*x**4 + 3*sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*x**2
 + 1)),x))/(sqrt(c)*c**3)
```

3.278 $\int x^m(d-c^2dx^2)^3\,(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 1312

$$\int x^m\left(d-c^2dx^2\right)^3(a+b\arccos(cx))^2\,dx=\text{Too large to display}$$

output
```
-12*b^2*c^4*d^3*x^(5+m)/(5+m)^3/(7+m)+48*d^3*x^(1+m)*(a+b*arccos(c*x))^2/(
5+m)/(7+m)/(m^2+4*m+3)+24*d^3*x^(1+m)*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(7+
m)/(m^2+8*m+15)+6*d^3*x^(1+m)*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2/(5+m)/(7+
m)+2*b^2*c^6*d^3*x^(7+m)/(7+m)^3-30*b*c*d^3*x^(2+m)*(a+b*arccos(c*x))*hype
rgeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/(5+m)/(7+m)^2/(m^2+5*m+6)+30*b^2*c
^2*d^3*x^(3+m)/(3+m)^2/(5+m)/(7+m)^2+36*b^2*c^2*d^3*x^(3+m)/(3+m)^2/(5+m)^
2/(7+m)+12*b^2*c^2*d^3*x^(3+m)/(3+m)/(5+m)^2/(7+m)+48*b^2*c^2*d^3*x^(3+m)/
(3+m)^3/(5+m)/(7+m)-2*b*c*d^3*x^(2+m)*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))
/(7+m)^2-10*b*c*d^3*x^(2+m)*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/(5+m)/(7+
m)^2-12*b*c*d^3*x^(2+m)*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/(5+m)^2/(7+m)
-36*b*c*d^3*x^(2+m)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/(3+m)/(5+m)^2/(7+
m)-48*b*c*d^3*x^(2+m)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/(3+m)^2/(5+m)/(
7+m)+2*b^2*c^2*d^3*x^(3+m)/(3+m)/(7+m)^2+10*b^2*c^2*d^3*x^(3+m)/(7+m)^2/(m
^2+8*m+15)-10*b^2*c^4*d^3*x^(5+m)/(5+m)^2/(7+m)^2-4*b^2*c^4*d^3*x^(5+m)/(5
+m)/(7+m)^2-30*b*c*d^3*x^(2+m)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/(7+m)^
2/(m^2+8*m+15)+d^3*x^(1+m)*(-c^2*x^2+1)^3*(a+b*arccos(c*x))^2/(7+m)-36*b*c
*d^3*x^(2+m)*(a+b*arccos(c*x))*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)
/(5+m)^2/(7+m)/(m^2+5*m+6)-96*b*c*d^3*x^(2+m)*(a+b*arccos(c*x))*hypergeom(
[1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/(5+m)/(7+m)/(m^3+6*m^2+11*m+6)+30*b^2*c^
2*d^3*x^(3+m)*hypergeom([1, 3/2+1/2*m, 3/2+1/2*m],[2+1/2*m, 5/2+1/2*m],...
```

Mathematica [F]

$$\int x^m \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx=\int x^m \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx$$

input
```
Integrate[x^m*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
Integrate[x^m*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2, x]
```

Rubi [A] (warning: unable to verify)

Time = 3.38 (sec) , antiderivative size = 1021, normalized size of antiderivative = 0.78, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5203, 27, 5203, 244, 2009, 5203, 244, 2009, 5139, 5199, 15, 5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(d-c^2 d x^2\right)^3 (a+b \arccos (c x))^2 \, dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{2 b c d^3 \int x^{m+1} \left(1-c^2 x^2\right)^{5/2} (a+b \arccos (c x)) dx}{m+7}+\frac{6 d \int d^2 x^m \left(1-c^2 x^2\right)^2 (a+b \arccos (c x))^2 dx}{m+7}+\frac{d^3 \left(1-c^2 x^2\right)^3 x^{m+1} (a+b \arccos (c x))^2}{m+7}$$

$$\Big\downarrow \text{27}$$

$$\frac{2 b c d^3 \int x^{m+1} \left(1-c^2 x^2\right)^{5/2} (a+b \arccos (c x)) dx}{m+7}+\frac{6 d^3 \int x^m \left(1-c^2 x^2\right)^2 (a+b \arccos (c x))^2 dx}{m+7}+\frac{d^3 \left(1-c^2 x^2\right)^3 x^{m+1} (a+b \arccos (c x))^2}{m+7}$$

$$\Big\downarrow \text{5203}$$

$$\frac{2 b c d^3 \left(\frac{5 \int x^{m+1} \left(1-c^2 x^2\right)^{3/2} (a+b \arccos (c x)) dx}{m+7}+\frac{b c \int x^{m+2} \left(1-c^2 x^2\right)^2 dx}{m+7}+\frac{\left(1-c^2 x^2\right)^{5/2} x^{m+2} (a+b \arccos (c x))}{m+7}\right)}{m+7}+\frac{6 d^3 \left(\frac{2 b c \int x^{m+1} \left(1-c^2 x^2\right)^{3/2} (a+b \arccos (c x)) dx}{m+5}+\frac{4 \int x^m \left(1-c^2 x^2\right) (a+b \arccos (c x))^2 dx}{m+5}+\frac{\left(1-c^2 x^2\right)^2 x^{m+1} (a+b \arccos (c x))^2}{m+5}\right)}{m+7}+\frac{d^3 \left(1-c^2 x^2\right)^3 x^{m+1} (a+b \arccos (c x))^2}{m+7}$$

$$\Big\downarrow \text{244}$$

$$\frac{6d^3\left(\frac{2bc\int x^{m+1}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx}{m+5}+\frac{4\int x^m\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx}{m+5}+\frac{\left(1-c^2x^2\right)^2x^{m+1}(a+b\arccos(cx))^2}{m+5}\right)}{m+7}+\frac{2bcd^3\left(\frac{5\int x^{m+1}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx}{m+7}+\frac{bc\int\left(x^{m+2}-2c^2x^{m+4}+c^4x^{m+6}\right)dx}{m+7}+\frac{\left(1-c^2x^2\right)^{5/2}x^{m+2}(a+b\arccos(cx))}{m+7}\right)}{m+7}+\frac{d^3\left(1-c^2x^2\right)^3x^{m+1}(a+b\arccos(cx))^2}{m+7}$$

$$\Big\downarrow \; 2009$$

$$\frac{6d^3\left(\frac{2bc\int x^{m+1}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx}{m+5}+\frac{4\int x^m\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx}{m+5}+\frac{\left(1-c^2x^2\right)^2x^{m+1}(a+b\arccos(cx))^2}{m+5}\right)}{m+7}+\frac{2bcd^3\left(\frac{5\int x^{m+1}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx}{m+7}+\frac{\left(1-c^2x^2\right)^{5/2}x^{m+2}(a+b\arccos(cx))}{m+7}+\frac{bc\left(\frac{c^4x^{m+7}}{m+7}-\frac{2c^2x^{m+5}}{m+5}+\frac{x^{m+3}}{m+3}\right)}{m+7}\right)}{m+7}+\frac{d^3\left(1-c^2x^2\right)^3x^{m+1}(a+b\arccos(cx))^2}{m+7}$$

$$\Big\downarrow \; 5203$$

$$\frac{6d^3\left(\frac{2bc\left(\frac{3\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+5}+\frac{bc\int x^{m+2}\left(1-c^2x^2\right)dx}{m+5}+\frac{\left(1-c^2x^2\right)^{3/2}x^{m+2}(a+b\arccos(cx))}{m+5}\right)}{m+5}+\frac{4\left(\frac{2bc\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos}{m+3}\right.}{}\right.}{m+7}+\frac{2bcd^3\left(\frac{5\left(\frac{3\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+5}+\frac{bc\int x^{m+2}\left(1-c^2x^2\right)dx}{m+5}+\frac{\left(1-c^2x^2\right)^{3/2}x^{m+2}(a+b\arccos(cx))}{m+5}\right)}{m+7}+\frac{\left(1-c^2x^2\right)^{5/2}x^{m+2}(a+b\arccos}{m+7}\right.}{m+7}$$

$$\frac{d^3\left(1-c^2x^2\right)^3x^{m+1}(a+b\arccos(cx))^2}{m+7}$$

$$\Big\downarrow \; 244$$

$$\frac{6d^3\left(\frac{2bc\left(\frac{3\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+5}+\frac{bc\int\left(x^{m+2}-c^2x^{m+4}\right)dx}{m+5}+\frac{\left(1-c^2x^2\right)^{3/2}x^{m+2}(a+b\arccos(cx))}{m+5}\right)}{m+5}+\frac{4\left(\frac{2bc\int x^{m+1}\sqrt{1-c^2x^2}(a+b\,\mathrm{ar}}{m+3}\right.}{}\right.}{m+7}$$

$$\frac{2bcd^3\left(\frac{5\left(\frac{3\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+5}+\frac{bc\int\left(x^{m+2}-c^2x^{m+4}\right)dx}{m+5}+\frac{\left(1-c^2x^2\right)^{3/2}x^{m+2}(a+b\arccos(cx))}{m+5}\right)}{m+7}+\frac{\left(1-c^2x^2\right)^{5/2}x^{m+2}(a+b\,\mathrm{arcc}}{m+7}\right.}{m+7}$$

$$\frac{d^3\left(1-c^2x^2\right)^3x^{m+1}(a+b\arccos(cx))^2}{m+7}$$

$$\Big\downarrow \ 2009$$

$$\frac{6d^3\left(\frac{2bc\left(\frac{3\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+5}+\frac{\left(1-c^2x^2\right)^{3/2}x^{m+2}(a+b\arccos(cx))}{m+5}+\frac{bc\left(\frac{x^{m+3}}{m+3}-\frac{c^2x^{m+5}}{m+5}\right)}{m+5}\right)}{m+5}+\frac{4\left(\frac{2bc\int x^{m+1}\sqrt{1-c^2x^2}(a+b\,\mathrm{arcc}}{m+3}\right.}{}\right.}{m+7}$$

$$\frac{2bcd^3\left(\frac{5\left(\frac{3\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+5}+\frac{\left(1-c^2x^2\right)^{3/2}x^{m+2}(a+b\arccos(cx))}{m+5}+\frac{bc\left(\frac{x^{m+3}}{m+3}-\frac{c^2x^{m+5}}{m+5}\right)}{m+5}\right)}{m+7}+\frac{\left(1-c^2x^2\right)^{5/2}x^{m+2}(a+b\,\mathrm{arccos}}{m+7}\right.}{m+7}$$

$$\frac{d^3\left(1-c^2x^2\right)^3x^{m+1}(a+b\arccos(cx))^2}{m+7}$$

$$\Big\downarrow \ 5139$$

$$6d^3\left(\frac{4\left(\frac{2\left(\frac{2bc\int\frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+1}+\frac{x^{m+1}(a+b\arccos(cx))^2}{m+1}\right)}{m+3}+\frac{2bc\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+3}+\frac{(1-c^2x^2)x^{m+1}(a+b\arccos(cx))^2}{m+3}\right)}{m+5}\right.$$

$$\frac{2bcd^3\left(\frac{5\left(\frac{3\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+5}+\frac{(1-c^2x^2)^{3/2}x^{m+2}(a+b\arccos(cx))}{m+5}+\frac{bc\left(\frac{x^{m+3}}{m+3}-\frac{c^2x^{m+5}}{m+5}\right)}{m+5}\right)}{m+7}+\frac{(1-c^2x^2)^{5/2}x^{m+2}(a+b\arccos}{m+7}\right.}{m+7}$$

$$\frac{d^3(1-c^2x^2)^3x^{m+1}(a+b\arccos(cx))^2}{m+7}$$

$$\downarrow 5199$$

$$6d^3\left(\frac{4\left(\frac{2\left(\frac{2bc\int\frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+1}+\frac{x^{m+1}(a+b\arccos(cx))^2}{m+1}\right)}{m+3}+\frac{2bc\left(\frac{\int\frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+3}+\frac{bc\int x^{m+2}dx}{m+3}+\frac{\sqrt{1-c^2x^2}x^{m+2}(a+b\arccos(cx}{m+3}\right.}{m+3}\right.}{m+5}\right.$$

$$\frac{2bcd^3\left(\frac{5\left(\frac{3\left(\frac{\int\frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+3}+\frac{bc\int x^{m+2}dx}{m+3}+\frac{\sqrt{1-c^2x^2}x^{m+2}(a+b\arccos(cx))}{m+3}\right)}{m+5}+\frac{(1-c^2x^2)^{3/2}x^{m+2}(a+b\arccos(cx))}{m+5}+\frac{bc\left(\frac{x^{m+3}}{m+3}-\frac{c^2x^{m+}}{m+5}\right.}{m+5}\right.}{m+7}\right.}{m+7}$$

$$\frac{d^3(1-c^2x^2)^3x^{m+1}(a+b\arccos(cx))^2}{m+7}$$

$$\Big\downarrow \; 15$$

$$6d^3\left(\frac{4\left(\frac{2\left(\frac{2bc\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+1}+\frac{x^{m+1}(a+b\arccos(cx))^2}{m+1}\right)}{m+3}+\frac{2bc\left(\frac{\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+3}+\frac{\sqrt{1-c^2x^2}\,x^{m+2}(a+b\arccos(cx))}{m+3}+\frac{bcx^{m+3}}{(m+3)^2}\right)}{m+3}\cdots\right)}{m+5}\cdots\right.$$

$$2bcd^3\left(\frac{5\left(\frac{3\left(\frac{\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+3}+\frac{\sqrt{1-c^2x^2}\,x^{m+2}(a+b\arccos(cx))}{m+3}+\frac{bcx^{m+3}}{(m+3)^2}\right)}{m+5}+\frac{\left(1-c^2x^2\right)^{3/2}x^{m+2}(a+b\arccos(cx))}{m+5}+\frac{bc\left(\frac{x^{m+3}}{m+3}-\frac{c^2x^{m+5}}{m+5}\right)}{m+5}\cdots\right)}{m+7}\cdots\right.$$

$$m+7$$

$$\frac{d^3\left(1-c^2x^2\right)^3x^{m+1}(a+b\arccos(cx))^2}{m+7}$$

$$\Big\downarrow \; 5221$$

$$\frac{d^3(1-c^2x^2)^3(a+b\arccos(cx))^2x^{m+1}}{m+7}+$$

$$6d^3\left(\frac{(1-c^2x^2)^2(a+b\arccos(cx))^2x^{m+1}}{m+5}+\frac{4\left(\frac{(1-c^2x^2)(a+b\arccos(cx))^2x^{m+1}}{m+3}+\frac{2\left(\frac{(a+b\arccos(cx))^2x^{m+1}}{m+1}+\frac{2bc\left(\frac{(a+b\arccos(cx))\text{ Hypergeometric}}{m\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$2bcd^3\left(\frac{(1-c^2x^2)^{5/2}(a+b\arccos(cx))x^{m+2}}{m+7}+\frac{bc\left(\frac{x^{m+3}}{m+3}-\frac{2c^2x^{m+5}}{m+5}+\frac{c^4x^{m+7}}{m+7}\right)}{m+7}+\frac{5\left(\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))x^{m+2}}{m+5}+\frac{bc\left(\frac{x^{m+3}}{m+3}-\frac{c^2x^{m+\cdots}}{m+5}\right.}{m+5}\right.}{\cdots}\right.$$

input
```
Int[x^m*(d - c^2*d*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^3*x^(1 + m)*(1 - c^2*x^2)^3*(a + b*ArcCos[c*x])^2)/(7 + m) + (6*d^3*((x
^(1 + m)*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/(5 + m) + (4*((x^(1 + m)*(
1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/(3 + m) + (2*((x^(1 + m)*(a + b*ArcCos
[c*x])^2)/(1 + m) + (2*b*c*((x^(2 + m)*(a + b*ArcCos[c*x])*Hypergeometric2
F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(2 + m) + (b*c*x^(3 + m)*Hypergeom
etricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, c^2*x^2])/(6 + 5
*m + m^2)))/(1 + m)))/(3 + m) + (2*b*c*((b*c*x^(3 + m))/(3 + m)^2 + (x^(2
+ m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(3 + m) + ((x^(2 + m)*(a + b*A
rcCos[c*x])*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(2 + m)
 + (b*c*x^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5
/2 + m/2}, c^2*x^2])/(6 + 5*m + m^2))/(3 + m)))/(3 + m)))/(5 + m) + (2*b*c
*((b*c*(x^(3 + m)/(3 + m) - (c^2*x^(5 + m))/(5 + m)))/(5 + m) + (x^(2 + m)
*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(5 + m) + (3*((b*c*x^(3 + m))/(3
 + m)^2 + (x^(2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(3 + m) + ((x^
(2 + m)*(a + b*ArcCos[c*x])*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c
^2*x^2])/(2 + m) + (b*c*x^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m
/2}, {2 + m/2, 5/2 + m/2}, c^2*x^2])/(6 + 5*m + m^2))/(3 + m)))/(5 + m)))/
(5 + m)))/(7 + m) + (2*b*c*d^3*((b*c*(x^(3 + m)/(3 + m) - (2*c^2*x^(5 + m)
)/(5 + m) + (c^4*x^(7 + m))/(7 + m)))/(7 + m) + (x^(2 + m)*(1 - c^2*x^2)^(
5/2)*(a + b*ArcCos[c*x]))/(7 + m) + (5*((b*c*(x^(3 + m)/(3 + m) - (c^2*...
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int x^m \left(-c^2 d\,x^2+d\right)^3 \left(a+b\arccos\left(cx\right)\right)^2 dx$$

input
```
int(x^m*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x)
```

output
```
int(x^m*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x)
```

Fricas [F]

$$\int x^m \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx=\int -\left(c^2dx^2-d\right)^3(b\arccos\left(cx\right)+a)^2x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*c^6*d^3*x^6 - 3*a^2*c^4*d^3*x^4 + 3*a^2*c^2*d^3*x^2 - a^2*d
^3 + (b^2*c^6*d^3*x^6 - 3*b^2*c^4*d^3*x^4 + 3*b^2*c^2*d^3*x^2 - b^2*d^3)*a
rccos(c*x)^2 + 2*(a*b*c^6*d^3*x^6 - 3*a*b*c^4*d^3*x^4 + 3*a*b*c^2*d^3*x^2
- a*b*d^3)*arccos(c*x))*x^m, x)
```

Sympy [F]

$$\begin{aligned}&\int x^m \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx\\&=-d^3\Bigg(\int\left(-a^2x^m\right)\,dx+\int\left(-b^2x^m\operatorname{acos}^2\left(cx\right)\right)\,dx+\int\left(-2abx^m\operatorname{acos}\left(cx\right)\right)\,dx\\&\quad+\int 3a^2c^2x^2x^m\,dx+\int\left(-3a^2c^4x^4x^m\right)\,dx+\int a^2c^6x^6x^m\,dx\\&\quad+\int 3b^2c^2x^2x^m\operatorname{acos}^2\left(cx\right)dx+\int\left(-3b^2c^4x^4x^m\operatorname{acos}^2\left(cx\right)\right)\,dx\\&\quad+\int b^2c^6x^6x^m\operatorname{acos}^2\left(cx\right)dx+\int 6abc^2x^2x^m\operatorname{acos}\left(cx\right)dx\\&\quad+\int\left(-6abc^4x^4x^m\operatorname{acos}\left(cx\right)\right)\,dx+\int 2abc^6x^6x^m\operatorname{acos}\left(cx\right)dx\Bigg)\end{aligned}$$

input
```
integrate(x**m*(-c**2*d*x**2+d)**3*(a+b*acos(c*x))**2,x)
```

output
```
-d**3*(Integral(-a**2*x**m, x) + Integral(-b**2*x**m*acos(c*x)**2, x) + In
tegral(-2*a*b*x**m*acos(c*x), x) + Integral(3*a**2*c**2*x**2*x**m, x) + In
tegral(-3*a**2*c**4*x**4*x**m, x) + Integral(a**2*c**6*x**6*x**m, x) + Int
egral(3*b**2*c**2*x**2*x**m*acos(c*x)**2, x) + Integral(-3*b**2*c**4*x**4*
x**m*acos(c*x)**2, x) + Integral(b**2*c**6*x**6*x**m*acos(c*x)**2, x) + In
tegral(6*a*b*c**2*x**2*x**m*acos(c*x), x) + Integral(-6*a*b*c**4*x**4*x**m
*acos(c*x), x) + Integral(2*a*b*c**6*x**6*x**m*acos(c*x), x))
```

Maxima [F]

$$\int x^m \left(d - c^2 d x^2\right)^3 (a + b \arccos(cx))^2 \, dx = \int -\left(c^2 d x^2 - d\right)^3 (b \arccos(cx) + a)^2 x^m \, dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-a^2*c^6*d^3*x^(m + 7)/(m + 7) + 3*a^2*c^4*d^3*x^(m + 5)/(m + 5) - 3*a^2*c
^2*d^3*x^(m + 3)/(m + 3) + a^2*d^3*x^(m + 1)/(m + 1) - (((b^2*c^6*d^3*m^3
+ 9*b^2*c^6*d^3*m^2 + 23*b^2*c^6*d^3*m + 15*b^2*c^6*d^3)*x^7 - 3*(b^2*c^4*
d^3*m^3 + 11*b^2*c^4*d^3*m^2 + 31*b^2*c^4*d^3*m + 21*b^2*c^4*d^3)*x^5 + 3*
(b^2*c^2*d^3*m^3 + 13*b^2*c^2*d^3*m^2 + 47*b^2*c^2*d^3*m + 35*b^2*c^2*d^3)
*x^3 - (b^2*d^3*m^3 + 15*b^2*d^3*m^2 + 71*b^2*d^3*m + 105*b^2*d^3)*x)*x^m*
arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 - (m^4 + 16*m^3 + 86*m^2 + 17
6*m + 105)*integrate(-2*(((b^2*c^7*d^3*m^3 + 9*b^2*c^7*d^3*m^2 + 23*b^2*c^
7*d^3*m + 15*b^2*c^7*d^3)*x^7 - 3*(b^2*c^5*d^3*m^3 + 11*b^2*c^5*d^3*m^2 +
31*b^2*c^5*d^3*m + 21*b^2*c^5*d^3)*x^5 + 3*(b^2*c^3*d^3*m^3 + 13*b^2*c^3*d
^3*m^2 + 47*b^2*c^3*d^3*m + 35*b^2*c^3*d^3)*x^3 - (b^2*c*d^3*m^3 + 15*b^2*
c*d^3*m^2 + 71*b^2*c*d^3*m + 105*b^2*c*d^3)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1
)*x^m*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - (a*b*d^3*m^4 + (a*b*c^8
*d^3*m^4 + 16*a*b*c^8*d^3*m^3 + 86*a*b*c^8*d^3*m^2 + 176*a*b*c^8*d^3*m + 1
05*a*b*c^8*d^3)*x^8 + 16*a*b*d^3*m^3 + 86*a*b*d^3*m^2 - 4*(a*b*c^6*d^3*m^4
 + 16*a*b*c^6*d^3*m^3 + 86*a*b*c^6*d^3*m^2 + 176*a*b*c^6*d^3*m + 105*a*b*c
^6*d^3)*x^6 + 176*a*b*d^3*m + 105*a*b*d^3 + 6*(a*b*c^4*d^3*m^4 + 16*a*b*c^
4*d^3*m^3 + 86*a*b*c^4*d^3*m^2 + 176*a*b*c^4*d^3*m + 105*a*b*c^4*d^3)*x^4
- 4*(a*b*c^2*d^3*m^4 + 16*a*b*c^2*d^3*m^3 + 86*a*b*c^2*d^3*m^2 + 176*a*b*c
^2*d^3*m + 105*a*b*c^2*d^3)*x^2)*x^m*arctan2(sqrt(c*x + 1)*sqrt(-c*x + ...
```

Giac [F]

$$\int x^m \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx=\int -\left(c^2dx^2-d\right)^3(b\arccos\left(cx\right)+a)^2x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(-(c^2*d*x^2 - d)^3*(b*arccos(c*x) + a)^2*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx=\int x^m\,\left(a+b\,\mathrm{acos}(c\,x)\right)^2\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

input
```
int(x^m*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^3,x)
```

output
```
int(x^m*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^3, x)
```

Reduce [F]

$$\int x^m \left(d-c^2dx^2\right)^3 (a+b\arccos(cx))^2\,dx=\text{Too large to display}$$

input
```
int(x^m*(-c^2*d*x^2+d)^3*(a+b*acos(c*x))^2,x)
```

output

```
(d**3*( - x**m*a**2*c**6*m**3*x**7 - 9*x**m*a**2*c**6*m**2*x**7 - 23*x**m*
a**2*c**6*m*x**7 - 15*x**m*a**2*c**6*x**7 + 3*x**m*a**2*c**4*m**3*x**5 + 3
3*x**m*a**2*c**4*m**2*x**5 + 93*x**m*a**2*c**4*m*x**5 + 63*x**m*a**2*c**4*
x**5 - 3*x**m*a**2*c**2*m**3*x**3 - 39*x**m*a**2*c**2*m**2*x**3 - 141*x**m
*a**2*c**2*m*x**3 - 105*x**m*a**2*c**2*x**3 + x**m*a**2*m**3*x + 15*x**m*a
**2*m**2*x + 71*x**m*a**2*m*x + 105*x**m*a**2*x - 2*int(x**m*acos(c*x)*x**
6,x)*a*b*c**6*m**4 - 32*int(x**m*acos(c*x)*x**6,x)*a*b*c**6*m**3 - 172*int
(x**m*acos(c*x)*x**6,x)*a*b*c**6*m**2 - 352*int(x**m*acos(c*x)*x**6,x)*a*b
*c**6*m - 210*int(x**m*acos(c*x)*x**6,x)*a*b*c**6 + 6*int(x**m*acos(c*x)*x
**4,x)*a*b*c**4*m**4 + 96*int(x**m*acos(c*x)*x**4,x)*a*b*c**4*m**3 + 516*i
nt(x**m*acos(c*x)*x**4,x)*a*b*c**4*m**2 + 1056*int(x**m*acos(c*x)*x**4,x)*
a*b*c**4*m + 630*int(x**m*acos(c*x)*x**4,x)*a*b*c**4 - 6*int(x**m*acos(c*x
)*x**2,x)*a*b*c**2*m**4 - 96*int(x**m*acos(c*x)*x**2,x)*a*b*c**2*m**3 - 51
6*int(x**m*acos(c*x)*x**2,x)*a*b*c**2*m**2 - 1056*int(x**m*acos(c*x)*x**2,
x)*a*b*c**2*m - 630*int(x**m*acos(c*x)*x**2,x)*a*b*c**2 + 2*int(x**m*acos(
c*x),x)*a*b*m**4 + 32*int(x**m*acos(c*x),x)*a*b*m**3 + 172*int(x**m*acos(c
*x),x)*a*b*m**2 + 352*int(x**m*acos(c*x),x)*a*b*m + 210*int(x**m*acos(c*x)
,x)*a*b - int(x**m*acos(c*x)**2*x**6,x)*b**2*c**6*m**4 - 16*int(x**m*acos(
c*x)**2*x**6,x)*b**2*c**6*m**3 - 86*int(x**m*acos(c*x)**2*x**6,x)*b**2*c**
6*m**2 - 176*int(x**m*acos(c*x)**2*x**6,x)*b**2*c**6*m - 105*int(x**m*a...
```

3.279 $\int x^m(d-c^2dx^2)^2(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 756

$$\int x^m \left(d-c^2 d x^2\right)^2 (a+b\arccos(cx))^2 \, dx$$
$$= \frac{6b^2c^2d^2x^{3+m}}{(3+m)^2(5+m)^2} + \frac{2b^2c^2d^2x^{3+m}}{(3+m)(5+m)^2} + \frac{8b^2c^2d^2x^{3+m}}{(3+m)^3(5+m)} - \frac{2b^2c^4d^2x^{5+m}}{(5+m)^3}$$
$$- \frac{6bcd^2x^{2+m}\sqrt{1-c^2x^2}(a+b\arccos(cx))}{(3+m)(5+m)^2} - \frac{8bcd^2x^{2+m}\sqrt{1-c^2x^2}(a+b\arccos(cx))}{(3+m)^2(5+m)}$$
$$- \frac{2bcd^2x^{2+m}(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{(5+m)^2} + \frac{8d^2x^{1+m}(a+b\arccos(cx))^2}{(5+m)(3+4m+m^2)}$$
$$+ \frac{4d^2x^{1+m}(1-c^2x^2)(a+b\arccos(cx))^2}{15+8m+m^2} + \frac{d^2x^{1+m}(1-c^2x^2)^2(a+b\arccos(cx))^2}{5+m}$$
$$- \frac{8bcd^2x^{2+m}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{(2+m)(3+m)^2(5+m)}$$
$$- \frac{6bcd^2x^{2+m}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{(5+m)^2(6+5m+m^2)}$$
$$- \frac{16bcd^2x^{2+m}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{(5+m)(6+11m+6m^2+m^3)}$$
$$+ \frac{6b^2c^2d^2x^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};c^2x^2\right)}{(2+m)(3+m)^2(5+m)^2}$$
$$+ \frac{8b^2c^2d^2x^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};c^2x^2\right)}{(2+m)(3+m)^3(5+m)}$$
$$+ \frac{16b^2c^2d^2x^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};c^2x^2\right)}{(3+m)^2(5+m)(2+3m+m^2)}$$

output
```
6*b^2*c^2*d^2*x^(3+m)/(3+m)^2/(5+m)^2+2*b^2*c^2*d^2*x^(3+m)/(3+m)/(5+m)^2+
8*b^2*c^2*d^2*x^(3+m)/(3+m)^3/(5+m)-2*b^2*c^4*d^2*x^(5+m)/(5+m)^3-6*b*c*d^
2*x^(2+m)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/(3+m)/(5+m)^2-8*b*c*d^2*x^(
2+m)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/(3+m)^2/(5+m)-2*b*c*d^2*x^(2+m)*
(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/(5+m)^2+8*d^2*x^(1+m)*(a+b*arccos(c*x
))^2/(5+m)/(m^2+4*m+3)+4*d^2*x^(1+m)*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(m^2
+8*m+15)+d^2*x^(1+m)*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2/(5+m)-8*b*c*d^2*x^
(2+m)*(a+b*arccos(c*x))*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/(2+m)/
(3+m)^2/(5+m)-6*b*c*d^2*x^(2+m)*(a+b*arccos(c*x))*hypergeom([1/2, 1+1/2*m]
,[2+1/2*m],c^2*x^2)/(5+m)^2/(m^2+5*m+6)-16*b*c*d^2*x^(2+m)*(a+b*arccos(c*x
))*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/(5+m)/(m^3+6*m^2+11*m+6)+6*
b^2*c^2*d^2*x^(3+m)*hypergeom([1, 3/2+1/2*m, 3/2+1/2*m],[2+1/2*m, 5/2+1/2*
m],c^2*x^2)/(2+m)/(3+m)^2/(5+m)^2+8*b^2*c^2*d^2*x^(3+m)*hypergeom([1, 3/2+
1/2*m, 3/2+1/2*m],[2+1/2*m, 5/2+1/2*m],c^2*x^2)/(2+m)/(3+m)^3/(5+m)+16*b^2
*c^2*d^2*x^(3+m)*hypergeom([1, 3/2+1/2*m, 3/2+1/2*m],[2+1/2*m, 5/2+1/2*m],
c^2*x^2)/(3+m)^2/(5+m)/(m^2+3*m+2)
```

Mathematica [F]

$$\int x^m \left(d-c^2 d x^2\right)^2 (a+b \arccos(c x))^2 \, dx = \int x^m \left(d-c^2 d x^2\right)^2 (a+b \arccos(c x))^2 \, dx$$

input
```
Integrate[x^m*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
Integrate[x^m*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2, x]
```

Rubi [A] (warning: unable to verify)

Time = 2.00 (sec) , antiderivative size = 644, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5203, 27, 5203, 244, 2009, 5139, 5199, 15, 5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2 dx$$

$$\Big\downarrow \ 5203$$

$$\frac{2bcd^2\int x^{m+1}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx}{m+5}+\frac{4d\int dx^m\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx}{m+5}+\frac{d^2\left(1-c^2x^2\right)^2x^{m+1}(a+b\arccos(cx))^2}{m+5}$$

$$\Big\downarrow \ 27$$

$$\frac{2bcd^2\int x^{m+1}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx}{m+5}+\frac{4d^2\int x^m\left(1-c^2x^2\right)(a+b\arccos(cx))^2dx}{m+5}+\frac{d^2\left(1-c^2x^2\right)^2x^{m+1}(a+b\arccos(cx))^2}{m+5}$$

$$\Big\downarrow \ 5203$$

$$\frac{2bcd^2\left(\frac{3\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+5}+\frac{bc\int x^{m+2}\left(1-c^2x^2\right)dx}{m+5}+\frac{\left(1-c^2x^2\right)^{3/2}x^{m+2}(a+b\arccos(cx))}{m+5}\right)}{m+5}+\frac{4d^2\left(\frac{2bc\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+3}+\frac{2\int x^m(a+b\arccos(cx))^2dx}{m+3}+\frac{\left(1-c^2x^2\right)x^{m+1}(a+b\arccos(cx))^2}{m+3}\right)}{m+5}+\frac{d^2\left(1-c^2x^2\right)^2x^{m+1}(a+b\arccos(cx))^2}{m+5}$$

$$\Big\downarrow \ 244$$

$$\frac{2bcd^2\left(\frac{3\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+5}+\frac{bc\int\left(x^{m+2}-c^2x^{m+4}\right)dx}{m+5}+\frac{\left(1-c^2x^2\right)^{3/2}x^{m+2}(a+b\arccos(cx))}{m+5}\right)}{m+5}+\frac{4d^2\left(\frac{2bc\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+3}+\frac{2\int x^m(a+b\arccos(cx))^2dx}{m+3}+\frac{\left(1-c^2x^2\right)x^{m+1}(a+b\arccos(cx))^2}{m+3}\right)}{m+5}+\frac{d^2\left(1-c^2x^2\right)^2x^{m+1}(a+b\arccos(cx))^2}{m+5}$$

$$\Big\downarrow \ 2009$$

$$\frac{2bcd^2\left(\frac{3\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+5}+\frac{\left(1-c^2x^2\right)^{3/2}x^{m+2}(a+b\arccos(cx))}{m+5}+\frac{bc\left(\frac{x^{m+3}}{m+3}-\frac{c^2x^{m+5}}{m+5}\right)}{m+5}\right)}{m+5}+\frac{4d^2\left(\frac{2bc\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+3}+\frac{2\int x^m(a+b\arccos(cx))^2dx}{m+3}+\frac{\left(1-c^2x^2\right)x^{m+1}(a+b\arccos(cx))^2}{m+3}\right)}{m+5}+\frac{d^2\left(1-c^2x^2\right)^2x^{m+1}(a+b\arccos(cx))^2}{m+5}$$

$$\Big\downarrow \text{5139}$$

$$\frac{4d^2\left(\frac{2\left(\frac{2bc\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+1}+\frac{x^{m+1}(a+b\arccos(cx))^2}{m+1}\right)}{m+3}+\frac{2bc\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+3}+\frac{(1-c^2x^2)x^{m+1}(a+b\arccos(c}{m+3}\right.}{m+5}$$

$$\frac{2bcd^2\left(\frac{3\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+5}+\frac{(1-c^2x^2)^{3/2}x^{m+2}(a+b\arccos(cx))}{m+5}+\frac{bc\left(\frac{x^{m+3}}{m+3}-\frac{c^2x^{m+5}}{m+5}\right)}{m+5}\right)}{m+5}+$$

$$\frac{d^2(1-c^2x^2)^2x^{m+1}(a+b\arccos(cx))^2}{m+5}$$

$$\Big\downarrow \text{5199}$$

$$\frac{4d^2\left(\frac{2\left(\frac{2bc\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+1}+\frac{x^{m+1}(a+b\arccos(cx))^2}{m+1}\right)}{m+3}+\frac{2bc\left(\frac{\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+3}+\frac{bc\int x^{m+2}dx}{m+3}+\frac{\sqrt{1-c^2x^2}x^{m+2}(a+b\arccos(c}{m+3}\right.}{m+3}\right.}{m+5}$$

$$\frac{2bcd^2\left(\frac{3\left(\frac{\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+3}+\frac{bc\int x^{m+2}dx}{m+3}+\frac{\sqrt{1-c^2x^2}x^{m+2}(a+b\arccos(cx))}{m+3}\right)}{m+5}+\frac{(1-c^2x^2)^{3/2}x^{m+2}(a+b\arccos(cx))}{m+5}+\frac{bc\left(\frac{x^{m+3}}{m+3}-c\right.}{m+5}\right.}{m+5}$$

$$\frac{d^2(1-c^2x^2)^2x^{m+1}(a+b\arccos(cx))^2}{m+5}$$

$$\Big\downarrow \text{15}$$

$$\frac{4d^2\left(\frac{2\left(\frac{2bc\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+1}+\frac{x^{m+1}(a+b\arccos(cx))^2}{m+1}\right)}{m+3}+\frac{2bc\left(\frac{\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+3}+\frac{\sqrt{1-c^2x^2}x^{m+2}(a+b\arccos(cx))}{m+3}+\frac{bcx^{m+3}}{(m+3)^2}\right)}{m+3}\right.}{m+5}$$

$$\frac{2bcd^2\left(\frac{3\left(\frac{\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+3}+\frac{\sqrt{1-c^2x^2}x^{m+2}(a+b\arccos(cx))}{m+3}+\frac{bcx^{m+3}}{(m+3)^2}\right)}{m+5}+\frac{(1-c^2x^2)^{3/2}x^{m+2}(a+b\arccos(cx))}{m+5}+\frac{bc\left(\frac{x^{m+3}}{m+3}-\frac{c^2x^m}{m+}\right.}{m+5}\right.}{m+5}$$

$$\frac{d^2(1-c^2x^2)^2x^{m+1}(a+b\arccos(cx))^2}{m+5}$$

$\downarrow$ 5221

$$\frac{4d^2\left(\frac{2\left(\frac{2bc\left(\frac{bcx^{m+3}{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};c^2x^2\right)}{m^2+5m+6}+\frac{x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)(a+b\arccos(cx))}{m+2}\right)}{m+1}+\frac{x^{m+1}(a+b\arccos(cx}{m+1}\right.}{m+3}\right.}{}$$

$$\frac{2bcd^2\left(\frac{3\left(\frac{\frac{bcx^{m+3}{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};c^2x^2\right)}{m^2+5m+6}+\frac{x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)(a+b\arccos(cx))}{m+2}}{m+3}+\frac{\sqrt{1-c^2x^2}x^{m+2}(a+b\arccos}{m+3}\right.}{m+5}\right.}{m+5}$$

$$\frac{d^2(1-c^2x^2)^2x^{m+1}(a+b\arccos(cx))^2}{m+5}$$

input
```
Int[x^m*(d - c^2*d*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*x^(1 + m)*(1 - c^2*x^2)^2*(a + b*ArcCos[c*x])^2)/(5 + m) + (4*d^2*((x
^(1 + m)*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/(3 + m) + (2*((x^(1 + m)*(a
+ b*ArcCos[c*x])^2)/(1 + m) + (2*b*c*((x^(2 + m)*(a + b*ArcCos[c*x])*Hyper
geometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(2 + m) + (b*c*x^(3 + m)
*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, c^2*x^
2])/(6 + 5*m + m^2)))/(1 + m)))/(3 + m) + (2*b*c*((b*c*x^(3 + m))/(3 + m)^
2 + (x^(2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(3 + m) + ((x^(2 + m
)*(a + b*ArcCos[c*x])*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2
])/(2 + m) + (b*c*x^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {
2 + m/2, 5/2 + m/2}, c^2*x^2])/(6 + 5*m + m^2))/(3 + m)))/(3 + m)))/(5 + m
) + (2*b*c*d^2*((b*c*(x^(3 + m)/(3 + m) - (c^2*x^(5 + m))/(5 + m)))/(5 + m
) + (x^(2 + m)*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(5 + m) + (3*((b*c
*x^(3 + m))/(3 + m)^2 + (x^(2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/
(3 + m) + ((x^(2 + m)*(a + b*ArcCos[c*x])*Hypergeometric2F1[1/2, (2 + m)/2
, (4 + m)/2, c^2*x^2])/(2 + m) + (b*c*x^(3 + m)*HypergeometricPFQ[{1, 3/2
+ m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, c^2*x^2])/(6 + 5*m + m^2))/(3 + m
)))/(5 + m)))/(5 + m)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int x^m\left(-c^2d\,x^2+d\right)^2\left(a+b\arccos\left(cx\right)\right)^2dx$$

input
```
int(x^m*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x)
```

output
```
int(x^m*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x)
```

Fricas [F]

$$\int x^m\left(d-c^2dx^2\right)^2\left(a+b\arccos(cx)\right)^2dx=\int\left(c^2dx^2-d\right)^2\left(b\arccos\left(cx\right)+a\right)^2x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arccos(c*x))*x^m, x)
```

Sympy [F]

$$\begin{aligned}\int x^m\left(d-c^2dx^2\right)^2\left(a+b\arccos(cx)\right)^2dx=d^2\Bigg(&\int a^2x^m\,dx+\int b^2x^m\operatorname{acos}^2\left(cx\right)dx\\&+\int 2abx^m\operatorname{acos}\left(cx\right)dx\\&+\int\left(-2a^2c^2x^2x^m\right)dx+\int a^2c^4x^4x^m\,dx\\&+\int\left(-2b^2c^2x^2x^m\operatorname{acos}^2\left(cx\right)\right)dx\\&+\int b^2c^4x^4x^m\operatorname{acos}^2\left(cx\right)dx\\&+\int\left(-4abc^2x^2x^m\operatorname{acos}\left(cx\right)\right)dx\\&+\int 2abc^4x^4x^m\operatorname{acos}\left(cx\right)dx\Bigg)\end{aligned}$$

input
```
integrate(x**m*(-c**2*d*x**2+d)**2*(a+b*acos(c*x))**2,x)
```

output
```
d**2*(Integral(a**2*x**m, x) + Integral(b**2*x**m*acos(c*x)**2, x) + Integ
ral(2*a*b*x**m*acos(c*x), x) + Integral(-2*a**2*c**2*x**2*x**m, x) + Integ
ral(a**2*c**4*x**4*x**m, x) + Integral(-2*b**2*c**2*x**2*x**m*acos(c*x)**2
, x) + Integral(b**2*c**4*x**4*x**m*acos(c*x)**2, x) + Integral(-4*a*b*c**
2*x**2*x**m*acos(c*x), x) + Integral(2*a*b*c**4*x**4*x**m*acos(c*x), x))
```

Maxima [F]

$$\int x^m \left(d-c^2dx^2\right)^2 (a+b\arccos(cx))^2\,dx=\int \left(c^2dx^2-d\right)^2(b\arccos(cx)+a)^2x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
a^2*c^4*d^2*x^(m + 5)/(m + 5) - 2*a^2*c^2*d^2*x^(m + 3)/(m + 3) + a^2*d^2*
x^(m + 1)/(m + 1) + (((b^2*c^4*d^2*m^2 + 4*b^2*c^4*d^2*m + 3*b^2*c^4*d^2)*
x^5 - 2*(b^2*c^2*d^2*m^2 + 6*b^2*c^2*d^2*m + 5*b^2*c^2*d^2)*x^3 + (b^2*d^2
*m^2 + 8*b^2*d^2*m + 15*b^2*d^2)*x)*x^m*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x)^2 - (m^3 + 9*m^2 + 23*m + 15)*integrate(-2*(((b^2*c^5*d^2*m^2 + 4
*b^2*c^5*d^2*m + 3*b^2*c^5*d^2)*x^5 - 2*(b^2*c^3*d^2*m^2 + 6*b^2*c^3*d^2*m
 + 5*b^2*c^3*d^2)*x^3 + (b^2*c*d^2*m^2 + 8*b^2*c*d^2*m + 15*b^2*c*d^2)*x)*
sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x
) + (a*b*d^2*m^3 - (a*b*c^6*d^2*m^3 + 9*a*b*c^6*d^2*m^2 + 23*a*b*c^6*d^2*m
 + 15*a*b*c^6*d^2)*x^6 + 9*a*b*d^2*m^2 + 23*a*b*d^2*m + 3*(a*b*c^4*d^2*m^3
 + 9*a*b*c^4*d^2*m^2 + 23*a*b*c^4*d^2*m + 15*a*b*c^4*d^2)*x^4 + 15*a*b*d^2
 - 3*(a*b*c^2*d^2*m^3 + 9*a*b*c^2*d^2*m^2 + 23*a*b*c^2*d^2*m + 15*a*b*c^2*
d^2)*x^2)*x^m*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/(m^3 - (c^2*m^3
+ 9*c^2*m^2 + 23*c^2*m + 15*c^2)*x^2 + 9*m^2 + 23*m + 15), x))/(m^3 + 9*m^
2 + 23*m + 15)
```

Giac [F]

$$\int x^m \left(d - c^2 d x^2\right)^2 (a + b \arccos(cx))^2 \, dx = \int \left(c^2 d x^2 - d\right)^2 (b \arccos(cx) + a)^2 x^m \, dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 - d)^2*(b*arccos(c*x) + a)^2*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(d - c^2 d x^2\right)^2 (a + b \arccos(cx))^2 \, dx = \int x^m (a + b \operatorname{acos}(c x))^2 \left(d - c^2 d x^2\right)^2 \, dx$$

input
```
int(x^m*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^2,x)
```

output
```
int(x^m*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^2, x)
```

Reduce [F]

$$\int x^m \left(d - c^2 d x^2\right)^2 (a + b \arccos(cx))^2 \, dx = \text{Too large to display}$$

input
```
int(x^m*(-c^2*d*x^2+d)^2*(a+b*acos(c*x))^2,x)
```

output

```
(d**2*(x**m*a**2*c**4*m**2*x**5 + 4*x**m*a**2*c**4*m*x**5 + 3*x**m*a**2*c*
*4*x**5 - 2*x**m*a**2*c**2*m**2*x**3 - 12*x**m*a**2*c**2*m*x**3 - 10*x**m*
a**2*c**2*x**3 + x**m*a**2*m**2*x + 8*x**m*a**2*m*x + 15*x**m*a**2*x + 2*i
nt(x**m*acos(c*x)*x**4,x)*a*b*c**4*m**3 + 18*int(x**m*acos(c*x)*x**4,x)*a*
b*c**4*m**2 + 46*int(x**m*acos(c*x)*x**4,x)*a*b*c**4*m + 30*int(x**m*acos(
c*x)*x**4,x)*a*b*c**4 - 4*int(x**m*acos(c*x)*x**2,x)*a*b*c**2*m**3 - 36*in
t(x**m*acos(c*x)*x**2,x)*a*b*c**2*m**2 - 92*int(x**m*acos(c*x)*x**2,x)*a*b
*c**2*m - 60*int(x**m*acos(c*x)*x**2,x)*a*b*c**2 + 2*int(x**m*acos(c*x),x)
*a*b*m**3 + 18*int(x**m*acos(c*x),x)*a*b*m**2 + 46*int(x**m*acos(c*x),x)*a
*b*m + 30*int(x**m*acos(c*x),x)*a*b + int(x**m*acos(c*x)**2*x**4,x)*b**2*c
**4*m**3 + 9*int(x**m*acos(c*x)**2*x**4,x)*b**2*c**4*m**2 + 23*int(x**m*ac
os(c*x)**2*x**4,x)*b**2*c**4*m + 15*int(x**m*acos(c*x)**2*x**4,x)*b**2*c**
4 - 2*int(x**m*acos(c*x)**2*x**2,x)*b**2*c**2*m**3 - 18*int(x**m*acos(c*x)
**2*x**2,x)*b**2*c**2*m**2 - 46*int(x**m*acos(c*x)**2*x**2,x)*b**2*c**2*m
- 30*int(x**m*acos(c*x)**2*x**2,x)*b**2*c**2 + int(x**m*acos(c*x)**2,x)*b*
*2*m**3 + 9*int(x**m*acos(c*x)**2,x)*b**2*m**2 + 23*int(x**m*acos(c*x)**2,
x)*b**2*m + 15*int(x**m*acos(c*x)**2,x)*b**2))/(m**3 + 9*m**2 + 23*m + 15)
```

3.280 $\int x^m(d-c^2dx^2)\left(a+b\arccos(cx)\right)^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 371

$$\begin{aligned}
&\int x^m(d-c^2dx^2)\left(a+b\arccos(cx)\right)^2\,dx \\
&= \frac{2b^2c^2dx^{3+m}}{(3+m)^3}-\frac{2bcdx^{2+m}\sqrt{1-c^2x^2}(a+b\arccos(cx))}{(3+m)^2} \\
&\quad+\frac{2dx^{1+m}(a+b\arccos(cx))^2}{3+4m+m^2}+\frac{dx^{1+m}(1-c^2x^2)\left(a+b\arccos(cx)\right)^2}{3+m} \\
&\quad-\frac{2bcdx^{2+m}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{(2+m)(3+m)^2} \\
&\quad-\frac{4bcdx^{2+m}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{6+11m+6m^2+m^3} \\
&\quad+\frac{2b^2c^2dx^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};c^2x^2\right)}{(2+m)(3+m)^3} \\
&\quad+\frac{4b^2c^2dx^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};c^2x^2\right)}{(3+m)^2\left(2+3m+m^2\right)}
\end{aligned}$$

output
```
2*b^2*c^2*d*x^(3+m)/(3+m)^3-2*b*c*d*x^(2+m)*(-c^2*x^2+1)^(1/2)*(a+b*arccos
(c*x))/(3+m)^2+2*d*x^(1+m)*(a+b*arccos(c*x))^2/(m^2+4*m+3)+d*x^(1+m)*(-c^2
*x^2+1)*(a+b*arccos(c*x))^2/(3+m)-2*b*c*d*x^(2+m)*(a+b*arccos(c*x))*hyperg
eom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/(2+m)/(3+m)^2-4*b*c*d*x^(2+m)*(a+b*a
rccos(c*x))*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/(m^3+6*m^2+11*m+6)
+2*b^2*c^2*d*x^(3+m)*hypergeom([1, 3/2+1/2*m, 3/2+1/2*m],[2+1/2*m, 5/2+1/2
*m],c^2*x^2)/(2+m)/(3+m)^3+4*b^2*c^2*d*x^(3+m)*hypergeom([1, 3/2+1/2*m, 3/
2+1/2*m],[2+1/2*m, 5/2+1/2*m],c^2*x^2)/(3+m)^2/(m^2+3*m+2)
```

Mathematica [F]

$$\int x^m \left(d-c^2dx^2\right) (a+b\arccos(cx))^2\,dx=\int x^m \left(d-c^2dx^2\right) (a+b\arccos(cx))^2\,dx$$

input
```
Integrate[x^m*(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
Integrate[x^m*(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2, x]
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5203, 5139, 5199, 15, 5221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(d-c^2dx^2\right) (a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{2bcd\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+3}+\frac{2d\int x^m(a+b\arccos(cx))^2dx}{m+3}+\frac{d\left(1-c^2x^2\right)x^{m+1}(a+b\arccos(cx))^2}{m+3}$$

$$\Big\downarrow \; 5139$$

$$\frac{2d\left(\frac{2bc\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+1}+\frac{x^{m+1}(a+b\arccos(cx))^2}{m+1}\right)}{m+3}+\frac{2bcd\int x^{m+1}\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{m+3}+\frac{d(1-c^2x^2)\,x^{m+1}(a+b\arccos(cx))^2}{m+3}$$

$$\Big\downarrow \; 5199$$

$$\frac{2d\left(\frac{2bc\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+1}+\frac{x^{m+1}(a+b\arccos(cx))^2}{m+1}\right)}{m+3}+\frac{2bcd\left(\frac{\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+3}+\frac{bc\int x^{m+2}dx}{m+3}+\frac{\sqrt{1-c^2x^2}x^{m+2}(a+b\arccos(cx))}{m+3}\right)}{m+3}+\frac{d(1-c^2x^2)\,x^{m+1}(a+b\arccos(cx))^2}{m+3}$$

$$\Big\downarrow \; 15$$

$$\frac{2d\left(\frac{2bc\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+1}+\frac{x^{m+1}(a+b\arccos(cx))^2}{m+1}\right)}{m+3}+\frac{2bcd\left(\frac{\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{m+3}+\frac{\sqrt{1-c^2x^2}x^{m+2}(a+b\arccos(cx))}{m+3}+\frac{bcx^{m+3}}{(m+3)^2}\right)}{m+3}+\frac{d(1-c^2x^2)\,x^{m+1}(a+b\arccos(cx))^2}{m+3}$$

$$\Big\downarrow \; 5221$$

$$\frac{2d\left(\frac{2bc\left(\frac{bcx^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};c^2x^2\right)}{m^2+5m+6}+\frac{x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)(a+b\arccos(cx))}{m+2}\right)}{m+1}+\frac{x^{m+1}(a+b\arccos(cx}{m+1}\right.}{m+3}$$

$$\frac{2bcd\left(\frac{\frac{bcx^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};c^2x^2\right)}{m^2+5m+6}+\frac{x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)(a+b\arccos(cx))}{m+2}}{m+3}+\frac{\sqrt{1-c^2x^2}x^{m+2}(a+b\arcc}{m+3}\right.}{m+3}$$

$$\frac{d(1-c^2x^2)\,x^{m+1}(a+b\arccos(cx))^2}{m+3}$$

input
```
Int[x^m*(d - c^2*d*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*x^(1 + m)*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/(3 + m) + (2*d*((x^(1 +
m)*(a + b*ArcCos[c*x])^2)/(1 + m) + (2*b*c*((x^(2 + m)*(a + b*ArcCos[c*x])
*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(2 + m) + (b*c*x^(
3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2},
c^2*x^2])/(6 + 5*m + m^2)))/(1 + m)))/(3 + m) + (2*b*c*d*((b*c*x^(3 + m))/
(3 + m)^2 + (x^(2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(3 + m) + ((
x^(2 + m)*(a + b*ArcCos[c*x])*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
 c^2*x^2])/(2 + m) + (b*c*x^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 +
 m/2}, {2 + m/2, 5/2 + m/2}, c^2*x^2])/(6 + 5*m + m^2))/(3 + m)))/(3 + m)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5221
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int x^m \left(-c^2 d\, x^2+d\right) \left(a+b \arccos \left(c x\right)\right)^2 dx$$

input
```
int(x^m*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x)
```

output
```
int(x^m*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x)
```

Fricas [F]

$$\int x^m \left(d-c^2 d x^2\right) (a+b \arccos(cx))^2 \, dx=\int -\left(c^2 d x^2-d\right) \left(b \arccos \left(cx\right)+a\right)^2 x^m \, dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccos(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccos(c*x))*x^m, x)
```

Sympy [F]

$$\int x^m(d-c^2dx^2)(a+b\arccos(cx))^2\,dx$$
$$= -d\bigg(\int(-a^2x^m)\ dx + \int(-b^2x^m\operatorname{acos}^2(cx))\ dx + \int(-2abx^m\operatorname{acos}(cx))\ dx + \int a^2c^2x^2x^m\,dx + \int b^2c^2x^2x^m\operatorname{acos}^2(cx)\,dx + \int 2abc^2x^2x^m\operatorname{acos}(cx)\,dx\bigg)$$

input
```
integrate(x**m*(-c**2*d*x**2+d)*(a+b*acos(c*x))**2,x)
```

output
```
-d*(Integral(-a**2*x**m, x) + Integral(-b**2*x**m*acos(c*x)**2, x) + Integ
ral(-2*a*b*x**m*acos(c*x), x) + Integral(a**2*c**2*x**2*x**m, x) + Integra
l(b**2*c**2*x**2*x**m*acos(c*x)**2, x) + Integral(2*a*b*c**2*x**2*x**m*aco
s(c*x), x))
```

Maxima [F]

$$\int x^m(d-c^2dx^2)(a+b\arccos(cx))^2\,dx = \int -(c^2dx^2-d)(b\arccos(cx)+a)^2x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-a^2*c^2*d*x^(m + 3)/(m + 3) + a^2*d*x^(m + 1)/(m + 1) - (((b^2*c^2*d*m +
b^2*c^2*d)*x^3 - (b^2*d*m + 3*b^2*d)*x)*x^m*arctan2(sqrt(c*x + 1)*sqrt(-c*
x + 1), c*x)^2 - (m^2 + 4*m + 3)*integrate(2*(((b^2*c^3*d*m + b^2*c^3*d)*x
^3 - (b^2*c*d*m + 3*b^2*c*d)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m*arctan2(s
qrt(c*x + 1)*sqrt(-c*x + 1), c*x) - (a*b*d*m^2 + (a*b*c^4*d*m^2 + 4*a*b*c^
4*d*m + 3*a*b*c^4*d)*x^4 + 4*a*b*d*m + 3*a*b*d - 2*(a*b*c^2*d*m^2 + 4*a*b*
c^2*d*m + 3*a*b*c^2*d)*x^2)*x^m*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)
)/((c^2*m^2 + 4*c^2*m + 3*c^2)*x^2 - m^2 - 4*m - 3), x))/(m^2 + 4*m + 3)
```

Giac [F]

$$\int x^m \left(d-c^2dx^2\right) (a+b\arccos(cx))^2 \, dx = \int -\left(c^2dx^2-d\right)\left(b\arccos\left(cx\right)+a\right)^2 x^m \, dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(-(c^2*d*x^2 - d)*(b*arccos(c*x) + a)^2*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(d-c^2dx^2\right) (a+b\arccos(cx))^2 \, dx = \int x^m \left(a+b\,\mathrm{acos}(c\,x)\right)^2 \left(d-c^2\,d\,x^2\right) \, dx$$

input
```
int(x^m*(a + b*acos(c*x))^2*(d - c^2*d*x^2),x)
```

output
```
int(x^m*(a + b*acos(c*x))^2*(d - c^2*d*x^2), x)
```

Reduce [F]

$$\int x^m \left(d-c^2dx^2\right) (a+b\arccos(cx))^2 \, dx = \frac{d\left(-x^m a^2 c^2 m\,x^3 - x^m a^2 c^2 x^3 + x^m a^2 m x + 3x^m a^2 x - 2\left(\int x^m acos(cx)\,x^2 dx\right) ab\,c^2 m^2 - 8\left(\int x^m acos(cx)\,x\right.}{}$$

input
```
int(x^m*(-c^2*d*x^2+d)*(a+b*acos(c*x))^2,x)
```

output

```
(d*( - x**m*a**2*c**2*m*x**3 - x**m*a**2*c**2*x**3 + x**m*a**2*m*x + 3*x**
m*a**2*x - 2*int(x**m*acos(c*x)*x**2,x)*a*b*c**2*m**2 - 8*int(x**m*acos(c*
x)*x**2,x)*a*b*c**2*m - 6*int(x**m*acos(c*x)*x**2,x)*a*b*c**2 + 2*int(x**m
*acos(c*x),x)*a*b*m**2 + 8*int(x**m*acos(c*x),x)*a*b*m + 6*int(x**m*acos(c
*x),x)*a*b - int(x**m*acos(c*x)**2*x**2,x)*b**2*c**2*m**2 - 4*int(x**m*aco
s(c*x)**2*x**2,x)*b**2*c**2*m - 3*int(x**m*acos(c*x)**2*x**2,x)*b**2*c**2
+ int(x**m*acos(c*x)**2,x)*b**2*m**2 + 4*int(x**m*acos(c*x)**2,x)*b**2*m +
 3*int(x**m*acos(c*x)**2,x)*b**2))/(m**2 + 4*m + 3)
```

3.281 $\int \frac{x^m(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \mathrm{Int}\left(\frac{x^m(a+b\arccos(cx))^2}{d-c^2dx^2},x\right)$$

output
```
Defer(Int)(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x)
```

Mathematica [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \int \frac{x^m(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2),x]
```

output
```
Integrate[(x^m*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arccos(cx))^2}{d-c^2dx^2}dx$$
$$\Big\downarrow \text{5235}$$
$$\int \frac{x^m(a+b\arccos(cx))^2}{d-c^2dx^2}dx$$

input
```
Int[(x^m*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arccos(cx))^2}{-c^2d\,x^2+d}dx$$

input
```
int(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x)
```

output
```
int(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{x^m(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx=\int -\frac{(b\arccos{(cx)}+a)^2x^m}{c^2dx^2-d}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*x^m/(c^2*d*x^2 - d
), x)
```

Sympy [N/A]

Not integrable

Time = 3.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int \frac{x^m(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx=-\frac{\int \frac{a^2x^m}{c^2x^2-1}\,dx+\int \frac{b^2x^m\operatorname{acos}^2{(cx)}}{c^2x^2-1}\,dx+\int \frac{2abx^m\operatorname{acos}{(cx)}}{c^2x^2-1}\,dx}{d}$$

input
```
integrate(x**m*(a+b*acos(c*x))**2/(-c**2*d*x**2+d),x)
```

output
```
-(Integral(a**2*x**m/(c**2*x**2 - 1), x) + Integral(b**2*x**m*acos(c*x)**2
/(c**2*x**2 - 1), x) + Integral(2*a*b*x**m*acos(c*x)/(c**2*x**2 - 1), x))/
d
```

Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{x^m(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx=\int -\frac{(b\arccos{(cx)}+a)^2x^m}{c^2dx^2-d}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="maxima")
```

output
```
-integrate((b*arccos(c*x) + a)^2*x^m/(c^2*d*x^2 - d), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx=\int \frac{x^m\,(a+b\,\mathrm{acos}(c\,x))^2}{d-c^2\,d\,x^2}\,dx$$

input
```
int((x^m*(a + b*acos(c*x))^2)/(d - c^2*d*x^2),x)
```

output
```
int((x^m*(a + b*acos(c*x))^2)/(d - c^2*d*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

$$\int \frac{x^m(a+b\arccos(cx))^2}{d-c^2dx^2}\,dx = \frac{-\left(\int \frac{x^m}{c^2x^2-1}dx\right)a^2-2\left(\int \frac{x^m acos(cx)}{c^2x^2-1}dx\right)ab-\left(\int \frac{x^m acos(cx)^2}{c^2x^2-1}dx\right)b^2}{d}$$

input
```
int(x^m*(a+b*acos(c*x))^2/(-c^2*d*x^2+d),x)
```

output
```
( - int(x**m/(c**2*x**2 - 1),x)*a**2 - 2*int((x**m*acos(c*x))/(c**2*x**2 -
 1),x)*a*b - int((x**m*acos(c*x)**2)/(c**2*x**2 - 1),x)*b**2)/d
```

3.282 $\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\begin{aligned}\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx &= -\frac{bcx^{2+m}(a+b\arccos(cx))}{d^2\sqrt{1-c^2x^2}}+\frac{x^{1+m}(a+b\arccos(cx))^2}{2d^2(1-c^2x^2)}\\&+\frac{bc(1+m)x^{2+m}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{d^2(2+m)}\\&+\frac{b^2c^2x^{3+m}\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{d^2(3+m)}\\&-\frac{b^2c^2(1+m)x^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};c^2x^2\right)}{d^2(6+5m+m^2)}\\&+\frac{(1-m)\text{Int}\left(\frac{x^m(a+b\arccos(cx))^2}{d-c^2dx^2},x\right)}{2d}\end{aligned}$$

output
```
-b*c*x^(2+m)*(a+b*arccos(c*x))/d^2/(-c^2*x^2+1)^(1/2)+1/2*x^(1+m)*(a+b*arc
cos(c*x))^2/d^2/(-c^2*x^2+1)+b*c*(1+m)*x^(2+m)*(a+b*arccos(c*x))*hypergeom
([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/d^2/(2+m)+b^2*c^2*x^(3+m)*hypergeom([1,
 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)/d^2/(3+m)-b^2*c^2*(1+m)*x^(3+m)*hypergeom
([1, 3/2+1/2*m, 3/2+1/2*m],[2+1/2*m, 5/2+1/2*m],c^2*x^2)/d^2/(m^2+5*m+6)+1
/2*(1-m)*Defer(Int)(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x)/d
```

Mathematica [N/A]

Not integrable

Time = 10.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx=\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^2,x]
```

output
```
Integrate[(x^m*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx$$

$$\Big\downarrow \text{5209}$$

$$\frac{bc\int \frac{x^{m+1}(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}+\frac{(1-m)\int \frac{x^m(a+b\arccos(cx))^2}{d(1-c^2x^2)}dx}{2d}+\frac{x^{m+1}(a+b\arccos(cx))^2}{2d^2\,(1-c^2x^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{x^{m+1}(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{d^2}+\frac{(1-m)\int \frac{x^m(a+b\arccos(cx))^2}{1-c^2x^2}dx}{2d^2}+\frac{x^{m+1}(a+b\arccos(cx))^2}{2d^2\,(1-c^2x^2)}$$

$$\Big\downarrow \text{5209}$$

$$\frac{bc\left(-(m+1)\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+bc\int \frac{x^{m+2}}{1-c^2x^2}dx+\frac{x^{m+2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)}{d^2}+$$
$$\frac{(1-m)\int \frac{x^m(a+b\arccos(cx))^2}{1-c^2x^2}dx}{2d^2}+\frac{x^{m+1}(a+b\arccos(cx))^2}{2d^2\left(1-c^2x^2\right)}$$

$\big\downarrow$ 278

$$\frac{bc\left(-(m+1)\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{x^{m+2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}+\frac{bcx^{m+3}\text{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{m+3}\right)}{d^2}+$$
$$\frac{(1-m)\int \frac{x^m(a+b\arccos(cx))^2}{1-c^2x^2}dx}{2d^2}+\frac{x^{m+1}(a+b\arccos(cx))^2}{2d^2\left(1-c^2x^2\right)}$$

$\big\downarrow$ 5221

$$\frac{(1-m)\int \frac{x^m(a+b\arccos(cx))^2}{1-c^2x^2}dx}{2d^2}+$$
$$\frac{bc\left(-(m+1)\left(\frac{bcx^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};c^2x^2\right)}{m^2+5m+6}+\frac{x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)(a+b\arccos(cx))}{m+2}\right)+x^{\ldots}\right.}{d^2}$$
$$\frac{x^{m+1}(a+b\arccos(cx))^2}{2d^2\left(1-c^2x^2\right)}$$

$\big\downarrow$ 5235

$$\frac{(1-m)\int \frac{x^m(a+b\arccos(cx))^2}{1-c^2x^2}dx}{2d^2}+$$
$$\frac{bc\left(-(m+1)\left(\frac{bcx^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};c^2x^2\right)}{m^2+5m+6}+\frac{x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)(a+b\arccos(cx))}{m+2}\right)+x^{\ldots}\right.}{d^2}$$
$$\frac{x^{m+1}(a+b\arccos(cx))^2}{2d^2\left(1-c^2x^2\right)}$$

input
```
Int[(x^m*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a+b\arccos(cx))^2}{(-c^2 d\, x^2+d)^2}dx$$

input
```
int(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x)
```

output
```
int(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)^2 x^m}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*x^m/(c^4*d^2*x^4 -
2*c^2*d^2*x^2 + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 14.63 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.41

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \frac{\int \frac{a^2x^m}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{b^2x^m \operatorname{acos}^2(cx)}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{2abx^m \operatorname{acos}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

input
```
integrate(x**m*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**2,x)
```

output
```
(Integral(a**2*x**m/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b**2*x**m
*acos(c*x)**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(2*a*b*x**m*acos
(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \int \frac{(b\arccos{(cx)}+a)^2x^m}{(c^2dx^2-d)^2}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^2*x^m/(c^2*d*x^2 - d)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \int \frac{x^m\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^2}\,dx$$

input
```
int((x^m*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^2,x)
```

output
```
int((x^m*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.78

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^2}\,dx = \frac{\left(\int \frac{x^m}{c^4x^4-2c^2x^2+1}dx\right)a^2+2\left(\int \frac{x^m\,acos(cx)}{c^4x^4-2c^2x^2+1}dx\right)ab+\left(\int \frac{x^m\,acos(cx)^2}{c^4x^4-2c^2x^2+1}dx\right)b^2}{d^2}$$

input
```
int(x^m*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^2,x)
```

output
```
(int(x**m/(c**4*x**4 - 2*c**2*x**2 + 1),x)*a**2 + 2*int((x**m*acos(c*x))/(
c**4*x**4 - 2*c**2*x**2 + 1),x)*a*b + int((x**m*acos(c*x)**2)/(c**4*x**4 -
 2*c**2*x**2 + 1),x)*b**2)/d**2
```

3.283 $\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$$
$$= -\frac{bcx^{2+m}(a+b\arccos(cx))}{6d^3(1-c^2x^2)^{3/2}} - \frac{bc(1-m)x^{2+m}(a+b\arccos(cx))}{6d^3\sqrt{1-c^2x^2}}$$
$$- \frac{bc(3-m)x^{2+m}(a+b\arccos(cx))}{4d^3\sqrt{1-c^2x^2}}$$
$$+ \frac{x^{1+m}(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2} + \frac{(3-m)x^{1+m}(a+b\arccos(cx))^2}{8d^3(1-c^2x^2)}$$
$$+ \frac{bc(1-m)(1+m)x^{2+m}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{6d^3(2+m)}$$
$$+ \frac{bc(3-m)(1+m)x^{2+m}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{4d^3(2+m)}$$
$$+ \frac{b^2c^2(1-m)x^{3+m}\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{6d^3(3+m)}$$
$$+ \frac{b^2c^2(3-m)x^{3+m}\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{4d^3(3+m)}$$
$$+ \frac{b^2c^2x^{3+m}\,\text{Hypergeometric2F1}\left(2,\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{6d^3(3+m)}$$
$$- \frac{b^2c^2(1-m)(1+m)x^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};c^2x^2\right)}{6d^3(6+5m+m^2)}$$
$$- \frac{b^2c^2(3-m)(1+m)x^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};c^2x^2\right)}{4d^3(6+5m+m^2)}$$
$$+ \frac{(1-m)(3-m)\text{Int}\left(\frac{x^m(a+b\arccos(cx))^2}{d-c^2dx^2},x\right)}{8d^2}$$

output
```
-1/6*b*c*x^(2+m)*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)^(3/2)-1/6*b*c*(1-m)*x^
(2+m)*(a+b*arccos(c*x))/d^3/(-c^2*x^2+1)^(1/2)-1/4*b*c*(3-m)*x^(2+m)*(a+b*
arccos(c*x))/d^3/(-c^2*x^2+1)^(1/2)+1/4*x^(1+m)*(a+b*arccos(c*x))^2/d^3/(-
c^2*x^2+1)^2+1/8*(3-m)*x^(1+m)*(a+b*arccos(c*x))^2/d^3/(-c^2*x^2+1)+1/6*b*
c*(1-m)*(1+m)*x^(2+m)*(a+b*arccos(c*x))*hypergeom([1/2, 1+1/2*m],[2+1/2*m]
,c^2*x^2)/d^3/(2+m)+1/4*b*c*(3-m)*(1+m)*x^(2+m)*(a+b*arccos(c*x))*hypergeo
m([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/d^3/(2+m)+1/6*b^2*c^2*(1-m)*x^(3+m)*hy
pergeom([1, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)/d^3/(3+m)+1/4*b^2*c^2*(3-m)*x^
(3+m)*hypergeom([1, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)/d^3/(3+m)+1/6*b^2*c^2*
x^(3+m)*hypergeom([2, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)/d^3/(3+m)-1/6*b^2*c^
2*(1-m)*(1+m)*x^(3+m)*hypergeom([1, 3/2+1/2*m, 3/2+1/2*m],[2+1/2*m, 5/2+1/
2*m],c^2*x^2)/d^3/(m^2+5*m+6)-1/4*b^2*c^2*(3-m)*(1+m)*x^(3+m)*hypergeom([1
, 3/2+1/2*m, 3/2+1/2*m],[2+1/2*m, 5/2+1/2*m],c^2*x^2)/d^3/(m^2+5*m+6)+1/8*
(1-m)*(3-m)*Defer(Int)(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d),x)/d^2
```

Mathematica [N/A]

Not integrable

Time = 15.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx=\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^3,x]
```

output
```
Integrate[(x^m*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^3, x]
```

Rubi [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$$

$$\Big\downarrow \text{5209}$$

$$\frac{bc\int \frac{x^{m+1}(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2d^3}+\frac{(3-m)\int \frac{x^m(a+b\arccos(cx))^2}{d^2(1-c^2x^2)^2}dx}{4d}+\frac{x^{m+1}(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{x^{m+1}(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{2d^3}+\frac{(3-m)\int \frac{x^m(a+b\arccos(cx))^2}{(1-c^2x^2)^2}dx}{4d^3}+\frac{x^{m+1}(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{5209}$$

$$\frac{bc\left(\frac{1}{3}(1-m)\int \frac{x^{m+1}(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx+\frac{1}{3}bc\int \frac{x^{m+2}}{(1-c^2x^2)^2}dx+\frac{x^{m+2}(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}\right)}{2d^3}+\frac{(3-m)\left(bc\int \frac{x^{m+1}(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx+\frac{1}{2}(1-m)\int \frac{x^m(a+b\arccos(cx))^2}{1-c^2x^2}dx+\frac{x^{m+1}(a+b\arccos(cx))^2}{2(1-c^2x^2)}\right)}{4d^3}+\frac{x^{m+1}(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{278}$$

$$\frac{bc\left(\frac{1}{3}(1-m)\int \frac{x^{m+1}(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx+\frac{x^{m+2}(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{bcx^{m+3}\text{Hypergeometric2F1}\left(2,\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{3(m+3)}\right)}{2d^3}+\frac{(3-m)\left(bc\int \frac{x^{m+1}(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx+\frac{1}{2}(1-m)\int \frac{x^m(a+b\arccos(cx))^2}{1-c^2x^2}dx+\frac{x^{m+1}(a+b\arccos(cx))^2}{2(1-c^2x^2)}\right)}{4d^3}+\frac{x^{m+1}(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{5209}$$

$$\frac{bc\left(\frac{1}{3}(1-m)\left(-(m+1)\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+bc\int \frac{x^{m+2}}{1-c^2x^2}dx+\frac{x^{m+2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)+\frac{x^{m+2}(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+bc\right.}{2d^3}$$
$$\frac{(3-m)\left(bc\left(-(m+1)\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+bc\int \frac{x^{m+2}}{1-c^2x^2}dx+\frac{x^{m+2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)+\frac{1}{2}(1-m)\int \frac{x^m(a+b\arccos(}{1-c^2x^2}\right.}{4d^3}$$
$$\frac{x^{m+1}(a+b\arccos(cx))^2}{4d^3(1-c^2x^2)^2}$$

$$\Big\downarrow \text{278}$$

$$\frac{bc\left(\frac{1}{3}(1-m)\left(-(m+1)\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{x^{m+2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}+\frac{bcx^{m+3}\text{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{m+3}\right)\right.}{2d^3}$$

$$\frac{(3-m)\left(bc\left(-(m+1)\int \frac{x^{m+1}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+\frac{x^{m+2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}+\frac{bcx^{m+3}\text{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{m+3}\right)+\right.}{4d^3}$$

$$\frac{x^{m+1}(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{5221}$$

$$\frac{(3-m)\left(\frac{1}{2}(1-m)\int \frac{x^m(a+b\arccos(cx))^2}{1-c^2x^2}dx+bc\left(-(m+1)\left(\frac{bcx^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};c^2x^2\right)}{m^2+5m+6}+\frac{x^{m+2}\text{Hypergeo}}{}\right.\right.\right.}{}$$

$$\frac{bc\left(\frac{1}{3}(1-m)\left(-(m+1)\left(\frac{bcx^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};c^2x^2\right)}{m^2+5m+6}+\frac{x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)(a+b\arccos}{m+2}\right.\right.\right.}{}$$

$$\frac{x^{m+1}(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}$$

$$\Big\downarrow \text{5235}$$

$$\frac{(3-m)\left(\frac{1}{2}(1-m)\int \frac{x^m(a+b\arccos(cx))^2}{1-c^2x^2}dx+bc\left(-(m+1)\left(\frac{bcx^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};c^2x^2\right)}{m^2+5m+6}+\frac{x^{m+2}\text{Hypergeo}}{}\right.\right.\right.}{}$$

$$\frac{bc\left(\frac{1}{3}(1-m)\left(-(m+1)\left(\frac{bcx^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};c^2x^2\right)}{m^2+5m+6}+\frac{x^{m+2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)(a+b\arccos}{m+2}\right.\right.\right.}{}$$

$$\frac{x^{m+1}(a+b\arccos(cx))^2}{4d^3\left(1-c^2x^2\right)^2}$$

input
```
Int[(x^m*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arccos{(cx)})^2}{(-c^2d\,x^2+d)^3}dx$$

input
```
int(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x)
```

output
```
int(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.56

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos{(cx)}+a)^2x^m}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*x^m/(c^6*d^3*x^6 -
 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [N/A]

Not integrable

Time = 98.59 (sec) , antiderivative size = 119, normalized size of antiderivative = 4.41

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx$$
$$= -\frac{\int \frac{a^2x^m}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int \frac{b^2x^m\operatorname{acos}^2{(cx)}}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int \frac{2abx^m\operatorname{acos}{(cx)}}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

input
```
integrate(x**m*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**3,x)
```

output
```
-(Integral(a**2*x**m/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Int
egral(b**2*x**m*acos(c*x)**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),
x) + Integral(2*a*b*x**m*acos(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2
- 1), x))/d**3
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \int -\frac{(b\arccos(cx)+a)^2x^m}{(c^2dx^2-d)^3}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

output
```
-integrate((b*arccos(c*x) + a)^2*x^m/(c^2*d*x^2 - d)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \int \frac{x^m\,(a+b\,\mathrm{acos}(c\,x))^2}{(d-c^2\,d\,x^2)^3}\,dx$$

input
```
int((x^m*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^3,x)
```

output
```
int((x^m*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 4.74

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^3}\,dx = \frac{-\left(\int \frac{x^m}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)a^2-2\left(\int \frac{x^m acos(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)ab-\left(\int \frac{x^m acos(cx)^2}{c^6x^6-3c^4x^4+3c^2x^2-1}dx\right)b^2}{d^3}$$

input
```
int(x^m*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^3,x)
```

output
```
( - int(x**m/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*a**2 - 2*int((
x**m*acos(c*x))/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*a*b - int((
x**m*acos(c*x)**2)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1),x)*b**2)/d*
*3
```

3.284 $\int x^m(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int x^m \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))^2\,dx = \frac{10b^2c^2d^2x^{3+m}\sqrt{d-c^2dx^2}}{(4+m)^3(6+m)}$$
$$+\frac{2b^2c^2d^2\left(52+15m+m^2\right)x^{3+m}\sqrt{d-c^2dx^2}}{(4+m)^2(6+m)^3}$$
$$-\frac{2b^2c^4d^2x^{5+m}\sqrt{d-c^2dx^2}}{(6+m)^3}-\frac{30bcd^2x^{2+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(2+m)^2(4+m)(6+m)\sqrt{1-c^2x^2}}$$
$$-\frac{10bcd^2x^{2+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(6+m)\left(8+6m+m^2\right)\sqrt{1-c^2x^2}}-\frac{2bcd^2x^{2+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{\left(12+8m+m^2\right)\sqrt{1-c^2x^2}}$$
$$+\frac{10bc^3d^2x^{4+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(4+m)^2(6+m)\sqrt{1-c^2x^2}}+\frac{4bc^3d^2x^{4+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(4+m)(6+m)\sqrt{1-c^2x^2}}$$
$$-\frac{2bc^5d^2x^{6+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(6+m)^2\sqrt{1-c^2x^2}}+\frac{15d^2x^{1+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{(6+m)\left(8+6m+m^2\right)}$$
$$+\frac{5dx^{1+m}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{(4+m)(6+m)}+\frac{x^{1+m}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{6+m}$$
$$+\frac{30b^2c^2d^2x^{3+m}\sqrt{d-c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(2+m)^2(3+m)(4+m)(6+m)\sqrt{1-c^2x^2}}$$
$$+\frac{10b^2c^2d^2(10+3m)x^{3+m}\sqrt{d-c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(2+m)(3+m)(4+m)^3(6+m)\sqrt{1-c^2x^2}}$$
$$+\frac{2b^2c^2d^2\left(264+130m+15m^2\right)x^{3+m}\sqrt{d-c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(2+m)(3+m)(4+m)^2(6+m)^3\sqrt{1-c^2x^2}}$$
$$+\frac{15d^3\text{Int}\left(\frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}},x\right)}{(6+m)\left(8+6m+m^2\right)}$$

output
```
10*b^2*c^2*d^2*x^(3+m)*(-c^2*d*x^2+d)^(1/2)/(4+m)^3/(6+m)+2*b^2*c^2*d^2*(m
^2+15*m+52)*x^(3+m)*(-c^2*d*x^2+d)^(1/2)/(4+m)^2/(6+m)^3-2*b^2*c^4*d^2*x^(
5+m)*(-c^2*d*x^2+d)^(1/2)/(6+m)^3-30*b*c*d^2*x^(2+m)*(-c^2*d*x^2+d)^(1/2)*
(a+b*arccos(c*x))/(2+m)^2/(4+m)/(6+m)/(-c^2*x^2+1)^(1/2)-10*b*c*d^2*x^(2+m
)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(6+m)/(m^2+6*m+8)/(-c^2*x^2+1)^(1
/2)-2*b*c*d^2*x^(2+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(m^2+8*m+12)/
(-c^2*x^2+1)^(1/2)+10*b*c^3*d^2*x^(4+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c
*x))/(4+m)^2/(6+m)/(-c^2*x^2+1)^(1/2)+4*b*c^3*d^2*x^(4+m)*(-c^2*d*x^2+d)^(
1/2)*(a+b*arccos(c*x))/(4+m)/(6+m)/(-c^2*x^2+1)^(1/2)-2*b*c^5*d^2*x^(6+m)*
(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(6+m)^2/(-c^2*x^2+1)^(1/2)+15*d^2*x
^(1+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/(6+m)/(m^2+6*m+8)+5*d*x^(1
+m)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/(4+m)/(6+m)+x^(1+m)*(-c^2*d*x
^2+d)^(5/2)*(a+b*arccos(c*x))^2/(6+m)+30*b^2*c^2*d^2*x^(3+m)*(-c^2*d*x^2+d
)^(1/2)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)/(2+m)^2/(3+m)/(4+m
)/(6+m)/(-c^2*x^2+1)^(1/2)+10*b^2*c^2*d^2*(10+3*m)*x^(3+m)*(-c^2*d*x^2+d)^
(1/2)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)/(2+m)/(3+m)/(4+m)^3/
(6+m)/(-c^2*x^2+1)^(1/2)+2*b^2*c^2*d^2*(15*m^2+130*m+264)*x^(3+m)*(-c^2*d*
x^2+d)^(1/2)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)/(2+m)/(3+m)/(
4+m)^2/(6+m)^3/(-c^2*x^2+1)^(1/2)+15*d^3*Defer(Int)(x^m*(a+b*arccos(c*x))^
2/(-c^2*d*x^2+d)^(1/2),x)/(6+m)/(m^2+6*m+8)
```

Mathematica [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int x^m \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))^2\,dx=\int x^m \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))^2\,dx$$

input
```
Integrate[x^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
Integrate[x^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 3.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))^2 \, dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\int x^{m+1}\left(1-c^2x^2\right)^2 (a+b\arccos(cx))dx}{(m+6)\sqrt{1-c^2x^2}}+\frac{5d\int x^m \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^2dx}{m+6}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))^2}{m+6}$$

$$\Big\downarrow \text{5193}$$

$$\frac{5d\int x^m \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^2dx}{m+6}+\frac{2bcd^2\sqrt{d-c^2dx^2}\left(bc\int \frac{x^{m+2}\left(\frac{c^4x^4}{m+6}-\frac{2c^2x^2}{m+4}+\frac{1}{m+2}\right)}{\sqrt{1-c^2x^2}}dx+\frac{c^4x^{m+6}(a+b\arccos(cx))}{m+6}-\frac{2c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}\right)}{(m+6)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))^2}{m+6}$$

$$\Big\downarrow \text{1590}$$

$$\frac{5d\int x^m \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^2dx}{m+6}+\frac{2bcd^2\sqrt{d-c^2dx^2}\left(bc\left(-\frac{\int -\frac{c^2x^{m+2}\left(\frac{m+6}{m+2}-\frac{c^2\left(m^2+15m+52\right)x^2}{(m+4)(m+6)}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+6)}-\frac{c^2\sqrt{1-c^2x^2}x^{m+5}}{(m+6)^2}\right)+\frac{c^4x^{m+6}(a+b\arccos(cx))}{m+6}-\frac{2c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}\right)}{(m+6)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))^2}{m+6}$$

$$\Big\downarrow \text{25}$$

$$\frac{5d\int x^m\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx}{m+6}+\frac{2bcd^2\sqrt{d-c^2dx^2}\left(bc\left(\frac{\int\frac{c^2x^{m+2}\left(\frac{m+6}{m+2}-\frac{c^2\left(m^2+15m+52\right)x^2}{(m+4)(m+6)}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+6)}-\frac{c^2\sqrt{1-c^2x^2}x^{m+5}}{(m+6)^2}\right)+\frac{c^4x^{m+6}(a+b\arccos(cx))}{m+6}-\frac{2c^2x^{m+4}(a+\cdots}{m\cdots}\cdots}{(m+6)\sqrt{1-c^2x^2}}$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{m+6}$$

$\big\downarrow$ 27

$$\frac{5d\int x^m\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx}{m+6}+\frac{2bcd^2\sqrt{d-c^2dx^2}\left(bc\left(\frac{\int\frac{x^{m+2}\left(\frac{m+6}{m+2}-\frac{c^2\left(m^2+15m+52\right)x^2}{(m+4)(m+6)}\right)}{\sqrt{1-c^2x^2}}dx}{m+6}-\frac{c^2\sqrt{1-c^2x^2}x^{m+5}}{(m+6)^2}\right)+\frac{c^4x^{m+6}(a+b\arccos(cx))}{m+6}-\frac{2c^2x^{m+4}(a+b\,a\cdots}{m+4\cdots}\cdots}{(m+6)\sqrt{1-c^2x^2}}$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{m+6}$$

$\big\downarrow$ 363

$$\frac{5d\int x^m\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx}{m+6}+\frac{2bcd^2\sqrt{d-c^2dx^2}\left(bc\left(\frac{\frac{\left(15m^2+130m+264\right)\int\frac{x^{m+2}}{\sqrt{1-c^2x^2}}dx}{(m+2)(m+4)^2(m+6)}+\frac{\left(m^2+15m+52\right)\sqrt{1-c^2x^2}x^{m+3}}{(m+4)^2(m+6)}}{m+6}-\frac{c^2\sqrt{1-c^2x^2}x^{m+5}}{(m+6)^2}\right)+\frac{c^4x^{m+6}(a+b\arccos(\cdots}{m+6}\cdots}{(m+6)\sqrt{1-c^2x^2}}$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{m+6}$$

$\big\downarrow$ 278

$$\frac{5d\int x^m\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2dx}{m+6}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{m+6}+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}(a+b\arccos(cx))}{m+6}-\frac{2c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+bc\left(\frac{\left(15m^2+130m+264\right)x^{m+3}\,\mathrm{Hy}\cdots}{(m+2)(m\cdots}\cdots\right.\right.}{(m+6)\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 5203

$$\frac{5d\left(\frac{2bcd\sqrt{d-c^2dx^2}\int x^{m+1}(1-c^2x^2)(a+b\arccos(cx))dx}{(m+4)\sqrt{1-c^2x^2}}+\frac{3d\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx}{m+4}+\frac{x^{m+1}(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^2}{m+4}\right.}{m+6}$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{m+6}+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}(a+b\arccos(cx))}{m+6}-\frac{2c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+bc\left(\frac{\left(15m^2+130m+264\right)x^{m+3}\text{Hy}}{(m+2)(m}\right.\right.}{(m+6)\sqrt{1-c^2x^2}}$$

$\downarrow$ 5193

$$\frac{5d\left(\frac{2bcd\sqrt{d-c^2dx^2}\left(bc\int\frac{x^{m+2}\left(\frac{1}{m+2}-\frac{c^2x^2}{m+4}\right)}{\sqrt{1-c^2x^2}}dx-\frac{c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}\right)}{(m+4)\sqrt{1-c^2x^2}}+\frac{3d\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx}{m+4}\right.}{m+6}$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{m+6}+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}(a+b\arccos(cx))}{m+6}-\frac{2c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+bc\left(\frac{\left(15m^2+130m+264\right)x^{m+3}\text{Hy}}{(m+2)(m}\right.\right.}{(m+6)\sqrt{1-c^2x^2}}$$

$\downarrow$ 363

$$\frac{5d\left(\frac{2bcd\sqrt{d-c^2dx^2}\left(bc\left(\frac{(3m+10)\int\frac{x^{m+2}}{\sqrt{1-c^2x^2}}dx}{(m+2)(m+4)^2}+\frac{\sqrt{1-c^2x^2}x^{m+3}}{(m+4)^2}\right)-\frac{c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}\right)}{(m+4)\sqrt{1-c^2x^2}}+\frac{3d\int x^m\sqrt{d-c^2dx^2}(a}{m+}\right.}{m+6}$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{m+6}+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}(a+b\arccos(cx))}{m+6}-\frac{2c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+bc\left(\frac{\left(15m^2+130m+264\right)x^{m+3}\text{Hy}}{(m+2)(m}\right.\right.}{(m+6)\sqrt{1-c^2x^2}}$$

$\downarrow$ 278

$$\frac{5d\left(\frac{3d\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx}{m+4}+\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+bc\left(\frac{(3m+10)x^{m+3}\text{ Hypergeon}}{(m+2)(}\right.\right.}{(m+4)\sqrt{1-c^2x^2}}\right.}{m+6}$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{m+6}+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}(a+b\arccos(cx))}{m+6}-\frac{2c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+bc\left(\frac{\left(15m^2+130m+264\right)x^{m+3}\text{ Hy}}{(m+2)(m}\right.\right.}{(m+6)\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 5203

$$5d\left(\frac{3d\left(\frac{2bc\sqrt{d-c^2dx^2}\int x^{m+1}(a+b\arccos(cx))dx}{(m+2)\sqrt{1-c^2x^2}}+\frac{d\int\frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{m+2}\right)}{m+4}+\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2x^{m+4}(}{}\right.}{}\right.$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{m+6}+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}(a+b\arccos(cx))}{m+6}-\frac{2c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+bc\left(\frac{\left(15m^2+130m+264\right)x^{m+3}\text{ Hy}}{(m+2)(m}\right.\right.}{(m+6)\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 5139

$$5d\left(\frac{3d\left(\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{bc\int\frac{x^{m+2}}{\sqrt{1-c^2x^2}}dx}{m+2}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}\right)}{(m+2)\sqrt{1-c^2x^2}}+\frac{d\int\frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{m+2}\right)}{m+4}+\frac{2bcd\sqrt{d-c}}{}\right.$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{m+6}+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}(a+b\arccos(cx))}{m+6}-\frac{2c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+bc\left(\frac{\left(15m^2+130m+264\right)x^{m+3}\text{Hy}}{(m+2)(m}\right.\right.}{(m+6)\sqrt{1-c^2x^2}}$$

$\downarrow$ 278

$$5d\left(\frac{3d\left(\frac{d\int\frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+\frac{bcx^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{(m+2)(m+3)}\right)}{(m+2)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a-}{m+2}\right.}{m+4}\right.$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{m+6}+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}(a+b\arccos(cx))}{m+6}-\frac{2c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+bc\left(\frac{\left(15m^2+130m+264\right)x^{m+3}\text{Hy}}{(m+2)(m}\right.\right.}{(m+6)\sqrt{1-c^2x^2}}$$

$\downarrow$ 5235

$$5d\left(\frac{3d\left(\frac{d\int\frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+\frac{bcx^{m+3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{(m+2)(m+3)}\right)}{(m+2)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a-}{m+2}\right.}{m+4}\right.$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^2}{m+6}+$$

$$\frac{2bcd^2\sqrt{d-c^2dx^2}\left(\frac{c^4x^{m+6}(a+b\arccos(cx))}{m+6}-\frac{2c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+bc\left(\frac{\left(15m^2+130m+264\right)x^{m+3}\,\text{Hy}}{(m+2)(m}\right.\right.}{(m+6)\sqrt{1-c^2x^2}}$$

input
```
Int[x^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 9.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int x^m\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}(a+b\arccos(cx))^2\,dx$$

input
```
int(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x)
```

output
```
int(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 4.62

$$\int x^m (d-c^2dx^2)^{5/2} (a+b\arccos(cx))^2\,dx = \int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\arccos(cx)+a)^2x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="frica
s")
```

output
```
integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
 - 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b
*c^2*d^2*x^2 + a*b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)*x^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m (d-c^2dx^2)^{5/2} (a+b\arccos(cx))^2\,dx = \text{Timed out}$$

input
```
integrate(x**m*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^m (d-c^2dx^2)^{5/2} (a+b\arccos(cx))^2\,dx = \int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\arccos(cx)+a)^2x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="maxim
a")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccos(c*x) + a)^2*x^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^m (d - c^2dx^2)^{5/2} (a + b\arccos(cx))^2\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^m (d - c^2dx^2)^{5/2} (a + b\arccos(cx))^2\, dx = \int x^m\, (a + b\,\mathrm{acos}(c\,x))^2\, (d - c^2\,d\,x^2)^{5/2}\, dx$$

input
```
int(x^m*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^m*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 264, normalized size of antiderivative = 9.10

$$\int x^m(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^2\,dx = \sqrt{d}\,d^2\Bigg(2\left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)\,x^4dx\right)ab\,c^4 - 4\left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)\,x^2dx\right)ab\,c^2 + 2\left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)\,dx\right)ab + \left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)^2\,x^4dx\right)b^2c^4 - 2\left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)^2\,x^2dx\right)b^2c^2 + \left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)^2\,dx\right)b^2 + \left(\int x^m\sqrt{-c^2x^2+1}\,x^4dx\right)a^2c^4 - 2\left(\int x^m\sqrt{-c^2x^2+1}\,x^2dx\right)a^2c^2 + \left(\int x^m\sqrt{-c^2x^2+1}dx\right)a^2\Bigg)$$

input
```
int(x^m*(-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))^2,x)
```

output
```
sqrt(d)*d**2*(2*int(x**m*sqrt( - c**2*x**2 + 1)*acos(c*x)*x**4,x)*a*b*c**4
 - 4*int(x**m*sqrt( - c**2*x**2 + 1)*acos(c*x)*x**2,x)*a*b*c**2 + 2*int(x*
*m*sqrt( - c**2*x**2 + 1)*acos(c*x),x)*a*b + int(x**m*sqrt( - c**2*x**2 +
1)*acos(c*x)**2*x**4,x)*b**2*c**4 - 2*int(x**m*sqrt( - c**2*x**2 + 1)*acos
(c*x)**2*x**2,x)*b**2*c**2 + int(x**m*sqrt( - c**2*x**2 + 1)*acos(c*x)**2,
x)*b**2 + int(x**m*sqrt( - c**2*x**2 + 1)*x**4,x)*a**2*c**4 - 2*int(x**m*s
qrt( - c**2*x**2 + 1)*x**2,x)*a**2*c**2 + int(x**m*sqrt( - c**2*x**2 + 1),
x)*a**2)
```

3.285 $\int x^m (d-c^2dx^2)^{3/2} (a+b\arccos(cx))^2 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$
\begin{aligned}
\int x^m \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^2 \, dx &= \frac{2b^2c^2dx^{3+m}\sqrt{d-c^2dx^2}}{(4+m)^3} \\
&- \frac{6bcdx^{2+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(2+m)^2(4+m)\sqrt{1-c^2x^2}} \\
&- \frac{2bcdx^{2+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(8+6m+m^2)\sqrt{1-c^2x^2}} \\
&+ \frac{2bc^3dx^{4+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(4+m)^2\sqrt{1-c^2x^2}} \\
&+ \frac{3dx^{1+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{8+6m+m^2} \\
&+ \frac{x^{1+m}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{4+m} \\
&+ \frac{6b^2c^2dx^{3+m}\sqrt{d-c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(2+m)^2(3+m)(4+m)\sqrt{1-c^2x^2}} \\
&+ \frac{2b^2c^2d(10+3m)x^{3+m}\sqrt{d-c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(2+m)(3+m)(4+m)^3\sqrt{1-c^2x^2}} \\
&+ \frac{3d^2\text{Int}\left(\frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}},x\right)}{8+6m+m^2}
\end{aligned}
$$

output
```
2*b^2*c^2*d*x^(3+m)*(-c^2*d*x^2+d)^(1/2)/(4+m)^3-6*b*c*d*x^(2+m)*(-c^2*d*x
^2+d)^(1/2)*(a+b*arccos(c*x))/(2+m)^2/(4+m)/(-c^2*x^2+1)^(1/2)-2*b*c*d*x^(
2+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(m^2+6*m+8)/(-c^2*x^2+1)^(1/2)
+2*b*c^3*d*x^(4+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(4+m)^2/(-c^2*x^
2+1)^(1/2)+3*d*x^(1+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/(m^2+6*m+8
)+x^(1+m)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2/(4+m)+6*b^2*c^2*d*x^(3+
m)*(-c^2*d*x^2+d)^(1/2)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)/(2
+m)^2/(3+m)/(4+m)/(-c^2*x^2+1)^(1/2)+2*b^2*c^2*d*(10+3*m)*x^(3+m)*(-c^2*d*
x^2+d)^(1/2)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)/(2+m)/(3+m)/(
4+m)^3/(-c^2*x^2+1)^(1/2)+3*d^2*Defer(Int)(x^m*(a+b*arccos(c*x))^2/(-c^2*d
*x^2+d)^(1/2),x)/(m^2+6*m+8)
```

Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int x^m \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^2\,dx=\int x^m \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^2\,dx$$

input
```
Integrate[x^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
Integrate[x^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^2 dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\int x^{m+1}\left(1-c^2x^2\right)(a+b\arccos(cx))dx}{(m+4)\sqrt{1-c^2x^2}}+\frac{3d\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx}{m+4}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{m+4}$$

$$\Big\downarrow \text{5193}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(bc\int \frac{x^{m+2}\left(\frac{1}{m+2}-\frac{c^2x^2}{m+4}\right)}{\sqrt{1-c^2x^2}}dx-\frac{c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}\right)}{(m+4)\sqrt{1-c^2x^2}}+\frac{3d\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx}{m+4}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{m+4}$$

$$\Big\downarrow \text{363}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(bc\left(\frac{(3m+10)\int \frac{x^{m+2}}{\sqrt{1-c^2x^2}}dx}{(m+2)(m+4)^2}+\frac{\sqrt{1-c^2x^2}x^{m+3}}{(m+4)^2}\right)-\frac{c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}\right)}{(m+4)\sqrt{1-c^2x^2}}+\frac{3d\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx}{m+4}+\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{m+4}$$

$$\Big\downarrow \text{278}$$

$$\frac{3d\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2dx}{m+4}+\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+bc\left(\frac{(3m+10)x^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{(m+2)(m+3)(m+4)^2}\right.\right.}{(m+4)\sqrt{1-c^2x^2}}$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{m+4}$$

$$\Big\downarrow \text{5203}$$

$$\frac{3d\left(\frac{2bc\sqrt{d-c^2dx^2}\int x^{m+1}(a+b\arccos(cx))dx}{(m+2)\sqrt{1-c^2x^2}}+\frac{d\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{m+2}\right)}{m+4}+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+bc\left(\frac{(3m+10)x^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{(m+2)(m+3)(m+4)^2}\right.\right.}{(m+4)\sqrt{1-c^2x^2}}$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{m+4}$$

$\big\downarrow$ 5139

$$\frac{3d\left(\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{bc\int\frac{x^{m+2}}{\sqrt{1-c^2x^2}}dx}{m+2}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}\right)}{(m+2)\sqrt{1-c^2x^2}}+\frac{d\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{m+2}\right)}{m+4}+$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+bc\left(\frac{(3m+10)x^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{(m+2)(m+3)(m+4)^2}\right.\right.}{(m+4)\sqrt{1-c^2x^2}}$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{m+4}$$

$\big\downarrow$ 278

$$\frac{3d\left(\frac{d\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+\frac{bcx^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{(m+2)(m+3)}\right)}{(m+2)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\sqrt{d-c^2dx^2}}{m}\right.}{m+4}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+bc\left(\frac{(3m+10)x^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{(m+2)(m+3)(m+4)^2}\right.\right.}{(m+4)\sqrt{1-c^2x^2}}$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{m+4}$$

$\big\downarrow$ 5235

$$\frac{3d\left(\frac{d\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+\frac{bcx^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{(m+2)(m+3)}\right)}{(m+2)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\sqrt{d-c^2dx^2}}{m}\right.}{m+4}$$

$$\frac{2bcd\sqrt{d-c^2dx^2}\left(-\frac{c^2x^{m+4}(a+b\arccos(cx))}{m+4}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+bc\left(\frac{(3m+10)x^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{(m+2)(m+3)(m+4)^2}\right.\right.}{(m+4)\sqrt{1-c^2x^2}}$$

$$\frac{x^{m+1}\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2}{m+4}$$

input
```
Int[x^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 3.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int x^m\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}(a+b\arccos(cx))^2\,dx$$

input
```
int(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x)
```

output
```
int(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.93

$$\int x^m\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^2\,dx=\int\left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\arccos(cx)+a)^2x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="frica
s")
```

output
```
integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccos(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)*x^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m (d - c^2dx^2)^{3/2} (a + b \arccos(cx))^2 \, dx = \text{Timed out}$$

input
```
integrate(x**m*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^m (d - c^2dx^2)^{3/2} (a + b \arccos(cx))^2 \, dx = \int (-c^2dx^2 + d)^{\frac{3}{2}} (b \arccos(cx) + a)^2 x^m \, dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="maxim
a")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccos(c*x) + a)^2*x^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^m \left(d - c^2 d x^2\right)^{3/2} (a + b \arccos(cx))^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^m \left(d - c^2 d x^2\right)^{3/2} (a + b \arccos(cx))^2 \, dx = \int x^m \, (a + b \operatorname{acos}(c\,x))^2 \, \left(d - c^2 \, d \, x^2\right)^{3/2} \, dx$$

input
```
int(x^m*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^m*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 5.90

$$\int x^m \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^2\,dx = \sqrt{d}\,d\left(-2\left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)\,x^2dx\right)ab\,c^2 + 2\left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)\,dx\right)ab - \left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)^2\,x^2dx\right)b^2c^2 + \left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)^2\,dx\right)b^2 - \left(\int x^m\sqrt{-c^2x^2+1}\,x^2dx\right)a^2c^2 + \left(\int x^m\sqrt{-c^2x^2+1}dx\right)a^2\right)$$

input
```
int(x^m*(-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))^2,x)
```

output
```
sqrt(d)*d*( - 2*int(x**m*sqrt( - c**2*x**2 + 1)*acos(c*x)*x**2,x)*a*b*c**2
 + 2*int(x**m*sqrt( - c**2*x**2 + 1)*acos(c*x),x)*a*b - int(x**m*sqrt( - c
**2*x**2 + 1)*acos(c*x)**2*x**2,x)*b**2*c**2 + int(x**m*sqrt( - c**2*x**2
+ 1)*acos(c*x)**2,x)*b**2 - int(x**m*sqrt( - c**2*x**2 + 1)*x**2,x)*a**2*c
**2 + int(x**m*sqrt( - c**2*x**2 + 1),x)*a**2)
```

3.286 $\int x^m \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\begin{aligned}&\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx\\&=-\frac{2bcx^{2+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(2+m)^2\sqrt{1-c^2x^2}}+\frac{x^{1+m}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2+m}\\&\quad+\frac{2b^2c^2x^{3+m}\sqrt{d-c^2dx^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(2+m)^2(3+m)\sqrt{1-c^2x^2}}\\&\quad+\frac{d\text{Int}\left(\frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}},x\right)}{2+m}\end{aligned}$$

output
```
-2*b*c*x^(2+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))/(2+m)^2/(-c^2*x^2+1)
^(1/2)+x^(1+m)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2/(2+m)+2*b^2*c^2*x^
(3+m)*(-c^2*d*x^2+d)^(1/2)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)
/(2+m)^2/(3+m)/(-c^2*x^2+1)^(1/2)+d*Defer(Int)(x^m*(a+b*arccos(c*x))^2/(-c
^2*d*x^2+d)^(1/2),x)/(2+m)
```

Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int x^m \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int x^m \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$$

input
```
Integrate[x^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
Integrate[x^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5203}$$

$$\frac{2bc\sqrt{d-c^2dx^2}\int x^{m+1}(a+b\arccos(cx))dx}{(m+2)\sqrt{1-c^2x^2}}+\frac{d\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{m+2}$$

$$\Big\downarrow \text{5139}$$

$$\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{bc\int \frac{x^{m+2}}{\sqrt{1-c^2x^2}}dx}{m+2}+\frac{x^{m+2}(a+b\arccos(cx))}{m+2}\right)}{(m+2)\sqrt{1-c^2x^2}}+\frac{d\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{m+2}$$

$$\Big\downarrow \text{278}$$

$$\frac{d\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+\frac{bcx^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{(m+2)(m+3)}\right)}{(m+2)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{m+2}$$

$$\Big\downarrow \text{5235}$$

$$\frac{d\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx}{m+2}+\frac{2bc\sqrt{d-c^2dx^2}\left(\frac{x^{m+2}(a+b\arccos(cx))}{m+2}+\frac{bcx^{m+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},c^2x^2\right)}{(m+2)(m+3)}\right)}{(m+2)\sqrt{1-c^2x^2}}+\frac{x^{m+1}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{m+2}$$

input
```
Int[x^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int x^m\sqrt{-c^2d\,x^2+d}\,(a+b\arccos(cx))^2\,dx$$

input
```
int(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x)
```

output
```
int(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int x^m \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int \sqrt{-c^2dx^2+d}(b\arccos(cx)+a)^2x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="frica
s")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
)*x^m, x)
```

Sympy [N/A]

Not integrable

Time = 18.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int x^m \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int x^m\sqrt{-d(cx-1)(cx+1)}(a+b\,\mathrm{acos}(cx))^2\,dx$$

input
```
integrate(x**m*(-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**m*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int \sqrt{-c^2dx^2+d}(b\arccos(cx)+a)^2x^m\,dx$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)^2*x^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\int x^m\,(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^m*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^m*(a + b*acos(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.66

$$\int x^m\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2\,dx=\sqrt{d}\left(2\left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)\,dx\right)ab+\left(\int x^m\sqrt{-c^2x^2+1}\,acos(cx)^2\,dx\right)b^2+\left(\int x^m\sqrt{-c^2x^2+1}dx\right)a^2\right)$$

input
```
int(x^m*(-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))^2,x)
```

output
```
sqrt(d)*(2*int(x**m*sqrt( - c**2*x**2 + 1)*acos(c*x),x)*a*b + int(x**m*sqr
t( - c**2*x**2 + 1)*acos(c*x)**2,x)*b**2 + int(x**m*sqrt( - c**2*x**2 + 1)
,x)*a**2)
```

3.287 $\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\text{Int}\left(\frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}},x\right)$$

output
```
Defer(Int)(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 3.47 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcCos[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
Integrate[(x^m*(a + b*ArcCos[c*x])^2)/Sqrt[d - c^2*d*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx$$

$$\Big\downarrow \text{ 5235}$$

$$\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}dx$$

input
```
Int[(x^m*(a + b*ArcCos[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{-c^2d\,x^2+d}}dx$$

input
```
int(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
int(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.93

$$\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x^m}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2)*x^m/(c^2*d*x^2 - d), x)
```

Sympy [N/A]

Not integrable

Time = 7.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{x^m(a+b\,\mathrm{acos}\,(cx))^2}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

input
```
integrate(x**m*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(x**m*(a + b*acos(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2x^m}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((b*arccos(c*x) + a)^2*x^m/sqrt(-c^2*d*x^2 + d), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{x^m\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((x^m*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((x^m*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.93

$$\int \frac{x^m(a+b\arccos(cx))^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}}dx\right)a^2+2\left(\int \frac{x^m\,acos(cx)}{\sqrt{-c^2x^2+1}}dx\right)ab+\left(\int \frac{x^m\,acos(cx)^2}{\sqrt{-c^2x^2+1}}dx\right)b^2}{\sqrt{d}}$$

input
```
int(x^m*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

output
```
(int(x**m/sqrt( - c**2*x**2 + 1),x)*a**2 + 2*int((x**m*acos(c*x))/sqrt( -
c**2*x**2 + 1),x)*a*b + int((x**m*acos(c*x)**2)/sqrt( - c**2*x**2 + 1),x)*
b**2)/sqrt(d)
```

3.288 $\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\text{Int}\left(\frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}},x\right)$$

output
```
Defer(Int)(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5235}$$

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}dx$$

input
```
Int[(x^m*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{x^m(a+b\arccos(cx))^2}{(-c^2d\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
int(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2x^m}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
)*x^m/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 10.80 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x^m(a+b\operatorname{acos}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

output
```
Integral(x**m*(a + b*acos(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \int \frac{(b\arccos{(cx)}+a)^2x^m}{(-c^2dx^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((b*arccos(c*x) + a)^2*x^m/(-c^2*d*x^2 + d)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\int \frac{x^m\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{3/2}}\,dx$$

input
```
int((x^m*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((x^m*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 5.28

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{3/2}}\,dx=\frac{-\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)a^2-2\left(\int \frac{x^m\,acos(cx)}{\sqrt{-c^2x^2+1}\,c^2x^2-\sqrt{-c^2x^2+1}}dx\right)ab-\left(\int\right.}{\sqrt{d}\,d}$$

input
```
int(x^m*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

output
```
( - int(x**m/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x
)*a**2 - 2*int((x**m*acos(c*x))/(sqrt( - c**2*x**2 + 1)*c**2*x**2 - sqrt(
- c**2*x**2 + 1)),x)*a*b - int((x**m*acos(c*x)**2)/(sqrt( - c**2*x**2 + 1)
*c**2*x**2 - sqrt( - c**2*x**2 + 1)),x)*b**2)/(sqrt(d)*d)
```

3.289 $\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\text{Int}\left(\frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}},x\right)$$

output
```
Defer(Int)(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx=\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
Integrate[(x^m*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5235}$$

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}dx$$

input
```
Int[(x^m*(a + b*ArcCos[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{x^m(a+b\arccos(cx))^2}{(-c^2d\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
int(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.83

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x^m}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^
2)*x^m/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a+b*acos(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x^m}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate((b*arccos(c*x) + a)^2*x^m/(-c^2*d*x^2 + d)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a+b*arccos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{x^m\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}{\left(d-c^2\,d\,x^2\right)^{5/2}} \,d x$$

input
```
int((x^m*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((x^m*(a + b*acos(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 202, normalized size of antiderivative = 6.97

$$\int \frac{x^m(a+b\arccos(cx))^2}{(d-c^2dx^2)^{5/2}}\,dx = \frac{\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)a^2+2\left(\int \frac{x^m\,acos(cx)}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)ab+\left(\int \frac{x^m\,acos(cx)^2}{\sqrt{-c^2x^2+1}\,c^4x^4-2\sqrt{-c^2x^2+1}\,c^2x^2+\sqrt{-c^2x^2+1}}dx\right)b^2}{\sqrt{d}\,d^2}$$

input
```
int(x^m*(a+b*acos(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

output
```
(int(x**m/(sqrt( - c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**
2*x**2 + sqrt( - c**2*x**2 + 1)),x)*a**2 + 2*int((x**m*acos(c*x))/(sqrt( -
 c**2*x**2 + 1)*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c
**2*x**2 + 1)),x)*a*b + int((x**m*acos(c*x)**2)/(sqrt( - c**2*x**2 + 1)*c*
*4*x**4 - 2*sqrt( - c**2*x**2 + 1)*c**2*x**2 + sqrt( - c**2*x**2 + 1)),x)*
b**2)/(sqrt(d)*d**2)
```

3.290 $\int \frac{x^m \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \text{Int}\left(\frac{x^m \arccos(ax)^2}{\sqrt{1-a^2x^2}},x\right)$$

output
```
Defer(Int)(x^m*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

input
```
Integrate[(x^m*ArcCos[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
Integrate[(x^m*ArcCos[a*x]^2)/Sqrt[1 - a^2*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arccos(ax)^2}{\sqrt{1-a^2x^2}} \, dx$$
$$\Big\downarrow \; 5235$$
$$\int \frac{x^m \arccos(ax)^2}{\sqrt{1-a^2x^2}}dx$$

input
```
Int[(x^m*ArcCos[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arccos\left(ax\right)^2}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^m*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(x^m*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{x^m \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m \arccos{(ax)}^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^m*arccos(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m \operatorname{acos}^2{(ax)}}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x**m*acos(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**m*acos(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m \arccos{(ax)}^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arccos(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m \arccos\left(ax\right)^2}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arccos(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*arccos(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m\,\mathrm{acos}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^m*acos(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^m*acos(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^m \arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m acos(ax)^2}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^m*acos(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((x**m*acos(a*x)**2)/sqrt( - a**2*x**2 + 1),x)
```

3.291 $\int (c-a^2cx^2)^3 \arccos(ax)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 370

$$\int (c-a^2cx^2)^3 \arccos(ax)^3\,dx = -\frac{413312c^3\sqrt{1-a^2x^2}}{128625a} - \frac{30256c^3(1-a^2x^2)^{3/2}}{385875a} - \frac{2664c^3(1-a^2x^2)^{5/2}}{214375a} - \frac{6c^3(1-a^2x^2)^{7/2}}{2401a} - \frac{4322c^3x\arccos(ax)}{1225} + \frac{1514a^2c^3x^3\arccos(ax)}{3675} - \frac{702a^4c^3x^5\arccos(ax)}{6125} + \frac{6}{343}a^6c^3x^7\arccos(ax) + \frac{48c^3\sqrt{1-a^2x^2}\arccos(ax)^2}{35a} + \frac{8c^3(1-a^2x^2)^{3/2}\arccos(ax)^2}{35a} + \frac{18c^3(1-a^2x^2)^{5/2}}{175a}$$

output
```
-413312/128625*c^3*(-a^2*x^2+1)^(1/2)/a-30256/385875*c^3*(-a^2*x^2+1)^(3/2
)/a-2664/214375*c^3*(-a^2*x^2+1)^(5/2)/a-6/2401*c^3*(-a^2*x^2+1)^(7/2)/a-4
322/1225*c^3*x*arccos(a*x)+1514/3675*a^2*c^3*x^3*arccos(a*x)-702/6125*a^4*
c^3*x^5*arccos(a*x)+6/343*a^6*c^3*x^7*arccos(a*x)+48/35*c^3*(-a^2*x^2+1)^(
1/2)*arccos(a*x)^2/a+8/35*c^3*(-a^2*x^2+1)^(3/2)*arccos(a*x)^2/a+18/175*c^
3*(-a^2*x^2+1)^(5/2)*arccos(a*x)^2/a+3/49*c^3*(-a^2*x^2+1)^(7/2)*arccos(a*
x)^2/a+16/35*c^3*x*arccos(a*x)^3+8/35*c^3*x*(-a^2*x^2+1)*arccos(a*x)^3+6/3
5*c^3*x*(-a^2*x^2+1)^2*arccos(a*x)^3+1/7*c^3*x*(-a^2*x^2+1)^3*arccos(a*x)^
3
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.46

$$\int \left(c-a^2cx^2\right)^3\arccos(ax)^3\,dx$$
$$=\frac{c^3\left(2\sqrt{1-a^2x^2}(22329151-747937a^2x^2+134541a^4x^4-16875a^6x^6)+210ax(-226905+26495a^2x^2-\right.}{}$$

input
```
Integrate[(c - a^2*c*x^2)^3*ArcCos[a*x]^3,x]
```

output
```
(c^3*(2*Sqrt[1 - a^2*x^2]*(22329151 - 747937*a^2*x^2 + 134541*a^4*x^4 - 16
875*a^6*x^6) + 210*a*x*(-226905 + 26495*a^2*x^2 - 7371*a^4*x^4 + 1125*a^6*
x^6)*ArcCos[a*x] + 11025*Sqrt[1 - a^2*x^2]*(-2161 + 757*a^2*x^2 - 351*a^4*
x^4 + 75*a^6*x^6)*ArcCos[a*x]^2 - 385875*a*x*(-35 + 35*a^2*x^2 - 21*a^4*x^
4 + 5*a^6*x^6)*ArcCos[a*x]^3))/(13505625*a)
```

Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.57, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {5159, 27, 5159, 5159, 5131, 5183, 5131, 241, 5155, 27, 353, 53, 1576, 1140, 2009, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^3\left(c-a^2cx^2\right)^3\,dx$$

$$\Big\downarrow \; 5159$$

$$\frac{3}{7}ac^3\int x\left(1-a^2x^2\right)^{5/2}\arccos(ax)^2dx+\frac{6}{7}c\int c^2\left(1-a^2x^2\right)^2\arccos(ax)^3dx+\frac{1}{7}c^3x\left(1-a^2x^2\right)^3\arccos(ax)^3$$

$$\Big\downarrow \; 27$$

$$\frac{3}{7}ac^3\int x\left(1-a^2x^2\right)^{5/2}\arccos(ax)^2dx+\frac{6}{7}c^3\int \left(1-a^2x^2\right)^2\arccos(ax)^3dx+\frac{1}{7}c^3x\left(1-a^2x^2\right)^3\arccos(ax)^3$$

$$\Big\downarrow \text{5159}$$

$$\frac{3}{7}ac^3\int x\left(1-a^2x^2\right)^{5/2}\arccos(ax)^2dx+\frac{6}{7}c^3\left(\frac{3}{5}a\int x\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2dx+\frac{4}{5}\int \left(1-a^2x^2\right)\arccos(ax)^3dx+\frac{1}{5}x\left(1-a^2x^2\right)^2\arccos(ax)^3\right)+\frac{1}{7}c^3x\left(1-a^2x^2\right)^3\arccos(ax)^3$$

$$\Big\downarrow \text{5159}$$

$$\frac{3}{7}ac^3\int x\left(1-a^2x^2\right)^{5/2}\arccos(ax)^2dx+\frac{6}{7}c^3\left(\frac{3}{5}a\int x\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2dx+\frac{4}{5}\left(a\int x\sqrt{1-a^2x^2}\arccos(ax)^2dx+\frac{2}{3}\int \arccos(ax)^3dx+\frac{1}{3}x(1-\right.\right.$$
$$\frac{1}{7}c^3x\left(1-a^2x^2\right)^3\arccos(ax)^3$$

$$\Big\downarrow \text{5131}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(3a\int \frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)^3\right)+a\int x\sqrt{1-a^2x^2}\arccos(ax)^2dx+\frac{1}{3}x\left(1-a^2x^2\right)\arccos(a\right.\right.$$
$$\frac{3}{7}ac^3\int x\left(1-a^2x^2\right)^{5/2}\arccos(ax)^2dx+\frac{1}{7}c^3x\left(1-a^2x^2\right)^3\arccos(ax)^3$$

$$\Big\downarrow \text{5183}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(3a\left(-\frac{2\int \arccos(ax)dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}\right)+x\arccos(ax)^3\right)+a\left(-\frac{2\int \left(1-a^2x^2\right)\arccos(}{3a}\right.\right.\right.$$
$$\frac{3}{7}ac^3\left(-\frac{2\int \left(1-a^2x^2\right)^3\arccos(ax)dx}{7a}-\frac{\left(1-a^2x^2\right)^{7/2}\arccos(ax)^2}{7a^2}\right)+$$
$$\frac{1}{7}c^3x\left(1-a^2x^2\right)^3\arccos(ax)^3$$

$$\Big\downarrow \text{5131}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(3a\left(-\frac{2\left(a\int\frac{x}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)\right)}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}\right)+x\arccos(ax)^3\right)+a\left(-\frac{2\int\right.\right.\right.$$

$$\frac{3}{7}ac^3\left(-\frac{2\int\left(1-a^2x^2\right)^3\arccos(ax)dx}{7a}-\frac{\left(1-a^2x^2\right)^{7/2}\arccos(ax)^2}{7a^2}\right)+$$

$$\frac{1}{7}c^3x\left(1-a^2x^2\right)^3\arccos(ax)^3$$

$$\Big\downarrow \ 241$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(a\left(-\frac{2\int\left(1-a^2x^2\right)\arccos(ax)dx}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2}{3a^2}\right)+\frac{1}{3}x\left(1-a^2x^2\right)\arccos(ax)^3+\frac{2}{3}\left(3a\right.\right.\right.$$

$$\frac{3}{7}ac^3\left(-\frac{2\int\left(1-a^2x^2\right)^3\arccos(ax)dx}{7a}-\frac{\left(1-a^2x^2\right)^{7/2}\arccos(ax)^2}{7a^2}\right)+$$

$$\frac{1}{7}c^3x\left(1-a^2x^2\right)^3\arccos(ax)^3$$

$$\Big\downarrow \ 5155$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(a\left(-\frac{2\left(a\int\frac{x(3-a^2x^2)}{3\sqrt{1-a^2x^2}}dx-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2}{3a^2}\right)+\frac{1}{3}x(1-\right.\right.$$

$$\frac{3}{7}ac^3\left(-\frac{2\left(a\int\frac{x(-5a^6x^6+21a^4x^4-35a^2x^2+35)}{35\sqrt{1-a^2x^2}}dx-\frac{1}{7}a^6x^7\arccos(ax)+\frac{3}{5}a^4x^5\arccos(ax)-a^2x^3\arccos(ax)+x\arccos\right.}{7a}\right.$$

$$\frac{1}{7}c^3x\left(1-a^2x^2\right)^3\arccos(ax)^3$$

$$\Big\downarrow \ 27$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(a\left(-\frac{2\left(\frac{1}{3}a\int\frac{x(3-a^2x^2)}{\sqrt{1-a^2x^2}}dx-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2}{3a^2}\right)+\frac{1}{3}x(1-\right.\right.$$

$$\frac{3}{7}ac^3\left(-\frac{2\left(\frac{1}{35}a\int\frac{x(-5a^6x^6+21a^4x^4-35a^2x^2+35)}{\sqrt{1-a^2x^2}}dx-\frac{1}{7}a^6x^7\arccos(ax)+\frac{3}{5}a^4x^5\arccos(ax)-a^2x^3\arccos(ax)+x\,\mathrm{ar}\right.}{7a}\right.$$

$$\frac{1}{7}c^3x\left(1-a^2x^2\right)^3\arccos(ax)^3$$

$$\Big\downarrow \ 353$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(a\left(-\frac{2\left(\frac{1}{6}a\int\frac{3-a^2x^2}{\sqrt{1-a^2x^2}}dx^2-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{(1-a^2x^2)^{3/2}\arccos(ax)^2}{3a^2}\right)+\frac{1}{3}x(1-\right.\right.$$
$$\frac{3}{7}ac^3\left(-\frac{2\left(\frac{1}{35}a\int\frac{x(-5a^6x^6+21a^4x^4-35a^2x^2+35)}{\sqrt{1-a^2x^2}}dx-\frac{1}{7}a^6x^7\arccos(ax)+\frac{3}{5}a^4x^5\arccos(ax)-a^2x^3\arccos(ax)+x\text{ ar}\right.}{7a}\right.$$
$$\frac{1}{7}c^3x(1-a^2x^2)^3\arccos(ax)^3$$

$\downarrow$ 53

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(a\left(-\frac{2\left(\frac{1}{6}a\int\left(\sqrt{1-a^2x^2}+\frac{2}{\sqrt{1-a^2x^2}}\right)dx^2-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{(1-a^2x^2)^{3/2}\arccos(}{3a^2}\right.\right.\right.$$
$$\frac{3}{7}ac^3\left(-\frac{2\left(\frac{1}{35}a\int\frac{x(-5a^6x^6+21a^4x^4-35a^2x^2+35)}{\sqrt{1-a^2x^2}}dx-\frac{1}{7}a^6x^7\arccos(ax)+\frac{3}{5}a^4x^5\arccos(ax)-a^2x^3\arccos(ax)+x\text{ ar}\right.}{7a}\right.$$
$$\frac{1}{7}c^3x(1-a^2x^2)^3\arccos(ax)^3$$

$\downarrow$ 1576

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(a\left(-\frac{2\left(\frac{1}{6}a\int\left(\sqrt{1-a^2x^2}+\frac{2}{\sqrt{1-a^2x^2}}\right)dx^2-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{(1-a^2x^2)^{3/2}\arccos(}{3a^2}\right.\right.\right.$$
$$\frac{3}{7}ac^3\left(-\frac{2\left(\frac{1}{35}a\int\frac{x(-5a^6x^6+21a^4x^4-35a^2x^2+35)}{\sqrt{1-a^2x^2}}dx-\frac{1}{7}a^6x^7\arccos(ax)+\frac{3}{5}a^4x^5\arccos(ax)-a^2x^3\arccos(ax)+x\text{ ar}\right.}{7a}\right.$$
$$\frac{1}{7}c^3x(1-a^2x^2)^3\arccos(ax)^3$$

$\downarrow$ 1140

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(a\left(-\frac{2\left(\frac{1}{6}a\int\left(\sqrt{1-a^2x^2}+\frac{2}{\sqrt{1-a^2x^2}}\right)dx^2-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{(1-a^2x^2)^{3/2}\arccos(}{3a^2}\right.\right.\right.$$
$$\frac{3}{7}ac^3\left(-\frac{2\left(\frac{1}{35}a\int\frac{x(-5a^6x^6+21a^4x^4-35a^2x^2+35)}{\sqrt{1-a^2x^2}}dx-\frac{1}{7}a^6x^7\arccos(ax)+\frac{3}{5}a^4x^5\arccos(ax)-a^2x^3\arccos(ax)+x\text{ ar}\right.}{7a}\right.$$
$$\frac{1}{7}c^3x(1-a^2x^2)^3\arccos(ax)^3$$

$\downarrow$ 2009

$$\frac{3}{7}ac^3\left(-\frac{2\left(\frac{1}{35}a\int\frac{x(-5a^6x^6+21a^4x^4-35a^2x^2+35)}{\sqrt{1-a^2x^2}}dx-\frac{1}{7}a^6x^7\arccos(ax)+\frac{3}{5}a^4x^5\arccos(ax)-a^2x^3\arccos(ax)+x\,\mathrm{ar}\right.}{7a}\right.$$

$$\frac{1}{7}c^3x(1-a^2x^2)^3\arccos(ax)^3+$$

$$\frac{6}{7}c^3\left(\frac{1}{5}x(1-a^2x^2)^2\arccos(ax)^3+\frac{4}{5}\left(\frac{1}{3}x(1-a^2x^2)\arccos(ax)^3+\frac{2}{3}\left(3a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}-\frac{2\left(x\,\mathrm{arc}\right.}{}\right.\right.\right.\right.$$

$\big\downarrow$ 2331

$$\frac{3}{7}ac^3\left(-\frac{2\left(\frac{1}{70}a\int\frac{-5a^6x^6+21a^4x^4-35a^2x^2+35}{\sqrt{1-a^2x^2}}dx^2-\frac{1}{7}a^6x^7\arccos(ax)+\frac{3}{5}a^4x^5\arccos(ax)-a^2x^3\arccos(ax)+x\,\mathrm{arcc}\right.}{7a}\right.$$

$$\frac{1}{7}c^3x(1-a^2x^2)^3\arccos(ax)^3+$$

$$\frac{6}{7}c^3\left(\frac{1}{5}x(1-a^2x^2)^2\arccos(ax)^3+\frac{4}{5}\left(\frac{1}{3}x(1-a^2x^2)\arccos(ax)^3+\frac{2}{3}\left(3a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}-\frac{2\left(x\,\mathrm{arc}\right.}{}\right.\right.\right.\right.$$

$\big\downarrow$ 2389

$$\frac{3}{7}ac^3\left(-\frac{2\left(\frac{1}{70}a\int\left(5(1-a^2x^2)^{5/2}+6(1-a^2x^2)^{3/2}+8\sqrt{1-a^2x^2}+\frac{16}{\sqrt{1-a^2x^2}}\right)dx^2-\frac{1}{7}a^6x^7\arccos(ax)+\frac{3}{5}a^4x^5\right.}{7a}\right.$$

$$\frac{1}{7}c^3x(1-a^2x^2)^3\arccos(ax)^3+$$

$$\frac{6}{7}c^3\left(\frac{1}{5}x(1-a^2x^2)^2\arccos(ax)^3+\frac{4}{5}\left(\frac{1}{3}x(1-a^2x^2)\arccos(ax)^3+\frac{2}{3}\left(3a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}-\frac{2\left(x\,\mathrm{arc}\right.}{}\right.\right.\right.\right.$$

$\big\downarrow$ 2009

$$\frac{1}{7}c^3x(1-a^2x^2)^3\arccos(ax)^3+$$

$$\frac{6}{7}c^3\left(\frac{1}{5}x(1-a^2x^2)^2\arccos(ax)^3+\frac{4}{5}\left(\frac{1}{3}x(1-a^2x^2)\arccos(ax)^3+\frac{2}{3}\left(3a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}-\frac{2\left(x\,\mathrm{arc}\right.}{}\right.\right.\right.\right.$$

$$\frac{3}{7}ac^3\left(-\frac{(1-a^2x^2)^{7/2}\arccos(ax)^2}{7a^2}-\frac{2\left(-\frac{1}{7}a^6x^7\arccos(ax)+\frac{3}{5}a^4x^5\arccos(ax)-a^2x^3\arccos(ax)+\frac{1}{70}a\left(-\frac{10(}{}\right.\right.}{7a}\right.$$

input
```
Int[(c - a^2*c*x^2)^3*ArcCos[a*x]^3,x]
```

output
```
(c^3*x*(1 - a^2*x^2)^3*ArcCos[a*x]^3)/7 + (3*a*c^3*(-1/7*((1 - a^2*x^2)^(7
/2)*ArcCos[a*x]^2)/a^2 - (2*((a*((-32*Sqrt[1 - a^2*x^2])/a^2 - (16*(1 - a^
2*x^2)^(3/2))/(3*a^2) - (12*(1 - a^2*x^2)^(5/2))/(5*a^2) - (10*(1 - a^2*x^
2)^(7/2))/(7*a^2)))/70 + x*ArcCos[a*x] - a^2*x^3*ArcCos[a*x] + (3*a^4*x^5*
ArcCos[a*x])/5 - (a^6*x^7*ArcCos[a*x])/7))/(7*a)))/7 + (6*c^3*((x*(1 - a^2
*x^2)^2*ArcCos[a*x]^3)/5 + (3*a*(-1/5*((1 - a^2*x^2)^(5/2)*ArcCos[a*x]^2)/
a^2 - (2*((a*((-16*Sqrt[1 - a^2*x^2])/a^2 - (8*(1 - a^2*x^2)^(3/2))/(3*a^2
) - (6*(1 - a^2*x^2)^(5/2))/(5*a^2)))/30 + x*ArcCos[a*x] - (2*a^2*x^3*ArcC
os[a*x])/3 + (a^4*x^5*ArcCos[a*x])/5))/(5*a)))/5 + (4*((x*(1 - a^2*x^2)*Ar
cCos[a*x]^3)/3 + a*(-1/3*((1 - a^2*x^2)^(3/2)*ArcCos[a*x]^2)/a^2 - (2*((a*
((-4*Sqrt[1 - a^2*x^2])/a^2 - (2*(1 - a^2*x^2)^(3/2))/(3*a^2)))/6 + x*ArcC
os[a*x] - (a^2*x^3*ArcCos[a*x])/3))/(3*a)) + (2*(x*ArcCos[a*x]^3 + 3*a*(-(
(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a^2) - (2*(-(Sqrt[1 - a^2*x^2]/a) + x*Ar
cCos[a*x]))/a)))/3))/5))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.75

method	result
derivativedivides	$-\frac{c^3\left(1929375 \arccos(ax)^3 a^7 x^7-826875\sqrt{-a^2x^2+1}\, \arccos(ax)^2 a^6 x^6-8103375 a^5 x^5 \arccos(ax)^3-236250 \arccos(ax)\right.}{}$
default	$-\frac{c^3\left(1929375 \arccos(ax)^3 a^7 x^7-826875\sqrt{-a^2x^2+1}\, \arccos(ax)^2 a^6 x^6-8103375 a^5 x^5 \arccos(ax)^3-236250 \arccos(ax)\right.}{}$
orering	$\frac{x\left(6215625 a^8 x^8-37489212 a^6 x^6+126346014 a^4 x^4-1949470892 a^2 x^2-879660415\right)\left(-a^2 c\, x^2+c\right)^3 \arccos(ax)^3}{13505625(ax-1)(ax+1)(a^2x^2-1)^3}-\frac{(366}{}$

input
```
int((-a^2*c*x^2+c)^3*arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/13505625/a*c^3*(1929375*arccos(a*x)^3*a^7*x^7-826875*(-a^2*x^2+1)^(1/2)
*arccos(a*x)^2*a^6*x^6-8103375*a^5*x^5*arccos(a*x)^3-236250*arccos(a*x)*a^
7*x^7+3869775*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2*a^4*x^4+33750*a^6*x^6*(-a^2
*x^2+1)^(1/2)+13505625*a^3*x^3*arccos(a*x)^3+1547910*a^5*x^5*arccos(a*x)-8
345925*x^2*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2*a^2-269082*a^4*x^4*(-a^2*x^2+1
)^(1/2)-13505625*a*x*arccos(a*x)^3-5563950*a^3*x^3*arccos(a*x)+23825025*ar
ccos(a*x)^2*(-a^2*x^2+1)^(1/2)+1495874*a^2*x^2*(-a^2*x^2+1)^(1/2)+47650050
*a*x*arccos(a*x)-44658302*(-a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.54

$$\int \left(c-a^2cx^2\right)^3 \arccos(ax)^3\,dx =$$
$$-\frac{385875\,(5\,a^7c^3x^7-21\,a^5c^3x^5+35\,a^3c^3x^3-35\,ac^3x)\arccos{(ax)}^3-210\,(1125\,a^7c^3x^7-7371\,a^5c^3x^5+$$

input
```
integrate((-a^2*c*x^2+c)^3*arccos(a*x)^3,x, algorithm="fricas")
```

output
```
-1/13505625*(385875*(5*a^7*c^3*x^7 - 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 - 35*
a*c^3*x)*arccos(a*x)^3 - 210*(1125*a^7*c^3*x^7 - 7371*a^5*c^3*x^5 + 26495*
a^3*c^3*x^3 - 226905*a*c^3*x)*arccos(a*x) + (33750*a^6*c^3*x^6 - 269082*a^
4*c^3*x^4 + 1495874*a^2*c^3*x^2 - 44658302*c^3 - 11025*(75*a^6*c^3*x^6 - 3
51*a^4*c^3*x^4 + 757*a^2*c^3*x^2 - 2161*c^3)*arccos(a*x)^2)*sqrt(-a^2*x^2
+ 1))/a
```

Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.98

$$\int \left(c-a^2cx^2\right)^3 \arccos(ax)^3\,dx$$
$$= \begin{cases} -\frac{a^6c^3x^7\operatorname{acos}^3{(ax)}}{7}+\frac{6a^6c^3x^7\operatorname{acos}{(ax)}}{343}+\frac{3a^5c^3x^6\sqrt{-a^2x^2+1}\operatorname{acos}^2{(ax)}}{49}-\frac{6a^5c^3x^6\sqrt{-a^2x^2+1}}{2401}+\frac{3a^4c^3x^5\operatorname{acos}^3{(ax)}}{5}-\frac{702a^4c^3x}{6} \\ \frac{\pi^3c^3x}{8} \end{cases}$$

input
```
integrate((-a**2*c*x**2+c)**3*acos(a*x)**3,x)
```

output
```
Piecewise((-a**6*c**3*x**7*acos(a*x)**3/7 + 6*a**6*c**3*x**7*acos(a*x)/343
 + 3*a**5*c**3*x**6*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/49 - 6*a**5*c**3*x**
6*sqrt(-a**2*x**2 + 1)/2401 + 3*a**4*c**3*x**5*acos(a*x)**3/5 - 702*a**4*c
**3*x**5*acos(a*x)/6125 - 351*a**3*c**3*x**4*sqrt(-a**2*x**2 + 1)*acos(a*x
)**2/1225 + 29898*a**3*c**3*x**4*sqrt(-a**2*x**2 + 1)/1500625 - a**2*c**3*
x**3*acos(a*x)**3 + 1514*a**2*c**3*x**3*acos(a*x)/3675 + 757*a*c**3*x**2*s
qrt(-a**2*x**2 + 1)*acos(a*x)**2/1225 - 1495874*a*c**3*x**2*sqrt(-a**2*x**
2 + 1)/13505625 + c**3*x*acos(a*x)**3 - 4322*c**3*x*acos(a*x)/1225 - 2161*
c**3*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/(1225*a) + 44658302*c**3*sqrt(-a**2
*x**2 + 1)/(13505625*a), Ne(a, 0)), (pi**3*c**3*x/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.77

$$\int \left(c-a^2cx^2\right)^3 \arccos(ax)^3\,dx$$
$$= \frac{1}{1225}\left(75\sqrt{-a^2x^2+1}a^4c^3x^6 - 351\sqrt{-a^2x^2+1}a^2c^3x^4 + 757\sqrt{-a^2x^2+1}c^3x^2 - \frac{2161\sqrt{-a^2x^2+1}c^3}{a^2}\right)\epsilon$$
$$- \frac{1}{35}\left(5\,a^6c^3x^7 - 21\,a^4c^3x^5 + 35\,a^2c^3x^3 - 35\,c^3x\right)\arccos\left(ax\right)^3$$
$$- \frac{2}{13505625}\left(16875\sqrt{-a^2x^2+1}a^4c^3x^6 - 134541\sqrt{-a^2x^2+1}a^2c^3x^4 + 747937\sqrt{-a^2x^2+1}c^3x^2 - \frac{2232}{}\right.$$

input
```
integrate((-a^2*c*x^2+c)^3*arccos(a*x)^3,x, algorithm="maxima")
```

output
```
1/1225*(75*sqrt(-a^2*x^2 + 1)*a^4*c^3*x^6 - 351*sqrt(-a^2*x^2 + 1)*a^2*c^3
*x^4 + 757*sqrt(-a^2*x^2 + 1)*c^3*x^2 - 2161*sqrt(-a^2*x^2 + 1)*c^3/a^2)*a
*arccos(a*x)^2 - 1/35*(5*a^6*c^3*x^7 - 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 - 3
5*c^3*x)*arccos(a*x)^3 - 2/13505625*(16875*sqrt(-a^2*x^2 + 1)*a^4*c^3*x^6
 - 134541*sqrt(-a^2*x^2 + 1)*a^2*c^3*x^4 + 747937*sqrt(-a^2*x^2 + 1)*c^3*x^
2 - 22329151*sqrt(-a^2*x^2 + 1)*c^3/a^2 - 105*(1125*a^6*c^3*x^7 - 7371*a^4
*c^3*x^5 + 26495*a^2*c^3*x^3 - 226905*c^3*x)*arccos(a*x)/a)*a
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.85

$$\int \left(c-a^2cx^2\right)^3 \arccos(ax)^3\,dx = -\frac{1}{7}\,a^6c^3x^7\arccos\left(ax\right)^3+\frac{6}{343}\,a^6c^3x^7\arccos\left(ax\right)+\frac{3}{49}\,\sqrt{-a^2x^2+1}a^5c^3x^6\arccos\left(ax\right)^2+\frac{3}{5}\,a^4c^3x^5\arccos\left(ax\right)^3-\frac{6}{2401}\,\sqrt{-a^2x^2+1}a^5c^3x^6-\frac{702}{6125}\,a^4c^3x^5\arccos\left(ax\right)-\frac{351}{1225}\,\sqrt{-a^2x^2+1}a^3c^3x^4\arccos\left(ax\right)^2-a^2c^3x^3\arccos\left(ax\right)^3+\frac{29898}{1500625}\,\sqrt{-a^2x^2+1}a^3c^3x^4+\frac{1514}{3675}\,a^2c^3x^3\arccos\left(ax\right)+\frac{757}{1225}\,\sqrt{-a^2x^2+1}ac^3x^2\arccos\left(ax\right)^2+c^3x\arccos\left(ax\right)^3-\frac{1495874}{13505625}\,\sqrt{-a^2x^2+1}ac^3x^2-\frac{4322}{1225}\,c^3x\arccos\left(ax\right)-\frac{2161\,\sqrt{-a^2x^2+1}c^3\arccos\left(ax\right)^2}{1225\,a}+\frac{44658302\,\sqrt{-a^2x^2+1}c^3}{13505625\,a}$$

input
```
integrate((-a^2*c*x^2+c)^3*arccos(a*x)^3,x, algorithm="giac")
```

output
```
-1/7*a^6*c^3*x^7*arccos(a*x)^3 + 6/343*a^6*c^3*x^7*arccos(a*x) + 3/49*sqrt
(-a^2*x^2 + 1)*a^5*c^3*x^6*arccos(a*x)^2 + 3/5*a^4*c^3*x^5*arccos(a*x)^3 -
 6/2401*sqrt(-a^2*x^2 + 1)*a^5*c^3*x^6 - 702/6125*a^4*c^3*x^5*arccos(a*x)
- 351/1225*sqrt(-a^2*x^2 + 1)*a^3*c^3*x^4*arccos(a*x)^2 - a^2*c^3*x^3*arcc
os(a*x)^3 + 29898/1500625*sqrt(-a^2*x^2 + 1)*a^3*c^3*x^4 + 1514/3675*a^2*c
^3*x^3*arccos(a*x) + 757/1225*sqrt(-a^2*x^2 + 1)*a*c^3*x^2*arccos(a*x)^2 +
 c^3*x*arccos(a*x)^3 - 1495874/13505625*sqrt(-a^2*x^2 + 1)*a*c^3*x^2 - 432
2/1225*c^3*x*arccos(a*x) - 2161/1225*sqrt(-a^2*x^2 + 1)*c^3*arccos(a*x)^2/
a + 44658302/13505625*sqrt(-a^2*x^2 + 1)*c^3/a
```

Mupad [F(-1)]

Timed out.

$$\int \left(c-a^2cx^2\right)^3 \arccos(ax)^3\,dx = \int \mathrm{acos}(a\,x)^3\,\left(c-a^2\,c\,x^2\right)^3\,dx$$

input
```
int(acos(a*x)^3*(c - a^2*c*x^2)^3,x)
```

output
```
int(acos(a*x)^3*(c - a^2*c*x^2)^3, x)
```

Reduce [F]

$$\int \left(c-a^2cx^2\right)^3 \arccos(ax)^3\,dx$$
$$= \frac{c^3\left(acos(ax)^3\,ax - 3\sqrt{-a^2x^2+1}\,acos(ax)^2 - 6acos(ax)\,ax + 6\sqrt{-a^2x^2+1} - \left(\int acos(ax)^3\,x^6dx\right)a^7 + 3\right.}{a}$$

input
```
int((-a^2*c*x^2+c)^3*acos(a*x)^3,x)
```

output
```
(c**3*(acos(a*x)**3*a*x - 3*sqrt( - a**2*x**2 + 1)*acos(a*x)**2 - 6*acos(a
*x)*a*x + 6*sqrt( - a**2*x**2 + 1) - int(acos(a*x)**3*x**6,x)*a**7 + 3*int
(acos(a*x)**3*x**4,x)*a**5 - 3*int(acos(a*x)**3*x**2,x)*a**3))/a
```

3.292 $\int (c-a^2cx^2)^2 \arccos(ax)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 273

$$\begin{aligned}\int (c-a^2cx^2)^2 \arccos(ax)^3\,dx &= -\frac{4144c^2\sqrt{1-a^2x^2}}{1125a}-\frac{272c^2(1-a^2x^2)^{3/2}}{3375a}-\frac{6c^2(1-a^2x^2)^{5/2}}{625a}-\frac{298}{75}c^2x\arccos(ax)\\&\quad+\frac{76}{225}a^2c^2x^3\arccos(ax)-\frac{6}{125}a^4c^2x^5\arccos(ax)+\frac{8c^2\sqrt{1-a^2x^2}\arccos(ax)^2}{5a}\\&\quad+\frac{4c^2(1-a^2x^2)^{3/2}\arccos(ax)^2}{15a}+\frac{3c^2(1-a^2x^2)^{5/2}\arccos(ax)^2}{25a}\\&\quad+\frac{8}{15}c^2x\arccos(ax)^3+\frac{4}{15}c^2x(1-a^2x^2)\arccos(ax)^3+\frac{1}{5}c^2x(1-a^2x^2)^2\arccos(ax)^3\end{aligned}$$

output

```
-4144/1125*c^2*(-a^2*x^2+1)^(1/2)/a-272/3375*c^2*(-a^2*x^2+1)^(3/2)/a-6/62
5*c^2*(-a^2*x^2+1)^(5/2)/a-298/75*c^2*x*arccos(a*x)+76/225*a^2*c^2*x^3*arc
cos(a*x)-6/125*a^4*c^2*x^5*arccos(a*x)+8/5*c^2*(-a^2*x^2+1)^(1/2)*arccos(a
*x)^2/a+4/15*c^2*(-a^2*x^2+1)^(3/2)*arccos(a*x)^2/a+3/25*c^2*(-a^2*x^2+1)^
(5/2)*arccos(a*x)^2/a+8/15*c^2*x*arccos(a*x)^3+4/15*c^2*x*(-a^2*x^2+1)*arc
cos(a*x)^3+1/5*c^2*x*(-a^2*x^2+1)^2*arccos(a*x)^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.51

$$\int \left(c-a^2cx^2\right)^2 \arccos(ax)^3\,dx = \frac{c^2\left(2\sqrt{1-a^2x^2}(31841-842a^2x^2+81a^4x^4)-30ax(2235-190a^2x^2+27a^4x^4)\arccos(ax)-225\sqrt{1-a\right.}{16875a}$$

input
```
Integrate[(c - a^2*c*x^2)^2*ArcCos[a*x]^3,x]
```

output
```
(c^2*(2*Sqrt[1 - a^2*x^2]*(31841 - 842*a^2*x^2 + 81*a^4*x^4) - 30*a*x*(223
5 - 190*a^2*x^2 + 27*a^4*x^4)*ArcCos[a*x] - 225*Sqrt[1 - a^2*x^2]*(149 - 3
8*a^2*x^2 + 9*a^4*x^4)*ArcCos[a*x]^2 + 1125*a*x*(15 - 10*a^2*x^2 + 3*a^4*x
^4)*ArcCos[a*x]^3))/(16875*a)
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.38, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5159, 27, 5159, 5131, 5183, 5131, 241, 5155, 27, 353, 53, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^3\left(c-a^2cx^2\right)^2\,dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{3}{5}ac^2\int x\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2dx+\frac{4}{5}c\int c\left(1-a^2x^2\right)\arccos(ax)^3dx+\frac{1}{5}c^2x\left(1-a^2x^2\right)^2\arccos(ax)^3$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{5}ac^2\int x\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2dx+\frac{4}{5}c^2\int \left(1-a^2x^2\right)\arccos(ax)^3dx+\frac{1}{5}c^2x\left(1-a^2x^2\right)^2\arccos(ax)^3$$

$$\Big\downarrow \text{5159}$$

$$\frac{3}{5}ac^2\int x\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2dx+$$
$$\frac{4}{5}c^2\left(a\int x\sqrt{1-a^2x^2}\arccos(ax)^2dx+\frac{2}{3}\int\arccos(ax)^3dx+\frac{1}{3}x\left(1-a^2x^2\right)\arccos(ax)^3\right)+$$
$$\frac{1}{5}c^2x\left(1-a^2x^2\right)^2\arccos(ax)^3$$

$$\Big\downarrow \text{5131}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(3a\int\frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)^3\right)+a\int x\sqrt{1-a^2x^2}\arccos(ax)^2dx+\frac{1}{3}x\left(1-a^2x^2\right)\arccos(ax)^3$$
$$\frac{3}{5}ac^2\int x\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2dx+\frac{1}{5}c^2x\left(1-a^2x^2\right)^2\arccos(ax)^3$$

$$\Big\downarrow \text{5183}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(3a\left(-\frac{2\int\arccos(ax)dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}\right)+x\arccos(ax)^3\right)+a\left(-\frac{2\int\left(1-a^2x^2\right)\arccos(ax)}{3a}\right.\right.$$
$$\frac{3}{5}ac^2\left(-\frac{2\int\left(1-a^2x^2\right)^2\arccos(ax)dx}{5a}-\frac{\left(1-a^2x^2\right)^{5/2}\arccos(ax)^2}{5a^2}\right)+$$
$$\frac{1}{5}c^2x\left(1-a^2x^2\right)^2\arccos(ax)^3$$

$$\Big\downarrow \text{5131}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(3a\left(-\frac{2\left(a\int\frac{x}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)\right)}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}\right)+x\arccos(ax)^3\right)+a\left(-\frac{2\int(1-}{}\right.\right.$$
$$\frac{3}{5}ac^2\left(-\frac{2\int\left(1-a^2x^2\right)^2\arccos(ax)dx}{5a}-\frac{\left(1-a^2x^2\right)^{5/2}\arccos(ax)^2}{5a^2}\right)+$$
$$\frac{1}{5}c^2x\left(1-a^2x^2\right)^2\arccos(ax)^3$$

$$\Big\downarrow \text{241}$$

$$\frac{4}{5}c^2\left(a\left(-\frac{2\int\left(1-a^2x^2\right)\arccos(ax)dx}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2}{3a^2}\right)+\frac{1}{3}x\left(1-a^2x^2\right)\arccos(ax)^3+\frac{2}{3}\left(3a\left(-\right.\right.\right.$$
$$\frac{3}{5}ac^2\left(-\frac{2\int\left(1-a^2x^2\right)^2\arccos(ax)dx}{5a}-\frac{\left(1-a^2x^2\right)^{5/2}\arccos(ax)^2}{5a^2}\right)+$$
$$\frac{1}{5}c^2x\left(1-a^2x^2\right)^2\arccos(ax)^3$$

$$\Big\downarrow \text{5155}$$

$$\frac{4}{5}c^2\left(a\left(-\frac{2\left(a\int\frac{x(3-a^2x^2)}{3\sqrt{1-a^2x^2}}dx-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{(1-a^2x^2)^{3/2}\arccos(ax)^2}{3a^2}\right)+\frac{1}{3}x(1-a^2x\right.$$

$$\frac{3}{5}ac^2\left(-\frac{2\left(a\int\frac{x(3a^4x^4-10a^2x^2+15)}{15\sqrt{1-a^2x^2}}dx+\frac{1}{5}a^4x^5\arccos(ax)-\frac{2}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{5a}-\frac{(1-a^2x^2)^{5/2}\text{ar}}{5a^2}\right.$$

$$\frac{1}{5}c^2x(1-a^2x^2)^2\arccos(ax)^3$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{5}c^2\left(a\left(-\frac{2\left(\frac{1}{3}a\int\frac{x(3-a^2x^2)}{\sqrt{1-a^2x^2}}dx-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{(1-a^2x^2)^{3/2}\arccos(ax)^2}{3a^2}\right)+\frac{1}{3}x(1-a^2\right.$$

$$\frac{3}{5}ac^2\left(-\frac{2\left(\frac{1}{15}a\int\frac{x(3a^4x^4-10a^2x^2+15)}{\sqrt{1-a^2x^2}}dx+\frac{1}{5}a^4x^5\arccos(ax)-\frac{2}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{5a}-\frac{(1-a^2x^2)^{5/2}}{5a}\right.$$

$$\frac{1}{5}c^2x(1-a^2x^2)^2\arccos(ax)^3$$

$$\Big\downarrow \text{353}$$

$$\frac{4}{5}c^2\left(a\left(-\frac{2\left(\frac{1}{6}a\int\frac{3-a^2x^2}{\sqrt{1-a^2x^2}}dx^2-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{(1-a^2x^2)^{3/2}\arccos(ax)^2}{3a^2}\right)+\frac{1}{3}x(1-a^2\right.$$

$$\frac{3}{5}ac^2\left(-\frac{2\left(\frac{1}{15}a\int\frac{x(3a^4x^4-10a^2x^2+15)}{\sqrt{1-a^2x^2}}dx+\frac{1}{5}a^4x^5\arccos(ax)-\frac{2}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{5a}-\frac{(1-a^2x^2)^{5/2}}{5a}\right.$$

$$\frac{1}{5}c^2x(1-a^2x^2)^2\arccos(ax)^3$$

$$\Big\downarrow \text{53}$$

$$\frac{4}{5}c^2\left(a\left(-\frac{2\left(\frac{1}{6}a\int\left(\sqrt{1-a^2x^2}+\frac{2}{\sqrt{1-a^2x^2}}\right)dx^2-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{(1-a^2x^2)^{3/2}\arccos(ax)}{3a^2}\right.\right.$$

$$\frac{3}{5}ac^2\left(-\frac{2\left(\frac{1}{15}a\int\frac{x(3a^4x^4-10a^2x^2+15)}{\sqrt{1-a^2x^2}}dx+\frac{1}{5}a^4x^5\arccos(ax)-\frac{2}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{5a}-\frac{(1-a^2x^2)^{5/2}}{5a}\right.$$

$$\frac{1}{5}c^2x(1-a^2x^2)^2\arccos(ax)^3$$

$$\Big\downarrow \text{1576}$$

$$\frac{4}{5}c^2\left(a\left(-\frac{2\left(\frac{1}{6}a\int\left(\sqrt{1-a^2x^2}+\frac{2}{\sqrt{1-a^2x^2}}\right)dx^2-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\arccos(ax)}{3a^2}\right.\right.$$

$$\frac{3}{5}ac^2\left(-\frac{2\left(\frac{1}{30}a\int\frac{3a^4x^4-10a^2x^2+15}{\sqrt{1-a^2x^2}}dx^2+\frac{1}{5}a^4x^5\arccos(ax)-\frac{2}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{5a}-\frac{\left(1-a^2x^2\right)^{5/2}a}{5a^2}\right.$$

$$\frac{1}{5}c^2x\left(1-a^2x^2\right)^2\arccos(ax)^3$$

$$\Big\downarrow \text{1140}$$

$$\frac{4}{5}c^2\left(a\left(-\frac{2\left(\frac{1}{6}a\int\left(\sqrt{1-a^2x^2}+\frac{2}{\sqrt{1-a^2x^2}}\right)dx^2-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\arccos(ax)}{3a^2}\right.\right.$$

$$\frac{3}{5}ac^2\left(-\frac{2\left(\frac{1}{30}a\int\left(3\left(1-a^2x^2\right)^{3/2}+4\sqrt{1-a^2x^2}+\frac{8}{\sqrt{1-a^2x^2}}\right)dx^2+\frac{1}{5}a^4x^5\arccos(ax)-\frac{2}{3}a^2x^3\arccos(ax)+x\,\text{arc}\right.}{5a}\right.$$

$$\frac{1}{5}c^2x\left(1-a^2x^2\right)^2\arccos(ax)^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}c^2x\left(1-a^2x^2\right)^2\arccos(ax)^3+$$

$$\frac{4}{5}c^2\left(\frac{1}{3}x\left(1-a^2x^2\right)\arccos(ax)^3+\frac{2}{3}\left(3a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}-\frac{2\left(x\arccos(ax)-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a}\right)+x\arccos\right.\right.$$

$$\frac{3}{5}ac^2\left(-\frac{\left(1-a^2x^2\right)^{5/2}\arccos(ax)^2}{5a^2}-\frac{2\left(\frac{1}{5}a^4x^5\arccos(ax)-\frac{2}{3}a^2x^3\arccos(ax)+\frac{1}{30}a\left(-\frac{6\left(1-a^2x^2\right)^{5/2}}{5a^2}-\frac{8\left(1-a^2x^2\right)}{3a^2}\right.\right.}{5a}\right.$$

input
```
Int[(c - a^2*c*x^2)^2*ArcCos[a*x]^3,x]
```

output
```
(c^2*x*(1 - a^2*x^2)^2*ArcCos[a*x]^3)/5 + (3*a*c^2*(-1/5*((1 - a^2*x^2)^(5
/2)*ArcCos[a*x]^2)/a^2 - (2*((a*((-16*Sqrt[1 - a^2*x^2])/a^2 - (8*(1 - a^2
*x^2)^(3/2))/(3*a^2) - (6*(1 - a^2*x^2)^(5/2))/(5*a^2)))/30 + x*ArcCos[a*x
] - (2*a^2*x^3*ArcCos[a*x])/3 + (a^4*x^5*ArcCos[a*x])/5))/(5*a)))/5 + (4*c
^2*((x*(1 - a^2*x^2)*ArcCos[a*x]^3)/3 + a*(-1/3*((1 - a^2*x^2)^(3/2)*ArcCo
s[a*x]^2)/a^2 - (2*((a*((-4*Sqrt[1 - a^2*x^2])/a^2 - (2*(1 - a^2*x^2)^(3/2
))/(3*a^2)))/6 + x*ArcCos[a*x] - (a^2*x^3*ArcCos[a*x])/3))/(3*a)) + (2*(x*
ArcCos[a*x]^3 + 3*a*(-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a^2) - (2*(-(Sqrt
[1 - a^2*x^2]/a) + x*ArcCos[a*x]))/a)))/3))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{c^2\left(3375a^5x^5\arccos(ax)^3-2025\sqrt{-a^2x^2+1}\,\arccos(ax)^2a^4x^4-11250a^3x^3\arccos(ax)^3-810a^5x^5\arccos(ax)+8550x^2\right.}{}$
default	$\frac{c^2\left(3375a^5x^5\arccos(ax)^3-2025\sqrt{-a^2x^2+1}\,\arccos(ax)^2a^4x^4-11250a^3x^3\arccos(ax)^3-810a^5x^5\arccos(ax)+8550x^2\right.}{}$
orering	$\frac{x(29889a^6x^6-179507a^4x^4+2768347a^2x^2+1732471)(-a^2c\,x^2+c)^2\arccos(ax)^3}{50625(a^2x^2-1)^3}-\frac{(7857a^6x^6-60788a^4x^4+144560}{}$

input
```
int((-a^2*c*x^2+c)^2*arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16875/a*c^2*(3375*a^5*x^5*arccos(a*x)^3-2025*(-a^2*x^2+1)^(1/2)*arccos(a
*x)^2*a^4*x^4-11250*a^3*x^3*arccos(a*x)^3-810*a^5*x^5*arccos(a*x)+8550*x^2
*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2*a^2+162*a^4*x^4*(-a^2*x^2+1)^(1/2)+16875
*a*x*arccos(a*x)^3+5700*a^3*x^3*arccos(a*x)-33525*arccos(a*x)^2*(-a^2*x^2+
1)^(1/2)-1684*a^2*x^2*(-a^2*x^2+1)^(1/2)-67050*a*x*arccos(a*x)+63682*(-a^2
*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.58

$$\int \left(c-a^2cx^2\right)^2 \arccos(ax)^3\,dx$$
$$=\frac{1125\,(3\,a^5c^2x^5-10\,a^3c^2x^3+15\,ac^2x)\arccos\,(ax)^3-30\,(27\,a^5c^2x^5-190\,a^3c^2x^3+2235\,ac^2x)\arccos\,(ax)}{168}$$

input
```
integrate((-a^2*c*x^2+c)^2*arccos(a*x)^3,x, algorithm="fricas")
```

output
```
1/16875*(1125*(3*a^5*c^2*x^5 - 10*a^3*c^2*x^3 + 15*a*c^2*x)*arccos(a*x)^3
- 30*(27*a^5*c^2*x^5 - 190*a^3*c^2*x^3 + 2235*a*c^2*x)*arccos(a*x) + (162*
a^4*c^2*x^4 - 1684*a^2*c^2*x^2 - 225*(9*a^4*c^2*x^4 - 38*a^2*c^2*x^2 + 149
*c^2)*arccos(a*x)^2 + 63682*c^2)*sqrt(-a^2*x^2 + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.99

$$\int \left(c-a^2cx^2\right)^2 \arccos(ax)^3\,dx$$
$$=\begin{cases} \frac{a^4c^2x^5\operatorname{acos}^3(ax)}{5}-\frac{6a^4c^2x^5\operatorname{acos}(ax)}{125}-\frac{3a^3c^2x^4\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{25}+\frac{6a^3c^2x^4\sqrt{-a^2x^2+1}}{625}-\frac{2a^2c^2x^3\operatorname{acos}^3(ax)}{3}+\frac{76a^2c^2x^3\operatorname{ac}}{225} \\ \frac{\pi^3c^2x}{8} \end{cases}$$

input
```
integrate((-a**2*c*x**2+c)**2*acos(a*x)**3,x)
```

output
```
Piecewise((a**4*c**2*x**5*acos(a*x)**3/5 - 6*a**4*c**2*x**5*acos(a*x)/125
- 3*a**3*c**2*x**4*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/25 + 6*a**3*c**2*x**4
*sqrt(-a**2*x**2 + 1)/625 - 2*a**2*c**2*x**3*acos(a*x)**3/3 + 76*a**2*c**2
*x**3*acos(a*x)/225 + 38*a*c**2*x**2*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/75
- 1684*a*c**2*x**2*sqrt(-a**2*x**2 + 1)/16875 + c**2*x*acos(a*x)**3 - 298*
c**2*x*acos(a*x)/75 - 149*c**2*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/(75*a) +
63682*c**2*sqrt(-a**2*x**2 + 1)/(16875*a), Ne(a, 0)), (pi**3*c**2*x/8, Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.79

$$\int \left(c-a^2cx^2\right)^2 \arccos(ax)^3\,dx =$$
$$-\frac{1}{75}\left(9\sqrt{-a^2x^2+1}a^2c^2x^4-38\sqrt{-a^2x^2+1}c^2x^2+\frac{149\sqrt{-a^2x^2+1}c^2}{a^2}\right)a\arccos(ax)^2$$
$$+\frac{1}{15}\left(3a^4c^2x^5-10a^2c^2x^3+15c^2x\right)\arccos(ax)^3$$
$$+\frac{2}{16875}\left(81\sqrt{-a^2x^2+1}a^2c^2x^4-842\sqrt{-a^2x^2+1}c^2x^2-\frac{15\left(27a^4c^2x^5-190a^2c^2x^3+2235c^2x\right)\operatorname{arcco}}{a}\right.$$

input
```
integrate((-a^2*c*x^2+c)^2*arccos(a*x)^3,x, algorithm="maxima")
```

output
```
-1/75*(9*sqrt(-a^2*x^2 + 1)*a^2*c^2*x^4 - 38*sqrt(-a^2*x^2 + 1)*c^2*x^2 +
149*sqrt(-a^2*x^2 + 1)*c^2/a^2)*a*arccos(a*x)^2 + 1/15*(3*a^4*c^2*x^5 - 10
*a^2*c^2*x^3 + 15*c^2*x)*arccos(a*x)^3 + 2/16875*(81*sqrt(-a^2*x^2 + 1)*a^
2*c^2*x^4 - 842*sqrt(-a^2*x^2 + 1)*c^2*x^2 - 15*(27*a^4*c^2*x^5 - 190*a^2*
c^2*x^3 + 2235*c^2*x)*arccos(a*x)/a + 31841*sqrt(-a^2*x^2 + 1)*c^2/a^2)*a
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \left(c-a^2cx^2\right)^2\arccos(ax)^3\,dx &= \frac{1}{5}\,a^4c^2x^5\arccos\left(ax\right)^3-\frac{6}{125}\,a^4c^2x^5\arccos\left(ax\right)\\&-\frac{3}{25}\,\sqrt{-a^2x^2+1}a^3c^2x^4\arccos\left(ax\right)^2\\&-\frac{2}{3}\,a^2c^2x^3\arccos\left(ax\right)^3+\frac{6}{625}\,\sqrt{-a^2x^2+1}a^3c^2x^4\\&+\frac{76}{225}\,a^2c^2x^3\arccos\left(ax\right)\\&+\frac{38}{75}\,\sqrt{-a^2x^2+1}ac^2x^2\arccos\left(ax\right)^2+c^2x\arccos\left(ax\right)^3\\&-\frac{1684}{16875}\,\sqrt{-a^2x^2+1}ac^2x^2-\frac{298}{75}\,c^2x\arccos\left(ax\right)\\&-\frac{149\,\sqrt{-a^2x^2+1}c^2\arccos\left(ax\right)^2}{75\,a}\\&+\frac{63682\,\sqrt{-a^2x^2+1}c^2}{16875\,a}\end{aligned}$$

input
```
integrate((-a^2*c*x^2+c)^2*arccos(a*x)^3,x, algorithm="giac")
```

output
```
1/5*a^4*c^2*x^5*arccos(a*x)^3 - 6/125*a^4*c^2*x^5*arccos(a*x) - 3/25*sqrt(
-a^2*x^2 + 1)*a^3*c^2*x^4*arccos(a*x)^2 - 2/3*a^2*c^2*x^3*arccos(a*x)^3 +
6/625*sqrt(-a^2*x^2 + 1)*a^3*c^2*x^4 + 76/225*a^2*c^2*x^3*arccos(a*x) + 38
/75*sqrt(-a^2*x^2 + 1)*a*c^2*x^2*arccos(a*x)^2 + c^2*x*arccos(a*x)^3 - 168
4/16875*sqrt(-a^2*x^2 + 1)*a*c^2*x^2 - 298/75*c^2*x*arccos(a*x) - 149/75*s
qrt(-a^2*x^2 + 1)*c^2*arccos(a*x)^2/a + 63682/16875*sqrt(-a^2*x^2 + 1)*c^2
/a
```

Mupad [F(-1)]

Timed out.

$$\int \left(c-a^2cx^2\right)^2 \arccos(ax)^3 \, dx = \int \mathrm{acos}(a\,x)^3 \left(c-a^2\,c\,x^2\right)^2 dx$$

input
```
int(acos(a*x)^3*(c - a^2*c*x^2)^2,x)
```

output
```
int(acos(a*x)^3*(c - a^2*c*x^2)^2, x)
```

Reduce [F]

$$\int \left(c-a^2cx^2\right)^2 \arccos(ax)^3 \, dx$$
$$= \frac{c^2\left(acos(ax)^3\,ax - 3\sqrt{-a^2x^2+1}\,acos(ax)^2 - 6acos(ax)\,ax + 6\sqrt{-a^2x^2+1} + \left(\int acos(ax)^3\,x^4dx\right)a^5 - 2\cdots\right.}{a}$$

input
```
int((-a^2*c*x^2+c)^2*acos(a*x)^3,x)
```

output
```
(c**2*(acos(a*x)**3*a*x - 3*sqrt( - a**2*x**2 + 1)*acos(a*x)**2 - 6*acos(a
*x)*a*x + 6*sqrt( - a**2*x**2 + 1) + int(acos(a*x)**3*x**4,x)*a**5 - 2*int
(acos(a*x)**3*x**2,x)*a**3))/a
```

3.293 $\int (c - a^2cx^2) \arccos(ax)^3 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 158

$$\int (c - a^2cx^2) \arccos(ax)^3 \, dx = -\frac{40c\sqrt{1-a^2x^2}}{9a} - \frac{2c(1-a^2x^2)^{3/2}}{27a} - \frac{14}{3}cx\arccos(ax) + \frac{2}{9}a^2cx^3\arccos(ax) + \frac{2c\sqrt{1-a^2x^2}\arccos(ax)^2}{a} + \frac{c(1-a^2x^2)^{3/2}\arccos(ax)^2}{3a} + \frac{2}{3}cx\arccos(ax)^3 + \frac{1}{3}cx(1-a^2x^2)\arccos(ax)^3$$

output
```
-40/9*c*(-a^2*x^2+1)^(1/2)/a-2/27*c*(-a^2*x^2+1)^(3/2)/a-14/3*c*x*arccos(a
*x)+2/9*a^2*c*x^3*arccos(a*x)+2*c*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2/a+1/3*c
*(-a^2*x^2+1)^(3/2)*arccos(a*x)^2/a+2/3*c*x*arccos(a*x)^3+1/3*c*x*(-a^2*x^
2+1)*arccos(a*x)^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.64

$$\int \left(c-a^2cx^2\right)\arccos(ax)^3\,dx$$
$$=\frac{c\left(-2\sqrt{1-a^2x^2}(-61+a^2x^2)+6ax(-21+a^2x^2)\arccos(ax)+9\sqrt{1-a^2x^2}(-7+a^2x^2)\arccos(ax)^2-9\right.}{27a}$$

input
```
Integrate[(c - a^2*c*x^2)*ArcCos[a*x]^3,x]
```

output
```
(c*(-2*Sqrt[1 - a^2*x^2]*(-61 + a^2*x^2) + 6*a*x*(-21 + a^2*x^2)*ArcCos[a*
x] + 9*Sqrt[1 - a^2*x^2]*(-7 + a^2*x^2)*ArcCos[a*x]^2 - 9*a*x*(-3 + a^2*x^
2)*ArcCos[a*x]^3))/(27*a)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.28, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5159, 5131, 5183, 5131, 241, 5155, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^3\left(c-a^2cx^2\right)\,dx$$

$$\Big\downarrow \text{5159}$$

$$ac\int x\sqrt{1-a^2x^2}\arccos(ax)^2dx+\frac{2}{3}c\int \arccos(ax)^3dx+\frac{1}{3}cx\left(1-a^2x^2\right)\arccos(ax)^3$$

$$\Big\downarrow \text{5131}$$

$$\frac{2}{3}c\left(3a\int \frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)^3\right)+ac\int x\sqrt{1-a^2x^2}\arccos(ax)^2dx+\frac{1}{3}cx\left(1-a^2x^2\right)\arccos(ax)^3$$

$$\Big\downarrow \text{5183}$$

$$\frac{2}{3}c\left(3a\left(-\frac{2\int \arccos(ax)dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}\right)+x\arccos(ax)^3\right)+$$
$$ac\left(-\frac{2\int \left(1-a^2x^2\right)\arccos(ax)dx}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2}{3a^2}\right)+\frac{1}{3}cx\left(1-a^2x^2\right)\arccos(ax)^3$$

$$\Big\downarrow \text{ 5131}$$

$$\frac{2}{3}c\left(3a\left(-\frac{2\left(a\int \frac{x}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)\right)}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}\right)+x\arccos(ax)^3\right)+$$
$$ac\left(-\frac{2\int \left(1-a^2x^2\right)\arccos(ax)dx}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2}{3a^2}\right)+\frac{1}{3}cx\left(1-a^2x^2\right)\arccos(ax)^3$$

$$\Big\downarrow \text{ 241}$$

$$ac\left(-\frac{2\int \left(1-a^2x^2\right)\arccos(ax)dx}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2}{3a^2}\right)+$$
$$\frac{1}{3}cx\left(1-a^2x^2\right)\arccos(ax)^3+$$
$$\frac{2}{3}c\left(3a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}-\frac{2\left(x\arccos(ax)-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a}\right)+x\arccos(ax)^3\right)$$

$$\Big\downarrow \text{ 5155}$$

$$ac\left(-\frac{2\left(a\int \frac{x\left(3-a^2x^2\right)}{3\sqrt{1-a^2x^2}}dx-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2}{3a^2}\right)+$$
$$\frac{1}{3}cx\left(1-a^2x^2\right)\arccos(ax)^3+$$
$$\frac{2}{3}c\left(3a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}-\frac{2\left(x\arccos(ax)-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a}\right)+x\arccos(ax)^3\right)$$

$$\Big\downarrow \text{ 27}$$

$$ac\left(-\frac{2\left(\frac{1}{3}a\int \frac{x\left(3-a^2x^2\right)}{\sqrt{1-a^2x^2}}dx-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2}{3a^2}\right)+$$
$$\frac{1}{3}cx\left(1-a^2x^2\right)\arccos(ax)^3+$$
$$\frac{2}{3}c\left(3a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}-\frac{2\left(x\arccos(ax)-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a}\right)+x\arccos(ax)^3\right)$$

$$\Big\downarrow \text{ 353}$$

$$ac\left(-\frac{2\left(\frac{1}{6}a\int\frac{3-a^2x^2}{\sqrt{1-a^2x^2}}dx^2-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2}{3a^2}\right)+\frac{1}{3}cx\left(1-a^2x^2\right)\arccos(ax)^3+\frac{2}{3}c\left(3a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}-\frac{2\left(x\arccos(ax)-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a}\right)+x\arccos(ax)^3\right)$$

$$\Big\downarrow 53$$

$$ac\left(-\frac{2\left(\frac{1}{6}a\int\left(\sqrt{1-a^2x^2}+\frac{2}{\sqrt{1-a^2x^2}}\right)dx^2-\frac{1}{3}a^2x^3\arccos(ax)+x\arccos(ax)\right)}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2}{3a^2}\right)+\frac{1}{3}cx\left(1-a^2x^2\right)\arccos(ax)^3+\frac{2}{3}c\left(3a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}-\frac{2\left(x\arccos(ax)-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a}\right)+x\arccos(ax)^3\right)$$

$$\Big\downarrow 2009$$

$$\frac{1}{3}cx\left(1-a^2x^2\right)\arccos(ax)^3+\frac{2}{3}c\left(3a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{a^2}-\frac{2\left(x\arccos(ax)-\frac{\sqrt{1-a^2x^2}}{a}\right)}{a}\right)+x\arccos(ax)^3\right)+ac\left(-\frac{\left(1-a^2x^2\right)^{3/2}\arccos(ax)^2}{3a^2}-\frac{2\left(-\frac{1}{3}a^2x^3\arccos(ax)+\frac{1}{6}a\left(-\frac{2\left(1-a^2x^2\right)^{3/2}}{3a^2}-\frac{4\sqrt{1-a^2x^2}}{a^2}\right)+x\arccos(ax)\right)}{3a}\right)$$

input
```
Int[(c - a^2*c*x^2)*ArcCos[a*x]^3,x]
```

output
```
(c*x*(1 - a^2*x^2)*ArcCos[a*x]^3)/3 + a*c*(-1/3*((1 - a^2*x^2)^(3/2)*ArcCo
s[a*x]^2)/a^2 - (2*((a*((-4*Sqrt[1 - a^2*x^2])/a^2 - (2*(1 - a^2*x^2)^(3/2
))/(3*a^2)))/6 + x*ArcCos[a*x] - (a^2*x^3*ArcCos[a*x])/3))/(3*a)) + (2*c*(
x*ArcCos[a*x]^3 + 3*a*(-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/a^2) - (2*(-(Sq
rt[1 - a^2*x^2]/a) + x*ArcCos[a*x]))/a)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{c\left(9a^3x^3\arccos(ax)^3-9x^2\sqrt{-a^2x^2+1}\,\arccos(ax)^2a^2-27ax\arccos(ax)^3-6a^3x^3\arccos(ax)+63\arccos(ax)^2\sqrt{-a^2x\ldots}\right.}{27a}$
default	$-\frac{c\left(9a^3x^3\arccos(ax)^3-9x^2\sqrt{-a^2x^2+1}\,\arccos(ax)^2a^2-27ax\arccos(ax)^3-6a^3x^3\arccos(ax)+63\arccos(ax)^2\sqrt{-a^2x\ldots}\right.}{27a}$
orering	$\frac{5x(13a^4x^4-194a^2x^2-179)(-a^2cx^2+c)\arccos(ax)^3}{81(a^2x^2-1)^2}-\frac{(25a^4x^4-683a^2x^2-242)\left(-2cxa^2\arccos(ax)^3-\frac{3(-a^2cx^2\ldots}{\sqrt{-\ldots}}\right.}{81a^2(a^2x^2-1)}$

input
```
int((-a^2*c*x^2+c)*arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/27/a*c*(9*a^3*x^3*arccos(a*x)^3-9*x^2*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2*
a^2-27*a*x*arccos(a*x)^3-6*a^3*x^3*arccos(a*x)+63*arccos(a*x)^2*(-a^2*x^2+
1)^(1/2)+2*a^2*x^2*(-a^2*x^2+1)^(1/2)+126*a*x*arccos(a*x)-122*(-a^2*x^2+1)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.59

$$\int \left(c-a^2cx^2\right)\arccos(ax)^3\,dx =$$
$$-\frac{9\left(a^3cx^3-3\,acx\right)\arccos\left(ax\right)^3-6\left(a^3cx^3-21\,acx\right)\arccos\left(ax\right)+\left(2\,a^2cx^2-9\left(a^2cx^2-7\,c\right)\arccos\left(ax\right.\right.}{27\,a}$$

input
```
integrate((-a^2*c*x^2+c)*arccos(a*x)^3,x, algorithm="fricas")
```

output
```
-1/27*(9*(a^3*c*x^3 - 3*a*c*x)*arccos(a*x)^3 - 6*(a^3*c*x^3 - 21*a*c*x)*ar
ccos(a*x) + (2*a^2*c*x^2 - 9*(a^2*c*x^2 - 7*c)*arccos(a*x)^2 - 122*c)*sqrt
(-a^2*x^2 + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99

$$\int \left(c-a^2cx^2\right)\arccos(ax)^3\,dx$$
$$=\begin{cases} -\frac{a^2cx^3\operatorname{acos}^3(ax)}{3}+\frac{2a^2cx^3\operatorname{acos}(ax)}{9}+\frac{acx^2\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{3}-\frac{2acx^2\sqrt{-a^2x^2+1}}{27}+cx\operatorname{acos}^3(ax)-\frac{14cx\operatorname{acos}(ax)}{3}- \\ \frac{\pi^3cx}{8} \end{cases}$$

input
```
integrate((-a**2*c*x**2+c)*acos(a*x)**3,x)
```

output
```
Piecewise((-a**2*c*x**3*acos(a*x)**3/3 + 2*a**2*c*x**3*acos(a*x)/9 + a*c*x
**2*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/3 - 2*a*c*x**2*sqrt(-a**2*x**2 + 1)/
27 + c*x**acos(a*x)**3 - 14*c*x*acos(a*x)/3 - 7*c*sqrt(-a**2*x**2 + 1)*acos
(a*x)**2/(3*a) + 122*c*sqrt(-a**2*x**2 + 1)/(27*a), Ne(a, 0)), (pi**3*c*x/
8, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.81

$$\int (c-a^2cx^2)\arccos(ax)^3\,dx = \frac{1}{3}\left(\sqrt{-a^2x^2+1}cx^2 - \frac{7\sqrt{-a^2x^2+1}c}{a^2}\right)a\arccos(ax)^2 - \frac{1}{3}\left(a^2cx^3-3cx\right)\arccos(ax)^3 - \frac{2}{27}\left(\sqrt{-a^2x^2+1}cx^2 - \frac{3\left(a^2cx^3-21cx\right)\arccos(ax)}{a} - \frac{61\sqrt{-a^2x^2+1}c}{a^2}\right)a$$

input
```
integrate((-a^2*c*x^2+c)*arccos(a*x)^3,x, algorithm="maxima")
```

output
```
1/3*(sqrt(-a^2*x^2 + 1)*c*x^2 - 7*sqrt(-a^2*x^2 + 1)*c/a^2)*a*arccos(a*x)^
2 - 1/3*(a^2*c*x^3 - 3*c*x)*arccos(a*x)^3 - 2/27*(sqrt(-a^2*x^2 + 1)*c*x^2
 - 3*(a^2*c*x^3 - 21*c*x)*arccos(a*x)/a - 61*sqrt(-a^2*x^2 + 1)*c/a^2)*a
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\int (c-a^2cx^2)\arccos(ax)^3\,dx = -\frac{1}{3}a^2cx^3\arccos(ax)^3 + \frac{2}{9}a^2cx^3\arccos(ax) + \frac{1}{3}\sqrt{-a^2x^2+1}acx^2\arccos(ax)^2 + cx\arccos(ax)^3 - \frac{2}{27}\sqrt{-a^2x^2+1}acx^2 - \frac{14}{3}cx\arccos(ax) - \frac{7\sqrt{-a^2x^2+1}c\arccos(ax)^2}{3a} + \frac{122\sqrt{-a^2x^2+1}c}{27a}$$

input
```
integrate((-a^2*c*x^2+c)*arccos(a*x)^3,x, algorithm="giac")
```

output
```
-1/3*a^2*c*x^3*arccos(a*x)^3 + 2/9*a^2*c*x^3*arccos(a*x) + 1/3*sqrt(-a^2*x
^2 + 1)*a*c*x^2*arccos(a*x)^2 + c*x*arccos(a*x)^3 - 2/27*sqrt(-a^2*x^2 + 1
)*a*c*x^2 - 14/3*c*x*arccos(a*x) - 7/3*sqrt(-a^2*x^2 + 1)*c*arccos(a*x)^2/
a + 122/27*sqrt(-a^2*x^2 + 1)*c/a
```

Mupad [F(-1)]

Timed out.

$$\int (c-a^2cx^2)\arccos(ax)^3\,dx=\int \mathrm{acos}(a\,x)^3\,\left(c-a^2\,c\,x^2\right)\,dx$$

input
```
int(acos(a*x)^3*(c - a^2*c*x^2),x)
```

output
```
int(acos(a*x)^3*(c - a^2*c*x^2), x)
```

Reduce [F]

$$\int (c-a^2cx^2)\arccos(ax)^3\,dx$$
$$=\frac{c\left(acos(ax)^3\,ax-3\sqrt{-a^2x^2+1}\,acos(ax)^2-6acos(ax)\,ax+6\sqrt{-a^2x^2+1}-\left(\int acos(ax)^3\,x^2dx\right)a^3\right)}{a}$$

input
```
int((-a^2*c*x^2+c)*acos(a*x)^3,x)
```

output
```
(c*(acos(a*x)**3*a*x - 3*sqrt( - a**2*x**2 + 1)*acos(a*x)**2 - 6*acos(a*x)
*a*x + 6*sqrt( - a**2*x**2 + 1) - int(acos(a*x)**3*x**2,x)*a**3))/a
```

3.294 $\int \frac{\arccos(ax)^3}{c-a^2cx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 200

$$\begin{aligned}\int \frac{\arccos(ax)^3}{c-a^2cx^2}\,dx = &-\frac{2i\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)}{ac} \\ &+\frac{3i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)}{ac} \\ &-\frac{3i\arccos(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)}{ac} \\ &-\frac{6\arccos(ax)\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)}{ac} \\ &+\frac{6\arccos(ax)\operatorname{PolyLog}\left(3,ie^{i\arccos(ax)}\right)}{ac} \\ &-\frac{6i\operatorname{PolyLog}\left(4,-ie^{i\arccos(ax)}\right)}{ac}+\frac{6i\operatorname{PolyLog}\left(4,ie^{i\arccos(ax)}\right)}{ac}\end{aligned}$$

output

```
-2*I*arccos(a*x)^3*arctan(a*x+I*(-a^2*x^2+1)^(1/2))/a/c+3*I*arccos(a*x)^2*
polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a/c-3*I*arccos(a*x)^2*polylog(2,I
*(a*x+I*(-a^2*x^2+1)^(1/2)))/a/c-6*arccos(a*x)*polylog(3,-I*(a*x+I*(-a^2*x
^2+1)^(1/2)))/a/c+6*arccos(a*x)*polylog(3,I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a/
c-6*I*polylog(4,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a/c+6*I*polylog(4,I*(a*x+I*
(-a^2*x^2+1)^(1/2)))/a/c
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.93

$$\int \frac{\arccos(ax)^3}{c-a^2cx^2}\,dx$$
$$= \frac{i\left(\pi^4-2\arccos(ax)^4+8i\arccos(ax)^3\log\left(1-e^{-i\arccos(ax)}\right)-8i\arccos(ax)^3\log\left(1+e^{i\arccos(ax)}\right)-24\,\mathrm{ar}\right.}{}$$

input
```
Integrate[ArcCos[a*x]^3/(c - a^2*c*x^2),x]
```

output
```
((I/8)*(Pi^4 - 2*ArcCos[a*x]^4 + (8*I)*ArcCos[a*x]^3*Log[1 - E^((-I)*ArcCo
s[a*x])] - (8*I)*ArcCos[a*x]^3*Log[1 + E^(I*ArcCos[a*x])] - 24*ArcCos[a*x]
^2*PolyLog[2, E^((-I)*ArcCos[a*x])] - 24*ArcCos[a*x]^2*PolyLog[2, -E^(I*Ar
cCos[a*x])] + (48*I)*ArcCos[a*x]*PolyLog[3, E^((-I)*ArcCos[a*x])] - (48*I)
*ArcCos[a*x]*PolyLog[3, -E^(I*ArcCos[a*x])] + 48*PolyLog[4, E^((-I)*ArcCos
[a*x])] + 48*PolyLog[4, -E^(I*ArcCos[a*x])]))/(a*c)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5165, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{c-a^2cx^2}\,dx$$
$$\Big\downarrow \text{5165}$$
$$-\frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}d\arccos(ax)}{ac}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{\int \arccos(ax)^3\csc(\arccos(ax))d\arccos(ax)}{ac}$$

$$\Big\downarrow \; 4671$$

$$-\frac{-3\int \arccos(ax)^2\log\left(1-e^{i\arccos(ax)}\right)d\arccos(ax)+3\int \arccos(ax)^2\log\left(1+e^{i\arccos(ax)}\right)d\arccos(ax)-2\,\mathrm{arcc}}{ac}$$

$$\Big\downarrow \; 3011$$

$$-\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\int \arccos(ax)\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)d\arccos(ax)\right)-3\left(i\,\mathrm{arcc}\right.}{}$$

$$\Big\downarrow \; 7163$$

$$-\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\left(i\int \operatorname{PolyLog}\left(3,-e^{i\arccos(ax)}\right)d\arccos(ax)-i\arccos(ax)\,\mathrm{PolyLo}\right.\right.}{}$$

$$\Big\downarrow \; 2720$$

$$-\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\left(\int e^{-i\arccos(ax)}\operatorname{PolyLog}\left(3,-e^{i\arccos(ax)}\right)de^{i\arccos(ax)}-i\arccos(\right.\right.}{}$$

$$\Big\downarrow \; 7143$$

$$-\frac{-2\arccos(ax)^3\operatorname{arctanh}\left(e^{i\arccos(ax)}\right)+3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-e^{i\arccos(ax)}\right.\right.\right.}{}$$

input
```
Int[ArcCos[a*x]^3/(c - a^2*c*x^2),x]
```

output
```
-((-2*ArcCos[a*x]^3*ArcTanh[E^(I*ArcCos[a*x])] + 3*(I*ArcCos[a*x]^2*PolyLo
g[2, -E^(I*ArcCos[a*x])] - (2*I)*((-I)*ArcCos[a*x]*PolyLog[3, -E^(I*ArcCos
[a*x])] + PolyLog[4, -E^(I*ArcCos[a*x])])) - 3*(I*ArcCos[a*x]^2*PolyLog[2,
 E^(I*ArcCos[a*x])] - (2*I)*((-I)*ArcCos[a*x]*PolyLog[3, E^(I*ArcCos[a*x])
] + PolyLog[4, E^(I*ArcCos[a*x])])))/(a*c))
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.26

method	result
derivativedivides	$-\frac{\arccos(ax)^3 \ln\left(1-ax-i\sqrt{-a^2x^2+1}\right)}{c}+\frac{3i \arccos(ax)^2 \operatorname{polylog}\left(2,ax+i\sqrt{-a^2x^2+1}\right)}{c}-\frac{6 \arccos(ax) \operatorname{polylog}\left(3,ax+i\sqrt{-a^2x^2+1}\right)}{c}-$
default	$-\frac{\arccos(ax)^3 \ln\left(1-ax-i\sqrt{-a^2x^2+1}\right)}{c}+\frac{3i \arccos(ax)^2 \operatorname{polylog}\left(2,ax+i\sqrt{-a^2x^2+1}\right)}{c}-\frac{6 \arccos(ax) \operatorname{polylog}\left(3,ax+i\sqrt{-a^2x^2+1}\right)}{c}-$

input
```
int(arccos(a*x)^3/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/c*arccos(a*x)^3*ln(1-a*x-I*(-a^2*x^2+1)^(1/2))+3*I/c*arccos(a*x)^2
*polylog(2,a*x+I*(-a^2*x^2+1)^(1/2))-6/c*arccos(a*x)*polylog(3,a*x+I*(-a^2
*x^2+1)^(1/2))-6*I/c*polylog(4,a*x+I*(-a^2*x^2+1)^(1/2))+1/c*arccos(a*x)^3
*ln(1+a*x+I*(-a^2*x^2+1)^(1/2))-3*I/c*arccos(a*x)^2*polylog(2,-a*x-I*(-a^2
*x^2+1)^(1/2))+6/c*arccos(a*x)*polylog(3,-a*x-I*(-a^2*x^2+1)^(1/2))+6*I/c*
polylog(4,-a*x-I*(-a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{c-a^2cx^2}\,dx = \int -\frac{\arccos\left(ax\right)^3}{a^2cx^2-c}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(-arccos(a*x)^3/(a^2*c*x^2 - c), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{c-a^2cx^2}\,dx = -\frac{\int \frac{\operatorname{acos}^3(ax)}{a^2x^2-1}\,dx}{c}$$

input
```
integrate(acos(a*x)**3/(-a**2*c*x**2+c),x)
```

output
```
-Integral(acos(a*x)**3/(a**2*x**2 - 1), x)/c
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{c-a^2cx^2}\,dx = \int -\frac{\arccos(ax)^3}{a^2cx^2-c}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/2*((log(a*x + 1) - log(-a*x + 1))*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
a*x)^3 - 2*a*c*integrate(3/2*sqrt(a*x + 1)*sqrt(-a*x + 1)*(log(a*x + 1) -
log(-a*x + 1))*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2/(a^2*c*x^2 - c
), x))/(a*c)
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{c-a^2cx^2}\,dx = \int -\frac{\arccos(ax)^3}{a^2cx^2-c}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(-arccos(a*x)^3/(a^2*c*x^2 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{c - a^2cx^2} \, dx = \int \frac{\mathrm{acos}(a\,x)^3}{c - a^2\,c\,x^2} \, dx$$

input
```
int(acos(a*x)^3/(c - a^2*c*x^2),x)
```

output
```
int(acos(a*x)^3/(c - a^2*c*x^2), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{c - a^2cx^2} \, dx = -\frac{\int \frac{acos(ax)^3}{a^2x^2-1}dx}{c}$$

input
```
int(acos(a*x)^3/(-a^2*c*x^2+c),x)
```

output
```
( - int(acos(a*x)**3/(a**2*x**2 - 1),x))/c
```

3.295 $\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 337

$$\begin{aligned}\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^2}\,dx = &-\frac{3\arccos(ax)^2}{2ac^2\sqrt{1-a^2x^2}}+\frac{x\arccos(ax)^3}{2c^2(1-a^2x^2)}\\&-\frac{6i\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right)}{ac^2}\\&-\frac{i\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)}{ac^2}+\frac{3i\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)}{ac^2}\\&+\frac{3i\arccos(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)}{2ac^2}\\&-\frac{3i\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right)}{ac^2}\\&-\frac{3i\arccos(ax)^2\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right)}{2ac^2}\\&-\frac{3\arccos(ax)\,\text{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)}{ac^2}\\&+\frac{3\arccos(ax)\,\text{PolyLog}\left(3,ie^{i\arccos(ax)}\right)}{ac^2}\\&-\frac{3i\,\text{PolyLog}\left(4,-ie^{i\arccos(ax)}\right)}{ac^2}+\frac{3i\,\text{PolyLog}\left(4,ie^{i\arccos(ax)}\right)}{ac^2}\end{aligned}$$

output
```
-3/2*arccos(a*x)^2/a/c^2/(-a^2*x^2+1)^(1/2)+1/2*x*arccos(a*x)^3/c^2/(-a^2*
x^2+1)-6*I*arccos(a*x)*arctan(a*x+I*(-a^2*x^2+1)^(1/2))/a/c^2-I*arccos(a*x
)^3*arctan(a*x+I*(-a^2*x^2+1)^(1/2))/a/c^2+3*I*polylog(2,-I*(a*x+I*(-a^2*x
^2+1)^(1/2)))/a/c^2+3/2*I*arccos(a*x)^2*polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(
1/2)))/a/c^2-3*I*polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a/c^2-3/2*I*arcco
s(a*x)^2*polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a/c^2-3*arccos(a*x)*polyl
og(3,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a/c^2+3*arccos(a*x)*polylog(3,I*(a*x+I
*(-a^2*x^2+1)^(1/2)))/a/c^2-3*I*polylog(4,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a
/c^2+3*I*polylog(4,I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a/c^2
```

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.95

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^2}\,dx = \frac{i\pi^4 - 2i\arccos(ax)^4 + 12\arccos(ax)^2\cot\left(\frac{1}{2}\arccos(ax)\right) + 2\arccos(ax)^3\csc^2\left(\frac{1}{2}\arccos(ax)\right) - 8\arccos(}{}$$

input
```
Integrate[ArcCos[a*x]^3/(c - a^2*c*x^2)^2,x]
```

output
```
(I*Pi^4 - (2*I)*ArcCos[a*x]^4 + 12*ArcCos[a*x]^2*Cot[ArcCos[a*x]/2] + 2*Ar
cCos[a*x]^3*Csc[ArcCos[a*x]/2]^2 - 8*ArcCos[a*x]^3*Log[1 - E^((-I)*ArcCos[
a*x])] - 48*ArcCos[a*x]*Log[1 - E^(I*ArcCos[a*x])] + 48*ArcCos[a*x]*Log[1
+ E^(I*ArcCos[a*x])] + 8*ArcCos[a*x]^3*Log[1 + E^(I*ArcCos[a*x])] - (24*I)
*ArcCos[a*x]^2*PolyLog[2, E^((-I)*ArcCos[a*x])] - (24*I)*(2 + ArcCos[a*x]^
2)*PolyLog[2, -E^(I*ArcCos[a*x])] + (48*I)*PolyLog[2, E^(I*ArcCos[a*x])] -
 48*ArcCos[a*x]*PolyLog[3, E^((-I)*ArcCos[a*x])] + 48*ArcCos[a*x]*PolyLog[
3, -E^(I*ArcCos[a*x])] + (48*I)*PolyLog[4, E^((-I)*ArcCos[a*x])] + (48*I)*
PolyLog[4, -E^(I*ArcCos[a*x])] - 2*ArcCos[a*x]^3*Sec[ArcCos[a*x]/2]^2 + 12
*ArcCos[a*x]^2*Tan[ArcCos[a*x]/2])/(16*a*c^2)
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.82, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5163, 27, 5165, 3042, 4671, 3011, 5183, 5165, 3042, 4671, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^2}\,dx$$

$$\Big\downarrow \; 5163$$

$$\frac{3a\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^{3/2}}dx}{2c^2}+\frac{\int \frac{\arccos(ax)^3}{c(1-a^2x^2)}dx}{2c}+\frac{x\arccos(ax)^3}{2c^2(1-a^2x^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{3a\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^{3/2}}dx}{2c^2}+\frac{\int \frac{\arccos(ax)^3}{1-a^2x^2}dx}{2c^2}+\frac{x\arccos(ax)^3}{2c^2(1-a^2x^2)}$$

$$\Big\downarrow \; 5165$$

$$\frac{3a\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^{3/2}}dx}{2c^2}-\frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}d\arccos(ax)}{2ac^2}+\frac{x\arccos(ax)^3}{2c^2(1-a^2x^2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{3a\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^{3/2}}dx}{2c^2}-\frac{\int \arccos(ax)^3\csc(\arccos(ax))d\arccos(ax)}{2ac^2}+\frac{x\arccos(ax)^3}{2c^2(1-a^2x^2)}$$

$$\Big\downarrow \; 4671$$

$$\frac{3a\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^{3/2}}dx}{2c^2}-\frac{-3\int \arccos(ax)^2\log\left(1-e^{i\arccos(ax)}\right)d\arccos(ax)+3\int \arccos(ax)^2\log\left(1+e^{i\arccos(ax)}\right)d\arccos(ax)-2\arccos}{2ac^2}$$
$$\frac{x\arccos(ax)^3}{2c^2(1-a^2x^2)}$$

$$\Big\downarrow \; 3011$$

$$\frac{\frac{3a\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^{3/2}}dx}{2c^2}-}{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\int \arccos(ax)\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)d\arccos(ax)\right)-3\left(i\arccos(\right.}$$

$$\frac{x\arccos(ax)^3}{2c^2(1-a^2x^2)}$$

$$\Big\downarrow \text{5183}$$

$$\frac{\frac{3a\left(\frac{2\int \frac{\arccos(ax)}{1-a^2x^2}dx}{a}+\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}\right)}{2c^2}-}{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\int \arccos(ax)\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)d\arccos(ax)\right)-3\left(i\arccos(\right.}$$

$$\frac{x\arccos(ax)^3}{2c^2(1-a^2x^2)}$$

$$\Big\downarrow \text{5165}$$

$$\frac{\frac{3a\left(\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}d\arccos(ax)}{a^2}\right)}{2c^2}-}{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\int \arccos(ax)\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)d\arccos(ax)\right)-3\left(i\arccos(\right.}$$

$$\frac{x\arccos(ax)^3}{2c^2(1-a^2x^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3a\left(\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\int \arccos(ax)\csc(\arccos(ax))d\arccos(ax)}{a^2}\right)}{2c^2}-}{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\int \arccos(ax)\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)d\arccos(ax)\right)-3\left(i\arccos(\right.}$$

$$\frac{x\arccos(ax)^3}{2c^2(1-a^2x^2)}$$

$$\Big\downarrow \text{4671}$$

$$\frac{\frac{3a\left(\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(-\int \log\left(1-e^{i\arccos(ax)}\right)d\arccos(ax)+\int \log\left(1+e^{i\arccos(ax)}\right)d\arccos(ax)-2\arccos(ax)\text{arctanh}\left(e^{i\arccos(ax)}\right)\right)}{a^2}\right)}{2c^2}-}{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\int \arccos(ax)\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)d\arccos(ax)\right)-3\left(i\arccos(\right.}$$

$$\frac{x\arccos(ax)^3}{2c^2(1-a^2x^2)}$$

$\Big\downarrow$ 2715

$$\frac{3a\left(\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(i\int e^{-i\arccos(ax)}\log\left(1-e^{i\arccos(ax)}\right)de^{i\arccos(ax)}-i\int e^{-i\arccos(ax)}\log\left(1+e^{i\arccos(ax)}\right)de^{i\arccos(ax)}-2\arccos(ax)\text{arc}\right.}{a^2}\right.}{2c^2}$$

$$\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\int\arccos(ax)\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)d\arccos(ax)\right)-3\left(i\arccos(\right.}{}$$

$$\frac{x\arccos(ax)^3}{2c^2\left(1-a^2x^2\right)}$$

$\Big\downarrow$ 2838

$$-\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\int\arccos(ax)\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)d\arccos(ax)\right)-3\left(i\arcc\right.}{}$$

$$\frac{3a\left(\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(-2\arccos(ax)\text{arctanh}\left(e^{i\arccos(ax)}\right)+i\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-i\operatorname{PolyLog}\left(2,e^{i\arccos(ax)}\right)\right)}{a^2}\right)}{2c^2}+$$

$$\frac{x\arccos(ax)^3}{2c^2\left(1-a^2x^2\right)}$$

$\Big\downarrow$ 7163

$$-\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-e^{i\arccos(ax)}\right)d\arccos(ax)-i\arccos(ax)\operatorname{PolyLo}\right.\right.}{}$$

$$\frac{3a\left(\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(-2\arccos(ax)\text{arctanh}\left(e^{i\arccos(ax)}\right)+i\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-i\operatorname{PolyLog}\left(2,e^{i\arccos(ax)}\right)\right)}{a^2}\right)}{2c^2}+$$

$$\frac{x\arccos(ax)^3}{2c^2\left(1-a^2x^2\right)}$$

$\Big\downarrow$ 2720

$$-\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\left(\int e^{-i\arccos(ax)}\operatorname{PolyLog}\left(3,-e^{i\arccos(ax)}\right)de^{i\arccos(ax)}-i\arccos(\right.\right.}{}$$

$$\frac{3a\left(\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(-2\arccos(ax)\text{arctanh}\left(e^{i\arccos(ax)}\right)+i\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-i\operatorname{PolyLog}\left(2,e^{i\arccos(ax)}\right)\right)}{a^2}\right)}{2c^2}+$$

$$\frac{x\arccos(ax)^3}{2c^2\left(1-a^2x^2\right)}$$

$\Big\downarrow$ 7143

$$\frac{3a\left(\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(-2\arccos(ax)\text{arctanh}\left(e^{i\arccos(ax)}\right)+i\,\text{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-i\,\text{PolyLog}\left(2,e^{i\arccos(ax)}\right)\right)}{a^2}\right)}{2c^2}+\frac{x\arccos(ax)^3}{2c^2\left(1-a^2x^2\right)}-\frac{-2\arccos(ax)^3\text{arctanh}\left(e^{i\arccos(ax)}\right)+3\left(i\arccos(ax)^2\,\text{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\left(\text{PolyLog}\left(4,-e^{i\arccos(ax)}\right)\right.\right.}{}$$

input
```
Int[ArcCos[a*x]^3/(c - a^2*c*x^2)^2,x]
```

output
```
(x*ArcCos[a*x]^3)/(2*c^2*(1 - a^2*x^2)) + (3*a*(ArcCos[a*x]^2/(a^2*Sqrt[1
- a^2*x^2]) - (2*(-2*ArcCos[a*x]*ArcTanh[E^(I*ArcCos[a*x])] + I*PolyLog[2,
 -E^(I*ArcCos[a*x])] - I*PolyLog[2, E^(I*ArcCos[a*x])]))/a^2))/(2*c^2) - (
-2*ArcCos[a*x]^3*ArcTanh[E^(I*ArcCos[a*x])] + 3*(I*ArcCos[a*x]^2*PolyLog[2
, -E^(I*ArcCos[a*x])] - (2*I)*((-I)*ArcCos[a*x]*PolyLog[3, -E^(I*ArcCos[a*
x])] + PolyLog[4, -E^(I*ArcCos[a*x])])) - 3*(I*ArcCos[a*x]^2*PolyLog[2, E^
(I*ArcCos[a*x])] - (2*I)*((-I)*ArcCos[a*x]*PolyLog[3, E^(I*ArcCos[a*x])] +
 PolyLog[4, E^(I*ArcCos[a*x])]))))/(2*a*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{-\frac{\arccos(ax)^2\left(ax\arccos(ax)+3\sqrt{-a^2x^2+1}\right)}{2\left(a^2x^2-1\right)c^2}-\frac{\arccos(ax)^3\ln\left(1-ax-i\sqrt{-a^2x^2+1}\right)}{2c^2}+\frac{3i\arccos(ax)^2\operatorname{polylog}\left(2,ax+i\sqrt{-a^2x^2+1}\right)}{2c^2}}{}$
default	$\frac{-\frac{\arccos(ax)^2\left(ax\arccos(ax)+3\sqrt{-a^2x^2+1}\right)}{2\left(a^2x^2-1\right)c^2}-\frac{\arccos(ax)^3\ln\left(1-ax-i\sqrt{-a^2x^2+1}\right)}{2c^2}+\frac{3i\arccos(ax)^2\operatorname{polylog}\left(2,ax+i\sqrt{-a^2x^2+1}\right)}{2c^2}}{}$

input
```
int(arccos(a*x)^3/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/2/(a^2*x^2-1)*arccos(a*x)^2*(a*x*arccos(a*x)+3*(-a^2*x^2+1)^(1/2))
/c^2-1/2/c^2*arccos(a*x)^3*ln(1-a*x-I*(-a^2*x^2+1)^(1/2))+3/2*I/c^2*arccos
(a*x)^2*polylog(2,a*x+I*(-a^2*x^2+1)^(1/2))-3/c^2*arccos(a*x)*polylog(3,a*
x+I*(-a^2*x^2+1)^(1/2))-3*I/c^2*polylog(4,a*x+I*(-a^2*x^2+1)^(1/2))+1/2/c^
2*arccos(a*x)^3*ln(1+a*x+I*(-a^2*x^2+1)^(1/2))-3/2*I/c^2*arccos(a*x)^2*pol
ylog(2,-a*x-I*(-a^2*x^2+1)^(1/2))+3/c^2*arccos(a*x)*polylog(3,-a*x-I*(-a^2
*x^2+1)^(1/2))+3*I/c^2*polylog(4,-a*x-I*(-a^2*x^2+1)^(1/2))-3/c^2*arccos(a
*x)*ln(1-a*x-I*(-a^2*x^2+1)^(1/2))+3*I/c^2*polylog(2,a*x+I*(-a^2*x^2+1)^(1
/2))+3/c^2*arccos(a*x)*ln(1+a*x+I*(-a^2*x^2+1)^(1/2))-3*I/c^2*polylog(2,-a
*x-I*(-a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^2}\,dx = \int \frac{\arccos(ax)^3}{(a^2cx^2-c)^2}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arccos(a*x)^3/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^2}\,dx = \frac{\int \frac{\operatorname{acos}^3(ax)}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(acos(a*x)**3/(-a**2*c*x**2+c)**2,x)
```

output
```
Integral(acos(a*x)**3/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^2}\,dx = \int \frac{\arccos(ax)^3}{(a^2cx^2-c)^2}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/4*((2*a*x - (a^2*x^2 - 1)*log(a*x + 1) + (a^2*x^2 - 1)*log(-a*x + 1))*a
rctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3 + 4*(a^3*c^2*x^2 - a*c^2)*inte
grate(-3/4*(2*a*x - (a^2*x^2 - 1)*log(a*x + 1) + (a^2*x^2 - 1)*log(-a*x +
1))*sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x
)^2/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2), x))/(a^3*c^2*x^2 - a*c^2)
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^2}\,dx = \int \frac{\arccos(ax)^3}{(a^2cx^2-c)^2}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^3/(a^2*c*x^2 - c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^2}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{(c-a^2\,c\,x^2)^2}\,dx$$

input
```
int(acos(a*x)^3/(c - a^2*c*x^2)^2,x)
```

output
```
int(acos(a*x)^3/(c - a^2*c*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^2}\,dx = \frac{\int \frac{acos(ax)^3}{a^4x^4-2a^2x^2+1}dx}{c^2}$$

input
```
int(acos(a*x)^3/(-a^2*c*x^2+c)^2,x)
```

output
```
int(acos(a*x)**3/(a**4*x**4 - 2*a**2*x**2 + 1),x)/c**2
```

3.296 $\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 455

$$\begin{aligned}\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^3}\,dx &= -\frac{1}{4ac^3\sqrt{1-a^2x^2}}+\frac{x\arccos(ax)}{4c^3(1-a^2x^2)}-\frac{\arccos(ax)^2}{4ac^3(1-a^2x^2)^{3/2}}\\
&-\frac{9\arccos(ax)^2}{8ac^3\sqrt{1-a^2x^2}}+\frac{x\arccos(ax)^3}{4c^3(1-a^2x^2)^2}\\
&+\frac{3x\arccos(ax)^3}{8c^3(1-a^2x^2)}-\frac{5i\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right)}{ac^3}\\
&-\frac{3i\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)}{4ac^3}+\frac{5i\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)}{2ac^3}\\
&+\frac{9i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)}{8ac^3}\\
&-\frac{5i\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)}{2ac^3}\\
&-\frac{9i\arccos(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)}{8ac^3}\\
&-\frac{9\arccos(ax)\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)}{4ac^3}\\
&+\frac{9\arccos(ax)\operatorname{PolyLog}\left(3,ie^{i\arccos(ax)}\right)}{4ac^3}\\
&-\frac{9i\operatorname{PolyLog}\left(4,-ie^{i\arccos(ax)}\right)}{4ac^3}+\frac{9i\operatorname{PolyLog}\left(4,ie^{i\arccos(ax)}\right)}{4ac^3}\end{aligned}$$

output
```
-1/4/a/c^3/(-a^2*x^2+1)^(1/2)+1/4*x*arccos(a*x)/c^3/(-a^2*x^2+1)-1/4*arcco
s(a*x)^2/a/c^3/(-a^2*x^2+1)^(3/2)-9/8*arccos(a*x)^2/a/c^3/(-a^2*x^2+1)^(1/
2)+1/4*x*arccos(a*x)^3/c^3/(-a^2*x^2+1)^2+3/8*x*arccos(a*x)^3/c^3/(-a^2*x^
2+1)+9/4*I*polylog(4,I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a/c^3-9/4*I*polylog(4,-
I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a/c^3-3/4*I*arccos(a*x)^3*arctan(a*x+I*(-a^2
*x^2+1)^(1/2))/a/c^3-5*I*arccos(a*x)*arctan(a*x+I*(-a^2*x^2+1)^(1/2))/a/c^
3-5/2*I*polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a/c^3-9/8*I*arccos(a*x)^2*
polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a/c^3-9/4*arccos(a*x)*polylog(3,-I
*(a*x+I*(-a^2*x^2+1)^(1/2)))/a/c^3+9/4*arccos(a*x)*polylog(3,I*(a*x+I*(-a^
2*x^2+1)^(1/2)))/a/c^3+5/2*I*polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a/c^
3+9/8*I*arccos(a*x)^2*polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))/a/c^3
```

Mathematica [A] (verified)

Time = 3.84 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.06

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^3}\,dx =$$
$$-\frac{-3i\pi^4+6i\arccos(ax)^4-8\cot\left(\frac{1}{2}\arccos(ax)\right)-40\arccos(ax)^2\cot\left(\frac{1}{2}\arccos(ax)\right)-4\arccos(ax)\csc^2}{}$$

input
```
Integrate[ArcCos[a*x]^3/(c - a^2*c*x^2)^3,x]
```

output
```
-1/64*((-3*I)*Pi^4 + (6*I)*ArcCos[a*x]^4 - 8*Cot[ArcCos[a*x]/2] - 40*ArcCo
s[a*x]^2*Cot[ArcCos[a*x]/2] - 4*ArcCos[a*x]*Csc[ArcCos[a*x]/2]^2 - 6*ArcCo
s[a*x]^3*Csc[ArcCos[a*x]/2]^2 - Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2*Csc[ArcCos
[a*x]/2]^4 - ArcCos[a*x]^3*Csc[ArcCos[a*x]/2]^4 + 24*ArcCos[a*x]^3*Log[1 -
 E^((-I)*ArcCos[a*x])] + 160*ArcCos[a*x]*Log[1 - E^(I*ArcCos[a*x])] - 160*
ArcCos[a*x]*Log[1 + E^(I*ArcCos[a*x])] - 24*ArcCos[a*x]^3*Log[1 + E^(I*Arc
Cos[a*x])] + (72*I)*ArcCos[a*x]^2*PolyLog[2, E^((-I)*ArcCos[a*x])] + (8*I)
*(20 + 9*ArcCos[a*x]^2)*PolyLog[2, -E^(I*ArcCos[a*x])] - (160*I)*PolyLog[2
, E^(I*ArcCos[a*x])] + 144*ArcCos[a*x]*PolyLog[3, E^((-I)*ArcCos[a*x])] -
144*ArcCos[a*x]*PolyLog[3, -E^(I*ArcCos[a*x])] - (144*I)*PolyLog[4, E^((-I
)*ArcCos[a*x])] - (144*I)*PolyLog[4, -E^(I*ArcCos[a*x])] + 4*ArcCos[a*x]*S
ec[ArcCos[a*x]/2]^2 + 6*ArcCos[a*x]^3*Sec[ArcCos[a*x]/2]^2 + ArcCos[a*x]^3
*Sec[ArcCos[a*x]/2]^4 - (16*ArcCos[a*x]^2*Sin[ArcCos[a*x]/2]^4)/(1 - a^2*x
^2)^(3/2) - 8*Tan[ArcCos[a*x]/2] - 40*ArcCos[a*x]^2*Tan[ArcCos[a*x]/2])/(a
*c^3)
```

Rubi [A] (verified)

Time = 2.89 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {5163, 27, 5163, 5165, 3042, 4671, 3011, 5183, 5163, 241, 5165, 3042, 4671, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^3} \, dx$$

$$\Big\downarrow \text{5163}$$

$$\frac{3a\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^{5/2}}dx}{4c^3}+\frac{3\int \frac{\arccos(ax)^3}{c^2(1-a^2x^2)^2}dx}{4c}+\frac{x\arccos(ax)^3}{4c^3(1-a^2x^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3a\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^{5/2}}dx}{4c^3}+\frac{3\int \frac{\arccos(ax)^3}{(1-a^2x^2)^2}dx}{4c^3}+\frac{x\arccos(ax)^3}{4c^3(1-a^2x^2)^2}$$

$$\Big\downarrow \text{5163}$$

$$\frac{3a\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^{5/2}}dx}{4c^3}+\frac{3\left(\frac{3}{2}a\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^{3/2}}dx+\frac{1}{2}\int \frac{\arccos(ax)^3}{1-a^2x^2}dx+\frac{x\arccos(ax)^3}{2(1-a^2x^2)}\right)}{4c^3}+\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

$$\Big\downarrow \text{ 5165}$$

$$\frac{3a\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^{5/2}}dx}{4c^3}+\frac{3\left(\frac{3}{2}a\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^{3/2}}dx-\frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}d\arccos(ax)}{2a}+\frac{x\arccos(ax)^3}{2(1-a^2x^2)}\right)}{4c^3}+\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3a\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^{5/2}}dx}{4c^3}+\frac{3\left(\frac{3}{2}a\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^{3/2}}dx-\frac{\int \arccos(ax)^3\csc(\arccos(ax))d\arccos(ax)}{2a}+\frac{x\arccos(ax)^3}{2(1-a^2x^2)}\right)}{4c^3}+\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

$$\Big\downarrow \text{ 4671}$$

$$\frac{3\left(\frac{3}{2}a\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^{3/2}}dx-\frac{-3\int \arccos(ax)^2\log\left(1-e^{i\arccos(ax)}\right)d\arccos(ax)+3\int \arccos(ax)^2\log\left(1+e^{i\arccos(ax)}\right)d\arccos(ax)-2\arccos(a\cdots}{2a}\right.}{4c^3}$$
$$\frac{3a\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^{5/2}}dx}{4c^3}+\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{3\left(\frac{3}{2}a\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^{3/2}}dx-\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\int \arccos(ax)\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)d\arccos(ax)\right)-3\left(i\arccos(a\cdots\right.}{\cdots}\right.}{4c^3}$$
$$\frac{3a\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^{5/2}}dx}{4c^3}+\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

$$\Big\downarrow \text{ 5183}$$

$$\frac{3\left(\frac{3}{2}a\left(\frac{2\int \frac{\arccos(ax)}{1-a^2x^2}dx}{a}+\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}\right)-\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\int \arccos(ax)\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)d\arccos(a\cdots\right.}{\cdots}\right.}{\cdots}$$
$$\frac{3a\left(\frac{2\int \frac{\arccos(ax)}{\left(1-a^2x^2\right)^2}dx}{3a}+\frac{\arccos(ax)^2}{3a^2\left(1-a^2x^2\right)^{3/2}}\right)}{4c^3}+\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

$\big\downarrow$ 5163

$$\frac{3\left(\frac{3}{2}a\left(\frac{2\int\frac{\arccos(ax)}{1-a^2x^2}dx}{a}+\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}\right)-\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\int\arccos(ax)\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)d\arccos(a\right.}{}\right.}{\frac{3a\left(\frac{2\left(\frac{1}{2}\int\frac{\arccos(ax)}{1-a^2x^2}dx+\frac{1}{2}a\int\frac{x}{\left(1-a^2x^2\right)^{3/2}}dx+\frac{x\arccos(ax)}{2\left(1-a^2x^2\right)}\right)}{3a}+\frac{\arccos(ax)^2}{3a^2(1-a^2x^2)^{3/2}}\right)}{4c^3}}+\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

$\big\downarrow$ 241

$$\frac{3\left(\frac{3}{2}a\left(\frac{2\int\frac{\arccos(ax)}{1-a^2x^2}dx}{a}+\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}\right)-\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\int\arccos(ax)\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)d\arccos(a\right.}{}\right.}{\frac{3a\left(\frac{2\left(\frac{1}{2}\int\frac{\arccos(ax)}{1-a^2x^2}dx+\frac{x\arccos(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{2a\sqrt{1-a^2x^2}}\right)}{3a}+\frac{\arccos(ax)^2}{3a^2(1-a^2x^2)^{3/2}}\right)}{4c^3}}+\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

$\big\downarrow$ 5165

$$\frac{3\left(\frac{3}{2}a\left(\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}d\arccos(ax)}{a^2}\right)-\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\int\arccos(ax)\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)\right.}{}\right.}{\frac{3a\left(\frac{2\left(-\frac{\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}d\arccos(ax)}{2a}+\frac{x\arccos(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{2a\sqrt{1-a^2x^2}}\right)}{3a}+\frac{\arccos(ax)^2}{3a^2(1-a^2x^2)^{3/2}}\right)}{4c^3}}+\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

$\big\downarrow$ 3042

$$\frac{3\left(\frac{3}{2}a\left(\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\int \arccos(ax)\csc(\arccos(ax))d\arccos(ax)}{a^2}\right)-\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\int \arccos(ax)\operatorname{PolyLog}(2\right.}{}}{}$$

$$\frac{3a\left(\frac{2\left(-\frac{\int \arccos(ax)\csc(\arccos(ax))d\arccos(ax)}{2a}+\frac{x\arccos(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{2a\sqrt{1-a^2x^2}}\right)}{3a}+\frac{\arccos(ax)^2}{3a^2(1-a^2x^2)^{3/2}}\right)}{4c^3}+$$

$$\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

$\Big\downarrow$ 4671

$$\frac{3\left(\frac{3}{2}a\left(\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(-\int \log\left(1-e^{i\arccos(ax)}\right)d\arccos(ax)+\int \log\left(1+e^{i\arccos(ax)}\right)d\arccos(ax)-2\arccos(ax)\operatorname{arctanh}\left(e^{i\arccos(ax)}\right)\right)}{a^2}\right)}{}$$

$$\frac{3a\left(\frac{\arccos(ax)^2}{3a^2(1-a^2x^2)^{3/2}}+\frac{2\left(-\frac{-\int \log\left(1-e^{i\arccos(ax)}\right)d\arccos(ax)+\int \log\left(1+e^{i\arccos(ax)}\right)d\arccos(ax)-2\arccos(ax)\operatorname{arctanh}\left(e^{i\arccos(ax)}\right)}{2a}+\frac{x\arccos}{2\left(1-a^2\right.}\right.}{3a}\right.}{4c^3}$$

$$\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

$\Big\downarrow$ 2715

$$\frac{3\left(\frac{3}{2}a\left(\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(i\int e^{-i\arccos(ax)}\log\left(1-e^{i\arccos(ax)}\right)de^{i\arccos(ax)}-i\int e^{-i\arccos(ax)}\log\left(1+e^{i\arccos(ax)}\right)de^{i\arccos(ax)}-2\arccos(ax)\right.}{a^2}\right.}{}$$

$$\frac{3a\left(\frac{\arccos(ax)^2}{3a^2(1-a^2x^2)^{3/2}}+\frac{2\left(-\frac{i\int e^{-i\arccos(ax)}\log\left(1-e^{i\arccos(ax)}\right)de^{i\arccos(ax)}-i\int e^{-i\arccos(ax)}\log\left(1+e^{i\arccos(ax)}\right)de^{i\arccos(ax)}-2\arccos(ax)\operatorname{ar}}{2a}\right.}{3a}\right.}{4c^3}$$

$$\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

$\Big\downarrow$ 2838

$$3\left(-\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\int\arccos(ax)\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)d\arccos(ax)\right)-3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,e^{i\arccos(ax}\right.\right.}{2a}\right.$$

$$3a\left(\frac{\arccos(ax)^2}{3a^2(1-a^2x^2)^{3/2}}+\frac{2\left(\frac{x\arccos(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{2a\sqrt{1-a^2x^2}}-\frac{-2\arccos(ax)\operatorname{arctanh}\left(e^{i\arccos(ax)}\right)+i\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-i\operatorname{PolyLog}\left(2,e^{i\arccos(ax}\right.}{2a}\right.}{3a}\right.$$

$$4c^3$$

$$\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

$\downarrow$ 7163

$$3\left(-\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-e^{i\arccos(ax)}\right)d\arccos(ax)-i\arccos(ax)\operatorname{PolyLog}\left(3,-e^{i\arccos(ax)}\right)\right)\right)-3\left(i\operatorname{ar}\right.}{}\right.$$

$$3a\left(\frac{\arccos(ax)^2}{3a^2(1-a^2x^2)^{3/2}}+\frac{2\left(\frac{x\arccos(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{2a\sqrt{1-a^2x^2}}-\frac{-2\arccos(ax)\operatorname{arctanh}\left(e^{i\arccos(ax)}\right)+i\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-i\operatorname{PolyLog}\left(2,e^{i\arccos(ax}\right.}{2a}\right.}{3a}\right.$$

$$4c^3$$

$$\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

$\downarrow$ 2720

$$3\left(-\frac{3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-2i\left(\int e^{-i\arccos(ax)}\operatorname{PolyLog}\left(3,-e^{i\arccos(ax)}\right)de^{i\arccos(ax)}-i\arccos(ax)\operatorname{PolyLog}\left(3,-e^{i\arccos(a}\right.\right.\right.}{}\right.$$

$$3a\left(\frac{\arccos(ax)^2}{3a^2(1-a^2x^2)^{3/2}}+\frac{2\left(\frac{x\arccos(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{2a\sqrt{1-a^2x^2}}-\frac{-2\arccos(ax)\operatorname{arctanh}\left(e^{i\arccos(ax)}\right)+i\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-i\operatorname{PolyLog}\left(2,e^{i\arccos(ax}\right.}{2a}\right.}{3a}\right.$$

$$4c^3$$

$$\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

$\downarrow$ 7143

$$\frac{3a\left(\frac{\arccos(ax)^2}{3a^2(1-a^2x^2)^{3/2}}+\frac{2\left(\frac{x\arccos(ax)}{2(1-a^2x^2)}+\frac{1}{2a\sqrt{1-a^2x^2}}-\frac{-2\arccos(ax)\operatorname{arctanh}\left(e^{i\arccos(ax)}\right)+i\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)-i\operatorname{PolyLog}\left(2,e^{i\arccos(ax}}{2a}}{3a}}{4c^3}$$

$$\frac{3\left(\frac{3}{2}a\left(\frac{\arccos(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\left(-2\arccos(ax)\operatorname{arctanh}(e^{i\arccos(ax)})+i\operatorname{PolyLog}(2,-e^{i\arccos(ax)})-i\operatorname{PolyLog}(2,e^{i\arccos(ax)})\right)}{a^2}\right)+\frac{x\arccos(ax)}{2(1-a^2x^2)}}{}$$

$$\frac{x\arccos(ax)^3}{4c^3\left(1-a^2x^2\right)^2}$$

input
```
Int[ArcCos[a*x]^3/(c - a^2*c*x^2)^3,x]
```

output
```
(x*ArcCos[a*x]^3)/(4*c^3*(1 - a^2*x^2)^2) + (3*a*(ArcCos[a*x]^2/(3*a^2*(1
- a^2*x^2)^(3/2)) + (2*(1/(2*a*Sqrt[1 - a^2*x^2]) + (x*ArcCos[a*x])/(2*(1
- a^2*x^2)) - (-2*ArcCos[a*x]*ArcTanh[E^(I*ArcCos[a*x])] + I*PolyLog[2, -E
^(I*ArcCos[a*x])] - I*PolyLog[2, E^(I*ArcCos[a*x])])/(2*a)))/(3*a)))/(4*c^
3) + (3*((x*ArcCos[a*x]^3)/(2*(1 - a^2*x^2)) + (3*a*(ArcCos[a*x]^2/(a^2*Sq
rt[1 - a^2*x^2]) - (2*(-2*ArcCos[a*x]*ArcTanh[E^(I*ArcCos[a*x])] + I*PolyL
og[2, -E^(I*ArcCos[a*x])] - I*PolyLog[2, E^(I*ArcCos[a*x])]))/a^2))/2 - (-
2*ArcCos[a*x]^3*ArcTanh[E^(I*ArcCos[a*x])] + 3*(I*ArcCos[a*x]^2*PolyLog[2,
 -E^(I*ArcCos[a*x])] - (2*I)*((-I)*ArcCos[a*x]*PolyLog[3, -E^(I*ArcCos[a*x
])] + PolyLog[4, -E^(I*ArcCos[a*x])])) - 3*(I*ArcCos[a*x]^2*PolyLog[2, E^(
I*ArcCos[a*x])] - (2*I)*((-I)*ArcCos[a*x]*PolyLog[3, E^(I*ArcCos[a*x])] +
PolyLog[4, E^(I*ArcCos[a*x])])))/(2*a)))/(4*c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{-\frac{3a^3x^3\arccos(ax)^3+9x^2\sqrt{-a^2x^2+1}\,\arccos(ax)^2a^2-5ax\arccos(ax)^3+2a^3x^3\arccos(ax)-11\arccos(ax)^2\sqrt{-a^2x^2+1}+2a^2x^2\sqrt{-a}}{8\left(a^4x^4-2a^2x^2+1\right)c^3}}{}$
default	$\frac{-\frac{3a^3x^3\arccos(ax)^3+9x^2\sqrt{-a^2x^2+1}\,\arccos(ax)^2a^2-5ax\arccos(ax)^3+2a^3x^3\arccos(ax)-11\arccos(ax)^2\sqrt{-a^2x^2+1}+2a^2x^2\sqrt{-a}}{8\left(a^4x^4-2a^2x^2+1\right)c^3}}{}$

input
```
int(arccos(a*x)^3/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/8*(3*a^3*x^3*arccos(a*x)^3+9*x^2*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2*
a^2-5*a*x*arccos(a*x)^3+2*a^3*x^3*arccos(a*x)-11*arccos(a*x)^2*(-a^2*x^2+1
)^(1/2)+2*a^2*x^2*(-a^2*x^2+1)^(1/2)-2*a*x*arccos(a*x)-2*(-a^2*x^2+1)^(1/2
))/(a^4*x^4-2*a^2*x^2+1)/c^3-5/2/c^3*arccos(a*x)*ln(1-a*x-I*(-a^2*x^2+1)^(
1/2))+5/2*I/c^3*polylog(2,a*x+I*(-a^2*x^2+1)^(1/2))+5/2/c^3*arccos(a*x)*ln
(1+a*x+I*(-a^2*x^2+1)^(1/2))-5/2*I/c^3*polylog(2,-a*x-I*(-a^2*x^2+1)^(1/2)
)-3/8/c^3*arccos(a*x)^3*ln(1-a*x-I*(-a^2*x^2+1)^(1/2))+9/8*I/c^3*arccos(a*
x)^2*polylog(2,a*x+I*(-a^2*x^2+1)^(1/2))-9/4/c^3*arccos(a*x)*polylog(3,a*x
+I*(-a^2*x^2+1)^(1/2))-9/4*I/c^3*polylog(4,a*x+I*(-a^2*x^2+1)^(1/2))+3/8/c
^3*arccos(a*x)^3*ln(1+a*x+I*(-a^2*x^2+1)^(1/2))-9/8*I/c^3*arccos(a*x)^2*po
lylog(2,-a*x-I*(-a^2*x^2+1)^(1/2))+9/4/c^3*arccos(a*x)*polylog(3,-a*x-I*(-
a^2*x^2+1)^(1/2))+9/4*I/c^3*polylog(4,-a*x-I*(-a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^3}\,dx = \int -\frac{\arccos{(ax)}^3}{(a^2cx^2-c)^3}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(-arccos(a*x)^3/(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3
), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^3}\,dx = -\frac{\int \frac{\operatorname{acos}^3(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

input
```
integrate(acos(a*x)**3/(-a**2*c*x**2+c)**3,x)
```

output
```
-Integral(acos(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)/c**
3
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^3}\,dx = \int -\frac{\arccos(ax)^3}{(a^2cx^2-c)^3}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/16*((6*a^3*x^3 - 10*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) + 3*
(a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1))*arctan2(sqrt(a*x + 1)*sqrt(-a*x +
 1), a*x)^3 + 16*(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)*integrate(-3/16*(6*
a^3*x^3 - 10*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) + 3*(a^4*x^4 -
 2*a^2*x^2 + 1)*log(-a*x + 1))*sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a
*x + 1)*sqrt(-a*x + 1), a*x)^2/(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^
2 - c^3), x))/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^3}\,dx = \int -\frac{\arccos(ax)^3}{(a^2cx^2-c)^3}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(-arccos(a*x)^3/(a^2*c*x^2 - c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^3}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{(c-a^2\,c\,x^2)^3}\,dx$$

input
```
int(acos(a*x)^3/(c - a^2*c*x^2)^3,x)
```

output
```
int(acos(a*x)^3/(c - a^2*c*x^2)^3, x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^3}\,dx = -\frac{\int \frac{acos(ax)^3}{a^6x^6-3a^4x^4+3a^2x^2-1}dx}{c^3}$$

input
```
int(acos(a*x)^3/(-a^2*c*x^2+c)^3,x)
```

output
```
( - int(acos(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1),x))/c**3
```

3.297 $\int (c-a^2cx^2)^{5/2} \arccos(ax)^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 530

$$\int (c-a^2cx^2)^{5/2} \arccos(ax)^3 \, dx = \frac{245ac^2x^2\sqrt{c-a^2cx^2}}{768\sqrt{1-a^2x^2}} - \frac{65c^2(1-a^2x^2)^{3/2}\sqrt{c-a^2cx^2}}{2304a} - \frac{c^2(1-a^2x^2)^{5/2}\sqrt{c-a^2cx^2}}{216a} - \frac{245}{384}c^2x\sqrt{c-a^2cx^2}\arccos(ax) - \frac{65}{576}c^2x(1-a^2x^2)\sqrt{c-a^2cx^2}\arccos(ax) - \frac{1}{36}c^2x(1-a^2x^2)^2\sqrt{c-a^2cx^2}$$

output
```
245/768*a*c^2*x^2*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-65/2304*c^2*(-a^
2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)/a-1/216*c^2*(-a^2*x^2+1)^(5/2)*(-a^2*c
*x^2+c)^(1/2)/a-245/384*c^2*x*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)-65/576*c^2*
x*(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)-1/36*c^2*x*(-a^2*x^2+1)^2*
(-a^2*c*x^2+c)^(1/2)*arccos(a*x)+115/768*c^2*(-a^2*c*x^2+c)^(1/2)*arccos(a
*x)^2/a/(-a^2*x^2+1)^(1/2)-15/32*a*c^2*x^2*(-a^2*c*x^2+c)^(1/2)*arccos(a*x
)^2/(-a^2*x^2+1)^(1/2)+5/32*c^2*(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)*ar
ccos(a*x)^2/a+1/12*c^2*(-a^2*x^2+1)^(5/2)*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)
^2/a+5/16*c^2*x*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^3+5/24*c*x*(-a^2*c*x^2+c)
^(3/2)*arccos(a*x)^3+1/6*x*(-a^2*c*x^2+c)^(5/2)*arccos(a*x)^3+5/64*c^2*(-a
^2*c*x^2+c)^(1/2)*arccos(a*x)^4/a/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.34

$$\int (c-a^2cx^2)^{5/2}\arccos(ax)^3\,dx = \frac{c^2\sqrt{c-a^2cx^2}(-4320\arccos(ax)^4-9720\cos(2\arccos(ax))+243\cos(4\arccos$$

input
```
Integrate[(c - a^2*c*x^2)^(5/2)*ArcCos[a*x]^3,x]
```

output
```
(c^2*Sqrt[c - a^2*c*x^2]*(-4320*ArcCos[a*x]^4 - 9720*Cos[2*ArcCos[a*x]] +
243*Cos[4*ArcCos[a*x]] - 8*Cos[6*ArcCos[a*x]] + 72*ArcCos[a*x]^2*(270*Cos[
2*ArcCos[a*x]] - 27*Cos[4*ArcCos[a*x]] + 2*Cos[6*ArcCos[a*x]]) + 288*ArcCo
s[a*x]^3*(45*Sin[2*ArcCos[a*x]] - 9*Sin[4*ArcCos[a*x]] + Sin[6*ArcCos[a*x]
]) - 12*ArcCos[a*x]*(1620*Sin[2*ArcCos[a*x]] - 81*Sin[4*ArcCos[a*x]] + 4*S
in[6*ArcCos[a*x]])))/(55296*a*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 4.08 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.15, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5159, 5159, 5157, 5139, 5153, 5183, 5159, 241, 244, 2009, 5157, 15, 5153, 5159, 244, 2009, 5157, 15, 5153, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)^2\arccos(ax)^2dx}{2\sqrt{1-a^2x^2}}+\frac{5}{6}c\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^3dx+\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \text{5159}$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)^2\arccos(ax)^2dx}{2\sqrt{1-a^2x^2}}+$$
$$\frac{5}{6}c\left(\frac{3ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\arccos(ax)^2dx}{4\sqrt{1-a^2x^2}}+\frac{3}{4}c\int\sqrt{c-a^2cx^2}\arccos(ax)^3dx+\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right.\right.$$
$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \text{5157}$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)^2\arccos(ax)^2dx}{2\sqrt{1-a^2x^2}}+$$
$$\frac{5}{6}c\left(\frac{3ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\arccos(ax)^2dx}{4\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\int x\arccos(ax)^2dx}{2\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int\frac{\arccos}{\sqrt{1-}}}{2\sqrt{1-a^2x^2}}\right.\right.$$
$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \text{5139}$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)^2\arccos(ax)^2dx}{2\sqrt{1-a^2x^2}}+$$
$$\frac{5}{6}c\left(\frac{3ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\arccos(ax)^2dx}{4\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}+\right.\right.$$
$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \text{5153}$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)^2\arccos(ax)^2dx}{2\sqrt{1-a^2x^2}}+$$
$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-}\right.\right.$$
$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \text{5183}$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\left(-\frac{\int\left(1-a^2x^2\right)^{5/2}\arccos(ax)dx}{3a}-\frac{\left(1-a^2x^2\right)^3\arccos(ax)^2}{6a^2}\right)}{2\sqrt{1-a^2x^2}}+$$
$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-}\right.\right.$$
$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \text{5159}$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\left(-\frac{\frac{5}{6}\int(1-a^2x^2)^{3/2}\arccos(ax)dx+\frac{1}{6}a\int x(1-a^2x^2)^2dx+\frac{1}{6}x(1-a^2x^2)^{5/2}\arccos(ax)}{3a}-\frac{(1-a^2x^2)^3\arccos(ax)^2}{6a^2}\right)}{2\sqrt{1-a^2x^2}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-\right.\right.$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \text{241}$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\left(-\frac{\frac{5}{6}\int(1-a^2x^2)^{3/2}\arccos(ax)dx+\frac{1}{6}x(1-a^2x^2)^{5/2}\arccos(ax)-\frac{(1-a^2x^2)^3}{36a}}{3a}-\frac{(1-a^2x^2)^3\arccos(ax)^2}{6a^2}\right)}{2\sqrt{1-a^2x^2}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-\right.\right.$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \text{244}$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\left(-\frac{\frac{5}{6}\int(1-a^2x^2)^{3/2}\arccos(ax)dx+\frac{1}{6}x(1-a^2x^2)^{5/2}\arccos(ax)-\frac{(1-a^2x^2)^3}{36a}}{3a}-\frac{(1-a^2x^2)^3\arccos(ax)^2}{6a^2}\right)}{2\sqrt{1-a^2x^2}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-\right.\right.$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\left(-\frac{\frac{5}{6}\int(1-a^2x^2)^{3/2}\arccos(ax)dx+\frac{1}{6}x(1-a^2x^2)^{5/2}\arccos(ax)-\frac{(1-a^2x^2)^3}{36a}}{3a}-\frac{(1-a^2x^2)^3\arccos(ax)^2}{6a^2}\right)}{2\sqrt{1-a^2x^2}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-\right.\right.$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \; 5157$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\left(-\frac{\frac{5}{6}\int(1-a^2x^2)^{3/2}\arccos(ax)dx+\frac{1}{6}x(1-a^2x^2)^{5/2}\arccos(ax)-\frac{(1-a^2x^2)^3}{36a}}{3a}-\frac{(1-a^2x^2)^3\arccos(ax)^2}{6a^2}\right)}{2\sqrt{1-a^2x^2}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-\right.\right.$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \; 15$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\left(-\frac{\frac{5}{6}\int(1-a^2x^2)^{3/2}\arccos(ax)dx+\frac{1}{6}x(1-a^2x^2)^{5/2}\arccos(ax)-\frac{(1-a^2x^2)^3}{36a}}{3a}-\frac{(1-a^2x^2)^3\arccos(ax)^2}{6a^2}\right)}{2\sqrt{1-a^2x^2}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-\right.\right.$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \; 5153$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\left(-\frac{\frac{5}{6}\int\left(1-a^2x^2\right)^{3/2}\arccos(ax)dx+\frac{1}{6}x\left(1-a^2x^2\right)^{5/2}\arccos(ax)-\frac{\left(1-a^2x^2\right)^3}{36a}}{3a}-\frac{\left(1-a^2x^2\right)^3\arccos(ax)^2}{6a^2}\right)}{2\sqrt{1-a^2x^2}}+$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-\right.\right.$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \text{5159}$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\left(-\frac{\frac{5}{6}\left(\frac{3}{4}\int\sqrt{1-a^2x^2}\arccos(ax)dx+\frac{1}{4}a\int x\left(1-a^2x^2\right)dx+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\arccos(ax)\right)+\frac{1}{6}x\left(1-a^2x^2\right)^{5/2}\arccos(ax)-\frac{\left(1-a^2\right.}{36a}}{3a}\right.}{2\sqrt{1-a^2x^2}}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-\right.\right.$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \text{244}$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\left(-\frac{\frac{5}{6}\left(\frac{3}{4}\int\sqrt{1-a^2x^2}\arccos(ax)dx+\frac{1}{4}a\int\left(x-a^2x^3\right)dx+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\arccos(ax)\right)+\frac{1}{6}x\left(1-a^2x^2\right)^{5/2}\arccos(ax)-\frac{\left(1-a^2x^2\right.}{36a}}{3a}\right.}{2\sqrt{1-a^2x^2}}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-\right.\right.$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ac^2\sqrt{c-a^2cx^2}\left(-\frac{\frac{5}{6}\left(\frac{3}{4}\int\sqrt{1-a^2x^2}\arccos(ax)dx+\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)\right)+\frac{1}{6}x(1-a^2x^2)^{5/2}\arccos(ax)-\frac{(1-a^2x^2)}{36a}}{3a}\right.}{2\sqrt{1-a^2x^2}}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-}\right.\right.$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$\downarrow$ 5157

$$\frac{ac^2\sqrt{c-a^2cx^2}\left(-\frac{\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{a\int xdx}{2}+\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)\right)+\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)\right)+\frac{1}{6}x(1-a^2}{3a}\right.}{2\sqrt{1-a^2x^2}}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-}\right.\right.$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$\downarrow$ 15

$$\frac{ac^2\sqrt{c-a^2cx^2}\left(-\frac{\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)+\frac{ax^2}{4}\right)+\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)\right)+\frac{1}{6}x(1-a^2x^2)}{3a}\right.}{2\sqrt{1-a^2x^2}}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-}\right.\right.$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$\downarrow$ 5153

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-\right.\right.$$

$$\frac{ac^2\left(-\frac{(1-a^2x^2)^3\arccos(ax)^2}{6a^2}-\frac{\frac{1}{6}x(1-a^2x^2)^{5/2}\arccos(ax)+\frac{5}{6}\left(\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)-\frac{\arccos(ax)^2}{4a}+\frac{ax}{4}\right.\right.}{3a}\right.}{2\sqrt{1-a^2x^2}}$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \text{5211}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\left(\frac{\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int xdx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-}{8a\sqrt{1-a^2x}}\right.\right.$$

$$\frac{ac^2\left(-\frac{(1-a^2x^2)^3\arccos(ax)^2}{6a^2}-\frac{\frac{1}{6}x(1-a^2x^2)^{5/2}\arccos(ax)+\frac{5}{6}\left(\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)-\frac{\arccos(ax)^2}{4a}+\frac{ax}{4}\right.\right.}{3a}\right.}{2\sqrt{1-a^2x^2}}$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \text{15}$$

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\left(\frac{\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2}{8a\sqrt{1-a^2x^2}}\right.\right.$$

$$\frac{ac^2\left(-\frac{(1-a^2x^2)^3\arccos(ax)^2}{6a^2}-\frac{\frac{1}{6}x(1-a^2x^2)^{5/2}\arccos(ax)+\frac{5}{6}\left(\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)-\frac{\arccos(ax)^2}{4a}+\frac{ax}{4}\right.\right.}{3a}\right.}{2\sqrt{1-a^2x^2}}$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}$$

$$\Big\downarrow \text{5153}$$

$$\frac{ac^2\left(-\frac{(1-a^2x^2)^3\arccos(ax)^2}{6a^2}-\frac{\frac{1}{6}x(1-a^2x^2)^{5/2}\arccos(ax)+\frac{5}{6}\left(\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)-\frac{\arccos(ax)^2}{4a}+\frac{ax}{4}\right.\right.}{3a}\right.}{2\sqrt{1-a^2x^2}}$$

$$\frac{1}{6}x\arccos(ax)^3\left(c-a^2cx^2\right)^{5/2}+$$

$$\frac{5}{6}c\left(\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}+\frac{3ac\left(-\frac{(1-a^2x^2)^2\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)-\right.}{2a}\right.}{4\sqrt{1-a^2x^2}}\right.$$

input
```
Int[(c - a^2*c*x^2)^(5/2)*ArcCos[a*x]^3,x]
```

output
```
(x*(c - a^2*c*x^2)^(5/2)*ArcCos[a*x]^3)/6 + (a*c^2*Sqrt[c - a^2*c*x^2]*(-1
/6*((1 - a^2*x^2)^3*ArcCos[a*x]^2)/a^2 - (-1/36*(1 - a^2*x^2)^3/a + (x*(1
- a^2*x^2)^(5/2)*ArcCos[a*x])/6 + (5*((a*(x^2/2 - (a^2*x^4)/4))/4 + (x*(1
- a^2*x^2)^(3/2)*ArcCos[a*x])/4 + (3*((a*x^2)/4 + (x*Sqrt[1 - a^2*x^2]*Arc
Cos[a*x])/2 - ArcCos[a*x]^2/(4*a)))/4))/6)/(3*a)))/(2*Sqrt[1 - a^2*x^2]) +
 (5*c*((x*(c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^3)/4 + (3*c*((x*Sqrt[c - a^2*c
*x^2]*ArcCos[a*x]^3)/2 - (Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^4)/(8*a*Sqrt[1 -
 a^2*x^2]) + (3*a*Sqrt[c - a^2*c*x^2]*((x^2*ArcCos[a*x]^2)/2 + a*(-1/4*x^2
/a - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a^2) - ArcCos[a*x]^2/(4*a^3))))/
(2*Sqrt[1 - a^2*x^2])))/4 + (3*a*c*Sqrt[c - a^2*c*x^2]*(-1/4*((1 - a^2*x^2
)^2*ArcCos[a*x]^2)/a^2 - ((a*(x^2/2 - (a^2*x^4)/4))/4 + (x*(1 - a^2*x^2)^(
3/2)*ArcCos[a*x])/4 + (3*((a*x^2)/4 + (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/2
- ArcCos[a*x]^2/(4*a)))/4)/(2*a)))/(4*Sqrt[1 - a^2*x^2])))/6
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 875, normalized size of antiderivative = 1.65

method	result
default	$\frac{5\sqrt{-c(a^2x^2-1)}\,\sqrt{-a^2x^2+1}\,\arccos(ax)^4 c^2}{64a(a^2x^2-1)}+\frac{\sqrt{-c(a^2x^2-1)}\,\left(32a^7x^7-64a^5x^5+32i\sqrt{-a^2x^2+1}\,a^6x^6+38a^3x^3-48i\sqrt{-a^2x^2+1}\,a\right.}{}$

input
```
int((-a^2*c*x^2+c)^(5/2)*arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
5/64*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a/(a^2*x^2-1)*arccos(a*x)^4
*c^2+1/13824*(-c*(a^2*x^2-1))^(1/2)*(32*a^7*x^7-64*a^5*x^5+32*I*(-a^2*x^2+
1)^(1/2)*a^6*x^6+38*a^3*x^3-48*I*(-a^2*x^2+1)^(1/2)*a^4*x^4-6*a*x+18*I*(-a
^2*x^2+1)^(1/2)*a^2*x^2-I*(-a^2*x^2+1)^(1/2))*(18*I*arccos(a*x)^2+36*arcco
s(a*x)^3-I-6*arccos(a*x))*c^2/a/(a^2*x^2-1)-3/4096*(-c*(a^2*x^2-1))^(1/2)*
(8*a^5*x^5-12*a^3*x^3+8*I*(-a^2*x^2+1)^(1/2)*a^4*x^4+4*a*x-8*I*(-a^2*x^2+1
)^(1/2)*a^2*x^2+I*(-a^2*x^2+1)^(1/2))*(24*I*arccos(a*x)^2+32*arccos(a*x)^3
-3*I-12*arccos(a*x))*c^2/a/(a^2*x^2-1)+15/512*(-c*(a^2*x^2-1))^(1/2)*(2*a^
3*x^3-2*a*x+2*I*(-a^2*x^2+1)^(1/2)*a^2*x^2-I*(-a^2*x^2+1)^(1/2))*(6*I*arcc
os(a*x)^2+4*arccos(a*x)^3-3*I-6*arccos(a*x))*c^2/a/(a^2*x^2-1)+15/512*(-c*
(a^2*x^2-1))^(1/2)*(-2*I*(-a^2*x^2+1)^(1/2)*a^2*x^2+2*a^3*x^3+I*(-a^2*x^2+
1)^(1/2)-2*a*x)*(-6*I*arccos(a*x)^2+4*arccos(a*x)^3+3*I-6*arccos(a*x))*c^2
/a/(a^2*x^2-1)-3/4096*(-c*(a^2*x^2-1))^(1/2)*(-8*I*(-a^2*x^2+1)^(1/2)*a^4*
x^4+8*a^5*x^5+8*I*(-a^2*x^2+1)^(1/2)*a^2*x^2-12*a^3*x^3-I*(-a^2*x^2+1)^(1/
2)+4*a*x)*(-24*I*arccos(a*x)^2+32*arccos(a*x)^3+3*I-12*arccos(a*x))*c^2/a/
(a^2*x^2-1)+1/13824*(-c*(a^2*x^2-1))^(1/2)*(-32*I*(-a^2*x^2+1)^(1/2)*a^6*x
^6+32*a^7*x^7+48*I*(-a^2*x^2+1)^(1/2)*a^4*x^4-64*a^5*x^5-18*I*(-a^2*x^2+1)
^(1/2)*a^2*x^2+38*a^3*x^3+I*(-a^2*x^2+1)^(1/2)-6*a*x)*(-18*I*arccos(a*x)^2
+36*arccos(a*x)^3+I-6*arccos(a*x))*c^2/a/(a^2*x^2-1)
```

Fricas [F]

$$\int \left(c-a^2cx^2\right)^{5/2}\arccos(ax)^3\,dx=\int \left(-a^2cx^2+c\right)^{\frac{5}{2}}\arccos\left(ax\right)^3\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*arccos(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*sqrt(-a^2*c*x^2 + c)*arccos(a
*x)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(c-a^2cx^2\right)^{5/2}\arccos(ax)^3\,dx=\text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(5/2)*acos(a*x)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(c-a^2cx^2\right)^{5/2}\arccos(ax)^3\,dx=\int \left(-a^2cx^2+c\right)^{\frac{5}{2}}\arccos\left(ax\right)^3\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*arccos(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((-a^2*c*x^2 + c)^(5/2)*arccos(a*x)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(c-a^2cx^2\right)^{5/2}\arccos(ax)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)*arccos(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(c-a^2cx^2\right)^{5/2}\arccos(ax)^3\,dx=\int \mathrm{acos}(a\,x)^3\,\left(c-a^2\,c\,x^2\right)^{5/2}\,dx$$

input
```
int(acos(a*x)^3*(c - a^2*c*x^2)^(5/2),x)
```

output
```
int(acos(a*x)^3*(c - a^2*c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \left(c-a^2cx^2\right)^{5/2}\arccos(ax)^3\,dx=\sqrt{c}\,c^2\left(\left(\int \sqrt{-a^2x^2+1}\,acos(ax)^3\,x^4dx\right)a^4-2\left(\int \sqrt{-a^2x^2+1}\,acos(ax)^3\,x^2dx\right)a^2+\int \sqrt{-a^2x^2+1}\,acos(ax)^3\,dx\right)$$

input
```
int((-a^2*c*x^2+c)^(5/2)*acos(a*x)^3,x)
```

output
```
sqrt(c)*c**2*(int(sqrt( - a**2*x**2 + 1)*acos(a*x)**3*x**4,x)*a**4 - 2*int
(sqrt( - a**2*x**2 + 1)*acos(a*x)**3*x**2,x)*a**2 + int(sqrt( - a**2*x**2
+ 1)*acos(a*x)**3,x))
```

3.298 $\int (c-a^2cx^2)^{3/2}\arccos(ax)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 362

$$\int (c-a^2cx^2)^{3/2}\arccos(ax)^3\,dx=\frac{45acx^2\sqrt{c-a^2cx^2}}{128\sqrt{1-a^2x^2}}-\frac{3c(1-a^2x^2)^{3/2}\sqrt{c-a^2cx^2}}{128a}-\frac{45}{64}cx\sqrt{c-a^2cx^2}\arccos(ax)-\frac{3}{32}cx(1-a^2x^2)\sqrt{c-a^2cx^2}\arccos(ax)+\frac{27c\sqrt{c-a^2cx^2}\arccos(ax)^2}{128a\sqrt{1-a^2x^2}}-\frac{9acx^2\sqrt{c-a^2cx^2}\arccos(ax)^2}{16\sqrt{1-a^2x^2}}+\frac{3c(1-a^2x^2)^{3/2}\sqrt{c-a^2cx^2}\arccos(ax)^2}{16a}+\frac{3}{8}cx\sqrt{c-a^2cx^2}\arccos(ax)^3+\frac{1}{4}x(c-a^2cx^2)^{3/2}\arccos(ax)^3+\frac{3c\sqrt{c-a^2cx^2}\arccos(ax)^4}{32a\sqrt{1-a^2x^2}}$$

output

```
45/128*a*c*x^2*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-3/128*c*(-a^2*x^2+1
)^(3/2)*(-a^2*c*x^2+c)^(1/2)/a-45/64*c*x*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)-
3/32*c*x*(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)+27/128*c*(-a^2*c*x^
2+c)^(1/2)*arccos(a*x)^2/a/(-a^2*x^2+1)^(1/2)-9/16*a*c*x^2*(-a^2*c*x^2+c)^
(1/2)*arccos(a*x)^2/(-a^2*x^2+1)^(1/2)+3/16*c*(-a^2*x^2+1)^(3/2)*(-a^2*c*x
^2+c)^(1/2)*arccos(a*x)^2/a+3/8*c*x*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^3+1/4
*x*(-a^2*c*x^2+c)^(3/2)*arccos(a*x)^3+3/32*c*(-a^2*c*x^2+c)^(1/2)*arccos(a
*x)^4/a/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.38

$$\int \left(c-a^2cx^2\right)^{3/2} \arccos(ax)^3 \, dx =$$
$$-\frac{c\sqrt{c-a^2cx^2}(96\arccos(ax)^4-3(-64\cos(2\arccos(ax))+\cos(4\arccos(ax)))+24\arccos(ax)^2(-16\cos(2$$

input
```
Integrate[(c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^3,x]
```

output
```
-1/1024*(c*Sqrt[c - a^2*c*x^2]*(96*ArcCos[a*x]^4 - 3*(-64*Cos[2*ArcCos[a*x
]] + Cos[4*ArcCos[a*x]]) + 24*ArcCos[a*x]^2*(-16*Cos[2*ArcCos[a*x]] + Cos[
4*ArcCos[a*x]]) - 12*ArcCos[a*x]*(-32*Sin[2*ArcCos[a*x]] + Sin[4*ArcCos[a*
x]]) + 32*ArcCos[a*x]^3*(-8*Sin[2*ArcCos[a*x]] + Sin[4*ArcCos[a*x]])))/(a*
Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {5159, 5157, 5139, 5153, 5183, 5159, 244, 2009, 5157, 15, 5153, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^3 \left(c-a^2cx^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 5159}$$

$$\frac{3ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\arccos(ax)^2dx}{4\sqrt{1-a^2x^2}}+\frac{3}{4}c\int \sqrt{c-a^2cx^2}\arccos(ax)^3dx+\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 5157}$$

$$\frac{3ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\arccos(ax)^2dx}{4\sqrt{1-a^2x^2}}+$$
$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\int x\arccos(ax)^2dx}{2\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int\frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}\right)+$$
$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$\Big\downarrow$ 5139

$$\frac{3ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\arccos(ax)^2dx}{4\sqrt{1-a^2x^2}}+$$
$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int\frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}\right)$$
$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$\Big\downarrow$ 5153

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}\right)$$
$$\frac{3ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\arccos(ax)^2dx}{4\sqrt{1-a^2x^2}}+\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$\Big\downarrow$ 5183

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}\right)$$
$$\frac{3ac\sqrt{c-a^2cx^2}\left(-\frac{\int\left(1-a^2x^2\right)^{3/2}\arccos(ax)dx}{2a}-\frac{\left(1-a^2x^2\right)^2\arccos(ax)^2}{4a^2}\right)}{4\sqrt{1-a^2x^2}}+$$
$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$\Big\downarrow$ 5159

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2c\right.$$
$$\frac{3ac\sqrt{c-a^2cx^2}\left(-\frac{\frac{3}{4}\int\sqrt{1-a^2x^2}\arccos(ax)dx+\frac{1}{4}a\int x(1-a^2x^2)dx+\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)}{2a}-\frac{(1-a^2x^2)^2\arccos(ax)^2}{4a^2}\right)}{4\sqrt{1-a^2x^2}}+$$
$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{244}$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2c\right.$$
$$\frac{3ac\sqrt{c-a^2cx^2}\left(-\frac{\frac{3}{4}\int\sqrt{1-a^2x^2}\arccos(ax)dx+\frac{1}{4}a\int(x-a^2x^3)dx+\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)}{2a}-\frac{(1-a^2x^2)^2\arccos(ax)^2}{4a^2}\right)}{4\sqrt{1-a^2x^2}}+$$
$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2c\right.$$
$$\frac{3ac\sqrt{c-a^2cx^2}\left(-\frac{\frac{3}{4}\int\sqrt{1-a^2x^2}\arccos(ax)dx+\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)}{2a}-\frac{(1-a^2x^2)^2\arccos(ax)^2}{4a^2}\right)}{4\sqrt{1-a^2x^2}}+$$
$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{5157}$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2c\right.$$
$$\frac{3ac\sqrt{c-a^2cx^2}\left(-\frac{\frac{3}{4}\left(\frac{1}{2}\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{a\int xdx}{2}+\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)\right)+\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)}{2a}-\frac{(1-a^2x^2)^2}{4a}\right.}{4\sqrt{1-a^2x^2}}$$
$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2c\ldots}\right.$$

$$\frac{3ac\sqrt{c-a^2cx^2}\left(-\frac{\frac{3}{4}\left(\frac{1}{2}\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)+\frac{ax^2}{4}\right)+\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)}{2a}-\frac{(1-a^2x^2)^2\operatorname{arc}\ldots}{4a^2}\ldots\right.}{4\sqrt{1-a^2x^2}}$$

$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}$$

$\Big\downarrow$ 5153

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2c\ldots}\right.$$

$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}+$$

$$\frac{3ac\left(-\frac{(1-a^2x^2)^2\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)-\frac{\arccos(ax)^2}{4a}+\frac{ax^2}{4}\right)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)}{2a}\right)\sqrt{c-c\ldots}}{4\sqrt{1-a^2x^2}}$$

$\Big\downarrow$ 5211

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\left(\frac{\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int xdx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx\ldots}}{8a\sqrt{1-a^2x^2}}\right.$$

$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}+$$

$$\frac{3ac\left(-\frac{(1-a^2x^2)^2\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)-\frac{\arccos(ax)^2}{4a}+\frac{ax^2}{4}\right)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)}{2a}\right)\sqrt{c-c\ldots}}{4\sqrt{1-a^2x^2}}$$

$\Big\downarrow$ 15

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(a\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}\right.$$

$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}+$$

$$\frac{3ac\left(-\frac{(1-a^2x^2)^2\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)-\frac{\arccos(ax)^2}{4a}+\frac{ax^2}{4}\right)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)}{2a}\right)\sqrt{c-c}}{4\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 5153}$$

$$\frac{1}{4}x\arccos(ax)^3\left(c-a^2cx^2\right)^{3/2}+$$

$$\frac{3ac\left(-\frac{(1-a^2x^2)^2\arccos(ax)^2}{4a^2}-\frac{\frac{1}{4}x(1-a^2x^2)^{3/2}\arccos(ax)+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\arccos(ax)-\frac{\arccos(ax)^2}{4a}+\frac{ax^2}{4}\right)+\frac{1}{4}a\left(\frac{x^2}{2}-\frac{a^2x^4}{4}\right)}{2a}\right)\sqrt{c-c}}{4\sqrt{1-a^2x^2}}$$

$$\frac{3}{4}c\left(-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}+\frac{3a\left(a\left(-\frac{\arccos(ax)^2}{4a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\right.}{2\sqrt{1-a^2x^2}}\right.$$

input
```
Int[(c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^3,x]
```

output
```
(x*(c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^3)/4 + (3*c*((x*Sqrt[c - a^2*c*x^2]*A
rcCos[a*x]^3)/2 - (Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^4)/(8*a*Sqrt[1 - a^2*x^
2]) + (3*a*Sqrt[c - a^2*c*x^2]*((x^2*ArcCos[a*x]^2)/2 + a*(-1/4*x^2/a - (x
*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a^2) - ArcCos[a*x]^2/(4*a^3))))/(2*Sqrt
[1 - a^2*x^2])))/4 + (3*a*c*Sqrt[c - a^2*c*x^2]*(-1/4*((1 - a^2*x^2)^2*Arc
Cos[a*x]^2)/a^2 - ((a*(x^2/2 - (a^2*x^4)/4))/4 + (x*(1 - a^2*x^2)^(3/2)*Ar
cCos[a*x])/4 + (3*((a*x^2)/4 + (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/2 - ArcCo
s[a*x]^2/(4*a)))/4)/(2*a)))/(4*Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.47

method	result
default	$\frac{3\sqrt{-c(a^2x^2-1)}\,\sqrt{-a^2x^2+1}\,\arccos(ax)^4 c}{32a(a^2x^2-1)}-\frac{\sqrt{-c(a^2x^2-1)}\left(8a^5x^5-12a^3x^3+8i\sqrt{-a^2x^2+1}\,a^4x^4+4ax-8i\sqrt{-a^2x^2+1}\,a^2x^2+i\sqrt{\right.}{2048a(a^2x^2-1)}$

input
```
int((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
3/32*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a/(a^2*x^2-1)*arccos(a*x)^4
*c-1/2048*(-c*(a^2*x^2-1))^(1/2)*(8*a^5*x^5-12*a^3*x^3+8*I*(-a^2*x^2+1)^(1
/2)*a^4*x^4+4*a*x-8*I*(-a^2*x^2+1)^(1/2)*a^2*x^2+I*(-a^2*x^2+1)^(1/2))*(24
*I*arccos(a*x)^2+32*arccos(a*x)^3-3*I-12*arccos(a*x))*c/a/(a^2*x^2-1)+1/32
*(-c*(a^2*x^2-1))^(1/2)*(2*a^3*x^3-2*a*x+2*I*(-a^2*x^2+1)^(1/2)*a^2*x^2-I*
(-a^2*x^2+1)^(1/2))*(6*I*arccos(a*x)^2+4*arccos(a*x)^3-3*I-6*arccos(a*x))*
c/a/(a^2*x^2-1)+1/32*(-c*(a^2*x^2-1))^(1/2)*(-2*I*(-a^2*x^2+1)^(1/2)*a^2*x
^2+2*a^3*x^3+I*(-a^2*x^2+1)^(1/2)-2*a*x)*(-6*I*arccos(a*x)^2+4*arccos(a*x)
^3+3*I-6*arccos(a*x))*c/a/(a^2*x^2-1)-1/2048*(-c*(a^2*x^2-1))^(1/2)*(-8*I*
(-a^2*x^2+1)^(1/2)*a^4*x^4+8*a^5*x^5+8*I*(-a^2*x^2+1)^(1/2)*a^2*x^2-12*a^3
*x^3-I*(-a^2*x^2+1)^(1/2)+4*a*x)*(-24*I*arccos(a*x)^2+32*arccos(a*x)^3+3*I
-12*arccos(a*x))*c/a/(a^2*x^2-1)
```

Fricas [F]

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^3\,dx=\int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\arccos\left(ax\right)^3\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^3,x, algorithm="fricas")
```

output
```
integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*arccos(a*x)^3, x)
```

Sympy [F]

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^3\,dx=\int \left(-c(ax-1)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{acos}^3\left(ax\right)dx$$

input
```
integrate((-a**2*c*x**2+c)**(3/2)*acos(a*x)**3,x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*acos(a*x)**3, x)
```

Maxima [F]

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^3\,dx=\int \left(-a^2cx^2+c\right)^{\frac{3}{2}}\arccos\left(ax\right)^3\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((-a^2*c*x^2 + c)^(3/2)*arccos(a*x)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^3\,dx=\int \mathrm{acos}(a\,x)^3\,\left(c-a^2\,c\,x^2\right)^{3/2}\,dx$$

input
```
int(acos(a*x)^3*(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(acos(a*x)^3*(c - a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^3\,dx=\sqrt{c}\,c\left(-\left(\int \sqrt{-a^2x^2+1}\,acos(ax)^3\,x^2dx\right)a^2+\int \sqrt{-a^2x^2+1}\,acos(ax)^3\,dx\right)$$

input
```
int((-a^2*c*x^2+c)^(3/2)*acos(a*x)^3,x)
```

output
```
sqrt(c)*c*( - int(sqrt( - a**2*x**2 + 1)*acos(a*x)**3*x**2,x)*a**2 + int(s
qrt( - a**2*x**2 + 1)*acos(a*x)**3,x))
```

3.299 $\int \sqrt{c-a^2cx^2} \arccos(ax)^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 215

$$\int \sqrt{c-a^2cx^2} \arccos(ax)^3 \, dx = \frac{3ax^2\sqrt{c-a^2cx^2}}{8\sqrt{1-a^2x^2}} - \frac{3}{4}x\sqrt{c-a^2cx^2}\arccos(ax) + \frac{3\sqrt{c-a^2cx^2}\arccos(ax)^2}{8a\sqrt{1-a^2x^2}} - \frac{3ax^2\sqrt{c-a^2cx^2}\arccos(ax)^2}{4\sqrt{1-a^2x^2}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\arccos(ax)^3 + \frac{\sqrt{c-a^2cx^2}\arccos(ax)^4}{8a\sqrt{1-a^2x^2}}$$

output
```
3/8*a*x^2*(-a^2*c*x^2+c)^(1/2)/(-a^2*x^2+1)^(1/2)-3/4*x*(-a^2*c*x^2+c)^(1/
2)*arccos(a*x)+3/8*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^2/a/(-a^2*x^2+1)^(1/2)
-3/4*a*x^2*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^2/(-a^2*x^2+1)^(1/2)+1/2*x*(-a
^2*c*x^2+c)^(1/2)*arccos(a*x)^3+1/8*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^4/a/(
-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.40

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx =$$
$$-\frac{\sqrt{c(1-a^2x^2)}((3-6\arccos(ax)^2)\cos(2\arccos(ax))+2\arccos(ax)\,(\arccos(ax)^3+(3-2\arccos(ax)^2}{16a\sqrt{1-a^2x^2}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^3,x]
```

output
```
-1/16*(Sqrt[c*(1 - a^2*x^2)]*((3 - 6*ArcCos[a*x]^2)*Cos[2*ArcCos[a*x]] + 2
*ArcCos[a*x]*(ArcCos[a*x]^3 + (3 - 2*ArcCos[a*x]^2)*Sin[2*ArcCos[a*x]])))/
(a*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5157, 5139, 5153, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^3\sqrt{c-a^2cx^2}\,dx$$

$$\Big\downarrow \text{5157}$$

$$\frac{3a\sqrt{c-a^2cx^2}\int x\arccos(ax)^2dx}{2\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{5139}$$

$$\frac{3a\sqrt{c-a^2cx^2}\left(a\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{5153}$$

$$\frac{3a\sqrt{c-a^2cx^2}\left(a\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{5211}$$

$$\frac{3a\sqrt{c-a^2cx^2}\left(a\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int xdx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{3a\sqrt{c-a^2cx^2}\left(a\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{5153}$$

$$-\frac{\arccos(ax)^4\sqrt{c-a^2cx^2}}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^3\sqrt{c-a^2cx^2}+\frac{3a\left(a\left(-\frac{\arccos(ax)^2}{4a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)\sqrt{c-a^2cx^2}}{2\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^3,x]
```

output
```
(x*Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^3)/2 - (Sqrt[c - a^2*c*x^2]*ArcCos[a*x]
^4)/(8*a*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - a^2*c*x^2]*((x^2*ArcCos[a*x]^2
)/2 + a*(-1/4*x^2/a - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a^2) - ArcCos[a
*x]^2/(4*a^3))))/(2*Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.00 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.21

method	result
default	$\frac{\sqrt{-c(a^2x^2-1)}\,\sqrt{-a^2x^2+1}\,\arccos(ax)^4}{8a(a^2x^2-1)}+\frac{\sqrt{-c(a^2x^2-1)}\left(2a^3x^3-2ax+2i\sqrt{-a^2x^2+1}\,a^2x^2-i\sqrt{-a^2x^2+1}\right)\left(6i\arccos(ax)^2+4\,\text{ar}\right.}{32a(a^2x^2-1)}$

input
```
int((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a/(a^2*x^2-1)*arccos(a*x)^4+
1/32*(-c*(a^2*x^2-1))^(1/2)*(2*a^3*x^3-2*a*x+2*I*(-a^2*x^2+1)^(1/2)*a^2*x^
2-I*(-a^2*x^2+1)^(1/2))*(6*I*arccos(a*x)^2+4*arccos(a*x)^3-3*I-6*arccos(a*
x))/a/(a^2*x^2-1)+1/32*(-c*(a^2*x^2-1))^(1/2)*(-2*I*(-a^2*x^2+1)^(1/2)*a^2
*x^2+2*a^3*x^3+I*(-a^2*x^2+1)^(1/2)-2*a*x)*(-6*I*arccos(a*x)^2+4*arccos(a*
x)^3+3*I-6*arccos(a*x))/a/(a^2*x^2-1)
```

Fricas [F]

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx=\int \sqrt{-a^2cx^2+c}\arccos{(ax)}^3\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*arccos(a*x)^3, x)
```

Sympy [F]

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx = \int \sqrt{-c\,(ax-1)\,(ax+1)}\operatorname{acos}^3{(ax)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*acos(a*x)**3,x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*acos(a*x)**3, x)
```

Maxima [F]

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx = \int \sqrt{-a^2cx^2+c}\arccos{(ax)}^3\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)*arccos(a*x)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx=\int \mathrm{acos}(a\,x)^3\,\sqrt{c-a^2\,c\,x^2}\,dx$$

input
```
int(acos(a*x)^3*(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(acos(a*x)^3*(c - a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^3\,dx=\sqrt{c}\left(\int \sqrt{-a^2x^2+1}\,acos(ax)^3\,dx\right)$$

input
```
int((-a^2*c*x^2+c)^(1/2)*acos(a*x)^3,x)
```

output
```
sqrt(c)*int(sqrt( - a**2*x**2 + 1)*acos(a*x)**3,x)
```

3.300 $\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 42

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}\,dx = \frac{\sqrt{1-a^2x^2}\arccos(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

output
```
1/4*(-a^2*x^2+1)^(1/2)*arccos(a*x)^4/a/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\arccos(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

input
```
Integrate[ArcCos[a*x]^3/Sqrt[c - a^2*c*x^2],x]
```

output
```
-1/4*(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^4)/(a*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{ 5153}$$

$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

input
```
Int[ArcCos[a*x]^3/Sqrt[c - a^2*c*x^2],x]
```

output
```
-1/4*(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^4)/(a*Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{\sqrt{-c(a^2x^2-1)}\,\sqrt{-a^2x^2+1}\,\arccos(ax)^4}{4ac(a^2x^2-1)}$	52

input
```
int(arccos(a*x)^3/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a/c/(a^2*x^2-1)*arccos(a*x)^
4
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\arccos{(ax)}^3}{\sqrt{-a^2cx^2+c}}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*c*x^2 + c)*arccos(a*x)^3/(a^2*c*x^2 - c), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\operatorname{acos}^3{(ax)}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

input
```
integrate(acos(a*x)**3/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(acos(a*x)**3/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.64

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}\,dx = \frac{\arccos(ax)^3 \arcsin(ax)}{a\sqrt{c}} + \frac{3\arccos(ax)^2 \arcsin(ax)^2}{2a\sqrt{c}} + \frac{\frac{4\arccos(ax)\arcsin(ax)^3}{a} + \frac{\arcsin(ax)^4}{a}}{4\sqrt{c}}$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
arccos(a*x)^3*arcsin(a*x)/(a*sqrt(c)) + 3/2*arccos(a*x)^2*arcsin(a*x)^2/(a
*sqrt(c)) + 1/4*(4*arccos(a*x)*arcsin(a*x)^3/a + arcsin(a*x)^4/a)/sqrt(c)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.33

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\arccos(ax)^4}{4a\sqrt{c}}$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*arccos(a*x)^4/(a*sqrt(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

input
```
int(acos(a*x)^3/(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(acos(a*x)^3/(c - a^2*c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.38

$$\int \frac{\arccos(ax)^3}{\sqrt{c-a^2cx^2}}\,dx = -\frac{\sqrt{c}\,acos(ax)^4}{4ac}$$

input
```
int(acos(a*x)^3/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
( - sqrt(c)*acos(a*x)**4)/(4*a*c)
```

3.301 $\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 238

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx = \frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}} - \frac{i\sqrt{1-a^2x^2}\arccos(ax)^3}{ac\sqrt{c-a^2cx^2}} + \frac{3\sqrt{1-a^2x^2}\arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right)}{ac\sqrt{c-a^2cx^2}} - \frac{3i\sqrt{1-a^2x^2}\arccos(ax)\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)}{ac\sqrt{c-a^2cx^2}} + \frac{3\sqrt{1-a^2x^2}\operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)}{2ac\sqrt{c-a^2cx^2}}$$

output
```
x*arccos(a*x)^3/c/(-a^2*c*x^2+c)^(1/2)-I*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/
a/c/(-a^2*c*x^2+c)^(1/2)+3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2*ln(1+(a*x+I*(-
a^2*x^2+1)^(1/2))^2)/a/c/(-a^2*c*x^2+c)^(1/2)-3*I*(-a^2*x^2+1)^(1/2)*arcco
s(a*x)*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c/(-a^2*c*x^2+c)^(1/2)+3
/2*(-a^2*x^2+1)^(1/2)*polylog(3,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c/(-a^2*c
*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.76

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx =$$
$$-\frac{-i\pi^3\sqrt{1-a^2x^2}-8ax\arccos(ax)^3+8i\sqrt{1-a^2x^2}\arccos(ax)^3+24\sqrt{1-a^2x^2}\arccos(ax)^2\log\left(1-e^{-2ia\right.}}{8ac\sqrt{c(1}}$$

input
```
Integrate[ArcCos[a*x]^3/(c - a^2*c*x^2)^(3/2),x]
```

output
```
-1/8*((-I)*Pi^3*Sqrt[1 - a^2*x^2] - 8*a*x*ArcCos[a*x]^3 + (8*I)*Sqrt[1 - a
^2*x^2]*ArcCos[a*x]^3 + 24*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2*Log[1 - E^((-2*
I)*ArcCos[a*x])] + (24*I)*Sqrt[1 - a^2*x^2]*ArcCos[a*x]*PolyLog[2, E^((-2*
I)*ArcCos[a*x])] + 12*Sqrt[1 - a^2*x^2]*PolyLog[3, E^((-2*I)*ArcCos[a*x])]
)/(a*c*Sqrt[c*(1 - a^2*x^2)])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {5161, 5181, 3042, 25, 4200, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{5161}$$

$$\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{1-a^2x^2}dx}{c\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{5181}$$

$$\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\int \frac{ax\arccos(ax)^2}{\sqrt{1-a^2x^2}}d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}$$

$\downarrow$ 3042

$$\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\int -\arccos(ax)^2\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}$$

$\downarrow$ 25

$$\frac{3\sqrt{1-a^2x^2}\int \arccos(ax)^2\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}$$

$\downarrow$ 4200

$$\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(2i\int -\frac{e^{2i\arccos(ax)}\arccos(ax)^2}{1-e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{3}i\arccos(ax)^3\right)}{ac\sqrt{c-a^2cx^2}}$$

$\downarrow$ 25

$$\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\int \frac{e^{2i\arccos(ax)}\arccos(ax)^2}{1-e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{3}i\arccos(ax)^3\right)}{ac\sqrt{c-a^2cx^2}}$$

$\downarrow$ 2620

$$\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\int \arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right)d\arccos(ax)\right)-\frac{1}{3}i\,\mathrm{a}}{ac\sqrt{c-a^2cx^2}}$$

$\downarrow$ 3011

$$\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\,\mathrm{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{2}i\int \mathrm{PolyLog}\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}$$

$\downarrow$ 2720

$$\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\,\mathrm{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\,\mathrm{arcco}}\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}$$

$\downarrow$ 7143

$$\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\,\mathrm{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\mathrm{PolyLog}\left(3\right.\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}$$

input
```
Int[ArcCos[a*x]^3/(c - a^2*c*x^2)^(3/2),x]
```

output
```
(x*ArcCos[a*x]^3)/(c*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2]*((-1/3*I)
*ArcCos[a*x]^3 - (2*I)*((I/2)*ArcCos[a*x]^2*Log[1 - E^((2*I)*ArcCos[a*x])]
 - I*((I/2)*ArcCos[a*x]*PolyLog[2, E^((2*I)*ArcCos[a*x])] - PolyLog[3, E^(
(2*I)*ArcCos[a*x])]/4))))/(a*c*Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.17

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\left(-i\sqrt{-a^2x^2+1}+ax\right)\arccos(ax)^3}{c^2a(a^2x^2-1)}-\frac{\sqrt{-a^2x^2+1}\,\sqrt{-c(a^2x^2-1)}\left(2i\arccos(ax)^3-3\arccos(ax)^2\ln\left(1+ax+i\sqrt{-\right.\right.}{}$

input
```
int(arccos(a*x)^3/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-c*(a^2*x^2-1))^(1/2)*(-I*(-a^2*x^2+1)^(1/2)+a*x)*arccos(a*x)^3/c^2/a/(a
^2*x^2-1)-(-a^2*x^2+1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*I*arccos(a*x)^3-3*a
rccos(a*x)^2*ln(1+a*x+I*(-a^2*x^2+1)^(1/2))-3*arccos(a*x)^2*ln(1-a*x-I*(-a
^2*x^2+1)^(1/2))+6*I*arccos(a*x)*polylog(2,-a*x-I*(-a^2*x^2+1)^(1/2))+6*I*
arccos(a*x)*polylog(2,a*x+I*(-a^2*x^2+1)^(1/2))-6*polylog(3,-a*x-I*(-a^2*x
^2+1)^(1/2))-6*polylog(3,a*x+I*(-a^2*x^2+1)^(1/2)))/c^2/a/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\arccos\left(ax\right)^3}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*arccos(a*x)^3/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 +
 c^2), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{acos}^3\left(ax\right)}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(acos(a*x)**3/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(acos(a*x)**3/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\arccos(ax)^3}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)^3/(-a^2*c*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\arccos(ax)^3}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^3/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{(c-a^2\,c\,x^2)^{3/2}}\,dx$$

input
```
int(acos(a*x)^3/(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(acos(a*x)^3/(c - a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{(c - a^2cx^2)^{3/2}}\,dx = -\frac{\int \frac{acos(ax)^3}{\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(acos(a*x)^3/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
( - int(acos(a*x)**3/(sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*x**2
 + 1)),x))/(sqrt(c)*c)
```

3.302 $\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 388

$$\begin{aligned}\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx &= \frac{x\arccos(ax)}{c^2\sqrt{c-a^2cx^2}}-\frac{\arccos(ax)^2}{2ac^2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}\\&\quad+\frac{x\arccos(ax)^3}{3c(c-a^2cx^2)^{3/2}}+\frac{2x\arccos(ax)^3}{3c^2\sqrt{c-a^2cx^2}}-\frac{2i\sqrt{1-a^2x^2}\arccos(ax)^3}{3ac^2\sqrt{c-a^2cx^2}}\\&\quad+\frac{2\sqrt{1-a^2x^2}\arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(1-a^2x^2)}{2ac^2\sqrt{c-a^2cx^2}}\\&\quad-\frac{2i\sqrt{1-a^2x^2}\arccos(ax)\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}}\\&\quad+\frac{\sqrt{1-a^2x^2}\operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}}\end{aligned}$$

output
```
x*arccos(a*x)/c^2/(-a^2*c*x^2+c)^(1/2)-1/2*arccos(a*x)^2/a/c^2/(-a^2*x^2+1
)^(1/2)/(-a^2*c*x^2+c)^(1/2)+1/3*x*arccos(a*x)^3/c/(-a^2*c*x^2+c)^(3/2)+2/
3*x*arccos(a*x)^3/c^2/(-a^2*c*x^2+c)^(1/2)-2/3*I*(-a^2*x^2+1)^(1/2)*arccos
(a*x)^3/a/c^2/(-a^2*c*x^2+c)^(1/2)+2*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2*ln(1
+(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c^2/(-a^2*c*x^2+c)^(1/2)+1/2*(-a^2*x^2+1)
^(1/2)*ln(-a^2*x^2+1)/a/c^2/(-a^2*c*x^2+c)^(1/2)-2*I*(-a^2*x^2+1)^(1/2)*ar
ccos(a*x)*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c^2/(-a^2*c*x^2+c)^(1
/2)+(-a^2*x^2+1)^(1/2)*polylog(3,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c^2/(-a^
2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.56

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{(1-a^2x^2)^{3/2}\left(-i\pi^3-\frac{12ax\arccos(ax)}{\sqrt{1-a^2x^2}}+\frac{6\arccos(ax)^2}{-1+a^2x^2}+8i\arccos(ax)^3-\frac{4ax\arccos(ax)^3}{(1-a^2x^2)^{3/2}}-\frac{8ax\arccos(ax)^3}{\sqrt{1-a^2x^2}}+24\,\mathrm{arc}\right.}{}$$

input
```
Integrate[ArcCos[a*x]^3/(c - a^2*c*x^2)^(5/2),x]
```

output
```
-1/12*((1 - a^2*x^2)^(3/2)*((-I)*Pi^3 - (12*a*x*ArcCos[a*x])/Sqrt[1 - a^2*
x^2] + (6*ArcCos[a*x]^2)/(-1 + a^2*x^2) + (8*I)*ArcCos[a*x]^3 - (4*a*x*Arc
Cos[a*x]^3)/(1 - a^2*x^2)^(3/2) - (8*a*x*ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2]
+ 24*ArcCos[a*x]^2*Log[1 - E^((-2*I)*ArcCos[a*x])] + 6*Log[1 - a^2*x^2] +
(24*I)*ArcCos[a*x]*PolyLog[2, E^((-2*I)*ArcCos[a*x])] + 12*PolyLog[3, E^((
-2*I)*ArcCos[a*x])]))/(a*c*(c - a^2*c*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.75, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5163, 5161, 5181, 3042, 25, 4200, 25, 2620, 3011, 2720, 5183, 5161, 240, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{5163}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{3/2}}dx}{3c}+\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{5161}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{1-a^2x^2}dx}{c\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{5181}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\int \frac{ax\arccos(ax)^2}{\sqrt{1-a^2x^2}}d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\int -\arccos(ax)^2\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{3\sqrt{1-a^2x^2}\int \arccos(ax)^2\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c\,(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 4200}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(2i\int -\frac{e^{2i\arccos(ax)}\arccos(ax)^2}{1-e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{3}i\arccos(ax)^3\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c\,(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\int \frac{e^{2i\arccos(ax)}\arccos(ax)^2}{1-e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{3}i\arccos(ax)^3\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c\,(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\int \arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right)d\arccos(ax)\right)-\frac{1}{3}i\arccos(ax)^3\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c\,(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+$$

$$\frac{2\Big(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}(-2i(\frac{1}{2}i\arccos(ax)^2\log(1-e^{2i\arccos(ax)})-i(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}(2,e^{2i\arccos(ax)})-\frac{1}{2}i\int \operatorname{PolyLog}(2,e^{2i\arccos}}{ac\sqrt{c-a^2cx^2}}}{3c}$$

$$\frac{x\arccos(ax)^3}{3c\,(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^2}dx}{c^2\sqrt{c-a^2cx^2}}+$$

$$\frac{2\Big(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}(-2i(\frac{1}{2}i\arccos(ax)^2\log(1-e^{2i\arccos(ax)})-i(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}(2,e^{2i\arccos(ax)})-\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyL}}{ac\sqrt{c-a^2cx^2}}}{3c}$$

$$\frac{x\arccos(ax)^3}{3c\,(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 5183}$$

$$\frac{a\sqrt{1-a^2x^2}\left(\frac{\int \frac{\arccos(ax)}{(1-a^2x^2)^{3/2}}dx}{a}+\frac{\arccos(ax)^2}{2a^2(1-a^2x^2)}\right)}{c^2\sqrt{c-a^2cx^2}}+$$

$$\frac{2\Big(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}(-2i(\frac{1}{2}i\arccos(ax)^2\log(1-e^{2i\arccos(ax)})-i(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}(2,e^{2i\arccos(ax)})-\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyL}}{ac\sqrt{c-a^2cx^2}}}{3c}$$

$$\frac{x\arccos(ax)^3}{3c\,(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 5161}$$

$$\frac{a\sqrt{1-a^2x^2}\left(\frac{a\int \frac{x}{1-a^2x^2}dx+\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}}{a}+\frac{\arccos(ax)^2}{2a^2(1-a^2x^2)}\right)}{c^2\sqrt{c-a^2cx^2}}+$$

$$\frac{2\Big(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}(-2i(\frac{1}{2}i\arccos(ax)^2\log(1-e^{2i\arccos(ax)})-i(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}(2,e^{2i\arccos(ax)})-\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyL}}{ac\sqrt{c-a^2cx^2}}}{3c}$$

$$\frac{x\arccos(ax)^3}{3c\,(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyL}\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}}{3c}$$

$$\frac{a\sqrt{1-a^2x^2}\left(\frac{\arccos(ax)^2}{2a^2(1-a^2x^2)}+\frac{\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}-\frac{\log\left(1-a^2x^2\right)}{2a}}{a}\right)}{c^2\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{7143}$$

$$\frac{a\sqrt{1-a^2x^2}\left(\frac{\arccos(ax)^2}{2a^2(1-a^2x^2)}+\frac{\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}-\frac{\log\left(1-a^2x^2\right)}{2a}}{a}\right)}{c^2\sqrt{c-a^2cx^2}}+$$

$$\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\operatorname{PolyLog}\left(3,e^{2i\arccos(a}\right.\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}}{3c}$$

$$\frac{x\arccos(ax)^3}{3c\left(c-a^2cx^2\right)^{3/2}}$$

input
```
Int[ArcCos[a*x]^3/(c - a^2*c*x^2)^(5/2),x]
```

output
```
(x*ArcCos[a*x]^3)/(3*c*(c - a^2*c*x^2)^(3/2)) + (a*Sqrt[1 - a^2*x^2]*(ArcC
os[a*x]^2/(2*a^2*(1 - a^2*x^2)) + ((x*ArcCos[a*x])/Sqrt[1 - a^2*x^2] - Log
[1 - a^2*x^2]/(2*a))/a))/(c^2*Sqrt[c - a^2*c*x^2]) + (2*((x*ArcCos[a*x]^3)
/(c*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2]*((-1/3*I)*ArcCos[a*x]^3 -
(2*I)*((I/2)*ArcCos[a*x]^2*Log[1 - E^((2*I)*ArcCos[a*x])] - I*((I/2)*ArcCo
s[a*x]*PolyLog[2, E^((2*I)*ArcCos[a*x])] - PolyLog[3, E^((2*I)*ArcCos[a*x]
)]/4))))/(a*c*Sqrt[c - a^2*c*x^2])))/(3*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.73

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\left(-2i\sqrt{-a^2x^2+1}\,a^2x^2+2a^3x^3+2i\sqrt{-a^2x^2+1}-3ax\right)\arccos(ax)\left(-6i\arccos(ax)a^4x^4+6\sqrt{-a^2x^2+1}\,\arccos(ax)a\right.}{}$

input
```
int(arccos(a*x)^3/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-c*(a^2*x^2-1))^(1/2)*(-2*I*(-a^2*x^2+1)^(1/2)*a^2*x^2+2*a^3*x^3+2*I
*(-a^2*x^2+1)^(1/2)-3*a*x)*arccos(a*x)*(-6*I*arccos(a*x)*a^4*x^4+6*(-a^2*x
^2+1)^(1/2)*arccos(a*x)*a^3*x^3-6*I*(-a^2*x^2+1)^(1/2)*a^3*x^3-6*a^4*x^4+6
*a^2*x^2*arccos(a*x)^2+12*I*arccos(a*x)*a^2*x^2-9*arccos(a*x)*(-a^2*x^2+1)
^(1/2)*a*x+6*I*(-a^2*x^2+1)^(1/2)*a*x+18*a^2*x^2-8*arccos(a*x)^2-6*I*arcco
s(a*x)-12)/c^3/(3*a^6*x^6-10*a^4*x^4+11*a^2*x^2-4)/a+(-c*(a^2*x^2-1))^(1/2
)*(-a^2*x^2+1)^(1/2)/a/c^3/(a^2*x^2-1)*ln(1+a*x+I*(-a^2*x^2+1)^(1/2))-2*(-
c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a/c^3/(a^2*x^2-1)*ln(a*x+I*(-a^2*x
^2+1)^(1/2))+(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a/c^3/(a^2*x^2-1)*l
n(I*(-a^2*x^2+1)^(1/2)+a*x-1)-2/3*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2
)*(2*I*arccos(a*x)^3-3*arccos(a*x)^2*ln(1+a*x+I*(-a^2*x^2+1)^(1/2))-3*arcc
os(a*x)^2*ln(1-a*x-I*(-a^2*x^2+1)^(1/2))+6*I*arccos(a*x)*polylog(2,-a*x-I*
(-a^2*x^2+1)^(1/2))+6*I*arccos(a*x)*polylog(2,a*x+I*(-a^2*x^2+1)^(1/2))-6*
polylog(3,-a*x-I*(-a^2*x^2+1)^(1/2))-6*polylog(3,a*x+I*(-a^2*x^2+1)^(1/2))
)/a/c^3/(a^2*x^2-1)
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\arccos(ax)^3}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*c*x^2 + c)*arccos(a*x)^3/(a^6*c^3*x^6 - 3*a^4*c^3*x^4
+ 3*a^2*c^3*x^2 - c^3), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{acos}^3(ax)}{(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(acos(a*x)**3/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(acos(a*x)**3/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\arccos\left(ax\right)^3}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)^3/(-a^2*c*x^2 + c)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{(c-a^2\,c\,x^2)^{5/2}}\,dx$$

input
```
int(acos(a*x)^3/(c - a^2*c*x^2)^(5/2),x)
```

output
```
int(acos(a*x)^3/(c - a^2*c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{acos(ax)^3}{\sqrt{-a^2x^2+1}\,a^4x^4-2\sqrt{-a^2x^2+1}\,a^2x^2+\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(acos(a*x)^3/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
int(acos(a*x)**3/(sqrt( - a**2*x**2 + 1)*a**4*x**4 - 2*sqrt( - a**2*x**2 +
 1)*a**2*x**2 + sqrt( - a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

3.303 $\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 547

$$
\begin{aligned}
\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{7/2}}\,dx = & -\frac{1}{20ac^3\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)}{c^3\sqrt{c-a^2cx^2}} \\
& +\frac{x\arccos(ax)}{10c^3(1-a^2x^2)\sqrt{c-a^2cx^2}}-\frac{3\arccos(ax)^2}{20ac^3(1-a^2x^2)^{3/2}\sqrt{c-a^2cx^2}} \\
& -\frac{2\arccos(ax)^2}{5ac^3\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^3}{5c(c-a^2cx^2)^{5/2}} \\
& +\frac{4x\arccos(ax)^3}{15c^2(c-a^2cx^2)^{3/2}}+\frac{8x\arccos(ax)^3}{15c^3\sqrt{c-a^2cx^2}}-\frac{8i\sqrt{1-a^2x^2}\arccos(ax)^3}{15ac^3\sqrt{c-a^2cx^2}} \\
& +\frac{8\sqrt{1-a^2x^2}\arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right)}{5ac^3\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-a^2x^2}\log(1-a^2x^2)}{2ac^3\sqrt{c-a^2cx^2}} \\
& -\frac{8i\sqrt{1-a^2x^2}\arccos(ax)\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)}{5ac^3\sqrt{c-a^2cx^2}} \\
& +\frac{4\sqrt{1-a^2x^2}\operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)}{5ac^3\sqrt{c-a^2cx^2}}
\end{aligned}
$$

output
```
-1/20/a/c^3/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2)+x*arccos(a*x)/c^3/(-a^
2*c*x^2+c)^(1/2)+1/10*x*arccos(a*x)/c^3/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2)-
3/20*arccos(a*x)^2/a/c^3/(-a^2*x^2+1)^(3/2)/(-a^2*c*x^2+c)^(1/2)-2/5*arcco
s(a*x)^2/a/c^3/(-a^2*x^2+1)^(1/2)/(-a^2*c*x^2+c)^(1/2)+1/5*x*arccos(a*x)^3
/c/(-a^2*c*x^2+c)^(5/2)+4/15*x*arccos(a*x)^3/c^2/(-a^2*c*x^2+c)^(3/2)+8/15
*x*arccos(a*x)^3/c^3/(-a^2*c*x^2+c)^(1/2)-8/15*I*(-a^2*x^2+1)^(1/2)*arccos
(a*x)^3/a/c^3/(-a^2*c*x^2+c)^(1/2)+8/5*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2*ln
(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c^3/(-a^2*c*x^2+c)^(1/2)+1/2*(-a^2*x^2+
1)^(1/2)*ln(-a^2*x^2+1)/a/c^3/(-a^2*c*x^2+c)^(1/2)-8/5*I*(-a^2*x^2+1)^(1/2
)*arccos(a*x)*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/c^3/(-a^2*c*x^2+c
)^(1/2)+4/5*(-a^2*x^2+1)^(1/2)*polylog(3,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)/a/
c^3/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.54

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{7/2}}\,dx =$$
$$-\frac{\sqrt{1-a^2x^2}\left(-4i\pi^3+\frac{3}{-1+a^2x^2}-\frac{6ax\arccos(ax)}{(1-a^2x^2)^{3/2}}-\frac{60ax\arccos(ax)}{\sqrt{1-a^2x^2}}-\frac{9\arccos(ax)^2}{(-1+a^2x^2)^2}+\frac{24\arccos(ax)^2}{-1+a^2x^2}+32i\arccos(ax)^3\right.}{}$$

input
```
Integrate[ArcCos[a*x]^3/(c - a^2*c*x^2)^(7/2),x]
```

output
```
-1/60*(Sqrt[1 - a^2*x^2]*((-4*I)*Pi^3 + 3/(-1 + a^2*x^2) - (6*a*x*ArcCos[a
*x])/(1 - a^2*x^2)^(3/2) - (60*a*x*ArcCos[a*x])/Sqrt[1 - a^2*x^2] - (9*Arc
Cos[a*x]^2)/(-1 + a^2*x^2)^2 + (24*ArcCos[a*x]^2)/(-1 + a^2*x^2) + (32*I)*
ArcCos[a*x]^3 - (12*a*x*ArcCos[a*x]^3)/(1 - a^2*x^2)^(5/2) - (16*a*x*ArcCo
s[a*x]^3)/(1 - a^2*x^2)^(3/2) - (32*a*x*ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2] +
 96*ArcCos[a*x]^2*Log[1 - E^((-2*I)*ArcCos[a*x])] + 30*Log[1 - a^2*x^2] +
(96*I)*ArcCos[a*x]*PolyLog[2, E^((-2*I)*ArcCos[a*x])] + 48*PolyLog[3, E^((
-2*I)*ArcCos[a*x])]))/(a*c^3*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 3.07 (sec) , antiderivative size = 485, normalized size of antiderivative = 0.89, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.864, Rules used = {5163, 5163, 5161, 5181, 3042, 25, 4200, 25, 2620, 3011, 2720, 5183, 5161, 240, 5163, 241, 5161, 240, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{7/2}}dx$$

$$\Big\downarrow \ 5163$$

$$\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{5/2}}dx}{5c}+\frac{x\arccos(ax)^3}{5c\,(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \ 5163$$

$$\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\int \frac{\arccos(ax)^3}{\left(c-a^2cx^2\right)^{3/2}}dx}{3c}+\frac{x\arccos(ax)^3}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{x\arccos(ax)^3}{5c\,(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \ 5161$$

$$\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{1-a^2x^2}dx}{c\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{x\arccos(ax)^3}{5c\,(c-a^2cx^2)^{5/2}}$$

$$\Big\downarrow \ 5181$$

$$\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\int \frac{ax\arccos(ax)^2}{\sqrt{1-a^2x^2}}d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\int -\arccos(ax)^2\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{3\sqrt{1-a^2x^2}\int \arccos(ax)^2\tan\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{ac\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)^3}{3c(c-a^2cx^2)^{3/2}}\right)}{5c}+\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 4200}$$

$$\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+$$

$$\frac{4\left(\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(2i\int -\frac{e^{2i\arccos(ax)}\arccos(ax)^2}{1-e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{3}i\arccos(ax)^3\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)}{3c(c-a^2cx^2)^3}\right.}{5c}$$

$$\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+$$

$$\frac{4\left(\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\int \frac{e^{2i\arccos(ax)}\arccos(ax)^2}{1-e^{2i\arccos(ax)}}d\arccos(ax)-\frac{1}{3}i\arccos(ax)^3\right)}{ac\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\arccos(ax)}{3c(c-a^2cx^2)^3}\right.}{5c}$$

$$\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 2620$$

$$\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+$$

$$\frac{4\left(\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\int \arccos(ax)\log\left(1-e^{2i\arccos(ax)}\right)d\,\text{arcc}\right.\right.}{ac\sqrt{c-a^2cx^2}}\right.}{3c}\right.}{5c}$$

$$\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 3011$$

$$\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}}{3c}\right.}{5c}$$

$$\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$\downarrow$ 2720

$$\frac{3a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{(1-a^2x^2)^3}dx}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^2}{\left(1-a^2x^2\right)^2}dx}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}}{3c}\right.}{5c}$$

$$\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$\downarrow$ 5183

$$\frac{3a\sqrt{1-a^2x^2}\left(\frac{\int \frac{\arccos(ax)}{\left(1-a^2x^2\right)^{5/2}}dx}{2a}+\frac{\arccos(ax)^2}{4a^2(1-a^2x^2)^2}\right)}{5c^3\sqrt{c-a^2cx^2}}+\frac{4\left(\frac{a\sqrt{1-a^2x^2}\left(\frac{\int \frac{\arccos(ax)}{\left(1-a^2x^2\right)^{3/2}}dx}{a}+\frac{\arccos(ax)^2}{2a^2\left(1-a^2x^2\right)}\right)}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\operatorname{Poly}\right.\right.\right.}{}\right.}{}\right.}{5c}$$

$$\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$\downarrow$ 5161

$$\frac{3a\sqrt{1-a^2x^2}\left(\frac{\int \frac{\arccos(ax)}{\left(1-a^2x^2\right)^{5/2}}dx}{2a}+\frac{\arccos(ax)^2}{4a^2(1-a^2x^2)^2}\right)}{5c^3\sqrt{c-a^2cx^2}}+$$

$$\frac{4\left(\frac{a\sqrt{1-a^2x^2}\left(\frac{a\int \frac{x}{1-a^2x^2}dx+\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}}{a}+\frac{\arccos(ax)^2}{2a^2\left(1-a^2x^2\right)}\right)}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos\right.\right.\right.}{}\right.}{}\right.}{5c}$$

$$\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \ 240$$

$$\frac{3a\sqrt{1-a^2x^2}\left(\frac{\int \frac{\arccos(ax)}{\left(1-a^2x^2\right)^{5/2}}dx}{2a}+\frac{\arccos(ax)^2}{4a^2(1-a^2x^2)^2}\right)}{5c^3\sqrt{c-a^2cx^2}}+$$

$$\frac{4\left(\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyLog}\left(2,e^{2i}\right.\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}\right.}{3c}\right.}{5c}$$

$$\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \ 5163$$

$$\frac{3a\sqrt{1-a^2x^2}\left(\frac{\frac{2}{3}\int \frac{\arccos(ax)}{\left(1-a^2x^2\right)^{3/2}}dx+\frac{1}{3}a\int \frac{x}{\left(1-a^2x^2\right)^2}dx+\frac{x\arccos(ax)}{3\left(1-a^2x^2\right)^{3/2}}}{2a}+\frac{\arccos(ax)^2}{4a^2(1-a^2x^2)^2}\right)}{5c^3\sqrt{c-a^2cx^2}}+$$

$$\frac{4\left(\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyLog}\left(2,e^{2i}\right.\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}\right.}{3c}\right.}{5c}$$

$$\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$$\Big\downarrow \ 241$$

$$\frac{3a\sqrt{1-a^2x^2}\left(\frac{\frac{2}{3}\int\frac{\arccos(ax)}{\left(1-a^2x^2\right)^{3/2}}dx+\frac{x\arccos(ax)}{3\left(1-a^2x^2\right)^{3/2}}+\frac{1}{6a\left(1-a^2x^2\right)}}{2a}+\frac{\arccos(ax)^2}{4a^2(1-a^2x^2)^2}\right)}{5c^3\sqrt{c-a^2cx^2}}+$$

$$\frac{4\left(\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyLog}\left(2,e^{2i\right.\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}}{3c}\right.}{5c}$$

$$\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$\downarrow$ 5161

$$\frac{3a\sqrt{1-a^2x^2}\left(\frac{\frac{2}{3}\left(a\int\frac{x}{1-a^2x^2}dx+\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}\right)+\frac{x\arccos(ax)}{3\left(1-a^2x^2\right)^{3/2}}+\frac{1}{6a\left(1-a^2x^2\right)}}{2a}+\frac{\arccos(ax)^2}{4a^2(1-a^2x^2)^2}\right)}{5c^3\sqrt{c-a^2cx^2}}+$$

$$\frac{4\left(\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyLog}\left(2,e^{2i\right.\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}}{3c}\right.}{5c}$$

$$\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$\downarrow$ 240

$$4\left(\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyLog}\left(2,e^{2i\cdots}\right.\right.\right.\right.}{ac\sqrt{c-a^2cx^2}}}{3c}\right.$$

$$\overline{\qquad\qquad 5c \qquad\qquad}$$

$$\frac{3a\sqrt{1-a^2x^2}\left(\frac{\arccos(ax)^2}{4a^2(1-a^2x^2)^2}+\frac{\frac{x\arccos(ax)}{3\left(1-a^2x^2\right)^{3/2}}+\frac{2}{3}\left(\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}-\frac{\log\left(1-a^2x^2\right)}{2a}\right)+\frac{1}{6a\left(1-a^2x^2\right)}}{2a}\right)}{5c^3\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

$\big\downarrow$ 7143

$$\frac{3a\sqrt{1-a^2x^2}\left(\frac{\arccos(ax)^2}{4a^2(1-a^2x^2)^2}+\frac{\frac{x\arccos(ax)}{3\left(1-a^2x^2\right)^{3/2}}+\frac{2}{3}\left(\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}-\frac{\log\left(1-a^2x^2\right)}{2a}\right)+\frac{1}{6a\left(1-a^2x^2\right)}}{2a}\right)}{5c^3\sqrt{c-a^2cx^2}}+$$

$$\frac{4\left(\frac{a\sqrt{1-a^2x^2}\left(\frac{\arccos(ax)^2}{2a^2\left(1-a^2x^2\right)}+\frac{\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}-\frac{\log\left(1-a^2x^2\right)}{2a}}{a}\right)}{c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{x\arccos(ax)^3}{c\sqrt{c-a^2cx^2}}-\frac{3\sqrt{1-a^2x^2}\left(-2i\left(\frac{1}{2}i\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right)-i\left(\frac{1}{2}i\arccos\cdots\right.\right.\right.}{\cdots}\right.}{3c\cdots}\right.}{5c}$$

$$\frac{x\arccos(ax)^3}{5c\left(c-a^2cx^2\right)^{5/2}}$$

input
```
Int[ArcCos[a*x]^3/(c - a^2*c*x^2)^(7/2),x]
```

output
```
(x*ArcCos[a*x]^3)/(5*c*(c - a^2*c*x^2)^(5/2)) + (3*a*Sqrt[1 - a^2*x^2]*(Ar
cCos[a*x]^2/(4*a^2*(1 - a^2*x^2)^2) + (1/(6*a*(1 - a^2*x^2)) + (x*ArcCos[a
*x])/(3*(1 - a^2*x^2)^(3/2)) + (2*((x*ArcCos[a*x])/Sqrt[1 - a^2*x^2] - Log
[1 - a^2*x^2]/(2*a)))/3)/(2*a)))/(5*c^3*Sqrt[c - a^2*c*x^2]) + (4*((x*ArcC
os[a*x]^3)/(3*c*(c - a^2*c*x^2)^(3/2)) + (a*Sqrt[1 - a^2*x^2]*(ArcCos[a*x]
^2/(2*a^2*(1 - a^2*x^2)) + ((x*ArcCos[a*x])/Sqrt[1 - a^2*x^2] - Log[1 - a^
2*x^2]/(2*a))/a))/(c^2*Sqrt[c - a^2*c*x^2]) + (2*((x*ArcCos[a*x]^3)/(c*Sqr
t[c - a^2*c*x^2]) - (3*Sqrt[1 - a^2*x^2]*((-1/3*I)*ArcCos[a*x]^3 - (2*I)*(
(I/2)*ArcCos[a*x]^2*Log[1 - E^((2*I)*ArcCos[a*x])] - I*((I/2)*ArcCos[a*x]*
PolyLog[2, E^((2*I)*ArcCos[a*x])] - PolyLog[3, E^((2*I)*ArcCos[a*x])]/4)))
)/(a*c*Sqrt[c - a^2*c*x^2])))/(3*c)))/(5*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(512) = 1024$.

Time = 0.87 (sec) , antiderivative size = 1029, normalized size of antiderivative = 1.88

method	result
default	$-\frac{\sqrt{-c(a^2x^2-1)}\left(-8i\sqrt{-a^2x^2+1}\,a^4x^4+8a^5x^5+16i\sqrt{-a^2x^2+1}\,a^2x^2-20a^3x^3-8i\sqrt{-a^2x^2+1}+15ax\right)\left(-1410a^2x^2\arccos(ax)+1\right.}{}$

input
```
int(arccos(a*x)^3/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*(-c*(a^2*x^2-1))^(1/2)*(-8*I*(-a^2*x^2+1)^(1/2)*a^4*x^4+8*a^5*x^5+16
*I*(-a^2*x^2+1)^(1/2)*a^2*x^2-20*a^3*x^3-8*I*(-a^2*x^2+1)^(1/2)+15*a*x)*(-
1410*a^2*x^2*arccos(a*x)+1590*a^4*x^4*arccos(a*x)-105*a^3*x^3*(-a^2*x^2+1)
^(1/2)+160*a^4*x^4*arccos(a*x)^3+84*(-a^2*x^2+1)^(1/2)*a^5*x^5+45*(-a^2*x^
2+1)^(1/2)*a*x+480*arccos(a*x)-1020*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)*a^3*x
^3+495*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)*a*x+256*arccos(a*x)^3+24*I-192*arc
cos(a*x)^2*(-a^2*x^2+1)^(1/2)*a^7*x^7+744*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)
*a^5*x^5+192*I*arccos(a*x)^2*a^8*x^8-840*I*arccos(a*x)^2*a^6*x^6+1368*I*ar
ccos(a*x)^2*a^4*x^4-984*I*arccos(a*x)^2*a^2*x^2+264*I*arccos(a*x)^2+192*I*
arccos(a*x)*(-a^2*x^2+1)^(1/2)*a^7*x^7-756*I*arccos(a*x)*(-a^2*x^2+1)^(1/2
)*a^5*x^5+936*I*arccos(a*x)*(-a^2*x^2+1)^(1/2)*a^3*x^3-372*I*arccos(a*x)*(
-a^2*x^2+1)^(1/2)*a*x-24*(-a^2*x^2+1)^(1/2)*a^7*x^7-96*I*x^2*a^2+144*I*a^4
*x^4+24*I*a^8*x^8-96*I*a^6*x^6+192*arccos(a*x)*a^8*x^8-852*arccos(a*x)*a^6
*x^6-380*arccos(a*x)^3*a^2*x^2)/c^4/(40*a^10*x^10-215*a^8*x^8+469*a^6*x^6-
517*a^4*x^4+287*a^2*x^2-64)/a+(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/c^
4/a/(a^2*x^2-1)*ln(1+a*x+I*(-a^2*x^2+1)^(1/2))-2*(-c*(a^2*x^2-1))^(1/2)*(-
a^2*x^2+1)^(1/2)/c^4/a/(a^2*x^2-1)*ln(a*x+I*(-a^2*x^2+1)^(1/2))+(-c*(a^2*x
^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/c^4/a/(a^2*x^2-1)*ln(I*(-a^2*x^2+1)^(1/2)+
a*x-1)-8/15*(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)*(2*I*arccos(a*x)^3-3
*arccos(a*x)^2*ln(1+a*x+I*(-a^2*x^2+1)^(1/2))-3*arccos(a*x)^2*ln(1-a*x-...
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\arccos\left(ax\right)^3}{(-a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a^2*c*x^2 + c)*arccos(a*x)^3/(a^8*c^4*x^8 - 4*a^6*c^4*x^6 +
 6*a^4*c^4*x^4 - 4*a^2*c^4*x^2 + c^4), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\operatorname{acos}^3(ax)}{(-c(ax-1)(ax+1))^{\frac{7}{2}}}\,dx$$

input
```
integrate(acos(a*x)**3/(-a**2*c*x**2+c)**(7/2),x)
```

output
```
Integral(acos(a*x)**3/(-c*(a*x - 1)*(a*x + 1))**(7/2), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\arccos(ax)^3}{(-a^2cx^2+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)^3/(-a^2*c*x^2 + c)^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^3/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{7/2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{(c-a^2\,c\,x^2)^{7/2}}\,dx$$

input
```
int(acos(a*x)^3/(c - a^2*c*x^2)^(7/2),x)
```

output
```
int(acos(a*x)^3/(c - a^2*c*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{\int \frac{acos(ax)^3}{\sqrt{-a^2x^2+1}\,a^6x^6-3\sqrt{-a^2x^2+1}\,a^4x^4+3\sqrt{-a^2x^2+1}\,a^2x^2-\sqrt{-a^2x^2+1}}dx}{\sqrt{c}\,c^3}$$

input
```
int(acos(a*x)^3/(-a^2*c*x^2+c)^(7/2),x)
```

output
```
( - int(acos(a*x)**3/(sqrt( - a**2*x**2 + 1)*a**6*x**6 - 3*sqrt( - a**2*x*
*2 + 1)*a**4*x**4 + 3*sqrt( - a**2*x**2 + 1)*a**2*x**2 - sqrt( - a**2*x**2
 + 1)),x))/(sqrt(c)*c**3)
```

3.304 $\int \frac{x^m \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \operatorname{Int}\left(\frac{x^m \arccos(ax)^3}{\sqrt{1-a^2x^2}},x\right)$$

output
```
Defer(Int)(x^m*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$

input
```
Integrate[(x^m*ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
Integrate[(x^m*ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arccos(ax)^3}{\sqrt{1-a^2x^2}} \, dx$$

$$\Big\downarrow \text{ 5235}$$

$$\int \frac{x^m \arccos(ax)^3}{\sqrt{1-a^2x^2}}dx$$

input
```
Int[(x^m*ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arccos\left(ax\right)^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^m*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(x^m*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{x^m \arccos(ax)^3}{\sqrt{1-a^2x^2}} \, dx = \int \frac{x^m \arccos{(ax)}^3}{\sqrt{-a^2x^2+1}} \, dx$$

input
```
integrate(x^m*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^m*arccos(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arccos(ax)^3}{\sqrt{1-a^2x^2}} \, dx = \int \frac{x^m \operatorname{acos}^3{(ax)}}{\sqrt{-(ax-1)(ax+1)}} \, dx$$

input
```
integrate(x**m*acos(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**m*acos(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arccos(ax)^3}{\sqrt{1-a^2x^2}} \, dx = \int \frac{x^m \arccos{(ax)}^3}{\sqrt{-a^2x^2+1}} \, dx$$

input
```
integrate(x^m*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arccos(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m \arccos{(ax)}^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*arccos(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m\,\mathrm{acos}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^m*acos(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^m*acos(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^m \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m acos(ax)^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^m*acos(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((x**m*acos(a*x)**3)/sqrt( - a**2*x**2 + 1),x)
```

3.305 $\int \frac{x^4 \arccos(ax)^3}{\sqrt{1-a^2x^2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 191

$$\int \frac{x^4 \arccos(ax)^3}{\sqrt{1-a^2x^2}} dx = -\frac{45x^2}{128a^3} - \frac{3x^4}{128a} + \frac{45x\sqrt{1-a^2x^2}\arccos(ax)}{64a^4} + \frac{3x^3\sqrt{1-a^2x^2}\arccos(ax)}{32a^2} - \frac{45\arccos(ax)^2}{128a^5} + \frac{9x^2\arccos(ax)^2}{16a^3} + \frac{3x^4\arccos(ax)^2}{16a} - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)^3}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2} + \frac{3\arccos(ax)^4}{32a^5}$$

output
```
-45/128*x^2/a^3-3/128*x^4/a+45/64*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^4+3/3
2*x^3*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^2-45/128*arccos(a*x)^2/a^5+9/16*x^2
*arccos(a*x)^2/a^3+3/16*x^4*arccos(a*x)^2/a-3/8*x*(-a^2*x^2+1)^(1/2)*arcco
s(a*x)^3/a^4-1/4*x^3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/a^2+3/32*arccos(a*x)
^4/a^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.65

$$\int \frac{x^4 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{3a^2x^2(15+a^2x^2)+6ax\sqrt{1-a^2x^2}(15+2a^2x^2)\arccos(ax)-3(-15+24a^2x^2+8a^4x^4)\arccos(ax)^2-1\ldots}{128a^5}$$

input
```
Integrate[(x^4*ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
(3*a^2*x^2*(15 + a^2*x^2) + 6*a*x*Sqrt[1 - a^2*x^2]*(15 + 2*a^2*x^2)*ArcCo
s[a*x] - 3*(-15 + 24*a^2*x^2 + 8*a^4*x^4)*ArcCos[a*x]^2 - 16*a*x*Sqrt[1 -
a^2*x^2]*(3 + 2*a^2*x^2)*ArcCos[a*x]^3 - 12*ArcCos[a*x]^4)/(128*a^5)
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.44, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5211, 5139, 5211, 15, 5139, 5153, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$
$$\Big\downarrow \text{5211}$$
$$\frac{3\int \frac{x^2\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{3\int x^3\arccos(ax)^2dx}{4a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}$$
$$\Big\downarrow \text{5139}$$
$$\frac{3\int \frac{x^2\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{3\left(\frac{1}{2}a\int \frac{x^4\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{4}x^4\arccos(ax)^2\right)}{4a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}$$
$$\Big\downarrow \text{5211}$$

$$\frac{3\left(\frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{3\int x\arccos(ax)^2dx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{2}a\left(\frac{3\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{\int x^3dx}{4a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}\right)+\frac{1}{4}x^4\arccos(ax)^2\right)}{4a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{3\left(\frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{3\int x\arccos(ax)^2dx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{2}a\left(\frac{3\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}\right)+\frac{1}{4}x^4\arccos(ax)^2\right)}{4a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}$$

$$\Big\downarrow \text{5139}$$

$$\frac{3\left(-\frac{3\left(a\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2a}+\frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}\right)}{4a^2}-\frac{3\left(\frac{1}{2}a\left(\frac{3\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}\right)+\frac{1}{4}x^4\arccos(ax)^2\right)}{4a}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}$$

$$\Big\downarrow \text{5153}$$

$$-\frac{3\left(\frac{1}{2}a\left(\frac{3\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}\right)+\frac{1}{4}x^4\arccos(ax)^2\right)}{4a}+\frac{3\left(-\frac{3\left(a\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2a}-\frac{\arccos(ax)^4}{8a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}$$

$$\Big\downarrow \text{5211}$$

$$-\frac{3\left(\frac{1}{2}a\left(\frac{3\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int xdx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}\right)+\frac{1}{4}x^4\arccos(ax)^2\right)}{4a}+\frac{3\left(-\frac{3\left(a\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int xdx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2a}-\frac{\arccos(ax)^4}{8a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}$$

$$\Big\downarrow \; 15$$

$$-\frac{3\left(\frac{1}{2}a\left(\frac{3\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}-\frac{x^4}{16a}\right)+\frac{1}{4}x^4\arccos(ax)^2\right)}{4a}+\frac{3\left(-\frac{3\left(a\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2a}-\frac{\arccos(ax)^4}{8a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}\right)}{4a^2}-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}$$

$$\Big\downarrow \; 5153$$

$$-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a^2}+\frac{3\left(-\frac{\arccos(ax)^4}{8a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}-\frac{3\left(a\left(-\frac{\arccos(ax)^2}{4a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2a}\right)}{4a^2}-\frac{3\left(\frac{1}{2}a\left(-\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{4a^2}+\frac{3\left(-\frac{\arccos(ax)^2}{4a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)}{4a^2}-\frac{x^4}{16a}\right)+\frac{1}{4}x^4\arccos(ax)^2\right)}{4a}$$

input

```
Int[(x^4*ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/4*(x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a^2 - (3*((x^4*ArcCos[a*x]^2)/4
 + (a*(-1/16*x^4/a - (x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(4*a^2) + (3*(-1/
4*x^2/a - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a^2) - ArcCos[a*x]^2/(4*a^3
)))/(4*a^2)))/2))/(4*a) + (3*(-1/2*(x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a^2
 - ArcCos[a*x]^4/(8*a^3) - (3*((x^2*ArcCos[a*x]^2)/2 + a*(-1/4*x^2/a - (x*
Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a^2) - ArcCos[a*x]^2/(4*a^3))))/(2*a)))/
(4*a^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.84

method	result
default	$-\frac{128 \arccos(ax)^3 \sqrt{-a^2x^2+1}\, a^3x^3+96a^4x^4 \arccos(ax)^2-48\sqrt{-a^2x^2+1}\, \arccos(ax)a^3x^3-12a^4x^4+192 \arccos(ax)^3 \sqrt{-a^2x^2+1}\, ax}{512a^5}$

input
```
int(x^4*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/512*(128*arccos(a*x)^3*(-a^2*x^2+1)^(1/2)*a^3*x^3+96*a^4*x^4*arccos(a*x
)^2-48*(-a^2*x^2+1)^(1/2)*arccos(a*x)*a^3*x^3-12*a^4*x^4+192*arccos(a*x)^3
*(-a^2*x^2+1)^(1/2)*a*x+288*a^2*x^2*arccos(a*x)^2+48*arccos(a*x)^4-360*arc
cos(a*x)*(-a^2*x^2+1)^(1/2)*a*x-180*a^2*x^2-180*arccos(a*x)^2+117)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.58

$$\int \frac{x^4 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\, dx = \frac{3\,a^4x^4+45\,a^2x^2-12\,\arccos\left(ax\right)^4-3\left(8\,a^4x^4+24\,a^2x^2-15\right)\arccos\left(ax\right)^2-2\,\sqrt{-a^2x^2+1}\left(8\left(2\,a^3x^3+\right.\right.}{128\,a^5}$$

input
```
integrate(x^4*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/128*(3*a^4*x^4 + 45*a^2*x^2 - 12*arccos(a*x)^4 - 3*(8*a^4*x^4 + 24*a^2*x
^2 - 15)*arccos(a*x)^2 - 2*sqrt(-a^2*x^2 + 1)*(8*(2*a^3*x^3 + 3*a*x)*arcco
s(a*x)^3 - 3*(2*a^3*x^3 + 15*a*x)*arccos(a*x)))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.01

$$\int \frac{x^4 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$
$$= \begin{cases} -\frac{3x^4 \operatorname{acos}^2(ax)}{16a} + \frac{3x^4}{128a} - \frac{x^3\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{4a^2} + \frac{3x^3\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{32a^2} - \frac{9x^2\operatorname{acos}^2(ax)}{16a^3} + \frac{45x^2}{128a^3} - \frac{3x\sqrt{-a^2x^2+1}\operatorname{acos}^3}{8a^4} \\ \frac{\pi^3x^5}{40} \end{cases}$$

input
```
integrate(x**4*acos(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-3*x**4*acos(a*x)**2/(16*a) + 3*x**4/(128*a) - x**3*sqrt(-a**2*
x**2 + 1)*acos(a*x)**3/(4*a**2) + 3*x**3*sqrt(-a**2*x**2 + 1)*acos(a*x)/(3
2*a**2) - 9*x**2*acos(a*x)**2/(16*a**3) + 45*x**2/(128*a**3) - 3*x*sqrt(-a
**2*x**2 + 1)*acos(a*x)**3/(8*a**4) + 45*x*sqrt(-a**2*x**2 + 1)*acos(a*x)/
(64*a**4) - 3*acos(a*x)**4/(32*a**5) + 45*acos(a*x)**2/(128*a**5), Ne(a, 0
)), (pi**3*x**5/40, True))
```

Maxima [F]

$$\int \frac{x^4 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4 \arccos(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^4*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4*arccos(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.88

$$\int \frac{x^4 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{3\,x^4 \arccos\left(ax\right)^2}{16\,a} - \frac{\sqrt{-a^2x^2+1}x^3 \arccos\left(ax\right)^3}{4\,a^2} + \frac{3\,x^4}{128\,a} + \frac{3\,\sqrt{-a^2x^2+1}x^3 \arccos\left(ax\right)}{32\,a^2} - \frac{9\,x^2 \arccos\left(ax\right)^2}{16\,a^3} - \frac{3\,\sqrt{-a^2x^2+1}x \arccos\left(ax\right)^3}{8\,a^4} + \frac{45\,x^2}{128\,a^3} - \frac{3\,\arccos\left(ax\right)^4}{32\,a^5} + \frac{45\,\sqrt{-a^2x^2+1}x \arccos\left(ax\right)}{64\,a^4} + \frac{45\,\arccos\left(ax\right)^2}{128\,a^5} - \frac{189}{1024\,a^5}$$

input
```
integrate(x^4*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-3/16*x^4*arccos(a*x)^2/a - 1/4*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)^3/a^2 +
 3/128*x^4/a + 3/32*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)/a^2 - 9/16*x^2*arcc
os(a*x)^2/a^3 - 3/8*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)^3/a^4 + 45/128*x^2/a^
3 - 3/32*arccos(a*x)^4/a^5 + 45/64*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)/a^4 +
45/128*arccos(a*x)^2/a^5 - 189/1024/a^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^4\,\mathrm{acos}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^4*acos(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^4*acos(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)^3\,x^4}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^4*acos(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acos(a*x)**3*x**4)/sqrt( - a**2*x**2 + 1),x)
```

3.306 $\int \frac{x^3 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 157

$$\int \frac{x^3 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{40x}{9a^3}-\frac{2x^3}{27a}+\frac{40\sqrt{1-a^2x^2}\arccos(ax)}{9a^4}+\frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)}{9a^2}+\frac{2x\arccos(ax)^2}{a^3}+\frac{x^3\arccos(ax)^2}{3a}-\frac{2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^4}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}$$

output

```
-40/9*x/a^3-2/27*x^3/a+40/9*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^4+2/9*x^2*(-a
^2*x^2+1)^(1/2)*arccos(a*x)/a^2+2*x*arccos(a*x)^2/a^3+1/3*x^3*arccos(a*x)^
2/a-2/3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/a^4-1/3*x^2*(-a^2*x^2+1)^(1/2)*ar
ccos(a*x)^3/a^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.64

$$\int \frac{x^3 \arccos(ax)^3}{\sqrt{1-a^2x^2}} dx$$
$$= \frac{2ax(60+a^2x^2) + 6\sqrt{1-a^2x^2}(20+a^2x^2)\arccos(ax) - 9ax(6+a^2x^2)\arccos(ax)^2 - 9\sqrt{1-a^2x^2}(2+a^2}{27a^4}$$

input
```
Integrate[(x^3*ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
(2*a*x*(60 + a^2*x^2) + 6*Sqrt[1 - a^2*x^2]*(20 + a^2*x^2)*ArcCos[a*x] - 9
*a*x*(6 + a^2*x^2)*ArcCos[a*x]^2 - 9*Sqrt[1 - a^2*x^2]*(2 + a^2*x^2)*ArcCo
s[a*x]^3)/(27*a^4)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.34, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5211, 5139, 5183, 5131, 5183, 24, 5211, 15, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arccos(ax)^3}{\sqrt{1-a^2x^2}} dx$$

$$\Big\downarrow \text{ 5211}$$

$$\frac{2\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{3a^2} - \frac{\int x^2 \arccos(ax)^2 dx}{a} - \frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}$$

$$\Big\downarrow \text{ 5139}$$

$$\frac{2\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{3a^2} - \frac{\frac{2}{3}a\int \frac{x^3 \arccos(ax)}{\sqrt{1-a^2x^2}}dx + \frac{1}{3}x^3\arccos(ax)^2}{a} - \frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}$$

$$\Big\downarrow \text{ 5183}$$

$$\frac{2\left(-\frac{3\int \arccos(ax)^2dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}\right)}{3a^2}-\frac{\frac{2}{3}a\int \frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}$$

$$\Big\downarrow \text{ 5131}$$

$$\frac{2\left(-\frac{3\left(2a\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)^2\right)}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}\right)}{3a^2}-\frac{\frac{2}{3}a\int \frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}$$

$$\Big\downarrow \text{ 5183}$$

$$\frac{2\left(-\frac{3\left(2a\left(-\frac{\int 1dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}\right)+x\arccos(ax)^2\right)}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}\right)}{3a^2}-\frac{\frac{2}{3}a\int \frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{\frac{2}{3}a\int \frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}-\frac{3\left(2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}-\frac{x}{a}\right)+x\arccos(ax)^2\right)}{a}\right)}{3a^2}$$

$$\Big\downarrow \text{ 5211}$$

$$-\frac{\frac{2}{3}a\left(\frac{2\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{\int x^2dx}{3a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}\right)+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}-\frac{3\left(2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}-\frac{x}{a}\right)+x\arccos(ax)^2\right)}{a}\right)}{3a^2}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\frac{2}{3}a\left(\frac{2\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}-\frac{x^3}{9a}\right)+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}-\frac{3\left(2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}-\frac{x}{a}\right)+x\arccos(ax)^2\right)}{a}\right)}{3a^2}$$

$\downarrow$ 5183

$$-\frac{\frac{2}{3}a\left(\frac{2\left(-\frac{\int 1dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}\right)}{3a^2}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}-\frac{x^3}{9a}\right)+\frac{1}{3}x^3\arccos(ax)^2}{a}-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}-\frac{3\left(2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}-\frac{x}{a}\right)+x\arccos(ax)^2\right)}{a}\right)}{3a^2}$$

$\downarrow$ 24

$$-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}-\frac{3\left(2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}-\frac{x}{a}\right)+x\arccos(ax)^2\right)}{a}\right)}{3a^2}-\frac{\frac{2}{3}a\left(-\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)}{3a^2}+\frac{2\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}-\frac{x}{a}\right)}{3a^2}-\frac{x^3}{9a}\right)+\frac{1}{3}x^3\arccos(ax)^2}{a}$$

input
```
Int[(x^3*ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/3*(x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a^2 - ((x^3*ArcCos[a*x]^2)/3 +
(2*a*(-1/9*x^3/a - (x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(3*a^2) + (2*(-(x/a
) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a^2))/(3*a^2)))/3)/a + (2*(-((Sqrt[1 -
 a^2*x^2]*ArcCos[a*x]^3)/a^2) - (3*(x*ArcCos[a*x]^2 + 2*a*(-(x/a) - (Sqrt[
1 - a^2*x^2]*ArcCos[a*x])/a^2)))/a))/(3*a^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.15

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\left(9a^4x^4\arccos(ax)^3+9\arccos(ax)^3a^2x^2-9\arccos(ax)^2\sqrt{-a^2x^2+1}\,a^3x^3-6a^4x^4\arccos(ax)-114a^2x^2\arccos(ax)+2\right.}{27a^4(a^2x^2-1)}$
orering	$\frac{5(13a^6x^6+144a^4x^4-936a^2x^2+864)\arccos(ax)^3}{81a^6x^2\sqrt{-a^2x^2+1}}-\frac{(25a^6x^6+578a^4x^4-2940a^2x^2+2520)\left(\frac{3x^2\arccos(ax)^3}{\sqrt{-a^2x^2+1}}-\frac{3x^3\arccos(ax)^2a}{-a^2x^2+1}+\frac{x^4}{}\right.}{81a^6x^4}$

input
```
int(x^3*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27/a^4*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)*(9*a^4*x^4*arccos(a*x)^3+9*arccos
(a*x)^3*a^2*x^2-9*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)*a^3*x^3-6*a^4*x^4*arcco
s(a*x)-114*a^2*x^2*arccos(a*x)+2*a^3*x^3*(-a^2*x^2+1)^(1/2)-18*arccos(a*x)
^3-54*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)*a*x+120*arccos(a*x)+120*(-a^2*x^2+1
)^(1/2)*a*x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.54

$$\int \frac{x^3\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{2\,a^3x^3-9\,(a^3x^3+6\,ax)\arccos{(ax)}^2+120\,ax-3\,\sqrt{-a^2x^2+1}\big(3\,(a^2x^2+2)\arccos{(ax)}^3-2\,(a^2x^2+20}{27\,a^4}$$

input
```
integrate(x^3*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/27*(2*a^3*x^3 - 9*(a^3*x^3 + 6*a*x)*arccos(a*x)^2 + 120*a*x - 3*sqrt(-a^
2*x^2 + 1)*(3*(a^2*x^2 + 2)*arccos(a*x)^3 - 2*(a^2*x^2 + 20)*arccos(a*x)))
/a^4
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.99

$$\int \frac{x^3 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$
$$= \begin{cases} -\frac{x^3\operatorname{acos}^2(ax)}{3a} + \frac{2x^3}{27a} - \frac{x^2\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{3a^2} + \frac{2x^2\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{9a^2} - \frac{2x\operatorname{acos}^2(ax)}{a^3} + \frac{40x}{9a^3} - \frac{2\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{3a^4} - \\ \frac{\pi^3x^4}{32} \end{cases}$$

input
```
integrate(x**3*acos(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-x**3*acos(a*x)**2/(3*a) + 2*x**3/(27*a) - x**2*sqrt(-a**2*x**2
 + 1)*acos(a*x)**3/(3*a**2) + 2*x**2*sqrt(-a**2*x**2 + 1)*acos(a*x)/(9*a**
2) - 2*x*acos(a*x)**2/a**3 + 40*x/(9*a**3) - 2*sqrt(-a**2*x**2 + 1)*acos(a
*x)**3/(3*a**4) + 40*sqrt(-a**2*x**2 + 1)*acos(a*x)/(9*a**4), Ne(a, 0)), (
pi**3*x**4/32, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{x^3 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\arccos(ax)^3 + \frac{2}{27}a\left(\frac{3\left(\sqrt{-a^2x^2+1}x^2 + \frac{20\sqrt{-a^2x^2+1}}{a^2}\right)\arccos(ax)}{a^3} + \frac{a^2x^3+60x}{a^4}\right) - \frac{(a^2x^3+6x)\arccos(ax)^2}{3a^3}$$

input
```
integrate(x^3*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arccos(a*x)^3
 + 2/27*a*(3*(sqrt(-a^2*x^2 + 1)*x^2 + 20*sqrt(-a^2*x^2 + 1)/a^2)*arccos(a
*x)/a^3 + (a^2*x^3 + 60*x)/a^4) - 1/3*(a^2*x^3 + 6*x)*arccos(a*x)^2/a^3
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3\,\mathrm{acos}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^3*acos(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^3*acos(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)^3\,x^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^3*acos(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acos(a*x)**3*x**3)/sqrt( - a**2*x**2 + 1),x)
```

3.307 $\int \frac{x^2 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{x^2 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{3x^2}{8a} + \frac{3x\sqrt{1-a^2x^2}\arccos(ax)}{4a^2} - \frac{3\arccos(ax)^2}{8a^3} + \frac{3x^2\arccos(ax)^2}{4a} - \frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2} + \frac{\arccos(ax)^4}{8a^3}$$

output
```
-3/8*x^2/a+3/4*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^2-3/8*arccos(a*x)^2/a^3+
3/4*x^2*arccos(a*x)^2/a-1/2*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/a^2+1/8*arc
cos(a*x)^4/a^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{x^2 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{3a^2x^2 + 6ax\sqrt{1-a^2x^2}\arccos(ax) + (3-6a^2x^2)\arccos(ax)^2 - 4ax\sqrt{1-a^2x^2}\arccos(ax)^3 - \arccos(ax)}{8a^3}$$

input
```
Integrate[(x^2*ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
(3*a^2*x^2 + 6*a*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x] + (3 - 6*a^2*x^2)*ArcCos[
a*x]^2 - 4*a*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3 - ArcCos[a*x]^4)/(8*a^3)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5211, 5139, 5153, 5211, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arccos(ax)^3}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{5211}$$

$$\frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{3\int x\arccos(ax)^2dx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}$$

$$\Big\downarrow \text{5139}$$

$$-\frac{3\left(a\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2a}+\frac{\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}$$

$$\Big\downarrow \text{5153}$$

$$-\frac{3\left(a\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^2\right)}{2a}-\frac{\arccos(ax)^4}{8a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}$$

$$\Big\downarrow \text{5211}$$

$$-\frac{3\left(a\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int xdx}{2a}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2a}-\frac{\arccos(ax)^4}{8a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\left(a\left(\frac{\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2a}-\frac{\arccos(ax)^4}{8a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}$$

$$\Big\downarrow \text{5153}$$

$$-\frac{\arccos(ax)^4}{8a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{2a^2}-\frac{3\left(a\left(-\frac{\arccos(ax)^2}{4a^3}-\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a^2}-\frac{x^2}{4a}\right)+\frac{1}{2}x^2\arccos(ax)^2\right)}{2a}$$

input
```
Int[(x^2*ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/2*(x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a^2 - ArcCos[a*x]^4/(8*a^3) - (3*
((x^2*ArcCos[a*x]^2)/2 + a*(-1/4*x^2/a - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])
/(2*a^2) - ArcCos[a*x]^2/(4*a^3))))/(2*a)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{4 \arccos(ax)^3 \sqrt{-a^2x^2+1}\, ax+6a^2x^2 \arccos(ax)^2+\arccos(ax)^4-6 \arccos(ax)\sqrt{-a^2x^2+1}\, ax-3a^2x^2-3 \arccos(ax)^2+3}{8a^3}$	86

input
```
int(x^2*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(4*arccos(a*x)^3*(-a^2*x^2+1)^(1/2)*a*x+6*a^2*x^2*arccos(a*x)^2+arcco
s(a*x)^4-6*arccos(a*x)*(-a^2*x^2+1)^(1/2)*a*x-3*a^2*x^2-3*arccos(a*x)^2+3)
/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int \frac{x^2 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\, dx = \frac{3\, a^2x^2 - \arccos\left(ax\right)^4 - 3\left(2\, a^2x^2 - 1\right)\arccos\left(ax\right)^2 - 2\left(2\, ax \arccos\left(ax\right)^3 - 3\, ax \arccos\left(ax\right)\right)\sqrt{-a^2x^2 + }}{8\, a^3}$$

input
```
integrate(x^2*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(3*a^2*x^2 - arccos(a*x)^4 - 3*(2*a^2*x^2 - 1)*arccos(a*x)^2 - 2*(2*a*
x*arccos(a*x)^3 - 3*a*x*arccos(a*x))*sqrt(-a^2*x^2 + 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} -\frac{3x^2\operatorname{acos}^2(ax)}{4a} + \frac{3x^2}{8a} - \frac{x\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{2a^2} + \frac{3x\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{4a^2} - \frac{\operatorname{acos}^4(ax)}{8a^3} + \frac{3\operatorname{acos}^2(ax)}{8a^3} & \text{for } a \neq 0 \\ \frac{\pi^3x^3}{24} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*acos(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-3*x**2*acos(a*x)**2/(4*a) + 3*x**2/(8*a) - x*sqrt(-a**2*x**2 +
 1)*acos(a*x)**3/(2*a**2) + 3*x*sqrt(-a**2*x**2 + 1)*acos(a*x)/(4*a**2) -
acos(a*x)**4/(8*a**3) + 3*acos(a*x)**2/(8*a**3), Ne(a, 0)), (pi**3*x**3/24
, True))
```

Maxima [F]

$$\int \frac{x^2 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2 \arccos(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arccos(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \frac{x^2 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{3\,x^2\arccos(ax)^2}{4\,a} - \frac{\sqrt{-a^2x^2+1}x\arccos(ax)^3}{2\,a^2} + \frac{3\,x^2}{8\,a} - \frac{\arccos(ax)^4}{8\,a^3} + \frac{3\sqrt{-a^2x^2+1}x\arccos(ax)}{4\,a^2} + \frac{3\arccos(ax)^2}{8\,a^3} - \frac{3}{16\,a^3}$$

input
```
integrate(x^2*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-3/4*x^2*arccos(a*x)^2/a - 1/2*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)^3/a^2 + 3/
8*x^2/a - 1/8*arccos(a*x)^4/a^3 + 3/4*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)/a^2
 + 3/8*arccos(a*x)^2/a^3 - 3/16/a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2 \operatorname{acos}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^2*acos(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^2*acos(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2 \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)^3 x^2}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^2*acos(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acos(a*x)**3*x**2)/sqrt( - a**2*x**2 + 1),x)
```

3.308 $\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}} \, dx = -\frac{6x}{a} + \frac{6\sqrt{1-a^2x^2} \arccos(ax)}{a^2} + \frac{3x \arccos(ax)^2}{a} - \frac{\sqrt{1-a^2x^2} \arccos(ax)^3}{a^2}$$

output
```
-6*x/a+6*(-a^2*x^2+1)^(1/2)*arccos(a*x)/a^2+3*x*arccos(a*x)^2/a-(-a^2*x^2+
1)^(1/2)*arccos(a*x)^3/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}} \, dx = \frac{6ax + 6\sqrt{1-a^2x^2} \arccos(ax) - 3ax \arccos(ax)^2 - \sqrt{1-a^2x^2} \arccos(ax)^3}{a^2}$$

input
```
Integrate[(x*ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
(6*a*x + 6*Sqrt[1 - a^2*x^2]*ArcCos[a*x] - 3*a*x*ArcCos[a*x]^2 - Sqrt[1 -
a^2*x^2]*ArcCos[a*x]^3)/a^2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5183, 5131, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{5183}$$

$$-\frac{3\int \arccos(ax)^2dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}$$

$$\Big\downarrow \text{5131}$$

$$-\frac{3\left(2a\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}dx+x\arccos(ax)^2\right)}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}$$

$$\Big\downarrow \text{5183}$$

$$-\frac{3\left(2a\left(-\frac{\int 1dx}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}\right)+x\arccos(ax)^2\right)}{a}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{a^2}-\frac{3\left(2a\left(-\frac{\sqrt{1-a^2x^2}\arccos(ax)}{a^2}-\frac{x}{a}\right)+x\arccos(ax)^2\right)}{a}$$

input
```
Int[(x*ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a^2) - (3*(x*ArcCos[a*x]^2 + 2*a*(-(x/
a) - (Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a^2)))/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.60

method	result
default	$-\frac{\sqrt{-a^2x^2+1}\left(\arccos(ax)^3a^2x^2-\arccos(ax)^3-3\arccos(ax)^2\sqrt{-a^2x^2+1}\,ax-6a^2x^2\arccos(ax)+6\arccos(ax)+6\sqrt{-a^2x^2+1}\,ax\right)}{a^2(a^2x^2-1)}$
orering	$\frac{(a^4x^4-8a^2x^2+8)\arccos(ax)^3}{a^4x^2\sqrt{-a^2x^2+1}}-\frac{(a^4x^4-6a^2x^2+8)\left(\frac{\arccos(ax)^3}{\sqrt{-a^2x^2+1}}-\frac{3x\arccos(ax)^2a}{-a^2x^2+1}+\frac{x^2\arccos(ax)^3a^2}{(-a^2x^2+1)^{\frac{3}{2}}}\right)}{x^2a^4}-\frac{2(ax-1)(ax+1)(a^2x^2}{}$

input
```
int(x*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(-a^2*x^2+1)^(1/2)/(a^2*x^2-1)*(arccos(a*x)^3*a^2*x^2-arccos(a*x)^3
-3*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)*a*x-6*a^2*x^2*arccos(a*x)+6*arccos(a*x
)+6*(-a^2*x^2+1)^(1/2)*a*x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{3\,ax \arccos{(ax)}^2 - 6\,ax + \sqrt{-a^2x^2+1}\left(\arccos{(ax)}^3 - 6\,\arccos{(ax)}\right)}{a^2}$$

input
```
integrate(x*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(3*a*x*arccos(a*x)^2 - 6*a*x + sqrt(-a^2*x^2 + 1)*(arccos(a*x)^3 - 6*arcc
os(a*x)))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} -\frac{3x\operatorname{acos}^2{(ax)}}{a} + \frac{6x}{a} - \frac{\sqrt{-a^2x^2+1}\operatorname{acos}^3{(ax)}}{a^2} + \frac{6\sqrt{-a^2x^2+1}\operatorname{acos}{(ax)}}{a^2} & \text{for}\ a \neq 0 \\ \frac{\pi^3x^2}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acos(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-3*x*acos(a*x)**2/a + 6*x/a - sqrt(-a**2*x**2 + 1)*acos(a*x)**3
/a**2 + 6*sqrt(-a**2*x**2 + 1)*acos(a*x)/a**2, Ne(a, 0)), (pi**3*x**2/16,
True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{3\,x\arccos{(ax)}^2}{a} - \frac{\sqrt{-a^2x^2+1}\arccos{(ax)}^3}{a^2} + \frac{6\left(x + \frac{\sqrt{-a^2x^2+1}\arccos(ax)}{a}\right)}{a}$$

input
```
integrate(x*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-3*x*arccos(a*x)^2/a - sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/a^2 + 6*(x + sqrt(
-a^2*x^2 + 1)*arccos(a*x)/a)/a
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{-a^2x^2+1}\arccos{(ax)}^3}{a^2} - \frac{3\left(x\arccos{(ax)}^2 - 2\,x - \frac{2\sqrt{-a^2x^2+1}\arccos(ax)}{a}\right)}{a}$$

input
```
integrate(x*arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/a^2 - 3*(x*arccos(a*x)^2 - 2*x - 2*sqrt(
-a^2*x^2 + 1)*arccos(a*x)/a)/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\mathrm{acos}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x*acos(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x*acos(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{x \arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{-\sqrt{-a^2x^2+1}\,acos(ax)^3-3acos(ax)^2\,ax+6\sqrt{-a^2x^2+1}\,acos(ax)+6ax}{a^2}$$

input
```
int(x*acos(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
( - sqrt( - a**2*x**2 + 1)*acos(a*x)**3 - 3*acos(a*x)**2*a*x + 6*sqrt( - a
**2*x**2 + 1)*acos(a*x) + 6*a*x)/a**2
```

3.309 $\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 13

$$\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\arccos(ax)^4}{4a}$$

output
```
1/4*arccos(a*x)^4/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos(ax)^4}{4a}$$

input
```
Integrate[ArcCos[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/4*ArcCos[a*x]^4/a
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{5153}$$

$$-\frac{\arccos(ax)^4}{4a}$$

input
```
Int[ArcCos[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

output
```
-1/4*ArcCos[a*x]^4/a
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{\arccos(ax)^4}{4a}$	12
default	$-\frac{\arccos(ax)^4}{4a}$	12

input
```
int(arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*arccos(a*x)^4/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos{(ax)}^4}{4\,a}$$

input
```
integrate(arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*arccos(a*x)^4/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} -\frac{\operatorname{acos}^4{(ax)}}{4a} & \text{for}\ a \neq 0 \\ \frac{\pi^3 x}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(acos(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((-acos(a*x)**4/(4*a), Ne(a, 0)), (pi**3*x/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos{(ax)}^4}{4\,a}$$

input
```
integrate(arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*arccos(a*x)^4/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos{(ax)}^4}{4\,a}$$

input
```
integrate(arccos(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/4*arccos(a*x)^4/a
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{\mathrm{acos}(a\,x)^4}{4\,a}$$

input
```
int(acos(a*x)^3/(1 - a^2*x^2)^(1/2),x)
```

output
```
-acos(a*x)^4/(4*a)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{acos(ax)^4}{4a}$$

input
```
int(acos(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
( - acos(a*x)**4)/(4*a)
```

3.310 $\int \frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 138

$$\int \frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = -2\arccos(ax)^3\operatorname{arctanh}\left(e^{i\arccos(ax)}\right) + 3i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right) - 3i\arccos(ax)^2\operatorname{PolyLog}\left(2,e^{i\arccos(ax)}\right) - 6\arccos(ax)\operatorname{PolyLog}\left(3,-e^{i\arccos(ax)}\right) + 6\arccos(ax)\operatorname{PolyLog}\left(3,e^{i\arccos(ax)}\right) - 6i\operatorname{PolyLog}\left(4,-e^{i\arccos(ax)}\right) + 6i\operatorname{PolyLog}\left(4,e^{i\arccos(ax)}\right)$$

output
```
-2*arccos(a*x)^3*arctanh(a*x+I*(-a^2*x^2+1)^(1/2))+3*I*arccos(a*x)^2*polyl
og(2,-a*x-I*(-a^2*x^2+1)^(1/2))-3*I*arccos(a*x)^2*polylog(2,a*x+I*(-a^2*x^
2+1)^(1/2))-6*arccos(a*x)*polylog(3,-a*x-I*(-a^2*x^2+1)^(1/2))+6*arccos(a*
x)*polylog(3,a*x+I*(-a^2*x^2+1)^(1/2))-6*I*polylog(4,-a*x-I*(-a^2*x^2+1)^(
1/2))+6*I*polylog(4,a*x+I*(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.13

$$\int \frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = i\left(2\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right) - 3\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) + 3\arccos(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right) - 6i\arccos(ax)\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right) + 6i\arccos(ax)\operatorname{PolyLog}\left(3,ie^{i\arccos(ax)}\right) + 6\operatorname{PolyLog}\left(4,-ie^{i\arccos(ax)}\right) - 6\operatorname{PolyLog}\left(4,ie^{i\arccos(ax)}\right)\right)$$

input
```
Integrate[ArcCos[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
I*(2*ArcCos[a*x]^3*ArcTan[E^(I*ArcCos[a*x])] - 3*ArcCos[a*x]^2*PolyLog[2,
(-I)*E^(I*ArcCos[a*x])] + 3*ArcCos[a*x]^2*PolyLog[2, I*E^(I*ArcCos[a*x])]
- (6*I)*ArcCos[a*x]*PolyLog[3, (-I)*E^(I*ArcCos[a*x])] + (6*I)*ArcCos[a*x]
*PolyLog[3, I*E^(I*ArcCos[a*x])] + 6*PolyLog[4, (-I)*E^(I*ArcCos[a*x])] -
6*PolyLog[4, I*E^(I*ArcCos[a*x])])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5219, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{5219}$$

$$-\int \frac{\arccos(ax)^3}{ax}\,d\arccos(ax)$$

$$\Big\downarrow \text{3042}$$

$$-\int \arccos(ax)^3 \csc\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)$$

$\downarrow$ 4669

$$3\int \arccos(ax)^2 \log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)-3\int \arccos(ax)^2 \log\left(1+ie^{i\arccos(ax)}\right)d\arccos(ax)+2i\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)$$

$\downarrow$ 3011

$$-3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\int \arccos(ax)\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)d\arccos(ax)\right)+3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)-2i\int \arccos(ax)\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)d\arccos(ax)\right)+2i\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)$$

$\downarrow$ 7163

$$-3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\left(i\int \operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)d\arccos(ax)-i\arccos(ax)\operatorname{P}\right.\right.$$
$$3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)-2i\left(i\int \operatorname{PolyLog}\left(3,ie^{i\arccos(ax)}\right)d\arccos(ax)-i\arccos(ax)\operatorname{PolyLo}\right.\right.$$
$$2i\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)$$

$\downarrow$ 2720

$$-3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\left(\int e^{-i\arccos(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)de^{i\arccos(ax)}-i\operatorname{ar}\right.\right.$$
$$3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)-2i\left(\int e^{-i\arccos(ax)}\operatorname{PolyLog}\left(3,ie^{i\arccos(ax)}\right)de^{i\arccos(ax)}-i\arccos(\right.\right.$$
$$2i\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)$$

$\downarrow$ 7143

$$2i\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)-$$
$$3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-ie^{i\arccos(ax)}\right)-i\arccos(ax)\operatorname{PolyLog}\left(3,-ie^{i\operatorname{arcc}}\right.\right.\right.$$
$$3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,ie^{i\arccos(ax)}\right)-i\arccos(ax)\operatorname{PolyLog}\left(3,ie^{i\arccos(ax)}\right)\right.\right.$$

input
```
Int[ArcCos[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
(2*I)*ArcCos[a*x]^3*ArcTan[E^(I*ArcCos[a*x])] - 3*(I*ArcCos[a*x]^2*PolyLog
[2, (-I)*E^(I*ArcCos[a*x])] - (2*I)*((-I)*ArcCos[a*x]*PolyLog[3, (-I)*E^(I
*ArcCos[a*x])] + PolyLog[4, (-I)*E^(I*ArcCos[a*x])])) + 3*(I*ArcCos[a*x]^2
*PolyLog[2, I*E^(I*ArcCos[a*x])] - (2*I)*((-I)*ArcCos[a*x]*PolyLog[3, I*E^
(I*ArcCos[a*x])] + PolyLog[4, I*E^(I*ArcCos[a*x])]))
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{\arccos\left(ax\right)^{3}}{x\sqrt{-a^{2}x^{2}+1}}dx$$

input
```
int(arccos(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(arccos(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos\left(ax\right)^{3}}{\sqrt{-a^{2}x^{2}+1}x}\,dx$$

input
```
integrate(arccos(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/(a^2*x^3 - x), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acos}^3(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acos(a*x)**3/x/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acos(a*x)**3/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos(ax)^3}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccos(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos(ax)^3}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccos(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acos(a*x)^3/(x*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acos(a*x)^3/(x*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)^3}{\sqrt{-a^2x^2+1}\,x}dx$$

input
```
int(acos(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acos(a*x)**3/(sqrt( - a**2*x**2 + 1)*x),x)
```

3.311 $\int \frac{\arccos(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{\arccos(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = -ia\arccos(ax)^3 - \frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x} + 3a\arccos(ax)^2\log\left(1-e^{2i\arccos(ax)}\right) - 3ia\arccos(ax)\,\text{PolyLog}\left(2,e^{2i\arccos(ax)}\right) + \frac{3}{2}a\,\text{PolyLog}\left(3,e^{2i\arccos(ax)}\right)$$

output
```
-I*a*arccos(a*x)^3-(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/x+3*a*arccos(a*x)^2*ln
(1-(a*x+I*(-a^2*x^2+1)^(1/2))^2)-3*I*a*arccos(a*x)*polylog(2,(a*x+I*(-a^2*
x^2+1)^(1/2))^2)+3/2*a*polylog(3,(a*x+I*(-a^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int \frac{\arccos(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos(ax)^2\left(\left(-iax+\sqrt{1-a^2x^2}\right)\arccos(ax)+3ax\log\left(1+e^{2i\arccos(ax)}\right)\right)}{x} + 3ia\arccos(ax)\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) - \frac{3}{2}a\,\text{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)$$

input
```
Integrate[ArcCos[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((ArcCos[a*x]^2*(((-I)*a*x + Sqrt[1 - a^2*x^2])*ArcCos[a*x] + 3*a*x*Log[1
 + E^((2*I)*ArcCos[a*x])]))/x) + (3*I)*a*ArcCos[a*x]*PolyLog[2, -E^((2*I)*
ArcCos[a*x])] - (3*a*PolyLog[3, -E^((2*I)*ArcCos[a*x])])/2
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5187, 5137, 3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{x^2\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{5187}$$

$$-3a\int \frac{\arccos(ax)^2}{x}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}$$

$$\Big\downarrow \text{5137}$$

$$3a\int \frac{\sqrt{1-a^2x^2}\arccos(ax)^2}{ax}d\arccos(ax)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}$$

$$\Big\downarrow \text{3042}$$

$$3a\int \arccos(ax)^2\tan(\arccos(ax))d\arccos(ax)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}$$

$$\Big\downarrow \text{4202}$$

$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}+3a\left(\frac{1}{3}i\arccos(ax)^3-2i\int \frac{e^{2i\arccos(ax)}\arccos(ax)^2}{1+e^{2i\arccos(ax)}}d\arccos(ax)\right)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}+$$
$$3a\left(\frac{1}{3}i\arccos(ax)^3-2i\left(i\int\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right)d\arccos(ax)-\frac{1}{2}i\arccos(ax)^2\log\left(1+e^{2i\arccos(ax}\right.\right.\right.$$

$\Big\downarrow$ 3011

$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}+$$
$$3a\left(\frac{1}{3}i\arccos(ax)^3-2i\left(i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{1}{2}i\int\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)d\arccos\right.\right.\right.$$

$\Big\downarrow$ 2720

$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}+$$
$$3a\left(\frac{1}{3}i\arccos(ax)^3-2i\left(i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{1}{4}\int e^{-2i\arccos(ax)}\operatorname{PolyLog}\left(2,-e^{2i\arccos}\right.\right.\right.\right.$$

$\Big\downarrow$ 7143

$$-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x}+$$
$$3a\left(\frac{1}{3}i\arccos(ax)^3-2i\left(i\left(\frac{1}{2}i\arccos(ax)\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)-\frac{1}{4}\operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)\right)-\frac{1}{2}i\operatorname{arcc}\right.\right.$$

input
```
Int[ArcCos[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/x) + 3*a*((I/3)*ArcCos[a*x]^3 - (2*I)*
((-1/2*I)*ArcCos[a*x]^2*Log[1 + E^((2*I)*ArcCos[a*x])] + I*((I/2)*ArcCos[a
*x]*PolyLog[2, -E^((2*I)*ArcCos[a*x])] - PolyLog[3, -E^((2*I)*ArcCos[a*x])
]/4)))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.36

method	result
default	$-\frac{\left(iax+\sqrt{-a^{2}x^{2}+1}\right)\arccos\left(ax\right)^{3}}{x}+\frac{a\left(4i\arccos\left(ax\right)^{3}-6\arccos\left(ax\right)^{2}\ln\left(1+\left(ax+i\sqrt{-a^{2}x^{2}+1}\right)^{2}\right)+6i\arccos\left(ax\right)\operatorname{polylog}\left(2,-\right.\right.}{2}$

input
```
int(arccos(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(I*a*x+(-a^2*x^2+1)^(1/2))/x*arccos(a*x)^3+1/2*a*(4*I*arccos(a*x)^3-6*arc
cos(a*x)^2*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)+6*I*arccos(a*x)*polylog(2,-(
a*x+I*(-a^2*x^2+1)^(1/2))^2)-3*polylog(3,-(a*x+I*(-a^2*x^2+1)^(1/2))^2))
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx=\int \frac{\arccos\left(ax\right)^3}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arccos(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/(a^2*x^4 - x^2), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx=\int \frac{\operatorname{acos}^3\left(ax\right)}{x^2\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(acos(a*x)**3/x**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acos(a*x)**3/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos(ax)^3}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arccos(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-(3*a^3*x*integrate(x*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2, x) + s
qrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3)/
x
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos(ax)^3}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arccos(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acos(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acos(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)^3}{\sqrt{-a^2x^2+1}\,x^2}dx$$

input
```
int(acos(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acos(a*x)**3/(sqrt( - a**2*x**2 + 1)*x**2),x)
```

3.312 $\int \frac{\arccos(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 264

$$\begin{aligned}\int \frac{\arccos(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = &-\frac{3a\arccos(ax)^2}{2x}-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{2x^2}\\ &-6a^2\arccos(ax)\operatorname{arctanh}\left(e^{i\arccos(ax)}\right)\\ &-a^2\arccos(ax)^3\operatorname{arctanh}\left(e^{i\arccos(ax)}\right)\\ &+3ia^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)\\ &+\frac{3}{2}ia^2\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arccos(ax)}\right)\\ &-3ia^2\operatorname{PolyLog}\left(2,e^{i\arccos(ax)}\right)\\ &-\frac{3}{2}ia^2\arccos(ax)^2\operatorname{PolyLog}\left(2,e^{i\arccos(ax)}\right)\\ &-3a^2\arccos(ax)\operatorname{PolyLog}\left(3,-e^{i\arccos(ax)}\right)\\ &+3a^2\arccos(ax)\operatorname{PolyLog}\left(3,e^{i\arccos(ax)}\right)\\ &-3ia^2\operatorname{PolyLog}\left(4,-e^{i\arccos(ax)}\right)+3ia^2\operatorname{PolyLog}\left(4,e^{i\arccos(ax)}\right)\end{aligned}$$

output
```
-3/2*a*arccos(a*x)^2/x-1/2*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3/x^2-6*a^2*arcc
os(a*x)*arctanh(a*x+I*(-a^2*x^2+1)^(1/2))-a^2*arccos(a*x)^3*arctanh(a*x+I*
(-a^2*x^2+1)^(1/2))+3*I*a^2*polylog(2,-a*x-I*(-a^2*x^2+1)^(1/2))+3/2*I*a^2
*arccos(a*x)^2*polylog(2,-a*x-I*(-a^2*x^2+1)^(1/2))-3*I*a^2*polylog(2,a*x+
I*(-a^2*x^2+1)^(1/2))-3/2*I*a^2*arccos(a*x)^2*polylog(2,a*x+I*(-a^2*x^2+1)
^(1/2))-3*a^2*arccos(a*x)*polylog(3,-a*x-I*(-a^2*x^2+1)^(1/2))+3*a^2*arcco
s(a*x)*polylog(3,a*x+I*(-a^2*x^2+1)^(1/2))-3*I*a^2*polylog(4,-a*x-I*(-a^2*
x^2+1)^(1/2))+3*I*a^2*polylog(4,a*x+I*(-a^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.93

$$\int \frac{\arccos(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \frac{1}{2}\left(\frac{3a\arccos(ax)^2}{x} - \frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{x^2} + 12ia^2\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right) + 2ia^2\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right) - 3ia^2\left(2+\arccos(ax)^2\right)\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) + 3ia^2\left(2+\arccos(ax)^2\right)\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right) + 6a^2\arccos(ax)\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right) - 6a^2\arccos(ax)\operatorname{PolyLog}\left(3,ie^{i\arccos(ax)}\right) + 6ia^2\operatorname{PolyLog}\left(4,-ie^{i\arccos(ax)}\right) - 6ia^2\operatorname{PolyLog}\left(4,ie^{i\arccos(ax)}\right)\right)$$

input
```
Integrate[ArcCos[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
((3*a*ArcCos[a*x]^2)/x - (Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/x^2 + (12*I)*a^
2*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])] + (2*I)*a^2*ArcCos[a*x]^3*ArcTan[E
^(I*ArcCos[a*x])] - (3*I)*a^2*(2 + ArcCos[a*x]^2)*PolyLog[2, (-I)*E^(I*Arc
Cos[a*x])] + (3*I)*a^2*(2 + ArcCos[a*x]^2)*PolyLog[2, I*E^(I*ArcCos[a*x])]
 + 6*a^2*ArcCos[a*x]*PolyLog[3, (-I)*E^(I*ArcCos[a*x])] - 6*a^2*ArcCos[a*x
]*PolyLog[3, I*E^(I*ArcCos[a*x])] + (6*I)*a^2*PolyLog[4, (-I)*E^(I*ArcCos[
a*x])] - (6*I)*a^2*PolyLog[4, I*E^(I*ArcCos[a*x])])/2
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5205, 5139, 5219, 3042, 4669, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^3}{x^3\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{5205}$$

$$\frac{1}{2}a^2\int \frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}dx-\frac{3}{2}a\int \frac{\arccos(ax)^2}{x^2}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{2x^2}$$

$$\Big\downarrow \text{5139}$$

$$-\frac{3}{2}a\left(-2a\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}dx-\frac{\arccos(ax)^2}{x}\right)+\frac{1}{2}a^2\int \frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{2x^2}$$

$$\Big\downarrow \text{5219}$$

$$-\frac{1}{2}a^2\int \frac{\arccos(ax)^3}{ax}d\arccos(ax)-\frac{3}{2}a\left(2a\int \frac{\arccos(ax)}{ax}d\arccos(ax)-\frac{\arccos(ax)^2}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{2x^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}a^2\int \arccos(ax)^3\csc\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)-\frac{3}{2}a\left(2a\int \arccos(ax)\csc\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)-\frac{\arccos(ax)^2}{x}\right)-\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{2x^2}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{1}{2}a^2\left(-3\int \arccos(ax)^2\log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)+3\int \arccos(ax)^2\log\left(1+ie^{i\arccos(ax)}\right)d\arccos(a\right.$$
$$\frac{3}{2}a\left(-\frac{\arccos(ax)^2}{x}+2a\left(-\int \log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)+\int \log\left(1+ie^{i\arccos(ax)}\right)d\arccos(ax)-2i\,\mathrm{ar}\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{2x^2}$$

$\downarrow$ 2715

$$-\frac{1}{2}a^2\left(-3\int \arccos(ax)^2\log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)+3\int \arccos(ax)^2\log\left(1+ie^{i\arccos(ax)}\right)d\arccos(a\right.$$
$$\frac{3}{2}a\left(-\frac{\arccos(ax)^2}{x}+2a\left(i\int e^{-i\arccos(ax)}\log\left(1-ie^{i\arccos(ax)}\right)de^{i\arccos(ax)}-i\int e^{-i\arccos(ax)}\log\left(1+ie^{i\arccos(\right.\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{2x^2}$$

$\downarrow$ 2838

$$-\frac{1}{2}a^2\left(-3\int \arccos(ax)^2\log\left(1-ie^{i\arccos(ax)}\right)d\arccos(ax)+3\int \arccos(ax)^2\log\left(1+ie^{i\arccos(ax)}\right)d\arccos(a\right.$$
$$\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{2x^2}-$$
$$\frac{3}{2}a\left(-\frac{\arccos(ax)^2}{x}+2a\left(-2i\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right)+i\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-i\,\mathrm{PolyLog}\left(2,ie^{i\,\mathrm{a}}\right.\right.\right.$$

$\downarrow$ 3011

$$-\frac{1}{2}a^2\left(3\left(i\arccos(ax)^2\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\int \arccos(ax)\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)d\arccos(ax)\right)\right.$$
$$\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{2x^2}-$$
$$\frac{3}{2}a\left(-\frac{\arccos(ax)^2}{x}+2a\left(-2i\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right)+i\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-i\,\mathrm{PolyLog}\left(2,ie^{i\,\mathrm{a}}\right.\right.\right.$$

$\downarrow$ 7163

$$-\frac{1}{2}a^2\left(3\left(i\arccos(ax)^2\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\left(i\int \mathrm{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)d\arccos(ax)-i\arccos(\right.\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{2x^2}-$$
$$\frac{3}{2}a\left(-\frac{\arccos(ax)^2}{x}+2a\left(-2i\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right)+i\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-i\,\mathrm{PolyLog}\left(2,ie^{i\,\mathrm{a}}\right.\right.\right.$$

$\downarrow$ 2720

$$-\frac{1}{2}a^2\left(3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\left(\int e^{-i\arccos(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)de^{i\arccos(ax)}\right.\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{2x^2}-$$
$$\frac{3}{2}a\left(-\frac{\arccos(ax)^2}{x}+2a\left(-2i\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right)+i\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-i\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right.\right.\right.$$

$\Big\downarrow$ 7143

$$-\frac{1}{2}a^2\left(-2i\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)+3\left(i\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-ie\right.\right.\right.\right.$$
$$\frac{\sqrt{1-a^2x^2}\arccos(ax)^3}{2x^2}-$$
$$\frac{3}{2}a\left(-\frac{\arccos(ax)^2}{x}+2a\left(-2i\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right)+i\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-i\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right.\right.\right.$$

input
```
Int[ArcCos[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

output
```
-1/2*(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/x^2 - (3*a*(-(ArcCos[a*x]^2/x) + 2*
a*((-2*I)*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])] + I*PolyLog[2, (-I)*E^(I*A
rcCos[a*x])] - I*PolyLog[2, I*E^(I*ArcCos[a*x])])))/2 - (a^2*((-2*I)*ArcCo
s[a*x]^3*ArcTan[E^(I*ArcCos[a*x])] + 3*(I*ArcCos[a*x]^2*PolyLog[2, (-I)*E^
(I*ArcCos[a*x])] - (2*I)*((-I)*ArcCos[a*x]*PolyLog[3, (-I)*E^(I*ArcCos[a*x
])] + PolyLog[4, (-I)*E^(I*ArcCos[a*x])])) - 3*(I*ArcCos[a*x]^2*PolyLog[2,
 I*E^(I*ArcCos[a*x])] - (2*I)*((-I)*ArcCos[a*x]*PolyLog[3, I*E^(I*ArcCos[a
*x])] + PolyLog[4, I*E^(I*ArcCos[a*x])]))))/2
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.66

method	result
default	$-\frac{\sqrt{-a^{2} x^{2}+1}\, \arccos \left(a x \right)^{2} \left(a^{2} x^{2} \arccos \left(a x \right)+3 \sqrt{-a^{2} x^{2}+1}\, a x -\arccos \left(a x \right)\right)}{2 x^{2} \left(a^{2} x^{2}-1\right)}-\frac{a^{2} \left(\arccos \left(a x \right)^{3} \ln \left(1-i \left(a x +i \sqrt{-a^{2} x^{2}+1}\right)\right)-\mathrm{arc}\right.}{}$

input
```
int(arccos(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-a^2*x^2+1)^(1/2)/x^2/(a^2*x^2-1)*arccos(a*x)^2*(a^2*x^2*arccos(a*x)
+3*(-a^2*x^2+1)^(1/2)*a*x-arccos(a*x))-1/2*a^2*(arccos(a*x)^3*ln(1-I*(a*x+
I*(-a^2*x^2+1)^(1/2)))-arccos(a*x)^3*ln(1+I*(a*x+I*(-a^2*x^2+1)^(1/2)))-3*
I*arccos(a*x)^2*polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))+3*I*arccos(a*x)^2*
polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))+6*arccos(a*x)*ln(1-I*(a*x+I*(-a^2
*x^2+1)^(1/2)))+6*arccos(a*x)*polylog(3,I*(a*x+I*(-a^2*x^2+1)^(1/2)))-6*ar
ccos(a*x)*ln(1+I*(a*x+I*(-a^2*x^2+1)^(1/2)))-6*arccos(a*x)*polylog(3,-I*(a
*x+I*(-a^2*x^2+1)^(1/2)))+6*I*polylog(4,I*(a*x+I*(-a^2*x^2+1)^(1/2)))+6*I*
dilog(1+I*(a*x+I*(-a^2*x^2+1)^(1/2)))-6*I*dilog(1-I*(a*x+I*(-a^2*x^2+1)^(1
/2)))-6*I*polylog(4,-I*(a*x+I*(-a^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos\left(ax\right)^3}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccos(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/(a^2*x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acos}^3\left(ax\right)}{x^3\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(acos(a*x)**3/x**3/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acos(a*x)**3/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos(ax)^3}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccos(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arccos(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos(ax)^3}{\sqrt{-a^2x^2+1}x^3}\,dx$$

input
```
integrate(arccos(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acos(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acos(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\arccos(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)^3}{\sqrt{-a^2x^2+1}\,x^3}dx$$

input
```
int(acos(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acos(a*x)**3/(sqrt( - a**2*x**2 + 1)*x**3),x)
```

3.313 $\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx = \frac{35c^3\,\text{CosIntegral}(\arccos(ax))}{64a} + \frac{21c^3\,\text{CosIntegral}(3\arccos(ax))}{64a} + \frac{7c^3\,\text{CosIntegral}(5\arccos(ax))}{64a} + \frac{c^3\,\text{CosIntegral}(7\arccos(ax))}{64a}$$

output
```
35/64*c^3*Ci(arccos(a*x))/a+21/64*c^3*Ci(3*arccos(a*x))/a+7/64*c^3*Ci(5*ar
ccos(a*x))/a+1/64*c^3*Ci(7*arccos(a*x))/a
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx = \frac{c^3(-35\text{Si}(\arccos(ax))+21\text{Si}(3\arccos(ax))-7\text{Si}(5\arccos(ax))+\text{Si}(7\arccos(ax)))}{64a}$$

input
```
Integrate[(c - a^2*c*x^2)^3/ArcCos[a*x],x]
```

output
```
(c^3*(-35*SinIntegral[ArcCos[a*x]] + 21*SinIntegral[3*ArcCos[a*x]] - 7*Sin
Integral[5*ArcCos[a*x]] + SinIntegral[7*ArcCos[a*x]]))/(64*a)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}dx$$

$$\Big\downarrow \text{5169}$$

$$-\frac{c^3\int \frac{(1-a^2x^2)^{7/2}}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{c^3\int \frac{\sin(\arccos(ax))^7}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{c^3\int \left(-\frac{21\sin(3\arccos(ax))}{64\arccos(ax)}+\frac{7\sin(5\arccos(ax))}{64\arccos(ax)}-\frac{\sin(7\arccos(ax))}{64\arccos(ax)}+\frac{35\sqrt{1-a^2x^2}}{64\arccos(ax)}\right)d\arccos(ax)}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^3\left(\frac{35}{64}\text{Si}(\arccos(ax))-\frac{21}{64}\text{Si}(3\arccos(ax))+\frac{7}{64}\text{Si}(5\arccos(ax))-\frac{1}{64}\text{Si}(7\arccos(ax))\right)}{a}$$

input
```
Int[(c - a^2*c*x^2)^3/ArcCos[a*x],x]
```

output
```
-((c^3*((35*SinIntegral[ArcCos[a*x]])/64 - (21*SinIntegral[3*ArcCos[a*x]])
/64 + (7*SinIntegral[5*ArcCos[a*x]])/64 - SinIntegral[7*ArcCos[a*x]]/64))/
a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\frac{c^{3}(21\ \mathrm{Si}(3\arccos(ax))-7\ \mathrm{Si}(5\arccos(ax))+\mathrm{Si}(7\arccos(ax))-35\ \mathrm{Si}(\arccos(ax)))}{64a}$	42
default	$\frac{c^{3}(21\ \mathrm{Si}(3\arccos(ax))-7\ \mathrm{Si}(5\arccos(ax))+\mathrm{Si}(7\arccos(ax))-35\ \mathrm{Si}(\arccos(ax)))}{64a}$	42

input
```
int((-a^2*c*x^2+c)^3/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/64/a*c^3*(21*Si(3*arccos(a*x))-7*Si(5*arccos(a*x))+Si(7*arccos(a*x))-35*
Si(arccos(a*x)))
```

Fricas [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx = \int -\frac{(a^2cx^2-c)^3}{\arccos(ax)}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^3/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)/arccos(a*x),
 x)
```

Sympy [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx = -c^3\left(\int \frac{3a^2x^2}{\operatorname{acos}(ax)}\,dx + \int \left(-\frac{3a^4x^4}{\operatorname{acos}(ax)}\right)dx + \int \frac{a^6x^6}{\operatorname{acos}(ax)}\,dx + \int \left(-\frac{1}{\operatorname{acos}(ax)}\right)dx\right)$$

input
```
integrate((-a**2*c*x**2+c)**3/acos(a*x),x)
```

output
```
-c**3*(Integral(3*a**2*x**2/acos(a*x), x) + Integral(-3*a**4*x**4/acos(a*x
), x) + Integral(a**6*x**6/acos(a*x), x) + Integral(-1/acos(a*x), x))
```

Maxima [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx = \int -\frac{(a^2cx^2-c)^3}{\arccos(ax)}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^3/arccos(a*x),x, algorithm="maxima")
```

output
```
-integrate((a^2*c*x^2 - c)^3/arccos(a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx = \frac{c^3\,\mathrm{Si}\,(7\arccos{(ax)})}{64\,a} - \frac{7\,c^3\,\mathrm{Si}\,(5\arccos{(ax)})}{64\,a} + \frac{21\,c^3\,\mathrm{Si}\,(3\arccos{(ax)})}{64\,a} - \frac{35\,c^3\,\mathrm{Si}\,(\arccos{(ax)})}{64\,a}$$

input
```
integrate((-a^2*c*x^2+c)^3/arccos(a*x),x, algorithm="giac")
```

output
```
1/64*c^3*sin_integral(7*arccos(a*x))/a - 7/64*c^3*sin_integral(5*arccos(a*
x))/a + 21/64*c^3*sin_integral(3*arccos(a*x))/a - 35/64*c^3*sin_integral(a
rccos(a*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx = \int \frac{(c-a^2\,c\,x^2)^3}{\mathrm{acos}\,(a\,x)}\,dx$$

input
```
int((c - a^2*c*x^2)^3/acos(a*x),x)
```

output
```
int((c - a^2*c*x^2)^3/acos(a*x), x)
```

Reduce [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)}\,dx = c^3\left(-\left(\int \frac{x^6}{acos\,(ax)}dx\right)a^6 + 3\left(\int \frac{x^4}{acos\,(ax)}dx\right)a^4 - 3\left(\int \frac{x^2}{acos\,(ax)}dx\right)a^2 + \int \frac{1}{acos\,(ax)}dx\right)$$

input
```
int((-a^2*c*x^2+c)^3/acos(a*x),x)
```

output
```
c**3*( - int(x**6/acos(a*x),x)*a**6 + 3*int(x**4/acos(a*x),x)*a**4 - 3*int
(x**2/acos(a*x),x)*a**2 + int(1/acos(a*x),x))
```

3.314 $\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx = \frac{5c^2\,\text{CosIntegral}(\arccos(ax))}{8a} + \frac{5c^2\,\text{CosIntegral}(3\arccos(ax))}{16a} + \frac{c^2\,\text{CosIntegral}(5\arccos(ax))}{16a}$$

output
```
5/8*c^2*Ci(arccos(a*x))/a+5/16*c^2*Ci(3*arccos(a*x))/a+1/16*c^2*Ci(5*arcco
s(a*x))/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx = -\frac{c^2(10\text{Si}(\arccos(ax)) - 5\text{Si}(3\arccos(ax)) + \text{Si}(5\arccos(ax)))}{16a}$$

input
```
Integrate[(c - a^2*c*x^2)^2/ArcCos[a*x],x]
```

output
```
-1/16*(c^2*(10*SinIntegral[ArcCos[a*x]] - 5*SinIntegral[3*ArcCos[a*x]] + S
inIntegral[5*ArcCos[a*x]]))/a
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx$$
$$\Big\downarrow \text{5169}$$
$$-\frac{c^2\int \frac{(1-a^2x^2)^{5/2}}{\arccos(ax)}d\arccos(ax)}{a}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{c^2\int \frac{\sin(\arccos(ax))^5}{\arccos(ax)}d\arccos(ax)}{a}$$
$$\Big\downarrow \text{3793}$$
$$-\frac{c^2\int \left(-\frac{5\sin(3\arccos(ax))}{16\arccos(ax)}+\frac{\sin(5\arccos(ax))}{16\arccos(ax)}+\frac{5\sqrt{1-a^2x^2}}{8\arccos(ax)}\right)d\arccos(ax)}{a}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{c^2\left(\frac{5}{8}\text{Si}(\arccos(ax))-\frac{5}{16}\text{Si}(3\arccos(ax))+\frac{1}{16}\text{Si}(5\arccos(ax))\right)}{a}$$

input
```
Int[(c - a^2*c*x^2)^2/ArcCos[a*x],x]
```

output
```
-((c^2*((5*SinIntegral[ArcCos[a*x]])/8 - (5*SinIntegral[3*ArcCos[a*x]])/16
 + SinIntegral[5*ArcCos[a*x]]/16))/a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{c^{2}\left(5\,\mathrm{Si}\left(3\arccos\left(ax\right)\right)-\mathrm{Si}\left(5\arccos\left(ax\right)\right)-10\,\mathrm{Si}\left(\arccos\left(ax\right)\right)\right)}{16a}$	35
default	$\frac{c^{2}\left(5\,\mathrm{Si}\left(3\arccos\left(ax\right)\right)-\mathrm{Si}\left(5\arccos\left(ax\right)\right)-10\,\mathrm{Si}\left(\arccos\left(ax\right)\right)\right)}{16a}$	35

input
```
int((-a^2*c*x^2+c)^2/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/16/a*c^2*(5*Si(3*arccos(a*x))-Si(5*arccos(a*x))-10*Si(arccos(a*x)))
```

Fricas [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx = \int \frac{(a^2cx^2-c)^2}{\arccos(ax)}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^2/arccos(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx = c^2\left(\int \left(-\frac{2a^2x^2}{\operatorname{acos}(ax)}\right)dx + \int \frac{a^4x^4}{\operatorname{acos}(ax)}\,dx + \int \frac{1}{\operatorname{acos}(ax)}\,dx\right)$$

input
```
integrate((-a**2*c*x**2+c)**2/acos(a*x),x)
```

output
```
c**2*(Integral(-2*a**2*x**2/acos(a*x), x) + Integral(a**4*x**4/acos(a*x),
x) + Integral(1/acos(a*x), x))
```

Maxima [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx = \int \frac{(a^2cx^2-c)^2}{\arccos(ax)}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^2/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 - c)^2/arccos(a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx = -\frac{c^2\,\mathrm{Si}\,(5\,\arccos\,(ax))}{16\,a}+\frac{5\,c^2\,\mathrm{Si}\,(3\,\arccos\,(ax))}{16\,a}-\frac{5\,c^2\,\mathrm{Si}\,(\arccos\,(ax))}{8\,a}$$

input
```
integrate((-a^2*c*x^2+c)^2/arccos(a*x),x, algorithm="giac")
```

output
```
-1/16*c^2*sin_integral(5*arccos(a*x))/a + 5/16*c^2*sin_integral(3*arccos(a
*x))/a - 5/8*c^2*sin_integral(arccos(a*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx = \int \frac{{\left(c-a^2\,c\,x^2\right)}^2}{\mathrm{acos}\,(a\,x)}\,dx$$

input
```
int((c - a^2*c*x^2)^2/acos(a*x),x)
```

output
```
int((c - a^2*c*x^2)^2/acos(a*x), x)
```

Reduce [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)}\,dx = c^2\left(\left(\int \frac{x^4}{acos\,(ax)}dx\right)a^4-2\left(\int \frac{x^2}{acos\,(ax)}dx\right)a^2+\int \frac{1}{acos\,(ax)}dx\right)$$

input
```
int((-a^2*c*x^2+c)^2/acos(a*x),x)
```

output
```
c**2*(int(x**4/acos(a*x),x)*a**4 - 2*int(x**2/acos(a*x),x)*a**2 + int(1/ac
os(a*x),x))
```

3.315 $\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx=\frac{3c\,\mathrm{CosIntegral}(\arccos(ax))}{4a}+\frac{c\,\mathrm{CosIntegral}(3\arccos(ax))}{4a}$$

output
```
3/4*c*Ci(arccos(a*x))/a+1/4*c*Ci(3*arccos(a*x))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx=\frac{c(-3\mathrm{Si}(\arccos(ax))+\mathrm{Si}(3\arccos(ax)))}{4a}$$

input
```
Integrate[(c - a^2*c*x^2)/ArcCos[a*x],x]
```

output
```
(c*(-3*SinIntegral[ArcCos[a*x]] + SinIntegral[3*ArcCos[a*x]]))/(4*a)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-a^2cx^2}{\arccos(ax)}dx$$
$$\Big\downarrow \text{5169}$$
$$-\frac{c\int \frac{\left(1-a^2x^2\right)^{3/2}}{\arccos(ax)}d\arccos(ax)}{a}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{c\int \frac{\sin(\arccos(ax))^3}{\arccos(ax)}d\arccos(ax)}{a}$$
$$\Big\downarrow \text{3793}$$
$$-\frac{c\int \left(\frac{3\sqrt{1-a^2x^2}}{4\arccos(ax)}-\frac{\sin(3\arccos(ax))}{4\arccos(ax)}\right)d\arccos(ax)}{a}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{c\left(\frac{3}{4}\text{Si}(\arccos(ax))-\frac{1}{4}\text{Si}(3\arccos(ax))\right)}{a}$$

input
```
Int[(c - a^2*c*x^2)/ArcCos[a*x],x]
```

output
```
-((c*((3*SinIntegral[ArcCos[a*x]])/4 - SinIntegral[3*ArcCos[a*x]]/4))/a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{c(\mathrm{Si}(3\arccos(ax))-3\ \mathrm{Si}(\arccos(ax)))}{4a}$	22
default	$\frac{c(\mathrm{Si}(3\arccos(ax))-3\ \mathrm{Si}(\arccos(ax)))}{4a}$	22

input
```
int((-a^2*c*x^2+c)/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*c*(Si(3*arccos(a*x))-3*Si(arccos(a*x)))
```

Fricas [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx = \int -\frac{a^2cx^2-c}{\arccos{(ax)}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(-(a^2*c*x^2 - c)/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx = -c\left(\int \frac{a^2x^2}{\operatorname{acos}{(ax)}}\,dx + \int \left(-\frac{1}{\operatorname{acos}{(ax)}}\right)\,dx\right)$$

input
```
integrate((-a**2*c*x**2+c)/acos(a*x),x)
```

output
```
-c*(Integral(a**2*x**2/acos(a*x), x) + Integral(-1/acos(a*x), x))
```

Maxima [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx = \int -\frac{a^2cx^2-c}{\arccos{(ax)}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)/arccos(a*x),x, algorithm="maxima")
```

output
```
-integrate((a^2*c*x^2 - c)/arccos(a*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx = \frac{c\,\mathrm{Si}\,(3\arccos{(ax)})}{4\,a} - \frac{3\,c\,\mathrm{Si}\,(\arccos{(ax)})}{4\,a}$$

input
```
integrate((-a^2*c*x^2+c)/arccos(a*x),x, algorithm="giac")
```

output
```
1/4*c*sin_integral(3*arccos(a*x))/a - 3/4*c*sin_integral(arccos(a*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx = \int \frac{c-a^2\,c\,x^2}{\mathrm{acos}\,(a\,x)}\,dx$$

input
```
int((c - a^2*c*x^2)/acos(a*x),x)
```

output
```
int((c - a^2*c*x^2)/acos(a*x), x)
```

Reduce [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)}\,dx = c\left(-\left(\int \frac{x^2}{acos\,(ax)}dx\right)a^2 + \int \frac{1}{acos\,(ax)}dx\right)$$

input
```
int((-a^2*c*x^2+c)/acos(a*x),x)
```

output
```
c*( - int(x**2/acos(a*x),x)*a**2 + int(1/acos(a*x),x))
```

3.316 $\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx = \text{Int}\left(\frac{1}{(c-a^2cx^2)\arccos(ax)},x\right)$$

output
```
Defer(Int)(1/(-a^2*c*x^2+c)/arccos(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx = \int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx$$

input
```
Integrate[1/((c - a^2*c*x^2)*ArcCos[a*x]),x]
```

output
```
Integrate[1/((c - a^2*c*x^2)*ArcCos[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)\left(c-a^2cx^2\right)}\,dx$$

$$\Big\downarrow \; 5175$$

$$\int \frac{1}{\arccos(ax)\left(c-a^2cx^2\right)}dx$$

input
```
Int[1/((c - a^2*c*x^2)*ArcCos[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(-a^{2} c \,x^{2}+c \right) \arccos \left(a x \right)}d x$$

input
```
int(1/(-a^2*c*x^2+c)/arccos(a*x),x)
```

output
```
int(1/(-a^2*c*x^2+c)/arccos(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx = \int -\frac{1}{(a^2cx^2-c)\arccos{(ax)}}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(-1/((a^2*c*x^2 - c)*arccos(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx = -\frac{\int \frac{1}{a^2x^2\operatorname{acos}{(ax)}-\operatorname{acos}{(ax)}}\,dx}{c}$$

input
```
integrate(1/(-a**2*c*x**2+c)/acos(a*x),x)
```

output
```
-Integral(1/(a**2*x**2*acos(a*x) - acos(a*x)), x)/c
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx = \int -\frac{1}{(a^2cx^2-c)\arccos{(ax)}}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)/arccos(a*x),x, algorithm="maxima")
```

output
```
-integrate(1/((a^2*c*x^2 - c)*arccos(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx = \int -\frac{1}{(a^2cx^2-c)\arccos(ax)}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)/arccos(a*x),x, algorithm="giac")
```

output
```
integrate(-1/((a^2*c*x^2 - c)*arccos(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx = \int \frac{1}{\mathrm{acos}(a\,x)\,\left(c-a^2\,c\,x^2\right)}\,dx$$

input
```
int(1/(acos(a*x)*(c - a^2*c*x^2)),x)
```

output
```
int(1/(acos(a*x)*(c - a^2*c*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)}\,dx = -\frac{\int \frac{1}{acos(ax)a^2x^2-acos(ax)}dx}{c}$$

input
```
int(1/(-a^2*c*x^2+c)/acos(a*x),x)
```

output
```
( - int(1/(acos(a*x)*a**2*x**2 - acos(a*x)),x))/c
```

3.317 $\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)} \, dx = \text{Int}\left(\frac{1}{(c-a^2cx^2)^2 \arccos(ax)}, x\right)$$

output
```
Defer(Int)(1/(-a^2*c*x^2+c)^2/arccos(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)} \, dx = \int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)} \, dx$$

input
```
Integrate[1/((c - a^2*c*x^2)^2*ArcCos[a*x]),x]
```

output
```
Integrate[1/((c - a^2*c*x^2)^2*ArcCos[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)\left(c-a^2cx^2\right)^2}\,dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{1}{\arccos(ax)\left(c-a^2cx^2\right)^2}dx$$

input
```
Int[1/((c - a^2*c*x^2)^2*ArcCos[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(-a^{2} c \,x^{2}+c \right)^{2} \arccos \left(a x \right)}d x$$

input
```
int(1/(-a^2*c*x^2+c)^2/arccos(a*x),x)
```

output
```
int(1/(-a^2*c*x^2+c)^2/arccos(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)}\,dx = \int \frac{1}{(a^2cx^2-c)^2 \arccos(ax)}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^2/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*arccos(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)}\,dx = \frac{\int \frac{1}{a^4x^4 \operatorname{acos}(ax)-2a^2x^2 \operatorname{acos}(ax)+\operatorname{acos}(ax)}\,dx}{c^2}$$

input
```
integrate(1/(-a**2*c*x**2+c)**2/acos(a*x),x)
```

output
```
Integral(1/(a**4*x**4*acos(a*x) - 2*a**2*x**2*acos(a*x) + acos(a*x)), x)/c
**2
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)}\,dx = \int \frac{1}{(a^2cx^2-c)^2 \arccos(ax)}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^2/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 - c)^2*arccos(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{1}{(c-a^2cx^2)^2\arccos(ax)}\,dx = \int \frac{1}{(a^2cx^2-c)^2\arccos{(ax)}}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^2/arccos(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 - c)^2*arccos(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)^2\arccos(ax)}\,dx = \int \frac{1}{\mathrm{acos}\left(a\,x\right)\,{\left(c-a^2\,c\,x^2\right)}^2}\,dx$$

input
```
int(1/(acos(a*x)*(c - a^2*c*x^2)^2),x)
```

output
```
int(1/(acos(a*x)*(c - a^2*c*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{\left(c-a^2cx^2\right)^2 \arccos(ax)}\,dx=\frac{\int \frac{1}{acos(ax)a^4x^4-2acos(ax)a^2x^2+acos(ax)}dx}{c^2}$$

input
```
int(1/(-a^2*c*x^2+c)^2/acos(a*x),x)
```

output
```
int(1/(acos(a*x)*a**4*x**4 - 2*acos(a*x)*a**2*x**2 + acos(a*x)),x)/c**2
```

3.318 $\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 206

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = -\frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{32bc^5} - \frac{\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{16bc^5} + \frac{\cos\left(\frac{6a}{b}\right)\text{CosIntegral}\left(\frac{6(a+b\arccos(cx))}{b}\right)}{32bc^5} + \frac{\log(a+b\arccos(cx))}{16bc^5} - \frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{32bc^5} - \frac{\sin\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{16bc^5} + \frac{\sin\left(\frac{6a}{b}\right)\text{Si}\left(\frac{6(a+b\arccos(cx))}{b}\right)}{32bc^5}$$

output
```
-1/32*cos(2*a/b)*Ci(2*(a+b*arccos(c*x))/b)/b/c^5-1/16*cos(4*a/b)*Ci(4*(a+b
*arccos(c*x))/b)/b/c^5+1/32*cos(6*a/b)*Ci(6*(a+b*arccos(c*x))/b)/b/c^5+1/1
6*ln(a+b*arccos(c*x))/b/c^5-1/32*sin(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b/c^
5-1/16*sin(4*a/b)*Si(4*(a+b*arccos(c*x))/b)/b/c^5+1/32*sin(6*a/b)*Si(6*(a+
b*arccos(c*x))/b)/b/c^5
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.74

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = -\frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)-2\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(4\left(\frac{a}{b}+\arccos(cx)\right)\right)-\cos\left(\frac{6a}{b}\right)\text{Cos}}{}$$

input
```
Integrate[(x^4*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]),x]
```

output
```
-1/32*(Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcCos[c*x])] - 2*Cos[(4*a)/b]*Co
sIntegral[4*(a/b + ArcCos[c*x])] - Cos[(6*a)/b]*CosIntegral[6*(a/b + ArcCo
s[c*x])] + 2*Log[a + b*ArcCos[c*x]] + Sin[(2*a)/b]*SinIntegral[2*(a/b + Ar
cCos[c*x])] - 2*Sin[(4*a)/b]*SinIntegral[4*(a/b + ArcCos[c*x])] - Sin[(6*a
)/b]*SinIntegral[6*(a/b + ArcCos[c*x])])/(b*c^5)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\int \frac{\cos^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^5}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \left(-\frac{\cos\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}-\frac{\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}+\frac{1}{16(a+b\arccos(cx))}\right)d(a+b\arccos}{bc^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{32}\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\frac{1}{16}\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)-\frac{1}{32}\cos\left(\frac{6a}{b}\right)\text{CosIntegral}}{}$$

input
```
Int[(x^4*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]),x]
```

output
```
-(((Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcCos[c*x]))/b])/32 - (Cos[(4*a)/b
]*CosIntegral[(4*(a + b*ArcCos[c*x]))/b])/16 - (Cos[(6*a)/b]*CosIntegral[(
6*(a + b*ArcCos[c*x]))/b])/32 + Log[a + b*ArcCos[c*x]]/16 + (Sin[(2*a)/b]*
SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/32 - (Sin[(4*a)/b]*SinIntegral[(4*
(a + b*ArcCos[c*x]))/b])/16 - (Sin[(6*a)/b]*SinIntegral[(6*(a + b*ArcCos[c
*x]))/b])/32)/(b*c^5))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\mathrm{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)+\mathrm{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)-\mathrm{Si}\left(6\arccos(cx)+\frac{6a}{b}\right)\sin\left(\frac{6a}{b}\right)-\mathrm{Ci}\left(6\arccos(cx)+\frac{6a}{b}\right)\cos\left(\frac{6a}{b}\right)-2\ \mathrm{S}}{32c^5b}$

input
```
int(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/32/c^5*(Si(2*arccos(c*x)+2*a/b)*sin(2*a/b)+Ci(2*arccos(c*x)+2*a/b)*cos(
2*a/b)-Si(6*arccos(c*x)+6*a/b)*sin(6*a/b)-Ci(6*arccos(c*x)+6*a/b)*cos(6*a/
b)-2*Si(4*arccos(c*x)+4*a/b)*sin(4*a/b)-2*Ci(4*arccos(c*x)+4*a/b)*cos(4*a/
b)+2*ln(a+b*arccos(c*x)))/b
```

Fricas [F]

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^4}{b\arccos\left(cx\right)+a}\,dx$$

input
```
integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^4/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{x^4\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{a+b\,\mathrm{acos}\left(cx\right)}\,dx$$

input
```
integrate(x**4*(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**4*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^4}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)*x^4/(b*arccos(c*x) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(192) = 384.

Time = 0.16 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.29

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
cos(a/b)^6*cos_integral(6*a/b + 6*arccos(c*x))/(b*c^5) + cos(a/b)^5*sin(a/
b)*sin_integral(6*a/b + 6*arccos(c*x))/(b*c^5) - 3/2*cos(a/b)^4*cos_integr
al(6*a/b + 6*arccos(c*x))/(b*c^5) + 1/2*cos(a/b)^4*cos_integral(4*a/b + 4*
arccos(c*x))/(b*c^5) - cos(a/b)^3*sin(a/b)*sin_integral(6*a/b + 6*arccos(c
*x))/(b*c^5) + 1/2*cos(a/b)^3*sin(a/b)*sin_integral(4*a/b + 4*arccos(c*x))
/(b*c^5) + 9/16*cos(a/b)^2*cos_integral(6*a/b + 6*arccos(c*x))/(b*c^5) - 1
/2*cos(a/b)^2*cos_integral(4*a/b + 4*arccos(c*x))/(b*c^5) - 1/16*cos(a/b)^
2*cos_integral(2*a/b + 2*arccos(c*x))/(b*c^5) + 3/16*cos(a/b)*sin(a/b)*sin
_integral(6*a/b + 6*arccos(c*x))/(b*c^5) - 1/4*cos(a/b)*sin(a/b)*sin_integ
ral(4*a/b + 4*arccos(c*x))/(b*c^5) - 1/16*cos(a/b)*sin(a/b)*sin_integral(2
*a/b + 2*arccos(c*x))/(b*c^5) - 1/32*cos_integral(6*a/b + 6*arccos(c*x))/(
b*c^5) + 1/16*cos_integral(4*a/b + 4*arccos(c*x))/(b*c^5) + 1/32*cos_integ
ral(2*a/b + 2*arccos(c*x))/(b*c^5) - 1/16*log(b*arccos(c*x) + a)/(b*c^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{x^4\,\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((x^4*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x)),x)
```

output
```
int((x^4*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}\,x^4}{acos\left(cx\right)b+a}dx$$

input
```
int(x^4*(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**4)/(acos(c*x)*b + a),x)
```

3.319 $\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 183

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = -\frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{8bc^4} - \frac{\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{16bc^4} + \frac{\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)\sin\left(\frac{5a}{b}\right)}{16bc^4} + \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8bc^4} + \frac{\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16bc^4} - \frac{\cos\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16bc^4}$$

output
```
-1/8*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c^4-1/16*Ci(3*(a+b*arccos(c*x))/b)
*sin(3*a/b)/b/c^4+1/16*Ci(5*(a+b*arccos(c*x))/b)*sin(5*a/b)/b/c^4+1/8*cos(
a/b)*Si((a+b*arccos(c*x))/b)/b/c^4+1/16*cos(3*a/b)*Si(3*(a+b*arccos(c*x))/
b)/b/c^4-1/16*cos(5*a/b)*Si(5*(a+b*arccos(c*x))/b)/b/c^4
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.73

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \frac{-2\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)+\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)+\cos\left(\frac{5a}{b}\right)\text{CosInte}}{}$$

input
```
Integrate[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]),x]
```

output
```
(-2*Cos[a/b]*CosIntegral[a/b + ArcCos[c*x]] + Cos[(3*a)/b]*CosIntegral[3*(
a/b + ArcCos[c*x])] + Cos[(5*a)/b]*CosIntegral[5*(a/b + ArcCos[c*x])] - 2*
Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] + Sin[(3*a)/b]*SinIntegral[3*(a/b
+ ArcCos[c*x])] + Sin[(5*a)/b]*SinIntegral[5*(a/b + ArcCos[c*x])])/(16*b*c
^4)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{ 5225}$$

$$-\frac{\int \frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^4}$$

$$\Big\downarrow \text{ 4906}$$

$$-\frac{\int \left(-\frac{\cos\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}-\frac{\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{bc^4}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{1}{8}\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{1}{16}\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)-\frac{1}{16}\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a}{}\right.}{bc}$$

input
```
Int[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]),x]
```

output
```
-(((Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/8 - (Cos[(3*a)/b]*CosInte
gral[(3*(a + b*ArcCos[c*x]))/b])/16 - (Cos[(5*a)/b]*CosIntegral[(5*(a + b*
ArcCos[c*x]))/b])/16 + (Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/8 - (
Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/16 - (Sin[(5*a)/b]*Si
nIntegral[(5*(a + b*ArcCos[c*x]))/b])/16)/(b*c^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]    Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.77

method	result
default	$-\frac{2\,\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)-\mathrm{Ci}\left(5\arccos(cx)+\frac{5a}{b}\right)\cos\left(\frac{5a}{b}\right)-\mathrm{Si}\left(5\arccos(cx)+\frac{5a}{b}\right)\sin\left(\frac{5a}{b}\right)-\mathrm{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)-\mathrm{Ci}(3}{16c^4b}$

input
```
int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/16/c^4*(2*Ci(arccos(c*x)+a/b)*cos(a/b)-Ci(5*arccos(c*x)+5*a/b)*cos(5*a/
b)-Si(5*arccos(c*x)+5*a/b)*sin(5*a/b)-Si(3*arccos(c*x)+3*a/b)*sin(3*a/b)-C
i(3*arccos(c*x)+3*a/b)*cos(3*a/b)+2*Si(arccos(c*x)+a/b)*sin(a/b))/b
```

Fricas [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{-c^2x^2+1}x^3}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^3/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx=\int \frac{x^3\sqrt{-(cx-1)(cx+1)}}{a+b\,\mathrm{acos}(cx)}\,dx$$

input
```
integrate(x**3*(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**3*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^3}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)*x^3/(b*arccos(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{x^3\,\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x)),x)
```

output
```
int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}\,x^3}{acos\,(cx)\,b+a}dx$$

input
```
int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**3)/(acos(c*x)*b + a),x)
```

3.320 $\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 82

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = -\frac{\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{8bc^3} + \frac{\log(a+b\arccos(cx))}{8bc^3} - \frac{\sin\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{8bc^3}$$

output
```
-1/8*cos(4*a/b)*Ci(4*(a+b*arccos(c*x))/b)/b/c^3+1/8*ln(a+b*arccos(c*x))/b/
c^3-1/8*sin(4*a/b)*Si(4*(a+b*arccos(c*x))/b)/b/c^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \frac{\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(4\left(\frac{a}{b}+\arccos(cx)\right)\right) - \log(8(a+b\arccos(cx))) + \sin\left(\frac{4a}{b}\right)\text{Si}\left(4\left(\frac{a}{b}+\arccos(cx)\right)\right)}{8bc^3}$$

input
```
Integrate[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]),x]
```

output
```
(Cos[(4*a)/b]*CosIntegral[4*(a/b + ArcCos[c*x])] - Log[8*(a + b*ArcCos[c*x
])] + Sin[(4*a)/b]*SinIntegral[4*(a/b + ArcCos[c*x])])/(8*b*c^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^3}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \left(\frac{1}{8(a+b\arccos(cx))}-\frac{\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{bc^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{8}\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)-\frac{1}{8}\sin\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{8}\log(a+b\arccos(cx))}{bc^3}$$

input
```
Int[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]),x]
```

output
```
-((-1/8*(Cos[(4*a)/b]*CosIntegral[(4*(a + b*ArcCos[c*x]))/b]) + Log[a + b*
ArcCos[c*x]]/8 - (Sin[(4*a)/b]*SinIntegral[(4*(a + b*ArcCos[c*x]))/b])/8)/
(b*c^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\mathrm{Si}\left(4 \arccos \left(c x \right)+\frac{4 a}{b}\right) \sin \left(\frac{4 a}{b}\right)+\mathrm{Ci}\left(4 \arccos \left(c x \right)+\frac{4 a}{b}\right) \cos \left(\frac{4 a}{b}\right)-\ln \left(4 b \arccos \left(c x \right)+4 a \right)}{8 c^{3} b}$	68

input
```
int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/8/c^3*(Si(4*arccos(c*x)+4*a/b)*sin(4*a/b)+Ci(4*arccos(c*x)+4*a/b)*cos(4*
a/b)-ln(4*b*arccos(c*x)+4*a))/b
```

Fricas [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^2}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^2/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{x^2\sqrt{-(cx-1)(cx+1)}}{a+b\operatorname{acos}{(cx)}}\,dx$$

input
```
integrate(x**2*(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**2*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^2}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)*x^2/(b*arccos(c*x) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(76) = 152.

Time = 0.16 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.05

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \frac{\cos\left(\frac{a}{b}\right)^4 \operatorname{Ci}\left(\frac{4a}{b}+4\arccos(cx)\right)}{bc^3} + \frac{\cos\left(\frac{a}{b}\right)^3 \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{4a}{b}+4\arccos(cx)\right)}{bc^3} - \frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{4a}{b}+4\arccos(cx)\right)}{bc^3} - \frac{\cos\left(\frac{a}{b}\right) \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{4a}{b}+4\arccos(cx)\right)}{2\,bc^3} + \frac{\operatorname{Ci}\left(\frac{4a}{b}+4\arccos(cx)\right)}{8\,bc^3} - \frac{\log(b\arccos(cx)+a)}{8\,bc^3}$$

input
```
integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
cos(a/b)^4*cos_integral(4*a/b + 4*arccos(c*x))/(b*c^3) + cos(a/b)^3*sin(a/
b)*sin_integral(4*a/b + 4*arccos(c*x))/(b*c^3) - cos(a/b)^2*cos_integral(4
*a/b + 4*arccos(c*x))/(b*c^3) - 1/2*cos(a/b)*sin(a/b)*sin_integral(4*a/b +
 4*arccos(c*x))/(b*c^3) + 1/8*cos_integral(4*a/b + 4*arccos(c*x))/(b*c^3)
- 1/8*log(b*arccos(c*x) + a)/(b*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{x^2\,\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x)),x)
```

output
```
int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}\,x^2}{a\cos(cx)\,b+a}dx$$

input
```
int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**2)/(acos(c*x)*b + a),x)
```

3.321 $\int \frac{x\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = -\frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{4bc^2} - \frac{\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{4bc^2} + \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4bc^2} + \frac{\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4bc^2}$$

output
```
-1/4*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c^2-1/4*Ci(3*(a+b*arccos(c*x))/b)*
sin(3*a/b)/b/c^2+1/4*cos(a/b)*Si((a+b*arccos(c*x))/b)/b/c^2+1/4*cos(3*a/b)
*Si(3*(a+b*arccos(c*x))/b)/b/c^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = -\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)-\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)+\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\text{arc}\right.}{4bc^2}$$

input
```
Integrate[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]),x]
```

output
```
-1/4*(Cos[a/b]*CosIntegral[a/b + ArcCos[c*x]] - Cos[(3*a)/b]*CosIntegral[3
*(a/b + ArcCos[c*x])] + Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] - Sin[(3*a
)/b]*SinIntegral[3*(a/b + ArcCos[c*x])])/(b*c^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^2}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \left(\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4(a+b\arccos(cx))}-\frac{\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{bc^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{4}\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{1}{4}\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-}{bc^2}$$

input
```
Int[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]),x]
```

output
```
-(((Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/4 - (Cos[(3*a)/b]*CosInte
gral[(3*(a + b*ArcCos[c*x]))/b])/4 + (Sin[a/b]*SinIntegral[(a + b*ArcCos[c
*x])/b])/4 - (Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/4)/(b*c
^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{\text{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)+\text{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)-\text{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)-\text{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)}{4c^2b}$	92

input
```
int(x*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/c^2*(Si(arccos(c*x)+a/b)*sin(a/b)+Ci(arccos(c*x)+a/b)*cos(a/b)-Si(3*a
rccos(c*x)+3*a/b)*sin(3*a/b)-Ci(3*arccos(c*x)+3*a/b)*cos(3*a/b))/b
```

Fricas [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}x}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{x\sqrt{-(cx-1)(cx+1)}}{a+b\,\mathrm{acos}(cx)}\,dx$$

input
```
integrate(x*(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}x}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)*x/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.41

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \frac{\cos\left(\frac{a}{b}\right)^3 \operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)}{bc^2} + \frac{\cos\left(\frac{a}{b}\right)^2\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{bc^2} - \frac{3\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)}{4bc^2} - \frac{\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)}{4bc^2} - \frac{\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{4bc^2} - \frac{\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{4bc^2}$$

input
```
integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
cos(a/b)^3*cos_integral(3*a/b + 3*arccos(c*x))/(b*c^2) + cos(a/b)^2*sin(a/
b)*sin_integral(3*a/b + 3*arccos(c*x))/(b*c^2) - 3/4*cos(a/b)*cos_integral
(3*a/b + 3*arccos(c*x))/(b*c^2) - 1/4*cos(a/b)*cos_integral(a/b + arccos(c
*x))/(b*c^2) - 1/4*sin(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b*c^2) -
1/4*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{x\,\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acos}\,(c\,x)}\,dx$$

input
```
int((x*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x)),x)
```

output
```
int((x*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}\,x}{acos\,(cx)\,b+a}dx$$

input
```
int(x*(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x)/(acos(c*x)*b + a),x)
```

3.322 $\int \frac{\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2bc} + \frac{\log(a+b\arccos(cx))}{2bc} + \frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2bc}$$

output
```
1/2*cos(2*a/b)*Ci(2*(a+b*arccos(c*x))/b)/b/c+1/2*ln(a+b*arccos(c*x))/b/c+1
/2*sin(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b/c
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right) - \log(a+b\arccos(cx)) + \sin\left(\frac{2a}{b}\right)\text{Si}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)}{2bc}$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(a + b*ArcCos[c*x]),x]
```

output
```
(Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcCos[c*x])] - Log[a + b*ArcCos[c*x]]
+ Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])])/(2*b*c)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \ 5169$$

$$-\frac{\int \frac{\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^2}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \ 3793$$

$$-\frac{\int \left(\frac{1}{2(a+b\arccos(cx))}-\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\frac{1}{2}\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\frac{1}{2}\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{1}{2}\log(a+b\arccos(cx))}{bc}$$

input
```
Int[Sqrt[1 - c^2*x^2]/(a + b*ArcCos[c*x]),x]
```

output
```
-((-1/2*(Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcCos[c*x]))/b]) + Log[a + b*
ArcCos[c*x]]/2 - (Sin[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/2)/
(b*c))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{-\operatorname{Si}\left(2\arccos\left(cx\right)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)-\operatorname{Ci}\left(2\arccos\left(cx\right)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)+\ln\left(a+b\arccos\left(cx\right)\right)}{2cb}$	65

input
```
int((-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c*(-Si(2*arccos(c*x)+2*a/b)*sin(2*a/b)-Ci(2*arccos(c*x)+2*a/b)*cos(2*
a/b)+ln(a+b*arccos(c*x)))/b
```

Fricas [F]

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{b\arccos\left(cx\right)+a}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{a+b\operatorname{acos}\left(cx\right)}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{b\arccos\left(cx\right)+a}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{2a}{b}+2\arccos(cx)\right)}{bc} + \frac{\cos\left(\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{2a}{b}+2\arccos(cx)\right)}{bc} - \frac{\operatorname{Ci}\left(\frac{2a}{b}+2\arccos(cx)\right)}{2bc} - \frac{\log\left(b\arccos(cx)+a\right)}{2bc}$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
cos(a/b)^2*cos_integral(2*a/b + 2*arccos(c*x))/(b*c) + cos(a/b)*sin(a/b)*s
in_integral(2*a/b + 2*arccos(c*x))/(b*c) - 1/2*cos_integral(2*a/b + 2*arcc
os(c*x))/(b*c) - 1/2*log(b*arccos(c*x) + a)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(a + b*acos(c*x)),x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)\,b+a}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b + a),x)
```

3.323 $\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))}\,dx=\frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{b}-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b}+\text{Int}\left(\frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))},x\right)$$

output
```
Ci((a+b*arccos(c*x))/b)*sin(a/b)/b-cos(a/b)*Si((a+b*arccos(c*x))/b)/b+Defe
r(Int)(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))}\,dx=\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))}\,dx$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))}\,dx$$
$$\Big\downarrow \ 5227$$
$$\int \left(\frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}-\frac{c^2x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx+\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b}+\frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b}$$

input
```
Int[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2x^2+1}}{x\left(a+b\arccos\left(cx\right)\right)}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x/(a+b*arccos(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(1/2)/x/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arccos\left(cx\right)+a)x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b*x*arccos(c*x) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x\left(a+b\,\mathrm{acos}\left(cx\right)\right)}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/x/(a+b*acos(c*x)),x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arccos{(cx)}+a)x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/((b*arccos(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{1-c^2\,x^2}}{x\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(x*(a + b*acos(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(x*(a + b*acos(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)\,bx+ax}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x/(a+b*acos(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b*x + a*x),x)
```

3.324 $\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))}\,dx = -\frac{c\log(a+b\arccos(cx))}{b} + \text{Int}\left(\frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))},x\right)$$

output
```
-c*ln(a+b*arccos(c*x))/b+Defer(Int)(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c
*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))}\,dx$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{5227}$$
$$\int \left(\frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}-\frac{c^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx+\frac{\log(a+b\arccos(cx))}{b^2}$$

input
```
Int[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2x^2+1}}{x^2\left(a+b\arccos\left(cx\right)\right)}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccos(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arccos{(cx)}+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b*x^2*arccos(c*x) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-(cx-1)(cx+1)}}{x^2\left(a+b\operatorname{acos}{(cx)}\right)}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/x**2/(a+b*acos(c*x)),x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**2*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arccos{(cx)}+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/((b*arccos(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arccos{(cx)}+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/((b*arccos(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{1-c^2\,x^2}}{x^2\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acos(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acos(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))}\,dx = \frac{\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)b\,x^2+\sqrt{-c^2x^2+1}\,a\,x^2}dx\right)b+\log(acos(cx)\,b+a)\,c}{b}$$

input
```
int((-c^2*x^2+1)^(1/2)/x^2/(a+b*acos(c*x)),x)
```

output
```
(int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*x**2 + sqrt( - c**2*x**2 + 1)*a
*x**2),x)*b + log(acos(c*x)*b + a)*c)/b
```

3.325 $\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 6.83 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))}\,dx$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))}\,dx$$
$$\Big\downarrow \text{5235}$$
$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))}dx$$

input
```
Int[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^{2}x^{2}+1}}{x^{3}\left(a+b\arccos\left(cx\right)\right)}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccos(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arccos{(cx)}+a)x^3}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b*x^3*arccos(c*x) + a*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-(cx-1)(cx+1)}}{x^3(a+b\operatorname{acos}{(cx)})}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/x**3/(a+b*acos(c*x)),x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**3*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arccos{(cx)}+a)x^3}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/((b*arccos(c*x) + a)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{1-c^2\,x^2}}{x^3\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*acos(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*acos(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)\,b\,x^3+a\,x^3}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x^3/(a+b*acos(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b*x**3 + a*x**3),x)
```

3.326 $\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))}\,dx=\operatorname{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))}\,dx=\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))}\,dx$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))}\,dx$$
$$\Big\downarrow \; 5235$$
$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))}dx$$

input
```
Int[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2x^2+1}}{x^4\left(a+b\arccos\left(cx\right)\right)}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccos(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{(b\arccos{(cx)}+a)x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b*x^4*arccos(c*x) + a*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))}\,dx=\int \frac{\sqrt{-(cx-1)\,(cx+1)}}{x^4\,(a+b\operatorname{acos}{(cx)})}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/x**4/(a+b*acos(c*x)),x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**4*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{(b\arccos{(cx)}+a)x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/((b*arccos(c*x) + a)*x^4), x)
```

Giac [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arccos{(cx)}+a)x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/((b*arccos(c*x) + a)*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{1-c^2\,x^2}}{x^4\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(x^4*(a + b*acos(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(x^4*(a + b*acos(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)\,b\,x^4+a\,x^4}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x^4/(a+b*acos(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b*x**4 + a*x**4),x)
```

3.327 $\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 245

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = -\frac{3\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{64bc^4} - \frac{3\,\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{64bc^4} + \frac{\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)\sin\left(\frac{5a}{b}\right)}{64bc^4} + \frac{\text{CosIntegral}\left(\frac{7(a+b\arccos(cx))}{b}\right)\sin\left(\frac{7a}{b}\right)}{64bc^4} + \frac{3\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{64bc^4} + \frac{3\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{64bc^4} - \frac{\cos\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{64bc^4} - \frac{\cos\left(\frac{7a}{b}\right)\text{Si}\left(\frac{7(a+b\arccos(cx))}{b}\right)}{64bc^4}$$

output

```
-3/64*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c^4-3/64*Ci(3*(a+b*arccos(c*x))/b
)*sin(3*a/b)/b/c^4+1/64*Ci(5*(a+b*arccos(c*x))/b)*sin(5*a/b)/b/c^4+1/64*Ci
(7*(a+b*arccos(c*x))/b)*sin(7*a/b)/b/c^4+3/64*cos(a/b)*Si((a+b*arccos(c*x)
)/b)/b/c^4+3/64*cos(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b/c^4-1/64*cos(5*a/b)
*Si(5*(a+b*arccos(c*x))/b)/b/c^4-1/64*cos(7*a/b)*Si(7*(a+b*arccos(c*x))/b)
/b/c^4
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.73

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \frac{-3\cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)+3\cos\left(\frac{3a}{b}\right)\operatorname{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(c\right.\right.}{}$$

input
```
Integrate[(x^3*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x]),x]
```

output
```
(-3*Cos[a/b]*CosIntegral[a/b + ArcCos[c*x]] + 3*Cos[(3*a)/b]*CosIntegral[3
*(a/b + ArcCos[c*x])] + Cos[(5*a)/b]*CosIntegral[5*(a/b + ArcCos[c*x])] -
Cos[(7*a)/b]*CosIntegral[7*(a/b + ArcCos[c*x])] - 3*Sin[a/b]*SinIntegral[a
/b + ArcCos[c*x]] + 3*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])] + Si
n[(5*a)/b]*SinIntegral[5*(a/b + ArcCos[c*x])] - Sin[(7*a)/b]*SinIntegral[7
*(a/b + ArcCos[c*x])])/(64*b*c^4)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(1-c^2x^2\right)^{3/2}}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{ 5225}$$

$$-\frac{\int \frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^4}$$

$$\Big\downarrow \text{ 4906}$$

$$-\frac{\int \left(\frac{\cos\left(\frac{7a}{b}-\frac{7(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}-\frac{\cos\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}-\frac{3\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}+\frac{3\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{64(a+b\arccos(cx))}\right)d(a+b\arc}{bc^4}$$

$\downarrow$ 2009

$$-\frac{\frac{3}{64}\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{3}{64}\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)-\frac{1}{64}\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5}{}\right.}{}$$

input
```
Int[(x^3*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x]),x]
```

output
```
-(((3*Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/64 - (3*Cos[(3*a)/b]*Co
sIntegral[(3*(a + b*ArcCos[c*x]))/b])/64 - (Cos[(5*a)/b]*CosIntegral[(5*(a
 + b*ArcCos[c*x]))/b])/64 + (Cos[(7*a)/b]*CosIntegral[(7*(a + b*ArcCos[c*x
]))/b])/64 + (3*Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/64 - (3*Sin[(
3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/64 - (Sin[(5*a)/b]*SinInte
gral[(5*(a + b*ArcCos[c*x]))/b])/64 + (Sin[(7*a)/b]*SinIntegral[(7*(a + b*
ArcCos[c*x]))/b])/64)/(b*c^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.75

method	result
default	$-\frac{3\,\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)+3\,\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)-\mathrm{Si}\left(5\arccos(cx)+\frac{5a}{b}\right)\sin\left(\frac{5a}{b}\right)-\mathrm{Ci}\left(5\arccos(cx)+\frac{5a}{b}\right)\cos\left(\frac{5a}{b}\right)-3\,\mathrm{Si}(3}{64c^4b}$

input
```
int(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/64/c^4*(3*Si(arccos(c*x)+a/b)*sin(a/b)+3*Ci(arccos(c*x)+a/b)*cos(a/b)-S
i(5*arccos(c*x)+5*a/b)*sin(5*a/b)-Ci(5*arccos(c*x)+5*a/b)*cos(5*a/b)-3*Si(
3*arccos(c*x)+3*a/b)*sin(3*a/b)-3*Ci(3*arccos(c*x)+3*a/b)*cos(3*a/b)+Si(7*
arccos(c*x)+7*a/b)*sin(7*a/b)+Ci(7*arccos(c*x)+7*a/b)*cos(7*a/b))/b
```

Fricas [F]

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^3}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-(c^2*x^5 - x^3)*sqrt(-c^2*x^2 + 1)/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{x^3(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\operatorname{acos}(cx)}\,dx$$

input
```
integrate(x**3*(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**3*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^3}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)*x^3/(b*arccos(c*x) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(229) = 458$.

Time = 0.16 (sec) , antiderivative size = 615, normalized size of antiderivative = 2.51

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-cos(a/b)^7*cos_integral(7*a/b + 7*arccos(c*x))/(b*c^4) - cos(a/b)^6*sin(a
/b)*sin_integral(7*a/b + 7*arccos(c*x))/(b*c^4) + 7/4*cos(a/b)^5*cos_integ
ral(7*a/b + 7*arccos(c*x))/(b*c^4) + 1/4*cos(a/b)^5*cos_integral(5*a/b + 5
*arccos(c*x))/(b*c^4) + 5/4*cos(a/b)^4*sin(a/b)*sin_integral(7*a/b + 7*arc
cos(c*x))/(b*c^4) + 1/4*cos(a/b)^4*sin(a/b)*sin_integral(5*a/b + 5*arccos(
c*x))/(b*c^4) - 7/8*cos(a/b)^3*cos_integral(7*a/b + 7*arccos(c*x))/(b*c^4)
 - 5/16*cos(a/b)^3*cos_integral(5*a/b + 5*arccos(c*x))/(b*c^4) + 3/16*cos(
a/b)^3*cos_integral(3*a/b + 3*arccos(c*x))/(b*c^4) - 3/8*cos(a/b)^2*sin(a/
b)*sin_integral(7*a/b + 7*arccos(c*x))/(b*c^4) - 3/16*cos(a/b)^2*sin(a/b)*
sin_integral(5*a/b + 5*arccos(c*x))/(b*c^4) + 3/16*cos(a/b)^2*sin(a/b)*sin
_integral(3*a/b + 3*arccos(c*x))/(b*c^4) + 7/64*cos(a/b)*cos_integral(7*a/
b + 7*arccos(c*x))/(b*c^4) + 5/64*cos(a/b)*cos_integral(5*a/b + 5*arccos(c
*x))/(b*c^4) - 9/64*cos(a/b)*cos_integral(3*a/b + 3*arccos(c*x))/(b*c^4) -
 3/64*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b*c^4) + 1/64*sin(a/b)*sin
_integral(7*a/b + 7*arccos(c*x))/(b*c^4) + 1/64*sin(a/b)*sin_integral(5*a/
b + 5*arccos(c*x))/(b*c^4) - 3/64*sin(a/b)*sin_integral(3*a/b + 3*arccos(c
*x))/(b*c^4) - 3/64*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b*c^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{x^3\,(1-c^2\,x^2)^{3/2}}{a+b\,\mathrm{acos}\,(c\,x)}\,dx$$

input
```
int((x^3*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x)),x)
```

output
```
int((x^3*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = -\left(\int \frac{\sqrt{-c^2x^2+1}\,x^5}{acos\,(cx)\,b+a}dx\right)c^2 + \int \frac{\sqrt{-c^2x^2+1}\,x^3}{acos\,(cx)\,b+a}dx$$

input
```
int(x^3*(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
 - int((sqrt( - c**2*x**2 + 1)*x**5)/(acos(c*x)*b + a),x)*c**2 + int((sqrt
( - c**2*x**2 + 1)*x**3)/(acos(c*x)*b + a),x)
```

3.328 $\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 206

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{32bc^3} - \frac{\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{16bc^3} - \frac{\cos\left(\frac{6a}{b}\right)\text{CosIntegral}\left(\frac{6(a+b\arccos(cx))}{b}\right)}{32bc^3} + \frac{\log(a+b\arccos(cx))}{16bc^3} + \frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{32bc^3} - \frac{\sin\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{16bc^3} - \frac{\sin\left(\frac{6a}{b}\right)\text{Si}\left(\frac{6(a+b\arccos(cx))}{b}\right)}{32bc^3}$$

output

```
1/32*cos(2*a/b)*Ci(2*(a+b*arccos(c*x))/b)/b/c^3-1/16*cos(4*a/b)*Ci(4*(a+b*
arccos(c*x))/b)/b/c^3-1/32*cos(6*a/b)*Ci(6*(a+b*arccos(c*x))/b)/b/c^3+1/16
*ln(a+b*arccos(c*x))/b/c^3+1/32*sin(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b/c^3
-1/16*sin(4*a/b)*Si(4*(a+b*arccos(c*x))/b)/b/c^3-1/32*sin(6*a/b)*Si(6*(a+b
*arccos(c*x))/b)/b/c^3
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)+2\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(4\left(\frac{a}{b}+\arccos(\right.\right.}{}$$

input
```
Integrate[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x]),x]
```

output
```
(Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcCos[c*x])] + 2*Cos[(4*a)/b]*CosInteg
ral[4*(a/b + ArcCos[c*x])] - Cos[(6*a)/b]*CosIntegral[6*(a/b + ArcCos[c*x]
)] + 2*Log[a + b*ArcCos[c*x]] - 4*Log[8*(a + b*ArcCos[c*x])] + Sin[(2*a)/b
]*SinIntegral[2*(a/b + ArcCos[c*x])] + 2*Sin[(4*a)/b]*SinIntegral[4*(a/b +
 ArcCos[c*x])] - Sin[(6*a)/b]*SinIntegral[6*(a/b + ArcCos[c*x])])/(32*b*c^
3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^3}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \left(\frac{\cos\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}-\frac{\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}-\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}+\frac{1}{16(a+b\arccos(cx))}\right)d(a+b\arccos(c}{bc^3}$$

$\big\downarrow$ 2009

$$-\frac{-\frac{1}{32}\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\frac{1}{16}\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{32}\cos\left(\frac{6a}{b}\right)\text{CosIntegr}}{}$$

input
```
Int[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x]),x]
```

output
```
-((-1/32*(Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcCos[c*x]))/b]) - (Cos[(4*a
)/b]*CosIntegral[(4*(a + b*ArcCos[c*x]))/b])/16 + (Cos[(6*a)/b]*CosIntegra
l[(6*(a + b*ArcCos[c*x]))/b])/32 + Log[a + b*ArcCos[c*x]]/16 - (Sin[(2*a)/
b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/32 - (Sin[(4*a)/b]*SinIntegral[
(4*(a + b*ArcCos[c*x]))/b])/16 + (Sin[(6*a)/b]*SinIntegral[(6*(a + b*ArcCo
s[c*x]))/b])/32)/(b*c^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.76

method	result
default	$\frac{2\,\mathrm{Si}\left(4\arccos(cx)+\frac{4a}{b}\right)\sin\left(\frac{4a}{b}\right)+2\,\mathrm{Ci}\left(4\arccos(cx)+\frac{4a}{b}\right)\cos\left(\frac{4a}{b}\right)+\mathrm{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)+\mathrm{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)-\mathrm{S}}{32c^3b}$

input
```
int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/32/c^3*(2*Si(4*arccos(c*x)+4*a/b)*sin(4*a/b)+2*Ci(4*arccos(c*x)+4*a/b)*c
os(4*a/b)+Si(2*arccos(c*x)+2*a/b)*sin(2*a/b)+Ci(2*arccos(c*x)+2*a/b)*cos(2
*a/b)-Si(6*arccos(c*x)+6*a/b)*sin(6*a/b)-Ci(6*arccos(c*x)+6*a/b)*cos(6*a/b
)-2*ln(a+b*arccos(c*x)))/b
```

Fricas [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^2}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-(c^2*x^4 - x^2)*sqrt(-c^2*x^2 + 1)/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{x^2(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\,\mathrm{acos}(cx)}\,dx$$

input
```
integrate(x**2*(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^2}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)*x^2/(b*arccos(c*x) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(192) = 384.

Time = 0.16 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.30

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-cos(a/b)^6*cos_integral(6*a/b + 6*arccos(c*x))/(b*c^3) - cos(a/b)^5*sin(a
/b)*sin_integral(6*a/b + 6*arccos(c*x))/(b*c^3) + 3/2*cos(a/b)^4*cos_integ
ral(6*a/b + 6*arccos(c*x))/(b*c^3) + 1/2*cos(a/b)^4*cos_integral(4*a/b + 4
*arccos(c*x))/(b*c^3) + cos(a/b)^3*sin(a/b)*sin_integral(6*a/b + 6*arccos(
c*x))/(b*c^3) + 1/2*cos(a/b)^3*sin(a/b)*sin_integral(4*a/b + 4*arccos(c*x)
)/(b*c^3) - 9/16*cos(a/b)^2*cos_integral(6*a/b + 6*arccos(c*x))/(b*c^3) -
1/2*cos(a/b)^2*cos_integral(4*a/b + 4*arccos(c*x))/(b*c^3) + 1/16*cos(a/b)
^2*cos_integral(2*a/b + 2*arccos(c*x))/(b*c^3) - 3/16*cos(a/b)*sin(a/b)*si
n_integral(6*a/b + 6*arccos(c*x))/(b*c^3) - 1/4*cos(a/b)*sin(a/b)*sin_inte
gral(4*a/b + 4*arccos(c*x))/(b*c^3) + 1/16*cos(a/b)*sin(a/b)*sin_integral(
2*a/b + 2*arccos(c*x))/(b*c^3) + 1/32*cos_integral(6*a/b + 6*arccos(c*x))/
(b*c^3) + 1/16*cos_integral(4*a/b + 4*arccos(c*x))/(b*c^3) - 1/32*cos_inte
gral(2*a/b + 2*arccos(c*x))/(b*c^3) - 1/16*log(b*arccos(c*x) + a)/(b*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{x^2\,(1-c^2\,x^2)^{3/2}}{a+b\,\mathrm{acos}\,(c\,x)}\,dx$$

input
```
int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x)),x)
```

output
```
int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = -\left(\int \frac{\sqrt{-c^2x^2+1}\,x^4}{acos\,(cx)\,b+a}dx\right)c^2 + \int \frac{\sqrt{-c^2x^2+1}\,x^2}{acos\,(cx)\,b+a}dx$$

input
```
int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
- int((sqrt( - c**2*x**2 + 1)*x**4)/(acos(c*x)*b + a),x)*c**2 + int((sqrt
( - c**2*x**2 + 1)*x**2)/(acos(c*x)*b + a),x)
```

3.329 $\int \frac{x(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 183

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = -\frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{8bc^2} - \frac{3\,\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{16bc^2} - \frac{\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)\sin\left(\frac{5a}{b}\right)}{16bc^2} + \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8bc^2} + \frac{3\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16bc^2} + \frac{\cos\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16bc^2}$$

output

```
-1/8*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c^2-3/16*Ci(3*(a+b*arccos(c*x))/b)
*sin(3*a/b)/b/c^2-1/16*Ci(5*(a+b*arccos(c*x))/b)*sin(5*a/b)/b/c^2+1/8*cos(
a/b)*Si((a+b*arccos(c*x))/b)/b/c^2+3/16*cos(3*a/b)*Si(3*(a+b*arccos(c*x))/
b)/b/c^2+1/16*cos(5*a/b)*Si(5*(a+b*arccos(c*x))/b)/b/c^2
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.74

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx =$$
$$-\frac{2\cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)-3\cos\left(\frac{3a}{b}\right)\operatorname{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)+\cos\left(\frac{5a}{b}\right)\operatorname{CosInteg}}{}$$

input
```
Integrate[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x]),x]
```

output
```
-1/16*(2*Cos[a/b]*CosIntegral[a/b + ArcCos[c*x]] - 3*Cos[(3*a)/b]*CosInteg
ral[3*(a/b + ArcCos[c*x])] + Cos[(5*a)/b]*CosIntegral[5*(a/b + ArcCos[c*x]
)] + 2*Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] - 3*Sin[(3*a)/b]*SinIntegra
l[3*(a/b + ArcCos[c*x])] + Sin[(5*a)/b]*SinIntegral[5*(a/b + ArcCos[c*x])]
)/(b*c^2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx$$
$$\Big\downarrow \ 5225$$
$$-\frac{\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^2}$$
$$\Big\downarrow \ 4906$$

$$-\frac{\int \left(\frac{\cos\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}-\frac{3\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{bc^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{8}\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{3}{16}\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{1}{16}\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a}{}\right.}{bc^{}}$$

input
```
Int[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x]),x]
```

output
```
-(((Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/8 - (3*Cos[(3*a)/b]*CosIn
tegral[(3*(a + b*ArcCos[c*x]))/b])/16 + (Cos[(5*a)/b]*CosIntegral[(5*(a +
b*ArcCos[c*x]))/b])/16 + (Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/8 -
 (3*Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/16 + (Sin[(5*a)/b
]*SinIntegral[(5*(a + b*ArcCos[c*x]))/b])/16)/(b*c^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.77

method	result
default	$\frac{3\,\mathrm{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)+3\,\mathrm{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)-\mathrm{Si}\left(5\arccos(cx)+\frac{5a}{b}\right)\sin\left(\frac{5a}{b}\right)-\mathrm{Ci}\left(5\arccos(cx)+\frac{5a}{b}\right)\cos\left(\frac{5a}{b}\right)-2}{16c^2b}$

input
```
int(x*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/16/c^2*(3*Si(3*arccos(c*x)+3*a/b)*sin(3*a/b)+3*Ci(3*arccos(c*x)+3*a/b)*c
os(3*a/b)-Si(5*arccos(c*x)+5*a/b)*sin(5*a/b)-Ci(5*arccos(c*x)+5*a/b)*cos(5
*a/b)-2*Si(arccos(c*x)+a/b)*sin(a/b)-2*Ci(arccos(c*x)+a/b)*cos(a/b))/b
```

Fricas [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-(c^2*x^3 - x)*sqrt(-c^2*x^2 + 1)/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{x(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\,\mathrm{acos}\,(cx)}\,dx$$

input
```
integrate(x*(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)*x/(b*arccos(c*x) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(171) = 342$.

Time = 0.16 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.97

$$\begin{aligned}\int \frac{x(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = &-\frac{\cos\left(\frac{a}{b}\right)^5 \operatorname{Ci}\left(\frac{5a}{b}+5\arccos(cx)\right)}{bc^2} \\ &-\frac{\cos\left(\frac{a}{b}\right)^4 \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{5a}{b}+5\arccos(cx)\right)}{bc^2} \\ &+\frac{5\cos\left(\frac{a}{b}\right)^3 \operatorname{Ci}\left(\frac{5a}{b}+5\arccos(cx)\right)}{4bc^2}+\frac{3\cos\left(\frac{a}{b}\right)^3 \operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)}{4bc^2} \\ &+\frac{3\cos\left(\frac{a}{b}\right)^2 \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{5a}{b}+5\arccos(cx)\right)}{4bc^2} \\ &+\frac{3\cos\left(\frac{a}{b}\right)^2 \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{4bc^2} \\ &-\frac{5\cos\left(\frac{a}{b}\right) \operatorname{Ci}\left(\frac{5a}{b}+5\arccos(cx)\right)}{16bc^2}-\frac{9\cos\left(\frac{a}{b}\right) \operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)}{16bc^2} \\ &-\frac{\cos\left(\frac{a}{b}\right) \operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)}{8bc^2}-\frac{\sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{5a}{b}+5\arccos(cx)\right)}{16bc^2} \\ &-\frac{3\sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{16bc^2}-\frac{\sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{8bc^2}\end{aligned}$$

input
```
integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-cos(a/b)^5*cos_integral(5*a/b + 5*arccos(c*x))/(b*c^2) - cos(a/b)^4*sin(a
/b)*sin_integral(5*a/b + 5*arccos(c*x))/(b*c^2) + 5/4*cos(a/b)^3*cos_integ
ral(5*a/b + 5*arccos(c*x))/(b*c^2) + 3/4*cos(a/b)^3*cos_integral(3*a/b + 3
*arccos(c*x))/(b*c^2) + 3/4*cos(a/b)^2*sin(a/b)*sin_integral(5*a/b + 5*arc
cos(c*x))/(b*c^2) + 3/4*cos(a/b)^2*sin(a/b)*sin_integral(3*a/b + 3*arccos(
c*x))/(b*c^2) - 5/16*cos(a/b)*cos_integral(5*a/b + 5*arccos(c*x))/(b*c^2)
- 9/16*cos(a/b)*cos_integral(3*a/b + 3*arccos(c*x))/(b*c^2) - 1/8*cos(a/b)
*cos_integral(a/b + arccos(c*x))/(b*c^2) - 1/16*sin(a/b)*sin_integral(5*a/
b + 5*arccos(c*x))/(b*c^2) - 3/16*sin(a/b)*sin_integral(3*a/b + 3*arccos(c
*x))/(b*c^2) - 1/8*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{x\,(1-c^2\,x^2)^{3/2}}{a+b\,\mathrm{acos}\,(c\,x)}\,dx$$

input
```
int((x*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x)),x)
```

output
```
int((x*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = -\left(\int \frac{\sqrt{-c^2x^2+1}\,x^3}{acos\,(cx)\,b+a}dx\right)c^2 + \int \frac{\sqrt{-c^2x^2+1}\,x}{acos\,(cx)\,b+a}dx$$

input
```
int(x*(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
- int((sqrt( - c**2*x**2 + 1)*x**3)/(acos(c*x)*b + a),x)*c**2 + int((sqrt
( - c**2*x**2 + 1)*x)/(acos(c*x)*b + a),x)
```

3.330 $\int \frac{(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 144

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx=\frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2bc}+\frac{\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{8bc}+\frac{3\log(a+b\arccos(cx))}{8bc}+\frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2bc}+\frac{\sin\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{8bc}$$

output
```
1/2*cos(2*a/b)*Ci(2*(a+b*arccos(c*x))/b)/b/c+1/8*cos(4*a/b)*Ci(4*(a+b*arcc
os(c*x))/b)/b/c+3/8*ln(a+b*arccos(c*x))/b/c+1/2*sin(2*a/b)*Si(2*(a+b*arcco
s(c*x))/b)/b/c+1/8*sin(4*a/b)*Si(4*(a+b*arccos(c*x))/b)/b/c
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \frac{4\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)-\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(4\left(\frac{a}{b}+\arccos(\right.\right.}{}$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(a + b*ArcCos[c*x]),x]
```

output
```
(4*Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcCos[c*x])] - Cos[(4*a)/b]*CosInteg
ral[4*(a/b + ArcCos[c*x])] - 4*Log[a + b*ArcCos[c*x]] + Log[8*(a + b*ArcCo
s[c*x])] + 4*Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])] - Sin[(4*a)/b
]*SinIntegral[4*(a/b + ArcCos[c*x])])/(8*b*c)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5169}$$

$$-\frac{\int \frac{\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^4}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8(a+b\arccos(cx))}-\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2(a+b\arccos(cx))}+\frac{3}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{bc}$$

$\big\downarrow$ 2009

$$-\frac{-\frac{1}{2}\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{1}{8}\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)-\frac{1}{2}\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arcco}{b}\right.}{bc}$$

input
```
Int[(1 - c^2*x^2)^(3/2)/(a + b*ArcCos[c*x]),x]
```

output
```
-((-1/2*(Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcCos[c*x]))/b]) + (Cos[(4*a)
/b]*CosIntegral[(4*(a + b*ArcCos[c*x]))/b])/8 + (3*Log[a + b*ArcCos[c*x]])
/8 - (Sin[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/2 + (Sin[(4*a)/
b]*SinIntegral[(4*(a + b*ArcCos[c*x]))/b])/8)/(b*c))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

method	result
default	$-\frac{\mathrm{Si}\left(4\arccos(cx)+\frac{4a}{b}\right)\sin\left(\frac{4a}{b}\right)+\mathrm{Ci}\left(4\arccos(cx)+\frac{4a}{b}\right)\cos\left(\frac{4a}{b}\right)-4\,\mathrm{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)-4\,\mathrm{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)}{8cb}$

input
```
int((-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8/c*(Si(4*arccos(c*x)+4*a/b)*sin(4*a/b)+Ci(4*arccos(c*x)+4*a/b)*cos(4*a
/b)-4*Si(2*arccos(c*x)+2*a/b)*sin(2*a/b)-4*Ci(2*arccos(c*x)+2*a/b)*cos(2*a
/b)+3*ln(a+b*arccos(c*x)))/b
```

Fricas [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{b\arccos(cx)+a}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\operatorname{acos}(cx)}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{b\arccos(cx)+a}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.76

$$\begin{aligned}\int \frac{(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx &= -\frac{\cos\left(\frac{a}{b}\right)^4 \operatorname{Ci}\left(\frac{4a}{b}+4\arccos(cx)\right)}{bc} \\ &- \frac{\cos\left(\frac{a}{b}\right)^3 \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{4a}{b}+4\arccos(cx)\right)}{bc} + \frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{4a}{b}+4\arccos(cx)\right)}{bc} \\ &+ \frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{2a}{b}+2\arccos(cx)\right)}{bc} + \frac{\cos\left(\frac{a}{b}\right) \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{4a}{b}+4\arccos(cx)\right)}{2bc} \\ &+ \frac{\cos\left(\frac{a}{b}\right) \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{2a}{b}+2\arccos(cx)\right)}{bc} - \frac{\operatorname{Ci}\left(\frac{4a}{b}+4\arccos(cx)\right)}{8bc} \\ &- \frac{\operatorname{Ci}\left(\frac{2a}{b}+2\arccos(cx)\right)}{2bc} - \frac{3\log(b\arccos(cx)+a)}{8bc}\end{aligned}$$

input
```
integrate((-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-cos(a/b)^4*cos_integral(4*a/b + 4*arccos(c*x))/(b*c) - cos(a/b)^3*sin(a/b
)*sin_integral(4*a/b + 4*arccos(c*x))/(b*c) + cos(a/b)^2*cos_integral(4*a/
b + 4*arccos(c*x))/(b*c) + cos(a/b)^2*cos_integral(2*a/b + 2*arccos(c*x))/
(b*c) + 1/2*cos(a/b)*sin(a/b)*sin_integral(4*a/b + 4*arccos(c*x))/(b*c) +
cos(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arccos(c*x))/(b*c) - 1/8*cos_inte
gral(4*a/b + 4*arccos(c*x))/(b*c) - 1/2*cos_integral(2*a/b + 2*arccos(c*x)
)/(b*c) - 3/8*log(b*arccos(c*x) + a)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{a+b\arccos(cx)}\,dx=\int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(a + b*acos(c*x)),x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{acos\left(cx\right)b+a}dx-\left(\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acos\left(cx\right)b+a}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b + a),x) - int((sqrt( - c**2*x**2 +
 1)*x**2)/(acos(c*x)*b + a),x)*c**2
```

3.331 $\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))}\,dx = \frac{5\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{4b} + \frac{\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{4b} - \frac{5\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b} - \frac{\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b} + \text{Int}\left(\frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))},x\right)$$

output
```
5/4*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b+1/4*Ci(3*(a+b*arccos(c*x))/b)*sin(3
*a/b)/b-5/4*cos(a/b)*Si((a+b*arccos(c*x))/b)/b-1/4*cos(3*a/b)*Si(3*(a+b*ar
ccos(c*x))/b)/b+Defer(Int)(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))}\,dx=\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))}\,dx$$
$$\Big\downarrow \text{5227}$$
$$\int \left(-\frac{2c^2x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}+\frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}+\frac{c^4x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx + \frac{5\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b} - \frac{\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b} + \frac{5\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b} - \frac{\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b}$$

input
```
Int[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x(a+b\arccos(cx))}dx$$

input
```
int((-c^2*x^2+1)^(3/2)/x/(a+b*arccos(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(3/2)/x/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos(cx)+a)x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b*x*arccos(c*x) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))}\,dx = \int \frac{(-(cx-1)\,(cx+1))^{\frac{3}{2}}}{x\,(a+b\,\mathrm{acos}\,(cx))}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/x/(a+b*acos(c*x)),x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos\,(cx)+a)x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/((b*arccos(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))}\,dx = \int \frac{(1-c^2\,x^2)^{3/2}}{x\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(x*(a + b*acos(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(x*(a + b*acos(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acos(cx)\,bx+ax}dx - \left(\int \frac{\sqrt{-c^2x^2+1}\,x}{acos(cx)\,b+a}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(3/2)/x/(a+b*acos(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b*x + a*x),x) - int((sqrt( - c**2*x*
*2 + 1)*x)/(acos(c*x)*b + a),x)*c**2
```

3.332 $\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))} \, dx = -\frac{c\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2b} - \frac{3c\log(a+b\arccos(cx))}{2b} - \frac{c\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2b} + \text{Int}\left(\frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))},x\right)$$

output
```
-1/2*c*cos(2*a/b)*Ci(2*(a+b*arccos(c*x))/b)/b-3/2*c*ln(a+b*arccos(c*x))/b-
1/2*c*sin(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b+Defer(Int)(1/x^2/(-c^2*x^2+1)
^(1/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))}\,dx = \int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))}\,dx$$
$$\Big\downarrow \text{5227}$$
$$\int \left(-\frac{2c^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}+\frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}+\frac{c^4x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx+\frac{2\log(a+b\arccos(cx))}{b^2}-\frac{c\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2b}-\frac{c\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2b}-\frac{c\log(a+b\arccos(cx))}{2b}$$

input
```
Int[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x^2\,(a+b\arccos{(cx)})}dx$$

input
```
int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccos(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos{(cx)}+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b*x^2*arccos(c*x) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))}\,dx = \int \frac{(-(cx-1)\,(cx+1))^{\frac{3}{2}}}{x^2\,(a+b\,\mathrm{acos}\,(cx))}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/x**2/(a+b*acos(c*x)),x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**2*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos\,(cx)+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/((b*arccos(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos\,(cx)+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/((b*arccos(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))}\,dx = \int \frac{\left(1-c^2\,x^2\right)^{3/2}}{x^2\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acos(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acos(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.18

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))}\,dx = \frac{-\left(\int \frac{\sqrt{-c^2x^2+1}}{acos(cx)b+a}dx\right)b\,c^2+\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)b\,x^2+\sqrt{-c^2x^2+1}\,a\,x^2}dx\right)b+\log(acos$$

input
```
int((-c^2*x^2+1)^(3/2)/x^2/(a+b*acos(c*x)),x)
```

output
```
( - int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b + a),x)*b*c**2 + int(1/(sqrt(
- c**2*x**2 + 1)*acos(c*x)*b*x**2 + sqrt( - c**2*x**2 + 1)*a*x**2),x)*b +
log(acos(c*x)*b + a)*c)/b
```

3.333 $\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))}\,dx=\text{Int}\left(\frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 6.82 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))}\,dx=\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^3(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{ 5235}$$
$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^3(a+b\arccos(cx))}dx$$

input
```
Int[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{x^3\left(a+b\arccos\left(cx\right)\right)}dx$$

input
```
int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccos(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos(cx)+a)x^3}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b*x^3*arccos(c*x) + a*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{x^3(a+b\operatorname{acos}(cx))}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/x**3/(a+b*acos(c*x)),x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**3*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos(cx)+a)x^3}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/((b*arccos(c*x) + a)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{(1-c^2\,x^2)^{3/2}}{x^3\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*acos(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*acos(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.29

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)\,b\,x^3+a\,x^3}dx - \left(\int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)\,bx+ax}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(3/2)/x^3/(a+b*acos(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b*x**3 + a*x**3),x) - int(sqrt( - c*
*2*x**2 + 1)/(acos(c*x)*b*x + a*x),x)*c**2
```

3.334 $\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))}\,dx = \int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{5235}$$
$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))}dx$$

input
```
Int[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x^4\left(a+b\arccos\left(cx\right)\right)}dx$$

input
```
int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccos(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos(cx)+a)x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b*x^4*arccos(c*x) + a*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))}\,dx = \int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{x^4(a+b\operatorname{acos}(cx))}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/x**4/(a+b*acos(c*x)),x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**4*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos(cx)+a)x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/((b*arccos(c*x) + a)*x^4), x)
```

Giac [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos(cx)+a)x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/((b*arccos(c*x) + a)*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))}\,dx = \int \frac{(1-c^2\,x^2)^{3/2}}{x^4\,(a+b\,\mathrm{acos}\,(c\,x))}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(x^4*(a + b*acos(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(x^4*(a + b*acos(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.89

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))}\,dx = \frac{\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)b\,x^4+\sqrt{-c^2x^2+1}\,a\,x^4}dx\right)b-2\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)b\,x^2+\sqrt{-c^2x^2+1}\,a\,x}\right.}{b}$$

input
```
int((-c^2*x^2+1)^(3/2)/x^4/(a+b*acos(c*x)),x)
```

output
```
(int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*x**4 + sqrt( - c**2*x**2 + 1)*a
*x**4),x)*b - 2*int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*x**2 + sqrt( - c
**2*x**2 + 1)*a*x**2),x)*b*c**2 - log(acos(c*x)*b + a)*c**3)/b
```

3.335 $\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 245

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = -\frac{3\,\mathrm{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{128bc^4} - \frac{\mathrm{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{32bc^4} + \frac{3\,\mathrm{CosIntegral}\left(\frac{7(a+b\arccos(cx))}{b}\right)\sin\left(\frac{7a}{b}\right)}{256bc^4} + \frac{\mathrm{CosIntegral}\left(\frac{9(a+b\arccos(cx))}{b}\right)\sin\left(\frac{9a}{b}\right)}{256bc^4} + \frac{3\cos\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{128bc^4} + \frac{\cos\left(\frac{3a}{b}\right)\mathrm{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{32bc^4} - \frac{3\cos\left(\frac{7a}{b}\right)\mathrm{Si}\left(\frac{7(a+b\arccos(cx))}{b}\right)}{256bc^4} - \frac{\cos\left(\frac{9a}{b}\right)\mathrm{Si}\left(\frac{9(a+b\arccos(cx))}{b}\right)}{256bc^4}$$

output

```
-3/128*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c^4-1/32*Ci(3*(a+b*arccos(c*x))/
b)*sin(3*a/b)/b/c^4+3/256*Ci(7*(a+b*arccos(c*x))/b)*sin(7*a/b)/b/c^4+1/256
*Ci(9*(a+b*arccos(c*x))/b)*sin(9*a/b)/b/c^4+3/128*cos(a/b)*Si((a+b*arccos(
c*x))/b)/b/c^4+1/32*cos(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b/c^4-3/256*cos(7
*a/b)*Si(7*(a+b*arccos(c*x))/b)/b/c^4-1/256*cos(9*a/b)*Si(9*(a+b*arccos(c*
x))/b)/b/c^4
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.73

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \frac{-6\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)+8\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(c\right.\right.}{}$$

input
```
Integrate[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x]),x]
```

output
```
(-6*Cos[a/b]*CosIntegral[a/b + ArcCos[c*x]] + 8*Cos[(3*a)/b]*CosIntegral[3
*(a/b + ArcCos[c*x])] - 3*Cos[(7*a)/b]*CosIntegral[7*(a/b + ArcCos[c*x])]
+ Cos[(9*a)/b]*CosIntegral[9*(a/b + ArcCos[c*x])] - 6*Sin[a/b]*SinIntegral
[a/b + ArcCos[c*x]] + 8*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])] -
3*Sin[(7*a)/b]*SinIntegral[7*(a/b + ArcCos[c*x])] + Sin[(9*a)/b]*SinIntegr
al[9*(a/b + ArcCos[c*x])])/(256*b*c^4)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\int \frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^6\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^4}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \left(-\frac{\cos\left(\frac{9a}{b}-\frac{9(a+b\arccos(cx))}{b}\right)}{256(a+b\arccos(cx))}+\frac{3\cos\left(\frac{7a}{b}-\frac{7(a+b\arccos(cx))}{b}\right)}{256(a+b\arccos(cx))}-\frac{\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}+\frac{3\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{128(a+b\arccos(cx))}\right)d(a+b\,a}{bc^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{3}{128}\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{1}{32}\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{3}{256}\cos\left(\frac{7a}{b}\right)\text{CosIntegral}\left(\right.}{}$$

input
```
Int[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x]),x]
```

output
```
-(((3*Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/128 - (Cos[(3*a)/b]*Cos
Integral[(3*(a + b*ArcCos[c*x]))/b])/32 + (3*Cos[(7*a)/b]*CosIntegral[(7*(
a + b*ArcCos[c*x]))/b])/256 - (Cos[(9*a)/b]*CosIntegral[(9*(a + b*ArcCos[c
*x]))/b])/256 + (3*Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/128 - (Sin
[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/32 + (3*Sin[(7*a)/b]*Sin
Integral[(7*(a + b*ArcCos[c*x]))/b])/256 - (Sin[(9*a)/b]*SinIntegral[(9*(a
 + b*ArcCos[c*x]))/b])/256)/(b*c^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.76

method	result
default	$-\frac{3\,\mathrm{Si}\left(7\arccos(cx)+\frac{7a}{b}\right)\sin\left(\frac{7a}{b}\right)+3\,\mathrm{Ci}\left(7\arccos(cx)+\frac{7a}{b}\right)\cos\left(\frac{7a}{b}\right)-\mathrm{Si}\left(9\arccos(cx)+\frac{9a}{b}\right)\sin\left(\frac{9a}{b}\right)-\mathrm{Ci}\left(9\arccos(cx)+\frac{9a}{b}\right)\cos\left(\frac{9a}{b}\right)}{256c}$

input
```
int(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/256/c^4*(3*Si(7*arccos(c*x)+7*a/b)*sin(7*a/b)+3*Ci(7*arccos(c*x)+7*a/b)
*cos(7*a/b)-Si(9*arccos(c*x)+9*a/b)*sin(9*a/b)-Ci(9*arccos(c*x)+9*a/b)*cos
(9*a/b)+6*Si(arccos(c*x)+a/b)*sin(a/b)+6*Ci(arccos(c*x)+a/b)*cos(a/b)-8*Si
(3*arccos(c*x)+3*a/b)*sin(3*a/b)-8*Ci(3*arccos(c*x)+3*a/b)*cos(3*a/b))/b
```

Fricas [F]

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^3}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^7 - 2*c^2*x^5 + x^3)*sqrt(-c^2*x^2 + 1)/(b*arccos(c*x) + a
), x)
```

Sympy [F]

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx=\int \frac{x^3(-(cx-1)(cx+1))^{\frac{5}{2}}}{a+b\,\mathrm{acos}(cx)}\,dx$$

input
```
integrate(x**3*(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**3*(-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^3}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)*x^3/(b*arccos(c*x) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. 2(229) = 458.

Time = 0.18 (sec) , antiderivative size = 745, normalized size of antiderivative = 3.04

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
cos(a/b)^9*cos_integral(9*a/b + 9*arccos(c*x))/(b*c^4) + cos(a/b)^8*sin(a/
b)*sin_integral(9*a/b + 9*arccos(c*x))/(b*c^4) - 9/4*cos(a/b)^7*cos_integr
al(9*a/b + 9*arccos(c*x))/(b*c^4) - 3/4*cos(a/b)^7*cos_integral(7*a/b + 7*
arccos(c*x))/(b*c^4) - 7/4*cos(a/b)^6*sin(a/b)*sin_integral(9*a/b + 9*arcc
os(c*x))/(b*c^4) - 3/4*cos(a/b)^6*sin(a/b)*sin_integral(7*a/b + 7*arccos(c
*x))/(b*c^4) + 27/16*cos(a/b)^5*cos_integral(9*a/b + 9*arccos(c*x))/(b*c^4
) + 21/16*cos(a/b)^5*cos_integral(7*a/b + 7*arccos(c*x))/(b*c^4) + 15/16*c
os(a/b)^4*sin(a/b)*sin_integral(9*a/b + 9*arccos(c*x))/(b*c^4) + 15/16*cos
(a/b)^4*sin(a/b)*sin_integral(7*a/b + 7*arccos(c*x))/(b*c^4) - 15/32*cos(a
/b)^3*cos_integral(9*a/b + 9*arccos(c*x))/(b*c^4) - 21/32*cos(a/b)^3*cos_i
ntegral(7*a/b + 7*arccos(c*x))/(b*c^4) + 1/8*cos(a/b)^3*cos_integral(3*a/b
 + 3*arccos(c*x))/(b*c^4) - 5/32*cos(a/b)^2*sin(a/b)*sin_integral(9*a/b +
9*arccos(c*x))/(b*c^4) - 9/32*cos(a/b)^2*sin(a/b)*sin_integral(7*a/b + 7*a
rccos(c*x))/(b*c^4) + 1/8*cos(a/b)^2*sin(a/b)*sin_integral(3*a/b + 3*arcco
s(c*x))/(b*c^4) + 9/256*cos(a/b)*cos_integral(9*a/b + 9*arccos(c*x))/(b*c^
4) + 21/256*cos(a/b)*cos_integral(7*a/b + 7*arccos(c*x))/(b*c^4) - 3/32*co
s(a/b)*cos_integral(3*a/b + 3*arccos(c*x))/(b*c^4) - 3/128*cos(a/b)*cos_in
tegral(a/b + arccos(c*x))/(b*c^4) + 1/256*sin(a/b)*sin_integral(9*a/b + 9*
arccos(c*x))/(b*c^4) + 3/256*sin(a/b)*sin_integral(7*a/b + 7*arccos(c*x))/
(b*c^4) - 1/32*sin(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b*c^4) - 3...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx=\int \frac{x^3\,(1-c^2\,x^2)^{5/2}}{a+b\,\mathrm{acos}\,(c\,x)}\,dx$$

input
```
int((x^3*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x)),x)
```

output
```
int((x^3*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \left(\int \frac{\sqrt{-c^2x^2+1}\,x^7}{acos\,(cx)\,b+a}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^5}{acos\,(cx)\,b+a}dx\right)c^2 + \int \frac{\sqrt{-c^2x^2+1}\,x^3}{acos\,(cx)\,b+a}dx$$

input
```
int(x^3*(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x)),x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**7)/(acos(c*x)*b + a),x)*c**4 - 2*int((sqrt(
 - c**2*x**2 + 1)*x**5)/(acos(c*x)*b + a),x)*c**2 + int((sqrt( - c**2*x**2
 + 1)*x**3)/(acos(c*x)*b + a),x)
```

3.336 $\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 268

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{32bc^3} - \frac{\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{32bc^3} - \frac{\cos\left(\frac{6a}{b}\right)\text{CosIntegral}\left(\frac{6(a+b\arccos(cx))}{b}\right)}{32bc^3} - \frac{\cos\left(\frac{8a}{b}\right)\text{CosIntegral}\left(\frac{8(a+b\arccos(cx))}{b}\right)}{128bc^3} + \frac{5\log(a+b\arccos(cx))}{128bc^3} + \frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{32bc^3} - \frac{\sin\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{32bc^3} - \frac{\sin\left(\frac{6a}{b}\right)\text{Si}\left(\frac{6(a+b\arccos(cx))}{b}\right)}{32bc^3} - \frac{\sin\left(\frac{8a}{b}\right)\text{Si}\left(\frac{8(a+b\arccos(cx))}{b}\right)}{128bc^3}$$

output

```
1/32*cos(2*a/b)*Ci(2*(a+b*arccos(c*x))/b)/b/c^3-1/32*cos(4*a/b)*Ci(4*(a+b*
arccos(c*x))/b)/b/c^3-1/32*cos(6*a/b)*Ci(6*(a+b*arccos(c*x))/b)/b/c^3-1/12
8*cos(8*a/b)*Ci(8*(a+b*arccos(c*x))/b)/b/c^3+5/128*ln(a+b*arccos(c*x))/b/c
^3+1/32*sin(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b/c^3-1/32*sin(4*a/b)*Si(4*(a
+b*arccos(c*x))/b)/b/c^3-1/32*sin(6*a/b)*Si(6*(a+b*arccos(c*x))/b)/b/c^3-1
/128*sin(8*a/b)*Si(8*(a+b*arccos(c*x))/b)/b/c^3
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.78

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \frac{4\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)+4\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(4\left(\frac{a}{b}+\text{arcco}\right.\right.}{}$$

input
```
Integrate[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x]),x]
```

output
```
(4*Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcCos[c*x])] + 4*Cos[(4*a)/b]*CosInt
egral[4*(a/b + ArcCos[c*x])] - 4*Cos[(6*a)/b]*CosIntegral[6*(a/b + ArcCos[
c*x])] + Cos[(8*a)/b]*CosIntegral[8*(a/b + ArcCos[c*x])] + 11*Log[a + b*Ar
cCos[c*x]] - 16*Log[8*(a + b*ArcCos[c*x])] + 4*Sin[(2*a)/b]*SinIntegral[2*
(a/b + ArcCos[c*x])] + 4*Sin[(4*a)/b]*SinIntegral[4*(a/b + ArcCos[c*x])] -
 4*Sin[(6*a)/b]*SinIntegral[6*(a/b + ArcCos[c*x])] + Sin[(8*a)/b]*SinInteg
ral[8*(a/b + ArcCos[c*x])])/(128*b*c^3)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{ 5225}$$

$$-\frac{\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^6\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^3}$$

$$\Big\downarrow \text{ 4906}$$

$$-\frac{\int \left(-\frac{\cos\left(\frac{8a}{b}-\frac{8(a+b\arccos(cx))}{b}\right)}{128(a+b\arccos(cx))}+\frac{\cos\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}-\frac{\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}-\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}+\frac{}{128(a+b}\right.}{bc^3}$$

$\downarrow$ 2009

$$-\frac{-\frac{1}{32}\cos\left(\frac{2a}{b}\right)\operatorname{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\frac{1}{32}\cos\left(\frac{4a}{b}\right)\operatorname{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{32}\cos\left(\frac{6a}{b}\right)\operatorname{CosIntegr}}{}$$

input
```
Int[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x]),x]
```

output
```
-((-1/32*(Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcCos[c*x]))/b]) - (Cos[(4*a
)/b]*CosIntegral[(4*(a + b*ArcCos[c*x]))/b])/32 + (Cos[(6*a)/b]*CosIntegra
l[(6*(a + b*ArcCos[c*x]))/b])/32 - (Cos[(8*a)/b]*CosIntegral[(8*(a + b*Arc
Cos[c*x]))/b])/128 + (5*Log[a + b*ArcCos[c*x]])/128 - (Sin[(2*a)/b]*SinInt
egral[(2*(a + b*ArcCos[c*x]))/b])/32 - (Sin[(4*a)/b]*SinIntegral[(4*(a + b
*ArcCos[c*x]))/b])/32 + (Sin[(6*a)/b]*SinIntegral[(6*(a + b*ArcCos[c*x]))/
b])/32 - (Sin[(8*a)/b]*SinIntegral[(8*(a + b*ArcCos[c*x]))/b])/128)/(b*c^3
))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.76

method	result
default	$\frac{4\ \mathrm{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)+\mathrm{Ci}\left(8\arccos(cx)+\frac{8a}{b}\right)\cos\left(\frac{8a}{b}\right)+4\ \mathrm{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)-4\ \mathrm{Si}\left(6\arccos(cx)+\frac{6a}{b}\right)\sin\left(\frac{6a}{b}\right)-}{}$

input
```
int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/128/c^3*(4*Si(2*arccos(c*x)+2*a/b)*sin(2*a/b)+Ci(8*arccos(c*x)+8*a/b)*co
s(8*a/b)+4*Ci(2*arccos(c*x)+2*a/b)*cos(2*a/b)-4*Si(6*arccos(c*x)+6*a/b)*si
n(6*a/b)-4*Ci(6*arccos(c*x)+6*a/b)*cos(6*a/b)+4*Si(4*arccos(c*x)+4*a/b)*si
n(4*a/b)+4*Ci(4*arccos(c*x)+4*a/b)*cos(4*a/b)+Si(8*arccos(c*x)+8*a/b)*sin(
8*a/b)-5*ln(a+b*arccos(c*x)))/b
```

Fricas [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^2}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^6 - 2*c^2*x^4 + x^2)*sqrt(-c^2*x^2 + 1)/(b*arccos(c*x) + a
), x)
```

Sympy [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{x^2(-(cx-1)(cx+1))^{\frac{5}{2}}}{a+b\,\mathrm{acos}\,(cx)}\,dx$$

input
```
integrate(x**2*(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**2*(-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^2}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)*x^2/(b*arccos(c*x) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(250) = 500$.

Time = 0.17 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.82

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
cos(a/b)^8*cos_integral(8*a/b + 8*arccos(c*x))/(b*c^3) + cos(a/b)^7*sin(a/
b)*sin_integral(8*a/b + 8*arccos(c*x))/(b*c^3) - 2*cos(a/b)^6*cos_integral
(8*a/b + 8*arccos(c*x))/(b*c^3) - cos(a/b)^6*cos_integral(6*a/b + 6*arccos
(c*x))/(b*c^3) - 3/2*cos(a/b)^5*sin(a/b)*sin_integral(8*a/b + 8*arccos(c*x
))/(b*c^3) - cos(a/b)^5*sin(a/b)*sin_integral(6*a/b + 6*arccos(c*x))/(b*c^
3) + 5/4*cos(a/b)^4*cos_integral(8*a/b + 8*arccos(c*x))/(b*c^3) + 3/2*cos(
a/b)^4*cos_integral(6*a/b + 6*arccos(c*x))/(b*c^3) + 1/4*cos(a/b)^4*cos_in
tegral(4*a/b + 4*arccos(c*x))/(b*c^3) + 5/8*cos(a/b)^3*sin(a/b)*sin_integr
al(8*a/b + 8*arccos(c*x))/(b*c^3) + cos(a/b)^3*sin(a/b)*sin_integral(6*a/b
 + 6*arccos(c*x))/(b*c^3) + 1/4*cos(a/b)^3*sin(a/b)*sin_integral(4*a/b + 4
*arccos(c*x))/(b*c^3) - 1/4*cos(a/b)^2*cos_integral(8*a/b + 8*arccos(c*x))
/(b*c^3) - 9/16*cos(a/b)^2*cos_integral(6*a/b + 6*arccos(c*x))/(b*c^3) - 1
/4*cos(a/b)^2*cos_integral(4*a/b + 4*arccos(c*x))/(b*c^3) + 1/16*cos(a/b)^
2*cos_integral(2*a/b + 2*arccos(c*x))/(b*c^3) - 1/16*cos(a/b)*sin(a/b)*sin
_integral(8*a/b + 8*arccos(c*x))/(b*c^3) - 3/16*cos(a/b)*sin(a/b)*sin_inte
gral(6*a/b + 6*arccos(c*x))/(b*c^3) - 1/8*cos(a/b)*sin(a/b)*sin_integral(4
*a/b + 4*arccos(c*x))/(b*c^3) + 1/16*cos(a/b)*sin(a/b)*sin_integral(2*a/b
+ 2*arccos(c*x))/(b*c^3) + 1/128*cos_integral(8*a/b + 8*arccos(c*x))/(b*c^
3) + 1/32*cos_integral(6*a/b + 6*arccos(c*x))/(b*c^3) + 1/32*cos_integral(
4*a/b + 4*arccos(c*x))/(b*c^3) - 1/32*cos_integral(2*a/b + 2*arccos(c*x...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx=\int \frac{x^2\,{\left(1-c^2\,x^2\right)}^{5/2}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x)),x)
```

output
```
int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \left(\int \frac{\sqrt{-c^2x^2+1}\,x^6}{acos\,(cx)\,b+a}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^4}{acos\,(cx)\,b+a}dx\right)c^2 + \int \frac{\sqrt{-c^2x^2+1}\,x^2}{acos\,(cx)\,b+a}dx$$

input
```
int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x)),x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**6)/(acos(c*x)*b + a),x)*c**4 - 2*int((sqrt(
 - c**2*x**2 + 1)*x**4)/(acos(c*x)*b + a),x)*c**2 + int((sqrt( - c**2*x**2
 + 1)*x**2)/(acos(c*x)*b + a),x)
```

3.337 $\int \frac{x(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 245

$$\begin{aligned}\int \frac{x(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = &-\frac{5\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{64bc^2}\\&-\frac{9\,\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{64bc^2}\\&-\frac{5\,\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)\sin\left(\frac{5a}{b}\right)}{64bc^2}-\frac{\text{CosIntegral}\left(\frac{7(a+b\arccos(cx))}{b}\right)\sin\left(\frac{7a}{b}\right)}{64bc^2}\\&+\frac{5\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{64bc^2}+\frac{9\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{64bc^2}\\&+\frac{5\cos\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{64bc^2}+\frac{\cos\left(\frac{7a}{b}\right)\text{Si}\left(\frac{7(a+b\arccos(cx))}{b}\right)}{64bc^2}\end{aligned}$$

output
```
-5/64*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c^2-9/64*Ci(3*(a+b*arccos(c*x))/b
)*sin(3*a/b)/b/c^2-5/64*Ci(5*(a+b*arccos(c*x))/b)*sin(5*a/b)/b/c^2-1/64*Ci
(7*(a+b*arccos(c*x))/b)*sin(7*a/b)/b/c^2+5/64*cos(a/b)*Si((a+b*arccos(c*x)
)/b)/b/c^2+9/64*cos(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b/c^2+5/64*cos(5*a/b)
*Si(5*(a+b*arccos(c*x))/b)/b/c^2+1/64*cos(7*a/b)*Si(7*(a+b*arccos(c*x))/b)
/b/c^2
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.73

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \frac{-5\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)+9\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(c\right.\right.}{}$$

input
```
Integrate[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x]),x]
```

output
```
(-5*Cos[a/b]*CosIntegral[a/b + ArcCos[c*x]] + 9*Cos[(3*a)/b]*CosIntegral[3
*(a/b + ArcCos[c*x])] - 5*Cos[(5*a)/b]*CosIntegral[5*(a/b + ArcCos[c*x])]
+ Cos[(7*a)/b]*CosIntegral[7*(a/b + ArcCos[c*x])] - 5*Sin[a/b]*SinIntegral
[a/b + ArcCos[c*x]] + 9*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])] -
5*Sin[(5*a)/b]*SinIntegral[5*(a/b + ArcCos[c*x])] + Sin[(7*a)/b]*SinIntegr
al[7*(a/b + ArcCos[c*x])])/(64*b*c^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^6\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^2}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \left(-\frac{\cos\left(\frac{7a}{b}-\frac{7(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}+\frac{5\cos\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}-\frac{9\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}+\frac{5\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{64(a+b\arccos(cx))}\right)d(a+b}{bc^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{5}{64}\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{9}{64}\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{5}{64}\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5}{}\right.}{}$$

input
```
Int[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x]),x]
```

output
```
-(((5*Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/64 - (9*Cos[(3*a)/b]*Co
sIntegral[(3*(a + b*ArcCos[c*x]))/b])/64 + (5*Cos[(5*a)/b]*CosIntegral[(5*
(a + b*ArcCos[c*x]))/b])/64 - (Cos[(7*a)/b]*CosIntegral[(7*(a + b*ArcCos[c
*x]))/b])/64 + (5*Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/64 - (9*Sin
[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/64 + (5*Sin[(5*a)/b]*Sin
Integral[(5*(a + b*ArcCos[c*x]))/b])/64 - (Sin[(7*a)/b]*SinIntegral[(7*(a
+ b*ArcCos[c*x]))/b])/64)/(b*c^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.76

method	result
default	$-\frac{5\,\mathrm{Si}\left(5\arccos(cx)+\frac{5a}{b}\right)\sin\left(\frac{5a}{b}\right)+5\,\mathrm{Ci}\left(5\arccos(cx)+\frac{5a}{b}\right)\cos\left(\frac{5a}{b}\right)-\mathrm{Si}\left(7\arccos(cx)+\frac{7a}{b}\right)\sin\left(\frac{7a}{b}\right)-\mathrm{Ci}\left(7\arccos(cx)+\frac{7a}{b}\right)\cos\left(\frac{7a}{b}\right)}{64c^2}$

input
```
int(x*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/64/c^2*(5*Si(5*arccos(c*x)+5*a/b)*sin(5*a/b)+5*Ci(5*arccos(c*x)+5*a/b)*
cos(5*a/b)-Si(7*arccos(c*x)+7*a/b)*sin(7*a/b)-Ci(7*arccos(c*x)+7*a/b)*cos(
7*a/b)-9*Si(3*arccos(c*x)+3*a/b)*sin(3*a/b)-9*Ci(3*arccos(c*x)+3*a/b)*cos(
3*a/b)+5*Si(arccos(c*x)+a/b)*sin(a/b)+5*Ci(arccos(c*x)+a/b)*cos(a/b))/b
```

Fricas [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^5 - 2*c^2*x^3 + x)*sqrt(-c^2*x^2 + 1)/(b*arccos(c*x) + a),
 x)
```

Sympy [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{x(-(cx-1)(cx+1))^{\frac{5}{2}}}{a+b\,\mathrm{acos}(cx)}\,dx$$

input
```
integrate(x*(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x*(-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)*x/(b*arccos(c*x) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(229) = 458$.

Time = 0.17 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.50

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \text{Too large to display}$$

input
```
integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
cos(a/b)^7*cos_integral(7*a/b + 7*arccos(c*x))/(b*c^2) + cos(a/b)^6*sin(a/
b)*sin_integral(7*a/b + 7*arccos(c*x))/(b*c^2) - 7/4*cos(a/b)^5*cos_integr
al(7*a/b + 7*arccos(c*x))/(b*c^2) - 5/4*cos(a/b)^5*cos_integral(5*a/b + 5*
arccos(c*x))/(b*c^2) - 5/4*cos(a/b)^4*sin(a/b)*sin_integral(7*a/b + 7*arcc
os(c*x))/(b*c^2) - 5/4*cos(a/b)^4*sin(a/b)*sin_integral(5*a/b + 5*arccos(c
*x))/(b*c^2) + 7/8*cos(a/b)^3*cos_integral(7*a/b + 7*arccos(c*x))/(b*c^2)
+ 25/16*cos(a/b)^3*cos_integral(5*a/b + 5*arccos(c*x))/(b*c^2) + 9/16*cos(
a/b)^3*cos_integral(3*a/b + 3*arccos(c*x))/(b*c^2) + 3/8*cos(a/b)^2*sin(a/
b)*sin_integral(7*a/b + 7*arccos(c*x))/(b*c^2) + 15/16*cos(a/b)^2*sin(a/b)
*sin_integral(5*a/b + 5*arccos(c*x))/(b*c^2) + 9/16*cos(a/b)^2*sin(a/b)*si
n_integral(3*a/b + 3*arccos(c*x))/(b*c^2) - 7/64*cos(a/b)*cos_integral(7*a
/b + 7*arccos(c*x))/(b*c^2) - 25/64*cos(a/b)*cos_integral(5*a/b + 5*arccos
(c*x))/(b*c^2) - 27/64*cos(a/b)*cos_integral(3*a/b + 3*arccos(c*x))/(b*c^2
) - 5/64*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b*c^2) - 1/64*sin(a/b)*
sin_integral(7*a/b + 7*arccos(c*x))/(b*c^2) - 5/64*sin(a/b)*sin_integral(5
*a/b + 5*arccos(c*x))/(b*c^2) - 9/64*sin(a/b)*sin_integral(3*a/b + 3*arcco
s(c*x))/(b*c^2) - 5/64*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{x\,(1-c^2\,x^2)^{5/2}}{a+b\,\mathrm{acos}\,(c\,x)}\,dx$$

input
```
int((x*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x)),x)
```

output
```
int((x*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \left(\int \frac{\sqrt{-c^2x^2+1}\,x^5}{acos\,(cx)\,b+a}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^3}{acos\,(cx)\,b+a}dx\right)c^2 + \int \frac{\sqrt{-c^2x^2+1}\,x}{acos\,(cx)\,b+a}dx$$

input
```
int(x*(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x)),x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**5)/(acos(c*x)*b + a),x)*c**4 - 2*int((sqrt(
 - c**2*x**2 + 1)*x**3)/(acos(c*x)*b + a),x)*c**2 + int((sqrt( - c**2*x**2
 + 1)*x)/(acos(c*x)*b + a),x)
```

3.338 $\int \frac{(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 206

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \frac{15\cos\left(\frac{2a}{b}\right)\operatorname{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{32bc} + \frac{3\cos\left(\frac{4a}{b}\right)\operatorname{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{16bc} + \frac{\cos\left(\frac{6a}{b}\right)\operatorname{CosIntegral}\left(\frac{6(a+b\arccos(cx))}{b}\right)}{32bc} + \frac{5\log(a+b\arccos(cx))}{16bc} + \frac{15\sin\left(\frac{2a}{b}\right)\operatorname{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{32bc} + \frac{3\sin\left(\frac{4a}{b}\right)\operatorname{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{16bc} + \frac{\sin\left(\frac{6a}{b}\right)\operatorname{Si}\left(\frac{6(a+b\arccos(cx))}{b}\right)}{32bc}$$

output

```
15/32*cos(2*a/b)*Ci(2*(a+b*arccos(c*x))/b)/b/c+3/16*cos(4*a/b)*Ci(4*(a+b*a
rccos(c*x))/b)/b/c+1/32*cos(6*a/b)*Ci(6*(a+b*arccos(c*x))/b)/b/c+5/16*ln(a
+b*arccos(c*x))/b/c+15/32*sin(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b/c+3/16*si
n(4*a/b)*Si(4*(a+b*arccos(c*x))/b)/b/c+1/32*sin(6*a/b)*Si(6*(a+b*arccos(c*
x))/b)/b/c
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \frac{15\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)-6\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(4\left(\frac{a}{b}+\text{arcc}\right.\right.}{}$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(a + b*ArcCos[c*x]),x]
```

output
```
(15*Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcCos[c*x])] - 6*Cos[(4*a)/b]*CosIn
tegral[4*(a/b + ArcCos[c*x])] + Cos[(6*a)/b]*CosIntegral[6*(a/b + ArcCos[c
*x])] - 18*Log[a + b*ArcCos[c*x]] + 8*Log[8*(a + b*ArcCos[c*x])] + 15*Sin[
(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])] - 6*Sin[(4*a)/b]*SinIntegral[4
*(a/b + ArcCos[c*x])] + Sin[(6*a)/b]*SinIntegral[6*(a/b + ArcCos[c*x])])/(
32*b*c)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5169}$$

$$-\frac{\int \frac{\sin^6\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^6}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(-\frac{\cos\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}+\frac{3\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}-\frac{15\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}+\frac{5}{16(a+b\arccos(cx))}\right)d(a+b\arc}{bc}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{15}{32}\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{3}{16}\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)-\frac{1}{32}\cos\left(\frac{6a}{b}\right)\text{CosIntegr}}{}$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(a + b*ArcCos[c*x]),x]
```

output
```
-(((-15*Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcCos[c*x]))/b])/32 + (3*Cos[(
4*a)/b]*CosIntegral[(4*(a + b*ArcCos[c*x]))/b])/16 - (Cos[(6*a)/b]*CosInte
gral[(6*(a + b*ArcCos[c*x]))/b])/32 + (5*Log[a + b*ArcCos[c*x]])/16 - (15*
Sin[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/32 + (3*Sin[(4*a)/b]*
SinIntegral[(4*(a + b*ArcCos[c*x]))/b])/16 - (Sin[(6*a)/b]*SinIntegral[(6*
(a + b*ArcCos[c*x]))/b])/32)/(b*c))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.77

method	result
default	$-\frac{-\operatorname{Si}\left(6\arccos(cx)+\frac{6a}{b}\right)\sin\left(\frac{6a}{b}\right)-\operatorname{Ci}\left(6\arccos(cx)+\frac{6a}{b}\right)\cos\left(\frac{6a}{b}\right)+6\operatorname{Si}\left(4\arccos(cx)+\frac{4a}{b}\right)\sin\left(\frac{4a}{b}\right)+6\operatorname{Ci}\left(4\arccos(cx)+\frac{4a}{b}\right)\cos\left(\frac{4a}{b}\right.}{32cb}$

input
```
int((-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/32/c*(-Si(6*arccos(c*x)+6*a/b)*sin(6*a/b)-Ci(6*arccos(c*x)+6*a/b)*cos(6
*a/b)+6*Si(4*arccos(c*x)+4*a/b)*sin(4*a/b)+6*Ci(4*arccos(c*x)+4*a/b)*cos(4
*a/b)-15*Si(2*arccos(c*x)+2*a/b)*sin(2*a/b)-15*Ci(2*arccos(c*x)+2*a/b)*cos
(2*a/b)+10*ln(a+b*arccos(c*x)))/b
```

Fricas [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{b\arccos(cx)+a}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*arccos(c*x) + a),
 x)
```

Sympy [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx=\int \frac{(-(cx-1)(cx+1))^{\frac{5}{2}}}{a+b\operatorname{acos}(cx)}\,dx$$

input
```
integrate((-c**2*x**2+1)**(5/2)/(a+b*acos(c*x)),x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{b\arccos(cx)+a}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/(b*arccos(c*x) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(192) = 384.

Time = 0.17 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.29

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
cos(a/b)^6*cos_integral(6*a/b + 6*arccos(c*x))/(b*c) + cos(a/b)^5*sin(a/b)
*sin_integral(6*a/b + 6*arccos(c*x))/(b*c) - 3/2*cos(a/b)^4*cos_integral(6
*a/b + 6*arccos(c*x))/(b*c) - 3/2*cos(a/b)^4*cos_integral(4*a/b + 4*arccos
(c*x))/(b*c) - cos(a/b)^3*sin(a/b)*sin_integral(6*a/b + 6*arccos(c*x))/(b*
c) - 3/2*cos(a/b)^3*sin(a/b)*sin_integral(4*a/b + 4*arccos(c*x))/(b*c) + 9
/16*cos(a/b)^2*cos_integral(6*a/b + 6*arccos(c*x))/(b*c) + 3/2*cos(a/b)^2*
cos_integral(4*a/b + 4*arccos(c*x))/(b*c) + 15/16*cos(a/b)^2*cos_integral(
2*a/b + 2*arccos(c*x))/(b*c) + 3/16*cos(a/b)*sin(a/b)*sin_integral(6*a/b +
 6*arccos(c*x))/(b*c) + 3/4*cos(a/b)*sin(a/b)*sin_integral(4*a/b + 4*arcco
s(c*x))/(b*c) + 15/16*cos(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arccos(c*x)
)/(b*c) - 1/32*cos_integral(6*a/b + 6*arccos(c*x))/(b*c) - 3/16*cos_integr
al(4*a/b + 4*arccos(c*x))/(b*c) - 15/32*cos_integral(2*a/b + 2*arccos(c*x)
)/(b*c) - 5/16*log(b*arccos(c*x) + a)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{(1-c^2\,x^2)^{5/2}}{a+b\,\mathrm{acos}\,(c\,x)}\,dx$$

input
```
int((1 - c^2*x^2)^(5/2)/(a + b*acos(c*x)),x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)\,b+a}dx + \left(\int \frac{\sqrt{-c^2x^2+1}\,x^4}{acos\,(cx)\,b+a}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acos\,(cx)\,b+a}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(5/2)/(a+b*acos(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b + a),x) + int((sqrt( - c**2*x**2 +
 1)*x**4)/(acos(c*x)*b + a),x)*c**4 - 2*int((sqrt( - c**2*x**2 + 1)*x**2)/
(acos(c*x)*b + a),x)*c**2
```

3.339 $\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))}\,dx = \frac{11\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{8b} + \frac{7\,\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{16b} + \frac{\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)\sin\left(\frac{5a}{b}\right)}{16b} - \frac{11\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b} - \frac{7\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b} - \frac{\cos\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b} + \text{Int}\left(\frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))},x\right)$$

output
```
11/8*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b+7/16*Ci(3*(a+b*arccos(c*x))/b)*sin
(3*a/b)/b+1/16*Ci(5*(a+b*arccos(c*x))/b)*sin(5*a/b)/b-11/8*cos(a/b)*Si((a+
b*arccos(c*x))/b)/b-7/16*cos(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b-1/16*cos(5
*a/b)*Si(5*(a+b*arccos(c*x))/b)/b+Defer(Int)(1/x/(-c^2*x^2+1)^(1/2)/(a+b*a
rccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))}\,dx = \int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))}\,dx$$

$$\Big\downarrow \ 5227$$

$$\int \left(-\frac{3c^2x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}+\frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}-\frac{c^6x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}+\frac{}{\sqrt{1-c^2x}}\right.$$

$$\Big\downarrow \ 2009$$

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx+\frac{11\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b}-\frac{7\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b}+\frac{\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b}+\frac{11\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b}-\frac{7\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b}+\frac{\sin\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b}$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x\,(a+b\arccos\,(cx))}dx$$

input
```
int((-c^2*x^2+1)^(5/2)/x/(a+b*arccos(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(5/2)/x/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos\,(cx)+a)x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x*arccos(c*x) + a
*x), x)
```

Sympy [N/A]

Not integrable

Time = 4.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))}\,dx = \int \frac{(-(cx-1)\,(cx+1))^{\frac{5}{2}}}{x\,(a+b\,\mathrm{acos}\,(cx))}\,dx$$

input
```
integrate((-c**2*x**2+1)**(5/2)/x/(a+b*acos(c*x)),x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos\,(cx)+a)x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/((b*arccos(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))}\,dx = \int \frac{{\left(1-c^2\,x^2\right)}^{5/2}}{x\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)} \,d x$$

input
```
int((1 - c^2*x^2)^(5/2)/(x*(a + b*acos(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(x*(a + b*acos(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.18

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-c^{2} x^{2}+1}}{acos\left(c x \right) b x +a x}d x \\ +\left(\int \frac{\sqrt{-c^{2} x^{2}+1}\, x^{3}}{acos\left(c x \right) b +a}d x \right) c^{4}-2 \left(\int \frac{\sqrt{-c^{2} x^{2}+1}\, x}{acos\left(c x \right) b +a}d x \right) c^{2}$$

input
```
int((-c^2*x^2+1)^(5/2)/x/(a+b*acos(c*x)),x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b*x + a*x),x) + int((sqrt( - c**2*x*
*2 + 1)*x**3)/(acos(c*x)*b + a),x)*c**4 - 2*int((sqrt( - c**2*x**2 + 1)*x)
/(acos(c*x)*b + a),x)*c**2
```

3.340 $\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))}\,dx = -\frac{c\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b} - \frac{c\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{8b} - \frac{15c\log(a+b\arccos(cx))}{8b} - \frac{c\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b} - \frac{c\sin\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{8b} + \text{Int}\left(\frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))},x\right)$$

output

```
-c*cos(2*a/b)*Ci(2*(a+b*arccos(c*x))/b)/b-1/8*c*cos(4*a/b)*Ci(4*(a+b*arcco
s(c*x))/b)/b-15/8*c*ln(a+b*arccos(c*x))/b-c*sin(2*a/b)*Si(2*(a+b*arccos(c*
x))/b)/b-1/8*c*sin(4*a/b)*Si(4*(a+b*arccos(c*x))/b)/b+Defer(Int)(1/x^2/(-c
^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.57 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))}\,dx=\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))}\,dx$$

$$\Big\downarrow \ 5227$$

$$\int \left(-\frac{3c^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}+\frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}-\frac{c^6x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}+\frac{}{\sqrt{1-c^2}}\right.$$

$$\Big\downarrow \ 2009$$

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx+\frac{3\log(a+b\arccos(cx))}{b^2}-\frac{c\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b}+\frac{c\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{8b}-\frac{c\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b}+\frac{c\sin\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{8b}-\frac{9c\log(a+b\arccos(cx))}{8b}$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x^2(a+b\arccos(cx))}dx$$

input
```
int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccos(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos(cx)+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x^2*arccos(c*x) +
 a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))}\,dx=\int \frac{(-(cx-1)\,(cx+1))^{\frac{5}{2}}}{x^2\,(a+b\,\mathrm{acos}\,(cx))}\,dx$$

input
```
integrate((-c**2*x**2+1)**(5/2)/x**2/(a+b*acos(c*x)),x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**2*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos\,(cx)+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/((b*arccos(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos(cx)+a)x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/((b*arccos(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))}\,dx = \int \frac{(1-c^2\,x^2)^{5/2}}{x^2\,(a+b\,\mathrm{acos}(c\,x))}\,dx$$

input
```
int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acos(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acos(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 4.32

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))}\,dx = \frac{-2\left(\int \frac{\sqrt{-c^2x^2+1}}{acos(cx)b+a}dx\right)b\,c^2+\left(\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acos(cx)b+a}dx\right)b\,c^4+\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)b\,x^2+\sqrt{-}}\right.}{b}$$

input
```
int((-c^2*x^2+1)^(5/2)/x^2/(a+b*acos(c*x)),x)
```

output
```
( - 2*int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b + a),x)*b*c**2 + int((sqrt(
- c**2*x**2 + 1)*x**2)/(acos(c*x)*b + a),x)*b*c**4 + int(1/(sqrt( - c**2*x
**2 + 1)*acos(c*x)*b*x**2 + sqrt( - c**2*x**2 + 1)*a*x**2),x)*b + log(acos
(c*x)*b + a)*c)/b
```

3.341 $\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))}\,dx = \operatorname{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 6.91 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{ 5235}$$
$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))}dx$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x^3\left(a+b\arccos\left(cx\right)\right)}dx$$

input
```
int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccos(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos{(cx)}+a)x^3}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x^3*arccos(c*x) +
 a*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 3.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{(-(cx-1)(cx+1))^{\frac{5}{2}}}{x^3(a+b\operatorname{acos}{(cx)})}\,dx$$

input
```
integrate((-c**2*x**2+1)**(5/2)/x**3/(a+b*acos(c*x)),x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**3*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos{(cx)}+a)x^3}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/((b*arccos(c*x) + a)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{(1-c^2\,x^2)^{5/2}}{x^3\,(a+b\,\mathrm{acos}\,(c\,x))}\,dx$$

input
```
int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*acos(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*acos(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.32

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)\,b\,x^3+a\,x^3}dx - 2\left(\int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)\,bx+ax}dx\right)c^2 + \left(\int \frac{\sqrt{-c^2x^2+1}\,x}{acos\,(cx)\,b+a}dx\right)c^4$$

input

```
int((-c^2*x^2+1)^(5/2)/x^3/(a+b*acos(c*x)),x)
```

output

```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b*x**3 + a*x**3),x) - 2*int(sqrt( -
c**2*x**2 + 1)/(acos(c*x)*b*x + a*x),x)*c**2 + int((sqrt( - c**2*x**2 + 1)
*x)/(acos(c*x)*b + a),x)*c**4
```

3.342 $\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))}\,dx = \int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{5235}$$
$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))}dx$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x^4\,(a+b\arccos\,(cx))}dx$$

input
```
int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccos(c*x)),x)
```

output
```
int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos(cx)+a)x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x^4*arccos(c*x) +
 a*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 4.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))}\,dx = \int \frac{(-(cx-1)(cx+1))^{\frac{5}{2}}}{x^4(a+b\,\mathrm{acos}(cx))}\,dx$$

input
```
integrate((-c**2*x**2+1)**(5/2)/x**4/(a+b*acos(c*x)),x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**4*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos(cx)+a)x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/((b*arccos(c*x) + a)*x^4), x)
```

Giac [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos{(cx)}+a)x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/((b*arccos(c*x) + a)*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))}\,dx = \int \frac{(1-c^2\,x^2)^{5/2}}{x^4\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}\,dx$$

input
```
int((1 - c^2*x^2)^(5/2)/(x^4*(a + b*acos(c*x))),x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(x^4*(a + b*acos(c*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 4.57

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))}\,dx = \frac{\left(\int \frac{\sqrt{-c^2x^2+1}}{acos(cx)b\,x^4+a\,x^4}dx\right)b+\left(\int \frac{\sqrt{-c^2x^2+1}}{acos(cx)b+a}dx\right)b\,c^4-2\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)b\,x^2+\sqrt{-c}}\right.}{b}$$

input
```
int((-c^2*x^2+1)^(5/2)/x^4/(a+b*acos(c*x)),x)
```

output
```
(int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*b*x**4 + a*x**4),x)*b + int(sqrt( -
 c**2*x**2 + 1)/(acos(c*x)*b + a),x)*b*c**4 - 2*int(1/(sqrt( - c**2*x**2 +
 1)*acos(c*x)*b*x**2 + sqrt( - c**2*x**2 + 1)*a*x**2),x)*b*c**2 - 2*log(ac
os(c*x)*b + a)*c**3)/b
```

3.343 $\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 41

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\text{CosIntegral}(2\arccos(ax))}{2a^5} + \frac{\text{CosIntegral}(4\arccos(ax))}{8a^5} + \frac{3\log(\arccos(ax))}{8a^5}$$

output
```
-1/2*Ci(2*arccos(a*x))/a^5+1/8*Ci(4*arccos(a*x))/a^5+3/8*ln(arccos(a*x))/a
^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{4\,\text{CosIntegral}(2\arccos(ax)) + \text{CosIntegral}(4\arccos(ax)) + 3\log(\arccos(ax))}{8a^5}$$

input
```
Integrate[x^4/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
-1/8*(4*CosIntegral[2*ArcCos[a*x]] + CosIntegral[4*ArcCos[a*x]] + 3*Log[Ar
cCos[a*x]])/a^5
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)}dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\int \frac{a^4x^4}{\arccos(ax)}d\arccos(ax)}{a^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^4}{\arccos(ax)}d\arccos(ax)}{a^5}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{\cos(2\arccos(ax))}{2\arccos(ax)}+\frac{\cos(4\arccos(ax))}{8\arccos(ax)}+\frac{3}{8\arccos(ax)}\right)d\arccos(ax)}{a^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\text{CosIntegral}(2\arccos(ax))+\frac{1}{8}\text{CosIntegral}(4\arccos(ax))+\frac{3}{8}\log(\arccos(ax))}{a^5}$$

input
```
Int[x^4/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
-((CosIntegral[2*ArcCos[a*x]]/2 + CosIntegral[4*ArcCos[a*x]]/8 + (3*Log[Ar
cCos[a*x]])/8)/a^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{3\ln(\arccos(ax))+4\ \mathrm{Ci}(2\arccos(ax))+\mathrm{Ci}(4\arccos(ax))}{8a^5}$	30

input
```
int(x^4/(-a^2*x^2+1)^(1/2)/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(3*ln(arccos(a*x))+4*Ci(2*arccos(a*x))+Ci(4*arccos(a*x)))/a^5
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^4}{\sqrt{-a^2x^2+1}\arccos(ax)}\,dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^4/((a^2*x^2 - 1)*arccos(a*x)), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^4}{\sqrt{-(ax-1)(ax+1)}\operatorname{acos}(ax)}\,dx$$

input
```
integrate(x**4/(-a**2*x**2+1)**(1/2)/acos(a*x),x)
```

output
```
Integral(x**4/(sqrt(-(a*x - 1)*(a*x + 1))*acos(a*x)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^4}{\sqrt{-a^2x^2+1}\arccos(ax)}\,dx$$

input
```
integrate(x^4/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(-a^2*x^2 + 1)*arccos(a*x)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\operatorname{Ci}(4\arccos(ax))}{8a^5} - \frac{\operatorname{Ci}(2\arccos(ax))}{2a^5} - \frac{3\log(\arccos(ax))}{8a^5}$$

input
```
integrate(x^4/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="giac")
```

output
```
-1/8*cos_integral(4*arccos(a*x))/a^5 - 1/2*cos_integral(2*arccos(a*x))/a^5
 - 3/8*log(arccos(a*x))/a^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^4}{\operatorname{acos}(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(x^4/(acos(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(x^4/(acos(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^4}{\sqrt{-a^2x^2+1}\,acos(ax)}dx$$

input
```
int(x^4/(-a^2*x^2+1)^(1/2)/acos(a*x),x)
```

output
```
int(x**4/(sqrt( - a**2*x**2 + 1)*acos(a*x)),x)
```

3.344 $\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \frac{3\mathrm{Si}(\arccos(ax))}{4a^4} - \frac{\mathrm{Si}(3\arccos(ax))}{4a^4}$$

output
```
3/4*Si(arccos(a*x))/a^4-1/4*Si(3*arccos(a*x))/a^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{3\,\mathrm{CosIntegral}(\arccos(ax)) + \mathrm{CosIntegral}(3\arccos(ax))}{4a^4}$$

input
```
Integrate[x^3/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
-1/4*(3*CosIntegral[ArcCos[a*x]] + CosIntegral[3*ArcCos[a*x]])/a^4
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)}dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\int \frac{a^3x^3}{\arccos(ax)}d\arccos(ax)}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^3}{\arccos(ax)}d\arccos(ax)}{a^4}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{3ax}{4\arccos(ax)}+\frac{\cos(3\arccos(ax))}{4\arccos(ax)}\right)d\arccos(ax)}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{3}{4}\text{CosIntegral}(\arccos(ax))+\frac{1}{4}\text{CosIntegral}(3\arccos(ax))}{a^4}$$

input
```
Int[x^3/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
-(((3*CosIntegral[ArcCos[a*x]])/4 + CosIntegral[3*ArcCos[a*x]]/4)/a^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{3\,\operatorname{Ci}\left(\arccos\left(a x\right)\right)+\operatorname{Ci}\left(3\arccos\left(a x\right)\right)}{4 a^{4}}$	21

input
```
int(x^3/(-a^2*x^2+1)^(1/2)/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(3*Ci(arccos(a*x))+Ci(3*arccos(a*x)))/a^4
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^3}{\sqrt{-a^2x^2+1}\arccos(ax)}\,dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^3/((a^2*x^2 - 1)*arccos(a*x)), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^3}{\sqrt{-(ax-1)(ax+1)}\operatorname{acos}(ax)}\,dx$$

input
```
integrate(x**3/(-a**2*x**2+1)**(1/2)/acos(a*x),x)
```

output
```
Integral(x**3/(sqrt(-(a*x - 1)*(a*x + 1))*acos(a*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^3}{\sqrt{-a^2x^2+1}\arccos(ax)}\,dx$$

input
```
integrate(x^3/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(-a^2*x^2 + 1)*arccos(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^3}{\mathrm{acos}\left(a\,x\right)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(x^3/(acos(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(x^3/(acos(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^3}{\sqrt{-a^2x^2+1}\,acos\,(ax)}dx$$

input
```
int(x^3/(-a^2*x^2+1)^(1/2)/acos(a*x),x)
```

output
```
int(x**3/(sqrt( - a**2*x**2 + 1)*acos(a*x)),x)
```

3.345 $\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\text{CosIntegral}(2\arccos(ax))}{2a^3}+\frac{\log(\arccos(ax))}{2a^3}$$

output
```
-1/2*Ci(2*arccos(a*x))/a^3+1/2*ln(arccos(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\text{CosIntegral}(2\arccos(ax))+\log(\arccos(ax))}{2a^3}$$

input
```
Integrate[x^2/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
-1/2*(CosIntegral[2*ArcCos[a*x]] + Log[ArcCos[a*x]])/a^3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\int \frac{a^2x^2}{\arccos(ax)}d\arccos(ax)}{a^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^2}{\arccos(ax)}d\arccos(ax)}{a^3}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{\cos(2\arccos(ax))}{2\arccos(ax)}+\frac{1}{2\arccos(ax)}\right)d\arccos(ax)}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\text{CosIntegral}(2\arccos(ax))+\frac{1}{2}\log(\arccos(ax))}{a^3}$$

input
```
Int[x^2/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
-((CosIntegral[2*ArcCos[a*x]]/2 + Log[ArcCos[a*x]]/2)/a^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{\ln(\arccos(ax))+\operatorname{Ci}(2\arccos(ax))}{2a^3}$	19

input
```
int(x^2/(-a^2*x^2+1)^(1/2)/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(arccos(a*x))+Ci(2*arccos(a*x)))/a^3
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^2}{\sqrt{-a^2x^2+1}\arccos(ax)}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^2/((a^2*x^2 - 1)*arccos(a*x)), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^2}{\sqrt{-(ax-1)(ax+1)}\operatorname{acos}(ax)}\,dx$$

input
```
integrate(x**2/(-a**2*x**2+1)**(1/2)/acos(a*x),x)
```

output
```
Integral(x**2/(sqrt(-(a*x - 1)*(a*x + 1))*acos(a*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^2}{\sqrt{-a^2x^2+1}\arccos(ax)}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(-a^2*x^2 + 1)*arccos(a*x)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\operatorname{Ci}(2\arccos(ax))}{2a^3} - \frac{\log(\arccos(ax))}{2a^3}$$

input
```
integrate(x^2/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="giac")
```

output
```
-1/2*cos_integral(2*arccos(a*x))/a^3 - 1/2*log(arccos(a*x))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^2}{\operatorname{acos}(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(x^2/(acos(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(x^2/(acos(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^2}{\sqrt{-a^2x^2+1}\,acos(ax)}dx$$

input
```
int(x^2/(-a^2*x^2+1)^(1/2)/acos(a*x),x)
```

output
```
int(x**2/(sqrt( - a**2*x**2 + 1)*acos(a*x)),x)
```

3.346 $\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\text{CosIntegral}(2\arccos(ax))}{2a^3} + \frac{\log(\arccos(ax))}{2a^3}$$

output
```
-1/2*Ci(2*arccos(a*x))/a^3+1/2*ln(arccos(a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\text{CosIntegral}(2\arccos(ax)) + \log(\arccos(ax))}{2a^3}$$

input
```
Integrate[x^2/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
-1/2*(CosIntegral[2*ArcCos[a*x]] + Log[ArcCos[a*x]])/a^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\int \frac{a^2x^2}{\arccos(ax)}d\arccos(ax)}{a^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^2}{\arccos(ax)}d\arccos(ax)}{a^3}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{\cos(2\arccos(ax))}{2\arccos(ax)}+\frac{1}{2\arccos(ax)}\right)d\arccos(ax)}{a^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\text{CosIntegral}(2\arccos(ax))+\frac{1}{2}\log(\arccos(ax))}{a^3}$$

input
```
Int[x^2/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
-((CosIntegral[2*ArcCos[a*x]]/2 + Log[ArcCos[a*x]]/2)/a^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{\ln(\arccos(ax))+\operatorname{Ci}(2\arccos(ax))}{2a^3}$	19

input
```
int(x^2/(-a^2*x^2+1)^(1/2)/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(arccos(a*x))+Ci(2*arccos(a*x)))/a^3
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^2}{\sqrt{-a^2x^2+1}\arccos(ax)}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^2/((a^2*x^2 - 1)*arccos(a*x)), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^2}{\sqrt{-(ax-1)(ax+1)}\operatorname{acos}(ax)}\,dx$$

input
```
integrate(x**2/(-a**2*x**2+1)**(1/2)/acos(a*x),x)
```

output
```
Integral(x**2/(sqrt(-(a*x - 1)*(a*x + 1))*acos(a*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^2}{\sqrt{-a^2x^2+1}\arccos(ax)}\,dx$$

input
```
integrate(x^2/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(-a^2*x^2 + 1)*arccos(a*x)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\text{Ci}\left(2\arccos\left(ax\right)\right)}{2\,a^3} - \frac{\log\left(\arccos\left(ax\right)\right)}{2\,a^3}$$

input
```
integrate(x^2/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="giac")
```

output
```
-1/2*cos_integral(2*arccos(a*x))/a^3 - 1/2*log(arccos(a*x))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^2}{\text{acos}\left(a\,x\right)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(x^2/(acos(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(x^2/(acos(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^2}{\sqrt{-a^2x^2+1}\,acos\left(ax\right)}dx$$

input
```
int(x^2/(-a^2*x^2+1)^(1/2)/acos(a*x),x)
```

output
```
int(x**2/(sqrt( - a**2*x**2 + 1)*acos(a*x)),x)
```

3.347 $\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 9

$$\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \frac{\text{Si}(\arccos(ax))}{a^2}$$

output
```
Si(arccos(a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\text{CosIntegral}(\arccos(ax))}{a^2}$$

input
```
Integrate[x/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
-(CosIntegral[ArcCos[a*x]]/a^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5225, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\int \frac{ax}{\arccos(ax)}d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)}{\arccos(ax)}d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{\text{CosIntegral}(\arccos(ax))}{a^2}$$

input
```
Int[x/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
-(CosIntegral[ArcCos[a*x]]/a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

method	result	size
default	$-\frac{\operatorname{Ci}(\arccos(ax))}{a^2}$	11

input
```
int(x/(-a^2*x^2+1)^(1/2)/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*Ci(arccos(a*x))
```

Fricas [F]

$$\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x}{\sqrt{-a^2x^2+1}\arccos(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x/((a^2*x^2 - 1)*arccos(a*x)), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x}{\sqrt{-(ax-1)(ax+1)}\operatorname{acos}(ax)}\,dx$$

input
```
integrate(x/(-a**2*x**2+1)**(1/2)/acos(a*x),x)
```

output
```
Integral(x/(sqrt(-(a*x - 1)*(a*x + 1))*acos(a*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x}{\sqrt{-a^2x^2+1}\arccos(ax)}\,dx$$

input
```
integrate(x/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-a^2*x^2 + 1)*arccos(a*x)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\operatorname{Ci}(\arccos(ax))}{a^2}$$

input
```
integrate(x/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="giac")
```

output
```
-cos_integral(arccos(a*x))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x}{\operatorname{acos}(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(x/(acos(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(x/(acos(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x}{\sqrt{-a^2x^2+1}\,acos(ax)}dx$$

input
```
int(x/(-a^2*x^2+1)^(1/2)/acos(a*x),x)
```

output
```
int(x/(sqrt( - a**2*x**2 + 1)*acos(a*x)),x)
```

3.348 $\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 9

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \frac{\log(\arccos(ax))}{a}$$

output
```
ln(arccos(a*x))/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\log(\arccos(ax))}{a}$$

input
```
Integrate[1/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
-(Log[ArcCos[a*x]]/a)
```

Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5151}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx$$
$$\Big\downarrow \; 5151$$
$$-\frac{\log(\arccos(ax))}{a^2}$$

input
```
Int[1/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
-(Log[ArcCos[a*x]]/a^2)
```

Defintions of rubi rules used

rule 5151
```
Int[1/(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sym
bol] :> Simp[(-(b*c)^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(Log[a +
 b*ArcCos[c*x]]/(b*c*Sqrt[d])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$-\frac{\ln(\arccos(ax))}{a}$	11
default	$-\frac{\ln(\arccos(ax))}{a}$	11

input
```
int(1/(-a^2*x^2+1)^(1/2)/arccos(a*x),x,method=_RETURNVERBOSE)
```

output
```
-ln(arccos(a*x))/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\log\left(\arccos\left(ax\right)\right)}{a}$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="fricas")
```

output
```
-log(arccos(a*x))/a
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.56

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \begin{cases} -\frac{\log\left(\operatorname{acos}\left(ax\right)\right)}{a} & \text{for}\: a \neq 0 \\ \frac{2x}{\pi} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-a**2*x**2+1)**(1/2)/acos(a*x),x)
```

output
```
Piecewise((-log(acos(a*x))/a, Ne(a, 0)), (2*x/pi, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\log(\arccos(ax))}{a}$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="maxima")
```

output
```
-log(arccos(a*x))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\log(|\arccos(ax)|)}{a}$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="giac")
```

output
```
-log(abs(arccos(a*x)))/a
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\ln(\mathrm{acos}(a\,x))}{a}$$

input
```
int(1/(acos(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
-log(acos(a*x))/a
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = -\frac{\log(acos(ax))}{a}$$

input
```
int(1/(-a^2*x^2+1)^(1/2)/acos(a*x),x)
```

output
```
( - log(acos(a*x)))/a
```

3.349 $\int \frac{1}{x\sqrt{1-a^2x^2}\arccos(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \text{Int}\left(\frac{1}{x\sqrt{1-a^2x^2}\arccos(ax)},x\right)$$

output
```
Defer(Int)(1/x/(-a^2*x^2+1)^(1/2)/arccos(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{1}{x\sqrt{1-a^2x^2}\arccos(ax)}\,dx$$

input
```
Integrate[1/(x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
Integrate[1/(x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arccos(ax)}dx$$
$$\Big\downarrow \text{ 5235}$$
$$\int \frac{1}{x\sqrt{1-a^2x^2}\arccos(ax)}dx$$

input
```
Int[1/(x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{-a^2x^2+1}\,\arccos\left(ax\right)}dx$$

input
```
int(1/x/(-a^2*x^2+1)^(1/2)/arccos(a*x),x)
```

output
```
int(1/x/(-a^2*x^2+1)^(1/2)/arccos(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}x\arccos{(ax)}}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^3 - x)*arccos(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{1}{x\sqrt{-(ax-1)(ax+1)}\operatorname{acos}{(ax)}}\,dx$$

input
```
integrate(1/x/(-a**2*x**2+1)**(1/2)/acos(a*x),x)
```

output
```
Integral(1/(x*sqrt(-(a*x - 1)*(a*x + 1))*acos(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}x\arccos{(ax)}}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*x*arccos(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}x\arccos(ax)}\,dx$$

input
```
integrate(1/x/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*x*arccos(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{1}{x\,\mathrm{acos}\left(a\,x\right)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(1/(x*acos(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(1/(x*acos(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,acos\,(ax)\,x}dx$$

input
```
int(1/x/(-a^2*x^2+1)^(1/2)/acos(a*x),x)
```

output
```
int(1/(sqrt( - a**2*x**2 + 1)*acos(a*x)*x),x)
```

3.350 $\int \frac{1}{x^2\sqrt{1-a^2x^2}\arccos(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \text{Int}\left(\frac{1}{x^2\sqrt{1-a^2x^2}\arccos(ax)},x\right)$$

output
```
Defer(Int)(1/x^2/(-a^2*x^2+1)^(1/2)/arccos(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{1}{x^2\sqrt{1-a^2x^2}\arccos(ax)}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
Integrate[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\arccos(ax)}dx$$
$$\Big\downarrow \text{ 5235}$$
$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\arccos(ax)}dx$$

input
```
Int[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\sqrt{-a^2x^2+1}\,\arccos\left(ax\right)}dx$$

input
```
int(1/x^2/(-a^2*x^2+1)^(1/2)/arccos(a*x),x)
```

output
```
int(1/x^2/(-a^2*x^2+1)^(1/2)/arccos(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}x^2\arccos{(ax)}}\,dx$$

input
```
integrate(1/x^2/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^4 - x^2)*arccos(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{1}{x^2\sqrt{-(ax-1)(ax+1)}\operatorname{acos}{(ax)}}\,dx$$

input
```
integrate(1/x**2/(-a**2*x**2+1)**(1/2)/acos(a*x),x)
```

output
```
Integral(1/(x**2*sqrt(-(a*x - 1)*(a*x + 1))*acos(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}x^2\arccos{(ax)}}\,dx$$

input
```
integrate(1/x^2/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}x^2\arccos{(ax)}}\,dx$$

input
```
integrate(1/x^2/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{1}{x^2\,\mathrm{acos}\left(a\,x\right)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(1/(x^2*acos(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*acos(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\,acos\,(ax)\,x^2}dx$$

input
```
int(1/x^2/(-a^2*x^2+1)^(1/2)/acos(a*x),x)
```

output
```
int(1/(sqrt( - a**2*x**2 + 1)*acos(a*x)*x**2),x)
```

3.351 $\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 183

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{5\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{8bc^6} + \frac{5\,\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{16bc^6} - \frac{\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)\sin\left(\frac{5a}{b}\right)}{16bc^6} + \frac{5\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8bc^6} - \frac{5\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16bc^6} + \frac{\cos\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16bc^6}$$

output
```
-5/8*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c^6+5/16*Ci(3*(a+b*arccos(c*x))/b)
*sin(3*a/b)/b/c^6-1/16*Ci(5*(a+b*arccos(c*x))/b)*sin(5*a/b)/b/c^6+5/8*cos(
a/b)*Si((a+b*arccos(c*x))/b)/b/c^6-5/16*cos(3*a/b)*Si(3*(a+b*arccos(c*x))/
b)/b/c^6+1/16*cos(5*a/b)*Si(5*(a+b*arccos(c*x))/b)/b/c^6
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx =$$
$$-\frac{10\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)+5\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)+\cos\left(\frac{5a}{b}\right)\text{CosI}}{}$$

input
```
Integrate[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
-1/16*(10*Cos[a/b]*CosIntegral[a/b + ArcCos[c*x]] + 5*Cos[(3*a)/b]*CosInte
gral[3*(a/b + ArcCos[c*x])] + Cos[(5*a)/b]*CosIntegral[5*(a/b + ArcCos[c*x
])] + 10*Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 5*Sin[(3*a)/b]*SinInteg
ral[3*(a/b + ArcCos[c*x])] + Sin[(5*a)/b]*SinIntegral[5*(a/b + ArcCos[c*x]
)])/(b*c^6)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$$
$$\Big\downarrow \text{5225}$$
$$-\frac{\int \frac{\cos^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^6}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)^5}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^6}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{\cos\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{5\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{5\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{bc^6}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{5}{8}\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{5}{16}\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{1}{16}\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(\ldots}{}\right.}{bc^{\ldots}}$$

input
```
Int[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
-(((5*Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/8 + (5*Cos[(3*a)/b]*Cos
Integral[(3*(a + b*ArcCos[c*x]))/b])/16 + (Cos[(5*a)/b]*CosIntegral[(5*(a
+ b*ArcCos[c*x]))/b])/16 + (5*Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])
/8 + (5*Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/16 + (Sin[(5*
a)/b]*SinIntegral[(5*(a + b*ArcCos[c*x]))/b])/16)/(b*c^6))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.75

method	result
default	$-\frac{\mathrm{Si}\left(5\arccos(cx)+\frac{5a}{b}\right)\sin\left(\frac{5a}{b}\right)+\mathrm{Ci}\left(5\arccos(cx)+\frac{5a}{b}\right)\cos\left(\frac{5a}{b}\right)+5\ \mathrm{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)+5\ \mathrm{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)}{16c^6b}$

input
```
int(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/16/c^6*(Si(5*arccos(c*x)+5*a/b)*sin(5*a/b)+Ci(5*arccos(c*x)+5*a/b)*cos(
5*a/b)+5*Si(3*arccos(c*x)+3*a/b)*sin(3*a/b)+5*Ci(3*arccos(c*x)+3*a/b)*cos(
3*a/b)+10*Si(arccos(c*x)+a/b)*sin(a/b)+10*Ci(arccos(c*x)+a/b)*cos(a/b))/b
```

Fricas [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^5}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^5/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccos(c*x)
- a), x)
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^5}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\mathrm{acos}\,(cx))}\,dx$$

input
```
integrate(x**5/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**5/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))), x)
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^5}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^5/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^5}{(a+b\,\mathrm{acos}\,(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^5/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^5/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^5}{\sqrt{-c^2x^2+1}\,acos\,(cx)\,b+\sqrt{-c^2x^2+1}\,a}dx$$

input
```
int(x^5/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int(x**5/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b + sqrt( - c**2*x**2 + 1)*a),x
)
```

3.352 $\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 144

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2bc^5} + \frac{\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{8bc^5} + \frac{3\log(a+b\arccos(cx))}{8bc^5} - \frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2bc^5} + \frac{\sin\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{8bc^5}$$

output
```
-1/2*cos(2*a/b)*Ci(2*(a+b*arccos(c*x))/b)/b/c^5+1/8*cos(4*a/b)*Ci(4*(a+b*a
rccos(c*x))/b)/b/c^5+3/8*ln(a+b*arccos(c*x))/b/c^5-1/2*sin(2*a/b)*Si(2*(a+
b*arccos(c*x))/b)/b/c^5+1/8*sin(4*a/b)*Si(4*(a+b*arccos(c*x))/b)/b/c^5
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{4\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)+\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(4\left(\frac{a}{b}+\arccos(cx)\right)\right)+3\log(a+b\,a}{8bc^5}$$

input
```
Integrate[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
-1/8*(4*Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcCos[c*x])] + Cos[(4*a)/b]*Cos
Integral[4*(a/b + ArcCos[c*x])] + 3*Log[a + b*ArcCos[c*x]] + 4*Sin[(2*a)/b
]*SinIntegral[2*(a/b + ArcCos[c*x])] + Sin[(4*a)/b]*SinIntegral[4*(a/b + A
rcCos[c*x])])/(b*c^5)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$$

$$\Big\downarrow \; 5225$$

$$-\frac{\int \frac{\cos^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^5}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)^4}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^5}$$

$$\Big\downarrow \; 3793$$

$$-\frac{\int \left(\frac{\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8(a+b\arccos(cx))}+\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2(a+b\arccos(cx))}+\frac{3}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{bc^5}$$

$\downarrow$ 2009

$$-\frac{\frac{1}{2}\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{1}{8}\cos\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{2}\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right.}{bc^5}$$

input

```
Int[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output

```
-(((Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcCos[c*x]))/b])/2 + (Cos[(4*a)/b]
*CosIntegral[(4*(a + b*ArcCos[c*x]))/b])/8 + (3*Log[a + b*ArcCos[c*x]])/8
+ (Sin[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/2 + (Sin[(4*a)/b]*
SinIntegral[(4*(a + b*ArcCos[c*x]))/b])/8)/(b*c^5))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5225

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

method	result
default	$-\frac{\mathrm{Si}\left(4\arccos(cx)+\frac{4a}{b}\right)\sin\left(\frac{4a}{b}\right)+\mathrm{Ci}\left(4\arccos(cx)+\frac{4a}{b}\right)\cos\left(\frac{4a}{b}\right)+4\,\mathrm{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)+4\,\mathrm{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)}{8c^5b}$

input
```
int(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8/c^5*(Si(4*arccos(c*x)+4*a/b)*sin(4*a/b)+Ci(4*arccos(c*x)+4*a/b)*cos(4
*a/b)+4*Si(2*arccos(c*x)+2*a/b)*sin(2*a/b)+4*Ci(2*arccos(c*x)+2*a/b)*cos(2
*a/b)+3*ln(a+b*arccos(c*x)))/b
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^4}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^4/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccos(c*x)
- a), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^4}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\mathrm{acos}(cx))}\,dx$$

input
```
integrate(x**4/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**4/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^4}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.77

$$\begin{aligned}\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = &-\frac{\cos\left(\frac{a}{b}\right)^4 \operatorname{Ci}\left(\frac{4a}{b}+4\arccos(cx)\right)}{bc^5} \\ &- \frac{\cos\left(\frac{a}{b}\right)^3 \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{4a}{b}+4\arccos(cx)\right)}{bc^5} \\ &+ \frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{4a}{b}+4\arccos(cx)\right)}{bc^5} \\ &- \frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{2a}{b}+2\arccos(cx)\right)}{bc^5} \\ &+ \frac{\cos\left(\frac{a}{b}\right) \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{4a}{b}+4\arccos(cx)\right)}{2\,bc^5} \\ &- \frac{\cos\left(\frac{a}{b}\right) \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{2a}{b}+2\arccos(cx)\right)}{bc^5} \\ &- \frac{\operatorname{Ci}\left(\frac{4a}{b}+4\arccos(cx)\right)}{8\,bc^5} \\ &+ \frac{\operatorname{Ci}\left(\frac{2a}{b}+2\arccos(cx)\right)}{2\,bc^5} \\ &- \frac{3\log\left(b\arccos(cx)+a\right)}{8\,bc^5}\end{aligned}$$

input
```
integrate(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-cos(a/b)^4*cos_integral(4*a/b + 4*arccos(c*x))/(b*c^5) - cos(a/b)^3*sin(a
/b)*sin_integral(4*a/b + 4*arccos(c*x))/(b*c^5) + cos(a/b)^2*cos_integral(
4*a/b + 4*arccos(c*x))/(b*c^5) - cos(a/b)^2*cos_integral(2*a/b + 2*arccos(
c*x))/(b*c^5) + 1/2*cos(a/b)*sin(a/b)*sin_integral(4*a/b + 4*arccos(c*x))/
(b*c^5) - cos(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arccos(c*x))/(b*c^5) -
1/8*cos_integral(4*a/b + 4*arccos(c*x))/(b*c^5) + 1/2*cos_integral(2*a/b +
 2*arccos(c*x))/(b*c^5) - 3/8*log(b*arccos(c*x) + a)/(b*c^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^4}{(a+b\,\mathrm{acos}\,(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^4/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^4/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^4}{\sqrt{-c^2x^2+1}\,acos\,(cx)\,b+\sqrt{-c^2x^2+1}\,a}dx$$

input
```
int(x^4/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int(x**4/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b + sqrt( - c**2*x**2 + 1)*a),x
)
```

3.353 $\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 121

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{3\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{4bc^4} + \frac{\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{4bc^4} + \frac{3\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4bc^4} - \frac{\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4bc^4}$$

output
```
-3/4*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c^4+1/4*Ci(3*(a+b*arccos(c*x))/b)*
sin(3*a/b)/b/c^4+3/4*cos(a/b)*Si((a+b*arccos(c*x))/b)/b/c^4-1/4*cos(3*a/b)
*Si(3*(a+b*arccos(c*x))/b)/b/c^4
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{3\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)+\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)+3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\right.}{4bc^4}$$

input
```
Integrate[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
-1/4*(3*Cos[a/b]*CosIntegral[a/b + ArcCos[c*x]] + Cos[(3*a)/b]*CosIntegral
[3*(a/b + ArcCos[c*x])] + 3*Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] + Sin[
(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])])/(b*c^4)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\int \frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)^3}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^4}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}+\frac{3\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{bc^4}$$

$\big\downarrow$ 2009

$$-\frac{\frac{3}{4}\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{1}{4}\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{3}{4}\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)+}{bc^4}$$

input
```
Int[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
-(((3*Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/4 + (Cos[(3*a)/b]*CosIn
tegral[(3*(a + b*ArcCos[c*x]))/b])/4 + (3*Sin[a/b]*SinIntegral[(a + b*ArcC
os[c*x])/b])/4 + (Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/4)/
(b*c^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{\operatorname{Si}\left(3\arccos\left(cx\right)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)+\operatorname{Ci}\left(3\arccos\left(cx\right)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)+3\operatorname{Si}\left(\arccos\left(cx\right)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)+3\operatorname{Ci}\left(\arccos\left(cx\right)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{4c^{4}b}$	92

input
```
int(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/c^4*(Si(3*arccos(c*x)+3*a/b)*sin(3*a/b)+Ci(3*arccos(c*x)+3*a/b)*cos(3
*a/b)+3*Si(arccos(c*x)+a/b)*sin(a/b)+3*Ci(arccos(c*x)+a/b)*cos(a/b))/b
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^3}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^3/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccos(c*x)
- a), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^3}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acos}(cx))}\,dx$$

input
```
integrate(x**3/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**3/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^3}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^3}{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^3/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^3/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^3}{\sqrt{-c^2x^2+1}\,acos\,(cx)\,b+\sqrt{-c^2x^2+1}\,a}dx$$

input
```
int(x^3/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int(x**3/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b + sqrt( - c**2*x**2 + 1)*a),x
)
```

3.354 $\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 82

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2bc^3} + \frac{\log(a+b\arccos(cx))}{2bc^3} - \frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2bc^3}$$

output
```
-1/2*cos(2*a/b)*Ci(2*(a+b*arccos(c*x))/b)/b/c^3+1/2*ln(a+b*arccos(c*x))/b/
c^3-1/2*sin(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b/c^3
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)+\log(a+b\arccos(cx))+\sin\left(\frac{2a}{b}\right)\text{Si}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)}{2bc^3}$$

input
```
Integrate[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
-1/2*(Cos[(2*a)/b]*CosIntegral[2*(a/b + ArcCos[c*x])] + Log[a + b*ArcCos[c
*x]] + Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])])/(b*c^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx$$

$$\Big\downarrow \text{ 5225}$$

$$-\frac{\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)^2}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^3}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{\int \left(\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2(a+b\arccos(cx))}+\frac{1}{2(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{bc^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{1}{2}\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{1}{2}\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{1}{2}\log(a+b\arccos(cx))}{bc^3}$$

input
```
Int[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
-(((Cos[(2*a)/b]*CosIntegral[(2*(a + b*ArcCos[c*x]))/b])/2 + Log[a + b*Arc
Cos[c*x]]/2 + (Sin[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/2)/(b*
c^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\operatorname{Ci}\left(2 \arccos \left(c x \right)+\frac{2 a}{b}\right) \cos \left(\frac{2 a}{b}\right)+\operatorname{Si}\left(2 \arccos \left(c x \right)+\frac{2 a}{b}\right) \sin \left(\frac{2 a}{b}\right)+\ln \left(a +b \arccos \left(c x \right)\right)}{2 c^{3} b}$	63

input
```
int(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^3*(Ci(2*arccos(c*x)+2*a/b)*cos(2*a/b)+Si(2*arccos(c*x)+2*a/b)*sin(2
*a/b)+ln(a+b*arccos(c*x)))/b
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^2}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^2/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccos(c*x)
- a), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^2}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acos}(cx))}\,dx$$

input
```
integrate(x**2/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**2/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^2}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{\cos\left(\frac{a}{b}\right)^2 \operatorname{Ci}\left(\frac{2a}{b}+2\arccos(cx)\right)}{bc^3} - \frac{\cos\left(\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{2a}{b}+2\arccos(cx)\right)}{bc^3} + \frac{\operatorname{Ci}\left(\frac{2a}{b}+2\arccos(cx)\right)}{2bc^3} - \frac{\log\left(b\arccos(cx)+a\right)}{2bc^3}$$

input

```
integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output

```
-cos(a/b)^2*cos_integral(2*a/b + 2*arccos(c*x))/(b*c^3) - cos(a/b)*sin(a/b
)*sin_integral(2*a/b + 2*arccos(c*x))/(b*c^3) + 1/2*cos_integral(2*a/b + 2
*arccos(c*x))/(b*c^3) - 1/2*log(b*arccos(c*x) + a)/(b*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^2}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)\,\sqrt{1-c^2\,x^2}}\,dx$$

input

```
int(x^2/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output

```
int(x^2/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^2}{\sqrt{-c^2x^2+1}\,acos\,(cx)\,b+\sqrt{-c^2x^2+1}\,a}dx$$

input

```
int(x^2/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x)),x)
```

output

```
int(x**2/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b + sqrt( - c**2*x**2 + 1)*a),x
)
```

3.355 $\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{bc^2} + \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc^2}$$

output
```
-Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c^2+cos(a/b)*Si((a+b*arccos(c*x))/b)/b
/c^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)+\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{bc^2}$$

input
```
Integrate[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
-((Cos[a/b]*CosIntegral[a/b + ArcCos[c*x]] + Sin[a/b]*SinIntegral[a/b + Ar
cCos[c*x]])/(b*c^2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5225, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^2}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc^2}$$

$$\Big\downarrow \text{ 3780}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))+\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc^2}$$

$$\Big\downarrow \text{ 3783}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc^2}$$

input
```
Int[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
-((Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b] + Sin[a/b]*SinIntegral[(a +
 b*ArcCos[c*x])/b])/(b*c^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{\operatorname{Si}\left(\arccos \left(c x \right)+\frac{a}{b}\right) \sin \left(\frac{a}{b}\right)+\operatorname{Ci}\left(\arccos \left(c x \right)+\frac{a}{b}\right) \cos \left(\frac{a}{b}\right)}{c^{2} b}$	46

input
```
int(x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*(Si(arccos(c*x)+a/b)*sin(a/b)+Ci(arccos(c*x)+a/b)*cos(a/b))/b
```

Fricas [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx=\int \frac{x}{\sqrt{-c^{2} x^{2}+1}\,(b \arccos \left(c x\right)+a)}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccos(c*x) -
a), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\mathrm{acos}\,(cx))}\,dx$$

input
```
integrate(x/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{\cos\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{a}{b}+\arccos(cx)\right)}{bc^2} - \frac{\sin\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{bc^2}$$

input
```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-cos(a/b)*cos_integral(a/b + arccos(c*x))/(b*c^2) - sin(a/b)*sin_integral(
a/b + arccos(c*x))/(b*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x}{(a+b\,\mathrm{acos}\,(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x}{\sqrt{-c^2x^2+1}\,acos\,(cx)\,b+\sqrt{-c^2x^2+1}\,a}dx$$

input
```
int(x/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int(x/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b + sqrt( - c**2*x**2 + 1)*a),x)
```

3.356 $\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 16

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \frac{\log(a+b\arccos(cx))}{bc}$$

output
```
ln(a+b*arccos(c*x))/b/c
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{\log(a+b\arccos(cx))}{bc}$$

input
```
Integrate[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
-(Log[a + b*ArcCos[c*x]]/(b*c))
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {5151}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$$

$$\Big\downarrow \text{5151}$$

$$-\frac{\log(a+b\arccos(cx))}{b^2c^2}$$

input
```
Int[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
-(Log[a + b*ArcCos[c*x]]/(b^2*c^2))
```

Defintions of rubi rules used

rule 5151
```
Int[1/(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sym
bol] :> Simp[(-(b*c)^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(Log[a +
 b*ArcCos[c*x]]/(b*c*Sqrt[d])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$-\frac{\ln(a+b\arccos(cx))}{bc}$	18
default	$-\frac{\ln(a+b\arccos(cx))}{bc}$	18

input
```
int(1/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(a+b*arccos(c*x))/b/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{\log\left(b\arccos\left(cx\right)+a\right)}{bc}$$

input
```
integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
-log(b*arccos(c*x) + a)/(b*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \begin{cases} \frac{x}{a} & \text{for}\: b = 0 \wedge c = 0 \\ \frac{\begin{cases} -\frac{i\operatorname{acosh}\left(cx\right)}{c} & \text{for}\: \left|c^2x^2\right| > 1 \\ \frac{\operatorname{asin}\left(cx\right)}{c} & \text{otherwise} \end{cases}}{a} & \text{for}\: b = 0 \\ \frac{x}{a+\frac{\pi b}{2}} & \text{for}\: c = 0 \\ -\frac{\log\left(\frac{a}{b}+\operatorname{acos}\left(cx\right)\right)}{bc} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Piecewise((x/a, Eq(b, 0) & Eq(c, 0)), (Piecewise((-I*acosh(c*x)/c, Abs(c**
2*x**2) > 1), (asin(c*x)/c, True))/a, Eq(b, 0)), (x/(a + pi*b/2), Eq(c, 0)
), (-log(a/b + acos(c*x))/(b*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{\log\left(b\arccos\left(cx\right)+a\right)}{bc}$$

input
```
integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
-log(b*arccos(c*x) + a)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{\log\left(\left|b\arccos\left(cx\right)+a\right|\right)}{bc}$$

input
```
integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
-log(abs(b*arccos(c*x) + a))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{\ln\left(a+b\,\mathrm{acos}(c\,x)\right)}{b\,c}$$

input
```
int(1/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
-log(a + b*acos(c*x))/(b*c)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = -\frac{\log(acos(cx)\,b+a)}{bc}$$

input
```
int(1/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
( - log(acos(c*x)*b + a))/(b*c)
```

3.357 $\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \mathrm{Int}\left(\frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 4.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{5235}$$
$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx$$

input
```
Int[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\sqrt{-c^2x^2+1}\,(a+b\arccos(cx))}dx$$

input
```
int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)/(a*c^2*x^3 - a*x + (b*c^2*x^3 - b*x)*arccos(c
*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{x\sqrt{-(cx-1)(cx+1)}\,(a+b\,\mathrm{acos}\,(cx))}\,dx$$

input
```
integrate(1/x/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/(x*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(1/(x*(a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(1/(x*(a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}\,acos\,(cx)\,bx+\sqrt{-c^2x^2+1}\,ax}dx$$

input
```
int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*x + sqrt( - c**2*x**2 + 1)*a*x),
x)
```

3.358 $\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \operatorname{Int}\left(\frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{ 5235}$$
$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx$$

input
```
Int[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\sqrt{-c^2x^2+1}\,(a+b\arccos(cx))}dx$$

input
```
int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)x^2}\,dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*x^2 + 1)/(a*c^2*x^4 - a*x^2 + (b*c^2*x^4 - b*x^2)*arcc
os(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{x^2\sqrt{-(cx-1)(cx+1)}\,(a+b\,\mathrm{acos}\,(cx))}\,dx$$

input
```
integrate(1/x**2/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/(x**2*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)x^2}\,dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)x^2}\,dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{x^2\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(1/(x^2*(a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$$
$$=\int \frac{1}{\sqrt{-c^2x^2+1}\,acos\,(cx)\,b\,x^2+\sqrt{-c^2x^2+1}\,a\,x^2}dx$$

input
```
int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*x**2 + sqrt( - c**2*x**2 + 1)*a*
x**2),x)
```

3.359 $\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}\,dx=\text{Int}\left(\frac{x^2}{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 9.91 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}\,dx=\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{5235}$$
$$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}dx$$

input
```
Int[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(-c^2x^2+1)^{\frac{3}{2}}(a+b\arccos(cx))}dx$$

input
```
int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

output
```
int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{x^2}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^2/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*
b*c^2*x^2 + b)*arccos(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{x^2}{(-(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))}\,dx$$

input
```
integrate(x**2/(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**2/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{x^2}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{x^2}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{x^2}{(a+b\,\mathrm{acos}(c\,x))\,(1-c^2\,x^2)^{3/2}}\,dx$$

input
```
int(x^2/((a + b*acos(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(x^2/((a + b*acos(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.57

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \frac{-\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)b\,c^2x^2-\sqrt{-c^2x^2+1}\,acos(cx)b+\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\right.}{b\,c^3}$$

input
```
int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
( - int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*c**2*x**2 - sqrt( - c**2*x**
2 + 1)*acos(c*x)*b + sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x**
2 + 1)*a),x)*b*c + log(acos(c*x)*b + a))/(b*c**3)
```

3.360 $\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx = \operatorname{Int}\left(\frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{ 5235}$$
$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}dx$$

input
```
Int[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x}{(-c^2x^2+1)^{\frac{3}{2}}(a+b\arccos(cx))}dx$$

input
```
int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

output
```
int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{x}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*
c^2*x^2 + b)*arccos(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{x}{(-(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))}\,dx$$

input
```
integrate(x/(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{x}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(x/((-c^2*x^2 + 1)^(3/2)*(b*arccos(c*x) + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{x}{(a+b\,\mathrm{acos}\,(c\,x))\,(1-c^2\,x^2)^{3/2}}\,dx$$

input
```
int(x/((a + b*acos(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(x/((a + b*acos(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.19

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx =$$
$$-\left(\int \frac{x}{\sqrt{-c^2x^2+1}\,acos(cx)\,b\,c^2x^2-\sqrt{-c^2x^2+1}\,acos(cx)\,b+\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a}dx\right)$$

input
```
int(x/(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
- int(x/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*c**2*x**2 - sqrt( - c**2*x**2
+ 1)*acos(c*x)*b + sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x**2
+ 1)*a),x)
```

3.361 $\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}dx$$

input
```
Int[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(a+b\arccos(cx))}dx$$

input
```
int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx=\int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c^
2*x^2 + b)*arccos(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx=\int \frac{1}{(-(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))}\,dx$$

input
```
integrate(1/(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx=\int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\,\left(1-c^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.24

$$\int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))}\,dx =$$
$$-\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acos\,(cx)\,b\,c^2x^2-\sqrt{-c^2x^2+1}\,acos\,(cx)\,b+\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a}dx\right)$$

input
```
int(1/(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
- int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*c**2*x**2 - sqrt( - c**2*x**2
+ 1)*acos(c*x)*b + sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x**2
+ 1)*a),x)
```

3.362 $\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 10.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{5235}$$
$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arccos(cx))}dx$$

input
```
Int[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x(-c^2x^2+1)^{\frac{3}{2}}(a+b\arccos(cx))}dx$$

input
```
int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.29

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}\,dx=\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}(b\arccos\left(cx\right)+a)x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^5 - 2*a*c^2*x^3 + a*x + (b*c^4*x^5 -
2*b*c^2*x^3 + b*x)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}\,dx=\int \frac{1}{x\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acos}\left(cx\right)\right)}\,dx$$

input
```
integrate(1/x/(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/(x*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}\,dx=\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}(b\arccos\left(cx\right)+a)x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccos(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\arccos(cx)\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\arccos(cx)\right)}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

input
```
int(1/(x*(a + b*acos(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(1/(x*(a + b*acos(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.96

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx =$$
$$-\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)\,b\,c^2x^3-\sqrt{-c^2x^2+1}\,acos(cx)\,bx+\sqrt{-c^2x^2+1}\,a\,c^2x^3-\sqrt{-c^2x^2+1}\,ax}dx\right)$$

input
```
int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
- int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*c**2*x**3 - sqrt( - c**2*x**2
 + 1)*acos(c*x)*b*x + sqrt( - c**2*x**2 + 1)*a*c**2*x**3 - sqrt( - c**2*x*
*2 + 1)*a*x),x)
```

3.363 $\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 10.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))}\,dx = \int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{ 5235}$$
$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))}dx$$

input
```
Int[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 (-c^2x^2+1)^{\frac{3}{2}} (a+b\arccos(cx))}dx$$

input
```
int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))} \, dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos{(cx)}+a)x^2} \, dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^6 - 2*a*c^2*x^4 + a*x^2 + (b*c^4*x^6
- 2*b*c^2*x^4 + b*x^2)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))} \, dx = \int \frac{1}{x^2 (-(cx-1)(cx+1))^{\frac{3}{2}} (a+b\operatorname{acos}{(cx)})} \, dx$$

input
```
integrate(1/x**2/(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))} \, dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos{(cx)}+a)x^2} \, dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccos(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 3.80 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))} \, dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos{(cx)}+a)x^2} \, dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccos(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))} \, dx = \int \frac{1}{x^2\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)\,{\left(1-c^2\,x^2\right)}^{3/2}} \,d x$$

input
```
int(1/(x^2*(a + b*acos(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*acos(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.11

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))} \, dx =$$
$$-\left(\int \frac{1}{\sqrt{-c^2x^2+1}\, acos(cx)\, b\, c^2x^4 - \sqrt{-c^2x^2+1}\, acos(cx)\, b\, x^2 + \sqrt{-c^2x^2+1}\, a\, c^2x^4 - \sqrt{-c^2x^2+1}\, a\, x^2} dx\right)$$

input
```
int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
- int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*c**2*x**4 - sqrt( - c**2*x**2
 + 1)*acos(c*x)*b*x**2 + sqrt( - c**2*x**2 + 1)*a*c**2*x**4 - sqrt( - c**2
*x**2 + 1)*a*x**2),x)
```

3.364 $\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{x^2}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 5.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \int \frac{x^2}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx$$

input
```
Integrate[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx$$

$$\Big\downarrow \text{ 5235}$$

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}dx$$

input
```
Int[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(-c^2x^2+1)^{\frac{5}{2}}\,(a+b\arccos(cx))}dx$$

input
```
int(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

output
```
int(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.07

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx=\int \frac{x^2}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^2/(a*c^6*x^6 - 3*a*c^4*x^4 + 3*a*c^2*x^2 +
(b*c^6*x^6 - 3*b*c^4*x^4 + 3*b*c^2*x^2 - b)*arccos(c*x) - a), x)
```

Sympy [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx=\int \frac{x^2}{(-(cx-1)(cx+1))^{\frac{5}{2}}(a+b\operatorname{acos}(cx))}\,dx$$

input
```
integrate(x**2/(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**2/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx=\int \frac{x^2}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^2/((-c^2*x^2 + 1)^(5/2)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 2.92 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{x^2}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(x^2/((-c^2*x^2 + 1)^(5/2)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{x^2}{(a+b\,\mathrm{acos}(c\,x))\,(1-c^2\,x^2)^{5/2}}\,dx$$

input
```
int(x^2/((a + b*acos(c*x))*(1 - c^2*x^2)^(5/2)),x)
```

output
```
int(x^2/((a + b*acos(c*x))*(1 - c^2*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 4.46

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}\,acos\,(cx)\,b\,c^4x^4-2\sqrt{-c^2x^2+1}\,acos\,(cx)\,b\,c^2x^2+\cdots}$$

input
```
int(x^2/(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x)),x)
```

output
```
int(x**2/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*c**4*x**4 - 2*sqrt( - c**2*x*
*2 + 1)*acos(c*x)*b*c**2*x**2 + sqrt( - c**2*x**2 + 1)*acos(c*x)*b + sqrt(
 - c**2*x**2 + 1)*a*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 + sqr
t( - c**2*x**2 + 1)*a),x)
```

3.365 $\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{x}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \int \frac{x}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx$$

input
```
Integrate[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{ 5235}$$
$$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}dx$$

input
```
Int[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x}{(-c^2x^2+1)^{\frac{5}{2}}(a+b\arccos(cx))}dx$$

input
```
int(x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

output
```
int(x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.23

$$\int \frac{x}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx=\int \frac{x}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x/(a*c^6*x^6 - 3*a*c^4*x^4 + 3*a*c^2*x^2 + (b
*c^6*x^6 - 3*b*c^4*x^4 + 3*b*c^2*x^2 - b)*arccos(c*x) - a), x)
```

Sympy [N/A]

Not integrable

Time = 1.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx=\int \frac{x}{(-(cx-1)(cx+1))^{\frac{5}{2}}\,(a+b\,\mathrm{acos}\,(cx))}\,dx$$

input
```
integrate(x/(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx=\int \frac{x}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(x/((-c^2*x^2 + 1)^(5/2)*(b*arccos(c*x) + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{x}{(a+b\,\mathrm{acos}\,(c\,x))\,{\left(1-c^2\,x^2\right)}^{5/2}}\,dx$$

input
```
int(x/((a + b*acos(c*x))*(1 - c^2*x^2)^(5/2)),x)
```

output
```
int(x/((a + b*acos(c*x))*(1 - c^2*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 4.73

$$\int \frac{x}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \int \frac{}{\sqrt{-c^2x^2+1}\,acos\,(cx)\,b\,c^4x^4-2\sqrt{-c^2x^2+1}\,acos\,(cx)\,b\,c^2x^2+}$$

input
```
int(x/(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x)),x)
```

output
```
int(x/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*c**4*x**4 - 2*sqrt( - c**2*x**2
+ 1)*acos(c*x)*b*c**2*x**2 + sqrt( - c**2*x**2 + 1)*acos(c*x)*b + sqrt( -
c**2*x**2 + 1)*a*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 + sqrt(
- c**2*x**2 + 1)*a),x)
```

3.366 $\int \frac{1}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx=\text{Int}\left(\frac{1}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx=\int \frac{1}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{5175}$$
$$\int \frac{1}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}dx$$

input
```
Int[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-c^2x^2+1)^{\frac{5}{2}}(a+b\arccos(cx))}dx$$

input
```
int(1/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.32

$$\int \frac{1}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)/(a*c^6*x^6 - 3*a*c^4*x^4 + 3*a*c^2*x^2 + (b*c
^6*x^6 - 3*b*c^4*x^4 + 3*b*c^2*x^2 - b)*arccos(c*x) - a), x)
```

Sympy [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(-(cx-1)(cx+1))^{\frac{5}{2}}(a+b\,\mathrm{acos}(cx))}\,dx$$

input
```
integrate(1/(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\,\left(1-c^2\,x^2\right)^{5/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(1 - c^2*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(1 - c^2*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 4.84

$$\int \frac{1}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}\,acos\,(cx)\,b\,c^4x^4-2\sqrt{-c^2x^2+1}\,acos\,(cx)\,b\,c^2x^2+ \cdots}$$

input
```
int(1/(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x)),x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*c**4*x**4 - 2*sqrt( - c**2*x**2
+ 1)*acos(c*x)*b*c**2*x**2 + sqrt( - c**2*x**2 + 1)*acos(c*x)*b + sqrt( -
c**2*x**2 + 1)*a*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 + sqrt(
- c**2*x**2 + 1)*a),x)
```

3.367 $\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 7.97 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{ 5235}$$
$$\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}dx$$

input
```
Int[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(-c^2x^2+1\right)^{\frac{5}{2}}\left(a+b\arccos\left(cx\right)\right)}dx$$

input
```
int(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.04

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)/(a*c^6*x^7 - 3*a*c^4*x^5 + 3*a*c^2*x^3 - a*x
+ (b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 4.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{x(-(cx-1)(cx+1))^{\frac{5}{2}}(a+b\operatorname{acos}(cx))}\,dx$$

input
```
integrate(1/x/(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/(x*(-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arccos(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(1-c^{2}x^{2}\right)^{5/2}\left(a+b\arccos(cx)\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-c^{2}x^{2}\right)^{5/2}\left(a+b\arccos(cx)\right)}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)\,{\left(1-c^{2}\,x^{2}\right)}^{5/2}}\,dx$$

input
```
int(1/(x*(a + b*acos(c*x))*(1 - c^2*x^2)^(5/2)),x)
```

output
```
int(1/(x*(a + b*acos(c*x))*(1 - c^2*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 4.39

$$\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}\left(a+b\arccos(cx)\right)}\,dx=\int \frac{}{\sqrt{-c^2x^2+1}\,acos\left(cx\right)b\,c^4x^5-2\sqrt{-c^2x^2+1}\,acos\left(cx\right)b\,c^2x^3+}$$

input
```
int(1/x/(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x)),x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*c**4*x**5 - 2*sqrt( - c**2*x**2
+ 1)*acos(c*x)*b*c**2*x**3 + sqrt( - c**2*x**2 + 1)*acos(c*x)*b*x + sqrt(
- c**2*x**2 + 1)*a*c**4*x**5 - 2*sqrt( - c**2*x**2 + 1)*a*c**2*x**3 + sqrt
( - c**2*x**2 + 1)*a*x),x)
```

3.368 $\int \frac{1}{x^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{x^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 9.75 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{x^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))} \, dx$$

$$\Big\downarrow \; 5235$$

$$\int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))} \, dx$$

input
```
Int[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 (-c^2x^2+1)^{\frac{5}{2}} (a+b\arccos(cx))} \, dx$$

input
```
int(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.18

$$\int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))}\,dx = \int \frac{1}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*x^2 + 1)/(a*c^6*x^8 - 3*a*c^4*x^6 + 3*a*c^2*x^4 - a*x^
2 + (b*c^6*x^8 - 3*b*c^4*x^6 + 3*b*c^2*x^4 - b*x^2)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))}\,dx = \int \frac{1}{x^2(-(cx-1)(cx+1))^{\frac{5}{2}}(a+b\operatorname{acos}(cx))}\,dx$$

input
```
integrate(1/x**2/(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/(x**2*(-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{5/2} (a + b \arccos(cx))} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arccos(cx) + a) x^2} dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="maxima"
)
```

output
```
integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arccos(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 7.77 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{5/2} (a + b \arccos(cx))} dx = \int \frac{1}{(-c^2 x^2 + 1)^{\frac{5}{2}} (b \arccos(cx) + a) x^2} dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arccos(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{5/2} (a + b \arccos(cx))} dx = \int \frac{1}{x^2 \left(a + b \operatorname{acos}(c x)\right) (1 - c^2 x^2)^{5/2}} dx$$

input
```
int(1/(x^2*(a + b*acos(c*x))*(1 - c^2*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*(a + b*acos(c*x))*(1 - c^2*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 4.54

$$\int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)\,b\,c^4x^6 - 2\sqrt{-c^2x^2+1}\,acos(cx)\,b\,c^2x^4}$$

input
```
int(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x)),x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*c**4*x**6 - 2*sqrt( - c**2*x**2
+ 1)*acos(c*x)*b*c**2*x**4 + sqrt( - c**2*x**2 + 1)*acos(c*x)*b*x**2 + sqr
t( - c**2*x**2 + 1)*a*c**4*x**6 - 2*sqrt( - c**2*x**2 + 1)*a*c**2*x**4 + s
qrt( - c**2*x**2 + 1)*a*x**2),x)
```

3.369 $\int \frac{x^m(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \operatorname{Int}\left(\frac{x^m(1-c^2x^2)^{5/2}}{a+b\arccos(cx)},x\right)$$

output
```
Defer(Int)(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{x^m(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx$$

input
```
Integrate[(x^m*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x]),x]
```

output
```
Integrate[(x^m*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-c^2x^2\right)^{5/2}x^m}{a+b\arccos(cx)}dx$$
$$\Big\downarrow \; 5235$$
$$\int \frac{\left(1-c^2x^2\right)^{5/2}x^m}{a+b\arccos(cx)}dx$$

input
```
Int[(x^m*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m(-c^2x^2+1)^{\frac{5}{2}}}{a+b\arccos{(cx)}}dx$$

input
```
int(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

output
```
int(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

$$\int \frac{x^m (1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^m}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)*x^m/(b*arccos(c*x) +
 a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m (1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m (1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^m}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)*x^m/(b*arccos(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \int \frac{x^m\,{\left(1-c^2\,x^2\right)}^{5/2}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((x^m*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x)),x)
```

output
```
int((x^m*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.46

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{a+b\arccos(cx)}\,dx = \left(\int \frac{x^m\sqrt{-c^2x^2+1}\,x^4}{acos\,(cx)\,b+a}dx\right)c^4 - 2\left(\int \frac{x^m\sqrt{-c^2x^2+1}\,x^2}{acos\,(cx)\,b+a}dx\right)c^2 + \int \frac{x^m\sqrt{-c^2x^2+1}}{acos\,(cx)\,b+a}dx$$

input

```
int(x^m*(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x)),x)
```

output

```
int((x**m*sqrt( - c**2*x**2 + 1)*x**4)/(acos(c*x)*b + a),x)*c**4 - 2*int((
x**m*sqrt( - c**2*x**2 + 1)*x**2)/(acos(c*x)*b + a),x)*c**2 + int((x**m*sq
rt( - c**2*x**2 + 1))/(acos(c*x)*b + a),x)
```

3.370 $\int \frac{x^m (1-c^2x^2)^{3/2}}{a+b \arccos(cx)} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m (1-c^2x^2)^{3/2}}{a+b \arccos(cx)} \, dx = \text{Int}\left(\frac{x^m (1-c^2x^2)^{3/2}}{a+b \arccos(cx)}, x\right)$$

output
```
Defer(Int)(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m (1-c^2x^2)^{3/2}}{a+b \arccos(cx)} \, dx = \int \frac{x^m (1-c^2x^2)^{3/2}}{a+b \arccos(cx)} \, dx$$

input
```
Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x]),x]
```

output
```
Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-c^2x^2\right)^{3/2}x^m}{a+b\arccos(cx)}dx$$

$$\Big\downarrow \text{5235}$$

$$\int \frac{\left(1-c^2x^2\right)^{3/2}x^m}{a+b\arccos(cx)}dx$$

input
```
Int[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m(-c^2x^2+1)^{\frac{3}{2}}}{a+b\arccos(cx)}dx$$

input
```
int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

output
```
int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^m}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*x^m/(b*arccos(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 13.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{x^m(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\,\mathrm{acos}(cx)}\,dx$$

input
```
integrate(x**m*(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**m*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acos(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^m}{b\arccos(cx)+a}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)*x^m/(b*arccos(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = \int \frac{x^m\,{\left(1-c^2\,x^2\right)}^{3/2}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((x^m*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x)),x)
```

output
```
int((x^m*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\arccos(cx)}\,dx = -\left(\int \frac{x^m\sqrt{-c^2x^2+1}\,x^2}{acos\,(cx)\,b+a}dx\right)c^2 + \int \frac{x^m\sqrt{-c^2x^2+1}}{acos\,(cx)\,b+a}dx$$

input
```
int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
- int((x**m*sqrt( - c**2*x**2 + 1)*x**2)/(acos(c*x)*b + a),x)*c**2 + int(
(x**m*sqrt( - c**2*x**2 + 1))/(acos(c*x)*b + a),x)
```

3.371 $\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \text{Int}\left(\frac{x^m\sqrt{1-c^2x^2}}{a+b\arccos(cx)},x\right)$$

output
```
Defer(Int)(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{x^m\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx$$

input
```
Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]),x]
```

output
```
Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}x^m}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \ 5235$$

$$\int \frac{\sqrt{1-c^2x^2}x^m}{a+b\arccos(cx)}dx$$

input
```
Int[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m\sqrt{-c^2x^2+1}}{a+b\arccos\left(cx\right)}dx$$

input
```
int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x)
```

output
```
int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^m}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^m/(b*arccos(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{x^m\sqrt{-(cx-1)(cx+1)}}{a+b\operatorname{acos}{(cx)}}\,dx$$

input
```
integrate(x**m*(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**m*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acos(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^m}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)*x^m/(b*arccos(c*x) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx = \int \frac{x^m\,\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((x^m*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x)),x)
```

output
```
int((x^m*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\arccos(cx)}\,dx=\int \frac{x^m\sqrt{-c^2x^2+1}}{acos\,(cx)\,b+a}dx$$

input
```
int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int((x**m*sqrt( - c**2*x**2 + 1))/(acos(c*x)*b + a),x)
```

3.372 $\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$$
$$\Big\downarrow \ 5235$$
$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx$$

input
```
Int[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{\sqrt{-c^2x^2+1}\,\left(a+b\arccos\left(cx\right)\right)}dx$$

input
```
int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x)
```

output
```
int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^m/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccos(c*x)
- a), x)
```

Sympy [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^m}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acos}{(cx)})}\,dx$$

input
```
integrate(x**m/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**m/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^m/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx=\int \frac{x^m}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(x^m/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx=\int \frac{x^m}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^m/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^m/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}\,acos\,(cx)\,b+\sqrt{-c^2x^2+1}\,a}dx$$

input
```
int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int(x**m/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b + sqrt( - c**2*x**2 + 1)*a),x
)
```

3.373 $\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))}\,dx$$

input
```
Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{ 5235}$$
$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}dx$$

input
```
Int[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(-c^2x^2+1)^{\frac{3}{2}}(a+b\arccos(cx))}dx$$

input
```
int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

output
```
int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^m/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*
b*c^2*x^2 + b)*arccos(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 9.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \int \frac{x^m}{(-(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acos}\,(cx))}\,dx$$

input
```
integrate(x**m/(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**m/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^m/((-c^2*x^2 + 1)^(3/2)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(x^m/((-c^2*x^2 + 1)^(3/2)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \int \frac{x^m}{(a+b\,\mathrm{acos}\left(c\,x\right))\,\left(1-c^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(x^m/((a + b*acos(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(x^m/((a + b*acos(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.04

$$\int \frac{x^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\arccos(cx)\right)}\,dx = -\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}\,acos\left(cx\right)b\,c^2x^2-\sqrt{-c^2x^2+1}\,acos\left(cx\right)b+\sqrt{-c^2x^2+1}\,a\,c^2x^2-\sqrt{-c^2x^2+1}\,a}dx\right)$$

input

```
int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x)),x)
```

output

```
 - int(x**m/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*c**2*x**2 - sqrt( - c**2*x
**2 + 1)*acos(c*x)*b + sqrt( - c**2*x**2 + 1)*a*c**2*x**2 - sqrt( - c**2*x
**2 + 1)*a),x)
```

3.374 $\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(1-c^2x^2)^{5/2} (a+b\arccos(cx))} \, dx$$
$$\Big\downarrow \; 5235$$
$$\int \frac{x^m}{(1-c^2x^2)^{5/2} (a+b\arccos(cx))}dx$$

input
```
Int[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(-c^2x^2+1)^{\frac{5}{2}} (a+b\arccos(cx))}dx$$

input
```
int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

output
```
int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.07

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^m/(a*c^6*x^6 - 3*a*c^4*x^4 + 3*a*c^2*x^2 +
(b*c^6*x^6 - 3*b*c^4*x^4 + 3*b*c^2*x^2 - b)*arccos(c*x) - a), x)
```

Sympy [N/A]

Not integrable

Time = 16.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{x^m}{(-(cx-1)(cx+1))^{\frac{5}{2}}(a+b\,\mathrm{acos}(cx))}\,dx$$

input
```
integrate(x**m/(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(x**m/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(x^m/((-c^2*x^2 + 1)^(5/2)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(x^m/((-c^2*x^2 + 1)^(5/2)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{x^m}{(a+b\,\mathrm{acos}\,(c\,x))\,(1-c^2\,x^2)^{5/2}}\,dx$$

input
```
int(x^m/((a + b*acos(c*x))*(1 - c^2*x^2)^(5/2)),x)
```

output
```
int(x^m/((a + b*acos(c*x))*(1 - c^2*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 4.46

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))}\,dx = \int \frac{}{\sqrt{-c^2x^2+1}\,acos\,(cx)\,b\,c^4x^4-2\sqrt{-c^2x^2+1}\,acos\,(cx)\,b\,c^2x^2+}$$

input
```
int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x)),x)
```

output
```
int(x**m/(sqrt( - c**2*x**2 + 1)*acos(c*x)*b*c**4*x**4 - 2*sqrt( - c**2*x*
*2 + 1)*acos(c*x)*b*c**2*x**2 + sqrt( - c**2*x**2 + 1)*acos(c*x)*b + sqrt(
 - c**2*x**2 + 1)*a*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*a*c**2*x**2 + sqr
t( - c**2*x**2 + 1)*a),x)
```

3.375 $\int \frac{x^m}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \operatorname{Int}\left(\frac{x^m}{\sqrt{1-a^2x^2}\arccos(ax)},x\right)$$

output
```
Defer(Int)(x^m/(-a^2*x^2+1)^(1/2)/arccos(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^m}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx$$

input
```
Integrate[x^m/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
Integrate[x^m/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{1-a^2x^2}\arccos(ax)}dx$$

$$\Big\downarrow \text{5235}$$

$$\int \frac{x^m}{\sqrt{1-a^2x^2}\arccos(ax)}dx$$

input
```
Int[x^m/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{-a^2x^2+1}\,\arccos\left(ax\right)}dx$$

input
```
int(x^m/(-a^2*x^2+1)^(1/2)/arccos(a*x),x)
```

output
```
int(x^m/(-a^2*x^2+1)^(1/2)/arccos(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{x^m}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^m}{\sqrt{-a^2x^2+1}\arccos{(ax)}}\,dx$$

input
```
integrate(x^m/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^m/((a^2*x^2 - 1)*arccos(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^m}{\sqrt{-(ax-1)(ax+1)}\operatorname{acos}{(ax)}}\,dx$$

input
```
integrate(x**m/(-a**2*x**2+1)**(1/2)/acos(a*x),x)
```

output
```
Integral(x**m/(sqrt(-(a*x - 1)*(a*x + 1))*acos(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^m}{\sqrt{-a^2x^2+1}\arccos{(ax)}}\,dx$$

input
```
integrate(x^m/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m/(sqrt(-a^2*x^2 + 1)*arccos(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^m}{\sqrt{-a^2x^2+1}\arccos(ax)}\,dx$$

input
```
integrate(x^m/(-a^2*x^2+1)^(1/2)/arccos(a*x),x, algorithm="giac")
```

output
```
integrate(x^m/(sqrt(-a^2*x^2 + 1)*arccos(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^m}{\mathrm{acos}\,(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(x^m/(acos(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(x^m/(acos(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^m}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx = \int \frac{x^m}{\sqrt{-a^2x^2+1}\,acos\,(ax)}dx$$

input
```
int(x^m/(-a^2*x^2+1)^(1/2)/acos(a*x),x)
```

output
```
int(x**m/(sqrt( - a**2*x**2 + 1)*acos(a*x)),x)
```

3.376 $\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 95

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = -\frac{c^3(1-a^2x^2)^{7/2}}{a\arccos(ax)} - \frac{35c^3\text{Si}(\arccos(ax))}{64a} - \frac{63c^3\text{Si}(3\arccos(ax))}{64a} - \frac{35c^3\text{Si}(5\arccos(ax))}{64a} - \frac{7c^3\text{Si}(7\arccos(ax))}{64a}$$

output
```
-c^3*(-a^2*x^2+1)^(7/2)/a/arccos(a*x)-35/64*c^3*Si(arccos(a*x))/a-63/64*c^
3*Si(3*arccos(a*x))/a-35/64*c^3*Si(5*arccos(a*x))/a-7/64*c^3*Si(7*arccos(a
*x))/a
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = \frac{c^3\left(64(1-a^2x^2)^{7/2} - 35\arccos(ax)\,\text{CosIntegral}(\arccos(ax)) + 63\arccos(ax)\,\text{CosIntegral}(3\arccos(ax))\right)}{64a\arccos(ax)}$$

input
```
Integrate[(c - a^2*c*x^2)^3/ArcCos[a*x]^2,x]
```

output
```
(c^3*(64*(1 - a^2*x^2)^(7/2) - 35*ArcCos[a*x]*CosIntegral[ArcCos[a*x]] + 6
3*ArcCos[a*x]*CosIntegral[3*ArcCos[a*x]] - 35*ArcCos[a*x]*CosIntegral[5*Ar
cCos[a*x]] + 7*ArcCos[a*x]*CosIntegral[7*ArcCos[a*x]]))/(64*a*ArcCos[a*x])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5167, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}dx$$

$$\Big\downarrow \text{ 5167}$$

$$7ac^3\int \frac{x(1-a^2x^2)^{5/2}}{\arccos(ax)}dx+\frac{c^3(1-a^2x^2)^{7/2}}{a\arccos(ax)}$$

$$\Big\downarrow \text{ 5225}$$

$$\frac{c^3(1-a^2x^2)^{7/2}}{a\arccos(ax)}-\frac{7c^3\int \frac{ax(1-a^2x^2)^3}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{c^3(1-a^2x^2)^{7/2}}{a\arccos(ax)}-\frac{7c^3\int \left(\frac{5ax}{64\arccos(ax)}-\frac{9\cos(3\arccos(ax))}{64\arccos(ax)}+\frac{5\cos(5\arccos(ax))}{64\arccos(ax)}-\frac{\cos(7\arccos(ax))}{64\arccos(ax)}\right)d\arccos(ax)}{a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^3(1-a^2x^2)^{7/2}}{a\arccos(ax)}-\frac{7c^3\left(\frac{5}{64}\text{CosIntegral}(\arccos(ax))-\frac{9}{64}\text{CosIntegral}(3\arccos(ax))+\frac{5}{64}\text{CosIntegral}(5\arccos(ax))-\frac{1}{64}\text{CosIntegral}\right.}{a}$$

input
```
Int[(c - a^2*c*x^2)^3/ArcCos[a*x]^2,x]
```

output
```
(c^3*(1 - a^2*x^2)^(7/2))/(a*ArcCos[a*x]) - (7*c^3*((5*CosIntegral[ArcCos[
a*x]])/64 - (9*CosIntegral[3*ArcCos[a*x]])/64 + (5*CosIntegral[5*ArcCos[a*
x]])/64 - CosIntegral[7*ArcCos[a*x]]/64))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{c^3\left(63\ \mathrm{Ci}(3\arccos(ax))\arccos(ax)-35\ \mathrm{Ci}(5\arccos(ax))\arccos(ax)+7\ \mathrm{Ci}(7\arccos(ax))\arccos(ax)-35\ \mathrm{Ci}(\arccos(ax)\right.}{64a\arccos(ax)}$
default	$\frac{c^3\left(63\ \mathrm{Ci}(3\arccos(ax))\arccos(ax)-35\ \mathrm{Ci}(5\arccos(ax))\arccos(ax)+7\ \mathrm{Ci}(7\arccos(ax))\arccos(ax)-35\ \mathrm{Ci}(\arccos(ax)\right.}{64a\arccos(ax)}$

input
```
int((-a^2*c*x^2+c)^3/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/64/a*c^3*(63*Ci(3*arccos(a*x))*arccos(a*x)-35*Ci(5*arccos(a*x))*arccos(a
*x)+7*Ci(7*arccos(a*x))*arccos(a*x)-35*Ci(arccos(a*x))*arccos(a*x)-21*sin(
3*arccos(a*x))+7*sin(5*arccos(a*x))-sin(7*arccos(a*x))+35*(-a^2*x^2+1)^(1/
2))/arccos(a*x)
```

Fricas [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = \int -\frac{(a^2cx^2-c)^3}{\arccos(ax)^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^3/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)/arccos(a*x)^
2, x)
```

Sympy [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = -c^3\left(\int \frac{3a^2x^2}{\operatorname{acos}^2(ax)}\,dx + \int \left(-\frac{3a^4x^4}{\operatorname{acos}^2(ax)}\right)dx + \int \frac{a^6x^6}{\operatorname{acos}^2(ax)}\,dx + \int \left(-\frac{1}{\operatorname{acos}^2(ax)}\right)dx\right)$$

input
```
integrate((-a**2*c*x**2+c)**3/acos(a*x)**2,x)
```

output
```
-c**3*(Integral(3*a**2*x**2/acos(a*x)**2, x) + Integral(-3*a**4*x**4/acos(
a*x)**2, x) + Integral(a**6*x**6/acos(a*x)**2, x) + Integral(-1/acos(a*x)*
*2, x))
```

Maxima [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = \int -\frac{(a^2cx^2-c)^3}{\arccos(ax)^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^3/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate(7*(a^5*c^3*x^5 - 2
*a^3*c^3*x^3 + a*c^3*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/arctan2(sqrt(a*x + 1)
*sqrt(-a*x + 1), a*x), x) - (a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 -
 c^3)*sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1
), a*x))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.78

$$\begin{aligned}\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = &-\frac{\sqrt{-a^2x^2+1}a^5c^3x^6}{\arccos(ax)} + \frac{3\sqrt{-a^2x^2+1}a^3c^3x^4}{\arccos(ax)} - \frac{3\sqrt{-a^2x^2+1}ac^3x^2}{\arccos(ax)} \\ &+ \frac{7c^3\,\mathrm{Ci}(7\arccos(ax))}{64a} - \frac{35c^3\,\mathrm{Ci}(5\arccos(ax))}{64a} \\ &+ \frac{63c^3\,\mathrm{Ci}(3\arccos(ax))}{64a} - \frac{35c^3\,\mathrm{Ci}(\arccos(ax))}{64a} + \frac{\sqrt{-a^2x^2+1}c^3}{a\arccos(ax)}\end{aligned}$$

input
```
integrate((-a^2*c*x^2+c)^3/arccos(a*x)^2,x, algorithm="giac")
```

output
```
-sqrt(-a^2*x^2 + 1)*a^5*c^3*x^6/arccos(a*x) + 3*sqrt(-a^2*x^2 + 1)*a^3*c^3
*x^4/arccos(a*x) - 3*sqrt(-a^2*x^2 + 1)*a*c^3*x^2/arccos(a*x) + 7/64*c^3*c
os_integral(7*arccos(a*x))/a - 35/64*c^3*cos_integral(5*arccos(a*x))/a + 6
3/64*c^3*cos_integral(3*arccos(a*x))/a - 35/64*c^3*cos_integral(arccos(a*x
))/a + sqrt(-a^2*x^2 + 1)*c^3/(a*arccos(a*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = \int \frac{(c-a^2\,c\,x^2)^3}{\mathrm{acos}\,(a\,x)^2}\,dx$$

input
```
int((c - a^2*c*x^2)^3/acos(a*x)^2,x)
```

output
```
int((c - a^2*c*x^2)^3/acos(a*x)^2, x)
```

Reduce [F]

$$\int \frac{(c-a^2cx^2)^3}{\arccos(ax)^2}\,dx = c^3\left(-\left(\int \frac{x^6}{acos\,(ax)^2}dx\right)a^6+3\left(\int \frac{x^4}{acos\,(ax)^2}dx\right)a^4 -3\left(\int \frac{x^2}{acos\,(ax)^2}dx\right)a^2+\int \frac{1}{acos\,(ax)^2}dx\right)$$

input
```
int((-a^2*c*x^2+c)^3/acos(a*x)^2,x)
```

output
```
c**3*( - int(x**6/acos(a*x)**2,x)*a**6 + 3*int(x**4/acos(a*x)**2,x)*a**4 -
 3*int(x**2/acos(a*x)**2,x)*a**2 + int(1/acos(a*x)**2,x))
```

3.377 $\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 78

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx = -\frac{c^2(1-a^2x^2)^{5/2}}{a\arccos(ax)} - \frac{5c^2\mathrm{Si}(\arccos(ax))}{8a} - \frac{15c^2\mathrm{Si}(3\arccos(ax))}{16a} - \frac{5c^2\mathrm{Si}(5\arccos(ax))}{16a}$$

output
```
-c^2*(-a^2*x^2+1)^(5/2)/a/arccos(a*x)-5/8*c^2*Si(arccos(a*x))/a-15/16*c^2*
Si(3*arccos(a*x))/a-5/16*c^2*Si(5*arccos(a*x))/a
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx = \frac{c^2\left(16(1-a^2x^2)^{5/2} - 10\arccos(ax)\,\mathrm{CosIntegral}(\arccos(ax)) + 15\arccos(ax)\,\mathrm{CosIntegral}(3\arccos(ax))\right.}{16a\arccos(ax)}$$

input
```
Integrate[(c - a^2*c*x^2)^2/ArcCos[a*x]^2,x]
```

output
```
(c^2*(16*(1 - a^2*x^2)^(5/2) - 10*ArcCos[a*x]*CosIntegral[ArcCos[a*x]] + 1
5*ArcCos[a*x]*CosIntegral[3*ArcCos[a*x]] - 5*ArcCos[a*x]*CosIntegral[5*Arc
Cos[a*x]]))/(16*a*ArcCos[a*x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5167, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}dx$$

$$\Big\downarrow \text{ 5167}$$

$$5ac^2\int \frac{x(1-a^2x^2)^{3/2}}{\arccos(ax)}dx+\frac{c^2(1-a^2x^2)^{5/2}}{a\arccos(ax)}$$

$$\Big\downarrow \text{ 5225}$$

$$\frac{c^2(1-a^2x^2)^{5/2}}{a\arccos(ax)}-\frac{5c^2\int \frac{ax(1-a^2x^2)^2}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{c^2(1-a^2x^2)^{5/2}}{a\arccos(ax)}-\frac{5c^2\int \left(\frac{ax}{8\arccos(ax)}-\frac{3\cos(3\arccos(ax))}{16\arccos(ax)}+\frac{\cos(5\arccos(ax))}{16\arccos(ax)}\right)d\arccos(ax)}{a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^2(1-a^2x^2)^{5/2}}{a\arccos(ax)}-\frac{5c^2\left(\frac{1}{8}\text{CosIntegral}(\arccos(ax))-\frac{3}{16}\text{CosIntegral}(3\arccos(ax))+\frac{1}{16}\text{CosIntegral}(5\arccos(ax))\right)}{a}$$

input
```
Int[(c - a^2*c*x^2)^2/ArcCos[a*x]^2,x]
```

output
```
(c^2*(1 - a^2*x^2)^(5/2))/(a*ArcCos[a*x]) - (5*c^2*(CosIntegral[ArcCos[a*x
]]/8 - (3*CosIntegral[3*ArcCos[a*x]])/16 + CosIntegral[5*ArcCos[a*x]]/16))
/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{c^2\left(15\ \mathrm{Ci}(3\arccos(ax))\arccos(ax)-5\ \mathrm{Ci}(5\arccos(ax))\arccos(ax)-10\ \mathrm{Ci}(\arccos(ax))\arccos(ax)-5\sin(3\arccos(ax))\right.}{16a\arccos(ax)}$
default	$\frac{c^2\left(15\ \mathrm{Ci}(3\arccos(ax))\arccos(ax)-5\ \mathrm{Ci}(5\arccos(ax))\arccos(ax)-10\ \mathrm{Ci}(\arccos(ax))\arccos(ax)-5\sin(3\arccos(ax))\right.}{16a\arccos(ax)}$

input
```
int((-a^2*c*x^2+c)^2/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16/a*c^2*(15*Ci(3*arccos(a*x))*arccos(a*x)-5*Ci(5*arccos(a*x))*arccos(a*
x)-10*Ci(arccos(a*x))*arccos(a*x)-5*sin(3*arccos(a*x))+sin(5*arccos(a*x))+
10*(-a^2*x^2+1)^(1/2))/arccos(a*x)
```

Fricas [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx = \int \frac{(a^2cx^2-c)^2}{\arccos(ax)^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^2/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)/arccos(a*x)^2, x)
```

Sympy [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx = c^2\left(\int \left(-\frac{2a^2x^2}{\operatorname{acos}^2(ax)}\right)dx + \int \frac{a^4x^4}{\operatorname{acos}^2(ax)}\,dx + \int \frac{1}{\operatorname{acos}^2(ax)}\,dx\right)$$

input
```
integrate((-a**2*c*x**2+c)**2/acos(a*x)**2,x)
```

output
```
c**2*(Integral(-2*a**2*x**2/acos(a*x)**2, x) + Integral(a**4*x**4/acos(a*x
)**2, x) + Integral(acos(a*x)**(-2), x))
```

Maxima [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx = \int \frac{(a^2cx^2-c)^2}{\arccos(ax)^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^2/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
-(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate(5*(a^3*c^2*x^3 -
a*c^2*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1)
, a*x), x) - (a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(-a*x +
 1))/(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.59

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx = \frac{\sqrt{-a^2x^2+1}a^3c^2x^4}{\arccos(ax)} - \frac{2\sqrt{-a^2x^2+1}ac^2x^2}{\arccos(ax)} - \frac{5c^2\operatorname{Ci}(5\arccos(ax))}{16a} + \frac{15c^2\operatorname{Ci}(3\arccos(ax))}{16a} - \frac{5c^2\operatorname{Ci}(\arccos(ax))}{8a} + \frac{\sqrt{-a^2x^2+1}c^2}{a\arccos(ax)}$$

input
```
integrate((-a^2*c*x^2+c)^2/arccos(a*x)^2,x, algorithm="giac")
```

output
```
sqrt(-a^2*x^2 + 1)*a^3*c^2*x^4/arccos(a*x) - 2*sqrt(-a^2*x^2 + 1)*a*c^2*x^
2/arccos(a*x) - 5/16*c^2*cos_integral(5*arccos(a*x))/a + 15/16*c^2*cos_int
egral(3*arccos(a*x))/a - 5/8*c^2*cos_integral(arccos(a*x))/a + sqrt(-a^2*x
^2 + 1)*c^2/(a*arccos(a*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx = \int \frac{\left(c-a^2\,c\,x^2\right)^2}{\mathrm{acos}\left(a\,x\right)^2}\,dx$$

input
```
int((c - a^2*c*x^2)^2/acos(a*x)^2,x)
```

output
```
int((c - a^2*c*x^2)^2/acos(a*x)^2, x)
```

Reduce [F]

$$\int \frac{(c-a^2cx^2)^2}{\arccos(ax)^2}\,dx = c^2\left(\left(\int \frac{x^4}{acos\left(ax\right)^2}dx\right)a^4 - 2\left(\int \frac{x^2}{acos\left(ax\right)^2}dx\right)a^2 + \int \frac{1}{acos\left(ax\right)^2}dx\right)$$

input
```
int((-a^2*c*x^2+c)^2/acos(a*x)^2,x)
```

output
```
c**2*(int(x**4/acos(a*x)**2,x)*a**4 - 2*int(x**2/acos(a*x)**2,x)*a**2 + in
t(1/acos(a*x)**2,x))
```

3.378 $\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 55

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = -\frac{c(1-a^2x^2)^{3/2}}{a\arccos(ax)} - \frac{3c\mathrm{Si}(\arccos(ax))}{4a} - \frac{3c\mathrm{Si}(3\arccos(ax))}{4a}$$

output
```
-c*(-a^2*x^2+1)^(3/2)/a/arccos(a*x)-3/4*c*Si(arccos(a*x))/a-3/4*c*Si(3*arc
cos(a*x))/a
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = \frac{c\left(4(1-a^2x^2)^{3/2} - 3\arccos(ax)\,\mathrm{CosIntegral}(\arccos(ax)) + 3\arccos(ax)\,\mathrm{CosIntegral}(3\arccos(ax))\right)}{4a\arccos(ax)}$$

input
```
Integrate[(c - a^2*c*x^2)/ArcCos[a*x]^2,x]
```

output
```
(c*(4*(1 - a^2*x^2)^(3/2) - 3*ArcCos[a*x]*CosIntegral[ArcCos[a*x]] + 3*Arc
Cos[a*x]*CosIntegral[3*ArcCos[a*x]]))/(4*a*ArcCos[a*x])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5167, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx$$

$$\Big\downarrow \text{5167}$$

$$3ac\int \frac{x\sqrt{1-a^2x^2}}{\arccos(ax)}dx+\frac{c(1-a^2x^2)^{3/2}}{a\arccos(ax)}$$

$$\Big\downarrow \text{5225}$$

$$\frac{c(1-a^2x^2)^{3/2}}{a\arccos(ax)}-\frac{3c\int \frac{ax(1-a^2x^2)}{\arccos(ax)}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{4906}$$

$$\frac{c(1-a^2x^2)^{3/2}}{a\arccos(ax)}-\frac{3c\int \left(\frac{ax}{4\arccos(ax)}-\frac{\cos(3\arccos(ax))}{4\arccos(ax)}\right)d\arccos(ax)}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(1-a^2x^2)^{3/2}}{a\arccos(ax)}-\frac{3c\left(\frac{1}{4}\text{CosIntegral}(\arccos(ax))-\frac{1}{4}\text{CosIntegral}(3\arccos(ax))\right)}{a}$$

input
```
Int[(c - a^2*c*x^2)/ArcCos[a*x]^2,x]
```

output
```
(c*(1 - a^2*x^2)^(3/2))/(a*ArcCos[a*x]) - (3*c*(CosIntegral[ArcCos[a*x]]/4
 - CosIntegral[3*ArcCos[a*x]]/4))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{c\left(3\ \mathrm{Ci}(3\arccos(ax))\arccos(ax)-3\ \mathrm{Ci}(\arccos(ax))\arccos(ax)-\sin(3\arccos(ax))+3\sqrt{-a^2x^2+1}\right)}{4a\arccos(ax)}$	61
default	$\frac{c\left(3\ \mathrm{Ci}(3\arccos(ax))\arccos(ax)-3\ \mathrm{Ci}(\arccos(ax))\arccos(ax)-\sin(3\arccos(ax))+3\sqrt{-a^2x^2+1}\right)}{4a\arccos(ax)}$	61

input
```
int((-a^2*c*x^2+c)/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/a*c*(3*Ci(3*arccos(a*x))*arccos(a*x)-3*Ci(arccos(a*x))*arccos(a*x)-sin
(3*arccos(a*x))+3*(-a^2*x^2+1)^(1/2))/arccos(a*x)
```

Fricas [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = \int -\frac{a^2cx^2-c}{\arccos{(ax)}^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-(a^2*c*x^2 - c)/arccos(a*x)^2, x)
```

Sympy [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = -c\left(\int \frac{a^2x^2}{\operatorname{acos}^2{(ax)}}\,dx + \int \left(-\frac{1}{\operatorname{acos}^2{(ax)}}\right)dx\right)$$

input
```
integrate((-a**2*c*x**2+c)/acos(a*x)**2,x)
```

output
```
-c*(Integral(a**2*x**2/acos(a*x)**2, x) + Integral(-1/acos(a*x)**2, x))
```

Maxima [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = \int -\frac{a^2cx^2-c}{\arccos{(ax)}^2}\,dx$$

input
```
integrate((-a^2*c*x^2+c)/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
(3*a^2*c*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate(sqrt(a*x + 1
)*sqrt(-a*x + 1)*x/arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x), x) - (a^2*c
*x^2 - c)*sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x
 + 1), a*x))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = -\frac{\sqrt{-a^2x^2+1}acx^2}{\arccos(ax)} + \frac{3\,c\,\mathrm{Ci}\left(3\,\arccos\left(ax\right)\right)}{4\,a} - \frac{3\,c\,\mathrm{Ci}\left(\arccos\left(ax\right)\right)}{4\,a} + \frac{\sqrt{-a^2x^2+1}c}{a\arccos\left(ax\right)}$$

input
```
integrate((-a^2*c*x^2+c)/arccos(a*x)^2,x, algorithm="giac")
```

output
```
-sqrt(-a^2*x^2 + 1)*a*c*x^2/arccos(a*x) + 3/4*c*cos_integral(3*arccos(a*x)
)/a - 3/4*c*cos_integral(arccos(a*x))/a + sqrt(-a^2*x^2 + 1)*c/(a*arccos(a
*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = \int \frac{c-a^2\,c\,x^2}{{\mathrm{acos}\left(a\,x\right)}^2}\,dx$$

input
```
int((c - a^2*c*x^2)/acos(a*x)^2,x)
```

output
```
int((c - a^2*c*x^2)/acos(a*x)^2, x)
```

Reduce [F]

$$\int \frac{c-a^2cx^2}{\arccos(ax)^2}\,dx = c\left(-\left(\int \frac{x^2}{acos\left(ax\right)^2}dx\right)a^2+\int \frac{1}{acos\left(ax\right)^2}dx\right)$$

input
```
int((-a^2*c*x^2+c)/acos(a*x)^2,x)
```

output
```
c*( - int(x**2/acos(a*x)**2,x)*a**2 + int(1/acos(a*x)**2,x))
```

3.379 $\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx = -\frac{1}{ac\sqrt{1-a^2x^2}\arccos(ax)} + \frac{a\mathrm{Int}\left(\frac{x}{(1-a^2x^2)^{3/2}\arccos(ax)},x\right)}{c}$$

output
```
-1/a/c/(-a^2*x^2+1)^(1/2)/arccos(a*x)+a*Defer(Int)(x/(-a^2*x^2+1)^(3/2)/ar
ccos(a*x),x)/c
```

Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx = \int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx$$

input
```
Integrate[1/((c - a^2*c*x^2)*ArcCos[a*x]^2),x]
```

output
```
Integrate[1/((c - a^2*c*x^2)*ArcCos[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^2 (c-a^2cx^2)} \, dx$$

$$\Big\downarrow \text{ 5167}$$

$$\frac{1}{ac\sqrt{1-a^2x^2}\arccos(ax)}-\frac{a\int \frac{x}{(1-a^2x^2)^{3/2}\arccos(ax)}dx}{c}$$

$$\Big\downarrow \text{ 5235}$$

$$\frac{1}{ac\sqrt{1-a^2x^2}\arccos(ax)}-\frac{a\int \frac{x}{(1-a^2x^2)^{3/2}\arccos(ax)}dx}{c}$$

input
```
Int[1/((c - a^2*c*x^2)*ArcCos[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-a^2c\,x^2+c)\arccos(ax)^2}dx$$

input
```
int(1/(-a^2*c*x^2+c)/arccos(a*x)^2,x)
```

output
```
int(1/(-a^2*c*x^2+c)/arccos(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx = \int -\frac{1}{(a^2cx^2-c)\arccos{(ax)}^2}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(-1/((a^2*c*x^2 - c)*arccos(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx = -\frac{\int \frac{1}{a^2x^2\operatorname{acos}^2{(ax)}-\operatorname{acos}^2{(ax)}}\,dx}{c}$$

input
```
integrate(1/(-a**2*c*x**2+c)/acos(a*x)**2,x)
```

output
```
-Integral(1/(a**2*x**2*acos(a*x)**2 - acos(a*x)**2), x)/c
```

Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 154, normalized size of antiderivative = 7.70

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx = \int -\frac{1}{(a^2cx^2-c)\arccos{(ax)}^2}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^4*c*x^2 - a^2*c)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate
(sqrt(a*x + 1)*sqrt(-a*x + 1)*x/((a^4*c*x^4 - 2*a^2*c*x^2 + c)*arctan2(sqr
t(a*x + 1)*sqrt(-a*x + 1), a*x)), x) + sqrt(a*x + 1)*sqrt(-a*x + 1))/((a^3
*c*x^2 - a*c)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))
```

Giac [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx = \int -\frac{1}{(a^2cx^2-c)\arccos{(ax)}^2}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)/arccos(a*x)^2,x, algorithm="giac")
```

output
```
integrate(-1/((a^2*c*x^2 - c)*arccos(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx=\int \frac{1}{\mathrm{acos}\left(a\,x\right)^2\,\left(c-a^2\,c\,x^2\right)}\,dx$$

input
```
int(1/(acos(a*x)^2*(c - a^2*c*x^2)),x)
```

output
```
int(1/(acos(a*x)^2*(c - a^2*c*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{(c-a^2cx^2)\arccos(ax)^2}\,dx=-\frac{\int \frac{1}{acos(ax)^2a^2x^2-acos(ax)^2}dx}{c}$$

input
```
int(1/(-a^2*c*x^2+c)/acos(a*x)^2,x)
```

output
```
( - int(1/(acos(a*x)**2*a**2*x**2 - acos(a*x)**2),x))/c
```

3.380 $\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx = -\frac{1}{ac^2(1-a^2x^2)^{3/2} \arccos(ax)} + \frac{3a\text{Int}\left(\frac{x}{(1-a^2x^2)^{5/2}\arccos(ax)}, x\right)}{c^2}$$

output
```
-1/a/c^2/(-a^2*x^2+1)^(3/2)/arccos(a*x)+3*a*Defer(Int)(x/(-a^2*x^2+1)^(5/2
)/arccos(a*x),x)/c^2
```

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx = \int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx$$

input
```
Integrate[1/((c - a^2*c*x^2)^2*ArcCos[a*x]^2),x]
```

output
```
Integrate[1/((c - a^2*c*x^2)^2*ArcCos[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^2 (c-a^2cx^2)^2}dx$$

$$\Big\downarrow \text{ 5167}$$

$$\frac{1}{ac^2 (1-a^2x^2)^{3/2}\arccos(ax)}-\frac{3a\int \frac{x}{(1-a^2x^2)^{5/2}\arccos(ax)}dx}{c^2}$$

$$\Big\downarrow \text{ 5235}$$

$$\frac{1}{ac^2 (1-a^2x^2)^{3/2}\arccos(ax)}-\frac{3a\int \frac{x}{(1-a^2x^2)^{5/2}\arccos(ax)}dx}{c^2}$$

input
```
Int[1/((c - a^2*c*x^2)^2*ArcCos[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(-a^{2} c \,x^{2}+c \right)^{2} \arccos \left(a x \right)^{2}}d x$$

input
```
int(1/(-a^2*c*x^2+c)^2/arccos(a*x)^2,x)
```

output
```
int(1/(-a^2*c*x^2+c)^2/arccos(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx=\int { \frac {1}{{\left (a^{2} c x^{2} - c\right)}^{2} \arccos \left (a x\right)^{2}} \,d x }$$

input
```
integrate(1/(-a^2*c*x^2+c)^2/arccos(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*arccos(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx=\frac{\int \frac{1}{a^{4} x^{4} \operatorname{acos}^{2}{\left(a x \right)} - 2 a^{2} x^{2} \operatorname{acos}^{2}{\left(a x \right)} + \operatorname{acos}^{2}{\left(a x \right)}}\, dx}{c^{2}}$$

input
```
integrate(1/(-a**2*c*x**2+c)**2/acos(a*x)**2,x)
```

output
```
Integral(1/(a**4*x**4*acos(a*x)**2 - 2*a**2*x**2*acos(a*x)**2 + acos(a*x)*
*2), x)/c**2
```

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 201, normalized size of antiderivative = 10.05

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx = \int \frac{1}{(a^2cx^2-c)^2 \arccos{(ax)}^2} \, dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^2/arccos(a*x)^2,x, algorithm="maxima")
```

output
```
(3*(a^6*c^2*x^4 - 2*a^4*c^2*x^2 + a^2*c^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x
 + 1), a*x)*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x/((a^6*c^2*x^6 - 3*a^4
*c^2*x^4 + 3*a^2*c^2*x^2 - c^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)
), x) + sqrt(a*x + 1)*sqrt(-a*x + 1))/((a^5*c^2*x^4 - 2*a^3*c^2*x^2 + a*c^
2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))
```

Giac [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx = \int \frac{1}{(a^2cx^2-c)^2 \arccos{(ax)}^2} \, dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^2/arccos(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 - c)^2*arccos(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx = \int \frac{1}{\mathrm{acos}\,(a\,x)^2\,(c-a^2\,c\,x^2)^2} \, dx$$

input
```
int(1/(acos(a*x)^2*(c - a^2*c*x^2)^2),x)
```

output
```
int(1/(acos(a*x)^2*(c - a^2*c*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{1}{(c-a^2cx^2)^2 \arccos(ax)^2} \, dx = \frac{\int \frac{1}{acos(ax)^2 a^4 x^4 - 2acos(ax)^2 a^2 x^2 + acos(ax)^2} dx}{c^2}$$

input
```
int(1/(-a^2*c*x^2+c)^2/acos(a*x)^2,x)
```

output
```
int(1/(acos(a*x)**2*a**4*x**4 - 2*acos(a*x)**2*a**2*x**2 + acos(a*x)**2),x
)/c**2
```

3.381 $\int \left(\frac{1}{(1-x^2)\arccos(x)^2}-\frac{x}{(1-x^2)^{3/2}\arccos(x)}\right)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 17

$$\int \left(\frac{1}{(1-x^2)\arccos(x)^2}-\frac{x}{(1-x^2)^{3/2}\arccos(x)}\right)\,dx=-\frac{1}{\sqrt{1-x^2}\arccos(x)}$$

output

```
-1/(-x^2+1)^(1/2)/arccos(x)
```

Mathematica [F]

$$\int \left(\frac{1}{(1-x^2)\arccos(x)^2}-\frac{x}{(1-x^2)^{3/2}\arccos(x)}\right)\,dx=\int \left(\frac{1}{(1-x^2)\arccos(x)^2}-\frac{x}{(1-x^2)^{3/2}\arccos(x)}\right)\,dx$$

input

```
Integrate[1/((1 - x^2)*ArcCos[x]^2) - x/((1 - x^2)^(3/2)*ArcCos[x]),x]
```

output

```
Integrate[1/((1 - x^2)*ArcCos[x]^2) - x/((1 - x^2)^(3/2)*ArcCos[x]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{(1-x^2)\arccos(x)^2}-\frac{x}{(1-x^2)^{3/2}\arccos(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{\sqrt{1-x^2}\arccos(x)}-2\int \frac{x}{(1-x^2)^{3/2}\arccos(x)}dx$$

input
```
Int[1/((1 - x^2)*ArcCos[x]^2) - x/((1 - x^2)^(3/2)*ArcCos[x]),x]
```

output
```
$Aborted
```

Maple [F]

$$\int \left(\frac{1}{(-x^2+1)\arccos(x)^2}-\frac{x}{(-x^2+1)^{\frac{3}{2}}\arccos(x)}\right)dx$$

input
```
int(1/(-x^2+1)/arccos(x)^2-x/(-x^2+1)^(3/2)/arccos(x),x)
```

output
```
int(1/(-x^2+1)/arccos(x)^2-x/(-x^2+1)^(3/2)/arccos(x),x)
```

Fricas [F]

$$\int \left(\frac{1}{(1-x^2)\arccos(x)^2}-\frac{x}{(1-x^2)^{3/2}\arccos(x)}\right)dx=\int -\frac{x}{(-x^2+1)^{\frac{3}{2}}\arccos(x)}-\frac{1}{(x^2-1)\arccos(x)^2}\,dx$$

input
```
integrate(1/(-x^2+1)/arccos(x)^2-x/(-x^2+1)^(3/2)/arccos(x),x, algorithm="
fricas")
```

output
```
integral(-(sqrt(-x^2 + 1)*x*arccos(x) + x^2 - 1)/((x^4 - 2*x^2 + 1)*arccos
(x)^2), x)
```

Sympy [F]

$$\int\left(\frac{1}{(1-x^2)\arccos(x)^2} - \frac{x}{(1-x^2)^{3/2}\arccos(x)}\right)dx = \int\frac{(x-1)(x+1)\left(x\,\mathrm{acos}\,(x)-\sqrt{1-x^2}\right)}{(-(x-1)(x+1))^{\frac{5}{2}}\,\mathrm{acos}^2(x)}\,dx$$

input
```
integrate(1/(-x**2+1)/acos(x)**2-x/(-x**2+1)**(3/2)/acos(x),x)
```

output
```
Integral((x - 1)*(x + 1)*(x*acos(x) - sqrt(1 - x**2))/((-(x - 1)*(x + 1))*
*(5/2)*acos(x)**2), x)
```

Maxima [F]

$$\int\left(\frac{1}{(1-x^2)\arccos(x)^2} - \frac{x}{(1-x^2)^{3/2}\arccos(x)}\right)dx = \int-\frac{x}{(-x^2+1)^{\frac{3}{2}}\arccos(x)} - \frac{1}{(x^2-1)\arccos(x)^2}\,dx$$

input
```
integrate(1/(-x^2+1)/arccos(x)^2-x/(-x^2+1)^(3/2)/arccos(x),x, algorithm="
maxima")
```

output
```
-(2*(x^2 - 1)*arctan2(sqrt(x + 1)*sqrt(-x + 1), x)*integrate(sqrt(x + 1)*x
*sqrt(-x + 1)/((x^4 - 2*x^2 + 1)*arctan2(sqrt(x + 1)*sqrt(-x + 1), x)), x)
 + sqrt(x + 1)*sqrt(-x + 1))/((x^2 - 1)*arctan2(sqrt(x + 1)*sqrt(-x + 1),
x))
```

Giac [F]

$$\int \left(\frac{1}{(1-x^2)\arccos(x)^2} - \frac{x}{(1-x^2)^{3/2}\arccos(x)}\right)dx = \int -\frac{x}{(-x^2+1)^{\frac{3}{2}}\arccos(x)} - \frac{1}{(x^2-1)\arccos(x)^2}\,dx$$

input
```
integrate(1/(-x^2+1)/arccos(x)^2-x/(-x^2+1)^(3/2)/arccos(x),x, algorithm="
giac")
```

output
```
integrate(-x/((-x^2 + 1)^(3/2)*arccos(x)) - 1/((x^2 - 1)*arccos(x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{1}{(1-x^2)\arccos(x)^2} - \frac{x}{(1-x^2)^{3/2}\arccos(x)}\right)dx = -\int \frac{1}{\mathrm{acos}(x)^2\,(x^2-1)} + \frac{x}{\mathrm{acos}(x)\,(1-x^2)^{3/2}}\,dx$$

input
```
int(- 1/(acos(x)^2*(x^2 - 1)) - x/(acos(x)*(1 - x^2)^(3/2)),x)
```

output
```
-int(1/(acos(x)^2*(x^2 - 1)) + x/(acos(x)*(1 - x^2)^(3/2)), x)
```

Reduce [F]

$$\int \left(\frac{1}{(1-x^2)\arccos(x)^2} - \frac{x}{(1-x^2)^{3/2}\arccos(x)}\right) dx = \frac{2\sqrt{-x^2+1}\,acos(x)\left(\int \frac{x}{\sqrt{-x^2+1}\,acos(x)x^2-\sqrt{-x^2+1}\,acos(x)}dx\right)+1}{\sqrt{-x^2+1}\,acos\,(x)}$$

input
```
int(1/(-x^2+1)/acos(x)^2-x/(-x^2+1)^(3/2)/acos(x),x)
```

output
```
(2*sqrt( - x**2 + 1)*acos(x)*int(x/(sqrt( - x**2 + 1)*acos(x)*x**2 - sqrt(
 - x**2 + 1)*acos(x)),x) + 1)/(sqrt( - x**2 + 1)*acos(x))
```

3.382 $\int \frac{x^m\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx=\text{Int}\left(\frac{x^m\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{x^m\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x])^2,x]
```

output
```
Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}x^m}{(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \text{ 5235}$$

$$\int \frac{\sqrt{1-c^2x^2}x^m}{(a+b\arccos(cx))^2}dx$$

input
```
Int[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m\sqrt{-c^2x^2+1}}{\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{x^m\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^m}{\left(b\arccos\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^m/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a
^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^m\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^m\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{\left(a+b\operatorname{acos}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(x**m*(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**m*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acos(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 4.96

$$\int \frac{x^m\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^m}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^2*x^2 - 1)*x^m - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) +
a*b*c)*integrate(((c^2*m + 2*c^2)*x^2 - m)*x^m/(b^2*c*x*arctan2(sqrt(c*x +
 1)*sqrt(-c*x + 1), c*x) + a*b*c*x), x))/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt
(-c*x + 1), c*x) + a*b*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^m\,\sqrt{1-c^2\,x^2}}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((x^m*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x))^2,x)
```

output
```
int((x^m*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{x^m\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^m\sqrt{-c^2x^2+1}}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx$$

input
```
int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int((x**m*sqrt( - c**2*x**2 + 1))/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a
**2),x)
```

3.383 $\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 214

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = -\frac{x^3(1-c^2x^2)}{bc(a+b\arccos(cx))} + \frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2c^4} + \frac{3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2c^4} - \frac{5\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2c^4} + \frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2c^4} + \frac{3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2c^4} - \frac{5\sin\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2c^4}$$

output
```
-x^3*(-c^2*x^2+1)/b/c/(a+b*arccos(c*x))+1/8*cos(a/b)*Ci((a+b*arccos(c*x))/
b)/b^2/c^4+3/16*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b^2/c^4-5/16*cos(5*a/
b)*Ci(5*(a+b*arccos(c*x))/b)/b^2/c^4+1/8*sin(a/b)*Si((a+b*arccos(c*x))/b)/
b^2/c^4+3/16*sin(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^2/c^4-5/16*sin(5*a/b)*
Si(5*(a+b*arccos(c*x))/b)/b^2/c^4
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.82

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \frac{\frac{16bc^3x^3}{a+b\arccos(cx)} - \frac{16bc^5x^5}{a+b\arccos(cx)} - 2\,\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right) + 3\,\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)$$

input
```
Integrate[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x])^2,x]
```

output
```
((16*b*c^3*x^3)/(a + b*ArcCos[c*x]) - (16*b*c^5*x^5)/(a + b*ArcCos[c*x]) -
 2*CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b] + 3*CosIntegral[3*(a/b + ArcCos
[c*x])]*Sin[(3*a)/b] + 5*CosIntegral[5*(a/b + ArcCos[c*x])]*Sin[(5*a)/b] +
 2*Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]] - 3*Cos[(3*a)/b]*SinIntegral[3*
(a/b + ArcCos[c*x])] - 5*Cos[(5*a)/b]*SinIntegral[5*(a/b + ArcCos[c*x])])/
(16*b^2*c^4)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5215, 5147, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \text{ 5215}$$

$$\frac{5c\int \frac{x^4}{a+b\arccos(cx)}dx}{b} - \frac{3\int \frac{x^2}{a+b\arccos(cx)}dx}{bc} + \frac{x^3(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{ 5147}$$

$$-\frac{5\int -\frac{\cos^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^4}+\frac{3\int -\frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^4}+\frac{x^3(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$\big\downarrow$ 25

$$\frac{5\int \frac{\cos^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^4}-\frac{3\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^4}+\frac{x^3(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$\big\downarrow$ 4906

$$\frac{5\int \left(\frac{\sin\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{3\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^4}-\frac{3\int \left(\frac{\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}+\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^4}+\frac{x^3(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$\big\downarrow$ 2009

$$\frac{3\left(-\frac{1}{4}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{1}{4}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)\right.}{b^2c^4}$$

$$\frac{5\left(-\frac{1}{8}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{3}{16}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)-\frac{1}{16}\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\right.\right.}{b^2}$$

$$\frac{x^3(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

input
```
Int[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x])^2,x]
```

output
```
(x^3*(1 - c^2*x^2))/(b*c*(a + b*ArcCos[c*x])) + (3*(-1/4*(CosIntegral[(a +
 b*ArcCos[c*x])/b]*Sin[a/b]) - (CosIntegral[(3*(a + b*ArcCos[c*x]))/b]*Sin
[(3*a)/b])/4 + (Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/4 + (Cos[(3*a
)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/4))/(b^2*c^4) - (5*(-1/8*(Cos
Integral[(a + b*ArcCos[c*x])/b]*Sin[a/b]) - (3*CosIntegral[(3*(a + b*ArcCo
s[c*x]))/b]*Sin[(3*a)/b])/16 - (CosIntegral[(5*(a + b*ArcCos[c*x]))/b]*Sin
[(5*a)/b])/16 + (Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/8 + (3*Cos[(
3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/16 + (Cos[(5*a)/b]*SinInte
gral[(5*(a + b*ArcCos[c*x]))/b])/16))/(b^2*c^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.59

method	result
default	$\frac{2\arccos(cx)\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)b-2\arccos(cx)\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)b-5\arccos(cx)\operatorname{Si}\left(5\arccos(cx)+\frac{5a}{b}\right)\cos\left(\frac{5a}{b}\right)b+5\,\mathrm{ar}}{}$

input
```
int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/16/c^4*(2*arccos(c*x)*Si(arccos(c*x)+a/b)*cos(a/b)*b-2*arccos(c*x)*Ci(ar
ccos(c*x)+a/b)*sin(a/b)*b-5*arccos(c*x)*Si(5*arccos(c*x)+5*a/b)*cos(5*a/b)
*b+5*arccos(c*x)*Ci(5*arccos(c*x)+5*a/b)*sin(5*a/b)*b-3*arccos(c*x)*Si(3*a
rccos(c*x)+3*a/b)*cos(3*a/b)*b+3*arccos(c*x)*Ci(3*arccos(c*x)+3*a/b)*sin(3
*a/b)*b+2*Si(arccos(c*x)+a/b)*cos(a/b)*a-2*Ci(arccos(c*x)+a/b)*sin(a/b)*a-
5*Si(5*arccos(c*x)+5*a/b)*cos(5*a/b)*a+5*Ci(5*arccos(c*x)+5*a/b)*sin(5*a/b
)*a-3*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)*a+3*Ci(3*arccos(c*x)+3*a/b)*sin(3
*a/b)*a+2*c*x*b-cos(5*arccos(c*x))*b-cos(3*arccos(c*x))*b)/(a+b*arccos(c*x
))/b^2
```

Fricas [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{-c^2x^2+1}x^3}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^3/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a
^2), x)
```

Sympy [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^3\sqrt{-(cx-1)(cx+1)}}{(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate(x**3*(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**3*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^3}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c^2*x^5 - x^3 - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*
c)*integrate((5*c^2*x^4 - 3*x^2)/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x) + a*b*c), x))/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) +
 a*b*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^3\,\sqrt{1-c^2\,x^2}}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x))^2,x)
```

output
```
int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x^3\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}\,x^3}{acos\left(cx\right)^2b^2+2acos\left(cx\right)ab+a^2}dx$$

input
```
int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**3)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a
**2),x)
```

3.384 $\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = -\frac{x^2(1-c^2x^2)}{bc(a+b\arccos(cx))} - \frac{\operatorname{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)\sin\left(\frac{4a}{b}\right)}{2b^2c^3} + \frac{\cos\left(\frac{4a}{b}\right)\operatorname{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{2b^2c^3}$$

output

```
-x^2*(-c^2*x^2+1)/b/c/(a+b*arccos(c*x))-1/2*Ci(4*(a+b*arccos(c*x))/b)*sin(
4*a/b)/b^2/c^3+1/2*cos(4*a/b)*Si(4*(a+b*arccos(c*x))/b)/b^2/c^3
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \frac{-\frac{2bc^2x^2(-1+c^2x^2)}{a+b\arccos(cx)} + \operatorname{CosIntegral}\left(4\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{4a}{b}\right) - \cos\left(\frac{4a}{b}\right)\operatorname{Si}\left(4\left(\frac{a}{b}+\arccos(cx)\right)\right)}{2b^2c^3}$$

input
```
Integrate[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x])^2,x]
```

output
```
((-2*b*c^2*x^2*(-1 + c^2*x^2))/(a + b*ArcCos[c*x]) + CosIntegral[4*(a/b +
ArcCos[c*x])]*Sin[(4*a)/b] - Cos[(4*a)/b]*SinIntegral[4*(a/b + ArcCos[c*x]
)])/(2*b^2*c^3)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 193 vs. $2(94) = 188$.

Time = 1.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5215, 5147, 25, 4906, 27, 2009, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5215}$$

$$\frac{4c\int \frac{x^3}{a+b\arccos(cx)}dx}{b}-\frac{2\int \frac{x}{a+b\arccos(cx)}dx}{bc}+\frac{x^2(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5147}$$

$$-\frac{4\int -\frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{2\int -\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{4\int \frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}-\frac{2\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{2\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2(a+b\arccos(cx))}d(a+b\arccos(cx))}{b^2c^3}+\frac{4\int \left(\frac{\sin\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8(a+b\arccos(cx))}+\frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{4\int \left(\frac{\sin\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8(a+b\arccos(cx))}+\frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}-\frac{4\left(-\frac{1}{4}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\frac{1}{8}\sin\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right.\right.}{b^2c^3}$$

$$\frac{x^2(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}-\frac{4\left(-\frac{1}{4}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\frac{1}{8}\sin\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right.\right.}{b^2c^3}$$

$$\frac{x^2(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{3784}$$

$$\frac{-\sin\left(\frac{2a}{b}\right)\int\frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{2a}{b}\right)\int-\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}-\frac{4\left(-\frac{1}{4}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\frac{1}{8}\sin\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos}{b}\right.\right.}{b^2c^3}$$

$$\frac{x^2(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$\downarrow$ 25

$$\frac{\cos\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int\frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}-\frac{4\left(-\frac{1}{4}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\frac{1}{8}\sin\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos}{b}\right.\right.}{b^2c^3}$$

$$\frac{x^2(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$\downarrow$ 3042

$$\frac{\cos\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}-\frac{4\left(-\frac{1}{4}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\frac{1}{8}\sin\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos}{b}\right.\right.}{b^2c^3}$$

$$\frac{x^2(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$\downarrow$ 3780

$$\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\sin\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}-\frac{4\left(-\frac{1}{4}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\frac{1}{8}\sin\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos}{b}\right.\right.}{b^2c^3}$$

$$\frac{x^2(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$\downarrow$ 3783

$$\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c^3}-$$
$$\frac{4\left(-\frac{1}{4}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\frac{1}{8}\sin\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos}{b}\right.\right.}{b^2c^3}$$
$$\frac{x^2(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

input
```
Int[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x])^2,x]
```

output
```
(x^2*(1 - c^2*x^2))/(b*c*(a + b*ArcCos[c*x])) + (-(CosIntegral[(2*(a + b*A
rcCos[c*x]))/b]*Sin[(2*a)/b]) + Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[
c*x]))/b])/(b^2*c^3) - (4*(-1/4*(CosIntegral[(2*(a + b*ArcCos[c*x]))/b]*Si
n[(2*a)/b]) - (CosIntegral[(4*(a + b*ArcCos[c*x]))/b]*Sin[(4*a)/b])/8 + (C
os[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/4 + (Cos[(4*a)/b]*SinI
ntegral[(4*(a + b*ArcCos[c*x]))/b])/8))/(b^2*c^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.45

method	result
default	$-\frac{4\arccos(cx)\,\mathrm{Si}\left(4\arccos(cx)+\frac{4a}{b}\right)\cos\left(\frac{4a}{b}\right)b-4\arccos(cx)\,\mathrm{Ci}\left(4\arccos(cx)+\frac{4a}{b}\right)\sin\left(\frac{4a}{b}\right)b+4\,\mathrm{Si}\left(4\arccos(cx)+\frac{4a}{b}\right)\cos\left(\frac{4a}{b}\right)a-4\,\mathrm{C}}{8c^3(a+b\arccos(cx))b^2}$

input
```
int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8/c^3*(4*arccos(c*x)*Si(4*arccos(c*x)+4*a/b)*cos(4*a/b)*b-4*arccos(c*x)
*Ci(4*arccos(c*x)+4*a/b)*sin(4*a/b)*b+4*Si(4*arccos(c*x)+4*a/b)*cos(4*a/b)
*a-4*Ci(4*arccos(c*x)+4*a/b)*sin(4*a/b)*a+cos(4*arccos(c*x))*b-b)/(a+b*arc
cos(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^2}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^2/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a
^2), x)
```

Sympy [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^2\sqrt{-(cx-1)(cx+1)}}{(a+b\,\mathrm{acos}(cx))^2}\,dx$$

input
```
integrate(x**2*(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**2*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}x^2}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c^2*x^4 - x^2 - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*
c)*integrate(2*(2*c^2*x^3 - x)/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1)
, c*x) + a*b*c), x))/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a
*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(88) = 176$.

Time = 0.23 (sec) , antiderivative size = 556, normalized size of antiderivative = 5.91

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-b*c^4*x^4/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 4*b*arccos(c*x)*cos(a/b)^3*
cos_integral(4*a/b + 4*arccos(c*x))*sin(a/b)/(b^3*c^3*arccos(c*x) + a*b^2*
c^3) - 4*b*arccos(c*x)*cos(a/b)^4*sin_integral(4*a/b + 4*arccos(c*x))/(b^3
*c^3*arccos(c*x) + a*b^2*c^3) + 4*a*cos(a/b)^3*cos_integral(4*a/b + 4*arcc
os(c*x))*sin(a/b)/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 4*a*cos(a/b)^4*sin_i
ntegral(4*a/b + 4*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + b*c^2*x
^2/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 2*b*arccos(c*x)*cos(a/b)*cos_integr
al(4*a/b + 4*arccos(c*x))*sin(a/b)/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 4*b
*arccos(c*x)*cos(a/b)^2*sin_integral(4*a/b + 4*arccos(c*x))/(b^3*c^3*arcco
s(c*x) + a*b^2*c^3) - 2*a*cos(a/b)*cos_integral(4*a/b + 4*arccos(c*x))*sin
(a/b)/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 4*a*cos(a/b)^2*sin_integral(4*a/
b + 4*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 1/2*b*arccos(c*x)*s
in_integral(4*a/b + 4*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 1/2
*a*sin_integral(4*a/b + 4*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{x^2\,\sqrt{1-c^2\,x^2}}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x))^2,x)
```

output
```
int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acos\left(cx\right)^2b^2+2acos\left(cx\right)ab+a^2}dx$$

input
```
int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**2)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a
**2),x)
```

3.385 $\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 150

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = -\frac{x(1-c^2x^2)}{bc(a+b\arccos(cx))} + \frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^2} + \frac{3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^2} + \frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^2} + \frac{3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^2}$$

output

```
-x*(-c^2*x^2+1)/b/c/(a+b*arccos(c*x))+1/4*cos(a/b)*Ci((a+b*arccos(c*x))/b)
/b^2/c^2+3/4*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b^2/c^2+1/4*sin(a/b)*Si(
(a+b*arccos(c*x))/b)/b^2/c^2+3/4*sin(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^2/
c^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx$$
$$=\frac{\frac{4bcx}{a+b\arccos(cx)}-\frac{4bc^3x^3}{a+b\arccos(cx)}-\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)+3\,\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)\text{s}}{4b^2c^2}$$

input
```
Integrate[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x])^2,x]
```

output
```
((4*b*c*x)/(a + b*ArcCos[c*x]) - (4*b*c^3*x^3)/(a + b*ArcCos[c*x]) - CosIn
tegral[a/b + ArcCos[c*x]]*Sin[a/b] + 3*CosIntegral[3*(a/b + ArcCos[c*x])]*
Sin[(3*a)/b] + Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]] - 3*Cos[(3*a)/b]*Si
nIntegral[3*(a/b + ArcCos[c*x])])/(4*b^2*c^2)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5215, 5135, 25, 3042, 3784, 25, 3042, 3780, 3783, 5147, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx$$
$$\Big\downarrow \text{ 5215}$$
$$\frac{3c\int \frac{x^2}{a+b\arccos(cx)}dx}{b}-\frac{\int \frac{1}{a+b\arccos(cx)}dx}{bc}+\frac{x(1-c^2x^2)}{bc(a+b\arccos(cx))}$$
$$\Big\downarrow \text{ 5135}$$
$$\frac{\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{3c\int \frac{x^2}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)}{bc(a+b\arccos(cx))}$$
$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{3c\int \frac{x^2}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{3c\int \frac{x^2}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{3784}$$

$$\frac{-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{3c\int \frac{x^2}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{3c\int \frac{x^2}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{3c\int \frac{x^2}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{3780}$$

$$\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{3c\int \frac{x^2}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{3783}$$

$$\frac{3c\int \frac{x^2}{a+b\arccos(cx)}dx}{b}+\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c^2}+\frac{x(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5147}$$

$$-\frac{3\int -\frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c^2}+\frac{x(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c^2}+\frac{x(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{4906}$$

$$\frac{3\int \left(\frac{\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}+\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^2}+\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c^2}+\frac{x(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c^2}-\frac{3\left(-\frac{1}{4}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{1}{4}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)\right)}{b^2c^2}+\frac{x(1-c^2x^2)}{bc(a+b\arccos(cx))}$$

input
```
Int[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x])^2,x]
```

output
```
(x*(1 - c^2*x^2))/(b*c*(a + b*ArcCos[c*x])) + (-(CosIntegral[(a + b*ArcCos
[c*x])/b]*Sin[a/b]) + Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/(b^2*c^
2) - (3*(-1/4*(CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b]) - (CosIntegral
[(3*(a + b*ArcCos[c*x]))/b]*Sin[(3*a)/b])/4 + (Cos[a/b]*SinIntegral[(a + b
*ArcCos[c*x])/b])/4 + (Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b]
)/4))/(b^2*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.49

method	result
default	$\arccos(cx)\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)b-\arccos(cx)\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)b-3\arccos(cx)\operatorname{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)b+3\operatorname{arcc}$

input
```
int(x*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/c^2*(arccos(c*x)*Si(arccos(c*x)+a/b)*cos(a/b)*b-arccos(c*x)*Ci(arccos(
c*x)+a/b)*sin(a/b)*b-3*arccos(c*x)*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)*b+3*
arccos(c*x)*Ci(3*arccos(c*x)+3*a/b)*sin(3*a/b)*b+Si(arccos(c*x)+a/b)*cos(a
/b)*a-Ci(arccos(c*x)+a/b)*sin(a/b)*a-3*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)*
a+3*Ci(3*arccos(c*x)+3*a/b)*sin(3*a/b)*a+c*x*b-cos(3*arccos(c*x))*b)/(a+b*
arccos(c*x))/b^2
```

Fricas [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}x}{\left(b\arccos\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
), x)
```

Sympy [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{\left(a+b\operatorname{acos}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(x*(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}x}{\left(b\arccos\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^2*x^3 - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*int
egrate((3*c^2*x^2 - 1)/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) +
 a*b*c), x) - x)/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(140) = 280$.

Time = 0.22 (sec) , antiderivative size = 630, normalized size of antiderivative = 4.20

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-b*c^3*x^3/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 3*b*arccos(c*x)*cos(a/b)^2*
cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*
c^2) - 3*b*arccos(c*x)*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(c*x))/(b^3
*c^2*arccos(c*x) + a*b^2*c^2) + 3*a*cos(a/b)^2*cos_integral(3*a/b + 3*arcc
os(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 3*a*cos(a/b)^3*sin_i
ntegral(3*a/b + 3*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 3/4*b*a
rccos(c*x)*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*
x) + a*b^2*c^2) - 1/4*b*arccos(c*x)*cos_integral(a/b + arccos(c*x))*sin(a/
b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 9/4*b*arccos(c*x)*cos(a/b)*sin_inte
gral(3*a/b + 3*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 1/4*b*arcc
os(c*x)*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c^2*arccos(c*x) + a*
b^2*c^2) + b*c*x/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 3/4*a*cos_integral(3*
a/b + 3*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 1/4*a*co
s_integral(a/b + arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) +
 9/4*a*cos(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^2*arccos(c*x) +
 a*b^2*c^2) + 1/4*a*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c^2*arcc
os(c*x) + a*b^2*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x\,\sqrt{1-c^2\,x^2}}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((x*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x))^2,x)
```

output
```
int((x*(1 - c^2*x^2)^(1/2))/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}\,x}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx$$

input
```
int(x*(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2
),x)
```

3.386 $\int \frac{\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 86

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = -\frac{1-c^2x^2}{bc(a+b\arccos(cx))} + \frac{\operatorname{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\sin\left(\frac{2a}{b}\right)}{b^2c} - \frac{\cos\left(\frac{2a}{b}\right)\operatorname{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c}$$

output
```
-(-c^2*x^2+1)/b/c/(a+b*arccos(c*x))+Ci(2*(a+b*arccos(c*x))/b)*sin(2*a/b)/b
^2/c-cos(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b^2/c
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \frac{\frac{b-bc^2x^2}{a+b\arccos(cx)} + \operatorname{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{2a}{b}\right) - \cos\left(\frac{2a}{b}\right)\operatorname{Si}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)}{b^2c}$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(a + b*ArcCos[c*x])^2,x]
```

output
```
((b - b*c^2*x^2)/(a + b*ArcCos[c*x]) + CosIntegral[2*(a/b + ArcCos[c*x])]*
Sin[(2*a)/b] - Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])])/(b^2*c)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5167, 5147, 25, 4906, 27, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5167}$$

$$\frac{2c\int \frac{x}{a+b\arccos(cx)}dx}{b}+\frac{1-c^2x^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5147}$$

$$\frac{1-c^2x^2}{bc(a+b\arccos(cx))}-\frac{2\int -\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}+\frac{1-c^2x^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{4906}$$

$$\frac{2\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2(a+b\arccos(cx))}d(a+b\arccos(cx))}{b^2c}+\frac{1-c^2x^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}+\frac{1-c^2x^2}{bc(a+b\arccos(cx))}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}+\frac{1-c^2x^2}{bc(a+b\arccos(cx))}$$

$\Big\downarrow$ 3784

$$\frac{1-c^2x^2}{bc(a+b\arccos(cx))}-\frac{-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{2a}{b}\right)\int -\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 25

$$\frac{1-c^2x^2}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 3042

$$\frac{1-c^2x^2}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 3780

$$\frac{1-c^2x^2}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 3783

$$\frac{1-c^2x^2}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c}$$

input
```
Int[Sqrt[1 - c^2*x^2]/(a + b*ArcCos[c*x])^2,x]
```

output
```
(1 - c^2*x^2)/(b*c*(a + b*ArcCos[c*x])) - (-(CosIntegral[(2*(a + b*ArcCos[
c*x]))/b]*Sin[(2*a)/b]) + Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))
/b])/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.58

method	result
default	$-\frac{2\arccos(cx)\,\mathrm{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)b-2\arccos(cx)\,\mathrm{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)b+2\,\mathrm{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)a-2\,\mathrm{C}}{2c\,b^2(a+b\arccos(cx))}$

input
```
int((-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/c*(2*arccos(c*x)*Si(2*arccos(c*x)+2*a/b)*cos(2*a/b)*b-2*arccos(c*x)*C
i(2*arccos(c*x)+2*a/b)*sin(2*a/b)*b+2*Si(2*arccos(c*x)+2*a/b)*cos(2*a/b)*a
-2*Ci(2*arccos(c*x)+2*a/b)*sin(2*a/b)*a+cos(2*arccos(c*x))*b-b)/b^2/(a+b*a
rccos(c*x))
```

Fricas [F]

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2),
 x)
```

Sympy [F]

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-(cx-1)(cx+1)}}{(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^2*x^2 - 2*(b^2*c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c^
2)*integrate(x/(b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b), x)
- 1)/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(84) = 168$.

Time = 0.22 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.58

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = -\frac{bc^2x^2}{b^3c\arccos(cx)+ab^2c} + \frac{2\,b\arccos(cx)\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{b^3c\arccos(cx)+ab^2c} - \frac{2\,b\arccos(cx)\cos\left(\frac{a}{b}\right)^2\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c} + \frac{2\,a\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{b^3c\arccos(cx)+ab^2c} - \frac{2\,a\cos\left(\frac{a}{b}\right)^2\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c} + \frac{b\arccos(cx)\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c} + \frac{a\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c} + \frac{b}{b^3c\arccos(cx)+ab^2c}$$

input

```
integrate((-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output

```
-b*c^2*x^2/(b^3*c*arccos(c*x) + a*b^2*c) + 2*b*arccos(c*x)*cos(a/b)*cos_in
tegral(2*a/b + 2*arccos(c*x))*sin(a/b)/(b^3*c*arccos(c*x) + a*b^2*c) - 2*b
*arccos(c*x)*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^3*c*arccos(
c*x) + a*b^2*c) + 2*a*cos(a/b)*cos_integral(2*a/b + 2*arccos(c*x))*sin(a/b
)/(b^3*c*arccos(c*x) + a*b^2*c) - 2*a*cos(a/b)^2*sin_integral(2*a/b + 2*ar
ccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) + b*arccos(c*x)*sin_integral(2*a/
b + 2*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) + a*sin_integral(2*a/b +
2*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) + b/(b^3*c*arccos(c*x) + a*b^
2*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{1-c^2\,x^2}}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(a + b*acos(c*x))^2,x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acos\left(cx\right)^2b^2+2acos\left(cx\right)ab+a^2}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)
```

3.387 $\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))^2}\,dx = -\frac{1-c^2x^2}{bcx(a+b\arccos(cx))}-\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2} - \frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2}-\frac{\text{Int}\left(\frac{1}{x^2(a+b\arccos(cx))},x\right)}{bc}$$

output
```
-(-c^2*x^2+1)/b/c/x/(a+b*arccos(c*x))-cos(a/b)*Ci((a+b*arccos(c*x))/b)/b^2
-sin(a/b)*Si((a+b*arccos(c*x))/b)/b^2-Defer(Int)(1/x^2/(a+b*arccos(c*x)),x
)/b/c
```

Mathematica [N/A]

Not integrable

Time = 13.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5215}$$

$$\frac{\int \frac{1}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{c\int \frac{1}{a+b\arccos(cx)}dx}{b}+\frac{1-c^2x^2}{bcx(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5135}$$

$$-\frac{\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2}+\frac{\int \frac{1}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{1-c^2x^2}{bcx(a+b\arccos(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2}+\frac{\int \frac{1}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{1-c^2x^2}{bcx(a+b\arccos(cx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2}+\frac{\int \frac{1}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{1-c^2x^2}{bcx(a+b\arccos(cx))}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2}+\frac{\int \frac{1}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{1-c^2x^2}{bcx(a+b\arccos(cx))}$$

$\Big\downarrow$ 25

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2}+\frac{\int \frac{1}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{1-c^2x^2}{bcx(a+b\arccos(cx))}$$

$\Big\downarrow$ 3042

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2}+\frac{\int \frac{1}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{1-c^2x^2}{bcx(a+b\arccos(cx))}$$

$\Big\downarrow$ 3780

$$-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2}+\frac{\int \frac{1}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{1-c^2x^2}{bcx(a+b\arccos(cx))}$$

$\Big\downarrow$ 3783

$$\frac{\int \frac{1}{x^2(a+b\arccos(cx))}dx}{bc}-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2}+\frac{1-c^2x^2}{bcx(a+b\arccos(cx))}$$

$\Big\downarrow$ 5149

$$\frac{\int \frac{1}{x^2(a+b\arccos(cx))}dx}{bc}-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2}+\frac{1-c^2x^2}{bcx(a+b\arccos(cx))}$$

input
```
Int[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2x^2+1}}{x\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x/(a+b*arccos(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(1/2)/x/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{\left(b\arccos\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b^2*x*arccos(c*x)^2 + 2*a*b*x*arccos(c*x) + a
^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{-(cx-1)(cx+1)}}{x(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/x/(a+b*acos(c*x))**2,x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 129, normalized size of antiderivative = 4.61

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{(b\arccos(cx)+a)^2x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^2*x^2 - (b^2*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x)
*integrate((c^2*x^2 + 1)/(b^2*c*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1),
c*x) + a*b*c*x^2), x) - 1)/(b^2*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1),
c*x) + a*b*c*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{1-c^2\,x^2}}{x\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(x*(a + b*acos(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(x*(a + b*acos(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{1-c^2x^2}}{x(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acos\left(cx\right)^2 b^2x+2acos\left(cx\right)abx+a^2x}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x/(a+b*acos(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2*x + 2*acos(c*x)*a*b*x + a**2
*x),x)
```

3.388 $\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))^2}\,dx = -\frac{1-c^2x^2}{bcx^2(a+b\arccos(cx))} - \frac{2\text{Int}\left(\frac{1}{x^3(a+b\arccos(cx))},x\right)}{bc}$$

output
```
-(-c^2*x^2+1)/b/c/x^2/(a+b*arccos(c*x))-2*Defer(Int)(1/x^3/(a+b*arccos(c*x
)),x)/b/c
```

Mathematica [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \text{ 5213}$$

$$\frac{2\int \frac{1}{x^3(a+b\arccos(cx))}dx}{bc}+\frac{1-c^2x^2}{bcx^2(a+b\arccos(cx))}$$

$$\Big\downarrow \text{ 5149}$$

$$\frac{2\int \frac{1}{x^3(a+b\arccos(cx))}dx}{bc}+\frac{1-c^2x^2}{bcx^2(a+b\arccos(cx))}$$

input
```
Int[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2x^2+1}}{x^2\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccos(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{\left(b\arccos\left(cx\right)+a\right)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b^2*x^2*arccos(c*x)^2 + 2*a*b*x^2*arccos(c*x)
 + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x^2\left(a+b\operatorname{acos}\left(cx\right)\right)^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/x**2/(a+b*acos(c*x))**2,x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**2*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 127, normalized size of antiderivative = 4.54

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arccos{(cx)}+a)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c^2*x^2 - 2*(b^2*c*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*
c*x^2)*integrate(1/(b^2*c*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) +
 a*b*c*x^3), x) - 1)/(b^2*c*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)
 + a*b*c*x^2)
```

Giac [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arccos{(cx)}+a)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/((b*arccos(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{1-c^2\,x^2}}{x^2\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acos(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acos(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 6.07

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\arccos(cx))^2}\,dx$$
$$= \frac{acos(cx)\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)^2b^2x^2+2\sqrt{-c^2x^2+1}\,acos(cx)ab\,x^2+\sqrt{-c^2x^2+1}\,a^2x^2}dx\right)ab+acos(cx)\,c+\left(\int \frac{}{\sqrt{-c^2x^2+1}\,aco}\right.}{a\left(acos\left(cx\right)b+a\right)}$$

input
```
int((-c^2*x^2+1)^(1/2)/x^2/(a+b*acos(c*x))^2,x)
```

output
```
(acos(c*x)*int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*x**2 + 2*sqrt(
- c**2*x**2 + 1)*acos(c*x)*a*b*x**2 + sqrt( - c**2*x**2 + 1)*a**2*x**2),x)
*a*b + acos(c*x)*c + int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*x**2
+ 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*x**2 + sqrt( - c**2*x**2 + 1)*a**
2*x**2),x)*a**2)/(a*(acos(c*x)*b + a))
```

3.389 $\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 19.74 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))^2} \, dx$$

$$\Big\downarrow \text{ 5235}$$

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))^2}dx$$

input
```
Int[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2x^2+1}}{x^3\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccos(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{(b\arccos{(cx)}+a)^2x^3}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b^2*x^3*arccos(c*x)^2 + 2*a*b*x^3*arccos(c*x)
 + a^2*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{-(cx-1)(cx+1)}}{x^3(a+b\operatorname{acos}{(cx)})^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/x**3/(a+b*acos(c*x))**2,x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**3*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 136, normalized size of antiderivative = 4.86

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arccos{(cx)}+a)^2x^3}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c^2*x^2 + (b^2*c*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*
x^3)*integrate((c^2*x^2 - 3)/(b^2*c*x^4*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x) + a*b*c*x^4), x) - 1)/(b^2*c*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x
+ 1), c*x) + a*b*c*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{1-c^2\,x^2}}{x^3\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*acos(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*acos(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acos\left(cx\right)^2 b^2x^3+2acos\left(cx\right)ab\,x^3+a^2x^3}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x^3/(a+b*acos(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2*x**3 + 2*acos(c*x)*a*b*x**3
+ a**2*x**3),x)
```

3.390 $\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))^2}\,dx$$
$$\Big\downarrow \ 5235$$
$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))^2}dx$$

input
```
Int[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2x^2+1}}{x^4\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccos(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{(b\arccos{(cx)}+a)^2x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(b^2*x^4*arccos(c*x)^2 + 2*a*b*x^4*arccos(c*x)
 + a^2*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x^4\left(a+b\operatorname{acos}{(cx)}\right)^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(1/2)/x**4/(a+b*acos(c*x))**2,x)
```

output
```
Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**4*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 137, normalized size of antiderivative = 4.89

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arccos(cx)+a)^2x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c^2*x^2 + (b^2*c*x^4*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*
x^4)*integrate(2*(c^2*x^2 - 2)/(b^2*c*x^5*arctan2(sqrt(c*x + 1)*sqrt(-c*x
+ 1), c*x) + a*b*c*x^5), x) - 1)/(b^2*c*x^4*arctan2(sqrt(c*x + 1)*sqrt(-c*
x + 1), c*x) + a*b*c*x^4)
```

Giac [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{(b\arccos(cx)+a)^2x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-c^2*x^2 + 1)/((b*arccos(c*x) + a)^2*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{1-c^2\,x^2}}{x^4\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((1 - c^2*x^2)^(1/2)/(x^4*(a + b*acos(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(1/2)/(x^4*(a + b*acos(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{1-c^2x^2}}{x^4(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{-c^2x^2+1}}{acos\left(cx\right)^2 b^2x^4+2acos\left(cx\right)ab\,x^4+a^2x^4}dx$$

input
```
int((-c^2*x^2+1)^(1/2)/x^4/(a+b*acos(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2*x**4 + 2*acos(c*x)*a*b*x**4
+ a**2*x**4),x)
```

3.391 $\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2} dx = \text{Int}\left(\frac{x^m(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}, x\right)$$

output
```
Defer(Int)(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2} dx = \int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2} dx$$

input
```
Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x])^2,x]
```

output
```
Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2} x^m}{(a+b\arccos(cx))^2} \, dx$$

$$\Big\downarrow \text{5235}$$

$$\int \frac{(1-c^2x^2)^{3/2} x^m}{(a+b\arccos(cx))^2} dx$$

input
```
Int[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m (-c^2x^2+1)^{\frac{3}{2}}}{(a+b\arccos(cx))^2} dx$$

input
```
int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^m}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(-(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*x^m/(b^2*arccos(c*x)^2 + 2*a*b*
arccos(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 31.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^m(-(cx-1)(cx+1))^{\frac{3}{2}}}{(a+b\,\mathrm{acos}(cx))^2}\,dx$$

input
```
integrate(x**m*(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**m*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acos(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 160, normalized size of antiderivative = 5.71

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^m}{(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
((c^4*x^4 - 2*c^2*x^2 + 1)*x^m - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x) + a*b*c)*integrate(((c^4*m + 4*c^4)*x^4 - 2*(c^2*m + 2*c^2)*x^2 +
 m)*x^m/(b^2*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x), x)
)/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^m\,{\left(1-c^2\,x^2\right)}^{3/2}}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((x^m*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x))^2,x)
```

output
```
int((x^m*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.25

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = -\left(\int \frac{x^m\sqrt{-c^2x^2+1}\,x^2}{acos(cx)^2\,b^2+2acos(cx)\,ab+a^2}dx\right)c^2 + \int \frac{x^m\sqrt{-c^2x^2+1}}{acos(cx)^2\,b^2+2acos(cx)\,ab+a^2}dx$$

input
```
int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
 - int((x**m*sqrt( - c**2*x**2 + 1)*x**2)/(acos(c*x)**2*b**2 + 2*acos(c*x)
*a*b + a**2),x)*c**2 + int((x**m*sqrt( - c**2*x**2 + 1))/(acos(c*x)**2*b**
2 + 2*acos(c*x)*a*b + a**2),x)
```

3.392 $\int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 278

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = -\frac{x^3(1-c^2x^2)^2}{bc(a+b\arccos(cx))} + \frac{3\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{64b^2c^4} + \frac{9\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{64b^2c^4} - \frac{5\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{64b^2c^4} - \frac{7\cos\left(\frac{7a}{b}\right)\text{CosIntegral}\left(\frac{7(a+b\arccos(cx))}{b}\right)}{64b^2c^4} + \frac{3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{64b^2c^4} + \frac{9\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{64b^2c^4} - \frac{5\sin\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{64b^2c^4} - \frac{7\sin\left(\frac{7a}{b}\right)\text{Si}\left(\frac{7(a+b\arccos(cx))}{b}\right)}{64b^2c^4}$$

output
```
-x^3*(-c^2*x^2+1)^2/b/c/(a+b*arccos(c*x))+3/64*cos(a/b)*Ci((a+b*arccos(c*x
))/b)/b^2/c^4+9/64*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b^2/c^4-5/64*cos(5
*a/b)*Ci(5*(a+b*arccos(c*x))/b)/b^2/c^4-7/64*cos(7*a/b)*Ci(7*(a+b*arccos(c
*x))/b)/b^2/c^4+3/64*sin(a/b)*Si((a+b*arccos(c*x))/b)/b^2/c^4+9/64*sin(3*a
/b)*Si(3*(a+b*arccos(c*x))/b)/b^2/c^4-5/64*sin(5*a/b)*Si(5*(a+b*arccos(c*x
))/b)/b^2/c^4-7/64*sin(7*a/b)*Si(7*(a+b*arccos(c*x))/b)/b^2/c^4
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.44

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \frac{64bc^3x^3-128bc^5x^5+64bc^7x^7-3(a+b\arccos(cx))\,\text{CosIntegral}\left(\frac{a}{b}+\arccos(c\right.}{}$$

input
```
Integrate[(x^3*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x])^2,x]
```

output
```
(64*b*c^3*x^3 - 128*b*c^5*x^5 + 64*b*c^7*x^7 - 3*(a + b*ArcCos[c*x])*CosIn
tegral[a/b + ArcCos[c*x]]*Sin[a/b] + 9*(a + b*ArcCos[c*x])*CosIntegral[3*(
a/b + ArcCos[c*x])]*Sin[(3*a)/b] + 5*a*CosIntegral[5*(a/b + ArcCos[c*x])]*
Sin[(5*a)/b] + 5*b*ArcCos[c*x]*CosIntegral[5*(a/b + ArcCos[c*x])]*Sin[(5*a
)/b] - 7*a*CosIntegral[7*(a/b + ArcCos[c*x])]*Sin[(7*a)/b] - 7*b*ArcCos[c*
x]*CosIntegral[7*(a/b + ArcCos[c*x])]*Sin[(7*a)/b] + 3*a*Cos[a/b]*SinInteg
ral[a/b + ArcCos[c*x]] + 3*b*ArcCos[c*x]*Cos[a/b]*SinIntegral[a/b + ArcCos
[c*x]] - 9*a*Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])] - 9*b*ArcCos[
c*x]*Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])] - 5*a*Cos[(5*a)/b]*Si
nIntegral[5*(a/b + ArcCos[c*x])] - 5*b*ArcCos[c*x]*Cos[(5*a)/b]*SinIntegra
l[5*(a/b + ArcCos[c*x])] + 7*a*Cos[(7*a)/b]*SinIntegral[7*(a/b + ArcCos[c*
x])] + 7*b*ArcCos[c*x]*Cos[(7*a)/b]*SinIntegral[7*(a/b + ArcCos[c*x])])/(6
4*b^2*c^4*(a + b*ArcCos[c*x]))
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.41, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5215, 5225, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5215}$$

$$-\frac{3\int \frac{x^2(1-c^2x^2)}{a+b\arccos(cx)}dx}{bc}+\frac{7c\int \frac{x^4(1-c^2x^2)}{a+b\arccos(cx)}dx}{b}+\frac{x^3(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5225}$$

$$-\frac{7\int -\frac{\cos^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^4}+\frac{3\int -\frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^4}+\frac{x^3(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{7\int \frac{\cos^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^4}-\frac{3\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^4}+\frac{x^3(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{4906}$$

$$\frac{7\int \left(-\frac{\sin\left(\frac{7a}{b}-\frac{7(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}-\frac{\sin\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}+\frac{3\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}+\frac{3\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{64(a+b\arccos(cx))}\right)d(a+b\,\text{ar}}{b^2c^4}$$

$$\frac{3\int \left(-\frac{\sin\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^4}+$$

$$\frac{x^3(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{1}{8}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{1}{16}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{1}{16}\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\right.\right.}{b^2}$$

$$7\left(-\frac{3}{64}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{3}{64}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{1}{64}\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\right.\right.$$

$$\frac{x^3(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

input
```
Int[(x^3*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x])^2,x]
```

output
```
(x^3*(1 - c^2*x^2)^2)/(b*c*(a + b*ArcCos[c*x])) + (3*(-1/8*(CosIntegral[(a
 + b*ArcCos[c*x])/b]*Sin[a/b]) - (CosIntegral[(3*(a + b*ArcCos[c*x]))/b]*S
in[(3*a)/b])/16 + (CosIntegral[(5*(a + b*ArcCos[c*x]))/b]*Sin[(5*a)/b])/16
 + (Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/8 + (Cos[(3*a)/b]*SinInte
gral[(3*(a + b*ArcCos[c*x]))/b])/16 - (Cos[(5*a)/b]*SinIntegral[(5*(a + b*
ArcCos[c*x]))/b])/16))/(b^2*c^4) - (7*((-3*CosIntegral[(a + b*ArcCos[c*x])
/b]*Sin[a/b])/64 - (3*CosIntegral[(3*(a + b*ArcCos[c*x]))/b]*Sin[(3*a)/b])
/64 + (CosIntegral[(5*(a + b*ArcCos[c*x]))/b]*Sin[(5*a)/b])/64 + (CosInteg
ral[(7*(a + b*ArcCos[c*x]))/b]*Sin[(7*a)/b])/64 + (3*Cos[a/b]*SinIntegral[
(a + b*ArcCos[c*x])/b])/64 + (3*Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[
c*x]))/b])/64 - (Cos[(5*a)/b]*SinIntegral[(5*(a + b*ArcCos[c*x]))/b])/64 -
 (Cos[(7*a)/b]*SinIntegral[(7*(a + b*ArcCos[c*x]))/b])/64))/(b^2*c^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.63

method	result
default	$3\arccos(cx)\,\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)b-3\arccos(cx)\,\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)b-5\arccos(cx)\,\mathrm{Si}\left(5\arccos(cx)+\frac{5a}{b}\right)\cos\left(\frac{5a}{b}\right)b+5\,\mathrm{ar}$

input
```
int(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/64/c^4*(3*arccos(c*x)*Si(arccos(c*x)+a/b)*cos(a/b)*b-3*arccos(c*x)*Ci(ar
ccos(c*x)+a/b)*sin(a/b)*b-5*arccos(c*x)*Si(5*arccos(c*x)+5*a/b)*cos(5*a/b)
*b+5*arccos(c*x)*Ci(5*arccos(c*x)+5*a/b)*sin(5*a/b)*b-9*arccos(c*x)*Si(3*a
rccos(c*x)+3*a/b)*cos(3*a/b)*b+9*arccos(c*x)*Ci(3*arccos(c*x)+3*a/b)*sin(3
*a/b)*b+7*arccos(c*x)*Si(7*arccos(c*x)+7*a/b)*cos(7*a/b)*b-7*arccos(c*x)*C
i(7*arccos(c*x)+7*a/b)*sin(7*a/b)*b+3*Si(arccos(c*x)+a/b)*cos(a/b)*a-3*Ci(
arccos(c*x)+a/b)*sin(a/b)*a-5*Si(5*arccos(c*x)+5*a/b)*cos(5*a/b)*a+5*Ci(5*
arccos(c*x)+5*a/b)*sin(5*a/b)*a-9*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)*a+9*C
i(3*arccos(c*x)+3*a/b)*sin(3*a/b)*a+7*Si(7*arccos(c*x)+7*a/b)*cos(7*a/b)*a
-7*Ci(7*arccos(c*x)+7*a/b)*sin(7*a/b)*a+3*c*x*b-cos(5*arccos(c*x))*b-3*cos
(3*arccos(c*x))*b+cos(7*arccos(c*x))*b)/(a+b*arccos(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^3}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(-(c^2*x^5 - x^3)*sqrt(-c^2*x^2 + 1)/(b^2*arccos(c*x)^2 + 2*a*b*ar
ccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^3(-(cx-1)(cx+1))^{\frac{3}{2}}}{(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate(x**3*(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**3*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^3}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
(c^4*x^7 - 2*c^2*x^5 + x^3 - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1),
c*x) + a*b*c)*integrate((7*c^4*x^6 - 10*c^2*x^4 + 3*x^2)/(b^2*c*arctan2(sq
rt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c), x))/(b^2*c*arctan2(sqrt(c*x + 1
)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2080 vs. $2(261) = 522$.

Time = 0.26 (sec) , antiderivative size = 2080, normalized size of antiderivative = 7.48

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
b*c^7*x^7/(b^3*c^4*arccos(c*x) + a*b^2*c^4) - 2*b*c^5*x^5/(b^3*c^4*arccos(
c*x) + a*b^2*c^4) - 7*b*arccos(c*x)*cos(a/b)^6*cos_integral(7*a/b + 7*arcc
os(c*x))*sin(a/b)/(b^3*c^4*arccos(c*x) + a*b^2*c^4) + 7*b*arccos(c*x)*cos(
a/b)^7*sin_integral(7*a/b + 7*arccos(c*x))/(b^3*c^4*arccos(c*x) + a*b^2*c^
4) - 7*a*cos(a/b)^6*cos_integral(7*a/b + 7*arccos(c*x))*sin(a/b)/(b^3*c^4*
arccos(c*x) + a*b^2*c^4) + 7*a*cos(a/b)^7*sin_integral(7*a/b + 7*arccos(c*
x))/(b^3*c^4*arccos(c*x) + a*b^2*c^4) + 35/4*b*arccos(c*x)*cos(a/b)^4*cos_
integral(7*a/b + 7*arccos(c*x))*sin(a/b)/(b^3*c^4*arccos(c*x) + a*b^2*c^4)
 + 5/4*b*arccos(c*x)*cos(a/b)^4*cos_integral(5*a/b + 5*arccos(c*x))*sin(a/
b)/(b^3*c^4*arccos(c*x) + a*b^2*c^4) - 49/4*b*arccos(c*x)*cos(a/b)^5*sin_i
ntegral(7*a/b + 7*arccos(c*x))/(b^3*c^4*arccos(c*x) + a*b^2*c^4) - 5/4*b*a
rccos(c*x)*cos(a/b)^5*sin_integral(5*a/b + 5*arccos(c*x))/(b^3*c^4*arccos(
c*x) + a*b^2*c^4) + b*c^3*x^3/(b^3*c^4*arccos(c*x) + a*b^2*c^4) + 35/4*a*c
os(a/b)^4*cos_integral(7*a/b + 7*arccos(c*x))*sin(a/b)/(b^3*c^4*arccos(c*x
) + a*b^2*c^4) + 5/4*a*cos(a/b)^4*cos_integral(5*a/b + 5*arccos(c*x))*sin(
a/b)/(b^3*c^4*arccos(c*x) + a*b^2*c^4) - 49/4*a*cos(a/b)^5*sin_integral(7*
a/b + 7*arccos(c*x))/(b^3*c^4*arccos(c*x) + a*b^2*c^4) - 5/4*a*cos(a/b)^5*
sin_integral(5*a/b + 5*arccos(c*x))/(b^3*c^4*arccos(c*x) + a*b^2*c^4) - 21
/8*b*arccos(c*x)*cos(a/b)^2*cos_integral(7*a/b + 7*arccos(c*x))*sin(a/b)/(
b^3*c^4*arccos(c*x) + a*b^2*c^4) - 15/16*b*arccos(c*x)*cos(a/b)^2*cos_i...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^3\,(1-c^2\,x^2)^{3/2}}{(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((x^3*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x))^2,x)
```

output
```
int((x^3*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x^3(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = -\left(\int \frac{\sqrt{-c^2x^2+1}\,x^5}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{-c^2x^2+1}\,x^3}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx$$

input
```
int(x^3*(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
 - int((sqrt( - c**2*x**2 + 1)*x**5)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b
+ a**2),x)*c**2 + int((sqrt( - c**2*x**2 + 1)*x**3)/(acos(c*x)**2*b**2 + 2
*acos(c*x)*a*b + a**2),x)
```

3.393 $\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 220

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = -\frac{x^2(1-c^2x^2)^2}{bc(a+b\arccos(cx))} + \frac{\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\sin\left(\frac{2a}{b}\right)}{16b^2c^3} - \frac{\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)\sin\left(\frac{4a}{b}\right)}{4b^2c^3} - \frac{3\,\text{CosIntegral}\left(\frac{6(a+b\arccos(cx))}{b}\right)\sin\left(\frac{6a}{b}\right)}{16b^2c^3} - \frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{16b^2c^3} + \frac{\cos\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{4b^2c^3} + \frac{3\cos\left(\frac{6a}{b}\right)\text{Si}\left(\frac{6(a+b\arccos(cx))}{b}\right)}{16b^2c^3}$$

output
```
-x^2*(-c^2*x^2+1)^2/b/c/(a+b*arccos(c*x))+1/16*Ci(2*(a+b*arccos(c*x))/b)*s
in(2*a/b)/b^2/c^3-1/4*Ci(4*(a+b*arccos(c*x))/b)*sin(4*a/b)/b^2/c^3-3/16*Ci
(6*(a+b*arccos(c*x))/b)*sin(6*a/b)/b^2/c^3-1/16*cos(2*a/b)*Si(2*(a+b*arcco
s(c*x))/b)/b^2/c^3+1/4*cos(4*a/b)*Si(4*(a+b*arccos(c*x))/b)/b^2/c^3+3/16*c
os(6*a/b)*Si(6*(a+b*arccos(c*x))/b)/b^2/c^3
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.39

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx =$$
$$-\frac{-16bc^2x^2+32bc^4x^4-16bc^6x^6-(a+b\arccos(cx))\,\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{2a}{b}\right)-4(a+b\,\text{a}}{}$$

input
```
Integrate[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/16*(-16*b*c^2*x^2 + 32*b*c^4*x^4 - 16*b*c^6*x^6 - (a + b*ArcCos[c*x])*C
osIntegral[2*(a/b + ArcCos[c*x])]*Sin[(2*a)/b] - 4*(a + b*ArcCos[c*x])*Cos
Integral[4*(a/b + ArcCos[c*x])]*Sin[(4*a)/b] + 3*a*CosIntegral[6*(a/b + Ar
cCos[c*x])]*Sin[(6*a)/b] + 3*b*ArcCos[c*x]*CosIntegral[6*(a/b + ArcCos[c*x
])]*Sin[(6*a)/b] + a*Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])] + b*A
rcCos[c*x]*Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])] + 4*a*Cos[(4*a)
/b]*SinIntegral[4*(a/b + ArcCos[c*x])] + 4*b*ArcCos[c*x]*Cos[(4*a)/b]*SinI
ntegral[4*(a/b + ArcCos[c*x])] - 3*a*Cos[(6*a)/b]*SinIntegral[6*(a/b + Arc
Cos[c*x])] - 3*b*ArcCos[c*x]*Cos[(6*a)/b]*SinIntegral[6*(a/b + ArcCos[c*x]
)])/(b^2*c^3*(a + b*ArcCos[c*x]))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5215, 5225, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5215}$$

$$-\frac{2\int \frac{x(1-c^2x^2)}{a+b\arccos(cx)}dx}{bc}+\frac{6c\int \frac{x^3(1-c^2x^2)}{a+b\arccos(cx)}dx}{b}+\frac{x^2(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5225}$$

$$-\frac{6\int -\frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{2\int -\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2\left(1-c^2x^2\right)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{6\int \frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}-\frac{2\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2\left(1-c^2x^2\right)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{4906}$$

$$\frac{6\int \left(\frac{3\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}-\frac{\sin\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^3}-\frac{2\int \left(\frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}-\frac{\sin\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2\left(1-c^2x^2\right)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{1}{4}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{1}{8}\sin\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right.\right.}{b^2c^3}$$
$$\frac{6\left(-\frac{3}{32}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{1}{32}\sin\left(\frac{6a}{b}\right)\text{CosIntegral}\left(\frac{6(a+b\arccos(cx))}{b}\right)+\frac{3}{32}\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\right.\right.}{b^2c^3}$$
$$\frac{x^2\left(1-c^2x^2\right)^2}{bc(a+b\arccos(cx))}$$

input
```
Int[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x])^2,x]
```

output
```
(x^2*(1 - c^2*x^2)^2)/(b*c*(a + b*ArcCos[c*x])) + (2*(-1/4*(CosIntegral[(2
*(a + b*ArcCos[c*x]))/b]*Sin[(2*a)/b]) + (CosIntegral[(4*(a + b*ArcCos[c*x
]))/b]*Sin[(4*a)/b])/8 + (Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))
/b])/4 - (Cos[(4*a)/b]*SinIntegral[(4*(a + b*ArcCos[c*x]))/b])/8))/(b^2*c^
3) - (6*((-3*CosIntegral[(2*(a + b*ArcCos[c*x]))/b]*Sin[(2*a)/b])/32 + (Co
sIntegral[(6*(a + b*ArcCos[c*x]))/b]*Sin[(6*a)/b])/32 + (3*Cos[(2*a)/b]*Si
nIntegral[(2*(a + b*ArcCos[c*x]))/b])/32 - (Cos[(6*a)/b]*SinIntegral[(6*(a
 + b*ArcCos[c*x]))/b])/32))/(b^2*c^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.65

method	result
default	$-\frac{8\arccos(cx)\,\mathrm{Si}\left(4\arccos(cx)+\frac{4a}{b}\right)\cos\left(\frac{4a}{b}\right)b-8\arccos(cx)\,\mathrm{Ci}\left(4\arccos(cx)+\frac{4a}{b}\right)\sin\left(\frac{4a}{b}\right)b+2\arccos(cx)\,\mathrm{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos(}{}$

input
```
int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32/c^3*(8*arccos(c*x)*Si(4*arccos(c*x)+4*a/b)*cos(4*a/b)*b-8*arccos(c*x
)*Ci(4*arccos(c*x)+4*a/b)*sin(4*a/b)*b+2*arccos(c*x)*Si(2*arccos(c*x)+2*a/
b)*cos(2*a/b)*b-2*arccos(c*x)*Ci(2*arccos(c*x)+2*a/b)*sin(2*a/b)*b-6*arcco
s(c*x)*Si(6*arccos(c*x)+6*a/b)*cos(6*a/b)*b+6*arccos(c*x)*Ci(6*arccos(c*x)
+6*a/b)*sin(6*a/b)*b+8*Si(4*arccos(c*x)+4*a/b)*cos(4*a/b)*a-8*Ci(4*arccos(
c*x)+4*a/b)*sin(4*a/b)*a+2*Si(2*arccos(c*x)+2*a/b)*cos(2*a/b)*a-2*Ci(2*arc
cos(c*x)+2*a/b)*sin(2*a/b)*a-6*Si(6*arccos(c*x)+6*a/b)*cos(6*a/b)*a+6*Ci(6
*arccos(c*x)+6*a/b)*sin(6*a/b)*a+2*cos(4*arccos(c*x))*b+cos(2*arccos(c*x))
*b-cos(6*arccos(c*x))*b-2*b)/(a+b*arccos(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^2}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(-(c^2*x^4 - x^2)*sqrt(-c^2*x^2 + 1)/(b^2*arccos(c*x)^2 + 2*a*b*ar
ccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^2(-(cx-1)(cx+1))^{\frac{3}{2}}}{(a+b\,\mathrm{acos}\,(cx))^2}\,dx$$

input
```
integrate(x**2*(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x^2}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
(c^4*x^6 - 2*c^2*x^4 + x^2 - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1),
c*x) + a*b*c)*integrate(2*(3*c^4*x^5 - 4*c^2*x^3 + x)/(b^2*c*arctan2(sqrt(
c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c), x))/(b^2*c*arctan2(sqrt(c*x + 1)*s
qrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1574 vs. $2(207) = 414$.

Time = 0.26 (sec) , antiderivative size = 1574, normalized size of antiderivative = 7.15

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
b*c^6*x^6/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 2*b*c^4*x^4/(b^3*c^3*arccos(
c*x) + a*b^2*c^3) - 6*b*arccos(c*x)*cos(a/b)^5*cos_integral(6*a/b + 6*arcc
os(c*x))*sin(a/b)/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 6*b*arccos(c*x)*cos(
a/b)^6*sin_integral(6*a/b + 6*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^
3) - 6*a*cos(a/b)^5*cos_integral(6*a/b + 6*arccos(c*x))*sin(a/b)/(b^3*c^3*
arccos(c*x) + a*b^2*c^3) + 6*a*cos(a/b)^6*sin_integral(6*a/b + 6*arccos(c*
x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 6*b*arccos(c*x)*cos(a/b)^3*cos_int
egral(6*a/b + 6*arccos(c*x))*sin(a/b)/(b^3*c^3*arccos(c*x) + a*b^2*c^3) +
2*b*arccos(c*x)*cos(a/b)^3*cos_integral(4*a/b + 4*arccos(c*x))*sin(a/b)/(b
^3*c^3*arccos(c*x) + a*b^2*c^3) - 9*b*arccos(c*x)*cos(a/b)^4*sin_integral(
6*a/b + 6*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 2*b*arccos(c*x)
*cos(a/b)^4*sin_integral(4*a/b + 4*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b
^2*c^3) + 6*a*cos(a/b)^3*cos_integral(6*a/b + 6*arccos(c*x))*sin(a/b)/(b^3
*c^3*arccos(c*x) + a*b^2*c^3) + 2*a*cos(a/b)^3*cos_integral(4*a/b + 4*arcc
os(c*x))*sin(a/b)/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 9*a*cos(a/b)^4*sin_i
ntegral(6*a/b + 6*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 2*a*cos
(a/b)^4*sin_integral(4*a/b + 4*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c
^3) + b*c^2*x^2/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 9/8*b*arccos(c*x)*cos(
a/b)*cos_integral(6*a/b + 6*arccos(c*x))*sin(a/b)/(b^3*c^3*arccos(c*x) + a
*b^2*c^3) - b*arccos(c*x)*cos(a/b)*cos_integral(4*a/b + 4*arccos(c*x))*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{x^2\,(1-c^2\,x^2)^{3/2}}{(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x))^2,x)
```

output
```
int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx=-\left(\int \frac{\sqrt{-c^2x^2+1}\,x^4}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2+\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx$$

input
```
int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
- int((sqrt( - c**2*x**2 + 1)*x**4)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b
+ a**2),x)*c**2 + int((sqrt( - c**2*x**2 + 1)*x**2)/(acos(c*x)**2*b**2 + 2
*acos(c*x)*a*b + a**2),x)
```

3.394 $\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 214

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = -\frac{x(1-c^2x^2)^2}{bc(a+b\arccos(cx))} + \frac{\cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2c^2} + \frac{9\cos\left(\frac{3a}{b}\right)\operatorname{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2c^2} + \frac{5\cos\left(\frac{5a}{b}\right)\operatorname{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2c^2} + \frac{\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2c^2} + \frac{9\sin\left(\frac{3a}{b}\right)\operatorname{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2c^2} + \frac{5\sin\left(\frac{5a}{b}\right)\operatorname{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2c^2}$$

output

```
-x*(-c^2*x^2+1)^2/b/c/(a+b*arccos(c*x))+1/8*cos(a/b)*Ci((a+b*arccos(c*x))/
b)/b^2/c^2+9/16*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b^2/c^2+5/16*cos(5*a/
b)*Ci(5*(a+b*arccos(c*x))/b)/b^2/c^2+1/8*sin(a/b)*Si((a+b*arccos(c*x))/b)/
b^2/c^2+9/16*sin(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^2/c^2+5/16*sin(5*a/b)*
Si(5*(a+b*arccos(c*x))/b)/b^2/c^2
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.38

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \frac{16bcx-32bc^3x^3+16bc^5x^5-2(a+b\arccos(cx))\,\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)}{}$$

input
```
Integrate[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x])^2,x]
```

output
```
(16*b*c*x - 32*b*c^3*x^3 + 16*b*c^5*x^5 - 2*(a + b*ArcCos[c*x])*CosIntegra
l[a/b + ArcCos[c*x]]*Sin[a/b] + 9*(a + b*ArcCos[c*x])*CosIntegral[3*(a/b +
 ArcCos[c*x])]*Sin[(3*a)/b] - 5*a*CosIntegral[5*(a/b + ArcCos[c*x])]*Sin[(
5*a)/b] - 5*b*ArcCos[c*x]*CosIntegral[5*(a/b + ArcCos[c*x])]*Sin[(5*a)/b]
+ 2*a*Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 2*b*ArcCos[c*x]*Cos[a/b]*S
inIntegral[a/b + ArcCos[c*x]] - 9*a*Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcC
os[c*x])] - 9*b*ArcCos[c*x]*Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])
] + 5*a*Cos[(5*a)/b]*SinIntegral[5*(a/b + ArcCos[c*x])] + 5*b*ArcCos[c*x]*
Cos[(5*a)/b]*SinIntegral[5*(a/b + ArcCos[c*x])])/(16*b^2*c^2*(a + b*ArcCos
[c*x]))
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.36, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5215, 5169, 25, 3042, 3793, 2009, 5225, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \text{5215}$$

$$-\frac{\int \frac{1-c^2x^2}{a+b\arccos(cx)}dx}{bc}+\frac{5c\int \frac{x^2(1-c^2x^2)}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5169}$$

$$\frac{\int -\frac{\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{5c\int \frac{x^2(1-c^2x^2)}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{5c\int \frac{x^2(1-c^2x^2)}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^3}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{5c\int \frac{x^2(1-c^2x^2)}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 3793$$

$$-\frac{\int \left(\frac{3\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4(a+b\arccos(cx))}-\frac{\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^2}+\frac{5c\int \frac{x^2(1-c^2x^2)}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 2009$$

$$\frac{5c\int \frac{x^2(1-c^2x^2)}{a+b\arccos(cx)}dx}{b}+\frac{-\frac{3}{4}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{1}{4}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{3}{4}\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-}{b^2c^2}$$
$$\frac{x(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 5225$$

$$-\frac{5\int -\frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+$$
$$\frac{-\frac{3}{4}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{1}{4}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{3}{4}\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-}{b^2c^2}$$
$$\frac{x(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 25$$

$$\frac{5\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+$$

$$\frac{-\frac{3}{4}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{1}{4}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{3}{4}\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-}{b^2c^2}$$

$$\frac{x(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 4906$$

$$\frac{5\int \left(-\frac{\sin\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^2}+$$

$$\frac{-\frac{3}{4}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{1}{4}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{3}{4}\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-}{b^2c^2}$$

$$\frac{x(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{3}{4}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{1}{4}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{3}{4}\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-}{b^2c^2}$$

$$\frac{5\left(-\frac{1}{8}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{1}{16}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{1}{16}\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\right.\right.}{b^2}$$

$$\frac{x(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

input
```
Int[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCos[c*x])^2,x]
```

output
```
(x*(1 - c^2*x^2)^2)/(b*c*(a + b*ArcCos[c*x])) + ((-3*CosIntegral[(a + b*Ar
cCos[c*x])/b]*Sin[a/b])/4 + (CosIntegral[(3*(a + b*ArcCos[c*x]))/b]*Sin[(3
*a)/b])/4 + (3*Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/4 - (Cos[(3*a)
/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/4)/(b^2*c^2) - (5*(-1/8*(CosIn
tegral[(a + b*ArcCos[c*x])/b]*Sin[a/b]) - (CosIntegral[(3*(a + b*ArcCos[c*
x]))/b]*Sin[(3*a)/b])/16 + (CosIntegral[(5*(a + b*ArcCos[c*x]))/b]*Sin[(5*
a)/b])/16 + (Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/8 + (Cos[(3*a)/b
]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/16 - (Cos[(5*a)/b]*SinIntegral[(
5*(a + b*ArcCos[c*x]))/b])/16))/(b^2*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.59

method	result
default	$-\frac{9\arccos(cx)\operatorname{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)b-9\arccos(cx)\operatorname{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)b-5\arccos(cx)\operatorname{Si}\left(5\arccos(cx)+\frac{5a}{b}\right)\cos(}{}$

input
```
int(x*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16/c^2*(9*arccos(c*x)*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)*b-9*arccos(c*x
)*Ci(3*arccos(c*x)+3*a/b)*sin(3*a/b)*b-5*arccos(c*x)*Si(5*arccos(c*x)+5*a/
b)*cos(5*a/b)*b+5*arccos(c*x)*Ci(5*arccos(c*x)+5*a/b)*sin(5*a/b)*b-2*arcco
s(c*x)*Si(arccos(c*x)+a/b)*cos(a/b)*b+2*arccos(c*x)*Ci(arccos(c*x)+a/b)*si
n(a/b)*b+9*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)*a-9*Ci(3*arccos(c*x)+3*a/b)*
sin(3*a/b)*a-5*Si(5*arccos(c*x)+5*a/b)*cos(5*a/b)*a+5*Ci(5*arccos(c*x)+5*a
/b)*sin(5*a/b)*a-2*Si(arccos(c*x)+a/b)*cos(a/b)*a+2*Ci(arccos(c*x)+a/b)*si
n(a/b)*a-2*c*x*b+3*cos(3*arccos(c*x))*b-cos(5*arccos(c*x))*b)/(a+b*arccos(
c*x))/b^2
```

Fricas [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(-(c^2*x^3 - x)*sqrt(-c^2*x^2 + 1)/(b^2*arccos(c*x)^2 + 2*a*b*arcc
os(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x(-(cx-1)\,(cx+1))^{\frac{3}{2}}}{(a+b\,\mathrm{acos}\,(cx))^2}\,dx$$

input
```
integrate(x*(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}x}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
(c^4*x^5 - 2*c^2*x^3 - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) +
 a*b*c)*integrate((5*c^4*x^4 - 6*c^2*x^2 + 1)/(b^2*c*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x) + a*b*c), x) + x)/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(
-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. $2(201) = 402$.

Time = 0.24 (sec) , antiderivative size = 1264, normalized size of antiderivative = 5.91

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
b*c^5*x^5/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 5*b*arccos(c*x)*cos(a/b)^4*c
os_integral(5*a/b + 5*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c
^2) + 5*b*arccos(c*x)*cos(a/b)^5*sin_integral(5*a/b + 5*arccos(c*x))/(b^3*
c^2*arccos(c*x) + a*b^2*c^2) - 2*b*c^3*x^3/(b^3*c^2*arccos(c*x) + a*b^2*c^
2) - 5*a*cos(a/b)^4*cos_integral(5*a/b + 5*arccos(c*x))*sin(a/b)/(b^3*c^2*
arccos(c*x) + a*b^2*c^2) + 5*a*cos(a/b)^5*sin_integral(5*a/b + 5*arccos(c*
x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 15/4*b*arccos(c*x)*cos(a/b)^2*cos_
integral(5*a/b + 5*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2)
 + 9/4*b*arccos(c*x)*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/
b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 25/4*b*arccos(c*x)*cos(a/b)^3*sin_i
ntegral(5*a/b + 5*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 9/4*b*a
rccos(c*x)*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^2*arccos(
c*x) + a*b^2*c^2) + 15/4*a*cos(a/b)^2*cos_integral(5*a/b + 5*arccos(c*x))*
sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 9/4*a*cos(a/b)^2*cos_integral
(3*a/b + 3*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 25/4*
a*cos(a/b)^3*sin_integral(5*a/b + 5*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*
b^2*c^2) - 9/4*a*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^2*a
rccos(c*x) + a*b^2*c^2) - 5/16*b*arccos(c*x)*cos_integral(5*a/b + 5*arccos
(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 9/16*b*arccos(c*x)*cos
_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x\,(1-c^2\,x^2)^{3/2}}{(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((x*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x))^2,x)
```

output
```
int((x*(1 - c^2*x^2)^(3/2))/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = -\left(\int \frac{\sqrt{-c^2x^2+1}\,x^3}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{-c^2x^2+1}\,x}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx$$

input
```
int(x*(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
 - int((sqrt( - c**2*x**2 + 1)*x**3)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b
+ a**2),x)*c**2 + int((sqrt( - c**2*x**2 + 1)*x)/(acos(c*x)**2*b**2 + 2*ac
os(c*x)*a*b + a**2),x)
```

3.395 $\int \frac{(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 150

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = -\frac{(1-c^2x^2)^2}{bc(a+b\arccos(cx))} + \frac{\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\sin\left(\frac{2a}{b}\right)}{b^2c} + \frac{\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)\sin\left(\frac{4a}{b}\right)}{2b^2c} - \frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c} - \frac{\cos\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{2b^2c}$$

output
```
-(-c^2*x^2+1)^2/b/c/(a+b*arccos(c*x))+Ci(2*(a+b*arccos(c*x))/b)*sin(2*a/b)
/b^2/c+1/2*Ci(4*(a+b*arccos(c*x))/b)*sin(4*a/b)/b^2/c-cos(2*a/b)*Si(2*(a+b
*arccos(c*x))/b)/b^2/c-1/2*cos(4*a/b)*Si(4*(a+b*arccos(c*x))/b)/b^2/c
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.81

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \frac{\frac{2b(-1+c^2x^2)^2}{a+b\arccos(cx)} + 2\,\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{2a}{b}\right) - \text{CosIntegral}\left(4\left(\frac{a}{b}+\right.\right.}{}$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
((2*b*(-1 + c^2*x^2)^2)/(a + b*ArcCos[c*x]) + 2*CosIntegral[2*(a/b + ArcCo
s[c*x])]*Sin[(2*a)/b] - CosIntegral[4*(a/b + ArcCos[c*x])]*Sin[(4*a)/b] -
2*Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])] + Cos[(4*a)/b]*SinIntegr
al[4*(a/b + ArcCos[c*x])])/(2*b^2*c)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5167, 5225, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \; 5167$$

$$\frac{4c\int \frac{x(1-c^2x^2)}{a+b\arccos(cx)}dx}{b} + \frac{(1-c^2x^2)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 5225$$

$$\frac{(1-c^2x^2)^2}{bc(a+b\arccos(cx))} - \frac{4\int -\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \; 25$$

$$\frac{4\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}+\frac{\left(1-c^2x^2\right)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{4\int \left(\frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}-\frac{\sin\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c}+\frac{\left(1-c^2x^2\right)^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(1-c^2x^2\right)^2}{bc(a+b\arccos(cx))}-\frac{4\left(-\frac{1}{4}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{1}{8}\sin\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos\ldots}{b}\right.\right.}{b^2c}$$

input
```
Int[(1 - c^2*x^2)^(3/2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
(1 - c^2*x^2)^2/(b*c*(a + b*ArcCos[c*x])) - (4*(-1/4*(CosIntegral[(2*(a +
b*ArcCos[c*x]))/b]*Sin[(2*a)/b]) + (CosIntegral[(4*(a + b*ArcCos[c*x]))/b]
*Sin[(4*a)/b])/8 + (Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/4
 - (Cos[(4*a)/b]*SinIntegral[(4*(a + b*ArcCos[c*x]))/b])/8))/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.67

method	result
default	$\frac{4\arccos(cx)\,\mathrm{Si}\left(4\arccos(cx)+\frac{4a}{b}\right)\cos\left(\frac{4a}{b}\right)b-4\arccos(cx)\,\mathrm{Ci}\left(4\arccos(cx)+\frac{4a}{b}\right)\sin\left(\frac{4a}{b}\right)b-8\arccos(cx)\,\mathrm{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right.}{}$

input
```
int((-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/c*(4*arccos(c*x)*Si(4*arccos(c*x)+4*a/b)*cos(4*a/b)*b-4*arccos(c*x)*Ci
(4*arccos(c*x)+4*a/b)*sin(4*a/b)*b-8*arccos(c*x)*Si(2*arccos(c*x)+2*a/b)*c
os(2*a/b)*b+8*arccos(c*x)*Ci(2*arccos(c*x)+2*a/b)*sin(2*a/b)*b+4*Si(4*arcc
os(c*x)+4*a/b)*cos(4*a/b)*a-4*Ci(4*arccos(c*x)+4*a/b)*sin(4*a/b)*a-8*Si(2*
arccos(c*x)+2*a/b)*cos(2*a/b)*a+8*Ci(2*arccos(c*x)+2*a/b)*sin(2*a/b)*a+cos
(4*arccos(c*x))*b-4*cos(2*arccos(c*x))*b+3*b)/b^2/(a+b*arccos(c*x))
```

Fricas [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2
), x)
```

Sympy [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
(c^4*x^4 - 2*c^2*x^2 - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) +
 a*b*c)*integrate(4*(c^3*x^3 - c*x)/(b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
 1), c*x) + a*b), x) + 1)/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x
) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(145) = 290$.

Time = 0.24 (sec) , antiderivative size = 788, normalized size of antiderivative = 5.25

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
b*c^4*x^4/(b^3*c*arccos(c*x) + a*b^2*c) - 4*b*arccos(c*x)*cos(a/b)^3*cos_i
ntegral(4*a/b + 4*arccos(c*x))*sin(a/b)/(b^3*c*arccos(c*x) + a*b^2*c) + 4*
b*arccos(c*x)*cos(a/b)^4*sin_integral(4*a/b + 4*arccos(c*x))/(b^3*c*arccos
(c*x) + a*b^2*c) - 4*a*cos(a/b)^3*cos_integral(4*a/b + 4*arccos(c*x))*sin(
a/b)/(b^3*c*arccos(c*x) + a*b^2*c) + 4*a*cos(a/b)^4*sin_integral(4*a/b + 4
*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) - 2*b*c^2*x^2/(b^3*c*arccos(c*
x) + a*b^2*c) + 2*b*arccos(c*x)*cos(a/b)*cos_integral(4*a/b + 4*arccos(c*x
))*sin(a/b)/(b^3*c*arccos(c*x) + a*b^2*c) + 2*b*arccos(c*x)*cos(a/b)*cos_i
ntegral(2*a/b + 2*arccos(c*x))*sin(a/b)/(b^3*c*arccos(c*x) + a*b^2*c) - 4*
b*arccos(c*x)*cos(a/b)^2*sin_integral(4*a/b + 4*arccos(c*x))/(b^3*c*arccos
(c*x) + a*b^2*c) - 2*b*arccos(c*x)*cos(a/b)^2*sin_integral(2*a/b + 2*arcco
s(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) + 2*a*cos(a/b)*cos_integral(4*a/b +
4*arccos(c*x))*sin(a/b)/(b^3*c*arccos(c*x) + a*b^2*c) + 2*a*cos(a/b)*cos_i
ntegral(2*a/b + 2*arccos(c*x))*sin(a/b)/(b^3*c*arccos(c*x) + a*b^2*c) - 4*
a*cos(a/b)^2*sin_integral(4*a/b + 4*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^
2*c) - 2*a*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^3*c*arccos(c*
x) + a*b^2*c) + 1/2*b*arccos(c*x)*sin_integral(4*a/b + 4*arccos(c*x))/(b^3
*c*arccos(c*x) + a*b^2*c) + b*arccos(c*x)*sin_integral(2*a/b + 2*arccos(c*
x))/(b^3*c*arccos(c*x) + a*b^2*c) + 1/2*a*sin_integral(4*a/b + 4*arccos(c*
x))/(b^3*c*arccos(c*x) + a*b^2*c) + a*sin_integral(2*a/b + 2*arccos(c*x...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(1-c^2\,x^2)^{3/2}}{(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(a + b*acos(c*x))^2,x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(1-c^2x^2)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx - \left(\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)
 - int((sqrt( - c**2*x**2 + 1)*x**2)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b
+ a**2),x)*c**2
```

3.396 $\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))^2}\,dx = -\frac{(1-c^2x^2)^2}{bcx(a+b\arccos(cx))} - \frac{9\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2} - \frac{3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2} - \frac{9\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2} - \frac{3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2} - \frac{\text{Int}\left(\frac{1-c^2x^2}{x^2(a+b\arccos(cx))},x\right)}{bc}$$

output
```
-(-c^2*x^2+1)^2/b/c/x/(a+b*arccos(c*x))-9/4*cos(a/b)*Ci((a+b*arccos(c*x))/
b)/b^2-3/4*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b^2-9/4*sin(a/b)*Si((a+b*a
rccos(c*x))/b)/b^2-3/4*sin(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^2-Defer(Int)
((-c^2*x^2+1)/x^2/(a+b*arccos(c*x)),x)/b/c
```

Mathematica [N/A]

Not integrable

Time = 10.85 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))^2}\,dx = \int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \text{5215}$$

$$\frac{3c\int \frac{1-c^2x^2}{a+b\arccos(cx)}dx}{b}+\frac{\int \frac{1-c^2x^2}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{(1-c^2x^2)^2}{bcx(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5169}$$

$$-\frac{3\int -\frac{\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2}+\frac{\int \frac{1-c^2x^2}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{(1-c^2x^2)^2}{bcx(a+b\arccos(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\int \frac{\sin^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2}+\frac{\int \frac{1-c^2x^2}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{\left(1-c^2x^2\right)^2}{bcx(a+b\arccos(cx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^3}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2}+\frac{\int \frac{1-c^2x^2}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{\left(1-c^2x^2\right)^2}{bcx(a+b\arccos(cx))}$$

$$\Big\downarrow \; 3793$$

$$\frac{3\int \left(\frac{3\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4(a+b\arccos(cx))}-\frac{\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2}+\frac{\int \frac{1-c^2x^2}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{\left(1-c^2x^2\right)^2}{bcx(a+b\arccos(cx))}$$

$$\Big\downarrow \; 2009$$

$$\frac{\int \frac{1-c^2x^2}{x^2(a+b\arccos(cx))}dx}{bc}-\frac{3\left(-\frac{3}{4}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{1}{4}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{3}{4}\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)\right.}{b^2} \frac{\left(1-c^2x^2\right)^2}{bcx(a+b\arccos(cx))}$$

$$\Big\downarrow \; 5235$$

$$\frac{\int \frac{1-c^2x^2}{x^2(a+b\arccos(cx))}dx}{bc}-\frac{3\left(-\frac{3}{4}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{1}{4}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{3}{4}\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)\right.}{b^2} \frac{\left(1-c^2x^2\right)^2}{bcx(a+b\arccos(cx))}$$

input
```
Int[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int((-c^2*x^2+1)^(3/2)/x/(a+b*arccos(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(3/2)/x/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{\left(b\arccos\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b^2*x*arccos(c*x)^2 + 2*a*b*x*arccos(c*x) +
 a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.95 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))^2}\,dx = \int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{x\left(a+b\operatorname{acos}\left(cx\right)\right)^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/x/(a+b*acos(c*x))**2,x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 145, normalized size of antiderivative = 5.18

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
(c^4*x^4 - 2*c^2*x^2 - (b^2*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)
 + a*b*c*x)*integrate((3*c^4*x^4 - 2*c^2*x^2 - 1)/(b^2*c*x^2*arctan2(sqrt(
c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^2), x) + 1)/(b^2*c*x*arctan2(sqrt(
c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))^2}\,dx = \int \frac{(1-c^2\,x^2)^{3/2}}{x\,(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(x*(a + b*acos(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(x*(a + b*acos(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.11

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)^2\,b^2x+2acos\,(cx)\,abx+a^2x}dx - \left(\int \frac{\sqrt{-c^2x^2+1}\,x}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(3/2)/x/(a+b*acos(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2*x + 2*acos(c*x)*a*b*x + a**2
*x),x) - int((sqrt( - c**2*x**2 + 1)*x)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a
*b + a**2),x)*c**2
```

3.397 $\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))^2}\,dx = -\frac{(1-c^2x^2)^2}{bcx^2(a+b\arccos(cx))} - \frac{2\text{Int}\left(\frac{1-c^2x^2}{x^3(a+b\arccos(cx))},x\right)}{bc} - \frac{2c\text{Int}\left(\frac{1-c^2x^2}{x(a+b\arccos(cx))},x\right)}{b}$$

output
```
-(-c^2*x^2+1)^2/b/c/x^2/(a+b*arccos(c*x))-2*Defer(Int)((-c^2*x^2+1)/x^3/(a
+b*arccos(c*x)),x)/b/c-2*c*Defer(Int)((-c^2*x^2+1)/x/(a+b*arccos(c*x)),x)/
b
```

Mathematica [N/A]

Not integrable

Time = 4.88 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \text{5215}$$

$$\frac{2c\int \frac{1-c^2x^2}{x(a+b\arccos(cx))}dx}{b}+\frac{2\int \frac{1-c^2x^2}{x^3(a+b\arccos(cx))}dx}{bc}+\frac{(1-c^2x^2)^2}{bcx^2(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5235}$$

$$\frac{2c\int \frac{1-c^2x^2}{x(a+b\arccos(cx))}dx}{b}+\frac{2\int \frac{1-c^2x^2}{x^3(a+b\arccos(cx))}dx}{bc}+\frac{(1-c^2x^2)^2}{bcx^2(a+b\arccos(cx))}$$

input
```
Int[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x^2 (a+b \arccos (cx))^2}dx$$

input
```
int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccos(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b \arccos (cx)+a)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b^2*x^2*arccos(c*x)^2 + 2*a*b*x^2*arccos(c*
x) + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{x^2 (a+b \operatorname{acos}(cx))^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/x**2/(a+b*acos(c*x))**2,x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**2*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 5.18

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
(c^4*x^4 - 2*c^2*x^2 - (b^2*c*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*
x) + a*b*c*x^2)*integrate(2*(c^4*x^4 - 1)/(b^2*c*x^3*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x) + a*b*c*x^3), x) + 1)/(b^2*c*x^2*arctan2(sqrt(c*x +
1)*sqrt(-c*x + 1), c*x) + a*b*c*x^2)
```

Giac [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/((b*arccos(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{\left(1-c^2\,x^2\right)^{3/2}}{x^2\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acos(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acos(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 265, normalized size of antiderivative = 9.46

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^2(a+b\arccos(cx))^2}\,dx = \frac{-acos(cx)\left(\int \frac{\sqrt{-c^2x^2+1}}{acos(cx)^2b^2+2acos(cx)ab+a^2}dx\right)ab\,c^2+acos(cx)\left(\int \frac{}{\sqrt{-c^2x^2+1}\,acos(cx)}\right.}{}$$

input
```
int((-c^2*x^2+1)^(3/2)/x^2/(a+b*acos(c*x))^2,x)
```

output
```
( - acos(c*x)*int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2 + 2*acos(c*x)*
a*b + a**2),x)*a*b*c**2 + acos(c*x)*int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x
)**2*b**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*x**2 + sqrt( - c**
2*x**2 + 1)*a**2*x**2),x)*a*b + acos(c*x)*c - int(sqrt( - c**2*x**2 + 1)/(
acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*a**2*c**2 + int(1/(sqrt( -
c**2*x**2 + 1)*acos(c*x)**2*b**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)
*a*b*x**2 + sqrt( - c**2*x**2 + 1)*a**2*x**2),x)*a**2)/(a*(acos(c*x)*b + a
))
```

3.398 $\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 19.88 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))^2}\,dx = \int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \; 5235$$

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))^2}dx$$

input
```
Int[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x^3\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccos(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2x^3}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b^2*x^3*arccos(c*x)^2 + 2*a*b*x^3*arccos(c*
x) + a^2*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 3.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))^2}\,dx = \int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{x^3(a+b\,\mathrm{acos}(cx))^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/x**3/(a+b*acos(c*x))**2,x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**3*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 152, normalized size of antiderivative = 5.43

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos{(cx)}+a)^2x^3}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
(c^4*x^4 - 2*c^2*x^2 - (b^2*c*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*
x) + a*b*c*x^3)*integrate((c^4*x^4 + 2*c^2*x^2 - 3)/(b^2*c*x^4*arctan2(sqr
t(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^4), x) + 1)/(b^2*c*x^3*arctan2(s
qrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))^2}\,dx = \int \frac{(1-c^2\,x^2)^{3/2}}{x^3\,(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*acos(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*acos(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 3.43

$$\int \frac{(1-c^2x^2)^{3/2}}{x^3(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)^2\,b^2x^3+2acos\,(cx)\,ab\,x^3+a^2x^3}dx - \left(\int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)^2\,b^2x+2acos\,(cx)\,abx+a^2x}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(3/2)/x^3/(a+b*acos(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2*x**3 + 2*acos(c*x)*a*b*x**3
+ a**2*x**3),x) - int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2*x + 2*acos
(c*x)*a*b*x + a**2*x),x)*c**2
```

3.399 $\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))^2}\,dx=-\frac{(1-c^2x^2)^2}{bcx^4(a+b\arccos(cx))}-\frac{4\text{Int}\left(\frac{1-c^2x^2}{x^5(a+b\arccos(cx))},x\right)}{bc}$$

output
```
-(-c^2*x^2+1)^2/b/c/x^4/(a+b*arccos(c*x))-4*Defer(Int)((-c^2*x^2+1)/x^5/(a
+b*arccos(c*x)),x)/b/c
```

Mathematica [N/A]

Not integrable

Time = 2.88 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))^2}\,dx=\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5213}$$

$$\frac{4\int \frac{1-c^2x^2}{x^5(a+b\arccos(cx))}dx}{bc}+\frac{(1-c^2x^2)^2}{bcx^4(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5235}$$

$$\frac{4\int \frac{1-c^2x^2}{x^5(a+b\arccos(cx))}dx}{bc}+\frac{(1-c^2x^2)^2}{bcx^4(a+b\arccos(cx))}$$

input
```
Int[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{x^4\,(a+b\arccos(cx))^2}dx$$

input
```
int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccos(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((-c^2*x^2 + 1)^(3/2)/(b^2*x^4*arccos(c*x)^2 + 2*a*b*x^4*arccos(c*
x) + a^2*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 5.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))^2}\,dx = \int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{x^4(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(3/2)/x**4/(a+b*acos(c*x))**2,x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**4*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 145, normalized size of antiderivative = 5.18

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
(c^4*x^4 - 2*c^2*x^2 - (b^2*c*x^4*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*
x) + a*b*c*x^4)*integrate(4*(c^2*x^2 - 1)/(b^2*c*x^5*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x) + a*b*c*x^5), x) + 1)/(b^2*c*x^4*arctan2(sqrt(c*x +
1)*sqrt(-c*x + 1), c*x) + a*b*c*x^4)
```

Giac [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(3/2)/((b*arccos(c*x) + a)^2*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))^2}\,dx = \int \frac{\left(1-c^2\,x^2\right)^{3/2}}{x^4\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((1 - c^2*x^2)^(3/2)/(x^4*(a + b*acos(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(3/2)/(x^4*(a + b*acos(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 330, normalized size of antiderivative = 11.79

$$\int \frac{(1-c^2x^2)^{3/2}}{x^4(a+b\arccos(cx))^2}\,dx = \frac{acos(cx)\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)^2b^2x^4+2\sqrt{-c^2x^2+1}\,acos(cx)ab\,x^4+\sqrt{-c^2x^2+1}\,a^2x^4}dx\right)ab -$$

input
```
int((-c^2*x^2+1)^(3/2)/x^4/(a+b*acos(c*x))^2,x)
```

output
```
(acos(c*x)*int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*x**4 + 2*sqrt(
- c**2*x**2 + 1)*acos(c*x)*a*b*x**4 + sqrt( - c**2*x**2 + 1)*a**2*x**4),x)
*a*b - 2*acos(c*x)*int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*x**2 +
2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*x**2 + sqrt( - c**2*x**2 + 1)*a**2*
x**2),x)*a*b*c**2 - acos(c*x)*c**3 + int(1/(sqrt( - c**2*x**2 + 1)*acos(c*
x)**2*b**2*x**4 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*x**4 + sqrt( - c*
*2*x**2 + 1)*a**2*x**4),x)*a**2 - 2*int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x
)**2*b**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*x**2 + sqrt( - c**
2*x**2 + 1)*a**2*x**2),x)*a**2*c**2)/(a*(acos(c*x)*b + a))
```

3.400 $\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx=\text{Int}\left(\frac{x^m(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[(x^m*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x])^2,x]
```

output
```
Integrate[(x^m*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2} x^m}{(a+b\arccos(cx))^2} dx$$
$$\Big\downarrow \text{5235}$$
$$\int \frac{(1-c^2x^2)^{5/2} x^m}{(a+b\arccos(cx))^2} dx$$

input
```
Int[(x^m*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m(-c^2x^2+1)^{\frac{5}{2}}}{(a+b\arccos(cx))^2} dx$$

input
```
int(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^m}{(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)*x^m/(b^2*arccos(c*x)
^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 6.68

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^m}{(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*x^m - (b^2*c*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x) + a*b*c)*integrate(((c^6*m + 6*c^6)*x^6 - 3*(c^4*m +
 4*c^4)*x^4 + 3*(c^2*m + 2*c^2)*x^2 - m)*x^m/(b^2*c*x*arctan2(sqrt(c*x + 1
)*sqrt(-c*x + 1), c*x) + a*b*c*x), x))/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-
c*x + 1), c*x) + a*b*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^m\,{\left(1-c^2\,x^2\right)}^{5/2}}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}\,dx$$

input
```
int((x^m*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x))^2,x)
```

output
```
int((x^m*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 4.96

$$\int \frac{x^m(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \left(\int \frac{x^m\sqrt{-c^2x^2+1}\,x^4}{acos\left(cx\right)^2b^2+2acos\left(cx\right)ab+a^2}dx\right)c^4 - 2\left(\int \frac{x^m\sqrt{-c^2x^2+1}\,x^2}{acos\left(cx\right)^2b^2+2acos\left(cx\right)ab+a^2}dx\right)c^2 + \int \frac{x^m\sqrt{-c^2x^2+1}}{acos\left(cx\right)^2b^2+2acos\left(cx\right)ab+a^2}dx$$

input
```
int(x^m*(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
int((x**m*sqrt( - c**2*x**2 + 1)*x**4)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*
b + a**2),x)*c**4 - 2*int((x**m*sqrt( - c**2*x**2 + 1)*x**2)/(acos(c*x)**2
*b**2 + 2*acos(c*x)*a*b + a**2),x)*c**2 + int((x**m*sqrt( - c**2*x**2 + 1)
)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)
```

3.401 $\int \frac{x^3(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 278

$$\begin{aligned}\int \frac{x^3(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx &= -\frac{x^3(1-c^2x^2)^3}{bc(a+b\arccos(cx))} \\ &+ \frac{3\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{128b^2c^4} + \frac{3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{32b^2c^4} \\ &- \frac{21\cos\left(\frac{7a}{b}\right)\text{CosIntegral}\left(\frac{7(a+b\arccos(cx))}{b}\right)}{256b^2c^4} \\ &- \frac{9\cos\left(\frac{9a}{b}\right)\text{CosIntegral}\left(\frac{9(a+b\arccos(cx))}{b}\right)}{256b^2c^4} \\ &+ \frac{3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{128b^2c^4} + \frac{3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{32b^2c^4} \\ &- \frac{21\sin\left(\frac{7a}{b}\right)\text{Si}\left(\frac{7(a+b\arccos(cx))}{b}\right)}{256b^2c^4} - \frac{9\sin\left(\frac{9a}{b}\right)\text{Si}\left(\frac{9(a+b\arccos(cx))}{b}\right)}{256b^2c^4}\end{aligned}$$

output
```
-x^3*(-c^2*x^2+1)^3/b/c/(a+b*arccos(c*x))+3/128*cos(a/b)*Ci((a+b*arccos(c*
x))/b)/b^2/c^4+3/32*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b^2/c^4-21/256*co
s(7*a/b)*Ci(7*(a+b*arccos(c*x))/b)/b^2/c^4-9/256*cos(9*a/b)*Ci(9*(a+b*arcc
os(c*x))/b)/b^2/c^4+3/128*sin(a/b)*Si((a+b*arccos(c*x))/b)/b^2/c^4+3/32*si
n(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^2/c^4-21/256*sin(7*a/b)*Si(7*(a+b*arc
cos(c*x))/b)/b^2/c^4-9/256*sin(9*a/b)*Si(9*(a+b*arccos(c*x))/b)/b^2/c^4
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.47

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \frac{256bc^3x^3 - 768bc^5x^5 + 768bc^7x^7 - 256bc^9x^9 - 6(a+b\arccos(cx))\,\text{CosIntegral}}{}$$

input
```
Integrate[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x])^2,x]
```

output
```
(256*b*c^3*x^3 - 768*b*c^5*x^5 + 768*b*c^7*x^7 - 256*b*c^9*x^9 - 6*(a + b*
ArcCos[c*x])*CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b] + 24*(a + b*ArcCos[c*
x])*CosIntegral[3*(a/b + ArcCos[c*x])]*Sin[(3*a)/b] - 21*a*CosIntegral[7*(
a/b + ArcCos[c*x])]*Sin[(7*a)/b] - 21*b*ArcCos[c*x]*CosIntegral[7*(a/b + A
rcCos[c*x])]*Sin[(7*a)/b] + 9*a*CosIntegral[9*(a/b + ArcCos[c*x])]*Sin[(9*
a)/b] + 9*b*ArcCos[c*x]*CosIntegral[9*(a/b + ArcCos[c*x])]*Sin[(9*a)/b] +
6*a*Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 6*b*ArcCos[c*x]*Cos[a/b]*Sin
Integral[a/b + ArcCos[c*x]] - 24*a*Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcCo
s[c*x])] - 24*b*ArcCos[c*x]*Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])
] + 21*a*Cos[(7*a)/b]*SinIntegral[7*(a/b + ArcCos[c*x])] + 21*b*ArcCos[c*x
]*Cos[(7*a)/b]*SinIntegral[7*(a/b + ArcCos[c*x])] - 9*a*Cos[(9*a)/b]*SinIn
tegral[9*(a/b + ArcCos[c*x])] - 9*b*ArcCos[c*x]*Cos[(9*a)/b]*SinIntegral[9
*(a/b + ArcCos[c*x])])/(256*b^2*c^4*(a + b*ArcCos[c*x]))
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {5215, 5225, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(1-c^2 x^2\right)^{5/2}}{(a+b \arccos (c x))^2} \, dx$$

$$\Big\downarrow \text{ 5215}$$

$$-\frac{3 \int \frac{x^2 \left(1-c^2 x^2\right)^2}{a+b \arccos (c x)}dx}{b c}+\frac{9 c \int \frac{x^4 \left(1-c^2 x^2\right)^2}{a+b \arccos (c x)}dx}{b}+\frac{x^3 \left(1-c^2 x^2\right)^3}{b c (a+b \arccos (c x))}$$

$$\Big\downarrow \text{ 5225}$$

$$-\frac{9 \int -\frac{\cos ^4\left(\frac{a}{b}-\frac{a+b \arccos (c x)}{b}\right) \sin ^5\left(\frac{a}{b}-\frac{a+b \arccos (c x)}{b}\right)}{a+b \arccos (c x)}d(a+b \arccos (c x))}{b^2 c^4}+\frac{3 \int -\frac{\cos ^2\left(\frac{a}{b}-\frac{a+b \arccos (c x)}{b}\right) \sin ^5\left(\frac{a}{b}-\frac{a+b \arccos (c x)}{b}\right)}{a+b \arccos (c x)}d(a+b \arccos (c x))}{b^2 c^4}+\frac{x^3 \left(1-c^2 x^2\right)^3}{b c (a+b \arccos (c x))}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{9 \int \frac{\cos ^4\left(\frac{a}{b}-\frac{a+b \arccos (c x)}{b}\right) \sin ^5\left(\frac{a}{b}-\frac{a+b \arccos (c x)}{b}\right)}{a+b \arccos (c x)}d(a+b \arccos (c x))}{b^2 c^4}-\frac{3 \int \frac{\cos ^2\left(\frac{a}{b}-\frac{a+b \arccos (c x)}{b}\right) \sin ^5\left(\frac{a}{b}-\frac{a+b \arccos (c x)}{b}\right)}{a+b \arccos (c x)}d(a+b \arccos (c x))}{b^2 c^4}+\frac{x^3 \left(1-c^2 x^2\right)^3}{b c (a+b \arccos (c x))}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{9 \int \left(\frac{\sin \left(\frac{9 a}{b}-\frac{9 (a+b \arccos (c x))}{b}\right)}{256 (a+b \arccos (c x))}-\frac{\sin \left(\frac{7 a}{b}-\frac{7 (a+b \arccos (c x))}{b}\right)}{256 (a+b \arccos (c x))}-\frac{\sin \left(\frac{5 a}{b}-\frac{5 (a+b \arccos (c x))}{b}\right)}{64 (a+b \arccos (c x))}+\frac{\sin \left(\frac{3 a}{b}-\frac{3 (a+b \arccos (c x))}{b}\right)}{64 (a+b \arccos (c x))}+\frac{3 \sin \left(\frac{a}{b}-\frac{a+\ldots}{}\right)}{128 (a+b \text{ ar}\ldots}\right.}{b^2 c^4}$$

$$\frac{3 \int \left(\frac{\sin \left(\frac{7 a}{b}-\frac{7 (a+b \arccos (c x))}{b}\right)}{64 (a+b \arccos (c x))}-\frac{3 \sin \left(\frac{5 a}{b}-\frac{5 (a+b \arccos (c x))}{b}\right)}{64 (a+b \arccos (c x))}+\frac{\sin \left(\frac{3 a}{b}-\frac{3 (a+b \arccos (c x))}{b}\right)}{64 (a+b \arccos (c x))}+\frac{5 \sin \left(\frac{a}{b}-\frac{a+b \arccos (c x)}{b}\right)}{64 (a+b \arccos (c x))}\right)d(a+b \text{ arc}\ldots}{b^2 c^4}$$

$$\frac{x^3 \left(1-c^2 x^2\right)^3}{b c (a+b \arccos (c x))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{5}{64}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{1}{64}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{3}{64}\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\right.\right.}{}$$

$$\frac{9\left(-\frac{3}{128}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{1}{64}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{1}{64}\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\right.}{}$$

$$\frac{x^3(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

input

```
Int[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x])^2,x]
```

output

```
(x^3*(1 - c^2*x^2)^3)/(b*c*(a + b*ArcCos[c*x])) + (3*((-5*CosIntegral[(a +
 b*ArcCos[c*x])/b]*Sin[a/b])/64 - (CosIntegral[(3*(a + b*ArcCos[c*x]))/b]*
Sin[(3*a)/b])/64 + (3*CosIntegral[(5*(a + b*ArcCos[c*x]))/b]*Sin[(5*a)/b])
/64 - (CosIntegral[(7*(a + b*ArcCos[c*x]))/b]*Sin[(7*a)/b])/64 + (5*Cos[a/
b]*SinIntegral[(a + b*ArcCos[c*x])/b])/64 + (Cos[(3*a)/b]*SinIntegral[(3*(
a + b*ArcCos[c*x]))/b])/64 - (3*Cos[(5*a)/b]*SinIntegral[(5*(a + b*ArcCos[
c*x]))/b])/64 + (Cos[(7*a)/b]*SinIntegral[(7*(a + b*ArcCos[c*x]))/b])/64))
/(b^2*c^4) - (9*((-3*CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/128 - (C
osIntegral[(3*(a + b*ArcCos[c*x]))/b]*Sin[(3*a)/b])/64 + (CosIntegral[(5*(
a + b*ArcCos[c*x]))/b]*Sin[(5*a)/b])/64 + (CosIntegral[(7*(a + b*ArcCos[c*
x]))/b]*Sin[(7*a)/b])/256 - (CosIntegral[(9*(a + b*ArcCos[c*x]))/b]*Sin[(9
*a)/b])/256 + (3*Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/128 + (Cos[(
3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/64 - (Cos[(5*a)/b]*SinInte
gral[(5*(a + b*ArcCos[c*x]))/b])/64 - (Cos[(7*a)/b]*SinIntegral[(7*(a + b*
ArcCos[c*x]))/b])/256 + (Cos[(9*a)/b]*SinIntegral[(9*(a + b*ArcCos[c*x]))/
b])/256))/(b^2*c^4)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.64

method	result
default	$\frac{6 \arccos(cx) \operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right) \cos\left(\frac{a}{b}\right) b-6 \arccos(cx) \operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right) \sin\left(\frac{a}{b}\right) b-24 \arccos(cx) \operatorname{Si}\left(3 \arccos(cx)+\frac{3a}{b}\right) \cos\left(\frac{3a}{b}\right) b+24}{}$

input
```
int(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/256/c^4*(6*arccos(c*x)*Si(arccos(c*x)+a/b)*cos(a/b)*b-6*arccos(c*x)*Ci(a
rccos(c*x)+a/b)*sin(a/b)*b-24*arccos(c*x)*Si(3*arccos(c*x)+3*a/b)*cos(3*a/
b)*b+24*arccos(c*x)*Ci(3*arccos(c*x)+3*a/b)*sin(3*a/b)*b+21*arccos(c*x)*Si
(7*arccos(c*x)+7*a/b)*cos(7*a/b)*b-21*arccos(c*x)*Ci(7*arccos(c*x)+7*a/b)*
sin(7*a/b)*b-9*arccos(c*x)*Si(9*arccos(c*x)+9*a/b)*cos(9*a/b)*b+9*arccos(c
*x)*Ci(9*arccos(c*x)+9*a/b)*sin(9*a/b)*b+6*Si(arccos(c*x)+a/b)*cos(a/b)*a-
6*Ci(arccos(c*x)+a/b)*sin(a/b)*a-24*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)*a+2
4*Ci(3*arccos(c*x)+3*a/b)*sin(3*a/b)*a+21*Si(7*arccos(c*x)+7*a/b)*cos(7*a/
b)*a-21*Ci(7*arccos(c*x)+7*a/b)*sin(7*a/b)*a-9*Si(9*arccos(c*x)+9*a/b)*cos
(9*a/b)*a+9*Ci(9*arccos(c*x)+9*a/b)*sin(9*a/b)*a+6*c*x*b-8*cos(3*arccos(c*
x))*b+3*cos(7*arccos(c*x))*b-cos(9*arccos(c*x))*b)/(a+b*arccos(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^3}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^4*x^7 - 2*c^2*x^5 + x^3)*sqrt(-c^2*x^2 + 1)/(b^2*arccos(c*x)^2
 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^3(-(cx-1)(cx+1))^{\frac{5}{2}}}{(a+b\,\mathrm{acos}(cx))^2}\,dx$$

input
```
integrate(x**3*(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**3*(-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^3}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c^6*x^9 - 3*c^4*x^7 + 3*c^2*x^5 - x^3 - (b^2*c*arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x) + a*b*c)*integrate(3*(3*c^6*x^8 - 7*c^4*x^6 + 5*c^2*x^4
- x^2)/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c), x))/(b^
2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2528 vs. $2(260) = 520$.

Time = 0.26 (sec) , antiderivative size = 2528, normalized size of antiderivative = 9.09

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output

```
-b*c^9*x^9/(b^3*c^4*arccos(c*x) + a*b^2*c^4) + 3*b*c^7*x^7/(b^3*c^4*arccos
(c*x) + a*b^2*c^4) + 9*b*arccos(c*x)*cos(a/b)^8*cos_integral(9*a/b + 9*arc
cos(c*x))*sin(a/b)/(b^3*c^4*arccos(c*x) + a*b^2*c^4) - 9*b*arccos(c*x)*cos
(a/b)^9*sin_integral(9*a/b + 9*arccos(c*x))/(b^3*c^4*arccos(c*x) + a*b^2*c
^4) - 3*b*c^5*x^5/(b^3*c^4*arccos(c*x) + a*b^2*c^4) + 9*a*cos(a/b)^8*cos_i
ntegral(9*a/b + 9*arccos(c*x))*sin(a/b)/(b^3*c^4*arccos(c*x) + a*b^2*c^4)
- 9*a*cos(a/b)^9*sin_integral(9*a/b + 9*arccos(c*x))/(b^3*c^4*arccos(c*x)
+ a*b^2*c^4) - 63/4*b*arccos(c*x)*cos(a/b)^6*cos_integral(9*a/b + 9*arccos
(c*x))*sin(a/b)/(b^3*c^4*arccos(c*x) + a*b^2*c^4) - 21/4*b*arccos(c*x)*cos
(a/b)^6*cos_integral(7*a/b + 7*arccos(c*x))*sin(a/b)/(b^3*c^4*arccos(c*x)
+ a*b^2*c^4) + 81/4*b*arccos(c*x)*cos(a/b)^7*sin_integral(9*a/b + 9*arccos
(c*x))/(b^3*c^4*arccos(c*x) + a*b^2*c^4) + 21/4*b*arccos(c*x)*cos(a/b)^7*s
in_integral(7*a/b + 7*arccos(c*x))/(b^3*c^4*arccos(c*x) + a*b^2*c^4) - 63/
4*a*cos(a/b)^6*cos_integral(9*a/b + 9*arccos(c*x))*sin(a/b)/(b^3*c^4*arcco
s(c*x) + a*b^2*c^4) - 21/4*a*cos(a/b)^6*cos_integral(7*a/b + 7*arccos(c*x)
)*sin(a/b)/(b^3*c^4*arccos(c*x) + a*b^2*c^4) + 81/4*a*cos(a/b)^7*sin_integ
ral(9*a/b + 9*arccos(c*x))/(b^3*c^4*arccos(c*x) + a*b^2*c^4) + 21/4*a*cos(
a/b)^7*sin_integral(7*a/b + 7*arccos(c*x))/(b^3*c^4*arccos(c*x) + a*b^2*c^
4) + 135/16*b*arccos(c*x)*cos(a/b)^4*cos_integral(9*a/b + 9*arccos(c*x))*s
in(a/b)/(b^3*c^4*arccos(c*x) + a*b^2*c^4) + 105/16*b*arccos(c*x)*cos(a/...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{x^3\,(1-c^2\,x^2)^{5/2}}{(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input

```
int((x^3*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x))^2,x)
```

output

```
int((x^3*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x^3(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \left(\int \frac{\sqrt{-c^2x^2+1}\,x^7}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^5}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{-c^2x^2+1}\,x^3}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx$$

input
```
int(x^3*(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**7)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a
**2),x)*c**4 - 2*int((sqrt( - c**2*x**2 + 1)*x**5)/(acos(c*x)**2*b**2 + 2*
acos(c*x)*a*b + a**2),x)*c**2 + int((sqrt( - c**2*x**2 + 1)*x**3)/(acos(c*
x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)
```

3.402 $\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 282

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = -\frac{x^2(1-c^2x^2)^3}{bc(a+b\arccos(cx))} + \frac{\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\sin\left(\frac{2a}{b}\right)}{16b^2c^3} - \frac{\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)\sin\left(\frac{4a}{b}\right)}{8b^2c^3} - \frac{3\,\text{CosIntegral}\left(\frac{6(a+b\arccos(cx))}{b}\right)\sin\left(\frac{6a}{b}\right)}{16b^2c^3} - \frac{\text{CosIntegral}\left(\frac{8(a+b\arccos(cx))}{b}\right)\sin\left(\frac{8a}{b}\right)}{16b^2c^3} - \frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{16b^2c^3} + \frac{\cos\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{8b^2c^3} + \frac{3\cos\left(\frac{6a}{b}\right)\text{Si}\left(\frac{6(a+b\arccos(cx))}{b}\right)}{16b^2c^3} + \frac{\cos\left(\frac{8a}{b}\right)\text{Si}\left(\frac{8(a+b\arccos(cx))}{b}\right)}{16b^2c^3}$$

output
```
-x^2*(-c^2*x^2+1)^3/b/c/(a+b*arccos(c*x))+1/16*Ci(2*(a+b*arccos(c*x))/b)*s
in(2*a/b)/b^2/c^3-1/8*Ci(4*(a+b*arccos(c*x))/b)*sin(4*a/b)/b^2/c^3-3/16*Ci
(6*(a+b*arccos(c*x))/b)*sin(6*a/b)/b^2/c^3-1/16*Ci(8*(a+b*arccos(c*x))/b)*
sin(8*a/b)/b^2/c^3-1/16*cos(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b^2/c^3+1/8*c
os(4*a/b)*Si(4*(a+b*arccos(c*x))/b)/b^2/c^3+3/16*cos(6*a/b)*Si(6*(a+b*arcc
os(c*x))/b)/b^2/c^3+1/16*cos(8*a/b)*Si(8*(a+b*arccos(c*x))/b)/b^2/c^3
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.46

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx =$$
$$-\frac{-16bc^2x^2+48bc^4x^4-48bc^6x^6+16bc^8x^8-(a+b\arccos(cx))\,\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{2a}{b}\right)}{}$$

input
```
Integrate[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/16*(-16*b*c^2*x^2 + 48*b*c^4*x^4 - 48*b*c^6*x^6 + 16*b*c^8*x^8 - (a + b
*ArcCos[c*x])*CosIntegral[2*(a/b + ArcCos[c*x])]*Sin[(2*a)/b] - 2*(a + b*A
rcCos[c*x])*CosIntegral[4*(a/b + ArcCos[c*x])]*Sin[(4*a)/b] + 3*a*CosInteg
ral[6*(a/b + ArcCos[c*x])]*Sin[(6*a)/b] + 3*b*ArcCos[c*x]*CosIntegral[6*(a
/b + ArcCos[c*x])]*Sin[(6*a)/b] - a*CosIntegral[8*(a/b + ArcCos[c*x])]*Sin
[(8*a)/b] - b*ArcCos[c*x]*CosIntegral[8*(a/b + ArcCos[c*x])]*Sin[(8*a)/b]
+ a*Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])] + b*ArcCos[c*x]*Cos[(2
*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])] + 2*a*Cos[(4*a)/b]*SinIntegral[4
*(a/b + ArcCos[c*x])] + 2*b*ArcCos[c*x]*Cos[(4*a)/b]*SinIntegral[4*(a/b +
ArcCos[c*x])] - 3*a*Cos[(6*a)/b]*SinIntegral[6*(a/b + ArcCos[c*x])] - 3*b*
ArcCos[c*x]*Cos[(6*a)/b]*SinIntegral[6*(a/b + ArcCos[c*x])] + a*Cos[(8*a)/
b]*SinIntegral[8*(a/b + ArcCos[c*x])] + b*ArcCos[c*x]*Cos[(8*a)/b]*SinInte
gral[8*(a/b + ArcCos[c*x])])/(b^2*c^3*(a + b*ArcCos[c*x]))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.42, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5215, 5225, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \text{5215}$$

$$-\frac{2\int \frac{x(1-c^2x^2)^2}{a+b\arccos(cx)}dx}{bc}+\frac{8c\int \frac{x^3(1-c^2x^2)^2}{a+b\arccos(cx)}dx}{b}+\frac{x^2(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5225}$$

$$-\frac{8\int -\frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{2\int -\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{8\int \frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}-\frac{2\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{4906}$$

$$\frac{8\int \left(\frac{\sin\left(\frac{8a}{b}-\frac{8(a+b\arccos(cx))}{b}\right)}{128(a+b\arccos(cx))}-\frac{\sin\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}-\frac{\sin\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}+\frac{3\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}\right)d(a+b\text{ar}}{b^2c^3}$$

$$\frac{2\int \left(\frac{\sin\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}-\frac{\sin\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8(a+b\arccos(cx))}+\frac{5\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{5}{32}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{1}{8}\sin\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)-\frac{1}{32}\sin\left(\frac{6a}{b}\right)\text{CosIntegra}\right.}{}$$

$$\frac{8\left(-\frac{3}{64}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{1}{64}\sin\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{64}\sin\left(\frac{6a}{b}\right)\text{CosIntegr}\right.}{}$$

$$\frac{x^2(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

input
```
Int[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x])^2,x]
```

output
```
(x^2*(1 - c^2*x^2)^3)/(b*c*(a + b*ArcCos[c*x])) + (2*((-5*CosIntegral[(2*(
a + b*ArcCos[c*x]))/b]*Sin[(2*a)/b])/32 + (CosIntegral[(4*(a + b*ArcCos[c*
x]))/b]*Sin[(4*a)/b])/8 - (CosIntegral[(6*(a + b*ArcCos[c*x]))/b]*Sin[(6*a
)/b])/32 + (5*Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/32 - (C
os[(4*a)/b]*SinIntegral[(4*(a + b*ArcCos[c*x]))/b])/8 + (Cos[(6*a)/b]*SinI
ntegral[(6*(a + b*ArcCos[c*x]))/b])/32))/(b^2*c^3) - (8*((-3*CosIntegral[(
2*(a + b*ArcCos[c*x]))/b]*Sin[(2*a)/b])/64 + (CosIntegral[(4*(a + b*ArcCos
[c*x]))/b]*Sin[(4*a)/b])/64 + (CosIntegral[(6*(a + b*ArcCos[c*x]))/b]*Sin[
(6*a)/b])/64 - (CosIntegral[(8*(a + b*ArcCos[c*x]))/b]*Sin[(8*a)/b])/128 +
 (3*Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/64 - (Cos[(4*a)/b
]*SinIntegral[(4*(a + b*ArcCos[c*x]))/b])/64 - (Cos[(6*a)/b]*SinIntegral[(
6*(a + b*ArcCos[c*x]))/b])/64 + (Cos[(8*a)/b]*SinIntegral[(8*(a + b*ArcCos
[c*x]))/b])/128))/(b^2*c^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.70

method	result
default	$-\frac{16\arccos(cx)\,\mathrm{Si}\left(4\arccos(cx)+\frac{4a}{b}\right)\cos\left(\frac{4a}{b}\right)b-16\arccos(cx)\,\mathrm{Ci}\left(4\arccos(cx)+\frac{4a}{b}\right)\sin\left(\frac{4a}{b}\right)b+8\arccos(cx)\,\mathrm{Si}\left(8\arccos(cx)+\frac{8a}{b}\right)\mathrm{co}}{}$

input
```
int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/128/c^3*(16*arccos(c*x)*Si(4*arccos(c*x)+4*a/b)*cos(4*a/b)*b-16*arccos(
c*x)*Ci(4*arccos(c*x)+4*a/b)*sin(4*a/b)*b+8*arccos(c*x)*Si(8*arccos(c*x)+8
*a/b)*cos(8*a/b)*b-8*arccos(c*x)*Ci(8*arccos(c*x)+8*a/b)*sin(8*a/b)*b+8*ar
ccos(c*x)*Si(2*arccos(c*x)+2*a/b)*cos(2*a/b)*b-8*arccos(c*x)*Ci(2*arccos(c
*x)+2*a/b)*sin(2*a/b)*b-24*arccos(c*x)*Si(6*arccos(c*x)+6*a/b)*cos(6*a/b)*
b+24*arccos(c*x)*Ci(6*arccos(c*x)+6*a/b)*sin(6*a/b)*b+16*Si(4*arccos(c*x)+
4*a/b)*cos(4*a/b)*a-16*Ci(4*arccos(c*x)+4*a/b)*sin(4*a/b)*a+8*Si(8*arccos(
c*x)+8*a/b)*cos(8*a/b)*a-8*Ci(8*arccos(c*x)+8*a/b)*sin(8*a/b)*a+8*Si(2*arc
cos(c*x)+2*a/b)*cos(2*a/b)*a-8*Ci(2*arccos(c*x)+2*a/b)*sin(2*a/b)*a-24*Si(
6*arccos(c*x)+6*a/b)*cos(6*a/b)*a+24*Ci(6*arccos(c*x)+6*a/b)*sin(6*a/b)*a+
4*cos(4*arccos(c*x))*b+cos(8*arccos(c*x))*b+4*cos(2*arccos(c*x))*b-4*cos(6
*arccos(c*x))*b-5*b)/(a+b*arccos(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^2}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^4*x^6 - 2*c^2*x^4 + x^2)*sqrt(-c^2*x^2 + 1)/(b^2*arccos(c*x)^2
 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{x^2(-(cx-1)(cx+1))^{\frac{5}{2}}}{(a+b\,\mathrm{acos}(cx))^2}\,dx$$

input
```
integrate(x**2*(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**2*(-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x^2}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c^6*x^8 - 3*c^4*x^6 + 3*c^2*x^4 - x^2 - (b^2*c*arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x) + a*b*c)*integrate(2*(4*c^6*x^7 - 9*c^4*x^5 + 6*c^2*x^3
- x)/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c), x))/(b^2*
c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2512 vs. $2(264) = 528$.

Time = 0.26 (sec) , antiderivative size = 2512, normalized size of antiderivative = 8.91

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-b*c^8*x^8/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 3*b*c^6*x^6/(b^3*c^3*arccos
(c*x) + a*b^2*c^3) + 8*b*arccos(c*x)*cos(a/b)^7*cos_integral(8*a/b + 8*arc
cos(c*x))*sin(a/b)/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 8*b*arccos(c*x)*cos
(a/b)^8*sin_integral(8*a/b + 8*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c
^3) + 8*a*cos(a/b)^7*cos_integral(8*a/b + 8*arccos(c*x))*sin(a/b)/(b^3*c^3
*arccos(c*x) + a*b^2*c^3) - 8*a*cos(a/b)^8*sin_integral(8*a/b + 8*arccos(c
*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 3*b*c^4*x^4/(b^3*c^3*arccos(c*x)
+ a*b^2*c^3) - 12*b*arccos(c*x)*cos(a/b)^5*cos_integral(8*a/b + 8*arccos(c
*x))*sin(a/b)/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 6*b*arccos(c*x)*cos(a/b)
^5*cos_integral(6*a/b + 6*arccos(c*x))*sin(a/b)/(b^3*c^3*arccos(c*x) + a*b
^2*c^3) + 16*b*arccos(c*x)*cos(a/b)^6*sin_integral(8*a/b + 8*arccos(c*x))/
(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 6*b*arccos(c*x)*cos(a/b)^6*sin_integra
l(6*a/b + 6*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 12*a*cos(a/b)
^5*cos_integral(8*a/b + 8*arccos(c*x))*sin(a/b)/(b^3*c^3*arccos(c*x) + a*b
^2*c^3) - 6*a*cos(a/b)^5*cos_integral(6*a/b + 6*arccos(c*x))*sin(a/b)/(b^3
*c^3*arccos(c*x) + a*b^2*c^3) + 16*a*cos(a/b)^6*sin_integral(8*a/b + 8*arc
cos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 6*a*cos(a/b)^6*sin_integral(
6*a/b + 6*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 5*b*arccos(c*x)
*cos(a/b)^3*cos_integral(8*a/b + 8*arccos(c*x))*sin(a/b)/(b^3*c^3*arccos(c
*x) + a*b^2*c^3) + 6*b*arccos(c*x)*cos(a/b)^3*cos_integral(6*a/b + 6*ar...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^2\,(1-c^2\,x^2)^{5/2}}{(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x))^2,x)
```

output
```
int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x^2(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \left(\int \frac{\sqrt{-c^2x^2+1}\,x^6}{a\cos(cx)^2 b^2+2a\cos(cx)\,ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^4}{a\cos(cx)^2 b^2+2a\cos(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{-c^2x^2+1}\,x^2}{a\cos(cx)^2 b^2+2a\cos(cx)\,ab+a^2}dx$$

input
```
int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**6)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a
**2),x)*c**4 - 2*int((sqrt( - c**2*x**2 + 1)*x**4)/(acos(c*x)**2*b**2 + 2*
acos(c*x)*a*b + a**2),x)*c**2 + int((sqrt( - c**2*x**2 + 1)*x**2)/(acos(c*
x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)
```

3.403 $\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 276

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = -\frac{x(1-c^2x^2)^3}{bc(a+b\arccos(cx))} + \frac{5\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{64b^2c^2} + \frac{27\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{64b^2c^2} + \frac{25\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{64b^2c^2} + \frac{7\cos\left(\frac{7a}{b}\right)\text{CosIntegral}\left(\frac{7(a+b\arccos(cx))}{b}\right)}{64b^2c^2} + \frac{5\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{64b^2c^2} + \frac{27\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{64b^2c^2} + \frac{25\sin\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{64b^2c^2} + \frac{7\sin\left(\frac{7a}{b}\right)\text{Si}\left(\frac{7(a+b\arccos(cx))}{b}\right)}{64b^2c^2}$$

output

```
-x*(-c^2*x^2+1)^3/b/c/(a+b*arccos(c*x))+5/64*cos(a/b)*Ci((a+b*arccos(c*x))
/b)/b^2/c^2+27/64*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b^2/c^2+25/64*cos(5
*a/b)*Ci(5*(a+b*arccos(c*x))/b)/b^2/c^2+7/64*cos(7*a/b)*Ci(7*(a+b*arccos(c
*x))/b)/b^2/c^2+5/64*sin(a/b)*Si((a+b*arccos(c*x))/b)/b^2/c^2+27/64*sin(3*
a/b)*Si(3*(a+b*arccos(c*x))/b)/b^2/c^2+25/64*sin(5*a/b)*Si(5*(a+b*arccos(c
*x))/b)/b^2/c^2+7/64*sin(7*a/b)*Si(7*(a+b*arccos(c*x))/b)/b^2/c^2
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.46

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \frac{64bcx-192bc^3x^3+192bc^5x^5-64bc^7x^7-5(a+b\arccos(cx))\operatorname{CosIntegral}\left(\frac{a}{b}\right.}{}$$

input
```
Integrate[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x])^2,x]
```

output
```
(64*b*c*x - 192*b*c^3*x^3 + 192*b*c^5*x^5 - 64*b*c^7*x^7 - 5*(a + b*ArcCos
[c*x])*CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b] + 27*(a + b*ArcCos[c*x])*Co
sIntegral[3*(a/b + ArcCos[c*x])]*Sin[(3*a)/b] - 25*a*CosIntegral[5*(a/b +
ArcCos[c*x])]*Sin[(5*a)/b] - 25*b*ArcCos[c*x]*CosIntegral[5*(a/b + ArcCos[
c*x])]*Sin[(5*a)/b] + 7*a*CosIntegral[7*(a/b + ArcCos[c*x])]*Sin[(7*a)/b]
+ 7*b*ArcCos[c*x]*CosIntegral[7*(a/b + ArcCos[c*x])]*Sin[(7*a)/b] + 5*a*Co
s[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 5*b*ArcCos[c*x]*Cos[a/b]*SinIntegr
al[a/b + ArcCos[c*x]] - 27*a*Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x]
)] - 27*b*ArcCos[c*x]*Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])] + 25
*a*Cos[(5*a)/b]*SinIntegral[5*(a/b + ArcCos[c*x])] + 25*b*ArcCos[c*x]*Cos[
(5*a)/b]*SinIntegral[5*(a/b + ArcCos[c*x])] - 7*a*Cos[(7*a)/b]*SinIntegral
[7*(a/b + ArcCos[c*x])] - 7*b*ArcCos[c*x]*Cos[(7*a)/b]*SinIntegral[7*(a/b
+ ArcCos[c*x])])/(64*b^2*c^2*(a + b*ArcCos[c*x]))
```

Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.41, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5215, 5169, 25, 3042, 3793, 2009, 5225, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \text{5215}$$

$$-\frac{\int \frac{(1-c^2x^2)^2}{a+b\arccos(cx)}dx}{bc}+\frac{7c\int \frac{x^2(1-c^2x^2)^2}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5169}$$

$$\frac{\int -\frac{\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{7c\int \frac{x^2(1-c^2x^2)^2}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{7c\int \frac{x^2(1-c^2x^2)^2}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^5}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{7c\int \frac{x^2(1-c^2x^2)^2}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{\sin\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}-\frac{5\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{5\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^2}+\frac{7c\int \frac{x^2(1-c^2x^2)^2}{a+b\arccos(cx)}dx}{b}+\frac{x(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{7c\int \frac{x^2(1-c^2x^2)^2}{a+b\arccos(cx)}dx}{b}+\frac{-\frac{5}{8}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{5}{16}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)-\frac{1}{16}\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a}{}\right.}{b^2c^2}+\frac{x(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5225}$$

$$-\frac{7\int -\frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+$$

$$\frac{-\frac{5}{8}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{5}{16}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)-\frac{1}{16}\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a}{}\right.}{b^2c^2}$$

$$\frac{x(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 25$$

$$\frac{7\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+$$

$$\frac{-\frac{5}{8}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{5}{16}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)-\frac{1}{16}\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a}{}\right.}{b^2c^2}$$

$$\frac{x(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 4906$$

$$\frac{7\int \left(\frac{\sin\left(\frac{7a}{b}-\frac{7(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}-\frac{3\sin\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}+\frac{\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{64(a+b\arccos(cx))}+\frac{5\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{64(a+b\arccos(cx))}\right)d(a+b\,\text{arc}}{b^2c^2}$$

$$\frac{-\frac{5}{8}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{5}{16}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)-\frac{1}{16}\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a}{}\right.}{b^2c^2}$$

$$\frac{x(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{5}{8}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{5}{16}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)-\frac{1}{16}\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a}{}\right.}{b^2c^2}$$

$$\frac{7\left(-\frac{5}{64}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{1}{64}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{3}{64}\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\right.\right.}{}$$

$$\frac{x(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

input
```
Int[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCos[c*x])^2,x]
```

output
```
(x*(1 - c^2*x^2)^3)/(b*c*(a + b*ArcCos[c*x])) + ((-5*CosIntegral[(a + b*Ar
cCos[c*x])/b]*Sin[a/b])/8 + (5*CosIntegral[(3*(a + b*ArcCos[c*x]))/b]*Sin[
(3*a)/b])/16 - (CosIntegral[(5*(a + b*ArcCos[c*x]))/b]*Sin[(5*a)/b])/16 +
(5*Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/8 - (5*Cos[(3*a)/b]*SinInt
egral[(3*(a + b*ArcCos[c*x]))/b])/16 + (Cos[(5*a)/b]*SinIntegral[(5*(a + b
*ArcCos[c*x]))/b])/16)/(b^2*c^2) - (7*((-5*CosIntegral[(a + b*ArcCos[c*x])
/b]*Sin[a/b])/64 - (CosIntegral[(3*(a + b*ArcCos[c*x]))/b]*Sin[(3*a)/b])/6
4 + (3*CosIntegral[(5*(a + b*ArcCos[c*x]))/b]*Sin[(5*a)/b])/64 - (CosInteg
ral[(7*(a + b*ArcCos[c*x]))/b]*Sin[(7*a)/b])/64 + (5*Cos[a/b]*SinIntegral[
(a + b*ArcCos[c*x])/b])/64 + (Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*
x]))/b])/64 - (3*Cos[(5*a)/b]*SinIntegral[(5*(a + b*ArcCos[c*x]))/b])/64 +
 (Cos[(7*a)/b]*SinIntegral[(7*(a + b*ArcCos[c*x]))/b])/64))/(b^2*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.65

method	result
default	$\frac{25\arccos(cx)\operatorname{Si}\left(5\arccos(cx)+\frac{5a}{b}\right)\cos\left(\frac{5a}{b}\right)b-25\arccos(cx)\operatorname{Ci}\left(5\arccos(cx)+\frac{5a}{b}\right)\sin\left(\frac{5a}{b}\right)b-7\arccos(cx)\operatorname{Si}\left(7\arccos(cx)+\frac{7a}{b}\right)\cos(}{}$

input
```
int(x*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/64/c^2*(25*arccos(c*x)*Si(5*arccos(c*x)+5*a/b)*cos(5*a/b)*b-25*arccos(c*
x)*Ci(5*arccos(c*x)+5*a/b)*sin(5*a/b)*b-7*arccos(c*x)*Si(7*arccos(c*x)+7*a
/b)*cos(7*a/b)*b+7*arccos(c*x)*Ci(7*arccos(c*x)+7*a/b)*sin(7*a/b)*b-27*arc
cos(c*x)*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)*b+27*arccos(c*x)*Ci(3*arccos(c
*x)+3*a/b)*sin(3*a/b)*b+5*arccos(c*x)*Si(arccos(c*x)+a/b)*cos(a/b)*b-5*arc
cos(c*x)*Ci(arccos(c*x)+a/b)*sin(a/b)*b+25*Si(5*arccos(c*x)+5*a/b)*cos(5*a
/b)*a-25*Ci(5*arccos(c*x)+5*a/b)*sin(5*a/b)*a-7*Si(7*arccos(c*x)+7*a/b)*co
s(7*a/b)*a+7*Ci(7*arccos(c*x)+7*a/b)*sin(7*a/b)*a-27*Si(3*arccos(c*x)+3*a/
b)*cos(3*a/b)*a+27*Ci(3*arccos(c*x)+3*a/b)*sin(3*a/b)*a+5*Si(arccos(c*x)+a
/b)*cos(a/b)*a-5*Ci(arccos(c*x)+a/b)*sin(a/b)*a+5*c*x*b+5*cos(5*arccos(c*x
))*b-cos(7*arccos(c*x))*b-9*cos(3*arccos(c*x))*b)/(a+b*arccos(c*x))/b^2
```

Fricas [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}x}{(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^4*x^5 - 2*c^2*x^3 + x)*sqrt(-c^2*x^2 + 1)/(b^2*arccos(c*x)^2 +
 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{x(-(cx-1)(cx+1))^{\frac{5}{2}}}{(a+b\operatorname{acos}{(cx)})^2}\,dx$$

input
```
integrate(x*(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x*(-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}x}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^6*x^7 - 3*c^4*x^5 + 3*c^2*x^3 - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x
 + 1), c*x) + a*b*c)*integrate((7*c^6*x^6 - 15*c^4*x^4 + 9*c^2*x^2 - 1)/(b
^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c), x) - x)/(b^2*c*a
rctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2108 vs. $2(258) = 516$.

Time = 0.27 (sec) , antiderivative size = 2108, normalized size of antiderivative = 7.64

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-b*c^7*x^7/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 3*b*c^5*x^5/(b^3*c^2*arccos
(c*x) + a*b^2*c^2) + 7*b*arccos(c*x)*cos(a/b)^6*cos_integral(7*a/b + 7*arc
cos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 7*b*arccos(c*x)*cos
(a/b)^7*sin_integral(7*a/b + 7*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c
^2) + 7*a*cos(a/b)^6*cos_integral(7*a/b + 7*arccos(c*x))*sin(a/b)/(b^3*c^2
*arccos(c*x) + a*b^2*c^2) - 7*a*cos(a/b)^7*sin_integral(7*a/b + 7*arccos(c
*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 35/4*b*arccos(c*x)*cos(a/b)^4*cos
_integral(7*a/b + 7*arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2
) - 25/4*b*arccos(c*x)*cos(a/b)^4*cos_integral(5*a/b + 5*arccos(c*x))*sin(
a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 49/4*b*arccos(c*x)*cos(a/b)^5*sin
_integral(7*a/b + 7*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 25/4*
b*arccos(c*x)*cos(a/b)^5*sin_integral(5*a/b + 5*arccos(c*x))/(b^3*c^2*arcc
os(c*x) + a*b^2*c^2) - 3*b*c^3*x^3/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 35/
4*a*cos(a/b)^4*cos_integral(7*a/b + 7*arccos(c*x))*sin(a/b)/(b^3*c^2*arcco
s(c*x) + a*b^2*c^2) - 25/4*a*cos(a/b)^4*cos_integral(5*a/b + 5*arccos(c*x)
)*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 49/4*a*cos(a/b)^5*sin_integ
ral(7*a/b + 7*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 25/4*a*cos(
a/b)^5*sin_integral(5*a/b + 5*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^
2) + 21/8*b*arccos(c*x)*cos(a/b)^2*cos_integral(7*a/b + 7*arccos(c*x))*sin
(a/b)/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + 75/16*b*arccos(c*x)*cos(a/b)^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{x\,(1-c^2\,x^2)^{5/2}}{(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((x*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x))^2,x)
```

output
```
int((x*(1 - c^2*x^2)^(5/2))/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{x(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \left(\int \frac{\sqrt{-c^2x^2+1}\,x^5}{acos(cx)^2 b^2+2acos(cx)\,ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^3}{acos(cx)^2 b^2+2acos(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{-c^2x^2+1}\,x}{acos(cx)^2 b^2+2acos(cx)\,ab+a^2}dx$$

input
```
int(x*(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
int((sqrt( - c**2*x**2 + 1)*x**5)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a
**2),x)*c**4 - 2*int((sqrt( - c**2*x**2 + 1)*x**3)/(acos(c*x)**2*b**2 + 2*
acos(c*x)*a*b + a**2),x)*c**2 + int((sqrt( - c**2*x**2 + 1)*x)/(acos(c*x)*
*2*b**2 + 2*acos(c*x)*a*b + a**2),x)
```

3.404 $\int \frac{(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = -\frac{(1-c^2x^2)^3}{bc(a+b\arccos(cx))} + \frac{15\,\mathrm{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\sin\left(\frac{2a}{b}\right)}{16b^2c} + \frac{3\,\mathrm{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)\sin\left(\frac{4a}{b}\right)}{4b^2c} + \frac{3\,\mathrm{CosIntegral}\left(\frac{6(a+b\arccos(cx))}{b}\right)\sin\left(\frac{6a}{b}\right)}{16b^2c} - \frac{15\cos\left(\frac{2a}{b}\right)\mathrm{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{16b^2c} - \frac{3\cos\left(\frac{4a}{b}\right)\mathrm{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{4b^2c} - \frac{3\cos\left(\frac{6a}{b}\right)\mathrm{Si}\left(\frac{6(a+b\arccos(cx))}{b}\right)}{16b^2c}$$

output
```
-(-c^2*x^2+1)^3/b/c/(a+b*arccos(c*x))+15/16*Ci(2*(a+b*arccos(c*x))/b)*sin(
2*a/b)/b^2/c+3/4*Ci(4*(a+b*arccos(c*x))/b)*sin(4*a/b)/b^2/c+3/16*Ci(6*(a+b
*arccos(c*x))/b)*sin(6*a/b)/b^2/c-15/16*cos(2*a/b)*Si(2*(a+b*arccos(c*x))/
b)/b^2/c-3/4*cos(4*a/b)*Si(4*(a+b*arccos(c*x))/b)/b^2/c-3/16*cos(6*a/b)*Si
(6*(a+b*arccos(c*x))/b)/b^2/c
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.43

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \frac{16b-48bc^2x^2+48bc^4x^4-16bc^6x^6+15(a+b\arccos(cx))\,\text{CosIntegral}\left(2\left(\frac{a}{b}+\right.\right.}{}$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
(16*b - 48*b*c^2*x^2 + 48*b*c^4*x^4 - 16*b*c^6*x^6 + 15*(a + b*ArcCos[c*x]
)*CosIntegral[2*(a/b + ArcCos[c*x])]*Sin[(2*a)/b] - 12*(a + b*ArcCos[c*x])
*CosIntegral[4*(a/b + ArcCos[c*x])]*Sin[(4*a)/b] + 3*a*CosIntegral[6*(a/b
+ ArcCos[c*x])]*Sin[(6*a)/b] + 3*b*ArcCos[c*x]*CosIntegral[6*(a/b + ArcCos
[c*x])]*Sin[(6*a)/b] - 15*a*Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])
] - 15*b*ArcCos[c*x]*Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])] + 12*
a*Cos[(4*a)/b]*SinIntegral[4*(a/b + ArcCos[c*x])] + 12*b*ArcCos[c*x]*Cos[(
4*a)/b]*SinIntegral[4*(a/b + ArcCos[c*x])] - 3*a*Cos[(6*a)/b]*SinIntegral[
6*(a/b + ArcCos[c*x])] - 3*b*ArcCos[c*x]*Cos[(6*a)/b]*SinIntegral[6*(a/b +
 ArcCos[c*x])])/(16*b^2*c*(a + b*ArcCos[c*x]))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5167, 5225, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \; 5167$$

$$\frac{6c\int \frac{x(1-c^2x^2)^2}{a+b\arccos(cx)}dx}{b}+\frac{(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 5225$$

$$\frac{(1-c^2x^2)^3}{bc(a+b\arccos(cx))}-\frac{6\int -\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{25}$$

$$\frac{6\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}+\frac{(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{4906}$$

$$\frac{6\int \left(\frac{\sin\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}-\frac{\sin\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8(a+b\arccos(cx))}+\frac{5\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{32(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c}+\frac{(1-c^2x^2)^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(1-c^2x^2)^3}{bc(a+b\arccos(cx))}-\frac{6\left(-\frac{5}{32}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)+\frac{1}{8}\sin\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)-\frac{1}{32}\sin\left(\frac{6a}{b}\right)\text{CosIntegra}\right.}{}$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
(1 - c^2*x^2)^3/(b*c*(a + b*ArcCos[c*x])) - (6*((-5*CosIntegral[(2*(a + b*
ArcCos[c*x]))/b]*Sin[(2*a)/b])/32 + (CosIntegral[(4*(a + b*ArcCos[c*x]))/b
]*Sin[(4*a)/b])/8 - (CosIntegral[(6*(a + b*ArcCos[c*x]))/b]*Sin[(6*a)/b])/
32 + (5*Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/32 - (Cos[(4*
a)/b]*SinIntegral[(4*(a + b*ArcCos[c*x]))/b])/8 + (Cos[(6*a)/b]*SinIntegra
l[(6*(a + b*ArcCos[c*x]))/b])/32))/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.68

method	result
default	$-\frac{6 \arccos(cx)\, \mathrm{Si}\left(6 \arccos(cx)+\frac{6a}{b}\right) \cos\left(\frac{6a}{b}\right) b-6 \arccos(cx)\, \mathrm{Ci}\left(6 \arccos(cx)+\frac{6a}{b}\right) \sin\left(\frac{6a}{b}\right) b-24 \arccos(cx)\, \mathrm{Si}\left(4 \arccos(cx)+\frac{4a}{b}\right) \cos}{}$

input
```
int((-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32/c*(6*arccos(c*x)*Si(6*arccos(c*x)+6*a/b)*cos(6*a/b)*b-6*arccos(c*x)*
Ci(6*arccos(c*x)+6*a/b)*sin(6*a/b)*b-24*arccos(c*x)*Si(4*arccos(c*x)+4*a/b
)*cos(4*a/b)*b+24*arccos(c*x)*Ci(4*arccos(c*x)+4*a/b)*sin(4*a/b)*b+30*arcc
os(c*x)*Si(2*arccos(c*x)+2*a/b)*cos(2*a/b)*b-30*arccos(c*x)*Ci(2*arccos(c*
x)+2*a/b)*sin(2*a/b)*b+6*Si(6*arccos(c*x)+6*a/b)*cos(6*a/b)*a-6*Ci(6*arcco
s(c*x)+6*a/b)*sin(6*a/b)*a-24*Si(4*arccos(c*x)+4*a/b)*cos(4*a/b)*a+24*Ci(4
*arccos(c*x)+4*a/b)*sin(4*a/b)*a+30*Si(2*arccos(c*x)+2*a/b)*cos(2*a/b)*a-3
0*Ci(2*arccos(c*x)+2*a/b)*sin(2*a/b)*a+cos(6*arccos(c*x))*b-6*cos(4*arccos
(c*x))*b+15*cos(2*arccos(c*x))*b-10*b)/b^2/(a+b*arccos(c*x))
```

Fricas [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*arccos(c*x)^2 +
 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(-(cx-1)(cx+1))^{\frac{5}{2}}}{(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x
 + 1), c*x) + a*b*c)*integrate(6*(c^5*x^5 - 2*c^3*x^3 + c*x)/(b^2*arctan2(
sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b), x) - 1)/(b^2*c*arctan2(sqrt(c*x
 + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1464 vs. $2(203) = 406$.

Time = 0.25 (sec) , antiderivative size = 1464, normalized size of antiderivative = 6.75

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx=\text{Too large to display}$$

input
```
integrate((-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-b*c^6*x^6/(b^3*c*arccos(c*x) + a*b^2*c) + 3*b*c^4*x^4/(b^3*c*arccos(c*x)
+ a*b^2*c) + 6*b*arccos(c*x)*cos(a/b)^5*cos_integral(6*a/b + 6*arccos(c*x)
)*sin(a/b)/(b^3*c*arccos(c*x) + a*b^2*c) - 6*b*arccos(c*x)*cos(a/b)^6*sin_
integral(6*a/b + 6*arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) + 6*a*cos(a/
b)^5*cos_integral(6*a/b + 6*arccos(c*x))*sin(a/b)/(b^3*c*arccos(c*x) + a*b
^2*c) - 6*a*cos(a/b)^6*sin_integral(6*a/b + 6*arccos(c*x))/(b^3*c*arccos(c
*x) + a*b^2*c) - 6*b*arccos(c*x)*cos(a/b)^3*cos_integral(6*a/b + 6*arccos(
c*x))*sin(a/b)/(b^3*c*arccos(c*x) + a*b^2*c) - 6*b*arccos(c*x)*cos(a/b)^3*
cos_integral(4*a/b + 4*arccos(c*x))*sin(a/b)/(b^3*c*arccos(c*x) + a*b^2*c)
 + 9*b*arccos(c*x)*cos(a/b)^4*sin_integral(6*a/b + 6*arccos(c*x))/(b^3*c*a
rccos(c*x) + a*b^2*c) + 6*b*arccos(c*x)*cos(a/b)^4*sin_integral(4*a/b + 4*
arccos(c*x))/(b^3*c*arccos(c*x) + a*b^2*c) - 6*a*cos(a/b)^3*cos_integral(6
*a/b + 6*arccos(c*x))*sin(a/b)/(b^3*c*arccos(c*x) + a*b^2*c) - 6*a*cos(a/b
)^3*cos_integral(4*a/b + 4*arccos(c*x))*sin(a/b)/(b^3*c*arccos(c*x) + a*b^
2*c) + 9*a*cos(a/b)^4*sin_integral(6*a/b + 6*arccos(c*x))/(b^3*c*arccos(c*
x) + a*b^2*c) + 6*a*cos(a/b)^4*sin_integral(4*a/b + 4*arccos(c*x))/(b^3*c*
arccos(c*x) + a*b^2*c) - 3*b*c^2*x^2/(b^3*c*arccos(c*x) + a*b^2*c) + 9/8*b
*arccos(c*x)*cos(a/b)*cos_integral(6*a/b + 6*arccos(c*x))*sin(a/b)/(b^3*c*
arccos(c*x) + a*b^2*c) + 3*b*arccos(c*x)*cos(a/b)*cos_integral(4*a/b + 4*a
rccos(c*x))*sin(a/b)/(b^3*c*arccos(c*x) + a*b^2*c) + 15/8*b*arccos(c*x)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(1-c^2\,x^2)^{5/2}}{(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((1 - c^2*x^2)^(5/2)/(a + b*acos(c*x))^2,x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(1-c^2x^2)^{5/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{acos(cx)^2 b^2+2acos(cx)ab+a^2}dx + \left(\int \frac{\sqrt{-c^2x^2+1}\,x^4}{acos(cx)^2 b^2+2acos(cx)ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x^2}{acos(cx)^2 b^2+2acos(cx)ab+a^2}dx\right)c^2$$

input
```
int((-c^2*x^2+1)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)
 + int((sqrt( - c**2*x**2 + 1)*x**4)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b
+ a**2),x)*c**4 - 2*int((sqrt( - c**2*x**2 + 1)*x**2)/(acos(c*x)**2*b**2 +
 2*acos(c*x)*a*b + a**2),x)*c**2
```

3.405 $\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\begin{aligned}\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))^2}\,dx = &-\frac{(1-c^2x^2)^3}{bcx(a+b\arccos(cx))} \\ &-\frac{25\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2} \\ &-\frac{25\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2} \\ &-\frac{5\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2} \\ &-\frac{25\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2}-\frac{25\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2} \\ &-\frac{5\sin\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2}-\frac{\text{Int}\left(\frac{(1-c^2x^2)^2}{x^2(a+b\arccos(cx))},x\right)}{bc}\end{aligned}$$

output

```
-(-c^2*x^2+1)^3/b/c/x/(a+b*arccos(c*x))-25/8*cos(a/b)*Ci((a+b*arccos(c*x))
/b)/b^2-25/16*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b^2-5/16*cos(5*a/b)*Ci(
5*(a+b*arccos(c*x))/b)/b^2-25/8*sin(a/b)*Si((a+b*arccos(c*x))/b)/b^2-25/16
*sin(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^2-5/16*sin(5*a/b)*Si(5*(a+b*arccos
(c*x))/b)/b^2-Defer(Int)((-c^2*x^2+1)^2/x^2/(a+b*arccos(c*x)),x)/b/c
```

Mathematica [N/A]

Not integrable

Time = 13.79 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))^2}\,dx=\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \text{5215}$$

$$\frac{5c\int \frac{(1-c^2x^2)^2}{a+b\arccos(cx)}dx}{b}+\frac{\int \frac{(1-c^2x^2)^2}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{(1-c^2x^2)^3}{bcx(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5169}$$

$$-\frac{5\int -\frac{\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2}+\frac{\int \frac{(1-c^2x^2)^2}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{(1-c^2x^2)^3}{bcx(a+b\arccos(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\int \frac{\sin^5\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2}+\frac{\int \frac{(1-c^2x^2)^2}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{(1-c^2x^2)^3}{bcx(a+b\arccos(cx))}$$

$\Big\downarrow$ 3042

$$\frac{5\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^5}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2}+\frac{\int \frac{(1-c^2x^2)^2}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{(1-c^2x^2)^3}{bcx(a+b\arccos(cx))}$$

$\Big\downarrow$ 3793

$$\frac{5\int \left(\frac{\sin\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}-\frac{5\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{5\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2}+\frac{\int \frac{(1-c^2x^2)^2}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{(1-c^2x^2)^3}{bcx(a+b\arccos(cx))}$$

$\Big\downarrow$ 2009

$$\frac{\int \frac{(1-c^2x^2)^2}{x^2(a+b\arccos(cx))}dx}{bc}-\frac{5\left(-\frac{5}{8}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{5}{16}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)-\frac{1}{16}\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\right.\right.}{b}\frac{(1-c^2x^2)^3}{bcx(a+b\arccos(cx))}$$

$\Big\downarrow$ 5235

$$\frac{\int \frac{(1-c^2x^2)^2}{x^2(a+b\arccos(cx))}dx}{bc}-\frac{5\left(-\frac{5}{8}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\frac{5}{16}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)-\frac{1}{16}\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\right.\right.}{b}\frac{(1-c^2x^2)^3}{bcx(a+b\arccos(cx))}$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x\,(a+b\arccos{(cx)})^2}dx$$

input
```
int((-c^2*x^2+1)^(5/2)/x/(a+b*arccos(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(5/2)/x/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos{(cx)}+a)^2x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x*arccos(c*x)^2
 + 2*a*b*x*arccos(c*x) + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 7.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))^2}\,dx = \int \frac{(-(cx-1)\,(cx+1))^{\frac{5}{2}}}{x\,(a+b\operatorname{acos}{(cx)})^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(5/2)/x/(a+b*acos(c*x))**2,x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 162, normalized size of antiderivative = 5.79

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos(cx)+a)^2x}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - (b^2*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c
*x + 1), c*x) + a*b*c*x)*integrate((5*c^6*x^6 - 9*c^4*x^4 + 3*c^2*x^2 + 1)
/(b^2*c*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^2), x) -
1)/(b^2*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))^2}\,dx = \int \frac{(1-c^2\,x^2)^{5/2}}{x\,(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((1 - c^2*x^2)^(5/2)/(x*(a + b*acos(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(x*(a + b*acos(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 4.71

$$\begin{aligned}\int \frac{(1-c^2x^2)^{5/2}}{x(a+b\arccos(cx))^2}\,dx &= \int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)^2\,b^2x+2acos\,(cx)\,abx+a^2x}dx \\ &+ \left(\int \frac{\sqrt{-c^2x^2+1}\,x^3}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^4 \\ &- 2\left(\int \frac{\sqrt{-c^2x^2+1}\,x}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2\end{aligned}$$

input
```
int((-c^2*x^2+1)^(5/2)/x/(a+b*acos(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2*x + 2*acos(c*x)*a*b*x + a**2
*x),x) + int((sqrt( - c**2*x**2 + 1)*x**3)/(acos(c*x)**2*b**2 + 2*acos(c*x
)*a*b + a**2),x)*c**4 - 2*int((sqrt( - c**2*x**2 + 1)*x)/(acos(c*x)**2*b**
2 + 2*acos(c*x)*a*b + a**2),x)*c**2
```

3.406 $\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))^2}\,dx = -\frac{(1-c^2x^2)^3}{bcx^2(a+b\arccos(cx))} - \frac{2\text{Int}\left(\frac{(1-c^2x^2)^2}{x^3(a+b\arccos(cx))},x\right)}{bc} - \frac{4c\text{Int}\left(\frac{(1-c^2x^2)^2}{x(a+b\arccos(cx))},x\right)}{b}$$

output
```
-(-c^2*x^2+1)^3/b/c/x^2/(a+b*arccos(c*x))-2*Defer(Int)((-c^2*x^2+1)^2/x^3/
(a+b*arccos(c*x)),x)/b/c-4*c*Defer(Int)((-c^2*x^2+1)^2/x/(a+b*arccos(c*x))
,x)/b
```

Mathematica [N/A]

Not integrable

Time = 4.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5215}$$

$$\frac{4c\int \frac{(1-c^2x^2)^2}{x(a+b\arccos(cx))}dx}{b}+\frac{2\int \frac{(1-c^2x^2)^2}{x^3(a+b\arccos(cx))}dx}{bc}+\frac{(1-c^2x^2)^3}{bcx^2(a+b\arccos(cx))}$$

$$\Big\downarrow \text{5235}$$

$$\frac{4c\int \frac{(1-c^2x^2)^2}{x(a+b\arccos(cx))}dx}{b}+\frac{2\int \frac{(1-c^2x^2)^2}{x^3(a+b\arccos(cx))}dx}{bc}+\frac{(1-c^2x^2)^3}{bcx^2(a+b\arccos(cx))}$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x^2 (a+b \arccos (cx))^2} dx$$

input
```
int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccos(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))^2}\, dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b \arccos (cx)+a)^2 x^2}\, dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x^2*arccos(c*x)
^2 + 2*a*b*x^2*arccos(c*x) + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 6.76 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))^2}\, dx = \int \frac{(-(cx-1)(cx+1))^{\frac{5}{2}}}{x^2 (a+b \operatorname{acos}(cx))^2}\, dx$$

input
```
integrate((-c**2*x**2+1)**(5/2)/x**2/(a+b*acos(c*x))**2,x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**2*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 5.82

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos(cx)+a)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - (b^2*c*x^2*arctan2(sqrt(c*x + 1)*sqrt(
-c*x + 1), c*x) + a*b*c*x^2)*integrate(2*(2*c^6*x^6 - 3*c^4*x^4 + 1)/(b^2*
c*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^3), x) - 1)/(b^
2*c*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^2)
```

Giac [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos(cx)+a)^2x^2}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/((b*arccos(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{(1-c^2\,x^2)^{5/2}}{x^2\,(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acos(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acos(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 364, normalized size of antiderivative = 13.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\arccos(cx))^2}\,dx = \frac{-2acos(cx)\left(\int \frac{\sqrt{-c^2x^2+1}}{acos(cx)^2b^2+2acos(cx)ab+a^2}dx\right)ab\,c^2+acos(cx)\left(\int \frac{\sqrt{-c^2x^2+}}{acos(cx)^2b^2+2aco}\right.}{}$$

input
```
int((-c^2*x^2+1)^(5/2)/x^2/(a+b*acos(c*x))^2,x)
```

output
```
( - 2*acos(c*x)*int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2 + 2*acos(c*x
)*a*b + a**2),x)*a*b*c**2 + acos(c*x)*int((sqrt( - c**2*x**2 + 1)*x**2)/(a
cos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*a*b*c**4 + acos(c*x)*int(1/(
sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*a
cos(c*x)*a*b*x**2 + sqrt( - c**2*x**2 + 1)*a**2*x**2),x)*a*b + acos(c*x)*c
 - 2*int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**
2),x)*a**2*c**2 + int((sqrt( - c**2*x**2 + 1)*x**2)/(acos(c*x)**2*b**2 + 2
*acos(c*x)*a*b + a**2),x)*a**2*c**4 + int(1/(sqrt( - c**2*x**2 + 1)*acos(c
*x)**2*b**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*x**2 + sqrt( - c
**2*x**2 + 1)*a**2*x**2),x)*a**2)/(a*(acos(c*x)*b + a))
```

3.407 $\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 20.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))^2}\,dx = \int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))^2} \, dx$$

$$\Big\downarrow \text{5235}$$

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))^2}dx$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x^3\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccos(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos(cx)+a)^2x^3}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x^3*arccos(c*x)
^2 + 2*a*b*x^3*arccos(c*x) + a^2*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 7.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))^2}\,dx = \int \frac{(-(cx-1)(cx+1))^{\frac{5}{2}}}{x^3(a+b\,\mathrm{acos}(cx))^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(5/2)/x**3/(a+b*acos(c*x))**2,x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**3*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 6.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos(cx)+a)^2x^3}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - (b^2*c*x^3*arctan2(sqrt(c*x + 1)*sqrt(
-c*x + 1), c*x) + a*b*c*x^3)*integrate(3*(c^6*x^6 - c^4*x^4 - c^2*x^2 + 1)
/(b^2*c*x^4*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^4), x) -
1)/(b^2*c*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))^2}\,dx = \int \frac{(1-c^2\,x^2)^{5/2}}{x^3\,(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*acos(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*acos(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 4.96

$$\begin{aligned}\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\arccos(cx))^2}\,dx &= \int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)^2\,b^2x^3+2acos\,(cx)\,ab\,x^3+a^2x^3}dx \\ &\quad - 2\left(\int \frac{\sqrt{-c^2x^2+1}}{acos\,(cx)^2\,b^2x+2acos\,(cx)\,abx+a^2x}dx\right)c^2 \\ &\quad + \left(\int \frac{\sqrt{-c^2x^2+1}\,x}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^4\end{aligned}$$

input
```
int((-c^2*x^2+1)^(5/2)/x^3/(a+b*acos(c*x))^2,x)
```

output
```
int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2*x**3 + 2*acos(c*x)*a*b*x**3
+ a**2*x**3),x) - 2*int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2*x + 2*ac
os(c*x)*a*b*x + a**2*x),x)*c**2 + int((sqrt( - c**2*x**2 + 1)*x)/(acos(c*x
)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*c**4
```

3.408 $\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))^2}\,dx = \int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))^2} dx$$

$$\Big\downarrow \text{5235}$$

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))^2}dx$$

input
```
Int[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{x^4\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccos(c*x))^2,x)
```

output
```
int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos{(cx)}+a)^2x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x^4*arccos(c*x)
^2 + 2*a*b*x^4*arccos(c*x) + a^2*x^4), x)
```

Sympy [N/A]

Not integrable

Time = 10.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))^2}\,dx = \int \frac{(-(cx-1)\,(cx+1))^{\frac{5}{2}}}{x^4\,(a+b\,\mathrm{acos}\,(cx))^2}\,dx$$

input
```
integrate((-c**2*x**2+1)**(5/2)/x**4/(a+b*acos(c*x))**2,x)
```

output
```
Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**4*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 5.79

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos{(cx)}+a)^2x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - (b^2*c*x^4*arctan2(sqrt(c*x + 1)*sqrt(
-c*x + 1), c*x) + a*b*c*x^4)*integrate(2*(c^6*x^6 - 3*c^2*x^2 + 2)/(b^2*c*
x^5*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^5), x) - 1)/(b^2*
c*x^4*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^4)
```

Giac [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))^2}\,dx = \int \frac{(-c^2x^2+1)^{\frac{5}{2}}}{(b\arccos{(cx)}+a)^2x^4}\,dx$$

input
```
integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate((-c^2*x^2 + 1)^(5/2)/((b*arccos(c*x) + a)^2*x^4), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))^2}\,dx = \int \frac{(1-c^2\,x^2)^{5/2}}{x^4\,(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((1 - c^2*x^2)^(5/2)/(x^4*(a + b*acos(c*x))^2),x)
```

output
```
int((1 - c^2*x^2)^(5/2)/(x^4*(a + b*acos(c*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 381, normalized size of antiderivative = 13.61

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\arccos(cx))^2}\,dx = \frac{acos(cx)\left(\int \frac{\sqrt{-c^2x^2+1}}{acos(cx)^2b^2x^4+2acos(cx)ab\,x^4+a^2x^4}dx\right)ab + acos(cx)\left(\int \frac{\sqrt{-c^2x^2}}{acos(cx)^2b^2+2aco}\right.}{}$$

input
```
int((-c^2*x^2+1)^(5/2)/x^4/(a+b*acos(c*x))^2,x)
```

output
```
(acos(c*x)*int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2*x**4 + 2*acos(c*x
)*a*b*x**4 + a**2*x**4),x)*a*b + acos(c*x)*int(sqrt( - c**2*x**2 + 1)/(aco
s(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*a*b*c**4 - 2*acos(c*x)*int(1/(
sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*a
cos(c*x)*a*b*x**2 + sqrt( - c**2*x**2 + 1)*a**2*x**2),x)*a*b*c**2 - 2*acos
(c*x)*c**3 + int(sqrt( - c**2*x**2 + 1)/(acos(c*x)**2*b**2*x**4 + 2*acos(c
*x)*a*b*x**4 + a**2*x**4),x)*a**2 + int(sqrt( - c**2*x**2 + 1)/(acos(c*x)*
*2*b**2 + 2*acos(c*x)*a*b + a**2),x)*a**2*c**4 - 2*int(1/(sqrt( - c**2*x**
2 + 1)*acos(c*x)**2*b**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*x**
2 + sqrt( - c**2*x**2 + 1)*a**2*x**2),x)*a**2*c**2)/(a*(acos(c*x)*b + a))
```

3.409 $\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = -\frac{x^m}{bc(a+b\arccos(cx))}+\frac{m\text{Int}\left(\frac{x^{-1+m}}{a+b\arccos(cx)},x\right)}{bc}$$

output
```
-x^m/b/c/(a+b*arccos(c*x))+m*Defer(Int)(x^(-1+m)/(a+b*arccos(c*x)),x)/b/c
```

Mathematica [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{ 5223}$$

$$\frac{x^m}{bc(a+b\arccos(cx))}-\frac{m\int \frac{x^{m-1}}{a+b\arccos(cx)}dx}{bc}$$

$$\Big\downarrow \text{ 5149}$$

$$\frac{x^m}{bc(a+b\arccos(cx))}-\frac{m\int \frac{x^{m-1}}{a+b\arccos(cx)}dx}{bc}$$

input
```
Int[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{\sqrt{-c^2x^2+1}\,\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^m/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccos
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 2.75 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^m}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acos}{(cx)})^2}\,dx$$

input
```
integrate(x**m/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**m/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.04

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)^2}\,dx$$

input

```
integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output

```
-((b^2*c*m*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*m)*integrate
(x^m/(b^2*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x), x) -
x^m)/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^m}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)^2}\,dx$$

input

```
integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output

```
integrate(x^m/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^m}{(a+b\,\mathrm{acos}\,(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^m/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^m/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \frac{x^m}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \frac{-acos(cx)\left(\int \frac{x^m}{acos(cx)bx+ax}dx\right)bm+x^m-\left(\int \frac{x^m}{acos(cx)bx+ax}dx\right)am}{bc\,(acos\,(cx)\,b+a)}$$

input
```
int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
( - acos(c*x)*int(x**m/(acos(c*x)*b*x + a*x),x)*b*m + x**m - int(x**m/(aco
s(c*x)*b*x + a*x),x)*a*m)/(b*c*(acos(c*x)*b + a))
```

3.410 $\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = -\frac{x^5}{bc(a+b\arccos(cx))} + \frac{5\cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2c^6} - \frac{15\cos\left(\frac{3a}{b}\right)\operatorname{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2c^6} + \frac{5\cos\left(\frac{5a}{b}\right)\operatorname{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2c^6} + \frac{5\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2c^6} - \frac{15\sin\left(\frac{3a}{b}\right)\operatorname{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2c^6} + \frac{5\sin\left(\frac{5a}{b}\right)\operatorname{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2c^6}$$

output
```
-x^5/b/c/(a+b*arccos(c*x))+5/8*cos(a/b)*Ci((a+b*arccos(c*x))/b)/b^2/c^6-15
/16*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b^2/c^6+5/16*cos(5*a/b)*Ci(5*(a+b
*arccos(c*x))/b)/b^2/c^6+5/8*sin(a/b)*Si((a+b*arccos(c*x))/b)/b^2/c^6-15/1
6*sin(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^2/c^6+5/16*sin(5*a/b)*Si(5*(a+b*a
rccos(c*x))/b)/b^2/c^6
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \frac{x^5}{bc(a+b\arccos(cx))} + \frac{5\left(-2\,\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)-3\,\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{3a}{b}\right)-\text{CosIntegra}\right.}{}$$

input
```
Integrate[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
x^5/(b*c*(a + b*ArcCos[c*x])) + (5*(-2*CosIntegral[a/b + ArcCos[c*x]]*Sin[
a/b] - 3*CosIntegral[3*(a/b + ArcCos[c*x])]*Sin[(3*a)/b] - CosIntegral[5*(
a/b + ArcCos[c*x])]*Sin[(5*a)/b] + 2*Cos[a/b]*SinIntegral[a/b + ArcCos[c*x
]] + 3*Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])] + Cos[(5*a)/b]*SinI
ntegral[5*(a/b + ArcCos[c*x])]))/(16*b^2*c^6)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5223, 5147, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{ 5223}$$

$$\frac{x^5}{bc(a+b\arccos(cx))}-\frac{5\int \frac{x^4}{a+b\arccos(cx)}dx}{bc}$$

$\downarrow$ 5147

$$\frac{5\int -\frac{\cos^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^6}+\frac{x^5}{bc(a+b\arccos(cx))}$$

$\downarrow$ 25

$$\frac{x^5}{bc(a+b\arccos(cx))}-\frac{5\int \frac{\cos^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^6}$$

$\downarrow$ 4906

$$\frac{x^5}{bc(a+b\arccos(cx))}-\frac{5\int \left(\frac{\sin\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{3\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{16(a+b\arccos(cx))}+\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{8(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^6}$$

$\downarrow$ 2009

$$\frac{5\left(-\frac{1}{8}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{3}{16}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)-\frac{1}{16}\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\right.\right.}{b^2}$$

$$\frac{x^5}{bc(a+b\arccos(cx))}$$

input
```
Int[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
x^5/(b*c*(a + b*ArcCos[c*x])) + (5*(-1/8*(CosIntegral[(a + b*ArcCos[c*x])/
b]*Sin[a/b]) - (3*CosIntegral[(3*(a + b*ArcCos[c*x]))/b]*Sin[(3*a)/b])/16
- (CosIntegral[(5*(a + b*ArcCos[c*x]))/b]*Sin[(5*a)/b])/16 + (Cos[a/b]*Sin
Integral[(a + b*ArcCos[c*x])/b])/8 + (3*Cos[(3*a)/b]*SinIntegral[(3*(a + b
*ArcCos[c*x]))/b])/16 + (Cos[(5*a)/b]*SinIntegral[(5*(a + b*ArcCos[c*x]))/
b])/16))/(b^2*c^6)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.67

method	result
default	$\frac{5 \arccos(cx) \operatorname{Si}\left(5 \arccos(cx)+\frac{5a}{b}\right) \cos\left(\frac{5a}{b}\right) b-5 \arccos(cx) \operatorname{Ci}\left(5 \arccos(cx)+\frac{5a}{b}\right) \sin\left(\frac{5a}{b}\right) b+15 \arccos(cx) \operatorname{Si}\left(3 \arccos(cx)+\frac{3a}{b}\right) \cos\left(\frac{3a}{b}\right.}{}$

input
```
int(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/16/c^6*(5*arccos(c*x)*Si(5*arccos(c*x)+5*a/b)*cos(5*a/b)*b-5*arccos(c*x)
*Ci(5*arccos(c*x)+5*a/b)*sin(5*a/b)*b+15*arccos(c*x)*Si(3*arccos(c*x)+3*a/
b)*cos(3*a/b)*b-15*arccos(c*x)*Ci(3*arccos(c*x)+3*a/b)*sin(3*a/b)*b+10*arc
cos(c*x)*Si(arccos(c*x)+a/b)*cos(a/b)*b-10*arccos(c*x)*Ci(arccos(c*x)+a/b)
*sin(a/b)*b+5*Si(5*arccos(c*x)+5*a/b)*cos(5*a/b)*a-5*Ci(5*arccos(c*x)+5*a/
b)*sin(5*a/b)*a+15*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)*a-15*Ci(3*arccos(c*x
)+3*a/b)*sin(3*a/b)*a+10*Si(arccos(c*x)+a/b)*cos(a/b)*a-10*Ci(arccos(c*x)+
a/b)*sin(a/b)*a+10*c*x*b+cos(5*arccos(c*x))*b+5*cos(3*arccos(c*x))*b)/(a+b
*arccos(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^5}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^5/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccos
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccos(c*x)), x)
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^5}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acos}{(cx)})^2}\,dx$$

input
```
integrate(x**5/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**5/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2), x)
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^5}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
(x^5 - 5*(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*integr
ate(x^4/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c), x))/(b
^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^5}{(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^5/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^5/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$$
$$=\int \frac{x^5}{\sqrt{-c^2x^2+1}\,acos\left(cx\right)^2b^2+2\sqrt{-c^2x^2+1}\,acos\left(cx\right)ab+\sqrt{-c^2x^2+1}\,a^2}dx$$

input
```
int(x^5/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(x**5/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 +
 1)*acos(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.411 $\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 141

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = -\frac{x^4}{bc(a+b\arccos(cx))} - \frac{\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\sin\left(\frac{2a}{b}\right)}{b^2c^5} + \frac{\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)\sin\left(\frac{4a}{b}\right)}{2b^2c^5} + \frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c^5} - \frac{\cos\left(\frac{4a}{b}\right)\text{Si}\left(\frac{4(a+b\arccos(cx))}{b}\right)}{2b^2c^5}$$

output
```
-x^4/b/c/(a+b*arccos(c*x))-Ci(2*(a+b*arccos(c*x))/b)*sin(2*a/b)/b^2/c^5+1/
2*Ci(4*(a+b*arccos(c*x))/b)*sin(4*a/b)/b^2/c^5+cos(2*a/b)*Si(2*(a+b*arccos
(c*x))/b)/b^2/c^5-1/2*cos(4*a/b)*Si(4*(a+b*arccos(c*x))/b)/b^2/c^5
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \frac{x^4}{bc(a+b\arccos(cx))} + \frac{-2\,\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{2a}{b}\right)-\text{CosIntegral}\left(4\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{4a}{b}\right)+2\cos\left(\frac{2a}{b}\right)\,\text{S}}{2b^2c^5}$$

input
```
Integrate[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
x^4/(b*c*(a + b*ArcCos[c*x])) + (-2*CosIntegral[2*(a/b + ArcCos[c*x])]*Sin
[(2*a)/b] - CosIntegral[4*(a/b + ArcCos[c*x])]*Sin[(4*a)/b] + 2*Cos[(2*a)/
b]*SinIntegral[2*(a/b + ArcCos[c*x])] + Cos[(4*a)/b]*SinIntegral[4*(a/b +
ArcCos[c*x])])/(2*b^2*c^5)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5223, 5147, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \text{5223}$$

$$\frac{x^4}{bc(a+b\arccos(cx))} - \frac{4\int \frac{x^3}{a+b\arccos(cx)}dx}{bc}$$

$$\Big\downarrow \text{5147}$$

$$\frac{4\int -\frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^5} + \frac{x^4}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^4}{bc(a+b\arccos(cx))}-\frac{4\int\frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^5}$$

$\Big\downarrow$ 4906

$$\frac{x^4}{bc(a+b\arccos(cx))}-\frac{4\int\left(\frac{\sin\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8(a+b\arccos(cx))}+\frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^5}$$

$\Big\downarrow$ 2009

$$\frac{4\left(-\frac{1}{4}\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\frac{1}{8}\sin\left(\frac{4a}{b}\right)\text{CosIntegral}\left(\frac{4(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos}{b}\right.\right.}{b^2c^5}$$

$$\frac{x^4}{bc(a+b\arccos(cx))}$$

input
```
Int[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
x^4/(b*c*(a + b*ArcCos[c*x])) + (4*(-1/4*(CosIntegral[(2*(a + b*ArcCos[c*x
]))/b]*Sin[(2*a)/b]) - (CosIntegral[(4*(a + b*ArcCos[c*x]))/b]*Sin[(4*a)/b
])/8 + (Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/4 + (Cos[(4*a
)/b]*SinIntegral[(4*(a + b*ArcCos[c*x]))/b])/8))/(b^2*c^5)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.77

method	result
default	$\frac{8 \arccos(cx) \operatorname{Si}\left(2 \arccos(cx)+\frac{2a}{b}\right) \cos\left(\frac{2a}{b}\right) b-8 \arccos(cx) \operatorname{Ci}\left(2 \arccos(cx)+\frac{2a}{b}\right) \sin\left(\frac{2a}{b}\right) b+4 \arccos(cx) \operatorname{Si}\left(4 \arccos(cx)+\frac{4a}{b}\right) \cos\left(\frac{4a}{b}\right.}{}$

input
```
int(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/c^5*(8*arccos(c*x)*Si(2*arccos(c*x)+2*a/b)*cos(2*a/b)*b-8*arccos(c*x)*
Ci(2*arccos(c*x)+2*a/b)*sin(2*a/b)*b+4*arccos(c*x)*Si(4*arccos(c*x)+4*a/b)
*cos(4*a/b)*b-4*arccos(c*x)*Ci(4*arccos(c*x)+4*a/b)*sin(4*a/b)*b+8*Si(2*ar
ccos(c*x)+2*a/b)*cos(2*a/b)*a-8*Ci(2*arccos(c*x)+2*a/b)*sin(2*a/b)*a+4*Si(
4*arccos(c*x)+4*a/b)*cos(4*a/b)*a-4*Ci(4*arccos(c*x)+4*a/b)*sin(4*a/b)*a+4
*cos(2*arccos(c*x))*b+cos(4*arccos(c*x))*b+3*b)/(a+b*arccos(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^4}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^4/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccos
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccos(c*x)), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^4}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\mathrm{acos}\,(cx))^2}\,dx$$

input
```
integrate(x**4/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**4/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^4}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
(x^4 - 4*(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*integr
ate(x^3/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c), x))/(b
^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(137) = 274$.

Time = 0.25 (sec) , antiderivative size = 811, normalized size of antiderivative = 5.75

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
b*c^4*x^4/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - 4*b*arccos(c*x)*cos(a/b)^3*c
os_integral(4*a/b + 4*arccos(c*x))*sin(a/b)/(b^3*c^5*arccos(c*x) + a*b^2*c
^5) + 4*b*arccos(c*x)*cos(a/b)^4*sin_integral(4*a/b + 4*arccos(c*x))/(b^3*
c^5*arccos(c*x) + a*b^2*c^5) - 4*a*cos(a/b)^3*cos_integral(4*a/b + 4*arcco
s(c*x))*sin(a/b)/(b^3*c^5*arccos(c*x) + a*b^2*c^5) + 4*a*cos(a/b)^4*sin_in
tegral(4*a/b + 4*arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) + 2*b*arcc
os(c*x)*cos(a/b)*cos_integral(4*a/b + 4*arccos(c*x))*sin(a/b)/(b^3*c^5*arc
cos(c*x) + a*b^2*c^5) - 2*b*arccos(c*x)*cos(a/b)*cos_integral(2*a/b + 2*ar
ccos(c*x))*sin(a/b)/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - 4*b*arccos(c*x)*co
s(a/b)^2*sin_integral(4*a/b + 4*arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2*
c^5) + 2*b*arccos(c*x)*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^3
*c^5*arccos(c*x) + a*b^2*c^5) + 2*a*cos(a/b)*cos_integral(4*a/b + 4*arccos
(c*x))*sin(a/b)/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - 2*a*cos(a/b)*cos_integ
ral(2*a/b + 2*arccos(c*x))*sin(a/b)/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - 4*
a*cos(a/b)^2*sin_integral(4*a/b + 4*arccos(c*x))/(b^3*c^5*arccos(c*x) + a*
b^2*c^5) + 2*a*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^3*c^5*arc
cos(c*x) + a*b^2*c^5) + 1/2*b*arccos(c*x)*sin_integral(4*a/b + 4*arccos(c*
x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - b*arccos(c*x)*sin_integral(2*a/b +
 2*arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) + 1/2*a*sin_integral(4*a
/b + 4*arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - a*sin_integral(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^4}{(a+b\,\mathrm{acos}\,(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^4/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^4/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^4}{\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2+2\sqrt{-c^2x^2+1}\,acos\,(cx)\,ab+\sqrt{-c^2x^2+1}\,a^2}dx$$

input
```
int(x^4/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(x**4/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 +
 1)*acos(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.412 $\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 142

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = -\frac{x^3}{bc(a+b\arccos(cx))} + \frac{3\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^4} - \frac{3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^4} + \frac{3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^4} - \frac{3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^4}$$

output
```
-x^3/b/c/(a+b*arccos(c*x))+3/4*cos(a/b)*Ci((a+b*arccos(c*x))/b)/b^2/c^4-3/
4*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b^2/c^4+3/4*sin(a/b)*Si((a+b*arccos
(c*x))/b)/b^2/c^4-3/4*sin(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^2/c^4
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \frac{x^3}{bc(a+b\arccos(cx))} + \frac{3\left(-\operatorname{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)-\operatorname{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{3a}{b}\right)+\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\right.\right.}{4b^2c^4}$$

input
```
Integrate[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
x^3/(b*c*(a + b*ArcCos[c*x])) + (3*(-(CosIntegral[a/b + ArcCos[c*x]]*Sin[a
/b]) - CosIntegral[3*(a/b + ArcCos[c*x])]*Sin[(3*a)/b] + Cos[a/b]*SinInteg
ral[a/b + ArcCos[c*x]] + Cos[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])]))
/(4*b^2*c^4)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5223, 5147, 25, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{ 5223}$$

$$\frac{x^3}{bc(a+b\arccos(cx))}-\frac{3\int \frac{x^2}{a+b\arccos(cx)}dx}{bc}$$

$$\Big\downarrow \text{ 5147}$$

$$\frac{3\int -\frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^4}+\frac{x^3}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{x^3}{bc(a+b\arccos(cx))}-\frac{3\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^4}$$

$$\Big\downarrow \; 4906$$

$$\frac{x^3}{bc(a+b\arccos(cx))}-\frac{3\int \left(\frac{\sin\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4(a+b\arccos(cx))}+\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4(a+b\arccos(cx))}\right)d(a+b\arccos(cx))}{b^2c^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{3\left(-\frac{1}{4}\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)-\frac{1}{4}\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)+\frac{1}{4}\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)\right.}{b^2c^4}$$

$$\frac{x^3}{bc(a+b\arccos(cx))}$$

input
```
Int[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
x^3/(b*c*(a + b*ArcCos[c*x])) + (3*(-1/4*(CosIntegral[(a + b*ArcCos[c*x])/
b]*Sin[a/b]) - (CosIntegral[(3*(a + b*ArcCos[c*x]))/b]*Sin[(3*a)/b])/4 + (
Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/4 + (Cos[(3*a)/b]*SinIntegral
[(3*(a + b*ArcCos[c*x]))/b])/4))/(b^2*c^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.59

method	result
default	$\frac{3 \arccos(cx) \operatorname{Si}\left(3 \arccos(cx)+\frac{3a}{b}\right) \cos\left(\frac{3a}{b}\right) b-3 \arccos(cx) \operatorname{Ci}\left(3 \arccos(cx)+\frac{3a}{b}\right) \sin\left(\frac{3a}{b}\right) b+3 \arccos(cx) \operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right) \cos\left(\frac{a}{b}\right) b-}{}$

input
```
int(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/c^4*(3*arccos(c*x)*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)*b-3*arccos(c*x)*
Ci(3*arccos(c*x)+3*a/b)*sin(3*a/b)*b+3*arccos(c*x)*Si(arccos(c*x)+a/b)*cos
(a/b)*b-3*arccos(c*x)*Ci(arccos(c*x)+a/b)*sin(a/b)*b+3*Si(3*arccos(c*x)+3*
a/b)*cos(3*a/b)*a-3*Ci(3*arccos(c*x)+3*a/b)*sin(3*a/b)*a+3*Si(arccos(c*x)+
a/b)*cos(a/b)*a-3*Ci(arccos(c*x)+a/b)*sin(a/b)*a+3*c*x*b+cos(3*arccos(c*x)
)*b)/(a+b*arccos(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^3}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^3/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccos
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccos(c*x)), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^3}{\sqrt{-(cx-1)(cx+1)}(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate(x**3/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**3/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^3}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
(x^3 - 3*(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*integr
ate(x^2/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c), x))/(b
^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^3}{(a+b\,\mathrm{acos}\,(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^3/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^3/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$$
$$= \int \frac{x^3}{\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2+2\sqrt{-c^2x^2+1}\,acos\,(cx)\,ab+\sqrt{-c^2x^2+1}\,a^2}dx$$

input
```
int(x^3/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(x**3/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 +
 1)*acos(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.413 $\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 79

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = -\frac{x^2}{bc(a+b\arccos(cx))} - \frac{\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\sin\left(\frac{2a}{b}\right)}{b^2c^3} + \frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c^3}$$

output
```
-x^2/b/c/(a+b*arccos(c*x))-Ci(2*(a+b*arccos(c*x))/b)*sin(2*a/b)/b^2/c^3+co
s(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b^2/c^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \frac{\frac{bc^2x^2}{a+b\arccos(cx)} - \text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{2a}{b}\right) + \cos\left(\frac{2a}{b}\right)\text{Si}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)}{b^2c^3}$$

input
```
Integrate[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
((b*c^2*x^2)/(a + b*ArcCos[c*x]) - CosIntegral[2*(a/b + ArcCos[c*x])]*Sin[
(2*a)/b] + Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])])/(b^2*c^3)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {5223, 5147, 25, 4906, 27, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5223}$$

$$\frac{x^2}{bc(a+b\arccos(cx))}-\frac{2\int \frac{x}{a+b\arccos(cx)}dx}{bc}$$

$$\Big\downarrow \text{5147}$$

$$\frac{2\int -\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^2}{bc(a+b\arccos(cx))}-\frac{2\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{x^2}{bc(a+b\arccos(cx))}-\frac{2\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2(a+b\arccos(cx))}d(a+b\arccos(cx))}{b^2c^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2}{bc(a+b\arccos(cx))}-\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{x^2}{bc(a+b\arccos(cx))}-\frac{\int \frac{\sin\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}$$

$$\Big\downarrow \; 3784$$

$$\frac{-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{2a}{b}\right)\int -\frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 25$$

$$\frac{\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 3780$$

$$\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2(a+b\arccos(cx))}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^3}+\frac{x^2}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 3783$$

$$\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)-\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c^3}+\frac{x^2}{bc(a+b\arccos(cx))}$$

input
```
Int[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
x^2/(b*c*(a + b*ArcCos[c*x])) + (-(CosIntegral[(2*(a + b*ArcCos[c*x]))/b]*
Sin[(2*a)/b]) + Cos[(2*a)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/(b^2*
c^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.70

method	result
default	$\frac{2\arccos(cx)\,\mathrm{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)b-2\arccos(cx)\,\mathrm{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)b+2\,\mathrm{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)a-2\,\mathrm{Ci}(2}{2c^3(a+b\arccos(cx))b^2}$

input
```
int(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/c^3*(2*arccos(c*x)*Si(2*arccos(c*x)+2*a/b)*cos(2*a/b)*b-2*arccos(c*x)*
Ci(2*arccos(c*x)+2*a/b)*sin(2*a/b)*b+2*Si(2*arccos(c*x)+2*a/b)*cos(2*a/b)*
a-2*Ci(2*arccos(c*x)+2*a/b)*sin(2*a/b)*a+cos(2*arccos(c*x))*b+b)/(a+b*arcc
os(c*x))/b^2
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccos
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccos(c*x)), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{\sqrt{-(cx-1)(cx+1)}(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate(x**2/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**2/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
(x^2 - 2*(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*integr
ate(x/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c), x))/(b^2
*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(79) = 158$.

Time = 0.23 (sec) , antiderivative size = 317, normalized size of antiderivative = 4.01

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \frac{bc^2x^2}{b^3c^3\arccos(cx)+ab^2c^3} - \frac{2\,b\arccos(cx)\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{b^3c^3\arccos(cx)+ab^2c^3} + \frac{2\,b\arccos(cx)\cos\left(\frac{a}{b}\right)^2\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)}{b^3c^3\arccos(cx)+ab^2c^3} - \frac{2\,a\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{b^3c^3\arccos(cx)+ab^2c^3} + \frac{2\,a\cos\left(\frac{a}{b}\right)^2\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)}{b^3c^3\arccos(cx)+ab^2c^3} - \frac{b\arccos(cx)\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)}{b^3c^3\arccos(cx)+ab^2c^3} - \frac{a\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arccos(cx)\right)}{b^3c^3\arccos(cx)+ab^2c^3}$$

input
```
integrate(x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
b*c^2*x^2/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 2*b*arccos(c*x)*cos(a/b)*cos
_integral(2*a/b + 2*arccos(c*x))*sin(a/b)/(b^3*c^3*arccos(c*x) + a*b^2*c^3
) + 2*b*arccos(c*x)*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^3*c^
3*arccos(c*x) + a*b^2*c^3) - 2*a*cos(a/b)*cos_integral(2*a/b + 2*arccos(c*
x))*sin(a/b)/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 2*a*cos(a/b)^2*sin_integr
al(2*a/b + 2*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - b*arccos(c*x
)*sin_integral(2*a/b + 2*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) -
a*sin_integral(2*a/b + 2*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x^2/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x^2/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$$
$$=\int \frac{x^2}{\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2+2\sqrt{-c^2x^2+1}\,acos\,(cx)\,ab+\sqrt{-c^2x^2+1}\,a^2}dx$$

input
```
int(x^2/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(x**2/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 +
 1)*acos(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.414 $\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 72

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = -\frac{x}{bc(a+b\arccos(cx))} + \frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c^2} + \frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c^2}$$

output
```
-x/b/c/(a+b*arccos(c*x))+cos(a/b)*Ci((a+b*arccos(c*x))/b)/b^2/c^2+sin(a/b)
*Si((a+b*arccos(c*x))/b)/b^2/c^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \frac{\frac{bcx}{a+b\arccos(cx)} - \text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right) + \cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^2c^2}$$

input
```
Integrate[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
((b*c*x)/(a + b*ArcCos[c*x]) - CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b] + C
os[a/b]*SinIntegral[a/b + ArcCos[c*x]])/(b^2*c^2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5223, 5135, 25, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5223}$$

$$\frac{x}{bc(a+b\arccos(cx))}-\frac{\int \frac{1}{a+b\arccos(cx)}dx}{bc}$$

$$\Big\downarrow \text{5135}$$

$$\frac{\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{x}{bc(a+b\arccos(cx))}-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{bc(a+b\arccos(cx))}-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}$$

$$\Big\downarrow \text{3784}$$

$$\frac{-\sin\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{a}{b}\right)\int-\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 25$$

$$\frac{\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 3780$$

$$\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \; 3783$$

$$\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c^2}+\frac{x}{bc(a+b\arccos(cx))}$$

input
```
Int[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
x/(b*c*(a + b*ArcCos[c*x])) + (-(CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/
b]) + Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/(b^2*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.51

method	result
default	$\frac{\arccos(cx)\,\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)b-\arccos(cx)\,\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)b+\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)a-\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin}{c^2(a+b\arccos(cx))b^2}$

input
```
int(x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(arccos(c*x)*Si(arccos(c*x)+a/b)*cos(a/b)*b-arccos(c*x)*Ci(arccos(c*
x)+a/b)*sin(a/b)*b+Si(arccos(c*x)+a/b)*cos(a/b)*a-Ci(arccos(c*x)+a/b)*sin(
a/b)*a+c*x*b)/(a+b*arccos(c*x))/b^2
```

Fricas [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx=\int \frac{x}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccos(c
*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccos(c*x)), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx=\int \frac{x}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\mathrm{acos}(cx))^2}\,dx$$

input
```
integrate(x/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-((b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*integrate(1/(
b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c), x) - x)/(b^2*c*
arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(72) = 144$.

Time = 0.22 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.79

$$\begin{aligned}\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = & -\frac{b\arccos(cx)\,\mathrm{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{b^3c^2\arccos(cx)+ab^2c^2} \\ & + \frac{b\arccos(cx)\cos\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c^2\arccos(cx)+ab^2c^2} \\ & + \frac{bcx}{b^3c^2\arccos(cx)+ab^2c^2} \\ & - \frac{a\,\mathrm{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{b^3c^2\arccos(cx)+ab^2c^2} \\ & + \frac{a\cos\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c^2\arccos(cx)+ab^2c^2}\end{aligned}$$

input
```
integrate(x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-b*arccos(c*x)*cos_integral(a/b + arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*
x) + a*b^2*c^2) + b*arccos(c*x)*cos(a/b)*sin_integral(a/b + arccos(c*x))/(
b^3*c^2*arccos(c*x) + a*b^2*c^2) + b*c*x/(b^3*c^2*arccos(c*x) + a*b^2*c^2)
 - a*cos_integral(a/b + arccos(c*x))*sin(a/b)/(b^3*c^2*arccos(c*x) + a*b^2
*c^2) + a*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c^2*arccos(c*x) +
a*b^2*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{x}{(a+b\,\mathrm{acos}\,(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(x/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(x/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$$
$$= \int \frac{x}{\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2+2\sqrt{-c^2x^2+1}\,acos\,(cx)\,ab+\sqrt{-c^2x^2+1}\,a^2}dx$$

input
```
int(x/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(x/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)
*acos(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.415 $\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 18

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = -\frac{1}{bc(a+b\arccos(cx))}$$

output
```
-1/b/c/(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \frac{1}{bc(a+b\arccos(cx))}$$

input
```
Integrate[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
1/(b*c*(a + b*ArcCos[c*x]))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$$
$$\Big\downarrow \text{5153}$$
$$\frac{1}{bc(a+b\arccos(cx))}$$

input
```
Int[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
1/(b*c*(a + b*ArcCos[c*x]))
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{1}{bc(a+b\arccos(cx))}$	18
default	$\frac{1}{bc(a+b\arccos(cx))}$	18

input
```
int(1/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/b/c/(a+b*arccos(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \frac{1}{b^2c\arccos{(cx)}+abc}$$

input
```
integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
1/(b^2*c*arccos(c*x) + a*b*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.11

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \begin{cases} \frac{x}{a^2} & \text{for}\: b=0 \wedge c=0 \\ \frac{\begin{cases} -\frac{i\operatorname{acosh}{(cx)}}{c} & \text{for}\: |c^2x^2|>1 \\ \frac{\operatorname{asin}{(cx)}}{c} & \text{otherwise} \end{cases}}{a^2} & \text{for}\: b=0 \\ \frac{x}{\left(a+\frac{\pi b}{2}\right)^2} & \text{for}\: c=0 \\ \frac{1}{abc+b^2c\operatorname{acos}{(cx)}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((x/a**2, Eq(b, 0) & Eq(c, 0)), (Piecewise((-I*acosh(c*x)/c, Abs(
c**2*x**2) > 1), (asin(c*x)/c, True))/a**2, Eq(b, 0)), (x/(a + pi*b/2)**2,
 Eq(c, 0)), (1/(a*b*c + b**2*c*acos(c*x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \frac{1}{(b\arccos(cx)+a)bc}$$

input
```
integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/((b*arccos(c*x) + a)*b*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \frac{1}{(b\arccos(cx)+a)bc}$$

input
```
integrate(1/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
1/((b*arccos(c*x) + a)*b*c)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \frac{1}{b\,c\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}$$

input
```
int(1/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
1/(b*c*(a + b*acos(c*x)))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = -\frac{acos(cx)}{ac\left(acos\left(cx\right)b+a\right)}$$

input
```
int(1/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
( - acos(c*x))/(a*c*(acos(c*x)*b + a))
```

3.416 $\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = -\frac{1}{bcx(a+b\arccos(cx))} - \frac{\text{Int}\left(\frac{1}{x^2(a+b\arccos(cx))},x\right)}{bc}$$

output
```
-1/b/c/x/(a+b*arccos(c*x))-Defer(Int)(1/x^2/(a+b*arccos(c*x)),x)/b/c
```

Mathematica [N/A]

Not integrable

Time = 7.77 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{ 5223}$$

$$\frac{\int \frac{1}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{1}{bcx(a+b\arccos(cx))}$$

$$\Big\downarrow \text{ 5149}$$

$$\frac{\int \frac{1}{x^2(a+b\arccos(cx))}dx}{bc}+\frac{1}{bcx(a+b\arccos(cx))}$$

input
```
Int[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\sqrt{-c^2x^2+1}\,(a+b\arccos(cx))^2}dx$$

input
```
int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)^2x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^2*x^3 - a^2*x + (b^2*c^2*x^3 - b^2*x)*
arccos(c*x)^2 + 2*(a*b*c^2*x^3 - a*b*x)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x\sqrt{-(cx-1)(cx+1)}(a+b\operatorname{acos}{(cx)})^2}\,dx$$

input
```
integrate(1/x/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/(x*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.93

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)^2x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
((b^2*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x)*integrate(
1/(b^2*c*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^2), x) +
 1)/(b^2*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x\,(a+b\,\mathrm{acos}\,(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(1/(x*(a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(1/(x*(a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$$
$$= \int \frac{1}{\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2x + 2\sqrt{-c^2x^2+1}\,acos\,(cx)\,abx + \sqrt{-c^2x^2+1}\,a^2x}dx$$

input
```
int(1/x/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*x + 2*sqrt( - c**2*x**2 +
1)*acos(c*x)*a*b*x + sqrt( - c**2*x**2 + 1)*a**2*x),x)
```

3.417 $\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = -\frac{1}{bcx^2(a+b\arccos(cx))}-\frac{2\text{Int}\left(\frac{1}{x^3(a+b\arccos(cx))},x\right)}{bc}$$

output
```
-1/b/c/x^2/(a+b*arccos(c*x))-2*Defer(Int)(1/x^3/(a+b*arccos(c*x)),x)/b/c
```

Mathematica [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}dx$$
$$\Big\downarrow \ 5223$$
$$\frac{2\int \frac{1}{x^3(a+b\arccos(cx))}dx}{bc}+\frac{1}{bcx^2(a+b\arccos(cx))}$$
$$\Big\downarrow \ 5149$$
$$\frac{2\int \frac{1}{x^3(a+b\arccos(cx))}dx}{bc}+\frac{1}{bcx^2(a+b\arccos(cx))}$$

input
```
Int[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\sqrt{-c^2x^2+1}\,\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.07

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)^2x^2}\,dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="frica
s")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^2*x^4 - a^2*x^2 + (b^2*c^2*x^4 - b^2*x
^2)*arccos(c*x)^2 + 2*(a*b*c^2*x^4 - a*b*x^2)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x^2\sqrt{-(cx-1)(cx+1)}(a+b\,\mathrm{acos}(cx))^2}\,dx$$

input
```
integrate(1/x**2/(-c**2*x**2+1)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/(x**2*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 119, normalized size of antiderivative = 4.25

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)^2x^2}\,dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxim
a")
```

output
```
(2*(b^2*c*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^2)*inte
grate(1/(b^2*c*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^3)
, x) + 1)/(b^2*c*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^
2)
```

Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{-c^2x^2+1}(b\arccos{(cx)}+a)^2x^2}\,dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac"
)
```

output
```
integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x^2\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(1/(x^2*(a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*acos(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx$$
$$=\int \frac{1}{\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2x^2+2\sqrt{-c^2x^2+1}\,acos\,(cx)\,ab\,x^2+\sqrt{-c^2x^2+1}\,a^2x^2}dx$$

input
```
int(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*x**2 + 2*sqrt( - c**2*x**2
 + 1)*acos(c*x)*a*b*x**2 + sqrt( - c**2*x**2 + 1)*a**2*x**2),x)
```

3.418 $\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{x^m}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx$$
$$\Big\downarrow \text{5235}$$
$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}dx$$

input
```
Int[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.79

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^m/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^
4 - 2*b^2*c^2*x^2 + b^2)*arccos(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*
x^2 + a*b)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 72.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^m}{(-(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate(x**m/(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**m/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 7.75

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{x^m}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
((a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-
c*x + 1), c*x))*integrate(((c^2*m - 2*c^2)*x^2 - m)*x^m/(a*b*c^5*x^5 - 2*a
*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*arctan2(sqr
t(c*x + 1)*sqrt(-c*x + 1), c*x)), x) - x^m)/(a*b*c^3*x^2 - a*b*c + (b^2*c^
3*x^2 - b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{x^m}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x^m/((-c^2*x^2 + 1)^(3/2)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx=\int \frac{x^m}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

input
```
int(x^m/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(x^m/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 5.04

$$\int \frac{x^m}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx =$$
$$-\left(\int \frac{x^m}{\sqrt{-c^2x^2+1}\,acos\left(cx\right)^2 b^2c^2x^2-\sqrt{-c^2x^2+1}\,acos\left(cx\right)^2 b^2+2\sqrt{-c^2x^2+1}\,acos\left(cx\right)ab\,c^2x^2-2\sqrt{-c}\right.$$

input
```
int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
 - int(x**m/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**2 - sqrt( -
c**2*x**2 + 1)*acos(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*
c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b + sqrt( - c**2*x**2 + 1
)*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.419 $\int \frac{x^3}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx=\mathrm{Int}\left(\frac{x^3}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 60.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx=\int \frac{x^3}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx$$
$$\Big\downarrow \text{ 5235}$$
$$\int \frac{x^3}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}dx$$

input
```
Int[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(-c^2x^2+1)^{\frac{3}{2}}(a+b\arccos(cx))^2}dx$$

input
```
int(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.79

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{x^3}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^3/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^
4 - 2*b^2*c^2*x^2 + b^2)*arccos(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*
x^2 + a*b)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{x^3}{(-(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate(x**3/(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**3/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 206, normalized size of antiderivative = 7.36

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x^3}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(x^3 - (a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x))*integrate((c^2*x^4 - 3*x^2)/(a*b*c^5*x^4 - 2*a*b*c^
3*x^2 + a*b*c + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*arctan2(sqrt(c*x + 1
)*sqrt(-c*x + 1), c*x)), x))/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*
arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x^3}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

input
```
int(x^3/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(x^3/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 141, normalized size of antiderivative = 5.04

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx =$$
$$-\left(\int \frac{x^3}{\sqrt{-c^2x^2+1}\,acos(cx)^2\,b^2c^2x^2-\sqrt{-c^2x^2+1}\,acos(cx)^2\,b^2+2\sqrt{-c^2x^2+1}\,acos(cx)\,ab\,c^2x^2-2\sqrt{-c}$$

input
```
int(x^3/(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
 - int(x**3/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**2 - sqrt( -
c**2*x**2 + 1)*acos(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*
c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b + sqrt( - c**2*x**2 + 1
)*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.420 $\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx = -\frac{x^2}{bc\,(1-c^2x^2)\,(a+b\arccos(cx))}+\frac{2\text{Int}\left(\frac{x}{(1-c^2x^2)^2(a+b\arccos(cx))},x\right)}{bc}$$

output
```
-x^2/b/c/(-c^2*x^2+1)/(a+b*arccos(c*x))+2*Defer(Int)(x/(-c^2*x^2+1)^2/(a+b
*arccos(c*x)),x)/b/c
```

Mathematica [N/A]

Not integrable

Time = 6.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5213}$$

$$\frac{x^2}{bc(1-c^2x^2)(a+b\arccos(cx))}-\frac{2\int \frac{x}{(1-c^2x^2)^2(a+b\arccos(cx))}dx}{bc}$$

$$\Big\downarrow \text{5235}$$

$$\frac{x^2}{bc(1-c^2x^2)(a+b\arccos(cx))}-\frac{2\int \frac{x}{(1-c^2x^2)^2(a+b\arccos(cx))}dx}{bc}$$

input
```
Int[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{\left(-c^2x^2+1\right)^{\frac{3}{2}} \left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.79

$$\int \frac{x^2}{(1-c^2x^2)^{3/2} (a+b\arccos(cx))^2} \, dx = \int \frac{x^2}{\left(-c^2x^2+1\right)^{\frac{3}{2}} \left(b\arccos\left(cx\right)+a\right)^2} \, dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^
4 - 2*b^2*c^2*x^2 + b^2)*arccos(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*
x^2 + a*b)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acos}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(x**2/(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**2/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 194, normalized size of antiderivative = 6.93

$$\int \frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\arccos\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-(x^2 + 2*(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(sqrt(c*x +
1)*sqrt(-c*x + 1), c*x))*integrate(x/(a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c
+ (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x)), x))/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(sqrt(c
*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2} (a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\arccos\left(cx\right)+a\right)^2}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2} (a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2\,\left(1-c^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(x^2/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(x^2/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 307, normalized size of antiderivative = 10.96

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2} (a+b\arccos(cx))^2}\,dx = \frac{-acos(cx)\left(\int \frac{}{\sqrt{-c^2x^2+1}\,acos(cx)^2b^2c^2x^2-\sqrt{-c^2x^2+1}\,acos(cx)^2b^2+2\sqrt{-c^2x^2+1}\,\cdots}\right.}{}$$

input
```
int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x))^2,x)
```

output

```
( - acos(c*x)*int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**2 -
sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c
*x)*a*b*c**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b + sqrt( - c**2*
x**2 + 1)*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a**2),x)*a*b*c + acos(c*
x) - int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**2 - sqrt( - c
**2*x**2 + 1)*acos(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*c
**2*x**2 - 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b + sqrt( - c**2*x**2 + 1)
*a**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a**2),x)*a**2*c)/(a*c**3*(acos(c*
x)*b + a))
```

3.421 $\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx=\operatorname{Int}\left(\frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 46.89 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx$$

$$\Big\downarrow \text{5235}$$

$$\int \frac{x}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}dx$$

input
```
Int[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x}{(-c^2x^2+1)^{\frac{3}{2}}\,(a+b\arccos(cx))^2}dx$$

input
```
int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.00

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{x}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)*x/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4
- 2*b^2*c^2*x^2 + b^2)*arccos(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x^
2 + a*b)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{x}{(-(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate(x/(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 199, normalized size of antiderivative = 7.65

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{x}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-((a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(
-c*x + 1), c*x))*integrate((c^2*x^2 + 1)/(a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*
b*c + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*
x + 1), c*x)), x) + x)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan
2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{x}{(a+b\,\mathrm{acos}\,(c\,x))^2\,(1-c^2\,x^2)^{3/2}}\,dx$$

input
```
int(x/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(x/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 5.35

$$\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx =$$

$$-\left(\int \frac{x}{\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2c^2x^2-\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2+2\sqrt{-c^2x^2+1}\,acos\,(cx)\,ab\,c^2x^2-2\sqrt{-c}\right.$$

input
```
int(x/(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
 - int(x/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**2 - sqrt( - c**
2*x**2 + 1)*acos(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*c**
2*x**2 - 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a
**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.422 $\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx = -\frac{1}{bc(1-c^2x^2)(a+b\arccos(cx))}+\frac{2c\text{Int}\left(\frac{x}{(1-c^2x^2)^2(a+b\arccos(cx))},x\right)}{b}$$

output
```
-1/b/c/(-c^2*x^2+1)/(a+b*arccos(c*x))+2*c*Defer(Int)(x/(-c^2*x^2+1)^2/(a+b
*arccos(c*x)),x)/b
```

Mathematica [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{ 5167}$$

$$\frac{1}{bc(1-c^2x^2)(a+b\arccos(cx))}-\frac{2c\int \frac{x}{(1-c^2x^2)^2(a+b\arccos(cx))}dx}{b}$$

$$\Big\downarrow \text{ 5235}$$

$$\frac{1}{bc(1-c^2x^2)(a+b\arccos(cx))}-\frac{2c\int \frac{x}{(1-c^2x^2)^2(a+b\arccos(cx))}dx}{b}$$

input
```
Int[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right)^{\frac{3}{2}} \left(a +b \arccos \left(c x \right)\right)^{2}}d x$$

input
```
int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.12

$$\int \frac{1}{(1-c^2x^2)^{3/2} (a+b\arccos(cx))^2} \, dx=\int { \frac {1}{{\left (-c^{2} x^{2} + 1\right)}^{\frac {3}{2}} {\left (b \arccos \left (c x\right) + a\right)}^{2}} \,d x }$$

input
```
integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4 -
2*b^2*c^2*x^2 + b^2)*arccos(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x^2
+ a*b)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.95 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1-c^2x^2)^{3/2} (a+b\arccos(cx))^2} \, dx=\int \frac{1}{\left(- \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{acos}{\left(c x \right)}\right)^{2}}\, dx$$

input
```
integrate(1/(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 193, normalized size of antiderivative = 7.72

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-(2*(a*b*c^4*x^2 - a*b*c^2 + (b^2*c^4*x^2 - b^2*c^2)*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x))*integrate(x/(a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b + (b
^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*
x)), x) + 1)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(sqrt(c*x
 + 1)*sqrt(-c*x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx=\int \frac{1}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.48

$$\int \frac{1}{(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}\,dx=$$
$$-\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acos(cx)^2 b^2c^2x^2-\sqrt{-c^2x^2+1}\,acos(cx)^2 b^2+2\sqrt{-c^2x^2+1}\,acos(cx)\,ab\,c^2x^2-2\sqrt{-c$$

input
```
int(1/(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
 - int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**2 - sqrt( - c**
2*x**2 + 1)*acos(c*x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*c**
2*x**2 - 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b + sqrt( - c**2*x**2 + 1)*a
**2*c**2*x**2 - sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.423 $\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{1}{x(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 46.97 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}dx$$
$$\Big\downarrow \text{5235}$$
$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}dx$$

input
```
Int[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.86

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\arccos(cx)\right)^2}\,dx=\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\arccos\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^5 - 2*a^2*c^2*x^3 + a^2*x + (b^2*c^
4*x^5 - 2*b^2*c^2*x^3 + b^2*x)*arccos(c*x)^2 + 2*(a*b*c^4*x^5 - 2*a*b*c^2*
x^3 + a*b*x)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 6.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\arccos(cx)\right)^2}\,dx=\int \frac{1}{x\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acos}\left(cx\right)\right)^2}\,dx$$

input
```
integrate(1/x/(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/(x*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 210, normalized size of antiderivative = 7.50

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)^2x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((a*b*c^3*x^3 - a*b*c*x + (b^2*c^3*x^3 - b^2*c*x)*arctan2(sqrt(c*x + 1)*s
qrt(-c*x + 1), c*x))*integrate((3*c^2*x^2 - 1)/(a*b*c^5*x^6 - 2*a*b*c^3*x^
4 + a*b*c*x^2 + (b^2*c^5*x^6 - 2*b^2*c^3*x^4 + b^2*c*x^2)*arctan2(sqrt(c*x
 + 1)*sqrt(-c*x + 1), c*x)), x) + 1)/(a*b*c^3*x^3 - a*b*c*x + (b^2*c^3*x^3
 - b^2*c*x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{1}{x\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2\,\left(1-c^2\,x^2\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a + b*acos(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(1/(x*(a + b*acos(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 5.00

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2}\,dx=$$
$$-\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acos\left(cx\right)^2 b^2c^2x^3-\sqrt{-c^2x^2+1}\,acos\left(cx\right)^2 b^2x+2\sqrt{-c^2x^2+1}\,acos\left(cx\right)ab\,c^2x^3-2\sqrt{-\right.$$

input
```
int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
 - int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**3 - sqrt( - c**
2*x**2 + 1)*acos(c*x)**2*b**2*x + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*c
**2*x**3 - 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*x + sqrt( - c**2*x**2 +
1)*a**2*c**2*x**3 - sqrt( - c**2*x**2 + 1)*a**2*x),x)
```

3.424 $\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2}\,dx=\text{Int}\left(\frac{1}{x^2\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 35.54 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))^2} dx$$
$$\Big\downarrow \; 5235$$
$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))^2} dx$$

input
```
Int[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 \left(-c^2x^2+1\right)^{\frac{3}{2}} \left(a+b\arccos\left(cx\right)\right)^2} dx$$

input
```
int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.07

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))^2}\,dx = \int \frac{1}{(-c^2x^2+1)^{\frac{3}{2}}(b\arccos(cx)+a)^2x^2}\,dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="frica
s")
```

output
```
integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^6 - 2*a^2*c^2*x^4 + a^2*x^2 + (b^2*
c^4*x^6 - 2*b^2*c^2*x^4 + b^2*x^2)*arccos(c*x)^2 + 2*(a*b*c^4*x^6 - 2*a*b*
c^2*x^4 + a*b*x^2)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 5.52 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))^2}\,dx = \int \frac{1}{x^2\left(-(cx-1)(cx+1)\right)^{\frac{3}{2}}(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate(1/x**2/(-c**2*x**2+1)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 219, normalized size of antiderivative = 7.82

$$\int \frac{1}{x^2 (1 - c^2x^2)^{3/2} (a + b \arccos(cx))^2} dx = \int \frac{1}{(-c^2x^2 + 1)^{\frac{3}{2}} (b \arccos(cx) + a)^2 x^2} dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxim
a")
```

output
```
-((a*b*c^3*x^4 - a*b*c*x^2 + (b^2*c^3*x^4 - b^2*c*x^2)*arctan2(sqrt(c*x +
1)*sqrt(-c*x + 1), c*x))*integrate(2*(2*c^2*x^2 - 1)/(a*b*c^5*x^7 - 2*a*b*
c^3*x^5 + a*b*c*x^3 + (b^2*c^5*x^7 - 2*b^2*c^3*x^5 + b^2*c*x^3)*arctan2(sq
rt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) + 1)/(a*b*c^3*x^4 - a*b*c*x^2 + (b^2
*c^3*x^4 - b^2*c*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 18.50 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1 - c^2x^2)^{3/2} (a + b \arccos(cx))^2} dx = \int \frac{1}{(-c^2x^2 + 1)^{\frac{3}{2}} (b \arccos(cx) + a)^2 x^2} dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac"
)
```

output
```
integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccos(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))^2} dx = \int \frac{1}{x^2 (a+b\,\mathrm{acos}\,(c\,x))^2 (1-c^2\,x^2)^{3/2}} dx$$

input
```
int(1/(x^2*(a + b*acos(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*acos(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 5.21

$$\int \frac{1}{x^2 (1-c^2x^2)^{3/2} (a+b\arccos(cx))^2} dx =$$
$$-\left(\int \frac{1}{\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2c^2x^4-\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2x^2+2\sqrt{-c^2x^2+1}\,acos\,(cx)\,ab\,c^2x^4-2\sqrt{-}$$

input
```
int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
- int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**4 - sqrt( - c**
2*x**2 + 1)*acos(c*x)**2*b**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*
b*c**2*x**4 - 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*x**2 + sqrt( - c**2*x
**2 + 1)*a**2*c**2*x**4 - sqrt( - c**2*x**2 + 1)*a**2*x**2),x)
```

3.425 $\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx$$
$$\Big\downarrow \; 5235$$
$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}dx$$

input
```
Int[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(-c^2x^2+1)^{\frac{5}{2}}(a+b\arccos(cx))^2}dx$$

input
```
int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 5.14

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^m/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*
x^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccos(c*x)^2 -
a^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccos(c*x)),
x)
```

Sympy [N/A]

Not integrable

Time = 73.79 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^m}{(-(cx-1)(cx+1))^{\frac{5}{2}}(a+b\,\mathrm{acos}(cx))^2}\,dx$$

input
```
integrate(x**m/(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**m/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 279, normalized size of antiderivative = 9.96

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
-((a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^
2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*integrate(((c^2*m - 4*c^2
)*x^2 - m)*x^m/(a*b*c^7*x^7 - 3*a*b*c^5*x^5 + 3*a*b*c^3*x^3 - a*b*c*x + (b
^2*c^7*x^7 - 3*b^2*c^5*x^5 + 3*b^2*c^3*x^3 - b^2*c*x)*arctan2(sqrt(c*x + 1
)*sqrt(-c*x + 1), c*x)), x) - x^m)/(a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c +
(b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1)
, c*x))
```

Giac [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x^m}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x^m/((-c^2*x^2 + 1)^(5/2)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x^m}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2\,{\left(1-c^2\,x^2\right)}^{5/2}}\,dx$$

input
```
int(x^m/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

output
```
int(x^m/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 212, normalized size of antiderivative = 7.57

$$\int \frac{x^m}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{}{\sqrt{-c^2x^2+1}\,acos\left(cx\right)^2 b^2c^4x^4 - 2\sqrt{-c^2x^2+1}\,acos\left(cx\right)^2 b^2c^2x^2}$$

input
```
int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(x**m/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**4*x**4 - 2*sqrt( - c
**2*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**2 + sqrt( - c**2*x**2 + 1)*acos(c*
x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*c**4*x**4 - 4*sqrt( -
c**2*x**2 + 1)*acos(c*x)*a*b*c**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x
)*a*b + sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*a
**2*c**2*x**2 + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.426 $\int \frac{x^3}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^3}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{x^3}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 100.90 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x^3}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[x^3/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[x^3/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-c^2x^2)^{5/2} (a+b\arccos(cx))^2} dx$$
$$\Big\downarrow \text{5235}$$
$$\int \frac{x^3}{(1-c^2x^2)^{5/2} (a+b\arccos(cx))^2} dx$$

input
```
Int[x^3/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 6.79 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{(-c^2x^2+1)^{\frac{5}{2}} (a+b\arccos(cx))^2} dx$$

input
```
int(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 5.14

$$\int \frac{x^3}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^3}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^3/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*
x^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccos(c*x)^2 -
a^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccos(c*x)),
x)
```

Sympy [N/A]

Not integrable

Time = 3.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^3}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^3}{(-(cx-1)(cx+1))^{\frac{5}{2}}(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate(x**3/(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**3/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 265, normalized size of antiderivative = 9.46

$$\int \frac{x^3}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x^3}{(-c^2x^2+1)^{\frac{5}{2}}\,(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
(x^3 + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 - 2*b^2*c^3*x^2
 + b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*integrate((c^2*x^4 +
 3*x^2)/(a*b*c^7*x^6 - 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 - a*b*c + (b^2*c^7*x^
6 - 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 - b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x
 + 1), c*x)), x))/(a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 - 2*
b^2*c^3*x^2 + b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(1-c^2x^2)^{5/2} (a+b\arccos(cx))^2}\,dx = \int \frac{x^3}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2\,\left(1-c^2\,x^2\right)^{5/2}}\,dx$$

input
```
int(x^3/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

output
```
int(x^3/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 212, normalized size of antiderivative = 7.57

$$\int \frac{x^3}{(1-c^2x^2)^{5/2} (a+b\arccos(cx))^2}\,dx = \int \frac{}{\sqrt{-c^2x^2+1}\,acos\left(cx\right)^2 b^2c^4x^4-2\sqrt{-c^2x^2+1}\,acos\left(cx\right)^2 b^2c^2x^2}$$

input
```
int(x^3/(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(x**3/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**4*x**4 - 2*sqrt( - c
**2*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**2 + sqrt( - c**2*x**2 + 1)*acos(c*
x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*c**4*x**4 - 4*sqrt( -
c**2*x**2 + 1)*acos(c*x)*a*b*c**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x
)*a*b + sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*a
**2*c**2*x**2 + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.427 $\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \operatorname{Int}\left(\frac{x^2}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 14.96 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-c^2x^2)^{5/2} (a+b\arccos(cx))^2} \, dx$$
$$\Big\downarrow \text{ 5235}$$
$$\int \frac{x^2}{(1-c^2x^2)^{5/2} (a+b\arccos(cx))^2}dx$$

input
```
Int[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 4.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{(-c^2x^2+1)^{\frac{5}{2}} (a+b\arccos(cx))^2}dx$$

input
```
int(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 5.14

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*
x^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccos(c*x)^2 -
a^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccos(c*x)),
x)
```

Sympy [N/A]

Not integrable

Time = 3.67 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acos}{\left(cx\right)}\right)^2}\,dx$$

input
```
integrate(x**2/(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**2/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 262, normalized size of antiderivative = 9.36

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
(x^2 + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 - 2*b^2*c^3*x^2
 + b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*integrate(2*(c^2*x^3
 + x)/(a*b*c^7*x^6 - 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 - a*b*c + (b^2*c^7*x^6
- 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 - b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
 1), c*x)), x))/(a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 - 2*b^
2*c^3*x^2 + b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 6.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(x^2/((-c^2*x^2 + 1)^(5/2)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{(a+b\,\mathrm{acos}\,(c\,x))^2\,(1-c^2\,x^2)^{5/2}}\,dx$$

input
```
int(x^2/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

output
```
int(x^2/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 212, normalized size of antiderivative = 7.57

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{}{\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2c^4x^4-2\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2c^2x^2}$$

input
```
int(x^2/(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(x**2/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**4*x**4 - 2*sqrt( - c
**2*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**2 + sqrt( - c**2*x**2 + 1)*acos(c*
x)**2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*c**4*x**4 - 4*sqrt( -
c**2*x**2 + 1)*acos(c*x)*a*b*c**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x
)*a*b + sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*a
**2*c**2*x**2 + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.428 $\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 93.86 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}dx$$
$$\Big\downarrow \text{5235}$$
$$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}dx$$

input
```
Int[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 4.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x}{(-c^2x^2+1)^{\frac{5}{2}}(a+b\arccos(cx))^2}dx$$

input
```
int(x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 5.46

$$\int \frac{x}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*x/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x^
2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccos(c*x)^2 - a^
2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{x}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{x}{(-(cx-1)\,(cx+1))^{\frac{5}{2}}\,(a+b\operatorname{acos}{(cx)})^2}\,dx$$

input
```
integrate(x/(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(x/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 260, normalized size of antiderivative = 10.00

$$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx=\int \frac{x}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2
*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*integrate((3*c^2*x^2 + 1)/
(a*b*c^7*x^6 - 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 - a*b*c + (b^2*c^7*x^6 - 3*b^
2*c^5*x^4 + 3*b^2*c^3*x^2 - b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c
*x)), x) + x)/(a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 - 2*b^2*
c^3*x^2 + b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{x}{(a+b\,\mathrm{acos}\,(c\,x))^2\,(1-c^2\,x^2)^{5/2}}\,dx$$

input
```
int(x/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

output
```
int(x/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 8.08

$$\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{}{\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2c^4x^4-2\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2c^2x^2}$$

input
```
int(x/(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(x/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**4*x**4 - 2*sqrt( - c**2
*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**2 + sqrt( - c**2*x**2 + 1)*acos(c*x)*
*2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*c**4*x**4 - 4*sqrt( - c**
2*x**2 + 1)*acos(c*x)*a*b*c**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a
*b + sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*a**2
*c**2*x**2 + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.429 $\int \frac{1}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = -\frac{1}{bc\,(1-c^2x^2)^2\,(a+b\arccos(cx))}+\frac{4c\text{Int}\left(\frac{x}{(1-c^2x^2)^3(a+b\arccos(cx))},x\right)}{b}$$

output
```
-1/b/c/(-c^2*x^2+1)^2/(a+b*arccos(c*x))+4*c*Defer(Int)(x/(-c^2*x^2+1)^3/(a
+b*arccos(c*x)),x)/b
```

Mathematica [N/A]

Not integrable

Time = 5.91 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \; 5167$$

$$\frac{1}{bc(1-c^2x^2)^2(a+b\arccos(cx))}-\frac{4c\int \frac{x}{(1-c^2x^2)^3(a+b\arccos(cx))}dx}{b}$$

$$\Big\downarrow \; 5235$$

$$\frac{1}{bc(1-c^2x^2)^2(a+b\arccos(cx))}-\frac{4c\int \frac{x}{(1-c^2x^2)^3(a+b\arccos(cx))}dx}{b}$$

input
```
Int[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right)^{\frac{5}{2}} \left(a +b \arccos \left(c x \right)\right)^{2}}d x$$

input
```
int(1/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 5.64

$$\int \frac{1}{(1-c^2x^2)^{5/2} (a+b\arccos(cx))^2} \, dx=\int \frac{1}{{\left(-c^{2} x^{2} + 1\right)}^{\frac{5}{2}} {\left(b \arccos\left(c x\right) + a\right)}^{2}} \,d x$$

input
```
integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
+ (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccos(c*x)^2 - a^2
+ 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.91 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1-c^2x^2)^{5/2} (a+b\arccos(cx))^2} \, dx=\int \frac{1}{\left(- \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{5}{2}} \left(a + b \operatorname{acos}{\left(c x \right)}\right)^{2}}\, dx$$

input
```
integrate(1/(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 254, normalized size of antiderivative = 10.16

$$\int \frac{1}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
(4*(a*b*c^6*x^4 - 2*a*b*c^4*x^2 + a*b*c^2 + (b^2*c^6*x^4 - 2*b^2*c^4*x^2 +
 b^2*c^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*integrate(x/(a*b*c^6
*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b + (b^2*c^6*x^6 - 3*b^2*c^4*x^4
+ 3*b^2*c^2*x^2 - b^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) + 1
)/(a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^
2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2\,\left(1-c^2\,x^2\right)^{5/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 208, normalized size of antiderivative = 8.32

$$\int \frac{1}{(1-c^2x^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{}{\sqrt{-c^2x^2+1}\,acos\left(cx\right)^2 b^2c^4x^4-2\sqrt{-c^2x^2+1}\,acos\left(cx\right)^2 b^2c^2x^2}$$

input
```
int(1/(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**4*x**4 - 2*sqrt( - c**2
*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**2 + sqrt( - c**2*x**2 + 1)*acos(c*x)*
*2*b**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*c**4*x**4 - 4*sqrt( - c**
2*x**2 + 1)*acos(c*x)*a*b*c**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a
*b + sqrt( - c**2*x**2 + 1)*a**2*c**4*x**4 - 2*sqrt( - c**2*x**2 + 1)*a**2
*c**2*x**2 + sqrt( - c**2*x**2 + 1)*a**2),x)
```

3.430 $\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 80.99 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}dx$$
$$\Big\downarrow \text{5235}$$
$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}dx$$

input
```
Int[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 9.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(-c^2x^2+1\right)^{\frac{5}{2}}\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 5.14

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(-c^2x^2+1)^{\frac{5}{2}}(b\arccos(cx)+a)^2x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas"
)
```

output
```
integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^6*x^7 - 3*a^2*c^4*x^5 + 3*a^2*c^2*x^3
- a^2*x + (b^2*c^6*x^7 - 3*b^2*c^4*x^5 + 3*b^2*c^2*x^3 - b^2*x)*arccos(c*x
)^2 + 2*(a*b*c^6*x^7 - 3*a*b*c^4*x^5 + 3*a*b*c^2*x^3 - a*b*x)*arccos(c*x))
, x)
```

Sympy [N/A]

Not integrable

Time = 8.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x(-(cx-1)(cx+1))^{\frac{5}{2}}(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate(1/x/(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/(x*(-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 270, normalized size of antiderivative = 9.64

$$\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\left(-c^2x^2+1\right)^{\frac{5}{2}}\left(b\arccos\left(cx\right)+a\right)^2x}\,dx$$

input
```
integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima"
)
```

output
```
((a*b*c^5*x^5 - 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^5*x^5 - 2*b^2*c^3*x^3 + b
^2*c*x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*integrate((5*c^2*x^2 -
 1)/(a*b*c^7*x^8 - 3*a*b*c^5*x^6 + 3*a*b*c^3*x^4 - a*b*c*x^2 + (b^2*c^7*x^
8 - 3*b^2*c^5*x^6 + 3*b^2*c^3*x^4 - b^2*c*x^2)*arctan2(sqrt(c*x + 1)*sqrt(
-c*x + 1), c*x)), x) + 1)/(a*b*c^5*x^5 - 2*a*b*c^3*x^3 + a*b*c*x + (b^2*c^
5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x
))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x\,(a+b\,\mathrm{acos}\,(c\,x))^2\,(1-c^2\,x^2)^{5/2}}\,dx$$

input
```
int(1/(x*(a + b*acos(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

output
```
int(1/(x*(a + b*acos(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 211, normalized size of antiderivative = 7.54

$$\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{}{\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2c^4x^5 - 2\sqrt{-c^2x^2+1}\,acos\,(cx)^2\,b^2c^2$$

input
```
int(1/x/(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**4*x**5 - 2*sqrt( - c**2
*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**3 + sqrt( - c**2*x**2 + 1)*acos(c*x)*
*2*b**2*x + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*c**4*x**5 - 4*sqrt( - c
**2*x**2 + 1)*acos(c*x)*a*b*c**2*x**3 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)
*a*b*x + sqrt( - c**2*x**2 + 1)*a**2*c**4*x**5 - 2*sqrt( - c**2*x**2 + 1)*
a**2*c**2*x**3 + sqrt( - c**2*x**2 + 1)*a**2*x),x)
```

3.431 $\int \frac{1}{x^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 23.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))^2}\,dx = \int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))^2} \, dx$$
$$\Big\downarrow \; 5235$$
$$\int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))^2} dx$$

input
```
Int[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 9.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2 (-c^2x^2+1)^{\frac{5}{2}} (a+b\arccos(cx))^2} dx$$

input
```
int(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 5.36

$$\int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))^2} \, dx = \int \frac{1}{(-c^2x^2+1)^{\frac{5}{2}} (b\arccos(cx)+a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="frica
s")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^6*x^8 - 3*a^2*c^4*x^6 + 3*a^2*c^2*x^4
- a^2*x^2 + (b^2*c^6*x^8 - 3*b^2*c^4*x^6 + 3*b^2*c^2*x^4 - b^2*x^2)*arccos
(c*x)^2 + 2*(a*b*c^6*x^8 - 3*a*b*c^4*x^6 + 3*a*b*c^2*x^4 - a*b*x^2)*arccos
(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 6.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))^2} \, dx = \int \frac{1}{x^2 \left(-(cx-1)(cx+1)\right)^{\frac{5}{2}} (a+b\operatorname{acos}(cx))^2} \, dx$$

input
```
integrate(1/x**2/(-c**2*x**2+1)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/(x**2*(-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 279, normalized size of antiderivative = 9.96

$$\int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))^2} \, dx = \int \frac{1}{(-c^2x^2+1)^{\frac{5}{2}} (b\arccos(cx)+a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxim
a")
```

output
```
((a*b*c^5*x^6 - 2*a*b*c^3*x^4 + a*b*c*x^2 + (b^2*c^5*x^6 - 2*b^2*c^3*x^4 +
 b^2*c*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*integrate(2*(3*c^2
*x^2 - 1)/(a*b*c^7*x^9 - 3*a*b*c^5*x^7 + 3*a*b*c^3*x^5 - a*b*c*x^3 + (b^2*
c^7*x^9 - 3*b^2*c^5*x^7 + 3*b^2*c^3*x^5 - b^2*c*x^3)*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x)), x) + 1)/(a*b*c^5*x^6 - 2*a*b*c^3*x^4 + a*b*c*x^2 +
 (b^2*c^5*x^6 - 2*b^2*c^3*x^4 + b^2*c*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x
 + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 36.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))^2} \, dx = \int \frac{1}{(-c^2x^2+1)^{\frac{5}{2}} (b\arccos(cx)+a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac"
)
```

output
```
integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arccos(c*x) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))^2} \, dx = \int \frac{1}{x^2 \left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2 \left(1-c^2\,x^2\right)^{5/2}} \, dx$$

input
```
int(1/(x^2*(a + b*acos(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*(a + b*acos(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 7.75

$$\int \frac{1}{x^2 (1-c^2x^2)^{5/2} (a+b\arccos(cx))^2} \, dx = \int \frac{}{\sqrt{-c^2x^2+1}\, acos(cx)^2 b^2c^4x^6 - 2\sqrt{-c^2x^2+1}\, acos(cx)^2 b^2c}$$

input
```
int(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(sqrt( - c**2*x**2 + 1)*acos(c*x)**2*b**2*c**4*x**6 - 2*sqrt( - c**2
*x**2 + 1)*acos(c*x)**2*b**2*c**2*x**4 + sqrt( - c**2*x**2 + 1)*acos(c*x)*
*2*b**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*a*b*c**4*x**6 - 4*sqrt(
- c**2*x**2 + 1)*acos(c*x)*a*b*c**2*x**4 + 2*sqrt( - c**2*x**2 + 1)*acos(c
*x)*a*b*x**2 + sqrt( - c**2*x**2 + 1)*a**2*c**4*x**6 - 2*sqrt( - c**2*x**2
 + 1)*a**2*c**2*x**4 + sqrt( - c**2*x**2 + 1)*a**2*x**2),x)
```

3.432 $\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 13

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = -\frac{1}{2a\arccos(ax)^2}$$

output
```
-1/2/a/arccos(a*x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = \frac{1}{2a\arccos(ax)^2}$$

input
```
Integrate[1/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3),x]
```

output
```
1/(2*a*ArcCos[a*x]^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx$$

$$\Big\downarrow \ 5153$$

$$\frac{1}{2a\arccos(ax)^2}$$

input
```
Int[1/(Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3),x]
```

output
```
1/(2*a*ArcCos[a*x]^2)
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{1}{2a\arccos(ax)^2}$	12
default	$\frac{1}{2a\arccos(ax)^2}$	12

input
```
int(1/(-a^2*x^2+1)^(1/2)/arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/a/arccos(a*x)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = \frac{1}{2\,a\arccos\left(ax\right)^2}$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arccos(a*x)^3,x, algorithm="fricas")
```

output
```
1/2/(a*arccos(a*x)^2)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = \begin{cases} \frac{1}{2a\operatorname{acos}^2\left(ax\right)} & \text{for}\ a \neq 0 \\ \frac{8x}{\pi^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-a**2*x**2+1)**(1/2)/acos(a*x)**3,x)
```

output
```
Piecewise((1/(2*a*acos(a*x)**2), Ne(a, 0)), (8*x/pi**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = \frac{1}{2\,a\arccos{(ax)}^2}$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arccos(a*x)^3,x, algorithm="maxima")
```

output
```
1/2/(a*arccos(a*x)^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = \frac{1}{2\,a\arccos{(ax)}^2}$$

input
```
integrate(1/(-a^2*x^2+1)^(1/2)/arccos(a*x)^3,x, algorithm="giac")
```

output
```
1/2/(a*arccos(a*x)^2)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = \frac{1}{2\,a\,\mathrm{acos}{(a\,x)}^2}$$

input
```
int(1/(acos(a*x)^3*(1 - a^2*x^2)^(1/2)),x)
```

output
```
1/(2*a*acos(a*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx = \frac{1}{2acos\left(ax\right)^2 a}$$

input
```
int(1/(-a^2*x^2+1)^(1/2)/acos(a*x)^3,x)
```

output
```
1/(2*acos(a*x)**2*a)
```

3.433 $\int \frac{x^3(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 251

$$\begin{aligned}\int \frac{x^3(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx &= -\frac{2dx^3(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}\\ &\quad-\frac{d\sqrt{3\pi}\cos\left(\frac{6a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^4}\\ &\quad+\frac{3d\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{8b^{3/2}c^4}\\ &\quad+\frac{3d\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{8b^{3/2}c^4}\\ &\quad-\frac{d\sqrt{3\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{6a}{b}\right)}{8b^{3/2}c^4}\end{aligned}$$

output
```
-2*d*x^3*(-c^2*x^2+1)^(3/2)/b/c/(a+b*arccos(c*x))^(1/2)-1/8*d*3^(1/2)*Pi^(
1/2)*cos(6*a/b)*FresnelC(2*3^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2
))/b^(3/2)/c^4+3/8*d*Pi^(1/2)*cos(2*a/b)*FresnelC(2*(a+b*arccos(c*x))^(1/2
)/b^(1/2)/Pi^(1/2))/b^(3/2)/c^4+3/8*d*Pi^(1/2)*FresnelS(2*(a+b*arccos(c*x)
)^(1/2)/b^(1/2)/Pi^(1/2))*sin(2*a/b)/b^(3/2)/c^4-1/8*d*3^(1/2)*Pi^(1/2)*Fr
esnelS(2*3^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(6*a/b)/b^(3
/2)/c^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.14

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = \frac{ide^{-\frac{6ia}{b}}\left(3\sqrt{2}e^{\frac{4ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},-\frac{2i(a+b\arccos(cx))}{b}\right)-3\sqrt{2}e^{\frac{8ia}{b}}\sqrt{\frac{i(a+b\,\mathrm{a}}{}}\right.}{}$$

input
```
Integrate[(x^3*(d - c^2*d*x^2))/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
((I/32)*d*(3*Sqrt[2]*E^(((4*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Ga
mma[1/2, ((-2*I)*(a + b*ArcCos[c*x]))/b] - 3*Sqrt[2]*E^(((8*I)*a)/b)*Sqrt[
(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((2*I)*(a + b*ArcCos[c*x]))/b] - Sqr
t[6]*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-6*I)*(a + b*ArcCos[c
*x]))/b] + Sqrt[6]*E^(((12*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[
1/2, ((6*I)*(a + b*ArcCos[c*x]))/b] - (6*I)*E^(((6*I)*a)/b)*Sin[2*ArcCos[c
*x]] + (2*I)*E^(((6*I)*a)/b)*Sin[6*ArcCos[c*x]]))/(b*c^4*E^(((6*I)*a)/b)*S
qrt[a + b*ArcCos[c*x]])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 507 vs. $2(251) = 502$.

Time = 1.37 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5215, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}dx$$

$$\Big\downarrow \text{5215}$$

$$-\frac{6d\int \frac{x^2\sqrt{1-c^2x^2}}{\sqrt{a+b\arccos(cx)}}dx}{bc}+\frac{12cd\int \frac{x^4\sqrt{1-c^2x^2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2dx^3(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{5225}$$

$$-\frac{12d\int \frac{\cos^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^4}+\frac{6d\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^4}+\frac{2dx^3(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{6d\int \left(\frac{1}{8\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{b^2c^4}-\frac{12d\int \left(-\frac{\cos\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)}{32\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{16\sqrt{a+b\arccos(cx)}}+\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{32\sqrt{a+b\arccos(cx)}}+\frac{1}{16\sqrt{a+b\arccos(cx)}}\right)d(a+b\,\text{arcc}}{b^2c^4}$$
$$\frac{2dx^3(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6d\left(-\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{4a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{}\right.}{b^2c^4}$$

$$\frac{12d\left(-\frac{1}{16}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{\frac{\pi}{3}}\sqrt{b}\cos\left(\frac{6a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\right.}{}$$

$$\frac{2dx^3(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

input
```
Int[(x^3*(d - c^2*d*x^2))/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(2*d*x^3*(1 - c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcCos[c*x]]) + (6*d*(Sqrt[a
 + b*ArcCos[c*x]]/4 - (Sqrt[b]*Sqrt[Pi/2]*Cos[(4*a)/b]*FresnelC[(2*Sqrt[2/
Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(2
*Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(4*a)/b])/8))/(b^2*c^4)
- (12*d*(Sqrt[a + b*ArcCos[c*x]]/8 - (Sqrt[b]*Sqrt[Pi/2]*Cos[(4*a)/b]*Fres
nelC[(2*Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/16 - (Sqrt[b]*Sqrt[P
i/3]*Cos[(6*a)/b]*FresnelC[(2*Sqrt[3/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]
)/32 + (Sqrt[b]*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])
/(Sqrt[b]*Sqrt[Pi])])/32 + (Sqrt[b]*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos
[c*x]])/(Sqrt[b]*Sqrt[Pi])]*Sin[(2*a)/b])/32 - (Sqrt[b]*Sqrt[Pi/2]*Fresnel
S[(2*Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(4*a)/b])/16 - (Sqrt
[b]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[3/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Si
n[(6*a)/b])/32))/(b^2*c^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.21

method	result
default	$-\frac{d\left(-\sqrt{-\frac{6}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{6a}{b}\right)\operatorname{FresnelC}\left(\frac{6\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{6}{b}}\,b}\right)+\sqrt{-\frac{6}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{6a}{b}\right)\operatorname{FresnelS}\right.}{}$

input
```
int(x^3*(-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*d/c^4/b*(-(-6/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(
6*a/b)*FresnelC(6*2^(1/2)/Pi^(1/2)/(-6/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)
+(-6/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(6*a/b)*FresnelS
(6*2^(1/2)/Pi^(1/2)/(-6/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+6*(-1/b)^(1/2)
*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(
-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-6*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos
(c*x))^(1/2)*sin(2*a/b)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arcc
os(c*x))^(1/2)/b)+3*sin(-2*(a+b*arccos(c*x))/b+2*a/b)-sin(-6*(a+b*arccos(c
*x))/b+6*a/b))/(a+b*arccos(c*x))^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = -d\left(\int \left(-\frac{x^3}{a\sqrt{a+b\operatorname{acos}(cx)}+b\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\right)dx + \int \frac{c^2x^5}{a\sqrt{a+b\operatorname{acos}(cx)}+b\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\,dx\right)$$

input
```
integrate(x**3*(-c**2*d*x**2+d)/(a+b*acos(c*x))**(3/2),x)
```

output
```
-d*(Integral(-x**3/(a*sqrt(a + b*acos(c*x)) + b*sqrt(a + b*acos(c*x))*acos
(c*x)), x) + Integral(c**2*x**5/(a*sqrt(a + b*acos(c*x)) + b*sqrt(a + b*ac
os(c*x))*acos(c*x)), x))
```

Maxima [F]

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = \int -\frac{(c^2dx^2-d)x^3}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima"
)
```

output
```
-integrate((c^2*d*x^2 - d)*x^3/(b*arccos(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = \int -\frac{(c^2dx^2-d)x^3}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(-(c^2*d*x^2 - d)*x^3/(b*arccos(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x^3\,(d-c^2\,d\,x^2)}{(a+b\,\mathrm{acos}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x^3*(d - c^2*d*x^2))/(a + b*acos(c*x))^(3/2),x)
```

output
```
int((x^3*(d - c^2*d*x^2))/(a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = d\left(-\left(\int \frac{\sqrt{acos\,(cx)\,b+a}\,x^5}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{acos\,(cx)\,b+a}\,x^3}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)$$

input
```
int(x^3*(-c^2*d*x^2+d)/(a+b*acos(c*x))^(3/2),x)
```

output
```
d*( - int((sqrt(acos(c*x)*b + a)*x**5)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*
b + a**2),x)*c**2 + int((sqrt(acos(c*x)*b + a)*x**3)/(acos(c*x)**2*b**2 +
2*acos(c*x)*a*b + a**2),x))
```

3.434 $\int \frac{x^2(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 591

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = -\frac{2dx^2(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{5d\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^3} + \frac{d\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} - \frac{5d\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} + \frac{d\sqrt{\frac{2\pi}{3}}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} + \frac{d\sqrt{\frac{5\pi}{2}}\cos\left(\frac{5a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} + \frac{5d\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{2b^{3/2}c^3} - \frac{d\sqrt{2\pi}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{b^{3/2}c^3} + \frac{5d\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{4b^{3/2}c^3} - \frac{d\sqrt{\frac{2\pi}{3}}\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{b^{3/2}c^3} - \frac{d\sqrt{\frac{5\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{5a}{b}\right)}{4b^{3/2}c^3}$$

output
```
-2*d*x^2*(-c^2*x^2+1)^(3/2)/b/c/(a+b*arccos(c*x))^(1/2)-1/4*d*2^(1/2)*Pi^(
1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b
^(3/2)/c^3+1/8*d*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelS(6^(1/2)/Pi^(1/2)*(a+
b*arccos(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3+1/8*d*10^(1/2)*Pi^(1/2)*cos(5*a/
b)*FresnelS(10^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3
+1/4*d*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/
b^(1/2))*sin(a/b)/b^(3/2)/c^3-1/8*d*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(
1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(3*a/b)/b^(3/2)/c^3-1/8*d*10^(1/2
)*Pi^(1/2)*FresnelC(10^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin
(5*a/b)/b^(3/2)/c^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 420, normalized size of antiderivative = 0.71

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = \frac{de^{-\frac{5ia}{b}}\left(4e^{\frac{5ia}{b}}\sqrt{1-c^2x^2}+2ie^{\frac{4ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(cx))}{b}\right)-2i\right.}{}$$

input
```
Integrate[(x^2*(d - c^2*d*x^2))/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(d*(4*E^(((5*I)*a)/b)*Sqrt[1 - c^2*x^2] + (2*I)*E^(((4*I)*a)/b)*Sqrt[((-I)
*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/b] - (2*I)*
E^(((6*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*ArcCo
s[c*x]))/b] + I*Sqrt[3]*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]
*Gamma[1/2, ((-3*I)*(a + b*ArcCos[c*x]))/b] - I*Sqrt[3]*E^(((8*I)*a)/b)*Sq
rt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((3*I)*(a + b*ArcCos[c*x]))/b] -
I*Sqrt[5]*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-5*I)*(a + b*Arc
Cos[c*x]))/b] + I*Sqrt[5]*E^(((10*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]
*Gamma[1/2, ((5*I)*(a + b*ArcCos[c*x]))/b] + 2*E^(((5*I)*a)/b)*Sin[3*ArcCo
s[c*x]] - 2*E^(((5*I)*a)/b)*Sin[5*ArcCos[c*x]]))/(16*b*c^3*E^(((5*I)*a)/b)
*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 585, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5215, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (d-c^2 d x^2)}{(a+b\arccos(cx))^{3/2}}dx$$

$$\Big\downarrow \text{5215}$$

$$-\frac{4d\int \frac{x\sqrt{1-c^2x^2}}{\sqrt{a+b\arccos(cx)}}dx}{bc}+\frac{10cd\int \frac{x^3\sqrt{1-c^2x^2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2dx^2(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{5225}$$

$$-\frac{10d\int \frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^3}+\frac{4d\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^3}+\frac{2dx^2(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{10d\int \left(-\frac{\cos\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{16\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{16\sqrt{a+b\arccos(cx)}}+\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{8\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{b^2c^3}+\frac{4d\int \left(\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{b^2c^3}+\frac{2dx^2(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4d\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\sqrt{b}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{}\right.}{b^2c^3}$$

$$10d\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{6}}\sqrt{b}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{10}}\right.$$

$$\frac{2dx^2(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

input
```
Int[(x^2*(d - c^2*d*x^2))/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(2*d*x^2*(1 - c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcCos[c*x]]) + (4*d*((Sqrt[
b]*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[
b]])/2 - (Sqrt[b]*Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*
ArcCos[c*x]])/Sqrt[b]])/2 + (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[
a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/2 - (Sqrt[b]*Sqrt[Pi/6]*FresnelS[(S
qrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/2))/(b^2*c^3) -
(10*d*((Sqrt[b]*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos
[c*x]])/Sqrt[b]])/4 - (Sqrt[b]*Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelC[(Sqrt[6/Pi
]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi/10]*Cos[(5*a)/b]
*FresnelC[(Sqrt[10/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqr
t[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/4
 - (Sqrt[b]*Sqrt[Pi/6]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[
b]]*Sin[(3*a)/b])/8 - (Sqrt[b]*Sqrt[Pi/10]*FresnelS[(Sqrt[10/Pi]*Sqrt[a +
b*ArcCos[c*x]])/Sqrt[b]]*Sin[(5*a)/b])/8))/(b^2*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.76

method	result
default	$-\frac{d\left(2\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)-2\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\right.\right.}{}$

input
```
int(x^2*(-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*d/c^3/b/(a+b*arccos(c*x))^(1/2)*(2*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b
*arccos(c*x))^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*a
rccos(c*x))^(1/2)/b)-2*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/
2)*sin(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)
/b)-(-5/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(5*a/b)*Fresn
elC(5*2^(1/2)/Pi^(1/2)/(-5/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+(-5/b)^(1/2
)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(5*a/b)*FresnelS(5*2^(1/2)/P
i^(1/2)/(-5/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+(-3/b)^(1/2)*Pi^(1/2)*2^(1
/2)*cos(3*a/b)*FresnelC(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^
(1/2)/b)*(a+b*arccos(c*x))^(1/2)-(-3/b)^(1/2)*Pi^(1/2)*2^(1/2)*sin(3*a/b)*
FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(a+b*a
rccos(c*x))^(1/2)+2*sin(-(a+b*arccos(c*x))/b+a/b)+sin(-3*(a+b*arccos(c*x))
/b+3*a/b)-sin(-5*(a+b*arccos(c*x))/b+5*a/b))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = -d\left(\int \left(-\frac{x^2}{a\sqrt{a+b\operatorname{acos}(cx)}+b\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\right)dx + \int \frac{c^2x^4}{a\sqrt{a+b\operatorname{acos}(cx)}+b\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\,dx\right)$$

input
```
integrate(x**2*(-c**2*d*x**2+d)/(a+b*acos(c*x))**(3/2),x)
```

output
```
-d*(Integral(-x**2/(a*sqrt(a + b*acos(c*x)) + b*sqrt(a + b*acos(c*x))*acos
(c*x)), x) + Integral(c**2*x**4/(a*sqrt(a + b*acos(c*x)) + b*sqrt(a + b*ac
os(c*x))*acos(c*x)), x))
```

Maxima [F]

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx=\int -\frac{(c^2dx^2-d)x^2}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima"
)
```

output
```
-integrate((c^2*d*x^2 - d)*x^2/(b*arccos(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx=\int -\frac{(c^2dx^2-d)x^2}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(-(c^2*d*x^2 - d)*x^2/(b*arccos(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{x^2\,(d-c^2\,d\,x^2)}{(a+b\,\mathrm{acos}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x^2*(d - c^2*d*x^2))/(a + b*acos(c*x))^(3/2),x)
```

output
```
int((x^2*(d - c^2*d*x^2))/(a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx=d\left(-\left(\int \frac{\sqrt{acos\,(cx)\,b+a}\,x^4}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2+\int \frac{\sqrt{acos\,(cx)\,b+a}\,x^2}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)$$

input
```
int(x^2*(-c^2*d*x^2+d)/(a+b*acos(c*x))^(3/2),x)
```

output
```
d*( - int((sqrt(acos(c*x)*b + a)*x**4)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*
b + a**2),x)*c**2 + int((sqrt(acos(c*x)*b + a)*x**2)/(acos(c*x)**2*b**2 +
2*acos(c*x)*a*b + a**2),x))
```

3.435 $\int \frac{x(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 241

$$\int \frac{x(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = -\frac{2dx(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}} + \frac{d\sqrt{\frac{\pi}{2}}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2} + \frac{d\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{b^{3/2}c^2} + \frac{d\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{b^{3/2}c^2} + \frac{d\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{4a}{b}\right)}{b^{3/2}c^2}$$

output

```
-2*d*x*(-c^2*x^2+1)^(3/2)/b/c/(a+b*arccos(c*x))^(1/2)+1/2*d*2^(1/2)*Pi^(1/
2)*cos(4*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))
/b^(3/2)/c^2+d*Pi^(1/2)*cos(2*a/b)*FresnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1
/2)/Pi^(1/2))/b^(3/2)/c^2+d*Pi^(1/2)*FresnelS(2*(a+b*arccos(c*x))^(1/2)/b^
(1/2)/Pi^(1/2))*sin(2*a/b)/b^(3/2)/c^2+1/2*d*2^(1/2)*Pi^(1/2)*FresnelS(2*2
^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(4*a/b)/b^(3/2)/c^2
```

Mathematica [F]

$$\int \frac{x(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{x(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx$$

input
```
Integrate[(x*(d - c^2*d*x^2))/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
Integrate[(x*(d - c^2*d*x^2))/(a + b*ArcCos[c*x])^(3/2), x]
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5215, 5169, 3042, 3793, 2009, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}dx$$

$$\Big\downarrow \text{5215}$$

$$-\frac{2d\int \frac{\sqrt{1-c^2x^2}}{\sqrt{a+b\arccos(cx)}}dx}{bc}+\frac{8cd\int \frac{x^2\sqrt{1-c^2x^2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2dx\left(1-c^2x^2\right)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{5169}$$

$$\frac{2d\int \frac{\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^2}+\frac{8cd\int \frac{x^2\sqrt{1-c^2x^2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2dx\left(1-c^2x^2\right)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^2}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^2}+\frac{8cd\int \frac{x^2\sqrt{1-c^2x^2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2dx\left(1-c^2x^2\right)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{2d\int \left(\frac{1}{2\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{b^2c^2}+\frac{8cd\int \frac{x^2\sqrt{1-c^2x^2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2dx(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\downarrow \text{2009}$$

$$\frac{8cd\int \frac{x^2\sqrt{1-c^2x^2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2d\left(-\frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)+\sqrt{a+b\arccos}\right.}{b^2c^2}$$

$$\frac{2dx(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\downarrow \text{5225}$$

$$-\frac{8d\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^2}+$$

$$\frac{2d\left(-\frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)+\sqrt{a+b\arccos}\right.}{b^2c^2}$$

$$\frac{2dx(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\downarrow \text{4906}$$

$$-\frac{8d\int \left(\frac{1}{8\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{b^2c^2}+$$

$$\frac{2d\left(-\frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)+\sqrt{a+b\arccos}\right.}{b^2c^2}$$

$$\frac{2dx(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\downarrow \text{2009}$$

$$\frac{2d\left(-\frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)+\sqrt{a+b\arccos}\right.}{b^2c^2}$$

$$\frac{8d\left(-\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{4a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\frac{1}{4}\sqrt{\right.}{b^2c^2}$$

$$\frac{2dx(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

input
```
Int[(x*(d - c^2*d*x^2))/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(2*d*x*(1 - c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcCos[c*x]]) + (2*d*(Sqrt[a +
 b*ArcCos[c*x]] - (Sqrt[b]*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*Ar
cCos[c*x]])/(Sqrt[b]*Sqrt[Pi])])/2 - (Sqrt[b]*Sqrt[Pi]*FresnelS[(2*Sqrt[a
+ b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])]*Sin[(2*a)/b])/2))/(b^2*c^2) - (8*d*(
Sqrt[a + b*ArcCos[c*x]]/4 - (Sqrt[b]*Sqrt[Pi/2]*Cos[(4*a)/b]*FresnelC[(2*S
qrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi/2]*Fresn
elS[(2*Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(4*a)/b])/8))/(b^2
*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.27

method	result
default	$-\frac{d\left(-2\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{4a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}+2\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{4a}{b}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{a+b}}{\sqrt{\pi}\,\sqrt{}}\right.\right.}{}$

input
```
int(x*(-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*d/c^2/b*(-2*(a+b*arccos(c*x))^(1/2)*cos(4*a/b)*FresnelC(2*2^(1/2)/Pi^
(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2
)+2*(a+b*arccos(c*x))^(1/2)*sin(4*a/b)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-1/b)^
(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)+4*(-1/b)^(1
/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-4*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arc
cos(c*x))^(1/2)*sin(2*a/b)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*a
rccos(c*x))^(1/2)/b)+2*sin(-2*(a+b*arccos(c*x))/b+2*a/b)-sin(-4*(a+b*arcco
s(c*x))/b+4*a/b))/(a+b*arccos(c*x))^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = -d\left(\int \left(-\frac{x}{a\sqrt{a+b\operatorname{acos}(cx)}+b\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\right)dx + \int \frac{c^2x^3}{a\sqrt{a+b\operatorname{acos}(cx)}+b\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\,dx\right)$$

input
```
integrate(x*(-c**2*d*x**2+d)/(a+b*acos(c*x))**(3/2),x)
```

output
```
-d*(Integral(-x/(a*sqrt(a + b*acos(c*x)) + b*sqrt(a + b*acos(c*x))*acos(c*
x)), x) + Integral(c**2*x**3/(a*sqrt(a + b*acos(c*x)) + b*sqrt(a + b*acos(
c*x))*acos(c*x)), x))
```

Maxima [F]

$$\int \frac{x(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = \int -\frac{(c^2dx^2-d)x}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
-integrate((c^2*d*x^2 - d)*x/(b*arccos(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = \int -\frac{(c^2dx^2-d)x}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(-(c^2*d*x^2 - d)*x/(b*arccos(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x\,(d-c^2\,d\,x^2)}{(a+b\,\mathrm{acos}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x*(d - c^2*d*x^2))/(a + b*acos(c*x))^(3/2),x)
```

output
```
int((x*(d - c^2*d*x^2))/(a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x(d-c^2dx^2)}{(a+b\arccos(cx))^{3/2}}\,dx = d\left(-\left(\int \frac{\sqrt{acos\,(cx)\,b+a}\,x^3}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{acos\,(cx)\,b+a}\,x}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)$$

input
```
int(x*(-c^2*d*x^2+d)/(a+b*acos(c*x))^(3/2),x)
```

output
```
d*( - int((sqrt(acos(c*x)*b + a)*x**3)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*
b + a**2),x)*c**2 + int((sqrt(acos(c*x)*b + a)*x)/(acos(c*x)**2*b**2 + 2*a
cos(c*x)*a*b + a**2),x))
```

3.436 $\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 253

$$\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^{3/2}}\,dx = -\frac{2d(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{3d\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} - \frac{d\sqrt{\frac{3\pi}{2}}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{3d\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{b^{3/2}c} + \frac{d\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{b^{3/2}c}$$

output
```
-2*d*(-c^2*x^2+1)^(3/2)/b/c/(a+b*arccos(c*x))^(1/2)-3/2*d*2^(1/2)*Pi^(1/2)
*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(3/
2)/c-1/2*d*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelS(6^(1/2)/Pi^(1/2)*(a+b*arcc
os(c*x))^(1/2)/b^(1/2))/b^(3/2)/c+3/2*d*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/
Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/b^(3/2)/c+1/2*d*6^(1/2)
*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(3
*a/b)/b^(3/2)/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.13

$$\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^{3/2}}\,dx = \frac{de^{-\frac{3ia}{b}}\left(6e^{\frac{3ia}{b}}\sqrt{1-c^2x^2}+3ie^{\frac{2ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(cx))}{b}\right)-3i\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(d*(6*E^(((3*I)*a)/b)*Sqrt[1 - c^2*x^2] + (3*I)*E^(((2*I)*a)/b)*Sqrt[((-I)
*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/b] - (3*I)*
E^(((4*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*ArcCo
s[c*x]))/b] - I*Sqrt[3]*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-3
*I)*(a + b*ArcCos[c*x]))/b] + I*Sqrt[3]*E^(((6*I)*a)/b)*Sqrt[(I*(a + b*Arc
Cos[c*x]))/b]*Gamma[1/2, ((3*I)*(a + b*ArcCos[c*x]))/b] - 2*E^(((3*I)*a)/b
)*Sin[3*ArcCos[c*x]]))/(4*b*c*E^(((3*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5167, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^{3/2}}\,dx$$

$\Big\downarrow$ 5167

$$\frac{6cd\int \frac{x\sqrt{1-c^2x^2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2d(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$\Big\downarrow$ 5225

$$\frac{2d(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{6d\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 4906

$$\frac{2d(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{6d\int \left(\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{4\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{4\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 2009

$$\frac{2d(1-c^2x^2)^{3/2}}{bc\sqrt{a+b\arccos(cx)}}-$$

$$\frac{6d\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\sqrt{b}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\sqrt{\right.}{b^2c}$$

input
```
Int[(d - c^2*d*x^2)/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(2*d*(1 - c^2*x^2)^(3/2))/(b*c*Sqrt[a + b*ArcCos[c*x]]) - (6*d*((Sqrt[b]*S
qrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])
/2 - (Sqrt[b]*Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcC
os[c*x]])/Sqrt[b]])/2 + (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a +
b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/2 - (Sqrt[b]*Sqrt[Pi/6]*FresnelS[(Sqrt[
6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/2))/(b^2*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.21

method	result
default	$-\frac{d\left(3\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)-3\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\right.}{}$

input
```
int((-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/c/b/(a+b*arccos(c*x))^(1/2)*(3*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*a
rccos(c*x))^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arc
cos(c*x))^(1/2)/b)-3*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)
*sin(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b
)-(-3/b)^(1/2)*Pi^(1/2)*2^(1/2)*cos(3*a/b)*FresnelC(3*2^(1/2)/Pi^(1/2)/(-3
/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(a+b*arccos(c*x))^(1/2)+(-3/b)^(1/2)*
Pi^(1/2)*2^(1/2)*sin(3*a/b)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*
arccos(c*x))^(1/2)/b)*(a+b*arccos(c*x))^(1/2)+3*sin(-(a+b*arccos(c*x))/b+a
/b)-sin(-3*(a+b*arccos(c*x))/b+3*a/b))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^{3/2}}\,dx = -d\left(\int \frac{c^2x^2}{a\sqrt{a+b\,\mathrm{acos}\,(cx)}+b\sqrt{a+b\,\mathrm{acos}\,(cx)}\,\mathrm{acos}\,(cx)}\,dx + \int \left(-\frac{1}{a\sqrt{a+b\,\mathrm{acos}\,(cx)}+b\sqrt{a+b\,\mathrm{acos}\,(cx)}\,\mathrm{acos}\,(cx)}\right)dx\right)$$

input
```
integrate((-c**2*d*x**2+d)/(a+b*acos(c*x))**(3/2),x)
```

output
```
-d*(Integral(c**2*x**2/(a*sqrt(a + b*acos(c*x)) + b*sqrt(a + b*acos(c*x))*
acos(c*x)), x) + Integral(-1/(a*sqrt(a + b*acos(c*x)) + b*sqrt(a + b*acos(
c*x))*acos(c*x)), x))
```

Maxima [F]

$$\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int -\frac{c^2dx^2-d}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
-integrate((c^2*d*x^2 - d)/(b*arccos(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int -\frac{c^2dx^2-d}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c^2*d*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(-(c^2*d*x^2 - d)/(b*arccos(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{d-c^2\,d\,x^2}{(a+b\,\mathrm{acos}\,(c\,x))^{3/2}}\,dx$$

input
```
int((d - c^2*d*x^2)/(a + b*acos(c*x))^(3/2),x)
```

output
```
int((d - c^2*d*x^2)/(a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{d-c^2dx^2}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((-c^2*d*x^2+d)/(a+b*acos(c*x))^(3/2),x)
```

output
```
(d*( - 12*sqrt(acos(c*x)*b + a)*sqrt( - c**2*x**2 + 1)*acos(c*x)*b + 12*ac
os(c*x)*int(sqrt(acos(c*x)*b + a)/(acos(c*x)**2*b**2*c**2*x**2 - acos(c*x)
**2*b**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b + a**2*c**2*x**2 -
a**2),x)*a*b**2*c - 12*acos(c*x)*int((sqrt(acos(c*x)*b + a)*x**4)/(acos(c*
x)**2*b**2*c**2*x**2 - acos(c*x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2*a
cos(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a*b**2*c**5 - 24*acos(c*x)*int((s
qrt(acos(c*x)*b + a)*sqrt( - c**2*x**2 + 1)*acos(c*x)*x**3)/(acos(c*x)**2*
b**2*c**2*x**2 - acos(c*x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*
x)*a*b + a**2*c**2*x**2 - a**2),x)*a*b**2*c**4 + 12*acos(c*x)*int((sqrt(ac
os(c*x)*b + a)*sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x)/(acos(c*x)**2*b**2*c
**2*x**2 - acos(c*x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b
 + a**2*c**2*x**2 - a**2),x)*b**3*c**2 - 24*acos(c*x)*int((sqrt(acos(c*x)*
b + a)*sqrt( - c**2*x**2 + 1)*x**3)/(acos(c*x)**2*b**2*c**2*x**2 - acos(c*
x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b + a**2*c**2*x**2
- a**2),x)*a**2*b*c**4 - 12*acos(c*x)*int((sqrt(acos(c*x)*b + a)*sqrt( - c
**2*x**2 + 1)*x)/(acos(c*x)**2*b**2*c**2*x**2 - acos(c*x)**2*b**2 + 2*acos
(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a**2*b*c
**2 - 3*acos(c*x)*int((sqrt(acos(c*x)*b + a)*sqrt( - c**2*x**2 + 1)*x)/(ac
os(c*x)**2*b**2*c**2*x**2 - acos(c*x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**2
- 2*acos(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*b**3*c**2 + 8*sqrt(acos(c...
```

3.437 $\int \frac{d-c^2dx^2}{x(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{d-c^2dx^2}{x(a+b\arccos(cx))^{3/2}}\,dx=-\frac{2d(1-c^2x^2)^{3/2}}{bcx\sqrt{a+b\arccos(cx)}}-\frac{2d\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{b^{3/2}}-\frac{2d\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{b^{3/2}}-\frac{2d\text{Int}\left(\frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}},x\right)}{bc}$$

output

```
-2*d*(-c^2*x^2+1)^(3/2)/b/c/x/(a+b*arccos(c*x))^(1/2)-2*d*Pi^(1/2)*cos(2*a
/b)*FresnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))/b^(3/2)-2*d*Pi^(1
/2)*FresnelS(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*sin(2*a/b)/b^(3/2
)-2*d*Defer(Int)(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^(1/2),x)/b/c
```

Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{d-c^2dx^2}{x(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{d-c^2dx^2}{x(a+b\arccos(cx))^{3/2}}\,dx$$

input
```
Integrate[(d - c^2*d*x^2)/(x*(a + b*ArcCos[c*x])^(3/2)),x]
```

output
```
Integrate[(d - c^2*d*x^2)/(x*(a + b*ArcCos[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-c^2dx^2}{x(a+b\arccos(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{5215}$$

$$\frac{4cd\int \frac{\sqrt{1-c^2x^2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2d\int \frac{\sqrt{1-c^2x^2}}{x^2\sqrt{a+b\arccos(cx)}}dx}{bc}+\frac{2d(1-c^2x^2)^{3/2}}{bcx\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{5169}$$

$$-\frac{4d\int \frac{\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2}+\frac{2d\int \frac{\sqrt{1-c^2x^2}}{x^2\sqrt{a+b\arccos(cx)}}dx}{bc}+\frac{2d(1-c^2x^2)^{3/2}}{bcx\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4d\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^2}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2}+\frac{2d\int \frac{\sqrt{1-c^2x^2}}{x^2\sqrt{a+b\arccos(cx)}}dx}{bc}+\frac{2d\left(1-c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{4d\int \left(\frac{1}{2\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{b^2}+\frac{2d\int \frac{\sqrt{1-c^2x^2}}{x^2\sqrt{a+b\arccos(cx)}}dx}{bc}+\frac{2d\left(1-c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2d\int \frac{\sqrt{1-c^2x^2}}{x^2\sqrt{a+b\arccos(cx)}}dx}{bc}-\frac{4d\left(-\frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)+\sqrt{a+b\arccos}\right.}{b^2}\frac{2d\left(1-c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{ 5227}$$

$$\frac{2d\int \left(\frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}-\frac{c^2}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}\right)dx}{bc}-\frac{4d\left(-\frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)+\sqrt{a+b\arccos}\right.}{b^2}\frac{2d\left(1-c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2d\left(\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}dx+\frac{2c\sqrt{a+b\arccos(cx)}}{b}\right)}{bc}-\frac{4d\left(-\frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)+\sqrt{a+b\arccos}\right.}{b^2}\frac{2d\left(1-c^2x^2\right)^{3/2}}{bcx\sqrt{a+b\arccos(cx)}}$$

input
```
Int[(d - c^2*d*x^2)/(x*(a + b*ArcCos[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{-c^2 d\, x^2+d}{x\left(a+b\arccos\left(cx\right)\right)^{\frac{3}{2}}}dx$$

input
```
int((-c^2*d*x^2+d)/x/(a+b*arccos(c*x))^(3/2),x)
```

output
```
int((-c^2*d*x^2+d)/x/(a+b*arccos(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d-c^2dx^2}{x(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)/x/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 4.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.22

$$\int \frac{d-c^2dx^2}{x(a+b\arccos(cx))^{3/2}}\,dx = -d\left(\int \frac{c^2x^2}{ax\sqrt{a+b\operatorname{acos}(cx)}+bx\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\,dx + \int \left(-\frac{1}{ax\sqrt{a+b\operatorname{acos}(cx)}+bx\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\right)dx\right)$$

input
```
integrate((-c**2*d*x**2+d)/x/(a+b*acos(c*x))**(3/2),x)
```

output
```
-d*(Integral(c**2*x**2/(a*x*sqrt(a + b*acos(c*x)) + b*x*sqrt(a + b*acos(c*
x))*acos(c*x)), x) + Integral(-1/(a*x*sqrt(a + b*acos(c*x)) + b*x*sqrt(a +
 b*acos(c*x))*acos(c*x)), x))
```

Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{d-c^2dx^2}{x(a+b\arccos(cx))^{3/2}}\,dx = \int -\frac{c^2dx^2-d}{(b\arccos(cx)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)/x/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
-integrate((c^2*d*x^2 - d)/((b*arccos(c*x) + a)^(3/2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d-c^2dx^2}{x(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)/x/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{d-c^2dx^2}{x(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{d-c^2\,d\,x^2}{x\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^{3/2}}\,dx$$

input
```
int((d - c^2*d*x^2)/(x*(a + b*acos(c*x))^(3/2)),x)
```

output
```
int((d - c^2*d*x^2)/(x*(a + b*acos(c*x))^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.15

$$\int \frac{d-c^2dx^2}{x(a+b\arccos(cx))^{3/2}}\,dx = d\left(\int \frac{\sqrt{acos\,(cx)\,b+a}}{acos\,(cx)^2\,b^2x+2acos\,(cx)\,abx+a^2x}dx - \left(\int \frac{\sqrt{acos\,(cx)\,b+a}\,x}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2\right)$$

input
```
int((-c^2*d*x^2+d)/x/(a+b*acos(c*x))^(3/2),x)
```

output
```
d*(int(sqrt(acos(c*x)*b + a)/(acos(c*x)**2*b**2*x + 2*acos(c*x)*a*b*x + a*
*2*x),x) - int((sqrt(acos(c*x)*b + a)*x)/(acos(c*x)**2*b**2 + 2*acos(c*x)*
a*b + a**2),x)*c**2)
```

3.438 $\int \frac{x^3 (d-c^2 d x^2)^2}{(a+b \arccos(cx))^{3/2}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 485

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = -\frac{2d^2x^3(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}} + \frac{d^2\sqrt{\frac{\pi}{2}}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^4} - \frac{d^2\sqrt{3\pi}\cos\left(\frac{6a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} + \frac{3d^2\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{16b^{3/2}c^4} - \frac{d^2\sqrt{\pi}\cos\left(\frac{8a}{b}\right)\text{FresnelC}\left(\frac{4\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{16b^{3/2}c^4} + \frac{3d^2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{16b^{3/2}c^4} + \frac{d^2\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{4a}{b}\right)}{8b^{3/2}c^4} - \frac{d^2\sqrt{3\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{6a}{b}\right)}{16b^{3/2}c^4} - \frac{d^2\sqrt{\pi}\,\text{FresnelS}\left(\frac{4\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{8a}{b}\right)}{16b^{3/2}c^4}$$

output
```
-2*d^2*x^3*(-c^2*x^2+1)^(5/2)/b/c/(a+b*arccos(c*x))^(1/2)+1/16*d^2*2^(1/2)
*Pi^(1/2)*cos(4*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b
^(1/2))/b^(3/2)/c^4-1/16*d^2*3^(1/2)*Pi^(1/2)*cos(6*a/b)*FresnelC(2*3^(1/2
)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^4+3/16*d^2*Pi^(1/2)*
cos(2*a/b)*FresnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))/b^(3/2)/c^
4-1/16*d^2*Pi^(1/2)*cos(8*a/b)*FresnelC(4*(a+b*arccos(c*x))^(1/2)/b^(1/2)/
Pi^(1/2))/b^(3/2)/c^4+3/16*d^2*Pi^(1/2)*FresnelS(2*(a+b*arccos(c*x))^(1/2)
/b^(1/2)/Pi^(1/2))*sin(2*a/b)/b^(3/2)/c^4+1/16*d^2*2^(1/2)*Pi^(1/2)*Fresne
lS(2*2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(4*a/b)/b^(3/2)/
c^4-1/16*d^2*3^(1/2)*Pi^(1/2)*FresnelS(2*3^(1/2)/Pi^(1/2)*(a+b*arccos(c*x)
)^(1/2)/b^(1/2))*sin(6*a/b)/b^(3/2)/c^4-1/16*d^2*Pi^(1/2)*FresnelS(4*(a+b*
arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*sin(8*a/b)/b^(3/2)/c^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.11

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx =$$

$$-\frac{id^2e^{-\frac{8ia}{b}}\left(-3\sqrt{2}e^{\frac{6ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},-\frac{2i(a+b\arccos(cx))}{b}\right)+3\sqrt{2}e^{\frac{10ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},\frac{2i(a+b\arccos(\right.\right.}{}$$

input
```
Integrate[(x^3*(d - c^2*d*x^2)^2)/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
((-1/64*I)*d^2*(-3*Sqrt[2]*E^(((6*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))
/b]*Gamma[1/2, ((-2*I)*(a + b*ArcCos[c*x]))/b] + 3*Sqrt[2]*E^(((10*I)*a)/b
)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((2*I)*(a + b*ArcCos[c*x]))/b
] + 2*E^(((4*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-4*I
)*(a + b*ArcCos[c*x]))/b] - 2*E^(((12*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x])
)/b]*Gamma[1/2, ((4*I)*(a + b*ArcCos[c*x]))/b] + Sqrt[6]*E^(((2*I)*a)/b)*S
qrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-6*I)*(a + b*ArcCos[c*x]))/
b] - Sqrt[6]*E^(((14*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (
(6*I)*(a + b*ArcCos[c*x]))/b] - Sqrt[2]*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]
*Gamma[1/2, ((-8*I)*(a + b*ArcCos[c*x]))/b] + Sqrt[2]*E^(((16*I)*a)/b)*Sqr
t[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((8*I)*(a + b*ArcCos[c*x]))/b] + (
6*I)*E^(((8*I)*a)/b)*Sin[2*ArcCos[c*x]] - (2*I)*E^(((8*I)*a)/b)*Sin[4*ArcC
os[c*x]] - (2*I)*E^(((8*I)*a)/b)*Sin[6*ArcCos[c*x]] + I*E^(((8*I)*a)/b)*Si
n[8*ArcCos[c*x]]))/(b*c^4*E^(((8*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {5215, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (d-c^2 d x^2)^2}{(a+b\arccos(cx))^{3/2}} \, dx$$

$$\Big\downarrow \text{5215}$$

$$-\frac{6d^2 \int \frac{x^2(1-c^2x^2)^{3/2}}{\sqrt{a+b\arccos(cx)}}dx}{bc} + \frac{16cd^2 \int \frac{x^4(1-c^2x^2)^{3/2}}{\sqrt{a+b\arccos(cx)}}dx}{b} + \frac{2d^2x^3(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{5225}$$

$$-\frac{16d^2 \int \frac{\cos^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^4} + \frac{6d^2 \int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^4} + \frac{2d^2x^3(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \; 4906$$

$$-\frac{16d^2\int \left(\frac{\cos\left(\frac{8a}{b}-\frac{8(a+b\arccos(cx))}{b}\right)}{128\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{32\sqrt{a+b\arccos(cx)}}+\frac{3}{128\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{b^2c^4}+$$

$$\frac{6d^2\int \left(\frac{\cos\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)}{32\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{16\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{32\sqrt{a+b\arccos(cx)}}+\frac{1}{16\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos}{b^2c^4}$$

$$\frac{2d^2x^3\left(1-c^2x^2\right)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \; 2009$$

$$\frac{6d^2\left(-\frac{1}{16}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\frac{\pi}{3}}\sqrt{b}\cos\left(\frac{6a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\right.}{}$$

$$\frac{16d^2\left(-\frac{1}{32}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\frac{1}{256}\sqrt{\pi}\sqrt{b}\cos\left(\frac{8a}{b}\right)\text{FresnelC}\left(\frac{4\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\frac{1}{32}\right.}{}$$

$$\frac{2d^2x^3\left(1-c^2x^2\right)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

input
```
Int[(x^3*(d - c^2*d*x^2)^2)/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(2*d^2*x^3*(1 - c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcCos[c*x]]) + (6*d^2*(Sq
rt[a + b*ArcCos[c*x]]/8 - (Sqrt[b]*Sqrt[Pi/2]*Cos[(4*a)/b]*FresnelC[(2*Sqr
t[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/16 + (Sqrt[b]*Sqrt[Pi/3]*Cos[(6
*a)/b]*FresnelC[(2*Sqrt[3/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/32 - (Sqr
t[b]*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*S
qrt[Pi])])/32 - (Sqrt[b]*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sq
rt[b]*Sqrt[Pi])]*Sin[(2*a)/b])/32 - (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[2
/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(4*a)/b])/16 + (Sqrt[b]*Sqrt[Pi
/3]*FresnelS[(2*Sqrt[3/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(6*a)/b])
/32))/(b^2*c^4) - (16*d^2*((3*Sqrt[a + b*ArcCos[c*x]])/64 - (Sqrt[b]*Sqrt[
Pi/2]*Cos[(4*a)/b]*FresnelC[(2*Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]
])/32 + (Sqrt[b]*Sqrt[Pi]*Cos[(8*a)/b]*FresnelC[(4*Sqrt[a + b*ArcCos[c*x]]
)/(Sqrt[b]*Sqrt[Pi])])/256 - (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[2/Pi]*Sq
rt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(4*a)/b])/32 + (Sqrt[b]*Sqrt[Pi]*Fresn
elS[(4*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])]*Sin[(8*a)/b])/256))/(b
^2*c^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.21

method	result
default	$-\frac{d^2\left(-4\sqrt{a+b\arccos(cx)}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{2}\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\sqrt{-\frac{1}{b}}\,b}\right)\sqrt{-\frac{1}{b}}\sqrt{\pi}\sqrt{2}+4\sqrt{a+b\arccos(cx)}\sin\left(\frac{4a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{2}\sqrt{a+}}{\sqrt{\pi}}\right.\right.}{}$

input
```
int(x^3*(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64*d^2/c^4/b/(a+b*arccos(c*x))^(1/2)*(-4*(a+b*arccos(c*x))^(1/2)*cos(4*
a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(
-1/b)^(1/2)*Pi^(1/2)*2^(1/2)+4*(a+b*arccos(c*x))^(1/2)*sin(4*a/b)*FresnelS
(2*2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-1/b)^(1/2)*P
i^(1/2)*2^(1/2)-2*(-6/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*co
s(6*a/b)*FresnelC(6*2^(1/2)/Pi^(1/2)/(-6/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/
b)+2*(-6/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(6*a/b)*Fres
nelS(6*2^(1/2)/Pi^(1/2)/(-6/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+12*(-1/b)^
(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1
/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-12*(-1/b)^(1/2)*Pi^(1/2)*(a+b*
arccos(c*x))^(1/2)*sin(2*a/b)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+
b*arccos(c*x))^(1/2)/b)+4*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*co
s(8*a/b)*FresnelC(4*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/
b)-4*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(8*a/b)*FresnelS(4*2
^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+6*sin(-2*(a+b*arcc
os(c*x))/b+2*a/b)-2*sin(-4*(a+b*arccos(c*x))/b+4*a/b)-2*sin(-6*(a+b*arccos
(c*x))/b+6*a/b)+sin(-8*(a+b*arccos(c*x))/b+8*a/b))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = d^2\left(\int \frac{x^3}{a\sqrt{a+b\operatorname{acos}(cx)}+b\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\,dx + \int \left(-\frac{2c^2x^5}{a\sqrt{a+b\operatorname{acos}(cx)}+b\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\right)dx + \int \frac{c^4x^7}{a\sqrt{a+b\operatorname{acos}(cx)}+b\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\,dx\right)$$

input
```
integrate(x**3*(-c**2*d*x**2+d)**2/(a+b*acos(c*x))**(3/2),x)
```

output
```
d**2*(Integral(x**3/(a*sqrt(a + b*acos(c*x)) + b*sqrt(a + b*acos(c*x))*aco
s(c*x)), x) + Integral(-2*c**2*x**5/(a*sqrt(a + b*acos(c*x)) + b*sqrt(a +
b*acos(c*x))*acos(c*x)), x) + Integral(c**4*x**7/(a*sqrt(a + b*acos(c*x))
+ b*sqrt(a + b*acos(c*x))*acos(c*x)), x))
```

Maxima [F]

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2-d)^2x^3}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 - d)^2*x^3/(b*arccos(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2-d)^2x^3}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x, algorithm="giac"
)
```

output
```
integrate((c^2*d*x^2 - d)^2*x^3/(b*arccos(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{x^3\,(d-c^2\,d\,x^2)^2}{(a+b\,\mathrm{acos}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x^3*(d - c^2*d*x^2)^2)/(a + b*acos(c*x))^(3/2),x)
```

output
```
int((x^3*(d - c^2*d*x^2)^2)/(a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = d^2\left(\left(\int \frac{\sqrt{acos\,(cx)\,b+a}\,x^7}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{acos\,(cx)\,b+a}\,x^5}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{acos\,(cx)\,b+a}\,x^3}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)$$

input
```
int(x^3*(-c^2*d*x^2+d)^2/(a+b*acos(c*x))^(3/2),x)
```

output
```
d**2*(int((sqrt(acos(c*x)*b + a)*x**7)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*
b + a**2),x)*c**4 - 2*int((sqrt(acos(c*x)*b + a)*x**5)/(acos(c*x)**2*b**2
+ 2*acos(c*x)*a*b + a**2),x)*c**2 + int((sqrt(acos(c*x)*b + a)*x**3)/(acos
(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x))
```

3.439 $\int \frac{x^2(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 511

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = -\frac{2d^2x^2(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{5d^2\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} + \frac{d^2\sqrt{\frac{3\pi}{2}}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} + \frac{3d^2\sqrt{\frac{5\pi}{2}}\cos\left(\frac{5a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} + \frac{d^2\sqrt{\frac{7\pi}{2}}\cos\left(\frac{7a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{14}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3} + \frac{5d^2\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{16b^{3/2}c^3} - \frac{d^2\sqrt{\frac{3\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{16b^{3/2}c^3} - \frac{3d^2\sqrt{\frac{5\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{5a}{b}\right)}{16b^{3/2}c^3} - \frac{d^2\sqrt{\frac{7\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{14}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{7a}{b}\right)}{16b^{3/2}c^3}$$

output
```
-2*d^2*x^2*(-c^2*x^2+1)^(5/2)/b/c/(a+b*arccos(c*x))^(1/2)-5/32*d^2*2^(1/2)
*Pi^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/
2))/b^(3/2)/c^3+1/32*d^2*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelS(6^(1/2)/Pi^(
1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3+3/32*d^2*10^(1/2)*Pi^(1/
2)*cos(5*a/b)*FresnelS(10^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/
b^(3/2)/c^3+1/32*d^2*14^(1/2)*Pi^(1/2)*cos(7*a/b)*FresnelS(14^(1/2)/Pi^(1/
2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3+5/32*d^2*2^(1/2)*Pi^(1/2)*
FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/b^(3/2
)/c^3-1/32*d^2*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x)
)^(1/2)/b^(1/2))*sin(3*a/b)/b^(3/2)/c^3-3/32*d^2*10^(1/2)*Pi^(1/2)*Fresnel
C(10^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(5*a/b)/b^(3/2)/c^
3-1/32*d^2*14^(1/2)*Pi^(1/2)*FresnelC(14^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^
(1/2)/b^(1/2))*sin(7*a/b)/b^(3/2)/c^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.09

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = \frac{d^2e^{-\frac{7ia}{b}}\left(10e^{\frac{7ia}{b}}\sqrt{1-c^2x^2}+5ie^{\frac{6ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(cx))}{b}\right)-\right.}{}$$

input
```
Integrate[(x^2*(d - c^2*d*x^2)^2)/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(d^2*(10*E^(((7*I)*a)/b)*Sqrt[1 - c^2*x^2] + (5*I)*E^(((6*I)*a)/b)*Sqrt[((
-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/b] - (5*
I)*E^(((8*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*Ar
cCos[c*x]))/b] + I*Sqrt[3]*E^(((4*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))
/b]*Gamma[1/2, ((-3*I)*(a + b*ArcCos[c*x]))/b] - I*Sqrt[3]*E^(((10*I)*a)/b
)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((3*I)*(a + b*ArcCos[c*x]))/b
] - (3*I)*Sqrt[5]*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma
[1/2, ((-5*I)*(a + b*ArcCos[c*x]))/b] + (3*I)*Sqrt[5]*E^(((12*I)*a)/b)*Sqr
t[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((5*I)*(a + b*ArcCos[c*x]))/b] + I
*Sqrt[7]*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-7*I)*(a + b*ArcC
os[c*x]))/b] - I*Sqrt[7]*E^(((14*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*
Gamma[1/2, ((7*I)*(a + b*ArcCos[c*x]))/b] + 2*E^(((7*I)*a)/b)*Sin[3*ArcCos
[c*x]] - 6*E^(((7*I)*a)/b)*Sin[5*ArcCos[c*x]] + 2*E^(((7*I)*a)/b)*Sin[7*Ar
cCos[c*x]]))/(64*b*c^3*E^(((7*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 803, normalized size of antiderivative = 1.57, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5215, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (d-c^2 d x^2)^2}{(a+b \arccos(cx))^{3/2}} \, dx$$

$$\Big\downarrow \ 5215$$

$$-\frac{4d^2 \int \frac{x(1-c^2x^2)^{3/2}}{\sqrt{a+b\arccos(cx)}}dx}{bc}+\frac{14cd^2 \int \frac{x^3(1-c^2x^2)^{3/2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2d^2x^2(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \ 5225$$

$$-\frac{14d^2 \int \frac{\cos^3\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^3}+\frac{4d^2 \int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^3}+\frac{2d^2x^2(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{14d^2\int\left(\frac{\cos\left(\frac{7a}{b}-\frac{7(a+b\arccos(cx))}{b}\right)}{64\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{64\sqrt{a+b\arccos(cx)}}-\frac{3\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{64\sqrt{a+b\arccos(cx)}}+\frac{3\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{64\sqrt{a+b\arccos(cx)}}\right)d(a+}{b^2c^3}$$
$$\frac{4d^2\int\left(\frac{\cos\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{16\sqrt{a+b\arccos(cx)}}-\frac{3\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{16\sqrt{a+b\arccos(cx)}}+\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{8\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{b^2c^3}+$$
$$\frac{2d^2x^2\left(1-c^2x^2\right)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d^2x^2\left(1-c^2x^2\right)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}+$$
$$\frac{4d^2\left(\frac{1}{4}\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{b}\sqrt{}\right.}{}$$
$$\frac{14d^2\left(\frac{3}{32}\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{32}\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{32}\right.}{}$$

input
```
Int[(x^2*(d - c^2*d*x^2)^2)/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(2*d^2*x^2*(1 - c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcCos[c*x]]) + (4*d^2*((S
qrt[b]*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/S
qrt[b]])/4 - (Sqrt[b]*Sqrt[(3*Pi)/2]*Cos[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqr
t[a + b*ArcCos[c*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi/10]*Cos[(5*a)/b]*Fres
nelC[(Sqrt[10/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi/
2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/4 - (S
qrt[b]*Sqrt[(3*Pi)/2]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b
]]*Sin[(3*a)/b])/8 + (Sqrt[b]*Sqrt[Pi/10]*FresnelS[(Sqrt[10/Pi]*Sqrt[a + b
*ArcCos[c*x]])/Sqrt[b]]*Sin[(5*a)/b])/8))/(b^2*c^3) - (14*d^2*((3*Sqrt[b]*
Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]
)/32 - (Sqrt[b]*Sqrt[(3*Pi)/2]*Cos[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a +
b*ArcCos[c*x]])/Sqrt[b]])/32 - (Sqrt[b]*Sqrt[Pi/10]*Cos[(5*a)/b]*FresnelC[
(Sqrt[10/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/32 + (Sqrt[b]*Sqrt[Pi/14]*
Cos[(7*a)/b]*FresnelC[(Sqrt[14/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/32 +
 (3*Sqrt[b]*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[
b]]*Sin[a/b])/32 - (Sqrt[b]*Sqrt[(3*Pi)/2]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b
*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/32 - (Sqrt[b]*Sqrt[Pi/10]*FresnelS[(
Sqrt[10/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(5*a)/b])/32 + (Sqrt[b]*
Sqrt[Pi/14]*FresnelS[(Sqrt[14/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(7
*a)/b])/32))/(b^2*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.16

method	result
default	$-\frac{d^{2}\left(5\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)-5\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\right.\right.}{}$

input
```
int(x^2*(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*d^2/c^3/b*(5*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*c
os(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-
5*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelS(
2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-3*(-5/b)^(1/2)*Pi
^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(5*a/b)*FresnelC(5*2^(1/2)/Pi^(1
/2)/(-5/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+3*(-5/b)^(1/2)*Pi^(1/2)*2^(1/2
)*(a+b*arccos(c*x))^(1/2)*sin(5*a/b)*FresnelS(5*2^(1/2)/Pi^(1/2)/(-5/b)^(1
/2)*(a+b*arccos(c*x))^(1/2)/b)+Pi^(1/2)*2^(1/2)*(-7/b)^(1/2)*(a+b*arccos(c
*x))^(1/2)*cos(7*a/b)*FresnelC(7*2^(1/2)/Pi^(1/2)/(-7/b)^(1/2)*(a+b*arccos
(c*x))^(1/2)/b)-Pi^(1/2)*2^(1/2)*(-7/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(
7*a/b)*FresnelS(7*2^(1/2)/Pi^(1/2)/(-7/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)
+(-3/b)^(1/2)*Pi^(1/2)*2^(1/2)*cos(3*a/b)*FresnelC(3*2^(1/2)/Pi^(1/2)/(-3/
b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(a+b*arccos(c*x))^(1/2)-(-3/b)^(1/2)*P
i^(1/2)*2^(1/2)*sin(3*a/b)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*a
rccos(c*x))^(1/2)/b)*(a+b*arccos(c*x))^(1/2)+5*sin(-(a+b*arccos(c*x))/b+a/
b)+sin(-3*(a+b*arccos(c*x))/b+3*a/b)-3*sin(-5*(a+b*arccos(c*x))/b+5*a/b)+s
in(-7*(a+b*arccos(c*x))/b+7*a/b))/(a+b*arccos(c*x))^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = d^2\left(\int \frac{x^2}{a\sqrt{a+b\,\mathrm{acos}\,(cx)}+b\sqrt{a+b\,\mathrm{acos}\,(cx)}\,\mathrm{acos}\,(cx)}\,dx + \int \left(-\frac{2c^2x^4}{a\sqrt{a+b\,\mathrm{acos}\,(cx)}+b\sqrt{a+b\,\mathrm{acos}\,(cx)}\,\mathrm{acos}\,(cx)}\right)dx + \int \frac{c^4x^6}{a\sqrt{a+b\,\mathrm{acos}\,(cx)}+b\sqrt{a+b\,\mathrm{acos}\,(cx)}\,\mathrm{acos}\,(cx)}\,dx\right)$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**2/(a+b*acos(c*x))**(3/2),x)
```

output
```
d**2*(Integral(x**2/(a*sqrt(a + b*acos(c*x)) + b*sqrt(a + b*acos(c*x))*aco
s(c*x)), x) + Integral(-2*c**2*x**4/(a*sqrt(a + b*acos(c*x)) + b*sqrt(a +
b*acos(c*x))*acos(c*x)), x) + Integral(c**4*x**6/(a*sqrt(a + b*acos(c*x))
 + b*sqrt(a + b*acos(c*x))*acos(c*x)), x))
```

Maxima [F]

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2-d)^2x^2}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((c^2*d*x^2 - d)^2*x^2/(b*arccos(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2-d)^2x^2}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x, algorithm="giac"
)
```

output
```
integrate((c^2*d*x^2 - d)^2*x^2/(b*arccos(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x^2\,(d-c^2\,d\,x^2)^2}{(a+b\,\mathrm{acos}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x^2*(d - c^2*d*x^2)^2)/(a + b*acos(c*x))^(3/2),x)
```

output
```
int((x^2*(d - c^2*d*x^2)^2)/(a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = d^2\left(\left(\left(\int \frac{\sqrt{acos\,(cx)\,b+a}\,x^6}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{acos\,(cx)\,b+a}\,x^4}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{acos\,(cx)\,b+a}\,x^2}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)\right.$$

input
```
int(x^2*(-c^2*d*x^2+d)^2/(a+b*acos(c*x))^(3/2),x)
```

output
```
d**2*(int((sqrt(acos(c*x)*b + a)*x**6)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*
b + a**2),x)*c**4 - 2*int((sqrt(acos(c*x)*b + a)*x**4)/(acos(c*x)**2*b**2
+ 2*acos(c*x)*a*b + a**2),x)*c**2 + int((sqrt(acos(c*x)*b + a)*x**2)/(acos
(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x))
```

3.440 $\int \frac{x(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 373

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = -\frac{2d^2x(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}} + \frac{d^2\sqrt{\frac{\pi}{2}}\cos\left(\frac{4a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2} + \frac{d^2\sqrt{3\pi}\cos\left(\frac{6a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^2} + \frac{5d^2\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{8b^{3/2}c^2} + \frac{5d^2\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{8b^{3/2}c^2} + \frac{d^2\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{4a}{b}\right)}{b^{3/2}c^2} + \frac{d^2\sqrt{3\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{6a}{b}\right)}{8b^{3/2}c^2}$$

output
```
-2*d^2*x*(-c^2*x^2+1)^(5/2)/b/c/(a+b*arccos(c*x))^(1/2)+1/2*d^2*2^(1/2)*Pi
^(1/2)*cos(4*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1
/2))/b^(3/2)/c^2+1/8*d^2*3^(1/2)*Pi^(1/2)*cos(6*a/b)*FresnelC(2*3^(1/2)/Pi
^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^2+5/8*d^2*Pi^(1/2)*cos(2
*a/b)*FresnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))/b^(3/2)/c^2+5/8
*d^2*Pi^(1/2)*FresnelS(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*sin(2*a
/b)/b^(3/2)/c^2+1/2*d^2*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*(a+b*
arccos(c*x))^(1/2)/b^(1/2))*sin(4*a/b)/b^(3/2)/c^2+1/8*d^2*3^(1/2)*Pi^(1/2
)*FresnelS(2*3^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(6*a/b)/
b^(3/2)/c^2
```

Mathematica [F]

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{x(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx$$

input
```
Integrate[(x*(d - c^2*d*x^2)^2)/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
Integrate[(x*(d - c^2*d*x^2)^2)/(a + b*ArcCos[c*x])^(3/2), x]
```

Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.62, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5215, 5169, 3042, 3793, 2009, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(d-c^2dx^2\right)^2}{(a+b\arccos(cx))^{3/2}}\,dx$$

$$\Big\downarrow \ 5215$$

$$-\frac{2d^2\int \frac{(1-c^2x^2)^{3/2}}{\sqrt{a+b\arccos(cx)}}dx}{bc}+\frac{12cd^2\int \frac{x^2(1-c^2x^2)^{3/2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2d^2x(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$\downarrow$ 5169

$$\frac{2d^2\int \frac{\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^2}+\frac{12cd^2\int \frac{x^2(1-c^2x^2)^{3/2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2d^2x(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$\downarrow$ 3042

$$\frac{2d^2\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^4}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^2}+\frac{12cd^2\int \frac{x^2(1-c^2x^2)^{3/2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2d^2x(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$\downarrow$ 3793

$$\frac{2d^2\int \left(\frac{\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2\sqrt{a+b\arccos(cx)}}+\frac{3}{8\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{b^2c^2}+$$
$$\frac{12cd^2\int \frac{x^2(1-c^2x^2)^{3/2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2d^2x(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$\downarrow$ 2009

$$\frac{12cd^2\int \frac{x^2(1-c^2x^2)^{3/2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+$$
$$\frac{2d^2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\,s}{b^2c}$$
$$\frac{2d^2x(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$\downarrow$ 5225

$$-\frac{12d^2\int \frac{\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c^2}+$$
$$\frac{2d^2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\,s}{b^2c}$$
$$\frac{2d^2x(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$\downarrow$ 4906

$$-\frac{12d^2\int\left(\frac{\cos\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)}{32\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{16\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{32\sqrt{a+b\arccos(cx)}}+\frac{1}{16\sqrt{a+b\arccos(cx)}}\right)d(a+b\,\mathrm{arc}}{b^2c^2}$$

$$\frac{2d^2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\,\mathrm{s}}{b^2c}$$

$$\frac{2d^2x\left(1-c^2x^2\right)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$\downarrow$ 2009

$$\frac{2d^2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\,\mathrm{s}}{b^2c}$$

$$\frac{12d^2\left(-\frac{1}{16}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\frac{1}{32}\sqrt{\frac{\pi}{3}}\sqrt{b}\cos\left(\frac{6a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-}{}$$

$$\frac{2d^2x\left(1-c^2x^2\right)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

input

```
Int[(x*(d - c^2*d*x^2)^2)/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(2*d^2*x*(1 - c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcCos[c*x]]) + (2*d^2*((3*S
qrt[a + b*ArcCos[c*x]])/4 + (Sqrt[b]*Sqrt[Pi/2]*Cos[(4*a)/b]*FresnelC[(2*S
qrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/8 - (Sqrt[b]*Sqrt[Pi]*Cos[(2*
a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])])/2 - (Sqrt[
b]*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])]*Sin[(
2*a)/b])/2 + (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[2/Pi]*Sqrt[a + b*ArcCos[
c*x]])/Sqrt[b]]*Sin[(4*a)/b])/8))/(b^2*c^2) - (12*d^2*(Sqrt[a + b*ArcCos[c
*x]]/8 - (Sqrt[b]*Sqrt[Pi/2]*Cos[(4*a)/b]*FresnelC[(2*Sqrt[2/Pi]*Sqrt[a +
b*ArcCos[c*x]])/Sqrt[b]])/16 + (Sqrt[b]*Sqrt[Pi/3]*Cos[(6*a)/b]*FresnelC[(
2*Sqrt[3/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/32 - (Sqrt[b]*Sqrt[Pi]*Cos
[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])])/32 - (
Sqrt[b]*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])]*
Sin[(2*a)/b])/32 - (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[2/Pi]*Sqrt[a + b*A
rcCos[c*x]])/Sqrt[b]]*Sin[(4*a)/b])/16 + (Sqrt[b]*Sqrt[Pi/3]*FresnelS[(2*S
qrt[3/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(6*a)/b])/32))/(b^2*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.20

method	result
default	$-\frac{d^2\left(\sqrt{-\frac{6}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{6a}{b}\right)\mathrm{FresnelC}\left(\frac{6\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{6}{b}}\,b}\right)-\sqrt{-\frac{6}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{6a}{b}\right)\mathrm{FresnelS}\right.}{}$

input
```
int(x*(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*d^2/c^2/b*((-6/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos
(6*a/b)*FresnelC(6*2^(1/2)/Pi^(1/2)/(-6/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b
)-(-6/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(6*a/b)*Fresnel
S(6*2^(1/2)/Pi^(1/2)/(-6/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-8*(a+b*arccos
(c*x))^(1/2)*cos(4*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arcc
os(c*x))^(1/2)/b)*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)+8*(a+b*arccos(c*x))^(1/2)*
sin(4*a/b)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2
)/b)*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)+10*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*
x))^(1/2)*cos(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(
c*x))^(1/2)/b)-10*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(2*a/b)
*FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+5*sin
(-2*(a+b*arccos(c*x))/b+2*a/b)-4*sin(-4*(a+b*arccos(c*x))/b+4*a/b)+sin(-6*
(a+b*arccos(c*x))/b+6*a/b))/(a+b*arccos(c*x))^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = d^2\left(\int \frac{x}{a\sqrt{a+b\operatorname{acos}(cx)}+b\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\,dx + \int \left(-\frac{2c^2x^3}{a\sqrt{a+b\operatorname{acos}(cx)}+b\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\right)dx + \int \frac{c^4x^5}{a\sqrt{a+b\operatorname{acos}(cx)}+b\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\,dx\right)$$

input
```
integrate(x*(-c**2*d*x**2+d)**2/(a+b*acos(c*x))**(3/2),x)
```

output
```
d**2*(Integral(x/(a*sqrt(a + b*acos(c*x)) + b*sqrt(a + b*acos(c*x))*acos(c
*x)), x) + Integral(-2*c**2*x**3/(a*sqrt(a + b*acos(c*x)) + b*sqrt(a + b*a
cos(c*x))*acos(c*x)), x) + Integral(c**4*x**5/(a*sqrt(a + b*acos(c*x)) + b
*sqrt(a + b*acos(c*x))*acos(c*x)), x))
```

Maxima [F]

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2-d)^2x}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((c^2*d*x^2 - d)^2*x/(b*arccos(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2-d)^2x}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 - d)^2*x/(b*arccos(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x\,(d-c^2\,d\,x^2)^2}{(a+b\,\mathrm{acos}\,(c\,x))^{3/2}}\,dx$$

input
```
int((x*(d - c^2*d*x^2)^2)/(a + b*acos(c*x))^(3/2),x)
```

output
```
int((x*(d - c^2*d*x^2)^2)/(a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = d^2\left(\left(\int \frac{\sqrt{acos\,(cx)\,b+a}\,x^5}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{acos\,(cx)\,b+a}\,x^3}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)c^2 + \int \frac{\sqrt{acos\,(cx)\,b+a}\,x}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)$$

input
```
int(x*(-c^2*d*x^2+d)^2/(a+b*acos(c*x))^(3/2),x)
```

output
```
d**2*(int((sqrt(acos(c*x)*b + a)*x**5)/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*
b + a**2),x)*c**4 - 2*int((sqrt(acos(c*x)*b + a)*x**3)/(acos(c*x)**2*b**2
+ 2*acos(c*x)*a*b + a**2),x)*c**2 + int((sqrt(acos(c*x)*b + a)*x)/(acos(c*
x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x))
```

3.441 $\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 390

$$\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = -\frac{2d^2(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{5d^2\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c} - \frac{5d^2\sqrt{\frac{3\pi}{2}}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} - \frac{d^2\sqrt{\frac{5\pi}{2}}\cos\left(\frac{5a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} + \frac{5d^2\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{2b^{3/2}c} + \frac{5d^2\sqrt{\frac{3\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{4b^{3/2}c} + \frac{d^2\sqrt{\frac{5\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{5a}{b}\right)}{4b^{3/2}c}$$

output
```
-2*d^2*(-c^2*x^2+1)^(5/2)/b/c/(a+b*arccos(c*x))^(1/2)-5/4*d^2*2^(1/2)*Pi^(
1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b
^(3/2)/c-5/8*d^2*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelS(6^(1/2)/Pi^(1/2)*(a+
b*arccos(c*x))^(1/2)/b^(1/2))/b^(3/2)/c-1/8*d^2*10^(1/2)*Pi^(1/2)*cos(5*a/
b)*FresnelS(10^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(3/2)/c+5
/4*d^2*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/
b^(1/2))*sin(a/b)/b^(3/2)/c+5/8*d^2*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(
1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(3*a/b)/b^(3/2)/c+1/8*d^2*10^(1/2
)*Pi^(1/2)*FresnelC(10^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin
(5*a/b)/b^(3/2)/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.08

$$\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = \frac{d^2e^{-\frac{5ia}{b}}\left(20e^{\frac{5ia}{b}}\sqrt{1-c^2x^2}+10ie^{\frac{4ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(cx))}{b}\right)-\right.}{}$$

input
```
Integrate[(d - c^2*d*x^2)^2/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(d^2*(20*E^(((5*I)*a)/b)*Sqrt[1 - c^2*x^2] + (10*I)*E^(((4*I)*a)/b)*Sqrt[(
(-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/b] - (1
0*I)*E^(((6*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*
ArcCos[c*x]))/b] - (5*I)*Sqrt[3]*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[
c*x]))/b]*Gamma[1/2, ((-3*I)*(a + b*ArcCos[c*x]))/b] + (5*I)*Sqrt[3]*E^(((
8*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((3*I)*(a + b*ArcCos
[c*x]))/b] + I*Sqrt[5]*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-5*
I)*(a + b*ArcCos[c*x]))/b] - I*Sqrt[5]*E^(((10*I)*a)/b)*Sqrt[(I*(a + b*Arc
Cos[c*x]))/b]*Gamma[1/2, ((5*I)*(a + b*ArcCos[c*x]))/b] - 10*E^(((5*I)*a)/
b)*Sin[3*ArcCos[c*x]] + 2*E^(((5*I)*a)/b)*Sin[5*ArcCos[c*x]]))/(16*b*c*E^(
((5*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5167, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}dx$$

$$\Big\downarrow \text{5167}$$

$$\frac{10cd^2\int \frac{x(1-c^2x^2)^{3/2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2d^2(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{5225}$$

$$\frac{2d^2(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{10d^2\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{4906}$$

$$\frac{2d^2(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{10d^2\int \left(\frac{\cos\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)}{16\sqrt{a+b\arccos(cx)}}-\frac{3\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)}{16\sqrt{a+b\arccos(cx)}}+\frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{8\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d^2(1-c^2x^2)^{5/2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{10d^2\left(\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{8}\sqrt{\frac{3\pi}{2}}\sqrt{b}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\frac{1}{8}\sqrt{\right.}$$

input
```
Int[(d - c^2*d*x^2)^2/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(2*d^2*(1 - c^2*x^2)^(5/2))/(b*c*Sqrt[a + b*ArcCos[c*x]]) - (10*d^2*((Sqrt
[b]*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt
[b]])/4 - (Sqrt[b]*Sqrt[(3*Pi)/2]*Cos[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a
 + b*ArcCos[c*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi/10]*Cos[(5*a)/b]*Fresnel
C[(Sqrt[10/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/8 + (Sqrt[b]*Sqrt[Pi/2]*
FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/4 - (Sqrt
[b]*Sqrt[(3*Pi)/2]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*
Sin[(3*a)/b])/8 + (Sqrt[b]*Sqrt[Pi/10]*FresnelS[(Sqrt[10/Pi]*Sqrt[a + b*Ar
cCos[c*x]])/Sqrt[b]]*Sin[(5*a)/b])/8))/(b^2*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.15

method	result
default	$-\frac{d^2\left(10\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)-10\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\text{Fresnel}}{}$

input
```
int((-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*d^2/c/b*(10*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos
(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-10
*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelS(2
^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-5*(-3/b)^(1/2)*Pi^
(1/2)*2^(1/2)*cos(3*a/b)*FresnelC(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arc
cos(c*x))^(1/2)/b)*(a+b*arccos(c*x))^(1/2)+5*(-3/b)^(1/2)*Pi^(1/2)*2^(1/2)
*sin(3*a/b)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/
2)/b)*(a+b*arccos(c*x))^(1/2)+(-5/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*
x))^(1/2)*cos(5*a/b)*FresnelC(5*2^(1/2)/Pi^(1/2)/(-5/b)^(1/2)*(a+b*arccos(
c*x))^(1/2)/b)-(-5/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(5
*a/b)*FresnelS(5*2^(1/2)/Pi^(1/2)/(-5/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+
10*sin(-(a+b*arccos(c*x))/b+a/b)-5*sin(-3*(a+b*arccos(c*x))/b+3*a/b)+sin(-
5*(a+b*arccos(c*x))/b+5*a/b))/(a+b*arccos(c*x))^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = d^2\left(\int\left(-\frac{2c^2x^2}{a\sqrt{a+b\operatorname{acos}(cx)}+b\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\right)dx + \int\frac{c^4x^4}{a\sqrt{a+b\operatorname{acos}(cx)}+b\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\,dx + \int\frac{1}{a\sqrt{a+b\operatorname{acos}(cx)}+b\sqrt{a+b\operatorname{acos}(cx)}\operatorname{acos}(cx)}\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2/(a+b*acos(c*x))**(3/2),x)
```

output
```
d**2*(Integral(-2*c**2*x**2/(a*sqrt(a + b*acos(c*x)) + b*sqrt(a + b*acos(c
*x))*acos(c*x)), x) + Integral(c**4*x**4/(a*sqrt(a + b*acos(c*x)) + b*sqrt
(a + b*acos(c*x))*acos(c*x)), x) + Integral(1/(a*sqrt(a + b*acos(c*x)) + b
*sqrt(a + b*acos(c*x))*acos(c*x)), x))
```

Maxima [F]

$$\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2-d)^2}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c^2*d*x^2 - d)^2/(b*arccos(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{(c^2dx^2-d)^2}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 - d)^2/(b*arccos(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{(d-c^2\,d\,x^2)^2}{(a+b\,\mathrm{acos}\,(c\,x))^{3/2}}\,dx$$

input
```
int((d - c^2*d*x^2)^2/(a + b*acos(c*x))^(3/2),x)
```

output
```
int((d - c^2*d*x^2)^2/(a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(d-c^2dx^2)^2}{(a+b\arccos(cx))^{3/2}}\,dx=\text{too large to display}$$

input
```
int((-c^2*d*x^2+d)^2/(a+b*acos(c*x))^(3/2),x)
```

output

```
(d**2*( - 20*sqrt(acos(c*x)*b + a)*sqrt( - c**2*x**2 + 1)*acos(c*x)*b + 20
*acos(c*x)*int(sqrt(acos(c*x)*b + a)/(acos(c*x)**2*b**2*c**2*x**2 - acos(c
*x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b + a**2*c**2*x**2
 - a**2),x)*a*b**2*c + 8*acos(c*x)*int((sqrt(acos(c*x)*b + a)*x**6)/(acos(
c*x)**2*b**2*c**2*x**2 - acos(c*x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2
*acos(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a*b**2*c**7 - 28*acos(c*x)*int(
(sqrt(acos(c*x)*b + a)*x**4)/(acos(c*x)**2*b**2*c**2*x**2 - acos(c*x)**2*b
**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b + a**2*c**2*x**2 - a**2)
,x)*a*b**2*c**5 - 24*acos(c*x)*int((sqrt(acos(c*x)*b + a)*sqrt( - c**2*x**
2 + 1)*acos(c*x)*x**3)/(acos(c*x)**2*b**2*c**2*x**2 - acos(c*x)**2*b**2 +
2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a*
b**2*c**4 + 20*acos(c*x)*int((sqrt(acos(c*x)*b + a)*sqrt( - c**2*x**2 + 1)
*acos(c*x)**2*x)/(acos(c*x)**2*b**2*c**2*x**2 - acos(c*x)**2*b**2 + 2*acos
(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*b**3*c**
2 - 24*acos(c*x)*int((sqrt(acos(c*x)*b + a)*sqrt( - c**2*x**2 + 1)*x**3)/(
acos(c*x)**2*b**2*c**2*x**2 - acos(c*x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**
2 - 2*acos(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a**2*b*c**4 - 20*acos(c*x)
*int((sqrt(acos(c*x)*b + a)*sqrt( - c**2*x**2 + 1)*x)/(acos(c*x)**2*b**2*c
**2*x**2 - acos(c*x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b
 + a**2*c**2*x**2 - a**2),x)*a**2*b*c**2 - 5*acos(c*x)*int((sqrt(acos(c...
```

3.442 $\int \frac{(d-c^2dx^2)^2}{x(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\begin{aligned}\int \frac{(d-c^2dx^2)^2}{x(a+b\arccos(cx))^{3/2}}\,dx &= -\frac{2d^2(1-c^2x^2)^{5/2}}{bcx\sqrt{a+b\arccos(cx)}} \\ &- \frac{d^2\sqrt{\frac{\pi}{2}}\cos\left(\frac{4a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}} \\ &- \frac{3d^2\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{b^{3/2}} \\ &- \frac{3d^2\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{b^{3/2}} \\ &- \frac{d^2\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{4a}{b}\right)}{b^{3/2}} \\ &- \frac{2d^2\operatorname{Int}\left(\frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}},x\right)}{bc}\end{aligned}$$

output
```
-2*d^2*(-c^2*x^2+1)^(5/2)/b/c/x/(a+b*arccos(c*x))^(1/2)-1/2*d^2*2^(1/2)*Pi
^(1/2)*cos(4*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1
/2))/b^(3/2)-3*d^2*Pi^(1/2)*cos(2*a/b)*FresnelC(2*(a+b*arccos(c*x))^(1/2)/
b^(1/2)/Pi^(1/2))/b^(3/2)-3*d^2*Pi^(1/2)*FresnelS(2*(a+b*arccos(c*x))^(1/2
)/b^(1/2)/Pi^(1/2))*sin(2*a/b)/b^(3/2)-1/2*d^2*2^(1/2)*Pi^(1/2)*FresnelS(2
*2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(4*a/b)/b^(3/2)-2*d^
2*Defer(Int)(1/x^2/(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^(1/2),x)/b/c
```

Mathematica [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{(d-c^2dx^2)^2}{x(a+b\arccos(cx))^{3/2}}\,dx$$

input
```
Integrate[(d - c^2*d*x^2)^2/(x*(a + b*ArcCos[c*x])^(3/2)),x]
```

output
```
Integrate[(d - c^2*d*x^2)^2/(x*(a + b*ArcCos[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\arccos(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{5215}$$

$$\frac{8cd^2\int \frac{(1-c^2x^2)^{3/2}}{\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2d^2\int \frac{(1-c^2x^2)^{3/2}}{x^2\sqrt{a+b\arccos(cx)}}dx}{bc}+\frac{2d^2(1-c^2x^2)^{5/2}}{bcx\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{5169}$$

$$-\frac{8d^2\int \frac{\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2}+\frac{2d^2\int \frac{(1-c^2x^2)^{3/2}}{x^2\sqrt{a+b\arccos(cx)}}dx}{bc}+\frac{2d^2(1-c^2x^2)^{5/2}}{bcx\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{8d^2\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^4}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2}+\frac{2d^2\int \frac{(1-c^2x^2)^{3/2}}{x^2\sqrt{a+b\arccos(cx)}}dx}{bc}+\frac{2d^2(1-c^2x^2)^{5/2}}{bcx\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{8d^2\int \left(\frac{\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)}{8\sqrt{a+b\arccos(cx)}}-\frac{\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)}{2\sqrt{a+b\arccos(cx)}}+\frac{3}{8\sqrt{a+b\arccos(cx)}}\right)d(a+b\arccos(cx))}{b^2}+\frac{2d^2\int \frac{(1-c^2x^2)^{3/2}}{x^2\sqrt{a+b\arccos(cx)}}dx}{bc}+\frac{2d^2(1-c^2x^2)^{5/2}}{bcx\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2d^2\int \frac{(1-c^2x^2)^{3/2}}{x^2\sqrt{a+b\arccos(cx)}}dx}{bc}-\frac{8d^2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\frac{1}{2}\sqrt{\pi}\sqrt{b}\,\text{s}\right.}{b^2}$$

$$\frac{2d^2(1-c^2x^2)^{5/2}}{bcx\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{5227}$$

$$\frac{2d^2 \int \left(\frac{x^2c^4}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}} - \frac{2c^2}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}} + \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}\right)dx}{bc} -$$

$$\frac{8d^2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right) - \frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right) - \frac{1}{2}\sqrt{\pi}\sqrt{b}\,\mathrm{s}}{b^2}$$

$$\frac{2d^2(1-c^2x^2)^{5/2}}{bcx\sqrt{a+b\arccos(cx)}}$$

$\downarrow$ 2009

$$\frac{2d^2\left(\int \frac{1}{x^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}dx - \frac{\sqrt{\pi}c\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{2\sqrt{b}} - \frac{\sqrt{\pi}c\sin\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{2\sqrt{b}} + \frac{3c\sqrt{}}{}\right.}{bc}$$

$$\frac{8d^2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{4a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right) - \frac{1}{2}\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right) - \frac{1}{2}\sqrt{\pi}\sqrt{b}\,\mathrm{s}}{b^2}$$

$$\frac{2d^2(1-c^2x^2)^{5/2}}{bcx\sqrt{a+b\arccos(cx)}}$$

input
```
Int[(d - c^2*d*x^2)^2/(x*(a + b*ArcCos[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2d\,x^2+d)^2}{x\,(a+b\arccos(cx))^{\frac{3}{2}}}dx$$

input
```
int((-c^2*d*x^2+d)^2/x/(a+b*arccos(c*x))^(3/2),x)
```

output
```
int((-c^2*d*x^2+d)^2/x/(a+b*arccos(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^2/x/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 5.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 4.59

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\arccos(cx))^{3/2}}\,dx = d^2\left(\int \left(-\frac{2c^2x^2}{ax\sqrt{a+b\,\mathrm{acos}\,(cx)}+bx\sqrt{a+b\,\mathrm{acos}\,(cx)}\,\mathrm{acos}\,(cx)}\right)dx + \int \frac{c^4x^4}{ax\sqrt{a+b\,\mathrm{acos}\,(cx)}+bx\sqrt{a+b\,\mathrm{acos}\,(cx)}\,\mathrm{acos}\,(cx)}\,dx + \int \frac{1}{ax\sqrt{a+b\,\mathrm{acos}\,(cx)}+bx\sqrt{a+b\,\mathrm{acos}\,(cx)}\,\mathrm{acos}\,(cx)}\,dx\right)$$

input
```
integrate((-c**2*d*x**2+d)**2/x/(a+b*acos(c*x))**(3/2),x)
```

output
```
d**2*(Integral(-2*c**2*x**2/(a*x*sqrt(a + b*acos(c*x)) + b*x*sqrt(a + b*ac
os(c*x))*acos(c*x)), x) + Integral(c**4*x**4/(a*x*sqrt(a + b*acos(c*x)) +
b*x*sqrt(a + b*acos(c*x))*acos(c*x)), x) + Integral(1/(a*x*sqrt(a + b*acos
(c*x)) + b*x*sqrt(a + b*acos(c*x))*acos(c*x)), x))
```

Maxima [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{(c^2dx^2-d)^2}{(b\arccos(cx)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^2/x/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((c^2*d*x^2 - d)^2/((b*arccos(c*x) + a)^(3/2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-c^2*d*x^2+d)^2/x/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{(d-c^2\,d\,x^2)^2}{x\,(a+b\,\mathrm{acos}\,(c\,x))^{3/2}}\,dx$$

input
```
int((d - c^2*d*x^2)^2/(x*(a + b*acos(c*x))^(3/2)),x)
```

output
```
int((d - c^2*d*x^2)^2/(x*(a + b*acos(c*x))^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 4.48

$$\int \frac{(d-c^2dx^2)^2}{x(a+b\arccos(cx))^{3/2}}\,dx = d^2\left(\int \frac{\sqrt{acos\left(cx\right)b+a}}{acos\left(cx\right)^2 b^2x+2acos\left(cx\right)abx+a^2x}dx + \left(\int \frac{\sqrt{acos\left(cx\right)b+a}\,x^3}{acos\left(cx\right)^2 b^2+2acos\left(cx\right)ab+a^2}dx\right)c^4 - 2\left(\int \frac{\sqrt{acos\left(cx\right)b+a}\,x}{acos\left(cx\right)^2 b^2+2acos\left(cx\right)ab+a^2}dx\right)c^2\right)$$

input
```
int((-c^2*d*x^2+d)^2/x/(a+b*acos(c*x))^(3/2),x)
```

output
```
d**2*(int(sqrt(acos(c*x)*b + a)/(acos(c*x)**2*b**2*x + 2*acos(c*x)*a*b*x +
 a**2*x),x) + int((sqrt(acos(c*x)*b + a)*x**3)/(acos(c*x)**2*b**2 + 2*acos
(c*x)*a*b + a**2),x)*c**4 - 2*int((sqrt(acos(c*x)*b + a)*x)/(acos(c*x)**2*
b**2 + 2*acos(c*x)*a*b + a**2),x)*c**2)
```

3.443 $\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arccos(x)}}+\frac{x\arccos(x)^{3/2}}{(1-x^2)^2}\right) dx$

Optimal result

Integrand size = 38, antiderivative size = 42

$$\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arccos(x)}}+\frac{x\arccos(x)^{3/2}}{(1-x^2)^2}\right) dx = -\frac{3x\sqrt{\arccos(x)}}{4\sqrt{1-x^2}}+\frac{\arccos(x)^{3/2}}{2(1-x^2)}$$

output
```
-3/4*x*arccos(x)^(1/2)/(-x^2+1)^(1/2)+arccos(x)^(3/2)/(-2*x^2+2)
```

Mathematica [F]

$$\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arccos(x)}}+\frac{x\arccos(x)^{3/2}}{(1-x^2)^2}\right) dx = \int \left(-\frac{3x}{8(1-x^2)\sqrt{\arccos(x)}}+\frac{x\arccos(x)^{3/2}}{(1-x^2)^2}\right) dx$$

input
```
Integrate[(-3*x)/(8*(1 - x^2)*Sqrt[ArcCos[x]]) + (x*ArcCos[x]^(3/2))/(1 - 
x^2)^2,x]
```

output
```
Integrate[(-3*x)/(8*(1 - x^2)*Sqrt[ArcCos[x]]) + (x*ArcCos[x]^(3/2))/(1 -
x^2)^2, x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x \arccos(x)^{3/2}}{(1-x^2)^2} - \frac{3x}{8(1-x^2)\sqrt{\arccos(x)}}\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\arccos(x)^{3/2}}{2(1-x^2)} + \frac{3x\sqrt{\arccos(x)}}{4\sqrt{1-x^2}}$$

input
```
Int[(-3*x)/(8*(1 - x^2)*Sqrt[ArcCos[x]]) + (x*ArcCos[x]^(3/2))/(1 - x^2)^2
,x]
```

output
```
(3*x*Sqrt[ArcCos[x]])/(4*Sqrt[1 - x^2]) + ArcCos[x]^(3/2)/(2*(1 - x^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{3i\sqrt{\arccos(x)}}{4}-\frac{\left(3ix^2+3x\sqrt{-x^2+1}-3i+2\arccos(x)\right)\sqrt{\arccos(x)}}{4(x^2-1)}$	47
parts	$\frac{3i\sqrt{\arccos(x)}}{4}-\frac{\left(3ix^2+3x\sqrt{-x^2+1}-3i+2\arccos(x)\right)\sqrt{\arccos(x)}}{4(x^2-1)}$	47

input
```
int(-3/8*x/(-x^2+1)/arccos(x)^(1/2)+x*arccos(x)^(3/2)/(-x^2+1)^2,x,method=
_RETURNVERBOSE)
```

output
```
3/4*I*arccos(x)^(1/2)-1/4*(3*I*x^2+3*x*(-x^2+1)^(1/2)-3*I+2*arccos(x))*arc
cos(x)^(1/2)/(x^2-1)
```

Fricas [F(-2)]

Exception generated.

$$\int\left(-\frac{3x}{8\left(1-x^2\right)\sqrt{\arccos(x)}}+\frac{x\arccos(x)^{3/2}}{\left(1-x^2\right)^2}\right)dx=\text{Exception raised: TypeError}$$

input
```
integrate(-3/8*x/(-x^2+1)/arccos(x)^(1/2)+x*arccos(x)^(3/2)/(-x^2+1)^2,x,
algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int\left(-\frac{3x}{8\left(1-x^{2}\right)\sqrt{\arccos(x)}}+\frac{x\arccos(x)^{3/2}}{\left(1-x^{2}\right)^{2}}\right)dx=\frac{\int\left(-\frac{3x}{x^{4}\sqrt{\operatorname{acos}(x)}-2x^{2}\sqrt{\operatorname{acos}(x)}+\sqrt{\operatorname{acos}(x)}}\right)dx+\int\frac{3x^{3}}{x^{4}\sqrt{\operatorname{acos}(x)}-2x^{2}\sqrt{\operatorname{acos}(x)}+\sqrt{\operatorname{acos}(x)}}\,dx+\int}{8}$$

input
```
integrate(-3/8*x/(-x**2+1)/acos(x)**(1/2)+x*acos(x)**(3/2)/(-x**2+1)**2,x)
```

output
```
(Integral(-3*x/(x**4*sqrt(acos(x)) - 2*x**2*sqrt(acos(x)) + sqrt(acos(x)))
, x) + Integral(3*x**3/(x**4*sqrt(acos(x)) - 2*x**2*sqrt(acos(x)) + sqrt(a
cos(x))), x) + Integral(8*x*acos(x)**2/(x**4*sqrt(acos(x)) - 2*x**2*sqrt(a
cos(x)) + sqrt(acos(x))), x))/8
```

Maxima [F(-2)]

Exception generated.

$$\int\left(-\frac{3x}{8\left(1-x^{2}\right)\sqrt{\arccos(x)}}+\frac{x\arccos(x)^{3/2}}{\left(1-x^{2}\right)^{2}}\right)dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(-3/8*x/(-x^2+1)/arccos(x)^(1/2)+x*arccos(x)^(3/2)/(-x^2+1)^2,x,
algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arccos(x)}}+\frac{x\arccos(x)^{3/2}}{(1-x^2)^2}\right)dx = -\frac{x^2\arccos{(x)}^{\frac{3}{2}}}{2(x^2-1)}+\frac{1}{2}\arccos{(x)}^{\frac{3}{2}}-\frac{3\sqrt{-x^2+1}x\sqrt{\arccos{(x)}}}{4(x^2-1)}$$

input
```
integrate(-3/8*x/(-x^2+1)/arccos(x)^(1/2)+x*arccos(x)^(3/2)/(-x^2+1)^2,x,
algorithm="giac")
```

output
```
-1/2*x^2*arccos(x)^(3/2)/(x^2 - 1) + 1/2*arccos(x)^(3/2) - 3/4*sqrt(-x^2 +
 1)*x*sqrt(arccos(x))/(x^2 - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arccos(x)}}+\frac{x\arccos(x)^{3/2}}{(1-x^2)^2}\right)dx = \int \frac{3x}{8\sqrt{\mathrm{acos}(x)}\,(x^2-1)}+\frac{x\,\mathrm{acos}(x)^{3/2}}{(x^2-1)^2}\,dx$$

input
```
int((3*x)/(8*acos(x)^(1/2)*(x^2 - 1)) + (x*acos(x)^(3/2))/(x^2 - 1)^2,x)
```

output
```
int((3*x)/(8*acos(x)^(1/2)*(x^2 - 1)) + (x*acos(x)^(3/2))/(x^2 - 1)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int \left(-\frac{3x}{8(1-x^2)\sqrt{\arccos(x)}} + \frac{x\arccos(x)^{3/2}}{(1-x^2)^2}\right) dx = \frac{\sqrt{acos(x)}\left(-2acos(x)-3\sqrt{-x^2+1}\,x\right)}{4x^2-4}$$

input
```
int(-3/8*x/(-x^2+1)/acos(x)^(1/2)+x*acos(x)^(3/2)/(-x^2+1)^2,x)
```

output
```
(sqrt(acos(x))*( - 2*acos(x) - 3*sqrt( - x**2 + 1)*x))/(4*(x**2 - 1))
```

3.444 $\int (c-a^2cx^2)^{3/2}\sqrt{\arccos(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 227

$$\int (c-a^2cx^2)^{3/2}\sqrt{\arccos(ax)}\,dx = \frac{3}{8}cx\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)} + \frac{1}{4}x(c-a^2cx^2)^{3/2}\sqrt{\arccos(ax)} + \frac{c\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}{4a\sqrt{1-a^2x^2}} - \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{64a\sqrt{1-a^2x^2}} - \frac{c\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a\sqrt{1-a^2x^2}}$$

output
```
3/8*c*x*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2)+1/4*x*(-a^2*c*x^2+c)^(3/2)*
arccos(a*x)^(1/2)+1/4*c*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(3/2)/a/(-a^2*x^2
+1)^(1/2)-1/128*c*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*FresnelS(2*2^(1/2)
/Pi^(1/2)*arccos(a*x)^(1/2))/a/(-a^2*x^2+1)^(1/2)-1/8*c*Pi^(1/2)*(-a^2*c*x
^2+c)^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))/a/(-a^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.65

$$\int \left(c-a^2cx^2\right)^{3/2}\sqrt{\arccos(ax)}\,dx =$$
$$-\frac{c\sqrt{c-a^2cx^2}\left(32\arccos(ax)^2+16\sqrt{\pi}\sqrt{\arccos(ax)}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\sqrt{-i\arccos(ax)}\Gamma\left(\frac{3}{2},-4i\arccos(ax)\right)\ldots\right)}{128a\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}$$

input
```
Integrate[(c - a^2*c*x^2)^(3/2)*Sqrt[ArcCos[a*x]],x]
```

output
```
-1/128*(c*Sqrt[c - a^2*c*x^2]*(32*ArcCos[a*x]^2 + 16*Sqrt[Pi]*Sqrt[ArcCos[
a*x]]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] + Sqrt[(-I)*ArcCos[a*x]]*Ga
mma[3/2, (-4*I)*ArcCos[a*x]] + Sqrt[I*ArcCos[a*x]]*Gamma[3/2, (4*I)*ArcCos
[a*x]] - 32*ArcCos[a*x]*Sin[2*ArcCos[a*x]]))/(a*Sqrt[1 - a^2*x^2]*Sqrt[Arc
Cos[a*x]])
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5159, 5157, 5147, 4906, 27, 3042, 3786, 3832, 5153, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arccos(ax)}\left(c-a^2cx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{ac\sqrt{c-a^2cx^2}\int \frac{x(1-a^2x^2)}{\sqrt{\arccos(ax)}}dx}{8\sqrt{1-a^2x^2}}+\frac{3}{4}c\int \sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}dx+\frac{1}{4}x\sqrt{\arccos(ax)}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{5157}$$

$$\frac{ac\sqrt{c-a^2cx^2}\int \frac{x\left(1-a^2x^2\right)}{\sqrt{\arccos(ax)}}dx}{8\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(\frac{a\sqrt{c-a^2cx^2}\int \frac{x}{\sqrt{\arccos(ax)}}dx}{4\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\sqrt{\arccos(ax)}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 5147}$$

$$\frac{ac\sqrt{c-a^2cx^2}\int \frac{x\left(1-a^2x^2\right)}{\sqrt{\arccos(ax)}}dx}{8\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(-\frac{\sqrt{c-a^2cx^2}\int \frac{ax\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\sqrt{\arccos(ax)}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{ac\sqrt{c-a^2cx^2}\int \frac{x\left(1-a^2x^2\right)}{\sqrt{\arccos(ax)}}dx}{8\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(\frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}\int \frac{\sin(2\arccos(ax))}{2\sqrt{\arccos(ax)}}d\arccos(ax)}{4a\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\sqrt{\arccos(ax)}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{ac\sqrt{c-a^2cx^2}\int \frac{x\left(1-a^2x^2\right)}{\sqrt{\arccos(ax)}}dx}{8\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(\frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\sqrt{\arccos(ax)}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{ac\sqrt{c-a^2cx^2}\int \frac{x(1-a^2x^2)}{\sqrt{\arccos(ax)}}dx}{8\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(\frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\sqrt{\arccos(ax)}\left(c-a^2cx^2\right)^{3/2}$$

$\big\downarrow$ 3786

$$\frac{ac\sqrt{c-a^2cx^2}\int \frac{x(1-a^2x^2)}{\sqrt{\arccos(ax)}}dx}{8\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(\frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}\int \sin(2\arccos(ax))d\sqrt{\arccos(ax)}}{4a\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\sqrt{\arccos(ax)}\left(c-a^2cx^2\right)^{3/2}$$

$\big\downarrow$ 3832

$$\frac{3}{4}c\left(\frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}-\frac{\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}\right)+\frac{ac\sqrt{c-a^2cx^2}\int \frac{x(1-a^2x^2)}{\sqrt{\arccos(ax)}}dx}{8\sqrt{1-a^2x^2}}+\frac{1}{4}x\sqrt{\arccos(ax)}\left(c-a^2cx^2\right)^{3/2}$$

$\big\downarrow$ 5153

$$\frac{ac\sqrt{c-a^2cx^2}\int \frac{x(1-a^2x^2)}{\sqrt{\arccos(ax)}}dx}{8\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(-\frac{\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\sqrt{\arccos(ax)}\left(c-a^2cx^2\right)^{3/2}$$

$\big\downarrow$ 5225

$$-\frac{c\sqrt{c-a^2cx^2}\int \frac{ax(1-a^2x^2)^{3/2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(-\frac{\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\sqrt{\arccos(ax)}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{c\sqrt{c-a^2cx^2}\int \left(\frac{\sin(2\arccos(ax))}{4\sqrt{\arccos(ax)}}-\frac{\sin(4\arccos(ax))}{8\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{8a\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(-\frac{\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\sqrt{\arccos(ax)}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\sqrt{c-a^2cx^2}\left(\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)\right)}{8a\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(-\frac{\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\sqrt{\arccos(ax)}\left(c-a^2cx^2\right)^{3/2}$$

input
```
Int[(c - a^2*c*x^2)^(3/2)*Sqrt[ArcCos[a*x]],x]
```

output
```
(x*(c - a^2*c*x^2)^(3/2)*Sqrt[ArcCos[a*x]])/4 - (c*Sqrt[c - a^2*c*x^2]*(-1
/8*(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]]) + (Sqrt[Pi]*Fresn
elS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/4))/(8*a*Sqrt[1 - a^2*x^2]) + (3*c*((
x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCos[a*x]])/2 - (Sqrt[c - a^2*c*x^2]*ArcCos[a
*x]^(3/2))/(3*a*Sqrt[1 - a^2*x^2]) - (Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Fresnel
S[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(8*a*Sqrt[1 - a^2*x^2])))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \left(-a^2 c\,x^2+c\right)^{\frac{3}{2}} \sqrt{\arccos\left(ax\right)}dx$$

input
```
int((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(1/2),x)
```

output
```
int((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(c-a^2cx^2\right)^{3/2}\sqrt{\arccos(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \left(c-a^2cx^2\right)^{3/2}\sqrt{\arccos(ax)}\,dx = \int \left(-c(ax-1)\,(ax+1)\right)^{\frac{3}{2}}\sqrt{\operatorname{acos}\left(ax\right)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(3/2)*acos(a*x)**(1/2),x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(c-a^2cx^2\right)^{3/2}\sqrt{\arccos(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(c-a^2cx^2\right)^{3/2}\sqrt{\arccos(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(c-a^2cx^2\right)^{3/2}\sqrt{\arccos(ax)}\,dx = \int \sqrt{\mathrm{acos}\left(a\,x\right)}\,{\left(c-a^2\,c\,x^2\right)}^{3/2} \,d x$$

input
```
int(acos(a*x)^(1/2)*(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(acos(a*x)^(1/2)*(c - a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(c-a^2cx^2\right)^{3/2}\sqrt{\arccos(ax)}\,dx = \sqrt{c}\,c\left(-\left(\int \sqrt{-a^2x^2+1}\,\sqrt{acos\left(ax\right)}\,x^2dx\right)a^2 + \int \sqrt{-a^2x^2+1}\,\sqrt{acos\left(ax\right)}dx\right)$$

input
```
int((-a^2*c*x^2+c)^(3/2)*acos(a*x)^(1/2),x)
```

output
```
sqrt(c)*c*( - int(sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**2,x)*a**2 + in
t(sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)),x))
```

3.445 $\int \sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}\,dx = \frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}+\frac{\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}{3a\sqrt{1-a^2x^2}} - \frac{\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a\sqrt{1-a^2x^2}}$$

output
```
1/2*x*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2)+1/3*(-a^2*c*x^2+c)^(1/2)*arcc
os(a*x)^(3/2)/a/(-a^2*x^2+1)^(1/2)-1/8*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*Fresn
elS(2*arccos(a*x)^(1/2)/Pi^(1/2))/a/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.67

$$\int \sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}\,dx = -\frac{\sqrt{c(1-a^2x^2)}\left(3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+2\sqrt{\arccos(ax)}(4\arccos(ax)-3\sin(2\arccos(ax)))\right)}{24a\sqrt{1-a^2x^2}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]*Sqrt[ArcCos[a*x]],x]
```

output
```
-1/24*(Sqrt[c*(1 - a^2*x^2)]*(3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sq
rt[Pi]] + 2*Sqrt[ArcCos[a*x]]*(4*ArcCos[a*x] - 3*Sin[2*ArcCos[a*x]])))/(a*
Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5157, 5147, 4906, 27, 3042, 3786, 3832, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}\,dx$$

$$\Big\downarrow \text{5157}$$

$$\frac{a\sqrt{c-a^2cx^2}\int \frac{x}{\sqrt{\arccos(ax)}}dx}{4\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{5147}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{ax\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}\int \frac{\sin(2\arccos(ax))}{2\sqrt{\arccos(ax)}}d\arccos(ax)}{4a\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}$$

$\Big\downarrow$ 3786

$$\frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}-\frac{\sqrt{c-a^2cx^2}\int \sin(2\arccos(ax))d\sqrt{\arccos(ax)}}{4a\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}$$

$\Big\downarrow$ 3832

$$\frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}-\frac{\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}$$

$\Big\downarrow$ 5153

$$-\frac{\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}$$

input
```
Int[Sqrt[c - a^2*c*x^2]*Sqrt[ArcCos[a*x]],x]
```

output
```
(x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCos[a*x]])/2 - (Sqrt[c - a^2*c*x^2]*ArcCos[
a*x]^(3/2))/(3*a*Sqrt[1 - a^2*x^2]) - (Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Fresne
lS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(8*a*Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Maple [F]

$$\int \sqrt{-a^2c\,x^2+c}\,\sqrt{\arccos\left(ax\right)}dx$$

input
```
int((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2),x)
```

output
```
int((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}\,dx = \int \sqrt{-c\left(ax-1\right)\left(ax+1\right)}\sqrt{\operatorname{acos}\left(ax\right)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*acos(a*x)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c - a^2cx^2}\sqrt{\arccos(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{c - a^2cx^2}\sqrt{\arccos(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c - a^2cx^2}\sqrt{\arccos(ax)}\,dx = \int \sqrt{\mathrm{acos}\left(a\,x\right)}\,\sqrt{c-a^2\,c\,x^2}\,dx$$

input
```
int(acos(a*x)^(1/2)*(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(acos(a*x)^(1/2)*(c - a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c-a^2cx^2}\,\sqrt{\arccos(ax)}\,dx = \sqrt{c}\left(\int \sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}dx\right)$$

input
```
int((-a^2*c*x^2+c)^(1/2)*acos(a*x)^(1/2),x)
```

output
```
sqrt(c)*int(sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)),x)
```

3.446 $\int \frac{\sqrt{\arccos(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 44

$$\int \frac{\sqrt{\arccos(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{3a\sqrt{c-a^2cx^2}}$$

output
```
2/3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(3/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arccos(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{3a\sqrt{c-a^2cx^2}}$$

input
```
Integrate[Sqrt[ArcCos[a*x]]/Sqrt[c - a^2*c*x^2],x]
```

output
```
(-2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(3*a*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arccos(ax)}}{\sqrt{c-a^2cx^2}}dx$$
$$\Big\downarrow \text{5153}$$
$$-\frac{2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{3a\sqrt{c-a^2cx^2}}$$

input
```
Int[Sqrt[ArcCos[a*x]]/Sqrt[c - a^2*c*x^2],x]
```

output
```
(-2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(3*a*Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{2\arccos(ax)^{\frac{3}{2}}\sqrt{-a^2x^2+1}}{3\sqrt{-c(a^2x^2-1)}\,a}$	38

input
```
int(arccos(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*arccos(a*x)^(3/2)/(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arccos(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{\arccos(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{acos}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acos(a*x)**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(acos(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arccos(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{\arccos(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2\,\arccos(ax)^{\frac{3}{2}}}{3\,a\sqrt{c}}$$

input
```
integrate(arccos(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-2/3*arccos(a*x)^(3/2)/(a*sqrt(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\arccos(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\sqrt{\mathrm{acos}\left(a\,x\right)}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

input
```
int(acos(a*x)^(1/2)/(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(acos(a*x)^(1/2)/(c - a^2*c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{\arccos(ax)}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2\sqrt{c}\,\sqrt{acos\,(ax)}\,acos(ax)}{3ac}$$

input
```
int(acos(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(acos(a*x))*acos(a*x))/(3*a*c)
```

3.447 $\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{x\sqrt{\arccos(ax)}}{c\sqrt{c-a^2cx^2}} - \frac{a\sqrt{1-a^2x^2}\text{Int}\left(\frac{x}{(1-a^2x^2)\sqrt{\arccos(ax)}},x\right)}{2c\sqrt{c-a^2cx^2}}$$

output
```
x*arccos(a*x)^(1/2)/c/(-a^2*c*x^2+c)^(1/2)-1/2*a*(-a^2*x^2+1)^(1/2)*Defer(
Int)(x/(-a^2*x^2+1)/arccos(a*x)^(1/2),x)/c/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 2.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[ArcCos[a*x]]/(c - a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[Sqrt[ArcCos[a*x]]/(c - a^2*c*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{3/2}}dx$$
$$\Big\downarrow \text{ 5161}$$
$$\frac{a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)\sqrt{\arccos(ax)}}dx}{2c\sqrt{c-a^2cx^2}}+\frac{x\sqrt{\arccos(ax)}}{c\sqrt{c-a^2cx^2}}$$
$$\Big\downarrow \text{ 5235}$$
$$\frac{a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)\sqrt{\arccos(ax)}}dx}{2c\sqrt{c-a^2cx^2}}+\frac{x\sqrt{\arccos(ax)}}{c\sqrt{c-a^2cx^2}}$$

input
```
Int[Sqrt[ArcCos[a*x]]/(c - a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arccos \left(a x \right)}}{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}d x$$

input
```
int(arccos(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arccos(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{acos}(ax)}}{(-c(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(acos(a*x)**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(acos(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\arccos(ax)}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arccos(a*x))/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\mathrm{acos}\left(a\,x\right)}}{{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

input
```
int(acos(a*x)^(1/2)/(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(acos(a*x)^(1/2)/(c - a^2*c*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}}{a^4x^4-2a^2x^2+1}dx\right)}{c^2}$$

input
```
int(acos(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/(a**4*x**4 - 2*a**2*
x**2 + 1),x))/c**2
```

3.448 $\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{x\sqrt{\arccos(ax)}}{3c(c-a^2cx^2)^{3/2}} + \frac{2x\sqrt{\arccos(ax)}}{3c^2\sqrt{c-a^2cx^2}} - \frac{a\sqrt{1-a^2x^2}\,\mathrm{Int}\left(\frac{x}{(1-a^2x^2)^2\sqrt{\arccos(ax)}},x\right)}{6c^2\sqrt{c-a^2cx^2}} - \frac{a\sqrt{1-a^2x^2}\,\mathrm{Int}\left(\frac{x}{(1-a^2x^2)\sqrt{\arccos(ax)}},x\right)}{3c^2\sqrt{c-a^2cx^2}}$$

output
```
1/3*x*arccos(a*x)^(1/2)/c/(-a^2*c*x^2+c)^(3/2)+2/3*x*arccos(a*x)^(1/2)/c^2
/(-a^2*c*x^2+c)^(1/2)-1/6*a*(-a^2*x^2+1)^(1/2)*Defer(Int)(x/(-a^2*x^2+1)^2
/arccos(a*x)^(1/2),x)/c^2/(-a^2*c*x^2+c)^(1/2)-1/3*a*(-a^2*x^2+1)^(1/2)*De
fer(Int)(x/(-a^2*x^2+1)/arccos(a*x)^(1/2),x)/c^2/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[ArcCos[a*x]]/(c - a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[Sqrt[ArcCos[a*x]]/(c - a^2*c*x^2)^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{5163}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)^2\sqrt{\arccos(ax)}}dx}{6c^2\sqrt{c-a^2cx^2}}+\frac{2\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{3/2}}dx}{3c}+\frac{x\sqrt{\arccos(ax)}}{3c(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{5161}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)^2\sqrt{\arccos(ax)}}dx}{6c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)\sqrt{\arccos(ax)}}dx}{2c\sqrt{c-a^2cx^2}}+\frac{x\sqrt{\arccos(ax)}}{c\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\sqrt{\arccos(ax)}}{3c(c-a^2cx^2)^{3/2}}$$

$$\Big\downarrow \text{5235}$$

$$\frac{a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)^2\sqrt{\arccos(ax)}}dx}{6c^2\sqrt{c-a^2cx^2}}+\frac{2\left(\frac{a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)\sqrt{\arccos(ax)}}dx}{2c\sqrt{c-a^2cx^2}}+\frac{x\sqrt{\arccos(ax)}}{c\sqrt{c-a^2cx^2}}\right)}{3c}+\frac{x\sqrt{\arccos(ax)}}{3c(c-a^2cx^2)^{3/2}}$$

input
```
Int[Sqrt[ArcCos[a*x]]/(c - a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arccos \left(a x\right)}}{\left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}d x$$

input
```
int(arccos(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arccos(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(arccos(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 21.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{acos}(ax)}}{(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(acos(a*x)**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(acos(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\arccos(ax)}}{(-a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arccos(a*x))/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\mathrm{acos}\left(a\,x\right)}}{{\left(c-a^2\,c\,x^2\right)}^{5/2}} \,d x$$

input
```
int(acos(a*x)^(1/2)/(c - a^2*c*x^2)^(5/2),x)
```

output
```
int(acos(a*x)^(1/2)/(c - a^2*c*x^2)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{\arccos(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{-a^{2} x^{2}+1}\,\sqrt{acos(ax)}}{a^{6} x^{6}-3 a^{4} x^{4}+3 a^{2} x^{2}-1}d x\right)}{c^{3}}$$

input
```
int(acos(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

output
```
( - sqrt(c)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/(a**6*x**6 - 3*a*
*4*x**4 + 3*a**2*x**2 - 1),x))/c**3
```

3.449 $\int (c-a^2cx^2)^{3/2} \arccos(ax)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 363

$$\int (c-a^2cx^2)^{3/2} \arccos(ax)^{3/2} \, dx = \frac{27c\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}{256a\sqrt{1-a^2x^2}} - \frac{9acx^2\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}{32\sqrt{1-a^2x^2}} + \frac{3c(1-a^2x^2)^{3/2}\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}{32a} + \frac{3}{8}cx\sqrt{c-a^2cx^2}\arccos(ax)^{3/2} + \frac{1}{4}x(c-a^2cx^2)^{3/2}\arccos(ax)^{3/2} + \frac{3c\sqrt{c-a^2cx^2}\arccos(ax)^{5/2}}{20a\sqrt{1-a^2x^2}} - \frac{3c\sqrt{\frac{\pi}{2}}\sqrt{c-}}{}$$

output

```
27/256*c*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2)/a/(-a^2*x^2+1)^(1/2)-9/32*
a*c*x^2*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2)/(-a^2*x^2+1)^(1/2)+3/32*c*(
-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2)/a+3/8*c*x*(-a^2*c
*x^2+c)^(1/2)*arccos(a*x)^(3/2)+1/4*x*(-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(3/
2)+3/20*c*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(5/2)/a/(-a^2*x^2+1)^(1/2)-3/10
24*c*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arc
cos(a*x)^(1/2))/a/(-a^2*x^2+1)^(1/2)-3/32*c*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*
FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))/a/(-a^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.51

$$\int (c-a^2cx^2)^{3/2} \arccos(ax)^{3/2}\, dx =$$
$$-\frac{c\sqrt{c-a^2cx^2}\left(240\sqrt{\pi}\sqrt{\arccos(ax)^2}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arccos(ax)}\left(5\sqrt{i\arccos(ax)}\Gamma\left(\frac{5}{2},-4i\arccos\right.\right.\right.}{}$$

input
```
Integrate[(c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^(3/2),x]
```

output
```
-1/2560*(c*Sqrt[c - a^2*c*x^2]*(240*Sqrt[Pi]*Sqrt[ArcCos[a*x]^2]*FresnelC[
(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] + Sqrt[ArcCos[a*x]]*(5*Sqrt[I*ArcCos[a*x]]
*Gamma[5/2, (-4*I)*ArcCos[a*x]] + 5*Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (4*I
)*ArcCos[a*x]] + 32*Sqrt[ArcCos[a*x]^2]*(12*ArcCos[a*x]^2 - 15*Cos[2*ArcCo
s[a*x]] - 20*ArcCos[a*x]*Sin[2*ArcCos[a*x]]))))/(a*Sqrt[1 - a^2*x^2]*Sqrt[
ArcCos[a*x]^2])
```

Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {5159, 5157, 5141, 5153, 5183, 5169, 3042, 3793, 2009, 5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^{3/2} \left(c-a^2cx^2\right)^{3/2} dx$$

$$\Big\downarrow \text{ 5159}$$

$$\frac{3ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\sqrt{\arccos(ax)}dx}{8\sqrt{1-a^2x^2}}+\frac{3}{4}c\int \sqrt{c-a^2cx^2}\arccos(ax)^{3/2}dx+\frac{1}{4}x\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{5157}$$

$$\frac{3ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\sqrt{\arccos(ax)}dx}{8\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\int x\sqrt{\arccos(ax)}dx}{4\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int \frac{\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{3/2}\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{5141}$$

$$\frac{3ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\sqrt{\arccos(ax)}dx}{8\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{4}a\int \frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{2}x^2\sqrt{\arccos(ax)}\right)}{4\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int \frac{\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(\right.$$
$$\frac{1}{4}x\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{5153}$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{4}a\int \frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{2}x^2\sqrt{\arccos(ax)}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax\right.$$
$$\frac{3ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\sqrt{\arccos(ax)}dx}{8\sqrt{1-a^2x^2}}+\frac{1}{4}x\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{5183}$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{4}a\int \frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{2}x^2\sqrt{\arccos(ax)}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax\right.$$
$$\frac{3ac\sqrt{c-a^2cx^2}\left(-\frac{\int \frac{\left(1-a^2x^2\right)^{3/2}}{\sqrt{\arccos(ax)}}dx}{8a}-\frac{\left(1-a^2x^2\right)^2\sqrt{\arccos(ax)}}{4a^2}\right)}{8\sqrt{1-a^2x^2}}+\frac{1}{4}x\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{5169}$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{4}a\int\frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{2}x^2\sqrt{\arccos(ax)}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax\right.$$

$$\frac{3ac\sqrt{c-a^2cx^2}\left(\frac{\int\frac{(1-a^2x^2)^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a^2}-\frac{(1-a^2x^2)^2\sqrt{\arccos(ax)}}{4a^2}\right)}{8\sqrt{1-a^2x^2}}+$$

$$\frac{1}{4}x\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{4}a\int\frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{2}x^2\sqrt{\arccos(ax)}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax\right.$$

$$\frac{3ac\sqrt{c-a^2cx^2}\left(\frac{\int\frac{\sin(\arccos(ax))^4}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a^2}-\frac{(1-a^2x^2)^2\sqrt{\arccos(ax)}}{4a^2}\right)}{8\sqrt{1-a^2x^2}}+$$

$$\frac{1}{4}x\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{4}a\int\frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{2}x^2\sqrt{\arccos(ax)}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax\right.$$

$$\frac{3ac\sqrt{c-a^2cx^2}\left(\frac{\int\left(-\frac{\cos(2\arccos(ax))}{2\sqrt{\arccos(ax)}}+\frac{\cos(4\arccos(ax))}{8\sqrt{\arccos(ax)}}+\frac{3}{8\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{8a^2}-\frac{(1-a^2x^2)^2\sqrt{\arccos(ax)}}{4a^2}\right)}{8\sqrt{1-a^2x^2}}+$$

$$\frac{1}{4}x\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{4}a\int\frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{2}x^2\sqrt{\arccos(ax)}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax\right.$$
$$\frac{3ac\sqrt{c-a^2cx^2}\left(\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arccos(ax)}}{8a^2}-\frac{(1-a^2x^2)^2\sqrt{\arccos(ax)}}{4a^2}\right)}{8\sqrt{1-a^2x^2}}+$$
$$\frac{1}{4}x\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 5225}$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\int\frac{a^2x^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{3/2}\right.$$
$$\frac{3ac\sqrt{c-a^2cx^2}\left(\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arccos(ax)}}{8a^2}-\frac{(1-a^2x^2)^2\sqrt{\arccos(ax)}}{4a^2}\right)}{8\sqrt{1-a^2x^2}}+$$
$$\frac{1}{4}x\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\int\frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(\right.$$
$$\frac{3ac\sqrt{c-a^2cx^2}\left(\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arccos(ax)}}{8a^2}-\frac{(1-a^2x^2)^2\sqrt{\arccos(ax)}}{4a^2}\right)}{8\sqrt{1-a^2x^2}}+$$
$$\frac{1}{4}x\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\int\left(\frac{\cos(2\arccos(ax))}{2\sqrt{\arccos(ax)}}+\frac{1}{2\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{4a^2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\right.$$

$$\frac{3ac\sqrt{c-a^2cx^2}\left(\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arccos(ax)}}{8a^2}-\frac{(1-a^2x^2)^2\sqrt{\arccos(ax)}}{4a^2}\right)}{8\sqrt{1-a^2x^2}}+$$

$$\frac{1}{4}x\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \; 2009$$

$$\frac{3ac\sqrt{c-a^2cx^2}\left(\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arccos(ax)}}{8a^2}-\frac{(1-a^2x^2)^2\sqrt{\arccos(ax)}}{4a^2}\right)}{8\sqrt{1-a^2x^2}}+$$

$$\frac{3}{4}c\left(\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arccos(ax)}}{4a^2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{1}{2}\right.$$

$$\frac{1}{4}x\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{3/2}$$

input
```
Int[(c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^(3/2),x]
```

output
```
(x*(c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^(3/2))/4 + (3*a*c*Sqrt[c - a^2*c*x^2]
*(-1/4*((1 - a^2*x^2)^2*Sqrt[ArcCos[a*x]])/a^2 + ((3*Sqrt[ArcCos[a*x]])/4
+ (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/8 - (Sqrt[Pi]*Fres
nelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/2)/(8*a^2)))/(8*Sqrt[1 - a^2*x^2]) +
 (3*c*((x*Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^(3/2))/2 - (Sqrt[c - a^2*c*x^2]*
ArcCos[a*x]^(5/2))/(5*a*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - a^2*c*x^2]*((x^
2*Sqrt[ArcCos[a*x]])/2 - (Sqrt[ArcCos[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[A
rcCos[a*x]])/Sqrt[Pi]])/2)/(4*a^2)))/(4*Sqrt[1 - a^2*x^2])))/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arccos \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(3/2),x)
```

output
```
int((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(c-a^2cx^2\right)^{3/2} \arccos(ax)^{3/2} \, dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(3/2)*acos(a*x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(c-a^2cx^2\right)^{3/2}\arccos(ax)^{3/2}\,dx=\int \mathrm{acos}(a\,x)^{3/2}\,\left(c-a^2\,c\,x^2\right)^{3/2}\,dx$$

input
```
int(acos(a*x)^(3/2)*(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(acos(a*x)^(3/2)*(c - a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int (c-a^2cx^2)^{3/2}\arccos(ax)^{3/2}\,dx=\sqrt{c}\,c\left(-\left(\int \sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}\,acos(ax)\,x^2dx\right)a^2+\int \sqrt{-a^2x^2+1}\,\sqrt{}$$

input
```
int((-a^2*c*x^2+c)^(3/2)*acos(a*x)^(3/2),x)
```

output
```
sqrt(c)*c*( - int(sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*acos(a*x)*x**2,x)
*a**2 + int(sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*acos(a*x),x))
```

3.450 $\int \sqrt{c-a^2cx^2} \arccos (ax)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 219

$$\int \sqrt{c-a^2cx^2} \arccos (ax)^{3/2} \, dx = \frac{3\sqrt{c-a^2cx^2}\sqrt{\arccos (ax)}}{16a\sqrt{1-a^2x^2}} - \frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\arccos (ax)}}{8\sqrt{1-a^2x^2}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\arccos (ax)^{3/2} + \frac{\sqrt{c-a^2cx^2}\arccos (ax)^{5/2}}{5a\sqrt{1-a^2x^2}} - \frac{3\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos (ax)}}{\sqrt{\pi}}\right)}{32a\sqrt{1-a^2x^2}}$$

output
```
3/16*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2)/a/(-a^2*x^2+1)^(1/2)-3/8*a*x^2
*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2)/(-a^2*x^2+1)^(1/2)+1/2*x*(-a^2*c*x
^2+c)^(1/2)*arccos(a*x)^(3/2)+1/5*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(5/2)/a
/(-a^2*x^2+1)^(1/2)-3/32*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*FresnelC(2*arccos(a
*x)^(1/2)/Pi^(1/2))/a/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.47

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^{3/2}\,dx = -\frac{\sqrt{c(1-a^2x^2)}\left(15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+2\sqrt{\arccos(ax)}(-15\cos(2\arccos(ax))+4\arccos(ax)(4a\ldots}{160a\sqrt{1-a^2x^2}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^(3/2),x]
```

output
```
-1/160*(Sqrt[c*(1 - a^2*x^2)]*(15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/
Sqrt[Pi]] + 2*Sqrt[ArcCos[a*x]]*(-15*Cos[2*ArcCos[a*x]] + 4*ArcCos[a*x]*(4
*ArcCos[a*x] - 5*Sin[2*ArcCos[a*x]]))))/(a*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5157, 5141, 5153, 5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^{3/2}\sqrt{c-a^2cx^2}\,dx$$

$$\Big\downarrow \text{5157}$$

$$\frac{3a\sqrt{c-a^2cx^2}\int x\sqrt{\arccos(ax)}dx}{4\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int \frac{\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{3/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{5141}$$

$$\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{4}a\int \frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{2}x^2\sqrt{\arccos(ax)}\right)}{4\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int \frac{\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{3/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \ 5153$$

$$\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{4}a\int \frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}dx+\frac{1}{2}x^2\sqrt{\arccos(ax)}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{3/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \ 5225$$

$$\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\int \frac{a^2x^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{3/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\int \frac{\sin\left(\arccos(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{3/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \ 3793$$

$$\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\int \left(\frac{\cos(2\arccos(ax))}{2\sqrt{\arccos(ax)}}+\frac{1}{2\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{4a^2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{3/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{3a\sqrt{c-a^2cx^2}\left(\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arccos(ax)}}{4a^2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}{5a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{3/2}\sqrt{c-a^2cx^2}$$

input
```
Int[Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^(3/2),x]
```

output
```
(x*Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^(3/2))/2 - (Sqrt[c - a^2*c*x^2]*ArcCos[
a*x]^(5/2))/(5*a*Sqrt[1 - a^2*x^2]) + (3*a*Sqrt[c - a^2*c*x^2]*((x^2*Sqrt[
ArcCos[a*x]])/2 - (Sqrt[ArcCos[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a
*x]])/Sqrt[Pi]])/2)/(4*a^2)))/(4*Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \sqrt{-a^{2} c\,x^{2}+c}\, \arccos \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(3/2),x)
```

output
```
int((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c-a^2cx^2} \arccos(ax)^{3/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^{3/2}\,dx=\int \sqrt{-c\,(ax-1)\,(ax+1)}\,\mathrm{acos}^{\frac{3}{2}}\,(ax)\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*acos(a*x)**(3/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^{3/2}\,dx=\int \mathrm{acos}(a\,x)^{3/2}\,\sqrt{c-a^2\,c\,x^2}\,dx$$

input
```
int(acos(a*x)^(3/2)*(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(acos(a*x)^(3/2)*(c - a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^{3/2}\,dx=\sqrt{c}\left(\int \sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}\,acos(ax)\,dx\right)$$

input
```
int((-a^2*c*x^2+c)^(1/2)*acos(a*x)^(3/2),x)
```

output
```
sqrt(c)*int(sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*acos(a*x),x)
```

3.451 $\int \frac{\arccos(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 44

$$\int \frac{\arccos(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}{5a\sqrt{c-a^2cx^2}}$$

output
```
2/5*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(5/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}{5a\sqrt{c-a^2cx^2}}$$

input
```
Integrate[ArcCos[a*x]^(3/2)/Sqrt[c - a^2*c*x^2],x]
```

output
```
(-2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(5/2))/(5*a*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx$$
$$\Big\downarrow \text{5153}$$
$$-\frac{2\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}{5a\sqrt{c-a^2cx^2}}$$

input
```
Int[ArcCos[a*x]^(3/2)/Sqrt[c - a^2*c*x^2],x]
```

output
```
(-2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(5/2))/(5*a*Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{2\arccos(ax)^{\frac{5}{2}}\sqrt{-a^2x^2+1}}{5\sqrt{-c(a^2x^2-1)}\,a}$	38

input
```
int(arccos(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*arccos(a*x)^(5/2)/(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\arccos(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\operatorname{acos}^{\frac{3}{2}}(ax)}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acos(a*x)**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(acos(a*x)**(3/2)/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.32

$$\int \frac{\arccos(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2\arccos(ax)^{\frac{5}{2}}}{5\,a\sqrt{c}}$$

input
```
integrate(arccos(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-2/5*arccos(a*x)^(5/2)/(a*sqrt(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^{3/2}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

input
```
int(acos(a*x)^(3/2)/(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(acos(a*x)^(3/2)/(c - a^2*c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int \frac{\arccos(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2\sqrt{c}\,\sqrt{acos\,(ax)}\,acos(ax)^2}{5ac}$$

input
```
int(acos(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(acos(a*x))*acos(a*x)**2)/(5*a*c)
```

3.452 $\int \frac{\arccos(ax)^{3/2}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arccos(ax)^{3/2}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{x\arccos(ax)^{3/2}}{c\sqrt{c-a^2cx^2}} - \frac{3a\sqrt{1-a^2x^2}\,\mathrm{Int}\left(\frac{x\sqrt{\arccos(ax)}}{1-a^2x^2},x\right)}{2c\sqrt{c-a^2cx^2}}$$

output
```
x*arccos(a*x)^(3/2)/c/(-a^2*c*x^2+c)^(1/2)-3/2*a*(-a^2*x^2+1)^(1/2)*Defer(
Int)(x*arccos(a*x)^(1/2)/(-a^2*x^2+1),x)/c/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 3.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arccos(ax)^{3/2}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\arccos(ax)^{3/2}}{(c-a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[ArcCos[a*x]^(3/2)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[ArcCos[a*x]^(3/2)/(c - a^2*c*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^{3/2}}{(c-a^2cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5161}$$

$$\frac{3a\sqrt{1-a^2x^2}\int \frac{x\sqrt{\arccos(ax)}}{1-a^2x^2}dx}{2c\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^{3/2}}{c\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 5235}$$

$$\frac{3a\sqrt{1-a^2x^2}\int \frac{x\sqrt{\arccos(ax)}}{1-a^2x^2}dx}{2c\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^{3/2}}{c\sqrt{c-a^2cx^2}}$$

input
```
Int[ArcCos[a*x]^(3/2)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arccos\left(ax\right)^{\frac{3}{2}}}{\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}dx$$

input
```
int(arccos(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arccos(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{3/2}}{(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 14.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arccos(ax)^{3/2}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{acos}^{\frac{3}{2}}(ax)}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(acos(a*x)**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(acos(a*x)**(3/2)/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{3/2}}{(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arccos(ax)^{3/2}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\arccos(ax)^{\frac{3}{2}}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arccos(ax)^{3/2}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^{3/2}}{(c-a^2\,c\,x^2)^{3/2}}\,dx$$

input
```
int(acos(a*x)^(3/2)/(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(acos(a*x)^(3/2)/(c - a^2*c*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.88

$$\int \frac{\arccos(ax)^{3/2}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(-2\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}\,acos(ax)\,x-3\left(\int \frac{\sqrt{acos(ax)}\,x}{a^2x^2-1}dx\right)a^3x^2+3\left(\int \frac{\sqrt{acos(}}{a^2x^2}\right.\right.}{2c^2\,(a^2x^2-1)}$$

input
```
int(acos(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*acos(a*x)*x - 3*int(
(sqrt(acos(a*x))*x)/(a**2*x**2 - 1),x)*a**3*x**2 + 3*int((sqrt(acos(a*x))*
x)/(a**2*x**2 - 1),x)*a))/(2*c**2*(a**2*x**2 - 1))
```

3.453 $\int \left(c-a^2cx^2\right)^{3/2} \arccos(ax)^{5/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 431

$$\int \left(c-a^2cx^2\right)^{3/2} \arccos(ax)^{5/2} \, dx = -\frac{225}{512}cx\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}$$
$$-\frac{15}{256}cx\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}+\frac{45c\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}{256a\sqrt{1-a^2x^2}}$$
$$-\frac{15acx^2\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}{32\sqrt{1-a^2x^2}}+\frac{5c(1-a^2x^2)^{3/2}\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}{32a}$$
$$+\frac{3}{8}cx\sqrt{c-a^2cx^2}\arccos(ax)^{5/2}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\arccos(ax)^{5/2}+\frac{3c\sqrt{c-a^2cx^2}\arccos(ax)^{7/2}}{28a\sqrt{1-a^2x^2}}+\frac{15c\sqrt{\frac{\pi}{2}}\sqrt{c}}{}$$

output
```
-225/512*c*x*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2)-15/256*c*x*(-a^2*x^2+1
)*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2)+45/256*c*(-a^2*c*x^2+c)^(1/2)*arc
cos(a*x)^(3/2)/a/(-a^2*x^2+1)^(1/2)-15/32*a*c*x^2*(-a^2*c*x^2+c)^(1/2)*arc
cos(a*x)^(3/2)/(-a^2*x^2+1)^(1/2)+5/32*c*(-a^2*x^2+1)^(3/2)*(-a^2*c*x^2+c)
^(1/2)*arccos(a*x)^(3/2)/a+3/8*c*x*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(5/2)+
1/4*x*(-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(5/2)+3/28*c*(-a^2*c*x^2+c)^(1/2)*a
rccos(a*x)^(7/2)/a/(-a^2*x^2+1)^(1/2)+15/8192*c*2^(1/2)*Pi^(1/2)*(-a^2*c*x
^2+c)^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a/(-a^2*x^2+1)^
(1/2)+15/128*c*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/
Pi^(1/2))/a/(-a^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.42

$$\int (c-a^2cx^2)^{3/2}\arccos(ax)^{5/2}\,dx = \frac{c\sqrt{c-a^2cx^2}\left(-1536\arccos(ax)^4+4480\arccos(ax)^2\cos(2\arccos(ax))+16\right.}{}$$

input
```
Integrate[(c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^(5/2),x]
```

output
```
(c*Sqrt[c - a^2*c*x^2]*(-1536*ArcCos[a*x]^4 + 4480*ArcCos[a*x]^2*Cos[2*Arc
Cos[a*x]] + 1680*Sqrt[Pi]*Sqrt[ArcCos[a*x]]*FresnelS[(2*Sqrt[ArcCos[a*x]])
/Sqrt[Pi]] + 7*Sqrt[(-I)*ArcCos[a*x]]*Gamma[7/2, (-4*I)*ArcCos[a*x]] + 7*S
qrt[I*ArcCos[a*x]]*Gamma[7/2, (4*I)*ArcCos[a*x]] - 3360*ArcCos[a*x]*Sin[2*
ArcCos[a*x]] + 3584*ArcCos[a*x]^3*Sin[2*ArcCos[a*x]]))/(14336*a*Sqrt[1 - a
^2*x^2]*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 5.26 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.10, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.042$, Rules used = {5159, 5157, 5141, 5153, 5183, 5159, 5157, 5147, 4906, 27, 3042, 3786, 3832, 5153, 5211, 5147, 4906, 27, 3042, 3786, 3832, 5153, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 5159}$$

$$\frac{5ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\arccos(ax)^{3/2}dx}{8\sqrt{1-a^2x^2}}+\frac{3}{4}c\int \sqrt{c-a^2cx^2}\arccos(ax)^{5/2}dx+\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \; 5157$$

$$\frac{5ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\arccos(ax)^{3/2}dx}{8\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\int x\arccos(ax)^{3/2}dx}{4\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int \frac{\arccos(ax)^{5/2}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}\right)+\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \; 5141$$

$$\frac{5ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\arccos(ax)^{3/2}dx}{8\sqrt{1-a^2x^2}}+\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\int \frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int \frac{\arccos(ax)^{5/2}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5\ldots}\right.$$
$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \; 5153$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\int \frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\ldots\right.$$
$$\frac{5ac\sqrt{c-a^2cx^2}\int x\left(1-a^2x^2\right)\arccos(ax)^{3/2}dx}{8\sqrt{1-a^2x^2}}+\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \; 5183$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\int \frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\ldots\right.$$
$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\int \left(1-a^2x^2\right)^{3/2}\sqrt{\arccos(ax)}dx}{8a}-\frac{\left(1-a^2x^2\right)^2\arccos(ax)^{3/2}}{4a^2}\right)}{8\sqrt{1-a^2x^2}}+$$
$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \; 5159$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\int\frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{\right.$$

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int\frac{x(1-a^2x^2)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\int\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}dx+\frac{1}{4}x(1-a^2x^2)^{3/2}\sqrt{\arccos(ax)}\right)}{8a}-\frac{(1-a^2x^2)^2\arccos(ax)^{3/2}}{4a^2}\right)}{8\sqrt{1-a^2x^2}}$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 5157}$$

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int\frac{x(1-a^2x^2)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{4}a\int\frac{x}{\sqrt{\arccos(ax)}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}\right)+\frac{1}{4}x(1-a^2x^2)^{3/2}\sqrt{\mathrm{ar}\right.}{8a}\right.}{8\sqrt{1-a^2x^2}}$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\int\frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{\right.$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 5147}$$

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int\frac{x(1-a^2x^2)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\left(-\frac{\int\frac{ax\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a}+\frac{1}{2}\int\frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}\right)+\frac{1}{4}x(1-a^2x^2\right.}{8a}\right.}{8\sqrt{1-a^2x^2}}$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\int\frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{\right.$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\int\frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{\right.$$

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int\frac{x(1-a^2x^2)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx-\frac{\int\frac{\sin(2\arccos(ax))}{2\sqrt{\arccos(ax)}}d\arccos(ax)}{4a}+\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}\right)+\frac{1}{4}x(1-a^2x\right.}{8a}}{8\sqrt{1-a^2x^2}}$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\int\frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{\right.$$

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int\frac{x(1-a^2x^2)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx-\frac{\int\frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a}+\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}\right)+\frac{1}{4}x(1-a^2x\right.}{8a}}{8\sqrt{1-a^2x^2}}$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \; 3042$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\int\frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{\right.$$

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int\frac{x(1-a^2x^2)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx-\frac{\int\frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a}+\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}\right)+\frac{1}{4}x(1-a^2x\right.}{8a}}{8\sqrt{1-a^2x^2}}$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \; 3786$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\int \frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{\right.$$

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int \frac{x\left(1-a^2x^2\right)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx-\frac{\int \sin(2\arccos(ax))d\sqrt{\arccos(ax)}}{4a}+\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}\right)+\frac{1}{4}x\left(1-a^2x\right.}{8a}\right.}{8\sqrt{1-a^2x^2}}$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$\downarrow$ 3832

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a}\right)+\frac{1}{8}a\int \frac{x\left(1-a^2x^2\right)}{\sqrt{\arccos(ax)}}dx+\frac{1}{4}x\left(1-a^2x^2\right.}{8a}\right.}{8\sqrt{1-a^2x^2}}$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\int \frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{\right.$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$\downarrow$ 5153

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int \frac{x\left(1-a^2x^2\right)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a}-\frac{\arccos(ax)^{3/2}}{3a}\right)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\sqrt{\right.}{8a}\right.}{8\sqrt{1-a^2x^2}}$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\int \frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{\right.$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$\downarrow$ 5211

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int \frac{x\left(1-a^2x^2\right)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a}-\frac{\arccos(ax)^{3/2}}{3a}\right)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\cdots}{8a}\right.}{8\sqrt{1-a^2x^2}}$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int \frac{x}{\sqrt{\arccos(ax)}}dx}{4a}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(\cdots}{7\cdots}\right.$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{5147}$$

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int \frac{x\left(1-a^2x^2\right)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a}-\frac{\arccos(ax)^{3/2}}{3a}\right)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\cdots}{8a}\right.}{8\sqrt{1-a^2x^2}}$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\int \frac{ax\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^3}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\cdots\right.$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{4906}$$

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int \frac{x\left(1-a^2x^2\right)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a}-\frac{\arccos(ax)^{3/2}}{3a}\right)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\cdots}{8a}\right.}{8\sqrt{1-a^2x^2}}$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\int \frac{\sin(2\arccos(ax))}{2\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^3}+\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}\cdots\right.$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int \frac{x\left(1-a^2x^2\right)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a}-\frac{\arccos(ax)^{3/2}}{3a}\right)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\right.}{8a}}{8\sqrt{1-a^2x^2}}$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a^3}+\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}\right.$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \ 3042$$

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int \frac{x\left(1-a^2x^2\right)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a}-\frac{\arccos(ax)^{3/2}}{3a}\right)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\right.}{8a}}{8\sqrt{1-a^2x^2}}$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a^3}+\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}\right.$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \ 3786$$

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int\frac{x\left(1-a^2x^2\right)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a}-\frac{\arccos(ax)^{3/2}}{3a}\right)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\sqrt{}\right.}{8a}\right.}{8\sqrt{1-a^2x^2}}$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\int\sin(2\arccos(ax))d\sqrt{\arccos(ax)}}{4a^3}+\frac{\int\frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}\right.}{4\sqrt{1-a^2x^2}}\right.$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{3832}$$

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int\frac{x\left(1-a^2x^2\right)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a}-\frac{\arccos(ax)^{3/2}}{3a}\right)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\sqrt{}\right.}{8a}\right.}{8\sqrt{1-a^2x^2}}$$

$$\frac{3}{4}c\left(\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\int\frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^3}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}\right.$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}$$

$$\Big\downarrow \text{5153}$$

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{1}{8}a\int\frac{x\left(1-a^2x^2\right)}{\sqrt{\arccos(ax)}}dx+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a}-\frac{\arccos(ax)^{3/2}}{3a}\right)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\sqrt{}\right.}{8a}\right.}{8\sqrt{1-a^2x^2}}$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}+$$

$$\frac{3}{4}c\left(-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}+\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^3}-\right.\right.}{}\right.$$

$\downarrow$ 5225

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(-\frac{\int\frac{ax\left(1-a^2x^2\right)^{3/2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a}+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a}-\frac{\arccos(ax)^{3/2}}{3a}\right)+\frac{1}{4}x(1-\right.}{8a}\right.}{8\sqrt{1-a^2x^2}}$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}+$$

$$\frac{3}{4}c\left(-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}+\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^3}-\right.\right.}{}\right.$$

$\downarrow$ 4906

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(-\frac{\int\left(\frac{\sin(2\arccos(ax))}{4\sqrt{\arccos(ax)}}-\frac{\sin(4\arccos(ax))}{8\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{8a}+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a}-\frac{\text{arcc}}{}\right.\right.}{8a}\right.}{8\sqrt{1-a^2x^2}}$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}+$$

$$\frac{3}{4}c\left(-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}+\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^3}-\right.\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{5ac\sqrt{c-a^2cx^2}\left(-\frac{3\left(\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a}-\frac{\arccos(ax)^{3/2}}{3a}\right)+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\sqrt{\arccos(ax)}-\frac{\frac{1}{4}\sqrt{\pi}\,\text{Fre}}{}\right.}{8a}\right.}{8\sqrt{1-a^2x^2}}$$

$$\frac{1}{4}x\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}+$$

$$\frac{3}{4}c\left(-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}+\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^3}-\right.\right.}{}\right.$$

input
```
Int[(c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^(5/2),x]
```

output
```
(x*(c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^(5/2))/4 + (3*c*((x*Sqrt[c - a^2*c*x^
2]*ArcCos[a*x]^(5/2))/2 - (Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^(7/2))/(7*a*Sqr
t[1 - a^2*x^2]) + (5*a*Sqrt[c - a^2*c*x^2]*((x^2*ArcCos[a*x]^(3/2))/2 + (3
*a*(-1/2*(x*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/a^2 - ArcCos[a*x]^(3/2)/(
3*a^3) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(8*a^3)))/4))
/(4*Sqrt[1 - a^2*x^2])))/4 + (5*a*c*Sqrt[c - a^2*c*x^2]*(-1/4*((1 - a^2*x^
2)^2*ArcCos[a*x]^(3/2))/a^2 - (3*((x*(1 - a^2*x^2)^(3/2)*Sqrt[ArcCos[a*x]]
)/4 - (-1/8*(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]]) + (Sqrt[
Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/4)/(8*a) + (3*((x*Sqrt[1 - a
^2*x^2]*Sqrt[ArcCos[a*x]])/2 - ArcCos[a*x]^(3/2)/(3*a) - (Sqrt[Pi]*Fresnel
S[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(8*a)))/4))/(8*a)))/(8*Sqrt[1 - a^2*x^2
])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arccos \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(5/2),x)
```

output
```
int((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(c-a^2cx^2\right)^{3/2} \arccos(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \left(c-a^2cx^2\right)^{3/2} \arccos(ax)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(3/2)*acos(a*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \left(c-a^2cx^2\right)^{3/2} \arccos(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \left(c-a^2cx^2\right)^{3/2} \arccos(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)*arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(c-a^2cx^2\right)^{3/2} \arccos(ax)^{5/2}\,dx = \int \mathrm{acos}(a\,x)^{5/2}\,\left(c-a^2\,c\,x^2\right)^{3/2}\,dx$$

input
```
int(acos(a*x)^(5/2)*(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(acos(a*x)^(5/2)*(c - a^2*c*x^2)^(3/2), x)
```

Reduce [F]

$$\int (c -a^2cx^2)^{3/2} \arccos(ax)^{5/2}\,dx = \sqrt{c}\,c\left(-\left(\int \sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}\,acos(ax)^2\,x^2dx\right)a^2+\int \sqrt{-a^2x^2+1}$$

input
```
int((-a^2*c*x^2+c)^(3/2)*acos(a*x)^(5/2),x)
```

output
```
sqrt(c)*c*( - int(sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*acos(a*x)**2*x**2
,x)*a**2 + int(sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*acos(a*x)**2,x))
```

3.454 $\int \sqrt{c-a^2cx^2}\arccos(ax)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 247

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^{5/2}\,dx=-\frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}+\frac{5\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}{16a\sqrt{1-a^2x^2}}-\frac{5ax^2\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}{8\sqrt{1-a^2x^2}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\arccos(ax)^{5/2}+\frac{\sqrt{c-a^2cx^2}\arccos(ax)^{7/2}}{7a\sqrt{1-a^2x^2}}+\frac{15\sqrt{\pi}\sqrt{c-a^2cx^2}\operatorname{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{128a\sqrt{1-a^2x^2}}$$

output
```
-15/32*x*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2)+5/16*(-a^2*c*x^2+c)^(1/2)*
arccos(a*x)^(3/2)/a/(-a^2*x^2+1)^(1/2)-5/8*a*x^2*(-a^2*c*x^2+c)^(1/2)*arcc
os(a*x)^(3/2)/(-a^2*x^2+1)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(5
/2)+1/7*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(7/2)/a/(-a^2*x^2+1)^(1/2)+15/128
*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))/a/(-
a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.45

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^{5/2}\,dx = -\frac{\sqrt{c(1-a^2x^2)}\left(-105\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+2\sqrt{\arccos(ax)}(64\arccos(ax)^3-140\arccos(ax)\cos(2}{896a\sqrt{1-a^2x^2}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^(5/2),x]
```

output
```
-1/896*(Sqrt[c*(1 - a^2*x^2)]*(-105*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]]
)/Sqrt[Pi]] + 2*Sqrt[ArcCos[a*x]]*(64*ArcCos[a*x]^3 - 140*ArcCos[a*x]*Cos[
2*ArcCos[a*x]] - 7*(-15 + 16*ArcCos[a*x]^2)*Sin[2*ArcCos[a*x]])))/(a*Sqrt[
1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.82, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5157, 5141, 5153, 5211, 5147, 4906, 27, 3042, 3786, 3832, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(ax)^{5/2}\sqrt{c-a^2cx^2}\,dx$$

$$\Big\downarrow \text{ 5157}$$

$$\frac{5a\sqrt{c-a^2cx^2}\int x\arccos(ax)^{3/2}dx}{4\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int \frac{\arccos(ax)^{5/2}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{ 5141}$$

$$\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\int\frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}+\frac{\sqrt{c-a^2cx^2}\int\frac{\arccos(ax)^{5/2}}{\sqrt{1-a^2x^2}}dx}{2\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{ 5153}$$

$$\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\int\frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{ 5211}$$

$$\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\int\frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{\int\frac{x}{\sqrt{\arccos(ax)}}dx}{4a}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{ 5147}$$

$$\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\int\frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\int\frac{ax\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^3}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\int\frac{\sin(2\arccos(ax))}{2\sqrt{\arccos(ax)}}d\arccos(ax)}{4a^3}+\frac{\int\frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a^3}+\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{8a^3}+\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \ 3786$$

$$\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\int \sin(2\arccos(ax))d\sqrt{\arccos(ax)}}{4a^3}+\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \ 3832$$

$$\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\int \frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}dx}{2a^2}+\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^3}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}$$

$$\Big\downarrow \ 5153$$

$$-\frac{\arccos(ax)^{7/2}\sqrt{c-a^2cx^2}}{7a\sqrt{1-a^2x^2}}+\frac{1}{2}x\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}+\frac{5a\sqrt{c-a^2cx^2}\left(\frac{3}{4}a\left(\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^3}-\frac{\arccos(ax)^{3/2}}{3a^3}-\frac{x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a^2}\right)+\frac{1}{2}x^2\arccos(ax)^{3/2}\right)}{4\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^(5/2),x]
```

output
```
(x*Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^(5/2))/2 - (Sqrt[c - a^2*c*x^2]*ArcCos[
a*x]^(7/2))/(7*a*Sqrt[1 - a^2*x^2]) + (5*a*Sqrt[c - a^2*c*x^2]*((x^2*ArcCo
s[a*x]^(3/2))/2 + (3*a*(-1/2*(x*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/a^2 -
 ArcCos[a*x]^(3/2)/(3*a^3) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt
[Pi]])/(8*a^3)))/4))/(4*Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

Maple [F]

$$\int \sqrt{-a^2c\,x^2+c}\,\arccos\left(ax\right)^{\frac{5}{2}}dx$$

input
```
int((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(5/2),x)
```

output
```
int((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c - a^2 c x^2} \arccos(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{c - a^2 c x^2} \arccos(ax)^{5/2} \, dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)*acos(a*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c - a^2 c x^2} \arccos(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^{5/2}\,dx = \int \mathrm{acos}(a\,x)^{5/2}\,\sqrt{c-a^2\,c\,x^2}\,dx$$

input
```
int(acos(a*x)^(5/2)*(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(acos(a*x)^(5/2)*(c - a^2*c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c-a^2cx^2}\arccos(ax)^{5/2}\,dx = \sqrt{c}\left(\int \sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,acos(ax)^2\,dx\right)$$

input
```
int((-a^2*c*x^2+c)^(1/2)*acos(a*x)^(5/2),x)
```

output
```
sqrt(c)*int(sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*acos(a*x)**2,x)
```

3.455 $\int \frac{\arccos(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 44

$$\int \frac{\arccos(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx = \frac{2\sqrt{1-a^2x^2}\arccos(ax)^{7/2}}{7a\sqrt{c-a^2cx^2}}$$

output
```
2/7*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(7/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2\sqrt{1-a^2x^2}\arccos(ax)^{7/2}}{7a\sqrt{c-a^2cx^2}}$$

input
```
Integrate[ArcCos[a*x]^(5/2)/Sqrt[c - a^2*c*x^2],x]
```

output
```
(-2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(7/2))/(7*a*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx$$

$$\Big\downarrow \text{5153}$$

$$-\frac{2\sqrt{1-a^2x^2}\arccos(ax)^{7/2}}{7a\sqrt{c-a^2cx^2}}$$

input
```
Int[ArcCos[a*x]^(5/2)/Sqrt[c - a^2*c*x^2],x]
```

output
```
(-2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(7/2))/(7*a*Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{2\arccos(ax)^{\frac{7}{2}}\sqrt{-a^2x^2+1}}{7\sqrt{-c(a^2x^2-1)}\,a}$	38

input
```
int(arccos(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*arccos(a*x)^(7/2)/(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(acos(a*x)**(5/2)/(-a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.32

$$\int \frac{\arccos(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2\,\arccos\left(ax\right)^{\frac{7}{2}}}{7\,a\sqrt{c}}$$

input
```
integrate(arccos(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-2/7*arccos(a*x)^(7/2)/(a*sqrt(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^{5/2}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

input
```
int(acos(a*x)^(5/2)/(c - a^2*c*x^2)^(1/2),x)
```

output
```
int(acos(a*x)^(5/2)/(c - a^2*c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int \frac{\arccos(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx = -\frac{2\sqrt{c}\,\sqrt{acos\,(ax)}\,acos(ax)^3}{7ac}$$

input
```
int(acos(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(acos(a*x))*acos(a*x)**3)/(7*a*c)
```

3.456 $\int \frac{\arccos(ax)^{5/2}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arccos(ax)^{5/2}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{x\arccos(ax)^{5/2}}{c\sqrt{c-a^2cx^2}} - \frac{5a\sqrt{1-a^2x^2}\,\mathrm{Int}\left(\frac{x\arccos(ax)^{3/2}}{1-a^2x^2},x\right)}{2c\sqrt{c-a^2cx^2}}$$

output
```
x*arccos(a*x)^(5/2)/c/(-a^2*c*x^2+c)^(1/2)-5/2*a*(-a^2*x^2+1)^(1/2)*Defer(
Int)(x*arccos(a*x)^(3/2)/(-a^2*x^2+1),x)/c/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 3.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arccos(ax)^{5/2}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\arccos(ax)^{5/2}}{(c-a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[ArcCos[a*x]^(5/2)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[ArcCos[a*x]^(5/2)/(c - a^2*c*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^{5/2}}{(c-a^2cx^2)^{3/2}}\,dx$$
$$\Big\downarrow \; 5161$$
$$\frac{5a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^{3/2}}{1-a^2x^2}dx}{2c\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^{5/2}}{c\sqrt{c-a^2cx^2}}$$
$$\Big\downarrow \; 5235$$
$$\frac{5a\sqrt{1-a^2x^2}\int \frac{x\arccos(ax)^{3/2}}{1-a^2x^2}dx}{2c\sqrt{c-a^2cx^2}}+\frac{x\arccos(ax)^{5/2}}{c\sqrt{c-a^2cx^2}}$$

input
```
Int[ArcCos[a*x]^(5/2)/(c - a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arccos\left(ax\right)^{\frac{5}{2}}}{\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}dx$$

input
```
int(arccos(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arccos(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{5/2}}{(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^{5/2}}{(c-a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(acos(a*x)**(5/2)/(-a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{5/2}}{(c-a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arccos(ax)^{5/2}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\arccos{(ax)}^{\frac{5}{2}}}{(-a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arccos(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^(5/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arccos(ax)^{5/2}}{(c-a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^{5/2}}{{(c-a^2\,c\,x^2)}^{3/2}}\,dx$$

input
```
int(acos(a*x)^(5/2)/(c - a^2*c*x^2)^(3/2),x)
```

output
```
int(acos(a*x)^(5/2)/(c - a^2*c*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.29

$$\int \frac{\arccos(ax)^{5/2}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(-2\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}\,acos(ax)^2\,x-5\left(\int\frac{\sqrt{acos(ax)}\,acos(ax)x}{a^2x^2-1}dx\right)a^3x^2+5\left(\right.}{2c^2\,(a^2x^2-1)}$$

input
```
int(acos(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*acos(a*x)**2*x - 5*i
nt((sqrt(acos(a*x))*acos(a*x)*x)/(a**2*x**2 - 1),x)*a**3*x**2 + 5*int((sqr
t(acos(a*x))*acos(a*x)*x)/(a**2*x**2 - 1),x)*a))/(2*c**2*(a**2*x**2 - 1))
```

3.457 $\int (a^2-x^2)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 226

$$\begin{aligned}\int (a^2-x^2)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}\,dx &= \frac{3}{8}a^2x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\\ &+\frac{1}{4}x(a^2-x^2)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}+\frac{a^3\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}}{4\sqrt{1-\frac{x^2}{a^2}}}\\ &-\frac{a^3\sqrt{\frac{\pi}{2}}\sqrt{a^2-x^2}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)}{64\sqrt{1-\frac{x^2}{a^2}}}\\ &-\frac{a^3\sqrt{\pi}\sqrt{a^2-x^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)}{8\sqrt{1-\frac{x^2}{a^2}}}\end{aligned}$$

output

```
3/8*a^2*x*(a^2-x^2)^(1/2)*arccos(x/a)^(1/2)+1/4*x*(a^2-x^2)^(3/2)*arccos(x
/a)^(1/2)+1/4*a^3*(a^2-x^2)^(1/2)*arccos(x/a)^(3/2)/(1-x^2/a^2)^(1/2)-1/12
8*a^3*2^(1/2)*Pi^(1/2)*(a^2-x^2)^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arccos(
x/a)^(1/2))/(1-x^2/a^2)^(1/2)-1/8*a^3*Pi^(1/2)*(a^2-x^2)^(1/2)*FresnelS(2*
arccos(x/a)^(1/2)/Pi^(1/2))/(1-x^2/a^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.73

$$\int \left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}\,dx = -\frac{a^3\sqrt{a^2-x^2}\left(32\arccos\left(\frac{x}{a}\right)^2+16\sqrt{\pi}\sqrt{\arccos\left(\frac{x}{a}\right)}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)+\sqrt{-i\arccos\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},-4i\arccos\left(\frac{x}{a}\right)\right)+\ldots\right)}{128\sqrt{1-\frac{x^2}{a^2}}\sqrt{\arccos\left(\frac{x}{a}\right)}}$$

input
```
Integrate[(a^2 - x^2)^(3/2)*Sqrt[ArcCos[x/a]],x]
```

output
```
-1/128*(a^3*Sqrt[a^2 - x^2]*(32*ArcCos[x/a]^2 + 16*Sqrt[Pi]*Sqrt[ArcCos[x/
a]]*FresnelS[(2*Sqrt[ArcCos[x/a]])/Sqrt[Pi]] + Sqrt[(-I)*ArcCos[x/a]]*Gamm
a[3/2, (-4*I)*ArcCos[x/a]] + Sqrt[I*ArcCos[x/a]]*Gamma[3/2, (4*I)*ArcCos[x
/a]] - 32*ArcCos[x/a]*Sin[2*ArcCos[x/a]]))/(Sqrt[1 - x^2/a^2]*Sqrt[ArcCos[
x/a]])
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {5159, 27, 5157, 5147, 4906, 27, 3042, 3786, 3832, 5153, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}\,dx$$

$$\Big\downarrow \ 5159$$

$$\frac{3}{4}a^2\int\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}dx+\frac{a\sqrt{a^2-x^2}\int\frac{x(a^2-x^2)}{a^2\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{8\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{4}a^2\int\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}dx+\frac{\sqrt{a^2-x^2}\int\frac{x(a^2-x^2)}{\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{8a\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 5157$$

$$\frac{3}{4}a^2\left(\frac{\sqrt{a^2-x^2}\int\frac{x}{\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{4a\sqrt{1-\frac{x^2}{a^2}}}+\frac{\sqrt{a^2-x^2}\int\frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)+\frac{\sqrt{a^2-x^2}\int\frac{x(a^2-x^2)}{\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{8a\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 5147$$

$$\frac{3}{4}a^2\left(-\frac{a\sqrt{a^2-x^2}\int\frac{x\sqrt{1-\frac{x^2}{a^2}}}{a\sqrt{\arccos\left(\frac{x}{a}\right)}}d\arccos\left(\frac{x}{a}\right)}{4\sqrt{1-\frac{x^2}{a^2}}}+\frac{\sqrt{a^2-x^2}\int\frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)+\frac{\sqrt{a^2-x^2}\int\frac{x(a^2-x^2)}{\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{8a\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \; 4906$$

$$\frac{\sqrt{a^2-x^2}\int \frac{x(a^2-x^2)}{\sqrt{\arccos(\frac{x}{a})}}dx}{8a\sqrt{1-\frac{x^2}{a^2}}}+\frac{3}{4}a^2\left(\frac{\sqrt{a^2-x^2}\int \frac{\sqrt{\arccos(\frac{x}{a})}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\int \frac{\sin(2\arccos(\frac{x}{a}))}{2\sqrt{\arccos(\frac{x}{a})}}d\arccos\left(\frac{x}{a}\right)}{4\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$\downarrow$ 27

$$\frac{\sqrt{a^2-x^2}\int \frac{x(a^2-x^2)}{\sqrt{\arccos(\frac{x}{a})}}dx}{8a\sqrt{1-\frac{x^2}{a^2}}}+\frac{3}{4}a^2\left(\frac{\sqrt{a^2-x^2}\int \frac{\sqrt{\arccos(\frac{x}{a})}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\int \frac{\sin(2\arccos(\frac{x}{a}))}{\sqrt{\arccos(\frac{x}{a})}}d\arccos\left(\frac{x}{a}\right)}{8\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$\downarrow$ 3042

$$\frac{\sqrt{a^2-x^2}\int \frac{x(a^2-x^2)}{\sqrt{\arccos(\frac{x}{a})}}dx}{8a\sqrt{1-\frac{x^2}{a^2}}}+\frac{3}{4}a^2\left(\frac{\sqrt{a^2-x^2}\int \frac{\sqrt{\arccos(\frac{x}{a})}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\int \frac{\sin(2\arccos(\frac{x}{a}))}{\sqrt{\arccos(\frac{x}{a})}}d\arccos\left(\frac{x}{a}\right)}{8\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$\downarrow$ 3786

$$\frac{\sqrt{a^2-x^2}\int \frac{x(a^2-x^2)}{\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{8a\sqrt{1-\frac{x^2}{a^2}}}+$$

$$\frac{3}{4}a^2\left(\frac{\sqrt{a^2-x^2}\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\int \sin\left(2\arccos\left(\frac{x}{a}\right)\right)d\sqrt{\arccos\left(\frac{x}{a}\right)}}{4\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)+$$

$$\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \ 3832$$

$$\frac{3}{4}a^2\left(\frac{\sqrt{a^2-x^2}\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}-\frac{\sqrt{\pi}a\sqrt{a^2-x^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)}{8\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)+$$

$$\frac{\sqrt{a^2-x^2}\int \frac{x(a^2-x^2)}{\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{8a\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \ 5153$$

$$\frac{\sqrt{a^2-x^2}\int \frac{x(a^2-x^2)}{\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{8a\sqrt{1-\frac{x^2}{a^2}}}+$$

$$\frac{3}{4}a^2\left(-\frac{\sqrt{\pi}a\sqrt{a^2-x^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)}{8\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)+$$

$$\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \ 5225$$

$$-\frac{a^3\sqrt{a^2-x^2}\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{a\sqrt{\arccos\left(\frac{x}{a}\right)}}d\arccos\left(\frac{x}{a}\right)}{8\sqrt{1-\frac{x^2}{a^2}}}+\frac{3}{4}a^2\left(-\frac{\sqrt{\pi}a\sqrt{a^2-x^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)}{8\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$\Big\downarrow$ 4906

$$-\frac{a^3\sqrt{a^2-x^2}\int \left(\frac{\sin\left(2\arccos\left(\frac{x}{a}\right)\right)}{4\sqrt{\arccos\left(\frac{x}{a}\right)}}-\frac{\sin\left(4\arccos\left(\frac{x}{a}\right)\right)}{8\sqrt{\arccos\left(\frac{x}{a}\right)}}\right)d\arccos\left(\frac{x}{a}\right)}{8\sqrt{1-\frac{x^2}{a^2}}}+\frac{3}{4}a^2\left(-\frac{\sqrt{\pi}a\sqrt{a^2-x^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)}{8\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$\Big\downarrow$ 2009

$$\frac{3}{4}a^2\left(-\frac{\sqrt{\pi}a\sqrt{a^2-x^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)}{8\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}-\frac{a^3\sqrt{a^2-x^2}\left(\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)\right)}{8\sqrt{1-\frac{x^2}{a^2}}}$$

input
```
Int[(a^2 - x^2)^(3/2)*Sqrt[ArcCos[x/a]],x]
```

output
```
(x*(a^2 - x^2)^(3/2)*Sqrt[ArcCos[x/a]])/4 - (a^3*Sqrt[a^2 - x^2]*(-1/8*(Sq
rt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcCos[x/a]]]) + (Sqrt[Pi]*FresnelS[(2
*Sqrt[ArcCos[x/a]])/Sqrt[Pi]])/4))/(8*Sqrt[1 - x^2/a^2]) + (3*a^2*((x*Sqrt
[a^2 - x^2]*Sqrt[ArcCos[x/a]])/2 - (a*Sqrt[a^2 - x^2]*ArcCos[x/a]^(3/2))/(
3*Sqrt[1 - x^2/a^2]) - (a*Sqrt[Pi]*Sqrt[a^2 - x^2]*FresnelS[(2*Sqrt[ArcCos
[x/a]])/Sqrt[Pi]])/(8*Sqrt[1 - x^2/a^2])))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \left(a^2 - x^2\right)^{\frac{3}{2}} \sqrt{\arccos\left(\frac{x}{a}\right)}dx$$

input
```
int((a^2-x^2)^(3/2)*arccos(x/a)^(1/2),x)
```

output
```
int((a^2-x^2)^(3/2)*arccos(x/a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a^2 - x^2\right)^{3/2} \sqrt{\arccos\left(\frac{x}{a}\right)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2-x^2)^(3/2)*arccos(x/a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \left(a^2 - x^2\right)^{3/2} \sqrt{\arccos\left(\frac{x}{a}\right)}\, dx = \int \left(-(-a + x)\,(a + x)\right)^{\frac{3}{2}} \sqrt{\operatorname{acos}\left(\frac{x}{a}\right)}\, dx$$

input
```
integrate((a**2-x**2)**(3/2)*acos(x/a)**(1/2),x)
```

output
```
Integral((-(-a + x)*(a + x))**(3/2)*sqrt(acos(x/a)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a^2 - x^2\right)^{3/2} \sqrt{\arccos\left(\frac{x}{a}\right)}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2-x^2)^(3/2)*arccos(x/a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}\,dx=\int \left(a^2-x^2\right)^{\frac{3}{2}}\sqrt{\arccos\left(\frac{x}{a}\right)}\,dx$$

input
```
integrate((a^2-x^2)^(3/2)*arccos(x/a)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2 - x^2)^(3/2)*sqrt(arccos(x/a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}\,dx=\int \sqrt{\operatorname{acos}\left(\frac{x}{a}\right)}\left(a^2-x^2\right)^{3/2}\,dx$$

input
```
int(acos(x/a)^(1/2)*(a^2 - x^2)^(3/2),x)
```

output
```
int(acos(x/a)^(1/2)*(a^2 - x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a^2-x^2\right)^{3/2}\sqrt{\arccos\left(\frac{x}{a}\right)}\,dx=-\left(\int \sqrt{a^2-x^2}\,\sqrt{acos\left(\frac{x}{a}\right)}\,x^2dx\right)+\left(\int \sqrt{a^2-x^2}\,\sqrt{acos\left(\frac{x}{a}\right)}dx\right)a^2$$

input
```
int((a^2-x^2)^(3/2)*acos(x/a)^(1/2),x)
```

output
```
- int(sqrt(a**2 - x**2)*sqrt(acos(x/a))*x**2,x) + int(sqrt(a**2 - x**2)*s
qrt(acos(x/a)),x)*a**2
```

3.458 $\int \sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\,dx = \frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}+\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{\pi}\sqrt{a^2-x^2}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)}{8\sqrt{1-\frac{x^2}{a^2}}}$$

output
```
1/2*x*(a^2-x^2)^(1/2)*arccos(x/a)^(1/2)+1/3*a*(a^2-x^2)^(1/2)*arccos(x/a)^
(3/2)/(1-x^2/a^2)^(1/2)-1/8*a*Pi^(1/2)*(a^2-x^2)^(1/2)*FresnelS(2*arccos(x
/a)^(1/2)/Pi^(1/2))/(1-x^2/a^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

$$\int \sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\,dx$$
$$=\frac{a\sqrt{a^2-x^2}\left(-3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)+2\sqrt{\arccos\left(\frac{x}{a}\right)}\left(-4\arccos\left(\frac{x}{a}\right)+3\sin\left(2\arccos\left(\frac{x}{a}\right)\right)\right)\right)}{24\sqrt{1-\frac{x^2}{a^2}}}$$

input
```
Integrate[Sqrt[a^2 - x^2]*Sqrt[ArcCos[x/a]],x]
```

output
```
(a*Sqrt[a^2 - x^2]*(-3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[x/a]])/Sqrt[Pi]] +
 2*Sqrt[ArcCos[x/a]]*(-4*ArcCos[x/a] + 3*Sin[2*ArcCos[x/a]])))/(24*Sqrt[1
- x^2/a^2])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5157, 5147, 4906, 27, 3042, 3786, 3832, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\,dx$$
$$\Big\downarrow \text{ 5157}$$
$$\frac{\sqrt{a^2-x^2}\int \frac{x}{\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{4a\sqrt{1-\frac{x^2}{a^2}}}+\frac{\sqrt{a^2-x^2}\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$
$$\Big\downarrow \text{ 5147}$$

$$-\frac{a\sqrt{a^2-x^2}\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{a\sqrt{\arccos\left(\frac{x}{a}\right)}}d\arccos\left(\frac{x}{a}\right)}{4\sqrt{1-\frac{x^2}{a^2}}}+\frac{\sqrt{a^2-x^2}\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{\sqrt{a^2-x^2}\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\int \frac{\sin\left(2\arccos\left(\frac{x}{a}\right)\right)}{2\sqrt{\arccos\left(\frac{x}{a}\right)}}d\arccos\left(\frac{x}{a}\right)}{4\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2-x^2}\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\int \frac{\sin\left(2\arccos\left(\frac{x}{a}\right)\right)}{\sqrt{\arccos\left(\frac{x}{a}\right)}}d\arccos\left(\frac{x}{a}\right)}{8\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a^2-x^2}\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\int \frac{\sin\left(2\arccos\left(\frac{x}{a}\right)\right)}{\sqrt{\arccos\left(\frac{x}{a}\right)}}d\arccos\left(\frac{x}{a}\right)}{8\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{ 3786}$$

$$\frac{\sqrt{a^2-x^2}\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\int \sin\left(2\arccos\left(\frac{x}{a}\right)\right)d\sqrt{\arccos\left(\frac{x}{a}\right)}}{4\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{ 3832}$$

$$\frac{\sqrt{a^2-x^2}\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}-\frac{\sqrt{\pi}a\sqrt{a^2-x^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)}{8\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

$$\Big\downarrow \text{ 5153}$$

$$-\frac{\sqrt{\pi}a\sqrt{a^2-x^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)}{8\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}$$

input
```
Int[Sqrt[a^2 - x^2]*Sqrt[ArcCos[x/a]],x]
```

output
```
(x*Sqrt[a^2 - x^2]*Sqrt[ArcCos[x/a]])/2 - (a*Sqrt[a^2 - x^2]*ArcCos[x/a]^(
3/2))/(3*Sqrt[1 - x^2/a^2]) - (a*Sqrt[Pi]*Sqrt[a^2 - x^2]*FresnelS[(2*Sqrt
[ArcCos[x/a]])/Sqrt[Pi]])/(8*Sqrt[1 - x^2/a^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

Maple [F]

$$\int \sqrt{a^2-x^2}\,\sqrt{\arccos\left(\frac{x}{a}\right)}dx$$

input
```
int((a^2-x^2)^(1/2)*arccos(x/a)^(1/2),x)
```

output
```
int((a^2-x^2)^(1/2)*arccos(x/a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2-x^2)^(1/2)*arccos(x/a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a^2 - x^2} \sqrt{\arccos\left(\frac{x}{a}\right)} \, dx = \int \sqrt{-(-a+x)(a+x)} \sqrt{\operatorname{acos}\left(\frac{x}{a}\right)} \, dx$$

input
```
integrate((a**2-x**2)**(1/2)*acos(x/a)**(1/2),x)
```

output
```
Integral(sqrt(-(-a + x)*(a + x))*sqrt(acos(x/a)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a^2 - x^2} \sqrt{\arccos\left(\frac{x}{a}\right)} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2-x^2)^(1/2)*arccos(x/a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \sqrt{a^2 - x^2} \sqrt{\arccos\left(\frac{x}{a}\right)} \, dx = \int \sqrt{a^2 - x^2} \sqrt{\arccos\left(\frac{x}{a}\right)} \, dx$$

input
```
integrate((a^2-x^2)^(1/2)*arccos(x/a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2 - x^2)*sqrt(arccos(x/a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\,dx = \int \sqrt{\mathrm{acos}\left(\frac{x}{a}\right)}\,\sqrt{a^2-x^2}\,dx$$

input
```
int(acos(x/a)^(1/2)*(a^2 - x^2)^(1/2),x)
```

output
```
int(acos(x/a)^(1/2)*(a^2 - x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}\,dx = \int \sqrt{a^2-x^2}\,\sqrt{acos\left(\frac{x}{a}\right)}dx$$

input
```
int((a^2-x^2)^(1/2)*acos(x/a)^(1/2),x)
```

output
```
int(sqrt(a**2 - x**2)*sqrt(acos(x/a)),x)
```

3.459 $\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 42

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx = \frac{2a\sqrt{1-\frac{x^2}{a^2}}\arccos\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2-x^2}}$$

output
```
2/3*a*(1-x^2/a^2)^(1/2)*arccos(x/a)^(3/2)/(a^2-x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx = -\frac{2a\sqrt{1-\frac{x^2}{a^2}}\arccos\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2-x^2}}$$

input
```
Integrate[Sqrt[ArcCos[x/a]]/Sqrt[a^2 - x^2],x]
```

output
```
(-2*a*Sqrt[1 - x^2/a^2]*ArcCos[x/a]^(3/2))/(3*Sqrt[a^2 - x^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{5153\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}dx$$
$$\Big\downarrow \text{5153}$$
$$-\frac{2a\sqrt{1-\frac{x^2}{a^2}}\arccos\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2-x^2}}$$

input
```
Int[Sqrt[ArcCos[x/a]]/Sqrt[a^2 - x^2],x]
```

output
```
(-2*a*Sqrt[1 - x^2/a^2]*ArcCos[x/a]^(3/2))/(3*Sqrt[a^2 - x^2])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{2\arccos\left(\frac{x}{a}\right)^{\frac{3}{2}}\sqrt{\frac{a^2-x^2}{a^2}}\,a}{3\sqrt{a^2-x^2}}$	38

input
```
int(arccos(x/a)^(1/2)/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*arccos(x/a)^(3/2)/(a^2-x^2)^(1/2)*((a^2-x^2)/a^2)^(1/2)*a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx = -\frac{2}{3}\arctan\left(\frac{\sqrt{a^2-x^2}}{x}\right)^{\frac{3}{2}}$$

input
```
integrate(arccos(x/a)^(1/2)/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*arctan(sqrt(a^2 - x^2)/x)^(3/2)
```

Sympy [F]

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx = \int \frac{\sqrt{\operatorname{acos}\left(\frac{x}{a}\right)}}{\sqrt{-\left(-a+x\right)\left(a+x\right)}}\,dx$$

input
```
integrate(acos(x/a)**(1/2)/(a**2-x**2)**(1/2),x)
```

output
```
Integral(sqrt(acos(x/a))/sqrt(-(-a + x)*(a + x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(x/a)^(1/2)/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx = -\frac{2\,|a|\arccos\left(\frac{x}{a}\right)^{\frac{3}{2}}}{3\,a}$$

input
```
integrate(arccos(x/a)^(1/2)/(a^2-x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/3*abs(a)*arccos(x/a)^(3/2)/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx = \int \frac{\sqrt{\mathrm{acos}\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx$$

input
```
int(acos(x/a)^(1/2)/(a^2 - x^2)^(1/2),x)
```

output
```
int(acos(x/a)^(1/2)/(a^2 - x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx = -\frac{2\sqrt{acos\left(\frac{x}{a}\right)}\,acos\left(\frac{x}{a}\right)}{3}$$

input
```
int(acos(x/a)^(1/2)/(a^2-x^2)^(1/2),x)
```

output
```
( - 2*sqrt(acos(x/a))*acos(x/a))/3
```

3.460 $\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{(a^2-x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{(a^2-x^2)^{3/2}}\,dx = \frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2-x^2}} - \frac{\sqrt{1-\frac{x^2}{a^2}}\,\mathrm{Int}\left(\frac{x}{\left(1-\frac{x^2}{a^2}\right)\sqrt{\arccos\left(\frac{x}{a}\right)}},x\right)}{2a^3\sqrt{a^2-x^2}}$$

output
```
x*arccos(x/a)^(1/2)/a^2/(a^2-x^2)^(1/2)-1/2*(1-x^2/a^2)^(1/2)*Defer(Int)(x
/(1-x^2/a^2)/arccos(x/a)^(1/2),x)/a^3/(a^2-x^2)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 4.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{(a^2-x^2)^{3/2}}\,dx = \int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{(a^2-x^2)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[ArcCos[x/a]]/(a^2 - x^2)^(3/2),x]
```

output
```
Integrate[Sqrt[ArcCos[x/a]]/(a^2 - x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{(a^2-x^2)^{3/2}}dx$$
$$\Big\downarrow \text{5161}$$
$$\frac{\sqrt{1-\frac{x^2}{a^2}}\int \frac{a^2x}{(a^2-x^2)\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{2a^3\sqrt{a^2-x^2}}+\frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2-x^2}}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sqrt{1-\frac{x^2}{a^2}}\int \frac{x}{(a^2-x^2)\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{2a\sqrt{a^2-x^2}}+\frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2-x^2}}$$
$$\Big\downarrow \text{5235}$$
$$\frac{\sqrt{1-\frac{x^2}{a^2}}\int \frac{x}{(a^2-x^2)\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{2a\sqrt{a^2-x^2}}+\frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2-x^2}}$$

input
```
Int[Sqrt[ArcCos[x/a]]/(a^2 - x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\left(a^{2}-x^{2}\right)^{\frac{3}{2}}}dx$$

input
```
int(arccos(x/a)^(1/2)/(a^2-x^2)^(3/2),x)
```

output
```
int(arccos(x/a)^(1/2)/(a^2-x^2)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\left(a^{2}-x^{2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(x/a)^(1/2)/(a^2-x^2)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 2.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\left(a^{2}-x^{2}\right)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{acos}\left(\frac{x}{a}\right)}}{\left(-\left(-a+x\right)\left(a+x\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(acos(x/a)**(1/2)/(a**2-x**2)**(3/2),x)
```

output
```
Integral(sqrt(acos(x/a))/(-(-a + x)*(a + x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{(a^2-x^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(x/a)^(1/2)/(a^2-x^2)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{(a^2-x^2)^{3/2}}\,dx = \int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{(a^2-x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arccos(x/a)^(1/2)/(a^2-x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arccos(x/a))/(a^2 - x^2)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\mathrm{acos}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

input
```
int(acos(x/a)^(1/2)/(a^2 - x^2)^(3/2),x)
```

output
```
int(acos(x/a)^(1/2)/(a^2 - x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx=\int \frac{\sqrt{a^2-x^2}\,\sqrt{acos\left(\frac{x}{a}\right)}}{a^4-2a^2x^2+x^4}dx$$

input
```
int(acos(x/a)^(1/2)/(a^2-x^2)^(3/2),x)
```

output
```
int((sqrt(a**2 - x**2)*sqrt(acos(x/a)))/(a**4 - 2*a**2*x**2 + x**4),x)
```

3.461 $\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx = \frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{3a^2\left(a^2-x^2\right)^{3/2}} + \frac{2x\sqrt{\arccos\left(\frac{x}{a}\right)}}{3a^4\sqrt{a^2-x^2}} - \frac{\sqrt{1-\frac{x^2}{a^2}}\,\text{Int}\left(\frac{x}{\left(1-\frac{x^2}{a^2}\right)^2\sqrt{\arccos\left(\frac{x}{a}\right)}},x\right)}{6a^5\sqrt{a^2-x^2}} - \frac{\sqrt{1-\frac{x^2}{a^2}}\,\text{Int}\left(\frac{x}{\left(1-\frac{x^2}{a^2}\right)\sqrt{\arccos\left(\frac{x}{a}\right)}},x\right)}{3a^5\sqrt{a^2-x^2}}$$

output
```
1/3*x*arccos(x/a)^(1/2)/a^2/(a^2-x^2)^(3/2)+2/3*x*arccos(x/a)^(1/2)/a^4/(a
^2-x^2)^(1/2)-1/6*(1-x^2/a^2)^(1/2)*Defer(Int)(x/(1-x^2/a^2)^2/arccos(x/a)
^(1/2),x)/a^5/(a^2-x^2)^(1/2)-1/3*(1-x^2/a^2)^(1/2)*Defer(Int)(x/(1-x^2/a^
2)/arccos(x/a)^(1/2),x)/a^5/(a^2-x^2)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{(a^2-x^2)^{5/2}}\,dx=\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{(a^2-x^2)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[ArcCos[x/a]]/(a^2 - x^2)^(5/2),x]
```

output
```
Integrate[Sqrt[ArcCos[x/a]]/(a^2 - x^2)^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{(a^2-x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5163}$$

$$\frac{2\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{(a^2-x^2)^{3/2}}dx}{3a^2}+\frac{\sqrt{1-\frac{x^2}{a^2}}\int \frac{a^4 x}{(a^2-x^2)^2\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{6a^5\sqrt{a^2-x^2}}+\frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{3a^2\left(a^2-x^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{1-\frac{x^2}{a^2}}\int \frac{x}{(a^2-x^2)^2\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{6a\sqrt{a^2-x^2}}+\frac{2\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{(a^2-x^2)^{3/2}}dx}{3a^2}+\frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{3a^2\left(a^2-x^2\right)^{3/2}}$$

$$\Big\downarrow \text{5161}$$

$$\frac{\sqrt{1-\frac{x^2}{a^2}}\int \frac{x}{(a^2-x^2)^2\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{6a\sqrt{a^2-x^2}}+\frac{2\left(\frac{\sqrt{1-\frac{x^2}{a^2}}\int \frac{a^2x}{\left(a^2-x^2\right)\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{2a^3\sqrt{a^2-x^2}}+\frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2-x^2}}\right)}{3a^2}+\frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{3a^2\left(a^2-x^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{1-\frac{x^2}{a^2}}\int \frac{x}{(a^2-x^2)^2\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{6a\sqrt{a^2-x^2}}+\frac{2\left(\frac{\sqrt{1-\frac{x^2}{a^2}}\int \frac{x}{\left(a^2-x^2\right)\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{2a\sqrt{a^2-x^2}}+\frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2-x^2}}\right)}{3a^2}+\frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{3a^2\left(a^2-x^2\right)^{3/2}}$$

$$\Big\downarrow \; 5235$$

$$\frac{\sqrt{1-\frac{x^2}{a^2}}\int \frac{x}{(a^2-x^2)^2\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{6a\sqrt{a^2-x^2}}+\frac{2\left(\frac{\sqrt{1-\frac{x^2}{a^2}}\int \frac{x}{\left(a^2-x^2\right)\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{2a\sqrt{a^2-x^2}}+\frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2-x^2}}\right)}{3a^2}+\frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{3a^2\left(a^2-x^2\right)^{3/2}}$$

input
```
Int[Sqrt[ArcCos[x/a]]/(a^2 - x^2)^(5/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{\frac{5}{2}}}dx$$

input
```
int(arccos(x/a)^(1/2)/(a^2-x^2)^(5/2),x)
```

output
```
int(arccos(x/a)^(1/2)/(a^2-x^2)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(x/a)^(1/2)/(a^2-x^2)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 21.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{acos}\left(\frac{x}{a}\right)}}{\left(-\left(-a+x\right)\left(a+x\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(acos(x/a)**(1/2)/(a**2-x**2)**(5/2),x)
```

output
```
Integral(sqrt(acos(x/a))/(-(-a + x)*(a + x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(x/a)^(1/2)/(a^2-x^2)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx=\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arccos(x/a)^(1/2)/(a^2-x^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arccos(x/a))/(a^2 - x^2)^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx=\int \frac{\sqrt{\mathrm{acos}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx$$

input
```
int(acos(x/a)^(1/2)/(a^2 - x^2)^(5/2),x)
```

output
```
int(acos(x/a)^(1/2)/(a^2 - x^2)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{\arccos\left(\frac{x}{a}\right)}}{(a^2-x^2)^{5/2}}\,dx = \int \frac{\sqrt{a^2-x^2}\,\sqrt{acos\left(\frac{x}{a}\right)}}{a^6-3a^4x^2+3a^2x^4-x^6}dx$$

input
```
int(acos(x/a)^(1/2)/(a^2-x^2)^(5/2),x)
```

output
```
int((sqrt(a**2 - x**2)*sqrt(acos(x/a)))/(a**6 - 3*a**4*x**2 + 3*a**2*x**4
- x**6),x)
```

3.462 $\int (a^2-x^2)^{3/2} \arccos\left(\frac{x}{a}\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 359

$$\int (a^2-x^2)^{3/2} \arccos\left(\frac{x}{a}\right)^{3/2} dx = \frac{27a^3\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}}{256\sqrt{1-\frac{x^2}{a^2}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}}{32\sqrt{1-\frac{x^2}{a^2}}} + \frac{3(a^2-x^2)^{5/2}\sqrt{\arccos\left(\frac{x}{a}\right)}}{32a\sqrt{1-\frac{x^2}{a^2}}}$$

$$+\frac{3}{8}a^2x\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}+\frac{1}{4}x(a^2-x^2)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}+\frac{3a^3\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{5/2}}{20\sqrt{1-\frac{x^2}{a^2}}}-\frac{3a^3\sqrt{\frac{\pi}{2}}\sqrt{a^2-}$$

output
```
27/256*a^3*(a^2-x^2)^(1/2)*arccos(x/a)^(1/2)/(1-x^2/a^2)^(1/2)-9/32*a*x^2*
(a^2-x^2)^(1/2)*arccos(x/a)^(1/2)/(1-x^2/a^2)^(1/2)+3/32*(a^2-x^2)^(5/2)*a
rccos(x/a)^(1/2)/a/(1-x^2/a^2)^(1/2)+3/8*a^2*x*(a^2-x^2)^(1/2)*arccos(x/a)
^(3/2)+1/4*x*(a^2-x^2)^(3/2)*arccos(x/a)^(3/2)+3/20*a^3*(a^2-x^2)^(1/2)*ar
ccos(x/a)^(5/2)/(1-x^2/a^2)^(1/2)-3/1024*a^3*2^(1/2)*Pi^(1/2)*(a^2-x^2)^(1
/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(x/a)^(1/2))/(1-x^2/a^2)^(1/2)-3/32*
a^3*Pi^(1/2)*(a^2-x^2)^(1/2)*FresnelC(2*arccos(x/a)^(1/2)/Pi^(1/2))/(1-x^2
/a^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.58

$$\int \left(a^2-x^2\right)^{3/2} \arccos \left(\frac{x}{a}\right)^{3/2} dx =$$

$$-\frac{a^3\sqrt{a^2-x^2}\left(240\sqrt{\pi}\sqrt{\arccos\left(\frac{x}{a}\right)^2}\text{FresnelC}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)+\sqrt{\arccos\left(\frac{x}{a}\right)}\left(5\sqrt{i\arccos\left(\frac{x}{a}\right)}\Gamma\left(\frac{5}{2},-4i\arccos\right.\right.\right.}{}$$

input
```
Integrate[(a^2 - x^2)^(3/2)*ArcCos[x/a]^(3/2),x]
```

output
```
-1/2560*(a^3*Sqrt[a^2 - x^2]*(240*Sqrt[Pi]*Sqrt[ArcCos[x/a]^2]*FresnelC[(2
*Sqrt[ArcCos[x/a]])/Sqrt[Pi]] + Sqrt[ArcCos[x/a]]*(5*Sqrt[I*ArcCos[x/a]]*G
amma[5/2, (-4*I)*ArcCos[x/a]] + 5*Sqrt[(-I)*ArcCos[x/a]]*Gamma[5/2, (4*I)*
ArcCos[x/a]] + 32*Sqrt[ArcCos[x/a]^2]*(12*ArcCos[x/a]^2 - 15*Cos[2*ArcCos[
x/a]] - 20*ArcCos[x/a]*Sin[2*ArcCos[x/a]]))))/(Sqrt[1 - x^2/a^2]*Sqrt[ArcC
os[x/a]^2])
```

Rubi [A] (verified)

Time = 2.44 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5159, 27, 5157, 5141, 5153, 5183, 5169, 3042, 3793, 2009, 5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2-x^2\right)^{3/2} \arccos \left(\frac{x}{a}\right)^{3/2} dx$$

$$\Big\downarrow \text{5159}$$

$$\frac{3}{4}a^2\int\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}dx+\frac{3a\sqrt{a^2-x^2}\int\frac{x(a^2-x^2)\sqrt{\arccos\left(\frac{x}{a}\right)}}{a^2}dx}{8\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{4}a^2\int\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}dx+\frac{3\sqrt{a^2-x^2}\int x\left(a^2-x^2\right)\sqrt{\arccos\left(\frac{x}{a}\right)}dx}{8a\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{5157}$$

$$\frac{3}{4}a^2\left(\frac{3\sqrt{a^2-x^2}\int x\sqrt{\arccos\left(\frac{x}{a}\right)}dx}{4a\sqrt{1-\frac{x^2}{a^2}}}+\frac{\sqrt{a^2-x^2}\int\frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}\right)+\frac{3\sqrt{a^2-x^2}\int x\left(a^2-x^2\right)\sqrt{\arccos\left(\frac{x}{a}\right)}dx}{8a\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{5141}$$

$$\frac{3}{4}a^2\left(\frac{3\sqrt{a^2-x^2}\left(\frac{\int\frac{x^2}{\sqrt{1-\frac{x^2}{a^2}}\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{4a}+\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}+\frac{\sqrt{a^2-x^2}\int\frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\right.\right.$$

$$\frac{3\sqrt{a^2-x^2}\int x\left(a^2-x^2\right)\sqrt{\arccos\left(\frac{x}{a}\right)}dx}{8a\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{5153}$$

$$\frac{3}{4}a^2\left(\frac{3\sqrt{a^2-x^2}\left(\frac{\int\frac{x^2}{\sqrt{1-\frac{x^2}{a^2}}\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{4a}+\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\right.\right.$$

$$\frac{3\sqrt{a^2-x^2}\int x\left(a^2-x^2\right)\sqrt{\arccos\left(\frac{x}{a}\right)}dx}{8a\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{5183}$$

$$\frac{3}{4}a^2\left(\frac{3\sqrt{a^2-x^2}\left(\frac{\int\frac{x^2}{\sqrt{1-\frac{x^2}{a^2}}\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{4a}+\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\right.\right.$$

$$\frac{3\sqrt{a^2-x^2}\left(-\frac{1}{8}a^3\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\sqrt{\arccos\left(\frac{x}{a}\right)}}dx-\frac{1}{4}\left(a^2-x^2\right)^2\sqrt{\arccos\left(\frac{x}{a}\right)}\right)}{8a\sqrt{1-\frac{x^2}{a^2}}}+$$

$$\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{5169}$$

$$\frac{3}{4}a^2\left(\frac{3\sqrt{a^2-x^2}\left(\frac{\int\frac{x^2}{\sqrt{1-\frac{x^2}{a^2}}\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{4a}+\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\right.\right.$$

$$\frac{3\sqrt{a^2-x^2}\left(\frac{1}{8}a^4\int\frac{\left(1-\frac{x^2}{a^2}\right)^2}{\sqrt{\arccos\left(\frac{x}{a}\right)}}d\arccos\left(\frac{x}{a}\right)-\frac{1}{4}\left(a^2-x^2\right)^2\sqrt{\arccos\left(\frac{x}{a}\right)}\right)}{8a\sqrt{1-\frac{x^2}{a^2}}}+$$

$$\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}a^2\left(\frac{3\sqrt{a^2-x^2}\left(\frac{\int \frac{x^2}{\sqrt{1-\frac{x^2}{a^2}}\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{4a}+\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\right.\right.$$

$$\frac{3\sqrt{a^2-x^2}\left(\frac{1}{8}a^4\int \frac{\sin\left(\arccos\left(\frac{x}{a}\right)\right)^4}{\sqrt{\arccos\left(\frac{x}{a}\right)}}d\arccos\left(\frac{x}{a}\right)-\frac{1}{4}\left(a^2-x^2\right)^2\sqrt{\arccos\left(\frac{x}{a}\right)}\right)}{8a\sqrt{1-\frac{x^2}{a^2}}}+$$

$$\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \; 3793$$

$$\frac{3}{4}a^2\left(\frac{3\sqrt{a^2-x^2}\left(\frac{\int \frac{x^2}{\sqrt{1-\frac{x^2}{a^2}}\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{4a}+\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\right.\right.$$

$$\frac{3\sqrt{a^2-x^2}\left(\frac{1}{8}a^4\int \left(-\frac{\cos\left(2\arccos\left(\frac{x}{a}\right)\right)}{2\sqrt{\arccos\left(\frac{x}{a}\right)}}+\frac{\cos\left(4\arccos\left(\frac{x}{a}\right)\right)}{8\sqrt{\arccos\left(\frac{x}{a}\right)}}+\frac{3}{8\sqrt{\arccos\left(\frac{x}{a}\right)}}\right)d\arccos\left(\frac{x}{a}\right)-\frac{1}{4}\left(a^2-x^2\right)^2\sqrt{\arccos\left(\frac{x}{a}\right)}\right)}{8a\sqrt{1-\frac{x^2}{a^2}}}$$

$$\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \; 2009$$

$$\frac{3}{4}a^2\left(\frac{3\sqrt{a^2-x^2}\left(\frac{\int \frac{x^2}{\sqrt{1-\frac{x^2}{a^2}}\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{4a}+\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\right.\right.$$

$$\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}+$$

$$\frac{3\sqrt{a^2-x^2}\left(\frac{1}{8}a^4\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)-\frac{1}{4}(a\right.}{8a\sqrt{1-\frac{x^2}{a^2}}}$$

$\big\downarrow$ 5225

$$\frac{3}{4}a^2\left(\frac{3\sqrt{a^2-x^2}\left(\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\int\frac{x^2}{a^2\sqrt{\arccos\left(\frac{x}{a}\right)}}d\arccos\left(\frac{x}{a}\right)\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-\right.$$

$$\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}+$$

$$\frac{3\sqrt{a^2-x^2}\left(\frac{1}{8}a^4\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)-\frac{1}{4}(a\right.}{8a\sqrt{1-\frac{x^2}{a^2}}}$$

$\big\downarrow$ 3042

$$\frac{3}{4}a^2\left(\frac{3\sqrt{a^2-x^2}\left(\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\int\frac{\sin\left(\arccos\left(\frac{x}{a}\right)+\frac{\pi}{2}\right)^2}{\sqrt{\arccos\left(\frac{x}{a}\right)}}d\arccos\left(\frac{x}{a}\right)\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2\right.$$

$$\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}+$$

$$\frac{3\sqrt{a^2-x^2}\left(\frac{1}{8}a^4\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)-\frac{1}{4}(a\right.}{8a\sqrt{1-\frac{x^2}{a^2}}}$$

$\big\downarrow$ 3793

$$\frac{3}{4}a^2\left(\frac{3\sqrt{a^2-x^2}\left(\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\int\left(\frac{\cos\left(2\arccos\left(\frac{x}{a}\right)\right)}{2\sqrt{\arccos\left(\frac{x}{a}\right)}}+\frac{1}{2\sqrt{\arccos\left(\frac{x}{a}\right)}}\right)d\arccos\left(\frac{x}{a}\right)\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\right.}{5\sqrt{1-\frac{x^2}{a^2}}}\right.$$

$$\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}+$$

$$\frac{3\sqrt{a^2-x^2}\left(\frac{1}{8}a^4\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)-\frac{1}{4}(a\right.}{8a\sqrt{1-\frac{x^2}{a^2}}}$$

$\downarrow$ 2009

$$\frac{3}{4}a^2\left(\frac{3\sqrt{a^2-x^2}\left(\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\left(\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)+\sqrt{\arccos\left(\frac{x}{a}\right)}\right)\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\operatorname{arcco}}{5\sqrt{1-\frac{x}{a}}}\right.$$

$$\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}+$$

$$\frac{3\sqrt{a^2-x^2}\left(\frac{1}{8}a^4\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arccos\left(\frac{x}{a}\right)}\right)-\frac{1}{4}(a\right.}{8a\sqrt{1-\frac{x^2}{a^2}}}$$

input
```
Int[(a^2 - x^2)^(3/2)*ArcCos[x/a]^(3/2),x]
```

output
```
(x*(a^2 - x^2)^(3/2)*ArcCos[x/a]^(3/2))/4 + (3*Sqrt[a^2 - x^2]*(-1/4*((a^2
 - x^2)^2*Sqrt[ArcCos[x/a]]) + (a^4*((3*Sqrt[ArcCos[x/a]])/4 + (Sqrt[Pi/2]
*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcCos[x/a]]])/8 - (Sqrt[Pi]*FresnelC[(2*Sqrt[
ArcCos[x/a]])/Sqrt[Pi]])/2))/8))/(8*a*Sqrt[1 - x^2/a^2]) + (3*a^2*((x*Sqrt
[a^2 - x^2]*ArcCos[x/a]^(3/2))/2 - (a*Sqrt[a^2 - x^2]*ArcCos[x/a]^(5/2))/(
5*Sqrt[1 - x^2/a^2]) + (3*Sqrt[a^2 - x^2]*((x^2*Sqrt[ArcCos[x/a]])/2 - (a^
2*(Sqrt[ArcCos[x/a]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[x/a]])/Sqrt[Pi]])
/2))/4))/(4*a*Sqrt[1 - x^2/a^2])))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int (a^2 - x^2)^{\frac{3}{2}} \arccos\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

input
```
int((a^2-x^2)^(3/2)*arccos(x/a)^(3/2),x)
```

output
```
int((a^2-x^2)^(3/2)*arccos(x/a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a^2-x^2\right)^{3/2} \arccos\left(\frac{x}{a}\right)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2-x^2)^(3/2)*arccos(x/a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a^2-x^2\right)^{3/2} \arccos\left(\frac{x}{a}\right)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate((a**2-x**2)**(3/2)*acos(x/a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a^2-x^2\right)^{3/2} \arccos\left(\frac{x}{a}\right)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2-x^2)^(3/2)*arccos(x/a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}\,dx=\int \left(a^2-x^2\right)^{\frac{3}{2}}\arccos\left(\frac{x}{a}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a^2-x^2)^(3/2)*arccos(x/a)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2 - x^2)^(3/2)*arccos(x/a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}\,dx=\int \mathrm{acos}\left(\frac{x}{a}\right)^{3/2}\left(a^2-x^2\right)^{3/2}\,dx$$

input
```
int(acos(x/a)^(3/2)*(a^2 - x^2)^(3/2),x)
```

output
```
int(acos(x/a)^(3/2)*(a^2 - x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a^2-x^2\right)^{3/2}\arccos\left(\frac{x}{a}\right)^{3/2}\,dx=-\left(\int \sqrt{a^2-x^2}\,\sqrt{acos\left(\frac{x}{a}\right)}\,acos\left(\frac{x}{a}\right)x^2dx\right)+\left(\int \sqrt{a^2-x^2}\,\sqrt{acos\left(\frac{x}{a}\right)}\,acos\left(\frac{x}{a}\right)dx\right)a^2$$

input
```
int((a^2-x^2)^(3/2)*acos(x/a)^(3/2),x)
```

output
```
- int(sqrt(a**2 - x**2)*sqrt(acos(x/a))*acos(x/a)*x**2,x) + int(sqrt(a**2
- x**2)*sqrt(acos(x/a))*acos(x/a),x)*a**2
```

3.463 $\int \sqrt{a^2-x^2} \arccos\left(\frac{x}{a}\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 215

$$\int \sqrt{a^2-x^2} \arccos\left(\frac{x}{a}\right)^{3/2} dx = \frac{3a\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}}{16\sqrt{1-\frac{x^2}{a^2}}} - \frac{3x^2\sqrt{a^2-x^2}\sqrt{\arccos\left(\frac{x}{a}\right)}}{8a\sqrt{1-\frac{x^2}{a^2}}} + \frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2} + \frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1-\frac{x^2}{a^2}}} - \frac{3a\sqrt{\pi}\sqrt{a^2-x^2}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)}{32\sqrt{1-\frac{x^2}{a^2}}}$$

output

```
3/16*a*(a^2-x^2)^(1/2)*arccos(x/a)^(1/2)/(1-x^2/a^2)^(1/2)-3/8*x^2*(a^2-x^
2)^(1/2)*arccos(x/a)^(1/2)/a/(1-x^2/a^2)^(1/2)+1/2*x*(a^2-x^2)^(1/2)*arcco
s(x/a)^(3/2)+1/5*a*(a^2-x^2)^(1/2)*arccos(x/a)^(5/2)/(1-x^2/a^2)^(1/2)-3/3
2*a*Pi^(1/2)*(a^2-x^2)^(1/2)*FresnelC(2*arccos(x/a)^(1/2)/Pi^(1/2))/(1-x^2
/a^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.51

$$\int \sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}\,dx=\frac{a\sqrt{a^2-x^2}\left(-15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)+2\sqrt{\arccos\left(\frac{x}{a}\right)}\left(-16\arccos\right.\right.}{160\sqrt{1-}}$$

input
```
Integrate[Sqrt[a^2 - x^2]*ArcCos[x/a]^(3/2),x]
```

output
```
(a*Sqrt[a^2 - x^2]*(-15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[x/a]])/Sqrt[Pi]]
+ 2*Sqrt[ArcCos[x/a]]*(-16*ArcCos[x/a]^2 + 15*Cos[2*ArcCos[x/a]] + 20*ArcC
os[x/a]*Sin[2*ArcCos[x/a]])))/(160*Sqrt[1 - x^2/a^2])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5157, 5141, 5153, 5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{5157}$$

$$\frac{3\sqrt{a^2-x^2}\int x\sqrt{\arccos\left(\frac{x}{a}\right)}dx}{4a\sqrt{1-\frac{x^2}{a^2}}}+\frac{\sqrt{a^2-x^2}\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{5141}$$

$$\frac{3\sqrt{a^2-x^2}\left(\frac{\int \frac{x^2}{\sqrt{1-\frac{x^2}{a^2}}\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{4a}+\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}+\frac{\sqrt{a^2-x^2}\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}dx}{2\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{ 5153}$$

$$\frac{3\sqrt{a^2-x^2}\left(\frac{\int \frac{x^2}{\sqrt{1-\frac{x^2}{a^2}}\sqrt{\arccos\left(\frac{x}{a}\right)}}dx}{4a}+\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{ 5225}$$

$$\frac{3\sqrt{a^2-x^2}\left(\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\int \frac{x^2}{a^2\sqrt{\arccos\left(\frac{x}{a}\right)}}d\arccos\left(\frac{x}{a}\right)\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\sqrt{a^2-x^2}\left(\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\int \frac{\sin\left(\arccos\left(\frac{x}{a}\right)+\frac{\pi}{2}\right)^2}{\sqrt{\arccos\left(\frac{x}{a}\right)}}d\arccos\left(\frac{x}{a}\right)\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{3\sqrt{a^2-x^2}\left(\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\int \left(\frac{\cos\left(2\arccos\left(\frac{x}{a}\right)\right)}{2\sqrt{\arccos\left(\frac{x}{a}\right)}}+\frac{1}{2\sqrt{\arccos\left(\frac{x}{a}\right)}}\right)d\arccos\left(\frac{x}{a}\right)\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3\sqrt{a^2-x^2}\left(\frac{1}{2}x^2\sqrt{\arccos\left(\frac{x}{a}\right)}-\frac{1}{4}a^2\left(\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos\left(\frac{x}{a}\right)}}{\sqrt{\pi}}\right)+\sqrt{\arccos\left(\frac{x}{a}\right)}\right)\right)}{4a\sqrt{1-\frac{x^2}{a^2}}}-\frac{a\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{2}x\sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}$$

input
```
Int[Sqrt[a^2 - x^2]*ArcCos[x/a]^(3/2),x]
```

output
```
(x*Sqrt[a^2 - x^2]*ArcCos[x/a]^(3/2))/2 - (a*Sqrt[a^2 - x^2]*ArcCos[x/a]^(
5/2))/(5*Sqrt[1 - x^2/a^2]) + (3*Sqrt[a^2 - x^2]*((x^2*Sqrt[ArcCos[x/a]])/
2 - (a^2*(Sqrt[ArcCos[x/a]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[x/a]])/Sqr
t[Pi]])/2))/4))/(4*a*Sqrt[1 - x^2/a^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5141
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Simp[b*c*(n/(m + 1))   Int[x
^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{
a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \sqrt{a^2-x^2}\, \arccos\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

input
```
int((a^2-x^2)^(1/2)*arccos(x/a)^(3/2),x)
```

output
```
int((a^2-x^2)^(1/2)*arccos(x/a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2-x^2)^(1/2)*arccos(x/a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}\,dx = \int \sqrt{-\left(-a+x\right)\left(a+x\right)}\operatorname{acos}^{\frac{3}{2}}\left(\frac{x}{a}\right)dx$$

input
```
integrate((a**2-x**2)**(1/2)*acos(x/a)**(3/2),x)
```

output
```
Integral(sqrt(-(-a + x)*(a + x))*acos(x/a)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2-x^2)^(1/2)*arccos(x/a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}\,dx=\int \sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a^2-x^2)^(1/2)*arccos(x/a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2 - x^2)*arccos(x/a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}\,dx=\int \mathrm{acos}\left(\frac{x}{a}\right)^{3/2}\sqrt{a^2-x^2}\,dx$$

input
```
int(acos(x/a)^(3/2)*(a^2 - x^2)^(1/2),x)
```

output
```
int(acos(x/a)^(3/2)*(a^2 - x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a^2-x^2}\arccos\left(\frac{x}{a}\right)^{3/2}\,dx=\int \sqrt{a^2-x^2}\,\sqrt{acos\left(\frac{x}{a}\right)}\,acos\left(\frac{x}{a}\right)dx$$

input
```
int((a^2-x^2)^(1/2)*acos(x/a)^(3/2),x)
```

output
```
int(sqrt(a**2 - x**2)*sqrt(acos(x/a))*acos(x/a),x)
```

3.464 $\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 42

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx=\frac{2a\sqrt{1-\frac{x^2}{a^2}}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

output
```
2/5*a*(1-x^2/a^2)^(1/2)*arccos(x/a)^(5/2)/(a^2-x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx=-\frac{2a\sqrt{1-\frac{x^2}{a^2}}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

input
```
Integrate[ArcCos[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```

output
```
(-2*a*Sqrt[1 - x^2/a^2]*ArcCos[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$$
$$\Big\downarrow \ 5153$$
$$-\frac{2a\sqrt{1-\frac{x^2}{a^2}}\arccos\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

input
```
Int[ArcCos[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```

output
```
(-2*a*Sqrt[1 - x^2/a^2]*ArcCos[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{2\arccos\left(\frac{x}{a}\right)^{\frac{5}{2}}\sqrt{\frac{a^2-x^2}{a^2}}\,a}{5\sqrt{a^2-x^2}}$	38

input
```
int(arccos(x/a)^(3/2)/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*arccos(x/a)^(5/2)/(a^2-x^2)^(1/2)*((a^2-x^2)/a^2)^(1/2)*a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.48

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = -\frac{2}{5}\arctan\left(\frac{\sqrt{a^2-x^2}}{x}\right)^{\frac{5}{2}}$$

input
```
integrate(arccos(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*arctan(sqrt(a^2 - x^2)/x)^(5/2)
```

Sympy [F]

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \int \frac{\operatorname{acos}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{-\left(-a+x\right)\left(a+x\right)}}\,dx$$

input
```
integrate(acos(x/a)**(3/2)/(a**2-x**2)**(1/2),x)
```

output
```
Integral(acos(x/a)**(3/2)/sqrt(-(-a + x)*(a + x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.36

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = -\frac{2\,|a|\arccos\left(\frac{x}{a}\right)^{\frac{5}{2}}}{5\,a}$$

input
```
integrate(arccos(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/5*abs(a)*arccos(x/a)^(5/2)/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = \int \frac{\mathrm{acos}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$$

input
```
int(acos(x/a)^(3/2)/(a^2 - x^2)^(1/2),x)
```

output
```
int(acos(x/a)^(3/2)/(a^2 - x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.40

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx = -\frac{2\sqrt{acos\left(\frac{x}{a}\right)}\,acos\left(\frac{x}{a}\right)^2}{5}$$

input
```
int(acos(x/a)^(3/2)/(a^2-x^2)^(1/2),x)
```

output
```
( - 2*sqrt(acos(x/a))*acos(x/a)**2)/5
```

3.465 $\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx=\frac{x\arccos\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2-x^2}}-\frac{3\sqrt{1-\frac{x^2}{a^2}}\operatorname{Int}\left(\frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{1-\frac{x^2}{a^2}},x\right)}{2a^3\sqrt{a^2-x^2}}$$

output
```
x*arccos(x/a)^(3/2)/a^2/(a^2-x^2)^(1/2)-3/2*(1-x^2/a^2)^(1/2)*Defer(Int)(x
*arccos(x/a)^(1/2)/(1-x^2/a^2),x)/a^3/(a^2-x^2)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 5.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx=\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

input
```
Integrate[ArcCos[x/a]^(3/2)/(a^2 - x^2)^(3/2),x]
```

output
```
Integrate[ArcCos[x/a]^(3/2)/(a^2 - x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{(a^2-x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5161}$$

$$\frac{3\sqrt{1-\frac{x^2}{a^2}}\int \frac{a^2x\sqrt{\arccos\left(\frac{x}{a}\right)}}{a^2-x^2}dx}{2a^3\sqrt{a^2-x^2}}+\frac{x\arccos\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2-x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\sqrt{1-\frac{x^2}{a^2}}\int \frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{a^2-x^2}dx}{2a\sqrt{a^2-x^2}}+\frac{x\arccos\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2-x^2}}$$

$$\Big\downarrow \text{5235}$$

$$\frac{3\sqrt{1-\frac{x^2}{a^2}}\int \frac{x\sqrt{\arccos\left(\frac{x}{a}\right)}}{a^2-x^2}dx}{2a\sqrt{a^2-x^2}}+\frac{x\arccos\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2-x^2}}$$

input
```
Int[ArcCos[x/a]^(3/2)/(a^2 - x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\left(a^{2}-x^{2}\right)^{\frac{3}{2}}}dx$$

input
```
int(arccos(x/a)^(3/2)/(a^2-x^2)^(3/2),x)
```

output
```
int(arccos(x/a)^(3/2)/(a^2-x^2)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\left(a^{2}-x^{2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(x/a)^(3/2)/(a^2-x^2)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 13.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\left(a^{2}-x^{2}\right)^{3/2}}\,dx = \int \frac{\operatorname{acos}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\left(-\left(-a+x\right)\left(a+x\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(acos(x/a)**(3/2)/(a**2-x**2)**(3/2),x)
```

output
```
Integral(acos(x/a)**(3/2)/(-(-a + x)*(a + x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{(a^2-x^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(x/a)^(3/2)/(a^2-x^2)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{(a^2-x^2)^{3/2}}\,dx = \int \frac{\arccos\left(\frac{x}{a}\right)^{\frac{3}{2}}}{(a^2-x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arccos(x/a)^(3/2)/(a^2-x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(arccos(x/a)^(3/2)/(a^2 - x^2)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx=\int \frac{\mathrm{acos}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

input
```
int(acos(x/a)^(3/2)/(a^2 - x^2)^(3/2),x)
```

output
```
int(acos(x/a)^(3/2)/(a^2 - x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.04

$$\int \frac{\arccos\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx=\frac{2\sqrt{a^2-x^2}\,\sqrt{acos\left(\frac{x}{a}\right)}\,acos\left(\frac{x}{a}\right)x+3\left(\int \frac{\sqrt{acos\left(\frac{x}{a}\right)}\,x}{a^2-x^2}dx\right)a^2-3\left(\int \frac{\sqrt{acos\left(\frac{x}{a}\right)}\,x}{a^2-x^2}dx\right)x^2}{2a^2\left(a^2-x^2\right)}$$

input
```
int(acos(x/a)^(3/2)/(a^2-x^2)^(3/2),x)
```

output
```
(2*sqrt(a**2 - x**2)*sqrt(acos(x/a))*acos(x/a)*x + 3*int((sqrt(acos(x/a))*
x)/(a**2 - x**2),x)*a**2 - 3*int((sqrt(acos(x/a))*x)/(a**2 - x**2),x)*x**2
)/(2*a**2*(a**2 - x**2))
```

3.466 $\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx = \sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(x)}\right)$$

output
```
2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arccos(x)^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx = \frac{i\left(\sqrt{-i\arccos(x)}\,\Gamma\left(\frac{1}{2},-i\arccos(x)\right)-\sqrt{i\arccos(x)}\,\Gamma\left(\frac{1}{2},i\arccos(x)\right)\right)}{2\sqrt{\arccos(x)}}$$

input
```
Integrate[x/(Sqrt[1 - x^2]*Sqrt[ArcCos[x]]),x]
```

output
```
((I/2)*(Sqrt[(-I)*ArcCos[x]]*Gamma[1/2, (-I)*ArcCos[x]] - Sqrt[I*ArcCos[x]
]*Gamma[1/2, I*ArcCos[x]]))/Sqrt[ArcCos[x]]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5225, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}dx$$

$$\Big\downarrow \text{5225}$$

$$-\int \frac{x}{\sqrt{\arccos(x)}}d\arccos(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sin\left(\arccos(x)+\frac{\pi}{2}\right)}{\sqrt{\arccos(x)}}d\arccos(x)$$

$$\Big\downarrow \text{3785}$$

$$-2\int xd\sqrt{\arccos(x)}$$

$$\Big\downarrow \text{3833}$$

$$-\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(x)}\right)$$

input
```
Int[x/(Sqrt[1 - x^2]*Sqrt[ArcCos[x]]),x]
```

output
```
-(Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[x]]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
default	$-\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos\left(x\right)}}{\sqrt{\pi}}\right)$	21

input
```
int(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(x)^(1/2))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx = \int \frac{x}{\sqrt{-(x-1)(x+1)}\sqrt{\operatorname{acos}(x)}}\,dx$$

input
```
integrate(x/(-x**2+1)**(1/2)/acos(x)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x - 1)*(x + 1))*sqrt(acos(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx = \left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(x)}\right) - \left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(x)}\right)$$

input
```
integrate(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x, algorithm="giac")
```

output
```
(1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(x)))
- (1/4*I - 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(x)
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx = \int \frac{x}{\sqrt{\operatorname{acos}(x)}\sqrt{1-x^2}}\,dx$$

input
```
int(x/(acos(x)^(1/2)*(1 - x^2)^(1/2)),x)
```

output
```
int(x/(acos(x)^(1/2)*(1 - x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx = -\left(\int \frac{\sqrt{-x^2+1}\,\sqrt{acos\,(x)}\,x}{acos\,(x)\,x^2-acos\,(x)}dx\right)$$

input
```
int(x/(-x^2+1)^(1/2)/acos(x)^(1/2),x)
```

output
```
- int((sqrt( - x**2 + 1)*sqrt(acos(x))*x)/(acos(x)*x**2 - acos(x)),x)
```

3.467 $\int \frac{(c-a^2cx^2)^{5/2}}{\sqrt{\arccos(ax)}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 244

$$\int \frac{(c-a^2cx^2)^{5/2}}{\sqrt{\arccos(ax)}}\,dx = \frac{5c^2\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}{8a\sqrt{1-a^2x^2}} + \frac{3c^2\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{16a\sqrt{1-a^2x^2}} + \frac{c^2\sqrt{\frac{\pi}{3}}\sqrt{c-a^2cx^2}\,\text{FresnelC}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\arccos(ax)}\right)}{32a\sqrt{1-a^2x^2}} + \frac{15c^2\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{32a\sqrt{1-a^2x^2}}$$

output
```
5/8*c^2*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2)/a/(-a^2*x^2+1)^(1/2)+3/32*c
^2*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arcco
s(a*x)^(1/2))/a/(-a^2*x^2+1)^(1/2)+1/96*c^2*3^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c
)^(1/2)*FresnelC(2*3^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a/(-a^2*x^2+1)^(1/2
)+15/32*c^2*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*FresnelC(2*arccos(a*x)^(1/2)/Pi^
(1/2))/a/(-a^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.14

$$\int \frac{(c-a^2cx^2)^{5/2}}{\sqrt{\arccos(ax)}}\,dx =$$
$$-\frac{c^2\sqrt{c-a^2cx^2}\left(-192\sqrt{\pi}\sqrt{\arccos(ax)^2}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arccos(ax)}\left(240\sqrt{\arccos(ax)^2}-3\sqrt{2}\right.\right.}{}$$

input
```
Integrate[(c - a^2*c*x^2)^(5/2)/Sqrt[ArcCos[a*x]],x]
```

output
```
-1/384*(c^2*Sqrt[c - a^2*c*x^2]*(-192*Sqrt[Pi]*Sqrt[ArcCos[a*x]^2]*Fresnel
C[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] + Sqrt[ArcCos[a*x]]*(240*Sqrt[ArcCos[a*x
]^2] - 3*Sqrt[2]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (-2*I)*ArcCos[a*x]] - 3*Sq
rt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (2*I)*ArcCos[a*x]] - 18*Sqrt[I*Arc
Cos[a*x]]*Gamma[1/2, (-4*I)*ArcCos[a*x]] - 18*Sqrt[(-I)*ArcCos[a*x]]*Gamma
[1/2, (4*I)*ArcCos[a*x]] + Sqrt[6]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (-6*I)*A
rcCos[a*x]] + Sqrt[6]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (6*I)*ArcCos[a*x]]
)))/(a*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.58, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-a^2cx^2)^{5/2}}{\sqrt{\arccos(ax)}}dx$$
$$\Big\downarrow \text{5169}$$

$$-\frac{c^2\sqrt{c-a^2cx^2}\int \frac{(1-a^2x^2)^3}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{c^2\sqrt{c-a^2cx^2}\int \frac{\sin(\arccos(ax))^6}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{c^2\sqrt{c-a^2cx^2}\int \left(-\frac{15\cos(2\arccos(ax))}{32\sqrt{\arccos(ax)}}+\frac{3\cos(4\arccos(ax))}{16\sqrt{\arccos(ax)}}-\frac{\cos(6\arccos(ax))}{32\sqrt{\arccos(ax)}}+\frac{5}{16\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2\sqrt{c-a^2cx^2}\left(\frac{3}{16}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{32}\sqrt{\frac{\pi}{3}}\text{FresnelC}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{15}{32}\sqrt{\pi}\text{FresnelC}\right.}{a\sqrt{1-a^2x^2}}$$

input
```
Int[(c - a^2*c*x^2)^(5/2)/Sqrt[ArcCos[a*x]],x]
```

output
```
-((c^2*Sqrt[c - a^2*c*x^2]*((5*Sqrt[ArcCos[a*x]])/8 + (3*Sqrt[Pi/2]*Fresne
lC[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/16 - (Sqrt[Pi/3]*FresnelC[2*Sqrt[3/Pi]
*Sqrt[ArcCos[a*x]]])/32 - (15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt
[Pi]])/32))/(a*Sqrt[1 - a^2*x^2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [F]

$$\int \frac{(-a^2 c\,x^2 + c)^{\frac{5}{2}}}{\sqrt{\arccos\left(ax\right)}}dx$$

input
```
int((-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(1/2),x)
```

output
```
int((-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c - a^2cx^2)^{5/2}}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c-a^2cx^2)^{5/2}}{\sqrt{\arccos(ax)}}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(5/2)/acos(a*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c-a^2cx^2)^{5/2}}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.65

$$\int \frac{(c-a^2cx^2)^{5/2}}{\sqrt{\arccos(ax)}}\,dx = -\frac{(i+1)\ \sqrt{3}\sqrt{\pi}c^{\frac{5}{2}}\operatorname{erf}\left((i-1)\ \sqrt{3}\sqrt{\arccos(ax)}\right)}{384\,a} + \frac{(i-1)\ \sqrt{3}\sqrt{\pi}c^{\frac{5}{2}}\operatorname{erf}\left(-(i+1)\ \sqrt{3}\sqrt{\arccos(ax)}\right)}{384\,a} + \frac{(3i+3)\ \sqrt{2}\sqrt{\pi}c^{\frac{5}{2}}\operatorname{erf}\left((i-1)\ \sqrt{2}\sqrt{\arccos(ax)}\right)}{128\,a} - \frac{(3i-3)\ \sqrt{2}\sqrt{\pi}c^{\frac{5}{2}}\operatorname{erf}\left(-(i+1)\ \sqrt{2}\sqrt{\arccos(ax)}\right)}{128\,a} - \frac{(15i+15)\ \sqrt{\pi}c^{\frac{5}{2}}\operatorname{erf}\left((i-1)\ \sqrt{\arccos(ax)}\right)}{128\,a} + \frac{(15i-15)\ \sqrt{\pi}c^{\frac{5}{2}}\operatorname{erf}\left(-(i+1)\ \sqrt{\arccos(ax)}\right)}{128\,a} - \frac{5\,c^{\frac{5}{2}}\sqrt{\arccos(ax)}}{8\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
-(1/384*I + 1/384)*sqrt(3)*sqrt(pi)*c^(5/2)*erf((I - 1)*sqrt(3)*sqrt(arcco
s(a*x)))/a + (1/384*I - 1/384)*sqrt(3)*sqrt(pi)*c^(5/2)*erf(-(I + 1)*sqrt(
3)*sqrt(arccos(a*x)))/a + (3/128*I + 3/128)*sqrt(2)*sqrt(pi)*c^(5/2)*erf((
I - 1)*sqrt(2)*sqrt(arccos(a*x)))/a - (3/128*I - 3/128)*sqrt(2)*sqrt(pi)*c
^(5/2)*erf(-(I + 1)*sqrt(2)*sqrt(arccos(a*x)))/a - (15/128*I + 15/128)*sqr
t(pi)*c^(5/2)*erf((I - 1)*sqrt(arccos(a*x)))/a + (15/128*I - 15/128)*sqrt(
pi)*c^(5/2)*erf(-(I + 1)*sqrt(arccos(a*x)))/a - 5/8*c^(5/2)*sqrt(arccos(a*
x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-a^2cx^2)^{5/2}}{\sqrt{\arccos(ax)}}\,dx = \int \frac{{\left(c-a^2\,c\,x^2\right)}^{5/2}}{\sqrt{\mathrm{acos}\left(a\,x\right)}}\,dx$$

input
```
int((c - a^2*c*x^2)^(5/2)/acos(a*x)^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(5/2)/acos(a*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c-a^2cx^2)^{5/2}}{\sqrt{\arccos(ax)}}\,dx = \sqrt{c}\,c^2\left(\left(\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}\,x^4}{acos\,(ax)}dx\right)a^4 - 2\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}\,x^2}{acos\,(ax)}dx\right)a^2 + \int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}}{acos\,(ax)}dx\right)\right.$$

input
```
int((-a^2*c*x^2+c)^(5/2)/acos(a*x)^(1/2),x)
```

output
```
sqrt(c)*c**2*(int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**4)/acos(a*x),
x)*a**4 - 2*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**2)/acos(a*x),x)
*a**2 + int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/acos(a*x),x))
```

3.468 $\int \frac{(c-a^2cx^2)^{3/2}}{\sqrt{\arccos(ax)}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{(c-a^2cx^2)^{3/2}}{\sqrt{\arccos(ax)}}\,dx = \frac{3c\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}{4a\sqrt{1-a^2x^2}} + \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a\sqrt{1-a^2x^2}} + \frac{c\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{2a\sqrt{1-a^2x^2}}$$

output
```
3/4*c*(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2)/a/(-a^2*x^2+1)^(1/2)+1/16*c*2
^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*
x)^(1/2))/a/(-a^2*x^2+1)^(1/2)+1/2*c*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*Fresnel
C(2*arccos(a*x)^(1/2)/Pi^(1/2))/a/(-a^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91

$$\int \frac{(c-a^2cx^2)^{3/2}}{\sqrt{\arccos(ax)}}\,dx = -\frac{c\sqrt{c-a^2cx^2}\left(-16\sqrt{\pi}\sqrt{\arccos(ax)^2}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arccos(ax)}\left(24\sqrt{\arccos(ax)^2}-\sqrt{i\arccos(ax)}\right.\right.}{32a\sqrt{1-a^2x^2}\sqrt{\arccos(ax)^2}}$$

input
```
Integrate[(c - a^2*c*x^2)^(3/2)/Sqrt[ArcCos[a*x]],x]
```

output
```
-1/32*(c*Sqrt[c - a^2*c*x^2]*(-16*Sqrt[Pi]*Sqrt[ArcCos[a*x]^2]*FresnelC[(2
*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] + Sqrt[ArcCos[a*x]]*(24*Sqrt[ArcCos[a*x]^2]
- Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (-4*I)*ArcCos[a*x]] - Sqrt[(-I)*ArcCos[a*
x]]*Gamma[1/2, (4*I)*ArcCos[a*x]])))/(a*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]
^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.62, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-a^2cx^2)^{3/2}}{\sqrt{\arccos(ax)}}\,dx$$

$$\Big\downarrow \text{5169}$$

$$-\frac{c\sqrt{c-a^2cx^2}\int \frac{(1-a^2x^2)^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{c\sqrt{c-a^2cx^2}\int \frac{\sin(\arccos(ax))^4}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{c\sqrt{c-a^2cx^2}\int \left(-\frac{\cos(2\arccos(ax))}{2\sqrt{\arccos(ax)}}+\frac{\cos(4\arccos(ax))}{8\sqrt{\arccos(ax)}}+\frac{3}{8\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{c\sqrt{c-a^2cx^2}\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)-\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arccos(ax)}\right)}{a\sqrt{1-a^2x^2}}$$

input
```
Int[(c - a^2*c*x^2)^(3/2)/Sqrt[ArcCos[a*x]],x]
```

output
```
-((c*Sqrt[c - a^2*c*x^2]*((3*Sqrt[ArcCos[a*x]])/4 + (Sqrt[Pi/2]*FresnelC[2
*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/8 - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]
])/Sqrt[Pi]])/2))/(a*Sqrt[1 - a^2*x^2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [F]

$$\int \frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{\sqrt{\arccos\left(ax\right)}}dx$$

input
```
int((-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(1/2),x)
```

output
```
int((-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c-a^2cx^2)^{3/2}}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(c-a^2cx^2)^{3/2}}{\sqrt{\arccos(ax)}}\,dx = \int \frac{(-c(ax-1)\,(ax+1))^{\frac{3}{2}}}{\sqrt{\operatorname{acos}\left(ax\right)}}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(3/2)/acos(a*x)**(1/2),x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c-a^2cx^2)^{3/2}}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.63

$$\int \frac{(c-a^2cx^2)^{3/2}}{\sqrt{\arccos(ax)}}\,dx = \frac{(i+1)\sqrt{2}\sqrt{\pi}c^{\frac{3}{2}}\operatorname{erf}\left((i-1)\sqrt{2}\sqrt{\arccos(ax)}\right)}{64\,a} - \frac{(i-1)\sqrt{2}\sqrt{\pi}c^{\frac{3}{2}}\operatorname{erf}\left(-(i+1)\sqrt{2}\sqrt{\arccos(ax)}\right)}{64\,a} - \frac{(i+1)\sqrt{\pi}c^{\frac{3}{2}}\operatorname{erf}\left((i-1)\sqrt{\arccos(ax)}\right)}{8\,a} + \frac{(i-1)\sqrt{\pi}c^{\frac{3}{2}}\operatorname{erf}\left(-(i+1)\sqrt{\arccos(ax)}\right)}{8\,a} - \frac{3\,c^{\frac{3}{2}}\sqrt{\arccos(ax)}}{4\,a}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
(1/64*I + 1/64)*sqrt(2)*sqrt(pi)*c^(3/2)*erf((I - 1)*sqrt(2)*sqrt(arccos(a
*x)))/a - (1/64*I - 1/64)*sqrt(2)*sqrt(pi)*c^(3/2)*erf(-(I + 1)*sqrt(2)*sq
rt(arccos(a*x)))/a - (1/8*I + 1/8)*sqrt(pi)*c^(3/2)*erf((I - 1)*sqrt(arcco
s(a*x)))/a + (1/8*I - 1/8)*sqrt(pi)*c^(3/2)*erf(-(I + 1)*sqrt(arccos(a*x))
)/a - 3/4*c^(3/2)*sqrt(arccos(a*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-a^2cx^2)^{3/2}}{\sqrt{\arccos(ax)}}\,dx=\int \frac{{\left(c-a^2\,c\,x^2\right)}^{3/2}}{\sqrt{\mathrm{acos}\left(a\,x\right)}}\,dx$$

input
```
int((c - a^2*c*x^2)^(3/2)/acos(a*x)^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(3/2)/acos(a*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c-a^2cx^2)^{3/2}}{\sqrt{\arccos(ax)}}\,dx=\sqrt{c}\,c\left(-\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x^2}{acos(ax)}dx\right)a^2+\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}}{acos(ax)}dx\right)$$

input
```
int((-a^2*c*x^2+c)^(3/2)/acos(a*x)^(1/2),x)
```

output
```
sqrt(c)*c*( - int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**2)/acos(a*x),
x)*a**2 + int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/acos(a*x),x))
```

3.469 $\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\arccos(ax)}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\arccos(ax)}}\,dx = \frac{\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}{a\sqrt{1-a^2x^2}} + \frac{\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{2a\sqrt{1-a^2x^2}}$$

output
```
(-a^2*c*x^2+c)^(1/2)*arccos(a*x)^(1/2)/a/(-a^2*x^2+1)^(1/2)+1/2*Pi^(1/2)*(
-a^2*c*x^2+c)^(1/2)*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))/a/(-a^2*x^2+1)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\arccos(ax)}}\,dx = -\frac{\sqrt{c(1-a^2x^2)}\left(2\sqrt{\arccos(ax)} - \sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/Sqrt[ArcCos[a*x]],x]
```

output
```
-1/2*(Sqrt[c*(1 - a^2*x^2)]*(2*Sqrt[ArcCos[a*x]] - Sqrt[Pi]*FresnelC[(2*Sq
rt[ArcCos[a*x]])/Sqrt[Pi]]))/(a*Sqrt[1 - a^2*x^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\arccos(ax)}}dx$$

$$\Big\downarrow \text{5169}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{1-a^2x^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \frac{\sin(\arccos(ax))^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{2\sqrt{\arccos(ax)}}-\frac{\cos(2\arccos(ax))}{2\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c-a^2cx^2}\left(\sqrt{\arccos(ax)}-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{a\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/Sqrt[ArcCos[a*x]],x]
```

output
```
-((Sqrt[c - a^2*c*x^2]*(Sqrt[ArcCos[a*x]] - (Sqrt[Pi]*FresnelC[(2*Sqrt[Arc
Cos[a*x]])/Sqrt[Pi]])/2))/(a*Sqrt[1 - a^2*x^2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [F]

$$\int \frac{\sqrt{-a^2c\,x^2+c}}{\sqrt{\arccos\left(ax\right)}}dx$$

input
```
int((-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(1/2),x)
```

output
```
int((-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\arccos(ax)}}\,dx=\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\operatorname{acos}{(ax)}}}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)/acos(a*x)**(1/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\arccos(ax)}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\arccos(ax)}}\,dx=$$

$$-\frac{1}{8}\sqrt{c}\left(\frac{(i+1)\,\sqrt{\pi}\operatorname{erf}\left((i-1)\,\sqrt{\arccos{(ax)}}\right)}{a}-\frac{(i-1)\,\sqrt{\pi}\operatorname{erf}\left(-(i+1)\,\sqrt{\arccos{(ax)}}\right)}{a}+\frac{8\,\sqrt{\operatorname{arcc}}}{a}\right.$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
-1/8*sqrt(c)*((I + 1)*sqrt(pi)*erf((I - 1)*sqrt(arccos(a*x)))/a - (I - 1)*
sqrt(pi)*erf(-(I + 1)*sqrt(arccos(a*x)))/a + 8*sqrt(arccos(a*x))/a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\arccos(ax)}}\,dx=\int \frac{\sqrt{c-a^2\,c\,x^2}}{\sqrt{\mathrm{acos}\left(a\,x\right)}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/acos(a*x)^(1/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/acos(a*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\arccos(ax)}}\,dx=\sqrt{c}\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos\left(ax\right)}}{acos\left(ax\right)}dx\right)$$

input
```
int((-a^2*c*x^2+c)^(1/2)/acos(a*x)^(1/2),x)
```

output
```
sqrt(c)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/acos(a*x),x)
```

3.470 $\int \frac{1}{\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 42

$$\int \frac{1}{\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}\,dx = \frac{2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a\sqrt{c-a^2cx^2}}$$

output
```
2*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}\,dx = -\frac{2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a\sqrt{c-a^2cx^2}}$$

input
```
Integrate[1/(Sqrt[c - a^2*c*x^2]*Sqrt[ArcCos[a*x]]),x]
```

output
```
(-2*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(a*Sqrt[c - a^2*c*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}}\,dx$$

$$\Big\downarrow \text{5153}$$

$$-\frac{2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{a\sqrt{c-a^2cx^2}}$$

input
```
Int[1/(Sqrt[c - a^2*c*x^2]*Sqrt[ArcCos[a*x]]),x]
```

output
```
(-2*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(a*Sqrt[c - a^2*c*x^2])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{2\sqrt{\arccos(ax)}\,\sqrt{-a^2x^2+1}}{\sqrt{-c(a^2x^2-1)}\,a}$	38

input
```
int(1/(-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arccos(a*x)^(1/2)/(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{\sqrt{-c(ax-1)(ax+1)}\sqrt{\operatorname{acos}(ax)}}\,dx$$

input
```
integrate(1/(-a**2*c*x**2+c)**(1/2)/acos(a*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(-c*(a*x - 1)*(a*x + 1))*sqrt(acos(a*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}\,dx = -\frac{2\sqrt{\arccos(ax)}}{a\sqrt{c}}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(arccos(a*x))/(a*sqrt(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{acos}(a\,x)}\,\sqrt{c-a^2\,c\,x^2}}\,dx$$

input
```
int(1/(acos(a*x)^(1/2)*(c - a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(acos(a*x)^(1/2)*(c - a^2*c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.36

$$\int \frac{1}{\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}\,dx = -\frac{2\sqrt{c}\,\sqrt{acos\,(ax)}}{ac}$$

input
```
int(1/(-a^2*c*x^2+c)^(1/2)/acos(a*x)^(1/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(acos(a*x)))/(a*c)
```

3.471 $\int \frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\arccos(ax)}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\arccos(ax)}}\,dx=\text{Int}\left(\frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\arccos(ax)}},x\right)$$

output
```
Defer(Int)(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 3.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\arccos(ax)}}\,dx=\int \frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\arccos(ax)}}\,dx$$

input
```
Integrate[1/((c - a^2*c*x^2)^(3/2)*Sqrt[ArcCos[a*x]]),x]
```

output
```
Integrate[1/((c - a^2*c*x^2)^(3/2)*Sqrt[ArcCos[a*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arccos(ax)}\,(c-a^2cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5175}$$

$$\int \frac{1}{\sqrt{\arccos(ax)}\,(c-a^2cx^2)^{3/2}}dx$$

input
```
Int[1/((c - a^2*c*x^2)^(3/2)*Sqrt[ArcCos[a*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}\sqrt{\arccos\left(ax\right)}}dx$$

input
```
int(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(1/2),x)
```

output
```
int(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 5.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}\sqrt{\operatorname{acos}(ax)}}\,dx$$

input
```
integrate(1/(-a**2*c*x**2+c)**(3/2)/acos(a*x)**(1/2),x)
```

output
```
Integral(1/((-c*(a*x - 1)*(a*x + 1))**(3/2)*sqrt(acos(a*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}\sqrt{\arccos(ax)}}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt(arccos(a*x))), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{acos}\left(a\,x\right)}\,{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

input
```
int(1/(acos(a*x)^(1/2)*(c - a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(acos(a*x)^(1/2)*(c - a^2*c*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.29

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\sqrt{\arccos(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}}{acos(ax)a^4x^4-2acos(ax)a^2x^2+acos(ax)}dx\right)}{c^2}$$

input
```
int(1/(-a^2*c*x^2+c)^(3/2)/acos(a*x)^(1/2),x)
```

output

```
(sqrt(c)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/(acos(a*x)*a**4*x**4
 - 2*acos(a*x)*a**2*x**2 + acos(a*x)),x))/c**2
```

3.472 $\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\arccos(ax)}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\arccos(ax)}}\,dx = \operatorname{Int}\left(\frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\arccos(ax)}},x\right)$$

output
```
Defer(Int)(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 2.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\arccos(ax)}}\,dx$$

input
```
Integrate[1/((c - a^2*c*x^2)^(5/2)*Sqrt[ArcCos[a*x]]),x]
```

output
```
Integrate[1/((c - a^2*c*x^2)^(5/2)*Sqrt[ArcCos[a*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arccos(ax)}\,(c-a^2cx^2)^{5/2}}\,dx$$
$$\Big\downarrow \; 5175$$
$$\int \frac{1}{\sqrt{\arccos(ax)}\,(c-a^2cx^2)^{5/2}}dx$$

input
```
Int[1/((c - a^2*c*x^2)^(5/2)*Sqrt[ArcCos[a*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(-a^{2} c\,x^{2}+c \right)^{\frac{5}{2}} \sqrt{\arccos \left(a x \right)}}d x$$

input
```
int(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(1/2),x)
```

output
```
int(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 53.91 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}\sqrt{\operatorname{acos}\,(ax)}}\,dx$$

input
```
integrate(1/(-a**2*c*x**2+c)**(5/2)/acos(a*x)**(1/2),x)
```

output
```
Integral(1/((-c*(a*x - 1)*(a*x + 1))**(5/2)*sqrt(acos(a*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\arccos(ax)}}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt(arccos(a*x))), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{acos}(a\,x)}\,(c-a^2\,c\,x^2)^{5/2}}\,dx$$

input
```
int(1/(acos(a*x)^(1/2)*(c - a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(acos(a*x)^(1/2)*(c - a^2*c*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.92

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\arccos(ax)}}\,dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}}{acos(ax)a^6x^6-3acos(ax)a^4x^4+3acos(ax)a^2x^2-acos(ax)}dx\right)}{c^3}$$

input
```
int(1/(-a^2*c*x^2+c)^(5/2)/acos(a*x)^(1/2),x)
```

output

```
( - sqrt(c)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/(acos(a*x)*a**6*x
**6 - 3*acos(a*x)*a**4*x**4 + 3*acos(a*x)*a**2*x**2 - acos(a*x)),x))/c**3
```

3.473 $\int \frac{(c-a^2cx^2)^{5/2}}{\arccos(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 237

$$\int \frac{(c-a^2cx^2)^{5/2}}{\arccos(ax)^{3/2}}\,dx = -\frac{2\sqrt{1-a^2x^2}(c-a^2cx^2)^{5/2}}{a\sqrt{\arccos(ax)}} - \frac{3c^2\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{2a\sqrt{1-a^2x^2}} - \frac{c^2\sqrt{3\pi}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\arccos(ax)}\right)}{8a\sqrt{1-a^2x^2}} - \frac{15c^2\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a\sqrt{1-a^2x^2}}$$

output
```
-2*(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(5/2)/a/arccos(a*x)^(1/2)-3/4*c^2*2^(
1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arccos(a*x)
^(1/2))/a/(-a^2*x^2+1)^(1/2)-1/8*c^2*3^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)
*FresnelS(2*3^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a/(-a^2*x^2+1)^(1/2)-15/8*
c^2*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))/a
/(-a^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.70

$$\int \frac{(c-a^2cx^2)^{5/2}}{\arccos(ax)^{3/2}}\,dx =$$

$$-\frac{c^2e^{-6i\arccos(ax)}\sqrt{c-a^2cx^2}\left(1-6e^{2i\arccos(ax)}+15e^{4i\arccos(ax)}-20e^{6i\arccos(ax)}+15e^{8i\arccos(ax)}-6e^{10i\arccos(}\right.}{}$$

input
```
Integrate[(c - a^2*c*x^2)^(5/2)/ArcCos[a*x]^(3/2),x]
```

output
```
-1/32*(c^2*Sqrt[c - a^2*c*x^2]*(1 - 6*E^((2*I)*ArcCos[a*x]) + 15*E^((4*I)*
ArcCos[a*x]) - 20*E^((6*I)*ArcCos[a*x]) + 15*E^((8*I)*ArcCos[a*x]) - 6*E^(
(10*I)*ArcCos[a*x]) + E^((12*I)*ArcCos[a*x]) + 64*E^((6*I)*ArcCos[a*x])*Sq
rt[Pi]*Sqrt[ArcCos[a*x]]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] + Sqrt[2
]*E^((6*I)*ArcCos[a*x])*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-2*I)*ArcCos[a*
x]] + Sqrt[2]*E^((6*I)*ArcCos[a*x])*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (2*I)*A
rcCos[a*x]] + 12*E^((6*I)*ArcCos[a*x])*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (
-4*I)*ArcCos[a*x]] + 12*E^((6*I)*ArcCos[a*x])*Sqrt[I*ArcCos[a*x]]*Gamma[1/
2, (4*I)*ArcCos[a*x]] - Sqrt[6]*E^((6*I)*ArcCos[a*x])*Sqrt[(-I)*ArcCos[a*x
]]*Gamma[1/2, (-6*I)*ArcCos[a*x]] - Sqrt[6]*E^((6*I)*ArcCos[a*x])*Sqrt[I*A
rcCos[a*x]]*Gamma[1/2, (6*I)*ArcCos[a*x]]))/(a*E^((6*I)*ArcCos[a*x])*Sqrt[
1 - a^2*x^2]*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5167, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-a^2cx^2)^{5/2}}{\arccos(ax)^{3/2}}\,dx$$

$$\Big\downarrow \text{5167}$$

$$\frac{12ac^2\sqrt{c-a^2cx^2}\int \frac{x(1-a^2x^2)^2}{\sqrt{\arccos(ax)}}dx}{\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^{5/2}}{a\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \text{5225}$$

$$\frac{2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^{5/2}}{a\sqrt{\arccos(ax)}}-\frac{12c^2\sqrt{c-a^2cx^2}\int \frac{ax(1-a^2x^2)^{5/2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^{5/2}}{a\sqrt{\arccos(ax)}}-\frac{12c^2\sqrt{c-a^2cx^2}\int \left(\frac{5\sin(2\arccos(ax))}{32\sqrt{\arccos(ax)}}-\frac{\sin(4\arccos(ax))}{8\sqrt{\arccos(ax)}}+\frac{\sin(6\arccos(ax))}{32\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^{5/2}}{a\sqrt{\arccos(ax)}}-\frac{12c^2\sqrt{c-a^2cx^2}\left(-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{1}{32}\sqrt{\frac{\pi}{3}}\,\text{FresnelS}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\arccos(ax)}\right)+\frac{5}{32}\sqrt{\pi}\,\text{FresnelS}\right.}{a\sqrt{1-a^2x^2}}$$

input
```
Int[(c - a^2*c*x^2)^(5/2)/ArcCos[a*x]^(3/2),x]
```

output
```
(2*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^(5/2))/(a*Sqrt[ArcCos[a*x]]) - (12*c^
2*Sqrt[c - a^2*c*x^2]*(-1/8*(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcCos[
a*x]]]) + (Sqrt[Pi/3]*FresnelS[2*Sqrt[3/Pi]*Sqrt[ArcCos[a*x]]])/32 + (5*Sq
rt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/32))/(a*Sqrt[1 - a^2*x^2]
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{(-a^2c\,x^2+c)^{\frac{5}{2}}}{\arccos{(ax)}^{\frac{3}{2}}}dx$$

input
```
int((-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(3/2),x)
```

output
```
int((-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c-a^2cx^2)^{5/2}}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c-a^2cx^2)^{5/2}}{\arccos(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((-a**2*c*x**2+c)**(5/2)/acos(a*x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c-a^2cx^2)^{5/2}}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{(c-a^2cx^2)^{5/2}}{\arccos(ax)^{3/2}}\,dx=\int \frac{(-a^2cx^2+c)^{\frac{5}{2}}}{\arccos(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^(5/2)/arccos(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-a^2cx^2)^{5/2}}{\arccos(ax)^{3/2}}\,dx=\int \frac{(c-a^2\,c\,x^2)^{5/2}}{\mathrm{acos}\,(a\,x)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(5/2)/acos(a*x)^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(5/2)/acos(a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c-a^2cx^2)^{5/2}}{\arccos(ax)^{3/2}}\,dx=\sqrt{c}\,c^2\left(\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}\,x^4}{acos\,(ax)^2}dx\right)a^4-2\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}\,x^2}{acos\,(ax)^2}\right.\right.$$

input
```
int((-a^2*c*x^2+c)^(5/2)/acos(a*x)^(3/2),x)
```

output
```
sqrt(c)*c**2*(int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**4)/acos(a*x)*
*2,x)*a**4 - 2*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**2)/acos(a*x)
**2,x)*a**2 + int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/acos(a*x)**2,x)
)
```

3.474 $\int \frac{(c-a^2cx^2)^{3/2}}{\arccos(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{(c-a^2cx^2)^{3/2}}{\arccos(ax)^{3/2}}\,dx = -\frac{2\sqrt{1-a^2x^2}(c-a^2cx^2)^{3/2}}{a\sqrt{\arccos(ax)}} - \frac{c\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a\sqrt{1-a^2x^2}} - \frac{2c\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a\sqrt{1-a^2x^2}}$$

output

```
-2*(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(3/2)/a/arccos(a*x)^(1/2)-1/2*c*2^(1/
2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(
1/2))/a/(-a^2*x^2+1)^(1/2)-2*c*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*FresnelS(2*ar
ccos(a*x)^(1/2)/Pi^(1/2))/a/(-a^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.29

$$\int \frac{(c-a^2cx^2)^{3/2}}{\arccos(ax)^{3/2}}\,dx = \frac{ce^{-4i\arccos(ax)}\sqrt{c-a^2cx^2}\left(1+6e^{4i\arccos(ax)}+e^{8i\arccos(ax)}-8e^{4i\arccos(ax)}\cos(2\arccos\right.}{}$$

input
```
Integrate[(c - a^2*c*x^2)^(3/2)/ArcCos[a*x]^(3/2),x]
```

output
```
(c*Sqrt[c - a^2*c*x^2]*(1 + 6*E^((4*I)*ArcCos[a*x]) + E^((8*I)*ArcCos[a*x]
) - 8*E^((4*I)*ArcCos[a*x])*Cos[2*ArcCos[a*x]] - 16*E^((4*I)*ArcCos[a*x])*
Sqrt[Pi]*Sqrt[ArcCos[a*x]]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] - 2*E^
((4*I)*ArcCos[a*x])*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-4*I)*ArcCos[a*x]]
- 2*E^((4*I)*ArcCos[a*x])*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (4*I)*ArcCos[a*x]
]))/(8*a*E^((4*I)*ArcCos[a*x])*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5167, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-a^2cx^2)^{3/2}}{\arccos(ax)^{3/2}}\,dx$$

$$\Big\downarrow \text{5167}$$

$$\frac{8ac\sqrt{c-a^2cx^2}\int \frac{x(1-a^2x^2)}{\sqrt{\arccos(ax)}}dx}{\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}(c-a^2cx^2)^{3/2}}{a\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \text{5225}$$

$$\frac{2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^{3/2}}{a\sqrt{\arccos(ax)}}-\frac{8c\sqrt{c-a^2cx^2}\int\frac{ax\left(1-a^2x^2\right)^{3/2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^{3/2}}{a\sqrt{\arccos(ax)}}-\frac{8c\sqrt{c-a^2cx^2}\int\left(\frac{\sin(2\arccos(ax))}{4\sqrt{\arccos(ax)}}-\frac{\sin(4\arccos(ax))}{8\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^{3/2}}{a\sqrt{\arccos(ax)}}-\frac{8c\sqrt{c-a^2cx^2}\left(\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)\right)}{a\sqrt{1-a^2x^2}}$$

input

```
Int[(c - a^2*c*x^2)^(3/2)/ArcCos[a*x]^(3/2),x]
```

output

```
(2*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^(3/2))/(a*Sqrt[ArcCos[a*x]]) - (8*c*S
qrt[c - a^2*c*x^2]*(-1/8*(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x
]]]) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/4))/(a*Sqrt[1 -
 a^2*x^2])
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{\arccos \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int((-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(3/2),x)
```

output
```
int((-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c-a^2cx^2)^{3/2}}{\arccos(ax)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(c - a^2cx^2)^{3/2}}{\arccos(ax)^{3/2}}\,dx = \int \frac{(-c(ax-1)\,(ax+1))^{\frac{3}{2}}}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(3/2)/acos(a*x)**(3/2),x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c - a^2cx^2)^{3/2}}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{(c - a^2cx^2)^{3/2}}{\arccos(ax)^{3/2}}\,dx = \int \frac{(-a^2cx^2 + c)^{\frac{3}{2}}}{\arccos(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^(3/2)/arccos(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-a^2cx^2)^{3/2}}{\arccos(ax)^{3/2}}\,dx = \int \frac{{\left(c-a^2\,c\,x^2\right)}^{3/2}}{{\mathrm{acos}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(3/2)/acos(a*x)^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(3/2)/acos(a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c-a^2cx^2)^{3/2}}{\arccos(ax)^{3/2}}\,dx = \sqrt{c}\,c\left(-\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}\,x^2}{acos(ax)^2}dx\right)a^2+\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}}{acos(ax)^2}dx\right)$$

input
```
int((-a^2*c*x^2+c)^(3/2)/acos(a*x)^(3/2),x)
```

output
```
sqrt(c)*c*( - int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**2)/acos(a*x)*
*2,x)*a**2 + int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/acos(a*x)**2,x))
```

3.475 $\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{3/2}}\,dx = -\frac{2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{a\sqrt{\arccos(ax)}}-\frac{2\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a\sqrt{1-a^2x^2}}$$

output
```
-2*(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/arccos(a*x)^(1/2)-2*Pi^(1/2)*
(-a^2*c*x^2+c)^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))/a/(-a^2*x^2+1)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{3/2}}\,dx = -\frac{\sqrt{c(1-a^2x^2)}\left(-1+\cos(2\arccos(ax))+2\sqrt{\pi}\sqrt{\arccos(ax)}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{a\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/ArcCos[a*x]^(3/2),x]
```

output
```
-((Sqrt[c*(1 - a^2*x^2)]*(-1 + Cos[2*ArcCos[a*x]] + 2*Sqrt[Pi]*Sqrt[ArcCos
[a*x]]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]]))/(a*Sqrt[1 - a^2*x^2]*Sqr
t[ArcCos[a*x]]))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5167, 5147, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{3/2}}\,dx$$

$$\Big\downarrow \text{5167}$$

$$\frac{4a\sqrt{c-a^2cx^2}\int \frac{x}{\sqrt{\arccos(ax)}}dx}{\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{a\sqrt{\arccos(ax)}}$$

$$\Big\downarrow \text{5147}$$

$$\frac{2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{a\sqrt{\arccos(ax)}}-\frac{4\sqrt{c-a^2cx^2}\int \frac{ax\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{a\sqrt{\arccos(ax)}}-\frac{4\sqrt{c-a^2cx^2}\int \frac{\sin(2\arccos(ax))}{2\sqrt{\arccos(ax)}}d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{a\sqrt{\arccos(ax)}}-\frac{2\sqrt{c-a^2cx^2}\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{a\sqrt{\arccos(ax)}}-\frac{2\sqrt{c-a^2cx^2}\int \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{3786}$$

$$\frac{2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{a\sqrt{\arccos(ax)}}-\frac{4\sqrt{c-a^2cx^2}\int\sin(2\arccos(ax))d\sqrt{\arccos(ax)}}{a\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{3832}$$

$$\frac{2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{a\sqrt{\arccos(ax)}}-\frac{2\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a\sqrt{1-a^2x^2}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/ArcCos[a*x]^(3/2),x]
```

output
```
(2*Sqrt[1 - a^2*x^2]*Sqrt[c - a^2*c*x^2])/(a*Sqrt[ArcCos[a*x]]) - (2*Sqrt[
Pi]*Sqrt[c - a^2*c*x^2]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(a*Sqrt[
1 - a^2*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5147
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(b*c^(m + 1))^(-1)   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x
, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

Maple [F]

$$\int \frac{\sqrt{-a^{2} c \,x^{2}+c}}{\arccos \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int((-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(3/2),x)
```

output
```
int((-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{3/2}}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)/acos(a*x)**(3/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{3/2}}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{\arccos(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/arccos(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{3/2}}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{\mathrm{acos}\,(a\,x)^{3/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/acos(a*x)^(3/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/acos(a*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{3/2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}}{acos\,(ax)^2}dx\right)$$

input
```
int((-a^2*c*x^2+c)^(1/2)/acos(a*x)^(3/2),x)
```

output
```
sqrt(c)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/acos(a*x)**2,x)
```

3.476 $\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 42

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}\,dx = -\frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}$$

output
```
-2*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}\,dx = \frac{2\sqrt{1-a^2x^2}}{a\sqrt{c-a^2cx^2}\sqrt{\arccos(ax)}}$$

input
```
Integrate[1/(Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^(3/2)),x]
```

output
```
(2*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCos[a*x]])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^{3/2}\sqrt{c-a^2cx^2}}\,dx$$
$$\Big\downarrow \text{5153}$$
$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}\sqrt{c-a^2cx^2}}$$

input
```
Int[1/(Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^(3/2)),x]
```

output
```
(2*Sqrt[1 - a^2*x^2])/(a*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCos[a*x]])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{2\sqrt{-a^2x^2+1}}{\sqrt{\arccos(ax)}\,\sqrt{-c(a^2x^2-1)}\,a}$	38

input
```
int(1/(-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/arccos(a*x)^(1/2)/(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}\,dx = -\frac{2\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{(a^3cx^2-ac)\sqrt{\arccos(ax)}}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/((a^3*c*x^2 - a*c)*sqrt(arccos(
a*x)))
```

Sympy [F]

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{-c(ax-1)(ax+1)}\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/(-a**2*c*x**2+c)**(1/2)/acos(a*x)**(3/2),x)
```

output
```
Integral(1/(sqrt(-c*(a*x - 1)*(a*x + 1))*acos(a*x)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}\,dx = \frac{2}{a\sqrt{c}\sqrt{\arccos{(ax)}}}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
2/(a*sqrt(c)*sqrt(arccos(a*x)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}\,dx = \int \frac{1}{\mathrm{acos}\left(a\,x\right)^{3/2}\,\sqrt{c-a^2\,c\,x^2}}\,dx$$

input
```
int(1/(acos(a*x)^(3/2)*(c - a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(acos(a*x)^(3/2)*(c - a^2*c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{acos\,(ax)}}{acos\,(ax)\,ac}$$

input
```
int(1/(-a^2*c*x^2+c)^(1/2)/acos(a*x)^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt(acos(a*x)))/(acos(a*x)*a*c)
```

3.477 $\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{3/2}}\,dx = -\frac{2\sqrt{1-a^2x^2}}{a\,(c-a^2cx^2)^{3/2}\sqrt{\arccos(ax)}} + \frac{4a\sqrt{1-a^2x^2}\text{Int}\left(\frac{x}{(1-a^2x^2)^2\sqrt{\arccos(ax)}},x\right)}{c\sqrt{c-a^2cx^2}}$$

output
```
-2*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(1/2)+4*a*(-a^2*x
^2+1)^(1/2)*Defer(Int)(x/(-a^2*x^2+1)^2/arccos(a*x)^(1/2),x)/c/(-a^2*c*x^2
+c)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{3/2}}\,dx = \int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{3/2}}\,dx$$

input
```
Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^(3/2)),x]
```

output
```
Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^{3/2} (c-a^2cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 5167}$$

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}(c-a^2cx^2)^{3/2}}-\frac{4a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)^2\sqrt{\arccos(ax)}}dx}{c\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{ 5235}$$

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}(c-a^2cx^2)^{3/2}}-\frac{4a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)^2\sqrt{\arccos(ax)}}dx}{c\sqrt{c-a^2cx^2}}$$

input
```
Int[1/((c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arccos \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(3/2),x)
```

output
```
int(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c-a^2cx^2)^{3/2} \arccos(ax)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 36.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{1}{(c-a^2cx^2)^{3/2} \arccos(ax)^{3/2}}\, dx = \int \frac{1}{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}} \operatorname{acos}^{\frac{3}{2}}{\left(a x \right)}}\, dx$$

input
```
integrate(1/(-a**2*c*x**2+c)**(3/2)/acos(a*x)**(3/2),x)
```

output
```
Integral(1/((-c*(a*x - 1)*(a*x + 1))**(3/2)*acos(a*x)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}\arccos{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*arccos(a*x)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{3/2}}\,dx = \int \frac{1}{\mathrm{acos}\left(a\,x\right)^{3/2}\,{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

input
```
int(1/(acos(a*x)^(3/2)*(c - a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(acos(a*x)^(3/2)*(c - a^2*c*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}}{acos(ax)^2a^4x^4-2acos(ax)^2a^2x^2+acos(ax)^2}dx\right)}{c^2}$$

input
```
int(1/(-a^2*c*x^2+c)^(3/2)/acos(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/(acos(a*x)**2*a**4*x
**4 - 2*acos(a*x)**2*a**2*x**2 + acos(a*x)**2),x))/c**2
```

3.478 $\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{3/2}}\,dx = -\frac{2\sqrt{1-a^2x^2}}{a(c-a^2cx^2)^{5/2}\sqrt{\arccos(ax)}} + \frac{8a\sqrt{1-a^2x^2}\mathrm{Int}\left(\frac{x}{(1-a^2x^2)^3\sqrt{\arccos(ax)}},x\right)}{c^2\sqrt{c-a^2cx^2}}$$

output
```
-2*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(1/2)+8*a*(-a^2*x
^2+1)^(1/2)*Defer(Int)(x/(-a^2*x^2+1)^3/arccos(a*x)^(1/2),x)/c^2/(-a^2*c*x
^2+c)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{3/2}}\,dx = \int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{3/2}}\,dx$$

input
```
Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCos[a*x]^(3/2)),x]
```

output
```
Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCos[a*x]^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{5167}$$

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}\left(c-a^2cx^2\right)^{5/2}}-\frac{8a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)^3\sqrt{\arccos(ax)}}dx}{c^2\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{5235}$$

$$\frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}\left(c-a^2cx^2\right)^{5/2}}-\frac{8a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)^3\sqrt{\arccos(ax)}}dx}{c^2\sqrt{c-a^2cx^2}}$$

input
```
Int[1/((c - a^2*c*x^2)^(5/2)*ArcCos[a*x]^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arccos \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(3/2),x)
```

output
```
int(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(-a**2*c*x**2+c)**(5/2)/acos(a*x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{3/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\arccos(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*arccos(a*x)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{3/2}}\,dx = \int \frac{1}{\mathrm{acos}(a\,x)^{3/2}\,(c-a^2\,c\,x^2)^{5/2}}\,dx$$

input
```
int(1/(acos(a*x)^(3/2)*(c - a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(acos(a*x)^(3/2)*(c - a^2*c*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.25

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{3/2}}\,dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}}{acos(ax)^2a^6x^6-3acos(ax)^2a^4x^4+3acos(ax)^2a^2x^2-acos(ax)^2}dx\right)}{c^3}$$

input
```
int(1/(-a^2*c*x^2+c)^(5/2)/acos(a*x)^(3/2),x)
```

output
```
( - sqrt(c)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/(acos(a*x)**2*a**
6*x**6 - 3*acos(a*x)**2*a**4*x**4 + 3*acos(a*x)**2*a**2*x**2 - acos(a*x)**
2),x))/c**3
```

3.479 $\int \frac{(c-a^2cx^2)^{3/2}}{\arccos(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\int \frac{(c-a^2cx^2)^{3/2}}{\arccos(ax)^{5/2}}\,dx = -\frac{2\sqrt{1-a^2x^2}(c-a^2cx^2)^{3/2}}{3a\arccos(ax)^{3/2}}+\frac{16cx(1-a^2x^2)\sqrt{c-a^2cx^2}}{3\sqrt{\arccos(ax)}} - \frac{4c\sqrt{2\pi}\sqrt{c-a^2cx^2}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a\sqrt{1-a^2x^2}} - \frac{8c\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{3a\sqrt{1-a^2x^2}}$$

output
```
-2/3*(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(3/2)/a/arccos(a*x)^(3/2)+16/3*c*x*
(-a^2*x^2+1)*(-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(1/2)-4/3*c*2^(1/2)*Pi^(1/2)
*(-a^2*c*x^2+c)^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))/a/(-a
^2*x^2+1)^(1/2)-8/3*c*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*FresnelC(2*arccos(a*x)
^(1/2)/Pi^(1/2))/a/(-a^2*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.30

$$\int \frac{(c-a^2cx^2)^{3/2}}{\arccos(ax)^{5/2}}\,dx = \frac{ce^{-4i\arccos(ax)}\sqrt{c-a^2cx^2}\left(1+14e^{4i\arccos(ax)}+e^{8i\arccos(ax)}-16a^2e^{4i\arccos(ax)}x^2-8i\right.}{}$$

input
```
Integrate[(c - a^2*c*x^2)^(3/2)/ArcCos[a*x]^(5/2),x]
```

output
```
(c*Sqrt[c - a^2*c*x^2]*(1 + 14*E^((4*I)*ArcCos[a*x]) + E^((8*I)*ArcCos[a*x
]) - 16*a^2*E^((4*I)*ArcCos[a*x])*x^2 - (8*I)*ArcCos[a*x] + (8*I)*E^((8*I)
*ArcCos[a*x])*ArcCos[a*x] + 64*a*E^((4*I)*ArcCos[a*x])*x*Sqrt[1 - a^2*x^2]
*ArcCos[a*x] - 64*E^((4*I)*ArcCos[a*x])*Sqrt[Pi]*ArcCos[a*x]^(3/2)*Fresnel
C[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] + 16*E^((4*I)*ArcCos[a*x])*((-I)*ArcCos[
a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcCos[a*x]] + 16*E^((4*I)*ArcCos[a*x])*(I*A
rcCos[a*x])^(3/2)*Gamma[1/2, (4*I)*ArcCos[a*x]]))/(24*a*E^((4*I)*ArcCos[a*
x])*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5167, 5215, 5169, 3042, 3793, 2009, 5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-a^2cx^2)^{3/2}}{\arccos(ax)^{5/2}}\,dx$$

$$\Big\downarrow \text{5167}$$

$$\frac{8ac\sqrt{c-a^2cx^2}\int \frac{x(1-a^2x^2)}{\arccos(ax)^{3/2}}dx}{3\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}(c-a^2cx^2)^{3/2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{5215}$$

$$\frac{8ac\sqrt{c-a^2cx^2}\left(-\frac{2\int \frac{\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}dx}{a}+8a\int \frac{x^2\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}dx+\frac{2x(1-a^2x^2)^{3/2}}{a\sqrt{\arccos(ax)}}\right)}{3\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}(c-a^2cx^2)^{3/2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \; 5169$$

$$\frac{8ac\sqrt{c-a^2cx^2}\left(8a\int \frac{x^2\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}dx+\frac{2\int \frac{1-a^2x^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}+\frac{2x(1-a^2x^2)^{3/2}}{a\sqrt{\arccos(ax)}}\right)}{3\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}(c-a^2cx^2)^{3/2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{8ac\sqrt{c-a^2cx^2}\left(8a\int \frac{x^2\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}dx+\frac{2\int \frac{\sin(\arccos(ax))^2}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}+\frac{2x(1-a^2x^2)^{3/2}}{a\sqrt{\arccos(ax)}}\right)}{3\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}(c-a^2cx^2)^{3/2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \; 3793$$

$$\frac{8ac\sqrt{c-a^2cx^2}\left(8a\int \frac{x^2\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}dx+\frac{2\int \left(\frac{1}{2\sqrt{\arccos(ax)}}-\frac{\cos(2\arccos(ax))}{2\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^2}+\frac{2x(1-a^2x^2)^{3/2}}{a\sqrt{\arccos(ax)}}\right)}{3\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}(c-a^2cx^2)^{3/2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{8ac\sqrt{c-a^2cx^2}\left(8a\int \frac{x^2\sqrt{1-a^2x^2}}{\sqrt{\arccos(ax)}}dx+\frac{2\left(\sqrt{\arccos(ax)}-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{a^2}+\frac{2x(1-a^2x^2)^{3/2}}{a\sqrt{\arccos(ax)}}\right)}{3\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}(c-a^2cx^2)^{3/2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \; 5225$$

$$\frac{8ac\sqrt{c-a^2cx^2}\left(-\frac{8\int \frac{a^2x^2(1-a^2x^2)}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}+\frac{2\left(\sqrt{\arccos(ax)}-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{a^2}+\frac{2x(1-a^2x^2)^{3/2}}{a\sqrt{\arccos(ax)}}\right)}{3\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^{3/2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \; 4906$$

$$\frac{8ac\sqrt{c-a^2cx^2}\left(-\frac{8\int \left(\frac{1}{8\sqrt{\arccos(ax)}}-\frac{\cos(4\arccos(ax))}{8\sqrt{\arccos(ax)}}\right)d\arccos(ax)}{a^2}+\frac{2\left(\sqrt{\arccos(ax)}-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{a^2}+\frac{2x(1-a^2x^2}{a\sqrt{\arccos(}}\right.}{3\sqrt{1-a^2x^2}}$$
$$\frac{2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^{3/2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{8ac\sqrt{c-a^2cx^2}\left(-\frac{8\left(\frac{1}{4}\sqrt{\arccos(ax)}-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)\right)}{a^2}+\frac{2\left(\sqrt{\arccos(ax)}-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{a^2}+\frac{2x(}{a\sqrt{}}\right.}{3\sqrt{1-a^2x^2}}$$
$$\frac{2\sqrt{1-a^2x^2}\left(c-a^2cx^2\right)^{3/2}}{3a\arccos(ax)^{3/2}}$$

input
```
Int[(c - a^2*c*x^2)^(3/2)/ArcCos[a*x]^(5/2),x]
```

output
```
(2*Sqrt[1 - a^2*x^2]*(c - a^2*c*x^2)^(3/2))/(3*a*ArcCos[a*x]^(3/2)) + (8*a
*c*Sqrt[c - a^2*c*x^2]*((2*x*(1 - a^2*x^2)^(3/2))/(a*Sqrt[ArcCos[a*x]]) -
(8*(Sqrt[ArcCos[a*x]]/4 - (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*
x]]])/8))/a^2 + (2*(Sqrt[ArcCos[a*x]] - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[
a*x]])/Sqrt[Pi]])/2))/a^2))/(3*Sqrt[1 - a^2*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

rule 5215
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(f*x)^m)*Sqrt[1 - c^2*x^2]*(d + e*x^2)
^p*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (Simp[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m - 1)*(1 - c^2*x^2)^(
p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x] - Simp[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x
^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p +
 1, 0] && IGtQ[m, -3]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{\arccos{(ax)}^{\frac{5}{2}}}dx$$

input
```
int((-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(5/2),x)
```

output
```
int((-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c-a^2cx^2)^{3/2}}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(c - a^2cx^2)^{3/2}}{\arccos(ax)^{5/2}}\,dx = \int \frac{(-c(ax-1)\,(ax+1))^{\frac{3}{2}}}{\operatorname{acos}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(3/2)/acos(a*x)**(5/2),x)
```

output
```
Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c - a^2cx^2)^{3/2}}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{(c-a^2cx^2)^{3/2}}{\arccos(ax)^{5/2}}\,dx = \int \frac{(-a^2cx^2+c)^{\frac{3}{2}}}{\arccos(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((-a^2*c*x^2 + c)^(3/2)/arccos(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-a^2cx^2)^{3/2}}{\arccos(ax)^{5/2}}\,dx = \int \frac{(c-a^2\,c\,x^2)^{3/2}}{\mathrm{acos}\,(a\,x)^{5/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(3/2)/acos(a*x)^(5/2),x)
```

output
```
int((c - a^2*c*x^2)^(3/2)/acos(a*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c-a^2cx^2)^{3/2}}{\arccos(ax)^{5/2}}\,dx = \sqrt{c}\,c\left(-\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}\,x^2}{acos\,(ax)^3}dx\right)a^2+\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos\,(ax)}}{acos\,(ax)^3}dx\right)$$

input
```
int((-a^2*c*x^2+c)^(3/2)/acos(a*x)^(5/2),x)
```

output
```
sqrt(c)*c*( - int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x))*x**2)/acos(a*x)*
*3,x)*a**2 + int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/acos(a*x)**3,x))
```

3.480 $\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{5/2}}\,dx = -\frac{2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{3a\arccos(ax)^{3/2}} + \frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\arccos(ax)}} - \frac{8\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{3a\sqrt{1-a^2x^2}}$$

output
```
-2/3*(-a^2*x^2+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/arccos(a*x)^(3/2)+8/3*x*(-a
^2*c*x^2+c)^(1/2)/arccos(a*x)^(1/2)-8/3*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)*Fres
nelC(2*arccos(a*x)^(1/2)/Pi^(1/2))/a/(-a^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{5/2}}\,dx = -\frac{2\sqrt{1-a^2x^2}\sqrt{c(1-a^2x^2)}\left(-1-\frac{4ax\arccos(ax)}{\sqrt{1-a^2x^2}}+\frac{4\sqrt{\pi}\arccos(ax)^{3/2}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{1-a^2x^2}\right)}{3a\arccos(ax)^{3/2}}$$

input
```
Integrate[Sqrt[c - a^2*c*x^2]/ArcCos[a*x]^(5/2),x]
```

output
```
(-2*Sqrt[1 - a^2*x^2]*Sqrt[c*(1 - a^2*x^2)]*(-1 - (4*a*x*ArcCos[a*x])/Sqrt
[1 - a^2*x^2] + (4*Sqrt[Pi]*ArcCos[a*x]^(3/2)*FresnelC[(2*Sqrt[ArcCos[a*x]
])/Sqrt[Pi]])/(1 - a^2*x^2)))/(3*a*ArcCos[a*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5167, 5143, 25, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{5/2}}dx$$

$$\Big\downarrow \text{5167}$$

$$\frac{4a\sqrt{c-a^2cx^2}\int \frac{x}{\arccos(ax)^{3/2}}dx}{3\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{5143}$$

$$\frac{4a\sqrt{c-a^2cx^2}\left(\frac{2\int -\frac{\cos(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}+\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}\right)}{3\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{4a\sqrt{c-a^2cx^2}\left(\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\int \frac{\cos(2\arccos(ax))}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}\right)}{3\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\sqrt{c-a^2cx^2}\left(\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\int \frac{\sin\left(2\arccos(ax)+\frac{\pi}{2}\right)}{\sqrt{\arccos(ax)}}d\arccos(ax)}{a^2}\right)}{3\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{3785}$$

$$\frac{4a\sqrt{c-a^2cx^2}\left(\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{4\int\cos(2\arccos(ax))d\sqrt{\arccos(ax)}}{a^2}\right)}{3\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{3a\arccos(ax)^{3/2}}$$

$$\Big\downarrow \text{3833}$$

$$\frac{4a\sqrt{c-a^2cx^2}\left(\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^2}\right)}{3\sqrt{1-a^2x^2}}+\frac{2\sqrt{1-a^2x^2}\sqrt{c-a^2cx^2}}{3a\arccos(ax)^{3/2}}$$

input
```
Int[Sqrt[c - a^2*c*x^2]/ArcCos[a*x]^(5/2),x]
```

output
```
(2*Sqrt[1 - a^2*x^2]*Sqrt[c - a^2*c*x^2])/(3*a*ArcCos[a*x]^(3/2)) + (4*a*S
qrt[c - a^2*c*x^2]*((2*x*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (2*Sqr
t[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/a^2))/(3*Sqrt[1 - a^2*x^2]
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5143
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - S
imp[1/(b^2*c^(m + 1)*(n + 1))   Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-
a/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos
[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

rule 5167
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(-Sqrt[1 - c^2*x^2])*(d + e*x^2)^p*((a + b*ArcCos[c*x])^(n
+ 1)/(b*c*(n + 1))), x] - Simp[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x^2)^p
/(1 - c^2*x^2)^p]   Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n +
1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -
1]
```

Maple [F]

$$\int \frac{\sqrt{-a^{2} c \,x^{2}+c}}{\arccos \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int((-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(5/2),x)
```

output
```
int((-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{5/2}}\,dx = \int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\operatorname{acos}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate((-a**2*c*x**2+c)**(1/2)/acos(a*x)**(5/2),x)
```

output
```
Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{5/2}}\,dx = \int \frac{\sqrt{-a^2cx^2+c}}{\arccos(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-a^2*c*x^2 + c)/arccos(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{5/2}}\,dx = \int \frac{\sqrt{c-a^2\,c\,x^2}}{\mathrm{acos}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c - a^2*c*x^2)^(1/2)/acos(a*x)^(5/2),x)
```

output
```
int((c - a^2*c*x^2)^(1/2)/acos(a*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c-a^2cx^2}}{\arccos(ax)^{5/2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos\left(ax\right)}}{acos\left(ax\right)^3}dx\right)$$

input
```
int((-a^2*c*x^2+c)^(1/2)/acos(a*x)^(5/2),x)
```

output
```
sqrt(c)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/acos(a*x)**3,x)
```

3.481 $\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 44

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{5/2}}\,dx = -\frac{2\sqrt{1-a^2x^2}}{3a\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}$$

output
```
-2/3*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{5/2}}\,dx = \frac{2\sqrt{1-a^2x^2}}{3a\sqrt{c-a^2cx^2}\arccos(ax)^{3/2}}$$

input
```
Integrate[1/(Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^(5/2)),x]
```

output
```
(2*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^{5/2}\sqrt{c-a^2cx^2}}dx$$

$$\Big\downarrow \text{ 5153}$$

$$\frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}\sqrt{c-a^2cx^2}}$$

input
```
Int[1/(Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^(5/2)),x]
```

output
```
(2*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[c - a^2*c*x^2]*ArcCos[a*x]^(3/2))
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\sqrt{-a^2x^2+1}}{3\arccos(ax)^{\frac{3}{2}}\sqrt{-c(a^2x^2-1)}\,a}$	38

input
```
int(1/(-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/arccos(a*x)^(3/2)/(-c*(a^2*x^2-1))^(1/2)*(-a^2*x^2+1)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{5/2}}\,dx = -\frac{2\sqrt{-a^2cx^2+c}\sqrt{-a^2x^2+1}}{3\left(a^3cx^2-ac\right)\arccos\left(ax\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(-a^2*c*x^2 + c)*sqrt(-a^2*x^2 + 1)/((a^3*c*x^2 - a*c)*arccos(a*x
)^(3/2))
```

Sympy [F]

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\operatorname{acos}^{\frac{5}{2}}\left(ax\right)}\,dx$$

input
```
integrate(1/(-a**2*c*x**2+c)**(1/2)/acos(a*x)**(5/2),x)
```

output
```
Integral(1/(sqrt(-c*(a*x - 1)*(a*x + 1))*acos(a*x)**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.32

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{5/2}}\,dx = \frac{2}{3\,a\sqrt{c}\arccos\left(ax\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(1/2)/arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
2/3/(a*sqrt(c)*arccos(a*x)^(3/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{5/2}}\,dx = \int \frac{1}{\mathrm{acos}\left(a\,x\right)^{5/2}\,\sqrt{c-a^2\,c\,x^2}}\,dx$$

input
```
int(1/(acos(a*x)^(5/2)*(c - a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(acos(a*x)^(5/2)*(c - a^2*c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{c-a^2cx^2}\arccos(ax)^{5/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{acos\,(ax)}}{3acos\,(ax)^2\,ac}$$

input
```
int(1/(-a^2*c*x^2+c)^(1/2)/acos(a*x)^(5/2),x)
```

output
```
(2*sqrt(c)*sqrt(acos(a*x)))/(3*acos(a*x)**2*a*c)
```

3.482 $\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{5/2}}\,dx = -\frac{2\sqrt{1-a^2x^2}}{3a\,(c-a^2cx^2)^{3/2}\arccos(ax)^{3/2}} + \frac{4a\sqrt{1-a^2x^2}\,\text{Int}\left(\frac{x}{(1-a^2x^2)^2\arccos(ax)^{3/2}},x\right)}{3c\sqrt{c-a^2cx^2}}$$

output
```
-2/3*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(3/2)+4/3*a*(-a
^2*x^2+1)^(1/2)*Defer(Int)(x/(-a^2*x^2+1)^2/arccos(a*x)^(3/2),x)/c/(-a^2*c
*x^2+c)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{5/2}}\,dx = \int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{5/2}}\,dx$$

input
```
Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^(5/2)),x]
```

output
```
Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^{5/2}\left(c-a^2cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{5167}$$

$$\frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{3/2}}-\frac{4a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)^2\arccos(ax)^{3/2}}dx}{3c\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{5235}$$

$$\frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}\left(c-a^2cx^2\right)^{3/2}}-\frac{4a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)^2\arccos(ax)^{3/2}}dx}{3c\sqrt{c-a^2cx^2}}$$

input
```
Int[1/((c - a^2*c*x^2)^(3/2)*ArcCos[a*x]^(5/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arccos \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(5/2),x)
```

output
```
int(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(-a**2*c*x**2+c)**(3/2)/acos(a*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{3}{2}}\arccos(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^(3/2)/arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(3/2)*arccos(a*x)^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{5/2}}\,dx = \int \frac{1}{\mathrm{acos}\left(a\,x\right)^{5/2}\,{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

input
```
int(1/(acos(a*x)^(5/2)*(c - a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(acos(a*x)^(5/2)*(c - a^2*c*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \frac{1}{(c-a^2cx^2)^{3/2}\arccos(ax)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}}{acos(ax)^3a^4x^4-2acos(ax)^3a^2x^2+acos(ax)^3}dx\right)}{c^2}$$

input
```
int(1/(-a^2*c*x^2+c)^(3/2)/acos(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/(acos(a*x)**3*a**4*x
**4 - 2*acos(a*x)**3*a**2*x**2 + acos(a*x)**3),x))/c**2
```

3.483 $\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{5/2}}\,dx = -\frac{2\sqrt{1-a^2x^2}}{3a(c-a^2cx^2)^{5/2}\arccos(ax)^{3/2}} + \frac{8a\sqrt{1-a^2x^2}\,\mathrm{Int}\left(\frac{x}{(1-a^2x^2)^3\arccos(ax)^{3/2}},x\right)}{3c^2\sqrt{c-a^2cx^2}}$$

output
```
-2/3*(-a^2*x^2+1)^(1/2)/a/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(3/2)+8/3*a*(-a
^2*x^2+1)^(1/2)*Defer(Int)(x/(-a^2*x^2+1)^3/arccos(a*x)^(3/2),x)/c^2/(-a^2
*c*x^2+c)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{5/2}}\,dx = \int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{5/2}}\,dx$$

input
```
Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCos[a*x]^(5/2)),x]
```

output
```
Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCos[a*x]^(5/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(ax)^{5/2} (c-a^2cx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{5167}$$

$$\frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2} (c-a^2cx^2)^{5/2}}-\frac{8a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)^3\arccos(ax)^{3/2}}dx}{3c^2\sqrt{c-a^2cx^2}}$$

$$\Big\downarrow \text{5235}$$

$$\frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2} (c-a^2cx^2)^{5/2}}-\frac{8a\sqrt{1-a^2x^2}\int \frac{x}{(1-a^2x^2)^3\arccos(ax)^{3/2}}dx}{3c^2\sqrt{c-a^2cx^2}}$$

input
```
Int[1/((c - a^2*c*x^2)^(5/2)*ArcCos[a*x]^(5/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arccos \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(5/2),x)
```

output
```
int(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(-a**2*c*x**2+c)**(5/2)/acos(a*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{5/2}}\,dx = \int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\arccos(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-a^2*c*x^2+c)^(5/2)/arccos(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-a^2*c*x^2 + c)^(5/2)*arccos(a*x)^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{5/2}}\,dx = \int \frac{1}{\mathrm{acos}(a\,x)^{5/2}\,(c-a^2\,c\,x^2)^{5/2}}\,dx$$

input
```
int(1/(acos(a*x)^(5/2)*(c - a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(acos(a*x)^(5/2)*(c - a^2*c*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.25

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\arccos(ax)^{5/2}}\,dx = -\frac{\sqrt{c}\left(\int \frac{\sqrt{-a^2x^2+1}\,\sqrt{acos(ax)}}{acos(ax)^3a^6x^6-3acos(ax)^3a^4x^4+3acos(ax)^3a^2x^2-acos(ax)^3}dx\right)}{c^3}$$

input
```
int(1/(-a^2*c*x^2+c)^(5/2)/acos(a*x)^(5/2),x)
```

output
```
( - sqrt(c)*int((sqrt( - a**2*x**2 + 1)*sqrt(acos(a*x)))/(acos(a*x)**3*a**
6*x**6 - 3*acos(a*x)**3*a**4*x**4 + 3*acos(a*x)**3*a**2*x**2 - acos(a*x)**
3),x))/c**3
```

3.484 $\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx$

Optimal result

Integrand size = 29, antiderivative size = 259

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^{1+n}}{8bc^3(1+n)\sqrt{1-c^2x^2}}$$
$$+\frac{i2^{-2(3+n)}e^{-\frac{4ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$
$$-\frac{i2^{-2(3+n)}e^{\frac{4ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{4i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$

output
```
1/8*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^(1+n)/b/c^3/(1+n)/(-c^2*x^2+1)^
(1/2)+I*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-4*I*(a+b*arcco
s(c*x))/b)/(2^(6+2*n))/c^3/exp(4*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcco
s(c*x))/b)^n)-I*exp(4*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMM
A(1+n,4*I*(a+b*arccos(c*x))/b)/(2^(6+2*n))/c^3/(-c^2*x^2+1)^(1/2)/((I*(a+b
*arccos(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.74

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx =$$
$$-\frac{d\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{8a+8b\arccos(cx)}{b+bn}+i4^{-n}e^{-\frac{4ia}{b}}\left(\frac{(a+b\arccos(cx))^2}{b^2}\right)^{-n}\left(\left(\frac{i(a+b\arccos(cx))}{b}\right)^n\Gamma\left(1\right.\right.\right.}{64c^3\sqrt{d(1-c^2x^2)}}$$

input
```
Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^n,x]
```

output
```
-1/64*(d*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^n*((8*a + 8*b*ArcCos[c*x])/
(b + b*n) + (I*(((I*(a + b*ArcCos[c*x]))/b)^n*Gamma[1 + n, ((-4*I)*(a + b*
ArcCos[c*x]))/b] - E^(((8*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n*Gamma[
1 + n, ((4*I)*(a + b*ArcCos[c*x]))/b]))/(4^n*E^(((4*I)*a)/b)*((a + b*ArcCo
s[c*x])^2/b^2)^n)))/(c^3*Sqrt[d*(1 - c^2*x^2)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\sqrt{d-c^2dx^2}\int(a+b\arccos(cx))^n\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)d(a+b\arccos(cx))}{bc^3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \left(\frac{1}{8}(a+b\arccos(cx))^n-\frac{1}{8}(a+b\arccos(cx))^n\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)\right)d(a+b\arccos(cx))}{bc^3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{(a+b\arccos(cx))^{n+1}}{8(n+1)}+ib2^{-2(n+3)}e^{-\frac{4ia}{b}}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,-\frac{4i(a+b\arccos(cx))}{b}\right.\right.}{bc^3\sqrt{1-c^2x^2}}$$

input
```
Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^n,x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*((a + b*ArcCos[c*x])^(1 + n)/(8*(1 + n)) + (I*b*(a
+ b*ArcCos[c*x])^n*Gamma[1 + n, ((-4*I)*(a + b*ArcCos[c*x]))/b])/(2^(2*(3
+ n))*E^(((4*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n) - (I*b*E^(((4*I)*a
)/b)*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((4*I)*(a + b*ArcCos[c*x]))/b])/(2
^(2*(3 + n))*((I*(a + b*ArcCos[c*x]))/b)^n)))/(b*c^3*Sqrt[1 - c^2*x^2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^2\sqrt{-c^2d\,x^2+d}\,(a+b\arccos(cx))^n\,dx$$

input
```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n,x)
```

output
```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n,x)
```

Fricas [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\int\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)^nx^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n,x, algorithm="frica
s")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)^n*x^2, x)
```

Sympy [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\int x^2\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\operatorname{acos}(cx))^n\,dx$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))**n,x)
```

output
```
Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**n, x)
```

Maxima [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\int\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)^nx^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)^n*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\int\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)^nx^2\,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n,x, algorithm="giac"
)
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)^n*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\int x^2\,(a+b\,\mathrm{acos}(c\,x))^n\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x^2*(a + b*acos(c*x))^n*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x^2*(a + b*acos(c*x))^n*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\sqrt{d}\left(\int(acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^2dx\right)$$

input
```
int(x^2*(-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))^n,x)
```

output
```
sqrt(d)*int((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**2,x)
```

3.485 $\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx$

Optimal result

Integrand size = 27, antiderivative size = 391

$$\begin{aligned}
&\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx \\
&= -\frac{e^{-\frac{ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{i(a+b\arccos(cx))}{b}\right)}{8c^2\sqrt{1-c^2x^2}} \\
&\quad-\frac{e^{\frac{ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{i(a+b\arccos(cx))}{b}\right)}{8c^2\sqrt{1-c^2x^2}} \\
&\quad-\frac{3^{-1-n}e^{-\frac{3ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3i(a+b\arccos(cx))}{b}\right)}{8c^2\sqrt{1-c^2x^2}} \\
&\quad-\frac{3^{-1-n}e^{\frac{3ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{3i(a+b\arccos(cx))}{b}\right)}{8c^2\sqrt{1-c^2x^2}}
\end{aligned}$$

output
```
-1/8*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-I*(a+b*arccos(c*x
))/b)/c^2/exp(I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)-1/8*e
xp(I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,I*(a+b*arccos
(c*x))/b)/c^2/(-c^2*x^2+1)^(1/2)/((I*(a+b*arccos(c*x))/b)^n)-1/8*3^(-1-n)*
(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-3*I*(a+b*arccos(c*x))/
b)/c^2/exp(3*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)-1/8*3^
(-1-n)*exp(3*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,3*I
*(a+b*arccos(c*x))/b)/c^2/(-c^2*x^2+1)^(1/2)/((I*(a+b*arccos(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.70

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx$$

$$= \frac{ide^{-\frac{3ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(3e^{\frac{2ia}{b}}\left(\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{i(a+b\arccos(cx))}{b}\right)-e^{\frac{2ia}{b}}\left(\frac{i(a+\right.\right.\right.}{}$$

input
```
Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^n,x]
```

output
```
((I/24)*d*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^n*(3*E^(((2*I)*a)/b)*(Gamm
a[1 + n, ((-I)*(a + b*ArcCos[c*x]))/b]/(((-I)*(a + b*ArcCos[c*x]))/b)^n -
(E^(((2*I)*a)/b)*Gamma[1 + n, (I*(a + b*ArcCos[c*x]))/b])/((I*(a + b*ArcCo
s[c*x]))/b)^n) + (-(((I*(a + b*ArcCos[c*x]))/b)^n*Gamma[1 + n, ((-3*I)*(a
+ b*ArcCos[c*x]))/b]) + E^(((6*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n*G
amma[1 + n, ((3*I)*(a + b*ArcCos[c*x]))/b])/(3^n*((a + b*ArcCos[c*x])^2/b^
2)^n)))/(c^2*E^(((3*I)*a)/b)*Sqrt[d*(1 - c^2*x^2)])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\sqrt{d-c^2dx^2}\int(a+b\arccos(cx))^n\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)d(a+b\arccos(cx))}{bc^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\sqrt{d-c^2dx^2}\int\left(\frac{1}{4}(a+b\arccos(cx))^n\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)-\frac{1}{4}(a+b\arccos(cx))^n\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)\right)}{bc^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{d-c^2dx^2}\left(-\frac{1}{8}ibe^{-\frac{ia}{b}}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,-\frac{i(a+b\arccos(cx))}{b}\right)+\frac{1}{8}ib3^{-n-1}e^{-\frac{3ia}{b}}(\right.}{}$$

input
```
Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^n,x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*(((-1/8*I)*b*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((-
I)*(a + b*ArcCos[c*x]))/b])/(E^((I*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n)
 + ((I/8)*b*E^((I*a)/b)*(a + b*ArcCos[c*x])^n*Gamma[1 + n, (I*(a + b*ArcCo
s[c*x]))/b])/((I*(a + b*ArcCos[c*x]))/b)^n + ((I/8)*3^(-1 - n)*b*(a + b*Ar
cCos[c*x])^n*Gamma[1 + n, ((-3*I)*(a + b*ArcCos[c*x]))/b])/(E^(((3*I)*a)/b
)*(((-I)*(a + b*ArcCos[c*x]))/b)^n) - ((I/8)*3^(-1 - n)*b*E^(((3*I)*a)/b)*
(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((3*I)*(a + b*ArcCos[c*x]))/b])/((I*(a
+ b*ArcCos[c*x]))/b)^n))/(b*c^2*Sqrt[1 - c^2*x^2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x\sqrt{-c^2d\,x^2+d}\,(a+b\arccos{(cx)})^n\,dx$$

input
```
int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n,x)
```

output
```
int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n,x)
```

Fricas [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\int\sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)^n x\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)^n*x, x)
```

Sympy [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\int x\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\text{acos}\,(cx))^n\,dx$$

input
```
integrate(x*(-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))**n,x)
```

output
```
Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**n, x)
```

Maxima [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\int \sqrt{-c^2dx^2+d}(b\arccos\,(cx)+a)^n x\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n,x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)^n*x, x)
```

Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\int x\,(a+b\,\mathrm{acos}(c\,x))^n\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int(x*(a + b*acos(c*x))^n*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(x*(a + b*acos(c*x))^n*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\sqrt{d}\left(\int(acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,xdx\right)$$

input
```
int(x*(-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))^n,x)
```

output
```
sqrt(d)*int((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x,x)
```

3.486 $\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx$

Optimal result

Integrand size = 26, antiderivative size = 259

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx = \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^{1+n}}{2bc(1+n)\sqrt{1-c^2x^2}} - \frac{i2^{-3-n}e^{-\frac{2ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2i(a+b\arccos(cx))}{b}\right)}{c\sqrt{1-c^2x^2}} + \frac{i2^{-3-n}e^{\frac{2ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{2i(a+b\arccos(cx))}{b}\right)}{c\sqrt{1-c^2x^2}}$$

output
```
1/2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^(1+n)/b/c/(1+n)/(-c^2*x^2+1)^(1
/2)-I*2^(-3-n)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-2*I*(a+
b*arccos(c*x))/b)/c/exp(2*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arccos(c*x))
/b)^n)+I*2^(-3-n)*exp(2*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GA
MMA(1+n,2*I*(a+b*arccos(c*x))/b)/c/(-c^2*x^2+1)^(1/2)/((I*(a+b*arccos(c*x)
)/b)^n)
```

Mathematica [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx$$

input
```
Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^n,x]
```

output
```
Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^n, x]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx$$

$$\Big\downarrow \text{5169}$$

$$-\frac{\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))^n\sin^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)d(a+b\arccos(cx))}{bc\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))^n\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^2d(a+b\arccos(cx))}{bc\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\sqrt{d-c^2dx^2}\int \left(\frac{1}{2}(a+b\arccos(cx))^n-\frac{1}{2}(a+b\arccos(cx))^n\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)\right)d(a+b\arccos(cx))}{bc\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{d-c^2dx^2}\left(\frac{(a+b\arccos(cx))^{n+1}}{2(n+1)}+ib2^{-n-3}e^{-\frac{2ia}{b}}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2i(a+b\arccos(}{b}\right.\right.}{bc\sqrt{1-c^2x^2}}$$

input
```
Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^n,x]
```

output
```
-((Sqrt[d - c^2*d*x^2]*((a + b*ArcCos[c*x])^(1 + n)/(2*(1 + n)) + (I*2^(-3
 - n)*b*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((-2*I)*(a + b*ArcCos[c*x]))/b]
)/(E^(((2*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n) - (I*2^(-3 - n)*b*E^(
((2*I)*a)/b)*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((2*I)*(a + b*ArcCos[c*x])
)/b])/((I*(a + b*ArcCos[c*x]))/b)^n))/(b*c*Sqrt[1 - c^2*x^2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [F]

$$\int \sqrt{-c^2d\,x^2+d}\,(a+b\arccos(cx))^n\,dx$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n,x)
```

output
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n,x)
```

Fricas [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx = \int \sqrt{-c^2dx^2+d}(b\arccos(cx)+a)^n\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n,x, algorithm="fricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)^n, x)
```

Sympy [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx = \int \sqrt{-d\,(cx-1)\,(cx+1)}(a+b\operatorname{acos}(cx))^n\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))**n,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**n, x)
```

Maxima [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\int \sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)^n\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n,x, algorithm="maxima")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)^n, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\int (a+b\,\mathrm{acos}(c\,x))^n\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\,dx=\sqrt{d}\left(\int(acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}dx\right)$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))^n,x)
```

output
```
sqrt(d)*int((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1),x)
```

3.487 $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x}\,dx = \frac{de^{-\frac{ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{i(a+b\arccos(cx))}{b}\right)}{2\sqrt{d-c^2dx^2}} + \frac{de^{\frac{ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{i(a+b\arccos(cx))}{b}\right)}{2\sqrt{d-c^2dx^2}} + d\text{Int}\left(\frac{(a+b\arccos(cx))^n}{x\sqrt{d-c^2dx^2}},x\right)$$

output
```
1/2*d*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-I*(a+b*arccos(c*x)
)/b)/exp(I*a/b)/(-c^2*d*x^2+d)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)+1/2*d*ex
p(I*a/b)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,I*(a+b*arccos(c*
x))/b)/(-c^2*d*x^2+d)^(1/2)/((I*(a+b*arccos(c*x))/b)^n)+d*Defer(Int)((a+b*
arccos(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x}\,dx=\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x}\,dx$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^n)/x,x]
```

output
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^n)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x}\,dx$$

$$\Big\downarrow \text{5227}$$

$$\int \left(\frac{d(a+b\arccos(cx))^n}{x\sqrt{d-c^2dx^2}}-\frac{c^2dx(a+b\arccos(cx))^n}{\sqrt{d-c^2dx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$d\int \frac{(a+b\arccos(cx))^n}{x\sqrt{d-c^2dx^2}}dx-\frac{ide^{-\frac{ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,-\frac{i(a+b\arccos(cx))}{b}\right)}{2\sqrt{d-c^2dx^2}}+\frac{ide^{\frac{ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,\frac{i(a+b\arccos(cx))}{b}\right)}{2\sqrt{d-c^2dx^2}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^n)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2 d\, x^2+d}\,(a+b\arccos{(cx)})^n}{x}dx$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n/x,x)
```

output
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)^n}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n/x,x, algorithm="fricas"
)
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)^n/x, x)
```

Sympy [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x}\,dx=\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))^n}{x}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))**n/x,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**n/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x}\,dx=\int \frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)^n}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n/x,x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)^n/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x}\,dx=\int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^n\,\sqrt{d-c^2\,d\,x^2}}{x}\,dx$$

input
```
int(((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(1/2))/x,x)
```

output
```
int(((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(1/2))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x}\,dx=\sqrt{d}\left(\int \frac{(acos(cx)\,b+a)^n\,\sqrt{-c^2x^2+1}}{x}dx\right)$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))^n/x,x)
```

output
```
sqrt(d)*int(((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1))/x,x)
```

3.488 $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x^2}\,dx = -\frac{cd\sqrt{1-c^2x^2}(a+b\arccos(cx))^{1+n}}{b(1+n)\sqrt{d-c^2dx^2}} + d\text{Int}\left(\frac{(a+b\arccos(cx))^n}{x^2\sqrt{d-c^2dx^2}},x\right)$$

output
```
-c*d*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^(1+n)/b/(1+n)/(-c^2*d*x^2+d)^(1/
2)+d*Defer(Int)((a+b*arccos(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x^2}\,dx = \int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x^2}\,dx$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^n)/x^2,x]
```

output
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^n)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x^2} \, dx$$

$$\Big\downarrow \text{5227}$$

$$\int \left(\frac{d(a+b\arccos(cx))^n}{x^2\sqrt{d-c^2dx^2}}-\frac{c^2d(a+b\arccos(cx))^n}{\sqrt{d-c^2dx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$d\int \frac{(a+b\arccos(cx))^n}{x^2\sqrt{d-c^2dx^2}}dx+\frac{cd\sqrt{1-c^2x^2}(a+b\arccos(cx))^{n+1}}{b(n+1)\sqrt{d-c^2dx^2}}$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x])^n)/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-c^2d\,x^2+d}\,(a+b\arccos(cx))^n}{x^2}dx$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n/x^2,x)
```

output
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x^2}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)^n}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n/x^2,x, algorithm="frica
s")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)^n/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x^2}\,dx = \int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\operatorname{acos}{(cx)})^n}{x^2}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(1/2)*(a+b*acos(c*x))**n/x**2,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))**n/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x^2}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\arccos{(cx)}+a)^n}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n/x^2,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)^n/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^n\,\sqrt{d-c^2\,d\,x^2}}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(1/2))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.62

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n}{x^2}\,dx$$
$$= \frac{\sqrt{d}\left(\left(acos(cx)\,b+a\right)^n acos(cx)\,bc + \left(acos(cx)\,b+a\right)^n ac + \left(\int \frac{(acos(cx)b+a)^n}{\sqrt{-c^2x^2+1}\,x^2}dx\right) bn + \left(\int \frac{(acos(cx)b+a)^n}{\sqrt{-c^2x^2+1}\,x^2}dx\right)\right.}{b\,(n+1)}$$

input
```
int((-c^2*d*x^2+d)^(1/2)*(a+b*acos(c*x))^n/x^2,x)
```

output
```
(sqrt(d)*((acos(c*x)*b + a)**n*acos(c*x)*b*c + (acos(c*x)*b + a)**n*a*c +
int((acos(c*x)*b + a)**n/(sqrt( - c**2*x**2 + 1)*x**2),x)*b*n + int((acos(
c*x)*b + a)**n/(sqrt( - c**2*x**2 + 1)*x**2),x)*b))/(b*(n + 1))
```

3.489 $\int x^2(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n\,dx$

Optimal result

Integrand size = 29, antiderivative size = 684

$$\int x^2(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n\,dx = \frac{d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^{1+n}}{16bc^3(1+n)\sqrt{1-c^2x^2}}$$
$$-\frac{i2^{-7-n}de^{-\frac{2ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$
$$+\frac{i2^{-7-n}de^{\frac{2ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{2i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$
$$+\frac{i2^{-7-2n}de^{-\frac{4ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$
$$-\frac{i2^{-7-2n}de^{\frac{4ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{4i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$
$$+\frac{i2^{-7-n}3^{-1-n}de^{-\frac{6ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{6i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$
$$-\frac{i2^{-7-n}3^{-1-n}de^{\frac{6ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{6i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$

output
```
1/16*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^(1+n)/b/c^3/(1+n)/(-c^2*x^2+
1)^(1/2)-I*2^(-7-n)*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-
2*I*(a+b*arccos(c*x))/b)/c^3/exp(2*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arc
cos(c*x))/b)^n)+I*2^(-7-n)*d*exp(2*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos
(c*x))^n*GAMMA(1+n,2*I*(a+b*arccos(c*x))/b)/c^3/(-c^2*x^2+1)^(1/2)/((I*(a+
b*arccos(c*x))/b)^n)+I*2^(-7-2*n)*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))
^n*GAMMA(1+n,-4*I*(a+b*arccos(c*x))/b)/c^3/exp(4*I*a/b)/(-c^2*x^2+1)^(1/2)
/((-I*(a+b*arccos(c*x))/b)^n)-I*2^(-7-2*n)*d*exp(4*I*a/b)*(-c^2*d*x^2+d)^(
1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,4*I*(a+b*arccos(c*x))/b)/c^3/(-c^2*x^2+
1)^(1/2)/((I*(a+b*arccos(c*x))/b)^n)+I*2^(-7-n)*3^(-1-n)*d*(-c^2*d*x^2+d)^
(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-6*I*(a+b*arccos(c*x))/b)/c^3/exp(6*I*
a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)-I*2^(-7-n)*3^(-1-n)*d
*exp(6*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,6*I*(a+b*
arccos(c*x))/b)/c^3/(-c^2*x^2+1)^(1/2)/((I*(a+b*arccos(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 3.01 (sec) , antiderivative size = 509, normalized size of antiderivative = 0.74

$$\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^n\,dx=\frac{d^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{24a}{b+bn}-\frac{24\arccos(cx)}{1+n}-3i2^{-n}e^{-\frac{2ia}{b}}\left(\frac{i(a+b\arccos(cx))}{b}\right)^n\right.}{}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^n,x]
```

output
```
(d^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^n*((-24*a)/(b + b*n) - (24*ArcC
os[c*x])/(1 + n) - ((3*I)*((I*(a + b*ArcCos[c*x]))/b)^n*Gamma[1 + n, ((-2*
I)*(a + b*ArcCos[c*x]))/b])/(2^n*E^(((2*I)*a)/b)*((a + b*ArcCos[c*x])^2/b^
2)^n) + ((3*I)*E^(((2*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n*Gamma[1 +
n, ((2*I)*(a + b*ArcCos[c*x]))/b])/(2^n*((a + b*ArcCos[c*x])^2/b^2)^n) - (
(3*I)*((I*(a + b*ArcCos[c*x]))/b)^n*Gamma[1 + n, ((-4*I)*(a + b*ArcCos[c*x
]))/b])/(4^n*E^(((4*I)*a)/b)*((a + b*ArcCos[c*x])^2/b^2)^n) + ((3*I)*E^(((
4*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n*Gamma[1 + n, ((4*I)*(a + b*Arc
Cos[c*x]))/b])/(4^n*((a + b*ArcCos[c*x])^2/b^2)^n) + (I*((I*(a + b*ArcCos[
c*x]))/b)^n*Gamma[1 + n, ((-6*I)*(a + b*ArcCos[c*x]))/b])/(6^n*E^(((6*I)*a
)/b)*((a + b*ArcCos[c*x])^2/b^2)^n) - (I*E^(((6*I)*a)/b)*(((-I)*(a + b*Arc
Cos[c*x]))/b)^n*Gamma[1 + n, ((6*I)*(a + b*ArcCos[c*x]))/b])/(6^n*((a + b*
ArcCos[c*x])^2/b^2)^n)))/(384*c^3*Sqrt[d - c^2*d*x^2])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^n\,dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{d\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))^n\cos^2\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)d(a+b\arccos(cx))}{bc^3\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{d\sqrt{d-c^2dx^2}\int \left(\frac{1}{32}\cos\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)(a+b\arccos(cx))^n-\frac{1}{16}\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)(a+b\arccos

$$-\frac{d\sqrt{d-c^2dx^2}\left(\frac{(a+b\arccos(cx))^{n+1}}{16(n+1)}+ib2^{-n-7}e^{-\frac{2ia}{b}}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2i(a+b\arccos(cx))}{b}\right.\right.}{}$$

input
```
Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^n,x]
```

output
```
-((d*Sqrt[d - c^2*d*x^2]*((a + b*ArcCos[c*x])^(1 + n)/(16*(1 + n)) + (I*2^
(-7 - n)*b*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((-2*I)*(a + b*ArcCos[c*x]))
/b])/(E^(((2*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n) - (I*2^(-7 - n)*b*
E^(((2*I)*a)/b)*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((2*I)*(a + b*ArcCos[c*
x]))/b])/((I*(a + b*ArcCos[c*x]))/b)^n + (I*2^(-7 - 2*n)*b*(a + b*ArcCos[c
*x])^n*Gamma[1 + n, ((-4*I)*(a + b*ArcCos[c*x]))/b])/(E^(((4*I)*a)/b)*(((-
I)*(a + b*ArcCos[c*x]))/b)^n) - (I*2^(-7 - 2*n)*b*E^(((4*I)*a)/b)*(a + b*A
rcCos[c*x])^n*Gamma[1 + n, ((4*I)*(a + b*ArcCos[c*x]))/b])/((I*(a + b*ArcC
os[c*x]))/b)^n - (I*2^(-7 - n)*3^(-1 - n)*b*(a + b*ArcCos[c*x])^n*Gamma[1
+ n, ((-6*I)*(a + b*ArcCos[c*x]))/b])/(E^(((6*I)*a)/b)*(((-I)*(a + b*ArcCo
s[c*x]))/b)^n) + (I*2^(-7 - n)*3^(-1 - n)*b*E^(((6*I)*a)/b)*(a + b*ArcCos[
c*x])^n*Gamma[1 + n, ((6*I)*(a + b*ArcCos[c*x]))/b])/((I*(a + b*ArcCos[c*x
]))/b)^n))/(b*c^3*Sqrt[1 - c^2*x^2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^2 \left(-c^2 d\,x^2+d\right)^{\frac{3}{2}} \left(a+b\arccos\left(cx\right)\right)^n dx$$

input
```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n,x)
```

output
```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n,x)
```

Fricas [F]

$$\int x^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\arccos(cx)\right)^n dx = \int \left(-c^2dx^2+d\right)^{\frac{3}{2}} \left(b\arccos\left(cx\right)+a\right)^n x^2\, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n,x, algorithm="frica
s")
```

output
```
integral(-(c^2*d*x^4 - d*x^2)*sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)^n,
x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\arccos(cx)\right)^n dx = \text{Timed out}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 (d - c^2 dx^2)^{3/2} (a + b \arccos(cx))^n \, dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arccos(cx) + a)^n x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n,x, algorithm="maxim
a")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccos(c*x) + a)^n*x^2, x)
```

Giac [F]

$$\int x^2 (d - c^2 dx^2)^{3/2} (a + b \arccos(cx))^n \, dx = \int (-c^2 dx^2 + d)^{\frac{3}{2}} (b \arccos(cx) + a)^n x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n,x, algorithm="giac"
)
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccos(c*x) + a)^n*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d - c^2 dx^2)^{3/2} (a + b \arccos(cx))^n \, dx = \int x^2 (a + b \operatorname{acos}(c x))^n (d - c^2 d x^2)^{3/2} \, dx$$

input
```
int(x^2*(a + b*acos(c*x))^n*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x^2*(a + b*acos(c*x))^n*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x^2\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^n\,dx=\sqrt{d}\,d\left(-\left(\int(acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^4dx\right)c^2+\int(acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^2dx\right)$$

input
```
int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))^n,x)
```

output
```
sqrt(d)*d*( - int((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**4,x)*c**2
 + int((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**2,x))
```

3.490 $\int x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n\,dx$

Optimal result

Integrand size = 27, antiderivative size = 595

$$\begin{aligned}
&\int x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n\,dx = \\
&-\frac{de^{-\frac{ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{i(a+b\arccos(cx))}{b}\right)}{16c^2\sqrt{1-c^2x^2}} \\
&-\frac{de^{\frac{ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{i(a+b\arccos(cx))}{b}\right)}{16c^2\sqrt{1-c^2x^2}} \\
&-\frac{3^{-n}de^{-\frac{3ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3i(a+b\arccos(cx))}{b}\right)}{32c^2\sqrt{1-c^2x^2}} \\
&-\frac{3^{-n}de^{\frac{3ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{3i(a+b\arccos(cx))}{b}\right)}{32c^2\sqrt{1-c^2x^2}} \\
&-\frac{5^{-1-n}de^{-\frac{5ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5i(a+b\arccos(cx))}{b}\right)}{32c^2\sqrt{1-c^2x^2}} \\
&-\frac{5^{-1-n}de^{\frac{5ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{5i(a+b\arccos(cx))}{b}\right)}{32c^2\sqrt{1-c^2x^2}}
\end{aligned}$$

output
```
-1/16*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-I*(a+b*arccos(
c*x))/b)/c^2/exp(I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)-1/
16*d*exp(I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,I*(a+b*
arccos(c*x))/b)/c^2/(-c^2*x^2+1)^(1/2)/((I*(a+b*arccos(c*x))/b)^n)-1/32*d*
(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-3*I*(a+b*arccos(c*x))/
b)/(3^n)/c^2/exp(3*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)-
1/32*d*exp(3*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,3*I
*(a+b*arccos(c*x))/b)/(3^n)/c^2/(-c^2*x^2+1)^(1/2)/((I*(a+b*arccos(c*x))/b
)^n)-1/32*5^(-1-n)*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-5
*I*(a+b*arccos(c*x))/b)/c^2/exp(5*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arcc
os(c*x))/b)^n)-1/32*5^(-1-n)*d*exp(5*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arcc
os(c*x))^n*GAMMA(1+n,5*I*(a+b*arccos(c*x))/b)/c^2/(-c^2*x^2+1)^(1/2)/((I*(
a+b*arccos(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 1.50 (sec) , antiderivative size = 468, normalized size of antiderivative = 0.79

$$\int x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n\,dx =$$

$$-\frac{i15^{-1-n}d^2e^{-\frac{5ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{(a+b\arccos(cx))^2}{b^2}\right)^{-3n}\left(-215^{1+n}e^{\frac{4ia}{b}}\left(\frac{i(a+b\arccos(cx))}{b}\right)^n\left(\frac{(a+b\,\mathrm{ar}}{}\right.\right.}{}$$

input
```
Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^n,x]
```

output
```
((-1/32*I)*15^(-1 - n)*d^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^n*(-2*15^
(1 + n)*E^(((4*I)*a)/b)*((I*(a + b*ArcCos[c*x]))/b)^n*((a + b*ArcCos[c*x])
^2/b^2)^(2*n)*Gamma[1 + n, ((-I)*(a + b*ArcCos[c*x]))/b] + (((-I)*(a + b*A
rcCos[c*x]))/b)^n*(2*15^(1 + n)*E^(((6*I)*a)/b)*((a + b*ArcCos[c*x])^2/b^2
)^(2*n)*Gamma[1 + n, (I*(a + b*ArcCos[c*x]))/b] + 3*(5^(1 + n)*E^(((2*I)*a
)/b)*((I*(a + b*ArcCos[c*x]))/b)^(2*n)*((a + b*ArcCos[c*x])^2/b^2)^n*Gamma
[1 + n, ((-3*I)*(a + b*ArcCos[c*x]))/b] - 5^(1 + n)*E^(((8*I)*a)/b)*((a +
b*ArcCos[c*x])^2/b^2)^(2*n)*Gamma[1 + n, ((3*I)*(a + b*ArcCos[c*x]))/b] +
3^n*(-((((-I)*(a + b*ArcCos[c*x]))/b)^n*((I*(a + b*ArcCos[c*x]))/b)^(3*n)*
Gamma[1 + n, ((-5*I)*(a + b*ArcCos[c*x]))/b]) + E^(((10*I)*a)/b)*((a + b*A
rcCos[c*x])^2/b^2)^(2*n)*Gamma[1 + n, ((5*I)*(a + b*ArcCos[c*x]))/b])))))/
(c^2*E^(((5*I)*a)/b)*Sqrt[d - c^2*d*x^2]*((a + b*ArcCos[c*x])^2/b^2)^(3*n)
)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^n\,dx$$

$$\Big\downarrow \text{ 5225}$$

$$-\frac{d\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))^n\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)d(a+b\arccos(cx))}{bc^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 4906}$$

$$-\frac{d\sqrt{d-c^2dx^2}\int\left(\frac{1}{16}\cos\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)(a+b\arccos(cx))^n-\frac{3}{16}\cos\left(\frac{3a}{b}-\frac{3(a+b\arccos(cx))}{b}\right)(a+b\arccos

$$-\frac{d\sqrt{d-c^2dx^2}\left(-\frac{1}{16}ibe^{-\frac{ia}{b}}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,-\frac{i(a+b\arccos(cx))}{b}\right)+\frac{1}{32}ib3^{-n}e^{-\frac{3ia}{b}}(\cdots$$

input
```
Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^n,x]
```

output
```
-((d*Sqrt[d - c^2*d*x^2]*(((-1/16*I)*b*(a + b*ArcCos[c*x])^n*Gamma[1 + n,
((-I)*(a + b*ArcCos[c*x]))/b])/(E^((I*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)
^n) + ((I/16)*b*E^((I*a)/b)*(a + b*ArcCos[c*x])^n*Gamma[1 + n, (I*(a + b*A
rcCos[c*x]))/b])/((I*(a + b*ArcCos[c*x]))/b)^n + ((I/32)*b*(a + b*ArcCos[c
*x])^n*Gamma[1 + n, ((-3*I)*(a + b*ArcCos[c*x]))/b])/(3^n*E^(((3*I)*a)/b)*
(((-I)*(a + b*ArcCos[c*x]))/b)^n) - ((I/32)*b*E^(((3*I)*a)/b)*(a + b*ArcCo
s[c*x])^n*Gamma[1 + n, ((3*I)*(a + b*ArcCos[c*x]))/b])/(3^n*((I*(a + b*Arc
Cos[c*x]))/b)^n) - ((I/32)*5^(-1 - n)*b*(a + b*ArcCos[c*x])^n*Gamma[1 + n,
 ((-5*I)*(a + b*ArcCos[c*x]))/b])/(E^(((5*I)*a)/b)*(((-I)*(a + b*ArcCos[c*
x]))/b)^n) + ((I/32)*5^(-1 - n)*b*E^(((5*I)*a)/b)*(a + b*ArcCos[c*x])^n*Ga
mma[1 + n, ((5*I)*(a + b*ArcCos[c*x]))/b])/((I*(a + b*ArcCos[c*x]))/b)^n))
/(b*c^2*Sqrt[1 - c^2*x^2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x\left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \arccos \left(c x \right)\right)^{n}d x$$

input
```
int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n,x)
```

output
```
int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n,x)
```

Fricas [F]

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^n\,dx=\int\left(-c^{2} d x^{2}+d\right)^{\frac{3}{2}}\left(b \arccos\left(c x\right)+a\right)^{n} x\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n,x, algorithm="fricas"
)
```

output
```
integral(-(c^2*d*x^3 - d*x)*sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^n\,dx=\text{Timed out}$$

input
```
integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))^n\,dx=\int\left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\arccos\left(cx\right)+a)^nx\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n,x, algorithm="maxima"
)
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccos(c*x) + a)^n*x, x)
```

Giac [F(-2)]

Exception generated.

$$\int x(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))^n\,dx=\int x\,(a+b\,\mathrm{acos}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int(x*(a + b*acos(c*x))^n*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int(x*(a + b*acos(c*x))^n*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int x(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n\,dx=\sqrt{d}\,d\left(-\left(\int(acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^3dx\right)c^2+\int(acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,xdx\right)$$

input
```
int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))^n,x)
```

output
```
sqrt(d)*d*( - int((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**3,x)*c**2
 + int((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x,x))
```

3.491 $\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^n\,dx$

Optimal result

Integrand size = 26, antiderivative size = 466

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^n\,dx=\frac{3d\sqrt{d-c^2dx^2}(a+b\arccos(cx))^{1+n}}{8bc(1+n)\sqrt{1-c^2x^2}}$$
$$-\frac{i2^{-3-n}de^{-\frac{2ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2i(a+b\arccos(cx))}{b}\right)}{c\sqrt{1-c^2x^2}}$$
$$+\frac{i2^{-3-n}de^{\frac{2ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{2i(a+b\arccos(cx))}{b}\right)}{c\sqrt{1-c^2x^2}}$$
$$-\frac{i2^{-2(3+n)}de^{-\frac{4ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4i(a+b\arccos(cx))}{b}\right)}{c\sqrt{1-c^2x^2}}$$
$$+\frac{i2^{-2(3+n)}de^{\frac{4ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{4i(a+b\arccos(cx))}{b}\right)}{c\sqrt{1-c^2x^2}}$$

output
```
3/8*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^(1+n)/b/c/(1+n)/(-c^2*x^2+1)^
(1/2)-I*2^(-3-n)*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-2*I
*(a+b*arccos(c*x))/b)/c/exp(2*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arccos(c
*x))/b)^n)+I*2^(-3-n)*d*exp(2*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x)
)^n*GAMMA(1+n,2*I*(a+b*arccos(c*x))/b)/c/(-c^2*x^2+1)^(1/2)/((I*(a+b*arcco
s(c*x))/b)^n)-I*d*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-4*I*
(a+b*arccos(c*x))/b)/(2^(6+2*n))/c/exp(4*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a
+b*arccos(c*x))/b)^n)+I*d*exp(4*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*
x))^n*GAMMA(1+n,4*I*(a+b*arccos(c*x))/b)/(2^(6+2*n))/c/(-c^2*x^2+1)^(1/2)/
((I*(a+b*arccos(c*x))/b)^n)
```

Mathematica [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^n\,dx=\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^n\,dx$$

input
```
Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^n,x]
```

output
```
Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^n, x]
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^n\,dx$$

$$\Big\downarrow \text{ 5169}$$

$$-\frac{d\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))^n\sin^4\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)d(a+b\arccos(cx))}{bc\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))^n\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^4d(a+b\arccos(cx))}{bc\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{d\sqrt{d-c^2dx^2}\int \left(\frac{1}{8}\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)(a+b\arccos(cx))^n-\frac{1}{2}\cos\left(\frac{2a}{b}-\frac{2(a+b\arccos(cx))}{b}\right)(a+b\arccos(c\right.}{bc\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\sqrt{d-c^2dx^2}\left(\frac{3(a+b\arccos(cx))^{n+1}}{8(n+1)}+ib2^{-n-3}e^{-\frac{2ia}{b}}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2i(a+b\arcc}{b}\right.\right.}{}$$

input
```
Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^n,x]
```

output
```
-((d*Sqrt[d - c^2*d*x^2]*((3*(a + b*ArcCos[c*x])^(1 + n))/(8*(1 + n)) + (I
*2^(-3 - n)*b*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((-2*I)*(a + b*ArcCos[c*x
]))/b])/(E^(((2*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n) - (I*2^(-3 - n)
*b*E^(((2*I)*a)/b)*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((2*I)*(a + b*ArcCos
[c*x]))/b])/((I*(a + b*ArcCos[c*x]))/b)^n - (I*b*(a + b*ArcCos[c*x])^n*Gam
ma[1 + n, ((-4*I)*(a + b*ArcCos[c*x]))/b])/(2^(2*(3 + n))*E^(((4*I)*a)/b)*
(((-I)*(a + b*ArcCos[c*x]))/b)^n) + (I*b*E^(((4*I)*a)/b)*(a + b*ArcCos[c*x
])^n*Gamma[1 + n, ((4*I)*(a + b*ArcCos[c*x]))/b])/(2^(2*(3 + n))*((I*(a +
b*ArcCos[c*x]))/b)^n)))/(b*c*Sqrt[1 - c^2*x^2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [F]

$$\int \left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \arccos \left(c x \right)\right)^{n}d x$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n,x)
```

output
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n,x)
```

Fricas [F]

$$\int \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))^n \, dx=\int { {\left(-c^{2} d x^{2} + d\right)}^{\frac{3}{2}} {\left(b \arccos \left(c x\right) + a\right)}^{n} \,d x}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n,x, algorithm="fricas")
```

output
```
integral((-c^2*d*x^2 + d)^(3/2)*(b*arccos(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^n\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^n\,dx=\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\arccos(cx)+a)^n\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n,x, algorithm="maxima")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccos(c*x) + a)^n, x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^n\,dx=\int (a+b\,\mathrm{acos}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(3/2), x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))^n\,dx=\sqrt{d}\,d\left(-\left(\int (acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^2dx\right)c^2+\int (acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}dx\right)$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))^n,x)
```

output
```
sqrt(d)*d*( - int((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**2,x)*c**2
 + int((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1),x))
```

3.492 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x}\,dx = \frac{5d^2e^{-\frac{ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\Big(1}{8\sqrt{d-c^2dx^2}}$$

$$+\frac{5d^2e^{\frac{ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{i(a+b\arccos(cx))}{b}\right)}{8\sqrt{d-c^2dx^2}}$$

$$+\frac{3^{-1-n}d^2e^{-\frac{3ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3i(a+b\arccos(cx))}{b}\right)}{8\sqrt{d-c^2dx^2}}$$

$$+\frac{3^{-1-n}d^2e^{\frac{3ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{3i(a+b\arccos(cx))}{b}\right)}{8\sqrt{d-c^2dx^2}}$$

$$+d^2\mathrm{Int}\left(\frac{(a+b\arccos(cx))^n}{x\sqrt{d-c^2dx^2}},x\right)$$

output
```
5/8*d^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-I*(a+b*arccos(c*
x))/b)/exp(I*a/b)/(-c^2*d*x^2+d)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)+5/8*d^
2*exp(I*a/b)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,I*(a+b*arcco
s(c*x))/b)/(-c^2*d*x^2+d)^(1/2)/((I*(a+b*arccos(c*x))/b)^n)+1/8*3^(-1-n)*d
^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-3*I*(a+b*arccos(c*x))
/b)/exp(3*I*a/b)/(-c^2*d*x^2+d)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)+1/8*3^(
-1-n)*d^2*exp(3*I*a/b)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,3*
I*(a+b*arccos(c*x))/b)/(-c^2*d*x^2+d)^(1/2)/((I*(a+b*arccos(c*x))/b)^n)+d^
2*Defer(Int)((a+b*arccos(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x}\,dx=\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x}\,dx$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^n)/x,x]
```

output
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^n)/x, x]
```

Rubi [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x}\,dx$$

$$\downarrow \text{5227}$$

$$\int \left(-\frac{2c^2d^2x(a+b\arccos(cx))^n}{\sqrt{d-c^2dx^2}}+\frac{d^2(a+b\arccos(cx))^n}{x\sqrt{d-c^2dx^2}}+\frac{c^4d^2x^3(a+b\arccos(cx))^n}{\sqrt{d-c^2dx^2}}\right)dx$$

$$\downarrow \text{2009}$$

$$d^2\int \frac{(a+b\arccos(cx))^n}{x\sqrt{d-c^2dx^2}}dx-\frac{5id^2e^{-\frac{ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,-\frac{i(a+b\arccos(cx))}{b}\right)}{8\sqrt{d-c^2dx^2}}+\frac{id^23^{-n-1}e^{-\frac{3ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3i(a+b\arccos(cx))}{b}\right)}{8\sqrt{d-c^2dx^2}}+\frac{5id^2e^{\frac{ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,\frac{i(a+b\arccos(cx))}{b}\right)}{8\sqrt{d-c^2dx^2}}-\frac{id^23^{-n-1}e^{\frac{3ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,\frac{3i(a+b\arccos(cx))}{b}\right)}{8\sqrt{d-c^2dx^2}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^n)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}\left(a+b\arccos\left(cx\right)\right)^n}{x}dx$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n/x,x)
```

output
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)^n}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n/x,x, algorithm="fricas"
)
```

output
```
integral((-c^2*d*x^2 + d)^(3/2)*(b*arccos(c*x) + a)^n/x, x)
```

Sympy [N/A]

Not integrable

Time = 102.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x}\,dx=\int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\,\mathrm{acos}(cx))^n}{x}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))**n/x,x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))**n/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)^n}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n/x,x, algorithm="maxima"
)
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccos(c*x) + a)^n/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^n\,(d-c^2\,d\,x^2)^{3/2}}{x}\,dx$$

input
```
int(((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(3/2))/x,x)
```

output
```
int(((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(3/2))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.14

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x}\,dx = \sqrt{d}\,d\left(\int \frac{(acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}}{x}dx - \left(\int (acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,xdx\right)c^2\right)$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))^n/x,x)
```

output
```
sqrt(d)*d*(int(((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1))/x,x) - int((a
cos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x,x)*c**2)
```

3.493 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x^2}\,dx = -\frac{3cd^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^{1+n}}{2b(1+n)\sqrt{d-c^2dx^2}} + \frac{i2^{-3-n}cd^2e^{-\frac{2ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2i(a+b\arccos(cx))}{b}\right)}{\sqrt{d-c^2dx^2}} - \frac{i2^{-3-n}cd^2e^{\frac{2ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{2i(a+b\arccos(cx))}{b}\right)}{\sqrt{d-c^2dx^2}} + d^2\text{Int}\left(\frac{(a+b\arccos(cx))^n}{x^2\sqrt{d-c^2dx^2}},x\right)$$

output
```
-3/2*c*d^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^(1+n)/b/(1+n)/(-c^2*d*x^2+
d)^(1/2)+I*2^(-3-n)*c*d^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n
,-2*I*(a+b*arccos(c*x))/b)/exp(2*I*a/b)/(-c^2*d*x^2+d)^(1/2)/((-I*(a+b*arc
cos(c*x))/b)^n)-I*2^(-3-n)*c*d^2*exp(2*I*a/b)*(-c^2*x^2+1)^(1/2)*(a+b*arcc
os(c*x))^n*GAMMA(1+n,2*I*(a+b*arccos(c*x))/b)/(-c^2*d*x^2+d)^(1/2)/((I*(a+
b*arccos(c*x))/b)^n)+d^2*Defer(Int)((a+b*arccos(c*x))^n/x^2/(-c^2*d*x^2+d)
^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x^2}\,dx=\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x^2}\,dx$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^n)/x^2,x]
```

output
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^n)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x^2}\,dx$$

$$\Big\downarrow \text{5227}$$

$$\int \left(-\frac{2c^2d^2(a+b\arccos(cx))^n}{\sqrt{d-c^2dx^2}}+\frac{d^2(a+b\arccos(cx))^n}{x^2\sqrt{d-c^2dx^2}}+\frac{c^4d^2x^2(a+b\arccos(cx))^n}{\sqrt{d-c^2dx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$d^2\int \frac{(a+b\arccos(cx))^n}{x^2\sqrt{d-c^2dx^2}}dx+\frac{3cd^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^{n+1}}{2b(n+1)\sqrt{d-c^2dx^2}}+\frac{icd^22^{-n-3}e^{-\frac{2ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2i(a+b\arccos(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}-\frac{icd^22^{-n-3}e^{\frac{2ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,\frac{2i(a+b\arccos(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x])^n)/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}\left(a+b\arccos\left(cx\right)\right)^n}{x^2}dx$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n/x^2,x)
```

output
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{3/2}\,(a+b\arccos(cx))^n}{x^2}\,dx=\int \frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}\left(b\arccos\left(cx\right)+a\right)^n}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n/x^2,x, algorithm="frica
s")
```

output
```
integral((-c^2*d*x^2 + d)^(3/2)*(b*arccos(c*x) + a)^n/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))**n/x**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x^2}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)^n}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n/x^2,x, algorithm="maxim
a")
```

output
```
integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccos(c*x) + a)^n/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))^n/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^n\,(d-c^2\,d\,x^2)^{3/2}}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(3/2))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 5.76

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))^n}{x^2}\,dx = \frac{\sqrt{d}\,d\Big((acos(cx)\,b+a)^n\,acos(cx)\,bc+(acos(cx)\,b+a)^n\,ac+\Big(\int$$

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*acos(c*x))^n/x^2,x)
```

output
```
(sqrt(d)*d*((acos(c*x)*b + a)**n*acos(c*x)*b*c + (acos(c*x)*b + a)**n*a*c
+ int((acos(c*x)*b + a)**n/(sqrt( - c**2*x**2 + 1)*x**2),x)*b*n + int((aco
s(c*x)*b + a)**n/(sqrt( - c**2*x**2 + 1)*x**2),x)*b - int((acos(c*x)*b + a
)**n*sqrt( - c**2*x**2 + 1),x)*b*c**2*n - int((acos(c*x)*b + a)**n*sqrt( -
 c**2*x**2 + 1),x)*b*c**2))/(b*(n + 1))
```

3.494 $\int x^2(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n\,dx$

Optimal result

Integrand size = 29, antiderivative size = 906

$$\int x^2\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^n\,dx=\frac{5d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^{1+n}}{128bc^3(1+n)\sqrt{1-c^2x^2}}$$
$$-\frac{i2^{-7-n}d^2e^{-\frac{2ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$
$$+\frac{i2^{-7-n}d^2e^{\frac{2ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{2i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$
$$+\frac{i2^{-2(4+n)}d^2e^{-\frac{4ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$
$$-\frac{i2^{-2(4+n)}d^2e^{\frac{4ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{4i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$
$$+\frac{i2^{-7-n}3^{-1-n}d^2e^{-\frac{6ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{6i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$
$$-\frac{i2^{-7-n}3^{-1-n}d^2e^{\frac{6ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{6i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$
$$+\frac{i2^{-11-3n}d^2e^{-\frac{8ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{8i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$
$$-\frac{i2^{-11-3n}d^2e^{\frac{8ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{8i(a+b\arccos(cx))}{b}\right)}{c^3\sqrt{1-c^2x^2}}$$

output
```
5/128*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^(1+n)/b/c^3/(1+n)/(-c^2*x
^2+1)^(1/2)-I*2^(-7-n)*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(
1+n,-2*I*(a+b*arccos(c*x))/b)/c^3/exp(2*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+
b*arccos(c*x))/b)^n)+I*2^(-7-n)*d^2*exp(2*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b
*arccos(c*x))^n*GAMMA(1+n,2*I*(a+b*arccos(c*x))/b)/c^3/(-c^2*x^2+1)^(1/2)/
((I*(a+b*arccos(c*x))/b)^n)+I*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n
*GAMMA(1+n,-4*I*(a+b*arccos(c*x))/b)/(2^(8+2*n))/c^3/exp(4*I*a/b)/(-c^2*x^
2+1)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)-I*d^2*exp(4*I*a/b)*(-c^2*d*x^2+d)^
(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,4*I*(a+b*arccos(c*x))/b)/(2^(8+2*n))/c
^3/(-c^2*x^2+1)^(1/2)/((I*(a+b*arccos(c*x))/b)^n)+I*2^(-7-n)*3^(-1-n)*d^2*
(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-6*I*(a+b*arccos(c*x))/
b)/c^3/exp(6*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)-I*2^(-
7-n)*3^(-1-n)*d^2*exp(6*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GA
MMA(1+n,6*I*(a+b*arccos(c*x))/b)/c^3/(-c^2*x^2+1)^(1/2)/((I*(a+b*arccos(c*
x))/b)^n)+I*2^(-11-3*n)*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA
(1+n,-8*I*(a+b*arccos(c*x))/b)/c^3/exp(8*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a
+b*arccos(c*x))/b)^n)-I*2^(-11-3*n)*d^2*exp(8*I*a/b)*(-c^2*d*x^2+d)^(1/2)*
(a+b*arccos(c*x))^n*GAMMA(1+n,8*I*(a+b*arccos(c*x))/b)/c^3/(-c^2*x^2+1)^(1
/2)/((I*(a+b*arccos(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 3.09 (sec) , antiderivative size = 990, normalized size of antiderivative = 1.09

$$\int x^2 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos (c x))^n \, dx = \text{Too large to display}$$

input
```
Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^n,x]
```

output
```
-((2^(-11 - 3*n)*3^(-1 - n)*d^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^n*(5
*2^(4 + 3*n)*3^(1 + n)*a*E^(((8*I)*a)/b)*((a + b*ArcCos[c*x])^2/b^2)^n + 5
*2^(4 + 3*n)*3^(1 + n)*b*E^(((8*I)*a)/b)*ArcCos[c*x]*((a + b*ArcCos[c*x])^
2/b^2)^n + I*3^(1 + n)*4^(2 + n)*b*E^(((6*I)*a)/b)*(1 + n)*((I*(a + b*ArcC
os[c*x]))/b)^n*Gamma[1 + n, ((-2*I)*(a + b*ArcCos[c*x]))/b] - I*3^(1 + n)*
4^(2 + n)*b*E^(((10*I)*a)/b)*(1 + n)*(((-I)*(a + b*ArcCos[c*x]))/b)^n*Gamm
a[1 + n, ((2*I)*(a + b*ArcCos[c*x]))/b] + I*2^(3 + n)*3^(1 + n)*b*E^(((4*I
)*a)/b)*((I*(a + b*ArcCos[c*x]))/b)^n*Gamma[1 + n, ((-4*I)*(a + b*ArcCos[c
*x]))/b] + I*2^(3 + n)*3^(1 + n)*b*E^(((4*I)*a)/b)*n*((I*(a + b*ArcCos[c*x
]))/b)^n*Gamma[1 + n, ((-4*I)*(a + b*ArcCos[c*x]))/b] - I*2^(3 + n)*3^(1 +
 n)*b*E^(((12*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n*Gamma[1 + n, ((4*I
)*(a + b*ArcCos[c*x]))/b] - I*2^(3 + n)*3^(1 + n)*b*E^(((12*I)*a)/b)*n*(((
-I)*(a + b*ArcCos[c*x]))/b)^n*Gamma[1 + n, ((4*I)*(a + b*ArcCos[c*x]))/b]
- I*4^(2 + n)*b*E^(((2*I)*a)/b)*((I*(a + b*ArcCos[c*x]))/b)^n*Gamma[1 + n,
 ((-6*I)*(a + b*ArcCos[c*x]))/b] - I*4^(2 + n)*b*E^(((2*I)*a)/b)*n*((I*(a
+ b*ArcCos[c*x]))/b)^n*Gamma[1 + n, ((-6*I)*(a + b*ArcCos[c*x]))/b] + I*4^
(2 + n)*b*E^(((14*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n*Gamma[1 + n, (
(6*I)*(a + b*ArcCos[c*x]))/b] + I*4^(2 + n)*b*E^(((14*I)*a)/b)*n*(((-I)*(a
 + b*ArcCos[c*x]))/b)^n*Gamma[1 + n, ((6*I)*(a + b*ArcCos[c*x]))/b] + I*3^
(1 + n)*b*((I*(a + b*ArcCos[c*x]))/b)^n*Gamma[1 + n, ((-8*I)*(a + b*Arc...
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 636, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos (c x))^n \, dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{d^2 \sqrt{d-c^2 d x^2} \int (a+b \arccos (c x))^n \cos ^2\left(\frac{a}{b}-\frac{a+b \arccos (c x)}{b}\right) \sin ^6\left(\frac{a}{b}-\frac{a+b \arccos (c x)}{b}\right)d(a+b \arccos (c x))}{b c^3 \sqrt{1-c^2 x^2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{d^2\sqrt{d-c^2dx^2}\int\left(-\frac{1}{128}\cos\left(\frac{8a}{b}-\frac{8(a+b\arccos(cx))}{b}\right)(a+b\arccos(cx))^n+\frac{1}{32}\cos\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)(a+b\arc$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2\sqrt{d-c^2dx^2}\left(\frac{5(a+b\arccos(cx))^{n+1}}{128(n+1)}+ib2^{-n-7}e^{-\frac{2ia}{b}}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2i(a+b\arc}{b}\right.\right.}{}$$

input
```
Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^n,x]
```

output
```
-((d^2*Sqrt[d - c^2*d*x^2]*((5*(a + b*ArcCos[c*x])^(1 + n))/(128*(1 + n))
+ (I*2^(-7 - n)*b*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((-2*I)*(a + b*ArcCos
[c*x]))/b])/(E^(((2*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n) - (I*2^(-7
- n)*b*E^(((2*I)*a)/b)*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((2*I)*(a + b*Ar
cCos[c*x]))/b])/((I*(a + b*ArcCos[c*x]))/b)^n + (I*b*(a + b*ArcCos[c*x])^n
*Gamma[1 + n, ((-4*I)*(a + b*ArcCos[c*x]))/b])/(2^(2*(4 + n))*E^(((4*I)*a)
/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n) - (I*b*E^(((4*I)*a)/b)*(a + b*ArcCos
[c*x])^n*Gamma[1 + n, ((4*I)*(a + b*ArcCos[c*x]))/b])/(2^(2*(4 + n))*((I*(
a + b*ArcCos[c*x]))/b)^n) - (I*2^(-7 - n)*3^(-1 - n)*b*(a + b*ArcCos[c*x])
^n*Gamma[1 + n, ((-6*I)*(a + b*ArcCos[c*x]))/b])/(E^(((6*I)*a)/b)*(((-I)*(
a + b*ArcCos[c*x]))/b)^n) + (I*2^(-7 - n)*3^(-1 - n)*b*E^(((6*I)*a)/b)*(a
+ b*ArcCos[c*x])^n*Gamma[1 + n, ((6*I)*(a + b*ArcCos[c*x]))/b])/((I*(a + b
*ArcCos[c*x]))/b)^n + (I*2^(-11 - 3*n)*b*(a + b*ArcCos[c*x])^n*Gamma[1 + n
, ((-8*I)*(a + b*ArcCos[c*x]))/b])/(E^(((8*I)*a)/b)*(((-I)*(a + b*ArcCos[c
*x]))/b)^n) - (I*2^(-11 - 3*n)*b*E^(((8*I)*a)/b)*(a + b*ArcCos[c*x])^n*Gam
ma[1 + n, ((8*I)*(a + b*ArcCos[c*x]))/b])/((I*(a + b*ArcCos[c*x]))/b)^n))/
(b*c^3*Sqrt[1 - c^2*x^2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x^2 \left(-c^2 d\,x^2+d\right)^{\frac{5}{2}} \left(a+b \arccos \left(c x\right)\right)^{n}dx$$

input
```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n,x)
```

output
```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n,x)
```

Fricas [F]

$$\int x^2 \left(d-c^2 d x^2\right)^{5/2} (a+b \arccos(cx))^n \, dx = \int \left(-c^2 d x^2+d\right)^{\frac{5}{2}} (b \arccos \left(c x\right)+a)^{n} x^{2} \,dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n,x, algorithm="frica
s")
```

output
```
integral((c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2)*sqrt(-c^2*d*x^2 + d)*(b*a
rccos(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 (d - c^2 d x^2)^{5/2} (a + b \arccos(cx))^n \, dx = \text{Timed out}$$

input
```
integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 (d - c^2 d x^2)^{5/2} (a + b \arccos(cx))^n \, dx = \int (-c^2 d x^2 + d)^{\frac{5}{2}} (b \arccos(cx) + a)^n x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n,x, algorithm="maxim
a")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccos(c*x) + a)^n*x^2, x)
```

Giac [F]

$$\int x^2 (d - c^2 d x^2)^{5/2} (a + b \arccos(cx))^n \, dx = \int (-c^2 d x^2 + d)^{\frac{5}{2}} (b \arccos(cx) + a)^n x^2 \, dx$$

input
```
integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n,x, algorithm="giac"
)
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccos(c*x) + a)^n*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b \arccos(cx))^n \, dx = \int x^2 \, (a + b \, \mathrm{acos}(c\, x))^n \, (d - c^2 \, d \, x^2)^{5/2} \, dx$$

input
```
int(x^2*(a + b*acos(c*x))^n*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x^2*(a + b*acos(c*x))^n*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int x^2 (d - c^2 dx^2)^{5/2} (a + b \arccos(cx))^n \, dx = \sqrt{d}\, d^2 \left(\left(\int (acos(cx)\, b + a)^n \sqrt{-c^2x^2 + 1}\, x^6 dx \right) c^4 - 2 \left(\int (acos(cx)\, b + a)^n \sqrt{-c^2x^2 + 1}\, x^4 dx \right) c^2 + \int (acos(cx)\, b + a)^n \sqrt{-c^2x^2 + 1}\, x^2 dx \right)$$

input
```
int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))^n,x)
```

output
```
sqrt(d)*d**2*(int((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**6,x)*c**4
 - 2*int((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**4,x)*c**2 + int((a
cos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**2,x))
```

3.495 $\int x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n\,dx$

Optimal result

Integrand size = 27, antiderivative size = 815

$$\int x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n\,dx =$$

$$-\frac{5d^2e^{-\frac{ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{i(a+b\arccos(cx))}{b}\right)}{128c^2\sqrt{1-c^2x^2}}$$

$$-\frac{5d^2e^{\frac{ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{i(a+b\arccos(cx))}{b}\right)}{128c^2\sqrt{1-c^2x^2}}$$

$$-\frac{3^{1-n}d^2e^{-\frac{3ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3i(a+b\arccos(cx))}{b}\right)}{128c^2\sqrt{1-c^2x^2}}$$

$$-\frac{3^{1-n}d^2e^{\frac{3ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{3i(a+b\arccos(cx))}{b}\right)}{128c^2\sqrt{1-c^2x^2}}$$

$$-\frac{5^{-n}d^2e^{-\frac{5ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5i(a+b\arccos(cx))}{b}\right)}{128c^2\sqrt{1-c^2x^2}}$$

$$-\frac{5^{-n}d^2e^{\frac{5ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{5i(a+b\arccos(cx))}{b}\right)}{128c^2\sqrt{1-c^2x^2}}$$

$$-\frac{7^{-1-n}d^2e^{-\frac{7ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{7i(a+b\arccos(cx))}{b}\right)}{128c^2\sqrt{1-c^2x^2}}$$

$$-\frac{7^{-1-n}d^2e^{\frac{7ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{7i(a+b\arccos(cx))}{b}\right)}{128c^2\sqrt{1-c^2x^2}}$$

output
```
-5/128*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-I*(a+b*arcc
os(c*x))/b)/c^2/exp(I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)
-5/128*d^2*exp(I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,I
*(a+b*arccos(c*x))/b)/c^2/(-c^2*x^2+1)^(1/2)/((I*(a+b*arccos(c*x))/b)^n)-1
/128*3^(1-n)*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-3*I*(
a+b*arccos(c*x))/b)/c^2/exp(3*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arccos(c
*x))/b)^n)-1/128*3^(1-n)*d^2*exp(3*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos
(c*x))^n*GAMMA(1+n,3*I*(a+b*arccos(c*x))/b)/c^2/(-c^2*x^2+1)^(1/2)/((I*(a+
b*arccos(c*x))/b)^n)-1/128*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GA
MMA(1+n,-5*I*(a+b*arccos(c*x))/b)/(5^n)/c^2/exp(5*I*a/b)/(-c^2*x^2+1)^(1/2
)/((-I*(a+b*arccos(c*x))/b)^n)-1/128*d^2*exp(5*I*a/b)*(-c^2*d*x^2+d)^(1/2)
*(a+b*arccos(c*x))^n*GAMMA(1+n,5*I*(a+b*arccos(c*x))/b)/(5^n)/c^2/(-c^2*x^
2+1)^(1/2)/((I*(a+b*arccos(c*x))/b)^n)-1/128*7^(-1-n)*d^2*(-c^2*d*x^2+d)^(
1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-7*I*(a+b*arccos(c*x))/b)/c^2/exp(7*I*a
/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)-1/128*7^(-1-n)*d^2*exp
(7*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,7*I*(a+b*arcc
os(c*x))/b)/c^2/(-c^2*x^2+1)^(1/2)/((I*(a+b*arccos(c*x))/b)^n)
```

Mathematica [A] (warning: unable to verify)

Time = 2.65 (sec) , antiderivative size = 608, normalized size of antiderivative = 0.75

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^n\,dx=\frac{i5^{-n}21^{-1-n}d^3e^{-\frac{7ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{(a+b\arccos(cx))^2}{b^2}\right)^{-3n}\left(105^{1+n}e^{\frac{6ia}{b}}\left(\frac{i(a}{}\right.\right.}{}$$

input
```
Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^n,x]
```

output
```
((I/128)*21^(-1 - n)*d^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^n*(105^(1 +
 n)*E^(((6*I)*a)/b)*((I*(a + b*ArcCos[c*x]))/b)^n*((a + b*ArcCos[c*x])^2/b
^2)^(2*n)*Gamma[1 + n, ((-I)*(a + b*ArcCos[c*x]))/b] + (((-I)*(a + b*ArcCo
s[c*x]))/b)^n*(-(105^(1 + n)*E^(((8*I)*a)/b)*((a + b*ArcCos[c*x])^2/b^2)^(
2*n)*Gamma[1 + n, (I*(a + b*ArcCos[c*x]))/b]) - 9*5^n*7^(1 + n)*E^(((4*I)*
a)/b)*((I*(a + b*ArcCos[c*x]))/b)^(2*n)*((a + b*ArcCos[c*x])^2/b^2)^n*Gamm
a[1 + n, ((-3*I)*(a + b*ArcCos[c*x]))/b] + 9*5^n*7^(1 + n)*E^(((10*I)*a)/b
)*((a + b*ArcCos[c*x])^2/b^2)^(2*n)*Gamma[1 + n, ((3*I)*(a + b*ArcCos[c*x]
))/b] + 3^(1 + n)*(7^(1 + n)*E^(((2*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b
)^n*((I*(a + b*ArcCos[c*x]))/b)^(3*n)*Gamma[1 + n, ((-5*I)*(a + b*ArcCos[c
*x]))/b] - 7^(1 + n)*E^(((12*I)*a)/b)*((a + b*ArcCos[c*x])^2/b^2)^(2*n)*Ga
mma[1 + n, ((5*I)*(a + b*ArcCos[c*x]))/b] + 5^n*(-((((-I)*(a + b*ArcCos[c*
x]))/b)^n*((I*(a + b*ArcCos[c*x]))/b)^(3*n)*Gamma[1 + n, ((-7*I)*(a + b*Ar
cCos[c*x]))/b]) + E^(((14*I)*a)/b)*((a + b*ArcCos[c*x])^2/b^2)^(2*n)*Gamma
[1 + n, ((7*I)*(a + b*ArcCos[c*x]))/b])))))/(5^n*c^2*E^(((7*I)*a)/b)*Sqrt[
d - c^2*d*x^2]*((a + b*ArcCos[c*x])^2/b^2)^(3*n))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 599, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5225, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^n\,dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{d^2\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))^n\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)\sin^6\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)d(a+b\arccos(cx))}{bc^2\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{d^2\sqrt{d-c^2dx^2}\int \left(-\frac{1}{64}\cos\left(\frac{7a}{b}-\frac{7(a+b\arccos(cx))}{b}\right)(a+b\arccos(cx))^n+\frac{5}{64}\cos\left(\frac{5a}{b}-\frac{5(a+b\arccos(cx))}{b}\right)(a+b\arccos(cx)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2\sqrt{d-c^2dx^2}\left(-\frac{5}{128}ibe^{-\frac{ia}{b}}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,-\frac{i(a+b\arccos(cx))}{b}\right)+\frac{1}{128}ib3^{1-n}e^{-}\right.}{}$$

input
```
Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^n,x]
```

output
```
-((d^2*Sqrt[d - c^2*d*x^2]*((((-5*I)/128)*b*(a + b*ArcCos[c*x])^n*Gamma[1
+ n, ((-I)*(a + b*ArcCos[c*x]))/b])/(E^((I*a)/b)*(((-I)*(a + b*ArcCos[c*x]
))/b)^n) + (((5*I)/128)*b*E^((I*a)/b)*(a + b*ArcCos[c*x])^n*Gamma[1 + n, (
I*(a + b*ArcCos[c*x]))/b])/((I*(a + b*ArcCos[c*x]))/b)^n + ((I/128)*3^(1 -
 n)*b*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((-3*I)*(a + b*ArcCos[c*x]))/b])/
(E^(((3*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n) - ((I/128)*3^(1 - n)*b*
E^(((3*I)*a)/b)*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((3*I)*(a + b*ArcCos[c*
x]))/b])/((I*(a + b*ArcCos[c*x]))/b)^n - ((I/128)*b*(a + b*ArcCos[c*x])^n*
Gamma[1 + n, ((-5*I)*(a + b*ArcCos[c*x]))/b])/(5^n*E^(((5*I)*a)/b)*(((-I)*
(a + b*ArcCos[c*x]))/b)^n) + ((I/128)*b*E^(((5*I)*a)/b)*(a + b*ArcCos[c*x]
)^n*Gamma[1 + n, ((5*I)*(a + b*ArcCos[c*x]))/b])/(5^n*((I*(a + b*ArcCos[c*
x]))/b)^n) + ((I/128)*7^(-1 - n)*b*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((-7
*I)*(a + b*ArcCos[c*x]))/b])/(E^(((7*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/
b)^n) - ((I/128)*7^(-1 - n)*b*E^(((7*I)*a)/b)*(a + b*ArcCos[c*x])^n*Gamma[
1 + n, ((7*I)*(a + b*ArcCos[c*x]))/b])/((I*(a + b*ArcCos[c*x]))/b)^n))/(b*
c^2*Sqrt[1 - c^2*x^2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int x\left(-c^{2}d\,x^{2}+d\right)^{\frac{5}{2}}\left(a+b\arccos\left(cx\right)\right)^{n}dx$$

input
```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n,x)
```

output
```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n,x)
```

Fricas [F]

$$\int x\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^n\,dx=\int\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}\left(b\arccos\left(cx\right)+a\right)^{n}x\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n,x, algorithm="fricas"
)
```

output
```
integral((c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x)*sqrt(-c^2*d*x^2 + d)*(b*arc
cos(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n\,dx=\text{Timed out}$$

input
```
integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n\,dx=\int(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)^nx\,dx$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n,x, algorithm="maxima"
)
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccos(c*x) + a)^n*x, x)
```

Giac [F(-2)]

Exception generated.

$$\int x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n\,dx=\int x\,(a+b\,\mathrm{acos}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int(x*(a + b*acos(c*x))^n*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int(x*(a + b*acos(c*x))^n*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\begin{aligned}&\int x(d-c^2dx^2)^{5/2}(a\\&+b\arccos(cx))^n\,dx=\sqrt{d}\,d^2\left(\left(\int(acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^5dx\right)c^4\right.\\&-2\left(\int(acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^3dx\right)c^2\\&\left.+\int(acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,xdx\right)\end{aligned}$$

input
```
int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))^n,x)
```

output
```
sqrt(d)*d**2*(int((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**5,x)*c**4
 - 2*int((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**3,x)*c**2 + int((a
cos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x,x))
```

3.496 $\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^n\,dx$

Optimal result

Integrand size = 26, antiderivative size = 698

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^n\,dx=\frac{5d^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^{1+n}}{16bc(1+n)\sqrt{1-c^2x^2}}$$
$$-\frac{15i2^{-7-n}d^2e^{-\frac{2ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2i(a+b\arccos(cx))}{b}\right)}{c\sqrt{1-c^2x^2}}$$
$$+\frac{15i2^{-7-n}d^2e^{\frac{2ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{2i(a+b\arccos(cx))}{b}\right)}{c\sqrt{1-c^2x^2}}$$
$$-\frac{3i2^{-7-2n}d^2e^{-\frac{4ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4i(a+b\arccos(cx))}{b}\right)}{c\sqrt{1-c^2x^2}}$$
$$+\frac{3i2^{-7-2n}d^2e^{\frac{4ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{4i(a+b\arccos(cx))}{b}\right)}{c\sqrt{1-c^2x^2}}$$
$$-\frac{i2^{-7-n}3^{-1-n}d^2e^{-\frac{6ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{6i(a+b\arccos(cx))}{b}\right)}{c\sqrt{1-c^2x^2}}$$
$$+\frac{i2^{-7-n}3^{-1-n}d^2e^{\frac{6ia}{b}}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{6i(a+b\arccos(cx))}{b}\right)}{c\sqrt{1-c^2x^2}}$$

output
```
5/16*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^(1+n)/b/c/(1+n)/(-c^2*x^2+
1)^(1/2)-15*I*2^(-7-n)*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(
1+n,-2*I*(a+b*arccos(c*x))/b)/c/exp(2*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*
arccos(c*x))/b)^n)+15*I*2^(-7-n)*d^2*exp(2*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+
b*arccos(c*x))^n*GAMMA(1+n,2*I*(a+b*arccos(c*x))/b)/c/(-c^2*x^2+1)^(1/2)/(
(I*(a+b*arccos(c*x))/b)^n)-3*I*2^(-7-2*n)*d^2*(-c^2*d*x^2+d)^(1/2)*(a+b*ar
ccos(c*x))^n*GAMMA(1+n,-4*I*(a+b*arccos(c*x))/b)/c/exp(4*I*a/b)/(-c^2*x^2+
1)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)+3*I*2^(-7-2*n)*d^2*exp(4*I*a/b)*(-c^
2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,4*I*(a+b*arccos(c*x))/b)/c/
(-c^2*x^2+1)^(1/2)/((I*(a+b*arccos(c*x))/b)^n)-I*2^(-7-n)*3^(-1-n)*d^2*(-c
^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-6*I*(a+b*arccos(c*x))/b)/
c/exp(6*I*a/b)/(-c^2*x^2+1)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)+I*2^(-7-n)*
3^(-1-n)*d^2*exp(6*I*a/b)*(-c^2*d*x^2+d)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1
+n,6*I*(a+b*arccos(c*x))/b)/c/(-c^2*x^2+1)^(1/2)/((I*(a+b*arccos(c*x))/b)^
n)
```

Mathematica [F]

$$\int (d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n\,dx=\int (d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n\,dx$$

input
```
Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^n,x]
```

output
```
Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^n, x]
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 496, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5169, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^n\,dx$$

$$\Big\downarrow \text{5169}$$

$$-\frac{d^2\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))^n\sin^6\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)d(a+b\arccos(cx))}{bc\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\sqrt{d-c^2dx^2}\int (a+b\arccos(cx))^n\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)^6d(a+b\arccos(cx))}{bc\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{d^2\sqrt{d-c^2dx^2}\int \left(-\frac{1}{32}\cos\left(\frac{6a}{b}-\frac{6(a+b\arccos(cx))}{b}\right)(a+b\arccos(cx))^n+\frac{3}{16}\cos\left(\frac{4a}{b}-\frac{4(a+b\arccos(cx))}{b}\right)(a+b\operatorname{arcc}\right.}{bc\sqrt{1-c}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2\sqrt{d-c^2dx^2}\left(\frac{5(a+b\arccos(cx))^{n+1}}{16(n+1)}+15ib2^{-n-7}e^{-\frac{2ia}{b}}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2i(a+b}{}\right.\right.}{}$$

input
```
Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^n,x]
```

output
```
-((d^2*Sqrt[d - c^2*d*x^2]*((5*(a + b*ArcCos[c*x])^(1 + n))/(16*(1 + n)) +
 ((15*I)*2^(-7 - n)*b*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((-2*I)*(a + b*Ar
cCos[c*x]))/b])/(E^(((2*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n) - ((15*
I)*2^(-7 - n)*b*E^(((2*I)*a)/b)*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((2*I)*
(a + b*ArcCos[c*x]))/b])/((I*(a + b*ArcCos[c*x]))/b)^n - ((3*I)*2^(-7 - 2*
n)*b*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((-4*I)*(a + b*ArcCos[c*x]))/b])/(
E^(((4*I)*a)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n) + ((3*I)*2^(-7 - 2*n)*b*
E^(((4*I)*a)/b)*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((4*I)*(a + b*ArcCos[c*
x]))/b])/((I*(a + b*ArcCos[c*x]))/b)^n + (I*2^(-7 - n)*3^(-1 - n)*b*(a + b
*ArcCos[c*x])^n*Gamma[1 + n, ((-6*I)*(a + b*ArcCos[c*x]))/b])/(E^(((6*I)*a
)/b)*(((-I)*(a + b*ArcCos[c*x]))/b)^n) - (I*2^(-7 - n)*3^(-1 - n)*b*E^(((6
*I)*a)/b)*(a + b*ArcCos[c*x])^n*Gamma[1 + n, ((6*I)*(a + b*ArcCos[c*x]))/b
])/((I*(a + b*ArcCos[c*x]))/b)^n))/(b*c*Sqrt[1 - c^2*x^2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5169
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(-(b*c)^(-1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Subst[
Int[x^n*Sin[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{
a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && IGtQ[2*p, 0]
```

Maple [F]

$$\int \left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}} \left(a +b \arccos \left(c x \right)\right)^{n}d x$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n,x)
```

output
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n,x)
```

Fricas [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^n\,dx=\int \left(-c^{2} d x^{2}+d\right)^{\frac{5}{2}} (b \arccos \left(c x\right)+a)^{n}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n,x, algorithm="fricas")
```

output
```
integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arcco
s(c*x) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^n\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^n\,dx=\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\arccos(cx)+a)^n\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n,x, algorithm="maxima")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccos(c*x) + a)^n, x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^n\,dx=\int (a+b\,\mathrm{acos}(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(5/2), x)
```

Reduce [F]

$$\int \left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^n\,dx=\sqrt{d}\,d^2\left(\left(\int (acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^4dx\right)c^4-2\left(\int (acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^2dx\right)c^2+\int (acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}dx\right)$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))^n,x)
```

output
```
sqrt(d)*d**2*(int((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**4,x)*c**4
 - 2*int((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**2,x)*c**2 + int((a
cos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1),x))
```

3.497 $\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^n}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n}{x}\,dx = \frac{11d^3e^{-\frac{ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1\right.}{16\sqrt{d-c^2dx^2}}$$
$$+\frac{11d^3e^{\frac{ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{i(a+b\arccos(cx))}{b}\right)}{16\sqrt{d-c^2dx^2}}$$
$$-\frac{5\ 3^{-1-n}d^3e^{-\frac{3ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3i(a+b\arccos(cx))}{b}\right)}{32\sqrt{d-c^2dx^2}}$$
$$+\frac{3^{-n}d^3e^{-\frac{3ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3i(a+b\arccos(cx))}{b}\right)}{8\sqrt{d-c^2dx^2}}$$
$$-\frac{5\ 3^{-1-n}d^3e^{\frac{3ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{3i(a+b\arccos(cx))}{b}\right)}{32\sqrt{d-c^2dx^2}}$$
$$+\frac{3^{-n}d^3e^{\frac{3ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{3i(a+b\arccos(cx))}{b}\right)}{8\sqrt{d-c^2dx^2}}$$
$$+\frac{5^{-1-n}d^3e^{-\frac{5ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{5i(a+b\arccos(cx))}{b}\right)}{32\sqrt{d-c^2dx^2}}$$
$$+\frac{5^{-1-n}d^3e^{\frac{5ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{5i(a+b\arccos(cx))}{b}\right)}{32\sqrt{d-c^2dx^2}}$$
$$+d^3\text{Int}\left(\frac{(a+b\arccos(cx))^n}{x\sqrt{d-c^2dx^2}},x\right)$$

output
```
11/16*d^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-I*(a+b*arccos(
c*x))/b)/exp(I*a/b)/(-c^2*d*x^2+d)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)+11/1
6*d^3*exp(I*a/b)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,I*(a+b*a
rccos(c*x))/b)/(-c^2*d*x^2+d)^(1/2)/((I*(a+b*arccos(c*x))/b)^n)-5/32*3^(-1
-n)*d^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-3*I*(a+b*arccos(
c*x))/b)/exp(3*I*a/b)/(-c^2*d*x^2+d)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n)+1/
8*d^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+n,-3*I*(a+b*arccos(c*
x))/b)/(3^n)/exp(3*I*a/b)/(-c^2*d*x^2+d)^(1/2)/((-I*(a+b*arccos(c*x))/b)^n
)-5/32*3^(-1-n)*d^3*exp(3*I*a/b)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GA
MMA(1+n,3*I*(a+b*arccos(c*x))/b)/(-c^2*d*x^2+d)^(1/2)/((I*(a+b*arccos(c*x)
)/b)^n)+1/8*d^3*exp(3*I*a/b)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(
1+n,3*I*(a+b*arccos(c*x))/b)/(3^n)/(-c^2*d*x^2+d)^(1/2)/((I*(a+b*arccos(c*
x))/b)^n)+1/32*5^(-1-n)*d^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1
+n,-5*I*(a+b*arccos(c*x))/b)/exp(5*I*a/b)/(-c^2*d*x^2+d)^(1/2)/((-I*(a+b*a
rccos(c*x))/b)^n)+1/32*5^(-1-n)*d^3*exp(5*I*a/b)*(-c^2*x^2+1)^(1/2)*(a+b*a
rccos(c*x))^n*GAMMA(1+n,5*I*(a+b*arccos(c*x))/b)/(-c^2*d*x^2+d)^(1/2)/((I*
(a+b*arccos(c*x))/b)^n)+d^3*Defer(Int)((a+b*arccos(c*x))^n/x/(-c^2*d*x^2+d
)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))^n}{x}\,dx=\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))^n}{x}\,dx$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^n)/x,x]
```

output
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^n)/x, x]
```

Rubi [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))^n}{x}\,dx$$

$$\Big\downarrow \text{5227}$$

$$\int \left(-\frac{3c^2d^3x(a+b\arccos(cx))^n}{\sqrt{d-c^2dx^2}}+\frac{d^3(a+b\arccos(cx))^n}{x\sqrt{d-c^2dx^2}}-\frac{c^6d^3x^5(a+b\arccos(cx))^n}{\sqrt{d-c^2dx^2}}+\frac{3c^4d^3x^3(a+b\arccos(c}{\sqrt{d-c^2dx^2}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{11id^3e^{-\frac{ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\Gamma\left(n+1,-\frac{i(a+b\arccos(cx))}{b}\right)\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}}{16\sqrt{d-c^2dx^2}}-\frac{5i3^{-n-1}d^3e^{-\frac{3ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\Gamma\left(n+1,-\frac{3i(a+b\arccos(cx))}{b}\right)\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}}{32\sqrt{d-c^2dx^2}}+\frac{i3^{-n}d^3e^{-\frac{3ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\Gamma\left(n+1,-\frac{3i(a+b\arccos(cx))}{b}\right)\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}}{8\sqrt{d-c^2dx^2}}-\frac{i5^{-n-1}d^3e^{-\frac{5ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\Gamma\left(n+1,-\frac{5i(a+b\arccos(cx))}{b}\right)\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}}{32\sqrt{d-c^2dx^2}}+\frac{11id^3e^{\frac{ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,\frac{i(a+b\arccos(cx))}{b}\right)}{16\sqrt{d-c^2dx^2}}+\frac{5i3^{-n-1}d^3e^{\frac{3ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,\frac{3i(a+b\arccos(cx))}{b}\right)}{32\sqrt{d-c^2dx^2}}-\frac{i3^{-n}d^3e^{\frac{3ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,\frac{3i(a+b\arccos(cx))}{b}\right)}{8\sqrt{d-c^2dx^2}}+\frac{i5^{-n-1}d^3e^{\frac{5ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,\frac{5i(a+b\arccos(cx))}{b}\right)}{32\sqrt{d-c^2dx^2}}+d^3\int \frac{(a+b\arccos(cx))^n}{x\sqrt{d-c^2dx^2}}dx$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^n)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}} \left(a+b \arccos \left(c x\right)\right)^{n}}{x}dx$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n/x,x)
```

output
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))^n}{x}\,dx=\int \frac{\left(-c^{2} d x^{2}+d\right)^{\frac{5}{2}} {\left(b \arccos \left(c x\right)+a\right)}^{n}}{x}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n/x,x, algorithm="fricas"
)
```

output
```
integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arcco
s(c*x) + a)^n/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d - c^2dx^2)^{5/2} (a + b\arccos(cx))^n}{x} \, dx = \text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))**n/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d - c^2dx^2)^{5/2} (a + b\arccos(cx))^n}{x} \, dx = \int \frac{(-c^2dx^2 + d)^{\frac{5}{2}} (b\arccos(cx) + a)^n}{x} \, dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n/x,x, algorithm="maxima"
)
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccos(c*x) + a)^n/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d - c^2dx^2)^{5/2} (a + b\arccos(cx))^n}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n}{x}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^n\,(d-c^2\,d\,x^2)^{5/2}}{x}\,dx$$

input
```
int(((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(5/2))/x,x)
```

output
```
int(((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(5/2))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.28

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n}{x}\,dx=\sqrt{d}\,d^2\left(\int \frac{(acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}}{x}dx+\left(\int (acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,x^3dx\right)c^4-2\left(\int (acos(cx)\,b+a)^n\sqrt{-c^2x^2+1}\,xdx\right)c^2\right)$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))^n/x,x)
```

output
```
sqrt(d)*d**2*(int(((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1))/x,x) + int
((acos(c*x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x**3,x)*c**4 - 2*int((acos(c*
x)*b + a)**n*sqrt( - c**2*x**2 + 1)*x,x)*c**2)
```

3.498 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\begin{aligned}\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n}{x^2}\,dx &= -\frac{15cd^3\sqrt{1-c^2x^2}(a+b\arccos(cx))^{1+n}}{8b(1+n)\sqrt{d-c^2dx^2}}\\
&+\frac{i2^{-2-n}cd^3e^{-\frac{2ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2i(a+b\arccos(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}\\
&-\frac{i2^{-2-n}cd^3e^{\frac{2ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{2i(a+b\arccos(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}\\
&+\frac{i2^{-2(3+n)}cd^3e^{-\frac{4ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,-\frac{4i(a+b\arccos(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}\\
&-\frac{i2^{-2(3+n)}cd^3e^{\frac{4ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(1+n,\frac{4i(a+b\arccos(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}\\
&+d^3\text{Int}\left(\frac{(a+b\arccos(cx))^n}{x^2\sqrt{d-c^2dx^2}},x\right)\end{aligned}$$

output
```
-15/8*c*d^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^(1+n)/b/(1+n)/(-c^2*d*x^2
+d)^(1/2)+I*2^(-2-n)*c*d^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GAMMA(1+
n,-2*I*(a+b*arccos(c*x))/b)/exp(2*I*a/b)/(-c^2*d*x^2+d)^(1/2)/((-I*(a+b*ar
ccos(c*x))/b)^n)-I*2^(-2-n)*c*d^3*exp(2*I*a/b)*(-c^2*x^2+1)^(1/2)*(a+b*arc
cos(c*x))^n*GAMMA(1+n,2*I*(a+b*arccos(c*x))/b)/(-c^2*d*x^2+d)^(1/2)/((I*(a
+b*arccos(c*x))/b)^n)+I*c*d^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^n*GAMMA
(1+n,-4*I*(a+b*arccos(c*x))/b)/(2^(6+2*n))/exp(4*I*a/b)/(-c^2*d*x^2+d)^(1/
2)/((-I*(a+b*arccos(c*x))/b)^n)-I*c*d^3*exp(4*I*a/b)*(-c^2*x^2+1)^(1/2)*(a
+b*arccos(c*x))^n*GAMMA(1+n,4*I*(a+b*arccos(c*x))/b)/(2^(6+2*n))/(-c^2*d*x
^2+d)^(1/2)/((I*(a+b*arccos(c*x))/b)^n)+d^3*Defer(Int)((a+b*arccos(c*x))^n
/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))^n}{x^2}\,dx=\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))^n}{x^2}\,dx$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^n)/x^2,x]
```

output
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^n)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n}{x^2}\,dx$$
$$\Big\downarrow \text{5227}$$
$$\int \left(-\frac{3c^2d^3(a+b\arccos(cx))^n}{\sqrt{d-c^2dx^2}}+\frac{d^3(a+b\arccos(cx))^n}{x^2\sqrt{d-c^2dx^2}}-\frac{c^6d^3x^4(a+b\arccos(cx))^n}{\sqrt{d-c^2dx^2}}+\frac{3c^4d^3x^2(a+b\arccos(cx}{\sqrt{d-c^2dx^2}}\right.$$
$$\Big\downarrow \text{2009}$$
$$d^3\int \frac{(a+b\arccos(cx))^n}{x^2\sqrt{d-c^2dx^2}}dx+\frac{15cd^3\sqrt{1-c^2x^2}(a+b\arccos(cx))^{n+1}}{8b(n+1)\sqrt{d-c^2dx^2}}+$$
$$\frac{icd^32^{-n-2}e^{-\frac{2ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2i(a+b\arccos(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}-$$
$$\frac{icd^32^{-2(n+3)}e^{-\frac{4ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(-\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,-\frac{4i(a+b\arccos(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}-$$
$$\frac{icd^32^{-n-2}e^{\frac{2ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,\frac{2i(a+b\arccos(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}+$$
$$\frac{icd^32^{-2(n+3)}e^{\frac{4ia}{b}}\sqrt{1-c^2x^2}(a+b\arccos(cx))^n\left(\frac{i(a+b\arccos(cx))}{b}\right)^{-n}\Gamma\left(n+1,\frac{4i(a+b\arccos(cx))}{b}\right)}{\sqrt{d-c^2dx^2}}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x])^n)/x^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(-c^2d\,x^2+d)^{\frac{5}{2}}(a+b\arccos(cx))^n}{x^2}dx$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n/x^2,x)
```

output
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n}{x^2}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)^n}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n/x^2,x, algorithm="frica
s")
```

output
```
integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arcco
s(c*x) + a)^n/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))**n/x**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))^n}{x^2}\,dx=\int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)^n}{x^2}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n/x^2,x, algorithm="maxim
a")
```

output
```
integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccos(c*x) + a)^n/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))^n}{x^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))^n/x^2,x, algorithm="giac"
)
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n}{x^2}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^n\,(d-c^2\,d\,x^2)^{5/2}}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(5/2))/x^2,x)
```

output
```
int(((a + b*acos(c*x))^n*(d - c^2*d*x^2)^(5/2))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 234, normalized size of antiderivative = 8.07

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))^n}{x^2}\,dx=\frac{\sqrt{d}\,d^2\Big((acos(cx)\,b+a)^n\,acos(cx)\,bc+(acos(cx)\,b+a)^n\,ac+\Big(\ldots}{}$$

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*acos(c*x))^n/x^2,x)
```

output
```
(sqrt(d)*d**2*((acos(c*x)*b + a)**n*acos(c*x)*b*c + (acos(c*x)*b + a)**n*a
*c + int((acos(c*x)*b + a)**n/(sqrt( - c**2*x**2 + 1)*x**2),x)*b*n + int((
acos(c*x)*b + a)**n/(sqrt( - c**2*x**2 + 1)*x**2),x)*b + int((acos(c*x)*b
+ a)**n*sqrt( - c**2*x**2 + 1)*x**2,x)*b*c**4*n + int((acos(c*x)*b + a)**n
*sqrt( - c**2*x**2 + 1)*x**2,x)*b*c**4 - 2*int((acos(c*x)*b + a)**n*sqrt(
- c**2*x**2 + 1),x)*b*c**2*n - 2*int((acos(c*x)*b + a)**n*sqrt( - c**2*x**
2 + 1),x)*b*c**2))/(b*(n + 1))
```

3.499 $\int \frac{x^m \arccos(ax)^n}{\sqrt{1-a^2x^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arccos(ax)^n}{\sqrt{1-a^2x^2}} \, dx = \text{Int}\left(\frac{x^m \arccos(ax)^n}{\sqrt{1-a^2x^2}}, x\right)$$

output
```
Defer(Int)(x^m*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arccos(ax)^n}{\sqrt{1-a^2x^2}} \, dx = \int \frac{x^m \arccos(ax)^n}{\sqrt{1-a^2x^2}} \, dx$$

input
```
Integrate[(x^m*ArcCos[a*x]^n)/Sqrt[1 - a^2*x^2],x]
```

output
```
Integrate[(x^m*ArcCos[a*x]^n)/Sqrt[1 - a^2*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arccos(ax)^n}{\sqrt{1-a^2x^2}} \, dx$$

$$\Big\downarrow \text{ 5235}$$

$$\int \frac{x^m \arccos(ax)^n}{\sqrt{1-a^2x^2}}dx$$

input
```
Int[(x^m*ArcCos[a*x]^n)/Sqrt[1 - a^2*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arccos\left(ax\right)^n}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^m*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(x^m*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{x^m \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m \arccos{(ax)}^n}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^m*arccos(a*x)^n/(a^2*x^2 - 1), x)
```

Sympy [N/A]

Not integrable

Time = 3.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m \operatorname{acos}^n{(ax)}}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x**m*acos(a*x)**n/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**m*acos(a*x)**n/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m \arccos\left(ax\right)^n}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^m*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*arccos(a*x)^n/sqrt(-a^2*x^2 + 1), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m \operatorname{acos}(a\,x)^n}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^m*acos(a*x)^n)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^m*acos(a*x)^n)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^m \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^m acos(ax)^n}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^m*acos(a*x)^n/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((x**m*acos(a*x)**n)/sqrt( - a**2*x**2 + 1),x)
```

3.500 $\int \frac{x^3 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{x^3 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = -\frac{3(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-i\arccos(ax))}{8a^4} - \frac{3(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,i\arccos(ax))}{8a^4} + \frac{3^{-1-n}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-3i\arccos(ax))}{8a^4} + \frac{3^{-1-n}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,3i\arccos(ax))}{8a^4}$$

output

```
-3/8*arccos(a*x)^n*GAMMA(1+n,-I*arccos(a*x))/a^4/((-I*arccos(a*x))^n)-3/8*
arccos(a*x)^n*GAMMA(1+n,I*arccos(a*x))/a^4/((I*arccos(a*x))^n)+1/8*3^(-1-n
)*arccos(a*x)^n*GAMMA(1+n,-3*I*arccos(a*x))/a^4/((-I*arccos(a*x))^n)+1/8*3
^(-1-n)*arccos(a*x)^n*GAMMA(1+n,3*I*arccos(a*x))/a^4/((I*arccos(a*x))^n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.84

$$\int \frac{x^3 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{i3^{-1-n}\arccos(ax)^n\left(\arccos(ax)^2\right)^{-n}\left(3^{2+n}(i\arccos(ax))^n\Gamma(1+n,-i\arccos(ax))-3^{2+n}(-i\arccos(ax))^n\right.}{}$$

input
```
Integrate[(x^3*ArcCos[a*x]^n)/Sqrt[1 - a^2*x^2],x]
```

output
```
((I/8)*3^(-1 - n)*ArcCos[a*x]^n*(3^(2 + n)*(I*ArcCos[a*x])^n*Gamma[1 + n,
(-I)*ArcCos[a*x]] - 3^(2 + n)*((-I)*ArcCos[a*x])^n*Gamma[1 + n, I*ArcCos[a
*x]] + (I*ArcCos[a*x])^n*Gamma[1 + n, (-3*I)*ArcCos[a*x]] - ((-I)*ArcCos[a
*x])^n*Gamma[1 + n, (3*I)*ArcCos[a*x]]))/(a^4*(ArcCos[a*x]^2)^n)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx$$
$$\Big\downarrow \; 5225$$
$$-\frac{\int a^3x^3\arccos(ax)^n d\arccos(ax)}{a^4}$$
$$\Big\downarrow \; 3042$$
$$-\frac{\int \arccos(ax)^n \sin\left(\arccos(ax)+\frac{\pi}{2}\right)^3 d\arccos(ax)}{a^4}$$
$$\Big\downarrow \; 3793$$

$$-\frac{\int \left(\frac{3}{4}ax\arccos(ax)^n+\frac{1}{4}\cos(3\arccos(ax))\arccos(ax)^n\right)d\arccos(ax)}{a^4}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\frac{3}{8}i\arccos(ax)^n(-i\arccos(ax))^{-n}\Gamma(n+1,-i\arccos(ax))-\frac{1}{8}i3^{-n-1}\arccos(ax)^n(-i\arccos(ax))^{-n}\Gamma(n+1,-$$

input
```
Int[(x^3*ArcCos[a*x]^n)/Sqrt[1 - a^2*x^2],x]
```

output
```
-(((((-3*I)/8)*ArcCos[a*x]^n*Gamma[1 + n, (-I)*ArcCos[a*x]])/((-I)*ArcCos[
a*x])^n + (((3*I)/8)*ArcCos[a*x]^n*Gamma[1 + n, I*ArcCos[a*x]])/(I*ArcCos[
a*x])^n - ((I/8)*3^(-1 - n)*ArcCos[a*x]^n*Gamma[1 + n, (-3*I)*ArcCos[a*x]]
)/((-I)*ArcCos[a*x])^n + ((I/8)*3^(-1 - n)*ArcCos[a*x]^n*Gamma[1 + n, (3*I
)*ArcCos[a*x]])/(I*ArcCos[a*x])^n)/a^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]    Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3 \arccos{(ax)}^n}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^3*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(x^3*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3 \arccos{(ax)}^n}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^3*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)^n/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^3 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3 \operatorname{acos}^n{(ax)}}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x**3*acos(a*x)**n/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**3*acos(a*x)**n/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^3 \operatorname{acos}(a\,x)^n}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^3*acos(a*x)^n)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^3*acos(a*x)^n)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)^n\,x^3}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^3*acos(a*x)^n/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acos(a*x)**n*x**3)/sqrt( - a**2*x**2 + 1),x)
```

3.501 $\int \frac{x^2 \arccos(ax)^n}{\sqrt{1-a^2x^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \frac{x^2 \arccos(ax)^n}{\sqrt{1-a^2x^2}} \, dx = \frac{\arccos(ax)^{1+n}}{2a^3(1+n)} + \frac{i2^{-3-n}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-2i\arccos(ax))}{a^3} - \frac{i2^{-3-n}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,2i\arccos(ax))}{a^3}$$

output
```
1/2*arccos(a*x)^(1+n)/a^3/(1+n)+I*2^(-3-n)*arccos(a*x)^n*GAMMA(1+n,-2*I*ar
ccos(a*x))/a^3/((-I*arccos(a*x))^n)-I*2^(-3-n)*arccos(a*x)^n*GAMMA(1+n,2*I
*arccos(a*x))/a^3/((I*arccos(a*x))^n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

$$\int \frac{x^2 \arccos(ax)^n}{\sqrt{1-a^2x^2}} \, dx = -\frac{2^{-3-n}\arccos(ax)^n\left(\arccos(ax)^2\right)^{-n}\left(2^{2+n}\arccos(ax)\left(\arccos(ax)^2\right)^n - i(1+n)(i\arccos(ax))^n\Gamma(1+n,\right.}{a^3(1+n)}$$

input
```
Integrate[(x^2*ArcCos[a*x]^n)/Sqrt[1 - a^2*x^2],x]
```

output
```
-((2^(-3 - n)*ArcCos[a*x]^n*(2^(2 + n)*ArcCos[a*x]*(ArcCos[a*x]^2)^n - I*(
1 + n)*(I*ArcCos[a*x])^n*Gamma[1 + n, (-2*I)*ArcCos[a*x]] + I*(1 + n)*((-I
)*ArcCos[a*x])^n*Gamma[1 + n, (2*I)*ArcCos[a*x]]))/(a^3*(1 + n)*(ArcCos[a*
x]^2)^n))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5225, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \ 5225$$

$$-\frac{\int a^2x^2 \arccos(ax)^n d\arccos(ax)}{a^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \arccos(ax)^n \sin\left(\arccos(ax)+\frac{\pi}{2}\right)^2 d\arccos(ax)}{a^3}$$

$$\Big\downarrow \ 3793$$

$$-\frac{\int \left(\frac{1}{2}\cos(2\arccos(ax))\arccos(ax)^n+\frac{1}{2}\arccos(ax)^n\right) d\arccos(ax)}{a^3}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{\arccos(ax)^{n+1}}{2(n+1)} - i2^{-n-3}\arccos(ax)^n(-i\arccos(ax))^{-n}\Gamma(n+1,-2i\arccos(ax)) + i2^{-n-3}(i\arccos(ax))^{-n}\arccos}{a^3}$$

input
```
Int[(x^2*ArcCos[a*x]^n)/Sqrt[1 - a^2*x^2],x]
```

output
```
-((ArcCos[a*x]^(1 + n)/(2*(1 + n)) - (I*2^(-3 - n)*ArcCos[a*x]^n*Gamma[1 +
 n, (-2*I)*ArcCos[a*x]])/((-I)*ArcCos[a*x])^n + (I*2^(-3 - n)*ArcCos[a*x]^
n*Gamma[1 + n, (2*I)*ArcCos[a*x]])/(I*ArcCos[a*x])^n)/a^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^2 \arccos(ax)^n}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^2*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(x^2*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2 \arccos\left(ax\right)^n}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x)^n/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^2 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2 \operatorname{acos}^n\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

input
```
integrate(x**2*acos(a*x)**n/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x**2*acos(a*x)**n/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x^2 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2 \arccos{(ax)}^n}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x^2*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arccos(a*x)^n/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x^2 \operatorname{acos}(a\,x)^n}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x^2*acos(a*x)^n)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x^2*acos(a*x)^n)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2 \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)^n\,x^2}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x^2*acos(a*x)^n/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acos(a*x)**n*x**2)/sqrt( - a**2*x**2 + 1),x)
```

3.502 $\int \frac{x \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{x \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = -\frac{(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-i\arccos(ax))}{2a^2} - \frac{(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,i\arccos(ax))}{2a^2}$$

output
```
-1/2*arccos(a*x)^n*GAMMA(1+n,-I*arccos(a*x))/a^2/((-I*arccos(a*x))^n)-1/2*
arccos(a*x)^n*GAMMA(1+n,I*arccos(a*x))/a^2/((I*arccos(a*x))^n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{x \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = -\frac{i\arccos(ax)^n\left(\arccos(ax)^2\right)^{-n}\left(-(i\arccos(ax))^n\Gamma(1+n,-i\arccos(ax))+(-i\arccos(ax))^n\Gamma(1+n,i\right.}{2a^2}$$

input
```
Integrate[(x*ArcCos[a*x]^n)/Sqrt[1 - a^2*x^2],x]
```

output
```
((-1/2*I)*ArcCos[a*x]^n*(-((I*ArcCos[a*x])^n*Gamma[1 + n, (-I)*ArcCos[a*x]
]) + ((-I)*ArcCos[a*x])^n*Gamma[1 + n, I*ArcCos[a*x]]))/(a^2*(ArcCos[a*x]^
2)^n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5225, 3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arccos(ax)^n}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{5225}$$

$$-\frac{\int ax \arccos(ax)^n d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \arccos(ax)^n \sin\left(\arccos(ax)+\frac{\pi}{2}\right)d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{3788}$$

$$-\frac{\frac{1}{2}i\int -ie^{-i\arccos(ax)}\arccos(ax)^n d\arccos(ax)-\frac{1}{2}i\int ie^{i\arccos(ax)}\arccos(ax)^n d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\frac{1}{2}\int e^{-i\arccos(ax)}\arccos(ax)^n d\arccos(ax)+\frac{1}{2}\int e^{i\arccos(ax)}\arccos(ax)^n d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{2612}$$

$$-\frac{\frac{1}{2}i(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(n+1,i\arccos(ax))-\frac{1}{2}i(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(n+1,-i\arccos(ax$$

input
```
Int[(x*ArcCos[a*x]^n)/Sqrt[1 - a^2*x^2],x]
```

output
```
-((((-1/2*I)*ArcCos[a*x]^n*Gamma[1 + n, (-I)*ArcCos[a*x]])/((-I)*ArcCos[a*
x])^n + ((I/2)*ArcCos[a*x]^n*Gamma[1 + n, I*ArcCos[a*x]])/(I*ArcCos[a*x])^
n)/a^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x \arccos{(ax)}^n}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(x*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x \arccos{(ax)}^n}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*x*arccos(a*x)^n/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x \operatorname{acos}^n{(ax)}}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(x*acos(a*x)**n/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(x*acos(a*x)**n/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [F]

$$\int \frac{x \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x \arccos\left(ax\right)^n}{\sqrt{-a^2x^2+1}}\,dx$$

input
```
integrate(x*arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*arccos(a*x)^n/sqrt(-a^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{x\,\mathrm{acos}(a\,x)^n}{\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int((x*acos(a*x)^n)/(1 - a^2*x^2)^(1/2),x)
```

output
```
int((x*acos(a*x)^n)/(1 - a^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x \arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)^n\,x}{\sqrt{-a^2x^2+1}}dx$$

input
```
int(x*acos(a*x)^n/(-a^2*x^2+1)^(1/2),x)
```

output
```
int((acos(a*x)**n*x)/sqrt( - a**2*x**2 + 1),x)
```

3.503 $\int \frac{\arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 17

$$\int \frac{\arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \frac{\arccos(ax)^{1+n}}{a(1+n)}$$

output
```
arccos(a*x)^(1+n)/a/(1+n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos(ax)^{1+n}}{a(1+n)}$$

input
```
Integrate[ArcCos[a*x]^n/Sqrt[1 - a^2*x^2],x]
```

output
```
-(ArcCos[a*x]^(1 + n)/(a*(1 + n)))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^n}{\sqrt{1-a^2x^2}}dx$$
$$\Big\downarrow \text{ 5153}$$
$$-\frac{\arccos(ax)^{n+1}}{a(n+1)}$$

input
```
Int[ArcCos[a*x]^n/Sqrt[1 - a^2*x^2],x]
```

output
```
-(ArcCos[a*x]^(1 + n)/(a*(1 + n)))
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$-\frac{\arccos(ax)^{1+n}}{a(1+n)}$	19
default	$-\frac{\arccos(ax)^{1+n}}{a(1+n)}$	19

input
```
int(arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arccos(a*x)^(1+n)/a/(1+n)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{\arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos{(ax)}^n\arccos{(ax)}}{an+a}$$

input
```
integrate(arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-arccos(a*x)^n*arccos(a*x)/(a*n + a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(12) = 24.

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.41

$$\int \frac{\arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} \frac{2x}{\pi} & \text{for } a=0 \wedge n=-1 \\ x\left(\frac{\pi}{2}\right)^n & \text{for } a=0 \\ -\frac{\log{(\operatorname{acos}{(ax)})}}{a} & \text{for } n=-1 \\ -\frac{\operatorname{acos}{(ax)}\operatorname{acos}^n{(ax)}}{an+a} & \text{otherwise} \end{cases}$$

input
```
integrate(acos(a*x)**n/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((2*x/pi, Eq(a, 0) & Eq(n, -1)), (x*(pi/2)**n, Eq(a, 0)), (-log(a
cos(a*x))/a, Eq(n, -1)), (-acos(a*x)*acos(a*x)**n/(a*n + a), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos(ax)^{n+1}}{a(n+1)}$$

input
```
integrate(arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-arccos(a*x)^(n + 1)/(a*(n + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = -\frac{\arccos(ax)^{n+1}}{a(n+1)}$$

input
```
integrate(arccos(a*x)^n/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-arccos(a*x)^(n + 1)/(a*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{\arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} -\frac{\ln(\mathrm{acos}(a\,x))}{a} & \text{if } n=-1 \\ -\frac{\mathrm{acos}(a\,x)^{n+1}}{a\,(n+1)} & \text{if } n\neq -1 \end{cases}$$

input
```
int(acos(a*x)^n/(1 - a^2*x^2)^(1/2),x)
```

output
```
piecewise(n == -1, -log(acos(a*x))/a, n ~= -1, -acos(a*x)^(n + 1)/(a*(n +
1)))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{\arccos(ax)^n}{\sqrt{1-a^2x^2}}\,dx = -\frac{acos(ax)^n\, acos(ax)}{a\,(n+1)}$$

input
```
int(acos(a*x)^n/(-a^2*x^2+1)^(1/2),x)
```

output
```
( - acos(a*x)**n*acos(a*x))/(a*(n + 1))
```

3.504 $\int \frac{\arccos(ax)^n}{x\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arccos(ax)^n}{x\sqrt{1-a^2x^2}}\,dx = \text{Int}\left(\frac{\arccos(ax)^n}{x\sqrt{1-a^2x^2}},x\right)$$

output
```
Defer(Int)(arccos(a*x)^n/x/(-a^2*x^2+1)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arccos(ax)^n}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos(ax)^n}{x\sqrt{1-a^2x^2}}\,dx$$

input
```
Integrate[ArcCos[a*x]^n/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
Integrate[ArcCos[a*x]^n/(x*Sqrt[1 - a^2*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^n}{x\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \text{ 5235}$$

$$\int \frac{\arccos(ax)^n}{x\sqrt{1-a^2x^2}}dx$$

input
```
Int[ArcCos[a*x]^n/(x*Sqrt[1 - a^2*x^2]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arccos\left(ax\right)^{n}}{x\sqrt{-a^{2}x^{2}+1}}dx$$

input
```
int(arccos(a*x)^n/x/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(arccos(a*x)^n/x/(-a^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\arccos(ax)^n}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos{(ax)}^n}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccos(a*x)^n/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccos(a*x)^n/(a^2*x^3 - x), x)
```

Sympy [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acos}^n{(ax)}}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acos(a*x)**n/x/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acos(a*x)**n/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^n}{x\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^n/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos{(ax)}^n}{\sqrt{-a^2x^2+1}x}\,dx$$

input
```
integrate(arccos(a*x)^n/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^n/(sqrt(-a^2*x^2 + 1)*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^n}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acos(a*x)^n/(x*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acos(a*x)^n/(x*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{\arccos(ax)^n}{x\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)^n}{\sqrt{-a^2x^2+1}\,x}dx$$

input
```
int(acos(a*x)^n/x/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acos(a*x)**n/(sqrt( - a**2*x**2 + 1)*x),x)
```

3.505 $\int \frac{\arccos(ax)^n}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arccos(ax)^n}{x^2\sqrt{1-a^2x^2}}\,dx = \text{Int}\left(\frac{\arccos(ax)^n}{x^2\sqrt{1-a^2x^2}},x\right)$$

output
```
Defer(Int)(arccos(a*x)^n/x^2/(-a^2*x^2+1)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arccos(ax)^n}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos(ax)^n}{x^2\sqrt{1-a^2x^2}}\,dx$$

input
```
Integrate[ArcCos[a*x]^n/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
Integrate[ArcCos[a*x]^n/(x^2*Sqrt[1 - a^2*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax)^n}{x^2\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \; 5235$$

$$\int \frac{\arccos(ax)^n}{x^2\sqrt{1-a^2x^2}}dx$$

input
```
Int[ArcCos[a*x]^n/(x^2*Sqrt[1 - a^2*x^2]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arccos\left(ax\right)^n}{x^2\sqrt{-a^2x^2+1}}dx$$

input
```
int(arccos(a*x)^n/x^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(arccos(a*x)^n/x^2/(-a^2*x^2+1)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\arccos(ax)^n}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos{(ax)}^n}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arccos(a*x)^n/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a^2*x^2 + 1)*arccos(a*x)^n/(a^2*x^4 - x^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arccos(ax)^n}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\operatorname{acos}^n{(ax)}}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

input
```
integrate(acos(a*x)**n/x**2/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(acos(a*x)**n/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^n}{x^2\sqrt{1-a^2x^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a*x)^n/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\arccos{(ax)}^n}{\sqrt{-a^2x^2+1}x^2}\,dx$$

input
```
integrate(arccos(a*x)^n/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x)^n/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^n}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(acos(a*x)^n/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(acos(a*x)^n/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{\arccos(ax)^n}{x^2\sqrt{1-a^2x^2}}\,dx = \int \frac{acos(ax)^n}{\sqrt{-a^2x^2+1}\,x^2}dx$$

input
```
int(acos(a*x)^n/x^2/(-a^2*x^2+1)^(1/2),x)
```

output
```
int(acos(a*x)**n/(sqrt( - a**2*x**2 + 1)*x**2),x)
```

3.506 $\int (d+cdx)^{5/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 376

$$\begin{aligned}\int (d+cdx)^{5/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx &= \frac{2bd^2x\sqrt{d+cdx}\sqrt{f-cfx}}{3\sqrt{1-c^2x^2}} \\ &- \frac{3bcd^2x^2\sqrt{d+cdx}\sqrt{f-cfx}}{16\sqrt{1-c^2x^2}} - \frac{2bc^2d^2x^3\sqrt{d+cdx}\sqrt{f-cfx}}{9\sqrt{1-c^2x^2}} \\ &- \frac{bc^3d^2x^4\sqrt{d+cdx}\sqrt{f-cfx}}{16\sqrt{1-c^2x^2}} + \frac{3}{8}d^2x\sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx)) \\ &+ \frac{1}{4}c^2d^2x^3\sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx)) \\ &- \frac{2d^2\sqrt{d+cdx}\sqrt{f-cfx}(1-c^2x^2)(a+b\arccos(cx))}{3c} \\ &+ \frac{5d^2\sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx))^2}{16bc\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
2/3*b*d^2*x*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)/(-c^2*x^2+1)^(1/2)-3/16*b*c*d
^2*x^2*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)/(-c^2*x^2+1)^(1/2)-2/9*b*c^2*d^2*x
^3*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)/(-c^2*x^2+1)^(1/2)-1/16*b*c^3*d^2*x^4*
(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)/(-c^2*x^2+1)^(1/2)+3/8*d^2*x*(c*d*x+d)^(1
/2)*(-c*f*x+f)^(1/2)*(a+b*arccos(c*x))+1/4*c^2*d^2*x^3*(c*d*x+d)^(1/2)*(-c
*f*x+f)^(1/2)*(a+b*arccos(c*x))-2/3*d^2*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)*(
-c^2*x^2+1)*(a+b*arccos(c*x))/c+5/16*d^2*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)*
(a+b*arccos(c*x))^2/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.78

$$\int (d+cdx)^{5/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx = \frac{-360bd^2\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx)^2-720ad^{5/2}\sqrt{f}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{f-}}{\sqrt{d}\sqrt{f}(-1+c^2}\right.}{}$$

input
```
Integrate[(d + c*d*x)^(5/2)*Sqrt[f - c*f*x]*(a + b*ArcCos[c*x]),x]
```

output
```
(-360*b*d^2*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]^2 - 720*a*d^(5/2)*
Sqrt[f]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sq
rt[d]*Sqrt[f]*(-1 + c^2*x^2))] + d^2*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(256*
b*c*x*(-3 + c^2*x^2) + 48*a*Sqrt[1 - c^2*x^2]*(-16 + 9*c*x + 16*c^2*x^2 +
6*c^3*x^3) + 144*b*Cos[2*ArcCos[c*x]] + 9*b*Cos[4*ArcCos[c*x]]) + 12*b*d^2
*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]*(-64*(1 - c^2*x^2)^(3/2) + 24
*Sin[2*ArcCos[c*x]] + 3*Sin[4*ArcCos[c*x]]))/(1152*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^{5/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \; 5179$$

$$\frac{\sqrt{cdx+d}\sqrt{f-cfx}\int d^2(cx+1)^2\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{d^2\sqrt{cdx+d}\sqrt{f-cfx}\int (cx+1)^2\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$\downarrow$ 5263

$$\frac{d^2\sqrt{cdx+d}\sqrt{f-cfx}\int \left(c^2\sqrt{1-c^2x^2}(a+b\arccos(cx))x^2+2c\sqrt{1-c^2x^2}(a+b\arccos(cx))x+\sqrt{1-c^2x^2}(a+b\right.}{\sqrt{1-c^2x^2}}$$

$\downarrow$ 2009

$$\frac{d^2\sqrt{cdx+d}\sqrt{f-cfx}\left(\frac{3}{8}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{2(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3c}+\frac{1}{4}c^2x^3\sqrt{1-c^2x^2}(a+b\operatorname{arcc}\right.}{\sqrt{1-c^2x^2}}$$

input
```
Int[(d + c*d*x)^(5/2)*Sqrt[f - c*f*x]*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*((-2*b*x)/3 + (3*b*c*x^2)/16 + (2*b*c
^2*x^3)/9 + (b*c^3*x^4)/16 + (3*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/8
 + (c^2*x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/4 - (2*(1 - c^2*x^2)^(3
/2)*(a + b*ArcCos[c*x]))/(3*c) - (5*(a + b*ArcCos[c*x])^2)/(16*b*c)))/Sqrt
[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 961, normalized size of antiderivative = 2.56

method	result
default	$-\frac{a(cdx+d)^{\frac{5}{2}}(-cfx+f)^{\frac{3}{2}}}{4cf}-\frac{5ad(cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{3}{2}}}{12cf}-\frac{5a\,d^{2}\sqrt{cdx+d}\,(-cfx+f)^{\frac{3}{2}}}{8cf}+\frac{5a\,d^{2}\sqrt{-cfx+f}\,\sqrt{cdx+d}}{8c}+\frac{5a\,d^{3}f}{}$
parts	$-\frac{a(cdx+d)^{\frac{5}{2}}(-cfx+f)^{\frac{3}{2}}}{4cf}-\frac{5ad(cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{3}{2}}}{12cf}-\frac{5a\,d^{2}\sqrt{cdx+d}\,(-cfx+f)^{\frac{3}{2}}}{8cf}+\frac{5a\,d^{2}\sqrt{-cfx+f}\,\sqrt{cdx+d}}{8c}+\frac{5a\,d^{3}f}{}$

input
```
int((c*d*x+d)^(5/2)*(-c*f*x+f)^(1/2)*(a+b*arccos(c*x)),x,method=_RETURNVER
BOSE)
```

output
```
-1/4*a/c/f*(c*d*x+d)^(5/2)*(-c*f*x+f)^(3/2)-5/12*a*d/c/f*(c*d*x+d)^(3/2)*(
-c*f*x+f)^(3/2)-5/8*a*d^2/c/f*(c*d*x+d)^(1/2)*(-c*f*x+f)^(3/2)+5/8*a*d^2/c
*(-c*f*x+f)^(1/2)*(c*d*x+d)^(1/2)+5/8*a*d^3*f*((-c*f*x+f)*(c*d*x+d))^(1/2)
/(-c*f*x+f)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*f)^(1/2)*arctan((c^2*d*f)^(1/2)*x
/(-c^2*d*f*x^2+d*f)^(1/2))+b*(5/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-
c^2*x^2+1)^(1/2)/c/(c^2*x^2-1)*arccos(c*x)^2*d^2+1/256*(d*(c*x+1))^(1/2)*(
-f*(c*x-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c
*x-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(I+4*arccos(c*x))*
d^2/c/(c^2*x^2-1)+1/36*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(4*c^4*x^4-5*c
^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-3*I*(-c^2*x^2+1)^(1/2)*c*x+1)*(I+3*a
rccos(c*x))*d^2/c/(c^2*x^2-1)-1/4*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I
*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)-I)*d^2/c/(c^2*x^2-1)+1/16*
(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^
3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))*d^2/c/(c^2*x^2-1)-1/2
56*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2
-1)*(17*I+28*arccos(c*x))*cos(3*arccos(c*x))*d^2/c/(c^2*x^2-1)-3/256*(d*(c
*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(5*I+
12*arccos(c*x))*sin(3*arccos(c*x))*d^2/c/(c^2*x^2-1)+1/9*(d*(c*x+1))^(1/2)
*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(2*I+3*arccos(c*
x))*cos(2*arccos(c*x))*d^2/c/(c^2*x^2-1)+1/18*(d*(c*x+1))^(1/2)*(-f*(c*...
```

Fricas [F]

$$\int (d+cdx)^{5/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx=\int (cdx+d)^{\frac{5}{2}}\sqrt{-cfx+f}(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*f*x+f)^(1/2)*(a+b*arccos(c*x)),x, algorithm=
"fricas")
```

output
```
integral((a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^2
*x + b*d^2)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+cdx)^{5/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx=\text{Timed out}$$

input
```
integrate((c*d*x+d)**(5/2)*(-c*f*x+f)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+cdx)^{5/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx=\int (cdx+d)^{\frac{5}{2}}\sqrt{-cfx+f}(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*f*x+f)^(1/2)*(a+b*arccos(c*x)),x, algorithm=
"maxima")
```

output
```
b*sqrt(d)*sqrt(f)*integrate((c^2*d^2*x^2 + 2*c*d^2*x + d^2)*sqrt(c*x + 1)*
sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x) + 1/24*(15*s
qrt(-c^2*d*f*x^2 + d*f)*d^2*x + 15*d^3*f*arcsin(c*x)/(sqrt(d*f)*c) - 6*(-c
^2*d*f*x^2 + d*f)^(3/2)*d*x/f - 16*(-c^2*d*f*x^2 + d*f)^(3/2)*d/(c*f))*a
```

Giac [F]

$$\int (d+cdx)^{5/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx=\int (cdx+d)^{\frac{5}{2}}\sqrt{-cfx+f}(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*f*x+f)^(1/2)*(a+b*arccos(c*x)),x, algorithm=
"giac")
```

output
```
integrate((c*d*x + d)^(5/2)*sqrt(-c*f*x + f)*(b*arccos(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)^{5/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,\left(d+c\,d\,x\right)^{5/2}\,\sqrt{f-c\,f\,x}\,dx$$

input
```
int((a + b*acos(c*x))*(d + c*d*x)^(5/2)*(f - c*f*x)^(1/2),x)
```

output
```
int((a + b*acos(c*x))*(d + c*d*x)^(5/2)*(f - c*f*x)^(1/2), x)
```

Reduce [F]

$$\int (d+cdx)^{5/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx=\frac{\sqrt{f}\,\sqrt{d}\,d^2\left(-30asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a+6\sqrt{cx+1}\,\sqrt{-cx+1}\,a\,c^3x^3+16\sqrt{cx+1}\,\sqrt{-cx+1}\,a$$

input
```
int((c*d*x+d)^(5/2)*(-c*f*x+f)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*d**2*( - 30*asin(sqrt( - c*x + 1)/sqrt(2))*a + 6*sqrt(c*x
 + 1)*sqrt( - c*x + 1)*a*c**3*x**3 + 16*sqrt(c*x + 1)*sqrt( - c*x + 1)*a*c
**2*x**2 + 9*sqrt(c*x + 1)*sqrt( - c*x + 1)*a*c*x - 16*sqrt(c*x + 1)*sqrt(
 - c*x + 1)*a + 24*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**2,x)*b*
c**3 + 48*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x,x)*b*c**2 + 24*in
t(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x),x)*b*c))/(24*c)
```

3.507 $\int (d+cdx)^{3/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 273

$$\int (d+cdx)^{3/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx = \frac{bdx\sqrt{d+cdx}\sqrt{f-cfx}}{3\sqrt{1-c^2x^2}} - \frac{bcdx^2\sqrt{d+cdx}\sqrt{f-cfx}}{4\sqrt{1-c^2x^2}} - \frac{bc^2dx^3\sqrt{d+cdx}\sqrt{f-cfx}}{9\sqrt{1-c^2x^2}} + \frac{1}{2}dx\sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx)) - \frac{d\sqrt{d+cdx}\sqrt{f-cfx}(1-c^2x^2)(a+b\arccos(cx))}{3c} + \frac{d\sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx))^2}{4bc\sqrt{1-c^2x^2}}$$

output
```
1/3*b*d*x*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)/(-c^2*x^2+1)^(1/2)-1/4*b*c*d*x^
2*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)/(-c^2*x^2+1)^(1/2)-1/9*b*c^2*d*x^3*(c*d
*x+d)^(1/2)*(-c*f*x+f)^(1/2)/(-c^2*x^2+1)^(1/2)+1/2*d*x*(c*d*x+d)^(1/2)*(-
c*f*x+f)^(1/2)*(a+b*arccos(c*x))-1/3*d*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)*(-
c^2*x^2+1)*(a+b*arccos(c*x))/c+1/4*d*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)*(a+b
*arccos(c*x))^2/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.95

$$\int (d+cdx)^{3/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx = \frac{-18bd\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx)^2-36ad^{3/2}\sqrt{f}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{f-cfx}}{\sqrt{d}\sqrt{f}(-1+c^2x^2)}\right.}{}$$

input
```
Integrate[(d + c*d*x)^(3/2)*Sqrt[f - c*f*x]*(a + b*ArcCos[c*x]),x]
```

output
```
(-18*b*d*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]^2 - 36*a*d^(3/2)*Sqrt
[f]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d
]*Sqrt[f]*(-1 + c^2*x^2))] + d*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(8*b*c*x*(-
3 + c^2*x^2) + 12*a*Sqrt[1 - c^2*x^2]*(-2 + 3*c*x + 2*c^2*x^2) + 9*b*Cos[2
*ArcCos[c*x]]) + 6*b*d*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]*(-4*(1
- c^2*x^2)^(3/2) + 3*Sin[2*ArcCos[c*x]]))/(72*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^{3/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\sqrt{cdx+d}\sqrt{f-cfx}\int d(cx+1)\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\sqrt{cdx+d}\sqrt{f-cfx}\int (cx+1)\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 5263$$

$$\frac{d\sqrt{cdx+d}\sqrt{f-cfx}\int \left(cx\sqrt{1-c^2x^2}(a+b\arccos(cx))+\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{d\sqrt{cdx+d}\sqrt{f-cfx}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3c}-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{9}bc^2x^3+\frac{1}{4}\right.}{\sqrt{1-c^2x^2}}$$

input
```
Int[(d + c*d*x)^(3/2)*Sqrt[f - c*f*x]*(a + b*ArcCos[c*x]),x]
```

output
```
(d*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(-1/3*(b*x) + (b*c*x^2)/4 + (b*c^2*x^3)
/9 + (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/2 - ((1 - c^2*x^2)^(3/2)*(a
 + b*ArcCos[c*x]))/(3*c) - (a + b*ArcCos[c*x])^2/(4*b*c)))/Sqrt[1 - c^2*x^
2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 726, normalized size of antiderivative = 2.66

method	result
default	$-\frac{a(cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{3}{2}}}{3cf}-\frac{ad\sqrt{cdx+d}\,(-cfx+f)^{\frac{3}{2}}}{2cf}+\frac{ad\sqrt{-cfx+f}\,\sqrt{cdx+d}}{2c}+\frac{a\,d^{2}f\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^{2}}}{\sqrt{-c^{2}d}}\right.}{2\sqrt{-cfx+f}\,\sqrt{cdx+d}\,\sqrt{c^{2}df}}$
parts	$-\frac{a(cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{3}{2}}}{3cf}-\frac{ad\sqrt{cdx+d}\,(-cfx+f)^{\frac{3}{2}}}{2cf}+\frac{ad\sqrt{-cfx+f}\,\sqrt{cdx+d}}{2c}+\frac{a\,d^{2}f\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^{2}}}{\sqrt{-c^{2}d}}\right.}{2\sqrt{-cfx+f}\,\sqrt{cdx+d}\,\sqrt{c^{2}df}}$

input
```
int((c*d*x+d)^(3/2)*(-c*f*x+f)^(1/2)*(a+b*arccos(c*x)),x,method=_RETURNVER
BOSE)
```

output
```
-1/3*a/c/f*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)-1/2*a*d/c/f*(c*d*x+d)^(1/2)*(-
c*f*x+f)^(3/2)+1/2*a*d/c*(-c*f*x+f)^(1/2)*(c*d*x+d)^(1/2)+1/2*a*d^2*f*((-c
*f*x+f)*(c*d*x+d))^(1/2)/(-c*f*x+f)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*f)^(1/2)*
arctan((c^2*d*f)^(1/2)*x/(-c^2*d*f*x^2+d*f)^(1/2))+b*(1/4*(d*(c*x+1))^(1/2
)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^2*d+1/72
*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*I*(-c^2*x^2+1
)^(1/2)*x^3*c^3-3*I*(-c^2*x^2+1)^(1/2)*c*x+1)*(I+3*arccos(c*x))*d/(c^2*x^2
-1)/c+1/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(2*c^3*x^3-2*c*x+2*I*(-c^2
*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(I+2*arccos(c*x))*d/(c^2*x^2-1
)/c-1/8*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^
2*x^2-1)*(arccos(c*x)-I)*d/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1
))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2
*c*x)*(-I+2*arccos(c*x))*d/(c^2*x^2-1)/c+1/18*(d*(c*x+1))^(1/2)*(-f*(c*x-1
))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(2*I+3*arccos(c*x))*cos(2*a
rccos(c*x))*d/(c^2*x^2-1)/c+1/36*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(I*c
^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(5*I+3*arccos(c*x))*sin(2*arccos(c*x))*d/
(c^2*x^2-1)/c)
```

Fricas [F]

$$\int (d+cdx)^{3/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx=\int (cdx+d)^{\frac{3}{2}}\sqrt{-cfx+f}(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*f*x+f)^(1/2)*(a+b*arccos(c*x)),x, algorithm=
"fricas")
```

output
```
integral((a*c*d*x + a*d + (b*c*d*x + b*d)*arccos(c*x))*sqrt(c*d*x + d)*sqr
t(-c*f*x + f), x)
```

Sympy [F]

$$\int (d+cdx)^{3/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx = \int \left(d(cx+1)\right)^{\frac{3}{2}}\sqrt{-f\,(cx-1)}(a+b\,\mathrm{acos}\,(cx))\;dx$$

input
```
integrate((c*d*x+d)**(3/2)*(-c*f*x+f)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
Integral((d*(c*x + 1))**(3/2)*sqrt(-f*(c*x - 1))*(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int (d+cdx)^{3/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx = \int (cdx+d)^{\frac{3}{2}}\sqrt{-cfx+f}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*f*x+f)^(1/2)*(a+b*arccos(c*x)),x, algorithm=
"maxima")
```

output
```
b*sqrt(d)*sqrt(f)*integrate((c*d*x + d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arcta
n2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x) + 1/6*(3*sqrt(-c^2*d*f*x^2 + d*f
)*d*x + 3*d^2*f*arcsin(c*x)/(sqrt(d*f)*c) - 2*(-c^2*d*f*x^2 + d*f)^(3/2)/(
c*f))*a
```

Giac [F]

$$\int (d+cdx)^{3/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx = \int (cdx+d)^{\frac{3}{2}}\sqrt{-cfx+f}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*f*x+f)^(1/2)*(a+b*arccos(c*x)),x, algorithm=
"giac")
```

output
```
integrate((c*d*x + d)^(3/2)*sqrt(-c*f*x + f)*(b*arccos(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)^{3/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx = \int (a+b\operatorname{acos}(c\,x))\,(d+c\,d\,x)^{3/2}\,\sqrt{f-c\,f\,x}\,dx$$

input
```
int((a + b*acos(c*x))*(d + c*d*x)^(3/2)*(f - c*f*x)^(1/2),x)
```

output
```
int((a + b*acos(c*x))*(d + c*d*x)^(3/2)*(f - c*f*x)^(1/2), x)
```

Reduce [F]

$$\int (d+cdx)^{3/2}\sqrt{f-cfx}(a+b\arccos(cx))\,dx = \frac{\sqrt{f}\,\sqrt{d}\,d\left(-6asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a+2\sqrt{cx+1}\,\sqrt{-cx+1}\,a\,c^2x^2+3\sqrt{cx+1}\,\sqrt{-cx+1}\,acx$$

input
```
int((c*d*x+d)^(3/2)*(-c*f*x+f)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*d*( - 6*asin(sqrt( - c*x + 1)/sqrt(2))*a + 2*sqrt(c*x + 1
)*sqrt( - c*x + 1)*a*c**2*x**2 + 3*sqrt(c*x + 1)*sqrt( - c*x + 1)*a*c*x -
2*sqrt(c*x + 1)*sqrt( - c*x + 1)*a + 6*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*
acos(c*x)*x,x)*b*c**2 + 6*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x),x)*
b*c))/(6*c)
```

3.508 $\int \sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 134

$$\int \sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx))\,dx = -\frac{bcx^2\sqrt{d+cdx}\sqrt{f-cfx}}{4\sqrt{1-c^2x^2}}+\frac{1}{2}x\sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx)) + \frac{\sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx))^2}{4bc\sqrt{1-c^2x^2}}$$

output
```
-1/4*b*c*x^2*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)/(-c^2*x^2+1)^(1/2)+1/2*x*(c*
d*x+d)^(1/2)*(-c*f*x+f)^(1/2)*(a+b*arccos(c*x))+1/4*(c*d*x+d)^(1/2)*(-c*f*
x+f)^(1/2)*(a+b*arccos(c*x))^2/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.56

$$\int \sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx))\,dx = \frac{1}{2}ax\sqrt{-f(-1+cx)}\sqrt{d(1+cx)}-\frac{a\sqrt{d}\sqrt{f}\arctan\left(\frac{cx\sqrt{-f(-1+cx)}\sqrt{d(1+cx)}}{\sqrt{d}\sqrt{f}(-1+cx)(1+cx)}\right)}{2c} + \frac{b\sqrt{d+cdx}\sqrt{f-cfx}\sqrt{-df\left(1-c^2x^2\right)}(\cos(2\arccos(cx))+2\arccos(cx)(-\arccos(cx)+\sin(2\arccos(c}{8c\sqrt{(-d-cdx)(f-cfx)}\sqrt{1-c^2x^2}}$$

input
```
Integrate[Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(a + b*ArcCos[c*x]),x]
```

output
```
(a*x*Sqrt[-(f*(-1 + c*x))]*Sqrt[d*(1 + c*x)])/2 - (a*Sqrt[d]*Sqrt[f]*ArcTa
n[(c*x*Sqrt[-(f*(-1 + c*x))]*Sqrt[d*(1 + c*x)])/(Sqrt[d]*Sqrt[f]*(-1 + c*x
)*(1 + c*x))])/(2*c) + (b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*Sqrt[-(d*f*(1 -
c^2*x^2))]*(Cos[2*ArcCos[c*x]] + 2*ArcCos[c*x]*(-ArcCos[c*x] + Sin[2*ArcCo
s[c*x]])))/(8*c*Sqrt[(-d - c*d*x)*(f - c*f*x)]*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cdx+d}\sqrt{f-cfx}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\sqrt{cdx+d}\sqrt{f-cfx}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5157}$$

$$\frac{\sqrt{cdx+d}\sqrt{f-cfx}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sqrt{cdx+d}\sqrt{f-cfx}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{4}bcx^2\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{\sqrt{cdx+d}\sqrt{f-cfx}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcx^2\right)}{\sqrt{1-c^2x^2}}$$

input
```
Int[Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(a + b*ArcCos[c*x]),x]
```

output
```
(Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*((b*c*x^2)/4 + (x*Sqrt[1 - c^2*x^2]*(a +
b*ArcCos[c*x]))/2 - (a + b*ArcCos[c*x])^2/(4*b*c)))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.73

method	result
default	$-\frac{a\sqrt{cdx+d}\,(-cfx+f)^{\frac{3}{2}}}{2cf}+\frac{a\sqrt{-cfx+f}\,\sqrt{cdx+d}}{2c}+\frac{adf\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2df}\,x}{\sqrt{-c^2df\,x^2+df}}\right)}{2\sqrt{-cfx+f}\,\sqrt{cdx+d}\,\sqrt{c^2df}}+b\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-}\right.$
parts	$-\frac{a\sqrt{cdx+d}\,(-cfx+f)^{\frac{3}{2}}}{2cf}+\frac{a\sqrt{-cfx+f}\,\sqrt{cdx+d}}{2c}+\frac{adf\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2df}\,x}{\sqrt{-c^2df\,x^2+df}}\right)}{2\sqrt{-cfx+f}\,\sqrt{cdx+d}\,\sqrt{c^2df}}+b\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-}\right.$

input
```
int((c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)*(a+b*arccos(c*x)),x,method=_RETURNVER
BOSE)
```

output
```
-1/2*a/c/f*(c*d*x+d)^(1/2)*(-c*f*x+f)^(3/2)+1/2*a/c*(-c*f*x+f)^(1/2)*(c*d*
x+d)^(1/2)+1/2*a*d*f*((-c*f*x+f)*(c*d*x+d))^(1/2)/(-c*f*x+f)^(1/2)/(c*d*x+
d)^(1/2)/(c^2*d*f)^(1/2)*arctan((c^2*d*f)^(1/2)*x/(-c^2*d*f*x^2+d*f)^(1/2)
)+b*(1/4*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-
1)/c*arccos(c*x)^2+1/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(2*c^3*x^3-2*
c*x+2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(I+2*arccos(c*x))
/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1
)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))/(
c^2*x^2-1)/c)
```

Fricas [F]

$$\int \sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx))\,dx = \int \sqrt{cdx+d}\sqrt{-cfx+f}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)*(a+b*arccos(c*x)),x, algorithm=
"fricas")
```

output
```
integral(sqrt(c*d*x + d)*sqrt(-c*f*x + f)*(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx))\,dx$$
$$=\int \sqrt{d\,(cx+1)}\sqrt{-f\,(cx-1)}(a+b\operatorname{acos}{(cx)})\;dx$$

input
```
integrate((c*d*x+d)**(1/2)*(-c*f*x+f)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
Integral(sqrt(d*(c*x + 1))*sqrt(-f*(c*x - 1))*(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx))\,dx$$
$$=\int \sqrt{cdx+d}\sqrt{-cfx+f}(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)*(a+b*arccos(c*x)),x, algorithm=
"maxima")
```

output
```
b*sqrt(d)*sqrt(f)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x
+ 1)*sqrt(-c*x + 1), c*x), x) + 1/2*(sqrt(-c^2*d*f*x^2 + d*f)*x + d*f*arcs
in(c*x)/(sqrt(d*f)*c))*a
```

Giac [F]

$$\int \sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx))\,dx = \int \sqrt{cdx+d}\sqrt{-cfx+f}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)*(a+b*arccos(c*x)),x, algorithm=
"giac")
```

output
```
integrate(sqrt(c*d*x + d)*sqrt(-c*f*x + f)*(b*arccos(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx))\,dx = \int (a+b\,\mathrm{acos}(c\,x))\,\sqrt{d+c\,d\,x}\,\sqrt{f-c\,f\,x}\,dx$$

input
```
int((a + b*acos(c*x))*(d + c*d*x)^(1/2)*(f - c*f*x)^(1/2),x)
```

output
```
int((a + b*acos(c*x))*(d + c*d*x)^(1/2)*(f - c*f*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx))\,dx = \frac{\sqrt{f}\,\sqrt{d}\left(-2asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a+\sqrt{cx+1}\,\sqrt{-cx+1}\,acx+2\left(\int \sqrt{cx+1}\,\sqrt{-cx+1}\,acos(cx)\,dx\right)bc\right)}{2c}$$

input
```
int((c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*( - 2*asin(sqrt( - c*x + 1)/sqrt(2))*a + sqrt(c*x + 1)*sq
rt( - c*x + 1)*a*c*x + 2*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x),x)*b
*c))/(2*c)
```

3.509 $\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 141

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=-\frac{bfx\sqrt{1-c^2x^2}}{\sqrt{d+cdx}\sqrt{f-cfx}}+\frac{f(1-c^2x^2)\,(a+b\arccos(cx))}{c\sqrt{d+cdx}\sqrt{f-cfx}}+\frac{f\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2bc\sqrt{d+cdx}\sqrt{f-cfx}}$$

output
```
-b*f*x*(-c^2*x^2+1)^(1/2)/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+f*(-c^2*x^2+1)*
(a+b*arccos(c*x))/c/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+1/2*f*(-c^2*x^2+1)^(1
/2)*(a+b*arccos(c*x))^2/b/c/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx$$

$$=\frac{\frac{2\sqrt{d+cdx}\sqrt{f-cfx}\left(bcx+a\sqrt{1-c^2x^2}\right)}{\sqrt{1-c^2x^2}}+2b\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx)-\frac{b\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx)^2}{\sqrt{1-c^2x^2}}-2a\sqrt{d}\sqrt{f}\text{ ar}}{2cd}$$

input
```
Integrate[(Sqrt[f - c*f*x]*(a + b*ArcCos[c*x]))/Sqrt[d + c*d*x],x]
```

output
```
((2*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(b*c*x + a*Sqrt[1 - c^2*x^2]))/Sqrt[1
- c^2*x^2] + 2*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x] - (b*Sqrt[d +
 c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]^2)/Sqrt[1 - c^2*x^2] - 2*a*Sqrt[d]*Sqr
t[f]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d]*Sqrt[f]*(-1 + c
^2*x^2))])/(2*c*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{\sqrt{cdx+d}}dx$$

$$\Big\downarrow \text{ 5179}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{f(1-cx)(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f\sqrt{1-c^2x^2}\int \frac{(1-cx)(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \text{ 5263}$$

$$\frac{f\sqrt{1-c^2x^2}\int \left(\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}-\frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f\sqrt{1-c^2x^2}\left(\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}-\frac{(a+b\arccos(cx))^2}{2bc}+bx\right)}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

input
```
Int[(Sqrt[f - c*f*x]*(a + b*ArcCos[c*x]))/Sqrt[d + c*d*x],x]
```

output
```
(f*Sqrt[1 - c^2*x^2]*(b*x + (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c - (a
 + b*ArcCos[c*x])^2/(2*b*c)))/(Sqrt[d + c*d*x]*Sqrt[f - c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.18

method	result
default	$\frac{a\sqrt{-cfx+f}\,\sqrt{cdx+d}}{dc}+\frac{af\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^{2}df}\,x}{\sqrt{-c^{2}df\,x^{2}+df}}\right)}{\sqrt{-cfx+f}\,\sqrt{cdx+d}\,\sqrt{c^{2}df}}+b\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^{2}x^{2}+1}\,\arccos(cx)}{2(cx+1)dc(cx-1)}\right.$
parts	$\frac{a\sqrt{-cfx+f}\,\sqrt{cdx+d}}{dc}+\frac{af\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^{2}df}\,x}{\sqrt{-c^{2}df\,x^{2}+df}}\right)}{\sqrt{-cfx+f}\,\sqrt{cdx+d}\,\sqrt{c^{2}df}}+b\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^{2}x^{2}+1}\,\arccos(cx)}{2(cx+1)dc(cx-1)}\right.$

input
```
int((-c*f*x+f)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
a/d/c*(-c*f*x+f)^(1/2)*(c*d*x+d)^(1/2)+a*f*((-c*f*x+f)*(c*d*x+d))^(1/2)/(-
c*f*x+f)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*f)^(1/2)*arctan((c^2*d*f)^(1/2)*x/(-
c^2*d*f*x^2+d*f)^(1/2))+b*(1/2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*
x^2+1)^(1/2)/(c*x+1)/d/c/(c*x-1)*arccos(c*x)^2+1/2*(d*(c*x+1))^(1/2)*(-f*(
c*x-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)+I)/(c*x+1)
/d/c/(c*x-1)+1/2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/
2)*x*c+c^2*x^2-1)*(arccos(c*x)-I)/(c*x+1)/d/c/(c*x-1))
```

Fricas [F]

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=\int \frac{\sqrt{-cfx+f}(b\arccos{(cx)}+a)}{\sqrt{cdx+d}}\,dx$$

input
```
integrate((-c*f*x+f)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(sqrt(-c*f*x + f)*(b*arccos(c*x) + a)/sqrt(c*d*x + d), x)
```

Sympy [F]

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=\int \frac{\sqrt{-f\,(cx-1)}(a+b\,\mathrm{acos}\,(cx))}{\sqrt{d\,(cx+1)}}\,dx$$

input
```
integrate((-c*f*x+f)**(1/2)*(a+b*acos(c*x))/(c*d*x+d)**(1/2),x)
```

output
```
Integral(sqrt(-f*(c*x - 1))*(a + b*acos(c*x))/sqrt(d*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=\int \frac{\sqrt{-cfx+f}(b\arccos\,(cx)+a)}{\sqrt{cdx+d}}\,dx$$

input
```
integrate((-c*f*x+f)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2),x, algorithm=
"maxima")
```

output
```
a*(f*arcsin(c*x)/(c*d*sqrt(f/d)) + sqrt(-c^2*d*f*x^2 + d*f)/(c*d)) + b*sqr
t(f)*integrate(sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/s
qrt(c*x + 1), x)/sqrt(d)
```

Giac [F]

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=\int \frac{\sqrt{-cfx+f}(b\arccos\,(cx)+a)}{\sqrt{cdx+d}}\,dx$$

input
```
integrate((-c*f*x+f)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(-c*f*x + f)*(b*arccos(c*x) + a)/sqrt(c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{f-c\,f\,x}}{\sqrt{d+c\,d\,x}}\,dx$$

input
```
int(((a + b*acos(c*x))*(f - c*f*x)^(1/2))/(d + c*d*x)^(1/2),x)
```

output
```
int(((a + b*acos(c*x))*(f - c*f*x)^(1/2))/(d + c*d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx = \frac{\sqrt{f}\left(-2asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a+\sqrt{cx+1}\,\sqrt{-cx+1}\,a+\left(\int \frac{\sqrt{-cx+1}\,acos(cx)}{\sqrt{cx+1}}dx\right)bc\right)}{\sqrt{d}\,c}$$

input
```
int((-c*f*x+f)^(1/2)*(a+b*acos(c*x))/(c*d*x+d)^(1/2),x)
```

output
```
(sqrt(f)*( - 2*asin(sqrt( - c*x + 1)/sqrt(2))*a + sqrt(c*x + 1)*sqrt( - c*
x + 1)*a + int((sqrt( - c*x + 1)*acos(c*x))/sqrt(c*x + 1),x)*b*c))/(sqrt(d
)*c)
```

3.510 $\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 162

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx = -\frac{2f^2(1-cx)\left(1-c^2x^2\right)(a+b\arccos(cx))}{c(d+cdx)^{3/2}(f-cfx)^{3/2}} - \frac{f^2\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2}{2bc(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{2bf^2\left(1-c^2x^2\right)^{3/2}\log(1+cx)}{c(d+cdx)^{3/2}(f-cfx)^{3/2}}$$

output
```
-2*f^2*(-c*x+1)*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(3/2)/(-c*f*x+f
)^(3/2)-1/2*f^2*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))^2/b/c/(c*d*x+d)^(3/2)
/(-c*f*x+f)^(3/2)+2*b*f^2*(-c^2*x^2+1)^(3/2)*ln(c*x+1)/c/(c*d*x+d)^(3/2)/(
-c*f*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx = -\frac{\frac{4a\sqrt{d+cdx}\sqrt{f-cfx}}{1+cx} - 2a\sqrt{d}\sqrt{f}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{f-cfx}}{\sqrt{d}\sqrt{f}(-1+c^2x^2)}\right) + \frac{b(-1+cx)\sqrt{d+cdx}\sqrt{f-cfx}\cot\left(\frac{1}{2}\arccos(cx)\right)\left(-4\arccos(cx)+\cot\left(\right.\right.}{(1+cx)\sqrt{1-c^2}}}{2cd^2}$$

input
```
Integrate[(Sqrt[f - c*f*x]*(a + b*ArcCos[c*x]))/(d + c*d*x)^(3/2),x]
```

output
```
-1/2*((4*a*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(1 + c*x) - 2*a*Sqrt[d]*Sqrt[f
]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d]*Sqrt[f]*(-1 + c^2*
x^2))] + (b*(-1 + c*x)*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*Cot[ArcCos[c*x]/2]*
(-4*ArcCos[c*x] + Cot[ArcCos[c*x]/2]*(ArcCos[c*x]^2 - 8*Log[Cos[ArcCos[c*x
]/2]])))/((1 + c*x)*Sqrt[1 - c^2*x^2]))/(c*d^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.65, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(cdx+d)^{3/2}}dx$$

$$\Big\downarrow \text{ 5179}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\int \frac{f^2(1-cx)^2(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f^2\left(1-c^2x^2\right)^{3/2}\int \frac{(1-cx)^2(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{ 5275}$$

$$\frac{f^2\left(1-c^2x^2\right)^{3/2}\int \left(\frac{2(1-cx)(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}-\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}\right)dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f^2\left(1-c^2x^2\right)^{3/2}\left(-\frac{2(1-cx)(a+b\arccos(cx))}{c\sqrt{1-c^2x^2}}+\frac{(a+b\arccos(cx))^2}{2bc}-\frac{2b\log(cx+1)}{c}\right)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

input
```
Int[(Sqrt[f - c*f*x]*(a + b*ArcCos[c*x]))/(d + c*d*x)^(3/2),x]
```

output
```
(f^2*(1 - c^2*x^2)^(3/2)*((-2*(1 - c*x)*(a + b*ArcCos[c*x]))/(c*Sqrt[1 - c
^2*x^2]) + (a + b*ArcCos[c*x])^2/(2*b*c) - (2*b*Log[1 + c*x])/c))/((d + c*
d*x)^(3/2)*(f - c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.10 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.99

method	result
default	$-\frac{\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,b\arccos(cx)^2}{2(cx+1)d^2(cx-1)c}-\frac{\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,a\arccos(cx)}{(cx+1)d^2(cx-1)c}-\frac{4i\sqrt{d(cx+1)}\,\sqrt{-f(c}{(cx+}$

input
```
int((-c*f*x+f)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x+1)/d^2/(
c*x-1)/c*b*arccos(c*x)^2-(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)
^(1/2)/(c*x+1)/d^2/(c*x-1)/c*a*arccos(c*x)-4*I*(d*(c*x+1))^(1/2)*(-f*(c*x-
1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x+1)/d^2/(c*x-1)/c*b*arccos(c*x)-2*(d*(c*x
+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x-1)*(a+b*arccos(c*
x))/(c*x+1)/d^2/(c*x-1)/c+4*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2
+1)^(1/2)/(c*x+1)/d^2/(c*x-1)/c*b*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx=\int \frac{\sqrt{-cfx+f}(b\arccos(cx)+a)}{(cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(3/2),x, algorithm=
"fricas")
```

output
```
integral(sqrt(c*d*x + d)*sqrt(-c*f*x + f)*(b*arccos(c*x) + a)/(c^2*d^2*x^2
 + 2*c*d^2*x + d^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx=\int \frac{\sqrt{-f(cx-1)}(a+b\operatorname{acos}(cx))}{(d(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)**(1/2)*(a+b*acos(c*x))/(c*d*x+d)**(3/2),x)
```

output
```
Integral(sqrt(-f*(c*x - 1))*(a + b*acos(c*x))/(d*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx = \int \frac{\sqrt{-cfx+f}(b\arccos{(cx)}+a)}{(cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(3/2),x, algorithm=
"maxima")
```

output
```
-a*(2*sqrt(-c^2*d*f*x^2 + d*f)/(c^2*d^2*x + c*d^2) + f*arcsin(c*x)/(c*d^2*
sqrt(f/d))) + b*sqrt(f)*integrate(sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x)/((c*d*x + d)*sqrt(c*x + 1)), x)/sqrt(d)
```

Giac [F]

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx = \int \frac{\sqrt{-cfx+f}(b\arccos{(cx)}+a)}{(cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(3/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(-c*f*x + f)*(b*arccos(c*x) + a)/(c*d*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{f-c\,f\,x}}{{(d+c\,d\,x)}^{3/2}}\,dx$$

input
```
int(((a + b*acos(c*x))*(f - c*f*x)^(1/2))/(d + c*d*x)^(3/2),x)
```

output
```
int(((a + b*acos(c*x))*(f - c*f*x)^(1/2))/(d + c*d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{f-cfx}\,(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx = \frac{\sqrt{f}\left(2\sqrt{cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a-2\sqrt{-cx+1}\,a+\sqrt{cx+1}\left(\int \frac{\sqrt{-cx+}}{\sqrt{cx+1}\,c}\right.\right.}{\sqrt{d}\,\sqrt{cx+1}\,cd}$$

input
```
int((-c*f*x+f)^(1/2)*(a+b*acos(c*x))/(c*d*x+d)^(3/2),x)
```

output
```
(sqrt(f)*(2*sqrt(c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a - 2*sqrt( - c*x
 + 1)*a + sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x))/(sqrt(c*x + 1)*c*
x + sqrt(c*x + 1)),x)*b*c))/(sqrt(d)*sqrt(c*x + 1)*c*d)
```

3.511 $\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 163

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx=-\frac{2bf^3(1-c^2x^2)^{5/2}}{3c(1+cx)(d+cdx)^{5/2}(f-cfx)^{5/2}}-\frac{f^3(1-cx)^3\,(1-c^2x^2)\,(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}}-\frac{bf^3(1-c^2x^2)^{5/2}\log(1+cx)}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}}$$

output
```
-2/3*b*f^3*(-c^2*x^2+1)^(5/2)/c/(c*x+1)/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)-1
/3*f^3*(-c*x+1)^3*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*f*x
+f)^(5/2)-1/3*b*f^3*(-c^2*x^2+1)^(5/2)*ln(c*x+1)/c/(c*d*x+d)^(5/2)/(-c*f*x
+f)^(5/2)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx=-\frac{f\sqrt{d+cdx}\left((-1+cx)\left(-a+acx+b\sqrt{1-c^2x^2}\right)+b(-1+cx)^2\arccos(cx)-b(1+cx)\sqrt{1-c^2x^2}\log(-\right.}{3cd^3(1+cx)^2\sqrt{f-cfx}}$$

input
```
Integrate[(Sqrt[f - c*f*x]*(a + b*ArcCos[c*x]))/(d + c*d*x)^(5/2),x]
```

output
```
-1/3*(f*Sqrt[d + c*d*x]*((-1 + c*x)*(-a + a*c*x + b*Sqrt[1 - c^2*x^2]) + b
*(-1 + c*x)^2*ArcCos[c*x] - b*(1 + c*x)*Sqrt[1 - c^2*x^2]*Log[-(f*(1 + c*x
))]))/(c*d^3*(1 + c*x)^2*Sqrt[f - c*f*x])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {5179, 27, 5261, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(cdx+d)^{5/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{5/2}\int \frac{f^3(1-cx)^3(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^3(1-c^2x^2)^{5/2}\int \frac{(1-cx)^3(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{5261}$$

$$\frac{f^3(1-c^2x^2)^{5/2}\left(bc\int -\frac{(1-cx)^3}{3c(1-c^2x^2)^2}dx-\frac{(1-cx)^3(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^3(1-c^2x^2)^{5/2}\left(-\frac{1}{3}b\int \frac{(1-cx)^3}{(1-c^2x^2)^2}dx-\frac{(1-cx)^3(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{456}$$

$$\frac{f^3\left(1-c^2x^2\right)^{5/2}\left(-\frac{1}{3}b\int\frac{1-cx}{(cx+1)^2}dx-\frac{(1-cx)^3(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{f^3\left(1-c^2x^2\right)^{5/2}\left(-\frac{1}{3}b\int\left(\frac{2}{(cx+1)^2}+\frac{1}{-cx-1}\right)dx-\frac{(1-cx)^3(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^3\left(1-c^2x^2\right)^{5/2}\left(-\frac{(1-cx)^3(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}-\frac{1}{3}b\left(-\frac{2}{c(cx+1)}-\frac{\log(cx+1)}{c}\right)\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

input
```
Int[(Sqrt[f - c*f*x]*(a + b*ArcCos[c*x]))/(d + c*d*x)^(5/2),x]
```

output
```
(f^3*(1 - c^2*x^2)^(5/2)*(-1/3*((1 - c*x)^3*(a + b*ArcCos[c*x]))/(c*(1 - c
^2*x^2)^(3/2)) - (b*(-2/(c*(1 + c*x)) - Log[1 + c*x]/c))/3))/((d + c*d*x)^
(5/2)*(f - c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5261
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p,
 x]}, Simp[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[1/Sqrt[1 - c^2*x^2]
   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IG
tQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.75

method	result
default	$a\left(-\frac{\sqrt{-cfx+f}}{dc\,(cdx+d)^{\frac{3}{2}}}-f\left(-\frac{\sqrt{-cfx+f}}{3fdc\,(cdx+d)^{\frac{3}{2}}}-\frac{\sqrt{-cfx+f}}{3cf\,d^{2}\sqrt{cdx+d}}\right)\right)+\frac{b\sqrt{-f(cx-1)}\,\sqrt{d(cx+1)}\,\left(c^{2}x^{2}-2cx+1+i\sqrt{-c^{2}x^{2}+1}\right.}{}$
parts	$a\left(-\frac{\sqrt{-cfx+f}}{dc\,(cdx+d)^{\frac{3}{2}}}-f\left(-\frac{\sqrt{-cfx+f}}{3fdc\,(cdx+d)^{\frac{3}{2}}}-\frac{\sqrt{-cfx+f}}{3cf\,d^{2}\sqrt{cdx+d}}\right)\right)+\frac{b\sqrt{-f(cx-1)}\,\sqrt{d(cx+1)}\,\left(c^{2}x^{2}-2cx+1+i\sqrt{-c^{2}x^{2}+1}\right.}{}$

input
```
int((-c*f*x+f)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
a*(-1/d/c*(-c*f*x+f)^(1/2)/(c*d*x+d)^(3/2)-f*(-1/3/f/d/c/(c*d*x+d)^(3/2)*(
-c*f*x+f)^(1/2)-1/3/c/f/d^2/(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)))+1/3*b*(-f*(
c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(c^2*x^2-2*c*x+1+I*(-c^2*x^2+1)^(1/2)*c*x+
I*(-c^2*x^2+1)^(1/2))*(I*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*x^3*c^3+(-c^2*x^2+
1)^(1/2)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*x^2*c^2+3*I*ln(1+c*x+I*(-c^2*x^2+1
)^(1/2))*x^2*c^2+3*c^2*x^2*arccos(c*x)+I*c^2*x^2+3*I*ln(1+c*x+I*(-c^2*x^2+
1)^(1/2))*x*c+c*x*(-c^2*x^2+1)^(1/2)+2*I*c*x-(-c^2*x^2+1)^(1/2)*ln(1+c*x+I
*(-c^2*x^2+1)^(1/2))+I*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+arccos(c*x)-(-c^2*x^
2+1)^(1/2)+I)/(3*c^4*x^4+6*c^3*x^3+4*c^2*x^2+2*c*x+1)/d^3/c/(c*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.19

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx = \left[\frac{(bc^3dx^3+bc^2dx^2-bcdx-bd)\sqrt{\frac{f}{d}}\log\left(\frac{c^6fx^6+4\,c^5fx^5+5\,c^4fx^4-4\,c^2fx^2-4}{}\right.}{}\right.$$

input
```
integrate((-c*f*x+f)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2),x, algorithm=
"fricas")
```

output
```
[1/6*((b*c^3*d*x^3 + b*c^2*d*x^2 - b*c*d*x - b*d)*sqrt(f/d)*log((c^6*f*x^6
 + 4*c^5*f*x^5 + 5*c^4*f*x^4 - 4*c^2*f*x^2 - 4*c*f*x - (c^4*x^4 + 4*c^3*x^
3 + 6*c^2*x^2 + 4*c*x)*sqrt(-c^2*x^2 + 1)*sqrt(c*d*x + d)*sqrt(-c*f*x + f)
*sqrt(f/d) - 2*f)/(c^4*x^4 + 2*c^3*x^3 - 2*c*x - 1)) + 2*(a*c^2*x^2 + 2*sq
rt(-c^2*x^2 + 1)*b*c*x - 2*a*c*x + (b*c^2*x^2 - 2*b*c*x + b)*arccos(c*x) +
 a)*sqrt(c*d*x + d)*sqrt(-c*f*x + f))/(c^4*d^3*x^3 + c^3*d^3*x^2 - c^2*d^3
*x - c*d^3), 1/3*((b*c^3*d*x^3 + b*c^2*d*x^2 - b*c*d*x - b*d)*sqrt(-f/d)*a
rctan((c^2*x^2 + 2*c*x + 2)*sqrt(-c^2*x^2 + 1)*sqrt(c*d*x + d)*sqrt(-c*f*x
 + f)*sqrt(-f/d)/(c^4*f*x^4 + 2*c^3*f*x^3 - c^2*f*x^2 - 2*c*f*x)) + (a*c^2
*x^2 + 2*sqrt(-c^2*x^2 + 1)*b*c*x - 2*a*c*x + (b*c^2*x^2 - 2*b*c*x + b)*ar
ccos(c*x) + a)*sqrt(c*d*x + d)*sqrt(-c*f*x + f))/(c^4*d^3*x^3 + c^3*d^3*x^
2 - c^2*d^3*x - c*d^3)]
```

Sympy [F]

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx = \int \frac{\sqrt{-f\,(cx-1)}\,(a+b\,\mathrm{acos}\,(cx))}{(d\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)**(1/2)*(a+b*acos(c*x))/(c*d*x+d)**(5/2),x)
```

output
```
Integral(sqrt(-f*(c*x - 1))*(a + b*acos(c*x))/(d*(c*x + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.32

$$\begin{aligned}\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx &= \frac{1}{3}\,bc\left(\frac{2\,\sqrt{f}}{c^3d^{\frac{5}{2}}x+c^2d^{\frac{5}{2}}}+\frac{\sqrt{f}\log\left(cx+1\right)}{c^2d^{\frac{5}{2}}}\right)\\ &\quad -\frac{1}{3}\,b\left(\frac{2\,\sqrt{-c^2dfx^2+df}}{c^3d^3x^2+2\,c^2d^3x+cd^3}-\frac{\sqrt{-c^2dfx^2+df}}{c^2d^3x+cd^3}\right)\arccos\left(cx\right)\\ &\quad -\frac{1}{3}\,a\left(\frac{2\,\sqrt{-c^2dfx^2+df}}{c^3d^3x^2+2\,c^2d^3x+cd^3}-\frac{\sqrt{-c^2dfx^2+df}}{c^2d^3x+cd^3}\right)\end{aligned}$$

input
```
integrate((-c*f*x+f)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2),x, algorithm=
"maxima")
```

output
```
1/3*b*c*(2*sqrt(f)/(c^3*d^(5/2)*x + c^2*d^(5/2)) + sqrt(f)*log(c*x + 1)/(c
^2*d^(5/2))) - 1/3*b*(2*sqrt(-c^2*d*f*x^2 + d*f)/(c^3*d^3*x^2 + 2*c^2*d^3*
x + c*d^3) - sqrt(-c^2*d*f*x^2 + d*f)/(c^2*d^3*x + c*d^3))*arccos(c*x) - 1
/3*a*(2*sqrt(-c^2*d*f*x^2 + d*f)/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3) - sqr
t(-c^2*d*f*x^2 + d*f)/(c^2*d^3*x + c*d^3))
```

Giac [F]

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx = \int \frac{\sqrt{-cfx+f}(b\arccos{(cx)}+a)}{(cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(-c*f*x + f)*(b*arccos(c*x) + a)/(c*d*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{f-c\,f\,x}}{(d+c\,d\,x)^{5/2}}\,dx$$

input
```
int(((a + b*acos(c*x))*(f - c*f*x)^(1/2))/(d + c*d*x)^(5/2),x)
```

output
```
int(((a + b*acos(c*x))*(f - c*f*x)^(1/2))/(d + c*d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{f-cfx}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx = \frac{\sqrt{f}\left(\sqrt{-cx+1}\,acx-\sqrt{-cx+1}\,a+3\sqrt{cx+1}\left(\int \frac{\sqrt{-cx+1}\,acos(c}{\sqrt{cx+1}\,c^2x^2+2\sqrt{cx+1}\,c}\right.\right.}{3\sqrt{d}\,\sqrt{cx+1}\,c}$$

input
```
int((-c*f*x+f)^(1/2)*(a+b*acos(c*x))/(c*d*x+d)^(5/2),x)
```

output
```
(sqrt(f)*(sqrt( - c*x + 1)*a*c*x - sqrt( - c*x + 1)*a + 3*sqrt(c*x + 1)*in
t((sqrt( - c*x + 1)*acos(c*x))/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*
c*x + sqrt(c*x + 1)),x)*b*c**2*x + 3*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*a
cos(c*x))/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),
x)*b*c))/(3*sqrt(d)*sqrt(c*x + 1)*c*d**2*(c*x + 1))
```

3.512 $\int (d+cdx)^{5/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 411

$$\int (d+cdx)^{5/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx = \frac{bdx(d+cdx)^{3/2}(f-cfx)^{3/2}}{5(1-c^2x^2)^{3/2}} - \frac{3bcdx^2(d+cdx)^{3/2}(f-cfx)^{3/2}}{16(1-c^2x^2)^{3/2}} - \frac{2bc^2dx^3(d+cdx)^{3/2}(f-cfx)^{3/2}}{15(1-c^2x^2)^{3/2}} + \frac{bc^4dx^5(d+cdx)^{3/2}(f-cfx)^{3/2}}{25(1-c^2x^2)^{3/2}} + \frac{bd(d+cdx)^{3/2}(f-cfx)^{3/2}\sqrt{1-c^2x^2}}{16c} + \frac{1}{4}dx(d+cdx)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx)) + \frac{3dx(d+cdx)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx))}{8(1-c^2x^2)} - \frac{d(d+cd}{}$$

output

```
1/5*b*d*x*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)/(-c^2*x^2+1)^(3/2)-3/16*b*c*d*x
^2*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)/(-c^2*x^2+1)^(3/2)-2/15*b*c^2*d*x^3*(c
*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)/(-c^2*x^2+1)^(3/2)+1/25*b*c^4*d*x^5*(c*d*x+
d)^(3/2)*(-c*f*x+f)^(3/2)/(-c^2*x^2+1)^(3/2)+1/16*b*d*(c*d*x+d)^(3/2)*(-c*
f*x+f)^(3/2)*(-c^2*x^2+1)^(1/2)/c+1/4*d*x*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)
*(a+b*arccos(c*x))+3*d*x*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x)
)/(-8*c^2*x^2+8)-1/5*d*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)*(-c^2*x^2+1)*(a+b*
arccos(c*x))/c+3/16*d*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x))^2
/b/c/(-c^2*x^2+1)^(3/2)
```

Mathematica [A] (verified)

Time = 2.53 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.74

$$\int (d+cdx)^{5/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx =$$
$$-\frac{d^2f\left(1800b\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx)^2+3600a\sqrt{d}\sqrt{f}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{f-cfx}}{\sqrt{d}\sqrt{f}(-1+c^2x^2)}\right)+\sqrt{d+c}\right.}{}$$

input
```
Integrate[(d + c*d*x)^(5/2)*(f - c*f*x)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
-1/9600*(d^2*f*(1800*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]^2 + 360
0*a*Sqrt[d]*Sqrt[f]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f -
 c*f*x])/(Sqrt[d]*Sqrt[f]*(-1 + c^2*x^2))] + Sqrt[d + c*d*x]*Sqrt[f - c*f*
x]*(128*b*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) + 240*a*Sqrt[1 - c^2*x^2]*(8 -
 25*c*x - 16*c^2*x^2 + 10*c^3*x^3 + 8*c^4*x^4) - 1200*b*Cos[2*ArcCos[c*x]]
 + 75*b*Cos[4*ArcCos[c*x]]) + 60*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[
c*x]*(32*(1 - c^2*x^2)^(5/2) - 40*Sin[2*ArcCos[c*x]] + 5*Sin[4*ArcCos[c*x]
])))/(c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^{5/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(cdx+d)^{3/2}(f-cfx)^{3/2}\int d(cx+1)\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d(cdx+d)^{3/2}(f-cfx)^{3/2}\int(cx+1)\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx}{(1-c^2x^2)^{3/2}}$$

$\Big\downarrow$ 5263

$$\frac{d(cdx+d)^{3/2}(f-cfx)^{3/2}\int\left(cx(a+b\arccos(cx))\left(1-c^2x^2\right)^{3/2}+(a+b\arccos(cx))\left(1-c^2x^2\right)^{3/2}\right)dx}{(1-c^2x^2)^{3/2}}$$

$\Big\downarrow$ 2009

$$\frac{d(cdx+d)^{3/2}(f-cfx)^{3/2}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{8}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(1-c^2x^2)^{5/2}(a}{5c}\right.}{(1-c^2x^2)^{3/2}}$$

input
```
Int[(d + c*d*x)^(5/2)*(f - c*f*x)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d*(d + c*d*x)^(3/2)*(f - c*f*x)^(3/2)*(-1/5*(b*x) + (5*b*c*x^2)/16 + (2*b
*c^2*x^3)/15 - (b*c^3*x^4)/16 - (b*c^4*x^5)/25 + (3*x*Sqrt[1 - c^2*x^2]*(a
 + b*ArcCos[c*x]))/8 + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 - ((1
 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c) - (3*(a + b*ArcCos[c*x])^2)/(
16*b*c)))/(1 - c^2*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 1191, normalized size of antiderivative = 2.90

method	result	size
default	Expression too large to display	1191
parts	Expression too large to display	1191

input
```
int((c*d*x+d)^(5/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVER
BOSE)
```

output
```
-1/5*a/c/f*(c*d*x+d)^(5/2)*(-c*f*x+f)^(5/2)-1/4*a*d/c/f*(c*d*x+d)^(3/2)*(-
c*f*x+f)^(5/2)-1/4*a*d^2/c/f*(c*d*x+d)^(1/2)*(-c*f*x+f)^(5/2)+1/8*a*d^2/c*
(-c*f*x+f)^(3/2)*(c*d*x+d)^(1/2)+3/8*a*d^2*f/c*(-c*f*x+f)^(1/2)*(c*d*x+d)^
(1/2)+3/8*a*d^3*f^2*((-c*f*x+f)*(c*d*x+d))^(1/2)/(-c*f*x+f)^(1/2)/(c*d*x+d
)^(1/2)/(c^2*d*f)^(1/2)*arctan((c^2*d*f)^(1/2)*x/(-c^2*d*f*x^2+d*f)^(1/2))
+b*(3/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-
1)/c*arccos(c*x)^2*d^2*f-1/800*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(16*c^
6*x^6-28*c^4*x^4+16*I*(-c^2*x^2+1)^(1/2)*x^5*c^5+13*c^2*x^2-20*I*(-c^2*x^2
+1)^(1/2)*x^3*c^3+5*I*(-c^2*x^2+1)^(1/2)*c*x-1)*(I+5*arccos(c*x))*d^2*f/(c
^2*x^2-1)/c-1/256*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(8*c^5*x^5-12*c^3*x
^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(
-c^2*x^2+1)^(1/2))*(I+4*arccos(c*x))*d^2*f/(c^2*x^2-1)/c-1/16*(d*(c*x+1))^
(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x
)-I)*d^2*f/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-2*I*(
-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arcc
os(c*x))*d^2*f/(c^2*x^2-1)/c-1/1200*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(
-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(11*I+45*arccos(c*x))*cos(4*arccos(c*
x))*d^2*f/(c^2*x^2-1)/c-1/600*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(I*c^2*
x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(7*I+15*arccos(c*x))*sin(4*arccos(c*x))*d^2*
f/(c^2*x^2-1)/c-3/256*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^...
```

Fricas [F]

$$\int (d+cdx)^{5/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx=\int (cdx+d)^{\frac{5}{2}}(-cfx+f)^{\frac{3}{2}}(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x)),x, algorithm=
"fricas")
```

output
```
integral(-(a*c^3*d^2*f*x^3 + a*c^2*d^2*f*x^2 - a*c*d^2*f*x - a*d^2*f + (b*
c^3*d^2*f*x^3 + b*c^2*d^2*f*x^2 - b*c*d^2*f*x - b*d^2*f)*arccos(c*x))*sqrt
(c*d*x + d)*sqrt(-c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+cdx)^{5/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+d)**(5/2)*(-c*f*x+f)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+cdx)^{5/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx = \int (cdx+d)^{\frac{5}{2}}(-cfx+f)^{\frac{3}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x)),x, algorithm=
"maxima")
```

output
```
b*sqrt(d)*sqrt(f)*integrate(-(c^3*d^2*f*x^3 + c^2*d^2*f*x^2 - c*d^2*f*x -
d^2*f)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1),
c*x), x) + 1/40*(15*sqrt(-c^2*d*f*x^2 + d*f)*d^2*f*x + 15*d^3*f^2*arcsin(c
*x)/(sqrt(d*f)*c) + 10*(-c^2*d*f*x^2 + d*f)^(3/2)*d*x - 8*(-c^2*d*f*x^2 +
d*f)^(5/2)/(c*f))*a
```

Giac [F]

$$\int (d+cdx)^{5/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx=\int (cdx+d)^{\frac{5}{2}}(-cfx+f)^{\frac{3}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x)),x, algorithm=
"giac")
```

output
```
integrate((c*d*x + d)^(5/2)*(-c*f*x + f)^(3/2)*(b*arccos(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)^{5/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,\left(d+c\,d\,x\right)^{5/2}\,\left(f-c\,f\,x\right)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))*(d + c*d*x)^(5/2)*(f - c*f*x)^(3/2),x)
```

output
```
int((a + b*acos(c*x))*(d + c*d*x)^(5/2)*(f - c*f*x)^(3/2), x)
```

Reduce [F]

$$\int (d+cdx)^{5/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx=\frac{\sqrt{f}\,\sqrt{d}\,d^2 f\left(-30\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a-8\sqrt{cx+1}\,\sqrt{-cx+1}\,a\,c^4x^4-10\sqrt{cx+\right.}{}$$

input
```
int((c*d*x+d)^(5/2)*(-c*f*x+f)^(3/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*d**2*f*( - 30*asin(sqrt( - c*x + 1)/sqrt(2))*a - 8*sqrt(c
*x + 1)*sqrt( - c*x + 1)*a*c**4*x**4 - 10*sqrt(c*x + 1)*sqrt( - c*x + 1)*a
*c**3*x**3 + 16*sqrt(c*x + 1)*sqrt( - c*x + 1)*a*c**2*x**2 + 25*sqrt(c*x +
 1)*sqrt( - c*x + 1)*a*c*x - 8*sqrt(c*x + 1)*sqrt( - c*x + 1)*a - 40*int(s
qrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**3,x)*b*c**4 - 40*int(sqrt(c*x +
 1)*sqrt( - c*x + 1)*acos(c*x)*x**2,x)*b*c**3 + 40*int(sqrt(c*x + 1)*sqrt(
 - c*x + 1)*acos(c*x)*x,x)*b*c**2 + 40*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*
acos(c*x),x)*b*c))/(40*c)
```

3.513 $\int (d+cdx)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 223

$$\int (d+cdx)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx = -\frac{3bcx^2(d+cdx)^{3/2}(f-cfx)^{3/2}}{16(1-c^2x^2)^{3/2}}+\frac{b(d+cdx)^{3/2}(f-cfx)^{3/2}\sqrt{1-c^2x^2}}{16c}$$
$$+\frac{1}{4}x(d+cdx)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx))+\frac{3x(d+cdx)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx))}{8(1-c^2x^2)}+\frac{3(d+cdx)^{3}}{}$$

output
```
-3/16*b*c*x^2*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)/(-c^2*x^2+1)^(3/2)+1/16*b*(
c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)*(-c^2*x^2+1)^(1/2)/c+1/4*x*(c*d*x+d)^(3/2)
*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x))+3*x*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)*(
a+b*arccos(c*x))/(-8*c^2*x^2+8)+3/16*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)*(a+b
*arccos(c*x))^2/b/c/(-c^2*x^2+1)^(3/2)
```

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.11

$$\int (d+cdx)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx = \frac{-24bdf\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx)^2-48ad^{3/2}f^{3/2}\sqrt{1-c^2x^2}\arctan\left(\right.}{}$$

input
```
Integrate[(d + c*d*x)^(3/2)*(f - c*f*x)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(-24*b*d*f*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]^2 - 48*a*d^(3/2)*f^
(3/2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt
[d]*Sqrt[f]*(-1 + c^2*x^2))] + d*f*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(16*a*c
*x*(5 - 2*c^2*x^2)*Sqrt[1 - c^2*x^2] + 16*b*Cos[2*ArcCos[c*x]] - b*Cos[4*A
rcCos[c*x]]) - 4*b*d*f*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]*(-8*Sin
[2*ArcCos[c*x]] + Sin[4*ArcCos[c*x]]))/(128*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.67, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {5179, 5159, 244, 2009, 5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5179}$$

$$\frac{(cdx+d)^{3/2}(f-cfx)^{3/2}\int \left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{ 5159}$$

$$\frac{(cdx+d)^{3/2}(f-cfx)^{3/2}\left(\frac{3}{4}\int\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int x(1-c^2x^2)\,dx+\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\,\mathrm{a}\right.}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{244}$$

$$\frac{(cdx+d)^{3/2}(f-cfx)^{3/2}\left(\frac{3}{4}\int\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int(x-c^2x^3)\,dx+\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\,\mathrm{ar}\right.}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(cdx+d)^{3/2}(f-cfx)^{3/2}\left(\frac{3}{4}\int\sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{x^2}{2}-\right.\right.}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{5157}$$

$$\frac{(cdx+d)^{3/2}(f-cfx)^{3/2}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)+\frac{1}{4}x(1-c^2x^2\right.}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{(cdx+d)^{3/2}(f-cfx)^{3/2}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{4}bcx^2\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right.}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{5153}$$

$$\frac{(cdx+d)^{3/2}(f-cfx)^{3/2}\left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{(a+b\arccos(c}{4bc}\right.\right.}{(1-c^2x^2)^{3/2}}$$

input
```
Int[(d + c*d*x)^(3/2)*(f - c*f*x)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
((d + c*d*x)^(3/2)*(f - c*f*x)^(3/2)*((b*c*(x^2/2 - (c^2*x^4)/4))/4 + (x*(
1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 + (3*((b*c*x^2)/4 + (x*Sqrt[1 -
c^2*x^2]*(a + b*ArcCos[c*x]))/2 - (a + b*ArcCos[c*x])^2/(4*b*c)))/4))/(1 -
 c^2*x^2)^(3/2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.77

method	result
default	$-\frac{a(cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{5}{2}}}{4cf}-\frac{ad\sqrt{cdx+d}\,(-cfx+f)^{\frac{5}{2}}}{4cf}+\frac{ad(-cfx+f)^{\frac{3}{2}}\sqrt{cdx+d}}{8c}+\frac{3adf\sqrt{-cfx+f}\,\sqrt{cdx+d}}{8c}+\frac{3a\,d^{2}f^{2}\sqrt{(-}}{}$
parts	$-\frac{a(cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{5}{2}}}{4cf}-\frac{ad\sqrt{cdx+d}\,(-cfx+f)^{\frac{5}{2}}}{4cf}+\frac{ad(-cfx+f)^{\frac{3}{2}}\sqrt{cdx+d}}{8c}+\frac{3adf\sqrt{-cfx+f}\,\sqrt{cdx+d}}{8c}+\frac{3a\,d^{2}f^{2}\sqrt{(-}}{}$

input
```
int((c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVER
BOSE)
```

output
```
-1/4*a/c/f*(c*d*x+d)^(3/2)*(-c*f*x+f)^(5/2)-1/4*a*d/c/f*(c*d*x+d)^(1/2)*(-
c*f*x+f)^(5/2)+1/8*a*d/c*(-c*f*x+f)^(3/2)*(c*d*x+d)^(1/2)+3/8*a*d*f/c*(-c*
f*x+f)^(1/2)*(c*d*x+d)^(1/2)+3/8*a*d^2*f^2*((-c*f*x+f)*(c*d*x+d))^(1/2)/(-
c*f*x+f)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*f)^(1/2)*arctan((c^2*d*f)^(1/2)*x/(-
c^2*d*f*x^2+d*f)^(1/2))+b*(3/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2
*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^2*d*f-1/256*(d*(c*x+1))^(1/2)*(-f*
(c*x-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-
8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(I+4*arccos(c*x))*d*f
/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1
)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))*d
*f/(c^2*x^2-1)/c-3/256*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+
1)^(1/2)*x*c+c^2*x^2-1)*(5*I+12*arccos(c*x))*cos(3*arccos(c*x))*d*f/(c^2*x
^2-1)/c-1/256*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^
2+1)^(1/2)-I)*(17*I+28*arccos(c*x))*sin(3*arccos(c*x))*d*f/(c^2*x^2-1)/c)
```

Fricas [F]

$$\int (d+cdx)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx = \int (cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{3}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x)),x, algorithm=
"fricas")
```

output
```
integral(-(a*c^2*d*f*x^2 - a*d*f + (b*c^2*d*f*x^2 - b*d*f)*arccos(c*x))*sq
rt(c*d*x + d)*sqrt(-c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+cdx)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+d)**(3/2)*(-c*f*x+f)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+cdx)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx = \int (cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{3}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x)),x, algorithm=
"maxima")
```

output
```
b*sqrt(d)*sqrt(f)*integrate(-(c^2*d*f*x^2 - d*f)*sqrt(c*x + 1)*sqrt(-c*x +
 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x) + 1/8*(3*sqrt(-c^2*d*f*
x^2 + d*f)*d*f*x + 3*d^2*f^2*arcsin(c*x)/(sqrt(d*f)*c) + 2*(-c^2*d*f*x^2 +
 d*f)^(3/2)*x)*a
```

Giac [F]

$$\int (d+cdx)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx=\int (cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{3}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x)),x, algorithm=
"giac")
```

output
```
integrate((c*d*x + d)^(3/2)*(-c*f*x + f)^(3/2)*(b*arccos(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,(d+c\,d\,x)^{3/2}\,(f-c\,f\,x)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))*(d + c*d*x)^(3/2)*(f - c*f*x)^(3/2),x)
```

output
```
int((a + b*acos(c*x))*(d + c*d*x)^(3/2)*(f - c*f*x)^(3/2), x)
```

Reduce [F]

$$\int (d+cdx)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx = \frac{\sqrt{f}\,\sqrt{d}\,df\left(-6asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a-2\sqrt{cx+1}\,\sqrt{-cx+1}\,a\,c^3x^3+5\sqrt{cx+1}\,\sqrt{}\right.}{}$$

input
```
int((c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*d*f*( - 6*asin(sqrt( - c*x + 1)/sqrt(2))*a - 2*sqrt(c*x +
 1)*sqrt( - c*x + 1)*a*c**3*x**3 + 5*sqrt(c*x + 1)*sqrt( - c*x + 1)*a*c*x
- 8*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**2,x)*b*c**3 + 8*int(sq
rt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x),x)*b*c))/(8*c)
```

3.514 $\int \sqrt{d+cdx}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 273

$$\int \sqrt{d+cdx}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx = -\frac{bfx\sqrt{d+cdx}\sqrt{f-cfx}}{3\sqrt{1-c^2x^2}}-\frac{bcfx^2\sqrt{d+cdx}\sqrt{f-cfx}}{4\sqrt{1-c^2x^2}}+\frac{bc^2fx^3\sqrt{d+cdx}\sqrt{f-cfx}}{9\sqrt{1-c^2x^2}}+\frac{1}{2}fx\sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx))+\frac{f\sqrt{d+cdx}\sqrt{f-cfx}(1-c^2x^2)(a+b\arccos(cx))}{3c}+\frac{f\sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx))^2}{4bc\sqrt{1-c^2x^2}}$$

output
```
-1/3*b*f*x*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)/(-c^2*x^2+1)^(1/2)-1/4*b*c*f*x
^2*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)/(-c^2*x^2+1)^(1/2)+1/9*b*c^2*f*x^3*(c*
d*x+d)^(1/2)*(-c*f*x+f)^(1/2)/(-c^2*x^2+1)^(1/2)+1/2*f*x*(c*d*x+d)^(1/2)*(
-c*f*x+f)^(1/2)*(a+b*arccos(c*x))+1/3*f*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)*(
-c^2*x^2+1)*(a+b*arccos(c*x))/c+1/4*f*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)*(a+
b*arccos(c*x))^2/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.95

$$\int \sqrt{d+cdx}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx = \frac{-18bf\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx)^2-36a\sqrt{d}f^{3/2}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{f-cfx}}{\sqrt{d}\sqrt{f}(-1+c^2x^2)}\right)}{}$$

input
```
Integrate[Sqrt[d + c*d*x]*(f - c*f*x)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(-18*b*f*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]^2 - 36*a*Sqrt[d]*f^(3
/2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d
]*Sqrt[f]*(-1 + c^2*x^2))] + f*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(12*a*(2 +
3*c*x - 2*c^2*x^2)*Sqrt[1 - c^2*x^2] - 8*b*c*x*(-3 + c^2*x^2) + 9*b*Cos[2*
ArcCos[c*x]]) + 6*b*f*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]*(4*(1 -
c^2*x^2)^(3/2) + 3*Sin[2*ArcCos[c*x]]))/(72*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cdx+d}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\sqrt{cdx+d}\sqrt{f-cfx}\int f(1-cx)\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\sqrt{cdx+d}\sqrt{f-cfx}\int (1-cx)\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5263

$$\frac{f\sqrt{cdx+d}\sqrt{f-cfx}\int\left(\sqrt{1-c^2x^2}(a+b\arccos(cx))-cx\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)dx}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 2009

$$\frac{f\sqrt{cdx+d}\sqrt{f-cfx}\left(\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3c}-\frac{(a+b\arccos(cx))^2}{4bc}-\frac{1}{9}bc^2x^3+\frac{1}{4}\right.}{\sqrt{1-c^2x^2}}$$

input
```
Int[Sqrt[d + c*d*x]*(f - c*f*x)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(f*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*((b*x)/3 + (b*c*x^2)/4 - (b*c^2*x^3)/9
+ (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/2 + ((1 - c^2*x^2)^(3/2)*(a +
b*ArcCos[c*x]))/(3*c) - (a + b*ArcCos[c*x])^2/(4*b*c)))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 722, normalized size of antiderivative = 2.64

method	result
default	$-\frac{a\sqrt{cdx+d}\,(-cfx+f)^{\frac{5}{2}}}{3cf}+\frac{a(-cfx+f)^{\frac{3}{2}}\sqrt{cdx+d}}{6c}+\frac{af\sqrt{-cfx+f}\,\sqrt{cdx+d}}{2c}+\frac{ad\,f^{2}\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^{2}df}\,x}{\sqrt{-c^{2}df\,x^{2}}}\right.}{2\sqrt{-cfx+f}\,\sqrt{cdx+d}\,\sqrt{c^{2}df}}$
parts	$-\frac{a\sqrt{cdx+d}\,(-cfx+f)^{\frac{5}{2}}}{3cf}+\frac{a(-cfx+f)^{\frac{3}{2}}\sqrt{cdx+d}}{6c}+\frac{af\sqrt{-cfx+f}\,\sqrt{cdx+d}}{2c}+\frac{ad\,f^{2}\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^{2}df}\,x}{\sqrt{-c^{2}df\,x^{2}}}\right.}{2\sqrt{-cfx+f}\,\sqrt{cdx+d}\,\sqrt{c^{2}df}}$

input
```
int((c*d*x+d)^(1/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVER
BOSE)
```

output
```
-1/3*a/c/f*(c*d*x+d)^(1/2)*(-c*f*x+f)^(5/2)+1/6*a/c*(-c*f*x+f)^(3/2)*(c*d*
x+d)^(1/2)+1/2*a*f/c*(-c*f*x+f)^(1/2)*(c*d*x+d)^(1/2)+1/2*a*d*f^2*((-c*f*x
+f)*(c*d*x+d))^(1/2)/(-c*f*x+f)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*f)^(1/2)*arct
an((c^2*d*f)^(1/2)*x/(-c^2*d*f*x^2+d*f)^(1/2))+b*(1/4*(d*(c*x+1))^(1/2)*(-
f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^2*f-1/72*(d*
(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*I*(-c^2*x^2+1)^(1
/2)*x^3*c^3-3*I*(-c^2*x^2+1)^(1/2)*c*x+1)*(I+3*arccos(c*x))*f/(c^2*x^2-1)/
c+1/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(2*c^3*x^3-2*c*x+2*I*(-c^2*x^2
+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(I+2*arccos(c*x))*f/(c^2*x^2-1)/c+
1/8*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^
2-1)*(arccos(c*x)-I)*f/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(
1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x
)*(-I+2*arccos(c*x))*f/(c^2*x^2-1)/c-1/18*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(
1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(2*I+3*arccos(c*x))*cos(2*arcco
s(c*x))*f/(c^2*x^2-1)/c-1/36*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(I*c^2*x
^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(5*I+3*arccos(c*x))*sin(2*arccos(c*x))*f/(c^2
*x^2-1)/c)
```

Fricas [F]

$$\int \sqrt{d+cdx}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx=\int \sqrt{cdx+d}(-cfx+f)^{\frac{3}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x)),x, algorithm=
"fricas")
```

output
```
integral(-(a*c*f*x - a*f + (b*c*f*x - b*f)*arccos(c*x))*sqrt(c*d*x + d)*sq
rt(-c*f*x + f), x)
```

Sympy [F]

$$\int \sqrt{d+cdx}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx=\int \sqrt{d\,(cx+1)}(-f(cx-1))^{\frac{3}{2}}\,(a+b\,\mathrm{acos}\,(cx))\,\,dx$$

input
```
integrate((c*d*x+d)**(1/2)*(-c*f*x+f)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Integral(sqrt(d*(c*x + 1))*(-f*(c*x - 1))**(3/2)*(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \sqrt{d+cdx}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx=\int \sqrt{cdx+d}(-cfx+f)^{\frac{3}{2}}(b\arccos\,(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x)),x, algorithm=
"maxima")
```

output
```
b*sqrt(d)*sqrt(f)*integrate(-(c*f*x - f)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arct
an2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x) + 1/6*(3*sqrt(-c^2*d*f*x^2 + d*
f)*f*x + 3*d*f^2*arcsin(c*x)/(sqrt(d*f)*c) + 2*(-c^2*d*f*x^2 + d*f)^(3/2)/
(c*d))*a
```

Giac [F]

$$\int \sqrt{d+cdx}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx = \int \sqrt{cdx+d}(-cfx+f)^{\frac{3}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x)),x, algorithm=
"giac")
```

output
```
integrate(sqrt(c*d*x + d)*(-c*f*x + f)^(3/2)*(b*arccos(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+cdx}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx = \int (a+b\,\mathrm{acos}(c\,x))\,\sqrt{d+c\,d\,x}\,(f-c\,f\,x)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))*(d + c*d*x)^(1/2)*(f - c*f*x)^(3/2),x)
```

output
```
int((a + b*acos(c*x))*(d + c*d*x)^(1/2)*(f - c*f*x)^(3/2), x)
```

Reduce [F]

$$\int \sqrt{d+cdx}(f-cfx)^{3/2}(a+b\arccos(cx))\,dx = \frac{\sqrt{f}\,\sqrt{d}\,f\left(-6asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a-2\sqrt{cx+1}\,\sqrt{-cx+1}\,a\,c^2x^2+3\sqrt{cx+1}\,\sqrt{-cx+1}\,acx\right.}{}$$

input
```
int((c*d*x+d)^(1/2)*(-c*f*x+f)^(3/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*f*( - 6*asin(sqrt( - c*x + 1)/sqrt(2))*a - 2*sqrt(c*x + 1
)*sqrt( - c*x + 1)*a*c**2*x**2 + 3*sqrt(c*x + 1)*sqrt( - c*x + 1)*a*c*x +
2*sqrt(c*x + 1)*sqrt( - c*x + 1)*a - 6*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*
acos(c*x)*x,x)*b*c**2 + 6*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x),x)*
b*c))/(6*c)
```

3.515 $\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 242

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx = -\frac{2bf^2x\sqrt{1-c^2x^2}}{\sqrt{d+cdx}\sqrt{f-cfx}} + \frac{bcf^2x^2\sqrt{1-c^2x^2}}{4\sqrt{d+cdx}\sqrt{f-cfx}} + \frac{2f^2(1-c^2x^2)\,(a+b\arccos(cx))}{c\sqrt{d+cdx}\sqrt{f-cfx}} - \frac{f^2x(1-c^2x^2)\,(a+b\arccos(cx))}{2\sqrt{d+cdx}\sqrt{f-cfx}} + \frac{3f^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4bc\sqrt{d+cdx}\sqrt{f-cfx}}$$

output

```
-2*b*f^2*x*(-c^2*x^2+1)^(1/2)/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+1/4*b*c*f^2
*x^2*(-c^2*x^2+1)^(1/2)/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+2*f^2*(-c^2*x^2+1
)*(a+b*arccos(c*x))/c/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)-1/2*f^2*x*(-c^2*x^2
+1)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+3/4*f^2*(-c^2*x^2+1
)^(1/2)*(a+b*arccos(c*x))^2/b/c/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)
```

Mathematica [A] (verified)

Time = 4.69 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.07

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx = \frac{f(-1+cx)\csc^2\left(\frac{1}{2}\arccos(cx)\right)\left(6b\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx)^2\right.}{}$$

input
```
Integrate[((f - c*f*x)^(3/2)*(a + b*ArcCos[c*x]))/Sqrt[d + c*d*x],x]
```

output
```
(f*(-1 + c*x)*Csc[ArcCos[c*x]/2]^2*(6*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*Ar
cCos[c*x]^2 + 12*a*Sqrt[d]*Sqrt[f]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d +
c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d]*Sqrt[f]*(-1 + c^2*x^2))] + Sqrt[d + c*d*x
]*Sqrt[f - c*f*x]*(-16*b*c*x + 4*a*(-4 + c*x)*Sqrt[1 - c^2*x^2] + b*Cos[2*
ArcCos[c*x]]) - 2*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]*(8*Sqrt[1
- c^2*x^2] - Sin[2*ArcCos[c*x]])))/(16*c*d*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{\sqrt{cdx+d}}\,dx$$

$$\Big\downarrow \; 5179$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{f^2(1-cx)^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \; 27$$

$$\frac{f^2\sqrt{1-c^2x^2}\int \frac{(1-cx)^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \; 5263$$

$$\frac{f^2\sqrt{1-c^2x^2}\int\left(\frac{c^2(a+b\arccos(cx))x^2}{\sqrt{1-c^2x^2}}-\frac{2c(a+b\arccos(cx))x}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}\right)dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f^2\sqrt{1-c^2x^2}\left(-\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}-\frac{3(a+b\arccos(cx))^2}{4bc}-\frac{1}{4}bcx^2+2bx\right)}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

input
```
Int[((f - c*f*x)^(3/2)*(a + b*ArcCos[c*x]))/Sqrt[d + c*d*x],x]
```

output
```
(f^2*Sqrt[1 - c^2*x^2]*(2*b*x - (b*c*x^2)/4 + (2*Sqrt[1 - c^2*x^2]*(a + b*
ArcCos[c*x]))/c - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/2 - (3*(a + b*
ArcCos[c*x])^2)/(4*b*c)))/(Sqrt[d + c*d*x]*Sqrt[f - c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 714, normalized size of antiderivative = 2.95

method	result
default	$\frac{a(-cfx+f)^{\frac{3}{2}}\sqrt{cdx+d}}{2dc}+\frac{3af\sqrt{-cfx+f}\,\sqrt{cdx+d}}{2dc}+\frac{3a\,f^{2}\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^{2}df}\,x}{\sqrt{-c^{2}df\,x^{2}+df}}\right)}{2\sqrt{-cfx+f}\,\sqrt{cdx+d}\,\sqrt{c^{2}df}}+b\left(\frac{3\sqrt{d(cx+1)}\,\sqrt{}}{}\right.$
parts	$\frac{a(-cfx+f)^{\frac{3}{2}}\sqrt{cdx+d}}{2dc}+\frac{3af\sqrt{-cfx+f}\,\sqrt{cdx+d}}{2dc}+\frac{3a\,f^{2}\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^{2}df}\,x}{\sqrt{-c^{2}df\,x^{2}+df}}\right)}{2\sqrt{-cfx+f}\,\sqrt{cdx+d}\,\sqrt{c^{2}df}}+b\left(\frac{3\sqrt{d(cx+1)}\,\sqrt{}}{}\right.$

input
```
int((-c*f*x+f)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/2*a/d/c*(-c*f*x+f)^(3/2)*(c*d*x+d)^(1/2)+3/2*a*f/d/c*(-c*f*x+f)^(1/2)*(c
*d*x+d)^(1/2)+3/2*a*f^2*((-c*f*x+f)*(c*d*x+d))^(1/2)/(-c*f*x+f)^(1/2)/(c*d
*x+d)^(1/2)/(c^2*d*f)^(1/2)*arctan((c^2*d*f)^(1/2)*x/(-c^2*d*f*x^2+d*f)^(1
/2))+b*(3/4*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x+1
)/d/c/(c*x-1)*arccos(c*x)^2*f-1/32*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-
2*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+4*c^3*x^3+1-2*I*(-c^2*x^2+1)^(1/2
)*c*x-I*(-c^2*x^2+1)^(1/2)-3*c*x)*(I+2*arccos(c*x))*f/(c*x+1)/d/c/(c*x-1)+
1/2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)+c*x-1)*(arc
cos(c*x)+I)*f/(c*x+1)/d/c/(c*x-1)+(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I
*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)-I)*f/(c*x+1)/d/c/(c*x-1)-1
/32*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^
2*x^2+I*(-c^2*x^2+1)^(1/2)-c*x-1)*(-I+2*arccos(c*x))*f/(c*x+1)/d/c/(c*x-1)
-1/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x-1)*(
7*I+8*arccos(c*x))*cos(2*arccos(c*x))*f/(c*x+1)/d/c/(c*x-1)-1/8*(d*(c*x+1)
)^(1/2)*(-f*(c*x-1))^(1/2)*(I*c*x-I+(-c^2*x^2+1)^(1/2))*(4*I+3*arccos(c*x)
)*sin(2*arccos(c*x))*f/(c*x+1)/d/c/(c*x-1))
```

Fricas [F]

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=\int \frac{(-cfx+f)^{\frac{3}{2}}(b\arccos(cx)+a)}{\sqrt{cdx+d}}\,dx$$

input
```
integrate((-c*f*x+f)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-(a*c*f*x - a*f + (b*c*f*x - b*f)*arccos(c*x))*sqrt(-c*f*x + f)/s
qrt(c*d*x + d), x)
```

Sympy [F]

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=\int \frac{(-f(cx-1))^{\frac{3}{2}}\,(a+b\operatorname{acos}(cx))}{\sqrt{d\,(cx+1)}}\,dx$$

input
```
integrate((-c*f*x+f)**(3/2)*(a+b*acos(c*x))/(c*d*x+d)**(1/2),x)
```

output
```
Integral((-f*(c*x - 1))**(3/2)*(a + b*acos(c*x))/sqrt(d*(c*x + 1)), x)
```

Maxima [F]

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=\int \frac{(-cfx+f)^{\frac{3}{2}}(b\arccos(cx)+a)}{\sqrt{cdx+d}}\,dx$$

input
```
integrate((-c*f*x+f)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/2*(sqrt(-c^2*d*f*x^2 + d*f)*f*x/d - 3*f^2*arcsin(c*x)/(sqrt(d*f)*c) - 4
*sqrt(-c^2*d*f*x^2 + d*f)*f/(c*d))*a - b*sqrt(f)*integrate((c*f*x - f)*sqr
t(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/sqrt(c*x + 1), x)/s
qrt(d)
```

Giac [F]

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=\int \frac{(-cfx+f)^{\frac{3}{2}}(b\arccos{(cx)}+a)}{\sqrt{cdx+d}}\,dx$$

input
```
integrate((-c*f*x+f)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((-c*f*x + f)^(3/2)*(b*arccos(c*x) + a)/sqrt(c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,(f-c\,f\,x)^{3/2}}{\sqrt{d+c\,d\,x}}\,dx$$

input
```
int(((a + b*acos(c*x))*(f - c*f*x)^(3/2))/(d + c*d*x)^(1/2),x)
```

output
```
int(((a + b*acos(c*x))*(f - c*f*x)^(3/2))/(d + c*d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx = \frac{\sqrt{f}\,f\left(-6asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a-\sqrt{cx+1}\,\sqrt{-cx+1}\,acx+4\sqrt{cx+1}\,\sqrt{}\right.}{2}$$

input
```
int((-c*f*x+f)^(3/2)*(a+b*acos(c*x))/(c*d*x+d)^(1/2),x)
```

output
```
(sqrt(f)*f*( - 6*asin(sqrt( - c*x + 1)/sqrt(2))*a - sqrt(c*x + 1)*sqrt( -
c*x + 1)*a*c*x + 4*sqrt(c*x + 1)*sqrt( - c*x + 1)*a - 2*int((sqrt( - c*x +
 1)*acos(c*x)*x)/sqrt(c*x + 1),x)*b*c**2 + 2*int((sqrt( - c*x + 1)*acos(c*
x))/sqrt(c*x + 1),x)*b*c))/(2*sqrt(d)*c)
```

3.516 $\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 252

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx = \frac{bf^3x(1-c^2x^2)^{3/2}}{(d+cdx)^{3/2}(f-cfx)^{3/2}} - \frac{4f^3(1-cx)\left(1-c^2x^2\right)(a+b\arccos(cx))}{c(d+cdx)^{3/2}(f-cfx)^{3/2}} - \frac{f^3\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{c(d+cdx)^{3/2}(f-cfx)^{3/2}} - \frac{3f^3\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2}{2bc(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{4bf^3\left(1-c^2x^2\right)^{3/2}\log(1+cx)}{c(d+cdx)^{3/2}(f-cfx)^{3/2}}$$

output
```
b*f^3*x*(-c^2*x^2+1)^(3/2)/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)-4*f^3*(-c*x+1)
*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)-f^3*(-c
^2*x^2+1)^2*(a+b*arccos(c*x))/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)-3/2*f^3*(
-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))^2/b/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)
+4*b*f^3*(-c^2*x^2+1)^(3/2)*ln(c*x+1)/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 5.23 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.12

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx = \frac{f(-1+cx)\csc^2\left(\frac{1}{2}\arccos(cx)\right)\left(2b(5+cx)\sqrt{d+cdx}\sqrt{f-cfx}\sqrt{1}\right.}{}$$

input
```
Integrate[((f - c*f*x)^(3/2)*(a + b*ArcCos[c*x]))/(d + c*d*x)^(3/2),x]
```

output
```
(f*(-1 + c*x)*Csc[ArcCos[c*x]/2]^2*(2*b*(5 + c*x)*Sqrt[d + c*d*x]*Sqrt[f -
 c*f*x]*Sqrt[1 - c^2*x^2]*ArcCos[c*x] - 3*b*(1 + c*x)*Sqrt[d + c*d*x]*Sqrt
[f - c*f*x]*ArcCos[c*x]^2 - 6*a*Sqrt[d]*Sqrt[f]*(1 + c*x)*Sqrt[1 - c^2*x^2
]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d]*Sqrt[f]*(-1 + c^2*
x^2))] + 2*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(b*c*x*(1 + c*x) + a*(5 + c*x)*
Sqrt[1 - c^2*x^2] + 8*b*(1 + c*x)*Log[Cos[ArcCos[c*x]/2]])))/(4*c*d^2*(1 +
 c*x)*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(cdx+d)^{3/2}}\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\int \frac{f^3(1-cx)^3(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^3\left(1-c^2x^2\right)^{3/2}\int \frac{(1-cx)^3(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{5275}$$

$$\frac{f^3(1-c^2x^2)^{3/2}\int\left(\frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}+\frac{4(1-cx)(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}\right)dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^3(1-c^2x^2)^{3/2}\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}-\frac{4(1-cx)(a+b\arccos(cx))}{c\sqrt{1-c^2x^2}}+\frac{3(a+b\arccos(cx))^2}{2bc}-\frac{4b\log(cx+1)}{c}-bx\right)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

input
```
Int[((f - c*f*x)^(3/2)*(a + b*ArcCos[c*x]))/(d + c*d*x)^(3/2),x]
```

output
```
(f^3*(1 - c^2*x^2)^(3/2)*(-(b*x) - (4*(1 - c*x)*(a + b*ArcCos[c*x]))/(c*Sq
rt[1 - c^2*x^2]) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c + (3*(a + b*A
rcCos[c*x])^2)/(2*b*c) - (4*b*Log[1 + c*x])/c))/((d + c*d*x)^(3/2)*(f - c*
f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 29.03 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.94

method	result
default	$-\frac{3\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,b\arccos(cx)^2 f}{2(cx+1)d^2(cx-1)c}-\frac{3\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,a\arccos(cx) f}{(cx+1)d^2(cx-1)c}-\frac{\sqrt{d(cx+1)}\,\sqrt{-}}{}$

input
```
int((-c*f*x+f)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-3/2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x+1)/d^2/(
c*x-1)/c*b*arccos(c*x)^2*f-3*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^
2+1)^(1/2)/(c*x+1)/d^2/(c*x-1)/c*a*arccos(c*x)*f-1/2*(d*(c*x+1))^(1/2)*(-f
*(c*x-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(b*arccos(c*x)+a+I*b)
*f/(c*x+1)/d^2/(c*x-1)/c-1/2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^
2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(b*arccos(c*x)+a-I*b)*f/(c*x+1)/d^2/(c*x-1)/
c-8*I*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x+1)/d^2/
(c*x-1)/c*b*arccos(c*x)*f-4*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2
*x^2+1)^(1/2)+c*x-1)*(a+b*arccos(c*x))*f/(c*x+1)/d^2/(c*x-1)/c+8*(d*(c*x+1
))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x+1)/d^2/(c*x-1)/c*b*ln(
1+c*x+I*(-c^2*x^2+1)^(1/2))*f
```

Fricas [F]

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx=\int \frac{(-cfx+f)^{\frac{3}{2}}(b\arccos(cx)+a)}{(cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(3/2),x, algorithm=
"fricas")
```

output
```
integral(-(a*c*f*x - a*f + (b*c*f*x - b*f)*arccos(c*x))*sqrt(c*d*x + d)*sq
rt(-c*f*x + f)/(c^2*d^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F]

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx=\int \frac{(-f(cx-1))^{\frac{3}{2}}\,(a+b\operatorname{acos}(cx))}{(d\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)**(3/2)*(a+b*acos(c*x))/(c*d*x+d)**(3/2),x)
```

output
```
Integral((-f*(c*x - 1))**(3/2)*(a + b*acos(c*x))/(d*(c*x + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx=\int \frac{(-cfx+f)^{\frac{3}{2}}(b\arccos(cx)+a)}{(cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(3/2),x, algorithm=
"maxima")
```

output
```
-b*sqrt(d)*sqrt(f)*integrate((c*f*x - f)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arct
an2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^2*d^2*x^2 + 2*c*d^2*x + d^2), x)
 + a*((-c^2*d*f*x^2 + d*f)^(3/2)/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3) - 6*s
qrt(-c^2*d*f*x^2 + d*f)*f/(c^2*d^2*x + c*d^2) - 3*f^2*arcsin(c*x)/(c*d^2*s
qrt(f/d)))
```

Giac [F]

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx=\int \frac{(-cfx+f)^{\frac{3}{2}}(b\arccos{(cx)}+a)}{(cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((-c*f*x + f)^(3/2)*(b*arccos(c*x) + a)/(c*d*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,{(f-c\,f\,x)}^{3/2}}{{(d+c\,d\,x)}^{3/2}}\,dx$$

input
```
int(((a + b*acos(c*x))*(f - c*f*x)^(3/2))/(d + c*d*x)^(3/2),x)
```

output
```
int(((a + b*acos(c*x))*(f - c*f*x)^(3/2))/(d + c*d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx = \frac{\sqrt{f}\,f\left(6\sqrt{cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a-\sqrt{-cx+1}\,acx-5\sqrt{-cx+1}\,a\right.}{\sqrt{d}}$$

input
```
int((-c*f*x+f)^(3/2)*(a+b*acos(c*x))/(c*d*x+d)^(3/2),x)
```

output
```
(sqrt(f)*f*(6*sqrt(c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a - sqrt( - c*x
 + 1)*a*c*x - 5*sqrt( - c*x + 1)*a - sqrt(c*x + 1)*int((sqrt( - c*x + 1)*a
cos(c*x)*x)/(sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b*c**2 + sqrt(c*x + 1)*
int((sqrt( - c*x + 1)*acos(c*x))/(sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b*
c))/(sqrt(d)*sqrt(c*x + 1)*c*d)
```

3.517 $\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 324

$$\begin{aligned}\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx &= -\frac{4bf^4(1-c^2x^2)^{5/2}}{3c(1+cx)(d+cdx)^{5/2}(f-cfx)^{5/2}}\\ &\quad-\frac{bf^4(1-c^2x^2)^{5/2}\arccos(cx)^2}{2c(d+cdx)^{5/2}(f-cfx)^{5/2}}-\frac{2f^4(1-cx)^3\,(1-c^2x^2)\,(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}}\\ &\quad+\frac{2f^4(1-cx)\,(1-c^2x^2)^2\,(a+b\arccos(cx))}{c(d+cdx)^{5/2}(f-cfx)^{5/2}}\\ &\quad+\frac{f^4(1-c^2x^2)^{5/2}\arccos(cx)(a+b\arccos(cx))}{c(d+cdx)^{5/2}(f-cfx)^{5/2}}-\frac{8bf^4(1-c^2x^2)^{5/2}\log(1+cx)}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}}\end{aligned}$$

output
```
-4/3*b*f^4*(-c^2*x^2+1)^(5/2)/c/(c*x+1)/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)-1
/2*b*f^4*(-c^2*x^2+1)^(5/2)*arccos(c*x)^2/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/
2)-2/3*f^4*(-c*x+1)^3*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c
*f*x+f)^(5/2)+2*f^4*(-c*x+1)*(-c^2*x^2+1)^2*(a+b*arccos(c*x))/c/(c*d*x+d)^
(5/2)/(-c*f*x+f)^(5/2)+f^4*(-c^2*x^2+1)^(5/2)*arccos(c*x)*(a+b*arccos(c*x)
)/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)-8/3*b*f^4*(-c^2*x^2+1)^(5/2)*ln(c*x+1
)/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)
```

Mathematica [A] (verified)

Time = 5.57 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.90

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx = \frac{f(-1+cx)\csc^2\left(\frac{1}{2}\arccos(cx)\right)\left(12a\sqrt{d}\sqrt{f}(1+cx)^2\sqrt{1-c^2x^2}\,\mathrm{arc}\right.}{}$$

input

```
Integrate[((f - c*f*x)^(3/2)*(a + b*ArcCos[c*x]))/(d + c*d*x)^(5/2),x]
```

output

```
(f*(-1 + c*x)*Csc[ArcCos[c*x]/2]^2*(12*a*Sqrt[d]*Sqrt[f]*(1 + c*x)^2*Sqrt[
1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d]*Sqrt[f]
*(-1 + c^2*x^2))] + 3*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]^2*(3 +
 4*c*x + Cos[2*ArcCos[c*x]]) - 16*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(b + b*c
*x + a*(1 + 2*c*x)*Sqrt[1 - c^2*x^2] + 4*b*(1 + c*x)^2*Log[Cos[ArcCos[c*x]
/2]]) - 16*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]*(Sqrt[1 - c^2*x^2
] + Sin[2*ArcCos[c*x]])))/(24*c*d^3*(1 + c*x)^2*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5179, 27, 5261, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(cdx+d)^{5/2}}\,dx$$

$$\Big\downarrow \; 5179$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\int \frac{f^4(1-cx)^4(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{f^4\left(1-c^2x^2\right)^{5/2}\int \frac{(1-cx)^4(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$\downarrow$ 5261

$$\frac{f^4(1-c^2x^2)^{5/2}\left(bc\int\left(-\frac{2(1-cx)^3}{3c(1-c^2x^2)^2}+\frac{2(1-cx)}{c(1-c^2x^2)}+\frac{\arcsin(cx)}{c\sqrt{1-c^2x^2}}\right)dx+\frac{\arcsin(cx)(a+b\arccos(cx))}{c}-\frac{2(1-cx)^3(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}\right.}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$\downarrow$ 2009

$$\frac{f^4(1-c^2x^2)^{5/2}\left(\frac{\arcsin(cx)(a+b\arccos(cx))}{c}-\frac{2(1-cx)^3(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}+\frac{2(1-cx)(a+b\arccos(cx))}{c\sqrt{1-c^2x^2}}+bc\left(\frac{\arcsin(cx)^2}{2c^2}+\frac{4}{3c^2(cx+1}\right.\right.}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

input
```
Int[((f - c*f*x)^(3/2)*(a + b*ArcCos[c*x]))/(d + c*d*x)^(5/2),x]
```

output
```
(f^4*(1 - c^2*x^2)^(5/2)*((-2*(1 - c*x)^3*(a + b*ArcCos[c*x]))/(3*c*(1 - c
^2*x^2)^(3/2)) + (2*(1 - c*x)*(a + b*ArcCos[c*x]))/(c*Sqrt[1 - c^2*x^2]) +
 ((a + b*ArcCos[c*x])*ArcSin[c*x])/c + b*c*(4/(3*c^2*(1 + c*x)) + ArcSin[c
*x]^2/(2*c^2) + (8*Log[1 + c*x])/(3*c^2))))/((d + c*d*x)^(5/2)*(f - c*f*x)
^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5261
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p,
 x]}, Simp[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[1/Sqrt[1 - c^2*x^2]
   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IG
tQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 17.13 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.07

method	result
default	$\frac{\sqrt{-f(cx-1)}\,\sqrt{d(cx+1)}\,\sqrt{-c^2x^2+1}\,\left(16i\arccos(cx)b+3\arccos(cx)^2b\,c^2x^2-16ia\,c^2x^2+6\arccos(cx)a\,c^2x^2-32\ln\left(1+cx+i\sqrt{-c^2x^2}\right.\right.}{}$

input
```
int((-c*f*x+f)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/6*(-f*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^3*x^3+3*c^2
*x^2+3*c*x+1)/d^3/(c*x-1)/c*(16*I*arccos(c*x)*b+3*arccos(c*x)^2*b*c^2*x^2-
16*I*a*c^2*x^2+6*arccos(c*x)*a*c^2*x^2-32*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*b
*c^2*x^2-16*arccos(c*x)*(-c^2*x^2+1)^(1/2)*b*c*x-32*I*a*c*x+6*b*c*x*arccos
(c*x)^2-16*(-c^2*x^2+1)^(1/2)*a*c*x+32*I*arccos(c*x)*b*c*x+12*arccos(c*x)*
a*c*x-64*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*b*c*x-8*arccos(c*x)*(-c^2*x^2+1)^(
1/2)*b-16*I*a+3*arccos(c*x)^2*b-8*c*x*b-8*(-c^2*x^2+1)^(1/2)*a+16*I*arccos
(c*x)*b*c^2*x^2+6*arccos(c*x)*a-32*b*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-8*b)*f
```

Fricas [F]

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx=\int \frac{(-cfx+f)^{\frac{3}{2}}(b\arccos{(cx)}+a)}{(cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2),x, algorithm=
"fricas")
```

output
```
integral(-(a*c*f*x - a*f + (b*c*f*x - b*f)*arccos(c*x))*sqrt(c*d*x + d)*sq
rt(-c*f*x + f)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

Sympy [F]

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx=\int \frac{(-f(cx-1))^{\frac{3}{2}}\,(a+b\,\mathrm{acos}\,(cx))}{(d\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)**(3/2)*(a+b*acos(c*x))/(c*d*x+d)**(5/2),x)
```

output
```
Integral((-f*(c*x - 1))**(3/2)*(a + b*acos(c*x))/(d*(c*x + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx=\int \frac{(-cfx+f)^{\frac{3}{2}}(b\arccos(cx)+a)}{(cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2),x, algorithm=
"maxima")
```

output
```
-b*sqrt(d)*sqrt(f)*integrate((c*f*x - f)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arct
an2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*
d^3*x + d^3), x) - 1/3*a*((-c^2*d*f*x^2 + d*f)^(3/2)/(c^4*d^4*x^3 + 3*c^3*
d^4*x^2 + 3*c^2*d^4*x + c*d^4) + 2*sqrt(-c^2*d*f*x^2 + d*f)*f/(c^3*d^3*x^2
 + 2*c^2*d^3*x + c*d^3) - 7*sqrt(-c^2*d*f*x^2 + d*f)*f/(c^2*d^3*x + c*d^3)
 - 3*f^2*arcsin(c*x)/(c*d^3*sqrt(f/d)))
```

Giac [F]

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx=\int \frac{(-cfx+f)^{\frac{3}{2}}(b\arccos{(cx)}+a)}{(cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((-c*f*x + f)^(3/2)*(b*arccos(c*x) + a)/(c*d*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,{(f-c\,f\,x)}^{3/2}}{{(d+c\,d\,x)}^{5/2}}\,dx$$

input
```
int(((a + b*acos(c*x))*(f - c*f*x)^(3/2))/(d + c*d*x)^(5/2),x)
```

output
```
int(((a + b*acos(c*x))*(f - c*f*x)^(3/2))/(d + c*d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(f-cfx)^{3/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx=\frac{\sqrt{f}\,f\left(-6\sqrt{cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)acx-6\sqrt{cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a\right.}{}$$

input
```
int((-c*f*x+f)^(3/2)*(a+b*acos(c*x))/(c*d*x+d)^(5/2),x)
```

output
```
(sqrt(f)*f*( - 6*sqrt(c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a*c*x - 6*sq
rt(c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a + 8*sqrt( - c*x + 1)*a*c*x +
4*sqrt( - c*x + 1)*a - 3*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)*x)/
(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b*c**3*
x - 3*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)*x)/(sqrt(c*x + 1)*c**2
*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b*c**2 + 3*sqrt(c*x + 1)*i
nt((sqrt( - c*x + 1)*acos(c*x))/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)
*c*x + sqrt(c*x + 1)),x)*b*c**2*x + 3*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*
acos(c*x))/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1))
,x)*b*c))/(3*sqrt(d)*sqrt(c*x + 1)*c*d**2*(c*x + 1))
```

3.518 $\int (d+cdx)^{5/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 312

$$\int (d+cdx)^{5/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx=-\frac{5bcx^2(d+cdx)^{5/2}(f-cfx)^{5/2}}{32\left(1-c^2x^2\right)^{5/2}}$$
$$+\frac{5b(d+cdx)^{5/2}(f-cfx)^{5/2}}{96c\sqrt{1-c^2x^2}}+\frac{b(d+cdx)^{5/2}(f-cfx)^{5/2}\sqrt{1-c^2x^2}}{36c}$$
$$+\frac{1}{6}x(d+cdx)^{5/2}(f-cfx)^{5/2}(a+b\arccos(cx))+\frac{5x(d+cdx)^{5/2}(f-cfx)^{5/2}(a+b\arccos(cx))}{16\left(1-c^2x^2\right)^2}+\frac{5x(d+cdx}{}$$

output
```
-5/32*b*c*x^2*(c*d*x+d)^(5/2)*(-c*f*x+f)^(5/2)/(-c^2*x^2+1)^(5/2)+5/96*b*(
c*d*x+d)^(5/2)*(-c*f*x+f)^(5/2)/c/(-c^2*x^2+1)^(1/2)+1/36*b*(c*d*x+d)^(5/2
)*(-c*f*x+f)^(5/2)*(-c^2*x^2+1)^(1/2)/c+1/6*x*(c*d*x+d)^(5/2)*(-c*f*x+f)^(
5/2)*(a+b*arccos(c*x))+5/16*x*(c*d*x+d)^(5/2)*(-c*f*x+f)^(5/2)*(a+b*arccos
(c*x))/(-c^2*x^2+1)^2+5*x*(c*d*x+d)^(5/2)*(-c*f*x+f)^(5/2)*(a+b*arccos(c*x
))/(-24*c^2*x^2+24)+5/32*(c*d*x+d)^(5/2)*(-c*f*x+f)^(5/2)*(a+b*arccos(c*x)
)^2/b/c/(-c^2*x^2+1)^(5/2)
```

Mathematica [A] (verified)

Time = 2.58 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.97

$$\int (d+cdx)^{5/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx = \frac{d^2f^2\left(-360b\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx)^2-720a\sqrt{d}\sqrt{f}\sqrt{1-c^2x^2}\,\text{arcta}\right.}{}$$

input
```
Integrate[(d + c*d*x)^(5/2)*(f - c*f*x)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*f^2*(-360*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]^2 - 720*a*Sqr
t[d]*Sqrt[f]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]
)/(Sqrt[d]*Sqrt[f]*(-1 + c^2*x^2))] + Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(158
4*a*c*x*Sqrt[1 - c^2*x^2] - 1248*a*c^3*x^3*Sqrt[1 - c^2*x^2] + 384*a*c^5*x
^5*Sqrt[1 - c^2*x^2] + 270*b*Cos[2*ArcCos[c*x]] - 27*b*Cos[4*ArcCos[c*x]]
+ 2*b*Cos[6*ArcCos[c*x]]) + 12*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*
x]*(45*Sin[2*ArcCos[c*x]] - 9*Sin[4*ArcCos[c*x]] + Sin[6*ArcCos[c*x]])))/(
2304*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.64, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5179, 5159, 241, 5159, 244, 2009, 5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^{5/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5179}$$

$$\frac{(cdx+d)^{5/2}(f-cfx)^{5/2}\int \left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx}{(1-c^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 5159}$$

$$\frac{(cdx+d)^{5/2}(f-cfx)^{5/2}\left(\frac{5}{6}\int \left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{6}bc\int x\left(1-c^2x^2\right)^2dx+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a\right.}{\left(1-c^2x^2\right)^{5/2}}$$

$\downarrow$ 241

$$\frac{(cdx+d)^{5/2}(f-cfx)^{5/2}\left(\frac{5}{6}\int \left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx+\frac{1}{6}x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))-\frac{b(1-c^2}{36c}\right.}{\left(1-c^2x^2\right)^{5/2}}$$

$\downarrow$ 5159

$$\frac{(cdx+d)^{5/2}(f-cfx)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int x\left(1-c^2x^2\right)dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+\right.\right.}{\left(1-c^2x^2\right)^{5/2}}$$

$\downarrow$ 244

$$\frac{(cdx+d)^{5/2}(f-cfx)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}bc\int \left(x-c^2x^3\right)dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+\right.\right.}{\left(1-c^2x^2\right)^{5/2}}$$

$\downarrow$ 2009

$$\frac{(cdx+d)^{5/2}(f-cfx)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))dx+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{x^2}{2}\right.\right.\right.}{\left(1-c^2x^2\right)^{5/2}}$$

$\downarrow$ 5157

$$\frac{(cdx+d)^{5/2}(f-cfx)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int xdx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)+\frac{1}{4}x\left(1-c^2\right.\right.\right.}{\left(1-c^2x^2\right)^{5/2}}$$

$\downarrow$ 15

$$\frac{(cdx+d)^{5/2}(f-cfx)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{1}{4}bcx^2\right)+\frac{1}{4}x\left(1-c^2x^2\right)^{3}\right.\right.}{\left(1-c^2x^2\right)^{5/2}}$$

$\downarrow$ 5153

$$\frac{(cdx+d)^{5/2}(f-cfx)^{5/2}\left(\frac{1}{6}x(1-c^2x^2)^{5/2}(a+b\arccos(cx))+\frac{5}{6}\left(\frac{1}{4}x(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\frac{3}{4}\left(\frac{1}{2}x\sqrt{\right.\right.\right.}}{(1-c^2x^2)^{5/2}}$$

input
```
Int[(d + c*d*x)^(5/2)*(f - c*f*x)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
((d + c*d*x)^(5/2)*(f - c*f*x)^(5/2)*(-1/36*(b*(1 - c^2*x^2)^3)/c + (x*(1
- c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/6 + (5*((b*c*(x^2/2 - (c^2*x^4)/4))/
4 + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 + (3*((b*c*x^2)/4 + (x*S
qrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/2 - (a + b*ArcCos[c*x])^2/(4*b*c)))/
4))/6))/(1 - c^2*x^2)^(5/2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 895, normalized size of antiderivative = 2.87

method	result
default	$-\frac{a(cdx+d)^{\frac{5}{2}}(-cfx+f)^{\frac{7}{2}}}{6cf}-\frac{ad(cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{7}{2}}}{6cf}-\frac{a\,d^{2}\sqrt{cdx+d}\,(-cfx+f)^{\frac{7}{2}}}{8cf}+\frac{a\,d^{2}(-cfx+f)^{\frac{5}{2}}\sqrt{cdx+d}}{24c}+\frac{5a\,d^{2}f(-$
parts	$-\frac{a(cdx+d)^{\frac{5}{2}}(-cfx+f)^{\frac{7}{2}}}{6cf}-\frac{ad(cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{7}{2}}}{6cf}-\frac{a\,d^{2}\sqrt{cdx+d}\,(-cfx+f)^{\frac{7}{2}}}{8cf}+\frac{a\,d^{2}(-cfx+f)^{\frac{5}{2}}\sqrt{cdx+d}}{24c}+\frac{5a\,d^{2}f(-$

input
```
int((c*d*x+d)^(5/2)*(-c*f*x+f)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVER
BOSE)
```

output
```
-1/6*a/c/f*(c*d*x+d)^(5/2)*(-c*f*x+f)^(7/2)-1/6*a*d/c/f*(c*d*x+d)^(3/2)*(-
c*f*x+f)^(7/2)-1/8*a*d^2/c/f*(c*d*x+d)^(1/2)*(-c*f*x+f)^(7/2)+1/24*a*d^2/c
*(-c*f*x+f)^(5/2)*(c*d*x+d)^(1/2)+5/48*a*d^2*f/c*(-c*f*x+f)^(3/2)*(c*d*x+d
)^(1/2)+5/16*a*d^2*f^2/c*(-c*f*x+f)^(1/2)*(c*d*x+d)^(1/2)+5/16*a*d^3*f^3*(
(-c*f*x+f)*(c*d*x+d))^(1/2)/(-c*f*x+f)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*f)^(1/
2)*arctan((c^2*d*f)^(1/2)*x/(-c^2*d*f*x^2+d*f)^(1/2))+b*(5/32*(d*(c*x+1))^
(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^2*d^
2*f^2+1/2304*(-f*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(32*c^7*x^7-64*c^5*x^5+3
2*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+38*c^3*x^3-48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-
6*c*x+18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(I+6*arccos(c*
x))*d^2*f^2/(c^2*x^2-1)/c+15/256*(-f*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-2*
I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*a
rccos(c*x))*d^2*f^2/(c^2*x^2-1)/c+5/4608*(-f*(c*x-1))^(1/2)*(d*(c*x+1))^(1
/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(5*I+24*arccos(c*x))*cos(5*arcco
s(c*x))*d^2*f^2/(c^2*x^2-1)/c+1/4608*(-f*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*
(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(29*I+96*arccos(c*x))*sin(5*arccos(c*
x))*d^2*f^2/(c^2*x^2-1)/c-9/512*(-f*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-I*(
-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(3*I+8*arccos(c*x))*cos(3*arccos(c*x))*d^
2*f^2/(c^2*x^2-1)/c-3/512*(-f*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(I*c^2*x^2+
c*x*(-c^2*x^2+1)^(1/2)-I)*(11*I+16*arccos(c*x))*sin(3*arccos(c*x))*d^2*...
```

Fricas [F]

$$\int (d+cdx)^{5/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx=\int (cdx+d)^{\frac{5}{2}}(-cfx+f)^{\frac{5}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*f*x+f)^(5/2)*(a+b*arccos(c*x)),x, algorithm=
"fricas")
```

output
```
integral((a*c^4*d^2*f^2*x^4 - 2*a*c^2*d^2*f^2*x^2 + a*d^2*f^2 + (b*c^4*d^2
*f^2*x^4 - 2*b*c^2*d^2*f^2*x^2 + b*d^2*f^2)*arccos(c*x))*sqrt(c*d*x + d)*s
qrt(-c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+cdx)^{5/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+d)**(5/2)*(-c*f*x+f)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+cdx)^{5/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx = \int (cdx+d)^{\frac{5}{2}}(-cfx+f)^{\frac{5}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*f*x+f)^(5/2)*(a+b*arccos(c*x)),x, algorithm=
"maxima")
```

output
```
b*sqrt(d)*sqrt(f)*integrate((c^4*d^2*f^2*x^4 - 2*c^2*d^2*f^2*x^2 + d^2*f^2
)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x),
 x) + 1/48*(15*sqrt(-c^2*d*f*x^2 + d*f)*d^2*f^2*x + 15*d^3*f^3*arcsin(c*x)
/(sqrt(d*f)*c) + 10*(-c^2*d*f*x^2 + d*f)^(3/2)*d*f*x + 8*(-c^2*d*f*x^2 + d
*f)^(5/2)*x)*a
```

Giac [F]

$$\int (d+cdx)^{5/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx=\int (cdx+d)^{\frac{5}{2}}(-cfx+f)^{\frac{5}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*f*x+f)^(5/2)*(a+b*arccos(c*x)),x, algorithm=
"giac")
```

output
```
integrate((c*d*x + d)^(5/2)*(-c*f*x + f)^(5/2)*(b*arccos(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)^{5/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,(d+c\,d\,x)^{5/2}\,(f-c\,f\,x)^{5/2}\,dx$$

input
```
int((a + b*acos(c*x))*(d + c*d*x)^(5/2)*(f - c*f*x)^(5/2),x)
```

output
```
int((a + b*acos(c*x))*(d + c*d*x)^(5/2)*(f - c*f*x)^(5/2), x)
```

Reduce [F]

$$\int (d+cdx)^{5/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx=\frac{\sqrt{f}\,\sqrt{d}\,d^2f^2\left(-30asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a+8\sqrt{cx+1}\,\sqrt{-cx+1}\,a\,c^5x^5-26\sqrt{cx}$$

input
```
int((c*d*x+d)^(5/2)*(-c*f*x+f)^(5/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*d**2*f**2*( - 30*asin(sqrt( - c*x + 1)/sqrt(2))*a + 8*sqr
t(c*x + 1)*sqrt( - c*x + 1)*a*c**5*x**5 - 26*sqrt(c*x + 1)*sqrt( - c*x + 1
)*a*c**3*x**3 + 33*sqrt(c*x + 1)*sqrt( - c*x + 1)*a*c*x + 48*int(sqrt(c*x
+ 1)*sqrt( - c*x + 1)*acos(c*x)*x**4,x)*b*c**5 - 96*int(sqrt(c*x + 1)*sqrt
( - c*x + 1)*acos(c*x)*x**2,x)*b*c**3 + 48*int(sqrt(c*x + 1)*sqrt( - c*x +
 1)*acos(c*x),x)*b*c))/(48*c)
```

3.519 $\int (d+cdx)^{3/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 411

$$\int (d+cdx)^{3/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx = -\frac{bfx(d+cdx)^{3/2}(f-cfx)^{3/2}}{5(1-c^2x^2)^{3/2}} - \frac{3bcfx^2(d+cdx)^{3/2}(f-cfx)^{3/2}}{16(1-c^2x^2)^{3/2}} + \frac{2bc^2fx^3(d+cdx)^{3/2}(f-cfx)^{3/2}}{15(1-c^2x^2)^{3/2}} - \frac{bc^4fx^5(d+cdx)^{3/2}(f-cfx)^{3/2}}{25(1-c^2x^2)^{3/2}} + \frac{bf(d+cdx)^{3/2}(f-cfx)^{3/2}\sqrt{1-c^2x^2}}{16c} + \frac{1}{4}fx(d+cdx)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx)) + \frac{3fx(d+cdx)^{3/2}(f-cfx)^{3/2}(a+b\arccos(cx))}{8(1-c^2x^2)} + \frac{f(d+cd}{}$$

output
```
-1/5*b*f*x*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)/(-c^2*x^2+1)^(3/2)-3/16*b*c*f*
x^2*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)/(-c^2*x^2+1)^(3/2)+2/15*b*c^2*f*x^3*(
c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)/(-c^2*x^2+1)^(3/2)-1/25*b*c^4*f*x^5*(c*d*x
+d)^(3/2)*(-c*f*x+f)^(3/2)/(-c^2*x^2+1)^(3/2)+1/16*b*f*(c*d*x+d)^(3/2)*(-c
*f*x+f)^(3/2)*(-c^2*x^2+1)^(1/2)/c+1/4*f*x*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2
)*(a+b*arccos(c*x))+3*f*x*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x
))/(-8*c^2*x^2+8)+1/5*f*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)*(-c^2*x^2+1)*(a+b
*arccos(c*x))/c+3/16*f*(c*d*x+d)^(3/2)*(-c*f*x+f)^(3/2)*(a+b*arccos(c*x))^
2/b/c/(-c^2*x^2+1)^(3/2)
```

Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.74

$$\int (d+cdx)^{3/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx = \frac{df^2\left(-1800b\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx)^2-3600a\sqrt{d}\sqrt{f}\sqrt{1-c^2x^2}\,\mathrm{arc}\right.}{}$$

input
```
Integrate[(d + c*d*x)^(3/2)*(f - c*f*x)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d*f^2*(-1800*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]^2 - 3600*a*Sqr
t[d]*Sqrt[f]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]
)/(Sqrt[d]*Sqrt[f]*(-1 + c^2*x^2))] + Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(128
*b*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) + 240*a*Sqrt[1 - c^2*x^2]*(8 + 25*c*x
 - 16*c^2*x^2 - 10*c^3*x^3 + 8*c^4*x^4) + 1200*b*Cos[2*ArcCos[c*x]] - 75*b
*Cos[4*ArcCos[c*x]]) + 60*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]*(3
2*(1 - c^2*x^2)^(5/2) + 40*Sin[2*ArcCos[c*x]] - 5*Sin[4*ArcCos[c*x]])))/(9
600*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^{3/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5179}$$

$$\frac{(cdx+d)^{3/2}(f-cfx)^{3/2}\int f(1-cx)\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f(cdx+d)^{3/2}(f-cfx)^{3/2}\int(1-cx)\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{ 5263}$$

$$\frac{f(cdx+d)^{3/2}(f-cfx)^{3/2}\int\left(\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))-cx\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))\right)dx}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f(cdx+d)^{3/2}(f-cfx)^{3/2}\left(\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{8}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{(1-c^2x^2)^{5/2}(a}{5}\right.}{(1-c^2x^2)^{3/2}}$$

input
```
Int[(d + c*d*x)^(3/2)*(f - c*f*x)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(f*(d + c*d*x)^(3/2)*(f - c*f*x)^(3/2)*((b*x)/5 + (5*b*c*x^2)/16 - (2*b*c^
2*x^3)/15 - (b*c^3*x^4)/16 + (b*c^4*x^5)/25 + (3*x*Sqrt[1 - c^2*x^2]*(a +
b*ArcCos[c*x]))/8 + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 + ((1 -
c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c) - (3*(a + b*ArcCos[c*x])^2)/(16*
b*c)))/(1 - c^2*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.86 (sec) , antiderivative size = 1185, normalized size of antiderivative = 2.88

method	result	size
default	Expression too large to display	1185
parts	Expression too large to display	1185

input
```
int((c*d*x+d)^(3/2)*(-c*f*x+f)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVER
BOSE)
```

output
```
-1/5*a/c/f*(c*d*x+d)^(3/2)*(-c*f*x+f)^(7/2)-3/20*a*d/c/f*(c*d*x+d)^(1/2)*(
-c*f*x+f)^(7/2)+1/20*a*d/c*(-c*f*x+f)^(5/2)*(c*d*x+d)^(1/2)+1/8*a*d*f/c*(-
c*f*x+f)^(3/2)*(c*d*x+d)^(1/2)+3/8*a*d*f^2/c*(-c*f*x+f)^(1/2)*(c*d*x+d)^(1
/2)+3/8*a*d^2*f^3*((-c*f*x+f)*(c*d*x+d))^(1/2)/(-c*f*x+f)^(1/2)/(c*d*x+d)^
(1/2)/(c^2*d*f)^(1/2)*arctan((c^2*d*f)^(1/2)*x/(-c^2*d*f*x^2+d*f)^(1/2))+b
*(3/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)
/c*arccos(c*x)^2*f^2*d+1/800*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(16*c^6*
x^6-28*c^4*x^4+16*I*(-c^2*x^2+1)^(1/2)*x^5*c^5+13*c^2*x^2-20*I*(-c^2*x^2+1
)^(1/2)*x^3*c^3+5*I*(-c^2*x^2+1)^(1/2)*c*x-1)*(I+5*arccos(c*x))*f^2*d/(c^2
*x^2-1)/c-1/256*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3
+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c
^2*x^2+1)^(1/2))*(I+4*arccos(c*x))*f^2*d/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1
/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)-
I)*f^2*d/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-2*I*(-c
^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos
(c*x))*f^2*d/(c^2*x^2-1)/c+1/1200*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I
*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(11*I+45*arccos(c*x))*cos(4*arccos(c*x)
)*f^2*d/(c^2*x^2-1)/c+1/600*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(I*c^2*x^
2+c*x*(-c^2*x^2+1)^(1/2)-I)*(7*I+15*arccos(c*x))*sin(4*arccos(c*x))*f^2*d/
(c^2*x^2-1)/c-3/256*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+...
```

Fricas [F]

$$\int (d+cdx)^{3/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx=\int (cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{5}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*f*x+f)^(5/2)*(a+b*arccos(c*x)),x, algorithm=
"fricas")
```

output
```
integral((a*c^3*d*f^2*x^3 - a*c^2*d*f^2*x^2 - a*c*d*f^2*x + a*d*f^2 + (b*c
^3*d*f^2*x^3 - b*c^2*d*f^2*x^2 - b*c*d*f^2*x + b*d*f^2)*arccos(c*x))*sqrt(
c*d*x + d)*sqrt(-c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+cdx)^{3/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx=\text{Timed out}$$

input
```
integrate((c*d*x+d)**(3/2)*(-c*f*x+f)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+cdx)^{3/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx=\int (cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{5}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*f*x+f)^(5/2)*(a+b*arccos(c*x)),x, algorithm=
"maxima")
```

output
```
b*sqrt(d)*sqrt(f)*integrate((c^3*d*f^2*x^3 - c^2*d*f^2*x^2 - c*d*f^2*x + d
*f^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c
*x), x) + 1/40*(15*sqrt(-c^2*d*f*x^2 + d*f)*d*f^2*x + 15*d^2*f^3*arcsin(c*
x)/(sqrt(d*f)*c) + 10*(-c^2*d*f*x^2 + d*f)^(3/2)*f*x + 8*(-c^2*d*f*x^2 + d
*f)^(5/2)/(c*d))*a
```

Giac [F]

$$\int (d+cdx)^{3/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx=\int (cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{5}{2}}(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*f*x+f)^(5/2)*(a+b*arccos(c*x)),x, algorithm=
"giac")
```

output
```
integrate((c*d*x + d)^(3/2)*(-c*f*x + f)^(5/2)*(b*arccos(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)^{3/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,\left(d+c\,d\,x\right)^{3/2}\,\left(f-c\,f\,x\right)^{5/2}\,dx$$

input
```
int((a + b*acos(c*x))*(d + c*d*x)^(3/2)*(f - c*f*x)^(5/2),x)
```

output
```
int((a + b*acos(c*x))*(d + c*d*x)^(3/2)*(f - c*f*x)^(5/2), x)
```

Reduce [F]

$$\int (d+cdx)^{3/2}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx=\frac{\sqrt{f}\,\sqrt{d}\,d\,f^2\left(-30 asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a+8\sqrt{cx+1}\,\sqrt{-cx+1}\,a\,c^4x^4-10\sqrt{cx-}\right.}{}$$

input
```
int((c*d*x+d)^(3/2)*(-c*f*x+f)^(5/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*d*f**2*( - 30*asin(sqrt( - c*x + 1)/sqrt(2))*a + 8*sqrt(c
*x + 1)*sqrt( - c*x + 1)*a*c**4*x**4 - 10*sqrt(c*x + 1)*sqrt( - c*x + 1)*a
*c**3*x**3 - 16*sqrt(c*x + 1)*sqrt( - c*x + 1)*a*c**2*x**2 + 25*sqrt(c*x +
 1)*sqrt( - c*x + 1)*a*c*x + 8*sqrt(c*x + 1)*sqrt( - c*x + 1)*a + 40*int(s
qrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**3,x)*b*c**4 - 40*int(sqrt(c*x +
 1)*sqrt( - c*x + 1)*acos(c*x)*x**2,x)*b*c**3 - 40*int(sqrt(c*x + 1)*sqrt(
 - c*x + 1)*acos(c*x)*x,x)*b*c**2 + 40*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*
acos(c*x),x)*b*c))/(40*c)
```

3.520 $\int \sqrt{d+cdx}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 376

$$\begin{aligned}\int \sqrt{d+cdx}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx &= -\frac{2bf^2x\sqrt{d+cdx}\sqrt{f-cfx}}{3\sqrt{1-c^2x^2}} \\ &- \frac{3bcf^2x^2\sqrt{d+cdx}\sqrt{f-cfx}}{16\sqrt{1-c^2x^2}} + \frac{2bc^2f^2x^3\sqrt{d+cdx}\sqrt{f-cfx}}{9\sqrt{1-c^2x^2}} \\ &- \frac{bc^3f^2x^4\sqrt{d+cdx}\sqrt{f-cfx}}{16\sqrt{1-c^2x^2}} + \frac{3}{8}f^2x\sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx)) \\ &+ \frac{1}{4}c^2f^2x^3\sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx)) \\ &+ \frac{2f^2\sqrt{d+cdx}\sqrt{f-cfx}(1-c^2x^2)\,(a+b\arccos(cx))}{3c} \\ &+ \frac{5f^2\sqrt{d+cdx}\sqrt{f-cfx}(a+b\arccos(cx))^2}{16bc\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-2/3*b*f^2*x*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)/(-c^2*x^2+1)^(1/2)-3/16*b*c*
f^2*x^2*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)/(-c^2*x^2+1)^(1/2)+2/9*b*c^2*f^2*
x^3*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)/(-c^2*x^2+1)^(1/2)-1/16*b*c^3*f^2*x^4
*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)/(-c^2*x^2+1)^(1/2)+3/8*f^2*x*(c*d*x+d)^(
1/2)*(-c*f*x+f)^(1/2)*(a+b*arccos(c*x))+1/4*c^2*f^2*x^3*(c*d*x+d)^(1/2)*(-
c*f*x+f)^(1/2)*(a+b*arccos(c*x))+2/3*f^2*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)*
(-c^2*x^2+1)*(a+b*arccos(c*x))/c+5/16*f^2*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)
*(a+b*arccos(c*x))^2/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.78

$$\int \sqrt{d+cdx}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx = \frac{-360bf^2\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx)^2-720a\sqrt{d}f^{5/2}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{f-}}{\sqrt{d}\sqrt{f}(-1+c^2}\right.}{}$$

input

```
Integrate[Sqrt[d + c*d*x]*(f - c*f*x)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output

```
(-360*b*f^2*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]^2 - 720*a*Sqrt[d]*
f^(5/2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sq
rt[d]*Sqrt[f]*(-1 + c^2*x^2))] + f^2*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(-256
*b*c*x*(-3 + c^2*x^2) + 48*a*Sqrt[1 - c^2*x^2]*(16 + 9*c*x - 16*c^2*x^2 +
6*c^3*x^3) + 144*b*Cos[2*ArcCos[c*x]] + 9*b*Cos[4*ArcCos[c*x]]) + 12*b*f^2
*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]*(64*(1 - c^2*x^2)^(3/2) + 24*
Sin[2*ArcCos[c*x]] + 3*Sin[4*ArcCos[c*x]]))/(1152*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cdx+d}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5179}$$

$$\frac{\sqrt{cdx+d}\sqrt{f-cfx}\int f^2(1-cx)^2\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f^2\sqrt{cdx+d}\sqrt{f-cfx}\int(1-cx)^2\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$\downarrow$ 5263

$$\frac{f^2\sqrt{cdx+d}\sqrt{f-cfx}\int\left(c^2\sqrt{1-c^2x^2}(a+b\arccos(cx))x^2-2c\sqrt{1-c^2x^2}(a+b\arccos(cx))x+\sqrt{1-c^2x^2}(a+\right.}{\sqrt{1-c^2x^2}}$$

$\downarrow$ 2009

$$\frac{f^2\sqrt{cdx+d}\sqrt{f-cfx}\left(\frac{3}{8}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{2(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3c}+\frac{1}{4}c^2x^3\sqrt{1-c^2x^2}(a+b\operatorname{arcc}\right.}{\sqrt{1-c^2x^2}}$$

input
```
Int[Sqrt[d + c*d*x]*(f - c*f*x)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(f^2*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*((2*b*x)/3 + (3*b*c*x^2)/16 - (2*b*c^
2*x^3)/9 + (b*c^3*x^4)/16 + (3*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/8
+ (c^2*x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/4 + (2*(1 - c^2*x^2)^(3/
2)*(a + b*ArcCos[c*x]))/(3*c) - (5*(a + b*ArcCos[c*x])^2)/(16*b*c)))/Sqrt[
1 - c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1)) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 952, normalized size of antiderivative = 2.53

method	result
default	$-\frac{a\sqrt{cdx+d}\,(-cfx+f)^{\frac{7}{2}}}{4cf}+\frac{a(-cfx+f)^{\frac{5}{2}}\sqrt{cdx+d}}{12c}+\frac{5af(-cfx+f)^{\frac{3}{2}}\sqrt{cdx+d}}{24c}+\frac{5a\,f^{2}\sqrt{-cfx+f}\,\sqrt{cdx+d}}{8c}+\frac{5ad\,f^{3}\sqrt{(-cf}}{8}$
parts	$-\frac{a\sqrt{cdx+d}\,(-cfx+f)^{\frac{7}{2}}}{4cf}+\frac{a(-cfx+f)^{\frac{5}{2}}\sqrt{cdx+d}}{12c}+\frac{5af(-cfx+f)^{\frac{3}{2}}\sqrt{cdx+d}}{24c}+\frac{5a\,f^{2}\sqrt{-cfx+f}\,\sqrt{cdx+d}}{8c}+\frac{5ad\,f^{3}\sqrt{(-cf}}{8}$

input
```
int((c*d*x+d)^(1/2)*(-c*f*x+f)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVER
BOSE)
```

output
```
-1/4*a/c/f*(c*d*x+d)^(1/2)*(-c*f*x+f)^(7/2)+1/12*a/c*(-c*f*x+f)^(5/2)*(c*d
*x+d)^(1/2)+5/24*a*f/c*(-c*f*x+f)^(3/2)*(c*d*x+d)^(1/2)+5/8*a*f^2/c*(-c*f*
x+f)^(1/2)*(c*d*x+d)^(1/2)+5/8*a*d*f^3*((-c*f*x+f)*(c*d*x+d))^(1/2)/(-c*f*
x+f)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*f)^(1/2)*arctan((c^2*d*f)^(1/2)*x/(-c^2*
d*f*x^2+d*f)^(1/2))+b*(5/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2
+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^2*f^2+1/256*(d*(c*x+1))^(1/2)*(-f*(c*x
-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-8*I*
(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(I+4*arccos(c*x))*f^2/(c^
2*x^2-1)/c-1/36*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+
4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-3*I*(-c^2*x^2+1)^(1/2)*c*x+1)*(I+3*arccos(c
*x))*f^2/(c^2*x^2-1)/c+1/4*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*
x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)-I)*f^2/(c^2*x^2-1)/c+1/16*(d*(c*x
+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I
*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))*f^2/(c^2*x^2-1)/c-1/256*(d*(
c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(17
*I+28*arccos(c*x))*cos(3*arccos(c*x))*f^2/(c^2*x^2-1)/c-3/256*(d*(c*x+1))^
(1/2)*(-f*(c*x-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(5*I+12*arcc
os(c*x))*sin(3*arccos(c*x))*f^2/(c^2*x^2-1)/c-1/9*(d*(c*x+1))^(1/2)*(-f*(c
*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(2*I+3*arccos(c*x))*cos
(2*arccos(c*x))*f^2/(c^2*x^2-1)/c-1/18*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(...
```

Fricas [F]

$$\int \sqrt{d+cdx}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx = \int \sqrt{cdx+d}(-cfx+f)^{\frac{5}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*f*x+f)^(5/2)*(a+b*arccos(c*x)),x, algorithm=
"fricas")
```

output
```
integral((a*c^2*f^2*x^2 - 2*a*c*f^2*x + a*f^2 + (b*c^2*f^2*x^2 - 2*b*c*f^2
*x + b*f^2)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*f*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d+cdx}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+d)**(1/2)*(-c*f*x+f)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{d+cdx}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx = \int \sqrt{cdx+d}(-cfx+f)^{\frac{5}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*f*x+f)^(5/2)*(a+b*arccos(c*x)),x, algorithm=
"maxima")
```

output
```
b*sqrt(d)*sqrt(f)*integrate((c^2*f^2*x^2 - 2*c*f^2*x + f^2)*sqrt(c*x + 1)*
sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x) + 1/24*(15*s
qrt(-c^2*d*f*x^2 + d*f)*f^2*x + 15*d*f^3*arcsin(c*x)/(sqrt(d*f)*c) - 6*(-c
^2*d*f*x^2 + d*f)^(3/2)*f*x/d + 16*(-c^2*d*f*x^2 + d*f)^(3/2)*f/(c*d))*a
```

Giac [F]

$$\int \sqrt{d+cdx}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx = \int \sqrt{cdx+d}(-cfx+f)^{\frac{5}{2}}(b\arccos(cx)+a)\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*f*x+f)^(5/2)*(a+b*arccos(c*x)),x, algorithm=
"giac")
```

output
```
integrate(sqrt(c*d*x + d)*(-c*f*x + f)^(5/2)*(b*arccos(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+cdx}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,\sqrt{d+c\,d\,x}\,(f-c\,f\,x)^{5/2}\,dx$$

input
```
int((a + b*acos(c*x))*(d + c*d*x)^(1/2)*(f - c*f*x)^(5/2),x)
```

output
```
int((a + b*acos(c*x))*(d + c*d*x)^(1/2)*(f - c*f*x)^(5/2), x)
```

Reduce [F]

$$\int \sqrt{d+cdx}(f-cfx)^{5/2}(a+b\arccos(cx))\,dx=\frac{\sqrt{f}\,\sqrt{d}\,f^2\left(-30asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a+6\sqrt{cx+1}\,\sqrt{-cx+1}\,a\,c^3x^3-16\sqrt{cx+1}\,\sqrt{-cx+1}\right.}{}$$

input
```
int((c*d*x+d)^(1/2)*(-c*f*x+f)^(5/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(f)*sqrt(d)*f**2*( - 30*asin(sqrt( - c*x + 1)/sqrt(2))*a + 6*sqrt(c*x
 + 1)*sqrt( - c*x + 1)*a*c**3*x**3 - 16*sqrt(c*x + 1)*sqrt( - c*x + 1)*a*c
**2*x**2 + 9*sqrt(c*x + 1)*sqrt( - c*x + 1)*a*c*x + 16*sqrt(c*x + 1)*sqrt(
 - c*x + 1)*a + 24*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**2,x)*b*
c**3 - 48*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x,x)*b*c**2 + 24*in
t(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x),x)*b*c))/(24*c)
```

3.521 $\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 345

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx = -\frac{11bf^3x\sqrt{1-c^2x^2}}{3\sqrt{d+cdx}\sqrt{f-cfx}} + \frac{3bcf^3x^2\sqrt{1-c^2x^2}}{4\sqrt{d+cdx}\sqrt{f-cfx}} - \frac{bc^2f^3x^3\sqrt{1-c^2x^2}}{9\sqrt{d+cdx}\sqrt{f-cfx}} + \frac{11f^3(1-c^2x^2)(a+b\arccos(cx))}{3c\sqrt{d+cdx}\sqrt{f-cfx}} - \frac{3f^3x(1-c^2x^2)(a+b\arccos(cx))}{2\sqrt{d+cdx}\sqrt{f-cfx}} + \frac{cf^3x^2(1-c^2x^2)(a+b\arccos(cx))}{3\sqrt{d+cdx}\sqrt{f-cfx}} + \frac{5f^3\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4bc\sqrt{d+cdx}\sqrt{f-cfx}}$$

output

```
-11/3*b*f^3*x*(-c^2*x^2+1)^(1/2)/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+3/4*b*c*
f^3*x^2*(-c^2*x^2+1)^(1/2)/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)-1/9*b*c^2*f^3*
x^3*(-c^2*x^2+1)^(1/2)/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+11/3*f^3*(-c^2*x^2
+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)-3/2*f^3*x*(-c^2*x
^2+1)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+1/3*c*f^3*x^2*(-c
^2*x^2+1)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+5/4*f^3*(-c^2
*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/b/c/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)
```

Mathematica [A] (verified)

Time = 5.94 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.83

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx = \frac{f^2(-1+cx)\csc^2\left(\frac{1}{2}\arccos(cx)\right)\left(90b\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx\right.}{}$$

input
```
Integrate[((f - c*f*x)^(5/2)*(a + b*ArcCos[c*x]))/Sqrt[d + c*d*x],x]
```

output
```
(f^2*(-1 + c*x)*Csc[ArcCos[c*x]/2]^2*(90*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]
*ArcCos[c*x]^2 + 180*a*Sqrt[d]*Sqrt[f]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[
d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d]*Sqrt[f]*(-1 + c^2*x^2))] + Sqrt[d + c
*d*x]*Sqrt[f - c*f*x]*(-8*b*c*x*(33 + c^2*x^2) - 12*a*Sqrt[1 - c^2*x^2]*(2
2 - 9*c*x + 2*c^2*x^2) + 27*b*Cos[2*ArcCos[c*x]]) - 6*b*Sqrt[d + c*d*x]*Sq
rt[f - c*f*x]*ArcCos[c*x]*(45*Sqrt[1 - c^2*x^2] - 9*Sin[2*ArcCos[c*x]] + S
in[3*ArcCos[c*x]])))/(144*c*d*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{\sqrt{cdx+d}}\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{f^3(1-cx)^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^3\sqrt{1-c^2x^2}\int \frac{(1-cx)^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \textcolor{blue}{5263}$$

$$\frac{f^3\sqrt{1-c^2x^2}\int\left(-\frac{c^3(a+b\arccos(cx))x^3}{\sqrt{1-c^2x^2}}+\frac{3c^2(a+b\arccos(cx))x^2}{\sqrt{1-c^2x^2}}-\frac{3c(a+b\arccos(cx))x}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}\right)dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{f^3\sqrt{1-c^2x^2}\left(\frac{1}{3}cx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{3}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{11\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c}-\frac{5(}{}\right.}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

input
```
Int[((f - c*f*x)^(5/2)*(a + b*ArcCos[c*x]))/Sqrt[d + c*d*x],x]
```

output
```
(f^3*Sqrt[1 - c^2*x^2]*((11*b*x)/3 - (3*b*c*x^2)/4 + (b*c^2*x^3)/9 + (11*S
qrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(3*c) - (3*x*Sqrt[1 - c^2*x^2]*(a +
b*ArcCos[c*x]))/2 + (c*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/3 - (5*(
a + b*ArcCos[c*x])^2)/(4*b*c)))/(Sqrt[d + c*d*x]*Sqrt[f - c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1)) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.13 (sec) , antiderivative size = 947, normalized size of antiderivative = 2.74

method	result
default	$\frac{a(-cfx+f)^{\frac{5}{2}}\sqrt{cdx+d}}{3dc}+\frac{5af(-cfx+f)^{\frac{3}{2}}\sqrt{cdx+d}}{6dc}+\frac{5a\,f^2\sqrt{-cfx+f}\,\sqrt{cdx+d}}{2dc}+\frac{5a\,f^3\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2 d}}{\sqrt{-c^2 df}}\right.}{2\sqrt{-cfx+f}\,\sqrt{cdx+d}\,\sqrt{c^2 df}}$
parts	$\frac{a(-cfx+f)^{\frac{5}{2}}\sqrt{cdx+d}}{3dc}+\frac{5af(-cfx+f)^{\frac{3}{2}}\sqrt{cdx+d}}{6dc}+\frac{5a\,f^2\sqrt{-cfx+f}\,\sqrt{cdx+d}}{2dc}+\frac{5a\,f^3\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2 d}}{\sqrt{-c^2 df}}\right.}{2\sqrt{-cfx+f}\,\sqrt{cdx+d}\,\sqrt{c^2 df}}$

input
```
int((-c*f*x+f)^(5/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/3*a/d/c*(-c*f*x+f)^(5/2)*(c*d*x+d)^(1/2)+5/6*a*f/d/c*(-c*f*x+f)^(3/2)*(c
*d*x+d)^(1/2)+5/2*a*f^2/d/c*(-c*f*x+f)^(1/2)*(c*d*x+d)^(1/2)+5/2*a*f^3*((-
c*f*x+f)*(c*d*x+d))^(1/2)/(-c*f*x+f)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*f)^(1/2)
*arctan((c^2*d*f)^(1/2)*x/(-c^2*d*f*x^2+d*f)^(1/2))+b*(5/4*(d*(c*x+1))^(1/
2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x+1)/d/c/(c*x-1)*arccos(c*x)^2
*f^2+1/144*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-4*c^3*x^3+8*I*(-c^2*x^2+
1)^(1/2)*x^3*c^3+8*c^4*x^4+3*c*x-4*I*(-c^2*x^2+1)^(1/2)*c^2*x^2-4*I*(-c^2*
x^2+1)^(1/2)*x*c-8*c^2*x^2+I*(-c^2*x^2+1)^(1/2)+1)*(I+3*arccos(c*x))*f^2/(
c*x+1)/d/c/(c*x-1)+15/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(I*(-c^2*x^2
+1)^(1/2)+c*x-1)*(arccos(c*x)+I)*f^2/(c*x+1)/d/c/(c*x-1)+15/8*(d*(c*x+1))^
(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x
)-I)*f^2/(c*x+1)/d/c/(c*x-1)-3/32*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-2
*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2+I*(-c^2*x^2+1)^(1/2)-c*x-1)*(-I+2*arcc
os(c*x))*f^2/(c*x+1)/d/c/(c*x-1)+1/288*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2
)*(-I*(-c^2*x^2+1)^(1/2)+c*x-1)*(23*I+54*arccos(c*x))*cos(3*arccos(c*x))*f
^2/(c*x+1)/d/c/(c*x-1)+1/96*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(I*c*x-I+
(-c^2*x^2+1)^(1/2))*(9*I+14*arccos(c*x))*sin(3*arccos(c*x))*f^2/(c*x+1)/d/
c/(c*x-1)-1/144*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2
)+c*x-1)*(109*I+132*arccos(c*x))*cos(2*arccos(c*x))*f^2/(c*x+1)/d/c/(c*x-1
)-1/72*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(I*c*x-I+(-c^2*x^2+1)^(1/2)...
```

Fricas [F]

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=\int \frac{(-cfx+f)^{\frac{5}{2}}(b\arccos{(cx)}+a)}{\sqrt{cdx+d}}\,dx$$

input
```
integrate((-c*f*x+f)^(5/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2),x, algorithm=
"fricas")
```

output
```
integral((a*c^2*f^2*x^2 - 2*a*c*f^2*x + a*f^2 + (b*c^2*f^2*x^2 - 2*b*c*f^2
*x + b*f^2)*arccos(c*x))*sqrt(-c*f*x + f)/sqrt(c*d*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=\text{Timed out}$$

input
```
integrate((-c*f*x+f)**(5/2)*(a+b*acos(c*x))/(c*d*x+d)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=\int \frac{(-cfx+f)^{\frac{5}{2}}(b\arccos(cx)+a)}{\sqrt{cdx+d}}\,dx$$

input
```
integrate((-c*f*x+f)^(5/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2),x, algorithm=
"maxima")
```

output
```
1/6*(2*sqrt(-c^2*d*f*x^2 + d*f)*c*f^2*x^2/d - 9*sqrt(-c^2*d*f*x^2 + d*f)*f
^2*x/d + 15*f^3*arcsin(c*x)/(sqrt(d*f)*c) + 22*sqrt(-c^2*d*f*x^2 + d*f)*f^
2/(c*d))*a + b*sqrt(f)*integrate((c^2*f^2*x^2 - 2*c*f^2*x + f^2)*sqrt(-c*x
 + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/sqrt(c*x + 1), x)/sqrt(d)
```

Giac [F]

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=\int \frac{(-cfx+f)^{\frac{5}{2}}(b\arccos(cx)+a)}{\sqrt{cdx+d}}\,dx$$

input
```
integrate((-c*f*x+f)^(5/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((-c*f*x + f)^(5/2)*(b*arccos(c*x) + a)/sqrt(c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(f-c\,f\,x\right)^{5/2}}{\sqrt{d+c\,d\,x}}\,dx$$

input
```
int(((a + b*acos(c*x))*(f - c*f*x)^(5/2))/(d + c*d*x)^(1/2),x)
```

output
```
int(((a + b*acos(c*x))*(f - c*f*x)^(5/2))/(d + c*d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{\sqrt{d+cdx}}\,dx=\frac{\sqrt{f}\,f^2\left(-30asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a+2\sqrt{cx+1}\,\sqrt{-cx+1}\,a\,c^2x^2-9\sqrt{cx}\right.}{}$$

input
```
int((-c*f*x+f)^(5/2)*(a+b*acos(c*x))/(c*d*x+d)^(1/2),x)
```

output
```
(sqrt(f)*f**2*( - 30*asin(sqrt( - c*x + 1)/sqrt(2))*a + 2*sqrt(c*x + 1)*sq
rt( - c*x + 1)*a*c**2*x**2 - 9*sqrt(c*x + 1)*sqrt( - c*x + 1)*a*c*x + 22*s
qrt(c*x + 1)*sqrt( - c*x + 1)*a + 6*int((sqrt( - c*x + 1)*acos(c*x)*x**2)/
sqrt(c*x + 1),x)*b*c**3 - 12*int((sqrt( - c*x + 1)*acos(c*x)*x)/sqrt(c*x +
 1),x)*b*c**2 + 6*int((sqrt( - c*x + 1)*acos(c*x))/sqrt(c*x + 1),x)*b*c))/
(6*sqrt(d)*c)
```

3.522 $\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 400

$$\begin{aligned}\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx &= \frac{4bf^4x(1-c^2x^2)^{3/2}}{(d+cdx)^{3/2}(f-cfx)^{3/2}} \\ &- \frac{bcf^4x^2(1-c^2x^2)^{3/2}}{4(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{15bf^4(1-c^2x^2)^{3/2}\arccos(cx)^2}{4c(d+cdx)^{3/2}(f-cfx)^{3/2}} \\ &- \frac{8f^4(1-cx)\,(1-c^2x^2)\,(a+b\arccos(cx))}{c(d+cdx)^{3/2}(f-cfx)^{3/2}} \\ &- \frac{4f^4(1-c^2x^2)^2\,(a+b\arccos(cx))}{c(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{f^4x(1-c^2x^2)^2\,(a+b\arccos(cx))}{2(d+cdx)^{3/2}(f-cfx)^{3/2}} \\ &- \frac{15f^4(1-c^2x^2)^{3/2}\arccos(cx)(a+b\arccos(cx))}{2c(d+cdx)^{3/2}(f-cfx)^{3/2}} \\ &+ \frac{8bf^4(1-c^2x^2)^{3/2}\log(1+cx)}{c(d+cdx)^{3/2}(f-cfx)^{3/2}}\end{aligned}$$

output
```
4*b*f^4*x*(-c^2*x^2+1)^(3/2)/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)-1/4*b*c*f^4*
x^2*(-c^2*x^2+1)^(3/2)/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)+15/4*b*f^4*(-c^2*x
^2+1)^(3/2)*arccos(c*x)^2/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)-8*f^4*(-c*x+1
)*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)-4*f^4*
(-c^2*x^2+1)^2*(a+b*arccos(c*x))/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)+1/2*f^
4*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)-15/2
*f^4*(-c^2*x^2+1)^(3/2)*arccos(c*x)*(a+b*arccos(c*x))/c/(c*d*x+d)^(3/2)/(-
c*f*x+f)^(3/2)+8*b*f^4*(-c^2*x^2+1)^(3/2)*ln(c*x+1)/c/(c*d*x+d)^(3/2)/(-c*
f*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 6.40 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.78

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx =$$
$$-\frac{f^2(-1+cx)\csc^2\left(\frac{1}{2}\arccos(cx)\right)\left(4b\sqrt{d+cdx}\sqrt{f-cfx}\sqrt{1-c^2x^2}(-24-7cx+c^2x^2)\arccos(cx)+30b(\right.}{}$$

input
```
Integrate[((f - c*f*x)^(5/2)*(a + b*ArcCos[c*x]))/(d + c*d*x)^(3/2),x]
```

output
```
-1/16*(f^2*(-1 + c*x)*Csc[ArcCos[c*x]/2]^2*(4*b*Sqrt[d + c*d*x]*Sqrt[f - c
*f*x]*Sqrt[1 - c^2*x^2]*(-24 - 7*c*x + c^2*x^2)*ArcCos[c*x] + 30*b*(1 + c*
x)*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]^2 + 60*a*Sqrt[d]*Sqrt[f]*(1
 + c*x)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sq
rt[d]*Sqrt[f]*(-1 + c^2*x^2))] + Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(4*a*Sqrt
[1 - c^2*x^2]*(-24 - 7*c*x + c^2*x^2) + b*(-1 - 33*c*x - 30*c^2*x^2 + 2*c^
3*x^3) - 128*b*(1 + c*x)*Log[Cos[ArcCos[c*x]/2]])))/(c*d^2*(1 + c*x)*Sqrt[
1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5179, 27, 5261, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(cdx+d)^{3/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{3/2}\int \frac{f^4(1-cx)^4(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^4(1-c^2x^2)^{3/2}\int \frac{(1-cx)^4(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{5261}$$

$$\frac{f^4(1-c^2x^2)^{3/2}\left(bc\int \left(\frac{x}{2}-\frac{15\arcsin(cx)}{2c\sqrt{1-c^2x^2}}-\frac{4}{c}-\frac{8(1-cx)}{c(1-c^2x^2)}\right)dx-\frac{15\arcsin(cx)(a+b\arccos(cx))}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\operatorname{arcc}\right.}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^4(1-c^2x^2)^{3/2}\left(-\frac{15\arcsin(cx)(a+b\arccos(cx))}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}-\frac{8(1-cx)(}{c\right.}}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

input
```
Int[((f - c*f*x)^(5/2)*(a + b*ArcCos[c*x]))/(d + c*d*x)^(3/2),x]
```

output
```
(f^4*(1 - c^2*x^2)^(3/2)*((-8*(1 - c*x)*(a + b*ArcCos[c*x]))/(c*Sqrt[1 - c
^2*x^2]) - (4*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c + (x*Sqrt[1 - c^2*x
^2]*(a + b*ArcCos[c*x]))/2 - (15*(a + b*ArcCos[c*x])*ArcSin[c*x])/(2*c) +
b*c*((-4*x)/c + x^2/4 - (15*ArcSin[c*x]^2)/(4*c^2) - (8*Log[1 + c*x])/c^2)
))/((d + c*d*x)^(3/2)*(f - c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5261
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p,
 x]}, Simp[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[1/Sqrt[1 - c^2*x^2]
   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IG
tQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 28.96 (sec) , antiderivative size = 907, normalized size of antiderivative = 2.27

method	result
default	$-\frac{15\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,b\arccos(cx)^2f^2}{4(cx+1)d^2(cx-1)c}-\frac{15\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,a\arccos(cx)f^2}{2(cx+1)d^2(cx-1)c}+\frac{\sqrt{d(cx+1)}}{}$

input
```
int((-c*f*x+f)^(5/2)*(a+b*arccos(c*x))/(c*d*x+d)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-15/4*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x+1)/d^2/
(c*x-1)/c*b*arccos(c*x)^2*f^2-15/2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-
c^2*x^2+1)^(1/2)/(c*x+1)/d^2/(c*x-1)/c*a*arccos(c*x)*f^2+1/32*(d*(c*x+1))^
(1/2)*(-f*(c*x-1))^(1/2)*(-2*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+4*c^3*
x^3+1-2*I*(-c^2*x^2+1)^(1/2)*c*x-I*(-c^2*x^2+1)^(1/2)-3*c*x)*(I*b+2*b*arcc
os(c*x)+2*a)*f^2/(c*x+1)/d^2/(c*x-1)/c-(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2
)*(I*(-c^2*x^2+1)^(1/2)+c*x-1)*(b*arccos(c*x)+a+I*b)*f^2/(c*x+1)/d^2/(c*x-
1)/c-2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2
*x^2-1)*(b*arccos(c*x)+a-I*b)*f^2/(c*x+1)/d^2/(c*x-1)/c+1/32*(d*(c*x+1))^(
1/2)*(-f*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2+I*(-c^2*x^2
+1)^(1/2)-c*x-1)*(-I*b+2*b*arccos(c*x)+2*a)*f^2/(c*x+1)/d^2/(c*x-1)/c-16*I
*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)/(c*x+1)/d^2/(c*x-
1)/c*b*arccos(c*x)*f^2-8*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^
2+1)^(1/2)+c*x-1)*(a+b*arccos(c*x))*f^2/(c*x+1)/d^2/(c*x-1)/c+16*(d*(c*x+1
))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x+1)/d^2/(c*x-1)/c*b*ln(
1+c*x+I*(-c^2*x^2+1)^(1/2))*f^2+1/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*
(-I*(-c^2*x^2+1)^(1/2)+c*x-1)*(15*I*b+16*b*arccos(c*x)+16*a)*cos(2*arccos(
c*x))*f^2/(c*x+1)/d^2/(c*x-1)/c+1/8*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(
I*c*x-I+(-c^2*x^2+1)^(1/2))*(8*I*b+7*b*arccos(c*x)+7*a)*sin(2*arccos(c*x))
*f^2/(c*x+1)/d^2/(c*x-1)/c
```

Fricas [F]

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx=\int \frac{(-cfx+f)^{\frac{5}{2}}(b\arccos{(cx)}+a)}{(cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(5/2)*(a+b*arccos(c*x))/(c*d*x+d)^(3/2),x, algorithm=
"fricas")
```

output
```
integral((a*c^2*f^2*x^2 - 2*a*c*f^2*x + a*f^2 + (b*c^2*f^2*x^2 - 2*b*c*f^2
*x + b*f^2)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*f*x + f)/(c^2*d^2*x^2 + 2
*c*d^2*x + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((-c*f*x+f)**(5/2)*(a+b*acos(c*x))/(c*d*x+d)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx=\int \frac{(-cfx+f)^{\frac{5}{2}}(b\arccos{(cx)}+a)}{(cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(5/2)*(a+b*arccos(c*x))/(c*d*x+d)^(3/2),x, algorithm=
"maxima")
```

output
```
-1/2*(c^2*f^3*x^3/(sqrt(-c^2*d*f*x^2 + d*f)*d) - 8*c*f^3*x^2/(sqrt(-c^2*d*
f*x^2 + d*f)*d) - 17*f^3*x/(sqrt(-c^2*d*f*x^2 + d*f)*d) + 15*f^3*arcsin(c*
x)/(sqrt(d*f)*c*d) + 24*f^3/(sqrt(-c^2*d*f*x^2 + d*f)*c*d))*a + b*sqrt(f)*
integrate((c^2*f^2*x^2 - 2*c*f^2*x + f^2)*sqrt(-c*x + 1)*arctan2(sqrt(c*x
+ 1)*sqrt(-c*x + 1), c*x)/((c*d*x + d)*sqrt(c*x + 1)), x)/sqrt(d)
```

Giac [F]

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx=\int \frac{(-cfx+f)^{\frac{5}{2}}(b\arccos{(cx)}+a)}{(cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(5/2)*(a+b*arccos(c*x))/(c*d*x+d)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((-c*f*x + f)^(5/2)*(b*arccos(c*x) + a)/(c*d*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(f-c\,f\,x\right)^{5/2}}{\left(d+c\,d\,x\right)^{3/2}}\,dx$$

input
```
int(((a + b*acos(c*x))*(f - c*f*x)^(5/2))/(d + c*d*x)^(3/2),x)
```

output
```
int(((a + b*acos(c*x))*(f - c*f*x)^(5/2))/(d + c*d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{3/2}}\,dx=\frac{\sqrt{f}\,f^2\left(30\sqrt{cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a+\sqrt{-cx+1}\,a\,c^2x^2-7\sqrt{-cx+}\right.}{}$$

input
```
int((-c*f*x+f)^(5/2)*(a+b*acos(c*x))/(c*d*x+d)^(3/2),x)
```

output
```
(sqrt(f)*f**2*(30*sqrt(c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a + sqrt( -
 c*x + 1)*a*c**2*x**2 - 7*sqrt( - c*x + 1)*a*c*x - 24*sqrt( - c*x + 1)*a +
 2*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)*x**2)/(sqrt(c*x + 1)*c*x
+ sqrt(c*x + 1)),x)*b*c**3 - 4*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*
x)*x)/(sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b*c**2 + 2*sqrt(c*x + 1)*int(
(sqrt( - c*x + 1)*acos(c*x))/(sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b*c))/
(2*sqrt(d)*sqrt(c*x + 1)*c*d)
```

3.523 $\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 420

$$\begin{aligned}\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx &= -\frac{bf^5x(1-c^2x^2)^{5/2}}{(d+cdx)^{5/2}(f-cfx)^{5/2}} \\ &-\frac{8bf^5(1-c^2x^2)^{5/2}}{3c(1+cx)(d+cdx)^{5/2}(f-cfx)^{5/2}}-\frac{5bf^5(1-c^2x^2)^{5/2}\arccos(cx)^2}{2c(d+cdx)^{5/2}(f-cfx)^{5/2}} \\ &-\frac{2f^5(1-cx)^4(1-c^2x^2)(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}} \\ &+\frac{20f^5(1-cx)(1-c^2x^2)^2(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}} \\ &+\frac{5f^5(1-c^2x^2)^3(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}} \\ &+\frac{5f^5(1-c^2x^2)^{5/2}\arccos(cx)(a+b\arccos(cx))}{c(d+cdx)^{5/2}(f-cfx)^{5/2}} \\ &-\frac{28bf^5(1-c^2x^2)^{5/2}\log(1+cx)}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}}\end{aligned}$$

output
```
-b*f^5*x*(-c^2*x^2+1)^(5/2)/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)-8/3*b*f^5*(-c
^2*x^2+1)^(5/2)/c/(c*x+1)/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)-5/2*b*f^5*(-c^2
*x^2+1)^(5/2)*arccos(c*x)^2/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)-2/3*f^5*(-c
*x+1)^4*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)+
20/3*f^5*(-c*x+1)*(-c^2*x^2+1)^2*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*f
*x+f)^(5/2)+5/3*f^5*(-c^2*x^2+1)^3*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c
*f*x+f)^(5/2)+5*f^5*(-c^2*x^2+1)^(5/2)*arccos(c*x)*(a+b*arccos(c*x))/c/(c*
d*x+d)^(5/2)/(-c*f*x+f)^(5/2)-28/3*b*f^5*(-c^2*x^2+1)^(5/2)*ln(c*x+1)/c/(c
*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)
```

Mathematica [A] (verified)

Time = 7.43 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.21

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx = \frac{f^2\left(4a\sqrt{d+cdx}\sqrt{f-cfx}\sqrt{1-c^2x^2}(23+34cx+3c^2x^2)-60a\sqrt{d}\right.}{}$$

input
```
Integrate[((f - c*f*x)^(5/2)*(a + b*ArcCos[c*x]))/(d + c*d*x)^(5/2),x]
```

output
```
(f^2*(4*a*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*Sqrt[1 - c^2*x^2]*(23 + 34*c*x +
 3*c^2*x^2) - 60*a*Sqrt[d]*Sqrt[f]*(1 + c*x)^2*Sqrt[1 - c^2*x^2]*ArcTan[(c
*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d]*Sqrt[f]*(-1 + c^2*x^2))] - 8*
b*(1 - c*x)*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*Cot[ArcCos[c*x]/2]*(-((5 + 7*c
*x)*ArcCos[c*x]) + 3*ArcCos[c*x]^2*Cos[ArcCos[c*x]/2]^2*Cot[ArcCos[c*x]/2]
 - 2*Cot[ArcCos[c*x]/2]*(1 + 7*(1 + c*x)*Log[Cos[ArcCos[c*x]/2]])) - 8*b*(
1 - c*x)*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*Cot[ArcCos[c*x]/2]*(-(Cot[ArcCos[
c*x]/2]*(1 + (1 + c*x)*Log[Cos[ArcCos[c*x]/2]])) + ArcCos[c*x]*Sin[ArcCos[
c*x]/2]^2) + b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*Sqrt[1 - c^2*x^2]*Csc[ArcCo
s[c*x]/2]*(-36*ArcCos[c*x]^2*Cos[ArcCos[c*x]/2]^3 + 4*Cos[ArcCos[c*x]/2]*(
2 + 3*c*x + 3*c^2*x^2 + 26*(1 + c*x)*Log[Cos[ArcCos[c*x]/2]]) + ArcCos[c*x
]*(24*Sin[ArcCos[c*x]/2] + 35*Sin[(3*ArcCos[c*x])/2] + 3*Sin[(5*ArcCos[c*x
])/2]))))/(12*c*d^3*(1 + c*x)^2*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.49, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5179, 27, 5261, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(cdx+d)^{5/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{5/2}\int \frac{f^5(1-cx)^5(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f^5(1-c^2x^2)^{5/2}\int \frac{(1-cx)^5(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{5261}$$

$$\frac{f^5(1-c^2x^2)^{5/2}\left(bc\int \left(-\frac{2(1-cx)^4}{3c(1-c^2x^2)^2}+\frac{20(1-cx)}{3c(1-c^2x^2)}+\frac{5\arcsin(cx)}{c\sqrt{1-c^2x^2}}+\frac{5}{3c}\right)dx+\frac{5\arcsin(cx)(a+b\arccos(cx))}{c}-\frac{2(1-cx)^4(a+b\text{ ar}}{3c(1-c^2x^2}\right.}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^5(1-c^2x^2)^{5/2}\left(\frac{5\arcsin(cx)(a+b\arccos(cx))}{c}-\frac{2(1-cx)^4(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}+\frac{20(1-cx)(a+b\arccos(cx))}{3c\sqrt{1-c^2x^2}}+\frac{5\sqrt{1-c^2x^2}(a+b\arccos(cx)}{3c}\right.}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

input
```
Int[((f - c*f*x)^(5/2)*(a + b*ArcCos[c*x]))/(d + c*d*x)^(5/2),x]
```

output
```
(f^5*(1 - c^2*x^2)^(5/2)*((-2*(1 - c*x)^4*(a + b*ArcCos[c*x]))/(3*c*(1 - c
^2*x^2)^(3/2)) + (20*(1 - c*x)*(a + b*ArcCos[c*x]))/(3*c*Sqrt[1 - c^2*x^2]
) + (5*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(3*c) + (5*(a + b*ArcCos[c*x
])*ArcSin[c*x])/c + b*c*(x/c + 8/(3*c^2*(1 + c*x)) + (5*ArcSin[c*x]^2)/(2*
c^2) + (28*Log[1 + c*x])/(3*c^2))))/((d + c*d*x)^(5/2)*(f - c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5261
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p,
 x]}, Simp[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[1/Sqrt[1 - c^2*x^2]
   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IG
tQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 17.72 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.99

method	result
default	$\frac{\sqrt{-f(cx-1)}\,\sqrt{d(cx+1)}\,\sqrt{-c^2x^2+1}\left(-46\arccos(cx)\sqrt{-c^2x^2+1}\,b-16b-46\sqrt{-c^2x^2+1}\,a-68\sqrt{-c^2x^2+1}\,acx+60\arccos(cx)acx-56\right.}{}$

input
```
int((-c*f*x+f)^(5/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/6*(-f*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^3*x^3+3*c^2
*x^2+3*c*x+1)/d^3/(c*x-1)/c*(-46*arccos(c*x)*(-c^2*x^2+1)^(1/2)*b-16*b-112
*I*a*c*x-46*(-c^2*x^2+1)^(1/2)*a-68*(-c^2*x^2+1)^(1/2)*a*c*x-112*ln(1+c*x+
I*(-c^2*x^2+1)^(1/2))*b*c^2*x^2+60*arccos(c*x)*a*c*x-224*ln(1+c*x+I*(-c^2*
x^2+1)^(1/2))*b*c*x-6*(-c^2*x^2+1)^(1/2)*arccos(c*x)*b*c^2*x^2-56*I*a*c^2*
x^2+15*arccos(c*x)^2*b*c^2*x^2+30*arccos(c*x)*a*c^2*x^2-6*(-c^2*x^2+1)^(1/
2)*a*c^2*x^2-22*c*x*b-6*b*c^3*x^3-12*x^2*c^2*b+30*b*c*x*arccos(c*x)^2+56*I
*arccos(c*x)*b*c^2*x^2+112*I*arccos(c*x)*b*c*x-68*arccos(c*x)*(-c^2*x^2+1)
^(1/2)*b*c*x-56*I*a-112*b*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+15*arccos(c*x)^2*
b+30*arccos(c*x)*a+56*I*b*arccos(c*x))*f^2
```

Fricas [F]

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx = \int \frac{(-cfx+f)^{\frac{5}{2}}(b\arccos(cx)+a)}{(cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(5/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2),x, algorithm=
"fricas")
```

output
```
integral((a*c^2*f^2*x^2 - 2*a*c*f^2*x + a*f^2 + (b*c^2*f^2*x^2 - 2*b*c*f^2
*x + b*f^2)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*f*x + f)/(c^3*d^3*x^3 + 3
*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((-c*f*x+f)**(5/2)*(a+b*acos(c*x))/(c*d*x+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx = \int \frac{(-cfx+f)^{\frac{5}{2}}(b\arccos{(cx)}+a)}{(cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(5/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2),x, algorithm=
"maxima")
```

output
```
1/3*(3*(-c^2*d*f*x^2 + d*f)^(5/2)/(c^5*d^5*x^4 + 4*c^4*d^5*x^3 + 6*c^3*d^5
*x^2 + 4*c^2*d^5*x + c*d^5) - 5*(-c^2*d*f*x^2 + d*f)^(3/2)*f/(c^4*d^4*x^3
+ 3*c^3*d^4*x^2 + 3*c^2*d^4*x + c*d^4) - 10*sqrt(-c^2*d*f*x^2 + d*f)*f^2/(
c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3) + 35*sqrt(-c^2*d*f*x^2 + d*f)*f^2/(c^2*
d^3*x + c*d^3) + 15*f^3*arcsin(c*x)/(c*d^3*sqrt(f/d)))*a + b*sqrt(f)*integ
rate((c^2*f^2*x^2 - 2*c*f^2*x + f^2)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*
sqrt(-c*x + 1), c*x)/((c^2*d^2*x^2 + 2*c*d^2*x + d^2)*sqrt(c*x + 1)), x)/s
qrt(d)
```

Giac [F]

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx = \int \frac{(-cfx+f)^{\frac{5}{2}}(b\arccos{(cx)}+a)}{(cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*f*x+f)^(5/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((-c*f*x + f)^(5/2)*(b*arccos(c*x) + a)/(c*d*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(f-c\,f\,x\right)^{5/2}}{\left(d+c\,d\,x\right)^{5/2}}\,dx$$

input
```
int(((a + b*acos(c*x))*(f - c*f*x)^(5/2))/(d + c*d*x)^(5/2),x)
```

output
```
int(((a + b*acos(c*x))*(f - c*f*x)^(5/2))/(d + c*d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(f-cfx)^{5/2}(a+b\arccos(cx))}{(d+cdx)^{5/2}}\,dx=\frac{\sqrt{f}\,f^2\left(-30\sqrt{cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)acx-30\sqrt{cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)\right.}{}$$

input
```
int((-c*f*x+f)^(5/2)*(a+b*acos(c*x))/(c*d*x+d)^(5/2),x)
```

output
```
(sqrt(f)*f**2*( - 30*sqrt(c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a*c*x -
30*sqrt(c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a + 3*sqrt( - c*x + 1)*a*c
**2*x**2 + 34*sqrt( - c*x + 1)*a*c*x + 23*sqrt( - c*x + 1)*a + 3*sqrt(c*x
+ 1)*int((sqrt( - c*x + 1)*acos(c*x)*x**2)/(sqrt(c*x + 1)*c**2*x**2 + 2*sq
rt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b*c**4*x + 3*sqrt(c*x + 1)*int((sqrt(
- c*x + 1)*acos(c*x)*x**2)/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x
+ sqrt(c*x + 1)),x)*b*c**3 - 6*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*
x)*x)/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b
*c**3*x - 6*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)*x)/(sqrt(c*x + 1
)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b*c**2 + 3*sqrt(c*x
+ 1)*int((sqrt( - c*x + 1)*acos(c*x))/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*
x + 1)*c*x + sqrt(c*x + 1)),x)*b*c**2*x + 3*sqrt(c*x + 1)*int((sqrt( - c*x
 + 1)*acos(c*x))/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x
 + 1)),x)*b*c))/(3*sqrt(d)*sqrt(c*x + 1)*c*d**2*(c*x + 1))
```

3.524 $\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 345

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx = \frac{11bd^3x\sqrt{1-c^2x^2}}{3\sqrt{d+cdx}\sqrt{f-cfx}} + \frac{3bcd^3x^2\sqrt{1-c^2x^2}}{4\sqrt{d+cdx}\sqrt{f-cfx}} + \frac{bc^2d^3x^3\sqrt{1-c^2x^2}}{9\sqrt{d+cdx}\sqrt{f-cfx}} - \frac{11d^3(1-c^2x^2)(a+b\arccos(cx))}{3c\sqrt{d+cdx}\sqrt{f-cfx}} - \frac{3d^3x(1-c^2x^2)(a+b\arccos(cx))}{2\sqrt{d+cdx}\sqrt{f-cfx}} - \frac{cd^3x^2(1-c^2x^2)(a+b\arccos(cx))}{3\sqrt{d+cdx}\sqrt{f-cfx}} + \frac{5d^3\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4bc\sqrt{d+cdx}\sqrt{f-cfx}}$$

output
```
11/3*b*d^3*x*(-c^2*x^2+1)^(1/2)/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+3/4*b*c*d
^3*x^2*(-c^2*x^2+1)^(1/2)/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+1/9*b*c^2*d^3*x
^3*(-c^2*x^2+1)^(1/2)/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)-11/3*d^3*(-c^2*x^2+
1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)-3/2*d^3*x*(-c^2*x^
2+1)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)-1/3*c*d^3*x^2*(-c^
2*x^2+1)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+5/4*d^3*(-c^2*
x^2+1)^(1/2)*(a+b*arccos(c*x))^2/b/c/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)
```

Mathematica [A] (verified)

Time = 3.39 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.83

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx =$$
$$-\frac{d^2(1+cx)\sec^2\left(\frac{1}{2}\arccos(cx)\right)\left(90b\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx)^2+180a\sqrt{d}\sqrt{f}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{}}{\sqrt{d}}\right.\right.}{}$$

input
```
Integrate[((d + c*d*x)^(5/2)*(a + b*ArcCos[c*x]))/Sqrt[f - c*f*x],x]
```

output
```
-1/144*(d^2*(1 + c*x)*Sec[ArcCos[c*x]/2]^2*(90*b*Sqrt[d + c*d*x]*Sqrt[f -
c*f*x]*ArcCos[c*x]^2 + 180*a*Sqrt[d]*Sqrt[f]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x
*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d]*Sqrt[f]*(-1 + c^2*x^2))] + Sqrt
[d + c*d*x]*Sqrt[f - c*f*x]*(8*b*c*x*(33 + c^2*x^2) + 12*a*Sqrt[1 - c^2*x^
2]*(22 + 9*c*x + 2*c^2*x^2) + 27*b*Cos[2*ArcCos[c*x]]) + 6*b*Sqrt[d + c*d*
x]*Sqrt[f - c*f*x]*ArcCos[c*x]*(45*Sqrt[1 - c^2*x^2] + 9*Sin[2*ArcCos[c*x]
] + Sin[3*ArcCos[c*x]])))/(c*f*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^{5/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx$$

$$\Big\downarrow \text{ 5179}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{d^3(cx+1)^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d^3\sqrt{1-c^2x^2}\int \frac{(cx+1)^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \text{ 5263}$$

$$\frac{d^3\sqrt{1-c^2x^2}\int \left(\frac{c^3(a+b\arccos(cx))x^3}{\sqrt{1-c^2x^2}}+\frac{3c^2(a+b\arccos(cx))x^2}{\sqrt{1-c^2x^2}}+\frac{3c(a+b\arccos(cx))x}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}\right)dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d^3\sqrt{1-c^2x^2}\left(-\frac{1}{3}cx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{3}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{11\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c}-\right.}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

input
```
Int[((d + c*d*x)^(5/2)*(a + b*ArcCos[c*x]))/Sqrt[f - c*f*x],x]
```

output
```
(d^3*Sqrt[1 - c^2*x^2]*((-11*b*x)/3 - (3*b*c*x^2)/4 - (b*c^2*x^3)/9 - (11*
Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(3*c) - (3*x*Sqrt[1 - c^2*x^2]*(a +
 b*ArcCos[c*x]))/2 - (c*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/3 - (5*
(a + b*ArcCos[c*x])^2)/(4*b*c)))/(Sqrt[d + c*d*x]*Sqrt[f - c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 946, normalized size of antiderivative = 2.74

method	result
default	$-\frac{a(cdx+d)^{\frac{5}{2}}\sqrt{-cfx+f}}{3cf}-\frac{5ad(cdx+d)^{\frac{3}{2}}\sqrt{-cfx+f}}{6cf}-\frac{5a\,d^{2}\sqrt{cdx+d}\,\sqrt{-cfx+f}}{2cf}+\frac{5a\,d^{3}\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c}}{\sqrt{-c^{2}}}\right.}{2\sqrt{cdx+d}\,\sqrt{-cfx+f}\,\sqrt{c^{2}df}}$
parts	$-\frac{a(cdx+d)^{\frac{5}{2}}\sqrt{-cfx+f}}{3cf}-\frac{5ad(cdx+d)^{\frac{3}{2}}\sqrt{-cfx+f}}{6cf}-\frac{5a\,d^{2}\sqrt{cdx+d}\,\sqrt{-cfx+f}}{2cf}+\frac{5a\,d^{3}\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c}}{\sqrt{-c^{2}}}\right.}{2\sqrt{cdx+d}\,\sqrt{-cfx+f}\,\sqrt{c^{2}df}}$

input
```
int((c*d*x+d)^(5/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/3*a/c/f*(c*d*x+d)^(5/2)*(-c*f*x+f)^(1/2)-5/6*a*d/c/f*(c*d*x+d)^(3/2)*(-
c*f*x+f)^(1/2)-5/2*a*d^2/c/f*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)+5/2*a*d^3*((
-c*f*x+f)*(c*d*x+d))^(1/2)/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)/(c^2*d*f)^(1/2
)*arctan((c^2*d*f)^(1/2)*x/(-c^2*d*f*x^2+d*f)^(1/2))+b*(5/4*(d*(c*x+1))^(1
/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x-1)/f/c/(c*x+1)*arccos(c*x)^
2*d^2-1/144*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(4*c^3*x^3+8*I*(-c^2*x^2+
1)^(1/2)*x^3*c^3+8*c^4*x^4-3*c*x+4*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-4*I*(-c^2*
x^2+1)^(1/2)*x*c-8*c^2*x^2-I*(-c^2*x^2+1)^(1/2)+1)*(I+3*arccos(c*x))*d^2/(
c*x-1)/f/c/(c*x+1)+15/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(1+c*x+I*(-c
^2*x^2+1)^(1/2))*(arccos(c*x)+I)*d^2/(c*x-1)/f/c/(c*x+1)-15/8*(d*(c*x+1))^
(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x
)-I)*d^2/(c*x-1)/f/c/(c*x+1)+3/32*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-2
*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-I*(-c^2*x^2+1)^(1/2)+c*x-1)*(-I+2*arcc
os(c*x))*d^2/(c*x-1)/f/c/(c*x+1)-1/288*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2
)*(-I*(-c^2*x^2+1)^(1/2)+c*x+1)*(23*I+54*arccos(c*x))*cos(3*arccos(c*x))*d
^2/(c*x-1)/f/c/(c*x+1)-1/96*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(I*c*x+(-
c^2*x^2+1)^(1/2)+I)*(9*I+14*arccos(c*x))*sin(3*arccos(c*x))*d^2/(c*x-1)/f/
c/(c*x+1)-1/144*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2
)+c*x+1)*(109*I+132*arccos(c*x))*cos(2*arccos(c*x))*d^2/(c*x-1)/f/c/(c*x+1
)-1/72*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)+I...
```

Fricas [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx=\int \frac{(cdx+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)}{\sqrt{-cfx+f}}\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-(a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^
2*x + b*d^2)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*f*x + f)/(c*f*x - f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+d)**(5/2)*(a+b*acos(c*x))/(-c*f*x+f)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx = \int \frac{(cdx+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{\sqrt{-cfx+f}}\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/6*(2*sqrt(-c^2*d*f*x^2 + d*f)*c*d^2*x^2/f + 9*sqrt(-c^2*d*f*x^2 + d*f)*
d^2*x/f - 15*d^3*arcsin(c*x)/(sqrt(d*f)*c) + 22*sqrt(-c^2*d*f*x^2 + d*f)*d
^2/(c*f))*a + b*sqrt(d)*integrate((c^2*d^2*x^2 + 2*c*d^2*x + d^2)*sqrt(c*x
 + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/sqrt(-c*x + 1), x)/sqrt(f
)
```

Giac [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx = \int \frac{(cdx+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{\sqrt{-cfx+f}}\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((c*d*x + d)^(5/2)*(b*arccos(c*x) + a)/sqrt(-c*f*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(d+c\,d\,x\right)^{5/2}}{\sqrt{f-c\,f\,x}}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + c*d*x)^(5/2))/(f - c*f*x)^(1/2),x)
```

output
```
int(((a + b*acos(c*x))*(d + c*d*x)^(5/2))/(f - c*f*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx=\frac{\sqrt{d}\,d^2\left(-30asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a-2\sqrt{cx+1}\,\sqrt{-cx+1}\,a\,c^2x^2-9\sqrt{cx+1}\right.}{}$$

input
```
int((c*d*x+d)^(5/2)*(a+b*acos(c*x))/(-c*f*x+f)^(1/2),x)
```

output
```
(sqrt(d)*d**2*( - 30*asin(sqrt( - c*x + 1)/sqrt(2))*a - 2*sqrt(c*x + 1)*sq
rt( - c*x + 1)*a*c**2*x**2 - 9*sqrt(c*x + 1)*sqrt( - c*x + 1)*a*c*x - 22*s
qrt(c*x + 1)*sqrt( - c*x + 1)*a + 6*int((sqrt(c*x + 1)*acos(c*x)*x**2)/sqr
t( - c*x + 1),x)*b*c**3 + 12*int((sqrt(c*x + 1)*acos(c*x)*x)/sqrt( - c*x +
 1),x)*b*c**2 + 6*int((sqrt(c*x + 1)*acos(c*x))/sqrt( - c*x + 1),x)*b*c))/
(6*sqrt(f)*c)
```

3.525 $\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 242

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx = \frac{2bd^2x\sqrt{1-c^2x^2}}{\sqrt{d+cdx}\sqrt{f-cfx}} + \frac{bcd^2x^2\sqrt{1-c^2x^2}}{4\sqrt{d+cdx}\sqrt{f-cfx}} - \frac{2d^2(1-c^2x^2)\,(a+b\arccos(cx))}{c\sqrt{d+cdx}\sqrt{f-cfx}} - \frac{d^2x(1-c^2x^2)\,(a+b\arccos(cx))}{2\sqrt{d+cdx}\sqrt{f-cfx}} + \frac{3d^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4bc\sqrt{d+cdx}\sqrt{f-cfx}}$$

output

```
2*b*d^2*x*(-c^2*x^2+1)^(1/2)/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+1/4*b*c*d^2*
x^2*(-c^2*x^2+1)^(1/2)/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)-2*d^2*(-c^2*x^2+1)
*(a+b*arccos(c*x))/c/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)-1/2*d^2*x*(-c^2*x^2+
1)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+3/4*d^2*(-c^2*x^2+1)
^(1/2)*(a+b*arccos(c*x))^2/b/c/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)
```

Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.06

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx =$$
$$-\frac{d(1+cx)\sec^2\left(\frac{1}{2}\arccos(cx)\right)\left(6b\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx)^2+12a\sqrt{d}\sqrt{f}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d+cd}}{\sqrt{d}\sqrt{f}(-}\right.\right.}{}$$

input
```
Integrate[((d + c*d*x)^(3/2)*(a + b*ArcCos[c*x]))/Sqrt[f - c*f*x],x]
```

output
```
-1/16*(d*(1 + c*x)*Sec[ArcCos[c*x]/2]^2*(6*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*
x]*ArcCos[c*x]^2 + 12*a*Sqrt[d]*Sqrt[f]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt
[d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d]*Sqrt[f]*(-1 + c^2*x^2))] + Sqrt[d +
c*d*x]*Sqrt[f - c*f*x]*(16*b*c*x + 4*a*(4 + c*x)*Sqrt[1 - c^2*x^2] + b*Cos
[2*ArcCos[c*x]]) + 2*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]*(8*Sqrt
[1 - c^2*x^2] + Sin[2*ArcCos[c*x]])))/(c*f*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^{3/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{d^2(cx+1)^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\sqrt{1-c^2x^2}\int \frac{(cx+1)^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$\Big\downarrow$ 5263

$$\frac{d^2\sqrt{1-c^2x^2}\int \left(\frac{c^2(a+b\arccos(cx))x^2}{\sqrt{1-c^2x^2}}+\frac{2c(a+b\arccos(cx))x}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}\right)dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$\Big\downarrow$ 2009

$$\frac{d^2\sqrt{1-c^2x^2}\left(-\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}-\frac{3(a+b\arccos(cx))^2}{4bc}-\frac{1}{4}bcx^2-2bx\right)}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

input
```
Int[((d + c*d*x)^(3/2)*(a + b*ArcCos[c*x]))/Sqrt[f - c*f*x],x]
```

output
```
(d^2*Sqrt[1 - c^2*x^2]*(-2*b*x - (b*c*x^2)/4 - (2*Sqrt[1 - c^2*x^2]*(a + b
*ArcCos[c*x]))/c - (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/2 - (3*(a + b
*ArcCos[c*x])^2)/(4*b*c)))/(Sqrt[d + c*d*x]*Sqrt[f - c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.01 (sec) , antiderivative size = 714, normalized size of antiderivative = 2.95

method	result
default	$-\frac{a(cdx+d)^{\frac{3}{2}}\sqrt{-cfx+f}}{2cf}-\frac{3ad\sqrt{cdx+d}\,\sqrt{-cfx+f}}{2cf}+\frac{3a\,d^{2}\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^{2}df}\,x}{\sqrt{-c^{2}df\,x^{2}+df}}\right)}{2\sqrt{cdx+d}\,\sqrt{-cfx+f}\,\sqrt{c^{2}df}}+b\Bigg(\frac{3\sqrt{d(cx+1)}}{}$
parts	$-\frac{a(cdx+d)^{\frac{3}{2}}\sqrt{-cfx+f}}{2cf}-\frac{3ad\sqrt{cdx+d}\,\sqrt{-cfx+f}}{2cf}+\frac{3a\,d^{2}\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^{2}df}\,x}{\sqrt{-c^{2}df\,x^{2}+df}}\right)}{2\sqrt{cdx+d}\,\sqrt{-cfx+f}\,\sqrt{c^{2}df}}+b\Bigg(\frac{3\sqrt{d(cx+1)}}{}$

input
```
int((c*d*x+d)^(3/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/2*a/c/f*(c*d*x+d)^(3/2)*(-c*f*x+f)^(1/2)-3/2*a*d/c/f*(c*d*x+d)^(1/2)*(-
c*f*x+f)^(1/2)+3/2*a*d^2*((-c*f*x+f)*(c*d*x+d))^(1/2)/(c*d*x+d)^(1/2)/(-c*
f*x+f)^(1/2)/(c^2*d*f)^(1/2)*arctan((c^2*d*f)^(1/2)*x/(-c^2*d*f*x^2+d*f)^(
1/2))+b*(3/4*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x-
1)/f/c/(c*x+1)*arccos(c*x)^2*d-1/32*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(
2*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+4*c^3*x^3-1+2*I*(-c^2*x^2+1)^(1/2
)*c*x-I*(-c^2*x^2+1)^(1/2)-3*c*x)*(I+2*arccos(c*x))*d/(c*x-1)/f/c/(c*x+1)+
1/2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(1+c*x+I*(-c^2*x^2+1)^(1/2))*(arc
cos(c*x)+I)*d/(c*x-1)/f/c/(c*x+1)-(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I
*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)-I)*d/(c*x-1)/f/c/(c*x+1)+1
/32*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^
2*x^2-I*(-c^2*x^2+1)^(1/2)+c*x-1)*(-I+2*arccos(c*x))*d/(c*x-1)/f/c/(c*x+1)
-1/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x+1)*(
7*I+8*arccos(c*x))*cos(2*arccos(c*x))*d/(c*x-1)/f/c/(c*x+1)-1/8*(d*(c*x+1)
)^(1/2)*(-f*(c*x-1))^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)+I)*(4*I+3*arccos(c*x)
)*sin(2*arccos(c*x))*d/(c*x-1)/f/c/(c*x+1))
```

Fricas [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx=\int \frac{(cdx+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)}{\sqrt{-cfx+f}}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-(a*c*d*x + a*d + (b*c*d*x + b*d)*arccos(c*x))*sqrt(c*d*x + d)*sq
rt(-c*f*x + f)/(c*f*x - f), x)
```

Sympy [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx=\int \frac{(d(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acos}\,(cx))}{\sqrt{-f\,(cx-1)}}\,dx$$

input
```
integrate((c*d*x+d)**(3/2)*(a+b*acos(c*x))/(-c*f*x+f)**(1/2),x)
```

output
```
Integral((d*(c*x + 1))**(3/2)*(a + b*acos(c*x))/sqrt(-f*(c*x - 1)), x)
```

Maxima [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx=\int \frac{(cdx+d)^{\frac{3}{2}}(b\arccos\,(cx)+a)}{\sqrt{-cfx+f}}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/2*(sqrt(-c^2*d*f*x^2 + d*f)*d*x/f - 3*d^2*arcsin(c*x)/(sqrt(d*f)*c) + 4
*sqrt(-c^2*d*f*x^2 + d*f)*d/(c*f))*a - b*sqrt(d)*integrate((c*d*x + d)*sqr
t(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c*x
- 1), x)/sqrt(f)
```

Giac [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx=\int \frac{(cdx+d)^{\frac{3}{2}}(b\arccos\,(cx)+a)}{\sqrt{-cfx+f}}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((c*d*x + d)^(3/2)*(b*arccos(c*x) + a)/sqrt(-c*f*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(d+c\,d\,x\right)^{3/2}}{\sqrt{f-c\,f\,x}}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + c*d*x)^(3/2))/(f - c*f*x)^(1/2),x)
```

output
```
int(((a + b*acos(c*x))*(d + c*d*x)^(3/2))/(f - c*f*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx=\frac{\sqrt{d}\,d\left(-6asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a-\sqrt{cx+1}\,\sqrt{-cx+1}\,acx-4\sqrt{cx+1}\,\sqrt{\cdot}\right.}{2\sqrt{\cdot}}$$

input
```
int((c*d*x+d)^(3/2)*(a+b*acos(c*x))/(-c*f*x+f)^(1/2),x)
```

output
```
(sqrt(d)*d*( - 6*asin(sqrt( - c*x + 1)/sqrt(2))*a - sqrt(c*x + 1)*sqrt( -
c*x + 1)*a*c*x - 4*sqrt(c*x + 1)*sqrt( - c*x + 1)*a + 2*int((sqrt(c*x + 1)
*acos(c*x)*x)/sqrt( - c*x + 1),x)*b*c**2 + 2*int((sqrt(c*x + 1)*acos(c*x))
/sqrt( - c*x + 1),x)*b*c))/(2*sqrt(f)*c)
```

3.526 $\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 141

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx = \frac{bdx\sqrt{1-c^2x^2}}{\sqrt{d+cdx}\sqrt{f-cfx}} - \frac{d(1-c^2x^2)(a+b\arccos(cx))}{c\sqrt{d+cdx}\sqrt{f-cfx}} + \frac{d\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2bc\sqrt{d+cdx}\sqrt{f-cfx}}$$

output
```
b*d*x*(-c^2*x^2+1)^(1/2)/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)-d*(-c^2*x^2+1)*(
a+b*arccos(c*x))/c/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+1/2*d*(-c^2*x^2+1)^(1/
2)*(a+b*arccos(c*x))^2/b/c/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx = -\frac{\frac{2\sqrt{d+cdx}\sqrt{f-cfx}\left(bcx+a\sqrt{1-c^2x^2}\right)}{\sqrt{1-c^2x^2}} + 2b\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx) + \frac{b\sqrt{d+cdx}\sqrt{f-cfx}\arccos(cx)^2}{\sqrt{1-c^2x^2}} + 2a\sqrt{d}\sqrt{f}}{2cf}$$

input
```
Integrate[(Sqrt[d + c*d*x]*(a + b*ArcCos[c*x]))/Sqrt[f - c*f*x],x]
```

output
```
-1/2*((2*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(b*c*x + a*Sqrt[1 - c^2*x^2]))/Sq
rt[1 - c^2*x^2] + 2*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x] + (b*Sqr
t[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c*x]^2)/Sqrt[1 - c^2*x^2] + 2*a*Sqrt[d
]*Sqrt[f]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d]*Sqrt[f]*(-
1 + c^2*x^2))])/(c*f)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.63, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cdx+d}(a+b\arccos(cx))}{\sqrt{f-cfx}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{d(cx+1)(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\sqrt{1-c^2x^2}\int \frac{(cx+1)(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{d\sqrt{1-c^2x^2}\int \left(\frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}+\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}\right)dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\sqrt{1-c^2x^2}\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}-\frac{(a+b\arccos(cx))^2}{2bc}-bx\right)}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

input
```
Int[(Sqrt[d + c*d*x]*(a + b*ArcCos[c*x]))/Sqrt[f - c*f*x],x]
```

output
```
(d*Sqrt[1 - c^2*x^2]*(-(b*x) - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c -
 (a + b*ArcCos[c*x])^2/(2*b*c)))/(Sqrt[d + c*d*x]*Sqrt[f - c*f*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.19

method	result
default	$-\frac{a\sqrt{cdx+d}\,\sqrt{-cfx+f}}{cf}+\frac{ad\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2df}\,x}{\sqrt{-c^2df\,x^2+df}}\right)}{\sqrt{cdx+d}\,\sqrt{-cfx+f}\,\sqrt{c^2df}}+b\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,\arccos(}{2(cx-1)fc(cx+1)}\right.$
parts	$-\frac{a\sqrt{cdx+d}\,\sqrt{-cfx+f}}{cf}+\frac{ad\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2df}\,x}{\sqrt{-c^2df\,x^2+df}}\right)}{\sqrt{cdx+d}\,\sqrt{-cfx+f}\,\sqrt{c^2df}}+b\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,\arccos(}{2(cx-1)fc(cx+1)}\right.$

input
```
int((c*d*x+d)^(1/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-a/c/f*(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)+a*d*((-c*f*x+f)*(c*d*x+d))^(1/2)/(
c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)/(c^2*d*f)^(1/2)*arctan((c^2*d*f)^(1/2)*x/(
-c^2*d*f*x^2+d*f)^(1/2))+b*(1/2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2
*x^2+1)^(1/2)/(c*x-1)/f/c/(c*x+1)*arccos(c*x)^2-1/2*(d*(c*x+1))^(1/2)*(-f*
(c*x-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)+I)/(c*x-1
)/f/c/(c*x+1)-1/2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1
/2)*x*c+c^2*x^2-1)*(arccos(c*x)-I)/(c*x-1)/f/c/(c*x+1))
```

Fricas [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx=\int \frac{\sqrt{cdx+d}(b\arccos{(cx)}+a)}{\sqrt{-cfx+f}}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-sqrt(c*d*x + d)*sqrt(-c*f*x + f)*(b*arccos(c*x) + a)/(c*f*x - f)
, x)
```

Sympy [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx=\int \frac{\sqrt{d\,(cx+1)}(a+b\operatorname{acos}{(cx)})}{\sqrt{-f\,(cx-1)}}\,dx$$

input
```
integrate((c*d*x+d)**(1/2)*(a+b*acos(c*x))/(-c*f*x+f)**(1/2),x)
```

output
```
Integral(sqrt(d*(c*x + 1))*(a + b*acos(c*x))/sqrt(-f*(c*x - 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx=\int \frac{\sqrt{cdx+d}(b\arccos{(cx)}+a)}{\sqrt{-cfx+f}}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(1/2),x, algorithm=
"maxima")
```

output
```
a*(d*arcsin(c*x)/(c*f*sqrt(d/f)) - sqrt(-c^2*d*f*x^2 + d*f)/(c*f)) + b*sqr
t(d)*integrate(sqrt(c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/sq
rt(-c*x + 1), x)/sqrt(f)
```

Giac [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx=\int \frac{\sqrt{cdx+d}(b\arccos{(cx)}+a)}{\sqrt{-cfx+f}}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(1/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(c*d*x + d)*(b*arccos(c*x) + a)/sqrt(-c*f*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d+c\,d\,x}}{\sqrt{f-c\,f\,x}}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + c*d*x)^(1/2))/(f - c*f*x)^(1/2),x)
```

output
```
int(((a + b*acos(c*x))*(d + c*d*x)^(1/2))/(f - c*f*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{\sqrt{f-cfx}}\,dx = \frac{\sqrt{d}\left(-2asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a-\sqrt{cx+1}\,\sqrt{-cx+1}\,a+\left(\int\frac{\sqrt{cx+1}\,acos(cx)}{\sqrt{-cx+1}}dx\right)bc\right)}{\sqrt{f}\,c}$$

input
```
int((c*d*x+d)^(1/2)*(a+b*acos(c*x))/(-c*f*x+f)^(1/2),x)
```

output
```
(sqrt(d)*( - 2*asin(sqrt( - c*x + 1)/sqrt(2))*a - sqrt(c*x + 1)*sqrt( - c*
x + 1)*a + int((sqrt(c*x + 1)*acos(c*x))/sqrt( - c*x + 1),x)*b*c))/(sqrt(f
)*c)
```

3.527 $\int \frac{a+b \arccos(cx)}{\sqrt{d+cdx}\sqrt{f-cfx}} dx$

Optimal result

Integrand size = 30, antiderivative size = 55

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}\sqrt{f-cfx}}\,dx=\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2bc\sqrt{d+cdx}\sqrt{f-cfx}}$$

output
```
1/2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/b/c/(c*d*x+d)^(1/2)/(-c*f*x+f)^
(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 112 vs. 2(55) = 110.

Time = 0.98 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.04

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}\sqrt{f-cfx}}\,dx=-\frac{b\sqrt{1-c^2x^2}\arccos(cx)^2}{2c\sqrt{d+cdx}\sqrt{f-cfx}}-\frac{a\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{f-cfx}}{\sqrt{d}\sqrt{f}(-1+c^2x^2)}\right)}{c\sqrt{d}\sqrt{f}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(Sqrt[d + c*d*x]*Sqrt[f - c*f*x]),x]
```

output
```
-1/2*(b*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2)/(c*Sqrt[d + c*d*x]*Sqrt[f - c*f*x
]) - (a*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d]*Sqrt[f]*(-1
+ c^2*x^2))])/(c*Sqrt[d]*Sqrt[f])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {5179, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{\sqrt{cdx+d}\sqrt{f-cfx}}dx$$

$$\Big\downarrow \text{ 5179}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{f-cfx}}$$

$$\Big\downarrow \text{ 5153}$$

$$-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2bc\sqrt{cdx+d}\sqrt{f-cfx}}$$

input
```
Int[(a + b*ArcCos[c*x])/(Sqrt[d + c*d*x]*Sqrt[f - c*f*x]),x]
```

output
```
-1/2*(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(b*c*Sqrt[d + c*d*x]*Sqrt[f
 - c*f*x])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(47) = 94$.

Time = 0.92 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.40

method	result	size
default	$\frac{a\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2df}\,x}{\sqrt{-c^2df\,x^2+df}}\right)}{\sqrt{cdx+d}\,\sqrt{-cfx+f}\,\sqrt{c^2df}}+\frac{b\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2}{2dfc(c^2x^2-1)}$	132
parts	$\frac{a\sqrt{(-cfx+f)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2df}\,x}{\sqrt{-c^2df\,x^2+df}}\right)}{\sqrt{cdx+d}\,\sqrt{-cfx+f}\,\sqrt{c^2df}}+\frac{b\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2}{2dfc(c^2x^2-1)}$	132

input
```
int((a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
a*((-c*f*x+f)*(c*d*x+d))^(1/2)/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)/(c^2*d*f)^
(1/2)*arctan((c^2*d*f)^(1/2)*x/(-c^2*d*f*x^2+d*f)^(1/2))+1/2*b*(d*(c*x+1))
^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/f/c/(c^2*x^2-1)*arccos(c*x)
^2
```

Fricas [F]

$$\int \frac{a + b\arccos(cx)}{\sqrt{d + cdx}\sqrt{f - cfx}}\,dx = \int \frac{b\arccos(cx) + a}{\sqrt{cdx + d}\sqrt{-cfx + f}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-sqrt(c*d*x + d)*sqrt(-c*f*x + f)*(b*arccos(c*x) + a)/(c^2*d*f*x^
2 - d*f), x)
```

Sympy [F]

$$\int \frac{a + b\arccos(cx)}{\sqrt{d + cdx}\sqrt{f - cfx}}\,dx = \int \frac{a + b\,\mathrm{acos}\,(cx)}{\sqrt{d\,(cx + 1)}\sqrt{-f\,(cx - 1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(c*d*x+d)**(1/2)/(-c*f*x+f)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))/(sqrt(d*(c*x + 1))*sqrt(-f*(c*x - 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{a + b\arccos(cx)}{\sqrt{d + cdx}\sqrt{f - cfx}}\,dx = \frac{b\arccos(cx)\arcsin(cx)}{\sqrt{df}c} + \frac{b\arcsin(cx)^2}{2\sqrt{df}c} + \frac{a\arcsin(cx)}{\sqrt{df}c}$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2),x, algorithm=
"maxima")
```

output
```
b*arccos(c*x)*arcsin(c*x)/(sqrt(d*f)*c) + 1/2*b*arcsin(c*x)^2/(sqrt(d*f)*c
) + a*arcsin(c*x)/(sqrt(d*f)*c)
```

Giac [F]

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}\sqrt{f-cfx}}\,dx=\int \frac{b\arccos{(cx)}+a}{\sqrt{cdx+d}\sqrt{-cfx+f}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((b*arccos(c*x) + a)/(sqrt(c*d*x + d)*sqrt(-c*f*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}\sqrt{f-cfx}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{\sqrt{d+c\,d\,x}\,\sqrt{f-c\,f\,x}}\,dx$$

input
```
int((a + b*acos(c*x))/((d + c*d*x)^(1/2)*(f - c*f*x)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))/((d + c*d*x)^(1/2)*(f - c*f*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}\sqrt{f-cfx}}\,dx=\frac{-2asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a+\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)bc}{\sqrt{f}\,\sqrt{d}\,c}$$

input
```
int((a+b*acos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2),x)
```

output
```
( - 2*asin(sqrt( - c*x + 1)/sqrt(2))*a + int(acos(c*x)/(sqrt(c*x + 1)*sqrt
( - c*x + 1)),x)*b*c)/(sqrt(f)*sqrt(d)*c)
```

3.528 $\int \frac{a+b \arccos(cx)}{(d+cdx)^{3/2}\sqrt{f-cfx}} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 99

$$\int \frac{a+b \arccos(cx)}{(d+cdx)^{3/2}\sqrt{f-cfx}} \, dx = -\frac{f(1-cx)\left(1-c^2x^2\right)(a+b \arccos(cx))}{c(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{bf\left(1-c^2x^2\right)^{3/2}\log(1+cx)}{c(d+cdx)^{3/2}(f-cfx)^{3/2}}$$

output
```
-f*(-c*x+1)*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3
/2)+b*f*(-c^2*x^2+1)^(3/2)*ln(c*x+1)/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.81

$$\int \frac{a+b \arccos(cx)}{(d+cdx)^{3/2}\sqrt{f-cfx}} \, dx = \frac{\sqrt{d+cdx}\left(a(-1+cx)+b(-1+cx)\arccos(cx)-b\sqrt{1-c^2x^2}\log(-f(1+\right.}{cd^2(1+cx)\sqrt{f-cfx}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/((d + c*d*x)^(3/2)*Sqrt[f - c*f*x]),x]
```

output
```
(Sqrt[d + c*d*x]*(a*(-1 + c*x) + b*(-1 + c*x)*ArcCos[c*x] - b*Sqrt[1 - c^2
*x^2]*Log[-(f*(1 + c*x))]))/(c*d^2*(1 + c*x)*Sqrt[f - c*f*x])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {5179, 27, 5261, 25, 27, 451, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(cdx+d)^{3/2}\sqrt{f-cfx}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{3/2}\int \frac{f(1-cx)(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(1-c^2x^2)^{3/2}\int \frac{(1-cx)(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{5261}$$

$$\frac{f(1-c^2x^2)^{3/2}\left(bc\int -\frac{1-cx}{c(1-c^2x^2)}dx-\frac{(1-cx)(a+b\arccos(cx))}{c\sqrt{1-c^2x^2}}\right)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{f(1-c^2x^2)^{3/2}\left(-bc\int \frac{1-cx}{c(1-c^2x^2)}dx-\frac{(1-cx)(a+b\arccos(cx))}{c\sqrt{1-c^2x^2}}\right)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(1-c^2x^2)^{3/2}\left(-b\int \frac{1-cx}{1-c^2x^2}dx-\frac{(1-cx)(a+b\arccos(cx))}{c\sqrt{1-c^2x^2}}\right)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{451}$$

$$\frac{f\left(1-c^2x^2\right)^{3/2}\left(-b\int \frac{1}{cx+1}dx-\frac{(1-cx)(a+b\arccos(cx))}{c\sqrt{1-c^2x^2}}\right)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \ 16$$

$$\frac{f\left(1-c^2x^2\right)^{3/2}\left(-\frac{(1-cx)(a+b\arccos(cx))}{c\sqrt{1-c^2x^2}}-\frac{b\log(cx+1)}{c}\right)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])/((d + c*d*x)^(3/2)*Sqrt[f - c*f*x]),x]
```

output
```
(f*(1 - c^2*x^2)^(3/2)*(-(((1 - c*x)*(a + b*ArcCos[c*x]))/(c*Sqrt[1 - c^2*
x^2])) - (b*Log[1 + c*x])/c))/((d + c*d*x)^(3/2)*(f - c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 451
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c^2/a   In
t[1/(c - d*x), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5261
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p,
 x]}, Simp[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[1/Sqrt[1 - c^2*x^2]
   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IG
tQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.71

method	result
default	$-\frac{a\sqrt{-cfx+f}}{fdc\sqrt{cdx+d}}+\frac{b\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\left(-i\arccos(cx)xc+2\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)xc-i\arccos(cx)+\arccos(cx)\right.}{(cx+1)^2d^2cf(cx-1)}$
parts	$-\frac{a\sqrt{-cfx+f}}{fdc\sqrt{cdx+d}}+\frac{b\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\left(-i\arccos(cx)xc+2\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)xc-i\arccos(cx)+\arccos(cx)\right.}{(cx+1)^2d^2cf(cx-1)}$

input
```
int((a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-a/f/d/c/(c*d*x+d)^(1/2)*(-c*f*x+f)^(1/2)+b*(d*(c*x+1))^(1/2)*(-f*(c*x-1))
^(1/2)*(-c^2*x^2+1)^(1/2)*(-I*arccos(c*x)*x*c+2*ln(1+c*x+I*(-c^2*x^2+1)^(1
/2))*x*c-I*arccos(c*x)+arccos(c*x)*(-c^2*x^2+1)^(1/2)+2*ln(1+c*x+I*(-c^2*x
^2+1)^(1/2)))/(c*x+1)^2/d^2/c/f/(c*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.51

$$\int\frac{a+b\arccos(cx)}{(d+cdx)^{3/2}\sqrt{f-cfx}}\,dx=\left[\frac{(bcx+b)\sqrt{df}\log\left(\frac{c^6dfx^6+4c^5dfx^5+5c^4dfx^4-4c^2dfx^2-4cdfx+(c^4x^4+4c^3x^3+6c^2x^2+4c}{c^4x^4+2c^3x^3-2cx-1}\right.}{2(c^2d^2}\right.$$

$$-\frac{(bcx+b)\sqrt{-df}\arctan\left(\frac{(c^2x^2+2cx+2)\sqrt{-c^2x^2+1}\sqrt{cdx+d}\sqrt{-cfx+f}\sqrt{-df}}{c^4dfx^4+2c^3dfx^3-c^2dfx^2-2cdfx}\right)+\sqrt{cdx+d}\sqrt{-cfx+f}(b\arccos(cx)+}{c^2d^2fx+cd^2f}$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(1/2),x, algorithm=
"fricas")
```

output
```
[1/2*((b*c*x + b)*sqrt(d*f)*log((c^6*d*f*x^6 + 4*c^5*d*f*x^5 + 5*c^4*d*f*x
^4 - 4*c^2*d*f*x^2 - 4*c*d*f*x + (c^4*x^4 + 4*c^3*x^3 + 6*c^2*x^2 + 4*c*x)
*sqrt(-c^2*x^2 + 1)*sqrt(c*d*x + d)*sqrt(-c*f*x + f)*sqrt(d*f) - 2*d*f)/(c
^4*x^4 + 2*c^3*x^3 - 2*c*x - 1)) - 2*sqrt(c*d*x + d)*sqrt(-c*f*x + f)*(b*a
rccos(c*x) + a))/(c^2*d^2*f*x + c*d^2*f), -((b*c*x + b)*sqrt(-d*f)*arctan(
(c^2*x^2 + 2*c*x + 2)*sqrt(-c^2*x^2 + 1)*sqrt(c*d*x + d)*sqrt(-c*f*x + f)*
sqrt(-d*f)/(c^4*d*f*x^4 + 2*c^3*d*f*x^3 - c^2*d*f*x^2 - 2*c*d*f*x)) + sqrt
(c*d*x + d)*sqrt(-c*f*x + f)*(b*arccos(c*x) + a))/(c^2*d^2*f*x + c*d^2*f)]
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}\sqrt{f-cfx}}\,dx = \int \frac{a+b\,\mathrm{acos}\,(cx)}{(d\,(cx+1))^{\frac{3}{2}}\,\sqrt{-f\,(cx-1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(c*d*x+d)**(3/2)/(-c*f*x+f)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))/((d*(c*x + 1))**(3/2)*sqrt(-f*(c*x - 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}\sqrt{f-cfx}}\,dx = -\frac{\sqrt{-c^2dfx^2+df}\,b\arccos(cx)}{c^2d^2fx+cd^2f} - \frac{\sqrt{-c^2dfx^2+df}\,a}{c^2d^2fx+cd^2f} - \frac{b\log(cx+1)}{cd^{\frac{3}{2}}\sqrt{f}}$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(1/2),x, algorithm=
"maxima")
```

output
```
-sqrt(-c^2*d*f*x^2 + d*f)*b*arccos(c*x)/(c^2*d^2*f*x + c*d^2*f) - sqrt(-c^
2*d*f*x^2 + d*f)*a/(c^2*d^2*f*x + c*d^2*f) - b*log(c*x + 1)/(c*d^(3/2)*sqr
t(f))
```

Giac [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}\sqrt{f-cfx}}\,dx=\int \frac{b\arccos\left(cx\right)+a}{(cdx+d)^{\frac{3}{2}}\sqrt{-cfx+f}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((b*arccos(c*x) + a)/((c*d*x + d)^(3/2)*sqrt(-c*f*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}\sqrt{f-cfx}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{{(d+c\,d\,x)}^{3/2}\,\sqrt{f-c\,f\,x}}\,dx$$

input
```
int((a + b*acos(c*x))/((d + c*d*x)^(3/2)*(f - c*f*x)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))/((d + c*d*x)^(3/2)*(f - c*f*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a + b \arccos(cx)}{(d + cdx)^{3/2}\sqrt{f - cfx}}\,dx = \frac{-\sqrt{-cx+1}\,a + \sqrt{cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,cx+\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)bc}{\sqrt{f}\,\sqrt{d}\,\sqrt{cx+1}\,cd}$$

input
```
int((a+b*acos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(1/2),x)
```

output
```
( - sqrt( - c*x + 1)*a + sqrt(c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt(
- c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b*c)/(sqrt(f)*sqrt(d)*
sqrt(c*x + 1)*c*d)
```

3.529 $\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}\sqrt{f-cfx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 265

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}\sqrt{f-cfx}}\,dx = -\frac{bf^2(1-c^2x^2)^{5/2}}{3c(1+cx)(d+cdx)^{5/2}(f-cfx)^{5/2}} - \frac{2f^2(1-cx)\left(1-c^2x^2\right)(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{f^2x(1-c^2x^2)^2(a+b\arccos(cx))}{3(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{bf^2(1-c^2x^2)^{5/2}\operatorname{arctanh}(cx)}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{bf^2(1-c^2x^2)^{5/2}\log\left(1-c^2x^2\right)}{6c(d+cdx)^{5/2}(f-cfx)^{5/2}}$$

output

```
-1/3*b*f^2*(-c^2*x^2+1)^(5/2)/c/(c*x+1)/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)-2
/3*f^2*(-c*x+1)*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*f*x+f
)^(5/2)+1/3*f^2*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x
+f)^(5/2)+1/3*b*f^2*(-c^2*x^2+1)^(5/2)*arctanh(c*x)/c/(c*d*x+d)^(5/2)/(-c*
f*x+f)^(5/2)+1/6*b*f^2*(-c^2*x^2+1)^(5/2)*ln(-c^2*x^2+1)/c/(c*d*x+d)^(5/2)
/(-c*f*x+f)^(5/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.45

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}\sqrt{f-cfx}}\,dx = \frac{\sqrt{d+cdx}\left((2+cx)\left(-a+acx+b\sqrt{1-c^2x^2}\right)+b(-2+cx+c^2x^2)\arccos(\right.}{3cd^3(1+cx)^2\sqrt{f-cfx}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/((d + c*d*x)^(5/2)*Sqrt[f - c*f*x]),x]
```

output
```
(Sqrt[d + c*d*x]*((2 + c*x)*(-a + a*c*x + b*Sqrt[1 - c^2*x^2]) + b*(-2 + c
*x + c^2*x^2)*ArcCos[c*x] - b*(1 + c*x)*Sqrt[1 - c^2*x^2]*Log[-(f*(1 + c*x
))]))/(3*c*d^3*(1 + c*x)^2*Sqrt[f - c*f*x])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5261, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(cdx+d)^{5/2}\sqrt{f-cfx}}\,dx$$

$$\Big\downarrow \ 5179$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\int \frac{f^2(1-cx)^2(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{f^2\left(1-c^2x^2\right)^{5/2}\int \frac{(1-cx)^2(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \ 5261$$

$$\frac{f^2\left(1-c^2x^2\right)^{5/2}\left(bc\int \left(\frac{x}{3(1-c^2x^2)}-\frac{2(1-cx)}{3c\left(1-c^2x^2\right)^2}\right)dx+\frac{x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}-\frac{2(1-cx)(a+b\arccos(cx))}{3c\left(1-c^2x^2\right)^{3/2}}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\downarrow \text{2009}$$

$$\frac{f^2(1-c^2x^2)^{5/2}\left(\frac{x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}-\frac{2(1-cx)(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}+bc\left(-\frac{\operatorname{arctanh}(cx)}{3c^2}+\frac{1-cx}{3c^2(1-c^2x^2)}-\frac{\log(1-c^2x^2)}{6c^2}\right)\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

input
```
Int[(a + b*ArcCos[c*x])/((d + c*d*x)^(5/2)*Sqrt[f - c*f*x]),x]
```

output
```
(f^2*(1 - c^2*x^2)^(5/2)*((-2*(1 - c*x)*(a + b*ArcCos[c*x]))/(3*c*(1 - c^2
*x^2)^(3/2)) + (x*(a + b*ArcCos[c*x]))/(3*Sqrt[1 - c^2*x^2]) + b*c*((1 - c
*x)/(3*c^2*(1 - c^2*x^2)) - ArcTanh[c*x]/(3*c^2) - Log[1 - c^2*x^2]/(6*c^2
))))/((d + c*d*x)^(5/2)*(f - c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5261
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p,
 x]}, Simp[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[1/Sqrt[1 - c^2*x^2]
   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IG
tQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.77 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.04

method	result
default	$a\left(-\frac{\sqrt{-cfx+f}}{3fdc(cdx+d)^{\frac{3}{2}}}-\frac{\sqrt{-cfx+f}}{3cf\,d^2\sqrt{cdx+d}}\right)-\frac{b\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,\left(i\arccos(cx)c^2x^2-2\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)\right.}{}$
parts	$a\left(-\frac{\sqrt{-cfx+f}}{3fdc(cdx+d)^{\frac{3}{2}}}-\frac{\sqrt{-cfx+f}}{3cf\,d^2\sqrt{cdx+d}}\right)-\frac{b\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,\left(i\arccos(cx)c^2x^2-2\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)\right.}{}$

input
```
int((a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
a*(-1/3/f/d/c/(c*d*x+d)^(3/2)*(-c*f*x+f)^(1/2)-1/3/c/f/d^2/(c*d*x+d)^(1/2)
*(-c*f*x+f)^(1/2))-1/3*b*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)
^(1/2)*(I*arccos(c*x)*x^2*c^2-2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*x^2*c^2+2*I
*arccos(c*x)*x*c-(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c-4*ln(1+c*x+I*(-c^2*x^2
+1)^(1/2))*x*c+I*arccos(c*x)-2*arccos(c*x)*(-c^2*x^2+1)^(1/2)+c*x-2*ln(1+c
*x+I*(-c^2*x^2+1)^(1/2))+1)/d^3/f/c/(c^4*x^4+2*c^3*x^3-2*c*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.98

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}\sqrt{f-cfx}}\,dx = \left[\frac{(bc^3x^3+bc^2x^2-bcx-b)\sqrt{df}\log\left(\frac{c^6dfx^6+4\,c^5dfx^5+5\,c^4dfx^4-4\,c^2dfx^2-4\,cdfx+(c^4x}{c^4x^4}\right.}{}\right.$$

$$-\frac{(bc^3x^3+bc^2x^2-bcx-b)\sqrt{-df}\arctan\left(\frac{(c^2x^2+2\,cx+2)\sqrt{-c^2x^2+1}\sqrt{cdx+d}\sqrt{-cfx+f}\sqrt{-df}}{c^4dfx^4+2\,c^3dfx^3-c^2dfx^2-2\,cdfx}\right)+\left(ac^2x^2-\sqrt{-c^2x^2+}\right.}{3\left(c^4d^3fx^3+c^3d^3fx^2-c^2d^3fx-c\right.}$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(1/2),x, algorithm=
"fricas")
```

output
```
[1/6*((b*c^3*x^3 + b*c^2*x^2 - b*c*x - b)*sqrt(d*f)*log((c^6*d*f*x^6 + 4*c
^5*d*f*x^5 + 5*c^4*d*f*x^4 - 4*c^2*d*f*x^2 - 4*c*d*f*x + (c^4*x^4 + 4*c^3*
x^3 + 6*c^2*x^2 + 4*c*x)*sqrt(-c^2*x^2 + 1)*sqrt(c*d*x + d)*sqrt(-c*f*x +
f)*sqrt(d*f) - 2*d*f)/(c^4*x^4 + 2*c^3*x^3 - 2*c*x - 1)) - 2*(a*c^2*x^2 -
sqrt(-c^2*x^2 + 1)*b*c*x + a*c*x + (b*c^2*x^2 + b*c*x - 2*b)*arccos(c*x) -
 2*a)*sqrt(c*d*x + d)*sqrt(-c*f*x + f))/(c^4*d^3*f*x^3 + c^3*d^3*f*x^2 - c
^2*d^3*f*x - c*d^3*f), -1/3*((b*c^3*x^3 + b*c^2*x^2 - b*c*x - b)*sqrt(-d*f
)*arctan((c^2*x^2 + 2*c*x + 2)*sqrt(-c^2*x^2 + 1)*sqrt(c*d*x + d)*sqrt(-c*
f*x + f)*sqrt(-d*f)/(c^4*d*f*x^4 + 2*c^3*d*f*x^3 - c^2*d*f*x^2 - 2*c*d*f*x
)) + (a*c^2*x^2 - sqrt(-c^2*x^2 + 1)*b*c*x + a*c*x + (b*c^2*x^2 + b*c*x -
2*b)*arccos(c*x) - 2*a)*sqrt(c*d*x + d)*sqrt(-c*f*x + f))/(c^4*d^3*f*x^3 +
 c^3*d^3*f*x^2 - c^2*d^3*f*x - c*d^3*f)]
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}\sqrt{f-cfx}}\,dx=\int \frac{a+b\,\mathrm{acos}\,(cx)}{\left(d\left(cx+1\right)\right)^{\frac{5}{2}}\sqrt{-f\left(cx-1\right)}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(c*d*x+d)**(5/2)/(-c*f*x+f)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))/((d*(c*x + 1))**(5/2)*sqrt(-f*(c*x - 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}\sqrt{f-cfx}}\,dx&=\frac{1}{3}\,bc\left(\frac{1}{c^3d^{\frac{5}{2}}\sqrt{f}x+c^2d^{\frac{5}{2}}\sqrt{f}}-\frac{\log\left(cx+1\right)}{c^2d^{\frac{5}{2}}\sqrt{f}}\right)\\&\quad-\frac{1}{3}\,b\left(\frac{\sqrt{-c^2dfx^2+df}}{c^3d^3fx^2+2\,c^2d^3fx+cd^3f}+\frac{\sqrt{-c^2dfx^2+df}}{c^2d^3fx+cd^3f}\right)\arccos\left(cx\right)\\&\quad-\frac{1}{3}\,a\left(\frac{\sqrt{-c^2dfx^2+df}}{c^3d^3fx^2+2\,c^2d^3fx+cd^3f}+\frac{\sqrt{-c^2dfx^2+df}}{c^2d^3fx+cd^3f}\right)\end{aligned}$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(1/2),x, algorithm=
"maxima")
```

output
```
1/3*b*c*(1/(c^3*d^(5/2)*sqrt(f)*x + c^2*d^(5/2)*sqrt(f)) - log(c*x + 1)/(c
^2*d^(5/2)*sqrt(f))) - 1/3*b*(sqrt(-c^2*d*f*x^2 + d*f)/(c^3*d^3*f*x^2 + 2*
c^2*d^3*f*x + c*d^3*f) + sqrt(-c^2*d*f*x^2 + d*f)/(c^2*d^3*f*x + c*d^3*f))
*arccos(c*x) - 1/3*a*(sqrt(-c^2*d*f*x^2 + d*f)/(c^3*d^3*f*x^2 + 2*c^2*d^3*
f*x + c*d^3*f) + sqrt(-c^2*d*f*x^2 + d*f)/(c^2*d^3*f*x + c*d^3*f))
```

Giac [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}\sqrt{f-cfx}}\,dx = \int \frac{b\arccos(cx)+a}{(cdx+d)^{\frac{5}{2}}\sqrt{-cfx+f}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((b*arccos(c*x) + a)/((c*d*x + d)^(5/2)*sqrt(-c*f*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}\sqrt{f-cfx}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{{(d+c\,d\,x)}^{5/2}\,\sqrt{f-c\,f\,x}}\,dx$$

input
```
int((a + b*acos(c*x))/((d + c*d*x)^(5/2)*(f - c*f*x)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))/((d + c*d*x)^(5/2)*(f - c*f*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}\sqrt{f-cfx}}\,dx = \frac{-\sqrt{-cx+1}\,acx-2\sqrt{-cx+1}\,a+3\sqrt{cx+1}\left(\int \frac{acos(cx}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2+2\sqrt{cx+1}\,\sqrt{-}}\right.}{3\sqrt{}}$$

input
```
int((a+b*acos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(1/2),x)
```

output
```
( - sqrt( - c*x + 1)*a*c*x - 2*sqrt( - c*x + 1)*a + 3*sqrt(c*x + 1)*int(ac
os(c*x)/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*sqrt(
- c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b*c**2*x + 3*sqrt(c*x
+ 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 + 2*sqrt(c*x
+ 1)*sqrt( - c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b*c)/(3*sqr
t(f)*sqrt(d)*sqrt(c*x + 1)*c*d**2*(c*x + 1))
```

3.530 $\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 400

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = -\frac{4bd^4x(1-c^2x^2)^{3/2}}{(d+cdx)^{3/2}(f-cfx)^{3/2}} - \frac{bcd^4x^2(1-c^2x^2)^{3/2}}{4(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{15bd^4(1-c^2x^2)^{3/2}\arccos(cx)^2}{4c(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{8d^4(1+cx)(1-c^2x^2)(a+b\arccos(cx))}{c(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{4d^4(1-c^2x^2)^2(a+b\arccos(cx))}{c(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{d^4x(1-c^2x^2)^2(a+b\arccos(cx))}{2(d+cdx)^{3/2}(f-cfx)^{3/2}} - \frac{15d^4(1-c^2x^2)^{3/2}\arccos(cx)(a+b\arccos(cx))}{2c(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{8bd^4(1-c^2x^2)^{3/2}\log(1-cx)}{c(d+cdx)^{3/2}(f-cfx)^{3/2}}$$

output
```
-4*b*d^4*x*(-c^2*x^2+1)^(3/2)/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)-1/4*b*c*d^4
*x^2*(-c^2*x^2+1)^(3/2)/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)+15/4*b*d^4*(-c^2*
x^2+1)^(3/2)*arccos(c*x)^2/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)+8*d^4*(c*x+1
)*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)+4*d^4*
(-c^2*x^2+1)^2*(a+b*arccos(c*x))/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)+1/2*d^
4*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)-15/2
*d^4*(-c^2*x^2+1)^(3/2)*arccos(c*x)*(a+b*arccos(c*x))/c/(c*d*x+d)^(3/2)/(-
c*f*x+f)^(3/2)+8*b*d^4*(-c^2*x^2+1)^(3/2)*ln(-c*x+1)/c/(c*d*x+d)^(3/2)/(-c
*f*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 4.15 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.78

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = \frac{d^2(1+cx)\left(4b\sqrt{d+cdx}\sqrt{f-cfx}\sqrt{1-c^2x^2}(-24+7cx+c^2x^2)\,\mathrm{ar}\right.}{}$$

input
```
Integrate[((d + c*d*x)^(5/2)*(a + b*ArcCos[c*x]))/(f - c*f*x)^(3/2),x]
```

output
```
(d^2*(1 + c*x)*(4*b*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*Sqrt[1 - c^2*x^2]*(-24
 + 7*c*x + c^2*x^2)*ArcCos[c*x] + 30*b*(-1 + c*x)*Sqrt[d + c*d*x]*Sqrt[f -
 c*f*x]*ArcCos[c*x]^2 + 60*a*Sqrt[d]*Sqrt[f]*(-1 + c*x)*Sqrt[1 - c^2*x^2]*
ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d]*Sqrt[f]*(-1 + c^2*x^
2))] + Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(4*a*Sqrt[1 - c^2*x^2]*(-24 + 7*c*x
 + c^2*x^2) + b*(1 - 33*c*x + 30*c^2*x^2 + 2*c^3*x^3) - 128*b*(-1 + c*x)*L
og[Sin[ArcCos[c*x]/2]]))*Sin[ArcCos[c*x]/2]^2)/(4*c*f^2*(-1 + c*x)*(1 - c^
2*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5261, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{3/2}\int \frac{d^4(cx+1)^4(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^4\left(1-c^2x^2\right)^{3/2}\int \frac{(cx+1)^4(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$\Big\downarrow$ 5261

$$\frac{d^4\left(1-c^2x^2\right)^{3/2}\left(bc\int \left(\frac{x}{2}-\frac{15\arcsin(cx)}{2c\sqrt{1-c^2x^2}}+\frac{4}{c}+\frac{8(cx+1)}{c(1-c^2x^2)}\right)dx-\frac{15\arcsin(cx)(a+b\arccos(cx))}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))\right.}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$\Big\downarrow$ 2009

$$\frac{d^4\left(1-c^2x^2\right)^{3/2}\left(-\frac{15\arcsin(cx)(a+b\arccos(cx))}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))+\frac{4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}+\frac{8(cx+1)(}{c\sqrt{}}\right.}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

input
```
Int[((d + c*d*x)^(5/2)*(a + b*ArcCos[c*x]))/(f - c*f*x)^(3/2),x]
```

output
```
(d^4*(1 - c^2*x^2)^(3/2)*((8*(1 + c*x)*(a + b*ArcCos[c*x]))/(c*Sqrt[1 - c^
2*x^2]) + (4*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c + (x*Sqrt[1 - c^2*x^
2]*(a + b*ArcCos[c*x]))/2 - (15*(a + b*ArcCos[c*x])*ArcSin[c*x])/(2*c) + b
*c*((4*x)/c + x^2/4 - (15*ArcSin[c*x]^2)/(4*c^2) - (8*Log[1 - c*x])/c^2)))
/((d + c*d*x)^(3/2)*(f - c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5261
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p,
 x]}, Simp[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[1/Sqrt[1 - c^2*x^2]
   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IG
tQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 36.72 (sec) , antiderivative size = 906, normalized size of antiderivative = 2.26

method	result
default	$-\frac{15\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,b\arccos(cx)^2d^2}{4(cx-1)f^2c(cx+1)}-\frac{15\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,a\arccos(cx)d^2}{2(cx-1)f^2c(cx+1)}+\frac{\sqrt{d(cx+1)}}{}$

input
```
int((c*d*x+d)^(5/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-15/4*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x-1)/f^2/
c/(c*x+1)*b*arccos(c*x)^2*d^2-15/2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-
c^2*x^2+1)^(1/2)/(c*x-1)/f^2/c/(c*x+1)*a*arccos(c*x)*d^2+1/32*(d*(c*x+1))^
(1/2)*(-f*(c*x-1))^(1/2)*(2*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+4*c^3*x
^3-1+2*I*(-c^2*x^2+1)^(1/2)*c*x-I*(-c^2*x^2+1)^(1/2)-3*c*x)*(I*b+2*b*arcco
s(c*x)+2*a)*d^2/(c*x-1)/f^2/c/(c*x+1)-(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)
*(1+c*x+I*(-c^2*x^2+1)^(1/2))*(b*arccos(c*x)+a+I*b)*d^2/(c*x-1)/f^2/c/(c*x
+1)+2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*
x^2-1)*(b*arccos(c*x)+a-I*b)*d^2/(c*x-1)/f^2/c/(c*x+1)-1/32*(d*(c*x+1))^(1
/2)*(-f*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-I*(-c^2*x^2+
1)^(1/2)+c*x-1)*(-I*b+2*b*arccos(c*x)+2*a)*d^2/(c*x-1)/f^2/c/(c*x+1)-16*I*
(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)/(c*x-1)/f^2/c/(c*x
+1)*b*arccos(c*x)*d^2-8*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2
+1)^(1/2)+c*x+1)*(a+b*arccos(c*x))*d^2/(c*x-1)/f^2/c/(c*x+1)+16*(d*(c*x+1)
)^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x-1)/f^2/c/(c*x+1)*b*ln(I
*(-c^2*x^2+1)^(1/2)+c*x-1)*d^2+1/16*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(
-I*(-c^2*x^2+1)^(1/2)+c*x+1)*(15*I*b+16*b*arccos(c*x)+16*a)*cos(2*arccos(c
*x))*d^2/(c*x-1)/f^2/c/(c*x+1)+1/8*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(I
*c*x+(-c^2*x^2+1)^(1/2)+I)*(8*I*b+7*b*arccos(c*x)+7*a)*sin(2*arccos(c*x))*
d^2/(c*x-1)/f^2/c/(c*x+1)
```

Fricas [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = \int \frac{(cdx+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{(-cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(3/2),x, algorithm=
"fricas")
```

output
```
integral((a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^2
*x + b*d^2)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*f*x + f)/(c^2*f^2*x^2 - 2
*c*f^2*x + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+d)**(5/2)*(a+b*acos(c*x))/(-c*f*x+f)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = \int \frac{(cdx+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{(-cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(3/2),x, algorithm=
"maxima")
```

output
```
-1/2*(c^2*d^3*x^3/(sqrt(-c^2*d*f*x^2 + d*f)*f) + 8*c*d^3*x^2/(sqrt(-c^2*d*
f*x^2 + d*f)*f) - 17*d^3*x/(sqrt(-c^2*d*f*x^2 + d*f)*f) + 15*d^3*arcsin(c*
x)/(sqrt(d*f)*c*f) - 24*d^3/(sqrt(-c^2*d*f*x^2 + d*f)*c*f))*a - b*sqrt(d)*
integrate((c^2*d^2*x^2 + 2*c*d^2*x + d^2)*sqrt(c*x + 1)*arctan2(sqrt(c*x +
 1)*sqrt(-c*x + 1), c*x)/((c*f*x - f)*sqrt(-c*x + 1)), x)/sqrt(f)
```

Giac [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = \int \frac{(cdx+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{(-cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((c*d*x + d)^(5/2)*(b*arccos(c*x) + a)/(-c*f*x + f)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = \int \frac{(a+b\operatorname{acos}(c\,x))\,\left(d+c\,d\,x\right)^{5/2}}{\left(f-c\,f\,x\right)^{3/2}}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + c*d*x)^(5/2))/(f - c*f*x)^(3/2),x)
```

output
```
int(((a + b*acos(c*x))*(d + c*d*x)^(5/2))/(f - c*f*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = \frac{\sqrt{d}\,d^2\left(30\sqrt{-cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a-2\sqrt{-cx+1}\left(\int \frac{\sqrt{cx+1}\,acos(c}{\sqrt{-cx+1}\,cx-\sqrt{-}}\right.\right.}{}$$

input
```
int((c*d*x+d)^(5/2)*(a+b*acos(c*x))/(-c*f*x+f)^(3/2),x)
```

output
```
(sqrt(d)*d**2*(30*sqrt( - c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a - 2*sq
rt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)*x**2)/(sqrt( - c*x + 1)*c*x -
sqrt( - c*x + 1)),x)*b*c**3 - 4*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c
*x)*x)/(sqrt( - c*x + 1)*c*x - sqrt( - c*x + 1)),x)*b*c**2 - 2*sqrt( - c*x
 + 1)*int((sqrt(c*x + 1)*acos(c*x))/(sqrt( - c*x + 1)*c*x - sqrt( - c*x +
1)),x)*b*c - sqrt(c*x + 1)*a*c**2*x**2 - 7*sqrt(c*x + 1)*a*c*x + 24*sqrt(c
*x + 1)*a))/(2*sqrt(f)*sqrt( - c*x + 1)*c*f)
```

3.531 $\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 252

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = -\frac{bd^3x(1-c^2x^2)^{3/2}}{(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{4d^3(1+cx)\left(1-c^2x^2\right)(a+b\arccos(cx))}{c(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{d^3(1-c^2x^2)^2(a+b\arccos(cx))}{c(d+cdx)^{3/2}(f-cfx)^{3/2}} - \frac{3d^3(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{2bc(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{4bd^3(1-c^2x^2)^{3/2}\log(1-cx)}{c(d+cdx)^{3/2}(f-cfx)^{3/2}}$$

output
```
-b*d^3*x*(-c^2*x^2+1)^(3/2)/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)+4*d^3*(c*x+1)
*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)+d^3*(-c
^2*x^2+1)^2*(a+b*arccos(c*x))/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)-3/2*d^3*(
-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))^2/b/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)
+4*b*d^3*(-c^2*x^2+1)^(3/2)*ln(-c*x+1)/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 3.63 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.10

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = \frac{d(1+cx)\left(2b(-5+cx)\sqrt{d+cdx}\sqrt{f-cfx}\sqrt{1-c^2x^2}\arccos(cx)+\right.}{}$$

input
```
Integrate[((d + c*d*x)^(3/2)*(a + b*ArcCos[c*x]))/(f - c*f*x)^(3/2),x]
```

output
```
(d*(1 + c*x)*(2*b*(-5 + c*x)*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*Sqrt[1 - c^2*
x^2]*ArcCos[c*x] + 3*b*(-1 + c*x)*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*ArcCos[c
*x]^2 + 6*a*Sqrt[d]*Sqrt[f]*(-1 + c*x)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[
d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d]*Sqrt[f]*(-1 + c^2*x^2))] + 2*Sqrt[d +
 c*d*x]*Sqrt[f - c*f*x]*(b*c*x*(-1 + c*x) + a*(-5 + c*x)*Sqrt[1 - c^2*x^2]
 - 8*b*(-1 + c*x)*Log[Sin[ArcCos[c*x]/2]]))*Sin[ArcCos[c*x]/2]^2)/(c*f^2*(
-1 + c*x)*(1 - c^2*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.53, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\int \frac{d^3(cx+1)^3(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3\left(1-c^2x^2\right)^{3/2}\int \frac{(cx+1)^3(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \; 5275$$

$$\frac{d^3(1-c^2x^2)^{3/2}\int\left(-\frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}+\frac{4(cx+1)(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}\right)dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{d^3(1-c^2x^2)^{3/2}\left(\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}+\frac{4(cx+1)(a+b\arccos(cx))}{c\sqrt{1-c^2x^2}}+\frac{3(a+b\arccos(cx))^2}{2bc}-\frac{4b\log(1-cx)}{c}+bx\right)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

input
```
Int[((d + c*d*x)^(3/2)*(a + b*ArcCos[c*x]))/(f - c*f*x)^(3/2),x]
```

output
```
(d^3*(1 - c^2*x^2)^(3/2)*(b*x + (4*(1 + c*x)*(a + b*ArcCos[c*x]))/(c*Sqrt[
1 - c^2*x^2]) + (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c + (3*(a + b*ArcC
os[c*x])^2)/(2*b*c) - (4*b*Log[1 - c*x])/c))/((d + c*d*x)^(3/2)*(f - c*f*x
)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 33.14 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.94

method	result
default	$-\frac{3\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,b\arccos(cx)^2 d}{2(cx-1)f^2c(cx+1)}-\frac{3\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,a\arccos(cx)d}{(cx-1)f^2c(cx+1)}+\frac{\sqrt{d(cx+1)}\,\sqrt{-\ldots}}{}$

input
```
int((c*d*x+d)^(3/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-3/2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x-1)/f^2/c
/(c*x+1)*b*arccos(c*x)^2*d-3*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^
2+1)^(1/2)/(c*x-1)/f^2/c/(c*x+1)*a*arccos(c*x)*d+1/2*(d*(c*x+1))^(1/2)*(-f
*(c*x-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(b*arccos(c*x)+a+I*b)
*d/(c*x-1)/f^2/c/(c*x+1)+1/2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^
2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(b*arccos(c*x)+a-I*b)*d/(c*x-1)/f^2/c/(c*x+1
)-8*I*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x-1)/f^2/
c/(c*x+1)*b*arccos(c*x)*d-4*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2
*x^2+1)^(1/2)+c*x+1)*(a+b*arccos(c*x))*d/(c*x-1)/f^2/c/(c*x+1)+8*(d*(c*x+1
))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x-1)/f^2/c/(c*x+1)*b*ln(
I*(-c^2*x^2+1)^(1/2)+c*x-1)*d
```

Fricas [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx=\int \frac{(cdx+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)}{(-cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(3/2),x, algorithm=
"fricas")
```

output
```
integral((a*c*d*x + a*d + (b*c*d*x + b*d)*arccos(c*x))*sqrt(c*d*x + d)*sqr
t(-c*f*x + f)/(c^2*f^2*x^2 - 2*c*f^2*x + f^2), x)
```

Sympy [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx=\int \frac{(d(cx+1))^{\frac{3}{2}}\,(a+b\operatorname{acos}{(cx)})}{(-f\,(cx-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)**(3/2)*(a+b*acos(c*x))/(-c*f*x+f)**(3/2),x)
```

output
```
Integral((d*(c*x + 1))**(3/2)*(a + b*acos(c*x))/(-f*(c*x - 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx=\int \frac{(cdx+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)}{(-cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(3/2),x, algorithm=
"maxima")
```

output
```
b*sqrt(d)*sqrt(f)*integrate((c*d*x + d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arcta
n2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^2*f^2*x^2 - 2*c*f^2*x + f^2), x)
- a*((-c^2*d*f*x^2 + d*f)^(3/2)/(c^3*f^3*x^2 - 2*c^2*f^3*x + c*f^3) + 6*sq
rt(-c^2*d*f*x^2 + d*f)*d/(c^2*f^2*x - c*f^2) + 3*d^2*arcsin(c*x)/(c*f^2*sq
rt(d/f)))
```

Giac [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx=\int \frac{(cdx+d)^{\frac{3}{2}}(b\arccos(cx)+a)}{(-cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((c*d*x + d)^(3/2)*(b*arccos(c*x) + a)/(-c*f*x + f)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d+c\,d\,x)^{3/2}}{(f-c\,f\,x)^{3/2}}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + c*d*x)^(3/2))/(f - c*f*x)^(3/2),x)
```

output
```
int(((a + b*acos(c*x))*(d + c*d*x)^(3/2))/(f - c*f*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = \frac{\sqrt{d}\,d\left(6\sqrt{-cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a-\sqrt{-cx+1}\left(\int\frac{\sqrt{cx+1}\,acos(cx)x}{\sqrt{-cx+1}\,cx-\sqrt{-cx+}}\right.\right.}{\sqrt{}}$$

input
```
int((c*d*x+d)^(3/2)*(a+b*acos(c*x))/(-c*f*x+f)^(3/2),x)
```

output
```
(sqrt(d)*d*(6*sqrt( - c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a - sqrt( -
c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)*x)/(sqrt( - c*x + 1)*c*x - sqrt( - c
*x + 1)),x)*b*c**2 - sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x))/(sqrt(
 - c*x + 1)*c*x - sqrt( - c*x + 1)),x)*b*c - sqrt(c*x + 1)*a*c*x + 5*sqrt(
c*x + 1)*a))/(sqrt(f)*sqrt( - c*x + 1)*c*f)
```

3.532 $\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 162

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = \frac{2d^2(1+cx)\left(1-c^2x^2\right)(a+b\arccos(cx))}{c(d+cdx)^{3/2}(f-cfx)^{3/2}} - \frac{d^2\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2}{2bc(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{2bd^2\left(1-c^2x^2\right)^{3/2}\log(1-cx)}{c(d+cdx)^{3/2}(f-cfx)^{3/2}}$$

output
```
2*d^2*(c*x+1)*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^
(3/2)-1/2*d^2*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))^2/b/c/(c*d*x+d)^(3/2)/(
-c*f*x+f)^(3/2)+2*b*d^2*(-c^2*x^2+1)^(3/2)*ln(-c*x+1)/c/(c*d*x+d)^(3/2)/(-
c*f*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 2.15 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = -\frac{\frac{4a\sqrt{d+cdx}\sqrt{f-cfx}}{-1+cx} - 2a\sqrt{d}\sqrt{f}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{f-cfx}}{\sqrt{d}\sqrt{f}(-1+c^2x^2)}\right) + \frac{b(1+cx)\sqrt{d+cdx}\sqrt{f-cfx}\tan\left(\frac{1}{2}\arccos(cx)\right)\left(4\arccos(cx)+(\arccos(\right.}{(-1+cx)\sqrt{1-c^2x}}}{2cf^2}$$

input
```
Integrate[(Sqrt[d + c*d*x]*(a + b*ArcCos[c*x]))/(f - c*f*x)^(3/2),x]
```

output
```
-1/2*((4*a*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(-1 + c*x) - 2*a*Sqrt[d]*Sqrt[
f]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d]*Sqrt[f]*(-1 + c^2
*x^2))] + (b*(1 + c*x)*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*Tan[ArcCos[c*x]/2]*
(4*ArcCos[c*x] + (ArcCos[c*x]^2 - 8*Log[Sin[ArcCos[c*x]/2]])*Tan[ArcCos[c*
x]/2]))/((-1 + c*x)*Sqrt[1 - c^2*x^2]))/(c*f^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.65, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cdx+d}(a+b\arccos(cx))}{(f-cfx)^{3/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\int \frac{d^2(cx+1)^2(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\left(1-c^2x^2\right)^{3/2}\int \frac{(cx+1)^2(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{5275}$$

$$\frac{d^2\left(1-c^2x^2\right)^{3/2}\int \left(\frac{2(cx+1)(a+b\arccos(cx))}{(1-c^2x^2)^{3/2}}-\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}\right)dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\left(1-c^2x^2\right)^{3/2}\left(\frac{2(cx+1)(a+b\arccos(cx))}{c\sqrt{1-c^2x^2}}+\frac{(a+b\arccos(cx))^2}{2bc}-\frac{2b\log(1-cx)}{c}\right)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

input
```
Int[(Sqrt[d + c*d*x]*(a + b*ArcCos[c*x]))/(f - c*f*x)^(3/2),x]
```

output
```
(d^2*(1 - c^2*x^2)^(3/2)*((2*(1 + c*x)*(a + b*ArcCos[c*x]))/(c*Sqrt[1 - c^
2*x^2]) + (a + b*ArcCos[c*x])^2/(2*b*c) - (2*b*Log[1 - c*x])/c))/((d + c*d
*x)^(3/2)*(f - c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 17.10 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.99

method	result
default	$-\frac{\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,b\arccos(cx)^2}{2(cx-1)f^2c(cx+1)}-\frac{\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,a\arccos(cx)}{(cx-1)f^2c(cx+1)}-\frac{4i\sqrt{d(cx+1)}\,\sqrt{-f(c}{(cx-}$

input
```
int((c*d*x+d)^(1/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x-1)/f^2/c
/(c*x+1)*b*arccos(c*x)^2-(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)
^(1/2)/(c*x-1)/f^2/c/(c*x+1)*a*arccos(c*x)-4*I*(d*(c*x+1))^(1/2)*(-f*(c*x-
1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x-1)/f^2/c/(c*x+1)*b*arccos(c*x)-2*(d*(c*x
+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x+1)*(a+b*arccos(c*
x))/(c*x-1)/f^2/c/(c*x+1)+4*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2
+1)^(1/2)/(c*x-1)/f^2/c/(c*x+1)*b*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)
```

Fricas [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx=\int \frac{\sqrt{cdx+d}(b\arccos{(cx)}+a)}{(-cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(3/2),x, algorithm=
"fricas")
```

output
```
integral(sqrt(c*d*x + d)*sqrt(-c*f*x + f)*(b*arccos(c*x) + a)/(c^2*f^2*x^2
 - 2*c*f^2*x + f^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx=\int \frac{\sqrt{d\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))}{(-f\,(cx-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)**(1/2)*(a+b*acos(c*x))/(-c*f*x+f)**(3/2),x)
```

output
```
Integral(sqrt(d*(c*x + 1))*(a + b*acos(c*x))/(-f*(c*x - 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = \int \frac{\sqrt{cdx+d}(b\arccos{(cx)}+a)}{(-cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(3/2),x, algorithm=
"maxima")
```

output
```
-a*(2*sqrt(-c^2*d*f*x^2 + d*f)/(c^2*f^2*x - c*f^2) + d*arcsin(c*x)/(c*f^2*
sqrt(d/f))) - b*sqrt(d)*integrate(sqrt(c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt
(-c*x + 1), c*x)/((c*f*x - f)*sqrt(-c*x + 1)), x)/sqrt(f)
```

Giac [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = \int \frac{\sqrt{cdx+d}(b\arccos{(cx)}+a)}{(-cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(3/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(c*d*x + d)*(b*arccos(c*x) + a)/(-c*f*x + f)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d+c\,d\,x}}{(f-c\,f\,x)^{3/2}}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + c*d*x)^(1/2))/(f - c*f*x)^(3/2),x)
```

output
```
int(((a + b*acos(c*x))*(d + c*d*x)^(1/2))/(f - c*f*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{3/2}}\,dx = \frac{\sqrt{d}\left(2\sqrt{-cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a-\sqrt{-cx+1}\left(\int \frac{\sqrt{cx+1}\,acos(cx)}{\sqrt{-cx+1}\,cx-\sqrt{-cx+1}}dx\right.\right.}{\sqrt{f}\,\sqrt{-cx+1}\,cf}$$

input
```
int((c*d*x+d)^(1/2)*(a+b*acos(c*x))/(-c*f*x+f)^(3/2),x)
```

output
```
(sqrt(d)*(2*sqrt( - c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a - sqrt( - c*
x + 1)*int((sqrt(c*x + 1)*acos(c*x))/(sqrt( - c*x + 1)*c*x - sqrt( - c*x +
 1)),x)*b*c + 2*sqrt(c*x + 1)*a))/(sqrt(f)*sqrt( - c*x + 1)*c*f)
```

3.533 $\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{3/2}}\,dx = \frac{d(1+cx)\left(1-c^2x^2\right)(a+b\arccos(cx))}{c(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{bd\left(1-c^2x^2\right)^{3/2}\log(1-cx)}{c(d+cdx)^{3/2}(f-cfx)^{3/2}}$$

output

```
d*(c*x+1)*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2
)+b*d*(-c^2*x^2+1)^(3/2)*ln(-c*x+1)/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{3/2}}\,dx = -\frac{\sqrt{d+cdx}\sqrt{f-cfx}\left(a\sqrt{1-c^2x^2}+b\sqrt{1-c^2x^2}\arccos(cx)+b(-1+cx)\log(f-cfx)\right)}{cdf^2(-1+cx)\sqrt{1-c^2x^2}}$$

input

```
Integrate[(a + b*ArcCos[c*x])/(Sqrt[d + c*d*x]*(f - c*f*x)^(3/2)),x]
```

output
```
-((Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(a*Sqrt[1 - c^2*x^2] + b*Sqrt[1 - c^2*x
^2]*ArcCos[c*x] + b*(-1 + c*x)*Log[f - c*f*x]))/(c*d*f^2*(-1 + c*x)*Sqrt[1
 - c^2*x^2]))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5179, 27, 5261, 27, 451, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{\sqrt{cdx+d}(f-cfx)^{3/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\int \frac{d(cx+1)(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(1-c^2x^2\right)^{3/2}\int \frac{(cx+1)(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{5261}$$

$$\frac{d\left(1-c^2x^2\right)^{3/2}\left(bc\int \frac{cx+1}{c\left(1-c^2x^2\right)}dx+\frac{(cx+1)(a+b\arccos(cx))}{c\sqrt{1-c^2x^2}}\right)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(1-c^2x^2\right)^{3/2}\left(b\int \frac{cx+1}{1-c^2x^2}dx+\frac{(cx+1)(a+b\arccos(cx))}{c\sqrt{1-c^2x^2}}\right)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{451}$$

$$\frac{d\left(1-c^2x^2\right)^{3/2}\left(b\int \frac{1}{1-cx}dx+\frac{(cx+1)(a+b\arccos(cx))}{c\sqrt{1-c^2x^2}}\right)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{d(1-c^2x^2)^{3/2}\left(\frac{(cx+1)(a+b\arccos(cx))}{c\sqrt{1-c^2x^2}}-\frac{b\log(1-cx)}{c}\right)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(Sqrt[d + c*d*x]*(f - c*f*x)^(3/2)),x]
```

output
```
(d*(1 - c^2*x^2)^(3/2)*(((1 + c*x)*(a + b*ArcCos[c*x]))/(c*Sqrt[1 - c^2*x^
2]) - (b*Log[1 - c*x])/c))/((d + c*d*x)^(3/2)*(f - c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 451
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c^2/a   In
t[1/(c - d*x), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5261
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p,
 x]}, Simp[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[1/Sqrt[1 - c^2*x^2]
   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IG
tQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.35

method	result
default	$\frac{a\sqrt{cdx+d}}{fdc\sqrt{-cfx+f}}+b\left(-\frac{2i\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)}{d\,f^2c(c^2x^2-1)}-\frac{\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\left(-i\sqrt{-c^2x^2+1}+cx+1\right)\text{ar}}{d\,f^2c(c^2x^2-1)}\right.$
parts	$\frac{a\sqrt{cdx+d}}{fdc\sqrt{-cfx+f}}+b\left(-\frac{2i\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)}{d\,f^2c(c^2x^2-1)}-\frac{\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\left(-i\sqrt{-c^2x^2+1}+cx+1\right)\text{ar}}{d\,f^2c(c^2x^2-1)}\right.$

input
```
int((a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
a/f/d/c/(-c*f*x+f)^(1/2)*(c*d*x+d)^(1/2)+b*(-2*I*(d*(c*x+1))^(1/2)*(-f*(c*
x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/f^2/c/(c^2*x^2-1)*arccos(c*x)-(d*(c*x+1))
^(1/2)*(-f*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x+1)*arccos(c*x)/d/f^2/
c/(c^2*x^2-1)+2*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/
f^2/c/(c^2*x^2-1)*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.60

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{3/2}}\,dx = \left[\frac{(bcx-b)\sqrt{df}\log\left(\frac{c^6dfx^6-4\,c^5dfx^5+5\,c^4dfx^4-4\,c^2dfx^2+4\,cdfx+(c^4x^4-4\,c^3x^3+6\,c^2x^2-4\,cx}{c^4x^4-2\,c^3x^3+2\,cx-1}\right.}{2\,(c^2df}\right.$$

$$-\frac{(bcx-b)\sqrt{-df}\arctan\left(\frac{(c^2x^2-2\,cx+2)\sqrt{-c^2x^2+1}\sqrt{cdx+d}\sqrt{-cfx+f}\sqrt{-df}}{c^4dfx^4-2\,c^3dfx^3-c^2dfx^2+2\,cdfx}\right)+\sqrt{cdx+d}\sqrt{-cfx+f}(b\arccos{(cx)}+}{c^2df^2x-cdf^2}$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(3/2),x, algorithm=
"fricas")
```

output
```
[1/2*((b*c*x - b)*sqrt(d*f)*log((c^6*d*f*x^6 - 4*c^5*d*f*x^5 + 5*c^4*d*f*x
^4 - 4*c^2*d*f*x^2 + 4*c*d*f*x + (c^4*x^4 - 4*c^3*x^3 + 6*c^2*x^2 - 4*c*x)
*sqrt(-c^2*x^2 + 1)*sqrt(c*d*x + d)*sqrt(-c*f*x + f)*sqrt(d*f) - 2*d*f)/(c
^4*x^4 - 2*c^3*x^3 + 2*c*x - 1)) - 2*sqrt(c*d*x + d)*sqrt(-c*f*x + f)*(b*a
rccos(c*x) + a))/(c^2*d*f^2*x - c*d*f^2), -((b*c*x - b)*sqrt(-d*f)*arctan(
(c^2*x^2 - 2*c*x + 2)*sqrt(-c^2*x^2 + 1)*sqrt(c*d*x + d)*sqrt(-c*f*x + f)*
sqrt(-d*f)/(c^4*d*f*x^4 - 2*c^3*d*f*x^3 - c^2*d*f*x^2 + 2*c*d*f*x)) + sqrt
(c*d*x + d)*sqrt(-c*f*x + f)*(b*arccos(c*x) + a))/(c^2*d*f^2*x - c*d*f^2)]
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{acos}\,(cx)}{\sqrt{d\,(cx+1)}\,(-f\,(cx-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(c*d*x+d)**(1/2)/(-c*f*x+f)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))/(sqrt(d*(c*x + 1))*(-f*(c*x - 1))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{3/2}}\,dx = -\frac{\sqrt{-c^2dfx^2+df}\,b\arccos(cx)}{c^2df^2x-cdf^2} - \frac{\sqrt{-c^2dfx^2+df}\,a}{c^2df^2x-cdf^2} - \frac{b\log(cx-1)}{c\sqrt{d}f^{\frac{3}{2}}}$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(3/2),x, algorithm=
"maxima")
```

output
```
-sqrt(-c^2*d*f*x^2 + d*f)*b*arccos(c*x)/(c^2*d*f^2*x - c*d*f^2) - sqrt(-c^
2*d*f*x^2 + d*f)*a/(c^2*d*f^2*x - c*d*f^2) - b*log(c*x - 1)/(c*sqrt(d)*f^(
3/2))
```

Giac [F]

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{3/2}}\,dx=\int \frac{b\arccos(cx)+a}{\sqrt{cdx+d}(-cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((b*arccos(c*x) + a)/(sqrt(c*d*x + d)*(-c*f*x + f)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{\sqrt{d+c\,d\,x}\,(f-c\,f\,x)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))/((d + c*d*x)^(1/2)*(f - c*f*x)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))/((d + c*d*x)^(1/2)*(f - c*f*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{3/2}}\,dx=\frac{-\sqrt{-cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,cx-\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)bc+\sqrt{cx+1}\,a}{\sqrt{f}\,\sqrt{d}\,\sqrt{-cx+1}\,cf}$$

input
```
int((a+b*acos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(3/2),x)
```

output
```
( - sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c*x - s
qrt(c*x + 1)*sqrt( - c*x + 1)),x)*b*c + sqrt(c*x + 1)*a)/(sqrt(f)*sqrt(d)*
sqrt( - c*x + 1)*c*f)
```

3.534 $\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{3/2}}\,dx = \frac{x(1-c^2x^2)\,(a+b\arccos(cx))}{(d+cdx)^{3/2}(f-cfx)^{3/2}} + \frac{b(1-c^2x^2)^{3/2}\log(1-c^2x^2)}{2c(d+cdx)^{3/2}(f-cfx)^{3/2}}$$

output
```
x*(-c^2*x^2+1)*(a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)+1/2*b*(-
c^2*x^2+1)^(3/2)*ln(-c^2*x^2+1)/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.11

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{3/2}}\,dx = \frac{\sqrt{d+cdx}\,(2acx+2bcx\arccos(cx)-b\sqrt{1-c^2x^2}\log(-f(1+cx))-b\sqrt{1}}{2cd^2f(1+cx)\sqrt{f-cfx}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/((d + c*d*x)^(3/2)*(f - c*f*x)^(3/2)),x]
```

output
```
(Sqrt[d + c*d*x]*(2*a*c*x + 2*b*c*x*ArcCos[c*x] - b*Sqrt[1 - c^2*x^2]*Log[
-(f*(1 + c*x))] - b*Sqrt[1 - c^2*x^2]*Log[f - c*f*x]))/(2*c*d^2*f*(1 + c*x
)*Sqrt[f - c*f*x])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5179, 5161, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{3/2}\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{5161}$$

$$\frac{(1-c^2x^2)^{3/2}\left(bc\int \frac{x}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

$$\Big\downarrow \text{240}$$

$$\frac{(1-c^2x^2)^{3/2}\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)}{(cdx+d)^{3/2}(f-cfx)^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])/((d + c*d*x)^(3/2)*(f - c*f*x)^(3/2)),x]
```

output
```
((1 - c^2*x^2)^(3/2)*((x*(a + b*ArcCos[c*x]))/Sqrt[1 - c^2*x^2] - (b*Log[1
 - c^2*x^2])/(2*c)))/((d + c*d*x)^(3/2)*(f - c*f*x)^(3/2))
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.34

method	result
default	$a\left(-\frac{1}{fdc\sqrt{cdx+d}\,\sqrt{-cfx+f}}+\frac{\sqrt{cdx+d}}{cf\,d^{2}\sqrt{-cfx+f}}\right)-\frac{b\sqrt{-c^{2}x^{2}+1}\,\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\left(i\arccos(cx)c^{2}x^{2}-\ln\left(\left(cx+i\sqrt{-c^{2}\right.\right.\right.}{(c^{2}x^{2}-}$
parts	$a\left(-\frac{1}{fdc\sqrt{cdx+d}\,\sqrt{-cfx+f}}+\frac{\sqrt{cdx+d}}{cf\,d^{2}\sqrt{-cfx+f}}\right)-\frac{b\sqrt{-c^{2}x^{2}+1}\,\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\left(i\arccos(cx)c^{2}x^{2}-\ln\left(\left(cx+i\sqrt{-c^{2}\right.\right.\right.}{(c^{2}x^{2}-}$

input
```
int((a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
a*(-1/f/d/c/(c*d*x+d)^(1/2)/(-c*f*x+f)^(1/2)+1/c/f/d^2/(-c*f*x+f)^(1/2)*(c
*d*x+d)^(1/2))-b*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(
I*arccos(c*x)*x^2*c^2-ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)*x^2*c^2-(-c^2*x^2
+1)^(1/2)*arccos(c*x)*x*c-I*arccos(c*x)+ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)
)/(c^2*x^2-1)/c/d^2/f^2/(c*x+1)/(c*x-1)
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{3/2}}\,dx = \int \frac{b\arccos(cx)+a}{(cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2),x, algorithm=
"fricas")
```

output
```
integral(sqrt(c*d*x + d)*sqrt(-c*f*x + f)*(b*arccos(c*x) + a)/(c^4*d^2*f^2
*x^4 - 2*c^2*d^2*f^2*x^2 + d^2*f^2), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{3/2}}\,dx = \int \frac{a+b\operatorname{acos}(cx)}{(d(cx+1))^{\frac{3}{2}}(-f(cx-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(c*d*x+d)**(3/2)/(-c*f*x+f)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))/((d*(c*x + 1))**(3/2)*(-f*(c*x - 1))**(3/2)), x
)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.90

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{3/2}}\,dx = \frac{bx\arccos(cx)}{\sqrt{-c^2dfx^2+df}\,df} + \frac{ax}{\sqrt{-c^2dfx^2+df}\,df} + \frac{b\sqrt{\frac{1}{df}}\log\left(x^2-\frac{1}{c^2}\right)}{2\,cdf}$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2),x, algorithm=
"maxima")
```

output
```
b*x*arccos(c*x)/(sqrt(-c^2*d*f*x^2 + d*f)*d*f) + a*x/(sqrt(-c^2*d*f*x^2 +
d*f)*d*f) + 1/2*b*sqrt(1/(d*f))*log(x^2 - 1/c^2)/(c*d*f)
```

Giac [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{3/2}}\,dx = \int \frac{b\arccos(cx)+a}{(cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((b*arccos(c*x) + a)/((c*d*x + d)^(3/2)*(-c*f*x + f)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{{(d+c\,d\,x)}^{3/2}\,{(f-c\,f\,x)}^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))/((d + c*d*x)^(3/2)*(f - c*f*x)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))/((d + c*d*x)^(3/2)*(f - c*f*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{3/2}}\,dx = \frac{-\sqrt{cx+1}\,\sqrt{-cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2-\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)b+ax}{\sqrt{f}\,\sqrt{d}\,\sqrt{cx+1}\,\sqrt{-cx+1}\,df}$$

input
```
int((a+b*acos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2),x)
```

output
```
( - sqrt(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*
x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b + a*x)/(sqrt(f)*sq
rt(d)*sqrt(c*x + 1)*sqrt( - c*x + 1)*d*f)
```

3.535 $\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 255

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{3/2}}\,dx = -\frac{bf(1-c^2x^2)^{5/2}}{6c(1+cx)(d+cdx)^{5/2}(f-cfx)^{5/2}} - \frac{f(1-cx)\left(1-c^2x^2\right)(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{2fx(1-c^2x^2)^2\,(a+b\arccos(cx))}{3(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{bf(1-c^2x^2)^{5/2}\operatorname{arctanh}(cx)}{6c(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{bf(1-c^2x^2)^{5/2}\log\left(1-c^2x^2\right)}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}}$$

output
```
-1/6*b*f*(-c^2*x^2+1)^(5/2)/c/(c*x+1)/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)-1/3
*f*(-c*x+1)*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5
/2)+2/3*f*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5
/2)+1/6*b*f*(-c^2*x^2+1)^(5/2)*arctanh(c*x)/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(
5/2)+1/3*b*f*(-c^2*x^2+1)^(5/2)*ln(-c^2*x^2+1)/c/(c*d*x+d)^(5/2)/(-c*f*x+f
)^(5/2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.71

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{3/2}}\,dx = \frac{\sqrt{d+cdx}\left(-4a+8acx+8ac^2x^2+2b\sqrt{1-c^2x^2}+4b(-1+2cx+2c^2x^2)\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/((d + c*d*x)^(5/2)*(f - c*f*x)^(3/2)),x]
```

output
```
(Sqrt[d + c*d*x]*(-4*a + 8*a*c*x + 8*a*c^2*x^2 + 2*b*Sqrt[1 - c^2*x^2] + 4
*b*(-1 + 2*c*x + 2*c^2*x^2)*ArcCos[c*x] - 5*b*(1 + c*x)*Sqrt[1 - c^2*x^2]*
Log[-(f*(1 + c*x))] - 3*b*Sqrt[1 - c^2*x^2]*Log[f - c*f*x] - 3*b*c*x*Sqrt[
1 - c^2*x^2]*Log[f - c*f*x]))/(12*c*d^3*f*(1 + c*x)^2*Sqrt[f - c*f*x])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5179, 27, 5261, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(cdx+d)^{5/2}(f-cfx)^{3/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\int \frac{f(1-cx)(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\left(1-c^2x^2\right)^{5/2}\int \frac{(1-cx)(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{5261}$$

$$\frac{f(1-c^2x^2)^{5/2}\left(bc\int\left(\frac{2x}{3(1-c^2x^2)}-\frac{1-cx}{3c(1-c^2x^2)^2}\right)dx+\frac{2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}-\frac{(1-cx)(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f(1-c^2x^2)^{5/2}\left(\frac{2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}-\frac{(1-cx)(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}+bc\left(-\frac{\operatorname{arctanh}(cx)}{6c^2}+\frac{1-cx}{6c^2(1-c^2x^2)}-\frac{\log(1-c^2x^2)}{3c^2}\right)\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

input
```
Int[(a + b*ArcCos[c*x])/((d + c*d*x)^(5/2)*(f - c*f*x)^(3/2)),x]
```

output
```
(f*(1 - c^2*x^2)^(5/2)*(-1/3*((1 - c*x)*(a + b*ArcCos[c*x]))/(c*(1 - c^2*x
^2)^(3/2)) + (2*x*(a + b*ArcCos[c*x]))/(3*Sqrt[1 - c^2*x^2]) + b*c*((1 - c
*x)/(6*c^2*(1 - c^2*x^2)) - ArcTanh[c*x]/(6*c^2) - Log[1 - c^2*x^2]/(3*c^2
))))/((d + c*d*x)^(5/2)*(f - c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5261
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p,
 x]}, Simp[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[1/Sqrt[1 - c^2*x^2]
   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IG
tQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.96

method	result
default	$a\left(-\frac{1}{3fdc(cdx+d)^{\frac{3}{2}}\sqrt{-cfx+f}}+\frac{-\frac{2}{3fdc\sqrt{cdx+d}\,\sqrt{-cfx+f}}+\frac{2\sqrt{cdx+d}}{3cf\,d^{2}\sqrt{-cfx+f}}}{d}\right)-$ $b\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^{2}x^{2}+1}\,\left(-4i\,\mathrm{ar}\right.$
parts	$a\left(-\frac{1}{3fdc(cdx+d)^{\frac{3}{2}}\sqrt{-cfx+f}}+\frac{-\frac{2}{3fdc\sqrt{cdx+d}\,\sqrt{-cfx+f}}+\frac{2\sqrt{cdx+d}}{3cf\,d^{2}\sqrt{-cfx+f}}}{d}\right)-$ $b\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^{2}x^{2}+1}\,\left(-4i\,\mathrm{ar}\right.$

input
```
int((a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
a*(-1/3/f/d/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(1/2)+2/3/d*(-1/f/d/c/(c*d*x+d)^(
1/2)/(-c*f*x+f)^(1/2)+1/c/f/d^2/(-c*f*x+f)^(1/2)*(c*d*x+d)^(1/2)))-1/6*b*(
d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(-4*I*arccos(c*x)*x
*c-3*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)*x^3*c^3-5*ln(1+c*x+I*(-c^2*x^2+1)^(1/2
))*x^3*c^3+4*I*arccos(c*x)*c^2*x^2-4*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^
2-3*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)*x^2*c^2-5*ln(1+c*x+I*(-c^2*x^2+1)^(1/2)
)*x^2*c^2-4*I*arccos(c*x)-4*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c+c^2*x^2+3*l
n(I*(-c^2*x^2+1)^(1/2)+c*x-1)*x*c+5*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*x*c+4*I
*arccos(c*x)*c^3*x^3+2*arccos(c*x)*(-c^2*x^2+1)^(1/2)+3*ln(I*(-c^2*x^2+1)^
(1/2)+c*x-1)+5*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-1)/d^3/f^2/c/(c^5*x^5+c^4*x^
4-2*c^3*x^3-2*c^2*x^2+c*x+1)
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{3/2}}\,dx = \int \frac{b\arccos(cx)+a}{(cdx+d)^{\frac{5}{2}}(-cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(3/2),x, algorithm=
"fricas")
```

output
```
integral(sqrt(c*d*x + d)*sqrt(-c*f*x + f)*(b*arccos(c*x) + a)/(c^5*d^3*f^2
*x^5 + c^4*d^3*f^2*x^4 - 2*c^3*d^3*f^2*x^3 - 2*c^2*d^3*f^2*x^2 + c*d^3*f^2
*x + d^3*f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acos(c*x))/(c*d*x+d)**(5/2)/(-c*f*x+f)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.92

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{3/2}}\,dx = \frac{1}{12}\,bc\left(\frac{2\,\sqrt{d}\sqrt{f}}{c^3d^3f^2x+c^2d^3f^2} - \frac{5\,\log\left(cx+1\right)}{c^2d^{\frac{5}{2}}f^{\frac{3}{2}}} - \frac{3\,\log\left(cx-1\right)}{c^2d^{\frac{5}{2}}f^{\frac{3}{2}}}\right) - \frac{1}{3}\,b\left(\frac{1}{\sqrt{-c^2dfx^2+df}c^2d^2fx+\sqrt{-c^2dfx^2+df}cd^2f} - \frac{2\,x}{\sqrt{-c^2dfx^2+df}d^2f}\right)\arccos\left(cx\right) - \frac{1}{3}\,a\left(\frac{1}{\sqrt{-c^2dfx^2+df}c^2d^2fx+\sqrt{-c^2dfx^2+df}cd^2f} - \frac{2\,x}{\sqrt{-c^2dfx^2+df}d^2f}\right)$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(3/2),x, algorithm=
"maxima")
```

output
```
1/12*b*c*(2*sqrt(d)*sqrt(f)/(c^3*d^3*f^2*x + c^2*d^3*f^2) - 5*log(c*x + 1)
/(c^2*d^(5/2)*f^(3/2)) - 3*log(c*x - 1)/(c^2*d^(5/2)*f^(3/2))) - 1/3*b*(1/
(sqrt(-c^2*d*f*x^2 + d*f)*c^2*d^2*f*x + sqrt(-c^2*d*f*x^2 + d*f)*c*d^2*f)
- 2*x/(sqrt(-c^2*d*f*x^2 + d*f)*d^2*f))*arccos(c*x) - 1/3*a*(1/(sqrt(-c^2*
d*f*x^2 + d*f)*c^2*d^2*f*x + sqrt(-c^2*d*f*x^2 + d*f)*c*d^2*f) - 2*x/(sqrt
(-c^2*d*f*x^2 + d*f)*d^2*f))
```

Giac [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{3/2}}\,dx = \int \frac{b\arccos{(cx)}+a}{(cdx+d)^{\frac{5}{2}}(-cfx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(3/2),x, algorithm=
"giac")
```

output
```
integrate((b*arccos(c*x) + a)/((c*d*x + d)^(5/2)*(-c*f*x + f)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{{(d+c\,d\,x)}^{5/2}\,{(f-c\,f\,x)}^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))/((d + c*d*x)^(5/2)*(f - c*f*x)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))/((d + c*d*x)^(5/2)*(f - c*f*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{3/2}}\,dx = \frac{-3\sqrt{cx+1}\,\sqrt{-cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^3x^3+\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2-\sqrt{cx+1}\,\sqrt{-cx+}}\right.}{}$$

input
```
int((a+b*acos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(3/2),x)
```

output
```
( - 3*sqrt(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( -
c*x + 1)*c**3*x**3 + sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 - sqrt(c*x +
 1)*sqrt( - c*x + 1)*c*x - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b*c**2*x - 3
*sqrt(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x +
 1)*c**3*x**3 + sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*s
qrt( - c*x + 1)*c*x - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b*c + 2*a*c**2*x*
*2 + 2*a*c*x - a)/(3*sqrt(f)*sqrt(d)*sqrt(c*x + 1)*sqrt( - c*x + 1)*c*d**2
*f*(c*x + 1))
```

3.536 $\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 419

$$\begin{aligned}\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx &= \frac{bd^5x(1-c^2x^2)^{5/2}}{(d+cdx)^{5/2}(f-cfx)^{5/2}} \\ &- \frac{8bd^5(1-c^2x^2)^{5/2}}{3c(1-cx)(d+cdx)^{5/2}(f-cfx)^{5/2}} - \frac{5bd^5(1-c^2x^2)^{5/2}\arccos(cx)^2}{2c(d+cdx)^{5/2}(f-cfx)^{5/2}} \\ &+ \frac{2d^5(1+cx)^4(1-c^2x^2)(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}} \\ &- \frac{20d^5(1+cx)(1-c^2x^2)^2(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}} - \frac{5d^5(1-c^2x^2)^3(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}} \\ &+ \frac{5d^5(1-c^2x^2)^{5/2}\arccos(cx)(a+b\arccos(cx))}{c(d+cdx)^{5/2}(f-cfx)^{5/2}} - \frac{28bd^5(1-c^2x^2)^{5/2}\log(1-cx)}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}}\end{aligned}$$

output

```
b*d^5*x*(-c^2*x^2+1)^(5/2)/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)-8/3*b*d^5*(-c^
2*x^2+1)^(5/2)/c/(-c*x+1)/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)-5/2*b*d^5*(-c^2
*x^2+1)^(5/2)*arccos(c*x)^2/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)+2/3*d^5*(c*
x+1)^4*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)-2
0/3*d^5*(c*x+1)*(-c^2*x^2+1)^2*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*f*x
+f)^(5/2)-5/3*d^5*(-c^2*x^2+1)^3*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*f
*x+f)^(5/2)+5*d^5*(-c^2*x^2+1)^(5/2)*arccos(c*x)*(a+b*arccos(c*x))/c/(c*d*
x+d)^(5/2)/(-c*f*x+f)^(5/2)-28/3*b*d^5*(-c^2*x^2+1)^(5/2)*ln(-c*x+1)/c/(c*
d*x+d)^(5/2)/(-c*f*x+f)^(5/2)
```

Mathematica [A] (verified)

Time = 6.87 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.68

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx = \frac{d^2\left(-15a\sqrt{d}\sqrt{f}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{f-cfx}}{\sqrt{d}\sqrt{f}(-1+c^2x^2)}\right) - \frac{\sqrt{d+cdx}\sqrt{f-cfx}\left(a(-1+cx)^2(\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((d + c*d*x)^(5/2)*(a + b*ArcCos[c*x]))/(f - c*f*x)^(5/2),x]
```

output
```
(d^2*(-15*a*Sqrt[d]*Sqrt[f]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(
Sqrt[d]*Sqrt[f]*(-1 + c^2*x^2))] - (Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(a*(-1
 + c*x)^2*(23 - 11*c*x - 31*c^2*x^2 + 3*c^3*x^3) + b*(-1 + c*x)^2*(23 - 11
*c*x - 31*c^2*x^2 + 3*c^3*x^3)*ArcCos[c*x] - 32*b*Sqrt[1 - c^2*x^2]*Sin[Ar
cCos[c*x]/2]^4 + 60*b*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2*Sin[ArcCos[c*x]/2]^6
 + 8*b*Sqrt[1 - c^2*x^2]*(3*c*x - 56*Log[Sin[ArcCos[c*x]/2]])*Sin[ArcCos[c
*x]/2]^6))/((-1 + c*x)^4*(1 + c*x))))/(3*c*f^3)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.50, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5179, 27, 5261, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{5/2}\int \frac{d^5(cx+1)^5(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^5(1-c^2x^2)^{5/2}\int \frac{(cx+1)^5(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$\downarrow$ 5261

$$\frac{d^5\left(1-c^2x^2\right)^{5/2}\left(bc\int\left(\frac{2(cx+1)^4}{3c(1-c^2x^2)^2}-\frac{20(cx+1)}{3c(1-c^2x^2)}+\frac{5\arcsin(cx)}{c\sqrt{1-c^2x^2}}-\frac{5}{3c}\right)dx+\frac{5\arcsin(cx)(a+b\arccos(cx))}{c}+\frac{2(cx+1)^4(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}\right.}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$\downarrow$ 2009

$$\frac{d^5\left(1-c^2x^2\right)^{5/2}\left(\frac{5\arcsin(cx)(a+b\arccos(cx))}{c}+\frac{2(cx+1)^4(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}-\frac{20(cx+1)(a+b\arccos(cx))}{3c\sqrt{1-c^2x^2}}-\frac{5\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c}\right.}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

input
```
Int[((d + c*d*x)^(5/2)*(a + b*ArcCos[c*x]))/(f - c*f*x)^(5/2),x]
```

output
```
(d^5*(1 - c^2*x^2)^(5/2)*((2*(1 + c*x)^4*(a + b*ArcCos[c*x]))/(3*c*(1 - c^
2*x^2)^(3/2)) - (20*(1 + c*x)*(a + b*ArcCos[c*x]))/(3*c*Sqrt[1 - c^2*x^2])
 - (5*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(3*c) + (5*(a + b*ArcCos[c*x]
)*ArcSin[c*x])/c + b*c*(-(x/c) + 8/(3*c^2*(1 - c*x)) + (5*ArcSin[c*x]^2)/(
2*c^2) + (28*Log[1 - c*x])/(3*c^2))))/((d + c*d*x)^(5/2)*(f - c*f*x)^(5/2)
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5261
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p,
 x]}, Simp[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[1/Sqrt[1 - c^2*x^2]
   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IG
tQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 18.77 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.99

method	result
default	$\frac{\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\,\left(46\arccos(cx)\sqrt{-c^2x^2+1}\,b-16b+46\sqrt{-c^2x^2+1}\,a+56i\arccos(cx)b\,c^2x^2-68\sqrt{-c^2x^2+1}\,acx-\right.}{}$

input
```
int((c*d*x+d)^(5/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/6*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^3*x^3-3*c^2
*x^2+3*c*x-1)/f^3/c/(c*x+1)*(46*arccos(c*x)*(-c^2*x^2+1)^(1/2)*b-16*b+46*(
-c^2*x^2+1)^(1/2)*a-68*(-c^2*x^2+1)^(1/2)*a*c*x-60*arccos(c*x)*a*c*x+6*(-c
^2*x^2+1)^(1/2)*arccos(c*x)*b*c^2*x^2-56*I*a*c^2*x^2+112*I*a*c*x+15*arccos
(c*x)^2*b*c^2*x^2+30*arccos(c*x)*a*c^2*x^2+6*(-c^2*x^2+1)^(1/2)*a*c^2*x^2+
22*c*x*b-112*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)*b*c^2*x^2+224*ln(I*(-c^2*x^2+1
)^(1/2)+c*x-1)*b*c*x+6*b*c^3*x^3-12*x^2*c^2*b-30*b*c*x*arccos(c*x)^2-112*I
*arccos(c*x)*b*c*x+56*I*arccos(c*x)*b*c^2*x^2-68*arccos(c*x)*(-c^2*x^2+1)^
(1/2)*b*c*x-112*b*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)-56*I*a+15*arccos(c*x)^2*b
+30*arccos(c*x)*a+56*I*b*arccos(c*x))*d^2
```

Fricas [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx = \int \frac{(cdx+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{(-cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(5/2),x, algorithm=
"fricas")
```

output
```
integral(-(a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^
2*x + b*d^2)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*f*x + f)/(c^3*f^3*x^3 -
3*c^2*f^3*x^2 + 3*c*f^3*x - f^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+d)**(5/2)*(a+b*acos(c*x))/(-c*f*x+f)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx = \int \frac{(cdx+d)^{\frac{5}{2}}(b\arccos(cx)+a)}{(-cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(5/2),x, algorithm=
"maxima")
```

output
```
-1/3*(3*(-c^2*d*f*x^2 + d*f)^(5/2)/(c^5*f^5*x^4 - 4*c^4*f^5*x^3 + 6*c^3*f^
5*x^2 - 4*c^2*f^5*x + c*f^5) + 5*(-c^2*d*f*x^2 + d*f)^(3/2)*d/(c^4*f^4*x^3
 - 3*c^3*f^4*x^2 + 3*c^2*f^4*x - c*f^4) - 10*sqrt(-c^2*d*f*x^2 + d*f)*d^2/
(c^3*f^3*x^2 - 2*c^2*f^3*x + c*f^3) - 35*sqrt(-c^2*d*f*x^2 + d*f)*d^2/(c^2
*f^3*x - c*f^3) - 15*d^3*arcsin(c*x)/(c*f^3*sqrt(d/f)))*a + b*sqrt(d)*inte
grate((c^2*d^2*x^2 + 2*c*d^2*x + d^2)*sqrt(c*x + 1)*arctan2(sqrt(c*x + 1)*
sqrt(-c*x + 1), c*x)/((c^2*f^2*x^2 - 2*c*f^2*x + f^2)*sqrt(-c*x + 1)), x)/
sqrt(f)
```

Giac [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx=\int \frac{(cdx+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)}{(-cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((c*d*x + d)^(5/2)*(b*arccos(c*x) + a)/(-c*f*x + f)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,{(d+c\,d\,x)}^{5/2}}{{(f-c\,f\,x)}^{5/2}}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + c*d*x)^(5/2))/(f - c*f*x)^(5/2),x)
```

output
```
int(((a + b*acos(c*x))*(d + c*d*x)^(5/2))/(f - c*f*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx = \frac{\sqrt{d}\,d^2\left(-30\sqrt{-cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)acx+30\sqrt{-cx+1}\,asin\left(\frac{\sqrt{-cx}}{\sqrt{2}}\right.\right.}{}$$

input
```
int((c*d*x+d)^(5/2)*(a+b*acos(c*x))/(-c*f*x+f)^(5/2),x)
```

output
```
(sqrt(d)*d**2*( - 30*sqrt( - c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a*c*x
 + 30*sqrt( - c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a + 3*sqrt( - c*x +
1)*int((sqrt(c*x + 1)*acos(c*x)*x**2)/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt
( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*b*c**4*x - 3*sqrt( - c*x + 1)*int(
(sqrt(c*x + 1)*acos(c*x)*x**2)/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x
 + 1)*c*x + sqrt( - c*x + 1)),x)*b*c**3 + 6*sqrt( - c*x + 1)*int((sqrt(c*x
 + 1)*acos(c*x)*x)/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x +
sqrt( - c*x + 1)),x)*b*c**3*x - 6*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos
(c*x)*x)/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*
x + 1)),x)*b*c**2 + 3*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x))/(sqrt
( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*b*c
**2*x - 3*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x))/(sqrt( - c*x + 1)
*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*b*c + 3*sqrt(c*
x + 1)*a*c**2*x**2 - 34*sqrt(c*x + 1)*a*c*x + 23*sqrt(c*x + 1)*a))/(3*sqrt
(f)*sqrt( - c*x + 1)*c*f**2*(c*x - 1))
```

3.537 $\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 324

$$\begin{aligned}\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx &= -\frac{4bd^4(1-c^2x^2)^{5/2}}{3c(1-cx)(d+cdx)^{5/2}(f-cfx)^{5/2}} \\ &- \frac{bd^4(1-c^2x^2)^{5/2}\arccos(cx)^2}{2c(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{2d^4(1+cx)^3(1-c^2x^2)(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}} \\ &- \frac{2d^4(1+cx)(1-c^2x^2)^2(a+b\arccos(cx))}{c(d+cdx)^{5/2}(f-cfx)^{5/2}} \\ &+ \frac{d^4(1-c^2x^2)^{5/2}\arccos(cx)(a+b\arccos(cx))}{c(d+cdx)^{5/2}(f-cfx)^{5/2}} - \frac{8bd^4(1-c^2x^2)^{5/2}\log(1-cx)}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}}\end{aligned}$$

output
```
-4/3*b*d^4*(-c^2*x^2+1)^(5/2)/c/(-c*x+1)/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)-
1/2*b*d^4*(-c^2*x^2+1)^(5/2)*arccos(c*x)^2/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5
/2)+2/3*d^4*(c*x+1)^3*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c
*f*x+f)^(5/2)-2*d^4*(c*x+1)*(-c^2*x^2+1)^2*(a+b*arccos(c*x))/c/(c*d*x+d)^(
5/2)/(-c*f*x+f)^(5/2)+d^4*(-c^2*x^2+1)^(5/2)*arccos(c*x)*(a+b*arccos(c*x))
/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)-8/3*b*d^4*(-c^2*x^2+1)^(5/2)*ln(-c*x+1
)/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)
```

Mathematica [A] (verified)

Time = 4.36 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.73

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx = -\frac{ad^{3/2}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{f-cfx}}{\sqrt{d}\sqrt{f}(-1+c^2x^2)}\right)}{cf^{5/2}} + \frac{2d\sqrt{d+cdx}\sqrt{f-cfx}\left(-4b(-1+2cx)\arccos(cx)\sin^2\left(\frac{1}{2}\arccos(cx)\right)+3b\arccos(cx)^2\sin^4\left(\frac{1}{2}\arccos(cx)\right)\right.}{}$$

input
```
Integrate[((d + c*d*x)^(3/2)*(a + b*ArcCos[c*x]))/(f - c*f*x)^(5/2),x]
```

output
```
-((a*d^(3/2)*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[f - c*f*x])/(Sqrt[d]*Sqrt[f]
*(-1 + c^2*x^2))])/(c*f^(5/2))) + (2*d*Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(-4
*b*(-1 + 2*c*x)*ArcCos[c*x]*Sin[ArcCos[c*x]/2]^2 + 3*b*ArcCos[c*x]^2*Sin[A
rcCos[c*x]/2]^4*Tan[ArcCos[c*x]/2] + 2*(a - 3*a*c*x + 2*a*c^2*x^2 - 16*b*L
og[Sin[ArcCos[c*x]/2]]*Sin[ArcCos[c*x]/2]^4*Tan[ArcCos[c*x]/2] - b*Sqrt[1
- c^2*x^2]*Tan[ArcCos[c*x]/2]^2)))/(3*c*f^3*(-1 + c*x)^3)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5261, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{5/2}\int \frac{d^4(cx+1)^4(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^4\left(1-c^2x^2\right)^{5/2}\int \frac{(cx+1)^4(a+b\arccos(cx))}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$\Big\downarrow$ 5261

$$\frac{d^4\left(1-c^2x^2\right)^{5/2}\left(bc\int\left(\frac{2(cx+1)^3}{3c(1-c^2x^2)^2}-\frac{2(cx+1)}{c(1-c^2x^2)}+\frac{\arcsin(cx)}{c\sqrt{1-c^2x^2}}\right)dx+\frac{\arcsin(cx)(a+b\arccos(cx))}{c}+\frac{2(cx+1)^3(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}-\right.}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$\Big\downarrow$ 2009

$$\frac{d^4\left(1-c^2x^2\right)^{5/2}\left(\frac{\arcsin(cx)(a+b\arccos(cx))}{c}+\frac{2(cx+1)^3(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}-\frac{2(cx+1)(a+b\arccos(cx))}{c\sqrt{1-c^2x^2}}+bc\left(\frac{\arcsin(cx)^2}{2c^2}+\frac{4}{3c^2(1-cx}\right.\right.}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

input
```
Int[((d + c*d*x)^(3/2)*(a + b*ArcCos[c*x]))/(f - c*f*x)^(5/2),x]
```

output
```
(d^4*(1 - c^2*x^2)^(5/2)*((2*(1 + c*x)^3*(a + b*ArcCos[c*x]))/(3*c*(1 - c^
2*x^2)^(3/2)) - (2*(1 + c*x)*(a + b*ArcCos[c*x]))/(c*Sqrt[1 - c^2*x^2]) +
((a + b*ArcCos[c*x])*ArcSin[c*x])/c + b*c*(4/(3*c^2*(1 - c*x)) + ArcSin[c*
x]^2/(2*c^2) + (8*Log[1 - c*x])/(3*c^2))))/((d + c*d*x)^(5/2)*(f - c*f*x)^
(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5261
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p,
 x]}, Simp[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[1/Sqrt[1 - c^2*x^2]
   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IG
tQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 18.48 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.07

method	result
default	$\frac{\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^2x^2+1}\left(16i\arccos(cx)b+3\arccos(cx)^2 b\,c^2x^2-16ia\,c^2x^2+6\arccos(cx)a\,c^2x^2-32\ln\left(i\sqrt{-c^2x^2+1}+cx\right.\right.}{}$

input
```
int((c*d*x+d)^(3/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/6*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^3*x^3-3*c^2
*x^2+3*c*x-1)/f^3/c/(c*x+1)*(16*I*arccos(c*x)*b+3*arccos(c*x)^2*b*c^2*x^2-
16*I*a*c^2*x^2+6*arccos(c*x)*a*c^2*x^2-32*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)*b
*c^2*x^2+32*I*a*c*x-16*arccos(c*x)*(-c^2*x^2+1)^(1/2)*b*c*x-6*b*c*x*arccos
(c*x)^2-32*I*arccos(c*x)*b*c*x-16*(-c^2*x^2+1)^(1/2)*a*c*x-12*arccos(c*x)*
a*c*x+64*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)*b*c*x-16*I*a+8*arccos(c*x)*(-c^2*x
^2+1)^(1/2)*b+3*arccos(c*x)^2*b+8*c*x*b+16*I*arccos(c*x)*b*c^2*x^2+8*(-c^2
*x^2+1)^(1/2)*a+6*arccos(c*x)*a-32*b*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)-8*b)*d
```

Fricas [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx=\int \frac{(cdx+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)}{(-cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(5/2),x, algorithm=
"fricas")
```

output
```
integral(-(a*c*d*x + a*d + (b*c*d*x + b*d)*arccos(c*x))*sqrt(c*d*x + d)*sq
rt(-c*f*x + f)/(c^3*f^3*x^3 - 3*c^2*f^3*x^2 + 3*c*f^3*x - f^3), x)
```

Sympy [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx=\int \frac{(d(cx+1))^{\frac{3}{2}}\,(a+b\operatorname{acos}{(cx)})}{(-f\,(cx-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)**(3/2)*(a+b*acos(c*x))/(-c*f*x+f)**(5/2),x)
```

output
```
Integral((d*(c*x + 1))**(3/2)*(a + b*acos(c*x))/(-f*(c*x - 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx=\int \frac{(cdx+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)}{(-cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(5/2),x, algorithm=
"maxima")
```

output
```
-b*sqrt(d)*sqrt(f)*integrate((c*d*x + d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arct
an2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^3*f^3*x^3 - 3*c^2*f^3*x^2 + 3*c*
f^3*x - f^3), x) - 1/3*a*((-c^2*d*f*x^2 + d*f)^(3/2)/(c^4*f^4*x^3 - 3*c^3*
f^4*x^2 + 3*c^2*f^4*x - c*f^4) - 2*sqrt(-c^2*d*f*x^2 + d*f)*d/(c^3*f^3*x^2
 - 2*c^2*f^3*x + c*f^3) - 7*sqrt(-c^2*d*f*x^2 + d*f)*d/(c^2*f^3*x - c*f^3)
 - 3*d^2*arcsin(c*x)/(c*f^3*sqrt(d/f)))
```

Giac [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx=\int \frac{(cdx+d)^{\frac{3}{2}}(b\arccos(cx)+a)}{(-cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((c*d*x + d)^(3/2)*(b*arccos(c*x) + a)/(-c*f*x + f)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d+c\,d\,x)^{3/2}}{(f-c\,f\,x)^{5/2}}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + c*d*x)^(3/2))/(f - c*f*x)^(5/2),x)
```

output
```
int(((a + b*acos(c*x))*(d + c*d*x)^(3/2))/(f - c*f*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx=\frac{\sqrt{d}\,d\left(-6\sqrt{-cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)acx+6\sqrt{-cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)\right.}{}$$

input
```
int((c*d*x+d)^(3/2)*(a+b*acos(c*x))/(-c*f*x+f)^(5/2),x)
```

output
```
(sqrt(d)*d*( - 6*sqrt( - c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a*c*x + 6
*sqrt( - c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a + 3*sqrt( - c*x + 1)*in
t((sqrt(c*x + 1)*acos(c*x)*x)/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x
+ 1)*c*x + sqrt( - c*x + 1)),x)*b*c**3*x - 3*sqrt( - c*x + 1)*int((sqrt(c*
x + 1)*acos(c*x)*x)/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x +
 sqrt( - c*x + 1)),x)*b*c**2 + 3*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(
c*x))/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x +
 1)),x)*b*c**2*x - 3*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x))/(sqrt(
 - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*b*c
- 8*sqrt(c*x + 1)*a*c*x + 4*sqrt(c*x + 1)*a))/(3*sqrt(f)*sqrt( - c*x + 1)*
c*f**2*(c*x - 1))
```

3.538 $\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 164

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx = -\frac{2bd^3(1-c^2x^2)^{5/2}}{3c(1-cx)(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{d^3(1+cx)^3(1-c^2x^2)(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}} - \frac{bd^3(1-c^2x^2)^{5/2}\log(1-cx)}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}}$$

output
```
-2/3*b*d^3*(-c^2*x^2+1)^(5/2)/c/(-c*x+1)/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)+
1/3*d^3*(c*x+1)^3*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*f*x
+f)^(5/2)-1/3*b*d^3*(-c^2*x^2+1)^(5/2)*ln(-c*x+1)/c/(c*d*x+d)^(5/2)/(-c*f*
x+f)^(5/2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx = \frac{\sqrt{d+cdx}\sqrt{f-cfx}\left(-\left((1+cx)\left(-b+bcx-a\sqrt{1-c^2x^2}\right)\right)+b(1+c\cdots}{3cf^3(-1+cx)^2\sqrt{1-\cdots}}$$

input
```
Integrate[(Sqrt[d + c*d*x]*(a + b*ArcCos[c*x]))/(f - c*f*x)^(5/2),x]
```

output
```
(Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(-((1 + c*x)*(-b + b*c*x - a*Sqrt[1 - c^2
*x^2])) + b*(1 + c*x)*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + b*(-1 + c*x)^2*Log[f
 - c*f*x]))/(3*c*f^3*(-1 + c*x)^2*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {5179, 27, 5261, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cdx+d}(a+b\arccos(cx))}{(f-cfx)^{5/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\int \frac{d^3(cx+1)^3(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3\left(1-c^2x^2\right)^{5/2}\int \frac{(cx+1)^3(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{5261}$$

$$\frac{d^3\left(1-c^2x^2\right)^{5/2}\left(bc\int \frac{(cx+1)^3}{3c\left(1-c^2x^2\right)^2}dx+\frac{(cx+1)^3(a+b\arccos(cx))}{3c\left(1-c^2x^2\right)^{3/2}}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3\left(1-c^2x^2\right)^{5/2}\left(\frac{1}{3}b\int \frac{(cx+1)^3}{\left(1-c^2x^2\right)^2}dx+\frac{(cx+1)^3(a+b\arccos(cx))}{3c\left(1-c^2x^2\right)^{3/2}}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{456}$$

$$\frac{d^3\left(1-c^2x^2\right)^{5/2}\left(\frac{1}{3}b\int \frac{cx+1}{(1-cx)^2}dx+\frac{(cx+1)^3(a+b\arccos(cx))}{3c\left(1-c^2x^2\right)^{3/2}}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{49}$$

$$\frac{d^3\left(1-c^2x^2\right)^{5/2}\left(\frac{1}{3}b\int\left(\frac{1}{cx-1}+\frac{2}{(cx-1)^2}\right)dx+\frac{(cx+1)^3(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{d^3\left(1-c^2x^2\right)^{5/2}\left(\frac{(cx+1)^3(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}+\frac{1}{3}b\left(\frac{2}{c(1-cx)}+\frac{\log(1-cx)}{c}\right)\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

input
```
Int[(Sqrt[d + c*d*x]*(a + b*ArcCos[c*x]))/(f - c*f*x)^(5/2),x]
```

output
```
(d^3*(1 - c^2*x^2)^(5/2)*(((1 + c*x)^3*(a + b*ArcCos[c*x]))/(3*c*(1 - c^2*
x^2)^(3/2)) + (b*(2/(c*(1 - c*x)) + Log[1 - c*x]/c))/3))/((d + c*d*x)^(5/2
)*(f - c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5261
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p,
 x]}, Simp[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[1/Sqrt[1 - c^2*x^2]
   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IG
tQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.97 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.75

method	result
default	$a\left(\frac{\sqrt{cdx+d}}{cf(-cfx+f)^{\frac{3}{2}}}-d\left(\frac{\sqrt{cdx+d}}{3fdc(-cfx+f)^{\frac{3}{2}}}+\frac{\sqrt{cdx+d}}{3dc\,f^{2}\sqrt{-cfx+f}}\right)\right)-\frac{b\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\left(c^{2}x^{2}+2cx+1+i\sqrt{-c^{2}x^{2}+1}\right.}{}$
parts	$a\left(\frac{\sqrt{cdx+d}}{cf(-cfx+f)^{\frac{3}{2}}}-d\left(\frac{\sqrt{cdx+d}}{3fdc(-cfx+f)^{\frac{3}{2}}}+\frac{\sqrt{cdx+d}}{3dc\,f^{2}\sqrt{-cfx+f}}\right)\right)-\frac{b\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\left(c^{2}x^{2}+2cx+1+i\sqrt{-c^{2}x^{2}+1}\right.}{}$

input
```
int((c*d*x+d)^(1/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
a*(1/c/f*(c*d*x+d)^(1/2)/(-c*f*x+f)^(3/2)-d*(1/3/f/d/c/(-c*f*x+f)^(3/2)*(c
*d*x+d)^(1/2)+1/3/d/c/f^2/(-c*f*x+f)^(1/2)*(c*d*x+d)^(1/2)))-1/3*b*(d*(c*x
+1))^(1/2)*(-f*(c*x-1))^(1/2)*(c^2*x^2+2*c*x+1+I*(-c^2*x^2+1)^(1/2)*c*x-I*
(-c^2*x^2+1)^(1/2))*(I*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)*x^3*c^3-3*I*ln(I*(-c
^2*x^2+1)^(1/2)+c*x-1)*x^2*c^2+ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)*(-c^2*x^2+1)
^(1/2)*x^2*c^2-I*c^2*x^2-3*c^2*x^2*arccos(c*x)+3*I*ln(I*(-c^2*x^2+1)^(1/2)
+c*x-1)*x*c+2*I*c*x-c*x*(-c^2*x^2+1)^(1/2)-I*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1
)-ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)*(-c^2*x^2+1)^(1/2)-I-(-c^2*x^2+1)^(1/2)-a
rccos(c*x))/(3*c^4*x^4-6*c^3*x^3+4*c^2*x^2-2*c*x+1)/f^3/c/(c*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.17

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx = \left[\frac{(bc^3fx^3-bc^2fx^2-bcfx+bf)\sqrt{\frac{d}{f}}\log\left(\frac{c^6dx^6-4\,c^5dx^5+5\,c^4dx^4-4\,c^2dx^2+4}{}\right.}{}\right.$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(5/2),x, algorithm=
"fricas")
```

output
```
[1/6*((b*c^3*f*x^3 - b*c^2*f*x^2 - b*c*f*x + b*f)*sqrt(d/f)*log((c^6*d*x^6
 - 4*c^5*d*x^5 + 5*c^4*d*x^4 - 4*c^2*d*x^2 + 4*c*d*x - (c^4*x^4 - 4*c^3*x^
3 + 6*c^2*x^2 - 4*c*x)*sqrt(-c^2*x^2 + 1)*sqrt(c*d*x + d)*sqrt(-c*f*x + f)
*sqrt(d/f) - 2*d)/(c^4*x^4 - 2*c^3*x^3 + 2*c*x - 1)) + 2*(a*c^2*x^2 + 2*sq
rt(-c^2*x^2 + 1)*b*c*x + 2*a*c*x + (b*c^2*x^2 + 2*b*c*x + b)*arccos(c*x) +
 a)*sqrt(c*d*x + d)*sqrt(-c*f*x + f))/(c^4*f^3*x^3 - c^3*f^3*x^2 - c^2*f^3
*x + c*f^3), 1/3*((b*c^3*f*x^3 - b*c^2*f*x^2 - b*c*f*x + b*f)*sqrt(-d/f)*a
rctan((c^2*x^2 - 2*c*x + 2)*sqrt(-c^2*x^2 + 1)*sqrt(c*d*x + d)*sqrt(-c*f*x
 + f)*sqrt(-d/f)/(c^4*d*x^4 - 2*c^3*d*x^3 - c^2*d*x^2 + 2*c*d*x)) + (a*c^2
*x^2 + 2*sqrt(-c^2*x^2 + 1)*b*c*x + 2*a*c*x + (b*c^2*x^2 + 2*b*c*x + b)*ar
ccos(c*x) + a)*sqrt(c*d*x + d)*sqrt(-c*f*x + f))/(c^4*f^3*x^3 - c^3*f^3*x^
2 - c^2*f^3*x + c*f^3)]
```

Sympy [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx = \int \frac{\sqrt{d\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))}{(-f\,(cx-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)**(1/2)*(a+b*acos(c*x))/(-c*f*x+f)**(5/2),x)
```

output
```
Integral(sqrt(d*(c*x + 1))*(a + b*acos(c*x))/(-f*(c*x - 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx = -\frac{1}{3}\,bc\left(\frac{2\sqrt{d}}{c^3f^{\frac{5}{2}}x-c^2f^{\frac{5}{2}}}-\frac{\sqrt{d}\log\left(cx-1\right)}{c^2f^{\frac{5}{2}}}\right) + \frac{1}{3}\,b\left(\frac{2\sqrt{-c^2dfx^2+df}}{c^3f^3x^2-2\,c^2f^3x+cf^3}+\frac{\sqrt{-c^2dfx^2+df}}{c^2f^3x-cf^3}\right)\arccos\left(cx\right) + \frac{1}{3}\,a\left(\frac{2\sqrt{-c^2dfx^2+df}}{c^3f^3x^2-2\,c^2f^3x+cf^3}+\frac{\sqrt{-c^2dfx^2+df}}{c^2f^3x-cf^3}\right)$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(5/2),x, algorithm=
"maxima")
```

output
```
-1/3*b*c*(2*sqrt(d)/(c^3*f^(5/2)*x - c^2*f^(5/2)) - sqrt(d)*log(c*x - 1)/(
c^2*f^(5/2))) + 1/3*b*(2*sqrt(-c^2*d*f*x^2 + d*f)/(c^3*f^3*x^2 - 2*c^2*f^3
*x + c*f^3) + sqrt(-c^2*d*f*x^2 + d*f)/(c^2*f^3*x - c*f^3))*arccos(c*x) +
1/3*a*(2*sqrt(-c^2*d*f*x^2 + d*f)/(c^3*f^3*x^2 - 2*c^2*f^3*x + c*f^3) + sq
rt(-c^2*d*f*x^2 + d*f)/(c^2*f^3*x - c*f^3))
```

Giac [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx=\int \frac{\sqrt{cdx+d}(b\arccos{(cx)}+a)}{(-cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))/(-c*f*x+f)^(5/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(c*d*x + d)*(b*arccos(c*x) + a)/(-c*f*x + f)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d+c\,d\,x}}{(f-c\,f\,x)^{5/2}}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + c*d*x)^(1/2))/(f - c*f*x)^(5/2),x)
```

output
```
int(((a + b*acos(c*x))*(d + c*d*x)^(1/2))/(f - c*f*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))}{(f-cfx)^{5/2}}\,dx=\frac{\sqrt{d}\left(3\sqrt{-cx+1}\left(\int \frac{\sqrt{cx+1}\,acos(cx)}{\sqrt{-cx+1}\,c^2x^2-2\sqrt{-cx+1}\,cx+\sqrt{-cx+1}}dx\right)b\,c^2x-3\sqrt{-cx}}{3\sqrt{f}\,\sqrt{-cx}}$$

input
```
int((c*d*x+d)^(1/2)*(a+b*acos(c*x))/(-c*f*x+f)^(5/2),x)
```

output
```
(sqrt(d)*(3*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x))/(sqrt( - c*x +
1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*b*c**2*x - 3*
sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x))/(sqrt( - c*x + 1)*c**2*x**2
 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*b*c - sqrt(c*x + 1)*a*c*x
 - sqrt(c*x + 1)*a))/(3*sqrt(f)*sqrt( - c*x + 1)*c*f**2*(c*x - 1))
```

3.539 $\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 265

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{5/2}}\,dx = -\frac{bd^2(1-c^2x^2)^{5/2}}{3c(1-cx)(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{2d^2(1+cx)\left(1-c^2x^2\right)(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{d^2x(1-c^2x^2)^2\,(a+b\arccos(cx))}{3(d+cdx)^{5/2}(f-cfx)^{5/2}} - \frac{bd^2(1-c^2x^2)^{5/2}\operatorname{arctanh}(cx)}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{bd^2(1-c^2x^2)^{5/2}\log\left(1-c^2x^2\right)}{6c(d+cdx)^{5/2}(f-cfx)^{5/2}}$$

output
```
-1/3*b*d^2*(-c^2*x^2+1)^(5/2)/c/(-c*x+1)/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)+
2/3*d^2*(c*x+1)*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*f*x+f
)^(5/2)+1/3*d^2*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x
+f)^(5/2)-1/3*b*d^2*(-c^2*x^2+1)^(5/2)*arctanh(c*x)/c/(c*d*x+d)^(5/2)/(-c*
f*x+f)^(5/2)+1/6*b*d^2*(-c^2*x^2+1)^(5/2)*ln(-c^2*x^2+1)/c/(c*d*x+d)^(5/2)
/(-c*f*x+f)^(5/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.49

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{5/2}}\,dx =$$
$$-\frac{\sqrt{d+cdx}\sqrt{f-cfx}\left(-\left((-2+cx)\left(-b+bcx-a\sqrt{1-c^2x^2}\right)\right)+b(-2+cx)\sqrt{1-c^2x^2}\arccos(cx)+b(-\right.}{3cdf^3(-1+cx)^2\sqrt{1-c^2x^2}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(Sqrt[d + c*d*x]*(f - c*f*x)^(5/2)),x]
```

output
```
-1/3*(Sqrt[d + c*d*x]*Sqrt[f - c*f*x]*(-((-2 + c*x)*(-b + b*c*x - a*Sqrt[1
 - c^2*x^2])) + b*(-2 + c*x)*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + b*(-1 + c*x)^
2*Log[f - c*f*x]))/(c*d*f^3*(-1 + c*x)^2*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5261, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{\sqrt{cdx+d}(f-cfx)^{5/2}}\,dx$$

$$\Big\downarrow \; 5179$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\int \frac{d^2(cx+1)^2(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{d^2\left(1-c^2x^2\right)^{5/2}\int \frac{(cx+1)^2(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \; 5261$$

$$\frac{d^2\left(1-c^2x^2\right)^{5/2}\left(bc\int\left(\frac{x}{3(1-c^2x^2)}+\frac{2(cx+1)}{3c(1-c^2x^2)^2}\right)dx+\frac{x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{2(cx+1)(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\left(1-c^2x^2\right)^{5/2}\left(\frac{x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{2(cx+1)(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}+bc\left(\frac{\text{arctanh}(cx)}{3c^2}+\frac{cx+1}{3c^2(1-c^2x^2)}-\frac{\log(1-c^2x^2)}{6c^2}\right)\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

input
```
Int[(a + b*ArcCos[c*x])/(Sqrt[d + c*d*x]*(f - c*f*x)^(5/2)),x]
```

output
```
(d^2*(1 - c^2*x^2)^(5/2)*((2*(1 + c*x)*(a + b*ArcCos[c*x]))/(3*c*(1 - c^2*
x^2)^(3/2)) + (x*(a + b*ArcCos[c*x]))/(3*Sqrt[1 - c^2*x^2]) + b*c*((1 + c*
x)/(3*c^2*(1 - c^2*x^2)) + ArcTanh[c*x]/(3*c^2) - Log[1 - c^2*x^2]/(6*c^2)
)))/((d + c*d*x)^(5/2)*(f - c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5261
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p,
 x]}, Simp[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[1/Sqrt[1 - c^2*x^2]
   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IG
tQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.93 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.04

method	result
default	$a\left(\frac{\sqrt{cdx+d}}{3fdc(-cfx+f)^{\frac{3}{2}}}+\frac{\sqrt{cdx+d}}{3dc\,f^{2}\sqrt{-cfx+f}}\right)+\frac{b\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^{2}x^{2}+1}\,\left(-i\arccos(cx)c^{2}x^{2}+2\ln\left(i\sqrt{-c^{2}x^{2}+1}+cx\cdots\right.\right.}{\cdots}$
parts	$a\left(\frac{\sqrt{cdx+d}}{3fdc(-cfx+f)^{\frac{3}{2}}}+\frac{\sqrt{cdx+d}}{3dc\,f^{2}\sqrt{-cfx+f}}\right)-\frac{b\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^{2}x^{2}+1}\,\left(i\arccos(cx)c^{2}x^{2}-2\ln\left(i\sqrt{-c^{2}x^{2}+1}+cx-\cdots\right.\right.}{\cdots}$

input
```
int((a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
a*(1/3/f/d/c/(-c*f*x+f)^(3/2)*(c*d*x+d)^(1/2)+1/3/d/c/f^2/(-c*f*x+f)^(1/2)
*(c*d*x+d)^(1/2))+1/3*b*(d*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^
(1/2)*(-I*arccos(c*x)*x^2*c^2+2*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)*x^2*c^2+(-c
^2*x^2+1)^(1/2)*arccos(c*x)*x*c+2*I*arccos(c*x)*x*c-4*ln(I*(-c^2*x^2+1)^(1
/2)+c*x-1)*x*c-2*arccos(c*x)*(-c^2*x^2+1)^(1/2)-I*arccos(c*x)+c*x+2*ln(I*(
-c^2*x^2+1)^(1/2)+c*x-1)-1)/f^3/d/(c^4*x^4-2*c^3*x^3+2*c*x-1)/c
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.99

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{5/2}}\,dx = \left[\frac{(bc^3x^3-bc^2x^2-bcx+b)\sqrt{df}\log\left(\frac{c^6dfx^6-4\,c^5dfx^5+5\,c^4dfx^4-4\,c^2dfx^2+4\,cdfx+(c^4x}{c^4x^4}\right.}{}\right.$$

$$-\frac{(bc^3x^3-bc^2x^2-bcx+b)\sqrt{-df}\arctan\left(\frac{(c^2x^2-2\,cx+2)\sqrt{-c^2x^2+1}\sqrt{cdx+d}\sqrt{-cfx+f}\sqrt{-df}}{c^4dfx^4-2\,c^3dfx^3-c^2dfx^2+2\,cdfx}\right)+\left(ac^2x^2-\sqrt{-c^2x^2+}\right.}{3\,(c^4df^3x^3-c^3df^3x^2-c^2df^3x+cd}$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(5/2),x, algorithm=
"fricas")
```

output
```
[1/6*((b*c^3*x^3 - b*c^2*x^2 - b*c*x + b)*sqrt(d*f)*log((c^6*d*f*x^6 - 4*c
^5*d*f*x^5 + 5*c^4*d*f*x^4 - 4*c^2*d*f*x^2 + 4*c*d*f*x + (c^4*x^4 - 4*c^3*
x^3 + 6*c^2*x^2 - 4*c*x)*sqrt(-c^2*x^2 + 1)*sqrt(c*d*x + d)*sqrt(-c*f*x +
f)*sqrt(d*f) - 2*d*f)/(c^4*x^4 - 2*c^3*x^3 + 2*c*x - 1)) - 2*(a*c^2*x^2 -
sqrt(-c^2*x^2 + 1)*b*c*x - a*c*x + (b*c^2*x^2 - b*c*x - 2*b)*arccos(c*x) -
 2*a)*sqrt(c*d*x + d)*sqrt(-c*f*x + f))/(c^4*d*f^3*x^3 - c^3*d*f^3*x^2 - c
^2*d*f^3*x + c*d*f^3), -1/3*((b*c^3*x^3 - b*c^2*x^2 - b*c*x + b)*sqrt(-d*f
)*arctan((c^2*x^2 - 2*c*x + 2)*sqrt(-c^2*x^2 + 1)*sqrt(c*d*x + d)*sqrt(-c*
f*x + f)*sqrt(-d*f)/(c^4*d*f*x^4 - 2*c^3*d*f*x^3 - c^2*d*f*x^2 + 2*c*d*f*x
)) + (a*c^2*x^2 - sqrt(-c^2*x^2 + 1)*b*c*x - a*c*x + (b*c^2*x^2 - b*c*x -
2*b)*arccos(c*x) - 2*a)*sqrt(c*d*x + d)*sqrt(-c*f*x + f))/(c^4*d*f^3*x^3 -
 c^3*d*f^3*x^2 - c^2*d*f^3*x + c*d*f^3)]
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{5/2}}\,dx = \int \frac{a+b\operatorname{acos}(cx)}{\sqrt{d\,(cx+1)}\,(-f\,(cx-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(c*d*x+d)**(1/2)/(-c*f*x+f)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))/(sqrt(d*(c*x + 1))*(-f*(c*x - 1))**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.86

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{5/2}}\,dx = -\frac{1}{3}\,bc\left(\frac{1}{c^3\sqrt{d}f^{\frac{5}{2}}x-c^2\sqrt{d}f^{\frac{5}{2}}}+\frac{\log\left(cx-1\right)}{c^2\sqrt{d}f^{\frac{5}{2}}}\right) + \frac{1}{3}\,b\left(\frac{\sqrt{-c^2dfx^2+df}}{c^3df^3x^2-2\,c^2df^3x+cdf^3}-\frac{\sqrt{-c^2dfx^2+df}}{c^2df^3x-cdf^3}\right)\arccos\left(cx\right) + \frac{1}{3}\,a\left(\frac{\sqrt{-c^2dfx^2+df}}{c^3df^3x^2-2\,c^2df^3x+cdf^3}-\frac{\sqrt{-c^2dfx^2+df}}{c^2df^3x-cdf^3}\right)$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(5/2),x, algorithm=
"maxima")
```

output
```
-1/3*b*c*(1/(c^3*sqrt(d)*f^(5/2)*x - c^2*sqrt(d)*f^(5/2)) + log(c*x - 1)/(
c^2*sqrt(d)*f^(5/2))) + 1/3*b*(sqrt(-c^2*d*f*x^2 + d*f)/(c^3*d*f^3*x^2 - 2
*c^2*d*f^3*x + c*d*f^3) - sqrt(-c^2*d*f*x^2 + d*f)/(c^2*d*f^3*x - c*d*f^3)
)*arccos(c*x) + 1/3*a*(sqrt(-c^2*d*f*x^2 + d*f)/(c^3*d*f^3*x^2 - 2*c^2*d*f
^3*x + c*d*f^3) - sqrt(-c^2*d*f*x^2 + d*f)/(c^2*d*f^3*x - c*d*f^3))
```

Giac [F]

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{5/2}}\,dx = \int \frac{b\arccos\left(cx\right)+a}{\sqrt{cdx+d}(-cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((b*arccos(c*x) + a)/(sqrt(c*d*x + d)*(-c*f*x + f)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{\sqrt{d+c\,d\,x}\,\left(f-c\,f\,x\right)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))/((d + c*d*x)^(1/2)*(f - c*f*x)^(5/2)),x)
```

output
```
int((a + b*acos(c*x))/((d + c*d*x)^(1/2)*(f - c*f*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+cdx}(f-cfx)^{5/2}}\,dx=\frac{3\sqrt{-cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2-2\sqrt{cx+1}\,\sqrt{-cx+1}\,cx+\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)b\,c^2x-3}{3\sqrt{f}}$$

input
```
int((a+b*acos(c*x))/(c*d*x+d)^(1/2)/(-c*f*x+f)^(5/2),x)
```

output
```
(3*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**
2 - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x + 1))
,x)*b*c**2*x - 3*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x
 + 1)*c**2*x**2 - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*c*x + sqrt(c*x + 1)*sqr
t( - c*x + 1)),x)*b*c + sqrt(c*x + 1)*a*c*x - 2*sqrt(c*x + 1)*a)/(3*sqrt(f
)*sqrt(d)*sqrt( - c*x + 1)*c*f**2*(c*x - 1))
```

3.540 $\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 255

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{5/2}}\,dx = -\frac{bd(1-c^2x^2)^{5/2}}{6c(1-cx)(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{d(1+cx)\left(1-c^2x^2\right)(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{2dx(1-c^2x^2)^2(a+b\arccos(cx))}{3(d+cdx)^{5/2}(f-cfx)^{5/2}} - \frac{bd(1-c^2x^2)^{5/2}\operatorname{arctanh}(cx)}{6c(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{bd(1-c^2x^2)^{5/2}\log\left(1-c^2x^2\right)}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}}$$

output
```
-1/6*b*d*(-c^2*x^2+1)^(5/2)/c/(-c*x+1)/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)+1/
3*d*(c*x+1)*(-c^2*x^2+1)*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5
/2)+2/3*d*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5
/2)-1/6*b*d*(-c^2*x^2+1)^(5/2)*arctanh(c*x)/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(
5/2)+1/3*b*d*(-c^2*x^2+1)^(5/2)*ln(-c^2*x^2+1)/c/(c*d*x+d)^(5/2)/(-c*f*x+f
)^(5/2)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.72

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{5/2}}\,dx = \frac{\sqrt{d+cdx}\left(-4a-8acx+8ac^2x^2-2b\sqrt{1-c^2x^2}+4b(-1-2cx+2c^2x^2\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/((d + c*d*x)^(3/2)*(f - c*f*x)^(5/2)),x]
```

output
```
(Sqrt[d + c*d*x]*(-4*a - 8*a*c*x + 8*a*c^2*x^2 - 2*b*Sqrt[1 - c^2*x^2] + 4
*b*(-1 - 2*c*x + 2*c^2*x^2)*ArcCos[c*x] - 3*b*(-1 + c*x)*Sqrt[1 - c^2*x^2]
*Log[-(f*(1 + c*x))] + 5*b*Sqrt[1 - c^2*x^2]*Log[f - c*f*x] - 5*b*c*x*Sqrt
[1 - c^2*x^2]*Log[f - c*f*x]))/(12*c*d^2*f^2*Sqrt[f - c*f*x]*(-1 + c^2*x^2
))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5179, 27, 5261, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(cdx+d)^{3/2}(f-cfx)^{5/2}}dx$$

$$\Big\downarrow \ 5179$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\int \frac{d(cx+1)(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{d\left(1-c^2x^2\right)^{5/2}\int \frac{(cx+1)(a+b\arccos(cx))}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \ 5261$$

$$\frac{d(1-c^2x^2)^{5/2}\left(bc\int\left(\frac{2x}{3(1-c^2x^2)}+\frac{cx+1}{3c(1-c^2x^2)^2}\right)dx+\frac{2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{(cx+1)(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(1-c^2x^2)^{5/2}\left(\frac{2x(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}+\frac{(cx+1)(a+b\arccos(cx))}{3c(1-c^2x^2)^{3/2}}+bc\left(\frac{\operatorname{arctanh}(cx)}{6c^2}+\frac{cx+1}{6c^2(1-c^2x^2)}-\frac{\log(1-c^2x^2)}{3c^2}\right)\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

input
```
Int[(a + b*ArcCos[c*x])/((d + c*d*x)^(3/2)*(f - c*f*x)^(5/2)),x]
```

output
```
(d*(1 - c^2*x^2)^(5/2)*(((1 + c*x)*(a + b*ArcCos[c*x]))/(3*c*(1 - c^2*x^2)
^(3/2)) + (2*x*(a + b*ArcCos[c*x]))/(3*Sqrt[1 - c^2*x^2]) + b*c*((1 + c*x)
/(6*c^2*(1 - c^2*x^2)) + ArcTanh[c*x]/(6*c^2) - Log[1 - c^2*x^2]/(3*c^2)))
)/((d + c*d*x)^(5/2)*(f - c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5261
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_) + (g_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> With[{u = IntHide[(f + g*x)^m*(d + e*x^2)^p,
 x]}, Simp[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[1/Sqrt[1 - c^2*x^2]
   u, x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IG
tQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d, 0] && (LtQ[m, -2*p - 1] || GtQ[m, 3]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.54 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.97

method	result
default	$a\left(-\frac{1}{fdc\sqrt{cdx+d}\,(-cfx+f)^{\frac{3}{2}}}+\frac{\frac{2\sqrt{cdx+d}}{3fdc(-cfx+f)^{\frac{3}{2}}}+\frac{2\sqrt{cdx+d}}{3dc\,f^{2}\sqrt{-cfx+f}}}{d}\right)-\frac{b\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^{2}x^{2}+1}\,\left(-4i\arccos(cx)\right.}{}$
parts	$a\left(-\frac{1}{fdc\sqrt{cdx+d}\,(-cfx+f)^{\frac{3}{2}}}+\frac{\frac{2\sqrt{cdx+d}}{3fdc(-cfx+f)^{\frac{3}{2}}}+\frac{2\sqrt{cdx+d}}{3dc\,f^{2}\sqrt{-cfx+f}}}{d}\right)-\frac{b\sqrt{d(cx+1)}\,\sqrt{-f(cx-1)}\,\sqrt{-c^{2}x^{2}+1}\,\left(-4i\arccos(cx)\right.}{}$

input
```
int((a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
a*(-1/f/d/c/(c*d*x+d)^(1/2)/(-c*f*x+f)^(3/2)+2/d*(1/3/f/d/c/(-c*f*x+f)^(3/
2)*(c*d*x+d)^(1/2)+1/3/d/c/f^2/(-c*f*x+f)^(1/2)*(c*d*x+d)^(1/2)))-1/6*b*(d
*(c*x+1))^(1/2)*(-f*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(-4*I*arccos(c*x)*x*
c-5*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)*x^3*c^3-3*ln(1+c*x+I*(-c^2*x^2+1)^(1/2)
)*x^3*c^3-4*I*arccos(c*x)*x^2*c^2-4*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^2
+5*ln(I*(-c^2*x^2+1)^(1/2)+c*x-1)*x^2*c^2+3*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))
*x^2*c^2+4*I*arccos(c*x)+4*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c-c^2*x^2+5*ln
(I*(-c^2*x^2+1)^(1/2)+c*x-1)*x*c+3*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*x*c+4*I*
arccos(c*x)*c^3*x^3+2*arccos(c*x)*(-c^2*x^2+1)^(1/2)-5*ln(I*(-c^2*x^2+1)^(
1/2)+c*x-1)-3*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+1)/d^2/f^3/c/(c^5*x^5-c^4*x^4
-2*c^3*x^3+2*c^2*x^2+c*x-1)
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{5/2}}\,dx=\int \frac{b\arccos\left(cx\right)+a}{(cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(5/2),x, algorithm=
"fricas")
```

output
```
integral(-sqrt(c*d*x + d)*sqrt(-c*f*x + f)*(b*arccos(c*x) + a)/(c^5*d^2*f^
3*x^5 - c^4*d^2*f^3*x^4 - 2*c^3*d^2*f^3*x^3 + 2*c^2*d^2*f^3*x^2 + c*d^2*f^
3*x - d^2*f^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acos(c*x))/(c*d*x+d)**(3/2)/(-c*f*x+f)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.93

$$\begin{aligned}
&\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{5/2}}\,dx= \\
&-\frac{1}{12}\,bc\left(\frac{2\sqrt{d}\sqrt{f}}{c^3d^2f^3x-c^2d^2f^3}+\frac{3\log\left(cx+1\right)}{c^2d^{\frac{3}{2}}f^{\frac{5}{2}}}+\frac{5\log\left(cx-1\right)}{c^2d^{\frac{3}{2}}f^{\frac{5}{2}}}\right) \\
&-\frac{1}{3}\,b\left(\frac{1}{\sqrt{-c^2dfx^2+df}\,c^2df^2x-\sqrt{-c^2dfx^2+df}\,cdf^2}-\frac{2\,x}{\sqrt{-c^2dfx^2+df}\,df^2}\right)\arccos\left(cx\right) \\
&-\frac{1}{3}\,a\left(\frac{1}{\sqrt{-c^2dfx^2+df}\,c^2df^2x-\sqrt{-c^2dfx^2+df}\,cdf^2}-\frac{2\,x}{\sqrt{-c^2dfx^2+df}\,df^2}\right)
\end{aligned}$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(5/2),x, algorithm=
"maxima")
```

output
```
-1/12*b*c*(2*sqrt(d)*sqrt(f)/(c^3*d^2*f^3*x - c^2*d^2*f^3) + 3*log(c*x + 1
)/(c^2*d^(3/2)*f^(5/2)) + 5*log(c*x - 1)/(c^2*d^(3/2)*f^(5/2))) - 1/3*b*(1
/(sqrt(-c^2*d*f*x^2 + d*f)*c^2*d*f^2*x - sqrt(-c^2*d*f*x^2 + d*f)*c*d*f^2)
 - 2*x/(sqrt(-c^2*d*f*x^2 + d*f)*d*f^2))*arccos(c*x) - 1/3*a*(1/(sqrt(-c^2
*d*f*x^2 + d*f)*c^2*d*f^2*x - sqrt(-c^2*d*f*x^2 + d*f)*c*d*f^2) - 2*x/(sqr
t(-c^2*d*f*x^2 + d*f)*d*f^2))
```

Giac [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{5/2}}\,dx=\int \frac{b\arccos{(cx)}+a}{(cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((b*arccos(c*x) + a)/((c*d*x + d)^(3/2)*(-c*f*x + f)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{{(d+c\,d\,x)}^{3/2}\,{(f-c\,f\,x)}^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))/((d + c*d*x)^(3/2)*(f - c*f*x)^(5/2)),x)
```

output
```
int((a + b*acos(c*x))/((d + c*d*x)^(3/2)*(f - c*f*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{3/2}(f-cfx)^{5/2}}\,dx = \frac{3\sqrt{cx+1}\,\sqrt{-cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^3x^3-\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2-\sqrt{cx+1}\,\sqrt{-cx+1}\,c}\right.}{}$$

input
```
int((a+b*acos(c*x))/(c*d*x+d)^(3/2)/(-c*f*x+f)^(5/2),x)
```

output
```
(3*sqrt(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x
 + 1)*c**3*x**3 - sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 - sqrt(c*x + 1)
*sqrt( - c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b*c**2*x - 3*sq
rt(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x + 1)
*c**3*x**3 - sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt
( - c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b*c + 2*a*c**2*x**2
- 2*a*c*x - a)/(3*sqrt(f)*sqrt(d)*sqrt(c*x + 1)*sqrt( - c*x + 1)*c*d*f**2*
(c*x - 1))
```

3.541 $\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 188

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{5/2}}\,dx = -\frac{b(1-c^2x^2)^{3/2}}{6c(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{x(1-c^2x^2)\,(a+b\arccos(cx))}{3(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{2x(1-c^2x^2)^2\,(a+b\arccos(cx))}{3(d+cdx)^{5/2}(f-cfx)^{5/2}} + \frac{b(1-c^2x^2)^{5/2}\log(1-c^2x^2)}{3c(d+cdx)^{5/2}(f-cfx)^{5/2}}$$

output
```
-1/6*b*(-c^2*x^2+1)^(3/2)/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)+1/3*x*(-c^2*x
^2+1)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)+2/3*x*(-c^2*x^2+1
)^2*(a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)+1/3*b*(-c^2*x^2+1)^
(5/2)*ln(-c^2*x^2+1)/c/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.95

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{5/2}}\,dx = \frac{\sqrt{d+cdx}\Big(-6acx+4ac^3x^3-b\sqrt{1-c^2x^2}+2bcx(-3+2c^2x^2)\arccos(cx$$

$$6cd^3$$

input
```
Integrate[(a + b*ArcCos[c*x])/((d + c*d*x)^(5/2)*(f - c*f*x)^(5/2)),x]
```

output
```
(Sqrt[d + c*d*x]*(-6*a*c*x + 4*a*c^3*x^3 - b*Sqrt[1 - c^2*x^2] + 2*b*c*x*(
-3 + 2*c^2*x^2)*ArcCos[c*x] + 2*b*(1 - c^2*x^2)^(3/2)*Log[-(f*(1 + c*x))]
+ 2*b*Sqrt[1 - c^2*x^2]*Log[f - c*f*x] - 2*b*c^2*x^2*Sqrt[1 - c^2*x^2]*Log
[f - c*f*x]))/(6*c*d^3*(-1 + c*x)*Sqrt[f - c*f*x]*(f + c*f*x)^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5179, 5163, 241, 5161, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{5163}$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\left(\frac{2}{3}\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{3/2}}dx+\frac{1}{3}bc\int \frac{x}{(1-c^2x^2)^2}dx+\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \text{241}$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\left(\frac{2}{3}\int \frac{a+b\arccos(cx)}{(1-c^2x^2)^{3/2}}dx+\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{b}{6c(1-c^2x^2)}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \; 5161$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\left(\frac{2}{3}\left(bc\int \frac{x}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)+\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{b}{6c(1-c^2x^2)}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

$$\Big\downarrow \; 240$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\left(\frac{x(a+b\arccos(cx))}{3(1-c^2x^2)^{3/2}}+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}-\frac{b\log(1-c^2x^2)}{2c}\right)+\frac{b}{6c(1-c^2x^2)}\right)}{(cdx+d)^{5/2}(f-cfx)^{5/2}}$$

input
```
Int[(a + b*ArcCos[c*x])/((d + c*d*x)^(5/2)*(f - c*f*x)^(5/2)),x]
```

output
```
((1 - c^2*x^2)^(5/2)*(b/(6*c*(1 - c^2*x^2)) + (x*(a + b*ArcCos[c*x]))/(3*(
1 - c^2*x^2)^(3/2)) + (2*((x*(a + b*ArcCos[c*x]))/Sqrt[1 - c^2*x^2] - (b*L
og[1 - c^2*x^2])/(2*c)))/3))/((d + c*d*x)^(5/2)*(f - c*f*x)^(5/2))
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 11.34 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.99

method	result
default	$a\left(-\frac{1}{3fdc(cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{3}{2}}}+\frac{-\frac{1}{fdc\sqrt{cdx+d}\,(-cfx+f)^{\frac{3}{2}}}+\frac{\frac{2\sqrt{cdx+d}}{3fdc(-cfx+f)^{\frac{3}{2}}}+\frac{2\sqrt{cdx+d}}{3dc\,f^{2}\sqrt{-cfx+f}}}{d}}{d}\right)+\frac{b\sqrt{d(cx+1)}\,\sqrt{-f(cx-}}{}$
parts	$a\left(-\frac{1}{3fdc(cdx+d)^{\frac{3}{2}}(-cfx+f)^{\frac{3}{2}}}+\frac{-\frac{1}{fdc\sqrt{cdx+d}\,(-cfx+f)^{\frac{3}{2}}}+\frac{\frac{2\sqrt{cdx+d}}{3fdc(-cfx+f)^{\frac{3}{2}}}+\frac{2\sqrt{cdx+d}}{3dc\,f^{2}\sqrt{-cfx+f}}}{d}}{d}\right)+\frac{b\sqrt{d(cx+1)}\,\sqrt{-f(cx-}}{}$

input
```
int((a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
a*(-1/3/f/d/c/(c*d*x+d)^(3/2)/(-c*f*x+f)^(3/2)+1/d*(-1/f/d/c/(c*d*x+d)^(1/
2)/(-c*f*x+f)^(3/2)+2/d*(1/3/f/d/c/(-c*f*x+f)^(3/2)*(c*d*x+d)^(1/2)+1/3/d/
c/f^2/(-c*f*x+f)^(1/2)*(c*d*x+d)^(1/2))))+1/6*b*(d*(c*x+1))^(1/2)*(-f*(c*x
-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(-4*I*arccos(c*x)*x^4*c^4+4*ln((c*x+I*(-c^2*
x^2+1)^(1/2))^2-1)*x^4*c^4+4*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^3*c^3+8*I*ar
ccos(c*x)*c^2*x^2-8*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)*x^2*c^2-6*(-c^2*x^2
+1)^(1/2)*arccos(c*x)*x*c+c^2*x^2-4*I*arccos(c*x)+4*ln((c*x+I*(-c^2*x^2+1)
^(1/2))^2-1)-1)/f^3/d^3/(c^6*x^6-3*c^4*x^4+3*c^2*x^2-1)/c
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{5/2}}\,dx=\int \frac{b\arccos{(cx)}+a}{(cdx+d)^{\frac{5}{2}}(-cfx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2),x, algorithm=
"fricas")
```

output
```
integral(-sqrt(c*d*x + d)*sqrt(-c*f*x + f)*(b*arccos(c*x) + a)/(c^6*d^3*f^
3*x^6 - 3*c^4*d^3*f^3*x^4 + 3*c^2*d^3*f^3*x^2 - d^3*f^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acos(c*x))/(c*d*x+d)**(5/2)/(-c*f*x+f)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.94

$$\int \frac{a + b\arccos(cx)}{(d + cdx)^{5/2}(f - cfx)^{5/2}}\,dx = -\frac{1}{6}\,bc\left(\frac{1}{c^4d^{\frac{5}{2}}f^{\frac{5}{2}}x^2 - c^2d^{\frac{5}{2}}f^{\frac{5}{2}}} + \frac{2\,\log\left(cx+1\right)}{c^2d^{\frac{5}{2}}f^{\frac{5}{2}}} + \frac{2\,\log\left(cx-1\right)}{c^2d^{\frac{5}{2}}f^{\frac{5}{2}}}\right) + \frac{1}{3}\,b\left(\frac{x}{\left(-c^2dfx^2+df\right)^{\frac{3}{2}}df} + \frac{2\,x}{\sqrt{-c^2dfx^2+df}\,d^2f^2}\right)\arccos\left(cx\right) + \frac{1}{3}\,a\left(\frac{x}{\left(-c^2dfx^2+df\right)^{\frac{3}{2}}df} + \frac{2\,x}{\sqrt{-c^2dfx^2+df}\,d^2f^2}\right)$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2),x, algorithm=
"maxima")
```

output
```
-1/6*b*c*(1/(c^4*d^(5/2)*f^(5/2)*x^2 - c^2*d^(5/2)*f^(5/2)) + 2*log(c*x +
1)/(c^2*d^(5/2)*f^(5/2)) + 2*log(c*x - 1)/(c^2*d^(5/2)*f^(5/2))) + 1/3*b*(
x/((-c^2*d*f*x^2 + d*f)^(3/2)*d*f) + 2*x/(sqrt(-c^2*d*f*x^2 + d*f)*d^2*f^2
))*arccos(c*x) + 1/3*a*(x/((-c^2*d*f*x^2 + d*f)^(3/2)*d*f) + 2*x/(sqrt(-c^
2*d*f*x^2 + d*f)*d^2*f^2))
```

Giac [F]

$$\int \frac{a + b\arccos(cx)}{(d + cdx)^{5/2}(f - cfx)^{5/2}}\,dx = \int \frac{b\arccos\left(cx\right) + a}{\left(cdx+d\right)^{\frac{5}{2}}\left(-cfx+f\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2),x, algorithm=
"giac")
```

output
```
integrate((b*arccos(c*x) + a)/((c*d*x + d)^(5/2)*(-c*f*x + f)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{(d+c\,d\,x)^{5/2}\,(f-c\,f\,x)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))/((d + c*d*x)^(5/2)*(f - c*f*x)^(5/2)),x)
```

output
```
int((a + b*acos(c*x))/((d + c*d*x)^(5/2)*(f - c*f*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d+cdx)^{5/2}(f-cfx)^{5/2}}\,dx=\frac{3\sqrt{cx+1}\,\sqrt{-cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^4x^4-2\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2+\sqrt{cx+1}\,\sqrt{-cx+1}}\right.}{3\sqrt{f}}$$

input
```
int((a+b*acos(c*x))/(c*d*x+d)^(5/2)/(-c*f*x+f)^(5/2),x)
```

output
```
(3*sqrt(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x
 + 1)*c**4*x**4 - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 + sqrt(c*x +
1)*sqrt( - c*x + 1)),x)*b*c**2*x**2 - 3*sqrt(c*x + 1)*sqrt( - c*x + 1)*int
(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**4*x**4 - 2*sqrt(c*x + 1)*sqr
t( - c*x + 1)*c**2*x**2 + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b + 2*a*c**2*
x**3 - 3*a*x)/(3*sqrt(f)*sqrt(d)*sqrt(c*x + 1)*sqrt( - c*x + 1)*d**2*f**2*
(c**2*x**2 - 1))
```

3.542 $\int (d+cdx)^{5/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 613

$$\int (d+cdx)^{5/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \frac{8b^2d^2\sqrt{d+cdx}\sqrt{e-cex}}{9c} - \frac{15}{64}b^2d^2x\sqrt{d+cdx}\sqrt{e-cex} - \frac{1}{32}b^2c^2d^2x^3\sqrt{d+cdx}\sqrt{e-cex} + \frac{4b^2d^2\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)}{27c} + \frac{15b^2d^2\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)}{64c\sqrt{1-c^2x^2}} + \frac{4bd^2x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}} - \frac{3bcd^2x^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{8\sqrt{1-c^2x^2}} - \frac{4bc^2d^2x^3\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{9\sqrt{1-c^2x^2}} - \frac{bc^3d^2x^4\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{8\sqrt{1-c^2x^2}} + \frac{3}{8}d^2x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2 + \frac{1}{4}c^2d^2x^3\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2 - \frac{2d^2\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)\,(a+b\arccos(cx))^2}{3c} + \frac{5d^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^3}{24bc\sqrt{1-c^2x^2}}$$

output
```
8/9*b^2*d^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)/c-15/64*b^2*d^2*x*(c*d*x+d)^(
1/2)*(-c*e*x+e)^(1/2)-1/32*b^2*c^2*d^2*x^3*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2
)+4/27*b^2*d^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)/c+15/64*b^2*d
^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*arccos(c*x)/c/(-c^2*x^2+1)^(1/2)+4/3*b
*d^2*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/
2)-3/8*b*c*d^2*x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))/(-c^
2*x^2+1)^(1/2)-4/9*b*c^2*d^2*x^3*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arc
cos(c*x))/(-c^2*x^2+1)^(1/2)-1/8*b*c^3*d^2*x^4*(c*d*x+d)^(1/2)*(-c*e*x+e)^
(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+3/8*d^2*x*(c*d*x+d)^(1/2)*(-c*e
*x+e)^(1/2)*(a+b*arccos(c*x))^2+1/4*c^2*d^2*x^3*(c*d*x+d)^(1/2)*(-c*e*x+e)
^(1/2)*(a+b*arccos(c*x))^2-2/3*d^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*
x^2+1)*(a+b*arccos(c*x))^2/c+5/24*d^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+
b*arccos(c*x))^3/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 3.14 (sec) , antiderivative size = 574, normalized size of antiderivative = 0.94

$$\int (d+cdx)^{5/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \frac{-1440b^2d^2\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)^3-4320a^2d^{5/2}\sqrt{e}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d+cd}}{\sqrt{d}\sqrt{e}(-}\right.}{}$$

input
```
Integrate[(d + c*d*x)^(5/2)*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(-1440*b^2*d^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^3 - 4320*a^2*d^
(5/2)*Sqrt[e]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x
])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] + 12*b*d^2*Sqrt[d + c*d*x]*Sqrt[e - c
*e*x]*ArcCos[c*x]*(-576*b*c*x - 768*a*Sqrt[1 - c^2*x^2] + 768*a*c^2*x^2*Sq
rt[1 - c^2*x^2] + 144*b*Cos[2*ArcCos[c*x]] + 64*b*Cos[3*ArcCos[c*x]] + 9*b
*Cos[4*ArcCos[c*x]] + 288*a*Sin[2*ArcCos[c*x]] + 36*a*Sin[4*ArcCos[c*x]])
+ 72*b*d^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^2*(-60*a - 32*b*Sqr
t[1 - c^2*x^2] + 32*b*Sqrt[1 - c^2*x^2]*Cos[2*ArcCos[c*x]] + 24*b*Sin[2*Ar
cCos[c*x]] + 3*b*Sin[4*ArcCos[c*x]]) + d^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]
*(-9216*a*b*c*x + 3072*a*b*c^3*x^3 - 4608*a^2*Sqrt[1 - c^2*x^2] + 6656*b^2
*Sqrt[1 - c^2*x^2] + 2592*a^2*c*x*Sqrt[1 - c^2*x^2] + 4608*a^2*c^2*x^2*Sqr
t[1 - c^2*x^2] + 1728*a^2*c^3*x^3*Sqrt[1 - c^2*x^2] + 64*b*(27*a - 8*b*Sqr
t[1 - c^2*x^2])*Cos[2*ArcCos[c*x]] + 108*a*b*Cos[4*ArcCos[c*x]] - 864*b^2*
Sin[2*ArcCos[c*x]] - 27*b^2*Sin[4*ArcCos[c*x]]))/(6912*c*Sqrt[1 - c^2*x^2]
)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^{5/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\int d^2(cx+1)^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\sqrt{cdx+d}\sqrt{e-cex}\int (cx+1)^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{d^2\sqrt{cdx+d}\sqrt{e-cex}\int\left(c^2x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2+2cx\sqrt{1-c^2x^2}(a+b\arccos(cx))^2+\sqrt{1-c^2x^2}(a+\right.}{\sqrt{1-c^2x^2}}$$

$\downarrow$ 2009

$$d^2\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{8}bc^3x^4(a+b\arccos(cx))+\frac{4}{9}bc^2x^3(a+b\arccos(cx))+\frac{3}{8}x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2-\right.$$

input
```
Int[(d + c*d*x)^(5/2)*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*((8*b^2*Sqrt[1 - c^2*x^2])/(9*c) - (1
5*b^2*x*Sqrt[1 - c^2*x^2])/64 - (b^2*c^2*x^3*Sqrt[1 - c^2*x^2])/32 + (4*b^
2*(1 - c^2*x^2)^(3/2))/(27*c) - (4*b*x*(a + b*ArcCos[c*x]))/3 + (3*b*c*x^2
*(a + b*ArcCos[c*x]))/8 + (4*b*c^2*x^3*(a + b*ArcCos[c*x]))/9 + (b*c^3*x^4
*(a + b*ArcCos[c*x]))/8 + (3*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/8
+ (c^2*x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/4 - (2*(1 - c^2*x^2)^(
3/2)*(a + b*ArcCos[c*x])^2)/(3*c) - (5*(a + b*ArcCos[c*x])^3)/(24*b*c) + (
15*b^2*ArcSin[c*x])/(64*c)))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.93 (sec) , antiderivative size = 1815, normalized size of antiderivative = 2.96

method	result	size
default	Expression too large to display	1815
parts	Expression too large to display	1815

input
```
int((c*d*x+d)^(5/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/4*a^2/c/e*(c*d*x+d)^(5/2)*(-c*e*x+e)^(3/2)-5/12*a^2*d/c/e*(c*d*x+d)^(3/
2)*(-c*e*x+e)^(3/2)-5/8*a^2*d^2/c/e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(3/2)+5/8*a
^2*d^2/c*(-c*e*x+e)^(1/2)*(c*d*x+d)^(1/2)+5/8*a^2*d^3*e*((-c*e*x+e)*(c*d*x
+d))^(1/2)/(-c*e*x+e)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*e)^(1/2)*arctan((c^2*d*
e)^(1/2)*x/(-c^2*d*e*x^2+d*e)^(1/2))+b^2*(5/24*(d*(c*x+1))^(1/2)*(-e*(c*x-
1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^3*d^2+1/512*(d*(c*x
+1))^(1/2)*(-e*(c*x-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)
*x^4*c^4+4*c*x-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(4*I*a
rccos(c*x)+8*arccos(c*x)^2-1)*d^2/(c^2*x^2-1)/c+1/108*(d*(c*x+1))^(1/2)*(-
e*(c*x-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-3*I*(
-c^2*x^2+1)^(1/2)*c*x+1)*(6*I*arccos(c*x)+9*arccos(c*x)^2-2)*d^2/(c^2*x^2-
1)/c-1/4*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c
^2*x^2-1)*(arccos(c*x)^2-2-2*I*arccos(c*x))*d^2/(c^2*x^2-1)/c+1/16*(d*(c*x
+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I
*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*arccos(c*x)^2-1-2*I*arccos(c*x))*d^2/(c^2*x^
2-1)/c-1/512*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x
*c+c^2*x^2-1)*(68*I*arccos(c*x)+56*arccos(c*x)^2-31)*cos(3*arccos(c*x))*d^
2/(c^2*x^2-1)/c-3/512*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*c^2*x^2+c*x*
(-c^2*x^2+1)^(1/2)-I)*(20*I*arccos(c*x)+24*arccos(c*x)^2-11)*sin(3*arccos(
c*x))*d^2/(c^2*x^2-1)/c+1/27*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(...
```

Fricas [F]

$$\int (d+cdx)^{5/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx=\int (cdx+d)^{\frac{5}{2}}\sqrt{-cex+e}(b\arccos{(cx)}+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algorith
m="fricas")
```

output
```
integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2
*b^2*c*d^2*x + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x
 + a*b*d^2)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+cdx)^{5/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+d)**(5/2)*(-c*e*x+e)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+cdx)^{5/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (d+cdx)^{5/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \int (cdx+d)^{\frac{5}{2}}\sqrt{-cex+e}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algorith
m="giac")
```

output
```
integrate((c*d*x + d)^(5/2)*sqrt(-c*e*x + e)*(b*arccos(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)^{5/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,(d+c\,d\,x)^{5/2}\,\sqrt{e-c\,e\,x}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(5/2)*(e - c*e*x)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(5/2)*(e - c*e*x)^(1/2), x)
```

Reduce [F]

$$\int (d+cdx)^{5/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx=\frac{\sqrt{e}\,\sqrt{d}\,d^2\left(-30asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2+6\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^3x^3+16\sqrt{cx+1}\,\sqrt{-cx+}\right.}{}$$

input
```
int((c*d*x+d)^(5/2)*(-c*e*x+e)^(1/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(e)*sqrt(d)*d**2*( - 30*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + 6*sqrt(
c*x + 1)*sqrt( - c*x + 1)*a**2*c**3*x**3 + 16*sqrt(c*x + 1)*sqrt( - c*x +
1)*a**2*c**2*x**2 + 9*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c*x - 16*sqrt(c*
x + 1)*sqrt( - c*x + 1)*a**2 + 48*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(
c*x)*x**2,x)*a*b*c**3 + 96*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x,
x)*a*b*c**2 + 48*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x),x)*a*b*c + 2
4*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2*x**2,x)*b**2*c**3 + 48*i
nt(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2*x,x)*b**2*c**2 + 24*int(sqr
t(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2,x)*b**2*c))/(24*c)
```

3.543 $\int (d+cdx)^{3/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 455

$$\begin{aligned}\int (d+cdx)^{3/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx &= \frac{4b^2d\sqrt{d+cdx}\sqrt{e-cex}}{9c}\\ &-\frac{1}{4}b^2dx\sqrt{d+cdx}\sqrt{e-cex}+\frac{2b^2d\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)}{27c}\\ &+\frac{b^2d\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)}{4c\sqrt{1-c^2x^2}}\\ &+\frac{2bdx\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}\\ &-\frac{bcdx^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{2\sqrt{1-c^2x^2}}\\ &-\frac{2bc^2dx^3\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{9\sqrt{1-c^2x^2}}\\ &+\frac{1}{2}dx\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\\ &-\frac{d\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)\,(a+b\arccos(cx))^2}{3c}\\ &+\frac{d\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
4/9*b^2*d*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)/c-1/4*b^2*d*x*(c*d*x+d)^(1/2)*(
-c*e*x+e)^(1/2)+2/27*b^2*d*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)/c
+1/4*b^2*d*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*arccos(c*x)/c/(-c^2*x^2+1)^(1/
2)+2/3*b*d*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+
1)^(1/2)-1/2*b*c*d*x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))/
(-c^2*x^2+1)^(1/2)-2/9*b*c^2*d*x^3*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*a
rccos(c*x))/(-c^2*x^2+1)^(1/2)+1/2*d*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a
+b*arccos(c*x))^2-1/3*d*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)*(a+b
*arccos(c*x))^2/c+1/6*d*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))
^3/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.00

$$\int (d+cdx)^{3/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \frac{-36b^2d\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)^3-108a^2d^{3/2}\sqrt{e}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{e-}}{\sqrt{d}\sqrt{e}(-1+c^2}\right.}{}$$

input
```
Integrate[(d + c*d*x)^(3/2)*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(-36*b^2*d*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^3 - 108*a^2*d^(3/2)
*Sqrt[e]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(S
qrt[d]*Sqrt[e]*(-1 + c^2*x^2))] + 18*b*d*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*A
rcCos[c*x]^2*(-6*a - 2*b*Sqrt[1 - c^2*x^2] + 2*b*Sqrt[1 - c^2*x^2]*Cos[2*A
rcCos[c*x]] + 3*b*Sin[2*ArcCos[c*x]]) + d*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*
(104*b^2*Sqrt[1 - c^2*x^2] + 48*a*b*c*x*(-3 + c^2*x^2) + 36*a^2*Sqrt[1 - c
^2*x^2]*(-2 + 3*c*x + 2*c^2*x^2) + 2*b*(27*a - 4*b*Sqrt[1 - c^2*x^2])*Cos[
2*ArcCos[c*x]] - 27*b^2*Sin[2*ArcCos[c*x]]) + 6*b*d*Sqrt[d + c*d*x]*Sqrt[e
 - c*e*x]*ArcCos[c*x]*(9*b*Cos[2*ArcCos[c*x]] + 2*(-9*b*c*x - 12*a*Sqrt[1
- c^2*x^2] + 12*a*c^2*x^2*Sqrt[1 - c^2*x^2] + b*Cos[3*ArcCos[c*x]] + 9*a*S
in[2*ArcCos[c*x]])))/(216*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^{3/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\int d(cx+1)\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\sqrt{cdx+d}\sqrt{e-cex}\int (cx+1)\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{d\sqrt{cdx+d}\sqrt{e-cex}\int \left(cx\sqrt{1-c^2x^2}(a+b\arccos(cx))^2+\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\right)dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{2}{9}bc^2x^3(a+b\arccos(cx))+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2-\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{3c}+\right.}{}$$

input
```
Int[(d + c*d*x)^(3/2)*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*((4*b^2*Sqrt[1 - c^2*x^2])/(9*c) - (b^2
*x*Sqrt[1 - c^2*x^2])/4 + (2*b^2*(1 - c^2*x^2)^(3/2))/(27*c) - (2*b*x*(a +
 b*ArcCos[c*x]))/3 + (b*c*x^2*(a + b*ArcCos[c*x]))/2 + (2*b*c^2*x^3*(a + b
*ArcCos[c*x]))/9 + (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/2 - ((1 - c
^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/(3*c) - (a + b*ArcCos[c*x])^3/(6*b*c)
 + (b^2*ArcSin[c*x])/(4*c)))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.44 (sec) , antiderivative size = 1358, normalized size of antiderivative = 2.98

method	result	size
default	Expression too large to display	1358
parts	Expression too large to display	1358

input
```
int((c*d*x+d)^(3/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/3*a^2/c/e*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)-1/2*a^2*d/c/e*(c*d*x+d)^(1/2
)*(-c*e*x+e)^(3/2)+1/2*a^2*d/c*(-c*e*x+e)^(1/2)*(c*d*x+d)^(1/2)+1/2*a^2*d^
2*e*((-c*e*x+e)*(c*d*x+d))^(1/2)/(-c*e*x+e)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*e
)^(1/2)*arctan((c^2*d*e)^(1/2)*x/(-c^2*d*e*x^2+d*e)^(1/2))+b^2*(1/6*(d*(c*
x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x
)^3*d+1/216*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*I*
(-c^2*x^2+1)^(1/2)*x^3*c^3-3*I*(-c^2*x^2+1)^(1/2)*c*x+1)*(6*I*arccos(c*x)+
9*arccos(c*x)^2-2)*d/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/
2)*(2*c^3*x^3-2*c*x+2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(
2*arccos(c*x)^2-1+2*I*arccos(c*x))*d/(c^2*x^2-1)/c-1/8*(d*(c*x+1))^(1/2)*(
-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)^2-2-2
*I*arccos(c*x))*d/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*
(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*
arccos(c*x)^2-1-2*I*arccos(c*x))*d/(c^2*x^2-1)/c+1/54*(d*(c*x+1))^(1/2)*(-
e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(12*I*arccos(c*x)+9
*arccos(c*x)^2-14)*cos(2*arccos(c*x))*d/(c^2*x^2-1)/c+1/108*(d*(c*x+1))^(1
/2)*(-e*(c*x-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(30*I*arccos(c
*x)+9*arccos(c*x)^2-26)*sin(2*arccos(c*x))*d/(c^2*x^2-1)/c)+2*a*b*(1/4*(d*
(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(
c*x)^2*d+1/72*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2...
```

Fricas [F]

$$\int (d+cdx)^{3/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx=\int (cdx+d)^{\frac{3}{2}}\sqrt{-cex+e}(b\arccos{(cx)}+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algorith
m="fricas")
```

output
```
integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arccos(c*x)^2 + 2*(a*b*c
*d*x + a*b*d)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e), x)
```

Sympy [F]

$$\int (d+cdx)^{3/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx=\int (d(cx+1))^{\frac{3}{2}}\sqrt{-e\,(cx-1)}(a+b\,\mathrm{acos}\,(cx))^2\,dx$$

input
```
integrate((c*d*x+d)**(3/2)*(-c*e*x+e)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral((d*(c*x + 1))**(3/2)*sqrt(-e*(c*x - 1))*(a + b*acos(c*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (d+cdx)^{3/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (d+cdx)^{3/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx=\int (cdx+d)^{\frac{3}{2}}\sqrt{-cex+e}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algorith
m="giac")
```

output
```
integrate((c*d*x + d)^(3/2)*sqrt(-c*e*x + e)*(b*arccos(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)^{3/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,(d+c\,d\,x)^{3/2}\,\sqrt{e-c\,e\,x}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(1/2), x)
```

Reduce [F]

$$\int (d+cdx)^{3/2}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx=\frac{\sqrt{e}\,\sqrt{d}\,d\left(-6asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2+2\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^2x^2+3\sqrt{cx+1}\,\sqrt{-cx+1}\,a$$

input
```
int((c*d*x+d)^(3/2)*(-c*e*x+e)^(1/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(e)*sqrt(d)*d*( - 6*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + 2*sqrt(c*x
+ 1)*sqrt( - c*x + 1)*a**2*c**2*x**2 + 3*sqrt(c*x + 1)*sqrt( - c*x + 1)*a*
*2*c*x - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2 + 12*int(sqrt(c*x + 1)*sqrt
( - c*x + 1)*acos(c*x)*x,x)*a*b*c**2 + 12*int(sqrt(c*x + 1)*sqrt( - c*x +
1)*acos(c*x),x)*a*b*c + 6*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2*
x,x)*b**2*c**2 + 6*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2,x)*b**2
*c))/(6*c)
```

3.544 $\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 222

$$\begin{aligned}\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx &= -\frac{1}{4}b^2x\sqrt{d+cdx}\sqrt{e-cex}+\frac{b^2\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)}{4c\sqrt{1-c^2x^2}}\\ &\quad-\frac{bcx^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{2\sqrt{1-c^2x^2}}\\ &\quad+\frac{1}{2}x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2+\frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-1/4*b^2*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)+1/4*b^2*(c*d*x+d)^(1/2)*(-c*e*
x+e)^(1/2)*arccos(c*x)/c/(-c^2*x^2+1)^(1/2)-1/2*b*c*x^2*(c*d*x+d)^(1/2)*(-
c*e*x+e)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+1/2*x*(c*d*x+d)^(1/2)*
(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2+1/6*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*
(a+b*arccos(c*x))^3/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.30

$$\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$
$$= \frac{-4b^2\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)^3-12a^2\sqrt{d}\sqrt{e}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{e-cex}}{\sqrt{d}\sqrt{e}(-1+c^2x^2)}\right)+6b\sqrt{d+cdx}\sqrt{e}}{}$$

input
```
Integrate[Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(-4*b^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^3 - 12*a^2*Sqrt[d]*Sqr
t[e]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[
d]*Sqrt[e]*(-1 + c^2*x^2))] + 6*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c
*x]*(b*Cos[2*ArcCos[c*x]] + 2*a*Sin[2*ArcCos[c*x]]) + 6*b*Sqrt[d + c*d*x]*
Sqrt[e - c*e*x]*ArcCos[c*x]^2*(-2*a + b*Sin[2*ArcCos[c*x]]) + 3*Sqrt[d + c
*d*x]*Sqrt[e - c*e*x]*(4*a^2*c*x*Sqrt[1 - c^2*x^2] + 2*a*b*Cos[2*ArcCos[c*
x]] - b^2*Sin[2*ArcCos[c*x]]))/(24*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.65, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5179, 5157, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cdx+d}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{ 5179}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 5157}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+bc\int x(a+b\arccos(cx))dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\right)}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5139

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(bc\left(\frac{1}{2}bc\int\frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)+\frac{1}{2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 262

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(bc\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)+\frac{1}{2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 223

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)+\frac{1}{2}x\sqrt{1-\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5153

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\right.}{\sqrt{1-c^2x^2}}$$

input
```
Int[Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*((x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])
^2)/2 - (a + b*ArcCos[c*x])^3/(6*b*c) + b*c*((x^2*(a + b*ArcCos[c*x]))/2 +
 (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2)))/Sqrt[1
- c^2*x^2]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 634, normalized size of antiderivative = 2.86

method	result
default	$-\frac{a^2\sqrt{cdx+d}\,(-cex+e)^{\frac{3}{2}}}{2ce}+\frac{a^2\sqrt{-cex+e}\,\sqrt{cdx+d}}{2c}+\frac{a^2de\sqrt{(-cex+e)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-c^2de\,x^2+de}}\right)}{2\sqrt{-cex+e}\,\sqrt{cdx+d}\,\sqrt{c^2de}}+b^2\left(\frac{\sqrt{d(cx+1)}\,\sqrt{}\right.$
parts	$-\frac{a^2\sqrt{cdx+d}\,(-cex+e)^{\frac{3}{2}}}{2ce}+\frac{a^2\sqrt{-cex+e}\,\sqrt{cdx+d}}{2c}+\frac{a^2de\sqrt{(-cex+e)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-c^2de\,x^2+de}}\right)}{2\sqrt{-cex+e}\,\sqrt{cdx+d}\,\sqrt{c^2de}}+b^2\left(\frac{\sqrt{d(cx+1)}\,\sqrt{}\right.$

input

```
int((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x,method=_RETURNV
ERBOSE)
```

output

```
-1/2*a^2/c/e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(3/2)+1/2*a^2/c*(-c*e*x+e)^(1/2)*(
c*d*x+d)^(1/2)+1/2*a^2*d*e*((-c*e*x+e)*(c*d*x+d))^(1/2)/(-c*e*x+e)^(1/2)/(
c*d*x+d)^(1/2)/(c^2*d*e)^(1/2)*arctan((c^2*d*e)^(1/2)*x/(-c^2*d*e*x^2+d*e)
^(1/2))+b^2*(1/6*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(
c^2*x^2-1)/c*arccos(c*x)^3+1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(2*c^
3*x^3-2*c*x+2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(2*arccos
(c*x)^2-1+2*I*arccos(c*x))/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1
))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2
*c*x)*(2*arccos(c*x)^2-1-2*I*arccos(c*x))/(c^2*x^2-1)/c)+2*a*b*(1/4*(d*(c*
x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x
)^2+1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(2*c^3*x^3-2*c*x+2*I*(-c^2*x
^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(I+2*arccos(c*x))/(c^2*x^2-1)/c+
1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2
+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))/(c^2*x^2-1)/c)
```

Fricas [F]

$$\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \int \sqrt{cdx+d}\sqrt{-cex+e}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algorith
m="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sqr
t(-c*e*x + e), x)
```

Sympy [F]

$$\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \int \sqrt{d(cx+1)}\sqrt{-e(cx-1)}(a+b\operatorname{acos}(cx))^2\,dx$$

input
```
integrate((c*d*x+d)**(1/2)*(-c*e*x+e)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral(sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))*(a + b*acos(c*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \int \sqrt{cdx+d}\sqrt{-cex+e}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algorith
m="giac")
```

output
```
integrate(sqrt(c*d*x + d)*sqrt(-c*e*x + e)*(b*arccos(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \int (a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d+c\,d\,x}\,\sqrt{e-c\,e\,x}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$
$$=\frac{\sqrt{e}\,\sqrt{d}\left(-2asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2+\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2cx+4\left(\int\sqrt{cx+1}\,\sqrt{-cx+1}\,acos(cx)\,dx\right)abc+2(\right.}{2c}$$

input
```
int((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(e)*sqrt(d)*( - 2*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + sqrt(c*x + 1)
*sqrt( - c*x + 1)*a**2*c*x + 4*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x
),x)*a*b*c + 2*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2,x)*b**2*c))
/(2*c)
```

3.545 $\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 230

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx = -\frac{2abex\sqrt{1-c^2x^2}}{\sqrt{d+cdx}\sqrt{e-cex}} - \frac{2b^2e(1-c^2x^2)}{c\sqrt{d+cdx}\sqrt{e-cex}} - \frac{2b^2ex\sqrt{1-c^2x^2}\arccos(cx)}{\sqrt{d+cdx}\sqrt{e-cex}} + \frac{e(1-c^2x^2)\left(a+b\arccos(cx)\right)^2}{c\sqrt{d+cdx}\sqrt{e-cex}} + \frac{e\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc\sqrt{d+cdx}\sqrt{e-cex}}$$

output
```
-2*a*b*e*x*(-c^2*x^2+1)^(1/2)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2*b^2*e*(-c
^2*x^2+1)/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2*b^2*e*x*(-c^2*x^2+1)^(1/2)*
arccos(c*x)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+e*(-c^2*x^2+1)*(a+b*arccos(c*
x))^2/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+1/3*e*(-c^2*x^2+1)^(1/2)*(a+b*arc
cos(c*x))^3/b/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx$$
$$= \frac{(-1+cx)\left(-3\sqrt{d+cdx}\sqrt{e-cex}\left(2abcx+a^2\sqrt{1-c^2x^2}-2b^2\sqrt{1-c^2x^2}\right)-6b\sqrt{d+cdx}\sqrt{e-cex}(bcx\right.}{}$$

input
```
Integrate[(Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2)/Sqrt[d + c*d*x],x]
```

output
```
((-1 + c*x)*(-3*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(2*a*b*c*x + a^2*Sqrt[1 -
c^2*x^2] - 2*b^2*Sqrt[1 - c^2*x^2]) - 6*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*
(b*c*x + a*Sqrt[1 - c^2*x^2])*ArcCos[c*x] + 3*b*Sqrt[d + c*d*x]*Sqrt[e - c
*e*x]*(a - b*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2 + b^2*Sqrt[d + c*d*x]*Sqrt[e
 - c*e*x]*ArcCos[c*x]^3 + 3*a^2*Sqrt[d]*Sqrt[e]*Sqrt[1 - c^2*x^2]*ArcTan[(
c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))])*Cs
c[ArcCos[c*x]/2]^2)/(6*c*d*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.53, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{\sqrt{cdx+d}}\,dx$$
$$\Big\downarrow \text{5179}$$
$$\frac{\sqrt{1-c^2x^2}\int \frac{e(1-cx)(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$
$$\Big\downarrow \text{27}$$

$$\frac{e\sqrt{1-c^2x^2}\int \frac{(1-cx)(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 5263}$$

$$\frac{e\sqrt{1-c^2x^2}\int \left(\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{cx(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e\sqrt{1-c^2x^2}\left(\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c}-\frac{(a+b\arccos(cx))^3}{3bc}+2abx+2b^2x\arccos(cx)-\frac{2b^2\sqrt{1-c^2x^2}}{c}\right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

input
```
Int[(Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2)/Sqrt[d + c*d*x],x]
```

output
```
(e*Sqrt[1 - c^2*x^2]*(2*a*b*x - (2*b^2*Sqrt[1 - c^2*x^2])/c + 2*b^2*x*ArcC
os[c*x] + (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c - (a + b*ArcCos[c*x]
)^3/(3*b*c)))/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.36

method	result
default	$\frac{a^2\sqrt{-cex+e}\,\sqrt{cdx+d}}{dc}+\frac{a^2e\sqrt{(-cex+e)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-c^2de\,x^2+de}}\right)}{\sqrt{-cex+e}\,\sqrt{cdx+d}\,\sqrt{c^2de}}+b^2\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,\arccos(c}{3(cx+1)dc(cx-1)}\right.$
parts	$\frac{a^2\sqrt{-cex+e}\,\sqrt{cdx+d}}{dc}+\frac{a^2e\sqrt{(-cex+e)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-c^2de\,x^2+de}}\right)}{\sqrt{-cex+e}\,\sqrt{cdx+d}\,\sqrt{c^2de}}+b^2\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,\arccos(c}{3(cx+1)dc(cx-1)}\right.$

input
```
int((-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
a^2/d/c*(-c*e*x+e)^(1/2)*(c*d*x+d)^(1/2)+a^2*e*((-c*e*x+e)*(c*d*x+d))^(1/2
)/(-c*e*x+e)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*e)^(1/2)*arctan((c^2*d*e)^(1/2)*
x/(-c^2*d*e*x^2+d*e)^(1/2))+b^2*(1/3*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*
(-c^2*x^2+1)^(1/2)/(c*x+1)/d/c/(c*x-1)*arccos(c*x)^3+1/2*(d*(c*x+1))^(1/2)
*(-e*(c*x-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)^2-2+
2*I*arccos(c*x))/(c*x+1)/d/c/(c*x-1)+1/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1
/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)^2-2-2*I*arccos(c*x)
)/(c*x+1)/d/c/(c*x-1))+2*a*b*(1/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c
^2*x^2+1)^(1/2)/(c*x+1)/d/c/(c*x-1)*arccos(c*x)^2+1/2*(d*(c*x+1))^(1/2)*(-
e*(c*x-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)+I)/(c*x
+1)/d/c/(c*x-1)+1/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^
(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)-I)/(c*x+1)/d/c/(c*x-1))
```

Fricas [F]

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx=\int \frac{\sqrt{-cex+e}(b\arccos{(cx)}+a)^2}{\sqrt{cdx+d}}\,dx$$

input
```
integrate((-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2),x, algorith
m="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(-c*e*x + e)/sq
rt(c*d*x + d), x)
```

Sympy [F]

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx=\int \frac{\sqrt{-e(cx-1)}(a+b\operatorname{acos}{(cx)})^2}{\sqrt{d(cx+1)}}\,dx$$

input
```
integrate((-c*e*x+e)**(1/2)*(a+b*acos(c*x))**2/(c*d*x+d)**(1/2),x)
```

output
```
Integral(sqrt(-e*(c*x - 1))*(a + b*acos(c*x))**2/sqrt(d*(c*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx=\int \frac{\sqrt{-cex+e}(b\arccos{(cx)}+a)^2}{\sqrt{cdx+d}}\,dx$$

input
```
integrate((-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(-c*e*x + e)*(b*arccos(c*x) + a)^2/sqrt(c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{e-c\,e\,x}}{\sqrt{d+c\,d\,x}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(1/2))/(d + c*d*x)^(1/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(1/2))/(d + c*d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx$$
$$=\frac{\sqrt{e}\left(-2asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2+\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2+2\left(\int\frac{\sqrt{-cx+1}\,acos(cx)}{\sqrt{cx+1}}dx\right)abc+\left(\int\frac{\sqrt{-cx+1}\,acos(cx)^2}{\sqrt{cx+1}}dx\right)\right.}{\sqrt{d}\,c}$$

input
```
int((-c*e*x+e)^(1/2)*(a+b*acos(c*x))^2/(c*d*x+d)^(1/2),x)
```

output
```
(sqrt(e)*( - 2*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + sqrt(c*x + 1)*sqrt( -
 c*x + 1)*a**2 + 2*int((sqrt( - c*x + 1)*acos(c*x))/sqrt(c*x + 1),x)*a*b*c
 + int((sqrt( - c*x + 1)*acos(c*x)**2)/sqrt(c*x + 1),x)*b**2*c))/(sqrt(d)*
c)
```

3.546 $\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 530

$$\begin{aligned}
\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx &= -\frac{2e^2(1-c^2x^2)(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}}+\frac{2e^2x(1-c^2x^2)(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} \\
&-\frac{2ie^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}}-\frac{e^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))^3}{3bc(d+cdx)^{3/2}(e-cex)^{3/2}} \\
&-\frac{8ibe^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\
&+\frac{4be^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\
&+\frac{4ib^2e^2(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\
&-\frac{4ib^2e^2(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\
&-\frac{2ib^2e^2(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}}
\end{aligned}$$

output
```
-2*e^2*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)
+2*e^2*x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)
-2*I*e^2*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+
e)^(3/2)-1/3*e^2*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))^3/b/c/(c*d*x+d)^(3/2
)/(-c*e*x+e)^(3/2)-8*I*b*e^2*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))*arctan(c
*x+I*(-c^2*x^2+1)^(1/2))/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+4*b*e^2*(-c^2*
x^2+1)^(3/2)*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x
+d)^(3/2)/(-c*e*x+e)^(3/2)+4*I*b^2*e^2*(-c^2*x^2+1)^(3/2)*polylog(2,-I*(c*
x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-4*I*b^2*e^2*(-
c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(3/2)
/(-c*e*x+e)^(3/2)-2*I*b^2*e^2*(-c^2*x^2+1)^(3/2)*polylog(2,-(c*x+I*(-c^2*x
^2+1)^(1/2))^2)/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)
```

Mathematica [A] (verified)

Time = 4.26 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx =$$

$$-\frac{2a^2\sqrt{-e(-1+cx)}\sqrt{d(1+cx)}}{cd^2(1+cx)}+\frac{a^2\sqrt{e}\arctan\left(\frac{cx\sqrt{-e(-1+cx)}\sqrt{d(1+cx)}}{\sqrt{d}\sqrt{e}(-1+cx)(1+cx)}\right)}{cd^{3/2}}$$

$$-\frac{ab(-1+cx)\sqrt{d+cdx}\sqrt{e-cex}\sqrt{-de\left(1-c^2x^2\right)}\cot\left(\frac{1}{2}\arccos(cx)\right)\left(-4\arccos(cx)+\cot\left(\frac{1}{2}\arccos(cx)\right)\right.}{cd^2(1+cx)\sqrt{(-d-cdx)(e-cex)}\sqrt{1-c^2x^2}}$$

$$+\frac{b^2\sqrt{d+cdx}\sqrt{e-cex}\sqrt{-de\left(1-c^2x^2\right)}\left(\arccos(cx)\left(-6\arccos(cx)+\cot\left(\frac{1}{2}\arccos(cx)\right)\right)\left(\arccos(cx)(6i+\right.\right.}{3cd^2(1+cx)\sqrt{(-d-cdx}}$$

input
```
Integrate[(Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2)/(d + c*d*x)^(3/2),x]
```

output
```
(-2*a^2*Sqrt[-(e*(-1 + c*x))]*Sqrt[d*(1 + c*x)])/(c*d^2*(1 + c*x)) + (a^2*
Sqrt[e]*ArcTan[(c*x*Sqrt[-(e*(-1 + c*x))]*Sqrt[d*(1 + c*x)])/(Sqrt[d]*Sqrt
[e]*(-1 + c*x)*(1 + c*x))])/(c*d^(3/2)) - (a*b*(-1 + c*x)*Sqrt[d + c*d*x]*
Sqrt[e - c*e*x]*Sqrt[-(d*e*(1 - c^2*x^2))]*Cot[ArcCos[c*x]/2]*(-4*ArcCos[c
*x] + Cot[ArcCos[c*x]/2]*(ArcCos[c*x]^2 - 8*Log[Cos[ArcCos[c*x]/2]])))/(c*
d^2*(1 + c*x)*Sqrt[(-d - c*d*x)*(e - c*e*x)]*Sqrt[1 - c^2*x^2]) + (b^2*Sqr
t[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[-(d*e*(1 - c^2*x^2))]*(ArcCos[c*x]*(-6*A
rcCos[c*x] + Cot[ArcCos[c*x]/2]*(ArcCos[c*x]*(6*I + ArcCos[c*x]) - 24*Log[
1 + E^(I*ArcCos[c*x])])) + (24*I)*Cot[ArcCos[c*x]/2]*PolyLog[2, -E^(I*ArcC
os[c*x])]))/(3*c*d^2*(1 + c*x)*Sqrt[(-d - c*d*x)*(e - c*e*x)])
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(cdx+d)^{3/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{3/2}\int \frac{e^2(1-cx)^2(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2(1-c^2x^2)^{3/2}\int \frac{(1-cx)^2(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{5275}$$

$$\frac{e^2(1-c^2x^2)^{3/2}\int \left(\frac{2(1-cx)(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}-\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2(1-c^2x^2)^{3/2}\left(-\frac{8b\,\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{c}+\frac{2x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2(a+b\arccos(cx))^2}{c\sqrt{1-c^2x^2}}+\frac{(a+b\arccos(cx))^3}{3bc}+2\right.}{(cd}$$

input
```
Int[(Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2)/(d + c*d*x)^(3/2),x]
```

output
```
(e^2*(1 - c^2*x^2)^(3/2)*(((2*I)*(a + b*ArcCos[c*x])^2)/c - (2*(a + b*ArcC
os[c*x])^2)/(c*Sqrt[1 - c^2*x^2]) + (2*x*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c
^2*x^2] + (a + b*ArcCos[c*x])^3/(3*b*c) - (8*b*(a + b*ArcCos[c*x])*ArcTanh
[E^(I*ArcCos[c*x])])/c - (4*b*(a + b*ArcCos[c*x])*Log[1 - E^((2*I)*ArcCos[
c*x])])/c + ((4*I)*b^2*PolyLog[2, -E^(I*ArcCos[c*x])])/c - ((4*I)*b^2*Poly
Log[2, E^(I*ArcCos[c*x])])/c + ((2*I)*b^2*PolyLog[2, E^((2*I)*ArcCos[c*x])
])/c))/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.60

method	result
default	$-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^{2}x^{2}+1}\,(a+b\arccos(cx))^{3}}{3(cx+1)d^{2}(cx-1)cb}-\frac{2\sqrt{-e(cx-1)}\,\sqrt{d(cx+1)}\,\left(-i\sqrt{-c^{2}x^{2}+1}+cx-1\right)\left(\arccos(cx)^{2}b^{2}+2a\right.}{(cx+1)d^{2}(cx-1)c}$

input
```
int((-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/3*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x+1)/d^2/(
c*x-1)/c*(a+b*arccos(c*x))^3/b-2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-I*
(-c^2*x^2+1)^(1/2)+c*x-1)*(arccos(c*x)^2*b^2+2*arccos(c*x)*a*b+a^2)/(c*x+1
)/d^2/(c*x-1)/c-4*I*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2
)/(c*x+1)/d^2/(c*x-1)/c*b*(arccos(c*x)^2*b+2*I*arccos(c*x)*ln(1+c*x+I*(-c^
2*x^2+1)^(1/2))*b+2*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))*b+2*I*ln(1+c*x+I*
(-c^2*x^2+1)^(1/2))*a-2*I*ln(c*x+I*(-c^2*x^2+1)^(1/2))*a)
```

Fricas [F]

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx=\int \frac{\sqrt{-cex+e}\left(b\arccos\left(cx\right)+a\right)^{2}}{\left(cdx+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2),x, algorith
m="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sqr
t(-c*e*x + e)/(c^2*d^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx = \int \frac{\sqrt{-e\,(cx-1)}(a+b\,\mathrm{acos}\,(cx))^2}{(d\,(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)**(1/2)*(a+b*acos(c*x))**2/(c*d*x+d)**(3/2),x)
```

output
```
Integral(sqrt(-e*(c*x - 1))*(a + b*acos(c*x))**2/(d*(c*x + 1))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx = \int \frac{\sqrt{-cex+e}(b\arccos\,(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(-c*e*x + e)*(b*arccos(c*x) + a)^2/(c*d*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{e-c\,e\,x}}{(d+c\,d\,x)^{3/2}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(1/2))/(d + c*d*x)^(3/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(1/2))/(d + c*d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx = \frac{\sqrt{e}\left(2\sqrt{cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2-2\sqrt{-cx+1}\,a^2+2\sqrt{cx+1}\left(\int \frac{\sqrt{-}}{\sqrt{cx}}\right.\right.}{\sqrt{d}\,\sqrt{cx+1}\,c}$$

input
```
int((-c*e*x+e)^(1/2)*(a+b*acos(c*x))^2/(c*d*x+d)^(3/2),x)
```

output
```
(sqrt(e)*(2*sqrt(c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - 2*sqrt( -
c*x + 1)*a**2 + 2*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x))/(sqrt(c*x
 + 1)*c*x + sqrt(c*x + 1)),x)*a*b*c + sqrt(c*x + 1)*int((sqrt( - c*x + 1)*
acos(c*x)**2)/(sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b**2*c))/(sqrt(d)*sqr
t(c*x + 1)*c*d)
```

3.547 $\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 486

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \frac{ie^3(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{4b^2e^3(1-c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{e^3(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{2be^3(1-c^2x^2)^{5/2}(a+b\arccos(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{e^3(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{4be^3(1-c^2x^2)^{5/2}(a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{4ib^2e^3(1-c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}$$

output
```
1/3*I*e^3*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x
+e)^(5/2)-4/3*b^2*e^3*(-c^2*x^2+1)^(5/2)*cot(1/4*Pi+1/2*arccos(c*x))/c/(c*
d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+1/3*e^3*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))
^2*cot(1/4*Pi+1/2*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-2/3*b*e^
3*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*csc(1/4*Pi+1/2*arccos(c*x))^2/c/(c*
d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3*e^3*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))
^2*cot(1/4*Pi+1/2*arccos(c*x))*csc(1/4*Pi+1/2*arccos(c*x))^2/c/(c*d*x+d)^(
5/2)/(-c*e*x+e)^(5/2)-4/3*b*e^3*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*ln(1-
I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+4/3*I*b^2
*e^3*(-c^2*x^2+1)^(5/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d
)^(5/2)/(-c*e*x+e)^(5/2)
```

Mathematica [A] (verified)

Time = 3.55 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \frac{(-1+cx)\sqrt{d+cdx}\sqrt{e-cex}\left(a^2\sqrt{1-c^2x^2}+8b^2\cos^2\left(\frac{1}{2}\arccos(cx)\right)\right.}{}$$

input
```
Integrate[(Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2)/(d + c*d*x)^(5/2),x]
```

output
```
((-1 + c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a^2*Sqrt[1 - c^2*x^2] + 8*b^2
*Cos[ArcCos[c*x]/2]^2*Cot[ArcCos[c*x]/2] - 4*a*b*Cot[ArcCos[c*x]/2]^2 + 2*
b^2*ArcCos[c*x]^2*Cot[ArcCos[c*x]/2]*(1 + I*Cos[ArcCos[c*x]/2]^2*(I + Cot[
ArcCos[c*x]/2])) - 4*b*ArcCos[c*x]*Cot[ArcCos[c*x]/2]*(-a + b*Cot[ArcCos[c
*x]/2] + Cos[ArcCos[c*x]/2]^2*(a + 2*b*Cot[ArcCos[c*x]/2]*Log[1 + E^(I*Arc
Cos[c*x])])) - 8*a*b*Cos[ArcCos[c*x]/2]^2*Cot[ArcCos[c*x]/2]^2*Log[Cos[Arc
Cos[c*x]/2]] + (8*I)*b^2*Cos[ArcCos[c*x]/2]^2*Cot[ArcCos[c*x]/2]^2*PolyLog
[2, -E^(I*ArcCos[c*x])]))/(3*c*d^3*(1 + c*x)^2*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(cdx+d)^{5/2}}dx$$

$$\Big\downarrow \text{ 5179}$$

$$\frac{(1-c^2x^2)^{5/2}\int \frac{e^3(1-cx)^3(a+b\arccos(cx))^2}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^3(1-c^2x^2)^{5/2}\int \frac{(1-cx)^3(a+b\arccos(cx))^2}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{ 5275}$$

$$\frac{e^3(1-c^2x^2)^{5/2}\int \left(\frac{(a+b\arccos(cx))^2}{(-cx-1)\sqrt{1-c^2x^2}}+\frac{2(a+b\arccos(cx))^2}{(cx+1)^2\sqrt{1-c^2x^2}}\right)dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^3(1-c^2x^2)^{5/2}\left(-\frac{i(a+b\arccos(cx))^2}{3c}+\frac{4b\log\left(1+e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{3c}+\frac{\tan\left(\frac{1}{2}\arccos(cx)\right)(a+b\arccos(cx))^2}{3c}+\frac{2b\sec^2\left(\frac{1}{2}\text{ar}\right.}{}\right.}{(cdx+d)^5}$$

input
```
Int[(Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2)/(d + c*d*x)^(5/2),x]
```

output
```
(e^3*(1 - c^2*x^2)^(5/2)*(((-1/3*I)*(a + b*ArcCos[c*x])^2)/c + (4*b*(a + b
*ArcCos[c*x])*Log[1 + E^(I*ArcCos[c*x])])/(3*c) - (((4*I)/3)*b^2*PolyLog[2
, -E^(I*ArcCos[c*x])])/c + (2*b*(a + b*ArcCos[c*x])*Sec[ArcCos[c*x]/2]^2)/
(3*c) - (4*b^2*Tan[ArcCos[c*x]/2])/(3*c) + ((a + b*ArcCos[c*x])^2*Tan[ArcC
os[c*x]/2])/(3*c) - ((a + b*ArcCos[c*x])^2*Sec[ArcCos[c*x]/2]^2*Tan[ArcCos
[c*x]/2])/(3*c)))/((d + c*d*x)^(5/2)*(e - c*e*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2820 vs. $2(424) = 848$.

Time = 4.16 (sec) , antiderivative size = 2821, normalized size of antiderivative = 5.80

method	result	size
default	Expression too large to display	2821
parts	Expression too large to display	2821

input
```
int((-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
4*I*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(3*c^4*x^4+6*c^3*x^3+4*c^2*x^
2+2*c*x+1)/d^3*c/(c*x-1)*(-c^2*x^2+1)^(1/2)*x^2+4/3*I*b^2*(d*(c*x+1))^(1/2
)*(-e*(c*x-1))^(1/2)/(3*c^4*x^4+6*c^3*x^3+4*c^2*x^2+2*c*x+1)/d^3*c/(c*x-1)
*arccos(c*x)*x^2-4*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(3*c^4*x^4+6*c
^3*x^3+4*c^2*x^2+2*c*x+1)/d^3*c/(c*x-1)*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x^2
-4/3*b^2*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(c*x+1)/d
^3/c/(c*x-1)*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+4/3*I*b^2*(-c^2*x^
2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(c*x+1)/d^3/c/(c*x-1)*poly
log(2,-c*x-I*(-c^2*x^2+1)^(1/2))-2/3*I*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^
(1/2)/(3*c^4*x^4+6*c^3*x^3+4*c^2*x^2+2*c*x+1)/d^3*c^3/(c*x-1)*arccos(c*x)*
x^4+4/3*I*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(3*c^4*x^4+6*c^3*x^3+4*
c^2*x^2+2*c*x+1)/d^3/c/(c*x-1)*(-c^2*x^2+1)^(1/2)+4/3*b^2*(d*(c*x+1))^(1/2
)*(-e*(c*x-1))^(1/2)/(3*c^4*x^4+6*c^3*x^3+4*c^2*x^2+2*c*x+1)/d^3*c/(c*x-1)
*(-c^2*x^2+1)*x^2-4/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(3*c^4*x^4+
6*c^3*x^3+4*c^2*x^2+2*c*x+1)/d^3/c/(c*x-1)*(-c^2*x^2+1)^(1/2)*arccos(c*x)-
2*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(3*c^4*x^4+6*c^3*x^3+4*c^2*x^2+
2*c*x+1)/d^3*c^2/(c*x-1)*arccos(c*x)^2*x^3+4/3*b^2*(d*(c*x+1))^(1/2)*(-e*(
c*x-1))^(1/2)/(3*c^4*x^4+6*c^3*x^3+4*c^2*x^2+2*c*x+1)/d^3*c/(c*x-1)*arccos
(c*x)^2*x^2-2/3*I*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(3*c^4*x^4+6*c^
3*x^3+4*c^2*x^2+2*c*x+1)/d^3/c/(c*x-1)*arccos(c*x)+4/3*I*b^2*(d*(c*x+1)...
```

Fricas [F]

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx=\int \frac{\sqrt{-cex+e}(b\arccos{(cx)}+a)^2}{(cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2),x, algorith
m="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sqr
t(-c*e*x + e)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \int \frac{\sqrt{-e\,(cx-1)}(a+b\operatorname{acos}(cx))^2}{(d\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)**(1/2)*(a+b*acos(c*x))**2/(c*d*x+d)**(5/2),x)
```

output
```
Integral(sqrt(-e*(c*x - 1))*(a + b*acos(c*x))**2/(d*(c*x + 1))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \int \frac{\sqrt{-cex+e}(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(-c*e*x + e)*(b*arccos(c*x) + a)^2/(c*d*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{e-c\,e\,x}}{(d+c\,d\,x)^{5/2}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(1/2))/(d + c*d*x)^(5/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(1/2))/(d + c*d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{e-cex}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \frac{\sqrt{e}\left(\sqrt{-cx+1}\,a^2cx-\sqrt{-cx+1}\,a^2+6\sqrt{cx+1}\left(\int \frac{\sqrt{-cx+1}\,acos}{\sqrt{cx+1}\,c^2x^2+2\sqrt{cx+}}\right.\right.}{}$$

input
```
int((-c*e*x+e)^(1/2)*(a+b*acos(c*x))^2/(c*d*x+d)^(5/2),x)
```

output
```
(sqrt(e)*(sqrt( - c*x + 1)*a**2*c*x - sqrt( - c*x + 1)*a**2 + 6*sqrt(c*x +
 1)*int((sqrt( - c*x + 1)*acos(c*x))/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x
 + 1)*c*x + sqrt(c*x + 1)),x)*a*b*c**2*x + 6*sqrt(c*x + 1)*int((sqrt( - c*
x + 1)*acos(c*x))/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*
x + 1)),x)*a*b*c + 3*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)**2)/(sq
rt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b**2*c**2*
x + 3*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)**2)/(sqrt(c*x + 1)*c**
2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b**2*c))/(3*sqrt(d)*sqrt(
c*x + 1)*c*d**2*(c*x + 1))
```

3.548 $\int (d+cdx)^{5/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 697

$$\int (d+cdx)^{5/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{8b^2d(d+cdx)^{3/2}(e-cex)^{3/2}}{225c}$$
$$-\frac{1}{32}b^2dx(d+cdx)^{3/2}(e-cex)^{3/2}+\frac{16b^2d(d+cdx)^{3/2}(e-cex)^{3/2}}{75c\,(1-c^2x^2)}-\frac{15b^2dx(d+cdx)^{3/2}(e-cex)^{3/2}}{64\,(1-c^2x^2)}+\frac{2b^2d(\ldots}{\ldots}$$

output
```
8/225*b^2*d*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)/c-1/32*b^2*d*x*(c*d*x+d)^(3/2
)*(-c*e*x+e)^(3/2)+16/75*b^2*d*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)/c/(-c^2*x^
2+1)-15*b^2*d*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)/(-64*c^2*x^2+64)+2/125*b^
2*d*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(-c^2*x^2+1)/c+9/64*b^2*d*(c*d*x+d)^(
3/2)*(-c*e*x+e)^(3/2)*arccos(c*x)/c/(-c^2*x^2+1)^(3/2)+2/5*b*d*x*(c*d*x+d)
^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(3/2)-3/8*b*c*d*x^2
*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(3/2)-4/1
5*b*c^2*d*x^3*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))/(-c^2*x^2
+1)^(3/2)+2/25*b*c^4*d*x^5*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*
x))/(-c^2*x^2+1)^(3/2)+1/8*b*d*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(-c^2*x^2+
1)^(1/2)*(a+b*arccos(c*x))/c+1/4*d*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b
*arccos(c*x))^2+3*d*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2
/(-8*c^2*x^2+8)-1/5*d*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(-c^2*x^2+1)*(a+b*a
rccos(c*x))^2/c+1/8*d*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^3
/b/c/(-c^2*x^2+1)^(3/2)
```

Mathematica [A] (verified)

Time = 3.96 (sec) , antiderivative size = 684, normalized size of antiderivative = 0.98

$$\int (d+cdx)^{5/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx =$$

$$-\frac{d^2e\left(108000b^2\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)^3+324000a^2\sqrt{d}\sqrt{e}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{e-cex}}{\sqrt{d}\sqrt{e}(-1+c^2x^2)}\right)+18\right.}{}$$

input
```
Integrate[(d + c*d*x)^(5/2)*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/864000*(d^2*e*(108000*b^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^3
 + 324000*a^2*Sqrt[d]*Sqrt[e]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x
]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] + 180*b*Sqrt[d + c*d*
x]*Sqrt[e - c*e*x]*ArcCos[c*x]*(-1200*b*Cos[2*ArcCos[c*x]] - 200*b*Cos[3*A
rcCos[c*x]] + 75*b*Cos[4*ArcCos[c*x]] + 24*b*Cos[5*ArcCos[c*x]] + 60*(4*(5
*b*c*x + 8*a*Sqrt[1 - c^2*x^2] - 16*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 8*a*c^4*
x^4*Sqrt[1 - c^2*x^2]) - 40*a*Sin[2*ArcCos[c*x]] + 5*a*Sin[4*ArcCos[c*x]])
) + 1800*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^2*(180*a + 20*b*Sqr
t[1 - c^2*x^2] - 80*b*Sqrt[1 - c^2*x^2]*Cos[2*ArcCos[c*x]] - 120*b*Sin[2*A
rcCos[c*x]] + 10*b*Sin[3*ArcCos[c*x]] + 15*b*Sin[4*ArcCos[c*x]] + 6*b*Sin[
5*ArcCos[c*x]]) + Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(345600*a*b*c*x - 230400
*a*b*c^3*x^3 + 69120*a*b*c^5*x^5 + 172800*a^2*Sqrt[1 - c^2*x^2] - 200000*b
^2*Sqrt[1 - c^2*x^2] - 540000*a^2*c*x*Sqrt[1 - c^2*x^2] - 345600*a^2*c^2*x
^2*Sqrt[1 - c^2*x^2] + 216000*a^2*c^3*x^3*Sqrt[1 - c^2*x^2] + 172800*a^2*c
^4*x^4*Sqrt[1 - c^2*x^2] + 8000*b*(-27*a + 4*b*Sqrt[1 - c^2*x^2])*Cos[2*Ar
cCos[c*x]] + 13500*a*b*Cos[4*ArcCos[c*x]] + 108000*b^2*Sin[2*ArcCos[c*x]]
- 4000*b^2*Sin[3*ArcCos[c*x]] - 3375*b^2*Sin[4*ArcCos[c*x]] - 864*b^2*Sin[
5*ArcCos[c*x]])))/(c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^{5/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\int d(cx+1)\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2dx}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d(cdx+d)^{3/2}(e-cex)^{3/2}\int (cx+1)\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2dx}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{d(cdx+d)^{3/2}(e-cex)^{3/2}\int \left(cx\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2+\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2\right)dx}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(cdx+d)^{3/2}(e-cex)^{3/2}\left(-\frac{2}{25}bc^4x^5(a+b\arccos(cx))+\frac{4}{15}bc^2x^3(a+b\arccos(cx))+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\,\text{ar}\right.}{}$$

input

```
Int[(d + c*d*x)^(5/2)*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output

```
(d*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*((16*b^2*Sqrt[1 - c^2*x^2])/(75*c)
- (15*b^2*x*Sqrt[1 - c^2*x^2])/64 + (8*b^2*(1 - c^2*x^2)^(3/2))/(225*c) -
(b^2*x*(1 - c^2*x^2)^(3/2))/32 + (2*b^2*(1 - c^2*x^2)^(5/2))/(125*c) - (2*
b*x*(a + b*ArcCos[c*x]))/5 + (3*b*c*x^2*(a + b*ArcCos[c*x]))/8 + (4*b*c^2*
x^3*(a + b*ArcCos[c*x]))/15 - (2*b*c^4*x^5*(a + b*ArcCos[c*x]))/25 - (b*(1
 - c^2*x^2)^2*(a + b*ArcCos[c*x]))/(8*c) + (3*x*Sqrt[1 - c^2*x^2]*(a + b*A
rcCos[c*x])^2)/8 + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/4 - ((1 -
 c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/(5*c) - (a + b*ArcCos[c*x])^3/(8*b*
c) + (9*b^2*ArcSin[c*x])/(64*c)))/(1 - c^2*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.94 (sec) , antiderivative size = 2264, normalized size of antiderivative = 3.25

method	result	size
default	Expression too large to display	2264
parts	Expression too large to display	2264

input
```
int((c*d*x+d)^(5/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/5*a^2/c/e*(c*d*x+d)^(5/2)*(-c*e*x+e)^(5/2)-1/4*a^2*d/c/e*(c*d*x+d)^(3/2
)*(-c*e*x+e)^(5/2)-1/4*a^2*d^2/c/e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(5/2)+1/8*a^
2*d^2/c*(-c*e*x+e)^(3/2)*(c*d*x+d)^(1/2)+3/8*a^2*d^2*e/c*(-c*e*x+e)^(1/2)*
(c*d*x+d)^(1/2)+3/8*a^2*d^3*e^2*((-c*e*x+e)*(c*d*x+d))^(1/2)/(-c*e*x+e)^(1
/2)/(c*d*x+d)^(1/2)/(c^2*d*e)^(1/2)*arctan((c^2*d*e)^(1/2)*x/(-c^2*d*e*x^2
+d*e)^(1/2))+b^2*(1/8*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1
/2)/(c^2*x^2-1)/c*arccos(c*x)^3*e*d^2-1/4000*(d*(c*x+1))^(1/2)*(-e*(c*x-1)
)^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*I*(-c^2*x^2+1)^(1/2)*x^5*c^5+13*c^2*x^2-
20*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+5*I*(-c^2*x^2+1)^(1/2)*c*x-1)*(10*I*arccos
(c*x)+25*arccos(c*x)^2-2)*e*d^2/(c^2*x^2-1)/c-1/512*(d*(c*x+1))^(1/2)*(-e*
(c*x-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-
8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(4*I*arccos(c*x)+8*ar
ccos(c*x)^2-1)*e*d^2/(c^2*x^2-1)/c-1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/
2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)^2-2-2*I*arccos(c*x))
*e*d^2/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*I*(-c^2
*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*arccos(c*x)
^2-1-2*I*arccos(c*x))*e*d^2/(c^2*x^2-1)/c-1/18000*(d*(c*x+1))^(1/2)*(-e*(c
*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(330*I*arccos(c*x)+675*
arccos(c*x)^2-134)*cos(4*arccos(c*x))*e*d^2/(c^2*x^2-1)/c-1/9000*(d*(c*x+1
))^(1/2)*(-e*(c*x-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(210*I...
```

Fricas [F]

$$\int (d+cdx)^{5/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\int (cdx+d)^{\frac{5}{2}}(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algorith
m="fricas")
```

output
```
integral(-(a^2*c^3*d^2*e*x^3 + a^2*c^2*d^2*e*x^2 - a^2*c*d^2*e*x - a^2*d^2
*e + (b^2*c^3*d^2*e*x^3 + b^2*c^2*d^2*e*x^2 - b^2*c*d^2*e*x - b^2*d^2*e)*a
rccos(c*x)^2 + 2*(a*b*c^3*d^2*e*x^3 + a*b*c^2*d^2*e*x^2 - a*b*c*d^2*e*x -
a*b*d^2*e)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+cdx)^{5/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\text{Timed out}$$

input
```
integrate((c*d*x+d)**(5/2)*(-c*e*x+e)**(3/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+cdx)^{5/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (d+cdx)^{5/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\int (cdx+d)^{\frac{5}{2}}(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algorith
m="giac")
```

output
```
integrate((c*d*x + d)^(5/2)*(-c*e*x + e)^(3/2)*(b*arccos(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)^{5/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,(d+c\,d\,x)^{5/2}\,(e-c\,e\,x)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(5/2)*(e - c*e*x)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(5/2)*(e - c*e*x)^(3/2), x)
```

Reduce [F]

$$\int (d+cdx)^{5/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{\sqrt{e}\,\sqrt{d}\,d^2e\left(-30asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2-8\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^4x^4-10\sqrt{cx}\right.}{}$$

input
```
int((c*d*x+d)^(5/2)*(-c*e*x+e)^(3/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(e)*sqrt(d)*d**2*e*( - 30*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - 8*sqr
t(c*x + 1)*sqrt( - c*x + 1)*a**2*c**4*x**4 - 10*sqrt(c*x + 1)*sqrt( - c*x
+ 1)*a**2*c**3*x**3 + 16*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c**2*x**2 + 2
5*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c*x - 8*sqrt(c*x + 1)*sqrt( - c*x +
1)*a**2 - 80*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**3,x)*a*b*c**4
 - 80*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**2,x)*a*b*c**3 + 80*i
nt(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x,x)*a*b*c**2 + 80*int(sqrt(c*
x + 1)*sqrt( - c*x + 1)*acos(c*x),x)*a*b*c - 40*int(sqrt(c*x + 1)*sqrt( -
c*x + 1)*acos(c*x)**2*x**3,x)*b**2*c**4 - 40*int(sqrt(c*x + 1)*sqrt( - c*x
 + 1)*acos(c*x)**2*x**2,x)*b**2*c**3 + 40*int(sqrt(c*x + 1)*sqrt( - c*x +
1)*acos(c*x)**2*x,x)*b**2*c**2 + 40*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*aco
s(c*x)**2,x)*b**2*c))/(40*c)
```

3.549 $\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 362

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx =$$
$$-\frac{1}{32}b^2x(d+cdx)^{3/2}(e-cex)^{3/2}-\frac{15b^2x(d+cdx)^{3/2}(e-cex)^{3/2}}{64\,(1-c^2x^2)}+\frac{9b^2(d+cdx)^{3/2}(e-cex)^{3/2}\arccos(cx)}{64c\,(1-c^2x^2)^{3/2}}-$$

output
```
-1/32*b^2*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)-15*b^2*x*(c*d*x+d)^(3/2)*(-c*
e*x+e)^(3/2)/(-64*c^2*x^2+64)+9/64*b^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*ar
ccos(c*x)/c/(-c^2*x^2+1)^(3/2)-3/8*b*c*x^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2
)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(3/2)+1/8*b*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3
/2)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+1/4*x*(c*d*x+d)^(3/2)*(-c*e*x+e
)^(3/2)*(a+b*arccos(c*x))^2+3*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcc
os(c*x))^2/(-8*c^2*x^2+8)+1/8*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos
(c*x))^3/b/c/(-c^2*x^2+1)^(3/2)
```

Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.03

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \frac{-32b^2de\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)^3-96a^2d^{3/2}e^{3/2}\sqrt{1-c^2x^2}\arctan\left(\ldots\right.}{\ldots}$$

input
```
Integrate[(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(-32*b^2*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^3 - 96*a^2*d^(3/2
)*e^(3/2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(
Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] - 8*b*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]
*ArcCos[c*x]^2*(12*a - 8*b*Sin[2*ArcCos[c*x]] + b*Sin[4*ArcCos[c*x]]) + d*
e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(160*a^2*c*x*Sqrt[1 - c^2*x^2] - 64*a^2*
c^3*x^3*Sqrt[1 - c^2*x^2] + 64*a*b*Cos[2*ArcCos[c*x]] - 4*a*b*Cos[4*ArcCos
[c*x]] - 32*b^2*Sin[2*ArcCos[c*x]] + b^2*Sin[4*ArcCos[c*x]]) - 4*b*d*e*Sqr
t[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]*(-16*b*Cos[2*ArcCos[c*x]] + b*Cos
[4*ArcCos[c*x]] + 4*a*(-8*Sin[2*ArcCos[c*x]] + Sin[4*ArcCos[c*x]])))/(256*
c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.76, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {5179, 5159, 5157, 5139, 262, 223, 5153, 5183, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \ 5179$$

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\int \left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2dx}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \; 5159$$

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\int x(1-c^2x^2)\,(a+b\arccos(cx))dx+\frac{3}{4}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx+\frac{1}{4}x(1}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \; 5157$$

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\int x(1-c^2x^2)\,(a+b\arccos(cx))dx+\frac{3}{4}\left(\frac{1}{2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+bc\int x(a+b\arcco}{(1-c^2x^2)^{3/2}}$$

$$\Big\downarrow \; 5139$$

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\int x(1-c^2x^2)\,(a+b\arccos(cx))dx+\frac{3}{4}\left(bc\left(\frac{1}{2}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(c}{(1-c^2x^2)}$$

$$\Big\downarrow \; 262$$

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\int x(1-c^2x^2)\,(a+b\arccos(cx))dx+\frac{3}{4}\left(bc\left(\frac{1}{2}bc\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2}{(1}$$

$$\Big\downarrow \; 223$$

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)+}{(1-}$$

$$\Big\downarrow \; 5153$$

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\int x(1-c^2x^2)\,(a+b\arccos(cx))dx+\frac{3}{4}\left(bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}\right.\right.\right.}{(1-c^2x}$$

$$\Big\downarrow \; 5183$$

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\left(-\frac{b\int (1-c^2x^2)^{3/2}dx}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)+\frac{3}{4}\left(bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\right.\right.\right.}{(1-}$$

$$\Big\downarrow \; 211$$

$$(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\left(-\frac{b\left(\frac{3}{4}\int\sqrt{1-c^2x^2}dx+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)+\frac{3}{4}\left(bc\left(\frac{1}{2}x^2(a+b\,\text{ar}\right.\right.\right.$$

$\downarrow$ 211

$$(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\left(-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)+\frac{3}{4}\left(\right.\right.$$

$\downarrow$ 223

$$(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\left(-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}\right)+\frac{3}{4}\left(bc\left(\frac{1}{2}x\right.\right.\right.$$

input
```
Int[(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*((x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos
[c*x])^2)/4 + (3*((x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/2 - (a + b*A
rcCos[c*x])^3/(6*b*c) + b*c*((x^2*(a + b*ArcCos[c*x]))/2 + (b*c*(-1/2*(x*S
qrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2)))/4 + (b*c*(-1/4*((1 - c^
2*x^2)^2*(a + b*ArcCos[c*x]))/c^2 - (b*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x
*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/(4*c)))/2))/(1 - c^2*x^2)^
(3/2)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.23 (sec) , antiderivative size = 1099, normalized size of antiderivative = 3.04

method	result	size
default	Expression too large to display	1099
parts	Expression too large to display	1099

input
```
int((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/4*a^2/c/e*(c*d*x+d)^(3/2)*(-c*e*x+e)^(5/2)-1/4*a^2*d/c/e*(c*d*x+d)^(1/2
)*(-c*e*x+e)^(5/2)+1/8*a^2*d/c*(-c*e*x+e)^(3/2)*(c*d*x+d)^(1/2)+3/8*a^2*d*
e/c*(-c*e*x+e)^(1/2)*(c*d*x+d)^(1/2)+3/8*a^2*d^2*e^2*((-c*e*x+e)*(c*d*x+d)
)^(1/2)/(-c*e*x+e)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*e)^(1/2)*arctan((c^2*d*e)^
(1/2)*x/(-c^2*d*e*x^2+d*e)^(1/2))+b^2*(1/8*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^
(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^3*d*e-1/512*(d*(c*x+1))
^(1/2)*(-e*(c*x-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4
*c^4+4*c*x-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(4*I*arcco
s(c*x)+8*arccos(c*x)^2-1)*d*e/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-e*(c*
x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2
)-2*c*x)*(2*arccos(c*x)^2-1-2*I*arccos(c*x))*d*e/(c^2*x^2-1)/c-3/512*(d*(c
*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(20*
I*arccos(c*x)+24*arccos(c*x)^2-11)*cos(3*arccos(c*x))*d*e/(c^2*x^2-1)/c-1/
512*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)
-I)*(68*I*arccos(c*x)+56*arccos(c*x)^2-31)*sin(3*arccos(c*x))*d*e/(c^2*x^2
-1)/c)+2*a*b*(3/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)
/(c^2*x^2-1)/c*arccos(c*x)^2*d*e-1/256*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2
)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-8*I*(-c^2*x^2
+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(I+4*arccos(c*x))*d*e/(c^2*x^2-1)/
c+1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^...
```

Fricas [F]

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \int (cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algorith
m="fricas")
```

output
```
integral(-(a^2*c^2*d*e*x^2 - a^2*d*e + (b^2*c^2*d*e*x^2 - b^2*d*e)*arccos(
c*x)^2 + 2*(a*b*c^2*d*e*x^2 - a*b*d*e)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-
c*e*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+d)**(3/2)*(-c*e*x+e)**(3/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \int (cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algorith
m="giac")
```

output
```
integrate((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)*(b*arccos(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,(d+c\,d\,x)^{3/2}\,(e-c\,e\,x)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2), x)
```

Reduce [F]

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{\sqrt{e}\,\sqrt{d}\,de\left(-6asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2-2\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^3x^3+5\sqrt{cx+1}\right.}{}$$

input
```
int((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(e)*sqrt(d)*d*e*( - 6*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - 2*sqrt(c*
x + 1)*sqrt( - c*x + 1)*a**2*c**3*x**3 + 5*sqrt(c*x + 1)*sqrt( - c*x + 1)*
a**2*c*x - 16*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**2,x)*a*b*c**
3 + 16*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x),x)*a*b*c - 8*int(sqrt(
c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2*x**2,x)*b**2*c**3 + 8*int(sqrt(c*x
+ 1)*sqrt( - c*x + 1)*acos(c*x)**2,x)*b**2*c))/(8*c)
```

3.550 $\int \sqrt{d+cdx}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 455

$$\begin{aligned}
&\int \sqrt{d+cdx}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \\
&-\frac{4b^2e\sqrt{d+cdx}\sqrt{e-cex}}{9c}-\frac{1}{4}b^2ex\sqrt{d+cdx}\sqrt{e-cex} \\
&-\frac{2b^2e\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)}{27c}+\frac{b^2e\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)}{4c\sqrt{1-c^2x^2}} \\
&-\frac{2bex\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}} \\
&-\frac{bcex^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{2\sqrt{1-c^2x^2}} \\
&+\frac{2bc^2ex^3\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{9\sqrt{1-c^2x^2}} \\
&+\frac{1}{2}ex\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2 \\
&+\frac{e\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)\,(a+b\arccos(cx))^2}{3c} \\
&+\frac{e\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}
\end{aligned}$$

output
```
-4/9*b^2*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)/c-1/4*b^2*e*x*(c*d*x+d)^(1/2)*
(-c*e*x+e)^(1/2)-2/27*b^2*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)/
c+1/4*b^2*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*arccos(c*x)/c/(-c^2*x^2+1)^(1
/2)-2/3*b*e*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2
+1)^(1/2)-1/2*b*c*e*x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))
/(-c^2*x^2+1)^(1/2)+2/9*b*c^2*e*x^3*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*
arccos(c*x))/(-c^2*x^2+1)^(1/2)+1/2*e*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(
a+b*arccos(c*x))^2+1/3*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)*(a+
b*arccos(c*x))^2/c+1/6*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x)
)^3/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 2.79 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.00

$$\int \sqrt{d+cdx}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \frac{-36b^2e\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)^3-108a^2\sqrt{d}e^{3/2}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{e-}}{\sqrt{d}\sqrt{e}(-1+c^2x}\right.}{}$$

input
```
Integrate[Sqrt[d + c*d*x]*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(-36*b^2*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^3 - 108*a^2*Sqrt[d]
*e^(3/2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(S
qrt[d]*Sqrt[e]*(-1 + c^2*x^2))] - 18*b*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*A
rcCos[c*x]^2*(6*a - 2*b*Sqrt[1 - c^2*x^2] + 2*b*Sqrt[1 - c^2*x^2]*Cos[2*Ar
cCos[c*x]] - 3*b*Sin[2*ArcCos[c*x]]) + e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(
-104*b^2*Sqrt[1 - c^2*x^2] - 48*a*b*c*x*(-3 + c^2*x^2) - 36*a^2*Sqrt[1 - c
^2*x^2]*(-2 - 3*c*x + 2*c^2*x^2) + 2*b*(27*a + 4*b*Sqrt[1 - c^2*x^2])*Cos[
2*ArcCos[c*x]] - 27*b^2*Sin[2*ArcCos[c*x]]) - 6*b*e*Sqrt[d + c*d*x]*Sqrt[e
 - c*e*x]*ArcCos[c*x]*(-9*b*Cos[2*ArcCos[c*x]] + 2*(-9*b*c*x - 12*a*Sqrt[1
 - c^2*x^2] + 12*a*c^2*x^2*Sqrt[1 - c^2*x^2] + b*Cos[3*ArcCos[c*x]] - 9*a*
Sin[2*ArcCos[c*x]])))/(216*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cdx+d}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\int e(1-cx)\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\sqrt{cdx+d}\sqrt{e-cex}\int (1-cx)\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{e\sqrt{cdx+d}\sqrt{e-cex}\int \left(\sqrt{1-c^2x^2}(a+b\arccos(cx))^2-cx\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\right)dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\sqrt{cdx+d}\sqrt{e-cex}\left(-\frac{2}{9}bc^2x^3(a+b\arccos(cx))+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2+\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{3c}\right.}{}$$

input
```
Int[Sqrt[d + c*d*x]*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*((-4*b^2*Sqrt[1 - c^2*x^2])/(9*c) - (b^
2*x*Sqrt[1 - c^2*x^2])/4 - (2*b^2*(1 - c^2*x^2)^(3/2))/(27*c) + (2*b*x*(a
+ b*ArcCos[c*x]))/3 + (b*c*x^2*(a + b*ArcCos[c*x]))/2 - (2*b*c^2*x^3*(a +
b*ArcCos[c*x]))/9 + (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/2 + ((1 -
c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/(3*c) - (a + b*ArcCos[c*x])^3/(6*b*c
) + (b^2*ArcSin[c*x])/(4*c)))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 1354, normalized size of antiderivative = 2.98

method	result	size
default	Expression too large to display	1354
parts	Expression too large to display	1354

input
```
int((c*d*x+d)^(1/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/3*a^2/c/e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(5/2)+1/6*a^2/c*(-c*e*x+e)^(3/2)*(
c*d*x+d)^(1/2)+1/2*a^2*e/c*(-c*e*x+e)^(1/2)*(c*d*x+d)^(1/2)+1/2*a^2*d*e^2*
((-c*e*x+e)*(c*d*x+d))^(1/2)/(-c*e*x+e)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*e)^(1
/2)*arctan((c^2*d*e)^(1/2)*x/(-c^2*d*e*x^2+d*e)^(1/2))+b^2*(1/6*(d*(c*x+1)
)^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^3*
e-1/216*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*I*(-c^
2*x^2+1)^(1/2)*x^3*c^3-3*I*(-c^2*x^2+1)^(1/2)*c*x+1)*(6*I*arccos(c*x)+9*ar
ccos(c*x)^2-2)*e/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(
2*c^3*x^3-2*c*x+2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(2*ar
ccos(c*x)^2-1+2*I*arccos(c*x))*e/(c^2*x^2-1)/c+1/8*(d*(c*x+1))^(1/2)*(-e*(
c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)^2-2-2*I*a
rccos(c*x))*e/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*
I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*arcc
os(c*x)^2-1-2*I*arccos(c*x))*e/(c^2*x^2-1)/c-1/54*(d*(c*x+1))^(1/2)*(-e*(c
*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(12*I*arccos(c*x)+9*arc
cos(c*x)^2-14)*cos(2*arccos(c*x))*e/(c^2*x^2-1)/c-1/108*(d*(c*x+1))^(1/2)*
(-e*(c*x-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(30*I*arccos(c*x)+
9*arccos(c*x)^2-26)*sin(2*arccos(c*x))*e/(c^2*x^2-1)/c)+2*a*b*(1/4*(d*(c*x
+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)
^2*e-1/72*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*I...
```

Fricas [F]

$$\int \sqrt{d+cdx}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \int \sqrt{cdx+d}(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algorith
m="fricas")
```

output
```
integral(-(a^2*c*e*x - a^2*e + (b^2*c*e*x - b^2*e)*arccos(c*x)^2 + 2*(a*b*
c*e*x - a*b*e)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e), x)
```

Sympy [F]

$$\int \sqrt{d+cdx}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \int \sqrt{d\,(cx+1)}(-e(cx-1))^{\frac{3}{2}}\,(a+b\,\mathrm{acos}\,(cx))^2\;dx$$

input
```
integrate((c*d*x+d)**(1/2)*(-c*e*x+e)**(3/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral(sqrt(d*(c*x + 1))*(-e*(c*x - 1))**(3/2)*(a + b*acos(c*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+cdx}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \sqrt{d+cdx}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \int \sqrt{cdx+d}(-cex+e)^{\frac{3}{2}}(b\arccos\,(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algorith
m="giac")
```

output
```
integrate(sqrt(c*d*x + d)*(-c*e*x + e)^(3/2)*(b*arccos(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+cdx}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d+c\,d\,x}\,(e-c\,e\,x)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(3/2), x)
```

Reduce [F]

$$\int \sqrt{d+cdx}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{\sqrt{e}\,\sqrt{d}\,e\left(-6asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2-2\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^2x^2+3\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2\right.}{}$$

input
```
int((c*d*x+d)^(1/2)*(-c*e*x+e)^(3/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(e)*sqrt(d)*e*( - 6*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - 2*sqrt(c*x
+ 1)*sqrt( - c*x + 1)*a**2*c**2*x**2 + 3*sqrt(c*x + 1)*sqrt( - c*x + 1)*a*
*2*c*x + 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2 - 12*int(sqrt(c*x + 1)*sqrt
( - c*x + 1)*acos(c*x)*x,x)*a*b*c**2 + 12*int(sqrt(c*x + 1)*sqrt( - c*x +
1)*acos(c*x),x)*a*b*c - 6*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2*
x,x)*b**2*c**2 + 6*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2,x)*b**2
*c))/(6*c)
```

3.551 $\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 398

$$\begin{aligned}\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx =& -\frac{4b^2e^2(1-c^2x^2)}{c\sqrt{d+cdx}\sqrt{e-cex}}+\frac{b^2e^2x(1-c^2x^2)}{4\sqrt{d+cdx}\sqrt{e-cex}}\\&-\frac{b^2e^2\sqrt{1-c^2x^2}\arccos(cx)}{4c\sqrt{d+cdx}\sqrt{e-cex}}-\frac{4be^2x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{\sqrt{d+cdx}\sqrt{e-cex}}\\&+\frac{bce^2x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2\sqrt{d+cdx}\sqrt{e-cex}}+\frac{2e^2(1-c^2x^2)\,(a+b\arccos(cx))^2}{c\sqrt{d+cdx}\sqrt{e-cex}}\\&-\frac{e^2x(1-c^2x^2)\,(a+b\arccos(cx))^2}{2\sqrt{d+cdx}\sqrt{e-cex}}+\frac{e^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{2bc\sqrt{d+cdx}\sqrt{e-cex}}\end{aligned}$$

output
```
-4*b^2*e^2*(-c^2*x^2+1)/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+1/4*b^2*e^2*x*(
-c^2*x^2+1)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-1/4*b^2*e^2*(-c^2*x^2+1)^(1/2
)*arccos(c*x)/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-4*b*e^2*x*(-c^2*x^2+1)^(1
/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+1/2*b*c*e^2*x^2*(-c
^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*e^2*(
-c^2*x^2+1)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-1/2*e^2
*x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+1/2*e
^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^3/b/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(
1/2)
```

Mathematica [A] (verified)

Time = 7.72 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.31

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx = \frac{24a^2e(4-cx)\sqrt{d+cdx}\sqrt{e-cex}\sqrt{1-c^2x^2}+48abe\sqrt{d+cdx}\sqrt{e-}}{}$$

input
```
Integrate[((e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2)/Sqrt[d + c*d*x],x]
```

output
```
(24*a^2*e*(4 - c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2] + 48
*a*b*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(2*c*x + 2*Sqrt[1 - c^2*x^2]*ArcCos
[c*x] - ArcCos[c*x]^2) - 72*a^2*Sqrt[d]*e^(3/2)*Sqrt[1 - c^2*x^2]*ArcTan[(
c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] - 8
*b^2*e*(1 - c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(6*Sqrt[1 - c^2*x^2] - 6*
c*x*ArcCos[c*x] - 3*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2 + ArcCos[c*x]^3)*Csc[A
rcCos[c*x]/2]^2 - b^2*e*(1 - c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Csc[ArcC
os[c*x]/2]^2*(48*Sqrt[1 - c^2*x^2] + 4*ArcCos[c*x]^3 + 6*ArcCos[c*x]*(-8*c
*x + Cos[2*ArcCos[c*x]]) - 3*Sin[2*ArcCos[c*x]] + 6*ArcCos[c*x]^2*(-4*Sqrt
[1 - c^2*x^2] + Sin[2*ArcCos[c*x]])) + 6*a*b*e*(1 - c*x)*Sqrt[d + c*d*x]*S
qrt[e - c*e*x]*Csc[ArcCos[c*x]/2]^2*(8*c*x - Cos[2*ArcCos[c*x]] - 2*ArcCos
[c*x]*(-4*Sqrt[1 - c^2*x^2] + ArcCos[c*x] + Sin[2*ArcCos[c*x]])))/(48*c*d*
Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.55, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5179, 27, 5273, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{\sqrt{cdx+d}}\,dx$$

$$\Big\downarrow \; 5179$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{e^2(1-cx)^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \; 27$$

$$\frac{e^2\sqrt{1-c^2x^2}\int \frac{(1-cx)^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \; 5273$$

$$-\frac{e^2\sqrt{1-c^2x^2}\int \left(c-c^2x\right)^2(a+b\arccos(cx))^2d\arccos(cx)}{c^3\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{e^2\sqrt{1-c^2x^2}\int (a+b\arccos(cx))^2\left(c-c\sin\left(\arccos(cx)+\frac{\pi}{2}\right)\right)^2d\arccos(cx)}{c^3\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \; 3798$$

$$-\frac{e^2\sqrt{1-c^2x^2}\int \left(x^2(a+b\arccos(cx))^2c^4-2x(a+b\arccos(cx))^2c^3+(a+b\arccos(cx))^2c^2\right)d\arccos(cx)}{c^3\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{e^2\sqrt{1-c^2x^2}\left(\frac{1}{2}bc^4x^2(a+b\arccos(cx))-4bc^3x(a+b\arccos(cx))-2c^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2+\frac{c^2(a+b a}{}\right.}{c^3\sqrt{cdx+d}\sqrt{}}$$

input
```
Int[((e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2)/Sqrt[d + c*d*x],x]
```

output
```
-((e^2*Sqrt[1 - c^2*x^2]*(4*b^2*c^2*Sqrt[1 - c^2*x^2] - (b^2*c^3*x*Sqrt[1
- c^2*x^2])/4 - (b^2*c^2*ArcCos[c*x])/4 - 4*b*c^3*x*(a + b*ArcCos[c*x]) +
(b*c^4*x^2*(a + b*ArcCos[c*x]))/2 - 2*c^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[
c*x])^2 + (c^3*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/2 + (c^2*(a + b*
ArcCos[c*x])^3)/(2*b)))/(c^3*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5273
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[-(c^(m + 1)*Sqrt[d])^(-1)   Subs
t[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &
& (GtQ[m, 0] || IGtQ[n, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.95 (sec) , antiderivative size = 1357, normalized size of antiderivative = 3.41

method	result	size
default	Expression too large to display	1357
parts	Expression too large to display	1357

input

```
int((-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2),x,method=_RETURNV
ERBOSE)
```

output

```
1/2*a^2/d/c*(-c*e*x+e)^(3/2)*(c*d*x+d)^(1/2)+3/2*a^2*e/d/c*(-c*e*x+e)^(1/2
)*(c*d*x+d)^(1/2)+3/2*a^2*e^2*((-c*e*x+e)*(c*d*x+d))^(1/2)/(-c*e*x+e)^(1/2
)/(c*d*x+d)^(1/2)/(c^2*d*e)^(1/2)*arctan((c^2*d*e)^(1/2)*x/(-c^2*d*e*x^2+d
*e)^(1/2))+b^2*(1/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2
)/(c*x+1)/d/c/(c*x-1)*arccos(c*x)^3*e-1/32*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^
(1/2)*(-2*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+4*c^3*x^3+1-2*I*(-c^2*x^2
+1)^(1/2)*c*x-I*(-c^2*x^2+1)^(1/2)-3*c*x)*(2*arccos(c*x)^2-1+2*I*arccos(c*
x))*e/(c*x+1)/d/c/(c*x-1)+1/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*(-c^
2*x^2+1)^(1/2)+c*x-1)*(arccos(c*x)^2-2+2*I*arccos(c*x))*e/(c*x+1)/d/c/(c*x
-1)+(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^
2-1)*(arccos(c*x)^2-2-2*I*arccos(c*x))*e/(c*x+1)/d/c/(c*x-1)-1/32*(d*(c*x+
1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2+I*(-c^
2*x^2+1)^(1/2)-c*x-1)*(2*arccos(c*x)^2-1-2*I*arccos(c*x))*e/(c*x+1)/d/c/(c
*x-1)-1/8*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x-
1)*(7*I*arccos(c*x)+4*arccos(c*x)^2-8)*cos(2*arccos(c*x))*e/(c*x+1)/d/c/(c
*x-1)-1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*c*x-I+(-c^2*x^2+1)^(1/2
))*(16*I*arccos(c*x)+6*arccos(c*x)^2-15)*sin(2*arccos(c*x))*e/(c*x+1)/d/c/
(c*x-1))+2*a*b*(3/4*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2
)/(c*x+1)/d/c/(c*x-1)*arccos(c*x)^2*e-1/32*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^
(1/2)*(-2*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+4*c^3*x^3+1-2*I*(-c^2*...
```

Fricas [F]

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx=\int \frac{(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{\sqrt{cdx+d}}\,dx$$

input
```
integrate((-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-(a^2*c*e*x - a^2*e + (b^2*c*e*x - b^2*e)*arccos(c*x)^2 + 2*(a*b*
c*e*x - a*b*e)*arccos(c*x))*sqrt(-c*e*x + e)/sqrt(c*d*x + d), x)
```

Sympy [F]

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx=\int \frac{(-e(cx-1))^{\frac{3}{2}}\,(a+b\operatorname{acos}(cx))^2}{\sqrt{d\,(cx+1)}}\,dx$$

input
```
integrate((-c*e*x+e)**(3/2)*(a+b*acos(c*x))**2/(c*d*x+d)**(1/2),x)
```

output
```
Integral((-e*(c*x - 1))**(3/2)*(a + b*acos(c*x))**2/sqrt(d*(c*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx=\int \frac{(-cex+e)^{\frac{3}{2}}(b\arccos{(cx)}+a)^2}{\sqrt{cdx+d}}\,dx$$

input
```
integrate((-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2),x, algorith
m="giac")
```

output
```
integrate((-c*e*x + e)^(3/2)*(b*arccos(c*x) + a)^2/sqrt(c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(e-c\,e\,x)^{3/2}}{\sqrt{d+c\,d\,x}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(3/2))/(d + c*d*x)^(1/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(3/2))/(d + c*d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx = \frac{\sqrt{e}\,e\left(-6asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2-\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2cx+4\sqrt{cx+1}\right.}{}$$

input
```
int((-c*e*x+e)^(3/2)*(a+b*acos(c*x))^2/(c*d*x+d)^(1/2),x)
```

output
```
(sqrt(e)*e*( - 6*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - sqrt(c*x + 1)*sqrt(
 - c*x + 1)*a**2*c*x + 4*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2 - 4*int((sqrt
( - c*x + 1)*acos(c*x)*x)/sqrt(c*x + 1),x)*a*b*c**2 + 4*int((sqrt( - c*x +
 1)*acos(c*x))/sqrt(c*x + 1),x)*a*b*c - 2*int((sqrt( - c*x + 1)*acos(c*x)*
*2*x)/sqrt(c*x + 1),x)*b**2*c**2 + 2*int((sqrt( - c*x + 1)*acos(c*x)**2)/s
qrt(c*x + 1),x)*b**2*c))/(2*sqrt(d)*c)
```

3.552 $\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 714

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx = \frac{2abe^3x(1-c^2x^2)^{3/2}}{(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{2b^2e^3(1-c^2x^2)^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{2b^2e^3x(1-c^2x^2)^{3/2}\arccos(cx)}{(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{4e^3(1-c^2x^2)(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{4e^3x(1-c^2x^2)(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{4ie^3(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{e^3(1-c^2x^2)^2(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{e^3(1-c^2x^2)^{3/2}(a+b\arccos(cx))^3}{bc(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{16ibe^3(1-c^2x^2)^{3/2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{8be^3(1-c^2x^2)^{3/2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{8ib^2e^3(1-c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{8ib^2e^3(1-c^2x^2)^{3/2}\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{4ib^2e^3(1-c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}}$$

output
```
2*a*b*e^3*x*(-c^2*x^2+1)^(3/2)/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+2*b^2*e^3*
(-c^2*x^2+1)^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+2*b^2*e^3*x*(-c^2*x^2+1)
^(3/2)*arccos(c*x)/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-4*e^3*(-c^2*x^2+1)*(a+
b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+4*e^3*x*(-c^2*x^2+1)*(
a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-4*I*e^3*(-c^2*x^2+1)^(
3/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-e^3*(-c^2*x^2+
1)^2*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-e^3*(-c^2*x^2+
1)^(3/2)*(a+b*arccos(c*x))^3/b/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-16*I*b*e
^3*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c
/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+8*b*e^3*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c
*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)
+8*I*b^2*e^3*(-c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c
/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-8*I*b^2*e^3*(-c^2*x^2+1)^(3/2)*polylog(2
,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-4*I*b^2*
e^3*(-c^2*x^2+1)^(3/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d
)^(3/2)/(-c*e*x+e)^(3/2)
```

Mathematica [A] (verified)

Time = 11.14 (sec) , antiderivative size = 653, normalized size of antiderivative = 0.91

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx = \frac{-3a^2e(5+cx)\sqrt{d+cdx}\sqrt{e-cex}\sqrt{1-c^2x^2}+9a^2\sqrt{d}e^{3/2}(1+cx)}{}$$

input
```
Integrate[((e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2)/(d + c*d*x)^(3/2),x]
```

output
```
(-3*a^2*e*(5 + c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2] + 9*
a^2*Sqrt[d]*e^(3/2)*(1 + c*x)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x
]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] - 3*a*b*e*(1 - c*x)*S
qrt[d + c*d*x]*Sqrt[e - c*e*x]*Cot[ArcCos[c*x]/2]*(4*ArcCos[c*x] - Cot[Arc
Cos[c*x]/2]*(ArcCos[c*x]^2 - 8*Log[Cos[ArcCos[c*x]/2]])) - 6*a*b*e*(1 - c*
x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Cot[ArcCos[c*x]/2]*(2*ArcCos[c*x] + Cot
[ArcCos[c*x]/2]*(c*x + Sqrt[1 - c^2*x^2]*ArcCos[c*x] - ArcCos[c*x]^2 + 4*L
og[Cos[ArcCos[c*x]/2]])) + (b^2*e*(1 - c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x
]*Cot[ArcCos[c*x]/2]*Csc[ArcCos[c*x]/2]^2*(6 - 6*c^2*x^2 + 3*(-3 + 2*c*x +
 c^2*x^2 + (2*I)*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2 + 2*Sqrt[1 - c^2*x^2]*Ar
cCos[c*x]^3 - 6*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*(c*x + 4*Log[1 + E^(I*ArcCos
[c*x])]) + (24*I)*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^(I*ArcCos[c*x])]))/2 + b
^2*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2]*(ArcCos[c*x]*(-6*Ar
cCos[c*x] + Cot[ArcCos[c*x]/2]*(ArcCos[c*x]*(6*I + ArcCos[c*x]) - 24*Log[1
 + E^(I*ArcCos[c*x])])) + (24*I)*Cot[ArcCos[c*x]/2]*PolyLog[2, -E^(I*ArcCo
s[c*x])]))/(3*c*d^2*(1 + c*x)*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(cdx+d)^{3/2}}dx$$

$$\Big\downarrow \text{ 5179}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\int \frac{e^3(1-cx)^3(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^3\left(1-c^2x^2\right)^{3/2}\int \frac{(1-cx)^3(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{ 5275}$$

$$\frac{e^3\left(1-c^2x^2\right)^{3/2}\int\left(\frac{cx(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}+\frac{4(1-cx)(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}\right)dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^3\left(1-c^2x^2\right)^{3/2}\left(-\frac{16b\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c}+\frac{4x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{4(a+b\arccos(}{c\sqrt{1-c^2x}}\right.}{}$$

input
```
Int[((e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2)/(d + c*d*x)^(3/2),x]
```

output
```
(e^3*(1 - c^2*x^2)^(3/2)*(-2*a*b*x + (2*b^2*Sqrt[1 - c^2*x^2])/c - 2*b^2*x
*ArcCos[c*x] + ((4*I)*(a + b*ArcCos[c*x])^2)/c - (4*(a + b*ArcCos[c*x])^2)
/(c*Sqrt[1 - c^2*x^2]) + (4*x*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2] - (
Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c + (a + b*ArcCos[c*x])^3/(b*c) -
 (16*b*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])])/c - (8*b*(a + b*Arc
Cos[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])])/c + ((8*I)*b^2*PolyLog[2, -E^(I*
ArcCos[c*x])])/c - ((8*I)*b^2*PolyLog[2, E^(I*ArcCos[c*x])])/c + ((4*I)*b^
2*PolyLog[2, E^((2*I)*ArcCos[c*x])])/c))/((d + c*d*x)^(3/2)*(e - c*e*x)^(3
/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 6.18 (sec) , antiderivative size = 540, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,(a+b\arccos(cx))^3 e}{(cx+1)d^2(cx-1)cb}-\frac{\sqrt{-e(cx-1)}\,\sqrt{d(cx+1)}\,\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\left(\arccos(cx)^2b^2+\right.}{2(cx+1)d^2(cx-1)c}$

input
```
int((-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x+1)/d^2/(c*x-
1)/c*(a+b*arccos(c*x))^3*e/b-1/2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(I*(
-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)^2*b^2+2*arccos(c*x)*a*b+a^2-
2*b^2+2*I*arccos(c*x)*b^2+2*I*a*b)*e/(c*x+1)/d^2/(c*x-1)/c-1/2*(-e*(c*x-1)
)^(1/2)*(d*(c*x+1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*
x)^2*b^2+2*arccos(c*x)*a*b+a^2-2*b^2-2*I*b^2*arccos(c*x)-2*I*a*b)*e/(c*x+1
)/d^2/(c*x-1)/c-4*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-I*(-c^2*x^2+1)^(1
/2)+c*x-1)*(arccos(c*x)^2*b^2+2*arccos(c*x)*a*b+a^2)*e/(c*x+1)/d^2/(c*x-1)
/c+8*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)*b*(-I*arccos(
c*x)^2*b+2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*arccos(c*x)*b-2*I*polylog(2,-c*x
-I*(-c^2*x^2+1)^(1/2))*b+2*a*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-2*a*ln(c*x+I*(
-c^2*x^2+1)^(1/2)))*e/(c*x+1)/d^2/(c*x-1)/c
```

Fricas [F]

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx=\int \frac{(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2),x, algorith
m="fricas")
```

output
```
integral(-(a^2*c*e*x - a^2*e + (b^2*c*e*x - b^2*e)*arccos(c*x)^2 + 2*(a*b*
c*e*x - a*b*e)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^2*d^2*x^2
+ 2*c*d^2*x + d^2), x)
```

Sympy [F]

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx=\int \frac{(-e(cx-1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))^2}{(d(cx+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)**(3/2)*(a+b*acos(c*x))**2/(c*d*x+d)**(3/2),x)
```

output
```
Integral((-e*(c*x - 1))**(3/2)*(a + b*acos(c*x))**2/(d*(c*x + 1))**(3/2),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx = \int \frac{(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2),x, algorith
m="giac")
```

output
```
integrate((-c*e*x + e)^(3/2)*(b*arccos(c*x) + a)^2/(c*d*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(e-c\,e\,x)^{3/2}}{(d+c\,d\,x)^{3/2}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(3/2))/(d + c*d*x)^(3/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(3/2))/(d + c*d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx = \frac{\sqrt{e}\,e\left(6\sqrt{cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2-\sqrt{-cx+1}\,a^2cx-5\sqrt{-cx+1}\right.}{}$$

input
```
int((-c*e*x+e)^(3/2)*(a+b*acos(c*x))^2/(c*d*x+d)^(3/2),x)
```

output
```
(sqrt(e)*e*(6*sqrt(c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - sqrt( -
c*x + 1)*a**2*c*x - 5*sqrt( - c*x + 1)*a**2 - 2*sqrt(c*x + 1)*int((sqrt( -
 c*x + 1)*acos(c*x)*x)/(sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*a*b*c**2 + 2
*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x))/(sqrt(c*x + 1)*c*x + sqrt(
c*x + 1)),x)*a*b*c - sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)**2*x)/(
sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b**2*c**2 + sqrt(c*x + 1)*int((sqrt(
 - c*x + 1)*acos(c*x)**2)/(sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b**2*c))/
(sqrt(d)*sqrt(c*x + 1)*c*d)
```

3.553 $\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 544

$$\begin{aligned}
\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx &= \frac{8ie^4(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+ \frac{e^4(1-c^2x^2)^{5/2}(a+b\arccos(cx))^3}{3bc(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{8b^2e^4(1-c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+ \frac{8e^4(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&- \frac{4be^4(1-c^2x^2)^{5/2}(a+b\arccos(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&- \frac{2e^4(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&- \frac{32be^4(1-c^2x^2)^{5/2}(a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+ \frac{32ib^2e^4(1-c^2x^2)^{5/2}\,\mathrm{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}
\end{aligned}$$

output
```
8/3*I*e^4*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x
+e)^(5/2)+1/3*e^4*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^3/b/c/(c*d*x+d)^(5/
2)/(-c*e*x+e)^(5/2)-8/3*b^2*e^4*(-c^2*x^2+1)^(5/2)*cot(1/4*Pi+1/2*arccos(c
*x))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+8/3*e^4*(-c^2*x^2+1)^(5/2)*(a+b*ar
ccos(c*x))^2*cot(1/4*Pi+1/2*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2
)-4/3*b*e^4*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*csc(1/4*Pi+1/2*arccos(c*x
))^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-2/3*e^4*(-c^2*x^2+1)^(5/2)*(a+b*ar
ccos(c*x))^2*cot(1/4*Pi+1/2*arccos(c*x))*csc(1/4*Pi+1/2*arccos(c*x))^2/c/(
c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-32/3*b*e^4*(-c^2*x^2+1)^(5/2)*(a+b*arccos(
c*x))*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2
)+32/3*I*b^2*e^4*(-c^2*x^2+1)^(5/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2))
)/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)
```

Mathematica [A] (verified)

Time = 13.93 (sec) , antiderivative size = 762, normalized size of antiderivative = 1.40

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2)/(d + c*d*x)^(5/2),x]
```

output
```
(32*a^2*e*(1 + 2*c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2] -
24*Sqrt[d]*e^(3/2)*(a + a*c*x)^2*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*
d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] - 16*a*b*e*(1 - c*
x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Cot[ArcCos[c*x]/2]*(-((5 + 7*c*x)*ArcCo
s[c*x]) + 3*ArcCos[c*x]^2*Cos[ArcCos[c*x]/2]^2*Cot[ArcCos[c*x]/2] - 2*Cot[
ArcCos[c*x]/2]*(1 + 7*(1 + c*x)*Log[Cos[ArcCos[c*x]/2]])) - 16*b^2*e*(1 -
c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Cot[ArcCos[c*x]/2]*(ArcCos[c*x]*(ArcC
os[c*x] - 2*Cot[ArcCos[c*x]/2]) + Cos[ArcCos[c*x]/2]^2*(4 - ArcCos[c*x]^2
+ I*ArcCos[c*x]*Cot[ArcCos[c*x]/2]*(ArcCos[c*x] + (4*I)*Log[1 + E^(I*ArcCo
s[c*x])])) + (4*I)*Cos[ArcCos[c*x]/2]^2*Cot[ArcCos[c*x]/2]*PolyLog[2, -E^(
I*ArcCos[c*x])]) - b^2*e*(1 - c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1
- c^2*x^2]*Csc[ArcCos[c*x]/2]^2*(8*ArcCos[c*x]^2 - 2*Sqrt[1 - c^2*x^2]*(-4
*Sqrt[1 - c^2*x^2] + 4*ArcCos[c*x] + 7*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2)*Cs
c[ArcCos[c*x]/2]^2 + (1 - c^2*x^2)^(3/2)*ArcCos[c*x]*Csc[ArcCos[c*x]/2]^4*
(ArcCos[c*x]*(7*I + ArcCos[c*x]) - 28*Log[1 + E^(I*ArcCos[c*x])]) + (28*I)
*(1 - c^2*x^2)^(3/2)*Csc[ArcCos[c*x]/2]^4*PolyLog[2, -E^(I*ArcCos[c*x])])
- 32*a*b*e*(1 - c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Cot[ArcCos[c*x]/2]*(-
(Cot[ArcCos[c*x]/2]*(1 + (1 + c*x)*Log[Cos[ArcCos[c*x]/2]])) + ArcCos[c*x]
*Sin[ArcCos[c*x]/2]^2))/(24*c*d^3*(1 + c*x)^2*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(cdx+d)^{5/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\int \frac{e^4(1-cx)^4(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^4(1-c^2x^2)^{5/2}\int \frac{(1-cx)^4(a+b\arccos(cx))^2}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{5275}$$

$$\frac{e^4(1-c^2x^2)^{5/2}\int \left(-\frac{4(a+b\arccos(cx))^2}{(cx+1)\sqrt{1-c^2x^2}}+\frac{4(a+b\arccos(cx))^2}{(cx+1)^2\sqrt{1-c^2x^2}}+\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^4(1-c^2x^2)^{5/2}\left(-\frac{(a+b\arccos(cx))^3}{3bc}-\frac{8i(a+b\arccos(cx))^2}{3c}+\frac{32b\log\left(1+e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{3c}+\frac{8\tan\left(\frac{1}{2}\arccos(cx)\right)(a+b\text{arc}}{3c}\right.}{}$$

input
```
Int[((e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2)/(d + c*d*x)^(5/2),x]
```

output
```
(e^4*(1 - c^2*x^2)^(5/2)*((((-8*I)/3)*(a + b*ArcCos[c*x])^2)/c - (a + b*Ar
cCos[c*x])^3/(3*b*c) + (32*b*(a + b*ArcCos[c*x])*Log[1 + E^(I*ArcCos[c*x])
])/(3*c) - (((32*I)/3)*b^2*PolyLog[2, -E^(I*ArcCos[c*x])])/c + (4*b*(a + b
*ArcCos[c*x])*Sec[ArcCos[c*x]/2]^2)/(3*c) - (8*b^2*Tan[ArcCos[c*x]/2])/(3*
c) + (8*(a + b*ArcCos[c*x])^2*Tan[ArcCos[c*x]/2])/(3*c) - (2*(a + b*ArcCos
[c*x])^2*Sec[ArcCos[c*x]/2]^2*Tan[ArcCos[c*x]/2])/(3*c)))/((d + c*d*x)^(5/
2)*(e - c*e*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,(a+b\arccos(cx))^3 e}{3(cx+1)d^3(cx-1)cb}+\frac{4\sqrt{-e(cx-1)}\,\sqrt{d(cx+1)}\,\left(-2i\sqrt{-c^2x^2+1}\,xc+2c^2x^2-2i\sqrt{-c^2x^2+1}-c\right.}{}$

input
```
int((-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/3*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x+1)/d^3/(c
*x-1)/c*(a+b*arccos(c*x))^3*e/b+4/3*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(
-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-2*I*(-c^2*x^2+1)^(1/2)-c*x-1)*(12*b^
2*c^2*x^2*arccos(c*x)^2-8*I*a*b*c*x+24*a*b*c^2*x^2*arccos(c*x)+2*I*(-c^2*x
^2+1)^(1/2)*b^2*c*x+15*arccos(c*x)^2*b^2*c*x-4*I*a*b+4*(-c^2*x^2+1)^(1/2)*
arccos(c*x)*b^2*c*x-4*I*arccos(c*x)*b^2+12*a^2*c^2*x^2-10*x^2*c^2*b^2+30*a
rccos(c*x)*a*b*c*x-8*I*arccos(c*x)*b^2*c*x+4*(-c^2*x^2+1)^(1/2)*a*b*c*x+5*
arccos(c*x)^2*b^2-4*I*arccos(c*x)*b^2*c^2*x^2+2*arccos(c*x)*(-c^2*x^2+1)^(
1/2)*b^2-4*I*a*b*c^2*x^2+15*a^2*c*x-16*c*x*b^2+10*arccos(c*x)*a*b+2*I*(-c^
2*x^2+1)^(1/2)*b^2+2*(-c^2*x^2+1)^(1/2)*a*b+5*a^2-6*b^2)*e/(12*c^4*x^4+39*
c^3*x^3+47*c^2*x^2+25*c*x+5)/d^3/(c*x-1)/c+16/3*I*(-c^2*x^2+1)^(1/2)*(d*(c
*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(c*x+1)/d^3/(c*x-1)/c*b*(arccos(c*x)^2*b+2
*I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*b+2*polylog(2,-c*x-I*(-c^2*x
^2+1)^(1/2))*b+2*I*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*a-2*I*ln(c*x+I*(-c^2*x^2
+1)^(1/2))*a)*e
```

Fricas [F]

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \int \frac{(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2),x, algorith
m="fricas")
```

output
```
integral(-(a^2*c*e*x - a^2*e + (b^2*c*e*x - b^2*e)*arccos(c*x)^2 + 2*(a*b*
c*e*x - a*b*e)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^3*d^3*x^3
+ 3*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

Sympy [F]

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \int \frac{(-e(cx-1))^{\frac{3}{2}}\,(a+b\operatorname{acos}(cx))^2}{(d\,(cx+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)**(3/2)*(a+b*acos(c*x))**2/(c*d*x+d)**(5/2),x)
```

output
```
Integral((-e*(c*x - 1))**(3/2)*(a + b*acos(c*x))**2/(d*(c*x + 1))**(5/2),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx=\int \frac{(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2),x, algorith
m="giac")
```

output
```
integrate((-c*e*x + e)^(3/2)*(b*arccos(c*x) + a)^2/(c*d*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(e-c\,e\,x)^{3/2}}{(d+c\,d\,x)^{5/2}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(3/2))/(d + c*d*x)^(5/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(3/2))/(d + c*d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(e-cex)^{3/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \frac{\sqrt{e}\,e\left(-6\sqrt{cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2cx-6\sqrt{cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a$$

input
```
int((-c*e*x+e)^(3/2)*(a+b*acos(c*x))^2/(c*d*x+d)^(5/2),x)
```

output
```
(sqrt(e)*e*( - 6*sqrt(c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a**2*c*x - 6
*sqrt(c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + 8*sqrt( - c*x + 1)*a*
*2*c*x + 4*sqrt( - c*x + 1)*a**2 - 6*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*a
cos(c*x)*x)/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)
),x)*a*b*c**3*x - 6*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)*x)/(sqrt
(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*a*b*c**2 + 6
*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x))/(sqrt(c*x + 1)*c**2*x**2 +
 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*a*b*c**2*x + 6*sqrt(c*x + 1)*int(
(sqrt( - c*x + 1)*acos(c*x))/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*
x + sqrt(c*x + 1)),x)*a*b*c - 3*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c
*x)**2*x)/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),
x)*b**2*c**3*x - 3*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)**2*x)/(sq
rt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b**2*c**2
+ 3*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)**2)/(sqrt(c*x + 1)*c**2*
x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b**2*c**2*x + 3*sqrt(c*x +
1)*int((sqrt( - c*x + 1)*acos(c*x)**2)/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c
*x + 1)*c*x + sqrt(c*x + 1)),x)*b**2*c))/(3*sqrt(d)*sqrt(c*x + 1)*c*d**2*(
c*x + 1))
```

3.554 $\int (d+cdx)^{5/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 502

$$\int (d+cdx)^{5/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx =$$

$$-\frac{1}{108}b^2x(d+cdx)^{5/2}(e-cex)^{5/2}-\frac{245b^2x(d+cdx)^{5/2}(e-cex)^{5/2}}{1152\,(1-c^2x^2)^2}-\frac{65b^2x(d+cdx)^{5/2}(e-cex)^{5/2}}{1728\,(1-c^2x^2)}+\frac{115b^2}{}$$

output

```
-1/108*b^2*x*(c*d*x+d)^(5/2)*(-c*e*x+e)^(5/2)-245/1152*b^2*x*(c*d*x+d)^(5/
2)*(-c*e*x+e)^(5/2)/(-c^2*x^2+1)^2-65*b^2*x*(c*d*x+d)^(5/2)*(-c*e*x+e)^(5/
2)/(-1728*c^2*x^2+1728)+115/1152*b^2*(c*d*x+d)^(5/2)*(-c*e*x+e)^(5/2)*arcc
os(c*x)/c/(-c^2*x^2+1)^(5/2)-5/16*b*c*x^2*(c*d*x+d)^(5/2)*(-c*e*x+e)^(5/2)
*(a+b*arccos(c*x))/(-c^2*x^2+1)^(5/2)+5/48*b*(c*d*x+d)^(5/2)*(-c*e*x+e)^(5
/2)*(a+b*arccos(c*x))/c/(-c^2*x^2+1)^(1/2)+1/18*b*(c*d*x+d)^(5/2)*(-c*e*x+
e)^(5/2)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+1/6*x*(c*d*x+d)^(5/2)*(-c*
e*x+e)^(5/2)*(a+b*arccos(c*x))^2+5/16*x*(c*d*x+d)^(5/2)*(-c*e*x+e)^(5/2)*(
a+b*arccos(c*x))^2/(-c^2*x^2+1)^2+5*x*(c*d*x+d)^(5/2)*(-c*e*x+e)^(5/2)*(a+
b*arccos(c*x))^2/(-24*c^2*x^2+24)+5/48*(c*d*x+d)^(5/2)*(-c*e*x+e)^(5/2)*(a
+b*arccos(c*x))^3/b/c/(-c^2*x^2+1)^(5/2)
```

Mathematica [A] (verified)

Time = 3.69 (sec) , antiderivative size = 450, normalized size of antiderivative = 0.90

$$\int (d+cdx)^{5/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx = \frac{d^2e^2\left(-1440b^2\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)^3-4320a^2\sqrt{d}\sqrt{e}\sqrt{1-c^2x^2}\,a\right.}{}$$

input
```
Integrate[(d + c*d*x)^(5/2)*(e - c*e*x)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*e^2*(-1440*b^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^3 - 4320*a
^2*Sqrt[d]*Sqrt[e]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e -
c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] + 12*b*Sqrt[d + c*d*x]*Sqrt[e -
c*e*x]*ArcCos[c*x]*(270*b*Cos[2*ArcCos[c*x]] - 27*b*Cos[4*ArcCos[c*x]] + 2
*b*Cos[6*ArcCos[c*x]] + 540*a*Sin[2*ArcCos[c*x]] - 108*a*Sin[4*ArcCos[c*x]
] + 12*a*Sin[6*ArcCos[c*x]]) + 72*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos
[c*x]^2*(-60*a + 45*b*Sin[2*ArcCos[c*x]] - 9*b*Sin[4*ArcCos[c*x]] + b*Sin[
6*ArcCos[c*x]]) + Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(9504*a^2*c*x*Sqrt[1 - c
^2*x^2] - 7488*a^2*c^3*x^3*Sqrt[1 - c^2*x^2] + 2304*a^2*c^5*x^5*Sqrt[1 - c
^2*x^2] + 3240*a*b*Cos[2*ArcCos[c*x]] - 324*a*b*Cos[4*ArcCos[c*x]] + 24*a*
b*Cos[6*ArcCos[c*x]] - 1620*b^2*Sin[2*ArcCos[c*x]] + 81*b^2*Sin[4*ArcCos[c
*x]] - 4*b^2*Sin[6*ArcCos[c*x]])))/(13824*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.86, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {5179, 5159, 5159, 5157, 5139, 262, 223, 5153, 5183, 211, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^{5/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(cdx+d)^{5/2}(e-cex)^{5/2}\int \left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))^2dx}{\left(1-c^2x^2\right)^{5/2}}$$

$\Big\downarrow$ 5159

$$\frac{(cdx+d)^{5/2}(e-cex)^{5/2}\left(\frac{1}{3}bc\int x\left(1-c^2x^2\right)^2(a+b\arccos(cx))dx+\frac{5}{6}\int \left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2dx+\frac{1}{6}\right.}{\left(1-c^2x^2\right)^{5/2}}$$

$\Big\downarrow$ 5159

$$\frac{(cdx+d)^{5/2}(e-cex)^{5/2}\left(\frac{1}{3}bc\int x\left(1-c^2x^2\right)^2(a+b\arccos(cx))dx+\frac{5}{6}\left(\frac{1}{2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx+\right.\right.}{(1-}$$

$\Big\downarrow$ 5157

$$(cdx+d)^{5/2}(e-cex)^{5/2}\left(\frac{1}{3}bc\int x\left(1-c^2x^2\right)^2(a+b\arccos(cx))dx+\frac{5}{6}\left(\frac{1}{2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx+\right.\right.$$

$\Big\downarrow$ 5139

$$(cdx+d)^{5/2}(e-cex)^{5/2}\left(\frac{1}{3}bc\int x\left(1-c^2x^2\right)^2(a+b\arccos(cx))dx+\frac{5}{6}\left(\frac{1}{2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx+\right.\right.$$

$\Big\downarrow$ 262

$$(cdx+d)^{5/2}(e-cex)^{5/2}\left(\frac{1}{3}bc\int x\left(1-c^2x^2\right)^2(a+b\arccos(cx))dx+\frac{5}{6}\left(\frac{1}{2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx\right.\right.$$

$\Big\downarrow$ 223

$$(cdx+d)^{5/2}(e-cex)^{5/2}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)\right.\right.\right.$$

$\Big\downarrow$ 5153

$$(cdx+d)^{5/2}(e-cex)^{5/2}\left(\frac{5}{6}\left(\frac{1}{2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx+\frac{3}{4}\left(bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(}{2c^3}\right.\right.\right.\right.\right.$$

$\downarrow$ 5183

$$(cdx+d)^{5/2}(e-cex)^{5/2}\left(\frac{5}{6}\left(\frac{1}{2}bc\left(-\frac{b\int(1-c^2x^2)^{3/2}dx}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)+\frac{3}{4}\left(bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\right.\right.\right.\right.$$

$\downarrow$ 211

$$(cdx+d)^{5/2}(e-cex)^{5/2}\left(\frac{5}{6}\left(\frac{1}{2}bc\left(-\frac{b\left(\frac{3}{4}\int\sqrt{1-c^2x^2}dx+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)+\frac{3}{4}\left(bc\left(\frac{1}{2}x^2(a+\right.\right.\right.\right.$$

$\downarrow$ 211

$$(cdx+d)^{5/2}(e-cex)^{5/2}\left(\frac{5}{6}\left(\frac{1}{2}bc\left(-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)+\right.\right.$$

$\downarrow$ 211

$$(cdx+d)^{5/2}(e-cex)^{5/2}\left(\frac{5}{6}\left(\frac{1}{2}bc\left(-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)+\right.\right.$$

$\downarrow$ 223

$$(cdx+d)^{5/2}(e-cex)^{5/2}\left(\frac{1}{3}bc\left(-\frac{(1-c^2x^2)^3(a+b\arccos(cx))}{6c^2}-\frac{b\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)+\frac{1}{6}x(1-c^2x^2)\right.}{6c}\right.\right.$$

input
```
Int[(d + c*d*x)^(5/2)*(e - c*e*x)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
((d + c*d*x)^(5/2)*(e - c*e*x)^(5/2)*((x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos
[c*x])^2)/6 + (b*c*(-1/6*((1 - c^2*x^2)^3*(a + b*ArcCos[c*x]))/c^2 - (b*((
x*(1 - c^2*x^2)^(5/2))/6 + (5*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x*Sqrt[1 -
 c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/6))/(6*c)))/3 + (5*((x*(1 - c^2*x^2
)^(3/2)*(a + b*ArcCos[c*x])^2)/4 + (3*((x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[
c*x])^2)/2 - (a + b*ArcCos[c*x])^3/(6*b*c) + b*c*((x^2*(a + b*ArcCos[c*x])
)/2 + (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2)))/4
+ (b*c*(-1/4*((1 - c^2*x^2)^2*(a + b*ArcCos[c*x]))/c^2 - (b*((x*(1 - c^2*x
^2)^(3/2))/4 + (3*((x*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/(4*c)
))/2))/6))/(1 - c^2*x^2)^(5/2)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.50 (sec) , antiderivative size = 1613, normalized size of antiderivative = 3.21

method	result	size
default	Expression too large to display	1613
parts	Expression too large to display	1613

input
```
int((c*d*x+d)^(5/2)*(-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/6*a^2/c/e*(c*d*x+d)^(5/2)*(-c*e*x+e)^(7/2)-1/6*a^2*d/c/e*(c*d*x+d)^(3/2
)*(-c*e*x+e)^(7/2)-1/8*a^2*d^2/c/e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(7/2)+1/24*a
^2*d^2/c*(-c*e*x+e)^(5/2)*(c*d*x+d)^(1/2)+5/48*a^2*d^2*e/c*(-c*e*x+e)^(3/2
)*(c*d*x+d)^(1/2)+5/16*a^2*d^2*e^2/c*(-c*e*x+e)^(1/2)*(c*d*x+d)^(1/2)+5/16
*a^2*d^3*e^3*((-c*e*x+e)*(c*d*x+d))^(1/2)/(-c*e*x+e)^(1/2)/(c*d*x+d)^(1/2)
/(c^2*d*e)^(1/2)*arctan((c^2*d*e)^(1/2)*x/(-c^2*d*e*x^2+d*e)^(1/2))+b^2*(5
/48*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*
arccos(c*x)^3*e^2*d^2+1/6912*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(32*c^7*
x^7-64*c^5*x^5+32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+38*c^3*x^3-48*I*(-c^2*x^2+1
)^(1/2)*x^4*c^4-6*c*x+18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2)
)*(6*I*arccos(c*x)+18*arccos(c*x)^2-1)*e^2*d^2/(c^2*x^2-1)/c+15/256*(d*(c*
x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+
I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*arccos(c*x)^2-1-2*I*arccos(c*x))*e^2*d^2/(c
^2*x^2-1)/c+5/27648*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^
(1/2)*x*c+c^2*x^2-1)*(60*I*arccos(c*x)+144*arccos(c*x)^2-17)*cos(5*arccos(
c*x))*e^2*d^2/(c^2*x^2-1)/c+1/27648*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(
I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(348*I*arccos(c*x)+576*arccos(c*x)^2-7
7)*sin(5*arccos(c*x))*e^2*d^2/(c^2*x^2-1)/c-9/1024*(d*(c*x+1))^(1/2)*(-e*(
c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(12*I*arccos(c*x)+16*a
rccos(c*x)^2-7)*cos(3*arccos(c*x))*e^2*d^2/(c^2*x^2-1)/c-3/1024*(d*(c*x...
```

Fricas [F]

$$\int (d+cdx)^{5/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx=\int (cdx+d)^{\frac{5}{2}}(-cex+e)^{\frac{5}{2}}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2,x, algorith
m="fricas")
```

output
```
integral((a^2*c^4*d^2*e^2*x^4 - 2*a^2*c^2*d^2*e^2*x^2 + a^2*d^2*e^2 + (b^2
*c^4*d^2*e^2*x^4 - 2*b^2*c^2*d^2*e^2*x^2 + b^2*d^2*e^2)*arccos(c*x)^2 + 2*
(a*b*c^4*d^2*e^2*x^4 - 2*a*b*c^2*d^2*e^2*x^2 + a*b*d^2*e^2)*arccos(c*x))*s
qrt(c*d*x + d)*sqrt(-c*e*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+cdx)^{5/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx=\text{Timed out}$$

input
```
integrate((c*d*x+d)**(5/2)*(-c*e*x+e)**(5/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+cdx)^{5/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (d+cdx)^{5/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx=\int (cdx+d)^{\frac{5}{2}}(-cex+e)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2,x, algorith
m="giac")
```

output
```
integrate((c*d*x + d)^(5/2)*(-c*e*x + e)^(5/2)*(b*arccos(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)^{5/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,(d+c\,d\,x)^{5/2}\,(e-c\,e\,x)^{5/2}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(5/2)*(e - c*e*x)^(5/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(5/2)*(e - c*e*x)^(5/2), x)
```

Reduce [F]

$$\int (d+cdx)^{5/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx = \frac{\sqrt{e}\,\sqrt{d}\,d^2e^2\left(-30asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2+8\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^5x^5-26\sqrt{cx}\right.}{}$$

input
```
int((c*d*x+d)^(5/2)*(-c*e*x+e)^(5/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(e)*sqrt(d)*d**2*e**2*( - 30*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + 8*
sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c**5*x**5 - 26*sqrt(c*x + 1)*sqrt( - c
*x + 1)*a**2*c**3*x**3 + 33*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c*x + 96*i
nt(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**4,x)*a*b*c**5 - 192*int(sqr
t(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**2,x)*a*b*c**3 + 96*int(sqrt(c*x +
 1)*sqrt( - c*x + 1)*acos(c*x),x)*a*b*c + 48*int(sqrt(c*x + 1)*sqrt( - c*x
 + 1)*acos(c*x)**2*x**4,x)*b**2*c**5 - 96*int(sqrt(c*x + 1)*sqrt( - c*x +
1)*acos(c*x)**2*x**2,x)*b**2*c**3 + 48*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*
acos(c*x)**2,x)*b**2*c))/(48*c)
```

3.555 $\int (d+cdx)^{3/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 697

$$\int (d+cdx)^{3/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx = -\frac{8b^2e(d+cdx)^{3/2}(e-cex)^{3/2}}{225c}$$
$$-\frac{1}{32}b^2ex(d+cdx)^{3/2}(e-cex)^{3/2}-\frac{16b^2e(d+cdx)^{3/2}(e-cex)^{3/2}}{75c(1-c^2x^2)}-\frac{15b^2ex(d+cdx)^{3/2}(e-cex)^{3/2}}{64(1-c^2x^2)}-\frac{2b^2e(a}{}$$

output
```
-8/225*b^2*e*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)/c-1/32*b^2*e*x*(c*d*x+d)^(3/
2)*(-c*e*x+e)^(3/2)-16/75*b^2*e*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)/c/(-c^2*x
^2+1)-15*b^2*e*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)/(-64*c^2*x^2+64)-2/125*b
^2*e*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(-c^2*x^2+1)/c+9/64*b^2*e*(c*d*x+d)^
(3/2)*(-c*e*x+e)^(3/2)*arccos(c*x)/c/(-c^2*x^2+1)^(3/2)-2/5*b*e*x*(c*d*x+d
)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(3/2)-3/8*b*c*e*x^
2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(3/2)+4/
15*b*c^2*e*x^3*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))/(-c^2*x^
2+1)^(3/2)-2/25*b*c^4*e*x^5*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c
*x))/(-c^2*x^2+1)^(3/2)+1/8*b*e*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(-c^2*x^2
+1)^(1/2)*(a+b*arccos(c*x))/c+1/4*e*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+
b*arccos(c*x))^2+3*e*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^
2/(-8*c^2*x^2+8)+1/5*e*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(-c^2*x^2+1)*(a+b*
arccos(c*x))^2/c+1/8*e*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^
3/b/c/(-c^2*x^2+1)^(3/2)
```

Mathematica [A] (verified)

Time = 3.91 (sec) , antiderivative size = 684, normalized size of antiderivative = 0.98

$$\int (d+cdx)^{3/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx = \frac{de^2\left(-108000b^2\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)^3-324000a^2\sqrt{d}\sqrt{e}\sqrt{1-c^2}\right.}{}$$

input
```
Integrate[(d + c*d*x)^(3/2)*(e - c*e*x)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*e^2*(-108000*b^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^3 - 324000
*a^2*Sqrt[d]*Sqrt[e]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e
- c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] + 180*b*Sqrt[d + c*d*x]*Sqrt[e
 - c*e*x]*ArcCos[c*x]*(1200*b*Cos[2*ArcCos[c*x]] - 200*b*Cos[3*ArcCos[c*x]
] - 75*b*Cos[4*ArcCos[c*x]] + 24*b*Cos[5*ArcCos[c*x]] + 60*(4*(5*b*c*x + 8
*a*Sqrt[1 - c^2*x^2] - 16*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 8*a*c^4*x^4*Sqrt[1
 - c^2*x^2]) + 40*a*Sin[2*ArcCos[c*x]] - 5*a*Sin[4*ArcCos[c*x]])) - 1800*b
*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^2*(180*a - 20*b*Sqrt[1 - c^2*
x^2] + 80*b*Sqrt[1 - c^2*x^2]*Cos[2*ArcCos[c*x]] - 120*b*Sin[2*ArcCos[c*x]
] - 10*b*Sin[3*ArcCos[c*x]] + 15*b*Sin[4*ArcCos[c*x]] - 6*b*Sin[5*ArcCos[c
*x]]) + Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(345600*a*b*c*x - 230400*a*b*c^3*x
^3 + 69120*a*b*c^5*x^5 + 172800*a^2*Sqrt[1 - c^2*x^2] - 200000*b^2*Sqrt[1
- c^2*x^2] + 540000*a^2*c*x*Sqrt[1 - c^2*x^2] - 345600*a^2*c^2*x^2*Sqrt[1
- c^2*x^2] - 216000*a^2*c^3*x^3*Sqrt[1 - c^2*x^2] + 172800*a^2*c^4*x^4*Sqr
t[1 - c^2*x^2] + 8000*b*(27*a + 4*b*Sqrt[1 - c^2*x^2])*Cos[2*ArcCos[c*x]]
- 13500*a*b*Cos[4*ArcCos[c*x]] - 108000*b^2*Sin[2*ArcCos[c*x]] - 4000*b^2*
Sin[3*ArcCos[c*x]] + 3375*b^2*Sin[4*ArcCos[c*x]] - 864*b^2*Sin[5*ArcCos[c*
x]])))/(864000*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^{3/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx$$

$\Big\downarrow$ 5179

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\int e(1-cx)\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2dx}{(1-c^2x^2)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{e(cdx+d)^{3/2}(e-cex)^{3/2}\int (1-cx)\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2dx}{(1-c^2x^2)^{3/2}}$$

$\Big\downarrow$ 5263

$$\frac{e(cdx+d)^{3/2}(e-cex)^{3/2}\int \left(\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2-cx\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2\right)dx}{(1-c^2x^2)^{3/2}}$$

$\Big\downarrow$ 2009

$$e(cdx+d)^{3/2}(e-cex)^{3/2}\Bigg(\frac{2}{25}bc^4x^5(a+b\arccos(cx))-\frac{4}{15}bc^2x^3(a+b\arccos(cx))+\frac{1}{4}x\left(1-c^2x^2\right)^{3/2}(a+b\arcc$$

input
```
Int[(d + c*d*x)^(3/2)*(e - c*e*x)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(e*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*((-16*b^2*Sqrt[1 - c^2*x^2])/(75*c)
 - (15*b^2*x*Sqrt[1 - c^2*x^2])/64 - (8*b^2*(1 - c^2*x^2)^(3/2))/(225*c) -
 (b^2*x*(1 - c^2*x^2)^(3/2))/32 - (2*b^2*(1 - c^2*x^2)^(5/2))/(125*c) + (2
*b*x*(a + b*ArcCos[c*x]))/5 + (3*b*c*x^2*(a + b*ArcCos[c*x]))/8 - (4*b*c^2
*x^3*(a + b*ArcCos[c*x]))/15 + (2*b*c^4*x^5*(a + b*ArcCos[c*x]))/25 - (b*(
1 - c^2*x^2)^2*(a + b*ArcCos[c*x]))/(8*c) + (3*x*Sqrt[1 - c^2*x^2]*(a + b*
ArcCos[c*x])^2)/8 + (x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/4 + ((1
- c^2*x^2)^(5/2)*(a + b*ArcCos[c*x])^2)/(5*c) - (a + b*ArcCos[c*x])^3/(8*b
*c) + (9*b^2*ArcSin[c*x])/(64*c)))/(1 - c^2*x^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.86 (sec) , antiderivative size = 2258, normalized size of antiderivative = 3.24

method	result	size
default	Expression too large to display	2258
parts	Expression too large to display	2258

input
```
int((c*d*x+d)^(3/2)*(-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/5*a^2/c/e*(c*d*x+d)^(3/2)*(-c*e*x+e)^(7/2)-3/20*a^2*d/c/e*(c*d*x+d)^(1/
2)*(-c*e*x+e)^(7/2)+1/20*a^2*d/c*(-c*e*x+e)^(5/2)*(c*d*x+d)^(1/2)+1/8*a^2*
d*e/c*(-c*e*x+e)^(3/2)*(c*d*x+d)^(1/2)+3/8*a^2*d*e^2/c*(-c*e*x+e)^(1/2)*(c
*d*x+d)^(1/2)+3/8*a^2*d^2*e^3*((-c*e*x+e)*(c*d*x+d))^(1/2)/(-c*e*x+e)^(1/2
)/(c*d*x+d)^(1/2)/(c^2*d*e)^(1/2)*arctan((c^2*d*e)^(1/2)*x/(-c^2*d*e*x^2+d
*e)^(1/2))+b^2*(1/8*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2
)/(c^2*x^2-1)/c*arccos(c*x)^3*e^2*d+1/4000*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^
(1/2)*(16*c^6*x^6-28*c^4*x^4+16*I*(-c^2*x^2+1)^(1/2)*x^5*c^5+13*c^2*x^2-20
*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+5*I*(-c^2*x^2+1)^(1/2)*c*x-1)*(10*I*arccos(c
*x)+25*arccos(c*x)^2-2)*e^2*d/(c^2*x^2-1)/c-1/512*(d*(c*x+1))^(1/2)*(-e*(c
*x-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-8*
I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(4*I*arccos(c*x)+8*arcc
os(c*x)^2-1)*e^2*d/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)
*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)^2-2-2*I*arccos(c*x))*e
^2*d/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*I*(-c^2*x
^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*arccos(c*x)^2
-1-2*I*arccos(c*x))*e^2*d/(c^2*x^2-1)/c+1/18000*(d*(c*x+1))^(1/2)*(-e*(c*x
-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(330*I*arccos(c*x)+675*ar
ccos(c*x)^2-134)*cos(4*arccos(c*x))*e^2*d/(c^2*x^2-1)/c+1/9000*(d*(c*x+1))
^(1/2)*(-e*(c*x-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(210*I*a...
```

Fricas [F]

$$\int (d+cdx)^{3/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx=\int (cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2,x, algorith
m="fricas")
```

output
```
integral((a^2*c^3*d*e^2*x^3 - a^2*c^2*d*e^2*x^2 - a^2*c*d*e^2*x + a^2*d*e^
2 + (b^2*c^3*d*e^2*x^3 - b^2*c^2*d*e^2*x^2 - b^2*c*d*e^2*x + b^2*d*e^2)*ar
ccos(c*x)^2 + 2*(a*b*c^3*d*e^2*x^3 - a*b*c^2*d*e^2*x^2 - a*b*c*d*e^2*x + a
*b*d*e^2)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+cdx)^{3/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+d)**(3/2)*(-c*e*x+e)**(5/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+cdx)^{3/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (d+cdx)^{3/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx = \int (cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{5}{2}}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2,x, algorith
m="giac")
```

output
```
integrate((c*d*x + d)^(3/2)*(-c*e*x + e)^(5/2)*(b*arccos(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)^{3/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,(d+c\,d\,x)^{3/2}\,(e-c\,e\,x)^{5/2}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(5/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(5/2), x)
```

Reduce [F]

$$\int (d+cdx)^{3/2}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx=\frac{\sqrt{e}\,\sqrt{d}\,d\,e^2\left(-30asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2+8\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^4x^4-10\sqrt{cx}\right.}{}$$

input
```
int((c*d*x+d)^(3/2)*(-c*e*x+e)^(5/2)*(a+b*acos(c*x))^2,x)
```

output

```
(sqrt(e)*sqrt(d)*d*e**2*( - 30*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + 8*sqr
t(c*x + 1)*sqrt( - c*x + 1)*a**2*c**4*x**4 - 10*sqrt(c*x + 1)*sqrt( - c*x
+ 1)*a**2*c**3*x**3 - 16*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c**2*x**2 + 2
5*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c*x + 8*sqrt(c*x + 1)*sqrt( - c*x +
1)*a**2 + 80*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**3,x)*a*b*c**4
 - 80*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**2,x)*a*b*c**3 - 80*i
nt(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x,x)*a*b*c**2 + 80*int(sqrt(c*
x + 1)*sqrt( - c*x + 1)*acos(c*x),x)*a*b*c + 40*int(sqrt(c*x + 1)*sqrt( -
c*x + 1)*acos(c*x)**2*x**3,x)*b**2*c**4 - 40*int(sqrt(c*x + 1)*sqrt( - c*x
 + 1)*acos(c*x)**2*x**2,x)*b**2*c**3 - 40*int(sqrt(c*x + 1)*sqrt( - c*x +
1)*acos(c*x)**2*x,x)*b**2*c**2 + 40*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*aco
s(c*x)**2,x)*b**2*c))/(40*c)
```

3.556 $\int \sqrt{d+cdx}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 613

$$\int \sqrt{d+cdx}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx = -\frac{8b^2e^2\sqrt{d+cdx}\sqrt{e-cex}}{9c} - \frac{15}{64}b^2e^2x\sqrt{d+cdx}\sqrt{e-cex} - \frac{1}{32}b^2c^2e^2x^3\sqrt{d+cdx}\sqrt{e-cex} - \frac{4b^2e^2\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)}{27c} + \frac{15b^2e^2\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)}{64c\sqrt{1-c^2x^2}} - \frac{4be^2x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}} - \frac{3bce^2x^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{8\sqrt{1-c^2x^2}} + \frac{4bc^2e^2x^3\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{9\sqrt{1-c^2x^2}} - \frac{bc^3e^2x^4\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{8\sqrt{1-c^2x^2}} + \frac{3}{8}e^2x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2 + \frac{1}{4}c^2e^2x^3\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2 + \frac{2e^2\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)(a+b\arccos(cx))^2}{3c} + \frac{5e^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^3}{24bc\sqrt{1-c^2x^2}}$$

output
```
-8/9*b^2*e^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)/c-15/64*b^2*e^2*x*(c*d*x+d)^
(1/2)*(-c*e*x+e)^(1/2)-1/32*b^2*c^2*e^2*x^3*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/
2)-4/27*b^2*e^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)/c+15/64*b^2*
e^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*arccos(c*x)/c/(-c^2*x^2+1)^(1/2)-4/3*
b*e^2*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1
/2)-3/8*b*c*e^2*x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))/(-c
^2*x^2+1)^(1/2)+4/9*b*c^2*e^2*x^3*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*ar
ccos(c*x))/(-c^2*x^2+1)^(1/2)-1/8*b*c^3*e^2*x^4*(c*d*x+d)^(1/2)*(-c*e*x+e)
^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+3/8*e^2*x*(c*d*x+d)^(1/2)*(-c*
e*x+e)^(1/2)*(a+b*arccos(c*x))^2+1/4*c^2*e^2*x^3*(c*d*x+d)^(1/2)*(-c*e*x+e
)^(1/2)*(a+b*arccos(c*x))^2+2/3*e^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2
*x^2+1)*(a+b*arccos(c*x))^2/c+5/24*e^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a
+b*arccos(c*x))^3/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 3.22 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.94

$$\int \sqrt{d+cdx}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx = \frac{-1440b^2e^2\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)^3-4320a^2\sqrt{d}e^{5/2}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d+cd}}{\sqrt{d}\sqrt{e}(-}\right.}{}$$

input
```
Integrate[Sqrt[d + c*d*x]*(e - c*e*x)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(-1440*b^2*e^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^3 - 4320*a^2*Sq
rt[d]*e^(5/2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x
])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] - 72*b*e^2*Sqrt[d + c*d*x]*Sqrt[e - c
*e*x]*ArcCos[c*x]^2*(60*a - 32*b*Sqrt[1 - c^2*x^2] + 32*b*Sqrt[1 - c^2*x^2
]*Cos[2*ArcCos[c*x]] - 24*b*Sin[2*ArcCos[c*x]] - 3*b*Sin[4*ArcCos[c*x]]) +
 e^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(9216*a*b*c*x - 3072*a*b*c^3*x^3 + 46
08*a^2*Sqrt[1 - c^2*x^2] - 6656*b^2*Sqrt[1 - c^2*x^2] + 2592*a^2*c*x*Sqrt[
1 - c^2*x^2] - 4608*a^2*c^2*x^2*Sqrt[1 - c^2*x^2] + 1728*a^2*c^3*x^3*Sqrt[
1 - c^2*x^2] + 64*b*(27*a + 8*b*Sqrt[1 - c^2*x^2])*Cos[2*ArcCos[c*x]] + 10
8*a*b*Cos[4*ArcCos[c*x]] - 864*b^2*Sin[2*ArcCos[c*x]] - 27*b^2*Sin[4*ArcCo
s[c*x]]) - 12*b*e^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]*(-144*b*Co
s[2*ArcCos[c*x]] + 64*b*Cos[3*ArcCos[c*x]] - 9*b*Cos[4*ArcCos[c*x]] - 12*(
48*b*c*x + 64*a*Sqrt[1 - c^2*x^2] - 64*a*c^2*x^2*Sqrt[1 - c^2*x^2] + 24*a*
Sin[2*ArcCos[c*x]] + 3*a*Sin[4*ArcCos[c*x]])))/(6912*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cdx+d}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\int e^2(1-cx)^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\sqrt{cdx+d}\sqrt{e-cex}\int (1-cx)^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{e^2\sqrt{cdx+d}\sqrt{e-cex}\int \left(c^2x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2-2cx\sqrt{1-c^2x^2}(a+b\arccos(cx))^2+\sqrt{1-c^2x^2}(a+\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{8}bc^3x^4(a+b\arccos(cx))-\frac{4}{9}bc^2x^3(a+b\arccos(cx))+\frac{3}{8}x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2+\right.}{}$$

input
```
Int[Sqrt[d + c*d*x]*(e - c*e*x)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(e^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*((-8*b^2*Sqrt[1 - c^2*x^2])/(9*c) - (
15*b^2*x*Sqrt[1 - c^2*x^2])/64 - (b^2*c^2*x^3*Sqrt[1 - c^2*x^2])/32 - (4*b
^2*(1 - c^2*x^2)^(3/2))/(27*c) + (4*b*x*(a + b*ArcCos[c*x]))/3 + (3*b*c*x^
2*(a + b*ArcCos[c*x]))/8 - (4*b*c^2*x^3*(a + b*ArcCos[c*x]))/9 + (b*c^3*x^
4*(a + b*ArcCos[c*x]))/8 + (3*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/8
 + (c^2*x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/4 + (2*(1 - c^2*x^2)^
(3/2)*(a + b*ArcCos[c*x])^2)/(3*c) - (5*(a + b*ArcCos[c*x])^3)/(24*b*c) +
(15*b^2*ArcSin[c*x])/(64*c)))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.57 (sec) , antiderivative size = 1806, normalized size of antiderivative = 2.95

method	result	size
default	Expression too large to display	1806
parts	Expression too large to display	1806

input
```
int((c*d*x+d)^(1/2)*(-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/4*a^2/c/e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(7/2)+1/12*a^2/c*(-c*e*x+e)^(5/2)*
(c*d*x+d)^(1/2)+5/24*a^2*e/c*(-c*e*x+e)^(3/2)*(c*d*x+d)^(1/2)+5/8*a^2*e^2/
c*(-c*e*x+e)^(1/2)*(c*d*x+d)^(1/2)+5/8*a^2*d*e^3*((-c*e*x+e)*(c*d*x+d))^(1
/2)/(-c*e*x+e)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*e)^(1/2)*arctan((c^2*d*e)^(1/2
)*x/(-c^2*d*e*x^2+d*e)^(1/2))+b^2*(5/24*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/
2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^3*e^2+1/512*(d*(c*x+1))^(1
/2)*(-e*(c*x-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^
4+4*c*x-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(4*I*arccos(c
*x)+8*arccos(c*x)^2-1)*e^2/(c^2*x^2-1)/c-1/108*(d*(c*x+1))^(1/2)*(-e*(c*x-
1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-3*I*(-c^2*x^
2+1)^(1/2)*c*x+1)*(6*I*arccos(c*x)+9*arccos(c*x)^2-2)*e^2/(c^2*x^2-1)/c+1/
4*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-
1)*(arccos(c*x)^2-2-2*I*arccos(c*x))*e^2/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1
/2)*(-e*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*
x^2+1)^(1/2)-2*c*x)*(2*arccos(c*x)^2-1-2*I*arccos(c*x))*e^2/(c^2*x^2-1)/c-
1/512*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*
x^2-1)*(68*I*arccos(c*x)+56*arccos(c*x)^2-31)*cos(3*arccos(c*x))*e^2/(c^2*
x^2-1)/c-3/512*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x
^2+1)^(1/2)-I)*(20*I*arccos(c*x)+24*arccos(c*x)^2-11)*sin(3*arccos(c*x))*e
^2/(c^2*x^2-1)/c-1/27*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^...
```

Fricas [F]

$$\int \sqrt{d+cdx}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx = \int \sqrt{cdx+d}(-cex+e)^{\frac{5}{2}}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2,x, algorith
m="fricas")
```

output
```
integral((a^2*c^2*e^2*x^2 - 2*a^2*c*e^2*x + a^2*e^2 + (b^2*c^2*e^2*x^2 - 2
*b^2*c*e^2*x + b^2*e^2)*arccos(c*x)^2 + 2*(a*b*c^2*e^2*x^2 - 2*a*b*c*e^2*x
 + a*b*e^2)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d+cdx}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx=\text{Timed out}$$

input
```
integrate((c*d*x+d)**(1/2)*(-c*e*x+e)**(5/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+cdx}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \sqrt{d+cdx}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx=\int \sqrt{cdx+d}(-cex+e)^{\frac{5}{2}}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2,x, algorith
m="giac")
```

output
```
integrate(sqrt(c*d*x + d)*(-c*e*x + e)^(5/2)*(b*arccos(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+cdx}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d+c\,d\,x}\,(e-c\,e\,x)^{5/2}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(5/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(5/2), x)
```

Reduce [F]

$$\int \sqrt{d+cdx}(e-cex)^{5/2}(a+b\arccos(cx))^2\,dx=\frac{\sqrt{e}\,\sqrt{d}\,e^2\left(-30asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2+6\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^3x^3-16\sqrt{cx+1}\,\sqrt{-cx+\cdots}\right.}{}$$

input
```
int((c*d*x+d)^(1/2)*(-c*e*x+e)^(5/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(e)*sqrt(d)*e**2*( - 30*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + 6*sqrt(
c*x + 1)*sqrt( - c*x + 1)*a**2*c**3*x**3 - 16*sqrt(c*x + 1)*sqrt( - c*x +
1)*a**2*c**2*x**2 + 9*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c*x + 16*sqrt(c*
x + 1)*sqrt( - c*x + 1)*a**2 + 48*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(
c*x)*x**2,x)*a*b*c**3 - 96*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x,
x)*a*b*c**2 + 48*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x),x)*a*b*c + 2
4*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2*x**2,x)*b**2*c**3 - 48*i
nt(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2*x,x)*b**2*c**2 + 24*int(sqr
t(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2,x)*b**2*c))/(24*c)
```

3.557 $\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 559

$$\begin{aligned}\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx &= -\frac{68b^2e^3(1-c^2x^2)}{9c\sqrt{d+cdx}\sqrt{e-cex}} \\ &+\frac{3b^2e^3x(1-c^2x^2)}{4\sqrt{d+cdx}\sqrt{e-cex}}+\frac{2b^2e^3(1-c^2x^2)^2}{27c\sqrt{d+cdx}\sqrt{e-cex}} \\ &-\frac{3b^2e^3\sqrt{1-c^2x^2}\arccos(cx)}{4c\sqrt{d+cdx}\sqrt{e-cex}}-\frac{22be^3x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3\sqrt{d+cdx}\sqrt{e-cex}} \\ &+\frac{3bce^3x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2\sqrt{d+cdx}\sqrt{e-cex}}-\frac{2bc^2e^3x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9\sqrt{d+cdx}\sqrt{e-cex}} \\ &+\frac{11e^3(1-c^2x^2)(a+b\arccos(cx))^2}{3c\sqrt{d+cdx}\sqrt{e-cex}}-\frac{3e^3x(1-c^2x^2)(a+b\arccos(cx))^2}{2\sqrt{d+cdx}\sqrt{e-cex}} \\ &+\frac{ce^3x^2(1-c^2x^2)(a+b\arccos(cx))^2}{3\sqrt{d+cdx}\sqrt{e-cex}}+\frac{5e^3\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{6bc\sqrt{d+cdx}\sqrt{e-cex}}\end{aligned}$$

output
```
-68/9*b^2*e^3*(-c^2*x^2+1)/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+3/4*b^2*e^3*
x*(-c^2*x^2+1)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2/27*b^2*e^3*(-c^2*x^2+1)^
2/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-3/4*b^2*e^3*(-c^2*x^2+1)^(1/2)*arccos
(c*x)/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-22/3*b*e^3*x*(-c^2*x^2+1)^(1/2)*(
a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+3/2*b*c*e^3*x^2*(-c^2*x^
2+1)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2/9*b*c^2*e^
3*x^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2
)+11/3*e^3*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(
1/2)-3/2*e^3*x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)
^(1/2)+1/3*c*e^3*x^2*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*
e*x+e)^(1/2)+5/6*e^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^3/b/c/(c*d*x+d)^
(1/2)/(-c*e*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 13.03 (sec) , antiderivative size = 473, normalized size of antiderivative = 0.85

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx = \frac{e^2(-1+cx)\csc^2\left(\frac{1}{2}\arccos(cx)\right)\left(180b^2\sqrt{d+cdx}\sqrt{e-cex}\arccos(c\right.}{}$$

input
```
Integrate[((e - c*e*x)^(5/2)*(a + b*ArcCos[c*x])^2)/Sqrt[d + c*d*x],x]
```

output
```
(e^2*(-1 + c*x)*Csc[ArcCos[c*x]/2]^2*(180*b^2*Sqrt[d + c*d*x]*Sqrt[e - c*e
*x]*ArcCos[c*x]^3 + 540*a^2*Sqrt[d]*Sqrt[e]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*
Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] - 18*b*
Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^2*(-30*a + 45*b*Sqrt[1 - c^2*x
^2] - 9*b*Sin[2*ArcCos[c*x]] + b*Sin[3*ArcCos[c*x]]) + Sqrt[d + c*d*x]*Sqr
t[e - c*e*x]*(-1620*a*b*c*x - 792*a^2*Sqrt[1 - c^2*x^2] + 1620*b^2*Sqrt[1
- c^2*x^2] + 324*a^2*c*x*Sqrt[1 - c^2*x^2] - 72*a^2*c^2*x^2*Sqrt[1 - c^2*x
^2] + 162*a*b*Cos[2*ArcCos[c*x]] - 12*a*b*Cos[3*ArcCos[c*x]] - 81*b^2*Sin[
2*ArcCos[c*x]] + 4*b^2*Sin[3*ArcCos[c*x]]) - 6*b*Sqrt[d + c*d*x]*Sqrt[e -
c*e*x]*ArcCos[c*x]*(-27*b*Cos[2*ArcCos[c*x]] + 2*b*Cos[3*ArcCos[c*x]] + 6*
(45*b*c*x + 45*a*Sqrt[1 - c^2*x^2] - 9*a*Sin[2*ArcCos[c*x]] + a*Sin[3*ArcC
os[c*x]]))))/(432*c*d*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.54, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5179, 27, 5273, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{\sqrt{cdx+d}}dx$$

$$\Big\downarrow \text{ 5179}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{e^3(1-cx)^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^3\sqrt{1-c^2x^2}\int \frac{(1-cx)^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 5273}$$

$$-\frac{e^3\sqrt{1-c^2x^2}\int \left(c-c^2x\right)^3(a+b\arccos(cx))^2d\arccos(cx)}{c^4\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{e^3\sqrt{1-c^2x^2}\int (a+b\arccos(cx))^2\left(c-c\sin\left(\arccos(cx)+\frac{\pi}{2}\right)\right)^3d\arccos(cx)}{c^4\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 3798}$$

$$-\frac{e^3\sqrt{1-c^2x^2}\int \left(-x^3(a+b\arccos(cx))^2c^6+3x^2(a+b\arccos(cx))^2c^5-3x(a+b\arccos(cx))^2c^4+(a+b\arccos$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{e^3\sqrt{1-c^2x^2}\left(-\frac{2}{9}bc^6x^3(a+b\arccos(cx))+\frac{3}{2}bc^5x^2(a+b\arccos(cx))-\frac{22}{3}bc^4x(a+b\arccos(cx))+\frac{5c^3(a+b\arccos$$

input
```
Int[((e - c*e*x)^(5/2)*(a + b*ArcCos[c*x])^2)/Sqrt[d + c*d*x],x]
```

output
```
-((e^3*Sqrt[1 - c^2*x^2]*((68*b^2*c^3*Sqrt[1 - c^2*x^2])/9 - (3*b^2*c^4*x*
Sqrt[1 - c^2*x^2])/4 - (2*b^2*c^3*(1 - c^2*x^2)^(3/2))/27 - (3*b^2*c^3*Arc
Cos[c*x])/4 - (22*b*c^4*x*(a + b*ArcCos[c*x]))/3 + (3*b*c^5*x^2*(a + b*Arc
Cos[c*x]))/2 - (2*b*c^6*x^3*(a + b*ArcCos[c*x]))/9 - (11*c^3*Sqrt[1 - c^2*
x^2]*(a + b*ArcCos[c*x])^2)/3 + (3*c^4*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c
*x])^2)/2 - (c^5*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/3 + (5*c^3*(
a + b*ArcCos[c*x])^3)/(6*b)))/(c^4*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5273
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[-(c^(m + 1)*Sqrt[d])^(-1)   Subs
t[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &
& (GtQ[m, 0] || IGtQ[n, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 18.08 (sec) , antiderivative size = 1812, normalized size of antiderivative = 3.24

method	result	size
default	Expression too large to display	1812
parts	Expression too large to display	1812

input
```
int((-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/3*a^2/d/c*(-c*e*x+e)^(5/2)*(c*d*x+d)^(1/2)+5/6*a^2*e/d/c*(-c*e*x+e)^(3/2
)*(c*d*x+d)^(1/2)+5/2*a^2*e^2/d/c*(-c*e*x+e)^(1/2)*(c*d*x+d)^(1/2)+5/2*a^2
*e^3*((-c*e*x+e)*(c*d*x+d))^(1/2)/(-c*e*x+e)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*
e)^(1/2)*arctan((c^2*d*e)^(1/2)*x/(-c^2*d*e*x^2+d*e)^(1/2))+b^2*(5/6*(d*(c
*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x+1)/d/c/(c*x-1)*arc
cos(c*x)^3*e^2+1/432*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-4*c^3*x^3+8*I*
(-c^2*x^2+1)^(1/2)*x^3*c^3+8*c^4*x^4+3*c*x-4*I*(-c^2*x^2+1)^(1/2)*c^2*x^2-
4*I*(-c^2*x^2+1)^(1/2)*x*c-8*c^2*x^2+I*(-c^2*x^2+1)^(1/2)+1)*(6*I*arccos(c
*x)+9*arccos(c*x)^2-2)*e^2/(c*x+1)/d/c/(c*x-1)+15/16*(d*(c*x+1))^(1/2)*(-e
*(c*x-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)+c*x-1)*(arccos(c*x)^2-2+2*I*arccos(c
*x))*e^2/(c*x+1)/d/c/(c*x-1)+15/8*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I
*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)^2-2-2*I*arccos(c*x))*e^2/(
c*x+1)/d/c/(c*x-1)-3/32*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*I*(-c^2*x
^2+1)^(1/2)*c*x+2*c^2*x^2+I*(-c^2*x^2+1)^(1/2)-c*x-1)*(2*arccos(c*x)^2-1-2
*I*arccos(c*x))*e^2/(c*x+1)/d/c/(c*x-1)+1/288*(d*(c*x+1))^(1/2)*(-e*(c*x-1
))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x-1)*(46*I*arccos(c*x)+54*arccos(c*x)^2-
27)*cos(3*arccos(c*x))*e^2/(c*x+1)/d/c/(c*x-1)+1/864*(d*(c*x+1))^(1/2)*(-e
*(c*x-1))^(1/2)*(I*c*x-I+(-c^2*x^2+1)^(1/2))*(162*I*arccos(c*x)+126*arccos
(c*x)^2-73)*sin(3*arccos(c*x))*e^2/(c*x+1)/d/c/(c*x-1)-1/216*(d*(c*x+1))^(
1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x-1)*(327*I*arccos(c*x...
```

Fricas [F]

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx=\int \frac{(-cex+e)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}{\sqrt{cdx+d}}\,dx$$

input
```
integrate((-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2),x, algorith
m="fricas")
```

output
```
integral((a^2*c^2*e^2*x^2 - 2*a^2*c*e^2*x + a^2*e^2 + (b^2*c^2*e^2*x^2 - 2
*b^2*c*e^2*x + b^2*e^2)*arccos(c*x)^2 + 2*(a*b*c^2*e^2*x^2 - 2*a*b*c*e^2*x
 + a*b*e^2)*arccos(c*x))*sqrt(-c*e*x + e)/sqrt(c*d*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx = \text{Timed out}$$

input
```
integrate((-c*e*x+e)**(5/2)*(a+b*acos(c*x))**2/(c*d*x+d)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx = \int \frac{(-cex+e)^{\frac{5}{2}}(b\arccos(cx)+a)^2}{\sqrt{cdx+d}}\,dx$$

input
```
integrate((-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2),x, algorith
m="giac")
```

output
```
integrate((-c*e*x + e)^(5/2)*(b*arccos(c*x) + a)^2/sqrt(c*d*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(e-c\,e\,x)^{5/2}}{\sqrt{d+c\,d\,x}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(5/2))/(d + c*d*x)^(1/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(5/2))/(d + c*d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}}\,dx = \frac{\sqrt{e}\,e^2\left(-30asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2+2\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^2x^2-9\sqrt{cx}\right.}{}$$

input
```
int((-c*e*x+e)^(5/2)*(a+b*acos(c*x))^2/(c*d*x+d)^(1/2),x)
```

output
```
(sqrt(e)*e**2*( - 30*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + 2*sqrt(c*x + 1)
*sqrt( - c*x + 1)*a**2*c**2*x**2 - 9*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c
*x + 22*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2 + 12*int((sqrt( - c*x + 1)*aco
s(c*x)*x**2)/sqrt(c*x + 1),x)*a*b*c**3 - 24*int((sqrt( - c*x + 1)*acos(c*x
)*x)/sqrt(c*x + 1),x)*a*b*c**2 + 12*int((sqrt( - c*x + 1)*acos(c*x))/sqrt(
c*x + 1),x)*a*b*c + 6*int((sqrt( - c*x + 1)*acos(c*x)**2*x**2)/sqrt(c*x +
1),x)*b**2*c**3 - 12*int((sqrt( - c*x + 1)*acos(c*x)**2*x)/sqrt(c*x + 1),x
)*b**2*c**2 + 6*int((sqrt( - c*x + 1)*acos(c*x)**2)/sqrt(c*x + 1),x)*b**2*
c))/(6*sqrt(d)*c)
```

3.558 $\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 918

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx = \frac{8abe^4x(1-c^2x^2)^{3/2}}{(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{8b^2e^4(1-c^2x^2)^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{b^2e^4x(1-c^2x^2)^2}{4(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{b^2e^4(1-c^2x^2)^{3/2}\arccos(cx)}{4c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{8b^2e^4x(1-c^2x^2)^{3/2}\arccos(cx)}{(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{bce^4x^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{2(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{8e^4(1-c^2x^2)(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{8e^4x(1-c^2x^2)(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{8ie^4(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{4e^4(1-c^2x^2)^2(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{e^4x(1-c^2x^2)^2(a+b\arccos(cx))^2}{2(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{5e^4(1-c^2x^2)^{3/2}(a+b\arccos(cx))^3}{2bc(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{32ibe^4(1-c^2x^2)^{3/2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{16be^4(1-c^2x^2)^{3/2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{16ib^2e^4(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{16ib^2e^4(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{8ib^2e^4(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}}$$

output
```
8*a*b*e^4*x*(-c^2*x^2+1)^(3/2)/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+8*b^2*e^4*
(-c^2*x^2+1)^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-1/4*b^2*e^4*x*(-c^2*x^2+
1)^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+1/4*b^2*e^4*(-c^2*x^2+1)^(3/2)*arcco
s(c*x)/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+8*b^2*e^4*x*(-c^2*x^2+1)^(3/2)*a
rccos(c*x)/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-1/2*b*c*e^4*x^2*(-c^2*x^2+1)^(
3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-8*e^4*(-c^2*x^2+1)
*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+8*e^4*x*(-c^2*x^2+
1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-16*I*b^2*e^4*(-c^2
*x^2+1)^(3/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(3/2)/(-
c*e*x+e)^(3/2)-4*e^4*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/
(-c*e*x+e)^(3/2)+1/2*e^4*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2/(c*d*x+d)^(3
/2)/(-c*e*x+e)^(3/2)-5/2*e^4*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))^3/b/c/(c
*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-32*I*b*e^4*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c
*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+1
6*b*e^4*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2
))^2)/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-8*I*b^2*e^4*(-c^2*x^2+1)^(3/2)*po
lylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-
8*I*e^4*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e
)^(3/2)+16*I*b^2*e^4*(-c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(
1/2)))/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 19.14 (sec) , antiderivative size = 999, normalized size of antiderivative = 1.09

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((e - c*e*x)^(5/2)*(a + b*ArcCos[c*x])^2)/(d + c*d*x)^(3/2),x]
```

output

```
(e^2*(24*a^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2]*(-24 - 7*c*
x + c^2*x^2) + 360*a^2*Sqrt[d]*Sqrt[e]*(1 + c*x)*Sqrt[1 - c^2*x^2]*ArcTan[
(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] -
48*a*b*(1 - c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Cot[ArcCos[c*x]/2]*(4*Arc
Cos[c*x] - Cot[ArcCos[c*x]/2]*(ArcCos[c*x]^2 - 8*Log[Cos[ArcCos[c*x]/2]]))
 - 192*a*b*(1 - c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Cot[ArcCos[c*x]/2]*(2
*ArcCos[c*x] + Cot[ArcCos[c*x]/2]*(c*x + Sqrt[1 - c^2*x^2]*ArcCos[c*x] - A
rcCos[c*x]^2 + 4*Log[Cos[ArcCos[c*x]/2]])) + 16*b^2*(1 - c*x)*Sqrt[d + c*d
*x]*Sqrt[e - c*e*x]*Cot[ArcCos[c*x]/2]*Csc[ArcCos[c*x]/2]^2*(6 - 6*c^2*x^2
 + 3*(-3 + 2*c*x + c^2*x^2 + (2*I)*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2 + 2*Sq
rt[1 - c^2*x^2]*ArcCos[c*x]^3 - 6*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*(c*x + 4*L
og[1 + E^(I*ArcCos[c*x])]) + (24*I)*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^(I*Arc
Cos[c*x])]) + 16*b^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2]*(Ar
cCos[c*x]*(-6*ArcCos[c*x] + Cot[ArcCos[c*x]/2]*(ArcCos[c*x]*(6*I + ArcCos[
c*x]) - 24*Log[1 + E^(I*ArcCos[c*x])])) + (24*I)*Cot[ArcCos[c*x]/2]*PolyLo
g[2, -E^(I*ArcCos[c*x])]) - b^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c
^2*x^2]*Csc[ArcCos[c*x]/2]*(90*ArcCos[c*x]*Cos[(3*ArcCos[c*x])/2] - 6*ArcC
os[c*x]*Cos[(5*ArcCos[c*x])/2] - 8*ArcCos[c*x]*Cos[ArcCos[c*x]/2]*(-12 + (
12*I)*ArcCos[c*x] + 5*ArcCos[c*x]^2 - 48*Log[1 + E^(I*ArcCos[c*x])]) - (38
4*I)*Cos[ArcCos[c*x]/2]*PolyLog[2, -E^(I*ArcCos[c*x])] + 6*(-31 - 30*c*...
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(cdx+d)^{3/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{3/2}\int \frac{e^4(1-cx)^4(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^4\left(1-c^2x^2\right)^{3/2}\int\frac{(1-cx)^4(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{ 5275}$$

$$\frac{e^4\left(1-c^2x^2\right)^{3/2}\int\left(-\frac{c^2x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}+\frac{4cx(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{7(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}+\frac{8(1-cx)(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}\right)dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^4\left(1-c^2x^2\right)^{3/2}\left(-\frac{32b\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{c}+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2-\frac{4\sqrt{1-c^2x^2}(a+b\arccos(ca}{c}\right.}{}$$

input
```
Int[((e - c*e*x)^(5/2)*(a + b*ArcCos[c*x])^2)/(d + c*d*x)^(3/2),x]
```

output
```
(e^4*(1 - c^2*x^2)^(3/2)*(-8*a*b*x + (8*b^2*Sqrt[1 - c^2*x^2])/c - (b^2*x*
Sqrt[1 - c^2*x^2])/4 - 8*b^2*x*ArcCos[c*x] + (b*c*x^2*(a + b*ArcCos[c*x]))
/2 + ((8*I)*(a + b*ArcCos[c*x])^2)/c - (8*(a + b*ArcCos[c*x])^2)/(c*Sqrt[1
 - c^2*x^2]) + (8*x*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2] - (4*Sqrt[1 -
 c^2*x^2]*(a + b*ArcCos[c*x])^2)/c + (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*
x])^2)/2 + (5*(a + b*ArcCos[c*x])^3)/(2*b*c) + (b^2*ArcSin[c*x])/(4*c) - (
32*b*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])])/c - (16*b*(a + b*ArcC
os[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])])/c + ((16*I)*b^2*PolyLog[2, -E^(I*
ArcCos[c*x])])/c - ((16*I)*b^2*PolyLog[2, E^(I*ArcCos[c*x])])/c + ((8*I)*b
^2*PolyLog[2, E^((2*I)*ArcCos[c*x])])/c))/((d + c*d*x)^(3/2)*(e - c*e*x)^(
3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 3.98 (sec) , antiderivative size = 1076, normalized size of antiderivative = 1.17

method	result	size
default	Expression too large to display	1076

input
```
int((-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-5/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x+1)/d^2/(
c*x-1)/c*(a+b*arccos(c*x))^3*e^2/b+1/32*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/
2)*(-2*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+4*c^3*x^3+1-2*I*(-c^2*x^2+1)
^(1/2)*c*x-I*(-c^2*x^2+1)^(1/2)-3*c*x)*(2*I*arccos(c*x)*b^2+2*arccos(c*x)^
2*b^2+2*I*a*b+4*arccos(c*x)*a*b+2*a^2-b^2)*e^2/(c*x+1)/d^2/(c*x-1)/c-(-e*(
c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)+c*x-1)*(arccos(c*x)^
2*b^2+2*arccos(c*x)*a*b+a^2-2*b^2+2*I*arccos(c*x)*b^2+2*I*a*b)*e^2/(c*x+1)
/d^2/(c*x-1)/c-2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-I*(-c^2*x^2+1)^(1/
2)*x*c+c^2*x^2-1)*(arccos(c*x)^2*b^2+2*arccos(c*x)*a*b+a^2-2*b^2-2*I*b^2*a
rccos(c*x)-2*I*a*b)*e^2/(c*x+1)/d^2/(c*x-1)/c+1/32*(-e*(c*x-1))^(1/2)*(d*(
c*x+1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2+I*(-c^2*x^2+1)^(1/2)-
c*x-1)*(-2*I*b^2*arccos(c*x)+2*arccos(c*x)^2*b^2-2*I*a*b+4*arccos(c*x)*a*b
+2*a^2-b^2)*e^2/(c*x+1)/d^2/(c*x-1)/c-8*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/
2)*(-I*(-c^2*x^2+1)^(1/2)+c*x-1)*(arccos(c*x)^2*b^2+2*arccos(c*x)*a*b+a^2)
*e^2/(c*x+1)/d^2/(c*x-1)/c-16*I*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(
c*x-1))^(1/2)/(c*x+1)/d^2/(c*x-1)/c*b*(arccos(c*x)^2*b+2*I*arccos(c*x)*ln(
1+c*x+I*(-c^2*x^2+1)^(1/2))*b+2*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))*b+2*I
*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*a-2*I*ln(c*x+I*(-c^2*x^2+1)^(1/2))*a)*e^2+
1/8*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x-1)*(15
*I*b^2*arccos(c*x)+8*arccos(c*x)^2*b^2+15*I*a*b+16*arccos(c*x)*a*b+8*a^...
```

Fricas [F]

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx=\int \frac{(-cex+e)^{\frac{5}{2}}(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2),x, algorith
m="fricas")
```

output
```
integral((a^2*c^2*e^2*x^2 - 2*a^2*c*e^2*x + a^2*e^2 + (b^2*c^2*e^2*x^2 - 2
*b^2*c*e^2*x + b^2*e^2)*arccos(c*x)^2 + 2*(a*b*c^2*e^2*x^2 - 2*a*b*c*e^2*x
 + a*b*e^2)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^2*d^2*x^2 + 2
*c*d^2*x + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((-c*e*x+e)**(5/2)*(a+b*acos(c*x))**2/(c*d*x+d)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx=\int \frac{(-cex+e)^{\frac{5}{2}}(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2),x, algorith
m="giac")
```

output
```
integrate((-c*e*x + e)^(5/2)*(b*arccos(c*x) + a)^2/(c*d*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx=\int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2\,\left(e-c\,e\,x\right)^{5/2}}{\left(d+c\,d\,x\right)^{3/2}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(5/2))/(d + c*d*x)^(3/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(5/2))/(d + c*d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{3/2}}\,dx=\frac{\sqrt{e}\,e^2\left(30\sqrt{cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2+\sqrt{-cx+1}\,a^2c^2x^2-7\sqrt{-cx}\right.}{}$$

input
```
int((-c*e*x+e)^(5/2)*(a+b*acos(c*x))^2/(c*d*x+d)^(3/2),x)
```

output
```
(sqrt(e)*e**2*(30*sqrt(c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + sqrt
( - c*x + 1)*a**2*c**2*x**2 - 7*sqrt( - c*x + 1)*a**2*c*x - 24*sqrt( - c*x
 + 1)*a**2 + 4*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)*x**2)/(sqrt(c
*x + 1)*c*x + sqrt(c*x + 1)),x)*a*b*c**3 - 8*sqrt(c*x + 1)*int((sqrt( - c*
x + 1)*acos(c*x)*x)/(sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*a*b*c**2 + 4*sq
rt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x))/(sqrt(c*x + 1)*c*x + sqrt(c*x
 + 1)),x)*a*b*c + 2*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)**2*x**2)
/(sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b**2*c**3 - 4*sqrt(c*x + 1)*int((s
qrt( - c*x + 1)*acos(c*x)**2*x)/(sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b**
2*c**2 + 2*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)**2)/(sqrt(c*x + 1
)*c*x + sqrt(c*x + 1)),x)*b**2*c))/(2*sqrt(d)*sqrt(c*x + 1)*c*d)
```

3.559 $\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 729

$$
\begin{aligned}
\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx &= -\frac{2abe^5x(1-c^2x^2)^{5/2}}{(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&- \frac{2b^2e^5(1-c^2x^2)^3}{c(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{2b^2e^5x(1-c^2x^2)^{5/2}\arccos(cx)}{(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+ \frac{28ie^5(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{e^5(1-c^2x^2)^3(a+b\arccos(cx))^2}{c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+ \frac{5e^5(1-c^2x^2)^{5/2}(a+b\arccos(cx))^3}{3bc(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{16b^2e^5(1-c^2x^2)^{5/2}\cot\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+ \frac{28e^5(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&- \frac{8be^5(1-c^2x^2)^{5/2}(a+b\arccos(cx))\csc^2\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&- \frac{4e^5(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2\cot\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)\csc^2\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&- \frac{112be^5(1-c^2x^2)^{5/2}(a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+ \frac{112ib^2e^5(1-c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}
\end{aligned}
$$

output
```
-2*a*b*e^5*x*(-c^2*x^2+1)^(5/2)/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-2*b^2*e^5
*(-c^2*x^2+1)^3/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-2*b^2*e^5*x*(-c^2*x^2+1
)^(5/2)*arccos(c*x)/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+28/3*I*e^5*(-c^2*x^2+
1)^(5/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+e^5*(-c^2*
x^2+1)^3*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+5/3*e^5*(-
c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^3/b/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-
16/3*b^2*e^5*(-c^2*x^2+1)^(5/2)*cot(1/4*Pi+1/2*arccos(c*x))/c/(c*d*x+d)^(5
/2)/(-c*e*x+e)^(5/2)+28/3*e^5*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^2*cot(1
/4*Pi+1/2*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-8/3*b*e^5*(-c^2*
x^2+1)^(5/2)*(a+b*arccos(c*x))*csc(1/4*Pi+1/2*arccos(c*x))^2/c/(c*d*x+d)^(
5/2)/(-c*e*x+e)^(5/2)-4/3*e^5*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^2*cot(1
/4*Pi+1/2*arccos(c*x))*csc(1/4*Pi+1/2*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c
*e*x+e)^(5/2)-112/3*b*e^5*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*ln(1-I*(c*x
+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+112/3*I*b^2*e^5
*(-c^2*x^2+1)^(5/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(5
/2)/(-c*e*x+e)^(5/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1526 vs. $2(729) = 1458$.

Time = 21.26 (sec) , antiderivative size = 1526, normalized size of antiderivative = 2.09

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((e - c*e*x)^(5/2)*(a + b*ArcCos[c*x])^2)/(d + c*d*x)^(5/2),x]
```

output
```
(Sqrt[-(e*(-1 + c*x))]*Sqrt[d*(1 + c*x)]*((a^2*e^2)/d^3 - (8*a^2*e^2)/(3*d
^3*(1 + c*x)^2) + (28*a^2*e^2)/(3*d^3*(1 + c*x))))/c - (5*a^2*e^(5/2)*ArcT
an[(c*x*Sqrt[-(e*(-1 + c*x))]*Sqrt[d*(1 + c*x)])/(Sqrt[d]*Sqrt[e]*(-1 + c*
x)*(1 + c*x))])/(c*d^(5/2)) + (4*a*b*e^2*(-1 + c*x)*Sqrt[d + c*d*x]*Sqrt[e
 - c*e*x]*Sqrt[-(d*e*(1 - c^2*x^2))]*Cot[ArcCos[c*x]/2]*(2*(ArcCos[c*x] -
Cot[ArcCos[c*x]/2]) + Cos[ArcCos[c*x]/2]^2*(-14*ArcCos[c*x] + Cot[ArcCos[c
*x]/2]*(3*ArcCos[c*x]^2 - 28*Log[Cos[ArcCos[c*x]/2]]))))/(3*c*d^3*(1 + c*x
)^2*Sqrt[(-d - c*d*x)*(e - c*e*x)]*Sqrt[1 - c^2*x^2]) - (4*a*b*e^2*(-1 + c
*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[-(d*e*(1 - c^2*x^2))]*Cot[ArcCos[
c*x]/2]*(-ArcCos[c*x] + Cot[ArcCos[c*x]/2] + Cos[ArcCos[c*x]/2]^2*(ArcCos[
c*x] + 2*Cot[ArcCos[c*x]/2]*Log[Cos[ArcCos[c*x]/2]])))/(3*c*d^3*(1 + c*x)^
2*Sqrt[(-d - c*d*x)*(e - c*e*x)]*Sqrt[1 - c^2*x^2]) + (2*b^2*e^2*(-1 + c*x
)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[-(d*e*(1 - c^2*x^2))]*Cot[ArcCos[c*
x]/2]*(ArcCos[c*x]*(ArcCos[c*x] - 2*Cot[ArcCos[c*x]/2]) + Cos[ArcCos[c*x]/
2]^2*(4 - ArcCos[c*x]^2 + I*ArcCos[c*x]*Cot[ArcCos[c*x]/2]*(ArcCos[c*x] +
(4*I)*Log[1 + E^(I*ArcCos[c*x])])) + (4*I)*Cos[ArcCos[c*x]/2]^2*Cot[ArcCos
[c*x]/2]*PolyLog[2, -E^(I*ArcCos[c*x])]))/(3*c*d^3*(1 + c*x)^2*Sqrt[(-d -
c*d*x)*(e - c*e*x)]*Sqrt[1 - c^2*x^2]) + (b^2*e^2*(-1 + c*x)*Sqrt[d + c*d*
x]*Sqrt[e - c*e*x]*(1 - c^2*x^2)^(3/2)*Sqrt[-(d*e*(1 - c^2*x^2))]*Csc[ArcC
os[c*x]/2]^2*((8*ArcCos[c*x]^2)/(1 - c^2*x^2)^(3/2) - (2*(-4 + (4*ArcCo...
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(cdx+d)^{5/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{5/2}\int \frac{e^5(1-cx)^5(a+b\arccos(cx))^2}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^5\left(1-c^2x^2\right)^{5/2}\int \frac{(1-cx)^5(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{5275}$$

$$\frac{e^5\left(1-c^2x^2\right)^{5/2}\int \left(-\frac{cx(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{12(a+b\arccos(cx))^2}{(cx+1)\sqrt{1-c^2x^2}}+\frac{8(a+b\arccos(cx))^2}{(cx+1)^2\sqrt{1-c^2x^2}}+\frac{5(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^5\left(1-c^2x^2\right)^{5/2}\left(\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c}-\frac{5(a+b\arccos(cx))^3}{3bc}-\frac{28i(a+b\arccos(cx))^2}{3c}+\frac{112b\log\left(1+e^{i\arccos(cx)}\right)(a+b\arccos(cx)}{3c}\right.}{}$$

input
```
Int[((e - c*e*x)^(5/2)*(a + b*ArcCos[c*x])^2)/(d + c*d*x)^(5/2),x]
```

output
```
(e^5*(1 - c^2*x^2)^(5/2)*(2*a*b*x - (2*b^2*Sqrt[1 - c^2*x^2])/c + 2*b^2*x*
ArcCos[c*x] - (((28*I)/3)*(a + b*ArcCos[c*x])^2)/c + (Sqrt[1 - c^2*x^2]*(a
 + b*ArcCos[c*x])^2)/c - (5*(a + b*ArcCos[c*x])^3)/(3*b*c) + (112*b*(a + b
*ArcCos[c*x])*Log[1 + E^(I*ArcCos[c*x])])/(3*c) - (((112*I)/3)*b^2*PolyLog
[2, -E^(I*ArcCos[c*x])])/c + (8*b*(a + b*ArcCos[c*x])*Sec[ArcCos[c*x]/2]^2
)/(3*c) - (16*b^2*Tan[ArcCos[c*x]/2])/(3*c) + (28*(a + b*ArcCos[c*x])^2*Ta
n[ArcCos[c*x]/2])/(3*c) - (4*(a + b*ArcCos[c*x])^2*Sec[ArcCos[c*x]/2]^2*Ta
n[ArcCos[c*x]/2])/(3*c)))/((d + c*d*x)^(5/2)*(e - c*e*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.20

method	result
default	$\frac{5\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,(a+b\arccos(cx))^3e^2}{3(cx+1)d^3(cx-1)cb}+\frac{\sqrt{-e(cx-1)}\,\sqrt{d(cx+1)}\,\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\left(\arccos(cx)^2b^2+\right.}{2(cx+1)d^3(cx-1)}$

input
```
int((-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
5/3*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x+1)/d^3/(c
*x-1)/c*(a+b*arccos(c*x))^3*e^2/b+1/2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)
*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)^2*b^2+2*arccos(c*x)*a*b
+a^2-2*b^2+2*I*arccos(c*x)*b^2+2*I*a*b)*e^2/(c*x+1)/d^3/(c*x-1)/c+1/2*(-e*
(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(ar
ccos(c*x)^2*b^2+2*arccos(c*x)*a*b+a^2-2*b^2-2*I*b^2*arccos(c*x)-2*I*a*b)*e
^2/(c*x+1)/d^3/(c*x-1)/c+4/3*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(7*c^2*x
^2-2*c*x-5-7*I*(-c^2*x^2+1)^(1/2)*c*x-7*I*(-c^2*x^2+1)^(1/2))*(63*b^2*c^2*
x^2*arccos(c*x)^2-28*I*a*b*c*x+126*a*b*c^2*x^2*arccos(c*x)+4*I*(-c^2*x^2+1
)^(1/2)*b^2*c*x+96*arccos(c*x)^2*b^2*c*x+14*(-c^2*x^2+1)^(1/2)*arccos(c*x)
*b^2*c*x-14*I*a*b-14*I*arccos(c*x)*b^2+63*a^2*c^2*x^2-32*x^2*c^2*b^2+192*a
rccos(c*x)*a*b*c*x+14*(-c^2*x^2+1)^(1/2)*a*b*c*x-28*I*arccos(c*x)*b^2*c*x+
37*arccos(c*x)^2*b^2+10*arccos(c*x)*(-c^2*x^2+1)^(1/2)*b^2-14*I*arccos(c*x
)*b^2*c^2*x^2-14*I*a*b*c^2*x^2+96*a^2*c*x-56*c*x*b^2+74*arccos(c*x)*a*b+10
*(-c^2*x^2+1)^(1/2)*a*b+4*I*(-c^2*x^2+1)^(1/2)*b^2+37*a^2-24*b^2)*e^2/(63*
c^4*x^4+222*c^3*x^3+292*c^2*x^2+170*c*x+37)/d^3/(c*x-1)/c+56/3*I*(-c^2*x^2
+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(c*x+1)/d^3/(c*x-1)/c*b*(ar
ccos(c*x)^2*b+2*I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*b+2*polylog(2
,-c*x-I*(-c^2*x^2+1)^(1/2))*b+2*I*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*a-2*I*ln(
c*x+I*(-c^2*x^2+1)^(1/2))*a)*e^2
```

Fricas [F]

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx=\int \frac{(-cex+e)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}{(cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2),x, algorith
m="fricas")
```

output
```
integral((a^2*c^2*e^2*x^2 - 2*a^2*c*e^2*x + a^2*e^2 + (b^2*c^2*e^2*x^2 - 2
*b^2*c*e^2*x + b^2*e^2)*arccos(c*x)^2 + 2*(a*b*c^2*e^2*x^2 - 2*a*b*c*e^2*x
 + a*b*e^2)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^3*d^3*x^3 + 3
*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((-c*e*x+e)**(5/2)*(a+b*acos(c*x))**2/(c*d*x+d)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx = \int \frac{(-cex+e)^{\frac{5}{2}}(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*e*x+e)^(5/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2),x, algorith
m="giac")
```

output
```
integrate((-c*e*x + e)^(5/2)*(b*arccos(c*x) + a)^2/(c*d*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx=\int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2\,\left(e-c\,e\,x\right)^{5/2}}{\left(d+c\,d\,x\right)^{5/2}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(5/2))/(d + c*d*x)^(5/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(e - c*e*x)^(5/2))/(d + c*d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(e-cex)^{5/2}(a+b\arccos(cx))^2}{(d+cdx)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((-c*e*x+e)^(5/2)*(a+b*acos(c*x))^2/(c*d*x+d)^(5/2),x)
```

output

```
(sqrt(e)*e**2*( - 30*sqrt(c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a**2*c*x
 - 30*sqrt(c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + 3*sqrt( - c*x +
1)*a**2*c**2*x**2 + 34*sqrt( - c*x + 1)*a**2*c*x + 23*sqrt( - c*x + 1)*a**
2 + 6*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)*x**2)/(sqrt(c*x + 1)*c
**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*a*b*c**4*x + 6*sqrt(c*x
 + 1)*int((sqrt( - c*x + 1)*acos(c*x)*x**2)/(sqrt(c*x + 1)*c**2*x**2 + 2*s
qrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*a*b*c**3 - 12*sqrt(c*x + 1)*int((sqrt
( - c*x + 1)*acos(c*x)*x)/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x +
 sqrt(c*x + 1)),x)*a*b*c**3*x - 12*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*aco
s(c*x)*x)/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),
x)*a*b*c**2 + 6*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x))/(sqrt(c*x +
 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*a*b*c**2*x + 6*sqr
t(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x))/(sqrt(c*x + 1)*c**2*x**2 + 2*s
qrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*a*b*c + 3*sqrt(c*x + 1)*int((sqrt( -
c*x + 1)*acos(c*x)**2*x**2)/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x
 + sqrt(c*x + 1)),x)*b**2*c**4*x + 3*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*a
cos(c*x)**2*x**2)/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*
x + 1)),x)*b**2*c**3 - 6*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)**2*
x)/(sqrt(c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*c*x + sqrt(c*x + 1)),x)*b**2
*c**3*x - 6*sqrt(c*x + 1)*int((sqrt( - c*x + 1)*acos(c*x)**2*x)/(sqrt(c...
```

3.560 $\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 559

$$\begin{aligned}
\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx &= \frac{68b^2d^3(1-c^2x^2)}{9c\sqrt{d+cdx}\sqrt{e-cex}} \\
&+ \frac{3b^2d^3x(1-c^2x^2)}{4\sqrt{d+cdx}\sqrt{e-cex}} - \frac{2b^2d^3(1-c^2x^2)^2}{27c\sqrt{d+cdx}\sqrt{e-cex}} \\
&- \frac{3b^2d^3\sqrt{1-c^2x^2}\arccos(cx)}{4c\sqrt{d+cdx}\sqrt{e-cex}} + \frac{22bd^3x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3\sqrt{d+cdx}\sqrt{e-cex}} \\
&+ \frac{3bcd^3x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2\sqrt{d+cdx}\sqrt{e-cex}} + \frac{2bc^2d^3x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9\sqrt{d+cdx}\sqrt{e-cex}} \\
&- \frac{11d^3(1-c^2x^2)\,(a+b\arccos(cx))^2}{3c\sqrt{d+cdx}\sqrt{e-cex}} - \frac{3d^3x(1-c^2x^2)\,(a+b\arccos(cx))^2}{2\sqrt{d+cdx}\sqrt{e-cex}} \\
&- \frac{cd^3x^2(1-c^2x^2)\,(a+b\arccos(cx))^2}{3\sqrt{d+cdx}\sqrt{e-cex}} + \frac{5d^3\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{6bc\sqrt{d+cdx}\sqrt{e-cex}}
\end{aligned}$$

output
```
68/9*b^2*d^3*(-c^2*x^2+1)/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+3/4*b^2*d^3*x
*(-c^2*x^2+1)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2/27*b^2*d^3*(-c^2*x^2+1)^2
/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-3/4*b^2*d^3*(-c^2*x^2+1)^(1/2)*arccos(
c*x)/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+22/3*b*d^3*x*(-c^2*x^2+1)^(1/2)*(a
+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+3/2*b*c*d^3*x^2*(-c^2*x^2
+1)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2/9*b*c^2*d^3
*x^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)
-11/3*d^3*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1
/2)-3/2*d^3*x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^
(1/2)-1/3*c*d^3*x^2*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e
*x+e)^(1/2)+5/6*d^3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^3/b/c/(c*d*x+d)^(
1/2)/(-c*e*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 8.74 (sec) , antiderivative size = 473, normalized size of antiderivative = 0.85

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx =$$

$$-\frac{d^2(1+cx)\sec^2\left(\frac{1}{2}\arccos(cx)\right)\left(180b^2\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)^3+540a^2\sqrt{d}\sqrt{e}\sqrt{1-c^2x^2}\arctan\left(\frac{cx}{\sqrt{}}\right.\right.}{}$$

input
```
Integrate[((d + c*d*x)^(5/2)*(a + b*ArcCos[c*x])^2)/Sqrt[e - c*e*x],x]
```

output
```
-1/432*(d^2*(1 + c*x)*Sec[ArcCos[c*x]/2]^2*(180*b^2*Sqrt[d + c*d*x]*Sqrt[e
 - c*e*x]*ArcCos[c*x]^3 + 540*a^2*Sqrt[d]*Sqrt[e]*Sqrt[1 - c^2*x^2]*ArcTan
[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] +
 18*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^2*(30*a + 45*b*Sqrt[1 -
c^2*x^2] + 9*b*Sin[2*ArcCos[c*x]] + b*Sin[3*ArcCos[c*x]]) + Sqrt[d + c*d*x
]*Sqrt[e - c*e*x]*(1620*a*b*c*x + 792*a^2*Sqrt[1 - c^2*x^2] - 1620*b^2*Sqr
t[1 - c^2*x^2] + 324*a^2*c*x*Sqrt[1 - c^2*x^2] + 72*a^2*c^2*x^2*Sqrt[1 - c
^2*x^2] + 162*a*b*Cos[2*ArcCos[c*x]] + 12*a*b*Cos[3*ArcCos[c*x]] - 81*b^2*
Sin[2*ArcCos[c*x]] - 4*b^2*Sin[3*ArcCos[c*x]]) + 6*b*Sqrt[d + c*d*x]*Sqrt[
e - c*e*x]*ArcCos[c*x]*(27*b*Cos[2*ArcCos[c*x]] + 2*b*Cos[3*ArcCos[c*x]] +
 6*(45*b*c*x + 45*a*Sqrt[1 - c^2*x^2] + 9*a*Sin[2*ArcCos[c*x]] + a*Sin[3*A
rcCos[c*x]]))))/(c*e*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.54, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5179, 27, 5273, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^{5/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{d^3(cx+1)^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3\sqrt{1-c^2x^2}\int \frac{(cx+1)^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{5273}$$

$$-\frac{d^3\sqrt{1-c^2x^2}\int \left(xc^2+c\right)^3(a+b\arccos(cx))^2d\arccos(cx)}{c^4\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^3\sqrt{1-c^2x^2}\int (a+b\arccos(cx))^2\left(\sin\left(\arccos(cx)+\frac{\pi}{2}\right)c+c\right)^3d\arccos(cx)}{c^4\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{3798}$$

$$-\frac{d^3\sqrt{1-c^2x^2}\int \left(x^3(a+b\arccos(cx))^2c^6+3x^2(a+b\arccos(cx))^2c^5+3x(a+b\arccos(cx))^2c^4+(a+b\arccos(c\right.}{c^4\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3\sqrt{1-c^2x^2}\left(\frac{2}{9}bc^6x^3(a+b\arccos(cx))+\frac{3}{2}bc^5x^2(a+b\arccos(cx))+\frac{22}{3}bc^4x(a+b\arccos(cx))+\frac{5c^3(a+b\arccos(c}{6b}\right.}{}$$

input
```
Int[((d + c*d*x)^(5/2)*(a + b*ArcCos[c*x])^2)/Sqrt[e - c*e*x],x]
```

output
```
-((d^3*Sqrt[1 - c^2*x^2]*((-68*b^2*c^3*Sqrt[1 - c^2*x^2])/9 - (3*b^2*c^4*x
*Sqrt[1 - c^2*x^2])/4 + (2*b^2*c^3*(1 - c^2*x^2)^(3/2))/27 - (3*b^2*c^3*Ar
cCos[c*x])/4 + (22*b*c^4*x*(a + b*ArcCos[c*x]))/3 + (3*b*c^5*x^2*(a + b*Ar
cCos[c*x]))/2 + (2*b*c^6*x^3*(a + b*ArcCos[c*x]))/9 + (11*c^3*Sqrt[1 - c^2
*x^2]*(a + b*ArcCos[c*x])^2)/3 + (3*c^4*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[
c*x])^2)/2 + (c^5*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/3 + (5*c^3*
(a + b*ArcCos[c*x])^3)/(6*b)))/(c^4*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5273
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[-(c^(m + 1)*Sqrt[d])^(-1)   Subs
t[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &
& (GtQ[m, 0] || IGtQ[n, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.01 (sec) , antiderivative size = 1810, normalized size of antiderivative = 3.24

method	result	size
default	Expression too large to display	1810
parts	Expression too large to display	1810

input
```
int((c*d*x+d)^(5/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/3*a^2/c/e*(c*d*x+d)^(5/2)*(-c*e*x+e)^(1/2)-5/6*a^2*d/c/e*(c*d*x+d)^(3/2
)*(-c*e*x+e)^(1/2)-5/2*a^2*d^2/c/e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)+5/2*a^
2*d^3*((-c*e*x+e)*(c*d*x+d))^(1/2)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)/(c^2*d
*e)^(1/2)*arctan((c^2*d*e)^(1/2)*x/(-c^2*d*e*x^2+d*e)^(1/2))+b^2*(5/6*(d*(
c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x-1)/e/c/(c*x+1)*ar
ccos(c*x)^3*d^2-1/432*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(4*c^3*x^3+8*I*
(-c^2*x^2+1)^(1/2)*x^3*c^3+8*c^4*x^4-3*c*x+4*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-
4*I*(-c^2*x^2+1)^(1/2)*x*c-8*c^2*x^2-I*(-c^2*x^2+1)^(1/2)+1)*(6*I*arccos(c
*x)+9*arccos(c*x)^2-2)*d^2/(c*x-1)/e/c/(c*x+1)+15/16*(d*(c*x+1))^(1/2)*(-e
*(c*x-1))^(1/2)*(1+c*x+I*(-c^2*x^2+1)^(1/2))*(arccos(c*x)^2-2+2*I*arccos(c
*x))*d^2/(c*x-1)/e/c/(c*x+1)-15/8*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I
*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)^2-2-2*I*arccos(c*x))*d^2/(
c*x-1)/e/c/(c*x+1)+3/32*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*I*(-c^2*x
^2+1)^(1/2)*c*x+2*c^2*x^2-I*(-c^2*x^2+1)^(1/2)+c*x-1)*(2*arccos(c*x)^2-1-2
*I*arccos(c*x))*d^2/(c*x-1)/e/c/(c*x+1)-1/288*(d*(c*x+1))^(1/2)*(-e*(c*x-1
))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x+1)*(46*I*arccos(c*x)+54*arccos(c*x)^2-
27)*cos(3*arccos(c*x))*d^2/(c*x-1)/e/c/(c*x+1)-1/864*(d*(c*x+1))^(1/2)*(-e
*(c*x-1))^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)+I)*(162*I*arccos(c*x)+126*arccos
(c*x)^2-73)*sin(3*arccos(c*x))*d^2/(c*x-1)/e/c/(c*x+1)-1/216*(d*(c*x+1))^(
1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x+1)*(327*I*arccos(c*x...
```

Fricas [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx = \int \frac{(cdx+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}{\sqrt{-cex+e}}\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-(a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 +
2*b^2*c*d^2*x + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*
x + a*b*d^2)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c*e*x - e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+d)**(5/2)*(a+b*acos(c*x))**2/(-c*e*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(1/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx = \int \frac{(cdx+d)^{\frac{5}{2}}(b\arccos(cx)+a)^2}{\sqrt{-cex+e}}\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(1/2),x, algorith
m="giac")
```

output
```
integrate((c*d*x + d)^(5/2)*(b*arccos(c*x) + a)^2/sqrt(-c*e*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx=\int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2\,\left(d+c\,d\,x\right)^{5/2}}{\sqrt{e-c\,e\,x}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(5/2))/(e - c*e*x)^(1/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(5/2))/(e - c*e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx=\frac{\sqrt{d}\,d^2\left(-30asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2-2\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^2x^2-9\sqrt{c}$$

input
```
int((c*d*x+d)^(5/2)*(a+b*acos(c*x))^2/(-c*e*x+e)^(1/2),x)
```

output
```
(sqrt(d)*d**2*( - 30*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - 2*sqrt(c*x + 1)
*sqrt( - c*x + 1)*a**2*c**2*x**2 - 9*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c
*x - 22*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2 + 12*int((sqrt(c*x + 1)*acos(c
*x)*x**2)/sqrt( - c*x + 1),x)*a*b*c**3 + 24*int((sqrt(c*x + 1)*acos(c*x)*x
)/sqrt( - c*x + 1),x)*a*b*c**2 + 12*int((sqrt(c*x + 1)*acos(c*x))/sqrt( -
c*x + 1),x)*a*b*c + 6*int((sqrt(c*x + 1)*acos(c*x)**2*x**2)/sqrt( - c*x +
1),x)*b**2*c**3 + 12*int((sqrt(c*x + 1)*acos(c*x)**2*x)/sqrt( - c*x + 1),x
)*b**2*c**2 + 6*int((sqrt(c*x + 1)*acos(c*x)**2)/sqrt( - c*x + 1),x)*b**2*
c))/(6*sqrt(e)*c)
```

3.561 $\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 398

$$\begin{aligned}\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx &= \frac{4b^2d^2(1-c^2x^2)}{c\sqrt{d+cdx}\sqrt{e-cex}} \\ &+ \frac{b^2d^2x(1-c^2x^2)}{4\sqrt{d+cdx}\sqrt{e-cex}} - \frac{b^2d^2\sqrt{1-c^2x^2}\arccos(cx)}{4c\sqrt{d+cdx}\sqrt{e-cex}} \\ &+ \frac{4bd^2x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{\sqrt{d+cdx}\sqrt{e-cex}} \\ &+ \frac{bcd^2x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2\sqrt{d+cdx}\sqrt{e-cex}} - \frac{2d^2(1-c^2x^2)(a+b\arccos(cx))^2}{c\sqrt{d+cdx}\sqrt{e-cex}} \\ &- \frac{d^2x(1-c^2x^2)(a+b\arccos(cx))^2}{2\sqrt{d+cdx}\sqrt{e-cex}} + \frac{d^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{2bc\sqrt{d+cdx}\sqrt{e-cex}}\end{aligned}$$

output
```
4*b^2*d^2*(-c^2*x^2+1)/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+1/4*b^2*d^2*x*(-
c^2*x^2+1)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-1/4*b^2*d^2*(-c^2*x^2+1)^(1/2)
*arccos(c*x)/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+4*b*d^2*x*(-c^2*x^2+1)^(1/
2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+1/2*b*c*d^2*x^2*(-c^
2*x^2+1)^(1/2)*(a+b*arccos(c*x))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2*d^2*(-
c^2*x^2+1)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-1/2*d^2*
x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+1/2*d^
2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^3/b/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1
/2)
```

Mathematica [A] (verified)

Time = 5.51 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.29

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx = \frac{-24a^2d(4+cx)\sqrt{d+cdx}\sqrt{e-cex}\sqrt{1-c^2x^2}-48abd\sqrt{d+cdx}\sqrt{}}{}$$

input
```
Integrate[((d + c*d*x)^(3/2)*(a + b*ArcCos[c*x])^2)/Sqrt[e - c*e*x],x]
```

output
```
(-24*a^2*d*(4 + c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2] - 4
8*a*b*d*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(2*c*x + 2*Sqrt[1 - c^2*x^2]*ArcCo
s[c*x] + ArcCos[c*x]^2) - 72*a^2*d^(3/2)*Sqrt[e]*Sqrt[1 - c^2*x^2]*ArcTan[
(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] -
8*b^2*d*(1 + c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-6*Sqrt[1 - c^2*x^2] +
6*c*x*ArcCos[c*x] + 3*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2 + ArcCos[c*x]^3)*Sec
[ArcCos[c*x]/2]^2 - b^2*d*(1 + c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sec[Ar
cCos[c*x]/2]^2*(4*ArcCos[c*x]^3 + 6*ArcCos[c*x]*(8*c*x + Cos[2*ArcCos[c*x]
]) + 6*ArcCos[c*x]^2*(4*Sqrt[1 - c^2*x^2] + Sin[2*ArcCos[c*x]]) - 3*(16*Sq
rt[1 - c^2*x^2] + Sin[2*ArcCos[c*x]])) - 6*a*b*d*(1 + c*x)*Sqrt[d + c*d*x]
*Sqrt[e - c*e*x]*Sec[ArcCos[c*x]/2]^2*(8*c*x + Cos[2*ArcCos[c*x]] + 2*ArcC
os[c*x]*(4*Sqrt[1 - c^2*x^2] + ArcCos[c*x] + Sin[2*ArcCos[c*x]])))/(48*c*e
*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.55, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5179, 27, 5273, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^{3/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx$$

$$\Big\downarrow \; 5179$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{d^2(cx+1)^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$\Big\downarrow$ 27

$$\frac{d^2\sqrt{1-c^2x^2}\int \frac{(cx+1)^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$\Big\downarrow$ 5273

$$-\frac{d^2\sqrt{1-c^2x^2}\int \left(xc^2+c\right)^2(a+b\arccos(cx))^2d\arccos(cx)}{c^3\sqrt{cdx+d}\sqrt{e-cex}}$$

$\Big\downarrow$ 3042

$$-\frac{d^2\sqrt{1-c^2x^2}\int (a+b\arccos(cx))^2\left(\sin\left(\arccos(cx)+\frac{\pi}{2}\right)c+c\right)^2d\arccos(cx)}{c^3\sqrt{cdx+d}\sqrt{e-cex}}$$

$\Big\downarrow$ 3798

$$-\frac{d^2\sqrt{1-c^2x^2}\int \left(x^2(a+b\arccos(cx))^2c^4+2x(a+b\arccos(cx))^2c^3+(a+b\arccos(cx))^2c^2\right)d\arccos(cx)}{c^3\sqrt{cdx+d}\sqrt{e-cex}}$$

$\Big\downarrow$ 2009

$$-\frac{d^2\sqrt{1-c^2x^2}\left(\frac{1}{2}bc^4x^2(a+b\arccos(cx))+4bc^3x(a+b\arccos(cx))+2c^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2+\frac{c^2(a+b\,a}{}\right.}{c^3\sqrt{cdx+d}\sqrt{}}$$

input
```
Int[((d + c*d*x)^(3/2)*(a + b*ArcCos[c*x])^2)/Sqrt[e - c*e*x],x]
```

output
```
-((d^2*Sqrt[1 - c^2*x^2]*(-4*b^2*c^2*Sqrt[1 - c^2*x^2] - (b^2*c^3*x*Sqrt[1
 - c^2*x^2])/4 - (b^2*c^2*ArcCos[c*x])/4 + 4*b*c^3*x*(a + b*ArcCos[c*x]) +
 (b*c^4*x^2*(a + b*ArcCos[c*x]))/2 + 2*c^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos
[c*x])^2 + (c^3*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/2 + (c^2*(a + b
*ArcCos[c*x])^3)/(2*b)))/(c^3*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5273
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[-(c^(m + 1)*Sqrt[d])^(-1)   Subs
t[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &
& (GtQ[m, 0] || IGtQ[n, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.24 (sec) , antiderivative size = 1357, normalized size of antiderivative = 3.41

method	result	size
default	Expression too large to display	1357
parts	Expression too large to display	1357

input

```
int((c*d*x+d)^(3/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(1/2),x,method=_RETURNV
ERBOSE)
```

output

```
-1/2*a^2/c/e*(c*d*x+d)^(3/2)*(-c*e*x+e)^(1/2)-3/2*a^2*d/c/e*(c*d*x+d)^(1/2
)*(-c*e*x+e)^(1/2)+3/2*a^2*d^2*((-c*e*x+e)*(c*d*x+d))^(1/2)/(c*d*x+d)^(1/2
)/(-c*e*x+e)^(1/2)/(c^2*d*e)^(1/2)*arctan((c^2*d*e)^(1/2)*x/(-c^2*d*e*x^2+
d*e)^(1/2))+b^2*(1/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/
2)/(c*x-1)/e/c/(c*x+1)*arccos(c*x)^3*d-1/32*(d*(c*x+1))^(1/2)*(-e*(c*x-1))
^(1/2)*(2*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+4*c^3*x^3-1+2*I*(-c^2*x^2
+1)^(1/2)*c*x-I*(-c^2*x^2+1)^(1/2)-3*c*x)*(2*arccos(c*x)^2-1+2*I*arccos(c*
x))*d/(c*x-1)/e/c/(c*x+1)+1/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(1+c*x+
I*(-c^2*x^2+1)^(1/2))*(arccos(c*x)^2-2+2*I*arccos(c*x))*d/(c*x-1)/e/c/(c*x
+1)-(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^
2-1)*(arccos(c*x)^2-2-2*I*arccos(c*x))*d/(c*x-1)/e/c/(c*x+1)+1/32*(d*(c*x+
1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-I*(-c^
2*x^2+1)^(1/2)+c*x-1)*(2*arccos(c*x)^2-1-2*I*arccos(c*x))*d/(c*x-1)/e/c/(c
*x+1)-1/8*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x+
1)*(7*I*arccos(c*x)+4*arccos(c*x)^2-8)*cos(2*arccos(c*x))*d/(c*x-1)/e/c/(c
*x+1)-1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*c*x+(-c^2*x^2+1)^(1/2)+
I)*(16*I*arccos(c*x)+6*arccos(c*x)^2-15)*sin(2*arccos(c*x))*d/(c*x-1)/e/c/
(c*x+1))+2*a*b*(3/4*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2
)/(c*x-1)/e/c/(c*x+1)*arccos(c*x)^2*d-1/32*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^
(1/2)*(2*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+4*c^3*x^3-1+2*I*(-c^2*x...
```

Fricas [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx = \int \frac{(cdx+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)^2}{\sqrt{-cex+e}}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-(a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arccos(c*x)^2 + 2*(a*b*
c*d*x + a*b*d)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c*e*x - e),
x)
```

Sympy [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx = \int \frac{(d(cx+1))^{\frac{3}{2}}\,(a+b\operatorname{acos}{(cx)})^2}{\sqrt{-e\,(cx-1)}}\,dx$$

input
```
integrate((c*d*x+d)**(3/2)*(a+b*acos(c*x))**2/(-c*e*x+e)**(1/2),x)
```

output
```
Integral((d*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2/sqrt(-e*(c*x - 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(1/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx = \int \frac{(cdx+d)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{\sqrt{-cex+e}}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(1/2),x, algorith
m="giac")
```

output
```
integrate((c*d*x + d)^(3/2)*(b*arccos(c*x) + a)^2/sqrt(-c*e*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d+c\,d\,x)^{3/2}}{\sqrt{e-c\,e\,x}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(3/2))/(e - c*e*x)^(1/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(3/2))/(e - c*e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx = \frac{\sqrt{d}\,d\left(-6asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2-\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2cx-4\sqrt{cx+1}\right.}{}$$

input
```
int((c*d*x+d)^(3/2)*(a+b*acos(c*x))^2/(-c*e*x+e)^(1/2),x)
```

output
```
(sqrt(d)*d*( - 6*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - sqrt(c*x + 1)*sqrt(
 - c*x + 1)*a**2*c*x - 4*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2 + 4*int((sqrt
(c*x + 1)*acos(c*x)*x)/sqrt( - c*x + 1),x)*a*b*c**2 + 4*int((sqrt(c*x + 1)
*acos(c*x))/sqrt( - c*x + 1),x)*a*b*c + 2*int((sqrt(c*x + 1)*acos(c*x)**2*
x)/sqrt( - c*x + 1),x)*b**2*c**2 + 2*int((sqrt(c*x + 1)*acos(c*x)**2)/sqrt
( - c*x + 1),x)*b**2*c))/(2*sqrt(e)*c)
```

3.562 $\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 231

$$\begin{aligned}\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx =& \frac{2abdx\sqrt{1-c^2x^2}}{\sqrt{d+cdx}\sqrt{e-cex}}+\frac{2b^2d(1-c^2x^2)}{c\sqrt{d+cdx}\sqrt{e-cex}}\\&+\frac{2b^2dx\sqrt{1-c^2x^2}\arccos(cx)}{\sqrt{d+cdx}\sqrt{e-cex}}\\&-\frac{d(1-c^2x^2)\,(a+b\arccos(cx))^2}{c\sqrt{d+cdx}\sqrt{e-cex}}\\&+\frac{d\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc\sqrt{d+cdx}\sqrt{e-cex}}\end{aligned}$$

output
```
2*a*b*d*x*(-c^2*x^2+1)^(1/2)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*b^2*d*(-c^
2*x^2+1)/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*b^2*d*x*(-c^2*x^2+1)^(1/2)*a
rccos(c*x)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-d*(-c^2*x^2+1)*(a+b*arccos(c*x
))^2/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+1/3*d*(-c^2*x^2+1)^(1/2)*(a+b*arcc
os(c*x))^3/b/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx =$$
$$-\frac{(1+cx)\left(3\sqrt{d+cdx}\sqrt{e-cex}\left(2abcx+a^2\sqrt{1-c^2x^2}-2b^2\sqrt{1-c^2x^2}\right)+6b\sqrt{d+cdx}\sqrt{e-cex}(bcx+$$

input
```
Integrate[(Sqrt[d + c*d*x]*(a + b*ArcCos[c*x])^2)/Sqrt[e - c*e*x],x]
```

output
```
-1/6*((1 + c*x)*(3*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(2*a*b*c*x + a^2*Sqrt[1
 - c^2*x^2] - 2*b^2*Sqrt[1 - c^2*x^2]) + 6*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*
x]*(b*c*x + a*Sqrt[1 - c^2*x^2])*ArcCos[c*x] + 3*b*Sqrt[d + c*d*x]*Sqrt[e
- c*e*x]*(a + b*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2 + b^2*Sqrt[d + c*d*x]*Sqr
t[e - c*e*x]*ArcCos[c*x]^3 + 3*a^2*Sqrt[d]*Sqrt[e]*Sqrt[1 - c^2*x^2]*ArcTa
n[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))])
*Sec[ArcCos[c*x]/2]^2)/(c*e*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cdx+d}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx$$
$$\Big\downarrow \; 5179$$
$$\frac{\sqrt{1-c^2x^2}\int \frac{d(cx+1)(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$
$$\Big\downarrow \; 27$$

$$\frac{d\sqrt{1-c^2x^2}\int \frac{(cx+1)(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 5263}$$

$$\frac{d\sqrt{1-c^2x^2}\int \left(\frac{cx(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}+\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\sqrt{1-c^2x^2}\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c}-\frac{(a+b\arccos(cx))^3}{3bc}-2abx-2b^2x\arccos(cx)+\frac{2b^2\sqrt{1-c^2x^2}}{c}\right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

input
```
Int[(Sqrt[d + c*d*x]*(a + b*ArcCos[c*x])^2)/Sqrt[e - c*e*x],x]
```

output
```
(d*Sqrt[1 - c^2*x^2]*(-2*a*b*x + (2*b^2*Sqrt[1 - c^2*x^2])/c - 2*b^2*x*Arc
Cos[c*x] - (Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c - (a + b*ArcCos[c*x
])^3/(3*b*c)))/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.35

method	result
default	$-\frac{a^2\sqrt{cdx+d}\,\sqrt{-cex+e}}{ce}+\frac{a^2d\sqrt{(-cex+e)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-c^2de\,x^2+de}}\right)}{\sqrt{cdx+d}\,\sqrt{-cex+e}\,\sqrt{c^2de}}+b^2\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,\text{arcco}}{3(cx-1)ec(cx+1)}\right.$
parts	$-\frac{a^2\sqrt{cdx+d}\,\sqrt{-cex+e}}{ce}+\frac{a^2d\sqrt{(-cex+e)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-c^2de\,x^2+de}}\right)}{\sqrt{cdx+d}\,\sqrt{-cex+e}\,\sqrt{c^2de}}+b^2\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,\text{arcco}}{3(cx-1)ec(cx+1)}\right.$

input
```
int((c*d*x+d)^(1/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-a^2/c/e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)+a^2*d*((-c*e*x+e)*(c*d*x+d))^(1/
2)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)/(c^2*d*e)^(1/2)*arctan((c^2*d*e)^(1/2)
*x/(-c^2*d*e*x^2+d*e)^(1/2))+b^2*(1/3*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)
*(-c^2*x^2+1)^(1/2)/(c*x-1)/e/c/(c*x+1)*arccos(c*x)^3-1/2*(d*(c*x+1))^(1/2
)*(-e*(c*x-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)^2-2
+2*I*arccos(c*x))/(c*x-1)/e/c/(c*x+1)-1/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(
1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)^2-2-2*I*arccos(c*x
))/(c*x-1)/e/c/(c*x+1))+2*a*b*(1/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-
c^2*x^2+1)^(1/2)/(c*x-1)/e/c/(c*x+1)*arccos(c*x)^2-1/2*(d*(c*x+1))^(1/2)*(
-e*(c*x-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)+I)/(c*
x-1)/e/c/(c*x+1)-1/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)
^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)-I)/(c*x-1)/e/c/(c*x+1))
```

Fricas [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx = \int \frac{\sqrt{cdx+d}(b\arccos{(cx)}+a)^2}{\sqrt{-cex+e}}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sq
rt(-c*e*x + e)/(c*e*x - e), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx = \int \frac{\sqrt{d\,(cx+1)}(a+b\operatorname{acos}{(cx)})^2}{\sqrt{-e\,(cx-1)}}\,dx$$

input
```
integrate((c*d*x+d)**(1/2)*(a+b*acos(c*x))**2/(-c*e*x+e)**(1/2),x)
```

output
```
Integral(sqrt(d*(c*x + 1))*(a + b*acos(c*x))**2/sqrt(-e*(c*x - 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(1/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx=\int \frac{\sqrt{cdx+d}(b\arccos(cx)+a)^2}{\sqrt{-cex+e}}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(1/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(c*d*x + d)*(b*arccos(c*x) + a)^2/sqrt(-c*e*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d+c\,d\,x}}{\sqrt{e-c\,e\,x}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(1/2))/(e - c*e*x)^(1/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(1/2))/(e - c*e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{\sqrt{e-cex}}\,dx$$
$$= \frac{\sqrt{d}\left(-2asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2-\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2+2\left(\int \frac{\sqrt{cx+1}\,acos(cx)}{\sqrt{-cx+1}}dx\right)abc+\left(\int \frac{\sqrt{cx+1}\,acos(cx)^2}{\sqrt{-cx+1}}dx\right)b^2c\right)}{\sqrt{e}\,c}$$

input
```
int((c*d*x+d)^(1/2)*(a+b*acos(c*x))^2/(-c*e*x+e)^(1/2),x)
```

output
```
(sqrt(d)*( - 2*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - sqrt(c*x + 1)*sqrt( -
c*x + 1)*a**2 + 2*int((sqrt(c*x + 1)*acos(c*x))/sqrt( - c*x + 1),x)*a*b*c
+ int((sqrt(c*x + 1)*acos(c*x)**2)/sqrt( - c*x + 1),x)*b**2*c))/(sqrt(e)*
c)
```

3.563 $\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc\sqrt{d+cdx}\sqrt{e-cex}}$$

output
```
1/3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^3/b/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^
(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 159 vs. 2(55) = 110.

Time = 2.03 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.89

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx = -\frac{\frac{3ab\sqrt{1-c^2x^2}\arccos(cx)^2}{\sqrt{d+cdx}\sqrt{e-cex}}+\frac{b^2\sqrt{1-c^2x^2}\arccos(cx)^3}{\sqrt{d+cdx}\sqrt{e-cex}}+\frac{3a^2\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{e-cex}}{\sqrt{d}\sqrt{e}\left(-1+c^2x^2\right)}\right)}{\sqrt{d}\sqrt{e}}}{3c}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]
```

output
```
-1/3*((3*a*b*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2)/(Sqrt[d + c*d*x]*Sqrt[e - c*
e*x]) + (b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^3)/(Sqrt[d + c*d*x]*Sqrt[e - c*
e*x]) + (3*a^2*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[
e]*(-1 + c^2*x^2))])/(Sqrt[d]*Sqrt[e]))/c
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {5179, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{cdx+d}\sqrt{e-cex}}dx$$

$$\Big\downarrow \; 5179$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \; 5153$$

$$-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc\sqrt{cdx+d}\sqrt{e-cex}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]
```

output
```
-1/3*(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^3)/(b*c*Sqrt[d + c*d*x]*Sqrt[e
 - c*e*x])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(47) = 94$.

Time = 1.21 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.56

method	result
default	$\frac{a^2\sqrt{(-cex+e)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-c^2de\,x^2+de}}\right)}{\sqrt{cdx+d}\,\sqrt{-cex+e}\,\sqrt{c^2de}}+\frac{b^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3}{3dec(c^2x^2-1)}+\frac{ab\sqrt{d(cx+1)}\,\sqrt{-e(cx}{dec}$
parts	$\frac{a^2\sqrt{(-cex+e)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-c^2de\,x^2+de}}\right)}{\sqrt{cdx+d}\,\sqrt{-cex+e}\,\sqrt{c^2de}}+\frac{b^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3}{3dec(c^2x^2-1)}+\frac{ab\sqrt{d(cx+1)}\,\sqrt{-e(cx}{dec}$

input
```
int((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
a^2*((-c*e*x+e)*(c*d*x+d))^(1/2)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)/(c^2*d*e
)^(1/2)*arctan((c^2*d*e)^(1/2)*x/(-c^2*d*e*x^2+d*e)^(1/2))+1/3*b^2*(d*(c*x
+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/e/c/(c^2*x^2-1)*arccos(
c*x)^3+a*b*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/e/c/(
c^2*x^2-1)*arccos(c*x)^2
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{\sqrt{cdx+d}\sqrt{-cex+e}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sq
rt(-c*e*x + e)/(c^2*d*e*x^2 - d*e), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(a+b\operatorname{acos}(cx))^2}{\sqrt{d(cx+1)}\sqrt{-e(cx-1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(c*d*x+d)**(1/2)/(-c*e*x+e)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2}{\sqrt{cdx+d}\sqrt{-cex+e}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorith
m="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/(sqrt(c*d*x + d)*sqrt(-c*e*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{\sqrt{d+c\,d\,x}\,\sqrt{e-c\,e\,x}}\,dx$$

input
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx$$
$$= \frac{-2asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2+2\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)abc+\left(\int \frac{acos(cx)^2}{\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)b^2c}{\sqrt{e}\,\sqrt{d}\,c}$$

input
```
int((a+b*acos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x)
```

output
```
( - 2*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + 2*int(acos(c*x)/(sqrt(c*x + 1)
*sqrt( - c*x + 1)),x)*a*b*c + int(acos(c*x)**2/(sqrt(c*x + 1)*sqrt( - c*x
+ 1)),x)*b**2*c)/(sqrt(e)*sqrt(d)*c)
```

3.564 $\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}\sqrt{e-cex}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 455

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}\sqrt{e-cex}}\,dx &= -\frac{e(1-c^2x^2)(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\ &+ \frac{ex(1-c^2x^2)(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{ie(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\ &- \frac{4ibe(1-c^2x^2)^{3/2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\ &+ \frac{2be(1-c^2x^2)^{3/2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\ &+ \frac{2ib^2e(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\ &- \frac{2ib^2e(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\ &- \frac{ib^2e(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}}\end{aligned}$$

output
```
-e*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+e*x
*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-I*e*(-c
^2*x^2+1)^(3/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-4*I
*b*e*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))
/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+2*b*e*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c
*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)
+2*I*b^2*e*(-c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(
c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-2*I*b^2*e*(-c^2*x^2+1)^(3/2)*polylog(2,I*(
c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-I*b^2*e*(-c^
2*x^2+1)^(3/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(3/2)/
(-c*e*x+e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.42

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}\sqrt{e-cex}}\,dx =$$

$$-\frac{\sqrt{e-cex}\left(b^2\left(-1+cx+i\sqrt{1-c^2x^2}\right)\arccos(cx)^2-2b\arccos(cx)\left(a-acx+2b\sqrt{1-c^2x^2}\log\left(1+e^{i\,\mathrm{arcc}}\right.\right.\right.}{cde(-1+}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/((d + c*d*x)^(3/2)*Sqrt[e - c*e*x]),x]
```

output
```
-((Sqrt[e - c*e*x]*(b^2*(-1 + c*x + I*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2 - 2
*b*ArcCos[c*x]*(a - a*c*x + 2*b*Sqrt[1 - c^2*x^2]*Log[1 + E^(I*ArcCos[c*x]
)]) + a*(-a + a*c*x - 4*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/2]]) + (4*
I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^(I*ArcCos[c*x])]))/(c*d*e*(-1 + c*x
)*Sqrt[d + c*d*x]))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(cdx+d)^{3/2}\sqrt{e-cex}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{3/2}\int \frac{e(1-cx)(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e(1-c^2x^2)^{3/2}\int \frac{(1-cx)(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{e(1-c^2x^2)^{3/2}\int \left(\frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}-\frac{cx(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}\right)dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e(1-c^2x^2)^{3/2}\left(-\frac{4b\,\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))}{c}+\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{(a+b\arccos(cx))^2}{c\sqrt{1-c^2x^2}}+\frac{i(a+b\arccos(cx))^2}{c}-\frac{2b\,\text{lo}}{}\right.}{(cdx+d)^{3/2}(e}$$

input
```
Int[(a + b*ArcCos[c*x])^2/((d + c*d*x)^(3/2)*Sqrt[e - c*e*x]),x]
```

output
```
(e*(1 - c^2*x^2)^(3/2)*((I*(a + b*ArcCos[c*x])^2)/c - (a + b*ArcCos[c*x])^
2/(c*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2] - (4
*b*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])])/c - (2*b*(a + b*ArcCos[
c*x])*Log[1 - E^((2*I)*ArcCos[c*x])])/c + ((2*I)*b^2*PolyLog[2, -E^(I*ArcC
os[c*x])])/c - ((2*I)*b^2*PolyLog[2, E^(I*ArcCos[c*x])])/c + (I*b^2*PolyLo
g[2, E^((2*I)*ArcCos[c*x])])/c))/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [A] (verified)

Time = 4.05 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.83

method	result
default	$-\frac{a^2\sqrt{-cex+e}}{dce\sqrt{cdx+d}}+\frac{b^2\sqrt{-c^2x^2+1}\,\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\left(-i\arccos(cx)^2xc+4\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)\arccos(cx)xc-4i\,\mathrm{polylog}\left(\right.\right.}{}$
parts	$-\frac{a^2\sqrt{-cex+e}}{dce\sqrt{cdx+d}}-\frac{b^2\sqrt{-c^2x^2+1}\,\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\left(i\arccos(cx)^2xc-4\ln\left(1+cx+i\sqrt{-c^2x^2+1}\right)\arccos(cx)xc+4i\,\mathrm{polylog}\left(2,\right.\right.}{}$

input
```
int((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-a^2/d/c/e/(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)+b^2*(-c^2*x^2+1)^(1/2)*(d*(c*x
+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*arccos(c*x)^2*x*c+4*ln(1+c*x+I*(-c^2*x^2
+1)^(1/2))*arccos(c*x)*x*c-4*I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))*x*c-I*
arccos(c*x)^2+arccos(c*x)^2*(-c^2*x^2+1)^(1/2)+4*arccos(c*x)*ln(1+c*x+I*(-
c^2*x^2+1)^(1/2))-4*I*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2)))/(c*x+1)^2/d^2/
c/e/(c*x-1)+2*a*b*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*
(-I*arccos(c*x)*x*c+2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*x*c-I*arccos(c*x)+arc
cos(c*x)*(-c^2*x^2+1)^(1/2)+2*ln(1+c*x+I*(-c^2*x^2+1)^(1/2)))/(c*x+1)^2/d^
2/c/e/(c*x-1)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}\sqrt{e-cex}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}\sqrt{-cex+e}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sq
rt(-c*e*x + e)/(c^3*d^2*e*x^3 + c^2*d^2*e*x^2 - c*d^2*e*x - d^2*e), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}\sqrt{e-cex}}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^2}{(d(cx+1))^{\frac{3}{2}}\sqrt{-e(cx-1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(c*d*x+d)**(3/2)/(-c*e*x+e)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/((d*(c*x + 1))**(3/2)*sqrt(-e*(c*x - 1))), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}\sqrt{e-cex}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(1/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}\sqrt{e-cex}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}\sqrt{-cex+e}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(1/2),x, algorith
m="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/((c*d*x + d)^(3/2)*sqrt(-c*e*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}\sqrt{e-cex}}\,dx = \int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{\left(d+c\,d\,x\right)^{3/2}\,\sqrt{e-c\,e\,x}}\,dx$$

input
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}\sqrt{e-cex}}\,dx = \frac{-\sqrt{-cx+1}\,a^2+2\sqrt{cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,cx+\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)abc+\sqrt{cx}}{\sqrt{e}\,\sqrt{d}\,\sqrt{cx+1}\,cd}$$

input
```
int((a+b*acos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(1/2),x)
```

output
```
( - sqrt( - c*x + 1)*a**2 + 2*sqrt(c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*s
qrt( - c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a*b*c + sqrt(c*x
+ 1)*int(acos(c*x)**2/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c*x + sqrt(c*x + 1)*
sqrt( - c*x + 1)),x)*b**2*c)/(sqrt(e)*sqrt(d)*sqrt(c*x + 1)*c*d)
```

3.565 $\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}\sqrt{e-cex}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 896

$$\begin{aligned}
&\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}\sqrt{e-cex}}\,dx = -\frac{2b^2e^2(1-c^2x^2)^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+\frac{2b^2e^2x(1-c^2x^2)^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}}-\frac{b^2e^2(1-c^2x^2)^{5/2}\arccos(cx)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&-\frac{be^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+\frac{2be^2x(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&-\frac{bce^2x^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&-\frac{2e^2(1-c^2x^2)(a+b\arccos(cx))^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}+\frac{e^2x(1-c^2x^2)(a+b\arccos(cx))^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+\frac{c^2e^2x^3(1-c^2x^2)(a+b\arccos(cx))^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+\frac{2e^2x(1-c^2x^2)^2(a+b\arccos(cx))^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&-\frac{ie^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&-\frac{4ibe^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+\frac{2be^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+\frac{2ib^2e^2(1-c^2x^2)^{5/2}\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&-\frac{2ib^2e^2(1-c^2x^2)^{5/2}\,\mathrm{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&-\frac{ib^2e^2(1-c^2x^2)^{5/2}\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}
\end{aligned}$$

output
```
-2/3*b^2*e^2*(-c^2*x^2+1)^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+2/3*b^2*e^2
*x*(-c^2*x^2+1)^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3*b^2*e^2*(-c^2*x^2+1
)^(5/2)*arccos(c*x)/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3*b*e^2*(-c^2*x^2
+1)^(3/2)*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+2/3*b*e^2*x
*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3
*b*c*e^2*x^2*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*e*x+
e)^(5/2)-2/3*e^2*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*
x+e)^(5/2)+1/3*e^2*x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*
e*x+e)^(5/2)+1/3*c^2*e^2*x^3*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5
/2)/(-c*e*x+e)^(5/2)+2/3*e^2*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2/(c*d*x+d
)^(5/2)/(-c*e*x+e)^(5/2)-4/3*I*b*e^2*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*
arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3*I*
e^2*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5
/2)+2/3*b*e^2*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1
)^(1/2))^2)/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3*I*b^2*e^2*(-c^2*x^2+1)^
(5/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(5/2)/(-c*e*x+e
)^(5/2)-2/3*I*b^2*e^2*(-c^2*x^2+1)^(5/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(
1/2)))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+2/3*I*b^2*e^2*(-c^2*x^2+1)^(5/2)
*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/
2)
```

Mathematica [A] (verified)

Time = 3.87 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.36

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}\sqrt{e-cex}}\,dx = \frac{\sqrt{e-cex}\Big(b^2\arccos(cx)^2\left(-8-4cx+i(1-c^2x^2)^{3/2}\csc^4\left(\frac{1}{2}\arccos(cx)\right)\right)+$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/((d + c*d*x)^(5/2)*Sqrt[e - c*e*x]),x]
```

output
```
(Sqrt[e - c*e*x]*(b^2*ArcCos[c*x]^2*(-8 - 4*c*x + I*(1 - c^2*x^2)^(3/2)*Cs
c[ArcCos[c*x]/2]^4) + 4*b*ArcCos[c*x]*(-2*a*(2 + c*x) + b*Sqrt[1 - c^2*x^2
]*Csc[ArcCos[c*x]/2]^2 - b*(1 - c^2*x^2)^(3/2)*Csc[ArcCos[c*x]/2]^4*Log[1
+ E^(I*ArcCos[c*x])]) - 4*(2*b^2*(1 + c*x) + a^2*(2 + c*x) - a*b*Sqrt[1 -
c^2*x^2]*Csc[ArcCos[c*x]/2]^2 + a*b*(1 - c^2*x^2)^(3/2)*Csc[ArcCos[c*x]/2]
^4*Log[(Sqrt[1 - c^2*x^2]*Csc[ArcCos[c*x]/2])/2]) + (4*I)*b^2*(1 - c^2*x^2
)^(3/2)*Csc[ArcCos[c*x]/2]^4*PolyLog[2, -E^(I*ArcCos[c*x])]))/(12*c*d^2*e*
(1 + c*x)*Sqrt[d + c*d*x])
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(cdx+d)^{5/2}\sqrt{e-cex}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\int \frac{e^2(1-cx)^2(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\left(1-c^2x^2\right)^{5/2}\int \frac{(1-cx)^2(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{e^2\left(1-c^2x^2\right)^{5/2}\int \left(\frac{c^2x^2(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}-\frac{2cx(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}+\frac{(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}\right)dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2\left(1-c^2x^2\right)^{5/2}\left(-\frac{4b\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{3c}+\frac{bcx^2(a+b\arccos(cx))}{3(1-c^2x^2)}+\frac{2x(a+b\arccos(cx))^2}{3\sqrt{1-c^2x^2}}+\frac{x(a+b\arccos(cx))^2}{3(1-c^2x^2)^{3/2}} -\right.}{}$$

input
```
Int[(a + b*ArcCos[c*x])^2/((d + c*d*x)^(5/2)*Sqrt[e - c*e*x]),x]
```

output
```
(e^2*(1 - c^2*x^2)^(5/2)*((-2*b^2)/(3*c*Sqrt[1 - c^2*x^2]) + (2*b^2*x)/(3*
Sqrt[1 - c^2*x^2]) + (b*(a + b*ArcCos[c*x]))/(3*c*(1 - c^2*x^2)) - (2*b*x*
(a + b*ArcCos[c*x]))/(3*(1 - c^2*x^2)) + (b*c*x^2*(a + b*ArcCos[c*x]))/(3*
(1 - c^2*x^2)) + ((I/3)*(a + b*ArcCos[c*x])^2)/c - (2*(a + b*ArcCos[c*x])^
2)/(3*c*(1 - c^2*x^2)^(3/2)) + (x*(a + b*ArcCos[c*x])^2)/(3*(1 - c^2*x^2)^
(3/2)) + (c^2*x^3*(a + b*ArcCos[c*x])^2)/(3*(1 - c^2*x^2)^(3/2)) + (2*x*(a
 + b*ArcCos[c*x])^2)/(3*Sqrt[1 - c^2*x^2]) - (b^2*ArcSin[c*x])/(3*c) - (4*
b*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])])/(3*c) - (2*b*(a + b*ArcC
os[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])])/(3*c) + (((2*I)/3)*b^2*PolyLog[2,
 -E^(I*ArcCos[c*x])])/c - (((2*I)/3)*b^2*PolyLog[2, E^(I*ArcCos[c*x])])/c
+ ((I/3)*b^2*PolyLog[2, E^((2*I)*ArcCos[c*x])])/c))/((d + c*d*x)^(5/2)*(e
- c*e*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2579 vs. $2(822) = 1644$.

Time = 4.27 (sec) , antiderivative size = 2580, normalized size of antiderivative = 2.88

method	result	size
default	Expression too large to display	2580
parts	Expression too large to display	2580

input
```
int((a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
10/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/e/d^3/(3*c^4*x^4+8*c^3*x^3+2
*c^2*x^2-8*c*x-5)/c*arccos(c*x)^2+1/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(
1/2)/e/d^3/(3*c^4*x^4+8*c^3*x^3+2*c^2*x^2-8*c*x-5)*arccos(c*x)^2*x-2/3*b^2
*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/e/d^3/(3*c^4*x^4+8*c^3*x^3+2*c^2*x^2
-8*c*x-5)/c*(-c^2*x^2+1)-10/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/e/d
^3/(3*c^4*x^4+8*c^3*x^3+2*c^2*x^2-8*c*x-5)*c^2*x^3-10/3*b^2*(d*(c*x+1))^(1
/2)*(-e*(c*x-1))^(1/2)/e/d^3/(3*c^4*x^4+8*c^3*x^3+2*c^2*x^2-8*c*x-5)*c*x^2
-4/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/e/d^3/(3*c^4*x^4+8*c^3*x^3+2
*c^2*x^2-8*c*x-5)*(-c^2*x^2+1)*x-2/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1
/2)/e/d^3/(3*c^4*x^4+8*c^3*x^3+2*c^2*x^2-8*c*x-5)*c^3*x^4+a^2*(-1/3/c/e/d/
(c*d*x+d)^(3/2)*(-c*e*x+e)^(1/2)-1/3/c/e/d^2/(c*d*x+d)^(1/2)*(-c*e*x+e)^(1
/2))-2*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/e/d^3/(3*c^4*x^4+8*c^3*x^3
+2*c^2*x^2-8*c*x-5)*(-c^2*x^2+1)^(1/2)*arccos(c*x)*x-2/3*b^2*(d*(c*x+1))^(
1/2)*(-e*(c*x-1))^(1/2)/e/d^3/(3*c^4*x^4+8*c^3*x^3+2*c^2*x^2-8*c*x-5)*c*(-
c^2*x^2+1)*x^2+10/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/e/d^3/(3*c^4*
x^4+8*c^3*x^3+2*c^2*x^2-8*c*x-5)*x+4*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1
/2)/e/d^3/(3*c^4*x^4+8*c^3*x^3+2*c^2*x^2-8*c*x-5)/c-10/3*b^2*(d*(c*x+1))^(
1/2)*(-e*(c*x-1))^(1/2)/e/d^3/(3*c^4*x^4+8*c^3*x^3+2*c^2*x^2-8*c*x-5)/c*(-
c^2*x^2+1)^(1/2)*arccos(c*x)-b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/e/d^
3/(3*c^4*x^4+8*c^3*x^3+2*c^2*x^2-8*c*x-5)*c^2*arccos(c*x)^2*x^3-8/3*b^2...
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}\sqrt{e-cex}}\,dx=\int \frac{\left(b\arccos\left(cx\right)+a\right)^2}{\left(cdx+d\right)^{\frac{5}{2}}\sqrt{-cex+e}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sq
rt(-c*e*x + e)/(c^4*d^3*e*x^4 + 2*c^3*d^3*e*x^3 - 2*c*d^3*e*x - d^3*e), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}\sqrt{e-cex}}\,dx=\int \frac{\left(a+b\operatorname{acos}\left(cx\right)\right)^2}{\left(d\left(cx+1\right)\right)^{\frac{5}{2}}\sqrt{-e\left(cx-1\right)}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(c*d*x+d)**(5/2)/(-c*e*x+e)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/((d*(c*x + 1))**(5/2)*sqrt(-e*(c*x - 1))), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}\sqrt{e-cex}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(1/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}\sqrt{e-cex}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{5}{2}}\sqrt{-cex+e}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(1/2),x, algorith
m="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/((c*d*x + d)^(5/2)*sqrt(-c*e*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}\sqrt{e-cex}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(d+c\,d\,x)^{5/2}\,\sqrt{e-c\,e\,x}}\,dx$$

input
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(5/2)*(e - c*e*x)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(5/2)*(e - c*e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}\sqrt{e-cex}}\,dx = \frac{-\sqrt{-cx+1}\,a^2cx-2\sqrt{-cx+1}\,a^2+6\sqrt{cx+1}\left(\int \frac{acos(c}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2+2\sqrt{cx+1}\,\sqrt{}}\right.}{}$$

input
```
int((a+b*acos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(1/2),x)
```

output
```
( - sqrt( - c*x + 1)*a**2*c*x - 2*sqrt( - c*x + 1)*a**2 + 6*sqrt(c*x + 1)*
int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*
sqrt( - c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a*b*c**2*x + 6*s
qrt(c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 + 2*s
qrt(c*x + 1)*sqrt( - c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a*b
*c + 3*sqrt(c*x + 1)*int(acos(c*x)**2/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2
*x**2 + 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x +
 1)),x)*b**2*c**2*x + 3*sqrt(c*x + 1)*int(acos(c*x)**2/(sqrt(c*x + 1)*sqrt
( - c*x + 1)*c**2*x**2 + 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*c*x + sqrt(c*x +
 1)*sqrt( - c*x + 1)),x)*b**2*c)/(3*sqrt(e)*sqrt(d)*sqrt(c*x + 1)*c*d**2*(
c*x + 1))
```

3.566 $\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 918

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = -\frac{8abd^4x(1-c^2x^2)^{3/2}}{(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{8b^2d^4(1-c^2x^2)^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{b^2d^4x(1-c^2x^2)^2}{4(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{b^2d^4(1-c^2x^2)^{3/2}\arccos(cx)}{4c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{8b^2d^4x(1-c^2x^2)^{3/2}\arccos(cx)}{(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{bcd^4x^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{2(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{8d^4(1-c^2x^2)(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{8d^4x(1-c^2x^2)(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{8id^4(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{4d^4(1-c^2x^2)^2(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{d^4x(1-c^2x^2)^2(a+b\arccos(cx))^2}{2(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{5d^4(1-c^2x^2)^{3/2}(a+b\arccos(cx))^3}{2bc(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{32ibd^4(1-c^2x^2)^{3/2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{16bd^4(1-c^2x^2)^{3/2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{16ib^2d^4(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{16ib^2d^4(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{8ib^2d^4(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}}$$

output
```
-8*a*b*d^4*x*(-c^2*x^2+1)^(3/2)/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-8*b^2*d^4
*(-c^2*x^2+1)^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-1/4*b^2*d^4*x*(-c^2*x^2
+1)^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+1/4*b^2*d^4*(-c^2*x^2+1)^(3/2)*arcc
os(c*x)/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-8*b^2*d^4*x*(-c^2*x^2+1)^(3/2)*
arccos(c*x)/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-1/2*b*c*d^4*x^2*(-c^2*x^2+1)^
(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+8*d^4*(-c^2*x^2+1
)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+8*d^4*x*(-c^2*x^2
+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-8*I*d^4*(-c^2*x^2
+1)^(3/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+4*d^4*(-c
^2*x^2+1)^2*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+1/2*d^4
*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-5/2
*d^4*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))^3/b/c/(c*d*x+d)^(3/2)/(-c*e*x+e)
^(3/2)-16*I*b^2*d^4*(-c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1
/2)))/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+16*b*d^4*(-c^2*x^2+1)^(3/2)*(a+b*
arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(3/2)/(-c*e*x+
e)^(3/2)+32*I*b*d^4*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))*arctan(c*x+I*(-c^
2*x^2+1)^(1/2))/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-8*I*b^2*d^4*(-c^2*x^2+1
)^(3/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(3/2)/(-c*e*x
+e)^(3/2)+16*I*b^2*d^4*(-c^2*x^2+1)^(3/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^
(1/2)))/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 13.72 (sec) , antiderivative size = 971, normalized size of antiderivative = 1.06

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((d + c*d*x)^(5/2)*(a + b*ArcCos[c*x])^2)/(e - c*e*x)^(3/2),x]
```

output
```
(d^2*(-24*a^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2]*(-24 + 7*c
*x + c^2*x^2) - 360*a^2*Sqrt[d]*Sqrt[e]*(-1 + c*x)*Sqrt[1 - c^2*x^2]*ArcTa
n[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))]
+ 6*a*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2]*Sec[ArcCos[c*x]/
2]*(-2*ArcCos[c*x]*(-24*Cos[ArcCos[c*x]/2] + 7*Cos[(3*ArcCos[c*x])/2] + Co
s[(5*ArcCos[c*x])/2]) + 20*ArcCos[c*x]^2*Sin[ArcCos[c*x]/2] + 2*(16*c*x +
Cos[2*ArcCos[c*x]] - 32*Log[Sin[ArcCos[c*x]/2]])*Sin[ArcCos[c*x]/2]) + b^2
*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2]*Sec[ArcCos[c*x]/2]*(-6*
ArcCos[c*x]^2*(-24*Cos[ArcCos[c*x]/2] + 7*Cos[(3*ArcCos[c*x])/2] + Cos[(5*
ArcCos[c*x])/2] - (16*I)*Sin[ArcCos[c*x]/2]) + 40*ArcCos[c*x]^3*Sin[ArcCos
[c*x]/2] + 12*ArcCos[c*x]*(16*c*x + Cos[2*ArcCos[c*x]] - 32*Log[1 - E^(I*A
rcCos[c*x])])*Sin[ArcCos[c*x]/2] + (384*I)*PolyLog[2, E^(I*ArcCos[c*x])]*S
in[ArcCos[c*x]/2] - 12*(33*Cos[ArcCos[c*x]/2] + Cos[(3*ArcCos[c*x])/2])*Si
n[ArcCos[c*x]/2]^2) + 48*a*b*(1 + c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Tan
[ArcCos[c*x]/2]*(4*ArcCos[c*x] + (ArcCos[c*x]^2 - 8*Log[Sin[ArcCos[c*x]/2]
])*Tan[ArcCos[c*x]/2]) + 192*a*b*(1 + c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]
*Tan[ArcCos[c*x]/2]*((3 - c*x)*ArcCos[c*x] + ArcCos[c*x]^2*Tan[ArcCos[c*x]
/2] + (c*x - 4*Log[Sin[ArcCos[c*x]/2]])*Tan[ArcCos[c*x]/2]) + 32*b^2*(1 +
c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Tan[ArcCos[c*x]/2]*(6*ArcCos[c*x]^2 +
 (6*c*x*ArcCos[c*x] + 3*Sqrt[1 - c^2*x^2]*(-2 + ArcCos[c*x]^2) + 2*ArcC...
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{3/2}\int \frac{d^4(cx+1)^4(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^4\left(1-c^2x^2\right)^{3/2}\int \frac{(cx+1)^4(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{5275}$$

$$\frac{d^4\left(1-c^2x^2\right)^{3/2}\int \left(-\frac{c^2x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{4cx(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{7(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}+\frac{8(cx+1)(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{3/2}}\right)dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$d^4\left(1-c^2x^2\right)^{3/2}\left(\frac{32b\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{c}+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2+\frac{4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}\right.$$

input
```
Int[((d + c*d*x)^(5/2)*(a + b*ArcCos[c*x])^2)/(e - c*e*x)^(3/2),x]
```

output
```
(d^4*(1 - c^2*x^2)^(3/2)*(8*a*b*x - (8*b^2*Sqrt[1 - c^2*x^2])/c - (b^2*x*S
qrt[1 - c^2*x^2])/4 + 8*b^2*x*ArcCos[c*x] + (b*c*x^2*(a + b*ArcCos[c*x]))/
2 + ((8*I)*(a + b*ArcCos[c*x])^2)/c + (8*(a + b*ArcCos[c*x])^2)/(c*Sqrt[1
- c^2*x^2]) + (8*x*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2] + (4*Sqrt[1 -
c^2*x^2]*(a + b*ArcCos[c*x])^2)/c + (x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x
])^2)/2 + (5*(a + b*ArcCos[c*x])^3)/(2*b*c) + (b^2*ArcSin[c*x])/(4*c) + (3
2*b*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])])/c - (16*b*(a + b*ArcCo
s[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])])/c - ((16*I)*b^2*PolyLog[2, -E^(I*A
rcCos[c*x])])/c + ((16*I)*b^2*PolyLog[2, E^(I*ArcCos[c*x])])/c + ((8*I)*b^
2*PolyLog[2, E^((2*I)*ArcCos[c*x])])/c))/((d + c*d*x)^(3/2)*(e - c*e*x)^(3
/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 11.06 (sec) , antiderivative size = 1172, normalized size of antiderivative = 1.28

method	result	size
default	Expression too large to display	1172

input
```
int((c*d*x+d)^(5/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-5/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x-1)/e^2/c
/(c*x+1)*(a+b*arccos(c*x))^3*d^2/b+1/32*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/
2)*(2*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+4*c^3*x^3-1+2*I*(-c^2*x^2+1)^
(1/2)*c*x-I*(-c^2*x^2+1)^(1/2)-3*c*x)*(2*I*arccos(c*x)*b^2+2*arccos(c*x)^2
*b^2+2*I*a*b+4*arccos(c*x)*a*b+2*a^2-b^2)*d^2/(c*x-1)/e^2/c/(c*x+1)-(-e*(c
*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(1+c*x+I*(-c^2*x^2+1)^(1/2))*(arccos(c*x)^2
*b^2+2*arccos(c*x)*a*b+a^2-2*b^2+2*I*arccos(c*x)*b^2+2*I*a*b)*d^2/(c*x-1)/
e^2/c/(c*x+1)+2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2
)*x*c+c^2*x^2-1)*(arccos(c*x)^2*b^2+2*arccos(c*x)*a*b+a^2-2*b^2-2*I*b^2*ar
ccos(c*x)-2*I*a*b)*d^2/(c*x-1)/e^2/c/(c*x+1)-1/32*(-e*(c*x-1))^(1/2)*(d*(c
*x+1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-I*(-c^2*x^2+1)^(1/2)+c
*x-1)*(-2*I*b^2*arccos(c*x)+2*arccos(c*x)^2*b^2-2*I*a*b+4*arccos(c*x)*a*b+
2*a^2-b^2)*d^2/(c*x-1)/e^2/c/(c*x+1)-8*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2
)*(-I*(-c^2*x^2+1)^(1/2)+c*x+1)*(arccos(c*x)^2*b^2+2*arccos(c*x)*a*b+a^2)*
d^2/(c*x-1)/e^2/c/(c*x+1)-16*I*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c
*x-1))^(1/2)/(c*x-1)/e^2/c/(c*x+1)*b*(arccos(c*x)^2*b+2*I*arccos(c*x)*ln((
c*x+I*(-c^2*x^2+1)^(1/2))^(1/2)+1)*b+2*I*arccos(c*x)*ln(1-(c*x+I*(-c^2*x^2
+1)^(1/2))^(1/2))*b+4*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2))*b+4*poly
log(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2))*b-2*I*ln(c*x+I*(-c^2*x^2+1)^(1/2)
)*a+2*I*ln((c*x+I*(-c^2*x^2+1)^(1/2))^(1/2)-1)*a+2*I*ln((c*x+I*(-c^2*x^...
```

Fricas [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx=\int \frac{(cdx+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}{(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(3/2),x, algorith
m="fricas")
```

output
```
integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2
*b^2*c*d^2*x + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x
 + a*b*d^2)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^2*e^2*x^2 - 2
*c*e^2*x + e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+d)**(5/2)*(a+b*acos(c*x))**2/(-c*e*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(3/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = \int \frac{(cdx+d)^{\frac{5}{2}}(b\arccos(cx)+a)^2}{(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(3/2),x, algorith
m="giac")
```

output
```
integrate((c*d*x + d)^(5/2)*(b*arccos(c*x) + a)^2/(-c*e*x + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d+c\,d\,x)^{5/2}}{(e-c\,e\,x)^{3/2}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(5/2))/(e - c*e*x)^(3/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(5/2))/(e - c*e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = \frac{\sqrt{d}\,d^2\left(30\sqrt{-cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2-4\sqrt{-cx+1}\left(\int \frac{\sqrt{cx+1}\,acos}{\sqrt{-cx+1}\,cx-}\right.\right.}{}$$

input
```
int((c*d*x+d)^(5/2)*(a+b*acos(c*x))^2/(-c*e*x+e)^(3/2),x)
```

output
```
(sqrt(d)*d**2*(30*sqrt( - c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - 4
*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)*x**2)/(sqrt( - c*x + 1)*c*x
 - sqrt( - c*x + 1)),x)*a*b*c**3 - 8*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*a
cos(c*x)*x)/(sqrt( - c*x + 1)*c*x - sqrt( - c*x + 1)),x)*a*b*c**2 - 4*sqrt
( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x))/(sqrt( - c*x + 1)*c*x - sqrt( -
 c*x + 1)),x)*a*b*c - 2*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)**2*x
**2)/(sqrt( - c*x + 1)*c*x - sqrt( - c*x + 1)),x)*b**2*c**3 - 4*sqrt( - c*
x + 1)*int((sqrt(c*x + 1)*acos(c*x)**2*x)/(sqrt( - c*x + 1)*c*x - sqrt( -
c*x + 1)),x)*b**2*c**2 - 2*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)**
2)/(sqrt( - c*x + 1)*c*x - sqrt( - c*x + 1)),x)*b**2*c - sqrt(c*x + 1)*a**
2*c**2*x**2 - 7*sqrt(c*x + 1)*a**2*c*x + 24*sqrt(c*x + 1)*a**2))/(2*sqrt(e
)*sqrt( - c*x + 1)*c*e)
```

3.567 $\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 713

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = -\frac{2abd^3x(1-c^2x^2)^{3/2}}{(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{2b^2d^3(1-c^2x^2)^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{2b^2d^3x(1-c^2x^2)^{3/2}\arccos(cx)}{(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{4d^3(1-c^2x^2)\,(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{4d^3x(1-c^2x^2)\,(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{4id^3(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{d^3(1-c^2x^2)^2\,(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{d^3(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^3}{bc(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{16ibd^3(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{8bd^3(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{8ib^2d^3(1-c^2x^2)^{3/2}\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{8ib^2d^3(1-c^2x^2)^{3/2}\,\mathrm{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{4ib^2d^3(1-c^2x^2)^{3/2}\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}}$$

output
```
-2*a*b*d^3*x*(-c^2*x^2+1)^(3/2)/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-2*b^2*d^3
*(-c^2*x^2+1)^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-2*b^2*d^3*x*(-c^2*x^2+1
)^(3/2)*arccos(c*x)/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+4*d^3*(-c^2*x^2+1)*(a
+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+4*d^3*x*(-c^2*x^2+1)*
(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-4*I*d^3*(-c^2*x^2+1)^
(3/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+d^3*(-c^2*x^2
+1)^2*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-d^3*(-c^2*x^2
+1)^(3/2)*(a+b*arccos(c*x))^3/b/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+16*I*b*
d^3*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/
c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+8*b*d^3*(-c^2*x^2+1)^(3/2)*(a+b*arccos(
c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2
)-8*I*b^2*d^3*(-c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/
c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+8*I*b^2*d^3*(-c^2*x^2+1)^(3/2)*polylog(
2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-4*I*b^2
*d^3*(-c^2*x^2+1)^(3/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+
d)^(3/2)/(-c*e*x+e)^(3/2)
```

Mathematica [A] (verified)

Time = 7.59 (sec) , antiderivative size = 731, normalized size of antiderivative = 1.03

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx =$$

$$-\frac{2d\left(-2abcx\sqrt{d+cdx}\sqrt{e-cex}+2abc^2x^2\sqrt{d+cdx}\sqrt{e-cex}-5a^2\sqrt{d+cdx}\sqrt{e-cex}\sqrt{1-c^2x^2}+2b^2\sqrt{\right.}{}$$

input
```
Integrate[((d + c*d*x)^(3/2)*(a + b*ArcCos[c*x])^2)/(e - c*e*x)^(3/2),x]
```

output
```
(-2*d*(-2*a*b*c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x] + 2*a*b*c^2*x^2*Sqrt[d +
 c*d*x]*Sqrt[e - c*e*x] - 5*a^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c
^2*x^2] + 2*b^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2] + a^2*c*
x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2] - 2*b^2*c*x*Sqrt[d + c
*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2] + b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x
]*(3*a*(-1 + c*x) + b*(-4*I - 5*Sqrt[1 - c^2*x^2] + c*x*(4*I + Sqrt[1 - c^
2*x^2])))*ArcCos[c*x]^2 + b^2*(-1 + c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*A
rcCos[c*x]^3 - 3*a^2*Sqrt[d]*Sqrt[e]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d
+ c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] + 3*a^2*c*Sqrt
[d]*Sqrt[e]*x*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x
])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] + 2*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]
*ArcCos[c*x]*(b*c*x*(-1 + c*x) + a*(-5 + c*x)*Sqrt[1 - c^2*x^2] - 8*b*(-1
+ c*x)*Log[1 - E^(I*ArcCos[c*x])]) + 16*a*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x
]*Log[Sin[ArcCos[c*x]/2]] - 16*a*b*c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Log
[Sin[ArcCos[c*x]/2]] + (16*I)*b^2*(-1 + c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*
x]*PolyLog[2, E^(I*ArcCos[c*x])]*Sin[ArcCos[c*x]/2]^2)/(c*e^2*(-1 + c*x)^
2*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{3/2}\int \frac{d^3(cx+1)^3(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3(1-c^2x^2)^{3/2}\int \frac{(cx+1)^3(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{5275}$$

$$\frac{d^3(1-c^2x^2)^{3/2}\int\left(-\frac{cx(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}+\frac{4(cx+1)(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}\right)dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3(1-c^2x^2)^{3/2}\left(\frac{16b\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{c}+\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c}+\frac{4x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}+\frac{4(a+b\arccos(cx}{c\sqrt{1-c^2x^2}}\right.}{}$$

input
```
Int[((d + c*d*x)^(3/2)*(a + b*ArcCos[c*x])^2)/(e - c*e*x)^(3/2),x]
```

output
```
(d^3*(1 - c^2*x^2)^(3/2)*(2*a*b*x - (2*b^2*Sqrt[1 - c^2*x^2])/c + 2*b^2*x*
ArcCos[c*x] + ((4*I)*(a + b*ArcCos[c*x])^2)/c + (4*(a + b*ArcCos[c*x])^2)/
(c*Sqrt[1 - c^2*x^2]) + (4*x*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2] + (S
qrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c + (a + b*ArcCos[c*x])^3/(b*c) +
(16*b*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])])/c - (8*b*(a + b*ArcC
os[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])])/c - ((8*I)*b^2*PolyLog[2, -E^(I*A
rcCos[c*x])])/c + ((8*I)*b^2*PolyLog[2, E^(I*ArcCos[c*x])])/c + ((4*I)*b^2
*PolyLog[2, E^((2*I)*ArcCos[c*x])])/c))/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/
2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 19.10 (sec) , antiderivative size = 636, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,(a+b\arccos(cx))^3 d}{(cx-1)e^2c(cx+1)b}+\frac{\sqrt{-e(cx-1)}\,\sqrt{d(cx+1)}\,\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\left(\arccos(cx)^2b^2+\right.}{2(cx-1)e^2c(cx+1)}$

input
```
int((c*d*x+d)^(3/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x-1)/e^2/c/(c*
x+1)*(a+b*arccos(c*x))^3*d/b+1/2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(I*(
-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)^2*b^2+2*arccos(c*x)*a*b+a^2-
2*b^2+2*I*arccos(c*x)*b^2+2*I*a*b)*d/(c*x-1)/e^2/c/(c*x+1)+1/2*(-e*(c*x-1)
)^(1/2)*(d*(c*x+1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*
x)^2*b^2+2*arccos(c*x)*a*b+a^2-2*b^2-2*I*b^2*arccos(c*x)-2*I*a*b)*d/(c*x-1
)/e^2/c/(c*x+1)-4*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-I*(-c^2*x^2+1)^(1
/2)+c*x+1)*(arccos(c*x)^2*b^2+2*arccos(c*x)*a*b+a^2)*d/(c*x-1)/e^2/c/(c*x+
1)+8*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)*b*(-I*arccos(
c*x)^2*b-4*I*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2))*b-4*I*polylog(2,-
(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2))*b+2*ln((c*x+I*(-c^2*x^2+1)^(1/2))^(1/2)+
1)*arccos(c*x)*b+2*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2))*arccos(c*x)*b+2*
a*ln((c*x+I*(-c^2*x^2+1)^(1/2))^(1/2)+1)-2*a*ln(c*x+I*(-c^2*x^2+1)^(1/2))+
2*a*ln((c*x+I*(-c^2*x^2+1)^(1/2))^(1/2)-1))*d/(c*x-1)/e^2/c/(c*x+1)
```

Fricas [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = \int \frac{(cdx+d)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(3/2),x, algorith
m="fricas")
```

output
```
integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arccos(c*x)^2 + 2*(a*b*c
*d*x + a*b*d)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^2*e^2*x^2 -
 2*c*e^2*x + e^2), x)
```

Sympy [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = \int \frac{(d(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))^2}{(-e(cx-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)**(3/2)*(a+b*acos(c*x))**2/(-c*e*x+e)**(3/2),x)
```

output
```
Integral((d*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2/(-e*(c*x - 1))**(3/2),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(3/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx=\int \frac{(cdx+d)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(3/2),x, algorith
m="giac")
```

output
```
integrate((c*d*x + d)^(3/2)*(b*arccos(c*x) + a)^2/(-c*e*x + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d+c\,d\,x)^{3/2}}{(e-c\,e\,x)^{3/2}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(3/2))/(e - c*e*x)^(3/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(3/2))/(e - c*e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = \frac{\sqrt{d}\,d\Big(6\sqrt{-cx+1}\,asin\Big(\frac{\sqrt{-cx+1}}{\sqrt{2}}\Big)\,a^2-2\sqrt{-cx+1}\,\Big(\int\frac{\sqrt{cx+1}\,acos(c}{\sqrt{-cx+1}\,cx-\sqrt{-}}$$

input
```
int((c*d*x+d)^(3/2)*(a+b*acos(c*x))^2/(-c*e*x+e)^(3/2),x)
```

output
```
(sqrt(d)*d*(6*sqrt( - c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - 2*sqr
t( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)*x)/(sqrt( - c*x + 1)*c*x - sqrt
( - c*x + 1)),x)*a*b*c**2 - 2*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x
))/(sqrt( - c*x + 1)*c*x - sqrt( - c*x + 1)),x)*a*b*c - sqrt( - c*x + 1)*i
nt((sqrt(c*x + 1)*acos(c*x)**2*x)/(sqrt( - c*x + 1)*c*x - sqrt( - c*x + 1)
),x)*b**2*c**2 - sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)**2)/(sqrt(
- c*x + 1)*c*x - sqrt( - c*x + 1)),x)*b**2*c - sqrt(c*x + 1)*a**2*c*x + 5*
sqrt(c*x + 1)*a**2))/(sqrt(e)*sqrt( - c*x + 1)*c*e)
```

3.568 $\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 530

$$\begin{aligned}\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx &= \frac{2d^2(1-c^2x^2)(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\
&+ \frac{2d^2x(1-c^2x^2)(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{2id^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\
&- \frac{d^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))^3}{3bc(d+cdx)^{3/2}(e-cex)^{3/2}} \\
&+ \frac{8ibd^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\
&+ \frac{4bd^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\
&- \frac{4ib^2d^2(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\
&+ \frac{4ib^2d^2(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\
&- \frac{2ib^2d^2(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}}\end{aligned}$$

output

```
2*d^2*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+
2*d^2*x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-
2*I*d^2*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e
)^(3/2)-1/3*d^2*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))^3/b/c/(c*d*x+d)^(3/2)
/(-c*e*x+e)^(3/2)+8*I*b*d^2*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))*arctan(c*
x+I*(-c^2*x^2+1)^(1/2))/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+4*b*d^2*(-c^2*x
^2+1)^(3/2)*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+
d)^(3/2)/(-c*e*x+e)^(3/2)-4*I*b^2*d^2*(-c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x
+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+4*I*b^2*d^2*(-c
^2*x^2+1)^(3/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(3/2)/
(-c*e*x+e)^(3/2)-2*I*b^2*d^2*(-c^2*x^2+1)^(3/2)*polylog(2,-(c*x+I*(-c^2*x^
2+1)^(1/2))^2)/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)
```

Mathematica [A] (verified)

Time = 3.66 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = \frac{-\frac{6a^2\sqrt{d+cdx}\sqrt{e-cex}}{-1+cx}+3a^2\sqrt{d}\sqrt{e}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{e-cex}}{\sqrt{d}\sqrt{e}(-1+c^2x^2)}\right)+\frac{3ab(1+cx)\sqrt{d+}\cdots}{\cdots}}{\cdots}$$

input

```
Integrate[(Sqrt[d + c*d*x]*(a + b*ArcCos[c*x])^2)/(e - c*e*x)^(3/2),x]
```

output

```
((-6*a^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(-1 + c*x) + 3*a^2*Sqrt[d]*Sqrt[
e]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2
*x^2))] + (3*a*b*(1 + c*x)*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Tan[ArcCos[c*x]
/2]*(4*ArcCos[c*x] + (ArcCos[c*x]^2 - 8*Log[Sin[ArcCos[c*x]/2]])*Tan[ArcCo
s[c*x]/2]))/((1 - c*x)*Sqrt[1 - c^2*x^2]) + (b^2*(1 + c*x)*Sqrt[d + c*d*x]
*Sqrt[e - c*e*x]*Tan[ArcCos[c*x]/2]*((24*I)*PolyLog[2, E^(I*ArcCos[c*x])]*
Tan[ArcCos[c*x]/2] + ArcCos[c*x]*(6*ArcCos[c*x] + (ArcCos[c*x]*(6*I + ArcC
os[c*x]) - 24*Log[1 - E^(I*ArcCos[c*x])])*Tan[ArcCos[c*x]/2])))/((1 - c*x)
*Sqrt[1 - c^2*x^2]))/(3*c*e^2)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cdx+d}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{3/2}\int \frac{d^2(cx+1)^2(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2(1-c^2x^2)^{3/2}\int \frac{(cx+1)^2(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{5275}$$

$$\frac{d^2(1-c^2x^2)^{3/2}\int \left(\frac{2(cx+1)(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}-\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2(1-c^2x^2)^{3/2}\left(\frac{8b\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))}{c}+\frac{2x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}+\frac{2(a+b\arccos(cx))^2}{c\sqrt{1-c^2x^2}}+\frac{(a+b\arccos(cx))^3}{3bc}+\frac{2i(}{}\right.}{(cdx}$$

input
```
Int[(Sqrt[d + c*d*x]*(a + b*ArcCos[c*x])^2)/(e - c*e*x)^(3/2),x]
```

output
```
(d^2*(1 - c^2*x^2)^(3/2)*(((2*I)*(a + b*ArcCos[c*x])^2)/c + (2*(a + b*ArcC
os[c*x])^2)/(c*Sqrt[1 - c^2*x^2]) + (2*x*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c
^2*x^2] + (a + b*ArcCos[c*x])^3/(3*b*c) + (8*b*(a + b*ArcCos[c*x])*ArcTanh
[E^(I*ArcCos[c*x])])/c - (4*b*(a + b*ArcCos[c*x])*Log[1 - E^((2*I)*ArcCos[
c*x])])/c - ((4*I)*b^2*PolyLog[2, -E^(I*ArcCos[c*x])])/c + ((4*I)*b^2*Poly
Log[2, E^(I*ArcCos[c*x])])/c + ((2*I)*b^2*PolyLog[2, E^((2*I)*ArcCos[c*x])
])/c))/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,(a+b\arccos(cx))^3}{3(cx-1)e^2c(cx+1)b}-\frac{2\sqrt{-e(cx-1)}\,\sqrt{d(cx+1)}\,\left(-i\sqrt{-c^2x^2+1}+cx+1\right)\left(\arccos(cx)^2b^2+2a\right.}{(cx-1)e^2c(cx+1)}$

input
```
int((c*d*x+d)^(1/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/3*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x-1)/e^2/c
/(c*x+1)*(a+b*arccos(c*x))^3/b-2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-I*
(-c^2*x^2+1)^(1/2)+c*x+1)*(arccos(c*x)^2*b^2+2*arccos(c*x)*a*b+a^2)/(c*x-1
)/e^2/c/(c*x+1)-4*I*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2
)/(c*x-1)/e^2/c/(c*x+1)*b*(arccos(c*x)^2*b+2*I*arccos(c*x)*ln((c*x+I*(-c^2
*x^2+1)^(1/2))^(1/2)+1)*b+2*I*arccos(c*x)*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^
(1/2))*b+4*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2))*b+4*polylog(2,-(c*x
+I*(-c^2*x^2+1)^(1/2))^(1/2))*b-2*I*ln(c*x+I*(-c^2*x^2+1)^(1/2))*a+2*I*ln(
(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2)-1)*a+2*I*ln((c*x+I*(-c^2*x^2+1)^(1/2))^(1
/2)+1)*a)
```

Fricas [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx=\int \frac{\sqrt{cdx+d}(b\arccos(cx)+a)^2}{(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(3/2),x, algorith
m="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sqr
t(-c*e*x + e)/(c^2*e^2*x^2 - 2*c*e^2*x + e^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = \int \frac{\sqrt{d\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))^2}{(-e\,(cx-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)**(1/2)*(a+b*acos(c*x))**2/(-c*e*x+e)**(3/2),x)
```

output
```
Integral(sqrt(d*(c*x + 1))*(a + b*acos(c*x))**2/(-e*(c*x - 1))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(3/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx = \int \frac{\sqrt{cdx+d}(b\arccos\,(cx)+a)^2}{(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(3/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(c*d*x + d)*(b*arccos(c*x) + a)^2/(-c*e*x + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2\,\sqrt{d+c\,d\,x}}{{\left(e-c\,e\,x\right)}^{3/2}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(1/2))/(e - c*e*x)^(3/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(1/2))/(e - c*e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{3/2}}\,dx=\frac{\sqrt{d}\left(2\sqrt{-cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2-2\sqrt{-cx+1}\left(\int\frac{\sqrt{cx+1}\,acos(cx)}{\sqrt{-cx+1}\,cx-\sqrt{-cx+}}\right.\right.}{\sqrt{e}\,\sqrt{-c}}$$

input
```
int((c*d*x+d)^(1/2)*(a+b*acos(c*x))^2/(-c*e*x+e)^(3/2),x)
```

output
```
(sqrt(d)*(2*sqrt( - c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - 2*sqrt(
 - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x))/(sqrt( - c*x + 1)*c*x - sqrt( -
c*x + 1)),x)*a*b*c - sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)**2)/(sq
rt( - c*x + 1)*c*x - sqrt( - c*x + 1)),x)*b**2*c + 2*sqrt(c*x + 1)*a**2))/
(sqrt(e)*sqrt( - c*x + 1)*c*e)
```

3.569 $\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 454

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{3/2}}\,dx &= \frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\ &+ \frac{dx(1-c^2x^2)(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{id(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\ &+ \frac{4ibd(1-c^2x^2)^{3/2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\ &+ \frac{2bd(1-c^2x^2)^{3/2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\ &- \frac{2ib^2d(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\ &+ \frac{2ib^2d(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} \\ &- \frac{ib^2d(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}}\end{aligned}$$

output
```
d*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+d*x*
(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-I*d*(-c^
2*x^2+1)^(3/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+4*I*
b*d*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/
c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+2*b*d*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*
x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-
2*I*b^2*d*(-c^2*x^2+1)^(3/2)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c
*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+2*I*b^2*d*(-c^2*x^2+1)^(3/2)*polylog(2,I*(c
*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-I*b^2*d*(-c^2
*x^2+1)^(3/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(3/2)/(
-c*e*x+e)^(3/2)
```

Mathematica [A] (verified)

Time = 3.53 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.54

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{3/2}}\,dx =$$

$$-\frac{\sqrt{d+cdx}\sqrt{e-cex}\left(b^2(-1+cx)\left(1+cx+i\sqrt{1-c^2x^2}\right)\arccos(cx)^2+a(-1+cx)\left(a+acx-4b\sqrt{1-c^2\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(Sqrt[d + c*d*x]*(e - c*e*x)^(3/2)),x]
```

output
```
-((Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(b^2*(-1 + c*x)*(1 + c*x + I*Sqrt[1 - c
^2*x^2])*ArcCos[c*x]^2 + a*(-1 + c*x)*(a + a*c*x - 4*b*Sqrt[1 - c^2*x^2]*L
og[Sin[ArcCos[c*x]/2]]) + b*ArcCos[c*x]*(-(a*(-1 + c*x)^2*(1 + c*x)*Csc[Ar
cCos[c*x]/2]^4) + 8*b*Sqrt[1 - c^2*x^2]*Log[1 - E^(I*ArcCos[c*x])])*Sin[Ar
cCos[c*x]/2]^2 - (8*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, E^(I*ArcCos[c*x])]
*Sin[ArcCos[c*x]/2]^2))/(c*d*e^2*(-1 + c*x)^2*(1 + c*x)))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{cdx+d}(e-cex)^{3/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{3/2}\int \frac{d(cx+1)(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d(1-c^2x^2)^{3/2}\int \frac{(cx+1)(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{d(1-c^2x^2)^{3/2}\int \left(\frac{cx(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}+\frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}\right)dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(1-c^2x^2)^{3/2}\left(\frac{4b\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{c}+\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}+\frac{(a+b\arccos(cx))^2}{c\sqrt{1-c^2x^2}}+\frac{i(a+b\arccos(cx))^2}{c}-\frac{2b\log}{}\right.}{(cdx+d)^{3/2}(e}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(Sqrt[d + c*d*x]*(e - c*e*x)^(3/2)),x]
```

output
```
(d*(1 - c^2*x^2)^(3/2)*((I*(a + b*ArcCos[c*x])^2)/c + (a + b*ArcCos[c*x])^
2/(c*Sqrt[1 - c^2*x^2]) + (x*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2] + (4
*b*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])])/c - (2*b*(a + b*ArcCos[
c*x])*Log[1 - E^((2*I)*ArcCos[c*x])])/c - ((2*I)*b^2*PolyLog[2, -E^(I*ArcC
os[c*x])])/c + ((2*I)*b^2*PolyLog[2, E^(I*ArcCos[c*x])])/c + (I*b^2*PolyLo
g[2, E^((2*I)*ArcCos[c*x])])/c))/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [A] (verified)

Time = 3.87 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.06

method	result
default	$\frac{a^2\sqrt{cdx+d}}{dce\sqrt{-cex+e}}+b^2\left(-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\left(-i\sqrt{-c^2x^2+1}+cx+1\right)\arccos(cx)^2}{d\,e^2c(c^2x^2-1)}-\frac{2i\sqrt{-c^2x^2+1}\,\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,(\right.$
parts	$\frac{a^2\sqrt{cdx+d}}{dce\sqrt{-cex+e}}+b^2\left(-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\left(-i\sqrt{-c^2x^2+1}+cx+1\right)\arccos(cx)^2}{d\,e^2c(c^2x^2-1)}-\frac{2i\sqrt{-c^2x^2+1}\,\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,(\right.$

input
```
int((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
a^2/d/c/e/(-c*e*x+e)^(1/2)*(c*d*x+d)^(1/2)+b^2*(-(d*(c*x+1))^(1/2)*(-e*(c*
x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)+c*x+1)*arccos(c*x)^2/d/e^2/c/(c^2*x^2-1
)-2*I*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(2*I*arccos(
c*x)*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2))+2*I*arccos(c*x)*ln((c*x+I*(-c^
2*x^2+1)^(1/2))^(1/2)+1)+arccos(c*x)^2+4*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/
2))^(1/2))+4*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2)))/d/e^2/c/(c^2*x^
2-1))+2*a*b*(-2*I*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/
(c^2*x^2-1)/c/d/e^2*arccos(c*x)-(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(
-c^2*x^2+1)^(1/2)+c*x+1)*arccos(c*x)/(c^2*x^2-1)/c/d/e^2+2*(d*(c*x+1))^(1/
2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c/d/e^2*ln(I*(-c^2*x^
2+1)^(1/2)+c*x-1))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{3/2}}\,dx=\int \frac{\left(b\arccos\left(cx\right)+a\right)^2}{\sqrt{cdx+d}(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(3/2),x, algorith
m="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sqr
t(-c*e*x + e)/(c^3*d*e^2*x^3 - c^2*d*e^2*x^2 - c*d*e^2*x + d*e^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{3/2}}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^2}{\sqrt{d(cx+1)}\,(-e(cx-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(c*d*x+d)**(1/2)/(-c*e*x+e)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(sqrt(d*(c*x + 1))*(-e*(c*x - 1))**(3/2)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(3/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{\sqrt{cdx+d}(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(3/2),x, algorith
m="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/(sqrt(c*d*x + d)*(-c*e*x + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{3/2}}\,dx=\int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{\sqrt{d+c\,d\,x}\,\left(e-c\,e\,x\right)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(1/2)*(e - c*e*x)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(1/2)*(e - c*e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{3/2}}\,dx=\frac{-2\sqrt{-cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,cx-\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)abc-\sqrt{-cx+1}\left(\int \frac{}{\sqrt{cx+}}\right.}{\sqrt{e}\,\sqrt{d}\,\sqrt{-cx+1}\,ce}$$

input
```
int((a+b*acos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(3/2),x)
```

output
```
( - 2*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c*x -
 sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a*b*c - sqrt( - c*x + 1)*int(acos(c*x)
**2/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c*x - sqrt(c*x + 1)*sqrt( - c*x + 1)),
x)*b**2*c + sqrt(c*x + 1)*a**2)/(sqrt(e)*sqrt(d)*sqrt( - c*x + 1)*c*e)
```

3.570 $\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 217

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \frac{x(1-c^2x^2)\,(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{i(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{2b(1-c^2x^2)^{3/2}\,(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{ib^2(1-c^2x^2)^{3/2}\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}}$$

output
```
x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-I*(-c^
2*x^2+1)^(3/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+2*b*
(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/
(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-I*b^2*(-c^2*x^2+1)^(3/2)*polylog(2,-(c*x+
I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)
```

Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \frac{a^2cx+2abcx\arccos(cx)+b^2cx\arccos(cx)^2+ib^2\sqrt{1-c^2x^2}\arccos(cx)^2-}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]
```

output
```
(a^2*c*x + 2*a*b*c*x*ArcCos[c*x] + b^2*c*x*ArcCos[c*x]^2 + I*b^2*Sqrt[1 -
c^2*x^2]*ArcCos[c*x]^2 - 2*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 - E^(I*
ArcCos[c*x])] - 2*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*
x])] - 2*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/2]] - 2*a*b*Sqrt[1 - c^
2*x^2]*Log[Sin[ArcCos[c*x]/2]] + (2*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, -E
^(I*ArcCos[c*x])] + (2*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, E^(I*ArcCos[c*x
])])/(c*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.65, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5179, 5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(cdx+d)^{3/2}(e-cex)^{3/2}}\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{3/2}\int \frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{5161}$$

$$\frac{(1-c^2x^2)^{3/2}\left(2bc\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{5181}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\int\frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c}\right)}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\int-\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{c}\right)}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(\frac{2b\int(a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c}+\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{4200}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(2i\int-\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c}\right)}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\int\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c}\right)}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+b\,\mathrm{ar}}{}\right.}{c}\right.}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\,\mathrm{arcc}}\right.\right.}{c}\right.}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{(1-c^2x^2)^{3/2}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c}\right)}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]
```

output
```
((1 - c^2*x^2)^(3/2)*((x*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2] - (2*b*(
((-1/2*I)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])*Log[
1 - E^((2*I)*ArcCos[c*x])] + (b*PolyLog[2, E^((2*I)*ArcCos[c*x])])/4)))/c)
)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(215) = 430$.

Time = 1.70 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.12

method	result
default	$a^2\left(-\frac{1}{dce\sqrt{cdx+d}\,\sqrt{-cex+e}}+\frac{\sqrt{cdx+d}}{ce\,d^2\sqrt{-cex+e}}\right)+b^2\left(-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\left(-i\sqrt{-c^2x^2+1}+cx\right)\arccos(cx)^2}{d^2e^2c(c^2x^2-1)}-\frac{2i\sqrt{}}{}\right.$
parts	$a^2\left(-\frac{1}{dce\sqrt{cdx+d}\,\sqrt{-cex+e}}+\frac{\sqrt{cdx+d}}{ce\,d^2\sqrt{-cex+e}}\right)+b^2\left(-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\left(-i\sqrt{-c^2x^2+1}+cx\right)\arccos(cx)^2}{d^2e^2c(c^2x^2-1)}-\frac{2i\sqrt{}}{}\right.$

input
```
int((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
a^2*(-1/d/c/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+1/c/e/d^2/(-c*e*x+e)^(1/2)*
(c*d*x+d)^(1/2))+b^2*(-(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+
1)^(1/2)+c*x)*arccos(c*x)^2/d^2/e^2/c/(c^2*x^2-1)-2*I*(-c^2*x^2+1)^(1/2)*(
d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)
^(1/2))+I*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+arccos(c*x)^2+polylog
(2,-c*x-I*(-c^2*x^2+1)^(1/2))+polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)))/d^2/e^2
/c/(c^2*x^2-1))+2*a*b*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1
/2)/(c^2*x^2-1)/c/d^2/e^2/(c*x+1)/(c*x-1)*(-I*arccos(c*x)*x^2*c^2+ln((c*x+
I*(-c^2*x^2+1)^(1/2))^2-1)*x^2*c^2+(-c^2*x^2+1)^(1/2)*arccos(c*x)*x*c+I*ar
ccos(c*x)-ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorith
m="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sqr
t(-c*e*x + e)/(c^4*d^2*e^2*x^4 - 2*c^2*d^2*e^2*x^2 + d^2*e^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{(a+b\operatorname{acos}(cx))^2}{(d(cx+1))^{\frac{3}{2}}(-e(cx-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(c*d*x+d)**(3/2)/(-c*e*x+e)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/((d*(c*x + 1))**(3/2)*(-e*(c*x - 1))**(3/2))
, x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorith
m="maxima")
```

output
```
-b^2*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2/((c^2*d*e*x^2
- d*e)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/(sqrt(d)*sqrt(e)) + 2*a*b*x*arcco
s(c*x)/(sqrt(-c^2*d*e*x^2 + d*e)*d*e) + a^2*x/(sqrt(-c^2*d*e*x^2 + d*e)*d*
e) + a*b*sqrt(1/(d*e))*log(x^2 - 1/c^2)/(c*d*e)
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorith
m="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(d+c\,d\,x)^{3/2}\,(e-c\,e\,x)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\frac{-2\sqrt{cx+1}\,\sqrt{-cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2-\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)ab-\sqrt{cx+}}{\sqrt{e}\,\sqrt{d}\,\sqrt{cx+1}\,\sqrt{-cx}}$$

input
```
int((a+b*acos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x)
```

output
```
( - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( -
c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a*b - sqrt(c*x + 1
)*sqrt( - c*x + 1)*int(acos(c*x)**2/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x
**2 - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b**2 + a**2*x)/(sqrt(e)*sqrt(d)*s
qrt(c*x + 1)*sqrt( - c*x + 1)*d*e)
```

3.571 $\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 709

$$
\begin{aligned}
\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{3/2}}\,dx &= -\frac{b^2e(1-c^2x^2)^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+ \frac{b^2ex(1-c^2x^2)^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{be(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+ \frac{bex(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{e(1-c^2x^2)(a+b\arccos(cx))^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+ \frac{ex(1-c^2x^2)(a+b\arccos(cx))^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{2ex(1-c^2x^2)^2(a+b\arccos(cx))^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&- \frac{2ie(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&- \frac{2ibe(1-c^2x^2)^{5/2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+ \frac{4be(1-c^2x^2)^{5/2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&+ \frac{ib^2e(1-c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&- \frac{ib^2e(1-c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
&- \frac{2ib^2e(1-c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}
\end{aligned}
$$

output
```
-1/3*b^2*e*(-c^2*x^2+1)^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+1/3*b^2*e*x*(
-c^2*x^2+1)^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3*b*e*(-c^2*x^2+1)^(3/2)*
(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+1/3*b*e*x*(-c^2*x^2+1
)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3*e*(-c^2*x^2
+1)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+1/3*e*x*(-c^2*x
^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+2/3*e*x*(-c^2*x
^2+1)^2*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-2/3*I*e*(-c^2
*x^2+1)^(5/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-2/3*I
*b*e*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))
/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+4/3*b*e*(-c^2*x^2+1)^(5/2)*(a+b*arccos
(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/
2)+1/3*I*b^2*e*(-c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))
/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3*I*b^2*e*(-c^2*x^2+1)^(5/2)*polylog
(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-2/3*I*
b^2*e*(-c^2*x^2+1)^(5/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x
+d)^(5/2)/(-c*e*x+e)^(5/2)
```

Mathematica [A] (verified)

Time = 3.49 (sec) , antiderivative size = 563, normalized size of antiderivative = 0.79

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{3/2}}\,dx = -\frac{\sqrt{d+cdx}\sqrt{e-cex}\left(-2a^2-b^2+4a^2cx+4a^2c^2x^2+2ab\sqrt{1-c^2x^2}+8abcx\arccos(cx)+2b^2\sqrt{1-c^2x^2}\,\mathrm{a}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/((d + c*d*x)^(5/2)*(e - c*e*x)^(3/2)),x]
```

output
```
-1/6*(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-2*a^2 - b^2 + 4*a^2*c*x + 4*a^2*c^
2*x^2 + 2*a*b*Sqrt[1 - c^2*x^2] + 8*a*b*c*x*ArcCos[c*x] + 2*b^2*Sqrt[1 - c
^2*x^2]*ArcCos[c*x] + 4*b^2*c*x*ArcCos[c*x]^2 + (4*I)*b^2*Sqrt[1 - c^2*x^2
]*ArcCos[c*x]^2 + b^2*Cos[2*ArcCos[c*x]] + 4*a*b*ArcCos[c*x]*Cos[2*ArcCos[
c*x]] + 2*b^2*ArcCos[c*x]^2*Cos[2*ArcCos[c*x]] - 6*b^2*Sqrt[1 - c^2*x^2]*A
rcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] - 10*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*
x]*Log[1 + E^(I*ArcCos[c*x])] - 10*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*
x]/2]] - 10*a*b*c*x*Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/2]] - 6*a*b*Sqrt
[1 - c^2*x^2]*Log[Sin[ArcCos[c*x]/2]] - 6*a*b*c*x*Sqrt[1 - c^2*x^2]*Log[Si
n[ArcCos[c*x]/2]] + (6*I)*b^2*(1 + c*x)*Sqrt[1 - c^2*x^2]*PolyLog[2, E^(I*
ArcCos[c*x])] + (2*I)*b^2*ArcCos[c*x]^2*Sin[2*ArcCos[c*x]] - 3*b^2*ArcCos[
c*x]*Log[1 - E^(I*ArcCos[c*x])]*Sin[2*ArcCos[c*x]] - 5*b^2*ArcCos[c*x]*Log
[1 + E^(I*ArcCos[c*x])]*Sin[2*ArcCos[c*x]] + (5*I)*b^2*PolyLog[2, -E^(I*Ar
cCos[c*x])]*(2*Sqrt[1 - c^2*x^2] + Sin[2*ArcCos[c*x]])))/(c*d^3*(-1 + c*x)
*(e + c*e*x)^2)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.53, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(cdx+d)^{5/2}(e-cex)^{3/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\int \frac{e(1-cx)(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\left(1-c^2x^2\right)^{5/2}\int \frac{(1-cx)(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{e(1-c^2x^2)^{5/2}\int\left(\frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^{5/2}}-\frac{cx(a+b\arccos(cx))^2}{(1-c^2x^2)^{5/2}}\right)dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$e(1-c^2x^2)^{5/2}\left(-\frac{2b\,\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))}{3c}-\frac{bx(a+b\arccos(cx))}{3(1-c^2x^2)}+\frac{b(a+b\arccos(cx))}{3c(1-c^2x^2)}+\frac{2x(a+b\arccos(cx))^2}{3\sqrt{1-c^2x^2}}+\frac{x(\right.$$

input
```
Int[(a + b*ArcCos[c*x])^2/((d + c*d*x)^(5/2)*(e - c*e*x)^(3/2)),x]
```

output
```
(e*(1 - c^2*x^2)^(5/2)*(-1/3*b^2/(c*Sqrt[1 - c^2*x^2]) + (b^2*x)/(3*Sqrt[1
 - c^2*x^2]) + (b*(a + b*ArcCos[c*x]))/(3*c*(1 - c^2*x^2)) - (b*x*(a + b*A
rcCos[c*x]))/(3*(1 - c^2*x^2)) + (((2*I)/3)*(a + b*ArcCos[c*x])^2)/c - (a
+ b*ArcCos[c*x])^2/(3*c*(1 - c^2*x^2)^(3/2)) + (x*(a + b*ArcCos[c*x])^2)/(
3*(1 - c^2*x^2)^(3/2)) + (2*x*(a + b*ArcCos[c*x])^2)/(3*Sqrt[1 - c^2*x^2])
 - (2*b*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])])/(3*c) - (4*b*(a +
b*ArcCos[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])])/(3*c) + ((I/3)*b^2*PolyLog[
2, -E^(I*ArcCos[c*x])])/c - ((I/3)*b^2*PolyLog[2, E^(I*ArcCos[c*x])])/c +
(((2*I)/3)*b^2*PolyLog[2, E^((2*I)*ArcCos[c*x])])/c))/((d + c*d*x)^(5/2)*(
e - c*e*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1999 vs. $2(657) = 1314$.

Time = 2.50 (sec) , antiderivative size = 2000, normalized size of antiderivative = 2.82

method	result	size
default	Expression too large to display	2000
parts	Expression too large to display	2000

input
```
int((a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
a^2*(-1/3/d/c/e/(c*d*x+d)^(3/2)/(-c*e*x+e)^(1/2)+2/3/d*(-1/d/c/e/(c*d*x+d)
^(1/2)/(-c*e*x+e)^(1/2)+1/c/e/d^2/(-c*e*x+e)^(1/2)*(c*d*x+d)^(1/2)))-1/6*b
^2/(-e*(c*x-1))^(1/2)/(c*x+1)/(d*(c*x+1))^(1/2)/e/d^2/c+5/3*b^2*(-e*(c*x-1
))^(1/2)*(d*(c*x+1))^(1/2)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*arccos(c*x)/d^3/
e^2/c/(c^3*x^3+c^2*x^2-c*x-1)*(-c^2*x^2+1)^(1/2)-I*b^2*(-e*(c*x-1))^(1/2)*
(d*(c*x+1))^(1/2)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/d^3/e^2/c/(c^3*x^3+c
^2*x^2-c*x-1)*(-c^2*x^2+1)^(1/2)-1/3*I*b^2/d^2/e/(d*(c*x+1))^(1/2)/(I*(-c^
2*x^2+1)^(1/2)*c*x+c^2*x^2+I*(-c^2*x^2+1)^(1/2)+c*x)/(-e*(c*x-1))^(1/2)*(-
c^2*x^2+1)^(1/2)*x-2/3*I*b^2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*arccos(c
*x)^2/d^3/e^2/c/(c^3*x^3+c^2*x^2-c*x-1)*(-c^2*x^2+1)^(1/2)+b^2*(-e*(c*x-1)
)^(1/2)*(d*(c*x+1))^(1/2)*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))/d^3/e
^2/c/(c^3*x^3+c^2*x^2-c*x-1)*(-c^2*x^2+1)^(1/2)-I*b^2*(-e*(c*x-1))^(1/2)*(
d*(c*x+1))^(1/2)*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/d^3/e^2/(c^3*x^3+c^2*
x^2-c*x-1)*(-c^2*x^2+1)^(1/2)*x-2/3*b^2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/
2)*arccos(c*x)^2/d^3/e^2*c/(c^3*x^3+c^2*x^2-c*x-1)*x^2-1/3*b^2*(-e*(c*x-1)
)^(1/2)*(d*(c*x+1))^(1/2)*arccos(c*x)/d^3/e^2/c/(c^3*x^3+c^2*x^2-c*x-1)*(-
c^2*x^2+1)^(1/2)+1/3*b^2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*arccos(c*x)^
2/d^3/e^2/c/(c^3*x^3+c^2*x^2-c*x-1)-2/3*b^2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))
^(1/2)*arccos(c*x)^2/d^3/e^2/(c^3*x^3+c^2*x^2-c*x-1)*x+1/6*b^2/(-e*(c*x-1)
)^(1/2)/(d*(c*x+1))^(1/2)/(2*c^3*x^3-c*x+2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2...
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{5}{2}}(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(3/2),x, algorith
m="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sqr
t(-c*e*x + e)/(c^5*d^3*e^2*x^5 + c^4*d^3*e^2*x^4 - 2*c^3*d^3*e^2*x^3 - 2*c
^2*d^3*e^2*x^2 + c*d^3*e^2*x + d^3*e^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acos(c*x))**2/(c*d*x+d)**(5/2)/(-c*e*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(3/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{5}{2}}(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(3/2),x, algorith
m="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/((c*d*x + d)^(5/2)*(-c*e*x + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(d+c\,d\,x)^{5/2}\,(e-c\,e\,x)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(5/2)*(e - c*e*x)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(5/2)*(e - c*e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{3/2}}\,dx = \frac{-6\sqrt{cx+1}\,\sqrt{-cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^3x^3+\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2-\sqrt{cx+1}\,\sqrt{-cx+1}\right.}{}$$

input
```
int((a+b*acos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(3/2),x)
```

output
```
( - 6*sqrt(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( -
c*x + 1)*c**3*x**3 + sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 - sqrt(c*x +
 1)*sqrt( - c*x + 1)*c*x - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a*b*c**2*x -
 6*sqrt(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x
 + 1)*c**3*x**3 + sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 - sqrt(c*x + 1)
*sqrt( - c*x + 1)*c*x - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a*b*c - 3*sqrt(
c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)**2/(sqrt(c*x + 1)*sqrt( - c*x + 1)
*c**3*x**3 + sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt
( - c*x + 1)*c*x - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b**2*c**2*x - 3*sqrt
(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)**2/(sqrt(c*x + 1)*sqrt( - c*x + 1
)*c**3*x**3 + sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqr
t( - c*x + 1)*c*x - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b**2*c + 2*a**2*c**
2*x**2 + 2*a**2*c*x - a**2)/(3*sqrt(e)*sqrt(d)*sqrt(c*x + 1)*sqrt( - c*x +
 1)*c*d**2*e*(c*x + 1))
```

3.572 $\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 730

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = \frac{2abd^5x(1-c^2x^2)^{5/2}}{(d+cdx)^{5/2}(e-cex)^{5/2}}$$
$$+\frac{2b^2d^5(1-c^2x^2)^3}{c(d+cdx)^{5/2}(e-cex)^{5/2}}+\frac{2b^2d^5x(1-c^2x^2)^{5/2}\arccos(cx)}{(d+cdx)^{5/2}(e-cex)^{5/2}}$$
$$-\frac{28id^5(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}$$
$$-\frac{d^5(1-c^2x^2)^3(a+b\arccos(cx))^2}{c(d+cdx)^{5/2}(e-cex)^{5/2}}+\frac{5d^5(1-c^2x^2)^{5/2}(a+b\arccos(cx))^3}{3bc(d+cdx)^{5/2}(e-cex)^{5/2}}$$
$$-\frac{112bd^5(1-c^2x^2)^{5/2}(a+b\arccos(cx))\log\left(1-ie^{-i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}$$
$$-\frac{112ib^2d^5(1-c^2x^2)^{5/2}\,\mathrm{PolyLog}\left(2,ie^{-i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}$$
$$-\frac{8bd^5(1-c^2x^2)^{5/2}(a+b\arccos(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}$$
$$+\frac{16b^2d^5(1-c^2x^2)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}$$
$$-\frac{28d^5(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}$$
$$+\frac{4d^5(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}$$

output
```
2*a*b*d^5*x*(-c^2*x^2+1)^(5/2)/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+2*b^2*d^5*
(-c^2*x^2+1)^3/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+2*b^2*d^5*x*(-c^2*x^2+1)
^(5/2)*arccos(c*x)/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-28/3*I*d^5*(-c^2*x^2+1
)^(5/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-d^5*(-c^2*x
^2+1)^3*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+5/3*d^5*(-c
^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^3/b/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1
12/3*b*d^5*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*ln(1-I/(c*x+I*(-c^2*x^2+1)
^(1/2)))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-112/3*I*b^2*d^5*(-c^2*x^2+1)^(
5/2)*polylog(2,I/(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^
(5/2)-8/3*b*d^5*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*sec(1/4*Pi+1/2*arccos
(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+16/3*b^2*d^5*(-c^2*x^2+1)^(5/2
)*tan(1/4*Pi+1/2*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-28/3*d^5*
(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^2*tan(1/4*Pi+1/2*arccos(c*x))/c/(c*d*
x+d)^(5/2)/(-c*e*x+e)^(5/2)+4/3*d^5*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^2
*sec(1/4*Pi+1/2*arccos(c*x))^2*tan(1/4*Pi+1/2*arccos(c*x))/c/(c*d*x+d)^(5/
2)/(-c*e*x+e)^(5/2)
```

Mathematica [A] (verified)

Time = 15.52 (sec) , antiderivative size = 568, normalized size of antiderivative = 0.78

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = \frac{d^2\sqrt{1-c^2x^2}\Big(15a^2\sqrt{d}\sqrt{e}(-1+cx)^3\sqrt{1-c^2x^2}\arctan\Big(\frac{cx\sqrt{d+cdx}\sqrt{e\cdot}}{\sqrt{d}\sqrt{e}(-1+c}$$

input
```
Integrate[((d + c*d*x)^(5/2)*(a + b*ArcCos[c*x])^2)/(e - c*e*x)^(5/2),x]
```

output
```
(d^2*Sqrt[1 - c^2*x^2]*(15*a^2*Sqrt[d]*Sqrt[e]*(-1 + c*x)^3*Sqrt[1 - c^2*x
^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^
2*x^2))] - (896*I)*b^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*PolyLog[2, E^(I*Arc
Cos[c*x])]*Sin[ArcCos[c*x]/2]^6 + Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a^2*Sqr
t[1 - c^2*x^2]*(-23 + 57*c*x - 37*c^2*x^2 + 3*c^3*x^3) - 2*b*(15*a*(-1 + c
*x)^2 + b*(28*I + 23*Sqrt[1 - c^2*x^2] + c^2*x^2*(28*I + 3*Sqrt[1 - c^2*x^
2]) - 2*c*x*(28*I + 17*Sqrt[1 - c^2*x^2])))*ArcCos[c*x]^2*Sin[ArcCos[c*x]/
2]^2 + 64*b*(a + b*Sqrt[1 - c^2*x^2])*Sin[ArcCos[c*x]/2]^4 - 40*b^2*ArcCos
[c*x]^3*Sin[ArcCos[c*x]/2]^6 + 16*b*(-3*a*c*x + 3*b*Sqrt[1 - c^2*x^2] + 56
*a*Log[Sin[ArcCos[c*x]/2]])*Sin[ArcCos[c*x]/2]^6 - 16*b*ArcCos[c*x]*Sin[Ar
cCos[c*x]/2]^2*(-2*a*Sqrt[1 - c^2*x^2] + (-4*b + 14*a*Sqrt[1 - c^2*x^2])*S
in[ArcCos[c*x]/2]^2 + (3*b*c*x + 3*a*Sqrt[1 - c^2*x^2] - 56*b*Log[1 - E^(I
*ArcCos[c*x])])*Sin[ArcCos[c*x]/2]^4))))/(3*c*e^3*(-1 + c*x)^4*(1 + c*x))
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}dx$$

$$\Big\downarrow \text{ 5179}$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\int \frac{d^5(cx+1)^5(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d^5\left(1-c^2x^2\right)^{5/2}\int \frac{(cx+1)^5(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{ 5275}$$

$$\frac{d^5\left(1-c^2x^2\right)^{5/2}\int \left(\frac{cx(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}+\frac{12(a+b\arccos(cx))^2}{(cx-1)\sqrt{1-c^2x^2}}+\frac{8(a+b\arccos(cx))^2}{(cx-1)^2\sqrt{1-c^2x^2}}+\frac{5(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$\downarrow$ 2009

$$\frac{d^5\left(1-c^2x^2\right)^{5/2}\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c}-\frac{5(a+b\arccos(cx))^3}{3bc}-\frac{28i(a+b\arccos(cx))^2}{3c}+\frac{112b\log\left(1-e^{i\arccos(cx)}\right)(a+b\arccos(c$$

input
```
Int[((d + c*d*x)^(5/2)*(a + b*ArcCos[c*x])^2)/(e - c*e*x)^(5/2),x]
```

output
```
(d^5*(1 - c^2*x^2)^(5/2)*(-2*a*b*x + (2*b^2*Sqrt[1 - c^2*x^2])/c - 2*b^2*x
*ArcCos[c*x] - (((28*I)/3)*(a + b*ArcCos[c*x])^2)/c - (Sqrt[1 - c^2*x^2]*(
a + b*ArcCos[c*x])^2)/c - (5*(a + b*ArcCos[c*x])^3)/(3*b*c) + (16*b^2*Cot[
ArcCos[c*x]/2])/(3*c) - (28*(a + b*ArcCos[c*x])^2*Cot[ArcCos[c*x]/2])/(3*c
) + (8*b*(a + b*ArcCos[c*x])*Csc[ArcCos[c*x]/2]^2)/(3*c) + (4*(a + b*ArcCo
s[c*x])^2*Cot[ArcCos[c*x]/2]*Csc[ArcCos[c*x]/2]^2)/(3*c) + (112*b*(a + b*A
rcCos[c*x])*Log[1 - E^(I*ArcCos[c*x])])/(3*c) - (((112*I)/3)*b^2*PolyLog[2
, E^(I*ArcCos[c*x])])/c))/((d + c*d*x)^(5/2)*(e - c*e*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 3.62 (sec) , antiderivative size = 974, normalized size of antiderivative = 1.33

method	result
default	$\frac{5\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,(a+b\arccos(cx))^3d^2}{3(cx-1)e^3c(cx+1)b}-\frac{\sqrt{-e(cx-1)}\,\sqrt{d(cx+1)}\left(i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\left(\arccos(cx)^2b^2+\right.}{2(cx-1)e^3c(cx+1}$

input
```
int((c*d*x+d)^(5/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
5/3*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x-1)/e^3/c/
(c*x+1)*(a+b*arccos(c*x))^3*d^2/b-1/2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)
*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)^2*b^2+2*arccos(c*x)*a*b
+a^2-2*b^2+2*I*arccos(c*x)*b^2+2*I*a*b)*d^2/(c*x-1)/e^3/c/(c*x+1)-1/2*(-e*
(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(ar
ccos(c*x)^2*b^2+2*arccos(c*x)*a*b+a^2-2*b^2-2*I*b^2*arccos(c*x)-2*I*a*b)*d
^2/(c*x-1)/e^3/c/(c*x+1)+4/3*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(7*c^2*x
^2+2*c*x-5-7*I*(-c^2*x^2+1)^(1/2)*c*x+7*I*(-c^2*x^2+1)^(1/2))*(63*b^2*c^2*
x^2*arccos(c*x)^2+28*I*a*b*c*x+126*a*b*c^2*x^2*arccos(c*x)+4*I*(-c^2*x^2+1
)^(1/2)*b^2*c*x-96*arccos(c*x)^2*b^2*c*x+14*(-c^2*x^2+1)^(1/2)*arccos(c*x)
*b^2*c*x-14*I*a*b-14*I*arccos(c*x)*b^2+63*a^2*c^2*x^2-32*x^2*c^2*b^2-192*a
rccos(c*x)*a*b*c*x+14*(-c^2*x^2+1)^(1/2)*a*b*c*x+28*I*arccos(c*x)*b^2*c*x+
37*arccos(c*x)^2*b^2-10*arccos(c*x)*(-c^2*x^2+1)^(1/2)*b^2-14*I*arccos(c*x
)*b^2*c^2*x^2-14*I*a*b*c^2*x^2-96*a^2*c*x+56*c*x*b^2+74*arccos(c*x)*a*b-10
*(-c^2*x^2+1)^(1/2)*a*b-4*I*(-c^2*x^2+1)^(1/2)*b^2+37*a^2-24*b^2)*d^2/(63*
c^4*x^4-222*c^3*x^3+292*c^2*x^2-170*c*x+37)/e^3/c/(c*x+1)+56/3*I*(-c^2*x^2
+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(c*x-1)/e^3/c/(c*x+1)*b*(ar
ccos(c*x)^2*b+2*I*arccos(c*x)*ln((c*x+I*(-c^2*x^2+1)^(1/2))^(1/2)+1)*b+2*I
*arccos(c*x)*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2))*b+4*polylog(2,(c*x+I*(
-c^2*x^2+1)^(1/2))^(1/2))*b+4*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^(1/...
```

Fricas [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = \int \frac{(cdx+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}{(-cex+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(5/2),x, algorith
m="fricas")
```

output
```
integral(-(a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 +
2*b^2*c*d^2*x + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*
x + a*b*d^2)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^3*e^3*x^3 -
3*c^2*e^3*x^2 + 3*c*e^3*x - e^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+d)**(5/2)*(a+b*acos(c*x))**2/(-c*e*x+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(5/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx=\int \frac{(cdx+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}{(-cex+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(5/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(5/2),x, algorith
m="giac")
```

output
```
integrate((c*d*x + d)^(5/2)*(b*arccos(c*x) + a)^2/(-c*e*x + e)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d+c\,d\,x)^{5/2}}{(e-c\,e\,x)^{5/2}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(5/2))/(e - c*e*x)^(5/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(5/2))/(e - c*e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+cdx)^{5/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((c*d*x+d)^(5/2)*(a+b*acos(c*x))^2/(-c*e*x+e)^(5/2),x)
```

output
```
(sqrt(d)*d**2*( - 30*sqrt( - c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a**2*
c*x + 30*sqrt( - c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + 6*sqrt( -
c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)*x**2)/(sqrt( - c*x + 1)*c**2*x**2 -
2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*a*b*c**4*x - 6*sqrt( - c*x +
 1)*int((sqrt(c*x + 1)*acos(c*x)*x**2)/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqr
t( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*a*b*c**3 + 12*sqrt( - c*x + 1)*in
t((sqrt(c*x + 1)*acos(c*x)*x)/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x
+ 1)*c*x + sqrt( - c*x + 1)),x)*a*b*c**3*x - 12*sqrt( - c*x + 1)*int((sqrt
(c*x + 1)*acos(c*x)*x)/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*
x + sqrt( - c*x + 1)),x)*a*b*c**2 + 6*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*
acos(c*x))/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( -
c*x + 1)),x)*a*b*c**2*x - 6*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x))
/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),
x)*a*b*c + 3*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)**2*x**2)/(sqrt(
 - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*b**2
*c**4*x - 3*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)**2*x**2)/(sqrt(
- c*x + 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*b**2*
c**3 + 6*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)**2*x)/(sqrt( - c*x
+ 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*b**2*c**3*x
 - 6*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)**2*x)/(sqrt( - c*x +...
```

3.573 $\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 544

$$\begin{aligned}\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx &= -\frac{8id^4(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}+\frac{d^4(1-c^2x^2)^{5/2}(a+b\arccos(cx))^3}{3bc(d+cdx)^{5/2}(e-cex)^{5/2}}\\
&-\frac{32bd^4(1-c^2x^2)^{5/2}(a+b\arccos(cx))\log\left(1-ie^{-i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}\\
&-\frac{32ib^2d^4(1-c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{-i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}\\
&-\frac{4bd^4(1-c^2x^2)^{5/2}(a+b\arccos(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}\\
&+\frac{8b^2d^4(1-c^2x^2)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}\\
&-\frac{8d^4(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}\\
&+\frac{2d^4(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}\end{aligned}$$

output
```
-8/3*I*d^4*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*
x+e)^(5/2)+1/3*d^4*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^3/b/c/(c*d*x+d)^(5
/2)/(-c*e*x+e)^(5/2)-32/3*b*d^4*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*ln(1-
I/(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-32/3*I*b^
2*d^4*(-c^2*x^2+1)^(5/2)*polylog(2,I/(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+
d)^(5/2)/(-c*e*x+e)^(5/2)-4/3*b*d^4*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*s
ec(1/4*Pi+1/2*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+8/3*b^2*d^
4*(-c^2*x^2+1)^(5/2)*tan(1/4*Pi+1/2*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*e*x
+e)^(5/2)-8/3*d^4*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^2*tan(1/4*Pi+1/2*ar
ccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+2/3*d^4*(-c^2*x^2+1)^(5/2)*(
a+b*arccos(c*x))^2*sec(1/4*Pi+1/2*arccos(c*x))^2*tan(1/4*Pi+1/2*arccos(c*x
))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 12.69 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.00

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = \frac{-3a^2d^{3/2}\sqrt{e}(-1+cx)^4(1+cx)\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{e-cex}}{\sqrt{d}\sqrt{e}(-1+c^2x^2)}\right)-256ib^2d}{}$$

input
```
Integrate[((d + c*d*x)^(3/2)*(a + b*ArcCos[c*x])^2)/(e - c*e*x)^(5/2),x]
```

output
```
(-3*a^2*d^(3/2)*Sqrt[e]*(-1 + c*x)^4*(1 + c*x)*ArcTan[(c*x*Sqrt[d + c*d*x]
*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] - (256*I)*b^2*d*Sqrt[d
 + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2]*PolyLog[2, E^(I*ArcCos[c*x])]*
Sin[ArcCos[c*x]/2]^6 + 4*d*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-2*b^2*Sqrt[1
- c^2*x^2]*ArcCos[c*x]^3*Sin[ArcCos[c*x]/2]^6 + b*ArcCos[c*x]^2*Sin[ArcCos
[c*x]/2]^2*(-2*b*(-1 + c^2*x^2) - (16*I)*b*Sqrt[1 - c^2*x^2]*Sin[ArcCos[c*
x]/2]^4 + (-1 + c^2*x^2)*Sin[ArcCos[c*x]/2]*(8*b*Cos[ArcCos[c*x]/2] + 3*a*
Sin[ArcCos[c*x]/2])*Tan[ArcCos[c*x]/2]) + 4*b*ArcCos[c*x]*Sin[ArcCos[c*x]/
2]^2*(4*a*(-1 + c^2*x^2)*Sin[ArcCos[c*x]/2]^2 + 16*b*Sqrt[1 - c^2*x^2]*Log
[1 - E^(I*ArcCos[c*x])]*Sin[ArcCos[c*x]/2]^4 - (-1 + c^2*x^2)*(a + b*Tan[A
rcCos[c*x]/2])) + (-1 + c^2*x^2)*(-8*b*Sin[ArcCos[c*x]/2]^4*(b + 4*a*Log[S
in[ArcCos[c*x]/2]]*Tan[ArcCos[c*x]/2]) + a*(a - 3*a*c*x + 2*a*c^2*x^2 - 2*
b*Sqrt[1 - c^2*x^2]*Tan[ArcCos[c*x]/2]^2))))/(3*c*e^3*(-1 + c*x)^4*(1 + c*
x))
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}dx$$

$$\Big\downarrow \textbf{5179}$$

$$\frac{(1-c^2x^2)^{5/2}\int \frac{d^4(cx+1)^4(a+b\arccos(cx))^2}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{d^4(1-c^2x^2)^{5/2}\int \frac{(cx+1)^4(a+b\arccos(cx))^2}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \textbf{5275}$$

$$\frac{d^4(1-c^2x^2)^{5/2}\int \left(\frac{4(a+b\arccos(cx))^2}{(cx-1)\sqrt{1-c^2x^2}}+\frac{4(a+b\arccos(cx))^2}{(cx-1)^2\sqrt{1-c^2x^2}}+\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{d^4(1-c^2x^2)^{5/2}\left(-\frac{(a+b\arccos(cx))^3}{3bc}-\frac{8i(a+b\arccos(cx))^2}{3c}+\frac{32b\log(1-e^{i\arccos(cx)})(a+b\arccos(cx))}{3c}-\frac{8\cot(\frac{1}{2}\arccos(cx))(a+b\text{arc}}{3c}\right.}{}$$

input
```
Int[((d + c*d*x)^(3/2)*(a + b*ArcCos[c*x])^2)/(e - c*e*x)^(5/2),x]
```

output
```
(d^4*(1 - c^2*x^2)^(5/2)*((((-8*I)/3)*(a + b*ArcCos[c*x])^2)/c - (a + b*Ar
cCos[c*x])^3/(3*b*c) + (8*b^2*Cot[ArcCos[c*x]/2])/(3*c) - (8*(a + b*ArcCos
[c*x])^2*Cot[ArcCos[c*x]/2])/(3*c) + (4*b*(a + b*ArcCos[c*x])*Csc[ArcCos[c
*x]/2]^2)/(3*c) + (2*(a + b*ArcCos[c*x])^2*Cot[ArcCos[c*x]/2]*Csc[ArcCos[c
*x]/2]^2)/(3*c) + (32*b*(a + b*ArcCos[c*x])*Log[1 - E^(I*ArcCos[c*x])])/(3
*c) - (((32*I)/3)*b^2*PolyLog[2, E^(I*ArcCos[c*x])])/c))/((d + c*d*x)^(5/2
)*(e - c*e*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.37

method	result
default	$\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,(a+b\arccos(cx))^3 d}{3(cx-1)e^3c(cx+1)b} + \frac{4\sqrt{-e(cx-1)}\,\sqrt{d(cx+1)}\left(-2i\sqrt{-c^2x^2+1}\,xc+2c^2x^2+2i\sqrt{-c^2x^2+1}+c \cdots\right.}{\cdots}$

input
```
int((c*d*x+d)^(3/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/3*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c*x-1)/e^3/c/
(c*x+1)*(a+b*arccos(c*x))^3*d/b+4/3*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)*(
-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2+2*I*(-c^2*x^2+1)^(1/2)+c*x-1)*(12*b^
2*c^2*x^2*arccos(c*x)^2+8*I*a*b*c*x+24*a*b*c^2*x^2*arccos(c*x)+2*I*(-c^2*x
^2+1)^(1/2)*b^2*c*x-15*arccos(c*x)^2*b^2*c*x+4*(-c^2*x^2+1)^(1/2)*arccos(c
*x)*b^2*c*x-4*I*a*b-4*I*arccos(c*x)*b^2+12*a^2*c^2*x^2-10*x^2*c^2*b^2-30*a
rccos(c*x)*a*b*c*x+4*(-c^2*x^2+1)^(1/2)*a*b*c*x+8*I*arccos(c*x)*b^2*c*x+5*
arccos(c*x)^2*b^2-2*arccos(c*x)*(-c^2*x^2+1)^(1/2)*b^2-4*I*arccos(c*x)*b^2
*c^2*x^2-4*I*a*b*c^2*x^2-15*a^2*c*x+16*c*x*b^2+10*arccos(c*x)*a*b-2*(-c^2*
x^2+1)^(1/2)*a*b-2*I*(-c^2*x^2+1)^(1/2)*b^2+5*a^2-6*b^2)*d/(12*c^4*x^4-39*
c^3*x^3+47*c^2*x^2-25*c*x+5)/e^3/c/(c*x+1)+16/3*I*(-c^2*x^2+1)^(1/2)*(d*(c
*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(c*x-1)/e^3/c/(c*x+1)*b*(arccos(c*x)^2*b+2
*I*arccos(c*x)*ln((c*x+I*(-c^2*x^2+1)^(1/2))^(1/2)+1)*b+2*I*arccos(c*x)*ln
(1-(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2))*b+4*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/
2))^(1/2))*b+4*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2))*b-2*I*ln(c*x+I
*(-c^2*x^2+1)^(1/2))*a+2*I*ln((c*x+I*(-c^2*x^2+1)^(1/2))^(1/2)-1)*a+2*I*ln
((c*x+I*(-c^2*x^2+1)^(1/2))^(1/2)+1)*a)*d
```

Fricas [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = \int \frac{(cdx+d)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{(-cex+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(5/2),x, algorith
m="fricas")
```

output
```
integral(-(a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arccos(c*x)^2 + 2*(a*b*
c*d*x + a*b*d)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^3*e^3*x^3
- 3*c^2*e^3*x^2 + 3*c*e^3*x - e^3), x)
```

Sympy [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = \int \frac{(d(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acos}\,(cx))^2}{(-e\,(cx-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)**(3/2)*(a+b*acos(c*x))**2/(-c*e*x+e)**(5/2),x)
```

output
```
Integral((d*(c*x + 1))**(3/2)*(a + b*acos(c*x))**2/(-e*(c*x - 1))**(5/2),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(5/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx=\int \frac{(cdx+d)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{(-cex+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(5/2),x, algorith
m="giac")
```

output
```
integrate((c*d*x + d)^(3/2)*(b*arccos(c*x) + a)^2/(-c*e*x + e)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d+c\,d\,x)^{3/2}}{(e-c\,e\,x)^{5/2}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(3/2))/(e - c*e*x)^(5/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(3/2))/(e - c*e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+cdx)^{3/2}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx=\frac{\sqrt{d}\,d\left(-6\sqrt{-cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2cx+6\sqrt{-cx+1}\,asin\left(\frac{\sqrt{-cx+}}{\sqrt{2}}\right.\right.}{}$$

input
```
int((c*d*x+d)^(3/2)*(a+b*acos(c*x))^2/(-c*e*x+e)^(5/2),x)
```

output

```
(sqrt(d)*d*( - 6*sqrt( - c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a**2*c*x
+ 6*sqrt( - c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + 6*sqrt( - c*x +
 1)*int((sqrt(c*x + 1)*acos(c*x)*x)/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt(
- c*x + 1)*c*x + sqrt( - c*x + 1)),x)*a*b*c**3*x - 6*sqrt( - c*x + 1)*int(
(sqrt(c*x + 1)*acos(c*x)*x)/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x +
1)*c*x + sqrt( - c*x + 1)),x)*a*b*c**2 + 6*sqrt( - c*x + 1)*int((sqrt(c*x
+ 1)*acos(c*x))/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqr
t( - c*x + 1)),x)*a*b*c**2*x - 6*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(
c*x))/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x +
 1)),x)*a*b*c + 3*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)**2*x)/(sqr
t( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*b*
*2*c**3*x - 3*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)**2*x)/(sqrt( -
 c*x + 1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*b**2*c
**2 + 3*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)**2)/(sqrt( - c*x + 1
)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*b**2*c**2*x -
3*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x)**2)/(sqrt( - c*x + 1)*c**2
*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*b**2*c - 8*sqrt(c*x
+ 1)*a**2*c*x + 4*sqrt(c*x + 1)*a**2))/(3*sqrt(e)*sqrt( - c*x + 1)*c*e**2*
(c*x - 1))
```

3.574 $\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 486

$$\begin{aligned}
\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = & -\frac{id^3(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
& -\frac{4bd^3(1-c^2x^2)^{5/2}(a+b\arccos(cx))\log\left(1-ie^{-i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
& -\frac{4ib^2d^3(1-c^2x^2)^{5/2}\,\text{PolyLog}\left(2,ie^{-i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
& -\frac{2bd^3(1-c^2x^2)^{5/2}(a+b\arccos(cx))\sec^2\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
& +\frac{4b^2d^3(1-c^2x^2)^{5/2}\tan\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
& -\frac{d^3(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2\tan\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\
& +\frac{d^3(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2\sec^2\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)\tan\left(\frac{\pi}{4}+\frac{1}{2}\arccos(cx)\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}
\end{aligned}$$

output
```
-1/3*I*d^3*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*
x+e)^(5/2)-4/3*b*d^3*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*ln(1-I/(c*x+I*(-
c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-4/3*I*b^2*d^3*(-c^2*
x^2+1)^(5/2)*polylog(2,I/(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(5/2)/(-c
*e*x+e)^(5/2)-2/3*b*d^3*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*sec(1/4*Pi+1/
2*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+4/3*b^2*d^3*(-c^2*x^2+
1)^(5/2)*tan(1/4*Pi+1/2*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/
3*d^3*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^2*tan(1/4*Pi+1/2*arccos(c*x))/c
/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+1/3*d^3*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c
*x))^2*sec(1/4*Pi+1/2*arccos(c*x))^2*tan(1/4*Pi+1/2*arccos(c*x))/c/(c*d*x+
d)^(5/2)/(-c*e*x+e)^(5/2)
```

Mathematica [A] (verified)

Time = 4.75 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = \frac{\sqrt{d+cdx}\sqrt{e-cex}\sqrt{1-c^2x^2}\left(a^2\sqrt{1-c^2x^2}-a^2c^2x^2\sqrt{1-c^2x^2}+2b\right.}{}$$

input
```
Integrate[(Sqrt[d + c*d*x]*(a + b*ArcCos[c*x])^2)/(e - c*e*x)^(5/2),x]
```

output
```
(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2]*(a^2*Sqrt[1 - c^2*x^2]
- a^2*c^2*x^2*Sqrt[1 - c^2*x^2] + 2*b^2*(-I - I*c^2*x^2 + Sqrt[1 - c^2*x^2
] + c*x*(2*I + Sqrt[1 - c^2*x^2]))*ArcCos[c*x]^2*Sin[ArcCos[c*x]/2]^2 + 16
*a*b*Sin[ArcCos[c*x]/2]^4 + 16*b^2*Sqrt[1 - c^2*x^2]*Sin[ArcCos[c*x]/2]^4
+ 32*a*b*Log[Sin[ArcCos[c*x]/2]]*Sin[ArcCos[c*x]/2]^6 - (32*I)*b^2*PolyLog
[2, E^(I*ArcCos[c*x])]*Sin[ArcCos[c*x]/2]^6 + 8*b*ArcCos[c*x]*Sin[ArcCos[c
*x]/2]^2*(a*Sqrt[1 - c^2*x^2] + (2*b - a*Sqrt[1 - c^2*x^2])*Sin[ArcCos[c*x
]/2]^2 + 4*b*Log[1 - E^(I*ArcCos[c*x])]*Sin[ArcCos[c*x]/2]^4))/(3*c*e^3*(
-1 + c*x)^4*(1 + c*x))
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5275, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cdx+d}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{5/2}\int \frac{d^3(cx+1)^3(a+b\arccos(cx))^2}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3(1-c^2x^2)^{5/2}\int \frac{(cx+1)^3(a+b\arccos(cx))^2}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{5275}$$

$$\frac{d^3(1-c^2x^2)^{5/2}\int \left(\frac{(a+b\arccos(cx))^2}{(cx-1)\sqrt{1-c^2x^2}}+\frac{2(a+b\arccos(cx))^2}{(cx-1)^2\sqrt{1-c^2x^2}}\right)dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3(1-c^2x^2)^{5/2}\left(-\frac{i(a+b\arccos(cx))^2}{3c}+\frac{4b\log\left(1-e^{i\arccos(cx)}\right)(a+b\arccos(cx))}{3c}-\frac{\cot\left(\frac{1}{2}\arccos(cx)\right)(a+b\arccos(cx))^2}{3c}+\frac{2b\csc^2\left(\frac{1}{2}\text{ar}\right.}{}\right.}{(cdx+d)^{5/}}$$

input
```
Int[(Sqrt[d + c*d*x]*(a + b*ArcCos[c*x])^2)/(e - c*e*x)^(5/2),x]
```

output
```
(d^3*(1 - c^2*x^2)^(5/2)*(((-1/3*I)*(a + b*ArcCos[c*x])^2)/c + (4*b^2*Cot[
ArcCos[c*x]/2])/(3*c) - ((a + b*ArcCos[c*x])^2*Cot[ArcCos[c*x]/2])/(3*c) +
 (2*b*(a + b*ArcCos[c*x])*Csc[ArcCos[c*x]/2]^2)/(3*c) + ((a + b*ArcCos[c*x
])^2*Cot[ArcCos[c*x]/2]*Csc[ArcCos[c*x]/2]^2)/(3*c) + (4*b*(a + b*ArcCos[c
*x])*Log[1 - E^(I*ArcCos[c*x])])/(3*c) - (((4*I)/3)*b^2*PolyLog[2, E^(I*Ar
cCos[c*x])])/c))/((d + c*d*x)^(5/2)*(e - c*e*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5275
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x]
)^n/Sqrt[d + e*x^2], (f + g*x)^m*(d + e*x^2)^(p + 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && ILtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2994 vs. $2(428) = 856$.

Time = 4.38 (sec) , antiderivative size = 2995, normalized size of antiderivative = 6.16

method	result	size
default	Expression too large to display	2995
parts	Expression too large to display	2995

input
```
int((c*d*x+d)^(1/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
a^2*(1/c/e*(c*d*x+d)^(1/2)/(-c*e*x+e)^(3/2)-d*(1/3/d/c/e/(-c*e*x+e)^(3/2)*
(c*d*x+d)^(1/2)+1/3/d/c/e^2/(-c*e*x+e)^(1/2)*(c*d*x+d)^(1/2)))+4/3*b^2*(d*
(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(3*c^4*x^4-6*c^3*x^3+4*c^2*x^2-2*c*x+1)/
e^3*c/(c*x+1)*(-c^2*x^2+1)*x^2+4/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2
)/(3*c^4*x^4-6*c^3*x^3+4*c^2*x^2-2*c*x+1)/e^3/c/(c*x+1)*(-c^2*x^2+1)^(1/2)
*arccos(c*x)+b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(3*c^4*x^4-6*c^3*x^3
+4*c^2*x^2-2*c*x+1)/e^3*c^3/(c*x+1)*arccos(c*x)^2*x^4+2*b^2*(d*(c*x+1))^(1
/2)*(-e*(c*x-1))^(1/2)/(3*c^4*x^4-6*c^3*x^3+4*c^2*x^2-2*c*x+1)/e^3*c^2/(c*
x+1)*arccos(c*x)^2*x^3+4/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(3*c^4
*x^4-6*c^3*x^3+4*c^2*x^2-2*c*x+1)/e^3*c/(c*x+1)*arccos(c*x)^2*x^2-2/3*I*b^
2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(3*c^4*x^4-6*c^3*x^3+4*c^2*x^2-2*c*
x+1)/e^3/c/(c*x+1)*arccos(c*x)+4/3*I*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1
/2)/(3*c^4*x^4-6*c^3*x^3+4*c^2*x^2-2*c*x+1)/e^3/(c*x+1)*(-c^2*x^2+1)^(1/2)
*x-4/3*I*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(3*c^4*x^4-6*c^3*x^3+4*c
^2*x^2-2*c*x+1)/e^3/c/(c*x+1)*(-c^2*x^2+1)^(1/2)+2/3*b^2*(d*(c*x+1))^(1/2)
*(-e*(c*x-1))^(1/2)/(3*c^4*x^4-6*c^3*x^3+4*c^2*x^2-2*c*x+1)/e^3/(c*x+1)*ar
ccos(c*x)^2*x-8/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(3*c^4*x^4-6*c^
3*x^3+4*c^2*x^2-2*c*x+1)/e^3*c^3/(c*x+1)*x^4+4/3*b^2*(d*(c*x+1))^(1/2)*(-e
*(c*x-1))^(1/2)/(3*c^4*x^4-6*c^3*x^3+4*c^2*x^2-2*c*x+1)/e^3/c/(c*x+1)*(-c^
2*x^2+1)-8/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(3*c^4*x^4-6*c^3*...
```

Fricas [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = \int \frac{\sqrt{cdx+d}\left(b\arccos\left(cx\right)+a\right)^2}{\left(-cex+e\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(5/2),x, algorith
m="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sq
rt(-c*e*x + e)/(c^3*e^3*x^3 - 3*c^2*e^3*x^2 + 3*c*e^3*x - e^3), x)
```

Sympy [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = \int \frac{\sqrt{d\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))^2}{(-e\,(cx-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)**(1/2)*(a+b*acos(c*x))**2/(-c*e*x+e)**(5/2),x)
```

output
```
Integral(sqrt(d*(c*x + 1))*(a + b*acos(c*x))**2/(-e*(c*x - 1))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(5/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx = \int \frac{\sqrt{cdx+d}(b\arccos\,(cx)+a)^2}{(-cex+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(a+b*arccos(c*x))^2/(-c*e*x+e)^(5/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(c*d*x + d)*(b*arccos(c*x) + a)^2/(-c*e*x + e)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d+c\,d\,x}}{(e-c\,e\,x)^{5/2}}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(1/2))/(e - c*e*x)^(5/2),x)
```

output
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(1/2))/(e - c*e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+cdx}(a+b\arccos(cx))^2}{(e-cex)^{5/2}}\,dx=\frac{\sqrt{d}\left(6\sqrt{-cx+1}\left(\int \frac{\sqrt{cx+1}\,acos(cx)}{\sqrt{-cx+1}\,c^2x^2-2\sqrt{-cx+1}\,cx+\sqrt{-cx+1}}dx\right)ab\,c^2x-6\sqrt{-}\right.}{}$$

input
```
int((c*d*x+d)^(1/2)*(a+b*acos(c*x))^2/(-c*e*x+e)^(5/2),x)
```

output
```
(sqrt(d)*(6*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x))/(sqrt( - c*x +
1)*c**2*x**2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*a*b*c**2*x -
6*sqrt( - c*x + 1)*int((sqrt(c*x + 1)*acos(c*x))/(sqrt( - c*x + 1)*c**2*x*
*2 - 2*sqrt( - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*a*b*c + 3*sqrt( - c*x +
 1)*int((sqrt(c*x + 1)*acos(c*x)**2)/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt(
 - c*x + 1)*c*x + sqrt( - c*x + 1)),x)*b**2*c**2*x - 3*sqrt( - c*x + 1)*in
t((sqrt(c*x + 1)*acos(c*x)**2)/(sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt( - c*x
 + 1)*c*x + sqrt( - c*x + 1)),x)*b**2*c - sqrt(c*x + 1)*a**2*c*x - sqrt(c*
x + 1)*a**2))/(3*sqrt(e)*sqrt( - c*x + 1)*c*e**2*(c*x - 1))
```

3.575 $\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 896

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{5/2}}\,dx = \frac{2b^2d^2(1-c^2x^2)^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{2b^2d^2x(1-c^2x^2)^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{b^2d^2(1-c^2x^2)^{5/2}\arccos(cx)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{bd^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{2bd^2x(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{bcd^2x^2(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{2d^2(1-c^2x^2)(a+b\arccos(cx))^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{d^2x(1-c^2x^2)(a+b\arccos(cx))^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{c^2d^2x^3(1-c^2x^2)(a+b\arccos(cx))^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{2d^2x(1-c^2x^2)^2(a+b\arccos(cx))^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{id^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{4ibd^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{2bd^2(1-c^2x^2)^{5/2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{2ib^2d^2(1-c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{2ib^2d^2(1-c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{ib^2d^2(1-c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}$$

output
```
2/3*b^2*d^2*(-c^2*x^2+1)^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+2/3*b^2*d^2*
x*(-c^2*x^2+1)^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3*b^2*d^2*(-c^2*x^2+1)
^(5/2)*arccos(c*x)/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3*b*d^2*(-c^2*x^2+
1)^(3/2)*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-2/3*b*d^2*x*
(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3*
b*c*d^2*x^2*(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*e*x+e
)^(5/2)+2/3*d^2*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x
+e)^(5/2)+1/3*d^2*x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e
*x+e)^(5/2)+1/3*c^2*d^2*x^3*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/
2)/(-c*e*x+e)^(5/2)+2/3*d^2*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2/(c*d*x+d)
^(5/2)/(-c*e*x+e)^(5/2)-1/3*I*d^2*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))^2/c
/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-2/3*I*b^2*d^2*(-c^2*x^2+1)^(5/2)*polylog
(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+2/3*b
*d^2*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^
2)/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3*I*b^2*d^2*(-c^2*x^2+1)^(5/2)*pol
ylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+4
/3*I*b*d^2*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^
(1/2))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+2/3*I*b^2*d^2*(-c^2*x^2+1)^(5/2)
*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2
)
```

Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.45

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{5/2}}\,dx =$$
$$-\frac{\sqrt{d+cdx}\sqrt{e-cex}\left(2a^2-a^2cx-2a^2c^2x^2+a^2c^3x^3+4b^2\sin^2\left(\frac{1}{2}\arccos(cx)\right)-4b^2c^2x^2\sin^2\left(\frac{1}{2}\arccos(cx\right.\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(Sqrt[d + c*d*x]*(e - c*e*x)^(5/2)),x]
```

output
```
-1/3*(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(2*a^2 - a^2*c*x - 2*a^2*c^2*x^2 + a
^2*c^3*x^3 + 4*b^2*Sin[ArcCos[c*x]/2]^2 - 4*b^2*c^2*x^2*Sin[ArcCos[c*x]/2]
^2 + 4*a*b*Sqrt[1 - c^2*x^2]*Sin[ArcCos[c*x]/2]^2 - 16*a*b*Sqrt[1 - c^2*x^
2]*Log[Sin[ArcCos[c*x]/2]]*Sin[ArcCos[c*x]/2]^4 + (16*I)*b^2*Sqrt[1 - c^2*
x^2]*PolyLog[2, E^(I*ArcCos[c*x])]*Sin[ArcCos[c*x]/2]^4 + b^2*ArcCos[c*x]^
2*(1 - c^2*x^2 + (2 - 2*c^2*x^2)*Sin[ArcCos[c*x]/2]^2 + (4*I)*Sqrt[1 - c^2
*x^2]*Sin[ArcCos[c*x]/2]^4) - 2*b*ArcCos[c*x]*(a*(-1 + c^2*x^2) - 2*(a - a
*c^2*x^2 + b*Sqrt[1 - c^2*x^2])*Sin[ArcCos[c*x]/2]^2 + 8*b*Sqrt[1 - c^2*x^
2]*Log[1 - E^(I*ArcCos[c*x])]*Sin[ArcCos[c*x]/2]^4)))/(c*d*e^3*(-1 + c*x)^
3*(1 + c*x))
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{cdx+d}(e-cex)^{5/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\int \frac{d^2(cx+1)^2(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\left(1-c^2x^2\right)^{5/2}\int \frac{(cx+1)^2(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{d^2\left(1-c^2x^2\right)^{5/2}\int \left(\frac{c^2x^2(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}+\frac{2cx(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}+\frac{(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{5/2}}\right)dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$d^2(1-c^2x^2)^{5/2}\left(\frac{4b\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))}{3c}+\frac{bcx^2(a+b\arccos(cx))}{3(1-c^2x^2)}+\frac{2x(a+b\arccos(cx))^2}{3\sqrt{1-c^2x^2}}+\frac{x(a+b\arccos(cx))^2}{3(1-c^2x^2)^{3/2}}+\right.$$

input
```
Int[(a + b*ArcCos[c*x])^2/(Sqrt[d + c*d*x]*(e - c*e*x)^(5/2)),x]
```

output
```
(d^2*(1 - c^2*x^2)^(5/2)*((2*b^2)/(3*c*Sqrt[1 - c^2*x^2]) + (2*b^2*x)/(3*S
qrt[1 - c^2*x^2]) + (b*(a + b*ArcCos[c*x]))/(3*c*(1 - c^2*x^2)) + (2*b*x*(
a + b*ArcCos[c*x]))/(3*(1 - c^2*x^2)) + (b*c*x^2*(a + b*ArcCos[c*x]))/(3*(
1 - c^2*x^2)) + ((I/3)*(a + b*ArcCos[c*x])^2)/c + (2*(a + b*ArcCos[c*x])^2
)/(3*c*(1 - c^2*x^2)^(3/2)) + (x*(a + b*ArcCos[c*x])^2)/(3*(1 - c^2*x^2)^(
3/2)) + (c^2*x^3*(a + b*ArcCos[c*x])^2)/(3*(1 - c^2*x^2)^(3/2)) + (2*x*(a
+ b*ArcCos[c*x])^2)/(3*Sqrt[1 - c^2*x^2]) - (b^2*ArcSin[c*x])/(3*c) + (4*b
*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])])/(3*c) - (2*b*(a + b*ArcCo
s[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])])/(3*c) - (((2*I)/3)*b^2*PolyLog[2,
-E^(I*ArcCos[c*x])])/c + (((2*I)/3)*b^2*PolyLog[2, E^(I*ArcCos[c*x])])/c +
 ((I/3)*b^2*PolyLog[2, E^((2*I)*ArcCos[c*x])])/c))/((d + c*d*x)^(5/2)*(e -
 c*e*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2750 vs. $2(822) = 1644$.

Time = 4.39 (sec) , antiderivative size = 2751, normalized size of antiderivative = 3.07

method	result	size
default	Expression too large to display	2751
parts	Expression too large to display	2751

input
```
int((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
a^2*(1/3/d/c/e/(-c*e*x+e)^(3/2)*(c*d*x+d)^(1/2)+1/3/d/c/e^2/(-c*e*x+e)^(1/
2)*(c*d*x+d)^(1/2))+2/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/e^3/d/(3*
c^4*x^4-8*c^3*x^3+2*c^2*x^2+8*c*x-5)*c^3*x^4-10/3*b^2*(d*(c*x+1))^(1/2)*(-
e*(c*x-1))^(1/2)/e^3/d/(3*c^4*x^4-8*c^3*x^3+2*c^2*x^2+8*c*x-5)*c^2*x^3+10/
3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/e^3/d/(3*c^4*x^4-8*c^3*x^3+2*c^
2*x^2+8*c*x-5)*c*x^2-10/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/e^3/d/(
3*c^4*x^4-8*c^3*x^3+2*c^2*x^2+8*c*x-5)/c*arccos(c*x)^2-4/3*b^2*(d*(c*x+1))
^(1/2)*(-e*(c*x-1))^(1/2)/e^3/d/(3*c^4*x^4-8*c^3*x^3+2*c^2*x^2+8*c*x-5)*(-
c^2*x^2+1)*x+2/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/e^3/d/(3*c^4*x^4
-8*c^3*x^3+2*c^2*x^2+8*c*x-5)/c*(-c^2*x^2+1)+1/3*b^2*(d*(c*x+1))^(1/2)*(-e
*(c*x-1))^(1/2)/e^3/d/(3*c^4*x^4-8*c^3*x^3+2*c^2*x^2+8*c*x-5)*arccos(c*x)^
2*x-b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/e^3/d/(3*c^4*x^4-8*c^3*x^3+2*
c^2*x^2+8*c*x-5)*c^2*arccos(c*x)^2*x^3+8/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-
1))^(1/2)/e^3/d/(3*c^4*x^4-8*c^3*x^3+2*c^2*x^2+8*c*x-5)*c*arccos(c*x)^2*x^
2+10/3*I*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/e^3/d/(3*c^4*x^4-8*c^3*x
^3+2*c^2*x^2+8*c*x-5)/c*(-c^2*x^2+1)^(1/2)-4/3*I*b^2*(d*(c*x+1))^(1/2)*(-e
*(c*x-1))^(1/2)/e^3/d/(3*c^4*x^4-8*c^3*x^3+2*c^2*x^2+8*c*x-5)/c*arccos(c*x
)+2/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/e^3/d/(3*c^4*x^4-8*c^3*x^3+
2*c^2*x^2+8*c*x-5)*c*(-c^2*x^2+1)*x^2+2*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))
^(1/2)/e^3/d/(3*c^4*x^4-8*c^3*x^3+2*c^2*x^2+8*c*x-5)*(-c^2*x^2+1)^(1/2)...
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{5/2}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2}{\sqrt{cdx+d}(-cex+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(5/2),x, algorith
m="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sq
rt(-c*e*x + e)/(c^4*d*e^3*x^4 - 2*c^3*d*e^3*x^3 + 2*c*d*e^3*x - d*e^3), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{5/2}}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^2}{\sqrt{d(cx+1)}\,(-e(cx-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(c*d*x+d)**(1/2)/(-c*e*x+e)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(sqrt(d*(c*x + 1))*(-e*(c*x - 1))**(5/2)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(5/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{\sqrt{cdx+d}(-cex+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(5/2),x, algorith
m="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/(sqrt(c*d*x + d)*(-c*e*x + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{5/2}}\,dx=\int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{\sqrt{d+c\,d\,x}\,{\left(e-c\,e\,x\right)}^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(1/2)*(e - c*e*x)^(5/2)),x)
```

output
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(1/2)*(e - c*e*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}(e-cex)^{5/2}}\,dx=\frac{6\sqrt{-cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2-2\sqrt{cx+1}\,\sqrt{-cx+1}\,cx+\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)ab\,c^2x-}{}$$

input
```
int((a+b*acos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(5/2),x)
```

output
```
(6*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**
2 - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x + 1))
,x)*a*b*c**2*x - 6*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c
*x + 1)*c**2*x**2 - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*c*x + sqrt(c*x + 1)*s
qrt( - c*x + 1)),x)*a*b*c + 3*sqrt( - c*x + 1)*int(acos(c*x)**2/(sqrt(c*x
+ 1)*sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*c*x + s
qrt(c*x + 1)*sqrt( - c*x + 1)),x)*b**2*c**2*x - 3*sqrt( - c*x + 1)*int(aco
s(c*x)**2/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 - 2*sqrt(c*x + 1)*sqrt
( - c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b**2*c + sqrt(c*x +
1)*a**2*c*x - 2*sqrt(c*x + 1)*a**2)/(3*sqrt(e)*sqrt(d)*sqrt( - c*x + 1)*c*
e**2*(c*x - 1))
```

3.576 $\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 709

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{5/2}}\,dx = \frac{b^2d(1-c^2x^2)^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{b^2dx(1-c^2x^2)^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{bd(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{bdx(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{d(1-c^2x^2)(a+b\arccos(cx))^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{dx(1-c^2x^2)(a+b\arccos(cx))^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{2dx(1-c^2x^2)^2(a+b\arccos(cx))^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{2id(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{2ibd(1-c^2x^2)^{5/2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{4bd(1-c^2x^2)^{5/2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{ib^2d(1-c^2x^2)^{5/2}\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{ib^2d(1-c^2x^2)^{5/2}\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{2ib^2d(1-c^2x^2)^{5/2}\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}$$

output
```
1/3*b^2*d*(-c^2*x^2+1)^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+1/3*b^2*d*x*(-
c^2*x^2+1)^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3*b*d*(-c^2*x^2+1)^(3/2)*(
a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3*b*d*x*(-c^2*x^2+1)
^(3/2)*(a+b*arccos(c*x))/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+1/3*d*(-c^2*x^2+
1)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+1/3*d*x*(-c^2*x^
2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+2/3*d*x*(-c^2*x^
2+1)^2*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-2/3*I*d*(-c^2*
x^2+1)^(5/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+2/3*I*
b*d*(-c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/
c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+4/3*b*d*(-c^2*x^2+1)^(5/2)*(a+b*arccos(
c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2
)-1/3*I*b^2*d*(-c^2*x^2+1)^(5/2)*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))/
c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+1/3*I*b^2*d*(-c^2*x^2+1)^(5/2)*polylog(
2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-2/3*I*b
^2*d*(-c^2*x^2+1)^(5/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+
d)^(5/2)/(-c*e*x+e)^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.34 (sec) , antiderivative size = 566, normalized size of antiderivative = 0.80

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{5/2}}\,dx = \frac{\sqrt{d+cdx}\sqrt{e-cex}\left(4a^2+4b^2+8a^2cx-8a^2c^2x^2-4b^2c^2x^2+4ab\sqrt{1-c\right.}{}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(5/2)),x]
```

output
```
(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(4*a^2 + 4*b^2 + 8*a^2*c*x - 8*a^2*c^2*x^
2 - 4*b^2*c^2*x^2 + 4*a*b*Sqrt[1 - c^2*x^2] + 16*a*b*c*x*ArcCos[c*x] + 4*b
^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + 5*b^2*ArcCos[c*x]^2 - 5*b^2*c^2*x^2*Arc
Cos[c*x]^2 + 4*b^2*ArcCos[c*x]^2*Cos[ArcCos[c*x]/2]^2 - 8*a*b*ArcCos[c*x]*
Cos[2*ArcCos[c*x]] - 12*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/2]] + 12
*a*b*c*x*Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/2]] - 20*a*b*Sqrt[1 - c^2*x
^2]*Log[Sin[ArcCos[c*x]/2]] + 20*a*b*c*x*Sqrt[1 - c^2*x^2]*Log[Sin[ArcCos[
c*x]/2]] + (16*I)*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2*Sin[ArcCos[c*x]/2]^2
 - 40*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])]*Sin[Arc
Cos[c*x]/2]^2 - 24*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c
*x])]*Sin[ArcCos[c*x]/2]^2 + (24*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^(I
*ArcCos[c*x])]*Sin[ArcCos[c*x]/2]^2 + (40*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog
[2, E^(I*ArcCos[c*x])]*Sin[ArcCos[c*x]/2]^2 - 12*b^2*ArcCos[c*x]^2*Sin[Arc
Cos[c*x]/2]^4))/(12*c*d^2*e^3*(-1 + c*x)^2*(1 + c*x))
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.53, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5179, 27, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(cdx+d)^{3/2}(e-cex)^{5/2}}\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(1-c^2x^2)^{5/2}\int \frac{d(cx+1)(a+b\arccos(cx))^2}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d(1-c^2x^2)^{5/2}\int \frac{(cx+1)(a+b\arccos(cx))^2}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{d(1-c^2x^2)^{5/2}\int \left(\frac{cx(a+b\arccos(cx))^2}{(1-c^2x^2)^{5/2}}+\frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^{5/2}}\right)dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$\Big\downarrow$ 2009

$$\frac{d(1-c^2x^2)^{5/2}\left(\frac{2b\text{arctanh}(e^{i\arccos(cx)})(a+b\arccos(cx))}{3c}+\frac{bx(a+b\arccos(cx))}{3(1-c^2x^2)}+\frac{b(a+b\arccos(cx))}{3c(1-c^2x^2)}+\frac{2x(a+b\arccos(cx))^2}{3\sqrt{1-c^2x^2}}+\frac{x(a+\cdots}{3(\cdots}\right.}{\cdots}$$

input
```
Int[(a + b*ArcCos[c*x])^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(5/2)),x]
```

output
```
(d*(1 - c^2*x^2)^(5/2)*(b^2/(3*c*Sqrt[1 - c^2*x^2]) + (b^2*x)/(3*Sqrt[1 -
c^2*x^2]) + (b*(a + b*ArcCos[c*x]))/(3*c*(1 - c^2*x^2)) + (b*x*(a + b*ArcC
os[c*x]))/(3*(1 - c^2*x^2)) + (((2*I)/3)*(a + b*ArcCos[c*x])^2)/c + (a + b
*ArcCos[c*x])^2/(3*c*(1 - c^2*x^2)^(3/2)) + (x*(a + b*ArcCos[c*x])^2)/(3*(
1 - c^2*x^2)^(3/2)) + (2*x*(a + b*ArcCos[c*x])^2)/(3*Sqrt[1 - c^2*x^2]) +
(2*b*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])])/(3*c) - (4*b*(a + b*A
rcCos[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])])/(3*c) - ((I/3)*b^2*PolyLog[2,
-E^(I*ArcCos[c*x])])/c + ((I/3)*b^2*PolyLog[2, E^(I*ArcCos[c*x])])/c + (((
2*I)/3)*b^2*PolyLog[2, E^((2*I)*ArcCos[c*x])])/c))/((d + c*d*x)^(5/2)*(e -
 c*e*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2019 vs. 2(657) = 1314.

Time = 13.38 (sec) , antiderivative size = 2020, normalized size of antiderivative = 2.85

method	result	size
default	Expression too large to display	2020
parts	Expression too large to display	2020

input
```
int((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
a^2*(-1/d/c/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(3/2)+2/d*(1/3/d/c/e/(-c*e*x+e)^(
3/2)*(c*d*x+d)^(1/2)+1/3/d/c/e^2/(-c*e*x+e)^(1/2)*(c*d*x+d)^(1/2)))+1/3*b^
2*c/d/e^2/(d*(c*x+1))^(1/2)/(-e*(c*x-1))^(1/2)/(I*(-c^2*x^2+1)^(1/2)*c*x+c
^2*x^2-I*(-c^2*x^2+1)^(1/2)-c*x)*x^2+1/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1)
)^(1/2)*arccos(c*x)^2/d^2/e^3/c/(c^3*x^3-c^2*x^2-c*x+1)-2/3*b^2*(d*(c*x+1)
)^(1/2)*(-e*(c*x-1))^(1/2)*arccos(c*x)^2/d^2/e^3*c/(c^3*x^3-c^2*x^2-c*x+1)
*x^2+2/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*arccos(c*x)^2/d^2/e^3/(c
^3*x^3-c^2*x^2-c*x+1)*x+1/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*arcco
s(c*x)/d^2/e^3/c/(c^3*x^3-c^2*x^2-c*x+1)*(-c^2*x^2+1)^(1/2)+1/3*I*b^2/d/e^
2/(d*(c*x+1))^(1/2)/(-e*(c*x-1))^(1/2)/(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-I
*(-c^2*x^2+1)^(1/2)-c*x)*(-c^2*x^2+1)^(1/2)*x-2/3*I*b^2*(d*(c*x+1))^(1/2)*
(-e*(c*x-1))^(1/2)*arccos(c*x)^2/d^2/e^3/(c^3*x^3-c^2*x^2-c*x+1)*(-c^2*x^2
+1)^(1/2)*x+I*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*polylog(2,-c*x-I*(-
c^2*x^2+1)^(1/2))/d^2/e^3/c/(c^3*x^3-c^2*x^2-c*x+1)*(-c^2*x^2+1)^(1/2)+5/3
*I*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*polylog(2,c*x+I*(-c^2*x^2+1)^(
1/2))/d^2/e^3/c/(c^3*x^3-c^2*x^2-c*x+1)*(-c^2*x^2+1)^(1/2)-b^2*(d*(c*x+1))
^(1/2)*(-e*(c*x-1))^(1/2)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*arccos(c*x)/d^2/e
^3/c/(c^3*x^3-c^2*x^2-c*x+1)*(-c^2*x^2+1)^(1/2)-5/3*b^2*(d*(c*x+1))^(1/2)*
(-e*(c*x-1))^(1/2)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))*arccos(c*x)/d^2/e^3/c/(c
^3*x^3-c^2*x^2-c*x+1)*(-c^2*x^2+1)^(1/2)-1/6*b^2/(c*x-1)/(-e*(c*x-1))^(...
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(5/2),x, algorith
m="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sq
rt(-c*e*x + e)/(c^5*d^2*e^3*x^5 - c^4*d^2*e^3*x^4 - 2*c^3*d^2*e^3*x^3 + 2*
c^2*d^2*e^3*x^2 + c*d^2*e^3*x - d^2*e^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acos(c*x))**2/(c*d*x+d)**(3/2)/(-c*e*x+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(5/2),x, algorith
m="maxima")
```

output
```
-1/6*a*b*c*(2*sqrt(d)*sqrt(e)/(c^3*d^2*e^3*x - c^2*d^2*e^3) + 3*log(c*x +
1)/(c^2*d^(3/2)*e^(5/2)) + 5*log(c*x - 1)/(c^2*d^(3/2)*e^(5/2))) - 2/3*a*b
*(1/(sqrt(-c^2*d*e*x^2 + d*e)*c^2*d*e^2*x - sqrt(-c^2*d*e*x^2 + d*e)*c*d*e
^2) - 2*x/(sqrt(-c^2*d*e*x^2 + d*e)*d*e^2))*arccos(c*x) - 1/3*a^2*(1/(sqrt
(-c^2*d*e*x^2 + d*e)*c^2*d*e^2*x - sqrt(-c^2*d*e*x^2 + d*e)*c*d*e^2) - 2*x
/(sqrt(-c^2*d*e*x^2 + d*e)*d*e^2)) + b^2*integrate(arctan2(sqrt(c*x + 1)*s
qrt(-c*x + 1), c*x)^2/((c^3*d*e^2*x^3 - c^2*d*e^2*x^2 - c*d*e^2*x + d*e^2)
*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/(sqrt(d)*sqrt(e))
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(5/2),x, algorith
m="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/((c*d*x + d)^(3/2)*(-c*e*x + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{5/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(d+c\,d\,x)^{3/2}\,(e-c\,e\,x)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(5/2)),x)
```

output
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{5/2}}\,dx=\frac{6\sqrt{cx+1}\,\sqrt{-cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^3x^3-\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2-\sqrt{cx+1}\,\sqrt{-cx+1}\,cx\right.}{}$$

input
```
int((a+b*acos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(5/2),x)
```

output

```
(6*sqrt(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x
 + 1)*c**3*x**3 - sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 - sqrt(c*x + 1)
*sqrt( - c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a*b*c**2*x - 6*
sqrt(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x +
1)*c**3*x**3 - sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sq
rt( - c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a*b*c + 3*sqrt(c*x
 + 1)*sqrt( - c*x + 1)*int(acos(c*x)**2/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c*
*3*x**3 - sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt( -
 c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b**2*c**2*x - 3*sqrt(c*
x + 1)*sqrt( - c*x + 1)*int(acos(c*x)**2/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c
**3*x**3 - sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt(
- c*x + 1)*c*x + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b**2*c + 2*a**2*c**2*x
**2 - 2*a**2*c*x - a**2)/(3*sqrt(e)*sqrt(d)*sqrt(c*x + 1)*sqrt( - c*x + 1)
*c*d*e**2*(c*x - 1))
```

3.577 $\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 366

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{5/2}}\,dx &= \frac{b^2x(1-c^2x^2)^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} \\ &- \frac{b(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} + \frac{x(1-c^2x^2)(a+b\arccos(cx))^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} \\ &+ \frac{2x(1-c^2x^2)^2(a+b\arccos(cx))^2}{3(d+cdx)^{5/2}(e-cex)^{5/2}} - \frac{2i(1-c^2x^2)^{5/2}(a+b\arccos(cx))^2}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\ &+ \frac{4b(1-c^2x^2)^{5/2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}} \\ &- \frac{2ib^2(1-c^2x^2)^{5/2}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{3c(d+cdx)^{5/2}(e-cex)^{5/2}}\end{aligned}$$

output
```
1/3*b^2*x*(-c^2*x^2+1)^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-1/3*b*(-c^2*x^2+
1)^(3/2)*(a+b*arccos(c*x))/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+1/3*x*(-c^2*
x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+2/3*x*(-c^2*x^
2+1)^2*(a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-2/3*I*(-c^2*x^
2+1)^(5/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)+4/3*b*(-
c^2*x^2+1)^(5/2)*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c
*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)-2/3*I*b^2*(-c^2*x^2+1)^(5/2)*polylog(2,-(c*
x+I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2)
```

Mathematica [A] (verified)

Time = 3.46 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.51

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{5/2}}\,dx = \frac{-12a^2cx - b^2cx + 8a^2c^3x^3 - 4ab\sqrt{1-c^2x^2} - 12abcx\arccos(cx) - 4b^2\sqrt{1}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/((d + c*d*x)^(5/2)*(e - c*e*x)^(5/2)),x]
```

output
```
(-12*a^2*c*x - b^2*c*x + 8*a^2*c^3*x^3 - 4*a*b*Sqrt[1 - c^2*x^2] - 12*a*b*
c*x*ArcCos[c*x] - 4*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x] - 6*b^2*c*x*ArcCos[c
*x]^2 - (6*I)*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2 + b^2*Cos[3*ArcCos[c*x]]
 + 4*a*b*ArcCos[c*x]*Cos[3*ArcCos[c*x]] + 2*b^2*ArcCos[c*x]^2*Cos[3*ArcCos
[c*x]] + 12*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] +
 12*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])] + 8*a*b*S
qrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/2]] - 8*a*b*Sqrt[1 - c^2*x^2]*Cos[2*A
rcCos[c*x]]*Log[Cos[ArcCos[c*x]/2]] + 8*a*b*Sqrt[1 - c^2*x^2]*Log[Sin[ArcC
os[c*x]/2]] - 8*a*b*Sqrt[1 - c^2*x^2]*Cos[2*ArcCos[c*x]]*Log[Sin[ArcCos[c*
x]/2]] - (16*I)*b^2*(1 - c^2*x^2)^(3/2)*PolyLog[2, -E^(I*ArcCos[c*x])] - (
16*I)*b^2*(1 - c^2*x^2)^(3/2)*PolyLog[2, E^(I*ArcCos[c*x])] + (2*I)*b^2*Ar
cCos[c*x]^2*Sin[3*ArcCos[c*x]] - 4*b^2*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x
])]*Sin[3*ArcCos[c*x]] - 4*b^2*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])]*Sin[
3*ArcCos[c*x]])/(12*c*d^2*e^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-1 + c^2*x^
2))
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.63, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {5179, 5163, 5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838, 5183, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(cdx+d)^{5/2}(e-cex)^{5/2}}\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\left(1-c^2x^2\right)^{5/2}\int \frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^{5/2}}dx}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$\big\downarrow$ 5163

$$\frac{\left(1-c^2x^2\right)^{5/2}\left(\frac{2}{3}bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx+\frac{2}{3}\int \frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx+\frac{x(a+b\arccos(cx))^2}{3(1-c^2x^2)^{3/2}}\right)}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$\big\downarrow$ 5161

$$\frac{\left(1-c^2x^2\right)^{5/2}\left(\frac{2}{3}bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx+\frac{2}{3}\left(2bc\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)+\frac{x(a+b\arccos(cx))^2}{3(1-c^2x^2)^{3/2}}\right)}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$\big\downarrow$ 5181

$$\frac{\left(1-c^2x^2\right)^{5/2}\left(\frac{2}{3}bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c}\right)+\frac{x(a+b\arccos(cx))^2}{3(1-c^2x^2)^{3/2}}\right)}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{\left(1-c^2x^2\right)^{5/2}\left(\frac{2}{3}bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{c}\right)+\right.}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$\big\downarrow$ 25

$$\frac{\left(1-c^2x^2\right)^{5/2}\left(\frac{2}{3}bc\int \frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx+\frac{2}{3}\left(\frac{2b\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c}+\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)+\frac{x(a-}{3(}\right.}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$\big\downarrow$ 4200

$$\frac{\left(1-c^2x^2\right)^{5/2}\left(\frac{2}{3}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c}\right)+\frac{2}{3}bc\int \frac{x(a+b\arc}{(1-c^2}\right.}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$\big\downarrow$ 25

$$\frac{(1-c^2x^2)^{5/2}\left(\frac{2}{3}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\int\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c}\right)+\frac{2}{3}bc\int\frac{x(a+b\,\mathrm{arc}}{(1-c^2}}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$\downarrow$ 2620

$$\frac{(1-c^2x^2)^{5/2}\left(\frac{2}{3}bc\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\log(1\right.\right.}{c}\right.}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$\downarrow$ 2715

$$\frac{(1-c^2x^2)^{5/2}\left(\frac{2}{3}bc\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\,\mathrm{a}}\right.\right.}{}\right.}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$\downarrow$ 2838

$$\frac{(1-c^2x^2)^{5/2}\left(\frac{2}{3}bc\int\frac{x(a+b\arccos(cx))}{(1-c^2x^2)^2}dx+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\mathrm{PolyLog}\right.\right.}{c}\right.}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$\downarrow$ 5183

$$\frac{(1-c^2x^2)^{5/2}\left(\frac{2}{3}bc\left(\frac{b\int\frac{1}{(1-c^2x^2)^{3/2}}dx}{2c}+\frac{a+b\arccos(cx)}{2c^2(1-c^2x^2)}\right)+\frac{2}{3}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\,\mathrm{arcco}\right.\right.}{}\right.}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

$\downarrow$ 208

$$\frac{(1-c^2x^2)^{5/2}\left(\frac{2}{3}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(c}{2b}\right.}{c}\right.}{(cdx+d)^{5/2}(e-cex)^{5/2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/((d + c*d*x)^(5/2)*(e - c*e*x)^(5/2)),x]
```

output
```
((1 - c^2*x^2)^(5/2)*((x*(a + b*ArcCos[c*x])^2)/(3*(1 - c^2*x^2)^(3/2)) +
(2*b*c*((b*x)/(2*c*Sqrt[1 - c^2*x^2]) + (a + b*ArcCos[c*x])/(2*c^2*(1 - c^
2*x^2))))/3 + (2*((x*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2] - (2*b*(((-1
/2*I)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])*Log[1 -
E^((2*I)*ArcCos[c*x])] + (b*PolyLog[2, E^((2*I)*ArcCos[c*x])])/4)))/c))/3)
)/((d + c*d*x)^(5/2)*(e - c*e*x)^(5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5163
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[(-x)*(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*d*(p + 1
))), x] + (Simp[(2*p + 3)/(2*d*(p + 1))   Int[(d + e*x^2)^(p + 1)*(a + b*Ar
cCos[c*x])^n, x], x] - Simp[b*c*(n/(2*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2
*x^2)^p]   Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p,
 -1] && NeQ[p, -3/2]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2002 vs. $2(336) = 672$.

Time = 16.05 (sec) , antiderivative size = 2003, normalized size of antiderivative = 5.47

method	result	size
default	Expression too large to display	2003
parts	Expression too large to display	2003

input
```
int((a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
a^2*(-1/3/d/c/e/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+1/d*(-1/d/c/e/(c*d*x+d)^(
1/2)/(-c*e*x+e)^(3/2)+2/d*(1/3/d/c/e/(-c*e*x+e)^(3/2)*(c*d*x+d)^(1/2)+1/3/
d/c/e^2/(-c*e*x+e)^(1/2)*(c*d*x+d)^(1/2)))+1/12*b^2/(d*(c*x+1))^(1/2)/(-e
*(c*x-1))^(1/2)/(4*c^5*x^5-7*c^3*x^3+4*I*(-c^2*x^2+1)^(1/2)*c^4*x^4+3*c*x-
5*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))/e^2/d^2/c-2/3*I*b^2*(
d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*arccos(c*x)^2/e^3/d^3*c/(c^4*x^4-2*c^2
*x^2+1)*(-c^2*x^2+1)^(1/2)*x^2+1/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2
)*arccos(c*x)/e^3/d^3/c/(c^4*x^4-2*c^2*x^2+1)*(-c^2*x^2+1)^(1/2)-2/3*b^2*(
d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*arccos(c*x)^2/e^3/d^3*c^2/(c^4*x^4-2*c
^2*x^2+1)*x^3+b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*arccos(c*x)^2/e^3/d
^3/(c^4*x^4-2*c^2*x^2+1)*x-1/6*b^2/(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-I*(-c
^2*x^2+1)^(1/2)-c*x)/(-e*(c*x-1))^(1/2)/(c*x+1)/(d*(c*x+1))^(1/2)/e^2/d^2/
c+2/3*I*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*arccos(c*x)^2/e^3/d^3/c/(
c^4*x^4-2*c^2*x^2+1)*(-c^2*x^2+1)^(1/2)+4/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x
-1))^(1/2)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))*arccos(c*x)/e^3/d^3*c/(c^4*x^4-2
*c^2*x^2+1)*(-c^2*x^2+1)^(1/2)*x^2+4/3*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^
(1/2)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*arccos(c*x)/e^3/d^3*c/(c^4*x^4-2*c^2*
x^2+1)*(-c^2*x^2+1)^(1/2)*x^2+1/3*I*b^2/(d*(c*x+1))^(1/2)/(-e*(c*x-1))^(1/
2)/(2*c^4*x^4-3*c^2*x^2+2*I*(-c^2*x^2+1)^(1/2)*c^3*x^3-2*I*(-c^2*x^2+1)^(1
/2)*c*x+1)/e^2/d^2*c*(-c^2*x^2+1)^(1/2)*x^2-1/12*I*b^2/(d*(c*x+1))^(1/2...
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{5}{2}}(-cex+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2),x, algorith
m="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sq
rt(-c*e*x + e)/(c^6*d^3*e^3*x^6 - 3*c^4*d^3*e^3*x^4 + 3*c^2*d^3*e^3*x^2 -
d^3*e^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acos(c*x))**2/(c*d*x+d)**(5/2)/(-c*e*x+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{5}{2}}(-cex+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2),x, algorith
m="maxima")
```

output
```
-1/3*a*b*c*(1/(c^4*d^(5/2)*e^(5/2)*x^2 - c^2*d^(5/2)*e^(5/2)) + 2*log(c*x
+ 1)/(c^2*d^(5/2)*e^(5/2)) + 2*log(c*x - 1)/(c^2*d^(5/2)*e^(5/2))) + 2/3*a
*b*(x/((-c^2*d*e*x^2 + d*e)^(3/2)*d*e) + 2*x/(sqrt(-c^2*d*e*x^2 + d*e)*d^2
*e^2))*arccos(c*x) + 1/3*a^2*(x/((-c^2*d*e*x^2 + d*e)^(3/2)*d*e) + 2*x/(sq
rt(-c^2*d*e*x^2 + d*e)*d^2*e^2)) + b^2*integrate(arctan2(sqrt(c*x + 1)*sqr
t(-c*x + 1), c*x)^2/((c^4*d^2*e^2*x^4 - 2*c^2*d^2*e^2*x^2 + d^2*e^2)*sqrt(
c*x + 1)*sqrt(-c*x + 1)), x)/(sqrt(d)*sqrt(e))
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{5}{2}}(-cex+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2),x, algorith
m="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/((c*d*x + d)^(5/2)*(-c*e*x + e)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{5/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(d+c\,d\,x)^{5/2}\,(e-c\,e\,x)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(5/2)*(e - c*e*x)^(5/2)),x)
```

output
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(5/2)*(e - c*e*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{5/2}(e-cex)^{5/2}}\,dx = \frac{6\sqrt{cx+1}\,\sqrt{-cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^4x^4-2\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2+\sqrt{cx+1}\,\sqrt{-cx+1}}\right.}{}$$

input
```
int((a+b*acos(c*x))^2/(c*d*x+d)^(5/2)/(-c*e*x+e)^(5/2),x)
```

output
```
(6*sqrt(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x
 + 1)*c**4*x**4 - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 + sqrt(c*x +
1)*sqrt( - c*x + 1)),x)*a*b*c**2*x**2 - 6*sqrt(c*x + 1)*sqrt( - c*x + 1)*i
nt(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**4*x**4 - 2*sqrt(c*x + 1)*s
qrt( - c*x + 1)*c**2*x**2 + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a*b + 3*sqr
t(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)**2/(sqrt(c*x + 1)*sqrt( - c*x +
1)*c**4*x**4 - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x**2 + sqrt(c*x + 1)*
sqrt( - c*x + 1)),x)*b**2*c**2*x**2 - 3*sqrt(c*x + 1)*sqrt( - c*x + 1)*int
(acos(c*x)**2/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**4*x**4 - 2*sqrt(c*x + 1)*
sqrt( - c*x + 1)*c**2*x**2 + sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b**2 + 2*a
**2*c**2*x**3 - 3*a**2*x)/(3*sqrt(e)*sqrt(d)*sqrt(c*x + 1)*sqrt( - c*x + 1
)*d**2*e**2*(c**2*x**2 - 1))
```

3.578 $\int x^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 351

$$\begin{aligned}
&\int x^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx\\
&=\frac{b^2x\sqrt{d+cdx}\sqrt{e-cex}}{64c^2}-\frac{1}{32}b^2x^3\sqrt{d+cdx}\sqrt{e-cex}\\
&\quad-\frac{b^2\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)}{64c^3\sqrt{1-c^2x^2}}+\frac{bx^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{8c\sqrt{1-c^2x^2}}\\
&\quad-\frac{bcx^4\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{8\sqrt{1-c^2x^2}}\\
&\quad-\frac{x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{8c^2}\\
&\quad+\frac{1}{4}x^3\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2+\frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^3}{24bc^3\sqrt{1-c^2x^2}}
\end{aligned}$$

output
```
1/64*b^2*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)/c^2-1/32*b^2*x^3*(c*d*x+d)^(1/
2)*(-c*e*x+e)^(1/2)-1/64*b^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*arccos(c*x)/
c^3/(-c^2*x^2+1)^(1/2)+1/8*b*x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arc
cos(c*x))/c/(-c^2*x^2+1)^(1/2)-1/8*b*c*x^4*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2
)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-1/8*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1
/2)*(a+b*arccos(c*x))^2/c^2+1/4*x^3*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*
arccos(c*x))^2+1/24*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^3/b
/c^3/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.85

$$\int x^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$
$$= \frac{-32b^2\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)^3-96a^2\sqrt{d}\sqrt{e}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{e-cex}}{\sqrt{d}\sqrt{e}(-1+c^2x^2)}\right)+12b\sqrt{d+cdx}\sqrt{}}{}$$

input
```
Integrate[x^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(-32*b^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^3 - 96*a^2*Sqrt[d]*Sq
rt[e]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt
[d]*Sqrt[e]*(-1 + c^2*x^2))] + 12*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos
[c*x]*(b*Cos[4*ArcCos[c*x]] + 4*a*Sin[4*ArcCos[c*x]]) + 24*b*Sqrt[d + c*d*
x]*Sqrt[e - c*e*x]*ArcCos[c*x]^2*(-4*a + b*Sin[4*ArcCos[c*x]]) + 3*Sqrt[d
+ c*d*x]*Sqrt[e - c*e*x]*(32*a^2*c*x*Sqrt[1 - c^2*x^2]*(-1 + 2*c^2*x^2) +
4*a*b*Cos[4*ArcCos[c*x]] - b^2*Sin[4*ArcCos[c*x]]))/(768*c^3*Sqrt[1 - c^2*
x^2])
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.80, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {5239, 5199, 5139, 262, 262, 223, 5211, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{cdx+d}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5239}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\int x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5199}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{4}\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int x^3(a+b\arccos(cx))dx+\frac{1}{4}x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\right)}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5139

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{4}\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\left(\frac{1}{4}bc\int \frac{x^4}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))\right)+\frac{1}{4}x^3\sqrt{1-c^2x^2}(a}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 262

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{4}\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\left(\frac{1}{4}bc\left(\frac{3\int \frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{4}x^4(a+b\arccos(cx))\right)\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 262

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{4}\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\left(\frac{1}{4}bc\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{4}x^4(a+b\,\mathrm{a}\right.\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 223

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{4}\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-}}{4c}\right.\right.\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5211

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{4}\left(\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int x(a+b\arccos(cx))dx}{c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)+\frac{1}{2}bc\left(\frac{1}{4}x^4(a+b\,\mathrm{ar}\right.\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5139

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{4}\left(-\frac{b\left(\frac{1}{2}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c}+\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)+\right.}{\sqrt{1-}}$$

$\downarrow$ 262

$$\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{4}\left(-\frac{b\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c}+\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\text{ ar}}{2c^2}\right.\right.$$

$\downarrow$ 223

$$\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{4}\left(\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx}{2c^2}\right.\right.$$

$\downarrow$ 5153

$$\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{4}\left(-\frac{b\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c}-\frac{(a+b\arccos(cx))^3}{6bc^3}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx)}{2c^2}\right.\right.$$

input
```
Int[x^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*((x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x
])^2)/4 + (b*c*((x^4*(a + b*ArcCos[c*x]))/4 + (b*c*(-1/4*(x^3*Sqrt[1 - c^2
*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4
*c^2)))/4))/2 + (-1/2*(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c^2 - (a
 + b*ArcCos[c*x])^3/(6*b*c^3) - (b*((x^2*(a + b*ArcCos[c*x]))/2 + (b*c*(-1
/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2))/c)/4))/Sqrt[1 - c
^2*x^2]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 13.33 (sec) , antiderivative size = 759, normalized size of antiderivative = 2.16

method	result
default	$\frac{a^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\left(2x^3c^2\sqrt{-de(c^2x^2-1)}\,\sqrt{c^2de}+\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-de\left(c^2x^2-1\right)}}\right)de-\sqrt{c^2de}\,\sqrt{-de(c^2x^2-1)}\,x\right)}{8\sqrt{-de(c^2x^2-1)}\,c^2\sqrt{c^2de}}+b^2\Bigg(\frac{\sqrt{d(}}{}$
parts	$\frac{a^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\left(2x^3c^2\sqrt{-de(c^2x^2-1)}\,\sqrt{c^2de}+\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-de\left(c^2x^2-1\right)}}\right)de-\sqrt{c^2de}\,\sqrt{-de(c^2x^2-1)}\,x\right)}{8\sqrt{-de(c^2x^2-1)}\,c^2\sqrt{c^2de}}+b^2\Bigg(\frac{\sqrt{d(}}{}$

input
```
int(x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/8*a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(2*x^3*c^2*(-d*e*(c^2*x^2-1))
^(1/2)*(c^2*d*e)^(1/2)+arctan((c^2*d*e)^(1/2)*x/(-d*e*(c^2*x^2-1))^(1/2))*
d*e-(c^2*d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2)*x)/(-d*e*(c^2*x^2-1))^(1/2)/c
^2/(c^2*d*e)^(1/2)+b^2*(1/24*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^
2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^3+1/512*(d*(c*x+1))^(1/2)*(-e*(c*x-
1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-8*I*(
-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(4*I*arccos(c*x)+8*arccos(
c*x)^2-1)/c^3/(c^2*x^2-1)+1/512*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-8*I
*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^
3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*(-4*I*arccos(c*x)+8*arccos(c*x)^2-1)/c^3
/(c^2*x^2-1))+2*a*b*(1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1
)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2+1/256*(d*(c*x+1))^(1/2)*(-e*(c*x-1))
^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-8*I*(-c^
2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(I+4*arccos(c*x))/c^3/(c^2*x^
2-1)+1/256*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x
^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^
(1/2)+4*c*x)*(-I+4*arccos(c*x))/c^3/(c^2*x^2-1))
```

Fricas [F]

$$\int x^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$
$$=\int\sqrt{cdx+d}\sqrt{-cex+e}(b\arccos(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algo
rithm="fricas")
```

output
```
integral((b^2*x^2*arccos(c*x)^2 + 2*a*b*x^2*arccos(c*x) + a^2*x^2)*sqrt(c*
d*x + d)*sqrt(-c*e*x + e), x)
```

Sympy [F]

$$\int x^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$
$$=\int x^2\sqrt{d\,(cx+1)}\sqrt{-e\,(cx-1)}(a+b\,\mathrm{acos}\,(cx))^2\;dx$$

input
```
integrate(x**2*(c*d*x+d)**(1/2)*(-c*e*x+e)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral(x**2*sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))*(a + b*acos(c*x))**2, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int x^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$
$$=\int \sqrt{cdx+d}\sqrt{-cex+e}(b\arccos{(cx)}+a)^2x^2\,dx$$

input
```
integrate(x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algo
rithm="giac")
```

output
```
integrate(sqrt(c*d*x + d)*sqrt(-c*e*x + e)*(b*arccos(c*x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$
$$=\int x^2\,(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d+c\,d\,x}\,\sqrt{e-c\,e\,x}\,dx$$

input
```
int(x^2*(a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2),x)
```

output
```
int(x^2*(a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$
$$=\frac{\sqrt{e}\,\sqrt{d}\,\left(-2asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2+2\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^3x^3-\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2cx+16\left(\int\sqrt{cx+1}\right.\right.}{8c^3}$$

input
```
int(x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*acos(c*x))^2,x)
```

output

```
(sqrt(e)*sqrt(d)*( - 2*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + 2*sqrt(c*x +
1)*sqrt( - c*x + 1)*a**2*c**3*x**3 - sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c
*x + 16*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**2,x)*a*b*c**3 + 8*
int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2*x**2,x)*b**2*c**3))/(8*c**
3)
```

3.579 $\int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 33, antiderivative size = 225

$$\begin{aligned}\int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx &= \frac{4b^2\sqrt{d+cdx}\sqrt{e-cex}}{9c^2}+\frac{2b^2\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)}{27c^2}\\ &\quad+\frac{2bx\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{3c\sqrt{1-c^2x^2}}\\ &\quad-\frac{2bcx^3\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{9\sqrt{1-c^2x^2}}\\ &\quad-\frac{\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)\,(a+b\arccos(cx))^2}{3c^2}\end{aligned}$$

output
```
4/9*b^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)/c^2+2/27*b^2*(c*d*x+d)^(1/2)*(-c*
e*x+e)^(1/2)*(-c^2*x^2+1)/c^2+2/3*b*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+
b*arccos(c*x))/c/(-c^2*x^2+1)^(1/2)-2/9*b*c*x^3*(c*d*x+d)^(1/2)*(-c*e*x+e)
^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-1/3*(c*d*x+d)^(1/2)*(-c*e*x+e)
^(1/2)*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/c^2
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.80

$$\int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$
$$=\frac{\sqrt{d+cdx}\sqrt{e-cex}\left(-6abcx\sqrt{1-c^2x^2}(-3+c^2x^2)+9a^2(-1+c^2x^2)^2-2b^2(7-8c^2x^2+c^4x^4)+6b\left(bc\right.\right.}{27c^2\left(-1+c^2x^2\right)}$$

input
```
Integrate[x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-6*a*b*c*x*Sqrt[1 - c^2*x^2]*(-3 + c^2*x
^2) + 9*a^2*(-1 + c^2*x^2)^2 - 2*b^2*(7 - 8*c^2*x^2 + c^4*x^4) + 6*b*(b*c*
x*Sqrt[1 - c^2*x^2]*(3 - c^2*x^2) + 3*a*(-1 + c^2*x^2)^2)*ArcCos[c*x] + 9*
b^2*(-1 + c^2*x^2)^2*ArcCos[c*x]^2))/(27*c^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.68, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {5239, 5183, 5155, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{cdx+d}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5239}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\int x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5183}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(-\frac{2b\int(1-c^2x^2)(a+b\arccos(cx))dx}{3c}-\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{3c^2}\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5155}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(-\frac{2b\left(bc\int \frac{x(3-c^2x^2)}{3\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{3c^2}\right)}{\sqrt{1-c^2x^2}}$$

$\downarrow$ 27

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(-\frac{2b\left(\frac{1}{3}bc\int \frac{x(3-c^2x^2)}{\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{3c^2}\right)}{\sqrt{1-c^2x^2}}$$

$\downarrow$ 353

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(-\frac{2b\left(\frac{1}{6}bc\int \frac{3-c^2x^2}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{3c^2}\right)}{\sqrt{1-c^2x^2}}$$

$\downarrow$ 53

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(-\frac{2b\left(\frac{1}{6}bc\int \left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{3c}-\frac{(1-c^2x^2)^{3/2}(a+b\arccos(}{3c^2}}{\sqrt{1-c^2x^2}}$$

$\downarrow$ 2009

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(-\frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{3c^2}-\frac{2b\left(-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{6}bc\left(-\frac{2(1-c^2x^2)^{3/2}}{3c^2}-\frac{4\sqrt{1}}{}\right.\right.}{3c}\right.}{\sqrt{1-c^2x^2}}$$

input
```
Int[x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-1/3*((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[
c*x])^2)/c^2 - (2*b*((b*c*((-4*Sqrt[1 - c^2*x^2])/c^2 - (2*(1 - c^2*x^2)^(
3/2))/(3*c^2)))/6 + x*(a + b*ArcCos[c*x]) - (c^2*x^3*(a + b*ArcCos[c*x]))/
3))/(3*c)))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(191) = 382$.

Time = 16.94 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.88

method	result
orering	$\frac{(19c^6x^6-71c^4x^4+48c^2x^2-14)\sqrt{cdx+d}\,\sqrt{-cex+e}\,(a+b\arccos(cx))^2}{27c^4x^2(cx-1)(cx+1)}-\frac{2(3c^4x^4-16c^2x^2+7)\left(\sqrt{cdx+d}\,\sqrt{-cex+e}\,(a+b\arccos(cx\right.}{}$
default	$\frac{a^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,(c^2x^2-1)}{3c^2}+b^2\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\left(4c^4x^4-5c^2x^2+4i\sqrt{-c^2x^2+1}\,x^3c^3-3i\sqrt{-c^2x^2+1}\,xc+1\right)\left(6\right.}{216c^2(c^2x^2-1)}\right.$
parts	$\frac{a^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,(c^2x^2-1)}{3c^2}+b^2\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\left(4c^4x^4-5c^2x^2+4i\sqrt{-c^2x^2+1}\,x^3c^3-3i\sqrt{-c^2x^2+1}\,xc+1\right)\left(6\right.}{216c^2(c^2x^2-1)}\right.$

input
```
int(x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/27*(19*c^6*x^6-71*c^4*x^4+48*c^2*x^2-14)/c^4/x^2/(c*x-1)/(c*x+1)*(c*d*x+
d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2-2/27*(3*c^4*x^4-16*c^2*x^2+7
)/c^4/x^2*((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2+1/2*x/(c*d
*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2*d*c-1/2*x*(c*d*x+d)^(1/2)
/(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2*c*e-2*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(
1/2)*(a+b*arccos(c*x))*b*c/(-c^2*x^2+1)^(1/2))+1/27*(c^2*x^2-7)/c^4/x*(c*x
-1)*(c*x+1)*(1/(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2*d*c-(c
*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2*c*e-4*(c*d*x+d)^(1/2)*(
-c*e*x+e)^(1/2)*(a+b*arccos(c*x))*b*c/(-c^2*x^2+1)^(1/2)-1/4*x/(c*d*x+d)^(
3/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2*d^2*c^2-1/2*x/(c*d*x+d)^(1/2)/(-
c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2*d*c^2*e-2*x/(c*d*x+d)^(1/2)*(-c*e*x+e)^
(1/2)*(a+b*arccos(c*x))*d*c^2*b/(-c^2*x^2+1)^(1/2)-1/4*x*(c*d*x+d)^(1/2)/(
-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2*c^2*e^2+2*x*(c*d*x+d)^(1/2)/(-c*e*x+e)
^(1/2)*(a+b*arccos(c*x))*c^2*e*b/(-c^2*x^2+1)^(1/2)+2*x*(c*d*x+d)^(1/2)*(-
c*e*x+e)^(1/2)*b^2*c^2/(-c^2*x^2+1)-2*x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)
*(a+b*arccos(c*x))*b*c^3/(-c^2*x^2+1)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.88

$$\int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$
$$= \frac{((9\,a^2-2\,b^2)c^4x^4-2\,(9\,a^2-8\,b^2)c^2x^2+9\,(b^2c^4x^4-2\,b^2c^2x^2+b^2)\arccos{(cx)}^2+9\,a^2-14\,b^2+18\,(abc^4}{}$$

input
```
integrate(x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algori
thm="fricas")
```

output
```
1/27*((9*a^2 - 2*b^2)*c^4*x^4 - 2*(9*a^2 - 8*b^2)*c^2*x^2 + 9*(b^2*c^4*x^4
 - 2*b^2*c^2*x^2 + b^2)*arccos(c*x)^2 + 9*a^2 - 14*b^2 + 18*(a*b*c^4*x^4 -
 2*a*b*c^2*x^2 + a*b)*arccos(c*x) - 6*(a*b*c^3*x^3 - 3*a*b*c*x + (b^2*c^3*
x^3 - 3*b^2*c*x)*arccos(c*x))*sqrt(-c^2*x^2 + 1))*sqrt(c*d*x + d)*sqrt(-c*
e*x + e)/(c^4*x^2 - c^2)
```

Sympy [F]

$$\int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \int x\sqrt{d\,(cx+1)}\,\sqrt{-e\,(cx-1)}(a+b\,\mathrm{acos}\,(cx))^2\;dx$$

input
```
integrate(x*(c*d*x+d)**(1/2)*(-c*e*x+e)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral(x*sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))*(a + b*acos(c*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \int \sqrt{cdx+d}\sqrt{-cex+e}(b\arccos\,(cx)+a)^2x\,dx$$

input
```
integrate(x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algori
thm="giac")
```

output
```
integrate(sqrt(c*d*x + d)*sqrt(-c*e*x + e)*(b*arccos(c*x) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \int x\,(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d+c\,d\,x}\,\sqrt{e-c\,e\,x}\,dx$$

input
```
int(x*(a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2),x)
```

output
```
int(x*(a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \frac{\sqrt{e}\,\sqrt{d}\,\left(\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^2x^2-\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2+6\left(\int\sqrt{cx+1}\,\sqrt{-cx+1}\,acos(cx)\,xdx\right)ab\,c}{3c^2}$$

input
```
int(x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(e)*sqrt(d)*(sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c**2*x**2 - sqrt(c*x
 + 1)*sqrt( - c*x + 1)*a**2 + 6*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*
x)*x,x)*a*b*c**2 + 3*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2*x,x)*
b**2*c**2))/(3*c**2)
```

3.580 $\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 222

$$\begin{aligned}\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx &= -\frac{1}{4}b^2x\sqrt{d+cdx}\sqrt{e-cex}+\frac{b^2\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)}{4c\sqrt{1-c^2x^2}}\\ &\quad-\frac{bcx^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{2\sqrt{1-c^2x^2}}\\ &\quad+\frac{1}{2}x\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2+\frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^3}{6bc\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-1/4*b^2*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)+1/4*b^2*(c*d*x+d)^(1/2)*(-c*e*
x+e)^(1/2)*arccos(c*x)/c/(-c^2*x^2+1)^(1/2)-1/2*b*c*x^2*(c*d*x+d)^(1/2)*(-
c*e*x+e)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+1/2*x*(c*d*x+d)^(1/2)*
(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2+1/6*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*
(a+b*arccos(c*x))^3/b/c/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.30

$$\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$
$$= \frac{-4b^2\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)^3 - 12a^2\sqrt{d}\sqrt{e}\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{e-cex}}{\sqrt{d}\sqrt{e}(-1+c^2x^2)}\right) + 6b\sqrt{d+cdx}\sqrt{e}}{}$$

input
```
Integrate[Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(-4*b^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^3 - 12*a^2*Sqrt[d]*Sqr
t[e]*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[
d]*Sqrt[e]*(-1 + c^2*x^2))] + 6*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c
*x]*(b*Cos[2*ArcCos[c*x]] + 2*a*Sin[2*ArcCos[c*x]]) + 6*b*Sqrt[d + c*d*x]*
Sqrt[e - c*e*x]*ArcCos[c*x]^2*(-2*a + b*Sin[2*ArcCos[c*x]]) + 3*Sqrt[d + c
*d*x]*Sqrt[e - c*e*x]*(4*a^2*c*x*Sqrt[1 - c^2*x^2] + 2*a*b*Cos[2*ArcCos[c*
x]] - b^2*Sin[2*ArcCos[c*x]]))/(24*c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.65, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5179, 5157, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cdx+d}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$
$$\Big\downarrow \text{5179}$$
$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\int \sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx}{\sqrt{1-c^2x^2}}$$
$$\Big\downarrow \text{5157}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+bc\int x(a+b\arccos(cx))dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\right)}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5139

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(bc\left(\frac{1}{2}bc\int\frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)+\frac{1}{2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 262

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(bc\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)+\frac{1}{2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 223

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)+\frac{1}{2}x\sqrt{1-\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5153

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\right.}{\sqrt{1-c^2x^2}}$$

input
```
Int[Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2,x]
```

output
```
(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*((x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])
^2)/2 - (a + b*ArcCos[c*x])^3/(6*b*c) + b*c*((x^2*(a + b*ArcCos[c*x]))/2 +
 (b*c*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2)))/Sqrt[1
- c^2*x^2]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 634, normalized size of antiderivative = 2.86

method	result
default	$-\frac{a^2\sqrt{cdx+d}\,(-cex+e)^{\frac{3}{2}}}{2ce}+\frac{a^2\sqrt{-cex+e}\,\sqrt{cdx+d}}{2c}+\frac{a^2de\sqrt{(-cex+e)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-c^2de\,x^2+de}}\right)}{2\sqrt{-cex+e}\,\sqrt{cdx+d}\,\sqrt{c^2de}}+b^2\left(\frac{\sqrt{d(cx+1)}\,\sqrt{}\right.$
parts	$-\frac{a^2\sqrt{cdx+d}\,(-cex+e)^{\frac{3}{2}}}{2ce}+\frac{a^2\sqrt{-cex+e}\,\sqrt{cdx+d}}{2c}+\frac{a^2de\sqrt{(-cex+e)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-c^2de\,x^2+de}}\right)}{2\sqrt{-cex+e}\,\sqrt{cdx+d}\,\sqrt{c^2de}}+b^2\left(\frac{\sqrt{d(cx+1)}\,\sqrt{}\right.$

input

```
int((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x,method=_RETURNV
ERBOSE)
```

output

```
-1/2*a^2/c/e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(3/2)+1/2*a^2/c*(-c*e*x+e)^(1/2)*(
c*d*x+d)^(1/2)+1/2*a^2*d*e*((-c*e*x+e)*(c*d*x+d))^(1/2)/(-c*e*x+e)^(1/2)/(
c*d*x+d)^(1/2)/(c^2*d*e)^(1/2)*arctan((c^2*d*e)^(1/2)*x/(-c^2*d*e*x^2+d*e)
^(1/2))+b^2*(1/6*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(
c^2*x^2-1)/c*arccos(c*x)^3+1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(2*c^
3*x^3-2*c*x+2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(2*arccos
(c*x)^2-1+2*I*arccos(c*x))/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1
))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2
*c*x)*(2*arccos(c*x)^2-1-2*I*arccos(c*x))/(c^2*x^2-1)/c)+2*a*b*(1/4*(d*(c*
x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x
)^2+1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(2*c^3*x^3-2*c*x+2*I*(-c^2*x
^2+1)^(1/2)*x^2*c^2-I*(-c^2*x^2+1)^(1/2))*(I+2*arccos(c*x))/(c^2*x^2-1)/c+
1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2
+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-I+2*arccos(c*x))/(c^2*x^2-1)/c)
```

Fricas [F]

$$\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \int \sqrt{cdx+d}\sqrt{-cex+e}(b\arccos{(cx)}+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algorith
m="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sqr
t(-c*e*x + e), x)
```

Sympy [F]

$$\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \int \sqrt{d\,(cx+1)}\sqrt{-e\,(cx-1)}(a+b\operatorname{acos}{(cx)})^2\,dx$$

input
```
integrate((c*d*x+d)**(1/2)*(-c*e*x+e)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral(sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))*(a + b*acos(c*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \int \sqrt{cdx+d}\sqrt{-cex+e}(b\arccos{(cx)}+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2,x, algorith
m="giac")
```

output
```
integrate(sqrt(c*d*x + d)*sqrt(-c*e*x + e)*(b*arccos(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx = \int (a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d+c\,d\,x}\,\sqrt{e-c\,e\,x}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\,dx$$
$$= \frac{\sqrt{e}\,\sqrt{d}\left(-2asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2+\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2cx+4\left(\int\sqrt{cx+1}\,\sqrt{-cx+1}\,acos(cx)\,dx\right)abc+2(\right.}{2c}$$

input
```
int((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(e)*sqrt(d)*( - 2*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + sqrt(c*x + 1)
*sqrt( - c*x + 1)*a**2*c*x + 4*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x
),x)*a*b*c + 2*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2,x)*b**2*c))
/(2*c)
```

3.581 $\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 432

$$
\begin{aligned}
\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x}\,dx
&= -2b^2\sqrt{d+cdx}\sqrt{e-cex} - \frac{2abcx\sqrt{d+cdx}\sqrt{e-cex}}{\sqrt{1-c^2x^2}} \\
&\quad - \frac{2b^2cx\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)}{\sqrt{1-c^2x^2}} + \sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2 \\
&\quad - \frac{2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&\quad + \frac{2ib\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&\quad - \frac{2ib\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&\quad - \frac{2b^2\sqrt{d+cdx}\sqrt{e-cex}\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&\quad + \frac{2b^2\sqrt{d+cdx}\sqrt{e-cex}\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}
\end{aligned}
$$

output
```
-2*b^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)-2*a*b*c*x*(c*d*x+d)^(1/2)*(-c*e*x+
e)^(1/2)/(-c^2*x^2+1)^(1/2)-2*b^2*c*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*arc
cos(c*x)/(-c^2*x^2+1)^(1/2)+(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c
*x))^2-2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2*arctanh(c*x+
I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*I*b*(c*d*x+d)^(1/2)*(-c*e*x+e)^
(1/2)*(a+b*arccos(c*x))*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^
(1/2)-2*I*b*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))*polylog(2,c
*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-2*b^2*(c*d*x+d)^(1/2)*(-c*e*x+
e)^(1/2)*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*b^2*(c*
d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^
2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x}\,dx = a^2\sqrt{d+cdx}\sqrt{e-cex}$$
$$+a^2\sqrt{d}\sqrt{e}\log(cx)-a^2\sqrt{d}\sqrt{e}\log\left(de+\sqrt{d}\sqrt{e}\sqrt{d+cdx}\sqrt{e-cex}\right)$$
$$+\frac{2ab\sqrt{d+cdx}\sqrt{e-cex}\left(cx+\sqrt{1-c^2x^2}\arccos(cx)-\arccos(cx)\log\left(1-ie^{i\arccos(cx)}\right)+\arccos(cx)\log\right.}{\sqrt{1-c^2x^2}}$$
$$+\frac{b^2\sqrt{d+cdx}\sqrt{e-cex}\left(-2\sqrt{1-c^2x^2}+2cx\arccos(cx)+\sqrt{1-c^2x^2}\arccos(cx)^2-\arccos(cx)^2\log\left(1-\right.\right.}{}$$

input
```
Integrate[(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
a^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x] + a^2*Sqrt[d]*Sqrt[e]*Log[c*x] - a^2*S
qrt[d]*Sqrt[e]*Log[d*e + Sqrt[d]*Sqrt[e]*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]]
 + (2*a*b*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(c*x + Sqrt[1 - c^2*x^2]*ArcCos[c
*x] - ArcCos[c*x]*Log[1 - I*E^(I*ArcCos[c*x])] + ArcCos[c*x]*Log[1 + I*E^(
I*ArcCos[c*x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + I*PolyLog[2, I*E^
(I*ArcCos[c*x])]))/Sqrt[1 - c^2*x^2] + (b^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x
]*(-2*Sqrt[1 - c^2*x^2] + 2*c*x*ArcCos[c*x] + Sqrt[1 - c^2*x^2]*ArcCos[c*x
]^2 - ArcCos[c*x]^2*Log[1 - I*E^(I*ArcCos[c*x])] + ArcCos[c*x]^2*Log[1 + I
*E^(I*ArcCos[c*x])] - (2*I)*ArcCos[c*x]*PolyLog[2, (-I)*E^(I*ArcCos[c*x])]
 + (2*I)*ArcCos[c*x]*PolyLog[2, I*E^(I*ArcCos[c*x])] + 2*PolyLog[3, (-I)*E
^(I*ArcCos[c*x])] - 2*PolyLog[3, I*E^(I*ArcCos[c*x])]))/Sqrt[1 - c^2*x^2]
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.52, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {5239, 5199, 2009, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cdx+d}\sqrt{e-cex}(a+b\arccos(cx))^2}{x}dx$$

$$\Big\downarrow \text{5239}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5199}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx+2bc\int (a+b\arccos(cx))dx+\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx+\sqrt{1-c^2x^2}(a+b\arccos(cx))^2+2bc\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5219}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(-\int \frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)+\sqrt{1-c^2x^2}(a+b\arccos(cx))^2+2bc\left(ax+bx\arccos(cx)-\right.\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(-\int (a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)+\sqrt{1-c^2x^2}(a+b\arccos(cx))^2+\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{4669}$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(2b\int (a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)-2b\int (a+b\arccos(cx))\log\left(1+ie\right.\right.}{}$$

$\downarrow$ 3011

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\Big(-2b\big(i\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\arccos$$

$\downarrow$ 2720

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\Big(-2b\big(i\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-ie^{i\arccos(}$$

$\downarrow$ 7143

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\Big(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+\sqrt{1-c^2x^2}(a+b\arccos(cx))^2+2bc\Big(ax+bx\operatorname{arc}$$

input
```
Int[(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2
+ 2*b*c*(a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]) + (2*I)*(a + b*A
rcCos[c*x])^2*ArcTan[E^(I*ArcCos[c*x])] - 2*b*(I*(a + b*ArcCos[c*x])*PolyL
og[2, (-I)*E^(I*ArcCos[c*x])] - b*PolyLog[3, (-I)*E^(I*ArcCos[c*x])]) + 2*
b*(I*(a + b*ArcCos[c*x])*PolyLog[2, I*E^(I*ArcCos[c*x])] - b*PolyLog[3, I*
E^(I*ArcCos[c*x])])))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 5.21 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.78

method	result
default	$\frac{a^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\left(-de\ln\left(\frac{2\sqrt{de}\,\sqrt{-de\left(c^2x^2-1\right)}+2de}{x}\right)+\sqrt{de}\,\sqrt{-de(c^2x^2-1)}\right)}{\sqrt{de}\,\sqrt{-de(c^2x^2-1)}}+b^2\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\left(i\sqrt{-c\right.}}{}\right.$
parts	$\frac{a^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\left(-de\ln\left(\frac{2\sqrt{de}\,\sqrt{-de\left(c^2x^2-1\right)}+2de}{x}\right)+\sqrt{de}\,\sqrt{-de(c^2x^2-1)}\right)}{\sqrt{de}\,\sqrt{-de(c^2x^2-1)}}+b^2\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\left(i\sqrt{-c\right.}}{}\right.$

input
```
int((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/x,x,method=_RETUR
NVERBOSE)
```

output
```
a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-d*e*ln(2*((d*e)^(1/2)*(-d*e*(c^
2*x^2-1))^(1/2)+d*e)/x)+(d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2))/(d*e)^(1/2)/
(-d*e*(c^2*x^2-1))^(1/2)+b^2*(1/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*
(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)^2-2+2*I*arccos(c*x))/(c^2*x
^2-1)+1/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+
c^2*x^2-1)*(arccos(c*x)^2-2-2*I*arccos(c*x))/(c^2*x^2-1)+(d*(c*x+1))^(1/2)
*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(-arccos(c*x)^2*ln(1+I*(c*x+I*(-c^2
*x^2+1)^(1/2)))+arccos(c*x)^2*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*polylog
(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*I*arccos(c*x)*polylog(2,-I*(c*x+I*(-c^2
*x^2+1)^(1/2)))-2*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*I*arccos(c*x)
*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2))))/(c^2*x^2-1))+2*a*b*(1/2*(d*(c*x+
1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(
c*x)+I)/(c^2*x^2-1)+1/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2
+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)-I)/(c^2*x^2-1)-(d*(c*x+1))^(1/2)*(-e
*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)
^(1/2)))-arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-I*polylog(2,-I*(c*
x+I*(-c^2*x^2+1)^(1/2)))+I*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))))/(c^2*x
^2-1))
```

Fricas [F]

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x}\,dx = \int \frac{\sqrt{cdx+d}\sqrt{-cex+e}(b\arccos(cx)+a)^2}{x}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/x,x, algori
thm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sqr
t(-c*e*x + e)/x, x)
```

Sympy [F]

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x}\,dx = \int \frac{\sqrt{d(cx+1)}\sqrt{-e(cx-1)}(a+b\operatorname{acos}(cx))^2}{x}\,dx$$

input
```
integrate((c*d*x+d)**(1/2)*(-c*e*x+e)**(1/2)*(a+b*acos(c*x))**2/x,x)
```

output
```
Integral(sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))*(a + b*acos(c*x))**2/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/x,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x}\,dx = \int \frac{\sqrt{cdx+d}\sqrt{-cex+e}(b\arccos(cx)+a)^2}{x}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/x,x, algori
thm="giac")
```

output
```
integrate(sqrt(c*d*x + d)*sqrt(-c*e*x + e)*(b*arccos(c*x) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d+c\,d\,x}\,\sqrt{e-c\,e\,x}}{x}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2))/x,x)
```

output
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2))/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x}\,dx$$
$$= \sqrt{e}\,\sqrt{d}\left(\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2+2\left(\int \frac{\sqrt{cx+1}\,\sqrt{-cx+1}\,acos(cx)}{x}dx\right)ab + \left(\int \frac{\sqrt{cx+1}\,\sqrt{-cx+1}\,acos(cx)^2}{x}dx\right)b^2 - \log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)}{2}\right)-1\right)a^2 + \log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)}{2}\right)+1\right)a^2 - \log\left(\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)}{2}\right)-1\right)a^2 + \log\left(\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)}{2}\right)+1\right)a^2\right)$$

input
```
int((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*acos(c*x))^2/x,x)
```

output
```
sqrt(e)*sqrt(d)*(sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2 + 2*int((sqrt(c*x + 1
)*sqrt( - c*x + 1)*acos(c*x))/x,x)*a*b + int((sqrt(c*x + 1)*sqrt( - c*x +
1)*acos(c*x)**2)/x,x)*b**2 - log( - sqrt(2) + tan(asin(sqrt( - c*x + 1)/sq
rt(2))/2) - 1)*a**2 + log( - sqrt(2) + tan(asin(sqrt( - c*x + 1)/sqrt(2))/
2) + 1)*a**2 - log(sqrt(2) + tan(asin(sqrt( - c*x + 1)/sqrt(2))/2) - 1)*a*
*2 + log(sqrt(2) + tan(asin(sqrt( - c*x + 1)/sqrt(2))/2) + 1)*a**2)
```

3.582 $\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 257

$$\begin{aligned}\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x^2}\,dx &= -\frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x} - \frac{ic\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}} \\ &\quad - \frac{c\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^3}{3b\sqrt{1-c^2x^2}} \\ &\quad + \frac{2bc\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\ &\quad - \frac{ib^2c\sqrt{d+cdx}\sqrt{e-cex}\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/x-I*c*(c*d*x+d)^(1/2
)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/(-c^2*x^2+1)^(1/2)-1/3*c*(c*d*x+d)^
(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^3/b/(-c^2*x^2+1)^(1/2)+2*b*c*(c*d
*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1
/2))^2)/(-c^2*x^2+1)^(1/2)-I*b^2*c*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*polylo
g(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x^2}\,dx$$
$$= -\frac{a^2\sqrt{-e(-1+cx)}\sqrt{d(1+cx)}}{x} + a^2c\sqrt{d}\sqrt{e}\arctan\left(\frac{cx\sqrt{-e(-1+cx)}\sqrt{d(1+cx)}}{\sqrt{d}\sqrt{e}(-1+cx)(1+cx)}\right)$$
$$-\frac{abc\sqrt{d+cdx}\sqrt{e-cex}\sqrt{-de(1-c^2x^2)}\left(\frac{2\arccos(cx)}{cx}-\frac{\arccos(cx)^2-2\log(cx)}{\sqrt{1-c^2x^2}}\right)}{\sqrt{(-d-cdx)(e-cex)}}$$
$$+\frac{b^2c\sqrt{d+cdx}\sqrt{e-cex}\sqrt{-de(1-c^2x^2)}\left(\arccos(cx)\left(-\frac{3\arccos(cx)}{cx}+\frac{\arccos(cx)(3i+\arccos(cx))-6\log(1+e^{2i\arccos(}}{\sqrt{1-c^2x^2}}\right.\right.}{3\sqrt{(-d-cdx)(e-cex)}}$$

input
```
Integrate[(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2)/x^2,x]
```

output
```
-((a^2*Sqrt[-(e*(-1 + c*x))]*Sqrt[d*(1 + c*x)])/x) + a^2*c*Sqrt[d]*Sqrt[e]
*ArcTan[(c*x*Sqrt[-(e*(-1 + c*x))]*Sqrt[d*(1 + c*x)])/(Sqrt[d]*Sqrt[e]*(-1
 + c*x)*(1 + c*x))] - (a*b*c*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[-(d*e*(1
 - c^2*x^2))]*((2*ArcCos[c*x])/(c*x) - (ArcCos[c*x]^2 - 2*Log[c*x])/Sqrt[1
 - c^2*x^2]))/Sqrt[(-d - c*d*x)*(e - c*e*x)] + (b^2*c*Sqrt[d + c*d*x]*Sqrt
[e - c*e*x]*Sqrt[-(d*e*(1 - c^2*x^2))]*(ArcCos[c*x]*((-3*ArcCos[c*x])/(c*x
) + (ArcCos[c*x]*(3*I + ArcCos[c*x]) - 6*Log[1 + E^((2*I)*ArcCos[c*x])])/S
qrt[1 - c^2*x^2]) + ((3*I)*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/Sqrt[1 - c^
2*x^2]))/(3*Sqrt[(-d - c*d*x)*(e - c*e*x)])
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.62, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {5239, 5197, 5137, 3042, 4202, 2620, 2715, 2838, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cdx+d}\sqrt{e-cex}(a+b\arccos(cx))^2}{x^2}\,dx$$

$$\Big\downarrow \; 5239$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x^2}dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 5197$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(c^2\left(-\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx\right)-2bc\int \frac{a+b\arccos(cx)}{x}dx-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 5137$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(c^2\left(-\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx\right)+2bc\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(c^2\left(-\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx\right)+2bc\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)-\frac{\sqrt{1-c^2x^2}(\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 4202$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(c^2\left(-\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx\right)+2bc\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)\right.\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 2620$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(c^2\left(-\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx\right)+2bc\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(c\right.\right.\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 2715$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(c^2\left(-\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx\right)+2bc\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx}\right.\right.\right.\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 2838$$

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(c^2\left(-\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx\right)-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}+2bc\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+\right.\right.\right.\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5153

$$\frac{\sqrt{cdx+d}\sqrt{e-cex}\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}+2bc\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(\right.\right.\right.}{\sqrt{1-c^2x^2}}$$

input
```
Int[(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(a + b*ArcCos[c*x])^2)/x^2,x]
```

output
```
(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-((Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])
^2)/x) + (c*(a + b*ArcCos[c*x])^3)/(3*b) + 2*b*c*(((I/2)*(a + b*ArcCos[c*x
])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCos[c*x]
)] - (b*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/4))))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5197
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (Simp[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^2
]/Sqrt[1 - c^2*x^2]]   Int[(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x
] + Simp[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 2)*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 583 vs. 2(247) = 494.

Time = 5.10 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.27

method	result
default	$\frac{a^2\left(-\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-de\left(c^2x^2-1\right)}}\right)x\,c^2de-\sqrt{c^2de}\,\sqrt{-de(c^2x^2-1)}\right)\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}}{\sqrt{-de(c^2x^2-1)}\,x\sqrt{c^2de}}+b^2\left(-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-}}{3(c^2x^2-}\right.$
parts	$\frac{a^2\left(-\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-de\left(c^2x^2-1\right)}}\right)x\,c^2de-\sqrt{c^2de}\,\sqrt{-de(c^2x^2-1)}\right)\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}}{\sqrt{-de(c^2x^2-1)}\,x\sqrt{c^2de}}+b^2\left(-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-}}{3(c^2x^2-}\right.$

input
```
int((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/x^2,x,method=_RET
URNVERBOSE)
```

output
```
a^2*(-arctan((c^2*d*e)^(1/2)*x/(-d*e*(c^2*x^2-1))^(1/2))*x*c^2*d*e-(c^2*d*
e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2))*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/(-
d*e*(c^2*x^2-1))^(1/2)/x/(c^2*d*e)^(1/2)+b^2*(-1/3*(d*(c*x+1))^(1/2)*(-e*(
c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)*arccos(c*x)^3*c/(c^2*x^2-1)-(d*(c*x+1))^(
1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*arccos(c*x)^
2/x/(c^2*x^2-1)-I*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)*
(2*I*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+2*arccos(c*x)^2+polylo
g(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2))*c/(c^2*x^2-1))+2*a*b*(-1/2*(d*(c*x+1))
^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)*arccos(c*x)^2*c/(c^2*x^2-1)-2
*I*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c/(
c^2*x^2-1)-(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c
+c^2*x^2-1)*arccos(c*x)/x/(c^2*x^2-1)+(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)
*(-c^2*x^2+1)^(1/2)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*c/(c^2*x^2-1))
```

Fricas [F]

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{\sqrt{cdx+d}\sqrt{-cex+e}(b\arccos(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/x^2,x, algo
rithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sqr
t(-c*e*x + e)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{\sqrt{d(cx+1)}\sqrt{-e(cx-1)}(a+b\operatorname{acos}(cx))^2}{x^2}\,dx$$

input
```
integrate((c*d*x+d)**(1/2)*(-c*e*x+e)**(1/2)*(a+b*acos(c*x))**2/x**2,x)
```

output
```
Integral(sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))*(a + b*acos(c*x))**2/x**2, x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/x^2,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{\sqrt{cdx+d}\sqrt{-cex+e}(b\arccos(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/x^2,x, algo
rithm="giac")
```

output
```
integrate(sqrt(c*d*x + d)*sqrt(-c*e*x + e)*(b*arccos(c*x) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d+c\,d\,x}\,\sqrt{e-c\,e\,x}}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2))/x^2,x)
```

output
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{x^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\left(2asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2cx-\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2+2\left(\int \frac{\sqrt{cx+1}\,\sqrt{-cx+1}\,acos(cx)}{x^2}dx\right)abx+\left(\int \frac{\sqrt{cx+1}\,\sqrt{-}}{}\right.\right.}{x}$$

input
```
int((c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*acos(c*x))^2/x^2,x)
```

output
```
(sqrt(e)*sqrt(d)*(2*asin(sqrt( - c*x + 1)/sqrt(2))*a**2*c*x - sqrt(c*x + 1
)*sqrt( - c*x + 1)*a**2 + 2*int((sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x))
/x**2,x)*a*b*x + int((sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2)/x**2,x)
*b**2*x))/x
```

3.583 $\int x^2(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 509

$$\begin{aligned}\int x^2(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = & -\frac{7b^2dex\sqrt{d+cdx}\sqrt{e-cex}}{1152c^2}-\frac{43b^2dex^3\sqrt{d+cdx}\sqrt{e-cex}}{1728} \\ & +\frac{1}{108}b^2c^2dex^5\sqrt{d+cdx}\sqrt{e-cex}+\frac{7b^2de\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)}{1152c^3\sqrt{1-c^2x^2}} \\ & +\frac{bdex^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{16c\sqrt{1-c^2x^2}} \\ & -\frac{7bcdex^4\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{48\sqrt{1-c^2x^2}} \\ & +\frac{bc^3dex^6\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{18\sqrt{1-c^2x^2}} \\ & -\frac{dex\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{16c^2} \\ & +\frac{1}{8}dex^3\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2 \\ & +\frac{1}{6}dex^3\sqrt{d+cdx}\sqrt{e-cex}\left(1-c^2x^2\right)(a+b\arccos(cx))^2 \\ & +\frac{de\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^3}{48bc^3\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
-7/1152*b^2*d*e*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)/c^2-43/1728*b^2*d*e*x^3
*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)+1/108*b^2*c^2*d*e*x^5*(c*d*x+d)^(1/2)*(-
c*e*x+e)^(1/2)+7/1152*b^2*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*arccos(c*x)
/c^3/(-c^2*x^2+1)^(1/2)+1/16*b*d*e*x^2*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a
+b*arccos(c*x))/c/(-c^2*x^2+1)^(1/2)-7/48*b*c*d*e*x^4*(c*d*x+d)^(1/2)*(-c*
e*x+e)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+1/18*b*c^3*d*e*x^6*(c*d*
x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-1/16*d*e*
x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/c^2+1/8*d*e*x^3*(c*
d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2+1/6*d*e*x^3*(c*d*x+d)^(1
/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)*(a+b*arccos(c*x))^2+1/48*d*e*(c*d*x+d)^(
1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^3/b/c^3/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 452, normalized size of antiderivative = 0.89

$$\int x^2(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \frac{-288b^2de\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)^3-864a^2d^{3/2}e^{3/2}\sqrt{1-c^2x^2}\,\text{arcta}}{}$$

input
```
Integrate[x^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(-288*b^2*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^3 - 864*a^2*d^(3
/2)*e^(3/2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])
/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] + 12*b*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e
*x]*ArcCos[c*x]*(18*b*Cos[2*ArcCos[c*x]] + 9*b*Cos[4*ArcCos[c*x]] - 2*b*Co
s[6*ArcCos[c*x]] + 36*a*Sin[2*ArcCos[c*x]] + 36*a*Sin[4*ArcCos[c*x]] - 12*
a*Sin[6*ArcCos[c*x]]) - 72*b*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*
x]^2*(12*a - 3*b*Sin[2*ArcCos[c*x]] - 3*b*Sin[4*ArcCos[c*x]] + b*Sin[6*Arc
Cos[c*x]]) + d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-864*a^2*c*x*Sqrt[1 - c^
2*x^2] + 4032*a^2*c^3*x^3*Sqrt[1 - c^2*x^2] - 2304*a^2*c^5*x^5*Sqrt[1 - c^
2*x^2] + 216*a*b*Cos[2*ArcCos[c*x]] + 108*a*b*Cos[4*ArcCos[c*x]] - 24*a*b*
Cos[6*ArcCos[c*x]] - 108*b^2*Sin[2*ArcCos[c*x]] - 27*b^2*Sin[4*ArcCos[c*x]
] + 4*b^2*Sin[6*ArcCos[c*x]]))/(13824*c^3*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 2.65 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.90, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {5239, 5203, 5193, 27, 363, 262, 262, 223, 5199, 5139, 262, 262, 223, 5211, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(cdx+d)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5239}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\int x^2\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5203}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\int x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx+\frac{1}{3}bc\int x^3\left(1-c^2x^2\right)(a+b\arccos(cx))dx+\frac{1}{6}x^3(1\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5193}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\int x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx+\frac{1}{3}bc\left(bc\int\frac{x^4(3-2c^2x^2)}{12\sqrt{1-c^2x^2}}dx-\frac{1}{6}c^2x^6(a+b\arccos(cx)\right.\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\int x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx+\frac{1}{3}bc\left(\frac{1}{12}bc\int\frac{x^4(3-2c^2x^2)}{\sqrt{1-c^2x^2}}dx-\frac{1}{6}c^2x^6(a+b\arccos(c\right.\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{363}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\int x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx+\frac{1}{3}bc\left(\frac{1}{12}bc\left(\frac{4}{3}\int\frac{x^4}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}x^5\sqrt{1-c^2x^2}\right)-\frac{1}{6}\right.\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\int x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx+\frac{1}{3}bc\left(\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\int\frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{3}x\right.\right.\right.}{\sqrt{1-c}}$$

$\downarrow$ 262

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\int x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx+\frac{1}{3}bc\left(\frac{1}{12}bc\left(\frac{4}{3}\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1}}{4}\right.\right.\right.\right.}{}$$

$\downarrow$ 223

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\int x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx+\frac{1}{3}bc\left(-\frac{1}{6}c^2x^6(a+b\arccos(cx))+\frac{1}{4}x^4(a+b\operatorname{arc}\right.\right.}{\sqrt{}}$$

$\downarrow$ 5199

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\left(\frac{1}{4}\int\frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\int x^3(a+b\arccos(cx))dx+\frac{1}{4}x^3\sqrt{1-c^2x^2}(a+b\arccos(c\right.\right.}{}$$

$\downarrow$ 5139

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\left(\frac{1}{4}\int\frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\left(\frac{1}{4}bc\int\frac{x^4}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}x^4(a+b\arccos(cx))\right)+\frac{1}{4}x^3\sqrt{1-}\right.\right.}{}$$

$\downarrow$ 262

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\left(\frac{1}{4}\int\frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\left(\frac{1}{4}bc\left(\frac{3\int\frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{4}x^4(a+b\arccos(c\right.\right.\right.}{}$$

$\downarrow$ 262

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\left(\frac{1}{4}\int\frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\left(\frac{1}{4}bc\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)+\frac{1}{4}x^4(a\right.\right.\right.$$

$\downarrow$ 223

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\left(\frac{1}{4}\int\frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}bc\left(\frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\right.\right.\right.\right.$$

$\downarrow$ 5211

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int x(a+b\arccos(cx))dx}{c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)+\frac{1}{2}bc\left(\frac{1}{4}x^4(a\right.\right.\right.$$

$\downarrow$ 5139

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\left(\frac{1}{4}\left(-\frac{b\left(\frac{1}{2}bc\int\frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c}+\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}\right.\right.\right.$$

$\downarrow$ 262

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\left(\frac{1}{4}\left(-\frac{b\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c}+\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{}\right.\right.\right.$$

$\downarrow$ 223

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c}-\frac{x\sqrt{1-c^2x^2}(a+b\text{ ar}}{2c^2}\right.\right.\right.$$

$\downarrow$ 5153

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{1}{2}\left(\frac{1}{4}\left(-\frac{b\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c}-\frac{(a+b\arccos(cx))^3}{6bc^3}-\frac{x\sqrt{1-c^2x^2}(a+b\arc$$

input
```
Int[x^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*((x^3*(1 - c^2*x^2)^(3/2)*(a + b*ArcC
os[c*x])^2)/6 + (b*c*((x^4*(a + b*ArcCos[c*x]))/4 - (c^2*x^6*(a + b*ArcCos
[c*x]))/6 + (b*c*((x^5*Sqrt[1 - c^2*x^2])/3 + (4*(-1/4*(x^3*Sqrt[1 - c^2*x
^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c
^2)))/3))/12))/3 + ((x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/4 + (b*c
*((x^4*(a + b*ArcCos[c*x]))/4 + (b*c*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (
3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/4))/2
+ (-1/2*(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c^2 - (a + b*ArcCos[c*
x])^3/(6*b*c^3) - (b*((x^2*(a + b*ArcCos[c*x]))/2 + (b*c*(-1/2*(x*Sqrt[1 -
 c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2))/c)/4)/2))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5193
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp[
(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c
^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && IGtQ[p, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 29.80 (sec) , antiderivative size = 1458, normalized size of antiderivative = 2.86

method	result	size
default	Expression too large to display	1458
parts	Expression too large to display	1458

input
```
int(x^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/48*a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*d*e*(-8*x^5*c^4*(-d*e*(c^2*x
^2-1))^(1/2)*(c^2*d*e)^(1/2)+14*x^3*c^2*(-d*e*(c^2*x^2-1))^(1/2)*(c^2*d*e)
^(1/2)+3*arctan((c^2*d*e)^(1/2)*x/(-d*e*(c^2*x^2-1))^(1/2))*d*e-3*(c^2*d*e
)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2)*x)/c^2/(-d*e*(c^2*x^2-1))^(1/2)/(c^2*d*e)
^(1/2)+b^2*(1/48*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c
^3/(c^2*x^2-1)*arccos(c*x)^3*d*e-1/6912*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/
2)*(32*c^7*x^7-64*c^5*x^5+32*I*(-c^2*x^2+1)^(1/2)*x^6*c^6+38*c^3*x^3-48*I*
(-c^2*x^2+1)^(1/2)*x^4*c^4-6*c*x+18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-I*(-c^2*x
^2+1)^(1/2))*(6*I*arccos(c*x)+18*arccos(c*x)^2-1)*d*e/c^3/(c^2*x^2-1)+1/25
6*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*
c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(2*arccos(c*x)^2-1-2*I*arccos(c*x))*d*
e/c^3/(c^2*x^2-1)-1/27648*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x
^2+1)^(1/2)*x*c+c^2*x^2-1)*(84*I*arccos(c*x)+288*arccos(c*x)^2-31)*cos(5*a
rccos(c*x))*d*e/c^3/(c^2*x^2-1)-1/27648*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/
2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(132*I*arccos(c*x)+144*arccos(c*x)
^2-23)*sin(5*arccos(c*x))*d*e/c^3/(c^2*x^2-1)-3/1024*(d*(c*x+1))^(1/2)*(-e
*(c*x-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(I+4*arccos(c*x))*cos
(3*arccos(c*x))*d*e/c^3/(c^2*x^2-1)-1/1024*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^
(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(4*I*arccos(c*x)+16*arccos(c*x)
^2-5)*sin(3*arccos(c*x))*d*e/c^3/(c^2*x^2-1))+2*a*b*(1/32*(d*(c*x+1))^(...
```

Fricas [F]

$$\int x^2(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\int(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algo
rithm="fricas")
```

output
```
integral(-(a^2*c^2*d*e*x^4 - a^2*d*e*x^2 + (b^2*c^2*d*e*x^4 - b^2*d*e*x^2)
*arccos(c*x)^2 + 2*(a*b*c^2*d*e*x^4 - a*b*d*e*x^2)*arccos(c*x))*sqrt(c*d*x
 + d)*sqrt(-c*e*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 (d + cdx)^{3/2} (e - cex)^{3/2} (a + b \arccos(cx))^2 \, dx = \text{Timed out}$$

input
```
integrate(x**2*(c*d*x+d)**(3/2)*(-c*e*x+e)**(3/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 (d + cdx)^{3/2} (e - cex)^{3/2} (a + b \arccos(cx))^2 \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int x^2 (d + cdx)^{3/2} (e - cex)^{3/2} (a + b \arccos(cx))^2 \, dx = \int (cdx + d)^{\frac{3}{2}} (-cex + e)^{\frac{3}{2}} (b \arccos(cx) + a)^2 x^2 \, dx$$

input
```
integrate(x^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algo
rithm="giac")
```

output
```
integrate((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)*(b*arccos(c*x) + a)^2*x^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\int x^2\,(a+b\,\mathrm{acos}(c\,x))^2\,(d+c\,d\,x)^{3/2}\,(e-c\,e\,x)^{3/2}\,dx$$

input
```
int(x^2*(a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2),x)
```

output
```
int(x^2*(a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2), x)
```

Reduce [F]

$$\int x^2(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{\sqrt{e}\,\sqrt{d}\,de\left(-6asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2-8\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^5x^5+14\sqrt{cx+}\right.}{}$$

input
```
int(x^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(e)*sqrt(d)*d*e*( - 6*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - 8*sqrt(c*
x + 1)*sqrt( - c*x + 1)*a**2*c**5*x**5 + 14*sqrt(c*x + 1)*sqrt( - c*x + 1)
*a**2*c**3*x**3 - 3*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c*x - 96*int(sqrt(
c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**4,x)*a*b*c**5 + 96*int(sqrt(c*x + 1
)*sqrt( - c*x + 1)*acos(c*x)*x**2,x)*a*b*c**3 - 48*int(sqrt(c*x + 1)*sqrt(
 - c*x + 1)*acos(c*x)**2*x**4,x)*b**2*c**5 + 48*int(sqrt(c*x + 1)*sqrt( -
c*x + 1)*acos(c*x)**2*x**2,x)*b**2*c**3))/(48*c**3)
```

3.584 $\int x(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 33, antiderivative size = 338

$$\int x(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{16b^2de\sqrt{d+cdx}\sqrt{e-cex}}{75c^2}+\frac{8b^2de\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)}{225c^2}+\frac{2b^2de\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)^2}{125c^2}+\frac{2bdex\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{5c\sqrt{1-c^2x^2}}-\frac{4bcdex^3\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{15\sqrt{1-c^2x^2}}+\frac{2bc^3dex^5\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{25\sqrt{1-c^2x^2}}-\frac{de\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)^2(a+b\arccos(cx))^2}{5c^2}$$

output

```
16/75*b^2*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)/c^2+8/225*b^2*d*e*(c*d*x+d)
^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)/c^2+2/125*b^2*d*e*(c*d*x+d)^(1/2)*(-c
*e*x+e)^(1/2)*(-c^2*x^2+1)^2/c^2+2/5*b*d*e*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1
/2)*(a+b*arccos(c*x))/c/(-c^2*x^2+1)^(1/2)-4/15*b*c*d*e*x^3*(c*d*x+d)^(1/2
)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+2/25*b*c^3*d*e*x^5
*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)-1/5
*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)^2*(a+b*arccos(c*x))^2/c
^2
```

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.61

$$\int x(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx =$$

$$-\frac{de\sqrt{d+cdx}\sqrt{e-cex}\left(225a^2(-1+c^2x^2)^3-30abcx\sqrt{1-c^2x^2}(15-10c^2x^2+3c^4x^4)+2b^2(149-187c^2x\right.}{}$$

input
```
Integrate[x*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/1125*(d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(225*a^2*(-1 + c^2*x^2)^3 - 3
0*a*b*c*x*Sqrt[1 - c^2*x^2]*(15 - 10*c^2*x^2 + 3*c^4*x^4) + 2*b^2*(149 - 1
87*c^2*x^2 + 47*c^4*x^4 - 9*c^6*x^6) - 30*b*(-15*a*(-1 + c^2*x^2)^3 + b*c*
x*Sqrt[1 - c^2*x^2]*(15 - 10*c^2*x^2 + 3*c^4*x^4))*ArcCos[c*x] + 225*b^2*(
-1 + c^2*x^2)^3*ArcCos[c*x]^2))/(c^2*(-1 + c^2*x^2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.57, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {5239, 5183, 5155, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(cdx+d)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5239}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\int x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5183}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(-\frac{2b\int\left(1-c^2x^2\right)^2(a+b\arccos(cx))dx}{5c}-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))^2}{5c^2}\right)}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5155

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(-\frac{2b\left(bc\int \frac{x\left(3c^4x^4-10c^2x^2+15\right)}{15\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{5c}-\frac{(1-}{}\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 27

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(-\frac{2b\left(\frac{1}{15}bc\int \frac{x\left(3c^4x^4-10c^2x^2+15\right)}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{5c}-\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 1576

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(-\frac{2b\left(\frac{1}{30}bc\int \frac{3c^4x^4-10c^2x^2+15}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))\right)}{5c}-\frac{(1-}{}\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 1140

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(-\frac{2b\left(\frac{1}{30}bc\int \left(3\left(1-c^2x^2\right)^{3/2}+4\sqrt{1-c^2x^2}+\frac{8}{\sqrt{1-c^2x^2}}\right)dx^2+\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a\right.}{5c}\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 2009

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(-\frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))^2}{5c^2}-\frac{2b\left(\frac{1}{5}c^4x^5(a+b\arccos(cx))-\frac{2}{3}c^2x^3(a+b\arccos(cx))+x(a+b\arccos(cx))+\frac{1}{30}\right.}{5c}\right.}{\sqrt{1-c^2x^2}}$$

input
```
Int[x*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-1/5*((1 - c^2*x^2)^(5/2)*(a + b*Arc
Cos[c*x])^2)/c^2 - (2*b*((b*c*((-16*Sqrt[1 - c^2*x^2])/c^2 - (8*(1 - c^2*x
^2)^(3/2))/(3*c^2) - (6*(1 - c^2*x^2)^(5/2))/(5*c^2)))/30 + x*(a + b*ArcCo
s[c*x]) - (2*c^2*x^3*(a + b*ArcCos[c*x]))/3 + (c^4*x^5*(a + b*ArcCos[c*x])
)/5))/(5*c)))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(290) = 580$.

Time = 27.66 (sec) , antiderivative size = 679, normalized size of antiderivative = 2.01

method	result
orering	$\frac{(549c^8x^8-1982c^6x^6+4355c^4x^4-1420c^2x^2+298)(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}(a+b\arccos(cx))^2}{1125c^4x^2(cx-1)^2(cx+1)^2} - \frac{2(54c^6x^6-217c^4x^4+672c^2x^2-149)}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/1125*(549*c^8*x^8-1982*c^6*x^6+4355*c^4*x^4-1420*c^2*x^2+298)/c^4/x^2/(c
*x-1)^2/(c*x+1)^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2-2/1
125*(54*c^6*x^6-217*c^4*x^4+672*c^2*x^2-149)/c^4/x^2/(c*x-1)/(c*x+1)*((c*d
*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2+3/2*x*(c*d*x+d)^(1/2)*(-c
*e*x+e)^(3/2)*(a+b*arccos(c*x))^2*d*c-3/2*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(1/
2)*(a+b*arccos(c*x))^2*c*e-2*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcco
s(c*x))*b*c/(-c^2*x^2+1)^(1/2))+1/1125*(9*c^4*x^4-38*c^2*x^2+149)/c^4/x*(3
*(c*d*x+d)^(1/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2*d*c-3*(c*d*x+d)^(3/2
)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2*c*e-4*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3
/2)*(a+b*arccos(c*x))*b*c/(-c^2*x^2+1)^(1/2)+3/4*x/(c*d*x+d)^(1/2)*(-c*e*x
+e)^(3/2)*(a+b*arccos(c*x))^2*d^2*c^2-9/2*c^2*d*e*x*(c*d*x+d)^(1/2)*(-c*e*
x+e)^(1/2)*(a+b*arccos(c*x))^2-6*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(3/2)*(a+b*a
rccos(c*x))*d*c^2*b/(-c^2*x^2+1)^(1/2)+3/4*x*(c*d*x+d)^(3/2)/(-c*e*x+e)^(1
/2)*(a+b*arccos(c*x))^2*c^2*e^2+6*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(1/2)*(a+b*
arccos(c*x))*c^2*e*b/(-c^2*x^2+1)^(1/2)+2*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/
2)*b^2*c^2/(-c^2*x^2+1)-2*x^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos
(c*x))*b*c^3/(-c^2*x^2+1)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.89

$$\int x(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx =$$

$$-\frac{(9\,(25\,a^2-2\,b^2)c^6dex^6-(675\,a^2-94\,b^2)c^4dex^4+(675\,a^2-374\,b^2)c^2dex^2-(225\,a^2-298\,b^2)de+225\,(}{}$$

input
```
integrate(x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algori
thm="fricas")
```

output
```
-1/1125*(9*(25*a^2 - 2*b^2)*c^6*d*e*x^6 - (675*a^2 - 94*b^2)*c^4*d*e*x^4 +
 (675*a^2 - 374*b^2)*c^2*d*e*x^2 - (225*a^2 - 298*b^2)*d*e + 225*(b^2*c^6*
d*e*x^6 - 3*b^2*c^4*d*e*x^4 + 3*b^2*c^2*d*e*x^2 - b^2*d*e)*arccos(c*x)^2 +
 450*(a*b*c^6*d*e*x^6 - 3*a*b*c^4*d*e*x^4 + 3*a*b*c^2*d*e*x^2 - a*b*d*e)*a
rccos(c*x) - 30*(3*a*b*c^5*d*e*x^5 - 10*a*b*c^3*d*e*x^3 + 15*a*b*c*d*e*x +
 (3*b^2*c^5*d*e*x^5 - 10*b^2*c^3*d*e*x^3 + 15*b^2*c*d*e*x)*arccos(c*x))*sq
rt(-c^2*x^2 + 1))*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^4*x^2 - c^2)
```

Sympy [F(-1)]

Timed out.

$$\int x(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\text{Timed out}$$

input
```
integrate(x*(c*d*x+d)**(3/2)*(-c*e*x+e)**(3/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int x(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\int(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2x\,dx$$

input
```
integrate(x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algori
thm="giac")
```

output
```
integrate((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)*(b*arccos(c*x) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\int x\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2\,{\left(d+c\,d\,x\right)}^{3/2}\,{\left(e-c\,e\,x\right)}^{3/2}\,dx$$

input
```
int(x*(a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2),x)
```

output
```
int(x*(a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2), x)
```

Reduce [F]

$$\int x(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{\sqrt{e}\,\sqrt{d}\,de\left(-\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^4x^4+2\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^2x^2-\sqrt{}\right.}{}$$

input
```
int(x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(e)*sqrt(d)*d*e*( - sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c**4*x**4 + 2
*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c**2*x**2 - sqrt(c*x + 1)*sqrt( - c*x
 + 1)*a**2 - 10*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**3,x)*a*b*c
**4 + 10*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x,x)*a*b*c**2 - 5*in
t(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2*x**3,x)*b**2*c**4 + 5*int(sq
rt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2*x,x)*b**2*c**2))/(5*c**2)
```

3.585 $\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 362

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx =$$

$$-\frac{1}{32}b^2x(d+cdx)^{3/2}(e-cex)^{3/2}-\frac{15b^2x(d+cdx)^{3/2}(e-cex)^{3/2}}{64(1-c^2x^2)}+\frac{9b^2(d+cdx)^{3/2}(e-cex)^{3/2}\arccos(cx)}{64c(1-c^2x^2)^{3/2}}-$$

output
```
-1/32*b^2*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)-15*b^2*x*(c*d*x+d)^(3/2)*(-c*
e*x+e)^(3/2)/(-64*c^2*x^2+64)+9/64*b^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*ar
ccos(c*x)/c/(-c^2*x^2+1)^(3/2)-3/8*b*c*x^2*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2
)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(3/2)+1/8*b*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3
/2)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+1/4*x*(c*d*x+d)^(3/2)*(-c*e*x+e
)^(3/2)*(a+b*arccos(c*x))^2+3*x*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arcc
os(c*x))^2/(-8*c^2*x^2+8)+1/8*(c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos
(c*x))^3/b/c/(-c^2*x^2+1)^(3/2)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.03

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \frac{-32b^2de\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)^3-96a^2d^{3/2}e^{3/2}\sqrt{1-c^2x^2}\arctan\left(\cdots\right.}{\cdots}$$

input
```
Integrate[(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(-32*b^2*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^3 - 96*a^2*d^(3/2
)*e^(3/2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(
Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))] - 8*b*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]
*ArcCos[c*x]^2*(12*a - 8*b*Sin[2*ArcCos[c*x]] + b*Sin[4*ArcCos[c*x]]) + d*
e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(160*a^2*c*x*Sqrt[1 - c^2*x^2] - 64*a^2*
c^3*x^3*Sqrt[1 - c^2*x^2] + 64*a*b*Cos[2*ArcCos[c*x]] - 4*a*b*Cos[4*ArcCos
[c*x]] - 32*b^2*Sin[2*ArcCos[c*x]] + b^2*Sin[4*ArcCos[c*x]]) - 4*b*d*e*Sqr
t[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]*(-16*b*Cos[2*ArcCos[c*x]] + b*Cos
[4*ArcCos[c*x]] + 4*a*(-8*Sin[2*ArcCos[c*x]] + Sin[4*ArcCos[c*x]])))/(256*
c*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.76, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {5179, 5159, 5157, 5139, 262, 223, 5153, 5183, 211, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx+d)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\int \left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2dx}{(1-c^2x^2)^{3/2}}$$

$\Big\downarrow$ 5159

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx+\frac{3}{4}\int\sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx+\frac{1}{4}x(1}{\left(1-c^2x^2\right)^{3/2}}$$

$\Big\downarrow$ 5157

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx+\frac{3}{4}\left(\frac{1}{2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+bc\int x(a+b\arcco}{\left(1-c^2x^2\right)^{3/2}}$$

$\Big\downarrow$ 5139

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx+\frac{3}{4}\left(bc\left(\frac{1}{2}bc\int\frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(c}{\left(1-c^2x^2\right)}$$

$\Big\downarrow$ 262

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx+\frac{3}{4}\left(bc\left(\frac{1}{2}bc\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2}{(1}$$

$\Big\downarrow$ 223

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)+\right.}{(1-}$$

$\Big\downarrow$ 5153

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\int x\left(1-c^2x^2\right)(a+b\arccos(cx))dx+\frac{3}{4}\left(bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}\right.\right.\right.}{(1-c^2x}$$

$\Big\downarrow$ 5183

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\left(-\frac{b\int\left(1-c^2x^2\right)^{3/2}dx}{4c}-\frac{\left(1-c^2x^2\right)^2(a+b\arccos(cx))}{4c^2}\right)+\frac{3}{4}\left(bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\right.\right.\right.}{(1-}$$

$\Big\downarrow$ 211

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\left(-\frac{b\left(\frac{3}{4}\int\sqrt{1-c^2x^2}dx+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)+\frac{3}{4}\left(bc\left(\frac{1}{2}x^2(a+b\,\mathrm{ar}\right.\right.}{}$$

$\downarrow$ 211

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\left(-\frac{b\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}\right)+\frac{3}{4}\left(\right.\right.}{}$$

$\downarrow$ 223

$$\frac{(cdx+d)^{3/2}(e-cex)^{3/2}\left(\frac{1}{2}bc\left(-\frac{(1-c^2x^2)^2(a+b\arccos(cx))}{4c^2}-\frac{b\left(\frac{3}{4}\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{4}x(1-c^2x^2)^{3/2}\right)}{4c}\right)+\frac{3}{4}\left(bc\left(\frac{1}{2}x\right.\right.\right.}{}$$

input
```
Int[(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*((x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos
[c*x])^2)/4 + (3*((x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/2 - (a + b*A
rcCos[c*x])^3/(6*b*c) + b*c*((x^2*(a + b*ArcCos[c*x]))/2 + (b*c*(-1/2*(x*S
qrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2)))/4 + (b*c*(-1/4*((1 - c^
2*x^2)^2*(a + b*ArcCos[c*x]))/c^2 - (b*((x*(1 - c^2*x^2)^(3/2))/4 + (3*((x
*Sqrt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/4))/(4*c)))/2))/(1 - c^2*x^2)^
(3/2)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5159
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcCos[c*x])^n/(2*p + 1)), x] + (S
imp[2*d*(p/(2*p + 1))   Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x],
x] + Simp[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[x*(1
- c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 1099, normalized size of antiderivative = 3.04

method	result	size
default	Expression too large to display	1099
parts	Expression too large to display	1099

input
```
int((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/4*a^2/c/e*(c*d*x+d)^(3/2)*(-c*e*x+e)^(5/2)-1/4*a^2*d/c/e*(c*d*x+d)^(1/2
)*(-c*e*x+e)^(5/2)+1/8*a^2*d/c*(-c*e*x+e)^(3/2)*(c*d*x+d)^(1/2)+3/8*a^2*d*
e/c*(-c*e*x+e)^(1/2)*(c*d*x+d)^(1/2)+3/8*a^2*d^2*e^2*((-c*e*x+e)*(c*d*x+d)
)^(1/2)/(-c*e*x+e)^(1/2)/(c*d*x+d)^(1/2)/(c^2*d*e)^(1/2)*arctan((c^2*d*e)^
(1/2)*x/(-c^2*d*e*x^2+d*e)^(1/2))+b^2*(1/8*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^
(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c*arccos(c*x)^3*d*e-1/512*(d*(c*x+1))
^(1/2)*(-e*(c*x-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4
*c^4+4*c*x-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(4*I*arcco
s(c*x)+8*arccos(c*x)^2-1)*d*e/(c^2*x^2-1)/c+1/16*(d*(c*x+1))^(1/2)*(-e*(c*
x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2
)-2*c*x)*(2*arccos(c*x)^2-1-2*I*arccos(c*x))*d*e/(c^2*x^2-1)/c-3/512*(d*(c
*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(20*
I*arccos(c*x)+24*arccos(c*x)^2-11)*cos(3*arccos(c*x))*d*e/(c^2*x^2-1)/c-1/
512*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)
-I)*(68*I*arccos(c*x)+56*arccos(c*x)^2-31)*sin(3*arccos(c*x))*d*e/(c^2*x^2
-1)/c)+2*a*b*(3/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)
/(c^2*x^2-1)/c*arccos(c*x)^2*d*e-1/256*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2
)*(8*c^5*x^5-12*c^3*x^3+8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+4*c*x-8*I*(-c^2*x^2
+1)^(1/2)*x^2*c^2+I*(-c^2*x^2+1)^(1/2))*(I+4*arccos(c*x))*d*e/(c^2*x^2-1)/
c+1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^...
```

Fricas [F]

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\int (cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}(b\arccos{(cx)}+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algorith
m="fricas")
```

output
```
integral(-(a^2*c^2*d*e*x^2 - a^2*d*e + (b^2*c^2*d*e*x^2 - b^2*d*e)*arccos(
c*x)^2 + 2*(a*b*c^2*d*e*x^2 - a*b*d*e)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-
c*e*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+d)**(3/2)*(-c*e*x+e)**(3/2)*(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx = \int (cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2,x, algorith
m="giac")
```

output
```
integrate((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)*(b*arccos(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,(d+c\,d\,x)^{3/2}\,(e-c\,e\,x)^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2), x)
```

Reduce [F]

$$\int (d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{\sqrt{e}\,\sqrt{d}\,de\left(-6asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2-2\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^3x^3+5\sqrt{cx+1}\right.}{}$$

input
```
int((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(e)*sqrt(d)*d*e*( - 6*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - 2*sqrt(c*
x + 1)*sqrt( - c*x + 1)*a**2*c**3*x**3 + 5*sqrt(c*x + 1)*sqrt( - c*x + 1)*
a**2*c*x - 16*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x**2,x)*a*b*c**
3 + 16*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x),x)*a*b*c - 8*int(sqrt(
c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2*x**2,x)*b**2*c**3 + 8*int(sqrt(c*x
+ 1)*sqrt( - c*x + 1)*acos(c*x)**2,x)*b**2*c))/(8*c)
```

3.586 $\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 647

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x}\,dx =$$
$$-\frac{22}{9}b^2de\sqrt{d+cdx}\sqrt{e-cex}-\frac{2abcdex\sqrt{d+cdx}\sqrt{e-cex}}{\sqrt{1-c^2x^2}}$$
$$-\frac{2}{27}b^2de\sqrt{d+cdx}\sqrt{e-cex}\left(1-c^2x^2\right)$$
$$-\frac{2b^2cdex\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)}{\sqrt{1-c^2x^2}}$$
$$-\frac{2bcdex\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{3\sqrt{1-c^2x^2}}$$
$$+\frac{2bc^3dex^3\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{9\sqrt{1-c^2x^2}}$$
$$+de\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2$$
$$+\frac{1}{3}de\sqrt{d+cdx}\sqrt{e-cex}\left(1-c^2x^2\right)(a+b\arccos(cx))^2$$
$$-\frac{2de\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}$$
$$+\frac{2ibde\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}$$
$$-\frac{2ibde\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}$$
$$-\frac{2b^2de\sqrt{d+cdx}\sqrt{e-cex}\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}$$
$$+\frac{2b^2de\sqrt{d+cdx}\sqrt{e-cex}\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}$$

output
```
-22/9*b^2*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)-2*a*b*c*d*e*x*(c*d*x+d)^(1/
2)*(-c*e*x+e)^(1/2)/(-c^2*x^2+1)^(1/2)-2/27*b^2*d*e*(c*d*x+d)^(1/2)*(-c*e*
x+e)^(1/2)*(-c^2*x^2+1)-2*b^2*c*d*e*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*arc
cos(c*x)/(-c^2*x^2+1)^(1/2)-2/3*b*c*d*e*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)
*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+2/9*b*c^3*d*e*x^3*(c*d*x+d)^(1/2)*(-
c*e*x+e)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+d*e*(c*d*x+d)^(1/2)*(-
c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2+1/3*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2
)*(-c^2*x^2+1)*(a+b*arccos(c*x))^2-2*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*
(a+b*arccos(c*x))^2*arctanh(c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2
*I*b*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))*polylog(2,-c*x
-I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)-2*I*b*d*e*(c*d*x+d)^(1/2)*(-c*e*
x+e)^(1/2)*(a+b*arccos(c*x))*polylog(2,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2
+1)^(1/2)-2*b^2*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*polylog(3,-c*x-I*(-c^
2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)+2*b^2*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1
/2)*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))/(-c^2*x^2+1)^(1/2)
```

Mathematica [A] (verified)

Time = 4.60 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.02

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x}\,dx =$$

$$-\frac{1}{3}a^2de\sqrt{d+cdx}\sqrt{e-cex}\left(-4+c^2x^2\right)$$

$$-\frac{2abde\sqrt{d+cdx}\sqrt{e-cex}\left(-3cx+c^3x^3-3\left(1-c^2x^2\right)^{3/2}\arccos(cx)\right)}{9\sqrt{1-c^2x^2}}$$

$$-\frac{b^2de\sqrt{d+cdx}\sqrt{e-cex}\left(-2\sqrt{1-c^2x^2}(-7+c^2x^2)+6cx(-3+c^2x^2)\arccos(cx)-9\left(1-c^2x^2\right)^{3/2}\arccos\right.}{27\sqrt{1-c^2x^2}}$$

$$+a^2d^{3/2}e^{3/2}\log(cx)-a^2d^{3/2}e^{3/2}\log\left(de+\sqrt{d}\sqrt{e}\sqrt{d+cdx}\sqrt{e-cex}\right)+\frac{2abde\sqrt{d+cdx}\sqrt{e-cex}\left(cx+\sqrt{1-\right.}}{}$$

input
```
Integrate[((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
-1/3*(a^2*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-4 + c^2*x^2)) - (2*a*b*d*e
*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-3*c*x + c^3*x^3 - 3*(1 - c^2*x^2)^(3/2)
*ArcCos[c*x]))/(9*Sqrt[1 - c^2*x^2]) - (b^2*d*e*Sqrt[d + c*d*x]*Sqrt[e - c
*e*x]*(-2*Sqrt[1 - c^2*x^2]*(-7 + c^2*x^2) + 6*c*x*(-3 + c^2*x^2)*ArcCos[c
*x] - 9*(1 - c^2*x^2)^(3/2)*ArcCos[c*x]^2))/(27*Sqrt[1 - c^2*x^2]) + a^2*d
^(3/2)*e^(3/2)*Log[c*x] - a^2*d^(3/2)*e^(3/2)*Log[d*e + Sqrt[d]*Sqrt[e]*Sq
rt[d + c*d*x]*Sqrt[e - c*e*x]] + (2*a*b*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x
]*(c*x + Sqrt[1 - c^2*x^2]*ArcCos[c*x] - ArcCos[c*x]*Log[1 - I*E^(I*ArcCos
[c*x])] + ArcCos[c*x]*Log[1 + I*E^(I*ArcCos[c*x])] - I*PolyLog[2, (-I)*E^(
I*ArcCos[c*x])] + I*PolyLog[2, I*E^(I*ArcCos[c*x])]))/Sqrt[1 - c^2*x^2] +
(b^2*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-2*Sqrt[1 - c^2*x^2] + 2*c*x*Arc
Cos[c*x] + Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2 - ArcCos[c*x]^2*Log[1 - I*E^(I*
ArcCos[c*x])] + ArcCos[c*x]^2*Log[1 + I*E^(I*ArcCos[c*x])] - (2*I)*ArcCos[
c*x]*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] + (2*I)*ArcCos[c*x]*PolyLog[2, I*E
^(I*ArcCos[c*x])] + 2*PolyLog[3, (-I)*E^(I*ArcCos[c*x])] - 2*PolyLog[3, I*
E^(I*ArcCos[c*x])]))/Sqrt[1 - c^2*x^2]
```

Rubi [A] (verified)

Time = 2.21 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.52, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5239, 5203, 5155, 27, 353, 53, 2009, 5199, 2009, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x}dx$$

$$\Big\downarrow \text{5239}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\int \frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{x}dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5203}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\frac{2}{3}bc\int \left(1-c^2x^2\right)(a+b\arccos(cx))dx+\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}dx+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5155

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}dx+\frac{2}{3}bc\left(bc\int \frac{x(3-c^2x^2)}{3\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\,\mathrm{ar}\right.\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 27

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}dx+\frac{2}{3}bc\left(\frac{1}{3}bc\int \frac{x(3-c^2x^2)}{\sqrt{1-c^2x^2}}dx-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\right.\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 353

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}dx+\frac{2}{3}bc\left(\frac{1}{6}bc\int \frac{3-c^2x^2}{\sqrt{1-c^2x^2}}dx^2-\frac{1}{3}c^2x^3(a+b\arccos(cx))+x(a+b\right.\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 53

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}dx+\frac{2}{3}bc\left(\frac{1}{6}bc\int \left(\sqrt{1-c^2x^2}+\frac{2}{\sqrt{1-c^2x^2}}\right)dx^2-\frac{1}{3}c^2x^3(a+b\,\mathrm{arccos}\right.\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 2009

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}dx+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2+\frac{2}{3}bc\left(-\frac{1}{3}c^2x^3(a+b\,\mathrm{arc}\right.\right.}{\sqrt{1-c^2x^2}}$$

$\Big\downarrow$ 5199

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx+2bc\int(a+b\arccos(cx))dx+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2+\sqrt{}\right.$$

$\Big\downarrow$ 2009

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2+\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\right.$$

$\Big\downarrow$ 5219

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(-\int \frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2+\sqrt{1-c^2x^2}(a+b\,\mathrm{a}\right.$$

$\downarrow$ 3042

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(-\int (a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(c\right.$$

$\downarrow$ 4669

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(2b\int (a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)-2b\int (a+b\arccos(cx))\log\left(1+\right.\right.$$

$\downarrow$ 3011

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(-2b\left(i\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\,\mathrm{arc}\right.\right.$$

$\downarrow$ 2720

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(-2b\left(i\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\mathrm{PolyLog}\left(2,-ie^{i\,\mathrm{arcc}}\right.\right.\right.$$

$\downarrow$ 7143

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+\frac{1}{3}\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))^2+\sqrt{1-c^2}\right.$$

input
```
Int[((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2)/x,x]
```

output
```
(d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]
)^2 + ((1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x])^2)/3 + 2*b*c*(a*x - (b*Sqrt
[1 - c^2*x^2])/c + b*x*ArcCos[c*x]) + (2*b*c*((b*c*((-4*Sqrt[1 - c^2*x^2])
/c^2 - (2*(1 - c^2*x^2)^(3/2))/(3*c^2)))/6 + x*(a + b*ArcCos[c*x]) - (c^2*
x^3*(a + b*ArcCos[c*x]))/3))/3 + (2*I)*(a + b*ArcCos[c*x])^2*ArcTan[E^(I*A
rcCos[c*x])] - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*ArcCos[c*x]
)] - b*PolyLog[3, (-I)*E^(I*ArcCos[c*x])]) + 2*b*(I*(a + b*ArcCos[c*x])*Po
lyLog[2, I*E^(I*ArcCos[c*x])] - b*PolyLog[3, I*E^(I*ArcCos[c*x])])))/Sqrt[
1 - c^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5155
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5199
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcC
os[c*x])^n/(f*(m + 2))), x] + (Simp[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt[1
 - c^2*x^2]]   Int[(f*x)^m*((a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x^2]), x], x
] + Simp[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[
(f*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

rule 5203
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 2*p + 1))), x] + (Simp[2*d*(p/(m + 2*p + 1))   Int[(f*x)
^m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2
*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 5.49 (sec) , antiderivative size = 1201, normalized size of antiderivative = 1.86

method	result	size
default	Expression too large to display	1201
parts	Expression too large to display	1201

input
```
int((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/x,x,method=_RETUR
NVERBOSE)
```

output
```
-1/3*a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*d*e*(x^2*c^2*(d*e)^(1/2)*(-d
*e*(c^2*x^2-1))^(1/2)+3*d*e*ln(2*((d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2)+d*e
)/x)-4*(d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2))/(d*e)^(1/2)/(-d*e*(c^2*x^2-1)
)^(1/2)+b^2*(-1/216*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(4*c^4*x^4-5*c^2*
x^2+4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-3*I*(-c^2*x^2+1)^(1/2)*c*x+1)*(6*I*arcc
os(c*x)+9*arccos(c*x)^2-2)*d*e/(c^2*x^2-1)+5/8*(d*(c*x+1))^(1/2)*(-e*(c*x-
1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)^2-2-2*I*arcco
s(c*x))*d*e/(c^2*x^2-1)-(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^
(1/2)/(c^2*x^2-1)*(arccos(c*x)^2*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-arccos
(c*x)^2*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*I*arccos(c*x)*polylog(2,-I*(c
*x+I*(-c^2*x^2+1)^(1/2)))+2*I*arccos(c*x)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^
(1/2)))+2*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*polylog(3,I*(c*x+I*(-
c^2*x^2+1)^(1/2))))*d*e-1/27*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^
2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(33*I*arccos(c*x)+18*arccos(c*x)^2-34)*cos(2
*arccos(c*x))*d*e/(c^2*x^2-1)-1/108*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(
I*c^2*x^2+c*x*(-c^2*x^2+1)^(1/2)-I)*(138*I*arccos(c*x)+63*arccos(c*x)^2-13
4)*sin(2*arccos(c*x))*d*e/(c^2*x^2-1))+2*a*b*(-1/72*(d*(c*x+1))^(1/2)*(-e*
(c*x-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-3*I*(-c
^2*x^2+1)^(1/2)*c*x+1)*(I+3*arccos(c*x))*d*e/(c^2*x^2-1)+5/8*(d*(c*x+1))^(
1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c...
```

Fricas [F]

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x}\,dx=\int \frac{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{x}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/x,x, algori
thm="fricas")
```

output
```
integral(-(a^2*c^2*d*e*x^2 - a^2*d*e + (b^2*c^2*d*e*x^2 - b^2*d*e)*arccos(
c*x)^2 + 2*(a*b*c^2*d*e*x^2 - a*b*d*e)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-
c*e*x + e)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x}\,dx = \text{Timed out}$$

input
```
integrate((c*d*x+d)**(3/2)*(-c*e*x+e)**(3/2)*(a+b*acos(c*x))**2/x,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/x,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x}\,dx = \int \frac{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{x}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/x,x, algori
thm="giac")
```

output
```
integrate((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)*(b*arccos(c*x) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d+c\,d\,x)^{3/2}\,(e-c\,e\,x)^{3/2}}{x}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2))/x,x)
```

output
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2))/x, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x}\,dx=\frac{\sqrt{e}\,\sqrt{d}\,de\left(-\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2c^2x^2+4\sqrt{cx+1}\,\sqrt{}\right.}{}$$

input
```
int((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*acos(c*x))^2/x,x)
```

output
```
(sqrt(e)*sqrt(d)*d*e*( - sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2*c**2*x**2 + 4
*sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2 + 6*int((sqrt(c*x + 1)*sqrt( - c*x +
1)*acos(c*x))/x,x)*a*b + 3*int((sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**
2)/x,x)*b**2 - 6*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)*x,x)*a*b*c**
2 - 3*int(sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2*x,x)*b**2*c**2 - 3*l
og( - sqrt(2) + tan(asin(sqrt( - c*x + 1)/sqrt(2))/2) - 1)*a**2 + 3*log( -
 sqrt(2) + tan(asin(sqrt( - c*x + 1)/sqrt(2))/2) + 1)*a**2 - 3*log(sqrt(2)
 + tan(asin(sqrt( - c*x + 1)/sqrt(2))/2) - 1)*a**2 + 3*log(sqrt(2) + tan(a
sin(sqrt( - c*x + 1)/sqrt(2))/2) + 1)*a**2))/3
```

3.587 $\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 505

$$\begin{aligned}
\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx &= \frac{1}{4}b^2c^2dex\sqrt{d+cdx}\sqrt{e-cex} \\
&- \frac{5b^2cde\sqrt{d+cdx}\sqrt{e-cex}\arccos(cx)}{4\sqrt{1-c^2x^2}} + \frac{3bc^3dex^2\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))}{2\sqrt{1-c^2x^2}} \\
&+ bcde\sqrt{d+cdx}\sqrt{e-cex}\sqrt{1-c^2x^2}(a+b\arccos(cx)) \\
&- \frac{3}{2}c^2dex\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2 \\
&- \frac{icde\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}} \\
&- \frac{de\sqrt{d+cdx}\sqrt{e-cex}(1-c^2x^2)\left(a+b\arccos(cx)\right)^2}{x} \\
&- \frac{cde\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))^3}{2b\sqrt{1-c^2x^2}} \\
&+ \frac{2bcde\sqrt{d+cdx}\sqrt{e-cex}(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}} \\
&- \frac{ib^2cde\sqrt{d+cdx}\sqrt{e-cex}\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{\sqrt{1-c^2x^2}}
\end{aligned}$$

output
```
1/4*b^2*c^2*d*e*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)-5/4*b^2*c*d*e*(c*d*x+d)
^(1/2)*(-c*e*x+e)^(1/2)*arccos(c*x)/(-c^2*x^2+1)^(1/2)+3/2*b*c^3*d*e*x^2*(
c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2)+b*c*d
*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))-3
/2*c^2*d*e*x*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2-I*c*d*e*
(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^2/(-c^2*x^2+1)^(1/2)-d*
e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/x-1/2*
c*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))^3/b/(-c^2*x^2+1)^
(1/2)+2*b*c*d*e*(c*d*x+d)^(1/2)*(-c*e*x+e)^(1/2)*(a+b*arccos(c*x))*ln(1-(c
*x+I*(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/2)-I*b^2*c*d*e*(c*d*x+d)^(1/2)
*(-c*e*x+e)^(1/2)*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/(-c^2*x^2+1)^(1/
2)
```

Mathematica [A] (verified)

Time = 2.74 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.07

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx = \frac{-8a^2de\sqrt{d+cdx}\sqrt{e-cex}\sqrt{1-c^2x^2}-4a^2c^2dex^2\sqrt{d}$$

input
```
Integrate[((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2)/x^2,
x]
```

output
```
(-8*a^2*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2] - 4*a^2*c^2*
d*e*x^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sqrt[1 - c^2*x^2] + 4*b^2*c*d*e*x*
Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]^3 + 12*a^2*c*d^(3/2)*e^(3/2)*x
*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*S
qrt[e]*(-1 + c^2*x^2))] - 2*a*b*c*d*e*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Co
s[2*ArcCos[c*x]] - 16*a*b*c*d*e*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Log[c*x]
 + (8*I)*b^2*c*d*e*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*PolyLog[2, -E^((2*I)*
ArcCos[c*x])] + b^2*c*d*e*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*Sin[2*ArcCos[c
*x]] - 2*b*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcCos[c*x]*(8*a*Sqrt[1 - c
^2*x^2] + b*c*x*Cos[2*ArcCos[c*x]] + 8*b*c*x*Log[1 + E^((2*I)*ArcCos[c*x])
] + 2*a*c*x*Sin[2*ArcCos[c*x]]) - 2*b*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*
ArcCos[c*x]^2*(-6*a*c*x - (4*I)*b*c*x + 4*b*Sqrt[1 - c^2*x^2] + b*c*x*Sin[
2*ArcCos[c*x]]))/(8*x*Sqrt[1 - c^2*x^2])
```

Rubi [A] (verified)

Time = 2.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.63, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.457, Rules used = {5239, 5201, 5157, 5139, 262, 223, 5153, 5189, 211, 223, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cdx+d)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x^2}dx$$

$$\Big\downarrow \text{5239}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\int \frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{x^2}dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5201}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(-3c^2\int \sqrt{1-c^2x^2}(a+b\arccos(cx))^2dx-2bc\int \frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx-\frac{(1-c^2x^2)^{3/2}(a+b}{x}\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5157}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(-3c^2\left(\frac{1}{2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+bc\int x(a+b\arccos(cx))dx+\frac{1}{2}x\sqrt{1-c^2x^2}(a+b\arccos(cx\right.\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5139}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(-3c^2\left(bc\left(\frac{1}{2}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)+\frac{1}{2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c\right.\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(-3c^2\left(bc\left(\frac{1}{2}bc\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)+\frac{1}{2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right.\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(-3c^2\left(\frac{1}{2}\int\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx+bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)+\right.}{\sqrt{1-c^2x^2}}$$

$\downarrow$ 5153

$$\frac{de\sqrt{cdx+d}\sqrt{e-cex}\left(-2bc\int\frac{(1-c^2x^2)(a+b\arccos(cx))}{x}dx-3c^2\left(bc\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c}}{2c^2}\right.\right.\right.\right.}{\sqrt{1-c^2x^2}}$$

$\downarrow$ 5189

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(-2bc\left(\int\frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\int\sqrt{1-c^2x^2}dx+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))\right)-3c^2\left(bc\right.\right.$$

$\downarrow$ 211

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(-2bc\left(\int\frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}bc\left(\frac{1}{2}\int\frac{1}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x\sqrt{1-c^2x^2}\right)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\,\mathrm{arcc}\right.\right.$$

$\downarrow$ 223

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(-2bc\left(\int\frac{a+b\arccos(cx)}{x}dx+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c}+\frac{1}{2}x\sqrt{1-c^2x^2}\right.\right.\right.$$

$\downarrow$ 5137

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(-2bc\left(-\int\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\mathrm{arcsi}}{2}\right.\right.\right.$$

$\downarrow$ 3042

$$de\sqrt{cdx+d}\sqrt{e-cex}\left(-2bc\left(-\int(a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx)\right.\right.$$

$\downarrow$ 4202

$$de\sqrt{cdx+d}\sqrt{e-cex}\Bigg(-2bc\Big(2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)+\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))-\frac{i(a+b\arccos(cx))}{}$$

$\downarrow$ 2620

$$de\sqrt{cdx+d}\sqrt{e-cex}\Bigg(-2bc\Big(2i\Big(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos($$

$\downarrow$ 2715

$$de\sqrt{cdx+d}\sqrt{e-cex}\Bigg(-2bc\Big(2i\Big(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)($$

$\downarrow$ 2838

$$de\sqrt{cdx+d}\sqrt{e-cex}\Bigg(-2bc\Big(\frac{1}{2}\left(1-c^2x^2\right)(a+b\arccos(cx))+2i\Big(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-$$

input
```
Int[((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)*(a + b*ArcCos[c*x])^2)/x^2,x]
```

output
```
(d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(-(((1 - c^2*x^2)^(3/2)*(a + b*ArcCos
[c*x])^2)/x) - 3*c^2*((x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/2 - (a +
 b*ArcCos[c*x])^3/(6*b*c) + b*c*((x^2*(a + b*ArcCos[c*x]))/2 + (b*c*(-1/2*
(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2)) - 2*b*c*(((1 - c^2*x
^2)*(a + b*ArcCos[c*x]))/2 - ((I/2)*(a + b*ArcCos[c*x])^2)/b + (b*c*((x*Sq
rt[1 - c^2*x^2])/2 + ArcSin[c*x]/(2*c)))/2 + (2*I)*((-1/2*I)*(a + b*ArcCos
[c*x])*Log[1 + E^((2*I)*ArcCos[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcCos[c*x
])])/4))))/Sqrt[1 - c^2*x^2]
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

rule 5189
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
x_Symbol] :> Simp[(d + e*x^2)^p*((a + b*ArcCos[c*x])/(2*p)), x] + (Simp[d
 Int[(d + e*x^2)^(p - 1)*((a + b*ArcCos[c*x])/x), x], x] + Simp[b*c*(d^p/(2
*p))   Int[(1 - c^2*x^2)^(p - 1/2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 5201
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*ArcC
os[c*x])^n/(f*(m + 1))), x] + (-Simp[2*e*(p/(f^2*(m + 1)))   Int[(f*x)^(m +
 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*c*(n/(f*(m +
 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)
^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.02

method	result
parts	$-\frac{a^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,de\left(3\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-de\left(c^2x^2-1\right)}}\right)x\,c^2de+x^2c^2\sqrt{-de(c^2x^2-1)}\,\sqrt{c^2de}+2\sqrt{c^2de}\,\sqrt{-de(c^2x^2-1)}\right)}{2\sqrt{-de(c^2x^2-1)}\,\sqrt{c^2de}\,x}-$
default	$-\frac{a^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,de\left(3\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-de\left(c^2x^2-1\right)}}\right)x\,c^2de+x^2c^2\sqrt{-de(c^2x^2-1)}\,\sqrt{c^2de}+2\sqrt{c^2de}\,\sqrt{-de(c^2x^2-1)}\right)}{2\sqrt{-de(c^2x^2-1)}\,\sqrt{c^2de}\,x}-$

input
```
int((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/x^2,x,method=_RET
URNVERBOSE)
```

output
```
-1/2*a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*d*e*(3*arctan((c^2*d*e)^(1/2
)*x/(-d*e*(c^2*x^2-1))^(1/2))*x*c^2*d*e+x^2*c^2*(-d*e*(c^2*x^2-1))^(1/2)*(
c^2*d*e)^(1/2)+2*(c^2*d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2))/(-d*e*(c^2*x^2-
1))^(1/2)/(c^2*d*e)^(1/2)/x-1/4*b^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(
-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/x*(-2*arccos(c*x)^2*(-c^2*x^2+1)^(1/2)*x^2*c
^2-2*c^3*x^3*arccos(c*x)+2*arccos(c*x)^3*c*x+4*I*arccos(c*x)^2*x*c+c^2*x^2
*(-c^2*x^2+1)^(1/2)-8*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*x*c+4
*I*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)*x*c-4*arccos(c*x)^2*(-c^2*x^2+
1)^(1/2)+c*x*arccos(c*x))*d*e-1/4*a*b*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)
*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/x*(-4*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x
^2-2*c^3*x^3+6*arccos(c*x)^2*c*x+8*I*arccos(c*x)*x*c-8*ln(1+(c*x+I*(-c^2*x
^2+1)^(1/2))^2)*x*c-8*arccos(c*x)*(-c^2*x^2+1)^(1/2)+c*x)*d*e
```

Fricas [F]

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx=\int \frac{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/x^2,x, algo
rithm="fricas")
```

output
```
integral(-(a^2*c^2*d*e*x^2 - a^2*d*e + (b^2*c^2*d*e*x^2 - b^2*d*e)*arccos(
c*x)^2 + 2*(a*b*c^2*d*e*x^2 - a*b*d*e)*arccos(c*x))*sqrt(c*d*x + d)*sqrt(-
c*e*x + e)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((c*d*x+d)**(3/2)*(-c*e*x+e)**(3/2)*(a+b*acos(c*x))**2/x**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/x^2,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}(b\arccos(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*arccos(c*x))^2/x^2,x, algo
rithm="giac")
```

output
```
integrate((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)*(b*arccos(c*x) + a)^2/x^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2\,(d+c\,d\,x)^{3/2}\,(e-c\,e\,x)^{3/2}}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2))/x^2,x)
```

output
```
int(((a + b*acos(c*x))^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(d+cdx)^{3/2}(e-cex)^{3/2}(a+b\arccos(cx))^2}{x^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,de\left(6asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2cx-\sqrt{cx+1}\,\sqrt{-cx+1}$$

input
```
int((c*d*x+d)^(3/2)*(-c*e*x+e)^(3/2)*(a+b*acos(c*x))^2/x^2,x)
```

output
```
(sqrt(e)*sqrt(d)*d*e*(6*asin(sqrt( - c*x + 1)/sqrt(2))*a**2*c*x - sqrt(c*x
 + 1)*sqrt( - c*x + 1)*a**2*c**2*x**2 - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*a
**2 + 4*int((sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x))/x**2,x)*a*b*x + 2*i
nt((sqrt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x)**2)/x**2,x)*b**2*x - 4*int(sq
rt(c*x + 1)*sqrt( - c*x + 1)*acos(c*x),x)*a*b*c**2*x - 2*int(sqrt(c*x + 1)
*sqrt( - c*x + 1)*acos(c*x)**2,x)*b**2*c**2*x))/(2*x)
```

3.588 $\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 250

$$\begin{aligned}\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx &= \frac{b^2x(1-c^2x^2)}{4c^2\sqrt{d+cdx}\sqrt{e-cex}} - \frac{b^2\sqrt{1-c^2x^2}\arccos(cx)}{4c^3\sqrt{d+cdx}\sqrt{e-cex}} \\ &+ \frac{bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c\sqrt{d+cdx}\sqrt{e-cex}} \\ &- \frac{x(1-c^2x^2)(a+b\arccos(cx))^2}{2c^2\sqrt{d+cdx}\sqrt{e-cex}} \\ &+ \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{6bc^3\sqrt{d+cdx}\sqrt{e-cex}}\end{aligned}$$

output
```
1/4*b^2*x*(-c^2*x^2+1)/c^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-1/4*b^2*(-c^2*
x^2+1)^(1/2)*arccos(c*x)/c^3/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+1/2*b*x^2*(-
c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-1/2*
x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/c^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+1/
6*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^3/b/c^3/(c*d*x+d)^(1/2)/(-c*e*x+e)^
(1/2)
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.97

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx$$
$$= \frac{12a^2cex(-1+cx)(d+cdx)-12a^2\sqrt{d}\sqrt{e}\sqrt{d+cdx}\sqrt{e-cex}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{e-cex}}{\sqrt{d}\sqrt{e}(-1+c^2x^2)}\right)-b^2de\sqrt{1-c^2x^2}(}{}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x])^2)/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]
```

output
```
(12*a^2*c*e*x*(-1 + c*x)*(d + c*d*x) - 12*a^2*Sqrt[d]*Sqrt[e]*Sqrt[d + c*d
*x]*Sqrt[e - c*e*x]*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*
Sqrt[e]*(-1 + c^2*x^2))] - b^2*d*e*Sqrt[1 - c^2*x^2]*(4*ArcCos[c*x]^3 + 6*
ArcCos[c*x]*Cos[2*ArcCos[c*x]] + (-3 + 6*ArcCos[c*x]^2)*Sin[2*ArcCos[c*x]]
) - 6*a*b*d*e*Sqrt[1 - c^2*x^2]*(Cos[2*ArcCos[c*x]] + 2*ArcCos[c*x]*(ArcCo
s[c*x] + Sin[2*ArcCos[c*x]])))/(24*c^3*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]
)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.60, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {5239, 5211, 5139, 262, 223, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{cdx+d}\sqrt{e-cex}}\,dx$$

$$\Big\downarrow \; 5239$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \; 5211$$

$$\frac{\sqrt{1-c^2x^2}\left(\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\int x(a+b\arccos(cx))dx}{c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$\Big\downarrow$ 5139

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{b\left(\frac{1}{2}bc\int \frac{x^2}{\sqrt{1-c^2x^2}}dx+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c}+\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$\Big\downarrow$ 262

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{b\left(\frac{1}{2}bc\left(\frac{\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)+\frac{1}{2}x^2(a+b\arccos(cx))\right)}{c}+\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$\Big\downarrow$ 223

$$\frac{\sqrt{1-c^2x^2}\left(\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{b\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$\Big\downarrow$ 5153

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{b\left(\frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}bc\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)\right)}{c}-\frac{(a+b\arccos(cx))^3}{6bc^3}-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2c^2}\right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x])^2)/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]
```

output
```
(Sqrt[1 - c^2*x^2]*(-1/2*(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c^2 -
 (a + b*ArcCos[c*x])^3/(6*b*c^3) - (b*((x^2*(a + b*ArcCos[c*x]))/2 + (b*c*
(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/2))/c))/(Sqrt[d +
c*d*x]*Sqrt[e - c*e*x])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5139
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Simp[b*c*(n
/(d*(m + 1)))   Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2
*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5211
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Simp[f^2*((m - 1)/(c^2*(m + 2*p
 + 1)))   Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - S
imp[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*
x)^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m
, 1] && NeQ[m + 2*p + 1, 0]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.23 (sec) , antiderivative size = 1016, normalized size of antiderivative = 4.06

method	result
default	$\frac{a^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\left(\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-de\left(c^2x^2-1\right)}}\right)de-\sqrt{c^2de}\,\sqrt{-de(c^2x^2-1)}\,x\right)}{2c^2\sqrt{-de(c^2x^2-1)}\,ed\sqrt{c^2de}}+b^2\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+}}{6de\,c^3(c^2x^2-1)}\right.$
parts	$\frac{a^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\left(\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-de\left(c^2x^2-1\right)}}\right)de-\sqrt{c^2de}\,\sqrt{-de(c^2x^2-1)}\,x\right)}{2c^2\sqrt{-de(c^2x^2-1)}\,ed\sqrt{c^2de}}+b^2\left(\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+}}{6de\,c^3(c^2x^2-1)}\right.$

input
```
int(x^2*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/2*a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(arctan((c^2*d*e)^(1/2)*x/(-d
*e*(c^2*x^2-1))^(1/2))*d*e-(c^2*d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2)*x)/c^2
/(-d*e*(c^2*x^2-1))^(1/2)/e/d/(c^2*d*e)^(1/2)+b^2*(1/6*(d*(c*x+1))^(1/2)*(
-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/e/c^3/(c^2*x^2-1)*arccos(c*x)^3-1/3
2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(2*c^2*x^2-1+2*I*(-c^2*x^2+1)^(1/2)
*c*x-I*(-c^2*x^2+1)^(1/2)-c*x)*(2*arccos(c*x)^2-1+2*I*arccos(c*x))/c^3/d/(
c*x+1)/e/(c*x-1)-1/32*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*c^2*x^2-4*I
*(-c^2*x^2+1)^(1/2)*c^2*x^2+4*c^3*x^3+1+2*I*(-c^2*x^2+1)^(1/2)*c*x+I*(-c^2
*x^2+1)^(1/2)-3*c*x)*(2*arccos(c*x)^2-1-2*I*arccos(c*x))/d/e/c^3/(c^2*x^2-
1)-1/8*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*c*x-I-(-c^2*x^2+1)^(1/2))*a
rccos(c*x)*cos(2*arccos(c*x))/c^3/d/(c*x+1)/e/(c*x-1)-1/16*(d*(c*x+1))^(1/
2)*(-e*(c*x-1))^(1/2)*(I*c*x-I-(-c^2*x^2+1)^(1/2))*(2*arccos(c*x)^2-1)*sin
(2*arccos(c*x))/c^3/d/(c*x+1)/e/(c*x-1))+2*a*b*(1/4*(d*(c*x+1))^(1/2)*(-e*
(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/e/c^3/(c^2*x^2-1)*arccos(c*x)^2-1/32*(
d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(2*c^2*x^2-1+2*I*(-c^2*x^2+1)^(1/2)*c*
x-I*(-c^2*x^2+1)^(1/2)-c*x)*(I+2*arccos(c*x))/c^3/d/(c*x+1)/e/(c*x-1)-1/32
*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*c^2*x^2-4*I*(-c^2*x^2+1)^(1/2)*c
^2*x^2+4*c^3*x^3+1+2*I*(-c^2*x^2+1)^(1/2)*c*x+I*(-c^2*x^2+1)^(1/2)-3*c*x)*
(-I+2*arccos(c*x))/d/e/c^3/(c^2*x^2-1)-1/16*(d*(c*x+1))^(1/2)*(-e*(c*x-1))
^(1/2)*(I*c*x-I-(-c^2*x^2+1)^(1/2))*cos(2*arccos(c*x))/c^3/d/(c*x+1)/e/...
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2x^2}{\sqrt{cdx+d}\sqrt{-cex+e}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-(b^2*x^2*arccos(c*x)^2 + 2*a*b*x^2*arccos(c*x) + a^2*x^2)*sqrt(c
*d*x + d)*sqrt(-c*e*x + e)/(c^2*d*e*x^2 - d*e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*acos(c*x))**2/(c*d*x+d)**(1/2)/(-c*e*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx = \int \frac{(b\arccos(cx)+a)^2x^2}{\sqrt{cdx+d}\sqrt{-cex+e}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2*x^2/(sqrt(c*d*x + d)*sqrt(-c*e*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{x^2\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}{\sqrt{d+c\,d\,x}\,\sqrt{e-c\,e\,x}}\,dx$$

input
```
int((x^2*(a + b*acos(c*x))^2)/((d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)),x)
```

output
```
int((x^2*(a + b*acos(c*x))^2)/((d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx$$
$$=\frac{-2asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2-\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2cx+4\left(\int \frac{acos(cx)x^2}{\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)ab\,c^3+2\left(\int \frac{acos(cx)^2x^2}{\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)b^2c}{2\sqrt{e}\,\sqrt{d}\,c^3}$$

input
```
int(x^2*(a+b*acos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x)
```

output
```
( - 2*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - sqrt(c*x + 1)*sqrt( - c*x + 1)
*a**2*c*x + 4*int((acos(c*x)*x**2)/(sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a*b
*c**3 + 2*int((acos(c*x)**2*x**2)/(sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b**2
*c**3)/(2*sqrt(e)*sqrt(d)*c**3)
```

3.589 $\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 177

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx = \frac{2abx\sqrt{1-c^2x^2}}{c\sqrt{d+cdx}\sqrt{e-cex}} + \frac{2b^2(1-c^2x^2)}{c^2\sqrt{d+cdx}\sqrt{e-cex}} + \frac{2b^2x\sqrt{1-c^2x^2}\arccos(cx)}{c\sqrt{d+cdx}\sqrt{e-cex}} - \frac{(1-c^2x^2)\,(a+b\arccos(cx))^2}{c^2\sqrt{d+cdx}\sqrt{e-cex}}$$

output
```
2*a*b*x*(-c^2*x^2+1)^(1/2)/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*b^2*(-c^2*
x^2+1)/c^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*b^2*x*(-c^2*x^2+1)^(1/2)*arc
cos(c*x)/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-(-c^2*x^2+1)*(a+b*arccos(c*x))
^2/c^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.85

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx = \frac{\sqrt{d+cdx}\sqrt{e-cex}\left(2abcx\sqrt{1-c^2x^2}+a^2(1-c^2x^2)+2b^2(-1+c^2x^2)+2b\left(a-ac^2x^2+bcx\sqrt{1-c^2x^2}\right)\right.}{c^2de(-1+cx)(1+cx)}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x])^2)/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]
```

output
```
(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*(2*a*b*c*x*Sqrt[1 - c^2*x^2] + a^2*(1 - c
^2*x^2) + 2*b^2*(-1 + c^2*x^2) + 2*b*(a - a*c^2*x^2 + b*c*x*Sqrt[1 - c^2*x
^2])*ArcCos[c*x] + b^2*(1 - c^2*x^2)*ArcCos[c*x]^2))/(c^2*d*e*(-1 + c*x)*(
1 + c*x))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.58, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5239, 5183, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{cdx+d}\sqrt{e-cex}}\,dx$$

$$\Big\downarrow \; 5239$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \; 5183$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{2b\int (a+b\arccos(cx))dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}\right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^2}-\frac{2b\left(ax+bx\arccos(cx)-\frac{b\sqrt{1-c^2x^2}}{c}\right)}{c}\right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

input
```
Int[(x*(a + b*ArcCos[c*x])^2)/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]
```

output
```
(Sqrt[1 - c^2*x^2]*(-((Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c^2) - (2*
b*(a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]))/c))/(Sqrt[d + c*d*x]*
Sqrt[e - c*e*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 16.71 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.05

method	result
default	$-\frac{a^{2}\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}}{ed\,c^{2}}+b^{2}\left(-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\left(i\sqrt{-c^{2}x^{2}+1}\,xc+c^{2}x^{2}-1\right)\left(\arccos(cx)^{2}-2+2i\arccos(cx)\right)}{2c^{2}d(cx+1)e(cx-1)}-\right.$
parts	$-\frac{a^{2}\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}}{ed\,c^{2}}+b^{2}\left(-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\left(i\sqrt{-c^{2}x^{2}+1}\,xc+c^{2}x^{2}-1\right)\left(\arccos(cx)^{2}-2+2i\arccos(cx)\right)}{2c^{2}d(cx+1)e(cx-1)}-\right.$
orering	$\frac{(c^{4}x^{4}-4c^{2}x^{2}+2)(a+b\arccos(cx))^{2}}{c^{4}x^{2}\sqrt{cdx+d}\,\sqrt{-cex+e}}+\frac{2(cx-1)(cx+1)\left(\frac{(a+b\arccos(cx))^{2}}{\sqrt{cdx+d}\,\sqrt{-cex+e}}-\frac{2x(a+b\arccos(cx))bc}{\sqrt{cdx+d}\,\sqrt{-cex+e}\,\sqrt{-c^{2}x^{2}+1}}-\frac{x(a+b\arccos(cx))^{2}dc}{2(cdx+d)^{\frac{3}{2}}\sqrt{-cex+e}}+\right.}{c^{4}x^{2}}$

input
```
int(x*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
-a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/e/d/c^2+b^2*(-1/2*(d*(c*x+1))^(1
/2)*(-e*(c*x-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)^2
-2+2*I*arccos(c*x))/c^2/d/(c*x+1)/e/(c*x-1)-1/2*(d*(c*x+1))^(1/2)*(-e*(c*x
-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*(arccos(c*x)^2-2-2*I*arcc
os(c*x))/(c^2*x^2-1)/c^2/d/e)+2*a*b*(-1/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(
1/2)*(I*(-c^2*x^2+1)^(1/2)*c*x+c^2*x^2-1)*(arccos(c*x)+I)/c^2/d/(c*x+1)/e/
(c*x-1)-1/2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*
c+c^2*x^2-1)*(arccos(c*x)-I)/(c^2*x^2-1)/c^2/d/e)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx = -\frac{\left((a^2-2\,b^2)c^2x^2+(b^2c^2x^2-b^2)\arccos{(cx)}^2-a^2+2\,b^2+2\,(abc^2x^2-ab)\arccos{(cx)}-2\,(b^2cx\arccos\right.}{c^4dex^2-c^2de}$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algori
thm="fricas")
```

output
```
-((a^2 - 2*b^2)*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccos(c*x)^2 - a^2 + 2*b^2
+ 2*(a*b*c^2*x^2 - a*b)*arccos(c*x) - 2*(b^2*c*x*arccos(c*x) + a*b*c*x)*sq
rt(-c^2*x^2 + 1))*sqrt(c*d*x + d)*sqrt(-c*e*x + e)/(c^4*d*e*x^2 - c^2*d*e)
```

Sympy [F]

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{x(a+b\,\mathrm{acos}\,(cx))^2}{\sqrt{d\,(cx+1)}\,\sqrt{-e\,(cx-1)}}\,dx$$

input
```
integrate(x*(a+b*acos(c*x))**2/(c*d*x+d)**(1/2)/(-c*e*x+e)**(1/2),x)
```

output
```
Integral(x*(a + b*acos(c*x))**2/(sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(b\arccos(cx)+a)^2x}{\sqrt{cdx+d}\sqrt{-cex+e}}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algori
thm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2*x/(sqrt(c*d*x + d)*sqrt(-c*e*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{x\,(a+b\,\mathrm{acos}(c\,x))^2}{\sqrt{d+c\,d\,x}\,\sqrt{e-c\,e\,x}}\,dx$$

input
```
int((x*(a + b*acos(c*x))^2)/((d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)),x)
```

output
```
int((x*(a + b*acos(c*x))^2)/((d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx = \frac{-\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2+2\left(\int \frac{acos(cx)x}{\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)ab\,c^2+\left(\int \frac{acos(cx)^2x}{\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)b^2c^2}{\sqrt{e}\,\sqrt{d}\,c^2}$$

input
```
int(x*(a+b*acos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x)
```

output
```
( - sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2 + 2*int((acos(c*x)*x)/(sqrt(c*x +
1)*sqrt( - c*x + 1)),x)*a*b*c**2 + int((acos(c*x)**2*x)/(sqrt(c*x + 1)*sqr
t( - c*x + 1)),x)*b**2*c**2)/(sqrt(e)*sqrt(d)*c**2)
```

3.590 $\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx = \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc\sqrt{d+cdx}\sqrt{e-cex}}$$

output
```
1/3*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^3/b/c/(c*d*x+d)^(1/2)/(-c*e*x+e)^
(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 159 vs. 2(55) = 110.

Time = 0.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.89

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx = -\frac{\frac{3ab\sqrt{1-c^2x^2}\arccos(cx)^2}{\sqrt{d+cdx}\sqrt{e-cex}} + \frac{b^2\sqrt{1-c^2x^2}\arccos(cx)^3}{\sqrt{d+cdx}\sqrt{e-cex}} + \frac{3a^2\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{e-cex}}{\sqrt{d}\sqrt{e}(-1+c^2x^2)}\right)}{\sqrt{d}\sqrt{e}}}{3c}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]
```

output
```
-1/3*((3*a*b*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2)/(Sqrt[d + c*d*x]*Sqrt[e - c*
e*x]) + (b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^3)/(Sqrt[d + c*d*x]*Sqrt[e - c*
e*x]) + (3*a^2*ArcTan[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[
e]*(-1 + c^2*x^2))])/(Sqrt[d]*Sqrt[e]))/c
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{5179, 5153\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{cdx+d}\sqrt{e-cex}}dx$$

$$\Big\downarrow \text{ 5179}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 5153}$$

$$-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc\sqrt{cdx+d}\sqrt{e-cex}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]
```

output
```
-1/3*(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^3)/(b*c*Sqrt[d + c*d*x]*Sqrt[e
 - c*e*x])
```

Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(47) = 94$.

Time = 1.03 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.56

method	result
default	$\frac{a^2\sqrt{(-cex+e)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-c^2de\,x^2+de}}\right)}{\sqrt{cdx+d}\,\sqrt{-cex+e}\,\sqrt{c^2de}}+\frac{b^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3}{3(c^2x^2-1)cde}+\frac{ab\sqrt{d(cx+1)}\,\sqrt{-e(cx}{(c^2}$
parts	$\frac{a^2\sqrt{(-cex+e)(cdx+d)}\,\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-c^2de\,x^2+de}}\right)}{\sqrt{cdx+d}\,\sqrt{-cex+e}\,\sqrt{c^2de}}+\frac{b^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^3}{3(c^2x^2-1)cde}+\frac{ab\sqrt{d(cx+1)}\,\sqrt{-e(cx}{(c^2}$

input
```
int((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
a^2*((-c*e*x+e)*(c*d*x+d))^(1/2)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)/(c^2*d*e
)^(1/2)*arctan((c^2*d*e)^(1/2)*x/(-c^2*d*e*x^2+d*e)^(1/2))+1/3*b^2*(d*(c*x
+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/c/d/e*arccos(
c*x)^3+a*b*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^
2-1)/c/d/e*arccos(c*x)^2
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{\sqrt{cdx+d}\sqrt{-cex+e}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sq
rt(-c*e*x + e)/(c^2*d*e*x^2 - d*e), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(a+b\operatorname{acos}(cx))^2}{\sqrt{d(cx+1)}\sqrt{-e(cx-1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(c*d*x+d)**(1/2)/(-c*e*x+e)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2}{\sqrt{cdx+d}\sqrt{-cex+e}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algorith
m="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/(sqrt(c*d*x + d)*sqrt(-c*e*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{\sqrt{d+c\,d\,x}\,\sqrt{e-c\,e\,x}}\,dx$$

input
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}}\,dx$$
$$= \frac{-2asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2+2\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)abc+\left(\int \frac{acos(cx)^2}{\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)b^2c}{\sqrt{e}\,\sqrt{d}\,c}$$

input
```
int((a+b*acos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x)
```

output
```
( - 2*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 + 2*int(acos(c*x)/(sqrt(c*x + 1)
*sqrt( - c*x + 1)),x)*a*b*c + int(acos(c*x)**2/(sqrt(c*x + 1)*sqrt( - c*x
+ 1)),x)*b**2*c)/(sqrt(e)*sqrt(d)*c)
```

3.591 $\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d+cdx}\sqrt{e-cex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 287

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d+cdx}\sqrt{e-cex}}\,dx = &-\frac{2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\text{arctanh}\left(e^{i\arccos(cx)}\right)}{\sqrt{d+cdx}\sqrt{e-cex}} \\ &+\frac{2ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{\sqrt{d+cdx}\sqrt{e-cex}} \\ &-\frac{2ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{\sqrt{d+cdx}\sqrt{e-cex}} \\ &-\frac{2b^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{\sqrt{d+cdx}\sqrt{e-cex}} \\ &+\frac{2b^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{\sqrt{d+cdx}\sqrt{e-cex}}\end{aligned}$$

output
```
-2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2*arctanh(c*x+I*(-c^2*x^2+1)^(1/2)
)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*
x))*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-
2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*polylog(2,c*x+I*(-c^2*x^2+1)^(1
/2))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2*b^2*(-c^2*x^2+1)^(1/2)*polylog(3,-
c*x-I*(-c^2*x^2+1)^(1/2))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*b^2*(-c^2*x^2
+1)^(1/2)*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))/(c*d*x+d)^(1/2)/(-c*e*x+e)^(
1/2)
```

Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.28

$$\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d+cdx}\sqrt{e-cex}}\,dx = \frac{a^2\log(cx)}{\sqrt{d}\sqrt{e}} - \frac{a^2\log\left(de+\sqrt{d}\sqrt{e}\sqrt{d+cdx}\sqrt{e-cex}\right)}{\sqrt{d}\sqrt{e}} - \frac{2ab\sqrt{1-c^2x^2}\left(\arccos(cx)\left(\log\left(1-ie^{i\arccos(cx)}\right)-\log\left(1+ie^{i\arccos(cx)}\right)\right)+i\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)\right.}{\sqrt{d+cdx}\sqrt{e-cex}} - \frac{b^2\sqrt{1-c^2x^2}\left(\arccos(cx)^2\log\left(1-ie^{i\arccos(cx)}\right)-\arccos(cx)^2\log\left(1+ie^{i\arccos(cx)}\right)+2i\arccos(cx)\,\mathrm{Poly}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]
```

output
```
(a^2*Log[c*x])/(Sqrt[d]*Sqrt[e]) - (a^2*Log[d*e + Sqrt[d]*Sqrt[e]*Sqrt[d +
 c*d*x]*Sqrt[e - c*e*x]])/(Sqrt[d]*Sqrt[e]) - (2*a*b*Sqrt[1 - c^2*x^2]*(Ar
cCos[c*x]*(Log[1 - I*E^(I*ArcCos[c*x])] - Log[1 + I*E^(I*ArcCos[c*x])]) +
I*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - I*PolyLog[2, I*E^(I*ArcCos[c*x])]))
/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]) - (b^2*Sqrt[1 - c^2*x^2]*(ArcCos[c*x]^2
*Log[1 - I*E^(I*ArcCos[c*x])] - ArcCos[c*x]^2*Log[1 + I*E^(I*ArcCos[c*x])]
 + (2*I)*ArcCos[c*x]*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - (2*I)*ArcCos[c*x
]*PolyLog[2, I*E^(I*ArcCos[c*x])] - 2*PolyLog[3, (-I)*E^(I*ArcCos[c*x])] +
 2*PolyLog[3, I*E^(I*ArcCos[c*x])]))/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.57, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5239, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x\sqrt{cdx+d}\sqrt{e-cex}}\,dx$$

$$\Big\downarrow \ 5239$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$\downarrow$ 5219

$$-\frac{\sqrt{1-c^2x^2}\int \frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$\downarrow$ 3042

$$-\frac{\sqrt{1-c^2x^2}\int (a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$\downarrow$ 4669

$$-\frac{\sqrt{1-c^2x^2}\left(-2b\int (a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)+2b\int (a+b\arccos(cx))\log\left(1+ie^{i\arccos\right.}\right.}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$\downarrow$ 3011

$$-\frac{\sqrt{1-c^2x^2}\left(2b\left(i\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int \operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\arccos(cx)\right)-\right.}{}$$

$\downarrow$ 2720

$$-\frac{\sqrt{1-c^2x^2}\left(2b\left(i\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)de^{i\,\mathrm{ar}}\right.\right.}{}$$

$\downarrow$ 7143

$$-\frac{\sqrt{1-c^2x^2}\left(-2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2+2b\left(i\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\right.\right.}{\sqrt{cdx+d}\sqrt{}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]
```

output
```
-((Sqrt[1 - c^2*x^2]*((-2*I)*(a + b*ArcCos[c*x])^2*ArcTan[E^(I*ArcCos[c*x]
)] + 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - b*Pol
yLog[3, (-I)*E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, I
*E^(I*ArcCos[c*x])] - b*PolyLog[3, I*E^(I*ArcCos[c*x])])))/(Sqrt[d + c*d*x
]*Sqrt[e - c*e*x]))
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 9.30 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.40

method	result
default	$\frac{i\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,\left(i\arccos(cx)^2\ln\left(1+i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)b^2-i\arccos(cx)^2\ln\left(1-i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)b^2+2i}{}$

input
```
int((a+b*arccos(c*x))^2/x/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
I*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(I*arccos(c*x)^2
*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b^2-I*arccos(c*x)^2*ln(1-I*(c*x+I*(-c^
2*x^2+1)^(1/2)))*b^2+2*I*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))*a*
b-2*I*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))*a*b+2*arccos(c*x)*pol
ylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b^2-2*arccos(c*x)*polylog(2,I*(c*x+I
*(-c^2*x^2+1)^(1/2)))*b^2-2*I*polylog(3,I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b^2+
2*I*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))*b^2+2*polylog(2,-I*(c*x+I*(-c
^2*x^2+1)^(1/2)))*a*b-2*a^2*arctan(c*x+I*(-c^2*x^2+1)^(1/2))-2*polylog(2,I
*(c*x+I*(-c^2*x^2+1)^(1/2)))*a*b)/d/e/(c^2*x^2-1)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{\sqrt{cdx+d}\sqrt{-cex+ex}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algori
thm="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sq
rt(-c*e*x + e)/(c^2*d*e*x^3 - d*e*x), x)
```

Sympy [F]

$$\int \frac{(a + b\arccos(cx))^2}{x\sqrt{d + cdx}\sqrt{e - cex}}\,dx = \int \frac{(a + b\operatorname{acos}(cx))^2}{x\sqrt{d(cx + 1)}\sqrt{-e(cx - 1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x/(c*d*x+d)**(1/2)/(-c*e*x+e)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(x*sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\arccos(cx))^2}{x\sqrt{d + cdx}\sqrt{e - cex}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/x/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{\sqrt{cdx+d}\sqrt{-cex+ex}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algori
thm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/(sqrt(c*d*x + d)*sqrt(-c*e*x + e)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x\,\sqrt{d+c\,d\,x}\,\sqrt{e-c\,e\,x}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x\sqrt{d+cdx}\sqrt{e-cex}}\,dx$$
$$=\frac{2\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,x}dx\right)ab+\left(\int \frac{acos(cx)^2}{\sqrt{cx+1}\,\sqrt{-cx+1}\,x}dx\right)b^2-\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)}{2}\right)-1\right)a^2+\log\left(\cdots\right.}{\cdots}$$

input
```
int((a+b*acos(c*x))^2/x/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x)
```

output
```
(2*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( - c*x + 1)*x),x)*a*b + int(acos(c*x)
**2/(sqrt(c*x + 1)*sqrt( - c*x + 1)*x),x)*b**2 - log( - sqrt(2) + tan(asin
(sqrt( - c*x + 1)/sqrt(2))/2) - 1)*a**2 + log( - sqrt(2) + tan(asin(sqrt(
- c*x + 1)/sqrt(2))/2) + 1)*a**2 - log(sqrt(2) + tan(asin(sqrt( - c*x + 1)
/sqrt(2))/2) - 1)*a**2 + log(sqrt(2) + tan(asin(sqrt( - c*x + 1)/sqrt(2))/
2) + 1)*a**2)/(sqrt(e)*sqrt(d))
```

3.592 $\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d+cdx}\sqrt{e-cex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 214

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d+cdx}\sqrt{e-cex}}\,dx = -\frac{ic\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{\sqrt{d+cdx}\sqrt{e-cex}} - \frac{(1-c^2x^2)\,(a+b\arccos(cx))^2}{x\sqrt{d+cdx}\sqrt{e-cex}} + \frac{2bc\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{\sqrt{d+cdx}\sqrt{e-cex}} - \frac{ib^2c\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{\sqrt{d+cdx}\sqrt{e-cex}}$$

output
```
-I*c*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/
2)-(-c^2*x^2+1)*(a+b*arccos(c*x))^2/x/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*b
*c*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)
/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-I*b^2*c*(-c^2*x^2+1)^(1/2)*polylog(2,(c*
x+I*(-c^2*x^2+1)^(1/2))^2)/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d+cdx}\sqrt{e-cex}}\,dx$$
$$= \frac{b^2\left(-1+c^2x^2+icx\sqrt{1-c^2x^2}\right)\arccos(cx)^2-2b\arccos(cx)\left(a-ac^2x^2+bcx\sqrt{1-c^2x^2}\log\left(1+e^{2i\arccos(}\right.\right.}{x\sqrt{d+cdx}\sqrt{e-}}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]
```

output
```
(b^2*(-1 + c^2*x^2 + I*c*x*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2 - 2*b*ArcCos[c
*x]*(a - a*c^2*x^2 + b*c*x*Sqrt[1 - c^2*x^2]*Log[1 + E^((2*I)*ArcCos[c*x])
]) + a*(-a + a*c^2*x^2 - 2*b*c*x*Sqrt[1 - c^2*x^2]*Log[c*x]) + I*b^2*c*x*S
qrt[1 - c^2*x^2]*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/(x*Sqrt[d + c*d*x]*Sq
rt[e - c*e*x])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.66, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {5239, 5187, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{cdx+d}\sqrt{e-cex}}\,dx$$

$$\Big\downarrow \text{5239}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{1-c^2x^2}}dx}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{5187}$$

$$\frac{\sqrt{1-c^2x^2}\left(-2bc\int \frac{a+b\arccos(cx)}{x}dx-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}\right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{5137}$$

$$\frac{\sqrt{1-c^2x^2}\left(2bc\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{cx}d\arccos(cx)-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}\right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{1-c^2x^2}\left(2bc\int (a+b\arccos(cx))\tan(\arccos(cx))d\arccos(cx)-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}\right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{4202}$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}+2bc\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1+e^{2i\arccos(cx)}}d\arccos(cx)\right)\right)}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}+2bc\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{2}ib\int \log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\frac{1}{2}i\log\right.\right.\right.}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}+2bc\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)de^{2i\arccos}\right.\right.\right.}{\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{x}+2bc\left(\frac{i(a+b\arccos(cx))^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}\right.\right.\right.}{\sqrt{cdx+d}\sqrt{e-cex}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]),x]
```

output
```
(Sqrt[1 - c^2*x^2]*(-((Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/x) + 2*b*c
*(((I/2)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((-1/2*I)*(a + b*ArcCos[c*x])*Lo
g[1 + E^((2*I)*ArcCos[c*x])] - (b*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/4)))
)/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x])
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5187
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Simp[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*A
rcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^
2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(212) = 424$.

Time = 2.07 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.04

method	result
default	$-\frac{a^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}}{dex}+b^2\left(-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\left(-i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\arccos(cx)^2}{edx(c^2x^2-1)}-\frac{i\sqrt{d(cx+1)}\,\sqrt{-e(c}{}\right.$
parts	$-\frac{a^2\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}}{dex}+b^2\left(-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\left(-i\sqrt{-c^2x^2+1}\,xc+c^2x^2-1\right)\arccos(cx)^2}{edx(c^2x^2-1)}-\frac{i\sqrt{d(cx+1)}\,\sqrt{-e(c}{}\right.$

input
```
int((a+b*arccos(c*x))^2/x^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-a^2*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/d/e/x+b^2*(-(d*(c*x+1))^(1/2)*(-
e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*arccos(c*x)^2/e/d/x
/(c^2*x^2-1)-I*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/e
/(c^2*x^2-1)*(2*I*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+2*arccos(
c*x)^2+polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2))*c)+2*a*b*(-2*I*(d*(c*x+1)
)^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/e/(c^2*x^2-1)*arccos(c*x)*
c-(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-
1)*arccos(c*x)/e/d/x/(c^2*x^2-1)+(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^
2*x^2+1)^(1/2)/d/e/(c^2*x^2-1)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)*c)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{\sqrt{cdx+d}\sqrt{-cex+ex^2}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(-(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sq
rt(-c*e*x + e)/(c^2*d*e*x^4 - d*e*x^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(a+b\operatorname{acos}(cx))^2}{x^2\sqrt{d(cx+1)}\sqrt{-e(cx-1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x**2/(c*d*x+d)**(1/2)/(-c*e*x+e)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(x**2*sqrt(d*(c*x + 1))*sqrt(-e*(c*x - 1))),
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2}{\sqrt{cdx+d}\sqrt{-cex+e}x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/(sqrt(c*d*x + d)*sqrt(-c*e*x + e)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d+cdx}\sqrt{e-cex}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x^2\,\sqrt{d+c\,d\,x}\,\sqrt{e-c\,e\,x}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x^2*(d + c*d*x)^(1/2)*(e - c*e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2\sqrt{d+cdx}\sqrt{e-cex}}\,dx$$
$$= \frac{-\sqrt{cx+1}\,\sqrt{-cx+1}\,a^2+2\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,x^2}dx\right)abx+\left(\int \frac{acos(cx)^2}{\sqrt{cx+1}\,\sqrt{-cx+1}\,x^2}dx\right)b^2x}{\sqrt{e}\,\sqrt{d}\,x}$$

input
```
int((a+b*acos(c*x))^2/x^2/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2),x)
```

output
```
( - sqrt(c*x + 1)*sqrt( - c*x + 1)*a**2 + 2*int(acos(c*x)/(sqrt(c*x + 1)*s
qrt( - c*x + 1)*x**2),x)*a*b*x + int(acos(c*x)**2/(sqrt(c*x + 1)*sqrt( - c
*x + 1)*x**2),x)*b**2*x)/(sqrt(e)*sqrt(d)*x)
```

3.593 $\int \frac{x^2(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 295

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \frac{x(a+b\arccos(cx))^2}{c^2de\sqrt{d+cdx}\sqrt{e-cex}} - \frac{i\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c^3de\sqrt{d+cdx}\sqrt{e-cex}} - \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{3bc^3de\sqrt{d+cdx}\sqrt{e-cex}} + \frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c^3de\sqrt{d+cdx}\sqrt{e-cex}} - \frac{ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c^3de\sqrt{d+cdx}\sqrt{e-cex}}$$

output
```
x*(a+b*arccos(c*x))^2/c^2/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-I*(-c^2*x^2
+1)^(1/2)*(a+b*arccos(c*x))^2/c^3/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-1/3
*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^3/b/c^3/d/e/(c*d*x+d)^(1/2)/(-c*e*x+
e)^(1/2)+2*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)
^(1/2))^2)/c^3/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-I*b^2*(-c^2*x^2+1)^(1/
2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c^3/d/e/(c*d*x+d)^(1/2)/(-c*e*
x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 2.75 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.19

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \frac{3a^2c\sqrt{d}ex + 3a^2\sqrt{e}\sqrt{d+cdx}\sqrt{e-cex}\arctan\left(\frac{cx\sqrt{d+cdx}\sqrt{e-cex}}{\sqrt{d}\sqrt{e}(-1+c^2x^2)}\right) + 3ab\sqrt{d}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x])^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)
),x]
```

output
```
(3*a^2*c*Sqrt[d]*e*x + 3*a^2*Sqrt[e]*Sqrt[d + c*d*x]*Sqrt[e - c*e*x]*ArcTa
n[(c*x*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(Sqrt[d]*Sqrt[e]*(-1 + c^2*x^2))]
+ 3*a*b*Sqrt[d]*e*(2*c*x*ArcCos[c*x] + Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2 - 2
*Sqrt[1 - c^2*x^2]*(Log[Cos[ArcCos[c*x]/2]] + Log[Sin[ArcCos[c*x]/2]])) +
b^2*Sqrt[d]*e*(ArcCos[c*x]*(3*c*x*ArcCos[c*x] + Sqrt[1 - c^2*x^2]*((3*I)*A
rcCos[c*x] + ArcCos[c*x]^2 - 6*(Log[1 - E^(I*ArcCos[c*x])] + Log[1 + E^(I*
ArcCos[c*x])]))) + (6*I)*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^(I*ArcCos[c*x])]
+ (6*I)*Sqrt[1 - c^2*x^2]*PolyLog[2, E^(I*ArcCos[c*x])]))/(3*c^3*d^(3/2)*e
^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.57, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {5239, 5207, 5153, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))^2}{(cdx+d)^{3/2}(e-cex)^{3/2}}\,dx$$

$$\Big\downarrow \text{5239}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{x^2(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{5207}$$

$$\frac{\sqrt{1-c^2x^2}\left(\frac{2b\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c}-\frac{\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}dx}{c^2}+\frac{x(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 5153}$$

$$\frac{\sqrt{1-c^2x^2}\left(\frac{2b\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx}{c}+\frac{(a+b\arccos(cx))^3}{3bc^3}+\frac{x(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 5181}$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{2b\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^3}+\frac{(a+b\arccos(cx))^3}{3bc^3}+\frac{x(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{2b\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{c^3}+\frac{(a+b\arccos(cx))^3}{3bc^3}+\frac{x(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{1-c^2x^2}\left(\frac{2b\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c^3}+\frac{(a+b\arccos(cx))^3}{3bc^3}+\frac{x(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 4200}$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{2b\left(2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c^3}+\frac{(a+b\arccos(cx))^3}{3bc^3}+\frac{x(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{2b\left(-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c^3}+\frac{(a+b\arccos(cx))^3}{3bc^3}+\frac{x(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c^3}+\frac{(a+b\arccos}{3bc}\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right.}{c^3}\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c^3}+\frac{(a+b\arccos(cx))^3}{3bc^3}+\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x])^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]
```

output
```
(Sqrt[1 - c^2*x^2]*((x*(a + b*ArcCos[c*x])^2)/(c^2*Sqrt[1 - c^2*x^2]) + (a
 + b*ArcCos[c*x])^3/(3*b*c^3) - (2*b*(((-1/2*I)*(a + b*ArcCos[c*x])^2)/b -
 (2*I)*((I/2)*(a + b*ArcCos[c*x])*Log[1 - E^((2*I)*ArcCos[c*x])] + (b*Poly
Log[2, E^((2*I)*ArcCos[c*x])])/4)))/c^3))/(d*e*Sqrt[d + c*d*x]*Sqrt[e - c*
e*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5207
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(2*e*(p + 1))), x] + (-Simp[f^2*((m - 1)/(2*e*(p + 1)))
 Int[(f*x)^(m - 2)*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp
[b*f*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m -
 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && IG
tQ[m, 1]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(285) = 570$.

Time = 8.75 (sec) , antiderivative size = 711, normalized size of antiderivative = 2.41

method	result
default	$\frac{a^2\left(-\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-(cx-1)ed(cx+1)}}\right)x^2c^2de+\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-(cx-1)ed(cx+1)}}\right)de-\sqrt{c^2de}\,\sqrt{-de(c^2x^2-1)}\,x\right)\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}}{d^2e^2(cx+1)\sqrt{c^2de}\,(cx-1)\sqrt{-de(c^2x^2-1)}\,c^2}+$
parts	$\frac{a^2\left(-\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-(cx-1)ed(cx+1)}}\right)x^2c^2de+\arctan\left(\frac{\sqrt{c^2de}\,x}{\sqrt{-(cx-1)ed(cx+1)}}\right)de-\sqrt{c^2de}\,\sqrt{-de(c^2x^2-1)}\,x\right)\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}}{d^2e^2(cx+1)\sqrt{c^2de}\,(cx-1)\sqrt{-de(c^2x^2-1)}\,c^2}+$

input
```
int(x^2*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x,method=_RET
URNVERBOSE)
```

output

```
a^2*(-arctan((c^2*d*e)^(1/2)*x/(-(c*x-1)*e*d*(c*x+1))^(1/2))*x^2*c^2*d*e+a
rctan((c^2*d*e)^(1/2)*x/(-(c*x-1)*e*d*(c*x+1))^(1/2))*d*e-(c^2*d*e)^(1/2)*
(-d*e*(c^2*x^2-1))^(1/2)*x)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/d^2/e^2/(
c*x+1)/(c^2*d*e)^(1/2)/(c*x-1)/(-d*e*(c^2*x^2-1))^(1/2)/c^2+b^2*(-1/3*(d*(
c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/e^2/d^2/c^3/(c^2*x^2-1
)*arccos(c*x)^3-(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+1)^(1/2
)+c*x)*arccos(c*x)^2/e^2/d^2/c^3/(c^2*x^2-1)-2*I*(-c^2*x^2+1)^(1/2)*(d*(c*
x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2
))+I*arccos(c*x)*ln((c*x+I*(-c^2*x^2+1)^(1/2))^(1/2)+1)+I*arccos(c*x)*ln(1
-(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2))+arccos(c*x)^2+polylog(2,-c*x-I*(-c^2*x^
2+1)^(1/2))+2*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2))+2*polylog(2,(c*
x+I*(-c^2*x^2+1)^(1/2))^(1/2)))/e^2/d^2/c^3/(c^2*x^2-1))-a*b*(-c^2*x^2+1)^
(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(2*I*arccos(c*x)*c^2*x^2+arccos
(c*x)^2*x^2*c^2-2*ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)*x^2*c^2-2*(-c^2*x^2+1
)^(1/2)*arccos(c*x)*x*c-2*I*arccos(c*x)-arccos(c*x)^2+2*ln((c*x+I*(-c^2*x^
2+1)^(1/2))^2-1))/(c^2*x^2-1)/d^2/e^2/c^3/(c*x+1)/(c*x-1)
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2x^2}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}}\,dx$$

input

```
integrate(x^2*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algo
rithm="fricas")
```

output

```
integral((b^2*x^2*arccos(c*x)^2 + 2*a*b*x^2*arccos(c*x) + a^2*x^2)*sqrt(c*
d*x + d)*sqrt(-c*e*x + e)/(c^4*d^2*e^2*x^4 - 2*c^2*d^2*e^2*x^2 + d^2*e^2),
 x)
```

Sympy [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \int \frac{x^2\left(a+b\operatorname{acos}\left(cx\right)\right)^2}{\left(d\left(cx+1\right)\right)^{\frac{3}{2}}\left(-e\left(cx-1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*acos(c*x))**2/(c*d*x+d)**(3/2)/(-c*e*x+e)**(3/2),x)
```

output
```
Integral(x**2*(a + b*acos(c*x))**2/((d*(c*x + 1))**(3/2)*(-e*(c*x - 1))**(
3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \int \frac{\left(b\arccos\left(cx\right)+a\right)^2x^2}{\left(cdx+d\right)^{\frac{3}{2}}\left(-cex+e\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2*x^2/((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{{\left(d+c\,d\,x\right)}^{3/2}\,{\left(e-c\,e\,x\right)}^{3/2}}\,dx$$

input
```
int((x^2*(a + b*acos(c*x))^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x)
```

output
```
int((x^2*(a + b*acos(c*x))^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\frac{2\sqrt{cx+1}\,\sqrt{-cx+1}\,asin\left(\frac{\sqrt{-cx+1}}{\sqrt{2}}\right)a^2-2\sqrt{cx+1}\,\sqrt{-cx+1}\left(\int\frac{}{\sqrt{cx+1}\,\sqrt{-}}\right.}{}$$

input
```
int(x^2*(a+b*acos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x)
```

output
```
(2*sqrt(c*x + 1)*sqrt( - c*x + 1)*asin(sqrt( - c*x + 1)/sqrt(2))*a**2 - 2*
sqrt(c*x + 1)*sqrt( - c*x + 1)*int((acos(c*x)*x**2)/(sqrt(c*x + 1)*sqrt( -
 c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a*b*c**3 - sqrt(c
*x + 1)*sqrt( - c*x + 1)*int((acos(c*x)**2*x**2)/(sqrt(c*x + 1)*sqrt( - c*
x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b**2*c**3 + a**2*c*x
)/(sqrt(e)*sqrt(d)*sqrt(c*x + 1)*sqrt( - c*x + 1)*c**3*d*e)
```

3.594 $\int \frac{x(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 244

$$\int \frac{x(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \frac{(a+b\arccos(cx))^2}{c^2de\sqrt{d+cdx}\sqrt{e-cex}} + \frac{4ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{c^2de\sqrt{d+cdx}\sqrt{e-cex}} - \frac{2ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{c^2de\sqrt{d+cdx}\sqrt{e-cex}} + \frac{2ib^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{c^2de\sqrt{d+cdx}\sqrt{e-cex}}$$

output

```
(a+b*arccos(c*x))^2/c^2/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+4*I*b*(-c^2*x
^2+1)^(1/2)*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/c^2/d/e/(c*
d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*(c*x
+I*(-c^2*x^2+1)^(1/2)))/c^2/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*I*b^2*(
-c^2*x^2+1)^(1/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/c^2/d/e/(c*d*x+d
)^(1/2)/(-c*e*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 2.96 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.03

$$\int \frac{x(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \frac{a^2+2ab\arccos(cx)+b^2\arccos(cx)^2-2b^2\sqrt{1-c^2x^2}\arccos(cx)\log\left(1-\right.}{}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x])^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),
x]
```

output
```
(a^2 + 2*a*b*ArcCos[c*x] + b^2*ArcCos[c*x]^2 - 2*b^2*Sqrt[1 - c^2*x^2]*Arc
Cos[c*x]*Log[1 - E^(I*ArcCos[c*x])] + 2*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*
Log[1 + E^(I*ArcCos[c*x])] + 2*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/2
]] - 2*a*b*Sqrt[1 - c^2*x^2]*Log[Sin[ArcCos[c*x]/2]] - (2*I)*b^2*Sqrt[1 -
c^2*x^2]*PolyLog[2, -E^(I*ArcCos[c*x])] + (2*I)*b^2*Sqrt[1 - c^2*x^2]*Poly
Log[2, E^(I*ArcCos[c*x])])/(c^2*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.55, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {5239, 5183, 5165, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))^2}{(cdx+d)^{3/2}(e-cex)^{3/2}}\,dx$$

$$\Big\downarrow \text{5239}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{x(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{5183}$$

$$\frac{\sqrt{1-c^2x^2}\left(\frac{2b\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx}{c}+\frac{(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \; 5165$$

$$\frac{\sqrt{1-c^2x^2}\left(\frac{(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}-\frac{2b\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c^2}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{1-c^2x^2}\left(\frac{(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}-\frac{2b\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)}{c^2}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \; 4671$$

$$\frac{\sqrt{1-c^2x^2}\left(\frac{(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}-\frac{2b\left(-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a\right.}{c^2}\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \; 2715$$

$$\frac{\sqrt{1-c^2x^2}\left(\frac{(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}-\frac{2b\left(ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos(cx)}\right.}{c^2}\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \; 2838$$

$$\frac{\sqrt{1-c^2x^2}\left(\frac{(a+b\arccos(cx))^2}{c^2\sqrt{1-c^2x^2}}-\frac{2b\left(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-ib\,\text{PolyLog}\left(2,e^{i\arccos(cx)}\right)\right)}{c^2}\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

input
```
Int[(x*(a + b*ArcCos[c*x])^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]
```

output
```
(Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^2/(c^2*Sqrt[1 - c^2*x^2]) - (2*b*(
-2*(a + b*ArcCos[c*x])*ArcTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(I*A
rcCos[c*x])] - I*b*PolyLog[2, E^(I*ArcCos[c*x])]))/c^2))/(d*e*Sqrt[d + c*d
*x]*Sqrt[e - c*e*x])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [A] (verified)

Time = 29.00 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.82

method	result
default	$\frac{a^2\sqrt{-e(cx-1)}\,\sqrt{d(cx+1)}}{(-cx-1)(cx-1)e^2d^2c^2}+b^2\left(-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\arccos(cx)^2}{d^2e^2c^2(cx+1)(cx-1)}+\frac{2i\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,\left(i\arccos(cx)\right.}{}\right.$
parts	$\frac{a^2\sqrt{-e(cx-1)}\,\sqrt{d(cx+1)}}{(-cx-1)(cx-1)e^2d^2c^2}+b^2\left(-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\arccos(cx)^2}{d^2e^2c^2(cx+1)(cx-1)}+\frac{2i\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\,\sqrt{-c^2x^2+1}\,\left(i\arccos(cx)\right.}{}\right.$

input
```
int(x*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
a^2/(-c*x-1)/(c*x-1)/e^2/d^2*(-e*(c*x-1))^(1/2)*(d*(c*x+1))^(1/2)/c^2+b^2*
(-(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/d^2/e^2/c^2/(c*x+1)/(c*x-1)*arccos(
c*x)^2+2*I*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(I*arcc
os(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-I*arccos(c*x)*ln((c*x+I*(-c^2*x^2+1
)^(1/2))^(1/2)+1)-I*arccos(c*x)*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2))+pol
ylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))-2*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^
(1/2))-2*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^(1/2)))/d^2/e^2/c^2/(c^2*x^2
-1))-2*a*b*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/d^2/e^2/c^2/(c*x+1)/(c*x-1
)*((-c^2*x^2+1)^(1/2)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))-ln(I*(-c^2*x^2+1)^(1/
2)+c*x-1)*(-c^2*x^2+1)^(1/2)+arccos(c*x))
```

Fricas [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2x}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algori
thm="fricas")
```

output
```
integral((b^2*x*arccos(c*x)^2 + 2*a*b*x*arccos(c*x) + a^2*x)*sqrt(c*d*x +
d)*sqrt(-c*e*x + e)/(c^4*d^2*e^2*x^4 - 2*c^2*d^2*e^2*x^2 + d^2*e^2), x)
```

Sympy [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \int \frac{x(a+b\operatorname{acos}(cx))^2}{(d(cx+1))^{\frac{3}{2}}(-e(cx-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*acos(c*x))**2/(c*d*x+d)**(3/2)/(-c*e*x+e)**(3/2),x)
```

output
```
Integral(x*(a + b*acos(c*x))**2/((d*(c*x + 1))**(3/2)*(-e*(c*x - 1))**(3/2
)), x)
```

Maxima [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algori
thm="maxima")
```

output
```
sqrt(d)*sqrt(e)*integrate((b^2*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x
)^2 + 2*a*b*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*sqrt(c*x + 1)*sq
rt(-c*x + 1)/(c^4*d^2*e^2*x^4 - 2*c^2*d^2*e^2*x^2 + d^2*e^2), x) + a^2/(sq
rt(-c^2*d*e*x^2 + d*e)*c^2*d*e)
```

Giac [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2x}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algori
thm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2*x/((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{x\,(a+b\,\mathrm{acos}(c\,x))^2}{(d+c\,d\,x)^{3/2}\,(e-c\,e\,x)^{3/2}}\,dx$$

input
```
int((x*(a + b*acos(c*x))^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x)
```

output
```
int((x*(a + b*acos(c*x))^2)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\frac{-2\sqrt{cx+1}\,\sqrt{-cx+1}\left(\int \frac{acos(cx)x}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2-\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)ab\,c^2-\sqrt{cx}}{\sqrt{e}\,\sqrt{d}\,\sqrt{cx+1}\,\sqrt{-c}}$$

input
```
int(x*(a+b*acos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x)
```

output
```
( - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*int((acos(c*x)*x)/(sqrt(c*x + 1)*sqrt
( - c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a*b*c**2 - sqr
t(c*x + 1)*sqrt( - c*x + 1)*int((acos(c*x)**2*x)/(sqrt(c*x + 1)*sqrt( - c*
x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b**2*c**2 + a**2)/(s
qrt(e)*sqrt(d)*sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*d*e)
```

3.595 $\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 217

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \frac{x(1-c^2x^2)(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{i(1-c^2x^2)^{3/2}(a+b\arccos(cx))^2}{c(d+cdx)^{3/2}(e-cex)^{3/2}} + \frac{2b(1-c^2x^2)^{3/2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}} - \frac{ib^2(1-c^2x^2)^{3/2}\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{c(d+cdx)^{3/2}(e-cex)^{3/2}}$$

output
```
x*(-c^2*x^2+1)*(a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-I*(-c^
2*x^2+1)^(3/2)*(a+b*arccos(c*x))^2/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)+2*b*
(-c^2*x^2+1)^(3/2)*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/c/
(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)-I*b^2*(-c^2*x^2+1)^(3/2)*polylog(2,-(c*x+
I*(-c^2*x^2+1)^(1/2))^2)/c/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \frac{a^2cx+2abcx\arccos(cx)+b^2cx\arccos(cx)^2+ib^2\sqrt{1-c^2x^2}\arccos(cx)^2-$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]
```

output
```
(a^2*c*x + 2*a*b*c*x*ArcCos[c*x] + b^2*c*x*ArcCos[c*x]^2 + I*b^2*Sqrt[1 -
c^2*x^2]*ArcCos[c*x]^2 - 2*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 - E^(I*
ArcCos[c*x])] - 2*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*
x])] - 2*a*b*Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/2]] - 2*a*b*Sqrt[1 - c^
2*x^2]*Log[Sin[ArcCos[c*x]/2]] + (2*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, -E
^(I*ArcCos[c*x])] + (2*I)*b^2*Sqrt[1 - c^2*x^2]*PolyLog[2, E^(I*ArcCos[c*x
])])/(c*d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.65, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5179, 5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(cdx+d)^{3/2}(e-cex)^{3/2}}dx$$

$$\Big\downarrow \text{5179}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\int \frac{(a+b\arccos(cx))^2}{\left(1-c^2x^2\right)^{3/2}}dx}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{5161}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(2bc\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{5181}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c}\right)}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{c}\right)}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(\frac{2b\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c}+\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{4200}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c}\right)}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c}\right)}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int \log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)\right)-\frac{i(a+b\,\mathrm{ar}}{}}{c}\right.}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\,\mathrm{arcco}}\right.\right.}{c}\right.}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\left(1-c^2x^2\right)^{3/2}\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\right)-\frac{i(a+b\arccos(cx))^2}{2b}\right.}{c}\right.}{(cdx+d)^{3/2}(e-cex)^{3/2}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]
```

output
```
((1 - c^2*x^2)^(3/2)*((x*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2] - (2*b*(
((-1/2*I)*(a + b*ArcCos[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])*Log[
1 - E^((2*I)*ArcCos[c*x])] + (b*PolyLog[2, E^((2*I)*ArcCos[c*x])])/4)))/c)
)/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5179
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(p_)*((f_)
 + (g_.)*(x_))^(q_), x_Symbol] :> Simp[(d + e*x)^q*((f + g*x)^q/(1 - c^2*x^
2)^q)   Int[(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, n}, x] && EqQ[e*f + d*g, 0] && EqQ[c^2*d^2
- e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 459 vs. 2(215) = 430.

Time = 1.60 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.12

method	result
default	$a^2\left(-\frac{1}{dce\sqrt{cdx+d}\sqrt{-cex+e}}+\frac{\sqrt{cdx+d}}{ce\,d^2\sqrt{-cex+e}}\right)+b^2\left(-\frac{\sqrt{d(cx+1)}\sqrt{-e(cx-1)}\left(-i\sqrt{-c^2x^2+1}+cx\right)\arccos(cx)^2}{(c^2x^2-1)c\,d^2e^2}-\frac{2i\sqrt{}}{}\right.$
parts	$a^2\left(-\frac{1}{dce\sqrt{cdx+d}\sqrt{-cex+e}}+\frac{\sqrt{cdx+d}}{ce\,d^2\sqrt{-cex+e}}\right)+b^2\left(-\frac{\sqrt{d(cx+1)}\sqrt{-e(cx-1)}\left(-i\sqrt{-c^2x^2+1}+cx\right)\arccos(cx)^2}{(c^2x^2-1)c\,d^2e^2}-\frac{2i\sqrt{}}{}\right.$

input
```
int((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
a^2*(-1/d/c/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+1/c/e/d^2/(-c*e*x+e)^(1/2)*
(c*d*x+d)^(1/2))+b^2*(-(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-I*(-c^2*x^2+
1)^(1/2)+c*x)*arccos(c*x)^2/(c^2*x^2-1)/c/d^2/e^2-2*I*(-c^2*x^2+1)^(1/2)*(
d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)
^(1/2))+I*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+1)^(1/2))+arccos(c*x)^2+polylog
(2,-c*x-I*(-c^2*x^2+1)^(1/2))+polylog(2,c*x+I*(-c^2*x^2+1)^(1/2)))/(c^2*x^
2-1)/c/d^2/e^2)-2*a*b*(-c^2*x^2+1)^(1/2)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1
/2)*(I*arccos(c*x)*x^2*c^2-ln((c*x+I*(-c^2*x^2+1)^(1/2))^2-1)*x^2*c^2-(-c^
2*x^2+1)^(1/2)*arccos(c*x)*x*c-I*arccos(c*x)+ln((c*x+I*(-c^2*x^2+1)^(1/2))
^2-1))/(c^2*x^2-1)/c/d^2/e^2/(c*x+1)/(c*x-1)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorith
m="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sqr
t(-c*e*x + e)/(c^4*d^2*e^2*x^4 - 2*c^2*d^2*e^2*x^2 + d^2*e^2), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{(a+b\operatorname{acos}{(cx)})^2}{(d(cx+1))^{\frac{3}{2}}(-e(cx-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(c*d*x+d)**(3/2)/(-c*e*x+e)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/((d*(c*x + 1))**(3/2)*(-e*(c*x - 1))**(3/2))
, x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \int \frac{(b\arccos{(cx)}+a)^2}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorith
m="maxima")
```

output
```
-b^2*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2/((c^2*d*e*x^2
- d*e)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)/(sqrt(d)*sqrt(e)) + 2*a*b*x*arcco
s(c*x)/(sqrt(-c^2*d*e*x^2 + d*e)*d*e) + a^2*x/(sqrt(-c^2*d*e*x^2 + d*e)*d*
e) + a*b*sqrt(1/(d*e))*log(x^2 - 1/c^2)/(c*d*e)
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \int \frac{(b\arccos{(cx)}+a)^2}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algorith
m="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(d+c\,d\,x)^{3/2}\,(e-c\,e\,x)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))^2/((d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \frac{-2\sqrt{cx+1}\,\sqrt{-cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^2-\sqrt{cx+1}\,\sqrt{-cx+1}}dx\right)ab-\sqrt{cx+}}{\sqrt{e}\,\sqrt{d}\,\sqrt{cx+1}\,\sqrt{-cx}}$$

input
```
int((a+b*acos(c*x))^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x)
```

output
```
( - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( -
c*x + 1)*c**2*x**2 - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*a*b - sqrt(c*x + 1
)*sqrt( - c*x + 1)*int(acos(c*x)**2/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2*x
**2 - sqrt(c*x + 1)*sqrt( - c*x + 1)),x)*b**2 + a**2*x)/(sqrt(e)*sqrt(d)*s
qrt(c*x + 1)*sqrt( - c*x + 1)*d*e)
```

3.596 $\int \frac{(a+b\arccos(cx))^2}{x(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 548

$$\begin{aligned}
\int \frac{(a+b\arccos(cx))^2}{x(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx &= \frac{(a+b\arccos(cx))^2}{de\sqrt{d+cdx}\sqrt{e-cex}} \\
&+ \frac{4ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right)}{de\sqrt{d+cdx}\sqrt{e-cex}} \\
&- \frac{2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2\operatorname{arctanh}\left(e^{i\arccos(cx)}\right)}{de\sqrt{d+cdx}\sqrt{e-cex}} \\
&+ \frac{2ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{i\arccos(cx)}\right)}{de\sqrt{d+cdx}\sqrt{e-cex}} \\
&- \frac{2ib^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)}{de\sqrt{d+cdx}\sqrt{e-cex}} \\
&+ \frac{2ib^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)}{de\sqrt{d+cdx}\sqrt{e-cex}} \\
&- \frac{2ib\sqrt{1-c^2x^2}(a+b\arccos(cx))\operatorname{PolyLog}\left(2,e^{i\arccos(cx)}\right)}{de\sqrt{d+cdx}\sqrt{e-cex}} \\
&- \frac{2b^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arccos(cx)}\right)}{de\sqrt{d+cdx}\sqrt{e-cex}} \\
&+ \frac{2b^2\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(3,e^{i\arccos(cx)}\right)}{de\sqrt{d+cdx}\sqrt{e-cex}}
\end{aligned}$$

output
```
(a+b*arccos(c*x))^2/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+4*I*b*(-c^2*x^2+1
)^(1/2)*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^(1/2))/d/e/(c*d*x+d)^(
1/2)/(-c*e*x+e)^(1/2)-2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^2*arctanh(c*x
+I*(-c^2*x^2+1)^(1/2))/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*I*b*(-c^2*x^
2+1)^(1/2)*(a+b*arccos(c*x))*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2))/d/e/(c*d
*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2*I*b^2*(-c^2*x^2+1)^(1/2)*polylog(2,-I*(c*x+
I*(-c^2*x^2+1)^(1/2)))/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*I*b^2*(-c^2*
x^2+1)^(1/2)*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))/d/e/(c*d*x+d)^(1/2)/(
-c*e*x+e)^(1/2)-2*I*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*polylog(2,c*x+I
*(-c^2*x^2+1)^(1/2))/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2*b^2*(-c^2*x^2+
1)^(1/2)*polylog(3,-c*x-I*(-c^2*x^2+1)^(1/2))/d/e/(c*d*x+d)^(1/2)/(-c*e*x+
e)^(1/2)+2*b^2*(-c^2*x^2+1)^(1/2)*polylog(3,c*x+I*(-c^2*x^2+1)^(1/2))/d/e/
(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 5.38 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\arccos(cx))^2}{x(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx =$$

$$-\frac{\frac{a^2\sqrt{d+cdx}\sqrt{e-cex}}{-1+c^2x^2} - a^2\sqrt{d}\sqrt{e}\log(cx) + a^2\sqrt{d}\sqrt{e}\log\left(de+\sqrt{d}\sqrt{e}\sqrt{d+cdx}\sqrt{e-cex}\right) + \frac{2abde\left(-\arccos(cx)+\sqrt{1\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]
```

output
```
-(((a^2*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])/(-1 + c^2*x^2) - a^2*Sqrt[d]*Sqrt
[e]*Log[c*x] + a^2*Sqrt[d]*Sqrt[e]*Log[d*e + Sqrt[d]*Sqrt[e]*Sqrt[d + c*d*
x]*Sqrt[e - c*e*x]] + (2*a*b*d*e*(-ArcCos[c*x] + Sqrt[1 - c^2*x^2]*ArcCos[
c*x]*Log[1 - I*E^(I*ArcCos[c*x])] - Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 +
I*E^(I*ArcCos[c*x])] - Sqrt[1 - c^2*x^2]*Log[Cos[ArcCos[c*x]/2]] + Sqrt[1
- c^2*x^2]*Log[Sin[ArcCos[c*x]/2]] + I*Sqrt[1 - c^2*x^2]*PolyLog[2, (-I)*E
^(I*ArcCos[c*x])] - I*Sqrt[1 - c^2*x^2]*PolyLog[2, I*E^(I*ArcCos[c*x])]))/
(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]) + (b^2*d*e*(-ArcCos[c*x]^2 + 2*Sqrt[1 -
c^2*x^2]*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c*x])] + Sqrt[1 - c^2*x^2]*ArcCos
[c*x]^2*Log[1 - I*E^(I*ArcCos[c*x])] - Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2*Log
[1 + I*E^(I*ArcCos[c*x])] - 2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 + E^(I*A
rcCos[c*x])] + (2*I)*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^(I*ArcCos[c*x])] + (2
*I)*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - (2*
I)*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*PolyLog[2, I*E^(I*ArcCos[c*x])] - (2*I)*S
qrt[1 - c^2*x^2]*PolyLog[2, E^(I*ArcCos[c*x])] - 2*Sqrt[1 - c^2*x^2]*PolyL
og[3, (-I)*E^(I*ArcCos[c*x])] + 2*Sqrt[1 - c^2*x^2]*PolyLog[3, I*E^(I*ArcC
os[c*x])]))/(Sqrt[d + c*d*x]*Sqrt[e - c*e*x]))/(d^2*e^2))
```

Rubi [A] (verified)

Time = 2.42 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.47, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {5239, 5209, 5165, 3042, 4671, 2715, 2838, 5219, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x(cdx+d)^{3/2}(e-cex)^{3/2}}dx$$

$$\Big\downarrow \text{ 5239}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{(a+b\arccos(cx))^2}{x(1-c^2x^2)^{3/2}}dx}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 5209}$$

$$\frac{\sqrt{1-c^2x^2}\left(2bc\int \frac{a+b\arccos(cx)}{1-c^2x^2}dx+\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx+\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 5165}$$

$$\frac{\sqrt{1-c^2x^2}\left(-2b\int \frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}d\arccos(cx)+\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx+\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$\downarrow$ 3042

$$\frac{\sqrt{1-c^2x^2}\left(\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx-2b\int (a+b\arccos(cx))\csc(\arccos(cx))d\arccos(cx)+\frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$\downarrow$ 4671

$$\frac{\sqrt{1-c^2x^2}\Big(-2b\big(-b\int \log\left(1-e^{i\arccos(cx)}\right)d\arccos(cx)+b\int \log\left(1+e^{i\arccos(cx)}\right)d\arccos(cx)-2\text{arctanh}\big(e^{i\arccos}}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$\downarrow$ 2715

$$\frac{\sqrt{1-c^2x^2}\Big(-2b\big(ib\int e^{-i\arccos(cx)}\log\left(1-e^{i\arccos(cx)}\right)de^{i\arccos(cx)}-ib\int e^{-i\arccos(cx)}\log\left(1+e^{i\arccos(cx)}\right)de^{i\arccos}}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$\downarrow$ 2838

$$\frac{\sqrt{1-c^2x^2}\Big(\int \frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}dx-2b\big(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i\arccos(cx)}\right)-}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$\downarrow$ 5219

$$\frac{\sqrt{1-c^2x^2}\Big(-\int \frac{(a+b\arccos(cx))^2}{cx}d\arccos(cx)-2b\big(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyLog}\left(2,-e^{i}\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$\downarrow$ 3042

$$\frac{\sqrt{1-c^2x^2}\Big(-\int (a+b\arccos(cx))^2\csc\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)-2b\big(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$\downarrow$ 4669

$$\frac{\sqrt{1-c^2x^2}\Big(2b\int (a+b\arccos(cx))\log\left(1-ie^{i\arccos(cx)}\right)d\arccos(cx)-2b\int (a+b\arccos(cx))\log\left(1+ie^{i\arccos(cx)}\right.}{}$$

$\downarrow$ 3011

$$\sqrt{1-c^2x^2}\Big(-2b\big(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-ib\int\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)d\arccos(cx)\big)+$$

$\big\downarrow$ 2720

$$\sqrt{1-c^2x^2}\Big(-2b\big(i\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))-b\int e^{-i\arccos(cx)}\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)de^{i\,\text{ar}}$$

$\big\downarrow$ 7143

$$\sqrt{1-c^2x^2}\Big(2i\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2-2b\big(-2\text{arctanh}\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))+ib\,\text{PolyL}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]
```

output
```
(Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^2/Sqrt[1 - c^2*x^2] + (2*I)*(a + b
*ArcCos[c*x])^2*ArcTan[E^(I*ArcCos[c*x])] - 2*b*(-2*(a + b*ArcCos[c*x])*Ar
cTanh[E^(I*ArcCos[c*x])] + I*b*PolyLog[2, -E^(I*ArcCos[c*x])] - I*b*PolyLo
g[2, E^(I*ArcCos[c*x])]) - 2*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I
*ArcCos[c*x])] - b*PolyLog[3, (-I)*E^(I*ArcCos[c*x])]) + 2*b*(I*(a + b*Arc
Cos[c*x])*PolyLog[2, I*E^(I*ArcCos[c*x])] - b*PolyLog[3, I*E^(I*ArcCos[c*x
])])))/(d*e*Sqrt[d + c*d*x]*Sqrt[e - c*e*x])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5165
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(c*d)^(-1)   Subst[Int[(a + b*x)^n*Csc[x], x], x, ArcCos[c*x]],
 x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5209
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x^2)^(p + 1)*((a
+ b*ArcCos[c*x])^n/(2*d*f*(p + 1))), x] + (Simp[(m + 2*p + 3)/(2*d*(p + 1))
   Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Simp[b*c
*(n/(2*f*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*
(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1])
```

rule 5219
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]]   Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 961, normalized size of antiderivative = 1.75

method	result
default	$\frac{a^2\left(-\ln\left(\frac{2\sqrt{de}\,\sqrt{-de\left(c^2x^2-1\right)}+2de}{x}\right)x^2c^2de+de\ln\left(\frac{2\sqrt{de}\,\sqrt{-de\left(c^2x^2-1\right)}+2de}{x}\right)-\sqrt{de}\,\sqrt{-de(c^2x^2-1)}\right)\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}}{d^2e^2(cx+1)(cx-1)\sqrt{de}\,\sqrt{-de(c^2x^2-1)}}$
parts	$\frac{a^2\left(-\ln\left(\frac{2\sqrt{de}\,\sqrt{-de\left(c^2x^2-1\right)}+2de}{x}\right)x^2c^2de+de\ln\left(\frac{2\sqrt{de}\,\sqrt{-de\left(c^2x^2-1\right)}+2de}{x}\right)-\sqrt{de}\,\sqrt{-de(c^2x^2-1)}\right)\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}}{d^2e^2(cx+1)(cx-1)\sqrt{de}\,\sqrt{-de(c^2x^2-1)}}$

input
```
int((a+b*arccos(c*x))^2/x/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
a^2*(-ln(2*((d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2)+d*e)/x)*x^2*c^2*d*e+d*e*l
n(2*((d*e)^(1/2)*(-d*e*(c^2*x^2-1))^(1/2)+d*e)/x)-(d*e)^(1/2)*(-d*e*(c^2*x
^2-1))^(1/2))*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/d^2/e^2/(c*x+1)/(c*x-1)
/(d*e)^(1/2)/(-d*e*(c^2*x^2-1))^(1/2)+b^2*(-(d*(c*x+1))^(1/2)*(-e*(c*x-1))
^(1/2)/(c*x-1)/(c*x+1)/e^2/d^2*arccos(c*x)^2+(d*(c*x+1))^(1/2)*(-e*(c*x-1)
)^(1/2)*(-c^2*x^2+1)^(1/2)*(arccos(c*x)^2*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)
))-arccos(c*x)^2*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*I*arccos(c*x)*polylo
g(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*I*arccos(c*x)*polylog(2,-I*(c*x+I*(-c^
2*x^2+1)^(1/2)))-2*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+2*I*dilog(c*
x+I*(-c^2*x^2+1)^(1/2))+2*I*dilog(1+c*x+I*(-c^2*x^2+1)^(1/2))+2*polylog(3,
I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2))))/
d^2/e^2/(c^2*x^2-1))+2*a*b*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-c^2*x^2+
1)^(1/2)*(I*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))*c^2*x^2-I*dilog(1-I*(c*x
+I*(-c^2*x^2+1)^(1/2)))*c^2*x^2+arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/
2)))*x^2*c^2-arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))*x^2*c^2+ln(I*(
-c^2*x^2+1)^(1/2)+c*x-1)*x^2*c^2-ln(1+c*x+I*(-c^2*x^2+1)^(1/2))*x^2*c^2-I*
dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+I*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2
)))+arccos(c*x)*(-c^2*x^2+1)^(1/2)-arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^
(1/2)))+arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-ln(I*(-c^2*x^2+1)^(
1/2)+c*x-1)+ln(1+c*x+I*(-c^2*x^2+1)^(1/2)))/(c*x-1)/(c*x+1)/e^2/d^2/(c^...
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{(b\arccos{(cx)}+a)^2}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algori
thm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sqr
t(-c*e*x + e)/(c^4*d^2*e^2*x^5 - 2*c^2*d^2*e^2*x^3 + d^2*e^2*x), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}\,(cx))^2}{x\,(d\,(cx+1))^{\frac{3}{2}}\,(-e\,(cx-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/x/(c*d*x+d)**(3/2)/(-c*e*x+e)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(x*(d*(c*x + 1))**(3/2)*(-e*(c*x - 1))**(3/2
)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/x/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{x(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{(b\arccos\,(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algori
thm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)*x),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x\,(d+c\,d\,x)^{3/2}\,(e-c\,e\,x)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\frac{-2\sqrt{cx+1}\,\sqrt{-cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^3-\sqrt{cx+1}\,\sqrt{-cx+1}\,x}dx\right)ab-\sqrt{cx}}{}$$

input
```
int((a+b*acos(c*x))^2/x/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x)
```

output
```
( - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( -
c*x + 1)*c**2*x**3 - sqrt(c*x + 1)*sqrt( - c*x + 1)*x),x)*a*b - sqrt(c*x +
 1)*sqrt( - c*x + 1)*int(acos(c*x)**2/(sqrt(c*x + 1)*sqrt( - c*x + 1)*c**2
*x**3 - sqrt(c*x + 1)*sqrt( - c*x + 1)*x),x)*b**2 - sqrt(c*x + 1)*sqrt( -
c*x + 1)*log( - sqrt(2) + tan(asin(sqrt( - c*x + 1)/sqrt(2))/2) - 1)*a**2
+ sqrt(c*x + 1)*sqrt( - c*x + 1)*log( - sqrt(2) + tan(asin(sqrt( - c*x + 1
)/sqrt(2))/2) + 1)*a**2 - sqrt(c*x + 1)*sqrt( - c*x + 1)*log(sqrt(2) + tan
(asin(sqrt( - c*x + 1)/sqrt(2))/2) - 1)*a**2 + sqrt(c*x + 1)*sqrt( - c*x +
 1)*log(sqrt(2) + tan(asin(sqrt( - c*x + 1)/sqrt(2))/2) + 1)*a**2 + a**2)/
(sqrt(e)*sqrt(d)*sqrt(c*x + 1)*sqrt( - c*x + 1)*d*e)
```

3.597 $\int \frac{(a+b\arccos(cx))^2}{x^2(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 396

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x^2(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx &= -\frac{(a+b\arccos(cx))^2}{dex\sqrt{d+cdx}\sqrt{e-cex}}\\
&+\frac{2c^2x(a+b\arccos(cx))^2}{de\sqrt{d+cdx}\sqrt{e-cex}}-\frac{2ic\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{de\sqrt{d+cdx}\sqrt{e-cex}}\\
&-\frac{4bc\sqrt{1-c^2x^2}(a+b\arccos(cx))\text{arctanh}\left(e^{2i\arccos(cx)}\right)}{de\sqrt{d+cdx}\sqrt{e-cex}}\\
&+\frac{4bc\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)}{de\sqrt{d+cdx}\sqrt{e-cex}}\\
&-\frac{ib^2c\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{de\sqrt{d+cdx}\sqrt{e-cex}}\\
&-\frac{ib^2c\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{de\sqrt{d+cdx}\sqrt{e-cex}}\end{aligned}$$

output
```
-(a+b*arccos(c*x))^2/d/e/x/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)+2*c^2*x*(a+b*a
rccos(c*x))^2/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-2*I*c*(-c^2*x^2+1)^(1/2
)*(a+b*arccos(c*x))^2/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-4*b*c*(-c^2*x^2
+1)^(1/2)*(a+b*arccos(c*x))*arctanh((c*x+I*(-c^2*x^2+1)^(1/2))^2)/d/e/(c*d
*x+d)^(1/2)/(-c*e*x+e)^(1/2)+4*b*c*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))*ln
(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)-I*b^
2*c*(-c^2*x^2+1)^(1/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d/e/(c*d*x
+d)^(1/2)/(-c*e*x+e)^(1/2)-I*b^2*c*(-c^2*x^2+1)^(1/2)*polylog(2,(c*x+I*(-c
^2*x^2+1)^(1/2))^2)/d/e/(c*d*x+d)^(1/2)/(-c*e*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \frac{-a^2+2a^2c^2x^2-b^2\arccos(cx)^2+2b^2c^2x^2\arccos(cx)^2+2ib^2cx\sqrt{1-c^2}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(x^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),
x]
```

output
```
(-a^2 + 2*a^2*c^2*x^2 - b^2*ArcCos[c*x]^2 + 2*b^2*c^2*x^2*ArcCos[c*x]^2 +
(2*I)*b^2*c*x*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2 + 2*a*b*ArcCos[c*x]*Cos[2*Ar
cCos[c*x]] - 2*b^2*c*x*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 - E^(I*ArcCos[c
*x])] - 2*b^2*c*x*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 + E^(I*ArcCos[c*x])]
 - 2*b^2*c*x*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])]
- 2*a*b*c*x*Sqrt[1 - c^2*x^2]*Log[c*x] - 2*a*b*c*x*Sqrt[1 - c^2*x^2]*Log[C
os[ArcCos[c*x]/2]] - 2*a*b*c*x*Sqrt[1 - c^2*x^2]*Log[Sin[ArcCos[c*x]/2]] +
 (2*I)*b^2*c*x*Sqrt[1 - c^2*x^2]*PolyLog[2, -E^(I*ArcCos[c*x])] + (2*I)*b^
2*c*x*Sqrt[1 - c^2*x^2]*PolyLog[2, E^(I*ArcCos[c*x])] + I*b^2*c*x*Sqrt[1 -
 c^2*x^2]*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/(d*e*x*Sqrt[d + c*d*x]*Sqrt[
e - c*e*x])
```

Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.62, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.486, Rules used = {5239, 5205, 5161, 5181, 3042, 25, 4200, 25, 2620, 2715, 2838, 5185, 4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{x^2(cdx+d)^{3/2}(e-cex)^{3/2}}\,dx$$

$$\Big\downarrow \text{5239}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{(a+b\arccos(cx))^2}{x^2(1-c^2x^2)^{3/2}}dx}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{5205}$$

$$\frac{\sqrt{1-c^2x^2}\left(2c^2\int \frac{(a+b\arccos(cx))^2}{(1-c^2x^2)^{3/2}}dx-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx-\frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{5161}$$

$$\frac{\sqrt{1-c^2x^2}\left(2c^2\left(2bc\int \frac{x(a+b\arccos(cx))}{1-c^2x^2}dx+\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx-\frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{5181}$$

$$\frac{\sqrt{1-c^2x^2}\left(2c^2\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\int \frac{cx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}d\arccos(cx)}{c}\right)-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx-\frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}\right)}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{1-c^2x^2}\left(-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+2c^2\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\int -\left((a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)\right)d\arccos(cx)}{c}\right)-\frac{(a+}{}\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{1-c^2x^2}\left(-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+2c^2\left(\frac{2b\int (a+b\arccos(cx))\tan\left(\arccos(cx)+\frac{\pi}{2}\right)d\arccos(cx)}{c}+\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\right)-\frac{(a+b\operatorname{ar}}{x\sqrt{1}}\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 4200}$$

$$\frac{\sqrt{1-c^2x^2}\left(2c^2\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(2i\int -\frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c}\right)-2bc\int \frac{a+b\arccos(c}{x(1-c^2x^2)}\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{1-c^2x^2}\left(2c^2\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\int \frac{e^{2i\arccos(cx)}(a+b\arccos(cx))}{1-e^{2i\arccos(cx)}}d\arccos(cx)-\frac{i(a+b\arccos(cx))^2}{2b}\right)}{c}\right)-2bc\int \frac{a+b\arccos(c}{x(1-c^2x^2)}\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{\sqrt{1-c^2x^2}\left(-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+2c^2\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{2}ib\int \log(1-e\right.\right.}{c}\right.\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{\sqrt{1-c^2x^2}\left(-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+2c^2\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))-\frac{1}{4}b\int e^{-2i\operatorname{arcco}}\right.\right.}{c}\right.\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\sqrt{1-c^2x^2}\left(-2bc\int \frac{a+b\arccos(cx)}{x(1-c^2x^2)}dx+2c^2\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\operatorname{PolyLog}(2,\right.\right.}{c}\right.\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$$\Big\downarrow \text{ 5185}$$

$$\frac{\sqrt{1-c^2x^2}\left(2bc\int \frac{a+b\arccos(cx)}{cx\sqrt{1-c^2x^2}}d\arccos(cx)+2c^2\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))+\frac{1}{4}b\,\mathrm{P}\right.}{c}\right.\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$\downarrow$ 4919

$$\frac{\sqrt{1-c^2x^2}\left(4bc\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)+2c^2\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos}\right.\right.\right.}{}\right.\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$\downarrow$ 3042

$$\frac{\sqrt{1-c^2x^2}\left(4bc\int(a+b\arccos(cx))\csc(2\arccos(cx))d\arccos(cx)+2c^2\left(\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}-\frac{2b\left(-2i\left(\frac{1}{2}i\log\left(1-e^{2i\arccos}\right.\right.\right.}{}\right.\right.}{de\sqrt{cdx+d}\sqrt{e-cex}}$$

$\downarrow$ 4671

$$\sqrt{1-c^2x^2}\left(4bc\left(-\frac{1}{2}b\int\log\left(1-e^{2i\arccos(cx)}\right)d\arccos(cx)+\frac{1}{2}b\int\log\left(1+e^{2i\arccos(cx)}\right)d\arccos(cx)-\left(\operatorname{arctanh}\left(e\right.\right.\right.\right.$$

$\downarrow$ 2715

$$\sqrt{1-c^2x^2}\left(4bc\left(\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1-e^{2i\arccos(cx)}\right)de^{2i\arccos(cx)}-\frac{1}{4}ib\int e^{-2i\arccos(cx)}\log\left(1+e^{2i\arccos(cx)}\right)\right.\right.$$

$\downarrow$ 2838

$$\sqrt{1-c^2x^2}\left(4bc\left(-\left(\operatorname{arctanh}\left(e^{2i\arccos(cx)}\right)(a+b\arccos(cx))\right)+\frac{1}{4}ib\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)-\frac{1}{4}ib\,\mathrm{PolyLog}\left(2,\right.\right.\right.$$

input
```
Int[(a + b*ArcCos[c*x])^2/(x^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x]
```

output
```
(Sqrt[1 - c^2*x^2]*(-((a + b*ArcCos[c*x])^2/(x*Sqrt[1 - c^2*x^2])) + 4*b*c
*(-((a + b*ArcCos[c*x])*ArcTanh[E^((2*I)*ArcCos[c*x])]) + (I/4)*b*PolyLog[
2, -E^((2*I)*ArcCos[c*x])] - (I/4)*b*PolyLog[2, E^((2*I)*ArcCos[c*x])]) +
2*c^2*((x*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2] - (2*b*(((-1/2*I)*(a +
b*ArcCos[c*x])^2)/b - (2*I)*((I/2)*(a + b*ArcCos[c*x])*Log[1 - E^((2*I)*Ar
cCos[c*x])] + (b*PolyLog[2, E^((2*I)*ArcCos[c*x])])/4)))/c)))/(d*e*Sqrt[d
+ c*d*x]*Sqrt[e - c*e*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 5161
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[x*((a + b*ArcCos[c*x])^n/(d*Sqrt[d + e*x^2])), x] + Simp[b
*c*(n/d)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[x*((a + b*ArcCos[c*x
])^(n - 1)/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

rule 5181
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[1/e   Subst[Int[(a + b*x)^n*Cot[x], x], x, ArcCos[c*x]],
x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

rule 5185
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[-d^(-1)   Subst[Int[(a + b*x)^n/(Cos[x]*Sin[x]), x], x, A
rcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n
, 0]
```

rule 5205
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Simp[c^2*((m + 2*p + 3)/(f^2*(m + 1))
)   Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Simp[b*
c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   Int[(f*x)^(m + 1)*(
1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

rule 5239
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((h_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(q_), x_Symbol] :> Simp[((-d^2)*(g/e))^In
tPart[q]*(d + e*x)^FracPart[q]*((f + g*x)^FracPart[q]/(1 - c^2*x^2)^FracPar
t[q])   Int[(h*x)^m*(d + e*x)^(p - q)*(1 - c^2*x^2)^q*(a + b*ArcCos[c*x])^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[e*f + d*g, 0] &
& EqQ[c^2*d^2 - e^2, 0] && HalfIntegerQ[p, q] && GeQ[p - q, 0]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.37

method	result
default	$\frac{a^2(-2c^2x^2+1)\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}}{d^2e^2(cx+1)(cx-1)x}+b^2\left(-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\left(-2i\sqrt{-c^2x^2+1}\,xc+2c^2x^2-1\right)\arccos(cx)^2}{d^2e^2x(c^2x^2-1)}-\frac{i\sqrt{d(c}}{}\right.$
parts	$\frac{a^2(-2c^2x^2+1)\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}}{d^2e^2(cx+1)(cx-1)x}+b^2\left(-\frac{\sqrt{d(cx+1)}\,\sqrt{-e(cx-1)}\left(-2i\sqrt{-c^2x^2+1}\,xc+2c^2x^2-1\right)\arccos(cx)^2}{d^2e^2x(c^2x^2-1)}-\frac{i\sqrt{d(c}}{}\right.$

input
```
int((a+b*arccos(c*x))^2/x^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
a^2*(-2*c^2*x^2+1)*(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)/d^2/e^2/(c*x+1)/(c
*x-1)/x+b^2*(-(d*(c*x+1))^(1/2)*(-e*(c*x-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2
)*c*x+2*c^2*x^2-1)*arccos(c*x)^2/d^2/e^2/x/(c^2*x^2-1)-I*(d*(c*x+1))^(1/2)
*(-e*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)/e^2/d^2*(2*I*arccos(c*x
)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)+2*I*arccos(c*x)*ln(1-c*x-I*(-c^2*x^2+
1)^(1/2))+2*I*arccos(c*x)*ln(1+c*x+I*(-c^2*x^2+1)^(1/2))+4*arccos(c*x)^2+p
olylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+2*polylog(2,c*x+I*(-c^2*x^2+1)^(1/
2))+2*polylog(2,-c*x-I*(-c^2*x^2+1)^(1/2)))*c)-2*a*b*(d*(c*x+1))^(1/2)*(-e
*(c*x-1))^(1/2)*(-c^2*x^2+1)^(1/2)*(2*I*arccos(c*x)*x^3*c^3-ln((c*x+I*(-c^
2*x^2+1)^(1/2))^4-1)*x^3*c^3-2*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^2-2*I*
arccos(c*x)*x*c+ln((c*x+I*(-c^2*x^2+1)^(1/2))^4-1)*x*c+arccos(c*x)*(-c^2*x
^2+1)^(1/2))/e^2/d^2/x/(c^4*x^4-2*c^2*x^2+1)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algo
rithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(c*d*x + d)*sqr
t(-c*e*x + e)/(c^4*d^2*e^2*x^6 - 2*c^2*d^2*e^2*x^4 + d^2*e^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acos(c*x))**2/x**2/(c*d*x+d)**(3/2)/(-c*e*x+e)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(cdx+d)^{\frac{3}{2}}(-cex+e)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/x^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x, algo
rithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2/((c*d*x + d)^(3/2)*(-c*e*x + e)^(3/2)*x^2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x^2\,(d+c\,d\,x)^{3/2}\,(e-c\,e\,x)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(x^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)),x)
```

output
```
int((a + b*acos(c*x))^2/(x^2*(d + c*d*x)^(3/2)*(e - c*e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2(d+cdx)^{3/2}(e-cex)^{3/2}}\,dx = \frac{-2\sqrt{cx+1}\,\sqrt{-cx+1}\left(\int \frac{acos(cx)}{\sqrt{cx+1}\,\sqrt{-cx+1}\,c^2x^4-\sqrt{cx+1}\,\sqrt{-cx+1}\,x^2}dx\right)abx - \sqrt{}}{\sqrt{e}\,\sqrt{d}\,\sqrt{cx}}$$

input

```
int((a+b*acos(c*x))^2/x^2/(c*d*x+d)^(3/2)/(-c*e*x+e)^(3/2),x)
```

output

```
( - 2*sqrt(c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)/(sqrt(c*x + 1)*sqrt( -
c*x + 1)*c**2*x**4 - sqrt(c*x + 1)*sqrt( - c*x + 1)*x**2),x)*a*b*x - sqrt(
c*x + 1)*sqrt( - c*x + 1)*int(acos(c*x)**2/(sqrt(c*x + 1)*sqrt( - c*x + 1)
*c**2*x**4 - sqrt(c*x + 1)*sqrt( - c*x + 1)*x**2),x)*b**2*x + 2*a**2*c**2*
x**2 - a**2)/(sqrt(e)*sqrt(d)*sqrt(c*x + 1)*sqrt( - c*x + 1)*d*e*x)
```

3.598 $\int x^4(d+ex^2)(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 152

$$\int x^4(d+ex^2)(a+b\arccos(cx))\,dx = \frac{b(7c^2d+5e)\sqrt{1-c^2x^2}}{35c^7} - \frac{b(14c^2d+15e)(1-c^2x^2)^{3/2}}{105c^7} + \frac{b(7c^2d+15e)(1-c^2x^2)^{5/2}}{175c^7} - \frac{be(1-c^2x^2)^{7/2}}{49c^7} + \frac{1}{5}dx^5(a+b\arccos(cx)) + \frac{1}{7}ex^7(a+b\arccos(cx))$$

output
```
1/35*b*(7*c^2*d+5*e)*(-c^2*x^2+1)^(1/2)/c^7-1/105*b*(14*c^2*d+15*e)*(-c^2*
x^2+1)^(3/2)/c^7+1/175*b*(7*c^2*d+15*e)*(-c^2*x^2+1)^(5/2)/c^7-1/49*b*e*(-
c^2*x^2+1)^(7/2)/c^7+1/5*d*x^5*(a+b*arccos(c*x))+1/7*e*x^7*(a+b*arccos(c*x
))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int x^4(d+ex^2)(a+b\arccos(cx))\,dx = \frac{1}{5}adx^5+\frac{1}{7}aex^7 + bd\sqrt{1-c^2x^2}\left(-\frac{8}{75c^5}-\frac{4x^2}{75c^3}-\frac{x^4}{25c}\right) + be\sqrt{1-c^2x^2}\left(-\frac{16}{245c^7}-\frac{8x^2}{245c^5}-\frac{6x^4}{245c^3}-\frac{x^6}{49c}\right) + \frac{1}{5}bdx^5\arccos(cx)+\frac{1}{7}bex^7\arccos(cx)$$

input
```
Integrate[x^4*(d + e*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(a*d*x^5)/5 + (a*e*x^7)/7 + b*d*Sqrt[1 - c^2*x^2]*(-8/(75*c^5) - (4*x^2)/(
75*c^3) - x^4/(25*c)) + b*e*Sqrt[1 - c^2*x^2]*(-16/(245*c^7) - (8*x^2)/(24
5*c^5) - (6*x^4)/(245*c^3) - x^6/(49*c)) + (b*d*x^5*ArcCos[c*x])/5 + (b*e*
x^7*ArcCos[c*x])/7
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5231, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(d+ex^2)(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5231}$$

$$bc\int \frac{x^5(5ex^2+7d)}{35\sqrt{1-c^2x^2}}dx+\frac{1}{5}dx^5(a+b\arccos(cx))+\frac{1}{7}ex^7(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{35}bc\int \frac{x^5(5ex^2+7d)}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}dx^5(a+b\arccos(cx))+\frac{1}{7}ex^7(a+b\arccos(cx))$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{70}bc\int \frac{x^4(5ex^2+7d)}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{5}dx^5(a+b\arccos(cx))+\frac{1}{7}ex^7(a+b\arccos(cx))$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{70}bc\int \left(-\frac{5e(1-c^2x^2)^{5/2}}{c^6}+\frac{(7dc^2+15e)(1-c^2x^2)^{3/2}}{c^6}+\frac{(-14dc^2-15e)\sqrt{1-c^2x^2}}{c^6}+\frac{7dc^2+5e}{c^6\sqrt{1-c^2x^2}}\right)dx^2+ \frac{1}{5}dx^5(a+b\arccos(cx))+\frac{1}{7}ex^7(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}dx^5(a+b\arccos(cx))+\frac{1}{7}ex^7(a+b\arccos(cx))+ \frac{1}{70}bc\left(-\frac{2(1-c^2x^2)^{5/2}(7c^2d+15e)}{5c^8}+\frac{2(1-c^2x^2)^{3/2}(14c^2d+15e)}{3c^8}-\frac{2\sqrt{1-c^2x^2}(7c^2d+5e)}{c^8}+\frac{10e(1-c^2x^2}{7c^8}\right.$$

input
```
Int[x^4*(d + e*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*((-2*(7*c^2*d + 5*e)*Sqrt[1 - c^2*x^2])/c^8 + (2*(14*c^2*d + 15*e)*(1
 - c^2*x^2)^(3/2))/(3*c^8) - (2*(7*c^2*d + 15*e)*(1 - c^2*x^2)^(5/2))/(5*c
^8) + (10*e*(1 - c^2*x^2)^(7/2))/(7*c^8)))/70 + (d*x^5*(a + b*ArcCos[c*x])
)/5 + (e*x^7*(a + b*ArcCos[c*x]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.28

method	result
parts	$a\left(\frac{1}{7}e\,x^7+\frac{1}{5}d\,x^5\right)+\frac{b\left(\frac{c^5\arccos(cx)e\,x^7}{7}+\frac{\arccos(cx)c^5x^5d}{5}+\frac{5e\left(-\frac{c^6x^6\sqrt{-c^2x^2+1}}{7}-\frac{6c^4x^4\sqrt{-c^2x^2+1}}{35}-\frac{8c^2x^2\sqrt{-c^2}\cdots}{35}\cdots\right.}{\cdots}\cdots\right.}{c^5}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\left(\frac{\arccos(cx)d\,c^7x^5}{5}+\frac{\arccos(cx)e\,c^7x^7}{7}+\frac{e\left(-\frac{c^6x^6\sqrt{-c^2x^2+1}}{7}-\frac{6c^4x^4\sqrt{-c^2x^2+1}}{35}-\frac{8c^2x^2\sqrt{-c^2x^2+1}}{35}-\cdots\right.}{7}\cdots\right.}{c^2}\cdots}{c^5}$
default	$\frac{\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\left(\frac{\arccos(cx)d\,c^7x^5}{5}+\frac{\arccos(cx)e\,c^7x^7}{7}+\frac{e\left(-\frac{c^6x^6\sqrt{-c^2x^2+1}}{7}-\frac{6c^4x^4\sqrt{-c^2x^2+1}}{35}-\frac{8c^2x^2\sqrt{-c^2x^2+1}}{35}-\cdots\right.}{7}\cdots\right.}{c^2}\cdots}{c^5}$
orering	$\frac{(975c^8e^2x^{10}+2442c^8de\,x^8+1323c^8d^2x^6+90x^8e^2c^6+354x^6e\,c^6d+196c^6d^2x^4+180e^2x^6c^4+1296c^4de\,x^4+784c^4d^2x^2+\cdots}{3675(e\,x^2+d)x\,c^8}$

input
```
int(x^4*(e*x^2+d)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e*x^7+1/5*d*x^5)+b/c^5*(1/7*c^5*arccos(c*x)*e*x^7+1/5*arccos(c*x)*c
^5*x^5*d+1/35/c^2*(5*e*(-1/7*c^6*x^6*(-c^2*x^2+1)^(1/2)-6/35*c^4*x^4*(-c^2
*x^2+1)^(1/2)-8/35*c^2*x^2*(-c^2*x^2+1)^(1/2)-16/35*(-c^2*x^2+1)^(1/2))+7*
d*c^2*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/1
5*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int x^4(d+ex^2)(a+b\arccos(cx))\,dx = \frac{525\,ac^7ex^7+735\,ac^7dx^5+105\,(5\,bc^7ex^7+7\,bc^7dx^5)\arccos(cx)-(75\,bc^6ex^6+3\,(49\,bc^6d+30\,bc^4e)x^4+}{3675\,c^7}$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/3675*(525*a*c^7*e*x^7 + 735*a*c^7*d*x^5 + 105*(5*b*c^7*e*x^7 + 7*b*c^7*d
*x^5)*arccos(c*x) - (75*b*c^6*e*x^6 + 3*(49*b*c^6*d + 30*b*c^4*e)*x^4 + 39
2*b*c^2*d + 4*(49*b*c^4*d + 30*b*c^2*e)*x^2 + 240*b*e)*sqrt(-c^2*x^2 + 1))
/c^7
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.50

$$\int x^4(d+ex^2)(a+b\arccos(cx))\,dx = \begin{cases} \frac{adx^5}{5}+\frac{aex^7}{7}+\frac{bdx^5\operatorname{acos}(cx)}{5}+\frac{bex^7\operatorname{acos}(cx)}{7}-\frac{bdx^4\sqrt{-c^2x^2+1}}{25c}-\frac{bex^6\sqrt{-c^2x^2+1}}{49c}-\frac{4bdx^2\sqrt{-c^2x^2+1}}{75c^3}-\frac{6bex^4\sqrt{-c^2x^2+1}}{245c^3}- \\ \left(a+\frac{\pi b}{2}\right)\left(\frac{dx^5}{5}+\frac{ex^7}{7}\right) \end{cases}$$

input
```
integrate(x**4*(e*x**2+d)*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d*x**5/5 + a*e*x**7/7 + b*d*x**5*acos(c*x)/5 + b*e*x**7*acos(
c*x)/7 - b*d*x**4*sqrt(-c**2*x**2 + 1)/(25*c) - b*e*x**6*sqrt(-c**2*x**2 +
 1)/(49*c) - 4*b*d*x**2*sqrt(-c**2*x**2 + 1)/(75*c**3) - 6*b*e*x**4*sqrt(-
c**2*x**2 + 1)/(245*c**3) - 8*b*d*sqrt(-c**2*x**2 + 1)/(75*c**5) - 8*b*e*x
**2*sqrt(-c**2*x**2 + 1)/(245*c**5) - 16*b*e*sqrt(-c**2*x**2 + 1)/(245*c**
7), Ne(c, 0)), ((a + pi*b/2)*(d*x**5/5 + e*x**7/7), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.22

$$\int x^4(d+ex^2)(a+b\arccos(cx))\,dx = \frac{1}{7}aex^7 + \frac{1}{5}adx^5 + \frac{1}{75}\left(15\,x^5\arccos(cx) - \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bd + \frac{1}{245}\left(35\,x^7\arccos(cx) - \left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\sqrt{-c^2x^2+1}}{c^8}\right.\right.$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/75*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 +
1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b
*d + 1/245*(35*x^7*arccos(c*x) - (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c
^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)
/c^8)*c)*b*e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.22

$$\int x^4(d+ex^2)(a+b\arccos(cx))\,dx = \frac{1}{7}\,bex^7\arccos(cx)+\frac{1}{7}\,aex^7+\frac{1}{5}\,bdx^5\arccos(cx) - \frac{\sqrt{-c^2x^2+1}bex^6}{49\,c}+\frac{1}{5}\,adx^5 - \frac{\sqrt{-c^2x^2+1}bdx^4}{25\,c}-\frac{6\,\sqrt{-c^2x^2+1}bex^4}{245\,c^3} - \frac{4\,\sqrt{-c^2x^2+1}bdx^2}{75\,c^3}-\frac{8\,\sqrt{-c^2x^2+1}bex^2}{245\,c^5} - \frac{8\,\sqrt{-c^2x^2+1}bd}{75\,c^5}-\frac{16\,\sqrt{-c^2x^2+1}be}{245\,c^7}$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/7*b*e*x^7*arccos(c*x) + 1/7*a*e*x^7 + 1/5*b*d*x^5*arccos(c*x) - 1/49*sqr
t(-c^2*x^2 + 1)*b*e*x^6/c + 1/5*a*d*x^5 - 1/25*sqrt(-c^2*x^2 + 1)*b*d*x^4/
c - 6/245*sqrt(-c^2*x^2 + 1)*b*e*x^4/c^3 - 4/75*sqrt(-c^2*x^2 + 1)*b*d*x^2
/c^3 - 8/245*sqrt(-c^2*x^2 + 1)*b*e*x^2/c^5 - 8/75*sqrt(-c^2*x^2 + 1)*b*d/
c^5 - 16/245*sqrt(-c^2*x^2 + 1)*b*e/c^7
```

Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex^2)(a+b\arccos(cx))\,dx = \int x^4\,(a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)\,dx$$

input
```
int(x^4*(a + b*acos(c*x))*(d + e*x^2),x)
```

output
```
int(x^4*(a + b*acos(c*x))*(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.26

$$\int x^4(d+ex^2)(a+b\arccos(cx))\,dx$$
$$=\frac{735acos(cx)\,b\,c^7d\,x^5+525acos(cx)\,b\,c^7e\,x^7-147\sqrt{-c^2x^2+1}\,b\,c^6d\,x^4-75\sqrt{-c^2x^2+1}\,b\,c^6e\,x^6-196\sqrt{-}}{}$$

input
```
int(x^4*(e*x^2+d)*(a+b*acos(c*x)),x)
```

output
```
(735*acos(c*x)*b*c**7*d*x**5 + 525*acos(c*x)*b*c**7*e*x**7 - 147*sqrt( - c
**2*x**2 + 1)*b*c**6*d*x**4 - 75*sqrt( - c**2*x**2 + 1)*b*c**6*e*x**6 - 19
6*sqrt( - c**2*x**2 + 1)*b*c**4*d*x**2 - 90*sqrt( - c**2*x**2 + 1)*b*c**4*
e*x**4 - 392*sqrt( - c**2*x**2 + 1)*b*c**2*d - 120*sqrt( - c**2*x**2 + 1)*
b*c**2*e*x**2 - 240*sqrt( - c**2*x**2 + 1)*b*e + 735*a*c**7*d*x**5 + 525*a
*c**7*e*x**7)/(3675*c**7)
```

3.599 $\int x^3(d+ex^2)(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 149

$$\int x^3(d+ex^2)(a+b\arccos(cx))\,dx = \frac{b(9c^2d+5e)\,x\sqrt{1-c^2x^2}}{96c^5} + \frac{b(9c^2d+5e)\,x^3\sqrt{1-c^2x^2}}{144c^3} + \frac{bex^5\sqrt{1-c^2x^2}}{36c} - \frac{b(9c^2d+5e)\arccos(cx)}{96c^6} + \frac{1}{4}dx^4(a+b\arccos(cx)) + \frac{1}{6}ex^6(a+b\arccos(cx))$$

output
```
1/96*b*(9*c^2*d+5*e)*x*(-c^2*x^2+1)^(1/2)/c^5+1/144*b*(9*c^2*d+5*e)*x^3*(-
c^2*x^2+1)^(1/2)/c^3+1/36*b*e*x^5*(-c^2*x^2+1)^(1/2)/c-1/96*b*(9*c^2*d+5*e
)*arccos(c*x)/c^6+1/4*d*x^4*(a+b*arccos(c*x))+1/6*e*x^6*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int x^3(d+ex^2)(a+b\arccos(cx))\,dx = \frac{1}{4}adx^4+\frac{1}{6}aex^6+bd\sqrt{1-c^2x^2}\left(-\frac{3x}{32c^3}-\frac{x^3}{16c}\right)+be\sqrt{1-c^2x^2}\left(-\frac{5x}{96c^5}-\frac{5x^3}{144c^3}-\frac{x^5}{36c}\right)+\frac{1}{4}bdx^4\arccos(cx)+\frac{1}{6}bex^6\arccos(cx)+\frac{3bd\arcsin(cx)}{32c^4}+\frac{5be\arcsin(cx)}{96c^6}$$

input
```
Integrate[x^3*(d + e*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(a*d*x^4)/4 + (a*e*x^6)/6 + b*d*Sqrt[1 - c^2*x^2]*((-3*x)/(32*c^3) - x^3/(
16*c)) + b*e*Sqrt[1 - c^2*x^2]*((-5*x)/(96*c^5) - (5*x^3)/(144*c^3) - x^5/
(36*c)) + (b*d*x^4*ArcCos[c*x])/4 + (b*e*x^6*ArcCos[c*x])/6 + (3*b*d*ArcSi
n[c*x])/(32*c^4) + (5*b*e*ArcSin[c*x])/(96*c^6)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5231, 27, 363, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex^2)(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5231}$$

$$bc\int \frac{x^4(2ex^2+3d)}{12\sqrt{1-c^2x^2}}dx+\frac{1}{4}dx^4(a+b\arccos(cx))+\frac{1}{6}ex^6(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}bc\int\frac{x^4(2ex^2+3d)}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}dx^4(a+b\arccos(cx))+\frac{1}{6}ex^6(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 363}$$

$$\frac{1}{12}bc\left(\frac{1}{3}\left(\frac{5e}{c^2}+9d\right)\int\frac{x^4}{\sqrt{1-c^2x^2}}dx-\frac{ex^5\sqrt{1-c^2x^2}}{3c^2}\right)+\frac{1}{4}dx^4(a+b\arccos(cx))+\frac{1}{6}ex^6(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{12}bc\left(\frac{1}{3}\left(\frac{5e}{c^2}+9d\right)\left(\frac{3\int\frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)-\frac{ex^5\sqrt{1-c^2x^2}}{3c^2}\right)+\frac{1}{4}dx^4(a+b\arccos(cx))+\frac{1}{6}ex^6(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{12}bc\left(\frac{1}{3}\left(\frac{5e}{c^2}+9d\right)\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)-\frac{ex^5\sqrt{1-c^2x^2}}{3c^2}\right)+\frac{1}{4}dx^4(a+b\arccos(cx))+\frac{1}{6}ex^6(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{4}dx^4(a+b\arccos(cx))+\frac{1}{6}ex^6(a+b\arccos(cx))+\frac{1}{12}bc\left(\frac{1}{3}\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)\left(\frac{5e}{c^2}+9d\right)-\frac{ex^5\sqrt{1-c^2x^2}}{3c^2}\right)$$

input
```
Int[x^3*(d + e*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d*x^4*(a + b*ArcCos[c*x]))/4 + (e*x^6*(a + b*ArcCos[c*x]))/6 + (b*c*(-1/3
*(e*x^5*Sqrt[1 - c^2*x^2])/c^2 + ((9*d + (5*e)/c^2)*(-1/4*(x^3*Sqrt[1 - c^
2*x^2])/c^2 + (3*(-1/2*(x*Sqrt[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(
4*c^2)))/3))/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.14

method	result
parts	$a\left(\frac{1}{6}e\,x^6+\frac{1}{4}d\,x^4\right)+\frac{b\left(\frac{c^4\arccos(cx)e\,x^6}{6}+\frac{\arccos(cx)c^4x^4d}{4}+\frac{2e\left(-\frac{c^5x^5\sqrt{-c^2x^2+1}}{6}-\frac{5c^3x^3\sqrt{-c^2x^2+1}}{24}-\frac{5cx\sqrt{-c^2x^2}}{16}\right.}{}\right.}{c^4}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{4}c^6d\,x^4+\frac{1}{6}c^6e\,x^6\right)}{c^2}+\frac{b\left(\frac{\arccos(cx)d\,c^6x^4}{4}+\frac{\arccos(cx)e\,c^6x^6}{6}+\frac{e\left(-\frac{c^5x^5\sqrt{-c^2x^2+1}}{6}-\frac{5c^3x^3\sqrt{-c^2x^2+1}}{24}-\frac{5cx\sqrt{-c^2x^2+1}}{16}+5\right.}{6}\right.}{c^2}}{c^4}$
default	$\frac{\frac{a\left(\frac{1}{4}c^6d\,x^4+\frac{1}{6}c^6e\,x^6\right)}{c^2}+\frac{b\left(\frac{\arccos(cx)d\,c^6x^4}{4}+\frac{\arccos(cx)e\,c^6x^6}{6}+\frac{e\left(-\frac{c^5x^5\sqrt{-c^2x^2+1}}{6}-\frac{5c^3x^3\sqrt{-c^2x^2+1}}{24}-\frac{5cx\sqrt{-c^2x^2+1}}{16}+5\right.}{6}\right.}{c^2}}{c^4}$
orering	$\frac{(88x^8e^2c^6+234x^6e\,c^6d+126c^6d^2x^4+10e^2x^6c^4+51c^4de\,x^4+27c^4d^2x^2+25c^2e^2x^4-147c^2de\,x^2-108c^2d^2-90e^2x^2-60d}{288(e\,x^2+d)c^6}$

input
```
int(x^3*(e*x^2+d)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/6*e*x^6+1/4*d*x^4)+b/c^4*(1/6*c^4*arccos(c*x)*e*x^6+1/4*arccos(c*x)*c
^4*x^4*d+1/12/c^2*(2*e*(-1/6*c^5*x^5*(-c^2*x^2+1)^(1/2)-5/24*c^3*x^3*(-c^2
*x^2+1)^(1/2)-5/16*c*x*(-c^2*x^2+1)^(1/2)+5/16*arcsin(c*x))+3*d*c^2*(-1/4*
c^3*x^3*(-c^2*x^2+1)^(1/2)-3/8*c*x*(-c^2*x^2+1)^(1/2)+3/8*arcsin(c*x))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^3\left(d+ex^2\right)\left(a+b\arccos(cx)\right)dx$$
$$=\frac{48\,ac^6ex^6+72\,ac^6dx^4+3\left(16\,bc^6ex^6+24\,bc^6dx^4-9\,bc^2d-5\,be\right)\arccos\left(cx\right)-\left(8\,bc^5ex^5+2\left(9\,bc^5d+5\right.\right.}{288\,c^6}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/288*(48*a*c^6*e*x^6 + 72*a*c^6*d*x^4 + 3*(16*b*c^6*e*x^6 + 24*b*c^6*d*x^
4 - 9*b*c^2*d - 5*b*e)*arccos(c*x) - (8*b*c^5*e*x^5 + 2*(9*b*c^5*d + 5*b*c
^3*e)*x^3 + 3*(9*b*c^3*d + 5*b*c*e)*x)*sqrt(-c^2*x^2 + 1))/c^6
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.42

$$\int x^3 (d+ex^2) (a+b\arccos(cx))\,dx$$
$$=\begin{cases} \frac{adx^4}{4}+\frac{aex^6}{6}+\frac{bdx^4\,\mathrm{acos}\,(cx)}{4}+\frac{bex^6\,\mathrm{acos}\,(cx)}{6}-\frac{bdx^3\sqrt{-c^2x^2+1}}{16c}-\frac{bex^5\sqrt{-c^2x^2+1}}{36c}-\frac{3bdx\sqrt{-c^2x^2+1}}{32c^3}-\frac{5bex^3\sqrt{-c^2x^2+1}}{144c^3}- \\ \left(a+\frac{\pi b}{2}\right)\left(\frac{dx^4}{4}+\frac{ex^6}{6}\right) \end{cases}$$

input
```
integrate(x**3*(e*x**2+d)*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*acos(c*x)/4 + b*e*x**6*acos(
c*x)/6 - b*d*x**3*sqrt(-c**2*x**2 + 1)/(16*c) - b*e*x**5*sqrt(-c**2*x**2 +
 1)/(36*c) - 3*b*d*x*sqrt(-c**2*x**2 + 1)/(32*c**3) - 5*b*e*x**3*sqrt(-c**
2*x**2 + 1)/(144*c**3) - 3*b*d*acos(c*x)/(32*c**4) - 5*b*e*x*sqrt(-c**2*x*
*2 + 1)/(96*c**5) - 5*b*e*acos(c*x)/(96*c**6), Ne(c, 0)), ((a + pi*b/2)*(d
*x**4/4 + e*x**6/6), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

$$\int x^3 (d+ex^2) (a+b\arccos(cx))\,dx=\frac{1}{6}\,aex^6+\frac{1}{4}\,adx^4$$
$$+\frac{1}{32}\left(8\,x^4\arccos\left(cx\right)-\left(\frac{2\,\sqrt{-c^2x^2+1}x^3}{c^2}+\frac{3\,\sqrt{-c^2x^2+1}x}{c^4}-\frac{3\,\arcsin\left(cx\right)}{c^5}\right)c\right)bd$$
$$+\frac{1}{288}\left(48\,x^6\arccos\left(cx\right)-\left(\frac{8\,\sqrt{-c^2x^2+1}x^5}{c^2}+\frac{10\,\sqrt{-c^2x^2+1}x^3}{c^4}+\frac{15\,\sqrt{-c^2x^2+1}x}{c^6}-\frac{15\,\arcsin\left(cx\right)}{c^7}\right.\right.$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/32*(8*x^4*arccos(c*x) - (2*sqrt(-c^2*x^2 + 1
)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*d + 1/288
*(48*x^6*arccos(c*x) - (8*sqrt(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 +
1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*b*e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

$$\int x^3(d+ex^2)(a+b\arccos(cx))\,dx = \frac{1}{6}bex^6\arccos(cx)+\frac{1}{6}aex^6+\frac{1}{4}bdx^4\arccos(cx) - \frac{\sqrt{-c^2x^2+1}bex^5}{36\,c}+\frac{1}{4}adx^4 - \frac{\sqrt{-c^2x^2+1}bdx^3}{16\,c}-\frac{5\sqrt{-c^2x^2+1}bex^3}{144\,c^3} - \frac{3\sqrt{-c^2x^2+1}bdx}{32\,c^3}-\frac{3\,bd\arccos(cx)}{32\,c^4} - \frac{5\sqrt{-c^2x^2+1}bex}{96\,c^5}-\frac{5\,be\arccos(cx)}{96\,c^6}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/6*b*e*x^6*arccos(c*x) + 1/6*a*e*x^6 + 1/4*b*d*x^4*arccos(c*x) - 1/36*sqr
t(-c^2*x^2 + 1)*b*e*x^5/c + 1/4*a*d*x^4 - 1/16*sqrt(-c^2*x^2 + 1)*b*d*x^3/
c - 5/144*sqrt(-c^2*x^2 + 1)*b*e*x^3/c^3 - 3/32*sqrt(-c^2*x^2 + 1)*b*d*x/c
^3 - 3/32*b*d*arccos(c*x)/c^4 - 5/96*sqrt(-c^2*x^2 + 1)*b*e*x/c^5 - 5/96*b
*e*arccos(c*x)/c^6
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)(a+b\arccos(cx))\,dx = \int x^3\,(a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)\,dx$$

input
```
int(x^3*(a + b*acos(c*x))*(d + e*x^2),x)
```

output
```
int(x^3*(a + b*acos(c*x))*(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.15

$$\int x^3(d+ex^2)(a+b\arccos(cx))\,dx$$
$$= \frac{72acos(cx)\,b\,c^6d\,x^4+48acos(cx)\,b\,c^6e\,x^6+27asin(cx)\,b\,c^2d+15asin(cx)\,be-18\sqrt{-c^2x^2+1}\,b\,c^5d\,x^3-}{}$$

input
```
int(x^3*(e*x^2+d)*(a+b*acos(c*x)),x)
```

output
```
(72*acos(c*x)*b*c**6*d*x**4 + 48*acos(c*x)*b*c**6*e*x**6 + 27*asin(c*x)*b*
c**2*d + 15*asin(c*x)*b*e - 18*sqrt( - c**2*x**2 + 1)*b*c**5*d*x**3 - 8*sq
rt( - c**2*x**2 + 1)*b*c**5*e*x**5 - 27*sqrt( - c**2*x**2 + 1)*b*c**3*d*x
- 10*sqrt( - c**2*x**2 + 1)*b*c**3*e*x**3 - 15*sqrt( - c**2*x**2 + 1)*b*c*
e*x + 72*a*c**6*d*x**4 + 48*a*c**6*e*x**6)/(288*c**6)
```

3.600 $\int x^2(d+ex^2)(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 120

$$\int x^2(d+ex^2)(a+b\arccos(cx))\,dx=\frac{b(5c^2d+3e)\sqrt{1-c^2x^2}}{15c^5}-\frac{b(5c^2d+6e)(1-c^2x^2)^{3/2}}{45c^5}+\frac{be(1-c^2x^2)^{5/2}}{25c^5}+\frac{1}{3}dx^3(a+b\arccos(cx))+\frac{1}{5}ex^5(a+b\arccos(cx))$$

output
```
1/15*b*(5*c^2*d+3*e)*(-c^2*x^2+1)^(1/2)/c^5-1/45*b*(5*c^2*d+6*e)*(-c^2*x^2
+1)^(3/2)/c^5+1/25*b*e*(-c^2*x^2+1)^(5/2)/c^5+1/3*d*x^3*(a+b*arccos(c*x))+
1/5*e*x^5*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.04

$$\int x^2(d+ex^2)(a+b\arccos(cx))\,dx=\frac{1}{3}adx^3+\frac{1}{5}aex^5+bd\left(-\frac{2}{9c^3}-\frac{x^2}{9c}\right)\sqrt{1-c^2x^2}+be\sqrt{1-c^2x^2}\left(-\frac{8}{75c^5}-\frac{4x^2}{75c^3}-\frac{x^4}{25c}\right)+\frac{1}{3}bdx^3\arccos(cx)+\frac{1}{5}bex^5\arccos(cx)$$

input
```
Integrate[x^2*(d + e*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(a*d*x^3)/3 + (a*e*x^5)/5 + b*d*(-2/(9*c^3) - x^2/(9*c))*Sqrt[1 - c^2*x^2]
 + b*e*Sqrt[1 - c^2*x^2]*(-8/(75*c^5) - (4*x^2)/(75*c^3) - x^4/(25*c)) + (
b*d*x^3*ArcCos[c*x])/3 + (b*e*x^5*ArcCos[c*x])/5
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5231, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d+ex^2\right)(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5231}$$

$$bc\int \frac{x^3\left(3ex^2+5d\right)}{15\sqrt{1-c^2x^2}}dx+\frac{1}{3}dx^3(a+b\arccos(cx))+\frac{1}{5}ex^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}bc\int \frac{x^3\left(3ex^2+5d\right)}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}dx^3(a+b\arccos(cx))+\frac{1}{5}ex^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{30}bc\int \frac{x^2\left(3ex^2+5d\right)}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{3}dx^3(a+b\arccos(cx))+\frac{1}{5}ex^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{30}bc\int \left(\frac{3e\left(1-c^2x^2\right)^{3/2}}{c^4}+\frac{\left(-5dc^2-6e\right)\sqrt{1-c^2x^2}}{c^4}+\frac{5dc^2+3e}{c^4\sqrt{1-c^2x^2}}\right)dx^2+\frac{1}{3}dx^3(a+b\arccos(cx))+\frac{1}{5}ex^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}dx^3(a+b\arccos(cx))+\frac{1}{5}ex^5(a+b\arccos(cx))+\frac{1}{30}bc\left(\frac{2(1-c^2x^2)^{3/2}(5c^2d+6e)}{3c^6}-\frac{2\sqrt{1-c^2x^2}(5c^2d+3e)}{c^6}-\frac{6e(1-c^2x^2)^{5/2}}{5c^6}\right)$$

input
```
Int[x^2*(d + e*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*((-2*(5*c^2*d + 3*e)*Sqrt[1 - c^2*x^2])/c^6 + (2*(5*c^2*d + 6*e)*(1 -
 c^2*x^2)^(3/2))/(3*c^6) - (6*e*(1 - c^2*x^2)^(5/2))/(5*c^6)))/30 + (d*x^3
*(a + b*ArcCos[c*x]))/3 + (e*x^5*(a + b*ArcCos[c*x]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.28

method	result
parts	$a\left(\frac{1}{5}e\,x^5+\frac{1}{3}x^3d\right)+\frac{b\left(\frac{c^3\arccos(cx)e\,x^5}{5}+\frac{\arccos(cx)c^3x^3d}{3}+\frac{3e\left(-\frac{c^4x^4\sqrt{-c^2x^2+1}}{5}-\frac{4c^2x^2\sqrt{-c^2x^2+1}}{15}-\frac{8\sqrt{-c^2x^2+1}}{15}\right)}{15c^2}\cdots\right.}{c^3}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\arccos(cx)d\,c^5x^3}{3}+\frac{\arccos(cx)e\,c^5x^5}{5}+\frac{e\left(-\frac{c^4x^4\sqrt{-c^2x^2+1}}{5}-\frac{4c^2x^2\sqrt{-c^2x^2+1}}{15}-\frac{8\sqrt{-c^2x^2+1}}{15}\right)}{5}+\cdots\right.}{c^2}}{c^3}$
default	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\arccos(cx)d\,c^5x^3}{3}+\frac{\arccos(cx)e\,c^5x^5}{5}+\frac{e\left(-\frac{c^4x^4\sqrt{-c^2x^2+1}}{5}-\frac{4c^2x^2\sqrt{-c^2x^2+1}}{15}-\frac{8\sqrt{-c^2x^2+1}}{15}\right)}{5}+\cdots\right.}{c^2}}{c^3}$
orering	$\frac{(81x^8e^2c^6+238x^6e\,c^6d+125c^6d^2x^4+12e^2x^6c^4+106c^4de\,x^4+50c^4d^2x^2+48c^2e^2x^4-176c^2de\,x^2-100c^2d^2-96e^2x^2-48\cdots}{225(e\,x^2+d)c^6x}$

input
```
int(x^2*(e*x^2+d)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e*x^5+1/3*x^3*d)+b/c^3*(1/5*c^3*arccos(c*x)*e*x^5+1/3*arccos(c*x)*c
^3*x^3*d+1/15/c^2*(3*e*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(-c^2
*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))+5*d*c^2*(-1/3*c^2*x^2*(-c^2*x^2+1)^
(1/2)-2/3*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.90

$$\int x^2(d+ex^2)(a+b\arccos(cx))\,dx$$
$$=\frac{45\,ac^5ex^5+75\,ac^5dx^3+15\,(3\,bc^5ex^5+5\,bc^5dx^3)\arccos{(cx)}-(9\,bc^4ex^4+50\,bc^2d+(25\,bc^4d+12\,bc^2e)x}{225\,c^5}$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/225*(45*a*c^5*e*x^5 + 75*a*c^5*d*x^3 + 15*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3
)*arccos(c*x) - (9*b*c^4*e*x^4 + 50*b*c^2*d + (25*b*c^4*d + 12*b*c^2*e)*x^
2 + 24*b*e)*sqrt(-c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.48

$$\int x^2(d+ex^2)(a+b\arccos(cx))\,dx$$
$$=\begin{cases}\frac{adx^3}{3}+\frac{aex^5}{5}+\frac{bdx^3\operatorname{acos}{(cx)}}{3}+\frac{bex^5\operatorname{acos}{(cx)}}{5}-\frac{bdx^2\sqrt{-c^2x^2+1}}{9c}-\frac{bex^4\sqrt{-c^2x^2+1}}{25c}-\frac{2bd\sqrt{-c^2x^2+1}}{9c^3}-\frac{4bex^2\sqrt{-c^2x^2+1}}{75c^3}-\\ \left(a+\frac{\pi b}{2}\right)\left(\frac{dx^3}{3}+\frac{ex^5}{5}\right)\end{cases}$$

input
```
integrate(x**2*(e*x**2+d)*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*acos(c*x)/3 + b*e*x**5*acos(
c*x)/5 - b*d*x**2*sqrt(-c**2*x**2 + 1)/(9*c) - b*e*x**4*sqrt(-c**2*x**2 +
1)/(25*c) - 2*b*d*sqrt(-c**2*x**2 + 1)/(9*c**3) - 4*b*e*x**2*sqrt(-c**2*x*
*2 + 1)/(75*c**3) - 8*b*e*sqrt(-c**2*x**2 + 1)/(75*c**5), Ne(c, 0)), ((a +
 pi*b/2)*(d*x**3/3 + e*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.20

$$\int x^2(d+ex^2)(a+b\arccos(cx))\,dx = \frac{1}{5}aex^5+\frac{1}{3}adx^3+\frac{1}{9}\left(3x^3\arccos(cx)-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bd + \frac{1}{75}\left(15x^5\arccos(cx)-\left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2}+\frac{4\sqrt{-c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)be$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)
*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d + 1/75*(15*x^5*arccos(c*x) - (3*
sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^
2 + 1)/c^6)*c)*b*e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.18

$$\int x^2(d+ex^2)(a+b\arccos(cx))\,dx = \frac{1}{5}bex^5\arccos(cx)+\frac{1}{5}aex^5+\frac{1}{3}bdx^3\arccos(cx) - \frac{\sqrt{-c^2x^2+1}bex^4}{25c}+\frac{1}{3}adx^3 - \frac{\sqrt{-c^2x^2+1}bdx^2}{9c}-\frac{4\sqrt{-c^2x^2+1}bex^2}{75c^3} - \frac{2\sqrt{-c^2x^2+1}bd}{9c^3}-\frac{8\sqrt{-c^2x^2+1}be}{75c^5}$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/5*b*e*x^5*arccos(c*x) + 1/5*a*e*x^5 + 1/3*b*d*x^3*arccos(c*x) - 1/25*sqr
t(-c^2*x^2 + 1)*b*e*x^4/c + 1/3*a*d*x^3 - 1/9*sqrt(-c^2*x^2 + 1)*b*d*x^2/c
 - 4/75*sqrt(-c^2*x^2 + 1)*b*e*x^2/c^3 - 2/9*sqrt(-c^2*x^2 + 1)*b*d/c^3 -
8/75*sqrt(-c^2*x^2 + 1)*b*e/c^5
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(d+ex^2\right)\left(a+b\arccos(cx)\right)dx=\int x^2\left(a+b\,\mathrm{acos}(c\,x)\right)\,\left(e\,x^2+d\right)\,dx$$

input
```
int(x^2*(a + b*acos(c*x))*(d + e*x^2),x)
```

output
```
int(x^2*(a + b*acos(c*x))*(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.25

$$\int x^2\left(d+ex^2\right)\left(a+b\arccos(cx)\right)dx = \frac{75acos(cx)\,b\,c^5d\,x^3+45acos(cx)\,b\,c^5e\,x^5-25\sqrt{-c^2x^2+1}\,b\,c^4d\,x^2-9\sqrt{-c^2x^2+1}\,b\,c^4e\,x^4-50\sqrt{-c^2x^2}}{225c^5}$$

input
```
int(x^2*(e*x^2+d)*(a+b*acos(c*x)),x)
```

output
```
(75*acos(c*x)*b*c**5*d*x**3 + 45*acos(c*x)*b*c**5*e*x**5 - 25*sqrt( - c**2
*x**2 + 1)*b*c**4*d*x**2 - 9*sqrt( - c**2*x**2 + 1)*b*c**4*e*x**4 - 50*sqr
t( - c**2*x**2 + 1)*b*c**2*d - 12*sqrt( - c**2*x**2 + 1)*b*c**2*e*x**2 - 2
4*sqrt( - c**2*x**2 + 1)*b*e + 75*a*c**5*d*x**3 + 45*a*c**5*e*x**5)/(225*c
**5)
```

3.601 $\int x(d+ex^2)(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 120

$$\int x\left(d+ex^2\right)(a+b\arccos(cx))\,dx=\frac{b(8c^2d+3e)\,x\sqrt{1-c^2x^2}}{32c^3}+\frac{bex^3\sqrt{1-c^2x^2}}{16c}-\frac{b(8c^4d^2+8c^2de+3e^2)\arccos(cx)}{32c^4e}+\frac{\left(d+ex^2\right)^2(a+b\arccos(cx))}{4e}$$

output

```
1/32*b*(8*c^2*d+3*e)*x*(-c^2*x^2+1)^(1/2)/c^3+1/16*b*e*x^3*(-c^2*x^2+1)^(1
/2)/c-1/32*b*(8*c^4*d^2+8*c^2*d*e+3*e^2)*arccos(c*x)/c^4/e+1/4*(e*x^2+d)^2
*(a+b*arccos(c*x))/e
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.09

$$\int x(d+ex^2)(a+b\arccos(cx))\,dx=\frac{1}{2}adx^2+\frac{1}{4}aex^4-\frac{bdx\sqrt{1-c^2x^2}}{4c}+be\sqrt{1-c^2x^2}\left(-\frac{3x}{32c^3}-\frac{x^3}{16c}\right)+\frac{1}{2}bdx^2\arccos(cx)+\frac{1}{4}bex^4\arccos(cx)+\frac{bd\arcsin(cx)}{4c^2}+\frac{3be\arcsin(cx)}{32c^4}$$

input
```
Integrate[x*(d + e*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(a*d*x^2)/2 + (a*e*x^4)/4 - (b*d*x*Sqrt[1 - c^2*x^2])/(4*c) + b*e*Sqrt[1 -
 c^2*x^2]*((-3*x)/(32*c^3) - x^3/(16*c)) + (b*d*x^2*ArcCos[c*x])/2 + (b*e*
x^4*ArcCos[c*x])/4 + (b*d*ArcSin[c*x])/(4*c^2) + (3*b*e*ArcSin[c*x])/(32*c
^4)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5229, 318, 25, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex^2)(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5229}$$

$$\frac{bc\int\frac{(ex^2+d)^2}{\sqrt{1-c^2x^2}}dx}{4e}+\frac{(d+ex^2)^2(a+b\arccos(cx))}{4e}$$

$$\Big\downarrow \text{318}$$

$$\frac{bc\left(-\frac{\int -\frac{3e(2dc^2+e)x^2+d(4dc^2+e)}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{ex\sqrt{1-c^2x^2}(d+ex^2)}{4c^2}\right)}{4e}+\frac{(d+ex^2)^2(a+b\arccos(cx))}{4e}$$

$$\Big\downarrow \ 25$$

$$\frac{bc\left(\frac{\int \frac{3e(2dc^2+e)x^2+d(4dc^2+e)}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{ex\sqrt{1-c^2x^2}(d+ex^2)}{4c^2}\right)}{4e}+\frac{(d+ex^2)^2(a+b\arccos(cx))}{4e}$$

$$\Big\downarrow \ 299$$

$$\frac{bc\left(\frac{\frac{(8c^4d^2+8c^2de+3e^2)\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{3ex\sqrt{1-c^2x^2}(2c^2d+e)}{2c^2}}{4c^2}-\frac{ex\sqrt{1-c^2x^2}(d+ex^2)}{4c^2}\right)}{4e}+\frac{(d+ex^2)^2(a+b\arccos(cx))}{4e}$$

$$\Big\downarrow \ 223$$

$$\frac{(d+ex^2)^2(a+b\arccos(cx))}{4e}+\frac{bc\left(\frac{\frac{\arcsin(cx)(8c^4d^2+8c^2de+3e^2)}{2c^3}-\frac{3ex\sqrt{1-c^2x^2}(2c^2d+e)}{2c^2}}{4c^2}-\frac{ex\sqrt{1-c^2x^2}(d+ex^2)}{4c^2}\right)}{4e}$$

input
```
Int[x*(d + e*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
((d + e*x^2)^2*(a + b*ArcCos[c*x]))/(4*e) + (b*c*(-1/4*(e*x*Sqrt[1 - c^2*x
^2]*(d + e*x^2))/c^2 + ((-3*e*(2*c^2*d + e)*x*Sqrt[1 - c^2*x^2])/(2*c^2) +
 ((8*c^4*d^2 + 8*c^2*d*e + 3*e^2)*ArcSin[c*x])/(2*c^3))/(4*c^2)))/(4*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 5229
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])/(2*e*(p + 1))), x]
 + Simp[b*(c/(2*e*(p + 1)))   Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x]
, x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.34

method	result
parts	$\frac{a(e x^2+d)^2}{4e}+\frac{b\left(\frac{c^2 e \arccos(cx) x^4}{4}+\frac{\arccos(cx) c^2 x^2 d}{2}+\frac{c^2 \arccos(cx) d^2}{4e}+\frac{c^4 d^2 \arcsin(cx)+e^2\left(-\frac{c^3 x^3\sqrt{-c^2x^2+1}}{4}-\frac{3cx\sqrt{-c^2}}{8}\right.}{}\right.}{c^2}$
derivativedivides	$\frac{\frac{a(c^2 e x^2+c^2 d)^2}{4c^2 e}+\frac{b\left(\frac{\arccos(cx) c^4 d^2}{4e}+\frac{\arccos(cx) c^4 d x^2}{2}+\frac{e \arccos(cx) c^4 x^4}{4}+\frac{c^4 d^2 \arcsin(cx)+e^2\left(-\frac{c^3x^3\sqrt{-c^2x^2+1}}{4}-\frac{3cx\sqrt{-c^2}}{8}\right.}{}\right.}{c^2}}{c^2}$
default	$\frac{\frac{a(c^2 e x^2+c^2 d)^2}{4c^2 e}+\frac{b\left(\frac{\arccos(cx) c^4 d^2}{4e}+\frac{\arccos(cx) c^4 d x^2}{2}+\frac{e \arccos(cx) c^4 x^4}{4}+\frac{c^4 d^2 \arcsin(cx)+e^2\left(-\frac{c^3x^3\sqrt{-c^2x^2+1}}{4}-\frac{3cx\sqrt{-c^2}}{8}\right.}{}\right.}{c^2}}{c^2}$
orering	$\frac{(14e^2x^6c^4+50c^4de\,x^4+24c^4d^2x^2+3c^2e^2x^4-31c^2de\,x^2-16c^2d^2-12e^2x^2-6de)(a+b\arccos(cx))}{32(e\,x^2+d)c^4}-\frac{(2c^2e\,x^2+8c^2d+}{}$

input
```
int(x*(e*x^2+d)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(e*x^2+d)^2/e+b/c^2*(1/4*c^2*e*arccos(c*x)*x^4+1/2*arccos(c*x)*c^2*x
^2*d+1/4*c^2/e*arccos(c*x)*d^2+1/4/c^2/e*(c^4*d^2*arcsin(c*x)+e^2*(-1/4*c^
3*x^3*(-c^2*x^2+1)^(1/2)-3/8*c*x*(-c^2*x^2+1)^(1/2)+3/8*arcsin(c*x))+2*d*c
^2*e*(-1/2*c*x*(-c^2*x^2+1)^(1/2)+1/2*arcsin(c*x))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.86

$$\int x(d+ex^2)(a+b\arccos(cx))\,dx$$
$$=\frac{8\,ac^4ex^4+16\,ac^4dx^2+(8\,bc^4ex^4+16\,bc^4dx^2-8\,bc^2d-3\,be)\arccos(cx)-(2\,bc^3ex^3+(8\,bc^3d+3\,bce)x}{32\,c^4}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/32*(8*a*c^4*e*x^4 + 16*a*c^4*d*x^2 + (8*b*c^4*e*x^4 + 16*b*c^4*d*x^2 - 8
*b*c^2*d - 3*b*e)*arccos(c*x) - (2*b*c^3*e*x^3 + (8*b*c^3*d + 3*b*c*e)*x)*
sqrt(-c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.32

$$\int x(d+ex^2)\,(a+b\arccos(cx))\,dx$$
$$=\begin{cases}\frac{adx^2}{2}+\frac{aex^4}{4}+\frac{bdx^2\operatorname{acos}(cx)}{2}+\frac{bex^4\operatorname{acos}(cx)}{4}-\frac{bdx\sqrt{-c^2x^2+1}}{4c}-\frac{bex^3\sqrt{-c^2x^2+1}}{16c}-\frac{bd\operatorname{acos}(cx)}{4c^2}-\frac{3bex\sqrt{-c^2x^2+1}}{32c^3}-\frac{3be\operatorname{ac}}{32} \\ \left(a+\frac{\pi b}{2}\right)\left(\frac{dx^2}{2}+\frac{ex^4}{4}\right)\end{cases}$$

input
```
integrate(x*(e*x**2+d)*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*acos(c*x)/2 + b*e*x**4*acos(
c*x)/4 - b*d*x*sqrt(-c**2*x**2 + 1)/(4*c) - b*e*x**3*sqrt(-c**2*x**2 + 1)/
(16*c) - b*d*acos(c*x)/(4*c**2) - 3*b*e*x*sqrt(-c**2*x**2 + 1)/(32*c**3) -
 3*b*e*acos(c*x)/(32*c**4), Ne(c, 0)), ((a + pi*b/2)*(d*x**2/2 + e*x**4/4)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03

$$\int x(d+ex^2)\,(a+b\arccos(cx))\,dx$$
$$=\frac{1}{4}aex^4+\frac{1}{2}adx^2+\frac{1}{4}\left(2x^2\arccos(cx)-c\left(\frac{\sqrt{-c^2x^2+1}x}{c^2}-\frac{\arcsin(cx)}{c^3}\right)\right)bd$$
$$+\frac{1}{32}\left(8x^4\arccos(cx)-\left(\frac{2\sqrt{-c^2x^2+1}x^3}{c^2}+\frac{3\sqrt{-c^2x^2+1}x}{c^4}-\frac{3\arcsin(cx)}{c^5}\right)c\right)be$$

input
```
integrate(x*(e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/4*(2*x^2*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)
*x/c^2 - arcsin(c*x)/c^3))*b*d + 1/32*(8*x^4*arccos(c*x) - (2*sqrt(-c^2*x^
2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01

$$\int x(d+ex^2)(a+b\arccos(cx))\,dx = \frac{1}{4}bex^4\arccos(cx) + \frac{1}{4}aex^4 + \frac{1}{2}bdx^2\arccos(cx) - \frac{\sqrt{-c^2x^2+1}bex^3}{16\,c} + \frac{1}{2}adx^2 - \frac{\sqrt{-c^2x^2+1}bdx}{4\,c} - \frac{bd\arccos(cx)}{4\,c^2} - \frac{3\,\sqrt{-c^2x^2+1}bex}{32\,c^3} - \frac{3\,be\arccos(cx)}{32\,c^4}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/4*b*e*x^4*arccos(c*x) + 1/4*a*e*x^4 + 1/2*b*d*x^2*arccos(c*x) - 1/16*sqr
t(-c^2*x^2 + 1)*b*e*x^3/c + 1/2*a*d*x^2 - 1/4*sqrt(-c^2*x^2 + 1)*b*d*x/c -
 1/4*b*d*arccos(c*x)/c^2 - 3/32*sqrt(-c^2*x^2 + 1)*b*e*x/c^3 - 3/32*b*e*ar
ccos(c*x)/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)(a+b\arccos(cx))\,dx = \int x\,(a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)\,dx$$

input
```
int(x*(a + b*acos(c*x))*(d + e*x^2),x)
```

output
```
int(x*(a + b*acos(c*x))*(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08

$$\int x(d+ex^2)(a+b\arccos(cx))\,dx$$
$$= \frac{16acos(cx)\,b\,c^4d\,x^2+8acos(cx)\,b\,c^4e\,x^4+8asin(cx)\,b\,c^2d+3asin(cx)\,be-8\sqrt{-c^2x^2+1}\,b\,c^3dx-2\sqrt{-c}}{32c^4}$$

input
```
int(x*(e*x^2+d)*(a+b*acos(c*x)),x)
```

output
```
(16*acos(c*x)*b*c**4*d*x**2 + 8*acos(c*x)*b*c**4*e*x**4 + 8*asin(c*x)*b*c*
*2*d + 3*asin(c*x)*b*e - 8*sqrt( - c**2*x**2 + 1)*b*c**3*d*x - 2*sqrt( - c
**2*x**2 + 1)*b*c**3*e*x**3 - 3*sqrt( - c**2*x**2 + 1)*b*c*e*x + 16*a*c**4
*d*x**2 + 8*a*c**4*e*x**4)/(32*c**4)
```

3.602 $\int (d+ex^2)(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int (d+ex^2)(a+b\arccos(cx))\,dx = \frac{b(3c^2d+e)\sqrt{1-c^2x^2}}{3c^3} - \frac{be(1-c^2x^2)^{3/2}}{9c^3} + dx(a+b\arccos(cx)) + \frac{1}{3}ex^3(a+b\arccos(cx))$$

output
```
1/3*b*(3*c^2*d+e)*(-c^2*x^2+1)^(1/2)/c^3-1/9*b*e*(-c^2*x^2+1)^(3/2)/c^3+d*
x*(a+b*arccos(c*x))+1/3*e*x^3*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int (d+ex^2)(a+b\arccos(cx))\,dx = adx + \frac{1}{3}aex^3 - \frac{bd\sqrt{1-c^2x^2}}{c} + be\left(-\frac{2}{9c^3} - \frac{x^2}{9c}\right)\sqrt{1-c^2x^2} + bdx\arccos(cx) + \frac{1}{3}bex^3\arccos(cx)$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
a*d*x + (a*e*x^3)/3 - (b*d*Sqrt[1 - c^2*x^2])/c + b*e*(-2/(9*c^3) - x^2/(9
*c))*Sqrt[1 - c^2*x^2] + b*d*x*ArcCos[c*x] + (b*e*x^3*ArcCos[c*x])/3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5171, 27, 353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5171}$$

$$bc\int \frac{x\left(ex^2+3d\right)}{3\sqrt{1-c^2x^2}}dx+dx(a+b\arccos(cx))+\frac{1}{3}ex^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bc\int \frac{x\left(ex^2+3d\right)}{\sqrt{1-c^2x^2}}dx+dx(a+b\arccos(cx))+\frac{1}{3}ex^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{6}bc\int \frac{ex^2+3d}{\sqrt{1-c^2x^2}}dx^2+dx(a+b\arccos(cx))+\frac{1}{3}ex^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{6}bc\int \left(\frac{3dc^2+e}{c^2\sqrt{1-c^2x^2}}-\frac{e\sqrt{1-c^2x^2}}{c^2}\right)dx^2+dx(a+b\arccos(cx))+\frac{1}{3}ex^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$dx(a+b\arccos(cx))+\frac{1}{3}ex^3(a+b\arccos(cx))+\frac{1}{6}bc\left(\frac{2e\left(1-c^2x^2\right)^{3/2}}{3c^4}-\frac{2\sqrt{1-c^2x^2}\left(3c^2d+e\right)}{c^4}\right)$$

input
```
Int[(d + e*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*((-2*(3*c^2*d + e)*Sqrt[1 - c^2*x^2])/c^4 + (2*e*(1 - c^2*x^2)^(3/2))
/(3*c^4)))/6 + d*x*(a + b*ArcCos[c*x]) + (e*x^3*(a + b*ArcCos[c*x]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5171
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2
, 0])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.23

method	result
parts	$a\left(\frac{1}{3}e\,x^3+dx\right)+\frac{b\left(\frac{c\arccos(cx)e\,x^3}{3}+\arccos(cx)dcx+\frac{e\left(-\frac{c^2x^2\sqrt{-c^2x^2+1}}{3}-\frac{2\sqrt{-c^2x^2+1}}{3}\right)-3d\,c^2\sqrt{-c^2x^2+1}}{3c^2}\right)}{c}$
derivativedivides	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\arccos(cx)d\,c^3x+\frac{\arccos(cx)e\,c^3x^3}{3}+\frac{e\left(-\frac{c^2x^2\sqrt{-c^2x^2+1}}{3}-\frac{2\sqrt{-c^2x^2+1}}{3}\right)}{3}-d\,c^2\sqrt{-c^2x^2+1}\right)}{c^2}}{c}$
default	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\arccos(cx)d\,c^3x+\frac{\arccos(cx)e\,c^3x^3}{3}+\frac{e\left(-\frac{c^2x^2\sqrt{-c^2x^2+1}}{3}-\frac{2\sqrt{-c^2x^2+1}}{3}\right)}{3}-d\,c^2\sqrt{-c^2x^2+1}\right)}{c^2}}{c}$
orering	$\frac{x\left(5e^2x^4c^4+30c^4de\,x^2+9c^4d^2+2c^2e^2x^2-18c^2de-4e^2\right)(a+b\arccos(cx))}{9(e\,x^2+d)c^4}-\frac{(c^2e\,x^2+9c^2d+2e)(cx-1)(cx+1)\left(2ex(a\right.}{9c^4(e\,x^2+d)}$

input
```
int((e*x^2+d)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e*x^3+d*x)+b/c*(1/3*c*arccos(c*x)*e*x^3+arccos(c*x)*d*c*x+1/3/c^2*(
e*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))-3*d*c^2*(-c^2*x
^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int\left(d+ex^2\right)(a+b\arccos(cx))\,dx$$
$$=\frac{3\,ac^3ex^3+9\,ac^3dx+3\,(bc^3ex^3+3\,bc^3dx)\arccos(cx)-(bc^2ex^2+9\,bc^2d+2\,be)\sqrt{-c^2x^2+1}}{9\,c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/9*(3*a*c^3*e*x^3 + 9*a*c^3*d*x + 3*(b*c^3*e*x^3 + 3*b*c^3*d*x)*arccos(c*
x) - (b*c^2*e*x^2 + 9*b*c^2*d + 2*b*e)*sqrt(-c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.41

$$\int (d+ex^2)(a+b\arccos(cx))\,dx = \begin{cases} adx+\frac{aex^3}{3}+bdx\operatorname{acos}(cx)+\frac{bex^3\operatorname{acos}(cx)}{3}-\frac{bd\sqrt{-c^2x^2+1}}{c}-\frac{bex^2\sqrt{-c^2x^2+1}}{9c}-\frac{2be\sqrt{-c^2x^2+1}}{9c^3} & \text{for } c\neq 0 \\ \left(a+\frac{\pi b}{2}\right)\left(dx+\frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d*x + a*e*x**3/3 + b*d*x*acos(c*x) + b*e*x**3*acos(c*x)/3 - b
*d*sqrt(-c**2*x**2 + 1)/c - b*e*x**2*sqrt(-c**2*x**2 + 1)/(9*c) - 2*b*e*sq
rt(-c**2*x**2 + 1)/(9*c**3), Ne(c, 0)), ((a + pi*b/2)*(d*x + e*x**3/3), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int (d+ex^2)(a+b\arccos(cx))\,dx = \frac{1}{3}aex^3+\frac{1}{9}\left(3x^3\arccos(cx)-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)be + adx+\frac{\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)bd}{c}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e*x^3 + 1/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*s
qrt(-c^2*x^2 + 1)/c^4))*b*e + a*d*x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1
))*b*d/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int (d+ex^2)(a+b\arccos(cx))\,dx = \frac{1}{3}bex^3\arccos(cx) + \frac{1}{3}aex^3 + bdx\arccos(cx) - \frac{\sqrt{-c^2x^2+1}bex^2}{9c} + adx - \frac{\sqrt{-c^2x^2+1}bd}{c} - \frac{2\sqrt{-c^2x^2+1}be}{9c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/3*b*e*x^3*arccos(c*x) + 1/3*a*e*x^3 + b*d*x*arccos(c*x) - 1/9*sqrt(-c^2*
x^2 + 1)*b*e*x^2/c + a*d*x - sqrt(-c^2*x^2 + 1)*b*d/c - 2/9*sqrt(-c^2*x^2
+ 1)*b*e/c^3
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)(a+b\arccos(cx))\,dx = \begin{cases} \frac{a\,x\,(e\,x^2+3\,d)}{3} - b\,e\left(\frac{\sqrt{\frac{1}{c^2}-x^2}\left(\frac{2}{c^2}+x^2\right)}{9} - \frac{x^3\,\mathrm{acos}(c\,x)}{3}\right) - \frac{b\,d\left(\sqrt{1-c^2\,x^2}-c\,x\,\mathrm{acos}(c\,x)\right)}{c} & \text{if } 0<c \\ \int (a+b\,\mathrm{acos}(c\,x))\,(e\,x^2+d)\,dx & \text{if } \neg 0<c \end{cases}$$

input
```
int((a + b*acos(c*x))*(d + e*x^2),x)
```

output
```
piecewise(0 < c, - b*e*(((1/c^2 - x^2)^(1/2)*(2/c^2 + x^2))/9 - (x^3*acos(
c*x))/3) + (a*x*(3*d + e*x^2))/3 - (b*d*((- c^2*x^2 + 1)^(1/2) - c*x*acos(
c*x)))/c, ~0 < c, int((a + b*acos(c*x))*(d + e*x^2), x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.28

$$\int \left(d+ex^2\right)\left(a+b\arccos(cx)\right)dx = \frac{9acos(cx)\,b\,c^3dx+3acos(cx)\,b\,c^3e\,x^3-9\sqrt{-c^2x^2+1}\,b\,c^2d-\sqrt{-c^2x^2+1}\,b\,c^2e\,x^2-2\sqrt{-c^2x^2+1}\,be+9}{9c^3}$$

input
```
int((e*x^2+d)*(a+b*acos(c*x)),x)
```

output
```
(9*acos(c*x)*b*c**3*d*x + 3*acos(c*x)*b*c**3*e*x**3 - 9*sqrt( - c**2*x**2
+ 1)*b*c**2*d - sqrt( - c**2*x**2 + 1)*b*c**2*e*x**2 - 2*sqrt( - c**2*x**2
 + 1)*b*e + 9*a*c**3*d*x + 3*a*c**3*e*x**3)/(9*c**3)
```

3.603 $\int \frac{(d+ex^2)(a+b\arccos(cx))}{x} dx$

Optimal result

Integrand size = 19, antiderivative size = 132

$$\begin{aligned}\int \frac{(d+ex^2)(a+b\arccos(cx))}{x} dx = & \frac{bex\sqrt{1-c^2x^2}}{4c} - \frac{be\arccos(cx)}{4c^2} \\ & - \frac{1}{2}ibd\arccos(cx)^2 + \frac{1}{2}ex^2(a+b\arccos(cx)) \\ & + bd\arccos(cx)\log\left(1-e^{2i\arccos(cx)}\right) \\ & - bd\arccos(cx)\log(x) + d(a+b\arccos(cx))\log(x) \\ & - \frac{1}{2}ibd\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\end{aligned}$$

output
```
1/4*b*e*x*(-c^2*x^2+1)^(1/2)/c-1/4*b*e*arccos(c*x)/c^2-1/2*I*b*d*arccos(c*
x)^2+1/2*e*x^2*(a+b*arccos(c*x))+b*d*arccos(c*x)*ln(1-(c*x+I*(-c^2*x^2+1)^
(1/2))^2)-b*d*arccos(c*x)*ln(x)+d*(a+b*arccos(c*x))*ln(x)-1/2*I*b*d*polylo
g(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x}\,dx = \frac{1}{2}aex^2 - \frac{bex\sqrt{1-c^2x^2}}{4c} + \frac{1}{2}bex^2\arccos(cx) - \frac{1}{2}ibd\arccos(cx)^2 + \frac{be\arcsin(cx)}{4c^2} + bd\arccos(cx)\log\left(1+e^{2i\arccos(cx)}\right) + ad\log(x) - \frac{1}{2}ibd\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCos[c*x]))/x,x]
```

output
```
(a*e*x^2)/2 - (b*e*x*Sqrt[1 - c^2*x^2])/(4*c) + (b*e*x^2*ArcCos[c*x])/2 -
(I/2)*b*d*ArcCos[c*x]^2 + (b*e*ArcSin[c*x])/(4*c^2) + b*d*ArcCos[c*x]*Log[
1 + E^((2*I)*ArcCos[c*x])] + a*d*Log[x] - (I/2)*b*d*PolyLog[2, -E^((2*I)*A
rcCos[c*x])]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5231, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x}\,dx$$

$$\Big\downarrow \text{5231}$$

$$bc\int \frac{ex^2+2d\log(x)}{2\sqrt{1-c^2x^2}}dx + d\log(x)(a+b\arccos(cx)) + \frac{1}{2}ex^2(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}bc\int \frac{ex^2+2d\log(x)}{\sqrt{1-c^2x^2}}dx + d\log(x)(a+b\arccos(cx)) + \frac{1}{2}ex^2(a+b\arccos(cx))$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{2}bc\int\left(\frac{ex^2}{\sqrt{1-c^2x^2}}+\frac{2d\log(x)}{\sqrt{1-c^2x^2}}\right)dx+d\log(x)(a+b\arccos(cx))+\frac{1}{2}ex^2(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$d\log(x)(a+b\arccos(cx))+\frac{1}{2}ex^2(a+b\arccos(cx))+$$
$$\frac{1}{2}bc\left(\frac{e\arcsin(cx)}{2c^3}+\frac{id\,\text{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{c}+\frac{id\arcsin(cx)^2}{c}-\frac{2d\arcsin(cx)\log\left(1-e^{2i\arcsin(cx)}\right)}{c}+\frac{2d\log(\right.$$

input
```
Int[((d + e*x^2)*(a + b*ArcCos[c*x]))/x,x]
```

output
```
(e*x^2*(a + b*ArcCos[c*x]))/2 + d*(a + b*ArcCos[c*x])*Log[x] + (b*c*(-1/2*
(e*x*Sqrt[1 - c^2*x^2])/c^2 + (e*ArcSin[c*x])/(2*c^3) + (I*d*ArcSin[c*x]^2
)/c - (2*d*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/c + (2*d*ArcSin[c*x
]*Log[x])/c + (I*d*PolyLog[2, E^((2*I)*ArcSin[c*x])])/c))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

method	result
parts	$\frac{ae\,x^2}{2}+ad\ln\left(x\right)-\frac{ibd\arccos(cx)^2}{2}-\frac{bex\sqrt{-c^2x^2+1}}{4c}+\frac{\arccos(cx)be\,x^2}{2}-\frac{be\arccos(cx)}{4c^2}+db\arccos\left(\right.$
derivativedivides	$\frac{ae\,x^2}{2}+ad\ln\left(cx\right)-\frac{ibd\arccos(cx)^2}{2}-\frac{bex\sqrt{-c^2x^2+1}}{4c}+\frac{\arccos(cx)be\,x^2}{2}-\frac{be\arccos(cx)}{4c^2}+db\arccos$
default	$\frac{ae\,x^2}{2}+ad\ln\left(cx\right)-\frac{ibd\arccos(cx)^2}{2}-\frac{bex\sqrt{-c^2x^2+1}}{4c}+\frac{\arccos(cx)be\,x^2}{2}-\frac{be\arccos(cx)}{4c^2}+db\arccos$

input
```
int((e*x^2+d)*(a+b*arccos(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*e*x^2+a*d*ln(x)-1/2*I*b*d*arccos(c*x)^2-1/4*b*e*x*(-c^2*x^2+1)^(1/2)
/c+1/2*arccos(c*x)*b*e*x^2-1/4*b*e*arccos(c*x)/c^2+d*b*arccos(c*x)*ln(1+(c
*x+I*(-c^2*x^2+1)^(1/2))^2)-1/2*I*d*b*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2)
)^2)
```

Fricas [F]

$$\int \frac{(d+ex^2)\,(a+b\arccos(cx))}{x}\,dx=\int \frac{(ex^2+d)(b\arccos\left(cx\right)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccos(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}\,(cx))\,(d+ex^2)}{x}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acos(c*x))/x,x)
```

output
```
Integral((a + b*acos(c*x))*(d + e*x**2)/x, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x}\,dx = \int \frac{(ex^2+d)(b\arccos{(cx)}+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))/x,x, algorithm="maxima")
```

output
```
1/2*a*e*x^2 + a*d*log(x) + integrate((b*e*x^2 + b*d)*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(e\,x^2+d)}{x}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + e*x^2))/x,x)
```

output
```
int(((a + b*acos(c*x))*(d + e*x^2))/x, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x}\,dx = \frac{2acos(cx)\,b\,c^2e\,x^2+asin(cx)\,be-\sqrt{-c^2x^2+1}\,bcex+4\left(\int \frac{acos(cx)}{x}dx\right)b\,c^2d+4\log(x)\,a\,c^2d+2a\,c^2e\,x^2}{4c^2}$$

input
```
int((e*x^2+d)*(a+b*acos(c*x))/x,x)
```

output
```
(2*acos(c*x)*b*c**2*e*x**2 + asin(c*x)*b*e - sqrt( - c**2*x**2 + 1)*b*c*e*
x + 4*int(acos(c*x)/x,x)*b*c**2*d + 4*log(x)*a*c**2*d + 2*a*c**2*e*x**2)/(
4*c**2)
```

3.604 $\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int \frac{(d+ex^2)\,(a+b\arccos(cx))}{x^2}\,dx = \frac{be\sqrt{1-c^2x^2}}{c} - \frac{d(a+b\arccos(cx))}{x} + ex(a+b\arccos(cx)) - bcd\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)$$

output
```
b*e*(-c^2*x^2+1)^(1/2)/c-d*(a+b*arccos(c*x))/x+e*x*(a+b*arccos(c*x))-b*c*d
*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)\,(a+b\arccos(cx))}{x^2}\,dx = -\frac{ad}{x} + aex - \frac{be\sqrt{1-c^2x^2}}{c} - \frac{bd\arccos(cx)}{x} + bex\arccos(cx) - bcd\log(x) + bcd\log\left(1+\sqrt{1-c^2x^2}\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
-((a*d)/x) + a*e*x - (b*e*Sqrt[1 - c^2*x^2])/c - (b*d*ArcCos[c*x])/x + b*e
*x*ArcCos[c*x] - b*c*d*Log[x] + b*c*d*Log[1 + Sqrt[1 - c^2*x^2]]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5231, 25, 354, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^2}dx$$

$$\Big\downarrow \; 5231$$

$$bc\int -\frac{d-ex^2}{x\sqrt{1-c^2x^2}}dx-\frac{d(a+b\arccos(cx))}{x}+ex(a+b\arccos(cx))$$

$$\Big\downarrow \; 25$$

$$-bc\int \frac{d-ex^2}{x\sqrt{1-c^2x^2}}dx-\frac{d(a+b\arccos(cx))}{x}+ex(a+b\arccos(cx))$$

$$\Big\downarrow \; 354$$

$$-\frac{1}{2}bc\int \frac{d-ex^2}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{d(a+b\arccos(cx))}{x}+ex(a+b\arccos(cx))$$

$$\Big\downarrow \; 90$$

$$-\frac{1}{2}bc\left(d\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2+\frac{2e\sqrt{1-c^2x^2}}{c^2}\right)-\frac{d(a+b\arccos(cx))}{x}+ex(a+b\arccos(cx))$$

$$\Big\downarrow \; 73$$

$$-\frac{1}{2}bc\left(\frac{2e\sqrt{1-c^2x^2}}{c^2}-\frac{2d\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2}\right)-\frac{d(a+b\arccos(cx))}{x}+ex(a+b\arccos(cx))$$

$$\Big\downarrow \; 221$$

$$-\frac{d(a+b\arccos(cx))}{x}+ex(a+b\arccos(cx))-\frac{1}{2}bc\left(\frac{2e\sqrt{1-c^2x^2}}{c^2}-2d\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
-((d*(a + b*ArcCos[c*x]))/x) + e*x*(a + b*ArcCos[c*x]) - (b*c*((2*e*Sqrt[1
 - c^2*x^2])/c^2 - 2*d*ArcTanh[Sqrt[1 - c^2*x^2]]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^{2}}+\frac{b\left(\arccos(cx)ecx-\frac{\arccos(cx)dc}{x}+d\,c^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^{2}x^{2}+1}}\right)-e\sqrt{-c^{2}x^{2}+1}\right)}{c^{2}}\right)$	79
default	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^{2}}+\frac{b\left(\arccos(cx)ecx-\frac{\arccos(cx)dc}{x}+d\,c^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^{2}x^{2}+1}}\right)-e\sqrt{-c^{2}x^{2}+1}\right)}{c^{2}}\right)$	79
parts	$a\left(ex-\frac{d}{x}\right)+bc\left(\frac{\arccos(cx)ex}{c}-\frac{\arccos(cx)d}{cx}+\frac{-e\sqrt{-c^{2}x^{2}+1}+d\,c^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^{2}x^{2}+1}}\right)}{c^{2}}\right)$	79

input
```
int((e*x^2+d)*(a+b*arccos(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
c*(a/c^2*(c*e*x-d*c/x)+b/c^2*(arccos(c*x)*e*c*x-arccos(c*x)*d*c/x+d*c^2*ar
ctanh(1/(-c^2*x^2+1)^(1/2))-e*(-c^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(62) = 124$.

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.35

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^2}\,dx$$
$$=\frac{bc^2dx\log\left(\sqrt{-c^2x^2+1}+1\right)-bc^2dx\log\left(\sqrt{-c^2x^2+1}-1\right)+2\,acex^2-2\sqrt{-c^2x^2+1}bex-2\,acd-2\,(b}{2\,cx}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/2*(b*c^2*d*x*log(sqrt(-c^2*x^2 + 1) + 1) - b*c^2*d*x*log(sqrt(-c^2*x^2 +
 1) - 1) + 2*a*c*e*x^2 - 2*sqrt(-c^2*x^2 + 1)*b*e*x - 2*a*c*d - 2*(b*c*d -
 b*c*e)*x*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + 2*(b*c*e*x^2 - b*
c*d + (b*c*d - b*c*e)*x)*arccos(c*x))/(c*x)
```

Sympy [A] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^2}\,dx = -\frac{ad}{x} + aex - bcd\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \frac{1}{|c^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases}\right) - \frac{bd\operatorname{acos}(cx)}{x} + be\left(\begin{cases} \frac{\pi x}{2} & \text{for } c = 0 \\ x\operatorname{acos}(cx) - \frac{\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x**2+d)*(a+b*acos(c*x))/x**2,x)
```

output
```
-a*d/x + a*e*x - b*c*d*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1),
(I*asin(1/(c*x)), True)) - b*d*acos(c*x)/x + b*e*Piecewise((pi*x/2, Eq(c,
0)), (x*acos(c*x) - sqrt(-c**2*x**2 + 1)/c, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^2}\,dx = \left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{\arccos(cx)}{x}\right)bd + aex + \frac{\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)be}{c} - \frac{ad}{x}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))/x^2,x, algorithm="maxima")
```

output
```
(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*b*d + a*e*
x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b*e/c - a*d/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(62) = 124$.

Time = 0.56 (sec) , antiderivative size = 859, normalized size of antiderivative = 13.02

$$\int \frac{(d + ex^2)(a + b\arccos(cx))}{x^2} dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))/x^2,x, algorithm="giac")
```

output
```
-b*c^2*d*arccos(c*x)/(c - (c^2*x^2 - 1)^2*c/(c*x + 1)^4) + b*c^2*d*log(abs
(c*x + sqrt(-c^2*x^2 + 1) + 1))/(c - (c^2*x^2 - 1)^2*c/(c*x + 1)^4) - b*c^
2*d*log(abs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/(c - (c^2*x^2 - 1)^2*c/(c*x +
1)^4) - a*c^2*d/(c - (c^2*x^2 - 1)^2*c/(c*x + 1)^4) + 2*(c^2*x^2 - 1)*b*c^
2*d*arccos(c*x)/((c*x + 1)^2*(c - (c^2*x^2 - 1)^2*c/(c*x + 1)^4)) + 2*(c^2
*x^2 - 1)*a*c^2*d/((c*x + 1)^2*(c - (c^2*x^2 - 1)^2*c/(c*x + 1)^4)) - (c^2
*x^2 - 1)^2*b*c^2*d*arccos(c*x)/((c*x + 1)^4*(c - (c^2*x^2 - 1)^2*c/(c*x +
 1)^4)) + b*e*arccos(c*x)/(c - (c^2*x^2 - 1)^2*c/(c*x + 1)^4) - (c^2*x^2 -
 1)^2*b*c^2*d*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/((c*x + 1)^4*(c - (c^
2*x^2 - 1)^2*c/(c*x + 1)^4)) + (c^2*x^2 - 1)^2*b*c^2*d*log(abs(-c*x + sqrt
(-c^2*x^2 + 1) - 1))/((c*x + 1)^4*(c - (c^2*x^2 - 1)^2*c/(c*x + 1)^4)) - (
c^2*x^2 - 1)^2*a*c^2*d/((c*x + 1)^4*(c - (c^2*x^2 - 1)^2*c/(c*x + 1)^4)) +
 a*e/(c - (c^2*x^2 - 1)^2*c/(c*x + 1)^4) + 2*(c^2*x^2 - 1)*b*e*arccos(c*x)
/((c*x + 1)^2*(c - (c^2*x^2 - 1)^2*c/(c*x + 1)^4)) - 2*sqrt(-c^2*x^2 + 1)*
b*e/((c*x + 1)*(c - (c^2*x^2 - 1)^2*c/(c*x + 1)^4)) + 2*(c^2*x^2 - 1)*a*e/
((c*x + 1)^2*(c - (c^2*x^2 - 1)^2*c/(c*x + 1)^4)) + (c^2*x^2 - 1)^2*b*e*ar
ccos(c*x)/((c*x + 1)^4*(c - (c^2*x^2 - 1)^2*c/(c*x + 1)^4)) + 2*(-c^2*x^2
+ 1)^(3/2)*b*e/((c*x + 1)^3*(c - (c^2*x^2 - 1)^2*c/(c*x + 1)^4)) + (c^2*x^
2 - 1)^2*a*e/((c*x + 1)^4*(c - (c^2*x^2 - 1)^2*c/(c*x + 1)^4))
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^2}\,dx = b\,c\,d\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-c^2\,x^2}}\right)-\frac{b\,d\,\mathrm{acos}(c\,x)}{x}-\frac{a\,(d-e\,x^2)}{x}-\frac{b\,e\,\left(\sqrt{1-c^2\,x^2}-c\,x\,\mathrm{acos}(c\,x)\right)}{c}$$

input
```
int(((a + b*acos(c*x))*(d + e*x^2))/x^2,x)
```

output
```
b*c*d*atanh(1/(1 - c^2*x^2)^(1/2)) - (b*d*acos(c*x))/x - (a*(d - e*x^2))/x
 - (b*e*((1 - c^2*x^2)^(1/2) - c*x*acos(c*x)))/c
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^2}\,dx = \frac{-acos(cx)\,bcd+acos(cx)\,bce\,x^2-\sqrt{-c^2x^2+1}\,bex-\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)b\,c^2dx-acd+ace\,x^2}{cx}$$

input
```
int((e*x^2+d)*(a+b*acos(c*x))/x^2,x)
```

output
```
( - acos(c*x)*b*c*d + acos(c*x)*b*c*e*x**2 - sqrt( - c**2*x**2 + 1)*b*e*x
- log(tan(asin(c*x)/2))*b*c**2*d*x - a*c*d + a*c*e*x**2)/(c*x)
```

3.605 $\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 119

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^3}\,dx = -\frac{bcd\sqrt{1-c^2x^2}}{2x} - \frac{1}{2}ibe\arccos(cx)^2 - \frac{d(a+b\arccos(cx))}{2x^2} + be\arccos(cx)\log\left(1-e^{2i\arccos(cx)}\right) - be\arccos(cx)\log(x) + e(a+b\arccos(cx))\log(x) - \frac{1}{2}ibe\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)$$

output
```
-1/2*b*c*d*(-c^2*x^2+1)^(1/2)/x-1/2*I*b*e*arccos(c*x)^2-1/2*d*(a+b*arccos(
c*x))/x^2+b*e*arccos(c*x)*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-b*e*arccos(c*
x)*ln(x)+e*(a+b*arccos(c*x))*ln(x)-1/2*I*b*e*polylog(2,(c*x+I*(-c^2*x^2+1)
^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^3}\,dx = -\frac{ad}{2x^2}+\frac{bcd\sqrt{1-c^2x^2}}{2x} - \frac{bd\arccos(cx)}{2x^2}-\frac{1}{2}ibe\arccos(cx)^2 + be\arccos(cx)\log\left(1+e^{2i\arccos(cx)}\right) + ae\log(x)-\frac{1}{2}ibe\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
-1/2*(a*d)/x^2 + (b*c*d*Sqrt[1 - c^2*x^2])/(2*x) - (b*d*ArcCos[c*x])/(2*x^
2) - (I/2)*b*e*ArcCos[c*x]^2 + b*e*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x
])] + a*e*Log[x] - (I/2)*b*e*PolyLog[2, -E^((2*I)*ArcCos[c*x])]
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5231, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^3}\,dx$$
$$\Big\downarrow \text{5231}$$
$$bc\int -\frac{\frac{d}{x^2}-2e\log(x)}{2\sqrt{1-c^2x^2}}dx-\frac{d(a+b\arccos(cx))}{2x^2}+e\log(x)(a+b\arccos(cx))$$
$$\Big\downarrow \text{27}$$
$$-\frac{1}{2}bc\int \frac{\frac{d}{x^2}-2e\log(x)}{\sqrt{1-c^2x^2}}dx-\frac{d(a+b\arccos(cx))}{2x^2}+e\log(x)(a+b\arccos(cx))$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{2}bc\int\left(\frac{d}{x^2\sqrt{1-c^2x^2}}-\frac{2e\log(x)}{\sqrt{1-c^2x^2}}\right)dx-\frac{d(a+b\arccos(cx))}{2x^2}+e\log(x)(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(a+b\arccos(cx))}{2x^2}+e\log(x)(a+b\arccos(cx))-$$
$$\frac{1}{2}bc\left(-\frac{ie\,\text{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{c}-\frac{ie\arcsin(cx)^2}{c}+\frac{2e\arcsin(cx)\log\left(1-e^{2i\arcsin(cx)}\right)}{c}-\frac{2e\log(x)\arcsin(cx)}{c}\right.$$

input
```
Int[((d + e*x^2)*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
-1/2*(d*(a + b*ArcCos[c*x]))/x^2 + e*(a + b*ArcCos[c*x])*Log[x] - (b*c*(-(
(d*Sqrt[1 - c^2*x^2])/x) - (I*e*ArcSin[c*x]^2)/c + (2*e*ArcSin[c*x]*Log[1
- E^((2*I)*ArcSin[c*x])])/c - (2*e*ArcSin[c*x]*Log[x])/c - (I*e*PolyLog[2,
 E^((2*I)*ArcSin[c*x])])/c))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

method	result
derivativedivides	$c^2\left(\frac{ae\ln(cx)}{c^2}-\frac{ad}{2c^2x^2}+\frac{b\left(-\frac{ie\arccos(cx)^2}{2}-\frac{d\left(-ic^2x^2-cx\sqrt{-c^2x^2+1}+\arccos(cx)\right)}{2x^2}+\ln\left(1+\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)e\right.}{c^2}\right.$
default	$c^2\left(\frac{ae\ln(cx)}{c^2}-\frac{ad}{2c^2x^2}+\frac{b\left(-\frac{ie\arccos(cx)^2}{2}-\frac{d\left(-ic^2x^2-cx\sqrt{-c^2x^2+1}+\arccos(cx)\right)}{2x^2}+\ln\left(1+\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)e\right.}{c^2}\right.$
parts	$ae\ln(x)-\frac{ad}{2x^2}+b\,c^2\left(-\frac{ie\arccos(cx)^2}{2c^2}-\frac{d\left(-ic^2x^2-cx\sqrt{-c^2x^2+1}+\arccos(cx)\right)}{2c^2x^2}+\frac{e\arccos(cx)\ln\left(1+\right.}{c}\right.$

input
```
int((e*x^2+d)*(a+b*arccos(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(a/c^2*e*ln(c*x)-1/2*a*d/c^2/x^2+b/c^2*(-1/2*I*e*arccos(c*x)^2-1/2*d*(
-I*c^2*x^2-c*x*(-c^2*x^2+1)^(1/2)+arccos(c*x))/x^2+ln(1+(c*x+I*(-c^2*x^2+1
)^(1/2))^2)*e*arccos(c*x)-1/2*I*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)*e
))
```

Fricas [F]

$$\int\frac{(d+ex^2)\,(a+b\arccos(cx))}{x^3}\,dx=\int\frac{(ex^2+d)(b\arccos(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccos(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)\,(a+b\arccos(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acos}\,(cx))\,(d+ex^2)}{x^3}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acos(c*x))/x**3,x)
```

output
```
Integral((a + b*acos(c*x))*(d + e*x**2)/x**3, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)\,(a+b\arccos(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\arccos\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*b*d*(sqrt(-c^2*x^2 + 1)*c/x - arccos(c*x)/x^2) + b*e*integrate(arctan2
(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x, x) + a*e*log(x) - 1/2*a*d/x^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)\,(a+b\arccos(cx))}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\ (e\,x^2+d)}{x^3}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + e*x^2))/x^3,x)
```

output
```
int(((a + b*acos(c*x))*(d + e*x^2))/x^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^3}\,dx = \frac{-acos(cx)\,bd+\sqrt{-c^2x^2+1}\,bcdx+2\left(\int \frac{acos(cx)}{x}dx\right)be\,x^2+2\log(x)\,ae\,x^2-ad}{2x^2}$$

input
```
int((e*x^2+d)*(a+b*acos(c*x))/x^3,x)
```

output
```
( - acos(c*x)*b*d + sqrt( - c**2*x**2 + 1)*b*c*d*x + 2*int(acos(c*x)/x,x)*
b*e*x**2 + 2*log(x)*a*e*x**2 - a*d)/(2*x**2)
```

3.606 $\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^4}\,dx = -\frac{bcd\sqrt{1-c^2x^2}}{6x^2}-\frac{d(a+b\arccos(cx))}{3x^3}-\frac{e(a+b\arccos(cx))}{x}-\frac{1}{6}bc(c^2d+6e)\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)$$

output
```
-1/6*b*c*d*(-c^2*x^2+1)^(1/2)/x^2-1/3*d*(a+b*arccos(c*x))/x^3-e*(a+b*arcco
s(c*x))/x-1/6*b*c*(c^2*d+6*e)*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^4}\,dx = -\frac{ad}{3x^3}-\frac{ae}{x}+\frac{bcd\sqrt{1-c^2x^2}}{6x^2}-\frac{bd\arccos(cx)}{3x^3}-\frac{be\arccos(cx)}{x}-\frac{1}{6}bc^3d\log(x)-bce\log(x)+\frac{1}{6}bc^3d\log\left(1+\sqrt{1-c^2x^2}\right)+bce\log\left(1+\sqrt{1-c^2x^2}\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
-1/3*(a*d)/x^3 - (a*e)/x + (b*c*d*Sqrt[1 - c^2*x^2])/(6*x^2) - (b*d*ArcCos
[c*x])/(3*x^3) - (b*e*ArcCos[c*x])/x - (b*c^3*d*Log[x])/6 - b*c*e*Log[x] +
 (b*c^3*d*Log[1 + Sqrt[1 - c^2*x^2]])/6 + b*c*e*Log[1 + Sqrt[1 - c^2*x^2]]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5231, 27, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^4}dx$$

$$\Big\downarrow \text{ 5231}$$

$$bc\int -\frac{3ex^2+d}{3x^3\sqrt{1-c^2x^2}}dx-\frac{d(a+b\arccos(cx))}{3x^3}-\frac{e(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{3}bc\int \frac{3ex^2+d}{x^3\sqrt{1-c^2x^2}}dx-\frac{d(a+b\arccos(cx))}{3x^3}-\frac{e(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{1}{6}bc\int \frac{3ex^2+d}{x^4\sqrt{1-c^2x^2}}dx^2-\frac{d(a+b\arccos(cx))}{3x^3}-\frac{e(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{1}{6}bc\left(\frac{1}{2}(c^2d+6e)\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{d\sqrt{1-c^2x^2}}{x^2}\right)-\frac{d(a+b\arccos(cx))}{3x^3}-\frac{e(a+b\arccos(cx))}{x}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{1}{6}bc\left(-\frac{(c^2d+6e)\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2}-\frac{d\sqrt{1-c^2x^2}}{x^2}\right)-\frac{d(a+b\arccos(cx))}{3x^3}-\frac{e(a+b\arccos(cx))}{x}$$

$\Big\downarrow$ 221

$$-\frac{d(a+b\arccos(cx))}{3x^3}-\frac{e(a+b\arccos(cx))}{x}-\frac{1}{6}bc\left(-\text{arctanh}\left(\sqrt{1-c^2x^2}\right)(c^2d+6e)-\frac{d\sqrt{1-c^2x^2}}{x^2}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
-1/3*(d*(a + b*ArcCos[c*x]))/x^3 - (e*(a + b*ArcCos[c*x]))/x - (b*c*(-((d*
Sqrt[1 - c^2*x^2])/x^2) - (c^2*d + 6*e)*ArcTanh[Sqrt[1 - c^2*x^2]]))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.33

method	result
parts	$a\left(-\frac{e}{x}-\frac{d}{3x^3}\right)+b\,c^3\left(-\frac{\arccos(cx)e}{c^3x}-\frac{\arccos(cx)d}{3c^3x^3}+\frac{-d\,c^2\left(-\frac{\sqrt{-c^2x^2+1}}{2c^2x^2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{2}\right)+3e\ \mathrm{ar}\ldots}{3c^2}\right.$
derivativedivides	$c^3\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3c\,x^3}\right)}{c^2}+\frac{b\left(-\frac{\arccos(cx)e}{cx}-\frac{\arccos(cx)d}{3c\,x^3}-\frac{d\,c^2\left(-\frac{\sqrt{-c^2x^2+1}}{2c^2x^2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{2}\right)}{3}+e\operatorname{arctanh}\left(\frac{\ldots}{\sqrt{-c\ldots}}\right.\right.}{c^2}\right.$
default	$c^3\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3c\,x^3}\right)}{c^2}+\frac{b\left(-\frac{\arccos(cx)e}{cx}-\frac{\arccos(cx)d}{3c\,x^3}-\frac{d\,c^2\left(-\frac{\sqrt{-c^2x^2+1}}{2c^2x^2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{2}\right)}{3}+e\operatorname{arctanh}\left(\frac{\ldots}{\sqrt{-c\ldots}}\right.\right.}{c^2}\right.$

input
```
int((e*x^2+d)*(a+b*arccos(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(-e/x-1/3*d/x^3)+b*c^3*(-1/c^3*arccos(c*x)*e/x-1/3*arccos(c*x)*d/c^3/x^3
+1/3/c^2*(-d*c^2*(-1/2/c^2/x^2*(-c^2*x^2+1)^(1/2)-1/2*arctanh(1/(-c^2*x^2+
1)^(1/2)))+3*e*arctanh(1/(-c^2*x^2+1)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(75) = 150$.

Time = 0.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.98

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^4}\,dx =$$

$$-\frac{4(bd+3be)x^3\arctan\left(\frac{\sqrt{-c^2x^2+1}cx}{c^2x^2-1}\right)-(bc^3d+6bce)x^3\log\left(\sqrt{-c^2x^2+1}+1\right)+(bc^3d+6bce)x^3\log\left(\sqrt{\ldots}\right.}{12\ldots}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/12*(4*(b*d + 3*b*e)*x^3*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) -
(b*c^3*d + 6*b*c*e)*x^3*log(sqrt(-c^2*x^2 + 1) + 1) + (b*c^3*d + 6*b*c*e)*
x^3*log(sqrt(-c^2*x^2 + 1) - 1) - 2*sqrt(-c^2*x^2 + 1)*b*c*d*x + 12*a*e*x^
2 + 4*a*d + 4*(3*b*e*x^2 - (b*d + 3*b*e)*x^3 + b*d)*arccos(c*x))/x^3
```

Sympy [A] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.00

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^4}\,dx$$
$$= -\frac{ad}{3x^3} - \frac{ae}{x} - \frac{bcd\left(\begin{cases} -\frac{c^2\operatorname{acosh}\left(\frac{1}{cx}\right)}{2} + \frac{c}{2x\sqrt{-1+\frac{1}{c^2x^2}}} - \frac{1}{2cx^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for}\ \frac{1}{|c^2x^2|} > 1 \\ \frac{ic^2\operatorname{asin}\left(\frac{1}{cx}\right)}{2} - \frac{ic\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise} \end{cases}\right)}{3}$$
$$- bce\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for}\ \frac{1}{|c^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases}\right) - \frac{bd\operatorname{acos}(cx)}{3x^3} - \frac{be\operatorname{acos}(cx)}{x}$$

input
```
integrate((e*x**2+d)*(a+b*acos(c*x))/x**4,x)
```

output
```
-a*d/(3*x**3) - a*e/x - b*c*d*Piecewise((-c**2*acosh(1/(c*x))/2 + c/(2*x*s
qrt(-1 + 1/(c**2*x**2))) - 1/(2*c*x**3*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c*
*2*x**2) > 1), (I*c**2*asin(1/(c*x))/2 - I*c*sqrt(1 - 1/(c**2*x**2))/(2*x)
, True))/3 - b*c*e*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*a
sin(1/(c*x)), True)) - b*d*acos(c*x)/(3*x**3) - b*e*acos(c*x)/x
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^4}\,dx = \frac{1}{6}\left(\left(c^2\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{\sqrt{-c^2x^2+1}}{x^2}\right)c-\frac{2\arccos(cx)}{x^3}\right)bd + \left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\arccos(cx)}{x}\right)be-\frac{ae}{x}-\frac{ad}{3x^3}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/6*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)
/x^2)*c - 2*arccos(c*x)/x^3)*b*d + (c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/
abs(x)) - arccos(c*x)/x)*b*e - a*e/x - 1/3*a*d/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3082 vs. 2(75) = 150.

Time = 121.07 (sec) , antiderivative size = 3082, normalized size of antiderivative = 36.26

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))/x^4,x, algorithm="giac")
```

output
```
-1/3*b*c^3*d*arccos(c*x)/(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/
(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) + 1/6*b*c^3*d*log(abs(c*x +
 sqrt(-c^2*x^2 + 1) + 1))/(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2
/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) - 1/6*b*c^3*d*log(abs(-c*x
 + sqrt(-c^2*x^2 + 1) - 1))/(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)
^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) - 1/3*a*c^3*d/(3*(c^2*x^
2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x
+ 1)^6 + 1) + (c^2*x^2 - 1)*b*c^3*d*arccos(c*x)/((c*x + 1)^2*(3*(c^2*x^2 -
 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1
)^6 + 1)) + 1/2*(c^2*x^2 - 1)*b*c^3*d*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1
))/((c*x + 1)^2*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)
^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) - 1/2*(c^2*x^2 - 1)*b*c^3*d*log(abs
(-c*x + sqrt(-c^2*x^2 + 1) - 1))/((c*x + 1)^2*(3*(c^2*x^2 - 1)/(c*x + 1)^2
 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + 1/3
*sqrt(-c^2*x^2 + 1)*b*c^3*d/((c*x + 1)*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c
^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + (c^2*x^2 -
 1)*a*c^3*d/((c*x + 1)^2*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/
(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) - (c^2*x^2 - 1)^2*b*c^3*d*
arccos(c*x)/((c*x + 1)^4*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/
(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) - b*c*e*arccos(c*x)/(3*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(e\,x^2+d)}{x^4}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + e*x^2))/x^4,x)
```

output
```
int(((a + b*acos(c*x))*(d + e*x^2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex^2)(a+b\arccos(cx))}{x^4}\,dx$$
$$= \frac{-2acos(cx)\,bd-6acos(cx)\,be\,x^2+\sqrt{-c^2x^2+1}\,bcdx-\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right)b\,c^3d\,x^3-6\log\left(\tan\left(\frac{asin(cx)}{2}\right)\right.}{6x^3}$$

input
```
int((e*x^2+d)*(a+b*acos(c*x))/x^4,x)
```

output
```
( - 2*acos(c*x)*b*d - 6*acos(c*x)*b*e*x**2 + sqrt( - c**2*x**2 + 1)*b*c*d*
x - log(tan(asin(c*x)/2))*b*c**3*d*x**3 - 6*log(tan(asin(c*x)/2))*b*c*e*x*
*3 - 2*a*d - 6*a*e*x**2)/(6*x**3)
```

3.607 $\int x^4(d+ex^2)^2(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 241

$$\begin{aligned}
&\int x^4(d+ex^2)^2(a+b\arccos(cx))\,dx \\
&= \frac{b(63c^4d^2+90c^2de+35e^2)\sqrt{1-c^2x^2}}{315c^9} - \frac{2b(63c^4d^2+135c^2de+70e^2)(1-c^2x^2)^{3/2}}{945c^9} \\
&\quad + \frac{b(21c^4d^2+90c^2de+70e^2)(1-c^2x^2)^{5/2}}{525c^9} \\
&\quad - \frac{2be(9c^2d+14e)(1-c^2x^2)^{7/2}}{441c^9} + \frac{be^2(1-c^2x^2)^{9/2}}{81c^9} \\
&\quad + \frac{1}{5}d^2x^5(a+b\arccos(cx)) + \frac{2}{7}dex^7(a+b\arccos(cx)) + \frac{1}{9}e^2x^9(a+b\arccos(cx))
\end{aligned}$$

output

```
1/315*b*(63*c^4*d^2+90*c^2*d*e+35*e^2)*(-c^2*x^2+1)^(1/2)/c^9-2/945*b*(63*
c^4*d^2+135*c^2*d*e+70*e^2)*(-c^2*x^2+1)^(3/2)/c^9+1/525*b*(21*c^4*d^2+90*
c^2*d*e+70*e^2)*(-c^2*x^2+1)^(5/2)/c^9-2/441*b*e*(9*c^2*d+14*e)*(-c^2*x^2+
1)^(7/2)/c^9+1/81*b*e^2*(-c^2*x^2+1)^(9/2)/c^9+1/5*d^2*x^5*(a+b*arccos(c*x
))+2/7*d*e*x^7*(a+b*arccos(c*x))+1/9*e^2*x^9*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.78

$$\int x^4\left(d+ex^2\right)^2\left(a+b\arccos(cx)\right)dx$$
$$=\frac{315ax^5(63d^2+90dex^2+35e^2x^4)-\frac{b\sqrt{1-c^2x^2}\left(4480e^2+160c^2e(81d+14ex^2)+24c^4(441d^2+270dex^2+70e^2x^4)+4c^6(1323d^2x^2+1\right.}{c^9}}{99225}$$

input
```
Integrate[x^4*(d + e*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(315*a*x^5*(63*d^2 + 90*d*e*x^2 + 35*e^2*x^4) - (b*Sqrt[1 - c^2*x^2]*(4480
*e^2 + 160*c^2*e*(81*d + 14*e*x^2) + 24*c^4*(441*d^2 + 270*d*e*x^2 + 70*e^
2*x^4) + 4*c^6*(1323*d^2*x^2 + 1215*d*e*x^4 + 350*e^2*x^6) + c^8*(3969*d^2
*x^4 + 4050*d*e*x^6 + 1225*e^2*x^8)))/c^9 + 315*b*x^5*(63*d^2 + 90*d*e*x^2
 + 35*e^2*x^4)*ArcCos[c*x])/99225
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5231, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(d+ex^2\right)^2\left(a+b\arccos(cx)\right)dx$$

$$\Big\downarrow \text{ 5231}$$

$$bc\int \frac{x^5\left(35e^2x^4+90dex^2+63d^2\right)}{315\sqrt{1-c^2x^2}}dx+\frac{1}{5}d^2x^5(a+b\arccos(cx))+\frac{2}{7}dex^7(a+b\arccos(cx))+\frac{1}{9}e^2x^9(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{315}bc\int \frac{x^5\left(35e^2x^4+90dex^2+63d^2\right)}{\sqrt{1-c^2x^2}}dx+\frac{1}{5}d^2x^5(a+b\arccos(cx))+\frac{2}{7}dex^7(a+b\arccos(cx))+\frac{1}{9}e^2x^9(a+b\arccos(cx))$$

$\Big\downarrow$ 1578

$$\frac{1}{630}bc\int \frac{x^4\left(35e^2x^4+90dex^2+63d^2\right)}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{5}d^2x^5(a+b\arccos(cx))+\frac{2}{7}dex^7(a+b\arccos(cx))+\frac{1}{9}e^2x^9(a+b\arccos(cx))$$

$\Big\downarrow$ 1195

$$\frac{1}{630}bc\int \left(\frac{35e^2\left(1-c^2x^2\right)^{7/2}}{c^8}-\frac{10e\left(9dc^2+14e\right)\left(1-c^2x^2\right)^{5/2}}{c^8}+\frac{3\left(21d^2c^4+90dec^2+70e^2\right)\left(1-c^2x^2\right)^{3/2}}{c^8}-\frac{2(\right.$$
$$\frac{1}{5}d^2x^5(a+b\arccos(cx))+\frac{2}{7}dex^7(a+b\arccos(cx))+\frac{1}{9}e^2x^9(a+b\arccos(cx))$$

$\Big\downarrow$ 2009

$$\frac{1}{5}d^2x^5(a+b\arccos(cx))+\frac{2}{7}dex^7(a+b\arccos(cx))+\frac{1}{9}e^2x^9(a+b\arccos(cx))+$$
$$\frac{1}{630}bc\left(\frac{20e\left(1-c^2x^2\right)^{7/2}\left(9c^2d+14e\right)}{7c^{10}}-\frac{70e^2\left(1-c^2x^2\right)^{9/2}}{9c^{10}}-\frac{6\left(1-c^2x^2\right)^{5/2}\left(21c^4d^2+90c^2de+70e^2\right)}{5c^{10}}+\frac{4(1-\right.$$

input
```
Int[x^4*(d + e*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*((-2*(63*c^4*d^2 + 90*c^2*d*e + 35*e^2)*Sqrt[1 - c^2*x^2])/c^10 + (4*
(63*c^4*d^2 + 135*c^2*d*e + 70*e^2)*(1 - c^2*x^2)^(3/2))/(3*c^10) - (6*(21
*c^4*d^2 + 90*c^2*d*e + 70*e^2)*(1 - c^2*x^2)^(5/2))/(5*c^10) + (20*e*(9*c
^2*d + 14*e)*(1 - c^2*x^2)^(7/2))/(7*c^10) - (70*e^2*(1 - c^2*x^2)^(9/2))/
(9*c^10)))/630 + (d^2*x^5*(a + b*ArcCos[c*x]))/5 + (2*d*e*x^7*(a + b*ArcCo
s[c*x]))/7 + (e^2*x^9*(a + b*ArcCos[c*x]))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.37

method	result
parts	$a\left(\frac{1}{9}e^2x^9+\frac{2}{7}de\,x^7+\frac{1}{5}d^2x^5\right)+\frac{b\left(\frac{c^5\arccos(cx)e^2x^9}{9}+\frac{2c^5\arccos(cx)de\,x^7}{7}+\frac{\arccos(cx)c^5x^5d^2}{5}+\frac{35e^2\left(-\frac{c^8x^8\sqrt{-}}{9}\right.}{}\right.}{}$
derivativedivides	$\frac{a\left(\frac{1}{5}d^2c^9x^5+\frac{2}{7}d\,c^9e\,x^7+\frac{1}{9}e^2c^9x^9\right)}{c^4}+\frac{b\left(\frac{\arccos(cx)d^2c^9x^5}{5}+\frac{2\arccos(cx)d\,c^9e\,x^7}{7}+\frac{\arccos(cx)e^2c^9x^9}{9}+e^2\left(-\frac{c^8x^8\sqrt{-c^2x^2+1}}{9}-8\right.\right.}{}$
default	$\frac{a\left(\frac{1}{5}d^2c^9x^5+\frac{2}{7}d\,c^9e\,x^7+\frac{1}{9}e^2c^9x^9\right)}{c^4}+\frac{b\left(\frac{\arccos(cx)d^2c^9x^5}{5}+\frac{2\arccos(cx)d\,c^9e\,x^7}{7}+\frac{\arccos(cx)e^2c^9x^9}{9}+e^2\left(-\frac{c^8x^8\sqrt{-c^2x^2+1}}{9}-8\right.\right.}{}$
orering	$\frac{(20825c^{10}e^3x^{12}+76675c^{10}d\,e^2x^{10}+96147c^{10}d^2e\,x^8+1400c^8e^3x^{10}+35721c^{10}d^3x^6+7180c^8d\,e^2x^8+13824c^8d^2e\,x^6+22}{}$

input
```
int(x^4*(e*x^2+d)^2*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/9*e^2*x^9+2/7*d*e*x^7+1/5*d^2*x^5)+b/c^5*(1/9*c^5*arccos(c*x)*e^2*x^9
+2/7*c^5*arccos(c*x)*d*e*x^7+1/5*arccos(c*x)*c^5*x^5*d^2+1/315/c^4*(35*e^2
*(-1/9*c^8*x^8*(-c^2*x^2+1)^(1/2)-8/63*c^6*x^6*(-c^2*x^2+1)^(1/2)-16/105*c
^4*x^4*(-c^2*x^2+1)^(1/2)-64/315*c^2*x^2*(-c^2*x^2+1)^(1/2)-128/315*(-c^2*
x^2+1)^(1/2))+63*d^2*c^4*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(-c
^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))+90*d*c^2*e*(-1/7*c^6*x^6*(-c^2*x^
2+1)^(1/2)-6/35*c^4*x^4*(-c^2*x^2+1)^(1/2)-8/35*c^2*x^2*(-c^2*x^2+1)^(1/2)
-16/35*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.91

$$\int x^4\left(d+ex^2\right)^2\left(a+b\arccos(cx)\right)dx$$
$$=\frac{11025\,ac^9e^2x^9+28350\,ac^9dex^7+19845\,ac^9d^2x^5+315\left(35\,bc^9e^2x^9+90\,bc^9dex^7+63\,bc^9d^2x^5\right)\arccos\left(cx\right)}{}$$

input
```
integrate(x^4*(e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/99225*(11025*a*c^9*e^2*x^9 + 28350*a*c^9*d*e*x^7 + 19845*a*c^9*d^2*x^5 +
 315*(35*b*c^9*e^2*x^9 + 90*b*c^9*d*e*x^7 + 63*b*c^9*d^2*x^5)*arccos(c*x)
- (1225*b*c^8*e^2*x^8 + 10584*b*c^4*d^2 + 50*(81*b*c^8*d*e + 28*b*c^6*e^2)
*x^6 + 12960*b*c^2*d*e + 3*(1323*b*c^8*d^2 + 1620*b*c^6*d*e + 560*b*c^4*e^
2)*x^4 + 4480*b*e^2 + 4*(1323*b*c^6*d^2 + 1620*b*c^4*d*e + 560*b*c^2*e^2)*
x^2)*sqrt(-c^2*x^2 + 1))/c^9
```

Sympy [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.74

$$\int x^4(d+ex^2)^2(a+b\arccos(cx))\,dx$$
$$= \begin{cases} \frac{ad^2x^5}{5} + \frac{2adex^7}{7} + \frac{ae^2x^9}{9} + \frac{bd^2x^5\operatorname{acos}(cx)}{5} + \frac{2bdex^7\operatorname{acos}(cx)}{7} + \frac{be^2x^9\operatorname{acos}(cx)}{9} - \frac{bd^2x^4\sqrt{-c^2x^2+1}}{25c} - \frac{2bdex^6\sqrt{-c^2x^2+1}}{49c} - b \\ \left(a+\frac{\pi b}{2}\right)\left(\frac{d^2x^5}{5} + \frac{2dex^7}{7} + \frac{e^2x^9}{9}\right) \end{cases}$$

input
```
integrate(x**4*(e*x**2+d)**2*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d**2*x**5/5 + 2*a*d*e*x**7/7 + a*e**2*x**9/9 + b*d**2*x**5*ac
os(c*x)/5 + 2*b*d*e*x**7*acos(c*x)/7 + b*e**2*x**9*acos(c*x)/9 - b*d**2*x*
*4*sqrt(-c**2*x**2 + 1)/(25*c) - 2*b*d*e*x**6*sqrt(-c**2*x**2 + 1)/(49*c)
- b*e**2*x**8*sqrt(-c**2*x**2 + 1)/(81*c) - 4*b*d**2*x**2*sqrt(-c**2*x**2
+ 1)/(75*c**3) - 12*b*d*e*x**4*sqrt(-c**2*x**2 + 1)/(245*c**3) - 8*b*e**2*
x**6*sqrt(-c**2*x**2 + 1)/(567*c**3) - 8*b*d**2*sqrt(-c**2*x**2 + 1)/(75*c
**5) - 16*b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(245*c**5) - 16*b*e**2*x**4*sqrt
(-c**2*x**2 + 1)/(945*c**5) - 32*b*d*e*sqrt(-c**2*x**2 + 1)/(245*c**7) - 6
4*b*e**2*x**2*sqrt(-c**2*x**2 + 1)/(2835*c**7) - 128*b*e**2*sqrt(-c**2*x**
2 + 1)/(2835*c**9), Ne(c, 0)), ((a + pi*b/2)*(d**2*x**5/5 + 2*d*e*x**7/7 +
 e**2*x**9/9), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.32

$$\int x^4 (d + ex^2)^2 (a + b \arccos(cx)) \, dx = \frac{1}{9} ae^2 x^9 + \frac{2}{7} adex^7 + \frac{1}{5} ad^2 x^5 + \frac{1}{75} \left(15 x^5 \arccos(cx) - \left(\frac{3 \sqrt{-c^2x^2 + 1}x^4}{c^2} + \frac{4 \sqrt{-c^2x^2 + 1}x^2}{c^4} + \frac{8 \sqrt{-c^2x^2 + 1}}{c^6}\right)c\right)bd^2 + \frac{2}{245} \left(35 x^7 \arccos(cx) - \left(\frac{5 \sqrt{-c^2x^2 + 1}x^6}{c^2} + \frac{6 \sqrt{-c^2x^2 + 1}x^4}{c^4} + \frac{8 \sqrt{-c^2x^2 + 1}x^2}{c^6} + \frac{16 \sqrt{-c^2x^2 + 1}}{c^8} \right.\right. + \frac{1}{2835} \left(315 x^9 \arccos(cx) - \left(\frac{35 \sqrt{-c^2x^2 + 1}x^8}{c^2} + \frac{40 \sqrt{-c^2x^2 + 1}x^6}{c^4} + \frac{48 \sqrt{-c^2x^2 + 1}x^4}{c^6} + \frac{64 \sqrt{-c^2x^2 + 1}x^2}{c^8} \right.\right.$$

input
```
integrate(x^4*(e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/9*a*e^2*x^9 + 2/7*a*d*e*x^7 + 1/5*a*d^2*x^5 + 1/75*(15*x^5*arccos(c*x) -
 (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^
2*x^2 + 1)/c^6)*c)*b*d^2 + 2/245*(35*x^7*arccos(c*x) - (5*sqrt(-c^2*x^2 +
1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 +
 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*d*e + 1/2835*(315*x^9*arccos(c*x) - (35*s
qrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*x
^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/c
^10)*c)*b*e^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.41

$$\int x^4(d+ex^2)^2(a+b\arccos(cx))\,dx = \frac{1}{9}be^2x^9\arccos(cx)+\frac{1}{9}ae^2x^9 + \frac{2}{7}bdex^7\arccos(cx)-\frac{\sqrt{-c^2x^2+1}be^2x^8}{81\,c} + \frac{2}{7}adex^7+\frac{1}{5}bd^2x^5\arccos(cx) - \frac{2\sqrt{-c^2x^2+1}bdex^6}{49\,c}+\frac{1}{5}ad^2x^5 - \frac{\sqrt{-c^2x^2+1}bd^2x^4}{25\,c}-\frac{8\sqrt{-c^2x^2+1}be^2x^6}{567\,c^3} - \frac{12\sqrt{-c^2x^2+1}bdex^4}{245\,c^3}-\frac{4\sqrt{-c^2x^2+1}bd^2x^2}{75\,c^3} - \frac{16\sqrt{-c^2x^2+1}be^2x^4}{945\,c^5}-\frac{16\sqrt{-c^2x^2+1}bdex^2}{245\,c^5} - \frac{8\sqrt{-c^2x^2+1}bd^2}{75\,c^5}-\frac{64\sqrt{-c^2x^2+1}be^2x^2}{2835\,c^7} - \frac{32\sqrt{-c^2x^2+1}bde}{245\,c^7}-\frac{128\sqrt{-c^2x^2+1}be^2}{2835\,c^9}$$

input
```
integrate(x^4*(e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/9*b*e^2*x^9*arccos(c*x) + 1/9*a*e^2*x^9 + 2/7*b*d*e*x^7*arccos(c*x) - 1/
81*sqrt(-c^2*x^2 + 1)*b*e^2*x^8/c + 2/7*a*d*e*x^7 + 1/5*b*d^2*x^5*arccos(c
*x) - 2/49*sqrt(-c^2*x^2 + 1)*b*d*e*x^6/c + 1/5*a*d^2*x^5 - 1/25*sqrt(-c^2
*x^2 + 1)*b*d^2*x^4/c - 8/567*sqrt(-c^2*x^2 + 1)*b*e^2*x^6/c^3 - 12/245*sq
rt(-c^2*x^2 + 1)*b*d*e*x^4/c^3 - 4/75*sqrt(-c^2*x^2 + 1)*b*d^2*x^2/c^3 - 1
6/945*sqrt(-c^2*x^2 + 1)*b*e^2*x^4/c^5 - 16/245*sqrt(-c^2*x^2 + 1)*b*d*e*x
^2/c^5 - 8/75*sqrt(-c^2*x^2 + 1)*b*d^2/c^5 - 64/2835*sqrt(-c^2*x^2 + 1)*b*
e^2*x^2/c^7 - 32/245*sqrt(-c^2*x^2 + 1)*b*d*e/c^7 - 128/2835*sqrt(-c^2*x^2
 + 1)*b*e^2/c^9
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d+ex^2)^2 (a+b\arccos(cx))\,dx = \int x^4 (a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int(x^4*(a + b*acos(c*x))*(d + e*x^2)^2,x)
```

output
```
int(x^4*(a + b*acos(c*x))*(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.44

$$\int x^4 (d+ex^2)^2 (a+b\arccos(cx))\,dx = \frac{19845acos(cx)\,b\,c^9d^2x^5 + 28350acos(cx)\,b\,c^9de\,x^7 + 11025acos(cx)\,b\,c^9e^2x^9 - 3969\sqrt{-c^2x^2+1}\,b\,c^8d^2x^4}{}$$

input
```
int(x^4*(e*x^2+d)^2*(a+b*acos(c*x)),x)
```

output
```
(19845*acos(c*x)*b*c**9*d**2*x**5 + 28350*acos(c*x)*b*c**9*d*e*x**7 + 1102
5*acos(c*x)*b*c**9*e**2*x**9 - 3969*sqrt( - c**2*x**2 + 1)*b*c**8*d**2*x**
4 - 4050*sqrt( - c**2*x**2 + 1)*b*c**8*d*e*x**6 - 1225*sqrt( - c**2*x**2 +
 1)*b*c**8*e**2*x**8 - 5292*sqrt( - c**2*x**2 + 1)*b*c**6*d**2*x**2 - 4860
*sqrt( - c**2*x**2 + 1)*b*c**6*d*e*x**4 - 1400*sqrt( - c**2*x**2 + 1)*b*c*
*6*e**2*x**6 - 10584*sqrt( - c**2*x**2 + 1)*b*c**4*d**2 - 6480*sqrt( - c**
2*x**2 + 1)*b*c**4*d*e*x**2 - 1680*sqrt( - c**2*x**2 + 1)*b*c**4*e**2*x**4
 - 12960*sqrt( - c**2*x**2 + 1)*b*c**2*d*e - 2240*sqrt( - c**2*x**2 + 1)*b
*c**2*e**2*x**2 - 4480*sqrt( - c**2*x**2 + 1)*b*e**2 + 19845*a*c**9*d**2*x
**5 + 28350*a*c**9*d*e*x**7 + 11025*a*c**9*e**2*x**9)/(99225*c**9)
```

3.608 $\int x^3(d+ex^2)^2 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 241

$$\begin{aligned}\int x^3\left(d+ex^2\right)^2(a+b\arccos(cx))\,dx &= \frac{b(288c^4d^2+320c^2de+105e^2)\,x\sqrt{1-c^2x^2}}{3072c^7} \\ &\quad+\frac{b(288c^4d^2+320c^2de+105e^2)\,x^3\sqrt{1-c^2x^2}}{4608c^5} \\ &\quad+\frac{be(64c^2d+21e)\,x^5\sqrt{1-c^2x^2}}{1152c^3} \\ &\quad+\frac{be^2x^7\sqrt{1-c^2x^2}}{64c} \\ &\quad-\frac{b(288c^4d^2+320c^2de+105e^2)\arccos(cx)}{3072c^8} \\ &\quad+\frac{1}{4}d^2x^4(a+b\arccos(cx)) \\ &\quad+\frac{1}{3}dex^6(a+b\arccos(cx)) \\ &\quad+\frac{1}{8}e^2x^8(a+b\arccos(cx))\end{aligned}$$

output
```
1/3072*b*(288*c^4*d^2+320*c^2*d*e+105*e^2)*x*(-c^2*x^2+1)^(1/2)/c^7+1/4608
*b*(288*c^4*d^2+320*c^2*d*e+105*e^2)*x^3*(-c^2*x^2+1)^(1/2)/c^5+1/1152*b*e
*(64*c^2*d+21*e)*x^5*(-c^2*x^2+1)^(1/2)/c^3+1/64*b*e^2*x^7*(-c^2*x^2+1)^(1
/2)/c-1/3072*b*(288*c^4*d^2+320*c^2*d*e+105*e^2)*arccos(c*x)/c^8+1/4*d^2*x
^4*(a+b*arccos(c*x))+1/3*d*e*x^6*(a+b*arccos(c*x))+1/8*e^2*x^8*(a+b*arccos
(c*x))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.81

$$\int x^3 (d+ex^2)^2 (a+b\arccos(cx))\,dx$$
$$= \frac{384ac^8x^4(6d^2+8dex^2+3e^2x^4) - bcx\sqrt{1-c^2x^2}(315e^2+30c^2e(32d+7ex^2)+8c^4(108d^2+80dex^2+21}{}$$

input
```
Integrate[x^3*(d + e*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(384*a*c^8*x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) - b*c*x*Sqrt[1 - c^2*x^2]*(
315*e^2 + 30*c^2*e*(32*d + 7*e*x^2) + 8*c^4*(108*d^2 + 80*d*e*x^2 + 21*e^2
*x^4) + 16*c^6*(36*d^2*x^2 + 32*d*e*x^4 + 9*e^2*x^6)) + 384*b*c^8*x^4*(6*d
^2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcCos[c*x] + 3*b*(288*c^4*d^2 + 320*c^2*d*e +
 105*e^2)*ArcSin[c*x])/(9216*c^8)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5231, 27, 1590, 25, 363, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (d+ex^2)^2 (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \; 5231$$

$$bc\int \frac{x^4(3e^2x^4+8dex^2+6d^2)}{24\sqrt{1-c^2x^2}}dx+\frac{1}{4}d^2x^4(a+b\arccos(cx))+\frac{1}{3}dex^6(a+b\arccos(cx))+\frac{1}{8}e^2x^8(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}bc\int \frac{x^4(3e^2x^4+8dex^2+6d^2)}{\sqrt{1-c^2x^2}}dx+\frac{1}{4}d^2x^4(a+b\arccos(cx))+\frac{1}{3}dex^6(a+b\arccos(cx))+\frac{1}{8}e^2x^8(a+b\arccos(cx))$$

$$\Big\downarrow \text{1590}$$

$$\frac{1}{24}bc\left(-\frac{\int -\frac{x^4(48c^2d^2+e(64dc^2+21e)x^2)}{\sqrt{1-c^2x^2}}dx}{8c^2}-\frac{3e^2x^7\sqrt{1-c^2x^2}}{8c^2}\right)+\frac{1}{4}d^2x^4(a+b\arccos(cx))+\frac{1}{3}dex^6(a+b\arccos(cx))+\frac{1}{8}e^2x^8(a+b\arccos(cx))$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{24}bc\left(\frac{\int \frac{x^4(48c^2d^2+e(64dc^2+21e)x^2)}{\sqrt{1-c^2x^2}}dx}{8c^2}-\frac{3e^2x^7\sqrt{1-c^2x^2}}{8c^2}\right)+\frac{1}{4}d^2x^4(a+b\arccos(cx))+\frac{1}{3}dex^6(a+b\arccos(cx))+\frac{1}{8}e^2x^8(a+b\arccos(cx))$$

$$\Big\downarrow \text{363}$$

$$\frac{1}{24}bc\left(\frac{\frac{(288c^4d^2+320c^2de+105e^2)\int \frac{x^4}{\sqrt{1-c^2x^2}}dx}{6c^2}-\frac{ex^5\sqrt{1-c^2x^2}(64c^2d+21e)}{6c^2}}{8c^2}-\frac{3e^2x^7\sqrt{1-c^2x^2}}{8c^2}\right)+\frac{1}{4}d^2x^4(a+b\arccos(cx))+\frac{1}{3}dex^6(a+b\arccos(cx))+\frac{1}{8}e^2x^8(a+b\arccos(cx))$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{24}bc\left(\frac{\frac{(288c^4d^2+320c^2de+105e^2)\left(\frac{3\int \frac{x^2}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)}{6c^2}-\frac{ex^5\sqrt{1-c^2x^2}(64c^2d+21e)}{6c^2}}{8c^2}-\frac{3e^2x^7\sqrt{1-c^2x^2}}{8c^2}\right)+\frac{1}{4}d^2x^4(a+b\arccos(cx))+\frac{1}{3}dex^6(a+b\arccos(cx))+\frac{1}{8}e^2x^8(a+b\arccos(cx))$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{24}bc\left(\frac{\frac{\left(288c^4d^2+320c^2de+105e^2\right)\left(\frac{3\left(\frac{\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)}{6c^2}-\frac{ex^5\sqrt{1-c^2x^2}\left(64c^2d+21e\right)}{6c^2}}{8c^2}-\frac{3e^2x^7\sqrt{1-c^2x^2}}{8c^2}\right)$$

$$\frac{1}{4}d^2x^4(a+b\arccos(cx))+\frac{1}{3}dex^6(a+b\arccos(cx))+\frac{1}{8}e^2x^8(a+b\arccos(cx))$$

$$\Big\downarrow 223$$

$$\frac{1}{4}d^2x^4(a+b\arccos(cx))+\frac{1}{3}dex^6(a+b\arccos(cx))+\frac{1}{8}e^2x^8(a+b\arccos(cx))+$$
$$\frac{1}{24}bc\left(\frac{\frac{\left(\frac{3\left(\frac{\arcsin(cx)}{2c^3}-\frac{x\sqrt{1-c^2x^2}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{1-c^2x^2}}{4c^2}\right)\left(288c^4d^2+320c^2de+105e^2\right)}{6c^2}-\frac{ex^5\sqrt{1-c^2x^2}\left(64c^2d+21e\right)}{6c^2}}{8c^2}-\frac{3e^2x^7\sqrt{1-c^2x^2}}{8c^2}\right)$$

input
```
Int[x^3*(d + e*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*x^4*(a + b*ArcCos[c*x]))/4 + (d*e*x^6*(a + b*ArcCos[c*x]))/3 + (e^2*x
^8*(a + b*ArcCos[c*x]))/8 + (b*c*((-3*e^2*x^7*Sqrt[1 - c^2*x^2])/(8*c^2) +
 (-1/6*(e*(64*c^2*d + 21*e)*x^5*Sqrt[1 - c^2*x^2])/c^2 + ((288*c^4*d^2 + 3
20*c^2*d*e + 105*e^2)*(-1/4*(x^3*Sqrt[1 - c^2*x^2])/c^2 + (3*(-1/2*(x*Sqrt
[1 - c^2*x^2])/c^2 + ArcSin[c*x]/(2*c^3)))/(4*c^2)))/(6*c^2))/(8*c^2)))/24
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.22

method	result
parts	$a\left(\frac{1}{8}e^2x^8+\frac{1}{3}de\,x^6+\frac{1}{4}d^2x^4\right)+\frac{b\left(\frac{c^4\arccos(cx)e^2x^8}{8}+\frac{c^4\arccos(cx)de\,x^6}{3}+\frac{\arccos(cx)c^4x^4d^2}{4}+\frac{3e^2\left(-\frac{c^7x^7\sqrt{-c^2}}{8}\right.}{}\right.}{}$
derivativedivides	$\frac{a\left(\frac{1}{4}c^8d^2x^4+\frac{1}{3}c^8de\,x^6+\frac{1}{8}e^2x^8c^8\right)}{c^4}+\frac{b\left(\frac{\arccos(cx)d^2c^8x^4}{4}+\frac{\arccos(cx)d\,c^8e\,x^6}{3}+\frac{\arccos(cx)e^2c^8x^8}{8}+\frac{e^2\left(-\frac{c^7x^7\sqrt{-c^2x^2+1}}{8}-\frac{7c^5}{}\right.}{}\right.}{}$
default	$\frac{a\left(\frac{1}{4}c^8d^2x^4+\frac{1}{3}c^8de\,x^6+\frac{1}{8}e^2x^8c^8\right)}{c^4}+\frac{b\left(\frac{\arccos(cx)d^2c^8x^4}{4}+\frac{\arccos(cx)d\,c^8e\,x^6}{3}+\frac{\arccos(cx)e^2c^8x^8}{8}+\frac{e^2\left(-\frac{c^7x^7\sqrt{-c^2x^2+1}}{8}-\frac{7c^5}{}\right.}{}\right.}{}$
orering	$\frac{(2160c^8e^3x^{10}+8240c^8d\,e^2x^8+10944c^8d^2e\,x^6+168x^8e^3c^6+4032c^8d^3x^4+968x^6e^2c^6d+2400x^4e\,c^6d^2+294x^6e^3c^4+864}{921}$

input
```
int(x^3*(e*x^2+d)^2*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/8*e^2*x^8+1/3*d*e*x^6+1/4*d^2*x^4)+b/c^4*(1/8*c^4*arccos(c*x)*e^2*x^8
+1/3*c^4*arccos(c*x)*d*e*x^6+1/4*arccos(c*x)*c^4*x^4*d^2+1/24/c^4*(3*e^2*(
-1/8*c^7*x^7*(-c^2*x^2+1)^(1/2)-7/48*c^5*x^5*(-c^2*x^2+1)^(1/2)-35/192*c^3
*x^3*(-c^2*x^2+1)^(1/2)-35/128*c*x*(-c^2*x^2+1)^(1/2)+35/128*arcsin(c*x))+
6*d^2*c^4*(-1/4*c^3*x^3*(-c^2*x^2+1)^(1/2)-3/8*c*x*(-c^2*x^2+1)^(1/2)+3/8*
arcsin(c*x))+8*d*c^2*e*(-1/6*c^5*x^5*(-c^2*x^2+1)^(1/2)-5/24*c^3*x^3*(-c^2
*x^2+1)^(1/2)-5/16*c*x*(-c^2*x^2+1)^(1/2)+5/16*arcsin(c*x))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.90

$$\int x^3\left(d+ex^2\right)^2\left(a+b\arccos(cx)\right)dx$$
$$=\frac{1152\,ac^8e^2x^8+3072\,ac^8dex^6+2304\,ac^8d^2x^4+3\left(384\,bc^8e^2x^8+1024\,bc^8dex^6+768\,bc^8d^2x^4-288\,bc^4d^2\right.}{}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/9216*(1152*a*c^8*e^2*x^8 + 3072*a*c^8*d*e*x^6 + 2304*a*c^8*d^2*x^4 + 3*(
384*b*c^8*e^2*x^8 + 1024*b*c^8*d*e*x^6 + 768*b*c^8*d^2*x^4 - 288*b*c^4*d^2
 - 320*b*c^2*d*e - 105*b*e^2)*arccos(c*x) - (144*b*c^7*e^2*x^7 + 8*(64*b*c
^7*d*e + 21*b*c^5*e^2)*x^5 + 2*(288*b*c^7*d^2 + 320*b*c^5*d*e + 105*b*c^3*
e^2)*x^3 + 3*(288*b*c^5*d^2 + 320*b*c^3*d*e + 105*b*c*e^2)*x)*sqrt(-c^2*x^
2 + 1))/c^8
```

Sympy [A] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.61

$$\int x^3 (d+ex^2)^2 (a+b\arccos(cx))\,dx$$
$$= \begin{cases} \frac{ad^2x^4}{4} + \frac{adex^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4\operatorname{acos}(cx)}{4} + \frac{bdex^6\operatorname{acos}(cx)}{3} + \frac{be^2x^8\operatorname{acos}(cx)}{8} - \frac{bd^2x^3\sqrt{-c^2x^2+1}}{16c} - \frac{bdex^5\sqrt{-c^2x^2+1}}{18c} - \frac{be^2x}{} \\ \left(a+\frac{\pi b}{2}\right)\left(\frac{d^2x^4}{4}+\frac{dex^6}{3}+\frac{e^2x^8}{8}\right) \end{cases}$$

input
```
integrate(x**3*(e*x**2+d)**2*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*acos
(c*x)/4 + b*d*e*x**6*acos(c*x)/3 + b*e**2*x**8*acos(c*x)/8 - b*d**2*x**3*s
qrt(-c**2*x**2 + 1)/(16*c) - b*d*e*x**5*sqrt(-c**2*x**2 + 1)/(18*c) - b*e*
*2*x**7*sqrt(-c**2*x**2 + 1)/(64*c) - 3*b*d**2*x*sqrt(-c**2*x**2 + 1)/(32*
c**3) - 5*b*d*e*x**3*sqrt(-c**2*x**2 + 1)/(72*c**3) - 7*b*e**2*x**5*sqrt(-
c**2*x**2 + 1)/(384*c**3) - 3*b*d**2*acos(c*x)/(32*c**4) - 5*b*d*e*x*sqrt(
-c**2*x**2 + 1)/(48*c**5) - 35*b*e**2*x**3*sqrt(-c**2*x**2 + 1)/(1536*c**5
) - 5*b*d*e*acos(c*x)/(48*c**6) - 35*b*e**2*x*sqrt(-c**2*x**2 + 1)/(1024*c
**7) - 35*b*e**2*acos(c*x)/(1024*c**8), Ne(c, 0)), ((a + pi*b/2)*(d**2*x**
4/4 + d*e*x**6/3 + e**2*x**8/8), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.19

$$\int x^3(d+ex^2)^2(a+b\arccos(cx))\,dx = \frac{1}{8}ae^2x^8 + \frac{1}{3}adex^6 + \frac{1}{4}ad^2x^4 + \frac{1}{32}\left(8x^4\arccos(cx) - \left(\frac{2\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\sqrt{-c^2x^2+1}x}{c^4} - \frac{3\arcsin(cx)}{c^5}\right)c\right)bd^2 + \frac{1}{144}\left(48x^6\arccos(cx) - \left(\frac{8\sqrt{-c^2x^2+1}x^5}{c^2} + \frac{10\sqrt{-c^2x^2+1}x^3}{c^4} + \frac{15\sqrt{-c^2x^2+1}x}{c^6} - \frac{15\arcsin(cx)}{c^7}\right. + \frac{1}{3072}\left(384x^8\arccos(cx) - \left(\frac{48\sqrt{-c^2x^2+1}x^7}{c^2} + \frac{56\sqrt{-c^2x^2+1}x^5}{c^4} + \frac{70\sqrt{-c^2x^2+1}x^3}{c^6} + \frac{105\sqrt{-c^2}}{c^8}\right.\right.$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/32*(8*x^4*arccos(c*x) -
(2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)
/c^5)*c)*b*d^2 + 1/144*(48*x^6*arccos(c*x) - (8*sqrt(-c^2*x^2 + 1)*x^5/c^2
 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/c^6 - 15*arcsin
(c*x)/c^7)*c)*b*d*e + 1/3072*(384*x^8*arccos(c*x) - (48*sqrt(-c^2*x^2 + 1)
*x^7/c^2 + 56*sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(-c^2*x^2 + 1)*x^3/c^6 +
 105*sqrt(-c^2*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9)*c)*b*e^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.29

$$\int x^3(d+ex^2)^2(a+b\arccos(cx))\,dx = \frac{1}{8}be^2x^8\arccos(cx)+\frac{1}{8}ae^2x^8+\frac{1}{3}bdex^6\arccos(cx)-\frac{\sqrt{-c^2x^2+1}be^2x^7}{64\,c}+\frac{1}{3}adex^6+\frac{1}{4}bd^2x^4\arccos(cx)-\frac{\sqrt{-c^2x^2+1}bdex^5}{18\,c}+\frac{1}{4}ad^2x^4-\frac{\sqrt{-c^2x^2+1}bd^2x^3}{16\,c}-\frac{7\sqrt{-c^2x^2+1}be^2x^5}{384\,c^3}-\frac{5\sqrt{-c^2x^2+1}bdex^3}{72\,c^3}-\frac{3\sqrt{-c^2x^2+1}bd^2x}{32\,c^3}-\frac{35\sqrt{-c^2x^2+1}be^2x^3}{1536\,c^5}-\frac{3\,bd^2\arccos(cx)}{32\,c^4}-\frac{5\sqrt{-c^2x^2+1}bdex}{48\,c^5}-\frac{5\,bde\arccos(cx)}{48\,c^6}-\frac{35\sqrt{-c^2x^2+1}be^2x}{1024\,c^7}-\frac{35\,be^2\arccos(cx)}{1024\,c^8}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/8*b*e^2*x^8*arccos(c*x) + 1/8*a*e^2*x^8 + 1/3*b*d*e*x^6*arccos(c*x) - 1/
64*sqrt(-c^2*x^2 + 1)*b*e^2*x^7/c + 1/3*a*d*e*x^6 + 1/4*b*d^2*x^4*arccos(c
*x) - 1/18*sqrt(-c^2*x^2 + 1)*b*d*e*x^5/c + 1/4*a*d^2*x^4 - 1/16*sqrt(-c^2
*x^2 + 1)*b*d^2*x^3/c - 7/384*sqrt(-c^2*x^2 + 1)*b*e^2*x^5/c^3 - 5/72*sqrt
(-c^2*x^2 + 1)*b*d*e*x^3/c^3 - 3/32*sqrt(-c^2*x^2 + 1)*b*d^2*x/c^3 - 35/15
36*sqrt(-c^2*x^2 + 1)*b*e^2*x^3/c^5 - 3/32*b*d^2*arccos(c*x)/c^4 - 5/48*sq
rt(-c^2*x^2 + 1)*b*d*e*x/c^5 - 5/48*b*d*e*arccos(c*x)/c^6 - 35/1024*sqrt(-
c^2*x^2 + 1)*b*e^2*x/c^7 - 35/1024*b*e^2*arccos(c*x)/c^8
```

Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)^2(a+b\arccos(cx))\,dx=\int x^3\,(a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int(x^3*(a + b*acos(c*x))*(d + e*x^2)^2,x)
```

output
```
int(x^3*(a + b*acos(c*x))*(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.32

$$\int x^3(d+ex^2)^2(a+b\arccos(cx))\,dx$$
$$=\frac{2304acos(cx)\,b\,c^8d^2x^4+3072acos(cx)\,b\,c^8de\,x^6+1152acos(cx)\,b\,c^8e^2x^8+864asin(cx)\,b\,c^4d^2+960asin(\ldots}{\ldots}$$

input
```
int(x^3*(e*x^2+d)^2*(a+b*acos(c*x)),x)
```

output
```
(2304*acos(c*x)*b*c**8*d**2*x**4 + 3072*acos(c*x)*b*c**8*d*e*x**6 + 1152*a
cos(c*x)*b*c**8*e**2*x**8 + 864*asin(c*x)*b*c**4*d**2 + 960*asin(c*x)*b*c*
*2*d*e + 315*asin(c*x)*b*e**2 - 576*sqrt( - c**2*x**2 + 1)*b*c**7*d**2*x**
3 - 512*sqrt( - c**2*x**2 + 1)*b*c**7*d*e*x**5 - 144*sqrt( - c**2*x**2 + 1
)*b*c**7*e**2*x**7 - 864*sqrt( - c**2*x**2 + 1)*b*c**5*d**2*x - 640*sqrt(
- c**2*x**2 + 1)*b*c**5*d*e*x**3 - 168*sqrt( - c**2*x**2 + 1)*b*c**5*e**2*
x**5 - 960*sqrt( - c**2*x**2 + 1)*b*c**3*d*e*x - 210*sqrt( - c**2*x**2 + 1
)*b*c**3*e**2*x**3 - 315*sqrt( - c**2*x**2 + 1)*b*c*e**2*x + 2304*a*c**8*d
**2*x**4 + 3072*a*c**8*d*e*x**6 + 1152*a*c**8*e**2*x**8)/(9216*c**8)
```

3.609 $\int x^2(d+ex^2)^2(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 198

$$\begin{aligned}\int x^2(d+ex^2)^2(a+b\arccos(cx))\,dx &= \frac{b(35c^4d^2+42c^2de+15e^2)\sqrt{1-c^2x^2}}{105c^7} - \frac{b(35c^4d^2+84c^2de+45e^2)(1-c^2x^2)^{3/2}}{315c^7} \\ &\quad + \frac{be(14c^2d+15e)(1-c^2x^2)^{5/2}}{175c^7} - \frac{be^2(1-c^2x^2)^{7/2}}{49c^7} \\ &\quad + \frac{1}{3}d^2x^3(a+b\arccos(cx)) + \frac{2}{5}dex^5(a+b\arccos(cx)) + \frac{1}{7}e^2x^7(a+b\arccos(cx))\end{aligned}$$

output
```
1/105*b*(35*c^4*d^2+42*c^2*d*e+15*e^2)*(-c^2*x^2+1)^(1/2)/c^7-1/315*b*(35*
c^4*d^2+84*c^2*d*e+45*e^2)*(-c^2*x^2+1)^(3/2)/c^7+1/175*b*e*(14*c^2*d+15*e
)*(-c^2*x^2+1)^(5/2)/c^7-1/49*b*e^2*(-c^2*x^2+1)^(7/2)/c^7+1/3*d^2*x^3*(a+
b*arccos(c*x))+2/5*d*e*x^5*(a+b*arccos(c*x))+1/7*e^2*x^7*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.80

$$\int x^2\left(d+ex^2\right)^2\left(a+b\arccos(cx)\right)dx$$
$$=\frac{105ax^3(35d^2+42dex^2+15e^2x^4)-\frac{b\sqrt{1-c^2x^2}(720e^2+24c^2e(98d+15ex^2)+2c^4(1225d^2+588dex^2+135e^2x^4)+c^6(1225d^2x^2+882}{c^7}}{11025}$$

input
```
Integrate[x^2*(d + e*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(105*a*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) - (b*Sqrt[1 - c^2*x^2]*(720*
e^2 + 24*c^2*e*(98*d + 15*e*x^2) + 2*c^4*(1225*d^2 + 588*d*e*x^2 + 135*e^2
*x^4) + c^6*(1225*d^2*x^2 + 882*d*e*x^4 + 225*e^2*x^6)))/c^7 + 105*b*x^3*(
35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcCos[c*x])/11025
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5231, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d+ex^2\right)^2\left(a+b\arccos(cx)\right)dx$$

$$\Big\downarrow \text{5231}$$

$$bc\int \frac{x^3\left(15e^2x^4+42dex^2+35d^2\right)}{105\sqrt{1-c^2x^2}}dx+\frac{1}{3}d^2x^3(a+b\arccos(cx))+\frac{2}{5}dex^5(a+b\arccos(cx))+\frac{1}{7}e^2x^7(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{105}bc\int \frac{x^3\left(15e^2x^4+42dex^2+35d^2\right)}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}d^2x^3(a+b\arccos(cx))+\frac{2}{5}dex^5(a+b\arccos(cx))+\frac{1}{7}e^2x^7(a+b\arccos(cx))$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{210}bc\int \frac{x^2(15e^2x^4+42dex^2+35d^2)}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{3}d^2x^3(a+b\arccos(cx))+\frac{2}{5}dex^5(a+b\arccos(cx))+\frac{1}{7}e^2x^7(a+b\arccos(cx))$$

$$\Big\downarrow \text{1195}$$

$$\frac{1}{210}bc\int \left(-\frac{15e^2(1-c^2x^2)^{5/2}}{c^6}+\frac{3e(14dc^2+15e)(1-c^2x^2)^{3/2}}{c^6}+\frac{(-35d^2c^4-84dec^2-45e^2)\sqrt{1-c^2x^2}}{c^6}+\frac{35\ldots}{}\right.$$
$$\frac{1}{3}d^2x^3(a+b\arccos(cx))+\frac{2}{5}dex^5(a+b\arccos(cx))+\frac{1}{7}e^2x^7(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}d^2x^3(a+b\arccos(cx))+\frac{2}{5}dex^5(a+b\arccos(cx))+\frac{1}{7}e^2x^7(a+b\arccos(cx))+$$
$$\frac{1}{210}bc\left(-\frac{6e(1-c^2x^2)^{5/2}(14c^2d+15e)}{5c^8}+\frac{30e^2(1-c^2x^2)^{7/2}}{7c^8}+\frac{2(1-c^2x^2)^{3/2}(35c^4d^2+84c^2de+45e^2)}{3c^8}-\frac{2\sqrt{\ldots}}{}\right.$$

input
```
Int[x^2*(d + e*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*((-2*(35*c^4*d^2 + 42*c^2*d*e + 15*e^2)*Sqrt[1 - c^2*x^2])/c^8 + (2*(
35*c^4*d^2 + 84*c^2*d*e + 45*e^2)*(1 - c^2*x^2)^(3/2))/(3*c^8) - (6*e*(14*
c^2*d + 15*e)*(1 - c^2*x^2)^(5/2))/(5*c^8) + (30*e^2*(1 - c^2*x^2)^(7/2))/
(7*c^8)))/210 + (d^2*x^3*(a + b*ArcCos[c*x]))/3 + (2*d*e*x^5*(a + b*ArcCos
[c*x]))/5 + (e^2*x^7*(a + b*ArcCos[c*x]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.36

method	result
parts	$a\left(\frac{1}{7}e^2x^7+\frac{2}{5}de\,x^5+\frac{1}{3}d^2x^3\right)+\frac{b\left(\frac{c^3\arccos(cx)e^2x^7}{7}+\frac{2c^3\arccos(cx)de\,x^5}{5}+\frac{\arccos(cx)c^3x^3d^2}{3}+\frac{15e^2\left(-\frac{c^6x^6\sqrt{-}}{7}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}dc^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\arccos(cx)d^2c^7x^3}{3}+\frac{2\arccos(cx)dc^7e\,x^5}{5}+\frac{\arccos(cx)e^2c^7x^7}{7}+\frac{e^2\left(-\frac{c^6x^6\sqrt{-c^2x^2+1}}{7}-6\right.}{}\right.}{}}{}$
default	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}dc^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\arccos(cx)d^2c^7x^3}{3}+\frac{2\arccos(cx)dc^7e\,x^5}{5}+\frac{\arccos(cx)e^2c^7x^7}{7}+\frac{e^2\left(-\frac{c^6x^6\sqrt{-c^2x^2+1}}{7}-6\right.}{}\right.}{}}{}$
orering	$\frac{(2925c^8e^3x^{10}+11727c^8d\,e^2x^8+17199c^8d^2e\,x^6+270x^8e^3c^6+6125c^8d^3x^4+1854x^6e^2c^6d+7938x^4e\,c^6d^2+540x^6e^3c^4+2}{1}$

input
```
int(x^2*(e*x^2+d)^2*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e^2*x^7+2/5*d*e*x^5+1/3*d^2*x^3)+b/c^3*(1/7*c^3*arccos(c*x)*e^2*x^7
+2/5*c^3*arccos(c*x)*d*e*x^5+1/3*arccos(c*x)*c^3*x^3*d^2+1/105/c^4*(15*e^2
*(-1/7*c^6*x^6*(-c^2*x^2+1)^(1/2)-6/35*c^4*x^4*(-c^2*x^2+1)^(1/2)-8/35*c^2
*x^2*(-c^2*x^2+1)^(1/2)-16/35*(-c^2*x^2+1)^(1/2))+35*d^2*c^4*(-1/3*c^2*x^2
*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))+42*d*c^2*e*(-1/5*c^4*x^4*(-c^2
*x^2+1)^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.94

$$\int x^2(d+ex^2)^2(a+b\arccos(cx))\,dx = \frac{1575\,ac^7e^2x^7+4410\,ac^7dex^5+3675\,ac^7d^2x^3+105\,(15\,bc^7e^2x^7+42\,bc^7dex^5+35\,bc^7d^2x^3)\arccos(cx)-}{}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/11025*(1575*a*c^7*e^2*x^7 + 4410*a*c^7*d*e*x^5 + 3675*a*c^7*d^2*x^3 + 10
5*(15*b*c^7*e^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*d^2*x^3)*arccos(c*x) - (
225*b*c^6*e^2*x^6 + 2450*b*c^4*d^2 + 2352*b*c^2*d*e + 18*(49*b*c^6*d*e + 1
5*b*c^4*e^2)*x^4 + 720*b*e^2 + (1225*b*c^6*d^2 + 1176*b*c^4*d*e + 360*b*c^
2*e^2)*x^2)*sqrt(-c^2*x^2 + 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.71

$$\int x^2(d+ex^2)^2(a+b\arccos(cx))\,dx = \begin{cases} \frac{ad^2x^3}{3}+\frac{2adex^5}{5}+\frac{ae^2x^7}{7}+\frac{bd^2x^3\operatorname{acos}(cx)}{3}+\frac{2bdex^5\operatorname{acos}(cx)}{5}+\frac{be^2x^7\operatorname{acos}(cx)}{7}-\frac{bd^2x^2\sqrt{-c^2x^2+1}}{9c}-\frac{2bdex^4\sqrt{-c^2x^2+1}}{25c}-b \\ \left(a+\frac{\pi b}{2}\right)\left(\frac{d^2x^3}{3}+\frac{2dex^5}{5}+\frac{e^2x^7}{7}\right) \end{cases}$$

input
```
integrate(x**2*(e*x**2+d)**2*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*ac
os(c*x)/3 + 2*b*d*e*x**5*acos(c*x)/5 + b*e**2*x**7*acos(c*x)/7 - b*d**2*x*
*2*sqrt(-c**2*x**2 + 1)/(9*c) - 2*b*d*e*x**4*sqrt(-c**2*x**2 + 1)/(25*c) -
 b*e**2*x**6*sqrt(-c**2*x**2 + 1)/(49*c) - 2*b*d**2*sqrt(-c**2*x**2 + 1)/(
9*c**3) - 8*b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(75*c**3) - 6*b*e**2*x**4*sqrt
(-c**2*x**2 + 1)/(245*c**3) - 16*b*d*e*sqrt(-c**2*x**2 + 1)/(75*c**5) - 8*
b*e**2*x**2*sqrt(-c**2*x**2 + 1)/(245*c**5) - 16*b*e**2*sqrt(-c**2*x**2 +
1)/(245*c**7), Ne(c, 0)), ((a + pi*b/2)*(d**2*x**3/3 + 2*d*e*x**5/5 + e**2
*x**7/7), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.29

$$\int x^2\left(d+ex^2\right)^2(a+b\arccos(cx))\,dx = \frac{1}{7}\,ae^2x^7 + \frac{2}{5}\,adex^5 + \frac{1}{3}\,ad^2x^3 + \frac{1}{9}\left(3\,x^3\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bd^2 + \frac{2}{75}\left(15\,x^5\arccos(cx) - \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bde + \frac{1}{245}\left(35\,x^7\arccos(cx) - \left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\sqrt{-c^2x^2+1}}{c^8}\right.\right.$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arccos(c*x) - c
*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d^2 + 2/75*(15
*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^
2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d*e + 1/245*(35*x^7*arccos(c*x) - (
5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*
x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*e^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.36

$$\int x^2(d+ex^2)^2(a+b\arccos(cx))\,dx = \frac{1}{7}be^2x^7\arccos(cx)+\frac{1}{7}ae^2x^7+\frac{2}{5}bdex^5\arccos(cx) - \frac{\sqrt{-c^2x^2+1}be^2x^6}{49\,c}+\frac{2}{5}adex^5 + \frac{1}{3}bd^2x^3\arccos(cx)-\frac{2\sqrt{-c^2x^2+1}bdex^4}{25\,c} + \frac{1}{3}ad^2x^3-\frac{\sqrt{-c^2x^2+1}bd^2x^2}{9\,c} - \frac{6\sqrt{-c^2x^2+1}be^2x^4}{245\,c^3}-\frac{8\sqrt{-c^2x^2+1}bdex^2}{75\,c^3} - \frac{2\sqrt{-c^2x^2+1}bd^2}{9\,c^3}-\frac{8\sqrt{-c^2x^2+1}be^2x^2}{245\,c^5} - \frac{16\sqrt{-c^2x^2+1}bde}{75\,c^5}-\frac{16\sqrt{-c^2x^2+1}be^2}{245\,c^7}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/7*b*e^2*x^7*arccos(c*x) + 1/7*a*e^2*x^7 + 2/5*b*d*e*x^5*arccos(c*x) - 1/
49*sqrt(-c^2*x^2 + 1)*b*e^2*x^6/c + 2/5*a*d*e*x^5 + 1/3*b*d^2*x^3*arccos(c
*x) - 2/25*sqrt(-c^2*x^2 + 1)*b*d*e*x^4/c + 1/3*a*d^2*x^3 - 1/9*sqrt(-c^2*
x^2 + 1)*b*d^2*x^2/c - 6/245*sqrt(-c^2*x^2 + 1)*b*e^2*x^4/c^3 - 8/75*sqrt(
-c^2*x^2 + 1)*b*d*e*x^2/c^3 - 2/9*sqrt(-c^2*x^2 + 1)*b*d^2/c^3 - 8/245*sqr
t(-c^2*x^2 + 1)*b*e^2*x^2/c^5 - 16/75*sqrt(-c^2*x^2 + 1)*b*d*e/c^5 - 16/24
5*sqrt(-c^2*x^2 + 1)*b*e^2/c^7
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)^2(a+b\arccos(cx))\,dx = \int x^2\,(a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int(x^2*(a + b*acos(c*x))*(d + e*x^2)^2,x)
```

output
```
int(x^2*(a + b*acos(c*x))*(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.41

$$\int x^2(d+ex^2)^2(a+b\arccos(cx))\,dx$$
$$= \frac{3675acos(cx)\,b\,c^7d^2x^3+4410acos(cx)\,b\,c^7de\,x^5+1575acos(cx)\,b\,c^7e^2x^7-1225\sqrt{-c^2x^2+1}\,b\,c^6d^2x^2-88}{}$$

input
```
int(x^2*(e*x^2+d)^2*(a+b*acos(c*x)),x)
```

output
```
(3675*acos(c*x)*b*c**7*d**2*x**3 + 4410*acos(c*x)*b*c**7*d*e*x**5 + 1575*a
cos(c*x)*b*c**7*e**2*x**7 - 1225*sqrt( - c**2*x**2 + 1)*b*c**6*d**2*x**2 -
 882*sqrt( - c**2*x**2 + 1)*b*c**6*d*e*x**4 - 225*sqrt( - c**2*x**2 + 1)*b
*c**6*e**2*x**6 - 2450*sqrt( - c**2*x**2 + 1)*b*c**4*d**2 - 1176*sqrt( - c
**2*x**2 + 1)*b*c**4*d*e*x**2 - 270*sqrt( - c**2*x**2 + 1)*b*c**4*e**2*x**
4 - 2352*sqrt( - c**2*x**2 + 1)*b*c**2*d*e - 360*sqrt( - c**2*x**2 + 1)*b*
c**2*e**2*x**2 - 720*sqrt( - c**2*x**2 + 1)*b*e**2 + 3675*a*c**7*d**2*x**3
 + 4410*a*c**7*d*e*x**5 + 1575*a*c**7*e**2*x**7)/(11025*c**7)
```

3.610 $\int x(d+ex^2)^2 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 177

$$\int x(d+ex^2)^2 (a+b\arccos(cx))\,dx = \frac{b(24c^4d^2+18c^2de+5e^2)\,x\sqrt{1-c^2x^2}}{96c^5} + \frac{be(18c^2d+5e)\,x^3\sqrt{1-c^2x^2}}{144c^3} + \frac{be^2x^5\sqrt{1-c^2x^2}}{36c} - \frac{b(2c^2d+e)\,(8c^4d^2+8c^2de+5e^2)\arccos(cx)}{96c^6e} + \frac{(d+ex^2)^3\,(a+b\arccos(cx))}{6e}$$

output
```
1/96*b*(24*c^4*d^2+18*c^2*d*e+5*e^2)*x*(-c^2*x^2+1)^(1/2)/c^5+1/144*b*e*(1
8*c^2*d+5*e)*x^3*(-c^2*x^2+1)^(1/2)/c^3+1/36*b*e^2*x^5*(-c^2*x^2+1)^(1/2)/
c-1/96*b*(2*c^2*d+e)*(8*c^4*d^2+8*c^2*d*e+5*e^2)*arccos(c*x)/c^6/e+1/6*(e*
x^2+d)^3*(a+b*arccos(c*x))/e
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.93

$$\int x\left(d+ex^2\right)^2(a+b\arccos(cx))\,dx$$
$$=\frac{cx\left(48ac^5x(3d^2+3dex^2+e^2x^4)-b\sqrt{1-c^2x^2}(15e^2+2c^2e(27d+5ex^2)+4c^4(18d^2+9dex^2+2e^2x^4))\right)}{288c^6}$$

input
```
Integrate[x*(d + e*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(c*x*(48*a*c^5*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) - b*Sqrt[1 - c^2*x^2]*(15*e
^2 + 2*c^2*e*(27*d + 5*e*x^2) + 4*c^4*(18*d^2 + 9*d*e*x^2 + 2*e^2*x^4))) +
 48*b*c^6*x^2*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcCos[c*x] + 3*b*(24*c^4*d^2
+ 18*c^2*d*e + 5*e^2)*ArcSin[c*x])/(288*c^6)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5229, 318, 25, 403, 25, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^2(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \; 5229$$

$$\frac{bc\int \frac{\left(ex^2+d\right)^3}{\sqrt{1-c^2x^2}}dx}{6e}+\frac{\left(d+ex^2\right)^3(a+b\arccos(cx))}{6e}$$

$$\Big\downarrow \; 318$$

$$\frac{bc\left(-\frac{\int -\frac{\left(ex^2+d\right)\left(5e\left(2dc^2+e\right)x^2+d\left(6dc^2+e\right)\right)}{\sqrt{1-c^2x^2}}dx}{6c^2}-\frac{ex\sqrt{1-c^2x^2}\left(d+ex^2\right)^2}{6c^2}\right)}{6e}+\frac{\left(d+ex^2\right)^3(a+b\arccos(cx))}{6e}$$

$$\Big\downarrow \; 25$$

$$\frac{bc\left(\frac{\int \frac{(ex^2+d)\left(5e(2dc^2+e)x^2+d(6dc^2+e)\right)}{\sqrt{1-c^2x^2}}dx}{6c^2}-\frac{ex\sqrt{1-c^2x^2}(d+ex^2)^2}{6c^2}\right)}{6e}+\frac{(d+ex^2)^3(a+b\arccos(cx))}{6e}$$

$\Big\downarrow$ 403

$$\frac{bc\left(\frac{-\frac{\int -\frac{e(44d^2c^4+44dec^2+15e^2)x^2+d(24d^2c^4+14dec^2+5e^2)}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{5ex\sqrt{1-c^2x^2}(2c^2d+e)(d+ex^2)}{4c^2}}{6c^2}-\frac{ex\sqrt{1-c^2x^2}(d+ex^2)^2}{6c^2}\right)}{6e}+\frac{(d+ex^2)^3(a+b\arccos(cx))}{6e}$$

$\Big\downarrow$ 25

$$\frac{bc\left(\frac{\frac{\int \frac{e(44d^2c^4+44dec^2+15e^2)x^2+d(24d^2c^4+14dec^2+5e^2)}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{5ex\sqrt{1-c^2x^2}(2c^2d+e)(d+ex^2)}{4c^2}}{6c^2}-\frac{ex\sqrt{1-c^2x^2}(d+ex^2)^2}{6c^2}\right)}{6e}+\frac{(d+ex^2)^3(a+b\arccos(cx))}{6e}$$

$\Big\downarrow$ 299

$$\frac{bc\left(\frac{\frac{\frac{3(2c^2d+e)(8c^4d^2+8c^2de+5e^2)\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{ex\sqrt{1-c^2x^2}(44c^4d^2+44c^2de+15e^2)}{2c^2}}{4c^2}-\frac{5ex\sqrt{1-c^2x^2}(2c^2d+e)(d+ex^2)}{4c^2}}{6c^2}-\frac{ex\sqrt{1-c^2x^2}(d+ex^2)^2}{6c^2}\right)}{6e}+\frac{(d+ex^2)^3(a+b\arccos(cx))}{6e}$$

$\Big\downarrow$ 223

$$\frac{(d+ex^2)^3(a+b\arccos(cx))}{6e}+\frac{bc\left(\frac{\frac{\frac{3\arcsin(cx)(2c^2d+e)(8c^4d^2+8c^2de+5e^2)}{2c^3}-\frac{ex\sqrt{1-c^2x^2}(44c^4d^2+44c^2de+15e^2)}{2c^2}}{4c^2}-\frac{5ex\sqrt{1-c^2x^2}(2c^2d+e)(d+ex^2)}{4c^2}}{6c^2}-\frac{ex\sqrt{1-c^2x^2}(d+ex^2)^2}{6c^2}\right)}{6e}$$

input
```
Int[x*(d + e*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
((d + e*x^2)^3*(a + b*ArcCos[c*x]))/(6*e) + (b*c*(-1/6*(e*x*Sqrt[1 - c^2*x
^2]*(d + e*x^2)^2)/c^2 + ((-5*e*(2*c^2*d + e)*x*Sqrt[1 - c^2*x^2]*(d + e*x
^2))/(4*c^2) + (-1/2*(e*(44*c^4*d^2 + 44*c^2*d*e + 15*e^2)*x*Sqrt[1 - c^2*
x^2])/c^2 + (3*(2*c^2*d + e)*(8*c^4*d^2 + 8*c^2*d*e + 5*e^2)*ArcSin[c*x])/
(2*c^3))/(4*c^2))/(6*c^2)))/(6*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5229
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])/(2*e*(p + 1))), x]
 + Simp[b*(c/(2*e*(p + 1)))   Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x]
, x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.43

method	result
parts	$\frac{a\left(e\,x^{2}+d\right)^{3}}{6e}+\frac{b\left(\frac{c^{2} e^{2} \arccos\left(c x\right) x^{6}}{6}+\frac{c^{2} e \arccos\left(c x\right) x^{4} d}{2}+\frac{\arccos\left(c x\right) c^{2} x^{2} d^{2}}{2}+\frac{c^{2} \arccos\left(c x\right) d^{3}}{6 e}+\frac{c^{6} d^{3} \arcsin\left(c x\right)+e^{3}\left(-\frac{c^{5} x^{5}}{}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a\left(c^{2} e\,x^{2}+c^{2} d\right)^{3}}{6 c^{4} e}+\frac{b\left(\frac{\arccos\left(c x\right) c^{6} d^{3}}{6 e}+\frac{\arccos\left(c x\right) c^{6} d^{2} x^{2}}{2}+\frac{e \arccos\left(c x\right) c^{6} d\,x^{4}}{2}+\frac{e^{2} \arccos\left(c x\right) c^{6} x^{6}}{6}+\frac{c^{6} d^{3} \arcsin\left(c x\right)+e^{3}\left(-\frac{c^{5} x^{5}}{}\right.}{}\right.}{}}{}$
default	$\frac{\frac{a\left(c^{2} e\,x^{2}+c^{2} d\right)^{3}}{6 c^{4} e}+\frac{b\left(\frac{\arccos\left(c x\right) c^{6} d^{3}}{6 e}+\frac{\arccos\left(c x\right) c^{6} d^{2} x^{2}}{2}+\frac{e \arccos\left(c x\right) c^{6} d\,x^{4}}{2}+\frac{e^{2} \arccos\left(c x\right) c^{6} x^{6}}{6}+\frac{c^{6} d^{3} \arcsin\left(c x\right)+e^{3}\left(-\frac{c^{5} x^{5}}{}\right.}{}\right.}{}}{}$
orering	$\frac{(88x^{8}e^{3}c^{6}+380x^{6}e^{2}c^{6}d+684x^{4}e\,c^{6}d^{2}+10x^{6}e^{3}c^{4}+216c^{6}d^{3}x^{2}+92x^{4}e^{2}c^{4}d-414x^{2}e\,c^{4}d^{2}+25x^{4}e^{3}c^{2}-144c^{4}d^{3}-319x^{2}e}{288(e\,x^{2}+d)c^{6}}$

input
```
int(x*(e*x^2+d)^2*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*(e*x^2+d)^3/e+b/c^2*(1/6*c^2*e^2*arccos(c*x)*x^6+1/2*c^2*e*arccos(c*
x)*x^4*d+1/2*arccos(c*x)*c^2*x^2*d^2+1/6*c^2/e*arccos(c*x)*d^3+1/6/c^4/e*(
c^6*d^3*arcsin(c*x)+e^3*(-1/6*c^5*x^5*(-c^2*x^2+1)^(1/2)-5/24*c^3*x^3*(-c^
2*x^2+1)^(1/2)-5/16*c*x*(-c^2*x^2+1)^(1/2)+5/16*arcsin(c*x))+3*d*c^2*e^2*(
-1/4*c^3*x^3*(-c^2*x^2+1)^(1/2)-3/8*c*x*(-c^2*x^2+1)^(1/2)+3/8*arcsin(c*x)
)+3*d^2*c^4*e*(-1/2*c*x*(-c^2*x^2+1)^(1/2)+1/2*arcsin(c*x))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.04

$$\int x(d+ex^2)^2(a+b\arccos(cx))\,dx$$
$$= \frac{48\,ac^6e^2x^6+144\,ac^6dex^4+144\,ac^6d^2x^2+3\,(16\,bc^6e^2x^6+48\,bc^6dex^4+48\,bc^6d^2x^2-24\,bc^4d^2-18\,bc^2de}{2}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/288*(48*a*c^6*e^2*x^6 + 144*a*c^6*d*e*x^4 + 144*a*c^6*d^2*x^2 + 3*(16*b*
c^6*e^2*x^6 + 48*b*c^6*d*e*x^4 + 48*b*c^6*d^2*x^2 - 24*b*c^4*d^2 - 18*b*c^
2*d*e - 5*b*e^2)*arccos(c*x) - (8*b*c^5*e^2*x^5 + 2*(18*b*c^5*d*e + 5*b*c^
3*e^2)*x^3 + 3*(24*b*c^5*d^2 + 18*b*c^3*d*e + 5*b*c*e^2)*x)*sqrt(-c^2*x^2
+ 1))/c^6
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.72

$$\int x(d+ex^2)^2(a+b\arccos(cx))\,dx$$
$$= \begin{cases} \frac{ad^2x^2}{2}+\frac{adex^4}{2}+\frac{ae^2x^6}{6}+\frac{bd^2x^2\operatorname{acos}(cx)}{2}+\frac{bdex^4\operatorname{acos}(cx)}{2}+\frac{be^2x^6\operatorname{acos}(cx)}{6}-\frac{bd^2x\sqrt{-c^2x^2+1}}{4c}-\frac{bdex^3\sqrt{-c^2x^2+1}}{8c}-\frac{be^2x^5}{} \\ \left(a+\frac{\pi b}{2}\right)\left(\frac{d^2x^2}{2}+\frac{dex^4}{2}+\frac{e^2x^6}{6}\right) \end{cases}$$

input
```
integrate(x*(e*x**2+d)**2*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*acos
(c*x)/2 + b*d*e*x**4*acos(c*x)/2 + b*e**2*x**6*acos(c*x)/6 - b*d**2*x*sqrt
(-c**2*x**2 + 1)/(4*c) - b*d*e*x**3*sqrt(-c**2*x**2 + 1)/(8*c) - b*e**2*x*
*5*sqrt(-c**2*x**2 + 1)/(36*c) - b*d**2*acos(c*x)/(4*c**2) - 3*b*d*e*x*sqr
t(-c**2*x**2 + 1)/(16*c**3) - 5*b*e**2*x**3*sqrt(-c**2*x**2 + 1)/(144*c**3
) - 3*b*d*e*acos(c*x)/(16*c**4) - 5*b*e**2*x*sqrt(-c**2*x**2 + 1)/(96*c**5
) - 5*b*e**2*acos(c*x)/(96*c**6), Ne(c, 0)), ((a + pi*b/2)*(d**2*x**2/2 +
d*e*x**4/2 + e**2*x**6/6), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.28

$$\int x(d+ex^2)^2(a+b\arccos(cx))\,dx = \frac{1}{6}\,ae^2x^6 + \frac{1}{2}\,adex^4 + \frac{1}{2}\,ad^2x^2 + \frac{1}{4}\left(2\,x^2\arccos\left(cx\right) - c\left(\frac{\sqrt{-c^2x^2+1}x}{c^2} - \frac{\arcsin\left(cx\right)}{c^3}\right)\right)bd^2 + \frac{1}{16}\left(8\,x^4\arccos\left(cx\right) - \left(\frac{2\,\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\,\sqrt{-c^2x^2+1}x}{c^4} - \frac{3\,\arcsin\left(cx\right)}{c^5}\right)c\right)bde + \frac{1}{288}\left(48\,x^6\arccos\left(cx\right) - \left(\frac{8\,\sqrt{-c^2x^2+1}x^5}{c^2} + \frac{10\,\sqrt{-c^2x^2+1}x^3}{c^4} + \frac{15\,\sqrt{-c^2x^2+1}x}{c^6} - \frac{15\,\arcsin\left(cx\right)}{c^7}\right.\right.$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/4*(2*x^2*arccos(c*x) - c
*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*x)/c^3))*b*d^2 + 1/16*(8*x^4*arccos(
c*x) - (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)*x/c^4 - 3*arcs
in(c*x)/c^5)*c)*b*d*e + 1/288*(48*x^6*arccos(c*x) - (8*sqrt(-c^2*x^2 + 1)*
x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 + 1)*x/c^6 - 15
*arcsin(c*x)/c^7)*c)*b*e^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.35

$$\int x(d+ex^2)^2 (a+b\arccos(cx))\,dx = \frac{1}{6}be^2x^6\arccos(cx)+\frac{1}{6}ae^2x^6+\frac{1}{2}bdex^4\arccos(cx) - \frac{\sqrt{-c^2x^2+1}be^2x^5}{36\,c} + \frac{1}{2}adex^4 + \frac{1}{2}bd^2x^2\arccos(cx) - \frac{\sqrt{-c^2x^2+1}bdex^3}{8\,c} + \frac{1}{2}ad^2x^2 - \frac{\sqrt{-c^2x^2+1}bd^2x}{4\,c} - \frac{5\sqrt{-c^2x^2+1}be^2x^3}{144\,c^3} - \frac{bd^2\arccos(cx)}{4\,c^2} - \frac{3\sqrt{-c^2x^2+1}bdex}{16\,c^3} - \frac{3\,bde\arccos(cx)}{16\,c^4} - \frac{5\sqrt{-c^2x^2+1}be^2x}{96\,c^5} - \frac{5\,be^2\arccos(cx)}{96\,c^6}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/6*b*e^2*x^6*arccos(c*x) + 1/6*a*e^2*x^6 + 1/2*b*d*e*x^4*arccos(c*x) - 1/
36*sqrt(-c^2*x^2 + 1)*b*e^2*x^5/c + 1/2*a*d*e*x^4 + 1/2*b*d^2*x^2*arccos(c
*x) - 1/8*sqrt(-c^2*x^2 + 1)*b*d*e*x^3/c + 1/2*a*d^2*x^2 - 1/4*sqrt(-c^2*x
^2 + 1)*b*d^2*x/c - 5/144*sqrt(-c^2*x^2 + 1)*b*e^2*x^3/c^3 - 1/4*b*d^2*arc
cos(c*x)/c^2 - 3/16*sqrt(-c^2*x^2 + 1)*b*d*e*x/c^3 - 3/16*b*d*e*arccos(c*x
)/c^4 - 5/96*sqrt(-c^2*x^2 + 1)*b*e^2*x/c^5 - 5/96*b*e^2*arccos(c*x)/c^6
```

Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^2 (a+b\arccos(cx))\,dx = \int x\,(a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int(x*(a + b*acos(c*x))*(d + e*x^2)^2,x)
```

output
```
int(x*(a + b*acos(c*x))*(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.42

$$\int x(d+ex^2)^2(a+b\arccos(cx))\,dx$$
$$= \frac{144acos(cx)\,b\,c^6d^2x^2+144acos(cx)\,b\,c^6de\,x^4+48acos(cx)\,b\,c^6e^2x^6+72asin(cx)\,b\,c^4d^2+54asin(cx)\,b\,c^2}{}$$

input
```
int(x*(e*x^2+d)^2*(a+b*acos(c*x)),x)
```

output
```
(144*acos(c*x)*b*c**6*d**2*x**2 + 144*acos(c*x)*b*c**6*d*e*x**4 + 48*acos(
c*x)*b*c**6*e**2*x**6 + 72*asin(c*x)*b*c**4*d**2 + 54*asin(c*x)*b*c**2*d*e
 + 15*asin(c*x)*b*e**2 - 72*sqrt( - c**2*x**2 + 1)*b*c**5*d**2*x - 36*sqrt
( - c**2*x**2 + 1)*b*c**5*d*e*x**3 - 8*sqrt( - c**2*x**2 + 1)*b*c**5*e**2*
x**5 - 54*sqrt( - c**2*x**2 + 1)*b*c**3*d*e*x - 10*sqrt( - c**2*x**2 + 1)*
b*c**3*e**2*x**3 - 15*sqrt( - c**2*x**2 + 1)*b*c*e**2*x + 144*a*c**6*d**2*
x**2 + 144*a*c**6*d*e*x**4 + 48*a*c**6*e**2*x**6)/(288*c**6)
```

3.611 $\int (d+ex^2)^2 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 150

$$\int (d+ex^2)^2 (a+b\arccos(cx))\,dx = \frac{b(15c^4d^2+10c^2de+3e^2)\sqrt{1-c^2x^2}}{15c^5} - \frac{2be(5c^2d+3e)\,(1-c^2x^2)^{3/2}}{45c^5} + \frac{be^2(1-c^2x^2)^{5/2}}{25c^5} + d^2x(a+b\arccos(cx)) + \frac{2}{3}dex^3(a+b\arccos(cx)) + \frac{1}{5}e^2x^5(a+b\arccos(cx))$$

output
```
1/15*b*(15*c^4*d^2+10*c^2*d*e+3*e^2)*(-c^2*x^2+1)^(1/2)/c^5-2/45*b*e*(5*c^
2*d+3*e)*(-c^2*x^2+1)^(3/2)/c^5+1/25*b*e^2*(-c^2*x^2+1)^(5/2)/c^5+d^2*x*(a
+b*arccos(c*x))+2/3*d*e*x^3*(a+b*arccos(c*x))+1/5*e^2*x^5*(a+b*arccos(c*x)
)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int (d+ex^2)^2 (a+b\arccos(cx))\,dx$$
$$=\frac{1}{225}\left(15ax(15d^2+10dex^2+3e^2x^4) - \frac{b\sqrt{1-c^2x^2}(24e^2+4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e^2x^4))}{c^5} + 15bx(15d^2+10dex^2+3e^2x^4)\arccos(cx)\right)$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(15*a*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - (b*Sqrt[1 - c^2*x^2]*(24*e^2 +
 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4)))/c^5 +
 15*b*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcCos[c*x])/225
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5171, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)^2 (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5171}$$

$$bc\int \frac{x(3e^2x^4+10dex^2+15d^2)}{15\sqrt{1-c^2x^2}}dx + d^2x(a+b\arccos(cx)) + \frac{2}{3}dex^3(a+b\arccos(cx)) + \frac{1}{5}e^2x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}bc\int\frac{x(3e^2x^4+10dex^2+15d^2)}{\sqrt{1-c^2x^2}}dx+d^2x(a+b\arccos(cx))+\frac{2}{3}dex^3(a+b\arccos(cx))+\frac{1}{5}e^2x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{30}bc\int\frac{3e^2x^4+10dex^2+15d^2}{\sqrt{1-c^2x^2}}dx^2+d^2x(a+b\arccos(cx))+\frac{2}{3}dex^3(a+b\arccos(cx))+\frac{1}{5}e^2x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{1140}$$

$$\frac{1}{30}bc\int\left(\frac{3(1-c^2x^2)^{3/2}e^2}{c^4}-\frac{2(5dc^2+3e)\sqrt{1-c^2x^2}e}{c^4}+\frac{15d^2c^4+10dec^2+3e^2}{c^4\sqrt{1-c^2x^2}}\right)dx^2+d^2x(a+b\arccos(cx))+\frac{2}{3}dex^3(a+b\arccos(cx))+\frac{1}{5}e^2x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$d^2x(a+b\arccos(cx))+\frac{2}{3}dex^3(a+b\arccos(cx))+\frac{1}{5}e^2x^5(a+b\arccos(cx))+\frac{1}{30}bc\left(\frac{4e(1-c^2x^2)^{3/2}(5c^2d+3e)}{3c^6}-\frac{6e^2(1-c^2x^2)^{5/2}}{5c^6}-\frac{2\sqrt{1-c^2x^2}(15c^4d^2+10c^2de+3e^2)}{c^6}\right)$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*((-2*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*Sqrt[1 - c^2*x^2])/c^6 + (4*e*
(5*c^2*d + 3*e)*(1 - c^2*x^2)^(3/2))/(3*c^6) - (6*e^2*(1 - c^2*x^2)^(5/2))
/(5*c^6)))/30 + d^2*x*(a + b*ArcCos[c*x]) + (2*d*e*x^3*(a + b*ArcCos[c*x])
)/3 + (e^2*x^5*(a + b*ArcCos[c*x]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5171
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2
, 0])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.29

method	result
parts	$a\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b\left(\frac{c\arccos(cx)e^2x^5}{5}+\frac{2c\arccos(cx)x^3de}{3}+\arccos(cx)cx\,d^2+\frac{3e^2\left(-\frac{c^4x^4\sqrt{-c^2x^2+1}}{5}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}dc^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\arccos(cx)d^2c^5x+\frac{2\arccos(cx)dc^5e\,x^3}{3}+\frac{\arccos(cx)e^2c^5x^5}{5}+\frac{e^2\left(-\frac{c^4x^4\sqrt{-c^2x^2+1}}{5}-\frac{4c^2x^2}{5}\right.}{}\right.}{c}}{c}$
default	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}dc^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\arccos(cx)d^2c^5x+\frac{2\arccos(cx)dc^5e\,x^3}{3}+\frac{\arccos(cx)e^2c^5x^5}{5}+\frac{e^2\left(-\frac{c^4x^4\sqrt{-c^2x^2+1}}{5}-\frac{4c^2x^2}{5}\right.}{}\right.}{c}}{c}$
orering	$\frac{x\left(81e^3x^6c^6+395c^6de^2x^4+1275c^6d^2e\,x^2+12c^4e^3x^4+225c^6d^3+200c^4de^2x^2-900c^4d^2e+48c^2e^3x^2-400c^2de^2-96e^3\right)(}{225(e\,x^2+d)c^6}$

input
```
int((e*x^2+d)^2*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e^2*x^5+2/3*d*e*x^3+d^2*x)+b/c*(1/5*c*arccos(c*x)*e^2*x^5+2/3*c*arc
cos(c*x)*x^3*d*e+arccos(c*x)*c*x*d^2+1/15/c^4*(3*e^2*(-1/5*c^4*x^4*(-c^2*x
^2+1)^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))-15*d^
2*c^4*(-c^2*x^2+1)^(1/2)+10*d*c^2*e*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(
-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx$$
$$=\frac{45\,ac^5e^2x^5+150\,ac^5dex^3+225\,ac^5d^2x+15\,(3\,bc^5e^2x^5+10\,bc^5dex^3+15\,bc^5d^2x)\arccos{(cx)}-(9\,bc^4e^2x}{225\,c^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/225*(45*a*c^5*e^2*x^5 + 150*a*c^5*d*e*x^3 + 225*a*c^5*d^2*x + 15*(3*b*c^
5*e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x)*arccos(c*x) - (9*b*c^4*e^2*
x^4 + 225*b*c^4*d^2 + 100*b*c^2*d*e + 24*b*e^2 + 2*(25*b*c^4*d*e + 6*b*c^2
*e^2)*x^2)*sqrt(-c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.63

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx$$
$$=\begin{cases} ad^2x+\frac{2adex^3}{3}+\frac{ae^2x^5}{5}+bd^2x\operatorname{acos}(cx)+\frac{2bdex^3\operatorname{acos}(cx)}{3}+\frac{be^2x^5\operatorname{acos}(cx)}{5}-\frac{bd^2\sqrt{-c^2x^2+1}}{c}-\frac{2bdex^2\sqrt{-c^2x^2+1}}{9c}- \\ \left(a+\frac{\pi b}{2}\right)\left(d^2x+\frac{2dex^3}{3}+\frac{e^2x^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*acos(c*x)
+ 2*b*d*e*x**3*acos(c*x)/3 + b*e**2*x**5*acos(c*x)/5 - b*d**2*sqrt(-c**2*x
**2 + 1)/c - 2*b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(9*c) - b*e**2*x**4*sqrt(-c
**2*x**2 + 1)/(25*c) - 4*b*d*e*sqrt(-c**2*x**2 + 1)/(9*c**3) - 4*b*e**2*x*
*2*sqrt(-c**2*x**2 + 1)/(75*c**3) - 8*b*e**2*sqrt(-c**2*x**2 + 1)/(75*c**5
), Ne(c, 0)), ((a + pi*b/2)*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.24

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx$$
$$=\frac{1}{5}ae^2x^5+\frac{2}{3}adex^3+\frac{2}{9}\left(3x^3\arccos(cx)-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bde$$
$$+\frac{1}{75}\left(15x^5\arccos(cx)-\left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2}+\frac{4\sqrt{-c^2x^2+1}x^2}{c^4}+\frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)be^2$$
$$+ad^2x+\frac{\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)bd^2}{c}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 2/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2
+ 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d*e + 1/75*(15*x^5*arccos(c*x)
 - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-
c^2*x^2 + 1)/c^6)*c)*b*e^2 + a*d^2*x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 +
1))*b*d^2/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.28

$$\int (d+ex^2)^2 (a+b\arccos(cx))\,dx = \frac{1}{5}\,be^2x^5\arccos(cx) + \frac{1}{5}\,ae^2x^5 + \frac{2}{3}\,bdex^3\arccos(cx) - \frac{\sqrt{-c^2x^2+1}be^2x^4}{25\,c} + \frac{2}{3}\,adex^3 + bd^2x\arccos(cx) - \frac{2\sqrt{-c^2x^2+1}bdex^2}{9\,c} + ad^2x - \frac{\sqrt{-c^2x^2+1}bd^2}{c} - \frac{4\sqrt{-c^2x^2+1}be^2x^2}{75\,c^3} - \frac{4\sqrt{-c^2x^2+1}bde}{9\,c^3} - \frac{8\sqrt{-c^2x^2+1}be^2}{75\,c^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/5*b*e^2*x^5*arccos(c*x) + 1/5*a*e^2*x^5 + 2/3*b*d*e*x^3*arccos(c*x) - 1/
25*sqrt(-c^2*x^2 + 1)*b*e^2*x^4/c + 2/3*a*d*e*x^3 + b*d^2*x*arccos(c*x) -
2/9*sqrt(-c^2*x^2 + 1)*b*d*e*x^2/c + a*d^2*x - sqrt(-c^2*x^2 + 1)*b*d^2/c
- 4/75*sqrt(-c^2*x^2 + 1)*b*e^2*x^2/c^3 - 4/9*sqrt(-c^2*x^2 + 1)*b*d*e/c^3
 - 8/75*sqrt(-c^2*x^2 + 1)*b*e^2/c^5
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int((a + b*acos(c*x))*(d + e*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))*(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.39

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx = \frac{225acos(cx)\,b\,c^5d^2x+150acos(cx)\,b\,c^5de\,x^3+45acos(cx)\,b\,c^5e^2x^5-225\sqrt{-c^2x^2+1}\,b\,c^4d^2-50\sqrt{-c^2x^2}$$

input
```
int((e*x^2+d)^2*(a+b*acos(c*x)),x)
```

output
```
(225*acos(c*x)*b*c**5*d**2*x + 150*acos(c*x)*b*c**5*d*e*x**3 + 45*acos(c*x
)*b*c**5*e**2*x**5 - 225*sqrt( - c**2*x**2 + 1)*b*c**4*d**2 - 50*sqrt( - c
**2*x**2 + 1)*b*c**4*d*e*x**2 - 9*sqrt( - c**2*x**2 + 1)*b*c**4*e**2*x**4
- 100*sqrt( - c**2*x**2 + 1)*b*c**2*d*e - 12*sqrt( - c**2*x**2 + 1)*b*c**2
*e**2*x**2 - 24*sqrt( - c**2*x**2 + 1)*b*e**2 + 225*a*c**5*d**2*x + 150*a*
c**5*d*e*x**3 + 45*a*c**5*e**2*x**5)/(225*c**5)
```

3.612 $\int \frac{(d+ex^2)^2(a+b\arccos(cx))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 229

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\arccos(cx))}{x}\,dx &= \frac{bdex\sqrt{1-c^2x^2}}{2c}+\frac{3be^2x\sqrt{1-c^2x^2}}{32c^3}\\&+\frac{be^2x^3\sqrt{1-c^2x^2}}{16c}-\frac{bde\arccos(cx)}{2c^2}\\&-\frac{3be^2\arccos(cx)}{32c^4}-\frac{1}{2}ibd^2\arccos(cx)^2\\&+dex^2(a+b\arccos(cx))+\frac{1}{4}e^2x^4(a+b\arccos(cx))\\&+bd^2\arccos(cx)\log\left(1-e^{2i\arccos(cx)}\right)\\&-bd^2\arccos(cx)\log(x)+d^2(a+b\arccos(cx))\log(x)\\&-\frac{1}{2}ibd^2\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\end{aligned}$$

output

```
1/2*b*d*e*x*(-c^2*x^2+1)^(1/2)/c+3/32*b*e^2*x*(-c^2*x^2+1)^(1/2)/c^3+1/16*
b*e^2*x^3*(-c^2*x^2+1)^(1/2)/c-1/2*b*d*e*arccos(c*x)/c^2-3/32*b*e^2*arccos
(c*x)/c^4-1/2*I*b*d^2*arccos(c*x)^2+d*e*x^2*(a+b*arccos(c*x))+1/4*e^2*x^4*
(a+b*arccos(c*x))+b*d^2*arccos(c*x)*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-b*d
^2*arccos(c*x)*ln(x)+d^2*(a+b*arccos(c*x))*ln(x)-1/2*I*b*d^2*polylog(2,(c*
x+I*(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x}\,dx$$
$$= adex^2 + \frac{1}{4}ae^2x^4 + bdex^2\arccos(cx) + \frac{1}{4}be^2x^4\arccos(cx) - \frac{be^2\left(cx\sqrt{1-c^2x^2}(3+2c^2x^2) - 6\arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right)\right)}{32c^4} + \frac{bde\left(-\frac{1}{2}cx\sqrt{1-c^2x^2} + \arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right)\right)}{c^2} + ad^2\log(x) - \frac{1}{2}ibd^2\left(\arccos(cx)\left(\arccos(cx) + 2i\log\left(1+e^{2i\arccos(cx)}\right)\right) + \text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCos[c*x]))/x,x]
```

output
```
a*d*e*x^2 + (a*e^2*x^4)/4 + b*d*e*x^2*ArcCos[c*x] + (b*e^2*x^4*ArcCos[c*x]
)/4 - (b*e^2*(c*x*Sqrt[1 - c^2*x^2]*(3 + 2*c^2*x^2) - 6*ArcTan[(c*x)/(-1 +
 Sqrt[1 - c^2*x^2])]))/(32*c^4) + (b*d*e*(-1/2*(c*x*Sqrt[1 - c^2*x^2]) + A
rcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]))/c^2 + a*d^2*Log[x] - (I/2)*b*d^2*(
ArcCos[c*x]*(ArcCos[c*x] + (2*I)*Log[1 + E^((2*I)*ArcCos[c*x])]) + PolyLog
[2, -E^((2*I)*ArcCos[c*x])])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5231, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x}\,dx$$

$$\Big\downarrow \text{5231}$$

$$bc\int \frac{e^2x^4+4dex^2+4d^2\log(x)}{4\sqrt{1-c^2x^2}}dx+d^2\log(x)(a+b\arccos(cx))+dex^2(a+b\arccos(cx))+\frac{1}{4}e^2x^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}bc\int \frac{e^2x^4+4dex^2+4d^2\log(x)}{\sqrt{1-c^2x^2}}dx+d^2\log(x)(a+b\arccos(cx))+dex^2(a+b\arccos(cx))+\frac{1}{4}e^2x^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{4}bc\int \left(\frac{e^2x^4}{\sqrt{1-c^2x^2}}+\frac{4dex^2}{\sqrt{1-c^2x^2}}+\frac{4d^2\log(x)}{\sqrt{1-c^2x^2}}\right)dx+d^2\log(x)(a+b\arccos(cx))+dex^2(a+b\arccos(cx))+\frac{1}{4}e^2x^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$d^2\log(x)(a+b\arccos(cx))+dex^2(a+b\arccos(cx))+\frac{1}{4}e^2x^4(a+b\arccos(cx))+\frac{1}{4}bc\left(\frac{3e^2\arcsin(cx)}{8c^5}+\frac{2de\arcsin(cx)}{c^3}+\frac{2id^2\operatorname{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{c}+\frac{2id^2\arcsin(cx)^2}{c}-\frac{4d^2\arcsin(cx)\log\left(1\right.}{c}\right.$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCos[c*x]))/x,x]
```

output
```
d*e*x^2*(a + b*ArcCos[c*x]) + (e^2*x^4*(a + b*ArcCos[c*x]))/4 + d^2*(a + b
*ArcCos[c*x])*Log[x] + (b*c*((-2*d*e*x*Sqrt[1 - c^2*x^2])/c^2 - (3*e^2*x*S
qrt[1 - c^2*x^2])/(8*c^4) - (e^2*x^3*Sqrt[1 - c^2*x^2])/(4*c^2) + (2*d*e*A
rcSin[c*x])/c^3 + (3*e^2*ArcSin[c*x])/(8*c^5) + ((2*I)*d^2*ArcSin[c*x]^2)/
c - (4*d^2*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/c + (4*d^2*ArcSin[c
*x]*Log[x])/c + ((2*I)*d^2*PolyLog[2, E^((2*I)*ArcSin[c*x])])/c))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.02

method	result
parts	$a\left(\frac{e^2x^4}{4}+de\,x^2+d^2\ln\left(x\right)\right)+b\arccos\left(cx\right)de\,x^2-\frac{b\,e^2x\sqrt{-c^2x^2+1}}{8c^3}+\frac{b\,e^2\arccos(cx)\cos(4\arccos$
derivativedivides	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln\left(cx\right)+b\,d^2\arccos\left(cx\right)\ln\left(1+\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)-\frac{ib\,d^2\,\mathrm{po}}{}$
default	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln\left(cx\right)+b\,d^2\arccos\left(cx\right)\ln\left(1+\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)-\frac{ib\,d^2\,\mathrm{po}}{}$

input
```
int((e*x^2+d)^2*(a+b*arccos(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*e^2*x^4+d*e*x^2+d^2*ln(x))+b*arccos(c*x)*d*e*x^2-1/8*b*e^2*x*(-c^2*
x^2+1)^(1/2)/c^3+1/32*b/c^4*e^2*arccos(c*x)*cos(4*arccos(c*x))+1/4*b/c^2*a
rccos(c*x)*e^2*x^2-1/2*b*d*e*arccos(c*x)/c^2-1/2*I*b*d^2*polylog(2,-(c*x+I
*(-c^2*x^2+1)^(1/2))^2)-1/128*b/c^4*e^2*sin(4*arccos(c*x))+b*d^2*arccos(c*
x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1/2*I*b*d^2*arccos(c*x)^2-1/2*b*d*e*
x*(-c^2*x^2+1)^(1/2)/c-1/8*b*e^2*arccos(c*x)/c^4
```

Fricas [F]

$$\int \frac{(d+ex^2)^2\,(a+b\arccos(cx))}{x}\,dx = \int \frac{(ex^2+d)^2(b\arccos{(cx)}+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arccos(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2\,(a+b\arccos(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}\,(cx))\,(d+ex^2)^2}{x}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acos(c*x))/x,x)
```

output
```
Integral((a + b*acos(c*x))*(d + e*x**2)**2/x, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x} \, dx = \int \frac{(ex^2+d)^2 (b\arccos(cx)+a)}{x} \, dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) + integrate((b*e^2*x^4 + 2*b*d*e*
x^2 + b*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x} \, dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(e\,x^2+d)^2}{x} \, dx$$

input
```
int(((a + b*acos(c*x))*(d + e*x^2)^2)/x,x)
```

output
```
int(((a + b*acos(c*x))*(d + e*x^2)^2)/x, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x}\,dx$$
$$= \frac{32acos(cx)\,b\,c^4de\,x^2 + 8acos(cx)\,b\,c^4e^2x^4 + 16asin(cx)\,b\,c^2de + 3asin(cx)\,b\,e^2 - 16\sqrt{-c^2x^2+1}\,b\,c^3dex -}{}$$

input
```
int((e*x^2+d)^2*(a+b*acos(c*x))/x,x)
```

output
```
(32*acos(c*x)*b*c**4*d*e*x**2 + 8*acos(c*x)*b*c**4*e**2*x**4 + 16*asin(c*x
)*b*c**2*d*e + 3*asin(c*x)*b*e**2 - 16*sqrt( - c**2*x**2 + 1)*b*c**3*d*e*x
 - 2*sqrt( - c**2*x**2 + 1)*b*c**3*e**2*x**3 - 3*sqrt( - c**2*x**2 + 1)*b*
c*e**2*x + 32*int(acos(c*x)/x,x)*b*c**4*d**2 + 32*log(x)*a*c**4*d**2 + 32*
a*c**4*d*e*x**2 + 8*a*c**4*e**2*x**4)/(32*c**4)
```

3.613 $\int \frac{(d+ex^2)^2(a+b\arccos(cx))}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{(d+ex^2)^2(a+b\arccos(cx))}{x^2}\,dx = \frac{be(6c^2d+e)\sqrt{1-c^2x^2}}{3c^3} - \frac{be^2(1-c^2x^2)^{3/2}}{9c^3} - \frac{d^2(a+b\arccos(cx))}{x} + 2dex(a+b\arccos(cx)) + \frac{1}{3}e^2x^3(a+b\arccos(cx)) - bcd^2\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)$$

output

```
1/3*b*e*(6*c^2*d+e)*(-c^2*x^2+1)^(1/2)/c^3-1/9*b*e^2*(-c^2*x^2+1)^(3/2)/c^
3-d^2*(a+b*arccos(c*x))/x+2*d*e*x*(a+b*arccos(c*x))+1/3*e^2*x^3*(a+b*arcco
s(c*x))-b*c*d^2*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^2}\,dx = -\frac{ad^2}{x}+2adex+\frac{1}{3}ae^2x^3 - \frac{be\sqrt{1-c^2x^2}(2e+c^2(18d+ex^2))}{9c^3} + \frac{b(-3d^2+6dex^2+e^2x^4)\arccos(cx)}{3x} - bcd^2\log(x)+bcd^2\log\left(1+\sqrt{1-c^2x^2}\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
-((a*d^2)/x) + 2*a*d*e*x + (a*e^2*x^3)/3 - (b*e*Sqrt[1 - c^2*x^2]*(2*e + c
^2*(18*d + e*x^2)))/(9*c^3) + (b*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcCos[c*x
])/(3*x) - b*c*d^2*Log[x] + b*c*d^2*Log[1 + Sqrt[1 - c^2*x^2]]
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5231, 27, 1578, 1192, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^2}\,dx$$

$$\Big\downarrow \text{5231}$$

$$bc\int -\frac{-e^2x^4-6dex^2+3d^2}{3x\sqrt{1-c^2x^2}}dx - \frac{d^2(a+b\arccos(cx))}{x}+2dex(a+b\arccos(cx))+\frac{1}{3}e^2x^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}bc\int \frac{-e^2x^4-6dex^2+3d^2}{x\sqrt{1-c^2x^2}}dx-\frac{d^2(a+b\arccos(cx))}{x}+2dex(a+b\arccos(cx))+\frac{1}{3}e^2x^3(a+b\arccos(cx))$$

$\downarrow$ 1578

$$-\frac{1}{6}bc\int \frac{-e^2x^4-6dex^2+3d^2}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{d^2(a+b\arccos(cx))}{x}+2dex(a+b\arccos(cx))+\frac{1}{3}e^2x^3(a+b\arccos(cx))$$

$\downarrow$ 1192

$$-\frac{b\int -\frac{-e^2x^8+2e(3dc^2+e)x^4+3c^4d^2-e^2-6c^2de}{1-x^4}d\sqrt{1-c^2x^2}}{3c^3}-\frac{d^2(a+b\arccos(cx))}{x}+2dex(a+b\arccos(cx))+\frac{1}{3}e^2x^3(a+b\arccos(cx))$$

$\downarrow$ 25

$$\frac{b\int \frac{-e^2x^8+2e(3dc^2+e)x^4+3c^4d^2-e^2-6c^2de}{1-x^4}d\sqrt{1-c^2x^2}}{3c^3}-\frac{d^2(a+b\arccos(cx))}{x}+2dex(a+b\arccos(cx))+\frac{1}{3}e^2x^3(a+b\arccos(cx))$$

$\downarrow$ 1467

$$\frac{b\int \left(\frac{3d^2c^4}{1-x^4}+e^2x^4-e\left(6dc^2+e\right)\right)d\sqrt{1-c^2x^2}}{3c^3}-\frac{d^2(a+b\arccos(cx))}{x}+2dex(a+b\arccos(cx))+\frac{1}{3}e^2x^3(a+b\arccos(cx))$$

$\downarrow$ 2009

$$-\frac{d^2(a+b\arccos(cx))}{x}+2dex(a+b\arccos(cx))+\frac{1}{3}e^2x^3(a+b\arccos(cx))-\frac{b\left(-3c^4d^2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+e\sqrt{1-c^2x^2}\left(6c^2d+e\right)-\frac{1}{3}e^2x^6\right)}{3c^3}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
-((d^2*(a + b*ArcCos[c*x]))/x) + 2*d*e*x*(a + b*ArcCos[c*x]) + (e^2*x^3*(a
 + b*ArcCos[c*x]))/3 - (b*(-1/3*(e^2*x^6) + e*(6*c^2*d + e)*Sqrt[1 - c^2*x
^2] - 3*c^4*d^2*ArcTanh[Sqrt[1 - c^2*x^2]]))/(3*c^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.25

method	result
parts	$a\left(\frac{e^2x^3}{3}+2dex-\frac{d^2}{x}\right)+bc\left(\frac{\arccos(cx)e^2x^3}{3c}+\frac{2\arccos(cx)xde}{c}-\frac{\arccos(cx)d^2}{cx}+\frac{e^2\left(-\frac{c^2x^2\sqrt{-c^2x^2+1}}{3}\right.}{}\right.$
derivativedivides	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b\left(2\arccos(cx)c^3dex+\frac{\arccos(cx)e^2c^3x^3}{3}-\frac{\arccos(cx)c^3d^2}{x}+\frac{e^2\left(-\frac{c^2x^2\sqrt{-c^2x^2+1}}{3}-2\right.}{3}\right.}{c^4}\right.$
default	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b\left(2\arccos(cx)c^3dex+\frac{\arccos(cx)e^2c^3x^3}{3}-\frac{\arccos(cx)c^3d^2}{x}+\frac{e^2\left(-\frac{c^2x^2\sqrt{-c^2x^2+1}}{3}-2\right.}{3}\right.}{c^4}\right.$

input
```
int((e*x^2+d)^2*(a+b*arccos(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e^2*x^3+2*d*e*x-d^2/x)+b*c*(1/3/c*arccos(c*x)*e^2*x^3+2/c*arccos(c*
x)*x*d*e-arccos(c*x)*d^2/c/x+1/3/c^4*(e^2*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)
-2/3*(-c^2*x^2+1)^(1/2))+3*c^4*d^2*arctanh(1/(-c^2*x^2+1)^(1/2))-6*c^2*d*e
*(-c^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(114) = 228.

Time = 0.15 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.05

$$\int \frac{(d+ex^2)^2(a+b\arccos(cx))}{x^2}\,dx$$
$$=\frac{6\,ac^3e^2x^4+9\,bc^4d^2x\log\left(\sqrt{-c^2x^2+1}+1\right)-9\,bc^4d^2x\log\left(\sqrt{-c^2x^2+1}-1\right)+36\,ac^3dex^2-18\,ac^3d^2-}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/18*(6*a*c^3*e^2*x^4 + 9*b*c^4*d^2*x*log(sqrt(-c^2*x^2 + 1) + 1) - 9*b*c^
4*d^2*x*log(sqrt(-c^2*x^2 + 1) - 1) + 36*a*c^3*d*e*x^2 - 18*a*c^3*d^2 - 6*
(3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c
^2*x^2 - 1)) + 6*(b*c^3*e^2*x^4 + 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2 + (3*b*c^3
*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x)*arccos(c*x) - 2*(b*c^2*e^2*x^3 + 2*(9*b
*c^2*d*e + b*e^2)*x)*sqrt(-c^2*x^2 + 1))/(c^3*x)
```

Sympy [A] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^2} dx = -\frac{ad^2}{x} + 2adex + \frac{ae^2x^3}{3} - bcd^2 \left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \frac{1}{|c^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases} \right) + \frac{bce^2 \left(\begin{cases} -\frac{x^2\sqrt{-c^2x^2+1}}{3c^2} - \frac{2\sqrt{-c^2x^2+1}}{3c^4} & \text{for } c^2 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases} \right)}{3} - \frac{bd^2\operatorname{acos}(cx)}{x} + 2bde \left(\begin{cases} \frac{\pi x}{2} & \text{for } c = 0 \\ x\operatorname{acos}(cx) - \frac{\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases} \right) + \frac{be^2x^3\operatorname{acos}(cx)}{3}$$

input
```
integrate((e*x**2+d)**2*(a+b*acos(c*x))/x**2,x)
```

output
```
-a*d**2/x + 2*a*d*e*x + a*e**2*x**3/3 - b*c*d**2*Piecewise((-acosh(1/(c*x)
), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)), True)) + b*c*e**2*Piecewise((-
x**2*sqrt(-c**2*x**2 + 1)/(3*c**2) - 2*sqrt(-c**2*x**2 + 1)/(3*c**4), Ne(c
**2, 0)), (x**4/4, True))/3 - b*d**2*acos(c*x)/x + 2*b*d*e*Piecewise((pi*x
/2, Eq(c, 0)), (x*acos(c*x) - sqrt(-c**2*x**2 + 1)/c, True)) + b*e**2*x**3
*acos(c*x)/3
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^2}\,dx = \frac{1}{3}\,ae^2x^3 + \left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|}+\frac{2}{|x|}\right) - \frac{\arccos(cx)}{x}\right)bd^2 + \frac{1}{9}\left(3\,x^3\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)be^2 + 2\,adex + \frac{2\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)bde}{c} - \frac{ad^2}{x}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 + (c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*
x)/x)*b*d^2 + 1/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*s
qrt(-c^2*x^2 + 1)/c^4))*b*e^2 + 2*a*d*e*x + 2*(c*x*arccos(c*x) - sqrt(-c^2
*x^2 + 1))*b*d*e/c - a*d^2/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4251 vs. 2(114) = 228.

Time = 2.24 (sec) , antiderivative size = 4251, normalized size of antiderivative = 33.74

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))/x^2,x, algorithm="giac")
```

output
```
-b*c^4*d^2*arccos(c*x)/(c^3 - 2*(c^2*x^2 - 1)*c^3/(c*x + 1)^2 + 2*(c^2*x^2
 - 1)^3*c^3/(c*x + 1)^6 - (c^2*x^2 - 1)^4*c^3/(c*x + 1)^8) + b*c^4*d^2*log
(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/(c^3 - 2*(c^2*x^2 - 1)*c^3/(c*x + 1)^2
 + 2*(c^2*x^2 - 1)^3*c^3/(c*x + 1)^6 - (c^2*x^2 - 1)^4*c^3/(c*x + 1)^8) -
b*c^4*d^2*log(abs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/(c^3 - 2*(c^2*x^2 - 1)*c
^3/(c*x + 1)^2 + 2*(c^2*x^2 - 1)^3*c^3/(c*x + 1)^6 - (c^2*x^2 - 1)^4*c^3/(
c*x + 1)^8) - a*c^4*d^2/(c^3 - 2*(c^2*x^2 - 1)*c^3/(c*x + 1)^2 + 2*(c^2*x^
2 - 1)^3*c^3/(c*x + 1)^6 - (c^2*x^2 - 1)^4*c^3/(c*x + 1)^8) + 4*(c^2*x^2 -
 1)*b*c^4*d^2*arccos(c*x)/((c^3 - 2*(c^2*x^2 - 1)*c^3/(c*x + 1)^2 + 2*(c^2
*x^2 - 1)^3*c^3/(c*x + 1)^6 - (c^2*x^2 - 1)^4*c^3/(c*x + 1)^8)*(c*x + 1)^2
) - 2*(c^2*x^2 - 1)*b*c^4*d^2*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/((c^3
 - 2*(c^2*x^2 - 1)*c^3/(c*x + 1)^2 + 2*(c^2*x^2 - 1)^3*c^3/(c*x + 1)^6 - (
c^2*x^2 - 1)^4*c^3/(c*x + 1)^8)*(c*x + 1)^2) + 2*(c^2*x^2 - 1)*b*c^4*d^2*l
og(abs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/((c^3 - 2*(c^2*x^2 - 1)*c^3/(c*x +
1)^2 + 2*(c^2*x^2 - 1)^3*c^3/(c*x + 1)^6 - (c^2*x^2 - 1)^4*c^3/(c*x + 1)^8
)*(c*x + 1)^2) + 4*(c^2*x^2 - 1)*a*c^4*d^2/((c^3 - 2*(c^2*x^2 - 1)*c^3/(c*
x + 1)^2 + 2*(c^2*x^2 - 1)^3*c^3/(c*x + 1)^6 - (c^2*x^2 - 1)^4*c^3/(c*x +
1)^8)*(c*x + 1)^2) - 6*(c^2*x^2 - 1)^2*b*c^4*d^2*arccos(c*x)/((c^3 - 2*(c^
2*x^2 - 1)*c^3/(c*x + 1)^2 + 2*(c^2*x^2 - 1)^3*c^3/(c*x + 1)^6 - (c^2*x^2
- 1)^4*c^3/(c*x + 1)^8)*(c*x + 1)^4) + 2*b*c^2*d*e*arccos(c*x)/(c^3 - 2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^2}\,dx$$
$$= \begin{cases} \frac{a\,(-3\,d^2+6\,d\,e\,x^2+e^2\,x^4)}{3\,x} - b\,e^2\left(\frac{\sqrt{\frac{1}{c^2}-x^2}\,\left(\frac{2}{c^2}+x^2\right)}{9} - \frac{x^3\,\mathrm{acos}(c\,x)}{3}\right) + b\,c\,d^2\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-c^2\,x^2}}\right) - \frac{b\,d^2\,\mathrm{acos}(c\,x)}{x} - \frac{2\,b\,d\,e \cdots}{} \\ \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(e\,x^2+d)^2}{x^2}\,dx \end{cases}$$

input
```
int(((a + b*acos(c*x))*(d + e*x^2)^2)/x^2,x)
```

output
```
piecewise(0 < c, (a*(- 3*d^2 + e^2*x^4 + 6*d*e*x^2))/(3*x) - b*e^2*(((1/c^
2 - x^2)^(1/2)*(2/c^2 + x^2))/9 - (x^3*acos(c*x))/3) + b*c*d^2*atanh(1/(-
c^2*x^2 + 1)^(1/2)) - (b*d^2*acos(c*x))/x - (2*b*d*e*((- c^2*x^2 + 1)^(1/2
) - c*x*acos(c*x)))/c, ~0 < c, int(((a + b*acos(c*x))*(d + e*x^2)^2)/x^2,
x))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^2}\,dx = \frac{-9acos(cx)\,b\,c^3d^2 + 18acos(cx)\,b\,c^3de\,x^2 + 3acos(cx)\,b\,c^3e^2x^4 - 18\sqrt{-c^2x^2+1}\,b\,c^2dex - \sqrt{-c^2x^2+1}\,b\,c}{9c^3x}$$

input
```
int((e*x^2+d)^2*(a+b*acos(c*x))/x^2,x)
```

output
```
( - 9*acos(c*x)*b*c**3*d**2 + 18*acos(c*x)*b*c**3*d*e*x**2 + 3*acos(c*x)*b
*c**3*e**2*x**4 - 18*sqrt( - c**2*x**2 + 1)*b*c**2*d*e*x - sqrt( - c**2*x*
*2 + 1)*b*c**2*e**2*x**3 - 2*sqrt( - c**2*x**2 + 1)*b*e**2*x - 9*log(tan(a
sin(c*x)/2))*b*c**4*d**2*x - 9*a*c**3*d**2 + 18*a*c**3*d*e*x**2 + 3*a*c**3
*e**2*x**4)/(9*c**3*x)
```

3.614 $\int \frac{(d+ex^2)^2(a+b\arccos(cx))}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 185

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\arccos(cx))}{x^3}\,dx =& -\frac{bcd^2\sqrt{1-c^2x^2}}{2x}+\frac{be^2x\sqrt{1-c^2x^2}}{4c}\\&-\frac{be^2\arccos(cx)}{4c^2}-ibde\arccos(cx)^2\\&-\frac{d^2(a+b\arccos(cx))}{2x^2}+\frac{1}{2}e^2x^2(a+b\arccos(cx))\\&+2bde\arccos(cx)\log\left(1-e^{2i\arccos(cx)}\right)\\&-2bde\arccos(cx)\log(x)\\&+2de(a+b\arccos(cx))\log(x)\\&-ibde\operatorname{PolyLog}\left(2,e^{2i\arccos(cx)}\right)\end{aligned}$$

output

```
-1/2*b*c*d^2*(-c^2*x^2+1)^(1/2)/x+1/4*b*e^2*x*(-c^2*x^2+1)^(1/2)/c-1/4*b*e
^2*arccos(c*x)/c^2-I*b*d*e*arccos(c*x)^2-1/2*d^2*(a+b*arccos(c*x))/x^2+1/2
*e^2*x^2*(a+b*arccos(c*x))+2*b*d*e*arccos(c*x)*ln(1-(c*x+I*(-c^2*x^2+1)^(1
/2))^2)-2*b*d*e*arccos(c*x)*ln(x)+2*d*e*(a+b*arccos(c*x))*ln(x)-I*b*d*e*po
lylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^3}\,dx = \frac{1}{4}\left(-\frac{2ad^2}{x^2}+2ae^2x^2+\frac{2bcd^2\sqrt{1-c^2x^2}}{x} - \frac{be^2x\sqrt{1-c^2x^2}}{c} - 4ibde\arccos(cx)^2 + \frac{2be^2\arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right)}{c^2} + 2b\arccos(cx)\left(-\frac{d^2}{x^2} + e^2x^2 + 4de\log\left(1+e^{2i\arccos(cx)}\right)\right) + 8ade\log(x) - 4ibde\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
((-2*a*d^2)/x^2 + 2*a*e^2*x^2 + (2*b*c*d^2*Sqrt[1 - c^2*x^2])/x - (b*e^2*x
*Sqrt[1 - c^2*x^2])/c - (4*I)*b*d*e*ArcCos[c*x]^2 + (2*b*e^2*ArcTan[(c*x)/
(-1 + Sqrt[1 - c^2*x^2])])/c^2 + 2*b*ArcCos[c*x]*(-(d^2/x^2) + e^2*x^2 + 4
*d*e*Log[1 + E^((2*I)*ArcCos[c*x])]) + 8*a*d*e*Log[x] - (4*I)*b*d*e*PolyLo
g[2, -E^((2*I)*ArcCos[c*x])])/4
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5231, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{5231}$$

$$bc\int -\frac{\frac{d^2}{x^2}-4e\log(x)d-e^2x^2}{2\sqrt{1-c^2x^2}}dx-\frac{d^2(a+b\arccos(cx))}{2x^2}+2de\log(x)(a+b\arccos(cx))+\frac{1}{2}e^2x^2(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}bc\int \frac{\frac{d^2}{x^2}-4e\log(x)d-e^2x^2}{\sqrt{1-c^2x^2}}dx-\frac{d^2(a+b\arccos(cx))}{2x^2}+2de\log(x)(a+b\arccos(cx))+\frac{1}{2}e^2x^2(a+b\arccos(cx))$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{2}bc\int \left(\frac{d^2}{x^2\sqrt{1-c^2x^2}}-\frac{4e\log(x)d}{\sqrt{1-c^2x^2}}-\frac{e^2x^2}{\sqrt{1-c^2x^2}}\right)dx-\frac{d^2(a+b\arccos(cx))}{2x^2}+2de\log(x)(a+b\arccos(cx))+\frac{1}{2}e^2x^2(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+b\arccos(cx))}{2x^2}+2de\log(x)(a+b\arccos(cx))+\frac{1}{2}e^2x^2(a+b\arccos(cx))-\frac{1}{2}bc\left(-\frac{e^2\arcsin(cx)}{2c^3}-\frac{2ide\operatorname{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{c}-\frac{2ide\arcsin(cx)^2}{c}+\frac{4de\arcsin(cx)\log\left(1-e^{2i\arcsin(cx)}\right)}{c}\right.$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
-1/2*(d^2*(a + b*ArcCos[c*x]))/x^2 + (e^2*x^2*(a + b*ArcCos[c*x]))/2 + 2*d
*e*(a + b*ArcCos[c*x])*Log[x] - (b*c*(-((d^2*Sqrt[1 - c^2*x^2])/x) + (e^2*
x*Sqrt[1 - c^2*x^2])/(2*c^2) - (e^2*ArcSin[c*x])/(2*c^3) - ((2*I)*d*e*ArcS
in[c*x]^2)/c + (4*d*e*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/c - (4*d
*e*ArcSin[c*x]*Log[x])/c - ((2*I)*d*e*PolyLog[2, E^((2*I)*ArcSin[c*x])])/c
))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.06

method	result
parts	$a\left(\frac{e^2x^2}{2}+2de\ln\left(x\right)-\frac{d^2}{2x^2}\right)-ibde\arccos\left(cx\right)^2+\frac{b\,e^2\arccos(cx)x^2}{2}-\frac{b\,e^2x\sqrt{-c^2x^2+1}}{4c}-\frac{b\,e^2\,\mathrm{arc}}{4}$
derivativedivides	$c^2\left(\frac{a\,x^2e^2}{2c^2}+\frac{2ade\ln(cx)}{c^2}-\frac{a\,d^2}{2c^2x^2}-\frac{ibde\arccos(cx)^2}{c^2}-\frac{b\,e^2x\sqrt{-c^2x^2+1}}{4c^3}+\frac{b\arccos(cx)e^2x^2}{2c^2}-\frac{b\,e^2\arccos(}{4c^4}\right.$
default	$c^2\left(\frac{a\,x^2e^2}{2c^2}+\frac{2ade\ln(cx)}{c^2}-\frac{a\,d^2}{2c^2x^2}-\frac{ibde\arccos(cx)^2}{c^2}-\frac{b\,e^2x\sqrt{-c^2x^2+1}}{4c^3}+\frac{b\arccos(cx)e^2x^2}{2c^2}-\frac{b\,e^2\arccos(}{4c^4}\right.$

input
```
int((e*x^2+d)^2*(a+b*arccos(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e^2*x^2+2*d*e*ln(x)-1/2*d^2/x^2)-I*b*d*e*arccos(c*x)^2+1/2*b*e^2*ar
ccos(c*x)*x^2-1/4*b*e^2*x*(-c^2*x^2+1)^(1/2)/c-1/4*b*e^2*arccos(c*x)/c^2+1
/2*I*b*c^2*d^2+1/2*b*c*d^2*(-c^2*x^2+1)^(1/2)/x-1/2*b*d^2/x^2*arccos(c*x)+
2*b*d*e*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-I*b*d*e*polylog(2,-
(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Fricas [F]

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^2(b\arccos(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arccos(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^3}\,dx = \int \frac{(a+b\operatorname{acos}(cx))(d+ex^2)^2}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acos(c*x))/x**3,x)
```

output
```
Integral((a + b*acos(c*x))*(d + e*x**2)**2/x**3, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^2 (b\arccos(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*a*e^2*x^2 + 1/2*b*d^2*(sqrt(-c^2*x^2 + 1)*c/x - arccos(c*x)/x^2) + 2*a
*d*e*log(x) - 1/2*a*d^2/x^2 + integrate((b*e^2*x^2 + 2*b*d*e)*arctan2(sqrt
(c*x + 1)*sqrt(-c*x + 1), c*x)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(e\,x^2+d)^2}{x^3}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + e*x^2)^2)/x^3,x)
```

output
```
int(((a + b*acos(c*x))*(d + e*x^2)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(d + ex^2)^2 (a + b\arccos(cx))}{x^3} dx$$
$$= \frac{-2acos(cx) b c^2 d^2 + 2acos(cx) b c^2 e^2 x^4 + asin(cx) b e^2 x^2 + 2\sqrt{-c^2x^2 + 1} b c^3 d^2 x - \sqrt{-c^2x^2 + 1} bc e^2 x^3 +}{4c^2x^2}$$

input
```
int((e*x^2+d)^2*(a+b*acos(c*x))/x^3,x)
```

output
```
( - 2*acos(c*x)*b*c**2*d**2 + 2*acos(c*x)*b*c**2*e**2*x**4 + asin(c*x)*b*e
**2*x**2 + 2*sqrt( - c**2*x**2 + 1)*b*c**3*d**2*x - sqrt( - c**2*x**2 + 1)
*b*c*e**2*x**3 + 8*int(acos(c*x)/x,x)*b*c**2*d*e*x**2 + 8*log(x)*a*c**2*d*
e*x**2 - 2*a*c**2*d**2 + 2*a*c**2*e**2*x**4)/(4*c**2*x**2)
```

3.615 $\int \frac{(d+ex^2)^2(a+b\arccos(cx))}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{(d+ex^2)^2(a+b\arccos(cx))}{x^4}\,dx = \frac{be^2\sqrt{1-c^2x^2}}{c} - \frac{bcd^2\sqrt{1-c^2x^2}}{6x^2} - \frac{d^2(a+b\arccos(cx))}{3x^3} - \frac{2de(a+b\arccos(cx))}{x} + e^2x(a+b\arccos(cx)) - \frac{1}{6}bcd(c^2d+12e)\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)$$

output
```
b*e^2*(-c^2*x^2+1)^(1/2)/c-1/6*b*c*d^2*(-c^2*x^2+1)^(1/2)/x^2-1/3*d^2*(a+b
*arccos(c*x))/x^3-2*d*e*(a+b*arccos(c*x))/x+e^2*x*(a+b*arccos(c*x))-1/6*b*
c*d*(c^2*d+12*e)*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^4} dx = \frac{1}{6}\left(-\frac{2ad^2}{x^3}-\frac{12ade}{x}+6ae^2x+6b\left(-\frac{e^2}{c}+\frac{cd^2}{6x^2}\right)\sqrt{1-c^2x^2}-\frac{2b(d^2+6dex^2-3e^2x^4)\arccos(cx)}{x^3}-bcd(c^2d+12e)\log(x)+bcd(c^2d+12e)\log\left(1+\sqrt{1-c^2x^2}\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
((-2*a*d^2)/x^3 - (12*a*d*e)/x + 6*a*e^2*x + 6*b*(-(e^2/c) + (c*d^2)/(6*x^
2))*Sqrt[1 - c^2*x^2] - (2*b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcCos[c*x])/x^
3 - b*c*d*(c^2*d + 12*e)*Log[x] + b*c*d*(c^2*d + 12*e)*Log[1 + Sqrt[1 - c^
2*x^2]])/6
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5231, 27, 1578, 1192, 1471, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^4} dx$$

$$\Big\downarrow \text{5231}$$

$$bc\int -\frac{-3e^2x^4+6dex^2+d^2}{3x^3\sqrt{1-c^2x^2}}dx-\frac{d^2(a+b\arccos(cx))}{3x^3}-\frac{2de(a+b\arccos(cx))}{x}+e^2x(a+b\arccos(cx))$$

$$\downarrow \text{27}$$

$$-\frac{1}{3}bc\int \frac{-3e^2x^4+6dex^2+d^2}{x^3\sqrt{1-c^2x^2}}dx-\frac{d^2(a+b\arccos(cx))}{3x^3}-\frac{2de(a+b\arccos(cx))}{x}+e^2x(a+b\arccos(cx))$$

$$\downarrow \text{1578}$$

$$-\frac{1}{6}bc\int \frac{-3e^2x^4+6dex^2+d^2}{x^4\sqrt{1-c^2x^2}}dx^2-\frac{d^2(a+b\arccos(cx))}{3x^3}-\frac{2de(a+b\arccos(cx))}{x}+e^2x(a+b\arccos(cx))$$

$$\downarrow \text{1192}$$

$$\frac{b\int \frac{-3e^2x^8-6(c^2d-e)ex^4+c^4d^2-3e^2+6c^2de}{(1-x^4)^2}d\sqrt{1-c^2x^2}}{3c}-\frac{d^2(a+b\arccos(cx))}{3x^3}-\frac{2de(a+b\arccos(cx))}{x}+e^2x(a+b\arccos(cx))$$

$$\downarrow \text{1471}$$

$$\frac{b\left(\frac{c^4d^2\sqrt{1-c^2x^2}}{2(1-x^4)}-\frac{1}{2}\int -\frac{d^2c^4+12dec^2+6e^2x^4-6e^2}{1-x^4}d\sqrt{1-c^2x^2}\right)}{3c}-\frac{d^2(a+b\arccos(cx))}{3x^3}-\frac{2de(a+b\arccos(cx))}{x}+e^2x(a+b\arccos(cx))$$

$$\downarrow \text{25}$$

$$\frac{b\left(\frac{1}{2}\int \frac{d^2c^4+12dec^2+6e^2x^4-6e^2}{1-x^4}d\sqrt{1-c^2x^2}+\frac{c^4d^2\sqrt{1-c^2x^2}}{2(1-x^4)}\right)}{3c}-\frac{d^2(a+b\arccos(cx))}{3x^3}-\frac{2de(a+b\arccos(cx))}{x}+e^2x(a+b\arccos(cx))$$

$$\downarrow \text{299}$$

$$\frac{b\left(\frac{1}{2}\left(c^2d(c^2d+12e)\int \frac{1}{1-x^4}d\sqrt{1-c^2x^2}-6e^2\sqrt{1-c^2x^2}\right)+\frac{c^4d^2\sqrt{1-c^2x^2}}{2(1-x^4)}\right)}{3c}-\frac{d^2(a+b\arccos(cx))}{3x^3}-\frac{2de(a+b\arccos(cx))}{x}+e^2x(a+b\arccos(cx))$$

$$\downarrow \text{219}$$

$$-\frac{d^2(a+b\arccos(cx))}{3x^3}-\frac{2de(a+b\arccos(cx))}{x}+e^2x(a+b\arccos(cx))+\frac{b\left(\frac{1}{2}\left(c^2d\text{arctanh}\left(\sqrt{1-c^2x^2}\right)(c^2d+12e)-6e^2\sqrt{1-c^2x^2}\right)+\frac{c^4d^2\sqrt{1-c^2x^2}}{2(1-x^4)}\right)}{3c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
-1/3*(d^2*(a + b*ArcCos[c*x]))/x^3 - (2*d*e*(a + b*ArcCos[c*x]))/x + e^2*x
*(a + b*ArcCos[c*x]) + (b*((c^4*d^2*Sqrt[1 - c^2*x^2])/(2*(1 - x^4)) + (-6
*e^2*Sqrt[1 - c^2*x^2] + c^2*d*(c^2*d + 12*e)*ArcTanh[Sqrt[1 - c^2*x^2]])/
2))/(3*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.25

method	result
derivativedivides	$c^3\left(\frac{a\left(e^2cx-\frac{2cde}{x}-\frac{c\,d^2}{3x^3}\right)}{c^4}+\frac{b\left(\arccos(cx)e^2cx-\frac{2\arccos(cx)cde}{x}-\frac{\arccos(cx)c\,d^2}{3x^3}-\frac{c^4d^2\left(-\frac{\sqrt{-c^2x^2+1}}{2c^2x^2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{2}\right.}{3}\right.}{c^4}\right.$
default	$c^3\left(\frac{a\left(e^2cx-\frac{2cde}{x}-\frac{c\,d^2}{3x^3}\right)}{c^4}+\frac{b\left(\arccos(cx)e^2cx-\frac{2\arccos(cx)cde}{x}-\frac{\arccos(cx)c\,d^2}{3x^3}-\frac{c^4d^2\left(-\frac{\sqrt{-c^2x^2+1}}{2c^2x^2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{2}\right.}{3}\right.}{c^4}\right.$
parts	$a\left(e^2x-\frac{2de}{x}-\frac{d^2}{3x^3}\right)+b\,c^3\left(\frac{\arccos(cx)x\,e^2}{c^3}-\frac{2\arccos(cx)de}{c^3x}-\frac{\arccos(cx)d^2}{3c^3x^3}+\frac{-3e^2\sqrt{-c^2x^2+1}-c^4d^2}{}\right.$

input

```
int((e*x^2+d)^2*(a+b*arccos(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output

```
c^3*(a/c^4*(e^2*c*x-2*c*d*e/x-1/3*c*d^2/x^3)+b/c^4*(arccos(c*x)*e^2*c*x-2*
arccos(c*x)*c*d*e/x-1/3*arccos(c*x)*c*d^2/x^3-1/3*c^4*d^2*(-1/2/c^2/x^2*(-
c^2*x^2+1)^(1/2)-1/2*arctanh(1/(-c^2*x^2+1)^(1/2)))-e^2*(-c^2*x^2+1)^(1/2)
+2*c^2*d*e*arctanh(1/(-c^2*x^2+1)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(114) = 228$.

Time = 0.17 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.98

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^4}\,dx = \frac{12\,ace^2x^4 - 24\,acdex^2 - 4\,(bcd^2 + 6\,bcde - 3\,bce^2)x^3\arctan\left(\frac{\sqrt{-c^2x^2+1}cx}{c^2x^2-1}\right) + (bc^4d^2 + 12\,bc^2de)x^3\log\left(\sqrt{\cdot}\right.}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/12*(12*a*c*e^2*x^4 - 24*a*c*d*e*x^2 - 4*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2
)*x^3*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + (b*c^4*d^2 + 12*b*c^2
*d*e)*x^3*log(sqrt(-c^2*x^2 + 1) + 1) - (b*c^4*d^2 + 12*b*c^2*d*e)*x^3*log
(sqrt(-c^2*x^2 + 1) - 1) - 4*a*c*d^2 + 4*(3*b*c*e^2*x^4 - 6*b*c*d*e*x^2 -
b*c*d^2 + (b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*arccos(c*x) + 2*(b*c^2*d^
2*x - 6*b*e^2*x^3)*sqrt(-c^2*x^2 + 1))/(c*x^3)
```

Sympy [A] (verification not implemented)

Time = 3.48 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.75

$$\begin{aligned}\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^4}\,dx &= -\frac{ad^2}{3x^3} - \frac{2ade}{x} + ae^2x \\ &\quad - \frac{bcd^2\left(\begin{cases} -\frac{c^2\operatorname{acosh}\left(\frac{1}{cx}\right)}{2} + \frac{c}{2x\sqrt{-1+\frac{1}{c^2x^2}}} - \frac{1}{2cx^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for}\ \frac{1}{|c^2x^2|} > 1 \\ \frac{ic^2\operatorname{asin}\left(\frac{1}{cx}\right)}{2} - \frac{ic\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise} \end{cases}\right)}{3} \\ &\quad - 2bcde\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for}\ \frac{1}{|c^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases}\right) - \frac{bd^2\operatorname{acos}(cx)}{3x^3} \\ &\quad - \frac{2bde\operatorname{acos}(cx)}{x} + be^2\left(\begin{cases} \frac{\pi x}{2} & \text{for}\ c=0 \\ x\operatorname{acos}(cx) - \frac{\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases}\right)\end{aligned}$$

input
```
integrate((e*x**2+d)**2*(a+b*acos(c*x))/x**4,x)
```

output
```
-a*d**2/(3*x**3) - 2*a*d*e/x + a*e**2*x - b*c*d**2*Piecewise((-c**2*acosh(
1/(c*x))/2 + c/(2*x*sqrt(-1 + 1/(c**2*x**2))) - 1/(2*c*x**3*sqrt(-1 + 1/(c
**2*x**2))), 1/Abs(c**2*x**2) > 1), (I*c**2*asin(1/(c*x))/2 - I*c*sqrt(1 -
 1/(c**2*x**2))/(2*x), True))/3 - 2*b*c*d*e*Piecewise((-acosh(1/(c*x)), 1/
Abs(c**2*x**2) > 1), (I*asin(1/(c*x)), True)) - b*d**2*acos(c*x)/(3*x**3)
- 2*b*d*e*acos(c*x)/x + b*e**2*Piecewise((pi*x/2, Eq(c, 0)), (x*acos(c*x)
- sqrt(-c**2*x**2 + 1)/c, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex^2)^2(a+b\arccos(cx))}{x^4}\,dx$$
$$= \frac{1}{6}\left(\left(c^2\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{\sqrt{-c^2x^2+1}}{x^2}\right)c-\frac{2\arccos(cx)}{x^3}\right)bd^2$$
$$+2\left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\arccos(cx)}{x}\right)bde$$
$$+ae^2x+\frac{\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)be^2}{c}-\frac{2\,ade}{x}-\frac{ad^2}{3\,x^3}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/6*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)
/x^2)*c - 2*arccos(c*x)/x^3)*b*d^2 + 2*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x)
+ 2/abs(x)) - arccos(c*x)/x)*b*d*e + a*e^2*x + (c*x*arccos(c*x) - sqrt(-c^
2*x^2 + 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4761 vs. $2(114) = 228$.

Time = 2.16 (sec) , antiderivative size = 4761, normalized size of antiderivative = 37.79

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))/x^4,x, algorithm="giac")
```

output
```
-1/3*b*c^4*d^2*arccos(c*x)/(c + 2*(c^2*x^2 - 1)*c/(c*x + 1)^2 - 2*(c^2*x^2
 - 1)^3*c/(c*x + 1)^6 - (c^2*x^2 - 1)^4*c/(c*x + 1)^8) + 1/6*b*c^4*d^2*log
(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/(c + 2*(c^2*x^2 - 1)*c/(c*x + 1)^2 - 2
*(c^2*x^2 - 1)^3*c/(c*x + 1)^6 - (c^2*x^2 - 1)^4*c/(c*x + 1)^8) - 1/6*b*c^
4*d^2*log(abs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/(c + 2*(c^2*x^2 - 1)*c/(c*x
+ 1)^2 - 2*(c^2*x^2 - 1)^3*c/(c*x + 1)^6 - (c^2*x^2 - 1)^4*c/(c*x + 1)^8)
- 1/3*a*c^4*d^2/(c + 2*(c^2*x^2 - 1)*c/(c*x + 1)^2 - 2*(c^2*x^2 - 1)^3*c/(
c*x + 1)^6 - (c^2*x^2 - 1)^4*c/(c*x + 1)^8) + 4/3*(c^2*x^2 - 1)*b*c^4*d^2*
arccos(c*x)/((c*x + 1)^2*(c + 2*(c^2*x^2 - 1)*c/(c*x + 1)^2 - 2*(c^2*x^2 -
 1)^3*c/(c*x + 1)^6 - (c^2*x^2 - 1)^4*c/(c*x + 1)^8)) + 1/3*(c^2*x^2 - 1)*
b*c^4*d^2*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/((c*x + 1)^2*(c + 2*(c^2*
x^2 - 1)*c/(c*x + 1)^2 - 2*(c^2*x^2 - 1)^3*c/(c*x + 1)^6 - (c^2*x^2 - 1)^4
*c/(c*x + 1)^8)) - 1/3*(c^2*x^2 - 1)*b*c^4*d^2*log(abs(-c*x + sqrt(-c^2*x^
2 + 1) - 1))/((c*x + 1)^2*(c + 2*(c^2*x^2 - 1)*c/(c*x + 1)^2 - 2*(c^2*x^2
- 1)^3*c/(c*x + 1)^6 - (c^2*x^2 - 1)^4*c/(c*x + 1)^8)) + 1/3*sqrt(-c^2*x^2
 + 1)*b*c^4*d^2/((c*x + 1)*(c + 2*(c^2*x^2 - 1)*c/(c*x + 1)^2 - 2*(c^2*x^2
 - 1)^3*c/(c*x + 1)^6 - (c^2*x^2 - 1)^4*c/(c*x + 1)^8)) + 4/3*(c^2*x^2 - 1
)*a*c^4*d^2/((c*x + 1)^2*(c + 2*(c^2*x^2 - 1)*c/(c*x + 1)^2 - 2*(c^2*x^2 -
 1)^3*c/(c*x + 1)^6 - (c^2*x^2 - 1)^4*c/(c*x + 1)^8)) - 2*(c^2*x^2 - 1)^2*
b*c^4*d^2*arccos(c*x)/((c*x + 1)^4*(c + 2*(c^2*x^2 - 1)*c/(c*x + 1)^2 -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(e\,x^2+d)^2}{x^4}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + e*x^2)^2)/x^4,x)
```

output
```
int(((a + b*acos(c*x))*(d + e*x^2)^2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)^2 (a+b\arccos(cx))}{x^4}\,dx = \frac{-2acos(cx)\,bc\,d^2-12acos(cx)\,bcde\,x^2+6acos(cx)\,bc\,e^2x^4+\sqrt{-c^2x^2+1}\,b\,c^2d^2x-6\sqrt{-c^2x^2+1}\,b\,e^2x^3}{6c\,x^3}$$

input
```
int((e*x^2+d)^2*(a+b*acos(c*x))/x^4,x)
```

output
```
( - 2*acos(c*x)*b*c*d**2 - 12*acos(c*x)*b*c*d*e*x**2 + 6*acos(c*x)*b*c*e**
2*x**4 + sqrt( - c**2*x**2 + 1)*b*c**2*d**2*x - 6*sqrt( - c**2*x**2 + 1)*b
*e**2*x**3 - log(tan(asin(c*x)/2))*b*c**4*d**2*x**3 - 12*log(tan(asin(c*x)
/2))*b*c**2*d*e*x**3 - 2*a*c*d**2 - 12*a*c*d*e*x**2 + 6*a*c*e**2*x**4)/(6*
c*x**3)
```

3.616 $\int x^4(d+ex^2)^3(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 341

$$\begin{aligned}
&\int x^4(d+ex^2)^3(a+b\arccos(cx))\,dx\\
&=\frac{b(231c^6d^3+495c^4d^2e+385c^2de^2+105e^3)\sqrt{1-c^2x^2}}{1155c^{11}}\\
&\quad-\frac{b(462c^6d^3+1485c^4d^2e+1540c^2de^2+525e^3)(1-c^2x^2)^{3/2}}{3465c^{11}}\\
&\quad+\frac{b(77c^6d^3+495c^4d^2e+770c^2de^2+350e^3)(1-c^2x^2)^{5/2}}{1925c^{11}}\\
&\quad-\frac{be(99c^4d^2+308c^2de+210e^2)(1-c^2x^2)^{7/2}}{1617c^{11}}\\
&\quad+\frac{be^2(11c^2d+15e)(1-c^2x^2)^{9/2}}{297c^{11}}-\frac{be^3(1-c^2x^2)^{11/2}}{121c^{11}}\\
&\quad+\frac{1}{5}d^3x^5(a+b\arccos(cx))+\frac{3}{7}d^2ex^7(a+b\arccos(cx))+\frac{1}{3}de^2x^9(a+b\arccos(cx))+\frac{1}{11}e^3x^{11}(a+b\arccos(cx
\end{aligned}$$

output
```
1/1155*b*(231*c^6*d^3+495*c^4*d^2*e+385*c^2*d*e^2+105*e^3)*(-c^2*x^2+1)^(1
/2)/c^11-1/3465*b*(462*c^6*d^3+1485*c^4*d^2*e+1540*c^2*d*e^2+525*e^3)*(-c^
2*x^2+1)^(3/2)/c^11+1/1925*b*(77*c^6*d^3+495*c^4*d^2*e+770*c^2*d*e^2+350*e
^3)*(-c^2*x^2+1)^(5/2)/c^11-1/1617*b*e*(99*c^4*d^2+308*c^2*d*e+210*e^2)*(-
c^2*x^2+1)^(7/2)/c^11+1/297*b*e^2*(11*c^2*d+15*e)*(-c^2*x^2+1)^(9/2)/c^11-
1/121*b*e^3*(-c^2*x^2+1)^(11/2)/c^11+1/5*d^3*x^5*(a+b*arccos(c*x))+3/7*d^2
*e*x^7*(a+b*arccos(c*x))+1/3*d*e^2*x^9*(a+b*arccos(c*x))+1/11*e^3*x^11*(a+
b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.80

$$\int x^4 (d+ex^2)^3 (a+b\arccos(cx))\,dx$$
$$= \frac{3465ax^5(231d^3+495d^2ex^2+385de^2x^4+105e^3x^6) - \frac{b\sqrt{1-c^2x^2}(134400e^3+4480c^2e^2(121d+15ex^2)+80c^4e(9801d^2+3388}{}}{}$$

input
```
Integrate[x^4*(d + e*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
(3465*a*x^5*(231*d^3 + 495*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6) - (b*S
qrt[1 - c^2*x^2]*(134400*e^3 + 4480*c^2*e^2*(121*d + 15*e*x^2) + 80*c^4*e*
(9801*d^2 + 3388*d*e*x^2 + 630*e^2*x^4) + 24*c^6*(17787*d^3 + 16335*d^2*e*
x^2 + 8470*d*e^2*x^4 + 1750*e^3*x^6) + c^10*x^4*(160083*d^3 + 245025*d^2*e
*x^2 + 148225*d*e^2*x^4 + 33075*e^3*x^6) + 2*c^8*(106722*d^3*x^2 + 147015*
d^2*e*x^4 + 84700*d*e^2*x^6 + 18375*e^3*x^8)))/c^11 + 3465*b*x^5*(231*d^3
+ 495*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6)*ArcCos[c*x])/4002075
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5231, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx$$

$\Big\downarrow$ 5231

$$bc\int \frac{x^5\left(105e^3x^6+385de^2x^4+495d^2ex^2+231d^3\right)}{1155\sqrt{1-c^2x^2}}dx+\frac{1}{5}d^3x^5(a+b\arccos(cx))+\frac{3}{7}d^2ex^7(a+b\arccos(cx))+\frac{1}{3}de^2x^9(a+b\arccos(cx))+\frac{1}{11}e^3x^{11}(a+b\arccos(cx))$$

$\Big\downarrow$ 27

$$\frac{bc\int \frac{x^5\left(105e^3x^6+385de^2x^4+495d^2ex^2+231d^3\right)}{\sqrt{1-c^2x^2}}dx}{1155}+\frac{1}{5}d^3x^5(a+b\arccos(cx))+\frac{3}{7}d^2ex^7(a+b\arccos(cx))+\frac{1}{3}de^2x^9(a+b\arccos(cx))+\frac{1}{11}e^3x^{11}(a+b\arccos(cx))$$

$\Big\downarrow$ 2331

$$\frac{bc\int \frac{x^4\left(105e^3x^6+385de^2x^4+495d^2ex^2+231d^3\right)}{\sqrt{1-c^2x^2}}dx^2}{2310}+\frac{1}{5}d^3x^5(a+b\arccos(cx))+\frac{3}{7}d^2ex^7(a+b\arccos(cx))+\frac{1}{3}de^2x^9(a+b\arccos(cx))+\frac{1}{11}e^3x^{11}(a+b\arccos(cx))$$

$\Big\downarrow$ 2123

$$\frac{bc\int \left(-\frac{105e^3\left(1-c^2x^2\right)^{9/2}}{c^{10}}+\frac{35e^2\left(11dc^2+15e\right)\left(1-c^2x^2\right)^{7/2}}{c^{10}}-\frac{5e\left(99d^2c^4+308dec^2+210e^2\right)\left(1-c^2x^2\right)^{5/2}}{c^{10}}+\frac{3\left(77d^3c^6+495d^2ec^4+770de^2\right.}{c^{10}}\right.}{2310}$$
$$\frac{1}{5}d^3x^5(a+b\arccos(cx))+\frac{3}{7}d^2ex^7(a+b\arccos(cx))+\frac{1}{3}de^2x^9(a+b\arccos(cx))+\frac{1}{11}e^3x^{11}(a+b\arccos(cx))$$

$\Big\downarrow$ 2009

$$\frac{1}{5}d^3x^5(a+b\arccos(cx))+\frac{3}{7}d^2ex^7(a+b\arccos(cx))+\frac{1}{3}de^2x^9(a+b\arccos(cx))+\frac{1}{11}e^3x^{11}(a+b\arccos(cx))+$$
$$\frac{bc\left(-\frac{70e^2\left(1-c^2x^2\right)^{9/2}\left(11c^2d+15e\right)}{9c^{12}}+\frac{210e^3\left(1-c^2x^2\right)^{11/2}}{11c^{12}}+\frac{10e\left(1-c^2x^2\right)^{7/2}\left(99c^4d^2+308c^2de+210e^2\right)}{7c^{12}}-\frac{6\left(1-c^2x^2\right)^{5/2}\left(77c^6d^3+495c^4\right.}{5c^{12}}\right.}{231}$$

input
```
Int[x^4*(d + e*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*((-2*(231*c^6*d^3 + 495*c^4*d^2*e + 385*c^2*d*e^2 + 105*e^3)*Sqrt[1 -
 c^2*x^2])/c^12 + (2*(462*c^6*d^3 + 1485*c^4*d^2*e + 1540*c^2*d*e^2 + 525*
e^3)*(1 - c^2*x^2)^(3/2))/(3*c^12) - (6*(77*c^6*d^3 + 495*c^4*d^2*e + 770*
c^2*d*e^2 + 350*e^3)*(1 - c^2*x^2)^(5/2))/(5*c^12) + (10*e*(99*c^4*d^2 + 3
08*c^2*d*e + 210*e^2)*(1 - c^2*x^2)^(7/2))/(7*c^12) - (70*e^2*(11*c^2*d +
15*e)*(1 - c^2*x^2)^(9/2))/(9*c^12) + (210*e^3*(1 - c^2*x^2)^(11/2))/(11*c
^12)))/2310 + (d^3*x^5*(a + b*ArcCos[c*x]))/5 + (3*d^2*e*x^7*(a + b*ArcCos
[c*x]))/7 + (d*e^2*x^9*(a + b*ArcCos[c*x]))/3 + (e^3*x^11*(a + b*ArcCos[c*
x]))/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.42

method	result
parts	$a\left(\frac{1}{11}e^3x^{11}+\frac{1}{3}d\,e^2x^9+\frac{3}{7}d^2e\,x^7+\frac{1}{5}d^3x^5\right)+\frac{b\left(\frac{c^5\arccos(cx)e^3x^{11}}{11}+\frac{c^5\arccos(cx)d\,e^2x^9}{3}+\frac{3c^5\arccos(cx)d^2}{7}\right.}{}$
derivativedivides	$\frac{a\left(\frac{1}{5}d^3c^{11}x^5+\frac{3}{7}d^2c^{11}e\,x^7+\frac{1}{3}d\,c^{11}e^2x^9+\frac{1}{11}e^3c^{11}x^{11}\right)}{c^6}+\frac{b\left(\frac{\arccos(cx)d^3c^{11}x^5}{5}+\frac{3\arccos(cx)d^2c^{11}e\,x^7}{7}+\frac{\arccos(cx)d\,c^{11}e^2x^9}{3}+\right.}{}$
default	$\frac{a\left(\frac{1}{5}d^3c^{11}x^5+\frac{3}{7}d^2c^{11}e\,x^7+\frac{1}{3}d\,c^{11}e^2x^9+\frac{1}{11}e^3c^{11}x^{11}\right)}{c^6}+\frac{b\left(\frac{\arccos(cx)d^3c^{11}x^5}{5}+\frac{3\arccos(cx)d^2c^{11}e\,x^7}{7}+\frac{\arccos(cx)d\,c^{11}e^2x^9}{3}+\right.}{}$
orering	$\frac{(694575c^{12}e^4x^{14}+3312400c^{12}d\,e^3x^{12}+6092350c^{12}d^2e^2x^{10}+36750c^{10}e^4x^{12}+5096520c^{12}d^3e\,x^8+226450c^{10}d\,e^3x^{10}+}{}$

input
```
int(x^4*(e*x^2+d)^3*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/11*e^3*x^11+1/3*d*e^2*x^9+3/7*d^2*e*x^7+1/5*d^3*x^5)+b/c^5*(1/11*c^5*
arccos(c*x)*e^3*x^11+1/3*c^5*arccos(c*x)*d*e^2*x^9+3/7*c^5*arccos(c*x)*d^2
*e*x^7+1/5*arccos(c*x)*c^5*x^5*d^3+1/1155/c^6*(105*e^3*(-1/11*c^10*x^10*(-
c^2*x^2+1)^(1/2)-10/99*c^8*x^8*(-c^2*x^2+1)^(1/2)-80/693*c^6*x^6*(-c^2*x^2
+1)^(1/2)-32/231*c^4*x^4*(-c^2*x^2+1)^(1/2)-128/693*c^2*x^2*(-c^2*x^2+1)^(
1/2)-256/693*(-c^2*x^2+1)^(1/2))+231*d^3*c^6*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1
/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))+385*d*c^2*e^2
*(-1/9*c^8*x^8*(-c^2*x^2+1)^(1/2)-8/63*c^6*x^6*(-c^2*x^2+1)^(1/2)-16/105*c
^4*x^4*(-c^2*x^2+1)^(1/2)-64/315*c^2*x^2*(-c^2*x^2+1)^(1/2)-128/315*(-c^2*
x^2+1)^(1/2))+495*d^2*c^4*e*(-1/7*c^6*x^6*(-c^2*x^2+1)^(1/2)-6/35*c^4*x^4*
(-c^2*x^2+1)^(1/2)-8/35*c^2*x^2*(-c^2*x^2+1)^(1/2)-16/35*(-c^2*x^2+1)^(1/2
))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.95

$$\int x^4 (d + ex^2)^3 (a + b \arccos(cx)) \, dx$$
$$= \frac{363825 \, ac^{11}e^3x^{11} + 1334025 \, ac^{11}de^2x^9 + 1715175 \, ac^{11}d^2ex^7 + 800415 \, ac^{11}d^3x^5 + 3465 \, (105 \, bc^{11}e^3x^{11} + 3}{}$$

input
```
integrate(x^4*(e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/4002075*(363825*a*c^11*e^3*x^11 + 1334025*a*c^11*d*e^2*x^9 + 1715175*a*c
^11*d^2*e*x^7 + 800415*a*c^11*d^3*x^5 + 3465*(105*b*c^11*e^3*x^11 + 385*b*
c^11*d*e^2*x^9 + 495*b*c^11*d^2*e*x^7 + 231*b*c^11*d^3*x^5)*arccos(c*x) -
(33075*b*c^10*e^3*x^10 + 426888*b*c^6*d^3 + 1225*(121*b*c^10*d*e^2 + 30*b*
c^8*e^3)*x^8 + 784080*b*c^4*d^2*e + 25*(9801*b*c^10*d^2*e + 6776*b*c^8*d*e
^2 + 1680*b*c^6*e^3)*x^6 + 542080*b*c^2*d*e^2 + 3*(53361*b*c^10*d^3 + 9801
0*b*c^8*d^2*e + 67760*b*c^6*d*e^2 + 16800*b*c^4*e^3)*x^4 + 134400*b*e^3 +
4*(53361*b*c^8*d^3 + 98010*b*c^6*d^2*e + 67760*b*c^4*d*e^2 + 16800*b*c^2*e
^3)*x^2)*sqrt(-c^2*x^2 + 1))/c^11
```

Sympy [A] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.87

$$\int x^4 (d + ex^2)^3 (a + b \arccos(cx)) \, dx$$
$$= \begin{cases} \frac{ad^3x^5}{5} + \frac{3ad^2ex^7}{7} + \frac{ade^2x^9}{3} + \frac{ae^3x^{11}}{11} + \frac{bd^3x^5 \operatorname{acos}(cx)}{5} + \frac{3bd^2ex^7 \operatorname{acos}(cx)}{7} + \frac{bde^2x^9 \operatorname{acos}(cx)}{3} + \frac{be^3x^{11} \operatorname{acos}(cx)}{11} - \frac{bd^3x^4\sqrt{-}}{25} \\ \left(a + \frac{\pi b}{2}\right) \left(\frac{d^3x^5}{5} + \frac{3d^2ex^7}{7} + \frac{de^2x^9}{3} + \frac{e^3x^{11}}{11}\right) \end{cases}$$

input
```
integrate(x**4*(e*x**2+d)**3*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d**3*x**5/5 + 3*a*d**2*e*x**7/7 + a*d*e**2*x**9/3 + a*e**3*x*
*11/11 + b*d**3*x**5*acos(c*x)/5 + 3*b*d**2*e*x**7*acos(c*x)/7 + b*d*e**2*
x**9*acos(c*x)/3 + b*e**3*x**11*acos(c*x)/11 - b*d**3*x**4*sqrt(-c**2*x**2
 + 1)/(25*c) - 3*b*d**2*e*x**6*sqrt(-c**2*x**2 + 1)/(49*c) - b*d*e**2*x**8
*sqrt(-c**2*x**2 + 1)/(27*c) - b*e**3*x**10*sqrt(-c**2*x**2 + 1)/(121*c) -
 4*b*d**3*x**2*sqrt(-c**2*x**2 + 1)/(75*c**3) - 18*b*d**2*e*x**4*sqrt(-c**
2*x**2 + 1)/(245*c**3) - 8*b*d*e**2*x**6*sqrt(-c**2*x**2 + 1)/(189*c**3) -
 10*b*e**3*x**8*sqrt(-c**2*x**2 + 1)/(1089*c**3) - 8*b*d**3*sqrt(-c**2*x**
2 + 1)/(75*c**5) - 24*b*d**2*e*x**2*sqrt(-c**2*x**2 + 1)/(245*c**5) - 16*b
*d*e**2*x**4*sqrt(-c**2*x**2 + 1)/(315*c**5) - 80*b*e**3*x**6*sqrt(-c**2*x
**2 + 1)/(7623*c**5) - 48*b*d**2*e*sqrt(-c**2*x**2 + 1)/(245*c**7) - 64*b*
d*e**2*x**2*sqrt(-c**2*x**2 + 1)/(945*c**7) - 32*b*e**3*x**4*sqrt(-c**2*x*
*2 + 1)/(2541*c**7) - 128*b*d*e**2*sqrt(-c**2*x**2 + 1)/(945*c**9) - 128*b
*e**3*x**2*sqrt(-c**2*x**2 + 1)/(7623*c**9) - 256*b*e**3*sqrt(-c**2*x**2 +
 1)/(7623*c**11), Ne(c, 0)), ((a + pi*b/2)*(d**3*x**5/5 + 3*d**2*e*x**7/7
+ d*e**2*x**9/3 + e**3*x**11/11), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.38

$$\int x^4 (d + ex^2)^3 (a + b \arccos(cx)) \, dx = \frac{1}{11} ae^3x^{11} + \frac{1}{3} ade^2x^9 + \frac{3}{7} ad^2ex^7 + \frac{1}{5} ad^3x^5$$
$$+ \frac{1}{75} \left(15 x^5 \arccos(cx) - \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6} \right) c \right) bd^3$$
$$+ \frac{3}{245} \left(35 x^7 \arccos(cx) - \left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\sqrt{-c^2x^2+1}}{c^8} \right.\right.$$
$$+ \frac{1}{945} \left(315 x^9 \arccos(cx) - \left(\frac{35\sqrt{-c^2x^2+1}x^8}{c^2} + \frac{40\sqrt{-c^2x^2+1}x^6}{c^4} + \frac{48\sqrt{-c^2x^2+1}x^4}{c^6} + \frac{64\sqrt{-c^2x^2}}{c^8} \right.\right.$$
$$+ \frac{1}{7623} \left(693 x^{11} \arccos(cx) - \left(\frac{63\sqrt{-c^2x^2+1}x^{10}}{c^2} + \frac{70\sqrt{-c^2x^2+1}x^8}{c^4} + \frac{80\sqrt{-c^2x^2+1}x^6}{c^6} + \frac{96\sqrt{-c}}{} \right.\right.$$

input
```
integrate(x^4*(e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output

```
1/11*a*e^3*x^11 + 1/3*a*d*e^2*x^9 + 3/7*a*d^2*e*x^7 + 1/5*a*d^3*x^5 + 1/75
*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1
)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d^3 + 3/245*(35*x^7*arccos(c*x)
 - (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-
c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*d^2*e + 1/945*(315*
x^9*arccos(c*x) - (35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*sqrt(-c^2*x^2 + 1)*x
^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x^2 + 1)*x^2/c^8 + 1
28*sqrt(-c^2*x^2 + 1)/c^10)*c)*b*d*e^2 + 1/7623*(693*x^11*arccos(c*x) - (6
3*sqrt(-c^2*x^2 + 1)*x^10/c^2 + 70*sqrt(-c^2*x^2 + 1)*x^8/c^4 + 80*sqrt(-c
^2*x^2 + 1)*x^6/c^6 + 96*sqrt(-c^2*x^2 + 1)*x^4/c^8 + 128*sqrt(-c^2*x^2 +
1)*x^2/c^10 + 256*sqrt(-c^2*x^2 + 1)/c^12)*c)*b*e^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.53

$$\int x^4(d+ex^2)^3(a+b\arccos(cx))\,dx = \frac{1}{11}\,be^3x^{11}\arccos(cx)+\frac{1}{11}\,ae^3x^{11}+\frac{1}{3}\,bde^2x^9\arccos(cx)-\frac{\sqrt{-c^2x^2+1}be^3x^{10}}{121\,c}+\frac{1}{3}\,ade^2x^9+\frac{3}{7}\,bd^2ex^7\arccos(cx)-\frac{\sqrt{-c^2x^2+1}bde^2x^8}{27\,c}+\frac{3}{7}\,ad^2ex^7+\frac{1}{5}\,bd^3x^5\arccos(cx)-\frac{3\sqrt{-c^2x^2+1}bd^2ex^6}{49\,c}-\frac{10\sqrt{-c^2x^2+1}be^3x^8}{1089\,c^3}+\frac{1}{5}\,ad^3x^5-\frac{\sqrt{-c^2x^2+1}bd^3x^4}{25\,c}-\frac{8\sqrt{-c^2x^2+1}bde^2x^6}{189\,c^3}-\frac{18\sqrt{-c^2x^2+1}bd^2ex^4}{245\,c^3}-\frac{80\sqrt{-c^2x^2+1}be^3x^6}{7623\,c^5}-\frac{4\sqrt{-c^2x^2+1}bd^3x^2}{75\,c^3}-\frac{16\sqrt{-c^2x^2+1}bde^2x^4}{315\,c^5}-\frac{24\sqrt{-c^2x^2+1}bd^2ex^2}{245\,c^5}-\frac{32\sqrt{-c^2x^2+1}be^3x^4}{2541\,c^7}-\frac{8\sqrt{-c^2x^2+1}bd^3}{75\,c^5}-\frac{64\sqrt{-c^2x^2+1}bde^2x^2}{945\,c^7}-\frac{48\sqrt{-c^2x^2+1}bd^2e}{245\,c^7}-\frac{128\sqrt{-c^2x^2+1}be^3x^2}{7623\,c^9}-\frac{128\sqrt{-c^2x^2+1}bde^2}{945\,c^9}-\frac{256\sqrt{-c^2x^2+1}be^3}{7623\,c^{11}}$$

input
```
integrate(x^4*(e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/11*b*e^3*x^11*arccos(c*x) + 1/11*a*e^3*x^11 + 1/3*b*d*e^2*x^9*arccos(c*x
) - 1/121*sqrt(-c^2*x^2 + 1)*b*e^3*x^10/c + 1/3*a*d*e^2*x^9 + 3/7*b*d^2*e*
x^7*arccos(c*x) - 1/27*sqrt(-c^2*x^2 + 1)*b*d*e^2*x^8/c + 3/7*a*d^2*e*x^7
+ 1/5*b*d^3*x^5*arccos(c*x) - 3/49*sqrt(-c^2*x^2 + 1)*b*d^2*e*x^6/c - 10/1
089*sqrt(-c^2*x^2 + 1)*b*e^3*x^8/c^3 + 1/5*a*d^3*x^5 - 1/25*sqrt(-c^2*x^2
+ 1)*b*d^3*x^4/c - 8/189*sqrt(-c^2*x^2 + 1)*b*d*e^2*x^6/c^3 - 18/245*sqrt(
-c^2*x^2 + 1)*b*d^2*e*x^4/c^3 - 80/7623*sqrt(-c^2*x^2 + 1)*b*e^3*x^6/c^5 -
 4/75*sqrt(-c^2*x^2 + 1)*b*d^3*x^2/c^3 - 16/315*sqrt(-c^2*x^2 + 1)*b*d*e^2
*x^4/c^5 - 24/245*sqrt(-c^2*x^2 + 1)*b*d^2*e*x^2/c^5 - 32/2541*sqrt(-c^2*x
^2 + 1)*b*e^3*x^4/c^7 - 8/75*sqrt(-c^2*x^2 + 1)*b*d^3/c^5 - 64/945*sqrt(-c
^2*x^2 + 1)*b*d*e^2*x^2/c^7 - 48/245*sqrt(-c^2*x^2 + 1)*b*d^2*e/c^7 - 128/
7623*sqrt(-c^2*x^2 + 1)*b*e^3*x^2/c^9 - 128/945*sqrt(-c^2*x^2 + 1)*b*d*e^2
/c^9 - 256/7623*sqrt(-c^2*x^2 + 1)*b*e^3/c^11
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (d+ex^2)^3 (a+b\arccos(cx))\,dx = \int x^4\,(a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int(x^4*(a + b*acos(c*x))*(d + e*x^2)^3,x)
```

output
```
int(x^4*(a + b*acos(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.55

$$\int x^4 (d+ex^2)^3 (a+b\arccos(cx))\,dx = \frac{800415 acos(cx)\,b\,c^{11}d^3x^5 + 363825 acos(cx)\,b\,c^{11}e^3x^{11} - 160083\sqrt{-c^2x^2+1}\,b\,c^{10}d^3x^4 - 33075\sqrt{-c^2x^2+}}{}$$

input
```
int(x^4*(e*x^2+d)^3*(a+b*acos(c*x)),x)
```

output
```
(800415*acos(c*x)*b*c**11*d**3*x**5 + 1715175*acos(c*x)*b*c**11*d**2*e*x**
7 + 1334025*acos(c*x)*b*c**11*d*e**2*x**9 + 363825*acos(c*x)*b*c**11*e**3*
x**11 - 160083*sqrt( - c**2*x**2 + 1)*b*c**10*d**3*x**4 - 245025*sqrt( - c
**2*x**2 + 1)*b*c**10*d**2*e*x**6 - 148225*sqrt( - c**2*x**2 + 1)*b*c**10*
d*e**2*x**8 - 33075*sqrt( - c**2*x**2 + 1)*b*c**10*e**3*x**10 - 213444*sqr
t( - c**2*x**2 + 1)*b*c**8*d**3*x**2 - 294030*sqrt( - c**2*x**2 + 1)*b*c**
8*d**2*e*x**4 - 169400*sqrt( - c**2*x**2 + 1)*b*c**8*d*e**2*x**6 - 36750*s
qrt( - c**2*x**2 + 1)*b*c**8*e**3*x**8 - 426888*sqrt( - c**2*x**2 + 1)*b*c
**6*d**3 - 392040*sqrt( - c**2*x**2 + 1)*b*c**6*d**2*e*x**2 - 203280*sqrt(
 - c**2*x**2 + 1)*b*c**6*d*e**2*x**4 - 42000*sqrt( - c**2*x**2 + 1)*b*c**6
*e**3*x**6 - 784080*sqrt( - c**2*x**2 + 1)*b*c**4*d**2*e - 271040*sqrt( -
c**2*x**2 + 1)*b*c**4*d*e**2*x**2 - 50400*sqrt( - c**2*x**2 + 1)*b*c**4*e*
*3*x**4 - 542080*sqrt( - c**2*x**2 + 1)*b*c**2*d*e**2 - 67200*sqrt( - c**2
*x**2 + 1)*b*c**2*e**3*x**2 - 134400*sqrt( - c**2*x**2 + 1)*b*e**3 + 80041
5*a*c**11*d**3*x**5 + 1715175*a*c**11*d**2*e*x**7 + 1334025*a*c**11*d*e**2
*x**9 + 363825*a*c**11*e**3*x**11)/(4002075*c**11)
```

3.617 $\int x^3(d+ex^2)^3(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 335

$$\begin{aligned}\int x^3(d+ex^2)^3(a+b\arccos(cx))\,dx &= \frac{b(480c^6d^3+800c^4d^2e+525c^2de^2+126e^3)\,x\sqrt{1-c^2x^2}}{5120c^9} \\ &+ \frac{b(480c^6d^3+800c^4d^2e+525c^2de^2+126e^3)\,x^3\sqrt{1-c^2x^2}}{7680c^7} \\ &+ \frac{be(800c^4d^2+525c^2de+126e^2)\,x^5\sqrt{1-c^2x^2}}{9600c^5} \\ &+ \frac{3be^2(25c^2d+6e)\,x^7\sqrt{1-c^2x^2}}{1600c^3}+\frac{be^3x^9\sqrt{1-c^2x^2}}{100c} \\ &+ \frac{b(128c^{10}d^5-480c^6d^3e^2-800c^4d^2e^3-525c^2de^4-126e^5)\arccos(cx)}{5120c^{10}e^2} \\ &- \frac{d(d+ex^2)^4(a+b\arccos(cx))}{8e^2}+\frac{(d+ex^2)^5(a+b\arccos(cx))}{10e^2}\end{aligned}$$

output
```
1/5120*b*(480*c^6*d^3+800*c^4*d^2*e+525*c^2*d*e^2+126*e^3)*x*(-c^2*x^2+1)^
(1/2)/c^9+1/7680*b*(480*c^6*d^3+800*c^4*d^2*e+525*c^2*d*e^2+126*e^3)*x^3*(
-c^2*x^2+1)^(1/2)/c^7+1/9600*b*e*(800*c^4*d^2+525*c^2*d*e+126*e^2)*x^5*(-c
^2*x^2+1)^(1/2)/c^5+3/1600*b*e^2*(25*c^2*d+6*e)*x^7*(-c^2*x^2+1)^(1/2)/c^3
+1/100*b*e^3*x^9*(-c^2*x^2+1)^(1/2)/c+1/5120*b*(128*c^10*d^5-480*c^6*d^3*e
^2-800*c^4*d^2*e^3-525*c^2*d*e^4-126*e^5)*arccos(c*x)/c^10/e^2-1/8*d*(e*x^
2+d)^4*(a+b*arccos(c*x))/e^2+1/10*(e*x^2+d)^5*(a+b*arccos(c*x))/e^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.82

$$\int x^3 \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx$$
$$= \frac{1920ax^4(10d^3+20d^2ex^2+15de^2x^4+4e^3x^6) - \frac{bx\sqrt{1-c^2x^2}\left(1890e^3+315c^2e^2\left(25d+4ex^2\right)+6c^4e\left(2000d^2+875dex^2+168e^2x^4\right.\right.}{}}{}$$

input
```
Integrate[x^3*(d + e*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
(1920*a*x^4*(10*d^3 + 20*d^2*e*x^2 + 15*d*e^2*x^4 + 4*e^3*x^6) - (b*x*Sqrt
[1 - c^2*x^2]*(1890*e^3 + 315*c^2*e^2*(25*d + 4*e*x^2) + 6*c^4*e*(2000*d^2
 + 875*d*e*x^2 + 168*e^2*x^4) + 8*c^6*(900*d^3 + 1000*d^2*e*x^2 + 525*d*e^
2*x^4 + 108*e^3*x^6) + 16*c^8*(300*d^3*x^2 + 400*d^2*e*x^4 + 225*d*e^2*x^6
 + 48*e^3*x^8)))/c^9 + 1920*b*x^4*(10*d^3 + 20*d^2*e*x^2 + 15*d*e^2*x^4 +
4*e^3*x^6)*ArcCos[c*x] + (15*b*(480*c^6*d^3 + 800*c^4*d^2*e + 525*c^2*d*e^
2 + 126*e^3)*ArcSin[c*x])/c^10)/76800
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5231, 27, 403, 27, 403, 25, 403, 25, 403, 25, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \; 5231$$

$$bc\int -\frac{\left(d-4ex^2\right)\left(ex^2+d\right)^4}{40e^2\sqrt{1-c^2x^2}}dx+\frac{\left(d+ex^2\right)^5 (a+b\arccos(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4 (a+b\arccos(cx))}{8e^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{bc\int \frac{\left(d-4ex^2\right)\left(ex^2+d\right)^4}{\sqrt{1-c^2x^2}}dx}{40e^2}+\frac{\left(d+ex^2\right)^5 (a+b\arccos(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4 (a+b\arccos(cx))}{8e^2}$$

$$\Big\downarrow \; 403$$

$$-\frac{bc\left(\frac{2ex\sqrt{1-c^2x^2}\left(d+ex^2\right)^4}{5c^2}-\frac{\int -\frac{2\left(ex^2+d\right)^3\left(d\left(5c^2d-2e\right)-e\left(11dc^2+18e\right)x^2\right)}{\sqrt{1-c^2x^2}}dx}{10c^2}\right)}{40e^2}+\frac{\left(d+ex^2\right)^5 (a+b\arccos(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4 (a+b\arccos(cx))}{8e^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{bc\left(\frac{\int \frac{\left(ex^2+d\right)^3\left(d\left(5c^2d-2e\right)-e\left(11dc^2+18e\right)x^2\right)}{\sqrt{1-c^2x^2}}dx}{5c^2}+\frac{2ex\sqrt{1-c^2x^2}\left(d+ex^2\right)^4}{5c^2}\right)}{40e^2}+\frac{\left(d+ex^2\right)^5 (a+b\arccos(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4 (a+b\arccos(cx))}{8e^2}$$

$$\Big\downarrow \; 403$$

$$-\frac{bc\left(\frac{\frac{ex\sqrt{1-c^2x^2}\left(11c^2d+18e\right)\left(d+ex^2\right)^3}{8c^2}-\frac{\int -\frac{\left(ex^2+d\right)^2\left(d\left(40d^2c^4-27dec^2-18e^2\right)-e\left(26d^2c^4+201dec^2+126e^2\right)x^2\right)}{\sqrt{1-c^2x^2}}dx}{8c^2}}{5c^2}+\frac{2ex\sqrt{1-c^2x^2}\left(d+ex^2\right)^4}{5c^2}\right)}{40e^2}+\frac{\left(d+ex^2\right)^5 (a+b\arccos(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4 (a+b\arccos(cx))}{8e^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{bc\left(\frac{\frac{\int \frac{(ex^2+d)^2\left(d\left(40d^2c^4-27dec^2-18e^2\right)-e\left(26d^2c^4+201dec^2+126e^2\right)x^2\right)}{\sqrt{1-c^2x^2}}dx}{8c^2}+\frac{ex\sqrt{1-c^2x^2}\left(11c^2d+18e\right)\left(d+ex^2\right)^3}{8c^2}}{5c^2}+\frac{2ex\sqrt{1-c^2x^2}\left(d+ex^2\right)^4}{5c^2}\right)}{40e^2}+$$

$$\frac{\left(d+ex^2\right)^5\left(a+b\arccos(cx)\right)}{10e^2}-\frac{d\left(d+ex^2\right)^4\left(a+b\arccos(cx)\right)}{8e^2}$$

$\downarrow$ 403

$$-\frac{bc\left(\frac{\frac{\frac{ex\sqrt{1-c^2x^2}\left(26c^4d^2+201c^2de+126e^2\right)\left(d+ex^2\right)^2}{6c^2}-\frac{\int -\frac{(ex^2+d)\left(e\left(136d^3c^6-1096d^2ec^4-1617de^2c^2-630e^3\right)x^2+d\left(240d^3c^6-188d^2ec^4-309de^2c^2-12\right.\right.}{\sqrt{1-c^2x^2}}}{6c^2}}{8c^2}}{5c^2}\right.}{40e^2}$$

$$\frac{\left(d+ex^2\right)^5\left(a+b\arccos(cx)\right)}{10e^2}-\frac{d\left(d+ex^2\right)^4\left(a+b\arccos(cx)\right)}{8e^2}$$

$\downarrow$ 25

$$-\frac{bc\left(\frac{\frac{\frac{\int \frac{(ex^2+d)\left(e\left(136d^3c^6-1096d^2ec^4-1617de^2c^2-630e^3\right)x^2+d\left(240d^3c^6-188d^2ec^4-309de^2c^2-126e^3\right)\right)}{\sqrt{1-c^2x^2}}dx}{6c^2}+\frac{ex\sqrt{1-c^2x^2}\left(26c^4d^2+201c^2de+126e^2\right)\left(d+\right.}{6c^2}}{8c^2}}{5c^2}\right.}{40e^2}$$

$$\frac{\left(d+ex^2\right)^5\left(a+b\arccos(cx)\right)}{10e^2}-\frac{d\left(d+ex^2\right)^4\left(a+b\arccos(cx)\right)}{8e^2}$$

$\downarrow$ 403

$$-\frac{bc\left(\frac{\frac{\frac{-\frac{\int -\frac{e\left(1232d^4c^8-2536d^3ec^6-7758d^2e^2c^4-6615de^3c^2-1890e^4\right)x^2+d\left(960d^4c^8-616d^3ec^6-2332d^2e^2c^4-2121de^3c^2-630e^4\right)}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{ex\sqrt{1-c^2x^2}\left(136c^6\right.}{}}{6c^2}}{8c^2}}{5c^2}\right.}{}$$

$$\frac{\left(d+ex^2\right)^5\left(a+b\arccos(cx)\right)}{10e^2}-\frac{d\left(d+ex^2\right)^4\left(a+b\arccos(cx)\right)}{8e^2}$$

$\downarrow$ 25

$$-\frac{bc\left(\frac{\frac{\frac{\int \frac{e\left(1232d^4c^8-2536d^3ec^6-7758d^2e^2c^4-6615de^3c^2-1890e^4\right)x^2+d\left(960d^4c^8-616d^3ec^6-2332d^2e^2c^4-2121de^3c^2-630e^4\right)}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{ex\sqrt{1-c^2x^2}\left(136c^6d^3-\right.}{}}{6c^2}}{8c^2}}{5c^2}\right.}{}$$

$$\frac{\left(d+ex^2\right)^5\left(a+b\arccos(cx)\right)}{10e^2}-\frac{d\left(d+ex^2\right)^4\left(a+b\arccos(cx)\right)}{8e^2}$$

$$\Big\downarrow \textbf{299}$$

$$-\frac{bc\left(\frac{\frac{\frac{\frac{15\left(128c^{10}d^5-480c^6d^3e^2-800c^4d^2e^3-525c^2de^4-126e^5\right)\int\frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{ex\sqrt{1-c^2x^2}\left(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-6615c^2de^3-1890e^4\right)}{2c^2}}{4c^2}-\frac{e}{}}{6c^2}}{8c^2}}{}\right.}{}$$

$$\frac{\left(d+ex^2\right)^5\left(a+b\arccos(cx)\right)}{10e^2}-\frac{d\left(d+ex^2\right)^4\left(a+b\arccos(cx)\right)}{8e^2}$$

$$\Big\downarrow \textbf{223}$$

$$\frac{\left(d+ex^2\right)^5\left(a+b\arccos(cx)\right)}{10e^2}-\frac{d\left(d+ex^2\right)^4\left(a+b\arccos(cx)\right)}{8e^2}-$$

$$\frac{bc\left(\frac{\frac{\frac{\frac{15\arcsin(cx)\left(128c^{10}d^5-480c^6d^3e^2-800c^4d^2e^3-525c^2de^4-126e^5\right)}{2c^3}-\frac{ex\sqrt{1-c^2x^2}\left(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-6615c^2de^3-1890e^4\right)}{2c^2}}{4c^2}-\frac{ex\sqrt{1-c^2}}{}}{6c^2}}{8c^2}}{5c^2}\right.}{}$$

input
```
Int[x^3*(d + e*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
-1/8*(d*(d + e*x^2)^4*(a + b*ArcCos[c*x]))/e^2 + ((d + e*x^2)^5*(a + b*Arc
Cos[c*x]))/(10*e^2) - (b*c*((2*e*x*Sqrt[1 - c^2*x^2]*(d + e*x^2)^4)/(5*c^2
) + ((e*(11*c^2*d + 18*e)*x*Sqrt[1 - c^2*x^2]*(d + e*x^2)^3)/(8*c^2) + ((e
*(26*c^4*d^2 + 201*c^2*d*e + 126*e^2)*x*Sqrt[1 - c^2*x^2]*(d + e*x^2)^2)/(
6*c^2) + (-1/4*(e*(136*c^6*d^3 - 1096*c^4*d^2*e - 1617*c^2*d*e^2 - 630*e^3
)*x*Sqrt[1 - c^2*x^2]*(d + e*x^2))/c^2 + (-1/2*(e*(1232*c^8*d^4 - 2536*c^6
*d^3*e - 7758*c^4*d^2*e^2 - 6615*c^2*d*e^3 - 1890*e^4)*x*Sqrt[1 - c^2*x^2]
)/c^2 + (15*(128*c^10*d^5 - 480*c^6*d^3*e^2 - 800*c^4*d^2*e^3 - 525*c^2*d*
e^4 - 126*e^5)*ArcSin[c*x])/(2*c^3))/(4*c^2))/(6*c^2))/(8*c^2))/(5*c^2)))/
(40*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.30

method	result
parts	$a\left(\frac{1}{10}e^3x^{10}+\frac{3}{8}d\,e^2x^8+\frac{1}{2}d^2e\,x^6+\frac{1}{4}d^3x^4\right)+\frac{b\left(\frac{c^4\arccos(cx)e^3x^{10}}{10}+\frac{3c^4\arccos(cx)d\,e^2x^8}{8}+\frac{c^4\arccos(cx)d^2}{2}\right.}{}$
derivativedivides	$\frac{a\left(\frac{1}{4}c^{10}d^3x^4+\frac{1}{2}c^{10}d^2e\,x^6+\frac{3}{8}c^{10}d\,e^2x^8+\frac{1}{10}e^3x^{10}c^{10}\right)}{c^6}+\frac{b\left(\frac{\arccos(cx)d^3c^{10}x^4}{4}+\frac{\arccos(cx)d^2c^{10}e\,x^6}{2}+\frac{3\arccos(cx)d\,c^{10}e^2x^8}{8}+\right.}{}$
default	$\frac{a\left(\frac{1}{4}c^{10}d^3x^4+\frac{1}{2}c^{10}d^2e\,x^6+\frac{3}{8}c^{10}d\,e^2x^8+\frac{1}{10}e^3x^{10}c^{10}\right)}{c^6}+\frac{b\left(\frac{\arccos(cx)d^3c^{10}x^4}{4}+\frac{\arccos(cx)d^2c^{10}e\,x^6}{2}+\frac{3\arccos(cx)d\,c^{10}e^2x^8}{8}+\right.}{}$
orering	$\frac{(4864c^{10}e^4x^{12}+23728c^{10}d\,e^3x^{10}+45200c^{10}d^2e^2x^8+288c^8e^4x^{10}+40000c^{10}d^3e\,x^6+1896c^8d\,e^3x^8+11200c^{10}d^4x^4+540}{}$

input
```
int(x^3*(e*x^2+d)^3*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/10*e^3*x^10+3/8*d*e^2*x^8+1/2*d^2*e*x^6+1/4*d^3*x^4)+b/c^4*(1/10*c^4*
arccos(c*x)*e^3*x^10+3/8*c^4*arccos(c*x)*d*e^2*x^8+1/2*c^4*arccos(c*x)*d^2
*e*x^6+1/4*arccos(c*x)*c^4*x^4*d^3+1/40/c^6*(4*e^3*(-1/10*c^9*x^9*(-c^2*x^
2+1)^(1/2)-9/80*c^7*x^7*(-c^2*x^2+1)^(1/2)-21/160*c^5*x^5*(-c^2*x^2+1)^(1/
2)-21/128*c^3*x^3*(-c^2*x^2+1)^(1/2)-63/256*c*x*(-c^2*x^2+1)^(1/2)+63/256*
arcsin(c*x))+10*d^3*c^6*(-1/4*c^3*x^3*(-c^2*x^2+1)^(1/2)-3/8*c*x*(-c^2*x^2
+1)^(1/2)+3/8*arcsin(c*x))+15*d*c^2*e^2*(-1/8*c^7*x^7*(-c^2*x^2+1)^(1/2)-7
/48*c^5*x^5*(-c^2*x^2+1)^(1/2)-35/192*c^3*x^3*(-c^2*x^2+1)^(1/2)-35/128*c*
x*(-c^2*x^2+1)^(1/2)+35/128*arcsin(c*x))+20*d^2*c^4*e*(-1/6*c^5*x^5*(-c^2*
x^2+1)^(1/2)-5/24*c^3*x^3*(-c^2*x^2+1)^(1/2)-5/16*c*x*(-c^2*x^2+1)^(1/2)+5
/16*arcsin(c*x))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.95

$$\int x^3(d+ex^2)^3(a+b\arccos(cx))\,dx$$
$$= \frac{7680\,ac^{10}e^3x^{10}+28800\,ac^{10}de^2x^8+38400\,ac^{10}d^2ex^6+19200\,ac^{10}d^3x^4+15\,(512\,bc^{10}e^3x^{10}+1920\,bc^{10}de}{}$$

input
```
integrate(x^3*(e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/76800*(7680*a*c^10*e^3*x^10 + 28800*a*c^10*d*e^2*x^8 + 38400*a*c^10*d^2*
e*x^6 + 19200*a*c^10*d^3*x^4 + 15*(512*b*c^10*e^3*x^10 + 1920*b*c^10*d*e^2
*x^8 + 2560*b*c^10*d^2*e*x^6 + 1280*b*c^10*d^3*x^4 - 480*b*c^6*d^3 - 800*b
*c^4*d^2*e - 525*b*c^2*d*e^2 - 126*b*e^3)*arccos(c*x) - (768*b*c^9*e^3*x^9
 + 144*(25*b*c^9*d*e^2 + 6*b*c^7*e^3)*x^7 + 8*(800*b*c^9*d^2*e + 525*b*c^7
*d*e^2 + 126*b*c^5*e^3)*x^5 + 10*(480*b*c^9*d^3 + 800*b*c^7*d^2*e + 525*b*
c^5*d*e^2 + 126*b*c^3*e^3)*x^3 + 15*(480*b*c^7*d^3 + 800*b*c^5*d^2*e + 525
*b*c^3*d*e^2 + 126*b*c*e^3)*x)*sqrt(-c^2*x^2 + 1))/c^10
```

Sympy [A] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.80

$$\int x^3(d+ex^2)^3(a+b\arccos(cx))\,dx$$
$$= \begin{cases} \frac{ad^3x^4}{4}+\frac{ad^2ex^6}{2}+\frac{3ade^2x^8}{8}+\frac{ae^3x^{10}}{10}+\frac{bd^3x^4\operatorname{acos}(cx)}{4}+\frac{bd^2ex^6\operatorname{acos}(cx)}{2}+\frac{3bde^2x^8\operatorname{acos}(cx)}{8}+\frac{be^3x^{10}\operatorname{acos}(cx)}{10}-\frac{bd^3x^3\sqrt{-}}{16} \\ \left(a+\frac{\pi b}{2}\right)\left(\frac{d^3x^4}{4}+\frac{d^2ex^6}{2}+\frac{3de^2x^8}{8}+\frac{e^3x^{10}}{10}\right) \end{cases}$$

input
```
integrate(x**3*(e*x**2+d)**3*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d**3*x**4/4 + a*d**2*e*x**6/2 + 3*a*d*e**2*x**8/8 + a*e**3*x*
*10/10 + b*d**3*x**4*acos(c*x)/4 + b*d**2*e*x**6*acos(c*x)/2 + 3*b*d*e**2*
x**8*acos(c*x)/8 + b*e**3*x**10*acos(c*x)/10 - b*d**3*x**3*sqrt(-c**2*x**2
 + 1)/(16*c) - b*d**2*e*x**5*sqrt(-c**2*x**2 + 1)/(12*c) - 3*b*d*e**2*x**7
*sqrt(-c**2*x**2 + 1)/(64*c) - b*e**3*x**9*sqrt(-c**2*x**2 + 1)/(100*c) -
3*b*d**3*x*sqrt(-c**2*x**2 + 1)/(32*c**3) - 5*b*d**2*e*x**3*sqrt(-c**2*x**
2 + 1)/(48*c**3) - 7*b*d*e**2*x**5*sqrt(-c**2*x**2 + 1)/(128*c**3) - 9*b*e
**3*x**7*sqrt(-c**2*x**2 + 1)/(800*c**3) - 3*b*d**3*acos(c*x)/(32*c**4) -
5*b*d**2*e*x*sqrt(-c**2*x**2 + 1)/(32*c**5) - 35*b*d*e**2*x**3*sqrt(-c**2*
x**2 + 1)/(512*c**5) - 21*b*e**3*x**5*sqrt(-c**2*x**2 + 1)/(1600*c**5) - 5
*b*d**2*e*acos(c*x)/(32*c**6) - 105*b*d*e**2*x*sqrt(-c**2*x**2 + 1)/(1024*
c**7) - 21*b*e**3*x**3*sqrt(-c**2*x**2 + 1)/(1280*c**7) - 105*b*d*e**2*aco
s(c*x)/(1024*c**8) - 63*b*e**3*x*sqrt(-c**2*x**2 + 1)/(2560*c**9) - 63*b*e
**3*acos(c*x)/(2560*c**10), Ne(c, 0)), ((a + pi*b/2)*(d**3*x**4/4 + d**2*e
*x**6/2 + 3*d*e**2*x**8/8 + e**3*x**10/10), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.28

$$\int x^3(d+ex^2)^3(a+b\arccos(cx))\,dx = \frac{1}{10}ae^3x^{10} + \frac{3}{8}ade^2x^8 + \frac{1}{2}ad^2ex^6 + \frac{1}{4}ad^3x^4$$
$$+ \frac{1}{32}\left(8\,x^4\arccos(cx) - \left(\frac{2\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\sqrt{-c^2x^2+1}x}{c^4} - \frac{3\arcsin(cx)}{c^5}\right)c\right)bd^3$$
$$+ \frac{1}{96}\left(48\,x^6\arccos(cx) - \left(\frac{8\sqrt{-c^2x^2+1}x^5}{c^2} + \frac{10\sqrt{-c^2x^2+1}x^3}{c^4} + \frac{15\sqrt{-c^2x^2+1}x}{c^6} - \frac{15\arcsin(cx)}{c^7}\right)\right.$$
$$+ \frac{1}{1024}\left(384\,x^8\arccos(cx) - \left(\frac{48\sqrt{-c^2x^2+1}x^7}{c^2} + \frac{56\sqrt{-c^2x^2+1}x^5}{c^4} + \frac{70\sqrt{-c^2x^2+1}x^3}{c^6} + \frac{105\sqrt{-c^2}}{c^8}\right.\right.$$
$$+ \frac{1}{12800}\left(1280\,x^{10}\arccos(cx) - \left(\frac{128\sqrt{-c^2x^2+1}x^9}{c^2} + \frac{144\sqrt{-c^2x^2+1}x^7}{c^4} + \frac{168\sqrt{-c^2x^2+1}x^5}{c^6} + \frac{21}{}\right.\right.$$

input
```
integrate(x^3*(e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output

```
1/10*a*e^3*x^10 + 3/8*a*d*e^2*x^8 + 1/2*a*d^2*e*x^6 + 1/4*a*d^3*x^4 + 1/32
*(8*x^4*arccos(c*x) - (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2 + 1)
*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*d^3 + 1/96*(48*x^6*arccos(c*x) - (8*sqrt(
-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^2*x^2 +
 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*b*d^2*e + 1/1024*(384*x^8*arccos(c*x) -
 (48*sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*sqrt(-c^2*x^2 + 1)*x^5/c^4 + 70*sqrt(
-c^2*x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2*x^2 + 1)*x/c^8 - 105*arcsin(c*x)/c^9
)*c)*b*d*e^2 + 1/12800*(1280*x^10*arccos(c*x) - (128*sqrt(-c^2*x^2 + 1)*x^
9/c^2 + 144*sqrt(-c^2*x^2 + 1)*x^7/c^4 + 168*sqrt(-c^2*x^2 + 1)*x^5/c^6 +
210*sqrt(-c^2*x^2 + 1)*x^3/c^8 + 315*sqrt(-c^2*x^2 + 1)*x/c^10 - 315*arcsi
n(c*x)/c^11)*c)*b*e^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.44

$$\int x^3(d+ex^2)^3(a+b\arccos(cx))\,dx = \frac{1}{10}\,be^3x^{10}\arccos(cx) + \frac{1}{10}\,ae^3x^{10} + \frac{3}{8}\,bde^2x^8\arccos(cx) - \frac{\sqrt{-c^2x^2+1}be^3x^9}{100\,c} + \frac{3}{8}\,ade^2x^8 + \frac{1}{2}\,bd^2ex^6\arccos(cx) - \frac{3\sqrt{-c^2x^2+1}bde^2x^7}{64\,c} + \frac{1}{2}\,ad^2ex^6 + \frac{1}{4}\,bd^3x^4\arccos(cx) - \frac{\sqrt{-c^2x^2+1}bd^2ex^5}{12\,c} - \frac{9\sqrt{-c^2x^2+1}be^3x^7}{800\,c^3} + \frac{1}{4}\,ad^3x^4 - \frac{\sqrt{-c^2x^2+1}bd^3x^3}{16\,c} - \frac{7\sqrt{-c^2x^2+1}bde^2x^5}{128\,c^3} - \frac{5\sqrt{-c^2x^2+1}bd^2ex^3}{48\,c^3} - \frac{21\sqrt{-c^2x^2+1}be^3x^5}{1600\,c^5} - \frac{3\sqrt{-c^2x^2+1}bd^3x}{32\,c^3} - \frac{35\sqrt{-c^2x^2+1}bde^2x^3}{512\,c^5} - \frac{3\,bd^3\arccos(cx)}{32\,c^4} - \frac{5\sqrt{-c^2x^2+1}bd^2ex}{32\,c^5} - \frac{21\sqrt{-c^2x^2+1}be^3x^3}{1280\,c^7} - \frac{5\,bd^2e\arccos(cx)}{32\,c^6} - \frac{105\sqrt{-c^2x^2+1}bde^2x}{1024\,c^7} - \frac{105\,bde^2\arccos(cx)}{1024\,c^8} - \frac{63\sqrt{-c^2x^2+1}be^3x}{2560\,c^9} - \frac{63\,be^3\arccos(cx)}{2560\,c^{10}}$$

input
```
integrate(x^3*(e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/10*b*e^3*x^10*arccos(c*x) + 1/10*a*e^3*x^10 + 3/8*b*d*e^2*x^8*arccos(c*x
) - 1/100*sqrt(-c^2*x^2 + 1)*b*e^3*x^9/c + 3/8*a*d*e^2*x^8 + 1/2*b*d^2*e*x
^6*arccos(c*x) - 3/64*sqrt(-c^2*x^2 + 1)*b*d*e^2*x^7/c + 1/2*a*d^2*e*x^6 +
 1/4*b*d^3*x^4*arccos(c*x) - 1/12*sqrt(-c^2*x^2 + 1)*b*d^2*e*x^5/c - 9/800
*sqrt(-c^2*x^2 + 1)*b*e^3*x^7/c^3 + 1/4*a*d^3*x^4 - 1/16*sqrt(-c^2*x^2 + 1
)*b*d^3*x^3/c - 7/128*sqrt(-c^2*x^2 + 1)*b*d*e^2*x^5/c^3 - 5/48*sqrt(-c^2*
x^2 + 1)*b*d^2*e*x^3/c^3 - 21/1600*sqrt(-c^2*x^2 + 1)*b*e^3*x^5/c^5 - 3/32
*sqrt(-c^2*x^2 + 1)*b*d^3*x/c^3 - 35/512*sqrt(-c^2*x^2 + 1)*b*d*e^2*x^3/c^
5 - 3/32*b*d^3*arccos(c*x)/c^4 - 5/32*sqrt(-c^2*x^2 + 1)*b*d^2*e*x/c^5 - 2
1/1280*sqrt(-c^2*x^2 + 1)*b*e^3*x^3/c^7 - 5/32*b*d^2*e*arccos(c*x)/c^6 - 1
05/1024*sqrt(-c^2*x^2 + 1)*b*d*e^2*x/c^7 - 105/1024*b*d*e^2*arccos(c*x)/c^
8 - 63/2560*sqrt(-c^2*x^2 + 1)*b*e^3*x/c^9 - 63/2560*b*e^3*arccos(c*x)/c^1
0
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (d+ex^2)^3 (a+b\arccos(cx))\,dx = \int x^3 \left(a+b\,\mathrm{acos}(c\,x)\right)\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int(x^3*(a + b*acos(c*x))*(d + e*x^2)^3,x)
```

output
```
int(x^3*(a + b*acos(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.47

$$\int x^3 (d+ex^2)^3 (a+b\arccos(cx))\,dx = \frac{7200 asin(cx)\,b\,c^6 d^3 + 19200 a\,c^{10} d^3 x^4 + 7680 a\,c^{10} e^3 x^{10} + 38400 acos(cx)\,b\,c^{10} d^2 e\,x^6 + 28800 acos(cx)\,b\,c^{1}}{}$$

input
```
int(x^3*(e*x^2+d)^3*(a+b*acos(c*x)),x)
```

output
```
(19200*acos(c*x)*b*c**10*d**3*x**4 + 38400*acos(c*x)*b*c**10*d**2*e*x**6 +
 28800*acos(c*x)*b*c**10*d*e**2*x**8 + 7680*acos(c*x)*b*c**10*e**3*x**10 +
 7200*asin(c*x)*b*c**6*d**3 + 12000*asin(c*x)*b*c**4*d**2*e + 7875*asin(c*
x)*b*c**2*d*e**2 + 1890*asin(c*x)*b*e**3 - 4800*sqrt( - c**2*x**2 + 1)*b*c
**9*d**3*x**3 - 6400*sqrt( - c**2*x**2 + 1)*b*c**9*d**2*e*x**5 - 3600*sqrt
( - c**2*x**2 + 1)*b*c**9*d*e**2*x**7 - 768*sqrt( - c**2*x**2 + 1)*b*c**9*
e**3*x**9 - 7200*sqrt( - c**2*x**2 + 1)*b*c**7*d**3*x - 8000*sqrt( - c**2*
x**2 + 1)*b*c**7*d**2*e*x**3 - 4200*sqrt( - c**2*x**2 + 1)*b*c**7*d*e**2*x
**5 - 864*sqrt( - c**2*x**2 + 1)*b*c**7*e**3*x**7 - 12000*sqrt( - c**2*x**
2 + 1)*b*c**5*d**2*e*x - 5250*sqrt( - c**2*x**2 + 1)*b*c**5*d*e**2*x**3 -
1008*sqrt( - c**2*x**2 + 1)*b*c**5*e**3*x**5 - 7875*sqrt( - c**2*x**2 + 1)
*b*c**3*d*e**2*x - 1260*sqrt( - c**2*x**2 + 1)*b*c**3*e**3*x**3 - 1890*sqr
t( - c**2*x**2 + 1)*b*c*e**3*x + 19200*a*c**10*d**3*x**4 + 38400*a*c**10*d
**2*e*x**6 + 28800*a*c**10*d*e**2*x**8 + 7680*a*c**10*e**3*x**10)/(76800*c
**10)
```

3.618 $\int x^2(d+ex^2)^3(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 287

$$\begin{aligned}\int x^2(d+ex^2)^3(a+b\arccos(cx))\,dx &= \frac{b(105c^6d^3+189c^4d^2e+135c^2de^2+35e^3)\sqrt{1-c^2x^2}}{315c^9}\\ &\quad-\frac{b(105c^6d^3+378c^4d^2e+405c^2de^2+140e^3)(1-c^2x^2)^{3/2}}{945c^9}\\ &\quad+\frac{be(63c^4d^2+135c^2de+70e^2)(1-c^2x^2)^{5/2}}{525c^9}\\ &\quad-\frac{be^2(27c^2d+28e)(1-c^2x^2)^{7/2}}{441c^9}+\frac{be^3(1-c^2x^2)^{9/2}}{81c^9}\\ &\quad+\frac{1}{3}d^3x^3(a+b\arccos(cx))+\frac{3}{5}d^2ex^5(a+b\arccos(cx))+\frac{3}{7}de^2x^7(a+b\arccos(cx))+\frac{1}{9}e^3x^9(a+b\arccos(cx))\end{aligned}$$

output
```
1/315*b*(105*c^6*d^3+189*c^4*d^2*e+135*c^2*d*e^2+35*e^3)*(-c^2*x^2+1)^(1/2
)/c^9-1/945*b*(105*c^6*d^3+378*c^4*d^2*e+405*c^2*d*e^2+140*e^3)*(-c^2*x^2+
1)^(3/2)/c^9+1/525*b*e*(63*c^4*d^2+135*c^2*d*e+70*e^2)*(-c^2*x^2+1)^(5/2)/
c^9-1/441*b*e^2*(27*c^2*d+28*e)*(-c^2*x^2+1)^(7/2)/c^9+1/81*b*e^3*(-c^2*x^
2+1)^(9/2)/c^9+1/3*d^3*x^3*(a+b*arccos(c*x))+3/5*d^2*e*x^5*(a+b*arccos(c*x
))+3/7*d*e^2*x^7*(a+b*arccos(c*x))+1/9*e^3*x^9*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.81

$$\int x^2\left(d+ex^2\right)^3\left(a+b\arccos(cx)\right)dx$$
$$=\frac{315ax^3(105d^3+189d^2ex^2+135de^2x^4+35e^3x^6)-\frac{b\sqrt{1-c^2x^2}\left(4480e^3+80c^2e^2\left(243d+28ex^2\right)+24c^4e\left(1323d^2+405dex^2+70\right.\right.}{}}{}$$

input
```
Integrate[x^2*(d + e*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
(315*a*x^3*(105*d^3 + 189*d^2*e*x^2 + 135*d*e^2*x^4 + 35*e^3*x^6) - (b*Sqr
t[1 - c^2*x^2]*(4480*e^3 + 80*c^2*e^2*(243*d + 28*e*x^2) + 24*c^4*e*(1323*
d^2 + 405*d*e*x^2 + 70*e^2*x^4) + 2*c^6*(11025*d^3 + 7938*d^2*e*x^2 + 3645
*d*e^2*x^4 + 700*e^3*x^6) + c^8*(11025*d^3*x^2 + 11907*d^2*e*x^4 + 6075*d*
e^2*x^6 + 1225*e^3*x^8)))/c^9 + 315*b*x^3*(105*d^3 + 189*d^2*e*x^2 + 135*d
*e^2*x^4 + 35*e^3*x^6)*ArcCos[c*x])/99225
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5231, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d+ex^2\right)^3\left(a+b\arccos(cx)\right)dx$$

$$\Big\downarrow \; 5231$$

$$bc\int \frac{x^3\left(35e^3x^6+135de^2x^4+189d^2ex^2+105d^3\right)}{315\sqrt{1-c^2x^2}}dx+\frac{1}{3}d^3x^3(a+b\arccos(cx))+\frac{3}{5}d^2ex^5(a+b\arccos(cx))+\frac{3}{7}de^2x^7(a+b\arccos(cx))+\frac{1}{9}e^3x^9(a+b\arccos(cx))$$

$$\Big\downarrow \; 27$$

$$\frac{1}{315}bc\int\frac{x^3(35e^3x^6+135de^2x^4+189d^2ex^2+105d^3)}{\sqrt{1-c^2x^2}}dx+\frac{1}{3}d^3x^3(a+b\arccos(cx))+\frac{3}{5}d^2ex^5(a+b\arccos(cx))+\frac{3}{7}de^2x^7(a+b\arccos(cx))+\frac{1}{9}e^3x^9(a+b\arccos(cx))$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{630}bc\int\frac{x^2(35e^3x^6+135de^2x^4+189d^2ex^2+105d^3)}{\sqrt{1-c^2x^2}}dx^2+\frac{1}{3}d^3x^3(a+b\arccos(cx))+\frac{3}{5}d^2ex^5(a+b\arccos(cx))+\frac{3}{7}de^2x^7(a+b\arccos(cx))+\frac{1}{9}e^3x^9(a+b\arccos(cx))$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{630}bc\int\left(\frac{35e^3(1-c^2x^2)^{7/2}}{c^8}-\frac{5e^2(27dc^2+28e)(1-c^2x^2)^{5/2}}{c^8}+\frac{3e(63d^2c^4+135dec^2+70e^2)(1-c^2x^2)^{3/2}}{c^8}+\right.$$
$$\frac{1}{3}d^3x^3(a+b\arccos(cx))+\frac{3}{5}d^2ex^5(a+b\arccos(cx))+\frac{3}{7}de^2x^7(a+b\arccos(cx))+\frac{1}{9}e^3x^9(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}d^3x^3(a+b\arccos(cx))+\frac{3}{5}d^2ex^5(a+b\arccos(cx))+\frac{3}{7}de^2x^7(a+b\arccos(cx))+\frac{1}{9}e^3x^9(a+b\arccos(cx))+$$
$$\frac{1}{630}bc\left(\frac{10e^2(1-c^2x^2)^{7/2}(27c^2d+28e)}{7c^{10}}-\frac{70e^3(1-c^2x^2)^{9/2}}{9c^{10}}-\frac{6e(1-c^2x^2)^{5/2}(63c^4d^2+135c^2de+70e^2)}{5c^{10}}+\right.$$

input
```
Int[x^2*(d + e*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*((-2*(105*c^6*d^3 + 189*c^4*d^2*e + 135*c^2*d*e^2 + 35*e^3)*Sqrt[1 -
c^2*x^2])/c^10 + (2*(105*c^6*d^3 + 378*c^4*d^2*e + 405*c^2*d*e^2 + 140*e^3
)*(1 - c^2*x^2)^(3/2))/(3*c^10) - (6*e*(63*c^4*d^2 + 135*c^2*d*e + 70*e^2)
*(1 - c^2*x^2)^(5/2))/(5*c^10) + (10*e^2*(27*c^2*d + 28*e)*(1 - c^2*x^2)^(
7/2))/(7*c^10) - (70*e^3*(1 - c^2*x^2)^(9/2))/(9*c^10)))/630 + (d^3*x^3*(a
 + b*ArcCos[c*x]))/3 + (3*d^2*e*x^5*(a + b*ArcCos[c*x]))/5 + (3*d*e^2*x^7*
(a + b*ArcCos[c*x]))/7 + (e^3*x^9*(a + b*ArcCos[c*x]))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.41

method	result
parts	$a\left(\frac{1}{9}e^3x^9+\frac{3}{7}d\,e^2x^7+\frac{3}{5}d^2e\,x^5+\frac{1}{3}d^3x^3\right)+\frac{b\left(\frac{c^3\arccos(cx)e^3x^9}{9}+\frac{3c^3\arccos(cx)d\,e^2x^7}{7}+\frac{3c^3\arccos(cx)d^2e\,x}{5}\right.}{}$
derivativedivides	$\frac{a\left(\frac{1}{3}d^3c^9x^3+\frac{3}{5}d^2c^9e\,x^5+\frac{3}{7}d\,c^9e^2x^7+\frac{1}{9}e^3c^9x^9\right)}{c^6}+\frac{b\left(\frac{\arccos(cx)d^3c^9x^3}{3}+\frac{3\arccos(cx)d^2c^9e\,x^5}{5}+\frac{3\arccos(cx)d\,c^9e^2x^7}{7}+\frac{\arccos(c}{}\right.}{}$
default	$\frac{a\left(\frac{1}{3}d^3c^9x^3+\frac{3}{5}d^2c^9e\,x^5+\frac{3}{7}d\,c^9e^2x^7+\frac{1}{9}e^3c^9x^9\right)}{c^6}+\frac{b\left(\frac{\arccos(cx)d^3c^9x^3}{3}+\frac{3\arccos(cx)d^2c^9e\,x^5}{5}+\frac{3\arccos(cx)d\,c^9e^2x^7}{7}+\frac{\arccos(c}{}\right.}{}$
orering	$\frac{(20825c^{10}e^4x^{12}+104600c^{10}d\,e^3x^{10}+209466c^{10}d^2e^2x^8+1400c^8e^4x^{10}+204624c^{10}d^3e\,x^6+10070c^8d\,e^3x^8+55125c^{10}d^4}{}$

input
```
int(x^2*(e*x^2+d)^3*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/9*e^3*x^9+3/7*d*e^2*x^7+3/5*d^2*e*x^5+1/3*d^3*x^3)+b/c^3*(1/9*c^3*arc
cos(c*x)*e^3*x^9+3/7*c^3*arccos(c*x)*d*e^2*x^7+3/5*c^3*arccos(c*x)*d^2*e*x
^5+1/3*arccos(c*x)*d^3*c^3*x^3+1/315/c^6*(35*e^3*(-1/9*c^8*x^8*(-c^2*x^2+1
)^(1/2)-8/63*c^6*x^6*(-c^2*x^2+1)^(1/2)-16/105*c^4*x^4*(-c^2*x^2+1)^(1/2)-
64/315*c^2*x^2*(-c^2*x^2+1)^(1/2)-128/315*(-c^2*x^2+1)^(1/2))+105*d^3*c^6*
(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))+135*d*c^2*e^2*(-1
/7*c^6*x^6*(-c^2*x^2+1)^(1/2)-6/35*c^4*x^4*(-c^2*x^2+1)^(1/2)-8/35*c^2*x^2
*(-c^2*x^2+1)^(1/2)-16/35*(-c^2*x^2+1)^(1/2))+189*d^2*c^4*e*(-1/5*c^4*x^4*
(-c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2)
)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.97

$$\int x^2\left(d+ex^2\right)^3\left(a+b\arccos(cx)\right)dx$$
$$=\frac{11025\,ac^9e^3x^9+42525\,ac^9de^2x^7+59535\,ac^9d^2ex^5+33075\,ac^9d^3x^3+315\left(35\,bc^9e^3x^9+135\,bc^9de^2x^7+1\right.}{}$$

input
```
integrate(x^2*(e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/99225*(11025*a*c^9*e^3*x^9 + 42525*a*c^9*d*e^2*x^7 + 59535*a*c^9*d^2*e*x
^5 + 33075*a*c^9*d^3*x^3 + 315*(35*b*c^9*e^3*x^9 + 135*b*c^9*d*e^2*x^7 + 1
89*b*c^9*d^2*e*x^5 + 105*b*c^9*d^3*x^3)*arccos(c*x) - (1225*b*c^8*e^3*x^8
+ 22050*b*c^6*d^3 + 31752*b*c^4*d^2*e + 25*(243*b*c^8*d*e^2 + 56*b*c^6*e^3
)*x^6 + 19440*b*c^2*d*e^2 + 3*(3969*b*c^8*d^2*e + 2430*b*c^6*d*e^2 + 560*b
*c^4*e^3)*x^4 + 4480*b*e^3 + (11025*b*c^8*d^3 + 15876*b*c^6*d^2*e + 9720*b
*c^4*d*e^2 + 2240*b*c^2*e^3)*x^2)*sqrt(-c^2*x^2 + 1))/c^9
```

Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.85

$$\int x^2(d+ex^2)^3(a+b\arccos(cx))\,dx$$
$$= \begin{cases} \frac{ad^3x^3}{3}+\frac{3ad^2ex^5}{5}+\frac{3ade^2x^7}{7}+\frac{ae^3x^9}{9}+\frac{bd^3x^3\operatorname{acos}(cx)}{3}+\frac{3bd^2ex^5\operatorname{acos}(cx)}{5}+\frac{3bde^2x^7\operatorname{acos}(cx)}{7}+\frac{be^3x^9\operatorname{acos}(cx)}{9}-\frac{bd^3x^2\sqrt{-}}{9} \\ \left(a+\frac{\pi b}{2}\right)\left(\frac{d^3x^3}{3}+\frac{3d^2ex^5}{5}+\frac{3de^2x^7}{7}+\frac{e^3x^9}{9}\right) \end{cases}$$

input
```
integrate(x**2*(e*x**2+d)**3*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d**3*x**3/3 + 3*a*d**2*e*x**5/5 + 3*a*d*e**2*x**7/7 + a*e**3*
x**9/9 + b*d**3*x**3*acos(c*x)/3 + 3*b*d**2*e*x**5*acos(c*x)/5 + 3*b*d*e**
2*x**7*acos(c*x)/7 + b*e**3*x**9*acos(c*x)/9 - b*d**3*x**2*sqrt(-c**2*x**2
 + 1)/(9*c) - 3*b*d**2*e*x**4*sqrt(-c**2*x**2 + 1)/(25*c) - 3*b*d*e**2*x**
6*sqrt(-c**2*x**2 + 1)/(49*c) - b*e**3*x**8*sqrt(-c**2*x**2 + 1)/(81*c) -
2*b*d**3*sqrt(-c**2*x**2 + 1)/(9*c**3) - 4*b*d**2*e*x**2*sqrt(-c**2*x**2 +
 1)/(25*c**3) - 18*b*d*e**2*x**4*sqrt(-c**2*x**2 + 1)/(245*c**3) - 8*b*e**
3*x**6*sqrt(-c**2*x**2 + 1)/(567*c**3) - 8*b*d**2*e*sqrt(-c**2*x**2 + 1)/(
25*c**5) - 24*b*d*e**2*x**2*sqrt(-c**2*x**2 + 1)/(245*c**5) - 16*b*e**3*x*
*4*sqrt(-c**2*x**2 + 1)/(945*c**5) - 48*b*d*e**2*sqrt(-c**2*x**2 + 1)/(245
*c**7) - 64*b*e**3*x**2*sqrt(-c**2*x**2 + 1)/(2835*c**7) - 128*b*e**3*sqrt
(-c**2*x**2 + 1)/(2835*c**9), Ne(c, 0)), ((a + pi*b/2)*(d**3*x**3/3 + 3*d*
*2*e*x**5/5 + 3*d*e**2*x**7/7 + e**3*x**9/9), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.35

$$\int x^2 (d+ex^2)^3 (a+b\arccos(cx))\,dx = \frac{1}{9}\,ae^3x^9 + \frac{3}{7}\,ade^2x^7 + \frac{3}{5}\,ad^2ex^5 + \frac{1}{3}\,ad^3x^3 + \frac{1}{9}\left(3\,x^3\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\,\sqrt{-c^2x^2+1}}{c^4}\right)\right)bd^3 + \frac{1}{25}\left(15\,x^5\arccos(cx) - \left(\frac{3\,\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\,\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bd^2e + \frac{3}{245}\left(35\,x^7\arccos(cx) - \left(\frac{5\,\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\,\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\,\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\,\sqrt{-c^2x^2+1}}{c^8}\right.\right. + \frac{1}{2835}\left(315\,x^9\arccos(cx) - \left(\frac{35\,\sqrt{-c^2x^2+1}x^8}{c^2} + \frac{40\,\sqrt{-c^2x^2+1}x^6}{c^4} + \frac{48\,\sqrt{-c^2x^2+1}x^4}{c^6} + \frac{64\,\sqrt{-c^2x}}{c^8}\right.\right.$$

input
```
integrate(x^2*(e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/9*a*e^3*x^9 + 3/7*a*d*e^2*x^7 + 3/5*a*d^2*e*x^5 + 1/3*a*d^3*x^3 + 1/9*(3
*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^
4))*b*d^3 + 1/25*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*s
qrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d^2*e + 3/245*(
35*x^7*arccos(c*x) - (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*
x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*d
*e^2 + 1/2835*(315*x^9*arccos(c*x) - (35*sqrt(-c^2*x^2 + 1)*x^8/c^2 + 40*s
qrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqrt(-c^2*x
^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/c^10)*c)*b*e^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.48

$$\int x^2(d+ex^2)^3(a+b\arccos(cx))\,dx = \frac{1}{9}\,be^3x^9\arccos{(cx)}+\frac{1}{9}\,ae^3x^9+\frac{3}{7}\,bde^2x^7\arccos{(cx)}-\frac{\sqrt{-c^2x^2+1}be^3x^8}{81\,c}+\frac{3}{7}\,ade^2x^7+\frac{3}{5}\,bd^2ex^5\arccos{(cx)}-\frac{3\,\sqrt{-c^2x^2+1}bde^2x^6}{49\,c}+\frac{3}{5}\,ad^2ex^5+\frac{1}{3}\,bd^3x^3\arccos{(cx)}-\frac{3\,\sqrt{-c^2x^2+1}bd^2ex^4}{25\,c}-\frac{8\,\sqrt{-c^2x^2+1}be^3x^6}{567\,c^3}+\frac{1}{3}\,ad^3x^3-\frac{\sqrt{-c^2x^2+1}bd^3x^2}{9\,c}-\frac{18\,\sqrt{-c^2x^2+1}bde^2x^4}{245\,c^3}-\frac{4\,\sqrt{-c^2x^2+1}bd^2ex^2}{25\,c^3}-\frac{16\,\sqrt{-c^2x^2+1}be^3x^4}{945\,c^5}-\frac{2\,\sqrt{-c^2x^2+1}bd^3}{9\,c^3}-\frac{24\,\sqrt{-c^2x^2+1}bde^2x^2}{245\,c^5}-\frac{8\,\sqrt{-c^2x^2+1}bd^2e}{25\,c^5}-\frac{64\,\sqrt{-c^2x^2+1}be^3x^2}{2835\,c^7}-\frac{48\,\sqrt{-c^2x^2+1}bde^2}{245\,c^7}-\frac{128\,\sqrt{-c^2x^2+1}be^3}{2835\,c^9}$$

input
```
integrate(x^2*(e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/9*b*e^3*x^9*arccos(c*x) + 1/9*a*e^3*x^9 + 3/7*b*d*e^2*x^7*arccos(c*x) -
1/81*sqrt(-c^2*x^2 + 1)*b*e^3*x^8/c + 3/7*a*d*e^2*x^7 + 3/5*b*d^2*e*x^5*ar
ccos(c*x) - 3/49*sqrt(-c^2*x^2 + 1)*b*d*e^2*x^6/c + 3/5*a*d^2*e*x^5 + 1/3*
b*d^3*x^3*arccos(c*x) - 3/25*sqrt(-c^2*x^2 + 1)*b*d^2*e*x^4/c - 8/567*sqrt
(-c^2*x^2 + 1)*b*e^3*x^6/c^3 + 1/3*a*d^3*x^3 - 1/9*sqrt(-c^2*x^2 + 1)*b*d^
3*x^2/c - 18/245*sqrt(-c^2*x^2 + 1)*b*d*e^2*x^4/c^3 - 4/25*sqrt(-c^2*x^2 +
 1)*b*d^2*e*x^2/c^3 - 16/945*sqrt(-c^2*x^2 + 1)*b*e^3*x^4/c^5 - 2/9*sqrt(-
c^2*x^2 + 1)*b*d^3/c^3 - 24/245*sqrt(-c^2*x^2 + 1)*b*d*e^2*x^2/c^5 - 8/25*
sqrt(-c^2*x^2 + 1)*b*d^2*e/c^5 - 64/2835*sqrt(-c^2*x^2 + 1)*b*e^3*x^2/c^7
- 48/245*sqrt(-c^2*x^2 + 1)*b*d*e^2/c^7 - 128/2835*sqrt(-c^2*x^2 + 1)*b*e^
3/c^9
```

Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)^3(a+b\arccos(cx))\,dx = \int x^2\,(a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int(x^2*(a + b*acos(c*x))*(d + e*x^2)^3,x)
```

output
```
int(x^2*(a + b*acos(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.52

$$\int x^2(d+ex^2)^3(a+b\arccos(cx))\,dx = \frac{33075acos(cx)\,b\,c^9d^3x^3+59535acos(cx)\,b\,c^9d^2e\,x^5+42525acos(cx)\,b\,c^9d\,e^2x^7+11025acos(cx)\,b\,c^9e^3x^9}{}$$

input
```
int(x^2*(e*x^2+d)^3*(a+b*acos(c*x)),x)
```

output
```
(33075*acos(c*x)*b*c**9*d**3*x**3 + 59535*acos(c*x)*b*c**9*d**2*e*x**5 + 4
2525*acos(c*x)*b*c**9*d*e**2*x**7 + 11025*acos(c*x)*b*c**9*e**3*x**9 - 110
25*sqrt( - c**2*x**2 + 1)*b*c**8*d**3*x**2 - 11907*sqrt( - c**2*x**2 + 1)*
b*c**8*d**2*e*x**4 - 6075*sqrt( - c**2*x**2 + 1)*b*c**8*d*e**2*x**6 - 1225
*sqrt( - c**2*x**2 + 1)*b*c**8*e**3*x**8 - 22050*sqrt( - c**2*x**2 + 1)*b*
c**6*d**3 - 15876*sqrt( - c**2*x**2 + 1)*b*c**6*d**2*e*x**2 - 7290*sqrt( -
 c**2*x**2 + 1)*b*c**6*d*e**2*x**4 - 1400*sqrt( - c**2*x**2 + 1)*b*c**6*e*
*3*x**6 - 31752*sqrt( - c**2*x**2 + 1)*b*c**4*d**2*e - 9720*sqrt( - c**2*x
**2 + 1)*b*c**4*d*e**2*x**2 - 1680*sqrt( - c**2*x**2 + 1)*b*c**4*e**3*x**4
 - 19440*sqrt( - c**2*x**2 + 1)*b*c**2*d*e**2 - 2240*sqrt( - c**2*x**2 + 1
)*b*c**2*e**3*x**2 - 4480*sqrt( - c**2*x**2 + 1)*b*e**3 + 33075*a*c**9*d**
3*x**3 + 59535*a*c**9*d**2*e*x**5 + 42525*a*c**9*d*e**2*x**7 + 11025*a*c**
9*e**3*x**9)/(99225*c**9)
```

3.619 $\int x(d+ex^2)^3 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 251

$$\begin{aligned}\int x\left(d+ex^2\right)^3(a+b\arccos(cx))\,dx &= \frac{b(256c^6d^3+288c^4d^2e+160c^2de^2+35e^3)\,x\sqrt{1-c^2x^2}}{1024c^7}\\ &+\frac{be(288c^4d^2+160c^2de+35e^2)\,x^3\sqrt{1-c^2x^2}}{1536c^5}\\ &+\frac{be^2(32c^2d+7e)\,x^5\sqrt{1-c^2x^2}}{384c^3}+\frac{be^3x^7\sqrt{1-c^2x^2}}{64c}\\ &-\frac{b(128c^8d^4+256c^6d^3e+288c^4d^2e^2+160c^2de^3+35e^4)\arccos(cx)}{1024c^8e}\\ &+\frac{\left(d+ex^2\right)^4(a+b\arccos(cx))}{8e}\end{aligned}$$

output
```
1/1024*b*(256*c^6*d^3+288*c^4*d^2*e+160*c^2*d*e^2+35*e^3)*x*(-c^2*x^2+1)^(
1/2)/c^7+1/1536*b*e*(288*c^4*d^2+160*c^2*d*e+35*e^2)*x^3*(-c^2*x^2+1)^(1/2
)/c^5+1/384*b*e^2*(32*c^2*d+7*e)*x^5*(-c^2*x^2+1)^(1/2)/c^3+1/64*b*e^3*x^7
*(-c^2*x^2+1)^(1/2)/c-1/1024*b*(128*c^8*d^4+256*c^6*d^3*e+288*c^4*d^2*e^2+
160*c^2*d*e^3+35*e^4)*arccos(c*x)/c^8/e+1/8*(e*x^2+d)^4*(a+b*arccos(c*x))/
e
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.95

$$\int x(d+ex^2)^3(a+b\arccos(cx))\,dx$$
$$= \frac{cx(384ac^7x(4d^3+6d^2ex^2+4de^2x^4+e^3x^6)-b\sqrt{1-c^2x^2}(105e^3+10c^2e^2(48d+7ex^2)+8c^4e(108d^2+4$$

input
```
Integrate[x*(d + e*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
(c*x*(384*a*c^7*x*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 + e^3*x^6) - b*Sqrt[1
 - c^2*x^2]*(105*e^3 + 10*c^2*e^2*(48*d + 7*e*x^2) + 8*c^4*e*(108*d^2 + 40
*d*e*x^2 + 7*e^2*x^4) + 16*c^6*(48*d^3 + 36*d^2*e*x^2 + 16*d*e^2*x^4 + 3*e
^3*x^6))) + 384*b*c^8*x^2*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 + e^3*x^6)*Ar
cCos[c*x] + 3*b*(256*c^6*d^3 + 288*c^4*d^2*e + 160*c^2*d*e^2 + 35*e^3)*Arc
Sin[c*x])/(3072*c^8)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5229, 318, 25, 403, 25, 403, 25, 299, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex^2)^3(a+b\arccos(cx))\,dx$$
$$\Big\downarrow \text{5229}$$
$$\frac{bc\int \frac{(ex^2+d)^4}{\sqrt{1-c^2x^2}}dx}{8e}+\frac{(d+ex^2)^4(a+b\arccos(cx))}{8e}$$
$$\Big\downarrow \text{318}$$

$$\frac{bc\left(-\frac{\int -\frac{\left(ex^2+d\right)^2\left(7e\left(2dc^2+e\right)x^2+d\left(8dc^2+e\right)\right)}{\sqrt{1-c^2x^2}}dx}{8c^2}-\frac{ex\sqrt{1-c^2x^2}\left(d+ex^2\right)^3}{8c^2}\right)}{8e}+\frac{\left(d+ex^2\right)^4\left(a+b\arccos(cx)\right)}{8e}$$

$$\Big\downarrow \ 25$$

$$\frac{bc\left(\frac{\int \frac{\left(ex^2+d\right)^2\left(7e\left(2dc^2+e\right)x^2+d\left(8dc^2+e\right)\right)}{\sqrt{1-c^2x^2}}dx}{8c^2}-\frac{ex\sqrt{1-c^2x^2}\left(d+ex^2\right)^3}{8c^2}\right)}{8e}+\frac{\left(d+ex^2\right)^4\left(a+b\arccos(cx)\right)}{8e}$$

$$\Big\downarrow \ 403$$

$$\frac{bc\left(\frac{-\frac{\int -\frac{\left(ex^2+d\right)\left(e\left(104d^2c^4+104dec^2+35e^2\right)x^2+d\left(48d^2c^4+20dec^2+7e^2\right)\right)}{\sqrt{1-c^2x^2}}dx}{6c^2}-\frac{7ex\sqrt{1-c^2x^2}\left(2c^2d+e\right)\left(d+ex^2\right)^2}{6c^2}}{8c^2}-\frac{ex\sqrt{1-c^2x^2}\left(d+ex^2\right)^3}{8c^2}\right)}{8e}+\frac{\left(d+ex^2\right)^4\left(a+b\arccos(cx)\right)}{8e}$$

$$\Big\downarrow \ 25$$

$$\frac{bc\left(\frac{\frac{\int \frac{\left(ex^2+d\right)\left(e\left(104d^2c^4+104dec^2+35e^2\right)x^2+d\left(48d^2c^4+20dec^2+7e^2\right)\right)}{\sqrt{1-c^2x^2}}dx}{6c^2}-\frac{7ex\sqrt{1-c^2x^2}\left(2c^2d+e\right)\left(d+ex^2\right)^2}{6c^2}}{8c^2}-\frac{ex\sqrt{1-c^2x^2}\left(d+ex^2\right)^3}{8c^2}\right)}{8e}+\frac{\left(d+ex^2\right)^4\left(a+b\arccos(cx)\right)}{8e}$$

$$\Big\downarrow \ 403$$

$$\frac{bc\left(\frac{\frac{-\frac{\int -\frac{5e\left(2dc^2+e\right)\left(40d^2c^4+40dec^2+21e^2\right)x^2+d\left(192d^3c^6+184d^2ec^4+132de^2c^2+35e^3\right)}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{ex\sqrt{1-c^2x^2}\left(104c^4d^2+104c^2de+35e^2\right)\left(d+ex^2\right)}{4c^2}}{6c^2}-\frac{7ex\sqrt{1-c\cdots}}{\cdots}}{8c^2}\cdots\right)}{8e}+\frac{\left(d+ex^2\right)^4\left(a+b\arccos(cx)\right)}{8e}$$

$$\Big\downarrow \ 25$$

$$\frac{bc\left(\frac{\frac{\frac{\int \frac{5e(2dc^2+e)(40d^2c^4+40dec^2+21e^2)x^2+d(192d^3c^6+184d^2ec^4+132de^2c^2+35e^3)}{\sqrt{1-c^2x^2}}dx}{4c^2}-\frac{ex\sqrt{1-c^2x^2}(104c^4d^2+104c^2de+35e^2)(d+ex^2)}{4c^2}}{6c^2}-\frac{7ex\sqrt{1-c^2x^2}}{}}{8c^2}\right.}{8e}$$

$$\frac{(d+ex^2)^4(a+b\arccos(cx))}{8e}$$

$$\Big\downarrow \text{299}$$

$$\frac{bc\left(\frac{\frac{\frac{\frac{3(128c^8d^4+256c^6d^3e+288c^4d^2e^2+160c^2de^3+35e^4)\int \frac{1}{\sqrt{1-c^2x^2}}dx}{2c^2}-\frac{5ex\sqrt{1-c^2x^2}(2c^2d+e)(40c^4d^2+40c^2de+21e^2)}{2c^2}}{4c^2}-\frac{ex\sqrt{1-c^2x^2}(104c^4d^2+104c^2de}{4c^2}}{6c^2}}{8c^2}\right.}{8e}$$

$$\frac{(d+ex^2)^4(a+b\arccos(cx))}{8e}$$

$$\Big\downarrow \text{223}$$

$$\frac{(d+ex^2)^4(a+b\arccos(cx))}{8e}+$$

$$\frac{bc\left(\frac{\frac{\frac{\frac{3\arcsin(cx)(128c^8d^4+256c^6d^3e+288c^4d^2e^2+160c^2de^3+35e^4)}{2c^3}-\frac{5ex\sqrt{1-c^2x^2}(2c^2d+e)(40c^4d^2+40c^2de+21e^2)}{2c^2}}{4c^2}-\frac{ex\sqrt{1-c^2x^2}(104c^4d^2+104c^2de+35e}{4c^2}}{6c^2}}{8c^2}\right.}{8e}$$

input
```
Int[x*(d + e*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
((d + e*x^2)^4*(a + b*ArcCos[c*x]))/(8*e) + (b*c*(-1/8*(e*x*Sqrt[1 - c^2*x
^2]*(d + e*x^2)^3)/c^2 + ((-7*e*(2*c^2*d + e)*x*Sqrt[1 - c^2*x^2]*(d + e*x
^2)^2)/(6*c^2) + (-1/4*(e*(104*c^4*d^2 + 104*c^2*d*e + 35*e^2)*x*Sqrt[1 -
c^2*x^2]*(d + e*x^2))/c^2 + ((-5*e*(2*c^2*d + e)*(40*c^4*d^2 + 40*c^2*d*e
+ 21*e^2)*x*Sqrt[1 - c^2*x^2])/(2*c^2) + (3*(128*c^8*d^4 + 256*c^6*d^3*e +
 288*c^4*d^2*e^2 + 160*c^2*d*e^3 + 35*e^4)*ArcSin[c*x])/(2*c^3))/(4*c^2))/
(6*c^2))/(8*c^2)))/(8*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5229
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])/(2*e*(p + 1))), x]
 + Simp[b*(c/(2*e*(p + 1)))   Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x]
, x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.45

method	result
parts	$\frac{a(e\,x^2+d)^4}{8e}+\frac{b\left(\frac{c^2 e^3 \arccos(cx)x^8}{8}+\frac{c^2 e^2 \arccos(cx)x^6 d}{2}+\frac{3c^2 e \arccos(cx)x^4 d^2}{4}+\frac{\arccos(cx)c^2 x^2 d^3}{2}+\frac{c^2 \arccos(cx)d^4}{8e}+\frac{c^8 d^4}{}\right.}{}$
derivativedivides	$\frac{a\left(c^2 e\,x^2+c^2 d\right)^4}{8c^6 e}+\frac{b\left(\frac{\arccos(cx)c^8 d^4}{8e}+\frac{\arccos(cx)c^8 d^3 x^2}{2}+\frac{3e\arccos(cx)c^8 d^2 x^4}{4}+\frac{e^2\arccos(cx)c^8 d\,x^6}{2}+\frac{e^3\arccos(cx)c^8 x^8}{8}+\frac{c^8 d^4}{}\right.}{}$
default	$\frac{a\left(c^2 e\,x^2+c^2 d\right)^4}{8c^6 e}+\frac{b\left(\frac{\arccos(cx)c^8 d^4}{8e}+\frac{\arccos(cx)c^8 d^3 x^2}{2}+\frac{3e\arccos(cx)c^8 d^2 x^4}{4}+\frac{e^2\arccos(cx)c^8 d\,x^6}{2}+\frac{e^3\arccos(cx)c^8 x^8}{8}+\frac{c^8 d^4}{}\right.}{}$
orering	$\frac{(720c^8e^4x^{10}+3760c^8d\,e^3x^8+8128c^8d^2e^2x^6+56c^6e^4x^8+9792c^8d^3e\,x^4+456c^6d\,e^3x^6+2304c^8d^4x^2+2080c^6d^2e^2x^4+98}{}$

input
```
int(x*(e*x^2+d)^3*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/8*a*(e*x^2+d)^4/e+b/c^2*(1/8*c^2*e^3*arccos(c*x)*x^8+1/2*c^2*e^2*arccos(
c*x)*x^6*d+3/4*c^2*e*arccos(c*x)*x^4*d^2+1/2*arccos(c*x)*c^2*x^2*d^3+1/8*c
^2/e*arccos(c*x)*d^4+1/8/c^6/e*(c^8*d^4*arcsin(c*x)+e^4*(-1/8*c^7*x^7*(-c^
2*x^2+1)^(1/2)-7/48*c^5*x^5*(-c^2*x^2+1)^(1/2)-35/192*c^3*x^3*(-c^2*x^2+1)
^(1/2)-35/128*c*x*(-c^2*x^2+1)^(1/2)+35/128*arcsin(c*x))+4*d*c^2*e^3*(-1/6
*c^5*x^5*(-c^2*x^2+1)^(1/2)-5/24*c^3*x^3*(-c^2*x^2+1)^(1/2)-5/16*c*x*(-c^2
*x^2+1)^(1/2)+5/16*arcsin(c*x))+6*d^2*c^4*e^2*(-1/4*c^3*x^3*(-c^2*x^2+1)^(
1/2)-3/8*c*x*(-c^2*x^2+1)^(1/2)+3/8*arcsin(c*x))+4*d^3*c^6*e*(-1/2*c*x*(-c
^2*x^2+1)^(1/2)+1/2*arcsin(c*x))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.10

$$\int x(d+ex^2)^3(a+b\arccos(cx))\,dx$$

$$=\frac{384\,ac^8e^3x^8+1536\,ac^8de^2x^6+2304\,ac^8d^2ex^4+1536\,ac^8d^3x^2+3\,(128\,bc^8e^3x^8+512\,bc^8de^2x^6+768\,bc^8d}{}$$

input
```
integrate(x*(e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/3072*(384*a*c^8*e^3*x^8 + 1536*a*c^8*d*e^2*x^6 + 2304*a*c^8*d^2*e*x^4 +
1536*a*c^8*d^3*x^2 + 3*(128*b*c^8*e^3*x^8 + 512*b*c^8*d*e^2*x^6 + 768*b*c^
8*d^2*e*x^4 + 512*b*c^8*d^3*x^2 - 256*b*c^6*d^3 - 288*b*c^4*d^2*e - 160*b*
c^2*d*e^2 - 35*b*e^3)*arccos(c*x) - (48*b*c^7*e^3*x^7 + 8*(32*b*c^7*d*e^2
+ 7*b*c^5*e^3)*x^5 + 2*(288*b*c^7*d^2*e + 160*b*c^5*d*e^2 + 35*b*c^3*e^3)*
x^3 + 3*(256*b*c^7*d^3 + 288*b*c^5*d^2*e + 160*b*c^3*d*e^2 + 35*b*c*e^3)*x
)*sqrt(-c^2*x^2 + 1))/c^8
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(238) = 476$.

Time = 1.00 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.94

$$\int x(d+ex^2)^3(a+b\arccos(cx))\,dx$$
$$= \begin{cases} \frac{ad^3x^2}{2} + \frac{3ad^2ex^4}{4} + \frac{ade^2x^6}{2} + \frac{ae^3x^8}{8} + \frac{bd^3x^2\operatorname{acos}(cx)}{2} + \frac{3bd^2ex^4\operatorname{acos}(cx)}{4} + \frac{bde^2x^6\operatorname{acos}(cx)}{2} + \frac{be^3x^8\operatorname{acos}(cx)}{8} - \frac{bd^3x\sqrt{-c^2}}{4c} \\ \left(a+\frac{\pi b}{2}\right)\left(\frac{d^3x^2}{2} + \frac{3d^2ex^4}{4} + \frac{de^2x^6}{2} + \frac{e^3x^8}{8}\right) \end{cases}$$

input
```
integrate(x*(e*x**2+d)**3*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d**3*x**2/2 + 3*a*d**2*e*x**4/4 + a*d*e**2*x**6/2 + a*e**3*x*
*8/8 + b*d**3*x**2*acos(c*x)/2 + 3*b*d**2*e*x**4*acos(c*x)/4 + b*d*e**2*x*
*6*acos(c*x)/2 + b*e**3*x**8*acos(c*x)/8 - b*d**3*x*sqrt(-c**2*x**2 + 1)/(
4*c) - 3*b*d**2*e*x**3*sqrt(-c**2*x**2 + 1)/(16*c) - b*d*e**2*x**5*sqrt(-c
**2*x**2 + 1)/(12*c) - b*e**3*x**7*sqrt(-c**2*x**2 + 1)/(64*c) - b*d**3*ac
os(c*x)/(4*c**2) - 9*b*d**2*e*x*sqrt(-c**2*x**2 + 1)/(32*c**3) - 5*b*d*e**
2*x**3*sqrt(-c**2*x**2 + 1)/(48*c**3) - 7*b*e**3*x**5*sqrt(-c**2*x**2 + 1)
/(384*c**3) - 9*b*d**2*e*acos(c*x)/(32*c**4) - 5*b*d*e**2*x*sqrt(-c**2*x**
2 + 1)/(32*c**5) - 35*b*e**3*x**3*sqrt(-c**2*x**2 + 1)/(1536*c**5) - 5*b*d
*e**2*acos(c*x)/(32*c**6) - 35*b*e**3*x*sqrt(-c**2*x**2 + 1)/(1024*c**7) -
 35*b*e**3*acos(c*x)/(1024*c**8), Ne(c, 0)), ((a + pi*b/2)*(d**3*x**2/2 +
3*d**2*e*x**4/4 + d*e**2*x**6/2 + e**3*x**8/8), True))
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.39

$$\int x(d+ex^2)^3(a+b\arccos(cx))\,dx = \frac{1}{8}ae^3x^8 + \frac{1}{2}ade^2x^6 + \frac{3}{4}ad^2ex^4 + \frac{1}{2}ad^3x^2 + \frac{1}{4}\left(2x^2\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x}{c^2} - \frac{\arcsin(cx)}{c^3}\right)\right)bd^3 + \frac{3}{32}\left(8x^4\arccos(cx) - \left(\frac{2\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\sqrt{-c^2x^2+1}x}{c^4} - \frac{3\arcsin(cx)}{c^5}\right)c\right)bd^2e + \frac{1}{96}\left(48x^6\arccos(cx) - \left(\frac{8\sqrt{-c^2x^2+1}x^5}{c^2} + \frac{10\sqrt{-c^2x^2+1}x^3}{c^4} + \frac{15\sqrt{-c^2x^2+1}x}{c^6} - \frac{15\arcsin(cx)}{c^7}\right. + \frac{1}{3072}\left(384x^8\arccos(cx) - \left(\frac{48\sqrt{-c^2x^2+1}x^7}{c^2} + \frac{56\sqrt{-c^2x^2+1}x^5}{c^4} + \frac{70\sqrt{-c^2x^2+1}x^3}{c^6} + \frac{105\sqrt{-c^2}}{c^8}\right.\right.$$

input
```
integrate(x*(e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e^3*x^8 + 1/2*a*d*e^2*x^6 + 3/4*a*d^2*e*x^4 + 1/2*a*d^3*x^2 + 1/4*(2
*x^2*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*x)/c^3))*b*d^3 +
 3/32*(8*x^4*arccos(c*x) - (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqrt(-c^2*x^2
 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b*d^2*e + 1/96*(48*x^6*arccos(c*x) - (
8*sqrt(-c^2*x^2 + 1)*x^5/c^2 + 10*sqrt(-c^2*x^2 + 1)*x^3/c^4 + 15*sqrt(-c^
2*x^2 + 1)*x/c^6 - 15*arcsin(c*x)/c^7)*c)*b*d*e^2 + 1/3072*(384*x^8*arccos
(c*x) - (48*sqrt(-c^2*x^2 + 1)*x^7/c^2 + 56*sqrt(-c^2*x^2 + 1)*x^5/c^4 + 7
0*sqrt(-c^2*x^2 + 1)*x^3/c^6 + 105*sqrt(-c^2*x^2 + 1)*x/c^8 - 105*arcsin(c
*x)/c^9)*c)*b*e^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.53

$$\int x(d+ex^2)^3(a+b\arccos(cx))\,dx = \frac{1}{8}be^3x^8\arccos(cx) + \frac{1}{8}ae^3x^8 + \frac{1}{2}bde^2x^6\arccos(cx) - \frac{\sqrt{-c^2x^2+1}be^3x^7}{64c} + \frac{1}{2}ade^2x^6 + \frac{3}{4}bd^2ex^4\arccos(cx) - \frac{\sqrt{-c^2x^2+1}bde^2x^5}{12c} + \frac{3}{4}ad^2ex^4 + \frac{1}{2}bd^3x^2\arccos(cx) - \frac{3\sqrt{-c^2x^2+1}bd^2ex^3}{16c} - \frac{7\sqrt{-c^2x^2+1}be^3x^5}{384c^3} + \frac{1}{2}ad^3x^2 - \frac{\sqrt{-c^2x^2+1}bd^3x}{4c} - \frac{5\sqrt{-c^2x^2+1}bde^2x^3}{48c^3} - \frac{bd^3\arccos(cx)}{4c^2} - \frac{9\sqrt{-c^2x^2+1}bd^2ex}{32c^3} - \frac{35\sqrt{-c^2x^2+1}be^3x^3}{1536c^5} - \frac{9bd^2e\arccos(cx)}{32c^4} - \frac{5\sqrt{-c^2x^2+1}bde^2x}{32c^5} - \frac{5bde^2\arccos(cx)}{32c^6} - \frac{35\sqrt{-c^2x^2+1}be^3x}{1024c^7} - \frac{35be^3\arccos(cx)}{1024c^8}$$

input
```
integrate(x*(e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/8*b*e^3*x^8*arccos(c*x) + 1/8*a*e^3*x^8 + 1/2*b*d*e^2*x^6*arccos(c*x) -
1/64*sqrt(-c^2*x^2 + 1)*b*e^3*x^7/c + 1/2*a*d*e^2*x^6 + 3/4*b*d^2*e*x^4*ar
ccos(c*x) - 1/12*sqrt(-c^2*x^2 + 1)*b*d*e^2*x^5/c + 3/4*a*d^2*e*x^4 + 1/2*
b*d^3*x^2*arccos(c*x) - 3/16*sqrt(-c^2*x^2 + 1)*b*d^2*e*x^3/c - 7/384*sqrt
(-c^2*x^2 + 1)*b*e^3*x^5/c^3 + 1/2*a*d^3*x^2 - 1/4*sqrt(-c^2*x^2 + 1)*b*d^
3*x/c - 5/48*sqrt(-c^2*x^2 + 1)*b*d*e^2*x^3/c^3 - 1/4*b*d^3*arccos(c*x)/c^
2 - 9/32*sqrt(-c^2*x^2 + 1)*b*d^2*e*x/c^3 - 35/1536*sqrt(-c^2*x^2 + 1)*b*e
^3*x^3/c^5 - 9/32*b*d^2*e*arccos(c*x)/c^4 - 5/32*sqrt(-c^2*x^2 + 1)*b*d*e^
2*x/c^5 - 5/32*b*d*e^2*arccos(c*x)/c^6 - 35/1024*sqrt(-c^2*x^2 + 1)*b*e^3*
x/c^7 - 35/1024*b*e^3*arccos(c*x)/c^8
```

Mupad [F(-1)]

Timed out.

$$\int x\left(d+ex^2\right)^3(a+b\arccos(cx))\,dx=\int x\left(a+b\,\mathrm{acos}(c\,x)\right)\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int(x*(a + b*acos(c*x))*(d + e*x^2)^3,x)
```

output
```
int(x*(a + b*acos(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.59

$$\int x\left(d+ex^2\right)^3(a+b\arccos(cx))\,dx$$
$$=\frac{1536acos(cx)\,b\,c^8d^3x^2+2304acos(cx)\,b\,c^8d^2e\,x^4+1536acos(cx)\,b\,c^8d\,e^2x^6+384acos(cx)\,b\,c^8e^3x^8+768}{}$$

input
```
int(x*(e*x^2+d)^3*(a+b*acos(c*x)),x)
```

output
```
(1536*acos(c*x)*b*c**8*d**3*x**2 + 2304*acos(c*x)*b*c**8*d**2*e*x**4 + 153
6*acos(c*x)*b*c**8*d*e**2*x**6 + 384*acos(c*x)*b*c**8*e**3*x**8 + 768*asin
(c*x)*b*c**6*d**3 + 864*asin(c*x)*b*c**4*d**2*e + 480*asin(c*x)*b*c**2*d*e
**2 + 105*asin(c*x)*b*e**3 - 768*sqrt( - c**2*x**2 + 1)*b*c**7*d**3*x - 57
6*sqrt( - c**2*x**2 + 1)*b*c**7*d**2*e*x**3 - 256*sqrt( - c**2*x**2 + 1)*b
*c**7*d*e**2*x**5 - 48*sqrt( - c**2*x**2 + 1)*b*c**7*e**3*x**7 - 864*sqrt(
 - c**2*x**2 + 1)*b*c**5*d**2*e*x - 320*sqrt( - c**2*x**2 + 1)*b*c**5*d*e*
*2*x**3 - 56*sqrt( - c**2*x**2 + 1)*b*c**5*e**3*x**5 - 480*sqrt( - c**2*x*
*2 + 1)*b*c**3*d*e**2*x - 70*sqrt( - c**2*x**2 + 1)*b*c**3*e**3*x**3 - 105
*sqrt( - c**2*x**2 + 1)*b*c*e**3*x + 1536*a*c**8*d**3*x**2 + 2304*a*c**8*d
**2*e*x**4 + 1536*a*c**8*d*e**2*x**6 + 384*a*c**8*e**3*x**8)/(3072*c**8)
```

3.620 $\int (d+ex^2)^3 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 225

$$\int (d+ex^2)^3 (a+b\arccos(cx))\,dx = \frac{b(35c^6d^3+35c^4d^2e+21c^2de^2+5e^3)\sqrt{1-c^2x^2}}{35c^7} - \frac{be(35c^4d^2+42c^2de+15e^2)(1-c^2x^2)^{3/2}}{105c^7} + \frac{3be^2(7c^2d+5e)(1-c^2x^2)^{5/2}}{175c^7} - \frac{be^3(1-c^2x^2)^{7/2}}{49c^7} + d^3x(a+b\arccos(cx)) + d^2ex^3(a+b\arccos(cx)) + \frac{3}{5}de^2x^5(a+b\arccos(cx)) + \frac{1}{7}e^3x^7(a+b\arccos(cx))$$

output

```
1/35*b*(35*c^6*d^3+35*c^4*d^2*e+21*c^2*d*e^2+5*e^3)*(-c^2*x^2+1)^(1/2)/c^7
-1/105*b*e*(35*c^4*d^2+42*c^2*d*e+15*e^2)*(-c^2*x^2+1)^(3/2)/c^7+3/175*b*e
^2*(7*c^2*d+5*e)*(-c^2*x^2+1)^(5/2)/c^7-1/49*b*e^3*(-c^2*x^2+1)^(7/2)/c^7+
d^3*x*(a+b*arccos(c*x))+d^2*e*x^3*(a+b*arccos(c*x))+3/5*d*e^2*x^5*(a+b*arc
cos(c*x))+1/7*e^3*x^7*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.84

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx = a\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)$$
$$-\frac{b\sqrt{1-c^2x^2}(240e^3+24c^2e^2(49d+5ex^2)+2c^4e(1225d^2+294dex^2+45e^2x^4)+c^6(3675d^3+1225d^2ex}{3675c^7}$$
$$+b\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)\arccos(cx)$$

input
```
Integrate[(d + e*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
a*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7) - (b*Sqrt[1 - c^2*x^
2]*(240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*c^4*e*(1225*d^2 + 294*d*e*x^
2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3*
x^6)))/(3675*c^7) + b*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7)*
ArcCos[c*x]
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5171, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5171}$$

$$bc\int \frac{x\left(5e^3x^6+21de^2x^4+35d^2ex^2+35d^3\right)}{35\sqrt{1-c^2x^2}}dx+d^3x(a+b\arccos(cx))+d^2ex^3(a+b\arccos(cx))+\frac{3}{5}de^2x^5(a+b\arccos(cx))+\frac{1}{7}e^3x^7(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{35}bc\int \frac{x(5e^3x^6+21de^2x^4+35d^2ex^2+35d^3)}{\sqrt{1-c^2x^2}}dx+d^3x(a+b\arccos(cx))+d^2ex^3(a+b\arccos(cx))+\frac{3}{5}de^2x^5(a+b\arccos(cx))+\frac{1}{7}e^3x^7(a+b\arccos(cx))$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{70}bc\int \frac{5e^3x^6+21de^2x^4+35d^2ex^2+35d^3}{\sqrt{1-c^2x^2}}dx^2+d^3x(a+b\arccos(cx))+d^2ex^3(a+b\arccos(cx))+\frac{3}{5}de^2x^5(a+b\arccos(cx))+\frac{1}{7}e^3x^7(a+b\arccos(cx))$$

$$\Big\downarrow \text{2389}$$

$$\frac{1}{70}bc\int \left(-\frac{5(1-c^2x^2)^{5/2}e^3}{c^6}+\frac{3(7dc^2+5e)(1-c^2x^2)^{3/2}e^2}{c^6}-\frac{(35d^2c^4+42dec^2+15e^2)\sqrt{1-c^2x^2}e}{c^6}+\frac{35d^3c^{6}}{}\right.$$
$$d^3x(a+b\arccos(cx))+d^2ex^3(a+b\arccos(cx))+\frac{3}{5}de^2x^5(a+b\arccos(cx))+\frac{1}{7}e^3x^7(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$d^3x(a+b\arccos(cx))+d^2ex^3(a+b\arccos(cx))+\frac{3}{5}de^2x^5(a+b\arccos(cx))+\frac{1}{7}e^3x^7(a+b\arccos(cx))+$$
$$\frac{1}{70}bc\left(-\frac{6e^2(1-c^2x^2)^{5/2}(7c^2d+5e)}{5c^8}+\frac{10e^3(1-c^2x^2)^{7/2}}{7c^8}+\frac{2e(1-c^2x^2)^{3/2}(35c^4d^2+42c^2de+15e^2)}{3c^8}-\frac{2\sqrt{1}}{}\right.$$

input
```
Int[(d + e*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*((-2*(35*c^6*d^3 + 35*c^4*d^2*e + 21*c^2*d*e^2 + 5*e^3)*Sqrt[1 - c^2*
x^2])/c^8 + (2*e*(35*c^4*d^2 + 42*c^2*d*e + 15*e^2)*(1 - c^2*x^2)^(3/2))/(
3*c^8) - (6*e^2*(7*c^2*d + 5*e)*(1 - c^2*x^2)^(5/2))/(5*c^8) + (10*e^3*(1
- c^2*x^2)^(7/2))/(7*c^8)))/70 + d^3*x*(a + b*ArcCos[c*x]) + d^2*e*x^3*(a
+ b*ArcCos[c*x]) + (3*d*e^2*x^5*(a + b*ArcCos[c*x]))/5 + (e^3*x^7*(a + b*A
rcCos[c*x]))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5171
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2
, 0])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.36

method	result
parts	$a\left(\frac{1}{7}e^3x^7+\frac{3}{5}d\,e^2x^5+d^2e\,x^3+d^3x\right)+\frac{b\left(\frac{c\arccos(cx)e^3x^7}{7}+\frac{3c\arccos(cx)d\,e^2x^5}{5}+c\arccos(cx)d^2e\,x^3+\arccos\right.}{}$
derivativedivides	$\frac{\frac{a\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\left(\arccos(cx)d^3c^7x+\arccos(cx)d^2c^7e\,x^3+\frac{3\arccos(cx)d\,c^7e^2x^5}{5}+\frac{\arccos(cx)e^3c^7x^7}{7}\right.}{}}{}$
default	$\frac{\frac{a\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\left(\arccos(cx)d^3c^7x+\arccos(cx)d^2c^7e\,x^3+\frac{3\arccos(cx)d\,c^7e^2x^5}{5}+\frac{\arccos(cx)e^3c^7x^7}{7}\right.}{}}{}$
orering	$\frac{x\left(325c^8e^4x^8+1792c^8d\,e^3x^6+4410c^8d^2e^2x^4+30c^6e^4x^6+9800c^8d^3e\,x^2+294c^6d\,e^3x^4+1225c^8d^4+2450c^6d^2e^2x^2+60c^4\right.}{1225(e\,x^2+d)c^8}$

input
```
int((e*x^2+d)^3*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e^3*x^7+3/5*d*e^2*x^5+d^2*e*x^3+d^3*x)+b/c*(1/7*c*arccos(c*x)*e^3*x
^7+3/5*c*arccos(c*x)*d*e^2*x^5+c*arccos(c*x)*d^2*e*x^3+arccos(c*x)*d^3*c*x
+1/35/c^6*(5*e^3*(-1/7*c^6*x^6*(-c^2*x^2+1)^(1/2)-6/35*c^4*x^4*(-c^2*x^2+1
)^(1/2)-8/35*c^2*x^2*(-c^2*x^2+1)^(1/2)-16/35*(-c^2*x^2+1)^(1/2))-35*d^3*c
^6*(-c^2*x^2+1)^(1/2)+21*d*c^2*e^2*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2)-4/15*c
^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))+35*d^2*c^4*e*(-1/3*c^2*
x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.02

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx$$
$$=\frac{525\,ac^7e^3x^7+2205\,ac^7de^2x^5+3675\,ac^7d^2ex^3+3675\,ac^7d^3x+105\,(5\,bc^7e^3x^7+21\,bc^7de^2x^5+35\,bc^7d^2ex}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/3675*(525*a*c^7*e^3*x^7 + 2205*a*c^7*d*e^2*x^5 + 3675*a*c^7*d^2*e*x^3 +
3675*a*c^7*d^3*x + 105*(5*b*c^7*e^3*x^7 + 21*b*c^7*d*e^2*x^5 + 35*b*c^7*d^
2*e*x^3 + 35*b*c^7*d^3*x)*arccos(c*x) - (75*b*c^6*e^3*x^6 + 3675*b*c^6*d^3
 + 2450*b*c^4*d^2*e + 1176*b*c^2*d*e^2 + 9*(49*b*c^6*d*e^2 + 10*b*c^4*e^3)
*x^4 + 240*b*e^3 + (1225*b*c^6*d^2*e + 588*b*c^4*d*e^2 + 120*b*c^2*e^3)*x^
2)*sqrt(-c^2*x^2 + 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.75

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx$$
$$= \begin{cases} ad^3x + ad^2ex^3 + \frac{3ade^2x^5}{5} + \frac{ae^3x^7}{7} + bd^3x\operatorname{acos}(cx) + bd^2ex^3\operatorname{acos}(cx) + \frac{3bde^2x^5\operatorname{acos}(cx)}{5} + \frac{be^3x^7\operatorname{acos}(cx)}{7} - b \\ \left(a+\frac{\pi b}{2}\right)\left(d^3x + d^2ex^3 + \frac{3de^2x^5}{5} + \frac{e^3x^7}{7}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**3*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 +
b*d**3*x*acos(c*x) + b*d**2*e*x**3*acos(c*x) + 3*b*d*e**2*x**5*acos(c*x)/5
 + b*e**3*x**7*acos(c*x)/7 - b*d**3*sqrt(-c**2*x**2 + 1)/c - b*d**2*e*x**2
*sqrt(-c**2*x**2 + 1)/(3*c) - 3*b*d*e**2*x**4*sqrt(-c**2*x**2 + 1)/(25*c)
- b*e**3*x**6*sqrt(-c**2*x**2 + 1)/(49*c) - 2*b*d**2*e*sqrt(-c**2*x**2 + 1
)/(3*c**3) - 4*b*d*e**2*x**2*sqrt(-c**2*x**2 + 1)/(25*c**3) - 6*b*e**3*x**
4*sqrt(-c**2*x**2 + 1)/(245*c**3) - 8*b*d*e**2*sqrt(-c**2*x**2 + 1)/(25*c*
*5) - 8*b*e**3*x**2*sqrt(-c**2*x**2 + 1)/(245*c**5) - 16*b*e**3*sqrt(-c**2
*x**2 + 1)/(245*c**7), Ne(c, 0)), ((a + pi*b/2)*(d**3*x + d**2*e*x**3 + 3*
d*e**2*x**5/5 + e**3*x**7/7), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.32

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx = \frac{1}{7}\,ae^3x^7 + \frac{3}{5}\,ade^2x^5 + ad^2ex^3 + \frac{1}{3}\left(3\,x^3\arccos\left(cx\right) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\,\sqrt{-c^2x^2+1}}{c^4}\right)\right)bd^2e + \frac{1}{25}\left(15\,x^5\arccos\left(cx\right) - \left(\frac{3\,\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\,\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\,\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bde^2 + \frac{1}{245}\left(35\,x^7\arccos\left(cx\right) - \left(\frac{5\,\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\,\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\,\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\,\sqrt{-c^2x^2+1}}{c^8}\right. + ad^3x + \frac{\left(cx\arccos\left(cx\right) - \sqrt{-c^2x^2+1}\right)bd^3}{c}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^3*x^7 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + 1/3*(3*x^3*arccos(c*x) - c
*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d^2*e + 1/25*(
15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*
x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d*e^2 + 1/245*(35*x^7*arccos(c*x)
 - (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-
c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8)*c)*b*e^3 + a*d^3*x + (c*
x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b*d^3/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.41

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx = \frac{1}{7}\,be^3x^7\arccos\left(cx\right)+\frac{1}{7}\,ae^3x^7+\frac{3}{5}\,bde^2x^5\arccos\left(cx\right) - \frac{\sqrt{-c^2x^2+1}be^3x^6}{49\,c}+\frac{3}{5}\,ade^2x^5 + bd^2ex^3\arccos\left(cx\right)-\frac{3\,\sqrt{-c^2x^2+1}bde^2x^4}{25\,c} + ad^2ex^3+bd^3x\arccos\left(cx\right)-\frac{\sqrt{-c^2x^2+1}bd^2ex^2}{3\,c} - \frac{6\,\sqrt{-c^2x^2+1}be^3x^4}{245\,c^3}+ad^3x - \frac{\sqrt{-c^2x^2+1}bd^3}{c}-\frac{4\,\sqrt{-c^2x^2+1}bde^2x^2}{25\,c^3} - \frac{2\,\sqrt{-c^2x^2+1}bd^2e}{3\,c^3}-\frac{8\,\sqrt{-c^2x^2+1}be^3x^2}{245\,c^5} - \frac{8\,\sqrt{-c^2x^2+1}bde^2}{25\,c^5}-\frac{16\,\sqrt{-c^2x^2+1}be^3}{245\,c^7}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/7*b*e^3*x^7*arccos(c*x) + 1/7*a*e^3*x^7 + 3/5*b*d*e^2*x^5*arccos(c*x) -
1/49*sqrt(-c^2*x^2 + 1)*b*e^3*x^6/c + 3/5*a*d*e^2*x^5 + b*d^2*e*x^3*arccos
(c*x) - 3/25*sqrt(-c^2*x^2 + 1)*b*d*e^2*x^4/c + a*d^2*e*x^3 + b*d^3*x*arcc
os(c*x) - 1/3*sqrt(-c^2*x^2 + 1)*b*d^2*e*x^2/c - 6/245*sqrt(-c^2*x^2 + 1)*
b*e^3*x^4/c^3 + a*d^3*x - sqrt(-c^2*x^2 + 1)*b*d^3/c - 4/25*sqrt(-c^2*x^2
+ 1)*b*d*e^2*x^2/c^3 - 2/3*sqrt(-c^2*x^2 + 1)*b*d^2*e/c^3 - 8/245*sqrt(-c^
2*x^2 + 1)*b*e^3*x^2/c^5 - 8/25*sqrt(-c^2*x^2 + 1)*b*d*e^2/c^5 - 16/245*sq
rt(-c^2*x^2 + 1)*b*e^3/c^7
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx = \int (a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int((a + b*acos(c*x))*(d + e*x^2)^3,x)
```

output
```
int((a + b*acos(c*x))*(d + e*x^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.50

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx$$
$$= \frac{3675 acos(cx)\,b\,c^7d^3x + 3675 acos(cx)\,b\,c^7d^2e\,x^3 + 2205 acos(cx)\,b\,c^7d\,e^2x^5 + 525 acos(cx)\,b\,c^7e^3x^7 - 3675}{}$$

input
```
int((e*x^2+d)^3*(a+b*acos(c*x)),x)
```

output
```
(3675*acos(c*x)*b*c**7*d**3*x + 3675*acos(c*x)*b*c**7*d**2*e*x**3 + 2205*a
cos(c*x)*b*c**7*d*e**2*x**5 + 525*acos(c*x)*b*c**7*e**3*x**7 - 3675*sqrt(
- c**2*x**2 + 1)*b*c**6*d**3 - 1225*sqrt( - c**2*x**2 + 1)*b*c**6*d**2*e*x
**2 - 441*sqrt( - c**2*x**2 + 1)*b*c**6*d*e**2*x**4 - 75*sqrt( - c**2*x**2
 + 1)*b*c**6*e**3*x**6 - 2450*sqrt( - c**2*x**2 + 1)*b*c**4*d**2*e - 588*s
qrt( - c**2*x**2 + 1)*b*c**4*d*e**2*x**2 - 90*sqrt( - c**2*x**2 + 1)*b*c**
4*e**3*x**4 - 1176*sqrt( - c**2*x**2 + 1)*b*c**2*d*e**2 - 120*sqrt( - c**2
*x**2 + 1)*b*c**2*e**3*x**2 - 240*sqrt( - c**2*x**2 + 1)*b*e**3 + 3675*a*c
**7*d**3*x + 3675*a*c**7*d**2*e*x**3 + 2205*a*c**7*d*e**2*x**5 + 525*a*c**
7*e**3*x**7)/(3675*c**7)
```

3.621 $\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 357

$$
\begin{aligned}
\int \frac{(d+ex^2)^3\,(a+b\arccos(cx))}{x}\,dx &= \frac{3bd^2ex\sqrt{1-c^2x^2}}{4c}+\frac{9bde^2x\sqrt{1-c^2x^2}}{32c^3} \\
&+\frac{5be^3x\sqrt{1-c^2x^2}}{96c^5}+\frac{3bde^2x^3\sqrt{1-c^2x^2}}{16c} \\
&+\frac{5be^3x^3\sqrt{1-c^2x^2}}{144c^3}+\frac{be^3x^5\sqrt{1-c^2x^2}}{36c} \\
&-\frac{3bd^2e\arccos(cx)}{4c^2}-\frac{9bde^2\arccos(cx)}{32c^4} \\
&-\frac{5be^3\arccos(cx)}{96c^6}-\frac{1}{2}ibd^3\arccos(cx)^2 \\
&+\frac{3}{2}d^2ex^2(a+b\arccos(cx)) \\
&+\frac{3}{4}de^2x^4(a+b\arccos(cx)) \\
&+\frac{1}{6}e^3x^6(a+b\arccos(cx)) \\
&+bd^3\arccos(cx)\log\left(1-e^{2i\arccos(cx)}\right) \\
&-bd^3\arccos(cx)\log(x)+d^3(a+b\arccos(cx))\log(x) \\
&-\frac{1}{2}ibd^3\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)
\end{aligned}
$$

output
```
3/4*b*d^2*e*x*(-c^2*x^2+1)^(1/2)/c+9/32*b*d*e^2*x*(-c^2*x^2+1)^(1/2)/c^3+5
/96*b*e^3*x*(-c^2*x^2+1)^(1/2)/c^5+3/16*b*d*e^2*x^3*(-c^2*x^2+1)^(1/2)/c+5
/144*b*e^3*x^3*(-c^2*x^2+1)^(1/2)/c^3+1/36*b*e^3*x^5*(-c^2*x^2+1)^(1/2)/c-
3/4*b*d^2*e*arccos(c*x)/c^2-9/32*b*d*e^2*arccos(c*x)/c^4-5/96*b*e^3*arccos
(c*x)/c^6-1/2*I*b*d^3*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3/2*d^2*e*x^
2*(a+b*arccos(c*x))+3/4*d*e^2*x^4*(a+b*arccos(c*x))+1/6*e^3*x^6*(a+b*arcco
s(c*x))+b*d^3*arccos(c*x)*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-b*d^3*arccos(
c*x)*ln(x)+d^3*(a+b*arccos(c*x))*ln(x)-1/2*I*b*d^3*arccos(c*x)^2
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x}\,dx$$

$$= \frac{1}{12}\left(18ad^2ex^2+9ade^2x^4+2ae^3x^6+18bd^2ex^2\arccos(cx)+9bde^2x^4\arccos(cx)+2be^3x^6\arccos(cx) - \frac{be^3\left(cx\sqrt{1-c^2x^2}(15+10c^2x^2+8c^4x^4)-30\arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right)\right)}{24c^6} - \frac{9bde^2\left(cx\sqrt{1-c^2x^2}(3+2c^2x^2)-6\arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right)\right)}{8c^4} - \frac{9bd^2e\left(cx\sqrt{1-c^2x^2}-2\arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right)\right)}{c^2} + 12ad^3\log(x) - 6ibd^3\left(\arccos(cx)\left(\arccos(cx)+2i\log\left(1+e^{2i\arccos(cx)}\right)\right)+\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\right)\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcCos[c*x]))/x,x]
```

output
```
(18*a*d^2*e*x^2 + 9*a*d*e^2*x^4 + 2*a*e^3*x^6 + 18*b*d^2*e*x^2*ArcCos[c*x]
 + 9*b*d*e^2*x^4*ArcCos[c*x] + 2*b*e^3*x^6*ArcCos[c*x] - (b*e^3*(c*x*Sqrt[
1 - c^2*x^2]*(15 + 10*c^2*x^2 + 8*c^4*x^4) - 30*ArcTan[(c*x)/(-1 + Sqrt[1
- c^2*x^2])]))/(24*c^6) - (9*b*d*e^2*(c*x*Sqrt[1 - c^2*x^2]*(3 + 2*c^2*x^2
) - 6*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]))/(8*c^4) - (9*b*d^2*e*(c*x*S
qrt[1 - c^2*x^2] - 2*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]))/c^2 + 12*a*d
^3*Log[x] - (6*I)*b*d^3*(ArcCos[c*x]*(ArcCos[c*x] + (2*I)*Log[1 + E^((2*I)
*ArcCos[c*x])]) + PolyLog[2, -E^((2*I)*ArcCos[c*x])]))/12
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5231, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x} dx$$

$$\Big\downarrow \text{5231}$$

$$bc\int \frac{2e^3x^6+9de^2x^4+18d^2ex^2+12d^3\log(x)}{12\sqrt{1-c^2x^2}}dx+d^3\log(x)(a+b\arccos(cx))+\frac{3}{2}d^2ex^2(a+b\arccos(cx))+\frac{3}{4}de^2x^4(a+b\arccos(cx))+\frac{1}{6}e^3x^6(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}bc\int \frac{2e^3x^6+9de^2x^4+18d^2ex^2+12d^3\log(x)}{\sqrt{1-c^2x^2}}dx+d^3\log(x)(a+b\arccos(cx))+\frac{3}{2}d^2ex^2(a+b\arccos(cx))+\frac{3}{4}de^2x^4(a+b\arccos(cx))+\frac{1}{6}e^3x^6(a+b\arccos(cx))$$

$$\Big\downarrow \text{7293}$$

$$\frac{1}{12}bc\int \left(\frac{2e^3x^6}{\sqrt{1-c^2x^2}}+\frac{9de^2x^4}{\sqrt{1-c^2x^2}}+\frac{18d^2ex^2}{\sqrt{1-c^2x^2}}+\frac{12d^3\log(x)}{\sqrt{1-c^2x^2}}\right)dx+d^3\log(x)(a+b\arccos(cx))+\frac{3}{2}d^2ex^2(a+b\arccos(cx))+\frac{3}{4}de^2x^4(a+b\arccos(cx))+\frac{1}{6}e^3x^6(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$d^3\log(x)(a+b\arccos(cx))+\frac{3}{2}d^2ex^2(a+b\arccos(cx))+\frac{3}{4}de^2x^4(a+b\arccos(cx))+\frac{1}{6}e^3x^6(a+b\arccos(cx))+\frac{1}{12}bc\left(\frac{5e^3\arcsin(cx)}{8c^7}+\frac{27de^2\arcsin(cx)}{8c^5}+\frac{9d^2e\arcsin(cx)}{c^3}+\frac{6id^3\operatorname{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{c}+\frac{6id^3\arcsin(cx)^2}{c}\right.$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcCos[c*x]))/x,x]
```

output
```
(3*d^2*e*x^2*(a + b*ArcCos[c*x]))/2 + (3*d*e^2*x^4*(a + b*ArcCos[c*x]))/4
+ (e^3*x^6*(a + b*ArcCos[c*x]))/6 + d^3*(a + b*ArcCos[c*x])*Log[x] + (b*c*
((-9*d^2*e*x*Sqrt[1 - c^2*x^2])/c^2 - (27*d*e^2*x*Sqrt[1 - c^2*x^2])/(8*c^
4) - (5*e^3*x*Sqrt[1 - c^2*x^2])/(8*c^6) - (9*d*e^2*x^3*Sqrt[1 - c^2*x^2])
/(4*c^2) - (5*e^3*x^3*Sqrt[1 - c^2*x^2])/(12*c^4) - (e^3*x^5*Sqrt[1 - c^2*
x^2])/(3*c^2) + (9*d^2*e*ArcSin[c*x])/c^3 + (27*d*e^2*ArcSin[c*x])/(8*c^5)
 + (5*e^3*ArcSin[c*x])/(8*c^7) + ((6*I)*d^3*ArcSin[c*x]^2)/c - (12*d^3*Arc
Sin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/c + (12*d^3*ArcSin[c*x]*Log[x])/c
 + ((6*I)*d^3*PolyLog[2, E^((2*I)*ArcSin[c*x])])/c))/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.08

method	result
parts	$a\left(\frac{e^{3}x^{6}}{6}+\frac{3d\,e^{2}x^{4}}{4}+\frac{3d^{2}e\,x^{2}}{2}+d^{3}\ln\left(x\right)\right)+b\left(-\frac{i d^{3}\arccos(cx)^{2}}{2}+\frac{e(48ic^{4}d^{2}+96c^{4}d^{2}\arccos(cx)+24ic^{2}}{}\right.$
derivativedivides	$\frac{a\left(\frac{3c^{6}d^{2}e\,x^{2}}{2}+\frac{3c^{6}d\,e^{2}x^{4}}{4}+\frac{e^{3}x^{6}c^{6}}{6}+c^{6}d^{3}\ln(cx)\right)}{c^{6}}+\frac{b\left(-\frac{ic^{6}d^{3}\arccos(cx)^{2}}{2}+\frac{e\left(48ic^{4}d^{2}+96c^{4}d^{2}\arccos(cx)+24ic^{2}de+48c^{2}d\right.}{}\right.}{}$
default	$\frac{a\left(\frac{3c^{6}d^{2}e\,x^{2}}{2}+\frac{3c^{6}d\,e^{2}x^{4}}{4}+\frac{e^{3}x^{6}c^{6}}{6}+c^{6}d^{3}\ln(cx)\right)}{c^{6}}+\frac{b\left(-\frac{ic^{6}d^{3}\arccos(cx)^{2}}{2}+\frac{e\left(48ic^{4}d^{2}+96c^{4}d^{2}\arccos(cx)+24ic^{2}de+48c^{2}d\right.}{}\right.}{}$

input
```
int((e*x^2+d)^3*(a+b*arccos(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*(1/6*e^3*x^6+3/4*d*e^2*x^4+3/2*d^2*e*x^2+d^3*ln(x))+b*(-1/2*I*d^3*arccos
(c*x)^2+1/256/c^6*e*(48*I*c^4*d^2+96*c^4*d^2*arccos(c*x)+24*I*c^2*d*e+48*c
^2*d*e*arccos(c*x)+5*I*e^2+10*e^2*arccos(c*x))*(2*c^2*x^2-1+2*I*(-c^2*x^2+
1)^(1/2)*c*x)+1/256*(-2*I*(-c^2*x^2+1)^(1/2)*c*x+2*c^2*x^2-1)*e*(-48*I*c^4
*d^2+96*c^4*d^2*arccos(c*x)-24*I*c^2*d*e+48*c^2*d*e*arccos(c*x)-5*I*e^2+10
*e^2*arccos(c*x))/c^6+d^3*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1
/2*I*d^3*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/192*arccos(c*x)*e^3/c^
6*cos(6*arccos(c*x))-1/1152*e^3/c^6*sin(6*arccos(c*x))+1/32*arccos(c*x)*e^
2*(3*c^2*d+e)/c^6*cos(4*arccos(c*x))-3/128*e^2/c^4*sin(4*arccos(c*x))*d-1/
128*e^3/c^6*sin(4*arccos(c*x)))
```

Fricas [F]

$$\int\frac{\left(d+ex^{2}\right)^{3}\left(a+b\arccos(cx)\right)}{x}\,dx=\int\frac{\left(ex^{2}+d\right)^{3}\left(b\arccos\left(cx\right)+a\right)}{x}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccos(c*x))/x, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x}\,dx = \int \frac{(a+b\operatorname{acos}(cx))(d+ex^2)^3}{x}\,dx$$

input
```
integrate((e*x**2+d)**3*(a+b*acos(c*x))/x,x)
```

output
```
Integral((a + b*acos(c*x))*(d + e*x**2)**3/x, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x}\,dx = \int \frac{(ex^2+d)^3 (b\arccos(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))/x,x, algorithm="maxima")
```

output
```
1/6*a*e^3*x^6 + 3/4*a*d*e^2*x^4 + 3/2*a*d^2*e*x^2 + a*d^3*log(x) + integra
te((b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan2(sqrt(c*x +
1)*sqrt(-c*x + 1), c*x)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(e\,x^2+d)^3}{x}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + e*x^2)^3)/x,x)
```

output
```
int(((a + b*acos(c*x))*(d + e*x^2)^3)/x, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x}\,dx = \frac{432acos(cx)\,b\,c^6d^2e\,x^2+216acos(cx)\,b\,c^6d\,e^2x^4+48acos(cx)\,b\,c^6e^3x^6+216asin(cx)\,b\,c^4d^2e+81asin(cx}{}$$

input
```
int((e*x^2+d)^3*(a+b*acos(c*x))/x,x)
```

output
```
(432*acos(c*x)*b*c**6*d**2*e*x**2 + 216*acos(c*x)*b*c**6*d*e**2*x**4 + 48*
acos(c*x)*b*c**6*e**3*x**6 + 216*asin(c*x)*b*c**4*d**2*e + 81*asin(c*x)*b*
c**2*d*e**2 + 15*asin(c*x)*b*e**3 - 216*sqrt( - c**2*x**2 + 1)*b*c**5*d**2
*e*x - 54*sqrt( - c**2*x**2 + 1)*b*c**5*d*e**2*x**3 - 8*sqrt( - c**2*x**2
+ 1)*b*c**5*e**3*x**5 - 81*sqrt( - c**2*x**2 + 1)*b*c**3*d*e**2*x - 10*sqr
t( - c**2*x**2 + 1)*b*c**3*e**3*x**3 - 15*sqrt( - c**2*x**2 + 1)*b*c*e**3*
x + 288*int(acos(c*x)/x,x)*b*c**6*d**3 + 288*log(x)*a*c**6*d**3 + 432*a*c*
*6*d**2*e*x**2 + 216*a*c**6*d*e**2*x**4 + 48*a*c**6*e**3*x**6)/(288*c**6)
```

3.622 $\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 190

$$\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x^2}\,dx = \frac{be(15c^4d^2+5c^2de+e^2)\sqrt{1-c^2x^2}}{5c^5} - \frac{be^2(5c^2d+2e)(1-c^2x^2)^{3/2}}{15c^5} + \frac{be^3(1-c^2x^2)^{5/2}}{25c^5} - \frac{d^3(a+b\arccos(cx))}{x} + 3d^2ex(a+b\arccos(cx)) + de^2x^3(a+b\arccos(cx)) + \frac{1}{5}e^3x^5(a+b\arccos(cx)) - bcd^3\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)$$

output
```
1/5*b*e*(15*c^4*d^2+5*c^2*d*e+e^2)*(-c^2*x^2+1)^(1/2)/c^5-1/15*b*e^2*(5*c^
2*d+2*e)*(-c^2*x^2+1)^(3/2)/c^5+1/25*b*e^3*(-c^2*x^2+1)^(5/2)/c^5-d^3*(a+b
*arccos(c*x))/x+3*d^2*e*x*(a+b*arccos(c*x))+d*e^2*x^3*(a+b*arccos(c*x))+1/
5*e^3*x^5*(a+b*arccos(c*x))-b*c*d^3*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.96

$$\begin{aligned}&\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x^2}\,dx\\&=-\frac{ad^3}{x}+3ad^2ex+ade^2x^3+\frac{1}{5}ae^3x^5\\&\quad-\frac{be\sqrt{1-c^2x^2}(8e^2+2c^2e(25d+2ex^2)+c^4(225d^2+25dex^2+3e^2x^4))}{75c^5}\\&\quad+\frac{b(-5d^3+15d^2ex^2+5de^2x^4+e^3x^6)\arccos(cx)}{5x}\\&\quad-bcd^3\log(x)+bcd^3\log\left(1+\sqrt{1-c^2x^2}\right)\end{aligned}$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcCos[c*x]))/x^2,x]
```

output
```
-((a*d^3)/x) + 3*a*d^2*e*x + a*d*e^2*x^3 + (a*e^3*x^5)/5 - (b*e*Sqrt[1 - c
^2*x^2]*(8*e^2 + 2*c^2*e*(25*d + 2*e*x^2) + c^4*(225*d^2 + 25*d*e*x^2 + 3*
e^2*x^4)))/(75*c^5) + (b*(-5*d^3 + 15*d^2*e*x^2 + 5*d*e^2*x^4 + e^3*x^6)*A
rcCos[c*x])/(5*x) - b*c*d^3*Log[x] + b*c*d^3*Log[1 + Sqrt[1 - c^2*x^2]]
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5231, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x^2}\,dx$$

$$\Big\downarrow \text{5231}$$

$$bc\int -\frac{-e^3x^6-5de^2x^4-15d^2ex^2+5d^3}{5x\sqrt{1-c^2x^2}}dx-\frac{d^3(a+b\arccos(cx))}{x}+3d^2ex(a+b\arccos(cx))+de^2x^3(a+b\arccos(cx))+\frac{1}{5}e^3x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}bc\int \frac{-e^3x^6-5de^2x^4-15d^2ex^2+5d^3}{x\sqrt{1-c^2x^2}}dx-\frac{d^3(a+b\arccos(cx))}{x}+3d^2ex(a+b\arccos(cx))+de^2x^3(a+b\arccos(cx))+\frac{1}{5}e^3x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{2331}$$

$$-\frac{1}{10}bc\int \frac{-e^3x^6-5de^2x^4-15d^2ex^2+5d^3}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{d^3(a+b\arccos(cx))}{x}+3d^2ex(a+b\arccos(cx))+de^2x^3(a+b\arccos(cx))+\frac{1}{5}e^3x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{2123}$$

$$-\frac{1}{10}bc\int \left(\frac{5d^3}{x^2\sqrt{1-c^2x^2}}-\frac{e^3(1-c^2x^2)^{3/2}}{c^4}+\frac{e^2(5dc^2+2e)\sqrt{1-c^2x^2}}{c^4}-\frac{e(15d^2c^4+5dec^2+e^2)}{c^4\sqrt{1-c^2x^2}}\right)dx^2-\frac{d^3(a+b\arccos(cx))}{x}+3d^2ex(a+b\arccos(cx))+de^2x^3(a+b\arccos(cx))+\frac{1}{5}e^3x^5(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\arccos(cx))}{x}+3d^2ex(a+b\arccos(cx))+de^2x^3(a+b\arccos(cx))+\frac{1}{5}e^3x^5(a+b\arccos(cx))-\frac{1}{10}bc\left(-10d^3\text{arctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{2e^2(1-c^2x^2)^{3/2}(5c^2d+2e)}{3c^6}+\frac{2e^3(1-c^2x^2)^{5/2}}{5c^6}+\frac{2e\sqrt{1-c^2x^2}(15c^4d^2+\right.$$

input

```
Int[((d + e*x^2)^3*(a + b*ArcCos[c*x]))/x^2,x]
```

output

```
-((d^3*(a + b*ArcCos[c*x]))/x) + 3*d^2*e*x*(a + b*ArcCos[c*x]) + d*e^2*x^3
*(a + b*ArcCos[c*x]) + (e^3*x^5*(a + b*ArcCos[c*x]))/5 - (b*c*((2*e*(15*c^
4*d^2 + 5*c^2*d*e + e^2)*Sqrt[1 - c^2*x^2])/c^6 - (2*e^2*(5*c^2*d + 2*e)*(
1 - c^2*x^2)^(3/2))/(3*c^6) + (2*e^3*(1 - c^2*x^2)^(5/2))/(5*c^6) - 10*d^3
*ArcTanh[Sqrt[1 - c^2*x^2]]))/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.32

method	result
parts	$a\left(\frac{e^3x^5}{5}+d\,e^2x^3+3d^2ex-\frac{d^3}{x}\right)+bc\left(\frac{\arccos(cx)e^3x^5}{5c}+\frac{\arccos(cx)d\,e^2x^3}{c}+\frac{3\arccos(cx)x\,d^2e}{c}-\right.$
derivativedivides	$c\left(\frac{a\left(3e\,c^5d^2x+c^5x^3d\,e^2+\frac{c^5x^5e^3}{5}-\frac{c^5d^3}{x}\right)}{c^6}+\frac{b\left(3\arccos(cx)c^5d^2ex+\arccos(cx)c^5d\,e^2x^3+\frac{\arccos(cx)e^3c^5x^5}{5}-\right.}{}\right.$
default	$c\left(\frac{a\left(3e\,c^5d^2x+c^5x^3d\,e^2+\frac{c^5x^5e^3}{5}-\frac{c^5d^3}{x}\right)}{c^6}+\frac{b\left(3\arccos(cx)c^5d^2ex+\arccos(cx)c^5d\,e^2x^3+\frac{\arccos(cx)e^3c^5x^5}{5}-\right.}{}\right.$

input
```
int((e*x^2+d)^3*(a+b*arccos(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e^3*x^5+d*e^2*x^3+3*d^2*e*x-d^3/x)+b*c*(1/5/c*arccos(c*x)*e^3*x^5+1
/c*arccos(c*x)*d*e^2*x^3+3/c*arccos(c*x)*x*d^2*e-arccos(c*x)*d^3/c/x+1/5/c
^6*(e^3*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8
/15*(-c^2*x^2+1)^(1/2))+5*c^6*d^3*arctanh(1/(-c^2*x^2+1)^(1/2))-15*c^4*d^2
*e*(-c^2*x^2+1)^(1/2)+5*c^2*d*e^2*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c
^2*x^2+1)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(174) = 348$.

Time = 0.17 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.84

$$\int \frac{(d+ex^2)^3\,(a+b\arccos(cx))}{x^2}\,dx$$
$$=\frac{30\,ac^5e^3x^6+150\,ac^5de^2x^4+75\,bc^6d^3x\log\left(\sqrt{-c^2x^2+1}+1\right)-75\,bc^6d^3x\log\left(\sqrt{-c^2x^2+1}-1\right)+450\,a}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/150*(30*a*c^5*e^3*x^6 + 150*a*c^5*d*e^2*x^4 + 75*b*c^6*d^3*x*log(sqrt(-c
^2*x^2 + 1) + 1) - 75*b*c^6*d^3*x*log(sqrt(-c^2*x^2 + 1) - 1) + 450*a*c^5*
d^2*e*x^2 - 150*a*c^5*d^3 - 30*(5*b*c^5*d^3 - 15*b*c^5*d^2*e - 5*b*c^5*d*e
^2 - b*c^5*e^3)*x*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + 30*(b*c^5
*e^3*x^6 + 5*b*c^5*d*e^2*x^4 + 15*b*c^5*d^2*e*x^2 - 5*b*c^5*d^3 + (5*b*c^5
*d^3 - 15*b*c^5*d^2*e - 5*b*c^5*d*e^2 - b*c^5*e^3)*x)*arccos(c*x) - 2*(3*b
*c^4*e^3*x^5 + (25*b*c^4*d*e^2 + 4*b*c^2*e^3)*x^3 + (225*b*c^4*d^2*e + 50*
b*c^2*d*e^2 + 8*b*e^3)*x)*sqrt(-c^2*x^2 + 1))/(c^5*x)
```

Sympy [A] (verification not implemented)

Time = 3.12 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x^2}\,dx = -\frac{ad^3}{x} + 3ad^2ex + ade^2x^3 + \frac{ae^3x^5}{5} - bcd^3\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \frac{1}{|c^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise}\end{cases}\right) + bcde^2\left(\begin{cases} -\frac{x^2\sqrt{-c^2x^2+1}}{3c^2} - \frac{2\sqrt{-c^2x^2+1}}{3c^4} & \text{for } c^2 \neq 0 \\ \frac{x^4}{4} & \text{otherwise}\end{cases}\right) + \frac{bce^3\left(\begin{cases} -\frac{x^4\sqrt{-c^2x^2+1}}{5c^2} - \frac{4x^2\sqrt{-c^2x^2+1}}{15c^4} - \frac{8\sqrt{-c^2x^2+1}}{15c^6} & \text{for } c^2 \neq 0 \\ \frac{x^6}{6} & \text{otherwise}\end{cases}\right)}{5} - \frac{bd^3\operatorname{acos}(cx)}{x} + 3bd^2e\left(\begin{cases} \frac{\pi x}{2} & \text{for } c = 0 \\ x\operatorname{acos}(cx) - \frac{\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right) + bde^2x^3\operatorname{acos}(cx) + \frac{be^3x^5\operatorname{acos}(cx)}{5}$$

input
```
integrate((e*x**2+d)**3*(a+b*acos(c*x))/x**2,x)
```

output
```
-a*d**3/x + 3*a*d**2*e*x + a*d*e**2*x**3 + a*e**3*x**5/5 - b*c*d**3*Piecew
ise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)), True)) + b*
c*d*e**2*Piecewise((-x**2*sqrt(-c**2*x**2 + 1)/(3*c**2) - 2*sqrt(-c**2*x**
2 + 1)/(3*c**4), Ne(c**2, 0)), (x**4/4, True)) + b*c*e**3*Piecewise((-x**4
*sqrt(-c**2*x**2 + 1)/(5*c**2) - 4*x**2*sqrt(-c**2*x**2 + 1)/(15*c**4) - 8
*sqrt(-c**2*x**2 + 1)/(15*c**6), Ne(c**2, 0)), (x**6/6, True))/5 - b*d**3*
acos(c*x)/x + 3*b*d**2*e*Piecewise((pi*x/2, Eq(c, 0)), (x*acos(c*x) - sqrt
(-c**2*x**2 + 1)/c, True)) + b*d*e**2*x**3*acos(c*x) + b*e**3*x**5*acos(c*
x)/5
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x^2}\,dx$$
$$= \frac{1}{5}ae^3x^5 + ade^2x^3 + \left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|}\right) - \frac{\arccos(cx)}{x}\right)bd^3 + \frac{1}{3}\left(3x^3\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bde^2 + \frac{1}{75}\left(15x^5\arccos(cx) - \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)be^3 + 3ad^2ex + \frac{3\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)bd^2e}{c} - \frac{ad^3}{x}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/5*a*e^3*x^5 + a*d*e^2*x^3 + (c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x
)) - arccos(c*x)/x)*b*d^3 + 1/3*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)
*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*d*e^2 + 1/75*(15*x^5*arccos(c*x) -
 (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^
2*x^2 + 1)/c^6)*c)*b*e^3 + 3*a*d^2*e*x + 3*(c*x*arccos(c*x) - sqrt(-c^2*x^
2 + 1))*b*d^2*e/c - a*d^3/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12827 vs. $2(174) = 348$.

Time = 12.06 (sec) , antiderivative size = 12827, normalized size of antiderivative = 67.51

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))/x^2,x, algorithm="giac")
```

output
```
-b*c^6*d^3*arccos(c*x)/(c^5 - 4*(c^2*x^2 - 1)*c^5/(c*x + 1)^2 + 5*(c^2*x^2
 - 1)^2*c^5/(c*x + 1)^4 - 5*(c^2*x^2 - 1)^4*c^5/(c*x + 1)^8 + 4*(c^2*x^2 -
 1)^5*c^5/(c*x + 1)^10 - (c^2*x^2 - 1)^6*c^5/(c*x + 1)^12) + b*c^6*d^3*log
(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/(c^5 - 4*(c^2*x^2 - 1)*c^5/(c*x + 1)^2
 + 5*(c^2*x^2 - 1)^2*c^5/(c*x + 1)^4 - 5*(c^2*x^2 - 1)^4*c^5/(c*x + 1)^8 +
 4*(c^2*x^2 - 1)^5*c^5/(c*x + 1)^10 - (c^2*x^2 - 1)^6*c^5/(c*x + 1)^12) -
b*c^6*d^3*log(abs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/(c^5 - 4*(c^2*x^2 - 1)*c
^5/(c*x + 1)^2 + 5*(c^2*x^2 - 1)^2*c^5/(c*x + 1)^4 - 5*(c^2*x^2 - 1)^4*c^5
/(c*x + 1)^8 + 4*(c^2*x^2 - 1)^5*c^5/(c*x + 1)^10 - (c^2*x^2 - 1)^6*c^5/(c
*x + 1)^12) - a*c^6*d^3/(c^5 - 4*(c^2*x^2 - 1)*c^5/(c*x + 1)^2 + 5*(c^2*x^
2 - 1)^2*c^5/(c*x + 1)^4 - 5*(c^2*x^2 - 1)^4*c^5/(c*x + 1)^8 + 4*(c^2*x^2
- 1)^5*c^5/(c*x + 1)^10 - (c^2*x^2 - 1)^6*c^5/(c*x + 1)^12) + 6*(c^2*x^2 -
 1)*b*c^6*d^3*arccos(c*x)/((c^5 - 4*(c^2*x^2 - 1)*c^5/(c*x + 1)^2 + 5*(c^2
*x^2 - 1)^2*c^5/(c*x + 1)^4 - 5*(c^2*x^2 - 1)^4*c^5/(c*x + 1)^8 + 4*(c^2*x
^2 - 1)^5*c^5/(c*x + 1)^10 - (c^2*x^2 - 1)^6*c^5/(c*x + 1)^12)*(c*x + 1)^2
) - 4*(c^2*x^2 - 1)*b*c^6*d^3*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/((c^5
 - 4*(c^2*x^2 - 1)*c^5/(c*x + 1)^2 + 5*(c^2*x^2 - 1)^2*c^5/(c*x + 1)^4 - 5
*(c^2*x^2 - 1)^4*c^5/(c*x + 1)^8 + 4*(c^2*x^2 - 1)^5*c^5/(c*x + 1)^10 - (c
^2*x^2 - 1)^6*c^5/(c*x + 1)^12)*(c*x + 1)^2) + 4*(c^2*x^2 - 1)*b*c^6*d^3*l
og(abs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/((c^5 - 4*(c^2*x^2 - 1)*c^5/(c*x...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(e\,x^2+d)^3}{x^2}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + e*x^2)^3)/x^2,x)
```

output
```
int(((a + b*acos(c*x))*(d + e*x^2)^3)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x^2}\,dx = \frac{-75acos(cx)\,b\,c^5d^3+225acos(cx)\,b\,c^5d^2e\,x^2+75acos(cx)\,b\,c^5d\,e^2x^4+15acos(cx)\,b\,c^5e^3x^6-225\sqrt{-c^2x^2}}{}$$

input
```
int((e*x^2+d)^3*(a+b*acos(c*x))/x^2,x)
```

output
```
( - 75*acos(c*x)*b*c**5*d**3 + 225*acos(c*x)*b*c**5*d**2*e*x**2 + 75*acos(
c*x)*b*c**5*d*e**2*x**4 + 15*acos(c*x)*b*c**5*e**3*x**6 - 225*sqrt( - c**2
*x**2 + 1)*b*c**4*d**2*e*x - 25*sqrt( - c**2*x**2 + 1)*b*c**4*d*e**2*x**3
- 3*sqrt( - c**2*x**2 + 1)*b*c**4*e**3*x**5 - 50*sqrt( - c**2*x**2 + 1)*b*
c**2*d*e**2*x - 4*sqrt( - c**2*x**2 + 1)*b*c**2*e**3*x**3 - 8*sqrt( - c**2
*x**2 + 1)*b*e**3*x - 75*log(tan(asin(c*x)/2))*b*c**6*d**3*x - 75*a*c**5*d
**3 + 225*a*c**5*d**2*e*x**2 + 75*a*c**5*d*e**2*x**4 + 15*a*c**5*e**3*x**6
)/(75*c**5*x)
```

3.623 $\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 262

$$\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x^3}\,dx = -\frac{bcd^3\sqrt{1-c^2x^2}}{2x}+\frac{3be^2(8c^2d+e)\,x\sqrt{1-c^2x^2}}{32c^3}+\frac{be^3x^3\sqrt{1-c^2x^2}}{16c}-\frac{3be^2(8c^2d+e)\arccos(cx)}{32c^4}-\frac{3}{2}ibd^2e\arccos(cx)^2-\frac{d^3(a+b\arccos(cx))}{2x^2}+\frac{3}{2}de^2x^2(a+b\arccos(cx))+\frac{1}{4}e^3x^4(a+b\arccos(cx))+3bd^2e\arccos(cx)\log\left(1-e^{2i\arccos(cx)}\right)-3bd^2e\arccos(cx)\log(x)+3d^2e(a+b\arccos(cx))\log(x)-\frac{3}{2}ibd^2e\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)$$

output
```
-1/2*b*c*d^3*(-c^2*x^2+1)^(1/2)/x+3/32*b*e^2*(8*c^2*d+e)*x*(-c^2*x^2+1)^(1
/2)/c^3+1/16*b*e^3*x^3*(-c^2*x^2+1)^(1/2)/c-3/32*b*e^2*(8*c^2*d+e)*arccos(
c*x)/c^4-3/2*I*b*d^2*e*arccos(c*x)^2-1/2*d^3*(a+b*arccos(c*x))/x^2+3/2*d*e
^2*x^2*(a+b*arccos(c*x))+1/4*e^3*x^4*(a+b*arccos(c*x))+3*b*d^2*e*arccos(c*
x)*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)-3*b*d^2*e*arccos(c*x)*ln(x)+3*d^2*e*
(a+b*arccos(c*x))*ln(x)-3/2*I*b*d^2*e*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))
^2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x^3}\,dx = \frac{1}{4}\left(-\frac{2ad^3}{x^2}+6ade^2x^2+ae^3x^4+\frac{2bd^3\left(cx\sqrt{1-c^2x^2}-\arccos(cx)\right)}{x^2}+6bde^2x^2\arccos(cx)+be^3x^4\arccos(cx)-\frac{be^3\left(cx\sqrt{1-c^2x^2}(3+2c^2x^2)-6\arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right)\right)}{8c^4}-\frac{3bde^2\left(cx\sqrt{1-c^2x^2}-2\arctan\left(\frac{cx}{-1+\sqrt{1-c^2x^2}}\right)\right)}{c^2}+12ad^2e\log(x)-6ibd^2e\left(\arccos(cx)\left(\arccos(cx)+2i\log\left(1+e^{2i\arccos(cx)}\right)\right)+\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\right)\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
((-2*a*d^3)/x^2 + 6*a*d*e^2*x^2 + a*e^3*x^4 + (2*b*d^3*(c*x*Sqrt[1 - c^2*x
^2] - ArcCos[c*x]))/x^2 + 6*b*d*e^2*x^2*ArcCos[c*x] + b*e^3*x^4*ArcCos[c*x
] - (b*e^3*(c*x*Sqrt[1 - c^2*x^2]*(3 + 2*c^2*x^2) - 6*ArcTan[(c*x)/(-1 + S
qrt[1 - c^2*x^2])]))/(8*c^4) - (3*b*d*e^2*(c*x*Sqrt[1 - c^2*x^2] - 2*ArcTa
n[(c*x)/(-1 + Sqrt[1 - c^2*x^2])]))/c^2 + 12*a*d^2*e*Log[x] - (6*I)*b*d^2*
e*(ArcCos[c*x]*(ArcCos[c*x] + (2*I)*Log[1 + E^((2*I)*ArcCos[c*x])]) + Poly
Log[2, -E^((2*I)*ArcCos[c*x])]))/4
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5231, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x^3}dx$$

$$\Big\downarrow \text{5231}$$

$$bc\int -\frac{-e^3x^6-6de^2x^4-12d^2e\log(x)x^2+2d^3}{4x^2\sqrt{1-c^2x^2}}dx-\frac{d^3(a+b\arccos(cx))}{2x^2}+3d^2e\log(x)(a+b\arccos(cx))+\frac{3}{2}de^2x^2(a+b\arccos(cx))+\frac{1}{4}e^3x^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}bc\int \frac{-e^3x^6-6de^2x^4-12d^2e\log(x)x^2+2d^3}{x^2\sqrt{1-c^2x^2}}dx-\frac{d^3(a+b\arccos(cx))}{2x^2}+3d^2e\log(x)(a+b\arccos(cx))+\frac{3}{2}de^2x^2(a+b\arccos(cx))+\frac{1}{4}e^3x^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{7293}$$

$$-\frac{1}{4}bc\int \left(\frac{-e^3x^6-6de^2x^4+2d^3}{x^2\sqrt{1-c^2x^2}}-\frac{12d^2e\log(x)}{\sqrt{1-c^2x^2}}\right)dx-\frac{d^3(a+b\arccos(cx))}{2x^2}+3d^2e\log(x)(a+b\arccos(cx))+\frac{3}{2}de^2x^2(a+b\arccos(cx))+\frac{1}{4}e^3x^4(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\arccos(cx))}{2x^2}+3d^2e\log(x)(a+b\arccos(cx))+\frac{3}{2}de^2x^2(a+b\arccos(cx))+\frac{1}{4}e^3x^4(a+b\arccos(cx))-\frac{1}{4}bc\left(-\frac{3e^2\arcsin(cx)\left(8c^2d+e\right)}{8c^5}-\frac{6id^2e\,\text{PolyLog}\left(2,e^{2i\arcsin(cx)}\right)}{c}-\frac{6id^2e\arcsin(cx)^2}{c}+\frac{12d^2e\arcsin(cx)\log\left(1\right.}{c}\right.$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcCos[c*x]))/x^3,x]
```

output
```
-1/2*(d^3*(a + b*ArcCos[c*x]))/x^2 + (3*d*e^2*x^2*(a + b*ArcCos[c*x]))/2 +
 (e^3*x^4*(a + b*ArcCos[c*x]))/4 + 3*d^2*e*(a + b*ArcCos[c*x])*Log[x] - (b
*c*((-2*d^3*Sqrt[1 - c^2*x^2])/x + (3*e^2*(8*c^2*d + e)*x*Sqrt[1 - c^2*x^2
])/(8*c^4) + (e^3*x^3*Sqrt[1 - c^2*x^2])/(4*c^2) - (3*e^2*(8*c^2*d + e)*Ar
cSin[c*x])/(8*c^5) - ((6*I)*d^2*e*ArcSin[c*x]^2)/c + (12*d^2*e*ArcSin[c*x]
*Log[1 - E^((2*I)*ArcSin[c*x])])/c - (12*d^2*e*ArcSin[c*x]*Log[x])/c - ((6
*I)*d^2*e*PolyLog[2, E^((2*I)*ArcSin[c*x])])/c))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.16

method	result
parts	$a\left(\frac{x^4e^3}{4}+\frac{3d\,e^2x^2}{2}+3d^2e\ln\left(x\right)-\frac{d^3}{2x^2}\right)-\frac{3ib\,d^2e\,\mathrm{polylog}\left(2,-\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)}{2}+\frac{3b\,e^2\arccos(cx)\ldots}{2}$
derivativedivides	$c^2\left(\frac{a\left(\frac{3c^4d\,e^2x^2}{2}+\frac{c^4e^3x^4}{4}+3c^4d^2e\ln(cx)-\frac{c^4d^3}{2x^2}\right)}{c^6}+\frac{b\,e^3\arccos(cx)\cos(4\arccos(cx))}{32c^6}-\frac{3bd\,e^2x\sqrt{-c^2x^2+1}}{4c^3}+\ldots\right.$
default	$c^2\left(\frac{a\left(\frac{3c^4d\,e^2x^2}{2}+\frac{c^4e^3x^4}{4}+3c^4d^2e\ln(cx)-\frac{c^4d^3}{2x^2}\right)}{c^6}+\frac{b\,e^3\arccos(cx)\cos(4\arccos(cx))}{32c^6}-\frac{3bd\,e^2x\sqrt{-c^2x^2+1}}{4c^3}+\ldots\right.$

input
```
int((e*x^2+d)^3*(a+b*arccos(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*x^4*e^3+3/2*d*e^2*x^2+3*d^2*e*ln(x)-1/2*d^3/x^2)-3/2*I*b*d^2*e*poly
log(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3/2*b*e^2*arccos(c*x)*x^2*d-1/8*b/c^3
*e^3*(-c^2*x^2+1)^(1/2)*x+1/32*b/c^4*arccos(c*x)*e^3*cos(4*arccos(c*x))+1/
4*b/c^2*e^3*arccos(c*x)*x^2-3/4*b/c^2*e^2*d*arccos(c*x)+1/2*I*b*c^2*d^3+1/
2*b*c*d^3*(-c^2*x^2+1)^(1/2)/x-1/2*b*d^3/x^2*arccos(c*x)+3*b*d^2*e*arccos(
c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-3/2*I*b*d^2*e*arccos(c*x)^2-3/4*b/
c*e^2*(-c^2*x^2+1)^(1/2)*x*d-1/128*b/c^4*e^3*sin(4*arccos(c*x))-1/8*b/c^4*
e^3*arccos(c*x)
```

Fricas [F]

$$\int \frac{(d+ex^2)^3\,(a+b\arccos(cx))}{x^3}\,dx=\int \frac{(ex^2+d)^3(b\arccos\left(cx\right)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccos(c*x))/x^3, x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acos}\,(cx))\,(d+ex^2)^3}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**3*(a+b*acos(c*x))/x**3,x)
```

output
```
Integral((a + b*acos(c*x))*(d + e*x**2)**3/x**3, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^3 (b\arccos(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/4*a*e^3*x^4 + 3/2*a*d*e^2*x^2 + 1/2*b*d^3*(sqrt(-c^2*x^2 + 1)*c/x - arcc
os(c*x)/x^2) + 3*a*d^2*e*log(x) - 1/2*a*d^3/x^2 + integrate((b*e^3*x^4 + 3
*b*d*e^2*x^2 + 3*b*d^2*e)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))/x^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)^3}{x^3}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + e*x^2)^3)/x^3,x)
```

output
```
int(((a + b*acos(c*x))*(d + e*x^2)^3)/x^3, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x^3}\,dx$$
$$= \frac{-16acos(cx)\,b\,c^4d^3+48acos(cx)\,b\,c^4d\,e^2x^4+8acos(cx)\,b\,c^4e^3x^6+24asin(cx)\,b\,c^2d\,e^2x^2+3asin(cx)\,b\,e^3}{}$$

input
```
int((e*x^2+d)^3*(a+b*acos(c*x))/x^3,x)
```

output
```
( - 16*acos(c*x)*b*c**4*d**3 + 48*acos(c*x)*b*c**4*d*e**2*x**4 + 8*acos(c*
x)*b*c**4*e**3*x**6 + 24*asin(c*x)*b*c**2*d*e**2*x**2 + 3*asin(c*x)*b*e**3
*x**2 + 16*sqrt( - c**2*x**2 + 1)*b*c**5*d**3*x - 24*sqrt( - c**2*x**2 + 1
)*b*c**3*d*e**2*x**3 - 2*sqrt( - c**2*x**2 + 1)*b*c**3*e**3*x**5 - 3*sqrt(
 - c**2*x**2 + 1)*b*c*e**3*x**3 + 96*int(acos(c*x)/x,x)*b*c**4*d**2*e*x**2
 + 96*log(x)*a*c**4*d**2*e*x**2 - 16*a*c**4*d**3 + 48*a*c**4*d*e**2*x**4 +
 8*a*c**4*e**3*x**6)/(32*c**4*x**2)
```

3.624 $\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 186

$$\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x^4}\,dx = \frac{be^2(9c^2d+e)\sqrt{1-c^2x^2}}{3c^3} - \frac{bcd^3\sqrt{1-c^2x^2}}{6x^2} - \frac{be^3(1-c^2x^2)^{3/2}}{9c^3} - \frac{d^3(a+b\arccos(cx))}{3x^3} - \frac{3d^2e(a+b\arccos(cx))}{x} + 3de^2x(a+b\arccos(cx)) + \frac{1}{3}e^3x^3(a+b\arccos(cx)) - \frac{1}{6}bcd^2(c^2d+18e)\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)$$

output
```
1/3*b*e^2*(9*c^2*d+e)*(-c^2*x^2+1)^(1/2)/c^3-1/6*b*c*d^3*(-c^2*x^2+1)^(1/2
)/x^2-1/9*b*e^3*(-c^2*x^2+1)^(3/2)/c^3-1/3*d^3*(a+b*arccos(c*x))/x^3-3*d^2
*e*(a+b*arccos(c*x))/x+3*d*e^2*x*(a+b*arccos(c*x))+1/3*e^3*x^3*(a+b*arccos
(c*x))-1/6*b*c*d^2*(c^2*d+18*e)*arctanh((-c^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x^4}\,dx$$
$$= \frac{1}{6}\left(-\frac{2ad^3}{x^3}-\frac{18ad^2e}{x}+18ade^2x+2ae^3x^3 - \frac{b\sqrt{1-c^2x^2}(-3c^4d^3+4e^3x^2+2c^2e^2x^2(27d+ex^2))}{3c^3x^2} + \frac{2b(-d^3-9d^2ex^2+9de^2x^4+e^3x^6)\arccos(cx)}{x^3} - bcd^2\left(c^2d+18e\right)\log(x) + bcd^2\left(c^2d+18e\right)\log\left(1+\sqrt{1-c^2x^2}\right)\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
((-2*a*d^3)/x^3 - (18*a*d^2*e)/x + 18*a*d*e^2*x + 2*a*e^3*x^3 - (b*Sqrt[1
- c^2*x^2]*(-3*c^4*d^3 + 4*e^3*x^2 + 2*c^2*e^2*x^2*(27*d + e*x^2)))/(3*c^3
*x^2) + (2*b*(-d^3 - 9*d^2*e*x^2 + 9*d*e^2*x^4 + e^3*x^6)*ArcCos[c*x])/x^3
 - b*c*d^2*(c^2*d + 18*e)*Log[x] + b*c*d^2*(c^2*d + 18*e)*Log[1 + Sqrt[1 -
 c^2*x^2]])/6
```

Rubi [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5231, 27, 2331, 2124, 27, 1192, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x^4}\,dx$$
$$\Big\downarrow \text{ 5231}$$

$$bc\int -\frac{-e^3x^6-9de^2x^4+9d^2ex^2+d^3}{3x^3\sqrt{1-c^2x^2}}dx-\frac{d^3(a+b\arccos(cx))}{3x^3}-\frac{3d^2e(a+b\arccos(cx))}{x}+3de^2x(a+b\arccos(cx))+\frac{1}{3}e^3x^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}bc\int \frac{-e^3x^6-9de^2x^4+9d^2ex^2+d^3}{x^3\sqrt{1-c^2x^2}}dx-\frac{d^3(a+b\arccos(cx))}{3x^3}-\frac{3d^2e(a+b\arccos(cx))}{x}+3de^2x(a+b\arccos(cx))+\frac{1}{3}e^3x^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{2331}$$

$$-\frac{1}{6}bc\int \frac{-e^3x^6-9de^2x^4+9d^2ex^2+d^3}{x^4\sqrt{1-c^2x^2}}dx^2-\frac{d^3(a+b\arccos(cx))}{3x^3}-\frac{3d^2e(a+b\arccos(cx))}{x}+3de^2x(a+b\arccos(cx))+\frac{1}{3}e^3x^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{2124}$$

$$-\frac{1}{6}bc\left(-\int -\frac{-2e^3x^4-18de^2x^2+d^2(dc^2+18e)}{2x^2\sqrt{1-c^2x^2}}dx^2-\frac{d^3\sqrt{1-c^2x^2}}{x^2}\right)-\frac{d^3(a+b\arccos(cx))}{3x^3}-\frac{3d^2e(a+b\arccos(cx))}{x}+3de^2x(a+b\arccos(cx))+\frac{1}{3}e^3x^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}bc\left(\frac{1}{2}\int \frac{-2e^3x^4-18de^2x^2+d^2(dc^2+18e)}{x^2\sqrt{1-c^2x^2}}dx^2-\frac{d^3\sqrt{1-c^2x^2}}{x^2}\right)-\frac{d^3(a+b\arccos(cx))}{3x^3}-\frac{3d^2e(a+b\arccos(cx))}{x}+3de^2x(a+b\arccos(cx))+\frac{1}{3}e^3x^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{1192}$$

$$-\frac{1}{6}bc\left(\frac{\int -\frac{-2e^3x^8+2e^2(9dc^2+2e)x^4+c^6d^3-2e^3-18c^2de^2+18c^4d^2e}{1-x^4}d\sqrt{1-c^2x^2}}{c^4}-\frac{d^3\sqrt{1-c^2x^2}}{x^2}\right)-\frac{d^3(a+b\arccos(cx))}{3x^3}-\frac{3d^2e(a+b\arccos(cx))}{x}+3de^2x(a+b\arccos(cx))+\frac{1}{3}e^3x^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{6}bc\left(-\frac{\int \frac{-2e^3x^8+2e^2(9dc^2+2e)x^4+c^6d^3-2e^3-18c^2de^2+18c^4d^2e}{1-x^4}d\sqrt{1-c^2x^2}}{c^4}-\frac{d^3\sqrt{1-c^2x^2}}{x^2}\right)-\frac{d^3(a+b\arccos(cx))}{3x^3}-\frac{3d^2e(a+b\arccos(cx))}{x}+3de^2x(a+b\arccos(cx))+\frac{1}{3}e^3x^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{1467}$$

$$-\frac{1}{6}bc\left(-\frac{\int\left(2e^3x^4-2e^2(9dc^2+e)+\frac{d^3c^6+18d^2ec^4}{1-x^4}\right)d\sqrt{1-c^2x^2}}{c^4}-\frac{d^3\sqrt{1-c^2x^2}}{x^2}\right)-\frac{d^3(a+b\arccos(cx))}{3x^3}-\frac{3d^2e(a+b\arccos(cx))}{x}+3de^2x(a+b\arccos(cx))+\frac{1}{3}e^3x^3(a+b\arccos(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\arccos(cx))}{3x^3}-\frac{3d^2e(a+b\arccos(cx))}{x}+3de^2x(a+b\arccos(cx))+\frac{1}{3}e^3x^3(a+b\arccos(cx))-\frac{1}{6}bc\left(\frac{-c^4d^2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)(c^2d+18e)+2e^2\sqrt{1-c^2x^2}(9c^2d+e)-\frac{2}{3}e^3x^6}{c^4}-\frac{d^3\sqrt{1-c^2x^2}}{x^2}\right)$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcCos[c*x]))/x^4,x]
```

output
```
-1/3*(d^3*(a + b*ArcCos[c*x]))/x^3 - (3*d^2*e*(a + b*ArcCos[c*x]))/x + 3*d
*e^2*x*(a + b*ArcCos[c*x]) + (e^3*x^3*(a + b*ArcCos[c*x]))/3 - (b*c*(-((d^
3*Sqrt[1 - c^2*x^2])/x^2) + ((-2*e^3*x^6)/3 + 2*e^2*(9*c^2*d + e)*Sqrt[1 -
 c^2*x^2] - c^4*d^2*(c^2*d + 18*e)*ArcTanh[Sqrt[1 - c^2*x^2]])/c^4))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.26

method	result
parts	$a\left(\frac{e^3x^3}{3}+3d\,e^2x-\frac{3d^2e}{x}-\frac{d^3}{3x^3}\right)+b\,c^3\left(\frac{\arccos(cx)e^3x^3}{3c^3}+\frac{3\arccos(cx)xd\,e^2}{c^3}-\frac{3\arccos(cx)d^2e}{c^3x}-\text{ar}\right.$
derivativedivides	$c^3\left(\frac{a\left(3e^2c^3dx+\frac{c^3x^3e^3}{3}-\frac{3c^3d^2e}{x}-\frac{c^3d^3}{3x^3}\right)}{c^6}+\frac{b\left(3\arccos(cx)c^3d\,e^2x+\frac{\arccos(cx)e^3c^3x^3}{3}-\frac{3\arccos(cx)c^3d^2e}{x}-\frac{\arccos(cx)}{3x^3}\right.}{}\right.$
default	$c^3\left(\frac{a\left(3e^2c^3dx+\frac{c^3x^3e^3}{3}-\frac{3c^3d^2e}{x}-\frac{c^3d^3}{3x^3}\right)}{c^6}+\frac{b\left(3\arccos(cx)c^3d\,e^2x+\frac{\arccos(cx)e^3c^3x^3}{3}-\frac{3\arccos(cx)c^3d^2e}{x}-\frac{\arccos(cx)}{3x^3}\right.}{}\right.$

input
```
int((e*x^2+d)^3*(a+b*arccos(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e^3*x^3+3*d*e^2*x-3*d^2*e/x-1/3*d^3/x^3)+b*c^3*(1/3/c^3*arccos(c*x)
*e^3*x^3+3/c^3*arccos(c*x)*x*d*e^2-3/c^3*arccos(c*x)*d^2*e/x-1/3*arccos(c*
x)*d^3/c^3/x^3+1/3/c^6*(e^3*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2
+1)^(1/2))-c^6*d^3*(-1/2/c^2/x^2*(-c^2*x^2+1)^(1/2)-1/2*arctanh(1/(-c^2*x^
2+1)^(1/2)))-9*c^2*d*e^2*(-c^2*x^2+1)^(1/2)+9*c^4*d^2*e*arctanh(1/(-c^2*x^
2+1)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(166) = 332.

Time = 0.22 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x^4}\,dx = \frac{12\,ac^3e^3x^6 + 108\,ac^3de^2x^4 - 108\,ac^3d^2ex^2 - 12\,ac^3d^3 - 12\,(bc^3d^3 + 9\,bc^3d^2e - 9\,bc^3de^2 - bc^3e^3)x^3\arctan}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/36*(12*a*c^3*e^3*x^6 + 108*a*c^3*d*e^2*x^4 - 108*a*c^3*d^2*e*x^2 - 12*a*
c^3*d^3 - 12*(b*c^3*d^3 + 9*b*c^3*d^2*e - 9*b*c^3*d*e^2 - b*c^3*e^3)*x^3*a
rctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + 3*(b*c^6*d^3 + 18*b*c^4*d^2*
e)*x^3*log(sqrt(-c^2*x^2 + 1) + 1) - 3*(b*c^6*d^3 + 18*b*c^4*d^2*e)*x^3*lo
g(sqrt(-c^2*x^2 + 1) - 1) + 12*(b*c^3*e^3*x^6 + 9*b*c^3*d*e^2*x^4 - 9*b*c^
3*d^2*e*x^2 - b*c^3*d^3 + (b*c^3*d^3 + 9*b*c^3*d^2*e - 9*b*c^3*d*e^2 - b*c
^3*e^3)*x^3)*arccos(c*x) - 2*(2*b*c^2*e^3*x^5 - 3*b*c^4*d^3*x + 2*(27*b*c^
2*d*e^2 + 2*b*e^3)*x^3)*sqrt(-c^2*x^2 + 1))/(c^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.69

$$\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x^4}\,dx$$
$$= -\frac{ad^3}{3x^3} - \frac{3ad^2e}{x} + 3ade^2x + \frac{ae^3x^3}{3} - \frac{bcd^3\left(\begin{cases} -\frac{c^2\operatorname{acosh}\left(\frac{1}{cx}\right)}{2} + \frac{c}{2x\sqrt{-1+\frac{1}{c^2x^2}}} - \frac{1}{2cx^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \frac{1}{|c^2x^2|} > 1 \\ \frac{ic^2\operatorname{asin}\left(\frac{1}{cx}\right)}{2} - \frac{ic\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise} \end{cases}\right)}{3}$$
$$- 3bcd^2e\left(\begin{cases} -\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for } \frac{1}{|c^2x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise} \end{cases}\right)$$
$$+ \frac{bce^3\left(\begin{cases} -\frac{x^2\sqrt{-c^2x^2+1}}{3c^2} - \frac{2\sqrt{-c^2x^2+1}}{3c^4} & \text{for } c^2 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}\right)}{3} - \frac{bd^3\operatorname{acos}(cx)}{3x^3} - \frac{3bd^2e\operatorname{acos}(cx)}{x}$$
$$+ 3bde^2\left(\begin{cases} \frac{\pi x}{2} & \text{for } c = 0 \\ x\operatorname{acos}(cx) - \frac{\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases}\right) + \frac{be^3x^3\operatorname{acos}(cx)}{3}$$

input
```
integrate((e*x**2+d)**3*(a+b*acos(c*x))/x**4,x)
```

output
```
-a*d**3/(3*x**3) - 3*a*d**2*e/x + 3*a*d*e**2*x + a*e**3*x**3/3 - b*c*d**3*
Piecewise((-c**2*acosh(1/(c*x))/2 + c/(2*x*sqrt(-1 + 1/(c**2*x**2))) - 1/(
2*c*x**3*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (I*c**2*asin(1/
(c*x))/2 - I*c*sqrt(1 - 1/(c**2*x**2))/(2*x), True))/3 - 3*b*c*d**2*e*Piec
ewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*x)), True)) +
b*c*e**3*Piecewise((-x**2*sqrt(-c**2*x**2 + 1)/(3*c**2) - 2*sqrt(-c**2*x**
2 + 1)/(3*c**4), Ne(c**2, 0)), (x**4/4, True))/3 - b*d**3*acos(c*x)/(3*x**
3) - 3*b*d**2*e*acos(c*x)/x + 3*b*d*e**2*Piecewise((pi*x/2, Eq(c, 0)), (x*
acos(c*x) - sqrt(-c**2*x**2 + 1)/c, True)) + b*e**3*x**3*acos(c*x)/3
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x^4}\,dx$$
$$= \frac{1}{3}ae^3x^3 + \frac{1}{6}\left(\left(c^2\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{\sqrt{-c^2x^2+1}}{x^2}\right)c-\frac{2\arccos(cx)}{x^3}\right)bd^3 + 3\left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\arccos(cx)}{x}\right)bd^2e + \frac{1}{9}\left(3x^3\arccos(cx)-c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2}+\frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)be^3 + 3ade^2x + \frac{3\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)bde^2}{c} - \frac{3ad^2e}{x} - \frac{ad^3}{3x^3}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/3*a*e^3*x^3 + 1/6*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sq
rt(-c^2*x^2 + 1)/x^2)*c - 2*arccos(c*x)/x^3)*b*d^3 + 3*(c*log(2*sqrt(-c^2*
x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*b*d^2*e + 1/9*(3*x^3*arccos(c
*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*b*e^3 + 3
*a*d*e^2*x + 3*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b*d*e^2/c - 3*a*d^2*
e/x - 1/3*a*d^3/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9504 vs. 2(166) = 332.

Time = 8.33 (sec) , antiderivative size = 9504, normalized size of antiderivative = 51.10

$$\int \frac{(d+ex^2)^3(a+b\arccos(cx))}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))/x^4,x, algorithm="giac")
```

output
```
-1/3*b*c^6*d^3*arccos(c*x)/(c^3 - 3*(c^2*x^2 - 1)^2*c^3/(c*x + 1)^4 + 3*(c
^2*x^2 - 1)^4*c^3/(c*x + 1)^8 - (c^2*x^2 - 1)^6*c^3/(c*x + 1)^12) + 1/6*b*
c^6*d^3*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/(c^3 - 3*(c^2*x^2 - 1)^2*c^
3/(c*x + 1)^4 + 3*(c^2*x^2 - 1)^4*c^3/(c*x + 1)^8 - (c^2*x^2 - 1)^6*c^3/(c
*x + 1)^12) - 1/6*b*c^6*d^3*log(abs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/(c^3 -
 3*(c^2*x^2 - 1)^2*c^3/(c*x + 1)^4 + 3*(c^2*x^2 - 1)^4*c^3/(c*x + 1)^8 - (
c^2*x^2 - 1)^6*c^3/(c*x + 1)^12) - 1/3*a*c^6*d^3/(c^3 - 3*(c^2*x^2 - 1)^2*
c^3/(c*x + 1)^4 + 3*(c^2*x^2 - 1)^4*c^3/(c*x + 1)^8 - (c^2*x^2 - 1)^6*c^3/
(c*x + 1)^12) + 2*(c^2*x^2 - 1)*b*c^6*d^3*arccos(c*x)/((c^3 - 3*(c^2*x^2 -
 1)^2*c^3/(c*x + 1)^4 + 3*(c^2*x^2 - 1)^4*c^3/(c*x + 1)^8 - (c^2*x^2 - 1)^
6*c^3/(c*x + 1)^12)*(c*x + 1)^2) + 1/3*sqrt(-c^2*x^2 + 1)*b*c^6*d^3/((c^3
- 3*(c^2*x^2 - 1)^2*c^3/(c*x + 1)^4 + 3*(c^2*x^2 - 1)^4*c^3/(c*x + 1)^8 -
(c^2*x^2 - 1)^6*c^3/(c*x + 1)^12)*(c*x + 1)) + 2*(c^2*x^2 - 1)*a*c^6*d^3/(
(c^3 - 3*(c^2*x^2 - 1)^2*c^3/(c*x + 1)^4 + 3*(c^2*x^2 - 1)^4*c^3/(c*x + 1)
^8 - (c^2*x^2 - 1)^6*c^3/(c*x + 1)^12)*(c*x + 1)^2) - 5*(c^2*x^2 - 1)^2*b*
c^6*d^3*arccos(c*x)/((c^3 - 3*(c^2*x^2 - 1)^2*c^3/(c*x + 1)^4 + 3*(c^2*x^2
 - 1)^4*c^3/(c*x + 1)^8 - (c^2*x^2 - 1)^6*c^3/(c*x + 1)^12)*(c*x + 1)^4) -
 3*b*c^4*d^2*e*arccos(c*x)/(c^3 - 3*(c^2*x^2 - 1)^2*c^3/(c*x + 1)^4 + 3*(c
^2*x^2 - 1)^4*c^3/(c*x + 1)^8 - (c^2*x^2 - 1)^6*c^3/(c*x + 1)^12) - 1/2*(c
^2*x^2 - 1)^2*b*c^6*d^3*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/((c^3 - ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(e\,x^2+d)^3}{x^4}\,dx$$

input
```
int(((a + b*acos(c*x))*(d + e*x^2)^3)/x^4,x)
```

output
```
int(((a + b*acos(c*x))*(d + e*x^2)^3)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex^2)^3 (a+b\arccos(cx))}{x^4}\,dx$$
$$= \frac{-6acos(cx)\,b\,c^3d^3 - 54acos(cx)\,b\,c^3d^2e\,x^2 + 54acos(cx)\,b\,c^3d\,e^2x^4 + 6acos(cx)\,b\,c^3e^3x^6 + 3\sqrt{-c^2x^2+1}\,b}{}$$

input
```
int((e*x^2+d)^3*(a+b*acos(c*x))/x^4,x)
```

output
```
( - 6*acos(c*x)*b*c**3*d**3 - 54*acos(c*x)*b*c**3*d**2*e*x**2 + 54*acos(c*
x)*b*c**3*d*e**2*x**4 + 6*acos(c*x)*b*c**3*e**3*x**6 + 3*sqrt( - c**2*x**2
 + 1)*b*c**4*d**3*x - 54*sqrt( - c**2*x**2 + 1)*b*c**2*d*e**2*x**3 - 2*sqr
t( - c**2*x**2 + 1)*b*c**2*e**3*x**5 - 4*sqrt( - c**2*x**2 + 1)*b*e**3*x**
3 - 3*log(tan(asin(c*x)/2))*b*c**6*d**3*x**3 - 54*log(tan(asin(c*x)/2))*b*
c**4*d**2*e*x**3 - 6*a*c**3*d**3 - 54*a*c**3*d**2*e*x**2 + 54*a*c**3*d*e**
2*x**4 + 6*a*c**3*e**3*x**6)/(18*c**3*x**3)
```

3.625 $\int (d+ex^2)^4 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 317

$$\begin{aligned}\int (d+ex^2)^4 (a+b\arccos(cx))\,dx &= \frac{b(315c^8d^4+420c^6d^3e+378c^4d^2e^2+180c^2de^3+35e^4)\sqrt{1-c^2x^2}}{315c^9} \\ &- \frac{4be(105c^6d^3+189c^4d^2e+135c^2de^2+35e^3)(1-c^2x^2)^{3/2}}{945c^9} \\ &+ \frac{2be^2(63c^4d^2+90c^2de+35e^2)(1-c^2x^2)^{5/2}}{525c^9} \\ &- \frac{4be^3(9c^2d+7e)(1-c^2x^2)^{7/2}}{441c^9} + \frac{be^4(1-c^2x^2)^{9/2}}{81c^9} \\ &+ d^4x(a+b\arccos(cx)) + \frac{4}{3}d^3ex^3(a+b\arccos(cx)) + \frac{6}{5}d^2e^2x^5(a+b\arccos(cx)) + \frac{4}{7}de^3x^7(a+b\arccos(cx)) -\end{aligned}$$

output
```
1/315*b*(315*c^8*d^4+420*c^6*d^3*e+378*c^4*d^2*e^2+180*c^2*d*e^3+35*e^4)*(
-c^2*x^2+1)^(1/2)/c^9-4/945*b*e*(105*c^6*d^3+189*c^4*d^2*e+135*c^2*d*e^2+3
5*e^3)*(-c^2*x^2+1)^(3/2)/c^9+2/525*b*e^2*(63*c^4*d^2+90*c^2*d*e+35*e^2)*(
-c^2*x^2+1)^(5/2)/c^9-4/441*b*e^3*(9*c^2*d+7*e)*(-c^2*x^2+1)^(7/2)/c^9+1/8
1*b*e^4*(-c^2*x^2+1)^(9/2)/c^9+d^4*x*(a+b*arccos(c*x))+4/3*d^3*e*x^3*(a+b*
arccos(c*x))+6/5*d^2*e^2*x^5*(a+b*arccos(c*x))+4/7*d*e^3*x^7*(a+b*arccos(c
*x))+1/9*e^4*x^9*(a+b*arccos(c*x))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.82

$$\int \left(d+ex^2\right)^4 (a+b\arccos(cx))\,dx$$
$$= \frac{315ax(315d^4+420d^3ex^2+378d^2e^2x^4+180de^3x^6+35e^4x^8)-\frac{b\sqrt{1-c^2x^2}(4480e^4+320c^2e^3(81d+7ex^2)+48c^4e^2(1323}{}}{}$$

input
```
Integrate[(d + e*x^2)^4*(a + b*ArcCos[c*x]),x]
```

output
```
(315*a*x*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e
^4*x^8) - (b*Sqrt[1 - c^2*x^2]*(4480*e^4 + 320*c^2*e^3*(81*d + 7*e*x^2) +
48*c^4*e^2*(1323*d^2 + 270*d*e*x^2 + 35*e^2*x^4) + 8*c^6*e*(11025*d^3 + 39
69*d^2*e*x^2 + 1215*d*e^2*x^4 + 175*e^3*x^6) + c^8*(99225*d^4 + 44100*d^3*
e*x^2 + 23814*d^2*e^2*x^4 + 8100*d*e^3*x^6 + 1225*e^4*x^8)))/c^9 + 315*b*x
*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^4*x^8)*
ArcCos[c*x])/99225
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5171, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^4 (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5171}$$

$$bc\int \frac{x(35e^4x^8+180de^3x^6+378d^2e^2x^4+420d^3ex^2+315d^4)}{315\sqrt{1-c^2x^2}}dx+d^4x(a+b\arccos(cx))+\frac{4}{3}d^3ex^3(a+b\arccos(cx))+\frac{6}{5}d^2e^2x^5(a+b\arccos(cx))+\frac{4}{7}de^3x^7(a+b\arccos(cx))+\frac{1}{9}e^4x^9(a+b\arccos(cx))$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{315}bc\int \frac{x\left(35e^4x^8+180de^3x^6+378d^2e^2x^4+420d^3ex^2+315d^4\right)}{\sqrt{1-c^2x^2}}dx+d^4x(a+b\arccos(cx))+\frac{4}{3}d^3ex^3(a+b\arccos(cx))+\frac{6}{5}d^2e^2x^5(a+b\arccos(cx))+\frac{4}{7}de^3x^7(a+b\arccos(cx))+\frac{1}{9}e^4x^9(a+b\arccos(cx))$$

$\Big\downarrow$ 2331

$$\frac{1}{630}bc\int \frac{35e^4x^8+180de^3x^6+378d^2e^2x^4+420d^3ex^2+315d^4}{\sqrt{1-c^2x^2}}dx^2+d^4x(a+b\arccos(cx))+\frac{4}{3}d^3ex^3(a+b\arccos(cx))+\frac{6}{5}d^2e^2x^5(a+b\arccos(cx))+\frac{4}{7}de^3x^7(a+b\arccos(cx))+\frac{1}{9}e^4x^9(a+b\arccos(cx))$$

$\Big\downarrow$ 2389

$$\frac{1}{630}bc\int \left(\frac{35\left(1-c^2x^2\right)^{7/2}e^4}{c^8}-\frac{20\left(9dc^2+7e\right)\left(1-c^2x^2\right)^{5/2}e^3}{c^8}+\frac{6\left(63d^2c^4+90dec^2+35e^2\right)\left(1-c^2x^2\right)^{3/2}e^2}{c^8}-\right.$$
$$d^4x(a+b\arccos(cx))+\frac{4}{3}d^3ex^3(a+b\arccos(cx))+\frac{6}{5}d^2e^2x^5(a+b\arccos(cx))+\frac{4}{7}de^3x^7(a+b\arccos(cx))+\frac{1}{9}e^4x^9(a+b\arccos(cx))$$

$\Big\downarrow$ 2009

$$d^4x(a+b\arccos(cx))+\frac{4}{3}d^3ex^3(a+b\arccos(cx))+\frac{6}{5}d^2e^2x^5(a+b\arccos(cx))+\frac{4}{7}de^3x^7(a+b\arccos(cx))+\frac{1}{9}e^4x^9(a+b\arccos(cx))+$$
$$\frac{1}{630}bc\left(\frac{40e^3\left(1-c^2x^2\right)^{7/2}\left(9c^2d+7e\right)}{7c^{10}}-\frac{70e^4\left(1-c^2x^2\right)^{9/2}}{9c^{10}}-\frac{12e^2\left(1-c^2x^2\right)^{5/2}\left(63c^4d^2+90c^2de+35e^2\right)}{5c^{10}}+\frac{8e\right.$$

input
```
Int[(d + e*x^2)^4*(a + b*ArcCos[c*x]),x]
```

output
```
(b*c*((-2*(315*c^8*d^4 + 420*c^6*d^3*e + 378*c^4*d^2*e^2 + 180*c^2*d*e^3 +
 35*e^4)*Sqrt[1 - c^2*x^2])/c^10 + (8*e*(105*c^6*d^3 + 189*c^4*d^2*e + 135
*c^2*d*e^2 + 35*e^3)*(1 - c^2*x^2)^(3/2))/(3*c^10) - (12*e^2*(63*c^4*d^2 +
 90*c^2*d*e + 35*e^2)*(1 - c^2*x^2)^(5/2))/(5*c^10) + (40*e^3*(9*c^2*d + 7
*e)*(1 - c^2*x^2)^(7/2))/(7*c^10) - (70*e^4*(1 - c^2*x^2)^(9/2))/(9*c^10))
)/630 + d^4*x*(a + b*ArcCos[c*x]) + (4*d^3*e*x^3*(a + b*ArcCos[c*x]))/3 +
(6*d^2*e^2*x^5*(a + b*ArcCos[c*x]))/5 + (4*d*e^3*x^7*(a + b*ArcCos[c*x]))/
7 + (e^4*x^9*(a + b*ArcCos[c*x]))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5171
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2
, 0])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.39

method	result
parts	$a\left(\frac{1}{9}e^4x^9+\frac{4}{7}d\,e^3x^7+\frac{6}{5}d^2e^2x^5+\frac{4}{3}d^3e\,x^3+d^4x\right)+\frac{b\left(\frac{c\arccos(cx)e^4x^9}{9}+\frac{4c\arccos(cx)d\,e^3x^7}{7}+\frac{6c\arccos\right.}{}$
derivativedivides	$\frac{a\left(d^4c^9x+\frac{4}{3}d^3c^9e\,x^3+\frac{6}{5}d^2c^9e^2x^5+\frac{4}{7}d\,c^9e^3x^7+\frac{1}{9}e^4c^9x^9\right)}{c^8}+\frac{b\left(\arccos(cx)d^4c^9x+\frac{4\arccos(cx)d^3c^9e\,x^3}{3}+\frac{6\arccos(cx)d^2c^9e^2x^5}{5}+\right.}{}$
default	$\frac{a\left(d^4c^9x+\frac{4}{3}d^3c^9e\,x^3+\frac{6}{5}d^2c^9e^2x^5+\frac{4}{7}d\,c^9e^3x^7+\frac{1}{9}e^4c^9x^9\right)}{c^8}+\frac{b\left(\arccos(cx)d^4c^9x+\frac{4\arccos(cx)d^3c^9e\,x^3}{3}+\frac{6\arccos(cx)d^2c^9e^2x^5}{5}+\right.}{}$
orering	$\frac{x\left(20825c^{10}e^5x^{10}+132525c^{10}d\,e^4x^8+366282c^{10}d^2e^3x^6+1400c^8e^5x^8+604170c^{10}d^3e^2x^4+12960c^8d\,e^4x^6+1025325c^{10}\right.}{}$

input
```
int((e*x^2+d)^4*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/9*e^4*x^9+4/7*d*e^3*x^7+6/5*d^2*e^2*x^5+4/3*d^3*e*x^3+d^4*x)+b/c*(1/9
*c*arccos(c*x)*e^4*x^9+4/7*c*arccos(c*x)*d*e^3*x^7+6/5*c*arccos(c*x)*x^5*d
^2*e^2+4/3*c*arccos(c*x)*d^3*e*x^3+arccos(c*x)*d^4*c*x+1/315/c^8*(35*e^4*(
-1/9*c^8*x^8*(-c^2*x^2+1)^(1/2)-8/63*c^6*x^6*(-c^2*x^2+1)^(1/2)-16/105*c^4
*x^4*(-c^2*x^2+1)^(1/2)-64/315*c^2*x^2*(-c^2*x^2+1)^(1/2)-128/315*(-c^2*x^
2+1)^(1/2))-315*d^4*c^8*(-c^2*x^2+1)^(1/2)+180*d*c^2*e^3*(-1/7*c^6*x^6*(-c
^2*x^2+1)^(1/2)-6/35*c^4*x^4*(-c^2*x^2+1)^(1/2)-8/35*c^2*x^2*(-c^2*x^2+1)^
(1/2)-16/35*(-c^2*x^2+1)^(1/2))+378*d^2*c^4*e^2*(-1/5*c^4*x^4*(-c^2*x^2+1)
^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))+420*d^3*c^
6*e*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.02

$$\int \left(d+ex^2\right)^4 (a+b\arccos(cx))\,dx$$
$$=\frac{11025\,ac^9e^4x^9+56700\,ac^9de^3x^7+119070\,ac^9d^2e^2x^5+132300\,ac^9d^3ex^3+99225\,ac^9d^4x+315\,(35\,bc^9e^4x}{}$$

input
```
integrate((e*x^2+d)^4*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
1/99225*(11025*a*c^9*e^4*x^9 + 56700*a*c^9*d*e^3*x^7 + 119070*a*c^9*d^2*e^
2*x^5 + 132300*a*c^9*d^3*e*x^3 + 99225*a*c^9*d^4*x + 315*(35*b*c^9*e^4*x^9
 + 180*b*c^9*d*e^3*x^7 + 378*b*c^9*d^2*e^2*x^5 + 420*b*c^9*d^3*e*x^3 + 315
*b*c^9*d^4*x)*arccos(c*x) - (1225*b*c^8*e^4*x^8 + 99225*b*c^8*d^4 + 88200*
b*c^6*d^3*e + 63504*b*c^4*d^2*e^2 + 25920*b*c^2*d*e^3 + 100*(81*b*c^8*d*e^
3 + 14*b*c^6*e^4)*x^6 + 4480*b*e^4 + 6*(3969*b*c^8*d^2*e^2 + 1620*b*c^6*d*
e^3 + 280*b*c^4*e^4)*x^4 + 4*(11025*b*c^8*d^3*e + 7938*b*c^6*d^2*e^2 + 324
0*b*c^4*d*e^3 + 560*b*c^2*e^4)*x^2)*sqrt(-c^2*x^2 + 1))/c^9
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.89

$$\int \left(d+ex^2\right)^4 (a+b\arccos(cx))\,dx$$
$$= \begin{cases} ad^4x + \frac{4ad^3ex^3}{3} + \frac{6ad^2e^2x^5}{5} + \frac{4ade^3x^7}{7} + \frac{ae^4x^9}{9} + bd^4x\operatorname{acos}(cx) + \frac{4bd^3ex^3\operatorname{acos}(cx)}{3} + \frac{6bd^2e^2x^5\operatorname{acos}(cx)}{5} + \frac{4bde^3x^7\operatorname{a}}{7} \\ \left(a+\frac{\pi b}{2}\right)\left(d^4x + \frac{4d^3ex^3}{3} + \frac{6d^2e^2x^5}{5} + \frac{4de^3x^7}{7} + \frac{e^4x^9}{9}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**4*(a+b*acos(c*x)),x)
```

output
```
Piecewise((a*d**4*x + 4*a*d**3*e*x**3/3 + 6*a*d**2*e**2*x**5/5 + 4*a*d*e**
3*x**7/7 + a*e**4*x**9/9 + b*d**4*x*acos(c*x) + 4*b*d**3*e*x**3*acos(c*x)/
3 + 6*b*d**2*e**2*x**5*acos(c*x)/5 + 4*b*d*e**3*x**7*acos(c*x)/7 + b*e**4*
x**9*acos(c*x)/9 - b*d**4*sqrt(-c**2*x**2 + 1)/c - 4*b*d**3*e*x**2*sqrt(-c
**2*x**2 + 1)/(9*c) - 6*b*d**2*e**2*x**4*sqrt(-c**2*x**2 + 1)/(25*c) - 4*b
*d*e**3*x**6*sqrt(-c**2*x**2 + 1)/(49*c) - b*e**4*x**8*sqrt(-c**2*x**2 + 1
)/(81*c) - 8*b*d**3*e*sqrt(-c**2*x**2 + 1)/(9*c**3) - 8*b*d**2*e**2*x**2*s
qrt(-c**2*x**2 + 1)/(25*c**3) - 24*b*d*e**3*x**4*sqrt(-c**2*x**2 + 1)/(245
*c**3) - 8*b*e**4*x**6*sqrt(-c**2*x**2 + 1)/(567*c**3) - 16*b*d**2*e**2*sq
rt(-c**2*x**2 + 1)/(25*c**5) - 32*b*d*e**3*x**2*sqrt(-c**2*x**2 + 1)/(245*
c**5) - 16*b*e**4*x**4*sqrt(-c**2*x**2 + 1)/(945*c**5) - 64*b*d*e**3*sqrt(
-c**2*x**2 + 1)/(245*c**7) - 64*b*e**4*x**2*sqrt(-c**2*x**2 + 1)/(2835*c**
7) - 128*b*e**4*sqrt(-c**2*x**2 + 1)/(2835*c**9), Ne(c, 0)), ((a + pi*b/2)
*(d**4*x + 4*d**3*e*x**3/3 + 6*d**2*e**2*x**5/5 + 4*d*e**3*x**7/7 + e**4*x
**9/9), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.36

$$\int \left(d+ex^2\right)^4 (a+b\arccos(cx))\,dx = \frac{1}{9}ae^4x^9 + \frac{4}{7}ade^3x^7 + \frac{6}{5}ad^2e^2x^5 + \frac{4}{3}ad^3ex^3 + \frac{4}{9}\left(3\,x^3\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)bd^3e + \frac{2}{25}\left(15\,x^5\arccos(cx) - \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)bd^2e^2 + \frac{4}{245}\left(35\,x^7\arccos(cx) - \left(\frac{5\sqrt{-c^2x^2+1}x^6}{c^2} + \frac{6\sqrt{-c^2x^2+1}x^4}{c^4} + \frac{8\sqrt{-c^2x^2+1}x^2}{c^6} + \frac{16\sqrt{-c^2x^2+1}}{c^8}\right.\right. + \frac{1}{2835}\left(315\,x^9\arccos(cx) - \left(\frac{35\sqrt{-c^2x^2+1}x^8}{c^2} + \frac{40\sqrt{-c^2x^2+1}x^6}{c^4} + \frac{48\sqrt{-c^2x^2+1}x^4}{c^6} + \frac{64\sqrt{-c^2x^2+1}}{c^8}\right.\right. + ad^4x + \frac{\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)bd^4}{c}$$

input
```
integrate((e*x^2+d)^4*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
1/9*a*e^4*x^9 + 4/7*a*d*e^3*x^7 + 6/5*a*d^2*e^2*x^5 + 4/3*a*d^3*e*x^3 + 4/
9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1
)/c^4))*b*d^3*e + 2/25*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2
 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*b*d^2*e^2 +
 4/245*(35*x^7*arccos(c*x) - (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6*sqrt(-c^2*x
^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*x^2 + 1)/c^8
)*c)*b*d*e^3 + 1/2835*(315*x^9*arccos(c*x) - (35*sqrt(-c^2*x^2 + 1)*x^8/c^
2 + 40*sqrt(-c^2*x^2 + 1)*x^6/c^4 + 48*sqrt(-c^2*x^2 + 1)*x^4/c^6 + 64*sqr
t(-c^2*x^2 + 1)*x^2/c^8 + 128*sqrt(-c^2*x^2 + 1)/c^10)*c)*b*e^4 + a*d^4*x
+ (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b*d^4/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.50

$$\int \left(d+ex^2\right)^4 (a+b\arccos(cx))\,dx = \frac{1}{9}\,be^4x^9\arccos\left(cx\right) + \frac{1}{9}\,ae^4x^9 + \frac{4}{7}\,bde^3x^7\arccos\left(cx\right) - \frac{\sqrt{-c^2x^2+1}be^4x^8}{81\,c} + \frac{4}{7}\,ade^3x^7 + \frac{6}{5}\,bd^2e^2x^5\arccos\left(cx\right) - \frac{4\,\sqrt{-c^2x^2+1}bde^3x^6}{49\,c} + \frac{6}{5}\,ad^2e^2x^5 + \frac{4}{3}\,bd^3ex^3\arccos\left(cx\right) - \frac{6\,\sqrt{-c^2x^2+1}bd^2e^2x^4}{25\,c} - \frac{8\,\sqrt{-c^2x^2+1}be^4x^6}{567\,c^3} + \frac{4}{3}\,ad^3ex^3 + bd^4x\arccos\left(cx\right) - \frac{4\,\sqrt{-c^2x^2+1}bd^3ex^2}{9\,c} - \frac{24\,\sqrt{-c^2x^2+1}bde^3x^4}{245\,c^3} + ad^4x - \frac{\sqrt{-c^2x^2+1}bd^4}{c} - \frac{8\,\sqrt{-c^2x^2+1}bd^2e^2x^2}{25\,c^3} - \frac{16\,\sqrt{-c^2x^2+1}be^4x^4}{945\,c^5} - \frac{8\,\sqrt{-c^2x^2+1}bd^3e}{9\,c^3} - \frac{32\,\sqrt{-c^2x^2+1}bde^3x^2}{245\,c^5} - \frac{16\,\sqrt{-c^2x^2+1}bd^2e^2}{25\,c^5} - \frac{64\,\sqrt{-c^2x^2+1}be^4x^2}{2835\,c^7} - \frac{64\,\sqrt{-c^2x^2+1}bde^3}{245\,c^7} - \frac{128\,\sqrt{-c^2x^2+1}be^4}{2835\,c^9}$$

input
```
integrate((e*x^2+d)^4*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
1/9*b*e^4*x^9*arccos(c*x) + 1/9*a*e^4*x^9 + 4/7*b*d*e^3*x^7*arccos(c*x) -
1/81*sqrt(-c^2*x^2 + 1)*b*e^4*x^8/c + 4/7*a*d*e^3*x^7 + 6/5*b*d^2*e^2*x^5*
arccos(c*x) - 4/49*sqrt(-c^2*x^2 + 1)*b*d*e^3*x^6/c + 6/5*a*d^2*e^2*x^5 +
4/3*b*d^3*e*x^3*arccos(c*x) - 6/25*sqrt(-c^2*x^2 + 1)*b*d^2*e^2*x^4/c - 8/
567*sqrt(-c^2*x^2 + 1)*b*e^4*x^6/c^3 + 4/3*a*d^3*e*x^3 + b*d^4*x*arccos(c*
x) - 4/9*sqrt(-c^2*x^2 + 1)*b*d^3*e*x^2/c - 24/245*sqrt(-c^2*x^2 + 1)*b*d*
e^3*x^4/c^3 + a*d^4*x - sqrt(-c^2*x^2 + 1)*b*d^4/c - 8/25*sqrt(-c^2*x^2 +
1)*b*d^2*e^2*x^2/c^3 - 16/945*sqrt(-c^2*x^2 + 1)*b*e^4*x^4/c^5 - 8/9*sqrt(
-c^2*x^2 + 1)*b*d^3*e/c^3 - 32/245*sqrt(-c^2*x^2 + 1)*b*d*e^3*x^2/c^5 - 16
/25*sqrt(-c^2*x^2 + 1)*b*d^2*e^2/c^5 - 64/2835*sqrt(-c^2*x^2 + 1)*b*e^4*x^
2/c^7 - 64/245*sqrt(-c^2*x^2 + 1)*b*d*e^3/c^7 - 128/2835*sqrt(-c^2*x^2 + 1
)*b*e^4/c^9
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^4 (a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,\left(e\,x^2+d\right)^4\,dx$$

input
```
int((a + b*acos(c*x))*(d + e*x^2)^4,x)
```

output
```
int((a + b*acos(c*x))*(d + e*x^2)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.56

$$\int \left(d+ex^2\right)^4 (a+b\arccos(cx))\,dx$$
$$=\frac{-99225\sqrt{-c^2x^2+1}\,b\,c^8d^4+99225a\,c^9d^4x+11025a\,c^9e^4x^9+99225acos(cx)\,b\,c^9d^4x+11025acos(cx)\,b\,c}{}$$

input
```
int((e*x^2+d)^4*(a+b*acos(c*x)),x)
```

output
```
(99225*acos(c*x)*b*c**9*d**4*x + 132300*acos(c*x)*b*c**9*d**3*e*x**3 + 119
070*acos(c*x)*b*c**9*d**2*e**2*x**5 + 56700*acos(c*x)*b*c**9*d*e**3*x**7 +
 11025*acos(c*x)*b*c**9*e**4*x**9 - 99225*sqrt( - c**2*x**2 + 1)*b*c**8*d*
*4 - 44100*sqrt( - c**2*x**2 + 1)*b*c**8*d**3*e*x**2 - 23814*sqrt( - c**2*
x**2 + 1)*b*c**8*d**2*e**2*x**4 - 8100*sqrt( - c**2*x**2 + 1)*b*c**8*d*e**
3*x**6 - 1225*sqrt( - c**2*x**2 + 1)*b*c**8*e**4*x**8 - 88200*sqrt( - c**2
*x**2 + 1)*b*c**6*d**3*e - 31752*sqrt( - c**2*x**2 + 1)*b*c**6*d**2*e**2*x
**2 - 9720*sqrt( - c**2*x**2 + 1)*b*c**6*d*e**3*x**4 - 1400*sqrt( - c**2*x
**2 + 1)*b*c**6*e**4*x**6 - 63504*sqrt( - c**2*x**2 + 1)*b*c**4*d**2*e**2
- 12960*sqrt( - c**2*x**2 + 1)*b*c**4*d*e**3*x**2 - 1680*sqrt( - c**2*x**2
 + 1)*b*c**4*e**4*x**4 - 25920*sqrt( - c**2*x**2 + 1)*b*c**2*d*e**3 - 2240
*sqrt( - c**2*x**2 + 1)*b*c**2*e**4*x**2 - 4480*sqrt( - c**2*x**2 + 1)*b*e
**4 + 99225*a*c**9*d**4*x + 132300*a*c**9*d**3*e*x**3 + 119070*a*c**9*d**2
*e**2*x**5 + 56700*a*c**9*d*e**3*x**7 + 11025*a*c**9*e**4*x**9)/(99225*c**
9)
```

3.626 $\int \frac{x^4(a+b\arccos(cx))}{d+ex^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 653

$$\begin{aligned}\int \frac{x^4(a+b\arccos(cx))}{d+ex^2}\,dx = & -\frac{adx}{e^2}-\frac{bd\sqrt{1-c^2x^2}}{ce^2}+\frac{b\sqrt{1-c^2x^2}}{3c^3e}-\frac{b(1-c^2x^2)^{3/2}}{9c^3e} \\ & -\frac{bdx\arccos(cx)}{e^2}+\frac{x^3(a+b\arccos(cx))}{3e} \\ & +\frac{(-d)^{3/2}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\ & -\frac{(-d)^{3/2}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\ & +\frac{(-d)^{3/2}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\ & -\frac{(-d)^{3/2}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\ & +\frac{ib(-d)^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\ & -\frac{ib(-d)^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\ & +\frac{ib(-d)^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{5/2}} \\ & -\frac{ib(-d)^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{5/2}}\end{aligned}$$

output
```
-a*d*x/e^2-b*d*(-c^2*x^2+1)^(1/2)/c/e^2+1/3*b*(-c^2*x^2+1)^(1/2)/c^3/e-1/9
*b*(-c^2*x^2+1)^(3/2)/c^3/e-b*d*x*arccos(c*x)/e^2+1/3*x^3*(a+b*arccos(c*x)
)/e+1/2*(-d)^(3/2)*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2
))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)-1/2*(-d)^(3/2)*(a+b*arccos(c*
x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2
)))/e^(5/2)+1/2*(-d)^(3/2)*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2
+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)-1/2*(-d)^(3/2)*(a+b*a
rccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d
+e)^(1/2)))/e^(5/2)+1/2*I*b*(-d)^(3/2)*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2
+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)-1/2*I*b*(-d)^(3/2)*po
lylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)
))/e^(5/2)+1/2*I*b*(-d)^(3/2)*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2)
)/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)-1/2*I*b*(-d)^(3/2)*polylog(2,e
^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2
)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 968, normalized size of antiderivative = 1.48

$$\int \frac{x^4(a+b\arccos(cx))}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^4*(a + b*ArcCos[c*x]))/(d + e*x^2),x]
```

output
```
-((a*d*x)/e^2) + (a*x^3)/(3*e) + (a*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e
^(5/2) + (b*((-4*e^(3/2)*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2))/(9*c^3) + (4*e^(
3/2)*x^3*ArcCos[c*x])/3 + (4*d*Sqrt[e]*(Sqrt[1 - c^2*x^2] - c*x*ArcCos[c*x
]))/c + d^(3/2)*(ArcCos[c*x]^2 - 8*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/
Sqrt[2]]*ArcTan[((c*Sqrt[d] + I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d +
e]] + (2*I)*ArcCos[c*x]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*A
rcCos[c*x]))/Sqrt[e]] + (4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[
2]]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]
] + (2*I)*ArcCos[c*x]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos
[c*x]))/Sqrt[e]] - (4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*L
og[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*Po
lyLog[2, (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] +
 2*PolyLog[2, ((-I)*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[
e]]) - d^(3/2)*(ArcCos[c*x]^2 - 8*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/S
qrt[2]]*ArcTan[((c*Sqrt[d] - I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e
]] + (2*I)*ArcCos[c*x]*Log[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*Ar
cCos[c*x]))/Sqrt[e]] + (4*I)*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2
]]*Log[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]]
 + (2*I)*ArcCos[c*x]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[
c*x]))/Sqrt[e]] - (4*I)*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]...
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arccos(cx))}{d+ex^2}dx$$

$$\Big\downarrow \text{5233}$$

$$\int \left(\frac{d^2(a+b\arccos(cx))}{e^2\left(d+ex^2\right)}-\frac{d(a+b\arccos(cx))}{e^2}+\frac{x^2(a+b\arccos(cx))}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(-d)^{3/2}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2e^{5/2}}-\frac{(-d)^{3/2}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2e^{5/2}}+\frac{(-d)^{3/2}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2e^{5/2}}-\frac{(-d)^{3/2}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2e^{5/2}}+\frac{x^3(a+b\arccos(cx))}{3e}-\frac{adx}{e^2}+\frac{ib(-d)^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2e^{5/2}}-\frac{ib(-d)^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2e^{5/2}}+\frac{ib(-d)^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2e^{5/2}}-\frac{ib(-d)^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2e^{5/2}}-\frac{bdx\arccos(cx)}{e^2}+\frac{bd\sqrt{1-c^2x^2}}{ce^2}+\frac{b(1-c^2x^2)^{3/2}}{9c^3e}-\frac{b\sqrt{1-c^2x^2}}{3c^3e}$$

input
```
Int[(x^4*(a + b*ArcCos[c*x]))/(d + e*x^2),x]
```

output
```
-((a*d*x)/e^2) + (b*d*Sqrt[1 - c^2*x^2])/(c*e^2) - (b*Sqrt[1 - c^2*x^2])/(
3*c^3*e) + (b*(1 - c^2*x^2)^(3/2))/(9*c^3*e) - (b*d*x*ArcCos[c*x])/e^2 + (
x^3*(a + b*ArcCos[c*x]))/(3*e) + ((-d)^(3/2)*(a + b*ArcCos[c*x])*Log[1 - (
Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*e^(5/2))
 - ((-d)^(3/2)*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*S
qrt[-d] - I*Sqrt[c^2*d + e])])/(2*e^(5/2)) + ((-d)^(3/2)*(a + b*ArcCos[c*x
])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/
(2*e^(5/2)) - ((-d)^(3/2)*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos
[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*e^(5/2)) + ((I/2)*b*(-d)^(3/
2)*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e
]))])/e^(5/2) - ((I/2)*b*(-d)^(3/2)*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))
/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/e^(5/2) + ((I/2)*b*(-d)^(3/2)*PolyLog[
2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))])/e^(5/
2) - ((I/2)*b*(-d)^(3/2)*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d
] + I*Sqrt[c^2*d + e])])/e^(5/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 137.58 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.57

method	result
parts	$\frac{a x^3}{3e} - \frac{adx}{e^2} + \frac{a d^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{e^2\sqrt{de}} + \frac{bd\sqrt{-c^2x^2+1}}{c e^2} - \frac{bdx\arccos(cx)}{e^2} - \frac{b\sqrt{-c^2x^2+1}}{4c^3 e} + \frac{b\arccos(cx)x}{4c^2 e} - \frac{ibc\,d \cdots}{\cdots}$
derivativedivides	$\frac{-\frac{a c^5 dx}{e^2}+\frac{a c^5 x^3}{3e}+\frac{a c^5 d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e^2\sqrt{de}}+\frac{b c^4\sqrt{-c^2x^2+1}\,d}{e^2}-\frac{b c^5\arccos(cx)dx}{e^2}-\frac{b c^2\sqrt{-c^2x^2+1}}{4e}+\frac{b c^3\arccos(cx)x}{4e}-\frac{i b c^6 d^2 (\cdots}{\cdots}}{\cdots}$
default	$\frac{-\frac{a c^5 dx}{e^2}+\frac{a c^5 x^3}{3e}+\frac{a c^5 d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e^2\sqrt{de}}+\frac{b c^4\sqrt{-c^2x^2+1}\,d}{e^2}-\frac{b c^5\arccos(cx)dx}{e^2}-\frac{b c^2\sqrt{-c^2x^2+1}}{4e}+\frac{b c^3\arccos(cx)x}{4e}-\frac{i b c^6 d^2 (\cdots}{\cdots}}{\cdots}$

input
```
int(x^4*(a+b*arccos(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/3*a/e*x^3-a*d*x/e^2+a*d^2/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b*d*(-
c^2*x^2+1)^(1/2)/c/e^2-b*d*x*arccos(c*x)/e^2-1/4*b*(-c^2*x^2+1)^(1/2)/c^3/
e+1/4*b/c^2/e*arccos(c*x)*x-1/2*I*b*c*d^2/e^2*sum(_R1/(_R1^2*e+2*c^2*d+e)*
(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c
^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/2*I*b*c*
d^2/e^2*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x
^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*
_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/12*b/c^3*arccos(c*x)/e*cos(3*arccos(c*x))-1/
36*b/c^3/e*sin(3*arccos(c*x))
```

Fricas [F]

$$\int \frac{x^4(a+b\arccos(cx))}{d+ex^2}\,dx = \int \frac{(b\arccos(cx)+a)x^4}{ex^2+d}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^4*arccos(c*x) + a*x^4)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\arccos(cx))}{d+ex^2}\,dx = \int \frac{x^4(a+b\operatorname{acos}(cx))}{d+ex^2}\,dx$$

input
```
integrate(x**4*(a+b*acos(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**4*(a + b*acos(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\arccos(cx))}{d + ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\arccos(cx))}{d + ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b\arccos(cx))}{d + ex^2}\,dx = \int \frac{x^4\,(a + b\,\mathrm{acos}(c\,x))}{e\,x^2 + d}\,dx$$

input
```
int((x^4*(a + b*acos(c*x)))/(d + e*x^2),x)
```

output
```
int((x^4*(a + b*acos(c*x)))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{x^4(a+b\arccos(cx))}{d+ex^2}\,dx$$
$$=\frac{3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad+3\left(\int\frac{acos(cx)x^4}{e\,x^2+d}dx\right)b\,e^3-3adex+a\,e^2x^3}{3e^3}$$

input
```
int(x^4*(a+b*acos(c*x))/(e*x^2+d),x)
```

output
```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d + 3*int((acos(c*x)*x*
*4)/(d + e*x**2),x)*b*e**3 - 3*a*d*e*x + a*e**2*x**3)/(3*e**3)
```

3.627 $\int \frac{x^3(a+b\arccos(cx))}{d+ex^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 559

$$\int \frac{x^3(a+b\arccos(cx))}{d+ex^2}\,dx = \frac{bx\sqrt{1-c^2x^2}}{4ce} - \frac{b\arccos(cx)}{4c^2e} + \frac{x^2(a+b\arccos(cx))}{2e} + \frac{id(a+b\arccos(cx))^2}{2be^2} - \frac{d(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^2} - \frac{d(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^2} - \frac{d(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^2} - \frac{d(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^2} + \frac{ibd\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{ibd\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{ibd\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^2} + \frac{ibd\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^2}$$

output
```
1/4*b*x*(-c^2*x^2+1)^(1/2)/c/e-1/4*b*arccos(c*x)/c^2/e+1/2*x^2*(a+b*arccos
(c*x))/e+1/2*I*d*(a+b*arccos(c*x))^2/b/e^2-1/2*d*(a+b*arccos(c*x))*ln(1-e^
(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2
*d*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/
2)-(c^2*d+e)^(1/2)))/e^2-1/2*d*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2
*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^2-1/2*d*(a+b*arccos(c*x
))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)
))/e^2+1/2*I*b*d*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(
1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*I*b*d*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)
^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*I*b*d*polylog(2,-e^(1/2)
*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*I*b*
d*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(
1/2)))/e^2
```

Mathematica [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 926, normalized size of antiderivative = 1.66

$$\int \frac{x^3(a + b \arccos(cx))}{d + ex^2} \, dx = \text{Too large to display}$$

input
```
Integrate[(x^3*(a + b*ArcCos[c*x]))/(d + e*x^2),x]
```

output
```
(2*a*c^2*e*x^2 - 2*a*c^2*d*Log[d + e*x^2] + b*(-(c*e*x*Sqrt[1 - c^2*x^2])
+ 2*c^2*e*x^2*ArcCos[c*x] + 2*e*ArcTan[(c*x)/(-1 + Sqrt[1 - c^2*x^2])] + I
*c^2*d*(ArcCos[c*x]^2 - 8*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*
ArcTan[((c*Sqrt[d] + I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (2*
I)*ArcCos[c*x]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x
]))/Sqrt[e]] + (4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1
 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)
*ArcCos[c*x]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/S
qrt[e]] - (4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I
*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2,
(I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLo
g[2, ((-I)*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]]) + I*
c^2*d*(ArcCos[c*x]^2 - 8*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*A
rcTan[((c*Sqrt[d] - I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (2*I
)*ArcCos[c*x]*Log[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]
))/Sqrt[e]] + (4*I)*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1
+ (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)*
ArcCos[c*x]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sq
rt[e]] - (4*I)*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*
(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2...
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arccos(cx))}{d+ex^2}\,dx$$

$$\Big\downarrow \ 5233$$

$$\int \left(\frac{x(a+b\arccos(cx))}{e}-\frac{dx(a+b\arccos(cx))}{e\,(d+ex^2)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{d(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2e^2}-\frac{d(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2e^2}-\frac{d(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2e^2}-\frac{d(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2e^2}+\frac{id(a+b\arccos(cx))^2}{2be^2}+\frac{x^2(a+b\arccos(cx))}{2e}+\frac{ibd\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2e^2}+\frac{ibd\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2e^2}+\frac{ibd\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2e^2}+\frac{ibd\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2e^2}+\frac{b\arcsin(cx)}{4c^2e}-\frac{bx\sqrt{1-c^2x^2}}{4ce}$$

input
```
Int[(x^3*(a + b*ArcCos[c*x]))/(d + e*x^2),x]
```

output
```
-1/4*(b*x*Sqrt[1 - c^2*x^2])/(c*e) + (x^2*(a + b*ArcCos[c*x]))/(2*e) + ((I
/2)*d*(a + b*ArcCos[c*x])^2)/(b*e^2) + (b*ArcSin[c*x])/(4*c^2*e) - (d*(a +
 b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c
^2*d + e])])/(2*e^2) - (d*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos
[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*e^2) - (d*(a + b*ArcCos[c*x]
)*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(
2*e^2) - (d*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqr
t[-d] + I*Sqrt[c^2*d + e])])/(2*e^2) + ((I/2)*b*d*PolyLog[2, -((Sqrt[e]*E^
(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e]))])/e^2 + ((I/2)*b*d*Poly
Log[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/e^2
+ ((I/2)*b*d*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt
[c^2*d + e]))])/e^2 + ((I/2)*b*d*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c
*Sqrt[-d] + I*Sqrt[c^2*d + e])])/e^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.83 (sec) , antiderivative size = 2128, normalized size of antiderivative = 3.81

method	result	size
derivativedivides	Expression too large to display	2128
default	Expression too large to display	2128
parts	Expression too large to display	2135

input
```
int(x^3*(a+b*arccos(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/2*a*c^4/e*x^2-1/2*a*c^4*d/e^2*ln(c^2*e*x^2+c^2*d)+b*c^2*(-1/2*I*(
-2*d*c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/
2)*e)*c^2*d*polylog(2,e*(c*x+I*(-c^2*x^2+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c
^2*d+e))^(1/2)-e))/e^3/(c^2*d+e)-1/4*I*(c^2*d*(c^2*d+e))^(1/2)/e^2*c^2*d/(
c^2*d+e)*polylog(2,e*(c*x+I*(-c^2*x^2+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*
d+e))^(1/2)-e))-I*(-2*d*c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^4*d^2+2*c^2*d*e-(c
^2*d*(c^2*d+e))^(1/2)*e)*c^4*d^2*arccos(c*x)^2/e^4/(c^2*d+e)+(-2*d*c^2*(c^
2*d*(c^2*d+e))^(1/2)+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)/e^3*c^
2*d/(c^2*d+e)*ln(1-e*(c*x+I*(-c^2*x^2+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*
d+e))^(1/2)-e))*arccos(c*x)-I*(-2*d*c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^4*d^2+
2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*c^2*d*arccos(c*x)^2/e^3/(c^2*d+e)-1/2
*I*(-2*d*c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))
^(1/2)*e)*c^4*d^2*polylog(2,e*(c*x+I*(-c^2*x^2+1)^(1/2))^2/(-2*c^2*d-2*(c^
2*d*(c^2*d+e))^(1/2)-e))/e^4/(c^2*d+e)+1/2*I*(2*c^2*d-2*(c^2*d*(c^2*d+e))^
(1/2)+e)*polylog(2,e*(c*x+I*(-c^2*x^2+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*
d+e))^(1/2)-e))*c^4*d^2/e^4-1/8*I*(c^2*d*(c^2*d+e))^(1/2)/e/(c^2*d+e)*poly
log(2,e*(c*x+I*(-c^2*x^2+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e
))-1/4*I*(-2*d*c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2
*d+e))^(1/2)*e)*arccos(c*x)^2/e^2/(c^2*d+e)-1/8*I*(-2*d*c^2*(c^2*d*(c^2*d+
e))^(1/2)+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*polylog(2,e*(c...
```

Fricas [F]

$$\int \frac{x^3(a+b\arccos(cx))}{d+ex^2}\,dx=\int \frac{(b\arccos{(cx)}+a)x^3}{ex^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^3*arccos(c*x) + a*x^3)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\arccos(cx))}{d+ex^2}\,dx=\int \frac{x^3(a+b\operatorname{acos}{(cx)})}{d+ex^2}\,dx$$

input
```
integrate(x**3*(a+b*acos(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**3*(a + b*acos(c*x))/(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x^3(a+b\arccos(cx))}{d+ex^2}\,dx=\int \frac{(b\arccos{(cx)}+a)x^3}{ex^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a*(x^2/e - d*log(e*x^2 + d)/e^2) + b*integrate(x^3*arctan2(sqrt(c*x +
1)*sqrt(-c*x + 1), c*x)/(e*x^2 + d), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arccos(cx))}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arccos(cx))}{d+ex^2}\,dx = \int \frac{x^3\,(a+b\,\mathrm{acos}(c\,x))}{e\,x^2+d}\,dx$$

input
```
int((x^3*(a + b*acos(c*x)))/(d + e*x^2),x)
```

output
```
int((x^3*(a + b*acos(c*x)))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{x^3(a+b\arccos(cx))}{d+ex^2}\,dx$$
$$= \frac{2acos(cx)\,b\,c^2e\,x^2 - 2acos(cx)\,be - asin(cx)\,be - \sqrt{-c^2x^2+1}\,bcex - 4\left(\int \frac{acos(cx)x}{e\,x^2+d}dx\right)b\,c^2de - 2\log(e\,x

output
```
(2*acos(c*x)*b*c**2*e*x**2 - 2*acos(c*x)*b*e - asin(c*x)*b*e - sqrt( - c**
2*x**2 + 1)*b*c*e*x - 4*int((acos(c*x)*x)/(d + e*x**2),x)*b*c**2*d*e - 2*l
og(d + e*x**2)*a*c**2*d + 2*a*c**2*e*x**2)/(4*c**2*e**2)
```

3.628 $\int \frac{x^2(a+b\arccos(cx))}{d+ex^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 579

$$
\begin{aligned}
\int \frac{x^2(a+b\arccos(cx))}{d+ex^2}\,dx &= \frac{ax}{e}+\frac{b\sqrt{1-c^2x^2}}{ce}+\frac{bx\arccos(cx)}{e} \\
&+\frac{\sqrt{-d}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&-\frac{\sqrt{-d}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&+\frac{\sqrt{-d}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&-\frac{\sqrt{-d}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&+\frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&-\frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&+\frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\
&-\frac{ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}
\end{aligned}
$$

output
```
a*x/e+b*(-c^2*x^2+1)^(1/2)/c/e+b*x*arccos(c*x)/e+1/2*(-d)^(1/2)*(a+b*arcco
s(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^
(1/2)))/e^(3/2)-1/2*(-d)^(1/2)*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2
*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)+1/2*(-d)^(1/2)*(a
+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c
^2*d+e)^(1/2)))/e^(3/2)-1/2*(-d)^(1/2)*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x
+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)+1/2*I*b*(
-d)^(1/2)*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c
^2*d+e)^(1/2)))/e^(3/2)-1/2*I*b*(-d)^(1/2)*polylog(2,e^(1/2)*(c*x+I*(-c^2*
x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)+1/2*I*b*(-d)^(1/2)
*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(
1/2)))/e^(3/2)-1/2*I*b*(-d)^(1/2)*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1
/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 909, normalized size of antiderivative = 1.57

$$\int \frac{x^2(a+b\arccos(cx))}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x]))/(d + e*x^2),x]
```

output
```
(a*x)/e - (a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + b*((-Sqrt[1 -
c^2*x^2] + c*x*ArcCos[c*x])/(c*e) - (Sqrt[d]*(ArcCos[c*x]^2 - 8*ArcSin[Sqr
t[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + I*Sqrt[e])*Tan[
ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (2*I)*ArcCos[c*x]*Log[1 - (I*(-(c*Sqrt[
d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (4*I)*ArcSin[Sqrt[1 +
 (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e
])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)*ArcCos[c*x]*Log[1 + (I*(c*Sqrt[d] +
 Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - (4*I)*ArcSin[Sqrt[1 + (I*c
*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*
ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2, (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*
E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2, ((-I)*(c*Sqrt[d] + Sqrt[c^2*d +
 e])*E^(I*ArcCos[c*x]))/Sqrt[e]]))/(4*e^(3/2)) + (Sqrt[d]*(ArcCos[c*x]^2 -
 8*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - I*
Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (2*I)*ArcCos[c*x]*Log[1 +
(I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (4*I)*Ar
cSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(-(c*Sqrt[d]) + S
qrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)*ArcCos[c*x]*Log[1 - (I
*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - (4*I)*ArcSin[
Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^2*
d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2, ((-I)*(-(c*Sqrt[d]) ...
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))}{d+ex^2}\,dx$$

$$\Big\downarrow \text{ 5233}$$

$$\int \left(\frac{a+b\arccos(cx)}{e}-\frac{d(a+b\arccos(cx))}{e\,(d+ex^2)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{-d}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2e^{3/2}}+\frac{\sqrt{-d}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2e^{3/2}}+\frac{ax}{e}+\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2e^{3/2}}-\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2e^{3/2}}+\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2e^{3/2}}-\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2e^{3/2}}+\frac{bx\arccos(cx)}{e}-\frac{b\sqrt{1-c^2x^2}}{ce}$$

input

```
Int[(x^2*(a + b*ArcCos[c*x]))/(d + e*x^2),x]
```

output

```
(a*x)/e - (b*Sqrt[1 - c^2*x^2])/(c*e) + (b*x*ArcCos[c*x])/e + (Sqrt[-d]*(a
 + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt
[c^2*d + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e
]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*e^(3/2)) + (Sqr
t[-d]*(a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d]
+ I*Sqrt[c^2*d + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCos[c*x])*Log[1 +
 (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*e^(3/2)
) + ((I/2)*b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d]
 - I*Sqrt[c^2*d + e]))])/e^(3/2) - ((I/2)*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E
^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/e^(3/2) + ((I/2)*b*Sq
rt[-d]*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d
 + e]))])/e^(3/2) - ((I/2)*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]
))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/e^(3/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 178.46 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.49

method	result
parts	$\frac{ax}{e}-\frac{ad\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}-\frac{b\sqrt{-c^2x^2+1}}{ce}+\frac{bx\arccos(cx)}{e}+\frac{ibcd\left(\sum_{\_R1=\text{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1(\ldots}{\ldots}\right.}{\ldots}$
derivativedivides	$\frac{\frac{a\,c^3x}{e}-\frac{a\,c^3d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}-\frac{b\,c^2\sqrt{-c^2x^2+1}}{e}+\frac{b\,c^3\arccos(cx)x}{e}+\frac{i\,b\,c^4d\left(\sum_{\_R1=\text{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(i\ldots\right.}{\ldots}\right.}{\ldots}}{\ldots}$
default	$\frac{\frac{a\,c^3x}{e}-\frac{a\,c^3d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}-\frac{b\,c^2\sqrt{-c^2x^2+1}}{e}+\frac{b\,c^3\arccos(cx)x}{e}+\frac{i\,b\,c^4d\left(\sum_{\_R1=\text{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(i\ldots\right.}{\ldots}\right.}{\ldots}}{\ldots}$

input
```
int(x^2*(a+b*arccos(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a*x/e-a*d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-b*(-c^2*x^2+1)^(1/2)/c/e+b
*x*arccos(c*x)/e+1/2*I*b*c*d/e*sum(_R1/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*
ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2)
)/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/2*I*b*c*d/e*sum(1/_R1/(
_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+d
ilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*
_Z^2+e))
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))}{d+ex^2}\,dx=\int \frac{(b\arccos(cx)+a)x^2}{ex^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^2*arccos(c*x) + a*x^2)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\arccos(cx))}{d+ex^2}\,dx=\int \frac{x^2(a+b\operatorname{acos}(cx))}{d+ex^2}\,dx$$

input
```
integrate(x**2*(a+b*acos(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**2*(a + b*acos(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arccos(cx))}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arccos(cx))}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))}{d+ex^2}\,dx = \int \frac{x^2\,(a+b\,\mathrm{acos}(c\,x))}{e\,x^2+d}\,dx$$

input
```
int((x^2*(a + b*acos(c*x)))/(d + e*x^2),x)
```

output
```
int((x^2*(a + b*acos(c*x)))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))}{d+ex^2}\,dx = \frac{acos(cx)\,bcex-\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ac-\sqrt{-c^2x^2+1}\,be-\left(\int\frac{acos(cx)}{e\,x^2+d}dx\right)bcde+acex}{c\,e^2}$$

input
```
int(x^2*(a+b*acos(c*x))/(e*x^2+d),x)
```

output
```
(acos(c*x)*b*c*e*x - sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c - s
qrt( - c**2*x**2 + 1)*b*e - int(acos(c*x)/(d + e*x**2),x)*b*c*d*e + a*c*e*
x)/(c*e**2)
```

3.629 $\int \frac{x(a+b\arccos(cx))}{d+ex^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 491

$$\begin{aligned}\int \frac{x(a+b\arccos(cx))}{d+ex^2}\,dx = & -\frac{i(a+b\arccos(cx))^2}{2be} \\ & + \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e} \\ & + \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e} \\ & + \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e} \\ & + \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e} \\ & - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e} \\ & - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e} \\ & - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e} \\ & - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e}\end{aligned}$$

output
```
-1/2*I*(a+b*arccos(c*x))^2/b/e+1/2*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(
-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*(a+b*arccos(c*x
))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)
))/e+1/2*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-
d)^(1/2)+(c^2*d+e)^(1/2)))/e+1/2*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c
^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e-1/2*I*b*polylog(2,-e^
(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e-1/2*I
*b*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^
(1/2)))/e-1/2*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^
(1/2)+(c^2*d+e)^(1/2)))/e-1/2*I*b*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1
/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 835, normalized size of antiderivative = 1.70

$$\int \frac{x(a+b\arccos(cx))}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x]))/(d + e*x^2),x]
```

output
```
((-I)*b*ArcCos[c*x]^2 + (4*I)*b*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqr
t[2]]*ArcTan[((c*Sqrt[d] - I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]]
 + (4*I)*b*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt
[d] + I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + b*ArcCos[c*x]*Log[
1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*b*
ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(-(c*Sqrt[d]) +
 Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + b*ArcCos[c*x]*Log[1 + (I*(
-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*b*ArcSin[S
qrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^
2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + b*ArcCos[c*x]*Log[1 - (I*(c*Sqrt[d
] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - 2*b*ArcSin[Sqrt[1 - (I*
c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I
*ArcCos[c*x]))/Sqrt[e]] + b*ArcCos[c*x]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d
 + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - 2*b*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqr
t[e]]/Sqrt[2]]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))
/Sqrt[e]] + a*Log[d + e*x^2] - I*b*PolyLog[2, ((-I)*(-(c*Sqrt[d]) + Sqrt[c
^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - I*b*PolyLog[2, (I*(-(c*Sqrt[d]) +
 Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - I*b*PolyLog[2, ((-I)*(c*Sq
rt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - I*b*PolyLog[2, (I*(
c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]])/(2*e)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))}{d+ex^2}\,dx$$

$$\Big\downarrow \text{ 5233}$$

$$\int \left(\frac{a+b\arccos(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+b\arccos(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2e}+\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2e}+\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2e}+\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2e}-\frac{i(a+b\arccos(cx))^2}{2be}-\frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2e}-\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2e}-\frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2e}-\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2e}$$

input
```
Int[(x*(a + b*ArcCos[c*x]))/(d + e*x^2),x]
```

output
```
((-1/2*I)*(a + b*ArcCos[c*x])^2)/(b*e) + ((a + b*ArcCos[c*x])*Log[1 - (Sqr
t[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*e) + ((a + b
*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2
*d + e])])/(2*e) + ((a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x])
)/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcCos[c*x])*Log[1 +
(Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*e) - ((I
/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d
+ e]))])/e - ((I/2)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] -
 I*Sqrt[c^2*d + e])])/e - ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x])
)/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))])/e - ((I/2)*b*PolyLog[2, (Sqrt[e]*E^(
I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/e
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.79

method	result
parts	$\frac{a\ln(e\,x^2+d)}{2e}-\frac{ib\arccos(cx)^2}{2e}+\frac{b\ln\left(\frac{-2c^2d-e\left(cx+i\sqrt{-c^2x^2+1}\right)^2+2\sqrt{c^4d^2+c^2de}-e}{-2c^2d+2\sqrt{c^4d^2+c^2de}-e}\right)\arccos(cx)}{2e}+\frac{b\ln\left(\frac{2c^2d+e(\cdots}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{\frac{a\,c^2\ln\left(c^2e\,x^2+c^2d\right)}{2e}+b\,c^2\left(-\frac{i\arccos(cx)^2}{2e}+\frac{\arccos(cx)\ln\left(\frac{-2c^2d-e\left(cx+i\sqrt{-c^2x^2+1}\right)^2+2\sqrt{c^4d^2+c^2de}-e}{-2c^2d+2\sqrt{c^4d^2+c^2de}-e}\right)}{2e}+\frac{\arccos(cx)\ln\left(\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{a\,c^2\ln\left(c^2e\,x^2+c^2d\right)}{2e}+b\,c^2\left(-\frac{i\arccos(cx)^2}{2e}+\frac{\arccos(cx)\ln\left(\frac{-2c^2d-e\left(cx+i\sqrt{-c^2x^2+1}\right)^2+2\sqrt{c^4d^2+c^2de}-e}{-2c^2d+2\sqrt{c^4d^2+c^2de}-e}\right)}{2e}+\frac{\arccos(cx)\ln\left(\cdots\right.}{\cdots}\right.}{\cdots}$

input

```
int(x*(a+b*arccos(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output

```
1/2*a/e*ln(e*x^2+d)-1/2*I*b/e*arccos(c*x)^2+1/2*b/e*ln((-2*c^2*d-e*(c*x+I*
(-c^2*x^2+1)^(1/2))^2+2*(c^4*d^2+c^2*d*e)^(1/2)-e)/(-2*c^2*d+2*(c^4*d^2+c^
2*d*e)^(1/2)-e))*arccos(c*x)+1/2*b/e*ln((2*c^2*d+e*(c*x+I*(-c^2*x^2+1)^(1/
2))^2+2*(c^4*d^2+c^2*d*e)^(1/2)+e)/(2*c^2*d+2*(c^4*d^2+c^2*d*e)^(1/2)+e))*
arccos(c*x)-1/4*I*b/e*dilog((-2*c^2*d-e*(c*x+I*(-c^2*x^2+1)^(1/2))^2+2*(c^
4*d^2+c^2*d*e)^(1/2)-e)/(-2*c^2*d+2*(c^4*d^2+c^2*d*e)^(1/2)-e))-1/4*I*b/e*
dilog((2*c^2*d+e*(c*x+I*(-c^2*x^2+1)^(1/2))^2+2*(c^4*d^2+c^2*d*e)^(1/2)+e)
/(2*c^2*d+2*(c^4*d^2+c^2*d*e)^(1/2)+e))
```

Fricas [F]

$$\int \frac{x(a+b\arccos(cx))}{d+ex^2}\,dx=\int \frac{(b\arccos{(cx)}+a)x}{ex^2+d}\,dx$$

input

```
integrate(x*(a+b*arccos(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output

```
integral((b*x*arccos(c*x) + a*x)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{x(a+b\arccos(cx))}{d+ex^2}\,dx = \int \frac{x(a+b\,\mathrm{acos}\,(cx))}{d+ex^2}\,dx$$

input
```
integrate(x*(a+b*acos(c*x))/(e*x**2+d),x)
```

output
```
Integral(x*(a + b*acos(c*x))/(d + e*x**2), x)
```

Maxima [F]

$$\int \frac{x(a+b\arccos(cx))}{d+ex^2}\,dx = \int \frac{(b\arccos(cx)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arccos(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
b*integrate(x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(e*x^2 + d), x) +
 1/2*a*log(e*x^2 + d)/e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arccos(cx))}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arccos(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\arccos(cx))}{d + ex^2} \, dx = \int \frac{x\,(a + b\,\mathrm{acos}(c\,x))}{e\,x^2 + d} \, dx$$

input
```
int((x*(a + b*acos(c*x)))/(d + e*x^2),x)
```

output
```
int((x*(a + b*acos(c*x)))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{x(a + b\arccos(cx))}{d + ex^2} \, dx = \frac{2\left(\int \frac{acos(cx)x}{e\,x^2+d}dx\right)be + \log(e\,x^2 + d)\,a}{2e}$$

input
```
int(x*(a+b*acos(c*x))/(e*x^2+d),x)
```

output
```
(2*int((acos(c*x)*x)/(d + e*x**2),x)*b*e + log(d + e*x**2)*a)/(2*e)
```

3.630 $\int \frac{a+b\arccos(cx)}{d+ex^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 541

$$\begin{aligned}\int \frac{a+b\arccos(cx)}{d+ex^2}\,dx &= \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &- \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &+ \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &- \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &+ \frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &- \frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &+ \frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &- \frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

output
```
1/2*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1
/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccos(c*x))*ln(1+e^(1/2
)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/
e^(1/2)+1/2*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c
*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccos(c*x))*ln(
1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d
)^(1/2)/e^(1/2)+1/2*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c
*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*I*b*polylog(2,e^(1/2)
*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e
^(1/2)+1/2*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/
2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*I*b*polylog(2,e^(1/2)*(c*x+I*(
-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 843, normalized size of antiderivative = 1.56

$$\int \frac{a+b\arccos(cx)}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d + e*x^2),x]
```

output
```
(2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + 4*b*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[
e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*
d + e]] - 4*b*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*S
qrt[d] + I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + I*b*ArcCos[c*x]
*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] +
 (2*I)*b*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(-(c*S
qrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - I*b*ArcCos[c*x]*L
og[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - (
2*I)*b*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(-(c*Sqr
t[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - I*b*ArcCos[c*x]*Log
[1 - (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)*
b*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(c*Sqrt[d] +
Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + I*b*ArcCos[c*x]*Log[1 + (I*
(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - (2*I)*b*ArcSin
[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2
*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - b*PolyLog[2, ((-I)*(-(c*Sqrt[d]) +
Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + b*PolyLog[2, (I*(-(c*Sqrt[d
]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + b*PolyLog[2, ((-I)*(c*
Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - b*PolyLog[2, (I*(
c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]])/(2*Sqrt[d]*Sq...
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{d+ex^2}\,dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(\frac{\sqrt{-d}(a+b\arccos(cx))}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}(a+b\arccos(cx))}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d + e*x^2),x]
```

output
```
((a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*S
qrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCos[c*x])*Log[1 + (Sqr
t[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqr
t[e]) + ((a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-
d] + I*Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCos[c*x])*Log[
1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*Sqrt
[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[
-d] - I*Sqrt[c^2*d + e]))])/(Sqrt[-d]*Sqrt[e]) - ((I/2)*b*PolyLog[2, (Sqrt
[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e
]) + ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqr
t[c^2*d + e]))])/(Sqrt[-d]*Sqrt[e]) - ((I/2)*b*PolyLog[2, (Sqrt[e]*E^(I*Ar
cCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.37 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.43

method	result
parts	$\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}-\frac{ibc\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(i\arccos(cx)\ln\left(\frac{\_R1-cx-i\sqrt{-c^2x^2+1}}{\_R1}\right)+\mathrm{dilo}\right.}{\_R1^2e+2c^2d+e}\right.}{2}$
derivativedivides	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(-\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(i\arccos(cx)\ln\left(\frac{\_R1-cx-i\sqrt{-c^2x^2+1}}{\_R1}\right)+\right.}{\_R1^2e+2c^2d+e}\right.}{2}\right.}{}$
default	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(-\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(i\arccos(cx)\ln\left(\frac{\_R1-cx-i\sqrt{-c^2x^2+1}}{\_R1}\right)+\right.}{\_R1^2e+2c^2d+e}\right.}{2}\right.}{}$

input
```
int((a+b*arccos(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/2*I*b*c*sum(_R1/(_R1^2*e+2*c^2*d+e
)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(
-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/2*I*b*
c*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^
(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(
4*c^2*d+2*e)*_Z^2+e))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{d+ex^2}\,dx=\int \frac{b\arccos{(cx)}+a}{ex^2+d}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{d+ex^2}\,dx=\int \frac{a+b\,\mathrm{acos}\,(cx)}{d+ex^2}\,dx$$

input
```
integrate((a+b*acos(c*x))/(e*x**2+d),x)
```

output
```
Integral((a + b*acos(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{d+ex^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arccos(cx)}{d + ex^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arccos(cx)}{d + ex^2} \, dx = \int \frac{a + b \operatorname{acos}(c\,x)}{e\,x^2 + d} \, dx$$

input
```
int((a + b*acos(c*x))/(d + e*x^2),x)
```

output
```
int((a + b*acos(c*x))/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{a + b \arccos(cx)}{d + ex^2} \, dx = \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) a + \left(\int \frac{acos(cx)}{e\,x^2+d} dx\right) bde}{de}$$

input
```
int((a+b*acos(c*x))/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a + int(acos(c*x)/(d + e*x*
*2),x)*b*d*e)/(d*e)
```

3.631 $\int \frac{a+b\arccos(cx)}{x(d+ex^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 518

$$\begin{aligned}\int \frac{a+b\arccos(cx)}{x(d+ex^2)}\,dx = &-\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d}\\&-\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d}\\&-\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d}\\&-\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d}\\&+\frac{(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{d}\\&+\frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d}\\&+\frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d}\\&+\frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d}\\&+\frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d}-\frac{ib\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{2d}\end{aligned}$$

output
```
-1/2*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(
1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x
^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*arccos(c*x))*ln(
1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d-1
/2*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/
2)+(c^2*d+e)^(1/2)))/d+(a+b*arccos(c*x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2
)/d+1/2*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-
(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(
I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2
*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,e^(1/
2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d-1/2*I*b*
polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.68

$$\int \frac{a+b\arccos(cx)}{x\left(d+ex^2\right)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x*(d + e*x^2)),x]
```

output

```
-1/2*((4*I)*b*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*S
qrt[d] - I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (4*I)*b*ArcSin[
Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + I*Sqrt[e])*T
an[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + b*ArcCos[c*x]*Log[1 - (I*(-(c*Sqrt[d
]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*b*ArcSin[Sqrt[1 + (I
*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*
E^(I*ArcCos[c*x]))/Sqrt[e]] + b*ArcCos[c*x]*Log[1 + (I*(-(c*Sqrt[d]) + Sqr
t[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*b*ArcSin[Sqrt[1 - (I*c*Sqrt[
d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*Arc
Cos[c*x]))/Sqrt[e]] + b*ArcCos[c*x]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^2*d + e
])*E^(I*ArcCos[c*x]))/Sqrt[e]] - 2*b*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]
]/Sqrt[2]]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqr
t[e]] + b*ArcCos[c*x]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos
[c*x]))/Sqrt[e]] - 2*b*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log
[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - 2*b*Ar
cCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] - 2*a*Log[x] + a*Log[d + e*x^2] -
 I*b*PolyLog[2, ((-I)*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/
Sqrt[e]] - I*b*PolyLog[2, (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[
c*x]))/Sqrt[e]] - I*b*PolyLog[2, ((-I)*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*
ArcCos[c*x]))/Sqrt[e]] - I*b*PolyLog[2, (I*(c*Sqrt[d] + Sqrt[c^2*d + e]...
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x(d+ex^2)}\,dx$$

$$\Big\downarrow \ 5233$$

$$\int \left(\frac{a+b\arccos(cx)}{dx}-\frac{ex(a+b\arccos(cx))}{d(d+ex^2)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2d}-\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2d}-\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2d}-\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2d}+\frac{\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))}{d}+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2d}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2d}+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2d}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2d}-\frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{2d}$$

input
```
Int[(a + b*ArcCos[c*x])/(x*(d + e*x^2)),x]
```

output
```
-1/2*((a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d]
- I*Sqrt[c^2*d + e])])/d - ((a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcC
os[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcCos[c*x])*
Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*
d) - ((a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d]
+ I*Sqrt[c^2*d + e])])/(2*d) + ((a + b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCo
s[c*x])])/d + ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d
] - I*Sqrt[c^2*d + e]))])/d + ((I/2)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x
]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/d + ((I/2)*b*PolyLog[2, -((Sqrt[e]*
E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))])/d + ((I/2)*b*PolyLo
g[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/d - ((
I/2)*b*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.90 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.75

method	result
parts	$\frac{a\ln(x)}{d}-\frac{a\ln(e\,x^2+d)}{2d}+b\left(\frac{\arccos(cx)\ln\left(1+i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)}{d}+\frac{\arccos(cx)\ln\left(1-i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)}{d}\right.$
derivativedivides	$\frac{a\ln(cx)}{d}-\frac{a\ln(c^2e\,x^2+c^2d)}{2d}+\frac{ib\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\left(\_R1^2+1\right)\left(i\arccos(cx)\ln\left(\frac{\_R1-c}{}\right.\right.}{}\right.}{4d}$
default	$\frac{a\ln(cx)}{d}-\frac{a\ln(c^2e\,x^2+c^2d)}{2d}+\frac{ib\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\left(\_R1^2+1\right)\left(i\arccos(cx)\ln\left(\frac{\_R1-c}{}\right.\right.}{}\right.}{4d}$

input
```
int((a+b*arccos(c*x))/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/d*ln(x)-1/2*a/d*ln(e*x^2+d)+b*(1/d*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1
)^(1/2)))+1/d*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-I/d*dilog(1+I
*(c*x+I*(-c^2*x^2+1)^(1/2)))-I/d*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+1/4
*I*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^
2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_
Z^4+(4*c^2*d+2*e)*_Z^2+e))*e/d+1/4*I*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^
2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*
x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))/d)
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x(d+ex^2)}\,dx=\int \frac{b\arccos(cx)+a}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(e*x^3 + d*x), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x(d+ex^2)}\,dx=\int \frac{a+b\,\mathrm{acos}\,(cx)}{x(d+ex^2)}\,dx$$

input
```
integrate((a+b*acos(c*x))/x/(e*x**2+d),x)
```

output
```
Integral((a + b*acos(c*x))/(x*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x(d+ex^2)}\,dx=\int \frac{b\arccos(cx)+a}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x)/(e*x^3 + d*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x(d+ex^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x(d+ex^2)}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x\,(e\,x^2+d)}\,dx$$

input
```
int((a + b*acos(c*x))/(x*(d + e*x^2)),x)
```

output
```
int((a + b*acos(c*x))/(x*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x(d+ex^2)}\,dx = \frac{2\left(\int \frac{acos(cx)}{e\,x^3+dx}dx\right)bd-\log(e\,x^2+d)\,a+2\log(x)\,a}{2d}$$

input
```
int((a+b*acos(c*x))/x/(e*x^2+d),x)
```

output
```
(2*int(acos(c*x)/(d*x + e*x**3),x)*b*d - log(d + e*x**2)*a + 2*log(x)*a)/(
2*d)
```

3.632 $\int \frac{a+b\arccos(cx)}{x^2(d+ex^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 579

$$\int \frac{a+b\arccos(cx)}{x^2(d+ex^2)}\,dx = -\frac{a+b\arccos(cx)}{dx} - \frac{b\text{carctanh}\left(\sqrt{1-c^2x^2}\right)}{d} + \frac{\sqrt{e}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{\sqrt{e}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{\sqrt{e}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{\sqrt{e}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}$$

output
```
-(a+b*arccos(c*x))/d/x-b*c*arctanh((-c^2*x^2+1)^(1/2))/d+1/2*e^(1/2)*(a+b*
arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*
d+e)^(1/2)))/(-d)^(3/2)-1/2*e^(1/2)*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*
(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)+1/2*e^(1/
2)*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/
2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)-1/2*e^(1/2)*(a+b*arccos(c*x))*ln(1+e^(1/2)
*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)+1
/2*I*b*e^(1/2)*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/
2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)-1/2*I*b*e^(1/2)*polylog(2,e^(1/2)*(c*x+I*(
-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)+1/2*I*b*e^
(1/2)*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d
+e)^(1/2)))/(-d)^(3/2)-1/2*I*b*e^(1/2)*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+
1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 912, normalized size of antiderivative = 1.58

$$\int \frac{a+b\arccos(cx)}{x^2(d+ex^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
(-4*a*Sqrt[d] - 4*a*Sqrt[e]*x*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - 4*b*Sqrt[d]*(A
rcCos[c*x] + c*x*(Log[x] - Log[1 + Sqrt[1 - c^2*x^2]])) - b*Sqrt[e]*x*(Arc
Cos[c*x]^2 - 8*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*
Sqrt[d] + I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (2*I)*ArcCos[c
*x]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]
] + (4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(-(c*
Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)*ArcCos[c*x
]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - (
4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(c*Sqrt[d]
 + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2, (I*(-(c*Sqr
t[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2, ((-I)*
(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]]) + b*Sqrt[e]*x*(
ArcCos[c*x]^2 - 8*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[(
(c*Sqrt[d] - I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (2*I)*ArcCo
s[c*x]*Log[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt
[e]] + (4*I)*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(-
(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)*ArcCos[
c*x]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]]
 - (4*I)*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(c*Sqrt
[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2, ((-I)...
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^2(d+ex^2)}\,dx$$

$$\Big\downarrow \text{ 5233}$$

$$\int \left(\frac{a+b\arccos(cx)}{dx^2}-\frac{e(a+b\arccos(cx))}{d(d+ex^2)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{e}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}-\frac{a+b\arccos(cx)}{dx}+\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}-\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}+\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}-\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}+\frac{bc\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
-((a + b*ArcCos[c*x])/(d*x)) + (b*c*ArcTanh[Sqrt[1 - c^2*x^2]])/d + (Sqrt[
e]*(a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I
*Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCos[c*x])*Log[1 +
(Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*(-d)^(3/
2)) + (Sqrt[e]*(a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*
Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCos[c*
x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])
/(2*(-d)^(3/2)) + ((I/2)*b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x])
)/(c*Sqrt[-d] - I*Sqrt[c^2*d + e]))])/(-d)^(3/2) - ((I/2)*b*Sqrt[e]*PolyLo
g[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(-d)^(
3/2) + ((I/2)*b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-
d] + I*Sqrt[c^2*d + e]))])/(-d)^(3/2) - ((I/2)*b*Sqrt[e]*PolyLog[2, (Sqrt[
e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(-d)^(3/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 186.84 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.58

method	result
parts	$-\frac{a}{dx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d\sqrt{de}}+bc\left(-\frac{\arccos(cx)}{dcx}+\frac{ie\left(\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\left(4\_R1^2c^2d+\_R\cdots\right.}{\cdots}\right.}{\cdots}\right.$
derivativedivides	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}-\frac{b\arccos(cx)}{cxd}-\frac{ibe\left(\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\left(\_R1^2e+4c^2d+e\cdots\right.}{\cdots}\right.}{\cdots}\right.$
default	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}-\frac{b\arccos(cx)}{cxd}-\frac{ibe\left(\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\left(\_R1^2e+4c^2d+e\cdots\right.}{\cdots}\right.}{\cdots}\right.$

input
```
int((a+b*arccos(c*x))/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-a/d/x-a*e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b*c*(-arccos(c*x)/d/c/x+1
/8*I/d^2*e*sum((4*_R1^2*c^2*d+_R1^2*e+e)/_R1/(_R1^2*e+2*c^2*d+e)*(I*arccos
(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)
^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))/c^2-1/8*I/d^2*e*sum
((_R1^2*e+4*c^2*d+e)/_R1/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*
(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=Ro
otOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))/c^2-2*I/d*arctan(c*x+I*(-c^2*x^2+1)^(1/
2)))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^2\left(d+ex^2\right)}\,dx=\int \frac{b\arccos\left(cx\right)+a}{(ex^2+d)x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(e*x^4 + d*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^2\left(d+ex^2\right)}\,dx=\int \frac{a+b\operatorname{acos}\left(cx\right)}{x^2\left(d+ex^2\right)}\,dx$$

input
```
integrate((a+b*acos(c*x))/x**2/(e*x**2+d),x)
```

output
```
Integral((a + b*acos(c*x))/(x**2*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^2(d+ex^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^2(d+ex^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^2(d+ex^2)}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x^2\,(e\,x^2+d)}\,dx$$

input
```
int((a + b*acos(c*x))/(x^2*(d + e*x^2)),x)
```

output
```
int((a + b*acos(c*x))/(x^2*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^2\left(d+ex^2\right)}\,dx=\frac{-\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ax+\left(\int \frac{acos(cx)}{e\,x^4+d\,x^2}dx\right)b\,d^2x-ad}{d^2x}$$

input
```
int((a+b*acos(c*x))/x^2/(e*x^2+d),x)
```

output
```
( - sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*x + int(acos(c*x)/(d*x
**2 + e*x**4),x)*b*d**2*x - a*d)/(d**2*x)
```

3.633 $\int \frac{a+b\arccos(cx)}{x^3(d+ex^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 573

$$\int \frac{a+b\arccos(cx)}{x^3(d+ex^2)}\,dx = -\frac{bc\sqrt{1-c^2x^2}}{2dx} - \frac{a+b\arccos(cx)}{2dx^2} + \frac{e(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^2} + \frac{e(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^2} + \frac{e(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^2} + \frac{e(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^2} - \frac{e(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{d^2} - \frac{ibe\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^2} - \frac{ibe\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^2} - \frac{ibe\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^2} - \frac{ibe\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ibe\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{2d^2}$$

output
```
-1/2*b*c*(-c^2*x^2+1)^(1/2)/d/x-1/2*(a+b*arccos(c*x))/d/x^2+1/2*e*(a+b*arc
cos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e
)^(1/2)))/d^2+1/2*e*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/
2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^2+1/2*e*(a+b*arccos(c*x))*ln(1-e^(
1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^2+1/2*
e*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2
)+(c^2*d+e)^(1/2)))/d^2-e*(a+b*arccos(c*x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2)
)^2)/d^2-1/2*I*b*e*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)
^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*I*b*e*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+
1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*I*b*e*polylog(2,-e^(1/
2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^2-1/2*I*
b*e*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)
^(1/2)))/d^2+1/2*I*b*e*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 966, normalized size of antiderivative = 1.69

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^3*(d + e*x^2)),x]
```

output
```
-1/2*a/(d*x^2) - (a*e*Log[x])/d^2 + (a*e*Log[d + e*x^2])/(2*d^2) + b*((c*x
*Sqrt[1 - c^2*x^2] - ArcCos[c*x])/(2*d*x^2) - ((I/4)*e*(ArcCos[c*x]^2 - 8*
ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + I*Sqr
t[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (2*I)*ArcCos[c*x]*Log[1 - (I*
(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (4*I)*ArcSi
n[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt
[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)*ArcCos[c*x]*Log[1 + (I*(c
*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - (4*I)*ArcSin[Sqr
t[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d +
 e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2, (I*(-(c*Sqrt[d]) + Sqrt[c^
2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2, ((-I)*(c*Sqrt[d] + Sq
rt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]]))/d^2 - ((I/4)*e*(ArcCos[c*x]^2
 - 8*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] -
I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (2*I)*ArcCos[c*x]*Log[1
+ (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (4*I)*
ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(-(c*Sqrt[d]) +
 Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)*ArcCos[c*x]*Log[1 -
(I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - (4*I)*ArcSi
n[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^
2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2, ((-I)*(-(c*Sqrt[d]...
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^3(d+ex^2)}\,dx$$

$$\Big\downarrow \text{5233}$$

$$\int \left(\frac{e^2x(a+b\arccos(cx))}{d^2(d+ex^2)}-\frac{e(a+b\arccos(cx))}{d^2x}+\frac{a+b\arccos(cx)}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2d^2}+\frac{e(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2d^2}+\frac{e(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2d^2}+\frac{e(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2d^2}-\frac{e\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))}{d^2}-\frac{a+b\arccos(cx)}{2dx^2}-\frac{ibe\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2d^2}-\frac{ibe\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2d^2}-\frac{ibe\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2d^2}-\frac{ibe\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ibe\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{2d^2}+\frac{bc\sqrt{1-c^2x^2}}{2dx}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^3*(d + e*x^2)),x]
```

output
```
(b*c*Sqrt[1 - c^2*x^2])/(2*d*x) - (a + b*ArcCos[c*x])/(2*d*x^2) + (e*(a +
b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^
2*d + e])])/(2*d^2) + (e*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[
c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*d^2) + (e*(a + b*ArcCos[c*x])
*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2
*d^2) + (e*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt
[-d] + I*Sqrt[c^2*d + e])])/(2*d^2) - (e*(a + b*ArcCos[c*x])*Log[1 + E^((2
*I)*ArcCos[c*x])])/d^2 - ((I/2)*b*e*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]
))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e]))])/d^2 - ((I/2)*b*e*PolyLog[2, (Sqrt[e
]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/d^2 - ((I/2)*b*e*P
olyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))]
)/d^2 - ((I/2)*b*e*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*
Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*e*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/d^
2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.95 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.82

method	result
parts	$a\left(-\frac{1}{2d\,x^2}-\frac{e\ln(x)}{d^2}+\frac{e\ln(e\,x^2+d)}{2d^2}\right)+b\,c^2\left(-\frac{-ic^2x^2-cx\sqrt{-c^2x^2+1}+\arccos(cx)}{2c^2x^2d}-\frac{e\arccos(cx)\ln\left(1+i\right.}{d^2}\right.$
derivativedivides	$c^2\left(\frac{ae\ln(c^2e\,x^2+c^2d)}{2c^2d^2}-\frac{a}{2d\,c^2x^2}-\frac{ae\ln(cx)}{c^2d^2}+b\,c^2\left(-\frac{-ic^2x^2-cx\sqrt{-c^2x^2+1}+\arccos(cx)}{2c^4x^2d}-\frac{ie^2\left(\_R1=\mathrm{R}\right.}{}\right.\right.$
default	$c^2\left(\frac{ae\ln(c^2e\,x^2+c^2d)}{2c^2d^2}-\frac{a}{2d\,c^2x^2}-\frac{ae\ln(cx)}{c^2d^2}+b\,c^2\left(-\frac{-ic^2x^2-cx\sqrt{-c^2x^2+1}+\arccos(cx)}{2c^4x^2d}-\frac{ie^2\left(\_R1=\mathrm{R}\right.}{}\right.\right.$

input
```
int((a+b*arccos(c*x))/x^3/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2/d/x^2-e/d^2*ln(x)+1/2*e/d^2*ln(e*x^2+d))+b*c^2*(-1/2*(-I*c^2*x^2-c
*x*(-c^2*x^2+1)^(1/2)+arccos(c*x))/c^2/x^2/d-e/d^2/c^2*arccos(c*x)*ln(1+I*
(c*x+I*(-c^2*x^2+1)^(1/2)))-e/d^2/c^2*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+
1)^(1/2)))+I*e/d^2/c^2*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+I*e/d^2/c^2*d
ilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-1/4*I*e^2/d^2*sum((_R1^2+1)/(_R1^2*e+
2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R
1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
/c^2-1/4*I*e/d^2*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x
)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/
2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))/c^2)
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^3\,(d+ex^2)}\,dx = \int \frac{b\arccos\left(cx\right)+a}{(ex^2+d)x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(e*x^5 + d*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^3\,(d+ex^2)}\,dx = \int \frac{a+b\operatorname{acos}\left(cx\right)}{x^3\,(d+ex^2)}\,dx$$

input
```
integrate((a+b*acos(c*x))/x**3/(e*x**2+d),x)
```

output
```
Integral((a + b*acos(c*x))/(x**3*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)}\,dx=\int \frac{b\arccos\left(cx\right)+a}{(ex^2+d)x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(e*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a*(e*log(e*x^2 + d)/d^2 - 2*e*log(x)/d^2 - 1/(d*x^2)) + b*integrate(ar
ctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(e*x^5 + d*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^3/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)}\,dx=\int \frac{a+b\,\mathrm{acos}\left(c\,x\right)}{x^3\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((a + b*acos(c*x))/(x^3*(d + e*x^2)),x)
```

output
```
int((a + b*acos(c*x))/(x^3*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^3\,(d+ex^2)}\,dx$$
$$=\frac{2\left(\int \frac{acos(cx)}{e\,x^5+d\,x^3}dx\right)b\,d^2x^2+\log(e\,x^2+d)\,ae\,x^2-2\log(x)\,ae\,x^2-ad}{2d^2x^2}$$

input
```
int((a+b*acos(c*x))/x^3/(e*x^2+d),x)
```

output
```
(2*int(acos(c*x)/(d*x**3 + e*x**5),x)*b*d**2*x**2 + log(d + e*x**2)*a*e*x*
*2 - 2*log(x)*a*e*x**2 - a*d)/(2*d**2*x**2)
```

3.634 $\int \frac{a+b \arccos(cx)}{x^4(d+ex^2)} dx$

Optimal result

Integrand size = 21, antiderivative size = 649

$$\int \frac{a+b\arccos(cx)}{x^4(d+ex^2)}\,dx = -\frac{bc\sqrt{1-c^2x^2}}{6dx^2} - \frac{a+b\arccos(cx)}{3dx^3} + \frac{e(a+b\arccos(cx))}{d^2x} - \frac{bc^3\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{6d} + \frac{bce\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d^2} + \frac{e^{3/2}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}} - \frac{e^{3/2}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}} + \frac{e^{3/2}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}} - \frac{e^{3/2}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}} + \frac{ibe^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}} - \frac{ibe^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}} + \frac{ibe^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}} - \frac{ibe^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}}$$

output
```
-1/6*b*c*(-c^2*x^2+1)^(1/2)/d/x^2-1/3*(a+b*arccos(c*x))/d/x^3+e*(a+b*arcco
s(c*x))/d^2/x-1/6*b*c^3*arctanh((-c^2*x^2+1)^(1/2))/d+b*c*e*arctanh((-c^2*
x^2+1)^(1/2))/d^2+1/2*e^(3/2)*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*
x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)-1/2*e^(3/2)*(a+
b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^
2*d+e)^(1/2)))/(-d)^(5/2)+1/2*e^(3/2)*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+
I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)-1/2*e^(
3/2)*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(
1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)+1/2*I*b*e^(3/2)*polylog(2,-e^(1/2)*(c*x+
I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)-1/2*I*b
*e^(3/2)*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2
*d+e)^(1/2)))/(-d)^(5/2)+1/2*I*b*e^(3/2)*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x
^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)-1/2*I*b*e^(3/2)*
polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/
2)))/(-d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 1002, normalized size of antiderivative = 1.54

$$\int \frac{a+b\arccos(cx)}{x^4(d+ex^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^4*(d + e*x^2)),x]
```

output
```
-1/3*a/(d*x^3) + (a*e)/(d^2*x) + (a*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d
^(5/2) + b*(-((e*(-(ArcCos[c*x]/x) - c*Log[x] + c*Log[1 + Sqrt[1 - c^2*x^2
]]))/d^2) + ((c*Sqrt[1 - c^2*x^2])/(6*x^2) - ArcCos[c*x]/(3*x^3) - (c^3*Lo
g[x])/6 + (c^3*Log[1 + Sqrt[1 - c^2*x^2]])/6)/d + (e^(3/2)*(ArcCos[c*x]^2
- 8*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + I
*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (2*I)*ArcCos[c*x]*Log[1 -
 (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (4*I)*A
rcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(-(c*Sqrt[d]) +
Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)*ArcCos[c*x]*Log[1 + (
I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - (4*I)*ArcSin
[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2
*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2, (I*(-(c*Sqrt[d]) + Sqr
t[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2, ((-I)*(c*Sqrt[d]
+ Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]]))/(4*d^(5/2)) - (e^(3/2)*(A
rcCos[c*x]^2 - 8*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((
c*Sqrt[d] - I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (2*I)*ArcCos
[c*x]*Log[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[
e]] + (4*I)*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(-(
c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)*ArcCos[c
*x]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]...
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^4\left(d+ex^2\right)}dx$$

$$\Big\downarrow \text{5233}$$

$$\int \left(\frac{e^2(a+b\arccos(cx))}{d^2\left(d+ex^2\right)}-\frac{e(a+b\arccos(cx))}{d^2x^2}+\frac{a+b\arccos(cx)}{dx^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{3/2}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}}-\frac{e^{3/2}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}}+\frac{e^{3/2}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}}-\frac{e^{3/2}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2(-d)^{5/2}}+\frac{e(a+b\arccos(cx))}{d^2x}-\frac{a+b\arccos(cx)}{3dx^3}+\frac{ibe^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2(-d)^{5/2}}-\frac{ibe^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2(-d)^{5/2}}+\frac{ibe^{3/2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2(-d)^{5/2}}-\frac{ibe^{3/2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2(-d)^{5/2}}-\frac{bce\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d^2}+\frac{bc^3\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{6d}+\frac{bc\sqrt{1-c^2x^2}}{6dx^2}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^4*(d + e*x^2)),x]
```

output
```
(b*c*Sqrt[1 - c^2*x^2])/(6*d*x^2) - (a + b*ArcCos[c*x])/(3*d*x^3) + (e*(a
+ b*ArcCos[c*x]))/(d^2*x) + (b*c^3*ArcTanh[Sqrt[1 - c^2*x^2]])/(6*d) - (b*
c*e*ArcTanh[Sqrt[1 - c^2*x^2]])/d^2 + (e^(3/2)*(a + b*ArcCos[c*x])*Log[1 -
 (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*(-d)^(5
/2)) - (e^(3/2)*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c
*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*(-d)^(5/2)) + (e^(3/2)*(a + b*ArcCos[c
*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])]
)/(2*(-d)^(5/2)) - (e^(3/2)*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcC
os[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*(-d)^(5/2)) + ((I/2)*b*e^(
3/2)*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d +
 e]))])/(-d)^(5/2) - ((I/2)*b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]
))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(-d)^(5/2) + ((I/2)*b*e^(3/2)*PolyLo
g[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))])/(-d
)^(5/2) - ((I/2)*b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[
-d] + I*Sqrt[c^2*d + e])])/(-d)^(5/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 219.78 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.66

method	result
parts	$a\left(-\frac{1}{3d\,x^3}+\frac{e}{d^2x}+\frac{e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d^2\sqrt{de}}\right)-\frac{ib\left(8\arctan\left(cx+i\sqrt{-c^2x^2+1}\right)c^7d^2x^3+4i\sqrt{-c^2x^2+1}\,c^5d^2x-8i\operatorname{arcc}\right.}{}$
derivativedivides	$c^3\left(-\frac{a}{3d\,c^3x^3}+\frac{ae}{c^3d^2x}+\frac{a\,e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{c^3d^2\sqrt{de}}-\frac{ib\left(8\arctan\left(cx+i\sqrt{-c^2x^2+1}\right)c^7d^2x^3+4i\sqrt{-c^2x^2+1}\,c^5d^2x-8i\right.}{}\right.$
default	$c^3\left(-\frac{a}{3d\,c^3x^3}+\frac{ae}{c^3d^2x}+\frac{a\,e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{c^3d^2\sqrt{de}}-\frac{ib\left(8\arctan\left(cx+i\sqrt{-c^2x^2+1}\right)c^7d^2x^3+4i\sqrt{-c^2x^2+1}\,c^5d^2x-8i\right.}{}\right.$

input
```
int((a+b*arccos(c*x))/x^4/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3/d/x^3+e/d^2/x+e^2/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-1/24*I*
b/c^4*(8*arctan(c*x+I*(-c^2*x^2+1)^(1/2))*c^7*d^2*x^3+4*I*(-c^2*x^2+1)^(1/
2)*c^5*d^2*x-8*I*arccos(c*x)*c^4*d^2+24*I*arccos(c*x)*c^4*d*e*x^2-48*arcta
n(c*x+I*(-c^2*x^2+1)^(1/2))*c^5*d*e*x^3+3*sum((4*_R1^2*c^2*d+_R1^2*e+e)/_R
1/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1
)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*
e)*_Z^2+e))*e^2*c^3*x^3-3*sum((_R1^2*e+4*c^2*d+e)/_R1/(_R1^2*e+2*c^2*d+e)*
(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c
^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))*e^2*c^3*x^
3)/x^3/d^3
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^4(d+ex^2)}\,dx=\int \frac{b\arccos(cx)+a}{(ex^2+d)x^4}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^4/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(e*x^6 + d*x^4), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^4(d+ex^2)}\,dx=\int \frac{a+b\operatorname{acos}(cx)}{x^4(d+ex^2)}\,dx$$

input
```
integrate((a+b*acos(c*x))/x**4/(e*x**2+d),x)
```

output
```
Integral((a + b*acos(c*x))/(x**4*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \arccos(cx)}{x^4 (d + ex^2)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))/x^4/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b \arccos(cx)}{x^4 (d + ex^2)} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/x^4/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arccos(cx)}{x^4 (d + ex^2)} \, dx = \int \frac{a + b \operatorname{acos}(c\,x)}{x^4\,\left(e\,x^2 + d\right)} \, dx$$

input
```
int((a + b*acos(c*x))/(x^4*(d + e*x^2)),x)
```

output
```
int((a + b*acos(c*x))/(x^4*(d + e*x^2)), x)
```

Reduce [F]

$$\int \frac{a + b\arccos(cx)}{x^4 (d + ex^2)}\,dx$$
$$= \frac{3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) ae\,x^3 + 3\left(\int \frac{acos(cx)}{e\,x^6 + d\,x^4}dx\right) b\,d^3x^3 - a\,d^2 + 3ade\,x^2}{3d^3x^3}$$

input
```
int((a+b*acos(c*x))/x^4/(e*x^2+d),x)
```

output
```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e*x**3 + 3*int(acos(c*x
)/(d*x**4 + e*x**6),x)*b*d**3*x**3 - a*d**2 + 3*a*d*e*x**2)/(3*d**3*x**3)
```

3.635 $\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 574

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \frac{d(a+b\arccos(cx))}{2e^2(d+ex^2)} - \frac{i(a+b\arccos(cx))^2}{2be^2} - \frac{bc\sqrt{d}\arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2e^2\sqrt{c^2d+e}} + \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^2} + \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^2}$$

output
```
1/2*d*(a+b*arccos(c*x))/e^2/(e*x^2+d)-1/2*I*(a+b*arccos(c*x))^2/b/e^2-1/2*
b*c*d^(1/2)*arctan((c^2*d+e)^(1/2)*x/d^(1/2)/(-c^2*x^2+1)^(1/2))/e^2/(c^2*
d+e)^(1/2)+1/2*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(
I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c
*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*ar
ccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+
e)^(1/2)))/e^2+1/2*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2
))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog(2,-e^(1/2)*(c*x+I
*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog
(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^
2-1/2*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c
^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(
I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^2
```

Mathematica [A] (warning: unable to verify)

Time = 1.29 (sec) , antiderivative size = 1100, normalized size of antiderivative = 1.92

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^3*(a + b*ArcCos[c*x]))/(d + e*x^2)^2,x]
```

output
```
((2*a*d)/(d + e*x^2) + 2*a*Log[d + e*x^2] - I*b*(2*ArcCos[c*x]^2 - 8*ArcSi
n[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - I*Sqrt[e])
*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] - 8*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sq
rt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c
^2*d + e]] + (2*I)*ArcCos[c*x]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])
*E^(I*ArcCos[c*x]))/Sqrt[e]] + (4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]
]/Sqrt[2]]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/
Sqrt[e]] + (2*I)*ArcCos[c*x]*Log[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E
^(I*ArcCos[c*x]))/Sqrt[e]] + (4*I)*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/
Sqrt[2]]*Log[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sq
rt[e]] + (2*I)*ArcCos[c*x]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*A
rcCos[c*x]))/Sqrt[e]] - (4*I)*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[
2]]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] +
 (2*I)*ArcCos[c*x]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*
x]))/Sqrt[e]] - (4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[
1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + Sqrt[d]
*(ArcCos[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) - (c*Log[(2*e*(Sqrt[e] - I*c^2*Sq
rt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c*Sqrt[c^2*d + e]*((-I)*Sqr
t[d] + Sqrt[e]*x))])/Sqrt[c^2*d + e]) + Sqrt[d]*(-(ArcCos[c*x]/(I*Sqrt[d]
+ Sqrt[e]*x)) + (c*Log[(-2*e*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + ...
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^2}\,dx$$
$$\Big\downarrow \text{5233}$$
$$\int \left(\frac{x(a+b\arccos(cx))}{e\,(d+ex^2)}-\frac{dx(a+b\arccos(cx))}{e\,(d+ex^2)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2e^2}+\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2e^2}+\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2e^2}+\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2e^2}+\frac{d(a+b\arccos(cx))}{2e^2(d+ex^2)}-\frac{i(a+b\arccos(cx))^2}{2be^2}-\frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2e^2}-\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2e^2}-\frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2e^2}-\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2e^2}+\frac{bc\sqrt{d}\arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2e^2\sqrt{c^2d+e}}$$

input
```
Int[(x^3*(a + b*ArcCos[c*x]))/(d + e*x^2)^2,x]
```

output
```
(d*(a + b*ArcCos[c*x]))/(2*e^2*(d + e*x^2)) - ((I/2)*(a + b*ArcCos[c*x])^2
)/(b*e^2) + (b*c*Sqrt[d]*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*
x^2])])/(2*e^2*Sqrt[c^2*d + e]) + ((a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^
(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcC
os[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d +
e])])/(2*e^2) + ((a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(
c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcCos[c*x])*Log[1 + (
Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*e^2) - ((
I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d
 + e]))])/e^2 - ((I/2)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d
] - I*Sqrt[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[
c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))])/e^2 - ((I/2)*b*PolyLog[2, (Sqrt
[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/e^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.00 (sec) , antiderivative size = 2151, normalized size of antiderivative = 3.75

method	result	size
derivativedivides	Expression too large to display	2151
default	Expression too large to display	2151
parts	Expression too large to display	2163

input
```
int(x^3*(a+b*arccos(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/2*a*c^6*d/e^2/(c^2*e*x^2+c^2*d)+1/2*a*c^4/e^2*ln(c^2*e*x^2+c^2*d)
+b*c^4*(-1/2*I*arccos(c*x)^2/e^2-1/4*I*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+
e)*polylog(2,e*(c*x+I*(-c^2*x^2+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^
(1/2)-e))/e^3+1/2*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)/e^3*ln(1-e*(c*x+I*
(-c^2*x^2+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccos(c*x)-
1/2*I*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*arccos(c*x)^2/e^3-1/2*I*(2*c^2
*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*polylog(2,e*(c*x+I*(-c^2*x^2+1)^(1/2))^2/(
-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*d*c^2/e^4-1/2*I/e^2*sum((_R1^2*e+4*
c^2*d+2*e)/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(
1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4
*c^2*d+2*e)*_Z^2+e))+1/2*arccos(c*x)*d*c^2/e^2/(c^2*e*x^2+c^2*d)-(-2*d*c^2
*(c^2*d*(c^2*d+e))^(1/2)+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)/e^
3/(c^2*d+e)*ln(1-e*(c*x+I*(-c^2*x^2+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+
e))^(1/2)-e))*arccos(c*x)+1/4*I*(c^2*d*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)*poly
log(2,e*(c*x+I*(-c^2*x^2+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e
))+I*(-2*d*c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e
))^(1/2)*e)*arccos(c*x)^2/e^3/(c^2*d+e)+1/2*I*(-2*d*c^2*(c^2*d*(c^2*d+e))^
(1/2)+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*polylog(2,e*(c*x+I*(-
c^2*x^2+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))/e^3/(c^2*d+e)-
1/2*(c^2*d*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)*arccos(c*x)*ln(1-e*(c*x+I*(-c...
```

Fricas [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3*arccos(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^3(a+b\,\mathrm{acos}\,(cx))}{(d+ex^2)^2}\,dx$$

input
```
integrate(x**3*(a+b*acos(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**3*(a + b*acos(c*x))/(d + e*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*arctan2
(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^3\,(a+b\,\mathrm{acos}(c\,x))}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^3*(a + b*acos(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((x^3*(a + b*acos(c*x)))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^2}\,dx$$
$$= \frac{2\left(\int \frac{acos(cx)x^3}{e^2x^4+2de\,x^2+d^2}dx\right)bd\,e^2+2\left(\int \frac{acos(cx)x^3}{e^2x^4+2de\,x^2+d^2}dx\right)b\,e^3x^2+\log(e\,x^2+d)\,ad+\log(e\,x^2+d)\,ae\,x^2-ae\,x}{2e^2\,(e\,x^2+d)}$$

input
```
int(x^3*(a+b*acos(c*x))/(e*x^2+d)^2,x)
```

output

```
(2*int((acos(c*x)*x**3)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*d*e**2 + 2*in
t((acos(c*x)*x**3)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*e**3*x**2 + log(d
+ e*x**2)*a*d + log(d + e*x**2)*a*e*x**2 - a*e*x**2)/(2*e**2*(d + e*x**2))
```

3.636 $\int \frac{x(a+b\arccos(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{x(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \frac{-a-b\arccos(cx)}{2e(d+ex^2)} + \frac{bc\arctan\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2\sqrt{d}e\sqrt{c^2d+e}}$$

output
```
1/2*(-a-b*arccos(c*x))/e/(e*x^2+d)+1/2*b*c*arctan((c^2*d+e)^(1/2)*x/d^(1/2
)/(-c^2*x^2+1)^(1/2))/d^(1/2)/e/(c^2*d+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{x(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = -\frac{\frac{a}{d+ex^2} + \frac{b\arccos(cx)}{d+ex^2} + \frac{bc\arctan\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{\sqrt{d}\sqrt{c^2d+e}}}{2e}$$

input
```
Integrate[(x*(a + b*ArcCos[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a/(d + e*x^2) + (b*ArcCos[c*x])/(d + e*x^2) + (b*c*ArcTan[(Sqrt[c^2*
d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(Sqrt[d]*Sqrt[c^2*d + e]))/e
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5229, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{ 5229}$$

$$-\frac{bc\int \frac{1}{\sqrt{1-c^2x^2}(ex^2+d)}dx}{2e}-\frac{a+b\arccos(cx)}{2e(d+ex^2)}$$

$$\Big\downarrow \text{ 291}$$

$$-\frac{bc\int \frac{1}{d-\frac{(-dc^2-e)x^2}{1-c^2x^2}}d\frac{x}{\sqrt{1-c^2x^2}}}{2e}-\frac{a+b\arccos(cx)}{2e(d+ex^2)}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{a+b\arccos(cx)}{2e(d+ex^2)}-\frac{bc\arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2\sqrt{d}e\sqrt{c^2d+e}}$$

input
```
Int[(x*(a + b*ArcCos[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a + b*ArcCos[c*x])/(e*(d + e*x^2)) - (b*c*ArcTan[(Sqrt[c^2*d + e]*x)
/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(2*Sqrt[d]*e*Sqrt[c^2*d + e])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 5229
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])/(2*e*(p + 1))), x]
 + Simp[b*(c/(2*e*(p + 1)))   Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x]
, x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(72) = 144$.

Time = 7.11 (sec) , antiderivative size = 404, normalized size of antiderivative = 4.70

method	result
parts	$-\frac{a}{2e(e\,x^2+d)}-\frac{b\,c^2\arccos(cx)}{2e(c^2e\,x^2+c^2d)}+\frac{b\,c^2\ln\left(\frac{\frac{2c^2d+2e}{e}-\frac{2\sqrt{-c^2de}\left(cx-\frac{\sqrt{-c^2de}}{e}\right)}{e}+2\sqrt{\frac{c^2d+e}{e}}\sqrt{-\left(cx-\frac{\sqrt{-c^2de}}{e}\right)^2-\frac{2\sqrt{\cdots}}{\cdots}}}{cx-\frac{\sqrt{-c^2de}}{e}}\right.}{4e\sqrt{-c^2de}\sqrt{\frac{c^2d+e}{e}}}\cdots$
derivativedivides	$\frac{-\frac{a\,c^4}{2e\left(c^2e\,x^2+c^2d\right)}+b\,c^4\left(-\frac{\arccos(cx)}{2e\left(c^2e\,x^2+c^2d\right)}-\frac{-\frac{\ln\left(\frac{\frac{2c^2d+2e}{e}-\frac{2\sqrt{-c^2de}\left(cx-\frac{\sqrt{-c^2de}}{e}\right)}{e}+2\sqrt{\frac{c^2d+e}{e}}\sqrt{-\left(cx-\frac{\sqrt{-c^2de}}{e}\right)^2-\cdots}}{cx-\frac{\sqrt{-c^2de}}{e}}\right.}{2\sqrt{-c^2de}\sqrt{\frac{c^2d+e}{e}}}\cdots}{\cdots}\right.}{\cdots}$
default	$\frac{-\frac{a\,c^4}{2e\left(c^2e\,x^2+c^2d\right)}+b\,c^4\left(-\frac{\arccos(cx)}{2e\left(c^2e\,x^2+c^2d\right)}-\frac{-\frac{\ln\left(\frac{\frac{2c^2d+2e}{e}-\frac{2\sqrt{-c^2de}\left(cx-\frac{\sqrt{-c^2de}}{e}\right)}{e}+2\sqrt{\frac{c^2d+e}{e}}\sqrt{-\left(cx-\frac{\sqrt{-c^2de}}{e}\right)^2-\cdots}}{cx-\frac{\sqrt{-c^2de}}{e}}\right.}{2\sqrt{-c^2de}\sqrt{\frac{c^2d+e}{e}}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x*(a+b*arccos(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/e/(e*x^2+d)-1/2*b*c^2/e/(c^2*e*x^2+c^2*d)*arccos(c*x)+1/4*b*c^2/e/(
-c^2*d*e)^(1/2)/((c^2*d+e)/e)^(1/2)*ln((2*(c^2*d+e)/e-2*(-c^2*d*e)^(1/2)/e
*(c*x-(-c^2*d*e)^(1/2)/e)+2*((c^2*d+e)/e)^(1/2)*(-(c*x-(-c^2*d*e)^(1/2)/e)
^2-2*(-c^2*d*e)^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1/2))/(c*x-
(-c^2*d*e)^(1/2)/e))-1/4*b*c^2/e/(-c^2*d*e)^(1/2)/((c^2*d+e)/e)^(1/2)*ln((
2*(c^2*d+e)/e+2*(-c^2*d*e)^(1/2)/e*(c*x+(-c^2*d*e)^(1/2)/e)+2*((c^2*d+e)/e
)^(1/2)*(-(c*x+(-c^2*d*e)^(1/2)/e)^2+2*(-c^2*d*e)^(1/2)/e*(c*x+(-c^2*d*e)^
(1/2)/e)+(c^2*d+e)/e)^(1/2))/(c*x+(-c^2*d*e)^(1/2)/e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(69) = 138$.

Time = 0.17 (sec) , antiderivative size = 520, normalized size of antiderivative = 6.05

$$\int \frac{x(a+b\arccos(cx))}{(d+ex^2)^2}\,dx$$
$$= \left[-\frac{4\,ac^2d^2-4\,(bc^2de+be^2)x^2\arccos(cx)+4\,ade+(bcex^2+bcd)\sqrt{-c^2d^2-de}\log\left(\frac{(8\,c^4d^2+8\,c^2de+e^2)x^4-2}{\cdots}\right.}{8\,(c^2d^3e+d^2e\cdots}\right.$$
$$-\frac{2\,ac^2d^2-2\,(bc^2de+be^2)x^2\arccos(cx)+2\,ade-(bcex^2+bcd)\sqrt{c^2d^2+de}\arctan\left(\frac{\sqrt{c^2d^2+de}\sqrt{-c^2x^2+1}((\cdots}{2\,((c^4d^2+c^2de)x^3-(\cdots}\right.}{4\,(c^2d^3e+d^2e^2+(c^2d^2e^2+de^3)x\cdots}$$

input
```
integrate(x*(a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*a*c^2*d^2 - 4*(b*c^2*d*e + b*e^2)*x^2*arccos(c*x) + 4*a*d*e + (b*
c*e*x^2 + b*c*d)*sqrt(-c^2*d^2 - d*e)*log(((8*c^4*d^2 + 8*c^2*d*e + e^2)*x
^4 - 2*(4*c^2*d^2 + 3*d*e)*x^2 + 4*sqrt(-c^2*d^2 - d*e)*sqrt(-c^2*x^2 + 1)
*((2*c^2*d + e)*x^3 - d*x) + d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 4*(b*c^2*
d^2 + b*d*e + (b*c^2*d*e + b*e^2)*x^2)*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*
x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 + d*e^3)*x^2), -1/4*(2*a*c^
2*d^2 - 2*(b*c^2*d*e + b*e^2)*x^2*arccos(c*x) + 2*a*d*e - (b*c*e*x^2 + b*c
*d)*sqrt(c^2*d^2 + d*e)*arctan(1/2*sqrt(c^2*d^2 + d*e)*sqrt(-c^2*x^2 + 1)*
((2*c^2*d + e)*x^2 - d)/((c^4*d^2 + c^2*d*e)*x^3 - (c^2*d^2 + d*e)*x)) + 2
*(b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*e^2)*x^2)*arctan(sqrt(-c^2*x^2 + 1)*c
*x/(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 + d*e^3)*x^2)]
```

Sympy [F]

$$\int \frac{x(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{x(a+b\,\mathrm{acos}\,(cx))}{(d+ex^2)^2}\,dx$$

input
```
integrate(x*(a+b*acos(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x*(a + b*acos(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arccos(cx))}{\left(d+ex^2\right)^2}\,dx=\int \frac{x\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^2} \,d x$$

input
```
int((x*(a + b*acos(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((x*(a + b*acos(c*x)))/(d + e*x^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1401, normalized size of antiderivative = 16.29

$$\int \frac{x(a+b\arccos(cx))}{\left(d+ex^2\right)^2}\,dx=\text{Too large to display}$$

input
```
int(x*(a+b*acos(c*x))/(e*x^2+d)^2,x)
```

output

```
( - 2*acos(c*x)*b*c**3*d**3 - 2*acos(c*x)*b*c*d**2*e - 2*sqrt(e)*sqrt(d)*s
qrt(c**2*d + e)*sqrt( - 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)*atan((t
an(asin(c*x)/2)*c*d)/(sqrt(d)*sqrt( - 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d
+ 2*e)))*b*d - 2*sqrt(e)*sqrt(d)*sqrt(c**2*d + e)*sqrt( - 2*sqrt(e)*sqrt(c
**2*d + e) + c**2*d + 2*e)*atan((tan(asin(c*x)/2)*c*d)/(sqrt(d)*sqrt( - 2*
sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)))*b*e*x**2 - 2*sqrt(d)*sqrt( - 2*
sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)*atan((tan(asin(c*x)/2)*c*d)/(sqrt
(d)*sqrt( - 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)))*b*c**2*d**2 - 2*s
qrt(d)*sqrt( - 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)*atan((tan(asin(c
*x)/2)*c*d)/(sqrt(d)*sqrt( - 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d + 2*e)))*
b*c**2*d*e*x**2 - 2*sqrt(d)*sqrt( - 2*sqrt(e)*sqrt(c**2*d + e) + c**2*d +
2*e)*atan((tan(asin(c*x)/2)*c*d)/(sqrt(d)*sqrt( - 2*sqrt(e)*sqrt(c**2*d +
e) + c**2*d + 2*e)))*b*d*e - 2*sqrt(d)*sqrt( - 2*sqrt(e)*sqrt(c**2*d + e)
+ c**2*d + 2*e)*atan((tan(asin(c*x)/2)*c*d)/(sqrt(d)*sqrt( - 2*sqrt(e)*sqr
t(c**2*d + e) + c**2*d + 2*e)))*b*e**2*x**2 - sqrt(e)*sqrt(d)*sqrt(c**2*d
+ e)*sqrt( - 2*sqrt(e)*sqrt(c**2*d + e) - c**2*d - 2*e)*log( - sqrt( - 2*s
qrt(e)*sqrt(c**2*d + e) - c**2*d - 2*e) + sqrt(d)*tan(asin(c*x)/2)*c)*b*d
- sqrt(e)*sqrt(d)*sqrt(c**2*d + e)*sqrt( - 2*sqrt(e)*sqrt(c**2*d + e) - c*
*2*d - 2*e)*log( - sqrt( - 2*sqrt(e)*sqrt(c**2*d + e) - c**2*d - 2*e) + sq
rt(d)*tan(asin(c*x)/2)*c)*b*e*x**2 + sqrt(e)*sqrt(d)*sqrt(c**2*d + e)*s...
```

3.637 $\int \frac{a+b\arccos(cx)}{x\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 597

$$\int \frac{a+b\arccos(cx)}{x(d+ex^2)^2}\,dx = \frac{a+b\arccos(cx)}{2d(d+ex^2)} - \frac{bc\arctan\left(\frac{\sqrt{c^2d+ex}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}} - \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^2} - \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^2} - \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^2} - \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^2} + \frac{(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{d^2} + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^2} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^2} - \frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{2d^2}$$

output
```
1/2*(a+b*arccos(c*x))/d/(e*x^2+d)-1/2*b*c*arctan((c^2*d+e)^(1/2)*x/d^(1/2)
/(-c^2*x^2+1)^(1/2))/d^(3/2)/(c^2*d+e)^(1/2)-1/2*(a+b*arccos(c*x))*ln(1-e^
(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2
*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)
-(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2
+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccos(c*x))*ln(
1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^2
+(a+b*arccos(c*x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^2+1/2*I*b*polylog(
2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^
2+1/2*I*b*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^
2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(
I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,e^(1/2)*(c*x+I*(-c^
2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^2-1/2*I*b*polylog(2,(c
*x+I*(-c^2*x^2+1)^(1/2))^2)/d^2
```

Mathematica [A] (warning: unable to verify)

Time = 1.18 (sec) , antiderivative size = 1145, normalized size of antiderivative = 1.92

$$\int \frac{a+b\arccos(cx)}{x\left(d+ex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
a/(2*d^2 + 2*d*e*x^2) + (a*Log[x])/d^2 - (a*Log[d + e*x^2])/(2*d^2) + (b*(
(Sqrt[d]*ArcCos[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (Sqrt[d]*ArcCos[c*x])/(Sqr
t[d] + I*Sqrt[e]*x) - (8*I)*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]
]*ArcTan[((c*Sqrt[d] - I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] - (
8*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] +
I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] - 2*ArcCos[c*x]*Log[1 - (I
*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - 4*ArcSin[S
qrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^
2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - 2*ArcCos[c*x]*Log[1 + (I*(-(c*Sqrt
[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - 4*ArcSin[Sqrt[1 - (I
*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*
E^(I*ArcCos[c*x]))/Sqrt[e]] - 2*ArcCos[c*x]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c
^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 4*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/S
qrt[e]]/Sqrt[2]]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]
))/Sqrt[e]] - 2*ArcCos[c*x]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*
ArcCos[c*x]))/Sqrt[e]] + 4*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]
*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 4*
ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] + (I*c*Sqrt[d]*Log[(2*e*(Sqrt[e
] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c*Sqrt[c^2*d +
e]*((-I)*Sqrt[d] + Sqrt[e]*x))])/Sqrt[c^2*d + e] - (I*c*Sqrt[d]*Log[(-2...
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{5233}$$

$$\int \left(-\frac{ex(a+b\arccos(cx))}{d^2\left(d+ex^2\right)}+\frac{a+b\arccos(cx)}{d^2x}-\frac{ex(a+b\arccos(cx))}{d\left(d+ex^2\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2d^2}-\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2d^2}-\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2d^2}-\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2d^2}+\frac{\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))}{d^2}+\frac{a+b\arccos(cx)}{2d(d+ex^2)}+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2d^2}-\frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{2d^2}+\frac{bc\arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}}$$

input
```
Int[(a + b*ArcCos[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
(a + b*ArcCos[c*x])/(2*d*(d + e*x^2)) + (b*c*ArcTan[(Sqrt[c^2*d + e]*x)/(S
qrt[d]*Sqrt[1 - c^2*x^2])])/(2*d^(3/2)*Sqrt[c^2*d + e]) - ((a + b*ArcCos[c
*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])]
)/(2*d^2) - ((a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sq
rt[-d] - I*Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcCos[c*x])*Log[1 - (Sqrt
[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*d^2) - ((a +
b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^
2*d + e])])/(2*d^2) + ((a + b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCos[c*x])])
/d^2 + ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*S
qrt[c^2*d + e]))])/d^2 + ((I/2)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(
c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(
I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))])/d^2 + ((I/2)*b*PolyLog
[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/d^2 - (
(I/2)*b*PolyLog[2, -E^((2*I)*ArcCos[c*x])])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.07 (sec) , antiderivative size = 512, normalized size of antiderivative = 0.86

method	result
parts	$\frac{a\ln(x)}{d^2} - \frac{a\ln(e\,x^2+d)}{2d^2} + \frac{a}{2d(e\,x^2+d)} + b\left(\frac{c^2\arccos(cx)}{2d(c^2e\,x^2+c^2d)} - \frac{i\,\mathrm{dilog}\left(1+i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)}{d^2} + \frac{i\left(\sum_{\_R1=\mathrm{R}}\right.}{}\right.$
derivativedivides	$\frac{a\ln(cx)}{d^2} + \frac{a\,c^2}{2d(c^2e\,x^2+c^2d)} - \frac{a\ln(c^2e\,x^2+c^2d)}{2d^2} + \frac{b\,c^2\arccos(cx)}{2d(c^2e\,x^2+c^2d)} + \frac{ib\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right.}\right.}{}$
default	$\frac{a\ln(cx)}{d^2} + \frac{a\,c^2}{2d(c^2e\,x^2+c^2d)} - \frac{a\ln(c^2e\,x^2+c^2d)}{2d^2} + \frac{b\,c^2\arccos(cx)}{2d(c^2e\,x^2+c^2d)} + \frac{ib\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right.}\right.}{}$

input
```
int((a+b*arccos(c*x))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a/d^2*ln(x)-1/2*a/d^2*ln(e*x^2+d)+1/2*a/d/(e*x^2+d)+b*(1/2*c^2*arccos(c*x)
/d/(c^2*e*x^2+c^2*d)-I/d^2*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+1/4*I/d^2
*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+
1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^
4+(4*c^2*d+2*e)*_Z^2+e))*e-1/2*I*(c^2*d*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*arc
tanh(1/4*(4*c^2*d+2*e*(c*x+I*(-c^2*x^2+1)^(1/2))^2+2*e)/(c^4*d^2+c^2*d*e)^
(1/2))-I/d^2*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+1/4*I/d^2*sum((_R1^2*e+
4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(
1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4
*c^2*d+2*e)*_Z^2+e))+1/d^2*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+
1/d^2*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x\left(d+ex^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acos(c*x))/x/(e*x**2+d)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x(d+ex^2)^2}\,dx = \int \frac{b\arccos(cx)+a}{(ex^2+d)^2x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + b*integrat
e(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(e^2*x^5 + 2*d*e*x^3 + d^2*x)
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arccos(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x(d+ex^2)^2}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{x\,(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*acos(c*x))/(x*(d + e*x^2)^2),x)
```

output
```
int((a + b*acos(c*x))/(x*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d+ex^2\right)^2}\,dx$$
$$= \frac{2\left(\int \frac{acos(cx)}{e^2x^5+2de\,x^3+d^2x}dx\right)b\,d^3+2\left(\int \frac{acos(cx)}{e^2x^5+2de\,x^3+d^2x}dx\right)b\,d^2e\,x^2-\log(e\,x^2+d)\,ad-\log(e\,x^2+d)\,ae\,x^2+2}{2d^2\left(e\,x^2+d\right)}$$

input
```
int((a+b*acos(c*x))/x/(e*x^2+d)^2,x)
```

output
```
(2*int(acos(c*x)/(d**2*x + 2*d*e*x**3 + e**2*x**5),x)*b*d**3 + 2*int(acos(
c*x)/(d**2*x + 2*d*e*x**3 + e**2*x**5),x)*b*d**2*e*x**2 - log(d + e*x**2)*
a*d - log(d + e*x**2)*a*e*x**2 + 2*log(x)*a*d + 2*log(x)*a*e*x**2 - a*e*x*
*2)/(2*d**2*(d + e*x**2))
```

3.638 $\int \frac{a+b \arccos(cx)}{x^3 (d+ex^2)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 632

$$\int \frac{a+b\arccos(cx)}{x^3(d+ex^2)^2}\,dx = -\frac{bc\sqrt{1-c^2x^2}}{2d^2x} - \frac{a+b\arccos(cx)}{2d^2x^2} - \frac{e(a+b\arccos(cx))}{2d^2(d+ex^2)} + \frac{bce\arctan\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2d^{5/2}\sqrt{c^2d+e}} + \frac{e(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{d^3} + \frac{e(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{d^3} + \frac{e(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{d^3} + \frac{e(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{d^3} - \frac{2e(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{d^3} - \frac{ibe\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{d^3} - \frac{ibe\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{d^3} - \frac{ibe\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{d^3} - \frac{ibe\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{d^3} + \frac{ibe\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{d^3}$$

output
```
-1/2*b*c*(-c^2*x^2+1)^(1/2)/d^2/x-1/2*(a+b*arccos(c*x))/d^2/x^2-1/2*e*(a+b
*arccos(c*x))/d^2/(e*x^2+d)+1/2*b*c*e*arctan((c^2*d+e)^(1/2)*x/d^(1/2)/(-c
^2*x^2+1)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)+e*(a+b*arccos(c*x))*ln(1-e^(1/2)*
(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^3+e*(a+b*ar
ccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+
e)^(1/2)))/d^3+e*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))
/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^3+e*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c
*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^3-2*e*(a+b*ar
ccos(c*x))*ln(1-(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^3-I*b*e*polylog(2,-e^(1/2)
*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^3-I*b*e*po
lylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)
))/d^3-I*b*e*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)
+(c^2*d+e)^(1/2)))/d^3-I*b*e*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/
(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^3+I*b*e*polylog(2,(c*x+I*(-c^2*x^2+1)^
(1/2))^2)/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 2.01 (sec) , antiderivative size = 1196, normalized size of antiderivative = 1.89

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^3*(d + e*x^2)^2),x]
```

output
```
-1/4*((2*a*d)/x^2 + (2*a*d*e)/(d + e*x^2) + 8*a*e*Log[x] - 4*a*e*Log[d + e
*x^2] + b*((-2*c*d*Sqrt[1 - c^2*x^2])/x + (2*d*ArcCos[c*x])/x^2 + (Sqrt[d]
*e*ArcCos[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (Sqrt[d]*e*ArcCos[c*x])/(Sqrt[d]
 + I*Sqrt[e]*x) - (16*I)*e*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]
*ArcTan[((c*Sqrt[d] - I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] - (1
6*I)*e*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d]
+ I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] - 4*e*ArcCos[c*x]*Log[1
- (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - 8*e*Ar
cSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(-(c*Sqrt[d]) + S
qrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - 4*e*ArcCos[c*x]*Log[1 + (I*(
-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - 8*e*ArcSin[S
qrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^
2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - 4*e*ArcCos[c*x]*Log[1 - (I*(c*Sqrt
[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 8*e*ArcSin[Sqrt[1 - (
I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^
(I*ArcCos[c*x]))/Sqrt[e]] - 4*e*ArcCos[c*x]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c
^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 8*e*ArcSin[Sqrt[1 + (I*c*Sqrt[d])
/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*
x]))/Sqrt[e]] + 8*e*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] + (I*c*Sqrt
[d]*e*Log[(2*e*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^...
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^2}\,dx$$

$$\Big\downarrow \text{5233}$$

$$\int \left(\frac{2e^2x(a+b\arccos(cx))}{d^3\left(d+ex^2\right)}-\frac{2e(a+b\arccos(cx))}{d^3x}+\frac{e^2x(a+b\arccos(cx))}{d^2\left(d+ex^2\right)^2}+\frac{a+b\arccos(cx)}{d^2x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{d^3}+\frac{e(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{d^3}+\frac{e(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{d^3}+\frac{e(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{d^3}-\frac{2e\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))}{d^3}-\frac{e(a+b\arccos(cx))}{2d^2(d+ex^2)}-\frac{a+b\arccos(cx)}{2d^2x^2}-\frac{ibe\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{d^3}-\frac{ibe\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{d^3}-\frac{ibe\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{d^3}-\frac{ibe\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{d^3}+\frac{ibe\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{d^3}-\frac{bce\arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2d^{5/2}\sqrt{c^2d+e}}+\frac{bc\sqrt{1-c^2x^2}}{2d^2x}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^3*(d + e*x^2)^2),x]
```

output
```
(b*c*Sqrt[1 - c^2*x^2])/(2*d^2*x) - (a + b*ArcCos[c*x])/(2*d^2*x^2) - (e*(
a + b*ArcCos[c*x]))/(2*d^2*(d + e*x^2)) - (b*c*e*ArcTan[(Sqrt[c^2*d + e]*x
)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(2*d^(5/2)*Sqrt[c^2*d + e]) + (e*(a + b*Ar
cCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d
+ e])])/d^3 + (e*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(
c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/d^3 + (e*(a + b*ArcCos[c*x])*Log[1 - (Sq
rt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/d^3 + (e*(a +
b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^
2*d + e])])/d^3 - (2*e*(a + b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCos[c*x])])
/d^3 - (I*b*e*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqr
t[c^2*d + e]))])/d^3 - (I*b*e*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sq
rt[-d] - I*Sqrt[c^2*d + e])])/d^3 - (I*b*e*PolyLog[2, -((Sqrt[e]*E^(I*ArcC
os[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))])/d^3 - (I*b*e*PolyLog[2, (Sqr
t[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/d^3 + (I*b*e*Po
lyLog[2, -E^((2*I)*ArcCos[c*x])])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.33 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.01

method	result
parts	$-\frac{a}{2d^2x^2}-\frac{2ae\ln(x)}{d^3}+\frac{ae\ln(e\,x^2+d)}{d^3}-\frac{ae}{2d^2(e\,x^2+d)}+b\,c^2\left(-\frac{-ic^4d\,x^2-ie\,c^4x^4-c^3dx\sqrt{-c^2x^2+1}-c^3e\,x^3}{2c^2x^2d^2(}\right.$
derivativedivides	$c^2\left(-\frac{a}{2d^2c^2x^2}-\frac{2ae\ln(cx)}{c^2d^3}+\frac{ae\ln(c^2e\,x^2+c^2d)}{c^2d^3}-\frac{ae}{2d^2(c^2e\,x^2+c^2d)}+b\,c^4\left(-\frac{-ic^4d\,x^2-ie\,c^4x^4-c^3dx\sqrt{}}{}\right.\right.$
default	$c^2\left(-\frac{a}{2d^2c^2x^2}-\frac{2ae\ln(cx)}{c^2d^3}+\frac{ae\ln(c^2e\,x^2+c^2d)}{c^2d^3}-\frac{ae}{2d^2(c^2e\,x^2+c^2d)}+b\,c^4\left(-\frac{-ic^4d\,x^2-ie\,c^4x^4-c^3dx\sqrt{}}{}\right.\right.$

input
```
int((a+b*arccos(c*x))/x^3/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d^2/x^2-2*a/d^3*e*ln(x)+a*e/d^3*ln(e*x^2+d)-1/2*a*e/d^2/(e*x^2+d)+b
*c^2*(-1/2*(-I*c^4*d*x^2-I*e*c^4*x^4-c^3*d*x*(-c^2*x^2+1)^(1/2)-c^3*e*x^3*
(-c^2*x^2+1)^(1/2)+c^2*d*arccos(c*x)+2*arccos(c*x)*e*c^2*x^2)/c^2/x^2/d^2/
(c^2*e*x^2+c^2*d)+1/2*I*(c^2*d*(c^2*d+e))^(1/2)/d^3/c^2/(c^2*d+e)*arctanh(
1/4*(4*c^2*d+2*e*(c*x+I*(-c^2*x^2+1)^(1/2))^2+2*e)/(c^4*d^2+c^2*d*e)^(1/2)
)*e-1/2*I*e^2/d^3*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1
-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1))
,_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))/c^2+2*I*e/d^3/c^2*dilog(1+I*(c*x
+I*(-c^2*x^2+1)^(1/2)))+2*I*e/d^3/c^2*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2))
)-1/2*I*e/d^3*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*l
n((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))
/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))/c^2-2*e/d^3/c^2*arccos(c*x
)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*e/d^3/c^2*arccos(c*x)*ln(1-I*(c*x+I
*(-c^2*x^2+1)^(1/2))))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^2}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(ex^2+d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\operatorname{acos}\left(cx\right)}{x^3\left(d+ex^2\right)^2}\,dx$$

input
```
integrate((a+b*acos(c*x))/x**3/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*acos(c*x))/(x**3*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^2}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(ex^2+d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*((2*e*x^2 + d)/(d^2*e*x^4 + d^3*x^2) - 2*e*log(e*x^2 + d)/d^3 + 4*e
*log(x)/d^3) + b*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(e^2
*x^7 + 2*d*e*x^5 + d^2*x^3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arccos(c*x))/x^3/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{x^3\,\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*acos(c*x))/(x^3*(d + e*x^2)^2),x)
```

output
```
int((a + b*acos(c*x))/(x^3*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^2}\,dx$$
$$= \frac{2\left(\int \frac{acos(cx)}{e^2x^7+2de\,x^5+d^2x^3}dx\right)b\,d^4x^2+2\left(\int \frac{acos(cx)}{e^2x^7+2de\,x^5+d^2x^3}dx\right)b\,d^3e\,x^4+2\log(e\,x^2+d)\,ade\,x^2+2\log(e\,x^2+a}{2d^3x^2\,(e\,x^2+d)}$$

input
```
int((a+b*acos(c*x))/x^3/(e*x^2+d)^2,x)
```

output
```
(2*int(acos(c*x)/(d**2*x**3 + 2*d*e*x**5 + e**2*x**7),x)*b*d**4*x**2 + 2*i
nt(acos(c*x)/(d**2*x**3 + 2*d*e*x**5 + e**2*x**7),x)*b*d**3*e*x**4 + 2*log
(d + e*x**2)*a*d*e*x**2 + 2*log(d + e*x**2)*a*e**2*x**4 - 4*log(x)*a*d*e*x
**2 - 4*log(x)*a*e**2*x**4 - a*d**2 + 2*a*e**2*x**4)/(2*d**3*x**2*(d + e*x
**2))
```

3.639 $\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 787

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \frac{ax}{e^2}+\frac{b\sqrt{1-c^2x^2}}{ce^2}+\frac{bx\arccos(cx)}{e^2}-\frac{d(a+b\arccos(cx))}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{d(a+b\arccos(cx))}{4e^{5/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bcd\,\text{arctanh}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{4e^{5/2}\sqrt{c^2d+e}}+\frac{bcd\,\text{arctanh}\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{4e^{5/2}\sqrt{c^2d+e}}+\frac{3\sqrt{-d}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}-\frac{3\sqrt{-d}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}+\frac{3\sqrt{-d}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}-\frac{3\sqrt{-d}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}+\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}-\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}+\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}-\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

output
```
a*x/e^2+b*(-c^2*x^2+1)^(1/2)/c/e^2+b*x*arccos(c*x)/e^2-1/4*d*(a+b*arccos(c
*x))/e^(5/2)/((-d)^(1/2)-e^(1/2)*x)+1/4*d*(a+b*arccos(c*x))/e^(5/2)/((-d)^
(1/2)+e^(1/2)*x)+1/4*b*c*d*arctanh((e^(1/2)-c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1
/2)/(-c^2*x^2+1)^(1/2))/e^(5/2)/(c^2*d+e)^(1/2)+1/4*b*c*d*arctanh((e^(1/2)
+c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/e^(5/2)/(c^2*d+e)^(
1/2)+3/4*(-d)^(1/2)*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/
2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)-3/4*(-d)^(1/2)*(a+b*arccos(c
*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/
2)))/e^(5/2)+3/4*(-d)^(1/2)*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^
2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)-3/4*(-d)^(1/2)*(a+b*
arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*
d+e)^(1/2)))/e^(5/2)+3/4*I*b*(-d)^(1/2)*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^
2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)-3/4*I*b*(-d)^(1/2)*p
olylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2
)))/e^(5/2)+3/4*I*b*(-d)^(1/2)*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2
))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)-3/4*I*b*(-d)^(1/2)*polylog(2,
e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^(5/
2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.44 (sec) , antiderivative size = 1171, normalized size of antiderivative = 1.49

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^4*(a + b*ArcCos[c*x]))/(d + e*x^2)^2,x]
```

output
```
(8*a*Sqrt[e]*x + (4*a*d*Sqrt[e]*x)/(d + e*x^2) - 12*a*Sqrt[d]*ArcTan[(Sqrt
[e]*x)/Sqrt[d]] + b*((2*d*ArcCos[c*x])/(I*Sqrt[d] + Sqrt[e]*x) + (8*Sqrt[e
]*(-Sqrt[1 - c^2*x^2] + c*x*ArcCos[c*x]))/c + 2*d*(ArcCos[c*x]/((-I)*Sqrt[
d] + Sqrt[e]*x) - (c*Log[(2*e*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]
*Sqrt[1 - c^2*x^2]))/(c*Sqrt[c^2*d + e]*((-I)*Sqrt[d] + Sqrt[e]*x))])/Sqrt
[c^2*d + e]) - (2*c*d*Log[(-2*e*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d +
e]*Sqrt[1 - c^2*x^2]))/(c*Sqrt[c^2*d + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[
c^2*d + e] - 3*Sqrt[d]*(ArcCos[c*x]^2 - 8*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sq
rt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c
^2*d + e]] + (2*I)*ArcCos[c*x]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])
*E^(I*ArcCos[c*x]))/Sqrt[e]] + (4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]
]/Sqrt[2]]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/
Sqrt[e]] + (2*I)*ArcCos[c*x]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I
*ArcCos[c*x]))/Sqrt[e]] - (4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqr
t[2]]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]]
 + 2*PolyLog[2, (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqr
t[e]] + 2*PolyLog[2, ((-I)*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x])
)/Sqrt[e]]) + 3*Sqrt[d]*(ArcCos[c*x]^2 - 8*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/S
qrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[
c^2*d + e]] + (2*I)*ArcCos[c*x]*Log[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d +...
```

Rubi [A] (verified)

Time = 2.70 (sec) , antiderivative size = 788, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{5233}$$

$$\int \left(\frac{d^2(a+b\arccos(cx))}{e^2(d+ex^2)^2}-\frac{2d(a+b\arccos(cx))}{e^2(d+ex^2)}+\frac{a+b\arccos(cx)}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
\frac{3\sqrt{-d}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{4e^{5/2}}-\frac{3\sqrt{-d}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{4e^{5/2}}+\frac{3\sqrt{-d}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{4e^{5/2}}-\frac{3\sqrt{-d}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{4e^{5/2}}-\frac{d(a+b\arccos(cx))}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{d(a+b\arccos(cx))}{4e^{5/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{ax}{e^2}+\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{4e^{5/2}}-\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{4e^{5/2}}+\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{4e^{5/2}}-\frac{3ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{4e^{5/2}}+\frac{bx\arccos(cx)}{e^2}-\frac{bcd\,\text{arctanh}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{1-c^2x^2}\sqrt{c^2d+e}}\right)}{4e^{5/2}\sqrt{c^2d+e}}-\frac{bcd\,\text{arctanh}\left(\frac{c^2\sqrt{-d}x+\sqrt{e}}{\sqrt{1-c^2x^2}\sqrt{c^2d+e}}\right)}{4e^{5/2}\sqrt{c^2d+e}}-\frac{b\sqrt{1-c^2x^2}}{ce^2}
$$

input
```
Int[(x^4*(a + b*ArcCos[c*x]))/(d + e*x^2)^2,x]
```

output
```
(a*x)/e^2 - (b*Sqrt[1 - c^2*x^2])/(c*e^2) + (b*x*ArcCos[c*x])/e^2 - (d*(a
+ b*ArcCos[c*x]))/(4*e^(5/2)*(Sqrt[-d] - Sqrt[e]*x)) + (d*(a + b*ArcCos[c*
x]))/(4*e^(5/2)*(Sqrt[-d] + Sqrt[e]*x)) - (b*c*d*ArcTanh[(Sqrt[e] - c^2*Sq
rt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(4*e^(5/2)*Sqrt[c^2*d + e]
) - (b*c*d*ArcTanh[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^
2*x^2])])/(4*e^(5/2)*Sqrt[c^2*d + e]) + (3*Sqrt[-d]*(a + b*ArcCos[c*x])*Lo
g[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(4*e^
(5/2)) - (3*Sqrt[-d]*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]
))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*Arc
Cos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d +
 e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I
*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(4*e^(5/2)) + (((3*I)/4)
*b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[
c^2*d + e]))])/e^(5/2) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^(I*Ar
cCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/e^(5/2) + (((3*I)/4)*b*Sqrt
[-d]*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d +
 e]))])/e^(5/2) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*
x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/e^(5/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.32 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.17

$$c^4 a \left(\frac{cx}{e^2} - \frac{c^2 d \left(-\frac{cx}{2(c^2 e x^2 + c^2 d)} + \frac{3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2c\sqrt{de}} \right)}{e^2} \right) + b c^4 \left(\frac{(\arccos(cx)+i)\left(cx + i\sqrt{-c^2x^2+1}\right)}{2e^2} + \frac{\left(-i\sqrt{-c^2x^2+1}+cx\right)(\arccos(cx)-i)}{2e^2} \right.$$

input
```
int(x^4*(a+b*arccos(c*x))/(e*x^2+d)^2,x)
```

output
```
1/c^5*(c^4*a*(1/e^2*c*x-1/e^2*c^2*d*(-1/2*c*x/(c^2*e*x^2+c^2*d)+3/2/c/(d*e
)^(1/2)*arctan(e*x/(d*e)^(1/2))))+b*c^4*(1/2*(arccos(c*x)+I)/e^2*(c*x+I*(-
c^2*x^2+1)^(1/2))+1/2*(-I*(-c^2*x^2+1)^(1/2)+c*x)*(arccos(c*x)-I)/e^2+1/2*
arccos(c*x)/e^2*c^3*d*x/(c^2*e*x^2+c^2*d)-1/2*I*((2*c^2*d+2*(c^2*d*(c^2*d+
e))^(1/2)+e)*e)^(1/2)*(-2*d*c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^4*d^2+2*c^2*d*
e-(c^2*d*(c^2*d+e))^(1/2)*e)*c^2*d*arctan(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((2
*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^5/(c^2*d+e)+1/2*I*((2*c^2*
d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)
+e)*arctan(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2
)+e)*e)^(1/2))*c^2*d/e^5-1/2*I*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^
(1/2)*(2*c^4*d^2+2*d*c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^2*d*e+(c^2*d*(c^2*d+e
))^(1/2)*e)*c^2*d*arctanh(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d
*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^5/(c^2*d+e)+1/2*I*(-(2*c^2*d-2*(c^2*d*(c^
2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctanh(e*
(c*x+I*(-c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2
))*c^2*d/e^5-3/4*I/e^2*c^2*d*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*
ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2)
)/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+3/4*I/e^2*c^2*d*sum(_R1/(
_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+d
ilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*...
```

Fricas [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)x^4}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4*arccos(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^4(a+b\operatorname{acos}(cx))}{(d+ex^2)^2}\,dx$$

input
```
integrate(x**4*(a+b*acos(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**4*(a + b*acos(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^4\,(a+b\,\mathrm{acos}(c\,x))}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^4*(a + b*acos(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((x^4*(a + b*acos(c*x)))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^2}\,dx$$
$$= \frac{-3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad - 3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ae\,x^2 + 2\left(\int \frac{acos(cx)x^4}{e^2x^4+2de\,x^2+d^2}dx\right)bd\,e^3 + 2\left(\int \frac{acos(cx)x^4}{e^2x^4+2de\,x^2+}\right.}{2e^3\,(e\,x^2+d)}$$

input
```
int(x^4*(a+b*acos(c*x))/(e*x^2+d)^2,x)
```

output
```
( - 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d - 3*sqrt(e)*sqrt(d
)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e*x**2 + 2*int((acos(c*x)*x**4)/(d**2 +
2*d*e*x**2 + e**2*x**4),x)*b*d*e**3 + 2*int((acos(c*x)*x**4)/(d**2 + 2*d*e
*x**2 + e**2*x**4),x)*b*e**4*x**2 + 3*a*d*e*x + 2*a*e**2*x**3)/(2*e**3*(d
+ e*x**2))
```

3.640 $\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 745

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \frac{a+b\arccos(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{a+b\arccos(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{b\text{carctanh}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{4e^{3/2}\sqrt{c^2d+e}} - \frac{b\text{carctanh}\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{4e^{3/2}\sqrt{c^2d+e}} + \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

output
```
1/4*(a+b*arccos(c*x))/e^(3/2)/((-d)^(1/2)-e^(1/2)*x)-1/4*(a+b*arccos(c*x))
/e^(3/2)/((-d)^(1/2)+e^(1/2)*x)-1/4*b*c*arctanh((e^(1/2)-c^2*(-d)^(1/2)*x)
/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/e^(3/2)/(c^2*d+e)^(1/2)-1/4*b*c*arcta
nh((e^(1/2)+c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/e^(3/2)/
(c^2*d+e)^(1/2)+1/4*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/
2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(3/2)-1/4*(a+b*arccos(c
*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/
2)))/(-d)^(1/2)/e^(3/2)+1/4*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^
2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(3/2)-1/4*(a+b*
arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*
d+e)^(1/2)))/(-d)^(1/2)/e^(3/2)+1/4*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^
2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(3/2)-1/4*I*b*p
olylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2
)))/(-d)^(1/2)/e^(3/2)+1/4*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2
))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(3/2)-1/4*I*b*polylog(2,
e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^
(1/2)/e^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.13 (sec) , antiderivative size = 1122, normalized size of antiderivative = 1.51

$$\int \frac{x^2(a+b\arccos(cx))}{\left(d+ex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x]))/(d + e*x^2)^2,x]
```

output
```
((-4*a*Sqrt[e]*x)/(d + e*x^2) + (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d]
+ b*((2*ArcCos[c*x])/(I*Sqrt[d] - Sqrt[e]*x) - (2*ArcCos[c*x])/(I*Sqrt[d]
+ Sqrt[e]*x) + (2*c*Log[(2*e*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*
Sqrt[1 - c^2*x^2]))/(c*Sqrt[c^2*d + e]*((-I)*Sqrt[d] + Sqrt[e]*x))])/Sqrt[
c^2*d + e] + (2*c*Log[(-2*e*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*S
qrt[1 - c^2*x^2]))/(c*Sqrt[c^2*d + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[c^2*
d + e] + (ArcCos[c*x]^2 - 8*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]
]*ArcTan[((c*Sqrt[d] + I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (
2*I)*ArcCos[c*x]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c
*x]))/Sqrt[e]] + (4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log
[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*
I)*ArcCos[c*x]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))
/Sqrt[e]] - (4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 +
(I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2
, (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*Poly
Log[2, ((-I)*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]])/Sq
rt[d] - (ArcCos[c*x]^2 - 8*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]
*ArcTan[((c*Sqrt[d] - I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (2
*I)*ArcCos[c*x]*Log[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*
x]))/Sqrt[e]] + (4*I)*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*L...
```

Rubi [A] (verified)

Time = 2.48 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{5233}$$

$$\int \left(\frac{a+b\arccos(cx)}{e(d+ex^2)}-\frac{d(a+b\arccos(cx))}{e(d+ex^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{a+b\arccos(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{a+b\arccos(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{bc\operatorname{arctanh}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{1-c^2x^2}\sqrt{c^2d+e}}\right)}{4e^{3/2}\sqrt{c^2d+e}}+\frac{bc\operatorname{arctanh}\left(\frac{c^2\sqrt{-d}x+\sqrt{e}}{\sqrt{1-c^2x^2}\sqrt{c^2d+e}}\right)}{4e^{3/2}\sqrt{c^2d+e}}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x]))/(d + e*x^2)^2,x]
```

output
```
(a + b*ArcCos[c*x])/(4*e^(3/2)*(Sqrt[-d] - Sqrt[e]*x)) - (a + b*ArcCos[c*x
])/(4*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*ArcTanh[(Sqrt[e] - c^2*Sqrt[-
d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(4*e^(3/2)*Sqrt[c^2*d + e]) +
(b*c*ArcTanh[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]
)])/(4*e^(3/2)*Sqrt[c^2*d + e]) + ((a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^
(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) -
 ((a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*
Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcCos[c*x])*Log[1 - (Sq
rt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^
(3/2)) - ((a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[
-d] + I*Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) + ((I/4)*b*PolyLog[2, -((S
qrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e]))])/(Sqrt[-d]*e^
(3/2)) - ((I/4)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*S
qrt[c^2*d + e])])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*PolyLog[2, -((Sqrt[e]*E^(I
*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))])/(Sqrt[-d]*e^(3/2)) - ((
I/4)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d +
 e])])/(Sqrt[-d]*e^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.58 (sec) , antiderivative size = 819, normalized size of antiderivative = 1.10

method	result
derivativedivides	$-\frac{a c^5 x}{2e\left(c^2 e x^2+c^2 d\right)}+\frac{a c^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b c^4\left(-\frac{\arccos(cx) c x}{2e\left(c^2 e x^2+c^2 d\right)}-\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(i\,\mathrm{ar}\right.}{}\right.}{}\right.$
default	$-\frac{a c^5 x}{2e\left(c^2 e x^2+c^2 d\right)}+\frac{a c^3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b c^4\left(-\frac{\arccos(cx) c x}{2e\left(c^2 e x^2+c^2 d\right)}-\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(i\,\mathrm{ar}\right.}{}\right.}{}\right.$
parts	$-\frac{ax}{2e(e x^2+d)}+\frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+\frac{b\left(-\frac{c^5 \arccos(cx) x}{2e\left(c^2 e x^2+c^2 d\right)}-\frac{i c^4\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(i\,\mathrm{arc}\right.}{}\right.}{}\right.}{}$

input
```
int(x^2*(a+b*arccos(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-1/2*a*c^5/e*x/(c^2*e*x^2+c^2*d)+1/2*a*c^3/e/(d*e)^(1/2)*arctan(e*x
/(d*e)^(1/2))+b*c^4*(-1/2*arccos(c*x)/e*c*x/(c^2*e*x^2+c^2*d)-1/4*I/e*sum(
_R1/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_
R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+
2*e)*_Z^2+e))+1/2*I*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(-2*d*
c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)
*arctan(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e
)*e)^(1/2))/e^4/(c^2*d+e)-1/2*I*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^
(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctan(e*(c*x+I*(-c^2*x^2+1)^(
1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4+1/2*I*(-(2*c^2*
d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^4*d^2+2*d*c^2*(c^2*d*(c^2*d+e
))^(1/2)+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1/2)*e)*arctanh(e*(c*x+I*(-c^2*x^2+1
)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)-1
/2*I*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)+e)*arctanh(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d
*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^4+1/4*I/e*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(
I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^
2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))))
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arccos{(cx)}+a)x^2}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2*arccos(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^2(a+b\operatorname{acos}(cx))}{(d+ex^2)^2}\,dx$$

input
```
integrate(x**2*(a+b*acos(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**2*(a + b*acos(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)x^2}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)*x^2/(e*x^2 + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^2\,(a+b\,\mathrm{acos}(c\,x))}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^2*(a + b*acos(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((x^2*(a + b*acos(c*x)))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad+\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ae\,x^2+2\left(\int \frac{acos(cx)x^2}{e^2x^4+2de\,x^2+d^2}dx\right)b\,d^2e^2+2\left(\int \frac{acos(cx)x^2}{e^2x^4+2de\,x^2+d^2}d\right.}{2d\,e^2\,(e\,x^2+d)}$$

input
```
int(x^2*(a+b*acos(c*x))/(e*x^2+d)^2,x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d + sqrt(e)*sqrt(d)*atan(
(e*x)/(sqrt(e)*sqrt(d)))*a*e*x**2 + 2*int((acos(c*x)*x**2)/(d**2 + 2*d*e*x
**2 + e**2*x**4),x)*b*d**2*e**2 + 2*int((acos(c*x)*x**2)/(d**2 + 2*d*e*x**
2 + e**2*x**4),x)*b*d*e**3*x**2 - a*d*e*x)/(2*d*e**2*(d + e*x**2))
```

3.641 $\int \frac{a+b\arccos(cx)}{\left(d+ex^2\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 757

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^2}\,dx = -\frac{a+b\arccos(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\arccos(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)} + \frac{b c \operatorname{arctanh}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{4d\sqrt{e}\sqrt{c^2d+e}} + \frac{b c \operatorname{arctanh}\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{4d\sqrt{e}\sqrt{c^2d+e}} - \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

output
```
-1/4*(a+b*arccos(c*x))/d/e^(1/2)/((-d)^(1/2)-e^(1/2)*x)+1/4*(a+b*arccos(c*
x))/d/e^(1/2)/((-d)^(1/2)+e^(1/2)*x)+1/4*b*c*arctanh((e^(1/2)-c^2*(-d)^(1/
2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d/e^(1/2)/(c^2*d+e)^(1/2)+1/4*b*
c*arctanh((e^(1/2)+c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d
/e^(1/2)/(c^2*d+e)^(1/2)-1/4*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x
^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b
*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2
*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I
*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+
1/4*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1
/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*I*b*polylog(2,-e^(1/2)*(c*x+I
*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+
1/4*I*b*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*
d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^
2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*I*b*p
olylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2
)))/(-d)^(3/2)/e^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.13 (sec) , antiderivative size = 1111, normalized size of antiderivative = 1.47

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d + e*x^2)^2,x]
```

output
```
((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
 + (b*(4*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d
] - I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] - 4*ArcSin[Sqrt[1 + (I
*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + I*Sqrt[e])*Tan[ArcCos[c
*x]/2])/Sqrt[c^2*d + e]] + I*ArcCos[c*x]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c
^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d
])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcC
os[c*x]))/Sqrt[e]] - I*ArcCos[c*x]*Log[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d +
 e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - (2*I)*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqr
t[e]]/Sqrt[2]]*Log[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x
]))/Sqrt[e]] - I*ArcCos[c*x]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I
*ArcCos[c*x]))/Sqrt[e]] + (2*I)*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqr
t[2]]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]]
 + I*ArcCos[c*x]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]
))/Sqrt[e]] - (2*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1
+ (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + Sqrt[d]*(
ArcCos[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) - (c*Log[(2*e*(Sqrt[e] - I*c^2*Sqrt
[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c*Sqrt[c^2*d + e]*((-I)*Sqrt[
d] + Sqrt[e]*x))])/Sqrt[c^2*d + e]) + Sqrt[d]*(ArcCos[c*x]/(I*Sqrt[d] + Sq
rt[e]*x) - (c*Log[(-2*e*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sq...
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 757, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^2}\,dx$$
$$\Big\downarrow \text{5173}$$
$$\int \left(-\frac{e(a+b\arccos(cx))}{2d\,(-de-e^2x^2)}-\frac{e(a+b\arccos(cx))}{4d\,\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e(a+b\arccos(cx))}{4d\,\left(\sqrt{-d}\sqrt{e}+ex\right)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{a+b\arccos(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\arccos(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{bc\operatorname{arctanh}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{1-c^2x^2}\sqrt{c^2d+e}}\right)}{4d\sqrt{e}\sqrt{c^2d+e}}-\frac{bc\operatorname{arctanh}\left(\frac{c^2\sqrt{-d}x+\sqrt{e}}{\sqrt{1-c^2x^2}\sqrt{c^2d+e}}\right)}{4d\sqrt{e}\sqrt{c^2d+e}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d + e*x^2)^2,x]
```

output
```
-1/4*(a + b*ArcCos[c*x])/(d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcCo
s[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) - (b*c*ArcTanh[(Sqrt[e] - c^2
*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(4*d*Sqrt[e]*Sqrt[c^2*d
 + e]) - (b*c*ArcTanh[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 -
 c^2*x^2])])/(4*d*Sqrt[e]*Sqrt[c^2*d + e]) - ((a + b*ArcCos[c*x])*Log[1 -
(Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(4*(-d)^(3/
2)*Sqrt[e]) + ((a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*
Sqrt[-d] - I*Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcCos[c*
x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])
/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCo
s[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((I/4
)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d +
e]))])/((-d)^(3/2)*Sqrt[e]) + ((I/4)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x
]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/((-d)^(3/2)*Sqrt[e]) - ((I/4)*b*Pol
yLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))])/
((-d)^(3/2)*Sqrt[e]) + ((I/4)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*
Sqrt[-d] + I*Sqrt[c^2*d + e])])/((-d)^(3/2)*Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.59 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.10

method	result
parts	$\frac{ax}{2d(e\,x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+\frac{b\left(\frac{c^3\arccos(cx)x}{2d\left(c^2e\,x^2+c^2d\right)}-\frac{i\sqrt{\left(2c^2d+2\sqrt{c^2d\left(c^2d+e\right)}+e\right)e}\left(-2d\,c^2\sqrt{c^2d\left(c^2d+e\right)}+2c^4d^2+2c^3\right.}{2e^3d\left(c\right.}\right.}{}$
derivativedivides	$\frac{\frac{a\,c^3x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\arccos(cx)x}{2cd\left(c^2e\,x^2+c^2d\right)}-\frac{i\sqrt{\left(2c^2d+2\sqrt{c^2d\left(c^2d+e\right)}+e\right)e}\left(-2d\,c^2\sqrt{c^2d\left(c^2d+e\right)}+2c^4d^2+\right.}{2d\,c^2\left(\right.}\right.}{}$
default	$\frac{\frac{a\,c^3x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\arccos(cx)x}{2cd\left(c^2e\,x^2+c^2d\right)}-\frac{i\sqrt{\left(2c^2d+2\sqrt{c^2d\left(c^2d+e\right)}+e\right)e}\left(-2d\,c^2\sqrt{c^2d\left(c^2d+e\right)}+2c^4d^2+\right.}{2d\,c^2\left(\right.}\right.}{}$

input
```
int((a+b*arccos(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x/d/(e*x^2+d)+1/2*a/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c*(1/2*c
^3*arccos(c*x)*x/d/(c^2*e*x^2+c^2*d)-1/2*I*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(
1/2)+e)*e)^(1/2)*(-2*d*c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^4*d^2+2*c^2*d*e-(c^
2*d*(c^2*d+e))^(1/2)*e)*c^2*arctan(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((2*c^2*d+
2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/d/(c^2*d+e)+1/2*I*((2*c^2*d+2*(
c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*a
rctan(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*
e)^(1/2))*c^2/d/e^3-1/2*I*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)
*(2*c^4*d^2+2*d*c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1
/2)*e)*c^2*arctanh(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d
+e))^(1/2)-e)*e)^(1/2))/e^3/d/(c^2*d+e)+1/2*I*(-(2*c^2*d-2*(c^2*d*(c^2*d+e
))^(1/2)+e)*e)^(1/2)*(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctanh(e*(c*x+
I*(-c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))*c^
2/d/e^3-1/4*I/d*c^2*sum(_R1/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x
-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1
=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/4*I/d*c^2*sum(1/_R1/(_R1^2*e+2*c^2
*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x
-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^2}\,dx=\int \frac{b\arccos{(cx)}+a}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F]

$$\int \frac{a + b\arccos(cx)}{(d + ex^2)^2}\,dx = \int \frac{a + b\operatorname{acos}(cx)}{(d + ex^2)^2}\,dx$$

input
```
integrate((a+b*acos(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*acos(c*x))/(d + e*x**2)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\arccos(cx)}{(d + ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\arccos(cx)}{(d + ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^2}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*acos(c*x))/(d + e*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))/(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^2}\,dx$$
$$=\frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ad+\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ae\,x^2+2\left(\int\frac{acos(cx)}{e^2x^4+2de\,x^2+d^2}dx\right)b\,d^3e+2\left(\int\frac{acos(cx)}{e^2x^4+2de\,x^2+d^2}dx\right.}{2d^2e\,(e\,x^2+d)}$$

input
```
int((a+b*acos(c*x))/(e*x^2+d)^2,x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d + sqrt(e)*sqrt(d)*atan(
(e*x)/(sqrt(e)*sqrt(d)))*a*e*x**2 + 2*int(acos(c*x)/(d**2 + 2*d*e*x**2 + e
**2*x**4),x)*b*d**3*e + 2*int(acos(c*x)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)
*b*d**2*e**2*x**2 + a*d*e*x)/(2*d**2*e*(d + e*x**2))
```

3.642 $\int \frac{a+b \arccos(cx)}{x^2 (d+ex^2)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 795

$$\int \frac{a+b\arccos(cx)}{x^2(d+ex^2)^2}\,dx = -\frac{a+b\arccos(cx)}{d^2x}+\frac{\sqrt{e}(a+b\arccos(cx))}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)} - \frac{\sqrt{e}(a+b\arccos(cx))}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)} - \frac{bc\sqrt{e}\,\mathrm{arctanh}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{4d^2\sqrt{c^2d+e}} - \frac{bc\sqrt{e}\,\mathrm{arctanh}\left(\frac{\sqrt{e}+c^2\sqrt{-d}x}{\sqrt{c^2d+e}\sqrt{1-c^2x^2}}\right)}{4d^2\sqrt{c^2d+e}} - \frac{b\,\mathrm{carctanh}\left(\sqrt{1-c^2x^2}\right)}{d^2} - \frac{3\sqrt{e}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} + \frac{3\sqrt{e}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3\sqrt{e}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} + \frac{3\sqrt{e}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} + \frac{3ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} - \frac{3ib\sqrt{e}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}} + \frac{3ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

output
```
-(a+b*arccos(c*x))/d^2/x+1/4*e^(1/2)*(a+b*arccos(c*x))/d^2/((-d)^(1/2)-e^(
1/2)*x)-1/4*e^(1/2)*(a+b*arccos(c*x))/d^2/((-d)^(1/2)+e^(1/2)*x)-1/4*b*c*e
^(1/2)*arctanh((e^(1/2)-c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/
2))/d^2/(c^2*d+e)^(1/2)-1/4*b*c*e^(1/2)*arctanh((e^(1/2)+c^2*(-d)^(1/2)*x)
/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d^2/(c^2*d+e)^(1/2)-b*c*arctanh((-c^2
*x^2+1)^(1/2))/d^2-3/4*e^(1/2)*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2
*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)+3/4*e^(1/2)*(a
+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c
^2*d+e)^(1/2)))/(-d)^(5/2)-3/4*e^(1/2)*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x
+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)+3/4*e^
(1/2)*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^
(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)-3/4*I*b*e^(1/2)*polylog(2,-e^(1/2)*(c*x
+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)+3/4*I*
b*e^(1/2)*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^
2*d+e)^(1/2)))/(-d)^(5/2)-3/4*I*b*e^(1/2)*polylog(2,-e^(1/2)*(c*x+I*(-c^2*
x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)+3/4*I*b*e^(1/2)
*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1
/2)))/(-d)^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.76 (sec) , antiderivative size = 1198, normalized size of antiderivative = 1.51

$$\int \frac{a+b\arccos(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^2*(d + e*x^2)^2),x]
```

output
```
((-8*a*Sqrt[d])/x - (4*a*Sqrt[d]*e*x)/(d + e*x^2) - 12*a*Sqrt[e]*ArcTan[(S
qrt[e]*x)/Sqrt[d]] + b*((-8*Sqrt[d]*(ArcCos[c*x] + c*x*(Log[x] - Log[1 + S
qrt[1 - c^2*x^2]])))/x - 2*Sqrt[d]*Sqrt[e]*(ArcCos[c*x]/((-I)*Sqrt[d] + Sq
rt[e]*x) - (c*Log[(2*e*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1
 - c^2*x^2]))/(c*Sqrt[c^2*d + e]*((-I)*Sqrt[d] + Sqrt[e]*x))])/Sqrt[c^2*d
+ e]) + 2*Sqrt[d]*Sqrt[e]*(-(ArcCos[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) + (c*Log
[(-2*e*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c
*Sqrt[c^2*d + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[c^2*d + e]) - 3*Sqrt[e]*(
ArcCos[c*x]^2 - 8*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[(
(c*Sqrt[d] + I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (2*I)*ArcCo
s[c*x]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt
[e]] + (4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(-
(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)*ArcCos[
c*x]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]]
- (4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(c*Sqrt
[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2, (I*(-(c*
Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 2*PolyLog[2, ((-
I)*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]]) + 3*Sqrt[e]*
(ArcCos[c*x]^2 - 8*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[
((c*Sqrt[d] - I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (2*I)*A...
```

Rubi [A] (verified)

Time = 2.62 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^2\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{5233}$$

$$\int \left(-\frac{e(a+b\arccos(cx))}{d^2\left(d+ex^2\right)}+\frac{a+b\arccos(cx)}{d^2x^2}-\frac{e(a+b\arccos(cx))}{d\left(d+ex^2\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{e}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{\sqrt{e}(a+b\arccos(cx))}{4d^2\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{\sqrt{e}(a+b\arccos(cx))}{4d^2\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+b\arccos(cx)}{d^2x}-\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}-\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+\frac{bc\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{1-c^2x^2}\sqrt{c^2d+e}}\right)}{4d^2\sqrt{c^2d+e}}+\frac{bc\sqrt{e}\operatorname{arctanh}\left(\frac{c^2\sqrt{-d}x+\sqrt{e}}{\sqrt{1-c^2x^2}\sqrt{c^2d+e}}\right)}{4d^2\sqrt{c^2d+e}}+\frac{b\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{d^2}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^2*(d + e*x^2)^2),x]
```

output
```
-((a + b*ArcCos[c*x])/(d^2*x)) + (Sqrt[e]*(a + b*ArcCos[c*x]))/(4*d^2*(Sqr
t[-d] - Sqrt[e]*x)) - (Sqrt[e]*(a + b*ArcCos[c*x]))/(4*d^2*(Sqrt[-d] + Sqr
t[e]*x)) + (b*c*Sqrt[e]*ArcTanh[(Sqrt[e] - c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e
]*Sqrt[1 - c^2*x^2])])/(4*d^2*Sqrt[c^2*d + e]) + (b*c*Sqrt[e]*ArcTanh[(Sqr
t[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(4*d^2*Sqrt[c
^2*d + e]) + (b*c*ArcTanh[Sqrt[1 - c^2*x^2]])/d^2 - (3*Sqrt[e]*(a + b*ArcC
os[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d +
e])]])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(
I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])]])/(4*(-d)^(5/2)) - (3*Sqr
t[e]*(a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] +
 I*Sqrt[c^2*d + e])]])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCos[c*x])*Log[
1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])]])/(4*(-d)
^(5/2)) - (((3*I)/4)*b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c
*Sqrt[-d] - I*Sqrt[c^2*d + e]))]])/(-d)^(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLo
g[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])]])/(-d)^(
5/2) - (((3*I)/4)*b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sq
rt[-d] + I*Sqrt[c^2*d + e]))]])/(-d)^(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2
, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])]])/(-d)^(5/2
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.76 (sec) , antiderivative size = 907, normalized size of antiderivative = 1.14

method	result
parts	$a\left(-\frac{1}{d^2x}-\frac{e\left(\frac{x}{2e\,x^2+2d}+\frac{3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}}\right)}{d^2}\right)+bc\left(-\frac{\arccos(cx)\left(3c^2e\,x^2+2c^2d\right)}{2cx\,d^2\left(c^2e\,x^2+c^2d\right)}-\frac{2i\arctan\left(cx+i\sqrt{-c^2x^2}\right.}{d^2}\right.$
derivativedivides	$c\left(-\frac{a}{d^2cx}-\frac{aecx}{2d^2\left(c^2e\,x^2+c^2d\right)}-\frac{3ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2c\,d^2\sqrt{de}}+b\,c^4\left(-\frac{\arccos(cx)\left(3c^2e\,x^2+2c^2d\right)}{2c^5x\left(c^2e\,x^2+c^2d\right)d^2}-\frac{i\sqrt{-\left(2c^2d-2\sqrt{\right.}}}{}\right.\right.$
default	$c\left(-\frac{a}{d^2cx}-\frac{aecx}{2d^2\left(c^2e\,x^2+c^2d\right)}-\frac{3ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2c\,d^2\sqrt{de}}+b\,c^4\left(-\frac{\arccos(cx)\left(3c^2e\,x^2+2c^2d\right)}{2c^5x\left(c^2e\,x^2+c^2d\right)d^2}-\frac{i\sqrt{-\left(2c^2d-2\sqrt{\right.}}}{}\right.\right.$

input
```
int((a+b*arccos(c*x))/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/d^2/x-e/d^2*(1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
))+b*c*(-1/2/c/x*arccos(c*x)*(3*c^2*e*x^2+2*c^2*d)/d^2/(c^2*e*x^2+c^2*d)-2
*I/d^2*arctan(c*x+I*(-c^2*x^2+1)^(1/2))+1/2*I*((2*c^2*d+2*(c^2*d*(c^2*d+e)
)^(1/2)+e)*e)^(1/2)*(-2*d*c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^4*d^2+2*c^2*d*e-
(c^2*d*(c^2*d+e))^(1/2)*e)*arctan(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((2*c^2*d+2
*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d^2/(c^2*d+e)/e^2+1/2*I*(-(2*c^2*d-2
*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^4*d^2+2*d*c^2*(c^2*d*(c^2*d+e))^
(1/2)+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1/2)*e)*arctanh(e*(c*x+I*(-c^2*x^2+1)^(
1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/d^2/(c^2*d+e)/e^2+
3/16*I/d^3*e*sum((4*_R1^2*c^2*d+_R1^2*e+e)/_R1/(_R1^2*e+2*c^2*d+e)*(I*arcc
os(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+
1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))/c^2-3/16*I/d^3*e*
sum((_R1^2*e+4*c^2*d+e)/_R1/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x
-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1
=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))/c^2-1/2*I*((2*c^2*d+2*(c^2*d*(c^2*d+
e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctan(e*(c*x+
I*(-c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d^2
/e^2-1/2*I*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d+2*(c^
2*d*(c^2*d+e))^(1/2)+e)*arctanh(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((-2*c^2*d+2*
(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/d^2/e^2)
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(ex^2+d\right)^2x^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\operatorname{acos}\left(cx\right)}{x^2\left(d+ex^2\right)^2}\,dx$$

input
```
integrate((a+b*acos(c*x))/x**2/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*acos(c*x))/(x**2*(d + e*x**2)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arccos(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{x^2\,\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*acos(c*x))/(x^2*(d + e*x^2)^2),x)
```

output
```
int((a + b*acos(c*x))/(x^2*(d + e*x^2)^2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \frac{-3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)adx-3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ae\,x^3+2\left(\int\frac{acos(cx)}{e^2x^6+2de\,x^4+d^2x^2}dx\right)b\,d^4x+2\left(\int\frac{acos(}{e^2x^6+2de}\right.}{2d^3x\left(e\,x^2+d\right)}$$

input
```
int((a+b*acos(c*x))/x^2/(e*x^2+d)^2,x)
```

output
```
( - 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*x - 3*sqrt(e)*sqrt
(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e*x**3 + 2*int(acos(c*x)/(d**2*x**2 +
2*d*e*x**4 + e**2*x**6),x)*b*d**4*x + 2*int(acos(c*x)/(d**2*x**2 + 2*d*e*x
**4 + e**2*x**6),x)*b*d**3*e*x**3 - 2*a*d**2 - 3*a*d*e*x**2)/(2*d**3*x*(d
+ e*x**2))
```

3.643 $\int \frac{x^5(a+b\arccos(cx))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 705

$$\int \frac{x^5(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \frac{bcdx\sqrt{1-c^2x^2}}{8e^2(c^2d+e)(d+ex^2)} - \frac{d^2(a+b\arccos(cx))}{4e^3(d+ex^2)^2} + \frac{d(a+b\arccos(cx))}{e^3(d+ex^2)} - \frac{i(a+b\arccos(cx))^2}{2be^3} - \frac{bc\sqrt{d}\arctan\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e^3\sqrt{c^2d+e}} + \frac{bc\sqrt{d}(2c^2d+e)\arctan\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8e^3(c^2d+e)^{3/2}} + \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^3} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2e^3}$$

output
```
1/8*b*c*d*x*(-c^2*x^2+1)^(1/2)/e^2/(c^2*d+e)/(e*x^2+d)-1/4*d^2*(a+b*arccos
(c*x))/e^3/(e*x^2+d)^2+d*(a+b*arccos(c*x))/e^3/(e*x^2+d)-1/2*I*(a+b*arccos
(c*x))^2/b/e^3-b*c*d^(1/2)*arctan((c^2*d+e)^(1/2)*x/d^(1/2)/(-c^2*x^2+1)^(
1/2))/e^3/(c^2*d+e)^(1/2)+1/8*b*c*d^(1/2)*(2*c^2*d+e)*arctan((c^2*d+e)^(1/
2)*x/d^(1/2)/(-c^2*x^2+1)^(1/2))/e^3/(c^2*d+e)^(3/2)+1/2*(a+b*arccos(c*x))
*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))
/e^3+1/2*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-
d)^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(
-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccos(c
*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/
2)))/e^3-1/2*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(
1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(
1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,-e^(1/2)*(c*
x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/e^3-1/2*I*b*poly
log(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))
/e^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1547 vs. $2(705) = 1410$.

Time = 6.66 (sec) , antiderivative size = 1547, normalized size of antiderivative = 2.19

$$\int \frac{x^5(a + b\arccos(cx))}{(d + ex^2)^3} \, dx = \text{Too large to display}$$

input
```
Integrate[(x^5*(a + b*ArcCos[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*
x^2])/(2*e^3) + b*((((-7*I)/16)*Sqrt[d]*(ArcCos[c*x]/((-I)*Sqrt[d] + Sqrt[
e]*x) - (c*Log[(2*e*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 -
c^2*x^2]))/(c*Sqrt[c^2*d + e]*((-I)*Sqrt[d] + Sqrt[e]*x))])/Sqrt[c^2*d + e
]))/e^3 - (d*((c*Sqrt[1 - c^2*x^2])/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x
)) - ArcCos[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + (I*c^3*Sqrt[d]*L
og[(-4*e*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt
[1 - c^2*x^2]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))])/(Sqrt[e]*(c^2*d + e)^(3/
2))))/(16*e^(5/2)) - (((7*I)/16)*Sqrt[d]*(-(ArcCos[c*x]/(I*Sqrt[d] + Sqrt[
e]*x)) + (c*Log[(-2*e*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1
- c^2*x^2]))/(c*Sqrt[c^2*d + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[c^2*d + e]
))/e^3 - (d*((c*Sqrt[1 - c^2*x^2])/((c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) -
 ArcCos[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - (I*c^3*Sqrt[d]*Log[(-4*
e*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^
2*x^2]))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x))])/(Sqrt[e]*(c^2*d + e)^(3/2))))/(
16*e^(5/2)) - ((I/4)*(ArcCos[c*x]^2 - 8*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt
[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2
*d + e]] + (2*I)*ArcCos[c*x]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E
^(I*ArcCos[c*x]))/Sqrt[e]] + (4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/
Sqrt[2]]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))...
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\arccos(cx))}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{5233}$$

$$\int \left(\frac{d^2x(a+b\arccos(cx))}{e^2(d+ex^2)^3}-\frac{2dx(a+b\arccos(cx))}{e^2(d+ex^2)^2}+\frac{x(a+b\arccos(cx))}{e^2(d+ex^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2e^3}+\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2e^3}+\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2e^3}+\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2e^3}-\frac{d^2(a+b\arccos(cx))}{4e^3(d+ex^2)^2}+\frac{d(a+b\arccos(cx))}{e^3(d+ex^2)}-\frac{i(a+b\arccos(cx))^2}{2be^3}-\frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2e^3}-\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2e^3}-\frac{ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2e^3}-\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2e^3}-\frac{bc\sqrt{d}(2c^2d+e)\arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8e^3(c^2d+e)^{3/2}}+\frac{bc\sqrt{d}\arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e^3\sqrt{c^2d+e}}-\frac{bcdx\sqrt{1-c^2x^2}}{8e^2(c^2d+e)(d+ex^2)}$$

input
```
Int[(x^5*(a + b*ArcCos[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/8*(b*c*d*x*Sqrt[1 - c^2*x^2])/(e^2*(c^2*d + e)*(d + e*x^2)) - (d^2*(a +
 b*ArcCos[c*x]))/(4*e^3*(d + e*x^2)^2) + (d*(a + b*ArcCos[c*x]))/(e^3*(d +
 e*x^2)) - ((I/2)*(a + b*ArcCos[c*x])^2)/(b*e^3) + (b*c*Sqrt[d]*ArcTan[(Sq
rt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(e^3*Sqrt[c^2*d + e]) - (b*
c*Sqrt[d]*(2*c^2*d + e)*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x
^2])])/(8*e^3*(c^2*d + e)^(3/2)) + ((a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E
^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*Arc
Cos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d +
 e])])/(2*e^3) + ((a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/
(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcCos[c*x])*Log[1 +
(Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*e^3) - (
(I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*
d + e]))])/e^3 - ((I/2)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-
d] - I*Sqrt[c^2*d + e])])/e^3 - ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos
[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))])/e^3 - ((I/2)*b*PolyLog[2, (Sqr
t[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/e^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.14 (sec) , antiderivative size = 3580, normalized size of antiderivative = 5.08

method	result	size
derivativedivides	Expression too large to display	3580
default	Expression too large to display	3580
parts	Expression too large to display	3585

input
```
int(x^5*(a+b*arccos(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^6*(a*c^6*(-1/4*c^4*d^2/e^3/(c^2*e*x^2+c^2*d)^2+1/2/e^3*ln(c^2*e*x^2+c^
2*d)+1/e^3*d*c^2/(c^2*e*x^2+c^2*d))+b*c^6*(1/8*d*c^2*(-I*c^4*d^2-2*I*c^4*d
*e*x^2-I*e^2*c^4*x^4+6*c^4*d^2*arccos(c*x)+8*arccos(c*x)*c^4*d*e*x^2-(-c^2
*x^2+1)^(1/2)*c^3*d*e*x-(-c^2*x^2+1)^(1/2)*e^2*c^3*x^3+6*c^2*d*e*arccos(c*
x)+8*arccos(c*x)*e^2*c^2*x^2)/e^3/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2+1/8*I*(-2*
d*c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*
e)*polylog(2,e*(c*x+I*(-c^2*x^2+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^
(1/2)-e))/d/c^2/e^2/(c^4*d^2+2*c^2*d*e+e^2)+1/4*I*(-2*d*c^2*(c^2*d*(c^2*d+
e))^(1/2)+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*arccos(c*x)^2/d/c
^2/e^2/(c^4*d^2+2*c^2*d*e+e^2)+1/8*I*(c^2*d*(c^2*d+e))^(1/2)/e/(c^2*d+e)^2
/d/c^2*polylog(2,e*(c*x+I*(-c^2*x^2+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+
e))^(1/2)-e))+1/4*I*(c^2*d*(c^2*d+e))^(1/2)/e/(c^2*d+e)^2/d/c^2*arccos(c*x
)^2-1/4*(-2*d*c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*
d+e))^(1/2)*e)/d/c^2/e^2/(c^4*d^2+2*c^2*d*e+e^2)*ln(1-e*(c*x+I*(-c^2*x^2+1
)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccos(c*x)-1/4*(c^2*d*
(c^2*d+e))^(1/2)/e/(c^2*d+e)^2/d/c^2*arccos(c*x)*ln(1-e*(c*x+I*(-c^2*x^2+1
)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e))+1/2*I*(-2*d*c^2*(c^2*d*
(c^2*d+e))^(1/2)+2*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*d^2*c^4*po
lylog(2,e*(c*x+I*(-c^2*x^2+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)
-e))/e^5/(c^4*d^2+2*c^2*d*e+e^2)+2*I*(-2*d*c^2*(c^2*d*(c^2*d+e))^(1/2)+...
```

Fricas [F]

$$\int \frac{x^5(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\arccos(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^5*arccos(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F]

$$\int \frac{x^5(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^5(a+b\,\mathrm{acos}\,(cx))}{(d+ex^2)^3}\,dx$$

input
```
integrate(x**5*(a+b*acos(c*x))/(e*x**2+d)**3,x)
```

output
```
Integral(x**5*(a + b*acos(c*x))/(d + e*x**2)**3, x)
```

Maxima [F]

$$\int \frac{x^5(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\arccos(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
 + d)/e^3) + b*integrate(x^5*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(e
^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^5*(a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\arccos(cx))}{(d+ex^2)^3}\,dx=\int \frac{x^5\,(a+b\,\mathrm{acos}(c\,x))}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^5*(a + b*acos(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((x^5*(a + b*acos(c*x)))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^5(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \frac{4\left(\int \frac{acos(cx)x^5}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)b\,d^2e^3+8\left(\int \frac{acos(cx)x^5}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)bd\,e^4x^2+4\left(\int \frac{acos(cx)x^5}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}d\right.}{4e^3\left(e^2x^4+2de\right.}$$

input
```
int(x^5*(a+b*acos(c*x))/(e*x^2+d)^3,x)
```

output
```
(4*int((acos(c*x)*x**5)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6)
,x)*b*d**2*e**3 + 8*int((acos(c*x)*x**5)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*
x**4 + e**3*x**6),x)*b*d*e**4*x**2 + 4*int((acos(c*x)*x**5)/(d**3 + 3*d**2
*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*e**5*x**4 + 2*log(d + e*x**2)*a*
d**2 + 4*log(d + e*x**2)*a*d*e*x**2 + 2*log(d + e*x**2)*a*e**2*x**4 + a*d*
*2 - 2*a*e**2*x**4)/(4*e**3*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.644 $\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 153

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = -\frac{bcx\sqrt{1-c^2x^2}}{8e(c^2d+e)(d+ex^2)} - \frac{b\arccos(cx)}{4de^2} + \frac{x^4(a+b\arccos(cx))}{4d(d+ex^2)^2} + \frac{bc(2c^2d+3e)\arctan\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8\sqrt{d}e^2(c^2d+e)^{3/2}}$$

output
```
-1/8*b*c*x*(-c^2*x^2+1)^(1/2)/e/(c^2*d+e)/(e*x^2+d)-1/4*b*arccos(c*x)/d/e^
2+1/4*x^4*(a+b*arccos(c*x))/d/(e*x^2+d)^2+1/8*b*c*(2*c^2*d+3*e)*arctan((c^
2*d+e)^(1/2)*x/d^(1/2)/(-c^2*x^2+1)^(1/2))/d^(1/2)/e^2/(c^2*d+e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \frac{\frac{\frac{bcex\sqrt{1-c^2x^2}(d+ex^2)}{c^2d+e}-2a(d+2ex^2)}{(d+ex^2)^2} - \frac{2b(d+2ex^2)\arccos(cx)}{(d+ex^2)^2} - \frac{bc(2c^2d+3e)\arctan\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{\sqrt{d}(c^2d+e)^{3/2}}}{8e^2}$$

input
```
Integrate[(x^3*(a + b*ArcCos[c*x]))/(d + e*x^2)^3,x]
```

output
```
(((b*c*e*x*Sqrt[1 - c^2*x^2]*(d + e*x^2))/(c^2*d + e) - 2*a*(d + 2*e*x^2))
/(d + e*x^2)^2 - (2*b*(d + 2*e*x^2)*ArcCos[c*x])/(d + e*x^2)^2 - (b*c*(2*c
^2*d + 3*e)*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(Sqrt
[d]*(c^2*d + e)^(3/2)))/(8*e^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5231, 27, 372, 398, 223, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{5231}$$

$$bc\int \frac{x^4}{4d\sqrt{1-c^2x^2}\,(ex^2+d)^2}dx+\frac{x^4(a+b\arccos(cx))}{4d\,(d+ex^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{x^4}{\sqrt{1-c^2x^2}(ex^2+d)^2}dx}{4d}+\frac{x^4(a+b\arccos(cx))}{4d\,(d+ex^2)^2}$$

$$\Big\downarrow \text{372}$$

$$\frac{bc\left(\frac{dx\sqrt{1-c^2x^2}}{2e(c^2d+e)(d+ex^2)}-\frac{\int \frac{d-2\left(dc^2+e\right)x^2}{\sqrt{1-c^2x^2}\left(ex^2+d\right)}dx}{2e(c^2d+e)}\right)}{4d}+\frac{x^4(a+b\arccos(cx))}{4d\,(d+ex^2)^2}$$

$$\Big\downarrow \text{398}$$

$$\frac{bc\left(\frac{dx\sqrt{1-c^2x^2}}{2e(c^2d+e)(d+ex^2)}-\frac{\frac{d\left(2c^2d+3e\right)\int\frac{1}{\sqrt{1-c^2x^2}\left(ex^2+d\right)}dx}{e}-\frac{2\left(c^2d+e\right)\int\frac{1}{\sqrt{1-c^2x^2}}dx}{e}}{2e(c^2d+e)}\right)}{4d}+\frac{x^4(a+b\arccos(cx))}{4d\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{bc\left(\frac{dx\sqrt{1-c^2x^2}}{2e(c^2d+e)(d+ex^2)}-\frac{\frac{d\left(2c^2d+3e\right)\int\frac{1}{\sqrt{1-c^2x^2}\left(ex^2+d\right)}dx}{e}-\frac{2\arcsin(cx)\left(c^2d+e\right)}{ce}}{2e(c^2d+e)}\right)}{4d}+\frac{x^4(a+b\arccos(cx))}{4d\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{bc\left(\frac{dx\sqrt{1-c^2x^2}}{2e(c^2d+e)(d+ex^2)}-\frac{\frac{d\left(2c^2d+3e\right)\int\frac{1}{d-\frac{\left(-dc^2-e\right)x^2}{1-c^2x^2}}d\frac{x}{\sqrt{1-c^2x^2}}}{e}-\frac{2\arcsin(cx)\left(c^2d+e\right)}{ce}}{2e(c^2d+e)}\right)}{4d}+\frac{x^4(a+b\arccos(cx))}{4d\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x^4(a+b\arccos(cx))}{4d\left(d+ex^2\right)^2}+\frac{bc\left(\frac{dx\sqrt{1-c^2x^2}}{2e(c^2d+e)(d+ex^2)}-\frac{\frac{\sqrt{d}\left(2c^2d+3e\right)\arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e\sqrt{c^2d+e}}-\frac{2\arcsin(cx)\left(c^2d+e\right)}{ce}}{2e(c^2d+e)}\right)}{4d}$$

input

```
Int[(x^3*(a + b*ArcCos[c*x]))/(d + e*x^2)^3,x]
```

output

```
(x^4*(a + b*ArcCos[c*x]))/(4*d*(d + e*x^2)^2) + (b*c*((d*x*Sqrt[1 - c^2*x^
2])/(2*e*(c^2*d + e)*(d + e*x^2)) - ((-2*(c^2*d + e)*ArcSin[c*x])/(c*e) +
(Sqrt[d]*(2*c^2*d + 3*e)*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*
x^2])])/(e*Sqrt[c^2*d + e]))/(2*e*(c^2*d + e))))/(4*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. 2(133) = 266.

Time = 0.24 (sec) , antiderivative size = 1020, normalized size of antiderivative = 6.67

method	result	size
parts	Expression too large to display	1020
derivativedivides	Expression too large to display	1038
default	Expression too large to display	1038

input
```
int(x^3*(a+b*arccos(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*d/e^2/(e*x^2+d)^2-1/2/(e*x^2+d)/e^2)+b/c^4*(-1/2*c^6*arccos(c*x)/(c
^2*e*x^2+c^2*d)/e^2+1/4*c^8*arccos(c*x)*d/e^2/(c^2*e*x^2+c^2*d)^2-1/4*c^6/
e^2*(1/4/e*(-1/(c^2*d+e)*e/(c*x-(-c^2*d*e)^(1/2)/e)*(-(c*x-(-c^2*d*e)^(1/2
)/e)^2-2*(-c^2*d*e)^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1/2)-(-
c^2*d*e)^(1/2)/(c^2*d+e)/((c^2*d+e)/e)^(1/2)*ln((2*(c^2*d+e)/e-2*(-c^2*d*e
)^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+2*((c^2*d+e)/e)^(1/2)*(-(c*x-(-c^2*d*e)
^(1/2)/e)^2-2*(-c^2*d*e)^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1/
2))/(c*x-(-c^2*d*e)^(1/2)/e)))+1/4/e*(-1/(c^2*d+e)*e/(c*x+(-c^2*d*e)^(1/2)
/e)*(-(c*x+(-c^2*d*e)^(1/2)/e)^2+2*(-c^2*d*e)^(1/2)/e*(c*x+(-c^2*d*e)^(1/2
)/e)+(c^2*d+e)/e)^(1/2)+(-c^2*d*e)^(1/2)/(c^2*d+e)/((c^2*d+e)/e)^(1/2)*ln(
(2*(c^2*d+e)/e+2*(-c^2*d*e)^(1/2)/e*(c*x+(-c^2*d*e)^(1/2)/e)+2*((c^2*d+e)/
e)^(1/2)*(-(c*x+(-c^2*d*e)^(1/2)/e)^2+2*(-c^2*d*e)^(1/2)/e*(c*x+(-c^2*d*e)
^(1/2)/e)+(c^2*d+e)/e)^(1/2))/(c*x+(-c^2*d*e)^(1/2)/e)))-3/4/(-c^2*d*e)^(1
/2)/((c^2*d+e)/e)^(1/2)*ln((2*(c^2*d+e)/e-2*(-c^2*d*e)^(1/2)/e*(c*x-(-c^2*
d*e)^(1/2)/e)+2*((c^2*d+e)/e)^(1/2)*(-(c*x-(-c^2*d*e)^(1/2)/e)^2-2*(-c^2*d
*e)^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1/2))/(c*x-(-c^2*d*e)^(
1/2)/e))+3/4/(-c^2*d*e)^(1/2)/((c^2*d+e)/e)^(1/2)*ln((2*(c^2*d+e)/e+2*(-c^
2*d*e)^(1/2)/e*(c*x+(-c^2*d*e)^(1/2)/e)+2*((c^2*d+e)/e)^(1/2)*(-(c*x+(-c^2
*d*e)^(1/2)/e)^2+2*(-c^2*d*e)^(1/2)/e*(c*x+(-c^2*d*e)^(1/2)/e)+(c^2*d+e)/e
)^(1/2))/(c*x+(-c^2*d*e)^(1/2)/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(133) = 266$.

Time = 0.26 (sec) , antiderivative size = 1102, normalized size of antiderivative = 7.20

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/32*(8*a*c^4*d^4 + 16*a*c^2*d^3*e - 8*(b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 +
b*e^4)*x^4*arccos(c*x) + 8*a*d^2*e^2 + 16*(a*c^4*d^3*e + 2*a*c^2*d^2*e^2 +
 a*d*e^3)*x^2 + (2*b*c^3*d^3 + 3*b*c*d^2*e + (2*b*c^3*d*e^2 + 3*b*c*e^3)*x
^4 + 2*(2*b*c^3*d^2*e + 3*b*c*d*e^2)*x^2)*sqrt(-c^2*d^2 - d*e)*log(((8*c^4
*d^2 + 8*c^2*d*e + e^2)*x^4 - 2*(4*c^2*d^2 + 3*d*e)*x^2 + 4*sqrt(-c^2*d^2
- d*e)*sqrt(-c^2*x^2 + 1)*((2*c^2*d + e)*x^3 - d*x) + d^2)/(e^2*x^4 + 2*d*
e*x^2 + d^2)) + 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2
+ 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)
*x^2)*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) - 4*sqrt(-c^2*x^2 + 1)*
((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x))/(c^4*d^
5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^
4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/16*(4*a*c^4*d^4 + 8
*a*c^2*d^3*e - 4*(b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4*arccos(c*x) +
 4*a*d^2*e^2 + 8*(a*c^4*d^3*e + 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 - (2*b*c^3*
d^3 + 3*b*c*d^2*e + (2*b*c^3*d*e^2 + 3*b*c*e^3)*x^4 + 2*(2*b*c^3*d^2*e + 3
*b*c*d*e^2)*x^2)*sqrt(c^2*d^2 + d*e)*arctan(1/2*sqrt(c^2*d^2 + d*e)*sqrt(-
c^2*x^2 + 1)*((2*c^2*d + e)*x^2 - d)/((c^4*d^2 + c^2*d*e)*x^3 - (c^2*d^2 +
 d*e)*x)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*
b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2
)*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) - 2*sqrt(-c^2*x^2 + 1)*(...
```

Sympy [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^3(a+b\operatorname{acos}(cx))}{(d+ex^2)^3}\,dx$$

input
```
integrate(x**3*(a+b*acos(c*x))/(e*x**2+d)**3,x)
```

output
```
Integral(x**3*(a + b*acos(c*x))/(d + e*x**2)**3, x)
```

Maxima [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\arccos(cx)+a)x^3}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2) - 1/4*((2*e*x^2 + d
)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - 4*(e^4*x^4 + 2*d*e^3*x^2 +
d^2*e^2)*integrate(1/4*(2*c*e*x^2 + c*d)*e^(1/2*log(c*x + 1) + 1/2*log(-c*
x + 1))/(c^4*e^4*x^8 - c^2*d^2*e^2*x^2 + (2*c^4*d*e^3 - c^2*e^4)*x^6 + (c^
4*d^2*e^2 - 2*c^2*d*e^3)*x^4 + (c^2*e^4*x^6 + (2*c^2*d*e^3 - e^4)*x^4 - d^
2*e^2 + (c^2*d^2*e^2 - 2*d*e^3)*x^2)*e^(log(c*x + 1) + log(-c*x + 1))), x)
)*b/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^3}\,dx=\int \frac{x^3\,(a+b\,\mathrm{acos}(c\,x))}{{(e\,x^2+d)}^3}\,dx$$

input
```
int((x^3*(a + b*acos(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((x^3*(a + b*acos(c*x)))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^3(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \frac{4\left(\int \frac{acos(cx)x^3}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)b\,d^3+8\left(\int \frac{acos(cx)x^3}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)b\,d^2e\,x^2+4\left(\int \frac{acos(cx)x^3}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right.}{4d\,(e^2x^4+2de\,x^2+d^2)}$$

input
```
int(x^3*(a+b*acos(c*x))/(e*x^2+d)^3,x)
```

output
```
(4*int((acos(c*x)*x**3)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6)
,x)*b*d**3 + 8*int((acos(c*x)*x**3)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4
+ e**3*x**6),x)*b*d**2*e*x**2 + 4*int((acos(c*x)*x**3)/(d**3 + 3*d**2*e*x*
*2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d*e**2*x**4 + a*x**4)/(4*d*(d**2 + 2*
d*e*x**2 + e**2*x**4))
```

3.645 $\int \frac{x(a+b\arccos(cx))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \frac{x(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \frac{bcx\sqrt{1-c^2x^2}}{8d(c^2d+e)(d+ex^2)} - \frac{a+b\arccos(cx)}{4e(d+ex^2)^2} + \frac{bc(2c^2d+e)\arctan\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8d^{3/2}e(c^2d+e)^{3/2}}$$

output
```
1/8*b*c*x*(-c^2*x^2+1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)-1/4*(a+b*arccos(c*x))/e
/(e*x^2+d)^2+1/8*b*c*(2*c^2*d+e)*arctan((c^2*d+e)^(1/2)*x/d^(1/2)/(-c^2*x^
2+1)^(1/2))/d^(3/2)/e/(c^2*d+e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \frac{x(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \frac{1}{8}\left(-\frac{\frac{2a}{e}+\frac{bcx\sqrt{1-c^2x^2}(d+ex^2)}{d(c^2d+e)}}{(d+ex^2)^2} - \frac{2b\arccos(cx)}{e(d+ex^2)^2} - \frac{bc(2c^2d+e)\arctan\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{d^{3/2}e(c^2d+e)^{3/2}}\right)$$

input
```
Integrate[(x*(a + b*ArcCos[c*x]))/(d + e*x^2)^3,x]
```

output
```
(-(((2*a)/e + (b*c*x*Sqrt[1 - c^2*x^2]*(d + e*x^2))/(d*(c^2*d + e)))/(d +
e*x^2)^2) - (2*b*ArcCos[c*x])/(e*(d + e*x^2)^2) - (b*c*(2*c^2*d + e)*ArcTa
n[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(d^(3/2)*e*(c^2*d + e)
^(3/2)))/8
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5229, 296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arccos(cx))}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{5229}$$

$$-\frac{bc\int \frac{1}{\sqrt{1-c^2x^2}(ex^2+d)^2}dx}{4e}-\frac{a+b\arccos(cx)}{4e(d+ex^2)^2}$$

$$\Big\downarrow \text{296}$$

$$-\frac{bc\left(\frac{(2c^2d+e)\int \frac{1}{\sqrt{1-c^2x^2}(ex^2+d)}dx}{2d(c^2d+e)}+\frac{ex\sqrt{1-c^2x^2}}{2d(c^2d+e)(d+ex^2)}\right)}{4e}-\frac{a+b\arccos(cx)}{4e(d+ex^2)^2}$$

$$\Big\downarrow \text{291}$$

$$-\frac{bc\left(\frac{(2c^2d+e)\int \frac{1}{d-\frac{(-dc^2-e)x^2}{1-c^2x^2}}d\frac{x}{\sqrt{1-c^2x^2}}}{2d(c^2d+e)}+\frac{ex\sqrt{1-c^2x^2}}{2d(c^2d+e)(d+ex^2)}\right)}{4e}-\frac{a+b\arccos(cx)}{4e(d+ex^2)^2}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a+b\arccos(cx)}{4e\left(d+ex^2\right)^2}-\frac{bc\left(\frac{(2c^2d+e)\arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2d^{3/2}(c^2d+e)^{3/2}}+\frac{ex\sqrt{1-c^2x^2}}{2d(c^2d+e)(d+ex^2)}\right)}{4e}$$

input
```
Int[(x*(a + b*ArcCos[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a + b*ArcCos[c*x])/(e*(d + e*x^2)^2) - (b*c*((e*x*Sqrt[1 - c^2*x^2])
/(2*d*(c^2*d + e)*(d + e*x^2)) + ((2*c^2*d + e)*ArcTan[(Sqrt[c^2*d + e]*x)
/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(2*d^(3/2)*(c^2*d + e)^(3/2))))/(4*e)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 5229
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])/(2*e*(p + 1))), x]
 + Simp[b*(c/(2*e*(p + 1)))   Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x]
, x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(115) = 230$.

Time = 0.20 (sec) , antiderivative size = 998, normalized size of antiderivative = 7.50

method	result
parts	$-\frac{a}{4e(e\,x^2+d)^2}+\frac{b\left(-\frac{c^6\arccos(cx)}{4e\left(c^2e\,x^2+c^2d\right)^2}-\frac{c^6\left(-\frac{-\frac{e\sqrt{-\left(cx-\frac{\sqrt{-c^2de}}{e}\right)^2-\frac{2\sqrt{-c^2de}\left(cx-\frac{\sqrt{-c^2de}}{e}\right)}{e}+\frac{c^2d+e}{e}}}{\left(c^2d+e\right)\left(cx-\frac{\sqrt{-c^2de}}{e}\right)}-\frac{\sqrt{-c^2de}\,\ln\cdots}{\cdots}}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x*(a+b*arccos(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/e/(e*x^2+d)^2+b/c^2*(-1/4*c^6/e/(c^2*e*x^2+c^2*d)^2*arccos(c*x)-1/4
*c^6/e*(-1/4/d/c^2/e*(-1/(c^2*d+e)*e/(c*x-(-c^2*d*e)^(1/2)/e)*(-(c*x-(-c^2
*d*e)^(1/2)/e)^2-2*(-c^2*d*e)^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+(c^2*d+e)/e
)^(1/2)-(-c^2*d*e)^(1/2)/(c^2*d+e)/((c^2*d+e)/e)^(1/2)*ln((2*(c^2*d+e)/e-2
*(-c^2*d*e)^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+2*((c^2*d+e)/e)^(1/2)*(-(c*x-
(-c^2*d*e)^(1/2)/e)^2-2*(-c^2*d*e)^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+(c^2*d
+e)/e)^(1/2))/(c*x-(-c^2*d*e)^(1/2)/e)))-1/4/d/c^2/e*(-1/(c^2*d+e)*e/(c*x+
(-c^2*d*e)^(1/2)/e)*(-(c*x+(-c^2*d*e)^(1/2)/e)^2+2*(-c^2*d*e)^(1/2)/e*(c*x
+(-c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1/2)+(-c^2*d*e)^(1/2)/(c^2*d+e)/((c^2*d
+e)/e)^(1/2)*ln((2*(c^2*d+e)/e+2*(-c^2*d*e)^(1/2)/e*(c*x+(-c^2*d*e)^(1/2)/
e)+2*((c^2*d+e)/e)^(1/2)*(-(c*x+(-c^2*d*e)^(1/2)/e)^2+2*(-c^2*d*e)^(1/2)/e
*(c*x+(-c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1/2))/(c*x+(-c^2*d*e)^(1/2)/e)))-1
/4/d/c^2/(-c^2*d*e)^(1/2)/((c^2*d+e)/e)^(1/2)*ln((2*(c^2*d+e)/e-2*(-c^2*d*
e)^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+2*((c^2*d+e)/e)^(1/2)*(-(c*x-(-c^2*d*e
)^(1/2)/e)^2-2*(-c^2*d*e)^(1/2)/e*(c*x-(-c^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1
/2))/(c*x-(-c^2*d*e)^(1/2)/e))+1/4/d/c^2/(-c^2*d*e)^(1/2)/((c^2*d+e)/e)^(1
/2)*ln((2*(c^2*d+e)/e+2*(-c^2*d*e)^(1/2)/e*(c*x+(-c^2*d*e)^(1/2)/e)+2*((c^
2*d+e)/e)^(1/2)*(-(c*x+(-c^2*d*e)^(1/2)/e)^2+2*(-c^2*d*e)^(1/2)/e*(c*x+(-c
^2*d*e)^(1/2)/e)+(c^2*d+e)/e)^(1/2))/(c*x+(-c^2*d*e)^(1/2)/e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(115) = 230$.

Time = 0.25 (sec) , antiderivative size = 1100, normalized size of antiderivative = 8.27

$$\int \frac{x(a + b\arccos(cx))}{(d + ex^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/32*(8*a*c^4*d^4 + 16*a*c^2*d^3*e + 8*a*d^2*e^2 + (2*b*c^3*d^3 + b*c*d^
2*e + (2*b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(2*b*c^3*d^2*e + b*c*d*e^2)*x^2)*s
qrt(-c^2*d^2 - d*e)*log(((8*c^4*d^2 + 8*c^2*d*e + e^2)*x^4 - 2*(4*c^2*d^2
+ 3*d*e)*x^2 + 4*sqrt(-c^2*d^2 - d*e)*sqrt(-c^2*x^2 + 1)*((2*c^2*d + e)*x^
3 - d*x) + d^2)/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 8*((b*c^4*d^2*e^2 + 2*b*c^2
*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arc
cos(c*x) + 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b
*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)
*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + 4*sqrt(-c^2*x^2 + 1)*((b*c
^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x))/(c^4*d^6*e +
 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2
*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4)*x^2), -1/16*(4*a*c^4*d^4 + 8*a*c^
2*d^3*e + 4*a*d^2*e^2 - (2*b*c^3*d^3 + b*c*d^2*e + (2*b*c^3*d*e^2 + b*c*e^
3)*x^4 + 2*(2*b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(c^2*d^2 + d*e)*arctan(1/2
*sqrt(c^2*d^2 + d*e)*sqrt(-c^2*x^2 + 1)*((2*c^2*d + e)*x^2 - d)/((c^4*d^2
+ c^2*d*e)*x^3 - (c^2*d^2 + d*e)*x)) - 4*((b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 +
 b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arccos(c*x)
 + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e
^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arctan(
sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + 2*sqrt(-c^2*x^2 + 1)*((b*c^3*d^...
```

Sympy [F]

$$\int \frac{x(a + b\arccos(cx))}{\left(d + ex^2\right)^3}\,dx = \int \frac{x(a + b\operatorname{acos}(cx))}{\left(d + ex^2\right)^3}\,dx$$

input
```
integrate(x*(a+b*acos(c*x))/(e*x**2+d)**3,x)
```

output
```
Integral(x*(a + b*acos(c*x))/(d + e*x**2)**3, x)
```

Maxima [F]

$$\int \frac{x(a + b\arccos(cx))}{(d + ex^2)^3} \, dx = \int \frac{(b\arccos(cx) + a)x}{(ex^2 + d)^3} \, dx$$

input
```
integrate(x*(a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*(4*(c*e^3*x^4 + 2*c*d*e^2*x^2 + c*d^2*e)*integrate(1/4*e^(1/2*log(c*x
+ 1) + 1/2*log(-c*x + 1))/(c^4*e^3*x^8 - c^2*d^2*e*x^2 + (2*c^4*d*e^2 - c^
2*e^3)*x^6 + (c^4*d^2*e - 2*c^2*d*e^2)*x^4 + (c^2*e^3*x^6 + (2*c^2*d*e^2 -
 e^3)*x^4 - d^2*e + (c^2*d^2*e - 2*d*e^2)*x^2)*e^(log(c*x + 1) + log(-c*x
+ 1))), x) - arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*b/(e^3*x^4 + 2*d*
e^2*x^2 + d^2*e) - 1/4*a/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a + b\arccos(cx))}{(d + ex^2)^3} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \int \frac{x\,(a+b\,\mathrm{acos}(c\,x))}{(e\,x^2+d)^3}\,dx$$

input
```
int((x*(a + b*acos(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((x*(a + b*acos(c*x)))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x(a+b\arccos(cx))}{(d+ex^2)^3}\,dx$$
$$= \frac{4\left(\int \frac{acos(cx)x}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)b\,d^2e+8\left(\int \frac{acos(cx)x}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)bd\,e^2x^2+4\left(\int \frac{acos(cx)x}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right.}{4e\,(e^2x^4+2de\,x^2+d^2)}$$

input
```
int(x*(a+b*acos(c*x))/(e*x^2+d)^3,x)
```

output
```
(4*int((acos(c*x)*x)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)
*b*d**2*e + 8*int((acos(c*x)*x)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e*
*3*x**6),x)*b*d*e**2*x**2 + 4*int((acos(c*x)*x)/(d**3 + 3*d**2*e*x**2 + 3*
d*e**2*x**4 + e**3*x**6),x)*b*e**3*x**4 - a)/(4*e*(d**2 + 2*d*e*x**2 + e**
2*x**4))
```

3.646 $\int \frac{a+b\arccos(cx)}{x(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 727

$$\int \frac{a+b\arccos(cx)}{x(d+ex^2)^3}\,dx = -\frac{bcex\sqrt{1-c^2x^2}}{8d^2(c^2d+e)(d+ex^2)} + \frac{a+b\arccos(cx)}{4d(d+ex^2)^2} + \frac{a+b\arccos(cx)}{2d^2(d+ex^2)} - \frac{bc\arctan\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2d^{5/2}\sqrt{c^2d+e}} - \frac{bc(2c^2d+e)\arctan\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8d^{5/2}(c^2d+e)^{3/2}} - \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^3} - \frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^3} + \frac{(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{d^3} + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^3} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^3} - \frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{2d^3}$$

output
```
-1/8*b*c*e*x*(-c^2*x^2+1)^(1/2)/d^2/(c^2*d+e)/(e*x^2+d)+1/4*(a+b*arccos(c*
x))/d/(e*x^2+d)^2+1/2*(a+b*arccos(c*x))/d^2/(e*x^2+d)-1/2*b*c*arctan((c^2*
d+e)^(1/2)*x/d^(1/2)/(-c^2*x^2+1)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)-1/8*b*c*(
2*c^2*d+e)*arctan((c^2*d+e)^(1/2)*x/d^(1/2)/(-c^2*x^2+1)^(1/2))/d^(5/2)/(c
^2*d+e)^(3/2)-1/2*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2)
)/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arccos(c*x))*ln(1+e^(1/2)
*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2*(a+b
*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2
*d+e)^(1/2)))/d^3-1/2*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(
1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^3+(a+b*arccos(c*x))*ln(1-(c*x+I*
(-c^2*x^2+1)^(1/2))^2)/d^3+1/2*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^
(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polylog(2,e^(1/2)*(c*
x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^3+1/2*I*b*poly
log(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2))
)/d^3+1/2*I*b*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)
+(c^2*d+e)^(1/2)))/d^3-1/2*I*b*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1533 vs. $2(727) = 1454$.

Time = 5.45 (sec) , antiderivative size = 1533, normalized size of antiderivative = 2.11

$$\int \frac{a+b\arccos(cx)}{x\left(d+ex^2\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x*(d + e*x^2)^3),x]
```

output
```
((b*c*d*Sqrt[e]*Sqrt[1 - c^2*x^2])/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)
) + (b*c*d*Sqrt[e]*Sqrt[1 - c^2*x^2])/((c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)
) + (4*a*d^2)/(d + e*x^2)^2 + (8*a*d)/(d + e*x^2) + (b*d*ArcCos[c*x])/(Sqr
t[d] - I*Sqrt[e]*x)^2 + (5*b*Sqrt[d]*ArcCos[c*x])/(Sqrt[d] - I*Sqrt[e]*x)
+ (b*d*ArcCos[c*x])/(Sqrt[d] + I*Sqrt[e]*x)^2 + (5*b*Sqrt[d]*ArcCos[c*x])/
(Sqrt[d] + I*Sqrt[e]*x) - (32*I)*b*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/
Sqrt[2]]*ArcTan[((c*Sqrt[d] - I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d +
e]] - (32*I)*b*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*
Sqrt[d] + I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] - 8*b*ArcCos[c*x
]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]]
- 16*b*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(-(c*Sqr
t[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - 8*b*ArcCos[c*x]*Log
[1 + (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - 16*
b*ArcSin[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(-(c*Sqrt[d])
 + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - 8*b*ArcCos[c*x]*Log[1 -
(I*(c*Sqrt[d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 16*b*ArcSin
[Sqrt[1 - (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(c*Sqrt[d] + Sqrt[c^2
*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] - 8*b*ArcCos[c*x]*Log[1 + (I*(c*Sqrt[
d] + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + 16*b*ArcSin[Sqrt[1 + (
I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 + (I*(c*Sqrt[d] + Sqrt[c^2*d + e])...
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x\left(d+ex^2\right)^3}\,dx$$

$$\Big\downarrow \text{5233}$$

$$\int \left(-\frac{ex(a+b\arccos(cx))}{d^3\left(d+ex^2\right)}+\frac{a+b\arccos(cx)}{d^3x}-\frac{ex(a+b\arccos(cx))}{d^2\left(d+ex^2\right)^2}-\frac{ex(a+b\arccos(cx))}{d\left(d+ex^2\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2d^3}-\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2d^3}-\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2d^3}-\frac{(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2d^3}+\frac{\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))}{d^3}+\frac{a+b\arccos(cx)}{2d^2(d+ex^2)}+\frac{a+b\arccos(cx)}{4d(d+ex^2)^2}+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2d^3}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2d^3}+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2d^3}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2d^3}-\frac{ib\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{2d^3}+\frac{bc(2c^2d+e)\arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8d^{5/2}(c^2d+e)^{3/2}}+\frac{bc\arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{2d^{5/2}\sqrt{c^2d+e}}+\frac{bcex\sqrt{1-c^2x^2}}{8d^2(c^2d+e)(d+ex^2)}$$

input

```
Int[(a + b*ArcCos[c*x])/(x*(d + e*x^2)^3),x]
```

output

```
(b*c*e*x*Sqrt[1 - c^2*x^2])/(8*d^2*(c^2*d + e)*(d + e*x^2)) + (a + b*ArcCo
s[c*x])/(4*d*(d + e*x^2)^2) + (a + b*ArcCos[c*x])/(2*d^2*(d + e*x^2)) + (b
*c*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(2*d^(5/2)*Sqr
t[c^2*d + e]) + (b*c*(2*c^2*d + e)*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqr
t[1 - c^2*x^2])])/(8*d^(5/2)*(c^2*d + e)^(3/2)) - ((a + b*ArcCos[c*x])*Log
[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*d^3
) - ((a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] -
 I*Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I
*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcCos
[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]
)])/(2*d^3) + ((a + b*ArcCos[c*x])*Log[1 + E^((2*I)*ArcCos[c*x])])/d^3 + (
(I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*
d + e]))])/d^3 + ((I/2)*b*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-
d] - I*Sqrt[c^2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos
[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))])/d^3 + ((I/2)*b*PolyLog[2, (Sqr
t[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/d^3 - ((I/2)*b*
PolyLog[2, -E^((2*I)*ArcCos[c*x])])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.45 (sec) , antiderivative size = 1238, normalized size of antiderivative = 1.70

method	result	size
parts	Expression too large to display	1238
derivativedivides	Expression too large to display	1291
default	Expression too large to display	1291

input
```
int((a+b*arccos(c*x))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a/d^3*ln(x)+1/4*a/d/(e*x^2+d)^2-1/2*a/d^3*ln(e*x^2+d)+1/2*a/d^2/(e*x^2+d)+
b*(1/8*c^2*(I*c^4*d^2+2*I*c^4*d*e*x^2+I*e^2*c^4*x^4+6*c^4*d^2*arccos(c*x)+
4*arccos(c*x)*c^4*d*e*x^2+(-c^2*x^2+1)^(1/2)*c^3*d*e*x+(-c^2*x^2+1)^(1/2)*
e^2*c^3*x^3+6*c^2*d*e*arccos(c*x)+4*arccos(c*x)*e^2*c^2*x^2)/d^2/(c^2*d+e)
/(c^2*e*x^2+c^2*d)^2-I/(c^2*d+e)*c^2/d^2*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/
2)))-I/(c^2*d+e)*c^2/d^2*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+1/(c^2*d+e)
*c^2/d^2*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+1/(c^2*d+e)*c^2/d^
2*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+1/(c^2*d+e)/d^3*e*arccos(
c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+1/(c^2*d+e)/d^3*e*arccos(c*x)*ln(1
-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+1/4*I/(c^2*d+e)*c^2/d^2*sum((_R1^2+1)/(_R1^
2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog
((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2
+e))*e-5/8*I*(c^2*d*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^3*arctanh(1/4*(4*c^2*d+
2*e*(c*x+I*(-c^2*x^2+1)^(1/2))^2+2*e)/(c^4*d^2+c^2*d*e)^(1/2))*e+1/4*I/(c^
2*d+e)*c^2/d^2*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*
ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2)
)/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-3/4*I*(c^2*d*(c^2*d+e))^(
1/2)/(c^2*d+e)^2/d^2*c^2*arctanh(1/4*(4*c^2*d+2*e*(c*x+I*(-c^2*x^2+1)^(1/2
))^2+2*e)/(c^4*d^2+c^2*d*e)^(1/2))-I/(c^2*d+e)/d^3*e*dilog(1+I*(c*x+I*(-c^
2*x^2+1)^(1/2)))-I/(c^2*d+e)/d^3*e*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2))...
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x)
, x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{a+b\operatorname{acos}\left(cx\right)}{x\left(d+ex^2\right)^3}\,dx$$

input
```
integrate((a+b*acos(c*x))/x/(e*x**2+d)**3,x)
```

output
```
Integral((a + b*acos(c*x))/(x*(d + e*x**2)**3), x)
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d
)/d^3 + 4*log(x)/d^3) + b*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1),
c*x)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x\left(d+ex^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arccos(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{x\,\left(e\,x^2+d\right)^3}\,dx$$

input
```
int((a + b*acos(c*x))/(x*(d + e*x^2)^3),x)
```

output
```
int((a + b*acos(c*x))/(x*(d + e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x\left(d+ex^2\right)^3}\,dx = \frac{4\left(\int \frac{acos(cx)}{e^3x^7+3d\,e^2x^5+3d^2e\,x^3+d^3x}dx\right)b\,d^5+8\left(\int \frac{acos(cx)}{e^3x^7+3d\,e^2x^5+3d^2e\,x^3+d^3x}dx\right)b\,d^4e\,x^2+4\left(\int \frac{acos(cx)}{e^3x^7+3d\,e^2x^5+3d^2e\,x^3+d^3x}\right.}{}$$

input
```
int((a+b*acos(c*x))/x/(e*x^2+d)^3,x)
```

output
```
(4*int(acos(c*x)/(d**3*x + 3*d**2*e*x**3 + 3*d*e**2*x**5 + e**3*x**7),x)*b
*d**5 + 8*int(acos(c*x)/(d**3*x + 3*d**2*e*x**3 + 3*d*e**2*x**5 + e**3*x**
7),x)*b*d**4*e*x**2 + 4*int(acos(c*x)/(d**3*x + 3*d**2*e*x**3 + 3*d*e**2*x
**5 + e**3*x**7),x)*b*d**3*e**2*x**4 - 2*log(d + e*x**2)*a*d**2 - 4*log(d
+ e*x**2)*a*d*e*x**2 - 2*log(d + e*x**2)*a*e**2*x**4 + 4*log(x)*a*d**2 + 8
*log(x)*a*d*e*x**2 + 4*log(x)*a*e**2*x**4 + 2*a*d**2 - a*e**2*x**4)/(4*d**
3*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.647 $\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 783

$$\int \frac{a+b\arccos(cx)}{x^3(d+ex^2)^3}\,dx = -\frac{bc\sqrt{1-c^2x^2}}{2d^3x}+\frac{bce^2x\sqrt{1-c^2x^2}}{8d^3(c^2d+e)(d+ex^2)}-\frac{a+b\arccos(cx)}{2d^3x^2}-\frac{e(a+b\arccos(cx))}{4d^2(d+ex^2)^2}-\frac{e(a+b\arccos(cx))}{d^3(d+ex^2)}+\frac{bce\arctan\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{d^{7/2}\sqrt{c^2d+e}}+\frac{bce(2c^2d+e)\arctan\left(\frac{\sqrt{c^2d+e}x}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8d^{7/2}(c^2d+e)^{3/2}}+\frac{3e(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^4}+\frac{3e(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^4}+\frac{3e(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^4}+\frac{3e(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^4}-\frac{3e(a+b\arccos(cx))\log\left(1-e^{2i\arccos(cx)}\right)}{d^4}-\frac{3ibe\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^4}-\frac{3ibe\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2d^4}-\frac{3ibe\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^4}-\frac{3ibe\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2d^4}+\frac{3ibe\,\text{PolyLog}\left(2,e^{2i\arccos(cx)}\right)}{2d^4}$$

output
```
-1/2*b*c*(-c^2*x^2+1)^(1/2)/d^3/x+1/8*b*c*e^2*x*(-c^2*x^2+1)^(1/2)/d^3/(c^
2*d+e)/(e*x^2+d)-1/2*(a+b*arccos(c*x))/d^3/x^2-1/4*e*(a+b*arccos(c*x))/d^2
/(e*x^2+d)^2-e*(a+b*arccos(c*x))/d^3/(e*x^2+d)+b*c*e*arctan((c^2*d+e)^(1/2
)*x/d^(1/2)/(-c^2*x^2+1)^(1/2))/d^(7/2)/(c^2*d+e)^(1/2)+1/8*b*c*e*(2*c^2*d
+e)*arctan((c^2*d+e)^(1/2)*x/d^(1/2)/(-c^2*x^2+1)^(1/2))/d^(7/2)/(c^2*d+e)
^(3/2)+3/2*e*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*
c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^4+3/2*e*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c
*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^4+3/2*e*(a+b*
arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*
d+e)^(1/2)))/d^4+3/2*e*(a+b*arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^
(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^4-3*e*(a+b*arccos(c*x))*ln(1-(c
*x+I*(-c^2*x^2+1)^(1/2))^2)/d^4-3/2*I*b*e*polylog(2,-e^(1/2)*(c*x+I*(-c^2*
x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/d^4-3/2*I*b*e*polylog(2,-e
^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/d^4-3/
2*I*b*e*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*
d+e)^(1/2)))/d^4+3/2*I*b*e*polylog(2,(c*x+I*(-c^2*x^2+1)^(1/2))^2)/d^4-3/2
*I*b*e*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d
+e)^(1/2)))/d^4
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1660 vs. $2(783) = 1566$.

Time = 6.06 (sec) , antiderivative size = 1660, normalized size of antiderivative = 2.12

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(x^3*(d + e*x^2)^3),x]
```

output
```
-1/2*a/(d^3*x^2) - (a*e)/(4*d^2*(d + e*x^2)^2) - (a*e)/(d^3*(d + e*x^2)) -
 (3*a*e*Log[x])/d^4 + (3*a*e*Log[d + e*x^2])/(2*d^4) + b*((c*x*Sqrt[1 - c^
2*x^2] - ArcCos[c*x])/(2*d^3*x^2) + (((9*I)/16)*e*(ArcCos[c*x]/((-I)*Sqrt[
d] + Sqrt[e]*x) - (c*Log[(2*e*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]
*Sqrt[1 - c^2*x^2]))/(c*Sqrt[c^2*d + e]*((-I)*Sqrt[d] + Sqrt[e]*x))])/Sqrt
[c^2*d + e]))/d^(7/2) - (e^(3/2)*((c*Sqrt[1 - c^2*x^2])/((c^2*d + e)*((-I)
*Sqrt[d] + Sqrt[e]*x)) - ArcCos[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2
) + (I*c^3*Sqrt[d]*Log[(-4*e*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x +
Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))])/(Sqr
t[e]*(c^2*d + e)^(3/2))))/(16*d^3) + (((9*I)/16)*e*(-(ArcCos[c*x]/(I*Sqrt[
d] + Sqrt[e]*x)) + (c*Log[(-2*e*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d +
e]*Sqrt[1 - c^2*x^2]))/(c*Sqrt[c^2*d + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[
c^2*d + e]))/d^(7/2) - (e^(3/2)*((c*Sqrt[1 - c^2*x^2])/((c^2*d + e)*(I*Sqr
t[d] + Sqrt[e]*x)) - ArcCos[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - (I*
c^3*Sqrt[d]*Log[(-4*e*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^
2*d + e]*Sqrt[1 - c^2*x^2]))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x))])/(Sqrt[e]*(c
^2*d + e)^(3/2))))/(16*d^3) - (((3*I)/4)*e*(ArcCos[c*x]^2 - 8*ArcSin[Sqrt[
1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + I*Sqrt[e])*Tan[Ar
cCos[c*x]/2])/Sqrt[c^2*d + e]] + (2*I)*ArcCos[c*x]*Log[1 - (I*(-(c*Sqrt[d]
) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (4*I)*ArcSin[Sqrt[1 ...
```

Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{5233}$$

$$\int \left(\frac{3e^2x(a+b\arccos(cx))}{d^4\left(d+ex^2\right)}-\frac{3e(a+b\arccos(cx))}{d^4x}+\frac{2e^2x(a+b\arccos(cx))}{d^3\left(d+ex^2\right)^2}+\frac{a+b\arccos(cx)}{d^3x^3}+\frac{e^2x(a+b\arc}{d^2(d+e}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3e(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2d^4}+\frac{3e(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{c^2d+e}}\right)}{2d^4}+\frac{3e(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2d^4}+\frac{3e(a+b\arccos(cx))\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}+i\sqrt{c^2d+e}}\right)}{2d^4}-\frac{3e\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))}{d^4}-\frac{e(a+b\arccos(cx))}{d^3(d+ex^2)}-\frac{a+b\arccos(cx)}{2d^3x^2}-\frac{e(a+b\arccos(cx))}{4d^2(d+ex^2)^2}-\frac{3ibe\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2d^4}-\frac{3ibe\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2d^4}-\frac{3ibe\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-dc}+i\sqrt{dc^2+e}}\right)}{2d^4}-\frac{3ibe\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-dc}+i\sqrt{dc^2+e}}\right)}{2d^4}+\frac{3ibe\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)}{2d^4}-\frac{bce(2c^2d+e)\arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{8d^{7/2}(c^2d+e)^{3/2}}-\frac{bce\arctan\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{d^{7/2}\sqrt{c^2d+e}}-\frac{bce^2x\sqrt{1-c^2x^2}}{8d^3(c^2d+e)(d+ex^2)}+\frac{bc\sqrt{1-c^2x^2}}{2d^3x}$$

input
```
Int[(a + b*ArcCos[c*x])/(x^3*(d + e*x^2)^3),x]
```

output
```
(b*c*Sqrt[1 - c^2*x^2])/(2*d^3*x) - (b*c*e^2*x*Sqrt[1 - c^2*x^2])/(8*d^3*(
c^2*d + e)*(d + e*x^2)) - (a + b*ArcCos[c*x])/(2*d^3*x^2) - (e*(a + b*ArcC
os[c*x]))/(4*d^2*(d + e*x^2)^2) - (e*(a + b*ArcCos[c*x]))/(d^3*(d + e*x^2)
) - (b*c*e*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(d^(7/
2)*Sqrt[c^2*d + e]) - (b*c*e*(2*c^2*d + e)*ArcTan[(Sqrt[c^2*d + e]*x)/(Sqr
t[d]*Sqrt[1 - c^2*x^2])])/(8*d^(7/2)*(c^2*d + e)^(3/2)) + (3*e*(a + b*ArcC
os[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d +
e])])/(2*d^4) + (3*e*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]
))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*d^4) + (3*e*(a + b*ArcCos[c*x])*L
og[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*d
^4) + (3*e*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt
[-d] + I*Sqrt[c^2*d + e])])/(2*d^4) - (3*e*(a + b*ArcCos[c*x])*Log[1 + E^(
(2*I)*ArcCos[c*x])])/d^4 - (((3*I)/2)*b*e*PolyLog[2, -((Sqrt[e]*E^(I*ArcCo
s[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e]))])/d^4 - (((3*I)/2)*b*e*PolyLog[
2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/d^4 - ((
(3*I)/2)*b*e*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt
[c^2*d + e]))])/d^4 - (((3*I)/2)*b*e*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x])
)/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/d^4 + (((3*I)/2)*b*e*PolyLog[2, -E^((
2*I)*ArcCos[c*x])])/d^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.70 (sec) , antiderivative size = 1459, normalized size of antiderivative = 1.86

method	result	size
parts	Expression too large to display	1459
derivativedivides	Expression too large to display	1519
default	Expression too large to display	1519

input
```
int((a+b*arccos(c*x))/x^3/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d^3/x^2-3*a/d^4*e*ln(x)-1/4*a*e/d^2/(e*x^2+d)^2+3/2*a*e/d^4*ln(e*x^
2+d)-a*e/d^3/(e*x^2+d)+b*c^2*(-1/8*(-4*I*c^8*d^3*x^2-4*I*c^8*d*e^2*x^6-8*I
*c^8*d^2*e*x^4-4*(-c^2*x^2+1)^(1/2)*c^7*d^3*x-8*(-c^2*x^2+1)^(1/2)*c^7*d^2
*e*x^3-4*(-c^2*x^2+1)^(1/2)*c^7*d*e^2*x^5-6*I*c^6*e^2*x^4*d-3*I*e^3*c^6*x^
6-3*I*c^6*e*x^2*d^2+4*c^6*arccos(c*x)*d^3+18*c^6*arccos(c*x)*e*x^2*d^2+12*
arccos(c*x)*c^6*d*e^2*x^4-4*c^5*(-c^2*x^2+1)^(1/2)*e*x*d^2-7*c^5*(-c^2*x^2
+1)^(1/2)*e^2*x^3*d-3*(-c^2*x^2+1)^(1/2)*e^3*c^5*x^5+4*c^4*e*arccos(c*x)*d
^2+18*c^4*arccos(c*x)*e^2*x^2*d+12*arccos(c*x)*e^3*c^4*x^4)/c^2/x^2/(c^2*d
+e)/(c^2*e*x^2+c^2*d)^2/d^3-3/(c^2*d+e)/d^3*e*arccos(c*x)*ln(1+I*(c*x+I*(-
c^2*x^2+1)^(1/2)))-3/(c^2*d+e)/d^3*e*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1
)^(1/2)))-3/(c^2*d+e)/d^4*e^2/c^2*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(
1/2)))-3/(c^2*d+e)/d^4*e^2/c^2*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2
)))+3*I/(c^2*d+e)/d^3*e*dilog(1+I*(c*x+I*(-c^2*x^2+1)^(1/2)))+3*I/(c^2*d+e
)/d^3*e*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-3/4*I/(c^2*d+e)/d^3*e*sum((_
R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x
^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*
_Z^4+(4*c^2*d+2*e)*_Z^2+e))-3/4*I/(c^2*d+e)/d^3*e^2*sum((_R1^2+1)/(_R1^2*e
+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_
R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e)
)+5/4*I*(c^2*d*(c^2*d+e))^(1/2)/(c^2*d+e)^2/d^3*arctanh(1/4*(4*c^2*d+2*...
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x^3\,(d+ex^2)^3}\,dx = \int \frac{b\arccos{(cx)}+a}{(ex^2+d)^3x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^
3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acos(c*x))/x**3/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^3}\,dx=\int \frac{b\arccos\left(cx\right)+a}{\left(ex^2+d\right)^3x^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/x^3/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*a*((6*e^2*x^4 + 9*d*e*x^2 + 2*d^2)/(d^3*e^2*x^6 + 2*d^4*e*x^4 + d^5*x
^2) - 6*e*log(e*x^2 + d)/d^4 + 12*e*log(x)/d^4) + b*integrate(arctan2(sqrt
(c*x + 1)*sqrt(-c*x + 1), c*x)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*
x^3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arccos(c*x))/x^3/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^3}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{x^3\,\left(e\,x^2+d\right)^3}\,dx$$

input
```
int((a + b*acos(c*x))/(x^3*(d + e*x^2)^3),x)
```

output
```
int((a + b*acos(c*x))/(x^3*(d + e*x^2)^3), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{x^3\left(d+ex^2\right)^3}\,dx$$
$$=\frac{4\left(\int \frac{acos(cx)}{e^3x^9+3d\,e^2x^7+3d^2e\,x^5+d^3x^3}dx\right)b\,d^6x^2+8\left(\int \frac{acos(cx)}{e^3x^9+3d\,e^2x^7+3d^2e\,x^5+d^3x^3}dx\right)b\,d^5e\,x^4+4\left(\int \frac{acos(cx)}{e^3x^9+3d\,e^2x^7+3d^2e\,x^5}\right.}{}$$

input
```
int((a+b*acos(c*x))/x^3/(e*x^2+d)^3,x)
```

output
```
(4*int(acos(c*x)/(d**3*x**3 + 3*d**2*e*x**5 + 3*d*e**2*x**7 + e**3*x**9),x
)*b*d**6*x**2 + 8*int(acos(c*x)/(d**3*x**3 + 3*d**2*e*x**5 + 3*d*e**2*x**7
 + e**3*x**9),x)*b*d**5*e*x**4 + 4*int(acos(c*x)/(d**3*x**3 + 3*d**2*e*x**
5 + 3*d*e**2*x**7 + e**3*x**9),x)*b*d**4*e**2*x**6 + 6*log(d + e*x**2)*a*d
**2*e*x**2 + 12*log(d + e*x**2)*a*d*e**2*x**4 + 6*log(d + e*x**2)*a*e**3*x
**6 - 12*log(x)*a*d**2*e*x**2 - 24*log(x)*a*d*e**2*x**4 - 12*log(x)*a*e**3
*x**6 - 2*a*d**3 - 6*a*d**2*e*x**2 + 3*a*e**3*x**6)/(4*d**4*x**2*(d**2 + 2
*d*e*x**2 + e**2*x**4))
```

3.648 $\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 1082

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

output
```
1/16*b*c*(-d)^(1/2)*(-c^2*x^2+1)^(1/2)/e^2/(c^2*d+e)/((-d)^(1/2)-e^(1/2)*x
)+1/16*b*c*(-d)^(1/2)*(-c^2*x^2+1)^(1/2)/e^2/(c^2*d+e)/((-d)^(1/2)+e^(1/2)
*x)-1/16*(-d)^(1/2)*(a+b*arccos(c*x))/e^(5/2)/((-d)^(1/2)-e^(1/2)*x)^2+5/1
6*(a+b*arccos(c*x))/e^(5/2)/((-d)^(1/2)-e^(1/2)*x)+1/16*(-d)^(1/2)*(a+b*ar
ccos(c*x))/e^(5/2)/((-d)^(1/2)+e^(1/2)*x)^2-5/16*(a+b*arccos(c*x))/e^(5/2)
/((-d)^(1/2)+e^(1/2)*x)+1/16*b*c^3*d*arctanh((e^(1/2)-c^2*(-d)^(1/2)*x)/(c
^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/e^(5/2)/(c^2*d+e)^(3/2)-5/16*b*c*arctanh
((e^(1/2)-c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/e^(5/2)/(c
^2*d+e)^(1/2)+1/16*b*c^3*d*arctanh((e^(1/2)+c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1
/2)/(-c^2*x^2+1)^(1/2))/e^(5/2)/(c^2*d+e)^(3/2)-5/16*b*c*arctanh((e^(1/2)+
c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/e^(5/2)/(c^2*d+e)^(1
/2)+3/16*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-
d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(5/2)-3/16*(a+b*arccos(c*x))*ln(1+
e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^
(1/2)/e^(5/2)+3/16*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2
))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(5/2)-3/16*(a+b*arccos(c
*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/
2)))/(-d)^(1/2)/e^(5/2)+3/16*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1
/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(5/2)-3/16*I*b*polylog
(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))...
```

Mathematica [A] (warning: unable to verify)

Time = 6.02 (sec) , antiderivative size = 1527, normalized size of antiderivative = 1.41

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^4*(a + b*ArcCos[c*x]))/(d + e*x^2)^3,x]
```

output
```
((8*a*d*Sqrt[e]*x)/(d + e*x^2)^2 - (20*a*Sqrt[e]*x)/(d + e*x^2) + (12*a*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - 10*b*(ArcCos[c*x]/((-I)*Sqrt[d] + Sqr
t[e]*x) - (c*Log[(2*e*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1
- c^2*x^2]))/(c*Sqrt[c^2*d + e]*((-I)*Sqrt[d] + Sqrt[e]*x))])/Sqrt[c^2*d +
 e]) + 2*b*Sqrt[d]*((I*c*Sqrt[e]*Sqrt[1 - c^2*x^2])/((c^2*d + e)*((-I)*Sqr
t[d] + Sqrt[e]*x)) + (I*ArcCos[c*x])/(Sqrt[d] + I*Sqrt[e]*x)^2 - (c^3*Sqrt
[d]*Log[(-4*e*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]
*Sqrt[1 - c^2*x^2]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))])/(c^2*d + e)^(3/2))
+ 10*b*(-(ArcCos[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) + (c*Log[(-2*e*(Sqrt[e] + I
*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c*Sqrt[c^2*d + e]*(I
*Sqrt[d] + Sqrt[e]*x))])/Sqrt[c^2*d + e]) + 2*b*Sqrt[d]*(((-I)*c*Sqrt[e]*S
qrt[1 - c^2*x^2])/((c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) + (I*ArcCos[c*x])/
(I*Sqrt[d] + Sqrt[e]*x)^2 - (c^3*Sqrt[d]*Log[(-4*e*Sqrt[c^2*d + e]*(Sqrt[e
] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c^3*(d - I*Sqrt
[d]*Sqrt[e]*x))])/(c^2*d + e)^(3/2)) + (3*b*(ArcCos[c*x]^2 - 8*ArcSin[Sqrt
[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + I*Sqrt[e])*Tan[A
rcCos[c*x]/2])/Sqrt[c^2*d + e]] + (2*I)*ArcCos[c*x]*Log[1 - (I*(-(c*Sqrt[d
]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (4*I)*ArcSin[Sqrt[1 +
(I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e]
)*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)*ArcCos[c*x]*Log[1 + (I*(c*Sqrt[d]...
```

Rubi [A] (verified)

Time = 4.11 (sec) , antiderivative size = 1082, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{5233}$$

$$\int \left(\frac{d^2(a+b\arccos(cx))}{e^2(d+ex^2)^3}-\frac{2d(a+b\arccos(cx))}{e^2(d+ex^2)^2}+\frac{a+b\arccos(cx)}{e^2(d+ex^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{bd\operatorname{arctanh}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c^3}{16e^{5/2}\left(dc^2+e\right)^{3/2}}-\frac{bd\operatorname{arctanh}\left(\frac{\sqrt{-d}xc^2+\sqrt{e}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c^3}{16e^{5/2}\left(dc^2+e\right)^{3/2}}+\frac{5b\operatorname{arctanh}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c}{16e^{5/2}\sqrt{dc^2+e}}+\frac{5b\operatorname{arctanh}\left(\frac{\sqrt{-d}xc^2+\sqrt{e}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c}{16e^{5/2}\sqrt{dc^2+e}}-\frac{b\sqrt{-d}\sqrt{1-c^2x^2}c}{16e^2\left(dc^2+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{b\sqrt{-d}\sqrt{1-c^2x^2}c}{16e^2\left(dc^2+e\right)\left(\sqrt{e}x+\sqrt{-d}\right)}+\frac{5(a+b\arccos(cx))}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{5(a+b\arccos(cx))}{16e^{5/2}\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{\sqrt{-d}(a+b\arccos(cx))}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}x\right)^2}+\frac{\sqrt{-d}(a+b\arccos(cx))}{16e^{5/2}\left(\sqrt{e}x+\sqrt{-d}\right)^2}+\frac{3(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3(a+b\arccos(cx))\log\left(\frac{e^{i\arccos(cx)}\sqrt{e}}{c\sqrt{-d}-i\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}}+\frac{3(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3(a+b\arccos(cx))\log\left(\frac{e^{i\arccos(cx)}\sqrt{e}}{\sqrt{-d}c+i\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}}+\frac{3ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}+\frac{3ib\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3ib\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

input
```
Int[(x^4*(a + b*ArcCos[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/16*(b*c*Sqrt[-d]*Sqrt[1 - c^2*x^2])/(e^2*(c^2*d + e)*(Sqrt[-d] - Sqrt[e
]*x)) - (b*c*Sqrt[-d]*Sqrt[1 - c^2*x^2])/(16*e^2*(c^2*d + e)*(Sqrt[-d] + S
qrt[e]*x)) - (Sqrt[-d]*(a + b*ArcCos[c*x]))/(16*e^(5/2)*(Sqrt[-d] - Sqrt[e
]*x)^2) + (5*(a + b*ArcCos[c*x]))/(16*e^(5/2)*(Sqrt[-d] - Sqrt[e]*x)) + (S
qrt[-d]*(a + b*ArcCos[c*x]))/(16*e^(5/2)*(Sqrt[-d] + Sqrt[e]*x)^2) - (5*(a
 + b*ArcCos[c*x]))/(16*e^(5/2)*(Sqrt[-d] + Sqrt[e]*x)) - (b*c^3*d*ArcTanh[
(Sqrt[e] - c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(16*e^(5/
2)*(c^2*d + e)^(3/2)) + (5*b*c*ArcTanh[(Sqrt[e] - c^2*Sqrt[-d]*x)/(Sqrt[c^
2*d + e]*Sqrt[1 - c^2*x^2])])/(16*e^(5/2)*Sqrt[c^2*d + e]) - (b*c^3*d*ArcT
anh[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(16*e
^(5/2)*(c^2*d + e)^(3/2)) + (5*b*c*ArcTanh[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqr
t[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(16*e^(5/2)*Sqrt[c^2*d + e]) + (3*(a + b
*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2
*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]
*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2
)) + (3*(a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d
] + I*Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcCos[c*x])*Lo
g[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(16*S
qrt[-d]*e^(5/2)) + (((3*I)/16)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/
(c*Sqrt[-d] - I*Sqrt[c^2*d + e]))])/(Sqrt[-d]*e^(5/2)) - (((3*I)/16)*b*...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.38 (sec) , antiderivative size = 1760, normalized size of antiderivative = 1.63

method	result	size
parts	Expression too large to display	1760
derivativedivides	Expression too large to display	1762
default	Expression too large to display	1762

input
```
int(x^4*(a+b*arccos(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*((-5/8/e*x^3-3/8*d/e^2*x)/(e*x^2+d)^2+3/8/e^2/(d*e)^(1/2)*arctan(e*x/(d*
e)^(1/2)))+b/c^5*(-1/8*c^6*(3*arccos(c*x)*d^2*c^5*x+5*arccos(c*x)*c^5*d*e*
x^3+d^2*c^4*(-c^2*x^2+1)^(1/2)+c^4*d*e*x^2*(-c^2*x^2+1)^(1/2)+3*arccos(c*x
)*c^3*d*e*x+5*arccos(c*x)*e^2*c^3*x^3)/e^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2-5
/8*I*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)+e)*arctanh(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d
*(c^2*d+e))^(1/2)-e)*e)^(1/2))*c^6/e^4/(c^2*d+e)-1/2*I*((2*c^2*d+2*(c^2*d*
(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*arctan(
e*(c*x+I*(-c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/
2))*c^8*d/e^5/(c^2*d+e)-3/16*I/(c^2*d+e)*c^6/e*sum(_R1/(_R1^2*e+2*c^2*d+e)
*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-
c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-3/16*I/(c
^2*d+e)*c^8/e^2*d*sum(_R1/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I
*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=R
ootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+3/16*I/(c^2*d+e)*c^6/e*sum(1/_R1/(_R1^
2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog
((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2
+e))+1/2*I*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^4*d^2+2*d
*c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1/2)*e)*c^8*d*ar
ctanh(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)...
```

Fricas [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\arccos(cx)+a)x^4}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^4*arccos(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^4(a+b\operatorname{acos}(cx))}{(d+ex^2)^3}\,dx$$

input
```
integrate(x**4*(a+b*acos(c*x))/(e*x**2+d)**3,x)
```

output
```
Integral(x**4*(a + b*acos(c*x))/(d + e*x**2)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\arccos(cx)+a)x^4}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)*x^4/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^3}\,dx=\int \frac{x^4\,(a+b\,\mathrm{acos}(c\,x))}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^4*(a + b*acos(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((x^4*(a + b*acos(c*x)))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^4(a+b\arccos(cx))}{(d+ex^2)^3}\,dx$$
$$=\frac{3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,d^2+6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ade\,x^2+3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^2x^4+8\left(\int \frac{acos}{e^3x^6+3d\,e^2x}\right.}{8d}$$

input
```
int(x^4*(a+b*acos(c*x))/(e*x^2+d)^3,x)
```

output
```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d**2 + 6*sqrt(e)*sqrt(d
)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e*x**2 + 3*sqrt(e)*sqrt(d)*atan((e*x)/
(sqrt(e)*sqrt(d)))*a*e**2*x**4 + 8*int((acos(c*x)*x**4)/(d**3 + 3*d**2*e*x
**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**3*e**3 + 16*int((acos(c*x)*x**4)/
(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**2*e**4*x**2 + 8
*int((acos(c*x)*x**4)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x
)*b*d*e**5*x**4 - 3*a*d**2*e*x - 5*a*d*e**2*x**3)/(8*d*e**3*(d**2 + 2*d*e*
x**2 + e**2*x**4))
```

3.649 $\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 1092

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^3} dx = \text{Too large to display}$$

output
```
1/16*b*c*(-c^2*x^2+1)^(1/2)/(-d)^(1/2)/e/(c^2*d+e)/((-d)^(1/2)-e^(1/2)*x)+
1/16*b*c*(-c^2*x^2+1)^(1/2)/(-d)^(1/2)/e/(c^2*d+e)/((-d)^(1/2)+e^(1/2)*x)-
1/16*(a+b*arccos(c*x))/(-d)^(1/2)/e^(3/2)/((-d)^(1/2)-e^(1/2)*x)^2-1/16*(a
+b*arccos(c*x))/d/e^(3/2)/((-d)^(1/2)-e^(1/2)*x)+1/16*(a+b*arccos(c*x))/(-
d)^(1/2)/e^(3/2)/((-d)^(1/2)+e^(1/2)*x)^2+1/16*(a+b*arccos(c*x))/d/e^(3/2)
/((-d)^(1/2)+e^(1/2)*x)-1/16*b*c^3*arctanh((e^(1/2)-c^2*(-d)^(1/2)*x)/(c^2
*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/e^(3/2)/(c^2*d+e)^(3/2)+1/16*b*c*arctanh((
e^(1/2)-c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d/e^(3/2)/(c
^2*d+e)^(1/2)-1/16*b*c^3*arctanh((e^(1/2)+c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2
)/(-c^2*x^2+1)^(1/2))/e^(3/2)/(c^2*d+e)^(3/2)+1/16*b*c*arctanh((e^(1/2)+c^
2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d/e^(3/2)/(c^2*d+e)^(1
/2)-1/16*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-
d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arccos(c*x))*ln(1+
e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^
(3/2)/e^(3/2)-1/16*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2
))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arccos(c
*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/
2)))/(-d)^(3/2)/e^(3/2)+1/16*I*b*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/
2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*I*b*polylog(
2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))...
```

Mathematica [A] (warning: unable to verify)

Time = 5.80 (sec) , antiderivative size = 1531, normalized size of antiderivative = 1.40

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcCos[c*x]))/(d + e*x^2)^3,x]
```

output
```
((-8*a*Sqrt[e]*x)/(d + e*x^2)^2 + (4*a*Sqrt[e]*x)/(d^2 + d*e*x^2) + (4*a*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*b*(ArcCos[c*x]/((-I)*Sqrt[d] + Sq
rt[e]*x) - (c*Log[(2*e*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1
 - c^2*x^2]))/(c*Sqrt[c^2*d + e]*((-I)*Sqrt[d] + Sqrt[e]*x))])/Sqrt[c^2*d
+ e]))/d + 2*b*(((-I)*c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[d]*(c^2*d + e)*((
-I)*Sqrt[d] + Sqrt[e]*x)) - (I*ArcCos[c*x])/(Sqrt[d]*(Sqrt[d] + I*Sqrt[e]*
x)^2) + (c^3*Log[(-4*e*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c
^2*d + e]*Sqrt[1 - c^2*x^2]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))])/(c^2*d + e
)^(3/2)) - (2*b*(-(ArcCos[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) + (c*Log[(-2*e*(Sq
rt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c*Sqrt[c^2*
d + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[c^2*d + e]))/d + 2*b*((I*c*Sqrt[e]*
Sqrt[1 - c^2*x^2])/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) + (I*ArcC
os[c*x])/(Sqrt[d]*(Sqrt[d] - I*Sqrt[e]*x)^2) + (c^3*Log[(-4*e*Sqrt[c^2*d +
 e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c^3*
(d - I*Sqrt[d]*Sqrt[e]*x))])/(c^2*d + e)^(3/2)) + (b*(ArcCos[c*x]^2 - 8*Ar
cSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + I*Sqrt[
e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]] + (2*I)*ArcCos[c*x]*Log[1 - (I*(-
(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (4*I)*ArcSin[
Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c
^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e]] + (2*I)*ArcCos[c*x]*Log[1 + (I*(...
```

Rubi [A] (verified)

Time = 3.19 (sec) , antiderivative size = 1092, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5233, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{5233}$$

$$\int \left(\frac{a+b\arccos(cx)}{e\,(d+ex^2)^2}-\frac{d(a+b\arccos(cx))}{e\,(d+ex^2)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\,\text{arctanh}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c^3}{16e^{3/2}\left(dc^2+e\right)^{3/2}}+\frac{b\,\text{arctanh}\left(\frac{\sqrt{-d}xc^2+\sqrt{e}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c^3}{16e^{3/2}\left(dc^2+e\right)^{3/2}}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c}{16de^{3/2}\sqrt{dc^2+e}}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{-d}xc^2+\sqrt{e}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c}{16de^{3/2}\sqrt{dc^2+e}}-\frac{b\sqrt{1-c^2x^2}c}{16\sqrt{-d}e\left(dc^2+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{b\sqrt{1-c^2x^2}c}{16\sqrt{-d}e\left(dc^2+e\right)\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{a+b\arccos(cx)}{16de^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\arccos(cx)}{16de^{3/2}\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{a+b\arccos(cx)}{16\sqrt{-d}e^{3/2}\left(\sqrt{-d}-\sqrt{e}x\right)^2}+\frac{a+b\arccos(cx)}{16\sqrt{-d}e^{3/2}\left(\sqrt{e}x+\sqrt{-d}\right)^2}-\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{(a+b\arccos(cx))\log\left(\frac{e^{i\arccos(cx)}\sqrt{e}}{c\sqrt{-d}-i\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}}-\frac{(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{(a+b\arccos(cx))\log\left(\frac{e^{i\arccos(cx)}\sqrt{e}}{\sqrt{-d}c+i\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}}-\frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}-\frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcCos[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/16*(b*c*Sqrt[1 - c^2*x^2])/(Sqrt[-d]*e*(c^2*d + e)*(Sqrt[-d] - Sqrt[e]*
x)) - (b*c*Sqrt[1 - c^2*x^2])/(16*Sqrt[-d]*e*(c^2*d + e)*(Sqrt[-d] + Sqrt[
e]*x)) - (a + b*ArcCos[c*x])/(16*Sqrt[-d]*e^(3/2)*(Sqrt[-d] - Sqrt[e]*x)^2
) - (a + b*ArcCos[c*x])/(16*d*e^(3/2)*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*Arc
Cos[c*x])/(16*Sqrt[-d]*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)^2) + (a + b*ArcCos[c
*x])/(16*d*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)) + (b*c^3*ArcTanh[(Sqrt[e] - c^2
*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(16*e^(3/2)*(c^2*d + e)
^(3/2)) - (b*c*ArcTanh[(Sqrt[e] - c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1
- c^2*x^2])])/(16*d*e^(3/2)*Sqrt[c^2*d + e]) + (b*c^3*ArcTanh[(Sqrt[e] + c
^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(16*e^(3/2)*(c^2*d +
e)^(3/2)) - (b*c*ArcTanh[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[
1 - c^2*x^2])])/(16*d*e^(3/2)*Sqrt[c^2*d + e]) - ((a + b*ArcCos[c*x])*Log[
1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(16*(-d
)^(3/2)*e^(3/2)) + ((a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x])
)/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*Arc
Cos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d +
 e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(
I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2))
 - ((I/16)*b*PolyLog[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt
[c^2*d + e]))])/((-d)^(3/2)*e^(3/2)) + ((I/16)*b*PolyLog[2, (Sqrt[e]*E^...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5233
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.10 (sec) , antiderivative size = 1226, normalized size of antiderivative = 1.12

method	result	size
parts	Expression too large to display	1226
derivativedivides	Expression too large to display	1241
default	Expression too large to display	1241

input
```
int(x^2*(a+b*arccos(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*((1/8/d*x^3-1/8/e*x)/(e*x^2+d)^2+1/8/d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1
/2)))+b/c^3*(1/8*c^4*(arccos(c*x)*c^5*d*e*x^3-arccos(c*x)*d^2*c^5*x+c^4*d*
e*x^2*(-c^2*x^2+1)^(1/2)+d^2*c^4*(-c^2*x^2+1)^(1/2)+arccos(c*x)*e^2*c^3*x^
3-arccos(c*x)*c^3*d*e*x)/e/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2-1/8*I*((2*c^2*d
+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(-2*d*c^2*(c^2*d*(c^2*d+e))^(1/2)+2
*c^4*d^2+2*c^2*d*e-(c^2*d*(c^2*d+e))^(1/2)*e)*c^4*arctan(e*(c*x+I*(-c^2*x^
2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/(c^2*d+e)^2/e
^3/d+1/8*I*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2
*d*(c^2*d+e))^(1/2)+e)*arctan(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^
2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))*c^4/e^3/d/(c^2*d+e)-1/8*I*(-(2*c^2*d-2*(
c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^4*d^2+2*d*c^2*(c^2*d*(c^2*d+e))^(1
/2)+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1/2)*e)*c^4*arctanh(e*(c*x+I*(-c^2*x^2+1)
^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/(c^2*d+e)^2/e^3/
d+1/8*I*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d+2*(c^2*d
*(c^2*d+e))^(1/2)+e)*arctanh(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^
2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))*c^4/e^3/d/(c^2*d+e)-1/16*I/(c^2*d+e)/d*c
^4*sum(_R1/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(
1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4
*c^2*d+2*e)*_Z^2+e))-1/16*I/(c^2*d+e)*c^6/e*sum(_R1/(_R1^2*e+2*c^2*d+e)*(I
*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-...
```

Fricas [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\arccos{(cx)}+a)x^2}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2*arccos(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*acos(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\arccos(cx)+a)x^2}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)*x^2/(e*x^2 + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^2\,(a+b\,\mathrm{acos}(c\,x))}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^2*(a + b*acos(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((x^2*(a + b*acos(c*x)))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^2(a+b\arccos(cx))}{(d+ex^2)^3}\,dx$$
$$= \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,d^2+2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ade\,x^2+\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^2x^4+8\left(\int \frac{acos(cx}{e^3x^6+3d\,e^2x^4+}\right.}{8d^2e^2}$$

input
```
int(x^2*(a+b*acos(c*x))/(e*x^2+d)^3,x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d**2 + 2*sqrt(e)*sqrt(d)*
atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e*x**2 + sqrt(e)*sqrt(d)*atan((e*x)/(sqr
t(e)*sqrt(d)))*a*e**2*x**4 + 8*int((acos(c*x)*x**2)/(d**3 + 3*d**2*e*x**2
+ 3*d*e**2*x**4 + e**3*x**6),x)*b*d**4*e**2 + 16*int((acos(c*x)*x**2)/(d**
3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**3*e**3*x**2 + 8*int
((acos(c*x)*x**2)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*
d**2*e**4*x**4 - a*d**2*e*x + a*d*e**2*x**3)/(8*d**2*e**2*(d**2 + 2*d*e*x*
*2 + e**2*x**4))
```

3.650 $\int \frac{a+b\arccos(cx)}{(d+ex^2)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1092

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

output
```
1/16*b*c*(-c^2*x^2+1)^(1/2)/(-d)^(3/2)/(c^2*d+e)/((-d)^(1/2)-e^(1/2)*x)+1/
16*b*c*(-c^2*x^2+1)^(1/2)/(-d)^(3/2)/(c^2*d+e)/((-d)^(1/2)+e^(1/2)*x)-1/16
*(a+b*arccos(c*x))/(-d)^(3/2)/e^(1/2)/((-d)^(1/2)-e^(1/2)*x)^2-3/16*(a+b*a
rccos(c*x))/d^2/e^(1/2)/((-d)^(1/2)-e^(1/2)*x)+1/16*(a+b*arccos(c*x))/(-d)
^(3/2)/e^(1/2)/((-d)^(1/2)+e^(1/2)*x)^2+3/16*(a+b*arccos(c*x))/d^2/e^(1/2)
/((-d)^(1/2)+e^(1/2)*x)+1/16*b*c^3*arctanh((e^(1/2)-c^2*(-d)^(1/2)*x)/(c^2
*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d/e^(1/2)/(c^2*d+e)^(3/2)+3/16*b*c*arctanh
((e^(1/2)-c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d^2/e^(1/2
)/(c^2*d+e)^(1/2)+1/16*b*c^3*arctanh((e^(1/2)+c^2*(-d)^(1/2)*x)/(c^2*d+e)^
(1/2)/(-c^2*x^2+1)^(1/2))/d/e^(1/2)/(c^2*d+e)^(3/2)+3/16*b*c*arctanh((e^(1
/2)+c^2*(-d)^(1/2)*x)/(c^2*d+e)^(1/2)/(-c^2*x^2+1)^(1/2))/d^2/e^(1/2)/(c^2
*d+e)^(1/2)+3/16*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))
/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arccos(c*x
))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)
))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arccos(c*x))*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2
+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*
arccos(c*x))*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*
d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*I*b*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x
^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*I*b
*polylog(2,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e...
```

Mathematica [A] (warning: unable to verify)

Time = 5.65 (sec) , antiderivative size = 1547, normalized size of antiderivative = 1.42

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d + e*x^2)^3,x]
```

output
```
((8*a*d^(3/2)*x)/(d + e*x^2)^2 + (12*a*Sqrt[d]*x)/(d + e*x^2) + (12*a*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (6*b*Sqrt[d]*(ArcCos[c*x]/((-I)*Sqrt[d]
 + Sqrt[e]*x) - (c*Log[(2*e*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*S
qrt[1 - c^2*x^2]))/(c*Sqrt[c^2*d + e]*((-I)*Sqrt[d] + Sqrt[e]*x))])/Sqrt[c
^2*d + e]))/Sqrt[e] + (2*I)*b*d*((c*Sqrt[1 - c^2*x^2])/((c^2*d + e)*((-I)*
Sqrt[d] + Sqrt[e]*x)) - ArcCos[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2)
 + (I*c^3*Sqrt[d]*Log[(-4*e*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + S
qrt[c^2*d + e]*Sqrt[1 - c^2*x^2]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))])/(Sqrt
[e]*(c^2*d + e)^(3/2))) - (6*b*Sqrt[d]*(-(ArcCos[c*x]/(I*Sqrt[d] + Sqrt[e]
*x)) + (c*Log[(-2*e*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[1 -
c^2*x^2]))/(c*Sqrt[c^2*d + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[c^2*d + e]))
/Sqrt[e] - (2*I)*b*d*((c*Sqrt[1 - c^2*x^2])/((c^2*d + e)*(I*Sqrt[d] + Sqrt
[e]*x)) - ArcCos[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - (I*c^3*Sqrt[d]
*Log[(-4*e*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sq
rt[1 - c^2*x^2]))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x))])/(Sqrt[e]*(c^2*d + e)^(
3/2))) + (3*b*(ArcCos[c*x]^2 - 8*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sq
rt[2]]*ArcTan[((c*Sqrt[d] + I*Sqrt[e])*Tan[ArcCos[c*x]/2])/Sqrt[c^2*d + e]
] + (2*I)*ArcCos[c*x]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*Arc
Cos[c*x]))/Sqrt[e]] + (4*I)*ArcSin[Sqrt[1 + (I*c*Sqrt[d])/Sqrt[e]]/Sqrt[2]
]*Log[1 - (I*(-(c*Sqrt[d]) + Sqrt[c^2*d + e])*E^(I*ArcCos[c*x]))/Sqrt[e...
```

Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 1092, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(-\frac{3e(a+b\arccos(cx))}{8d^2(-de-e^2x^2)}-\frac{3e(a+b\arccos(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{3e(a+b\arccos(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}+ex\right)^2}-\frac{e^{3/2}(a+b\arccos(cx))}{8(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-ex\right)^3}-\frac{e}{8(}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\,\text{arctanh}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c^3}{16d\sqrt{e}\,(dc^2+e)^{3/2}}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{-d}xc^2+\sqrt{e}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c^3}{16d\sqrt{e}\,(dc^2+e)^{3/2}}-\frac{3b\,\text{arctanh}\left(\frac{\sqrt{e}-c^2\sqrt{-d}x}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c}{16d^2\sqrt{e}\sqrt{dc^2+e}}-\frac{3b\,\text{arctanh}\left(\frac{\sqrt{-d}xc^2+\sqrt{e}}{\sqrt{dc^2+e}\sqrt{1-c^2x^2}}\right)c}{16d^2\sqrt{e}\sqrt{dc^2+e}}-\frac{b\sqrt{1-c^2x^2}c}{16(-d)^{3/2}\,(dc^2+e)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{b\sqrt{1-c^2x^2}c}{16(-d)^{3/2}\,(dc^2+e)\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{3(a+b\arccos(cx))}{16d^2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{3(a+b\arccos(cx))}{16d^2\sqrt{e}\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{a+b\arccos(cx)}{16(-d)^{3/2}\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)^2}+\frac{a+b\arccos(cx)}{16(-d)^{3/2}\sqrt{e}\left(\sqrt{e}x+\sqrt{-d}\right)^2}+\frac{3(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3(a+b\arccos(cx))\log\left(\frac{e^{i\arccos(cx)}\sqrt{e}}{c\sqrt{-d}-i\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3(a+b\arccos(cx))\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-dc}+i\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3(a+b\arccos(cx))\log\left(\frac{e^{i\arccos(cx)}\sqrt{e}}{\sqrt{-dc}+i\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-dc}+i\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-dc}+i\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d + e*x^2)^3,x]
```

output
```
-1/16*(b*c*Sqrt[1 - c^2*x^2])/((-d)^(3/2)*(c^2*d + e)*(Sqrt[-d] - Sqrt[e]*
x)) - (b*c*Sqrt[1 - c^2*x^2])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d] + Sqrt[
e]*x)) - (a + b*ArcCos[c*x])/(16*(-d)^(3/2)*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)
^2) - (3*(a + b*ArcCos[c*x]))/(16*d^2*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a
 + b*ArcCos[c*x])/(16*(-d)^(3/2)*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)^2) + (3*(a
 + b*ArcCos[c*x]))/(16*d^2*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) - (b*c^3*ArcTan
h[(Sqrt[e] - c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(16*d*S
qrt[e]*(c^2*d + e)^(3/2)) - (3*b*c*ArcTanh[(Sqrt[e] - c^2*Sqrt[-d]*x)/(Sqr
t[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(16*d^2*Sqrt[e]*Sqrt[c^2*d + e]) - (b*c^
3*ArcTanh[(Sqrt[e] + c^2*Sqrt[-d]*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])
/(16*d*Sqrt[e]*(c^2*d + e)^(3/2)) - (3*b*c*ArcTanh[(Sqrt[e] + c^2*Sqrt[-d]
*x)/(Sqrt[c^2*d + e]*Sqrt[1 - c^2*x^2])])/(16*d^2*Sqrt[e]*Sqrt[c^2*d + e])
 + (3*(a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d]
- I*Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCos[c*x])*Lo
g[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(16*(
-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCos[c*x])*Log[1 - (Sqrt[e]*E^(I*ArcCos[c
*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a +
 b*ArcCos[c*x])*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c
^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (((3*I)/16)*b*PolyLog[2, -((Sqrt[e]
*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e]))])/((-d)^(5/2)*Sqr...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.82 (sec) , antiderivative size = 1786, normalized size of antiderivative = 1.64

method	result	size
parts	Expression too large to display	1786
derivativedivides	Expression too large to display	1811
default	Expression too large to display	1811

input
```
int((a+b*arccos(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x/d/(e*x^2+d)^2+3/8*a/d^2*x/(e*x^2+d)+3/8*a/d^2/(d*e)^(1/2)*arctan(e
*x/(d*e)^(1/2))+b/c*(1/8*c^2*(5*arccos(c*x)*d^2*c^5*x+3*arccos(c*x)*c^5*d*
e*x^3-d^2*c^4*(-c^2*x^2+1)^(1/2)-c^4*d*e*x^2*(-c^2*x^2+1)^(1/2)+5*arccos(c
*x)*c^3*d*e*x+3*arccos(c*x)*e^2*c^3*x^3)/d^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2
+3/8*I*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d+2*(c^2*d*
(c^2*d+e))^(1/2)+e)*arctanh(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2
*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))*c^2/d^2/(c^2*d+e)/e^2+1/2*I*((2*c^2*d+2*(
c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*a
rctan(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*
e)^(1/2))*c^4/e^3/d/(c^2*d+e)-3/16*I/(c^2*d+e)*c^2/d^2*e*sum(_R1/(_R1^2*e+
2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R
1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
-3/16*I/(c^2*d+e)/d*c^4*sum(_R1/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1
-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1))
,_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+3/16*I/(c^2*d+e)*c^2/d^2*e*sum(1
/_R1/(_R1^2*e+2*c^2*d+e)*(I*arccos(c*x)*ln((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/
_R1)+dilog((_R1-c*x-I*(-c^2*x^2+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d
+2*e)*_Z^2+e))-1/2*I*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*(2*c
^4*d^2+2*d*c^2*(c^2*d*(c^2*d+e))^(1/2)+2*c^2*d*e+(c^2*d*(c^2*d+e))^(1/2)*e
)*c^4*arctanh(e*(c*x+I*(-c^2*x^2+1)^(1/2))/((-2*c^2*d+2*(c^2*d*(c^2*d+e...
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^3}\,dx=\int \frac{b\arccos(cx)+a}{(ex^2+d)^3}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3),
x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^3}\,dx=\int \frac{a+b\operatorname{acos}(cx)}{(d+ex^2)^3}\,dx$$

input
```
integrate((a+b*acos(c*x))/(e*x**2+d)**3,x)
```

output
```
Integral((a + b*acos(c*x))/(d + e*x**2)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^3}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{(e\,x^2+d)^3}\,dx$$

input
```
int((a + b*acos(c*x))/(d + e*x^2)^3,x)
```

output
```
int((a + b*acos(c*x))/(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^3}\,dx = \frac{3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,d^2+6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ade\,x^2+3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a\,e^2x^4+8\left(\int \frac{aco}{e^3x^6+3d\,e^2x}\right.}{8d}$$

input
```
int((a+b*acos(c*x))/(e*x^2+d)^3,x)
```

output
```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d**2 + 6*sqrt(e)*sqrt(d
)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e*x**2 + 3*sqrt(e)*sqrt(d)*atan((e*x)/
(sqrt(e)*sqrt(d)))*a*e**2*x**4 + 8*int(acos(c*x)/(d**3 + 3*d**2*e*x**2 + 3
*d*e**2*x**4 + e**3*x**6),x)*b*d**5*e + 16*int(acos(c*x)/(d**3 + 3*d**2*e*
x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**4*e**2*x**2 + 8*int(acos(c*x)/(d
**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*d**3*e**3*x**4 + 5*a
*d**2*e*x + 3*a*d*e**2*x**3)/(8*d**3*e*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.651 $\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx = \text{Int}\left(\sqrt{d+ex^2}(a+b\arccos(cx)),x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)*(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx = \int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcCos[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5175}$$

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))dx$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \sqrt{e\,x^2+d}\,(a+b\arccos\,(cx))\,dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arccos(c*x)),x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx=\int \sqrt{ex^2+d}(b\arccos(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccos(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 10.61 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}\,(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*acos(c*x)),x)
```

output
```
Integral((a + b*acos(c*x))*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx=\int \sqrt{ex^2+d}(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccos(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((a + b*acos(c*x))*(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))*(d + e*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.05

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))\,dx$$
$$= \frac{\sqrt{e\,x^2+d}\,aex+\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)ad+2\left(\int\sqrt{e\,x^2+d}\,acos(cx)\,dx\right)be}{2e}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*acos(c*x)),x)
```

output
```
(sqrt(d + e*x**2)*a*e*x + sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(
d))*a*d + 2*int(sqrt(d + e*x**2)*acos(c*x),x)*b*e)/(2*e)
```

3.652 $\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{a+b\arccos(cx)}{\sqrt{d+ex^2}},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))/(e*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(a + b*ArcCos[c*x])/Sqrt[d + e*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{5175}$$

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcCos[c*x])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\arccos\left(cx\right)}{\sqrt{e\,x^2+d}}dx$$

input
```
int((a+b*arccos(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccos(c*x))/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\arccos{(cx)}+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 5.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\operatorname{acos}{(cx)}}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acos(c*x))/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))/(d + e*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{a+b\arccos(cx)}{\sqrt{d+ex^2}}\,dx=\frac{\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a+\left(\int\frac{acos(cx)}{\sqrt{e\,x^2+d}}dx\right)be}{e}$$

input
```
int((a+b*acos(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a + int(acos(c*x)/sqr
t(d + e*x**2),x)*b*e)/e
```

3.653 $\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x(a+b\arccos(cx))}{d\sqrt{d+ex^2}}+\frac{b\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

output
```
x*(a+b*arccos(c*x))/d/(e*x^2+d)^(1/2)+b*arctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/
c/(e*x^2+d)^(1/2))/d/e^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x\left(bcx\sqrt{1+\frac{ex^2}{d}}\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{ex^2}{d}\right)+2(a+b\arccos(cx))\right)}{2d\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(b*c*x*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/
d)] + 2*(a + b*ArcCos[c*x])))/(2*d*Sqrt[d + e*x^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5171, 27, 353, 66, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 5171$$

$$bc\int \frac{x}{d\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx+\frac{x(a+b\arccos(cx))}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{bc\int \frac{x}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx}{d}+\frac{x(a+b\arccos(cx))}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 353$$

$$\frac{bc\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2}{2d}+\frac{x(a+b\arccos(cx))}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 66$$

$$\frac{bc\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}}{d}+\frac{x(a+b\arccos(cx))}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \; 218$$

$$\frac{x(a+b\arccos(cx))}{d\sqrt{d+ex^2}}-\frac{b\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}}$$

input
```
Int[(a + b*ArcCos[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCos[c*x]))/(d*Sqrt[d + e*x^2]) - (b*ArcTan[(Sqrt[e]*Sqrt[1 -
c^2*x^2])/(c*Sqrt[d + e*x^2])])/(d*Sqrt[e])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 5171
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2
, 0])
```

Maple [F]

$$\int \frac{a + b\arccos(cx)}{(e\,x^2 + d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccos(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccos(c*x))/(e*x^2+d)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(60) = 120$.

Time = 0.15 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.20

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx = \left[-\frac{(bex^2+bd)\sqrt{-e}\log\left(8\,c^4e^2x^4+c^4d^2-6\,c^2de+8\,(c^4de-c^2e^2)x^2-4\,(2\,c^3ex^2\right.}{4\,(de^2}\right.$$

$$\left.-\frac{(bex^2+bd)\sqrt{e}\arctan\left(\frac{(2\,c^2ex^2+c^2d-e)\sqrt{-c^2x^2+1}\sqrt{ex^2+d}\sqrt{e}}{2\,(c^3e^2x^4-cde+(c^3de-ce^2)x^2)}\right)-2\,(bex\arccos(cx)+aex)\sqrt{ex^2+d}}{2\,(de^2x^2+d^2e)}\right]$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((b*e*x^2 + b*d)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e +
8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(-c^2*x^2 +
1)*sqrt(e*x^2 + d)*sqrt(-e) + e^2) - 4*(b*e*x*arccos(c*x) + a*e*x)*sqrt(e*
x^2 + d))/(d*e^2*x^2 + d^2*e), -1/2*((b*e*x^2 + b*d)*sqrt(e)*arctan(1/2*(2
*c^2*e*x^2 + c^2*d - e)*sqrt(-c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(e)/(c^3*e^
2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(b*e*x*arccos(c*x) + a*e*x)*sq
rt(e*x^2 + d))/(d*e^2*x^2 + d^2*e)]
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{acos}\,(cx)}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c^2*d>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))/(d + e*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,aex+\sqrt{e}\,ad+\sqrt{e}\,ae\,x^2+\left(\int \frac{acos(cx)}{\sqrt{e\,x^2+d}\,d+\sqrt{e\,x^2+d}\,e\,x^2}dx\right)b\,d^2e+\left(\int \frac{}{\sqrt{e\,x^2+}}\right.}{de\,(e\,x^2+d)}$$

input
```
int((a+b*acos(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*a*e*x + sqrt(e)*a*d + sqrt(e)*a*e*x**2 + int(acos(c*x)/(
sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*d**2*e + int(acos(c*x)/
(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*d*e**2*x**2)/(d*e*(d +
 e*x**2))
```

3.654 $\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 146

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx=\frac{bc\sqrt{1-c^2x^2}}{3d(c^2d+e)\sqrt{d+ex^2}}+\frac{x(a+b\arccos(cx))}{3d(d+ex^2)^{3/2}}+\frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d+ex^2}}+\frac{2b\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}}$$

output
```
1/3*b*c*(-c^2*x^2+1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)^(1/2)+1/3*x*(a+b*arccos(c
*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arccos(c*x))/d^2/(e*x^2+d)^(1/2)+2/3*b*a
rctan(e^(1/2)*(-c^2*x^2+1)^(1/2)/c/(e*x^2+d)^(1/2))/d^2/e^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.30

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx = -\frac{bc\sqrt{1-c^2x^2}}{3d(c^2d+e)\sqrt{d+ex^2}} + \sqrt{d+ex^2}\left(\frac{ax}{3d(d+ex^2)^2}+\frac{2ax}{3d^2(d+ex^2)}\right) + \frac{bcx^2\sqrt{\frac{d+ex^2}{d}}\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{ex^2}{d}\right)}{3d^2\sqrt{d+ex^2}} + \frac{bx(3d+2ex^2)\arccos(cx)}{3d^2(d+ex^2)^{3/2}}$$

input

```
Integrate[(a + b*ArcCos[c*x])/(d + e*x^2)^(5/2),x]
```

output

```
-1/3*(b*c*Sqrt[1 - c^2*x^2])/(d*(c^2*d + e)*Sqrt[d + e*x^2]) + Sqrt[d + e*
x^2]*((a*x)/(3*d*(d + e*x^2)^2) + (2*a*x)/(3*d^2*(d + e*x^2))) + (b*c*x^2*
Sqrt[(d + e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/(3*d^
2*Sqrt[d + e*x^2]) + (b*x*(3*d + 2*e*x^2)*ArcCos[c*x])/(3*d^2*(d + e*x^2)^
(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5171, 27, 435, 87, 66, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 5171}$$

$$bc\int \frac{x(2ex^2+3d)}{3d^2\sqrt{1-c^2x^2}(ex^2+d)^{3/2}}dx + \frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d+ex^2}} + \frac{x(a+b\arccos(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bc\int \frac{x(2ex^2+3d)}{\sqrt{1-c^2x^2}(ex^2+d)^{3/2}}dx}{3d^2}+\frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arccos(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{435}$$

$$\frac{bc\int \frac{2ex^2+3d}{\sqrt{1-c^2x^2}(ex^2+d)^{3/2}}dx^2}{6d^2}+\frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arccos(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{bc\left(2\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2-\frac{2d\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6d^2}+\frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arccos(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{66}$$

$$\frac{bc\left(4\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}-\frac{2d\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6d^2}+\frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arccos(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{2x(a+b\arccos(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arccos(cx))}{3d(d+ex^2)^{3/2}}+\frac{bc\left(-\frac{4\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{2d\sqrt{1-c^2x^2}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6d^2}$$

input
```
Int[(a + b*ArcCos[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcCos[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcCos[c*x])
)/(3*d^2*Sqrt[d + e*x^2]) + (b*c*((-2*d*Sqrt[1 - c^2*x^2])/((c^2*d + e)*Sq
rt[d + e*x^2]) - (4*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])
])/(c*Sqrt[e])))/(6*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5171
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2
, 0])
```

Maple [F]

$$\int \frac{a+b\arccos(cx)}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccos(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arccos(c*x))/(e*x^2+d)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(122) = 244.

Time = 0.17 (sec) , antiderivative size = 686, normalized size of antiderivative = 4.70

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx = \left[-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{-e}\log\left(8\,c^4e^2x^4+c\right.}{}\right.$$

$$-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{e}\arctan\left(\frac{(2\,c^2ex^2+c^2d-e)\sqrt{-c^2x^2+1}\sqrt{ex^2+d}\sqrt{e}}{2\,(c^3e^2x^4-cde+(c^3de-ce^2)x^2)}\right)-(2}{3\,(c^2d^5e+d^4}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e +
 b*d*e^2)*x^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d
*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(-c^2*x^2 + 1)*sqrt(
e*x^2 + d)*sqrt(-e) + e^2) - 2*(2*(a*c^2*d*e^2 + a*e^3)*x^3 + 3*(a*c^2*d^2
*e + a*d*e^2)*x + (2*(b*c^2*d*e^2 + b*e^3)*x^3 + 3*(b*c^2*d^2*e + b*d*e^2)
*x)*arccos(c*x) - (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(-c^2*x^2 + 1))*sqrt(e*x
^2 + d))/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e
^2 + d^3*e^3)*x^2), -1/3*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e
 + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d
- e)*sqrt(-c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(e)/(c^3*e^2*x^4 - c*d*e + (c^
3*d*e - c*e^2)*x^2)) - (2*(a*c^2*d*e^2 + a*e^3)*x^3 + 3*(a*c^2*d^2*e + a*d
*e^2)*x + (2*(b*c^2*d*e^2 + b*e^3)*x^3 + 3*(b*c^2*d^2*e + b*d*e^2)*x)*arcc
os(c*x) - (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(-c^2*x^2 + 1))*sqrt(e*x^2 + d))
/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3
*e^3)*x^2)]
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acos}\,(cx)}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))/(d + e*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx=\int \frac{b\arccos\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(
arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/((e^2*x^4 + 2*d*e*x^2 + d^2)*sq
rt(e*x^2 + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*acos(c*x))/(d + e*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{3\sqrt{e\,x^2+d}\,adex+2\sqrt{e\,x^2+d}\,a\,e^2x^3-2\sqrt{e}\,a\,d^2-4\sqrt{e}\,ade\,x^2-2\sqrt{e}\,a\,e^2x^4+3\big(}{}$$

input
```
int((a+b*acos(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
(3*sqrt(d + e*x**2)*a*d*e*x + 2*sqrt(d + e*x**2)*a*e**2*x**3 - 2*sqrt(e)*a
*d**2 - 4*sqrt(e)*a*d*e*x**2 - 2*sqrt(e)*a*e**2*x**4 + 3*int(acos(c*x)/(sq
rt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*
x**4),x)*b*d**4*e + 6*int(acos(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*
x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*d**3*e**2*x**2 + 3*int(a
cos(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e
*x**2)*e**2*x**4),x)*b*d**2*e**3*x**4)/(3*d**2*e*(d**2 + 2*d*e*x**2 + e**2
*x**4))
```

3.655 $\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 226

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx = \frac{bc\sqrt{1-c^2x^2}}{15d(c^2d+e)(d+ex^2)^{3/2}} + \frac{2bc(3c^2d+2e)\sqrt{1-c^2x^2}}{15d^2(c^2d+e)^2\sqrt{d+ex^2}} + \frac{x(a+b\arccos(cx))}{5d(d+ex^2)^{5/2}} + \frac{4x(a+b\arccos(cx))}{15d^2(d+ex^2)^{3/2}} + \frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}} + \frac{8b\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{15d^3\sqrt{e}}$$

output
```
1/15*b*c*(-c^2*x^2+1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)^(3/2)+2/15*b*c*(3*c^2*d+
2*e)*(-c^2*x^2+1)^(1/2)/d^2/(c^2*d+e)^2/(e*x^2+d)^(1/2)+1/5*x*(a+b*arccos(
c*x))/d/(e*x^2+d)^(5/2)+4/15*x*(a+b*arccos(c*x))/d^2/(e*x^2+d)^(3/2)+8/15*
x*(a+b*arccos(c*x))/d^3/(e*x^2+d)^(1/2)+8/15*b*arctan(e^(1/2)*(-c^2*x^2+1)
^(1/2)/c/(e*x^2+d)^(1/2))/d^3/e^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.84

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx = \frac{ax(15d^2+20dex^2+8e^2x^4)-\frac{bcd\sqrt{1-c^2x^2}(d+ex^2)\left(e(5d+4ex^2)+c^2d(7d+6ex^2)\right)}{(c^2d+e)^2}+4bcx^2(d+}{15d}$$

input
```
Integrate[(a + b*ArcCos[c*x])/(d + e*x^2)^(7/2),x]
```

output
```
(a*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4) - (b*c*d*Sqrt[1 - c^2*x^2]*(d + e*x
^2)*(e*(5*d + 4*e*x^2) + c^2*d*(7*d + 6*e*x^2)))/(c^2*d + e)^2 + 4*b*c*x^2
*(d + e*x^2)^2*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*
x^2)/d)] + b*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4)*ArcCos[c*x])/(15*d^3*(d +
 e*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5171, 27, 7266, 1193, 27, 87, 66, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx$$

$$\Big\downarrow \; 5171$$

$$bc\int \frac{x(8e^2x^4+20dex^2+15d^2)}{15d^3\sqrt{1-c^2x^2}\,(ex^2+d)^{5/2}}dx+\frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\arccos(cx))}{15d^2\,(d+ex^2)^{3/2}}+\frac{x(a+b\arccos(cx))}{5d\,(d+ex^2)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{bc\int \frac{x(8e^2x^4+20dex^2+15d^2)}{\sqrt{1-c^2x^2}(ex^2+d)^{5/2}}dx}{15d^3}+\frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\arccos(cx))}{15d^2(d+ex^2)^{3/2}}+\frac{x(a+b\arccos(cx))}{5d(d+ex^2)^{5/2}}$$

$$\Big\downarrow \; 7266$$

$$\frac{bc\int \frac{8e^2x^4+20dex^2+15d^2}{\sqrt{1-c^2x^2}(ex^2+d)^{5/2}}dx^2}{30d^3}+\frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\arccos(cx))}{15d^2(d+ex^2)^{3/2}}+\frac{x(a+b\arccos(cx))}{5d(d+ex^2)^{5/2}}$$

$$\Big\downarrow \; 1193$$

$$\frac{bc\left(-\frac{2\int -\frac{3\left(4e\left(dc^2+e\right)x^2+d\left(7dc^2+6e\right)\right)}{\sqrt{1-c^2x^2}\left(ex^2+d\right)^{3/2}}dx^2}{3(c^2d+e)}-\frac{2d^2\sqrt{1-c^2x^2}}{(c^2d+e)(d+ex^2)^{3/2}}\right)}{30d^3}+\frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\arccos(cx))}{15d^2(d+ex^2)^{3/2}}+\frac{x(a+b\arccos(cx))}{5d(d+ex^2)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{bc\left(\frac{2\int \frac{4e\left(dc^2+e\right)x^2+d\left(7dc^2+6e\right)}{\sqrt{1-c^2x^2}\left(ex^2+d\right)^{3/2}}dx^2}{c^2d+e}-\frac{2d^2\sqrt{1-c^2x^2}}{(c^2d+e)(d+ex^2)^{3/2}}\right)}{30d^3}+\frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\arccos(cx))}{15d^2(d+ex^2)^{3/2}}+\frac{x(a+b\arccos(cx))}{5d(d+ex^2)^{5/2}}$$

$$\Big\downarrow \; 87$$

$$\frac{bc\left(\frac{2\left(4(c^2d+e)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{ex^2+d}}dx^2-\frac{2d\sqrt{1-c^2x^2}\left(3c^2d+2e\right)}{\left(c^2d+e\right)\sqrt{d+ex^2}}\right)}{c^2d+e}-\frac{2d^2\sqrt{1-c^2x^2}}{(c^2d+e)(d+ex^2)^{3/2}}\right)}{30d^3}+\frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\arccos(cx))}{15d^2(d+ex^2)^{3/2}}+\frac{x(a+b\arccos(cx))}{5d(d+ex^2)^{5/2}}$$

$$\Big\downarrow \; 66$$

$$\frac{bc\left(\frac{2\left(8(c^2d+e)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{1-c^2x^2}}{\sqrt{ex^2+d}}-\frac{2d\sqrt{1-c^2x^2}\left(3c^2d+2e\right)}{\left(c^2d+e\right)\sqrt{d+ex^2}}\right)}{c^2d+e}-\frac{2d^2\sqrt{1-c^2x^2}}{(c^2d+e)(d+ex^2)^{3/2}}\right)}{30d^3}+\frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\arccos(cx))}{15d^2(d+ex^2)^{3/2}}+\frac{x(a+b\arccos(cx))}{5d(d+ex^2)^{5/2}}$$

$$\Big\downarrow \; 218$$

$$\frac{8x(a+b\arccos(cx))}{15d^3\sqrt{d+ex^2}}+\frac{4x(a+b\arccos(cx))}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x(a+b\arccos(cx))}{5d\left(d+ex^2\right)^{5/2}}+\frac{bc\left(\frac{2\left(-\frac{8\left(c^2d+e\right)\arctan\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{2d\sqrt{1-c^2x^2}\left(3c^2d+2e\right)}{\left(c^2d+e\right)\sqrt{d+ex^2}}\right)}{c^2d+e}-\frac{2d^2\sqrt{1-c^2x^2}}{(c^2d+e)(d+ex^2)^{3/2}}\right)}{30d^3}$$

input
```
Int[(a + b*ArcCos[c*x])/(d + e*x^2)^(7/2),x]
```

output
```
(x*(a + b*ArcCos[c*x]))/(5*d*(d + e*x^2)^(5/2)) + (4*x*(a + b*ArcCos[c*x])
)/(15*d^2*(d + e*x^2)^(3/2)) + (8*x*(a + b*ArcCos[c*x]))/(15*d^3*Sqrt[d +
e*x^2]) + (b*c*((-2*d^2*Sqrt[1 - c^2*x^2])/((c^2*d + e)*(d + e*x^2)^(3/2))
 + (2*((-2*d*(3*c^2*d + 2*e)*Sqrt[1 - c^2*x^2])/((c^2*d + e)*Sqrt[d + e*x^
2]) - (8*(c^2*d + e)*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2]
)])/(c*Sqrt[e])))/(c^2*d + e)))/(30*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 5171
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCos[c*x])   u, x
] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2
, 0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [F]

$$\int \frac{a+b\arccos(cx)}{(e\,x^2+d)^{\frac{7}{2}}}dx$$

input
```
int((a+b*arccos(c*x))/(e*x^2+d)^(7/2),x)
```

output
```
int((a+b*arccos(c*x))/(e*x^2+d)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(192) = 384$.

Time = 0.24 (sec) , antiderivative size = 1324, normalized size of antiderivative = 5.86

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx = \text{Too large to display}$$

input

```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

output

```
[-1/15*(2*(b*c^4*d^5 + 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 + 2*b*c^2*d*e^4 + b*
e^5)*x^6 + b*d^3*e^2 + 3*(b*c^4*d^3*e^2 + 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 +
 3*(b*c^4*d^4*e + 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(-e)*log(8*c^4*e^2
*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 +
c^3*d - c*e)*sqrt(-c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(-e) + e^2) - (8*(a*c^
4*d^2*e^3 + 2*a*c^2*d*e^4 + a*e^5)*x^5 + 20*(a*c^4*d^3*e^2 + 2*a*c^2*d^2*e
^3 + a*d*e^4)*x^3 + 15*(a*c^4*d^4*e + 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x + (8*
(b*c^4*d^2*e^3 + 2*b*c^2*d*e^4 + b*e^5)*x^5 + 20*(b*c^4*d^3*e^2 + 2*b*c^2*
d^2*e^3 + b*d*e^4)*x^3 + 15*(b*c^4*d^4*e + 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x)
*arccos(c*x) - (7*b*c^3*d^4*e + 5*b*c*d^3*e^2 + 2*(3*b*c^3*d^2*e^3 + 2*b*c
*d*e^4)*x^4 + (13*b*c^3*d^3*e^2 + 9*b*c*d^2*e^3)*x^2)*sqrt(-c^2*x^2 + 1))*
sqrt(e*x^2 + d))/(c^4*d^8*e + 2*c^2*d^7*e^2 + d^6*e^3 + (c^4*d^5*e^4 + 2*c
^2*d^4*e^5 + d^3*e^6)*x^6 + 3*(c^4*d^6*e^3 + 2*c^2*d^5*e^4 + d^4*e^5)*x^4
+ 3*(c^4*d^7*e^2 + 2*c^2*d^6*e^3 + d^5*e^4)*x^2), -1/15*(4*(b*c^4*d^5 + 2*
b*c^2*d^4*e + (b*c^4*d^2*e^3 + 2*b*c^2*d*e^4 + b*e^5)*x^6 + b*d^3*e^2 + 3*
(b*c^4*d^3*e^2 + 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 + 3*(b*c^4*d^4*e + 2*b*c^2
*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sq
rt(-c^2*x^2 + 1)*sqrt(e*x^2 + d)*sqrt(e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e -
 c*e^2)*x^2)) - (8*(a*c^4*d^2*e^3 + 2*a*c^2*d*e^4 + a*e^5)*x^5 + 20*(a*c^4
*d^3*e^2 + 2*a*c^2*d^2*e^3 + a*d*e^4)*x^3 + 15*(a*c^4*d^4*e + 2*a*c^2*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acos(c*x))/(e*x**2+d)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx = \int \frac{b\arccos(cx)+a}{(ex^2+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

output
```
1/15*a*(8*x/(sqrt(e*x^2 + d)*d^3) + 4*x/((e*x^2 + d)^(3/2)*d^2) + 3*x/((e*
x^2 + d)^(5/2)*d)) + b*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x
)/((e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3)*sqrt(e*x^2 + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))/(e*x^2+d)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{(e\,x^2+d)^{7/2}}\,dx$$

input
```
int((a + b*acos(c*x))/(d + e*x^2)^(7/2),x)
```

output
```
int((a + b*acos(c*x))/(d + e*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{a+b\arccos(cx)}{(d+ex^2)^{7/2}}\,dx=\frac{15\sqrt{e\,x^2+d}\,a\,d^2ex+20\sqrt{e\,x^2+d}\,ad\,e^2x^3+8\sqrt{e\,x^2+d}\,a\,e^3x^5-8\sqrt{e}\,a\,d^3-24\sqrt{}}{}$$

input
```
int((a+b*acos(c*x))/(e*x^2+d)^(7/2),x)
```

output
```
(15*sqrt(d + e*x**2)*a*d**2*e*x + 20*sqrt(d + e*x**2)*a*d*e**2*x**3 + 8*sq
rt(d + e*x**2)*a*e**3*x**5 - 8*sqrt(e)*a*d**3 - 24*sqrt(e)*a*d**2*e*x**2 -
 24*sqrt(e)*a*d*e**2*x**4 - 8*sqrt(e)*a*e**3*x**6 + 15*int(acos(c*x)/(sqrt
(d + e*x**2)*d**3 + 3*sqrt(d + e*x**2)*d**2*e*x**2 + 3*sqrt(d + e*x**2)*d*
e**2*x**4 + sqrt(d + e*x**2)*e**3*x**6),x)*b*d**6*e + 45*int(acos(c*x)/(sq
rt(d + e*x**2)*d**3 + 3*sqrt(d + e*x**2)*d**2*e*x**2 + 3*sqrt(d + e*x**2)*
d*e**2*x**4 + sqrt(d + e*x**2)*e**3*x**6),x)*b*d**5*e**2*x**2 + 45*int(aco
s(c*x)/(sqrt(d + e*x**2)*d**3 + 3*sqrt(d + e*x**2)*d**2*e*x**2 + 3*sqrt(d
+ e*x**2)*d*e**2*x**4 + sqrt(d + e*x**2)*e**3*x**6),x)*b*d**4*e**3*x**4 +
15*int(acos(c*x)/(sqrt(d + e*x**2)*d**3 + 3*sqrt(d + e*x**2)*d**2*e*x**2 +
 3*sqrt(d + e*x**2)*d*e**2*x**4 + sqrt(d + e*x**2)*e**3*x**6),x)*b*d**3*e*
*4*x**6)/(15*d**3*e*(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6))
```

3.656 $\int (fx)^m \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 484

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx$$
$$= \frac{be\left(3c^2de(7+m)^2\left(12+7m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)^2+e^2\left(360+342m+119m^2+18m^3+m^4\right)\right.}{c^5f^2(3+m)^2(5+m)^2(7+m)^2}$$
$$+\frac{be^2\left(3c^2d(7+m)^2+e\left(30+11m+m^2\right)\right)(fx)^{4+m}\sqrt{1-c^2x^2}}{c^3f^4(5+m)^2(7+m)^2}+\frac{be^3(fx)^{6+m}\sqrt{1-c^2x^2}}{cf^6(7+m)^2}$$
$$+\frac{d^3(fx)^{1+m}(a+b\arccos(cx))}{f(1+m)}+\frac{3d^2e(fx)^{3+m}(a+b\arccos(cx))}{f^3(3+m)}$$
$$+\frac{3de^2(fx)^{5+m}(a+b\arccos(cx))}{f^5(5+m)}+\frac{e^3(fx)^{7+m}(a+b\arccos(cx))}{f^7(7+m)}$$
$$-\frac{b\left(\frac{c^6d^3(3+m)(5+m)(7+m)}{1+m}+\frac{e(2+m)\left(3c^2de(7+m)^2\left(12+7m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)^2+e^2\left(360+342m+119m^2+18m^3+m^4\right)\right)}{(3+m)(5+m)(7+m)}\right)}{c^5f^2(2+m)(3+m)(5+m)(7+m)}$$

output
```
b*e*(3*c^2*d*e*(7+m)^2*(m^2+7*m+12)+3*c^4*d^2*(m^2+12*m+35)^2+e^2*(m^4+18*
m^3+119*m^2+342*m+360))*(f*x)^(2+m)*(-c^2*x^2+1)^(1/2)/c^5/f^2/(3+m)^2/(5+
m)^2/(7+m)^2+b*e^2*(3*c^2*d*(7+m)^2+e*(m^2+11*m+30))*(f*x)^(4+m)*(-c^2*x^2
+1)^(1/2)/c^3/f^4/(5+m)^2/(7+m)^2+b*e^3*(f*x)^(6+m)*(-c^2*x^2+1)^(1/2)/c/f
^6/(7+m)^2+d^3*(f*x)^(1+m)*(a+b*arccos(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(
a+b*arccos(c*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arccos(c*x))/f^5/(5+m)
+e^3*(f*x)^(7+m)*(a+b*arccos(c*x))/f^7/(7+m)-b*(c^6*d^3*(3+m)*(5+m)*(7+m)/
(1+m)+e*(2+m)*(3*c^2*d*e*(7+m)^2*(m^2+7*m+12)+3*c^4*d^2*(m^2+12*m+35)^2+e^
2*(m^4+18*m^3+119*m^2+342*m+360))/(3+m)/(5+m)/(7+m))*(f*x)^(2+m)*hypergeom
([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/c^5/f^2/(2+m)/(3+m)/(5+m)/(7+m)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.64

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx$$
$$= x(fx)^m \left(\frac{ad^3}{1+m}+\frac{3ad^2ex^2}{3+m}+\frac{3ade^2x^4}{5+m}+\frac{ae^3x^6}{7+m}+\frac{bd^3\arccos(cx)}{1+m}+\frac{3bd^2ex^2\arccos(cx)}{3+m}+\frac{3bde^2x^4\arccos(cx)}{5+m}+\frac{be^3x^6\arccos(cx)}{7+m}+\frac{bcd^3x\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{m}{2},2+\frac{m}{2},c^2x^2\right)}{2+3m+m^2}+\frac{3bcd^2ex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},2+\frac{m}{2},3+\frac{m}{2},c^2x^2\right)}{12+7m+m^2}+\frac{3bcde^2x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2},3+\frac{m}{2},4+\frac{m}{2},c^2x^2\right)}{(5+m)(6+m)}+\frac{bce^3x^7\,\text{Hypergeometric2F1}\left(\frac{1}{2},4+\frac{m}{2},5+\frac{m}{2},c^2x^2\right)}{(7+m)(8+m)}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
x*(f*x)^m*((a*d^3)/(1 + m) + (3*a*d^2*e*x^2)/(3 + m) + (3*a*d*e^2*x^4)/(5
+ m) + (a*e^3*x^6)/(7 + m) + (b*d^3*ArcCos[c*x])/(1 + m) + (3*b*d^2*e*x^2*
ArcCos[c*x])/(3 + m) + (3*b*d*e^2*x^4*ArcCos[c*x])/(5 + m) + (b*e^3*x^6*Ar
cCos[c*x])/(7 + m) + (b*c*d^3*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, c
^2*x^2])/(2 + 3*m + m^2) + (3*b*c*d^2*e*x^3*Hypergeometric2F1[1/2, 2 + m/2
, 3 + m/2, c^2*x^2])/(12 + 7*m + m^2) + (3*b*c*d*e^2*x^5*Hypergeometric2F1
[1/2, 3 + m/2, 4 + m/2, c^2*x^2])/((5 + m)*(6 + m)) + (b*c*e^3*x^7*Hyperge
ometric2F1[1/2, 4 + m/2, 5 + m/2, c^2*x^2])/((7 + m)*(8 + m)))
```

Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5231, 27, 2340, 25, 1590, 25, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^3 (fx)^m (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5231}$$

$$bc\int \frac{(fx)^{m+1}\left(\frac{e^3x^6}{m+7}+\frac{3de^2x^4}{m+5}+\frac{3d^2ex^2}{m+3}+\frac{d^3}{m+1}\right)}{f\sqrt{1-c^2x^2}}dx+\frac{d^3(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\arccos(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\arccos(cx))}{f^7(m+7)}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{(fx)^{m+1}\left(\frac{e^3x^6}{m+7}+\frac{3de^2x^4}{m+5}+\frac{3d^2ex^2}{m+3}+\frac{d^3}{m+1}\right)}{\sqrt{1-c^2x^2}}dx}{f}+\frac{d^3(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\arccos(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\arccos(cx))}{f^7(m+7)}$$

$$\Big\downarrow \text{2340}$$

$$\frac{bc\left(-\frac{\int -\frac{(fx)^{m+1}\left(\frac{e^2\left(3c^2d(m+7)^2+e\left(m^2+11m+30\right)\right)x^4}{(m+5)(m+7)}+\frac{3c^2d^2e(m+7)x^2}{m+3}+\frac{c^2d^3(m+7)}{m+1}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+7)}-\frac{e^3\sqrt{1-c^2x^2}(fx)^{m+6}}{c^2f^5(m+7)^2}\right)}{f}+\frac{d^3(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\arccos(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\arccos(cx))}{f^7(m+7)}$$

$$\Big\downarrow \; 25$$

$$\frac{bc\left(\frac{\int \frac{(fx)^{m+1}\left(\frac{e^2\left(3c^2d(m+7)^2+e\left(m^2+11m+30\right)\right)x^4}{(m+5)(m+7)}+\frac{3c^2d^2e(m+7)x^2}{m+3}+\frac{c^2d^3(m+7)}{m+1}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+7)}-\frac{e^3\sqrt{1-c^2x^2}(fx)^{m+6}}{c^2f^5(m+7)^2}\right)}{f}+\frac{d^3(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\arccos(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\arccos(cx))}{f^7(m+7)}$$

$$\Big\downarrow \; 1590$$

$$\frac{bc\left(\frac{-\frac{\int -\frac{(fx)^{m+1}\left(\frac{d^3(m+5)(m+7)c^4}{m+1}+\frac{e\left(3d^2\left(m^2+12m+35\right)^2c^4+3de(m+7)^2\left(m^2+7m+12\right)c^2+e^2\left(m^4+18m^3+119m^2+342m+360\right)\right)x^2}{(m+3)(m+5)(m+7)}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+5)}-\frac{e^2\sqrt{1-c^2 \cdots}}{\cdots}}{c^2(m+7)} \cdots\right)}{f}+\frac{d^3(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\arccos(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\arccos(cx))}{f^7(m+7)}$$

$$\Big\downarrow \; 25$$

$$bc\left(\frac{\frac{\int \frac{(fx)^{m+1}\left(\frac{d^3(m+5)(m+7)c^4}{m+1}+\frac{e\left(3d^2\left(m^2+12m+35\right)^2c^4+3de(m+7)^2\left(m^2+7m+12\right)c^2+e^2\left(m^4+18m^3+119m^2+342m+360\right)\right)x^2}{(m+3)(m+5)(m+7)}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+5)}-\frac{e^2\sqrt{1-c^2x^2}(}{}}{c^2(m+7)}\right.$$

$$\frac{}{f}$$

$$\frac{d^3(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\arccos(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\arccos(cx))}{f^7(m+7)}$$

$\downarrow$ 363

$$bc\left(\frac{\left(\frac{c^4d^3(m+5)(m+7)}{m+1}+\frac{e(m+2)\left(3c^4d^2\left(m^2+12m+35\right)^2+3c^2de(m+7)^2\left(m^2+7m+12\right)+e^2\left(m^4+18m^3+119m^2+342m+360\right)\right)}{c^2(m+3)^2(m+5)(m+7)}\right)\int \frac{(fx)^{m+1}}{\sqrt{1-c^2x^2}}dx-\frac{e\sqrt{1-c^2x}}{}}{c^2(m+5)}\right.$$

$$\frac{d^3(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\arccos(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\arccos(cx))}{f^7(m+7)}$$

$\downarrow$ 278

$$\frac{d^3(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{3d^2e(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}(a+b\arccos(cx))}{f^5(m+5)}+\frac{e^3(fx)^{m+7}(a+b\arccos(cx))}{f^7(m+7)}+$$

$$bc\left(\frac{\frac{(fx)^{m+2}\left(\frac{c^4d^3(m+5)(m+7)}{m+1}+\frac{e(m+2)\left(3c^4d^2\left(m^2+12m+35\right)^2+3c^2de(m+7)^2\left(m^2+7m+12\right)+e^2\left(m^4+18m^3+119m^2+342m+360\right)\right)}{c^2(m+3)^2(m+5)(m+7)}\right)\text{Hypergeometric2}}{f(m+2)}}{c^2(m+5)}\right.$$

input
```
Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCos[c*x]),x]
```

output
```
(d^3*(f*x)^(1 + m)*(a + b*ArcCos[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 +
m)*(a + b*ArcCos[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*ArcC
os[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCos[c*x]))/(f^7*(7
+ m)) + (b*c*(-((e^3*(f*x)^(6 + m)*Sqrt[1 - c^2*x^2])/(c^2*f^5*(7 + m)^2))
 + (-((e^2*(3*c^2*d*(7 + m)^2 + e*(30 + 11*m + m^2))*(f*x)^(4 + m)*Sqrt[1
- c^2*x^2])/(c^2*f^3*(5 + m)^2*(7 + m))) + (-((e*(3*c^2*d*e*(7 + m)^2*(12
+ 7*m + m^2) + 3*c^4*d^2*(35 + 12*m + m^2)^2 + e^2*(360 + 342*m + 119*m^2
+ 18*m^3 + m^4))*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2])/(c^2*f*(3 + m)^2*(5 + m)
*(7 + m))) + (((c^4*d^3*(5 + m)*(7 + m))/(1 + m) + (e*(2 + m)*(3*c^2*d*e*(
7 + m)^2*(12 + 7*m + m^2) + 3*c^4*d^2*(35 + 12*m + m^2)^2 + e^2*(360 + 342
*m + 119*m^2 + 18*m^3 + m^4)))/(c^2*(3 + m)^2*(5 + m)*(7 + m)))*(f*x)^(2 +
 m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f*(2 + m)))/(c
^2*(5 + m)))/(c^2*(7 + m))))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^3 (a+b\arccos (cx))\,dx$$

input
```
int((f*x)^m*(e*x^2+d)^3*(a+b*arccos(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^3*(a+b*arccos(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx=\int \left(ex^2+d\right)^3(b\arccos(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccos(c*x))*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx=\text{Timed out}$$

input
```
integrate((f*x)**m*(e*x**2+d)**3*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx=\int \left(ex^2+d\right)^3(b\arccos(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
a*e^3*f^m*x^7*x^m/(m + 7) + 3*a*d*e^2*f^m*x^5*x^m/(m + 5) + 3*a*d^2*e*f^m*
x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) + (((b*e^3*f^m*m^3 + 9*b
*e^3*f^m*m^2 + 23*b*e^3*f^m*m + 15*b*e^3*f^m)*x^7 + 3*(b*d*e^2*f^m*m^3 + 1
1*b*d*e^2*f^m*m^2 + 31*b*d*e^2*f^m*m + 21*b*d*e^2*f^m)*x^5 + 3*(b*d^2*e*f^
m*m^3 + 13*b*d^2*e*f^m*m^2 + 47*b*d^2*e*f^m*m + 35*b*d^2*e*f^m)*x^3 + (b*d
^3*f^m*m^3 + 15*b*d^3*f^m*m^2 + 71*b*d^3*f^m*m + 105*b*d^3*f^m)*x)*x^m*arc
tan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - (m^4 + 16*m^3 + 86*m^2 + 176*m +
 105)*integrate(-((b*c*e^3*f^m*m^3 + 9*b*c*e^3*f^m*m^2 + 23*b*c*e^3*f^m*m
+ 15*b*c*e^3*f^m)*x^7 + 3*(b*c*d*e^2*f^m*m^3 + 11*b*c*d*e^2*f^m*m^2 + 31*b
*c*d*e^2*f^m*m + 21*b*c*d*e^2*f^m)*x^5 + 3*(b*c*d^2*e*f^m*m^3 + 13*b*c*d^2
*e*f^m*m^2 + 47*b*c*d^2*e*f^m*m + 35*b*c*d^2*e*f^m)*x^3 + (b*c*d^3*f^m*m^3
 + 15*b*c*d^3*f^m*m^2 + 71*b*c*d^3*f^m*m + 105*b*c*d^3*f^m)*x)*sqrt(c*x +
1)*sqrt(-c*x + 1)*x^m/(m^4 + 16*m^3 - (c^2*m^4 + 16*c^2*m^3 + 86*c^2*m^2 +
 176*c^2*m + 105*c^2)*x^2 + 86*m^2 + 176*m + 105), x))/(m^4 + 16*m^3 + 86*
m^2 + 176*m + 105)
```

Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx=\int \left(ex^2+d\right)^3(b\arccos\left(cx\right)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arccos(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,\left(f\,x\right)^m\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int((a + b*acos(c*x))*(f*x)^m*(d + e*x^2)^3,x)
```

output
```
int((a + b*acos(c*x))*(f*x)^m*(d + e*x^2)^3, x)
```

Reduce [F]

$$\int (fx)^m \left(d+ex^2\right)^3 (a+b\arccos(cx))\,dx = \text{Too large to display}$$

input
```
int((f*x)^m*(e*x^2+d)^3*(a+b*acos(c*x)),x)
```

output
```
(f**m*(x**m*a*d**3*m**3*x + 15*x**m*a*d**3*m**2*x + 71*x**m*a*d**3*m*x + 1
05*x**m*a*d**3*x + 3*x**m*a*d**2*e*m**3*x**3 + 39*x**m*a*d**2*e*m**2*x**3
+ 141*x**m*a*d**2*e*m*x**3 + 105*x**m*a*d**2*e*x**3 + 3*x**m*a*d*e**2*m**3
*x**5 + 33*x**m*a*d*e**2*m**2*x**5 + 93*x**m*a*d*e**2*m*x**5 + 63*x**m*a*d
*e**2*x**5 + x**m*a*e**3*m**3*x**7 + 9*x**m*a*e**3*m**2*x**7 + 23*x**m*a*e
**3*m*x**7 + 15*x**m*a*e**3*x**7 + int(x**m*acos(c*x)*x**6,x)*b*e**3*m**4
+ 16*int(x**m*acos(c*x)*x**6,x)*b*e**3*m**3 + 86*int(x**m*acos(c*x)*x**6,x
)*b*e**3*m**2 + 176*int(x**m*acos(c*x)*x**6,x)*b*e**3*m + 105*int(x**m*aco
s(c*x)*x**6,x)*b*e**3 + 3*int(x**m*acos(c*x)*x**4,x)*b*d*e**2*m**4 + 48*in
t(x**m*acos(c*x)*x**4,x)*b*d*e**2*m**3 + 258*int(x**m*acos(c*x)*x**4,x)*b*
d*e**2*m**2 + 528*int(x**m*acos(c*x)*x**4,x)*b*d*e**2*m + 315*int(x**m*aco
s(c*x)*x**4,x)*b*d*e**2 + 3*int(x**m*acos(c*x)*x**2,x)*b*d**2*e*m**4 + 48*
int(x**m*acos(c*x)*x**2,x)*b*d**2*e*m**3 + 258*int(x**m*acos(c*x)*x**2,x)*
b*d**2*e*m**2 + 528*int(x**m*acos(c*x)*x**2,x)*b*d**2*e*m + 315*int(x**m*a
cos(c*x)*x**2,x)*b*d**2*e + int(x**m*acos(c*x),x)*b*d**3*m**4 + 16*int(x**
m*acos(c*x),x)*b*d**3*m**3 + 86*int(x**m*acos(c*x),x)*b*d**3*m**2 + 176*in
t(x**m*acos(c*x),x)*b*d**3*m + 105*int(x**m*acos(c*x),x)*b*d**3))/(m**4 +
16*m**3 + 86*m**2 + 176*m + 105)
```

3.657 $\int (fx)^m \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 293

$$\int (fx)^m \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx = \frac{be(2c^2d(5+m)^2+e(12+7m+m^2))(fx)^{2+m}\sqrt{1-c^2x^2}}{c^3f^2(3+m)^2(5+m)^2} + \frac{be^2(fx)^{4+m}\sqrt{1-c^2x^2}}{cf^4(5+m)^2} + \frac{d^2(fx)^{1+m}(a+b\arccos(cx))}{f(1+m)} + \frac{2de(fx)^{3+m}(a+b\arccos(cx))}{f^3(3+m)} + \frac{e^2(fx)^{5+m}(a+b\arccos(cx))}{f^5(5+m)} - \frac{b\left(\frac{c^4d^2(3+m)(5+m)}{1+m}+\frac{e(2+m)(2c^2d(5+m)^2+e(12+7m+m^2))}{(3+m)(5+m)}\right)(fx)^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{c^3f^2(2+m)(3+m)(5+m)}$$

output
```
b*e*(2*c^2*d*(5+m)^2+e*(m^2+7*m+12))*(f*x)^(2+m)*(-c^2*x^2+1)^(1/2)/c^3/f^
2/(3+m)^2/(5+m)^2+b*e^2*(f*x)^(4+m)*(-c^2*x^2+1)^(1/2)/c/f^4/(5+m)^2+d^2*(
f*x)^(1+m)*(a+b*arccos(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arccos(c*x))/f
^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arccos(c*x))/f^5/(5+m)-b*(c^4*d^2*(3+m)*(5+m
)/(1+m)+e*(2+m)*(2*c^2*d*(5+m)^2+e*(m^2+7*m+12))/(3+m)/(5+m))*(f*x)^(2+m)*
hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/c^3/f^2/(2+m)/(3+m)/(5+m)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.76

$$\int (fx)^m \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx$$
$$= x(fx)^m \left(\frac{ad^2}{1+m}+\frac{2adex^2}{3+m}+\frac{ae^2x^4}{5+m}+\frac{bd^2\arccos(cx)}{1+m}+\frac{2bdex^2\arccos(cx)}{3+m}+\frac{be^2x^4\arccos(cx)}{5+m}+\frac{bcd^2x\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{m}{2},2+\frac{m}{2},c^2x^2\right)}{2+3m+m^2}+\frac{2bcdex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},2+\frac{m}{2},3+\frac{m}{2},c^2x^2\right)}{12+7m+m^2}+\frac{bce^2x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2},3+\frac{m}{2},4+\frac{m}{2},c^2x^2\right)}{(5+m)(6+m)}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
x*(f*x)^m*((a*d^2)/(1 + m) + (2*a*d*e*x^2)/(3 + m) + (a*e^2*x^4)/(5 + m) +
 (b*d^2*ArcCos[c*x])/(1 + m) + (2*b*d*e*x^2*ArcCos[c*x])/(3 + m) + (b*e^2*
x^4*ArcCos[c*x])/(5 + m) + (b*c*d^2*x*Hypergeometric2F1[1/2, 1 + m/2, 2 +
m/2, c^2*x^2])/(2 + 3*m + m^2) + (2*b*c*d*e*x^3*Hypergeometric2F1[1/2, 2 +
 m/2, 3 + m/2, c^2*x^2])/(12 + 7*m + m^2) + (b*c*e^2*x^5*Hypergeometric2F1
[1/2, 3 + m/2, 4 + m/2, c^2*x^2])/((5 + m)*(6 + m)))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5231, 27, 1590, 25, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (fx)^m (a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5231}$$

$$bc\int \frac{(fx)^{m+1}\left(\frac{e^2x^4}{m+5}+\frac{2dex^2}{m+3}+\frac{d^2}{m+1}\right)}{f\sqrt{1-c^2x^2}}dx+\frac{d^2(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{2de(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}+\frac{e^2(fx)^{m+5}(a+b\arccos(cx))}{f^5(m+5)}$$

$$\Big\downarrow \; 27$$

$$\frac{bc\int \frac{(fx)^{m+1}\left(\frac{e^2x^4}{m+5}+\frac{2dex^2}{m+3}+\frac{d^2}{m+1}\right)}{\sqrt{1-c^2x^2}}dx}{f}+\frac{d^2(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{2de(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}+\frac{e^2(fx)^{m+5}(a+b\arccos(cx))}{f^5(m+5)}$$

$$\Big\downarrow \; 1590$$

$$\frac{bc\left(-\frac{\int -\frac{(fx)^{m+1}\left(\frac{c^2(m+5)d^2}{m+1}+\frac{e\left(2c^2d(m+5)^2+e\left(m^2+7m+12\right)\right)x^2}{(m+3)(m+5)}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+5)}-\frac{e^2\sqrt{1-c^2x^2}(fx)^{m+4}}{c^2f^3(m+5)^2}\right)}{f}+\frac{d^2(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{2de(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}+\frac{e^2(fx)^{m+5}(a+b\arccos(cx))}{f^5(m+5)}$$

$$\Big\downarrow \; 25$$

$$\frac{bc\left(\frac{\int \frac{(fx)^{m+1}\left(\frac{c^2(m+5)d^2}{m+1}+\frac{e\left(2c^2d(m+5)^2+e\left(m^2+7m+12\right)\right)x^2}{(m+3)(m+5)}\right)}{\sqrt{1-c^2x^2}}dx}{c^2(m+5)}-\frac{e^2\sqrt{1-c^2x^2}(fx)^{m+4}}{c^2f^3(m+5)^2}\right)}{f}+\frac{d^2(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{2de(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}+\frac{e^2(fx)^{m+5}(a+b\arccos(cx))}{f^5(m+5)}$$

$$\Big\downarrow \; 363$$

$$\frac{bc\Bigg(\frac{\left(\frac{c^2d^2(m+5)}{m+1}+\frac{e(m+2)\left(2c^2d(m+5)^2+e\left(m^2+7m+12\right)\right)}{c^2(m+3)^2(m+5)}\right)\int \frac{(fx)^{m+1}}{\sqrt{1-c^2x^2}}dx-\frac{e\sqrt{1-c^2x^2}(fx)^{m+2}\left(2c^2d(m+5)^2+e\left(m^2+7m+12\right)\right)}{c^2f(m+3)^2(m+5)}}{c^2(m+5)}-\frac{e^2\sqrt{1-c^2x^2}(fx}{c^2f^3(m+5)}}{f}+\frac{d^2(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{2de(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}+\frac{e^2(fx)^{m+5}(a+b\arccos(cx))}{f^5(m+5)}$$

$$\Big\downarrow \; 278$$

$$\frac{d^2(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{2de(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}+\frac{e^2(fx)^{m+5}(a+b\arccos(cx))}{f^5(m+5)}+$$

$$\frac{bc\left(\frac{\frac{(fx)^{m+2}\left(\frac{c^2d^2(m+5)}{m+1}+\frac{e(m+2)\left(2c^2d(m+5)^2+e\left(m^2+7m+12\right)\right)}{c^2(m+3)^2(m+5)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{f(m+2)}}{c^2(m+5)}-\frac{e\sqrt{1-c^2x^2}(fx)^{m+2}\left(2c^2d(m+5)^2+e\left(m \cdots\right.\right.}{c^2f(m+3)^2(m+5)} \cdots\right.}{f}$$

input
```
Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*(f*x)^(1 + m)*(a + b*ArcCos[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)
*(a + b*ArcCos[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcCos[c*x
]))/(f^5*(5 + m)) + (b*c*(-((e^2*(f*x)^(4 + m)*Sqrt[1 - c^2*x^2])/(c^2*f^3
*(5 + m)^2)) + (-((e*(2*c^2*d*(5 + m)^2 + e*(12 + 7*m + m^2))*(f*x)^(2 + m
)*Sqrt[1 - c^2*x^2])/(c^2*f*(3 + m)^2*(5 + m))) + (((c^2*d^2*(5 + m))/(1 +
 m) + (e*(2 + m)*(2*c^2*d*(5 + m)^2 + e*(12 + 7*m + m^2)))/(c^2*(3 + m)^2*
(5 + m)))*(f*x)^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x
^2])/(f*(2 + m)))/(c^2*(5 + m))))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^2 \left(a+b\arccos\left(cx\right)\right) dx$$

input
```
int((f*x)^m*(e*x^2+d)^2*(a+b*arccos(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^2*(a+b*arccos(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx = \int \left(ex^2+d\right)^2 (b\arccos(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arccos(c*x))*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx = \int (fx)^m (a+b\operatorname{acos}(cx)) \left(d+ex^2\right)^2\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)**2*(a+b*acos(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*acos(c*x))*(d + e*x**2)**2, x)
```

Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx = \int \left(ex^2+d\right)^2 (b\arccos(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
a*e^2*f^m*x^5*x^m/(m + 5) + 2*a*d*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*
d^2/(f*(m + 1)) + (((b*e^2*f^m*m^2 + 4*b*e^2*f^m*m + 3*b*e^2*f^m)*x^5 + 2*
(b*d*e*f^m*m^2 + 6*b*d*e*f^m*m + 5*b*d*e*f^m)*x^3 + (b*d^2*f^m*m^2 + 8*b*d
^2*f^m*m + 15*b*d^2*f^m)*x)*x^m*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)
 - (m^3 + 9*m^2 + 23*m + 15)*integrate(-((b*c*e^2*f^m*m^2 + 4*b*c*e^2*f^m*
m + 3*b*c*e^2*f^m)*x^5 + 2*(b*c*d*e*f^m*m^2 + 6*b*c*d*e*f^m*m + 5*b*c*d*e*
f^m)*x^3 + (b*c*d^2*f^m*m^2 + 8*b*c*d^2*f^m*m + 15*b*c*d^2*f^m)*x)*sqrt(c*
x + 1)*sqrt(-c*x + 1)*x^m/(m^3 - (c^2*m^3 + 9*c^2*m^2 + 23*c^2*m + 15*c^2)
*x^2 + 9*m^2 + 23*m + 15), x))/(m^3 + 9*m^2 + 23*m + 15)
```

Giac [F]

$$\int (fx)^m (d+ex^2)^2 (a+b\arccos(cx))\,dx = \int (ex^2+d)^2 (b\arccos(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccos(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^2)^2 (a+b\arccos(cx))\,dx = \int (a+b\,\mathrm{acos}(c\,x))\,(f\,x)^m\,(e\,x^2+d)^2\,dx$$

input
```
int((a + b*acos(c*x))*(f*x)^m*(d + e*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))*(f*x)^m*(d + e*x^2)^2, x)
```

Reduce [F]

$$\int (fx)^m \left(d+ex^2\right)^2 (a+b\arccos(cx))\,dx$$
$$=\frac{f^m\left(x^m a\,d^2m^2x+8x^m a\,d^2mx+15x^m a\,d^2x+2x^m ade\,m^2x^3+12x^m adem\,x^3+10x^m ade\,x^3+x^m a\,e^2m^2x\right.}{}$$

input
```
int((f*x)^m*(e*x^2+d)^2*(a+b*acos(c*x)),x)
```

output
```
(f**m*(x**m*a*d**2*m**2*x + 8*x**m*a*d**2*m*x + 15*x**m*a*d**2*x + 2*x**m*
a*d*e*m**2*x**3 + 12*x**m*a*d*e*m*x**3 + 10*x**m*a*d*e*x**3 + x**m*a*e**2*
m**2*x**5 + 4*x**m*a*e**2*m*x**5 + 3*x**m*a*e**2*x**5 + int(x**m*acos(c*x)
*x**4,x)*b*e**2*m**3 + 9*int(x**m*acos(c*x)*x**4,x)*b*e**2*m**2 + 23*int(x
**m*acos(c*x)*x**4,x)*b*e**2*m + 15*int(x**m*acos(c*x)*x**4,x)*b*e**2 + 2*
int(x**m*acos(c*x)*x**2,x)*b*d*e*m**3 + 18*int(x**m*acos(c*x)*x**2,x)*b*d*
e*m**2 + 46*int(x**m*acos(c*x)*x**2,x)*b*d*e*m + 30*int(x**m*acos(c*x)*x**
2,x)*b*d*e + int(x**m*acos(c*x),x)*b*d**2*m**3 + 9*int(x**m*acos(c*x),x)*b
*d**2*m**2 + 23*int(x**m*acos(c*x),x)*b*d**2*m + 15*int(x**m*acos(c*x),x)*
b*d**2))/(m**3 + 9*m**2 + 23*m + 15)
```

3.658 $\int (fx)^m (d+ex^2) (a+b\arccos(cx)) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 161

$$\begin{aligned}&\int (fx)^m (d+ex^2) (a+b\arccos(cx)) \, dx\\&=\frac{be(fx)^{2+m}\sqrt{1-c^2x^2}}{cf^2(3+m)^2}+\frac{d(fx)^{1+m}(a+b\arccos(cx))}{f(1+m)}+\frac{e(fx)^{3+m}(a+b\arccos(cx))}{f^3(3+m)}\\&\quad-\frac{b(e(1+m)(2+m)+c^2d(3+m)^2)(fx)^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{cf^2(1+m)(2+m)(3+m)^2}\end{aligned}$$

output

```
b*e*(f*x)^(2+m)*(-c^2*x^2+1)^(1/2)/c/f^2/(3+m)^2+d*(f*x)^(1+m)*(a+b*arccos
(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arccos(c*x))/f^3/(3+m)-b*(e*(1+m)*(2+m)+
c^2*d*(3+m)^2)*(f*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)/c/f
^2/(1+m)/(2+m)/(3+m)^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.75

$$\int (fx)^m \left(d+ex^2\right) (a+b\arccos(cx))\,dx$$
$$= x(fx)^m \left(\frac{bcdx\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{m}{2},2+\frac{m}{2},c^2x^2\right)}{2+3m+m^2} + \frac{\frac{(d(3+m)+e(1+m)x^2)(a+b\arccos(cx))}{1+m}+\frac{bcex^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},2+\frac{m}{2},3+\frac{m}{2},c^2x^2\right)}{4+m}}{3+m}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
x*(f*x)^m*((b*c*d*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m/2, c^2*x^2])/(2
+ 3*m + m^2) + (((d*(3 + m) + e*(1 + m)*x^2)*(a + b*ArcCos[c*x]))/(1 + m)
+ (b*c*e*x^3*Hypergeometric2F1[1/2, 2 + m/2, 3 + m/2, c^2*x^2])/(4 + m))/(
3 + m))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5231, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right) (fx)^m (a+b\arccos(cx))\,dx$$
$$\Big\downarrow \text{5231}$$
$$bc\int \frac{(fx)^{m+1}\left(e(m+1)x^2+d(m+3)\right)}{f\left(m^2+4m+3\right)\sqrt{1-c^2x^2}}dx+\frac{d(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{e(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{(fx)^{m+1}(e(m+1)x^2+d(m+3))}{\sqrt{1-c^2x^2}}dx}{f\left(m^2+4m+3\right)}+\frac{d(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{e(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}$$

$$\Big\downarrow \text{363}$$

$$\frac{bc\left(\left(\frac{e(m+1)(m+2)}{c^2(m+3)}+d(m+3)\right)\int \frac{(fx)^{m+1}}{\sqrt{1-c^2x^2}}dx-\frac{e(m+1)\sqrt{1-c^2x^2}(fx)^{m+2}}{c^2f(m+3)}\right)}{f\left(m^2+4m+3\right)}+\frac{d(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{e(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}$$

$$\Big\downarrow \text{278}$$

$$\frac{d(fx)^{m+1}(a+b\arccos(cx))}{f(m+1)}+\frac{e(fx)^{m+3}(a+b\arccos(cx))}{f^3(m+3)}+\frac{bc\left(\frac{(fx)^{m+2}\left(\frac{e(m+1)(m+2)}{c^2(m+3)}+d(m+3)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{f(m+2)}-\frac{e(m+1)\sqrt{1-c^2x^2}(fx)^{m+2}}{c^2f(m+3)}\right)}{f\left(m^2+4m+3\right)}$$

input
```
Int[(f*x)^m*(d + e*x^2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d*(f*x)^(1 + m)*(a + b*ArcCos[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(a +
b*ArcCos[c*x]))/(f^3*(3 + m)) + (b*c*(-((e*(1 + m)*(f*x)^(2 + m)*Sqrt[1 -
c^2*x^2])/(c^2*f*(3 + m))) + (((e*(1 + m)*(2 + m))/(c^2*(3 + m)) + d*(3 +
m))*(f*x)^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(
f*(2 + m))))/(f*(3 + 4*m + m^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5231
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcCos[c*x])   u, x] + Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right) \left(a+b\arccos \left(cx\right)\right) dx$$

input
```
int((f*x)^m*(e*x^2+d)*(a+b*arccos(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)*(a+b*arccos(c*x)),x)
```

Fricas [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\arccos(cx)\right) dx = \int \left(ex^2+d\right)\left(b\arccos\left(cx\right)+a\right)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccos(c*x))*(f*x)^m, x)
```

Sympy [F]

$$\int (fx)^m (d+ex^2) (a+b\arccos(cx))\,dx = \int (fx)^m (a+b\operatorname{acos}(cx)) (d+ex^2)\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)*(a+b*acos(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*acos(c*x))*(d + e*x**2), x)
```

Maxima [F]

$$\int (fx)^m (d+ex^2) (a+b\arccos(cx))\,dx = \int (ex^2+d)(b\arccos(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/(f*(m + 1)) + (((b*e*f^m*m + b
*e*f^m)*x^3 + (b*d*f^m*m + 3*b*d*f^m)*x)*x^m*arctan2(sqrt(c*x + 1)*sqrt(-c
*x + 1), c*x) - (m^2 + 4*m + 3)*integrate(((b*c*e*f^m*m + b*c*e*f^m)*x^3 +
 (b*c*d*f^m*m + 3*b*c*d*f^m)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^m/((c^2*m^2
 + 4*c^2*m + 3*c^2)*x^2 - m^2 - 4*m - 3), x))/(m^2 + 4*m + 3)
```

Giac [F]

$$\int (fx)^m (d+ex^2) (a+b\arccos(cx))\,dx = \int (ex^2+d)(b\arccos(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccos(c*x) + a)*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)(a+b\arccos(cx))\,dx=\int (a+b\,\mathrm{acos}(c\,x))\,(f\,x)^m\,\left(e\,x^2+d\right)\,dx$$

input
```
int((a + b*acos(c*x))*(f*x)^m*(d + e*x^2),x)
```

output
```
int((a + b*acos(c*x))*(f*x)^m*(d + e*x^2), x)
```

Reduce [F]

$$\int (fx)^m \left(d+ex^2\right)(a+b\arccos(cx))\,dx$$
$$=\frac{f^m\left(x^m adm x+3x^m adx+x^m aem\,x^3+x^m ae\,x^3+\left(\int x^m acos(cx)\,x^2dx\right)be\,m^2+4\left(\int x^m acos(cx)\,x^2dx\right)b}{m}$$

input
```
int((f*x)^m*(e*x^2+d)*(a+b*acos(c*x)),x)
```

output
```
(f**m*(x**m*a*d*m*x + 3*x**m*a*d*x + x**m*a*e*m*x**3 + x**m*a*e*x**3 + int
(x**m*acos(c*x)*x**2,x)*b*e*m**2 + 4*int(x**m*acos(c*x)*x**2,x)*b*e*m + 3*
int(x**m*acos(c*x)*x**2,x)*b*e + int(x**m*acos(c*x),x)*b*d*m**2 + 4*int(x*
*m*acos(c*x),x)*b*d*m + 3*int(x**m*acos(c*x),x)*b*d))/(m**2 + 4*m + 3)
```

3.659 $\int \frac{(fx)^m(a+b\arccos(cx))}{d+ex^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m(a+b\arccos(cx))}{d+ex^2}\,dx = \text{Int}\left(\frac{(fx)^m(a+b\arccos(cx))}{d+ex^2},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccos(c*x))/(e*x^2+d),x)
```

Mathematica [N/A]

Not integrable

Time = 3.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\arccos(cx))}{d+ex^2}\,dx = \int \frac{(fx)^m(a+b\arccos(cx))}{d+ex^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCos[c*x]))/(d + e*x^2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCos[c*x]))/(d + e*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\arccos(cx))}{d+ex^2}dx$$
$$\Big\downarrow \text{ 5235}$$
$$\int \frac{(fx)^m(a+b\arccos(cx))}{d+ex^2}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCos[c*x]))/(d + e*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\,(a+b\arccos\,(cx))}{e\,x^2+d}dx$$

input
```
int((f*x)^m*(a+b*arccos(c*x))/(e*x^2+d),x)
```

output
```
int((f*x)^m*(a+b*arccos(c*x))/(e*x^2+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\arccos(cx))}{d+ex^2}\,dx=\int \frac{(b\arccos(cx)+a)(fx)^m}{ex^2+d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccos(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 10.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(fx)^m(a+b\arccos(cx))}{d+ex^2}\,dx=\int \frac{(fx)^m\,(a+b\,\mathrm{acos}\,(cx))}{d+ex^2}\,dx$$

input
```
integrate((f*x)**m*(a+b*acos(c*x))/(e*x**2+d),x)
```

output
```
Integral((f*x)**m*(a + b*acos(c*x))/(d + e*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\arccos(cx))}{d+ex^2}\,dx=\int \frac{(b\arccos(cx)+a)(fx)^m}{ex^2+d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccos(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(fx)^m(a+b\arccos(cx))}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x)^m*(a+b*arccos(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\arccos(cx))}{d+ex^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(f\,x)^m}{e\,x^2+d}\,dx$$

input
```
int(((a + b*acos(c*x))*(f*x)^m)/(d + e*x^2),x)
```

output
```
int(((a + b*acos(c*x))*(f*x)^m)/(d + e*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{(fx)^m(a+b\arccos(cx))}{d+ex^2}\,dx = f^m\left(\left(\int \frac{x^m}{e\,x^2+d}dx\right)a+\left(\int \frac{x^m\,acos(cx)}{e\,x^2+d}dx\right)b\right)$$

input
```
int((f*x)^m*(a+b*acos(c*x))/(e*x^2+d),x)
```

output
```
f**m*(int(x**m/(d + e*x**2),x)*a + int((x**m*acos(c*x))/(d + e*x**2),x)*b)
```

3.660 $\int \frac{(fx)^m(a+b\arccos(cx))}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \text{Int}\left(\frac{(fx)^m(a+b\arccos(cx))}{(d+ex^2)^2},x\right)$$

output
```
Defer(Int)((f*x)^m*(a+b*arccos(c*x))/(e*x^2+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 5.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{(fx)^m(a+b\arccos(cx))}{(d+ex^2)^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCos[c*x]))/(d + e*x^2)^2,x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCos[c*x]))/(d + e*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m(a+b\arccos(cx))}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{ 5235}$$

$$\int \frac{(fx)^m(a+b\arccos(cx))}{(d+ex^2)^2}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCos[c*x]))/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m\left(a+b\arccos\left(cx\right)\right)}{\left(e\,x^2+d\right)^2}dx$$

input
```
int((f*x)^m*(a+b*arccos(c*x))/(e*x^2+d)^2,x)
```

output
```
int((f*x)^m*(a+b*arccos(c*x))/(e*x^2+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(fx)^m(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)(fx)^m}{(ex^2+d)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccos(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*acos(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)(fx)^m}{(ex^2+d)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(fx)^m(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x)^m*(a+b*arccos(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(f\,x)^m}{(e\,x^2+d)^2}\,dx$$

input
```
int(((a + b*acos(c*x))*(f*x)^m)/(d + e*x^2)^2,x)
```

output
```
int(((a + b*acos(c*x))*(f*x)^m)/(d + e*x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.83

$$\int \frac{(fx)^m(a+b\arccos(cx))}{(d+ex^2)^2}\,dx = f^m\left(\left(\int \frac{x^m}{e^2x^4+2de\,x^2+d^2}dx\right)a + \left(\int \frac{x^m\,acos(cx)}{e^2x^4+2de\,x^2+d^2}dx\right)b\right)$$

input
```
int((f*x)^m*(a+b*acos(c*x))/(e*x^2+d)^2,x)
```

output
```
f**m*(int(x**m/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*a + int((x**m*acos(c*x))
/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b)
```

3.661 $\int (d+ex^2)^3 (a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 569

$$\int (d+ex^2)^3 (a+b\arccos(cx))^2\,dx = -2b^2d^3x-\frac{4b^2d^2ex}{3c^2}-\frac{16b^2de^2x}{25c^4}-\frac{32b^2e^3x}{245c^6} -\frac{2}{9}b^2d^2ex^3-\frac{8b^2de^2x^3}{75c^2}-\frac{16b^2e^3x^3}{735c^4} -\frac{6}{125}b^2de^2x^5-\frac{12b^2e^3x^5}{1225c^2}-\frac{2}{343}b^2e^3x^7 +\frac{2bd^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c} +\frac{4bd^2e\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^3} +\frac{16bde^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c^5} +\frac{32be^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{245c^7} +\frac{2bd^2ex^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c} +\frac{8bde^2x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c^3} +\frac{16be^3x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{245c^5} +\frac{6bde^2x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c} +\frac{12be^3x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{245c^3} +\frac{2be^3x^6\sqrt{1-c^2x^2}(a+b\arccos(cx))}{49c} +d^3x(a+b\arccos(cx))^2+d^2ex^3(a+b\arccos(cx))^2 +\frac{3}{5}de^2x^5(a+b\arccos(cx))^2 +\frac{1}{7}e^3x^7(a+b\arccos(cx))^2$$

output
```
-2*b^2*d^3*x-4/3*b^2*d^2*e*x/c^2-16/25*b^2*d*e^2*x/c^4-32/245*b^2*e^3*x/c^
6-2/9*b^2*d^2*e*x^3-8/75*b^2*d*e^2*x^3/c^2-16/735*b^2*e^3*x^3/c^4-6/125*b^
2*d*e^2*x^5-12/1225*b^2*e^3*x^5/c^2-2/343*b^2*e^3*x^7+2*b*d^3*(-c^2*x^2+1)
^(1/2)*(a+b*arccos(c*x))/c+4/3*b*d^2*e*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x)
)/c^3+16/25*b*d*e^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^5+32/245*b*e^3*
(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^7+2/3*b*d^2*e*x^2*(-c^2*x^2+1)^(1/2
)*(a+b*arccos(c*x))/c+8/25*b*d*e^2*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x)
)/c^3+16/245*b*e^3*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^5+6/25*b*d*e
^2*x^4*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+12/245*b*e^3*x^4*(-c^2*x^2+1
)^(1/2)*(a+b*arccos(c*x))/c^3+2/49*b*e^3*x^6*(-c^2*x^2+1)^(1/2)*(a+b*arcco
s(c*x))/c+d^3*x*(a+b*arccos(c*x))^2+d^2*e*x^3*(a+b*arccos(c*x))^2+3/5*d*e^
2*x^5*(a+b*arccos(c*x))^2+1/7*e^3*x^7*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.78

$$\int (d+ex^2)^3 (a+b\arccos(cx))^2 \, dx$$
$$= \frac{11025a^2c^7x(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6)-210ab\sqrt{1-c^2x^2}(240e^3+24c^2e^2(49d+5ex^2)+2c^4e(}{}$$

input
```
Integrate[(d + e*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
(11025*a^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) - 210*
a*b*Sqrt[1 - c^2*x^2]*(240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*c^4*e*(12
25*d^2 + 294*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*
d*e^2*x^4 + 75*e^3*x^6)) - 2*b^2*c*x*(25200*e^3 + 840*c^2*e^2*(147*d + 5*e
*x^2) + 210*c^4*e*(1225*d^2 + 98*d*e*x^2 + 9*e^2*x^4) + c^6*(385875*d^3 +
42875*d^2*e*x^2 + 9261*d*e^2*x^4 + 1125*e^3*x^6)) - 210*b*(-105*a*c^7*x*(3
5*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) + b*Sqrt[1 - c^2*x^2]*(24
0*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*c^4*e*(1225*d^2 + 294*d*e*x^2 + 45
*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 + 75*e^3*x^6)))
*ArcCos[c*x] + 11025*b^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e
^3*x^6)*ArcCos[c*x]^2)/(385875*c^7)
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)^3 (a+b\arccos(cx))^2 dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(d^3(a+b\arccos(cx))^2+3d^2ex^2(a+b\arccos(cx))^2+3de^2x^4(a+b\arccos(cx))^2+e^3x^6(a+b\arccos(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2bd^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}-\frac{2bd^2ex^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c}-\frac{6bde^2x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c}-\frac{2be^3x^6\sqrt{1-c^2x^2}(a+b\arccos(cx))}{49c}-\frac{32be^3\sqrt{1-c^2x^2}(a+b\arccos(cx))}{245c^7}-\frac{16bde^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c^5}-\frac{16be^3x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{245c^5}-\frac{4bd^2e\sqrt{1-c^2x^2}(a+b\arccos(cx))}{3c^3}-\frac{8bde^2x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c^3}-\frac{12be^3x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{245c^3}+d^3x(a+b\arccos(cx))^2+d^2ex^3(a+b\arccos(cx))^2+\frac{3}{5}de^2x^5(a+b\arccos(cx))^2+\frac{1}{7}e^3x^7(a+b\arccos(cx))^2-\frac{32b^2e^3x}{245c^6}-\frac{16b^2de^2x}{25c^4}-\frac{16b^2e^3x^3}{735c^4}-\frac{4b^2d^2ex}{3c^2}-\frac{8b^2de^2x^3}{75c^2}-\frac{12b^2e^3x^5}{1225c^2}-2b^2d^3x-\frac{2}{9}b^2d^2ex^3-\frac{6}{125}b^2de^2x^5-\frac{2}{343}b^2e^3x^7$$

input
```
Int[(d + e*x^2)^3*(a + b*ArcCos[c*x])^2,x]
```

output
```
-2*b^2*d^3*x - (4*b^2*d^2*e*x)/(3*c^2) - (16*b^2*d*e^2*x)/(25*c^4) - (32*b
^2*e^3*x)/(245*c^6) - (2*b^2*d^2*e*x^3)/9 - (8*b^2*d*e^2*x^3)/(75*c^2) - (
16*b^2*e^3*x^3)/(735*c^4) - (6*b^2*d*e^2*x^5)/125 - (12*b^2*e^3*x^5)/(1225
*c^2) - (2*b^2*e^3*x^7)/343 - (2*b*d^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x
]))/c - (4*b*d^2*e*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(3*c^3) - (16*b*
d*e^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(25*c^5) - (32*b*e^3*Sqrt[1 -
 c^2*x^2]*(a + b*ArcCos[c*x]))/(245*c^7) - (2*b*d^2*e*x^2*Sqrt[1 - c^2*x^2
]*(a + b*ArcCos[c*x]))/(3*c) - (8*b*d*e^2*x^2*Sqrt[1 - c^2*x^2]*(a + b*Arc
Cos[c*x]))/(25*c^3) - (16*b*e^3*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))
/(245*c^5) - (6*b*d*e^2*x^4*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(25*c)
- (12*b*e^3*x^4*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(245*c^3) - (2*b*e^
3*x^6*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(49*c) + d^3*x*(a + b*ArcCos[
c*x])^2 + d^2*e*x^3*(a + b*ArcCos[c*x])^2 + (3*d*e^2*x^5*(a + b*ArcCos[c*x
])^2)/5 + (e^3*x^7*(a + b*ArcCos[c*x])^2)/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 702, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{a^2\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b^2\left(c^6d^3\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+\frac{c^4d^2e\left(9\arccos(cx)^2c^3x^3\right.}{}\right.}{}}{}$
default	$\frac{\frac{a^2\left(d^3c^7x+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b^2\left(c^6d^3\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+\frac{c^4d^2e\left(9\arccos(cx)^2c^3x^3\right.}{}\right.}{}}{}$
parts	$a^2\left(\frac{1}{7}e^3x^7+\frac{3}{5}d\,e^2x^5+d^2e\,x^3+d^3x\right)+\frac{b^2\left(55125\arccos(cx)^2c^7x^7e^3+231525\arccos(cx)^2c^7x^5d\,e^2+3858\right.}{}$
orering	$\frac{x\left(47625c^8e^5x^{10}+328917c^8d\,e^4x^8+1128666c^8d^2e^3x^6+10080c^6e^5x^8+5951050c^8d^3e^2x^4+146016c^6d\,e^4x^6-385875c^8d\right.}{}$

input
```
int((e*x^2+d)^3*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2/c^6*(d^3*c^7*x+d^2*c^7*e*x^3+3/5*d*c^7*e^2*x^5+1/7*e^3*c^7*x^7)+b
^2/c^6*(c^6*d^3*(arccos(c*x)^2*c*x-2*c*x-2*arccos(c*x)*(-c^2*x^2+1)^(1/2))
+1/9*c^4*d^2*e*(9*arccos(c*x)^2*c^3*x^3-6*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c
^2*x^2-2*c^3*x^3-12*arccos(c*x)*(-c^2*x^2+1)^(1/2)-12*c*x)+1/375*c^2*d*e^2
*(225*arccos(c*x)^2*c^5*x^5-90*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^4*x^4-18*c
^5*x^5-120*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^2-40*c^3*x^3-240*arccos(c*
x)*(-c^2*x^2+1)^(1/2)-240*c*x)+1/25725*e^3*(3675*arccos(c*x)^2*c^7*x^7-105
0*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^6*x^6-150*c^7*x^7-1260*(-c^2*x^2+1)^(1/
2)*arccos(c*x)*c^4*x^4-252*c^5*x^5-1680*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2
*x^2-560*c^3*x^3-3360*arccos(c*x)*(-c^2*x^2+1)^(1/2)-3360*c*x))+2*a*b/c^6*
(arccos(c*x)*d^3*c^7*x+arccos(c*x)*d^2*c^7*e*x^3+3/5*arccos(c*x)*d*c^7*e^2
*x^5+1/7*arccos(c*x)*e^3*c^7*x^7+1/7*e^3*(-1/7*c^6*x^6*(-c^2*x^2+1)^(1/2)-
6/35*c^4*x^4*(-c^2*x^2+1)^(1/2)-8/35*c^2*x^2*(-c^2*x^2+1)^(1/2)-16/35*(-c^
2*x^2+1)^(1/2))-d^3*c^6*(-c^2*x^2+1)^(1/2)+3/5*d*c^2*e^2*(-1/5*c^4*x^4*(-c
^2*x^2+1)^(1/2)-4/15*c^2*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))+d
^2*c^4*e*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 555, normalized size of antiderivative = 0.98

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))^2\,dx$$
$$= \frac{1125\,(49\,a^2-2\,b^2)c^7e^3x^7+189\,(49\,(25\,a^2-2\,b^2)c^7de^2-20\,b^2c^5e^3)x^5+35\,(1225\,(9\,a^2-2\,b^2)c^7d^2e-117}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
1/385875*(1125*(49*a^2 - 2*b^2)*c^7*e^3*x^7 + 189*(49*(25*a^2 - 2*b^2)*c^7
*d*e^2 - 20*b^2*c^5*e^3)*x^5 + 35*(1225*(9*a^2 - 2*b^2)*c^7*d^2*e - 1176*b
^2*c^5*d*e^2 - 240*b^2*c^3*e^3)*x^3 + 11025*(5*b^2*c^7*e^3*x^7 + 21*b^2*c^
7*d*e^2*x^5 + 35*b^2*c^7*d^2*e*x^3 + 35*b^2*c^7*d^3*x)*arccos(c*x)^2 + 105
*(3675*(a^2 - 2*b^2)*c^7*d^3 - 4900*b^2*c^5*d^2*e - 2352*b^2*c^3*d*e^2 - 4
80*b^2*c*e^3)*x + 22050*(5*a*b*c^7*e^3*x^7 + 21*a*b*c^7*d*e^2*x^5 + 35*a*b
*c^7*d^2*e*x^3 + 35*a*b*c^7*d^3*x)*arccos(c*x) - 210*(75*a*b*c^6*e^3*x^6 +
 3675*a*b*c^6*d^3 + 2450*a*b*c^4*d^2*e + 1176*a*b*c^2*d*e^2 + 240*a*b*e^3
+ 9*(49*a*b*c^6*d*e^2 + 10*a*b*c^4*e^3)*x^4 + (1225*a*b*c^6*d^2*e + 588*a*
b*c^4*d*e^2 + 120*a*b*c^2*e^3)*x^2 + (75*b^2*c^6*e^3*x^6 + 3675*b^2*c^6*d^
3 + 2450*b^2*c^4*d^2*e + 1176*b^2*c^2*d*e^2 + 240*b^2*e^3 + 9*(49*b^2*c^6*
d*e^2 + 10*b^2*c^4*e^3)*x^4 + (1225*b^2*c^6*d^2*e + 588*b^2*c^4*d*e^2 + 12
0*b^2*c^2*e^3)*x^2)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^7
```

Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 994, normalized size of antiderivative = 1.75

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x**2+d)**3*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*d**3*x + a**2*d**2*e*x**3 + 3*a**2*d*e**2*x**5/5 + a**2*e*
*3*x**7/7 + 2*a*b*d**3*x*acos(c*x) + 2*a*b*d**2*e*x**3*acos(c*x) + 6*a*b*d
*e**2*x**5*acos(c*x)/5 + 2*a*b*e**3*x**7*acos(c*x)/7 - 2*a*b*d**3*sqrt(-c*
*2*x**2 + 1)/c - 2*a*b*d**2*e*x**2*sqrt(-c**2*x**2 + 1)/(3*c) - 6*a*b*d*e*
*2*x**4*sqrt(-c**2*x**2 + 1)/(25*c) - 2*a*b*e**3*x**6*sqrt(-c**2*x**2 + 1)
/(49*c) - 4*a*b*d**2*e*sqrt(-c**2*x**2 + 1)/(3*c**3) - 8*a*b*d*e**2*x**2*s
qrt(-c**2*x**2 + 1)/(25*c**3) - 12*a*b*e**3*x**4*sqrt(-c**2*x**2 + 1)/(245
*c**3) - 16*a*b*d*e**2*sqrt(-c**2*x**2 + 1)/(25*c**5) - 16*a*b*e**3*x**2*s
qrt(-c**2*x**2 + 1)/(245*c**5) - 32*a*b*e**3*sqrt(-c**2*x**2 + 1)/(245*c**
7) + b**2*d**3*x*acos(c*x)**2 - 2*b**2*d**3*x + b**2*d**2*e*x**3*acos(c*x)
**2 - 2*b**2*d**2*e*x**3/9 + 3*b**2*d*e**2*x**5*acos(c*x)**2/5 - 6*b**2*d*
e**2*x**5/125 + b**2*e**3*x**7*acos(c*x)**2/7 - 2*b**2*e**3*x**7/343 - 2*b
**2*d**3*sqrt(-c**2*x**2 + 1)*acos(c*x)/c - 2*b**2*d**2*e*x**2*sqrt(-c**2*
x**2 + 1)*acos(c*x)/(3*c) - 6*b**2*d*e**2*x**4*sqrt(-c**2*x**2 + 1)*acos(c
*x)/(25*c) - 2*b**2*e**3*x**6*sqrt(-c**2*x**2 + 1)*acos(c*x)/(49*c) - 4*b*
*2*d**2*e*x/(3*c**2) - 8*b**2*d*e**2*x**3/(75*c**2) - 12*b**2*e**3*x**5/(1
225*c**2) - 4*b**2*d**2*e*sqrt(-c**2*x**2 + 1)*acos(c*x)/(3*c**3) - 8*b**2
*d*e**2*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(25*c**3) - 12*b**2*e**3*x**4*
sqrt(-c**2*x**2 + 1)*acos(c*x)/(245*c**3) - 16*b**2*d*e**2*x/(25*c**4) - 1
6*b**2*e**3*x**3/(735*c**4) - 16*b**2*d*e**2*sqrt(-c**2*x**2 + 1)*acos(...
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.23

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/7*b^2*e^3*x^7*arccos(c*x)^2 + 1/7*a^2*e^3*x^7 + 3/5*b^2*d*e^2*x^5*arccos
(c*x)^2 + 3/5*a^2*d*e^2*x^5 + b^2*d^2*e*x^3*arccos(c*x)^2 + a^2*d^2*e*x^3
+ b^2*d^3*x*arccos(c*x)^2 + 2/3*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)
*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*d^2*e - 2/9*(3*c*(sqrt(-c^2*x^2
+ 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x) + (c^2*x^3 + 6*x)/c^2
)*b^2*d^2*e + 2/25*(15*x^5*arccos(c*x) - (3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4
*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c)*a*b*d*e^2 - 2/3
75*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sq
rt(-c^2*x^2 + 1)/c^6)*c*arccos(c*x) + (9*c^4*x^5 + 20*c^2*x^3 + 120*x)/c^4
)*b^2*d*e^2 + 2/245*(35*x^7*arccos(c*x) - (5*sqrt(-c^2*x^2 + 1)*x^6/c^2 +
6*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2
*x^2 + 1)/c^8)*c)*a*b*e^3 - 2/25725*(105*(5*sqrt(-c^2*x^2 + 1)*x^6/c^2 + 6
*sqrt(-c^2*x^2 + 1)*x^4/c^4 + 8*sqrt(-c^2*x^2 + 1)*x^2/c^6 + 16*sqrt(-c^2*
x^2 + 1)/c^8)*c*arccos(c*x) + (75*c^6*x^7 + 126*c^4*x^5 + 280*c^2*x^3 + 16
80*x)/c^6)*b^2*e^3 - 2*b^2*d^3*(x + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + a^
2*d^3*x + 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a*b*d^3/c
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.45

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^3*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
1/7*b^2*e^3*x^7*arccos(c*x)^2 + 2/7*a*b*e^3*x^7*arccos(c*x) + 1/7*a^2*e^3*
x^7 - 2/343*b^2*e^3*x^7 + 3/5*b^2*d*e^2*x^5*arccos(c*x)^2 - 2/49*sqrt(-c^2
*x^2 + 1)*b^2*e^3*x^6*arccos(c*x)/c + 6/5*a*b*d*e^2*x^5*arccos(c*x) - 2/49
*sqrt(-c^2*x^2 + 1)*a*b*e^3*x^6/c + 3/5*a^2*d*e^2*x^5 - 6/125*b^2*d*e^2*x^
5 + b^2*d^2*e*x^3*arccos(c*x)^2 - 6/25*sqrt(-c^2*x^2 + 1)*b^2*d*e^2*x^4*ar
ccos(c*x)/c + 2*a*b*d^2*e*x^3*arccos(c*x) - 6/25*sqrt(-c^2*x^2 + 1)*a*b*d*
e^2*x^4/c + a^2*d^2*e*x^3 - 2/9*b^2*d^2*e*x^3 - 12/1225*b^2*e^3*x^5/c^2 +
b^2*d^3*x*arccos(c*x)^2 - 2/3*sqrt(-c^2*x^2 + 1)*b^2*d^2*e*x^2*arccos(c*x)
/c - 12/245*sqrt(-c^2*x^2 + 1)*b^2*e^3*x^4*arccos(c*x)/c^3 + 2*a*b*d^3*x*a
rccos(c*x) - 2/3*sqrt(-c^2*x^2 + 1)*a*b*d^2*e*x^2/c - 12/245*sqrt(-c^2*x^2
 + 1)*a*b*e^3*x^4/c^3 + a^2*d^3*x - 2*b^2*d^3*x - 8/75*b^2*d*e^2*x^3/c^2 -
 2*sqrt(-c^2*x^2 + 1)*b^2*d^3*arccos(c*x)/c - 8/25*sqrt(-c^2*x^2 + 1)*b^2*
d*e^2*x^2*arccos(c*x)/c^3 - 2*sqrt(-c^2*x^2 + 1)*a*b*d^3/c - 8/25*sqrt(-c^
2*x^2 + 1)*a*b*d*e^2*x^2/c^3 - 4/3*b^2*d^2*e*x/c^2 - 16/735*b^2*e^3*x^3/c^
4 - 4/3*sqrt(-c^2*x^2 + 1)*b^2*d^2*e*arccos(c*x)/c^3 - 16/245*sqrt(-c^2*x^
2 + 1)*b^2*e^3*x^2*arccos(c*x)/c^5 - 4/3*sqrt(-c^2*x^2 + 1)*a*b*d^2*e/c^3
- 16/245*sqrt(-c^2*x^2 + 1)*a*b*e^3*x^2/c^5 - 16/25*b^2*d*e^2*x/c^4 - 16/2
5*sqrt(-c^2*x^2 + 1)*b^2*d*e^2*arccos(c*x)/c^5 - 16/25*sqrt(-c^2*x^2 + 1)*
a*b*d*e^2/c^5 - 32/245*b^2*e^3*x/c^6 - 32/245*sqrt(-c^2*x^2 + 1)*b^2*e^3*a
rccos(c*x)/c^7 - 32/245*sqrt(-c^2*x^2 + 1)*a*b*e^3/c^7
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^3 (a+b\arccos(cx))^2\,dx = \int (a+b\,\mathrm{acos}(c\,x))^2\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + e*x^2)^3,x)
```

output
```
int((a + b*acos(c*x))^2*(d + e*x^2)^3, x)
```

Reduce [F]

$$\int \left(d+ex^2\right)^3 (a+b\arccos(cx))^2\,dx$$
$$= \frac{3675 acos(cx)^2 b^2 c^7 d^3 x - 7350\sqrt{-c^2x^2+1}\, acos(cx)\, b^2 c^6 d^3 + 7350 acos(cx)\, ab\, c^7 d^3 x + 7350 acos(cx)\, ab\, c^7}{}$$

input
```
int((e*x^2+d)^3*(a+b*acos(c*x))^2,x)
```

output
```
(3675*acos(c*x)**2*b**2*c**7*d**3*x - 7350*sqrt( - c**2*x**2 + 1)*acos(c*x
)*b**2*c**6*d**3 + 7350*acos(c*x)*a*b*c**7*d**3*x + 7350*acos(c*x)*a*b*c**
7*d**2*e*x**3 + 4410*acos(c*x)*a*b*c**7*d*e**2*x**5 + 1050*acos(c*x)*a*b*c
**7*e**3*x**7 - 7350*sqrt( - c**2*x**2 + 1)*a*b*c**6*d**3 - 2450*sqrt( - c
**2*x**2 + 1)*a*b*c**6*d**2*e*x**2 - 882*sqrt( - c**2*x**2 + 1)*a*b*c**6*d
*e**2*x**4 - 150*sqrt( - c**2*x**2 + 1)*a*b*c**6*e**3*x**6 - 4900*sqrt( -
c**2*x**2 + 1)*a*b*c**4*d**2*e - 1176*sqrt( - c**2*x**2 + 1)*a*b*c**4*d*e*
*2*x**2 - 180*sqrt( - c**2*x**2 + 1)*a*b*c**4*e**3*x**4 - 2352*sqrt( - c**
2*x**2 + 1)*a*b*c**2*d*e**2 - 240*sqrt( - c**2*x**2 + 1)*a*b*c**2*e**3*x**
2 - 480*sqrt( - c**2*x**2 + 1)*a*b*e**3 + 3675*int(acos(c*x)**2*x**6,x)*b*
*2*c**7*e**3 + 11025*int(acos(c*x)**2*x**4,x)*b**2*c**7*d*e**2 + 11025*int
(acos(c*x)**2*x**2,x)*b**2*c**7*d**2*e + 3675*a**2*c**7*d**3*x + 3675*a**2
*c**7*d**2*e*x**3 + 2205*a**2*c**7*d*e**2*x**5 + 525*a**2*c**7*e**3*x**7 -
 7350*b**2*c**7*d**3*x)/(3675*c**7)
```

3.662 $\int (d+ex^2)^2 (a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 335

$$\int (d+ex^2)^2 (a+b\arccos(cx))^2\,dx = -2b^2d^2x-\frac{8b^2dex}{9c^2}-\frac{16b^2e^2x}{75c^4}-\frac{4}{27}b^2dex^3-\frac{8b^2e^2x^3}{225c^2}-\frac{2}{125}b^2e^2x^5+\frac{2bd^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}+\frac{8bde\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^3}+\frac{16be^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{75c^5}+\frac{4bdex^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c}+\frac{8be^2x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{75c^3}+\frac{2be^2x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c}+d^2x(a+b\arccos(cx))^2+\frac{2}{3}dex^3(a+b\arccos(cx))^2+\frac{1}{5}e^2x^5(a+b\arccos(cx))^2$$

output
```
-2*b^2*d^2*x-8/9*b^2*d*e*x/c^2-16/75*b^2*e^2*x/c^4-4/27*b^2*d*e*x^3-8/225*
b^2*e^2*x^3/c^2-2/125*b^2*e^2*x^5+2*b*d^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c
*x))/c+8/9*b*d*e*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3+16/75*b*e^2*(-c^
2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^5+4/9*b*d*e*x^2*(-c^2*x^2+1)^(1/2)*(a+b
*arccos(c*x))/c+8/75*b*e^2*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3+2/
25*b*e^2*x^4*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+d^2*x*(a+b*arccos(c*x)
)^2+2/3*d*e*x^3*(a+b*arccos(c*x))^2+1/5*e^2*x^5*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.87

$$\int (d+ex^2)^2 (a+b\arccos(cx))^2\,dx$$
$$= \frac{225a^2c^5x(15d^2+10dex^2+3e^2x^4)-30ab\sqrt{1-c^2x^2}(24e^2+4c^2e(25d+3ex^2)+c^4(225d^2+50dex^2+9e}{}$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
(225*a^2*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - 30*a*b*Sqrt[1 - c^2*x^2
]*(24*e^2 + 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x
^4)) - 2*b^2*c*x*(360*e^2 + 60*c^2*e*(25*d + e*x^2) + c^4*(3375*d^2 + 250*
d*e*x^2 + 27*e^2*x^4)) - 30*b*(-15*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^
4) + b*Sqrt[1 - c^2*x^2]*(24*e^2 + 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2
 + 50*d*e*x^2 + 9*e^2*x^4)))*ArcCos[c*x] + 225*b^2*c^5*x*(15*d^2 + 10*d*e*
x^2 + 3*e^2*x^4)*ArcCos[c*x]^2)/(3375*c^5)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(d^2(a+b\arccos(cx))^2+2dex^2(a+b\arccos(cx))^2+e^2x^4(a+b\arccos(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2bd^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}-\frac{4bdex^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c}-\frac{2be^2x^4\sqrt{1-c^2x^2}(a+b\arccos(cx))}{25c}-\frac{16be^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{75c^5}-\frac{8bde\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^3}-\frac{8be^2x^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{75c^3}+d^2x(a+b\arccos(cx))^2+\frac{2}{3}dex^3(a+b\arccos(cx))^2+\frac{1}{5}e^2x^5(a+b\arccos(cx))^2-\frac{16b^2e^2x}{75c^4}-\frac{8b^2dex}{9c^2}-\frac{8b^2e^2x^3}{225c^2}-2b^2d^2x-\frac{4}{27}b^2dex^3-\frac{2}{125}b^2e^2x^5$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcCos[c*x])^2,x]
```

output
```
-2*b^2*d^2*x - (8*b^2*d*e*x)/(9*c^2) - (16*b^2*e^2*x)/(75*c^4) - (4*b^2*d*
e*x^3)/27 - (8*b^2*e^2*x^3)/(225*c^2) - (2*b^2*e^2*x^5)/125 - (2*b*d^2*Sqr
t[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c - (8*b*d*e*Sqrt[1 - c^2*x^2]*(a + b*
ArcCos[c*x]))/(9*c^3) - (16*b*e^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(
75*c^5) - (4*b*d*e*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(9*c) - (8*b
*e^2*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(75*c^3) - (2*b*e^2*x^4*Sq
rt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(25*c) + d^2*x*(a + b*ArcCos[c*x])^2
+ (2*d*e*x^3*(a + b*ArcCos[c*x])^2)/3 + (e^2*x^5*(a + b*ArcCos[c*x])^2)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{a^2\left(c^5d^2x+\frac{2}{3}c^5de\,x^3+\frac{1}{5}c^5e^2x^5\right)}{c^4}+\frac{b^2\left(c^4d^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+\frac{2c^2de\left(9\arccos(cx)^2c^3x^3-6\sqrt{-c^2x^2}\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{a^2\left(c^5d^2x+\frac{2}{3}c^5de\,x^3+\frac{1}{5}c^5e^2x^5\right)}{c^4}+\frac{b^2\left(c^4d^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+\frac{2c^2de\left(9\arccos(cx)^2c^3x^3-6\sqrt{-c^2x^2}\cdots\right.}{\cdots}\right.}{\cdots}$
parts	$a^2\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b^2\left(675\arccos(cx)^2c^5x^5e^2+2250\arccos(cx)^2c^5x^3de+3375\arccos(cx)^2c^5x\,d^2-\cdots\right.}{\cdots}$
orering	$\frac{x\left(1647c^6e^4x^8+10924c^6d\,e^3x^6+77050c^6d^2e^2x^4+600c^4e^4x^6-4500c^6d^3e\,x^2+21808c^4d\,e^3x^4+3375c^6d^4-89000c^4d^2e^2x\cdots\right.}{3375(e\,x^2+d)^2c^6}$

input
```
int((e*x^2+d)^2*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2/c^4*(c^5*d^2*x+2/3*c^5*d*e*x^3+1/5*c^5*e^2*x^5)+b^2/c^4*(c^4*d^2*
(arccos(c*x)^2*c*x-2*c*x-2*arccos(c*x)*(-c^2*x^2+1)^(1/2))+2/27*c^2*d*e*(9
*arccos(c*x)^2*c^3*x^3-6*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^2*x^2-2*c^3*x^3-
12*arccos(c*x)*(-c^2*x^2+1)^(1/2)-12*c*x)+1/1125*e^2*(225*arccos(c*x)^2*c^
5*x^5-90*(-c^2*x^2+1)^(1/2)*arccos(c*x)*c^4*x^4-18*c^5*x^5-120*(-c^2*x^2+1
)^(1/2)*arccos(c*x)*c^2*x^2-40*c^3*x^3-240*arccos(c*x)*(-c^2*x^2+1)^(1/2)-
240*c*x))+2*a*b/c^4*(arccos(c*x)*d^2*c^5*x+2/3*arccos(c*x)*c^5*d*e*x^3+1/5
*arccos(c*x)*e^2*c^5*x^5+1/5*e^2*(-1/5*c^4*x^4*(-c^2*x^2+1)^(1/2)-4/15*c^2
*x^2*(-c^2*x^2+1)^(1/2)-8/15*(-c^2*x^2+1)^(1/2))-d^2*c^4*(-c^2*x^2+1)^(1/2
)+2/3*d*c^2*e*(-1/3*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.04

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))^2\,dx = \frac{27\,(25\,a^2-2\,b^2)c^5e^2x^5+10\,(25\,(9\,a^2-2\,b^2)c^5de-12\,b^2c^3e^2)x^3+225\,(3\,b^2c^5e^2x^5+10\,b^2c^5dex^3+15\,b^2c$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
1/3375*(27*(25*a^2 - 2*b^2)*c^5*e^2*x^5 + 10*(25*(9*a^2 - 2*b^2)*c^5*d*e -
 12*b^2*c^3*e^2)*x^3 + 225*(3*b^2*c^5*e^2*x^5 + 10*b^2*c^5*d*e*x^3 + 15*b^
2*c^5*d^2*x)*arccos(c*x)^2 + 15*(225*(a^2 - 2*b^2)*c^5*d^2 - 200*b^2*c^3*d
*e - 48*b^2*c*e^2)*x + 450*(3*a*b*c^5*e^2*x^5 + 10*a*b*c^5*d*e*x^3 + 15*a*
b*c^5*d^2*x)*arccos(c*x) - 30*(9*a*b*c^4*e^2*x^4 + 225*a*b*c^4*d^2 + 100*a
*b*c^2*d*e + 24*a*b*e^2 + 2*(25*a*b*c^4*d*e + 6*a*b*c^2*e^2)*x^2 + (9*b^2*
c^4*e^2*x^4 + 225*b^2*c^4*d^2 + 100*b^2*c^2*d*e + 24*b^2*e^2 + 2*(25*b^2*c
^4*d*e + 6*b^2*c^2*e^2)*x^2)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.79

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))^2\,dx = \begin{cases} a^2d^2x+\frac{2a^2dex^3}{3}+\frac{a^2e^2x^5}{5}+2abd^2x\operatorname{acos}(cx)+\frac{4abdex^3\operatorname{acos}(cx)}{3}+\frac{2abe^2x^5\operatorname{acos}(cx)}{5}-\frac{2abd^2\sqrt{-c^2x^2+1}}{c}-\frac{4abdex^2\sqrt{}}{9c} \\ \left(a+\frac{\pi b}{2}\right)^2\left(d^2x+\frac{2dex^3}{3}+\frac{e^2x^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*d**2*x + 2*a**2*d*e*x**3/3 + a**2*e**2*x**5/5 + 2*a*b*d**2
*x*acos(c*x) + 4*a*b*d*e*x**3*acos(c*x)/3 + 2*a*b*e**2*x**5*acos(c*x)/5 -
2*a*b*d**2*sqrt(-c**2*x**2 + 1)/c - 4*a*b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(9
*c) - 2*a*b*e**2*x**4*sqrt(-c**2*x**2 + 1)/(25*c) - 8*a*b*d*e*sqrt(-c**2*x
**2 + 1)/(9*c**3) - 8*a*b*e**2*x**2*sqrt(-c**2*x**2 + 1)/(75*c**3) - 16*a*
b*e**2*sqrt(-c**2*x**2 + 1)/(75*c**5) + b**2*d**2*x*acos(c*x)**2 - 2*b**2*
d**2*x + 2*b**2*d*e*x**3*acos(c*x)**2/3 - 4*b**2*d*e*x**3/27 + b**2*e**2*x
**5*acos(c*x)**2/5 - 2*b**2*e**2*x**5/125 - 2*b**2*d**2*sqrt(-c**2*x**2 +
1)*acos(c*x)/c - 4*b**2*d*e*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(9*c) - 2*
b**2*e**2*x**4*sqrt(-c**2*x**2 + 1)*acos(c*x)/(25*c) - 8*b**2*d*e*x/(9*c**
2) - 8*b**2*e**2*x**3/(225*c**2) - 8*b**2*d*e*sqrt(-c**2*x**2 + 1)*acos(c*
x)/(9*c**3) - 8*b**2*e**2*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(75*c**3) -
16*b**2*e**2*x/(75*c**4) - 16*b**2*e**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(75
*c**5), Ne(c, 0)), ((a + pi*b/2)**2*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.31

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))^2\,dx$$
$$= \frac{1}{5}b^2e^2x^5\arccos(cx)^2 + \frac{1}{5}a^2e^2x^5 + \frac{2}{3}b^2dex^3\arccos(cx)^2 + \frac{2}{3}a^2dex^3 + b^2d^2x\arccos(cx)^2$$
$$+ \frac{4}{9}\left(3\,x^3\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)abde$$
$$- \frac{4}{27}\left(3\,c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\arccos(cx) + \frac{c^2x^3+6\,x}{c^2}\right)b^2de$$
$$+ \frac{2}{75}\left(15\,x^5\arccos(cx) - \left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\right)abe^2$$
$$- \frac{2}{1125}\left(15\left(\frac{3\sqrt{-c^2x^2+1}x^4}{c^2} + \frac{4\sqrt{-c^2x^2+1}x^2}{c^4} + \frac{8\sqrt{-c^2x^2+1}}{c^6}\right)c\arccos(cx) + \frac{9\,c^4x^5+20\,c^2x^3+}{c^4}\right.$$
$$- 2\,b^2d^2\left(x + \frac{\sqrt{-c^2x^2+1}\arccos(cx)}{c}\right)$$
$$+ a^2d^2x + \frac{2\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)abd^2}{c}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*b^2*e^2*x^5*arccos(c*x)^2 + 1/5*a^2*e^2*x^5 + 2/3*b^2*d*e*x^3*arccos(c
*x)^2 + 2/3*a^2*d*e*x^3 + b^2*d^2*x*arccos(c*x)^2 + 4/9*(3*x^3*arccos(c*x)
 - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b*d*e - 4/
27*(3*c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x
) + (c^2*x^3 + 6*x)/c^2)*b^2*d*e + 2/75*(15*x^5*arccos(c*x) - (3*sqrt(-c^2
*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^
6)*c)*a*b*e^2 - 2/1125*(15*(3*sqrt(-c^2*x^2 + 1)*x^4/c^2 + 4*sqrt(-c^2*x^2
 + 1)*x^2/c^4 + 8*sqrt(-c^2*x^2 + 1)/c^6)*c*arccos(c*x) + (9*c^4*x^5 + 20*
c^2*x^3 + 120*x)/c^4)*b^2*e^2 - 2*b^2*d^2*(x + sqrt(-c^2*x^2 + 1)*arccos(c
*x)/c) + a^2*d^2*x + 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a*b*d^2/c
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.47

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))^2\,dx = \frac{1}{5}\,b^2e^2x^5\arccos\left(cx\right)^2+\frac{2}{5}\,abe^2x^5\arccos\left(cx\right)+\frac{1}{5}\,a^2e^2x^5-\frac{2}{125}\,b^2e^2x^5+\frac{2}{3}\,b^2dex^3\arccos\left(cx\right)^2-\frac{2\sqrt{-c^2x^2+1}b^2e^2x^4\arccos\left(cx\right)}{25\,c}+\frac{4}{3}\,abdex^3\arccos\left(cx\right)-\frac{2\sqrt{-c^2x^2+1}abe^2x^4}{25\,c}+\frac{2}{3}\,a^2dex^3-\frac{4}{27}\,b^2dex^3+b^2d^2x\arccos\left(cx\right)^2-\frac{4\sqrt{-c^2x^2+1}b^2dex^2\arccos\left(cx\right)}{9\,c}+2\,abd^2x\arccos\left(cx\right)-\frac{4\sqrt{-c^2x^2+1}abdex^2}{9\,c}+a^2d^2x-2\,b^2d^2x-\frac{8\,b^2e^2x^3}{225\,c^2}-\frac{2\sqrt{-c^2x^2+1}b^2d^2\arccos\left(cx\right)}{c}-\frac{8\sqrt{-c^2x^2+1}b^2e^2x^2\arccos\left(cx\right)}{75\,c^3}-\frac{2\sqrt{-c^2x^2+1}abd^2}{c}-\frac{8\sqrt{-c^2x^2+1}abe^2x^2}{75\,c^3}-\frac{8\,b^2dex}{9\,c^2}-\frac{8\sqrt{-c^2x^2+1}b^2de\arccos\left(cx\right)}{9\,c^3}-\frac{8\sqrt{-c^2x^2+1}abde}{9\,c^3}-\frac{16\,b^2e^2x}{75\,c^4}-\frac{16\sqrt{-c^2x^2+1}b^2e^2\arccos\left(cx\right)}{75\,c^5}-\frac{16\sqrt{-c^2x^2+1}abe^2}{75\,c^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output

```
1/5*b^2*e^2*x^5*arccos(c*x)^2 + 2/5*a*b*e^2*x^5*arccos(c*x) + 1/5*a^2*e^2*
x^5 - 2/125*b^2*e^2*x^5 + 2/3*b^2*d*e*x^3*arccos(c*x)^2 - 2/25*sqrt(-c^2*x
^2 + 1)*b^2*e^2*x^4*arccos(c*x)/c + 4/3*a*b*d*e*x^3*arccos(c*x) - 2/25*sqr
t(-c^2*x^2 + 1)*a*b*e^2*x^4/c + 2/3*a^2*d*e*x^3 - 4/27*b^2*d*e*x^3 + b^2*d
^2*x*arccos(c*x)^2 - 4/9*sqrt(-c^2*x^2 + 1)*b^2*d*e*x^2*arccos(c*x)/c + 2*
a*b*d^2*x*arccos(c*x) - 4/9*sqrt(-c^2*x^2 + 1)*a*b*d*e*x^2/c + a^2*d^2*x -
 2*b^2*d^2*x - 8/225*b^2*e^2*x^3/c^2 - 2*sqrt(-c^2*x^2 + 1)*b^2*d^2*arccos
(c*x)/c - 8/75*sqrt(-c^2*x^2 + 1)*b^2*e^2*x^2*arccos(c*x)/c^3 - 2*sqrt(-c^
2*x^2 + 1)*a*b*d^2/c - 8/75*sqrt(-c^2*x^2 + 1)*a*b*e^2*x^2/c^3 - 8/9*b^2*d
*e*x/c^2 - 8/9*sqrt(-c^2*x^2 + 1)*b^2*d*e*arccos(c*x)/c^3 - 8/9*sqrt(-c^2*
x^2 + 1)*a*b*d*e/c^3 - 16/75*b^2*e^2*x/c^4 - 16/75*sqrt(-c^2*x^2 + 1)*b^2*
e^2*arccos(c*x)/c^5 - 16/75*sqrt(-c^2*x^2 + 1)*a*b*e^2/c^5
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,\left(e\,x^2+d\right)^2\,dx$$

input

```
int((a + b*acos(c*x))^2*(d + e*x^2)^2,x)
```

output

```
int((a + b*acos(c*x))^2*(d + e*x^2)^2, x)
```

Reduce [F]

$$\int \left(d+ex^2\right)^2 (a+b\arccos(cx))^2\,dx$$
$$=\frac{225acos(cx)^2\,b^2c^5d^2x-450\sqrt{-c^2x^2+1}\,acos(cx)\,b^2c^4d^2+450acos(cx)\,ab\,c^5d^2x+300acos(cx)\,ab\,c^5de\,x^3}{}$$

input

```
int((e*x^2+d)^2*(a+b*acos(c*x))^2,x)
```

output
```
(225*acos(c*x)**2*b**2*c**5*d**2*x - 450*sqrt( - c**2*x**2 + 1)*acos(c*x)*
b**2*c**4*d**2 + 450*acos(c*x)*a*b*c**5*d**2*x + 300*acos(c*x)*a*b*c**5*d*
e*x**3 + 90*acos(c*x)*a*b*c**5*e**2*x**5 - 450*sqrt( - c**2*x**2 + 1)*a*b*
c**4*d**2 - 100*sqrt( - c**2*x**2 + 1)*a*b*c**4*d*e*x**2 - 18*sqrt( - c**2
*x**2 + 1)*a*b*c**4*e**2*x**4 - 200*sqrt( - c**2*x**2 + 1)*a*b*c**2*d*e -
24*sqrt( - c**2*x**2 + 1)*a*b*c**2*e**2*x**2 - 48*sqrt( - c**2*x**2 + 1)*a
*b*e**2 + 225*int(acos(c*x)**2*x**4,x)*b**2*c**5*e**2 + 450*int(acos(c*x)*
*2*x**2,x)*b**2*c**5*d*e + 225*a**2*c**5*d**2*x + 150*a**2*c**5*d*e*x**3 +
 45*a**2*c**5*e**2*x**5 - 450*b**2*c**5*d**2*x)/(225*c**5)
```

3.663 $\int (d+ex^2)(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 156

$$\int (d+ex^2)(a+b\arccos(cx))^2\,dx = -2b^2dx-\frac{4b^2ex}{9c^2}-\frac{2}{27}b^2ex^3+\frac{2bd\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}+\frac{4be\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^3}+\frac{2bex^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c}+dx(a+b\arccos(cx))^2+\frac{1}{3}ex^3(a+b\arccos(cx))^2$$

output
```
-2*b^2*d*x-4/9*b^2*e*x/c^2-2/27*b^2*e*x^3+2*b*d*(-c^2*x^2+1)^(1/2)*(a+b*ar
ccos(c*x))/c+4/9*b*e*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c^3+2/9*b*e*x^2*
(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+d*x*(a+b*arccos(c*x))^2+1/3*e*x^3*(
a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.06

$$\int (d+ex^2)(a+b\arccos(cx))^2\,dx$$
$$=\frac{9a^2c^3x(3d+ex^2)-6ab\sqrt{1-c^2x^2}(2e+c^2(9d+ex^2))-2b^2cx(6e+c^2(27d+ex^2))-6b(-3ac^3x(3d+e\ldots}{27c^3}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output
```
(9*a^2*c^3*x*(3*d + e*x^2) - 6*a*b*Sqrt[1 - c^2*x^2]*(2*e + c^2*(9*d + e*x
^2)) - 2*b^2*c*x*(6*e + c^2*(27*d + e*x^2)) - 6*b*(-3*a*c^3*x*(3*d + e*x^2
) + b*Sqrt[1 - c^2*x^2]*(2*e + c^2*(9*d + e*x^2)))*ArcCos[c*x] + 9*b^2*c^3
*x*(3*d + e*x^2)*ArcCos[c*x]^2)/(27*c^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)(a+b\arccos(cx))^2\,dx$$
$$\Big\downarrow \text{5173}$$
$$\int \left(d(a+b\arccos(cx))^2+ex^2(a+b\arccos(cx))^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2bd\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}-\frac{2bex^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c}-\frac{4be\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^3}+dx(a+b\arccos(cx))^2+\frac{1}{3}ex^3(a+b\arccos(cx))^2-\frac{4b^2ex}{9c^2}-2b^2dx-\frac{2}{27}b^2ex^3$$

input

```
Int[(d + e*x^2)*(a + b*ArcCos[c*x])^2,x]
```

output

```
-2*b^2*d*x - (4*b^2*e*x)/(9*c^2) - (2*b^2*e*x^3)/27 - (2*b*d*Sqrt[1 - c^2*
x^2]*(a + b*ArcCos[c*x]))/c - (4*b*e*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])
)/(9*c^3) - (2*b*e*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(9*c) + d*x*
(a + b*ArcCos[c*x])^2 + (e*x^3*(a + b*ArcCos[c*x])^2)/3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.42

method	result
parts	$a^2\left(\frac{1}{3}e\,x^3+dx\right)+\frac{b^2\Big(\frac{e\left(9\arccos(cx)^2c^3x^3-6\sqrt{-c^2x^2+1}\,\arccos(cx)c^2x^2-2c^3x^3-12\arccos(cx)\sqrt{-c^2x^2+1}-12cx\right)}{27c^2}+d\Big(\ldots}{c}$
derivativedivides	$\frac{\frac{a^2\left(c^3dx+\frac{1}{3}c^3e\,x^3\right)}{c^2}+\frac{b^2\Big(d\,c^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+\frac{e\left(9\arccos(cx)^2c^3x^3-6\sqrt{-c^2x^2+1}\,\arccos(cx)c^2x^2 \ldots\right.}{27}\ldots}{c^2}}{\ldots}$
default	$\frac{\frac{a^2\left(c^3dx+\frac{1}{3}c^3e\,x^3\right)}{c^2}+\frac{b^2\Big(d\,c^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+\frac{e\left(9\arccos(cx)^2c^3x^3-6\sqrt{-c^2x^2+1}\,\arccos(cx)c^2x^2 \ldots\right.}{27}\ldots}{c^2}}{\ldots}$
orering	$\frac{x\left(19x^6e^3c^4+209x^4e^2d\,c^4+9x^2e\,d^2c^4+24x^4e^3c^2+27d^3c^4-232x^2e^2d\,c^2-48x^2e^3\right)(a+b\arccos(cx))^2}{27(e\,x^2+d)^2c^4}-\frac{\left(6x^6c^4e^2+11 \ldots\right.}{\ldots}$

input
```
int((e*x^2+d)*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/3*e*x^3+d*x)+b^2/c*(1/27*e*(9*arccos(c*x)^2*c^3*x^3-6*(-c^2*x^2+1)^
(1/2)*arccos(c*x)*c^2*x^2-2*c^3*x^3-12*arccos(c*x)*(-c^2*x^2+1)^(1/2)-12*c
*x)/c^2+d*(arccos(c*x)^2*c*x-2*c*x-2*arccos(c*x)*(-c^2*x^2+1)^(1/2)))+2*a*
b/c*(1/3*c*arccos(c*x)*e*x^3+arccos(c*x)*d*c*x+1/3/c^2*(e*(-1/3*c^2*x^2*(-
c^2*x^2+1)^(1/2)-2/3*(-c^2*x^2+1)^(1/2))-3*d*c^2*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13

$$\int \left(d+ex^2\right)(a+b\arccos(cx))^2\,dx$$
$$=\frac{(9\,a^2-2\,b^2)c^3ex^3+9\,(b^2c^3ex^3+3\,b^2c^3dx)\arccos{(cx)}^2+3\,(9\,(a^2-2\,b^2)c^3d-4\,b^2ce)x+18\,(abc^3ex^3+3}{27}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
1/27*((9*a^2 - 2*b^2)*c^3*e*x^3 + 9*(b^2*c^3*e*x^3 + 3*b^2*c^3*d*x)*arccos
(c*x)^2 + 3*(9*(a^2 - 2*b^2)*c^3*d - 4*b^2*c*e)*x + 18*(a*b*c^3*e*x^3 + 3*
a*b*c^3*d*x)*arccos(c*x) - 6*(a*b*c^2*e*x^2 + 9*a*b*c^2*d + 2*a*b*e + (b^2
*c^2*e*x^2 + 9*b^2*c^2*d + 2*b^2*e)*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.82

$$\int \left(d+ex^2\right)(a+b\arccos(cx))^2\,dx$$
$$=\begin{cases} a^2dx+\frac{a^2ex^3}{3}+2abdx\operatorname{acos}{(cx)}+\frac{2abex^3\operatorname{acos}{(cx)}}{3}-\frac{2abd\sqrt{-c^2x^2+1}}{c}-\frac{2abex^2\sqrt{-c^2x^2+1}}{9c}-\frac{4abe\sqrt{-c^2x^2+1}}{9c^3}+b^2dx\,\mathrm{a} \\ \left(a+\frac{\pi b}{2}\right)^2\left(dx+\frac{ex^3}{3}\right) \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*d*x + a**2*e*x**3/3 + 2*a*b*d*x*acos(c*x) + 2*a*b*e*x**3*a
cos(c*x)/3 - 2*a*b*d*sqrt(-c**2*x**2 + 1)/c - 2*a*b*e*x**2*sqrt(-c**2*x**2
 + 1)/(9*c) - 4*a*b*e*sqrt(-c**2*x**2 + 1)/(9*c**3) + b**2*d*x*acos(c*x)**
2 - 2*b**2*d*x + b**2*e*x**3*acos(c*x)**2/3 - 2*b**2*e*x**3/27 - 2*b**2*d*
sqrt(-c**2*x**2 + 1)*acos(c*x)/c - 2*b**2*e*x**2*sqrt(-c**2*x**2 + 1)*acos
(c*x)/(9*c) - 4*b**2*e*x/(9*c**2) - 4*b**2*e*sqrt(-c**2*x**2 + 1)*acos(c*x
)/(9*c**3), Ne(c, 0)), ((a + pi*b/2)**2*(d*x + e*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.42

$$\begin{aligned}
&\int \left(d+ex^2\right)(a+b\arccos(cx))^2\,dx \\
&= \frac{1}{3}b^2ex^3\arccos(cx)^2 + \frac{1}{3}a^2ex^3 + b^2dx\arccos(cx)^2 \\
&\quad + \frac{2}{9}\left(3x^3\arccos(cx) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)abe \\
&\quad - \frac{2}{27}\left(3c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\arccos(cx) + \frac{c^2x^3+6x}{c^2}\right)b^2e \\
&\quad - 2b^2d\left(x + \frac{\sqrt{-c^2x^2+1}\arccos(cx)}{c}\right) \\
&\quad + a^2dx + \frac{2\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)abd}{c}
\end{aligned}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*e*x^3*arccos(c*x)^2 + 1/3*a^2*e*x^3 + b^2*d*x*arccos(c*x)^2 + 2/9*
(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/
c^4))*a*b*e - 2/27*(3*c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)
/c^4)*arccos(c*x) + (c^2*x^3 + 6*x)/c^2)*b^2*e - 2*b^2*d*(x + sqrt(-c^2*x^
2 + 1)*arccos(c*x)/c) + a^2*d*x + 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))
*a*b*d/c
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.49

$$\int (d+ex^2)(a+b\arccos(cx))^2\,dx = \frac{1}{3}b^2ex^3\arccos(cx)^2 + \frac{2}{3}abex^3\arccos(cx) + \frac{1}{3}a^2ex^3 - \frac{2}{27}b^2ex^3 + b^2dx\arccos(cx)^2 - \frac{2\sqrt{-c^2x^2+1}b^2ex^2\arccos(cx)}{9c} + 2abdx\arccos(cx) - \frac{2\sqrt{-c^2x^2+1}abex^2}{9c} + a^2dx - 2b^2dx - \frac{2\sqrt{-c^2x^2+1}b^2d\arccos(cx)}{c} - \frac{2\sqrt{-c^2x^2+1}abd}{c} - \frac{4b^2ex}{9c^2} - \frac{4\sqrt{-c^2x^2+1}b^2e\arccos(cx)}{9c^3} - \frac{4\sqrt{-c^2x^2+1}abe}{9c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
1/3*b^2*e*x^3*arccos(c*x)^2 + 2/3*a*b*e*x^3*arccos(c*x) + 1/3*a^2*e*x^3 -
2/27*b^2*e*x^3 + b^2*d*x*arccos(c*x)^2 - 2/9*sqrt(-c^2*x^2 + 1)*b^2*e*x^2*
arccos(c*x)/c + 2*a*b*d*x*arccos(c*x) - 2/9*sqrt(-c^2*x^2 + 1)*a*b*e*x^2/c
 + a^2*d*x - 2*b^2*d*x - 2*sqrt(-c^2*x^2 + 1)*b^2*d*arccos(c*x)/c - 2*sqrt
(-c^2*x^2 + 1)*a*b*d/c - 4/9*b^2*e*x/c^2 - 4/9*sqrt(-c^2*x^2 + 1)*b^2*e*ar
ccos(c*x)/c^3 - 4/9*sqrt(-c^2*x^2 + 1)*a*b*e/c^3
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)(a+b\arccos(cx))^2\,dx = \int (a+b\,\mathrm{acos}(c\,x))^2\,(e\,x^2+d)\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + e*x^2),x)
```

output
```
int((a + b*acos(c*x))^2*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2)(a+b\arccos(cx))^2\,dx$$
$$= \frac{9acos(cx)^2 b^2c^3dx - 18\sqrt{-c^2x^2+1}\,acos(cx)\,b^2c^2d + 18acos(cx)\,ab\,c^3dx + 6acos(cx)\,ab\,c^3e\,x^3 - 18\sqrt{-c^2}}{}$$

input
```
int((e*x^2+d)*(a+b*acos(c*x))^2,x)
```

output
```
(9*acos(c*x)**2*b**2*c**3*d*x - 18*sqrt( - c**2*x**2 + 1)*acos(c*x)*b**2*c
**2*d + 18*acos(c*x)*a*b*c**3*d*x + 6*acos(c*x)*a*b*c**3*e*x**3 - 18*sqrt(
 - c**2*x**2 + 1)*a*b*c**2*d - 2*sqrt( - c**2*x**2 + 1)*a*b*c**2*e*x**2 -
4*sqrt( - c**2*x**2 + 1)*a*b*e + 9*int(acos(c*x)**2*x**2,x)*b**2*c**3*e +
9*a**2*c**3*d*x + 3*a**2*c**3*e*x**3 - 18*b**2*c**3*d*x)/(9*c**3)
```

3.664 $\int (a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int (a+b\arccos(cx))^2\,dx=-2b^2x+\frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}+x(a+b\arccos(cx))^2$$

output
```
-2*b^2*x+2*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))/c+x*(a+b*arccos(c*x))^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int (a+b\arccos(cx))^2\,dx=\left(a^2-2b^2\right)x-\frac{2ab\sqrt{1-c^2x^2}}{c}+\frac{2b\left(acx-b\sqrt{1-c^2x^2}\right)\arccos(cx)}{c}+b^2x\arccos(cx)^2$$

input
```
Integrate[(a + b*ArcCos[c*x])^2,x]
```

output
```
(a^2 - 2*b^2)*x - (2*a*b*Sqrt[1 - c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 - c
^2*x^2])*ArcCos[c*x])/c + b^2*x*ArcCos[c*x]^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5131, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \ 5131$$

$$2bc\int \frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx+x(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 5183$$

$$2bc\left(-\frac{b\int 1dx}{c}-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}\right)+x(a+b\arccos(cx))^2$$

$$\Big\downarrow \ 24$$

$$2bc\left(-\frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{bx}{c}\right)+x(a+b\arccos(cx))^2$$

input
```
Int[(a + b*ArcCos[c*x])^2,x]
```

output
```
x*(a + b*ArcCos[c*x])^2 + 2*b*c*(-((b*x)/c) - (Sqrt[1 - c^2*x^2]*(a + b*Ar
cCos[c*x]))/c^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{a^2cx+b^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+2ab\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	74
default	$\frac{a^2cx+b^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+2ab\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	74
parts	$x\,a^2+\frac{b^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)}{c}+\frac{2ab\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	75
orering	$x(a+b\arccos(cx))^2-\frac{2(a+b\arccos(cx))b}{c\sqrt{-c^2x^2+1}}+\frac{x(cx-1)(cx+1)\left(\frac{2b^2c^2}{-c^2x^2+1}-\frac{2(a+b\arccos(cx))b\,c^3x}{\left(-c^2x^2+1\right)^{\frac{3}{2}}}\right)}{c^2}$	103

input
```
int((a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2*c*x+b^2*(arccos(c*x)^2*c*x-2*c*x-2*arccos(c*x)*(-c^2*x^2+1)^(1/2)
)+2*a*b*(c*x*arccos(c*x)-(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int(a+b\arccos(cx))^2\,dx = \frac{b^2cx\arccos(cx)^2+2\,abcx\arccos(cx)+(a^2-2\,b^2)cx-2\,\sqrt{-c^2x^2+1}(b^2\arccos(cx)+ab)}{c}$$

input
```
integrate((a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
(b^2*c*x*arccos(c*x)^2 + 2*a*b*c*x*arccos(c*x) + (a^2 - 2*b^2)*c*x - 2*sqr
t(-c^2*x^2 + 1)*(b^2*arccos(c*x) + a*b))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(42) = 84$.

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.85

$$\int (a + b\arccos(cx))^2 \, dx = \begin{cases} a^2x + 2abx\operatorname{acos}(cx) - \frac{2ab\sqrt{-c^2x^2+1}}{c} + b^2x\operatorname{acos}^2(cx) - 2b^2x - \frac{2b^2\sqrt{-c^2x^2+1}\operatorname{acos}(cx)}{c} & \text{for } c \neq 0 \\ x\left(a + \frac{\pi b}{2}\right)^2 & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*acos(c*x))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*x*acos(c*x) - 2*a*b*sqrt(-c**2*x**2 + 1)/c + b**
2*x*acos(c*x)**2 - 2*b**2*x - 2*b**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/c, Ne(
c, 0)), (x*(a + pi*b/2)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int (a + b\arccos(cx))^2 \, dx = b^2x\arccos(cx)^2 - 2b^2\left(x + \frac{\sqrt{-c^2x^2+1}\arccos(cx)}{c}\right) + a^2x + \frac{2\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)ab}{c}$$

input
```
integrate((a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
b^2*x*arccos(c*x)^2 - 2*b^2*(x + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + a^2*x
 + 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a*b/c
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int (a+b\arccos(cx))^2\,dx = b^2x\arccos{(cx)}^2 + 2\,abx\arccos{(cx)} + a^2x - 2\,b^2x - \frac{2\,\sqrt{-c^2x^2+1}b^2\arccos{(cx)}}{c} - \frac{2\,\sqrt{-c^2x^2+1}ab}{c}$$

input
```
integrate((a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
b^2*x*arccos(c*x)^2 + 2*a*b*x*arccos(c*x) + a^2*x - 2*b^2*x - 2*sqrt(-c^2*
x^2 + 1)*b^2*arccos(c*x)/c - 2*sqrt(-c^2*x^2 + 1)*a*b/c
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.04

$$\int (a+b\arccos(cx))^2\,dx = \begin{cases} x\left(a^2+\pi\,a\,b+\frac{\pi^2\,b^2}{4}\right) & \text{if}\ \ c=0\\ a^2\,x+b^2\,x\left(\mathrm{acos}(c\,x)^2-2\right)-\frac{2\,b^2\,\mathrm{acos}(c\,x)\,\sqrt{1-c^2\,x^2}}{c}-\frac{2\,a\,b\left(\sqrt{1-c^2\,x^2}-c\,x\,\mathrm{acos}(c\,x)\right)}{c} & \text{if}\ \ c\neq 0 \end{cases}$$

input
```
int((a + b*acos(c*x))^2,x)
```

output
```
piecewise(c == 0, x*(a^2 + (b^2*pi^2)/4 + a*b*pi), c ~= 0, a^2*x + b^2*x*(
acos(c*x)^2 - 2) - (2*b^2*acos(c*x)*(- c^2*x^2 + 1)^(1/2))/c - (2*a*b*((-
c^2*x^2 + 1)^(1/2) - c*x*acos(c*x)))/c)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int (a+b\arccos(cx))^2\,dx$$
$$=\frac{acos(cx)^2\,b^2cx-2\sqrt{-c^2x^2+1}\,acos(cx)\,b^2+2acos(cx)\,abcx-2\sqrt{-c^2x^2+1}\,ab+a^2cx-2b^2cx}{c}$$

input
```
int((a+b*acos(c*x))^2,x)
```

output
```
(acos(c*x)**2*b**2*c*x - 2*sqrt( - c**2*x**2 + 1)*acos(c*x)*b**2 + 2*acos(
c*x)*a*b*c*x - 2*sqrt( - c**2*x**2 + 1)*a*b + a**2*c*x - 2*b**2*c*x)/c
```

3.665 $\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 821

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx = \frac{(a+b\arccos(cx))^2\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\arccos(cx))^2\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a+b\arccos(cx))^2\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a+b\arccos(cx))^2\log\left(1+\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2\operatorname{PolyLog}\left(3,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}-\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} - \frac{b^2\operatorname{PolyLog}\left(3,-\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}} + \frac{b^2\operatorname{PolyLog}\left(3,\frac{\sqrt{e}e^{i\arccos(cx)}}{ic\sqrt{-d}+\sqrt{c^2d+e}}\right)}{\sqrt{-d}\sqrt{e}}$$

output
```
1/2*(a+b*arccos(c*x))^2*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^
(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccos(c*x))^2*ln(1+e^
(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1
/2)/e^(1/2)+1/2*(a+b*arccos(c*x))^2*ln(1-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2)
)/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccos(c*x
))^2*ln(1+e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/
2)))/(-d)^(1/2)/e^(1/2)+I*b*(a+b*arccos(c*x))*polylog(2,-e^(1/2)*(c*x+I*(-
c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-I*b
*(a+b*arccos(c*x))*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^
(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+I*b*(a+b*arccos(c*x))*polylog(2
,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d
)^(1/2)/e^(1/2)-I*b*(a+b*arccos(c*x))*polylog(2,e^(1/2)*(c*x+I*(-c^2*x^2+1
)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b^2*polylog(
3,-e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)-(c^2*d+e)^(1/2)))/(-
d)^(1/2)/e^(1/2)+b^2*polylog(3,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d
)^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b^2*polylog(3,-e^(1/2)*(c*x+I
*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+
b^2*polylog(3,e^(1/2)*(c*x+I*(-c^2*x^2+1)^(1/2))/(I*c*(-d)^(1/2)+(c^2*d+e)
^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Mathematica [F]

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx=\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d + e*x^2),x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^2/(d + e*x^2), x]
```

Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(\frac{\sqrt{-d}(a+b\arccos(cx))^2}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}(a+b\arccos(cx))^2}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)b^2}{\sqrt{-d}\sqrt{e}}+\frac{\text{PolyLog}\left(3,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)b^2}{\sqrt{-d}\sqrt{e}}-\frac{\text{PolyLog}\left(3,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)b^2}{\sqrt{-d}\sqrt{e}}+\frac{\text{PolyLog}\left(3,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)b^2}{\sqrt{-d}\sqrt{e}}+\frac{i(a+b\arccos(cx))\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)b}{\sqrt{-d}\sqrt{e}}-\frac{i(a+b\arccos(cx))\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)b}{\sqrt{-d}\sqrt{e}}+\frac{i(a+b\arccos(cx))\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)b}{\sqrt{-d}\sqrt{e}}-\frac{i(a+b\arccos(cx))\text{PolyLog}\left(2,\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)b}{\sqrt{-d}\sqrt{e}}+\frac{(a+b\arccos(cx))^2\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{c\sqrt{-d}-i\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\arccos(cx))^2\log\left(\frac{e^{i\arccos(cx)}\sqrt{e}}{c\sqrt{-d}-i\sqrt{dc^2+e}}+1\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arccos(cx))^2\log\left(1-\frac{\sqrt{e}e^{i\arccos(cx)}}{\sqrt{-d}c+i\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\arccos(cx))^2\log\left(\frac{e^{i\arccos(cx)}\sqrt{e}}{\sqrt{-d}c+i\sqrt{dc^2+e}}+1\right)}{2\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d + e*x^2),x]
```

output
```
((a + b*ArcCos[c*x])^2*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I
*Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCos[c*x])^2*Log[1 +
(Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e])])/(2*Sqrt[-d]
*Sqrt[e]) + ((a + b*ArcCos[c*x])^2*Log[1 - (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*
Sqrt[-d] + I*Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCos[c*x]
)^2*Log[1 + (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])
/(2*Sqrt[-d]*Sqrt[e]) + (I*b*(a + b*ArcCos[c*x])*PolyLog[2, -((Sqrt[e]*E^(
I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e]))])/(Sqrt[-d]*Sqrt[e]) - (
I*b*(a + b*ArcCos[c*x])*PolyLog[2, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d]
 - I*Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) + (I*b*(a + b*ArcCos[c*x])*Poly
Log[2, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))])/(
Sqrt[-d]*Sqrt[e]) - (I*b*(a + b*ArcCos[c*x])*PolyLog[2, (Sqrt[e]*E^(I*ArcC
os[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) - (b^2*Pol
yLog[3, -((Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] - I*Sqrt[c^2*d + e]))])/
(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-
d] - I*Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, -((Sqrt[e]*
E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d + e]))])/(Sqrt[-d]*Sqrt[e])
+ (b^2*PolyLog[3, (Sqrt[e]*E^(I*ArcCos[c*x]))/(c*Sqrt[-d] + I*Sqrt[c^2*d +
 e])])/(Sqrt[-d]*Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [F]

$$\int \frac{(a+b\arccos(cx))^2}{e\,x^2+d}dx$$

input
```
int((a+b*arccos(c*x))^2/(e*x^2+d),x)
```

output
```
int((a+b*arccos(c*x))^2/(e*x^2+d),x)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx=\int \frac{(b\arccos{(cx)}+a)^2}{ex^2+d}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/(e*x^2 + d), x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx=\int \frac{(a+b\operatorname{acos}{(cx)})^2}{d+ex^2}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(e*x**2+d),x)
```

output
```
Integral((a + b*acos(c*x))**2/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{e\,x^2+d}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d + e*x^2),x)
```

output
```
int((a + b*acos(c*x))^2/(d + e*x^2), x)
```

Reduce [F]

$$\int \frac{(a+b\arccos(cx))^2}{d+ex^2}\,dx$$
$$= \frac{\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2+2\left(\int \frac{acos(cx)}{e\,x^2+d}dx\right)abde+\left(\int \frac{acos(cx)^2}{e\,x^2+d}dx\right)b^2de}{de}$$

input
```
int((a+b*acos(c*x))^2/(e*x^2+d),x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2 + 2*int(acos(c*x)/(d +
 e*x**2),x)*a*b*d*e + int(acos(c*x)**2/(d + e*x**2),x)*b**2*d*e)/(d*e)
```

3.666 $\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx = \text{Int}\Big(\sqrt{d+ex^2}(a+b\arccos(cx))^2,x\Big)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx = \int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx$$

$$\Big\downarrow \text{5175}$$

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2dx$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sqrt{e\,x^2+d}\,(a+b\arccos(cx))^2\,dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx = \int \sqrt{ex^2+d}(b\arccos{(cx)}+a)^2\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 10.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx = \int (a+b\operatorname{acos}{(cx)})^2\sqrt{d+ex^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*acos(c*x))**2,x)
```

output
```
Integral((a + b*acos(c*x))**2*sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx=\int \sqrt{ex^2+d}(b\arccos{(cx)}+a)^2\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccos(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx=\int (a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((a + b*acos(c*x))^2*(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2*(d + e*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.05

$$\int \sqrt{d+ex^2}(a+b\arccos(cx))^2\,dx$$
$$= \frac{\sqrt{e\,x^2+d}\,a^2ex+\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a^2d+4\left(\int\sqrt{e\,x^2+d}\,acos(cx)\,dx\right)abe+2\left(\int\sqrt{e\,x^2+d}\,acos(cx)^2\right.}{2e}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*acos(c*x))^2,x)
```

output
```
(sqrt(d + e*x**2)*a**2*e*x + sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sq
rt(d))*a**2*d + 4*int(sqrt(d + e*x**2)*acos(c*x),x)*a*b*e + 2*int(sqrt(d +
 e*x**2)*acos(c*x)**2,x)*b**2*e)/(2*e)
```

3.667 $\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^2/(e*x^2+d)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx=\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^2/Sqrt[d + e*x^2], x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{5175}$$

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcCos[c*x])^2/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{e\,x^2+d}}dx$$

input
```
int((a+b*arccos(c*x))^2/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccos(c*x))^2/(e*x^2+d)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\arccos{(cx)}+a)^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/sqrt(e*x^2 + d), x)
```

Sympy [N/A]

Not integrable

Time = 7.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx = \int \frac{(a+b\,\mathrm{acos}\,(cx))^2}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/sqrt(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx = \int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*acos(c*x))^2/(d + e*x^2)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.45

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a^2+2\left(\int \frac{acos(cx)}{\sqrt{e\,x^2+d}}dx\right)abe+\left(\int \frac{acos(cx)^2}{\sqrt{e\,x^2+d}}dx\right)b^2e}{e}$$

input
```
int((a+b*acos(c*x))^2/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a**2 + 2*int(acos(c*x
)/sqrt(d + e*x**2),x)*a*b*e + int(acos(c*x)**2/sqrt(d + e*x**2),x)*b**2*e)
/e
```

3.668 $\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx=\text{Int}\left(\frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^2/(e*x^2+d)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx=\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^2/(d + e*x^2)^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}dx$$
$$\Big\downarrow \text{5175}$$
$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}dx$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arccos(cx))^2}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccos(c*x))^2/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccos(c*x))^2/(e*x^2+d)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(e*x^2 + d)/(e^
2*x^4 + 2*d*e*x^2 + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}\,(cx))^2}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(d + e*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c^2*d>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx = \int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

input
```
int((a + b*acos(c*x))^2/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*acos(c*x))^2/(d + e*x^2)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 222, normalized size of antiderivative = 10.09

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{3/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,a^2ex+\sqrt{e}\,a^2d+\sqrt{e}\,a^2e\,x^2+2\left(\int \frac{acos(cx)}{\sqrt{e\,x^2+d}\,d+\sqrt{e\,x^2+d}\,e\,x^2}dx\right)ab\,d^2e+2}{}$$

input
```
int((a+b*acos(c*x))^2/(e*x^2+d)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*a**2*e*x + sqrt(e)*a**2*d + sqrt(e)*a**2*e*x**2 + 2*int(
acos(c*x)/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*a*b*d**2*e + 2
*int(acos(c*x)/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*a*b*d*e**
2*x**2 + int(acos(c*x)**2/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x
)*b**2*d**2*e + int(acos(c*x)**2/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*
x**2),x)*b**2*d*e**2*x**2)/(d*e*(d + e*x**2))
```

3.669 $\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\,dx=\text{Int}\left(\frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^2/(e*x^2+d)^(5/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\,dx=\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^2/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^2/(d + e*x^2)^(5/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5175}$$

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}dx$$

input
```
Int[(a + b*ArcCos[c*x])^2/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arccos(cx))^2}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccos(c*x))^2/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arccos(c*x))^2/(e*x^2+d)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(e*x^2 + d)/(e^
3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

Sympy [N/A]

Not integrable

Time = 67.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(cx))^2}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2/(e*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*acos(c*x))**2/(d + e*x**2)**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 130, normalized size of antiderivative = 5.91

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a^2*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + integrate(
(b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b*arctan2(sqrt(c*x
 + 1)*sqrt(-c*x + 1), c*x))*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2
*e*x^2 + d^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + b\arccos(cx))^2}{(d + ex^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\arccos(cx))^2}{(d + ex^2)^{5/2}}\,dx = \int \frac{(a + b\,\mathrm{acos}(c\,x))^2}{(e\,x^2 + d)^{5/2}}\,dx$$

input
```
int((a + b*acos(c*x))^2/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*acos(c*x))^2/(d + e*x^2)^(5/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 488, normalized size of antiderivative = 22.18

$$\int \frac{(a+b\arccos(cx))^2}{(d+ex^2)^{5/2}}\,dx = \frac{3\sqrt{e\,x^2+d}\,a^2dex+2\sqrt{e\,x^2+d}\,a^2e^2x^3-2\sqrt{e}\,a^2d^2-4\sqrt{e}\,a^2de\,x^2-2\sqrt{e}\,a^2e^2x}{}$$

input
```
int((a+b*acos(c*x))^2/(e*x^2+d)^(5/2),x)
```

output
```
(3*sqrt(d + e*x**2)*a**2*d*e*x + 2*sqrt(d + e*x**2)*a**2*e**2*x**3 - 2*sqr
t(e)*a**2*d**2 - 4*sqrt(e)*a**2*d*e*x**2 - 2*sqrt(e)*a**2*e**2*x**4 + 6*in
t(acos(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d
+ e*x**2)*e**2*x**4),x)*a*b*d**4*e + 12*int(acos(c*x)/(sqrt(d + e*x**2)*d*
*2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*a*b*d**3
*e**2*x**2 + 6*int(acos(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d
*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*a*b*d**2*e**3*x**4 + 3*int(acos(c
*x)**2/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x
**2)*e**2*x**4),x)*b**2*d**4*e + 6*int(acos(c*x)**2/(sqrt(d + e*x**2)*d**2
 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b**2*d**3*
e**2*x**2 + 3*int(acos(c*x)**2/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)
*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b**2*d**2*e**3*x**4)/(3*d**2*e*
(d**2 + 2*d*e*x**2 + e**2*x**4))
```

3.670 $\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 387

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx = \frac{d^2\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc} + \frac{de\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{2bc^3} + \frac{e^2\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{8bc^5} - \frac{de\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{2bc^3} - \frac{3e^2\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16bc^5} + \frac{e^2\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16bc^5} + \frac{d^2\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc} + \frac{de\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{2bc^3} + \frac{e^2\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8bc^5} - \frac{de\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{2bc^3} - \frac{3e^2\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16bc^5} + \frac{e^2\sin\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16bc^5}$$

output

```
d^2*cos(a/b)*Ci((a+b*arccos(c*x))/b)/b/c+1/2*d*e*cos(a/b)*Ci((a+b*arccos(c
*x))/b)/b/c^3+1/8*e^2*cos(a/b)*Ci((a+b*arccos(c*x))/b)/b/c^5-1/2*d*e*cos(3
*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b/c^3-3/16*e^2*cos(3*a/b)*Ci(3*(a+b*arccos
(c*x))/b)/b/c^5+1/16*e^2*cos(5*a/b)*Ci(5*(a+b*arccos(c*x))/b)/b/c^5+d^2*si
n(a/b)*Si((a+b*arccos(c*x))/b)/b/c+1/2*d*e*sin(a/b)*Si((a+b*arccos(c*x))/b
)/b/c^3+1/8*e^2*sin(a/b)*Si((a+b*arccos(c*x))/b)/b/c^5-1/2*d*e*sin(3*a/b)*
Si(3*(a+b*arccos(c*x))/b)/b/c^3-3/16*e^2*sin(3*a/b)*Si(3*(a+b*arccos(c*x))
/b)/b/c^5+1/16*e^2*sin(5*a/b)*Si(5*(a+b*arccos(c*x))/b)/b/c^5
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx = -\frac{-2(8c^4d^2+4c^2de+e^2)\,\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)-e(8c^2d+3e)\,\text{CosIntegral}\left(3\left(\frac{a}{b}+\text{arccc}\right.\right.}{}$$

input
```
Integrate[(d + e*x^2)^2/(a + b*ArcCos[c*x]),x]
```

output
```
-1/16*(-2*(8*c^4*d^2 + 4*c^2*d*e + e^2)*CosIntegral[a/b + ArcCos[c*x]]*Sin
[a/b] - e*(8*c^2*d + 3*e)*CosIntegral[3*(a/b + ArcCos[c*x])]*Sin[(3*a)/b]
- e^2*CosIntegral[5*(a/b + ArcCos[c*x])]*Sin[(5*a)/b] + 16*c^4*d^2*Cos[a/b
]*SinIntegral[a/b + ArcCos[c*x]] + 8*c^2*d*e*Cos[a/b]*SinIntegral[a/b + Ar
cCos[c*x]] + 2*e^2*Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 8*c^2*d*e*Cos
[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])] + 3*e^2*Cos[(3*a)/b]*SinInteg
ral[3*(a/b + ArcCos[c*x])] + e^2*Cos[(5*a)/b]*SinIntegral[5*(a/b + ArcCos[
c*x])])/(b*c^5)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(\frac{d^2}{a+b\arccos(cx)}+\frac{2dex^2}{a+b\arccos(cx)}+\frac{e^2x^4}{a+b\arccos(cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{8bc^5}+\frac{3e^2\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16bc^5}+\frac{e^2\sin\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16bc^5}-\frac{e^2\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8bc^5}-\frac{3e^2\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16bc^5}-\frac{e^2\cos\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16bc^5}+\frac{de\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{2bc^3}+\frac{de\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{2bc^3}-\frac{de\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{2bc^3}-\frac{de\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{2bc^3}+\frac{d^2\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}-\frac{d^2\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}$$

input
```
Int[(d + e*x^2)^2/(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/(b*c) + (d*e*CosIntegral
[(a + b*ArcCos[c*x])/b]*Sin[a/b])/(2*b*c^3) + (e^2*CosIntegral[(a + b*ArcC
os[c*x])/b]*Sin[a/b])/(8*b*c^5) + (d*e*CosIntegral[(3*(a + b*ArcCos[c*x]))
/b]*Sin[(3*a)/b])/(2*b*c^3) + (3*e^2*CosIntegral[(3*(a + b*ArcCos[c*x]))/b
]*Sin[(3*a)/b])/(16*b*c^5) + (e^2*CosIntegral[(5*(a + b*ArcCos[c*x]))/b]*S
in[(5*a)/b])/(16*b*c^5) - (d^2*Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b]
)/(b*c) - (d*e*Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/(2*b*c^3) - (e
^2*Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/(8*b*c^5) - (d*e*Cos[(3*a)
/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/(2*b*c^3) - (3*e^2*Cos[(3*a)/b
]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/(16*b*c^5) - (e^2*Cos[(5*a)/b]*S
inIntegral[(5*(a + b*ArcCos[c*x]))/b])/(16*b*c^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{16\,\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)c^4d^2-16\,\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)c^4d^2+8\,\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)c^2de-8\,\mathrm{Ci}\left(\arccos\right.}{}$
default	$-\frac{16\,\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)c^4d^2-16\,\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)c^4d^2+8\,\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)c^2de-8\,\mathrm{Ci}\left(\arccos\right.}{}$

input
```
int((e*x^2+d)^2/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/16/c^5*(16*Si(arccos(c*x)+a/b)*cos(a/b)*c^4*d^2-16*Ci(arccos(c*x)+a/b)*
sin(a/b)*c^4*d^2+8*Si(arccos(c*x)+a/b)*cos(a/b)*c^2*d*e-8*Ci(arccos(c*x)+a
/b)*sin(a/b)*c^2*d*e+8*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)*c^2*d*e-8*Ci(3*a
rccos(c*x)+3*a/b)*sin(3*a/b)*c^2*d*e+2*Si(arccos(c*x)+a/b)*cos(a/b)*e^2-2*
Ci(arccos(c*x)+a/b)*sin(a/b)*e^2+3*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)*e^2-
3*Ci(3*arccos(c*x)+3*a/b)*sin(3*a/b)*e^2+Si(5*arccos(c*x)+5*a/b)*cos(5*a/b
)*e^2-Ci(5*arccos(c*x)+5*a/b)*sin(5*a/b)*e^2)/b
```

Fricas [F]

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx = \int \frac{(ex^2+d)^2}{b\arccos(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx = \int \frac{(d+ex^2)^2}{a+b\,\mathrm{acos}\,(cx)}\,dx$$

input
```
integrate((e*x**2+d)**2/(a+b*acos(c*x)),x)
```

output
```
Integral((d + e*x**2)**2/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx = \int \frac{(ex^2+d)^2}{b\arccos(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
e^2*cos(a/b)^4*cos_integral(5*a/b + 5*arccos(c*x))*sin(a/b)/(b*c^5) + 2*d*
e*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c^3) + d^2*co
s_integral(a/b + arccos(c*x))*sin(a/b)/(b*c) - e^2*cos(a/b)^5*sin_integral
(5*a/b + 5*arccos(c*x))/(b*c^5) - 2*d*e*cos(a/b)^3*sin_integral(3*a/b + 3*
arccos(c*x))/(b*c^3) - d^2*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b*c)
- 3/4*e^2*cos(a/b)^2*cos_integral(5*a/b + 5*arccos(c*x))*sin(a/b)/(b*c^5)
- 1/2*d*e*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c^3) + 3/4*e^2*c
os(a/b)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c^5) + 1/2*d*e*c
os_integral(a/b + arccos(c*x))*sin(a/b)/(b*c^3) + 5/4*e^2*cos(a/b)^3*sin_i
ntegral(5*a/b + 5*arccos(c*x))/(b*c^5) + 3/2*d*e*cos(a/b)*sin_integral(3*a
/b + 3*arccos(c*x))/(b*c^3) - 3/4*e^2*cos(a/b)^3*sin_integral(3*a/b + 3*ar
ccos(c*x))/(b*c^5) - 1/2*d*e*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b*c
^3) + 1/16*e^2*cos_integral(5*a/b + 5*arccos(c*x))*sin(a/b)/(b*c^5) - 3/16
*e^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c^5) + 1/8*e^2*cos_in
tegral(a/b + arccos(c*x))*sin(a/b)/(b*c^5) - 5/16*e^2*cos(a/b)*sin_integra
l(5*a/b + 5*arccos(c*x))/(b*c^5) + 9/16*e^2*cos(a/b)*sin_integral(3*a/b +
3*arccos(c*x))/(b*c^5) - 1/8*e^2*cos(a/b)*sin_integral(a/b + arccos(c*x))/
(b*c^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx=\int \frac{{\left(e\,x^2+d\right)}^2}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((d + e*x^2)^2/(a + b*acos(c*x)),x)
```

output
```
int((d + e*x^2)^2/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2}{a+b\arccos(cx)}\,dx = \left(\int \frac{x^4}{acos\,(cx)\,b+a}dx\right)e^2 + 2\left(\int \frac{x^2}{acos\,(cx)\,b+a}dx\right)de + \left(\int \frac{1}{acos\,(cx)\,b+a}dx\right)d^2$$

input
```
int((e*x^2+d)^2/(a+b*acos(c*x)),x)
```

output
```
int(x**4/(acos(c*x)*b + a),x)*e**2 + 2*int(x**2/(acos(c*x)*b + a),x)*d*e +
 int(1/(acos(c*x)*b + a),x)*d**2
```

3.671 $\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 179

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx = \frac{d\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc} + \frac{e\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4bc^3} - \frac{e\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4bc^3} + \frac{d\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc} + \frac{e\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4bc^3} - \frac{e\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4bc^3}$$

output
```
d*cos(a/b)*Ci((a+b*arccos(c*x))/b)/b/c+1/4*e*cos(a/b)*Ci((a+b*arccos(c*x))
/b)/b/c^3-1/4*e*cos(3*a/b)*Ci(3*(a+b*arccos(c*x))/b)/b/c^3+d*sin(a/b)*Si((
a+b*arccos(c*x))/b)/b/c+1/4*e*sin(a/b)*Si((a+b*arccos(c*x))/b)/b/c^3-1/4*e
*sin(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b/c^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx = \frac{(4c^2d+e)\,\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)+e\,\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{3a}{b}\right)-4c^2d\cos}{4bc^3}$$

input
```
Integrate[(d + e*x^2)/(a + b*ArcCos[c*x]),x]
```

output
```
((4*c^2*d + e)*CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b] + e*CosIntegral[3*(
a/b + ArcCos[c*x])]*Sin[(3*a)/b] - 4*c^2*d*Cos[a/b]*SinIntegral[a/b + ArcC
os[c*x]] - e*Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]] - e*Cos[(3*a)/b]*SinI
ntegral[3*(a/b + ArcCos[c*x])])/(4*b*c^3)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(\frac{d}{a+b\arccos(cx)}+\frac{ex^2}{a+b\arccos(cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4bc^3}+\frac{e\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4bc^3}-\frac{e\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4bc^3}-\frac{e\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4bc^3}+\frac{d\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}-\frac{d\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}$$

input
```
Int[(d + e*x^2)/(a + b*ArcCos[c*x]),x]
```

output
```
(d*CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/(b*c) + (e*CosIntegral[(a
+ b*ArcCos[c*x])/b]*Sin[a/b])/(4*b*c^3) + (e*CosIntegral[(3*(a + b*ArcCos[
c*x]))/b]*Sin[(3*a)/b])/(4*b*c^3) - (d*Cos[a/b]*SinIntegral[(a + b*ArcCos[
c*x])/b])/(b*c) - (e*Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/(4*b*c^3
) - (e*Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/(4*b*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{-\frac{d\left(\text{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)-\text{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\right)}{b}-\frac{e\left(\text{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)-\text{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)\right)}{4c^2b}}{c}$
default	$\frac{-\frac{d\left(\text{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)-\text{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\right)}{b}-\frac{e\left(\text{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)-\text{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)\right)}{4c^2b}}{c}$

input
```
int((e*x^2+d)/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-d*(Si(arccos(c*x)+a/b)*cos(a/b)-Ci(arccos(c*x)+a/b)*sin(a/b))/b-1/4*
e/c^2*(Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)-Ci(3*arccos(c*x)+3*a/b)*sin(3*a/
b))/b-1/4*e/c^2*(Si(arccos(c*x)+a/b)*cos(a/b)-Ci(arccos(c*x)+a/b)*sin(a/b)
)/b)
```

Fricas [F]

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx = \int \frac{ex^2+d}{b\arccos(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx = \int \frac{d+ex^2}{a+b\operatorname{acos}(cx)}\,dx$$

input
```
integrate((e*x**2+d)/(a+b*acos(c*x)),x)
```

output
```
Integral((d + e*x**2)/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx = \int \frac{ex^2+d}{b\arccos(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx &= \frac{e\cos\left(\frac{a}{b}\right)^2\operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{bc^3} \\ &+ \frac{d\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{bc} \\ &- \frac{e\cos\left(\frac{a}{b}\right)^3\operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{bc^3} \\ &- \frac{d\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{bc} \\ &- \frac{e\operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{4bc^3} \\ &+ \frac{e\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{4bc^3} \\ &+ \frac{3e\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{4bc^3} \\ &- \frac{e\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{4bc^3}\end{aligned}$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
e*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c^3) + d*cos_
integral(a/b + arccos(c*x))*sin(a/b)/(b*c) - e*cos(a/b)^3*sin_integral(3*a
/b + 3*arccos(c*x))/(b*c^3) - d*cos(a/b)*sin_integral(a/b + arccos(c*x))/(
b*c) - 1/4*e*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c^3) + 1/4*e*
cos_integral(a/b + arccos(c*x))*sin(a/b)/(b*c^3) + 3/4*e*cos(a/b)*sin_inte
gral(3*a/b + 3*arccos(c*x))/(b*c^3) - 1/4*e*cos(a/b)*sin_integral(a/b + ar
ccos(c*x))/(b*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx=\int \frac{e\,x^2+d}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((d + e*x^2)/(a + b*acos(c*x)),x)
```

output
```
int((d + e*x^2)/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{a+b\arccos(cx)}\,dx=\left(\int \frac{x^2}{acos\left(cx\right)b+a}dx\right)e+\left(\int \frac{1}{acos\left(cx\right)b+a}dx\right)d$$

input
```
int((e*x^2+d)/(a+b*acos(c*x)),x)
```

output
```
int(x**2/(acos(c*x)*b + a),x)*e + int(1/(acos(c*x)*b + a),x)*d
```

3.672 $\int \frac{1}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \frac{1}{a+b\arccos(cx)}\,dx = \frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc} + \frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}$$

output
```
cos(a/b)*Ci((a+b*arccos(c*x))/b)/b/c+sin(a/b)*Si((a+b*arccos(c*x))/b)/b/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{1}{a+b\arccos(cx)}\,dx = -\frac{-\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)+\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{bc}$$

input
```
Integrate[(a + b*ArcCos[c*x])^(-1),x]
```

output
```
-((-(CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b]) + Cos[a/b]*SinIntegral[a/b + 
 ArcCos[c*x]])/(b*c))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5135, 25, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\arccos(cx)}dx$$

$$\Big\downarrow \text{5135}$$

$$-\frac{\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\cos\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)-\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}$$

input
```
Int[(a + b*ArcCos[c*x])^(-1),x]
```

output
```
-((-(CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b]) + Cos[a/b]*SinIntegral[(
a + b*ArcCos[c*x])/b])/(b*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{-\frac{\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b}+\frac{\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)}{b}}{c}$	49
default	$\frac{-\frac{\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b}+\frac{\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)}{b}}{c}$	49

input
```
int(1/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-Si(arccos(c*x)+a/b)*cos(a/b)/b+Ci(arccos(c*x)+a/b)*sin(a/b)/b)
```

Fricas [F]

$$\int \frac{1}{a+b\arccos(cx)}\,dx=\int \frac{1}{b\arccos\left(cx\right)+a}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{1}{a+b\arccos(cx)}\,dx = \int \frac{1}{a+b\,\mathrm{acos}\,(cx)}\,dx$$

input
```
integrate(1/(a+b*acos(c*x)),x)
```

output
```
Integral(1/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{1}{a+b\arccos(cx)}\,dx = \int \frac{1}{b\arccos(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+b\arccos(cx)}\,dx = \frac{\mathrm{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{bc} - \frac{\cos\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{bc}$$

input
```
integrate(1/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
cos_integral(a/b + arccos(c*x))*sin(a/b)/(b*c) - cos(a/b)*sin_integral(a/b
 + arccos(c*x))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\arccos(cx)}\,dx=\int \frac{1}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int(1/(a + b*acos(c*x)),x)
```

output
```
int(1/(a + b*acos(c*x)), x)
```

Reduce [F]

$$\int \frac{1}{a+b\arccos(cx)}\,dx=\int \frac{1}{acos\left(cx\right)b+a}dx$$

input
```
int(1/(a+b*acos(c*x)),x)
```

output
```
int(1/(acos(c*x)*b + a),x)
```

3.673 $\int \frac{1}{(d+ex^2)(a+b\arccos(cx))} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))} dx = \text{Int}\left(\frac{1}{(d+ex^2)(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))} dx = \int \frac{1}{(d+ex^2)(a+b\arccos(cx))} dx$$

input
```
Integrate[1/((d + e*x^2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{5175}$$
$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))}dx$$

input
```
Int[1/((d + e*x^2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)\,(a+b\arccos(cx))}dx$$

input
```
int(1/(e*x^2+d)/(a+b*arccos(c*x)),x)
```

output
```
int(1/(e*x^2+d)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(ex^2+d)(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 3.65 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(cx))\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/((a + b*acos(c*x))*(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))}\,dx = \int \frac{1}{(ex^2+d)(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))}\,dx=\int \frac{1}{(ex^2+d)(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))}\,dx=\int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\,(e\,x^2+d)}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d + e*x^2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(d + e*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))}\,dx = \int \frac{1}{acos\,(cx)\,bd + acos\,(cx)\,be\,x^2 + ad + ae\,x^2}dx$$

input
```
int(1/(e*x^2+d)/(a+b*acos(c*x)),x)
```

output
```
int(1/(acos(c*x)*b*d + acos(c*x)*b*e*x**2 + a*d + a*e*x**2),x)
```

3.674 $\int \frac{1}{(d+ex^2)^2(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^2 (a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^2/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))}\,dx = \int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))} dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))} dx$$

input
```
Int[1/((d + e*x^2)^2*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)^2 (a+b\arccos(cx))} dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*arccos(c*x)),x)
```

output
```
int(1/(e*x^2+d)^2/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))} \, dx = \int \frac{1}{(ex^2+d)^2 (b\arccos(cx)+a)} \, dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b
*d^2)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 109.96 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))} \, dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(cx)) (d+ex^2)^2} \, dx$$

input
```
integrate(1/(e*x**2+d)**2/(a+b*acos(c*x)),x)
```

output
```
Integral(1/((a + b*acos(c*x))*(d + e*x**2)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))} \, dx = \int \frac{1}{(ex^2+d)^2 (b\arccos(cx)+a)} \, dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 6.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))} dx = \int \frac{1}{(ex^2+d)^2(b\arccos(cx)+a)} dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))} dx = \int \frac{1}{(a+b\,\mathrm{acos}(c\,x))\,(e\,x^2+d)^2} dx$$

input
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^2),x)
```

output
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.95

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))} dx$$
$$= \int \frac{1}{acos(cx)\, b\, d^2 + 2acos(cx)\, bde\, x^2 + acos(cx)\, b\, e^2x^4 + a\, d^2 + 2ade\, x^2 + a\, e^2x^4} dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*acos(c*x)),x)
```

output
```
int(1/(acos(c*x)*b*d**2 + 2*acos(c*x)*b*d*e*x**2 + acos(c*x)*b*e**2*x**4 +
 a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4),x)
```

3.675 $\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}}{a+b\arccos(cx)},x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcCos[c*x]),x]
```

output
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcCos[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{ 5175}$$

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}dx$$

input
```
Int[Sqrt[d + e*x^2]/(a + b*ArcCos[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^2+d}}{a+b\arccos\left(cx\right)}dx$$

input
```
int((e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x)
```

output
```
int((e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{ex^2+d}}{b\arccos\left(cx\right)+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(b*arccos(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{d+ex^2}}{a+b\,\mathrm{acos}\left(cx\right)}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(sqrt(d + e*x**2)/(a + b*acos(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{ex^2+d}}{b\arccos\left(cx\right)+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)/(b*arccos(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{ex^2+d}}{b\arccos{(cx)}+a}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)/(b*arccos(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{e\,x^2+d}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a + b*acos(c*x)),x)
```

output
```
int((d + e*x^2)^(1/2)/(a + b*acos(c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 200.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{e\,x^2+d}}{acos\,(cx)\,b+a}dx$$

input
```
int((e*x^2+d)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int((e*x^2+d)^(1/2)/(a+b*acos(c*x)),x)
```

3.676 $\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}dx$$

input
```
Int[1/(Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{e\,x^{2}+d}\,\left(a+b\arccos\left(cx\right)\right)}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(cx))\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/((a + b*acos(c*x))*sqrt(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(e*x^2 + d)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx=\int \frac{1}{\sqrt{ex^2+d}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*x^2 + d)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx=\int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,acos\,(cx)\,b+\sqrt{e\,x^2+d}\,a}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(a+b*acos(c*x)),x)
```

output
```
int(1/(sqrt(d + e*x**2)*acos(c*x)*b + sqrt(d + e*x**2)*a),x)
```

3.677 $\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(d+ex^2\right)^{3/2} (a+b\arccos(cx))} \, dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{1}{\left(d+ex^2\right)^{3/2} (a+b\arccos(cx))}dx$$

input
```
Int[1/((d + e*x^2)^(3/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\arccos\left(c x\right)\right)}d x$$

input
```
int(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2
*b*d*e*x^2 + b*d^2)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\operatorname{acos}(cx))(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/((a + b*acos(c*x))*(d + e*x**2)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^(3/2)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(3/2)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}(c\,x))\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{1}{(d+ex^2)^{3/2}\,(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,acos\,(cx)\,bd+\sqrt{e\,x^2+d}\,acos\,(cx)\,be\,x^2+\sqrt{e\,x^2+d}\,a}$$

input
```
int(1/(e*x^2+d)^(3/2)/(a+b*acos(c*x)),x)
```

output
```
int(1/(sqrt(d + e*x**2)*acos(c*x)*b*d + sqrt(d + e*x**2)*acos(c*x)*b*e*x**
2 + sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e*x**2),x)
```

3.678 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCos[c*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{5175}$$
$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}dx$$

input
```
Int[1/((d + e*x^2)^(5/2)*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{5}{2}}(a+b\arccos(cx))}dx$$

input
```
int(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x)),x)
```

output
```
int(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.95

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^
3 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 9.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\operatorname{acos}(cx))(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(5/2)/(a+b*acos(c*x)),x)
```

output
```
Integral(1/((a + b*acos(c*x))*(d + e*x**2)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^(5/2)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\arccos(cx))}\,dx=\int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(5/2)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\arccos(cx))}\,dx=\int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*acos(c*x))*(d + e*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 107, normalized size of antiderivative = 4.86

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,acos\,(cx)\,b\,d^2+2\sqrt{e\,x^2+d}\,acos\,(cx)\,bde\,x^2+\sqrt{e\,x^2+}$$

input

```
int(1/(e*x^2+d)^(5/2)/(a+b*acos(c*x)),x)
```

output

```
int(1/(sqrt(d + e*x**2)*acos(c*x)*b*d**2 + 2*sqrt(d + e*x**2)*acos(c*x)*b*
d*e*x**2 + sqrt(d + e*x**2)*acos(c*x)*b*e**2*x**4 + sqrt(d + e*x**2)*a*d**
2 + 2*sqrt(d + e*x**2)*a*d*e*x**2 + sqrt(d + e*x**2)*a*e**2*x**4),x)
```

3.679 $\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 498

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx = -\frac{d^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} - \frac{2dex^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} - \frac{e^2x^4\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} + \frac{d^2\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{b^2c} + \frac{de\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{2b^2c^3} + \frac{e^2\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{8b^2c^5} - \frac{3de\,\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{2b^2c^3} - \frac{9e^2\,\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{16b^2c^5} + \frac{5e^2\,\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)\sin\left(\frac{5a}{b}\right)}{16b^2c^5} - \frac{d^2\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c} - \frac{de\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{2b^2c^3} - \frac{e^2\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2c^5} + \frac{3de\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{2b^2c^3} + \frac{9e^2\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2c^5} - \frac{5e^2\cos\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2c^5}$$

output
```
-d^2*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))-2*d*e*x^2*(-c^2*x^2+1)^(1/2)
/b/c/(a+b*arccos(c*x))-e^2*x^4*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))+d^
2*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b^2/c+1/2*d*e*Ci((a+b*arccos(c*x))/b)*s
in(a/b)/b^2/c^3+1/8*e^2*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b^2/c^5-3/2*d*e*C
i(3*(a+b*arccos(c*x))/b)*sin(3*a/b)/b^2/c^3-9/16*e^2*Ci(3*(a+b*arccos(c*x)
)/b)*sin(3*a/b)/b^2/c^5+5/16*e^2*Ci(5*(a+b*arccos(c*x))/b)*sin(5*a/b)/b^2/
c^5-d^2*cos(a/b)*Si((a+b*arccos(c*x))/b)/b^2/c-1/2*d*e*cos(a/b)*Si((a+b*ar
ccos(c*x))/b)/b^2/c^3-1/8*e^2*cos(a/b)*Si((a+b*arccos(c*x))/b)/b^2/c^5+3/2
*d*e*cos(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^2/c^3+9/16*e^2*cos(3*a/b)*Si(3
*(a+b*arccos(c*x))/b)/b^2/c^5-5/16*e^2*cos(5*a/b)*Si(5*(a+b*arccos(c*x))/b
)/b^2/c^5
```

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx = -\frac{-\frac{16bc^4d^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)} - \frac{32bc^4dex^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)} - \frac{16bc^4e^2x^4\sqrt{1-c^2x^2}}{a+b\arccos(cx)} + 2(8c^4d^2+4c^2de+e^2)\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\right.}{}$$

input
```
Integrate[(d + e*x^2)^2/(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/16*((-16*b*c^4*d^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) - (32*b*c^4*d
*e*x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) - (16*b*c^4*e^2*x^4*Sqrt[1 -
 c^2*x^2])/(a + b*ArcCos[c*x]) + 2*(8*c^4*d^2 + 4*c^2*d*e + e^2)*Cos[a/b]*
CosIntegral[a/b + ArcCos[c*x]] + 3*e*(8*c^2*d + 3*e)*Cos[(3*a)/b]*CosInteg
ral[3*(a/b + ArcCos[c*x])] + 5*e^2*Cos[(5*a)/b]*CosIntegral[5*(a/b + ArcCo
s[c*x])] + 16*c^4*d^2*Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 8*c^2*d*e*
Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 2*e^2*Sin[a/b]*SinIntegral[a/b +
 ArcCos[c*x]] + 24*c^2*d*e*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])]
 + 9*e^2*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])] + 5*e^2*Sin[(5*a)
/b]*SinIntegral[5*(a/b + ArcCos[c*x])])/(b^2*c^5)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(\frac{d^2}{(a+b\arccos(cx))^2}+\frac{2dex^2}{(a+b\arccos(cx))^2}+\frac{e^2x^4}{(a+b\arccos(cx))^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^2\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2c^5}-\frac{9e^2\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2c^5}-\frac{5e^2\cos\left(\frac{5a}{b}\right)\text{CosIntegral}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2c^5}-\frac{e^2\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^2c^5}-\frac{9e^2\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{16b^2c^5}-\frac{5e^2\sin\left(\frac{5a}{b}\right)\text{Si}\left(\frac{5(a+b\arccos(cx))}{b}\right)}{16b^2c^5}-\frac{de\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{2b^2c^3}-\frac{3de\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{2b^2c^3}-\frac{de\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{2b^2c^3}-\frac{3de\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{2b^2c^3}-\frac{d^2\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}-\frac{d^2\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}+\frac{d^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}+\frac{2dex^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}+\frac{e^2x^4\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

input
```
Int[(d + e*x^2)^2/(a + b*ArcCos[c*x])^2,x]
```

output
```
(d^2*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCos[c*x])) + (2*d*e*x^2*Sqrt[1 - c^
2*x^2])/(b*c*(a + b*ArcCos[c*x])) + (e^2*x^4*Sqrt[1 - c^2*x^2])/(b*c*(a +
b*ArcCos[c*x])) - (d^2*Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/(b^2*c
) - (d*e*Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/(2*b^2*c^3) - (e^2*C
os[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/(8*b^2*c^5) - (3*d*e*Cos[(3*a)
/b]*CosIntegral[(3*(a + b*ArcCos[c*x]))/b])/(2*b^2*c^3) - (9*e^2*Cos[(3*a)
/b]*CosIntegral[(3*(a + b*ArcCos[c*x]))/b])/(16*b^2*c^5) - (5*e^2*Cos[(5*a
)/b]*CosIntegral[(5*(a + b*ArcCos[c*x]))/b])/(16*b^2*c^5) - (d^2*Sin[a/b]*
SinIntegral[(a + b*ArcCos[c*x])/b])/(b^2*c) - (d*e*Sin[a/b]*SinIntegral[(a
 + b*ArcCos[c*x])/b])/(2*b^2*c^3) - (e^2*Sin[a/b]*SinIntegral[(a + b*ArcCo
s[c*x])/b])/(8*b^2*c^5) - (3*d*e*Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos
[c*x]))/b])/(2*b^2*c^3) - (9*e^2*Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos
[c*x]))/b])/(16*b^2*c^5) - (5*e^2*Sin[(5*a)/b]*SinIntegral[(5*(a + b*ArcCo
s[c*x]))/b])/(16*b^2*c^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 796, normalized size of antiderivative = 1.60

method	result	size
derivativedivides	Expression too large to display	796
default	Expression too large to display	796

input
```
int((e*x^2+d)^2/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16/c^5*(-2*(-c^2*x^2+1)^(1/2)*b*e^2-8*sin(3*arccos(c*x))*b*c^2*d*e+9*ar
ccos(c*x)*Si(3*arccos(c*x)+3*a/b)*sin(3*a/b)*b*e^2+9*arccos(c*x)*Ci(3*arcc
os(c*x)+3*a/b)*cos(3*a/b)*b*e^2+2*arccos(c*x)*Si(arccos(c*x)+a/b)*sin(a/b)
*b*e^2+2*arccos(c*x)*Ci(arccos(c*x)+a/b)*cos(a/b)*b*e^2+5*arccos(c*x)*Si(5
*arccos(c*x)+5*a/b)*sin(5*a/b)*b*e^2+5*arccos(c*x)*Ci(5*arccos(c*x)+5*a/b)
*cos(5*a/b)*b*e^2+16*Si(arccos(c*x)+a/b)*sin(a/b)*a*c^4*d^2+16*Ci(arccos(c
*x)+a/b)*cos(a/b)*a*c^4*d^2-8*(-c^2*x^2+1)^(1/2)*b*c^2*d*e+16*arccos(c*x)*
Si(arccos(c*x)+a/b)*sin(a/b)*b*c^4*d^2+16*arccos(c*x)*Ci(arccos(c*x)+a/b)*
cos(a/b)*b*c^4*d^2+24*Si(3*arccos(c*x)+3*a/b)*sin(3*a/b)*a*c^2*d*e+24*Ci(3
*arccos(c*x)+3*a/b)*cos(3*a/b)*a*c^2*d*e+8*Si(arccos(c*x)+a/b)*sin(a/b)*a*
c^2*d*e+8*Ci(arccos(c*x)+a/b)*cos(a/b)*a*c^2*d*e+24*arccos(c*x)*Si(3*arcco
s(c*x)+3*a/b)*sin(3*a/b)*b*c^2*d*e+24*arccos(c*x)*Ci(3*arccos(c*x)+3*a/b)*
cos(3*a/b)*b*c^2*d*e+8*arccos(c*x)*Si(arccos(c*x)+a/b)*sin(a/b)*b*c^2*d*e+
8*arccos(c*x)*Ci(arccos(c*x)+a/b)*cos(a/b)*b*c^2*d*e-sin(5*arccos(c*x))*b*
e^2-3*sin(3*arccos(c*x))*b*e^2-16*(-c^2*x^2+1)^(1/2)*b*c^4*d^2+9*Si(3*arcc
os(c*x)+3*a/b)*sin(3*a/b)*a*e^2+9*Ci(3*arccos(c*x)+3*a/b)*cos(3*a/b)*a*e^2
+2*Si(arccos(c*x)+a/b)*sin(a/b)*a*e^2+2*Ci(arccos(c*x)+a/b)*cos(a/b)*a*e^2
+5*Si(5*arccos(c*x)+5*a/b)*sin(5*a/b)*a*e^2+5*Ci(5*arccos(c*x)+5*a/b)*cos(
5*a/b)*a*e^2)/(a+b*arccos(c*x))/b^2
```

Fricas [F]

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{(ex^2+d)^2}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x
) + a^2), x)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{(d+ex^2)^2}{(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate((e*x**2+d)**2/(a+b*acos(c*x))**2,x)
```

output
```
Integral((d + e*x**2)**2/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{(ex^2+d)^2}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^2*c*arctan2
(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*integrate((5*c^2*e^2*x^5 + 2*
(3*c^2*d*e - 2*e^2)*x^3 + (c^2*d^2 - 4*d*e)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1
)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(
-c*x + 1), c*x)), x))/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) +
a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2213 vs. $2(472) = 944$.

Time = 0.22 (sec) , antiderivative size = 2213, normalized size of antiderivative = 4.44

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
sqrt(-c^2*x^2 + 1)*b*c^4*e^2*x^4/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - 5*b*e
^2*arccos(c*x)*cos(a/b)^5*cos_integral(5*a/b + 5*arccos(c*x))/(b^3*c^5*arc
cos(c*x) + a*b^2*c^5) - 6*b*c^2*d*e*arccos(c*x)*cos(a/b)^3*cos_integral(3*
a/b + 3*arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - b*c^4*d^2*arccos(
c*x)*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2
*c^5) - 5*b*e^2*arccos(c*x)*cos(a/b)^4*sin(a/b)*sin_integral(5*a/b + 5*arc
cos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - 6*b*c^2*d*e*arccos(c*x)*cos(
a/b)^2*sin(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^5*arccos(c*x) +
 a*b^2*c^5) - b*c^4*d^2*arccos(c*x)*sin(a/b)*sin_integral(a/b + arccos(c*x
))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) + 2*sqrt(-c^2*x^2 + 1)*b*c^4*d*e*x^2/
(b^3*c^5*arccos(c*x) + a*b^2*c^5) - 5*a*e^2*cos(a/b)^5*cos_integral(5*a/b
+ 5*arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - 6*a*c^2*d*e*cos(a/b)^
3*cos_integral(3*a/b + 3*arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) -
a*c^4*d^2*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b^3*c^5*arccos(c*x) +
a*b^2*c^5) - 5*a*e^2*cos(a/b)^4*sin(a/b)*sin_integral(5*a/b + 5*arccos(c*x
))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - 6*a*c^2*d*e*cos(a/b)^2*sin(a/b)*sin
_integral(3*a/b + 3*arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) - a*c^4
*d^2*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c^5*arccos(c*x) + a*b^2
*c^5) + 25/4*b*e^2*arccos(c*x)*cos(a/b)^3*cos_integral(5*a/b + 5*arccos(c*
x))/(b^3*c^5*arccos(c*x) + a*b^2*c^5) + 9/2*b*c^2*d*e*arccos(c*x)*cos(a...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx=\int \frac{(e\,x^2+d)^2}{(a+b\,\mathrm{acos}\,(c\,x))^2}\,dx$$

input
```
int((d + e*x^2)^2/(a + b*acos(c*x))^2,x)
```

output
```
int((d + e*x^2)^2/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2}{(a+b\arccos(cx))^2}\,dx = \left(\int \frac{x^4}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)e^2 + 2\left(\int \frac{x^2}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)de + \left(\int \frac{1}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)d^2$$

input
```
int((e*x^2+d)^2/(a+b*acos(c*x))^2,x)
```

output
```
int(x**4/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*e**2 + 2*int(x**2
/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*d*e + int(1/(acos(c*x)**2
*b**2 + 2*acos(c*x)*a*b + a**2),x)*d**2
```

3.680 $\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 249

$$\begin{aligned}\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx = &-\frac{d\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{ex^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}\\&+\frac{d\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{b^2c}\\&+\frac{e\,\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{4b^2c^3}\\&-\frac{3e\,\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{4b^2c^3}\\&-\frac{d\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}-\frac{e\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^3}\\&+\frac{3e\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^3}\end{aligned}$$

output

```
-d*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))-e*x^2*(-c^2*x^2+1)^(1/2)/b/c/(
a+b*arccos(c*x))+d*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b^2/c+1/4*e*Ci((a+b*ar
ccos(c*x))/b)*sin(a/b)/b^2/c^3-3/4*e*Ci(3*(a+b*arccos(c*x))/b)*sin(3*a/b)/
b^2/c^3-d*cos(a/b)*Si((a+b*arccos(c*x))/b)/b^2/c-1/4*e*cos(a/b)*Si((a+b*ar
ccos(c*x))/b)/b^2/c^3+3/4*e*cos(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^2/c^3
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.76

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx = -\frac{-\frac{4bc^2d\sqrt{1-c^2x^2}}{a+b\arccos(cx)} - \frac{4bc^2ex^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)} + (4c^2d+e)\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right) + 3e\cos\left(\frac{3a}{b}\right)\text{CosInte}$$

input
```
Integrate[(d + e*x^2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
-1/4*((-4*b*c^2*d*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) - (4*b*c^2*e*x^2*
Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) + (4*c^2*d + e)*Cos[a/b]*CosIntegra
l[a/b + ArcCos[c*x]] + 3*e*Cos[(3*a)/b]*CosIntegral[3*(a/b + ArcCos[c*x])]
 + 4*c^2*d*Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] + e*Sin[a/b]*SinIntegra
l[a/b + ArcCos[c*x]] + 3*e*Sin[(3*a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])]
)/(b^2*c^3)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(\frac{d}{(a+b\arccos(cx))^2}+\frac{ex^2}{(a+b\arccos(cx))^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^3}-\frac{3e\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^3}-\frac{e\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^3}-\frac{3e\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^3}-\frac{d\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}-\frac{d\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}+\frac{d\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}+\frac{ex^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

input
```
Int[(d + e*x^2)/(a + b*ArcCos[c*x])^2,x]
```

output
```
(d*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCos[c*x])) + (e*x^2*Sqrt[1 - c^2*x^2]
)/(b*c*(a + b*ArcCos[c*x])) - (d*Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/
b])/(b^2*c) - (e*Cos[a/b]*CosIntegral[(a + b*ArcCos[c*x])/b])/(4*b^2*c^3)
 - (3*e*Cos[(3*a)/b]*CosIntegral[(3*(a + b*ArcCos[c*x]))/b])/(4*b^2*c^3) -
(d*Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/(b^2*c) - (e*Sin[a/b]*SinI
ntegral[(a + b*ArcCos[c*x])/b])/(4*b^2*c^3) - (3*e*Sin[(3*a)/b]*SinIntegra
l[(3*(a + b*ArcCos[c*x]))/b])/(4*b^2*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.51

method	result
derivativedivides	$-\frac{d\left(\arccos(cx)\,\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)b+\arccos(cx)\,\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)b+\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)a+\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right.}{(a+b\arccos(cx))b^2}$
default	$-\frac{d\left(\arccos(cx)\,\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)b+\arccos(cx)\,\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)b+\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)a+\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right.}{(a+b\arccos(cx))b^2}$

input
```
int((e*x^2+d)/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-d*(arccos(c*x)*Si(arccos(c*x)+a/b)*sin(a/b)*b+arccos(c*x)*Ci(arccos(
c*x)+a/b)*cos(a/b)*b+Si(arccos(c*x)+a/b)*sin(a/b)*a+Ci(arccos(c*x)+a/b)*co
s(a/b)*a-(-c^2*x^2+1)^(1/2)*b)/(a+b*arccos(c*x))/b^2-1/4*e/c^2*(3*arccos(c
*x)*cos(3*a/b)*Ci(3*arccos(c*x)+3*a/b)*b+3*arccos(c*x)*sin(3*a/b)*Si(3*arc
cos(c*x)+3*a/b)*b+3*cos(3*a/b)*Ci(3*arccos(c*x)+3*a/b)*a+3*sin(3*a/b)*Si(3
*arccos(c*x)+3*a/b)*a-sin(3*arccos(c*x))*b)/(a+b*arccos(c*x))/b^2-1/4*e/c^
2*(arccos(c*x)*Si(arccos(c*x)+a/b)*sin(a/b)*b+arccos(c*x)*Ci(arccos(c*x)+a
/b)*cos(a/b)*b+Si(arccos(c*x)+a/b)*sin(a/b)*a+Ci(arccos(c*x)+a/b)*cos(a/b)
*a-(-c^2*x^2+1)^(1/2)*b)/(a+b*arccos(c*x))/b^2)
```

Fricas [F]

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx=\int \frac{ex^2+d}{(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral((e*x^2 + d)/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{d+ex^2}{(a+b\,\mathrm{acos}\,(cx))^2}\,dx$$

input
```
integrate((e*x**2+d)/(a+b*acos(c*x))**2,x)
```

output
```
Integral((d + e*x**2)/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{ex^2+d}{(b\arccos\,(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^2*c*arctan2(sqrt(c*x + 1)*s
qrt(-c*x + 1), c*x) + a*b*c)*integrate((3*c^2*e*x^3 + (c^2*d - 2*e)*x)*sqr
t(c*x + 1)*sqrt(-c*x + 1)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arc
tan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x))/(b^2*c*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(237) = 474$.

Time = 0.20 (sec) , antiderivative size = 860, normalized size of antiderivative = 3.45

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-3*b*e*arccos(c*x)*cos(a/b)^3*cos_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3
*arccos(c*x) + a*b^2*c^3) - b*c^2*d*arccos(c*x)*cos(a/b)*cos_integral(a/b
+ arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 3*b*e*arccos(c*x)*cos(a
/b)^2*sin(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) +
a*b^2*c^3) - b*c^2*d*arccos(c*x)*sin(a/b)*sin_integral(a/b + arccos(c*x))/
(b^3*c^3*arccos(c*x) + a*b^2*c^3) + sqrt(-c^2*x^2 + 1)*b*c^2*e*x^2/(b^3*c^
3*arccos(c*x) + a*b^2*c^3) - 3*a*e*cos(a/b)^3*cos_integral(3*a/b + 3*arcco
s(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - a*c^2*d*cos(a/b)*cos_integral(
a/b + arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 3*a*e*cos(a/b)^2*si
n(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^
3) - a*c^2*d*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c^3*arccos(c*x)
 + a*b^2*c^3) + 9/4*b*e*arccos(c*x)*cos(a/b)*cos_integral(3*a/b + 3*arccos
(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 1/4*b*e*arccos(c*x)*cos(a/b)*co
s_integral(a/b + arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 3/4*b*e*
arccos(c*x)*sin(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c
*x) + a*b^2*c^3) - 1/4*b*e*arccos(c*x)*sin(a/b)*sin_integral(a/b + arccos(
c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + sqrt(-c^2*x^2 + 1)*b*c^2*d/(b^3*
c^3*arccos(c*x) + a*b^2*c^3) + 9/4*a*e*cos(a/b)*cos_integral(3*a/b + 3*arc
cos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 1/4*a*e*cos(a/b)*cos_integra
l(a/b + arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 3/4*a*e*sin(a/...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{e\,x^2+d}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

input
```
int((d + e*x^2)/(a + b*acos(c*x))^2,x)
```

output
```
int((d + e*x^2)/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^2}\,dx = \left(\int \frac{x^2}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)e + \left(\int \frac{1}{acos\,(cx)^2\,b^2+2acos\,(cx)\,ab+a^2}dx\right)d$$

input
```
int((e*x^2+d)/(a+b*acos(c*x))^2,x)
```

output
```
int(x**2/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*e + int(1/(acos(c
*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)*d
```

3.681 $\int \frac{1}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 86

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = -\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} + \frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{b^2c} - \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}$$

output
```
-(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))+Ci((a+b*arccos(c*x))/b)*sin(a/b)
/b^2/c-cos(a/b)*Si((a+b*arccos(c*x))/b)/b^2/c
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \frac{\frac{b\sqrt{1-c^2x^2}}{a+b\arccos(cx)} - \cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right) - \sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^2c}$$

input
```
Integrate[(a + b*ArcCos[c*x])^(-2),x]
```

output
```
((b*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) - Cos[a/b]*CosIntegral[a/b + Ar
cCos[c*x]] - Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]])/(b^2*c)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5133, 5225, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{ 5133}$$

$$\frac{c\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx}{b}+\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

$$\Big\downarrow \text{ 5225}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{ 3784}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))}{b^2c}$$

$\Big\downarrow$ 3780

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{a+b\arccos(cx)}d(a+b\arccos(cx))+\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}$$

$\Big\downarrow$ 3783

$$\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)+\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}$$

input
```
Int[(a + b*ArcCos[c*x])^(-2),x]
```

output
```
Sqrt[1 - c^2*x^2]/(b*c*(a + b*ArcCos[c*x])) - (Cos[a/b]*CosIntegral[(a + b
*ArcCos[c*x])/b] + Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{\sqrt{-c^2x^2+1}}{(a+b\arccos(cx))b}-\frac{\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)+\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b^2}}{c}$	74
default	$\frac{\frac{\sqrt{-c^2x^2+1}}{(a+b\arccos(cx))b}-\frac{\mathrm{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)+\mathrm{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b^2}}{c}$	74

input
```
int(1/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*((-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))/b-(Si(arccos(c*x)+a/b)*sin(a/b)+
Ci(arccos(c*x)+a/b)*cos(a/b))/b^2)
```

Fricas [F]

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\operatorname{acos}(cx))^2}\,dx$$

input
```
integrate(1/(a+b*acos(c*x))**2,x)
```

output
```
Integral((a + b*acos(c*x))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-((b^2*c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c^2)*integrate
(sqrt(c*x + 1)*sqrt(-c*x + 1)*x/(a*b*c^2*x^2 - a*b + (b^2*c^2*x^2 - b^2)*a
rctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) - sqrt(c*x + 1)*sqrt(-c*x +
 1))/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(84) = 168$.

Time = 0.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.24

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = -\frac{b\arccos(cx)\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c} - \frac{b\arccos(cx)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c} - \frac{a\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c} - \frac{a\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c} + \frac{\sqrt{-c^2x^2+1}\,b}{b^3c\arccos(cx)+ab^2c}$$

input
```
integrate(1/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
-b*arccos(c*x)*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b^3*c*arccos(c*x)
 + a*b^2*c) - b*arccos(c*x)*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*
c*arccos(c*x) + a*b^2*c) - a*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b^3
*c*arccos(c*x) + a*b^2*c) - a*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^
3*c*arccos(c*x) + a*b^2*c) + sqrt(-c^2*x^2 + 1)*b/(b^3*c*arccos(c*x) + a*b
^2*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

input
```
int(1/(a + b*acos(c*x))^2,x)
```

output
```
int(1/(a + b*acos(c*x))^2, x)
```

Reduce [F]

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{acos\left(cx\right)^2 b^2+2acos\left(cx\right)ab+a^2}dx$$

input
```
int(1/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(acos(c*x)**2*b**2 + 2*acos(c*x)*a*b + a**2),x)
```

3.682 $\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}\,dx = \operatorname{Int}\left(\frac{1}{(d+ex^2)(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x^2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}dx$$

$$\Big\downarrow \text{5175}$$

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}dx$$

input
```
Int[1/((d + e*x^2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)\,(a+b\arccos\,(cx))^2}dx$$

input
```
int(1/(e*x^2+d)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.85

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arccos(c*x)^2 + 2*(a*b
*e*x^2 + a*b*d)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 45.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\operatorname{acos}(cx))^2(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/((a + b*acos(c*x))**2*(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 319, normalized size of antiderivative = 15.95

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((a*b*c*e*x^2 + a*b*c*d + (b^2*c*e*x^2 + b^2*c*d)*arctan2(sqrt(c*x + 1)*sq
rt(-c*x + 1), c*x))*integrate((c^2*e*x^3 - (c^2*d + 2*e)*x)*sqrt(c*x + 1)*
sqrt(-c*x + 1)/(a*b*c^3*e^2*x^6 - a*b*c*d^2 + (2*a*b*c^3*d*e - a*b*c*e^2)*
x^4 + (a*b*c^3*d^2 - 2*a*b*c*d*e)*x^2 + (b^2*c^3*e^2*x^6 - b^2*c*d^2 + (2*
b^2*c^3*d*e - b^2*c*e^2)*x^4 + (b^2*c^3*d^2 - 2*b^2*c*d*e)*x^2)*arctan2(sq
rt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) + sqrt(c*x + 1)*sqrt(-c*x + 1))/(a*b
*c*e*x^2 + a*b*c*d + (b^2*c*e*x^2 + b^2*c*d)*arctan2(sqrt(c*x + 1)*sqrt(-c
*x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\left(c\,x\right))^2\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.20

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^2}\,dx$$
$$=\int \frac{1}{acos(cx)^2 b^2 d+acos(cx)^2 b^2 e x^2+2acos(cx)abd+2acos(cx)abe x^2+a^2 d+a^2 e x^2}dx$$

input
```
int(1/(e*x^2+d)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(acos(c*x)**2*b**2*d + acos(c*x)**2*b**2*e*x**2 + 2*acos(c*x)*a*b*d
+ 2*acos(c*x)*a*b*e*x**2 + a**2*d + a**2*e*x**2),x)
```

3.683 $\int \frac{1}{(d+ex^2)^2(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(d+ex^2)^2(a+b\arccos(cx))^2}\,dx = \operatorname{Int}\left(\frac{1}{(d+ex^2)^2(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^2/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(d+ex^2)^2(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2} \, dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2} dx$$

input
```
Int[1/((d + e*x^2)^2*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e\,x^2+d)^2\,(a+b\arccos\,(cx))^2}dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^2/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 4.90

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^2(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d
*e*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2
)*arccos(c*x)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**2/(a+b*acos(c*x))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 3.11 (sec) , antiderivative size = 438, normalized size of antiderivative = 21.90

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^2(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((a*b*c*e^2*x^4 + 2*a*b*c*d*e*x^2 + a*b*c*d^2 + (b^2*c*e^2*x^4 + 2*b^2*c*d
*e*x^2 + b^2*c*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*integrate(
(3*c^2*e*x^3 - (c^2*d + 4*e)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(a*b*c^3*e^3*
x^8 + (3*a*b*c^3*d*e^2 - a*b*c*e^3)*x^6 - a*b*c*d^3 + 3*(a*b*c^3*d^2*e - a
*b*c*d*e^2)*x^4 + (a*b*c^3*d^3 - 3*a*b*c*d^2*e)*x^2 + (b^2*c^3*e^3*x^8 + (
3*b^2*c^3*d*e^2 - b^2*c*e^3)*x^6 - b^2*c*d^3 + 3*(b^2*c^3*d^2*e - b^2*c*d*
e^2)*x^4 + (b^2*c^3*d^3 - 3*b^2*c*d^2*e)*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-
c*x + 1), c*x)), x) + sqrt(c*x + 1)*sqrt(-c*x + 1))/(a*b*c*e^2*x^4 + 2*a*b
*c*d*e*x^2 + a*b*c*d^2 + (b^2*c*e^2*x^4 + 2*b^2*c*d*e*x^2 + b^2*c*d^2)*arc
tan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Not invertible Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))^2\,(e\,x^2+d)^2}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^2),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 5.75

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^2}\,dx$$
$$= \int \frac{1}{acos\,(cx)^2 b^2d^2 + 2acos\,(cx)^2 b^2de\,x^2 + acos\,(cx)^2 b^2e^2x^4 + 2acos\,(cx)\,ab\,d^2 + 4acos\,(cx)\,abde\,x^2 + 2ac$$

input
```
int(1/(e*x^2+d)^2/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(acos(c*x)**2*b**2*d**2 + 2*acos(c*x)**2*b**2*d*e*x**2 + acos(c*x)**
2*b**2*e**2*x**4 + 2*acos(c*x)*a*b*d**2 + 4*acos(c*x)*a*b*d*e*x**2 + 2*aco
s(c*x)*a*b*e**2*x**4 + a**2*d**2 + 2*a**2*d*e*x**2 + a**2*e**2*x**4),x)
```

3.684 $\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx=\text{Int}\left(\frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcCos[c*x])^2,x]
```

output
```
Integrate[Sqrt[d + e*x^2]/(a + b*ArcCos[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2} \, dx$$

$$\Big\downarrow \text{ 5175}$$

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}dx$$

input
```
Int[Sqrt[d + e*x^2]/(a + b*ArcCos[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^{2}+d}}{\left(a+b\arccos\left(cx\right)\right)^{2}}dx$$

input
```
int((e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int((e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{ex^2+d}}{(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{d+ex^2}}{(a+b\operatorname{acos}{(cx)})^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(sqrt(d + e*x**2)/(a + b*acos(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 10.77

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{ex^2+d}}{(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-((b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*integrate((2*
c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(a
*b*c^3*e*x^4 - a*b*c*d + (a*b*c^3*d - a*b*c*e)*x^2 + (b^2*c^3*e*x^4 - b^2*
c*d + (b^2*c^3*d - b^2*c*e)*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x
)), x) - sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*x + 1))/(b^2*c*arctan2(sqrt
(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{ex^2+d}}{(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)/(b*arccos(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{e\,x^2+d}}{(a+b\,\mathrm{acos}\left(c\,x\right))^2}\,dx$$

input
```
int((d + e*x^2)^(1/2)/(a + b*acos(c*x))^2,x)
```

output
```
int((d + e*x^2)^(1/2)/(a + b*acos(c*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{e\,x^2+d}}{\left(acos\left(cx\right)b+a\right)^2}dx$$

input
```
int((e*x^2+d)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int((e*x^2+d)^(1/2)/(a+b*acos(c*x))^2,x)
```

3.685 $\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2} \, dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}dx$$

input
```
Int[1/(Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{e\,x^2+d}\,\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.05

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arccos(c
*x)^2 + 2*(a*b*e*x^2 + a*b*d)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\operatorname{acos}{(cx)})^2\sqrt{d+ex^2}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(1/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/((a + b*acos(c*x))**2*sqrt(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 365, normalized size of antiderivative = 16.59

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
-((a*b*c^3*d^2 + a*b*c*d*e + (a*b*c^3*d*e + a*b*c*e^2)*x^2 + (b^2*c^3*d^2
+ b^2*c*d*e + (b^2*c^3*d*e + b^2*c*e^2)*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c
*x + 1), c*x))*integrate(sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x/(a
*b*c^3*e^2*x^6 - a*b*c*d^2 + (2*a*b*c^3*d*e - a*b*c*e^2)*x^4 + (a*b*c^3*d^
2 - 2*a*b*c*d*e)*x^2 + (b^2*c^3*e^2*x^6 - b^2*c*d^2 + (2*b^2*c^3*d*e - b^2
*c*e^2)*x^4 + (b^2*c^3*d^2 - 2*b^2*c*d*e)*x^2)*arctan2(sqrt(c*x + 1)*sqrt(
-c*x + 1), c*x)), x) - sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*x + 1))/(a*b*
c*e*x^2 + a*b*c*d + (b^2*c*e*x^2 + b^2*c*d)*arctan2(sqrt(c*x + 1)*sqrt(-c*
x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{ex^2+d}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*x^2 + d)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))^2\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex^2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{e\,x^2+d}\,\left(acos\left(cx\right)b+a\right)^2}dx$$

input
```
int(1/(e*x^2+d)^(1/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^(1/2)/(a+b*acos(c*x))^2,x)
```

3.686 $\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx$$

input
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{3/2} (a+b\arccos(cx))^2} dx$$
$$\Big\downarrow \text{5175}$$
$$\int \frac{1}{(d+ex^2)^{3/2} (a+b\arccos(cx))^2} dx$$

input
```
Int[1/((d + e*x^2)^(3/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{3}{2}} (a+b\arccos(cx))^2} dx$$

input
```
int(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.91

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2
*x^4 + 2*b^2*d*e*x^2 + b^2*d^2)*arccos(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e
*x^2 + a*b*d^2)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 5.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{1}{(a+b\,\mathrm{acos}\,(cx))^2\,(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(3/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/((a + b*acos(c*x))**2*(d + e*x**2)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 3.24 (sec) , antiderivative size = 456, normalized size of antiderivative = 20.73

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((a*b*c*e^2*x^4 + 2*a*b*c*d*e*x^2 + a*b*c*d^2 + (b^2*c*e^2*x^4 + 2*b^2*c*d
*e*x^2 + b^2*c*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))*integrate(
(2*c^2*e*x^3 - (c^2*d + 3*e)*x)*sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*x +
1)/(a*b*c^3*e^3*x^8 + (3*a*b*c^3*d*e^2 - a*b*c*e^3)*x^6 - a*b*c*d^3 + 3*(a
*b*c^3*d^2*e - a*b*c*d*e^2)*x^4 + (a*b*c^3*d^3 - 3*a*b*c*d^2*e)*x^2 + (b^2
*c^3*e^3*x^8 + (3*b^2*c^3*d*e^2 - b^2*c*e^3)*x^6 - b^2*c*d^3 + 3*(b^2*c^3*
d^2*e - b^2*c*d*e^2)*x^4 + (b^2*c^3*d^3 - 3*b^2*c*d^2*e)*x^2)*arctan2(sqrt
(c*x + 1)*sqrt(-c*x + 1), c*x)), x) + sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-
c*x + 1))/(a*b*c*e^2*x^4 + 2*a*b*c*d*e*x^2 + a*b*c*d^2 + (b^2*c*e^2*x^4 +
2*b^2*c*d*e*x^2 + b^2*c*d^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{3}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(3/2)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(acos\left(cx\right)b+a\right)^2}dx$$

input
```
int(1/(e*x^2+d)^(3/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^(3/2)/(a+b*acos(c*x))^2,x)
```

3.687 $\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2} dx = \text{Int}\left(\frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2}, x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2} dx = \int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2} dx$$

input
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2}dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2}dx$$

input
```
Int[1/((d + e*x^2)^(5/2)*(a + b*ArcCos[c*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^{\frac{5}{2}}\left(a+b\arccos\left(cx\right)\right)^2}dx$$

input
```
int(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x)
```

output
```
int(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 6.77

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)/(a^2*e^3*x^6 + 3*a^2*d*e^2*x^4 + 3*a^2*d^2*e*x^2
+ a^2*d^3 + (b^2*e^3*x^6 + 3*b^2*d*e^2*x^4 + 3*b^2*d^2*e*x^2 + b^2*d^3)*ar
ccos(c*x)^2 + 2*(a*b*e^3*x^6 + 3*a*b*d*e^2*x^4 + 3*a*b*d^2*e*x^2 + a*b*d^3
)*arccos(c*x)), x)
```

Sympy [N/A]

Not integrable

Time = 30.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\operatorname{acos}{(cx)})^2\,(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x**2+d)**(5/2)/(a+b*acos(c*x))**2,x)
```

output
```
Integral(1/((a + b*acos(c*x))**2*(d + e*x**2)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 4.92 (sec) , antiderivative size = 578, normalized size of antiderivative = 26.27

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

output
```
((a*b*c*e^3*x^6 + 3*a*b*c*d*e^2*x^4 + 3*a*b*c*d^2*e*x^2 + a*b*c*d^3 + (b^2
*c*e^3*x^6 + 3*b^2*c*d*e^2*x^4 + 3*b^2*c*d^2*e*x^2 + b^2*c*d^3)*arctan2(sq
rt(c*x + 1)*sqrt(-c*x + 1), c*x))*integrate((4*c^2*e*x^3 - (c^2*d + 5*e)*x
)*sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(a*b*c^3*e^4*x^10 + (4*a*b*
c^3*d*e^3 - a*b*c*e^4)*x^8 - a*b*c*d^4 + 2*(3*a*b*c^3*d^2*e^2 - 2*a*b*c*d*
e^3)*x^6 + 2*(2*a*b*c^3*d^3*e - 3*a*b*c*d^2*e^2)*x^4 + (a*b*c^3*d^4 - 4*a*
b*c*d^3*e)*x^2 + (b^2*c^3*e^4*x^10 + (4*b^2*c^3*d*e^3 - b^2*c*e^4)*x^8 - b
^2*c*d^4 + 2*(3*b^2*c^3*d^2*e^2 - 2*b^2*c*d*e^3)*x^6 + 2*(2*b^2*c^3*d^3*e
- 3*b^2*c*d^2*e^2)*x^4 + (b^2*c^3*d^4 - 4*b^2*c*d^3*e)*x^2)*arctan2(sqrt(c
*x + 1)*sqrt(-c*x + 1), c*x)), x) + sqrt(e*x^2 + d)*sqrt(c*x + 1)*sqrt(-c*
x + 1))/(a*b*c*e^3*x^6 + 3*a*b*c*d*e^2*x^4 + 3*a*b*c*d^2*e*x^2 + a*b*c*d^3
 + (b^2*c*e^3*x^6 + 3*b^2*c*d*e^2*x^4 + 3*b^2*c*d^2*e*x^2 + b^2*c*d^3)*arc
tan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(ex^2+d)^{\frac{5}{2}}(b\arccos(cx)+a)^2}\,dx$$

input
```
integrate(1/(e*x^2+d)^(5/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^(5/2)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*acos(c*x))^2*(d + e*x^2)^(5/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 187, normalized size of antiderivative = 8.50

$$\int \frac{1}{(d+ex^2)^{5/2}\,(a+b\arccos(cx))^2}\,dx = \int \frac{}{\sqrt{e\,x^2+d}\,acos\left(cx\right)^2 b^2d^2+2\sqrt{e\,x^2+d}\,acos\left(cx\right)^2 b^2de\,x^2+\sqrt{e}}$$

input
```
int(1/(e*x^2+d)^(5/2)/(a+b*acos(c*x))^2,x)
```

output
```
int(1/(sqrt(d + e*x**2)*acos(c*x)**2*b**2*d**2 + 2*sqrt(d + e*x**2)*acos(c
*x)**2*b**2*d*e*x**2 + sqrt(d + e*x**2)*acos(c*x)**2*b**2*e**2*x**4 + 2*sq
rt(d + e*x**2)*acos(c*x)*a*b*d**2 + 4*sqrt(d + e*x**2)*acos(c*x)*a*b*d*e*x
**2 + 2*sqrt(d + e*x**2)*acos(c*x)*a*b*e**2*x**4 + sqrt(d + e*x**2)*a**2*d
**2 + 2*sqrt(d + e*x**2)*a**2*d*e*x**2 + sqrt(d + e*x**2)*a**2*e**2*x**4),
x)
```

3.688 $\int (d+ex^2)^2 \sqrt{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 754

$$\int (d+ex^2)^2 \sqrt{a+b\arccos(cx)}\,dx = \text{Too large to display}$$

output

```
d^2*x*(a+b*arccos(c*x))^(1/2)+2/3*d*e*x^3*(a+b*arccos(c*x))^(1/2)+1/5*e^2*
x^5*(a+b*arccos(c*x))^(1/2)-1/2*b^(1/2)*d^2*2^(1/2)*Pi^(1/2)*cos(a/b)*Fres
nelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/c-1/4*b^(1/2)*d*e*2
^(1/2)*Pi^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)
/b^(1/2))/c^3-1/16*b^(1/2)*e^2*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelS(2^(1/2)/
Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/c^5+1/36*b^(1/2)*d*e*6^(1/2)*Pi^
(1/2)*cos(3*a/b)*FresnelS(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2)
)/c^3+1/96*b^(1/2)*e^2*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelS(6^(1/2)/Pi^(1/
2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/c^5-1/800*b^(1/2)*e^2*10^(1/2)*Pi^(1/2
)*cos(5*a/b)*FresnelS(10^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/c
^5+1/2*b^(1/2)*d^2*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(
c*x))^(1/2)/b^(1/2))*sin(a/b)/c+1/4*b^(1/2)*d*e*2^(1/2)*Pi^(1/2)*FresnelC(
2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/c^3+1/16*b^(1/2
)*e^2*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b
^(1/2))*sin(a/b)/c^5-1/36*b^(1/2)*d*e*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi
^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(3*a/b)/c^3-1/96*b^(1/2)*e^2*6^
(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*
sin(3*a/b)/c^5+1/800*b^(1/2)*e^2*10^(1/2)*Pi^(1/2)*FresnelC(10^(1/2)/Pi^(1
/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(5*a/b)/c^5
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.53

$$\int (d+ex^2)^2 \sqrt{a+b\arccos(cx)}\,dx$$
$$= \frac{ibe^{-\frac{5ia}{b}}\left(450(8c^4d^2+4c^2de+e^2)\,e^{\frac{4ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},-\frac{i(a+b\arccos(cx))}{b}\right)-450(8c^4d^2+4c^2de+e^2)\right.}{}$$

input
```
Integrate[(d + e*x^2)^2*Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
((I/7200)*b*(450*(8*c^4*d^2 + 4*c^2*d*e + e^2)*E^(((4*I)*a)/b)*Sqrt[((-I)*
(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-I)*(a + b*ArcCos[c*x]))/b] - 450*(8*
c^4*d^2 + 4*c^2*d*e + e^2)*E^(((6*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]
*Gamma[3/2, (I*(a + b*ArcCos[c*x]))/b] + e*(25*Sqrt[3]*(8*c^2*d + 3*e)*E^(
((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-3*I)*(a + b*
ArcCos[c*x]))/b] - 25*Sqrt[3]*(8*c^2*d + 3*e)*E^(((8*I)*a)/b)*Sqrt[(I*(a +
 b*ArcCos[c*x]))/b]*Gamma[3/2, ((3*I)*(a + b*ArcCos[c*x]))/b] + 9*Sqrt[5]*
e*(Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-5*I)*(a + b*ArcCos[c*x
]))/b] - E^(((10*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((5*I
)*(a + b*ArcCos[c*x]))/b]))))/(c^5*E^(((5*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]]
)
```

Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)^2 \sqrt{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(d^2\sqrt{a+b\arccos(cx)}+2dex^2\sqrt{a+b\arccos(cx)}+e^2x^4\sqrt{a+b\arccos(cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^2\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8c^5}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{b}e^2\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{16c^5}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{b}e^2\cos\left(\frac{5a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{80c^5}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^2\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8c^5}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{b}e^2\sin\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{16c^5}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{b}e^2\sin\left(\frac{5a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{80c^5}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}de\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c^3}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{b}de\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{6c^3}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}de\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c^3}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{b}de\sin\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{6c^3}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}d^2\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{c}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}d^2\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{c}+d^2x\sqrt{a+b\arccos(cx)}+\frac{2}{3}dex^3\sqrt{a+b\arccos(cx)}+\frac{1}{5}e^2x^5\sqrt{a+b\arccos(cx)}$$

input
```
Int[(d + e*x^2)^2*Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
d^2*x*Sqrt[a + b*ArcCos[c*x]] + (2*d*e*x^3*Sqrt[a + b*ArcCos[c*x]])/3 + (e
^2*x^5*Sqrt[a + b*ArcCos[c*x]])/5 - (Sqrt[b]*d^2*Sqrt[Pi/2]*Cos[a/b]*Fresn
elC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/c - (Sqrt[b]*d*e*Sqrt[P
i/2]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(2*c
^3) - (Sqrt[b]*e^2*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*Arc
Cos[c*x]])/Sqrt[b]])/(8*c^5) - (Sqrt[b]*d*e*Sqrt[Pi/6]*Cos[(3*a)/b]*Fresne
lC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(6*c^3) - (Sqrt[b]*e^2*S
qrt[Pi/6]*Cos[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[
b]])/(16*c^5) - (Sqrt[b]*e^2*Sqrt[Pi/10]*Cos[(5*a)/b]*FresnelC[(Sqrt[10/Pi
]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(80*c^5) - (Sqrt[b]*d^2*Sqrt[Pi/2]*Fr
esnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/c - (Sqrt[b
]*d*e*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Si
n[a/b])/(2*c^3) - (Sqrt[b]*e^2*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*
ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(8*c^5) - (Sqrt[b]*d*e*Sqrt[Pi/6]*Fresnel
S[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(6*c^3) - (S
qrt[b]*e^2*Sqrt[Pi/6]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b
]]*Sin[(3*a)/b])/(16*c^5) - (Sqrt[b]*e^2*Sqrt[Pi/10]*FresnelS[(Sqrt[10/Pi]
*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(5*a)/b])/(80*c^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 1155, normalized size of antiderivative = 1.53

method	result	size
default	Expression too large to display	1155

input
```
int((e*x^2+d)^2*(a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7200/c^5*(-9*2^(1/2)*(-5/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(5
*a/b)*FresnelC(5*2^(1/2)/Pi^(1/2)/(-5/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*
b*e^2+9*2^(1/2)*(-5/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(5*a/b)*F
resnelS(5*2^(1/2)/Pi^(1/2)/(-5/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b*e^2+9
0*arccos(c*x)*cos(-5*(a+b*arccos(c*x))/b+5*a/b)*b*e^2+90*cos(-5*(a+b*arcco
s(c*x))/b+5*a/b)*a*e^2-3600*2^(1/2)*cos(a/b)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*ar
ccos(c*x))^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^
(1/2)/b)*b*c^4*d^2+3600*2^(1/2)*sin(a/b)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos
(c*x))^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2
)/b)*b*c^4*d^2-1800*2^(1/2)*cos(a/b)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x
))^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)
*b*c^2*d*e+1800*2^(1/2)*sin(a/b)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(
1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b*c
^2*d*e+7200*arccos(c*x)*cos(-(a+b*arccos(c*x))/b+a/b)*b*c^4*d^2-450*2^(1/2
)*cos(a/b)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*FresnelC(2^(1/2)/
Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b*e^2+450*2^(1/2)*sin(a/b
)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/
(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b*e^2+7200*cos(-(a+b*arccos(c*x))/
b+a/b)*a*c^4*d^2+3600*arccos(c*x)*cos(-(a+b*arccos(c*x))/b+a/b)*b*c^2*d*e+
3600*cos(-(a+b*arccos(c*x))/b+a/b)*a*c^2*d*e+900*arccos(c*x)*cos(-(a+b*...
```

Fricas [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\arccos(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (d+ex^2)^2 \sqrt{a+b\arccos(cx)}\,dx = \int \sqrt{a+b\operatorname{acos}(cx)}\,(d+ex^2)^2\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acos(c*x))**(1/2),x)
```

output
```
Integral(sqrt(a + b*acos(c*x))*(d + e*x**2)**2, x)
```

Maxima [F]

$$\int (d+ex^2)^2 \sqrt{a+b\arccos(cx)}\,dx = \int (ex^2+d)^2 \sqrt{b\arccos(cx)+a}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2*sqrt(b*arccos(c*x) + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 3222, normalized size of antiderivative = 4.27

$$\int (d+ex^2)^2 \sqrt{a+b\arccos(cx)}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output
```
-1/480*(240*I*sqrt(2)*sqrt(pi)*a*b*c^4*d^2*erf(-1/2*I*sqrt(2)*sqrt(b*arcco
s(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b)
)/b)*e^(I*a/b)/(I*b^2/sqrt(abs(b)) + b*sqrt(abs(b))) - 120*sqrt(2)*sqrt(pi
)*b^2*c^4*d^2*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/
2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(I*b^2/sqrt(ab
s(b)) + b*sqrt(abs(b))) - 240*I*sqrt(2)*sqrt(pi)*a*b*c^4*d^2*erf(1/2*I*sqr
t(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x)
 + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b))) -
120*sqrt(2)*sqrt(pi)*b^2*c^4*d^2*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)
/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*
a/b)/(-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b))) - 480*I*sqrt(pi)*a*c^4*d^2*erf
(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*
arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(I*sqrt(2)*b/sqrt(abs(b)) + sqr
t(2)*sqrt(abs(b))) + 480*I*sqrt(pi)*a*c^4*d^2*erf(1/2*I*sqrt(2)*sqrt(b*arc
cos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(
b))/b)*e^(-I*a/b)/(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b))) + 120
*I*sqrt(2)*sqrt(pi)*a*b*c^2*d*e*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)
/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a
/b)/(I*b^2/sqrt(abs(b)) + b*sqrt(abs(b))) - 60*sqrt(2)*sqrt(pi)*b^2*c^2*d*
e*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)...
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\arccos(cx)}\,dx=\int \sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int((a + b*acos(c*x))^(1/2)*(d + e*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))^(1/2)*(d + e*x^2)^2, x)
```

Reduce [F]

$$\int (d+ex^2)^2 \sqrt{a+b\arccos(cx)}\,dx = \left(\int \sqrt{acos\,(cx)\,b+a}dx\right)d^2 + \left(\int \sqrt{acos\,(cx)\,b+a}\,x^4dx\right)e^2 + 2\left(\int \sqrt{acos\,(cx)\,b+a}\,x^2dx\right)de$$

input
```
int((e*x^2+d)^2*(a+b*acos(c*x))^(1/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a),x)*d**2 + int(sqrt(acos(c*x)*b + a)*x**4,x)*e**2
 + 2*int(sqrt(acos(c*x)*b + a)*x**2,x)*d*e
```

3.689 $\int (d+ex^2)\sqrt{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 369

$$\begin{aligned}\int (d+ex^2)\sqrt{a+b\arccos(cx)}\,dx &= dx\sqrt{a+b\arccos(cx)}+\frac{1}{3}ex^3\sqrt{a+b\arccos(cx)} \\ &\quad-\frac{\sqrt{b}d\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{c} \\ &\quad-\frac{\sqrt{b}e\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4c^3} \\ &\quad+\frac{\sqrt{b}e\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{12c^3} \\ &\quad+\frac{\sqrt{b}d\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{c} \\ &\quad+\frac{\sqrt{b}e\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{4c^3} \\ &\quad-\frac{\sqrt{b}e\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{12c^3}\end{aligned}$$

output
```
d*x*(a+b*arccos(c*x))^(1/2)+1/3*e*x^3*(a+b*arccos(c*x))^(1/2)-1/2*b^(1/2)*
d*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1
/2)/b^(1/2))/c-1/8*b^(1/2)*e*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi
^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/c^3+1/72*b^(1/2)*e*6^(1/2)*Pi^(1/2
)*cos(3*a/b)*FresnelS(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/c^
3+1/2*b^(1/2)*d*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x
))^(1/2)/b^(1/2))*sin(a/b)/c+1/8*b^(1/2)*e*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/
2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/c^3-1/72*b^(1/2)*e*6
^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))
*sin(3*a/b)/c^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.67

$$\int (d+ex^2)\sqrt{a+b\arccos(cx)}\,dx$$
$$= \frac{ibe^{-\frac{3ia}{b}}\left(9(4c^2d+e)\,e^{\frac{2ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},-\frac{i(a+b\arccos(cx))}{b}\right)-9(4c^2d+e)\,e^{\frac{4ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},\right.\right.}{72c^3\sqrt{a}}$$

input
```
Integrate[(d + e*x^2)*Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
((I/72)*b*(9*(4*c^2*d + e)*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))
/b]*Gamma[3/2, ((-I)*(a + b*ArcCos[c*x]))/b] - 9*(4*c^2*d + e)*E^(((4*I)*a
)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, (I*(a + b*ArcCos[c*x]))/b]
 + Sqrt[3]*e*(Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-3*I)*(a + b
*ArcCos[c*x]))/b] - E^(((6*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[
3/2, ((3*I)*(a + b*ArcCos[c*x]))/b])))/(c^3*E^(((3*I)*a)/b)*Sqrt[a + b*Arc
Cos[c*x]])
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)\sqrt{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(d\sqrt{a+b\arccos(cx)}+ex^2\sqrt{a+b\arccos(cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4c^3}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{b}e\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{12c^3}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4c^3}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{b}e\sin\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{12c^3}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}d\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{c}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}d\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{c}+dx\sqrt{a+b\arccos(cx)}+\frac{1}{3}ex^3\sqrt{a+b\arccos(cx)}$$

input
```
Int[(d + e*x^2)*Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
d*x*Sqrt[a + b*ArcCos[c*x]] + (e*x^3*Sqrt[a + b*ArcCos[c*x]])/3 - (Sqrt[b]
*d*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[
b]])/c - (Sqrt[b]*e*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*Ar
cCos[c*x]])/Sqrt[b]])/(4*c^3) - (Sqrt[b]*e*Sqrt[Pi/6]*Cos[(3*a)/b]*Fresnel
C[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(12*c^3) - (Sqrt[b]*d*Sqr
t[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/c
 - (Sqrt[b]*e*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqr
t[b]]*Sin[a/b])/(4*c^3) - (Sqrt[b]*e*Sqrt[Pi/6]*FresnelS[(Sqrt[6/Pi]*Sqrt[
a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(12*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.51

method	result
default	$-36\sqrt{2}\sqrt{-\frac{1}{b}}\sqrt{\pi}\sqrt{a+b\arccos(cx)}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\sqrt{-\frac{1}{b}}\,b}\right)\cos\left(\frac{a}{b}\right)b\,c^2d+36\sqrt{2}\sqrt{-\frac{1}{b}}\sqrt{\pi}\sqrt{a+b\arccos(cx)}\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\cdots}{\cdots}\right.$

input
```
int((e*x^2+d)*(a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/72/c^3*(-36*2^(1/2)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*Fresne
lC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*cos(a/b)*b*c^2
*d+36*2^(1/2)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*FresnelS(2^(1/
2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(a/b)*b*c^2*d-2^(1/
2)*(-3/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*FresnelC(3*2^(1/2)/Pi^(1/
2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*cos(3*a/b)*b*e+2^(1/2)*(-3/b)^(
1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(
1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(3*a/b)*b*e-9*2^(1/2)*(-1/b)^(1/2)*Pi^(
1/2)*(a+b*arccos(c*x))^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*a
rccos(c*x))^(1/2)/b)*cos(a/b)*b*e+9*2^(1/2)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arc
cos(c*x))^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(
1/2)/b)*sin(a/b)*b*e+72*arccos(c*x)*cos(-(a+b*arccos(c*x))/b+a/b)*b*c^2*d+
72*cos(-(a+b*arccos(c*x))/b+a/b)*a*c^2*d+6*arccos(c*x)*cos(-3*(a+b*arccos(
c*x))/b+3*a/b)*b*e+18*arccos(c*x)*cos(-(a+b*arccos(c*x))/b+a/b)*b*e+6*cos(
-3*(a+b*arccos(c*x))/b+3*a/b)*a*e+18*cos(-(a+b*arccos(c*x))/b+a/b)*a*e)/(a
+b*arccos(c*x))^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int (d+ex^2)\sqrt{a+b\arccos(cx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (d+ex^2)\sqrt{a+b\arccos(cx)}\,dx = \int \sqrt{a+b\operatorname{acos}(cx)}\,(d+ex^2)\ dx$$

input
```
integrate((e*x**2+d)*(a+b*acos(c*x))**(1/2),x)
```

output
```
Integral(sqrt(a + b*acos(c*x))*(d + e*x**2), x)
```

Maxima [F]

$$\int \left(d+ex^2\right)\sqrt{a+b\arccos(cx)}\,dx=\int \left(ex^2+d\right)\sqrt{b\arccos\left(cx\right)+a}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)*sqrt(b*arccos(c*x) + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 1661, normalized size of antiderivative = 4.50

$$\int \left(d+ex^2\right)\sqrt{a+b\arccos(cx)}\,dx=\text{Too large to display}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output
```
-1/2*I*sqrt(2)*sqrt(pi)*a*b^2*d*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)
/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a
/b)/((I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) + 1/4*sqrt(2)*sqrt(pi)*b^3
*d*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*s
qrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^3/sqrt(abs(b)) + b^
2*sqrt(abs(b)))*c) + 1/2*I*sqrt(2)*sqrt(pi)*a*b^2*d*erf(1/2*I*sqrt(2)*sqrt
(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqr
t(abs(b))/b)*e^(-I*a/b)/((-I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) + 1/4
*sqrt(2)*sqrt(pi)*b^3*d*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs
(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I
*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) + I*sqrt(pi)*a*b*d*erf(-1/2*I*sqr
t(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x)
 + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*sqrt(2)*b^2/sqrt(abs(b)) + sqrt(2)*b*s
qrt(abs(b)))*c) - I*sqrt(pi)*a*b*d*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) +
a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-
I*a/b)/((-I*sqrt(2)*b^2/sqrt(abs(b)) + sqrt(2)*b*sqrt(abs(b)))*c) - 1/8*I*
sqrt(2)*sqrt(pi)*a*b^2*e*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(a
bs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I
*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c^3) + 1/16*sqrt(2)*sqrt(pi)*b^3*e*e
rf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sq...
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)\,\sqrt{a+b\arccos(cx)}\,dx=\int \sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}\,\left(e\,x^2+d\right)\,dx$$

input
```
int((a + b*acos(c*x))^(1/2)*(d + e*x^2),x)
```

output
```
int((a + b*acos(c*x))^(1/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int (d+ex^2)\sqrt{a+b\arccos(cx)}\,dx = \left(\int \sqrt{acos\,(cx)\,b+a}dx\right)d + \left(\int \sqrt{acos\,(cx)\,b+a}\,x^2dx\right)e$$

input
```
int((e*x^2+d)*(a+b*acos(c*x))^(1/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a),x)*d + int(sqrt(acos(c*x)*b + a)*x**2,x)*e
```

3.690 $\int \sqrt{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 120

$$\int \sqrt{a+b\arccos(cx)}\,dx = x\sqrt{a+b\arccos(cx)} - \frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{c} + \frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{c}$$

output
```
x*(a+b*arccos(c*x))^(1/2)-1/2*b^(1/2)*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelS(2
^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/c+1/2*b^(1/2)*2^(1/2)*Pi^
(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/
c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02

$$\int \sqrt{a+b\arccos(cx)}\,dx = -\frac{ibe^{-\frac{ia}{b}}\left(-\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+e^{\frac{2ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},\frac{i(a+b\arccos(cx))}{b}\right)\right)}{2c\sqrt{a+b\arccos(cx)}}$$

input
```
Integrate[Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
((-1/2*I)*b*(-(Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-I)*(a + b*
ArcCos[c*x]))/b]) + E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[
3/2, (I*(a + b*ArcCos[c*x]))/b]))/(c*E^((I*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5131, 5225, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\arccos(cx)}\,dx$$

$$\Big\downarrow \text{5131}$$

$$\frac{1}{2}bc\int \frac{x}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}dx + x\sqrt{a+b\arccos(cx)}$$

$$\Big\downarrow \text{5225}$$

$$x\sqrt{a+b\arccos(cx)} - \frac{\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c}$$

$$\Big\downarrow \text{3042}$$

$$x\sqrt{a+b\arccos(cx)}-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c}$$

$$\Big\downarrow \text{3787}$$

$$x\sqrt{a+b\arccos(cx)}-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c}$$

$$\Big\downarrow \text{25}$$

$$x\sqrt{a+b\arccos(cx)}-\frac{\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c}$$

$$\Big\downarrow \text{3042}$$

$$x\sqrt{a+b\arccos(cx)}-\frac{\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c}$$

$$\Big\downarrow \text{3785}$$

$$x\sqrt{a+b\arccos(cx)}-\frac{\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))+2\cos\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c}$$

$$\Big\downarrow \text{3786}$$

$$x\sqrt{a+b\arccos(cx)}-\frac{2\sin\left(\frac{a}{b}\right)\int \sin\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}+2\cos\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c}$$

$$\Big\downarrow \text{3832}$$

$$x\sqrt{a+b\arccos(cx)}-\frac{2\cos\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}+\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c}$$

$$\Big\downarrow \text{3833}$$

$$\frac{x\sqrt{a+b\arccos(cx)}-\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c}$$

input
```
Int[Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
x*Sqrt[a + b*ArcCos[c*x]] - (Sqrt[b]*Sqrt[2*Pi]*Cos[a/b]*FresnelC[(Sqrt[2/
Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]] + Sqrt[b]*Sqrt[2*Pi]*FresnelS[(Sqrt[
2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.55

method	result
default	$\frac{-\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)b+\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{}}{}\right.}{2c\sqrt{a+b\arccos(cx)}}$

input
```
int((a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/c/(a+b*arccos(c*x))^(1/2)*(-2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(
c*x))^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*
x))^(1/2)/b)*b+2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a
/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b+2*
arccos(c*x)*cos(-(a+b*arccos(c*x))/b+a/b)*b+2*cos(-(a+b*arccos(c*x))/b+a/b
)*a)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a + b \arccos(cx)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a + b \arccos(cx)}\, dx = \int \sqrt{a + b \operatorname{acos}(cx)}\, dx$$

input
```
integrate((a+b*acos(c*x))**(1/2),x)
```

output
```
Integral(sqrt(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \sqrt{a + b \arccos(cx)}\, dx = \int \sqrt{b \arccos(cx) + a}\, dx$$

input
```
integrate((a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arccos(c*x) + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 531, normalized size of antiderivative = 4.42

$$\int \sqrt{a+b\arccos(cx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output
```
-1/2*I*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqr
t(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/
((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) + 1/4*sqrt(2)*sqrt(pi)*b^2*erf(-
1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*ar
ccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs
(b)))*c) + 1/2*I*sqrt(2)*sqrt(pi)*a*b*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x)
 + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e
^(-I*a/b)/((-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) + 1/4*sqrt(2)*sqrt(pi
)*b^2*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)
*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^2/sqrt(abs(b))
+ b*sqrt(abs(b)))*c) + I*sqrt(pi)*a*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x)
+ a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^
(I*a/b)/(c*(I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - I*sqrt(pi)
*a*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sq
rt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(c*(-I*sqrt(2)*b/sqrt(abs
(b)) + sqrt(2)*sqrt(abs(b)))) + 1/2*sqrt(b*arccos(c*x) + a)*e^(I*arccos(c*
x))/c + 1/2*sqrt(b*arccos(c*x) + a)*e^(-I*arccos(c*x))/c
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\arccos(cx)}\,dx = \int \sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

input
```
int((a + b*acos(c*x))^(1/2),x)
```

output
```
int((a + b*acos(c*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\arccos(cx)}\,dx = \int \sqrt{acos\,(cx)\,b+a}dx$$

input
```
int((a+b*acos(c*x))^(1/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a),x)
```

3.691 $\int \frac{\sqrt{a+b\arccos(cx)}}{d+ex^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{a+b\arccos(cx)}}{d+ex^2}\,dx = \operatorname{Int}\left(\frac{\sqrt{a+b\arccos(cx)}}{d+ex^2},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^(1/2)/(e*x^2+d),x)
```

Mathematica [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+b\arccos(cx)}}{d+ex^2}\,dx = \int \frac{\sqrt{a+b\arccos(cx)}}{d+ex^2}\,dx$$

input
```
Integrate[Sqrt[a + b*ArcCos[c*x]]/(d + e*x^2),x]
```

output
```
Integrate[Sqrt[a + b*ArcCos[c*x]]/(d + e*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\arccos(cx)}}{d+ex^2} \, dx$$

$$\Big\downarrow \; 5175$$

$$\int \frac{\sqrt{a+b\arccos(cx)}}{d+ex^2}dx$$

input
```
Int[Sqrt[a + b*ArcCos[c*x]]/(d + e*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\arccos\left(cx\right)}}{e\,x^{2}+d}dx$$

input
```
int((a+b*arccos(c*x))^(1/2)/(e*x^2+d),x)
```

output
```
int((a+b*arccos(c*x))^(1/2)/(e*x^2+d),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\arccos(cx)}}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+b\arccos(cx)}}{d+ex^2}\,dx = \int \frac{\sqrt{a+b\,\mathrm{acos}\,(cx)}}{d+ex^2}\,dx$$

input
```
integrate((a+b*acos(c*x))**(1/2)/(e*x**2+d),x)
```

output
```
Integral(sqrt(a + b*acos(c*x))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\arccos(cx)}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\arccos(cx)}}{d+ex^2}\,dx = \int \frac{\sqrt{b\arccos(cx)+a}}{ex^2+d}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(1/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arccos(c*x) + a)/(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\arccos(cx)}}{d+ex^2}\,dx = \int \frac{\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}}{e\,x^2+d}\,dx$$

input
```
int((a + b*acos(c*x))^(1/2)/(d + e*x^2),x)
```

output
```
int((a + b*acos(c*x))^(1/2)/(d + e*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 12.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+b\arccos(cx)}}{d+ex^2}\,dx=\int \frac{\sqrt{acos\left(cx\right)b+a}}{e\,x^2+d}dx$$

input
```
int((a+b*acos(c*x))^(1/2)/(e*x^2+d),x)
```

output
```
int(sqrt(acos(c*x)*b + a)/(d + e*x**2),x)
```

3.692 $\int \frac{\sqrt{a+b\arccos(cx)}}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sqrt{a+b\arccos(cx)}}{(d+ex^2)^2}\,dx = \text{Int}\left(\frac{\sqrt{a+b\arccos(cx)}}{(d+ex^2)^2},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^(1/2)/(e*x^2+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 20.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+b\arccos(cx)}}{(d+ex^2)^2}\,dx = \int \frac{\sqrt{a+b\arccos(cx)}}{(d+ex^2)^2}\,dx$$

input
```
Integrate[Sqrt[a + b*ArcCos[c*x]]/(d + e*x^2)^2,x]
```

output
```
Integrate[Sqrt[a + b*ArcCos[c*x]]/(d + e*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\arccos(cx)}}{(d+ex^2)^2}\,dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{\sqrt{a+b\arccos(cx)}}{(d+ex^2)^2}dx$$

input
```
Int[Sqrt[a + b*ArcCos[c*x]]/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+b\arccos(cx)}}{(e\,x^2+d)^2}dx$$

input
```
int((a+b*arccos(c*x))^(1/2)/(e*x^2+d)^2,x)
```

output
```
int((a+b*arccos(c*x))^(1/2)/(e*x^2+d)^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + b \arccos(cx)}}{(d + ex^2)^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 14.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a + b \arccos(cx)}}{(d + ex^2)^2} dx = \int \frac{\sqrt{a + b \operatorname{acos}(cx)}}{(d + ex^2)^2} dx$$

input
```
integrate((a+b*acos(c*x))**(1/2)/(e*x**2+d)**2,x)
```

output
```
Integral(sqrt(a + b*acos(c*x))/(d + e*x**2)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b \arccos(cx)}}{(d + ex^2)^2} dx = \int \frac{\sqrt{b \arccos(cx) + a}}{(ex^2 + d)^2} dx$$

input
```
integrate((a+b*arccos(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arccos(c*x) + a)/(e*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\arccos(cx)}}{(d+ex^2)^2}\,dx = \int \frac{\sqrt{b\arccos{(cx)}+a}}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*arccos(c*x) + a)/(e*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\arccos(cx)}}{(d+ex^2)^2}\,dx = \int \frac{\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}}{{\left(e\,x^2+d\right)}^2}\,dx$$

input
```
int((a + b*acos(c*x))^(1/2)/(d + e*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))^(1/2)/(d + e*x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\arccos(cx)}}{(d+ex^2)^2}\,dx = \int \frac{\sqrt{acos\left(cx\right)b+a}}{\left(e\,x^2+d\right)^2}dx$$

input
```
int((a+b*acos(c*x))^(1/2)/(e*x^2+d)^2,x)
```

output
```
int((a+b*acos(c*x))^(1/2)/(e*x^2+d)^2,x)
```

3.693 $\int \left(d+ex^2\right) (a+b\arccos(cx))^{3/2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 482

$$\int (d+ex^2)(a+b\arccos(cx))^{3/2}\,dx = \frac{3bd\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{2c} + \frac{be\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{3c^3} + \frac{bex^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{6c} + dx(a+b\arccos(cx))^{3/2} + \frac{1}{3}ex^3(a+b\arccos(cx))^{3/2} - \frac{3b^{3/2}d\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c} - \frac{3b^{3/2}e\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8c^3} + \frac{b^{3/2}e\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{24c^3} - \frac{3b^{3/2}d\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{2c} - \frac{3b^{3/2}e\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{8c^3} + \frac{b^{3/2}e\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{24c^3}$$

output
```
3/2*b*d*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^(1/2)/c+1/3*b*e*(-c^2*x^2+1)^
(1/2)*(a+b*arccos(c*x))^(1/2)/c^3+1/6*b*e*x^2*(-c^2*x^2+1)^(1/2)*(a+b*arcc
os(c*x))^(1/2)/c+d*x*(a+b*arccos(c*x))^(3/2)+1/3*e*x^3*(a+b*arccos(c*x))^(
3/2)-3/4*b^(3/2)*d*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)*(a+
b*arccos(c*x))^(1/2)/b^(1/2))/c-3/16*b^(3/2)*e*2^(1/2)*Pi^(1/2)*cos(a/b)*F
resnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/c^3+1/144*b^(3/2
)*e*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x)
)^(1/2)/b^(1/2))/c^3-3/4*b^(3/2)*d*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1
/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/c-3/16*b^(3/2)*e*2^(1/2)*Pi^
(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/
c^3+1/144*b^(3/2)*e*6^(1/2)*Pi^(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*(a+b*arccos
(c*x))^(1/2)/b^(1/2))*sin(3*a/b)/c^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.27 (sec) , antiderivative size = 851, normalized size of antiderivative = 1.77

$$\int (d+ex^2) (a+b\arccos(cx))^{3/2} \, dx = \text{Too large to display}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
((-1/2*I)*a*b*d*(-(Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-I)*(a
+ b*ArcCos[c*x]))/b]) + E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Ga
mma[3/2, (I*(a + b*ArcCos[c*x]))/b]))/(c*E^((I*a)/b)*Sqrt[a + b*ArcCos[c*x
]]) - ((I/72)*a*b*e*(-9*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]
*Gamma[3/2, ((-I)*(a + b*ArcCos[c*x]))/b] + 9*E^(((4*I)*a)/b)*Sqrt[(I*(a +
 b*ArcCos[c*x]))/b]*Gamma[3/2, (I*(a + b*ArcCos[c*x]))/b] + Sqrt[3]*(-(Sqr
t[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-3*I)*(a + b*ArcCos[c*x]))/b]
) + E^(((6*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((3*I)*(a +
 b*ArcCos[c*x]))/b])))/(c^3*E^(((3*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]]) - (Sq
rt[b]*d*(2*Sqrt[b]*Sqrt[a + b*ArcCos[c*x]]*(3*Sqrt[1 - c^2*x^2] - 2*c*x*Ar
cCos[c*x]) - Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt
[b]]*(3*b*Cos[a/b] + 2*a*Sin[a/b]) - Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[
a + b*ArcCos[c*x]])/Sqrt[b]]*(2*a*Cos[a/b] - 3*b*Sin[a/b])))/(4*c) - (Sqrt
[b]*e*(18*Sqrt[b]*Sqrt[a + b*ArcCos[c*x]]*(3*Sqrt[1 - c^2*x^2] - 2*c*x*Arc
Cos[c*x]) - 9*Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqr
t[b]]*(3*b*Cos[a/b] + 2*a*Sin[a/b]) - 9*Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sq
rt[a + b*ArcCos[c*x]])/Sqrt[b]]*(2*a*Cos[a/b] - 3*b*Sin[a/b]) - Sqrt[6*Pi]
*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*(b*Cos[(3*a)/b] +
2*a*Sin[(3*a)/b]) - Sqrt[6*Pi]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]
])/Sqrt[b]]*(2*a*Cos[(3*a)/b] - b*Sin[(3*a)/b]) + 6*Sqrt[b]*Sqrt[a + b*...
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)(a+b\arccos(cx))^{3/2}\,dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(d(a+b\arccos(cx))^{3/2}+ex^2(a+b\arccos(cx))^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8c^3}-\frac{\sqrt{\frac{\pi}{6}}b^{3/2}e\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{24c^3}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8c^3}+\frac{\sqrt{\frac{\pi}{6}}b^{3/2}e\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{24c^3}-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}d\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}d\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c}-\frac{3bd\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{2c}-\frac{bex^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{6c}-\frac{be\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{3c^3}+dx(a+b\arccos(cx))^{3/2}+\frac{1}{3}ex^3(a+b\arccos(cx))^{3/2}$$

input
```
Int[(d + e*x^2)*(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(-3*b*d*Sqrt[1 - c^2*x^2]*Sqrt[a + b*ArcCos[c*x]])/(2*c) - (b*e*Sqrt[1 - c
^2*x^2]*Sqrt[a + b*ArcCos[c*x]])/(3*c^3) - (b*e*x^2*Sqrt[1 - c^2*x^2]*Sqrt
[a + b*ArcCos[c*x]])/(6*c) + d*x*(a + b*ArcCos[c*x])^(3/2) + (e*x^3*(a + b
*ArcCos[c*x])^(3/2))/3 + (3*b^(3/2)*d*Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2
/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(2*c) + (3*b^(3/2)*e*Sqrt[Pi/2]*Co
s[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(8*c^3) + (
b^(3/2)*e*Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c
*x]])/Sqrt[b]])/(24*c^3) - (3*b^(3/2)*d*Sqrt[Pi/2]*FresnelC[(Sqrt[2/Pi]*Sq
rt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(2*c) - (3*b^(3/2)*e*Sqrt[Pi/2]*
FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(8*c^3) -
 (b^(3/2)*e*Sqrt[Pi/6]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[
b]]*Sin[(3*a)/b])/(24*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(374) = 748$.

Time = 0.22 (sec) , antiderivative size = 851, normalized size of antiderivative = 1.77

method	result	size
default	Expression too large to display	851

input
```
int((e*x^2+d)*(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/144/c^3*(-108*2^(1/2)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*Fres
nelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(a/b)*b^2
*c^2*d-108*2^(1/2)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*FresnelS(
2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*cos(a/b)*b^2*c^2*
d-2^(1/2)*(-3/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*FresnelC(3*2^(1/2)
/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(3*a/b)*b^2*e-2^(1/2)
*(-3/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*FresnelS(3*2^(1/2)/Pi^(1/2)
/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*cos(3*a/b)*b^2*e-27*2^(1/2)*(-1/b
)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^
(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(a/b)*b^2*e-27*2^(1/2)*(-1/b)^(1/2)*Pi
^(1/2)*(a+b*arccos(c*x))^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b
*arccos(c*x))^(1/2)/b)*cos(a/b)*b^2*e+144*arccos(c*x)^2*cos(-(a+b*arccos(c
*x))/b+a/b)*b^2*c^2*d+216*arccos(c*x)*sin(-(a+b*arccos(c*x))/b+a/b)*b^2*c^
2*d+288*arccos(c*x)*cos(-(a+b*arccos(c*x))/b+a/b)*a*b*c^2*d+12*arccos(c*x)
^2*cos(-3*(a+b*arccos(c*x))/b+3*a/b)*b^2*e+36*arccos(c*x)^2*cos(-(a+b*arcc
os(c*x))/b+a/b)*b^2*e+216*sin(-(a+b*arccos(c*x))/b+a/b)*a*b*c^2*d+144*cos(
-(a+b*arccos(c*x))/b+a/b)*a^2*c^2*d+24*arccos(c*x)*cos(-3*(a+b*arccos(c*x)
)/b+3*a/b)*a*b*e+6*arccos(c*x)*sin(-3*(a+b*arccos(c*x))/b+3*a/b)*b^2*e+54*
arccos(c*x)*sin(-(a+b*arccos(c*x))/b+a/b)*b^2*e+72*arccos(c*x)*cos(-(a+b*a
rccos(c*x))/b+a/b)*a*b*e+12*cos(-3*(a+b*arccos(c*x))/b+3*a/b)*a^2*e+6*s...
```

Fricas [F(-2)]

Exception generated.

$$\int (d+ex^2) (a+b\arccos(cx))^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (d+ex^2) (a+b\arccos(cx))^{3/2}\,dx = \int (a+b\operatorname{acos}(cx))^{\frac{3}{2}} (d+ex^2)\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acos(c*x))**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**(3/2)*(d + e*x**2), x)
```

Maxima [F]

$$\int (d+ex^2) (a+b\arccos(cx))^{3/2}\,dx = \int (ex^2+d)(b\arccos(cx)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)*(b*arccos(c*x) + a)^(3/2), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 3270, normalized size of antiderivative = 6.78

$$\int (d+ex^2) (a+b\arccos(cx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)*(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
-1/96*(96*I*sqrt(2)*sqrt(pi)*a^2*b*c^2*d*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(
c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/
b)*e^(I*a/b)/(I*b^2/sqrt(abs(b)) + b*sqrt(abs(b))) - 48*sqrt(2)*sqrt(pi)*a
*b^2*c^2*d*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*s
qrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(I*b^2/sqrt(abs(b
)) + b*sqrt(abs(b))) - 96*I*sqrt(2)*sqrt(pi)*a^2*b*c^2*d*erf(1/2*I*sqrt(2)
*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a
)*sqrt(abs(b))/b)*e^(-I*a/b)/(-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b))) - 48*s
qrt(2)*sqrt(pi)*a*b^2*c^2*d*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt
(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/
(-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b))) - 48*I*sqrt(2)*sqrt(pi)*a^2*c^2*d*e
rf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(
b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(I*b/sqrt(abs(b)) + sqrt(abs(
b))) + 48*sqrt(2)*sqrt(pi)*a*b*c^2*d*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x)
 + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e
^(I*a/b)/(I*b/sqrt(abs(b)) + sqrt(abs(b))) + 36*I*sqrt(2)*sqrt(pi)*b^2*c^2
*d*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*s
qrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(I*b/sqrt(abs(b)) + sqrt(
abs(b))) + 48*I*sqrt(2)*sqrt(pi)*a^2*c^2*d*erf(1/2*I*sqrt(2)*sqrt(b*arccos
(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(...
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)(a+b\arccos(cx))^{3/2}\,dx=\int (a+b\,\mathrm{acos}(c\,x))^{3/2}\left(e\,x^2+d\right)\,dx$$

input
```
int((a + b*acos(c*x))^(3/2)*(d + e*x^2),x)
```

output
```
int((a + b*acos(c*x))^(3/2)*(d + e*x^2), x)
```

Reduce [F]

$$\int \left(d+ex^2\right)\left(a+b\arccos(cx)\right)^{3/2}dx=\left(\int \sqrt{acos\left(cx\right)b+a}dx\right)ad$$
$$+\left(\int \sqrt{acos\left(cx\right)b+a}\,acos(cx)\,x^2dx\right)be$$
$$+\left(\int \sqrt{acos\left(cx\right)b+a}\,acos(cx)\,dx\right)bd+\left(\int \sqrt{acos\left(cx\right)b+a}\,x^2dx\right)ae$$

input
```
int((e*x^2+d)*(a+b*acos(c*x))^(3/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a),x)*a*d + int(sqrt(acos(c*x)*b + a)*acos(c*x)*x**
2,x)*b*e + int(sqrt(acos(c*x)*b + a)*acos(c*x),x)*b*d + int(sqrt(acos(c*x)
*b + a)*x**2,x)*a*e
```

3.694 $\int (a+b\arccos(cx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 159

$$\int (a+b\arccos(cx))^{3/2}\,dx=\frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{2c}+x(a+b\arccos(cx))^{3/2}-\frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c}-\frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{2c}$$

output
```
3/2*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^(1/2)/c+x*(a+b*arccos(c*x))^(3/
2)-3/4*b^(3/2)*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*ar
ccos(c*x))^(1/2)/b^(1/2))/c-3/4*b^(3/2)*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/
Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.82

$$\int (a + b \arccos(cx))^{3/2}\, dx = \frac{\sqrt{b}\left(2\sqrt{b}\sqrt{a+b\arccos(cx)}\left(-3\sqrt{1-c^2x^2}+2cx\arccos(cx)\right)+\frac{2ia\sqrt{b}e^{-\frac{ia}{b}}\left(\sqrt{-\frac{i(a+b\,\mathrm{arcc}}{b}}\right.}{}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(Sqrt[b]*(2*Sqrt[b]*Sqrt[a + b*ArcCos[c*x]]*(-3*Sqrt[1 - c^2*x^2] + 2*c*x*
ArcCos[c*x]) + ((2*I)*a*Sqrt[b]*(Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[
3/2, ((-I)*(a + b*ArcCos[c*x]))/b] - E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos
[c*x]))/b]*Gamma[3/2, (I*(a + b*ArcCos[c*x]))/b]))/(E^((I*a)/b)*Sqrt[a + b
*ArcCos[c*x]]) + Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/
Sqrt[b]]*(3*b*Cos[a/b] + 2*a*Sin[a/b]) + Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*S
qrt[a + b*ArcCos[c*x]])/Sqrt[b]]*(2*a*Cos[a/b] - 3*b*Sin[a/b])))/(4*c)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5131, 5183, 5135, 25, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arccos(cx))^{3/2}\, dx$$

$$\Big\downarrow \; 5131$$

$$\frac{3}{2}bc\int \frac{x\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}dx + x(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{5183}$$

$$\frac{3}{2}bc\left(-\frac{b\int \frac{1}{\sqrt{a+b\arccos(cx)}}dx}{2c}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)+x(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{5135}$$

$$\frac{3}{2}bc\left(\frac{\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)+x(a+ b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{2}bc\left(-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)+x(a+ b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}bc\left(-\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)+x(a+ b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{3787}$$

$$\frac{3}{2}bc\left(\frac{-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\cos\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)+ x(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{2}bc\left(\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{c^2}\right)+ x(a+b\arccos(cx))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}bc\left(\frac{\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{2c^2}-\frac{\sqrt{1-c^2x}}{}\right.$$

$$x(a+b\arccos(cx))^{3/2}$$

$\big\downarrow$ 3785

$$\frac{3}{2}bc\left(\frac{\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c^2}-\frac{\sqrt{1-}}{}\right.$$

$$x(a+b\arccos(cx))^{3/2}$$

$\big\downarrow$ 3786

$$\frac{3}{2}bc\left(\frac{2\cos\left(\frac{a}{b}\right)\int\sin\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}-2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c^2}-\frac{\sqrt{}}{}\right.$$

$$x(a+b\arccos(cx))^{3/2}$$

$\big\downarrow$ 3832

$$\frac{3}{2}bc\left(\frac{\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{2c^2}-\frac{\sqrt{1-}}{}\right.$$

$$x(a+b\arccos(cx))^{3/2}$$

$\big\downarrow$ 3833

$$\frac{3}{2}bc\left(\frac{\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c^2}-\frac{\sqrt{1-c^2x}}{}\right.$$

$$x(a+b\arccos(cx))^{3/2}$$

input

```
Int[(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
x*(a + b*ArcCos[c*x])^(3/2) + (3*b*c*(-((Sqrt[1 - c^2*x^2]*Sqrt[a + b*ArcC
os[c*x]])/c^2) + (Sqrt[b]*Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a
+ b*ArcCos[c*x]])/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[
a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(2*c^2)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(123) = 246$.

Time = 0.05 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.75

method	result
default	$\frac{-3\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)b^2-3\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\mathrm{FresnelC}\left(\sqrt{}\right.}{}$

input
```
int((a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/c/(a+b*arccos(c*x))^(1/2)*(-3*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arcco
s(c*x))^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(
c*x))^(1/2)/b)*b^2-3*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)
*sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b
)*b^2+4*arccos(c*x)^2*cos(-(a+b*arccos(c*x))/b+a/b)*b^2+8*arccos(c*x)*cos(
-(a+b*arccos(c*x))/b+a/b)*a*b+6*arccos(c*x)*sin(-(a+b*arccos(c*x))/b+a/b)*
b^2+4*cos(-(a+b*arccos(c*x))/b+a/b)*a^2+6*sin(-(a+b*arccos(c*x))/b+a/b)*a*
b)
```

Fricas [F(-2)]

Exception generated.

$$\int (a + b \arccos(cx))^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (a + b \arccos(cx))^{3/2} \, dx = \int (a + b \operatorname{acos}(cx))^{\frac{3}{2}} \, dx$$

input
```
integrate((a+b*acos(c*x))**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**(3/2), x)
```

Maxima [F]

$$\int (a + b \arccos(cx))^{3/2} \, dx = \int (b \arccos(cx) + a)^{\frac{3}{2}} \, dx$$

input
```
integrate((a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^(3/2), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 1159, normalized size of antiderivative = 7.29

$$\int (a + b \arccos(cx))^{3/2} \, dx = \text{Too large to display}$$

input
```
integrate((a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
-I*sqrt(2)*sqrt(pi)*a^2*b^2*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqr
t(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/
((I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) + 1/2*sqrt(2)*sqrt(pi)*a*b^3*e
rf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(
b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^3/sqrt(abs(b)) + b^2*sq
rt(abs(b)))*c) + I*sqrt(2)*sqrt(pi)*a^2*b^2*erf(1/2*I*sqrt(2)*sqrt(b*arcco
s(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b)
)/b)*e^(-I*a/b)/((-I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) + 1/2*sqrt(2)
*sqrt(pi)*a*b^3*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1
/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^3/sqr
t(abs(b)) + b^2*sqrt(abs(b)))*c) + 1/2*I*sqrt(2)*sqrt(pi)*a^2*b*erf(-1/2*I
*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(
c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))
*c) - 1/2*sqrt(2)*sqrt(pi)*a*b^2*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a
)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*
a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) - 3/8*I*sqrt(2)*sqrt(pi)*b^
3*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sq
rt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(b)) + b*s
qrt(abs(b)))*c) - 1/2*I*sqrt(2)*sqrt(pi)*a^2*b*erf(1/2*I*sqrt(2)*sqrt(b*ar
ccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(...
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\arccos(cx))^{3/2}\,dx=\int (a+b\,\mathrm{acos}(c\,x))^{3/2}\,dx$$

input
```
int((a + b*acos(c*x))^(3/2),x)
```

output
```
int((a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int (a+b\arccos(cx))^{3/2}\,dx=\left(\int \sqrt{acos\,(cx)\,b+a}dx\right)a+\left(\int \sqrt{acos\,(cx)\,b+a}\,acos(cx)\,dx\right)b$$

input
```
int((a+b*acos(c*x))^(3/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a),x)*a + int(sqrt(acos(c*x)*b + a)*acos(c*x),x)*b
```

3.695 $\int \frac{(a+b\arccos(cx))^{3/2}}{d+ex^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\arccos(cx))^{3/2}}{d+ex^2}\,dx = \text{Int}\left(\frac{(a+b\arccos(cx))^{3/2}}{d+ex^2},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^(3/2)/(e*x^2+d),x)
```

Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\arccos(cx))^{3/2}}{d+ex^2}\,dx = \int \frac{(a+b\arccos(cx))^{3/2}}{d+ex^2}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^(3/2)/(d + e*x^2),x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^(3/2)/(d + e*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^{3/2}}{d+ex^2}dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{(a+b\arccos(cx))^{3/2}}{d+ex^2}dx$$

input
```
Int[(a + b*ArcCos[c*x])^(3/2)/(d + e*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arccos(cx))^{\frac{3}{2}}}{e\,x^2+d}dx$$

input
```
int((a+b*arccos(c*x))^(3/2)/(e*x^2+d),x)
```

output
```
int((a+b*arccos(c*x))^(3/2)/(e*x^2+d),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^{3/2}}{d+ex^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 11.67 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\arccos(cx))^{3/2}}{d+ex^2}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^{\frac{3}{2}}}{d+ex^2}\,dx$$

input
```
integrate((a+b*acos(c*x))**(3/2)/(e*x**2+d),x)
```

output
```
Integral((a + b*acos(c*x))**(3/2)/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^{3/2}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{d+ex^2}\,dx = \int \frac{(b\arccos(cx)+a)^{\frac{3}{2}}}{ex^2+d}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(3/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^(3/2)/(e*x^2 + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{d+ex^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^{3/2}}{e\,x^2+d}\,dx$$

input
```
int((a + b*acos(c*x))^(3/2)/(d + e*x^2),x)
```

output
```
int((a + b*acos(c*x))^(3/2)/(d + e*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{d+ex^2}\,dx = \int \frac{(acos(cx)\,b+a)^{\frac{3}{2}}}{e\,x^2+d}dx$$

input
```
int((a+b*acos(c*x))^(3/2)/(e*x^2+d),x)
```

output
```
int((a+b*acos(c*x))^(3/2)/(e*x^2+d),x)
```

3.696 $\int \frac{(a+b\arccos(cx))^{3/2}}{(d+ex^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\arccos(cx))^{3/2}}{(d+ex^2)^2}\,dx=\text{Int}\left(\frac{(a+b\arccos(cx))^{3/2}}{(d+ex^2)^2},x\right)$$

output
```
Defer(Int)((a+b*arccos(c*x))^(3/2)/(e*x^2+d)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 12.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\arccos(cx))^{3/2}}{(d+ex^2)^2}\,dx=\int \frac{(a+b\arccos(cx))^{3/2}}{(d+ex^2)^2}\,dx$$

input
```
Integrate[(a + b*ArcCos[c*x])^(3/2)/(d + e*x^2)^2,x]
```

output
```
Integrate[(a + b*ArcCos[c*x])^(3/2)/(d + e*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^{3/2}}{(d+ex^2)^2} \, dx$$

$$\Big\downarrow \text{ 5175}$$

$$\int \frac{(a+b\arccos(cx))^{3/2}}{(d+ex^2)^2}dx$$

input
```
Int[(a + b*ArcCos[c*x])^(3/2)/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arccos(cx))^{\frac{3}{2}}}{(e\,x^2+d)^2}dx$$

input
```
int((a+b*arccos(c*x))^(3/2)/(e*x^2+d)^2,x)
```

output
```
int((a+b*arccos(c*x))^(3/2)/(e*x^2+d)^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^{3/2}}{(d+ex^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 92.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arccos(cx))^{3/2}}{(d+ex^2)^2}\,dx = \int \frac{(a+b\,\mathrm{acos}\,(cx))^{\frac{3}{2}}}{(d+ex^2)^2}\,dx$$

input
```
integrate((a+b*acos(c*x))**(3/2)/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*acos(c*x))**(3/2)/(d + e*x**2)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{(d+ex^2)^2}\,dx = \int \frac{(b\arccos(cx)+a)^{\frac{3}{2}}}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{(d+ex^2)^2}\,dx = \int \frac{(b\arccos{(cx)}+a)^{\frac{3}{2}}}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{(d+ex^2)^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^{3/2}}{(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*acos(c*x))^(3/2)/(d + e*x^2)^2,x)
```

output
```
int((a + b*acos(c*x))^(3/2)/(d + e*x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{(d+ex^2)^2}\,dx=\int \frac{(acos(cx)\,b+a)^{\frac{3}{2}}}{(e\,x^2+d)^2}dx$$

input
```
int((a+b*acos(c*x))^(3/2)/(e*x^2+d)^2,x)
```

output
```
int((a+b*acos(c*x))^(3/2)/(e*x^2+d)^2,x)
```

3.697 $\int \frac{(d+ex^2)^2}{\sqrt{a+b\arccos(cx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 679

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\arccos(cx)}}\,dx = \text{Too large to display}$$

output
```
1/2*d*e*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*
x))^(1/2)/b^(1/2))/b^(1/2)/c^3+1/8*e^2*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelC(
2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(1/2)/c^5+d^2*2^(1/2)*
Pi^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2
))/b^(1/2)/c-1/6*d*e*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelC(6^(1/2)/Pi^(1/2)
*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(1/2)/c^3-1/16*e^2*6^(1/2)*Pi^(1/2)*co
s(3*a/b)*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(1/2
)/c^5+1/80*e^2*10^(1/2)*Pi^(1/2)*cos(5*a/b)*FresnelC(10^(1/2)/Pi^(1/2)*(a+
b*arccos(c*x))^(1/2)/b^(1/2))/b^(1/2)/c^5+1/2*d*e*2^(1/2)*Pi^(1/2)*Fresnel
S(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/b^(1/2)/c^3+1
/8*e^2*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/
b^(1/2))*sin(a/b)/b^(1/2)/c^5+d^2*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/
2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/b^(1/2)/c-1/6*d*e*6^(1/2)*Pi^
(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(3*a/b
)/b^(1/2)/c^3-1/16*e^2*6^(1/2)*Pi^(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*(a+b*arc
cos(c*x))^(1/2)/b^(1/2))*sin(3*a/b)/b^(1/2)/c^5+1/80*e^2*10^(1/2)*Pi^(1/2)
*FresnelS(10^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(5*a/b)/b^
(1/2)/c^5
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.59

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{e^{-\frac{5ia}{b}}\left(30(8c^4d^2+4c^2de+e^2)\,e^{\frac{4ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\,\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+30(8c^4d^2+4c^2de+e^2)\,e^{\frac{6ia}{b}}\right.}{}$$

input
```
Integrate[(d + e*x^2)^2/Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
(30*(8*c^4*d^2 + 4*c^2*d*e + e^2)*E^(((4*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos
[c*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/b] + 30*(8*c^4*d^2 + 4*c^
2*d*e + e^2)*E^(((6*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I
*(a + b*ArcCos[c*x]))/b] + e*(5*Sqrt[3]*(8*c^2*d + 3*e)*E^(((2*I)*a)/b)*Sq
rt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-3*I)*(a + b*ArcCos[c*x]))/b
] + 5*Sqrt[3]*(8*c^2*d + 3*e)*E^(((8*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))
/b]*Gamma[1/2, ((3*I)*(a + b*ArcCos[c*x]))/b] + 3*Sqrt[5]*e*(Sqrt[((-I)*(a
 + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-5*I)*(a + b*ArcCos[c*x]))/b] + E^(((10
*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((5*I)*(a + b*ArcCos[
c*x]))/b])))/(480*c^5*E^(((5*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\arccos(cx)}}\,dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(\frac{d^2}{\sqrt{a+b\arccos(cx)}}+\frac{2dex^2}{\sqrt{a+b\arccos(cx)}}+\frac{e^2x^4}{\sqrt{a+b\arccos(cx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}e^2\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^5}+\frac{\sqrt{\frac{3\pi}{2}}e^2\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^5}+\frac{\sqrt{\frac{\pi}{10}}e^2\sin\left(\frac{5a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^5}-\frac{\sqrt{\frac{\pi}{2}}e^2\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^5}-\frac{\sqrt{\frac{3\pi}{2}}e^2\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^5}-\frac{\sqrt{\frac{\pi}{10}}e^2\cos\left(\frac{5a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^5}+\frac{\sqrt{\frac{\pi}{2}}de\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{6}}de\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c^3}-\frac{\sqrt{\frac{\pi}{2}}de\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c^3}-\frac{\sqrt{\frac{\pi}{6}}de\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c^3}+\frac{\sqrt{2\pi}d^2\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c}-\frac{\sqrt{2\pi}d^2\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c}$$

input
```
Int[(d + e*x^2)^2/Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
-((d*e*Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/S
qrt[b]])/(Sqrt[b]*c^3)) - (e^2*Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sq
rt[a + b*ArcCos[c*x]])/Sqrt[b]])/(4*Sqrt[b]*c^5) - (d^2*Sqrt[2*Pi]*Cos[a/b
]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(Sqrt[b]*c) - (d
*e*Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/S
qrt[b]])/(Sqrt[b]*c^3) - (e^2*Sqrt[(3*Pi)/2]*Cos[(3*a)/b]*FresnelS[(Sqrt[6
/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(8*Sqrt[b]*c^5) - (e^2*Sqrt[Pi/10]
*Cos[(5*a)/b]*FresnelS[(Sqrt[10/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(8*
Sqrt[b]*c^5) + (d*e*Sqrt[Pi/2]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]
])/Sqrt[b]]*Sin[a/b])/(Sqrt[b]*c^3) + (e^2*Sqrt[Pi/2]*FresnelC[(Sqrt[2/Pi]
*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(4*Sqrt[b]*c^5) + (d^2*Sqrt[2
*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(Sqr
t[b]*c) + (d*e*Sqrt[Pi/6]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sq
rt[b]]*Sin[(3*a)/b])/(Sqrt[b]*c^3) + (e^2*Sqrt[(3*Pi)/2]*FresnelC[(Sqrt[6/
Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(8*Sqrt[b]*c^5) + (e^2
*Sqrt[Pi/10]*FresnelC[(Sqrt[10/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(
5*a)/b])/(8*Sqrt[b]*c^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 664, normalized size of antiderivative = 0.98

method	result
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{5}{b}}\left(-48\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\sin\left(\frac{a}{b}\right)\sqrt{-\frac{1}{b}}\,\sqrt{-\frac{5}{b}}\,b\,c^4 d^2-48\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\cos\left(\frac{a}{b}\right)\sqrt{-\frac{1}{b}}\,\sqrt{-}$

input
```
int((e*x^2+d)^2/(a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/240/c^5*2^(1/2)*Pi^(1/2)*(-5/b)^(1/2)*(-48*FresnelC(2^(1/2)/Pi^(1/2)/(-1
/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(a/b)*(-1/b)^(1/2)*(-5/b)^(1/2)*b*
c^4*d^2-48*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/
b)*cos(a/b)*(-1/b)^(1/2)*(-5/b)^(1/2)*b*c^4*d^2-8*FresnelC(3*2^(1/2)/Pi^(1
/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(3*a/b)*(-3/b)^(1/2)*(-5/b)
^(1/2)*b*c^2*d*e-8*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*
x))^(1/2)/b)*cos(3*a/b)*(-3/b)^(1/2)*(-5/b)^(1/2)*b*c^2*d*e-24*FresnelC(2^
(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(a/b)*(-1/b)^(1/
2)*(-5/b)^(1/2)*b*c^2*d*e-24*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*a
rccos(c*x))^(1/2)/b)*cos(a/b)*(-1/b)^(1/2)*(-5/b)^(1/2)*b*c^2*d*e-3*Fresne
lC(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(3*a/b)*(
-3/b)^(1/2)*(-5/b)^(1/2)*b*e^2-3*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*
(a+b*arccos(c*x))^(1/2)/b)*cos(3*a/b)*(-3/b)^(1/2)*(-5/b)^(1/2)*b*e^2-6*Fr
esnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(a/b)*(
-1/b)^(1/2)*(-5/b)^(1/2)*b*e^2-6*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a
+b*arccos(c*x))^(1/2)/b)*cos(a/b)*(-1/b)^(1/2)*(-5/b)^(1/2)*b*e^2+3*Fresne
lC(5*2^(1/2)/Pi^(1/2)/(-5/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(5*a/b)*e
^2+3*FresnelS(5*2^(1/2)/Pi^(1/2)/(-5/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*c
os(5*a/b)*e^2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\arccos(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)^2/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{(d+ex^2)^2}{\sqrt{a+b\operatorname{acos}(cx)}}\,dx$$

input
```
integrate((e*x**2+d)**2/(a+b*acos(c*x))**(1/2),x)
```

output
```
Integral((d + e*x**2)**2/sqrt(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{(ex^2+d)^2}{\sqrt{b\arccos(cx)+a}}\,dx$$

input
```
integrate((e*x^2+d)^2/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^2/sqrt(b*arccos(c*x) + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 981, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\arccos(cx)}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output
```
I*sqrt(pi)*d^2*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1
/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(c*(I*sqrt(2)
*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - I*sqrt(pi)*d^2*erf(1/2*I*sqrt(2
)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) +
a)*sqrt(abs(b))/b)*e^(-I*a/b)/(c*(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt
(abs(b)))) + 1/2*I*sqrt(pi)*d*e*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/s
qrt(b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)
/((sqrt(6)*sqrt(b) + I*sqrt(6)*b^(3/2)/abs(b))*c^3) + 1/2*I*sqrt(pi)*d*e*e
rf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(
b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(c^3*(I*sqrt(2)*b/sqrt(abs(b)
) + sqrt(2)*sqrt(abs(b)))) - 1/2*I*sqrt(pi)*d*e*erf(1/2*I*sqrt(2)*sqrt(b*a
rccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(ab
s(b))/b)*e^(-I*a/b)/(c^3*(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b))
)) - 1/2*I*sqrt(pi)*d*e*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) +
 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/((sqrt
(6)*sqrt(b) - I*sqrt(6)*b^(3/2)/abs(b))*c^3) + 1/16*I*sqrt(pi)*e^2*erf(-1/
2*sqrt(10)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sqrt(10)*sqrt(b*arccos(
c*x) + a)*sqrt(b)/abs(b))*e^(5*I*a/b)/((sqrt(10)*sqrt(b) + I*sqrt(10)*b^(3
/2)/abs(b))*c^5) + 3/16*I*sqrt(pi)*e^2*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x)
 + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{{\left(e\,x^2+d\right)}^2}{\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}}\,dx$$

input
```
int((d + e*x^2)^2/(a + b*acos(c*x))^(1/2),x)
```

output
```
int((d + e*x^2)^2/(a + b*acos(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\arccos(cx)}}\,dx = \left(\int \frac{\sqrt{acos\left(cx\right)b+a}}{acos\left(cx\right)b+a}dx\right)d^2 + \left(\int \frac{\sqrt{acos\left(cx\right)b+a}\,x^4}{acos\left(cx\right)b+a}dx\right)e^2 + 2\left(\int \frac{\sqrt{acos\left(cx\right)b+a}\,x^2}{acos\left(cx\right)b+a}dx\right)de$$

input

```
int((e*x^2+d)^2/(a+b*acos(c*x))^(1/2),x)
```

output

```
int(sqrt(acos(c*x)*b + a)/(acos(c*x)*b + a),x)*d**2 + int((sqrt(acos(c*x)*
b + a)*x**4)/(acos(c*x)*b + a),x)*e**2 + 2*int((sqrt(acos(c*x)*b + a)*x**2
)/(acos(c*x)*b + a),x)*d*e
```

3.698 $\int \frac{d+ex^2}{\sqrt{a+b\arccos(cx)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 329

$$\int \frac{d+ex^2}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{e\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3} + \frac{d\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c} - \frac{e\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3} + \frac{e\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{2\sqrt{b}c^3} + \frac{d\sqrt{2\pi}\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{\sqrt{b}c} - \frac{e\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{2\sqrt{b}c^3}$$

output
```
1/4*e*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x)
)^(1/2)/b^(1/2))/b^(1/2)/c^3+d*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelC(2^(1/2)/
Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(1/2)/c-1/12*e*6^(1/2)*Pi^(1/2
)*cos(3*a/b)*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^
(1/2)/c^3+1/4*e*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x
))^(1/2)/b^(1/2))*sin(a/b)/b^(1/2)/c^3+d*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)
/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/b^(1/2)/c-1/12*e*6^(1/
2)*Pi^(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin
(3*a/b)/b^(1/2)/c^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.74

$$\int \frac{d+ex^2}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{e^{-\frac{3ia}{b}}\left(3(4c^2d+e)\,e^{\frac{2ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+3(4c^2d+e)\,e^{\frac{4ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},\frac{i(a\cdots}{}\right.\right.}{24c^3\sqrt{a\cdots}}$$

input
```
Integrate[(d + e*x^2)/Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
(3*(4*c^2*d + e)*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[
1/2, ((-I)*(a + b*ArcCos[c*x]))/b] + 3*(4*c^2*d + e)*E^(((4*I)*a)/b)*Sqrt[
(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*ArcCos[c*x]))/b] + Sqrt[3]
*e*(Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-3*I)*(a + b*ArcCos[c*
x]))/b] + E^(((6*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((3*I
)*(a + b*ArcCos[c*x]))/b]))/(24*c^3*E^(((3*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]
])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{\sqrt{a+b\arccos(cx)}}\,dx$$

$$\Big\downarrow \text{5173}$$

$$\int \left(\frac{d}{\sqrt{a+b\arccos(cx)}}+\frac{ex^2}{\sqrt{a+b\arccos(cx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}e\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3}+\frac{\sqrt{\frac{\pi}{6}}e\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3}-\frac{\sqrt{\frac{\pi}{2}}e\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3}-\frac{\sqrt{\frac{\pi}{6}}e\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3}+\frac{\sqrt{2\pi}d\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c}-\frac{\sqrt{2\pi}d\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c}$$

input
```
Int[(d + e*x^2)/Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
-1/2*(e*Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/
Sqrt[b]])/(Sqrt[b]*c^3) - (d*Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt
[a + b*ArcCos[c*x]])/Sqrt[b]])/(Sqrt[b]*c) - (e*Sqrt[Pi/6]*Cos[(3*a)/b]*Fr
esnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(2*Sqrt[b]*c^3) + (e
*Sqrt[Pi/2]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b
])/(2*Sqrt[b]*c^3) + (d*Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[
c*x]])/Sqrt[b]]*Sin[a/b])/(Sqrt[b]*c) + (e*Sqrt[Pi/6]*FresnelC[(Sqrt[6/Pi]
*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(2*Sqrt[b]*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.94

method	result
default	$-\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{3}{b}}\,\left(4\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\sin\left(\frac{a}{b}\right)\sqrt{-\frac{1}{b}}\,\sqrt{-\frac{3}{b}}\,b\,c^{2}d+4\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\cos\left(\frac{a}{b}\right)\sqrt{-\frac{1}{b}}\,\sqrt{-\frac{3}{b}}\right.}{}$

input
```
int((e*x^2+d)/(a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/c^3*2^(1/2)*Pi^(1/2)*(-3/b)^(1/2)*(4*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b
)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(a/b)*(-1/b)^(1/2)*(-3/b)^(1/2)*b*c^
2*d+4*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*co
s(a/b)*(-1/b)^(1/2)*(-3/b)^(1/2)*b*c^2*d+FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^
(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(a/b)*(-1/b)^(1/2)*(-3/b)^(1/2)*b*e+Fr
esnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*cos(a/b)*(
-1/b)^(1/2)*(-3/b)^(1/2)*b*e-e*sin(3*a/b)*FresnelC(3*2^(1/2)/Pi^(1/2)/(-3/
b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-e*cos(3*a/b)*FresnelS(3*2^(1/2)/Pi^(1/
2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d+ex^2}{\sqrt{a+b\arccos(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{d+ex^2}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{d+ex^2}{\sqrt{a+b\,\mathrm{acos}\,(cx)}}\,dx$$

input
```
integrate((e*x**2+d)/(a+b*acos(c*x))**(1/2),x)
```

output
```
Integral((d + e*x**2)/sqrt(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{ex^2+d}{\sqrt{b\arccos\,(cx)+a}}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/sqrt(b*arccos(c*x) + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.46

$$\int \frac{d+ex^2}{\sqrt{a+b\arccos(cx)}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output
```
I*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2
*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(c*(I*sqrt(2)*b
/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - I*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sq
rt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*s
qrt(abs(b))/b)*e^(-I*a/b)/(c*(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs
(b)))) + 1/4*I*sqrt(pi)*e*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b)
 - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/((sqr
t(6)*sqrt(b) + I*sqrt(6)*b^(3/2)/abs(b))*c^3) + 1/4*I*sqrt(pi)*e*erf(-1/2*
I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos
(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(c^3*(I*sqrt(2)*b/sqrt(abs(b)) + sqrt
(2)*sqrt(abs(b)))) - 1/4*I*sqrt(pi)*e*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x)
 + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e
^(-I*a/b)/(c^3*(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - 1/4*I
*sqrt(pi)*e*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(
6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)*sqrt(b)
- I*sqrt(6)*b^(3/2)/abs(b))*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{e\,x^2+d}{\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}}\,dx$$

input
```
int((d + e*x^2)/(a + b*acos(c*x))^(1/2),x)
```

output
```
int((d + e*x^2)/(a + b*acos(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{d+ex^2}{\sqrt{a+b\arccos(cx)}}\,dx = \left(\int \frac{\sqrt{acos\,(cx)\,b+a}}{acos\,(cx)\,b+a}dx\right)d+\left(\int \frac{\sqrt{acos\,(cx)\,b+a}\,x^2}{acos\,(cx)\,b+a}dx\right)e$$

input
```
int((e*x^2+d)/(a+b*acos(c*x))^(1/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)/(acos(c*x)*b + a),x)*d + int((sqrt(acos(c*x)*b +
 a)*x**2)/(acos(c*x)*b + a),x)*e
```

3.699 $\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c} + \frac{\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{\sqrt{b}c}$$

output
```
2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2
)/b^(1/2))/b^(1/2)/c+2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arcco
s(c*x))^(1/2)/b^(1/2))*sin(a/b)/b^(1/2)/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{e^{-\frac{ia}{b}}\left(\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+e^{\frac{2ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},\frac{i(a+b\arccos(cx))}{b}\right)\right)}{2c\sqrt{a+b\arccos(cx)}}$$

input
```
Integrate[1/Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
(Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/
b] + E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*
ArcCos[c*x]))/b])/(2*c*E^((I*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5135, 25, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}dx$$

$$\Big\downarrow \text{5135}$$

$$-\frac{\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3787}$$

$$-\frac{-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\cos\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{bc}$$

$$\Big\downarrow \text{3785}$$

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-2\sin\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{bc}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{2\cos\left(\frac{a}{b}\right)\int \sin\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}-2\sin\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{bc}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-2\sin\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}}{bc}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)-\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{bc}$$

input
```
Int[1/Sqrt[a + b*ArcCos[c*x]],x]
```

output
```
-((Sqrt[b]*Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]
])/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*
x]])/Sqrt[b]]*Sin[a/b])/(b*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\left(\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos\left(cx\right)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)+\sin\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos\left(cx\right)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\right)}{c}$	89

input
```
int(1/(a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(
1/2)*(a+b*arccos(c*x))^(1/2)/b)+sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^
(1/2)*(a+b*arccos(c*x))^(1/2)/b))/c
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{acos}\,(cx)}}\,dx$$

input
```
integrate(1/(a+b*acos(c*x))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{1}{\sqrt{b\arccos(cx)+a}}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arccos(c*x) + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx=\frac{i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2b}\right)e^{\left(\frac{ia}{b}\right)}}{c\left(\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)}-\frac{i\sqrt{\pi}\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2b}\right)e^{\left(-\frac{ia}{b}\right)}}{c\left(-\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)}$$

input
```
integrate(1/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output
```
I*sqrt(pi)*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*s
qrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(c*(I*sqrt(2)*b/s
qrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - I*sqrt(pi)*erf(1/2*I*sqrt(2)*sqrt(b
*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(
abs(b))/b)*e^(-I*a/b)/(c*(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b))
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}}\,dx$$

input
```
int(1/(a + b*acos(c*x))^(1/2),x)
```

output
```
int(1/(a + b*acos(c*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{\sqrt{acos\left(cx\right)b+a}}{acos\left(cx\right)b+a}dx$$

input
```
int(1/(a+b*acos(c*x))^(1/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)/(acos(c*x)*b + a),x)
```

3.700 $\int \frac{1}{(d+ex^2)\sqrt{a+b\arccos(cx)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\arccos(cx)}}\,dx=\text{Int}\left(\frac{1}{(d+ex^2)\sqrt{a+b\arccos(cx)}},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)/(a+b*arccos(c*x))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{1}{(d+ex^2)\sqrt{a+b\arccos(cx)}}\,dx$$

input
```
Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcCos[c*x]]),x]
```

output
```
Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcCos[c*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\arccos(cx)}}\,dx$$
$$\Big\downarrow \text{5175}$$
$$\int \frac{1}{(d+ex^2)\sqrt{a+b\arccos(cx)}}dx$$

input
```
Int[1/((d + e*x^2)*Sqrt[a + b*ArcCos[c*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)\sqrt{a+b\arccos\left(cx\right)}}dx$$

input
```
int(1/(e*x^2+d)/(a+b*arccos(c*x))^(1/2),x)
```

output
```
int(1/(e*x^2+d)/(a+b*arccos(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\arccos(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\mathrm{acos}\,(cx)}\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*acos(c*x))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*acos(c*x))*(d + e*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\arccos(cx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{1}{(ex^2+d)\sqrt{b\arccos{(cx)}+a}}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*sqrt(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/((a + b*acos(c*x))^(1/2)*(d + e*x^2)),x)
```

output
```
int(1/((a + b*acos(c*x))^(1/2)*(d + e*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{\sqrt{acos\,(cx)\,b+a}}{acos\,(cx)\,bd+acos\,(cx)\,be\,x^2+ad+ae\,x^2}dx$$

input
```
int(1/(e*x^2+d)/(a+b*acos(c*x))^(1/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)/(acos(c*x)*b*d + acos(c*x)*b*e*x**2 + a*d + a*e*
x**2),x)
```

3.701 $\int \frac{1}{(d+ex^2)^2 \sqrt{a+b \arccos(cx)}} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\arccos(cx)}}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)^2 \sqrt{a+b\arccos(cx)}},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^2/(a+b*arccos(c*x))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{(d+ex^2)^2 \sqrt{a+b\arccos(cx)}}\,dx$$

input
```
Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcCos[c*x]]),x]
```

output
```
Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcCos[c*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\arccos(cx)}}dx$$
$$\Big\downarrow \text{5175}$$
$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\arccos(cx)}}dx$$

input
```
Int[1/((d + e*x^2)^2*Sqrt[a + b*ArcCos[c*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^2 \sqrt{a+b\arccos\left(cx\right)}}dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*arccos(c*x))^(1/2),x)
```

output
```
int(1/(e*x^2+d)^2/(a+b*arccos(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\arccos(cx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 39.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\mathrm{acos}\,(cx)}\,(d+ex^2)^2}\,dx$$

input
```
integrate(1/(e*x**2+d)**2/(a+b*acos(c*x))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*acos(c*x))*(d + e*x**2)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{(ex^2+d)^2\sqrt{b\arccos(cx)+a}}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^2*sqrt(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{1}{(ex^2+d)^2\sqrt{b\arccos{(cx)}+a}}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^2*sqrt(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}\,{\left(e\,x^2+d\right)}^2}\,dx$$

input
```
int(1/((a + b*acos(c*x))^(1/2)*(d + e*x^2)^2),x)
```

output
```
int(1/((a + b*acos(c*x))^(1/2)*(d + e*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.14

$$\int \frac{1}{(d+ex^2)^2 \sqrt{a+b\arccos(cx)}}\,dx$$
$$= \int \frac{\sqrt{acos\,(cx)\,b+a}}{acos\,(cx)\,b\,d^2+2acos\,(cx)\,bde\,x^2+acos\,(cx)\,b\,e^2x^4+a\,d^2+2ade\,x^2+a\,e^2x^4}dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*acos(c*x))^(1/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)/(acos(c*x)*b*d**2 + 2*acos(c*x)*b*d*e*x**2 + aco
s(c*x)*b*e**2*x**4 + a*d**2 + 2*a*d*e*x**2 + a*e**2*x**4),x)
```

3.702 $\int \frac{d+ex^2}{(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 394

$$\begin{aligned}\int \frac{d+ex^2}{(a+b\arccos(cx))^{3/2}}\,dx &= -\frac{2d\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} \\
&- \frac{2ex^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{e\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} \\
&- \frac{2d\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} \\
&+ \frac{e\sqrt{\frac{3\pi}{2}}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} \\
&+ \frac{e\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{b^{3/2}c^3} \\
&+ \frac{2d\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{b^{3/2}c} \\
&- \frac{e\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{b^{3/2}c^3}\end{aligned}$$

output
```
-2*d*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^(1/2)-2*e*x^2*(-c^2*x^2+1)^(
1/2)/b/c/(a+b*arccos(c*x))^(1/2)-1/2*e*2^(1/2)*Pi^(1/2)*cos(a/b)*FresnelS(
2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3-2*d*2^(1/2)*
Pi^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2
))/b^(3/2)/c+1/2*e*6^(1/2)*Pi^(1/2)*cos(3*a/b)*FresnelS(6^(1/2)/Pi^(1/2)*(
a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(3/2)/c^3+1/2*e*2^(1/2)*Pi^(1/2)*Fresnel
C(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)/b^(3/2)/c^3+2
*d*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1
/2))*sin(a/b)/b^(3/2)/c-1/2*e*6^(1/2)*Pi^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*(
a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(3*a/b)/b^(3/2)/c^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.82

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^{3/2}}\,dx = \frac{e^{-\frac{3ia}{b}}\left(8c^2de^{\frac{3ia}{b}}\sqrt{1-c^2x^2}+8c^2ee^{\frac{3ia}{b}}x^2\sqrt{1-c^2x^2}+i(4c^2d+e)\,e^{\frac{2ia}{b}}\sqrt{-\frac{i(a+b}{}}\right.}{}$$

input
```
Integrate[(d + e*x^2)/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(8*c^2*d*E^(((3*I)*a)/b)*Sqrt[1 - c^2*x^2] + 8*c^2*e*E^(((3*I)*a)/b)*x^2*S
qrt[1 - c^2*x^2] + I*(4*c^2*d + e)*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCo
s[c*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/b] - I*(4*c^2*d + e)*E^(
((4*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*ArcCos[c
*x]))/b] + I*Sqrt[3]*e*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-3*
I)*(a + b*ArcCos[c*x]))/b] - I*Sqrt[3]*e*E^(((6*I)*a)/b)*Sqrt[(I*(a + b*Ar
cCos[c*x]))/b]*Gamma[1/2, ((3*I)*(a + b*ArcCos[c*x]))/b])/(4*b*c^3*E^(((3*
I)*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5173, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^{3/2}}\,dx$$

$$\Big\downarrow \ 5173$$

$$\int \left(\frac{d}{(a+b\arccos(cx))^{3/2}}+\frac{ex^2}{(a+b\arccos(cx))^{3/2}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sqrt{\frac{\pi}{2}}e\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}-\frac{\sqrt{\frac{3\pi}{2}}e\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}-\frac{\sqrt{\frac{\pi}{2}}e\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}-\frac{\sqrt{\frac{3\pi}{2}}e\sin\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}-\frac{2\sqrt{2\pi}d\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{2\pi}d\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{2d\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}+\frac{2ex^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

input
```
Int[(d + e*x^2)/(a + b*ArcCos[c*x])^(3/2),x]
```

output
```
(2*d*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[a + b*ArcCos[c*x]]) + (2*e*x^2*Sqrt[1 -
c^2*x^2])/(b*c*Sqrt[a + b*ArcCos[c*x]]) - (e*Sqrt[Pi/2]*Cos[a/b]*FresnelC[
(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(b^(3/2)*c^3) - (2*d*Sqrt[2
*Pi]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(b^(
3/2)*c) - (e*Sqrt[(3*Pi)/2]*Cos[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*A
rcCos[c*x]])/Sqrt[b]])/(b^(3/2)*c^3) - (e*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*
Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(b^(3/2)*c^3) - (2*d*Sqrt[2*Pi
]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(b^(3/2
)*c) - (e*Sqrt[(3*Pi)/2]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqr
t[b]]*Sin[(3*a)/b])/(b^(3/2)*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5173
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCos[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.16

method	result
default	$-\frac{4\sqrt{2}\,\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\cos\left(\frac{a}{b}\right)c^{2}d-4\sqrt{2}\,\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+}}{\sqrt{\pi}}\right.}{}$

input
```
int((e*x^2+d)/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^3/b*(4*2^(1/2)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*Fresne
lC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*cos(a/b)*c^2*d
-4*2^(1/2)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*FresnelS(2^(1/2)/
Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(a/b)*c^2*d+2^(1/2)*(-
3/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*FresnelC(3*2^(1/2)/Pi^(1/2)/(-
3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*cos(3*a/b)*e-2^(1/2)*(-3/b)^(1/2)*Pi
^(1/2)*(a+b*arccos(c*x))^(1/2)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a
+b*arccos(c*x))^(1/2)/b)*sin(3*a/b)*e+2^(1/2)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*a
rccos(c*x))^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))
^(1/2)/b)*cos(a/b)*e-2^(1/2)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)
*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(a/b
)*e+4*sin(-(a+b*arccos(c*x))/b+a/b)*c^2*d+e*sin(-3*(a+b*arccos(c*x))/b+3*a
/b)+e*sin(-(a+b*arccos(c*x))/b+a/b))/(a+b*arccos(c*x))^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{d+ex^2}{(a+b\operatorname{acos}(cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x**2+d)/(a+b*acos(c*x))**(3/2),x)
```

output
```
Integral((d + e*x**2)/(a + b*acos(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{ex^2+d}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)/(b*arccos(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{ex^2+d}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)/(b*arccos(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex^2}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{e\,x^2+d}{(a+b\,\mathrm{acos}(c\,x))^{3/2}}\,dx$$

input
```
int((d + e*x^2)/(a + b*acos(c*x))^(3/2),x)
```

output
```
int((d + e*x^2)/(a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{d + ex^2}{(a + b \arccos(cx))^{3/2}} \, dx = \text{too large to display}$$

input
```
int((e*x^2+d)/(a+b*acos(c*x))^(3/2),x)
```

output
```
( - 4*sqrt(acos(c*x)*b + a)*sqrt( - c**2*x**2 + 1)*acos(c*x)*b*c**2*d + 8*
sqrt(acos(c*x)*b + a)*sqrt( - c**2*x**2 + 1)*acos(c*x)*b*e + 4*acos(c*x)*i
nt(sqrt(acos(c*x)*b + a)/(acos(c*x)**2*b**2*c**2*x**2 - acos(c*x)**2*b**2
+ 2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*
a*b**2*c**3*d - 8*acos(c*x)*int(sqrt(acos(c*x)*b + a)/(acos(c*x)**2*b**2*c
**2*x**2 - acos(c*x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b
 + a**2*c**2*x**2 - a**2),x)*a*b**2*c*e - 4*acos(c*x)*int((sqrt(acos(c*x)*
b + a)*x**4)/(acos(c*x)**2*b**2*c**2*x**2 - acos(c*x)**2*b**2 + 2*acos(c*x
)*a*b*c**2*x**2 - 2*acos(c*x)*a*b + a**2*c**2*x**2 - a**2),x)*a*b**2*c**7*
d + 8*acos(c*x)*int((sqrt(acos(c*x)*b + a)*x**4)/(acos(c*x)**2*b**2*c**2*x
**2 - acos(c*x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b + a*
*2*c**2*x**2 - a**2),x)*a*b**2*c**5*e - 24*acos(c*x)*int((sqrt(acos(c*x)*b
 + a)*sqrt( - c**2*x**2 + 1)*acos(c*x)*x**3)/(acos(c*x)**2*b**2*c**2*x**2
- acos(c*x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b + a**2*c
**2*x**2 - a**2),x)*a*b**2*c**6*d + 4*acos(c*x)*int((sqrt(acos(c*x)*b + a)
*sqrt( - c**2*x**2 + 1)*acos(c*x)**2*x)/(acos(c*x)**2*b**2*c**2*x**2 - aco
s(c*x)**2*b**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b + a**2*c**2*x
**2 - a**2),x)*b**3*c**4*d - 8*acos(c*x)*int((sqrt(acos(c*x)*b + a)*sqrt(
- c**2*x**2 + 1)*acos(c*x)**2*x)/(acos(c*x)**2*b**2*c**2*x**2 - acos(c*x)*
*2*b**2 + 2*acos(c*x)*a*b*c**2*x**2 - 2*acos(c*x)*a*b + a**2*c**2*x**2 ...
```

3.703 $\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 137

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx = -\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{2\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{2\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{b^{3/2}c}$$

output
```
-2*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^(1/2)-2*2^(1/2)*Pi^(1/2)*cos(a
/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))/b^(3/2)/c+2
*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2
))*sin(a/b)/b^(3/2)/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx =$$

$$-\frac{ie^{-\frac{ia}{b}}\left(2ie^{\frac{ia}{b}}\sqrt{1-c^2x^2}-\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+e^{\frac{2ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},\frac{i(a+b\arccos(cx}{b}\right.\right.}{bc\sqrt{a+b\arccos(cx)}}$$

input
```
Integrate[(a + b*ArcCos[c*x])^(-3/2),x]
```

output
```
((-I)*((2*I)*E^((I*a)/b)*Sqrt[1 - c^2*x^2] - Sqrt[((-I)*(a + b*ArcCos[c*x]
))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/b] + E^(((2*I)*a)/b)*Sqrt[(I*(
a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*ArcCos[c*x]))/b]))/(b*c*E^((I*
a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5133, 5225, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx$$

$$\Big\downarrow \text{5133}$$

$$\frac{2c\int \frac{x}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}dx}{b}+\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

$$\Big\downarrow \text{5225}$$

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\int \frac{\cos\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))}{b^2c}$$

$$\Big\downarrow \text{ 3787}$$

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\left(\cos\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))-\sin\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\left(\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\left(\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))+\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))\right)}{b^2c}$$

$$\Big\downarrow \text{ 3785}$$

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\left(\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(cx)}{b}\right)}{\sqrt{a+b\arccos(cx)}}d(a+b\arccos(cx))+2\cos\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c}$$

$$\Big\downarrow \text{ 3786}$$

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\left(2\sin\left(\frac{a}{b}\right)\int\sin\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}+2\cos\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}\right)}{b^2c}$$

$\Big\downarrow$ 3832

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\left(2\cos\left(\frac{a}{b}\right)\int\cos\left(\frac{a+b\arccos(cx)}{b}\right)d\sqrt{a+b\arccos(cx)}+\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\right)}{b^2c}$$

$\Big\downarrow$ 3833

$$\frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\left(\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)+\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\right)}{b^2c}$$

input
```
Int[(a + b*ArcCos[c*x])^(-3/2),x]
```

output
```
(2*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[a + b*ArcCos[c*x]]) - (2*(Sqrt[b]*Sqrt[2*P
i]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]] + Sqrt[
b]*Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a
/b]))/(b^2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
default	$-\frac{2\left(\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)-\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\right.}{}\right.}{cb\sqrt{a+b\arccos(cx)}}$

input
```
int(1/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/c/b*((-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(a/b)*Fre
snelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-(-1/b)^(1/2
)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelS(2^(1/2)/Pi^(1
/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+sin(-(a+b*arccos(c*x))/b+a/b))
/(a+b*arccos(c*x))^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\operatorname{acos}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*acos(c*x))**(3/2),x)
```

output
```
Integral((a + b*acos(c*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{\left(b\arccos\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{\left(b\arccos\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*acos(c*x))^(3/2),x)
```

output
```
int(1/(a + b*acos(c*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx = \frac{-2acos(cx)\left(\int \frac{\sqrt{acos(cx)b+a}\,\sqrt{-c^2x^2+1}\,x}{acos(cx)b\,c^2x^2-acos(cx)b+a\,c^2x^2-a}dx\right)b\,c^2+2\sqrt{acos\left(cx\right)b+a}\,\sqrt{-}}{bc\left(acos\left(cx\right)b+a\right)}$$

input
```
int(1/(a+b*acos(c*x))^(3/2),x)
```

output
```
(2*( - acos(c*x)*int((sqrt(acos(c*x)*b + a)*sqrt( - c**2*x**2 + 1)*x)/(aco
s(c*x)*b*c**2*x**2 - acos(c*x)*b + a*c**2*x**2 - a),x)*b*c**2 + sqrt(acos(
c*x)*b + a)*sqrt( - c**2*x**2 + 1) - int((sqrt(acos(c*x)*b + a)*sqrt( - c*
*2*x**2 + 1)*x)/(acos(c*x)*b*c**2*x**2 - acos(c*x)*b + a*c**2*x**2 - a),x)
*a*c**2))/(b*c*(acos(c*x)*b + a))
```

3.704 $\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{(d+ex^2)(a+b\arccos(cx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)/(a+b*arccos(c*x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{(d+ex^2)(a+b\arccos(cx))^{3/2}}\,dx$$

input
```
Integrate[1/((d + e*x^2)*(a + b*ArcCos[c*x])^(3/2)),x]
```

output
```
Integrate[1/((d + e*x^2)*(a + b*ArcCos[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^{3/2}} \, dx$$
$$\Big\downarrow \text{ 5175}$$
$$\int \frac{1}{(d+ex^2)(a+b\arccos(cx))^{3/2}}dx$$

input
```
Int[1/((d + e*x^2)*(a + b*ArcCos[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)\,(a+b\arccos\,(cx))^{\frac{3}{2}}}dx$$

input
```
int(1/(e*x^2+d)/(a+b*arccos(c*x))^(3/2),x)
```

output
```
int(1/(e*x^2+d)/(a+b*arccos(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 9.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(cx))^{\frac{3}{2}}\,(d+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d)/(a+b*acos(c*x))**(3/2),x)
```

output
```
Integral(1/((a + b*acos(c*x))**(3/2)*(d + e*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{(ex^2+d)(b\arccos\,(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)*(b*arccos(c*x) + a)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{(ex^2+d)(b\arccos{(cx)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)*(b*arccos(c*x) + a)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\left(c\,x\right))^{3/2}\,\left(e\,x^2+d\right)}\,dx$$

input
```
int(1/((a + b*acos(c*x))^(3/2)*(d + e*x^2)),x)
```

output
```
int(1/((a + b*acos(c*x))^(3/2)*(d + e*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.36

$$\int \frac{1}{(d+ex^2)\,(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{\sqrt{acos\,(cx)\,b+a}}{acos\,(cx)^2\,b^2d+acos\,(cx)^2\,b^2e\,x^2+2acos\,(cx)\,abd+2acos\,(cx)\,a}$$

input
```
int(1/(e*x^2+d)/(a+b*acos(c*x))^(3/2),x)
```

output
```
int(sqrt(acos(c*x)*b + a)/(acos(c*x)**2*b**2*d + acos(c*x)**2*b**2*e*x**2
+ 2*acos(c*x)*a*b*d + 2*acos(c*x)*a*b*e*x**2 + a**2*d + a**2*e*x**2),x)
```

3.705 $\int \frac{1}{(d+ex^2)^2(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^{3/2}}\,dx = \mathrm{Int}\left(\frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/(e*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^{3/2}}\,dx$$

input
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcCos[c*x])^(3/2)),x]
```

output
```
Integrate[1/((d + e*x^2)^2*(a + b*ArcCos[c*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^{3/2}} \, dx$$
$$\Big\downarrow \text{5175}$$
$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^{3/2}}dx$$

input
```
Int[1/((d + e*x^2)^2*(a + b*ArcCos[c*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(e\,x^2+d)^2 \left(a+b\arccos\left(cx\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(e*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x)
```

output
```
int(1/(e*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x**2+d)**2/(a+b*acos(c*x))**(3/2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{(ex^2+d)^2 (b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arccos(c*x) + a)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 3.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{(ex^2+d)^2(b\arccos{(cx)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x^2+d)^2/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((e*x^2 + d)^2*(b*arccos(c*x) + a)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))^{3/2}\,(e\,x^2+d)^2}\,dx$$

input
```
int(1/((a + b*acos(c*x))^(3/2)*(d + e*x^2)^2),x)
```

output
```
int(1/((a + b*acos(c*x))^(3/2)*(d + e*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 125, normalized size of antiderivative = 5.68

$$\int \frac{1}{(d+ex^2)^2 (a+b\arccos(cx))^{3/2}}\,dx = \int \frac{}{acos\,(cx)^2\,b^2d^2+2acos\,(cx)^2\,b^2de\,x^2+acos\,(cx)^2\,b^2e^2x^4+2aco}$$

input
```
int(1/(e*x^2+d)^2/(a+b*acos(c*x))^(3/2),x)
```

output

```
int(sqrt(acos(c*x)*b + a)/(acos(c*x)**2*b**2*d**2 + 2*acos(c*x)**2*b**2*d*
e*x**2 + acos(c*x)**2*b**2*e**2*x**4 + 2*acos(c*x)*a*b*d**2 + 4*acos(c*x)*
a*b*d*e*x**2 + 2*acos(c*x)*a*b*e**2*x**4 + a**2*d**2 + 2*a**2*d*e*x**2 + a
**2*e**2*x**4),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

def is_atom(expn):
  try:
    if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
      return True
    else:
      return False

  except AttributeError as error:
    return False

def expnType(expn):
  debug=False
  if debug:
    print("expn=",expn,"type(expn)=",type(expn))

  if is_atom(expn):
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
  elif is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):  #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])  #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file