Computer Algebra Independent Integration Tests

Summer 2024

5-Inverse-trig-functions/5.3-Inverse-tangent/277-5.3.2

Nasser M. Abbasi

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CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**166**]. This is test number [277].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (166)	0.00 (0)
Mathematica	98.19 (163)	1.81 (3)
Maple	90.96 (151)	9.04 (15)
Mupad	65.06 (108)	34.94 (58)
Reduce	62.05 (103)	37.95 (63)
Giac	59.04 (98)	40.96 (68)
Maxima	56.02 (93)	43.98 (73)
Sympy	56.02 (93)	43.98 (73)
Fricas	55.42 (92)	44.58 (74)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

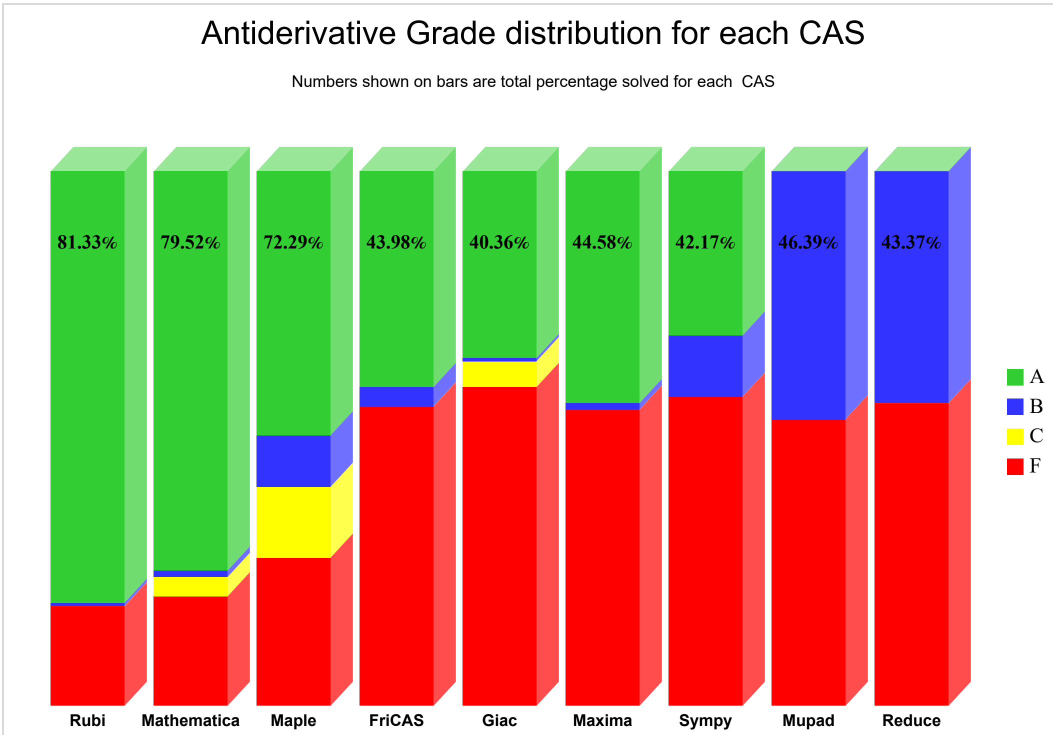
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

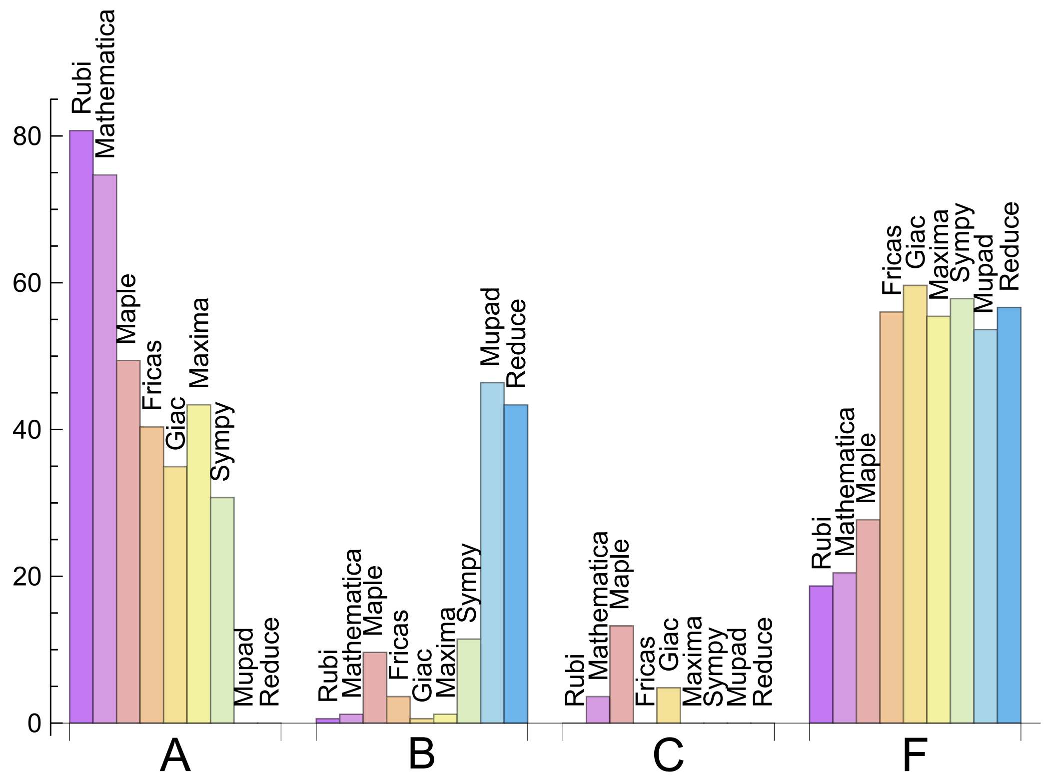
System	% A grade	% B grade	% C grade	% F grade
Rubi	80.723	0.602	0.000	18.675
Mathematica	74.699	1.205	3.614	20.482
Maple	49.398	9.639	13.253	27.711
Maxima	43.373	1.205	0.000	55.422
Fricas	40.361	3.614	0.000	56.024
Giac	34.940	0.602	4.819	59.639
Sympy	30.723	11.446	0.000	57.831
Mupad	0.000	46.386	0.000	53.614
Reduce	0.000	43.373	0.000	56.627

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	3	100.00	0.00	0.00
Maple	15	100.00	0.00	0.00
Mupad	58	0.00	100.00	0.00
Maxima	73	82.19	1.37	16.44
Fricas	74	83.78	0.00	16.22
Reduce	63	100.00	0.00	0.00
Sympy	73	82.19	17.81	0.00
Giac	68	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.10
Giac	0.15
Reduce	0.19
Maxima	0.33
Rubi	0.58
Mathematica	0.59
Maple	0.62
Mupad	0.75
Sympy	12.27

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	50.70	0.99	38.00	0.99
Giac	58.65	1.14	45.50	1.11
Reduce	69.63	1.76	50.00	1.20
Fricas	70.30	1.18	45.50	1.06
Maxima	88.04	2.57	51.00	1.08
Sympy	91.25	1.45	58.00	1.12
Rubi	133.44	1.09	75.50	1.00
Mathematica	161.80	1.22	65.00	1.13
Maple	516.22	4.07	60.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

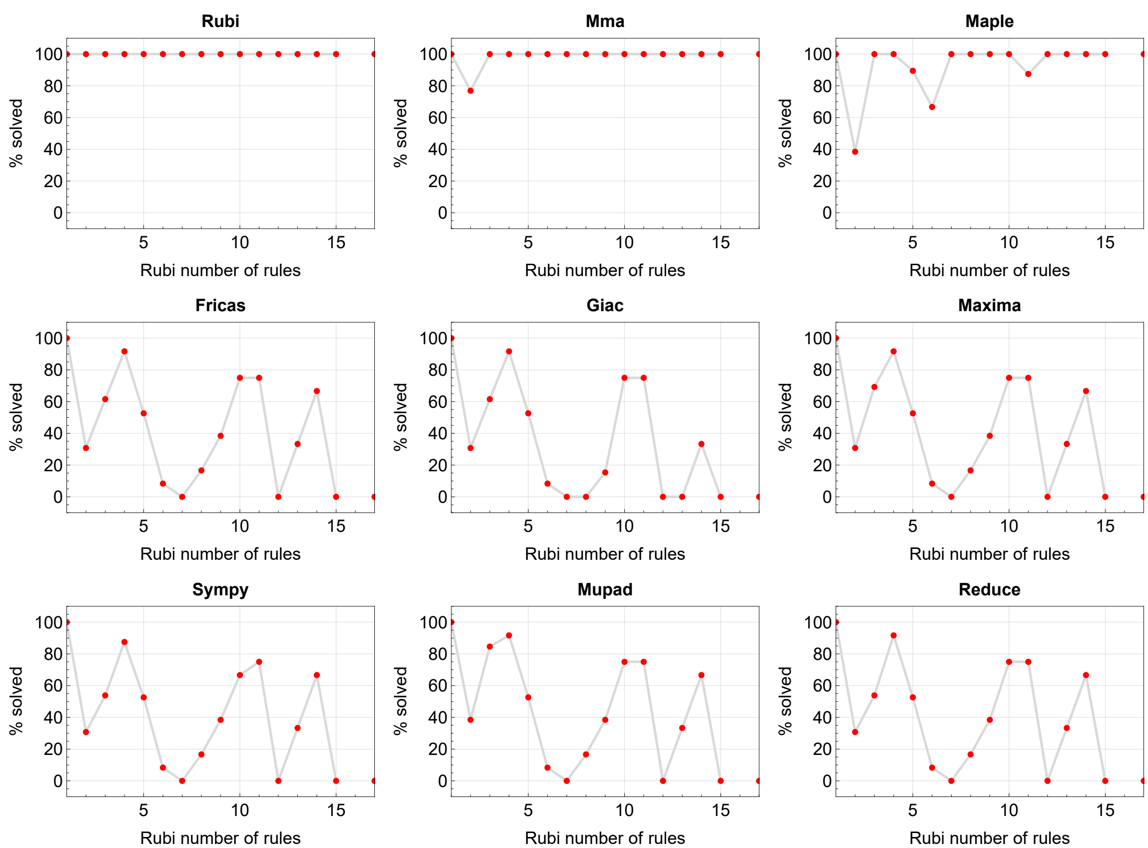


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

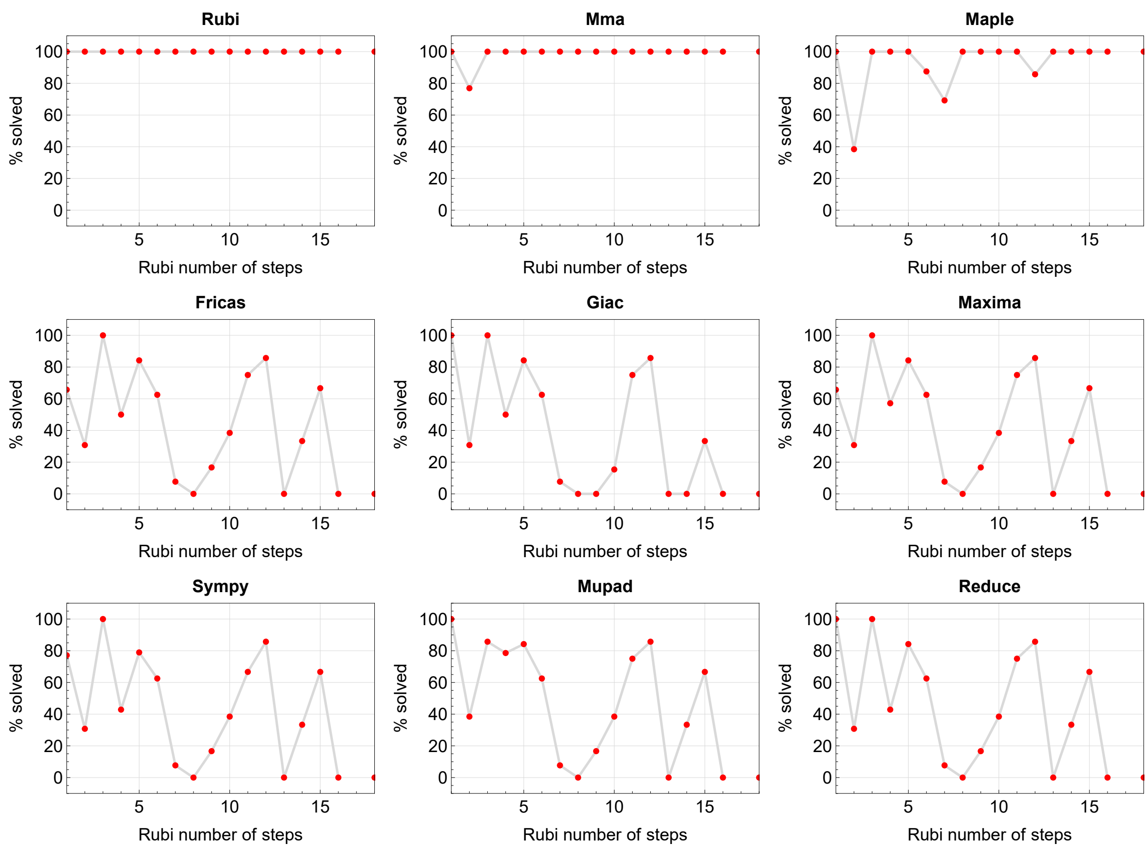


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

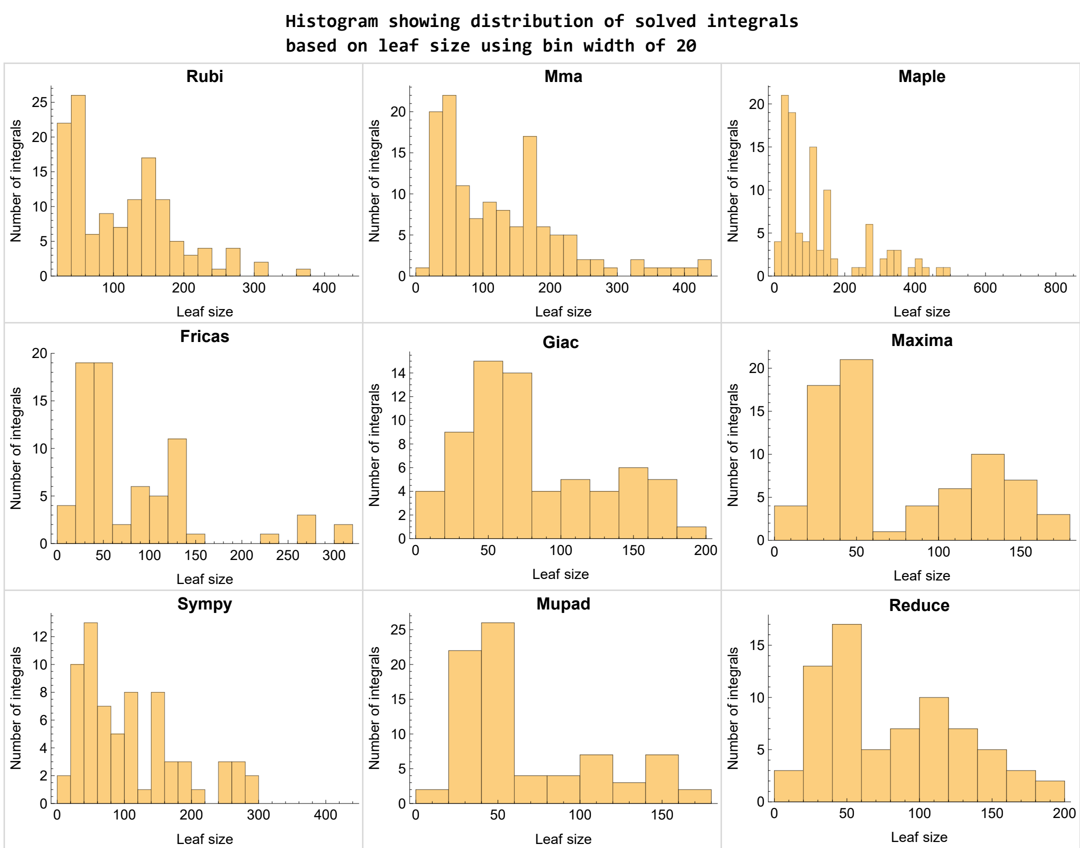


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

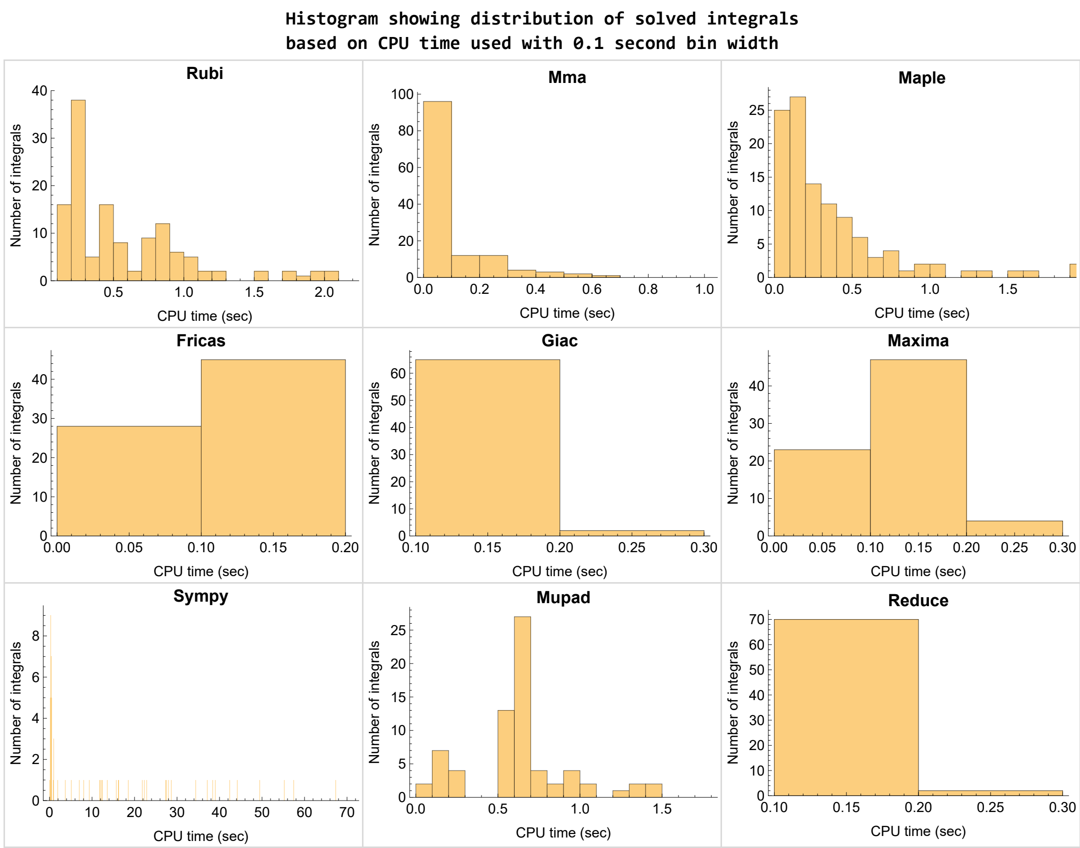


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

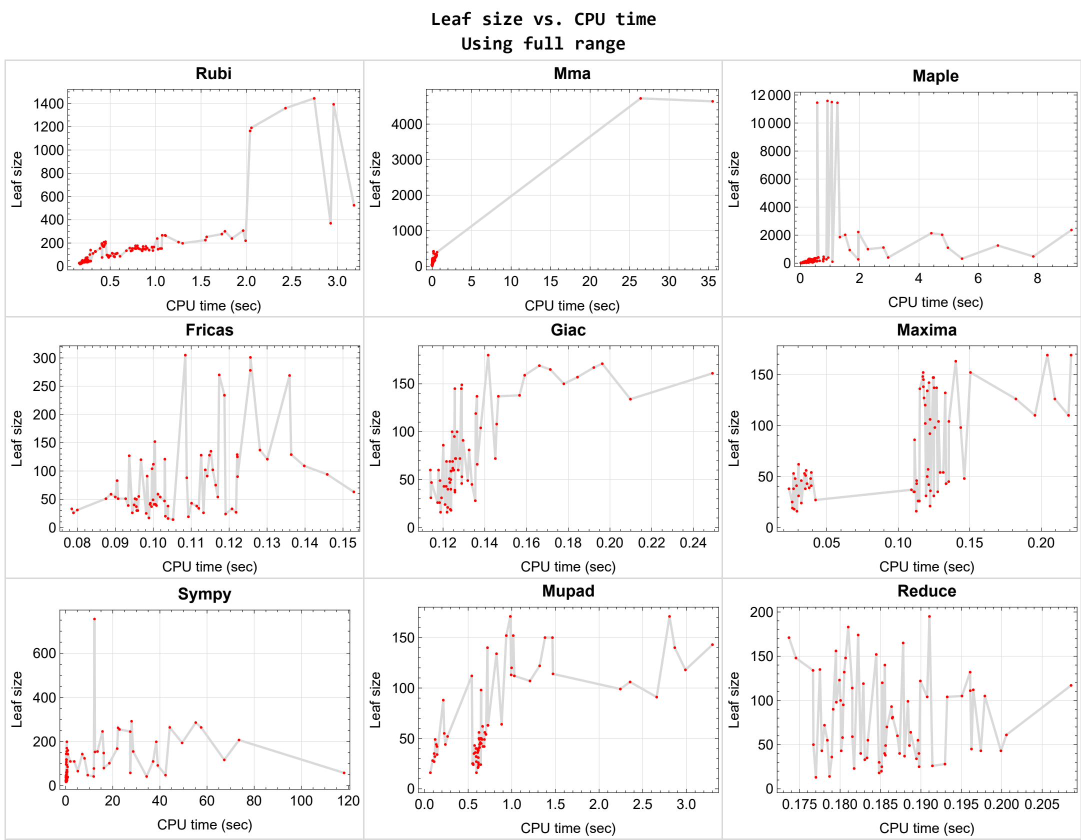


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 57, 58, 59, 91, 92, 94, 95, 127, 128, 130, 131\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{140\}$

Mathematica $\{82, 83\}$

Maple $\{19, 25, 27, 30, 31, 33, 75, 79, 86, 90, 114, 118, 120, 122, 126, 144, 148, 150, 151\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

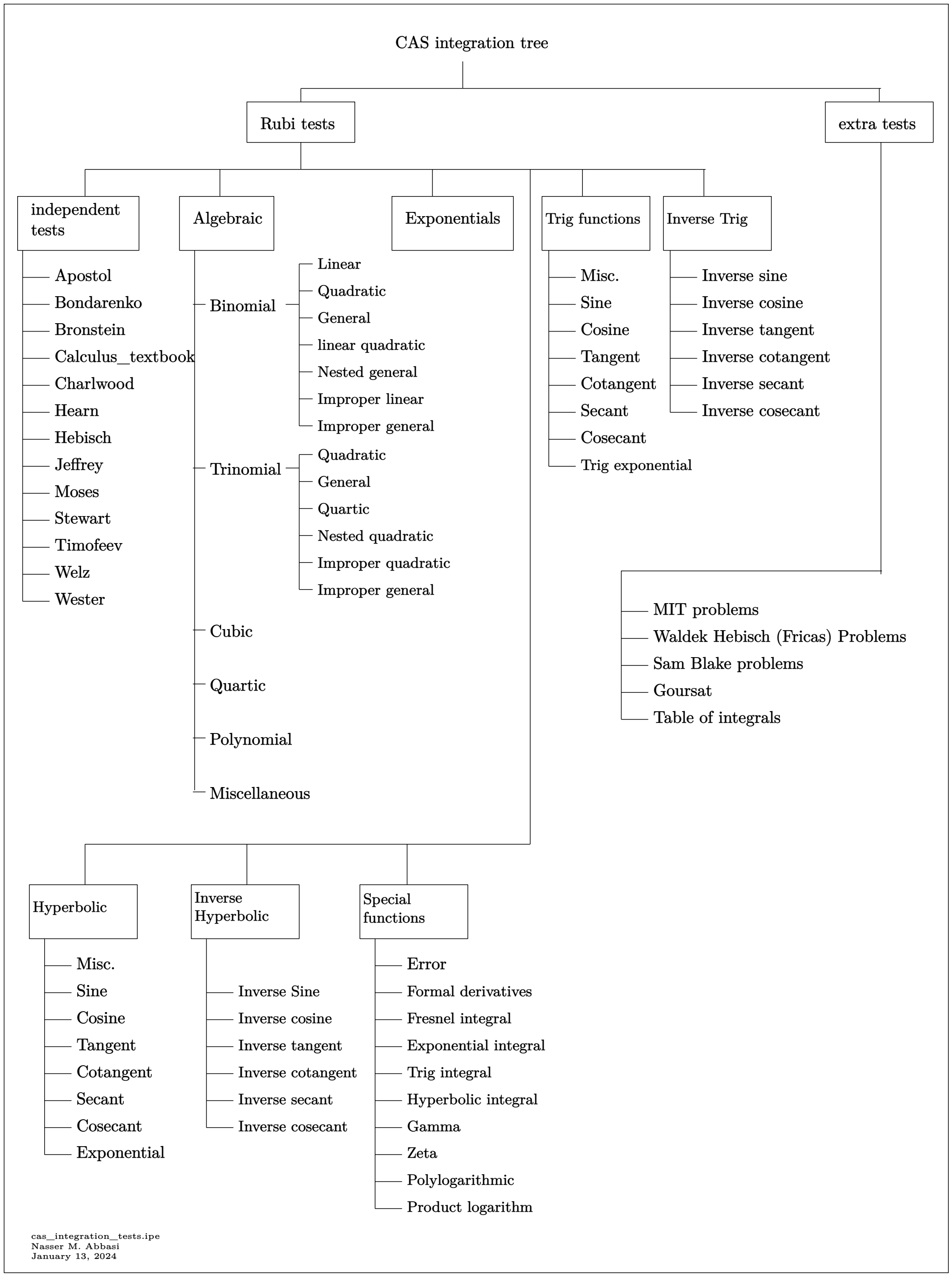
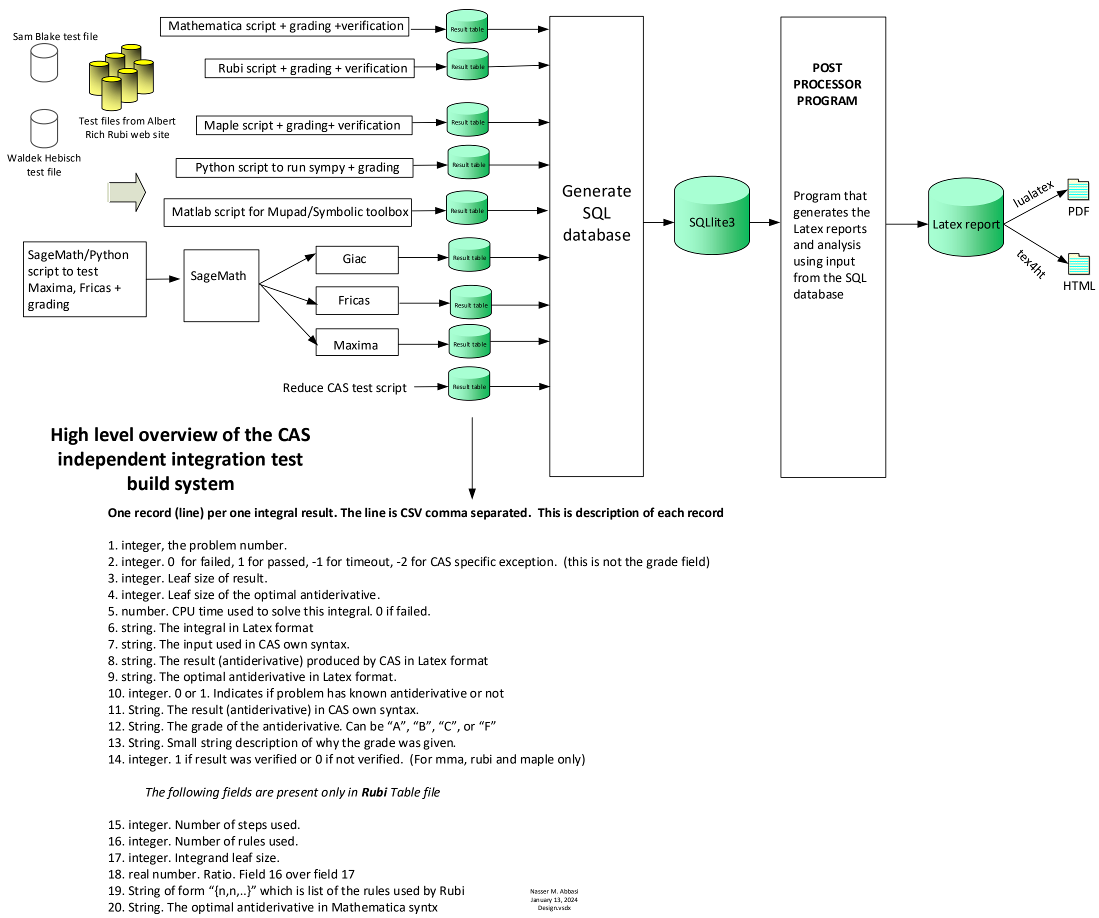


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 28
2.2 Detailed conclusion table per each integral for all CAS systems 33
2.3 Detailed conclusion table specific for Rubi results 75

2.1 List of integrals sorted by grade for each CAS

Rubi 28
Mma 29
Maple 29
Fricas 30
Maxima 30
Giac 31
Mupad 31
Sympy 32
Reduce 32

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 53, 56, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166 }

B grade { 24 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 53, 56, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 86, 87, 88, 89, 90, 93, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166 }

B grade { 82, 83 }

C grade { 9, 11, 66, 102, 158, 159 }

F normal fail { 81, 84, 85 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 23, 24, 26, 53, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 80, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 119, 132, 133, 134, 135, 137, 138, 139, 140, 142, 145, 146, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164 }

B grade { 20, 22, 28, 29, 32, 34, 87, 123, 136, 141, 143, 147, 149, 152, 153, 166 }

C grade { 19, 25, 27, 30, 31, 33, 64, 75, 79, 86, 90, 100, 114, 118, 120, 122, 126, 144, 148, 150, 151, 165 }

F normal fail { 56, 78, 81, 82, 83, 84, 85, 88, 89, 93, 117, 121, 124, 125, 129 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 53, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 80, 96, 97, 98, 99, 101, 102, 103, 105, 107, 109, 111, 113, 115, 119, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade { 104, 106, 108, 110, 112, 166 }

C grade { }

F normal fail { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165 }

F(-1) timedout fail { }

F(-2) exception fail { 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 53, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 80, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 119, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade { 157, 165 }

C grade { }

F normal fail { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 166 }

F(-1) timedout fail { 34 }

F(-2) exception fail { 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 8, 10, 12, 13, 15, 17, 53, 60, 61, 62, 63, 65, 67, 68, 69, 70, 71, 72, 73, 74, 76, 96, 97, 98, 99, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 133, 135, 137, 139, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade { 136 }

C grade { 9, 11, 66, 102, 132, 134, 138, 146 }

F normal fail { 7, 14, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 64, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165, 166 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 53, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 80, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 119, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 146, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165 }

C grade { }

F normal fail { }

F(-1) timedout fail { 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 161, 166 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 53, 60, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 96, 98, 102, 105, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 161, 162, 163 }

B grade { 61, 80, 97, 99, 101, 104, 106, 107, 108, 109, 110, 111, 112, 115, 119, 158, 159, 160, 164 }

C grade { }

F normal fail { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 122, 123, 124, 125, 126, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165, 166 }

F(-1) timedout fail { 59, 91, 94, 95, 103, 113, 120, 121, 127, 128, 129, 130, 131 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 53, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 80, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 119, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

C grade { }

F normal fail { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165, 166 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	57	64	52	57	54	63	66	58	52
N.S.	1	0.97	1.08	0.88	0.97	0.92	1.07	1.12	0.98	0.88
time (sec)	N/A	0.226	0.004	0.259	0.121	0.090	0.573	0.136	0.180	0.264

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	55	61	54	56	59	60	59	59	54
N.S.	1	0.98	1.09	0.96	1.00	1.05	1.07	1.05	1.05	0.96
time (sec)	N/A	0.236	0.010	0.086	0.036	0.089	0.363	0.124	0.182	0.705

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	47	53	44	48	47	53	49	49	44
N.S.	1	0.98	1.10	0.92	1.00	0.98	1.10	1.02	1.02	0.92
time (sec)	N/A	0.225	0.003	0.086	0.146	0.103	0.408	0.132	0.186	0.239

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	50	46	46	49	49	49	49	42
N.S.	1	1.00	1.11	1.02	1.02	1.09	1.09	1.09	1.09	0.93
time (sec)	N/A	0.220	0.007	0.072	0.032	0.100	0.275	0.119	0.189	0.684

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	42	36	37	34	42	45	37	34
N.S.	1	1.00	1.14	0.97	1.00	0.92	1.14	1.22	1.00	0.92
time (sec)	N/A	0.192	0.003	0.081	0.109	0.112	0.293	0.134	0.188	0.144

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	31	33	26	31	33	27
N.S.	1	1.00	1.00	0.97	1.07	1.14	0.90	1.07	1.14	0.93
time (sec)	N/A	0.162	0.002	0.046	0.030	0.079	0.133	0.119	0.183	0.111

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	34	0	0	0	0	17	28
N.S.	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	0.49	0.80
time (sec)	N/A	0.212	0.003	0.099	0.000	0.000	0.000	0.000	0.176	0.093

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	37	38	39	39	37	37	37	38	36
N.S.	1	1.06	1.09	1.11	1.11	1.06	1.06	1.06	1.09	1.03
time (sec)	N/A	0.202	0.003	0.079	0.038	0.100	0.274	0.126	0.186	0.610

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	35	46	39	31	26	37	53	34	42
N.S.	1	0.95	1.24	1.05	0.84	0.70	1.00	1.43	0.92	1.14
time (sec)	N/A	0.198	0.004	0.085	0.119	0.079	0.245	0.129	0.189	0.143

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	51	54	52	51	50	61	50	50	46
N.S.	1	0.96	1.02	0.98	0.96	0.94	1.15	0.94	0.94	0.87
time (sec)	N/A	0.234	0.011	0.078	0.035	0.096	0.355	0.124	0.177	0.640

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	46	44	46	41	46	62	43	42
N.S.	1	1.00	0.96	0.92	0.96	0.85	0.96	1.29	0.90	0.88
time (sec)	N/A	0.214	0.003	0.095	0.113	0.099	0.286	0.125	0.180	0.670

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	60	69	60	62	59	71	59	60	56
N.S.	1	0.94	1.08	0.94	0.97	0.92	1.11	0.92	0.94	0.88
time (sec)	N/A	0.244	0.008	0.087	0.030	0.101	0.521	0.124	0.187	0.691

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	198	138	146	163	152	199	180	171	171
N.S.	1	1.38	0.96	1.01	1.13	1.06	1.38	1.25	1.19	1.19
time (sec)	N/A	1.297	0.089	0.186	0.140	0.100	0.514	0.142	0.174	0.983

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	208	169	266	0	0	0	0	165	0
N.S.	1	1.22	0.99	1.56	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	1.252	0.348	0.421	0.000	0.000	0.000	0.000	0.182	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	133	111	118	136	121	155	134	134	134
N.S.	1	1.19	0.99	1.05	1.21	1.08	1.38	1.20	1.20	1.20
time (sec)	N/A	0.852	0.053	0.146	0.115	0.130	0.422	0.210	0.177	0.825

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	151	131	238	0	0	0	0	125	0
N.S.	1	1.09	0.95	1.72	0.00	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	0.818	0.202	0.256	0.000	0.000	0.000	0.000	0.179	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	88	104	83	107	104	95	88
N.S.	1	1.00	0.99	1.16	1.37	1.09	1.41	1.37	1.25	1.16
time (sec)	N/A	0.411	0.088	0.135	0.135	0.090	0.288	0.138	0.180	0.215

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	97	90	123	0	0	0	0	48	0
N.S.	1	1.17	1.08	1.48	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.464	0.076	0.305	0.000	0.000	0.000	0.000	0.199	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	132	157	179	1000	0	0	0	0	37	0
N.S.	1	1.19	1.36	7.58	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.715	0.102	2.277	0.000	0.000	0.000	0.000	0.178	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	93	102	270	0	0	0	0	78	0
N.S.	1	1.13	1.24	3.29	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.479	0.107	0.374	0.000	0.000	0.000	0.000	0.172	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	76	90	98	98	94	119	0	112	140
N.S.	1	0.96	1.14	1.24	1.24	1.19	1.51	0.00	1.42	1.77
time (sec)	N/A	0.484	0.047	0.165	0.144	0.146	0.336	0.000	0.196	2.867

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	137	153	310	0	0	0	0	133	0
N.S.	1	0.98	1.09	2.21	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.743	0.267	0.322	0.000	0.000	0.000	0.000	0.182	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	135	128	132	152	135	170	0	148	171
N.S.	1	1.16	1.10	1.14	1.31	1.16	1.47	0.00	1.28	1.47
time (sec)	N/A	0.850	0.063	0.185	0.150	0.115	0.457	0.000	0.175	2.807

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	525	291	402	0	0	0	0	348	0
N.S.	1	2.06	1.14	1.58	0.00	0.00	0.00	0.00	1.36	0.00
time (sec)	N/A	3.183	0.552	0.609	0.000	0.000	0.000	0.000	0.171	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	271	370	396	1261	0	0	0	0	323	0
N.S.	1	1.37	1.46	4.65	0.00	0.00	0.00	0.00	1.19	0.00
time (sec)	N/A	2.926	0.613	6.659	0.000	0.000	0.000	0.000	0.190	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	307	225	340	0	0	0	0	260	0
N.S.	1	1.58	1.16	1.75	0.00	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	1.963	0.361	0.474	0.000	0.000	0.000	0.000	0.164	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	206	225	269	1118	0	0	0	0	240	0
N.S.	1	1.09	1.31	5.43	0.00	0.00	0.00	0.00	1.17	0.00
time (sec)	N/A	1.548	0.394	2.800	0.000	0.000	0.000	0.000	0.184	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	146	152	276	0	0	0	0	174	0
N.S.	1	1.11	1.16	2.11	0.00	0.00	0.00	0.00	1.33	0.00
time (sec)	N/A	0.889	0.245	0.387	0.000	0.000	0.000	0.000	0.184	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	134	192	240	0	0	0	0	70	0
N.S.	1	1.13	1.61	2.02	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.678	0.088	0.493	0.000	0.000	0.000	0.000	0.177	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	206	239	368	2024	0	0	0	0	57	0
N.S.	1	1.16	1.79	9.83	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	1.018	0.174	1.511	0.000	0.000	0.000	0.000	0.189	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	116	138	214	1860	0	0	0	0	125	0
N.S.	1	1.19	1.84	16.03	0.00	0.00	0.00	0.00	1.08	0.00
time (sec)	N/A	0.850	0.245	1.332	0.000	0.000	0.000	0.000	0.181	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	135	176	354	0	0	0	0	223	0
N.S.	1	1.02	1.32	2.66	0.00	0.00	0.00	0.00	1.68	0.00
time (sec)	N/A	0.971	0.217	0.506	0.000	0.000	0.000	0.000	0.185	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	213	220	321	2138	0	0	0	0	268	0
N.S.	1	1.03	1.51	10.04	0.00	0.00	0.00	0.00	1.26	0.00
time (sec)	N/A	1.990	0.581	4.415	0.000	0.000	0.000	0.000	0.195	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	277	265	423	0	0	0	0	290	0
N.S.	1	1.40	1.34	2.14	0.00	0.00	0.00	0.00	1.46	0.00
time (sec)	N/A	1.729	0.488	0.606	0.000	0.000	0.000	0.000	0.197	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	10	8	10	10	7	10	10	10
N.S.	1	1.00	1.25	1.00	1.25	1.25	0.88	1.25	1.25	1.25
time (sec)	N/A	0.173	0.418	0.067	0.089	0.078	0.274	0.114	0.180	0.537

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	8	6	8	8	7	8	38	8
N.S.	1	1.00	1.33	1.00	1.33	1.33	1.17	1.33	6.33	1.33
time (sec)	N/A	0.159	0.011	0.055	0.098	0.079	0.275	0.112	0.184	0.538

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20	1.20
time (sec)	N/A	0.184	0.290	0.204	0.085	0.085	0.332	0.118	0.198	0.502

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	10	8	45	10	8	10	10	10
N.S.	1	1.00	1.25	1.00	5.62	1.25	1.00	1.25	1.25	1.25
time (sec)	N/A	0.168	0.400	0.065	0.103	0.083	0.283	0.114	0.182	0.521

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	8	6	39	8	8	8	8	8
N.S.	1	1.00	1.33	1.00	6.50	1.33	1.33	1.33	1.33	1.33
time (sec)	N/A	0.160	0.592	0.058	0.074	0.088	0.307	0.114	0.196	0.527

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	51	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	5.10	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.175	0.680	0.165	0.121	0.077	0.394	0.110	0.185	0.537

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	0	0	10	10	43	10
N.S.	1	1.00	1.20	0.80	0.00	0.00	1.00	1.00	4.30	1.00
time (sec)	N/A	0.165	0.820	0.103	0.000	0.000	0.409	0.188	0.185	0.560

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	10	6	0	0	8	8	38	8
N.S.	1	1.00	1.25	0.75	0.00	0.00	1.00	1.00	4.75	1.00
time (sec)	N/A	0.160	0.976	0.072	0.000	0.000	0.321	0.126	0.186	0.552

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	10	12	11	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	0.83	1.00	0.92	1.00
time (sec)	N/A	0.171	0.629	0.076	0.000	0.000	0.372	0.158	0.196	0.538

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	0	0	10	10	75	10
N.S.	1	1.00	1.20	0.80	0.00	0.00	1.00	1.00	7.50	1.00
time (sec)	N/A	0.163	0.486	0.095	0.000	0.000	1.459	0.213	0.188	0.538

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	10	6	0	0	8	8	12	8
N.S.	1	1.00	1.25	0.75	0.00	0.00	1.00	1.00	1.50	1.00
time (sec)	N/A	0.161	0.980	0.069	0.000	0.000	0.937	0.156	0.187	0.555

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	10	12	15	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	0.83	1.00	1.25	1.00
time (sec)	N/A	0.173	0.523	0.078	0.000	0.000	0.926	0.183	0.187	0.539

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	0	0	10	10	15	10
N.S.	1	1.00	1.20	0.80	0.00	0.00	1.00	1.00	1.50	1.00
time (sec)	N/A	0.172	0.523	0.085	0.000	0.000	0.377	0.165	0.183	0.522

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	10	6	0	0	10	8	14	8
N.S.	1	1.00	1.25	0.75	0.00	0.00	1.25	1.00	1.75	1.00
time (sec)	N/A	0.165	0.004	0.057	0.000	0.000	0.330	0.136	0.183	0.544

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	12	12	17	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.00	1.00	1.42	1.00
time (sec)	N/A	0.170	0.764	0.076	0.000	0.000	0.539	0.171	0.194	0.541

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	0	0	10	10	15	10
N.S.	1	1.00	1.20	0.80	0.00	0.00	1.00	1.00	1.50	1.00
time (sec)	N/A	0.164	0.613	0.098	0.000	0.000	0.708	0.133	0.183	0.554

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	10	6	0	0	10	8	14	8
N.S.	1	1.00	1.25	0.75	0.00	0.00	1.25	1.00	1.75	1.00
time (sec)	N/A	0.172	0.985	0.068	0.000	0.000	0.659	0.132	0.184	0.571

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	12	12	17	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.00	1.00	1.42	1.00
time (sec)	N/A	0.177	1.294	0.078	0.000	0.000	0.980	0.132	0.176	0.596

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	128	108	69	86	75	110	86	64	49
N.S.	1	1.36	1.15	0.73	0.91	0.80	1.17	0.91	0.68	0.52
time (sec)	N/A	0.336	0.018	0.101	0.111	0.116	1.977	0.120	0.189	0.122

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	387	44	15	18	203	18
N.S.	1	1.00	1.12	1.00	24.19	2.75	0.94	1.12	12.69	1.12
time (sec)	N/A	0.192	2.801	0.200	4.177	0.093	7.320	0.174	0.196	0.903

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	295	30	15	18	153	18
N.S.	1	1.00	1.12	1.00	18.44	1.88	0.94	1.12	9.56	1.12
time (sec)	N/A	0.196	1.803	0.188	2.289	0.107	4.690	0.157	0.186	0.863

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	106	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	1.45	0.00
time (sec)	N/A	0.233	0.022	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	20	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.25	1.12
time (sec)	N/A	0.202	0.205	0.180	0.107	0.102	1.489	0.119	0.196	0.498

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.178	0.305	0.061	0.212	0.111	2.000	0.200	0.185	0.621

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	0	18	20	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.00	1.12	1.25	1.12
time (sec)	N/A	0.202	0.249	0.221	0.395	0.105	0.000	0.751	0.195	0.646

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	53	59	50	54	51	58	60	55	49
N.S.	1	0.98	1.09	0.93	1.00	0.94	1.07	1.11	1.02	0.91
time (sec)	N/A	0.257	0.007	0.152	0.129	0.095	27.446	0.125	0.190	0.629

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	52	45	48	51	80	47	72	44
N.S.	1	1.00	1.11	0.96	1.02	1.09	1.70	1.00	1.53	0.94
time (sec)	N/A	0.258	0.011	0.097	0.039	0.091	16.224	0.123	0.178	0.135

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	48	41	43	38	48	43	43	40
N.S.	1	1.00	1.12	0.95	1.00	0.88	1.12	1.00	1.00	0.93
time (sec)	N/A	0.226	0.006	0.099	0.113	0.111	9.355	0.122	0.197	0.594

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	36	41	36	38	39	66	40	61	35
N.S.	1	0.88	1.00	0.88	0.93	0.95	1.61	0.98	1.49	0.85
time (sec)	N/A	0.199	0.012	0.071	0.024	0.101	5.156	0.124	0.201	0.567

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	45	39	63	0	0	0	0	19	32
N.S.	1	1.15	1.00	1.62	0.00	0.00	0.00	0.00	0.49	0.82
time (sec)	N/A	0.287	0.005	0.127	0.000	0.000	0.000	0.000	0.187	0.119

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	41	44	39	41	43	78	60	70	38
N.S.	1	1.05	1.13	1.00	1.05	1.10	2.00	1.54	1.79	0.97
time (sec)	N/A	0.227	0.006	0.109	0.029	0.099	11.977	0.118	0.186	0.613

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	39	48	39	35	30	42	72	40	41
N.S.	1	0.95	1.17	0.95	0.85	0.73	1.02	1.76	0.98	1.00
time (sec)	N/A	0.216	0.006	0.136	0.111	0.096	11.883	0.128	0.183	0.628

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	53	60	53	53	54	92	69	81	50
N.S.	1	0.96	1.09	0.96	0.96	0.98	1.67	1.25	1.47	0.91
time (sec)	N/A	0.244	0.010	0.157	0.027	0.102	39.087	0.122	0.187	0.652

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	180	179	121	147	128	153	169	122	64
N.S.	1	1.42	1.41	0.95	1.16	1.01	1.20	1.33	0.96	0.50
time (sec)	N/A	0.436	0.040	0.273	0.125	0.113	12.441	0.166	0.190	0.883

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	169	177	118	145	120	143	165	117	62
N.S.	1	1.35	1.42	0.94	1.16	0.96	1.14	1.32	0.94	0.50
time (sec)	N/A	0.430	0.033	0.182	0.118	0.097	7.056	0.171	0.209	0.665

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	140	107	103	127	104	110	149	104	49
N.S.	1	1.36	1.04	1.00	1.23	1.01	1.07	1.45	1.01	0.48
time (sec)	N/A	0.285	0.031	0.106	0.118	0.100	3.642	0.129	0.193	0.686

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	154	158	107	132	112	124	138	104	55
N.S.	1	1.45	1.49	1.01	1.25	1.06	1.17	1.30	0.98	0.52
time (sec)	N/A	0.388	0.038	0.158	0.133	0.100	8.004	0.157	0.191	0.227

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	175	177	115	142	127	148	159	123	63
N.S.	1	1.40	1.42	0.92	1.14	1.02	1.18	1.27	0.98	0.50
time (sec)	N/A	0.418	0.043	0.224	0.121	0.094	16.133	0.159	0.180	0.727

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	171	177	118	138	128	155	150	132	63
N.S.	1	1.37	1.42	0.94	1.10	1.02	1.24	1.20	1.06	0.50
time (sec)	N/A	0.423	0.044	0.321	0.118	0.115	28.658	0.178	0.181	0.726

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	149	121	151	169	129	199	145	174	150
N.S.	1	1.20	0.98	1.22	1.36	1.04	1.60	1.17	1.40	1.21
time (sec)	N/A	0.874	0.064	0.322	0.204	0.122	38.457	0.126	0.182	1.380

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	154	171	141	333	0	0	0	0	165	0
N.S.	1	1.11	0.92	2.16	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	0.862	0.071	0.791	0.000	0.000	0.000	0.000	0.190	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	110	126	91	155	100	132	112
N.S.	1	1.00	0.94	1.22	1.40	1.01	1.72	1.11	1.47	1.24
time (sec)	N/A	0.493	0.044	0.266	0.182	0.114	13.523	0.124	0.196	1.029

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	113	107	140	0	0	0	0	83	0
N.S.	1	1.12	1.06	1.39	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.522	0.071	0.427	0.000	0.000	0.000	0.000	0.187	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	177	201	0	0	0	0	0	41	0
N.S.	1	1.17	1.33	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.779	0.100	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	97	107	127	339	0	0	0	0	118	0
N.S.	1	1.10	1.31	3.49	0.00	0.00	0.00	0.00	1.22	0.00
time (sec)	N/A	0.567	0.106	0.474	0.000	0.000	0.000	0.000	0.186	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	86	98	118	110	102	168	0	156	152
N.S.	1	0.99	1.13	1.36	1.26	1.17	1.93	0.00	1.79	1.75
time (sec)	N/A	0.609	0.054	0.244	0.196	0.114	21.858	0.000	0.179	0.937

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1393	1393	0	0	0	0	0	0	274	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	2.959	0.000	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1191	1191	4723	0	0	0	0	0	127	0
N.S.	1	1.00	3.97	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.055	26.391	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1164	1164	4640	0	0	0	0	0	131	0
N.S.	1	1.00	3.99	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.040	35.508	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1360	1360	0	0	0	0	0	0	153	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.429	0.000	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1444	1444	0	0	0	0	0	0	162	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.745	0.000	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	149	166	170	399	0	0	0	0	225	0
N.S.	1	1.11	1.14	2.68	0.00	0.00	0.00	0.00	1.51	0.00
time (sec)	N/A	0.982	0.205	0.941	0.000	0.000	0.000	0.000	0.183	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	154	224	275	0	0	0	0	109	0
N.S.	1	1.07	1.56	1.91	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.768	0.091	0.878	0.000	0.000	0.000	0.000	0.199	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	229	267	417	0	0	0	0	0	63	0
N.S.	1	1.17	1.82	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	1.105	0.174	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	156	239	0	0	0	0	0	172	0
N.S.	1	1.13	1.73	0.00	0.00	0.00	0.00	0.00	1.25	0.00
time (sec)	N/A	0.908	0.277	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	149	153	196	465	0	0	0	0	284	0
N.S.	1	1.03	1.32	3.12	0.00	0.00	0.00	0.00	1.91	0.00
time (sec)	N/A	1.066	0.220	0.783	0.000	0.000	0.000	0.000	0.197	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	406	50	0	20	133	20
N.S.	1	1.00	1.11	1.00	22.56	2.78	0.00	1.11	7.39	1.11
time (sec)	N/A	0.211	1.309	0.026	5.040	0.098	0.000	0.274	0.182	0.552

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	303	34	17	20	88	20
N.S.	1	1.00	1.11	1.00	16.83	1.89	0.94	1.11	4.89	1.11
time (sec)	N/A	0.230	0.865	0.020	2.545	0.090	102.448	0.357	0.183	0.545

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	0	0	0	45	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.248	0.052	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	0	20	22	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.00	1.11	1.22	1.11
time (sec)	N/A	0.212	0.277	0.021	0.112	0.087	0.000	0.128	0.174	0.516

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	124	36	0	20	38	20
N.S.	1	1.00	1.11	1.00	6.89	2.00	0.00	1.11	2.11	1.11
time (sec)	N/A	0.205	0.286	0.022	0.290	0.087	0.000	0.135	0.176	0.575

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	53	59	50	54	51	58	60	55	49
N.S.	1	0.98	1.09	0.93	1.00	0.94	1.07	1.11	1.02	0.91
time (sec)	N/A	0.251	0.007	0.418	0.131	0.093	118.174	0.127	0.178	0.659

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	52	45	48	51	117	47	90	44
N.S.	1	1.00	1.11	0.96	1.02	1.09	2.49	1.00	1.91	0.94
time (sec)	N/A	0.245	0.010	0.260	0.028	0.088	67.326	0.114	0.179	0.630

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	48	41	43	38	48	43	43	40
N.S.	1	1.00	1.12	0.95	1.00	0.88	1.12	1.00	1.00	0.93
time (sec)	N/A	0.210	0.005	0.195	0.133	0.104	42.376	0.120	0.178	0.618

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	36	41	36	38	39	102	40	80	35
N.S.	1	0.88	1.00	0.88	0.93	0.95	2.49	0.98	1.95	0.85
time (sec)	N/A	0.193	0.012	0.125	0.035	0.093	18.531	0.122	0.186	0.111

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	45	39	63	0	0	0	0	19	32
N.S.	1	1.15	1.00	1.62	0.00	0.00	0.00	0.00	0.49	0.82
time (sec)	N/A	0.278	0.005	0.157	0.000	0.000	0.000	0.000	0.191	0.594

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	41	44	39	41	43	110	60	93	38
N.S.	1	1.05	1.13	1.00	1.05	1.10	2.82	1.54	2.38	0.97
time (sec)	N/A	0.208	0.006	0.123	0.039	0.110	37.105	0.114	0.186	0.617

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	39	48	39	35	30	42	72	40	41
N.S.	1	0.95	1.17	0.95	0.85	0.73	1.02	1.76	0.98	1.00
time (sec)	N/A	0.209	0.006	0.204	0.111	0.096	34.356	0.126	0.187	0.631

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	53	60	53	53	54	0	69	105	50
N.S.	1	0.96	1.09	0.96	0.96	0.98	0.00	1.25	1.91	0.91
time (sec)	N/A	0.234	0.010	0.284	0.034	0.117	0.000	0.125	0.198	0.642

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	197	179	155	148	269	255	167	99	114
N.S.	1	1.45	1.32	1.14	1.09	1.98	1.88	1.23	0.73	0.84
time (sec)	N/A	0.449	0.052	0.366	0.117	0.136	22.830	0.192	0.188	1.472

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	131	98	92	234	755	95	119	91
N.S.	1	1.00	1.30	0.97	0.91	2.32	7.48	0.94	1.18	0.90
time (sec)	N/A	0.276	0.027	0.148	0.122	0.119	12.230	0.125	0.183	2.659

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	186	170	144	137	270	245	137	105	107
N.S.	1	1.46	1.34	1.13	1.08	2.13	1.93	1.08	0.83	0.84
time (sec)	N/A	0.412	0.047	0.286	0.127	0.117	27.388	0.147	0.195	1.208

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	129	183	105	102	121	286	108	165	118
N.S.	1	1.12	1.59	0.91	0.89	1.05	2.49	0.94	1.43	1.03
time (sec)	N/A	0.336	0.037	0.460	0.119	0.103	55.205	0.146	0.188	2.989

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	212	181	167	152	305	264	171	114	122
N.S.	1	1.54	1.31	1.21	1.10	2.21	1.91	1.24	0.83	0.88
time (sec)	N/A	0.449	0.076	0.651	0.117	0.108	57.491	0.196	0.182	1.319

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	126	185	113	106	137	292	119	148	106
N.S.	1	1.08	1.58	0.97	0.91	1.17	2.50	1.02	1.26	0.91
time (sec)	N/A	0.339	0.027	0.358	0.122	0.128	27.960	0.136	0.181	2.355

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	196	170	153	137	278	246	157	100	113
N.S.	1	1.54	1.34	1.20	1.08	2.19	1.94	1.24	0.79	0.89
time (sec)	N/A	0.408	0.026	0.243	0.125	0.126	15.632	0.184	0.180	0.993

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	111	170	104	98	90	262	91	135	99
N.S.	1	1.07	1.63	1.00	0.94	0.87	2.52	0.88	1.30	0.95
time (sec)	N/A	0.311	0.034	0.196	0.126	0.122	22.238	0.130	0.178	2.243

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	207	179	164	147	301	264	161	111	120
N.S.	1	1.52	1.32	1.21	1.08	2.21	1.94	1.18	0.82	0.88
time (sec)	N/A	0.431	0.043	0.401	0.124	0.126	44.183	0.249	0.196	0.998

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	149	121	151	169	129	0	145	195	150
N.S.	1	1.20	0.98	1.22	1.36	1.04	0.00	1.17	1.57	1.21
time (sec)	N/A	0.856	0.066	0.567	0.221	0.136	0.000	0.129	0.191	1.466

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	154	171	141	11449	0	0	0	0	186	0
N.S.	1	1.11	0.92	74.34	0.00	0.00	0.00	0.00	1.21	0.00
time (sec)	N/A	0.893	0.073	1.250	0.000	0.000	0.000	0.000	0.191	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	110	126	91	194	100	152	112
N.S.	1	1.00	0.94	1.22	1.40	1.01	2.16	1.11	1.69	1.24
time (sec)	N/A	0.502	0.042	0.387	0.209	0.098	49.418	0.127	0.184	0.542

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	113	107	140	0	0	0	0	105	0
N.S.	1	1.09	1.03	1.35	0.00	0.00	0.00	0.00	1.01	0.00
time (sec)	N/A	0.578	0.038	0.530	0.000	0.000	0.000	0.000	0.183	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	177	201	0	0	0	0	0	41	0
N.S.	1	1.15	1.31	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.800	0.105	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	100	107	125	11455	0	0	0	0	142	0
N.S.	1	1.07	1.25	114.55	0.00	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	0.557	0.110	0.567	0.000	0.000	0.000	0.000	0.189	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	86	98	118	110	102	207	0	183	152
N.S.	1	0.99	1.13	1.36	1.26	1.17	2.38	0.00	2.10	1.75
time (sec)	N/A	0.539	0.054	0.319	0.219	0.116	73.570	0.000	0.181	1.017

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	154	155	167	11496	0	0	0	0	200	0
N.S.	1	1.01	1.08	74.65	0.00	0.00	0.00	0.00	1.30	0.00
time (sec)	N/A	0.808	0.274	1.060	0.000	0.000	0.000	0.000	0.199	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	253	346	0	0	0	0	0	372	0
N.S.	1	1.05	1.44	0.00	0.00	0.00	0.00	0.00	1.55	0.00
time (sec)	N/A	1.562	0.354	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	147	166	170	935	0	0	0	0	247	0
N.S.	1	1.13	1.16	6.36	0.00	0.00	0.00	0.00	1.68	0.00
time (sec)	N/A	0.960	0.084	1.664	0.000	0.000	0.000	0.000	0.197	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	154	224	275	0	0	0	0	135	0
N.S.	1	1.11	1.61	1.98	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.743	0.040	0.772	0.000	0.000	0.000	0.000	0.191	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	232	267	421	0	0	0	0	0	63	0
N.S.	1	1.15	1.81	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	1.073	0.176	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	156	240	0	0	0	0	0	198	0
N.S.	1	1.17	1.80	0.00	0.00	0.00	0.00	0.00	1.49	0.00
time (sec)	N/A	0.804	0.270	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	146	153	196	11581	0	0	0	0	312	0
N.S.	1	1.05	1.34	79.32	0.00	0.00	0.00	0.00	2.14	0.00
time (sec)	N/A	1.050	0.229	0.914	0.000	0.000	0.000	0.000	0.198	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	407	50	0	20	133	20
N.S.	1	1.00	1.11	1.00	22.61	2.78	0.00	1.11	7.39	1.11
time (sec)	N/A	0.185	1.313	0.027	4.428	0.111	0.000	0.296	0.198	0.558

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	304	34	0	20	88	20
N.S.	1	1.00	1.11	1.00	16.89	1.89	0.00	1.11	4.89	1.11
time (sec)	N/A	0.195	0.878	0.026	2.541	0.106	0.000	0.365	0.207	0.548

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	0	0	0	45	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.246	0.052	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	0	20	22	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.00	1.11	1.22	1.11
time (sec)	N/A	0.212	0.275	0.023	0.116	0.100	0.000	0.129	0.200	0.512

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	132	36	0	20	38	20
N.S.	1	1.00	1.11	1.00	7.33	2.00	0.00	1.11	2.11	1.11
time (sec)	N/A	0.207	0.292	0.025	0.287	0.097	0.000	0.131	0.197	0.581

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	48	55	46	45	41	46	81	45	45
N.S.	1	0.96	1.10	0.92	0.90	0.82	0.92	1.62	0.90	0.90
time (sec)	N/A	0.243	0.015	0.250	0.135	0.101	0.156	0.132	0.196	0.650

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	41	48	47	43	40	41	69	40	40
N.S.	1	0.95	1.12	1.09	1.00	0.93	0.95	1.60	0.93	0.93
time (sec)	N/A	0.236	0.012	0.100	0.036	0.095	0.136	0.123	0.190	0.590

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	35	44	37	36	31	36	72	36	36
N.S.	1	0.90	1.13	0.95	0.92	0.79	0.92	1.85	0.92	0.92
time (sec)	N/A	0.221	0.013	0.115	0.123	0.080	0.114	0.126	0.179	0.582

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	27	27	25	22	46	25	25
N.S.	1	1.00	1.00	1.00	1.00	0.93	0.81	1.70	0.93	0.93
time (sec)	N/A	0.160	0.002	0.108	0.042	0.098	0.087	0.129	0.185	0.550

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	42	39	87	0	0	0	72	19	32
N.S.	1	1.08	1.00	2.23	0.00	0.00	0.00	1.85	0.49	0.82
time (sec)	N/A	0.265	0.005	0.156	0.000	0.000	0.000	0.145	0.177	0.592

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	37	36	38	41	36	39	40	43
N.S.	1	1.00	1.09	1.06	1.12	1.21	1.06	1.15	1.18	1.26
time (sec)	N/A	0.208	0.013	0.100	0.027	0.100	0.274	0.126	0.186	0.574

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	44	48	41	42	37	44	60	43	50
N.S.	1	1.02	1.12	0.95	0.98	0.86	1.02	1.40	1.00	1.16
time (sec)	N/A	0.214	0.015	0.122	0.121	0.096	0.337	0.125	0.200	0.624

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	54	60	45	54	55	60	51	55	56
N.S.	1	1.15	1.28	0.96	1.15	1.17	1.28	1.09	1.17	1.19
time (sec)	N/A	0.235	0.015	0.108	0.039	0.096	0.415	0.123	0.183	0.625

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	139	111	141	134	125	144	0	140	140
N.S.	1	1.14	0.91	1.16	1.10	1.02	1.18	0.00	1.15	1.15
time (sec)	N/A	0.929	0.056	0.566	0.120	0.122	0.227	0.000	0.186	0.722

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	149	152	358	0	0	0	0	134	0
N.S.	1	0.98	1.00	2.36	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.825	0.233	0.561	0.000	0.000	0.000	0.000	0.184	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	81	73	105	104	88	97	0	98	98
N.S.	1	0.99	0.89	1.28	1.27	1.07	1.18	0.00	1.20	1.20
time (sec)	N/A	0.549	0.033	0.456	0.128	0.109	0.211	0.000	0.180	0.648

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	89	105	308	0	0	0	0	43	0
N.S.	1	1.07	1.27	3.71	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.491	0.071	0.284	0.000	0.000	0.000	0.000	0.196	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	148	173	203	1103	0	0	0	0	41	0
N.S.	1	1.17	1.37	7.45	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.784	0.098	4.974	0.000	0.000	0.000	0.000	0.181	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	110	107	142	0	0	0	0	87	0
N.S.	1	1.15	1.11	1.48	0.00	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	0.527	0.069	0.771	0.000	0.000	0.000	0.000	0.178	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	85	99	103	120	109	117	137	120	143
N.S.	1	1.01	1.18	1.23	1.43	1.30	1.39	1.63	1.43	1.70
time (sec)	N/A	0.477	0.045	1.081	0.119	0.140	0.345	0.136	0.185	3.299

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	301	253	480	0	0	0	0	287	0
N.S.	1	1.41	1.18	2.24	0.00	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	1.763	0.427	7.852	0.000	0.000	0.000	0.000	0.184	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	229	239	330	2367	0	0	0	0	254	0
N.S.	1	1.04	1.44	10.34	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	1.840	0.469	9.138	0.000	0.000	0.000	0.000	0.175	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	147	174	405	0	0	0	0	186	0
N.S.	1	1.01	1.20	2.79	0.00	0.00	0.00	0.00	1.28	0.00
time (sec)	N/A	1.027	0.176	2.960	0.000	0.000	0.000	0.000	0.187	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	119	132	215	2028	0	0	0	0	64	0
N.S.	1	1.11	1.81	17.04	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.728	0.169	4.779	0.000	0.000	0.000	0.000	0.178	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	230	263	422	2225	0	0	0	0	63	0
N.S.	1	1.14	1.83	9.67	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	1.110	0.167	1.951	0.000	0.000	0.000	0.000	0.194	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	151	222	275	0	0	0	0	142	0
N.S.	1	1.11	1.63	2.02	0.00	0.00	0.00	0.00	1.04	0.00
time (sec)	N/A	0.783	0.069	1.950	0.000	0.000	0.000	0.000	0.181	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	162	178	321	0	0	0	0	246	0
N.S.	1	1.10	1.21	2.18	0.00	0.00	0.00	0.00	1.67	0.00
time (sec)	N/A	0.965	0.185	5.451	0.000	0.000	0.000	0.000	0.192	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	52	34	30	31	27	39	31	30	31
N.S.	1	1.02	0.67	0.59	0.61	0.53	0.76	0.61	0.59	0.61
time (sec)	N/A	0.195	0.010	0.074	0.125	0.122	0.910	0.114	0.185	0.616

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	43	28	25	26	20	32	26	23	26
N.S.	1	1.02	0.67	0.60	0.62	0.48	0.76	0.62	0.55	0.62
time (sec)	N/A	0.187	0.008	0.019	0.114	0.103	0.569	0.118	0.182	0.616

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	29	18	17	16	14	19	16	13	16
N.S.	1	1.32	0.82	0.77	0.73	0.64	0.86	0.73	0.59	0.73
time (sec)	N/A	0.176	0.012	0.014	0.113	0.105	0.381	0.119	0.177	0.067

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	37	31	24	35	0	0	0	9	24
N.S.	1	1.19	1.00	0.77	1.13	0.00	0.00	0.00	0.29	0.77
time (sec)	N/A	0.244	0.005	0.086	0.128	0.000	0.000	0.000	0.191	0.559

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	32	30	19	21	17	94	21	20	21
N.S.	1	1.19	1.11	0.70	0.78	0.63	3.48	0.78	0.74	0.78
time (sec)	N/A	0.182	0.007	0.022	0.122	0.099	0.487	0.122	0.185	0.613

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	43	34	27	26	26	160	26	28	24
N.S.	1	1.02	0.81	0.64	0.62	0.62	3.81	0.62	0.67	0.57
time (sec)	N/A	0.189	0.009	0.025	0.115	0.113	0.931	0.117	0.193	0.621

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	35	30	25	24	24	85	24	25	24
N.S.	1	0.97	0.83	0.69	0.67	0.67	2.36	0.67	0.69	0.67
time (sec)	N/A	0.195	0.012	0.020	0.032	0.119	0.801	0.121	0.190	0.645

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	28	25	20	19	19	24	19	18	0
N.S.	1	0.97	0.86	0.69	0.66	0.66	0.83	0.66	0.62	0.00
time (sec)	N/A	0.190	0.008	0.016	0.026	0.109	0.562	0.123	0.185	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	17	16	14	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.85	0.80	0.70	0.80
time (sec)	N/A	0.168	0.005	0.024	0.029	0.104	0.142	0.122	0.179	0.595

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	26	20	18	26	22
N.S.	1	1.00	1.00	0.86	0.82	1.18	0.91	0.82	1.18	1.00
time (sec)	N/A	0.179	0.009	0.032	0.027	0.094	0.364	0.124	0.191	0.616

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	34	31	26	25	33	143	28	35	27
N.S.	1	0.92	0.84	0.70	0.68	0.89	3.86	0.76	0.95	0.73
time (sec)	N/A	0.194	0.013	0.037	0.026	0.121	0.952	0.135	0.183	0.594

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	37	33	57	50	0	0	0	12	25
N.S.	1	1.12	1.00	1.73	1.52	0.00	0.00	0.00	0.36	0.76
time (sec)	N/A	0.242	0.006	0.282	0.120	0.000	0.000	0.000	0.178	0.556

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	37	32	75	0	63	0	0	12	0
N.S.	1	0.95	0.82	1.92	0.00	1.62	0.00	0.00	0.31	0.00
time (sec)	N/A	0.256	0.010	0.362	0.000	0.153	0.000	0.000	0.181	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [53] had the largest ratio of [1.37500000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	0.97	12	0.250
2	A	5	4	0.98	12	0.333
3	A	3	3	0.98	12	0.250
4	A	5	4	1.00	12	0.333
5	A	3	3	1.00	10	0.300
6	A	1	1	1.00	8	0.125
7	A	2	2	1.00	12	0.167
8	A	6	5	1.06	12	0.417
9	A	3	3	0.95	12	0.250
10	A	5	4	0.96	12	0.333
11	A	4	4	1.00	12	0.333
12	A	5	4	0.94	12	0.333
13	A	15	14	1.38	14	1.000
14	A	14	13	1.22	14	0.929
15	A	10	9	1.19	14	0.643
16	A	10	9	1.09	14	0.643
17	A	4	4	1.00	12	0.333
18	A	6	5	1.17	10	0.500
19	A	4	4	1.19	14	0.286
20	A	4	4	1.13	14	0.286
21	A	9	8	0.96	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	8	8	0.98	14	0.571
23	A	14	13	1.16	14	0.929
24	B	18	17	2.06	14	1.214
25	A	16	15	1.37	14	1.071
26	A	14	13	1.58	14	0.929
27	A	10	10	1.09	14	0.714
28	A	9	8	1.11	12	0.667
29	A	5	5	1.13	10	0.500
30	A	5	5	1.16	14	0.357
31	A	5	5	1.19	14	0.357
32	A	7	7	1.02	14	0.500
33	A	15	14	1.03	14	1.000
34	A	11	11	1.40	14	0.786
35	N/A	1	0	1.00	8	0.000
36	N/A	1	0	1.00	6	0.000
37	N/A	1	0	1.00	10	0.000
38	N/A	1	0	1.00	8	0.000
39	N/A	1	0	1.00	6	0.000
40	N/A	1	0	1.00	10	0.000
41	N/A	1	0	1.00	10	0.000
42	N/A	1	0	1.00	8	0.000
43	N/A	1	0	1.00	12	0.000
44	N/A	1	0	1.00	10	0.000
45	N/A	1	0	1.00	8	0.000
46	N/A	1	0	1.00	12	0.000
47	N/A	1	0	1.00	10	0.000
48	N/A	1	0	1.00	8	0.000
49	N/A	1	0	1.00	12	0.000
50	N/A	1	0	1.00	10	0.000
51	N/A	1	0	1.00	8	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	N/A	1	0	1.00	12	0.000
53	A	12	11	1.36	8	1.375
54	N/A	1	0	1.00	16	0.000
55	N/A	1	0	1.00	16	0.000
56	A	2	2	1.00	14	0.143
57	N/A	1	0	1.00	16	0.000
58	N/A	1	0	1.00	10	0.000
59	N/A	1	0	1.00	16	0.000
60	A	5	4	0.98	14	0.286
61	A	5	4	1.00	14	0.286
62	A	5	4	1.00	14	0.286
63	A	2	2	0.88	12	0.167
64	A	4	3	1.15	14	0.214
65	A	6	5	1.05	14	0.357
66	A	5	4	0.95	14	0.286
67	A	5	4	0.96	14	0.286
68	A	11	10	1.42	14	0.714
69	A	11	10	1.35	14	0.714
70	A	1	1	1.36	10	0.100
71	A	10	9	1.45	14	0.643
72	A	11	10	1.40	14	0.714
73	A	11	10	1.37	14	0.714
74	A	11	10	1.20	16	0.625
75	A	11	10	1.11	16	0.625
76	A	6	5	1.00	16	0.312
77	A	7	6	1.12	14	0.429
78	A	6	5	1.17	16	0.312
79	A	6	5	1.10	16	0.312
80	A	10	9	0.99	16	0.562
81	A	2	2	1.00	16	0.125
82	A	2	2	1.00	12	0.167
83	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
84	A	2	2	1.00	16	0.125
85	A	2	2	1.00	16	0.125
86	A	10	9	1.11	16	0.562
87	A	7	6	1.07	14	0.429
88	A	7	6	1.17	16	0.375
89	A	7	6	1.13	16	0.375
90	A	9	8	1.03	16	0.500
91	N/A	1	0	1.00	18	0.000
92	N/A	1	0	1.00	18	0.000
93	A	2	2	1.00	16	0.125
94	N/A	1	0	1.00	18	0.000
95	N/A	1	0	1.00	18	0.000
96	A	5	4	0.98	14	0.286
97	A	5	4	1.00	14	0.286
98	A	5	4	1.00	14	0.286
99	A	2	2	0.88	14	0.143
100	A	4	3	1.15	14	0.214
101	A	6	5	1.05	14	0.357
102	A	5	4	0.95	14	0.286
103	A	5	4	0.96	14	0.286
104	A	12	11	1.45	14	0.786
105	A	1	1	1.00	10	0.100
106	A	11	10	1.46	14	0.714
107	A	12	11	1.12	14	0.786
108	A	12	11	1.54	14	0.786
109	A	12	11	1.08	14	0.786
110	A	11	10	1.54	12	0.833
111	A	11	10	1.07	14	0.714
112	A	12	11	1.52	14	0.786
113	A	11	10	1.20	16	0.625
114	A	11	10	1.11	16	0.625
115	A	6	5	1.00	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	7	6	1.09	16	0.375
117	A	6	5	1.15	16	0.312
118	A	6	5	1.07	16	0.312
119	A	10	9	0.99	16	0.562
120	A	10	9	1.01	16	0.562
121	A	12	11	1.05	16	0.688
122	A	10	9	1.13	16	0.562
123	A	7	6	1.11	16	0.375
124	A	7	6	1.15	16	0.375
125	A	7	6	1.17	16	0.375
126	A	9	8	1.05	16	0.500
127	N/A	1	0	1.00	18	0.000
128	N/A	1	0	1.00	18	0.000
129	A	2	2	1.00	16	0.125
130	N/A	1	0	1.00	18	0.000
131	N/A	1	0	1.00	18	0.000
132	A	4	4	0.96	14	0.286
133	A	6	5	0.95	14	0.357
134	A	4	4	0.90	12	0.333
135	A	1	1	1.00	10	0.100
136	A	4	3	1.08	14	0.214
137	A	2	2	1.00	14	0.143
138	A	4	4	1.02	14	0.286
139	A	6	5	1.15	14	0.357
140	A	15	14	1.14	16	0.875
141	A	10	9	0.98	16	0.562
142	A	10	9	0.99	14	0.643
143	A	10	9	1.07	12	0.750
144	A	6	5	1.17	16	0.312
145	A	7	6	1.15	16	0.375
146	A	6	5	1.01	16	0.312
147	A	13	12	1.41	16	0.750

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
148	A	16	15	1.04	16	0.938
149	A	9	8	1.01	14	0.571
150	A	9	9	1.11	12	0.750
151	A	7	6	1.14	16	0.375
152	A	7	6	1.11	16	0.375
153	A	10	9	1.10	16	0.562
154	A	7	6	1.02	10	0.600
155	A	6	5	1.02	8	0.625
156	A	5	4	1.32	6	0.667
157	A	4	3	1.19	10	0.300
158	A	5	4	1.19	10	0.400
159	A	6	5	1.02	10	0.500
160	A	3	3	0.97	12	0.250
161	A	3	3	0.97	12	0.250
162	A	2	2	1.00	12	0.167
163	A	4	4	1.00	12	0.333
164	A	3	3	0.92	12	0.250
165	A	4	3	1.12	10	0.300
166	A	4	3	0.95	10	0.300

CHAPTER 3

Listing of integrals

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3.1 $\int x^5(a+b\arctan(cx))\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^5(a+b\arctan(cx))\,dx=-\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}+\frac{b\arctan(cx)}{6c^6}+\frac{1}{6}x^6(a+b\arctan(cx))$$

output
```
-1/6*b*x/c^5+1/18*b*x^3/c^3-1/30*b*x^5/c+1/6*b*arctan(c*x)/c^6+1/6*x^6*(a+
b*arctan(c*x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int x^5(a+b\arctan(cx))\,dx=-\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}+\frac{ax^6}{6}+\frac{b\arctan(cx)}{6c^6}+\frac{1}{6}bx^6\arctan(cx)$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x]),x]
```

output
```
-1/6*(b*x)/c^5 + (b*x^3)/(18*c^3) - (b*x^5)/(30*c) + (a*x^6)/6 + (b*ArcTan
[c*x])/(6*c^6) + (b*x^6*ArcTan[c*x])/6
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5361, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))-\frac{1}{6}bc\int\frac{x^6}{c^2x^2+1}dx$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))-\frac{1}{6}bc\int\left(\frac{x^4}{c^2}-\frac{x^2}{c^4}-\frac{1}{c^6\left(c^2x^2+1\right)}+\frac{1}{c^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))-\frac{1}{6}bc\left(-\frac{\arctan(cx)}{c^7}+\frac{x}{c^6}-\frac{x^3}{3c^4}+\frac{x^5}{5c^2}\right)$$

input
```
Int[x^5*(a + b*ArcTan[c*x]),x]
```

output
```
(x^6*(a + b*ArcTan[c*x]))/6 - (b*c*(x/c^6 - x^3/(3*c^4) + x^5/(5*c^2) - Ar
cTan[c*x]/c^7))/6
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

method	result	siz
parts	$\frac{x^{6} a}{6}+\frac{b \left(\frac{c^{6} x^{6} \arctan \left(c x \right)}{6}-\frac{c^{5} x^{5}}{30}+\frac{c^{3} x^{3}}{18}-\frac{c x}{6}+\frac{\arctan \left(c x \right)}{6}\right)}{c^{6}}$	52
derivativedivides	$\frac{\frac{a \,c^{6} x^{6}}{6}+b \left(\frac{c^{6} x^{6} \arctan \left(c x \right)}{6}-\frac{c^{5} x^{5}}{30}+\frac{c^{3} x^{3}}{18}-\frac{c x}{6}+\frac{\arctan \left(c x \right)}{6}\right)}{c^{6}}$	56
default	$\frac{\frac{a \,c^{6} x^{6}}{6}+b \left(\frac{c^{6} x^{6} \arctan \left(c x \right)}{6}-\frac{c^{5} x^{5}}{30}+\frac{c^{3} x^{3}}{18}-\frac{c x}{6}+\frac{\arctan \left(c x \right)}{6}\right)}{c^{6}}$	56
parallelrisch	$\frac{15 b \arctan \left(c x \right) x^{6} c^{6}+15 a \,c^{6} x^{6}-3 b \,c^{5} x^{5}+5 b \,c^{3} x^{3}-15 b c x +15 b \arctan \left(c x \right)}{90 c^{6}}$	59
risch	$-\frac{i x^{6} b \ln \left(i c x +1\right)}{12}+\frac{x^{6} a}{6}+\frac{i x^{6} b \ln \left(-i c x +1\right)}{12}-\frac{b \,x^{5}}{30 c}+\frac{b \,x^{3}}{18 c^{3}}-\frac{b x}{6 c^{5}}+\frac{b \arctan \left(c x \right)}{6 c^{6}}$	73
orering	$\frac{\left(3 c^{6} x^{6}-c^{4} x^{4}+5 c^{2} x^{2}+9\right) \left(a +b \arctan \left(c x \right)\right)}{9 c^{6}}-\frac{\left(3 c^{4} x^{4}-5 c^{2} x^{2}+15\right) \left(c^{2} x^{2}+1\right) \left(5 x^{4} \left(a +b \arctan \left(c x \right)\right)+\frac{x^{5} b c}{c^{2} x^{2}+1}\right)}{90 x^{4} c^{6}}$	10

input
```
int(x^5*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*a+b/c^6*(1/6*c^6*x^6*arctan(c*x)-1/30*c^5*x^5+1/18*c^3*x^3-1/6*c*x
+1/6*arctan(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^5(a+b\arctan(cx))\,dx = \frac{15\,ac^6x^6-3\,bc^5x^5+5\,bc^3x^3-15\,bcx+15\,(bc^6x^6+b)\arctan(cx)}{90\,c^6}$$

input
```
integrate(x^5*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/90*(15*a*c^6*x^6 - 3*b*c^5*x^5 + 5*b*c^3*x^3 - 15*b*c*x + 15*(b*c^6*x^6
+ b)*arctan(c*x))/c^6
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int x^5(a+b\arctan(cx))\,dx = \begin{cases} \frac{ax^6}{6} + \frac{bx^6\operatorname{atan}(cx)}{6} - \frac{bx^5}{30c} + \frac{bx^3}{18c^3} - \frac{bx}{6c^5} + \frac{b\operatorname{atan}(cx)}{6c^6} & \text{for } c \neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*x**6/6 + b*x**6*atan(c*x)/6 - b*x**5/(30*c) + b*x**3/(18*c**3
) - b*x/(6*c**5) + b*atan(c*x)/(6*c**6), Ne(c, 0)), (a*x**6/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x^5(a+b\arctan(cx))\,dx = \frac{1}{6}ax^6 + \frac{1}{90}\left(15\,x^6\arctan(cx) - c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\,\arctan(cx)}{c^7}\right)\right)b$$

input
```
integrate(x^5*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/90*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c
^6 - 15*arctan(c*x)/c^7))*b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int x^5(a+b\arctan(cx))\,dx = \frac{15\,bc^6x^6\arctan(cx)+15\,ac^6x^6-3\,bc^5x^5+5\,bc^3x^3-15\,\pi b\text{sgn}(c)\,\text{sgn}(x)-15\,bcx+15\,b\arctan(cx)}{90\,c^6}$$

input
```
integrate(x^5*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/90*(15*b*c^6*x^6*arctan(c*x) + 15*a*c^6*x^6 - 3*b*c^5*x^5 + 5*b*c^3*x^3
- 15*pi*b*sgn(c)*sgn(x) - 15*b*c*x + 15*b*arctan(c*x))/c^6
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int x^5(a+b\arctan(cx))\,dx = \frac{\frac{b\,\text{atan}(c\,x)}{6}+\frac{b\,c^3\,x^3}{18}-\frac{b\,c^5\,x^5}{30}-\frac{b\,c\,x}{6}}{c^6}+\frac{a\,x^6}{6}+\frac{b\,x^6\,\text{atan}(c\,x)}{6}$$

input
```
int(x^5*(a + b*atan(c*x)),x)
```

output
```
((b*atan(c*x))/6 + (b*c^3*x^3)/18 - (b*c^5*x^5)/30 - (b*c*x)/6)/c^6 + (a*x
^6)/6 + (b*x^6*atan(c*x))/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int x^5(a+b\arctan(cx))\,dx = \frac{15atan(cx)\,b\,c^6x^6+15atan(cx)\,b+15a\,c^6x^6-3b\,c^5x^5+5b\,c^3x^3-15bcx}{90c^6}$$

input
```
int(x^5*(a+b*atan(c*x)),x)
```

output
```
(15*atan(c*x)*b*c**6*x**6 + 15*atan(c*x)*b + 15*a*c**6*x**6 - 3*b*c**5*x**
5 + 5*b*c**3*x**3 - 15*b*c*x)/(90*c**6)
```

3.2 $\int x^4(a+b\arctan(cx))\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 56

$$\int x^4(a+b\arctan(cx))\,dx=\frac{bx^2}{10c^3}-\frac{bx^4}{20c}+\frac{1}{5}x^5(a+b\arctan(cx))-\frac{b\log(1+c^2x^2)}{10c^5}$$

output
```
1/10*b*x^2/c^3-1/20*b*x^4/c+1/5*x^5*(a+b*arctan(c*x))-1/10*b*ln(c^2*x^2+1)
/c^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int x^4(a+b\arctan(cx))\,dx=\frac{bx^2}{10c^3}-\frac{bx^4}{20c}+\frac{ax^5}{5}+\frac{1}{5}bx^5\arctan(cx)-\frac{b\log(1+c^2x^2)}{10c^5}$$

input
```
Integrate[x^4*(a + b*ArcTan[c*x]),x]
```

output
```
(b*x^2)/(10*c^3) - (b*x^4)/(20*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x])/5 - (b
*Log[1 + c^2*x^2])/(10*c^5)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5361, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{5}bc\int\frac{x^5}{c^2x^2+1}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\int\frac{x^4}{c^2x^2+1}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\int\left(\frac{x^2}{c^2}+\frac{1}{c^4(c^2x^2+1)}-\frac{1}{c^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)$$

input
```
Int[x^4*(a + b*ArcTan[c*x]),x]
```

output
```
(x^5*(a + b*ArcTan[c*x]))/5 - (b*c*(-(x^2/c^4) + x^4/(2*c^2) + Log[1 + c^2
*x^2]/c^6))/10
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

method	result	size
parts	$\frac{a\,x^{5}}{5}+\frac{b\left(\frac{c^{5} x^{5} \arctan\left(c x\right)}{5}-\frac{c^{4} x^{4}}{20}+\frac{c^{2} x^{2}}{10}-\frac{\ln\left(c^{2} x^{2}+1\right)}{10}\right)}{c^{5}}$	54
derivativedivides	$\frac{\frac{c^{5} x^{5} a}{5}+b\left(\frac{c^{5} x^{5} \arctan\left(c x\right)}{5}-\frac{c^{4} x^{4}}{20}+\frac{c^{2} x^{2}}{10}-\frac{\ln\left(c^{2} x^{2}+1\right)}{10}\right)}{c^{5}}$	58
default	$\frac{\frac{c^{5} x^{5} a}{5}+b\left(\frac{c^{5} x^{5} \arctan\left(c x\right)}{5}-\frac{c^{4} x^{4}}{20}+\frac{c^{2} x^{2}}{10}-\frac{\ln\left(c^{2} x^{2}+1\right)}{10}\right)}{c^{5}}$	58
parallelrisch	$-\frac{-4 b \arctan\left(c x\right) x^{5} c^{5}-4 c^{5} x^{5} a+b\,c^{4} x^{4}-2 b\,c^{2} x^{2}+2 b \ln\left(c^{2} x^{2}+1\right)+2 b}{20 c^{5}}$	62
risch	$-\frac{i x^{5} b \ln\left(i c x+1\right)}{10}+\frac{i x^{5} b \ln\left(-i c x+1\right)}{10}+\frac{a\,x^{5}}{5}-\frac{b\,x^{4}}{20 c}+\frac{b\,x^{2}}{10 c^{3}}-\frac{b \ln\left(-c^{2} x^{2}-1\right)}{10 c^{5}}$	73

input
```
int(x^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+b/c^5*(1/5*c^5*x^5*arctan(c*x)-1/20*c^4*x^4+1/10*c^2*x^2-1/10*ln
(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05

$$\int x^4(a+b\arctan(cx))\,dx = \frac{4\,bc^5x^5\arctan(cx)+4\,ac^5x^5-bc^4x^4+2\,bc^2x^2-2\,b\log(c^2x^2+1)}{20\,c^5}$$

input
```
integrate(x^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/20*(4*b*c^5*x^5*arctan(c*x) + 4*a*c^5*x^5 - b*c^4*x^4 + 2*b*c^2*x^2 - 2*
b*log(c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int x^4(a+b\arctan(cx))\,dx = \begin{cases} \frac{ax^5}{5}+\frac{bx^5\operatorname{atan}(cx)}{5}-\frac{bx^4}{20c}+\frac{bx^2}{10c^3}-\frac{b\log\left(x^2+\frac{1}{c^2}\right)}{10c^5} & \text{for } c\neq 0 \\ \frac{ax^5}{5} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*x**5/5 + b*x**5*atan(c*x)/5 - b*x**4/(20*c) + b*x**2/(10*c**3
) - b*log(x**2 + c**(-2))/(10*c**5), Ne(c, 0)), (a*x**5/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int x^4(a+b\arctan(cx))\,dx$$
$$=\frac{1}{5}\,ax^5+\frac{1}{20}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)b$$

input
```
integrate(x^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2
*x^2 + 1)/c^6))*b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05

$$\int x^4(a+b\arctan(cx))\,dx$$
$$=\frac{4\,bc^5x^5\arctan\left(cx\right)+4\,ac^5x^5-bc^4x^4+2\,bc^2x^2-2\,b\log\left(c^2x^2+1\right)}{20\,c^5}$$

input
```
integrate(x^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/20*(4*b*c^5*x^5*arctan(c*x) + 4*a*c^5*x^5 - b*c^4*x^4 + 2*b*c^2*x^2 - 2*
b*log(c^2*x^2 + 1))/c^5
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int x^4(a+b\arctan(cx))\,dx=\frac{a\,x^5}{5}-\frac{\frac{b\,\ln\left(c^2\,x^2+1\right)}{10}-\frac{b\,c^2\,x^2}{10}+\frac{b\,c^4\,x^4}{20}}{c^5}+\frac{b\,x^5\,\mathrm{atan}\left(c\,x\right)}{5}$$

input
```
int(x^4*(a + b*atan(c*x)),x)
```

output
```
(a*x^5)/5 - ((b*log(c^2*x^2 + 1))/10 - (b*c^2*x^2)/10 + (b*c^4*x^4)/20)/c^
5 + (b*x^5*atan(c*x))/5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05

$$\int x^4(a+b\arctan(cx))\,dx$$
$$=\frac{4atan(cx)\,b\,c^5x^5-2\log(c^2x^2+1)\,b+4a\,c^5x^5-b\,c^4x^4+2b\,c^2x^2}{20c^5}$$

input
```
int(x^4*(a+b*atan(c*x)),x)
```

output
```
(4*atan(c*x)*b*c**5*x**5 - 2*log(c**2*x**2 + 1)*b + 4*a*c**5*x**5 - b*c**4
*x**4 + 2*b*c**2*x**2)/(20*c**5)
```

3.3 $\int x^3(a + b\arctan(cx))\,dx$

Optimal result 98
Mathematica [A] (verified) 98
Rubi [A] (verified) 99
Maple [A] (verified) 100
Fricas [A] (verification not implemented) 100
Sympy [A] (verification not implemented) 101
Maxima [A] (verification not implemented) 101
Giac [A] (verification not implemented) 102
Mupad [B] (verification not implemented) 102
Reduce [B] (verification not implemented) 102

Optimal result

Integrand size = 12, antiderivative size = 48

$$\int x^3(a + b\arctan(cx))\,dx = \frac{bx}{4c^3} - \frac{bx^3}{12c} - \frac{b\arctan(cx)}{4c^4} + \frac{1}{4}x^4(a + b\arctan(cx))$$

output
```
1/4*b*x/c^3-1/12*b*x^3/c-1/4*b*arctan(c*x)/c^4+1/4*x^4*(a+b*arctan(c*x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int x^3(a + b\arctan(cx))\,dx = \frac{bx}{4c^3} - \frac{bx^3}{12c} + \frac{ax^4}{4} - \frac{b\arctan(cx)}{4c^4} + \frac{1}{4}bx^4\arctan(cx)$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x]),x]
```

output
```
(b*x)/(4*c^3) - (b*x^3)/(12*c) + (a*x^4)/4 - (b*ArcTan[c*x])/(4*c^4) + (b*
x^4*ArcTan[c*x])/4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5361, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\int\frac{x^4}{c^2x^2+1}dx$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\int\left(\frac{x^2}{c^2}+\frac{1}{c^4\left(c^2x^2+1\right)}-\frac{1}{c^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)$$

input
```
Int[x^3*(a + b*ArcTan[c*x]),x]
```

output
```
(x^4*(a + b*ArcTan[c*x]))/4 - (b*c*(-(x/c^4) + x^3/(3*c^2) + ArcTan[c*x]/c
^5))/4
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

method	result	size
parts	$\frac{a\,x^{4}}{4}+\frac{b\left(\frac{c^{4} x^{4} \arctan \left(c x \right)}{4}-\frac{c^{3} x^{3}}{12}+\frac{c x}{4}-\frac{\arctan \left(c x \right)}{4}\right)}{c^{4}}$	44
derivativedivides	$\frac{\frac{c^{4} x^{4} a}{4}+b \left(\frac{c^{4} x^{4} \arctan \left(c x \right)}{4}-\frac{c^{3} x^{3}}{12}+\frac{c x}{4}-\frac{\arctan \left(c x \right)}{4}\right)}{c^{4}}$	48
default	$\frac{\frac{c^{4} x^{4} a}{4}+b \left(\frac{c^{4} x^{4} \arctan \left(c x \right)}{4}-\frac{c^{3} x^{3}}{12}+\frac{c x}{4}-\frac{\arctan \left(c x \right)}{4}\right)}{c^{4}}$	48
parallelrisch	$\frac{3 b \arctan \left(c x \right) x^{4} c^{4}+3 c^{4} x^{4} a -b \,c^{3} x^{3}+3 b c x -3 b \arctan \left(c x \right)}{12 c^{4}}$	50
risch	$-\frac{i x^{4} b \ln \left(i c x +1\right)}{8}+\frac{i x^{4} b \ln \left(-i c x +1\right)}{8}+\frac{a\,x^{4}}{4}-\frac{b\,x^{3}}{12 c}+\frac{b x}{4 c^{3}}-\frac{b \arctan \left(c x \right)}{4 c^{4}}$	64
orering	$\frac{\left(c^{4} x^{4}-c^{2} x^{2}-2\right) \left(a +b \arctan \left(c x \right)\right)}{2 c^{4}}-\frac{\left(c^{2} x^{2}-3\right) \left(c^{2} x^{2}+1\right) \left(3 x^{2} \left(a +b \arctan \left(c x \right)\right)+\frac{x^{3} b c}{c^{2} x^{2}+1}\right)}{12 x^{2} c^{4}}$	89

input
```
int(x^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+b/c^4*(1/4*c^4*x^4*arctan(c*x)-1/12*c^3*x^3+1/4*c*x-1/4*arctan(c
*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int x^3(a+b\arctan(cx))\,dx=\frac{3\,ac^4x^4-bc^3x^3+3\,bcx+3\,(bc^4x^4-b)\arctan{(cx)}}{12\,c^4}$$

input
```
integrate(x^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*a*c^4*x^4 - b*c^3*x^3 + 3*b*c*x + 3*(b*c^4*x^4 - b)*arctan(c*x))/c
^4
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int x^3(a+b\arctan(cx))\,dx = \begin{cases} \frac{ax^4}{4} + \frac{bx^4\operatorname{atan}(cx)}{4} - \frac{bx^3}{12c} + \frac{bx}{4c^3} - \frac{b\operatorname{atan}(cx)}{4c^4} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*x**4/4 + b*x**4*atan(c*x)/4 - b*x**3/(12*c) + b*x/(4*c**3) -
b*atan(c*x)/(4*c**4), Ne(c, 0)), (a*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int x^3(a+b\arctan(cx))\,dx = \frac{1}{4}ax^4 + \frac{1}{12}\left(3x^4\arctan(cx) - c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)b$$

input
```
integrate(x^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*
x)/c^5))*b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int x^3(a+b\arctan(cx))\,dx = \frac{3\,bc^4x^4\arctan(cx)+3\,ac^4x^4-bc^3x^3+3\,bcx-3\,b\arctan(cx)}{12\,c^4}$$

input
```
integrate(x^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/12*(3*b*c^4*x^4*arctan(c*x) + 3*a*c^4*x^4 - b*c^3*x^3 + 3*b*c*x - 3*b*ar
ctan(c*x))/c^4
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int x^3(a+b\arctan(cx))\,dx = \frac{a\,x^4}{4}-\frac{\frac{b\,\mathrm{atan}(c\,x)}{4}+\frac{b\,c^3\,x^3}{12}-\frac{b\,c\,x}{4}}{c^4}+\frac{b\,x^4\,\mathrm{atan}(c\,x)}{4}$$

input
```
int(x^3*(a + b*atan(c*x)),x)
```

output
```
(a*x^4)/4 - ((b*atan(c*x))/4 + (b*c^3*x^3)/12 - (b*c*x)/4)/c^4 + (b*x^4*at
an(c*x))/4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int x^3(a+b\arctan(cx))\,dx = \frac{3atan(cx)\,b\,c^4x^4-3atan(cx)\,b+3a\,c^4x^4-b\,c^3x^3+3bcx}{12c^4}$$

input
```
int(x^3*(a+b*atan(c*x)),x)
```

output
```
(3*atan(c*x)*b*c**4*x**4 - 3*atan(c*x)*b + 3*a*c**4*x**4 - b*c**3*x**3 + 3
*b*c*x)/(12*c**4)
```

3.4 $\int x^2(a+b\arctan(cx))\,dx$

Optimal result 104
Mathematica [A] (verified) 104
Rubi [A] (verified) 105
Maple [A] (verified) 106
Fricas [A] (verification not implemented) 107
Sympy [A] (verification not implemented) 107
Maxima [A] (verification not implemented) 108
Giac [A] (verification not implemented) 108
Mupad [B] (verification not implemented) 108
Reduce [B] (verification not implemented) 109

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int x^2(a+b\arctan(cx))\,dx = -\frac{bx^2}{6c}+\frac{1}{3}x^3(a+b\arctan(cx))+\frac{b\log(1+c^2x^2)}{6c^3}$$

output
```
-1/6*b*x^2/c+1/3*x^3*(a+b*arctan(c*x))+1/6*b*ln(c^2*x^2+1)/c^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int x^2(a+b\arctan(cx))\,dx = -\frac{bx^2}{6c}+\frac{ax^3}{3}+\frac{1}{3}bx^3\arctan(cx)+\frac{b\log(1+c^2x^2)}{6c^3}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x]),x]
```

output
```
-1/6*(b*x^2)/c + (a*x^3)/3 + (b*x^3*ArcTan[c*x])/3 + (b*Log[1 + c^2*x^2])/
(6*c^3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5361, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \ 5361$$

$$\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{3}bc\int \frac{x^3}{c^2x^2+1}dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int \frac{x^2}{c^2x^2+1}dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int \left(\frac{1}{c^2}-\frac{1}{c^2\left(c^2x^2+1\right)}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)$$

input
```
Int[x^2*(a + b*ArcTan[c*x]),x]
```

output
```
(x^3*(a + b*ArcTan[c*x]))/3 - (b*c*(x^2/c^2 - Log[1 + c^2*x^2]/c^4))/6
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
parts	$\frac{a\,x^{3}}{3}+\frac{b\left(\frac{c^{3}x^{3}\arctan\left(cx\right)}{3}-\frac{c^{2}x^{2}}{6}+\frac{\ln\left(c^{2}x^{2}+1\right)}{6}\right)}{c^{3}}$	46
derivativedivides	$\frac{\frac{c^{3}x^{3}a}{3}+b\left(\frac{c^{3}x^{3}\arctan\left(cx\right)}{3}-\frac{c^{2}x^{2}}{6}+\frac{\ln\left(c^{2}x^{2}+1\right)}{6}\right)}{c^{3}}$	50
default	$\frac{\frac{c^{3}x^{3}a}{3}+b\left(\frac{c^{3}x^{3}\arctan\left(cx\right)}{3}-\frac{c^{2}x^{2}}{6}+\frac{\ln\left(c^{2}x^{2}+1\right)}{6}\right)}{c^{3}}$	50
parallelrisch	$\frac{2x^{3}\arctan\left(cx\right)b\,c^{3}+2c^{3}x^{3}a-b\,c^{2}x^{2}+b\ln\left(c^{2}x^{2}+1\right)}{6c^{3}}$	50
risch	$-\frac{i x^{3}b\ln\left(icx+1\right)}{6}+\frac{i x^{3}b\ln\left(-icx+1\right)}{6}+\frac{a\,x^{3}}{3}-\frac{b\,x^{2}}{6c}+\frac{b\ln\left(-c^{2}x^{2}-1\right)}{6c^{3}}$	64

input
```
int(x^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+b/c^3*(1/3*c^3*x^3*arctan(c*x)-1/6*c^2*x^2+1/6*ln(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int x^2(a+b\arctan(cx))\,dx=\frac{2\,bc^3x^3\arctan\left(cx\right)+2\,ac^3x^3-bc^2x^2+b\log\left(c^2x^2+1\right)}{6\,c^3}$$

input
```
integrate(x^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*b*c^3*x^3*arctan(c*x) + 2*a*c^3*x^3 - b*c^2*x^2 + b*log(c^2*x^2 + 1
))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int x^2(a+b\arctan(cx))\,dx=\begin{cases}\frac{ax^3}{3}+\frac{bx^3\operatorname{atan}\left(cx\right)}{3}-\frac{bx^2}{6c}+\frac{b\log\left(x^2+\frac{1}{c^2}\right)}{6c^3} & \text{for}\ c\neq 0\\ \frac{ax^3}{3} & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*x**3/3 + b*x**3*atan(c*x)/3 - b*x**2/(6*c) + b*log(x**2 + c**
(-2))/(6*c**3), Ne(c, 0)), (a*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int x^2(a+b\arctan(cx))\,dx=\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)b$$

input
```
integrate(x^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int x^2(a+b\arctan(cx))\,dx=\frac{2\,bc^3x^3\arctan\left(cx\right)+2\,ac^3x^3-bc^2x^2+b\log\left(c^2x^2+1\right)}{6\,c^3}$$

input
```
integrate(x^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/6*(2*b*c^3*x^3*arctan(c*x) + 2*a*c^3*x^3 - b*c^2*x^2 + b*log(c^2*x^2 + 1
))/c^3
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x^2(a+b\arctan(cx))\,dx=\frac{a\,x^3}{3}+\frac{b\,x^3\,\mathrm{atan}(c\,x)}{3}+\frac{b\,\ln\left(c^2\,x^2+1\right)}{6\,c^3}-\frac{b\,x^2}{6\,c}$$

input
```
int(x^2*(a + b*atan(c*x)),x)
```

output
```
(a*x^3)/3 + (b*x^3*atan(c*x))/3 + (b*log(c^2*x^2 + 1))/(6*c^3) - (b*x^2)/(
6*c)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int x^2(a+b\arctan(cx))\,dx = \frac{2atan(cx)\,b\,c^3x^3+\log(c^2x^2+1)\,b+2a\,c^3x^3-b\,c^2x^2}{6c^3}$$

input
```
int(x^2*(a+b*atan(c*x)),x)
```

output
```
(2*atan(c*x)*b*c**3*x**3 + log(c**2*x**2 + 1)*b + 2*a*c**3*x**3 - b*c**2*x
**2)/(6*c**3)
```

3.5 $\int x(a+b\arctan(cx))\,dx$

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Mathematica [A] (verified) 110
Rubi [A] (verified) 111
Maple [A] (verified) 112
Fricas [A] (verification not implemented) 113
Sympy [A] (verification not implemented) 113
Maxima [A] (verification not implemented) 113
Giac [A] (verification not implemented) 114
Mupad [B] (verification not implemented) 114
Reduce [B] (verification not implemented) 114

Optimal result

Integrand size = 10, antiderivative size = 37

$$\int x(a+b\arctan(cx))\,dx=-\frac{bx}{2c}+\frac{b\arctan(cx)}{2c^2}+\frac{1}{2}x^2(a+b\arctan(cx))$$

output
```
-1/2*b*x/c+1/2*b*arctan(c*x)/c^2+1/2*x^2*(a+b*arctan(c*x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int x(a+b\arctan(cx))\,dx=-\frac{bx}{2c}+\frac{ax^2}{2}+\frac{b\arctan(cx)}{2c^2}+\frac{1}{2}bx^2\arctan(cx)$$

input
```
Integrate[x*(a + b*ArcTan[c*x]),x]
```

output
```
-1/2*(b*x)/c + (a*x^2)/2 + (b*ArcTan[c*x])/(2*c^2) + (b*x^2*ArcTan[c*x])/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5361, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int\frac{x^2}{c^2x^2+1}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int\frac{1}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)$$

input
```
Int[x*(a + b*ArcTan[c*x]),x]
```

output
```
(x^2*(a + b*ArcTan[c*x]))/2 - (b*c*(x/c^2 - ArcTan[c*x]/c^3))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
parts	$\frac{a\,x^{2}}{2}+\frac{b\left(\frac{c^{2} x^{2} \arctan\left(c x\right)}{2}-\frac{c x}{2}+\frac{\arctan\left(c x\right)}{2}\right)}{c^{2}}$	36
parallelrisch	$\frac{\arctan\left(c x\right) b\,c^{2} x^{2}+a\,c^{2} x^{2}-b c x+b \arctan\left(c x\right)}{2 c^{2}}$	38
derivativedivides	$\frac{\frac{a\,c^{2} x^{2}}{2}+b\left(\frac{c^{2} x^{2} \arctan\left(c x\right)}{2}-\frac{c x}{2}+\frac{\arctan\left(c x\right)}{2}\right)}{c^{2}}$	40
default	$\frac{\frac{a\,c^{2} x^{2}}{2}+b\left(\frac{c^{2} x^{2} \arctan\left(c x\right)}{2}-\frac{c x}{2}+\frac{\arctan\left(c x\right)}{2}\right)}{c^{2}}$	40
risch	$-\frac{i x^{2} b \ln\left(i c x+1\right)}{4}+\frac{a\,x^{2}}{2}+\frac{i x^{2} b \ln\left(-i c x+1\right)}{4}-\frac{b x}{2 c}+\frac{b \arctan\left(c x\right)}{2 c^{2}}$	55
orering	$\frac{\left(c^{2} x^{2}+1\right)\left(a+b \arctan\left(c x\right)\right)}{c^{2}}-\frac{\left(c^{2} x^{2}+1\right)\left(a+b \arctan\left(c x\right)+\frac{x b c}{c^{2} x^{2}+1}\right)}{2 c^{2}}$	60

input
```
int(x*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+b/c^2*(1/2*c^2*x^2*arctan(c*x)-1/2*c*x+1/2*arctan(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x(a+b\arctan(cx))\,dx=\frac{ac^2x^2-bcx+(bc^2x^2+b)\arctan(cx)}{2\,c^2}$$

input
```
integrate(x*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/2*(a*c^2*x^2 - b*c*x + (b*c^2*x^2 + b)*arctan(c*x))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int x(a+b\arctan(cx))\,dx=\begin{cases}\frac{ax^2}{2}+\frac{bx^2\operatorname{atan}(cx)}{2}-\frac{bx}{2c}+\frac{b\operatorname{atan}(cx)}{2c^2} & \text{for } c\neq 0\\ \frac{ax^2}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*x**2/2 + b*x**2*atan(c*x)/2 - b*x/(2*c) + b*atan(c*x)/(2*c**2
), Ne(c, 0)), (a*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x(a+b\arctan(cx))\,dx=\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)b$$

input
```
integrate(x*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int x(a+b\arctan(cx))\,dx$$
$$=\frac{bc^2x^2\arctan(cx)+ac^2x^2-\pi b\text{sgn}(c)\,\text{sgn}(x)-bcx+b\arctan(cx)}{2\,c^2}$$

input
```
integrate(x*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/2*(b*c^2*x^2*arctan(c*x) + a*c^2*x^2 - pi*b*sgn(c)*sgn(x) - b*c*x + b*ar
ctan(c*x))/c^2
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x(a+b\arctan(cx))\,dx=\frac{a\,x^2}{2}+\frac{b\,\text{atan}(c\,x)}{2\,c^2}+\frac{b\,x^2\,\text{atan}(c\,x)}{2}-\frac{b\,x}{2\,c}$$

input
```
int(x*(a + b*atan(c*x)),x)
```

output
```
(a*x^2)/2 + (b*atan(c*x))/(2*c^2) + (b*x^2*atan(c*x))/2 - (b*x)/(2*c)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x(a+b\arctan(cx))\,dx=\frac{atan(cx)\,b\,c^2x^2+atan(cx)\,b+a\,c^2x^2-bcx}{2c^2}$$

input
```
int(x*(a+b*atan(c*x)),x)
```

output
```
(atan(c*x)*b*c**2*x**2 + atan(c*x)*b + a*c**2*x**2 - b*c*x)/(2*c**2)
```

3.6 $\int (a+b\arctan(cx))\,dx$

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Rubi [A] (verified) 116
Maple [A] (verified) 117
Fricas [A] (verification not implemented) 117
Sympy [A] (verification not implemented) 118
Maxima [A] (verification not implemented) 118
Giac [A] (verification not implemented) 118
Mupad [B] (verification not implemented) 119
Reduce [B] (verification not implemented) 119

Optimal result

Integrand size = 8, antiderivative size = 29

$$\int (a+b\arctan(cx))\,dx = ax+bx\arctan(cx)-\frac{b\log(1+c^2x^2)}{2c}$$

output
```
a*x+b*x*arctan(c*x)-1/2*b*ln(c^2*x^2+1)/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (a+b\arctan(cx))\,dx = ax+bx\arctan(cx)-\frac{b\log(1+c^2x^2)}{2c}$$

input
```
Integrate[a + b*ArcTan[c*x],x]
```

output
```
a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\arctan(cx))\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax + bx\arctan(cx) - \frac{b\log(c^2x^2+1)}{2c}$$

input
```
Int[a + b*ArcTan[c*x],x]
```

output
```
a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

method	result	size
default	$ax + bx\arctan(cx) - \frac{b\ln(c^2x^2+1)}{2c}$	28
parts	$ax + bx\arctan(cx) - \frac{b\ln(c^2x^2+1)}{2c}$	28
parallelrisch	$-\frac{b(-2cx\arctan(cx)+\ln(c^2x^2+1))}{2c} + ax$	30
derivativedivides	$\frac{acx+b\left(cx\arctan(cx)-\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{c}$	32
risch	$ax - \frac{ibx\ln(icx+1)}{2} + \frac{ibx\ln(-icx+1)}{2} - \frac{b\ln(-c^2x^2-1)}{2c}$	48

input
```
int(a+b*arctan(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arctan(c*x)-1/2*b*ln(c^2*x^2+1)/c
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int (a + b\arctan(cx))\, dx = \frac{2\, bcx\arctan(cx) + 2\, acx - b\log(c^2x^2 + 1)}{2\, c}$$

input
```
integrate(a+b*arctan(c*x),x, algorithm="fricas")
```

output
```
1/2*(2*b*c*x*arctan(c*x) + 2*a*c*x - b*log(c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int (a+b\arctan(cx))\,dx = ax + b\left(\begin{cases} x\,\mathrm{atan}\,(cx) - \frac{\log\,(c^2x^2+1)}{2c} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*atan(c*x),x)
```

output
```
a*x + b*Piecewise((x*atan(c*x) - log(c**2*x**2 + 1)/(2*c), Ne(c, 0)), (0,
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int (a+b\arctan(cx))\,dx = ax + \frac{(2\,cx\arctan\,(cx) - \log\,(c^2x^2+1))b}{2\,c}$$

input
```
integrate(a+b*arctan(c*x),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int (a+b\arctan(cx))\,dx = ax + \frac{(2\,cx\arctan\,(cx) - \log\,(c^2x^2+1))b}{2\,c}$$

input
```
integrate(a+b*arctan(c*x),x, algorithm="giac")
```

output
```
a*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b/c
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int (a + b\arctan(cx))\,dx = a\,x - \frac{b\,\ln\left(c^2\,x^2 + 1\right)}{2\,c} + b\,x\,\mathrm{atan}(c\,x)$$

input
```
int(a + b*atan(c*x),x)
```

output
```
a*x - (b*log(c^2*x^2 + 1))/(2*c) + b*x*atan(c*x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int (a + b\arctan(cx))\,dx = \frac{2atan(cx)\,bcx - \log(c^2x^2 + 1)\,b + 2acx}{2c}$$

input
```
int(a+b*atan(c*x),x)
```

output
```
(2*atan(c*x)*b*c*x - log(c**2*x**2 + 1)*b + 2*a*c*x)/(2*c)
```

3.7 $\int \frac{a+b\arctan(cx)}{x}\,dx$

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Rubi [A] (verified) 121
Maple [A] (verified) 122
Fricas [F] 122
Sympy [F] 122
Maxima [F] 123
Giac [F] 123
Mupad [B] (verification not implemented) 123
Reduce [F] 124

Optimal result

Integrand size = 12, antiderivative size = 35

$$\int \frac{a+b\arctan(cx)}{x}\,dx = a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)$$

output
```
a*ln(x)+1/2*I*b*polylog(2,-I*c*x)-1/2*I*b*polylog(2,I*c*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x}\,dx = a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)$$

input
```
Integrate[(a + b*ArcTan[c*x])/x,x]
```

output
```
a*Log[x] + (I/2)*b*PolyLog[2, (-I)*c*x] - (I/2)*b*PolyLog[2, I*c*x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x}\,dx$$

$$\Big\downarrow \; 5355$$

$$\frac{1}{2}ib\int \frac{\log(1-icx)}{x}dx-\frac{1}{2}ib\int \frac{\log(icx+1)}{x}dx+a\log(x)$$

$$\Big\downarrow \; 2838$$

$$a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)$$

input
```
Int[(a + b*ArcTan[c*x])/x,x]
```

output
```
a*Log[x] + (I/2)*b*PolyLog[2, (-I)*c*x] - (I/2)*b*PolyLog[2, I*c*x]
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{ib\,\mathrm{dilog}(-icx+1)}{2}+a\ln\left(-icx\right)+\frac{ib\,\mathrm{dilog}(icx+1)}{2}$
parts	$a\ln\left(x\right)+b\Big(\ln\left(cx\right)\arctan\left(cx\right)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln(-icx+1)}{2}+\frac{i\,\mathrm{dilog}(icx+1)}{2}-$ $i\,\mathrm{dilo}$
derivativedivides	$a\ln\left(cx\right)+b\Big(\ln\left(cx\right)\arctan\left(cx\right)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln(-icx+1)}{2}+\frac{i\,\mathrm{dilog}(icx+1)}{2}-$ $i\,\mathrm{dil}$
default	$a\ln\left(cx\right)+b\Big(\ln\left(cx\right)\arctan\left(cx\right)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln(-icx+1)}{2}+\frac{i\,\mathrm{dilog}(icx+1)}{2}-$ $i\,\mathrm{dil}$

input
```
int((a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*b*dilog(1-I*c*x)+a*ln(-I*c*x)+1/2*I*b*dilog(1+I*c*x)
```

Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x}\,dx=\int \frac{b\arctan\left(cx\right)+a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/x, x)
```

Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x}\,dx=\int \frac{a+b\,\mathrm{atan}\left(cx\right)}{x}\,dx$$

input
```
integrate((a+b*atan(c*x))/x,x)
```

output
```
Integral((a + b*atan(c*x))/x, x)
```

Maxima [F]

$$\int \frac{a + b\arctan(cx)}{x}\,dx = \int \frac{b\arctan{(cx)} + a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
b*integrate(arctan(c*x)/x, x) + a*log(x)
```

Giac [F]

$$\int \frac{a + b\arctan(cx)}{x}\,dx = \int \frac{b\arctan{(cx)} + a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{a + b\arctan(cx)}{x}\,dx = a\,\ln{(x)} - \frac{b\,(\mathrm{Li}_2(1 - c\,x\,1\mathrm{i}) - \mathrm{Li}_2(1 + c\,x\,1\mathrm{i}))\,1\mathrm{i}}{2}$$

input
```
int((a + b*atan(c*x))/x,x)
```

output
```
a*log(x) - (b*(dilog(1 - c*x*1i) - dilog(c*x*1i + 1))*1i)/2
```

Reduce [F]

$$\int \frac{a + b \arctan(cx)}{x} \, dx = \left(\int \frac{atan(cx)}{x} dx \right) b + \log(x) \, a$$

input
```
int((a+b*atan(c*x))/x,x)
```

output
```
int(atan(c*x)/x,x)*b + log(x)*a
```

3.8 $\int \frac{a+b\arctan(cx)}{x^2}\,dx$

Optimal result 125
Mathematica [A] (verified) 125
Rubi [A] (verified) 126
Maple [A] (verified) 127
Fricas [A] (verification not implemented) 128
Sympy [A] (verification not implemented) 128
Maxima [A] (verification not implemented) 129
Giac [A] (verification not implemented) 129
Mupad [B] (verification not implemented) 129
Reduce [B] (verification not implemented) 130

Optimal result

Integrand size = 12, antiderivative size = 35

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx=-\frac{a+b\arctan(cx)}{x}+bc\log(x)-\frac{1}{2}bc\log\left(1+c^2x^2\right)$$

output
```
-(a+b*arctan(c*x))/x+b*c*ln(x)-1/2*b*c*ln(c^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx=-\frac{a}{x}-\frac{b\arctan(cx)}{x}+bc\log(x)-\frac{1}{2}bc\log\left(1+c^2x^2\right)$$

input
```
Integrate[(a + b*ArcTan[c*x])/x^2,x]
```

output
```
-(a/x) - (b*ArcTan[c*x])/x + b*c*Log[x] - (b*c*Log[1 + c^2*x^2])/2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5361, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx$$

$$\Big\downarrow \text{5361}$$

$$bc\int \frac{1}{x\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{x}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}bc\int \frac{1}{x^2\left(c^2x^2+1\right)}dx^2-\frac{a+b\arctan(cx)}{x}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}$$

input
```
Int[(a + b*ArcTan[c*x])/x^2,x]
```

output
```
-((a + b*ArcTan[c*x])/x) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

method	result	size
parallelrisch	$\frac{2bc\ln(x)x-bc\ln(c^2x^2+1)x-2b\arctan(cx)-2a}{2x}$	39
parts	$-\frac{a}{x}+bc\left(-\frac{\arctan(cx)}{cx}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)$	40
derivativedivides	$c\left(-\frac{a}{cx}+b\left(-\frac{\arctan(cx)}{cx}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)\right)$	44
default	$c\left(-\frac{a}{cx}+b\left(-\frac{\arctan(cx)}{cx}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)\right)$	44
risch	$\frac{ib\ln(icx+1)}{2x}-\frac{-2bc\ln(x)x+bc\ln(-c^2x^2-1)x+ib\ln(-icx+1)+2a}{2x}$	60

input
```
int((a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*b*c*ln(x)*x-b*c*ln(c^2*x^2+1)*x-2*b*arctan(c*x)-2*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx = -\frac{bcx\log\left(c^2x^2+1\right)-2\,bcx\log\left(x\right)+2\,b\arctan\left(cx\right)+2\,a}{2\,x}$$

input
```
integrate((a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
-1/2*(b*c*x*log(c^2*x^2 + 1) - 2*b*c*x*log(x) + 2*b*arctan(c*x) + 2*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx = \begin{cases} -\frac{a}{x}+bc\log\left(x\right)-\frac{bc\log\left(x^2+\frac{1}{c^2}\right)}{2}-\frac{b\,\mathrm{atan}\left(cx\right)}{x} & \text{for}\ c\neq 0\\ -\frac{a}{x} & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*atan(c*x))/x**2,x)
```

output
```
Piecewise((-a/x + b*c*log(x) - b*c*log(x**2 + c**(-2))/2 - b*atan(c*x)/x,
Ne(c, 0)), (-a/x, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{a + b\arctan(cx)}{x^2}\,dx = -\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan(cx)}{x}\right)b - \frac{a}{x}$$

input
```
integrate((a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b - a/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{a + b\arctan(cx)}{x^2}\,dx = -\frac{bcx\log\left(c^2x^2+1\right) - 2\,bcx\log(x) + 2\,b\arctan(cx) + 2\,a}{2\,x}$$

input
```
integrate((a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
-1/2*(b*c*x*log(c^2*x^2 + 1) - 2*b*c*x*log(x) + 2*b*arctan(c*x) + 2*a)/x
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{a + b\arctan(cx)}{x^2}\,dx = b\,c\,\ln(x) - \frac{a}{x} - \frac{b\,\mathrm{atan}(c\,x)}{x} - \frac{b\,c\,\ln\left(c^2\,x^2+1\right)}{2}$$

input
```
int((a + b*atan(c*x))/x^2,x)
```

output
```
b*c*log(x) - a/x - (b*atan(c*x))/x - (b*c*log(c^2*x^2 + 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx = \frac{-2atan(cx)\,b-\log(c^2x^2+1)\,bcx+2\log(x)\,bcx-2a}{2x}$$

input
```
int((a+b*atan(c*x))/x^2,x)
```

output
```
( - 2*atan(c*x)*b - log(c**2*x**2 + 1)*b*c*x + 2*log(x)*b*c*x - 2*a)/(2*x)
```

3.9 $\int \frac{a+b\arctan(cx)}{x^3}\,dx$

Optimal result 131
Mathematica [C] (verified) 131
Rubi [A] (verified) 132
Maple [A] (verified) 133
Fricas [A] (verification not implemented) 134
Sympy [A] (verification not implemented) 134
Maxima [A] (verification not implemented) 134
Giac [C] (verification not implemented) 135
Mupad [B] (verification not implemented) 135
Reduce [B] (verification not implemented) 136

Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx = -\frac{bc}{2x} - \frac{1}{2}bc^2\arctan(cx) - \frac{a+b\arctan(cx)}{2x^2}$$

output
```
-1/2*b*c/x-1/2*b*c^2*arctan(c*x)-1/2*(a+b*arctan(c*x))/x^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx = -\frac{a}{2x^2} - \frac{b\arctan(cx)}{2x^2} - \frac{bc\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2x}$$

input
```
Integrate[(a + b*ArcTan[c*x])/x^3,x]
```

output
```
-1/2*a/x^2 - (b*ArcTan[c*x])/(2*x^2) - (b*c*Hypergeometric2F1[-1/2, 1, 1/2
, -(c^2*x^2)])/(2*x)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5361, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}bc\int \frac{1}{x^2\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{2x^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}bc\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}$$

input
```
Int[(a + b*ArcTan[c*x])/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c*x])/x^2 + (b*c*(-x^(-1) - c*ArcTan[c*x]))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$-\frac{\arctan(cx) b\,c^2x^2-a\,c^2x^2+bcx+b\arctan(cx)+a}{2x^2}$	39
parts	$-\frac{a}{2x^2}+b\,c^2\left(-\frac{\arctan(cx)}{2c^2x^2}-\frac{1}{2cx}-\frac{\arctan(cx)}{2}\right)$	40
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\arctan(cx)}{2c^2x^2}-\frac{1}{2cx}-\frac{\arctan(cx)}{2}\right)\right)$	44
default	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\arctan(cx)}{2c^2x^2}-\frac{1}{2cx}-\frac{\arctan(cx)}{2}\right)\right)$	44
orering	$\frac{(-2x^3c^2-2x)(a+b\arctan(cx))}{x^3}-\frac{(c^2x^2+1)x^2\left(\frac{bc}{(c^2x^2+1)x^3}-\frac{3(a+b\arctan(cx))}{x^4}\right)}{2}$	71
risch	$\frac{ib\ln(icx+1)}{4x^2}-\frac{-ib\,c^2\ln(-cx+i)x^2+ib\,c^2\ln(-cx-i)x^2+ib\ln(-icx+1)+2bcx+2a}{4x^2}$	79

input
```
int((a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctan(c*x)*b*c^2*x^2-a*c^2*x^2+b*c*x+b*arctan(c*x)+a)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{a + b\arctan(cx)}{x^3}\,dx = -\frac{bcx + (bc^2x^2 + b)\arctan(cx) + a}{2\,x^2}$$

input
```
integrate((a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
-1/2*(b*c*x + (b*c^2*x^2 + b)*arctan(c*x) + a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{a + b\arctan(cx)}{x^3}\,dx = -\frac{a}{2x^2} - \frac{bc^2\,\mathrm{atan}(cx)}{2} - \frac{bc}{2x} - \frac{b\,\mathrm{atan}(cx)}{2x^2}$$

input
```
integrate((a+b*atan(c*x))/x**3,x)
```

output
```
-a/(2*x**2) - b*c**2*atan(c*x)/2 - b*c/(2*x) - b*atan(c*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{a + b\arctan(cx)}{x^3}\,dx = -\frac{1}{2}\left(\left(c\arctan(cx) + \frac{1}{x}\right)c + \frac{\arctan(cx)}{x^2}\right)b - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
-1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b - 1/2*a/x^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx = -\frac{-i\,bc^2x^2\log\left(i\,cx+1\right)+i\,bc^2x^2\log\left(-i\,cx+1\right)+2\,bcx+2\,b\arctan\left(cx\right)+2\,a}{4\,x^2}$$

input
```
integrate((a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
-1/4*(-I*b*c^2*x^2*log(I*c*x + 1) + I*b*c^2*x^2*log(-I*c*x + 1) + 2*b*c*x
+ 2*b*arctan(c*x) + 2*a)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx = -\frac{\frac{a}{2}+\frac{b\,\mathrm{atan}(c\,x)}{2}+\frac{b\,c\,x}{2}}{x^2}-\frac{b\,c\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,\sqrt{c^2}}{2}$$

input
```
int((a + b*atan(c*x))/x^3,x)
```

output
```
- (a/2 + (b*atan(c*x))/2 + (b*c*x)/2)/x^2 - (b*c*atan((c^2*x)/(c^2)^(1/2))
*(c^2)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx = \frac{-atan(cx)\,b\,c^2x^2 - atan(cx)\,b - a - bcx}{2x^2}$$

input
```
int((a+b*atan(c*x))/x^3,x)
```

output
```
( - (atan(c*x)*b*c**2*x**2 + atan(c*x)*b + a + b*c*x))/(2*x**2)
```

3.10 $\int \frac{a+b\arctan(cx)}{x^4}\,dx$

Optimal result 137
Mathematica [A] (verified) 137
Rubi [A] (verified) 138
Maple [A] (verified) 139
Fricas [A] (verification not implemented) 140
Sympy [A] (verification not implemented) 140
Maxima [A] (verification not implemented) 141
Giac [A] (verification not implemented) 141
Mupad [B] (verification not implemented) 142
Reduce [B] (verification not implemented) 142

Optimal result

Integrand size = 12, antiderivative size = 53

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = -\frac{bc}{6x^2}-\frac{a+b\arctan(cx)}{3x^3}-\frac{1}{3}bc^3\log(x)+\frac{1}{6}bc^3\log\left(1+c^2x^2\right)$$

output
```
-1/6*b*c/x^2-1/3*(a+b*arctan(c*x))/x^3-1/3*b*c^3*ln(x)+1/6*b*c^3*ln(c^2*x^
2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{b\arctan(cx)}{3x^3}+\frac{1}{6}bc\left(-\frac{1}{x^2}-2c^2\log(x)+c^2\log\left(1+c^2x^2\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x])/x^4,x]
```

output
```
-1/3*a/x^3 - (b*ArcTan[c*x])/(3*x^3) + (b*c*(-x^(-2) - 2*c^2*Log[x] + c^2*
Log[1 + c^2*x^2]))/6
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5361, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx$$

$$\Big\downarrow \; 5361$$

$$\frac{1}{3}bc\int \frac{1}{x^3\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{3x^3}$$

$$\Big\downarrow \; 243$$

$$\frac{1}{6}bc\int \frac{1}{x^4\left(c^2x^2+1\right)}dx^2-\frac{a+b\arctan(cx)}{3x^3}$$

$$\Big\downarrow \; 54$$

$$\frac{1}{6}bc\int \left(\frac{c^4}{c^2x^2+1}-\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{a+b\arctan(cx)}{3x^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}$$

input
```
Int[(a + b*ArcTan[c*x])/x^4,x]
```

output
```
-1/3*(a + b*ArcTan[c*x])/x^3 + (b*c*(-x^(-2) - c^2*Log[x^2] + c^2*Log[1 +
c^2*x^2]))/6
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result	size
parts	$-\frac{a}{3x^{3}}+b\,c^{3}\left(-\frac{\arctan\left(cx\right)}{3c^{3}x^{3}}+\frac{\ln\left(c^{2}x^{2}+1\right)}{6}-\frac{1}{6c^{2}x^{2}}-\frac{\ln\left(cx\right)}{3}\right)$	52
derivativedivides	$c^{3}\left(-\frac{a}{3c^{3}x^{3}}+b\left(-\frac{\arctan\left(cx\right)}{3c^{3}x^{3}}+\frac{\ln\left(c^{2}x^{2}+1\right)}{6}-\frac{1}{6c^{2}x^{2}}-\frac{\ln\left(cx\right)}{3}\right)\right)$	56
default	$c^{3}\left(-\frac{a}{3c^{3}x^{3}}+b\left(-\frac{\arctan\left(cx\right)}{3c^{3}x^{3}}+\frac{\ln\left(c^{2}x^{2}+1\right)}{6}-\frac{1}{6c^{2}x^{2}}-\frac{\ln\left(cx\right)}{3}\right)\right)$	56
parallelrisch	$-\frac{2b\,c^{3}\ln\left(x\right)x^{3}-b\,c^{3}\ln\left(c^{2}x^{2}+1\right)x^{3}-b\,c^{3}x^{3}+bcx+2b\arctan\left(cx\right)+2a}{6x^{3}}$	60
risch	$\frac{ib\ln\left(icx+1\right)}{6x^{3}}-\frac{2b\,c^{3}\ln\left(x\right)x^{3}-b\,c^{3}\ln\left(c^{2}x^{2}+1\right)x^{3}+ib\ln\left(-icx+1\right)+bcx+2a}{6x^{3}}$	72

input
```
int((a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*c^3*(-1/3/c^3/x^3*arctan(c*x)+1/6*ln(c^2*x^2+1)-1/6/c^2/x^2-1
/3*ln(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = \frac{bc^3x^3\log\left(c^2x^2+1\right)-2\,bc^3x^3\log\left(x\right)-bcx-2\,b\arctan\left(cx\right)-2\,a}{6\,x^3}$$

input
```
integrate((a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/6*(b*c^3*x^3*log(c^2*x^2 + 1) - 2*b*c^3*x^3*log(x) - b*c*x - 2*b*arctan(
c*x) - 2*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = \begin{cases} -\frac{a}{3x^3}-\frac{bc^3\log\left(x\right)}{3}+\frac{bc^3\log\left(x^2+\frac{1}{c^2}\right)}{6}-\frac{bc}{6x^2}-\frac{b\,\mathrm{atan}\left(cx\right)}{3x^3} & \text{for } c\neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atan(c*x))/x**4,x)
```

output
```
Piecewise((-a/(3*x**3) - b*c**3*log(x)/3 + b*c**3*log(x**2 + c**(-2))/6 -
b*c/(6*x**2) - b*atan(c*x)/(3*x**3), Ne(c, 0)), (-a/(3*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)b - \frac{a}{3\,x^3}$$

input
```
integrate((a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*
b - 1/3*a/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = \frac{bc^3x^3\log\left(c^2x^2+1\right)-2\,bc^3x^3\log\left(x\right)-bcx-2\,b\arctan\left(cx\right)-2\,a}{6\,x^3}$$

input
```
integrate((a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
1/6*(b*c^3*x^3*log(c^2*x^2 + 1) - 2*b*c^3*x^3*log(x) - b*c*x - 2*b*arctan(
c*x) - 2*a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = \frac{b\,c^3\,\ln\left(c^2\,x^2+1\right)}{6} - \frac{\frac{a}{3}+\frac{b\,\mathrm{atan}(c\,x)}{3}+\frac{b\,c\,x}{6}}{x^3} - \frac{b\,c^3\,\ln\left(x\right)}{3}$$

input
```
int((a + b*atan(c*x))/x^4,x)
```

output
```
(b*c^3*log(c^2*x^2 + 1))/6 - (a/3 + (b*atan(c*x))/3 + (b*c*x)/6)/x^3 - (b*
c^3*log(x))/3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = \frac{-2atan(cx)\,b+\log(c^2x^2+1)\,b\,c^3x^3-2\log(x)\,b\,c^3x^3-2a-bcx}{6x^3}$$

input
```
int((a+b*atan(c*x))/x^4,x)
```

output
```
( - 2*atan(c*x)*b + log(c**2*x**2 + 1)*b*c**3*x**3 - 2*log(x)*b*c**3*x**3
- 2*a - b*c*x)/(6*x**3)
```

3.11 $\int \frac{a+b\arctan(cx)}{x^5}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 48

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx = -\frac{bc}{12x^3}+\frac{bc^3}{4x}+\frac{1}{4}bc^4\arctan(cx)-\frac{a+b\arctan(cx)}{4x^4}$$

output
```
-1/12*b*c/x^3+1/4*b*c^3/x+1/4*b*c^4*arctan(c*x)-1/4*(a+b*arctan(c*x))/x^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx = -\frac{a}{4x^4}-\frac{b\arctan(cx)}{4x^4}-\frac{bc\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3}$$

input
```
Integrate[(a + b*ArcTan[c*x])/x^5,x]
```

output
```
-1/4*a/x^4 - (b*ArcTan[c*x])/(4*x^4) - (b*c*Hypergeometric2F1[-3/2, 1, -1/
2, -(c^2*x^2)])/(12*x^3)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5361, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}bc\int \frac{1}{x^4\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{4x^4}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{4}bc\left(c^2\left(-\int \frac{1}{x^2\left(c^2x^2+1\right)}dx\right)-\frac{1}{3x^3}\right)-\frac{a+b\arctan(cx)}{4x^4}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{4}bc\left(-\left(c^2\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)\right)-\frac{1}{3x^3}\right)-\frac{a+b\arctan(cx)}{4x^4}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}bc\left(-\left(c^2\left(-c\arctan(cx)-\frac{1}{x}\right)\right)-\frac{1}{3x^3}\right)-\frac{a+b\arctan(cx)}{4x^4}$$

input
```
Int[(a + b*ArcTan[c*x])/x^5,x]
```

output
```
-1/4*(a + b*ArcTan[c*x])/x^4 + (b*c*(-1/3*1/x^3 - c^2*(-x^(-1) - c*ArcTan[
c*x])))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{3b\arctan(cx)x^4c^4+3b\,c^3x^3-bcx-3b\arctan(cx)-3a}{12x^4}$	44
parts	$-\frac{a}{4x^4}+b\,c^4\left(-\frac{\arctan(cx)}{4c^4x^4}+\frac{\arctan(cx)}{4}-\frac{1}{12c^3x^3}+\frac{1}{4cx}\right)$	48
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\arctan(cx)}{4c^4x^4}+\frac{\arctan(cx)}{4}-\frac{1}{12c^3x^3}+\frac{1}{4cx}\right)\right)$	52
default	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\arctan(cx)}{4c^4x^4}+\frac{\arctan(cx)}{4}-\frac{1}{12c^3x^3}+\frac{1}{4cx}\right)\right)$	52
risch	$\frac{ib\ln(icx+1)}{8x^4}-\frac{3ib\,c^4\ln(-cx+i)x^4-3ib\,c^4\ln(-cx-i)x^4-6b\,c^3x^3+3ib\ln(-icx+1)+2bcx+6a}{24x^4}$	88
orering	$\frac{\left(\frac{3}{2}c^4x^5+\frac{5}{6}x^3c^2-\frac{2}{3}x\right)(a+b\arctan(cx))}{x^5}+\frac{(3c^2x^2-1)\,(c^2x^2+1)\,x^2\left(\frac{bc}{(c^2x^2+1)x^5}-\frac{5(a+b\arctan(cx))}{x^6}\right)}{12}$	89

input
```
int((a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*b*arctan(c*x)*x^4*c^4+3*b*c^3*x^3-b*c*x-3*b*arctan(c*x)-3*a)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx=\frac{3\,bc^3x^3-bcx+3\,(bc^4x^4-b)\arctan{(cx)}-3\,a}{12\,x^4}$$

input
```
integrate((a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
1/12*(3*b*c^3*x^3 - b*c*x + 3*(b*c^4*x^4 - b)*arctan(c*x) - 3*a)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx=-\frac{a}{4x^4}+\frac{bc^4\,\mathrm{atan}\,(cx)}{4}+\frac{bc^3}{4x}-\frac{bc}{12x^3}-\frac{b\,\mathrm{atan}\,(cx)}{4x^4}$$

input
```
integrate((a+b*atan(c*x))/x**5,x)
```

output
```
-a/(4*x**4) + b*c**4*atan(c*x)/4 + b*c**3/(4*x) - b*c/(12*x**3) - b*atan(c
*x)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx=\frac{1}{12}\left(\left(3\,c^3\arctan{(cx)}+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\,\arctan{(cx)}}{x^4}\right)b-\frac{a}{4\,x^4}$$

input
```
integrate((a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b -
 1/4*a/x^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx = -\frac{-3i\,bc^4x^4\log\left(i\,cx-1\right)+3i\,bc^4x^4\log\left(-i\,cx-1\right)-6\,bc^3x^3+2\,bcx+6\,b\arctan\left(cx\right)+6\,a}{24\,x^4}$$

input
```
integrate((a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
-1/24*(-3*I*b*c^4*x^4*log(I*c*x - 1) + 3*I*b*c^4*x^4*log(-I*c*x - 1) - 6*b
*c^3*x^3 + 2*b*c*x + 6*b*arctan(c*x) + 6*a)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx = \frac{b\,c^4\,\mathrm{atan}(c\,x)}{4}-\frac{-b\,c^3\,x^3+\frac{b\,c\,x}{3}+a}{4\,x^4}-\frac{b\,\mathrm{atan}(c\,x)}{4\,x^4}$$

input
```
int((a + b*atan(c*x))/x^5,x)
```

output
```
(b*c^4*atan(c*x))/4 - (a - b*c^3*x^3 + (b*c*x)/3)/(4*x^4) - (b*atan(c*x))/
(4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx = \frac{3atan(cx)\,b\,c^4x^4 - 3atan(cx)\,b - 3a + 3b\,c^3x^3 - bcx}{12x^4}$$

input
```
int((a+b*atan(c*x))/x^5,x)
```

output
```
(3*atan(c*x)*b*c**4*x**4 - 3*atan(c*x)*b - 3*a + 3*b*c**3*x**3 - b*c*x)/(1
2*x**4)
```

3.12 $\int \frac{a+b\arctan(cx)}{x^6}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 64

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx = -\frac{bc}{20x^4}+\frac{bc^3}{10x^2}-\frac{a+b\arctan(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1+c^2x^2\right)$$

output
```
-1/20*b*c/x^4+1/10*b*c^3/x^2-1/5*(a+b*arctan(c*x))/x^5+1/5*b*c^5*ln(x)-1/1
0*b*c^5*ln(c^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx = -\frac{a}{5x^5}-\frac{bc}{20x^4}+\frac{bc^3}{10x^2}-\frac{b\arctan(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1+c^2x^2\right)$$

input
```
Integrate[(a + b*ArcTan[c*x])/x^6,x]
```

output
```
-1/5*a/x^5 - (b*c)/(20*x^4) + (b*c^3)/(10*x^2) - (b*ArcTan[c*x])/(5*x^5) +
 (b*c^5*Log[x])/5 - (b*c^5*Log[1 + c^2*x^2])/10
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5361, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx$$

$$\Big\downarrow \ 5361$$

$$\frac{1}{5}bc\int \frac{1}{x^5\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{5x^5}$$

$$\Big\downarrow \ 243$$

$$\frac{1}{10}bc\int \frac{1}{x^6\left(c^2x^2+1\right)}dx^2-\frac{a+b\arctan(cx)}{5x^5}$$

$$\Big\downarrow \ 54$$

$$\frac{1}{10}bc\int \left(-\frac{c^6}{c^2x^2+1}+\frac{c^4}{x^2}-\frac{c^2}{x^4}+\frac{1}{x^6}\right)dx^2-\frac{a+b\arctan(cx)}{5x^5}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{10}bc\left(c^4\log\left(x^2\right)+\frac{c^2}{x^2}-c^4\log\left(c^2x^2+1\right)-\frac{1}{2x^4}\right)-\frac{a+b\arctan(cx)}{5x^5}$$

input
```
Int[(a + b*ArcTan[c*x])/x^6,x]
```

output
```
-1/5*(a + b*ArcTan[c*x])/x^5 + (b*c*(-1/2*1/x^4 + c^2/x^2 + c^4*Log[x^2] -
 c^4*Log[1 + c^2*x^2]))/10
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

method	result	size
parts	$-\frac{a}{5x^5}+b\,c^5\left(-\frac{\arctan(cx)}{5c^5x^5}-\frac{\ln(c^2x^2+1)}{10}-\frac{1}{20c^4x^4}+\frac{\ln(cx)}{5}+\frac{1}{10c^2x^2}\right)$	60
derivativedivides	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\arctan(cx)}{5c^5x^5}-\frac{\ln(c^2x^2+1)}{10}-\frac{1}{20c^4x^4}+\frac{\ln(cx)}{5}+\frac{1}{10c^2x^2}\right)\right)$	64
default	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\arctan(cx)}{5c^5x^5}-\frac{\ln(c^2x^2+1)}{10}-\frac{1}{20c^4x^4}+\frac{\ln(cx)}{5}+\frac{1}{10c^2x^2}\right)\right)$	64
parallelrisch	$\frac{4b\,c^5\ln(x)x^5-2b\,c^5\ln(c^2x^2+1)x^5-2b\,c^5x^5+2b\,c^3x^3-bcx-4b\arctan(cx)-4a}{20x^5}$	70
risch	$\frac{ib\ln(icx+1)}{10x^5}-\frac{-4b\,c^5\ln(x)x^5+2b\,c^5\ln(-c^2x^2-1)x^5-2b\,c^3x^3+2ib\ln(-icx+1)+bcx+4a}{20x^5}$	82

input
```
int((a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5+b*c^5*(-1/5/c^5/x^5*arctan(c*x)-1/10*ln(c^2*x^2+1)-1/20/c^4/x^4
+1/5*ln(c*x)+1/10/c^2/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx = -\frac{2\,bc^5x^5\log\left(c^2x^2+1\right)-4\,bc^5x^5\log\left(x\right)-2\,bc^3x^3+bcx+4\,b\arctan\left(cx\right)+4\,a}{20\,x^5}$$

input
```
integrate((a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/20*(2*b*c^5*x^5*log(c^2*x^2 + 1) - 4*b*c^5*x^5*log(x) - 2*b*c^3*x^3 + b
*c*x + 4*b*arctan(c*x) + 4*a)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx = \begin{cases} -\frac{a}{5x^5}+\frac{bc^5\log\left(x\right)}{5}-\frac{bc^5\log\left(x^2+\frac{1}{c^2}\right)}{10}+\frac{bc^3}{10x^2}-\frac{bc}{20x^4}-\frac{b\operatorname{atan}\left(cx\right)}{5x^5} & \text{for } c\neq 0 \\ -\frac{a}{5x^5} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atan(c*x))/x**6,x)
```

output
```
Piecewise((-a/(5*x**5) + b*c**5*log(x)/5 - b*c**5*log(x**2 + c**(-2))/10 +
 b*c**3/(10*x**2) - b*c/(20*x**4) - b*atan(c*x)/(5*x**5), Ne(c, 0)), (-a/(
5*x**5), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{a + b\arctan(cx)}{x^6}\,dx$$
$$= -\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right) - 2\,c^4\log\left(x^2\right) - \frac{2\,c^2x^2-1}{x^4}\right)c + \frac{4\,\arctan\left(cx\right)}{x^5}\right)b - \frac{a}{5\,x^5}$$

input
```
integrate((a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
 4*arctan(c*x)/x^5)*b - 1/5*a/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{a + b\arctan(cx)}{x^6}\,dx$$
$$= -\frac{2\,bc^5x^5\log\left(c^2x^2+1\right) - 4\,bc^5x^5\log\left(x\right) - 2\,bc^3x^3 + bcx + 4\,b\arctan\left(cx\right) + 4\,a}{20\,x^5}$$

input
```
integrate((a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
-1/20*(2*b*c^5*x^5*log(c^2*x^2 + 1) - 4*b*c^5*x^5*log(x) - 2*b*c^3*x^3 + b
*c*x + 4*b*arctan(c*x) + 4*a)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx = \frac{b\,c^5\,\ln(x)}{5} - \frac{b\,\mathrm{atan}(c\,x)}{5\,x^5} - \frac{b\,c^5\,\ln(c^2\,x^2+1)}{10} - \frac{-\frac{b\,c^3\,x^3}{2}+\frac{b\,c\,x}{4}+a}{5\,x^5}$$

input
```
int((a + b*atan(c*x))/x^6,x)
```

output
```
(b*c^5*log(x))/5 - (b*atan(c*x))/(5*x^5) - (b*c^5*log(c^2*x^2 + 1))/10 - (
a - (b*c^3*x^3)/2 + (b*c*x)/4)/(5*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx = \frac{-4atan(cx)\,b - 2\log(c^2x^2+1)\,b\,c^5x^5 + 4\log(x)\,b\,c^5x^5 - 4a + 2b\,c^3x^3 - bcx}{20x^5}$$

input
```
int((a+b*atan(c*x))/x^6,x)
```

output
```
( - 4*atan(c*x)*b - 2*log(c**2*x**2 + 1)*b*c**5*x**5 + 4*log(x)*b*c**5*x**
5 - 4*a + 2*b*c**3*x**3 - b*c*x)/(20*x**5)
```

3.13 $\int x^5(a+b\arctan(cx))^2\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 144

$$\int x^5(a+b\arctan(cx))^2\,dx = -\frac{abx}{3c^5}-\frac{4b^2x^2}{45c^4}+\frac{b^2x^4}{60c^2}-\frac{b^2x\arctan(cx)}{3c^5}+\frac{bx^3(a+b\arctan(cx))}{9c^3}-\frac{bx^5(a+b\arctan(cx))}{15c}+\frac{(a+b\arctan(cx))^2}{6c^6}+\frac{1}{6}x^6(a+b\arctan(cx))^2+\frac{23b^2\log(1+c^2x^2)}{90c^6}$$

output
```
-1/3*a*b*x/c^5-4/45*b^2*x^2/c^4+1/60*b^2*x^4/c^2-1/3*b^2*x*arctan(c*x)/c^5
+1/9*b*x^3*(a+b*arctan(c*x))/c^3-1/15*b*x^5*(a+b*arctan(c*x))/c+1/6*(a+b*a
rctan(c*x))^2/c^6+1/6*x^6*(a+b*arctan(c*x))^2+23/90*b^2*ln(c^2*x^2+1)/c^6
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.96

$$\int x^5(a+b\arctan(cx))^2\,dx$$
$$=\frac{cx(30a^2c^5x^5+b^2cx(-16+3c^2x^2)-4ab(15-5c^2x^2+3c^4x^4))+4b(bcx(-15+5c^2x^2-3c^4x^4)+15a(1}{180c^6}$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x])^2,x]
```

output
```
(c*x*(30*a^2*c^5*x^5 + b^2*c*x*(-16 + 3*c^2*x^2) - 4*a*b*(15 - 5*c^2*x^2 +
 3*c^4*x^4)) + 4*b*(b*c*x*(-15 + 5*c^2*x^2 - 3*c^4*x^4) + 15*a*(1 + c^6*x^
6))*ArcTan[c*x] + 30*b^2*(1 + c^6*x^6)*ArcTan[c*x]^2 + 46*b^2*Log[1 + c^2*
x^2])/(180*c^6)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.38, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5361, 5451, 5361, 243, 49, 2009, 5451, 5361, 243, 49, 2009, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\int\frac{x^6(a+b\arctan(cx))}{c^2x^2+1}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\int x^4(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{5}bc\int\frac{x^5}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \; 243$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\int\frac{x^4}{c^2x^2+1}dx^2}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \; 49$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\int\left(\frac{x^2}{c^2}+\frac{1}{c^4(c^2x^2+1)}-\frac{1}{c^4}\right)dx^2}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \; 5451$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2}-\frac{\frac{\int x^2(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}}{c^2}\right)$$

$$\Big\downarrow \; 5361$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2}-\frac{\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{3}bc\int\frac{x^3}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x^2(a+b\arctan(c}{c^2x^2+1}}{c^2}}{c^2}\right.$$

$$\Big\downarrow \; 243$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-$$
$$\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2}-\frac{\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int\frac{x^2}{c^2x^2+1}dx^2}{c^2}-\frac{\int\frac{x^2(a+b\arctan(}{c^2x^2+1}}{c^2}}{c^2}\right.$$

$\downarrow$ 49

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-$$
$$\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2}-\frac{\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int\left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^2+1)}\right)dx^2}{c^2}-\int\frac{x}{}}{c^2}\right.$$

$\downarrow$ 2009

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-$$
$$\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2}-\frac{\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\frac{\int\frac{x^2(a+}{}}{c}}{c^2}\right.$$

$\downarrow$ 5451

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-$$
$$\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2}-\frac{\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\frac{\int(a+b\,\mathrm{ar}}{}}{c^2}\right.$$

$\downarrow$ 2009

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-$$
$$\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2}-\frac{\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\frac{ax+bx\,\mathrm{ar}}{}}{c^2}\right.$$

$\downarrow$ 5419

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-$$
$$\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2}-\frac{\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\frac{ax+bx\,\mathrm{ar}}{}}{c^2}\right.$$

input
```
Int[x^5*(a + b*ArcTan[c*x])^2,x]
```

output
```
(x^6*(a + b*ArcTan[c*x])^2)/6 - (b*c*(((x^5*(a + b*ArcTan[c*x]))/5 - (b*c*
(-(x^2/c^4) + x^4/(2*c^2) + Log[1 + c^2*x^2]/c^6))/10)/c^2 - (((x^3*(a + b
*ArcTan[c*x]))/3 - (b*c*(x^2/c^2 - Log[1 + c^2*x^2]/c^4))/6)/c^2 - (-1/2*(
a + b*ArcTan[c*x])^2/(b*c^3) + (a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2
])/(2*c))/c^2)/c^2)/c^2))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

method	result
parts	$\frac{x^6a^2}{6}+\frac{b^2\left(\frac{c^6x^6\arctan(cx)^2}{6}-\frac{c^5x^5\arctan(cx)}{15}+\frac{c^3x^3\arctan(cx)}{9}-\frac{cx\arctan(cx)}{3}+\frac{\arctan(cx)^2}{6}+\frac{c^4x^4}{60}-\frac{4c^2x^2}{45}+\frac{23\ln\left(c^2x^2\right.}{90}\right.}{c^6}$
derivativedivides	$\frac{\frac{a^2c^6x^6}{6}+b^2\left(\frac{c^6x^6\arctan(cx)^2}{6}-\frac{c^5x^5\arctan(cx)}{15}+\frac{c^3x^3\arctan(cx)}{9}-\frac{cx\arctan(cx)}{3}+\frac{\arctan(cx)^2}{6}+\frac{c^4x^4}{60}-\frac{4c^2x^2}{45}+\frac{23\ln\left(c^2x^2-\right.}{90}\right.}{c^6}$
default	$\frac{\frac{a^2c^6x^6}{6}+b^2\left(\frac{c^6x^6\arctan(cx)^2}{6}-\frac{c^5x^5\arctan(cx)}{15}+\frac{c^3x^3\arctan(cx)}{9}-\frac{cx\arctan(cx)}{3}+\frac{\arctan(cx)^2}{6}+\frac{c^4x^4}{60}-\frac{4c^2x^2}{45}+\frac{23\ln\left(c^2x^2-\right.}{90}\right.}{c^6}$
parallelrisch	$\frac{30b^2\arctan(cx)^2x^6c^6+60ab\arctan(cx)x^6c^6+30a^2c^6x^6-12b^2\arctan(cx)x^5c^5-12ab\,c^5x^5+3b^2c^4x^4+20b^2\arctan(cx)x}{180}$
risch	$-\frac{b^2\left(x^6c^6+1\right)\ln(icx+1)^2}{24c^6}-\frac{ib^2x\ln(-icx+1)}{6c^5}-\frac{ib^2x^5\ln(-icx+1)}{30c}-\frac{b^2x^6\ln(-icx+1)^2}{24}+\frac{ib^2x^3\ln(-icx+1)}{18c^3}+$

input
```
int(x^5*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*a^2+b^2/c^6*(1/6*c^6*x^6*arctan(c*x)^2-1/15*c^5*x^5*arctan(c*x)+1/
9*c^3*x^3*arctan(c*x)-1/3*c*x*arctan(c*x)+1/6*arctan(c*x)^2+1/60*c^4*x^4-4
/45*c^2*x^2+23/90*ln(c^2*x^2+1))+2*a*b/c^6*(1/6*c^6*x^6*arctan(c*x)-1/30*c
^5*x^5+1/18*c^3*x^3-1/6*c*x+1/6*arctan(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

$$\int x^5(a + b\arctan(cx))^2\,dx$$
$$= \frac{30\,a^2c^6x^6 - 12\,abc^5x^5 + 3\,b^2c^4x^4 + 20\,abc^3x^3 - 16\,b^2c^2x^2 - 60\,abcx + 30\,(b^2c^6x^6 + b^2)\arctan(cx)^2 + 46}{180\,c^6}$$

input
```
integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/180*(30*a^2*c^6*x^6 - 12*a*b*c^5*x^5 + 3*b^2*c^4*x^4 + 20*a*b*c^3*x^3 -
16*b^2*c^2*x^2 - 60*a*b*c*x + 30*(b^2*c^6*x^6 + b^2)*arctan(c*x)^2 + 46*b^
2*log(c^2*x^2 + 1) + 4*(15*a*b*c^6*x^6 - 3*b^2*c^5*x^5 + 5*b^2*c^3*x^3 - 1
5*b^2*c*x + 15*a*b)*arctan(c*x))/c^6
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.38

$$\int x^5(a + b\arctan(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2x^6}{6} + \frac{abx^6\operatorname{atan}(cx)}{3} - \frac{abx^5}{15c} + \frac{abx^3}{9c^3} - \frac{abx}{3c^5} + \frac{ab\operatorname{atan}(cx)}{3c^6} + \frac{b^2x^6\operatorname{atan}^2(cx)}{6} - \frac{b^2x^5\operatorname{atan}(cx)}{15c} + \frac{b^2x^4}{60c^2} + \frac{b^2x^3\operatorname{atan}(cx)}{9c^3} - \\ \frac{a^2x^6}{6} \end{cases}$$

input
```
integrate(x**5*(a+b*atan(c*x))**2,x)
```

output
```
Piecewise((a**2*x**6/6 + a*b*x**6*atan(c*x)/3 - a*b*x**5/(15*c) + a*b*x**3
/(9*c**3) - a*b*x/(3*c**5) + a*b*atan(c*x)/(3*c**6) + b**2*x**6*atan(c*x)*
*2/6 - b**2*x**5*atan(c*x)/(15*c) + b**2*x**4/(60*c**2) + b**2*x**3*atan(c
*x)/(9*c**3) - 4*b**2*x**2/(45*c**4) - b**2*x*atan(c*x)/(3*c**5) + 23*b**2
*log(x**2 + c**(-2))/(90*c**6) + b**2*atan(c*x)**2/(6*c**6), Ne(c, 0)), (a
**2*x**6/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.13

$$\int x^5(a+b\arctan(cx))^2\,dx = \frac{1}{6}\,b^2x^6\arctan\left(cx\right)^2+\frac{1}{6}\,a^2x^6$$
$$+\frac{1}{45}\left(15\,x^6\arctan\left(cx\right)-c\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)ab$$
$$-\frac{1}{180}\left(4\,c\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\arctan\left(cx\right)-\frac{3\,c^4x^4-16\,c^2x^2-30\,\arctan\left(cx\right)^2}{c^6}\right.$$

input
```
integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/6*b^2*x^6*arctan(c*x)^2 + 1/6*a^2*x^6 + 1/45*(15*x^6*arctan(c*x) - c*((3
*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*a*b - 1/180*(4*c*(
(3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7)*arctan(c*x) - (3*
c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(c^2*x^2 + 1))/c^6)*b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.25

$$\int x^5(a+b\arctan(cx))^2\,dx$$
$$=\frac{30\,b^2c^6x^6\arctan\left(cx\right)^2+60\,abc^6x^6\arctan\left(cx\right)+30\,a^2c^6x^6-12\,b^2c^5x^5\arctan\left(cx\right)-12\,abc^5x^5+3\,b^2c^4x^4}{}$$

input
```
integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
1/180*(30*b^2*c^6*x^6*arctan(c*x)^2 + 60*a*b*c^6*x^6*arctan(c*x) + 30*a^2*
c^6*x^6 - 12*b^2*c^5*x^5*arctan(c*x) - 12*a*b*c^5*x^5 + 3*b^2*c^4*x^4 + 20
*b^2*c^3*x^3*arctan(c*x) + 20*a*b*c^3*x^3 - 16*b^2*c^2*x^2 - 60*b^2*c*x*ar
ctan(c*x) - 60*pi*a*b*sgn(c)*sgn(x) - 60*a*b*c*x + 30*b^2*arctan(c*x)^2 +
60*a*b*arctan(c*x) + 46*b^2*log(c^2*x^2 + 1))/c^6
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.19

$$\int x^5(a+b\arctan(cx))^2\,dx$$
$$=\frac{30\,b^2\,\mathrm{atan}(c\,x)^2+46\,b^2\,\ln\left(c^2\,x^2+1\right)+30\,a^2\,c^6\,x^6-16\,b^2\,c^2\,x^2+3\,b^2\,c^4\,x^4+60\,a\,b\,\mathrm{atan}(c\,x)+20\,b^2\,c^3\,x}{}$$

input
```
int(x^5*(a + b*atan(c*x))^2,x)
```

output
```
(30*b^2*atan(c*x)^2 + 46*b^2*log(c^2*x^2 + 1) + 30*a^2*c^6*x^6 - 16*b^2*c^
2*x^2 + 3*b^2*c^4*x^4 + 60*a*b*atan(c*x) + 20*b^2*c^3*x^3*atan(c*x) - 12*b
^2*c^5*x^5*atan(c*x) - 60*b^2*c*x*atan(c*x) + 30*b^2*c^6*x^6*atan(c*x)^2 +
 20*a*b*c^3*x^3 - 12*a*b*c^5*x^5 - 60*a*b*c*x + 60*a*b*c^6*x^6*atan(c*x))/
(180*c^6)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.19

$$\int x^5(a+b\arctan(cx))^2\,dx$$
$$=\frac{30atan(cx)^2\,b^2c^6x^6+30atan(cx)^2\,b^2+60atan(cx)\,ab\,c^6x^6+60atan(cx)\,ab-12atan(cx)\,b^2c^5x^5+20ata}{}$$

input
```
int(x^5*(a+b*atan(c*x))^2,x)
```

output
```
(30*atan(c*x)**2*b**2*c**6*x**6 + 30*atan(c*x)**2*b**2 + 60*atan(c*x)*a*b*
c**6*x**6 + 60*atan(c*x)*a*b - 12*atan(c*x)*b**2*c**5*x**5 + 20*atan(c*x)*
b**2*c**3*x**3 - 60*atan(c*x)*b**2*c*x + 46*log(c**2*x**2 + 1)*b**2 + 30*a
**2*c**6*x**6 - 12*a*b*c**5*x**5 + 20*a*b*c**3*x**3 - 60*a*b*c*x + 3*b**2*
c**4*x**4 - 16*b**2*c**2*x**2)/(180*c**6)
```

3.14 $\int x^4(a+b\arctan(cx))^2\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 170

$$\begin{aligned}\int x^4(a+b\arctan(cx))^2\,dx=&-\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{3b^2\arctan(cx)}{10c^5}\\&+\frac{bx^2(a+b\arctan(cx))}{5c^3}-\frac{bx^4(a+b\arctan(cx))}{10c}\\&+\frac{i(a+b\arctan(cx))^2}{5c^5}+\frac{1}{5}x^5(a+b\arctan(cx))^2\\&+\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^5}\\&+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}\end{aligned}$$

output
```
-3/10*b^2*x/c^4+1/30*b^2*x^3/c^2+3/10*b^2*arctan(c*x)/c^5+1/5*b*x^2*(a+b*a
rctan(c*x))/c^3-1/10*b*x^4*(a+b*arctan(c*x))/c+1/5*I*(a+b*arctan(c*x))^2/c
^5+1/5*x^5*(a+b*arctan(c*x))^2+2/5*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5
+1/5*I*b^2*polylog(2,1-2/(1+I*c*x))/c^5
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int x^4(a+b\arctan(cx))^2\,dx$$
$$=\frac{9ab-9b^2cx+6abc^2x^2+b^2c^3x^3-3abc^4x^4+6a^2c^5x^5+6b^2(-i+c^5x^5)\arctan(cx)^2-3b\arctan(cx)\,(-4}{}$$

input
```
Integrate[x^4*(a + b*ArcTan[c*x])^2,x]
```

output
```
(9*a*b - 9*b^2*c*x + 6*a*b*c^2*x^2 + b^2*c^3*x^3 - 3*a*b*c^4*x^4 + 6*a^2*c
^5*x^5 + 6*b^2*(-I + c^5*x^5)*ArcTan[c*x]^2 - 3*b*ArcTan[c*x]*(-4*a*c^5*x^
5 + b*(-3 - 2*c^2*x^2 + c^4*x^4) - 4*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - 6
*a*b*Log[1 + c^2*x^2] - (6*I)*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(30*
c^5)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {5361, 5451, 5361, 254, 2009, 5451, 5361, 262, 216, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\arctan(cx))}{c^2x^2+1}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\int x^3(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\int\frac{x^4}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{ 254}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\int\left(\frac{x^2}{c^2}+\frac{1}{c^4(c^2x^2+1)}-\frac{1}{c^4}\right)dx}{c^2}-\frac{\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}}{c^2}\right)$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int\frac{x^2}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}}{c^2}\right)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int\frac{1}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(}{c^2x^2+1}}{c^2}}{c^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2 - \frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}}{c^2}}{c^2}\right.$$

$\downarrow$ 5455

$$\frac{1}{5}x^5(a+b\arctan(cx))^2 - \frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\int\frac{a+b\arctan(cx)}{i-cx}}{c}}{}}{c^2}\right.$$

$\downarrow$ 5379

$$\frac{1}{5}x^5(a+b\arctan(cx))^2 - \frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\log\left(\frac{2}{1+icx}\right)(a+b}{c}}{}}{c^2}\right.$$

$\downarrow$ 2849

$$\frac{1}{5}x^5(a+b\arctan(cx))^2 - \frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}}{c}}{}}{c^2}\right.$$

$\downarrow$ 2752

$$\frac{1}{5}x^5(a+b\arctan(cx))^2 - \frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{i(a+b\arctan(cx))}{2bc^2}}{}}{c^2}\right.$$

input
```
Int[x^4*(a + b*ArcTan[c*x])^2,x]
```

output
```
(x^5*(a + b*ArcTan[c*x])^2)/5 - (2*b*c*(((x^4*(a + b*ArcTan[c*x]))/4 - (b*
c*(-(x/c^4) + x^3/(3*c^2) + ArcTan[c*x]/c^5))/4)/c^2 - (((x^2*(a + b*ArcTa
n[c*x]))/2 - (b*c*(x/c^2 - ArcTan[c*x]/c^3))/2)/c^2 - (((-1/2*I)*(a + b*Ar
cTan[c*x])^2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/
2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/c)/c)/c^2)/c^2))/5
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.56

method	result
parts	$\frac{a^2x^5}{5}+\frac{b^2\left(\frac{c^5x^5\arctan(cx)^2}{5}-\frac{c^4x^4\arctan(cx)}{10}+\frac{c^2x^2\arctan(cx)}{5}-\frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{5}+\frac{c^3x^3}{30}-\frac{3cx}{10}+\frac{3\arctan(cx)}{10}-\frac{i\left(\ln\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{c^5x^5a^2}{5}+b^2\left(\frac{c^5x^5\arctan(cx)^2}{5}-\frac{c^4x^4\arctan(cx)}{10}+\frac{c^2x^2\arctan(cx)}{5}-\frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{5}+\frac{c^3x^3}{30}-\frac{3cx}{10}+\frac{3\arctan(cx)}{10}-\frac{i\left(\ln\right.}{}\right.}{}$
default	$\frac{\frac{c^5x^5a^2}{5}+b^2\left(\frac{c^5x^5\arctan(cx)^2}{5}-\frac{c^4x^4\arctan(cx)}{10}+\frac{c^2x^2\arctan(cx)}{5}-\frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{5}+\frac{c^3x^3}{30}-\frac{3cx}{10}+\frac{3\arctan(cx)}{10}-\frac{i\left(\ln\right.}{}\right.}{}$
risch	$\frac{ib^2\ln(icx+1)x^4}{20c}-\frac{ib^2\ln(icx+1)x^2}{10c^3}-\frac{ib^2\ln(-icx+1)x^4}{20c}+\frac{ib^2\ln(-icx+1)x^2}{10c^3}+\frac{ib^2\ln(icx+1)\ln(-icx+1)}{10c^5}-\frac{i}{}$

input
```
int(x^4*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*x^5+b^2/c^5*(1/5*c^5*x^5*arctan(c*x)^2-1/10*c^4*x^4*arctan(c*x)+1/
5*c^2*x^2*arctan(c*x)-1/5*arctan(c*x)*ln(c^2*x^2+1)+1/30*c^3*x^3-3/10*c*x+
3/10*arctan(c*x)-1/10*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/
2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))+1/10*I*(ln(c*x+I)*ln(c^2*x^2+1)
-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))))+2*a*b/
c^5*(1/5*c^5*x^5*arctan(c*x)-1/20*c^4*x^4+1/10*c^2*x^2-1/10*ln(c^2*x^2+1))
```

Fricas [F]

$$\int x^4(a+b\arctan(cx))^2\,dx=\int(b\arctan(cx)+a)^2x^4\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^4*arctan(c*x)^2 + 2*a*b*x^4*arctan(c*x) + a^2*x^4, x)
```

Sympy [F]

$$\int x^4(a+b\arctan(cx))^2\,dx=\int x^4(a+b\,\mathrm{atan}\,(cx))^2\;dx$$

input
```
integrate(x**4*(a+b*atan(c*x))**2,x)
```

output
```
Integral(x**4*(a + b*atan(c*x))**2, x)
```

Maxima [F]

$$\int x^4(a+b\arctan(cx))^2\,dx=\int (b\arctan(cx)+a)^2x^4\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*x^5 + 1/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c
^2*x^2 + 1)/c^6))*a*b + 1/80*(4*x^5*arctan(c*x)^2 - x^5*log(c^2*x^2 + 1)^2
 + 80*integrate(1/80*(4*c^2*x^6*log(c^2*x^2 + 1) - 8*c*x^5*arctan(c*x) + 6
0*(c^2*x^6 + x^4)*arctan(c*x)^2 + 5*(c^2*x^6 + x^4)*log(c^2*x^2 + 1)^2)/(c
^2*x^2 + 1), x))*b^2
```

Giac [F]

$$\int x^4(a+b\arctan(cx))^2\,dx=\int (b\arctan(cx)+a)^2x^4\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a+b\arctan(cx))^2\,dx=\int x^4\,(a+b\,\mathrm{atan}(c\,x))^2\,dx$$

input
```
int(x^4*(a + b*atan(c*x))^2,x)
```

output
```
int(x^4*(a + b*atan(c*x))^2, x)
```

Reduce [F]

$$\int x^4(a+b\arctan(cx))^2\,dx$$
$$=\frac{6atan(cx)^2\,b^2c^5x^5-6atan(cx)^2\,b^2cx+12atan(cx)\,ab\,c^5x^5-3atan(cx)\,b^2c^4x^4+6atan(cx)\,b^2c^2x^2+9ata}{30}$$

input
```
int(x^4*(a+b*atan(c*x))^2,x)
```

output
```
(6*atan(c*x)**2*b**2*c**5*x**5 - 6*atan(c*x)**2*b**2*c*x + 12*atan(c*x)*a*
b*c**5*x**5 - 3*atan(c*x)*b**2*c**4*x**4 + 6*atan(c*x)*b**2*c**2*x**2 + 9*
atan(c*x)*b**2 + 6*int(atan(c*x)**2,x)*b**2*c - 6*log(c**2*x**2 + 1)*a*b +
 6*a**2*c**5*x**5 - 3*a*b*c**4*x**4 + 6*a*b*c**2*x**2 + b**2*c**3*x**3 - 9
*b**2*c*x)/(30*c**5)
```

3.15 $\int x^3(a+b\arctan(cx))^2\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 112

$$\int x^3(a+b\arctan(cx))^2\,dx=\frac{abx}{2c^3}+\frac{b^2x^2}{12c^2}+\frac{b^2x\arctan(cx)}{2c^3}-\frac{bx^3(a+b\arctan(cx))}{6c}-\frac{(a+b\arctan(cx))^2}{4c^4}+\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{b^2\log(1+c^2x^2)}{3c^4}$$

output
```
1/2*a*b*x/c^3+1/12*b^2*x^2/c^2+1/2*b^2*x*arctan(c*x)/c^3-1/6*b*x^3*(a+b*ar
ctan(c*x))/c-1/4*(a+b*arctan(c*x))^2/c^4+1/4*x^4*(a+b*arctan(c*x))^2-1/3*b
^2*ln(c^2*x^2+1)/c^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

$$\int x^3(a+b\arctan(cx))^2\,dx=\frac{cx(6ab+b^2cx-2abc^2x^2+3a^2c^3x^3)-2b(bcx(-3+c^2x^2)+a(3-3c^4x^4))\arctan(cx)+3b^2(-1+c^4x^4)}{12c^4}$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
(c*x*(6*a*b + b^2*c*x - 2*a*b*c^2*x^2 + 3*a^2*c^3*x^3) - 2*b*(b*c*x*(-3 +
c^2*x^2) + a*(3 - 3*c^4*x^4))*ArcTan[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTan[c*
x]^2 - 4*b^2*Log[1 + c^2*x^2])/(12*c^4)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5361, 5451, 5361, 243, 49, 2009, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\int\frac{x^4(a+b\arctan(cx))}{c^2x^2+1}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\int x^2(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{3}bc\int\frac{x^3}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int\frac{x^2}{c^2x^2+1}dx^2}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int\left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^2+1)}\right)dx^2}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\frac{\frac{\int(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}}{c^2}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}}{c^2}\right)$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}}{c^2}\right)$$

input

```
Int[x^3*(a + b*ArcTan[c*x])^2,x]
```

output

```
(x^4*(a + b*ArcTan[c*x])^2)/4 - (b*c*(((x^3*(a + b*ArcTan[c*x]))/3 - (b*c*
(x^2/c^2 - Log[1 + c^2*x^2]/c^4))/6)/c^2 - (-1/2*(a + b*ArcTan[c*x])^2/(b*
c^3) + (a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c))/c^2)/c^2))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05

method	result
parts	$\frac{a^2x^4}{4}+\frac{b^2\left(\frac{c^4x^4\arctan(cx)^2}{4}-\frac{c^3x^3\arctan(cx)}{6}+\frac{cx\arctan(cx)}{2}-\frac{\arctan(cx)^2}{4}+\frac{c^2x^2}{12}-\frac{\ln(c^2x^2+1)}{3}\right)}{c^4}+\frac{2ab\left(\frac{c^4x^4\arctan(\right.}{4}$
derivativedivides	$\frac{\frac{x^4c^4a^2}{4}+b^2\left(\frac{c^4x^4\arctan(cx)^2}{4}-\frac{c^3x^3\arctan(cx)}{6}+\frac{cx\arctan(cx)}{2}-\frac{\arctan(cx)^2}{4}+\frac{c^2x^2}{12}-\frac{\ln(c^2x^2+1)}{3}\right)+2ab\left(\frac{c^4x^4\arctan(cx)}{4}\right.}{c^4}$
default	$\frac{\frac{x^4c^4a^2}{4}+b^2\left(\frac{c^4x^4\arctan(cx)^2}{4}-\frac{c^3x^3\arctan(cx)}{6}+\frac{cx\arctan(cx)}{2}-\frac{\arctan(cx)^2}{4}+\frac{c^2x^2}{12}-\frac{\ln(c^2x^2+1)}{3}\right)+2ab\left(\frac{c^4x^4\arctan(cx)}{4}\right.}{c^4}$
parallelrisch	$-\frac{-3b^2\arctan(cx)^2x^4c^4-6ab\arctan(cx)x^4c^4-3x^4c^4a^2+2b^2\arctan(cx)x^3c^3+2ab\,c^3x^3-b^2c^2x^2-6b^2\arctan(cx)xc-6}{12c^4}$
risch	$-\frac{b^2(c^4x^4-1)\ln(icx+1)^2}{16c^4}-\frac{ib(6c^4x^4a+3ib\,c^4x^4\ln(-icx+1)-2b\,c^3x^3+6bcx-3ib\ln(-icx+1))\ln(icx+1)}{24c^4}-\frac{b^2x^4\ln}{}$

input
```
int(x^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+b^2/c^4*(1/4*c^4*x^4*arctan(c*x)^2-1/6*c^3*x^3*arctan(c*x)+1/2
*c*x*arctan(c*x)-1/4*arctan(c*x)^2+1/12*c^2*x^2-1/3*ln(c^2*x^2+1))+2*a*b/c
^4*(1/4*c^4*x^4*arctan(c*x)-1/12*c^3*x^3+1/4*c*x-1/4*arctan(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

$$\int x^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{3\,a^2c^4x^4-2\,abc^3x^3+b^2c^2x^2+6\,abcx+3\,(b^2c^4x^4-b^2)\arctan{(cx)}^2-4\,b^2\log{(c^2x^2+1)}+2\,(3\,abc^4x^4-}{12\,c^4}$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/12*(3*a^2*c^4*x^4 - 2*a*b*c^3*x^3 + b^2*c^2*x^2 + 6*a*b*c*x + 3*(b^2*c^4
*x^4 - b^2)*arctan(c*x)^2 - 4*b^2*log(c^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 - b^
2*c^3*x^3 + 3*b^2*c*x - 3*a*b)*arctan(c*x))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.38

$$\int x^3(a+b\arctan(cx))^2\,dx$$
$$=\begin{cases} \frac{a^2x^4}{4}+\frac{abx^4\operatorname{atan}(cx)}{2}-\frac{abx^3}{6c}+\frac{abx}{2c^3}-\frac{ab\operatorname{atan}(cx)}{2c^4}+\frac{b^2x^4\operatorname{atan}^2(cx)}{4}-\frac{b^2x^3\operatorname{atan}(cx)}{6c}+\frac{b^2x^2}{12c^2}+\frac{b^2x\operatorname{atan}(cx)}{2c^3}-\frac{b^2\log\left(x^2+\right.}{3c^4} \\ \frac{a^2x^4}{4} \end{cases}$$

input
```
integrate(x**3*(a+b*atan(c*x))**2,x)
```

output
```
Piecewise((a**2*x**4/4 + a*b*x**4*atan(c*x)/2 - a*b*x**3/(6*c) + a*b*x/(2*
c**3) - a*b*atan(c*x)/(2*c**4) + b**2*x**4*atan(c*x)**2/4 - b**2*x**3*atan
(c*x)/(6*c) + b**2*x**2/(12*c**2) + b**2*x*atan(c*x)/(2*c**3) - b**2*log(x
**2 + c**(-2))/(3*c**4) - b**2*atan(c*x)**2/(4*c**4), Ne(c, 0)), (a**2*x**
4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.21

$$\int x^3(a+b\arctan(cx))^2\,dx=\frac{1}{4}\,b^2x^4\arctan(cx)^2+\frac{1}{4}\,a^2x^4$$
$$+\frac{1}{6}\left(3\,x^4\arctan(cx)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan(cx)}{c^5}\right)\right)ab$$
$$-\frac{1}{12}\left(2\,c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan(cx)}{c^5}\right)\arctan(cx)-\frac{c^2x^2+3\,\arctan(cx)^2-4\,\log(c^2x^2+1)}{c^4}\right)b^2$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*arctan(c*x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arctan(c*x) - c*((c^2
*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4
 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2
*x^2 + 1))/c^4)*b^2
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.20

$$\int x^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{3\,b^2c^4x^4\arctan\left(cx\right)^2+6\,abc^4x^4\arctan\left(cx\right)+3\,a^2c^4x^4-2\,b^2c^3x^3\arctan\left(cx\right)-2\,abc^3x^3+b^2c^2x^2+6\,b^2c}{12\,c^4}$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
1/12*(3*b^2*c^4*x^4*arctan(c*x)^2 + 6*a*b*c^4*x^4*arctan(c*x) + 3*a^2*c^4*
x^4 - 2*b^2*c^3*x^3*arctan(c*x) - 2*a*b*c^3*x^3 + b^2*c^2*x^2 + 6*b^2*c*x*
arctan(c*x) + 6*a*b*c*x - 3*b^2*arctan(c*x)^2 - 6*a*b*arctan(c*x) - 4*b^2*
log(c^2*x^2 + 1))/c^4
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.20

$$\int x^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{3\,a^2\,c^4\,x^4-4\,b^2\,\ln\left(c^2\,x^2+1\right)-3\,b^2\,\mathrm{atan}(c\,x)^2+b^2\,c^2\,x^2-6\,a\,b\,\mathrm{atan}(c\,x)-2\,b^2\,c^3\,x^3\,\mathrm{atan}(c\,x)+6\,b^2\,c\,x}{12\,c^4}$$

input
```
int(x^3*(a + b*atan(c*x))^2,x)
```

output
```
(3*a^2*c^4*x^4 - 4*b^2*log(c^2*x^2 + 1) - 3*b^2*atan(c*x)^2 + b^2*c^2*x^2
- 6*a*b*atan(c*x) - 2*b^2*c^3*x^3*atan(c*x) + 6*b^2*c*x*atan(c*x) + 3*b^2*
c^4*x^4*atan(c*x)^2 - 2*a*b*c^3*x^3 + 6*a*b*c*x + 6*a*b*c^4*x^4*atan(c*x))
/(12*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.20

$$\int x^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{3atan(cx)^2\,b^2c^4x^4-3atan(cx)^2\,b^2+6atan(cx)\,ab\,c^4x^4-6atan(cx)\,ab-2atan(cx)\,b^2c^3x^3+6atan(cx)\,b}{12c^4}$$

input
```
int(x^3*(a+b*atan(c*x))^2,x)
```

output
```
(3*atan(c*x)**2*b**2*c**4*x**4 - 3*atan(c*x)**2*b**2 + 6*atan(c*x)*a*b*c**
4*x**4 - 6*atan(c*x)*a*b - 2*atan(c*x)*b**2*c**3*x**3 + 6*atan(c*x)*b**2*c
*x - 4*log(c**2*x**2 + 1)*b**2 + 3*a**2*c**4*x**4 - 2*a*b*c**3*x**3 + 6*a*
b*c*x + b**2*c**2*x**2)/(12*c**4)
```

3.16 $\int x^2(a+b\arctan(cx))^2\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 138

$$\begin{aligned}\int x^2(a+b\arctan(cx))^2\,dx &= \frac{b^2x}{3c^2}-\frac{b^2\arctan(cx)}{3c^3}-\frac{bx^2(a+b\arctan(cx))}{3c}\\&\quad-\frac{i(a+b\arctan(cx))^2}{3c^3}+\frac{1}{3}x^3(a+b\arctan(cx))^2\\&\quad-\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3}\\&\quad-\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}\end{aligned}$$

output
```
1/3*b^2*x/c^2-1/3*b^2*arctan(c*x)/c^3-1/3*b*x^2*(a+b*arctan(c*x))/c-1/3*I*
(a+b*arctan(c*x))^2/c^3+1/3*x^3*(a+b*arctan(c*x))^2-2/3*b*(a+b*arctan(c*x)
)*ln(2/(1+I*c*x))/c^3-1/3*I*b^2*polylog(2,1-2/(1+I*c*x))/c^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.95

$$\int x^2(a+b\arctan(cx))^2\,dx$$
$$=\frac{b^2cx-abc^2x^2+a^2c^3x^3+b^2(i+c^3x^3)\arctan(cx)^2-b\arctan(cx)\left(b+bc^2x^2-2ac^3x^3+2b\log\left(1+e^{2i\,\mathrm{ar}}\right.\right.}{3c^3}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(b^2*c*x - a*b*c^2*x^2 + a^2*c^3*x^3 + b^2*(I + c^3*x^3)*ArcTan[c*x]^2 - b
*ArcTan[c*x]*(b + b*c^2*x^2 - 2*a*c^3*x^3 + 2*b*Log[1 + E^((2*I)*ArcTan[c*
x])]) + a*b*Log[1 + c^2*x^2] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(
3*c^3)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5361, 5451, 5361, 262, 216, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int\frac{x^2}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 262

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int\frac{1}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 216

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 5455

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}}{c^2}\right)$$

$\downarrow$ 5379

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}}{c^2}\right)$$

$\downarrow$ 2849

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}}{c^2}\right.$$

$\downarrow$ 2752

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-$$
$$\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+}\right.}{2c}}{c}}{c^2}\right.$$

input
```
Int[x^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(x^3*(a + b*ArcTan[c*x])^2)/3 - (2*b*c*(((x^2*(a + b*ArcTan[c*x]))/2 - (b*
c*(x/c^2 - ArcTan[c*x]/c^3))/2)/c^2 - (((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b
*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2,
1 - 2/(1 + I*c*x)])/c)/c)/c^2))/3
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.72

method	result
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\frac{c^3x^3\arctan(cx)^2}{3}-\frac{c^2x^2\arctan(cx)}{3}+\frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{cx}{3}-\frac{\arctan(cx)}{3}+\frac{i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-}{2}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{c^3x^3a^2}{3}+b^2\left(\frac{c^3x^3\arctan(cx)^2}{3}-\frac{c^2x^2\arctan(cx)}{3}+\frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{cx}{3}-\frac{\arctan(cx)}{3}+\frac{i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-}{2}\right.}{}\right.}{}$
default	$\frac{\frac{c^3x^3a^2}{3}+b^2\left(\frac{c^3x^3\arctan(cx)^2}{3}-\frac{c^2x^2\arctan(cx)}{3}+\frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{cx}{3}-\frac{\arctan(cx)}{3}+\frac{i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-}{2}\right.}{}\right.}{}$
risch	$-\frac{ab\,x^2}{3c}+\frac{b^2x}{3c^2}-\frac{b^2\arctan(cx)}{6c^3}-\frac{17ib^2}{54c^3}-\frac{b^2\ln(-icx+1)^2x^3}{12}-\frac{b^2\ln(icx+1)^2x^3}{12}-\frac{ia^2}{3c^3}+\frac{a^2x^3}{3}-\frac{ib^2\ln(i}{12}$

input
```
int(x^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+b^2/c^3*(1/3*c^3*x^3*arctan(c*x)^2-1/3*c^2*x^2*arctan(c*x)+1/3
*arctan(c*x)*ln(c^2*x^2+1)+1/3*c*x-1/3*arctan(c*x)+1/6*I*(ln(c*x-I)*ln(c^2
*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I))
)-1/6*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c
*x+I)*ln(1/2*I*(c*x-I))))+2*a*b/c^3*(1/3*c^3*x^3*arctan(c*x)-1/6*c^2*x^2+1
/6*ln(c^2*x^2+1))
```

Fricas [F]

$$\int x^2(a+b\arctan(cx))^2\,dx=\int(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2, x)
```

Sympy [F]

$$\int x^2(a+b\arctan(cx))^2\,dx=\int x^2(a+b\,\mathrm{atan}\,(cx))^2\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))**2,x)
```

output
```
Integral(x**2*(a + b*atan(c*x))**2, x)
```

Maxima [F]

$$\int x^2(a+b\arctan(cx))^2\,dx=\int (b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))
*a*b + 1/48*(4*x^3*arctan(c*x)^2 - x^3*log(c^2*x^2 + 1)^2 + 48*integrate(1
/48*(4*c^2*x^4*log(c^2*x^2 + 1) - 8*c*x^3*arctan(c*x) + 36*(c^2*x^4 + x^2)
*arctan(c*x)^2 + 3*(c^2*x^4 + x^2)*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x))*
b^2
```

Giac [F]

$$\int x^2(a+b\arctan(cx))^2\,dx=\int (b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arctan(cx))^2\,dx=\int x^2\,(a+b\,\mathrm{atan}(c\,x))^2\,dx$$

input
```
int(x^2*(a + b*atan(c*x))^2,x)
```

output
```
int(x^2*(a + b*atan(c*x))^2, x)
```

Reduce [F]

$$\int x^2(a+b\arctan(cx))^2\,dx = \frac{atan(cx)^2\,b^2c^3x^3+atan(cx)^2\,b^2cx+2atan(cx)\,ab\,c^3x^3-atan(cx)\,b^2c^2x^2-atan(cx)\,b^2-\left(\int atan(cx)^2\,c\right.}{3c^3}$$

input
```
int(x^2*(a+b*atan(c*x))^2,x)
```

output
```
(atan(c*x)**2*b**2*c**3*x**3 + atan(c*x)**2*b**2*c*x + 2*atan(c*x)*a*b*c**
3*x**3 - atan(c*x)*b**2*c**2*x**2 - atan(c*x)*b**2 - int(atan(c*x)**2,x)*b
**2*c + log(c**2*x**2 + 1)*a*b + a**2*c**3*x**3 - a*b*c**2*x**2 + b**2*c*x
)/(3*c**3)
```

3.17 $\int x(a+b\arctan(cx))^2\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 76

$$\int x(a+b\arctan(cx))^2\,dx = -\frac{abx}{c}-\frac{b^2x\arctan(cx)}{c}+\frac{(a+b\arctan(cx))^2}{2c^2}+\frac{1}{2}x^2(a+b\arctan(cx))^2+\frac{b^2\log(1+c^2x^2)}{2c^2}$$

output
```
-a*b*x/c-b^2*x*arctan(c*x)/c+1/2*(a+b*arctan(c*x))^2/c^2+1/2*x^2*(a+b*arct
an(c*x))^2+1/2*b^2*ln(c^2*x^2+1)/c^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int x(a+b\arctan(cx))^2\,dx = \frac{acx(-2b+acx)+2b(a-bcx+ac^2x^2)\arctan(cx)+b^2(1+c^2x^2)\arctan(cx)^2+b^2\log(1+c^2x^2)}{2c^2}$$

input
```
Integrate[x*(a + b*ArcTan[c*x])^2,x]
```

output
```
(a*c*x*(-2*b + a*c*x) + 2*b*(a - b*c*x + a*c^2*x^2)*ArcTan[c*x] + b^2*(1 +
 c^2*x^2)*ArcTan[c*x]^2 + b^2*Log[1 + c^2*x^2])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5361, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{\int(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)$$

input
```
Int[x*(a + b*ArcTan[c*x])^2,x]
```

output
```
(x^2*(a + b*ArcTan[c*x])^2)/2 - b*c*(-1/2*(a + b*ArcTan[c*x])^2/(b*c^3) +
(a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c))/c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

method	result
parts	$\frac{a^2x^2}{2}+\frac{b^2\left(\frac{c^2x^2\arctan(cx)^2}{2}+\frac{\arctan(cx)^2}{2}-cx\arctan(cx)+\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{c^2}+ab\arctan\left(cx\right)x^2-\frac{abx}{c}+$
derivativedivides	$\frac{\frac{x^2c^2a^2}{2}+b^2\left(\frac{c^2x^2\arctan(cx)^2}{2}+\frac{\arctan(cx)^2}{2}-cx\arctan(cx)+\frac{\ln\left(c^2x^2+1\right)}{2}\right)+2ab\left(\frac{c^2x^2\arctan(cx)}{2}-\frac{cx}{2}+\frac{\arctan(cx)}{2}\right)}{c^2}$
default	$\frac{\frac{x^2c^2a^2}{2}+b^2\left(\frac{c^2x^2\arctan(cx)^2}{2}+\frac{\arctan(cx)^2}{2}-cx\arctan(cx)+\frac{\ln\left(c^2x^2+1\right)}{2}\right)+2ab\left(\frac{c^2x^2\arctan(cx)}{2}-\frac{cx}{2}+\frac{\arctan(cx)}{2}\right)}{c^2}$
parallelrisch	$\frac{b^2\arctan(cx)^2x^2c^2+2ab\arctan(cx)x^2c^2+x^2c^2a^2-2b^2\arctan(cx)xc-2abcx+b^2\arctan(cx)^2+b^2\ln(c^2x^2+1)+2ab\,\mathrm{arct}}{2c^2}$
risch	$-\frac{b^2\left(c^2x^2+1\right)\ln(icx+1)^2}{8c^2}-\frac{ib\left(2a\,c^2x^2+ib\,c^2x^2\ln(-icx+1)-2bcx+ib\ln(-icx+1)\right)\ln(icx+1)}{4c^2}+$

input
```
int(x*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+b^2/c^2*(1/2*c^2*x^2*arctan(c*x)^2+1/2*arctan(c*x)^2-c*x*arcta
n(c*x)+1/2*ln(c^2*x^2+1))+a*b*arctan(c*x)*x^2-a*b*x/c+1/c^2*a*b*arctan(c*x
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int x(a+b\arctan(cx))^2\,dx = \frac{a^2c^2x^2-2\,abcx+\left(b^2c^2x^2+b^2\right)\arctan\left(cx\right)^2+b^2\log\left(c^2x^2+1\right)+2\left(abc^2x^2-b^2cx+ab\right)\arctan\left(cx\right)}{2\,c^2}$$

input
```
integrate(x*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/2*(a^2*c^2*x^2 - 2*a*b*c*x + (b^2*c^2*x^2 + b^2)*arctan(c*x)^2 + b^2*log
(c^2*x^2 + 1) + 2*(a*b*c^2*x^2 - b^2*c*x + a*b)*arctan(c*x))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.41

$$\int x(a+b\arctan(cx))^2\,dx = \begin{cases} \frac{a^2x^2}{2}+abx^2\operatorname{atan}\left(cx\right)-\frac{abx}{c}+\frac{ab\operatorname{atan}\left(cx\right)}{c^2}+\frac{b^2x^2\operatorname{atan}^2\left(cx\right)}{2}-\frac{b^2x\operatorname{atan}\left(cx\right)}{c}+\frac{b^2\log\left(x^2+\frac{1}{c^2}\right)}{2c^2}+\frac{b^2\operatorname{atan}^2\left(cx\right)}{2c^2} & \text{for } c \\ \frac{a^2x^2}{2} & \text{other} \end{cases}$$

input
```
integrate(x*(a+b*atan(c*x))**2,x)
```

output
```
Piecewise((a**2*x**2/2 + a*b*x**2*atan(c*x) - a*b*x/c + a*b*atan(c*x)/c**2
 + b**2*x**2*atan(c*x)**2/2 - b**2*x*atan(c*x)/c + b**2*log(x**2 + c**(-2)
)/(2*c**2) + b**2*atan(c*x)**2/(2*c**2), Ne(c, 0)), (a**2*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

$$\int x(a+b\arctan(cx))^2\,dx = \frac{1}{2}\,b^2x^2\arctan\left(cx\right)^2 + \frac{1}{2}\,a^2x^2 + \left(x^2\arctan\left(cx\right) - c\left(\frac{x}{c^2} - \frac{\arctan\left(cx\right)}{c^3}\right)\right)ab - \frac{1}{2}\left(2\,c\left(\frac{x}{c^2} - \frac{\arctan\left(cx\right)}{c^3}\right)\arctan\left(cx\right) + \frac{\arctan\left(cx\right)^2 - \log\left(c^2x^2+1\right)}{c^2}\right)b^2$$

input
```
integrate(x*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*arctan(c*x)^2 + 1/2*a^2*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - ar
ctan(c*x)/c^3))*a*b - 1/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x) + (ar
ctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

$$\int x(a+b\arctan(cx))^2\,dx = \frac{b^2c^2x^2\arctan\left(cx\right)^2 + 2\,abc^2x^2\arctan\left(cx\right) + a^2c^2x^2 - 2\,b^2cx\arctan\left(cx\right) - 2\,\pi ab\text{sgn}(c)\,\text{sgn}(x) - 2\,abcx +}{2\,c^2}$$

input
```
integrate(x*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
1/2*(b^2*c^2*x^2*arctan(c*x)^2 + 2*a*b*c^2*x^2*arctan(c*x) + a^2*c^2*x^2 -
 2*b^2*c*x*arctan(c*x) - 2*pi*a*b*sgn(c)*sgn(x) - 2*a*b*c*x + b^2*arctan(c
*x)^2 + 2*a*b*arctan(c*x) + b^2*log(c^2*x^2 + 1))/c^2
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\int x(a+b\arctan(cx))^2\,dx$$
$$=\frac{\frac{b^2\,\mathrm{atan}(c\,x)^2}{2}+\frac{b^2\,\ln(c^2\,x^2+1)}{2}-c\,(x\,\mathrm{atan}(c\,x)\;b^2+a\,x\,b)+a\,b\,\mathrm{atan}(c\,x)}{c^2}$$
$$+\frac{a^2\,x^2}{2}+\frac{b^2\,x^2\,\mathrm{atan}(c\,x)^2}{2}+a\,b\,x^2\,\mathrm{atan}(c\,x)$$

input
```
int(x*(a + b*atan(c*x))^2,x)
```

output
```
((b^2*atan(c*x)^2)/2 + (b^2*log(c^2*x^2 + 1))/2 - c*(b^2*x*atan(c*x) + a*b
*x) + a*b*atan(c*x))/c^2 + (a^2*x^2)/2 + (b^2*x^2*atan(c*x)^2)/2 + a*b*x^2
*atan(c*x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.25

$$\int x(a+b\arctan(cx))^2\,dx$$
$$=\frac{atan(cx)^2\,b^2c^2x^2+atan(cx)^2\,b^2+2atan(cx)\,ab\,c^2x^2+2atan(cx)\,ab-2atan(cx)\,b^2cx+\log(c^2x^2+1)\,b^2}{2c^2}$$

input
```
int(x*(a+b*atan(c*x))^2,x)
```

output
```
(atan(c*x)**2*b**2*c**2*x**2 + atan(c*x)**2*b**2 + 2*atan(c*x)*a*b*c**2*x*
*2 + 2*atan(c*x)*a*b - 2*atan(c*x)*b**2*c*x + log(c**2*x**2 + 1)*b**2 + a*
*2*c**2*x**2 - 2*a*b*c*x)/(2*c**2)
```

3.18 $\int (a+b\arctan(cx))^2\,dx$

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Mupad [F(-1)] 200
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Optimal result

Integrand size = 10, antiderivative size = 83

$$\int (a+b\arctan(cx))^2\,dx = \frac{i(a+b\arctan(cx))^2}{c} + x(a+b\arctan(cx))^2 + \frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c} + \frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}$$

output
```
I*(a+b*arctan(c*x))^2/c+x*(a+b*arctan(c*x))^2+2*b*(a+b*arctan(c*x))*ln(2/(
1+I*c*x))/c+I*b^2*polylog(2,1-2/(1+I*c*x))/c
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

$$\int (a+b\arctan(cx))^2\,dx = \frac{b^2(-i+cx)\arctan(cx)^2 + 2b\arctan(cx)\left(acx + b\log\left(1+e^{2i\arctan(cx)}\right)\right) + a(acx - b\log(1+c^2x^2)) - ib}{c}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2,x]
```

output
```
(b^2*(-I + c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(a*c*x + b*Log[1 + E^((2*I
)*ArcTan[c*x])]) + a*(a*c*x - b*Log[1 + c^2*x^2]) - I*b^2*PolyLog[2, -E^((
2*I)*ArcTan[c*x])])/c
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5345}$$

$$x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx$$

$$\Big\downarrow \text{5455}$$

$$x(a+b\arctan(cx))^2-2bc\left(-\frac{\int \frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)$$

$$\Big\downarrow \text{5379}$$

$$x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int \frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)$$

$$\Big\downarrow \text{2849}$$

$$x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{ib\int \frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)$$

$$\Big\downarrow \text{2752}$$

$$x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}}{c}\right)$$

input
```
Int[(a + b*ArcTan[c*x])^2,x]
```

output
```
x*(a + b*ArcTan[c*x])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b*c^2)
- (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2, 1 - 2/
(1 + I*c*x)])/c)/c)
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{cx\,a^2 - i\arctan(cx)^2 b^2 + cx\,b^2\arctan(cx)^2 + 2\arctan(cx)\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)b^2 - i\,\mathrm{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)b^2 + 2cxab\,\mathrm{arcta}}{c}$
default	$\frac{cx\,a^2 - i\arctan(cx)^2 b^2 + cx\,b^2\arctan(cx)^2 + 2\arctan(cx)\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)b^2 - i\,\mathrm{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)b^2 + 2cxab\,\mathrm{arcta}}{c}$
parts	$a^2x + b^2\arctan(cx)^2 x - \frac{ib^2\arctan(cx)^2}{c} + \frac{2b^2\arctan(cx)\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)}{c} - \frac{ib^2\,\mathrm{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right.}{c}$
risch	$a^2x - \frac{b^2\arctan(cx)}{c} - \frac{b^2\ln(icx+1)^2x}{4} - \frac{\ln(-icx+1)^2b^2x}{4} + i\ln(-icx+1)\,abx + \frac{ia^2}{c} + \frac{ib^2\ln(icx+1}{4c}$

input
```
int((a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(c*x*a^2-I*arctan(c*x)^2*b^2+c*x*b^2*arctan(c*x)^2+2*arctan(c*x)*ln(1+
(1+I*c*x)^2/(c^2*x^2+1))*b^2-I*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))*b^2+2*c
*x*a*b*arctan(c*x)-a*b*ln(c^2*x^2+1))
```

Fricas [F]

$$\int (a+b\arctan(cx))^2\,dx = \int (b\arctan(cx)+a)^2\,dx$$

input
```
integrate((a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2, x)
```

Sympy [F]

$$\int (a+b\arctan(cx))^2\,dx = \int (a+b\,\mathrm{atan}\,(cx))^2\;dx$$

input
```
integrate((a+b*atan(c*x))**2,x)
```

output
```
Integral((a + b*atan(c*x))**2, x)
```

Maxima [F]

$$\int (a+b\arctan(cx))^2\,dx = \int (b\arctan(cx)+a)^2\,dx$$

input
```
integrate((a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/16*(4*x*arctan(c*x)^2 + 192*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^
2 + 1), x) + 16*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x
) + 64*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - x*log(c
^2*x^2 + 1)^2 + 4*arctan(c*x)^3/c - 128*c*integrate(1/16*x*arctan(c*x)/(c^
2*x^2 + 1), x) + 16*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x))*b
^2 + a^2*x + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b/c
```

Giac [F]

$$\int (a+b\arctan(cx))^2\,dx = \int (b\arctan(cx)+a)^2\,dx$$

input
```
integrate((a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\arctan(cx))^2\,dx = \int \left(a + b\,\mathrm{atan}(c\,x)\right)^2\,dx$$

input
```
int((a + b*atan(c*x))^2,x)
```

output
```
int((a + b*atan(c*x))^2, x)
```

Reduce [F]

$$\int (a + b\arctan(cx))^2\,dx$$
$$= \frac{2atan(cx)\,abcx + \left(\int atan(cx)^2\,dx\right)b^2c - \log(c^2x^2+1)\,ab + a^2cx}{c}$$

input
```
int((a+b*atan(c*x))^2,x)
```

output
```
(2*atan(c*x)*a*b*c*x + int(atan(c*x)**2,x)*b**2*c - log(c**2*x**2 + 1)*a*b
 + a**2*c*x)/c
```

3.19 $\int \frac{(a+b\arctan(cx))^2}{x}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 132

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x}\,dx = {} & 2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) \\ & - ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) \\ & + ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) \\ & - \frac{1}{2}b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right) \\ & + \frac{1}{2}b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\end{aligned}$$

output
```
-2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))-I*b*(a+b*arctan(c*x))*polyl
og(2,1-2/(1+I*c*x))+I*b*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*b^
2*polylog(3,1-2/(1+I*c*x))+1/2*b^2*polylog(3,-1+2/(1+I*c*x))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.36

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = a^2\log(cx)+iab(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx)) + b^2\left(-\frac{i\pi^3}{24}+\frac{2}{3}i\arctan(cx)^3 + \arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - \arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) + i\arctan(cx)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) + i\arctan(cx)\,\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) + \frac{1}{2}\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - \frac{1}{2}\text{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/x,x]
```

output
```
a^2*Log[c*x] + I*a*b*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + b^2*((-1
/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + ArcTan[c*x]^2*Log[1 - E^((-2*I)*Ar
cTan[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]
*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, -E^((2*I)*A
rcTan[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 - PolyLog[3, -E^((2*I)
*ArcTan[c*x])]/2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5357, 5523, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx$$

$\Big\downarrow$ 5357

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-4bc\int\frac{(a+b\arctan(cx))\text{arctanh}\left(1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx$$

$\Big\downarrow$ 5523

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-$$
$$4bc\left(\frac{1}{2}\int\frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{1}{2}\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)$$

$\Big\downarrow$ 5529

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-$$
$$4bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}ib\int\frac{\text{PolyLog}}{c}\right.\right.$$

$\Big\downarrow$ 7164

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-$$
$$4bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}+\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{4c}\right)+\frac{1}{2}\left(-\frac{i\,\text{PolyLog}\left(2,\frac{2}{icx+1}-\right.}{2}\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x])^2/x,x]
```

output
```
2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] - 4*b*c*((((I/2)*(a + b
*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c + (b*PolyLog[3, 1 - 2/(1 +
I*c*x)])/(4*c))/2 + (((-1/2*I)*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 +
I*c*x)])/c - (b*PolyLog[3, -1 + 2/(1 + I*c*x)])/(4*c))/2)
```

Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.28 (sec) , antiderivative size = 1000, normalized size of antiderivative = 7.58

method	result	size
parts	Expression too large to display	1000
derivativedivides	Expression too large to display	1002
default	Expression too large to display	1002

input
```
int((a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(x)+b^2*(ln(c*x)*arctan(c*x)^2+I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/
(c^2*x^2+1))-1/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-arctan(c*x)^2*ln((1+I
*c*x)^2/(c^2*x^2+1)-1)+arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I
*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,(1+I*c*x)/
(c^2*x^2+1)^(1/2))+arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arc
tan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*c*x)/(c
^2*x^2+1)^(1/2))+1/2*I*Pi*(csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)
^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+
1)))-csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(I*((1+I*c*x)
^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*
x^2+1)))^2-csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^
2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/
(1+(1+I*c*x)^2/(c^2*x^2+1)))^3-csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*
c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*
x^2+1)))^2+csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3
+1)*arctan(c*x)^2)+2*a*b*(ln(c*x)*arctan(c*x)+1/2*I*ln(c*x)*ln(1+I*c*x)-1/
2*I*ln(c*x)*ln(1-I*c*x)+1/2*I*dilog(1+I*c*x)-1/2*I*dilog(1-I*c*x))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx=\int \frac{(b\arctan{(cx)}+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(a+b\operatorname{atan}(cx))^2}{x}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/x,x)
```

output
```
Integral((a + b*atan(c*x))**2/x, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x)^2 + b^2*log(c^2*x^2 + 1)^2
 + 32*a*b*arctan(c*x))/x, x)
```

Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{x}\,dx$$

input
```
int((a + b*atan(c*x))^2/x,x)
```

output
```
int((a + b*atan(c*x))^2/x, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = 2\left(\int \frac{atan(cx)}{x}dx\right)ab + \left(\int \frac{atan(cx)^2}{x}dx\right)b^2 + \log(x)\,a^2$$

input
```
int((a+b*atan(c*x))^2/x,x)
```

output
```
2*int(atan(c*x)/x,x)*a*b + int(atan(c*x)**2/x,x)*b**2 + log(x)*a**2
```

3.20 $\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx$

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Giac [F] 212
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Optimal result

Integrand size = 14, antiderivative size = 82

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx = &-ic(a+b\arctan(cx))^2-\frac{(a+b\arctan(cx))^2}{x}\\ &+2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\ &-ib^2c\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\end{aligned}$$

output
```
-I*c*(a+b*arctan(c*x))^2-(a+b*arctan(c*x))^2/x+2*b*c*(a+b*arctan(c*x))*ln(
2-2/(1-I*c*x))-I*b^2*c*polylog(2,-1+2/(1-I*c*x))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx$$
$$=\frac{b^2(-1-icx)\arctan(cx)^2+2b\arctan(cx)\left(-a+bcx\log\left(1-e^{2i\arctan(cx)}\right)\right)-a(a-2bcx\log(cx)+bcx\,\mathrm{lo}}{x}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/x^2,x]
```

output
```
(b^2*(-1 - I*c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(-a + b*c*x*Log[1 - E^((
2*I)*ArcTan[c*x])]) - a*(a - 2*b*c*x*Log[c*x] + b*c*x*Log[1 + c^2*x^2]) -
I*b^2*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5361, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx$$

$$\Big\downarrow \text{5361}$$

$$2bc\int \frac{a+b\arctan(cx)}{x\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))^2}{x}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^2}{2b}\right)$$

$$\Big\downarrow \text{5403}$$

$$-\frac{(a+b\arctan(cx))^2}{x}+ 2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)$$

$$\Big\downarrow \text{2897}$$

$$-\frac{(a+b\arctan(cx))^2}{x}+ 2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)$$

input
```
Int[(a + b*ArcTan[c*x])^2/x^2,x]
```

output
```
-((a + b*ArcTan[c*x])^2/x) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b + I
*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 + 2/(
1 - I*c*x)])/2))
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(78) = 156.

Time = 0.37 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.29

method	result
parts	$-\frac{a^2}{x}+b^2c\left(-\frac{\arctan(cx)^2}{cx}-\arctan(cx)\ln(c^2x^2+1)+2\ln(cx)\arctan(cx)-\frac{i\left(\ln(cx-i)\ln\right.}{}\right.$
derivativedivides	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\arctan(cx)^2}{cx}-\arctan(cx)\ln(c^2x^2+1)+2\ln(cx)\arctan(cx)-\frac{i\left(\ln(cx-i\right.}{}\right.\right.$
default	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\arctan(cx)^2}{cx}-\arctan(cx)\ln(c^2x^2+1)+2\ln(cx)\arctan(cx)-\frac{i\left(\ln(cx-i\right.}{}\right.\right.$

input
```
int((a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*a^2+b^2*c*(-1/c/x*arctan(c*x)^2-arctan(c*x)*ln(c^2*x^2+1)+2*ln(c*x)*a
rctan(c*x)-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*
x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))+1/2*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(
c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I)))+I*ln(c*x)*ln(1+
I*c*x)-I*ln(c*x)*ln(1-I*c*x)+I*dilog(1+I*c*x)-I*dilog(1-I*c*x))+2*a*b*c*(-
1/c/x*arctan(c*x)-1/2*ln(c^2*x^2+1)+ln(c*x))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx=\int \frac{(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x^2, x)
```

Sympy [F]

$$\int \frac{(a + b \arctan(cx))^2}{x^2} \, dx = \int \frac{(a + b \operatorname{atan}(cx))^2}{x^2} \, dx$$

input
```
integrate((a+b*atan(c*x))**2/x**2,x)
```

output
```
Integral((a + b*atan(c*x))**2/x**2, x)
```

Maxima [F]

$$\int \frac{(a + b \arctan(cx))^2}{x^2} \, dx = \int \frac{(b \arctan(cx) + a)^2}{x^2} \, dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
-(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b + 1/16*(4*(c*arct
an(c*x)^3 + 4*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x
) - 16*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) + 32*c*
integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2), x) + 48*integrate(1/16*arcta
n(c*x)^2/(c^2*x^4 + x^2), x) + 4*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^
4 + x^2), x))*x - 4*arctan(c*x)^2 + log(c^2*x^2 + 1)^2)*b^2/x - a^2/x
```

Giac [F]

$$\int \frac{(a + b \arctan(cx))^2}{x^2} \, dx = \int \frac{(b \arctan(cx) + a)^2}{x^2} \, dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{x^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/x^2,x)
```

output
```
int((a + b*atan(c*x))^2/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx$$
$$= \frac{-atan(cx)^2\,b^2 - 2atan(cx)\,ab + 2\left(\int \frac{atan(cx)}{c^2x^3+x}dx\right)b^2cx - \log(c^2x^2+1)\,abcx + 2\log(x)\,abcx - a^2}{x}$$

input
```
int((a+b*atan(c*x))^2/x^2,x)
```

output
```
( - atan(c*x)**2*b**2 - 2*atan(c*x)*a*b + 2*int(atan(c*x)/(c**2*x**3 + x),
x)*b**2*c*x - log(c**2*x**2 + 1)*a*b*c*x + 2*log(x)*a*b*c*x - a**2)/x
```

3.21 $\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx = -\frac{bc(a+b\arctan(cx))}{x} - \frac{1}{2}c^2(a+b\arctan(cx))^2 - \frac{(a+b\arctan(cx))^2}{2x^2} + b^2c^2\log(x) - \frac{1}{2}b^2c^2\log\left(1+c^2x^2\right)$$

output
```
-b*c*(a+b*arctan(c*x))/x-1/2*c^2*(a+b*arctan(c*x))^2-1/2*(a+b*arctan(c*x))
^2/x^2+b^2*c^2*ln(x)-1/2*b^2*c^2*ln(c^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx = -\frac{a^2+2abcx+2b(a+bcx+ac^2x^2)\arctan(cx)+b^2(1+c^2x^2)\arctan(cx)^2-2b^2c^2x^2\log(x)+b^2c^2x^2\log}{2x^2}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/x^3,x]
```

output
```
-1/2*(a^2 + 2*a*b*c*x + 2*b*(a + b*c*x + a*c^2*x^2)*ArcTan[c*x] + b^2*(1 +
 c^2*x^2)*ArcTan[c*x]^2 - 2*b^2*c^2*x^2*Log[x] + b^2*c^2*x^2*Log[1 + c^2*x
^2])/x^2
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5361, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx$$

$$\Big\downarrow \text{5361}$$

$$bc\int \frac{a+b\arctan(cx)}{x^2\,(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{2x^2}$$

$$\Big\downarrow \text{5453}$$

$$bc\left(\int \frac{a+b\arctan(cx)}{x^2}dx-c^2\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{(a+b\arctan(cx))^2}{2x^2}$$

$$\Big\downarrow \text{5361}$$

$$bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x\,(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2\,(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}$$

$$\Big\downarrow \text{47}$$

$$bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}$$

$$\Big\downarrow \ 14$$

$$bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}$$

$$\Big\downarrow \ 16$$

$$bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}$$

$$\Big\downarrow \ 5419$$

$$bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}$$

input
```
Int[(a + b*ArcTan[c*x])^2/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c*x])^2/x^2 + b*c*(-((a + b*ArcTan[c*x])/x) - (c*(a + b
*ArcTan[c*x])^2)/(2*b) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.24

method	result
parts	$-\frac{a^2}{2x^2}+b^2c^2\left(-\frac{\arctan(cx)^2}{2c^2x^2}-\frac{\arctan(cx)}{cx}-\frac{\arctan(cx)^2}{2}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)-\frac{ab\arctan(cx)}{x^2}-$
derivativedivides	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\arctan(cx)^2}{2c^2x^2}-\frac{\arctan(cx)}{cx}-\frac{\arctan(cx)^2}{2}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)-\frac{ab\arctan(c}{c^2x^2}\right.$
default	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\arctan(cx)^2}{2c^2x^2}-\frac{\arctan(cx)}{cx}-\frac{\arctan(cx)^2}{2}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)-\frac{ab\arctan(c}{c^2x^2}\right.$
parallelrisch	$\frac{-b^2\arctan(cx)^2x^2c^2+2b^2c^2\ln(x)x^2-b^2c^2\ln(c^2x^2+1)x^2-2ab\arctan(cx)x^2c^2+x^2c^2a^2-2b^2\arctan(cx)xc-2abcx-b^2}{2x^2}$
risch	$\frac{b^2(c^2x^2+1)\ln(icx+1)^2}{8x^2}+\frac{ib(ib\,c^2x^2\ln(-icx+1)+2bcx+2a+ib\ln(-icx+1))\ln(icx+1)}{4x^2}-\frac{4i\ln((3ibc-ac)x-3b-ia}{}$

input
```
int((a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2+b^2*c^2*(-1/2/c^2/x^2*arctan(c*x)^2-1/c/x*arctan(c*x)-1/2*arc
tan(c*x)^2-1/2*ln(c^2*x^2+1)+ln(c*x))-a*b*arctan(c*x)/x^2-a*b*c/x-a*b*c^2*
arctan(c*x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int\frac{(a+b\arctan(cx))^2}{x^3}\,dx=$$
$$-\frac{b^2c^2x^2\log(c^2x^2+1)-2\,b^2c^2x^2\log(x)+2\,abcx+(b^2c^2x^2+b^2)\arctan(cx)^2+a^2+2\,(abc^2x^2+b^2cx-}{2\,x^2}$$

input
```
integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
-1/2*(b^2*c^2*x^2*log(c^2*x^2 + 1) - 2*b^2*c^2*x^2*log(x) + 2*a*b*c*x + (b
^2*c^2*x^2 + b^2)*arctan(c*x)^2 + a^2 + 2*(a*b*c^2*x^2 + b^2*c*x + a*b)*ar
ctan(c*x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.51

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx$$
$$= \begin{cases} -\frac{a^2}{2x^2} - abc^2\operatorname{atan}(cx) - \frac{abc}{x} - \frac{ab\operatorname{atan}(cx)}{x^2} + b^2c^2\log(x) - \frac{b^2c^2\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{b^2c^2\operatorname{atan}^2(cx)}{2} - \frac{b^2c\operatorname{atan}(cx)}{x} - b^2 \\ -\frac{a^2}{2x^2} \end{cases}$$

input
```
integrate((a+b*atan(c*x))**2/x**3,x)
```

output
```
Piecewise((-a**2/(2*x**2) - a*b*c**2*atan(c*x) - a*b*c/x - a*b*atan(c*x)/x
**2 + b**2*c**2*log(x) - b**2*c**2*log(x**2 + c**(-2))/2 - b**2*c**2*atan(
c*x)**2/2 - b**2*c*atan(c*x)/x - b**2*atan(c*x)**2/(2*x**2), Ne(c, 0)), (-
a**2/(2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.24

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx = -\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)ab$$
$$+\frac{1}{2}\left(\left(\arctan(cx)^2-\log\left(c^2x^2+1\right)+2\log(x)\right)c^2-2\left(c\arctan(cx)+\frac{1}{x}\right)c\arctan(cx)\right)b^2$$
$$-\frac{b^2\arctan(cx)^2}{2x^2}-\frac{a^2}{2x^2}$$

input
```
integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b + 1/2*((arctan(c*x)^2 - l
og(c^2*x^2 + 1) + 2*log(x))*c^2 - 2*(c*arctan(c*x) + 1/x)*c*arctan(c*x))*b
^2 - 1/2*b^2*arctan(c*x)^2/x^2 - 1/2*a^2/x^2
```

Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx=\int \frac{(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 2.87 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.77

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx=\;& b^2\,c^2\,\ln(x)-\frac{a^2}{2\,x^2}-\frac{b^2\,c^2\,\mathrm{atan}(c\,x)^2}{2}\\&-\frac{b^2\,c^2\,\ln(c\,x+1\mathrm{i})}{2}-\frac{b^2\,c^2\,\ln(1+c\,x\,1\mathrm{i})}{2}\\&-\frac{b^2\,\mathrm{atan}(c\,x)^2}{2\,x^2}-\frac{a\,b\,c}{x}-\frac{a\,b\,\mathrm{atan}(c\,x)}{x^2}-\frac{b^2\,c\,\mathrm{atan}(c\,x)}{x}\\&-\frac{a\,b\,c^2\,\ln(c\,x+1\mathrm{i})\,1\mathrm{i}}{2}+\frac{a\,b\,c^2\,\ln(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}\end{aligned}$$

input
```
int((a + b*atan(c*x))^2/x^3,x)
```

output
```
b^2*c^2*log(x) - a^2/(2*x^2) - (b^2*c^2*atan(c*x)^2)/2 - (b^2*c^2*log(c*x
+ 1i))/2 - (b^2*c^2*log(c*x*1i + 1))/2 - (b^2*atan(c*x)^2)/(2*x^2) - (a*b*
c)/x - (a*b*atan(c*x))/x^2 - (a*b*c^2*log(c*x + 1i)*1i)/2 + (a*b*c^2*log(c
*x*1i + 1)*1i)/2 - (b^2*c*atan(c*x))/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.42

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx$$
$$= \frac{-atan(cx)^2 b^2c^2x^2 - atan(cx)^2 b^2 - 2atan(cx)\,ab\,c^2x^2 - 2atan(cx)\,ab - 2atan(cx)\,b^2cx - \log(c^2x^2+1)}{2x^2}$$

input
```
int((a+b*atan(c*x))^2/x^3,x)
```

output
```
( - atan(c*x)**2*b**2*c**2*x**2 - atan(c*x)**2*b**2 - 2*atan(c*x)*a*b*c**2
*x**2 - 2*atan(c*x)*a*b - 2*atan(c*x)*b**2*c*x - log(c**2*x**2 + 1)*b**2*c
**2*x**2 + 2*log(x)*b**2*c**2*x**2 - a**2 - 2*a*b*c*x)/(2*x**2)
```

3.22 $\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 140

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = &-\frac{b^2c^2}{3x}-\frac{1}{3}b^2c^3\arctan(cx)-\frac{bc(a+b\arctan(cx))}{3x^2}\\&+\frac{1}{3}ic^3(a+b\arctan(cx))^2-\frac{(a+b\arctan(cx))^2}{3x^3}\\&-\frac{2}{3}bc^3(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\&+\frac{1}{3}ib^2c^3\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\end{aligned}$$

output
```
-1/3*b^2*c^2/x-1/3*b^2*c^3*arctan(c*x)-1/3*b*c*(a+b*arctan(c*x))/x^2+1/3*I
*c^3*(a+b*arctan(c*x))^2-1/3*(a+b*arctan(c*x))^2/x^3-2/3*b*c^3*(a+b*arctan
(c*x))*ln(2-2/(1-I*c*x))+1/3*I*b^2*c^3*polylog(2,-1+2/(1-I*c*x))
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = -\frac{a^2+abcx+b^2c^2x^2+b^2(1-ic^3x^3)\arctan(cx)^2+b\arctan(cx)\left(2a+bcx+bc^3x^3+2bc^3x^3\log\left(1-e^{2i\right.}\right.}{3x^3}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/x^4,x]
```

output
```
-1/3*(a^2 + a*b*c*x + b^2*c^2*x^2 + b^2*(1 - I*c^3*x^3)*ArcTan[c*x]^2 + b*
ArcTan[c*x]*(2*a + b*c*x + b*c^3*x^3 + 2*b*c^3*x^3*Log[1 - E^((2*I)*ArcTan
[c*x])]) + 2*a*b*c^3*x^3*Log[c*x] - a*b*c^3*x^3*Log[1 + c^2*x^2] - I*b^2*c
^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x^3
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5361, 5453, 5361, 264, 216, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3\,(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{2}{3}bc\left(\int \frac{a+b\arctan(cx)}{x^3}dx-c^2\int \frac{a+b\arctan(cx)}{x\,(c^2x^2+1)}dx\right)-\frac{(a+b\arctan(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x\left(c^2x^2+1\right)}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{2x^2}\right)-\frac{(a+b\arctan(cx))^2}{3x^3}$$

$\Big\downarrow$ 264

$$\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x\left(c^2x^2+1\right)}dx\right)+\frac{1}{2}bc\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}\right)-\frac{(a+b\arctan(cx))^2}{3x^3}$$

$\Big\downarrow$ 216

$$\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x\left(c^2x^2+1\right)}dx\right)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)-\frac{(a+b\arctan(cx))^2}{3x^3}$$

$\Big\downarrow$ 5459

$$-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^2}{2b}\right)\right)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right.$$

$\Big\downarrow$ 5403

$$-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)\right)\right.$$

$\Big\downarrow$ 2897

$$-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right.\right.\right.$$

input

```
Int[(a + b*ArcTan[c*x])^2/x^4,x]
```

output
```
-1/3*(a + b*ArcTan[c*x])^2/x^3 + (2*b*c*(-1/2*(a + b*ArcTan[c*x])/x^2 + (b
*c*(-x^(-1) - c*ArcTan[c*x]))/2 - c^2*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b
+ I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 +
2/(1 - I*c*x)])/2))))/3
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(122) = 244$.

Time = 0.32 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.21

method	result
parts	$-\frac{a^2}{3x^3}+b^2c^3\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{\arctan(cx)\ln(c^2x^2+1)}{3}-\frac{\arctan(cx)}{3c^2x^2}-\frac{2\ln(cx)\arctan(cx)}{3}+\frac{i\left(\ln(cx-i)\ln\right.}{}\right.$
derivativedivides	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{\arctan(cx)\ln(c^2x^2+1)}{3}-\frac{\arctan(cx)}{3c^2x^2}-\frac{2\ln(cx)\arctan(cx)}{3}+\frac{i\left(\ln(cx\right.}{}\right.\right.$
default	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{\arctan(cx)\ln(c^2x^2+1)}{3}-\frac{\arctan(cx)}{3c^2x^2}-\frac{2\ln(cx)\arctan(cx)}{3}+\frac{i\left(\ln(cx\right.}{}\right.\right.$

input
```
int((a+b*arctan(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3+b^2*c^3*(-1/3/c^3/x^3*arctan(c*x)^2+1/3*arctan(c*x)*ln(c^2*x^
2+1)-1/3/c^2/x^2*arctan(c*x)-2/3*ln(c*x)*arctan(c*x)+1/6*I*(ln(c*x-I)*ln(c
^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I
)))-1/6*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln
(c*x+I)*ln(1/2*I*(c*x-I)))-1/3/c/x-1/3*arctan(c*x)-1/3*I*ln(c*x)*ln(1+I*c*
x)+1/3*I*ln(c*x)*ln(1-I*c*x)-1/3*I*dilog(1+I*c*x)+1/3*I*dilog(1-I*c*x))+2*
a*b*c^3*(-1/3/c^3/x^3*arctan(c*x)+1/6*ln(c^2*x^2+1)-1/6/c^2/x^2-1/3*ln(c*x
))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx=\int \frac{(b\arctan{(cx)}+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x^4, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx=\int \frac{(a+b\operatorname{atan}{(cx)})^2}{x^4}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/x**4,x)
```

output
```
Integral((a + b*atan(c*x))**2/x**4, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(b\arctan(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
1/3*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*
a*b + 1/48*(48*x^3*integrate(-1/48*(4*c^2*x^2*log(c^2*x^2 + 1) - 8*c*x*arc
tan(c*x) - 36*(c^2*x^2 + 1)*arctan(c*x)^2 - 3*(c^2*x^2 + 1)*log(c^2*x^2 +
1)^2)/(c^2*x^6 + x^4), x) - 4*arctan(c*x)^2 + log(c^2*x^2 + 1)^2)*b^2/x^3
- 1/3*a^2/x^3
```

Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(b\arctan(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^4}\,dx$$

input
```
int((a + b*atan(c*x))^2/x^4,x)
```

output
```
int((a + b*atan(c*x))^2/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx$$
$$= \frac{-atan(cx)^2 b^2 - 2atan(cx)\,ab - atan(cx)\,b^2c^3x^3 - atan(cx)\,b^2cx - 2\left(\int \frac{atan(cx)}{c^2x^3+x}dx\right)b^2c^3x^3 + \log(c^2x^2 +}{3x^3}$$

input
```
int((a+b*atan(c*x))^2/x^4,x)
```

output
```
( - atan(c*x)**2*b**2 - 2*atan(c*x)*a*b - atan(c*x)*b**2*c**3*x**3 - atan(
c*x)*b**2*c*x - 2*int(atan(c*x)/(c**2*x**3 + x),x)*b**2*c**3*x**3 + log(c*
*2*x**2 + 1)*a*b*c**3*x**3 - 2*log(x)*a*b*c**3*x**3 - a**2 - a*b*c*x - b**
2*c**2*x**2)/(3*x**3)
```

3.23 $\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx$

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Reduce [B] (verification not implemented) 238

Optimal result

Integrand size = 14, antiderivative size = 116

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx = -\frac{b^2c^2}{12x^2} - \frac{bc(a+b\arctan(cx))}{6x^3} + \frac{bc^3(a+b\arctan(cx))}{2x} + \frac{1}{4}c^4(a+b\arctan(cx))^2 - \frac{(a+b\arctan(cx))^2}{4x^4} - \frac{2}{3}b^2c^4\log(x) + \frac{1}{3}b^2c^4\log\left(1+c^2x^2\right)$$

output
```
-1/12*b^2*c^2/x^2-1/6*b*c*(a+b*arctan(c*x))/x^3+1/2*b*c^3*(a+b*arctan(c*x)
)/x+1/4*c^4*(a+b*arctan(c*x))^2-1/4*(a+b*arctan(c*x))^2/x^4-2/3*b^2*c^4*ln
(x)+1/3*b^2*c^4*ln(c^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx = \frac{-3a^2 - 2abcx - b^2c^2x^2 + 6abc^3x^3 + 2b(bcx(-1+3c^2x^2) + 3a(-1+c^4x^4))\arctan(cx) + 3b^2(-1+c^4x^4)}{12x^4}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/x^5,x]
```

output
```
(-3*a^2 - 2*a*b*c*x - b^2*c^2*x^2 + 6*a*b*c^3*x^3 + 2*b*(b*c*x*(-1 + 3*c^2
*x^2) + 3*a*(-1 + c^4*x^4))*ArcTan[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTan[c*x]
^2 - 8*b^2*c^4*x^4*Log[x] + 4*b^2*c^4*x^4*Log[1 + c^2*x^2])/(12*x^4)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {5361, 5453, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}bc\int \frac{a+b\arctan(cx)}{x^4\,(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{4x^4}$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{2}bc\left(\int \frac{a+b\arctan(cx)}{x^4}dx-c^2\int \frac{a+b\arctan(cx)}{x^2\,(c^2x^2+1)}dx\right)-\frac{(a+b\arctan(cx))^2}{4x^4}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x^2\,(c^2x^2+1)}dx\right)+\frac{1}{3}bc\int \frac{1}{x^3\,(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{3x^3}\right)-\frac{(a+b\arctan(cx))^2}{4x^4}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x^2\,(c^2x^2+1)}dx\right)+\frac{1}{6}bc\int \frac{1}{x^4\,(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{3x^3}\right)-\frac{(a+b\arctan(cx))^2}{4x^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx\right)+\frac{1}{6}bc\int\left(\frac{c^4}{c^2x^2+1}-\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{a+b\arctan(cx)}{3x^3}\right)-\frac{(a+b\arctan(cx))^2}{4x^4}$$

$\Big\downarrow$ 2009

$$\frac{1}{2}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx\right)-\frac{a+b\arctan(cx)}{3x^3}+\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)\right)-\frac{(a+b\arctan(cx))^2}{4x^4}$$

$\Big\downarrow$ 5453

$$\frac{1}{2}bc\left(-\left(c^2\left(\int \frac{a+b\arctan(cx)}{x^2}dx-c^2\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)\right)-\frac{a+b\arctan(cx)}{3x^3}+\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c\right.\right.$$
$$\frac{(a+b\arctan(cx))^2}{4x^4}$$

$\Big\downarrow$ 5361

$$\frac{1}{2}bc\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a+b\arctan(cx)}{3x^3}+\frac{1}{6}\right.$$
$$\frac{(a+b\arctan(cx))^2}{4x^4}$$

$\Big\downarrow$ 243

$$\frac{1}{2}bc\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2\left(c^2x^2+1\right)}dx^2-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a+b\arctan(cx)}{3x^3}\right.$$
$$\frac{(a+b\arctan(cx))^2}{4x^4}$$

$\Big\downarrow$ 47

$$\frac{1}{2}bc\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a+}{}\right.$$
$$\frac{(a+b\arctan(cx))^2}{4x^4}$$

$\Big\downarrow$ 14

$$\frac{1}{2}bc\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a+b}{}\right.$$
$$\frac{(a+b\arctan(cx))^2}{4x^4}$$

$\Big\downarrow$ 16

$$\frac{1}{2}bc\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)\right)-\frac{a+b\,\mathrm{arct}}{3x^3}\right.$$
$$\frac{(a+b\arctan(cx))^2}{4x^4}$$

$\Big\downarrow$ 5419

$$\frac{1}{2}bc\left(-\left(c^2\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)\right)-\frac{a+b\arctan(cx)}{3x^3}\right.$$
$$\frac{(a+b\arctan(cx))^2}{4x^4}$$

input
```
Int[(a + b*ArcTan[c*x])^2/x^5,x]
```

output
```
-1/4*(a + b*ArcTan[c*x])^2/x^4 + (b*c*(-1/3*(a + b*ArcTan[c*x])/x^3 - c^2*
(-((a + b*ArcTan[c*x])/x) - (c*(a + b*ArcTan[c*x])^2)/(2*b) + (b*c*(Log[x^
2] - Log[1 + c^2*x^2]))/2) + (b*c*(-x^(-2) - c^2*Log[x^2] + c^2*Log[1 + c^
2*x^2]))/6))/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.14

method	result
parts	$-\frac{a^2}{4x^4}+b^2c^4\left(-\frac{\arctan(cx)^2}{4c^4x^4}+\frac{\arctan(cx)^2}{4}-\frac{\arctan(cx)}{6c^3x^3}+\frac{\arctan(cx)}{2cx}+\frac{\ln(c^2x^2+1)}{3}-\frac{1}{12c^2x^2}-\frac{2\ln(}{3}\right.$
derivativedivides	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\arctan(cx)^2}{4c^4x^4}+\frac{\arctan(cx)^2}{4}-\frac{\arctan(cx)}{6c^3x^3}+\frac{\arctan(cx)}{2cx}+\frac{\ln(c^2x^2+1)}{3}-\frac{1}{12c^2x^2}-\right.\right.$
default	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\arctan(cx)^2}{4c^4x^4}+\frac{\arctan(cx)^2}{4}-\frac{\arctan(cx)}{6c^3x^3}+\frac{\arctan(cx)}{2cx}+\frac{\ln(c^2x^2+1)}{3}-\frac{1}{12c^2x^2}-\right.\right.$
parallelrisch	$-\frac{-3b^2\arctan(cx)^2x^4c^4+8b^2c^4\ln(x)x^4-4b^2c^4\ln(c^2x^2+1)x^4-6ab\arctan(cx)x^4c^4-b^2c^4x^4-6b^2\arctan(cx)x^3c^3-6a}{12x^4}$
risch	$-\frac{b^2(c^4x^4-1)\ln(icx+1)^2}{16x^4}+\frac{ib(-3ib\,c^4x^4\ln(-icx+1)-6b\,c^3x^3+2bcx+6a+3ib\ln(-icx+1))\ln(icx+1)}{24x^4}-\frac{12i\ln((-}{}$

input
```
int((a+b*arctan(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/x^4+b^2*c^4*(-1/4/c^4/x^4*arctan(c*x)^2+1/4*arctan(c*x)^2-1/6/c^3
/x^3*arctan(c*x)+1/2/c/x*arctan(c*x)+1/3*ln(c^2*x^2+1)-1/12/c^2/x^2-2/3*ln
(c*x))+2*a*b*c^4*(-1/4/c^4/x^4*arctan(c*x)+1/4*arctan(c*x)-1/12/c^3/x^3+1/
4/c/x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16

$$\int\frac{(a+b\arctan(cx))^2}{x^5}\,dx = \frac{4\,b^2c^4x^4\log\left(c^2x^2+1\right)-8\,b^2c^4x^4\log\left(x\right)+6\,abc^3x^3-b^2c^2x^2-2\,abcx+3\left(b^2c^4x^4-b^2\right)\arctan\left(cx\right)^2-3}{12\,x^4}$$

input
```
integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="fricas")
```

output
```
1/12*(4*b^2*c^4*x^4*log(c^2*x^2 + 1) - 8*b^2*c^4*x^4*log(x) + 6*a*b*c^3*x^
3 - b^2*c^2*x^2 - 2*a*b*c*x + 3*(b^2*c^4*x^4 - b^2)*arctan(c*x)^2 - 3*a^2
+ 2*(3*a*b*c^4*x^4 + 3*b^2*c^3*x^3 - b^2*c*x - 3*a*b)*arctan(c*x))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx$$
$$= \begin{cases} -\frac{a^2}{4x^4} + \frac{abc^4\operatorname{atan}(cx)}{2} + \frac{abc^3}{2x} - \frac{abc}{6x^3} - \frac{ab\operatorname{atan}(cx)}{2x^4} - \frac{2b^2c^4\log(x)}{3} + \frac{b^2c^4\log\left(x^2+\frac{1}{c^2}\right)}{3} + \frac{b^2c^4\operatorname{atan}^2(cx)}{4} + \frac{b^2c^3\operatorname{atan}(cx)}{2x} - \\ -\frac{a^2}{4x^4} \end{cases}$$

input
```
integrate((a+b*atan(c*x))**2/x**5,x)
```

output
```
Piecewise((-a**2/(4*x**4) + a*b*c**4*atan(c*x)/2 + a*b*c**3/(2*x) - a*b*c/
(6*x**3) - a*b*atan(c*x)/(2*x**4) - 2*b**2*c**4*log(x)/3 + b**2*c**4*log(x
**2 + c**(-2))/3 + b**2*c**4*atan(c*x)**2/4 + b**2*c**3*atan(c*x)/(2*x) -
b**2*c**2/(12*x**2) - b**2*c*atan(c*x)/(6*x**3) - b**2*atan(c*x)**2/(4*x**
4), Ne(c, 0)), (-a**2/(4*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx = \frac{1}{6}\left(\left(3\,c^3\arctan(cx) + \frac{3\,c^2x^2-1}{x^3}\right)c - \frac{3\arctan(cx)}{x^4}\right)ab$$
$$+ \frac{1}{12}\left(2\left(3\,c^3\arctan(cx) + \frac{3\,c^2x^2-1}{x^3}\right)c\arctan(cx) - \frac{\left(3\,c^2x^2\arctan(cx)^2 - 4\,c^2x^2\log(c^2x^2+1) + 8\right.}{x^2}\right.$$
$$- \frac{b^2\arctan(cx)^2}{4\,x^4} - \frac{a^2}{4\,x^4}$$

input
```
integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="maxima")
```

output
```
1/6*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*a*b
+ 1/12*(2*(3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c*arctan(c*x) - (3*c^2
*x^2*arctan(c*x)^2 - 4*c^2*x^2*log(c^2*x^2 + 1) + 8*c^2*x^2*log(x) + 1)*c^
2/x^2)*b^2 - 1/4*b^2*arctan(c*x)^2/x^4 - 1/4*a^2/x^4
```

Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx=\int \frac{(b\arctan(cx)+a)^2}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx=\frac{b^2\,c^4\,\mathrm{atan}(c\,x)^2}{4}-\frac{2\,b^2\,c^4\,\ln(x)}{3} -\frac{\frac{b^2\,\mathrm{atan}(c\,x)^2}{4}+\frac{a^2}{4}+x\left(\frac{c\,\mathrm{atan}(c\,x)\,b^2}{6}+\frac{a\,c\,b}{6}\right)-x^3\left(\frac{b^2\,c^3\,\mathrm{atan}(c\,x)}{2}+\frac{a\,b\,c^3}{2}\right)+\frac{b^2\,c^2\,x^2}{12}+\frac{a\,b\,\mathrm{atan}(c\,x)}{2}}{x^4} +\frac{b^2\,c^4\,\ln(c\,x+1\mathrm{i})}{3}+\frac{b^2\,c^4\,\ln(1+c\,x\,1\mathrm{i})}{3} +\frac{a\,b\,c^4\,\ln(c\,x+1\mathrm{i})\,1\mathrm{i}}{4}-\frac{a\,b\,c^4\,\ln(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{4}$$

input
```
int((a + b*atan(c*x))^2/x^5,x)
```

output
```
(b^2*c^4*atan(c*x)^2)/4 - (2*b^2*c^4*log(x))/3 - ((b^2*atan(c*x)^2)/4 + a^
2/4 + x*((b^2*c*atan(c*x))/6 + (a*b*c)/6) - x^3*((b^2*c^3*atan(c*x))/2 + (
a*b*c^3)/2) + (b^2*c^2*x^2)/12 + (a*b*atan(c*x))/2)/x^4 + (b^2*c^4*log(c*x
 + 1i))/3 + (b^2*c^4*log(c*x*1i + 1))/3 + (a*b*c^4*log(c*x + 1i)*1i)/4 - (
a*b*c^4*log(c*x*1i + 1)*1i)/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.28

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx$$
$$= \frac{3atan(cx)^2\,b^2c^4x^4 - 3atan(cx)^2\,b^2 + 6atan(cx)\,ab\,c^4x^4 - 6atan(cx)\,ab + 6atan(cx)\,b^2c^3x^3 - 2atan(cx)\,b}{12x^4}$$

input
```
int((a+b*atan(c*x))^2/x^5,x)
```

output
```
(3*atan(c*x)**2*b**2*c**4*x**4 - 3*atan(c*x)**2*b**2 + 6*atan(c*x)*a*b*c**
4*x**4 - 6*atan(c*x)*a*b + 6*atan(c*x)*b**2*c**3*x**3 - 2*atan(c*x)*b**2*c
*x + 4*log(c**2*x**2 + 1)*b**2*c**4*x**4 - 8*log(x)*b**2*c**4*x**4 - 3*a**
2 + 6*a*b*c**3*x**3 - 2*a*b*c*x - b**2*c**2*x**2)/(12*x**4)
```

3.24 $\int x^5(a+b\arctan(cx))^3\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 255

$$\begin{aligned}\int x^5(a+b\arctan(cx))^3\,dx=&\frac{19b^3x}{60c^5}-\frac{b^3x^3}{60c^3}-\frac{19b^3\arctan(cx)}{60c^6}\\&-\frac{4b^2x^2(a+b\arctan(cx))}{15c^4}+\frac{b^2x^4(a+b\arctan(cx))}{20c^2}\\&-\frac{23ib(a+b\arctan(cx))^2}{30c^6}-\frac{bx(a+b\arctan(cx))^2}{2c^5}\\&+\frac{bx^3(a+b\arctan(cx))^2}{6c^3}-\frac{bx^5(a+b\arctan(cx))^2}{10c}\\&+\frac{(a+b\arctan(cx))^3}{6c^6}+\frac{1}{6}x^6(a+b\arctan(cx))^3\\&-\frac{23b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{15c^6}\\&-\frac{23ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{30c^6}\end{aligned}$$

output

```
19/60*b^3*x/c^5-1/60*b^3*x^3/c^3-19/60*b^3*arctan(c*x)/c^6-4/15*b^2*x^2*(a
+b*arctan(c*x))/c^4+1/20*b^2*x^4*(a+b*arctan(c*x))/c^2-23/30*I*b*(a+b*arct
an(c*x))^2/c^6-1/2*b*x*(a+b*arctan(c*x))^2/c^5+1/6*b*x^3*(a+b*arctan(c*x))
^2/c^3-1/10*b*x^5*(a+b*arctan(c*x))^2/c+1/6*(a+b*arctan(c*x))^3/c^6+1/6*x^
6*(a+b*arctan(c*x))^3-23/15*b^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^6-23/3
0*I*b^3*polylog(2,1-2/(1+I*c*x))/c^6
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.14

$$\int x^5(a+b\arctan(cx))^3\,dx$$
$$=\frac{-19ab^2-30a^2bcx+19b^3cx-16ab^2c^2x^2+10a^2bc^3x^3-b^3c^3x^3+3ab^2c^4x^4-6a^2bc^5x^5+10a^3c^6x^6+2b^2}{}$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x])^3,x]
```

output
```
(-19*a*b^2 - 30*a^2*b*c*x + 19*b^3*c*x - 16*a*b^2*c^2*x^2 + 10*a^2*b*c^3*x
^3 - b^3*c^3*x^3 + 3*a*b^2*c^4*x^4 - 6*a^2*b*c^5*x^5 + 10*a^3*c^6*x^6 + 2*
b^2*(b*(23*I - 15*c*x + 5*c^3*x^3 - 3*c^5*x^5) + 15*a*(1 + c^6*x^6))*ArcTa
n[c*x]^2 + 10*b^3*(1 + c^6*x^6)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(b^2*(-19 -
16*c^2*x^2 + 3*c^4*x^4) - 4*a*b*c*x*(15 - 5*c^2*x^2 + 3*c^4*x^4) + 30*a^2*
(1 + c^6*x^6) - 92*b^2*Log[1 + E^((2*I)*ArcTan[c*x])]) + 46*a*b^2*Log[1 +
c^2*x^2] + (46*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(60*c^6)
```

Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 525 vs. 2(255) = 510.

Time = 3.18 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.06, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.214, Rules used = {5361, 5451, 5361, 5451, 5361, 254, 2009, 5451, 5345, 5361, 262, 216, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a+b\arctan(cx))^3\,dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\int\frac{x^6(a+b\arctan(cx))^2}{c^2x^2+1}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\int x^4(a+b\arctan(cx))^2dx}{c^2}-\frac{\int \frac{x^4(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\int \frac{x^5(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{\int \frac{x^4(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\int x^3(a+b\arctan(cx))dx}{c^2}-\frac{\int \frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{\int x^2(a+b\arctan(cx))^2dx}{c^2}-\int \frac{x^2(a+b}{c}}{c^2}\right.$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\int \frac{x^4}{c^2x^2+1}dx}{c^2}-\frac{\int \frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-}{}\right.$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\int \left(\frac{x^2}{c^2}+\frac{1}{c^4(c^2x^2+1)}-\frac{1}{c^4}\right)dx}{c^2}-\frac{\int \frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{1}{3}x^3(}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{1}{3}x^3(a+b\,\mathrm{arc}\ldots}{\ldots}\right.$$

$\downarrow$ 5451

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}d\ldots}{c^2}}{c^2}\ldots\right.}{c^2}\ldots\right.$$

$\downarrow$ 5345

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}d\ldots}{c^2}}{c^2}\ldots\right.}{c^2}\ldots\right.$$

$\downarrow$ 5361

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int\frac{x^2}{c^2x^2+1}dx}{c^2}-\frac{\int x\ldots}{\ldots}}{c^2}\ldots\right.}{c^2}\ldots\right.$$

$\downarrow$ 262

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-$$
$$\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int\frac{1}{c^2x^2+1}dx}{c^2}\right.}{c^2}}{c^2}\right.}{c^2}\right.$$

$\Big\downarrow$ 216

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-$$
$$\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}}{c^2}\right.}{c^2}\right.$$

$\Big\downarrow$ 5419

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-$$
$$\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}}{c^2}\right.}{c^2}\right.$$

$\Big\downarrow$ 5455

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-$$
$$\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}}{c}\right.}{c^2}\right.$$

$\Big\downarrow$ 5379

$$\frac{1}{6}x^6(a+b\arctan(cx))^3 - \frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2 - \frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2} - \frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}\right.}{c^2}\right.$$

$$\Big\downarrow \ 2849$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3 - \frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2 - \frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2} - \frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}\right.}{c^2}\right.$$

$$\Big\downarrow \ 2752$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3 - \frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2 - \frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2} - \frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}\right.}{c^2}\right.$$

input
```
Int[x^5*(a + b*ArcTan[c*x])^3,x]
```

output
```
(x^6*(a + b*ArcTan[c*x])^3)/6 - (b*c*(((x^5*(a + b*ArcTan[c*x])^2)/5 - (2*
b*c*(((x^4*(a + b*ArcTan[c*x]))/4 - (b*c*(-(x/c^4) + x^3/(3*c^2) + ArcTan[
c*x]/c^5))/4)/c^2 - (((x^2*(a + b*ArcTan[c*x]))/2 - (b*c*(x/c^2 - ArcTan[c
*x]/c^3))/2)/c^2 - (((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b*c^2) - (((a + b*Ar
cTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])
/c)/c)/c^2)/c^2))/5)/c^2 - (((x^3*(a + b*ArcTan[c*x])^2)/3 - (2*b*c*(((x^2
*(a + b*ArcTan[c*x]))/2 - (b*c*(x/c^2 - ArcTan[c*x]/c^3))/2)/c^2 - (((-1/2
*I)*(a + b*ArcTan[c*x])^2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*
x)])/c + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/c)/c)/c^2))/3)/c^2 - (-1/
3*(a + b*ArcTan[c*x])^3/(b*c^3) + (x*(a + b*ArcTan[c*x])^2 - 2*b*c*(((-1/2
*I)*(a + b*ArcTan[c*x])^2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*
x)])/c + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/c)/c))/c^2)/c^2)/c^2))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.58

method	result
derivativedivides	$\frac{\frac{a^3c^6x^6}{6}+b^3\left(\frac{c^6x^6\arctan(cx)^3}{6}-\frac{c^5x^5\arctan(cx)^2}{10}+\frac{c^3x^3\arctan(cx)^2}{6}-\frac{cx\arctan(cx)^2}{2}+\frac{\arctan(cx)^3}{6}+\frac{c^4x^4\arctan(cx)}{20}-\frac{4c^2x^2}{}\right.}{}$
default	$\frac{\frac{a^3c^6x^6}{6}+b^3\left(\frac{c^6x^6\arctan(cx)^3}{6}-\frac{c^5x^5\arctan(cx)^2}{10}+\frac{c^3x^3\arctan(cx)^2}{6}-\frac{cx\arctan(cx)^2}{2}+\frac{\arctan(cx)^3}{6}+\frac{c^4x^4\arctan(cx)}{20}-\frac{4c^2x^2}{}\right.}{}$
parts	$\frac{x^6a^3}{6}+\frac{b^3\left(\frac{c^6x^6\arctan(cx)^3}{6}-\frac{c^5x^5\arctan(cx)^2}{10}+\frac{c^3x^3\arctan(cx)^2}{6}-\frac{cx\arctan(cx)^2}{2}+\frac{\arctan(cx)^3}{6}+\frac{c^4x^4\arctan(cx)}{20}-\frac{4c^2x^2}{}\right.}{}$
risch	Expression too large to display

input
```
int(x^5*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^6*(1/6*a^3*c^6*x^6+b^3*(1/6*c^6*x^6*arctan(c*x)^3-1/10*c^5*x^5*arctan(
c*x)^2+1/6*c^3*x^3*arctan(c*x)^2-1/2*c*x*arctan(c*x)^2+1/6*arctan(c*x)^3+1
/20*c^4*x^4*arctan(c*x)-4/15*c^2*x^2*arctan(c*x)+23/30*arctan(c*x)*ln(c^2*
x^2+1)-1/60*c^3*x^3+19/60*c*x-19/60*arctan(c*x)+23/60*I*(ln(c*x-I)*ln(c^2*
x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))
-23/60*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(
c*x+I)*ln(1/2*I*(c*x-I))))+3*a*b^2*(1/6*c^6*x^6*arctan(c*x)^2-1/15*c^5*x^5
*arctan(c*x)+1/9*c^3*x^3*arctan(c*x)-1/3*c*x*arctan(c*x)+1/6*arctan(c*x)^2
+1/60*c^4*x^4-4/45*c^2*x^2+23/90*ln(c^2*x^2+1))+3*a^2*b*(1/6*c^6*x^6*arcta
n(c*x)-1/30*c^5*x^5+1/18*c^3*x^3-1/6*c*x+1/6*arctan(c*x)))
```

Fricas [F]

$$\int x^5(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x^5\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^5*arctan(c*x)^3 + 3*a*b^2*x^5*arctan(c*x)^2 + 3*a^2*b*x^5*a
rctan(c*x) + a^3*x^5, x)
```

Sympy [F]

$$\int x^5(a+b\arctan(cx))^3\,dx=\int x^5(a+b\,\mathrm{atan}\,(cx))^3\,dx$$

input
```
integrate(x**5*(a+b*atan(c*x))**3,x)
```

output
```
Integral(x**5*(a + b*atan(c*x))**3, x)
```

Maxima [F]

$$\int x^5(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x^5\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
1/2*a*b^2*x^6*arctan(c*x)^2 + 1/6*a^3*x^6 + 1/30*(15*x^6*arctan(c*x) - c*(
(3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*a^2*b - 1/60*(4*
c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7)*arctan(c*x) -
(3*c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(c^2*x^2 + 1))/c^6)*a*b
^2 + 1/480*(20*(5760*c^7*integrate(1/480*x^7*arctan(c*x)^3/(c^7*x^2 + c^5)
, x) - 1440*c^6*integrate(1/480*x^6*arctan(c*x)^2/(c^7*x^2 + c^5), x) - 36
0*c^6*integrate(1/480*x^6*log(c^2*x^2 + 1)^2/(c^7*x^2 + c^5), x) - 288*c^6
*integrate(1/480*x^6*log(c^2*x^2 + 1)/(c^7*x^2 + c^5), x) + 5760*c^5*integ
rate(1/480*x^5*arctan(c*x)^3/(c^7*x^2 + c^5), x) + 576*c^5*integrate(1/480
*x^5*arctan(c*x)/(c^7*x^2 + c^5), x) + 480*c^4*integrate(1/480*x^4*log(c^2
*x^2 + 1)/(c^7*x^2 + c^5), x) - 960*c^3*integrate(1/480*x^3*arctan(c*x)/(c
^7*x^2 + c^5), x) - 1440*c^2*integrate(1/480*x^2*log(c^2*x^2 + 1)/(c^7*x^2
 + c^5), x) + 2880*c*integrate(1/480*x*arctan(c*x)/(c^7*x^2 + c^5), x) - a
rctan(c*x)^3/c^6 - 360*integrate(1/480*log(c^2*x^2 + 1)^2/(c^7*x^2 + c^5),
 x))*c^6 + 40*(c^6*x^6 + 1)*arctan(c*x)^3 - 4*(3*c^5*x^5 - 5*c^3*x^3 + 15*
c*x)*arctan(c*x)^2 + (3*c^5*x^5 - 5*c^3*x^3 + 15*c*x)*log(c^2*x^2 + 1)^2)*
b^3/c^6
```

Giac [F]

$$\int x^5(a+b\arctan(cx))^3\,dx=\int (b\arctan{(cx)}+a)^3x^5\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5(a+b\arctan(cx))^3\,dx=\int x^5\,{\left(a+b\,\mathrm{atan}(c\,x)\right)}^3\,dx$$

input
```
int(x^5*(a + b*atan(c*x))^3,x)
```

output
```
int(x^5*(a + b*atan(c*x))^3, x)
```

Reduce [F]

$$\int x^5(a+b\arctan(cx))^3\,dx$$
$$=\frac{92\left(\int \frac{atan(cx)x}{c^2x^2+1}dx\right)b^3c^2+10atan(cx)^2b^3c^3x^3-30atan(cx)^2b^3cx+3atan(cx)b^3c^4x^4-16atan(cx)b^3c^2x^2}{}$$

input
```
int(x^5*(a+b*atan(c*x))^3,x)
```

output
```
(10*atan(c*x)**3*b**3*c**6*x**6 + 10*atan(c*x)**3*b**3 + 30*atan(c*x)**2*a
*b**2*c**6*x**6 + 30*atan(c*x)**2*a*b**2 - 6*atan(c*x)**2*b**3*c**5*x**5 +
 10*atan(c*x)**2*b**3*c**3*x**3 - 30*atan(c*x)**2*b**3*c*x + 30*atan(c*x)*
a**2*b*c**6*x**6 + 30*atan(c*x)*a**2*b - 12*atan(c*x)*a*b**2*c**5*x**5 + 2
0*atan(c*x)*a*b**2*c**3*x**3 - 60*atan(c*x)*a*b**2*c*x + 3*atan(c*x)*b**3*
c**4*x**4 - 16*atan(c*x)*b**3*c**2*x**2 - 19*atan(c*x)*b**3 + 92*int((atan
(c*x)*x)/(c**2*x**2 + 1),x)*b**3*c**2 + 46*log(c**2*x**2 + 1)*a*b**2 + 10*
a**3*c**6*x**6 - 6*a**2*b*c**5*x**5 + 10*a**2*b*c**3*x**3 - 30*a**2*b*c*x
+ 3*a*b**2*c**4*x**4 - 16*a*b**2*c**2*x**2 - b**3*c**3*x**3 + 19*b**3*c*x)
/(60*c**6)
```

3.25 $\int x^4(a+b\arctan(cx))^3\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 271

$$\int x^4(a+b\arctan(cx))^3\,dx = -\frac{9ab^2x}{10c^4}-\frac{b^3x^2}{20c^3}-\frac{9b^3x\arctan(cx)}{10c^4} + \frac{b^2x^3(a+b\arctan(cx))}{10c^2}+\frac{9b(a+b\arctan(cx))^2}{20c^5} + \frac{3bx^2(a+b\arctan(cx))^2}{10c^3}-\frac{3bx^4(a+b\arctan(cx))^2}{20c} + \frac{i(a+b\arctan(cx))^3}{5c^5}+\frac{1}{5}x^5(a+b\arctan(cx))^3 + \frac{3b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{5c^5}+\frac{b^3\log(1+c^2x^2)}{2c^5} + \frac{3ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5} + \frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{10c^5}$$

output
```
-9/10*a*b^2*x/c^4-1/20*b^3*x^2/c^3-9/10*b^3*x*arctan(c*x)/c^4+1/10*b^2*x^3
*(a+b*arctan(c*x))/c^2+9/20*b*(a+b*arctan(c*x))^2/c^5+3/10*b*x^2*(a+b*arct
an(c*x))^2/c^3-3/20*b*x^4*(a+b*arctan(c*x))^2/c+1/5*I*(a+b*arctan(c*x))^3/
c^5+1/5*x^5*(a+b*arctan(c*x))^3+3/5*b*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/
c^5+1/2*b^3*ln(c^2*x^2+1)/c^5+3/5*I*b^2*(a+b*arctan(c*x))*polylog(2,1-2/(1
+I*c*x))/c^5+3/10*b^3*polylog(3,1-2/(1+I*c*x))/c^5
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.46

$$\int x^4(a+b\arctan(cx))^3\,dx$$
$$= \frac{-b^3-18ab^2cx+6a^2bc^2x^2-b^3c^2x^2+2ab^2c^3x^3-3a^2bc^4x^4+4a^3c^5x^5+18ab^2\arctan(cx)-18b^3cx\,\text{arcta}}{}$$

input
```
Integrate[x^4*(a + b*ArcTan[c*x])^3,x]
```

output
```
(-b^3 - 18*a*b^2*c*x + 6*a^2*b*c^2*x^2 - b^3*c^2*x^2 + 2*a*b^2*c^3*x^3 - 3
*a^2*b*c^4*x^4 + 4*a^3*c^5*x^5 + 18*a*b^2*ArcTan[c*x] - 18*b^3*c*x*ArcTan[
c*x] + 12*a*b^2*c^2*x^2*ArcTan[c*x] + 2*b^3*c^3*x^3*ArcTan[c*x] - 6*a*b^2*
c^4*x^4*ArcTan[c*x] + 12*a^2*b*c^5*x^5*ArcTan[c*x] - (12*I)*a*b^2*ArcTan[c
*x]^2 + 9*b^3*ArcTan[c*x]^2 + 6*b^3*c^2*x^2*ArcTan[c*x]^2 - 3*b^3*c^4*x^4*
ArcTan[c*x]^2 + 12*a*b^2*c^5*x^5*ArcTan[c*x]^2 - (4*I)*b^3*ArcTan[c*x]^3 +
 4*b^3*c^5*x^5*ArcTan[c*x]^3 + 24*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTa
n[c*x])] + 12*b^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 6*a^2*b*L
og[1 + c^2*x^2] + 10*b^3*Log[1 + c^2*x^2] - (12*I)*b^2*(a + b*ArcTan[c*x])
*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 6*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*
x])])/(20*c^5)
```

Rubi [A] (verified)

Time = 2.93 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.37, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {5361, 5451, 5361, 5451, 5361, 243, 49, 2009, 5451, 2009, 5419, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+b\arctan(cx))^3\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\int\frac{x^5(a+b\arctan(cx))^2}{c^2x^2+1}dx$$

$\downarrow$ 5451

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\int x^3(a+b\arctan(cx))^2dx}{c^2}-\frac{\int\frac{x^3(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 5361

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\int\frac{x^4(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x^3(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 5451

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\int x^2(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{\int x(a+b\arctan(cx))^2dx}{c^2}-\frac{\int\frac{x(a+b\,\mathrm{ar}}{c^2}}{c^2}$$

$\downarrow$ 5361

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{3}bc\int\frac{x^3}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2}{}$$

$\downarrow$ 243

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int\frac{x^2}{c^2x^2+1}dx^2}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2}{}$$

$\downarrow$ 49

$$\frac{1}{5}x^5(a+b\arctan(cx))^3 - \frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int\left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^2+1)}\right)dx^2}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{1}{2}x^2(a+b\ldots}{\ldots}\right.$$

$\downarrow$ 2009

$$\frac{1}{5}x^5(a+b\arctan(cx))^3 - \frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{1}{2}x^2(a+b\,\mathrm{arct}\ldots}{\ldots}\right.$$

$\downarrow$ 5451

$$\frac{1}{5}x^5(a+b\arctan(cx))^3 - \frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\frac{\frac{\int(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}}{c^2}\right)}{c^2}\ldots\right.$$

$\downarrow$ 2009

$$\frac{1}{5}x^5(a+b\arctan(cx))^3 - \frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{\int\frac{a+b\,\mathrm{arct}\ldots}{c^2x^2\ldots}}{c^2}}{c^2}\ldots\right.\right.$$

$\downarrow$ 5419

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan}{2bc^3}}{c^2}\right.}{c^2}\right.$$

$\downarrow$ 5455

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan}{2bc^3}}{c^2}\right.}{c^2}\right.$$

$\downarrow$ 5379

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan}{2bc^3}}{c^2}\right.}{c^2}\right.$$

$\downarrow$ 5529

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan}{2bc^3}}{c^2}\right.}{c^2}\right.$$

$\downarrow$ 7164

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan}{2bc^3}}{c^2}}{c^2}$$

input
```
Int[x^4*(a + b*ArcTan[c*x])^3,x]
```

output
```
(x^5*(a + b*ArcTan[c*x])^3)/5 - (3*b*c*(((x^4*(a + b*ArcTan[c*x])^2)/4 - (
b*c*(((x^3*(a + b*ArcTan[c*x]))/3 - (b*c*(x^2/c^2 - Log[1 + c^2*x^2]/c^4))
/6)/c^2 - (-1/2*(a + b*ArcTan[c*x])^2/(b*c^3) + (a*x + b*x*ArcTan[c*x] - (
b*Log[1 + c^2*x^2])/(2*c))/c^2)/c^2))/2)/c^2 - (((x^2*(a + b*ArcTan[c*x])^
2)/2 - b*c*(-1/2*(a + b*ArcTan[c*x])^2/(b*c^3) + (a*x + b*x*ArcTan[c*x] -
(b*Log[1 + c^2*x^2])/(2*c))/c^2))/c^2 - (((-1/3*I)*(a + b*ArcTan[c*x])^3)/
(b*c^2) - (((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/c - 2*b*(((-1/2*I)*(
a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/
(1 + I*c*x)])/(4*c)))/c)/c^2)/c^2))/5
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.66 (sec) , antiderivative size = 1261, normalized size of antiderivative = 4.65

method	result	size
derivativedivides	Expression too large to display	1261
default	Expression too large to display	1261
parts	Expression too large to display	1319

input
```
int(x^4*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(-3/20*I*b^3*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*
x)^2/(c^2*x^2+1))*arctan(c*x)^2-3/10*I*b^3*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x
^2+1)))*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2+3/20*I*b^3*P
i*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))
^2)*arctan(c*x)^2+3/20*I*b^3*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn
(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2+
3/10*I*b^3*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*
x^2+1))^2*arctan(c*x)^2+3/20*I*b^3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn
(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2-
1/20*b^3+1/5*c^5*x^5*a^3+3*a^2*b*(1/5*c^5*x^5*arctan(c*x)-1/20*c^4*x^4+1/1
0*c^2*x^2-1/10*ln(c^2*x^2+1))-3/10*b^3*arctan(c*x)^2*ln(c^2*x^2+1)+I*b^3*a
rctan(c*x)+3/5*b^3*arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+3/5*b^3*l
n(2)*arctan(c*x)^2-1/5*I*b^3*arctan(c*x)^3+3*a*b^2*(1/5*c^5*x^5*arctan(c*x
)^2-1/10*c^4*x^4*arctan(c*x)+1/5*c^2*x^2*arctan(c*x)-1/5*arctan(c*x)*ln(c^
2*x^2+1)+1/30*c^3*x^3-3/10*c*x+3/10*arctan(c*x)-1/10*I*(ln(c*x-I)*ln(c^2*x
^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))+
1/10*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*
x+I)*ln(1/2*I*(c*x-I))))-1/20*b^3*c^2*x^2-3/20*I*b^3*Pi*csgn(I/(1+(1+I*c*x
)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^
2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2+3/10*b^3*polylog(...
```

Fricas [F]

$$\int x^4(a+b\arctan(cx))^3\,dx=\int\left(b\arctan\left(cx\right)+a\right)^3x^4\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^4*arctan(c*x)^3 + 3*a*b^2*x^4*arctan(c*x)^2 + 3*a^2*b*x^4*a
rctan(c*x) + a^3*x^4, x)
```

Sympy [F]

$$\int x^4(a+b\arctan(cx))^3\,dx=\int x^4(a+b\,\mathrm{atan}\,(cx))^3\,dx$$

input
```
integrate(x**4*(a+b*atan(c*x))**3,x)
```

output
```
Integral(x**4*(a + b*atan(c*x))**3, x)
```

Maxima [F]

$$\int x^4(a+b\arctan(cx))^3\,dx=\int\left(b\arctan\left(cx\right)+a\right)^3x^4\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
1/40*b^3*x^5*arctan(c*x)^3 - 3/160*b^3*x^5*arctan(c*x)*log(c^2*x^2 + 1)^2
+ 1/5*a^3*x^5 + 3/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log
(c^2*x^2 + 1)/c^6))*a^2*b + integrate(1/160*(12*b^3*c^2*x^6*arctan(c*x)*lo
g(c^2*x^2 + 1) + 140*(b^3*c^2*x^6 + b^3*x^4)*arctan(c*x)^3 + 12*(40*a*b^2*
c^2*x^6 - b^3*c*x^5 + 40*a*b^2*x^4)*arctan(c*x)^2 + 3*(b^3*c*x^5 + 5*(b^3*
c^2*x^6 + b^3*x^4)*arctan(c*x))*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x)
```

Giac [F]

$$\int x^4(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x^4\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a+b\arctan(cx))^3\,dx=\int x^4\,(a+b\,\mathrm{atan}(c\,x))^3\,dx$$

input
```
int(x^4*(a + b*atan(c*x))^3,x)
```

output
```
int(x^4*(a + b*atan(c*x))^3, x)
```

Reduce [F]

$$\int x^4(a+b\arctan(cx))^3\,dx$$
$$=\frac{4atan(cx)^3\,b^3c^5x^5-4atan(cx)^3\,b^3cx+12atan(cx)^2\,a\,b^2c^5x^5-12atan(cx)^2\,a\,b^2cx-3atan(cx)^2\,b^3c^4x^4+}{}$$

input
```
int(x^4*(a+b*atan(c*x))^3,x)
```

output

```
(4*atan(c*x)**3*b**3*c**5*x**5 - 4*atan(c*x)**3*b**3*c*x + 12*atan(c*x)**2
*a*b**2*c**5*x**5 - 12*atan(c*x)**2*a*b**2*c*x - 3*atan(c*x)**2*b**3*c**4*
x**4 + 6*atan(c*x)**2*b**3*c**2*x**2 + 9*atan(c*x)**2*b**3 + 12*atan(c*x)*
a**2*b*c**5*x**5 - 6*atan(c*x)*a*b**2*c**4*x**4 + 12*atan(c*x)*a*b**2*c**2
*x**2 + 18*atan(c*x)*a*b**2 + 2*atan(c*x)*b**3*c**3*x**3 - 18*atan(c*x)*b*
*3*c*x + 4*int(atan(c*x)**3,x)*b**3*c + 12*int(atan(c*x)**2,x)*a*b**2*c -
6*log(c**2*x**2 + 1)*a**2*b + 10*log(c**2*x**2 + 1)*b**3 + 4*a**3*c**5*x**
5 - 3*a**2*b*c**4*x**4 + 6*a**2*b*c**2*x**2 + 2*a*b**2*c**3*x**3 - 18*a*b*
*2*c*x - b**3*c**2*x**2)/(20*c**5)
```

3.26 $\int x^3(a+b\arctan(cx))^3\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 194

$$\begin{aligned}\int x^3(a+b\arctan(cx))^3\,dx = &-\frac{b^3x}{4c^3}+\frac{b^3\arctan(cx)}{4c^4}+\frac{b^2x^2(a+b\arctan(cx))}{4c^2}\\&+\frac{ib(a+b\arctan(cx))^2}{c^4}+\frac{3bx(a+b\arctan(cx))^2}{4c^3}\\&-\frac{bx^3(a+b\arctan(cx))^2}{4c}-\frac{(a+b\arctan(cx))^3}{4c^4}\\&+\frac{1}{4}x^4(a+b\arctan(cx))^3\\&+\frac{2b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4}\\&+\frac{ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4}\end{aligned}$$

output
```
-1/4*b^3*x/c^3+1/4*b^3*arctan(c*x)/c^4+1/4*b^2*x^2*(a+b*arctan(c*x))/c^2+I
*b*(a+b*arctan(c*x))^2/c^4+3/4*b*x*(a+b*arctan(c*x))^2/c^3-1/4*b*x^3*(a+b*
arctan(c*x))^2/c-1/4*(a+b*arctan(c*x))^3/c^4+1/4*x^4*(a+b*arctan(c*x))^3+2
*b^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4+I*b^3*polylog(2,1-2/(1+I*c*x))/
c^4
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.16

$$\int x^3(a+b\arctan(cx))^3\,dx$$
$$= \frac{ab^2+3a^2bcx-b^3cx+ab^2c^2x^2-a^2bc^3x^3+a^3c^4x^4-b^2(b(4i-3cx+c^3x^3)+a(3-3c^4x^4))\arctan(cx)^2}{}$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x])^3,x]
```

output
```
(a*b^2 + 3*a^2*b*c*x - b^3*c*x + a*b^2*c^2*x^2 - a^2*b*c^3*x^3 + a^3*c^4*x
^4 - b^2*(b*(4*I - 3*c*x + c^3*x^3) + a*(3 - 3*c^4*x^4))*ArcTan[c*x]^2 + b
^3*(-1 + c^4*x^4)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(-2*a*b*c*x*(-3 + c^2*x^2)
 + b^2*(1 + c^2*x^2) + 3*a^2*(-1 + c^4*x^4) + 8*b^2*Log[1 + E^((2*I)*ArcTa
n[c*x])]) - 4*a*b^2*Log[1 + c^2*x^2] - (4*I)*b^3*PolyLog[2, -E^((2*I)*ArcT
an[c*x])])/(4*c^4)
```

Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.58, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {5361, 5451, 5361, 5451, 5345, 5361, 262, 216, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\arctan(cx))^3\,dx$$
$$\Big\downarrow \text{5361}$$
$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\int\frac{x^4(a+b\arctan(cx))^2}{c^2x^2+1}dx$$
$$\Big\downarrow \text{5451}$$
$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\int x^2(a+b\arctan(cx))^2dx}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \; 5361$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \; 5451$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{\int(a+b\arctan(cx))^2dx}{c^2}-\frac{\int\frac{(a+b\arctan(c}{c^2x^2+1}}{c^2}}{c^2}\right.$$

$$\Big\downarrow \; 5345$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(c}{c^2x^2+1}}{c^2}}{c^2}\right.$$

$$\Big\downarrow \; 5361$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int\frac{x^2}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{x(a+b\arctan(cx))^2-2bc}{c^2}\right.$$

$$\Big\downarrow \; 262$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int\frac{1}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{x(a+b\arctan(c}{}\right.$$

$$\Big\downarrow \; 216$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^3 - \frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2} - \frac{x(a+b\arctan(cx))^2}{}\right.$$

$\downarrow$ 5419

$$\frac{1}{4}x^4(a+b\arctan(cx))^3 - \frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2} - \frac{x(a+b\arctan(cx))^2}{}\right.$$

$\downarrow$ 5455

$$\frac{1}{4}x^4(a+b\arctan(cx))^3 - \frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}}{c^2}\right)}{c^2} - \right.$$

$\downarrow$ 5379

$$\frac{1}{4}x^4(a+b\arctan(cx))^3 - \frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}}{c^2}\right.}{c^2}\right.$$

$\downarrow$ 2849

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan}{c}}{c}}{c^2}\right.}{c^2}\right.$$

$\downarrow$ 2752

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}}{c^2}\right.}{c^2}\right.$$

input
```
Int[x^3*(a + b*ArcTan[c*x])^3,x]
```

output
```
(x^4*(a + b*ArcTan[c*x])^3)/4 - (3*b*c*(((x^3*(a + b*ArcTan[c*x])^2)/3 - (
2*b*c*(((x^2*(a + b*ArcTan[c*x]))/2 - (b*c*(x/c^2 - ArcTan[c*x]/c^3))/2)/c
^2 - (((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log
[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/c)/c)/c^2))/3
)/c^2 - (-1/3*(a + b*ArcTan[c*x])^3/(b*c^3) + (x*(a + b*ArcTan[c*x])^2 - 2
*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log
[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/c)/c))/c^2)/c
^2))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.75

method	result
derivativedivides	$\frac{\frac{c^4x^4a^3}{4}+b^3\left(\frac{c^4x^4\arctan(cx)^3}{4}-\frac{c^3x^3\arctan(cx)^2}{4}+\frac{3cx\arctan(cx)^2}{4}-\frac{\arctan(cx)^3}{4}+\frac{c^2x^2\arctan(cx)}{4}-\arctan(cx)\ln(c^2x^2+1\right.}{}$
default	$\frac{\frac{c^4x^4a^3}{4}+b^3\left(\frac{c^4x^4\arctan(cx)^3}{4}-\frac{c^3x^3\arctan(cx)^2}{4}+\frac{3cx\arctan(cx)^2}{4}-\frac{\arctan(cx)^3}{4}+\frac{c^2x^2\arctan(cx)}{4}-\arctan(cx)\ln(c^2x^2+1\right.}{}$
parts	$\frac{a^3x^4}{4}+\frac{b^3\left(\frac{c^4x^4\arctan(cx)^3}{4}-\frac{c^3x^3\arctan(cx)^2}{4}+\frac{3cx\arctan(cx)^2}{4}-\frac{\arctan(cx)^3}{4}+\frac{c^2x^2\arctan(cx)}{4}-\arctan(cx)\ln(c^2x^2+1\right.}{}$
risch	Expression too large to display

input
```
int(x^3*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/4*c^4*x^4*a^3+b^3*(1/4*c^4*x^4*arctan(c*x)^3-1/4*c^3*x^3*arctan(c
*x)^2+3/4*c*x*arctan(c*x)^2-1/4*arctan(c*x)^3+1/4*c^2*x^2*arctan(c*x)-arct
an(c*x)*ln(c^2*x^2+1)-1/4*c*x+1/4*arctan(c*x)-1/2*I*(ln(c*x-I)*ln(c^2*x^2+
1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))+1/2
*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)
*ln(1/2*I*(c*x-I))))+3*a*b^2*(1/4*c^4*x^4*arctan(c*x)^2-1/6*c^3*x^3*arctan
(c*x)+1/2*c*x*arctan(c*x)-1/4*arctan(c*x)^2+1/12*c^2*x^2-1/3*ln(c^2*x^2+1)
)+3*a^2*b*(1/4*c^4*x^4*arctan(c*x)-1/12*c^3*x^3+1/4*c*x-1/4*arctan(c*x)))
```

Fricas [F]

$$\int x^3(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*arctan(c*x)^3 + 3*a*b^2*x^3*arctan(c*x)^2 + 3*a^2*b*x^3*a
rctan(c*x) + a^3*x^3, x)
```

Sympy [F]

$$\int x^3(a+b\arctan(cx))^3\,dx=\int x^3(a+b\operatorname{atan}(cx))^3\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))**3,x)
```

output
```
Integral(x**3*(a + b*atan(c*x))**3, x)
```

Maxima [F]

$$\int x^3(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
3/4*a*b^2*x^4*arctan(c*x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arctan(c*x) - c*((c
^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a^2*b - 1/4*(2*c*((c^2*x^3 - 3*x)/
c^4 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(
c^2*x^2 + 1))/c^4)*a*b^2 + 1/64*(4*(512*c^5*integrate(1/64*x^5*arctan(c*x)
^3/(c^5*x^2 + c^3), x) - 192*c^4*integrate(1/64*x^4*arctan(c*x)^2/(c^5*x^2
 + c^3), x) - 48*c^4*integrate(1/64*x^4*log(c^2*x^2 + 1)^2/(c^5*x^2 + c^3)
, x) - 64*c^4*integrate(1/64*x^4*log(c^2*x^2 + 1)/(c^5*x^2 + c^3), x) + 51
2*c^3*integrate(1/64*x^3*arctan(c*x)^3/(c^5*x^2 + c^3), x) + 128*c^3*integ
rate(1/64*x^3*arctan(c*x)/(c^5*x^2 + c^3), x) + 192*c^2*integrate(1/64*x^2
*log(c^2*x^2 + 1)/(c^5*x^2 + c^3), x) - 384*c*integrate(1/64*x*arctan(c*x)
/(c^5*x^2 + c^3), x) + arctan(c*x)^3/c^4 + 48*integrate(1/64*log(c^2*x^2 +
 1)^2/(c^5*x^2 + c^3), x))*c^4 + 8*(c^4*x^4 - 1)*arctan(c*x)^3 - 4*(c^3*x^
3 - 3*c*x)*arctan(c*x)^2 + (c^3*x^3 - 3*c*x)*log(c^2*x^2 + 1)^2)*b^3/c^4
```

Giac [F]

$$\int x^3(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\arctan(cx))^3\,dx=\int x^3\,(a+b\,\mathrm{atan}(c\,x))^3\,dx$$

input
```
int(x^3*(a + b*atan(c*x))^3,x)
```

output
```
int(x^3*(a + b*atan(c*x))^3, x)
```

Reduce [F]

$$\int x^3(a+b\arctan(cx))^3\,dx$$
$$=\frac{atan(cx)^3\,b^3c^4x^4-atan(cx)^3\,b^3+3atan(cx)^2\,a\,b^2c^4x^4-3atan(cx)^2\,a\,b^2-atan(cx)^2\,b^3c^3x^3+3atan(cx)}{}$$

input
```
int(x^3*(a+b*atan(c*x))^3,x)
```

output
```
(atan(c*x)**3*b**3*c**4*x**4 - atan(c*x)**3*b**3 + 3*atan(c*x)**2*a*b**2*c
**4*x**4 - 3*atan(c*x)**2*a*b**2 - atan(c*x)**2*b**3*c**3*x**3 + 3*atan(c*
x)**2*b**3*c*x + 3*atan(c*x)*a**2*b*c**4*x**4 - 3*atan(c*x)*a**2*b - 2*ata
n(c*x)*a*b**2*c**3*x**3 + 6*atan(c*x)*a*b**2*c*x + atan(c*x)*b**3*c**2*x**
2 + atan(c*x)*b**3 - 8*int((atan(c*x)*x)/(c**2*x**2 + 1),x)*b**3*c**2 - 4*
log(c**2*x**2 + 1)*a*b**2 + a**3*c**4*x**4 - a**2*b*c**3*x**3 + 3*a**2*b*c
*x + a*b**2*c**2*x**2 - b**3*c*x)/(4*c**4)
```

3.27 $\int x^2(a+b\arctan(cx))^3\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 206

$$\int x^2(a+b\arctan(cx))^3\,dx=\frac{ab^2x}{c^2}+\frac{b^3x\arctan(cx)}{c^2}-\frac{b(a+b\arctan(cx))^2}{2c^3}-\frac{bx^2(a+b\arctan(cx))^2}{2c}-\frac{i(a+b\arctan(cx))^3}{3c^3}+\frac{1}{3}x^3(a+b\arctan(cx))^3-\frac{b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3}-\frac{b^3\log(1+c^2x^2)}{2c^3}-\frac{ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3}-\frac{b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3}$$

output
```
a*b^2*x/c^2+b^3*x*arctan(c*x)/c^2-1/2*b*(a+b*arctan(c*x))^2/c^3-1/2*b*x^2*
(a+b*arctan(c*x))^2/c-1/3*I*(a+b*arctan(c*x))^3/c^3+1/3*x^3*(a+b*arctan(c*
x))^3-b*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^3-1/2*b^3*ln(c^2*x^2+1)/c^3-
I*b^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^3-1/2*b^3*polylog(3,1-2
/(1+I*c*x))/c^3
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.31

$$\int x^2(a+b\arctan(cx))^3\,dx$$
$$=\frac{-3a^2bc^2x^2+2a^3c^3x^3+6a^2bc^3x^3\arctan(cx)+3a^2b\log(1+c^2x^2)+6ab^2\left(cx+(i+c^3x^3)\arctan(cx)^2-\right.}{}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x])^3,x]
```

output
```
(-3*a^2*b*c^2*x^2 + 2*a^3*c^3*x^3 + 6*a^2*b*c^3*x^3*ArcTan[c*x] + 3*a^2*b*
Log[1 + c^2*x^2] + 6*a*b^2*(c*x + (I + c^3*x^3)*ArcTan[c*x]^2 - ArcTan[c*x
]*(1 + c^2*x^2 + 2*Log[1 + E^((2*I)*ArcTan[c*x])]) + I*PolyLog[2, -E^((2*I
)*ArcTan[c*x])]) + b^3*(6*c*x*ArcTan[c*x] - 3*ArcTan[c*x]^2 - 3*c^2*x^2*Ar
cTan[c*x]^2 + (2*I)*ArcTan[c*x]^3 + 2*c^3*x^3*ArcTan[c*x]^3 - 6*ArcTan[c*x
]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 3*Log[1 + c^2*x^2] + (6*I)*ArcTan[c*x
]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - 3*PolyLog[3, -E^((2*I)*ArcTan[c*x])
]))/(6*c^3)
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.714$, Rules used = {5361, 5451, 5361, 5451, 2009, 5419, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\arctan(cx))^3\,dx$$
$$\Big\downarrow \text{5361}$$
$$\frac{1}{3}x^3(a+b\arctan(cx))^3-bc\int\frac{x^3(a+b\arctan(cx))^2}{c^2x^2+1}dx$$
$$\Big\downarrow \text{5451}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^3-bc\left(\frac{\int x(a+b\arctan(cx))^2dx}{c^2}-\frac{\int \frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 5361

$$\frac{1}{3}x^3(a+b\arctan(cx))^3-bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{\int \frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 5451

$$\frac{1}{3}x^3(a+b\arctan(cx))^3-bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{\int (a+b\arctan(cx))dx}{c^2}-\frac{\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\int \frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 2009

$$\frac{1}{3}x^3(a+b\arctan(cx))^3-bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\int \frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 5419

$$\frac{1}{3}x^3(a+b\arctan(cx))^3-bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{c^2}-\frac{\int \frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 5455

$$\frac{1}{3}x^3(a+b\arctan(cx))^3-bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{c^2}-\frac{-\frac{\int \frac{(a+b\arctan(cx))^2}{i-cx}dx}{c}-\frac{i(a+b\operatorname{arc}}{3b}}{c^2}\right.$$

$$\Big\downarrow \text{5379}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^3 - bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2 - bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2} - \frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{c^2} - \frac{-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\int}{c}}{}\right.$$

$$\Big\downarrow \text{5529}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^3 - bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2 - bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2} - \frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{c^2} - \frac{-\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\Big(}{}\right.$$

$$\Big\downarrow \text{7164}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^3 - bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2 - bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2} - \frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{c^2} - \frac{-\frac{i(a+b\arctan(cx))^3}{3bc^2} - \frac{\log\left(\frac{2}{1+icx}\right)}{}}{}\right.$$

input
```
Int[x^2*(a + b*ArcTan[c*x])^3,x]
```

output
```
(x^3*(a + b*ArcTan[c*x])^3)/3 - b*c*(((x^2*(a + b*ArcTan[c*x])^2)/2 - b*c*
(-1/2*(a + b*ArcTan[c*x])^2/(b*c^3) + (a*x + b*x*ArcTan[c*x] - (b*Log[1 +
c^2*x^2])/(2*c))/c^2))/c^2 - (((-1/3*I)*(a + b*ArcTan[c*x])^3)/(b*c^2) - (
((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/c - 2*b*(((-1/2*I)*(a + b*ArcTa
n[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/(1 + I*c*x)
])/(4*c)))/c)/c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.80 (sec) , antiderivative size = 1118, normalized size of antiderivative = 5.43

method	result	size
derivativedivides	Expression too large to display	1118
default	Expression too large to display	1118
parts	Expression too large to display	1120

input
```
int(x^2*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/3*c^3*x^3*a^3+b^3*(1/3*c^3*x^3*arctan(c*x)^3-1/2*c^2*x^2*arctan(c
*x)^2+1/2*arctan(c*x)^2*ln(c^2*x^2+1)+1/4*I*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2
+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*arctan(c*x)^2-1/2*polylog(3,-(1+I*c*x
)^2/(c^2*x^2+1))-arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*
x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/2*I*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+
1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*arctan(c*x)^2+ln(1+(1+I*c*x)^2
/(c^2*x^2+1))-1/4*I*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+
I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2-1/2*arctan(c*x)^2+1/4*I*Pi*csgn(I/(
1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2+1/3*I*arct
an(c*x)^3+1/4*I*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*arctan(c*x)^2+1/2*I*P
i*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2
)^2*arctan(c*x)^2+arctan(c*x)*(c*x-I)+1/4*I*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1
)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2-1/4*I*Pi*csgn(I/(
1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^
2/(c^2*x^2+1))^2)^2*arctan(c*x)^2-1/4*I*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))
*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*
x)^2-1/4*I*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*arctan(c*x)^2-ln(2)*
arctan(c*x)^2)+3*a*b^2*(1/3*c^3*x^3*arctan(c*x)^2-1/3*c^2*x^2*arctan(c*x)+
1/3*arctan(c*x)*ln(c^2*x^2+1)+1/3*c*x-1/3*arctan(c*x)+1/6*I*(ln(c*x-I)*...
```

Fricas [F]

$$\int x^2(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*arctan(c*x)^3 + 3*a*b^2*x^2*arctan(c*x)^2 + 3*a^2*b*x^2*a
rctan(c*x) + a^3*x^2, x)
```

Sympy [F]

$$\int x^2(a+b\arctan(cx))^3\,dx=\int x^2(a+b\,\mathrm{atan}\,(cx))^3\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))**3,x)
```

output
```
Integral(x**2*(a + b*atan(c*x))**3, x)
```

Maxima [F]

$$\int x^2(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
1/24*b^3*x^3*arctan(c*x)^3 - 1/32*b^3*x^3*arctan(c*x)*log(c^2*x^2 + 1)^2 +
 1/3*a^3*x^3 + 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4)
)*a^2*b + integrate(1/32*(4*b^3*c^2*x^4*arctan(c*x)*log(c^2*x^2 + 1) + 28*
(b^3*c^2*x^4 + b^3*x^2)*arctan(c*x)^3 + 4*(24*a*b^2*c^2*x^4 - b^3*c*x^3 +
24*a*b^2*x^2)*arctan(c*x)^2 + (b^3*c*x^3 + 3*(b^3*c^2*x^4 + b^3*x^2)*arcta
n(c*x))*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x)
```

Giac [F]

$$\int x^2(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arctan(cx))^3\,dx=\int x^2\,(a+b\,\mathrm{atan}(c\,x))^3\,dx$$

input
```
int(x^2*(a + b*atan(c*x))^3,x)
```

output
```
int(x^2*(a + b*atan(c*x))^3, x)
```

Reduce [F]

$$\int x^2(a+b\arctan(cx))^3\,dx = \frac{2atan(cx)^3\,b^3c^3x^3+2atan(cx)^3\,b^3cx+6atan(cx)^2\,a\,b^2c^3x^3+6atan(cx)^2\,a\,b^2cx-3atan(cx)^2\,b^3c^2x^2-3}{}$$

input
```
int(x^2*(a+b*atan(c*x))^3,x)
```

output
```
(2*atan(c*x)**3*b**3*c**3*x**3 + 2*atan(c*x)**3*b**3*c*x + 6*atan(c*x)**2*
a*b**2*c**3*x**3 + 6*atan(c*x)**2*a*b**2*c*x - 3*atan(c*x)**2*b**3*c**2*x*
*2 - 3*atan(c*x)**2*b**3 + 6*atan(c*x)*a**2*b*c**3*x**3 - 6*atan(c*x)*a*b*
*2*c**2*x**2 - 6*atan(c*x)*a*b**2 + 6*atan(c*x)*b**3*c*x - 2*int(atan(c*x)
**3,x)*b**3*c - 6*int(atan(c*x)**2,x)*a*b**2*c + 3*log(c**2*x**2 + 1)*a**2
*b - 3*log(c**2*x**2 + 1)*b**3 + 2*a**3*c**3*x**3 - 3*a**2*b*c**2*x**2 + 6
*a*b**2*c*x)/(6*c**3)
```

3.28 $\int x(a+b\arctan(cx))^3\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 131

$$\int x(a+b\arctan(cx))^3\,dx = -\frac{3ib(a+b\arctan(cx))^2}{2c^2}-\frac{3bx(a+b\arctan(cx))^2}{2c}+\frac{(a+b\arctan(cx))^3}{2c^2}+\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2}-\frac{3ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2}$$

output
```
-3/2*I*b*(a+b*arctan(c*x))^2/c^2-3/2*b*x*(a+b*arctan(c*x))^2/c+1/2*(a+b*ar
ctan(c*x))^3/c^2+1/2*x^2*(a+b*arctan(c*x))^3-3*b^2*(a+b*arctan(c*x))*ln(2/
(1+I*c*x))/c^2-3/2*I*b^3*polylog(2,1-2/(1+I*c*x))/c^2
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.16

$$\int x(a+b\arctan(cx))^3\,dx$$
$$=\frac{3b^2(a+ac^2x^2+b(i-cx))\arctan(cx)^2+b^3(1+c^2x^2)\arctan(cx)^3+3b\arctan(cx)\left(a(a-2bcx+ac^2x^2)\right.}{2c}$$

input
```
Integrate[x*(a + b*ArcTan[c*x])^3,x]
```

output
```
(3*b^2*(a + a*c^2*x^2 + b*(I - c*x))*ArcTan[c*x]^2 + b^3*(1 + c^2*x^2)*Arc
Tan[c*x]^3 + 3*b*ArcTan[c*x]*(a*(a - 2*b*c*x + a*c^2*x^2) - 2*b^2*Log[1 +
E^((2*I)*ArcTan[c*x])]) + a*(a*c*x*(-3*b + a*c*x) + 3*b^2*Log[1 + c^2*x^2]
) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5361, 5451, 5345, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arctan(cx))^3\,dx$$
$$\Big\downarrow \text{5361}$$
$$\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\int\frac{x^2(a+b\arctan(cx))^2}{c^2x^2+1}dx$$
$$\Big\downarrow \text{5451}$$
$$\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(\frac{\int(a+b\arctan(cx))^2dx}{c^2}-\frac{\int\frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$
$$\Big\downarrow \text{5345}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(\frac{x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 5419

$$\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(\frac{x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{(a+b\arctan(cx))^3}{3bc^3}\right)$$

$\downarrow$ 5455

$$\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)}{c^2}\right)$$

$\downarrow$ 5379

$$\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(a+b\arctan(c}{2bc^2}\right.}{c^2}\right.$$

$\downarrow$ 2849

$$\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}-\frac{i(a+b\arct}{2b}\right.}{c^2}\right.$$

$\downarrow$ 2752

$$\frac{1}{2}x^2(a+b\arctan(cx))^3 - \frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\,\text{PolyLog}\left(2,1-\right.}{2c}}{c}\right.}{c^2}\right.$$

input
```
Int[x*(a + b*ArcTan[c*x])^3,x]
```

output
```
(x^2*(a + b*ArcTan[c*x])^3)/2 - (3*b*c*(-1/3*(a + b*ArcTan[c*x])^3/(b*c^3)
 + (x*(a + b*ArcTan[c*x])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b*c
^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2, 1
- 2/(1 + I*c*x)])/c)/c))/c^2))/2
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(117) = 234$.

Time = 0.39 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.11

method	result
derivativedivides	$\frac{\frac{c^2x^2a^3}{2}+b^3\left(\frac{c^2x^2\arctan(cx)^3}{2}+\frac{\arctan(cx)^3}{2}-\frac{3cx\arctan(cx)^2}{2}+\frac{3\arctan(cx)\ln\left(c^2x^2+1\right)}{2}+\frac{3i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-i}{2}\right.}{}\right.}{}$
default	$\frac{\frac{c^2x^2a^3}{2}+b^3\left(\frac{c^2x^2\arctan(cx)^3}{2}+\frac{\arctan(cx)^3}{2}-\frac{3cx\arctan(cx)^2}{2}+\frac{3\arctan(cx)\ln\left(c^2x^2+1\right)}{2}+\frac{3i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-i}{2}\right.}{}\right.}{}$
parts	$\frac{a^3x^2}{2}+\frac{b^3\left(\frac{c^2x^2\arctan(cx)^3}{2}+\frac{\arctan(cx)^3}{2}-\frac{3cx\arctan(cx)^2}{2}+\frac{3\arctan(cx)\ln\left(c^2x^2+1\right)}{2}+\frac{3i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-i}{2}\right.}{}\right.}{}$
risch	$\frac{21a\,b^2\ln(c^2x^2+1)}{16c^2}-\frac{i\ln(-icx+1)^3b^3x^2}{16}-\frac{3ib^3\ln(-icx+1)x^2}{32}+\frac{3ib^3\ln(-icx+1)^2x^2}{32}-\frac{ib^3\ln(-icx+1)^3}{16c^2}+9$

input
```
int(x*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/2*c^2*x^2*a^3+b^3*(1/2*c^2*x^2*arctan(c*x)^3+1/2*arctan(c*x)^3-3/
2*c*x*arctan(c*x)^2+3/2*arctan(c*x)*ln(c^2*x^2+1)+3/4*I*(ln(c*x-I)*ln(c^2*
x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))
-3/4*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*
x+I)*ln(1/2*I*(c*x-I))))+3*a*b^2*(1/2*c^2*x^2*arctan(c*x)^2+1/2*arctan(c*x
)^2-c*x*arctan(c*x)+1/2*ln(c^2*x^2+1))+3*a^2*b*(1/2*c^2*x^2*arctan(c*x)-1/
2*c*x+1/2*arctan(c*x)))
```

Fricas [F]

$$\int x(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*arctan(c*x)^3 + 3*a*b^2*x*arctan(c*x)^2 + 3*a^2*b*x*arctan(
c*x) + a^3*x, x)
```

Sympy [F]

$$\int x(a+b\arctan(cx))^3\,dx=\int x(a+b\,\mathrm{atan}\,(cx))^3\ dx$$

input
```
integrate(x*(a+b*atan(c*x))**3,x)
```

output
```
Integral(x*(a + b*atan(c*x))**3, x)
```

Maxima [F]

$$\int x(a+b\arctan(cx))^3\,dx=\int (b\arctan\,(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
3/2*a*b^2*x^2*arctan(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arctan(c*x) - c*(x/c^
2 - arctan(c*x)/c^3))*a^2*b - 3/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*
x) + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*a*b^2 - 1/32*(12*c*x*arctan(c
*x)^2 - 8*(c^2*x^2 + 1)*arctan(c*x)^3 - 3*c*x*log(c^2*x^2 + 1)^2 - 4*(128*
c^3*integrate(1/32*x^3*arctan(c*x)^3/(c^3*x^2 + c), x) - 96*c^2*integrate(
1/32*x^2*arctan(c*x)^2/(c^3*x^2 + c), x) - 24*c^2*integrate(1/32*x^2*log(c
^2*x^2 + 1)^2/(c^3*x^2 + c), x) - 96*c^2*integrate(1/32*x^2*log(c^2*x^2 +
1)/(c^3*x^2 + c), x) + 128*c*integrate(1/32*x*arctan(c*x)^3/(c^3*x^2 + c),
 x) + 192*c*integrate(1/32*x*arctan(c*x)/(c^3*x^2 + c), x) - arctan(c*x)^3
/c^2 - 24*integrate(1/32*log(c^2*x^2 + 1)^2/(c^3*x^2 + c), x))*c^2)*b^3/c^
2
```

Giac [F]

$$\int x(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\arctan(cx))^3\,dx=\int x\,(a+b\,\mathrm{atan}(c\,x))^3\,dx$$

input
```
int(x*(a + b*atan(c*x))^3,x)
```

output
```
int(x*(a + b*atan(c*x))^3, x)
```

Reduce [F]

$$\int x(a+b\arctan(cx))^3\,dx=\frac{atan(cx)^3\,b^3c^2x^2+atan(cx)^3\,b^3+3atan(cx)^2\,a\,b^2c^2x^2+3atan(cx)^2\,a\,b^2-3atan(cx)^2\,b^3cx+3atan(cx)}{}$$

input
```
int(x*(a+b*atan(c*x))^3,x)
```

output
```
(atan(c*x)**3*b**3*c**2*x**2 + atan(c*x)**3*b**3 + 3*atan(c*x)**2*a*b**2*c
**2*x**2 + 3*atan(c*x)**2*a*b**2 - 3*atan(c*x)**2*b**3*c*x + 3*atan(c*x)*a
**2*b*c**2*x**2 + 3*atan(c*x)*a**2*b - 6*atan(c*x)*a*b**2*c*x + 6*int((ata
n(c*x)*x)/(c**2*x**2 + 1),x)*b**3*c**2 + 3*log(c**2*x**2 + 1)*a*b**2 + a**
3*c**2*x**2 - 3*a**2*b*c*x)/(2*c**2)
```

3.29 $\int (a+b\arctan(cx))^3 \, dx$

Optimal result 289
Mathematica [A] (verified) 290
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Giac [F] 295
Mupad [F(-1)] 295
Reduce [F] 295

Optimal result

Integrand size = 10, antiderivative size = 119

$$\int (a+b\arctan(cx))^3 \, dx = \frac{i(a+b\arctan(cx))^3}{c} + x(a+b\arctan(cx))^3 + \frac{3b(a+b\arctan(cx))^2 \log\left(\frac{2}{1+icx}\right)}{c} + \frac{3ib^2(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c} + \frac{3b^3\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c}$$

output
```
I*(a+b*arctan(c*x))^3/c+x*(a+b*arctan(c*x))^3+3*b*(a+b*arctan(c*x))^2*ln(2
/(1+I*c*x))/c+3*I*b^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c+3/2*b^3
*polylog(3,1-2/(1+I*c*x))/c
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.61

$$\int (a+b\arctan(cx))^3\,dx = a^3x+3a^2bx\arctan(cx)-\frac{3a^2b\log(1+c^2x^2)}{2c}$$
$$+\frac{3ab^2\left(-i\arctan(cx)^2+cx\arctan(cx)^2+2\arctan(cx)\log\left(1+e^{2i\arctan(cx)}\right)-i\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(c}\right.\right.}{c}$$
$$+\frac{b^3\left(-i\arctan(cx)^3+cx\arctan(cx)^3+3\arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right)-3i\arctan(cx)\,\mathrm{PolyLog}\left(2,\right.\right.}{c}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3,x]
```

output
```
a^3*x + 3*a^2*b*x*ArcTan[c*x] - (3*a^2*b*Log[1 + c^2*x^2])/(2*c) + (3*a*b^
2*((-I)*ArcTan[c*x]^2 + c*x*ArcTan[c*x]^2 + 2*ArcTan[c*x]*Log[1 + E^((2*I)
*ArcTan[c*x])] - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c + (b^3*((-I)*Arc
Tan[c*x]^3 + c*x*ArcTan[c*x]^3 + 3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c
*x])] - (3*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (3*PolyLog[
3, -E^((2*I)*ArcTan[c*x])])/2))/c
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5345, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arctan(cx))^3\,dx$$

$$\Big\downarrow \; 5345$$

$$x(a+b\arctan(cx))^3-3bc\int \frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx$$

$$\Big\downarrow \; 5455$$

$$x(a+b\arctan(cx))^3-3bc\left(-\frac{\int\frac{(a+b\arctan(cx))^2}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^3}{3bc^2}\right)$$

$\big\downarrow$ 5379

$$x(a+b\arctan(cx))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(a+b\arctan(cx))^3}{3bc^2}\right)$$

$\big\downarrow$ 5529

$$x(a+b\arctan(cx))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\left(\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}\right)}{c}-\frac{i(a+b\,a\ldots}{3\ldots}\right.$$

$\big\downarrow$ 7164

$$x(a+b\arctan(cx))^3-3bc\left(-\frac{i(a+b\arctan(cx))^3}{3bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\left(-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{b\,\text{PolyLog}\left(3,1-\frac{\ldots}{ic\ldots}\right.}{4c}\right.}{c}\right.$$

input
```
Int[(a + b*ArcTan[c*x])^3,x]
```

output
```
x*(a + b*ArcTan[c*x])^3 - 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x])^3)/(b*c^2)
- (((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/c - 2*b*(((-1/2*I)*(a + b*Ar
cTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/(1 + I*c
*x)])/(4*c)))/c)
```

Defintions of rubi rules used

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(112) = 224$.

Time = 0.49 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{c\,a^3x+b^3\left(\arctan(cx)^3(cx+i)-2i\arctan(cx)^3+3\arctan(cx)^2\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)-3i\arctan(cx)\operatorname{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)\right.}{}$
default	$\frac{c\,a^3x+b^3\left(\arctan(cx)^3(cx+i)-2i\arctan(cx)^3+3\arctan(cx)^2\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)-3i\arctan(cx)\operatorname{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)\right.}{}$
parts	$a^3x+\frac{b^3\left(\arctan(cx)^3(cx+i)-2i\arctan(cx)^3+3\arctan(cx)^2\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)-3i\arctan(cx)\operatorname{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)\right.}{c}$

input
```
int((a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(c*a^3*x+b^3*(arctan(c*x)^3*(c*x+I)-2*I*arctan(c*x)^3+3*arctan(c*x)^2*
ln(1+(1+I*c*x)^2/(c^2*x^2+1))-3*I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*
x^2+1))+3/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1)))+3*a*b^2*(arctan(c*x)^2*(c
*x+I)+2*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))-2*I*arctan(c*x)^2-I*poly
log(2,-(1+I*c*x)^2/(c^2*x^2+1)))+3*a^2*b*(c*x*arctan(c*x)-1/2*ln(c^2*x^2+1
)))
```

Fricas [F]

$$\int (a+b\arctan(cx))^3\,dx=\int (b\arctan{(cx)}+a)^3\,dx$$

input
```
integrate((a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3, x)
```

Sympy [F]

$$\int (a+b\arctan(cx))^3\,dx = \int (a+b\,\mathrm{atan}\,(cx))^3\,dx$$

input
```
integrate((a+b*atan(c*x))**3,x)
```

output
```
Integral((a + b*atan(c*x))**3, x)
```

Maxima [F]

$$\int (a+b\arctan(cx))^3\,dx = \int (b\arctan(cx)+a)^3\,dx$$

input
```
integrate((a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
1/8*b^3*x*arctan(c*x)^3 - 3/32*b^3*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 7/32
*b^3*arctan(c*x)^4/c + 28*b^3*c^2*integrate(1/32*x^2*arctan(c*x)^3/(c^2*x^
2 + 1), x) + 3*b^3*c^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(
c^2*x^2 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^2*arctan(c*x)^2/(c^2*x^2
+ 1), x) + 12*b^3*c^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2
*x^2 + 1), x) + a*b^2*arctan(c*x)^3/c - 12*b^3*c*integrate(1/32*x*arctan(c
*x)^2/(c^2*x^2 + 1), x) + 3*b^3*c*integrate(1/32*x*log(c^2*x^2 + 1)^2/(c^2
*x^2 + 1), x) + a^3*x + 3*b^3*integrate(1/32*arctan(c*x)*log(c^2*x^2 + 1)^
2/(c^2*x^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a^2*b/c
```

Giac [F]

$$\int (a+b\arctan(cx))^3\,dx = \int (b\arctan(cx)+a)^3\,dx$$

input
```
integrate((a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\arctan(cx))^3\,dx = \int (a+b\,\mathrm{atan}(c\,x))^3\,dx$$

input
```
int((a + b*atan(c*x))^3,x)
```

output
```
int((a + b*atan(c*x))^3, x)
```

Reduce [F]

$$\int (a+b\arctan(cx))^3\,dx = \frac{6atan(cx)\,a^2bcx+2\left(\int atan(cx)^3\,dx\right)b^3c+6\left(\int atan(cx)^2\,dx\right)a\,b^2c-3\log(c^2x^2+1)\,a^2b+2a^3cx}{2c}$$

input
```
int((a+b*atan(c*x))^3,x)
```

output
```
(6*atan(c*x)*a**2*b*c*x + 2*int(atan(c*x)**3,x)*b**3*c + 6*int(atan(c*x)**
2,x)*a*b**2*c - 3*log(c**2*x**2 + 1)*a**2*b + 2*a**3*c*x)/(2*c)
```

3.30 $\int \frac{(a+b\arctan(cx))^3}{x}\,dx$

Optimal result 296
Mathematica [A] (verified) 297
Rubi [A] (verified) 298
Maple [C] (warning: unable to verify) 300
Fricas [F] 301
Sympy [F] 302
Maxima [F] 302
Giac [F] 302
Mupad [F(-1)] 303
Reduce [F] 303

Optimal result

Integrand size = 14, antiderivative size = 206

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x}\,dx =& 2(a+b\arctan(cx))^3\text{arctanh}\left(1-\frac{2}{1+icx}\right)\\ &-\frac{3}{2}ib(a+b\arctan(cx))^2\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\ &+\frac{3}{2}ib(a+b\arctan(cx))^2\,\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\\ &-\frac{3}{2}b^2(a+b\arctan(cx))\,\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\\ &+\frac{3}{2}b^2(a+b\arctan(cx))\,\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\\ &+\frac{3}{4}ib^3\,\text{PolyLog}\left(4,1-\frac{2}{1+icx}\right)\\ &-\frac{3}{4}ib^3\,\text{PolyLog}\left(4,-1+\frac{2}{1+icx}\right)\end{aligned}$$

output
```
-2*(a+b*arctan(c*x))^3*arctanh(-1+2/(1+I*c*x))-3/2*I*b*(a+b*arctan(c*x))^2
*polylog(2,1-2/(1+I*c*x))+3/2*I*b*(a+b*arctan(c*x))^2*polylog(2,-1+2/(1+I*
c*x))-3/2*b^2*(a+b*arctan(c*x))*polylog(3,1-2/(1+I*c*x))+3/2*b^2*(a+b*arct
an(c*x))*polylog(3,-1+2/(1+I*c*x))+3/4*I*b^3*polylog(4,1-2/(1+I*c*x))-3/4*
I*b^3*polylog(4,-1+2/(1+I*c*x))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.79

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx = a^3\log(cx)+\frac{3}{2}ia^2b(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx)) + 3ab^2\Bigg(-\frac{i\pi^3}{24}+\frac{2}{3}i\arctan(cx)^3 + \arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - \arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) + i\arctan(cx)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) + i\arctan(cx)\,\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) + \frac{1}{2}\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - \frac{1}{2}\text{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\Bigg) - \frac{1}{64}ib^3\big(\pi^4 - 32\arctan(cx)^4 + 64i\arctan(cx)^3\log\left(1-e^{-2i\arctan(cx)}\right) - 64i\arctan(cx)^3\log\left(1+e^{2i\arctan(cx)}\right) - 96\arctan(cx)^2\,\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) - 96\arctan(cx)^2\,\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) + 96i\arctan(cx)\,\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - 96i\arctan(cx)\,\text{PolyLog}\left(3,-e^{2i\arctan(cx)}\right) + 48\,\text{PolyLog}\left(4,e^{-2i\arctan(cx)}\right) + 48\,\text{PolyLog}\left(4,-e^{2i\arctan(cx)}\right)\big)$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/x,x]
```

output
```
a^3*Log[c*x] + ((3*I)/2)*a^2*b*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x])
+ 3*a*b^2*((-1/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + ArcTan[c*x]^2*Log[1
- E^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] +
 I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[
2, -E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 - PolyLo
g[3, -E^((2*I)*ArcTan[c*x])]/2) - (I/64)*b^3*(Pi^4 - 32*ArcTan[c*x]^4 + (6
4*I)*ArcTan[c*x]^3*Log[1 - E^((-2*I)*ArcTan[c*x])] - (64*I)*ArcTan[c*x]^3*
Log[1 + E^((2*I)*ArcTan[c*x])] - 96*ArcTan[c*x]^2*PolyLog[2, E^((-2*I)*Arc
Tan[c*x])] - 96*ArcTan[c*x]^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (96*I)*
ArcTan[c*x]*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - (96*I)*ArcTan[c*x]*PolyLo
g[3, -E^((2*I)*ArcTan[c*x])] + 48*PolyLog[4, E^((-2*I)*ArcTan[c*x])] + 48*
PolyLog[4, -E^((2*I)*ArcTan[c*x])])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5357, 5523, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx$$

$$\Big\downarrow \text{5357}$$

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-6bc\int \frac{(a+b\arctan(cx))^2\text{arctanh}\left(1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx$$

$$\Big\downarrow \text{5523}$$

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-$$
$$6bc\left(\frac{1}{2}\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{1}{2}\int \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5529}$$

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-$$
$$6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}-ib\int\frac{(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)+\frac{1}{2}\right.$$

$\Big\downarrow$ 5533

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-$$
$$6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}-ib\left(\frac{i\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\right.\right.\right.$$

$\Big\downarrow$ 7164

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-$$
$$6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}-ib\left(\frac{i\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}+\frac{b\,\text{Poly}}{}\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x])^3/x,x]
```

output
```
2*(a + b*ArcTan[c*x])^3*ArcTanh[1 - 2/(1 + I*c*x)] - 6*b*c*((((I/2)*(a + b
*ArcTan[c*x])^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - I*b*(((I/2)*(a + b*ArcT
an[c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/c + (b*PolyLog[4, 1 - 2/(1 + I*c*x
)])/(4*c)))/2 + (((-1/2*I)*(a + b*ArcTan[c*x])^2*PolyLog[2, -1 + 2/(1 + I*
c*x)])/c + I*b*(((I/2)*(a + b*ArcTan[c*x])*PolyLog[3, -1 + 2/(1 + I*c*x)])
/c + (b*PolyLog[4, -1 + 2/(1 + I*c*x)])/(4*c)))/2)
```

Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.51 (sec) , antiderivative size = 2024, normalized size of antiderivative = 9.83

method	result	size
parts	Expression too large to display	2024
derivativedivides	Expression too large to display	2026
default	Expression too large to display	2026

input
```
int((a+b*arctan(c*x))^3/x,x,method=_RETURNVERBOSE)
```

output
```
a^3*ln(x)+b^3*(ln(c*x)*arctan(c*x)^3-arctan(c*x)^3*ln((1+I*c*x)^2/(c^2*x^2
+1)-1)+arctan(c*x)^3*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*arctan(c*x)^2*p
olylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*arctan(c*x)*polylog(3,(1+I*c*x)/(c
^2*x^2+1)^(1/2))+6*I*polylog(4,(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c*x)^3*
ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*arctan(c*x)^2*polylog(2,-(1+I*c*x)/(
c^2*x^2+1)^(1/2))+6*arctan(c*x)*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*
I*polylog(4,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*Pi*(csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)
/(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x
)^2/(c^2*x^2+1)))^2+csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x
)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x
^2+1)))-csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*cs
gn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3-csgn(I*((1+I*c*x)^
2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1
)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1
+I*c*x)^2/(c^2*x^2+1)))^3+1)*arctan(c*x)^3+3/2*I*arctan(c*x)^2*polylog(2,-
(1+I*c*x)^2/(c^2*x^2+1))-3/2*arctan(c*x)*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1
))-3/4*I*polylog(4,-(1+I*c*x)^2/(c^2*x^2+1)))+3*a^2*b*(ln(c*x)*arctan(c...
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx=\int \frac{(b\arctan(cx)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x)
+ a^3)/x, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))^3}{x}\,dx$$

input
```
integrate((a+b*atan(c*x))**3/x,x)
```

output
```
Integral((a + b*atan(c*x))**3/x, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx = \int \frac{(b\arctan\,(cx)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x)^3 + 3*b^3*arctan(c*x)*log(
c^2*x^2 + 1)^2 + 96*a*b^2*arctan(c*x)^2 + 96*a^2*b*arctan(c*x))/x, x)
```

Giac [F]

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx = \int \frac{(b\arctan\,(cx)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x}\,dx$$

input
```
int((a + b*atan(c*x))^3/x,x)
```

output
```
int((a + b*atan(c*x))^3/x, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx=3\left(\int \frac{atan(cx)}{x}dx\right)a^2b+\left(\int \frac{atan(cx)^3}{x}dx\right)b^3+3\left(\int \frac{atan(cx)^2}{x}dx\right)a\,b^2+\log(x)\,a^3$$

input
```
int((a+b*atan(c*x))^3/x,x)
```

output
```
3*int(atan(c*x)/x,x)*a**2*b + int(atan(c*x)**3/x,x)*b**3 + 3*int(atan(c*x)
**2/x,x)*a*b**2 + log(x)*a**3
```

3.31 $\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 116

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx = &-ic(a+b\arctan(cx))^3-\frac{(a+b\arctan(cx))^3}{x}\\&+3bc(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right)\\&-3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\\&+\frac{3}{2}b^3c\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)\end{aligned}$$

output
```
-I*c*(a+b*arctan(c*x))^3-(a+b*arctan(c*x))^3/x+3*b*c*(a+b*arctan(c*x))^2*l
n(2-2/(1-I*c*x))-3*I*b^2*c*(a+b*arctan(c*x))*polylog(2,-1+2/(1-I*c*x))+3/2
*b^3*c*polylog(3,-1+2/(1-I*c*x))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.84

$$\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx = -\frac{a^3}{x}-\frac{3a^2b\arctan(cx)}{x}+3a^2bc\log(x)-\frac{3}{2}a^2bc\log\left(1+c^2x^2\right)+3ab^2c\left(-\frac{\arctan(cx)^2}{cx}+2\arctan(cx)\log\left(1-e^{2i\arctan(cx)}\right)-i\left(\arctan(cx)^2+\text{PolyLog}\left(2,e^{2i\arctan(cx)}\right)\right)\right)+b^3c\left(-\frac{i\pi^3}{8}+i\arctan(cx)^3-\frac{\arctan(cx)^3}{cx}+3\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right)+3i\arctan(cx)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right)+\frac{3}{2}\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/x^2,x]
```

output
```
-(a^3/x) - (3*a^2*b*ArcTan[c*x])/x + 3*a^2*b*c*Log[x] - (3*a^2*b*c*Log[1 +
 c^2*x^2])/2 + 3*a*b^2*c*(-(ArcTan[c*x]^2/(c*x)) + 2*ArcTan[c*x]*Log[1 - E
^((2*I)*ArcTan[c*x])] - I*(ArcTan[c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x]
)])) + b^3*c*((-1/8*I)*Pi^3 + I*ArcTan[c*x]^3 - ArcTan[c*x]^3/(c*x) + 3*Ar
cTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (3*I)*ArcTan[c*x]*PolyLog[2,
 E^((-2*I)*ArcTan[c*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c*x])])/2)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5361, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx$$

$$\Big\downarrow \text{5361}$$

$$3bc\int \frac{(a+b\arctan(cx))^2}{x\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))^3}{x}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{(a+b\arctan(cx))^3}{x}+3bc\left(i\int \frac{(a+b\arctan(cx))^2}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^3}{3b}\right)$$

$$\Big\downarrow \text{5403}$$

$$-\frac{(a+b\arctan(cx))^3}{x}+$$
$$3bc\left(i\left(2ibc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))^2\right)-\frac{i(a+b\arcta}{3b}\right.$$

$$\Big\downarrow \text{5527}$$

$$-\frac{(a+b\arctan(cx))^3}{x}+$$
$$3bc\left(i\left(2ibc\left(\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2-\frac{}{1-}\right.\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$-\frac{(a+b\arctan(cx))^3}{x}+$$
$$3bc\left(i\left(2ibc\left(\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{b\,\text{PolyLog}\left(3,\frac{2}{1-icx}-1\right)}{4c}\right)-i\log\left(2-\frac{2}{1-icx}\right)(\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x])^3/x^2,x]
```

output
```
-((a + b*ArcTan[c*x])^3/x) + 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x])^3)/b + I
*((-I)*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1 - I*c*x)] + (2*I)*b*c*(((I/2)*(a
 + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 - I*c*x)])/c - (b*PolyLog[3, -1 + 2
/(1 - I*c*x)])/(4*c))))
```

Defintions of rubi rules used

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.33 (sec) , antiderivative size = 1860, normalized size of antiderivative = 16.03

method	result	size
parts	Expression too large to display	1860
derivativedivides	Expression too large to display	1862
default	Expression too large to display	1862

input
```
int((a+b*arctan(c*x))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*a^3+b^3*c*(-1/c/x*arctan(c*x)^3-3/2*arctan(c*x)^2*ln(c^2*x^2+1)+3*ln(
c*x)*arctan(c*x)^2+3*arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-3*arcta
n(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-I*arctan(c*x)^3+3/4*(2*I*Pi*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2
/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-2*I*Pi*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)
/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+I*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)
^3-I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)+I*Pi*csg
n(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)+2
*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1
)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))+I*Pi*c
sgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+
I*c*x)^2/(c^2*x^2+1))^2)^2-I*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn
(I*(1+I*c*x)^2/(c^2*x^2+1))+2*I*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csg
n(I*(1+I*c*x)^2/(c^2*x^2+1))^2+I*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I
*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2+2*I*Pi*csgn(((1+
I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3-2*I*Pi*csgn(I/(1+(1
+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/
(c^2*x^2+1)))^2-2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I...
```

Fricas [F]

$$\int \frac{(a + b\arctan(cx))^3}{x^2}\,dx = \int \frac{(b\arctan(cx) + a)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x)
+ a^3)/x^2, x)
```

Sympy [F]

$$\int \frac{(a + b\arctan(cx))^3}{x^2}\,dx = \int \frac{(a + b\,\mathrm{atan}(cx))^3}{x^2}\,dx$$

input
```
integrate((a+b*atan(c*x))**3/x**2,x)
```

output
```
Integral((a + b*atan(c*x))**3/x**2, x)
```

Maxima [F]

$$\int \frac{(a + b\arctan(cx))^3}{x^2}\,dx = \int \frac{(b\arctan(cx) + a)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="maxima")
```

output
```
-3/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a^2*b - a^3/x - 1
/32*(4*b^3*arctan(c*x)^3 - 3*b^3*arctan(c*x)*log(c^2*x^2 + 1)^2 - (7*b^3*c
*arctan(c*x)^4 + 32*a*b^2*c*arctan(c*x)^3 + 96*b^3*c^2*integrate(1/32*x^2*
arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) - 384*b^3*c^2*integrate
(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) + 384*b^3*c*int
egrate(1/32*x*arctan(c*x)^2/(c^2*x^4 + x^2), x) - 96*b^3*c*integrate(1/32*
x*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) + 896*b^3*integrate(1/32*arctan(c
*x)^3/(c^2*x^4 + x^2), x) + 96*b^3*integrate(1/32*arctan(c*x)*log(c^2*x^2
+ 1)^2/(c^2*x^4 + x^2), x) + 3072*a*b^2*integrate(1/32*arctan(c*x)^2/(c^2*
x^4 + x^2), x))*x)/x
```

Giac [F]

$$\int \frac{(a + b\arctan(cx))^3}{x^2}\,dx = \int \frac{(b\arctan(cx) + a)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx))^3}{x^2}\,dx = \int \frac{(a + b\,\mathrm{atan}(c\,x))^3}{x^2}\,dx$$

input
```
int((a + b*atan(c*x))^3/x^2,x)
```

output
```
int((a + b*atan(c*x))^3/x^2, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx$$
$$= \frac{-2atan(cx)^3 b^3 - 6atan(cx)^2 a b^2 - 6atan(cx) a^2 b + 12\left(\int \frac{atan(cx)}{c^2x^3+x}dx\right) a b^2 cx + 6\left(\int \frac{atan(cx)^2}{c^2x^3+x}dx\right) b^3 cx - 3}{2x}$$

input
```
int((a+b*atan(c*x))^3/x^2,x)
```

output
```
( - 2*atan(c*x)**3*b**3 - 6*atan(c*x)**2*a*b**2 - 6*atan(c*x)*a**2*b + 12*
int(atan(c*x)/(c**2*x**3 + x),x)*a*b**2*c*x + 6*int(atan(c*x)**2/(c**2*x**
3 + x),x)*b**3*c*x - 3*log(c**2*x**2 + 1)*a**2*b*c*x + 6*log(x)*a**2*b*c*x
 - 2*a**3)/(2*x)
```

3.32 $\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx$

Optimal result 312
Mathematica [A] (verified) 313
Rubi [A] (verified) 313
Maple [B] (verified) 316
Fricas [F] 316
Sympy [F] 317
Maxima [F] 317
Giac [F] 318
Mupad [F(-1)] 318
Reduce [F] 318

Optimal result

Integrand size = 14, antiderivative size = 133

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx &= -\frac{3}{2}ibc^2(a+b\arctan(cx))^2-\frac{3bc(a+b\arctan(cx))^2}{2x}\\ &\quad -\frac{1}{2}c^2(a+b\arctan(cx))^3-\frac{(a+b\arctan(cx))^3}{2x^2}\\ &\quad +3b^2c^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\ &\quad -\frac{3}{2}ib^3c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\end{aligned}$$

output
```
-3/2*I*b*c^2*(a+b*arctan(c*x))^2-3/2*b*c*(a+b*arctan(c*x))^2/x-1/2*c^2*(a+
b*arctan(c*x))^3-1/2*(a+b*arctan(c*x))^3/x^2+3*b^2*c^2*(a+b*arctan(c*x))*l
n(2-2/(1-I*c*x))-3/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx =$$

$$-\frac{3b^2(a+ac^2x^2+bcx(1+icx))\arctan(cx)^2+b^3(1+c^2x^2)\arctan(cx)^3+3b\arctan(cx)\left(a(a+2bcx+\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/x^3,x]
```

output
```
-1/2*(3*b^2*(a + a*c^2*x^2 + b*c*x*(1 + I*c*x))*ArcTan[c*x]^2 + b^3*(1 + c
^2*x^2)*ArcTan[c*x]^3 + 3*b*ArcTan[c*x]*(a*(a + 2*b*c*x + a*c^2*x^2) - 2*b
^2*c^2*x^2*Log[1 - E^((2*I)*ArcTan[c*x])]) + a*(a*(a + 3*b*c*x) - 6*b^2*c^
2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]) + (3*I)*b^3*c^2*x^2*PolyLog[2, E^((2*I
)*ArcTan[c*x])])/x^2
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5361, 5453, 5361, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x^3}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{3}{2}bc\int \frac{(a+b\arctan(cx))^2}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{2x^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{3}{2}bc\left(\int \frac{(a+b\arctan(cx))^2}{x^2}dx-c^2\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)-\frac{(a+b\arctan(cx))^3}{2x^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{3}{2}bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)+2bc\int \frac{a+b\arctan(cx)}{x\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))^2}{x}\right)-\frac{(a+b\arctan(cx))^3}{2x^2}$$

$$\Big\downarrow \text{5419}$$

$$\frac{3}{2}bc\left(2bc\int \frac{a+b\arctan(cx)}{x\left(c^2x^2+1\right)}dx-\frac{c(a+b\arctan(cx))^3}{3b}-\frac{(a+b\arctan(cx))^2}{x}\right)-\frac{(a+b\arctan(cx))^3}{2x^2}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{3}{2}bc\left(2bc\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^2}{2b}\right)-\frac{c(a+b\arctan(cx))^3}{3b}-\frac{(a+b\arctan(cx))^2}{x}\right)$$

$$\Big\downarrow \text{5403}$$

$$-\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{3}{2}bc\left(2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)-\frac{c(\ldots$$

$$\Big\downarrow \text{2897}$$

$$-\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)\ldots$$

input
```
Int[(a + b*ArcTan[c*x])^3/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c*x])^3/x^2 + (3*b*c*(-((a + b*ArcTan[c*x])^2/x) - (c*(
a + b*ArcTan[c*x])^3)/(3*b) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b +
I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 + 2/
(1 - I*c*x)])/2))))/2
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(119) = 238$.

Time = 0.51 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.66

method	result
derivativedivides	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(-\frac{\arctan(cx)^3}{2c^2x^2}-\frac{3\arctan(cx)^2}{2cx}-\frac{\arctan(cx)^3}{2}-\frac{3\arctan(cx)\ln(c^2x^2+1)}{2}+3\ln(cx\right.\right.$
default	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(-\frac{\arctan(cx)^3}{2c^2x^2}-\frac{3\arctan(cx)^2}{2cx}-\frac{\arctan(cx)^3}{2}-\frac{3\arctan(cx)\ln(c^2x^2+1)}{2}+3\ln(cx\right.\right.$
parts	$-\frac{a^3}{2x^2}+b^3c^2\left(-\frac{\arctan(cx)^3}{2c^2x^2}-\frac{3\arctan(cx)^2}{2cx}-\frac{\arctan(cx)^3}{2}-\frac{3\arctan(cx)\ln(c^2x^2+1)}{2}+3\ln(cx)\,\mathrm{arc}\right.$

input
```
int((a+b*arctan(c*x))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a^3/c^2/x^2+b^3*(-1/2/c^2/x^2*arctan(c*x)^3-3/2/c/x*arctan(c*x)^
2-1/2*arctan(c*x)^3-3/2*arctan(c*x)*ln(c^2*x^2+1)+3*ln(c*x)*arctan(c*x)-3/
4*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-
I)*ln(-1/2*I*(c*x+I)))+3/4*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilo
g(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I)))+3/2*I*ln(c*x)*ln(1+I*c*x)-3/
2*I*ln(c*x)*ln(1-I*c*x)+3/2*I*dilog(1+I*c*x)-3/2*I*dilog(1-I*c*x))+3*a*b^2
*(-1/2/c^2/x^2*arctan(c*x)^2-1/c/x*arctan(c*x)-1/2*arctan(c*x)^2-1/2*ln(c^
2*x^2+1)+ln(c*x))+3*a^2*b*(-1/2/c^2/x^2*arctan(c*x)-1/2/c/x-1/2*arctan(c*x
)))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx = \int \frac{(b\arctan(cx)+a)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x)
+ a^3)/x^3, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx=\int \frac{(a+b\,\mathrm{atan}\,(cx))^3}{x^3}\,dx$$

input
```
integrate((a+b*atan(c*x))**3/x**3,x)
```

output
```
Integral((a + b*atan(c*x))**3/x**3, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx=\int \frac{(b\arctan(cx)+a)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="maxima")
```

output
```
-3/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a^2*b + 3/2*((arctan(c*x)
^2 - log(c^2*x^2 + 1) + 2*log(x))*c^2 - 2*(c*arctan(c*x) + 1/x)*c*arctan(c
*x))*a*b^2 - 3/2*a*b^2*arctan(c*x)^2/x^2 - 1/32*(12*c*x*arctan(c*x)^2 + 8*
(c^2*x^2 + 1)*arctan(c*x)^3 - 3*c*x*log(c^2*x^2 + 1)^2 - 4*(c^2*arctan(c*x
)^3 + 24*c^3*integrate(1/32*x^3*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) - 9
6*c^3*integrate(1/32*x^3*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 128*c^2*in
tegrate(1/32*x^2*arctan(c*x)^3/(c^2*x^5 + x^3), x) + 192*c^2*integrate(1/3
2*x^2*arctan(c*x)/(c^2*x^5 + x^3), x) + 96*c*integrate(1/32*x*arctan(c*x)^
2/(c^2*x^5 + x^3), x) + 24*c*integrate(1/32*x*log(c^2*x^2 + 1)^2/(c^2*x^5
+ x^3), x) + 128*integrate(1/32*arctan(c*x)^3/(c^2*x^5 + x^3), x))*x^2)*b^
3/x^2 - 1/2*a^3/x^2
```

Giac [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx = \int \frac{(b\arctan(cx)+a)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x^3}\,dx$$

input
```
int((a + b*atan(c*x))^3/x^3,x)
```

output
```
int((a + b*atan(c*x))^3/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx$$
$$= \frac{-atan(cx)^3 b^3c^2x^2 - atan(cx)^3 b^3 - 3atan(cx)^2 a b^2c^2x^2 - 3atan(cx)^2 a b^2 - 3atan(cx)^2 b^3cx - 3atan(cx}{}$$

input
```
int((a+b*atan(c*x))^3/x^3,x)
```

output
```
( - atan(c*x)**3*b**3*c**2*x**2 - atan(c*x)**3*b**3 - 3*atan(c*x)**2*a*b**
2*c**2*x**2 - 3*atan(c*x)**2*a*b**2 - 3*atan(c*x)**2*b**3*c*x - 3*atan(c*x
)*a**2*b*c**2*x**2 - 3*atan(c*x)*a**2*b - 6*atan(c*x)*a*b**2*c*x - 3*atan(
c*x)*b**3*c**2*x**2 - 3*atan(c*x)*b**3 - 6*int(atan(c*x)/(c**2*x**5 + x**3
),x)*b**3*x**2 - 3*log(c**2*x**2 + 1)*a*b**2*c**2*x**2 + 6*log(x)*a*b**2*c
**2*x**2 - a**3 - 3*a**2*b*c*x - 3*b**3*c*x)/(2*x**2)
```

3.33 $\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx$

Optimal result 320
Mathematica [A] (verified) 321
Rubi [A] (verified) 321
Maple [C] (warning: unable to verify) 326
Fricas [F] 327
Sympy [F] 327
Maxima [F] 327
Giac [F] 328
Mupad [F(-1)] 328
Reduce [F] 328

Optimal result

Integrand size = 14, antiderivative size = 213

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx = &-\frac{b^2c^2(a+b\arctan(cx))}{x}-\frac{1}{2}bc^3(a+b\arctan(cx))^2\\&-\frac{bc(a+b\arctan(cx))^2}{2x^2}+\frac{1}{3}ic^3(a+b\arctan(cx))^3\\&-\frac{(a+b\arctan(cx))^3}{3x^3}+b^3c^3\log(x)-\frac{1}{2}b^3c^3\log\left(1+c^2x^2\right)\\&-bc^3(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right)\\&+ib^2c^3(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\\&-\frac{1}{2}b^3c^3\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)\end{aligned}$$

output
```
-b^2*c^2*(a+b*arctan(c*x))/x-1/2*b*c^3*(a+b*arctan(c*x))^2-1/2*b*c*(a+b*ar
ctan(c*x))^2/x^2+1/3*I*c^3*(a+b*arctan(c*x))^3-1/3*(a+b*arctan(c*x))^3/x^3
+b^3*c^3*ln(x)-1/2*b^3*c^3*ln(c^2*x^2+1)-b*c^3*(a+b*arctan(c*x))^2*ln(2-2/
(1-I*c*x))+I*b^2*c^3*(a+b*arctan(c*x))*polylog(2,-1+2/(1-I*c*x))-1/2*b^3*c
^3*polylog(3,-1+2/(1-I*c*x))
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.51

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx$$
$$= -\frac{a^3}{3x^3} - \frac{a^2bc}{2x^2} - \frac{a^2b\arctan(cx)}{x^3} - a^2bc^3\log(x) + \frac{1}{2}a^2bc^3\log\left(1+c^2x^2\right)$$
$$+ \frac{iab^2\left(ic^2x^2 + (i+c^3x^3)\arctan(cx)^2 + icx\arctan(cx)\left(1+c^2x^2+2c^2x^2\log\left(1-e^{2i\arctan(cx)}\right)\right) + c^3x^3\,\mathrm{P}\right.}{x^3}$$
$$+ \frac{1}{24}b^3c^3\left(i\pi^3 - \frac{24\arctan(cx)}{cx} - 12\arctan(cx)^2 - \frac{12\arctan(cx)^2}{c^2x^2} - 8i\arctan(cx)^3\right.$$
$$- \frac{8\arctan(cx)^3}{c^3x^3} - 24\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) + 24\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)$$
$$\left.- 24i\arctan(cx)\,\mathrm{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) - 12\,\mathrm{PolyLog}\left(3,e^{-2i\arctan(cx)}\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/x^4,x]
```

output
```
-1/3*a^3/x^3 - (a^2*b*c)/(2*x^2) - (a^2*b*ArcTan[c*x])/x^3 - a^2*b*c^3*Log
[x] + (a^2*b*c^3*Log[1 + c^2*x^2])/2 + (I*a*b^2*(I*c^2*x^2 + (I + c^3*x^3)
*ArcTan[c*x]^2 + I*c*x*ArcTan[c*x]*(1 + c^2*x^2 + 2*c^2*x^2*Log[1 - E^((2*
I)*ArcTan[c*x])]) + c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/x^3 + (b^3
*c^3*(I*Pi^3 - (24*ArcTan[c*x])/(c*x) - 12*ArcTan[c*x]^2 - (12*ArcTan[c*x]
^2)/(c^2*x^2) - (8*I)*ArcTan[c*x]^3 - (8*ArcTan[c*x]^3)/(c^3*x^3) - 24*Arc
Tan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + 24*Log[(c*x)/Sqrt[1 + c^2*x^2
]] - (24*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3,
 E^((-2*I)*ArcTan[c*x])]))/24
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5361, 5453, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x^4}dx$$

$$\Big\downarrow \text{ 5361}$$

$$bc\int \frac{(a+b\arctan(cx))^2}{x^3(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{3x^3}$$

$$\Big\downarrow \text{ 5453}$$

$$bc\left(\int \frac{(a+b\arctan(cx))^2}{x^3}dx-c^2\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx\right)-\frac{(a+b\arctan(cx))^3}{3x^3}$$

$$\Big\downarrow \text{ 5361}$$

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx\right)+bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{2x^2}\right)-\frac{(a+b\arctan(cx))^3}{3x^3}$$

$$\Big\downarrow \text{ 5453}$$

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx\right)+bc\left(\int \frac{a+b\arctan(cx)}{x^2}dx-c^2\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{(a+b\arctan(c}{2x^2}\right.$$
$$\frac{(a+b\arctan(cx))^3}{3x^3}$$

$$\Big\downarrow \text{ 5361}$$

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx\right)+bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(}{x}\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{3x^3}$$

$$\Big\downarrow \text{ 243}$$

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx\right)+bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx^2-\frac{a+b\arct}{x}\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{3x^3}$$

$$\Big\downarrow \text{ 47}$$

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx\right)+bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{3x^3}$$

$\downarrow$ 14

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x\left(c^2x^2+1\right)}dx\right)+bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{3x^3}$$

$\downarrow$ 16

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x\left(c^2x^2+1\right)}dx\right)+bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc(\log\left(x^2\right)-\text{lo}\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{3x^3}$$

$\downarrow$ 5419

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x\left(c^2x^2+1\right)}dx\right)+bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc(\log\left(x^2\right)-\log\left(c^2x^2\right.\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{3x^3}$$

$\downarrow$ 5459

$$-\frac{(a+b\arctan(cx))^3}{3x^3}+$$
$$bc\left(-\left(c^2\left(i\int \frac{(a+b\arctan(cx))^2}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^3}{3b}\right)\right)+bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}\right.\right.$$

$\downarrow$ 5403

$$-\frac{(a+b\arctan(cx))^3}{3x^3}+$$
$$bc\left(-\left(c^2\left(i\left(2ibc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))^2\right)-\frac{i(a+}{}\right.\right.\right.$$

$\downarrow$ 5527

$$-\frac{(a+b\arctan(cx))^3}{3x^3}+$$
$$bc\left(-\left(c^2\left(i\left(2ibc\left(\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2\right.\right.\right.\right.\right.$$

$\downarrow$ 7164

$$-\frac{(a+b\arctan(cx))^3}{3x^3}+$$
$$bc\left(-\left(c^2\left(i\left(2ibc\left(\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{b\,\text{PolyLog}\left(3,\frac{2}{1-icx}-1\right)}{4c}\right)-i\log\left(2-\frac{2}{1-\right.\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x])^3/x^4,x]
```

output
```
-1/3*(a + b*ArcTan[c*x])^3/x^3 + b*c*(-1/2*(a + b*ArcTan[c*x])^2/x^2 + b*c
*(-((a + b*ArcTan[c*x])/x) - (c*(a + b*ArcTan[c*x])^2)/(2*b) + (b*c*(Log[x
^2] - Log[1 + c^2*x^2]))/2) - c^2*(((-1/3*I)*(a + b*ArcTan[c*x])^3)/b + I*
((-I)*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1 - I*c*x)] + (2*I)*b*c*(((I/2)*(a
+ b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 - I*c*x)])/c - (b*PolyLog[3, -1 + 2/
(1 - I*c*x)])/(4*c)))))
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.42 (sec) , antiderivative size = 2138, normalized size of antiderivative = 10.04

method	result	size
derivativedivides	Expression too large to display	2138
default	Expression too large to display	2138
parts	Expression too large to display	2140

input
```
int((a+b*arctan(c*x))^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/3*a^3/c^3/x^3+b^3*(-1/2/c^2/x^2*arctan(c*x)^2+1/4*I*Pi*csgn(I/(1+(
1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x
)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2-1/2*I*Pi*csgn
(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-1/
2*arctan(c*x)^2-1/2*arctan(c*x)*(I*c*x-(c^2*x^2+1)^(1/2)+1)/c/x-1/2*I*Pi*c
sgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*arcta
n(c*x)^2-1/4*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2
/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2-1/4*I*Pi*csgn(
I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arc
tan(c*x)^2-1/4*I*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^
2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2+1/4*I*Pi*csgn(I*(1
+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2
+1/2*I*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+(1+I*c*x)^2/(c^2*x
^2+1))^2)^2*arctan(c*x)^2+1/4*I*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*arcta
n(c*x)^2-ln(c*x)*arctan(c*x)^2+arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)
-arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-arctan(c*x)^2*ln(1+(1+I*c
*x)/(c^2*x^2+1)^(1/2))-1/2*I*Pi*arctan(c*x)^2+2*I*arctan(c*x)*polylog(2,(1
+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1
)^(1/2))+ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)-2*polylog(3,(1+I*c*x)/(c^2*x...
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx=\int \frac{(b\arctan(cx)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x)
+ a^3)/x^4, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx=\int \frac{(a+b\,\mathrm{atan}(cx))^3}{x^4}\,dx$$

input
```
integrate((a+b*atan(c*x))**3/x**4,x)
```

output
```
Integral((a + b*atan(c*x))**3/x**4, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx=\int \frac{(b\arctan(cx)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="maxima")
```

output
```
1/2*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*
a^2*b - 1/3*a^3/x^3 - 1/96*(4*b^3*arctan(c*x)^3 - 3*b^3*arctan(c*x)*log(c^
2*x^2 + 1)^2 - 96*x^3*integrate(-1/32*(4*b^3*c^2*x^2*arctan(c*x)*log(c^2*x
^2 + 1) - 28*(b^3*c^2*x^2 + b^3)*arctan(c*x)^3 - 4*(24*a*b^2*c^2*x^2 + b^3
*c*x + 24*a*b^2)*arctan(c*x)^2 + (b^3*c*x - 3*(b^3*c^2*x^2 + b^3)*arctan(c
*x))*log(c^2*x^2 + 1)^2)/(c^2*x^6 + x^4), x))/x^3
```

Giac [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx = \int \frac{(b\arctan(cx)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x^4}\,dx$$

input
```
int((a + b*atan(c*x))^3/x^4,x)
```

output
```
int((a + b*atan(c*x))^3/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx$$
$$= \frac{-2atan(cx)^3 b^3 - 6atan(cx)^2 a b^2 - 3atan(cx)^2 b^3c^3x^3 - 3atan(cx)^2 b^3cx - 6atan(cx)\, a^2b - 6atan(cx)\, a}{}$$

input
```
int((a+b*atan(c*x))^3/x^4,x)
```

output
```
( - 2*atan(c*x)**3*b**3 - 6*atan(c*x)**2*a*b**2 - 3*atan(c*x)**2*b**3*c**3
*x**3 - 3*atan(c*x)**2*b**3*c*x - 6*atan(c*x)*a**2*b - 6*atan(c*x)*a*b**2*
c**3*x**3 - 6*atan(c*x)*a*b**2*c*x - 6*atan(c*x)*b**3*c**2*x**2 - 12*int(a
tan(c*x)/(c**2*x**3 + x),x)*a*b**2*c**3*x**3 - 6*int(atan(c*x)**2/(c**2*x*
*3 + x),x)*b**3*c**3*x**3 + 3*log(c**2*x**2 + 1)*a**2*b*c**3*x**3 - 3*log(
c**2*x**2 + 1)*b**3*c**3*x**3 - 6*log(x)*a**2*b*c**3*x**3 + 6*log(x)*b**3*
c**3*x**3 - 2*a**3 - 3*a**2*b*c*x - 6*a*b**2*c**2*x**2)/(6*x**3)
```

3.34 $\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx$

Optimal result 330
Mathematica [A] (verified) 331
Rubi [A] (verified) 331
Maple [B] (verified) 335
Fricas [F] 336
Sympy [F] 336
Maxima [F(-1)] 337
Giac [F] 337
Mupad [F(-1)] 337
Reduce [F] 338

Optimal result

Integrand size = 14, antiderivative size = 198

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx = -\frac{b^3c^3}{4x}-\frac{1}{4}b^3c^4\arctan(cx)-\frac{b^2c^2(a+b\arctan(cx))}{4x^2} + ibc^4(a+b\arctan(cx))^2-\frac{bc(a+b\arctan(cx))^2}{4x^3} + \frac{3bc^3(a+b\arctan(cx))^2}{4x} + \frac{1}{4}c^4(a+b\arctan(cx))^3-\frac{(a+b\arctan(cx))^3}{4x^4} - 2b^2c^4(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) + ib^3c^4\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)$$

output
```
-1/4*b^3*c^3/x-1/4*b^3*c^4*arctan(c*x)-1/4*b^2*c^2*(a+b*arctan(c*x))/x^2+I
*b*c^4*(a+b*arctan(c*x))^2-1/4*b*c*(a+b*arctan(c*x))^2/x^3+3/4*b*c^3*(a+b*
arctan(c*x))^2/x+1/4*c^4*(a+b*arctan(c*x))^3-1/4*(a+b*arctan(c*x))^3/x^4-2
*b^2*c^4*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))+I*b^3*c^4*polylog(2,-1+2/(1-I
*c*x))
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx =$$
$$-\frac{a^3+a^2bcx+ab^2c^2x^2-3a^2bc^3x^3+b^3c^3x^3+ab^2c^4x^4+b^2(bcx(1-3c^2x^2-4ic^3x^3)+a(3-3c^4x^4))\,\mathrm{arc}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/x^5,x]
```

output
```
-1/4*(a^3 + a^2*b*c*x + a*b^2*c^2*x^2 - 3*a^2*b*c^3*x^3 + b^3*c^3*x^3 + a*
b^2*c^4*x^4 + b^2*(b*c*x*(1 - 3*c^2*x^2 - (4*I)*c^3*x^3) + a*(3 - 3*c^4*x^
4))*ArcTan[c*x]^2 - b^3*(-1 + c^4*x^4)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(b^2*
c^2*x^2*(1 + c^2*x^2) + a*b*(2*c*x - 6*c^3*x^3) + a^2*(3 - 3*c^4*x^4) + 8*
b^2*c^4*x^4*Log[1 - E^((2*I)*ArcTan[c*x])]) + 8*a*b^2*c^4*x^4*Log[(c*x)/Sq
rt[1 + c^2*x^2]] - (4*I)*b^3*c^4*x^4*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x^
4
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.40, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {5361, 5453, 5361, 5453, 5361, 264, 216, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x^5}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{3}{4}bc\int \frac{(a+b\arctan(cx))^2}{x^4\,(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{4x^4}$$

$$\Big\downarrow \text{5453}$$

$$\frac{3}{4}bc\left(\int \frac{(a+b\arctan(cx))^2}{x^4}dx-c^2\int \frac{(a+b\arctan(cx))^2}{x^2\,(c^2x^2+1)}dx\right)-\frac{(a+b\arctan(cx))^3}{4x^4}$$

$$\Big\downarrow \text{5361}$$

$$\frac{3}{4}bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x^2\,(c^2x^2+1)}dx\right)+\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3\,(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{3x^3}\right)-\frac{(a+b\arctan(cx))^3}{4x^4}$$

$$\Big\downarrow \text{5453}$$

$$\frac{3}{4}bc\left(-\left(c^2\left(\int \frac{(a+b\arctan(cx))^2}{x^2}dx-c^2\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)\right)+\frac{2}{3}bc\left(\int \frac{a+b\arctan(cx)}{x^3}dx-c^2\int \frac{a\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{4x^4}$$

$$\Big\downarrow \text{5361}$$

$$\frac{3}{4}bc\left(-\left(c^2\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)+2bc\int \frac{a+b\arctan(cx)}{x\,(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}\right)\right)+\frac{2}{3}bc\left(c^2\left(\right.\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{4x^4}$$

$$\Big\downarrow \text{264}$$

$$\frac{3}{4}bc\left(-\left(c^2\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)+2bc\int \frac{a+b\arctan(cx)}{x\,(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}\right)\right)+\frac{2}{3}bc\left(c^2\left(\right.\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{4x^4}$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{4}bc\left(-\left(c^2\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)+2bc\int \frac{a+b\arctan(cx)}{x\,(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}\right)\right)+\frac{2}{3}bc\left(c^2\left(\right.\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{4x^4}$$

$$\Big\downarrow \text{5419}$$

$$\frac{3}{4}bc\left(-\left(c^2\left(2bc\int \frac{a+b\arctan(cx)}{x\,(c^2x^2+1)}dx-\frac{c(a+b\arctan(cx))^3}{3b}-\frac{(a+b\arctan(cx))^2}{x}\right)\right)+\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\,\mathrm{a}}{x\,(c^2\right.\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{4x^4}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{(a+b\arctan(cx))^3}{4x^4}+$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(-\left(c^2\left(i\int\frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^2}{2b}\right)\right)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\right.\right.\right.$$

$\big\downarrow$ 5403

$$-\frac{(a+b\arctan(cx))^3}{4x^4}+$$
$$\frac{3}{4}bc\left(-\left(c^2\left(2bc\left(i\left(ibc\int\frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right.\right.\right.\right.$$

$\big\downarrow$ 2897

$$-\frac{(a+b\arctan(cx))^3}{4x^4}+$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan}{2b}\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x])^3/x^5,x]
```

output
```
-1/4*(a + b*ArcTan[c*x])^3/x^4 + (3*b*c*(-1/3*(a + b*ArcTan[c*x])^2/x^3 -
c^2*(-((a + b*ArcTan[c*x])^2/x) - (c*(a + b*ArcTan[c*x])^3)/(3*b) + 2*b*c*
(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b + I*((-I)*(a + b*ArcTan[c*x])*Log[2 -
2/(1 - I*c*x)] - (b*PolyLog[2, -1 + 2/(1 - I*c*x)])/2))) + (2*b*c*(-1/2*(a
 + b*ArcTan[c*x])/x^2 + (b*c*(-x^(-1) - c*ArcTan[c*x]))/2 - c^2*(((-1/2*I)
*(a + b*ArcTan[c*x])^2)/b + I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c
*x)] - (b*PolyLog[2, -1 + 2/(1 - I*c*x)])/2))))/3))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(180) = 360$.

Time = 0.61 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.14

method	result
derivativedivides	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\arctan(cx)^3}{4c^4x^4}+\frac{\arctan(cx)^3}{4}-\frac{\arctan(cx)^2}{4c^3x^3}+\frac{3\arctan(cx)^2}{4cx}+\arctan(cx)\ln(c^2\right.\right.$
default	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\arctan(cx)^3}{4c^4x^4}+\frac{\arctan(cx)^3}{4}-\frac{\arctan(cx)^2}{4c^3x^3}+\frac{3\arctan(cx)^2}{4cx}+\arctan(cx)\ln(c^2\right.\right.$
parts	$-\frac{a^3}{4x^4}+b^3c^4\left(-\frac{\arctan(cx)^3}{4c^4x^4}+\frac{\arctan(cx)^3}{4}-\frac{\arctan(cx)^2}{4c^3x^3}+\frac{3\arctan(cx)^2}{4cx}+\arctan(cx)\ln(c^2x^2\right.$

input
```
int((a+b*arctan(c*x))^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
c^4*(-1/4*a^3/c^4/x^4+b^3*(-1/4/c^4/x^4*arctan(c*x)^3+1/4*arctan(c*x)^3-1/
4/c^3/x^3*arctan(c*x)^2+3/4/c/x*arctan(c*x)^2+arctan(c*x)*ln(c^2*x^2+1)-1/
4/c^2/x^2*arctan(c*x)-2*ln(c*x)*arctan(c*x)-1/4/c/x-1/4*arctan(c*x)+1/2*I*
(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*l
n(-1/2*I*(c*x+I)))-1/2*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/
2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I)))-I*ln(c*x)*ln(1+I*c*x)+I*ln(c*x)*
ln(1-I*c*x)-I*dilog(1+I*c*x)+I*dilog(1-I*c*x))+3*a*b^2*(-1/4/c^4/x^4*arcta
n(c*x)^2+1/4*arctan(c*x)^2-1/6/c^3/x^3*arctan(c*x)+1/2/c/x*arctan(c*x)+1/3
*ln(c^2*x^2+1)-1/12/c^2/x^2-2/3*ln(c*x))+3*a^2*b*(-1/4/c^4/x^4*arctan(c*x)
+1/4*arctan(c*x)-1/12/c^3/x^3+1/4/c/x))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx = \int \frac{(b\arctan(cx)+a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x)
+ a^3)/x^5, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))^3}{x^5}\,dx$$

input
```
integrate((a+b*atan(c*x))**3/x**5,x)
```

output
```
Integral((a + b*atan(c*x))**3/x**5, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx = \int \frac{(b\arctan(cx)+a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x^5}\,dx$$

input
```
int((a + b*atan(c*x))^3/x^5,x)
```

output
```
int((a + b*atan(c*x))^3/x^5, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx$$
$$= \frac{atan(cx)^3 b^3c^4x^4 - atan(cx)^3 b^3 + 3atan(cx)^2 a b^2c^4x^4 - 3atan(cx)^2 a b^2 + 3atan(cx)^2 b^3c^3x^3 - atan(cx)}{}$$

input
```
int((a+b*atan(c*x))^3/x^5,x)
```

output
```
(atan(c*x)**3*b**3*c**4*x**4 - atan(c*x)**3*b**3 + 3*atan(c*x)**2*a*b**2*c
**4*x**4 - 3*atan(c*x)**2*a*b**2 + 3*atan(c*x)**2*b**3*c**3*x**3 - atan(c*
x)**2*b**3*c*x + 3*atan(c*x)*a**2*b*c**4*x**4 - 3*atan(c*x)*a**2*b + 6*ata
n(c*x)*a*b**2*c**3*x**3 - 2*atan(c*x)*a*b**2*c*x - atan(c*x)*b**3*c**4*x**
4 - atan(c*x)*b**3*c**2*x**2 - 8*int(atan(c*x)/(c**2*x**3 + x),x)*b**3*c**
4*x**4 + 4*log(c**2*x**2 + 1)*a*b**2*c**4*x**4 - 8*log(x)*a*b**2*c**4*x**4
 - a**3 + 3*a**2*b*c**3*x**3 - a**2*b*c*x - a*b**2*c**2*x**2 - b**3*c**3*x
**3)/(4*x**4)
```

3.35 $\int \frac{x}{\arctan(ax)}\,dx$

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Mathematica [N/A] 339
Rubi [N/A] 340
Maple [N/A] 340
Fricas [N/A] 341
Sympy [N/A] 341
Maxima [N/A] 341
Giac [N/A] 342
Mupad [N/A] 342
Reduce [N/A] 343

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{x}{\arctan(ax)}\,dx = \operatorname{Int}\left(\frac{x}{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x/arctan(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\arctan(ax)}\,dx$$

input
```
Integrate[x/ArcTan[a*x],x]
```

output
```
Integrate[x/ArcTan[a*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)} \, dx$$
$$\Big\downarrow \text{5377}$$
$$\int \frac{x}{\arctan(ax)} dx$$

input
```
Int[x/ArcTan[a*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan(ax)} dx$$

input
```
int(x/arctan(a*x),x)
```

output
```
int(x/arctan(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\arctan{(ax)}}\,dx$$

input
```
integrate(x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x/arctan(a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate(x/atan(a*x),x)
```

output
```
Integral(x/atan(a*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\arctan{(ax)}}\,dx$$

input
```
integrate(x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x/arctan(a*x), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\arctan(ax)}\,dx$$

input
```
integrate(x/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x/arctan(a*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\mathrm{atan}(a\,x)}\,dx$$

input
```
int(x/atan(a*x),x)
```

output
```
int(x/atan(a*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{atan\,(ax)}dx$$

input
```
int(x/atan(a*x),x)
```

output
```
int(x/atan(a*x),x)
```

3.36 $\int \frac{1}{\arctan(ax)}\,dx$

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Rubi [N/A] 345
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Optimal result

Integrand size = 6, antiderivative size = 6

$$\int \frac{1}{\arctan(ax)}\,dx = \text{Int}\left(\frac{1}{\arctan(ax)},x\right)$$

output
```
Defer(Int)(1/arctan(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\arctan(ax)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(-1),x]
```

output
```
Integrate[ArcTan[a*x]^(-1), x]
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)} dx$$

$$\Big\downarrow \; 5353$$

$$\int \frac{1}{\arctan(ax)} dx$$

input
```
Int[ArcTan[a*x]^(-1),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arctan(ax)} dx$$

input
```
int(1/arctan(a*x),x)
```

output
```
int(1/arctan(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\arctan(ax)}\,dx$$

input
```
integrate(1/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/arctan(a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(1/atan(a*x),x)
```

output
```
Integral(1/atan(a*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\arctan(ax)}\,dx$$

input
```
integrate(1/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/arctan(a*x), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/arctan(a*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int(1/atan(a*x),x)
```

output
```
int(1/atan(a*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 6.33

$$\int \frac{1}{\arctan(ax)}\,dx = \frac{\left(\int \frac{x^2}{atan(ax)a^2x^2+atan(ax)}dx\right)a^3 + \log(atan(ax))}{a}$$

input
```
int(1/atan(a*x),x)
```

output
```
(int(x**2/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 + log(atan(a*x)))/a
```

3.37 $\int \frac{1}{x \arctan(ax)} dx$

Optimal result . . . 349
Mathematica [N/A] . . . 349
Rubi [N/A] . . . 350
Maple [N/A] . . . 350
Fricas [N/A] . . . 351
Sympy [N/A] . . . 351
Maxima [N/A] . . . 351
Giac [N/A] . . . 352
Mupad [N/A] . . . 352
Reduce [N/A] . . . 353

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x \arctan(ax)} dx = \text{Int}\left(\frac{1}{x \arctan(ax)}, x\right)$$

output
```
Defer(Int)(1/x/arctan(a*x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)} dx = \int \frac{1}{x \arctan(ax)} dx$$

input
```
Integrate[1/(x*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x*ArcTan[a*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)} \, dx$$
$$\Big\downarrow \ 5377$$
$$\int \frac{1}{x \arctan(ax)}dx$$

input
```
Int[1/(x*ArcTan[a*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arctan \left(a x \right)}dx$$

input
```
int(1/x/arctan(a*x),x)
```

output
```
int(1/x/arctan(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\arctan(ax)}\,dx = \int \frac{1}{x\arctan{(ax)}}\,dx$$

input
```
integrate(1/x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/(x*arctan(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\arctan(ax)}\,dx = \int \frac{1}{x\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate(1/x/atan(a*x),x)
```

output
```
Integral(1/(x*atan(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\arctan(ax)}\,dx = \int \frac{1}{x\arctan{(ax)}}\,dx$$

input
```
integrate(1/x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(x*arctan(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)} \, dx = \int \frac{1}{x \arctan{(ax)}} \, dx$$

input
```
integrate(1/x/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/(x*arctan(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)} \, dx = \int \frac{1}{x \, \mathrm{atan}\left(a\, x\right)} \, dx$$

input
```
int(1/(x*atan(a*x)),x)
```

output
```
int(1/(x*atan(a*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)} \, dx = \int \frac{1}{atan\,(ax)\,x} dx$$

input
```
int(1/x/atan(a*x),x)
```

output
```
int(1/(atan(a*x)*x),x)
```

3.38 $\int \frac{x}{\arctan(ax)^2}\,dx$

Optimal result 354
Mathematica [N/A] 354
Rubi [N/A] 355
Maple [N/A] 355
Fricas [N/A] 356
Sympy [N/A] 356
Maxima [N/A] 356
Giac [N/A] 357
Mupad [N/A] 357
Reduce [N/A] 358

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{x}{\arctan(ax)^2}\,dx = \text{Int}\left(\frac{x}{\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(x/arctan(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)^2}\,dx = \int \frac{x}{\arctan(ax)^2}\,dx$$

input
```
Integrate[x/ArcTan[a*x]^2,x]
```

output
```
Integrate[x/ArcTan[a*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^2} \, dx$$
$$\Big\downarrow \; 5377$$
$$\int \frac{x}{\arctan(ax)^2}dx$$

input
```
Int[x/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan \left(ax\right)^{2}}dx$$

input
```
int(x/arctan(a*x)^2,x)
```

output
```
int(x/arctan(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)^2}\,dx = \int \frac{x}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x/arctan(a*x)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan(ax)^2}\,dx = \int \frac{x}{\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(x/atan(a*x)**2,x)
```

output
```
Integral(x/atan(a*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 5.62

$$\int \frac{x}{\arctan(ax)^2}\,dx = \int \frac{x}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^2*x^3 - arctan(a*x)*integrate((3*a^2*x^2 + 1)/arctan(a*x), x) + x)/(a*
arctan(a*x))
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)^2} \, dx = \int \frac{x}{\arctan{(ax)}^2} \, dx$$

input
```
integrate(x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x/arctan(a*x)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)^2} \, dx = \int \frac{x}{\mathrm{atan}{(a\,x)}^2} \, dx$$

input
```
int(x/atan(a*x)^2,x)
```

output
```
int(x/atan(a*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)^2} \, dx = \int \frac{x}{atan\,(ax)^2} dx$$

input
```
int(x/atan(a*x)^2,x)
```

output
```
int(x/atan(a*x)**2,x)
```

3.39 $\int \frac{1}{\arctan(ax)^2}\,dx$

Optimal result 359
Mathematica [N/A] 359
Rubi [N/A] 360
Maple [N/A] 360
Fricas [N/A] 361
Sympy [N/A] 361
Maxima [N/A] 361
Giac [N/A] 362
Mupad [N/A] 362
Reduce [N/A] 363

Optimal result

Integrand size = 6, antiderivative size = 6

$$\int \frac{1}{\arctan(ax)^2}\,dx = \text{Int}\left(\frac{1}{\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(1/arctan(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{\arctan(ax)^2}\,dx$$

input
```
Integrate[ArcTan[a*x]^(-2),x]
```

output
```
Integrate[ArcTan[a*x]^(-2), x]
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^2} \, dx$$

$$\Big\downarrow \text{5353}$$

$$\int \frac{1}{\arctan(ax)^2}dx$$

input
```
Int[ArcTan[a*x]^(-2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arctan \left(a x \right)^{2}}d x$$

input
```
int(1/arctan(a*x)^2,x)
```

output
```
int(1/arctan(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^(-2), x)
```

Sympy [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(1/atan(a*x)**2,x)
```

output
```
Integral(atan(a*x)**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 6.50

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^2*x^2 - 2*a^2*arctan(a*x)*integrate(x/arctan(a*x), x) + 1)/(a*arctan(a
*x))
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(1/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{\mathrm{atan}{(a\,x)}^2}\,dx$$

input
```
int(1/atan(a*x)^2,x)
```

output
```
int(1/atan(a*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{atan\left(ax\right)^2}dx$$

input
```
int(1/atan(a*x)^2,x)
```

output
```
int(1/atan(a*x)**2,x)
```

3.40 $\int \frac{1}{x \arctan(ax)^2} dx$

Optimal result . . . 364
Mathematica [N/A] . . . 364
Rubi [N/A] . . . 365
Maple [N/A] . . . 365
Fricas [N/A] . . . 366
Sympy [N/A] . . . 366
Maxima [N/A] . . . 366
Giac [N/A] . . . 367
Mupad [N/A] . . . 367
Reduce [N/A] . . . 368

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x \arctan(ax)^2} dx = \text{Int}\left(\frac{1}{x \arctan(ax)^2}, x\right)$$

output
```
Defer(Int)(1/x/arctan(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)^2} dx = \int \frac{1}{x \arctan(ax)^2} dx$$

input
```
Integrate[1/(x*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x*ArcTan[a*x]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^2} \, dx$$
$$\Big\downarrow \; 5377$$
$$\int \frac{1}{x \arctan(ax)^2} dx$$

input
```
Int[1/(x*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arctan(ax)^2} dx$$

input
```
int(1/x/arctan(a*x)^2,x)
```

output
```
int(1/x/arctan(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)^2} \, dx = \int \frac{1}{x \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/(x*arctan(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arctan(ax)^2} \, dx = \int \frac{1}{x \operatorname{atan}^2{(ax)}} \, dx$$

input
```
integrate(1/x/atan(a*x)**2,x)
```

output
```
Integral(1/(x*atan(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 5.10

$$\int \frac{1}{x \arctan(ax)^2} \, dx = \int \frac{1}{x \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^2*x^2 - x*arctan(a*x)*integrate((a^2*x^2 - 1)/(x^2*arctan(a*x)), x) +
1)/(a*x*arctan(a*x))
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)^2} \, dx = \int \frac{1}{x \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/(x*arctan(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)^2} \, dx = \int \frac{1}{x \operatorname{atan}{(a\,x)}^2} \, dx$$

input
```
int(1/(x*atan(a*x)^2),x)
```

output
```
int(1/(x*atan(a*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)^2} \, dx = \int \frac{1}{atan\,(ax)^2\,x} dx$$

input
```
int(1/x/atan(a*x)^2,x)
```

output
```
int(1/(atan(a*x)**2*x),x)
```

3.41 $\int x\sqrt{\arctan(ax)}\,dx$

Optimal result . . . 369
Mathematica [N/A] . . . 369
Rubi [N/A] . . . 370
Maple [N/A] . . . 370
Fricas [F(-2)] . . . 371
Sympy [N/A] . . . 371
Maxima [F(-2)] . . . 371
Giac [N/A] . . . 372
Mupad [N/A] . . . 372
Reduce [N/A] . . . 372

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x\sqrt{\arctan(ax)}\,dx=\text{Int}\left(x\sqrt{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x*arctan(a*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x\sqrt{\arctan(ax)}\,dx=\int x\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x*Sqrt[ArcTan[a*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5377}$$

$$\int x\sqrt{\arctan(ax)}dx$$

input
```
Int[x*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sqrt{\arctan\left(ax\right)}dx$$

input
```
int(x*arctan(a*x)^(1/2),x)
```

output
```
int(x*arctan(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x\sqrt{\arctan(ax)}\,dx = \int x\sqrt{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x*atan(a*x)**(1/2),x)
```

output
```
Integral(x*sqrt(atan(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x\sqrt{\arctan(ax)}\,dx = \int x\sqrt{\arctan{(ax)}}\,dx$$

input
```
integrate(x*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(arctan(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x\sqrt{\arctan(ax)}\,dx = \int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int(x*atan(a*x)^(1/2),x)
```

output
```
int(x*atan(a*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 4.30

$$\int x\sqrt{\arctan(ax)}\,dx = \frac{\sqrt{atan\left(ax\right)}\,x^2}{2} - \frac{\left(\int \frac{\sqrt{atan(ax)}\,x^2}{atan(ax)a^2x^2+atan(ax)}dx\right)a}{4}$$

input
```
int(x*atan(a*x)^(1/2),x)
```

output
```
(2*sqrt(atan(a*x))*x**2 - int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**2*x**2
+ atan(a*x)),x)*a)/4
```

3.42 $\int \sqrt{\arctan(ax)}\,dx$

Optimal result 374
Mathematica [N/A] 374
Rubi [N/A] 375
Maple [N/A] 375
Fricas [F(-2)] 376
Sympy [N/A] 376
Maxima [F(-2)] 376
Giac [N/A] 377
Mupad [N/A] 377
Reduce [N/A] 377

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \sqrt{\arctan(ax)}\,dx = \mathrm{Int}\left(\sqrt{\arctan(ax)},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \sqrt{\arctan(ax)}\,dx = \int \sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5353}$$

$$\int \sqrt{\arctan(ax)}dx$$

input
```
Int[Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sqrt{\arctan\left(ax\right)}dx$$

input
```
int(arctan(a*x)^(1/2),x)
```

output
```
int(arctan(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sqrt{\arctan(ax)}\,dx = \int \sqrt{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(atan(a*x)**(1/2),x)
```

output
```
Integral(sqrt(atan(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sqrt{\arctan(ax)}\,dx = \int \sqrt{\arctan(ax)}\,dx$$

input
```
integrate(arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sqrt{\arctan(ax)}\,dx = \int \sqrt{\mathrm{atan}(a\,x)}\,dx$$

input
```
int(atan(a*x)^(1/2),x)
```

output
```
int(atan(a*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 4.75

$$\int \sqrt{\arctan(ax)}\,dx = \sqrt{atan(ax)}\,x - \frac{\left(\int \frac{\sqrt{atan(ax)}\,x}{atan(ax)a^2x^2+atan(ax)}dx\right)a}{2}$$

input
```
int(atan(a*x)^(1/2),x)
```

output
```
(2*sqrt(atan(a*x))*x - int((sqrt(atan(a*x))*x)/(atan(a*x)*a**2*x**2 + atan
(a*x)),x)*a)/2
```

3.43 $\int \frac{\sqrt{\arctan(ax)}}{x} \, dx$

Optimal result 379
Mathematica [N/A] 379
Rubi [N/A] 380
Maple [N/A] 380
Fricas [F(-2)] 381
Sympy [N/A] 381
Maxima [F(-2)] 381
Giac [N/A] 382
Mupad [N/A] 382
Reduce [N/A] 382

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sqrt{\arctan(ax)}}{x} \, dx = \text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x}, x\right)$$

output
```
Defer(Int)(arctan(a*x)^(1/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{\arctan(ax)}}{x} \, dx = \int \frac{\sqrt{\arctan(ax)}}{x} \, dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/x,x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x} \, dx$$

$$\Big\downarrow \text{ 5377}$$

$$\int \frac{\sqrt{\arctan(ax)}}{x} dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x} dx$$

input
```
int(arctan(a*x)^(1/2)/x,x)
```

output
```
int(arctan(a*x)^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x} dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arctan(ax)}}{x} dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x} dx$$

input
```
integrate(atan(a*x)**(1/2)/x,x)
```

output
```
Integral(sqrt(atan(a*x))/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x}\,dx$$

input
```
int(atan(a*x)^(1/2)/x,x)
```

output
```
int(atan(a*x)^(1/2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{atan\left(ax\right)}}{x}dx$$

input
```
int(atan(a*x)^(1/2)/x,x)
```

output

```
int(sqrt(atan(a*x))/x,x)
```

3.44 $\int x \arctan(ax)^{3/2} \, dx$

Optimal result . . . 384
Mathematica [N/A] . . . 384
Rubi [N/A] . . . 385
Maple [N/A] . . . 385
Fricas [F(-2)] . . . 386
Sympy [N/A] . . . 386
Maxima [F(-2)] . . . 386
Giac [N/A] . . . 387
Mupad [N/A] . . . 387
Reduce [N/A] . . . 387

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x \arctan(ax)^{3/2} \, dx = \operatorname{Int}\left(x \arctan(ax)^{3/2}, x\right)$$

output
```
Defer(Int)(x*arctan(a*x)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x \arctan(ax)^{3/2} \, dx = \int x \arctan(ax)^{3/2} \, dx$$

input
```
Integrate[x*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x*ArcTan[a*x]^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{3/2} \, dx$$

$$\Big\downarrow \text{5377}$$

$$\int x \arctan(ax)^{3/2} dx$$

input
```
Int[x*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x \arctan\left(ax\right)^{\frac{3}{2}} dx$$

input
```
int(x*arctan(a*x)^(3/2),x)
```

output
```
int(x*arctan(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x\arctan(ax)^{3/2}\,dx = \int x\,\text{atan}^{\frac{3}{2}}\,(ax)\,dx$$

input
```
integrate(x*atan(a*x)**(3/2),x)
```

output
```
Integral(x*atan(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x\arctan(ax)^{3/2}\,dx = \int x\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x\arctan(ax)^{3/2}\,dx = \int x\,\mathrm{atan}(a\,x)^{3/2}\,dx$$

input
```
int(x*atan(a*x)^(3/2),x)
```

output
```
int(x*atan(a*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 7.50

$$\int x\arctan(ax)^{3/2}\,dx = \frac{4\sqrt{atan\left(ax\right)}\,atan(ax)\,a^2x^2 + 4\sqrt{atan\left(ax\right)}\,atan(ax) - 6\sqrt{atan\left(ax\right)}\,ax + 3\Big(\int \frac{}{ata}}{8a^2}$$

input
```
int(x*atan(a*x)^(3/2),x)
```

output
```
(4*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 + 4*sqrt(atan(a*x))*atan(a*x) - 6*s
qrt(atan(a*x))*a*x + 3*int((sqrt(atan(a*x))*x)/(atan(a*x)*a**2*x**2 + atan
(a*x)),x)*a**2)/(8*a**2)
```

3.45 $\int \arctan(ax)^{3/2}\,dx$

Optimal result 389
Mathematica [N/A] 389
Rubi [N/A] 390
Maple [N/A] 390
Fricas [F(-2)] 391
Sympy [N/A] 391
Maxima [F(-2)] 391
Giac [N/A] 392
Mupad [N/A] 392
Reduce [N/A] 392

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \arctan(ax)^{3/2}\,dx = \operatorname{Int}\left(\arctan(ax)^{3/2},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \arctan(ax)^{3/2}\,dx = \int \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{3/2} \, dx$$

$$\Big\downarrow \text{ 5353}$$

$$\int \arctan(ax)^{3/2}dx$$

input
```
Int[ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \arctan\left(ax\right)^{\frac{3}{2}} dx$$

input
```
int(arctan(a*x)^(3/2),x)
```

output
```
int(arctan(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \arctan(ax)^{3/2}\,dx = \int \operatorname{atan}^{\frac{3}{2}}(ax)\,dx$$

input
```
integrate(atan(a*x)**(3/2),x)
```

output
```
Integral(atan(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \arctan(ax)^{3/2} \, dx = \int \arctan(ax)^{\frac{3}{2}} \, dx$$

input
```
integrate(arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \arctan(ax)^{3/2} \, dx = \int \mathrm{atan}(a\,x)^{3/2} \, dx$$

input
```
int(atan(a*x)^(3/2),x)
```

output
```
int(atan(a*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \arctan(ax)^{3/2} \, dx = \int \sqrt{atan(ax)}\, atan(ax) \, dx$$

input
```
int(atan(a*x)^(3/2),x)
```

output
```
int(sqrt(atan(a*x))*atan(a*x),x)
```

3.46 $\int \frac{\arctan(ax)^{3/2}}{x}\,dx$

Optimal result . . . 394
Mathematica [N/A] . . . 394
Rubi [N/A] . . . 395
Maple [N/A] . . . 395
Fricas [F(-2)] . . . 396
Sympy [N/A] . . . 396
Maxima [F(-2)] . . . 396
Giac [N/A] . . . 397
Mupad [N/A] . . . 397
Reduce [N/A] . . . 397

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \text{Int}\left(\frac{\arctan(ax)^{3/2}}{x},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(3/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\arctan(ax)^{3/2}}{x}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/x,x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x} \, dx$$

$$\Big\downarrow \text{ 5377}$$

$$\int \frac{\arctan(ax)^{3/2}}{x} dx$$

input
```
Int[ArcTan[a*x]^(3/2)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x}dx$$

input
```
int(arctan(a*x)^(3/2)/x,x)
```

output
```
int(arctan(a*x)^(3/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x,x)
```

output
```
Integral(atan(a*x)**(3/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\arctan{(ax)}^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x}\,dx$$

input
```
int(atan(a*x)^(3/2)/x,x)
```

output
```
int(atan(a*x)^(3/2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\sqrt{atan\,(ax)}\;atan(ax)}{x}dx$$

input
```
int(atan(a*x)^(3/2)/x,x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x))/x,x)
```

3.47 $\int \frac{x}{\sqrt{\arctan(ax)}}\,dx$

Optimal result 399
Mathematica [N/A] 399
Rubi [N/A] 400
Maple [N/A] 400
Fricas [F(-2)] 401
Sympy [N/A] 401
Maxima [F(-2)] 401
Giac [N/A] 402
Mupad [N/A] 402
Reduce [N/A] 402

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x/arctan(a*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x/Sqrt[ArcTan[a*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\arctan(ax)}} \, dx$$

$$\Big\downarrow \text{5377}$$

$$\int \frac{x}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[x/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x/arctan(a*x)^(1/2),x)
```

output
```
int(x/arctan(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x/atan(a*x)**(1/2),x)
```

output
```
Integral(x/sqrt(atan(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(arctan(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int(x/atan(a*x)^(1/2),x)
```

output
```
int(x/atan(a*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{atan\,(ax)}\,x}{atan\,(ax)}dx$$

input
```
int(x/atan(a*x)^(1/2),x)
```

output
```
int((sqrt(atan(a*x))*x)/atan(a*x),x)
```

3.48 $\int \frac{1}{\sqrt{\arctan(ax)}}\,dx$

Optimal result . . . 404
Mathematica [N/A] . . . 404
Rubi [N/A] . . . 405
Maple [N/A] . . . 405
Fricas [F(-2)] . . . 406
Sympy [N/A] . . . 406
Maxima [F(-2)] . . . 406
Giac [N/A] . . . 407
Mupad [N/A] . . . 407
Reduce [N/A] . . . 407

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(1/arctan(a*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[1/Sqrt[ArcTan[a*x]], x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arctan(ax)}} \, dx$$

$$\Big\downarrow \ 5353$$

$$\int \frac{1}{\sqrt{\arctan(ax)}} dx$$

input
```
Int[1/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{\arctan (ax)}} dx$$

input
```
int(1/arctan(a*x)^(1/2),x)
```

output
```
int(1/arctan(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(1/atan(a*x)**(1/2),x)
```

output
```
Integral(1/sqrt(atan(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(1/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(arctan(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int(1/atan(a*x)^(1/2),x)
```

output
```
int(1/atan(a*x)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{atan\left(ax\right)}}{atan\left(ax\right)}dx$$

input
```
int(1/atan(a*x)^(1/2),x)
```

output
```
int(sqrt(atan(a*x))/atan(a*x),x)
```

3.49 $\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx$

Optimal result 409
Mathematica [N/A] 409
Rubi [N/A] 410
Maple [N/A] 410
Fricas [F(-2)] 411
Sympy [N/A] 411
Maxima [F(-2)] 411
Giac [N/A] 412
Mupad [N/A] 412
Reduce [N/A] 412

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{1}{x\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(1/x/arctan(a*x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx$$
$$\Big\downarrow \ 5377$$
$$\int \frac{1}{x\sqrt{\arctan(ax)}}dx$$

input
```
Int[1/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(1/x/arctan(a*x)^(1/2),x)
```

output
```
int(1/x/arctan(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(1/x/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(atan(a*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(1/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sqrt(arctan(a*x))), x)
```

Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int(1/(x*atan(a*x)^(1/2)),x)
```

output
```
int(1/(x*atan(a*x)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{atan\left(ax\right)}}{atan\left(ax\right)x}dx$$

input
```
int(1/x/atan(a*x)^(1/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)*x),x)
```

3.50 $\int \frac{x}{\arctan (ax)^{3/2}} \, dx$

Optimal result 414
Mathematica [N/A] 414
Rubi [N/A] 415
Maple [N/A] 415
Fricas [F(-2)] 416
Sympy [N/A] 416
Maxima [F(-2)] 416
Giac [N/A] 417
Mupad [N/A] 417
Reduce [N/A] 417

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{x}{\arctan (ax)^{3/2}} \, dx = \text{Int}\left(\frac{x}{\arctan (ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x/arctan(a*x)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x}{\arctan (ax)^{3/2}} \, dx = \int \frac{x}{\arctan (ax)^{3/2}} \, dx$$

input
```
Integrate[x/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x/ArcTan[a*x]^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 5377}$$

$$\int \frac{x}{\arctan(ax)^{3/2}}dx$$

input
```
Int[x/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{\arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(x/arctan(a*x)^(3/2),x)
```

output
```
int(x/arctan(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x/atan(a*x)**(3/2),x)
```

output
```
Integral(x/atan(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/arctan(a*x)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(x/atan(a*x)^(3/2),x)
```

output
```
int(x/atan(a*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{atan\left(ax\right)}\,x}{atan\left(ax\right)^{2}}dx$$

input
```
int(x/atan(a*x)^(3/2),x)
```

output
```
int((sqrt(atan(a*x))*x)/atan(a*x)**2,x)
```

3.51 $\int \frac{1}{\arctan(ax)^{3/2}}\,dx$

Optimal result 419
Mathematica [N/A] 419
Rubi [N/A] 420
Maple [N/A] 420
Fricas [F(-2)] 421
Sympy [N/A] 421
Maxima [F(-2)] 421
Giac [N/A] 422
Mupad [N/A] 422
Reduce [N/A] 422

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{1}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(1/arctan(a*x)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(-3/2),x]
```

output
```
Integrate[ArcTan[a*x]^(-3/2), x]
```

Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{3/2}} \, dx$$
$$\Big\downarrow \text{5353}$$
$$\int \frac{1}{\arctan(ax)^{3/2}}dx$$

input
```
Int[ArcTan[a*x]^(-3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(1/arctan(a*x)^(3/2),x)
```

output
```
int(1/arctan(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/atan(a*x)**(3/2),x)
```

output
```
Integral(atan(a*x)**(-3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(-3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(1/atan(a*x)^(3/2),x)
```

output
```
int(1/atan(a*x)^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{atan(ax)}}{atan(ax)^{2}}dx$$

input
```
int(1/atan(a*x)^(3/2),x)
```

output
```
int(sqrt(atan(a*x))/atan(a*x)**2,x)
```

3.52 $\int \frac{1}{x\arctan(ax)^{3/2}}\,dx$

Optimal result 424
Mathematica [N/A] 424
Rubi [N/A] 425
Maple [N/A] 425
Fricas [F(-2)] 426
Sympy [N/A] 426
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Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{1}{x\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(1/x/arctan(a*x)^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 5377}$$

$$\int \frac{1}{x \arctan(ax)^{3/2}} dx$$

input
```
Int[1/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x \arctan \left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/arctan(a*x)^(3/2),x)
```

output
```
int(1/x/arctan(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\,\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/x/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(x*atan(a*x)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arctan(ax)^{3/2}}\,dx = \int \frac{1}{x \arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(x*arctan(a*x)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^(3/2)),x)
```

output
```
int(1/(x*atan(a*x)^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{x \arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{atan\left(ax\right)}}{atan\left(ax\right)^{2} x}dx$$

input
```
int(1/x/atan(a*x)^(3/2),x)
```

output

```
int(sqrt(atan(a*x))/(atan(a*x)**2*x),x)
```

3.53 $\int \sqrt{x} \arctan(x) \, dx$

Optimal result 429
Mathematica [A] (verified) 429
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Maxima [A] (verification not implemented) 435
Giac [A] (verification not implemented) 436
Mupad [B] (verification not implemented) 436
Reduce [B] (verification not implemented) 437

Optimal result

Integrand size = 8, antiderivative size = 94

$$\int \sqrt{x} \arctan(x) \, dx = -\frac{4\sqrt{x}}{3} - \frac{1}{3}\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{x}\right) + \frac{1}{3}\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right) + \frac{2}{3}x^{3/2}\arctan(x) + \frac{1}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)$$

output
```
-4/3*x^(1/2)+1/3*2^(1/2)*arctan(-1+2^(1/2)*x^(1/2))+1/3*2^(1/2)*arctan(1+2
^(1/2)*x^(1/2))+2/3*x^(3/2)*arctan(x)+1/3*2^(1/2)*arctanh(2^(1/2)*x^(1/2)/
(1+x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

$$\int \sqrt{x} \arctan(x) \, dx = \frac{1}{6}\left(-8\sqrt{x} - 2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{x}\right) + 2\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right) + 4x^{3/2}\arctan(x) - \sqrt{2}\log\left(1-\sqrt{2}\sqrt{x}+x\right) + \sqrt{2}\log\left(1+\sqrt{2}\sqrt{x}+x\right)\right)$$

input
```
Integrate[Sqrt[x]*ArcTan[x],x]
```

output
```
(-8*Sqrt[x] - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 2*Sqrt[2]*ArcTan[1 +
 Sqrt[2]*Sqrt[x]] + 4*x^(3/2)*ArcTan[x] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[x]
+ x] + Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/6
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.375$, Rules used = {5361, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\arctan(x)\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\int \frac{x^{3/2}}{x^2+1}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-\int \frac{1}{\sqrt{x}\,(x^2+1)}dx\right)$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\int \frac{1}{x^2+1}d\sqrt{x}\right)$$

$$\Big\downarrow \text{755}$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)\right)\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\int\frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\int\frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)\right)\right)$$

$$\Big\downarrow \; 217$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\int\frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right)$$

$$\Big\downarrow \; 1479$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right)$$

$$\Big\downarrow \; 25$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right)$$

$$\Big\downarrow \; 27$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right)$$

$$\Big\downarrow \; 1103$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)\right)$$

input
```
Int[Sqrt[x]*ArcTan[x],x]
```

output
```
(2*x^(3/2)*ArcTan[x])/3 - (2*(2*Sqrt[x] - 2*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]
]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2
]*Sqrt[x] + x]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2)))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{2x^{\frac{3}{2}}\arctan(x)}{3}-\frac{4\sqrt{x}}{3}+\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{6}$
default	$\frac{2x^{\frac{3}{2}}\arctan(x)}{3}-\frac{4\sqrt{x}}{3}+\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{6}$
meijerg	$-\frac{4\sqrt{x}}{3}+\frac{\sqrt{x}\left(-\frac{\sqrt{2}\ln\left(1-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}\right)}{\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\ln\left(1+\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(\cdots\right.}{\left(x^{2}\cdots\right.}\cdots\right.}{3}$

input
```
int(x^(1/2)*arctan(x),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)*arctan(x)-4/3*x^(1/2)+1/6*2^(1/2)*(ln((x+2^(1/2)*x^(1/2)+1)/(x
-2^(1/2)*x^(1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2}{3}\,(x\arctan(x)-2)\sqrt{x}+\frac{1}{3}\,\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right) + \frac{1}{3}\,\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)+\frac{1}{6}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{6}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

input
```
integrate(x^(1/2)*arctan(x),x, algorithm="fricas")
```

output
```
2/3*(x*arctan(x) - 2)*sqrt(x) + 1/3*sqrt(2)*arctan(sqrt(2)*sqrt(x) + 1) +
1/3*sqrt(2)*arctan(sqrt(2)*sqrt(x) - 1) + 1/6*sqrt(2)*log(sqrt(2)*sqrt(x)
+ x + 1) - 1/6*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Sympy [A] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.17

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2x^{\frac{3}{2}}\operatorname{atan}(x)}{3} - \frac{4\sqrt{x}}{3} - \frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{6} + \frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{6} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{3} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{3}$$

input
```
integrate(x**(1/2)*atan(x),x)
```

output
```
2*x**(3/2)*atan(x)/3 - 4*sqrt(x)/3 - sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x
+ 4)/6 + sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/6 + sqrt(2)*atan(sqrt(2)
*sqrt(x) - 1)/3 + sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2}{3}x^{\frac{3}{2}}\arctan(x) + \frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{1}{3}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{1}{6}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{6}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{4}{3}\sqrt{x}$$

input
```
integrate(x^(1/2)*arctan(x),x, algorithm="maxima")
```

output
```
2/3*x^(3/2)*arctan(x) + 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x
))) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/6*sqrt(2)
*log(sqrt(2)*sqrt(x) + x + 1) - 1/6*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
- 4/3*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\arctan\left(x\right) + \frac{1}{3}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right) + \frac{1}{3}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right) + \frac{1}{6}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{6}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{4}{3}\,\sqrt{x}$$

input
```
integrate(x^(1/2)*arctan(x),x, algorithm="giac")
```

output
```
2/3*x^(3/2)*arctan(x) + 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x
))) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/6*sqrt(2)
*log(sqrt(2)*sqrt(x) + x + 1) - 1/6*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
- 4/3*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.52

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2\,x^{3/2}\,\mathrm{atan}(x)}{3} - \frac{4\,\sqrt{x}}{3} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)$$

input
```
int(x^(1/2)*atan(x),x)
```

output
```
(2*x^(3/2)*atan(x))/3 + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/3 +
1i/3) + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/3 - 1i/3) - (4*x^(1/
2))/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.68

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)}{3} + \frac{2\sqrt{x}\,atan(x)\,x}{3} - \frac{\sqrt{2}\,atan(x)}{3} - \frac{4\sqrt{x}}{3} - \frac{\sqrt{2}\log\left(-\sqrt{x}\,\sqrt{2}+x+1\right)}{6} + \frac{\sqrt{2}\log\left(\sqrt{x}\,\sqrt{2}+x+1\right)}{6}$$

input
```
int(x^(1/2)*atan(x),x)
```

output
```
(4*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2)) + 4*sqrt(x)*atan(x)*x - 2*s
qrt(2)*atan(x) - 8*sqrt(x) - sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1) + sqr
t(2)*log(sqrt(x)*sqrt(2) + x + 1))/6
```

3.54 $\int (dx)^m(a+b\arctan(cx))^3\,dx$

Optimal result . . . 438
Mathematica [N/A] . . . 438
Rubi [N/A] . . . 439
Maple [N/A] . . . 439
Fricas [N/A] . . . 440
Sympy [N/A] . . . 440
Maxima [N/A] . . . 440
Giac [N/A] . . . 441
Mupad [N/A] . . . 441
Reduce [N/A] . . . 442

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m(a+b\arctan(cx))^3\,dx=\text{Int}\left((dx)^m(a+b\arctan(cx))^3,x\right)$$

output
```
Defer(Int)((d*x)^m*(a+b*arctan(c*x))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^3\,dx=\int (dx)^m(a+b\arctan(cx))^3\,dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x])^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m (a + b\arctan(cx))^3 \, dx$$

$$\Big\downarrow \text{5377}$$

$$\int (dx)^m (a + b\arctan(cx))^3 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x])^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a + b\arctan\left(cx\right)\right)^3 dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x))^3,x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int (dx)^m(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x)
+ a^3)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 7.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m(a+b\arctan(cx))^3\,dx=\int (dx)^m\,(a+b\,\mathrm{atan}\,(cx))^3\,dx$$

input
```
integrate((d*x)**m*(a+b*atan(c*x))**3,x)
```

output
```
Integral((d*x)**m*(a + b*atan(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 4.18 (sec) , antiderivative size = 387, normalized size of antiderivative = 24.19

$$\int (dx)^m(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/32*(4*b^3*d^m*x*x^m*arctan(c*x)^3 - 3*b^
3*d^m*x*x^m*arctan(c*x)*log(c^2*x^2 + 1)^2 + 32*(m + 1)*integrate(1/32*(12
*b^3*c^2*d^m*x^2*x^m*arctan(c*x)*log(c^2*x^2 + 1) + 28*(b^3*d^m*m + b^3*d^
m + (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^2)*x^m*arctan(c*x)^3 - 12*(b^3*c*d^m*x
 - 8*a*b^2*d^m*m - 8*a*b^2*d^m - 8*(a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^2)*
x^m*arctan(c*x)^2 + 96*(a^2*b*d^m*m + a^2*b*d^m + (a^2*b*c^2*d^m*m + a^2*b
*c^2*d^m)*x^2)*x^m*arctan(c*x) + 3*(b^3*c*d^m*x*x^m + (b^3*d^m*m + b^3*d^m
 + (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^2)*x^m*arctan(c*x))*log(c^2*x^2 + 1)^2)
/((c^2*m + c^2)*x^2 + m + 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^3\,dx=\int (b\arctan{(cx)}+a)^3(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^3\,dx=\int {\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^3\,{\left(d\,x\right)}^m\,dx$$

input
```
int((a + b*atan(c*x))^3*(d*x)^m,x)
```

output
```
int((a + b*atan(c*x))^3*(d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 203, normalized size of antiderivative = 12.69

$$\int (dx)^m(a+b\arctan(cx))^3\,dx$$
$$= \frac{d^m\left(3x^m atan(cx)\,a^2bcmx + x^m a^3cmx - 3x^m a^2b + 3\left(\int \frac{x^m}{c^2m\,x^3+c^2x^3+mx+x}dx\right)a^2b\,m^2 + 3\left(\int \frac{x^m}{c^2m\,x^3+c^2x^3+mx+}\right.\right.}{}$$

input
```
int((d*x)^m*(a+b*atan(c*x))^3,x)
```

output
```
(d**m*(3*x**m*atan(c*x)*a**2*b*c*m*x + x**m*a**3*c*m*x - 3*x**m*a**2*b + 3
*int(x**m/(c**2*m*x**3 + c**2*x**3 + m*x + x),x)*a**2*b*m**2 + 3*int(x**m/
(c**2*m*x**3 + c**2*x**3 + m*x + x),x)*a**2*b*m + int(x**m*atan(c*x)**3,x)
*b**3*c*m**2 + int(x**m*atan(c*x)**3,x)*b**3*c*m + 3*int(x**m*atan(c*x)**2
,x)*a*b**2*c*m**2 + 3*int(x**m*atan(c*x)**2,x)*a*b**2*c*m))/(c*m*(m + 1))
```

3.55 $\int (dx)^m (a+b\arctan(cx))^2\,dx$

Optimal result . . . 443
Mathematica [N/A] . . . 443
Rubi [N/A] . . . 444
Maple [N/A] . . . 444
Fricas [N/A] . . . 445
Sympy [N/A] . . . 445
Maxima [N/A] . . . 445
Giac [N/A] . . . 446
Mupad [N/A] . . . 446
Reduce [N/A] . . . 447

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m (a+b\arctan(cx))^2\,dx = \operatorname{Int}\left((dx)^m (a+b\arctan(cx))^2, x\right)$$

output
```
Defer(Int)((d*x)^m*(a+b*arctan(c*x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m (a+b\arctan(cx))^2\,dx = \int (dx)^m (a+b\arctan(cx))^2\,dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5377}$$

$$\int (dx)^m(a+b\arctan(cx))^2dx$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m\,(a+b\arctan\,(cx))^2\,dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x))^2,x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int (dx)^m(a+b\arctan(cx))^2\,dx=\int (b\arctan{(cx)}+a)^2(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 4.69 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m(a+b\arctan(cx))^2\,dx=\int (dx)^m\,(a+b\operatorname{atan}{(cx)})^2\,dx$$

input
```
integrate((d*x)**m*(a+b*atan(c*x))**2,x)
```

output
```
Integral((d*x)**m*(a + b*atan(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 295, normalized size of antiderivative = 18.44

$$\int (dx)^m(a+b\arctan(cx))^2\,dx=\int (b\arctan{(cx)}+a)^2(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/16*(4*b^2*d^m*x*x^m*arctan(c*x)^2 - b^2*
d^m*x*x^m*log(c^2*x^2 + 1)^2 + 16*(m + 1)*integrate(1/16*(4*b^2*c^2*d^m*x^
2*x^m*log(c^2*x^2 + 1) + 12*(b^2*d^m*m + b^2*d^m + (b^2*c^2*d^m*m + b^2*c^
2*d^m)*x^2)*x^m*arctan(c*x)^2 + (b^2*d^m*m + b^2*d^m + (b^2*c^2*d^m*m + b^
2*c^2*d^m)*x^2)*x^m*log(c^2*x^2 + 1)^2 - 8*(b^2*c*d^m*x - 4*a*b*d^m*m - 4*
a*b*d^m - 4*(a*b*c^2*d^m*m + a*b*c^2*d^m)*x^2)*x^m*arctan(c*x))/((c^2*m +
c^2)*x^2 + m + 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^2\,dx=\int(b\arctan(cx)+a)^2(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^2\,dx=\int(a+b\,\mathrm{atan}(c\,x))^2\,(d\,x)^m\,dx$$

input
```
int((a + b*atan(c*x))^2*(d*x)^m,x)
```

output
```
int((a + b*atan(c*x))^2*(d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 9.56

$$\int (dx)^m (a+b\arctan(cx))^2\,dx = \frac{d^m\left(2x^m atan(cx)\,abcmx + x^m a^2cmx - 2x^m ab + 2\left(\int \frac{x^m}{c^2m\,x^3+c^2x^3+mx+x}dx\right)ab\,m^2 + 2\left(\int \frac{x^m}{c^2m\,x^3+c^2x^3+mx+x}\right.\right.}{cm\,(m+1)}$$

input
```
int((d*x)^m*(a+b*atan(c*x))^2,x)
```

output
```
(d**m*(2*x**m*atan(c*x)*a*b*c*m*x + x**m*a**2*c*m*x - 2*x**m*a*b + 2*int(x
**m/(c**2*m*x**3 + c**2*x**3 + m*x + x),x)*a*b*m**2 + 2*int(x**m/(c**2*m*x
**3 + c**2*x**3 + m*x + x),x)*a*b*m + int(x**m*atan(c*x)**2,x)*b**2*c*m**2
 + int(x**m*atan(c*x)**2,x)*b**2*c*m))/(c*m*(m + 1))
```

3.56 $\int (dx)^m(a+b\arctan(cx))\,dx$

Optimal result 448
Mathematica [A] (verified) 448
Rubi [A] (verified) 449
Maple [F] 450
Fricas [F] 450
Sympy [F] 451
Maxima [F] 451
Giac [F] 451
Mupad [F(-1)] 452
Reduce [F] 452

Optimal result

Integrand size = 14, antiderivative size = 73

$$\int (dx)^m(a+b\arctan(cx))\,dx = \frac{(dx)^{1+m}(a+b\arctan(cx))}{d(1+m)} - \frac{bc(dx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{d^2(1+m)(2+m)}$$

output
```
(d*x)^(1+m)*(a+b*arctan(c*x))/d/(1+m)-b*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*
m],[2+1/2*m],-c^2*x^2)/d^2/(1+m)/(2+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int (dx)^m(a+b\arctan(cx))\,dx = -\frac{x(dx)^m\left(-((2+m)(a+b\arctan(cx)))+bcx\,\text{Hypergeometric2F1}\left(1,1+\frac{m}{2},2+\frac{m}{2},-c^2x^2\right)\right)}{(1+m)(2+m)}$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x]),x]
```

output
```
-((x*(d*x)^m*(-((2 + m)*(a + b*ArcTan[c*x])) + b*c*x*Hypergeometric2F1[1,
1 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 + m)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5373, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \; 5373$$

$$\frac{(dx)^{m+1}(a+b\arctan(cx))}{d(m+1)}-\frac{bc\int \frac{(dx)^{m+1}}{c^2x^2+1}dx}{d(m+1)}$$

$$\Big\downarrow \; 278$$

$$\frac{(dx)^{m+1}(a+b\arctan(cx))}{d(m+1)}-\frac{bc(dx)^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{d^2(m+1)(m+2)}$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x]),x]
```

output
```
((d*x)^(1 + m)*(a + b*ArcTan[c*x]))/(d*(1 + m)) - (b*c*(d*x)^(2 + m)*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(d^2*(1 + m)*(2 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 5373
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :>
 Simp[(d*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n/(
d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Maple [F]

$$\int (dx)^m (a + b\arctan(cx))\, dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x)),x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x)),x)
```

Fricas [F]

$$\int (dx)^m(a + b\arctan(cx))\, dx = \int (b\arctan(cx) + a)(dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m(a + b\arctan(cx))\, dx = \int (dx)^m\,(a + b\,\mathrm{atan}\,(cx))\; dx$$

input
```
integrate((d*x)**m*(a+b*atan(c*x)),x)
```

output
```
Integral((d*x)**m*(a + b*atan(c*x)), x)
```

Maxima [F]

$$\int (dx)^m(a + b\arctan(cx))\, dx = \int (b\arctan(cx) + a)(dx)^m\; dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
(d^m*x*x^m*arctan(c*x) - (c*d^m*m + c*d^m)*integrate(x*x^m/((c^2*m + c^2)*
x^2 + m + 1), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int (dx)^m(a + b\arctan(cx))\, dx = \int (b\arctan(cx) + a)(dx)^m\; dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m (a + b \arctan(cx))\, dx = \int (a + b\,\mathrm{atan}(c\,x))\, (d\,x)^m\, dx$$

input
```
int((a + b*atan(c*x))*(d*x)^m,x)
```

output
```
int((a + b*atan(c*x))*(d*x)^m, x)
```

Reduce [F]

$$\int (dx)^m (a + b \arctan(cx))\, dx = \frac{d^m \left(x^m atan(cx)\, bcmx + x^m acmx - x^m b + \left(\int \frac{x^m}{c^2 m\, x^3 + c^2 x^3 + mx + x} dx\right) b\, m^2 + \left(\int \frac{x^m}{c^2 m\, x^3 + c^2 x^3 + mx + x} dx\right) bm\right)}{cm\, (m + 1)}$$

input
```
int((d*x)^m*(a+b*atan(c*x)),x)
```

output
```
(d**m*(x**m*atan(c*x)*b*c*m*x + x**m*a*c*m*x - x**m*b + int(x**m/(c**2*m*x
**3 + c**2*x**3 + m*x + x),x)*b*m**2 + int(x**m/(c**2*m*x**3 + c**2*x**3 +
 m*x + x),x)*b*m))/(c*m*(m + 1))
```

3.57 $\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx$

Optimal result 453
Mathematica [N/A] 453
Rubi [N/A] 454
Maple [N/A] 454
Fricas [N/A] 455
Sympy [N/A] 455
Maxima [N/A] 455
Giac [N/A] 456
Mupad [N/A] 456
Reduce [N/A] 457

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx = \text{Int}\left(\frac{(dx)^m}{a+b\arctan(cx)},x\right)$$

output
```
Defer(Int)((d*x)^m/(a+b*arctan(c*x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx = \int \frac{(dx)^m}{a+b\arctan(cx)}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx$$

$$\Big\downarrow \text{5377}$$

$$\int \frac{(dx)^m}{a+b\arctan(cx)}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\arctan(cx)}dx$$

input
```
int((d*x)^m/(a+b*arctan(c*x)),x)
```

output
```
int((d*x)^m/(a+b*arctan(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx=\int \frac{(dx)^m}{b\arctan(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arctan(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx=\int \frac{(dx)^m}{a+b\,\mathrm{atan}\,(cx)}\,dx$$

input
```
integrate((d*x)**m/(a+b*atan(c*x)),x)
```

output
```
Integral((d*x)**m/(a + b*atan(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx=\int \frac{(dx)^m}{b\arctan(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arctan(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx=\int \frac{(dx)^m}{b\arctan(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctan(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx=\int \frac{(d\,x)^m}{a+b\,\mathrm{atan}\left(c\,x\right)}\,dx$$

input
```
int((d*x)^m/(a + b*atan(c*x)),x)
```

output
```
int((d*x)^m/(a + b*atan(c*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{(dx)^m}{a + b\arctan(cx)}\,dx = d^m\left(\int \frac{x^m}{atan\,(cx)\,b + a}dx\right)$$

input
```
int((d*x)^m/(a+b*atan(c*x)),x)
```

output
```
d**m*int(x**m/(atan(c*x)*b + a),x)
```

3.58 $\int (a+b\arctan(cx))^p\,dx$

Optimal result 458
Mathematica [N/A] 458
Rubi [N/A] 459
Maple [N/A] 459
Fricas [N/A] 460
Sympy [N/A] 460
Maxima [N/A] 460
Giac [N/A] 461
Mupad [N/A] 461
Reduce [N/A] 462

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int (a+b\arctan(cx))^p\,dx = \text{Int}((a+b\arctan(cx))^p, x)$$

output
```
Defer(Int)((a+b*arctan(c*x))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a+b\arctan(cx))^p\,dx = \int (a+b\arctan(cx))^p\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])^p,x]
```

output
```
Integrate[(a + b*ArcTan[c*x])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arctan(cx))^p\,dx$$

$$\Big\downarrow \text{ 5353}$$

$$\int (a+b\arctan(cx))^p dx$$

input
```
Int[(a + b*ArcTan[c*x])^p,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (a+b\arctan(cx))^p\,dx$$

input
```
int((a+b*arctan(c*x))^p,x)
```

output
```
int((a+b*arctan(c*x))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a + b \arctan(cx))^p \, dx = \int (b \arctan(cx) + a)^p \, dx$$

input
```
integrate((a+b*arctan(c*x))^p,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (a + b \arctan(cx))^p \, dx = \int (a + b \operatorname{atan}(cx))^p \, dx$$

input
```
integrate((a+b*atan(c*x))**p,x)
```

output
```
Integral((a + b*atan(c*x))**p, x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a + b \arctan(cx))^p \, dx = \int (b \arctan(cx) + a)^p \, dx$$

input
```
integrate((a+b*arctan(c*x))^p,x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a+b\arctan(cx))^p\,dx=\int (b\arctan{(cx)}+a)^p\,dx$$

input
```
integrate((a+b*arctan(c*x))^p,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a+b\arctan(cx))^p\,dx=\int (a+b\,\mathrm{atan}(c\,x))^p\,dx$$

input
```
int((a + b*atan(c*x))^p,x)
```

output
```
int((a + b*atan(c*x))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a+b\arctan(cx))^p\,dx = \int (atan(cx)\,b+a)^p\,dx$$

input
```
int((a+b*atan(c*x))^p,x)
```

output
```
int((atan(c*x)*b + a)**p,x)
```

3.59 $\int (dx)^m(a+b\arctan(cx))^p\,dx$

Optimal result 463
Mathematica [N/A] 463
Rubi [N/A] 464
Maple [N/A] 464
Fricas [N/A] 465
Sympy [F(-1)] 465
Maxima [N/A] 465
Giac [N/A] 466
Mupad [N/A] 466
Reduce [N/A] 466

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\text{Int}\left((dx)^m(a+b\arctan(cx))^p,x\right)$$

output
```
Defer(Int)((d*x)^m*(a+b*arctan(c*x))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\int (dx)^m(a+b\arctan(cx))^p\,dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x])^p,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m (a + b\arctan(cx))^p \, dx$$

$$\Big\downarrow \text{5377}$$

$$\int (dx)^m (a + b\arctan(cx))^p dx$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x])^p,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m (a + b\arctan(cx))^p \, dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x))^p,x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\int (dx)^m\,(b\arctan(cx)+a)^p\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="fricas")
```

output
```
integral((d*x)^m*(b*arctan(c*x) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*atan(c*x))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\int (dx)^m\,(b\arctan(cx)+a)^p\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="maxima")
```

output
```
integrate((d*x)^m*(b*arctan(c*x) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\int (dx)^m\,(b\arctan{(cx)}+a)^p\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="giac")
```

output
```
integrate((d*x)^m*(b*arctan(c*x) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\int (a+b\,\mathrm{atan}(c\,x))^p\,(d\,x)^m\,dx$$

input
```
int((a + b*atan(c*x))^p*(d*x)^m,x)
```

output
```
int((a + b*atan(c*x))^p*(d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=d^m\left(\int x^m(atan(cx)\,b+a)^p\,dx\right)$$

input
```
int((d*x)^m*(a+b*atan(c*x))^p,x)
```

output
```
d**m*int(x**m*(atan(c*x)*b + a)**p,x)
```

3.60 $\int x^7(a+b\arctan(cx^2))\,dx$

Optimal result 468
Mathematica [A] (verified) 468
Rubi [A] (verified) 469
Maple [A] (verified) 470
Fricas [A] (verification not implemented) 471
Sympy [A] (verification not implemented) 471
Maxima [A] (verification not implemented) 472
Giac [A] (verification not implemented) 472
Mupad [B] (verification not implemented) 472
Reduce [B] (verification not implemented) 473

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{bx^2}{8c^3}-\frac{bx^6}{24c}-\frac{b\arctan\left(cx^2\right)}{8c^4}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)$$

output
```
1/8*b*x^2/c^3-1/24*b*x^6/c-1/8*b*arctan(c*x^2)/c^4+1/8*x^8*(a+b*arctan(c*x
^2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{bx^2}{8c^3}-\frac{bx^6}{24c}+\frac{ax^8}{8}-\frac{b\arctan\left(cx^2\right)}{8c^4}+\frac{1}{8}bx^8\arctan\left(cx^2\right)$$

input
```
Integrate[x^7*(a + b*ArcTan[c*x^2]),x]
```

output
```
(b*x^2)/(8*c^3) - (b*x^6)/(24*c) + (a*x^8)/8 - (b*ArcTan[c*x^2])/(8*c^4) +
 (b*x^8*ArcTan[c*x^2])/8
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 807, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+b \arctan \left(c x^2\right)\right) \, dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{8} x^8 \left(a+b \arctan \left(c x^2\right)\right)-\frac{1}{4} b c \int \frac{x^9}{c^2 x^4+1}dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{8} x^8 \left(a+b \arctan \left(c x^2\right)\right)-\frac{1}{8} b c \int \frac{x^8}{c^2 x^4+1}dx^2$$

$$\Big\downarrow \text{ 254}$$

$$\frac{1}{8} x^8 \left(a+b \arctan \left(c x^2\right)\right)-\frac{1}{8} b c \int \left(\frac{x^4}{c^2}+\frac{1}{c^4 \left(c^2 x^4+1\right)}-\frac{1}{c^4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8} x^8 \left(a+b \arctan \left(c x^2\right)\right)-\frac{1}{8} b c \left(\frac{\arctan \left(c x^2\right)}{c^5}-\frac{x^2}{c^4}+\frac{x^6}{3 c^2}\right)$$

input
```
Int[x^7*(a + b*ArcTan[c*x^2]),x]
```

output
```
(x^8*(a + b*ArcTan[c*x^2]))/8 - (b*c*(-(x^2/c^4) + x^6/(3*c^2) + ArcTan[c*
x^2]/c^5))/8
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{a\,x^{8}}{8}+\frac{b\,x^{8}\arctan\left(c\,x^{2}\right)}{8}-\frac{b\,x^{6}}{24c}+\frac{b\,x^{2}}{8c^{3}}-\frac{b\arctan\left(c\,x^{2}\right)}{8c^{4}}$	50
parts	$\frac{a\,x^{8}}{8}+\frac{b\,x^{8}\arctan\left(c\,x^{2}\right)}{8}-\frac{b\,x^{6}}{24c}+\frac{b\,x^{2}}{8c^{3}}-\frac{b\arctan\left(c\,x^{2}\right)}{8c^{4}}$	50
parallelrisch	$\frac{3b\arctan\left(c\,x^{2}\right)x^{8}c^{4}+3a\,c^{4}x^{8}-b\,c^{3}x^{6}+3bc\,x^{2}-3b\arctan\left(c\,x^{2}\right)}{24c^{4}}$	56
risch	$-\frac{i x^{8}b\ln\left(i c\,x^{2}+1\right)}{16}+\frac{a\,x^{8}}{8}+\frac{i x^{8}b\ln\left(-i c\,x^{2}+1\right)}{16}-\frac{b\,x^{6}}{24c}+\frac{b\,x^{2}}{8c^{3}}-\frac{b\arctan\left(c\,x^{2}\right)}{8c^{4}}$	72
orering	$\frac{\left(13c^{4}x^{8}-14c^{2}x^{4}-27\right)\left(a+b\arctan\left(c\,x^{2}\right)\right)}{48c^{4}}-\frac{\left(c^{2}x^{4}-3\right)\left(c^{2}x^{4}+1\right)\left(7x^{6}\left(a+b\arctan\left(c\,x^{2}\right)\right)+\frac{2x^{8}bc}{c^{2}x^{4}+1}\right)}{48x^{6}c^{4}}$	95

input
```
int(x^7*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/8*a*x^8+1/8*b*x^8*arctan(c*x^2)-1/24*b*x^6/c+1/8*b*x^2/c^3-1/8*b*arctan(
c*x^2)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{3\,ac^4x^8-bc^3x^6+3\,bcx^2+3\,\left(bc^4x^8-b\right)\arctan\left(cx^2\right)}{24\,c^4}$$

input
```
integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
1/24*(3*a*c^4*x^8 - b*c^3*x^6 + 3*b*c*x^2 + 3*(b*c^4*x^8 - b)*arctan(c*x^2
))/c^4
```

Sympy [A] (verification not implemented)

Time = 27.45 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)\,dx=\begin{cases}\frac{ax^8}{8}+\frac{bx^8\,\mathrm{atan}\left(cx^2\right)}{8}-\frac{bx^6}{24c}+\frac{bx^2}{8c^3}-\frac{b\,\mathrm{atan}\left(cx^2\right)}{8c^4} & \text{for } c\neq 0\\ \frac{ax^8}{8} & \text{otherwise}\end{cases}$$

input
```
integrate(x**7*(a+b*atan(c*x**2)),x)
```

output
```
Piecewise((a*x**8/8 + b*x**8*atan(c*x**2)/8 - b*x**6/(24*c) + b*x**2/(8*c*
*3) - b*atan(c*x**2)/(8*c**4), Ne(c, 0)), (a*x**8/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^7(a+b\arctan(cx^2))\,dx = \frac{1}{8}ax^8+\frac{1}{24}\left(3x^8\arctan(cx^2)-c\left(\frac{c^2x^6-3x^2}{c^4}+\frac{3\arctan(cx^2)}{c^5}\right)\right)b$$

input
```
integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
1/8*a*x^8 + 1/24*(3*x^8*arctan(c*x^2) - c*((c^2*x^6 - 3*x^2)/c^4 + 3*arcta
n(c*x^2)/c^5))*b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int x^7(a+b\arctan(cx^2))\,dx = \frac{3acx^8+\left(3cx^8\arctan(cx^2)-\frac{3\arctan(cx^2)}{c^3}-\frac{c^9x^6-3c^7x^2}{c^9}\right)b}{24c}$$

input
```
integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
1/24*(3*a*c*x^8 + (3*c*x^8*arctan(c*x^2) - 3*arctan(c*x^2)/c^3 - (c^9*x^6
- 3*c^7*x^2)/c^9)*b)/c
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int x^7(a+b\arctan(cx^2))\,dx = \frac{a\,x^8}{8}+\frac{b\,x^2}{8\,c^3}-\frac{b\,x^6}{24\,c}-\frac{b\,\mathrm{atan}(c\,x^2)}{8\,c^4}+\frac{b\,x^8\,\mathrm{atan}(c\,x^2)}{8}$$

input
```
int(x^7*(a + b*atan(c*x^2)),x)
```

output
```
(a*x^8)/8 + (b*x^2)/(8*c^3) - (b*x^6)/(24*c) - (b*atan(c*x^2))/(8*c^4) + (
b*x^8*atan(c*x^2))/8
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)\,dx$$
$$=\frac{3atan(c\,x^2)\,b\,c^4x^8-3atan(c\,x^2)\,b+3a\,c^4x^8-b\,c^3x^6+3bc\,x^2}{24c^4}$$

input
```
int(x^7*(a+b*atan(c*x^2)),x)
```

output
```
(3*atan(c*x**2)*b*c**4*x**8 - 3*atan(c*x**2)*b + 3*a*c**4*x**8 - b*c**3*x*
*6 + 3*b*c*x**2)/(24*c**4)
```

3.61 $\int x^5(a+b\arctan(cx^2))\,dx$

Optimal result 474
Mathematica [A] (verified) 474
Rubi [A] (verified) 475
Maple [A] (verified) 476
Fricas [A] (verification not implemented) 477
Sympy [B] (verification not implemented) 477
Maxima [A] (verification not implemented) 478
Giac [A] (verification not implemented) 478
Mupad [B] (verification not implemented) 478
Reduce [B] (verification not implemented) 479

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)\,dx=-\frac{bx^4}{12c}+\frac{1}{6}x^6\left(a+b\arctan\left(cx^2\right)\right)+\frac{b\log\left(1+c^2x^4\right)}{12c^3}$$

output
```
-1/12*b*x^4/c+1/6*x^6*(a+b*arctan(c*x^2))+1/12*b*ln(c^2*x^4+1)/c^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)\,dx=-\frac{bx^4}{12c}+\frac{ax^6}{6}+\frac{1}{6}bx^6\arctan\left(cx^2\right)+\frac{b\log\left(1+c^2x^4\right)}{12c^3}$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x^2]),x]
```

output
```
-1/12*(b*x^4)/c + (a*x^6)/6 + (b*x^6*ArcTan[c*x^2])/6 + (b*Log[1 + c^2*x^4
])/(12*c^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b \arctan \left(c x^2\right)\right) \, dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{6} x^6 \left(a+b \arctan \left(c x^2\right)\right)-\frac{1}{3} b c \int \frac{x^7}{c^2 x^4+1}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{6} x^6 \left(a+b \arctan \left(c x^2\right)\right)-\frac{1}{12} b c \int \frac{x^4}{c^2 x^4+1}dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{6} x^6 \left(a+b \arctan \left(c x^2\right)\right)-\frac{1}{12} b c \int \left(\frac{1}{c^2}-\frac{1}{c^2 \left(c^2 x^4+1\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6} x^6 \left(a+b \arctan \left(c x^2\right)\right)-\frac{1}{12} b c \left(\frac{x^4}{c^2}-\frac{\log \left(c^2 x^4+1\right)}{c^4}\right)$$

input
```
Int[x^5*(a + b*ArcTan[c*x^2]),x]
```

output
```
(x^6*(a + b*ArcTan[c*x^2]))/6 - (b*c*(x^4/c^2 - Log[1 + c^2*x^4]/c^4))/12
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{x^{6} a}{6}+\frac{b \arctan\left(c \,x^{2}\right) x^{6}}{6}-\frac{b \,x^{4}}{12 c}+\frac{b \ln\left(c^{2} x^{4}+1\right)}{12 c^{3}}$	45
parts	$\frac{x^{6} a}{6}+\frac{b \arctan\left(c \,x^{2}\right) x^{6}}{6}-\frac{b \,x^{4}}{12 c}+\frac{b \ln\left(c^{2} x^{4}+1\right)}{12 c^{3}}$	45
parallelrisch	$\frac{2 x^{6} \arctan\left(c \,x^{2}\right) b \,c^{3}+2 a \,c^{3} x^{6}-b \,c^{2} x^{4}+b \ln\left(c^{2} x^{4}+1\right)}{12 c^{3}}$	52
risch	$-\frac{i x^{6} b \ln\left(i c \,x^{2}+1\right)}{12}+\frac{i x^{6} b \ln\left(-i c \,x^{2}+1\right)}{12}+\frac{x^{6} a}{6}-\frac{b \,x^{4}}{12 c}+\frac{b \ln\left(-c^{2} x^{4}-1\right)}{12 c^{3}}$	68

input
```
int(x^5*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*a+1/6*b*arctan(c*x^2)*x^6-1/12*b*x^4/c+1/12*b*ln(c^2*x^4+1)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int x^5(a+b\arctan(cx^2))\,dx=\frac{2\,bc^3x^6\arctan(cx^2)+2\,ac^3x^6-bc^2x^4+b\log(c^2x^4+1)}{12\,c^3}$$

input
```
integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
1/12*(2*b*c^3*x^6*arctan(c*x^2) + 2*a*c^3*x^6 - b*c^2*x^4 + b*log(c^2*x^4
+ 1))/c^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(39) = 78.

Time = 16.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int x^5(a+b\arctan(cx^2))\,dx$$
$$=\begin{cases}\frac{ax^6}{6}+\frac{bx^6\operatorname{atan}(cx^2)}{6}-\frac{bx^4}{12c}+\frac{b\sqrt{-\frac{1}{c^2}}\operatorname{atan}(cx^2)}{6c^2}+\frac{b\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6c^3} & \text{for } c\neq 0\\ \frac{ax^6}{6} & \text{otherwise}\end{cases}$$

input
```
integrate(x**5*(a+b*atan(c*x**2)),x)
```

output
```
Piecewise((a*x**6/6 + b*x**6*atan(c*x**2)/6 - b*x**4/(12*c) + b*sqrt(-1/c*
*2)*atan(c*x**2)/(6*c**2) + b*log(x**2 + sqrt(-1/c**2))/(6*c**3), Ne(c, 0)
), (a*x**6/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int x^5(a+b\arctan(cx^2))\,dx = \frac{1}{6}ax^6 + \frac{1}{12}\left(2x^6\arctan(cx^2) - \left(\frac{x^4}{c^2} - \frac{\log(c^2x^4+1)}{c^4}\right)c\right)b$$

input
```
integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/12*(2*x^6*arctan(c*x^2) - (x^4/c^2 - log(c^2*x^4 + 1)/c^4)*c
)*b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int x^5(a+b\arctan(cx^2))\,dx = \frac{2acx^6 + \left(2cx^6\arctan(cx^2) - x^4 + \frac{\log(c^2x^4+1)}{c^2}\right)b}{12c}$$

input
```
integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
1/12*(2*a*c*x^6 + (2*c*x^6*arctan(c*x^2) - x^4 + log(c^2*x^4 + 1)/c^2)*b)/
c
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int x^5(a+b\arctan(cx^2))\,dx = \frac{a\,x^6}{6} + \frac{b\,\ln(c^2\,x^4+1)}{12\,c^3} - \frac{b\,x^4}{12\,c} + \frac{b\,x^6\,\mathrm{atan}(c\,x^2)}{6}$$

input
```
int(x^5*(a + b*atan(c*x^2)),x)
```

output
```
(a*x^6)/6 + (b*log(c^2*x^4 + 1))/(12*c^3) - (b*x^4)/(12*c) + (b*x^6*atan(c
*x^2))/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)\,dx$$
$$=\frac{2atan(c\,x^2)\,b\,c^3x^6+\log\left(-\sqrt{c}\,\sqrt{2}\,x+c\,x^2+1\right)b+\log\left(\sqrt{c}\,\sqrt{2}\,x+c\,x^2+1\right)b+2a\,c^3x^6-b\,c^2x^4}{12c^3}$$

input
```
int(x^5*(a+b*atan(c*x^2)),x)
```

output
```
(2*atan(c*x**2)*b*c**3*x**6 + log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b + l
og(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b + 2*a*c**3*x**6 - b*c**2*x**4)/(12*c*
*3)
```

3.62 $\int x^3(a+b\arctan(cx^2))\,dx$

Optimal result 480
Mathematica [A] (verified) 480
Rubi [A] (verified) 481
Maple [A] (verified) 482
Fricas [A] (verification not implemented) 483
Sympy [A] (verification not implemented) 483
Maxima [A] (verification not implemented) 484
Giac [A] (verification not implemented) 484
Mupad [B] (verification not implemented) 484
Reduce [B] (verification not implemented) 485

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int x^3(a+b\arctan(cx^2))\,dx=-\frac{bx^2}{4c}+\frac{b\arctan(cx^2)}{4c^2}+\frac{1}{4}x^4(a+b\arctan(cx^2))$$

output
```
-1/4*b*x^2/c+1/4*b*arctan(c*x^2)/c^2+1/4*x^4*(a+b*arctan(c*x^2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^3(a+b\arctan(cx^2))\,dx=-\frac{bx^2}{4c}+\frac{ax^4}{4}+\frac{b\arctan(cx^2)}{4c^2}+\frac{1}{4}bx^4\arctan(cx^2)$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x^2]),x]
```

output
```
-1/4*(b*x^2)/c + (a*x^4)/4 + (b*ArcTan[c*x^2])/(4*c^2) + (b*x^4*ArcTan[c*x
^2])/4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 807, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b \arctan \left(c x^2\right)\right) \, dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4} x^4 \left(a+b \arctan \left(c x^2\right)\right)-\frac{1}{2} b c \int \frac{x^5}{c^2 x^4+1}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{4} x^4 \left(a+b \arctan \left(c x^2\right)\right)-\frac{1}{4} b c \int \frac{x^4}{c^2 x^4+1}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{4} x^4 \left(a+b \arctan \left(c x^2\right)\right)-\frac{1}{4} b c \left(\frac{x^2}{c^2}-\frac{\int \frac{1}{c^2 x^4+1}dx^2}{c^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4} x^4 \left(a+b \arctan \left(c x^2\right)\right)-\frac{1}{4} b c \left(\frac{x^2}{c^2}-\frac{\arctan \left(c x^2\right)}{c^3}\right)$$

input
```
Int[x^3*(a + b*ArcTan[c*x^2]),x]
```

output
```
(x^4*(a + b*ArcTan[c*x^2]))/4 - (b*c*(x^2/c^2 - ArcTan[c*x^2]/c^3))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{a\,x^{4}}{4}+\frac{b \arctan\left(c\,x^{2}\right) x^{4}}{4}-\frac{b\,x^{2}}{4c}+\frac{b \arctan\left(c\,x^{2}\right)}{4c^{2}}$	41
parts	$\frac{a\,x^{4}}{4}+\frac{b \arctan\left(c\,x^{2}\right) x^{4}}{4}-\frac{b\,x^{2}}{4c}+\frac{b \arctan\left(c\,x^{2}\right)}{4c^{2}}$	41
parallelrisch	$\frac{\arctan\left(c\,x^{2}\right) b\,c^{2} x^{4}+a\,c^{2} x^{4}-b c\,x^{2}+b \arctan\left(c\,x^{2}\right)}{4c^{2}}$	44
risch	$-\frac{i x^{4} b \ln\left(i c\,x^{2}+1\right)}{8}+\frac{i x^{4} b \ln\left(-i c\,x^{2}+1\right)}{8}+\frac{a\,x^{4}}{4}-\frac{b\,x^{2}}{4c}+\frac{b \arctan\left(c\,x^{2}\right)}{4c^{2}}+\frac{b^{2}}{16 a\,c^{2}}$	74
orering	$\frac{5\left(c^{2} x^{4}+1\right)\left(a+b \arctan\left(c\,x^{2}\right)\right)}{8c^{2}}-\frac{\left(c^{2} x^{4}+1\right)\left(3 x^{2}\left(a+b \arctan\left(c\,x^{2}\right)\right)+\frac{2 x^{4} b c}{c^{2} x^{4}+1}\right)}{8 c^{2} x^{2}}$	77

input
```
int(x^3*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+1/4*b*arctan(c*x^2)*x^4-1/4*b*x^2/c+1/4*b*arctan(c*x^2)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int x^3 (a + b \arctan(cx^2)) \, dx = \frac{ac^2x^4 - bcx^2 + (bc^2x^4 + b) \arctan(cx^2)}{4\,c^2}$$

input
```
integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
1/4*(a*c^2*x^4 - b*c*x^2 + (b*c^2*x^4 + b)*arctan(c*x^2))/c^2
```

Sympy [A] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^3 (a + b \arctan(cx^2)) \, dx = \begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{atan}(cx^2)}{4} - \frac{bx^2}{4c} + \frac{b \operatorname{atan}(cx^2)}{4c^2} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(a+b*atan(c*x**2)),x)
```

output
```
Piecewise((a*x**4/4 + b*x**4*atan(c*x**2)/4 - b*x**2/(4*c) + b*atan(c*x**2
)/(4*c**2), Ne(c, 0)), (a*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{1}{4}\,ax^4+\frac{1}{4}\left(x^4\arctan\left(cx^2\right)-c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\right)b$$

input
```
integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/4*(x^4*arctan(c*x^2) - c*(x^2/c^2 - arctan(c*x^2)/c^3))*b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{acx^4+\frac{\left(c^2x^4\arctan\left(cx^2\right)-cx^2+\arctan\left(cx^2\right)\right)b}{c}}{4\,c}$$

input
```
integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
1/4*(a*c*x^4 + (c^2*x^4*arctan(c*x^2) - c*x^2 + arctan(c*x^2))*b/c)/c
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{a\,x^4}{4}-\frac{b\,x^2}{4\,c}+\frac{b\,\mathrm{atan}(c\,x^2)}{4\,c^2}+\frac{b\,x^4\,\mathrm{atan}(c\,x^2)}{4}$$

input
```
int(x^3*(a + b*atan(c*x^2)),x)
```

output
```
(a*x^4)/4 - (b*x^2)/(4*c) + (b*atan(c*x^2))/(4*c^2) + (b*x^4*atan(c*x^2))/
4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^3(a+b\arctan(cx^2))\,dx=\frac{atan(c\,x^2)\,b\,c^2x^4+atan(c\,x^2)\,b+a\,c^2x^4-bc\,x^2}{4c^2}$$

input
```
int(x^3*(a+b*atan(c*x^2)),x)
```

output
```
(atan(c*x**2)*b*c**2*x**4 + atan(c*x**2)*b + a*c**2*x**4 - b*c*x**2)/(4*c*
*2)
```

3.63 $\int x(a+b\arctan(cx^2))\,dx$

Optimal result 486
Mathematica [A] (verified) 486
Rubi [A] (verified) 487
Maple [A] (verified) 488
Fricas [A] (verification not implemented) 488
Sympy [A] (verification not implemented) 489
Maxima [A] (verification not implemented) 489
Giac [A] (verification not implemented) 489
Mupad [B] (verification not implemented) 490
Reduce [B] (verification not implemented) 490

Optimal result

Integrand size = 12, antiderivative size = 41

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{ax^2}{2}+\frac{1}{2}bx^2\arctan\left(cx^2\right)-\frac{b\log\left(1+c^2x^4\right)}{4c}$$

output
```
1/2*a*x^2+1/2*b*x^2*arctan(c*x^2)-1/4*b*ln(c^2*x^4+1)/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{ax^2}{2}+\frac{1}{2}bx^2\arctan\left(cx^2\right)-\frac{b\log\left(1+c^2x^4\right)}{4c}$$

input
```
Integrate[x*(a + b*ArcTan[c*x^2]),x]
```

output
```
(a*x^2)/2 + (b*x^2*ArcTan[c*x^2])/2 - (b*Log[1 + c^2*x^4])/(4*c)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5361, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arctan(cx^2))\,dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{2}x^2(a+b\arctan(cx^2))-bc\int\frac{x^3}{c^2x^4+1}dx$$

$$\Big\downarrow \text{ 792}$$

$$\frac{1}{2}x^2(a+b\arctan(cx^2))-\frac{b\log(c^2x^4+1)}{4c}$$

input
```
Int[x*(a + b*ArcTan[c*x^2]),x]
```

output
```
(x^2*(a + b*ArcTan[c*x^2]))/2 - (b*Log[1 + c^2*x^4])/(4*c)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
parts	$\frac{a x^{2}}{2}+\frac{b x^{2} \arctan \left(c x^{2}\right)}{2}-\frac{b \ln \left(c^{2} x^{4}+1\right)}{4 c}$	36
derivativedivides	$\frac{a c x^{2}+b\left(c x^{2} \arctan \left(c x^{2}\right)-\frac{\ln \left(c^{2} x^{4}+1\right)}{2}\right)}{2 c}$	39
default	$\frac{a c x^{2}+b\left(c x^{2} \arctan \left(c x^{2}\right)-\frac{\ln \left(c^{2} x^{4}+1\right)}{2}\right)}{2 c}$	39
parallelrisch	$-\frac{-2 x^{2} \arctan \left(c x^{2}\right) b c-2 a c x^{2}+b \ln \left(c^{2} x^{4}+1\right)}{4 c}$	39
risch	$-\frac{i x^{2} b \ln \left(i c x^{2}+1\right)}{4}+\frac{i b x^{2} \ln \left(-i c x^{2}+1\right)}{4}+\frac{a x^{2}}{2}-\frac{b \ln \left(-c^{2} x^{4}-1\right)}{4 c}$	59

input
```
int(x*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/2*b*x^2*arctan(c*x^2)-1/4*b*ln(c^2*x^4+1)/c
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{2\,bcx^2\arctan\left(cx^2\right)+2\,acx^2-b\log\left(c^2x^4+1\right)}{4\,c}$$

input
```
integrate(x*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
1/4*(2*b*c*x^2*arctan(c*x^2) + 2*a*c*x^2 - b*log(c^2*x^4 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx$$
$$=\begin{cases}\frac{ax^2}{2}+\frac{bx^2\operatorname{atan}\left(cx^2\right)}{2}-\frac{b\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^2\right)}{2}-\frac{b\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2c} & \text{for } c\neq 0\\ \frac{ax^2}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(a+b*atan(c*x**2)),x)
```

output
```
Piecewise((a*x**2/2 + b*x**2*atan(c*x**2)/2 - b*sqrt(-1/c**2)*atan(c*x**2)
/2 - b*log(x**2 + sqrt(-1/c**2))/(2*c), Ne(c, 0)), (a*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{1}{2}\,ax^2+\frac{\left(2\,cx^2\arctan\left(cx^2\right)-\log\left(c^2x^4+1\right)\right)b}{4\,c}$$

input
```
integrate(x*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/4*(2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*b/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{2\,acx^2+\left(2\,cx^2\arctan\left(cx^2\right)-\log\left(c^2x^4+1\right)\right)b}{4\,c}$$

input
```
integrate(x*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
1/4*(2*a*c*x^2 + (2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*b)/c
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{a\,x^2}{2}-\frac{b\,\ln\left(c^2\,x^4+1\right)}{4\,c}+\frac{b\,x^2\,\mathrm{atan}\left(c\,x^2\right)}{2}$$

input
```
int(x*(a + b*atan(c*x^2)),x)
```

output
```
(a*x^2)/2 - (b*log(c^2*x^4 + 1))/(4*c) + (b*x^2*atan(c*x^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx$$
$$=\frac{2atan\left(c\,x^2\right)bc\,x^2-\log\left(-\sqrt{c}\,\sqrt{2}\,x+c\,x^2+1\right)b-\log\left(\sqrt{c}\,\sqrt{2}\,x+c\,x^2+1\right)b+2ac\,x^2}{4c}$$

input
```
int(x*(a+b*atan(c*x^2)),x)
```

output
```
(2*atan(c*x**2)*b*c*x**2 - log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b - log(
sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b + 2*a*c*x**2)/(4*c)
```

3.64 $\int \frac{a+b\arctan(cx^2)}{x}\,dx$

Optimal result . . . 491
Mathematica [A] (verified) . . . 491
Rubi [A] (verified) . . . 492
Maple [C] (verified) . . . 493
Fricas [F] . . . 494
Sympy [F] . . . 494
Maxima [F] . . . 494
Giac [F] . . . 495
Mupad [B] (verification not implemented) . . . 495
Reduce [F] . . . 495

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx = a\log(x) + \frac{1}{4}ib\,\text{PolyLog}\left(2,-icx^2\right) - \frac{1}{4}ib\,\text{PolyLog}\left(2,icx^2\right)$$

output
```
a*ln(x)+1/4*I*b*polylog(2,-I*c*x^2)-1/4*I*b*polylog(2,I*c*x^2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx = a\log(x) + \frac{1}{4}ib\,\text{PolyLog}\left(2,-icx^2\right) - \frac{1}{4}ib\,\text{PolyLog}\left(2,icx^2\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^2])/x,x]
```

output
```
a*Log[x] + (I/4)*b*PolyLog[2, (-I)*c*x^2] - (I/4)*b*PolyLog[2, I*c*x^2]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5359, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^2\right)}{x}dx$$

$$\Big\downarrow \text{5359}$$

$$\frac{1}{2}\int \frac{a+b\arctan\left(cx^2\right)}{x^2}dx^2$$

$$\Big\downarrow \text{5355}$$

$$\frac{1}{2}\left(\frac{1}{2}ib\int \frac{\log\left(1-icx^2\right)}{x^2}dx^2-\frac{1}{2}ib\int \frac{\log\left(icx^2+1\right)}{x^2}dx^2+a\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}\left(a\log\left(x^2\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,-icx^2\right)-\frac{1}{2}ib\,\text{PolyLog}\left(2,icx^2\right)\right)$$

input
```
Int[(a + b*ArcTan[c*x^2])/x,x]
```

output
```
(a*Log[x^2] + (I/2)*b*PolyLog[2, (-I)*c*x^2] - (I/2)*b*PolyLog[2, I*c*x^2]
)/2
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

method	result
default	$a\ln\left(x\right)+b\ln\left(x\right)\arctan\left(c\,x^{2}\right)-\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(c^{2}\_Z^{4}+1\right)}\frac{\ln\left(x\right)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^{2}}\right)}{2c}$
parts	$a\ln\left(x\right)+b\ln\left(x\right)\arctan\left(c\,x^{2}\right)-\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(c^{2}\_Z^{4}+1\right)}\frac{\ln\left(x\right)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^{2}}\right)}{2c}$
risch	$\frac{i\ln\left(-ic\,x^{2}+1\right)\ln\left(x\right)b}{2}-\frac{i\ln\left(x\right)\ln\left(1-ix\sqrt{-ic}\right)b}{2}-\frac{i\ln\left(x\right)\ln\left(1+ix\sqrt{-ic}\right)b}{2}-\frac{i\,\mathrm{dilog}\left(1-ix\sqrt{-ic}\right)b}{2}-\frac{i\,\mathrm{dilog}\left(1+ix\sqrt{-ic}\right)b}{2}+$

input
```
int((a+b*arctan(c*x^2))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*ln(x)*arctan(c*x^2)-1/2*b/c*sum(1/_R1^2*(ln(x)*ln((_R1-x)/_R1)+d
ilog((_R1-x)/_R1)),_R1=RootOf(_Z^4*c^2+1))
```

Fricas [F]

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx = \int \frac{b\arctan(cx^2)+a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))/x,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x^2) + a)/x, x)
```

Sympy [F]

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx = \int \frac{a+b\,\mathrm{atan}(cx^2)}{x}\,dx$$

input
```
integrate((a+b*atan(c*x**2))/x,x)
```

output
```
Integral((a + b*atan(c*x**2))/x, x)
```

Maxima [F]

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx = \int \frac{b\arctan(cx^2)+a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))/x,x, algorithm="maxima")
```

output
```
b*integrate(arctan(c*x^2)/x, x) + a*log(x)
```

Giac [F]

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx=\int \frac{b\arctan(cx^2)+a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx=a\ln(x)-\frac{b\left(\mathrm{Li}_2(1-c\,x^2\,1\mathrm{i})-\mathrm{Li}_2(1\mathrm{i}\,c\,x^2+1)\right)\,1\mathrm{i}}{4}$$

input
```
int((a + b*atan(c*x^2))/x,x)
```

output
```
a*log(x) - (b*(dilog(1 - c*x^2*1i) - dilog(c*x^2*1i + 1))*1i)/4
```

Reduce [F]

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx=\left(\int \frac{atan(c\,x^2)}{x}dx\right)b+\log(x)\,a$$

input
```
int((a+b*atan(c*x^2))/x,x)
```

output
```
int(atan(c*x**2)/x,x)*b + log(x)*a
```

3.65 $\int \frac{a+b\arctan(cx^2)}{x^3}\,dx$

Optimal result 496
Mathematica [A] (verified) 496
Rubi [A] (verified) 497
Maple [A] (verified) 498
Fricas [A] (verification not implemented) 499
Sympy [A] (verification not implemented) 499
Maxima [A] (verification not implemented) 500
Giac [A] (verification not implemented) 500
Mupad [B] (verification not implemented) 500
Reduce [B] (verification not implemented) 501

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx = -\frac{a+b\arctan(cx^2)}{2x^2}+bc\log(x)-\frac{1}{4}bc\log\left(1+c^2x^4\right)$$

output
```
-1/2*(a+b*arctan(c*x^2))/x^2+b*c*ln(x)-1/4*b*c*ln(c^2*x^4+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx = -\frac{a}{2x^2}-\frac{b\arctan(cx^2)}{2x^2}+bc\log(x)-\frac{1}{4}bc\log\left(1+c^2x^4\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^2])/x^3,x]
```

output
```
-1/2*a/x^2 - (b*ArcTan[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 + c^2*x^4
])/4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5361, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx$$

$$\Big\downarrow \text{5361}$$

$$bc\int \frac{1}{x(c^2x^4+1)}dx-\frac{a+b\arctan(cx^2)}{2x^2}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}bc\int \frac{1}{x^4(c^2x^4+1)}dx^4-\frac{a+b\arctan(cx^2)}{2x^2}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{4}bc\left(\int \frac{1}{x^4}dx^4-c^2\int \frac{1}{c^2x^4+1}dx^4\right)-\frac{a+b\arctan(cx^2)}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{4}bc\left(\log(x^4)-c^2\int \frac{1}{c^2x^4+1}dx^4\right)-\frac{a+b\arctan(cx^2)}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{4}bc(\log(x^4)-\log(c^2x^4+1))-\frac{a+b\arctan(cx^2)}{2x^2}$$

input
```
Int[(a + b*ArcTan[c*x^2])/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c*x^2])/x^2 + (b*c*(Log[x^4] - Log[1 + c^2*x^4]))/4
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{a}{2x^2}+b\left(-\frac{\arctan\left(c\,x^2\right)}{2x^2}+c\left(-\frac{\ln\left(c^2x^4+1\right)}{4}+\ln\left(x\right)\right)\right)$	39
parts	$-\frac{a}{2x^2}+b\left(-\frac{\arctan\left(c\,x^2\right)}{2x^2}+c\left(-\frac{\ln\left(c^2x^4+1\right)}{4}+\ln\left(x\right)\right)\right)$	39
parallelrisch	$\frac{4bc\ln(x)x^2-bc\ln\left(c^2x^4+1\right)x^2-2b\arctan\left(c\,x^2\right)-2a}{4x^2}$	45
risch	$\frac{ib\ln\left(ic\,x^2+1\right)}{4x^2}-\frac{-4bc\ln(x)x^2+bc\ln\left(-c^2x^4-1\right)x^2+ib\ln\left(-ic\,x^2+1\right)+2a}{4x^2}$	68

input
```
int((a+b*arctan(c*x^2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+b*(-1/2/x^2*arctan(c*x^2)+c*(-1/4*ln(c^2*x^4+1)+ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx = -\frac{bcx^2\log(c^2x^4+1)-4\,bcx^2\log(x)+2\,b\arctan(cx^2)+2\,a}{4\,x^2}$$

input
```
integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="fricas")
```

output
```
-1/4*(b*c*x^2*log(c^2*x^4 + 1) - 4*b*c*x^2*log(x) + 2*b*arctan(c*x^2) + 2*
a)/x^2
```

Sympy [A] (verification not implemented)

Time = 11.98 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.00

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx$$
$$= \begin{cases} -\frac{a}{2x^2} - \frac{bc^2\sqrt{-\frac{1}{c^2}}\operatorname{atan}(cx^2)}{2} + bc\log(x) - \frac{bc\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2} - \frac{b\operatorname{atan}(cx^2)}{2x^2} & \text{for } c \neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atan(c*x**2))/x**3,x)
```

output
```
Piecewise((-a/(2*x**2) - b*c**2*sqrt(-1/c**2)*atan(c*x**2)/2 + b*c*log(x)
- b*c*log(x**2 + sqrt(-1/c**2))/2 - b*atan(c*x**2)/(2*x**2), Ne(c, 0)), (-
a/(2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx=-\frac{1}{4}\left(c\left(\log\left(c^2x^4+1\right)-\log\left(x^4\right)\right)+\frac{2\arctan(cx^2)}{x^2}\right)b-\frac{a}{2\,x^2}$$

input
```
integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="maxima")
```

output
```
-1/4*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*b - 1/2*a/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx$$
$$=-\frac{bc^3x^2\log\left(c^2x^4+1\right)-2\,bc^3x^2\log\left(cx^2\right)+2\,bc^2\arctan\left(cx^2\right)+2\,ac^2}{4\,c^2x^2}$$

input
```
integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="giac")
```

output
```
-1/4*(b*c^3*x^2*log(c^2*x^4 + 1) - 2*b*c^3*x^2*log(c*x^2) + 2*b*c^2*arctan
(c*x^2) + 2*a*c^2)/(c^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx=b\,c\,\ln(x)-\frac{a}{2\,x^2}-\frac{b\,\mathrm{atan}(c\,x^2)}{2\,x^2}-\frac{b\,c\,\ln\left(c^2\,x^4+1\right)}{4}$$

input
```
int((a + b*atan(c*x^2))/x^3,x)
```

output
```
b*c*log(x) - a/(2*x^2) - (b*atan(c*x^2))/(2*x^2) - (b*c*log(c^2*x^4 + 1))/
4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.79

$$\int \frac{a + b\arctan(cx^2)}{x^3}\,dx$$
$$= \frac{-2atan(c\,x^2)\,b - \log\left(-\sqrt{c}\,\sqrt{2}\,x + c\,x^2 + 1\right) bc\,x^2 - \log\left(\sqrt{c}\,\sqrt{2}\,x + c\,x^2 + 1\right) bc\,x^2 + 4\log(x)\,bc\,x^2 - 2a}{4x^2}$$

input
```
int((a+b*atan(c*x^2))/x^3,x)
```

output
```
( - 2*atan(c*x**2)*b - log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b*c*x**2 - l
og(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b*c*x**2 + 4*log(x)*b*c*x**2 - 2*a)/(4*
x**2)
```

3.66 $\int \frac{a+b\arctan(cx^2)}{x^5}\,dx$

Optimal result 502
Mathematica [C] (verified) 502
Rubi [A] (verified) 503
Maple [A] (verified) 504
Fricas [A] (verification not implemented) 505
Sympy [A] (verification not implemented) 505
Maxima [A] (verification not implemented) 506
Giac [C] (verification not implemented) 506
Mupad [B] (verification not implemented) 506
Reduce [B] (verification not implemented) 507

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int \frac{a+b\arctan(cx^2)}{x^5}\,dx = -\frac{bc}{4x^2} - \frac{1}{4}bc^2\arctan\left(cx^2\right) - \frac{a+b\arctan(cx^2)}{4x^4}$$

output
```
-1/4*b*c/x^2-1/4*b*c^2*arctan(c*x^2)-1/4*(a+b*arctan(c*x^2))/x^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \frac{a+b\arctan(cx^2)}{x^5}\,dx = -\frac{a}{4x^4} - \frac{b\arctan(cx^2)}{4x^4} - \frac{bc\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^4\right)}{4x^2}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])/x^5,x]
```

output
```
-1/4*a/x^4 - (b*ArcTan[c*x^2])/(4*x^4) - (b*c*Hypergeometric2F1[-1/2, 1, 1
/2, -(c^2*x^4)])/(4*x^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 807, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^5}dx$$

$$\Big\downarrow \ 5361$$

$$\frac{1}{2}bc\int \frac{1}{x^3\left(c^2x^4+1\right)}dx-\frac{a+b\arctan\left(cx^2\right)}{4x^4}$$

$$\Big\downarrow \ 807$$

$$\frac{1}{4}bc\int \frac{1}{x^4\left(c^2x^4+1\right)}dx^2-\frac{a+b\arctan\left(cx^2\right)}{4x^4}$$

$$\Big\downarrow \ 264$$

$$\frac{1}{4}bc\left(c^2\left(-\int \frac{1}{c^2x^4+1}dx^2\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^2\right)}{4x^4}$$

$$\Big\downarrow \ 216$$

$$\frac{1}{4}bc\left(-c\arctan\left(cx^2\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^2\right)}{4x^4}$$

input
```
Int[(a + b*ArcTan[c*x^2])/x^5,x]
```

output
```
-1/4*(a + b*ArcTan[c*x^2])/x^4 + (b*c*(-x^(-2) - c*ArcTan[c*x^2]))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{a}{4x^{4}}-\frac{b \arctan\left(c\,x^{2}\right)}{4x^{4}}-\frac{bc}{4x^{2}}-\frac{b\,c^{2} \arctan\left(c\,x^{2}\right)}{4}$	39
parts	$-\frac{a}{4x^{4}}-\frac{b \arctan\left(c\,x^{2}\right)}{4x^{4}}-\frac{bc}{4x^{2}}-\frac{b\,c^{2} \arctan\left(c\,x^{2}\right)}{4}$	39
parallelrisch	$-\frac{\arctan\left(c\,x^{2}\right) b\,c^{2} x^{4}-a\,c^{2} x^{4}+bc\,x^{2}+b \arctan\left(c\,x^{2}\right)+a}{4x^{4}}$	45
orering	$\frac{\left(-\frac{7}{8}x^{5} c^{2}-\frac{7}{8}x\right)\left(a+b \arctan\left(c\,x^{2}\right)\right)}{x^{5}}-\frac{\left(c^{2} x^{4}+1\right) x^{2} \left(\frac{2bc}{x^{4}\left(c^{2} x^{4}+1\right)}-\frac{5\left(a+b \arctan\left(c\,x^{2}\right)\right)}{x^{6}}\right)}{8}$	76
risch	$\frac{ib \ln\left(ic\,x^{2}+1\right)}{8x^{4}}-\frac{-ib\,c^{2} \ln\left(c\,x^{2}-i\right) x^{4}+ib\,c^{2} \ln\left(c\,x^{2}+i\right) x^{4}+2bc\,x^{2}+ib \ln\left(-ic\,x^{2}+1\right)+2a}{8x^{4}}$	87

input
```
int((a+b*arctan(c*x^2))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/x^4-1/4*b*arctan(c*x^2)/x^4-1/4*b*c/x^2-1/4*b*c^2*arctan(c*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{a+b\arctan(cx^2)}{x^5}\,dx = -\frac{bcx^2 + (bc^2x^4 + b)\arctan(cx^2) + a}{4\,x^4}$$

input
```
integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="fricas")
```

output
```
-1/4*(b*c*x^2 + (b*c^2*x^4 + b)*arctan(c*x^2) + a)/x^4
```

Sympy [A] (verification not implemented)

Time = 11.88 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arctan(cx^2)}{x^5}\,dx = -\frac{a}{4x^4} - \frac{bc^2\operatorname{atan}(cx^2)}{4} - \frac{bc}{4x^2} - \frac{b\operatorname{atan}(cx^2)}{4x^4}$$

input
```
integrate((a+b*atan(c*x**2))/x**5,x)
```

output
```
-a/(4*x**4) - b*c**2*atan(c*x**2)/4 - b*c/(4*x**2) - b*atan(c*x**2)/(4*x**
4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^5}\,dx = -\frac{1}{4}\left(\left(c\arctan\left(cx^2\right)+\frac{1}{x^2}\right)c+\frac{\arctan\left(cx^2\right)}{x^4}\right)b-\frac{a}{4\,x^4}$$

input
```
integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="maxima")
```

output
```
-1/4*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*b - 1/4*a/x^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^5}\,dx = \frac{i\,bc^5x^4\log\left(i\,cx^2+1\right)-i\,bc^5x^4\log\left(-i\,cx^2+1\right)-2\,bc^4x^2-2\,bc^3\arctan\left(cx^2\right)-2\,ac^3}{8\,c^3x^4}$$

input
```
integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="giac")
```

output
```
1/8*(I*b*c^5*x^4*log(I*c*x^2 + 1) - I*b*c^5*x^4*log(-I*c*x^2 + 1) - 2*b*c^
4*x^2 - 2*b*c^3*arctan(c*x^2) - 2*a*c^3)/(c^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^5}\,dx = -\frac{\frac{b\,c\,x^2}{2}+\frac{a}{2}}{2\,x^4}-\frac{b\,c^2\,\mathrm{atan}\left(c\,x^2\right)}{4}-\frac{b\,\mathrm{atan}\left(c\,x^2\right)}{4\,x^4}$$

input
```
int((a + b*atan(c*x^2))/x^5,x)
```

output
```
- (a/2 + (b*c*x^2)/2)/(2*x^4) - (b*c^2*atan(c*x^2))/4 - (b*atan(c*x^2))/(4
*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{a + b\arctan(cx^2)}{x^5}\,dx = \frac{-atan(c\,x^2)\,b\,c^2x^4 - atan(c\,x^2)\,b - a - bc\,x^2}{4x^4}$$

input
```
int((a+b*atan(c*x^2))/x^5,x)
```

output
```
( - (atan(c*x**2)*b*c**2*x**4 + atan(c*x**2)*b + a + b*c*x**2))/(4*x**4)
```

3.67 $\int \frac{a+b\arctan(cx^2)}{x^7}\,dx$

Optimal result 508
Mathematica [A] (verified) 508
Rubi [A] (verified) 509
Maple [A] (verified) 510
Fricas [A] (verification not implemented) 511
Sympy [A] (verification not implemented) 511
Maxima [A] (verification not implemented) 512
Giac [A] (verification not implemented) 512
Mupad [B] (verification not implemented) 513
Reduce [B] (verification not implemented) 513

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx = -\frac{bc}{12x^4}-\frac{a+b\arctan(cx^2)}{6x^6}-\frac{1}{3}bc^3\log(x)+\frac{1}{12}bc^3\log(1+c^2x^4)$$

output
```
-1/12*b*c/x^4-1/6*(a+b*arctan(c*x^2))/x^6-1/3*b*c^3*ln(x)+1/12*b*c^3*ln(c^
2*x^4+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx = -\frac{a}{6x^6}-\frac{bc}{12x^4}-\frac{b\arctan(cx^2)}{6x^6} -\frac{1}{3}bc^3\log(x)+\frac{1}{12}bc^3\log(1+c^2x^4)$$

input
```
Integrate[(a + b*ArcTan[c*x^2])/x^7,x]
```

output
```
-1/6*a/x^6 - (b*c)/(12*x^4) - (b*ArcTan[c*x^2])/(6*x^6) - (b*c^3*Log[x])/3
 + (b*c^3*Log[1 + c^2*x^4])/12
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5361, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^7}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}bc\int \frac{1}{x^5\left(c^2x^4+1\right)}dx-\frac{a+b\arctan\left(cx^2\right)}{6x^6}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{12}bc\int \frac{1}{x^8\left(c^2x^4+1\right)}dx^4-\frac{a+b\arctan\left(cx^2\right)}{6x^6}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{12}bc\int \left(\frac{c^4}{c^2x^4+1}-\frac{c^2}{x^4}+\frac{1}{x^8}\right)dx^4-\frac{a+b\arctan\left(cx^2\right)}{6x^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{12}bc\left(c^2\left(-\log\left(x^4\right)\right)+c^2\log\left(c^2x^4+1\right)-\frac{1}{x^4}\right)-\frac{a+b\arctan\left(cx^2\right)}{6x^6}$$

input
```
Int[(a + b*ArcTan[c*x^2])/x^7,x]
```

output
```
-1/6*(a + b*ArcTan[c*x^2])/x^6 + (b*c*(-x^(-4) - c^2*Log[x^4] + c^2*Log[1
+ c^2*x^4]))/12
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a}{6x^{6}}+b\left(-\frac{\arctan\left(c\,x^{2}\right)}{6x^{6}}+\frac{c\left(\frac{c^{2}\ln\left(c^{2}x^{4}+1\right)}{4}-\frac{1}{4x^{4}}-c^{2}\ln\left(x\right)\right)}{3}\right)$	53
parts	$-\frac{a}{6x^{6}}+b\left(-\frac{\arctan\left(c\,x^{2}\right)}{6x^{6}}+\frac{c\left(\frac{c^{2}\ln\left(c^{2}x^{4}+1\right)}{4}-\frac{1}{4x^{4}}-c^{2}\ln\left(x\right)\right)}{3}\right)$	53
parallelrisch	$-\frac{4b\,c^{3}\ln\left(x\right)x^{6}-b\,c^{3}\ln\left(c^{2}x^{4}+1\right)x^{6}-b\,c^{3}x^{6}+bc\,x^{2}+2b\arctan\left(c\,x^{2}\right)+2a}{12x^{6}}$	64
risch	$\frac{ib\ln\left(ic\,x^{2}+1\right)}{12x^{6}}-\frac{4b\,c^{3}\ln\left(x\right)x^{6}-b\,c^{3}\ln\left(c^{2}x^{4}+1\right)x^{6}+bc\,x^{2}+ib\ln\left(-ic\,x^{2}+1\right)+2a}{12x^{6}}$	78

input
```
int((a+b*arctan(c*x^2))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a/x^6+b*(-1/6/x^6*arctan(c*x^2)+1/3*c*(1/4*c^2*ln(c^2*x^4+1)-1/4/x^4-
c^2*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx = \frac{bc^3x^6\log(c^2x^4+1)-4\,bc^3x^6\log(x)-bcx^2-2\,b\arctan(cx^2)-2\,a}{12\,x^6}$$

input
```
integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="fricas")
```

output
```
1/12*(b*c^3*x^6*log(c^2*x^4 + 1) - 4*b*c^3*x^6*log(x) - b*c*x^2 - 2*b*arct
an(c*x^2) - 2*a)/x^6
```

Sympy [A] (verification not implemented)

Time = 39.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.67

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx = \begin{cases} -\frac{a}{6x^6}+\frac{bc^4\sqrt{-\frac{1}{c^2}}\operatorname{atan}(cx^2)}{6}-\frac{bc^3\log(x)}{3}+\frac{bc^3\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6}-\frac{bc}{12x^4}-\frac{b\operatorname{atan}(cx^2)}{6x^6} & \text{for } c\neq 0 \\ -\frac{a}{6x^6} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atan(c*x**2))/x**7,x)
```

output
```
Piecewise((-a/(6*x**6) + b*c**4*sqrt(-1/c**2)*atan(c*x**2)/6 - b*c**3*log(
x)/3 + b*c**3*log(x**2 + sqrt(-1/c**2))/6 - b*c/(12*x**4) - b*atan(c*x**2)
/(6*x**6), Ne(c, 0)), (-a/(6*x**6), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx$$
$$=\frac{1}{12}\left(\left(c^2\log(c^2x^4+1)-c^2\log(x^4)-\frac{1}{x^4}\right)c-\frac{2\arctan(cx^2)}{x^6}\right)b-\frac{a}{6\,x^6}$$

input
```
integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="maxima")
```

output
```
1/12*((c^2*log(c^2*x^4 + 1) - c^2*log(x^4) - 1/x^4)*c - 2*arctan(c*x^2)/x^
6)*b - 1/6*a/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx$$
$$=\frac{bc^7x^6\log(c^2x^4+1)-2\,bc^7x^6\log(cx^2)-bc^5x^2-2\,bc^4\arctan(cx^2)-2\,ac^4}{12\,c^4x^6}$$

input
```
integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="giac")
```

output
```
1/12*(b*c^7*x^6*log(c^2*x^4 + 1) - 2*b*c^7*x^6*log(c*x^2) - b*c^5*x^2 - 2*
b*c^4*arctan(c*x^2) - 2*a*c^4)/(c^4*x^6)
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx = \frac{b\,c^3\,\ln(c^2\,x^4+1)}{12}-\frac{a}{6\,x^6}-\frac{b\,c^3\,\ln(x)}{3}-\frac{b\,\mathrm{atan}(c\,x^2)}{6\,x^6}-\frac{b\,c}{12\,x^4}$$

input
```
int((a + b*atan(c*x^2))/x^7,x)
```

output
```
(b*c^3*log(c^2*x^4 + 1))/12 - a/(6*x^6) - (b*c^3*log(x))/3 - (b*atan(c*x^2
))/(6*x^6) - (b*c)/(12*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.47

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx$$
$$= \frac{-2atan(c\,x^2)\,b+\log\left(-\sqrt{c}\,\sqrt{2}\,x+c\,x^2+1\right)b\,c^3x^6+\log\left(\sqrt{c}\,\sqrt{2}\,x+c\,x^2+1\right)b\,c^3x^6-4\log(x)\,b\,c^3x^6-2}{12x^6}$$

input
```
int((a+b*atan(c*x^2))/x^7,x)
```

output
```
( - 2*atan(c*x**2)*b + log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b*c**3*x**6
+ log(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b*c**3*x**6 - 4*log(x)*b*c**3*x**6 -
 2*a - b*c*x**2)/(12*x**6)
```

3.68 $\int x^4(a + b\arctan(cx^2))\, dx$

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Reduce [B] (verification not implemented) 523

Optimal result

Integrand size = 14, antiderivative size = 127

$$\int x^4(a+b\arctan(cx^2))\, dx = -\frac{2bx^3}{15c}+\frac{1}{5}x^5(a+b\arctan(cx^2))-\frac{b\arctan(1-\sqrt{2}\sqrt{c}x)}{5\sqrt{2}c^{5/2}}+\frac{b\arctan(1+\sqrt{2}\sqrt{c}x)}{5\sqrt{2}c^{5/2}}-\frac{b\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{1+cx^2}\right)}{5\sqrt{2}c^{5/2}}$$

output
```
-2/15*b*x^3/c+1/5*x^5*(a+b*arctan(c*x^2))+1/10*b*arctan(-1+2^(1/2)*c^(1/2)
*x)*2^(1/2)/c^(5/2)+1/10*b*arctan(1+2^(1/2)*c^(1/2)*x)*2^(1/2)/c^(5/2)-1/1
0*b*arctanh(2^(1/2)*c^(1/2)*x/(c*x^2+1))*2^(1/2)/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.41

$$\int x^4(a+b\arctan(cx^2))\, dx = -\frac{2bx^3}{15c}+\frac{ax^5}{5}+\frac{1}{5}bx^5\arctan(cx^2)+\frac{b\arctan\left(\frac{-\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{5\sqrt{2}c^{5/2}}+\frac{b\arctan\left(\frac{\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{5\sqrt{2}c^{5/2}}+\frac{b\log(1-\sqrt{2}\sqrt{c}x+cx^2)}{10\sqrt{2}c^{5/2}}-\frac{b\log(1+\sqrt{2}\sqrt{c}x+cx^2)}{10\sqrt{2}c^{5/2}}$$

input
```
Integrate[x^4*(a + b*ArcTan[c*x^2]),x]
```

output
```
(-2*b*x^3)/(15*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x^2])/5 + (b*ArcTan[(-Sqrt
[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]*c^(5/2)) + (b*ArcTan[(Sqrt[2] + 2*
Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]*c^(5/2)) + (b*Log[1 - Sqrt[2]*Sqrt[c]*x +
c*x^2])/(10*Sqrt[2]*c^(5/2)) - (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*
Sqrt[2]*c^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.42, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5361, 843, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+b \arctan \left(c x^2\right)\right) \, dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{5} x^5 \left(a+b \arctan \left(c x^2\right)\right)-\frac{2}{5} b c \int \frac{x^6}{c^2 x^4+1}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{5} x^5 \left(a+b \arctan \left(c x^2\right)\right)-\frac{2}{5} b c \left(\frac{x^3}{3 c^2}-\frac{\int \frac{x^2}{c^2 x^4+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{826}$$

$$\frac{1}{5} x^5 \left(a+b \arctan \left(c x^2\right)\right)-\frac{2}{5} b c \left(\frac{x^3}{3 c^2}-\frac{\frac{\int \frac{c x^2+1}{c^2 x^4+1}dx}{2 c}-\frac{\int \frac{1-c x^2}{c^2 x^4+1}dx}{2 c}}{c^2}\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{5}bc\left(\frac{x^3}{3c^2}-\frac{\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}}{2c}-\frac{\int \frac{1-cx^2}{c^2x^4+1}dx}{2c}}{c^2}\right)$$

$\big\downarrow$ 1082

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{5}bc\left(\frac{x^3}{3c^2}-\frac{\frac{\frac{\int \frac{1}{-\left(1-\sqrt{2}\sqrt{c}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{-\left(\sqrt{2}\sqrt{c}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\int \frac{1-cx^2}{c^2x^4+1}dx}{2c}}{c^2}\right)$$

$\big\downarrow$ 217

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{5}bc\left(\frac{x^3}{3c^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\int \frac{1-cx^2}{c^2x^4+1}dx}{2c}}{c^2}\right)$$

$\big\downarrow$ 1479

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{5}bc\left(\frac{x^3}{3c^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{-\frac{\int -\frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}}{2c}}{c^2}\right)$$

$\big\downarrow$ 25

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{5}bc\left(\frac{x^3}{3c^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\frac{\int \frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}}{2c}}{c^2}\right)$$

$\big\downarrow$ 27

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)-$$
$$\frac{2}{5}bc\left(\frac{x^3}{3c^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\frac{\int\frac{\sqrt{2}-2\sqrt{c}x}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2\sqrt{2}c}+\frac{\int\frac{\sqrt{2}\sqrt{c}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}}{2c}}{c^2}\right)$$

$\big\downarrow$ 1103

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)-$$
$$\frac{2}{5}bc\left(\frac{x^3}{3c^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\frac{\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}-\frac{\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}}{2c}}{c^2}\right)$$

input
```
Int[x^4*(a + b*ArcTan[c*x^2]),x]
```

output
```
(x^5*(a + b*ArcTan[c*x^2]))/5 - (2*b*c*(x^3/(3*c^2) - ((-(ArcTan[1 - Sqrt[
2]*Sqrt[c]*x]/(Sqrt[2]*Sqrt[c])) + ArcTan[1 + Sqrt[2]*Sqrt[c]*x]/(Sqrt[2]*
Sqrt[c]))/(2*c) - (-1/2*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2]/(Sqrt[2]*Sqrt[c
]) + Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2]/(2*Sqrt[2]*Sqrt[c]))/(2*c))/c^2))/
5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

method	result
default	$\frac{a\,x^{5}}{5}+b\left(\frac{x^{5}\arctan\left(c\,x^{2}\right)}{5}-\frac{2c\left(\frac{x^{3}}{3c^{2}}-\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^{4}\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}\right)}{5}\right)$
parts	$\frac{a\,x^{5}}{5}+b\left(\frac{x^{5}\arctan\left(c\,x^{2}\right)}{5}-\frac{2c\left(\frac{x^{3}}{3c^{2}}-\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^{4}\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}\right)}{5}\right)$

input
```
int(x^4*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+b*(1/5*x^5*arctan(c*x^2)-2/5*c*(1/3/c^2*x^3-1/8/c^4/(1/c^2)^(1/4
)*2^(1/2)*(ln((x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/
4)*x*2^(1/2)+(1/c^2)^(1/2)))+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(
2^(1/2)/(1/c^2)^(1/4)*x-1))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.01

$$\int x^4\left(a+b\arctan\left(cx^2\right)\right)\,dx$$

$$= \frac{12\,bc^2x^5\arctan\left(cx^2\right)+12\,ac^2x^5-8\,bcx^3+\frac{6\sqrt{2}b\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}}+\frac{6\sqrt{2}b\arctan\left(\sqrt{2}\sqrt{c}x-1\right)}{\sqrt{c}}-\frac{3\sqrt{2}b\log\left(cx^2+\sqrt{2}\sqrt{c}\right.}{\sqrt{c}}}{60\,c^2}$$

input
```
integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
1/60*(12*b*c^2*x^5*arctan(c*x^2) + 12*a*c^2*x^5 - 8*b*c*x^3 + 6*sqrt(2)*b*
arctan(sqrt(2)*sqrt(c)*x + 1)/sqrt(c) + 6*sqrt(2)*b*arctan(sqrt(2)*sqrt(c)
*x - 1)/sqrt(c) - 3*sqrt(2)*b*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) +
 3*sqrt(2)*b*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/sqrt(c))/c^2
```

Sympy [A] (verification not implemented)

Time = 12.44 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

$$\int x^4\left(a+b\arctan\left(cx^2\right)\right)\,dx$$

$$= \begin{cases} \frac{ax^5}{5}+\frac{bx^5\operatorname{atan}\left(cx^2\right)}{5}-\frac{2bx^3}{15c}-\frac{b\sqrt[4]{-\frac{1}{c^2}}\operatorname{atan}\left(cx^2\right)}{5c^2}+\frac{b\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{5c^3\sqrt[4]{-\frac{1}{c^2}}}-\frac{b\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{10c^3\sqrt[4]{-\frac{1}{c^2}}}+\frac{b\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{5c^3\sqrt[4]{-\frac{1}{c^2}}} & \text{for} \\ \frac{ax^5}{5} & \text{oth} \end{cases}$$

input
```
integrate(x**4*(a+b*atan(c*x**2)),x)
```

output
```
Piecewise((a*x**5/5 + b*x**5*atan(c*x**2)/5 - 2*b*x**3/(15*c) - b*(-1/c**2
)**(1/4)*atan(c*x**2)/(5*c**2) + b*log(x - (-1/c**2)**(1/4))/(5*c**3*(-1/c
**2)**(1/4)) - b*log(x**2 + sqrt(-1/c**2))/(10*c**3*(-1/c**2)**(1/4)) + b*
atan(x/(-1/c**2)**(1/4))/(5*c**3*(-1/c**2)**(1/4)), Ne(c, 0)), (a*x**5/5,
True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.16

$$\int x^4\left(a+b\arctan\left(cx^2\right)\right)\,dx = \frac{1}{5}ax^5 + \frac{1}{60}\left(12\,x^5\arctan\left(cx^2\right) - c\left(\frac{8\,x^3}{c^2} - \frac{3\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx+\sqrt{2}\sqrt{c}\right)}{2\,\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx-\sqrt{2}\sqrt{c}\right)}{2\,\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{\sqrt{2}\log\left(cx^2 \cdots\right.}{c \cdots}\right.}{c^2}\right.\right.$$

input
```
integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/60*(12*x^5*arctan(c*x^2) - c*(8*x^3/c^2 - 3*(2*sqrt(2)*arcta
n(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c^(3/2) + 2*sqrt(2)*arcta
n(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c^(3/2) - sqrt(2)*log(c*x
^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*
x + 1)/c^(3/2))/c^2))*b
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.33

$$\int x^4 \left(a + b \arctan \left(cx^2\right)\right) \, dx$$
$$= \frac{1}{20} bc^9 \left(\frac{2\sqrt{2}\sqrt{|c|} \arctan \left(\frac{1}{2}\sqrt{2}\left(2x + \frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^{12}} + \frac{2\sqrt{2}\sqrt{|c|} \arctan \left(\frac{1}{2}\sqrt{2}\left(2x - \frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^{12}} - \frac{\sqrt{2}1}{} \right.$$
$$+ \frac{3\,bcx^5 \arctan \left(cx^2\right) + 3\,acx^5 - 2\,bx^3}{15\,c}$$

input
```
integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
1/20*b*c^9*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(
abs(c)))*sqrt(abs(c)))/c^12 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^12 - sqrt(2)*log(x^2 + sqrt(2)*
x/sqrt(abs(c)) + 1/abs(c))/(c^10*abs(c)^(3/2)) + sqrt(2)*sqrt(abs(c))*log(
x^2 - sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^12) + 1/15*(3*b*c*x^5*arctan(c*
x^2) + 3*a*c*x^5 - 2*b*x^3)/c
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.50

$$\int x^4 \left(a + b \arctan \left(cx^2\right)\right) \, dx = \frac{a\,x^5}{5} - \frac{2\,b\,x^3}{15\,c} + \frac{b\,x^5\,\mathrm{atan}(c\,x^2)}{5} + \frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)}{5\,c^{5/2}} + \frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{5\,c^{5/2}}$$

input
```
int(x^4*(a + b*atan(c*x^2)),x)
```

output
```
(a*x^5)/5 - (2*b*x^3)/(15*c) + (b*x^5*atan(c*x^2))/5 + ((-1)^(1/4)*b*atan(
(-1)^(1/4)*c^(1/2)*x))/(5*c^(5/2)) + ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)
*x*1i)*1i)/(5*c^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

$$\int x^4 \left(a + b\arctan\left(cx^2\right)\right)\,dx$$

$$= \frac{-12\sqrt{c}\,\sqrt{2}\,atan\left(\frac{\sqrt{c}\,\sqrt{2}-2cx}{\sqrt{c}\,\sqrt{2}}\right)b - 6\sqrt{c}\,\sqrt{2}\,atan(c\,x^2)\,b + 12atan(c\,x^2)\,b\,c^3x^5 + 3\sqrt{c}\,\sqrt{2}\log\left(-\sqrt{c}\,\sqrt{2}\,x + c\right.}{60c^3}$$

input
```
int(x^4*(a+b*atan(c*x^2)),x)
```

output
```
( - 12*sqrt(c)*sqrt(2)*atan((sqrt(c)*sqrt(2) - 2*c*x)/(sqrt(c)*sqrt(2)))*b
 - 6*sqrt(c)*sqrt(2)*atan(c*x**2)*b + 12*atan(c*x**2)*b*c**3*x**5 + 3*sqrt
(c)*sqrt(2)*log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b - 3*sqrt(c)*sqrt(2)*l
og(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b + 12*a*c**3*x**5 - 8*b*c**2*x**3)/(60
*c**3)
```

3.69 $\int x^2(a+b\arctan(cx^2))\,dx$

Optimal result 524
Mathematica [A] (verified) 524
Rubi [A] (verified) 525
Maple [A] (verified) 529
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Giac [A] (verification not implemented) 532
Mupad [B] (verification not implemented) 533
Reduce [B] (verification not implemented) 533

Optimal result

Integrand size = 14, antiderivative size = 125

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)\,dx=-\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{b\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}}+\frac{b\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{1+cx^2}\right)}{3\sqrt{2}c^{3/2}}$$

output
```
-2/3*b*x/c+1/3*x^3*(a+b*arctan(c*x^2))+1/6*b*arctan(-1+2^(1/2)*c^(1/2)*x)*
2^(1/2)/c^(3/2)+1/6*b*arctan(1+2^(1/2)*c^(1/2)*x)*2^(1/2)/c^(3/2)+1/6*b*ar
ctanh(2^(1/2)*c^(1/2)*x/(c*x^2+1))*2^(1/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.42

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)\,dx=-\frac{2bx}{3c}+\frac{ax^3}{3}+\frac{1}{3}bx^3\arctan\left(cx^2\right)+\frac{b\arctan\left(\frac{-\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{3\sqrt{2}c^{3/2}}+\frac{b\arctan\left(\frac{\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{3\sqrt{2}c^{3/2}}-\frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}}+\frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x^2]),x]
```

output
```
(-2*b*x)/(3*c) + (a*x^3)/3 + (b*x^3*ArcTan[c*x^2])/3 + (b*ArcTan[(-Sqrt[2]
 + 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]*c^(3/2)) + (b*ArcTan[(Sqrt[2] + 2*Sqr
t[c]*x)/Sqrt[2]])/(3*Sqrt[2]*c^(3/2)) - (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x
^2])/(6*Sqrt[2]*c^(3/2)) + (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[
2]*c^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.35, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5361, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\int\frac{x^4}{c^2x^4+1}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\left(\frac{x}{c^2}-\frac{\int\frac{1}{c^2x^4+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\left(\frac{x}{c^2}-\frac{\frac{1}{2}\int\frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\int\frac{cx^2+1}{c^2x^4+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\left(\frac{x}{c^2}-\frac{\frac{1}{2}\int\frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\int\frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}\right)}{c^2}\right)$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\left(\frac{x}{c^2}-\frac{\frac{1}{2}\int\frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\int\frac{1}{-\left(1-\sqrt{2}\sqrt{c}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{\int\frac{1}{-\left(\sqrt{2}\sqrt{c}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}\right)}{c^2}\right)$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\left(\frac{x}{c^2}-\frac{\frac{1}{2}\int\frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)}{c^2}\right)$$

$$\Big\downarrow \; 1479$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\left(\frac{x}{c^2}-\frac{\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)}{c^2}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\left(\frac{x}{c^2}-\frac{\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)}{c^2}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\left(\frac{x}{c^2}-\frac{\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{c}x}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2\sqrt{2}c}+\frac{\int\frac{\sqrt{2}\sqrt{c}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)}{c^2}\right)$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\left(\frac{x}{c^2}-\frac{\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}-\frac{\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}\right)}{c^2}\right)$$

input
```
Int[x^2*(a + b*ArcTan[c*x^2]),x]
```

output
```
(x^3*(a + b*ArcTan[c*x^2]))/3 - (2*b*c*(x/c^2 - ((-(ArcTan[1 - Sqrt[2]*Sqr
t[c]*x]/(Sqrt[2]*Sqrt[c])) + ArcTan[1 + Sqrt[2]*Sqrt[c]*x]/(Sqrt[2]*Sqrt[c
]))/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2]/(Sqrt[2]*Sqrt[c]) + Log[1
 + Sqrt[2]*Sqrt[c]*x + c*x^2]/(2*Sqrt[2]*Sqrt[c]))/2)/c^2))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

method	result
default	$\frac{a\,x^3}{3}+b\left(\frac{x^3\arctan(c\,x^2)}{3}-\frac{2c\left(\frac{x}{c^2}-\frac{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^2}\right)}{3}\right.$
parts	$\frac{a\,x^3}{3}+b\left(\frac{x^3\arctan(c\,x^2)}{3}-\frac{2c\left(\frac{x}{c^2}-\frac{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^2}\right)}{3}\right.$

input
```
int(x^2*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+b*(1/3*x^3*arctan(c*x^2)-2/3*c*(1/c^2*x-1/8/c^2*(1/c^2)^(1/4)*2^
(1/2)*(ln((x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x
*2^(1/2)+(1/c^2)^(1/2)))+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1
/2)/(1/c^2)^(1/4)*x-1))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)\,dx$$

$$=\frac{4\,bcx^3\arctan\left(cx^2\right)+4\,acx^3-8\,bx+\frac{2\sqrt{2}b\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}}+\frac{2\sqrt{2}b\arctan\left(\sqrt{2}\sqrt{c}x-1\right)}{\sqrt{c}}+\frac{\sqrt{2}b\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}}-}{12\,c}$$

input
```
integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
1/12*(4*b*c*x^3*arctan(c*x^2) + 4*a*c*x^3 - 8*b*x + 2*sqrt(2)*b*arctan(sqr
t(2)*sqrt(c)*x + 1)/sqrt(c) + 2*sqrt(2)*b*arctan(sqrt(2)*sqrt(c)*x - 1)/sq
rt(c) + sqrt(2)*b*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*b*l
og(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/sqrt(c))/c
```

Sympy [A] (verification not implemented)

Time = 7.06 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.14

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)\,dx$$
$$=\begin{cases} \frac{ax^3}{3}+\frac{bx^3\operatorname{atan}\left(cx^2\right)}{3}+\frac{b\left(-\frac{1}{c^2}\right)^{\frac{3}{4}}\operatorname{atan}\left(cx^2\right)}{3}-\frac{2bx}{3c}-\frac{b\sqrt[4]{-\frac{1}{c^2}}\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{3c}+\frac{b\sqrt[4]{-\frac{1}{c^2}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6c}+\frac{b\sqrt[4]{-\frac{1}{c^2}}\ldots}{} \\ \frac{ax^3}{3} \end{cases}$$

input
```
integrate(x**2*(a+b*atan(c*x**2)),x)
```

output
```
Piecewise((a*x**3/3 + b*x**3*atan(c*x**2)/3 + b*(-1/c**2)**(3/4)*atan(c*x*
*2)/3 - 2*b*x/(3*c) - b*(-1/c**2)**(1/4)*log(x - (-1/c**2)**(1/4))/(3*c) +
 b*(-1/c**2)**(1/4)*log(x**2 + sqrt(-1/c**2))/(6*c) + b*(-1/c**2)**(1/4)*a
tan(x/(-1/c**2)**(1/4))/(3*c), Ne(c, 0)), (a*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.16

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{1}{3}\,ax^3$$
$$+\frac{1}{12}\left(4\,x^3\arctan\left(cx^2\right)-c\left(\frac{8\,x}{c^2}-\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}\ldots\right)}{\sqrt{c}}\ldots}{c^2}\right.\right.$$

input
```
integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/12*(4*x^3*arctan(c*x^2) - c*(8*x/c^2 - (2*sqrt(2)*arctan(1/2
*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + 2*sqrt(2)*arctan(1/2
*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + sqrt(2)*log(c*x^2 +
sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1
)/sqrt(c))/c^2))*b
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.32

$$\int x^2 \left(a + b \arctan \left(cx^2\right)\right) \, dx$$
$$= \frac{1}{12} bc^5 \left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x + \frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^6\sqrt{|c|}} + \frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x - \frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^6\sqrt{|c|}} + \frac{\sqrt{2}\log\left(x^2 + \right.}{c^6\sqrt{|}} \right.$$
$$+ \frac{bcx^3 \arctan\left(cx^2\right) + acx^3 - 2\,bx}{3\,c}$$

input
```
integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
1/12*b*c^5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt
(abs(c)))/(c^6*sqrt(abs(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
/sqrt(abs(c)))*sqrt(abs(c)))/(c^6*sqrt(abs(c))) + sqrt(2)*log(x^2 + sqrt(2
)*x/sqrt(abs(c)) + 1/abs(c))/(c^6*sqrt(abs(c))) - sqrt(2)*log(x^2 - sqrt(2
)*x/sqrt(abs(c)) + 1/abs(c))/(c^6*sqrt(abs(c)))) + 1/3*(b*c*x^3*arctan(c*x
^2) + a*c*x^3 - 2*b*x)/c
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.50

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{a\,x^3}{3}+\frac{b\,x^3\,\mathrm{atan}(c\,x^2)}{3}-\frac{2\,b\,x}{3\,c}-\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)\,1\mathrm{i}}{3\,c^{3/2}}-\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)}{3\,c^{3/2}}$$

input
```
int(x^2*(a + b*atan(c*x^2)),x)
```

output
```
(a*x^3)/3 + (b*x^3*atan(c*x^2))/3 - (2*b*x)/(3*c) - ((-1)^(1/4)*b*atan((-1
)^(1/4)*c^(1/2)*x)*1i)/(3*c^(3/2)) - ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)
*x*1i))/(3*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)\,dx$$
$$=\frac{-4\sqrt{c}\,\sqrt{2}\,atan\left(\frac{\sqrt{c}\,\sqrt{2}-2cx}{\sqrt{c}\,\sqrt{2}}\right)b-2\sqrt{c}\,\sqrt{2}\,atan(c\,x^2)\,b+4atan(c\,x^2)\,b\,c^2x^3-\sqrt{c}\,\sqrt{2}\log\left(-\sqrt{c}\,\sqrt{2}\,x+c\,x^2\right.}{12c^2}$$

input
```
int(x^2*(a+b*atan(c*x^2)),x)
```

output
```
( - 4*sqrt(c)*sqrt(2)*atan((sqrt(c)*sqrt(2) - 2*c*x)/(sqrt(c)*sqrt(2)))*b
- 2*sqrt(c)*sqrt(2)*atan(c*x**2)*b + 4*atan(c*x**2)*b*c**2*x**3 - sqrt(c)*
sqrt(2)*log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b + sqrt(c)*sqrt(2)*log(sqr
t(c)*sqrt(2)*x + c*x**2 + 1)*b + 4*a*c**2*x**3 - 8*b*c*x)/(12*c**2)
```

3.70 $\int (a+b\arctan(cx^2))\,dx$

Optimal result 534
Mathematica [A] (verified) 534
Rubi [A] (verified) 535
Maple [A] (verified) 536
Fricas [A] (verification not implemented) 536
Sympy [A] (verification not implemented) 537
Maxima [A] (verification not implemented) 538
Giac [A] (verification not implemented) 538
Mupad [B] (verification not implemented) 539
Reduce [B] (verification not implemented) 539

Optimal result

Integrand size = 10, antiderivative size = 103

$$\int (a+b\arctan(cx^2))\,dx = ax+bx\arctan(cx^2)+\frac{b\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}} - \frac{b\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}} + \frac{b\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{1+cx^2}\right)}{\sqrt{2}\sqrt{c}}$$

output
```
a*x+b*x*arctan(c*x^2)-1/2*b*arctan(-1+2^(1/2)*c^(1/2)*x)*2^(1/2)/c^(1/2)-1
/2*b*arctan(1+2^(1/2)*c^(1/2)*x)*2^(1/2)/c^(1/2)+1/2*b*arctanh(2^(1/2)*c^(
1/2)*x/(c*x^2+1))*2^(1/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04

$$\int (a+b\arctan(cx^2))\,dx = ax+bx\arctan(cx^2) - \frac{b\left(-2\arctan\left(1-\sqrt{2}\sqrt{c}x\right)+2\arctan\left(1+\sqrt{2}\sqrt{c}x\right)+\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)-\log\left(1+\sqrt{2}\sqrt{c}x+c\right.\right.}{2\sqrt{2}\sqrt{c}}$$

input
```
Integrate[a + b*ArcTan[c*x^2],x]
```

output
```
a*x + b*x*ArcTan[c*x^2] - (b*(-2*ArcTan[1 - Sqrt[2]*Sqrt[c]*x] + 2*ArcTan[
1 + Sqrt[2]*Sqrt[c]*x] + Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2] - Log[1 + Sqrt
[2]*Sqrt[c]*x + c*x^2]))/(2*Sqrt[2]*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.36, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\arctan\left(cx^2\right)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+bx\arctan\left(cx^2\right)+\frac{b\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{b\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}+\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}$$

input
```
Int[a + b*ArcTan[c*x^2],x]
```

output
```
a*x + b*x*ArcTan[c*x^2] + (b*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[
c]) - (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c]) - (b*Log[1 - Sqr
t[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]*Sqrt[c]) + (b*Log[1 + Sqrt[2]*Sqrt[c]*
x + c*x^2])/(2*Sqrt[2]*Sqrt[c])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

method	result	si
default	$ax+b\left(x\arctan\left(c\,x^2\right)-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}\right)$	1
parts	$ax+b\left(x\arctan\left(c\,x^2\right)-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}\right)$	1

input
```
int(a+b*arctan(c*x^2),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*(x*arctan(c*x^2)-1/4/c/(1/c^2)^(1/4)*2^(1/2)*(ln((x^2-(1/c^2)^(1/4)*
x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))+2*ar
ctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int\left(a+b\arctan\left(cx^2\right)\right)\,dx=bx\arctan\left(cx^2\right)+ax-\frac{\sqrt{2}b\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{c}}-\frac{\sqrt{2}b\arctan\left(\sqrt{2}\sqrt{c}x-1\right)}{2\sqrt{c}}+\frac{\sqrt{2}b\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{4\sqrt{c}}-\frac{\sqrt{2}b\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{4\sqrt{c}}$$

input
```
integrate(a+b*arctan(c*x^2),x, algorithm="fricas")
```

output
```
b*x*arctan(c*x^2) + a*x - 1/2*sqrt(2)*b*arctan(sqrt(2)*sqrt(c)*x + 1)/sqrt
(c) - 1/2*sqrt(2)*b*arctan(sqrt(2)*sqrt(c)*x - 1)/sqrt(c) + 1/4*sqrt(2)*b*
log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - 1/4*sqrt(2)*b*log(c*x^2 - sqr
t(2)*sqrt(c)*x + 1)/sqrt(c)
```

Sympy [A] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07

$$\int \left(a+b\arctan\left(cx^2\right)\right)\,dx = ax + b\left(\begin{cases} x\,\mathrm{atan}\left(cx^2\right)+\sqrt[4]{-\frac{1}{c^2}}\,\mathrm{atan}\left(cx^2\right)-\frac{\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{c\sqrt[4]{-\frac{1}{c^2}}}+\frac{\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2c\sqrt[4]{-\frac{1}{c^2}}}-\frac{\mathrm{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{c\sqrt[4]{-\frac{1}{c^2}}} & \text{for } c\neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*atan(c*x**2),x)
```

output
```
a*x + b*Piecewise((x*atan(c*x**2) + (-1/c**2)**(1/4)*atan(c*x**2) - log(x
- (-1/c**2)**(1/4))/(c*(-1/c**2)**(1/4)) + log(x**2 + sqrt(-1/c**2))/(2*c*
(-1/c**2)**(1/4)) - atan(x/(-1/c**2)**(1/4))/(c*(-1/c**2)**(1/4)), Ne(c, 0
)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.23

$$\int \left(a + b\arctan\left(cx^2\right)\right)\,dx = -\frac{1}{4}\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{c^{\frac{3}{2}}}\right.\right.$$
$$+ ax$$

input
```
integrate(a+b*arctan(c*x^2),x, algorithm="maxima")
```

output
```
-1/4*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c
^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c
^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(
c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) - 4*x*arctan(c*x^2))*b + a*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.45

$$\int \left(a + b\arctan\left(cx^2\right)\right)\,dx = -\frac{1}{4}\left(c\left(\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2} + \frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2} - \right.\right.$$
$$+ ax$$

input
```
integrate(a+b*arctan(c*x^2),x, algorithm="giac")
```

output
```
-1/4*(c*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs
(c)))*sqrt(abs(c)))/c^2 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^2 - sqrt(2)*sqrt(abs(c))*log(x^2 +
sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2 + sqrt(2)*sqrt(abs(c))*log(x^2 - sq
rt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2) - 4*x*arctan(c*x^2))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.48

$$\int \left(a+b\arctan \left(cx^2\right)\right)\,dx=a\,x+b\,x\,\mathrm{atan}\left(c\,x^2\right)-\frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{c}\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{\sqrt{c}}$$

input
```
int(a + b*atan(c*x^2),x)
```

output
```
a*x + b*x*atan(c*x^2) - ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x))/c^(1/2)
- ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x*1i)*1i)/c^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \left(a+b\arctan \left(cx^2\right)\right)\,dx$$
$$=\frac{4\sqrt{c}\,\sqrt{2}\,atan\left(\frac{\sqrt{c}\,\sqrt{2}-2cx}{\sqrt{c}\,\sqrt{2}}\right)b+2\sqrt{c}\,\sqrt{2}\,atan\left(c\,x^2\right)b+4atan\left(c\,x^2\right)bcx-\sqrt{c}\,\sqrt{2}\,\log\left(-\sqrt{c}\,\sqrt{2}\,x+c\,x^2+1\right)}{4c}$$

input
```
int(a+b*atan(c*x^2),x)
```

output
```
(4*sqrt(c)*sqrt(2)*atan((sqrt(c)*sqrt(2) - 2*c*x)/(sqrt(c)*sqrt(2)))*b + 2
*sqrt(c)*sqrt(2)*atan(c*x**2)*b + 4*atan(c*x**2)*b*c*x - sqrt(c)*sqrt(2)*l
og( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b + sqrt(c)*sqrt(2)*log(sqrt(c)*sqrt
(2)*x + c*x**2 + 1)*b + 4*a*c*x)/(4*c)
```

3.71 $\int \frac{a+b\arctan(cx^2)}{x^2}\,dx$

Optimal result 540
Mathematica [A] (verified) 540
Rubi [A] (verified) 541
Maple [A] (verified) 545
Fricas [A] (verification not implemented) 545
Sympy [A] (verification not implemented) 546
Maxima [A] (verification not implemented) 546
Giac [A] (verification not implemented) 547
Mupad [B] (verification not implemented) 547
Reduce [B] (verification not implemented) 548

Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{a+b\arctan(cx^2)}{x^2}\,dx=-\frac{a+b\arctan(cx^2)}{x}-\frac{b\sqrt{c}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}}+\frac{b\sqrt{c}\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}}+\frac{b\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{1+cx^2}\right)}{\sqrt{2}}$$

output
```
-(a+b*arctan(c*x^2))/x+1/2*b*c^(1/2)*arctan(-1+2^(1/2)*c^(1/2)*x)*2^(1/2)+
1/2*b*c^(1/2)*arctan(1+2^(1/2)*c^(1/2)*x)*2^(1/2)+1/2*b*c^(1/2)*arctanh(2^
(1/2)*c^(1/2)*x/(c*x^2+1))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.49

$$\int \frac{a+b\arctan(cx^2)}{x^2}\,dx=-\frac{a}{x}-\frac{b\arctan(cx^2)}{x}+\frac{b\sqrt{c}\arctan\left(\frac{-\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{b\sqrt{c}\arctan\left(\frac{\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{b\sqrt{c}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}}+\frac{b\sqrt{c}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])/x^2,x]
```

output
```
-(a/x) - (b*ArcTan[c*x^2])/x + (b*Sqrt[c]*ArcTan[(-Sqrt[2] + 2*Sqrt[c]*x)/
Sqrt[2]])/Sqrt[2] + (b*Sqrt[c]*ArcTan[(Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/Sq
rt[2] - (b*Sqrt[c]*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]) + (b*Sq
rt[c]*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.45, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5361, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^2}dx$$

$$\Big\downarrow \text{ 5361}$$

$$2bc\int \frac{1}{c^2x^4+1}dx-\frac{a+b\arctan\left(cx^2\right)}{x}$$

$$\Big\downarrow \text{ 755}$$

$$2bc\left(\frac{1}{2}\int \frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\int \frac{cx^2+1}{c^2x^4+1}dx\right)-\frac{a+b\arctan\left(cx^2\right)}{x}$$

$$\Big\downarrow \text{ 1476}$$

$$2bc\left(\frac{1}{2}\int \frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}\right)\right)-\frac{a+b\arctan\left(cx^2\right)}{x}$$

$$\Big\downarrow \text{ 1082}$$

$$2bc\left(\frac{1}{2}\int \frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}\sqrt{c}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{-\left(\sqrt{2}\sqrt{c}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}\right)\right)-\frac{a+b\arctan\left(cx^2\right)}{x}$$

$$\Big\downarrow \ 217$$

$$2bc\left(\frac{1}{2}\int \frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)\right)-\frac{a+b\arctan\left(cx^2\right)}{x}$$

$$\Big\downarrow \ 1479$$

$$2bc\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)\right)-\frac{a+b\arctan\left(cx^2\right)}{x}$$

$$\Big\downarrow \ 25$$

$$2bc\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)\right)-\frac{a+b\arctan\left(cx^2\right)}{x}$$

$$\Big\downarrow \ 27$$

$$2bc\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{c}x}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2\sqrt{2}c}+\frac{\int \frac{\sqrt{2}\sqrt{c}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)\right)-\frac{a+b\arctan\left(cx^2\right)}{x}$$

$$\Big\downarrow \ 1103$$

$$2bc\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}-\frac{\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}\right.\right.$$
$$\frac{a+b\arctan\left(cx^2\right)}{x}$$

input
```
Int[(a + b*ArcTan[c*x^2])/x^2,x]
```

output
```
-((a + b*ArcTan[c*x^2])/x) + 2*b*c*((-(ArcTan[1 - Sqrt[2]*Sqrt[c]*x]/(Sqrt
[2]*Sqrt[c])) + ArcTan[1 + Sqrt[2]*Sqrt[c]*x]/(Sqrt[2]*Sqrt[c]))/2 + (-1/2
*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2]/(Sqrt[2]*Sqrt[c]) + Log[1 + Sqrt[2]*Sq
rt[c]*x + c*x^2]/(2*Sqrt[2]*Sqrt[c]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01

method	result
default	$-\frac{a}{x}+b\left(-\frac{\arctan\left(c\,x^{2}\right)}{x}+\frac{c\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4}\right)$
parts	$-\frac{a}{x}+b\left(-\frac{\arctan\left(c\,x^{2}\right)}{x}+\frac{c\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4}\right)$

input
```
int((a+b*arctan(c*x^2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+b*(-1/x*arctan(c*x^2)+1/4*c*(1/c^2)^(1/4)*2^(1/2)*(ln((x^2+(1/c^2)^(1
/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))+
2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

$$\int \frac{a+b\arctan\left(cx^{2}\right)}{x^{2}}\,dx$$
$$=\frac{2\sqrt{2}b\sqrt{c}x\arctan\left(\sqrt{2}\sqrt{c}x+1\right)+2\sqrt{2}b\sqrt{c}x\arctan\left(\sqrt{2}\sqrt{c}x-1\right)+\sqrt{2}b\sqrt{c}x\log\left(cx^{2}+\sqrt{2}\sqrt{c}x+1\right)-}{4\,x}$$

input
```
integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*b*sqrt(c)*x*arctan(sqrt(2)*sqrt(c)*x + 1) + 2*sqrt(2)*b*sqr
t(c)*x*arctan(sqrt(2)*sqrt(c)*x - 1) + sqrt(2)*b*sqrt(c)*x*log(c*x^2 + sqr
t(2)*sqrt(c)*x + 1) - sqrt(2)*b*sqrt(c)*x*log(c*x^2 - sqrt(2)*sqrt(c)*x +
1) - 4*b*arctan(c*x^2) - 4*a)/x
```

Sympy [A] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.17

$$\int \frac{a + b \arctan (cx^2)}{x^2} \, dx = \begin{cases} -\frac{a}{x} + bc^2 \left(-\frac{1}{c^2}\right)^{\frac{3}{4}} \operatorname{atan}(cx^2) - bc\sqrt[4]{-\frac{1}{c^2}} \log\left(x - \sqrt[4]{-\frac{1}{c^2}}\right) + \frac{bc\sqrt[4]{-\frac{1}{c^2}} \log\left(x^2 + \sqrt{-\frac{1}{c^2}}\right)}{2} + bc\sqrt[4]{-\frac{1}{c^2}} \operatorname{atan}\left(\frac{}{\sqrt[4]{}}\right. \\ -\frac{a}{x} \end{cases}$$

input
```
integrate((a+b*atan(c*x**2))/x**2,x)
```

output
```
Piecewise((-a/x + b*c**2*(-1/c**2)**(3/4)*atan(c*x**2) - b*c*(-1/c**2)**(1
/4)*log(x - (-1/c**2)**(1/4)) + b*c*(-1/c**2)**(1/4)*log(x**2 + sqrt(-1/c*
*2))/2 + b*c*(-1/c**2)**(1/4)*atan(x/(-1/c**2)**(1/4)) - b*atan(c*x**2)/x,
 Ne(c, 0)), (-a/x, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.25

$$\int \frac{a + b \arctan (cx^2)}{x^2} \, dx = \frac{1}{4} \left(c \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\,cx + \sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\,cx - \sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}} + \frac{\sqrt{2} \log\left(cx^2 + \sqrt{2}\sqrt{c}x + 1\right)}{\sqrt{c}} - \right.\right.$$
$$-\frac{a}{x}$$

input
```
integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="maxima")
```

output
```
1/4*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sq
rt(c) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sq
rt(c) + sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c
*x^2 - sqrt(2)*sqrt(c)*x + 1)/sqrt(c)) - 4*arctan(c*x^2)/x)*b - a/x
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.30

$$\int \frac{a+b\arctan(cx^2)}{x^2}\,dx = \frac{1}{4}\,bc\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}}{\sqrt{|}}\right.}{\sqrt{|c|}} \right. - \frac{b\arctan(cx^2)+a}{x}$$

input
```
integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="giac")
```

output
```
1/4*b*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt(ab
s(c)))/sqrt(abs(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs
(c)))*sqrt(abs(c)))/sqrt(abs(c)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(c)
) + 1/abs(c))/sqrt(abs(c)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(c)) + 1/
abs(c))/sqrt(abs(c))) - (b*arctan(c*x^2) + a)/x
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.52

$$\int \frac{a+b\arctan(cx^2)}{x^2}\,dx = -\frac{a}{x}-\frac{b\,\mathrm{atan}(c\,x^2)}{x}-(-1)^{1/4}\,b\,\sqrt{c}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)\,1\mathrm{i} - (-1)^{1/4}\,b\,\sqrt{c}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)$$

input
```
int((a + b*atan(c*x^2))/x^2,x)
```

output
```
- a/x - (b*atan(c*x^2))/x - (-1)^(1/4)*b*c^(1/2)*atan((-1)^(1/4)*c^(1/2)*x
)*1i - (-1)^(1/4)*b*c^(1/2)*atan((-1)^(1/4)*c^(1/2)*x*1i)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

$$\int \frac{a + b \arctan(cx^2)}{x^2} \, dx$$
$$= \frac{-4\sqrt{c}\,\sqrt{2}\,atan\left(\frac{\sqrt{c}\,\sqrt{2}-2cx}{\sqrt{c}\,\sqrt{2}}\right) bx - 2\sqrt{c}\,\sqrt{2}\,atan(c\,x^2)\,bx - 4atan(c\,x^2)\,b - \sqrt{c}\,\sqrt{2}\log\left(-\sqrt{c}\,\sqrt{2}\,x + c\,x^2 + 1\right.}{4x}$$

input
```
int((a+b*atan(c*x^2))/x^2,x)
```

output
```
( - 4*sqrt(c)*sqrt(2)*atan((sqrt(c)*sqrt(2) - 2*c*x)/(sqrt(c)*sqrt(2)))*b*
x - 2*sqrt(c)*sqrt(2)*atan(c*x**2)*b*x - 4*atan(c*x**2)*b - sqrt(c)*sqrt(2
)*log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b*x + sqrt(c)*sqrt(2)*log(sqrt(c)
*sqrt(2)*x + c*x**2 + 1)*b*x - 4*a)/(4*x)
```

3.72 $\int \frac{a+b\arctan(cx^2)}{x^4}\,dx$

Optimal result . . . 549
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Optimal result

Integrand size = 14, antiderivative size = 125

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx = -\frac{2bc}{3x}-\frac{a+b\arctan(cx^2)}{3x^3}+\frac{bc^{3/2}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}}-\frac{bc^{3/2}\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}}+\frac{bc^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{1+cx^2}\right)}{3\sqrt{2}}$$

output
```
-2/3*b*c/x-1/3*(a+b*arctan(c*x^2))/x^3-1/6*b*c^(3/2)*arctan(-1+2^(1/2)*c^(
1/2)*x)*2^(1/2)-1/6*b*c^(3/2)*arctan(1+2^(1/2)*c^(1/2)*x)*2^(1/2)+1/6*b*c^
(3/2)*arctanh(2^(1/2)*c^(1/2)*x/(c*x^2+1))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.42

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{2bc}{3x}-\frac{b\arctan(cx^2)}{3x^3}-\frac{bc^{3/2}\arctan\left(\frac{-\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{3\sqrt{2}}-\frac{bc^{3/2}\arctan\left(\frac{\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{3\sqrt{2}}-\frac{bc^{3/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])/x^4,x]
```

output
```
-1/3*a/x^3 - (2*b*c)/(3*x) - (b*ArcTan[c*x^2])/(3*x^3) - (b*c^(3/2)*ArcTan
[(-Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]) - (b*c^(3/2)*ArcTan[(Sqrt[
2] + 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]) - (b*c^(3/2)*Log[1 - Sqrt[2]*Sqrt[
c]*x + c*x^2])/(6*Sqrt[2]) + (b*c^(3/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2]
)/(6*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.40, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5361, 847, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^4}\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{3}bc\int \frac{1}{x^2\left(c^2x^4+1\right)}dx-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$$\Big\downarrow \text{847}$$

$$\frac{2}{3}bc\left(c^2\left(-\int \frac{x^2}{c^2x^4+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$$\Big\downarrow \text{826}$$

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\int \frac{cx^2+1}{c^2x^4+1}dx}{2c}-\frac{\int \frac{1-cx^2}{c^2x^4+1}dx}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}}{2c}-\frac{\int \frac{1-cx^2}{c^2x^4+1}dx}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\frac{\int \frac{1}{-\left(1-\sqrt{2}\sqrt{c}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{-\left(\sqrt{2}\sqrt{c}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\int \frac{1-cx^2}{c^2x^4+1}dx}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$$\Big\downarrow \text{217}$$

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\int \frac{1-cx^2}{c^2x^4+1}dx}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{-\frac{\int -\frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\frac{\int \frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\frac{\int\frac{\sqrt{2}-2\sqrt{c}x}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2\sqrt{2}c}+\frac{\int\frac{\sqrt{2}\sqrt{c}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\frac{\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}-\frac{\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

input
```
Int[(a + b*ArcTan[c*x^2])/x^4,x]
```

output
```
-1/3*(a + b*ArcTan[c*x^2])/x^3 + (2*b*c*(-x^(-1) - c^2*((-(ArcTan[1 - Sqrt
[2]*Sqrt[c]*x]/(Sqrt[2]*Sqrt[c])) + ArcTan[1 + Sqrt[2]*Sqrt[c]*x]/(Sqrt[2]
*Sqrt[c]))/(2*c) - (-1/2*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2]/(Sqrt[2]*Sqrt[
c]) + Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2]/(2*Sqrt[2]*Sqrt[c]))/(2*c))))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a}{3x^3}+b\left(-\frac{\arctan(c\,x^2)}{3x^3}+\frac{2c\left(-\frac{1}{x}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}\right)}{3}\right)$
parts	$-\frac{a}{3x^3}+b\left(-\frac{\arctan(c\,x^2)}{3x^3}+\frac{2c\left(-\frac{1}{x}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}\right)}{3}\right)$

input
```
int((a+b*arctan(c*x^2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*(-1/3/x^3*arctan(c*x^2)+2/3*c*(-1/x-1/8/(1/c^2)^(1/4)*2^(1/2)
*(ln((x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1
/2)+(1/c^2)^(1/2)))+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(
1/c^2)^(1/4)*x-1))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx = -\frac{2\sqrt{2}bc^{\frac{3}{2}}x^3\arctan\left(\sqrt{2}\sqrt{c}x+1\right)+2\sqrt{2}bc^{\frac{3}{2}}x^3\arctan\left(\sqrt{2}\sqrt{c}x-1\right)-\sqrt{2}bc^{\frac{3}{2}}x^3\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{12\,x^3}$$

input
```
integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="fricas")
```

output
```
-1/12*(2*sqrt(2)*b*c^(3/2)*x^3*arctan(sqrt(2)*sqrt(c)*x + 1) + 2*sqrt(2)*b
*c^(3/2)*x^3*arctan(sqrt(2)*sqrt(c)*x - 1) - sqrt(2)*b*c^(3/2)*x^3*log(c*x
^2 + sqrt(2)*sqrt(c)*x + 1) + sqrt(2)*b*c^(3/2)*x^3*log(c*x^2 - sqrt(2)*sq
rt(c)*x + 1) + 8*b*c*x^2 + 4*b*arctan(c*x^2) + 4*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 16.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.18

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx = \begin{cases} -\frac{a}{3x^3}+\frac{bc^2\sqrt[4]{-\frac{1}{c^2}}\operatorname{atan}(cx^2)}{3}-\frac{bc\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{3\sqrt[4]{-\frac{1}{c^2}}}+\frac{bc\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6\sqrt[4]{-\frac{1}{c^2}}}-\frac{bc\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{3\sqrt[4]{-\frac{1}{c^2}}}-\frac{2bc}{3x}-\frac{b\operatorname{atan}(cx^2)}{3x^3} \\ -\frac{a}{3x^3} \end{cases}$$

input
```
integrate((a+b*atan(c*x**2))/x**4,x)
```

output
```
Piecewise((-a/(3*x**3) + b*c**2*(-1/c**2)**(1/4)*atan(c*x**2)/3 - b*c*log(
x - (-1/c**2)**(1/4))/(3*(-1/c**2)**(1/4)) + b*c*log(x**2 + sqrt(-1/c**2))
/(6*(-1/c**2)**(1/4)) - b*c*atan(x/(-1/c**2)**(1/4))/(3*(-1/c**2)**(1/4))
- 2*b*c/(3*x) - b*atan(c*x**2)/(3*x**3), Ne(c, 0)), (-a/(3*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14

$$\int \frac{a + b\arctan(cx^2)}{x^4}\,dx = -\frac{1}{12}\left(\left(\left(c^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+\right.}{c^{\frac{3}{2}}}\right.\right.\right.\right. - \frac{a}{3\,x^3}$$

input
```
integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="maxima")
```

output
```
-1/12*((c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c
))/c^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c
))/c^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*
log(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) + 8/x)*c + 4*arctan(c*x^2)/x^3
)*b - 1/3*a/x^3
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.27

$$\int \frac{a + b\arctan(cx^2)}{x^4}\,dx = -\frac{1}{12}\,bc^3\left(\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2} + \frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2} - \right. - \frac{2\,bcx^2 + b\arctan(cx^2) + a}{3\,x^3}$$

input
```
integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="giac")
```

output
```
-1/12*b*c^3*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt
(abs(c)))*sqrt(abs(c)))/c^2 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^2 - sqrt(2)*sqrt(abs(c))*log(x^
2 + sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2 + sqrt(2)*sqrt(abs(c))*log(x^2
- sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2) - 1/3*(2*b*c*x^2 + b*arctan(c*x^
2) + a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.50

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx = -\frac{2\,b\,c\,x^2+a}{3\,x^3}-\frac{b\,\mathrm{atan}(c\,x^2)}{3\,x^3} - \frac{(-1)^{1/4}\,b\,c^{3/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)}{3} - \frac{(-1)^{1/4}\,b\,c^{3/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{3}$$

input
```
int((a + b*atan(c*x^2))/x^4,x)
```

output
```
- (a + 2*b*c*x^2)/(3*x^3) - (b*atan(c*x^2))/(3*x^3) - ((-1)^(1/4)*b*c^(3/2
)*atan((-1)^(1/4)*c^(1/2)*x))/3 - ((-1)^(1/4)*b*c^(3/2)*atan((-1)^(1/4)*c^
(1/2)*x*1i)*1i)/3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx = \frac{4\sqrt{c}\,\sqrt{2}\,atan\left(\frac{\sqrt{c}\,\sqrt{2}-2cx}{\sqrt{c}\,\sqrt{2}}\right)bc\,x^3+2\sqrt{c}\,\sqrt{2}\,atan(c\,x^2)\,bc\,x^3-4atan(c\,x^2)\,b-\sqrt{c}\,\sqrt{2}\log\left(-\sqrt{c}\,\sqrt{2}\,x+c\,x^2\right.}{12x^3}$$

input
```
int((a+b*atan(c*x^2))/x^4,x)
```

output
```
(4*sqrt(c)*sqrt(2)*atan((sqrt(c)*sqrt(2) - 2*c*x)/(sqrt(c)*sqrt(2)))*b*c*x
**3 + 2*sqrt(c)*sqrt(2)*atan(c*x**2)*b*c*x**3 - 4*atan(c*x**2)*b - sqrt(c)
*sqrt(2)*log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b*c*x**3 + sqrt(c)*sqrt(2)
*log(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b*c*x**3 - 4*a - 8*b*c*x**2)/(12*x**3
)
```

3.73 $\int \frac{a+b\arctan(cx^2)}{x^6}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 125

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{2bc}{15x^3}-\frac{a+b\arctan(cx^2)}{5x^5}+\frac{bc^{5/2}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}} -\frac{bc^{5/2}\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}}-\frac{bc^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{1+cx^2}\right)}{5\sqrt{2}}$$

output
```
-2/15*b*c/x^3-1/5*(a+b*arctan(c*x^2))/x^5-1/10*b*c^(5/2)*arctan(-1+2^(1/2)
*c^(1/2)*x)*2^(1/2)-1/10*b*c^(5/2)*arctan(1+2^(1/2)*c^(1/2)*x)*2^(1/2)-1/1
0*b*c^(5/2)*arctanh(2^(1/2)*c^(1/2)*x/(c*x^2+1))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.42

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{a}{5x^5}-\frac{2bc}{15x^3}-\frac{b\arctan(cx^2)}{5x^5}-\frac{bc^{5/2}\arctan\left(\frac{-\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{5\sqrt{2}} -\frac{bc^{5/2}\arctan\left(\frac{\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{5\sqrt{2}}+\frac{bc^{5/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}} -\frac{bc^{5/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])/x^6,x]
```

output
```
-1/5*a/x^5 - (2*b*c)/(15*x^3) - (b*ArcTan[c*x^2])/(5*x^5) - (b*c^(5/2)*Arc
Tan[(-Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]) - (b*c^(5/2)*ArcTan[(Sq
rt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]) + (b*c^(5/2)*Log[1 - Sqrt[2]*Sq
rt[c]*x + c*x^2])/(10*Sqrt[2]) - (b*c^(5/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*
x^2])/(10*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.37, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5361, 847, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^6}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{5}bc\int \frac{1}{x^4\left(c^2x^4+1\right)}dx-\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \text{847}$$

$$\frac{2}{5}bc\left(c^2\left(-\int \frac{1}{c^2x^4+1}dx\right)-\frac{1}{3x^3}\right)-\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \text{755}$$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\int \frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\int \frac{cx^2+1}{c^2x^4+1}dx\right)\right)-\frac{1}{3x^3}\right)-\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\int \frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}\right)\right)\right)-\frac{1}{3x^3}\right)-\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \ 1082$$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\int\frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\int\frac{1}{-\left(1-\sqrt{2}\sqrt{c}x\right)^2-1}d(1-\sqrt{2}\sqrt{c}x)}{\sqrt{2}\sqrt{c}}-\frac{\int\frac{1}{-\left(\sqrt{2}\sqrt{c}x+1\right)^2-1}d(\sqrt{2}\sqrt{c}x+1)}{\sqrt{2}\sqrt{c}}\right)\right)\right)\right)$$
$$\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \ 217$$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\int\frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)\right)\right)-\frac{1}{3x^3}\right)-$$
$$\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \ 1479$$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{}\right.}{\sqrt{2}\sqrt{c}}\right.\right.\right.\right.$$
$$\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \ 25$$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right.\right.\right.\right.$$
$$\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \ 27$$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{c}x}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2\sqrt{2}c}+\frac{\int\frac{\sqrt{2}\sqrt{c}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)\right)\right)-\right.$$
$$\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \ 1103$$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}-\frac{\log\left(cx^2-\sqrt{2}\sqrt{c}\right.}{2\sqrt{2}\sqrt{c}}\right.\right.\right.\right.$$
$$\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

input
```
Int[(a + b*ArcTan[c*x^2])/x^6,x]
```

output
```
-1/5*(a + b*ArcTan[c*x^2])/x^5 + (2*b*c*(-1/3*1/x^3 - c^2*((-(ArcTan[1 - S
qrt[2]*Sqrt[c]*x]/(Sqrt[2]*Sqrt[c])) + ArcTan[1 + Sqrt[2]*Sqrt[c]*x]/(Sqrt
[2]*Sqrt[c]))/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2]/(Sqrt[2]*Sqrt[c
]) + Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2]/(2*Sqrt[2]*Sqrt[c]))/2)))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5361

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

method	result
default	$-\frac{a}{5x^5}+b\left(-\frac{\arctan(c\,x^2)}{5x^5}+\frac{2c\left(-\frac{1}{3x^3}-\frac{c^2\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}\right.\right.}{8}\right.}{5}\right.$
parts	$-\frac{a}{5x^5}+b\left(-\frac{\arctan(c\,x^2)}{5x^5}+\frac{2c\left(-\frac{1}{3x^3}-\frac{c^2\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}\right.\right.}{8}\right.}{5}\right.$

input
```
int((a+b*arctan(c*x^2))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5+b*(-1/5/x^5*arctan(c*x^2)+2/5*c*(-1/3/x^3-1/8*c^2*(1/c^2)^(1/4)
*2^(1/2)*(ln((x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4
)*x*2^(1/2)+(1/c^2)^(1/2)))+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2
^(1/2)/(1/c^2)^(1/4)*x-1))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^6}\,dx =$$
$$-\frac{6\sqrt{2}bc^{\frac{5}{2}}x^5\arctan\left(\sqrt{2}\sqrt{c}x+1\right)+6\sqrt{2}bc^{\frac{5}{2}}x^5\arctan\left(\sqrt{2}\sqrt{c}x-1\right)+3\sqrt{2}bc^{\frac{5}{2}}x^5\log\left(cx^2+\sqrt{2}\sqrt{c}x+\right.}{60\,x^5}$$

input
```
integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="fricas")
```

output
```
-1/60*(6*sqrt(2)*b*c^(5/2)*x^5*arctan(sqrt(2)*sqrt(c)*x + 1) + 6*sqrt(2)*b
*c^(5/2)*x^5*arctan(sqrt(2)*sqrt(c)*x - 1) + 3*sqrt(2)*b*c^(5/2)*x^5*log(c
*x^2 + sqrt(2)*sqrt(c)*x + 1) - 3*sqrt(2)*b*c^(5/2)*x^5*log(c*x^2 - sqrt(2
)*sqrt(c)*x + 1) + 8*b*c*x^2 + 12*b*arctan(c*x^2) + 12*a)/x^5
```

Sympy [A] (verification not implemented)

Time = 28.66 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.24

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^6}\,dx$$
$$=\begin{cases} -\frac{a}{5x^5}-\frac{bc^4\left(-\frac{1}{c^2}\right)^{\frac{3}{4}}\operatorname{atan}\left(cx^2\right)}{5}+\frac{bc^3\sqrt[4]{-\frac{1}{c^2}}\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{5}-\frac{bc^3\sqrt[4]{-\frac{1}{c^2}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{10}-\frac{bc^3\sqrt[4]{-\frac{1}{c^2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right.}{5} \\ -\frac{a}{5x^5} \end{cases}$$

input
```
integrate((a+b*atan(c*x**2))/x**6,x)
```

output
```
Piecewise((-a/(5*x**5) - b*c**4*(-1/c**2)**(3/4)*atan(c*x**2)/5 + b*c**3*(
-1/c**2)**(1/4)*log(x - (-1/c**2)**(1/4))/5 - b*c**3*(-1/c**2)**(1/4)*log(
x**2 + sqrt(-1/c**2))/10 - b*c**3*(-1/c**2)**(1/4)*atan(x/(-1/c**2)**(1/4)
)/5 - 2*b*c/(15*x**3) - b*atan(c*x**2)/(5*x**5), Ne(c, 0)), (-a/(5*x**5),
True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{1}{60}\left(\left(6\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}(2\,cx+\sqrt{2}\sqrt{c})}{2\sqrt{c}}\right)+6\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}(2\,cx-\sqrt{2}\sqrt{c})}{2\sqrt{c}}\right)+3\sqrt{2}c^{\frac{3}{2}}\log\left(cx\right.\right.\right.$$
$$-\frac{a}{5\,x^5}$$

input
```
integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="maxima")
```

output
```
-1/60*((6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqr
t(c)) + 6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqr
t(c)) + 3*sqrt(2)*c^(3/2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1) - 3*sqrt(2)*c
^(3/2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1) + 8/x^3)*c + 12*arctan(c*x^2)/x^
5)*b - 1/5*a/x^5
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.20

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{1}{20}\,bc^3\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\sqrt{2}\log\left(x^2\right.}{\sqrt{}}\right.$$
$$-\frac{2\,bcx^2+3\,b\arctan(cx^2)+3\,a}{15\,x^5}$$

input
```
integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="giac")
```

output
```
-1/20*b*c^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqr
t(abs(c)))/sqrt(abs(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt
(abs(c)))*sqrt(abs(c)))/sqrt(abs(c)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(ab
s(c)) + 1/abs(c))/sqrt(abs(c)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(c))
+ 1/abs(c))/sqrt(abs(c))) - 1/15*(2*b*c*x^2 + 3*b*arctan(c*x^2) + 3*a)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.50

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{\frac{2\,b\,c\,x^2}{3}+a}{5\,x^5} - \frac{b\,\mathrm{atan}(c\,x^2)}{5\,x^5} + \frac{(-1)^{1/4}\,b\,c^{5/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)\,1\mathrm{i}}{5} + \frac{(-1)^{1/4}\,b\,c^{5/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)}{5}$$

input
```
int((a + b*atan(c*x^2))/x^6,x)
```

output
```
((-1)^(1/4)*b*c^(5/2)*atan((-1)^(1/4)*c^(1/2)*x)*1i)/5 - (b*atan(c*x^2))/(
5*x^5) - (a + (2*b*c*x^2)/3)/(5*x^5) + ((-1)^(1/4)*b*c^(5/2)*atan((-1)^(1/
4)*c^(1/2)*x*1i))/5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = \frac{12\sqrt{c}\,\sqrt{2}\,atan\left(\frac{\sqrt{c}\,\sqrt{2}-2cx}{\sqrt{c}\,\sqrt{2}}\right)b\,c^2x^5 + 6\sqrt{c}\,\sqrt{2}\,atan(c\,x^2)\,b\,c^2x^5 - 12atan(c\,x^2)\,b + 3\sqrt{c}\,\sqrt{2}\log\left(-\sqrt{c}\,\sqrt{2}\,x + \right.}{60x^5}$$

input
```
int((a+b*atan(c*x^2))/x^6,x)
```

output
```
(12*sqrt(c)*sqrt(2)*atan((sqrt(c)*sqrt(2) - 2*c*x)/(sqrt(c)*sqrt(2)))*b*c*
*2*x**5 + 6*sqrt(c)*sqrt(2)*atan(c*x**2)*b*c**2*x**5 - 12*atan(c*x**2)*b +
 3*sqrt(c)*sqrt(2)*log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b*c**2*x**5 - 3*
sqrt(c)*sqrt(2)*log(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b*c**2*x**5 - 12*a - 8
*b*c*x**2)/(60*x**5)
```

3.74 $\int x^7(a+b\arctan(cx^2))^2\,dx$

Optimal result 569
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Optimal result

Integrand size = 16, antiderivative size = 124

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\frac{abx^2}{4c^3}+\frac{b^2x^4}{24c^2}+\frac{b^2x^2\arctan(cx^2)}{4c^3}-\frac{bx^6(a+b\arctan(cx^2))}{12c}-\frac{(a+b\arctan(cx^2))^2}{8c^4}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{b^2\log(1+c^2x^4)}{6c^4}$$

output
```
1/4*a*b*x^2/c^3+1/24*b^2*x^4/c^2+1/4*b^2*x^2*arctan(c*x^2)/c^3-1/12*b*x^6*
(a+b*arctan(c*x^2))/c-1/8*(a+b*arctan(c*x^2))^2/c^4+1/8*x^8*(a+b*arctan(c*
x^2))^2-1/6*b^2*ln(c^2*x^4+1)/c^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\frac{cx^2(6ab+b^2cx^2-2abc^2x^4+3a^2c^3x^6)-2b(bcx^2(-3+c^2x^4)+a(3-3c^4x^8))\arctan(cx^2)+3b^2(-1+c}{24c^4}$$

input
```
Integrate[x^7*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
(c*x^2*(6*a*b + b^2*c*x^2 - 2*a*b*c^2*x^4 + 3*a^2*c^3*x^6) - 2*b*(b*c*x^2*
(-3 + c^2*x^4) + a*(3 - 3*c^4*x^8))*ArcTan[c*x^2] + 3*b^2*(-1 + c^4*x^8)*A
rcTan[c*x^2]^2 - 4*b^2*Log[1 + c^2*x^4])/(24*c^4)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5363, 5361, 5451, 5361, 243, 49, 2009, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 (a + b\arctan(cx^2))^2 \, dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{2}\int x^6 (a + b\arctan(cx^2))^2 \, dx^2$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8 (a + b\arctan(cx^2))^2 - \frac{1}{2}bc\int \frac{x^8 (a + b\arctan(cx^2))}{c^2x^4 + 1}dx^2\right)$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8 (a + b\arctan(cx^2))^2 - \frac{1}{2}bc\left(\frac{\int x^4 (a + b\arctan(cx^2))\, dx^2}{c^2} - \frac{\int \frac{x^4 (a+b\arctan(cx^2))}{c^2x^4+1}dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8 (a + b\arctan(cx^2))^2 - \frac{1}{2}bc\left(\frac{\frac{1}{3}x^6 (a + b\arctan(cx^2)) - \frac{1}{3}bc\int \frac{x^6}{c^2x^4+1}dx^2}{c^2} - \frac{\int \frac{x^4 (a+b\arctan(cx^2))}{c^2x^4+1}dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{6}bc\int\frac{x^4}{c^2x^4+1}dx^4}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx^2))}{c^2x^4+1}dx^2}{c^2}\right)\right)$$

$\Big\downarrow$ 49

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{6}bc\int\left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^4+1)}\right)dx^4}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx^2}{c^2x^4+1}}{c^2}\right.\right.$$

$\Big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{6}bc\left(\frac{x^4}{c^2}-\frac{\log(c^2x^4+1)}{c^4}\right)}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx^2))}{c^2x^4+1}dx^2}{c^2}\right.\right.$$

$\Big\downarrow$ 5451

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{6}bc\left(\frac{x^4}{c^2}-\frac{\log(c^2x^4+1)}{c^4}\right)}{c^2}-\frac{\frac{\int(a+b\arctan(cx^2))dx^2}{c^2}-}{c^2}\right.\right.$$

$\Big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{6}bc\left(\frac{x^4}{c^2}-\frac{\log(c^2x^4+1)}{c^4}\right)}{c^2}-\frac{\frac{ax^2+bx^2\arctan(cx^2)-\frac{b\log}{}}{c^2}}{}\right.\right.$$

$\Big\downarrow$ 5419

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{6}bc\left(\frac{x^4}{c^2}-\frac{\log(c^2x^4+1)}{c^4}\right)}{c^2}-\frac{\frac{ax^2+bx^2\arctan(cx^2)-\frac{b\log}{}}{c^2}}{}\right.\right.$$

input
```
Int[x^7*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
((x^8*(a + b*ArcTan[c*x^2])^2)/4 - (b*c*(((x^6*(a + b*ArcTan[c*x^2]))/3 -
(b*c*(x^4/c^2 - Log[1 + c^2*x^4]/c^4))/6)/c^2 - (-1/2*(a + b*ArcTan[c*x^2]
)^2/(b*c^3) + (a*x^2 + b*x^2*ArcTan[c*x^2] - (b*Log[1 + c^2*x^4])/(2*c))/c
^2)/c^2))/2)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

method	result
default	$\frac{a^2x^8}{8}+\frac{b^2x^8\arctan(cx^2)^2}{8}-\frac{b^2\arctan(cx^2)x^6}{12c}+\frac{b^2x^2\arctan(cx^2)}{4c^3}-\frac{b^2\arctan(cx^2)^2}{8c^4}+\frac{b^2x^4}{24c^2}-\frac{b^2\ln(c^2x^4+1)}{6c^4}$
parts	$\frac{a^2x^8}{8}+\frac{b^2x^8\arctan(cx^2)^2}{8}-\frac{b^2\arctan(cx^2)x^6}{12c}+\frac{b^2x^2\arctan(cx^2)}{4c^3}-\frac{b^2\arctan(cx^2)^2}{8c^4}+\frac{b^2x^4}{24c^2}-\frac{b^2\ln(c^2x^4+1)}{6c^4}$
parallelrisch	$-\frac{-3b^2\arctan(cx^2)^2x^8c^4-6ab\arctan(cx^2)x^8c^4-3c^4a^2x^8+2b^2\arctan(cx^2)x^6c^3+2abc^3x^6-x^4b^2c^2-6b^2\arctan(cx^2)x^2c-}{24c^4}$
risch	$-\frac{b^2(x^8c^4-1)\ln(icx^2+1)^2}{32c^4}-\frac{ib(6ac^4x^8+3ibc^4x^8\ln(-icx^2+1)-2bc^3x^6+6bcx^2-3ib\ln(-icx^2+1))\ln(icx^2+1)}{48c^4}+\frac{iab}{}$

input
```
int(x^7*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*a^2*x^8+1/8*b^2*x^8*arctan(c*x^2)^2-1/12*b^2*arctan(c*x^2)/c*x^6+1/4*b
^2*x^2*arctan(c*x^2)/c^3-1/8*b^2/c^4*arctan(c*x^2)^2+1/24*b^2*x^4/c^2-1/6*
b^2*ln(c^2*x^4+1)/c^4+1/4*a*b*x^8*arctan(c*x^2)-1/12*a*b/c*x^6+1/4*a*b*x^2
/c^3-1/4*a*b/c^4*arctan(c*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$=\frac{3\,a^2c^4x^8-2\,abc^3x^6+b^2c^2x^4+6\,abcx^2+3\,(b^2c^4x^8-b^2)\arctan\left(cx^2\right)^2-4\,b^2\log\left(c^2x^4+1\right)+2\,(3\,abc^4x^8}{24\,c^4}$$

input
```
integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
1/24*(3*a^2*c^4*x^8 - 2*a*b*c^3*x^6 + b^2*c^2*x^4 + 6*a*b*c*x^2 + 3*(b^2*c
^4*x^8 - b^2)*arctan(c*x^2)^2 - 4*b^2*log(c^2*x^4 + 1) + 2*(3*a*b*c^4*x^8
- b^2*c^3*x^6 + 3*b^2*c*x^2 - 3*a*b)*arctan(c*x^2))/c^4
```

Sympy [A] (verification not implemented)

Time = 38.46 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.60

$$\int x^7 (a + b \arctan(cx^2))^2 \, dx = \begin{cases} \frac{a^2x^8}{8} + \frac{abx^8 \operatorname{atan}(cx^2)}{4} - \frac{abx^6}{12c} + \frac{abx^2}{4c^3} - \frac{ab \operatorname{atan}(cx^2)}{4c^4} + \frac{b^2x^8 \operatorname{atan}^2(cx^2)}{8} - \frac{b^2x^6 \operatorname{atan}(cx^2)}{12c} + \frac{b^2x^4}{24c^2} + \frac{b^2x^2 \operatorname{atan}(cx^2)}{4c^3} - \\ \frac{a^2x^8}{8} \end{cases}$$

input
```
integrate(x**7*(a+b*atan(c*x**2))**2,x)
```

output
```
Piecewise((a**2*x**8/8 + a*b*x**8*atan(c*x**2)/4 - a*b*x**6/(12*c) + a*b*x
**2/(4*c**3) - a*b*atan(c*x**2)/(4*c**4) + b**2*x**8*atan(c*x**2)**2/8 - b
**2*x**6*atan(c*x**2)/(12*c) + b**2*x**4/(24*c**2) + b**2*x**2*atan(c*x**2
)/(4*c**3) - b**2*sqrt(-1/c**2)*atan(c*x**2)/(3*c**3) - b**2*log(x**2 + sq
rt(-1/c**2))/(3*c**4) - b**2*atan(c*x**2)**2/(8*c**4), Ne(c, 0)), (a**2*x*
*8/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.36

$$\int x^7 (a + b \arctan(cx^2))^2 \, dx = \frac{1}{8} b^2 x^8 \arctan(cx^2)^2 + \frac{1}{8} a^2 x^8 + \frac{1}{12} \left(3x^8 \arctan(cx^2) - c \left(\frac{c^2x^6 - 3x^2}{c^4} + \frac{3 \arctan(cx^2)}{c^5} \right) \right) ab - \frac{1}{24} \left(2c \left(\frac{c^2x^6 - 3x^2}{c^4} + \frac{3 \arctan(cx^2)}{c^5} \right) \arctan(cx^2) - \frac{c^2x^4 + 3 \arctan(cx^2)^2 - 3 \log(12c^7x^4 + 12c}{c^4} \right.$$

input
```
integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/8*b^2*x^8*arctan(c*x^2)^2 + 1/8*a^2*x^8 + 1/12*(3*x^8*arctan(c*x^2) - c*
((c^2*x^6 - 3*x^2)/c^4 + 3*arctan(c*x^2)/c^5))*a*b - 1/24*(2*c*((c^2*x^6 -
 3*x^2)/c^4 + 3*arctan(c*x^2)/c^5)*arctan(c*x^2) - (c^2*x^4 + 3*arctan(c*x
^2)^2 - 3*log(12*c^7*x^4 + 12*c^5) - log(c^2*x^4 + 1))/c^4)*b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

$$\int x^7 \left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$= \frac{3\,a^2cx^8 + 2\left(3\,cx^8\arctan\left(cx^2\right) - \frac{3\,\arctan\left(cx^2\right)}{c^3} - \frac{c^9x^6-3\,c^7x^2}{c^9}\right)ab + \left(3\,cx^8\arctan\left(cx^2\right)^2 - \frac{2\,c^3x^6\arctan\left(cx^2\right)-c^2x}{}\right.}{24\,c}$$

input
```
integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
1/24*(3*a^2*c*x^8 + 2*(3*c*x^8*arctan(c*x^2) - 3*arctan(c*x^2)/c^3 - (c^9*
x^6 - 3*c^7*x^2)/c^9)*a*b + (3*c*x^8*arctan(c*x^2)^2 - (2*c^3*x^6*arctan(c
*x^2) - c^2*x^4 - 6*c*x^2*arctan(c*x^2) + 3*arctan(c*x^2)^2 + 4*log(c^2*x^
4 + 1))/c^3)*b^2)/c
```

Mupad [B] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\int x^7 \left(a+b\arctan\left(cx^2\right)\right)^2\,dx = \frac{a^2\,x^8}{8} - \frac{b^2\,\mathrm{atan}(c\,x^2)^2}{8\,c^4} + \frac{b^2\,x^8\,\mathrm{atan}(c\,x^2)^2}{8} - \frac{b^2\,\ln\left(c^2\,x^4+1\right)}{6\,c^4} + \frac{b^2\,x^4}{24\,c^2} + \frac{b^2\,x^2\,\mathrm{atan}(c\,x^2)}{4\,c^3} - \frac{b^2\,x^6\,\mathrm{atan}(c\,x^2)}{12\,c} + \frac{a\,b\,x^2}{4\,c^3} - \frac{a\,b\,x^6}{12\,c} - \frac{a\,b\,\mathrm{atan}(c\,x^2)}{4\,c^4} + \frac{a\,b\,x^8\,\mathrm{atan}(c\,x^2)}{4}$$

input
```
int(x^7*(a + b*atan(c*x^2))^2,x)
```

output
```
(a^2*x^8)/8 - (b^2*atan(c*x^2)^2)/(8*c^4) + (b^2*x^8*atan(c*x^2)^2)/8 - (b
^2*log(c^2*x^4 + 1))/(6*c^4) + (b^2*x^4)/(24*c^2) + (b^2*x^2*atan(c*x^2))/
(4*c^3) - (b^2*x^6*atan(c*x^2))/(12*c) + (a*b*x^2)/(4*c^3) - (a*b*x^6)/(12
*c) - (a*b*atan(c*x^2))/(4*c^4) + (a*b*x^8*atan(c*x^2))/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.40

$$\int x^7 (a + b \arctan(cx^2))^2 \, dx$$
$$= \frac{3atan(c\,x^2)^2\,b^2c^4x^8 - 3atan(c\,x^2)^2\,b^2 + 6atan(c\,x^2)\,ab\,c^4x^8 - 6atan(c\,x^2)\,ab - 2atan(c\,x^2)\,b^2c^3x^6 + 6ata}{}$$

input
```
int(x^7*(a+b*atan(c*x^2))^2,x)
```

output
```
(3*atan(c*x**2)**2*b**2*c**4*x**8 - 3*atan(c*x**2)**2*b**2 + 6*atan(c*x**2
)*a*b*c**4*x**8 - 6*atan(c*x**2)*a*b - 2*atan(c*x**2)*b**2*c**3*x**6 + 6*a
tan(c*x**2)*b**2*c*x**2 - 4*log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b**2 -
4*log(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b**2 + 3*a**2*c**4*x**8 - 2*a*b*c**3
*x**6 + 6*a*b*c*x**2 + b**2*c**2*x**4)/(24*c**4)
```

3.75 $\int x^5(a+b\arctan(cx^2))^2\,dx$

Optimal result 577
Mathematica [A] (verified) 578
Rubi [A] (verified) 578
Maple [C] (warning: unable to verify) 582
Fricas [F] 583
Sympy [F] 583
Maxima [F] 583
Giac [F] 584
Mupad [F(-1)] 584
Reduce [F] 584

Optimal result

Integrand size = 16, antiderivative size = 154

$$\int x^5(a+b\arctan(cx^2))^2\,dx=\frac{b^2x^2}{6c^2}-\frac{b^2\arctan(cx^2)}{6c^3}-\frac{bx^4(a+b\arctan(cx^2))}{6c}-\frac{i(a+b\arctan(cx^2))^2}{6c^3}+\frac{1}{6}x^6(a+b\arctan(cx^2))^2-\frac{b(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{3c^3}-\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{6c^3}$$

output

```
1/6*b^2*x^2/c^2-1/6*b^2*arctan(c*x^2)/c^3-1/6*b*x^4*(a+b*arctan(c*x^2))/c-
1/6*I*(a+b*arctan(c*x^2))^2/c^3+1/6*x^6*(a+b*arctan(c*x^2))^2-1/3*b*(a+b*a
rctan(c*x^2))*ln(2/(1+I*c*x^2))/c^3-1/6*I*b^2*polylog(2,1-2/(1+I*c*x^2))/c
^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\int x^5 (a + b \arctan(cx^2))^2 \, dx$$
$$= \frac{b^2cx^2 - abc^2x^4 + a^2c^3x^6 + b^2(i + c^3x^6)\arctan(cx^2)^2 - b\arctan(cx^2)\left(b + bc^2x^4 - 2ac^3x^6 + 2b\log\left(1 +\right.\right.}{6c^3}$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
(b^2*c*x^2 - a*b*c^2*x^4 + a^2*c^3*x^6 + b^2*(I + c^3*x^6)*ArcTan[c*x^2]^2
 - b*ArcTan[c*x^2]*(b + b*c^2*x^4 - 2*a*c^3*x^6 + 2*b*Log[1 + E^((2*I)*Arc
Tan[c*x^2])]) + a*b*Log[1 + c^2*x^4] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c
*x^2])])/(6*c^3)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5363, 5361, 5451, 5361, 262, 216, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (a + b \arctan(cx^2))^2 \, dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{2}\int x^4 (a + b \arctan(cx^2))^2 \, dx^2$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\frac{1}{3}x^6 (a + b \arctan(cx^2))^2 - \frac{2}{3}bc\int \frac{x^6 (a + b \arctan(cx^2))}{c^2x^4 + 1}dx^2\right)$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{2}{3}bc\left(\frac{\int x^2\left(a+b\arctan\left(cx^2\right)\right)dx^2}{c^2}-\frac{\int\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)}{c^2x^4+1}dx^2}{c^2}\right)\right)$$

$\big\downarrow$ 5361

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}bc\int\frac{x^4}{c^2x^4+1}dx^2}{c^2}-\frac{\int\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)}{c^2x^4+1}dx^2}{c^2}\right)\right)$$

$\big\downarrow$ 262

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}bc\left(\frac{x^2}{c^2}-\frac{\int\frac{1}{c^2x^4+1}dx^2}{c^2}\right)}{c^2}-\frac{\int\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)}{c^2x^4+1}dx^2}{c^2}\right)\right)$$

$\big\downarrow$ 216

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}bc\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)}{c^2}-\frac{\int\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)}{c^2x^4+1}dx^2}{c^2}\right)\right)$$

$\big\downarrow$ 5455

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}bc\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)}{c^2}-\frac{-\frac{\int\frac{a+b\arctan\left(cx^2\right)}{i-cx^2}dx^2}{c}-\ldots}{c^2}\right)\right)$$

$\big\downarrow$ 5379

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}bc\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)}{c^2}-\frac{-\frac{\log\left(\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(c\ldots\right.\right.}{c}\ldots}{\ldots}\right.\right.$$

$\big\downarrow$ 2849

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}bc\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)}{c^2}-\frac{-\frac{ib\int\frac{\log\left(\frac{2}{icx^2+1}\right)}{1-\frac{2}{icx^2+1}}d\frac{1}{icx^2+1}}{c}\cdots}{\cdots}\right.\right.$$

$\downarrow$ 2752

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}bc\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)}{c^2}-\frac{-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^2}{2bc^2}-\cdots}{\cdots}\right.\right.$$

input
```
Int[x^5*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
((x^6*(a + b*ArcTan[c*x^2])^2)/3 - (2*b*c*(((x^4*(a + b*ArcTan[c*x^2]))/2
- (b*c*(x^2/c^2 - ArcTan[c*x^2]/c^3))/2)/c^2 - (((-1/2*I)*(a + b*ArcTan[c*
x^2])^2)/(b*c^2) - (((a + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/c + ((I/2
)*b*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c)/c)/c^2))/3)/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.79 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.16

method	result
default	$\frac{x^6 a^2}{6}+\frac{b^2 \arctan(c\,x^2)^2 x^6}{6}-\frac{b^2 \arctan(c\,x^2) x^4}{6c}+\frac{b^2 \arctan(c\,x^2)\ln(c^2x^4+1)}{6c^3}+\frac{b^2 x^2}{6c^2}-\frac{b^2\arctan(c\,x^2)}{6c^3}-\frac{b^2\left(\sum_{\_\alpha=\mathrm{Root}} \right.}{}$
parts	$\frac{x^6 a^2}{6}+\frac{b^2 \arctan(c\,x^2)^2 x^6}{6}-\frac{b^2 \arctan(c\,x^2) x^4}{6c}+\frac{b^2 \arctan(c\,x^2)\ln(c^2x^4+1)}{6c^3}+\frac{b^2 x^2}{6c^2}-\frac{b^2\arctan(c\,x^2)}{6c^3}-\frac{b^2\left(\sum_{\_\alpha=\mathrm{Root}} \right.}{}$
risch	$\frac{b^2\ln(ic\,x^2+1)\ln(-ic\,x^2+1)x^6}{12}-\frac{17ib^2}{108c^3}-\frac{iba\,x^6\ln(ic\,x^2+1)}{6}-\frac{ib^2x^4\ln(-ic\,x^2+1)}{12c}+\frac{ib^2\ln\left(\frac{1}{2}+\frac{ic\,x^2}{2}\right)\ln(-ic\,x^2+1)}{6c^3}-$

input
```
int(x^5*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*a^2+1/6*b^2*arctan(c*x^2)^2*x^6-1/6*b^2*arctan(c*x^2)/c*x^4+1/6*b^
2*arctan(c*x^2)/c^3*ln(c^2*x^4+1)+1/6*b^2*x^2/c^2-1/6*b^2*arctan(c*x^2)/c^
3-1/24*b^2/c^4*sum(1/_alpha^2*(2*ln(x-_alpha)*ln(c^2*x^4+1)-c*(1/c/_alpha^
3*ln(x-_alpha)^2+2/_alpha*ln(x-_alpha)*(_alpha^2*ln(1/2*(x+_alpha)/_alpha)
*c+ln((_alpha^3*c-x)/_alpha/(_alpha^2*c-1))-ln((_alpha^3*c+x)/_alpha/(_alp
ha^2*c+1)))+2/_alpha*(_alpha^2*dilog(1/2*(x+_alpha)/_alpha)*c+dilog((_alph
a^3*c-x)/_alpha/(_alpha^2*c-1))-dilog((_alpha^3*c+x)/_alpha/(_alpha^2*c+1)
)))),_alpha=RootOf(_Z^4*c^2+1))+1/3*a*b*arctan(c*x^2)*x^6-1/6/c*a*b*x^4+1/
6*a*b/c^3*ln(c^2*x^4+1)
```

Fricas [F]

$$\int x^5 \left(a + b \arctan \left(c x^2\right)\right)^2 \, dx = \int \left(b \arctan \left(c x^2\right) + a\right)^2 x^5 \, dx$$

input
```
integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^5*arctan(c*x^2)^2 + 2*a*b*x^5*arctan(c*x^2) + a^2*x^5, x)
```

Sympy [F]

$$\int x^5 \left(a + b \arctan \left(c x^2\right)\right)^2 \, dx = \int x^5 \left(a + b \operatorname{atan}\left(c x^2\right)\right)^2 \, dx$$

input
```
integrate(x**5*(a+b*atan(c*x**2))**2,x)
```

output
```
Integral(x**5*(a + b*atan(c*x**2))**2, x)
```

Maxima [F]

$$\int x^5 \left(a + b \arctan \left(c x^2\right)\right)^2 \, dx = \int \left(b \arctan \left(c x^2\right) + a\right)^2 x^5 \, dx$$

input
```
integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/6*a^2*x^6 + 1/6*(2*x^6*arctan(c*x^2) - (x^4/c^2 - log(c^2*x^4 + 1)/c^4)*
c)*a*b + 1/96*(4*x^6*arctan(c*x^2)^2 - x^6*log(c^2*x^4 + 1)^2 + 96*integra
te(1/48*(4*c^2*x^9*log(c^2*x^4 + 1) - 8*c*x^7*arctan(c*x^2) + 36*(c^2*x^9
+ x^5)*arctan(c*x^2)^2 + 3*(c^2*x^9 + x^5)*log(c^2*x^4 + 1)^2)/(c^2*x^4 +
1), x))*b^2
```

Giac [F]

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)^2x^5\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int x^5\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2\,dx$$

input
```
int(x^5*(a + b*atan(c*x^2))^2,x)
```

output
```
int(x^5*(a + b*atan(c*x^2))^2, x)
```

Reduce [F]

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$=\frac{atan(c\,x^2)^2\,b^2c^3x^6+atan(c\,x^2)^2\,b^2c\,x^2+2atan(c\,x^2)\,ab\,c^3x^6-atan(c\,x^2)\,b^2c^2x^4-atan(c\,x^2)\,b^2-2\Big(\int a}{}$$

input
```
int(x^5*(a+b*atan(c*x^2))^2,x)
```

output
```
(atan(c*x**2)**2*b**2*c**3*x**6 + atan(c*x**2)**2*b**2*c*x**2 + 2*atan(c*x
**2)*a*b*c**3*x**6 - atan(c*x**2)*b**2*c**2*x**4 - atan(c*x**2)*b**2 - 2*i
nt(atan(c*x**2)**2*x,x)*b**2*c + log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a*
b + log(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a*b + a**2*c**3*x**6 - a*b*c**2*x*
*4 + b**2*c*x**2)/(6*c**3)
```

3.76 $\int x^3(a+b\arctan(cx^2))^2\,dx$

Optimal result 586
Mathematica [A] (verified) 586
Rubi [A] (verified) 587
Maple [A] (verified) 589
Fricas [A] (verification not implemented) 589
Sympy [A] (verification not implemented) 590
Maxima [A] (verification not implemented) 590
Giac [A] (verification not implemented) 591
Mupad [B] (verification not implemented) 591
Reduce [B] (verification not implemented) 592

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=-\frac{abx^2}{2c}-\frac{b^2x^2\arctan\left(cx^2\right)}{2c}+\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{4c^2}+\frac{1}{4}x^4\left(a+b\arctan\left(cx^2\right)\right)^2+\frac{b^2\log\left(1+c^2x^4\right)}{4c^2}$$

output
```
-1/2*a*b*x^2/c-1/2*b^2*x^2*arctan(c*x^2)/c+1/4*(a+b*arctan(c*x^2))^2/c^2+1
/4*x^4*(a+b*arctan(c*x^2))^2+1/4*b^2*ln(c^2*x^4+1)/c^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\frac{acx^2(-2b+acx^2)+2b(a-bcx^2+ac^2x^4)\arctan\left(cx^2\right)+b^2(1+c^2x^4)\arctan\left(cx^2\right)^2+b^2\log\left(1+c^2x^4\right)}{4c^2}$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
(a*c*x^2*(-2*b + a*c*x^2) + 2*b*(a - b*c*x^2 + a*c^2*x^4)*ArcTan[c*x^2] +
b^2*(1 + c^2*x^4)*ArcTan[c*x^2]^2 + b^2*Log[1 + c^2*x^4])/(4*c^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5363, 5361, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)^2dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{2}\int x^2\left(a+b\arctan\left(cx^2\right)\right)^2dx^2$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^2-bc\int\frac{x^4\left(a+b\arctan\left(cx^2\right)\right)}{c^2x^4+1}dx^2\right)$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^2-bc\left(\frac{\int\left(a+b\arctan\left(cx^2\right)\right)dx^2}{c^2}-\frac{\int\frac{a+b\arctan(cx^2)}{c^2x^4+1}dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^2-bc\left(\frac{ax^2+bx^2\arctan\left(cx^2\right)-\frac{b\log(c^2x^4+1)}{2c}}{c^2}-\frac{\int\frac{a+b\arctan(cx^2)}{c^2x^4+1}dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^2-bc\left(\frac{ax^2+bx^2\arctan\left(cx^2\right)-\frac{b\log(c^2x^4+1)}{2c}}{c^2}-\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{2bc^3}\right)\right)$$

input
```
Int[x^3*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
((x^4*(a + b*ArcTan[c*x^2])^2)/2 - b*c*(-1/2*(a + b*ArcTan[c*x^2])^2/(b*c^
3) + (a*x^2 + b*x^2*ArcTan[c*x^2] - (b*Log[1 + c^2*x^4])/(2*c))/c^2))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22

method	result
parallelrisch	$\frac{b^2 \arctan(c x^2)^2 x^4 c^2+2ab \arctan(c x^2) x^4 c^2+a^2 c^2 x^4-2b^2 \arctan(c x^2) x^2 c-2abc x^2+b^2 \arctan(c x^2)^2+b^2 \ln(c^2 x^4+1)+2ab}{4c^2}$
default	$\frac{a^2 x^4}{4}+\frac{b^2 \arctan(c x^2)^2 x^4}{4}-\frac{b^2 x^2 \arctan(c x^2)}{2c}+\frac{b^2 \arctan(c x^2)^2}{4c^2}+\frac{b^2 \ln(c^2 x^4+1)}{4c^2}+\frac{ab \arctan(c x^2) x^4}{2}-\frac{ab x^2}{2c}$
parts	$\frac{a^2 x^4}{4}+\frac{b^2 \arctan(c x^2)^2 x^4}{4}-\frac{b^2 x^2 \arctan(c x^2)}{2c}+\frac{b^2 \arctan(c x^2)^2}{4c^2}+\frac{b^2 \ln(c^2 x^4+1)}{4c^2}+\frac{ab \arctan(c x^2) x^4}{2}-\frac{ab x^2}{2c}$
risch	$-\frac{b^2 (c^2 x^4+1) \ln(ic x^2+1)^2}{16c^2}-\frac{ib(4a^2 c^2 x^4+2i x^4 b \ln(-ic x^2+1) a c^2-4abc x^2+b^2+2ib \ln(-ic x^2+1) a) \ln(ic x^2+1)}{16a c^2}+\frac{ib^3}{}$

input
```
int(x^3*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b^2*arctan(c*x^2)^2*x^4*c^2+2*a*b*arctan(c*x^2)*x^4*c^2+a^2*c^2*x^4-2
*b^2*arctan(c*x^2)*x^2*c-2*a*b*c*x^2+b^2*arctan(c*x^2)^2+b^2*ln(c^2*x^4+1)
+2*a*b*arctan(c*x^2))/c^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int x^3 (a+b\arctan(cx^2))^2 \, dx = \frac{a^2c^2x^4-2\,abcx^2+(b^2c^2x^4+b^2)\arctan(cx^2)^2+b^2\log(c^2x^4+1)+2\,(abc^2x^4-b^2cx^2+ab)\arctan(cx^2)}{4\,c^2}$$

input
```
integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
1/4*(a^2*c^2*x^4 - 2*a*b*c*x^2 + (b^2*c^2*x^4 + b^2)*arctan(c*x^2)^2 + b^2
*log(c^2*x^4 + 1) + 2*(a*b*c^2*x^4 - b^2*c*x^2 + a*b)*arctan(c*x^2))/c^2
```

Sympy [A] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.72

$$\int x^3 \left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$=\begin{cases} \frac{a^2x^4}{4}+\frac{abx^4\operatorname{atan}\left(cx^2\right)}{2}-\frac{abx^2}{2c}+\frac{ab\operatorname{atan}\left(cx^2\right)}{2c^2}+\frac{b^2x^4\operatorname{atan}^2\left(cx^2\right)}{4}-\frac{b^2x^2\operatorname{atan}\left(cx^2\right)}{2c}+\frac{b^2\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^2\right)}{2c}+\frac{b^2\log\left(x^2+\sqrt{-\right.}}{2c^2} \\ \frac{a^2x^4}{4} \end{cases}$$

input
```
integrate(x**3*(a+b*atan(c*x**2))**2,x)
```

output
```
Piecewise((a**2*x**4/4 + a*b*x**4*atan(c*x**2)/2 - a*b*x**2/(2*c) + a*b*at
an(c*x**2)/(2*c**2) + b**2*x**4*atan(c*x**2)**2/4 - b**2*x**2*atan(c*x**2)
/(2*c) + b**2*sqrt(-1/c**2)*atan(c*x**2)/(2*c) + b**2*log(x**2 + sqrt(-1/c
**2))/(2*c**2) + b**2*atan(c*x**2)**2/(4*c**2), Ne(c, 0)), (a**2*x**4/4, T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.40

$$\int x^3 \left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$=\frac{1}{4}\,b^2x^4\arctan\left(cx^2\right)^2+\frac{1}{4}\,a^2x^4+\frac{1}{2}\left(x^4\arctan\left(cx^2\right)-c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\right)ab$$
$$-\frac{1}{4}\left(2\,c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\arctan\left(cx^2\right)+\frac{\arctan\left(cx^2\right)^2-\log\left(4\,c^5x^4+4\,c^3\right)}{c^2}\right)b^2$$

input
```
integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*arctan(c*x^2)^2 + 1/4*a^2*x^4 + 1/2*(x^4*arctan(c*x^2) - c*(x^
2/c^2 - arctan(c*x^2)/c^3))*a*b - 1/4*(2*c*(x^2/c^2 - arctan(c*x^2)/c^3)*a
rctan(c*x^2) + (arctan(c*x^2)^2 - log(4*c^5*x^4 + 4*c^3))/c^2)*b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int x^3 \left(a+b \arctan \left(cx^2\right)\right)^2 \, dx$$
$$= \frac{a^2cx^4 + \frac{2\left(c^2x^4\arctan(cx^2)-cx^2+\arctan(cx^2)\right)ab}{c} + \frac{\left(c^2x^4\arctan(cx^2)^2-2\,cx^2\arctan(cx^2)+\arctan(cx^2)^2+\log(c^2x^4+1)\right)b^2}{c}}{4\,c}$$

input
```
integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
1/4*(a^2*c*x^4 + 2*(c^2*x^4*arctan(c*x^2) - c*x^2 + arctan(c*x^2))*a*b/c +
 (c^2*x^4*arctan(c*x^2)^2 - 2*c*x^2*arctan(c*x^2) + arctan(c*x^2)^2 + log(
c^2*x^4 + 1))*b^2/c)/c
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.24

$$\int x^3 \left(a+b \arctan \left(cx^2\right)\right)^2 \, dx = \frac{a^2\,x^4}{4} + \frac{b^2\,\mathrm{atan}(c\,x^2)^2}{4\,c^2} + \frac{b^2\,x^4\,\mathrm{atan}(c\,x^2)^2}{4} + \frac{b^2\,\ln\left(c^2\,x^4+1\right)}{4\,c^2} - \frac{b^2\,x^2\,\mathrm{atan}(c\,x^2)}{2\,c} - \frac{a\,b\,x^2}{2\,c} + \frac{a\,b\,\mathrm{atan}(c\,x^2)}{2\,c^2} + \frac{a\,b\,x^4\,\mathrm{atan}(c\,x^2)}{2}$$

input
```
int(x^3*(a + b*atan(c*x^2))^2,x)
```

output
```
(a^2*x^4)/4 + (b^2*atan(c*x^2)^2)/(4*c^2) + (b^2*x^4*atan(c*x^2)^2)/4 + (b
^2*log(c^2*x^4 + 1))/(4*c^2) - (b^2*x^2*atan(c*x^2))/(2*c) - (a*b*x^2)/(2*
c) + (a*b*atan(c*x^2))/(2*c^2) + (a*b*x^4*atan(c*x^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.47

$$\int x^3 (a + b \arctan(cx^2))^2 \, dx$$

$$= \frac{atan(c\,x^2)^2\,b^2c^2x^4 + atan(c\,x^2)^2\,b^2 + 2atan(c\,x^2)\,ab\,c^2x^4 + 2atan(c\,x^2)\,ab - 2atan(c\,x^2)\,b^2c\,x^2 + \log(-\sqrt{}}{4c^2}$$

input
```
int(x^3*(a+b*atan(c*x^2))^2,x)
```

output
```
(atan(c*x**2)**2*b**2*c**2*x**4 + atan(c*x**2)**2*b**2 + 2*atan(c*x**2)*a*
b*c**2*x**4 + 2*atan(c*x**2)*a*b - 2*atan(c*x**2)*b**2*c*x**2 + log( - sqr
t(c)*sqrt(2)*x + c*x**2 + 1)*b**2 + log(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b*
*2 + a**2*c**2*x**4 - 2*a*b*c*x**2)/(4*c**2)
```

3.77 $\int x(a+b\arctan(cx^2))^2\,dx$

Optimal result 593
Mathematica [A] (verified) 593
Rubi [A] (verified) 594
Maple [A] (verified) 596
Fricas [F] 597
Sympy [F] 597
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Reduce [F] 599

Optimal result

Integrand size = 14, antiderivative size = 101

$$\int x(a+b\arctan(cx^2))^2\,dx=\frac{i(a+b\arctan(cx^2))^2}{2c}+\frac{1}{2}x^2(a+b\arctan(cx^2))^2+\frac{b(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{c}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{2c}$$

output
```
1/2*I*(a+b*arctan(c*x^2))^2/c+1/2*x^2*(a+b*arctan(c*x^2))^2+b*(a+b*arctan(
c*x^2))*ln(2/(1+I*c*x^2))/c+1/2*I*b^2*polylog(2,1-2/(1+I*c*x^2))/c
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int x(a+b\arctan(cx^2))^2\,dx=\frac{b^2(-i+cx^2)\arctan(cx^2)^2+2b\arctan(cx^2)\left(acx^2+b\log\left(1+e^{2i\arctan(cx^2)}\right)\right)+a(acx^2-b\log(1+c^2x}{2c}$$

input
```
Integrate[x*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
(b^2*(-I + c*x^2)*ArcTan[c*x^2]^2 + 2*b*ArcTan[c*x^2]*(a*c*x^2 + b*Log[1 +
 E^((2*I)*ArcTan[c*x^2])]) + a*(a*c*x^2 - b*Log[1 + c^2*x^4]) - I*b^2*Poly
Log[2, -E^((2*I)*ArcTan[c*x^2])])/(2*c)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5363, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{2}\int \left(a+b\arctan\left(cx^2\right)\right)^2dx^2$$

$$\Big\downarrow \text{5345}$$

$$\frac{1}{2}\left(x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\int \frac{x^2\left(a+b\arctan\left(cx^2\right)\right)}{c^2x^4+1}dx^2\right)$$

$$\Big\downarrow \text{5455}$$

$$\frac{1}{2}\left(x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{\int \frac{a+b\arctan(cx^2)}{i-cx^2}dx^2}{c}-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^2}{2bc^2}\right)\right)$$

$$\Big\downarrow \text{5379}$$

$$\frac{1}{2}\left(x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx^2}\right)\left(a+b\arctan(cx^2)\right)}{c}-b\int \frac{\log\left(\frac{2}{icx^2+1}\right)}{c^2x^4+1}dx^2}{c}-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^2}{2bc^2}\right)\right)$$

$$\Big\downarrow \text{2849}$$

$$\frac{1}{2}\left(x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{-\frac{ib\int\frac{\log\left(\frac{2}{icx^2+1}\right)}{1-\frac{2}{icx^2+1}}d\frac{1}{icx^2+1}}{c}+\frac{\log\left(\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)}{c}}{c}-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^2}{2bc^2}\right.\right.$$

$$\Big\downarrow \; 2752$$

$$\frac{1}{2}\left(x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)}{c}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)}{2c}}{c}\right.\right.$$

input
```
Int[x*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
(x^2*(a + b*ArcTan[c*x^2])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^2])^2)/(
b*c^2) - (((a + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/c + ((I/2)*b*PolyLo
g[2, 1 - 2/(1 + I*c*x^2)])/c)/c))/2
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.39

method	result
parts	$\frac{a^2x^2}{2}+\frac{b^2\left(\arctan(c\,x^2)^2(c\,x^2+i)+2\arctan(c\,x^2)\ln\left(1+\frac{(ic\,x^2+1)^2}{c^2x^4+1}\right)-2i\arctan(c\,x^2)^2-i\,\mathrm{polylog}\left(2,-\frac{(ic\,x^2+1)^2}{c^2x^4+1}\right.\right.}{2c}$
derivativedivides	$\frac{c\,x^2a^2-i\arctan(c\,x^2)^2b^2+\arctan(c\,x^2)^2b^2c\,x^2-i\,\mathrm{polylog}\left(2,-\frac{(ic\,x^2+1)^2}{c^2x^4+1}\right)b^2+2\arctan(c\,x^2)\ln\left(1+\frac{(ic\,x^2+1)^2}{c^2x^4+1}\right)b^2-}{2c}$
default	$\frac{c\,x^2a^2-i\arctan(c\,x^2)^2b^2+\arctan(c\,x^2)^2b^2c\,x^2-i\,\mathrm{polylog}\left(2,-\frac{(ic\,x^2+1)^2}{c^2x^4+1}\right)b^2+2\arctan(c\,x^2)\ln\left(1+\frac{(ic\,x^2+1)^2}{c^2x^4+1}\right)b^2-}{2c}$
risch	$-\frac{\ln(-ic\,x^2+1)ab}{2c}-\frac{ib^2\ln(ic\,x^2+1)}{2c}+\frac{ib^2\,\mathrm{dilog}\left(\frac{1}{2}-\frac{ic\,x^2}{2}\right)}{2c}+\frac{b^2\ln(ic\,x^2+1)\ln(-ic\,x^2+1)x^2}{4}-\frac{ba\ln(ic\,x^2+1)}{2c}$

input
```
int(x*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+1/2*b^2/c*(arctan(c*x^2)^2*(c*x^2+I)+2*arctan(c*x^2)*ln(1+(1+I
*c*x^2)^2/(c^2*x^4+1))-2*I*arctan(c*x^2)^2-I*polylog(2,-(1+I*c*x^2)^2/(c^2
*x^4+1)))+a*b/c*(c*x^2*arctan(c*x^2)-1/2*ln(c^2*x^4+1))
```

Fricas [F]

$$\int x(a+b\arctan(cx^2))^2\,dx=\int (b\arctan(cx^2)+a)^2x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x*arctan(c*x^2)^2 + 2*a*b*x*arctan(c*x^2) + a^2*x, x)
```

Sympy [F]

$$\int x(a+b\arctan(cx^2))^2\,dx=\int x(a+b\operatorname{atan}(cx^2))^2\,dx$$

input
```
integrate(x*(a+b*atan(c*x**2))**2,x)
```

output
```
Integral(x*(a + b*atan(c*x**2))**2, x)
```

Maxima [F]

$$\int x(a+b\arctan(cx^2))^2\,dx=\int (b\arctan(cx^2)+a)^2x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*x^2 + 1/32*(4*x^2*arctan(c*x^2)^2 - x^2*log(c^2*x^4 + 1)^2 + 384*c
^2*integrate(1/16*x^5*arctan(c*x^2)^2/(c^2*x^4 + 1), x) + 32*c^2*integrate
(1/16*x^5*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 128*c^2*integrate(1/16*x^
5*log(c^2*x^4 + 1)/(c^2*x^4 + 1), x) + 4*arctan(c*x^2)^3/c - 256*c*integra
te(1/16*x^3*arctan(c*x^2)/(c^2*x^4 + 1), x) + 32*integrate(1/16*x*log(c^2*
x^4 + 1)^2/(c^2*x^4 + 1), x))*b^2 + 1/2*(2*c*x^2*arctan(c*x^2) - log(c^2*x
^4 + 1))*a*b/c
```

Giac [F]

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int x\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2\,dx$$

input
```
int(x*(a + b*atan(c*x^2))^2,x)
```

output
```
int(x*(a + b*atan(c*x^2))^2, x)
```

Reduce [F]

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$=\frac{2atan(c\,x^2)\,abc\,x^2+2\left(\int atan(c\,x^2)^2\,xdx\right)b^2c-\log\left(-\sqrt{c}\,\sqrt{2}\,x+c\,x^2+1\right)ab-\log\left(\sqrt{c}\,\sqrt{2}\,x+c\,x^2+1\right.}{2c}$$

input

```
int(x*(a+b*atan(c*x^2))^2,x)
```

output

```
(2*atan(c*x**2)*a*b*c*x**2 + 2*int(atan(c*x**2)**2*x,x)*b**2*c - log( - sq
rt(c)*sqrt(2)*x + c*x**2 + 1)*a*b - log(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a*
b + a**2*c*x**2)/(2*c)
```

3.78 $\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx$

Optimal result 600
Mathematica [A] (verified) 601
Rubi [A] (verified) 601
Maple [F] 603
Fricas [F] 604
Sympy [F] 604
Maxima [F] 604
Giac [F] 605
Mupad [F(-1)] 605
Reduce [F] 605

Optimal result

Integrand size = 16, antiderivative size = 151

$$\begin{aligned}\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = {} & (a+b\arctan(cx^2))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right) \\ & -\frac{1}{2}ib(a+b\arctan(cx^2))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right) \\ & +\frac{1}{2}ib(a+b\arctan(cx^2))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx^2}\right) \\ & -\frac{1}{4}b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^2}\right) \\ & +\frac{1}{4}b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx^2}\right)\end{aligned}$$

output
```
-(a+b*arctan(c*x^2))^2*arctanh(-1+2/(1+I*c*x^2))-1/2*I*b*(a+b*arctan(c*x^2
))*polylog(2,1-2/(1+I*c*x^2))+1/2*I*b*(a+b*arctan(c*x^2))*polylog(2,-1+2/(
1+I*c*x^2))-1/4*b^2*polylog(3,1-2/(1+I*c*x^2))+1/4*b^2*polylog(3,-1+2/(1+I
*c*x^2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.33

$$\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = a^2\log(x) + \frac{1}{2}iab\left(\text{PolyLog}\left(2,-icx^2\right) - \text{PolyLog}\left(2,icx^2\right)\right) + \frac{1}{48}b^2\left(-i\pi^3 + 16i\arctan\left(cx^2\right)^3 + 24\arctan\left(cx^2\right)^2\log\left(1-e^{-2i\arctan(cx^2)}\right) - 24\arctan\left(cx^2\right)^2\log\left(1+e^{2i\arctan(cx^2)}\right) + 24i\arctan\left(cx^2\right)\text{PolyLog}\left(2,e^{-2i\arctan(cx^2)}\right) + 24i\arctan\left(cx^2\right)\text{PolyLog}\left(2,-e^{2i\arctan(cx^2)}\right) + 12\,\text{PolyLog}\left(3,e^{-2i\arctan(cx^2)}\right) - 12\,\text{PolyLog}\left(3,-e^{2i\arctan(cx^2)}\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^2/x,x]
```

output
```
a^2*Log[x] + (I/2)*a*b*(PolyLog[2, (-I)*c*x^2] - PolyLog[2, I*c*x^2]) + (b
^2*((-I)*Pi^3 + (16*I)*ArcTan[c*x^2]^3 + 24*ArcTan[c*x^2]^2*Log[1 - E^((-2
*I)*ArcTan[c*x^2])] - 24*ArcTan[c*x^2]^2*Log[1 + E^((2*I)*ArcTan[c*x^2])]
 + (24*I)*ArcTan[c*x^2]*PolyLog[2, E^((-2*I)*ArcTan[c*x^2])] + (24*I)*ArcTa
n[c*x^2]*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])] + 12*PolyLog[3, E^((-2*I)*Ar
cTan[c*x^2])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c*x^2])]))/48
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5359, 5357, 5523, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x}\,dx$$

$\Big\downarrow$ 5359

$$\frac{1}{2}\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2}dx^2$$

$\Big\downarrow$ 5357

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^2-4bc\int \frac{\left(a+b\arctan\left(cx^2\right)\right)\text{arctanh}\left(1-\frac{2}{icx^2+1}\right)}{c^2x^4+1}dx^2\right)$$

$\Big\downarrow$ 5523

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^2-4bc\left(\frac{1}{2}\int \frac{\left(a+b\arctan\left(cx^2\right)\right)\log\left(2-\frac{2}{icx^2+1}\right)}{c^2x^4+1}dx^2-\frac{1}{2}\int\right.\right.$$

$\Big\downarrow$ 5529

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^2-4bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)\left(a+b\arctan\left(cx^2\right)\right)}{2c}-\right.\right.\right.$$

$\Big\downarrow$ 7164

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^2-4bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)\left(a+b\arctan\left(cx^2\right)\right)}{2c}+\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^2])^2/x,x]
```

output
```
(2*(a + b*ArcTan[c*x^2])^2*ArcTanh[1 - 2/(1 + I*c*x^2)] - 4*b*c*((((I/2)*(
a + b*ArcTan[c*x^2])*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c + (b*PolyLog[3, 1
- 2/(1 + I*c*x^2)])/(4*c))/2 + (((-1/2*I)*(a + b*ArcTan[c*x^2])*PolyLog[2,
 -1 + 2/(1 + I*c*x^2)])/c - (b*PolyLog[3, -1 + 2/(1 + I*c*x^2)])/(4*c))/2)
)/2
```

Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [F]

$$\int \frac{\left(a+b\arctan\left(c\,x^{2}\right)\right)^{2}}{x}dx$$

input
```
int((a+b*arctan(c*x^2))^2/x,x)
```

output
```
int((a+b*arctan(c*x^2))^2/x,x)
```

Fricas [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x}\,dx = \int \frac{(b\arctan(cx^2) + a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x}\,dx = \int \frac{(a + b\,\mathrm{atan}(cx^2))^2}{x}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**2/x,x)
```

output
```
Integral((a + b*atan(c*x**2))**2/x, x)
```

Maxima [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x}\,dx = \int \frac{(b\arctan(cx^2) + a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x^2)^2 + b^2*log(c^2*x^4 + 1)
^2 + 32*a*b*arctan(c*x^2))/x, x)
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^2))^2}{x}\,dx$$

input
```
int((a + b*atan(c*x^2))^2/x,x)
```

output
```
int((a + b*atan(c*x^2))^2/x, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = 2\left(\int \frac{atan(c\,x^2)}{x}dx\right)ab+\left(\int \frac{atan(c\,x^2)^2}{x}dx\right)b^2+\log(x)\,a^2$$

input
```
int((a+b*atan(c*x^2))^2/x,x)
```

output
```
2*int(atan(c*x**2)/x,x)*a*b + int(atan(c*x**2)**2/x,x)*b**2 + log(x)*a**2
```

3.79 $\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx$

Optimal result 606
Mathematica [A] (verified) 607
Rubi [A] (verified) 607
Maple [C] (warning: unable to verify) 609
Fricas [F] 610
Sympy [F] 611
Maxima [F] 611
Giac [F] 612
Mupad [F(-1)] 612
Reduce [F] 612

Optimal result

Integrand size = 16, antiderivative size = 97

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = -\frac{1}{2}ic(a+b\arctan(cx^2))^2 - \frac{(a+b\arctan(cx^2))^2}{2x^2} + bc(a+b\arctan(cx^2))\log\left(2-\frac{2}{1-icx^2}\right) - \frac{1}{2}ib^2c\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx^2}\right)$$

output
```
-1/2*I*c*(a+b*arctan(c*x^2))^2-1/2*(a+b*arctan(c*x^2))^2/x^2+b*c*(a+b*arct
an(c*x^2))*ln(2-2/(1-I*c*x^2))-1/2*I*b^2*c*polylog(2,-1+2/(1-I*c*x^2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = -\frac{a^2}{2x^2}+abc\left(-\frac{\arctan(cx^2)}{cx^2}+\log(cx^2)-\frac{1}{2}\log(1+c^2x^4)\right) +\frac{1}{2}b^2c\left(-\frac{\arctan(cx^2)^2}{cx^2} +2\arctan(cx^2)\log\left(1-e^{2i\arctan(cx^2)}\right) -i\left(\arctan(cx^2)^2+\text{PolyLog}\left(2,e^{2i\arctan(cx^2)}\right)\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^2/x^3,x]
```

output
```
-1/2*a^2/x^2 + a*b*c*(-(ArcTan[c*x^2]/(c*x^2)) + Log[c*x^2] - Log[1 + c^2*
x^4]/2) + (b^2*c*(-(ArcTan[c*x^2]^2/(c*x^2)) + 2*ArcTan[c*x^2]*Log[1 - E^(
(2*I)*ArcTan[c*x^2])] - I*(ArcTan[c*x^2]^2 + PolyLog[2, E^((2*I)*ArcTan[c*
x^2])])))/2
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5363, 5361, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx$$

$$\Big\downarrow \; 5363$$

$$\frac{1}{2}\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx^2$$

$$\Big\downarrow \; 5361$$

$$\frac{1}{2}\left(2bc\int \frac{a+b\arctan\left(cx^2\right)}{x^2\left(c^2x^4+1\right)}dx^2-\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2}\right)$$

$\Big\downarrow$ 5459

$$\frac{1}{2}\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2}+2bc\left(i\int \frac{a+b\arctan\left(cx^2\right)}{x^2\left(cx^2+i\right)}dx^2-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^2}{2b}\right)\right)$$

$\Big\downarrow$ 5403

$$\frac{1}{2}\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2}+2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx^2}\right)}{c^2x^4+1}dx^2-i\log\left(2-\frac{2}{1-icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)\right)\right.\right.$$

$\Big\downarrow$ 2897

$$\frac{1}{2}\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2}+2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}b\operatorname{PolyLog}\left(2,\frac{2}{1-icx^2}-1\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^2])^2/x^3,x]
```

output
```
(-((a + b*ArcTan[c*x^2])^2/x^2) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^2])^2
)/b + I*((-I)*(a + b*ArcTan[c*x^2])*Log[2 - 2/(1 - I*c*x^2)] - (b*PolyLog[
2, -1 + 2/(1 - I*c*x^2)])/2)))/2
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.49

method	result
default	$-\frac{a^2}{2x^2}-\frac{b^2\arctan(c\,x^2)^2}{2x^2}-\frac{b^2\arctan(c\,x^2)\ln(c^2x^4+1)c}{2}+2b^2c\ln(x)\arctan(c\,x^2)+\frac{b^2\left(\sum_{\_\alpha=\mathrm{RootOf}(c^2\_Z^4+1)}{}^{2} -\right.}{}$
parts	$-\frac{a^2}{2x^2}-\frac{b^2\arctan(c\,x^2)^2}{2x^2}-\frac{b^2\arctan(c\,x^2)\ln(c^2x^4+1)c}{2}+2b^2c\ln(x)\arctan(c\,x^2)+\frac{b^2\left(\sum_{\_\alpha=\mathrm{RootOf}(c^2\_Z^4+1)}{}^{2} -\right.}{}$

input
```
int((a+b*arctan(c*x^2))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2-1/2*b^2*arctan(c*x^2)^2/x^2-1/2*b^2*arctan(c*x^2)*ln(c^2*x^4+
1)*c+2*b^2*c*ln(x)*arctan(c*x^2)+1/8*b^2*sum(1/_alpha^2*(2*ln(x-_alpha)*ln
(c^2*x^4+1)-c*(1/c/_alpha^3*ln(x-_alpha)^2+2/_alpha*ln(x-_alpha)*(_alpha^2
*ln(1/2*(x+_alpha)/_alpha)*c+ln((_alpha^3*c-x)/_alpha/(_alpha^2*c-1))-ln((
_alpha^3*c+x)/_alpha/(_alpha^2*c+1)))+2/_alpha*(_alpha^2*dilog(1/2*(x+_alp
ha)/_alpha)*c+dilog((_alpha^3*c-x)/_alpha/(_alpha^2*c-1))-dilog((_alpha^3*
c+x)/_alpha/(_alpha^2*c+1))))),_alpha=RootOf(_Z^4*c^2+1))-b^2*sum(1/_R1^2*
(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1)),_R1=RootOf(_Z^4*c^2+1))+2*a*b*(
-1/2/x^2*arctan(c*x^2)+c*(-1/4*ln(c^2*x^4+1)+ln(x)))
```

Fricas [F]

$$\int\frac{(a+b\arctan(cx^2))^2}{x^3}\,dx=\int\frac{(b\arctan(cx^2)+a)^2}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^3, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = \int \frac{(a+b\operatorname{atan}(cx^2))^2}{x^3}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**2/x**3,x)
```

output
```
Integral((a + b*atan(c*x**2))**2/x**3, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*a*b + 1/32*(3
2*x^2*integrate(-1/16*(4*c^2*x^4*log(c^2*x^4 + 1) - 8*c*x^2*arctan(c*x^2)
- 12*(c^2*x^4 + 1)*arctan(c*x^2)^2 - (c^2*x^4 + 1)*log(c^2*x^4 + 1)^2)/(c^
2*x^7 + x^3), x) - 4*arctan(c*x^2)^2 + log(c^2*x^4 + 1)^2)*b^2/x^2 - 1/2*a
^2/x^2
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^2))^2}{x^3}\,dx$$

input
```
int((a + b*atan(c*x^2))^2/x^3,x)
```

output
```
int((a + b*atan(c*x^2))^2/x^3, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = \frac{-atan(c\,x^2)^2\,b^2-2atan(c\,x^2)\,ab+4\left(\int \frac{atan(c\,x^2)}{c^2x^5+x}dx\right)b^2c\,x^2-\log\left(-\sqrt{c}\,\sqrt{2}\,x+c\,x^2+1\right)abc\,x^2-\log\left(\sqrt{c}\right.}{2x^2}$$

input
```
int((a+b*atan(c*x^2))^2/x^3,x)
```

output
```
( - atan(c*x**2)**2*b**2 - 2*atan(c*x**2)*a*b + 4*int(atan(c*x**2)/(c**2*x
**5 + x),x)*b**2*c*x**2 - log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a*b*c*x**
2 - log(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a*b*c*x**2 + 4*log(x)*a*b*c*x**2 -
 a**2)/(2*x**2)
```

3.80 $\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx$

Optimal result . . . 613
Mathematica [A] (verified) . . . 613
Rubi [A] (verified) . . . 614
Maple [A] (verified) . . . 617
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Maxima [A] (verification not implemented) . . . 618
Giac [F] . . . 619
Mupad [B] (verification not implemented) . . . 619
Reduce [B] (verification not implemented) . . . 620

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx = -\frac{bc(a+b\arctan(cx^2))}{2x^2} - \frac{1}{4}c^2(a+b\arctan(cx^2))^2 - \frac{(a+b\arctan(cx^2))^2}{4x^4} + b^2c^2\log(x) - \frac{1}{4}b^2c^2\log(1+c^2x^4)$$

output
```
-1/2*b*c*(a+b*arctan(c*x^2))/x^2-1/4*c^2*(a+b*arctan(c*x^2))^2-1/4*(a+b*ar
ctan(c*x^2))^2/x^4+b^2*c^2*ln(x)-1/4*b^2*c^2*ln(c^2*x^4+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx = -\frac{a^2+2abcx^2+2b(a+bcx^2+ac^2x^4)\arctan(cx^2)+b^2(1+c^2x^4)\arctan(cx^2)^2-4b^2c^2x^4\log(x)+b^2c^2x}{4x^4}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^2/x^5,x]
```

output
```
-1/4*(a^2 + 2*a*b*c*x^2 + 2*b*(a + b*c*x^2 + a*c^2*x^4)*ArcTan[c*x^2] + b^
2*(1 + c^2*x^4)*ArcTan[c*x^2]^2 - 4*b^2*c^2*x^4*Log[x] + b^2*c^2*x^4*Log[1
 + c^2*x^4])/x^4
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5363, 5361, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{2}\int \frac{(a+b\arctan(cx^2))^2}{x^6}dx^2$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(bc\int \frac{a+b\arctan(cx^2)}{x^4(c^2x^4+1)}dx^2-\frac{(a+b\arctan(cx^2))^2}{2x^4}\right)$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{2}\left(bc\left(\int \frac{a+b\arctan(cx^2)}{x^4}dx^2-c^2\int \frac{a+b\arctan(cx^2)}{c^2x^4+1}dx^2\right)-\frac{(a+b\arctan(cx^2))^2}{2x^4}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(bc\left(c^2\left(-\int \frac{a+b\arctan(cx^2)}{c^2x^4+1}dx^2\right)+bc\int \frac{1}{x^2(c^2x^4+1)}dx^2-\frac{a+b\arctan(cx^2)}{x^2}\right)-\frac{(a+b\arctan(cx^2))^2}{2x^4}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(bc\left(c^2\left(-\int \frac{a+b\arctan(cx^2)}{c^2x^4+1}dx^2\right)+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^4+1)}dx^4-\frac{a+b\arctan(cx^2)}{x^2}\right)-\frac{(a+b\arctan(cx^2))^2}{2x^4}\right)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(bc\left(c^2\left(-\int \frac{a+b\arctan(cx^2)}{c^2x^4+1}dx^2\right)+\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^4-c^2\int \frac{1}{c^2x^4+1}dx^4\right)-\frac{a+b\arctan(cx^2)}{x^2}\right)-\frac{(a+b\right.$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(bc\left(c^2\left(-\int \frac{a+b\arctan(cx^2)}{c^2x^4+1}dx^2\right)+\frac{1}{2}bc\left(\log(x^4)-c^2\int \frac{1}{c^2x^4+1}dx^4\right)-\frac{a+b\arctan(cx^2)}{x^2}\right)-\frac{(a+b\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(bc\left(c^2\left(-\int \frac{a+b\arctan(cx^2)}{c^2x^4+1}dx^2\right)-\frac{a+b\arctan(cx^2)}{x^2}+\frac{1}{2}bc(\log(x^4)-\log(c^2x^4+1))\right)-\frac{(a+b\arctan}{2x^4}\right.$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{2}\left(bc\left(-\frac{c(a+b\arctan(cx^2))^2}{2b}-\frac{a+b\arctan(cx^2)}{x^2}+\frac{1}{2}bc(\log(x^4)-\log(c^2x^4+1))\right)-\frac{(a+b\arctan(cx^2))}{2x^4}\right.$$

input
```
Int[(a + b*ArcTan[c*x^2])^2/x^5,x]
```

output
```
(-1/2*(a + b*ArcTan[c*x^2])^2/x^4 + b*c*(-((a + b*ArcTan[c*x^2])/x^2) - (c
*(a + b*ArcTan[c*x^2])^2)/(2*b) + (b*c*(Log[x^4] - Log[1 + c^2*x^4]))/2))/
2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

method	result
default	$-\frac{a^2}{4x^4}-\frac{b^2\arctan(cx^2)^2}{4x^4}-\frac{b^2c\arctan(cx^2)}{2x^2}-\frac{b^2\arctan(cx^2)^2c^2}{4}-\frac{b^2c^2\ln(c^2x^4+1)}{4}+b^2c^2\ln(x)-\frac{ab\arctan}{2x}$
parts	$-\frac{a^2}{4x^4}-\frac{b^2\arctan(cx^2)^2}{4x^4}-\frac{b^2c\arctan(cx^2)}{2x^2}-\frac{b^2\arctan(cx^2)^2c^2}{4}-\frac{b^2c^2\ln(c^2x^4+1)}{4}+b^2c^2\ln(x)-\frac{ab\arctan}{2x}$
parallelrisch	$\frac{-b^2\arctan(cx^2)^2x^4c^2+4b^2c^2\ln(x)x^4-b^2c^2\ln(c^2x^4+1)x^4-2ab\arctan(cx^2)x^4c^2+a^2c^2x^4-2b^2\arctan(cx^2)x^2c-2abcx^2-}{4x^4}$
risch	$\frac{b^2(c^2x^4+1)\ln(icx^2+1)^2}{16x^4}+\frac{ib(ibc^2x^4\ln(-icx^2+1)+2bcx^2+2a+ib\ln(-icx^2+1))\ln(icx^2+1)}{8x^4}-\frac{4i\ln((-5ibc+ac)x^2+}{}$

input
```
int((a+b*arctan(c*x^2))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/x^4-1/4*b^2*arctan(c*x^2)^2/x^4-1/2*b^2*c*arctan(c*x^2)/x^2-1/4*b
^2*arctan(c*x^2)^2*c^2-1/4*b^2*c^2*ln(c^2*x^4+1)+b^2*c^2*ln(x)-1/2*a*b*arc
tan(c*x^2)/x^4-1/2*a*b*c/x^2-1/2*a*b*arctan(c*x^2)*c^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx =$$
$$-\frac{b^2c^2x^4\log(c^2x^4+1)-4\,b^2c^2x^4\log(x)+2\,abcx^2+(b^2c^2x^4+b^2)\arctan(cx^2)^2+a^2+2\,(abc^2x^4+b^2cx}{4\,x^4}$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="fricas")
```

output
```
-1/4*(b^2*c^2*x^4*log(c^2*x^4 + 1) - 4*b^2*c^2*x^4*log(x) + 2*a*b*c*x^2 +
(b^2*c^2*x^4 + b^2)*arctan(c*x^2)^2 + a^2 + 2*(a*b*c^2*x^4 + b^2*c*x^2 + a
*b)*arctan(c*x^2))/x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(80) = 160$.

Time = 21.86 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.93

$$\int \frac{(a + b\arctan(cx^2))^2}{x^5}\,dx$$
$$= \begin{cases} -\frac{a^2}{4x^4} - \frac{abc^2\operatorname{atan}(cx^2)}{2} - \frac{abc}{2x^2} - \frac{ab\operatorname{atan}(cx^2)}{2x^4} - \frac{b^2c^3\sqrt{-\frac{1}{c^2}}\operatorname{atan}(cx^2)}{2} + b^2c^2\log(x) - \frac{b^2c^2\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2} - \frac{b^2c^2\operatorname{atan}}{4} \\ -\frac{a^2}{4x^4} \end{cases}$$

input
```
integrate((a+b*atan(c*x**2))**2/x**5,x)
```

output
```
Piecewise((-a**2/(4*x**4) - a*b*c**2*atan(c*x**2)/2 - a*b*c/(2*x**2) - a*b
*atan(c*x**2)/(2*x**4) - b**2*c**3*sqrt(-1/c**2)*atan(c*x**2)/2 + b**2*c**
2*log(x) - b**2*c**2*log(x**2 + sqrt(-1/c**2))/2 - b**2*c**2*atan(c*x**2)*
*2/4 - b**2*c*atan(c*x**2)/(2*x**2) - b**2*atan(c*x**2)**2/(4*x**4), Ne(c,
 0)), (-a**2/(4*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

$$\int \frac{(a + b\arctan(cx^2))^2}{x^5}\,dx = -\frac{1}{2}\left(\left(c\arctan(cx^2) + \frac{1}{x^2}\right)c + \frac{\arctan(cx^2)}{x^4}\right)ab$$
$$+\frac{1}{4}\left(\left(\arctan(cx^2)^2 - \log(c^2x^4 + 1) + 4\log(x)\right)c^2 - 2\left(c\arctan(cx^2) + \frac{1}{x^2}\right)c\arctan(cx^2)\right)b^2$$
$$-\frac{b^2\arctan(cx^2)^2}{4x^4} - \frac{a^2}{4x^4}$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="maxima")
```

output
```
-1/2*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*a*b + 1/4*((arctan(
c*x^2)^2 - log(c^2*x^4 + 1) + 4*log(x))*c^2 - 2*(c*arctan(c*x^2) + 1/x^2)*
c*arctan(c*x^2))*b^2 - 1/4*b^2*arctan(c*x^2)^2/x^4 - 1/4*a^2/x^4
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.75

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx = b^2 c^2 \ln(x) - \frac{b^2 c^2 \operatorname{atan}(c\,x^2)^2}{4} - \frac{b^2 \operatorname{atan}(c\,x^2)^2}{4\,x^4} - \frac{b^2 c^2 \ln(c^2 x^4+1)}{4} - \frac{a^2}{4\,x^4} - \frac{b^2 c \operatorname{atan}(c\,x^2)}{2\,x^2} - \frac{a\,b\,c}{2\,x^2} - \frac{a\,b\,c^2 \operatorname{atan}\left(\frac{a^2\,c\,x^2}{a^2+25\,b^2}+\frac{25\,b^2\,c\,x^2}{a^2+25\,b^2}\right)}{2} - \frac{a\,b \operatorname{atan}(c\,x^2)}{2\,x^4}$$

input
```
int((a + b*atan(c*x^2))^2/x^5,x)
```

output
```
b^2*c^2*log(x) - (b^2*c^2*atan(c*x^2)^2)/4 - (b^2*atan(c*x^2)^2)/(4*x^4) -
 (b^2*c^2*log(c^2*x^4 + 1))/4 - a^2/(4*x^4) - (b^2*c*atan(c*x^2))/(2*x^2)
- (a*b*c)/(2*x^2) - (a*b*c^2*atan((a^2*c*x^2)/(a^2 + 25*b^2) + (25*b^2*c*x
^2)/(a^2 + 25*b^2)))/2 - (a*b*atan(c*x^2))/(2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.79

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx$$
$$= \frac{-atan(c\,x^2)^2\,b^2c^2x^4 - atan(c\,x^2)^2\,b^2 - 2atan(c\,x^2)\,ab\,c^2x^4 - 2atan(c\,x^2)\,ab - 2atan(c\,x^2)\,b^2c\,x^2 - \log(-}{4x^4}$$

input
```
int((a+b*atan(c*x^2))^2/x^5,x)
```

output
```
( - atan(c*x**2)**2*b**2*c**2*x**4 - atan(c*x**2)**2*b**2 - 2*atan(c*x**2)
*a*b*c**2*x**4 - 2*atan(c*x**2)*a*b - 2*atan(c*x**2)*b**2*c*x**2 - log( -
sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b**2*c**2*x**4 - log(sqrt(c)*sqrt(2)*x + c
*x**2 + 1)*b**2*c**2*x**4 + 4*log(x)*b**2*c**2*x**4 - a**2 - 2*a*b*c*x**2)
/(4*x**4)
```

3.81 $\int x^2(a+b\arctan(cx^2))^2\,dx$

Optimal result 621
Mathematica [F] 622
Rubi [A] (verified) 623
Maple [F] 626
Fricas [F] 626
Sympy [F] 626
Maxima [F] 627
Giac [F] 627
Mupad [F(-1)] 627
Reduce [F] 628

Optimal result

Integrand size = 16, antiderivative size = 1393

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\text{Too large to display}$$

output
```
1/12*x^3*(2*a+I*b*ln(1-I*c*x^2))^2-1/6*(-1)^(3/4)*b^2*polylog(2,1+2^(1/2)*
((-1)^(3/4)+c^(1/2)*x)/(1+(-1)^(3/4)*c^(1/2)*x))/c^(3/2)-1/6*(-1)^(1/4)*b^
2*polylog(2,1-2^(1/2)*((-1)^(1/4)+c^(1/2)*x)/(1+(-1)^(1/4)*c^(1/2)*x))/c^(
3/2)-1/6*(-1)^(1/4)*b^2*polylog(2,1+(-1+I)*(1+(-1)^(3/4)*c^(1/2)*x)/(1+(-1
)^(1/4)*c^(1/2)*x))/c^(3/2)-1/6*(-1)^(3/4)*b^2*polylog(2,1-(1+I)*(1+(-1)^(
1/4)*c^(1/2)*x)/(1+(-1)^(3/4)*c^(1/2)*x))/c^(3/2)+1/3*(-1)^(3/4)*b^2*polyl
og(2,1-2/(1+(-1)^(3/4)*c^(1/2)*x))/c^(3/2)+1/3*(-1)^(3/4)*b^2*polylog(2,1-
2/(1-(-1)^(3/4)*c^(1/2)*x))/c^(3/2)+1/3*(-1)^(1/4)*b^2*polylog(2,1-2/(1+(-
1)^(1/4)*c^(1/2)*x))/c^(3/2)+1/3*(-1)^(1/4)*b^2*polylog(2,1-2/(1-(-1)^(1/4
)*c^(1/2)*x))/c^(3/2)-4/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*c^(1/2)*x)/c^(
3/2)-1/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*c^(1/2)*x)^2/c^(3/2)+4/3*(-1)^(
3/4)*b^2*arctan((-1)^(3/4)*c^(1/2)*x)/c^(3/2)+1/3*(-1)^(1/4)*b^2*arctan((-
1)^(3/4)*c^(1/2)*x)^2/c^(3/2)-1/3*I*a*b*x^3*ln(1+I*c*x^2)-2/3*I*b^2*x*ln(1
-I*c*x^2)/c-4/3*a*b*x/c+2/9*I*a*b*x^3-1/12*b^2*x^3*ln(1+I*c*x^2)^2-1/9*b^2
*x^3*ln(1-I*c*x^2)-2/3*(-1)^(1/4)*a*b*arctanh((-1)^(3/4)*c^(1/2)*x)/c^(3/2
)+1/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*c^(1/2)*x)*ln(-2^(1/2)*((-1)^(3/4)
+c^(1/2)*x)/(1+(-1)^(3/4)*c^(1/2)*x))/c^(3/2)+1/3*(-1)^(3/4)*b^2*arctanh((
-1)^(3/4)*c^(1/2)*x)*ln((1+I)*(1+(-1)^(1/4)*c^(1/2)*x)/(1+(-1)^(3/4)*c^(1/
2)*x))/c^(3/2)-2/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*c^(1/2)*x)*ln(2/(1+(-
1)^(3/4)*c^(1/2)*x))/c^(3/2)+2/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*c^(1...
```

Mathematica [F]

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int x^2\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
Integrate[x^2*(a + b*ArcTan[c*x^2])^2, x]
```

Rubi [A] (verified)

Time = 2.96 (sec) , antiderivative size = 1393, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5365, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx$$

$$\Big\downarrow \text{5365}$$

$$\int \left(\frac{1}{4}x^2\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx^2\log\left(1+icx^2\right)\left(b\log\left(1-icx^2\right)-2ia\right)-\frac{1}{4}b^2x^2\log^2\left(1+icx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
\frac{1}{12}\left(2a+ib\log\left(1-icx^2\right)\right)^2x^3-\frac{1}{12}b^2\log^2\left(icx^2+1\right)x^3+\frac{2}{9}iabx^3-\frac{1}{9}b^2\log\left(1-icx^2\right)x^3-\frac{1}{9}ib\left(2a+ib\log\left(1-icx^2\right)\right)x^3-\frac{1}{3}iab\log\left(icx^2+1\right)x^3+\frac{1}{6}b^2\log\left(1-icx^2\right)\log\left(icx^2+1\right)x^3-\frac{2ib^2\log\left(1-icx^2\right)x}{3c}+\frac{2ib^2\log\left(icx^2+1\right)x}{3c}-\frac{4abx}{3c}+\frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}}-\frac{(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}}+\frac{4(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}-\frac{4(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}-\frac{2\sqrt[4]{-1}ab\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}-\frac{2(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)}{3c^{3/2}}+\frac{2(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{3c^{3/2}}-\frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{\sqrt{2}\left(\sqrt{c}x+\sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{3c^{3/2}}+\frac{2(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)}{3c^{3/2}}-\frac{2(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{(-1)^{3/4}\sqrt{c}x+1}\right)}{3c^{3/2}}+\frac{(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(-\frac{\sqrt{2}\left(\sqrt{c}x+(-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{c}x+1}\right)}{3c^{3/2}}+\frac{(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{c}x+1\right)}{(-1)^{3/4}\sqrt{c}x+1}\right)}{3c^{3/2}}-\frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1-i)\left((-1)^{3/4}\sqrt{c}x+1\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{3c^{3/2}}-\frac{(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)}{3c^{3/2}}-\frac{\sqrt[4]{-1}b\arctan\left((-1)^{3/4}\sqrt{c}x\right)\left(2a+ib\log\left(1-icx^2\right)\right)}{3c^{3/2}}+\frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(icx^2+1\right)}{3c^{3/2}}+\frac{(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(icx^2+1\right)}{3c^{3/2}}+\frac{\sqrt[4]{-1}b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)}{3c^{3/2}}+\frac{\sqrt[4]{-1}b^2\operatorname{PolyLog}\left(2,1-\frac{2}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{3c^{3/2}}-\frac{\sqrt[4]{-1}b^2\operatorname{PolyLog}\left(2,1-\frac{\sqrt{2}\left(\sqrt{c}x+\sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{6c^{3/2}}+\frac{(-1)^{3/4}b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)}{3c^{3/2}}+\frac{(-1)^{3/4}b^2\operatorname{PolyLog}\left(2,1-\frac{2}{(-1)^{3/4}\sqrt{c}x+1}\right)}{3c^{3/2}}-\frac{(-1)^{3/4}b^2\operatorname{PolyLog}\left(2,\frac{\sqrt{2}\left(\sqrt{c}x+(-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{c}x+1}+1\right)}{6c^{3/2}}-
$$

input
```
Int[x^2*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
(-4*a*b*x)/(3*c) + ((2*I)/9)*a*b*x^3 + (4*(-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)
*Sqrt[c]*x])/(3*c^(3/2)) + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)
/(3*c^(3/2)) - (2*(-1)^(1/4)*a*b*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/(3*c^(3/2)
) - (4*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/(3*c^(3/2)) - ((-1)^(
3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]^2)/(3*c^(3/2)) - (2*(-1)^(3/4)*b^2*
ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)
) + (2*(-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*S
qrt[c]*x)])/(3*c^(3/2)) - ((-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log
[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2
)) + (2*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)
*Sqrt[c]*x)])/(3*c^(3/2)) - (2*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x
]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*ArcTanh
[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^
(3/4)*Sqrt[c]*x))]])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[
c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)]
)/(3*c^(3/2)) - ((-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*
(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)]])/(3*c^(3/2)) - (((
2*I)/3)*b^2*x*Log[1 - I*c*x^2])/c - (b^2*x^3*Log[1 - I*c*x^2])/9 - ((-1)^(
3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/(3*c^(3/2)) - (I/
9)*b*x^3*(2*a + I*b*Log[1 - I*c*x^2]) - ((-1)^(1/4)*b*ArcTan[(-1)^(3/4)...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5365
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

Maple [F]

$$\int x^2 \left(a + b \arctan \left(c\,x^2\right)\right)^2 dx$$

input
```
int(x^2*(a+b*arctan(c*x^2))^2,x)
```

output
```
int(x^2*(a+b*arctan(c*x^2))^2,x)
```

Fricas [F]

$$\int x^2 \left(a + b \arctan \left(cx^2\right)\right)^2 \, dx = \int \left(b \arctan \left(cx^2\right) + a\right)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arctan(c*x^2)^2 + 2*a*b*x^2*arctan(c*x^2) + a^2*x^2, x)
```

Sympy [F]

$$\int x^2 \left(a + b \arctan \left(cx^2\right)\right)^2 \, dx = \int x^2 \left(a + b \operatorname{atan}\left(cx^2\right)\right)^2 \, dx$$

input
```
integrate(x**2*(a+b*atan(c*x**2))**2,x)
```

output
```
Integral(x**2*(a + b*atan(c*x**2))**2, x)
```

Maxima [F]

$$\int x^2 (a+b\arctan(cx^2))^2 dx = \int (b\arctan(cx^2)+a)^2 x^2 dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/6*(4*x^3*arctan(c*x^2) - c*(8*x/c^2 - (2*sqrt(2)*arctan(1/
2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + 2*sqrt(2)*arctan(1/
2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + sqrt(2)*log(c*x^2 +
 sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x +
1)/sqrt(c))/c^2))*a*b + 1/48*(4*x^3*arctan(c*x^2)^2 - x^3*log(c^2*x^4 + 1)
^2 + 48*integrate(1/48*(8*c^2*x^6*log(c^2*x^4 + 1) - 16*c*x^4*arctan(c*x^2
) + 36*(c^2*x^6 + x^2)*arctan(c*x^2)^2 + 3*(c^2*x^6 + x^2)*log(c^2*x^4 + 1
)^2)/(c^2*x^4 + 1), x))*b^2
```

Giac [F]

$$\int x^2 (a+b\arctan(cx^2))^2 dx = \int (b\arctan(cx^2)+a)^2 x^2 dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a+b\arctan(cx^2))^2 dx = \int x^2 (a+b\,\mathrm{atan}(c\,x^2))^2 dx$$

input
```
int(x^2*(a + b*atan(c*x^2))^2,x)
```

output
```
int(x^2*(a + b*atan(c*x^2))^2, x)
```

Reduce [F]

$$\int x^2 \left(a + b \arctan\left(cx^2\right)\right)^2 \, dx$$
$$= -4\sqrt{c}\,\sqrt{2}\, atan\left(\frac{\sqrt{c}\,\sqrt{2}-2cx}{\sqrt{c}\,\sqrt{2}}\right) ab - 8\sqrt{c}\,\sqrt{2}\, atan\left(\frac{\sqrt{c}\,\sqrt{2}-2cx}{\sqrt{c}\,\sqrt{2}}\right) b^2 + 2 atan(c\,x^2)^2\, b^2 c^2 x^3 - 2\sqrt{c}\,\sqrt{2}\, atan(c\,x^2) \cdots$$

input
```
int(x^2*(a+b*atan(c*x^2))^2,x)
```

output
```
( - 4*sqrt(c)*sqrt(2)*atan((sqrt(c)*sqrt(2) - 2*c*x)/(sqrt(c)*sqrt(2)))*a*
b - 8*sqrt(c)*sqrt(2)*atan((sqrt(c)*sqrt(2) - 2*c*x)/(sqrt(c)*sqrt(2)))*b*
*2 + 2*atan(c*x**2)**2*b**2*c**2*x**3 - 2*sqrt(c)*sqrt(2)*atan(c*x**2)*a*b
 - 4*sqrt(c)*sqrt(2)*atan(c*x**2)*b**2 + 4*atan(c*x**2)*a*b*c**2*x**3 - 8*
atan(c*x**2)*b**2*c*x - sqrt(c)*sqrt(2)*log( - sqrt(c)*sqrt(2)*x + c*x**2
+ 1)*a*b + 2*sqrt(c)*sqrt(2)*log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b**2 +
 sqrt(c)*sqrt(2)*log(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a*b - 2*sqrt(c)*sqrt(
2)*log(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*b**2 + 8*int(atan(c*x**2)/(c**2*x**
4 + 1),x)*b**2*c + 2*a**2*c**2*x**3 - 8*a*b*c*x)/(6*c**2)
```

3.82 $\int (a+b\arctan(cx^2))^2\,dx$

Optimal result . . . 629
Mathematica [B] (warning: unable to verify) . . . 630
Rubi [A] (verified) . . . 631
Maple [F] . . . 634
Fricas [F] . . . 634
Sympy [F] . . . 634
Maxima [F] . . . 635
Giac [F] . . . 635
Mupad [F(-1)] . . . 635
Reduce [F] . . . 636

Optimal result

Integrand size = 12, antiderivative size = 1191

$$\int (a+b\arctan(cx^2))^2\,dx = \text{Too large to display}$$

output
```
-1/4*b^2*x*ln(1+I*c*x^2)^2-1/4*b^2*x*ln(1-I*c*x^2)^2+(-1)^(3/4)*b^2*arctan
((-1)^(3/4)*c^(1/2)*x)^2/c^(1/2)+(-1)^(1/4)*b^2*polylog(2,1-2/(1+(-1)^(3/4
)*c^(1/2)*x))/c^(1/2)+(-1)^(1/4)*b^2*polylog(2,1-2/(1-(-1)^(3/4)*c^(1/2)*x
))/c^(1/2)+(-1)^(3/4)*b^2*polylog(2,1-2/(1+(-1)^(1/4)*c^(1/2)*x))/c^(1/2)+
(-1)^(3/4)*b^2*polylog(2,1-2/(1-(-1)^(1/4)*c^(1/2)*x))/c^(1/2)-(-1)^(1/4)*
b^2*arctanh((-1)^(3/4)*c^(1/2)*x)^2/c^(1/2)-1/2*(-1)^(1/4)*b^2*polylog(2,1
+2^(1/2)*((-1)^(3/4)+c^(1/2)*x)/(1+(-1)^(3/4)*c^(1/2)*x))/c^(1/2)-1/2*(-1)
^(3/4)*b^2*polylog(2,1-2^(1/2)*((-1)^(1/4)+c^(1/2)*x)/(1+(-1)^(1/4)*c^(1/2
)*x))/c^(1/2)-1/2*(-1)^(3/4)*b^2*polylog(2,1+(-1+I)*(1+(-1)^(3/4)*c^(1/2)*
x)/(1+(-1)^(1/4)*c^(1/2)*x))/c^(1/2)-1/2*(-1)^(1/4)*b^2*polylog(2,1-(1+I)*
(1+(-1)^(1/4)*c^(1/2)*x)/(1+(-1)^(3/4)*c^(1/2)*x))/c^(1/2)-I*a*b*x*ln(1+I*
c*x^2)+I*a*b*x*ln(1-I*c*x^2)+a^2*x+(-1)^(1/4)*b^2*arctan((-1)^(3/4)*c^(1/2
)*x)*ln(1-I*c*x^2)/c^(1/2)+(-1)^(1/4)*b^2*arctanh((-1)^(3/4)*c^(1/2)*x)*ln
(-2^(1/2)*((-1)^(3/4)+c^(1/2)*x)/(1+(-1)^(3/4)*c^(1/2)*x))/c^(1/2)+(-1)^(1
/4)*b^2*arctanh((-1)^(3/4)*c^(1/2)*x)*ln((1+I)*(1+(-1)^(1/4)*c^(1/2)*x)/(1
+(-1)^(3/4)*c^(1/2)*x))/c^(1/2)+(-1)^(1/4)*b^2*arctan((-1)^(3/4)*c^(1/2)*x
)*ln(2^(1/2)*((-1)^(1/4)+c^(1/2)*x)/(1+(-1)^(1/4)*c^(1/2)*x))/c^(1/2)+(-1)
^(1/4)*b^2*arctan((-1)^(3/4)*c^(1/2)*x)*ln((1-I)*(1+(-1)^(3/4)*c^(1/2)*x)/
(1+(-1)^(1/4)*c^(1/2)*x))/c^(1/2)+2*(-1)^(1/4)*b^2*arctan((-1)^(3/4)*c^(1/
2)*x)*ln(2/(1-(-1)^(1/4)*c^(1/2)*x))/c^(1/2)+2*(-1)^(3/4)*a*b*arctanh((...
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4723 vs. $2(1191) = 2382$.

Time = 26.39 (sec) , antiderivative size = 4723, normalized size of antiderivative = 3.97

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^2,x]
```

output
```
a^2*x + (a*b*Sqrt[c*x^2]*(2*Sqrt[c*x^2]*ArcTan[c*x^2] - Sqrt[2]*(ArcTan[(-
1 + c*x^2)/(Sqrt[2]*Sqrt[c*x^2])] - ArcTanh[(Sqrt[2]*Sqrt[c*x^2])/(1 + c*x
^2)])))/(c*x) + (b^2*Sqrt[c*x^2]*(2*Sqrt[c*x^2]*ArcTan[c*x^2]^2 + Sqrt[2]*
ArcTan[c*x^2]*(2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTan[1 + Sqrt[2]*Sq
rt[c*x^2]] - Log[1 + c*x^2 - Sqrt[2]*Sqrt[c*x^2]] + Log[1 + c*x^2 + Sqrt[2
]*Sqrt[c*x^2]]) - Sqrt[2]*((ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + ArcTan[1 + S
qrt[2]*Sqrt[c*x^2]])*Log[1 + c*x^2 - Sqrt[2]*Sqrt[c*x^2]] - (ArcTan[1 - Sq
rt[2]*Sqrt[c*x^2]] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])*Log[1 + c*x^2 + Sqrt
[2]*Sqrt[c*x^2]] + (Sqrt[c*x^2]*(1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2)^(3/2)*(2
*(-5*ArcTan[2 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + 4*ArcTan[1 - Sqrt[2]*
Sqrt[c*x^2]]^2 + ((1 + 2*I)*Sqrt[1 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]^2)
/E^(I*ArcTan[2 + I]) + ((1 - 2*I)*Sqrt[1 - I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^
2]]^2)/E^ArcTanh[1 + 2*I] - (5*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*ArcTanh[
1 + 2*I] + (5*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]])*Log[1
- E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] + 5*((-I)*
ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + ArcTanh[1 + 2*I])*Log[1 - E^((2*I)*ArcTa
n[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] + (5*I)*ArcTan[2 + I]*Lo
g[-Sin[ArcTan[2 + I] - ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]]] - 5*ArcTanh[1 + 2
*I]*Log[Sin[ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + I*ArcTanh[1 + 2*I]]]) + 5*Po
lyLog[2, E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] ...
```

Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 1191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5347, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2dx$$

$$\Big\downarrow \text{5347}$$

$$\int \left(a^2+iab\log\left(1-icx^2\right)-iab\log\left(1+icx^2\right)-\frac{1}{4}b^2\log^2\left(1-icx^2\right)-\frac{1}{4}b^2\log^2\left(1+icx^2\right)+\frac{1}{2}b^2\log\left(1-icx^2\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$xa^2 - \frac{2(-1)^{3/4} b \arctan\left((-1)^{3/4}\sqrt{c}x\right) a}{\sqrt{c}} + \frac{2(-1)^{3/4} b \operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) a}{\sqrt{c}} + ibx \log\left(1 - icx^2\right) a - ibx \log\left(icx^2 + 1\right) a + \frac{(-1)^{3/4} b^2 \arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}} - \frac{\sqrt[4]{-1} b^2 \operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}} - \frac{1}{4} b^2 x \log^2\left(1 - icx^2\right) - \frac{1}{4} b^2 x \log^2\left(icx^2 + 1\right) + \frac{2\sqrt[4]{-1} b^2 \arctan\left((-1)^{3/4}\sqrt{c}x\right) \log\left(\frac{2}{1 - \sqrt[4]{-1}\sqrt{c}x}\right)}{\sqrt{c}} - \frac{2\sqrt[4]{-1} b^2 \arctan\left((-1)^{3/4}\sqrt{c}x\right) \log\left(\frac{2}{\sqrt[4]{-1}\sqrt{c}x + 1}\right)}{\sqrt{c}} + \frac{\sqrt[4]{-1} b^2 \arctan\left((-1)^{3/4}\sqrt{c}x\right) \log\left(\frac{\sqrt{2}\left(\sqrt{c}x + \sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{c}x + 1}\right)}{\sqrt{c}} + \frac{2\sqrt[4]{-1} b^2 \operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) \log\left(\frac{2}{1 - (-1)^{3/4}\sqrt{c}x}\right)}{\sqrt{c}} - \frac{2\sqrt[4]{-1} b^2 \operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) \log\left(\frac{2}{(-1)^{3/4}\sqrt{c}x + 1}\right)}{\sqrt{c}} + \frac{\sqrt[4]{-1} b^2 \operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) \log\left(-\frac{\sqrt{2}\left(\sqrt{c}x + (-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{c}x + 1}\right)}{\sqrt{c}} + \frac{\sqrt[4]{-1} b^2 \operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) \log\left(\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{c}x + 1\right)}{(-1)^{3/4}\sqrt{c}x + 1}\right)}{\sqrt{c}} + \frac{\sqrt[4]{-1} b^2 \arctan\left((-1)^{3/4}\sqrt{c}x\right) \log\left(\frac{(1-i)\left((-1)^{3/4}\sqrt{c}x + 1\right)}{\sqrt[4]{-1}\sqrt{c}x + 1}\right)}{\sqrt{c}} + \frac{\sqrt[4]{-1} b^2 \arctan\left((-1)^{3/4}\sqrt{c}x\right) \log\left(1 - icx^2\right)}{\sqrt{c}} - \frac{\sqrt[4]{-1} b^2 \operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) \log\left(1 - icx^2\right)}{\sqrt{c}} - \frac{\sqrt[4]{-1} b^2 \arctan\left((-1)^{3/4}\sqrt{c}x\right) \log\left(icx^2 + 1\right)}{\sqrt{c}} + \frac{\sqrt[4]{-1} b^2 \operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) \log\left(icx^2 + 1\right)}{\sqrt{c}} + \frac{1}{2} b^2 x \log\left(1 - icx^2\right) \log\left(icx^2 + 1\right) + \frac{(-1)^{3/4} b^2 \operatorname{PolyLog}\left(2, 1 - \frac{2}{1 - \sqrt[4]{-1}\sqrt{c}x}\right)}{\sqrt{c}} + \frac{(-1)^{3/4} b^2 \operatorname{PolyLog}\left(2, 1 - \frac{2}{\sqrt[4]{-1}\sqrt{c}x + 1}\right)}{\sqrt{c}} - \frac{(-1)^{3/4} b^2 \operatorname{PolyLog}\left(2, 1 - \frac{\sqrt{2}\left(\sqrt{c}x + \sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{c}x + 1}\right)}{2\sqrt{c}} + \frac{\sqrt[4]{-1} b^2 \operatorname{PolyLog}\left(2, 1 - \frac{2}{1 - (-1)^{3/4}\sqrt{c}x}\right)}{\sqrt{c}} + \frac{\sqrt[4]{-1} b^2 \operatorname{PolyLog}\left(2, 1 - \frac{2}{(-1)^{3/4}\sqrt{c}x + 1}\right)}{\sqrt{c}} - \frac{\sqrt[4]{-1} b^2 \operatorname{PolyLog}\left(2, \frac{\sqrt{2}\left(\sqrt{c}x + (-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{c}x + 1} + 1\right)}{2\sqrt{c}} - \frac{\sqrt[4]{-1} b^2 \operatorname{PolyLog}\left(2, 1 - \frac{(1+i)\left(\sqrt[4]{-1}\sqrt{c}x + 1\right)}{(-1)^{3/4}\sqrt{c}x + 1}\right)}{2\sqrt{c}} - (-1)^{3/4} b^2 \operatorname{PolyLog}\left(2, 1 - \frac{(1-i)\left((-1)^{3/4}\sqrt{c}x + 1\right)}{\sqrt[4]{-1}\sqrt{c}x + 1}\right)$$

input
```
Int[(a + b*ArcTan[c*x^2])^2,x]
```

output
```
a^2*x - (2*(-1)^(3/4)*a*b*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] + ((-1)^(3
/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/Sqrt[c] + (2*(-1)^(3/4)*a*b*ArcTan
h[(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] - ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt
[c]*x]^2)/Sqrt[c] + (2*(-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(
1 - (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*
Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*Ar
cTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1
)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + (2*(-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c
]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*ArcTan
h[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)
^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt
[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x))])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTanh[(-
1)^(3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/
4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log
[((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c]
 + I*a*b*x*Log[1 - I*c*x^2] + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]
*Log[1 - I*c*x^2])/Sqrt[c] - ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]
*Log[1 - I*c*x^2])/Sqrt[c] - (b^2*x*Log[1 - I*c*x^2]^2)/4 - I*a*b*x*Log[1
+ I*c*x^2] - ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2]
)/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5347
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[ExpandIn
tegrand[(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n])/2)^p, x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Maple [F]

$$\int (a+b\arctan (c\,x^{2}))^{2}dx$$

input
```
int((a+b*arctan(c*x^2))^2,x)
```

output
```
int((a+b*arctan(c*x^2))^2,x)
```

Fricas [F]

$$\int (a+b\arctan (cx^2))^2 \, dx = \int (b\arctan (cx^2) + a)^2 \, dx$$

input
```
integrate((a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2, x)
```

Sympy [F]

$$\int (a+b\arctan (cx^2))^2 \, dx = \int (a+b\operatorname{atan}(cx^2))^2 \, dx$$

input
```
integrate((a+b*atan(c*x**2))**2,x)
```

output
```
Integral((a + b*atan(c*x**2))**2, x)
```

Maxima [F]

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
-1/2*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c
^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c
^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(
c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) - 4*x*arctan(c*x^2))*a*b + 1/16*(4
*x*arctan(c*x^2)^2 - x*log(c^2*x^4 + 1)^2 + 16*integrate(1/16*(8*c^2*x^4*l
og(c^2*x^4 + 1) - 16*c*x^2*arctan(c*x^2) + 12*(c^2*x^4 + 1)*arctan(c*x^2)^
2 + (c^2*x^4 + 1)*log(c^2*x^4 + 1)^2)/(c^2*x^4 + 1), x))*b^2 + a^2*x
```

Giac [F]

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2\,dx$$

input
```
int((a + b*atan(c*x^2))^2,x)
```

output
```
int((a + b*atan(c*x^2))^2, x)
```

Reduce [F]

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$=\frac{4\sqrt{c}\,\sqrt{2}\,atan\left(\frac{\sqrt{c}\,\sqrt{2}-2cx}{\sqrt{c}\,\sqrt{2}}\right)ab+2\sqrt{c}\,\sqrt{2}\,atan(c\,x^2)\,ab+4atan(c\,x^2)\,abcx-\sqrt{c}\,\sqrt{2}\log\left(-\sqrt{c}\,\sqrt{2}\,x+c\,x^2+\right.}{2c}$$

input
```
int((a+b*atan(c*x^2))^2,x)
```

output
```
(4*sqrt(c)*sqrt(2)*atan((sqrt(c)*sqrt(2) - 2*c*x)/(sqrt(c)*sqrt(2)))*a*b +
 2*sqrt(c)*sqrt(2)*atan(c*x**2)*a*b + 4*atan(c*x**2)*a*b*c*x - sqrt(c)*sqr
t(2)*log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a*b + sqrt(c)*sqrt(2)*log(sqrt
(c)*sqrt(2)*x + c*x**2 + 1)*a*b + 2*int(atan(c*x**2)**2,x)*b**2*c + 2*a**2
*c*x)/(2*c)
```

3.83 $\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2}\,dx$

Optimal result . . . 637
Mathematica [B] (warning: unable to verify) . . . 638
Rubi [A] (verified) . . . 639
Maple [F] . . . 642
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Giac [F] . . . 643
Mupad [F(-1)] . . . 643
Reduce [F] . . . 644

Optimal result

Integrand size = 16, antiderivative size = 1164

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2}\,dx = \text{Too large to display}$$

output
```
1/4*b^2*ln(1+I*c*x^2)^2/x+(-1)^(1/4)*b^2*c^(1/2)*polylog(2,1-2/(1-(-1)^(1/
4)*c^(1/2)*x))-(-1)^(3/4)*b^2*c^(1/2)*arctanh((-1)^(3/4)*c^(1/2)*x)^2+(-1)
^(1/4)*b^2*c^(1/2)*polylog(2,1-2/(1+(-1)^(1/4)*c^(1/2)*x))+(-1)^(1/4)*b^2*
c^(1/2)*arctan((-1)^(3/4)*c^(1/2)*x)^2-1/4*(2*a+I*b*ln(1-I*c*x^2))^2/x-(-1
)^(3/4)*b^2*c^(1/2)*arctanh((-1)^(3/4)*c^(1/2)*x)*ln(1-I*c*x^2)-(-1)^(1/4)
*b*c^(1/2)*arctan((-1)^(3/4)*c^(1/2)*x)*(2*a+I*b*ln(1-I*c*x^2))+(-1)^(3/4)
*b^2*c^(1/2)*arctanh((-1)^(3/4)*c^(1/2)*x)*ln(-2^(1/2)*((-1)^(3/4)+c^(1/2)
*x)/(1+(-1)^(3/4)*c^(1/2)*x))+(-1)^(3/4)*b^2*c^(1/2)*arctanh((-1)^(3/4)*c^
(1/2)*x)*ln((1+I)*(1+(-1)^(1/4)*c^(1/2)*x)/(1+(-1)^(3/4)*c^(1/2)*x))+(-1)^
(3/4)*b^2*c^(1/2)*polylog(2,1-2/(1-(-1)^(3/4)*c^(1/2)*x))+(-1)^(3/4)*b^2*c
^(1/2)*polylog(2,1-2/(1+(-1)^(3/4)*c^(1/2)*x))-1/2*(-1)^(3/4)*b^2*c^(1/2)*
polylog(2,1+2^(1/2)*((-1)^(3/4)+c^(1/2)*x)/(1+(-1)^(3/4)*c^(1/2)*x))-1/2*(
-1)^(1/4)*b^2*c^(1/2)*polylog(2,1-2^(1/2)*((-1)^(1/4)+c^(1/2)*x)/(1+(-1)^(
1/4)*c^(1/2)*x))-1/2*(-1)^(1/4)*b^2*c^(1/2)*polylog(2,1+(-1+I)*(1+(-1)^(3/
4)*c^(1/2)*x)/(1+(-1)^(1/4)*c^(1/2)*x))-1/2*(-1)^(3/4)*b^2*c^(1/2)*polylog
(2,1-(1+I)*(1+(-1)^(1/4)*c^(1/2)*x)/(1+(-1)^(3/4)*c^(1/2)*x))-2*(-1)^(3/4)
*b^2*c^(1/2)*arctanh((-1)^(3/4)*c^(1/2)*x)*ln(2/(1+(-1)^(3/4)*c^(1/2)*x))+
2*(-1)^(3/4)*b^2*c^(1/2)*arctanh((-1)^(3/4)*c^(1/2)*x)*ln(2/(1-(-1)^(3/4)*
c^(1/2)*x))-2*(-1)^(1/4)*a*b*c^(1/2)*arctanh((-1)^(3/4)*c^(1/2)*x)+2*(-1)^
(3/4)*b^2*c^(1/2)*arctan((-1)^(3/4)*c^(1/2)*x)*ln(2/(1+(-1)^(1/4)*c^(1/...
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4640 vs. 2(1164) = 2328.

Time = 35.51 (sec) , antiderivative size = 4640, normalized size of antiderivative = 3.99

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^2/x^2,x]
```

output
```
-(a^2/x) + (a*b*(c*x^2)^(3/2)*((-2*ArcTan[c*x^2])/Sqrt[c*x^2] + Sqrt[2]*(A
rcTan[(-1 + c*x^2)/(Sqrt[2]*Sqrt[c*x^2])] + ArcTanh[(Sqrt[2]*Sqrt[c*x^2])/
(1 + c*x^2)])))/(c*x^3) + (b^2*(c*x^2)^(3/2)*((-2*ArcTan[c*x^2]^2)/Sqrt[c*
x^2] + 4*((ArcTan[c*x^2]*(-2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + 2*ArcTan[1
+ Sqrt[2]*Sqrt[c*x^2]] - Log[1 + c*x^2 - Sqrt[2]*Sqrt[c*x^2]] + Log[1 + c*
x^2 + Sqrt[2]*Sqrt[c*x^2]]))/(2*Sqrt[2]) - ((ArcTan[1 - Sqrt[2]*Sqrt[c*x^2
]] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])*Log[1 + c*x^2 - Sqrt[2]*Sqrt[c*x^2]]
 - (ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])*Log
[1 + c*x^2 + Sqrt[2]*Sqrt[c*x^2]] - ((1/40 + I/40)*c*x^2*(1 + (1 - Sqrt[2]
*Sqrt[c*x^2])^2)*((5 + 5*I)*Pi*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + 10*ArcTan
[2 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + (4 - 4*I)*ArcTan[1 - Sqrt[2]*Sqr
t[c*x^2]]^2 - ((2 + 4*I)*Sqrt[1 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]^2)/E^
(I*ArcTan[2 + I]) + ((4 + 2*I)*Sqrt[1 - I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]
^2)/E^ArcTanh[1 + 2*I] + 10*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*ArcTanh[1 + 2*
I] + (5 - 5*I)*Pi*Log[1 + E^((-2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]])] + (1
0*I)*ArcTan[2 + I]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*S
qrt[c*x^2]]))] - (10*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*(
-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] + 10*ArcTan[1 - Sqrt[2
]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTan
h[1 + 2*I])] + (10*I)*ArcTanh[1 + 2*I]*Log[1 - E^((2*I)*ArcTan[1 - Sqrt...
```

Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 1164, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5365, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx$$

$$\Big\downarrow \text{5365}$$

$$\int \left(\frac{(2a+ib\log(1-icx^2))^2}{4x^2}+\frac{b\log(1+icx^2)\,(b\log(1-icx^2)-2ia)}{2x^2}-\frac{b^2\log^2(1+icx^2)}{4x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt[4]{-1}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 b^2 - (-1)^{3/4}\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2 b^2 +$$
$$\frac{\log^2\left(icx^2+1\right)b^2}{4x} - 2(-1)^{3/4}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)b^2 +$$
$$2(-1)^{3/4}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{\sqrt[4]{-1}\sqrt{c}x+1}\right)b^2 -$$
$$(-1)^{3/4}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{\sqrt{2}\left(\sqrt{c}x+\sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)b^2 +$$
$$2(-1)^{3/4}\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)b^2 -$$
$$2(-1)^{3/4}\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{(-1)^{3/4}\sqrt{c}x+1}\right)b^2 +$$
$$(-1)^{3/4}\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(-\frac{\sqrt{2}\left(\sqrt{c}x+(-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{c}x+1}\right)b^2 +$$
$$(-1)^{3/4}\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{c}x+1\right)}{(-1)^{3/4}\sqrt{c}x+1}\right)b^2 -$$
$$(-1)^{3/4}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1-i)\left((-1)^{3/4}\sqrt{c}x+1\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)b^2 -$$
$$(-1)^{3/4}\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)b^2 +$$
$$(-1)^{3/4}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(icx^2+1\right)b^2 +$$
$$(-1)^{3/4}\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(icx^2+1\right)b^2 - \frac{\log\left(1-icx^2\right)\log\left(icx^2+1\right)b^2}{2x} +$$
$$\sqrt[4]{-1}\sqrt{c}\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)b^2 + \sqrt[4]{-1}\sqrt{c}\,\mathrm{PolyLog}\left(2,1-\frac{2}{\sqrt[4]{-1}\sqrt{c}x+1}\right)b^2 -$$
$$\frac{1}{2}\sqrt[4]{-1}\sqrt{c}\,\mathrm{PolyLog}\left(2,1-\frac{\sqrt{2}\left(\sqrt{c}x+\sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)b^2 +$$
$$(-1)^{3/4}\sqrt{c}\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)b^2 +$$
$$(-1)^{3/4}\sqrt{c}\,\mathrm{PolyLog}\left(2,1-\frac{2}{(-1)^{3/4}\sqrt{c}x+1}\right)b^2 -$$
$$\frac{1}{2}(-1)^{3/4}\sqrt{c}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{2}\left(\sqrt{c}x+(-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{c}x+1}+1\right)b^2 -$$
$$\frac{1}{2}(-1)^{3/4}\sqrt{c}\,\mathrm{PolyLog}\left(2,1-\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{c}x+1\right)}{(-1)^{3/4}\sqrt{c}x+1}\right)b^2 -$$
$$\frac{1}{2}\sqrt[4]{-1}\sqrt{c}\,\mathrm{PolyLog}\left(2,1-\frac{(1-i)\left((-1)^{3/4}\sqrt{c}x+1\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)b^2 -$$
$$2\sqrt[4]{-1}a\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)b - \sqrt[4]{-1}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\left(2a+ib\log\left(1-icx^2\right)\right)b +$$
$$\frac{ia\log\left(icx^2+1\right)b}{x} - \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x}$$

input
```
Int[(a + b*ArcTan[c*x^2])^2/x^2,x]
```

output
```
(-1)^(1/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2 - 2*(-1)^(1/4)*a*b*S
qrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x] - (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)
^(3/4)*Sqrt[c]*x]^2 - 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x
]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)] + 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)
^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)] - (-1)^(3/4)*b^2*Sqrt[
c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1
+ (-1)^(1/4)*Sqrt[c]*x)] + 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqr
t[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)] - 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTa
nh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)] + (-1)^(3/4)*b^
2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[
c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x))] + (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^
(3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*
Sqrt[c]*x)] - (-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1
- I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)] - (-1)^(3/4)*
b^2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2] - (-1)^(1/4)*b*
Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*Log[1 - I*c*x^2]) - (2*a +
 I*b*Log[1 - I*c*x^2])^2/(4*x) + (I*a*b*Log[1 + I*c*x^2])/x + (-1)^(3/4)*b
^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2] + (-1)^(3/4)*b^2*
Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2] - (b^2*Log[1 - I*c*
x^2]*Log[1 + I*c*x^2])/(2*x) + (b^2*Log[1 + I*c*x^2]^2)/(4*x) + (-1)^(1...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5365
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

Maple [F]

$$\int \frac{(a + b\arctan(c\,x^2))^2}{x^2}dx$$

input
```
int((a+b*arctan(c*x^2))^2/x^2,x)
```

output
```
int((a+b*arctan(c*x^2))^2/x^2,x)
```

Fricas [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x^2}\,dx = \int \frac{(b\arctan(cx^2) + a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^2, x)
```

Sympy [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x^2}\,dx = \int \frac{(a + b\,\mathrm{atan}(cx^2))^2}{x^2}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**2/x**2,x)
```

output
```
Integral((a + b*atan(c*x**2))**2/x**2, x)
```

Maxima [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x^2}\,dx = \int \frac{(b\arctan(cx^2) + a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="maxima")
```

output
```
1/2*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sq
rt(c) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sq
rt(c) + sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c
*x^2 - sqrt(2)*sqrt(c)*x + 1)/sqrt(c)) - 4*arctan(c*x^2)/x)*a*b - 1/16*(4*
arctan(c*x^2)^2 - 16*x*integrate(-1/16*(8*c^2*x^4*log(c^2*x^4 + 1) - 16*c*
x^2*arctan(c*x^2) - 12*(c^2*x^4 + 1)*arctan(c*x^2)^2 - (c^2*x^4 + 1)*log(c
^2*x^4 + 1)^2)/(c^2*x^6 + x^2), x) - log(c^2*x^4 + 1)^2)*b^2/x - a^2/x
```

Giac [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x^2}\,dx = \int \frac{(b\arctan(cx^2) + a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx^2))^2}{x^2}\,dx = \int \frac{(a + b\,\mathrm{atan}(c\,x^2))^2}{x^2}\,dx$$

input
```
int((a + b*atan(c*x^2))^2/x^2,x)
```

output
```
int((a + b*atan(c*x^2))^2/x^2, x)
```

Reduce [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x^2}\,dx = \frac{-4\sqrt{c}\,\sqrt{2}\,atan\left(\frac{\sqrt{c}\,\sqrt{2}-2cx}{\sqrt{c}\,\sqrt{2}}\right)abx - 2\sqrt{c}\,\sqrt{2}\,atan(c\,x^2)\,abx - 4atan(c\,x^2)\,ab - \sqrt{c}\,\sqrt{2}\log(-\sqrt{c}\,\sqrt{2}\,x + c\,x^2}{2x}$$

input
```
int((a+b*atan(c*x^2))^2/x^2,x)
```

output
```
( - 4*sqrt(c)*sqrt(2)*atan((sqrt(c)*sqrt(2) - 2*c*x)/(sqrt(c)*sqrt(2)))*a*
b*x - 2*sqrt(c)*sqrt(2)*atan(c*x**2)*a*b*x - 4*atan(c*x**2)*a*b - sqrt(c)*
sqrt(2)*log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a*b*x + sqrt(c)*sqrt(2)*log
(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a*b*x + 2*int(atan(c*x**2)**2/x**2,x)*b**
2*x - 2*a**2)/(2*x)
```

3.84 $\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx$

Optimal result 645
Mathematica [F] 646
Rubi [A] (verified) 647
Maple [F] 650
Fricas [F] 650
Sympy [F] 650
Maxima [F] 651
Giac [F] 651
Mupad [F(-1)] 651
Reduce [F] 652

Optimal result

Integrand size = 16, antiderivative size = 1360

$$\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx = \text{Too large to display}$$

output
```
1/12*b^2*ln(1+I*c*x^2)^2/x^3-1/12*(2*a+I*b*ln(1-I*c*x^2))^2/x^3-1/3*I*b^2*
c*ln(1-I*c*x^2)/x+2/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*c^(1/2)*x)
*ln(2/(1-(-1)^(3/4)*c^(1/2)*x))+1/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3
/4)*c^(1/2)*x)*ln(-2^(1/2)*((-1)^(3/4)+c^(1/2)*x)/(1+(-1)^(3/4)*c^(1/2)*x)
)+1/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*c^(1/2)*x)*ln((1+I)*(1+(-1
)^(1/4)*c^(1/2)*x)/(1+(-1)^(3/4)*c^(1/2)*x))-2/3*(-1)^(1/4)*b^2*c^(3/2)*ar
ctanh((-1)^(3/4)*c^(1/2)*x)*ln(2/(1+(-1)^(3/4)*c^(1/2)*x))+2/3*(-1)^(3/4)*
a*b*c^(3/2)*arctanh((-1)^(3/4)*c^(1/2)*x)-2/3*(-1)^(1/4)*b^2*c^(3/2)*arcta
n((-1)^(3/4)*c^(1/2)*x)*ln(2/(1+(-1)^(1/4)*c^(1/2)*x))+2/3*(-1)^(1/4)*b^2*
c^(3/2)*arctan((-1)^(3/4)*c^(1/2)*x)*ln(2/(1-(-1)^(1/4)*c^(1/2)*x))+1/3*(-
1)^(1/4)*b^2*c^(3/2)*arctan((-1)^(3/4)*c^(1/2)*x)*ln(2^(1/2)*((-1)^(1/4)+c
^(1/2)*x)/(1+(-1)^(1/4)*c^(1/2)*x))+1/3*(-1)^(1/4)*b^2*c^(3/2)*arctan((-1)
^(3/4)*c^(1/2)*x)*ln((1-I)*(1+(-1)^(3/4)*c^(1/2)*x)/(1+(-1)^(1/4)*c^(1/2)*
x))+1/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*c^(1/2)*x)*ln(1+I*c*x^2)
-1/3*(-1)^(1/4)*b^2*c^(3/2)*arctan((-1)^(3/4)*c^(1/2)*x)*ln(1+I*c*x^2)-1/3
*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*c^(1/2)*x)*ln(1-I*c*x^2)-1/3*(-
1)^(3/4)*b*c^(3/2)*arctan((-1)^(3/4)*c^(1/2)*x)*(2*a+I*b*ln(1-I*c*x^2))-4/
3*(-1)^(1/4)*b^2*c^(3/2)*arctan((-1)^(3/4)*c^(1/2)*x)+1/3*(-1)^(3/4)*b^2*c
^(3/2)*arctan((-1)^(3/4)*c^(1/2)*x)^2-4/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((
-1)^(3/4)*c^(1/2)*x)-1/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*c^(1...
```

Mathematica [F]

$$\int \frac{\left(a + b\arctan\left(cx^2\right)\right)^2}{x^4}\,dx = \int \frac{\left(a + b\arctan\left(cx^2\right)\right)^2}{x^4}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^2/x^4,x]
```

output
```
Integrate[(a + b*ArcTan[c*x^2])^2/x^4, x]
```

Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 1360, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5365, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^4}dx$$

$$\Big\downarrow \text{5365}$$

$$\int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x^4}+\frac{b\log\left(1+icx^2\right)\left(b\log\left(1-icx^2\right)-2ia\right)}{2x^4}-\frac{b^2\log^2\left(1+icx^2\right)}{4x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}(-1)^{3/4}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)^2 b^2-\frac{1}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)^2 b^2+$$
$$\frac{\log^2\left(icx^2+1\right)b^2}{12x^3}-\frac{4}{3}\sqrt[4]{-1}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)b^2-\frac{4}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)b^2+$$
$$\frac{2}{3}\sqrt[4]{-1}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{cx}}\right)b^2-$$
$$\frac{2}{3}\sqrt[4]{-1}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{\sqrt[4]{-1}\sqrt{cx}+1}\right)b^2+$$
$$\frac{1}{3}\sqrt[4]{-1}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{\sqrt{2}\left(\sqrt{cx}+\sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{cx}+1}\right)b^2+$$
$$\frac{2}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{cx}}\right)b^2-$$
$$\frac{2}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{(-1)^{3/4}\sqrt{cx}+1}\right)b^2+$$
$$\frac{1}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(-\frac{\sqrt{2}\left(\sqrt{cx}+(-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{cx}+1}\right)b^2+$$
$$\frac{1}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{cx}+1\right)}{(-1)^{3/4}\sqrt{cx}+1}\right)b^2+$$
$$\frac{1}{3}\sqrt[4]{-1}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{(1-i)\left((-1)^{3/4}\sqrt{cx}+1\right)}{\sqrt[4]{-1}\sqrt{cx}+1}\right)b^2-$$
$$\frac{1}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(1-icx^2\right)b^2-\frac{ic\log\left(1-icx^2\right)b^2}{3x}-$$
$$\frac{1}{3}\sqrt[4]{-1}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(icx^2+1\right)b^2+$$
$$\frac{1}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(icx^2+1\right)b^2-\frac{\log\left(1-icx^2\right)\log\left(icx^2+1\right)b^2}{6x^3}+$$
$$\frac{2ic\log\left(icx^2+1\right)b^2}{3x}+\frac{1}{3}(-1)^{3/4}c^{3/2}\,\text{PolyLog}\left(2,1-\frac{2}{1-\sqrt[4]{-1}\sqrt{cx}}\right)b^2+$$
$$\frac{1}{3}(-1)^{3/4}c^{3/2}\,\text{PolyLog}\left(2,1-\frac{2}{\sqrt[4]{-1}\sqrt{cx}+1}\right)b^2-$$
$$\frac{1}{6}(-1)^{3/4}c^{3/2}\,\text{PolyLog}\left(2,1-\frac{\sqrt{2}\left(\sqrt{cx}+\sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{cx}+1}\right)b^2+$$
$$\frac{1}{3}\sqrt[4]{-1}c^{3/2}\,\text{PolyLog}\left(2,1-\frac{2}{1-(-1)^{3/4}\sqrt{cx}}\right)b^2+$$
$$\frac{1}{3}\sqrt[4]{-1}c^{3/2}\,\text{PolyLog}\left(2,1-\frac{2}{(-1)^{3/4}\sqrt{cx}+1}\right)b^2-$$
$$\frac{1}{6}\sqrt[4]{-1}c^{3/2}\,\text{PolyLog}\left(2,\frac{\sqrt{2}\left(\sqrt{cx}+(-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{cx}+1}+1\right)b^2-$$
$$\frac{1}{6}\sqrt[4]{-1}c^{3/2}\,\text{PolyLog}\left(2,1-\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{cx}+1\right)}{(-1)^{3/4}\sqrt{cx}+1}\right)b^2-$$
$$\frac{1}{6}(-1)^{3/4}c^{3/2}\,\text{PolyLog}\left(2,1-\frac{(1-i)\left((-1)^{3/4}\sqrt{cx}+1\right)}{\sqrt[4]{-1}\sqrt{cx}+1}\right)b^2+$$
$$\frac{2}{3}(-1)^{3/4}ac^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)b-$$
$$\frac{1}{3}(-1)^{3/4}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)\left(2a+ib\log\left(1-icx^2\right)\right)b-\frac{c\left(2a+ib\log\left(1-icx^2\right)\right)b}{3x}+$$
$$\frac{ia\log\left(icx^2+1\right)b}{3x^3}-\frac{2acb}{3x}-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{12x^3}$$

input
```
Int[(a + b*ArcTan[c*x^2])^2/x^4,x]
```

output
```
(-2*a*b*c)/(3*x) - (4*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x])
/3 + ((-1)^(3/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/3 + (2*(-1)^(
3/4)*a*b*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/3 - (4*(-1)^(1/4)*b^2*c^(3
/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/3 - ((-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1
)^(3/4)*Sqrt[c]*x]^2)/3 + (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt
[c]*x]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/3 - (2*(-1)^(1/4)*b^2*c^(3/2)*Ar
cTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 + ((-1)^(1
/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sq
rt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 + (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTa
nh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/3 - (2*(-1)^(1
/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c
]*x)])/3 + ((-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sq
rt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x))])/3 + ((-1)^(1
/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)
*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2*c^(3/2)*ArcT
an[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1
)^(1/4)*Sqrt[c]*x)])/3 - ((I/3)*b^2*c*Log[1 - I*c*x^2])/x - ((-1)^(1/4)*b^
2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/3 - (b*c*(2*a +
I*b*Log[1 - I*c*x^2]))/(3*x) - ((-1)^(3/4)*b*c^(3/2)*ArcTan[(-1)^(3/4)*Sqr
t[c]*x]*(2*a + I*b*Log[1 - I*c*x^2]))/3 - (2*a + I*b*Log[1 - I*c*x^2])^...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5365
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

Maple [F]

$$\int \frac{(a+b\arctan(c\,x^2))^2}{x^4}dx$$

input
```
int((a+b*arctan(c*x^2))^2/x^4,x)
```

output
```
int((a+b*arctan(c*x^2))^2/x^4,x)
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^4, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx^2))^2}{x^4}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**2/x**4,x)
```

output
```
Integral((a + b*atan(c*x**2))**2/x**4, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="maxima")
```

output
```
-1/6*((c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c)
)/c^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c)
)/c^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*l
og(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) + 8/x)*c + 4*arctan(c*x^2)/x^3)
*a*b + 1/48*(48*x^3*integrate(-1/48*(8*c^2*x^4*log(c^2*x^4 + 1) - 16*c*x^2
*arctan(c*x^2) - 36*(c^2*x^4 + 1)*arctan(c*x^2)^2 - 3*(c^2*x^4 + 1)*log(c^
2*x^4 + 1)^2)/(c^2*x^8 + x^4), x) - 4*arctan(c*x^2)^2 + log(c^2*x^4 + 1)^2
)*b^2/x^3 - 1/3*a^2/x^3
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^2))^2}{x^4}\,dx$$

input
```
int((a + b*atan(c*x^2))^2/x^4,x)
```

output
```
int((a + b*atan(c*x^2))^2/x^4, x)
```

Reduce [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x^4} dx = \frac{4\sqrt{c}\,\sqrt{2}\,atan\left(\frac{\sqrt{c}\,\sqrt{2}-2cx}{\sqrt{c}\,\sqrt{2}}\right) abc\,x^3 + 2\sqrt{c}\,\sqrt{2}\,atan(c\,x^2)\,abc\,x^3 - 4atan(c\,x^2)\,ab - \sqrt{c}\,\sqrt{2}\log\left(-\sqrt{c}\,\sqrt{2}\,x + c\right.}{6x^3}$$

input
```
int((a+b*atan(c*x^2))^2/x^4,x)
```

output
```
(4*sqrt(c)*sqrt(2)*atan((sqrt(c)*sqrt(2) - 2*c*x)/(sqrt(c)*sqrt(2)))*a*b*c
*x**3 + 2*sqrt(c)*sqrt(2)*atan(c*x**2)*a*b*c*x**3 - 4*atan(c*x**2)*a*b - s
qrt(c)*sqrt(2)*log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a*b*c*x**3 + sqrt(c)
*sqrt(2)*log(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a*b*c*x**3 + 6*int(atan(c*x**
2)**2/x**4,x)*b**2*x**3 - 2*a**2 - 8*a*b*c*x**2)/(6*x**3)
```

3.85 $\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx$

Optimal result 653
Mathematica [F] 654
Rubi [A] (verified) 655
Maple [F] 658
Fricas [F] 658
Sympy [F] 658
Maxima [F] 659
Giac [F] 659
Mupad [F(-1)] 659
Reduce [F] 660

Optimal result

Integrand size = 16, antiderivative size = 1444

$$\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx = \text{Too large to display}$$

output
```
1/20*b^2*ln(1+I*c*x^2)^2/x^5-1/20*(2*a+I*b*ln(1-I*c*x^2))^2/x^5-1/15*I*b^2
*c*ln(1-I*c*x^2)/x^3-1/5*I*b*c^2*(2*a+I*b*ln(1-I*c*x^2))/x-1/5*(-1)^(3/4)*
b^2*c^(5/2)*arctanh((-1)^(3/4)*c^(1/2)*x)*ln(-2^(1/2)*((-1)^(3/4)+c^(1/2)*
x)/(1+(-1)^(3/4)*c^(1/2)*x))-1/5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)
*c^(1/2)*x)*ln((1+I)*(1+(-1)^(1/4)*c^(1/2)*x)/(1+(-1)^(3/4)*c^(1/2)*x))+2/
5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*c^(1/2)*x)*ln(2/(1+(-1)^(3/4)*
c^(1/2)*x))-2/5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*c^(1/2)*x)*ln(2/
(1-(-1)^(3/4)*c^(1/2)*x))+2/5*(-1)^(1/4)*a*b*c^(5/2)*arctanh((-1)^(3/4)*c^
(1/2)*x)+1/5*(-1)^(3/4)*b^2*c^(5/2)*arctan((-1)^(3/4)*c^(1/2)*x)*ln(2^(1/2
)*((-1)^(1/4)+c^(1/2)*x)/(1+(-1)^(1/4)*c^(1/2)*x))+1/5*(-1)^(3/4)*b^2*c^(5
/2)*arctan((-1)^(3/4)*c^(1/2)*x)*ln((1-I)*(1+(-1)^(3/4)*c^(1/2)*x)/(1+(-1)
^(1/4)*c^(1/2)*x))-2/5*(-1)^(3/4)*b^2*c^(5/2)*arctan((-1)^(3/4)*c^(1/2)*x)
*ln(2/(1+(-1)^(1/4)*c^(1/2)*x))+2/5*(-1)^(3/4)*b^2*c^(5/2)*arctan((-1)^(3/
4)*c^(1/2)*x)*ln(2/(1-(-1)^(1/4)*c^(1/2)*x))-1/5*(-1)^(3/4)*b^2*c^(5/2)*ar
ctanh((-1)^(3/4)*c^(1/2)*x)*ln(1+I*c*x^2)-1/5*(-1)^(3/4)*b^2*c^(5/2)*arcta
n((-1)^(3/4)*c^(1/2)*x)*ln(1+I*c*x^2)+1/5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((
-1)^(3/4)*c^(1/2)*x)*ln(1-I*c*x^2)+1/5*(-1)^(1/4)*b*c^(5/2)*arctan((-1)^(3
/4)*c^(1/2)*x)*(2*a+I*b*ln(1-I*c*x^2))+1/5*I*a*b*ln(1+I*c*x^2)/x^5+2/15*I*
b^2*c*ln(1+I*c*x^2)/x^3+4/15*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*c^(
1/2)*x)+1/5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*c^(1/2)*x)^2-4/15...
```

Mathematica [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x^6}\,dx = \int \frac{(a + b\arctan(cx^2))^2}{x^6}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^2/x^6,x]
```

output
```
Integrate[(a + b*ArcTan[c*x^2])^2/x^6, x]
```

Rubi [A] (verified)

Time = 2.75 (sec) , antiderivative size = 1444, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5365, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx$$

$$\Big\downarrow \; 5365$$

$$\int \left(\frac{(2a+ib\log(1-icx^2))^2}{4x^6}+\frac{b\log(1+icx^2)\,(b\log(1-icx^2)-2ia)}{2x^6}-\frac{b^2\log^2(1+icx^2)}{4x^6}\right)dx$$

$$\Big\downarrow \; 2009$$

$$
-\frac{1}{5}\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{cx}\right)^2c^{5/2}+\frac{1}{5}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)^2c^{5/2}-
\frac{4}{15}(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{cx}\right)c^{5/2}+\frac{4}{15}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)c^{5/2}+
\frac{2}{5}\sqrt[4]{-1}ab\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)c^{5/2}+
\frac{2}{5}(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{cx}}\right)c^{5/2}-
\frac{2}{5}(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{\sqrt[4]{-1}\sqrt{cx}+1}\right)c^{5/2}+
\frac{1}{5}(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{\sqrt{2}(\sqrt{cx}+\sqrt[4]{-1})}{\sqrt[4]{-1}\sqrt{cx}+1}\right)c^{5/2}-
\frac{2}{5}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{cx}}\right)c^{5/2}+
\frac{2}{5}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{(-1)^{3/4}\sqrt{cx}+1}\right)c^{5/2}-
\frac{1}{5}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(-\frac{\sqrt{2}(\sqrt{cx}+(-1)^{3/4})}{(-1)^{3/4}\sqrt{cx}+1}\right)c^{5/2}-
\frac{1}{5}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{cx}+1\right)}{(-1)^{3/4}\sqrt{cx}+1}\right)c^{5/2}+
\frac{1}{5}(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{(1-i)\left((-1)^{3/4}\sqrt{cx}+1\right)}{\sqrt[4]{-1}\sqrt{cx}+1}\right)c^{5/2}+
\frac{1}{5}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(1-icx^2\right)c^{5/2}+
\frac{1}{5}\sqrt[4]{-1}b\arctan\left((-1)^{3/4}\sqrt{cx}\right)\left(2a+ib\log\left(1-icx^2\right)\right)c^{5/2}-
\frac{1}{5}(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(icx^2+1\right)c^{5/2}-
\frac{1}{5}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(icx^2+1\right)c^{5/2}-
\frac{1}{5}\sqrt[4]{-1}b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\sqrt[4]{-1}\sqrt{cx}}\right)c^{5/2}-
\frac{1}{5}\sqrt[4]{-1}b^2\,\text{PolyLog}\left(2,1-\frac{2}{\sqrt[4]{-1}\sqrt{cx}+1}\right)c^{5/2}+
\frac{1}{10}\sqrt[4]{-1}b^2\,\text{PolyLog}\left(2,1-\frac{\sqrt{2}(\sqrt{cx}+\sqrt[4]{-1})}{\sqrt[4]{-1}\sqrt{cx}+1}\right)c^{5/2}-
\frac{1}{5}(-1)^{3/4}b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-(-1)^{3/4}\sqrt{cx}}\right)c^{5/2}-
\frac{1}{5}(-1)^{3/4}b^2\,\text{PolyLog}\left(2,1-\frac{2}{(-1)^{3/4}\sqrt{cx}+1}\right)c^{5/2}+
\frac{1}{10}(-1)^{3/4}b^2\,\text{PolyLog}\left(2,\frac{\sqrt{2}(\sqrt{cx}+(-1)^{3/4})}{(-1)^{3/4}\sqrt{cx}+1}+1\right)c^{5/2}+
\frac{1}{10}(-1)^{3/4}b^2\,\text{PolyLog}\left(2,1-\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{cx}+1\right)}{(-1)^{3/4}\sqrt{cx}+1}\right)c^{5/2}+
\frac{1}{10}\sqrt[4]{-1}b^2\,\text{PolyLog}\left(2,1-\frac{(1-i)\left((-1)^{3/4}\sqrt{cx}+1\right)}{\sqrt[4]{-1}\sqrt{cx}+1}\right)c^{5/2}-\frac{b^2\log\left(1-icx^2\right)c^2}{5x}-
\frac{ib(2a+ib\log\left(1-icx^2\right))c^2}{5x}-\frac{8b^2c^2}{15x}+\frac{2iabc^2}{5x}-\frac{ib^2\log\left(1-icx^2\right)c}{15x^3}-
\frac{b(2a+ib\log\left(1-icx^2\right))c}{15x^3}+\frac{2ib^2\log\left(icx^2+1\right)c}{15x^3}-\frac{2abc}{15x^3}-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{20x^5}+
$$

input
```
Int[(a + b*ArcTan[c*x^2])^2/x^6,x]
```

output
```
(-2*a*b*c)/(15*x^3) + (((2*I)/5)*a*b*c^2)/x - (8*b^2*c^2)/(15*x) - (4*(-1)
^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/15 - ((-1)^(1/4)*b^2*c^(5
/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/5 + (2*(-1)^(1/4)*a*b*c^(5/2)*ArcTanh[
(-1)^(3/4)*Sqrt[c]*x])/5 + (4*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sq
rt[c]*x])/15 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]^2)/5
+ (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(
1/4)*Sqrt[c]*x)])/5 - (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*
x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTan[(
-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4
)*Sqrt[c]*x)])/5 - (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]
*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/5 + (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[
(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/5 - ((-1)^(3/4)*b
^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt
[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x))])/5 - ((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[
(-1)^(3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(
3/4)*Sqrt[c]*x)])/5 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]
*Log[((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 -
 ((I/15)*b^2*c*Log[1 - I*c*x^2])/x^3 - (b^2*c^2*Log[1 - I*c*x^2])/(5*x) +
((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/5
- (b*c*(2*a + I*b*Log[1 - I*c*x^2]))/(15*x^3) - ((I/5)*b*c^2*(2*a + I*b...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5365
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

Maple [F]

$$\int \frac{(a+b\arctan(c\,x^2))^2}{x^6}dx$$

input
```
int((a+b*arctan(c*x^2))^2/x^6,x)
```

output
```
int((a+b*arctan(c*x^2))^2/x^6,x)
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^6}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^6, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx^2))^2}{x^6}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**2/x**6,x)
```

output
```
Integral((a + b*atan(c*x**2))**2/x**6, x)
```

Maxima [F]

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^6}\,dx=\int \frac{\left(b\arctan\left(cx^2\right)+a\right)^2}{x^6}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="maxima")
```

output
```
-1/30*((6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqr
t(c)) + 6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqr
t(c)) + 3*sqrt(2)*c^(3/2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1) - 3*sqrt(2)*c
^(3/2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1) + 8/x^3)*c + 12*arctan(c*x^2)/x^
5)*a*b + 1/80*(80*x^5*integrate(-1/80*(8*c^2*x^4*log(c^2*x^4 + 1) - 16*c*x
^2*arctan(c*x^2) - 60*(c^2*x^4 + 1)*arctan(c*x^2)^2 - 5*(c^2*x^4 + 1)*log(
c^2*x^4 + 1)^2)/(c^2*x^10 + x^6), x) - 4*arctan(c*x^2)^2 + log(c^2*x^4 + 1
)^2)*b^2/x^5 - 1/5*a^2/x^5
```

Giac [F]

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^6}\,dx=\int \frac{\left(b\arctan\left(cx^2\right)+a\right)^2}{x^6}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^6}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2}{x^6}\,dx$$

input
```
int((a + b*atan(c*x^2))^2/x^6,x)
```

output
```
int((a + b*atan(c*x^2))^2/x^6, x)
```

Reduce [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x^6}\,dx$$
$$= \frac{12\sqrt{c}\,\sqrt{2}\,atan\left(\frac{\sqrt{c}\,\sqrt{2}-2cx}{\sqrt{c}\,\sqrt{2}}\right)ab\,c^2x^5 + 6\sqrt{c}\,\sqrt{2}\,atan(c\,x^2)\,ab\,c^2x^5 - 12atan(c\,x^2)\,ab + 3\sqrt{c}\,\sqrt{2}\log\left(-\sqrt{c}\,\sqrt{2}\right.}{3}$$

input
```
int((a+b*atan(c*x^2))^2/x^6,x)
```

output
```
(12*sqrt(c)*sqrt(2)*atan((sqrt(c)*sqrt(2) - 2*c*x)/(sqrt(c)*sqrt(2)))*a*b*
c**2*x**5 + 6*sqrt(c)*sqrt(2)*atan(c*x**2)*a*b*c**2*x**5 - 12*atan(c*x**2)
*a*b + 3*sqrt(c)*sqrt(2)*log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a*b*c**2*x
**5 - 3*sqrt(c)*sqrt(2)*log(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a*b*c**2*x**5
+ 30*int(atan(c*x**2)**2/x**6,x)*b**2*x**5 - 6*a**2 - 8*a*b*c*x**2)/(30*x*
*5)
```

3.86 $\int x^3(a+b\arctan(cx^2))^3\,dx$

Optimal result . . . 661
Mathematica [A] (verified) . . . 662
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Maple [C] (warning: unable to verify) . . . 666
Fricas [F] . . . 667
Sympy [F] . . . 667
Maxima [F] . . . 668
Giac [F] . . . 668
Mupad [F(-1)] . . . 668
Reduce [F] . . . 669

Optimal result

Integrand size = 16, antiderivative size = 149

$$\int x^3(a+b\arctan(cx^2))^3\,dx = -\frac{3ib(a+b\arctan(cx^2))^2}{4c^2} - \frac{3bx^2(a+b\arctan(cx^2))^2}{4c} + \frac{(a+b\arctan(cx^2))^3}{4c^2} + \frac{1}{4}x^4(a+b\arctan(cx^2))^3 - \frac{3b^2(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{2c^2} - \frac{3ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{4c^2}$$

output
```
-3/4*I*b*(a+b*arctan(c*x^2))^2/c^2-3/4*b*x^2*(a+b*arctan(c*x^2))^2/c+1/4*(
a+b*arctan(c*x^2))^3/c^2+1/4*x^4*(a+b*arctan(c*x^2))^3-3/2*b^2*(a+b*arctan
(c*x^2))*ln(2/(1+I*c*x^2))/c^2-3/4*I*b^3*polylog(2,1-2/(1+I*c*x^2))/c^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.14

$$\int x^3 \left(a + b \arctan\left(cx^2\right)\right)^3 dx$$
$$= \frac{3b^2(a + ac^2x^4 + b(i - cx^2)) \arctan\left(cx^2\right)^2 + b^3(1 + c^2x^4) \arctan\left(cx^2\right)^3 + 3b \arctan\left(cx^2\right) \left(a(a - 2bcx^2 + \right.}{}$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x^2])^3,x]
```

output
```
(3*b^2*(a + a*c^2*x^4 + b*(I - c*x^2))*ArcTan[c*x^2]^2 + b^3*(1 + c^2*x^4)
*ArcTan[c*x^2]^3 + 3*b*ArcTan[c*x^2]*(a*(a - 2*b*c*x^2 + a*c^2*x^4) - 2*b^
2*Log[1 + E^((2*I)*ArcTan[c*x^2])]) + a*(a*c*x^2*(-3*b + a*c*x^2) + 3*b^2*
Log[1 + c^2*x^4]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])])/(4*c^2
)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5363, 5361, 5451, 5345, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a + b \arctan\left(cx^2\right)\right)^3 dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{2} \int x^2 \left(a + b \arctan\left(cx^2\right)\right)^3 dx^2$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4 \left(a + b \arctan\left(cx^2\right)\right)^3 - \frac{3}{2}bc \int \frac{x^4 \left(a + b \arctan\left(cx^2\right)\right)^2}{c^2x^4 + 1} dx^2\right)$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(\frac{\int\left(a+b\arctan\left(cx^2\right)\right)^2dx^2}{c^2}-\frac{\int\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{c^2x^4+1}dx^2}{c^2}\right)\right)$$

$\downarrow$ 5345

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\int\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)}{c^2x^4+1}dx^2}{c^2}-\frac{\int\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{c^2x^4+1}}{c^2}\right.\right.$$

$\downarrow$ 5419

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\int\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)}{c^2x^4+1}dx^2}{c^2}-\frac{\left(a+b\arctan\left(cx^2\right.\right.}{3bc^3}\right.\right.$$

$\downarrow$ 5455

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{3bc^3}+\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{\int\frac{a+b\arctan\left(cx^2\right)}{i-cx^2}}{c}\right.}{c^2}\right.\right.$$

$\downarrow$ 5379

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{3bc^3}+\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{\log\left(\frac{2}{1+icx^2}\right)\left(a+b\right.}{c}\right.}{c}\right.\right.$$

$\downarrow$ 2849

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{3bc^3}+\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{ib\int\frac{\log\left(\frac{2}{icx^2+1}\right)}{1-\frac{2}{icx^2+1}}d}{c}\right.}{}\right.\right.$$

$\downarrow$ 2752

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{3bc^3}+\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{i\left(a+b\arctan\left(cx^2\right)\right)}{2bc^2}\right.}{}\right.\right.$$

input
```
Int[x^3*(a + b*ArcTan[c*x^2])^3,x]
```

output
```
((x^4*(a + b*ArcTan[c*x^2])^3)/2 - (3*b*c*(-1/3*(a + b*ArcTan[c*x^2])^3/(b
*c^3) + (x^2*(a + b*ArcTan[c*x^2])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^
2])^2)/(b*c^2) - (((a + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/c + ((I/2)*
b*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c)/c))/c^2))/2)/2
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.94 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.68

method	result
default	$\frac{a^3x^4}{4}+\frac{b^3x^4\arctan(cx^2)^3}{4}-\frac{3b^3\arctan(cx^2)^2x^2}{4c}+\frac{b^3\arctan(cx^2)^3}{4c^2}+\frac{3b^3\ln(c^2x^4+1)\arctan(cx^2)}{4c^2}-\frac{3b^3\left(\sum_{\_\alpha=\mathrm{RootOf}(}\right.}{}$
parts	$\frac{a^3x^4}{4}+\frac{b^3x^4\arctan(cx^2)^3}{4}-\frac{3b^3\arctan(cx^2)^2x^2}{4c}+\frac{b^3\arctan(cx^2)^3}{4c^2}+\frac{3b^3\ln(c^2x^4+1)\arctan(cx^2)}{4c^2}-\frac{3b^3\left(\sum_{\_\alpha=\mathrm{RootOf}(}\right.}{}$
risch	$-\frac{3b^2(ibc^2x^4\ln(-icx^2+1)+2ac^2x^4-2bcx^2+ib\ln(-icx^2+1)+2ib+2a)\ln(icx^2+1)^2}{32c^2}+\frac{a^3x^4}{4}-\frac{3a^2bx^2}{4c}+\frac{3a^2b\arctan(cx^2)}{4c^2}$

input
```
int(x^3*(a+b*arctan(c*x^2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^3*x^4+1/4*b^3*x^4*arctan(c*x^2)^3-3/4*b^3*arctan(c*x^2)^2/c*x^2+1/4*
b^3*arctan(c*x^2)^3/c^2+3/4*b^3/c^2*ln(c^2*x^4+1)*arctan(c*x^2)-3/16*b^3/c
^3*sum(1/_alpha^2*(2*ln(x-_alpha)*ln(c^2*x^4+1)-c*(1/c/_alpha^3*ln(x-_alph
a)^2+2/_alpha*ln(x-_alpha)*(_alpha^2*ln(1/2*(x+_alpha)/_alpha)*c+ln((_alph
a^3*c-x)/_alpha/(_alpha^2*c-1))-ln((_alpha^3*c+x)/_alpha/(_alpha^2*c+1)))+
2/_alpha*(_alpha^2*dilog(1/2*(x+_alpha)/_alpha)*c+dilog((_alpha^3*c-x)/_al
pha/(_alpha^2*c-1))-dilog((_alpha^3*c+x)/_alpha/(_alpha^2*c+1))))),_alpha=
RootOf(_Z^4*c^2+1))+3/4*a*b^2*x^4*arctan(c*x^2)^2-3/2*a*b^2*arctan(c*x^2)/
c*x^2+3/4*a*b^2/c^2*arctan(c*x^2)^2+3/4*a*b^2/c^2*ln(c^2*x^4+1)+3/4*a^2*b*
x^4*arctan(c*x^2)-3/4*a^2*b/c*x^2+3/4*a^2*b/c^2*arctan(c*x^2)
```

Fricas [F]

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*arctan(c*x^2)^3 + 3*a*b^2*x^3*arctan(c*x^2)^2 + 3*a^2*b*x
^3*arctan(c*x^2) + a^3*x^3, x)
```

Sympy [F]

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\int x^3\left(a+b\,\mathrm{atan}\left(cx^2\right)\right)^3\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x**2))**3,x)
```

output
```
Integral(x**3*(a + b*atan(c*x**2))**3, x)
```

Maxima [F]

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

output
```
3/4*a*b^2*x^4*arctan(c*x^2)^2 + 1/4*a^3*x^4 + 3/4*(x^4*arctan(c*x^2) - c*(
x^2/c^2 - arctan(c*x^2)/c^3))*a^2*b - 3/4*(2*c*(x^2/c^2 - arctan(c*x^2)/c^
3)*arctan(c*x^2) + (arctan(c*x^2)^2 - log(4*c^5*x^4 + 4*c^3))/c^2)*a*b^2 +
 1/128*(4*x^4*arctan(c*x^2)^3 - 3*x^4*arctan(c*x^2)*log(c^2*x^4 + 1)^2 + 1
28*integrate(1/64*(12*c^2*x^7*arctan(c*x^2)*log(c^2*x^4 + 1) - 12*c*x^5*ar
ctan(c*x^2)^2 + 56*(c^2*x^7 + x^3)*arctan(c*x^2)^3 + 3*(c*x^5 + 2*(c^2*x^7
 + x^3)*arctan(c*x^2))*log(c^2*x^4 + 1)^2)/(c^2*x^4 + 1), x))*b^3
```

Giac [F]

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^3*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\int x^3\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^3\,dx$$

input
```
int(x^3*(a + b*atan(c*x^2))^3,x)
```

output
```
int(x^3*(a + b*atan(c*x^2))^3, x)
```

Reduce [F]

$$\int x^3 \left(a + b \arctan\left(cx^2\right)\right)^3 dx$$
$$= \frac{atan(c\,x^2)^3\,b^3c^2x^4 + atan(c\,x^2)^3\,b^3 + 3atan(c\,x^2)^2\,a\,b^2c^2x^4 + 3atan(c\,x^2)^2\,a\,b^2 - 3atan(c\,x^2)^2\,b^3c\,x^2 + 3a}{}$$

input
```
int(x^3*(a+b*atan(c*x^2))^3,x)
```

output
```
(atan(c*x**2)**3*b**3*c**2*x**4 + atan(c*x**2)**3*b**3 + 3*atan(c*x**2)**2
*a*b**2*c**2*x**4 + 3*atan(c*x**2)**2*a*b**2 - 3*atan(c*x**2)**2*b**3*c*x*
*2 + 3*atan(c*x**2)*a**2*b*c**2*x**4 + 3*atan(c*x**2)*a**2*b - 6*atan(c*x*
*2)*a*b**2*c*x**2 + 12*int((atan(c*x**2)*x**3)/(c**2*x**4 + 1),x)*b**3*c**
2 + 3*log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a*b**2 + 3*log(sqrt(c)*sqrt(2
)*x + c*x**2 + 1)*a*b**2 + a**3*c**2*x**4 - 3*a**2*b*c*x**2)/(4*c**2)
```

3.87 $\int x(a+b\arctan(cx^2))^3\,dx$

Optimal result 670
Mathematica [A] (verified) 671
Rubi [A] (verified) 671
Maple [B] (verified) 674
Fricas [F] 674
Sympy [F] 675
Maxima [F] 675
Giac [F] 676
Mupad [F(-1)] 676
Reduce [F] 676

Optimal result

Integrand size = 14, antiderivative size = 144

$$\int x(a+b\arctan(cx^2))^3\,dx = \frac{i(a+b\arctan(cx^2))^3}{2c}+\frac{1}{2}x^2(a+b\arctan(cx^2))^3 + \frac{3b(a+b\arctan(cx^2))^2\log\left(\frac{2}{1+icx^2}\right)}{2c} + \frac{3ib^2(a+b\arctan(cx^2))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{2c} + \frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^2}\right)}{4c}$$

output
```
1/2*I*(a+b*arctan(c*x^2))^3/c+1/2*x^2*(a+b*arctan(c*x^2))^3+3/2*b*(a+b*arc
tan(c*x^2))^2*ln(2/(1+I*c*x^2))/c+3/2*I*b^2*(a+b*arctan(c*x^2))*polylog(2,
1-2/(1+I*c*x^2))/c+3/4*b^3*polylog(3,1-2/(1+I*c*x^2))/c
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.56

$$\int x(a+b\arctan(cx^2))^3\,dx$$
$$=\frac{2a^3cx^2+6a^2bcx^2\arctan(cx^2)-6iab^2\arctan(cx^2)^2+6ab^2cx^2\arctan(cx^2)^2-2ib^3\arctan(cx^2)^3+2b^3cx}{}$$

input
```
Integrate[x*(a + b*ArcTan[c*x^2])^3,x]
```

output
```
(2*a^3*c*x^2 + 6*a^2*b*c*x^2*ArcTan[c*x^2] - (6*I)*a*b^2*ArcTan[c*x^2]^2 +
 6*a*b^2*c*x^2*ArcTan[c*x^2]^2 - (2*I)*b^3*ArcTan[c*x^2]^3 + 2*b^3*c*x^2*A
rcTan[c*x^2]^3 + 12*a*b^2*ArcTan[c*x^2]*Log[1 + E^((2*I)*ArcTan[c*x^2])] +
 6*b^3*ArcTan[c*x^2]^2*Log[1 + E^((2*I)*ArcTan[c*x^2])] - 3*a^2*b*Log[1 +
c^2*x^4] - (6*I)*b^2*(a + b*ArcTan[c*x^2])*PolyLog[2, -E^((2*I)*ArcTan[c*x
^2])] + 3*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x^2])])/(4*c)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5363, 5345, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arctan(cx^2))^3\,dx$$
$$\Big\downarrow \text{5363}$$
$$\frac{1}{2}\int (a+b\arctan(cx^2))^3\,dx^2$$
$$\Big\downarrow \text{5345}$$
$$\frac{1}{2}\left(x^2(a+b\arctan(cx^2))^3-3bc\int\frac{x^2(a+b\arctan(cx^2))^2}{c^2x^4+1}dx^2\right)$$

$$\Big\downarrow \ 5455$$

$$\frac{1}{2}\left(x^2(a+b\arctan(cx^2))^3-3bc\left(-\frac{\int\frac{(a+b\arctan(cx^2))^2}{i-cx^2}dx^2}{c}-\frac{i(a+b\arctan(cx^2))^3}{3bc^2}\right)\right)$$

$$\Big\downarrow \ 5379$$

$$\frac{1}{2}\left(x^2(a+b\arctan(cx^2))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx^2}\right)(a+b\arctan(cx^2))^2}{c}-2b\int\frac{(a+b\arctan(cx^2))\log\left(\frac{2}{icx^2+1}\right)}{c^2x^4+1}dx^2}{c}-\frac{i(a+b\mathrm{a}$$

$$\Big\downarrow \ 5529$$

$$\frac{1}{2}\left(x^2(a+b\arctan(cx^2))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx^2}\right)(a+b\arctan(cx^2))^2}{c}-2b\left(\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)}{c^2x^4+1}dx^2-\frac{i\,\text{PolyLog}\left(2,1\right.}{}\right.}{c}\right.\right.$$

$$\Big\downarrow \ 7164$$

$$\frac{1}{2}\left(x^2(a+b\arctan(cx^2))^3-3bc\left(-\frac{i(a+b\arctan(cx^2))^3}{3bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx^2}\right)(a+b\arctan(cx^2))^2}{c}-2b\left(-\frac{i\,\text{PolyLog}\left(2,1-\frac{}{ic}\right.}{}\right.}{c}\right.\right.$$

input
```
Int[x*(a + b*ArcTan[c*x^2])^3,x]
```

output
```
(x^2*(a + b*ArcTan[c*x^2])^3 - 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x^2])^3)/(
b*c^2) - (((a + b*ArcTan[c*x^2])^2*Log[2/(1 + I*c*x^2)])/c - 2*b*(((-1/2*I
)*(a + b*ArcTan[c*x^2])*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c - (b*PolyLog[3,
 1 - 2/(1 + I*c*x^2)])/(4*c)))/c))/2
```

Defintions of rubi rules used

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(129) = 258$.

Time = 0.88 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.91

method	result
derivativedivides	$\frac{a^3cx^2+b^3\left(\arctan(cx^2)^3(cx^2+i)-2i\arctan(cx^2)^3+3\arctan(cx^2)^2\ln\left(1+\frac{(icx^2+1)^2}{c^2x^4+1}\right)-3i\arctan(cx^2)\operatorname{polylog}\left(\cdots\right.\right.}{\cdots}$
default	$\frac{a^3cx^2+b^3\left(\arctan(cx^2)^3(cx^2+i)-2i\arctan(cx^2)^3+3\arctan(cx^2)^2\ln\left(1+\frac{(icx^2+1)^2}{c^2x^4+1}\right)-3i\arctan(cx^2)\operatorname{polylog}\left(\cdots\right.\right.}{\cdots}$
parts	$\frac{a^3x^2}{2}+\frac{b^3\left(\arctan(cx^2)^3(cx^2+i)-2i\arctan(cx^2)^3+3\arctan(cx^2)^2\ln\left(1+\frac{(icx^2+1)^2}{c^2x^4+1}\right)-3i\arctan(cx^2)\operatorname{polylog}\left(\cdots\right.\right.}{2c}$

input
```
int(x*(a+b*arctan(c*x^2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/c*(a^3*c*x^2+b^3*(arctan(c*x^2)^3*(c*x^2+I)-2*I*arctan(c*x^2)^3+3*arct
an(c*x^2)^2*ln(1+(1+I*c*x^2)^2/(c^2*x^4+1))-3*I*arctan(c*x^2)*polylog(2,-(
1+I*c*x^2)^2/(c^2*x^4+1))+3/2*polylog(3,-(1+I*c*x^2)^2/(c^2*x^4+1)))+3*a*b
^2*(arctan(c*x^2)^2*(c*x^2+I)+2*arctan(c*x^2)*ln(1+(1+I*c*x^2)^2/(c^2*x^4+
1))-2*I*arctan(c*x^2)^2-I*polylog(2,-(1+I*c*x^2)^2/(c^2*x^4+1)))+3*a^2*b*(
c*x^2*arctan(c*x^2)-1/2*ln(c^2*x^4+1)))
```

Fricas [F]

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*arctan(c*x^2)^3 + 3*a*b^2*x*arctan(c*x^2)^2 + 3*a^2*b*x*arc
tan(c*x^2) + a^3*x, x)
```

Sympy [F]

$$\int x(a+b\arctan(cx^2))^3\,dx=\int x(a+b\operatorname{atan}(cx^2))^3\,dx$$

input
```
integrate(x*(a+b*atan(c*x**2))**3,x)
```

output
```
Integral(x*(a + b*atan(c*x**2))**3, x)
```

Maxima [F]

$$\int x(a+b\arctan(cx^2))^3\,dx=\int (b\arctan(cx^2)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

output
```
1/16*b^3*x^2*arctan(c*x^2)^3 - 3/64*b^3*x^2*arctan(c*x^2)*log(c^2*x^4 + 1)
^2 + 7/64*b^3*arctan(c*x^2)^4/c + 28*b^3*c^2*integrate(1/32*x^5*arctan(c*x
^2)^3/(c^2*x^4 + 1), x) + 3*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2)*log(c
^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^5*arctan(c
*x^2)^2/(c^2*x^4 + 1), x) + 12*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2)*lo
g(c^2*x^4 + 1)/(c^2*x^4 + 1), x) + 1/2*a^3*x^2 + 1/2*a*b^2*arctan(c*x^2)^3
/c - 12*b^3*c*integrate(1/32*x^3*arctan(c*x^2)^2/(c^2*x^4 + 1), x) + 3*b^3
*c*integrate(1/32*x^3*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 3*b^3*integra
te(1/32*x*arctan(c*x^2)*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 3/4*(2*c*x^
2*arctan(c*x^2) - log(c^2*x^4 + 1))*a^2*b/c
```

Giac [F]

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^3*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\int x\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^3\,dx$$

input
```
int(x*(a + b*atan(c*x^2))^3,x)
```

output
```
int(x*(a + b*atan(c*x^2))^3, x)
```

Reduce [F]

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^3\,dx$$
$$=\frac{6atan(c\,x^2)\,a^2bc\,x^2+4\left(\int atan(c\,x^2)^3\,xdx\right)b^3c+12\left(\int atan(c\,x^2)^2\,xdx\right)a\,b^2c-3\log\left(-\sqrt{c}\,\sqrt{2}\,x+c\,x^2\right.}{4c}$$

input
```
int(x*(a+b*atan(c*x^2))^3,x)
```

output
```
(6*atan(c*x**2)*a**2*b*c*x**2 + 4*int(atan(c*x**2)**3*x,x)*b**3*c + 12*int
(atan(c*x**2)**2*x,x)*a*b**2*c - 3*log( - sqrt(c)*sqrt(2)*x + c*x**2 + 1)*
a**2*b - 3*log(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a**2*b + 2*a**3*c*x**2)/(4*
c)
```

3.88 $\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx$

Optimal result 677
Mathematica [A] (verified) 678
Rubi [A] (verified) 679
Maple [F] 682
Fricas [F] 682
Sympy [F] 682
Maxima [F] 683
Giac [F] 683
Mupad [F(-1)] 683
Reduce [F] 684

Optimal result

Integrand size = 16, antiderivative size = 229

$$\begin{aligned}\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx = {} & (a+b\arctan(cx^2))^3\operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right) \\ & -\frac{3}{4}ib(a+b\arctan(cx^2))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right) \\ & +\frac{3}{4}ib(a+b\arctan(cx^2))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx^2}\right) \\ & -\frac{3}{4}b^2(a+b\arctan(cx^2))\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^2}\right) \\ & +\frac{3}{4}b^2(a+b\arctan(cx^2))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx^2}\right) \\ & +\frac{3}{8}ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+icx^2}\right) \\ & -\frac{3}{8}ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx^2}\right)\end{aligned}$$

output

```
-(a+b*arctan(c*x^2))^3*arctanh(-1+2/(1+I*c*x^2))-3/4*I*b*(a+b*arctan(c*x^2
))^2*polylog(2,1-2/(1+I*c*x^2))+3/4*I*b*(a+b*arctan(c*x^2))^2*polylog(2,-1
+2/(1+I*c*x^2))-3/4*b^2*(a+b*arctan(c*x^2))*polylog(3,1-2/(1+I*c*x^2))+3/4
*b^2*(a+b*arctan(c*x^2))*polylog(3,-1+2/(1+I*c*x^2))+3/8*I*b^3*polylog(4,1
-2/(1+I*c*x^2))-3/8*I*b^3*polylog(4,-1+2/(1+I*c*x^2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.82

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx = a^3\log(x)+\frac{3}{4}ia^2b\left(\text{PolyLog}\left(2,-icx^2\right)-\text{PolyLog}\left(2,icx^2\right)\right)$$
$$+\frac{1}{16}ab^2\Big(-i\pi^3+16i\arctan\left(cx^2\right)^3$$
$$+24\arctan\left(cx^2\right)^2\log\left(1-e^{-2i\arctan(cx^2)}\right)$$
$$-24\arctan\left(cx^2\right)^2\log\left(1+e^{2i\arctan(cx^2)}\right)$$
$$+24i\arctan\left(cx^2\right)\text{PolyLog}\left(2,e^{-2i\arctan(cx^2)}\right)$$
$$+24i\arctan\left(cx^2\right)\text{PolyLog}\left(2,-e^{2i\arctan(cx^2)}\right)$$
$$+12\,\text{PolyLog}\left(3,e^{-2i\arctan(cx^2)}\right)$$
$$-12\,\text{PolyLog}\left(3,-e^{2i\arctan(cx^2)}\right)\Big)$$
$$-\frac{1}{128}ib^3\Big(\pi^4-32\arctan\left(cx^2\right)^4$$
$$+64i\arctan\left(cx^2\right)^3\log\left(1-e^{-2i\arctan(cx^2)}\right)$$
$$-64i\arctan\left(cx^2\right)^3\log\left(1+e^{2i\arctan(cx^2)}\right)$$
$$-96\arctan\left(cx^2\right)^2\text{PolyLog}\left(2,e^{-2i\arctan(cx^2)}\right)$$
$$-96\arctan\left(cx^2\right)^2\text{PolyLog}\left(2,-e^{2i\arctan(cx^2)}\right)$$
$$+96i\arctan\left(cx^2\right)\text{PolyLog}\left(3,e^{-2i\arctan(cx^2)}\right)$$
$$-96i\arctan\left(cx^2\right)\text{PolyLog}\left(3,-e^{2i\arctan(cx^2)}\right)$$
$$+48\,\text{PolyLog}\left(4,e^{-2i\arctan(cx^2)}\right)$$
$$+48\,\text{PolyLog}\left(4,-e^{2i\arctan(cx^2)}\right)\Big)$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^3/x,x]
```

output
```
a^3*Log[x] + ((3*I)/4)*a^2*b*(PolyLog[2, (-I)*c*x^2] - PolyLog[2, I*c*x^2]
) + (a*b^2*((-I)*Pi^3 + (16*I)*ArcTan[c*x^2]^3 + 24*ArcTan[c*x^2]^2*Log[1
- E^((-2*I)*ArcTan[c*x^2])] - 24*ArcTan[c*x^2]^2*Log[1 + E^((2*I)*ArcTan[c
*x^2])] + (24*I)*ArcTan[c*x^2]*PolyLog[2, E^((-2*I)*ArcTan[c*x^2])] + (24*
I)*ArcTan[c*x^2]*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])] + 12*PolyLog[3, E^((
-2*I)*ArcTan[c*x^2])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c*x^2])]))/16 - (I/
128)*b^3*(Pi^4 - 32*ArcTan[c*x^2]^4 + (64*I)*ArcTan[c*x^2]^3*Log[1 - E^((-
2*I)*ArcTan[c*x^2])] - (64*I)*ArcTan[c*x^2]^3*Log[1 + E^((2*I)*ArcTan[c*x^
2])] - 96*ArcTan[c*x^2]^2*PolyLog[2, E^((-2*I)*ArcTan[c*x^2])] - 96*ArcTan
[c*x^2]^2*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])] + (96*I)*ArcTan[c*x^2]*Poly
Log[3, E^((-2*I)*ArcTan[c*x^2])] - (96*I)*ArcTan[c*x^2]*PolyLog[3, -E^((2*
I)*ArcTan[c*x^2])] + 48*PolyLog[4, E^((-2*I)*ArcTan[c*x^2])] + 48*PolyLog[
4, -E^((2*I)*ArcTan[c*x^2])])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5359, 5357, 5523, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x}dx$$

$$\Big\downarrow \text{5359}$$

$$\frac{1}{2}\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^2}dx^2$$

$$\Big\downarrow \text{5357}$$

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^3-6bc\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2\text{arctanh}\left(1-\frac{2}{icx^2+1}\right)}{c^2x^4+1}dx^2\right)$$

$$\Big\downarrow \text{5523}$$

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^3-6bc\left(\frac{1}{2}\int\frac{\left(a+b\arctan\left(cx^2\right)\right)^2\log\left(2-\frac{2}{icx^2+1}\right)}{c^2x^4+1}dx^2-\frac{1}{2}\int\right.\right.$$

$\downarrow$ 5529

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^3-6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)\left(a+b\arctan\left(cx^2\right)\right)^2}{2c}-\right.\right.\right.$$

$\downarrow$ 5533

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^3-6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)\left(a+b\arctan\left(cx^2\right)\right)^2}{2c}-\right.\right.\right.$$

$\downarrow$ 7164

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^3-6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)\left(a+b\arctan\left(cx^2\right)\right)^2}{2c}-\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^2])^3/x,x]
```

output
```
(2*(a + b*ArcTan[c*x^2])^3*ArcTanh[1 - 2/(1 + I*c*x^2)] - 6*b*c*((((I/2)*(
a + b*ArcTan[c*x^2])^2*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c - I*b*(((I/2)*(a
 + b*ArcTan[c*x^2])*PolyLog[3, 1 - 2/(1 + I*c*x^2)])/c + (b*PolyLog[4, 1 -
 2/(1 + I*c*x^2)])/(4*c)))/2 + (((-1/2*I)*(a + b*ArcTan[c*x^2])^2*PolyLog[
2, -1 + 2/(1 + I*c*x^2)])/c + I*b*(((I/2)*(a + b*ArcTan[c*x^2])*PolyLog[3,
 -1 + 2/(1 + I*c*x^2)])/c + (b*PolyLog[4, -1 + 2/(1 + I*c*x^2)])/(4*c)))/2
))/2
```

Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a + 
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, 
n}, x] && IGtQ[p, 0]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && 
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && 
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v, 
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [F]

$$\int \frac{\left(a+b\arctan\left(c\,x^{2}\right)\right)^{3}}{x}dx$$

input
```
int((a+b*arctan(c*x^2))^3/x,x)
```

output
```
int((a+b*arctan(c*x^2))^3/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\arctan\left(cx^{2}\right)\right)^{3}}{x}\,dx=\int \frac{\left(b\arctan\left(cx^{2}\right)+a\right)^{3}}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c
*x^2) + a^3)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\arctan\left(cx^{2}\right)\right)^{3}}{x}\,dx=\int \frac{\left(a+b\operatorname{atan}\left(cx^{2}\right)\right)^{3}}{x}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**3/x,x)
```

output
```
Integral((a + b*atan(c*x**2))**3/x, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx=\int \frac{(b\arctan(cx^2)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x^2)^3 + 3*b^3*arctan(c*x^2)*
log(c^2*x^4 + 1)^2 + 96*a*b^2*arctan(c*x^2)^2 + 96*a^2*b*arctan(c*x^2))/x,
 x)
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx=\int \frac{(b\arctan(cx^2)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x^2))^3}{x}\,dx$$

input
```
int((a + b*atan(c*x^2))^3/x,x)
```

output
```
int((a + b*atan(c*x^2))^3/x, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx = 3\left(\int \frac{atan(c\,x^2)}{x}dx\right)a^2b + \left(\int \frac{atan(c\,x^2)^3}{x}dx\right)b^3 + 3\left(\int \frac{atan(c\,x^2)^2}{x}dx\right)a\,b^2 + \log(x)\,a^3$$

input
```
int((a+b*atan(c*x^2))^3/x,x)
```

output
```
3*int(atan(c*x**2)/x,x)*a**2*b + int(atan(c*x**2)**3/x,x)*b**3 + 3*int(ata
n(c*x**2)**2/x,x)*a*b**2 + log(x)*a**3
```

3.89 $\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx$

Optimal result 685
Mathematica [A] (verified) 686
Rubi [A] (verified) 686
Maple [F] 689
Fricas [F] 689
Sympy [F] 689
Maxima [F] 690
Giac [F] 690
Mupad [F(-1)] 690
Reduce [F] 691

Optimal result

Integrand size = 16, antiderivative size = 138

$$\begin{aligned}\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx = & -\frac{1}{2}ic\big(a+b\arctan(cx^2)\big)^3-\frac{(a+b\arctan(cx^2))^3}{2x^2}\\ & +\frac{3}{2}bc\big(a+b\arctan(cx^2)\big)^2\log\left(2-\frac{2}{1-icx^2}\right)\\ & -\frac{3}{2}ib^2c\big(a+b\arctan(cx^2)\big)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx^2}\right)\\ & +\frac{3}{4}b^3c\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx^2}\right)\end{aligned}$$

output
```
-1/2*I*c*(a+b*arctan(c*x^2))^3-1/2*(a+b*arctan(c*x^2))^3/x^2+3/2*b*c*(a+b*
arctan(c*x^2))^2*ln(2-2/(1-I*c*x^2))-3/2*I*b^2*c*(a+b*arctan(c*x^2))*polyl
og(2,-1+2/(1-I*c*x^2))+3/4*b^3*c*polylog(3,-1+2/(1-I*c*x^2))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.73

$$\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx$$
$$=\frac{1}{4}\left(-\frac{2a^3}{x^2}-\frac{6a^2b\arctan(cx^2)}{x^2}+12a^2bc\log(x)-3a^2bc\log\left(1+c^2x^4\right)+6ab^2c\left(\arctan(cx^2)\left(\left(-i-\frac{1}{cx^2}\right)\arctan(cx^2)+2\log\left(1-e^{2i\arctan(cx^2)}\right)\right)-i\,\text{PolyLog}\left(2,e^{2i\arctan(cx^2)}\right)\right)+2b^3c\left(-\frac{i\pi^3}{8}+i\arctan(cx^2)^3-\frac{\arctan(cx^2)^3}{cx^2}+3\arctan(cx^2)^2\log\left(1-e^{-2i\arctan(cx^2)}\right)+3i\arctan(cx^2)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx^2)}\right)+\frac{3}{2}\text{PolyLog}\left(3,e^{-2i\arctan(cx^2)}\right)\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^3/x^3,x]
```

output
```
((-2*a^3)/x^2 - (6*a^2*b*ArcTan[c*x^2])/x^2 + 12*a^2*b*c*Log[x] - 3*a^2*b*
c*Log[1 + c^2*x^4] + 6*a*b^2*c*(ArcTan[c*x^2]*((-I - 1/(c*x^2))*ArcTan[c*x
^2] + 2*Log[1 - E^((2*I)*ArcTan[c*x^2])]) - I*PolyLog[2, E^((2*I)*ArcTan[c
*x^2])]) + 2*b^3*c*((-1/8*I)*Pi^3 + I*ArcTan[c*x^2]^3 - ArcTan[c*x^2]^3/(c
*x^2) + 3*ArcTan[c*x^2]^2*Log[1 - E^((-2*I)*ArcTan[c*x^2])] + (3*I)*ArcTan
[c*x^2]*PolyLog[2, E^((-2*I)*ArcTan[c*x^2])] + (3*PolyLog[3, E^((-2*I)*Arc
Tan[c*x^2])])/2))/4
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5363, 5361, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^3}\,dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{2}\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^4}dx^2$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(3bc\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2\left(c^2x^4+1\right)}dx^2-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^2}\right)$$

$$\Big\downarrow \text{5459}$$

$$\frac{1}{2}\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^2}+3bc\left(i\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2\left(cx^2+i\right)}dx^2-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^3}{3b}\right)\right)$$

$$\Big\downarrow \text{5403}$$

$$\frac{1}{2}\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^2}+3bc\left(i\left(2ibc\int \frac{\left(a+b\arctan\left(cx^2\right)\right)\log\left(2-\frac{2}{1-icx^2}\right)}{c^2x^4+1}dx^2-i\log\left(2-\frac{2}{1-icx^2}\right)\right.\right.\right.$$

$$\Big\downarrow \text{5527}$$

$$\frac{1}{2}\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^2}+3bc\left(i\left(2ibc\left(\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-icx^2}-1\right)\left(a+b\arctan\left(cx^2\right)\right)}{2c}-\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2\right.}{c^2x^4}\right.\right.\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$\frac{1}{2}\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^2}+3bc\left(i\left(2ibc\left(\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-icx^2}-1\right)\left(a+b\arctan\left(cx^2\right)\right)}{2c}-\frac{b\,\text{PolyLog}\left(3,\frac{2}{1-ic}\right.}{4c}\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^2])^3/x^3,x]
```

output
```
(-((a + b*ArcTan[c*x^2])^3/x^2) + 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x^2])^3
)/b + I*((-I)*(a + b*ArcTan[c*x^2])^2*Log[2 - 2/(1 - I*c*x^2)] + (2*I)*b*c
*(((I/2)*(a + b*ArcTan[c*x^2])*PolyLog[2, -1 + 2/(1 - I*c*x^2)])/c - (b*Po
lyLog[3, -1 + 2/(1 - I*c*x^2)])/(4*c)))))/2
```

Defintions of rubi rules used

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [F]

$$\int \frac{(a+b\arctan(c\,x^2))^3}{x^3}dx$$

input
```
int((a+b*arctan(c*x^2))^3/x^3,x)
```

output
```
int((a+b*arctan(c*x^2))^3/x^3,x)
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx = \int \frac{(b\arctan(cx^2)+a)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c
*x^2) + a^3)/x^3, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx^2))^3}{x^3}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**3/x**3,x)
```

output
```
Integral((a + b*atan(c*x**2))**3/x**3, x)
```

Maxima [F]

$$\int \frac{(a + b \arctan(cx^2))^3}{x^3} \, dx = \int \frac{(b \arctan(cx^2) + a)^3}{x^3} \, dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="maxima")
```

output
```
-3/4*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*a^2*b - 1/2*a
^3/x^2 - 1/64*(4*b^3*arctan(c*x^2)^3 - 3*b^3*arctan(c*x^2)*log(c^2*x^4 + 1
)^2 - 64*x^2*integrate(-1/32*(12*b^3*c^2*x^4*arctan(c*x^2)*log(c^2*x^4 + 1
) - 28*(b^3*c^2*x^4 + b^3)*arctan(c*x^2)^3 - 12*(8*a*b^2*c^2*x^4 + b^3*c*x
^2 + 8*a*b^2)*arctan(c*x^2)^2 + 3*(b^3*c*x^2 - (b^3*c^2*x^4 + b^3)*arctan(
c*x^2))*log(c^2*x^4 + 1)^2)/(c^2*x^7 + x^3), x))/x^2
```

Giac [F]

$$\int \frac{(a + b \arctan(cx^2))^3}{x^3} \, dx = \int \frac{(b \arctan(cx^2) + a)^3}{x^3} \, dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arctan(cx^2))^3}{x^3} \, dx = \int \frac{(a + b \operatorname{atan}(c x^2))^3}{x^3} \, dx$$

input
```
int((a + b*atan(c*x^2))^3/x^3,x)
```

output
```
int((a + b*atan(c*x^2))^3/x^3, x)
```

Reduce [F]

$$\int \frac{(a + b\arctan(cx^2))^3}{x^3}\,dx$$

$$= \frac{-2atan(c\,x^2)^3\,b^3 - 6atan(c\,x^2)^2\,a\,b^2 - 6atan(c\,x^2)\,a^2b + 24\left(\int \frac{atan(c\,x^2)}{c^2x^5+x}dx\right)a\,b^2c\,x^2 + 12\left(\int \frac{atan(c\,x^2)^2}{c^2x^5+x}dx\right)$$

input

```
int((a+b*atan(c*x^2))^3/x^3,x)
```

output

```
( - 2*atan(c*x**2)**3*b**3 - 6*atan(c*x**2)**2*a*b**2 - 6*atan(c*x**2)*a**
2*b + 24*int(atan(c*x**2)/(c**2*x**5 + x),x)*a*b**2*c*x**2 + 12*int(atan(c
*x**2)**2/(c**2*x**5 + x),x)*b**3*c*x**2 - 3*log( - sqrt(c)*sqrt(2)*x + c*
x**2 + 1)*a**2*b*c*x**2 - 3*log(sqrt(c)*sqrt(2)*x + c*x**2 + 1)*a**2*b*c*x
**2 + 12*log(x)*a**2*b*c*x**2 - 2*a**3)/(4*x**2)
```

3.90 $\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx$

Optimal result 692
Mathematica [A] (verified) 693
Rubi [A] (verified) 693
Maple [C] (warning: unable to verify) 696
Fricas [F] 697
Sympy [F] 697
Maxima [F] 698
Giac [F] 698
Mupad [F(-1)] 698
Reduce [F] 699

Optimal result

Integrand size = 16, antiderivative size = 149

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx = -\frac{3}{4}ibc^2(a+b\arctan(cx^2))^2 - \frac{3bc(a+b\arctan(cx^2))^2}{4x^2} - \frac{1}{4}c^2(a+b\arctan(cx^2))^3 - \frac{(a+b\arctan(cx^2))^3}{4x^4} + \frac{3}{2}b^2c^2(a+b\arctan(cx^2))\log\left(2-\frac{2}{1-icx^2}\right) - \frac{3}{4}ib^3c^2\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx^2}\right)$$

output

```
-3/4*I*b*c^2*(a+b*arctan(c*x^2))^2-3/4*b*c*(a+b*arctan(c*x^2))^2/x^2-1/4*c
^2*(a+b*arctan(c*x^2))^3-1/4*(a+b*arctan(c*x^2))^3/x^4+3/2*b^2*c^2*(a+b*ar
ctan(c*x^2))*ln(2-2/(1-I*c*x^2))-3/4*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x^2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx =$$
$$-\frac{3b^2(a+ac^2x^4+bcx^2(1+icx^2))\arctan(cx^2)^2+b^3(1+c^2x^4)\arctan(cx^2)^3+3b\arctan(cx^2)\Big(a(a+2\ldots}{}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^3/x^5,x]
```

output
```
-1/4*(3*b^2*(a + a*c^2*x^4 + b*c*x^2*(1 + I*c*x^2))*ArcTan[c*x^2]^2 + b^3*
(1 + c^2*x^4)*ArcTan[c*x^2]^3 + 3*b*ArcTan[c*x^2]*(a*(a + 2*b*c*x^2 + a*c^
2*x^4) - 2*b^2*c^2*x^4*Log[1 - E^((2*I)*ArcTan[c*x^2])]) + a*(a*(a + 3*b*c
*x^2) - 6*b^2*c^2*x^4*Log[(c*x^2)/Sqrt[1 + c^2*x^4]]) + (3*I)*b^3*c^2*x^4*
PolyLog[2, E^((2*I)*ArcTan[c*x^2])])/x^4
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5363, 5361, 5453, 5361, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}dx$$
$$\Big\downarrow \text{ 5363}$$
$$\frac{1}{2}\int \frac{(a+b\arctan(cx^2))^3}{x^6}dx^2$$
$$\Big\downarrow \text{ 5361}$$
$$\frac{1}{2}\left(\frac{3}{2}bc\int \frac{(a+b\arctan(cx^2))^2}{x^4(c^2x^4+1)}dx^2-\frac{(a+b\arctan(cx^2))^3}{2x^4}\right)$$

$$\downarrow \text{5453}$$

$$\frac{1}{2}\left(\frac{3}{2}bc\left(\int \frac{(a+b\arctan(cx^2))^2}{x^4}dx^2-c^2\int \frac{(a+b\arctan(cx^2))^2}{c^2x^4+1}dx^2\right)-\frac{(a+b\arctan(cx^2))^3}{2x^4}\right)$$

$$\downarrow \text{5361}$$

$$\frac{1}{2}\left(\frac{3}{2}bc\left(c^2\left(-\int \frac{(a+b\arctan(cx^2))^2}{c^2x^4+1}dx^2\right)+2bc\int \frac{a+b\arctan(cx^2)}{x^2(c^2x^4+1)}dx^2-\frac{(a+b\arctan(cx^2))^2}{x^2}\right)-\frac{(a+b$$

$$\downarrow \text{5419}$$

$$\frac{1}{2}\left(\frac{3}{2}bc\left(2bc\int \frac{a+b\arctan(cx^2)}{x^2(c^2x^4+1)}dx^2-\frac{c(a+b\arctan(cx^2))^3}{3b}-\frac{(a+b\arctan(cx^2))^2}{x^2}\right)-\frac{(a+b\arctan(cx^2)}{2x^4}$$

$$\downarrow \text{5459}$$

$$\frac{1}{2}\left(-\frac{(a+b\arctan(cx^2))^3}{2x^4}+\frac{3}{2}bc\left(2bc\left(i\int \frac{a+b\arctan(cx^2)}{x^2(cx^2+i)}dx^2-\frac{i(a+b\arctan(cx^2))^2}{2b}\right)-\frac{c(a+b\arctan(}{3b}$$

$$\downarrow \text{5403}$$

$$\frac{1}{2}\left(-\frac{(a+b\arctan(cx^2))^3}{2x^4}+\frac{3}{2}bc\left(2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx^2}\right)}{c^2x^4+1}dx^2-i\log\left(2-\frac{2}{1-icx^2}\right)(a+b\arctan(cx$$

$$\downarrow \text{2897}$$

$$\frac{1}{2}\left(-\frac{(a+b\arctan(cx^2))^3}{2x^4}+\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx^2}\right)(a+b\arctan(cx^2))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx$$

input
```
Int[(a + b*ArcTan[c*x^2])^3/x^5,x]
```

output
```
(-1/2*(a + b*ArcTan[c*x^2])^3/x^4 + (3*b*c*(-((a + b*ArcTan[c*x^2])^2/x^2)
 - (c*(a + b*ArcTan[c*x^2])^3)/(3*b) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^
2])^2)/b + I*((-I)*(a + b*ArcTan[c*x^2])*Log[2 - 2/(1 - I*c*x^2)] - (b*Pol
yLog[2, -1 + 2/(1 - I*c*x^2)])/2))))/2)/2
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.78 (sec) , antiderivative size = 465, normalized size of antiderivative = 3.12

method	result
default	$-\frac{a^3}{4x^4}-\frac{\arctan(c\,x^2)^3 b^3}{4x^4}-\frac{3b^3 c\arctan(c\,x^2)^2}{4x^2}-\frac{b^3\arctan(c\,x^2)^3 c^2}{4}-\frac{3b^3c^2\arctan(c\,x^2)\ln(c^2x^4+1)}{4}+3b^3c^2\ln(x)$
parts	$-\frac{a^3}{4x^4}-\frac{\arctan(c\,x^2)^3 b^3}{4x^4}-\frac{3b^3 c\arctan(c\,x^2)^2}{4x^2}-\frac{b^3\arctan(c\,x^2)^3 c^2}{4}-\frac{3b^3c^2\arctan(c\,x^2)\ln(c^2x^4+1)}{4}+3b^3c^2\ln(x)$

input
```
int((a+b*arctan(c*x^2))^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^3/x^4-1/4*arctan(c*x^2)^3*b^3/x^4-3/4*b^3*c*arctan(c*x^2)^2/x^2-1/4
*b^3*arctan(c*x^2)^3*c^2-3/4*b^3*c^2*arctan(c*x^2)*ln(c^2*x^4+1)+3*b^3*c^2
*ln(x)*arctan(c*x^2)+3/16*b^3*c*sum(1/_alpha^2*(2*ln(x-_alpha)*ln(c^2*x^4+
1)-c*(1/c/_alpha^3*ln(x-_alpha)^2+2/_alpha*ln(x-_alpha)*(_alpha^2*ln(1/2*(
x+_alpha)/_alpha)*c+ln((_alpha^3*c-x)/_alpha/(_alpha^2*c-1))-ln((_alpha^3*
c+x)/_alpha/(_alpha^2*c+1)))+2/_alpha*(_alpha^2*dilog(1/2*(x+_alpha)/_alph
a)*c+dilog((_alpha^3*c-x)/_alpha/(_alpha^2*c-1))-dilog((_alpha^3*c+x)/_alp
ha/(_alpha^2*c+1))))),_alpha=RootOf(_Z^4*c^2+1))-3/2*b^3*c*sum(1/_R1^2*(ln
(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1)),_R1=RootOf(_Z^4*c^2+1))-3/4*arctan
(c*x^2)^2*a*b^2/x^4-3/2*a*b^2*c/x^2*arctan(c*x^2)-3/4*a*b^2*arctan(c*x^2)^
2*c^2-3/4*a*b^2*c^2*ln(c^2*x^4+1)+3*a*b^2*c^2*ln(x)-3/4*arctan(c*x^2)*a^2*
b/x^4-3/4*a^2*b*c/x^2-3/4*a^2*b*c^2*arctan(c*x^2)
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx=\int \frac{(b\arctan(cx^2)+a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c
*x^2) + a^3)/x^5, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx=\int \frac{(a+b\operatorname{atan}(cx^2))^3}{x^5}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**3/x**5,x)
```

output
```
Integral((a + b*atan(c*x**2))**3/x**5, x)
```

Maxima [F]

$$\int \frac{(a + b\arctan(cx^2))^3}{x^5}\,dx = \int \frac{(b\arctan(cx^2) + a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="maxima")
```

output
```
-3/4*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*a^2*b + 3/4*((arcta
n(c*x^2)^2 - log(c^2*x^4 + 1) + 4*log(x))*c^2 - 2*(c*arctan(c*x^2) + 1/x^2
)*c*arctan(c*x^2))*a*b^2 - 3/4*a*b^2*arctan(c*x^2)^2/x^4 + 1/128*(128*x^4*
integrate(-1/64*(12*c^2*x^4*arctan(c*x^2)*log(c^2*x^4 + 1) - 12*c*x^2*arct
an(c*x^2)^2 - 56*(c^2*x^4 + 1)*arctan(c*x^2)^3 + 3*(c*x^2 - 2*(c^2*x^4 + 1
)*arctan(c*x^2))*log(c^2*x^4 + 1)^2)/(c^2*x^9 + x^5), x) - 4*arctan(c*x^2)
^3 + 3*arctan(c*x^2)*log(c^2*x^4 + 1)^2)*b^3/x^4 - 1/4*a^3/x^4
```

Giac [F]

$$\int \frac{(a + b\arctan(cx^2))^3}{x^5}\,dx = \int \frac{(b\arctan(cx^2) + a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^3/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx^2))^3}{x^5}\,dx = \int \frac{(a + b\,\mathrm{atan}(c\,x^2))^3}{x^5}\,dx$$

input
```
int((a + b*atan(c*x^2))^3/x^5,x)
```

output
```
int((a + b*atan(c*x^2))^3/x^5, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx$$
$$= \frac{-atan(c\,x^2)^3\,b^3c^2x^4 - atan(c\,x^2)^3\,b^3 - 3atan(c\,x^2)^2\,a\,b^2c^2x^4 - 3atan(c\,x^2)^2\,a\,b^2 - 3atan(c\,x^2)^2\,b^3c\,x^2 - 3}{}$$

input
```
int((a+b*atan(c*x^2))^3/x^5,x)
```

output
```
( - atan(c*x**2)**3*b**3*c**2*x**4 - atan(c*x**2)**3*b**3 - 3*atan(c*x**2)
**2*a*b**2*c**2*x**4 - 3*atan(c*x**2)**2*a*b**2 - 3*atan(c*x**2)**2*b**3*c
*x**2 - 3*atan(c*x**2)*a**2*b*c**2*x**4 - 3*atan(c*x**2)*a**2*b - 6*atan(c
*x**2)*a*b**2*c*x**2 - 3*atan(c*x**2)*b**3*c**2*x**4 - 3*atan(c*x**2)*b**3
 - 12*int(atan(c*x**2)/(c**2*x**9 + x**5),x)*b**3*x**4 - 3*log( - sqrt(c)*
sqrt(2)*x + c*x**2 + 1)*a*b**2*c**2*x**4 - 3*log(sqrt(c)*sqrt(2)*x + c*x**
2 + 1)*a*b**2*c**2*x**4 + 12*log(x)*a*b**2*c**2*x**4 - a**3 - 3*a**2*b*c*x
**2 - 3*b**3*c*x**2)/(4*x**4)
```

3.91 $\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx$

Optimal result 700
Mathematica [N/A] 700
Rubi [N/A] 701
Maple [N/A] 701
Fricas [N/A] 702
Sympy [F(-1)] 702
Maxima [N/A] 702
Giac [N/A] 703
Mupad [N/A] 703
Reduce [N/A] 704

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx = \text{Int}\left((dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3, x\right)$$

output
```
Defer(Int)((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx = \int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 dx$$
$$\Big\downarrow \text{5377}$$
$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x^2])^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^2\right)\right)^3 dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \,dx = \int \left(b\arctan\left(cx^2\right)+a\right)^3 (dx)^m \,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c
*x^2) + a^3)*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \,dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*atan(c*x**2))**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 5.04 (sec) , antiderivative size = 406, normalized size of antiderivative = 22.56

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \,dx = \int \left(b\arctan\left(cx^2\right)+a\right)^3 (dx)^m \,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/32*(4*b^3*d^m*x*x^m*arctan(c*x^2)^3 - 3*
b^3*d^m*x*x^m*arctan(c*x^2)*log(c^2*x^4 + 1)^2 + 32*(m + 1)*integrate(1/32
*(24*b^3*c^2*d^m*x^4*x^m*arctan(c*x^2)*log(c^2*x^4 + 1) + 28*(b^3*d^m*m +
(b^3*c^2*d^m*m + b^3*c^2*d^m)*x^4 + b^3*d^m)*x^m*arctan(c*x^2)^3 - 24*(b^3
*c*d^m*x^2 - 4*a*b^2*d^m*m - 4*(a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^4 - 4*a
*b^2*d^m)*x^m*arctan(c*x^2)^2 + 96*(a^2*b*d^m*m + (a^2*b*c^2*d^m*m + a^2*b
*c^2*d^m)*x^4 + a^2*b*d^m)*x^m*arctan(c*x^2) + 3*(2*b^3*c*d^m*x^2*x^m + (b
^3*d^m*m + (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^4 + b^3*d^m)*x^m*arctan(c*x^2))
*log(c^2*x^4 + 1)^2)/((c^2*m + c^2)*x^4 + m + 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan \left(cx^2\right)\right)^3 \, dx=\int \left(b\arctan \left(cx^2\right)+a\right)^3(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^3*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan \left(cx^2\right)\right)^3 \, dx=\int {\left(d\,x\right)}^m\,{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)}^3 \,d x$$

input
```
int((d*x)^m*(a + b*atan(c*x^2))^3,x)
```

output
```
int((d*x)^m*(a + b*atan(c*x^2))^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 7.39

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 dx$$
$$= \frac{d^m \left(x^m a^3 x + 3\left(\int x^m atan(c\,x^2)\,dx\right) a^2 b m + 3\left(\int x^m atan(c\,x^2)\,dx\right) a^2 b + \left(\int x^m atan(c\,x^2)^3\,dx\right) b^3 m + \left(\int \right.\right.}{m+1}$$

input
```
int((d*x)^m*(a+b*atan(c*x^2))^3,x)
```

output
```
(d**m*(x**m*a**3*x + 3*int(x**m*atan(c*x**2),x)*a**2*b*m + 3*int(x**m*atan
(c*x**2),x)*a**2*b + int(x**m*atan(c*x**2)**3,x)*b**3*m + int(x**m*atan(c*
x**2)**3,x)*b**3 + 3*int(x**m*atan(c*x**2)**2,x)*a*b**2*m + 3*int(x**m*ata
n(c*x**2)**2,x)*a*b**2))/(m + 1)
```

3.92 $\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 dx$

Optimal result 705
Mathematica [N/A] 705
Rubi [N/A] 706
Maple [N/A] 706
Fricas [N/A] 707
Sympy [N/A] 707
Maxima [N/A] 707
Giac [N/A] 708
Mupad [N/A] 708
Reduce [N/A] 709

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 dx = \text{Int}\left((dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2, x\right)$$

output
```
Defer(Int)((d*x)^m*(a+b*arctan(c*x^2))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 dx = \int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 dx$$

$$\Big\downarrow \text{5377}$$

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^2\right)\right)^2 dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x^2))^2,x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx = \int \left(b\arctan\left(cx^2\right)+a\right)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 102.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx = \int (dx)^m \left(a+b\,\mathrm{atan}\left(cx^2\right)\right)^2 \, dx$$

input
```
integrate((d*x)**m*(a+b*atan(c*x**2))**2,x)
```

output
```
Integral((d*x)**m*(a + b*atan(c*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 303, normalized size of antiderivative = 16.83

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx = \int \left(b\arctan\left(cx^2\right)+a\right)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/16*(4*b^2*d^m*x*x^m*arctan(c*x^2)^2 - b^
2*d^m*x*x^m*log(c^2*x^4 + 1)^2 + 16*(m + 1)*integrate(1/16*(8*b^2*c^2*d^m*
x^4*x^m*log(c^2*x^4 + 1) + 12*((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^4 + b^2*d^m
*m + b^2*d^m)*x^m*arctan(c*x^2)^2 + ((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^4 + b
^2*d^m*m + b^2*d^m)*x^m*log(c^2*x^4 + 1)^2 - 16*(b^2*c*d^m*x^2 - 2*(a*b*c^
2*d^m*m + a*b*c^2*d^m)*x^4 - 2*a*b*d^m*m - 2*a*b*d^m)*x^m*arctan(c*x^2))/(
(c^2*m + c^2)*x^4 + m + 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx = \int \left(b\arctan\left(cx^2\right)+a\right)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx = \int (d\,x)^m\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2 \,dx$$

input
```
int((d*x)^m*(a + b*atan(c*x^2))^2,x)
```

output
```
int((d*x)^m*(a + b*atan(c*x^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.89

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 dx$$

$$= \frac{d^m \left(x^m a^2 x + 2\left(\int x^m atan(c\,x^2)\,dx\right) abm + 2\left(\int x^m atan(c\,x^2)\,dx\right) ab + \left(\int x^m atan(c\,x^2)^2\,dx\right) b^2 m + \left(\int x\right.\right.}{m+1}$$

input
```
int((d*x)^m*(a+b*atan(c*x^2))^2,x)
```

output
```
(d**m*(x**m*a**2*x + 2*int(x**m*atan(c*x**2),x)*a*b*m + 2*int(x**m*atan(c*
x**2),x)*a*b + int(x**m*atan(c*x**2)**2,x)*b**2*m + int(x**m*atan(c*x**2)*
*2,x)*b**2))/(m + 1)
```

3.93 $\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)\,dx$

Optimal result 710
Mathematica [A] (verified) 710
Rubi [A] (verified) 711
Maple [F] 712
Fricas [F] 712
Sympy [F] 713
Maxima [F] 713
Giac [F] 713
Mupad [F(-1)] 714
Reduce [F] 714

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)\,dx = \frac{(dx)^{1+m}\left(a+b\arctan\left(cx^2\right)\right)}{d(1+m)} - \frac{2bc(dx)^{3+m}\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{4},\frac{7+m}{4},-c^2x^4\right)}{d^3(1+m)(3+m)}$$

output
```
(d*x)^(1+m)*(a+b*arctan(c*x^2))/d/(1+m)-2*b*c*(d*x)^(3+m)*hypergeom([1, 3/
4+1/4*m],[7/4+1/4*m],-c^2*x^4)/d^3/(1+m)/(3+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)\,dx = -\frac{x(dx)^m\left(-\left((3+m)\left(a+b\arctan\left(cx^2\right)\right)\right)+2bcx^2\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{4},\frac{7+m}{4},-c^2x^4\right)\right)}{(1+m)(3+m)}$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^2]),x]
```

output
```
-((x*(d*x)^m*(-((3 + m)*(a + b*ArcTan[c*x^2])) + 2*b*c*x^2*Hypergeometric2
F1[1, (3 + m)/4, (7 + m)/4, -(c^2*x^4)]))/((1 + m)*(3 + m)))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5373, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right) dx$$

$$\Big\downarrow \text{5373}$$

$$\frac{(dx)^{m+1} \left(a+b\arctan\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc\int \frac{(dx)^{m+2}}{c^2x^4+1}dx}{d^2(m+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{(dx)^{m+1} \left(a+b\arctan\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc(dx)^{m+3}\,\text{Hypergeometric2F1}\left(1,\frac{m+3}{4},\frac{m+7}{4},-c^2x^4\right)}{d^3(m+1)(m+3)}$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x^2]),x]
```

output
```
((d*x)^(1 + m)*(a + b*ArcTan[c*x^2]))/(d*(1 + m)) - (2*b*c*(d*x)^(3 + m)*H
ypergeometric2F1[1, (3 + m)/4, (7 + m)/4, -(c^2*x^4)])/(d^3*(1 + m)*(3 + m
))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5373
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :>
 Simp[(d*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n/(
d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Maple [F]

$$\int (dx)^m \left(a + b \arctan\left(c\,x^2\right)\right) dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x^2)),x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x^2)),x)
```

Fricas [F]

$$\int (dx)^m \left(a + b \arctan\left(cx^2\right)\right)\,dx = \int \left(b \arctan\left(cx^2\right) + a\right)(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x^2) + a)*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \left(a + b \arctan\left(cx^2\right)\right) \, dx = \int (dx)^m \left(a + b \operatorname{atan}\left(cx^2\right)\right) \, dx$$

input
```
integrate((d*x)**m*(a+b*atan(c*x**2)),x)
```

output
```
Integral((d*x)**m*(a + b*atan(c*x**2)), x)
```

Maxima [F]

$$\int (dx)^m \left(a + b \arctan\left(cx^2\right)\right) \, dx = \int \left(b \arctan\left(cx^2\right) + a\right) (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
(d^m*x*x^m*arctan(c*x^2) - 2*(c*d^m*m + c*d^m)*integrate(x^2*x^m/((c^2*m +
 c^2)*x^4 + m + 1), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int (dx)^m \left(a + b \arctan\left(cx^2\right)\right) \, dx = \int \left(b \arctan\left(cx^2\right) + a\right) (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right) \, dx=\int (d\,x)^m \left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right) \, dx$$

input
```
int((d*x)^m*(a + b*atan(c*x^2)),x)
```

output
```
int((d*x)^m*(a + b*atan(c*x^2)), x)
```

Reduce [F]

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right) \, dx = \frac{d^m \left(x^m a x+\left(\int x^m atan(c\,x^2)\,dx\right) b m+\left(\int x^m atan(c\,x^2)\,dx\right) b\right)}{m+1}$$

input
```
int((d*x)^m*(a+b*atan(c*x^2)),x)
```

output
```
(d**m*(x**m*a*x + int(x**m*atan(c*x**2),x)*b*m + int(x**m*atan(c*x**2),x)*
b))/(m + 1)
```

3.94 $\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx$

Optimal result . . . 715
Mathematica [N/A] . . . 715
Rubi [N/A] . . . 716
Maple [N/A] . . . 716
Fricas [N/A] . . . 717
Sympy [F(-1)] . . . 717
Maxima [N/A] . . . 717
Giac [N/A] . . . 718
Mupad [N/A] . . . 718
Reduce [N/A] . . . 718

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx = \text{Int}\left(\frac{(dx)^m}{a+b\arctan(cx^2)},x\right)$$

output
```
Defer(Int)((d*x)^m/(a+b*arctan(c*x^2)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx = \int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^2]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^2]), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx$$

$$\Big\downarrow \text{5377}$$

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTan[c*x^2]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\arctan(c\,x^2)}dx$$

input
```
int((d*x)^m/(a+b*arctan(c*x^2)),x)
```

output
```
int((d*x)^m/(a+b*arctan(c*x^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a + b\arctan(cx^2)} \, dx = \int \frac{(dx)^m}{b\arctan(cx^2) + a} \, dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arctan(c*x^2) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a + b\arctan(cx^2)} \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*atan(c*x**2)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a + b\arctan(cx^2)} \, dx = \int \frac{(dx)^m}{b\arctan(cx^2) + a} \, dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arctan(c*x^2) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx = \int \frac{(dx)^m}{b\arctan(cx^2)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctan(c*x^2) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx = \int \frac{(d\,x)^m}{a+b\,\mathrm{atan}\left(c\,x^2\right)}\,dx$$

input
```
int((d*x)^m/(a + b*atan(c*x^2)),x)
```

output
```
int((d*x)^m/(a + b*atan(c*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx = d^m\left(\int \frac{x^m}{atan\left(c\,x^2\right)b+a}dx\right)$$

input
```
int((d*x)^m/(a+b*atan(c*x^2)),x)
```

output
```
d**m*int(x**m/(atan(c*x**2)*b + a),x)
```

3.95 $\int \frac{(dx)^m}{\left(a+b\arctan\left(cx^2\right)\right)^2}\,dx$

Optimal result 720
Mathematica [N/A] 720
Rubi [N/A] 721
Maple [N/A] 721
Fricas [N/A] 722
Sympy [F(-1)] 722
Maxima [N/A] 722
Giac [N/A] 723
Mupad [N/A] 723
Reduce [N/A] 724

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{\left(a+b\arctan\left(cx^2\right)\right)^2}\,dx=\text{Int}\left(\frac{(dx)^m}{\left(a+b\arctan\left(cx^2\right)\right)^2},x\right)$$

output
```
Defer(Int)((d*x)^m/(a+b*arctan(c*x^2))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{\left(a+b\arctan\left(cx^2\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\arctan\left(cx^2\right)\right)^2}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^2])^2,x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^2])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}dx$$
$$\Big\downarrow \text{5377}$$
$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTan[c*x^2])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{(a+b\arctan(c\,x^2))^2}dx$$

input
```
int((d*x)^m/(a+b*arctan(c*x^2))^2,x)
```

output
```
int((d*x)^m/(a+b*arctan(c*x^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^2)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*atan(c*x**2))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 6.89

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^2)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
-1/2*((c^2*d^m*x^4 + d^m)*x^m - 2*(b^2*c*x*arctan(c*x^2) + a*b*c*x)*integr
ate(1/2*((c^2*d^m*m + 3*c^2*d^m)*x^4 + d^m*m - d^m)*x^m/(b^2*c*x^2*arctan(
c*x^2) + a*b*c*x^2), x))/(b^2*c*x*arctan(c*x^2) + a*b*c*x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a + b\arctan(cx^2))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^2) + a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctan(c*x^2) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a + b\arctan(cx^2))^2}\,dx = \int \frac{(d\,x)^m}{(a + b\,\mathrm{atan}(c\,x^2))^2}\,dx$$

input
```
int((d*x)^m/(a + b*atan(c*x^2))^2,x)
```

output
```
int((d*x)^m/(a + b*atan(c*x^2))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = d^m\left(\int \frac{x^m}{atan(c\,x^2)^2\,b^2+2atan(c\,x^2)\,ab+a^2}dx\right)$$

input
```
int((d*x)^m/(a+b*atan(c*x^2))^2,x)
```

output
```
d**m*int(x**m/(atan(c*x**2)**2*b**2 + 2*atan(c*x**2)*a*b + a**2),x)
```

3.96 $\int x^{11}(a+b\arctan(cx^3))\,dx$

Optimal result 725
Mathematica [A] (verified) 725
Rubi [A] (verified) 726
Maple [A] (verified) 727
Fricas [A] (verification not implemented) 728
Sympy [A] (verification not implemented) 728
Maxima [A] (verification not implemented) 729
Giac [A] (verification not implemented) 729
Mupad [B] (verification not implemented) 730
Reduce [B] (verification not implemented) 730

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{bx^3}{12c^3}-\frac{bx^9}{36c}-\frac{b\arctan\left(cx^3\right)}{12c^4}+\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)$$

output
```
1/12*b*x^3/c^3-1/36*b*x^9/c-1/12*b*arctan(c*x^3)/c^4+1/12*x^12*(a+b*arctan
(c*x^3))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{bx^3}{12c^3}-\frac{bx^9}{36c}+\frac{ax^{12}}{12}-\frac{b\arctan\left(cx^3\right)}{12c^4}+\frac{1}{12}bx^{12}\arctan\left(cx^3\right)$$

input
```
Integrate[x^11*(a + b*ArcTan[c*x^3]),x]
```

output
```
(b*x^3)/(12*c^3) - (b*x^9)/(36*c) + (a*x^12)/12 - (b*ArcTan[c*x^3])/(12*c^
4) + (b*x^12*ArcTan[c*x^3])/12
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 807, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{4}bc\int \frac{x^{14}}{c^2x^6+1}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{12}bc\int \frac{x^{12}}{c^2x^6+1}dx^3$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{12}bc\int \left(\frac{x^6}{c^2}+\frac{1}{c^4\left(c^2x^6+1\right)}-\frac{1}{c^4}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{12}bc\left(\frac{\arctan\left(cx^3\right)}{c^5}-\frac{x^3}{c^4}+\frac{x^9}{3c^2}\right)$$

input
```
Int[x^11*(a + b*ArcTan[c*x^3]),x]
```

output
```
(x^12*(a + b*ArcTan[c*x^3]))/12 - (b*c*(-(x^3/c^4) + x^9/(3*c^2) + ArcTan[
c*x^3]/c^5))/12
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{a\,x^{12}}{12}+\frac{b\,x^{12}\arctan\left(c\,x^{3}\right)}{12}-\frac{b\,x^{9}}{36c}+\frac{b\,x^{3}}{12c^{3}}-\frac{b\arctan\left(c\,x^{3}\right)}{12c^{4}}$	50
parts	$\frac{a\,x^{12}}{12}+\frac{b\,x^{12}\arctan\left(c\,x^{3}\right)}{12}-\frac{b\,x^{9}}{36c}+\frac{b\,x^{3}}{12c^{3}}-\frac{b\arctan\left(c\,x^{3}\right)}{12c^{4}}$	50
parallelrisch	$\frac{3b\arctan\left(c\,x^{3}\right)x^{12}c^{4}+3a\,c^{4}x^{12}-b\,c^{3}x^{9}+3bc\,x^{3}-3b\arctan\left(c\,x^{3}\right)}{36c^{4}}$	56
risch	$-\frac{i x^{12} b\ln\left(ic\,x^{3}+1\right)}{24}+\frac{a\,x^{12}}{12}+\frac{i x^{12} b\ln\left(-ic\,x^{3}+1\right)}{24}-\frac{b\,x^{9}}{36c}+\frac{b\,x^{3}}{12c^{3}}-\frac{b\arctan\left(c\,x^{3}\right)}{12c^{4}}$	72
orering	$\frac{\left(10c^{4}x^{12}-11c^{2}x^{6}-21\right)\left(a+b\arctan\left(c\,x^{3}\right)\right)}{54c^{4}}-\frac{\left(c^{2}x^{6}-3\right)\left(c^{2}x^{6}+1\right)\left(11x^{10}\left(a+b\arctan\left(c\,x^{3}\right)\right)+\frac{3x^{13}bc}{c^{2}x^{6}+1}\right)}{108x^{10}c^{4}}$	95

input
```
int(x^11*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/12*a*x^12+1/12*b*x^12*arctan(c*x^3)-1/36*b*x^9/c+1/12*b*x^3/c^3-1/12*b*a
rctan(c*x^3)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{3\,ac^4x^{12}-bc^3x^9+3\,bcx^3+3\left(bc^4x^{12}-b\right)\arctan\left(cx^3\right)}{36\,c^4}$$

input
```
integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/36*(3*a*c^4*x^12 - b*c^3*x^9 + 3*b*c*x^3 + 3*(b*c^4*x^12 - b)*arctan(c*x
^3))/c^4
```

Sympy [A] (verification not implemented)

Time = 118.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)\,dx=\begin{cases}\frac{ax^{12}}{12}+\frac{bx^{12}\operatorname{atan}\left(cx^3\right)}{12}-\frac{bx^9}{36c}+\frac{bx^3}{12c^3}-\frac{b\operatorname{atan}\left(cx^3\right)}{12c^4} & \text{for } c\neq 0\\ \frac{ax^{12}}{12} & \text{otherwise}\end{cases}$$

input
```
integrate(x**11*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**12/12 + b*x**12*atan(c*x**3)/12 - b*x**9/(36*c) + b*x**3/(
12*c**3) - b*atan(c*x**3)/(12*c**4), Ne(c, 0)), (a*x**12/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^{11}(a+b\arctan(cx^3))\,dx = \frac{1}{12}ax^{12} + \frac{1}{36}\left(3x^{12}\arctan(cx^3) - c\left(\frac{c^2x^9-3x^3}{c^4} + \frac{3\arctan(cx^3)}{c^5}\right)\right)b$$

input
```
integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/12*a*x^12 + 1/36*(3*x^12*arctan(c*x^3) - c*((c^2*x^9 - 3*x^3)/c^4 + 3*ar
ctan(c*x^3)/c^5))*b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int x^{11}(a+b\arctan(cx^3))\,dx = \frac{3\,acx^{12} + \left(3\,cx^{12}\arctan(cx^3) - \frac{3\arctan(cx^3)}{c^3} - \frac{c^9x^9-3\,c^7x^3}{c^9}\right)b}{36\,c}$$

input
```
integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
1/36*(3*a*c*x^12 + (3*c*x^12*arctan(c*x^3) - 3*arctan(c*x^3)/c^3 - (c^9*x^
9 - 3*c^7*x^3)/c^9)*b)/c
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{a\,x^{12}}{12}+\frac{b\,x^3}{12\,c^3}-\frac{b\,x^9}{36\,c}-\frac{b\,\mathrm{atan}(c\,x^3)}{12\,c^4}+\frac{b\,x^{12}\,\mathrm{atan}(c\,x^3)}{12}$$

input
```
int(x^11*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^12)/12 + (b*x^3)/(12*c^3) - (b*x^9)/(36*c) - (b*atan(c*x^3))/(12*c^4)
 + (b*x^12*atan(c*x^3))/12
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{3atan(c\,x^3)\,b\,c^4x^{12}-3atan(c\,x^3)\,b+3a\,c^4x^{12}-b\,c^3x^9+3bc\,x^3}{36c^4}$$

input
```
int(x^11*(a+b*atan(c*x^3)),x)
```

output
```
(3*atan(c*x**3)*b*c**4*x**12 - 3*atan(c*x**3)*b + 3*a*c**4*x**12 - b*c**3*
x**9 + 3*b*c*x**3)/(36*c**4)
```

3.97 $\int x^8(a+b\arctan(cx^3))\,dx$

Optimal result 731
Mathematica [A] (verified) 731
Rubi [A] (verified) 732
Maple [A] (verified) 733
Fricas [A] (verification not implemented) 734
Sympy [B] (verification not implemented) 734
Maxima [A] (verification not implemented) 735
Giac [A] (verification not implemented) 735
Mupad [B] (verification not implemented) 735
Reduce [B] (verification not implemented) 736

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int x^8(a+b\arctan(cx^3))\,dx=-\frac{bx^6}{18c}+\frac{1}{9}x^9(a+b\arctan(cx^3))+\frac{b\log(1+c^2x^6)}{18c^3}$$

output
```
-1/18*b*x^6/c+1/9*x^9*(a+b*arctan(c*x^3))+1/18*b*ln(c^2*x^6+1)/c^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int x^8(a+b\arctan(cx^3))\,dx=-\frac{bx^6}{18c}+\frac{ax^9}{9}+\frac{1}{9}bx^9\arctan(cx^3)+\frac{b\log(1+c^2x^6)}{18c^3}$$

input
```
Integrate[x^8*(a + b*ArcTan[c*x^3]),x]
```

output
```
-1/18*(b*x^6)/c + (a*x^9)/9 + (b*x^9*ArcTan[c*x^3])/9 + (b*Log[1 + c^2*x^6
])/(18*c^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+b \arctan \left(c x^3\right)\right) \, dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{9} x^9 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{3} b c \int \frac{x^{11}}{c^2 x^6+1}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{9} x^9 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{18} b c \int \frac{x^6}{c^2 x^6+1}dx^6$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{9} x^9 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{18} b c \int \left(\frac{1}{c^2}-\frac{1}{c^2 \left(c^2 x^6+1\right)}\right)dx^6$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9} x^9 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{18} b c \left(\frac{x^6}{c^2}-\frac{\log \left(c^2 x^6+1\right)}{c^4}\right)$$

input
```
Int[x^8*(a + b*ArcTan[c*x^3]),x]
```

output
```
(x^9*(a + b*ArcTan[c*x^3]))/9 - (b*c*(x^6/c^2 - Log[1 + c^2*x^6]/c^4))/18
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{a\,x^{9}}{9}+\frac{b\,x^{9}\arctan\left(c\,x^{3}\right)}{9}-\frac{b\,x^{6}}{18c}+\frac{b\ln\left(c^{2}x^{6}+1\right)}{18c^{3}}$	45
parts	$\frac{a\,x^{9}}{9}+\frac{b\,x^{9}\arctan\left(c\,x^{3}\right)}{9}-\frac{b\,x^{6}}{18c}+\frac{b\ln\left(c^{2}x^{6}+1\right)}{18c^{3}}$	45
parallelrisch	$\frac{2x^{9}\arctan\left(c\,x^{3}\right)b\,c^{3}+2a\,c^{3}x^{9}-c^{2}b\,x^{6}+b\ln\left(c^{2}x^{6}+1\right)}{18c^{3}}$	52
risch	$-\frac{i x^{9}b\ln\left(i c\,x^{3}+1\right)}{18}+\frac{i x^{9}b\ln\left(-i c\,x^{3}+1\right)}{18}+\frac{a\,x^{9}}{9}-\frac{b\,x^{6}}{18c}+\frac{b\ln\left(-c^{2}x^{6}-1\right)}{18c^{3}}$	68

input
```
int(x^8*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/9*a*x^9+1/9*b*x^9*arctan(c*x^3)-1/18*b*x^6/c+1/18*b*ln(c^2*x^6+1)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int x^8 (a + b \arctan(cx^3)) \, dx = \frac{2 bc^3 x^9 \arctan(cx^3) + 2 ac^3 x^9 - bc^2 x^6 + b \log(c^2 x^6 + 1)}{18 c^3}$$

input
```
integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/18*(2*b*c^3*x^9*arctan(c*x^3) + 2*a*c^3*x^9 - b*c^2*x^6 + b*log(c^2*x^6
+ 1))/c^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(39) = 78.

Time = 67.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.49

$$\int x^8 (a + b \arctan(cx^3)) \, dx$$
$$= \begin{cases} \frac{ax^9}{9} + \frac{bx^9 \operatorname{atan}(cx^3)}{9} - \frac{bx^6}{18c} - \frac{b\sqrt{-\frac{1}{c^2}} \operatorname{atan}(cx^3)}{9c^2} + \frac{b \log\left(x - \sqrt[6]{-\frac{1}{c^2}}\right)}{9c^3} + \frac{b \log\left(4x^2 + 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{9c^3} & \text{for } c \neq 0 \\ \frac{ax^9}{9} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**9/9 + b*x**9*atan(c*x**3)/9 - b*x**6/(18*c) - b*sqrt(-1/c*
*2)*atan(c*x**3)/(9*c**2) + b*log(x - (-1/c**2)**(1/6))/(9*c**3) + b*log(4
*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(9*c**3), Ne(c, 0)), (a
*x**9/9, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int x^8 \left(a + b \arctan \left(cx^3\right)\right) \, dx = \frac{1}{9} a x^9 + \frac{1}{18} \left(2 x^9 \arctan \left(cx^3\right) - \left(\frac{x^6}{c^2} - \frac{\log \left(c^2 x^6 + 1\right)}{c^4}\right) c\right) b$$

input
```
integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/9*a*x^9 + 1/18*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^4)*c
)*b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int x^8 \left(a + b \arctan \left(cx^3\right)\right) \, dx = \frac{2 \, a c x^9 + \left(2 \, c x^9 \arctan \left(cx^3\right) - x^6 + \frac{\log \left(c^2 x^6 + 1\right)}{c^2}\right) b}{18 \, c}$$

input
```
integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
1/18*(2*a*c*x^9 + (2*c*x^9*arctan(c*x^3) - x^6 + log(c^2*x^6 + 1)/c^2)*b)/
c
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int x^8 \left(a + b \arctan \left(cx^3\right)\right) \, dx = \frac{a \, x^9}{9} + \frac{b \, \ln \left(c^2 \, x^6 + 1\right)}{18 \, c^3} - \frac{b \, x^6}{18 \, c} + \frac{b \, x^9 \, \mathrm{atan}(c \, x^3)}{9}$$

input
```
int(x^8*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^9)/9 + (b*log(c^2*x^6 + 1))/(18*c^3) - (b*x^6)/(18*c) + (b*x^9*atan(c
*x^3))/9
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.91

$$\int x^8 \left(a + b \arctan\left(cx^3\right)\right) \, dx$$

$$= \frac{2 atan(c\,x^3)\, b\, c^3 x^9 + \log\left(c^{\frac{2}{3}} x^2 - c^{\frac{1}{3}} \sqrt{3}\, x + 1\right) b + \log\left(c^{\frac{2}{3}} x^2 + c^{\frac{1}{3}} \sqrt{3}\, x + 1\right) b + \log\left(c^{\frac{2}{3}} x^2 + 1\right) b + 2a\, c^3 x}{18c^3}$$

input
```
int(x^8*(a+b*atan(c*x^3)),x)
```

output
```
(2*atan(c*x**3)*b*c**3*x**9 + log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x + 1)*
b + log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x + 1)*b + log(c**(2/3)*x**2 + 1)
*b + 2*a*c**3*x**9 - b*c**2*x**6)/(18*c**3)
```

3.98 $\int x^5(a + b\arctan(cx^3))\, dx$

Optimal result . . . 737
Mathematica [A] (verified) . . . 737
Rubi [A] (verified) . . . 738
Maple [A] (verified) . . . 739
Fricas [A] (verification not implemented) . . . 740
Sympy [A] (verification not implemented) . . . 740
Maxima [A] (verification not implemented) . . . 741
Giac [A] (verification not implemented) . . . 741
Mupad [B] (verification not implemented) . . . 741
Reduce [B] (verification not implemented) . . . 742

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int x^5(a + b\arctan(cx^3))\, dx = -\frac{bx^3}{6c} + \frac{b\arctan(cx^3)}{6c^2} + \frac{1}{6}x^6(a + b\arctan(cx^3))$$

output
```
-1/6*b*x^3/c+1/6*b*arctan(c*x^3)/c^2+1/6*x^6*(a+b*arctan(c*x^3))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^5(a + b\arctan(cx^3))\, dx = -\frac{bx^3}{6c} + \frac{ax^6}{6} + \frac{b\arctan(cx^3)}{6c^2} + \frac{1}{6}bx^6\arctan(cx^3)$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x^3]),x]
```

output
```
-1/6*(b*x^3)/c + (a*x^6)/6 + (b*ArcTan[c*x^3])/(6*c^2) + (b*x^6*ArcTan[c*x
^3])/6
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 807, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b \arctan \left(c x^3\right)\right) \, dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{6} x^6 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{2} b c \int \frac{x^8}{c^2 x^6+1}dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{6} x^6 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{6} b c \int \frac{x^6}{c^2 x^6+1}dx^3$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{6} x^6 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{6} b c \left(\frac{x^3}{c^2}-\frac{\int \frac{1}{c^2 x^6+1}dx^3}{c^2}\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{6} x^6 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{6} b c \left(\frac{x^3}{c^2}-\frac{\arctan \left(c x^3\right)}{c^3}\right)$$

input
```
Int[x^5*(a + b*ArcTan[c*x^3]),x]
```

output
```
(x^6*(a + b*ArcTan[c*x^3]))/6 - (b*c*(x^3/c^2 - ArcTan[c*x^3]/c^3))/6
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{x^{6} a}{6}+\frac{b\,x^{6} \arctan \left(c\,x^{3}\right)}{6}-\frac{b\,x^{3}}{6 c}+\frac{b \arctan \left(c\,x^{3}\right)}{6 c^{2}}$	41
parts	$\frac{x^{6} a}{6}+\frac{b\,x^{6} \arctan \left(c\,x^{3}\right)}{6}-\frac{b\,x^{3}}{6 c}+\frac{b \arctan \left(c\,x^{3}\right)}{6 c^{2}}$	41
parallelrisch	$\frac{\arctan \left(c\,x^{3}\right) b\,c^{2} x^{6}+a\,c^{2} x^{6}-b c\,x^{3}+b \arctan \left(c\,x^{3}\right)}{6 c^{2}}$	44
risch	$-\frac{i x^{6} b \ln \left(i c\,x^{3}+1\right)}{12}+\frac{i x^{6} b \ln \left(-i c\,x^{3}+1\right)}{12}+\frac{x^{6} a}{6}-\frac{b\,x^{3}}{6 c}+\frac{b \arctan \left(c\,x^{3}\right)}{6 c^{2}}+\frac{b^{2}}{24 a\,c^{2}}$	74
orering	$\frac{4 \left(c^{2} x^{6}+1\right) \left(a +b \arctan \left(c\,x^{3}\right)\right)}{9 c^{2}}-\frac{\left(c^{2} x^{6}+1\right) \left(5 x^{4} \left(a +b \arctan \left(c\,x^{3}\right)\right)+\frac{3 x^{7} b c}{c^{2} x^{6}+1}\right)}{18 c^{2} x^{4}}$	77

input
```
int(x^5*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*a+1/6*b*x^6*arctan(c*x^3)-1/6*b*x^3/c+1/6*b*arctan(c*x^3)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{ac^2x^6-bcx^3+\left(bc^2x^6+b\right)\arctan\left(cx^3\right)}{6\,c^2}$$

input
```
integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/6*(a*c^2*x^6 - b*c*x^3 + (b*c^2*x^6 + b)*arctan(c*x^3))/c^2
```

Sympy [A] (verification not implemented)

Time = 42.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)\,dx=\begin{cases}\frac{ax^6}{6}+\frac{bx^6\operatorname{atan}\left(cx^3\right)}{6}-\frac{bx^3}{6c}+\frac{b\operatorname{atan}\left(cx^3\right)}{6c^2} & \text{for } c\neq 0\\ \frac{ax^6}{6} & \text{otherwise}\end{cases}$$

input
```
integrate(x**5*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**6/6 + b*x**6*atan(c*x**3)/6 - b*x**3/(6*c) + b*atan(c*x**3
)/(6*c**2), Ne(c, 0)), (a*x**6/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^5 \left(a + b \arctan\left(cx^3\right)\right) \, dx = \frac{1}{6} a x^6 + \frac{1}{6} \left(x^6 \arctan\left(cx^3\right) - c\left(\frac{x^3}{c^2} - \frac{\arctan\left(cx^3\right)}{c^3}\right)\right) b$$

input
```
integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/6*(x^6*arctan(c*x^3) - c*(x^3/c^2 - arctan(c*x^3)/c^3))*b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^5 \left(a + b \arctan\left(cx^3\right)\right) \, dx = \frac{acx^6 + \frac{\left(c^2 x^6 \arctan\left(cx^3\right) - cx^3 + \arctan\left(cx^3\right)\right) b}{c}}{6 c}$$

input
```
integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
1/6*(a*c*x^6 + (c^2*x^6*arctan(c*x^3) - c*x^3 + arctan(c*x^3))*b/c)/c
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^5 \left(a + b \arctan\left(cx^3\right)\right) \, dx = \frac{a\, x^6}{6} - \frac{b\, x^3}{6\, c} + \frac{b\, \mathrm{atan}(c\, x^3)}{6\, c^2} + \frac{b\, x^6\, \mathrm{atan}(c\, x^3)}{6}$$

input
```
int(x^5*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^6)/6 - (b*x^3)/(6*c) + (b*atan(c*x^3))/(6*c^2) + (b*x^6*atan(c*x^3))/
6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^5 \left(a + b \arctan \left(c x^3\right)\right) \, dx = \frac{atan(c\, x^3)\, b\, c^2 x^6 + atan(c\, x^3)\, b + a\, c^2 x^6 - bc\, x^3}{6c^2}$$

input
```
int(x^5*(a+b*atan(c*x^3)),x)
```

output
```
(atan(c*x**3)*b*c**2*x**6 + atan(c*x**3)*b + a*c**2*x**6 - b*c*x**3)/(6*c*
*2)
```

3.99 $\int x^2(a+b\arctan(cx^3))\,dx$

Optimal result 743
Mathematica [A] (verified) 743
Rubi [A] (verified) 744
Maple [A] (verified) 745
Fricas [A] (verification not implemented) 745
Sympy [B] (verification not implemented) 746
Maxima [A] (verification not implemented) 746
Giac [A] (verification not implemented) 747
Mupad [B] (verification not implemented) 747
Reduce [B] (verification not implemented) 747

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int x^2(a+b\arctan(cx^3))\,dx=\frac{ax^3}{3}+\frac{1}{3}bx^3\arctan(cx^3)-\frac{b\log(1+c^2x^6)}{6c}$$

output
```
1/3*a*x^3+1/3*b*x^3*arctan(c*x^3)-1/6*b*ln(c^2*x^6+1)/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int x^2(a+b\arctan(cx^3))\,dx=\frac{ax^3}{3}+\frac{1}{3}bx^3\arctan(cx^3)-\frac{b\log(1+c^2x^6)}{6c}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x^3]),x]
```

output
```
(a*x^3)/3 + (b*x^3*ArcTan[c*x^3])/3 - (b*Log[1 + c^2*x^6])/(6*c)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5361, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)-bc\int\frac{x^5}{c^2x^6+1}dx$$

$$\Big\downarrow \text{792}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)-\frac{b\log\left(c^2x^6+1\right)}{6c}$$

input
```
Int[x^2*(a + b*ArcTan[c*x^3]),x]
```

output
```
(x^3*(a + b*ArcTan[c*x^3]))/3 - (b*Log[1 + c^2*x^6])/(6*c)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
parts	$\frac{a\,x^{3}}{3}+\frac{b\,x^{3}\arctan\left(c\,x^{3}\right)}{3}-\frac{b\ln\left(c^{2}x^{6}+1\right)}{6c}$	36
derivativedivides	$\frac{ac\,x^{3}+b\left(c\,x^{3}\arctan\left(c\,x^{3}\right)-\frac{\ln\left(c^{2}x^{6}+1\right)}{2}\right)}{3c}$	39
default	$\frac{ac\,x^{3}+b\left(c\,x^{3}\arctan\left(c\,x^{3}\right)-\frac{\ln\left(c^{2}x^{6}+1\right)}{2}\right)}{3c}$	39
parallelrisch	$-\frac{-2x^{3}\arctan\left(c\,x^{3}\right)bc-2ac\,x^{3}+b\ln\left(c^{2}x^{6}+1\right)}{6c}$	39
risch	$-\frac{i x^{3} b\ln\left(ic\,x^{3}+1\right)}{6}+\frac{ib\,x^{3}\ln\left(-ic\,x^{3}+1\right)}{6}+\frac{a\,x^{3}}{3}-\frac{b\ln\left(-c^{2}x^{6}-1\right)}{6c}$	59

input
```
int(x^2*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+1/3*b*x^3*arctan(c*x^3)-1/6*b*ln(c^2*x^6+1)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int x^{2}\left(a+b\arctan\left(cx^{3}\right)\right)\,dx=\frac{2\,bcx^{3}\arctan\left(cx^{3}\right)+2\,acx^{3}-b\log\left(c^{2}x^{6}+1\right)}{6\,c}$$

input
```
integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/6*(2*b*c*x^3*arctan(c*x^3) + 2*a*c*x^3 - b*log(c^2*x^6 + 1))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(34) = 68$.

Time = 18.53 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.49

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)\,dx$$
$$=\begin{cases}\frac{ax^3}{3}+\frac{bx^3\operatorname{atan}\left(cx^3\right)}{3}+\frac{b\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^3\right)}{3}-\frac{b\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{3c}-\frac{b\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{3c} & \text{for } c\neq 0\\ \frac{ax^3}{3} & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**3/3 + b*x**3*atan(c*x**3)/3 + b*sqrt(-1/c**2)*atan(c*x**3)
/3 - b*log(x - (-1/c**2)**(1/6))/(3*c) - b*log(4*x**2 + 4*x*(-1/c**2)**(1/
6) + 4*(-1/c**2)**(1/3))/(3*c), Ne(c, 0)), (a*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{3}\,ax^3+\frac{\left(2\,cx^3\arctan\left(cx^3\right)-\log\left(c^2x^6+1\right)\right)b}{6\,c}$$

input
```
integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/6*(2*c*x^3*arctan(c*x^3) - log(c^2*x^6 + 1))*b/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{2\,acx^3+\left(2\,cx^3\arctan\left(cx^3\right)-\log\left(c^2x^6+1\right)\right)b}{6\,c}$$

input
```
integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
1/6*(2*a*c*x^3 + (2*c*x^3*arctan(c*x^3) - log(c^2*x^6 + 1))*b)/c
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{a\,x^3}{3}-\frac{b\,\ln\left(c^2\,x^6+1\right)}{6\,c}+\frac{b\,x^3\,\mathrm{atan}(c\,x^3)}{3}$$

input
```
int(x^2*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^3)/3 - (b*log(c^2*x^6 + 1))/(6*c) + (b*x^3*atan(c*x^3))/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.95

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)\,dx$$
$$=\frac{2atan(c\,x^3)\,bc\,x^3-\log\left(c^{\frac{2}{3}}x^2-c^{\frac{1}{3}}\sqrt{3}\,x+1\right)b-\log\left(c^{\frac{2}{3}}x^2+c^{\frac{1}{3}}\sqrt{3}\,x+1\right)b-\log\left(c^{\frac{2}{3}}x^2+1\right)b+2ac\,x^3}{6c}$$

input
```
int(x^2*(a+b*atan(c*x^3)),x)
```

output
```
(2*atan(c*x**3)*b*c*x**3 - log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x + 1)*b -
 log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x + 1)*b - log(c**(2/3)*x**2 + 1)*b
+ 2*a*c*x**3)/(6*c)
```

3.100 $\int \frac{a+b\arctan(cx^3)}{x}\,dx$

Optimal result 749
Mathematica [A] (verified) 749
Rubi [A] (verified) 750
Maple [C] (verified) 751
Fricas [F] 752
Sympy [F] 752
Maxima [F] 752
Giac [F] 753
Mupad [B] (verification not implemented) 753
Reduce [F] 753

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{a+b\arctan(cx^3)}{x}\,dx = a\log(x) + \frac{1}{6}ib\,\text{PolyLog}\left(2,-icx^3\right) - \frac{1}{6}ib\,\text{PolyLog}\left(2,icx^3\right)$$

output
```
a*ln(x)+1/6*I*b*polylog(2,-I*c*x^3)-1/6*I*b*polylog(2,I*c*x^3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx^3)}{x}\,dx = a\log(x) + \frac{1}{6}ib\,\text{PolyLog}\left(2,-icx^3\right) - \frac{1}{6}ib\,\text{PolyLog}\left(2,icx^3\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x,x]
```

output
```
a*Log[x] + (I/6)*b*PolyLog[2, (-I)*c*x^3] - (I/6)*b*PolyLog[2, I*c*x^3]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5359, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx^3)}{x}\,dx$$

$$\Big\downarrow \; 5359$$

$$\frac{1}{3}\int \frac{a+b\arctan(cx^3)}{x^3}dx^3$$

$$\Big\downarrow \; 5355$$

$$\frac{1}{3}\left(\frac{1}{2}ib\int \frac{\log(1-icx^3)}{x^3}dx^3-\frac{1}{2}ib\int \frac{\log(icx^3+1)}{x^3}dx^3+a\log(x^3)\right)$$

$$\Big\downarrow \; 2838$$

$$\frac{1}{3}\left(a\log(x^3)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx^3)-\frac{1}{2}ib\,\text{PolyLog}(2,icx^3)\right)$$

input
```
Int[(a + b*ArcTan[c*x^3])/x,x]
```

output
```
(a*Log[x^3] + (I/2)*b*PolyLog[2, (-I)*c*x^3] - (I/2)*b*PolyLog[2, I*c*x^3]
)/3
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

method	result
default	$a \ln(x) + b \ln(x) \arctan(c\,x^3) - \frac{b \left(\sum_{\_R1 = \mathrm{RootOf}\left(c^2 \_Z^6+1\right)} \frac{\ln(x) \ln\left(\frac{\_R1 - x}{\_R1}\right) + \mathrm{dilog}\left(\frac{\_R1 - x}{\_R1}\right)}{\_R1^3}\right)}{2c}$
parts	$a \ln(x) + b \ln(x) \arctan(c\,x^3) - \frac{b \left(\sum_{\_R1 = \mathrm{RootOf}\left(c^2 \_Z^6+1\right)} \frac{\ln(x) \ln\left(\frac{\_R1 - x}{\_R1}\right) + \mathrm{dilog}\left(\frac{\_R1 - x}{\_R1}\right)}{\_R1^3}\right)}{2c}$
risch	$-\frac{i \left(\sum_{\_R1 = \mathrm{RootOf}\left(c\_Z^3 + \mathrm{RootOf}\left(\_Z^2+1, \mathrm{index}=1\right)\right)} \left(\ln(x) \ln\left(\frac{\_R1 - x}{\_R1}\right) + \mathrm{dilog}\left(\frac{\_R1 - x}{\_R1}\right)\right)\right) b}{2} + \frac{i \ln(x) \ln(-i c x^3+1) b}{2} +$

input
```
int((a+b*arctan(c*x^3))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*ln(x)*arctan(c*x^3)-1/2*b/c*sum(1/_R1^3*(ln(x)*ln((_R1-x)/_R1)+d
ilog((_R1-x)/_R1)),_R1=RootOf(_Z^6*c^2+1))
```

Fricas [F]

$$\int \frac{a + b\arctan(cx^3)}{x}\,dx = \int \frac{b\arctan(cx^3) + a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))/x,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x^3) + a)/x, x)
```

Sympy [F]

$$\int \frac{a + b\arctan(cx^3)}{x}\,dx = \int \frac{a + b\operatorname{atan}(cx^3)}{x}\,dx$$

input
```
integrate((a+b*atan(c*x**3))/x,x)
```

output
```
Integral((a + b*atan(c*x**3))/x, x)
```

Maxima [F]

$$\int \frac{a + b\arctan(cx^3)}{x}\,dx = \int \frac{b\arctan(cx^3) + a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))/x,x, algorithm="maxima")
```

output
```
b*integrate(arctan(c*x^3)/x, x) + a*log(x)
```

Giac [F]

$$\int \frac{a+b\arctan(cx^3)}{x}\,dx=\int \frac{b\arctan(cx^3)+a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{a+b\arctan(cx^3)}{x}\,dx=a\,\ln(x)-\frac{b\,\left(\mathrm{Li}_2(1-c\,x^3\,1\mathrm{i})-\mathrm{Li}_2(1\mathrm{i}\,c\,x^3+1)\right)\,1\mathrm{i}}{6}$$

input
```
int((a + b*atan(c*x^3))/x,x)
```

output
```
a*log(x) - (b*(dilog(1 - c*x^3*1i) - dilog(c*x^3*1i + 1))*1i)/6
```

Reduce [F]

$$\int \frac{a+b\arctan(cx^3)}{x}\,dx=\left(\int \frac{atan(c\,x^3)}{x}dx\right)b+\log(x)\,a$$

input
```
int((a+b*atan(c*x^3))/x,x)
```

output
```
int(atan(c*x**3)/x,x)*b + log(x)*a
```

3.101 $\int \frac{a+b\arctan(cx^3)}{x^4}\,dx$

Optimal result 754
Mathematica [A] (verified) 754
Rubi [A] (verified) 755
Maple [A] (verified) 756
Fricas [A] (verification not implemented) 757
Sympy [B] (verification not implemented) 757
Maxima [A] (verification not implemented) 758
Giac [A] (verification not implemented) 758
Mupad [B] (verification not implemented) 758
Reduce [B] (verification not implemented) 759

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = -\frac{a+b\arctan(cx^3)}{3x^3} + bc\log(x) - \frac{1}{6}bc\log\left(1+c^2x^6\right)$$

output
```
-1/3*(a+b*arctan(c*x^3))/x^3+b*c*ln(x)-1/6*b*c*ln(c^2*x^6+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = -\frac{a}{3x^3} - \frac{b\arctan(cx^3)}{3x^3} + bc\log(x) - \frac{1}{6}bc\log\left(1+c^2x^6\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x^4,x]
```

output
```
-1/3*a/x^3 - (b*ArcTan[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 + c^2*x^6
])/6
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5361, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^3\right)}{x^4}dx$$

$$\Big\downarrow \text{5361}$$

$$bc\int \frac{1}{x\left(c^2x^6+1\right)}dx-\frac{a+b\arctan\left(cx^3\right)}{3x^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{6}bc\int \frac{1}{x^6\left(c^2x^6+1\right)}dx^6-\frac{a+b\arctan\left(cx^3\right)}{3x^3}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{6}bc\left(\int \frac{1}{x^6}dx^6-c^2\int \frac{1}{c^2x^6+1}dx^6\right)-\frac{a+b\arctan\left(cx^3\right)}{3x^3}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{6}bc\left(\log\left(x^6\right)-c^2\int \frac{1}{c^2x^6+1}dx^6\right)-\frac{a+b\arctan\left(cx^3\right)}{3x^3}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{6}bc\left(\log\left(x^6\right)-\log\left(c^2x^6+1\right)\right)-\frac{a+b\arctan\left(cx^3\right)}{3x^3}$$

input
```
Int[(a + b*ArcTan[c*x^3])/x^4,x]
```

output
```
-1/3*(a + b*ArcTan[c*x^3])/x^3 + (b*c*(Log[x^6] - Log[1 + c^2*x^6]))/6
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{a}{3x^{3}}+b\left(-\frac{\arctan\left(c\,x^{3}\right)}{3x^{3}}+c\left(-\frac{\ln\left(c^{2}x^{6}+1\right)}{6}+\ln\left(x\right)\right)\right)$	39
parts	$-\frac{a}{3x^{3}}+b\left(-\frac{\arctan\left(c\,x^{3}\right)}{3x^{3}}+c\left(-\frac{\ln\left(c^{2}x^{6}+1\right)}{6}+\ln\left(x\right)\right)\right)$	39
parallelrisch	$\frac{6bc\ln\left(x\right)x^{3}-bc\ln\left(c^{2}x^{6}+1\right)x^{3}-2b\arctan\left(c\,x^{3}\right)-2a}{6x^{3}}$	45
risch	$\frac{ib\ln\left(ic\,x^{3}+1\right)}{6x^{3}}-\frac{-6bc\ln\left(x\right)x^{3}+bc\ln\left(-c^{2}x^{6}-1\right)x^{3}+ib\ln\left(-ic\,x^{3}+1\right)+2a}{6x^{3}}$	68

input
```
int((a+b*arctan(c*x^3))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*(-1/3/x^3*arctan(c*x^3)+c*(-1/6*ln(c^2*x^6+1)+ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = -\frac{bcx^3\log(c^2x^6+1) - 6\,bcx^3\log(x) + 2\,b\arctan(cx^3) + 2\,a}{6\,x^3}$$

input
```
integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="fricas")
```

output
```
-1/6*(b*c*x^3*log(c^2*x^6 + 1) - 6*b*c*x^3*log(x) + 2*b*arctan(c*x^3) + 2*
a)/x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(39) = 78.

Time = 37.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.82

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = \begin{cases} -\frac{a}{3x^3} + bc\log(x) - \frac{bc\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{3} - \frac{bc\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{3} - \frac{b\,\mathrm{atan}(cx^3)}{3\sqrt{-\frac{1}{c^2}}} - \frac{b\,\mathrm{atan}(cx^3)}{3x^3} & \text{for } c \neq \\ -\frac{a}{3x^3} & \text{otherw} \end{cases}$$

input
```
integrate((a+b*atan(c*x**3))/x**4,x)
```

output
```
Piecewise((-a/(3*x**3) + b*c*log(x) - b*c*log(x - (-1/c**2)**(1/6))/3 - b*
c*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/3 - b*atan(c*x**
3)/(3*sqrt(-1/c**2)) - b*atan(c*x**3)/(3*x**3), Ne(c, 0)), (-a/(3*x**3), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = -\frac{1}{6}\left(c\left(\log\left(c^2x^6+1\right)-\log\left(x^6\right)\right)+\frac{2\arctan(cx^3)}{x^3}\right)b-\frac{a}{3\,x^3}$$

input
```
integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="maxima")
```

output
```
-1/6*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*b - 1/3*a/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = -\frac{bc^3x^3\log\left(c^2x^6+1\right)-2\,bc^3x^3\log\left(cx^3\right)+2\,bc^2\arctan\left(cx^3\right)+2\,ac^2}{6\,c^2x^3}$$

input
```
integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="giac")
```

output
```
-1/6*(b*c^3*x^3*log(c^2*x^6 + 1) - 2*b*c^3*x^3*log(c*x^3) + 2*b*c^2*arctan
(c*x^3) + 2*a*c^2)/(c^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = b\,c\,\ln(x)-\frac{a}{3\,x^3}-\frac{b\,\mathrm{atan}(c\,x^3)}{3\,x^3}-\frac{b\,c\,\ln\left(c^2\,x^6+1\right)}{6}$$

input
```
int((a + b*atan(c*x^3))/x^4,x)
```

output
```
b*c*log(x) - a/(3*x^3) - (b*atan(c*x^3))/(3*x^3) - (b*c*log(c^2*x^6 + 1))/
6
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.38

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx$$
$$= \frac{-2atan(c\,x^3)\,b-\log\left(c^{\frac{2}{3}}x^2-c^{\frac{1}{3}}\sqrt{3}\,x+1\right)bc\,x^3-\log\left(c^{\frac{2}{3}}x^2+c^{\frac{1}{3}}\sqrt{3}\,x+1\right)bc\,x^3-\log\left(c^{\frac{2}{3}}x^2+1\right)bc\,x^3}{6x^3}$$

input
```
int((a+b*atan(c*x^3))/x^4,x)
```

output
```
( - 2*atan(c*x**3)*b - log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x + 1)*b*c*x**
3 - log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x + 1)*b*c*x**3 - log(c**(2/3)*x*
*2 + 1)*b*c*x**3 + 6*log(x)*b*c*x**3 - 2*a)/(6*x**3)
```

3.102 $\int \frac{a+b\arctan(cx^3)}{x^7}\,dx$

Optimal result 760
Mathematica [C] (verified) 760
Rubi [A] (verified) 761
Maple [A] (verified) 762
Fricas [A] (verification not implemented) 763
Sympy [A] (verification not implemented) 763
Maxima [A] (verification not implemented) 764
Giac [C] (verification not implemented) 764
Mupad [B] (verification not implemented) 764
Reduce [B] (verification not implemented) 765

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = -\frac{bc}{6x^3} - \frac{1}{6}bc^2\arctan(cx^3) - \frac{a+b\arctan(cx^3)}{6x^6}$$

output
```
-1/6*b*c/x^3-1/6*b*c^2*arctan(c*x^3)-1/6*(a+b*arctan(c*x^3))/x^6
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = -\frac{a}{6x^6} - \frac{b\arctan(cx^3)}{6x^6} - \frac{bc\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^6\right)}{6x^3}$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x^7,x]
```

output
```
-1/6*a/x^6 - (b*ArcTan[c*x^3])/(6*x^6) - (b*c*Hypergeometric2F1[-1/2, 1, 1
/2, -(c^2*x^6)])/(6*x^3)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5361, 807, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^3\right)}{x^7}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}bc\int \frac{1}{x^4\left(c^2x^6+1\right)}dx-\frac{a+b\arctan\left(cx^3\right)}{6x^6}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{6}bc\int \frac{1}{x^6\left(c^2x^6+1\right)}dx^3-\frac{a+b\arctan\left(cx^3\right)}{6x^6}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{6}bc\left(c^2\left(-\int \frac{1}{c^2x^6+1}dx^3\right)-\frac{1}{x^3}\right)-\frac{a+b\arctan\left(cx^3\right)}{6x^6}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{6}bc\left(-c\arctan\left(cx^3\right)-\frac{1}{x^3}\right)-\frac{a+b\arctan\left(cx^3\right)}{6x^6}$$

input
```
Int[(a + b*ArcTan[c*x^3])/x^7,x]
```

output
```
-1/6*(a + b*ArcTan[c*x^3])/x^6 + (b*c*(-x^(-3) - c*ArcTan[c*x^3]))/6
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{a}{6x^{6}}-\frac{b\arctan\left(c\,x^{3}\right)}{6x^{6}}-\frac{bc}{6x^{3}}-\frac{b\,c^{2}\arctan\left(c\,x^{3}\right)}{6}$	39
parts	$-\frac{a}{6x^{6}}-\frac{b\arctan\left(c\,x^{3}\right)}{6x^{6}}-\frac{bc}{6x^{3}}-\frac{b\,c^{2}\arctan\left(c\,x^{3}\right)}{6}$	39
parallelrisch	$-\frac{\arctan\left(c\,x^{3}\right)b\,c^{2}x^{6}-a\,c^{2}x^{6}+bc\,x^{3}+b\arctan\left(c\,x^{3}\right)+a}{6x^{6}}$	45
orering	$\frac{\left(-\frac{5}{9}x^{7}c^{2}-\frac{5}{9}x\right)\left(a+b\arctan\left(c\,x^{3}\right)\right)}{x^{7}}-\frac{\left(c^{2}x^{6}+1\right)x^{2}\left(\frac{3bc}{x^{5}\left(c^{2}x^{6}+1\right)}-\frac{7\left(a+b\arctan\left(c\,x^{3}\right)\right)}{x^{8}}\right)}{18}$	76
risch	$\frac{ib\ln\left(ic\,x^{3}+1\right)}{12x^{6}}-\frac{-ib\,c^{2}\ln\left(c\,x^{3}-i\right)x^{6}+ib\,c^{2}\ln\left(c\,x^{3}+i\right)x^{6}+2bc\,x^{3}+ib\ln\left(-ic\,x^{3}+1\right)+2a}{12x^{6}}$	87

input
```
int((a+b*arctan(c*x^3))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a/x^6-1/6*b/x^6*arctan(c*x^3)-1/6*b*c/x^3-1/6*b*c^2*arctan(c*x^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = -\frac{bcx^3+(bc^2x^6+b)\arctan(cx^3)+a}{6\,x^6}$$

input
```
integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="fricas")
```

output
```
-1/6*(b*c*x^3 + (b*c^2*x^6 + b)*arctan(c*x^3) + a)/x^6
```

Sympy [A] (verification not implemented)

Time = 34.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = -\frac{a}{6x^6}-\frac{bc^2\operatorname{atan}(cx^3)}{6}-\frac{bc}{6x^3}-\frac{b\operatorname{atan}(cx^3)}{6x^6}$$

input
```
integrate((a+b*atan(c*x**3))/x**7,x)
```

output
```
-a/(6*x**6) - b*c**2*atan(c*x**3)/6 - b*c/(6*x**3) - b*atan(c*x**3)/(6*x**
6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = -\frac{1}{6}\left(\left(c\arctan(cx^3)+\frac{1}{x^3}\right)c+\frac{\arctan(cx^3)}{x^6}\right)b-\frac{a}{6\,x^6}$$

input
```
integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="maxima")
```

output
```
-1/6*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*b - 1/6*a/x^6
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = \frac{i\,bc^5x^6\log(i\,cx^3+1)-i\,bc^5x^6\log(-i\,cx^3+1)-2\,bc^4x^3-2\,bc^3\arctan(cx^3)-2\,ac^3}{12\,c^3x^6}$$

input
```
integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="giac")
```

output
```
1/12*(I*b*c^5*x^6*log(I*c*x^3 + 1) - I*b*c^5*x^6*log(-I*c*x^3 + 1) - 2*b*c
^4*x^3 - 2*b*c^3*arctan(c*x^3) - 2*a*c^3)/(c^3*x^6)
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = -\frac{\frac{b\,c\,x^3}{3}+\frac{a}{3}}{2\,x^6}-\frac{b\,c^2\,\mathrm{atan}(c\,x^3)}{6}-\frac{b\,\mathrm{atan}(c\,x^3)}{6\,x^6}$$

input
```
int((a + b*atan(c*x^3))/x^7,x)
```

output
```
- (a/3 + (b*c*x^3)/3)/(2*x^6) - (b*c^2*atan(c*x^3))/6 - (b*atan(c*x^3))/(6
*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{a + b \arctan(cx^3)}{x^7} \, dx = \frac{-atan(c\,x^3)\,b\,c^2x^6 - atan(c\,x^3)\,b - a - bc\,x^3}{6x^6}$$

input
```
int((a+b*atan(c*x^3))/x^7,x)
```

output
```
( - (atan(c*x**3)*b*c**2*x**6 + atan(c*x**3)*b + a + b*c*x**3))/(6*x**6)
```

3.103 $\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx$

Optimal result 766
Mathematica [A] (verified) 766
Rubi [A] (verified) 767
Maple [A] (verified) 768
Fricas [A] (verification not implemented) 769
Sympy [F(-1)] 769
Maxima [A] (verification not implemented) 770
Giac [A] (verification not implemented) 770
Mupad [B] (verification not implemented) 771
Reduce [B] (verification not implemented) 771

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx = -\frac{bc}{18x^6} - \frac{a+b\arctan(cx^3)}{9x^9} - \frac{1}{3}bc^3\log(x) + \frac{1}{18}bc^3\log(1+c^2x^6)$$

output
```
-1/18*b*c/x^6-1/9*(a+b*arctan(c*x^3))/x^9-1/3*b*c^3*ln(x)+1/18*b*c^3*ln(c^
2*x^6+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx = -\frac{a}{9x^9} - \frac{bc}{18x^6} - \frac{b\arctan(cx^3)}{9x^9} - \frac{1}{3}bc^3\log(x) + \frac{1}{18}bc^3\log(1+c^2x^6)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x^10,x]
```

output
```
-1/9*a/x^9 - (b*c)/(18*x^6) - (b*ArcTan[c*x^3])/(9*x^9) - (b*c^3*Log[x])/3
 + (b*c^3*Log[1 + c^2*x^6])/18
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5361, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}bc\int \frac{1}{x^7(c^2x^6+1)}dx-\frac{a+b\arctan(cx^3)}{9x^9}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{18}bc\int \frac{1}{x^{12}(c^2x^6+1)}dx^6-\frac{a+b\arctan(cx^3)}{9x^9}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{18}bc\int \left(\frac{c^4}{c^2x^6+1}-\frac{c^2}{x^6}+\frac{1}{x^{12}}\right)dx^6-\frac{a+b\arctan(cx^3)}{9x^9}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{18}bc\left(c^2(-\log(x^6))+c^2\log(c^2x^6+1)-\frac{1}{x^6}\right)-\frac{a+b\arctan(cx^3)}{9x^9}$$

input
```
Int[(a + b*ArcTan[c*x^3])/x^10,x]
```

output
```
-1/9*(a + b*ArcTan[c*x^3])/x^9 + (b*c*(-x^(-6) - c^2*Log[x^6] + c^2*Log[1
+ c^2*x^6]))/18
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a}{9x^9}+b\left(-\frac{\arctan\left(c\,x^3\right)}{9x^9}+\frac{c\left(\frac{c^2\ln\left(c^2x^6+1\right)}{6}-\frac{1}{6x^6}-c^2\ln\left(x\right)\right)}{3}\right)$	53
parts	$-\frac{a}{9x^9}+b\left(-\frac{\arctan\left(c\,x^3\right)}{9x^9}+\frac{c\left(\frac{c^2\ln\left(c^2x^6+1\right)}{6}-\frac{1}{6x^6}-c^2\ln\left(x\right)\right)}{3}\right)$	53
parallelrisch	$-\frac{6b\,c^3\ln\left(x\right)x^9-b\,c^3\ln\left(c^2x^6+1\right)x^9-b\,c^3x^9+bc\,x^3+2b\arctan\left(c\,x^3\right)+2a}{18x^9}$	64
risch	$\frac{ib\ln\left(ic\,x^3+1\right)}{18x^9}-\frac{6b\,c^3\ln\left(x\right)x^9-b\,c^3\ln\left(c^2x^6+1\right)x^9+bc\,x^3+ib\ln\left(-ic\,x^3+1\right)+2a}{18x^9}$	78

input
```
int((a+b*arctan(c*x^3))/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a/x^9+b*(-1/9/x^9*arctan(c*x^3)+1/3*c*(1/6*c^2*ln(c^2*x^6+1)-1/6/x^6-
c^2*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx = \frac{bc^3x^9\log(c^2x^6+1)-6\,bc^3x^9\log(x)-bcx^3-2\,b\arctan(cx^3)-2\,a}{18\,x^9}$$

input
```
integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="fricas")
```

output
```
1/18*(b*c^3*x^9*log(c^2*x^6 + 1) - 6*b*c^3*x^9*log(x) - b*c*x^3 - 2*b*arct
an(c*x^3) - 2*a)/x^9
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x**3))/x**10,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{a + b \arctan(cx^3)}{x^{10}} \, dx = \frac{1}{18} \left(\left(c^2 \log(c^2 x^6 + 1) - c^2 \log(x^6) - \frac{1}{x^6} \right) c - \frac{2 \arctan(cx^3)}{x^9} \right) b - \frac{a}{9 x^9}$$

input
```
integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="maxima")
```

output
```
1/18*((c^2*log(c^2*x^6 + 1) - c^2*log(x^6) - 1/x^6)*c - 2*arctan(c*x^3)/x^
9)*b - 1/9*a/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a + b \arctan(cx^3)}{x^{10}} \, dx = \frac{bc^7 x^9 \log(c^2 x^6 + 1) - 2 bc^7 x^9 \log(cx^3) - bc^5 x^3 - 2 bc^4 \arctan(cx^3) - 2 ac^4}{18 c^4 x^9}$$

input
```
integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="giac")
```

output
```
1/18*(b*c^7*x^9*log(c^2*x^6 + 1) - 2*b*c^7*x^9*log(c*x^3) - b*c^5*x^3 - 2*
b*c^4*arctan(c*x^3) - 2*a*c^4)/(c^4*x^9)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx=\frac{b\,c^3\,\ln(c^2\,x^6+1)}{18}-\frac{a}{9\,x^9}-\frac{b\,c^3\,\ln(x)}{3}-\frac{b\,\mathrm{atan}(c\,x^3)}{9\,x^9}-\frac{b\,c}{18\,x^6}$$

input
```
int((a + b*atan(c*x^3))/x^10,x)
```

output
```
(b*c^3*log(c^2*x^6 + 1))/18 - a/(9*x^9) - (b*c^3*log(x))/3 - (b*atan(c*x^3
))/(9*x^9) - (b*c)/(18*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.91

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx$$
$$=\frac{-2atan(c\,x^3)\,b+\log\left(c^{\frac{2}{3}}x^2-c^{\frac{1}{3}}\sqrt{3}\,x+1\right)b\,c^3x^9+\log\left(c^{\frac{2}{3}}x^2+c^{\frac{1}{3}}\sqrt{3}\,x+1\right)b\,c^3x^9+\log\left(c^{\frac{2}{3}}x^2+1\right)b\,c^3}{18x^9}$$

input
```
int((a+b*atan(c*x^3))/x^10,x)
```

output
```
( - 2*atan(c*x**3)*b + log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x + 1)*b*c**3*
x**9 + log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x + 1)*b*c**3*x**9 + log(c**(2
/3)*x**2 + 1)*b*c**3*x**9 - 6*log(x)*b*c**3*x**9 - 2*a - b*c*x**3)/(18*x**
9)
```

3.104 $\int x^3(a+b\arctan(cx^3))\,dx$

Optimal result 772
Mathematica [A] (verified) 773
Rubi [A] (verified) 773
Maple [A] (verified) 778
Fricas [B] (verification not implemented) 779
Sympy [B] (verification not implemented) 779
Maxima [A] (verification not implemented) 780
Giac [A] (verification not implemented) 781
Mupad [B] (verification not implemented) 781
Reduce [B] (verification not implemented) 782

Optimal result

Integrand size = 14, antiderivative size = 136

$$\int x^3\left(a+b\arctan\left(cx^3\right)\right)\,dx=-\frac{3bx}{4c}+\frac{b\arctan\left(\sqrt[3]{c}x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{8c^{4/3}}+\frac{b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{8c^{4/3}}+\frac{\sqrt{3}b\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[3]{c}x}{1+c^{2/3}x^2}\right)}{8c^{4/3}}$$

output
```
-3/4*b*x/c+1/4*b*arctan(c^(1/3)*x)/c^(4/3)+1/4*x^4*(a+b*arctan(c*x^3))+1/8
*b*arctan(-3^(1/2)+2*c^(1/3)*x)/c^(4/3)+1/8*b*arctan(3^(1/2)+2*c^(1/3)*x)/
c^(4/3)+1/8*3^(1/2)*b*arctanh(3^(1/2)*c^(1/3)*x/(1+c^(2/3)*x^2))/c^(4/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.32

$$\int x^3\left(a+b\arctan\left(cx^3\right)\right)\,dx=-\frac{3bx}{4c}+\frac{ax^4}{4}+\frac{b\arctan\left(\sqrt[3]{c}x\right)}{4c^{4/3}}+\frac{1}{4}bx^4\arctan\left(cx^3\right)-\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{8c^{4/3}}+\frac{b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{8c^{4/3}}-\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{16c^{4/3}}+\frac{\sqrt{3}b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{16c^{4/3}}$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x^3]),x]
```

output
```
(-3*b*x)/(4*c) + (a*x^4)/4 + (b*ArcTan[c^(1/3)*x])/(4*c^(4/3)) + (b*x^4*Ar
cTan[c*x^3])/4 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(8*c^(4/3)) + (b*ArcTan
[Sqrt[3] + 2*c^(1/3)*x])/(8*c^(4/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*
x + c^(2/3)*x^2])/(16*c^(4/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^
(2/3)*x^2])/(16*c^(4/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.45, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5361, 843, 753, 27, 216, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\arctan\left(cx^3\right)\right)\,dx$$

$$\Big\downarrow \ 5361$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{4}bc\int\frac{x^6}{c^2x^6+1}dx$$

$$\Big\downarrow \ 843$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\int \frac{1}{c^2x^6+1}dx}{c^2}\right)$$

$$\Big\downarrow \ 753$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-$$
$$\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{3}\int \frac{1}{c^{2/3}x^2+1}dx+\frac{1}{3}\int \frac{2-\sqrt{3}\sqrt[3]{c}x}{2\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}dx+\frac{1}{3}\int \frac{\sqrt{3}\sqrt[3]{c}x+2}{2\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}dx}{c^2}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-$$
$$\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{3}\int \frac{1}{c^{2/3}x^2+1}dx+\frac{1}{6}\int \frac{2-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{1}{6}\int \frac{\sqrt{3}\sqrt[3]{c}x+2}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{c^2}\right)$$

$$\Big\downarrow \ 216$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-$$
$$\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{6}\int \frac{2-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{1}{6}\int \frac{\sqrt{3}\sqrt[3]{c}x+2}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\arctan\left(\sqrt[3]{c}x\right)}{3\sqrt[3]{c}}}{c^2}\right)$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-$$
$$\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\sqrt{3}\int -\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\sqrt{3}\int \frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x \cdots\right.}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{\cdots}}}{2\sqrt[3]{c}} \cdots}{c^2}\right.$$

$$\Big\downarrow \ 25$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+\ldots\right)}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\ldots}}{2\sqrt[3]{c}}\ldots}{c^2}\ldots\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{\ldots}}{c^{2/3}x^2+\ldots}\ldots\right.}{c^2}\ldots\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\int\frac{1}{-\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\int\ldots}{\ldots}\ldots\right.}{c^2}\ldots\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\arctan\left(\sqrt{3}\left(\ldots\right)\right)}{\ldots}\ldots\right.}{c^2}\ldots\right.$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{6}\left(-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}-\frac{\sqrt{3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)\right)}{\sqrt[3]{c}}+\frac{\sqrt{3}\log\left(c^{2/3}x^2+\sqrt{\ldots}\right.}{2\sqrt[3]{c}}\right.}{c^2}\right.$$

input
```
Int[x^3*(a + b*ArcTan[c*x^3]),x]
```

output
```
(x^4*(a + b*ArcTan[c*x^3]))/4 - (3*b*c*(x/c^2 - (ArcTan[c^(1/3)*x]/(3*c^(1
/3)) + (-(ArcTan[Sqrt[3]*(1 - (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3)) - (Sqrt[3]*
Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(2*c^(1/3)))/6 + (ArcTan[Sqrt[3]
*(1 + (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3) + (Sqrt[3]*Log[1 + Sqrt[3]*c^(1/3)*x
 + c^(2/3)*x^2])/(2*c^(1/3)))/6)/c^2))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.14

method	result
default	$\frac{a x^4}{4} + b\left(\frac{x^4 \arctan(c x^3)}{4} - \frac{3c\left(\frac{x}{c^2} - \frac{-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}} \ln\left(x^2-\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}} x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12} + \frac{\left(\frac{1}{c^2}\right)^{\frac{1}{6}} \arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6} + \frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}} \ln}{c^2}}{}\right)}{4}\right.$
parts	$\frac{a x^4}{4} + b\left(\frac{x^4 \arctan(c x^3)}{4} - \frac{3c\left(\frac{x}{c^2} - \frac{-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}} \ln\left(x^2-\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}} x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12} + \frac{\left(\frac{1}{c^2}\right)^{\frac{1}{6}} \arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6} + \frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}} \ln}{c^2}}{}\right)}{4}\right.$

input
```
int(x^3*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+b*(1/4*x^4*arctan(c*x^3)-3/4*c*(1/c^2*x-(-1/12*3^(1/2)*(1/c^2)^(
1/6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6*(1/c^2)^(1/6)*arcta
n(2*x/(1/c^2)^(1/6)-3^(1/2))+1/12*3^(1/2)*(1/c^2)^(1/6)*ln(x^2+3^(1/2)*(1/
c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6*(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)+3^(
1/2))+1/3*(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6)))/c^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(100) = 200$.

Time = 0.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.98

$$\int x^3\left(a+b\arctan\left(cx^3\right)\right)\,dx = \frac{4\,bcx^4\arctan\left(cx^3\right)+4\,acx^4+\left(\sqrt{-3}c+c\right)\left(-\frac{b^6}{c^8}\right)^{\frac{1}{6}}\log\left(bx+\frac{1}{2}\left(\sqrt{-3}c+c\right)\left(-\frac{b^6}{c^8}\right)^{\frac{1}{6}}\right)-\left(\sqrt{-3}c+c\right)\left(-\right.}{}$$

input
```
integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/16*(4*b*c*x^4*arctan(c*x^3) + 4*a*c*x^4 + (sqrt(-3)*c + c)*(-b^6/c^8)^(1
/6)*log(b*x + 1/2*(sqrt(-3)*c + c)*(-b^6/c^8)^(1/6)) - (sqrt(-3)*c + c)*(-
b^6/c^8)^(1/6)*log(b*x - 1/2*(sqrt(-3)*c + c)*(-b^6/c^8)^(1/6)) + (sqrt(-3
)*c - c)*(-b^6/c^8)^(1/6)*log(b*x + 1/2*(sqrt(-3)*c - c)*(-b^6/c^8)^(1/6))
 - (sqrt(-3)*c - c)*(-b^6/c^8)^(1/6)*log(b*x - 1/2*(sqrt(-3)*c - c)*(-b^6/
c^8)^(1/6)) + 2*c*(-b^6/c^8)^(1/6)*log(b*x + c*(-b^6/c^8)^(1/6)) - 2*c*(-b
^6/c^8)^(1/6)*log(b*x - c*(-b^6/c^8)^(1/6)) - 12*b*x)/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(124) = 248$.

Time = 22.83 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.88

$$\int x^3\left(a+b\arctan\left(cx^3\right)\right)\,dx = \begin{cases} \frac{ax^4}{4}+\frac{bx^4\operatorname{atan}\left(cx^3\right)}{4}-\frac{3bx}{4c}-\frac{3b\sqrt[6]{-\frac{1}{c^2}}\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{16c}+\frac{3b\sqrt[6]{-\frac{1}{c^2}}\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{16c}+ \\ \frac{ax^4}{4} \end{cases}$$

input
```
integrate(x**3*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**4/4 + b*x**4*atan(c*x**3)/4 - 3*b*x/(4*c) - 3*b*(-1/c**2)*
*(1/6)*log(4*x**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(16*c) + 3*
b*(-1/c**2)**(1/6)*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))
/(16*c) + sqrt(3)*b*(-1/c**2)**(1/6)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6))
 - sqrt(3)/3)/(8*c) + sqrt(3)*b*(-1/c**2)**(1/6)*atan(2*sqrt(3)*x/(3*(-1/c
**2)**(1/6)) + sqrt(3)/3)/(8*c) + b*atan(c*x**3)/(4*c**2*(-1/c**2)**(1/3))
, Ne(c, 0)), (a*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.09

$$\int x^3\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{4}\,ax^4+\frac{1}{16}\left(4\,x^4\arctan\left(cx^3\right)+c\left(\frac{\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}-\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}+\frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{1}{3}}}+\frac{2\arctan\left(\frac{2c^{\frac{2}{3}}x+}{c^{\frac{1}{3}}}\right.}{c^{\frac{1}{3}}}}{c^2}\right.\right.$$

input
```
integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/16*(4*x^4*arctan(c*x^3) + c*((sqrt(3)*log(c^(2/3)*x^2 + sqrt
(3)*c^(1/3)*x + 1)/c^(1/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x +
 1)/c^(1/3) + 4*arctan(c^(1/3)*x)/c^(1/3) + 2*arctan((2*c^(2/3)*x + sqrt(3
)*c^(1/3))/c^(1/3))/c^(1/3) + 2*arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(
1/3))/c^(1/3))/c^2 - 12*x/c^2))*b
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.23

$$\int x^3(a+b\arctan(cx^3))\,dx$$
$$=\frac{1}{16}bc^7\left(\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^8|c|^{\frac{1}{3}}}-\frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^8|c|^{\frac{1}{3}}}+\frac{2\arctan\left(\left(2x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^8|c|^{\frac{1}{3}}}+\frac{2\text{ ar}}{}\right.$$
$$+\frac{bcx^4\arctan(cx^3)+acx^4-3bx}{4c}$$

input
```
integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
1/16*b*c^7*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/(c^
8*abs(c)^(1/3)) - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3
))/(c^8*abs(c)^(1/3)) + 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3)
)/(c^8*abs(c)^(1/3)) + 2*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))
/(c^8*abs(c)^(1/3)) + 4*arctan(x*abs(c)^(1/3))/(c^8*abs(c)^(1/3))) + 1/4*(
b*c*x^4*arctan(c*x^3) + a*c*x^4 - 3*b*x)/c
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int x^3(a+b\arctan(cx^3))\,dx$$
$$=\frac{a\,x^4}{4}$$
$$-\frac{b\left(\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)-\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)+2\,\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{8\,c^{4/3}}$$
$$+\frac{b\,x^4\,\mathrm{atan}(c\,x^3)}{4}-\frac{3\,b\,x}{4\,c}-\frac{\sqrt{3}\,b\left(\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)+\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)\right)\,1\mathrm{i}}{8\,c^{4/3}}$$

input
```
int(x^3*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^4)/4 - (b*(atan((-1)^(2/3)*c^(1/3)*x) - atan((c^(1/3)*x*(3^(1/2)*1i +
 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/(8*c^(4/3))
+ (b*x^4*atan(c*x^3))/4 - (3*b*x)/(4*c) - (3^(1/2)*b*(atan((c^(1/3)*x*(3^(
1/2)*1i + 1))/2) + atan((-1)^(2/3)*c^(1/3)*x))*1i)/(8*c^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.73

$$\int x^3(a+b\arctan(cx^3))\,dx$$
$$=\frac{6atan\left(c^{\frac{1}{3}}x\right)b+4c^{\frac{4}{3}}atan(c\,x^3)\,b\,x^4+2atan(c\,x^3)\,b+4c^{\frac{4}{3}}a\,x^4-12c^{\frac{1}{3}}bx-\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-c^{\frac{1}{3}}\sqrt{3}\,x+1\right)b}{16c^{\frac{4}{3}}}$$

input
```
int(x^3*(a+b*atan(c*x^3)),x)
```

output
```
(6*atan(c**(1/3)*x)*b + 4*c**(1/3)*atan(c*x**3)*b*c*x**4 + 2*atan(c*x**3)*
b + 4*c**(1/3)*a*c*x**4 - 12*c**(1/3)*b*x - sqrt(3)*log(c**(2/3)*x**2 - c*
*(1/3)*sqrt(3)*x + 1)*b + sqrt(3)*log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x +
 1)*b)/(16*c**(1/3)*c)
```

3.105 $\int (a+b\arctan(cx^3))\,dx$

Optimal result 783
Mathematica [A] (verified) 783
Rubi [A] (verified) 784
Maple [A] (verified) 785
Fricas [A] (verification not implemented) 785
Sympy [A] (verification not implemented) 786
Maxima [A] (verification not implemented) 787
Giac [A] (verification not implemented) 788
Mupad [B] (verification not implemented) 788
Reduce [B] (verification not implemented) 789

Optimal result

Integrand size = 10, antiderivative size = 101

$$\int (a+b\arctan(cx^3))\,dx = ax+bx\arctan(cx^3)+\frac{\sqrt{3}b\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+\frac{b\log(1+c^{2/3}x^2)}{2\sqrt[3]{c}}-\frac{b\log(1-c^{2/3}x^2+c^{4/3}x^4)}{4\sqrt[3]{c}}$$

output
```
a*x+b*x*arctan(c*x^3)+1/2*3^(1/2)*b*arctan(1/3*(1-2*c^(2/3)*x^2)*3^(1/2))/
c^(1/3)+1/2*b*ln(1+c^(2/3)*x^2)/c^(1/3)-1/4*b*ln(1-c^(2/3)*x^2+c^(4/3)*x^4
)/c^(1/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.30

$$\int (a+b\arctan(cx^3))\,dx = ax+bx\arctan(cx^3) - \frac{b\left(-2\sqrt{3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)-2\sqrt{3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)-2\log\left(1+c^{2/3}x^2\right)+\log\left(1-\sqrt{3}\sqrt[3]{c}x+c\right.\right.}{4\sqrt[3]{c}}$$

input
```
Integrate[a + b*ArcTan[c*x^3],x]
```

output
```
a*x + b*x*ArcTan[c*x^3] - (b*(-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*c^(1/3)*x] - 2
*Sqrt[3]*ArcTan[Sqrt[3] + 2*c^(1/3)*x] - 2*Log[1 + c^(2/3)*x^2] + Log[1 -
Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2] + Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2
]))/(4*c^(1/3))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\arctan(cx^3))\, dx$$

$$\Big\downarrow \text{2009}$$

$$ax + \frac{\sqrt{3}b\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}} + bx\arctan(cx^3) + \frac{b\log(c^{2/3}x^2+1)}{2\sqrt[3]{c}} - \frac{b\log(c^{4/3}x^4 - c^{2/3}x^2 + 1)}{4\sqrt[3]{c}}$$

input
```
Int[a + b*ArcTan[c*x^3],x]
```

output
```
a*x + b*x*ArcTan[c*x^3] + (Sqrt[3]*b*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/
(2*c^(1/3)) + (b*Log[1 + c^(2/3)*x^2])/(2*c^(1/3)) - (b*Log[1 - c^(2/3)*x^
2 + c^(4/3)*x^4])/(4*c^(1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

method	result	si
default	$ax + bx\arctan\left(c\,x^{3}\right) + \frac{b\ln\left(x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}} - \frac{b\ln\left(x^{4}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}} - \frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}$	9
parts	$ax + bx\arctan\left(c\,x^{3}\right) + \frac{b\ln\left(x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}} - \frac{b\ln\left(x^{4}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}} - \frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}$	9

input
```
int(a+b*arctan(c*x^3),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arctan(c*x^3)+1/2*b/c/(1/c^2)^(1/3)*ln(x^2+(1/c^2)^(1/3))-1/4*b/c/
(1/c^2)^(1/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))-1/2*b*3^(1/2)/c/(1/c
^2)^(1/3)*arctan(1/3*3^(1/2)*(2*x^2/(1/c^2)^(1/3)-1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.32

$$\int \left(a + b\arctan\left(cx^{3}\right)\right)\,dx$$

$$= \left[\frac{4\,bcx\arctan\left(cx^{3}\right) + \sqrt{3}bc\sqrt{-\frac{1}{c^{\frac{2}{3}}}}\log\left(\frac{2\,c^{2}x^{6}-3\,c^{\frac{2}{3}}x^{2}-\sqrt{3}\left(2\,c^{\frac{5}{3}}x^{4}+cx^{2}-c^{\frac{1}{3}}\right)\sqrt{-\frac{1}{c^{\frac{2}{3}}}}-1}{c^{2}x^{6}+1}\right) + 4\,acx - bc^{\frac{2}{3}}\log\left(c^{2}x^{4}\right.}{4\,c}\right.$$

input
```
integrate(a+b*arctan(c*x^3),x, algorithm="fricas")
```

output
```
[1/4*(4*b*c*x*arctan(c*x^3) + sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*c^2*x^6
- 3*c^(2/3)*x^2 - sqrt(3)*(2*c^(5/3)*x^4 + c*x^2 - c^(1/3))*sqrt(-1/c^(2/3
)) - 1)/(c^2*x^6 + 1)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 - c^(4/3)*x^2 + c
^(2/3)) + 2*b*c^(2/3)*log(c*x^2 + c^(1/3)))/c, 1/4*(4*b*c*x*arctan(c*x^3)
+ 2*sqrt(3)*b*c^(2/3)*arctan(-1/3*sqrt(3)*(2*c*x^2 - c^(1/3))/c^(1/3)) + 4
*a*c*x - b*c^(2/3)*log(c^2*x^4 - c^(4/3)*x^2 + c^(2/3)) + 2*b*c^(2/3)*log(
c*x^2 + c^(1/3)))/c]
```

Sympy [A] (verification not implemented)

Time = 12.23 (sec) , antiderivative size = 755, normalized size of antiderivative = 7.48

$$\int \left(a + b \arctan\left(cx^3\right)\right) dx = \text{Too large to display}$$

input
```
integrate(a+b*atan(c*x**3),x)
```

output
```
a*x + b*Piecewise((0, Eq(c, 0)), (-oo*I*x, Eq(c, -I/x**3)), (oo*I*x, Eq(c,
 I/x**3)), (-4*c**4*x**6*(-1/c**2)**(5/3)*log(x - (-1/c**2)**(1/6))/(4*c*x
**6 + 4/c) + 3*c**4*x**6*(-1/c**2)**(5/3)*log(4*x**2 - 4*x*(-1/c**2)**(1/6
) + 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) - c**4*x**6*(-1/c**2)**(5/3)*log(
4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) - 2*s
qrt(3)*c**4*x**6*(-1/c**2)**(5/3)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) -
sqrt(3)/3)/(4*c*x**6 + 4/c) + 2*sqrt(3)*c**4*x**6*(-1/c**2)**(5/3)*atan(2*
sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(4*c*x**6 + 4/c) - 4*c**4*x**6
*(-1/c**2)**(5/3)*log(2)/(4*c*x**6 + 4/c) - 4*c**3*x**6*(-1/c**2)**(7/6)*a
tan(c*x**3)/(4*c*x**6 + 4/c) - 4*c**2*(-1/c**2)**(5/3)*log(x - (-1/c**2)**
(1/6))/(4*c*x**6 + 4/c) + 3*c**2*(-1/c**2)**(5/3)*log(4*x**2 - 4*x*(-1/c**
2)**(1/6) + 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) - c**2*(-1/c**2)**(5/3)*l
og(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) -
2*sqrt(3)*c**2*(-1/c**2)**(5/3)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sq
rt(3)/3)/(4*c*x**6 + 4/c) + 2*sqrt(3)*c**2*(-1/c**2)**(5/3)*atan(2*sqrt(3)
*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(4*c*x**6 + 4/c) - 4*c**2*(-1/c**2)**
(5/3)*log(2)/(4*c*x**6 + 4/c) + 4*c*x**7*atan(c*x**3)/(4*c*x**6 + 4/c) - 4
*c*(-1/c**2)**(7/6)*atan(c*x**3)/(4*c*x**6 + 4/c) + 4*x*atan(c*x**3)/(4*c*
*2*x**6 + 4), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \left(a + b\arctan\left(cx^3\right)\right) dx =$$

$$-\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2 - c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}} + \frac{\log\left(c^{\frac{4}{3}}x^4 - c^{\frac{2}{3}}x^2 + 1\right)}{c^{\frac{4}{3}}} - \frac{2\log\left(\frac{c^{\frac{2}{3}}x^2 + 1}{c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}\right) - 4x\arctan\left(cx^3\right)\right) + ax$$

input
```
integrate(a+b*arctan(c*x^3),x, algorithm="maxima")
```

output
```
-1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2/3))/c
^(4/3) + log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1)/c^(4/3) - 2*log((c^(2/3)*x^2 +
 1)/c^(2/3))/c^(4/3)) - 4*x*arctan(c*x^3))*b + a*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \left(a+b\arctan\left(cx^3\right)\right)\,dx = -\frac{1}{4}\left(c\left(\frac{2\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2}+\frac{|c|^{\frac{2}{3}}\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2}-\frac{2\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{4}{3}}}\right.\right.$$
$$+ax$$

input
```
integrate(a+b*arctan(c*x^3),x, algorithm="giac")
```

output
```
-1/4*(c*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3)
)*abs(c)^(2/3))/c^2 + abs(c)^(2/3)*log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(
4/3))/c^2 - 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(4/3)) - 4*x*arctan(c*x^3))
*b + a*x
```

Mupad [B] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int \left(a+b\arctan\left(cx^3\right)\right)\,dx = a\,x+b\,x\,\mathrm{atan}\left(c\,x^3\right)+\frac{b\,\ln\left(c^{2/3}\,x^2+1\right)}{2\,c^{1/3}} - \frac{\ln\left(2-4\,c^{2/3}\,x^2+\sqrt{3}\,2\mathrm{i}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{4\,c^{1/3}} - \frac{\ln\left(4\,c^{2/3}\,x^2-2+\sqrt{3}\,2\mathrm{i}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{4\,c^{1/3}}$$

input
```
int(a + b*atan(c*x^3),x)
```

output
```
a*x + b*x*atan(c*x^3) + (b*log(c^(2/3)*x^2 + 1))/(2*c^(1/3)) - (log(3^(1/2
)*2i - 4*c^(2/3)*x^2 + 2)*(b - 3^(1/2)*b*1i))/(4*c^(1/3)) - (log(3^(1/2)*2
i + 4*c^(2/3)*x^2 - 2)*(b + 3^(1/2)*b*1i))/(4*c^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.18

$$\int \left(a+b\arctan\left(cx^3\right)\right)\,dx$$

$$= \frac{-4\sqrt{3}\,atan\left(2c^{\frac{1}{3}}x-\sqrt{3}\right)b+2\sqrt{3}\,atan\left(c^{\frac{1}{3}}x\right)b+4c^{\frac{1}{3}}\,atan(c\,x^3)\,bx+2\sqrt{3}\,atan(c\,x^3)\,b+4c^{\frac{1}{3}}ax-\log\left(c^{\frac{2}{3}}\right.}{4c^{\frac{1}{3}}}$$

input
```
int(a+b*atan(c*x^3),x)
```

output
```
( - 4*sqrt(3)*atan(2*c**(1/3)*x - sqrt(3))*b + 2*sqrt(3)*atan(c**(1/3)*x)*
b + 4*c**(1/3)*atan(c*x**3)*b*x + 2*sqrt(3)*atan(c*x**3)*b + 4*c**(1/3)*a*
x - log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x + 1)*b - log(c**(2/3)*x**2 + c*
*(1/3)*sqrt(3)*x + 1)*b + 2*log(c**(2/3)*x**2 + 1)*b)/(4*c**(1/3))
```

3.106 $\int \frac{a+b\arctan(cx^3)}{x^3}\,dx$

Optimal result 790
Mathematica [A] (verified) 790
Rubi [A] (verified) 791
Maple [A] (verified) 795
Fricas [B] (verification not implemented) 796
Sympy [B] (verification not implemented) 796
Maxima [A] (verification not implemented) 797
Giac [A] (verification not implemented) 798
Mupad [B] (verification not implemented) 798
Reduce [B] (verification not implemented) 799

Optimal result

Integrand size = 14, antiderivative size = 127

$$\int \frac{a+b\arctan(cx^3)}{x^3}\,dx = \frac{1}{2}bc^{2/3}\arctan\left(\sqrt[3]{c}x\right)-\frac{a+b\arctan(cx^3)}{2x^2}-\frac{1}{4}bc^{2/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)+\frac{1}{4}bc^{2/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)+\frac{1}{4}\sqrt{3}bc^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[3]{c}x}{1+c^{2/3}x^2}\right)$$

output
```
1/2*b*c^(2/3)*arctan(c^(1/3)*x)-1/2*(a+b*arctan(c*x^3))/x^2+1/4*b*c^(2/3)*
arctan(-3^(1/2)+2*c^(1/3)*x)+1/4*b*c^(2/3)*arctan(3^(1/2)+2*c^(1/3)*x)+1/4
*3^(1/2)*b*c^(2/3)*arctanh(3^(1/2)*c^(1/3)*x/(1+c^(2/3)*x^2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.34

$$\int \frac{a+b\arctan(cx^3)}{x^3}\,dx = -\frac{a}{2x^2}+\frac{1}{2}bc^{2/3}\arctan\left(\sqrt[3]{c}x\right)-\frac{b\arctan(cx^3)}{2x^2}-\frac{1}{4}bc^{2/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)+\frac{1}{4}bc^{2/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)-\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x^3,x]
```

output
```
-1/2*a/x^2 + (b*c^(2/3)*ArcTan[c^(1/3)*x])/2 - (b*ArcTan[c*x^3])/(2*x^2) -
 (b*c^(2/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/4 + (b*c^(2/3)*ArcTan[Sqrt[3] +
 2*c^(1/3)*x])/4 - (Sqrt[3]*b*c^(2/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*
x^2])/8 + (Sqrt[3]*b*c^(2/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.46, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5361, 753, 27, 216, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^3\right)}{x^3}dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{3}{2}bc\int \frac{1}{c^2x^6+1}dx-\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow \text{ 753}$$

$$\frac{3}{2}bc\left(\frac{1}{3}\int \frac{1}{c^{2/3}x^2+1}dx+\frac{1}{3}\int \frac{2-\sqrt{3}\sqrt[3]{c}x}{2\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}dx+\frac{1}{3}\int \frac{\sqrt{3}\sqrt[3]{c}x+2}{2\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}dx\right)-\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{2}bc\left(\frac{1}{3}\int \frac{1}{c^{2/3}x^2+1}dx+\frac{1}{6}\int \frac{2-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{1}{6}\int \frac{\sqrt{3}\sqrt[3]{c}x+2}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx\right)-\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{3}{2}bc\left(\frac{1}{6}\int \frac{2-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{1}{6}\int \frac{\sqrt{3}\sqrt[3]{c}x+2}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\arctan\left(\sqrt[3]{c}x\right)}{3\sqrt[3]{c}}\right)-\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$\big\downarrow$ 1142

$$\frac{3}{2}bc\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\sqrt{3}\int -\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\sqrt{3}\ldots}{\ldots}\right.\right.$$
$$\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$\big\downarrow$ 25

$$\frac{3}{2}bc\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\sqrt{3}\int \frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\sqrt{3}\int\ldots}{\ldots}\right.\right.$$
$$\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$\big\downarrow$ 27

$$\frac{3}{2}bc\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\ldots\right.\right.$$
$$\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$\big\downarrow$ 1082

$$\frac{3}{2}bc\left(\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\int \frac{1}{-\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x\ldots}\right.\right.$$
$$\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$\big\downarrow$ 217

$$\frac{3}{2}bc\left(\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3}{2}bc\left(\frac{1}{6}\left(-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}-\frac{\sqrt{3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)\right)}{\sqrt[3]{c}}+\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

input
```
Int[(a + b*ArcTan[c*x^3])/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c*x^3])/x^2 + (3*b*c*(ArcTan[c^(1/3)*x]/(3*c^(1/3)) + (
-(ArcTan[Sqrt[3]*(1 - (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3)) - (Sqrt[3]*Log[1 -
Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(2*c^(1/3)))/6 + (ArcTan[Sqrt[3]*(1 + (2
*c^(1/3)*x)/Sqrt[3])]/c^(1/3) + (Sqrt[3]*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/
3)*x^2])/(2*c^(1/3)))/6))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.13

method	result
default	$-\frac{a}{2x^2}+b\left(-\frac{\arctan(c\,x^3)}{2x^2}+\frac{3c\left(-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(x^2-\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6}+\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(\cdots\right.}{\cdots}\cdots\right)}{2}\cdots\right.$
parts	$-\frac{a}{2x^2}+b\left(-\frac{\arctan(c\,x^3)}{2x^2}+\frac{3c\left(-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(x^2-\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6}+\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(\cdots\right.}{\cdots}\cdots\right)}{2}\cdots\right.$

input
```
int((a+b*arctan(c*x^3))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+b*(-1/2/x^2*arctan(c*x^3)+3/2*c*(-1/12*3^(1/2)*(1/c^2)^(1/6)*ln
(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6*(1/c^2)^(1/6)*arctan(2*x/(
1/c^2)^(1/6)-3^(1/2))+1/12*3^(1/2)*(1/c^2)^(1/6)*ln(x^2+3^(1/2)*(1/c^2)^(1
/6)*x+(1/c^2)^(1/3))+1/6*(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))+1
/3*(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(93) = 186$.

Time = 0.12 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.13

$$\int \frac{a + b \arctan(cx^3)}{x^3} \, dx = \frac{2(-b^6c^4)^{\frac{1}{6}} x^2 \log\left(bcx + (-b^6c^4)^{\frac{1}{6}}\right) - 2(-b^6c^4)^{\frac{1}{6}} x^2 \log\left(bcx - (-b^6c^4)^{\frac{1}{6}}\right) + (-b^6c^4)^{\frac{1}{6}} \left(\sqrt{-3}x^2 + x^2\right) \log}{}$$

input
```
integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="fricas")
```

output
```
1/8*(2*(-b^6*c^4)^(1/6)*x^2*log(b*c*x + (-b^6*c^4)^(1/6)) - 2*(-b^6*c^4)^(
1/6)*x^2*log(b*c*x - (-b^6*c^4)^(1/6)) + (-b^6*c^4)^(1/6)*(sqrt(-3)*x^2 +
x^2)*log(2*b*c*x + (-b^6*c^4)^(1/6)*(sqrt(-3) + 1)) - (-b^6*c^4)^(1/6)*(sq
rt(-3)*x^2 + x^2)*log(2*b*c*x - (-b^6*c^4)^(1/6)*(sqrt(-3) + 1)) + (-b^6*c
^4)^(1/6)*(sqrt(-3)*x^2 - x^2)*log(2*b*c*x + (-b^6*c^4)^(1/6)*(sqrt(-3) -
1)) - (-b^6*c^4)^(1/6)*(sqrt(-3)*x^2 - x^2)*log(2*b*c*x - (-b^6*c^4)^(1/6)
*(sqrt(-3) - 1)) - 4*b*arctan(c*x^3) - 4*a)/x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(117) = 234$.

Time = 27.39 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.93

$$\int \frac{a + b \arctan(cx^3)}{x^3} \, dx = \begin{cases} -\frac{a}{2x^2} + \frac{b \operatorname{atan}(cx^3)}{2\sqrt[3]{-\frac{1}{c^2}}} - \frac{b \operatorname{atan}(cx^3)}{2x^2} + \frac{3b \log\left(4x^2 - 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{8c\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}} - \frac{3b \log\left(4x^2 + 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{8c\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}} - \frac{\sqrt{3}b \operatorname{ata}}{} \\ -\frac{a}{2x^2} \end{cases}$$

input
```
integrate((a+b*atan(c*x**3))/x**3,x)
```

output
```
Piecewise((-a/(2*x**2) + b*atan(c*x**3)/(2*(-1/c**2)**(1/3)) - b*atan(c*x*
*3)/(2*x**2) + 3*b*log(4*x**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))
/(8*c*(-1/c**2)**(5/6)) - 3*b*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c*
*2)**(1/3))/(8*c*(-1/c**2)**(5/6)) - sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**
2)**(1/6)) - sqrt(3)/3)/(4*c*(-1/c**2)**(5/6)) - sqrt(3)*b*atan(2*sqrt(3)*
x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(4*c*(-1/c**2)**(5/6)), Ne(c, 0)), (-a
/(2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\int \frac{a + b\arctan(cx^3)}{x^3}\,dx$$

$$= \frac{1}{8}\left(\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2 + \sqrt{3}c^{\frac{1}{3}}x + 1\right)}{c^{\frac{1}{3}}} - \frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2 - \sqrt{3}c^{\frac{1}{3}}x + 1\right)}{c^{\frac{1}{3}}} + \frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{1}{3}}} + \frac{2\arctan\left(\frac{2c^{\frac{2}{3}}x}{\cdots}\right.}{c^{\frac{1}{3}}}\right.\right.$$

$$- \frac{a}{2x^2}$$

input
```
integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="maxima")
```

output
```
1/8*((sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(1/3) - sqrt(3)*l
og(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/c^(1/3) + 4*arctan(c^(1/3)*x)/c^(1
/3) + 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(1/3) + 2*arctan
((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/c^(1/3))*c - 4*arctan(c*x^3)/x^2
)*b - 1/2*a/x^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\int \frac{a+b\arctan(cx^3)}{x^3}\,dx$$
$$=\frac{1}{8}\left(\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}-\frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\arctan\left(\left(2x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\arctan\right.$$
$$-\frac{b\arctan(cx^3)+a}{2x^2}$$

input
```
integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="giac")
```

output
```
1/8*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/
3) - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/
3) + 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(1/3) + 2*
arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(1/3) + 4*arctan(
x*abs(c)^(1/3))/abs(c)^(1/3))*b*c - 1/2*(b*arctan(c*x^3) + a)/x^2
```

Mupad [B] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int \frac{a+b\arctan(cx^3)}{x^3}\,dx$$
$$=-\frac{a}{2x^2}$$
$$-\frac{b\,c^{2/3}\left(\frac{\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)}{2}-\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)}{2}+\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{2}$$
$$-\frac{b\,\mathrm{atan}(c\,x^3)}{2\,x^2}-\frac{\sqrt{3}\,b\,c^{2/3}\left(\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)+\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)\right)\,1\mathrm{i}}{4}$$

input
```
int((a + b*atan(c*x^3))/x^3,x)
```

output
```
- a/(2*x^2) - (b*c^(2/3)*(atan((-1)^(2/3)*c^(1/3)*x)/2 - atan((c^(1/3)*x*(
3^(1/2)*1i + 1))/2)/2 + atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/
2 - (b*atan(c*x^3))/(2*x^2) - (3^(1/2)*b*c^(2/3)*(atan((c^(1/3)*x*(3^(1/2)
*1i + 1))/2) + atan((-1)^(2/3)*c^(1/3)*x))*1i)/4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \frac{a+b\arctan(cx^3)}{x^3}\,dx$$
$$= \frac{6atan\left(c^{\frac{1}{3}}x\right)bc\,x^2 - 4c^{\frac{1}{3}}atan(c\,x^3)\,b + 2atan(c\,x^3)\,bc\,x^2 - 4c^{\frac{1}{3}}a - \sqrt{3}\log\left(c^{\frac{2}{3}}x^2 - c^{\frac{1}{3}}\sqrt{3}\,x+1\right)bc\,x^2 + \cdots}{8c^{\frac{1}{3}}x^2}$$

input
```
int((a+b*atan(c*x^3))/x^3,x)
```

output
```
(6*atan(c**(1/3)*x)*b*c*x**2 - 4*c**(1/3)*atan(c*x**3)*b + 2*atan(c*x**3)*
b*c*x**2 - 4*c**(1/3)*a - sqrt(3)*log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x +
 1)*b*c*x**2 + sqrt(3)*log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x + 1)*b*c*x**
2)/(8*c**(1/3)*x**2)
```

3.107 $\int \frac{a+b\arctan(cx^3)}{x^6}\,dx$

Optimal result . . . 800
Mathematica [A] (verified) . . . 800
Rubi [A] (verified) . . . 801
Maple [A] (verified) . . . 805
Fricas [A] (verification not implemented) . . . 806
Sympy [B] (verification not implemented) . . . 806
Maxima [A] (verification not implemented) . . . 807
Giac [A] (verification not implemented) . . . 807
Mupad [B] (verification not implemented) . . . 808
Reduce [B] (verification not implemented) . . . 808

Optimal result

Integrand size = 14, antiderivative size = 115

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = -\frac{3bc}{10x^2}-\frac{a+b\arctan(cx^3)}{5x^5}+\frac{1}{10}\sqrt{3}bc^{5/3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)-\frac{1}{20}bc^{5/3}\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)$$

output
```
-3/10*b*c/x^2-1/5*(a+b*arctan(c*x^3))/x^5+1/10*3^(1/2)*b*c^(5/3)*arctan(1/
3*(1-2*c^(2/3)*x^2)*3^(1/2))+1/10*b*c^(5/3)*ln(1+c^(2/3)*x^2)-1/20*b*c^(5/
3)*ln(1-c^(2/3)*x^2+c^(4/3)*x^4)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.59

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = -\frac{a}{5x^5}-\frac{3bc}{10x^2}-\frac{b\arctan(cx^3)}{5x^5}+\frac{1}{10}\sqrt{3}bc^{5/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)+\frac{1}{10}\sqrt{3}bc^{5/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)-\frac{1}{20}bc^{5/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)-\frac{1}{20}bc^{5/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x^6,x]
```

output
```
-1/5*a/x^5 - (3*b*c)/(10*x^2) - (b*ArcTan[c*x^3])/(5*x^5) + (Sqrt[3]*b*c^(
5/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/10 + (Sqrt[3]*b*c^(5/3)*ArcTan[Sqrt[3]
 + 2*c^(1/3)*x])/10 + (b*c^(5/3)*Log[1 + c^(2/3)*x^2])/10 - (b*c^(5/3)*Log
[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/20 - (b*c^(5/3)*Log[1 + Sqrt[3]*c^(
1/3)*x + c^(2/3)*x^2])/20
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {5361, 807, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^3\right)}{x^6}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{3}{5}bc\int \frac{1}{x^3\left(c^2x^6+1\right)}dx-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

$$\Big\downarrow \text{807}$$

$$\frac{3}{10}bc\int \frac{1}{x^4\left(c^2x^6+1\right)}dx^2-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

$$\Big\downarrow \text{847}$$

$$\frac{3}{10}bc\left(c^2\left(-\int \frac{x^2}{c^2x^6+1}dx^2\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

$$\Big\downarrow \text{821}$$

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{\int \frac{c^{2/3}x^2+1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{3c^{2/3}}-\frac{\int \frac{1}{c^{2/3}x^2+1}dx^2}{3c^{2/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

$$\Big\downarrow \text{16}$$

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{\int \frac{c^{2/3}x^2+1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{3c^{2/3}}-\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{4/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{\frac{3}{2}\int \frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2+\frac{\int -\frac{c^{2/3}\left(1-2c^{2/3}x^2\right)}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{2c^{2/3}}}{3c^{2/3}}-\frac{\log \left(c^{2/3}x^2+1\right)}{3c^{4/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan \left(cx^3\right)}{5x^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{\frac{3}{2}\int \frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2-\frac{\int \frac{c^{2/3}\left(1-2c^{2/3}x^2\right)}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{2c^{2/3}}}{3c^{2/3}}-\frac{\log \left(c^{2/3}x^2+1\right)}{3c^{4/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan \left(cx^3\right)}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{\frac{3}{2}\int \frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2-\frac{1}{2}\int \frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{3c^{2/3}}-\frac{\log \left(c^{2/3}x^2+1\right)}{3c^{4/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan \left(cx^3\right)}{5x^5}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{\frac{3\int \frac{1}{-x^4-3}d\left(1-2c^{2/3}x^2\right)}{c^{2/3}}-\frac{1}{2}\int \frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{3c^{2/3}}-\frac{\log \left(c^{2/3}x^2+1\right)}{3c^{4/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan \left(cx^3\right)}{5x^5}$$

$$\Big\downarrow \text{217}$$

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{-\frac{1}{2}\int \frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2-\frac{\sqrt{3}\arctan \left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{c^{2/3}}}{3c^{2/3}}-\frac{\log \left(c^{2/3}x^2+1\right)}{3c^{4/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan \left(cx^3\right)}{5x^5}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{\frac{\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{2c^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{c^{2/3}}}{3c^{2/3}}-\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{4/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

input
```
Int[(a + b*ArcTan[c*x^3])/x^6,x]
```

output
```
-1/5*(a + b*ArcTan[c*x^3])/x^5 + (3*b*c*(-x^(-2) - c^2*(-1/3*Log[1 + c^(2/
3)*x^2]/c^(4/3) + (-((Sqrt[3]*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/c^(2/3)
) + Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4]/(2*c^(2/3)))/(3*c^(2/3)))))/10
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a}{5x^5}-\frac{b\arctan(c\,x^3)}{5x^5}-\frac{3bc}{10x^2}+\frac{bc\ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right.}{3}\right.}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}$
parts	$-\frac{a}{5x^5}-\frac{b\arctan(c\,x^3)}{5x^5}-\frac{3bc}{10x^2}+\frac{bc\ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right.}{3}\right.}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}$
risch	$\frac{ib\ln(ic\,x^3+1)}{10x^5}-\frac{ib\ln(-ic\,x^3+1)}{10x^5}-\frac{3bc}{10x^2}-\frac{bc\ln\left(x+\left(\frac{i}{c}\right)^{\frac{1}{3}}\right)}{10\left(\frac{i}{c}\right)^{\frac{2}{3}}}+\frac{bc\ln\left(x^2-\left(\frac{i}{c}\right)^{\frac{1}{3}}x+\left(\frac{i}{c}\right)^{\frac{2}{3}}\right)}{20\left(\frac{i}{c}\right)^{\frac{2}{3}}}-\frac{bc\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{i}{c}\right)^{\frac{1}{3}}}-1\right.}{3}\right.}{10\left(\frac{i}{c}\right)^{\frac{2}{3}}}$

input

```
int((a+b*arctan(c*x^3))/x^6,x,method=_RETURNVERBOSE)
```

output

```
-1/5*a/x^5-1/5*b*arctan(c*x^3)/x^5-3/10*b*c/x^2+1/10*b*c/(1/c^2)^(1/3)*ln(
x^2+(1/c^2)^(1/3))-1/20*b*c/(1/c^2)^(1/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)
^(2/3))-1/10*b*c*3^(1/2)/(1/c^2)^(1/3)*arctan(1/3*3^(1/2)*(2*x^2/(1/c^2)^(
1/3)-1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{a + b \arctan(cx^3)}{x^6} \, dx =$$
$$-\frac{2\sqrt{3}b(c^2)^{\frac{1}{3}}cx^5 \arctan\left(\frac{2}{3}\sqrt{3}(c^2)^{\frac{1}{3}}x^2 - \frac{1}{3}\sqrt{3}\right) + b(c^2)^{\frac{1}{3}}cx^5 \log\left(c^2x^4 - (c^2)^{\frac{2}{3}}x^2 + (c^2)^{\frac{1}{3}}\right) - 2b(c^2)^{\frac{1}{3}}cx^5 \mathrm{l}}{20\,x^5}$$

input
```
integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="fricas")
```

output
```
-1/20*(2*sqrt(3)*b*(c^2)^(1/3)*c*x^5*arctan(2/3*sqrt(3)*(c^2)^(1/3)*x^2 -
1/3*sqrt(3)) + b*(c^2)^(1/3)*c*x^5*log(c^2*x^4 - (c^2)^(2/3)*x^2 + (c^2)^(
1/3)) - 2*b*(c^2)^(1/3)*c*x^5*log(c^2*x^2 + (c^2)^(2/3)) + 6*b*c*x^3 + 4*b
*arctan(c*x^3) + 4*a)/x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(110) = 220$.

Time = 55.21 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.49

$$\int \frac{a + b \arctan(cx^3)}{x^6} \, dx$$
$$= \begin{cases} -\frac{a}{5x^5} + \frac{bc^2\sqrt[6]{-\frac{1}{c^2}}\operatorname{atan}(cx^3)}{5} - \frac{bc\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{5\sqrt[3]{-\frac{1}{c^2}}} + \frac{3bc\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{20\sqrt[3]{-\frac{1}{c^2}}} - \frac{bc\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{\cdots}\right.}{20\sqrt[3]{-\frac{1}{c^2}}} \\ -\frac{a}{5x^5} \end{cases}$$

input
```
integrate((a+b*atan(c*x**3))/x**6,x)
```

output
```
Piecewise((-a/(5*x**5) + b*c**2*(-1/c**2)**(1/6)*atan(c*x**3)/5 - b*c*log(
x - (-1/c**2)**(1/6))/(5*(-1/c**2)**(1/3)) + 3*b*c*log(4*x**2 - 4*x*(-1/c*
*2)**(1/6) + 4*(-1/c**2)**(1/3))/(20*(-1/c**2)**(1/3)) - b*c*log(4*x**2 +
4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(20*(-1/c**2)**(1/3)) - sqrt(3)
*b*c*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/(10*(-1/c**2)**(1/
3)) + sqrt(3)*b*c*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(10*(
-1/c**2)**(1/3)) - 3*b*c/(10*x**2) - b*atan(c*x**3)/(5*x**5), Ne(c, 0)), (
-a/(5*x**5), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = -\frac{1}{20}\left(\left(2\sqrt{3}c^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)+c^{\frac{2}{3}}\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)-2c^{\frac{2}{3}}\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)+\frac{6}{x^2}\right.\right.$$
$$-\frac{a}{5x^5}$$

input
```
integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="maxima")
```

output
```
-1/20*((2*sqrt(3)*c^(2/3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(
2/3)) + c^(2/3)*log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1) - 2*c^(2/3)*log((c^(2/3
)*x^2 + 1)/c^(2/3)) + 6/x^2)*c + 4*arctan(c*x^3)/x^5)*b - 1/5*a/x^5
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = -\frac{1}{20}bc^3\left(\frac{2\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2}+\frac{|c|^{\frac{2}{3}}\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2}-\frac{2\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{4}{3}}}\right.$$
$$-\frac{3bcx^3+2b\arctan(cx^3)+2a}{10x^5}$$

input
```
integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="giac")
```

output
```
-1/20*b*c^3*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(
2/3))*abs(c)^(2/3))/c^2 + abs(c)^(2/3)*log(x^4 - x^2/abs(c)^(2/3) + 1/abs(
c)^(4/3))/c^2 - 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(4/3)) - 1/10*(3*b*c*x^
3 + 2*b*arctan(c*x^3) + 2*a)/x^5
```

Mupad [B] (verification not implemented)

Time = 2.99 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = \frac{b\,c^{5/3}\,\ln\left(c^{2/3}\,x^2+1\right)}{10} - \frac{\frac{3\,b\,c\,x^3}{2}+a}{5\,x^5} - \frac{b\,\mathrm{atan}(c\,x^3)}{5\,x^5} - \frac{b\,c^{5/3}\,\ln\left(\sqrt{3}\,c^{2/3}\,x^2 - c^{2/3}\,x^2\,1\mathrm{i}+2\mathrm{i}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{20} + \frac{b\,c^{5/3}\,\ln\left(-c^{2/3}\,x^2\,1\mathrm{i}-\sqrt{3}\,c^{2/3}\,x^2+2\mathrm{i}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{20}$$

input
```
int((a + b*atan(c*x^3))/x^6,x)
```

output
```
(b*c^(5/3)*log(c^(2/3)*x^2 + 1))/10 - (a + (3*b*c*x^3)/2)/(5*x^5) - (b*ata
n(c*x^3))/(5*x^5) - (b*c^(5/3)*log(3^(1/2)*c^(2/3)*x^2 - c^(2/3)*x^2*1i +
2i)*(3^(1/2)*1i + 1))/20 + (b*c^(5/3)*log(2i - 3^(1/2)*c^(2/3)*x^2 - c^(2/
3)*x^2*1i)*(3^(1/2)*1i - 1))/20
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.43

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = \frac{-4\sqrt{3}\,atan\left(2c^{\frac{1}{3}}x-\sqrt{3}\right)b\,c^2x^5 + 2\sqrt{3}\,atan\left(c^{\frac{1}{3}}x\right)b\,c^2x^5 - 4c^{\frac{1}{3}}\,atan(c\,x^3)\,b + 2\sqrt{3}\,atan(c\,x^3)\,b\,c^2x^5 - 4c}{20}$$

input
```
int((a+b*atan(c*x^3))/x^6,x)
```

output
```
( - 4*sqrt(3)*atan(2*c**(1/3)*x - sqrt(3))*b*c**2*x**5 + 2*sqrt(3)*atan(c*
*(1/3)*x)*b*c**2*x**5 - 4*c**(1/3)*atan(c*x**3)*b + 2*sqrt(3)*atan(c*x**3)
*b*c**2*x**5 - 4*c**(1/3)*a - 6*c**(1/3)*b*c*x**3 - log(c**(2/3)*x**2 - c*
*(1/3)*sqrt(3)*x + 1)*b*c**2*x**5 - log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x
 + 1)*b*c**2*x**5 + 2*log(c**(2/3)*x**2 + 1)*b*c**2*x**5)/(20*c**(1/3)*x**
5)
```

3.108 $\int x^7(a+b\arctan(cx^3))\,dx$

Optimal result 810
Mathematica [A] (verified) 811
Rubi [A] (verified) 811
Maple [A] (verified) 816
Fricas [B] (verification not implemented) 817
Sympy [B] (verification not implemented) 817
Maxima [A] (verification not implemented) 818
Giac [A] (verification not implemented) 819
Mupad [B] (verification not implemented) 820
Reduce [B] (verification not implemented) 820

Optimal result

Integrand size = 14, antiderivative size = 138

$$\int x^7(a+b\arctan(cx^3))\,dx = -\frac{3bx^5}{40c}+\frac{b\arctan(\sqrt[3]{c}x)}{8c^{8/3}}+\frac{1}{8}x^8(a+b\arctan(cx^3))-\frac{b\arctan(\sqrt{3}-2\sqrt[3]{c}x)}{16c^{8/3}}+\frac{b\arctan(\sqrt{3}+2\sqrt[3]{c}x)}{16c^{8/3}}-\frac{\sqrt{3}b\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[3]{c}x}{1+c^{2/3}x^2}\right)}{16c^{8/3}}$$

output
```
-3/40*b*x^5/c+1/8*b*arctan(c^(1/3)*x)/c^(8/3)+1/8*x^8*(a+b*arctan(c*x^3))+
1/16*b*arctan(-3^(1/2)+2*c^(1/3)*x)/c^(8/3)+1/16*b*arctan(3^(1/2)+2*c^(1/3
)*x)/c^(8/3)-1/16*3^(1/2)*b*arctanh(3^(1/2)*c^(1/3)*x/(1+c^(2/3)*x^2))/c^(
8/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.31

$$\int x^7\left(a+b\arctan\left(cx^3\right)\right)\,dx=-\frac{3bx^5}{40c}+\frac{ax^8}{8}+\frac{b\arctan\left(\sqrt[3]{c}x\right)}{8c^{8/3}}+\frac{1}{8}bx^8\arctan\left(cx^3\right)-\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{16c^{8/3}}+\frac{b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{16c^{8/3}}+\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}}-\frac{\sqrt{3}b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}}$$

input
```
Integrate[x^7*(a + b*ArcTan[c*x^3]),x]
```

output
```
(-3*b*x^5)/(40*c) + (a*x^8)/8 + (b*ArcTan[c^(1/3)*x])/(8*c^(8/3)) + (b*x^8
*ArcTan[c*x^3])/8 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(16*c^(8/3)) + (b*Ar
cTan[Sqrt[3] + 2*c^(1/3)*x])/(16*c^(8/3)) + (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(
1/3)*x + c^(2/3)*x^2])/(32*c^(8/3)) - (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x
 + c^(2/3)*x^2])/(32*c^(8/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.54, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5361, 843, 824, 27, 216, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\left(a+b\arctan\left(cx^3\right)\right)\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\int\frac{x^{10}}{c^2x^6+1}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{\int \frac{x^4}{c^2x^6+1}dx}{c^2}\right)$$

$\Big\downarrow$ 824

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{\frac{\int \frac{1}{c^{2/3}x^2+1}dx}{3c^{4/3}}+\frac{\int -\frac{1-\sqrt{3}\sqrt[3]{c}x}{2\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}+\frac{\int -\frac{\sqrt{3}\sqrt[3]{c}x+1}{2\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}}{c^2}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{\frac{\int \frac{1}{c^{2/3}x^2+1}dx}{3c^{4/3}}-\frac{\int \frac{1-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int \frac{\sqrt{3}\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}}{c^2}\right)$$

$\Big\downarrow$ 216

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{-\frac{\int \frac{1-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int \frac{\sqrt{3}\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\arctan\left(\sqrt[3]{c}x\right)}{3c^{5/3}}}{c^2}\right)$$

$\Big\downarrow$ 1142

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{-\frac{-\frac{1}{2}\int \frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\sqrt{3}\int -\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int \frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\text{arct}\cdots}{c^2}\right.$$

$\Big\downarrow$ 25

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{-\frac{\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\cdots}{c^2}\right.$$

$$\downarrow 27$$

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\cdots}{c^2}\right.$$

$$\downarrow 1082$$

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\int\frac{1}{-\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\int\frac{1}{-\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)^2-\frac{1}{3}}d\cdots}{\sqrt{3}\sqrt[3]{c}}}{6c^{4/3}}\cdots}{c^2}\right.$$

$$\downarrow 217$$

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)\right)}{\sqrt[3]{c}}}{6c^{4/3}}+\cdots}{c^2}\right.$$

$$\downarrow 1103$$

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-$$

$$\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}-\frac{\sqrt{3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}-\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)\right)}{\sqrt[3]{c}}}{6c^{4/3}}+\frac{\arctan}{3c}}{c^2}\right.$$

input
```
Int[x^7*(a + b*ArcTan[c*x^3]),x]
```

output
```
(x^8*(a + b*ArcTan[c*x^3]))/8 - (3*b*c*(x^5/(5*c^2) - (ArcTan[c^(1/3)*x]/(
3*c^(5/3)) - (ArcTan[Sqrt[3]*(1 - (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3) - (Sqrt[
3]*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(2*c^(1/3)))/(6*c^(4/3)) - (-
(ArcTan[Sqrt[3]*(1 + (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3)) + (Sqrt[3]*Log[1 + S
qrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(2*c^(1/3)))/(6*c^(4/3)))/c^2))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.21

method	result
default	$\frac{a\,x^{8}}{8}+b\left(\frac{x^{8}\arctan\left(c\,x^{3}\right)}{8}-\frac{3c\left(\frac{x^{5}}{5c^{2}}-\frac{\frac{\sqrt{3}\left(\frac{1}{c^{2}}\right)^{\frac{5}{6}}\ln\left(x^{2}-\sqrt{3}\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6c^{2}\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{1}{c^{2}}\right)^{\frac{5}{6}}\ln\left(x^{2}+\sqrt{3}\right.}{1}}{c^{2}}\right.}{8}\right.$
parts	$\frac{a\,x^{8}}{8}+b\left(\frac{x^{8}\arctan\left(c\,x^{3}\right)}{8}-\frac{3c\left(\frac{x^{5}}{5c^{2}}-\frac{\frac{\sqrt{3}\left(\frac{1}{c^{2}}\right)^{\frac{5}{6}}\ln\left(x^{2}-\sqrt{3}\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6c^{2}\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{1}{c^{2}}\right)^{\frac{5}{6}}\ln\left(x^{2}+\sqrt{3}\right.}{1}}{c^{2}}\right.}{8}\right.$

input
```
int(x^7*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/8*a*x^8+b*(1/8*x^8*arctan(c*x^3)-3/8*c*(1/5/c^2*x^5-(1/12*3^(1/2)*(1/c^2
)^(5/6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6
)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))-1/12*3^(1/2)*(1/c^2)^(5/6)*ln(x^2+3^(1
/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)*arctan(2*x/(1/c^2
)^(1/6)+3^(1/2))+1/3/c^2/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6)))/c^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(102) = 204$.

Time = 0.11 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.21

$$\int x^7(a + b\arctan(cx^3))\,dx = \frac{20\,bcx^8\arctan(cx^3) + 20\,acx^8 - 12\,bx^5 + 10\,c\left(-\frac{b^6}{c^{16}}\right)^{\frac{1}{6}}\log\left(c^{13}\left(-\frac{b^6}{c^{16}}\right)^{\frac{5}{6}} + b^5x\right) - 10\,c\left(-\frac{b^6}{c^{16}}\right)^{\frac{1}{6}}\log\left(-c\right.}{}$$

input
```
integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/160*(20*b*c*x^8*arctan(c*x^3) + 20*a*c*x^8 - 12*b*x^5 + 10*c*(-b^6/c^16)
^(1/6)*log(c^13*(-b^6/c^16)^(5/6) + b^5*x) - 10*c*(-b^6/c^16)^(1/6)*log(-c
^13*(-b^6/c^16)^(5/6) + b^5*x) - 5*(sqrt(-3)*c - c)*(-b^6/c^16)^(1/6)*log(
b^5*x + 1/2*(sqrt(-3)*c^13 + c^13)*(-b^6/c^16)^(5/6)) + 5*(sqrt(-3)*c - c)
*(-b^6/c^16)^(1/6)*log(b^5*x - 1/2*(sqrt(-3)*c^13 + c^13)*(-b^6/c^16)^(5/6
)) - 5*(sqrt(-3)*c + c)*(-b^6/c^16)^(1/6)*log(b^5*x + 1/2*(sqrt(-3)*c^13 -
 c^13)*(-b^6/c^16)^(5/6)) + 5*(sqrt(-3)*c + c)*(-b^6/c^16)^(1/6)*log(b^5*x
 - 1/2*(sqrt(-3)*c^13 - c^13)*(-b^6/c^16)^(5/6)))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(126) = 252$.

Time = 57.49 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.91

$$\int x^7 \left(a + b \arctan\left(cx^3\right)\right) \, dx$$
$$= \begin{cases} \frac{ax^8}{8} + \frac{bx^8 \operatorname{atan}\left(cx^3\right)}{8} - \frac{3bx^5}{40c} + \frac{3b \log\left(4x^2 - 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{32c^3\sqrt[6]{-\frac{1}{c^2}}} - \frac{3b \log\left(4x^2 + 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{32c^3\sqrt[6]{-\frac{1}{c^2}}} + \frac{\sqrt{3}b \operatorname{atan}\left(\frac{2 \cdots}{3\sqrt[6]{\cdots}}\right)}{16c^3\sqrt[6]{\cdots}} \\ \frac{ax^8}{8} \end{cases}$$

input
```
integrate(x**7*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**8/8 + b*x**8*atan(c*x**3)/8 - 3*b*x**5/(40*c) + 3*b*log(4*
x**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(32*c**3*(-1/c**2)**(1/6
)) - 3*b*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(32*c**3*
(-1/c**2)**(1/6)) + sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt
(3)/3)/(16*c**3*(-1/c**2)**(1/6)) + sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**2
)**(1/6)) + sqrt(3)/3)/(16*c**3*(-1/c**2)**(1/6)) - b*atan(c*x**3)/(8*c**4
*(-1/c**2)**(2/3)), Ne(c, 0)), (a*x**8/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.10

$$\int x^7 \left(a + b \arctan\left(cx^3\right)\right) \, dx = \frac{1}{8}ax^8$$
$$+ \frac{1}{160}\left(20\,x^8 \arctan\left(cx^3\right) - \left(\frac{12\,x^5}{c^2} + \frac{5\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2 + \sqrt{3}c^{\frac{1}{3}}x + 1\right)}{c^{\frac{5}{3}}} - \frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2 - \sqrt{3}c^{\frac{1}{3}}x + 1\right)}{c^{\frac{5}{3}}} - \frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{5}{3}}} - \cdots\right)}{c^2} \cdots\right.\right.$$

input
```
integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/8*a*x^8 + 1/160*(20*x^8*arctan(c*x^3) - (12*x^5/c^2 + 5*(sqrt(3)*log(c^(
2/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt
(3)*c^(1/3)*x + 1)/c^(5/3) - 4*arctan(c^(1/3)*x)/c^(5/3) - 2*arctan((2*c^(
2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3) - 2*arctan((2*c^(2/3)*x - sqrt(
3)*c^(1/3))/c^(1/3))/c^(5/3))/c^2)*c)*b
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.24

$$\int x^7(a+b\arctan(cx^3))\,dx = -\frac{1}{32}bc^{15}\left(\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{18}}-\frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{16}|c|^{\frac{5}{3}}}-\frac{2\arctan\left(\left(2x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^{16}|c|^{\frac{5}{3}}}\right. + \frac{5bcx^8\arctan(cx^3)+5acx^8-3bx^5}{40c}$$

input
```
integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
-1/32*b*c^15*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/ab
s(c)^(2/3))/c^18 - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/
3))/(c^16*abs(c)^(5/3)) - 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/
3))/(c^16*abs(c)^(5/3)) - 2*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/
3))/(c^16*abs(c)^(5/3)) - 4*arctan(x*abs(c)^(1/3))/(c^16*abs(c)^(5/3))) +
1/40*(5*b*c*x^8*arctan(c*x^3) + 5*a*c*x^8 - 3*b*x^5)/c
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.88

$$\int x^7 (a + b \arctan (cx^3)) \, dx = \frac{a\,x^8}{8} - \frac{3\,b\,x^5}{40\,c} - \frac{b\left(\operatorname{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right) + \operatorname{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right) + 2\operatorname{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{16\,c^{8/3}} + \frac{b\,x^8\operatorname{atan}(c\,x^3)}{8} + \frac{\sqrt{3}\,b\left(\operatorname{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right) - \operatorname{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)\,1\mathrm{i}}{16\,c^{8/3}}$$

input
```
int(x^7*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^8)/8 - (3*b*x^5)/(40*c) - (b*(atan((-1)^(2/3)*c^(1/3)*x) + atan(((-1)
^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1
/2)*1i + 1))/2)))/(16*c^(8/3)) + (b*x^8*atan(c*x^3))/8 + (3^(1/2)*b*(atan(
(-1)^(2/3)*c^(1/3)*x) - atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2))*1
i)/(16*c^(8/3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int x^7 (a + b \arctan (cx^3)) \, dx = \frac{30c^{\frac{1}{3}}\,atan\left(c^{\frac{1}{3}}x\right)b + 10c^{\frac{1}{3}}\,atan(c\,x^3)\,b + 20atan(c\,x^3)\,b\,c^3x^8 + 5c^{\frac{1}{3}}\sqrt{3}\log\left(c^{\frac{2}{3}}x^2 - c^{\frac{1}{3}}\sqrt{3}\,x + 1\right)b - 5c^{\frac{1}{3}}\sqrt{3}}{160c^3}$$

input
```
int(x^7*(a+b*atan(c*x^3)),x)
```

output
```
(30*c**(1/3)*atan(c**(1/3)*x)*b + 10*c**(1/3)*atan(c*x**3)*b + 20*atan(c*x
**3)*b*c**3*x**8 + 5*c**(1/3)*sqrt(3)*log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)
*x + 1)*b - 5*c**(1/3)*sqrt(3)*log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x + 1)
*b + 20*a*c**3*x**8 - 12*b*c**2*x**5)/(160*c**3)
```

3.109 $\int x^4(a+b\arctan(cx^3))\,dx$

Optimal result 821
Mathematica [A] (verified) 821
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Optimal result

Integrand size = 14, antiderivative size = 117

$$\int x^4(a+b\arctan(cx^3))\,dx = -\frac{3bx^2}{10c}+\frac{1}{5}x^5(a+b\arctan(cx^3))-\frac{\sqrt{3}b\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}} + \frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{20c^{5/3}}$$

output
```
-3/10*b*x^2/c+1/5*x^5*(a+b*arctan(c*x^3))-1/10*3^(1/2)*b*arctan(1/3*(1-2*c
^(2/3)*x^2)*3^(1/2))/c^(5/3)+1/10*b*ln(1+c^(2/3)*x^2)/c^(5/3)-1/20*b*ln(1-
c^(2/3)*x^2+c^(4/3)*x^4)/c^(5/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.58

$$\int x^4(a+b\arctan(cx^3))\,dx = -\frac{3bx^2}{10c}+\frac{ax^5}{5}+\frac{1}{5}bx^5\arctan(cx^3) - \frac{\sqrt{3}b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{10c^{5/3}}-\frac{\sqrt{3}b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{10c^{5/3}} + \frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{20c^{5/3}} - \frac{b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{20c^{5/3}}$$

input
```
Integrate[x^4*(a + b*ArcTan[c*x^3]),x]
```

output
```
(-3*b*x^2)/(10*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x^3])/5 - (Sqrt[3]*b*ArcTa
n[Sqrt[3] - 2*c^(1/3)*x])/(10*c^(5/3)) - (Sqrt[3]*b*ArcTan[Sqrt[3] + 2*c^(
1/3)*x])/(10*c^(5/3)) + (b*Log[1 + c^(2/3)*x^2])/(10*c^(5/3)) - (b*Log[1 -
 Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(20*c^(5/3)) - (b*Log[1 + Sqrt[3]*c^(1/
3)*x + c^(2/3)*x^2])/(20*c^(5/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5361, 807, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+b \arctan \left(c x^3\right)\right) \, dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{5} x^5 \left(a+b \arctan \left(c x^3\right)\right)-\frac{3}{5} b c \int \frac{x^7}{c^2 x^6+1}dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{5} x^5 \left(a+b \arctan \left(c x^3\right)\right)-\frac{3}{10} b c \int \frac{x^6}{c^2 x^6+1}dx^2$$

$$\Big\downarrow \text{ 843}$$

$$\frac{1}{5} x^5 \left(a+b \arctan \left(c x^3\right)\right)-\frac{3}{10} b c \left(\frac{x^2}{c^2}-\frac{\int \frac{1}{c^2 x^6+1}dx^2}{c^2}\right)$$

$$\Big\downarrow \text{ 750}$$

$$\frac{1}{5} x^5 \left(a+b \arctan \left(c x^3\right)\right)-\frac{3}{10} b c \left(\frac{x^2}{c^2}-\frac{\frac{1}{3} \int \frac{1}{c^{2/3} x^2+1}dx^2+\frac{1}{3} \int \frac{2-c^{2/3} x^2}{c^{4/3} x^4-c^{2/3} x^2+1}dx^2}{c^2}\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\frac{1}{3}\int\frac{2-c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}}{c^2}\right)$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2-\frac{\int-\frac{c^{2/3}\left(1-2c^{2/3}x^2\right)}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{2c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}}{c^2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2+\frac{\int\frac{c^{2/3}\left(1-2c^{2/3}x^2\right)}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{2c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}}{c^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2+\frac{1}{2}\int\frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}}{c^2}\right)$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\frac{1}{3}\left(\frac{3\int\frac{1}{-x^4-3}d\left(1-2c^{2/3}x^2\right)}{c^{2/3}}+\frac{1}{2}\int\frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}}{c^2}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\frac{1}{3}\left(\frac{1}{2}\int\frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2-\frac{\sqrt{3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}}{c^2}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{c^{2/3}}-\frac{\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{2c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}}{c^2}\right)$$

input
```
Int[x^4*(a + b*ArcTan[c*x^3]),x]
```

output
```
(x^5*(a + b*ArcTan[c*x^3]))/5 - (3*b*c*(x^2/c^2 - (Log[1 + c^(2/3)*x^2]/(3
*c^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/c^(2/3)) - Lo
g[1 - c^(2/3)*x^2 + c^(4/3)*x^4]/(2*c^(2/3)))/3)/c^2))/10
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

method	result
default	$\frac{a x^5}{5}+\frac{b x^5 \arctan\left(c x^3\right)}{5}-\frac{3 b x^2}{10 c}+\frac{b \ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10 c^3 \left(\frac{1}{c^2}\right)^{\frac{2}{3}}}-\frac{b \ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}} x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20 c^3 \left(\frac{1}{c^2}\right)^{\frac{2}{3}}}+\frac{b \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\frac{2 x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{10 c^3 \left(\frac{1}{c^2}\right)^{\frac{2}{3}}}$
parts	$\frac{a x^5}{5}+\frac{b x^5 \arctan\left(c x^3\right)}{5}-\frac{3 b x^2}{10 c}+\frac{b \ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10 c^3 \left(\frac{1}{c^2}\right)^{\frac{2}{3}}}-\frac{b \ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}} x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20 c^3 \left(\frac{1}{c^2}\right)^{\frac{2}{3}}}+\frac{b \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\frac{2 x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{10 c^3 \left(\frac{1}{c^2}\right)^{\frac{2}{3}}}$
risch	$-\frac{i x^5 b \ln\left(i c x^3+1\right)}{10}+\frac{i b x^5 \ln\left(-i c x^3+1\right)}{10}-\frac{3 b x^2}{10 c}-\frac{i b \ln\left(x+\left(\frac{i}{c}\right)^{\frac{1}{3}}\right)}{10 c^2 \left(\frac{i}{c}\right)^{\frac{1}{3}}}+\frac{i b \ln\left(x^2-\left(\frac{i}{c}\right)^{\frac{1}{3}} x+\left(\frac{i}{c}\right)^{\frac{2}{3}}\right)}{20 c^2 \left(\frac{i}{c}\right)^{\frac{1}{3}}}+\frac{i b \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\cdots\right)}{\cdots}\right)}{10 c^2 \left(\frac{i}{c}\right)^{\frac{1}{3}}}$

input
```
int(x^4*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+1/5*b*x^5*arctan(c*x^3)-3/10*b*x^2/c+1/10*b/c^3/(1/c^2)^(2/3)*ln
(x^2+(1/c^2)^(1/3))-1/20*b/c^3/(1/c^2)^(2/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c
^2)^(2/3))+1/10*b/c^3/(1/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*x^2/(1/c
^2)^(1/3)-1))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.17

$$\int x^4(a+b\arctan(cx^3))\,dx$$
$$=\frac{4\,bc^3x^5\arctan(cx^3)+4\,ac^3x^5-6\,bc^2x^2+2\,\sqrt{3}b(c^2)^{\frac{1}{6}}c\arctan\left(\frac{\sqrt{3}\left(2\,(c^2)^{\frac{2}{3}}x^2-(c^2)^{\frac{1}{3}}\right)(c^2)^{\frac{1}{6}}}{3\,c}\right)-b(c^2)^{\frac{2}{3}}\log\left(c\right.}{20\,c^3}$$

input
```
integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/20*(4*b*c^3*x^5*arctan(c*x^3) + 4*a*c^3*x^5 - 6*b*c^2*x^2 + 2*sqrt(3)*b*
(c^2)^(1/6)*c*arctan(1/3*sqrt(3)*(2*(c^2)^(2/3)*x^2 - (c^2)^(1/3))*(c^2)^(
1/6)/c) - b*(c^2)^(2/3)*log(c^2*x^4 - (c^2)^(2/3)*x^2 + (c^2)^(1/3)) + 2*b
*(c^2)^(2/3)*log(c^2*x^2 + (c^2)^(2/3)))/c^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(109) = 218$.

Time = 27.96 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.50

$$\int x^4(a+b\arctan(cx^3))\,dx$$
$$=\begin{cases}\frac{ax^5}{5}-\frac{bc^3\left(-\frac{1}{c^2}\right)^{\frac{7}{3}}\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{5}+\frac{3bc^3\left(-\frac{1}{c^2}\right)^{\frac{7}{3}}\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{20}-\frac{bc^3\left(-\frac{1}{c^2}\right)^{\frac{7}{3}}\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{\right.}\right.}{20} \\ \frac{ax^5}{5}\end{cases}$$

input
```
integrate(x**4*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**5/5 - b*c**3*(-1/c**2)**(7/3)*log(x - (-1/c**2)**(1/6))/5
+ 3*b*c**3*(-1/c**2)**(7/3)*log(4*x**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2
)**(1/3))/20 - b*c**3*(-1/c**2)**(7/3)*log(4*x**2 + 4*x*(-1/c**2)**(1/6) +
 4*(-1/c**2)**(1/3))/20 + sqrt(3)*b*c**3*(-1/c**2)**(7/3)*atan(2*sqrt(3)*x
/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/10 - sqrt(3)*b*c**3*(-1/c**2)**(7/3)*at
an(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/10 - b*c**2*(-1/c**2)**(1
1/6)*atan(c*x**3)/5 + b*x**5*atan(c*x**3)/5 - 3*b*x**2/(10*c), Ne(c, 0)),
(a*x**5/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int x^4\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{5}\,ax^5+\frac{1}{20}\left(4\,x^5\arctan\left(cx^3\right)-c\left(\frac{6\,x^2}{c^2}-\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3\,c^{\frac{2}{3}}}\right)}{c^{\frac{8}{3}}}+\frac{\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{8}{3}}}-\frac{2\,\log\left(\right.}{c}\right.\right.$$

input
```
integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/20*(4*x^5*arctan(c*x^3) - c*(6*x^2/c^2 - 2*sqrt(3)*arctan(1/
3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2/3))/c^(8/3) + log(c^(4/3)*x^4 - c
^(2/3)*x^2 + 1)/c^(8/3) - 2*log((c^(2/3)*x^2 + 1)/c^(2/3))/c^(8/3)))*b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^4\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{20}\,bc^9\left(\frac{2\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^{10}|c|^{\frac{2}{3}}}-\frac{\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}}+\frac{2\,\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}}\right)+\frac{2\,bcx^5\arctan\left(cx^3\right)+2\,acx^5-3\,bx^2}{10\,c}$$

input
```
integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
1/20*b*c^9*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))*abs(c)^(
2/3))/(c^10*abs(c)^(2/3)) - log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/(
c^10*abs(c)^(2/3)) + 2*log(x^2 + 1/abs(c)^(2/3))/(c^10*abs(c)^(2/3))) + 1/
10*(2*b*c*x^5*arctan(c*x^3) + 2*a*c*x^5 - 3*b*x^2)/c
```

Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int x^4\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{a\,x^5}{5}+\frac{b\,\ln\left(c^{2/3}\,x^2+1\right)}{10\,c^{5/3}}-\frac{3\,b\,x^2}{10\,c}-\frac{\ln\left(1-2\,c^{2/3}\,x^2+\sqrt{3}\,1\mathrm{i}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{20\,c^{5/3}}-\frac{\ln\left(2\,c^{2/3}\,x^2-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{20\,c^{5/3}}+\frac{b\,x^5\,\mathrm{atan}(c\,x^3)}{5}$$

input
```
int(x^4*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^5)/5 + (b*log(c^(2/3)*x^2 + 1))/(10*c^(5/3)) - (3*b*x^2)/(10*c) - (lo
g(3^(1/2)*1i - 2*c^(2/3)*x^2 + 1)*(b + 3^(1/2)*b*1i))/(20*c^(5/3)) - (log(
3^(1/2)*1i + 2*c^(2/3)*x^2 - 1)*(b - 3^(1/2)*b*1i))/(20*c^(5/3)) + (b*x^5*
atan(c*x^3))/5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.26

$$\int x^4\left(a+b\arctan\left(cx^3\right)\right)\,dx$$
$$=\frac{4c^{\frac{1}{3}}\sqrt{3}\,atan\left(2c^{\frac{1}{3}}x-\sqrt{3}\right)b-2c^{\frac{1}{3}}\sqrt{3}\,atan\left(c^{\frac{1}{3}}x\right)b-2c^{\frac{1}{3}}\sqrt{3}\,atan(c\,x^3)\,b+4atan(c\,x^3)\,b\,c^2x^5-c^{\frac{1}{3}}\log\left(c^{\frac{2}{3}}\right.}{20c^2}$$

input
```
int(x^4*(a+b*atan(c*x^3)),x)
```

output
```
(4*c**(1/3)*sqrt(3)*atan(2*c**(1/3)*x - sqrt(3))*b - 2*c**(1/3)*sqrt(3)*at
an(c**(1/3)*x)*b - 2*c**(1/3)*sqrt(3)*atan(c*x**3)*b + 4*atan(c*x**3)*b*c*
*2*x**5 - c**(1/3)*log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x + 1)*b - c**(1/3
)*log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x + 1)*b + 2*c**(1/3)*log(c**(2/3)*
x**2 + 1)*b + 4*a*c**2*x**5 - 6*b*c*x**2)/(20*c**2)
```

3.110 $\int x(a+b\arctan(cx^3))\,dx$

Optimal result . . . 831
Mathematica [A] (verified) . . . 832
Rubi [A] (verified) . . . 832
Maple [A] (verified) . . . 836
Fricas [B] (verification not implemented) . . . 838
Sympy [B] (verification not implemented) . . . 839
Maxima [A] (verification not implemented) . . . 840
Giac [A] (verification not implemented) . . . 840
Mupad [B] (verification not implemented) . . . 841
Reduce [B] (verification not implemented) . . . 841

Optimal result

Integrand size = 12, antiderivative size = 127

$$\int x(a+b\arctan(cx^3))\,dx = -\frac{b\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}} + \frac{1}{2}x^2(a+b\arctan(cx^3)) + \frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{4c^{2/3}} - \frac{b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{4c^{2/3}} + \frac{\sqrt{3}b\,\text{arctanh}\left(\frac{\sqrt{3}\sqrt[3]{c}x}{1+c^{2/3}x^2}\right)}{4c^{2/3}}$$

output
```
-1/2*b*arctan(c^(1/3)*x)/c^(2/3)+1/2*x^2*(a+b*arctan(c*x^3))-1/4*b*arctan(
-3^(1/2)+2*c^(1/3)*x)/c^(2/3)-1/4*b*arctan(3^(1/2)+2*c^(1/3)*x)/c^(2/3)+1/
4*3^(1/2)*b*arctanh(3^(1/2)*c^(1/3)*x/(1+c^(2/3)*x^2))/c^(2/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.34

$$\int x\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{ax^2}{2}-\frac{b\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}bx^2\arctan\left(cx^3\right)+\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}+\frac{\sqrt{3}b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}$$

input
```
Integrate[x*(a + b*ArcTan[c*x^3]),x]
```

output
```
(a*x^2)/2 - (b*ArcTan[c^(1/3)*x])/(2*c^(2/3)) + (b*x^2*ArcTan[c*x^3])/2 +
(b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(4*c^(2/3)) - (b*ArcTan[Sqrt[3] + 2*c^(1
/3)*x])/(4*c^(2/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])
/(8*c^(2/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(
2/3))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.54, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5361, 824, 27, 216, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\arctan\left(cx^3\right)\right)\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{2}bc\int\frac{x^4}{c^2x^6+1}dx$$

$$\Big\downarrow \text{824}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{2}bc\left(\frac{\int\frac{1}{c^{2/3}x^2+1}dx}{3c^{4/3}}+\frac{\int-\frac{1-\sqrt{3}\sqrt[3]{c}x}{2\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}+\frac{\int-\frac{\sqrt{3}\sqrt[3]{c}x+1}{2\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{2}bc\left(\frac{\int\frac{1}{c^{2/3}x^2+1}dx}{3c^{4/3}}-\frac{\int\frac{1-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int\frac{\sqrt{3}\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{2}bc\left(-\frac{\int\frac{1-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int\frac{\sqrt{3}\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\arctan\left(\sqrt[3]{c}x\right)}{3c^{5/3}}\right)$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{2}bc\left(-\frac{-\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\sqrt{3}\int-\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\right.$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{2}bc\left(-\frac{\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\text{arc}}{}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-$$
$$\frac{3}{2}bc\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}d\right.$$

$\downarrow$ 1082

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-$$
$$\frac{3}{2}bc\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\int\frac{1}{-\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\int\frac{1}{-\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)^2}}{\sqrt{3}}}{6c^{4/3}}\right.$$

$\downarrow$ 217

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-$$
$$\frac{3}{2}bc\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)\right)}{\sqrt[3]{c}}}{6c^{4/3}}\right.$$

$\downarrow$ 1103

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-$$
$$\frac{3}{2}bc\left(-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}-\frac{\sqrt{3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}-\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)\right)}{\sqrt[3]{c}}}{6c^{4/3}}+\frac{\mathrm{a}}{}\right.$$

input
```
Int[x*(a + b*ArcTan[c*x^3]),x]
```

output
```
(x^2*(a + b*ArcTan[c*x^3]))/2 - (3*b*c*(ArcTan[c^(1/3)*x]/(3*c^(5/3)) - (A
rcTan[Sqrt[3]*(1 - (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3) - (Sqrt[3]*Log[1 - Sqrt
[3]*c^(1/3)*x + c^(2/3)*x^2])/(2*c^(1/3)))/(6*c^(4/3)) - (-(ArcTan[Sqrt[3]
*(1 + (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3)) + (Sqrt[3]*Log[1 + Sqrt[3]*c^(1/3)*
x + c^(2/3)*x^2])/(2*c^(1/3)))/(6*c^(4/3))))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

method	result
default	$\frac{a\,x^{2}}{2}+b\left(\frac{x^{2}\arctan\left(c\,x^{3}\right)}{2}-\frac{3c\left(\frac{\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{5}{6}}\ln\left(x^{2}-\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6c^{2}\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{5}{6}}\ln\left(x^{2}+\sqrt{3}\,\left(\frac{1}{c^{2}}\right.\right.}{12}\right.}{2}\right.$
parts	$\frac{a\,x^{2}}{2}+b\left(\frac{x^{2}\arctan\left(c\,x^{3}\right)}{2}-\frac{3c\left(\frac{\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{5}{6}}\ln\left(x^{2}-\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6c^{2}\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{5}{6}}\ln\left(x^{2}+\sqrt{3}\,\left(\frac{1}{c^{2}}\right.\right.}{12}\right.}{2}\right.$

input
```
int(x*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+b*(1/2*x^2*arctan(c*x^3)-3/2*c*(1/12*3^(1/2)*(1/c^2)^(5/6)*ln(x^
2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)*arctan(2*x/
(1/c^2)^(1/6)-3^(1/2))-1/12*3^(1/2)*(1/c^2)^(5/6)*ln(x^2+3^(1/2)*(1/c^2)^(
1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/
2))+1/3/c^2/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(93) = 186$.

Time = 0.13 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.19

$$\int x\left(a+b\arctan\left(cx^3\right)\right)\,dx = \frac{1}{2}\,bx^2\arctan\left(cx^3\right)+\frac{1}{2}\,ax^2 + \frac{1}{8}\left(-\frac{b^6}{c^4}\right)^{\frac{1}{6}}\left(\sqrt{-3}-1\right)\log\left(b^5x + \frac{1}{2}\left(\sqrt{-3}c^3+c^3\right)\left(-\frac{b^6}{c^4}\right)^{\frac{5}{6}}\right) - \frac{1}{8}\left(-\frac{b^6}{c^4}\right)^{\frac{1}{6}}\left(\sqrt{-3}-1\right)\log\left(b^5x - \frac{1}{2}\left(\sqrt{-3}c^3+c^3\right)\left(-\frac{b^6}{c^4}\right)^{\frac{5}{6}}\right) + \frac{1}{8}\left(-\frac{b^6}{c^4}\right)^{\frac{1}{6}}\left(\sqrt{-3}+1\right)\log\left(b^5x + \frac{1}{2}\left(\sqrt{-3}c^3-c^3\right)\left(-\frac{b^6}{c^4}\right)^{\frac{5}{6}}\right) - \frac{1}{8}\left(-\frac{b^6}{c^4}\right)^{\frac{1}{6}}\left(\sqrt{-3}+1\right)\log\left(b^5x - \frac{1}{2}\left(\sqrt{-3}c^3-c^3\right)\left(-\frac{b^6}{c^4}\right)^{\frac{5}{6}}\right) - \frac{1}{4}\left(-\frac{b^6}{c^4}\right)^{\frac{1}{6}}\log\left(b^5x+\left(-\frac{b^6}{c^4}\right)^{\frac{5}{6}}c^3\right) + \frac{1}{4}\left(-\frac{b^6}{c^4}\right)^{\frac{1}{6}}\log\left(b^5x-\left(-\frac{b^6}{c^4}\right)^{\frac{5}{6}}c^3\right)$$

input
```
integrate(x*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/2*b*x^2*arctan(c*x^3) + 1/2*a*x^2 + 1/8*(-b^6/c^4)^(1/6)*(sqrt(-3) - 1)*
log(b^5*x + 1/2*(sqrt(-3)*c^3 + c^3)*(-b^6/c^4)^(5/6)) - 1/8*(-b^6/c^4)^(1
/6)*(sqrt(-3) - 1)*log(b^5*x - 1/2*(sqrt(-3)*c^3 + c^3)*(-b^6/c^4)^(5/6))
+ 1/8*(-b^6/c^4)^(1/6)*(sqrt(-3) + 1)*log(b^5*x + 1/2*(sqrt(-3)*c^3 - c^3)
*(-b^6/c^4)^(5/6)) - 1/8*(-b^6/c^4)^(1/6)*(sqrt(-3) + 1)*log(b^5*x - 1/2*(
sqrt(-3)*c^3 - c^3)*(-b^6/c^4)^(5/6)) - 1/4*(-b^6/c^4)^(1/6)*log(b^5*x + (
-b^6/c^4)^(5/6)*c^3) + 1/4*(-b^6/c^4)^(1/6)*log(b^5*x - (-b^6/c^4)^(5/6)*c
^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(116) = 232$.

Time = 15.63 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.94

$$\int x\left(a+b\arctan\left(cx^3\right)\right)\,dx$$
$$=\begin{cases}\frac{ax^2}{2}+\frac{bx^2\operatorname{atan}\left(cx^3\right)}{2}-\frac{3b\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{8c\sqrt[6]{-\frac{1}{c^2}}}+\frac{3b\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{8c\sqrt[6]{-\frac{1}{c^2}}}-\frac{\sqrt{3}b\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[6]{-\frac{1}{c^2}}}-\ldots\right.}{4c\sqrt[6]{-\frac{1}{c^2}}} \\ \frac{ax^2}{2}\end{cases}$$

input
```
integrate(x*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**2/2 + b*x**2*atan(c*x**3)/2 - 3*b*log(4*x**2 - 4*x*(-1/c**
2)**(1/6) + 4*(-1/c**2)**(1/3))/(8*c*(-1/c**2)**(1/6)) + 3*b*log(4*x**2 +
4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(8*c*(-1/c**2)**(1/6)) - sqrt(3
)*b*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/(4*c*(-1/c**2)**(1/
6)) - sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(4*c*(-
1/c**2)**(1/6)) + b*atan(c*x**3)/(2*c**2*(-1/c**2)**(2/3)), Ne(c, 0)), (a*
x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\int x(a+b\arctan(cx^3))\,dx=\frac{1}{2}ax^2+\frac{1}{8}\left(4x^2\arctan(cx^3)+c\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{4\arctan\left(\right.}{c^{\frac{5}{3}}}\right.\right.$$

input
```
integrate(x*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/8*(4*x^2*arctan(c*x^3) + c*(sqrt(3)*log(c^(2/3)*x^2 + sqrt(3
)*c^(1/3)*x + 1)/c^(5/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1
)/c^(5/3) - 4*arctan(c^(1/3)*x)/c^(5/3) - 2*arctan((2*c^(2/3)*x + sqrt(3)*
c^(1/3))/c^(1/3))/c^(5/3) - 2*arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/
3))/c^(5/3)))*b
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.24

$$\int x(a+b\arctan(cx^3))\,dx=\frac{1}{8}bc^5\left(\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^6}-\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^6}-\frac{2\arctan\left(\left(2x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^4|c|^{\frac{5}{3}}}\right.$$
$$+\frac{1}{2}bx^2\arctan(cx^3)+\frac{1}{2}ax^2$$

input
```
integrate(x*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
1/8*b*c^5*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c
)^(2/3))/c^6 - sqrt(3)*abs(c)^(1/3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/a
bs(c)^(2/3))/c^6 - 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(c^
4*abs(c)^(5/3)) - 2*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(c^4
*abs(c)^(5/3)) - 4*arctan(x*abs(c)^(1/3))/(c^4*abs(c)^(5/3))) + 1/2*b*x^2*
arctan(c*x^3) + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.89

$$\int x\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{a\,x^2}{2}+\frac{b\left(\mathrm{atan}\left({\left(-1\right)}^{2/3}\,c^{1/3}\,x\right)+\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)+2\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\right)}{4\,c^{2/3}}+\frac{b\,x^2\,\mathrm{atan}\left(c\,x^3\right)}{2}-\frac{\sqrt{3}\,b\,\left(\mathrm{atan}\left({\left(-1\right)}^{2/3}\,c^{1/3}\,x\right)-\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\right)\,1{}\mathrm{i}}{4\,c^{2/3}}$$

input
```
int(x*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^2)/2 + (b*(atan((-1)^(2/3)*c^(1/3)*x) + atan(((-1)^(2/3)*c^(1/3)*x*(3
^(1/2)*1i - 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/(
4*c^(2/3)) + (b*x^2*atan(c*x^3))/2 - (3^(1/2)*b*(atan((-1)^(2/3)*c^(1/3)*x
) - atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2))*1i)/(4*c^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.79

$$\int x\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{-6c^{\frac{1}{3}}\,atan\left(c^{\frac{1}{3}}x\right)b-2c^{\frac{1}{3}}\,atan\left(c\,x^3\right)b+4atan\left(c\,x^3\right)bc\,x^2-c^{\frac{1}{3}}\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-c^{\frac{1}{3}}\sqrt{3}\,x+1\right)b+c^{\frac{1}{3}}\sqrt{3}\log\left(\right.}{8c}$$

input
```
int(x*(a+b*atan(c*x^3)),x)
```

output
```
( - 6*c**(1/3)*atan(c**(1/3)*x)*b - 2*c**(1/3)*atan(c*x**3)*b + 4*atan(c*x
**3)*b*c*x**2 - c**(1/3)*sqrt(3)*log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x +
1)*b + c**(1/3)*sqrt(3)*log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x + 1)*b + 4*
a*c*x**2)/(8*c)
```

3.111 $\int \frac{a+b\arctan(cx^3)}{x^2}\,dx$

Optimal result 843
Mathematica [A] (verified) 843
Rubi [A] (verified) 844
Maple [A] (verified) 847
Fricas [A] (verification not implemented) 848
Sympy [B] (verification not implemented) 848
Maxima [A] (verification not implemented) 849
Giac [A] (verification not implemented) 850
Mupad [B] (verification not implemented) 850
Reduce [B] (verification not implemented) 851

Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{a+b\arctan(cx^3)}{x^2}\,dx = -\frac{a+b\arctan(cx^3)}{x} - \frac{1}{2}\sqrt{3}b\sqrt[3]{c}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right) + \frac{1}{2}b\sqrt[3]{c}\log\left(1+c^{2/3}x^2\right) - \frac{1}{4}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)$$

output
```
-(a+b*arctan(c*x^3))/x-1/2*3^(1/2)*b*c^(1/3)*arctan(1/3*(1-2*c^(2/3)*x^2)*
3^(1/2))+1/2*b*c^(1/3)*ln(1+c^(2/3)*x^2)-1/4*b*c^(1/3)*ln(1-c^(2/3)*x^2+c^
(4/3)*x^4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.63

$$\int \frac{a+b\arctan(cx^3)}{x^2}\,dx = -\frac{a}{x} - \frac{b\arctan(cx^3)}{x} - \frac{1}{2}\sqrt{3}b\sqrt[3]{c}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right) - \frac{1}{2}\sqrt{3}b\sqrt[3]{c}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right) + \frac{1}{2}b\sqrt[3]{c}\log\left(1+c^{2/3}x^2\right) - \frac{1}{4}b\sqrt[3]{c}\log\left(1-\sqrt{3}\sqrt[3]{c}x + c^{2/3}x^2\right) - \frac{1}{4}b\sqrt[3]{c}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x^2,x]
```

output
```
-(a/x) - (b*ArcTan[c*x^3])/x - (Sqrt[3]*b*c^(1/3)*ArcTan[Sqrt[3] - 2*c^(1/
3)*x])/2 - (Sqrt[3]*b*c^(1/3)*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/2 + (b*c^(1/3
)*Log[1 + c^(2/3)*x^2])/2 - (b*c^(1/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)
*x^2])/4 - (b*c^(1/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5361, 807, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^3\right)}{x^2}dx$$

$$\Big\downarrow \text{5361}$$

$$3bc\int \frac{x}{c^2x^6+1}dx-\frac{a+b\arctan\left(cx^3\right)}{x}$$

$$\Big\downarrow \text{807}$$

$$\frac{3}{2}bc\int \frac{1}{c^2x^6+1}dx^2-\frac{a+b\arctan\left(cx^3\right)}{x}$$

$$\Big\downarrow \text{750}$$

$$\frac{3}{2}bc\left(\frac{1}{3}\int \frac{1}{c^{2/3}x^2+1}dx^2+\frac{1}{3}\int \frac{2-c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2\right)-\frac{a+b\arctan\left(cx^3\right)}{x}$$

$$\Big\downarrow \text{16}$$

$$\frac{3}{2}bc\left(\frac{1}{3}\int \frac{2-c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}\right)-\frac{a+b\arctan\left(cx^3\right)}{x}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2-\frac{\int -\frac{c^{2/3}\left(1-2c^{2/3}x^2\right)}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{2c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}\right)-\frac{a+b\arctan\left(cx^3\right)}{x}$$

$$\Big\downarrow \; 25$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2+\frac{\int \frac{c^{2/3}\left(1-2c^{2/3}x^2\right)}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{2c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}\right)-\frac{a+b\arctan\left(cx^3\right)}{x}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2+\frac{1}{2}\int \frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}\right)-\frac{a+b\arctan\left(cx^3\right)}{x}$$

$$\Big\downarrow \; 1082$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(\frac{3\int \frac{1}{-x^4-3}d\left(1-2c^{2/3}x^2\right)}{c^{2/3}}+\frac{1}{2}\int \frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}\right)-\frac{a+b\arctan\left(cx^3\right)}{x}$$

$$\Big\downarrow \; 217$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2-\frac{\sqrt{3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}\right)-\frac{a+b\arctan\left(cx^3\right)}{x}$$

$$\Big\downarrow \; 1103$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{c^{2/3}}-\frac{\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{2c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}\right)-\frac{a+b\arctan\left(cx^3\right)}{x}$$

input
```
Int[(a + b*ArcTan[c*x^3])/x^2,x]
```

output
```
-((a + b*ArcTan[c*x^3])/x) + (3*b*c*(Log[1 + c^(2/3)*x^2]/(3*c^(2/3)) + (-
((Sqrt[3]*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/c^(2/3)) - Log[1 - c^(2/3)*
x^2 + c^(4/3)*x^4]/(2*c^(2/3)))/3))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

method	result
default	$-\frac{a}{x}-\frac{b\arctan\left(c\,x^{3}\right)}{x}+\frac{b\ln\left(x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^{4}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}$
parts	$-\frac{a}{x}-\frac{b\arctan\left(c\,x^{3}\right)}{x}+\frac{b\ln\left(x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^{4}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}$
risch	$\frac{ib\ln\left(ic\,x^{3}+1\right)}{2x}-\frac{ib\ln\left(-ic\,x^{3}+1\right)}{2x}-\frac{ib\ln\left(x+\left(\frac{i}{c}\right)^{\frac{1}{3}}\right)}{2\left(\frac{i}{c}\right)^{\frac{1}{3}}}+\frac{ib\ln\left(x^{2}-\left(\frac{i}{c}\right)^{\frac{1}{3}}x+\left(\frac{i}{c}\right)^{\frac{2}{3}}\right)}{4\left(\frac{i}{c}\right)^{\frac{1}{3}}}+\frac{ib\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{i}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2\left(\frac{i}{c}\right)^{\frac{1}{3}}}-\frac{a}{x}$

input
```
int((a+b*arctan(c*x^3))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x-b/x*arctan(c*x^3)+1/2*b/c/(1/c^2)^(2/3)*ln(x^2+(1/c^2)^(1/3))-1/4*b/c
/(1/c^2)^(2/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))+1/2*b/c/(1/c^2)^(2/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*x^2/(1/c^2)^(1/3)-1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{a + b\arctan(cx^3)}{x^2}\,dx = \frac{2\sqrt{3}bc^{\frac{1}{3}}x\arctan\left(\frac{2}{3}\sqrt{3}c^{\frac{2}{3}}x^2 - \frac{1}{3}\sqrt{3}\right) - bc^{\frac{1}{3}}x\log\left(c^2x^4 - c^{\frac{4}{3}}x^2 + c^{\frac{2}{3}}\right) + 2\,bc^{\frac{1}{3}}x\log\left(cx^2 + c^{\frac{1}{3}}\right) - 4\,b\arcta}{4\,x}$$

input
```
integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(3)*b*c^(1/3)*x*arctan(2/3*sqrt(3)*c^(2/3)*x^2 - 1/3*sqrt(3)) -
 b*c^(1/3)*x*log(c^2*x^4 - c^(4/3)*x^2 + c^(2/3)) + 2*b*c^(1/3)*x*log(c*x^
2 + c^(1/3)) - 4*b*arctan(c*x^3) - 4*a)/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(97) = 194.

Time = 22.24 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.52

$$\int \frac{a + b\arctan(cx^3)}{x^2}\,dx = \begin{cases} -\frac{a}{x} + bc^2\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}\operatorname{atan}(cx^3) - bc\sqrt[3]{-\frac{1}{c^2}}\log\left(x - \sqrt[6]{-\frac{1}{c^2}}\right) + \frac{3bc\sqrt[3]{-\frac{1}{c^2}}\log\left(4x^2 - 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{4} - \frac{bc\sqrt[3]{}}{} \\ -\frac{a}{x} \end{cases}$$

input
```
integrate((a+b*atan(c*x**3))/x**2,x)
```

output
```
Piecewise((-a/x + b*c**2*(-1/c**2)**(5/6)*atan(c*x**3) - b*c*(-1/c**2)**(1
/3)*log(x - (-1/c**2)**(1/6)) + 3*b*c*(-1/c**2)**(1/3)*log(4*x**2 - 4*x*(-
1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/4 - b*c*(-1/c**2)**(1/3)*log(4*x**2 +
 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/4 + sqrt(3)*b*c*(-1/c**2)**(1/
3)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/2 - sqrt(3)*b*c*(-1/
c**2)**(1/3)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/2 - b*atan
(c*x**3)/x, Ne(c, 0)), (-a/x, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arctan(cx^3)}{x^2}\,dx = \frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}}-\frac{\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{2}{3}}}+\frac{2\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}}\right)-\frac{4\arctan(cx^3)}{x}\right)b - \frac{a}{x}$$

input
```
integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="maxima")
```

output
```
1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2/3))/c^
(2/3) - log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1)/c^(2/3) + 2*log((c^(2/3)*x^2 +
1)/c^(2/3))/c^(2/3)) - 4*arctan(c*x^3)/x)*b - a/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{a+b\arctan(cx^3)}{x^2}\,dx$$
$$=\frac{1}{4}bc\left(\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{|c|^{\frac{2}{3}}}-\frac{\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{|c|^{\frac{2}{3}}}+\frac{2\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{2}{3}}}\right)$$
$$-\frac{b\arctan(cx^3)+a}{x}$$

input
```
integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="giac")
```

output
```
1/4*b*c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))*abs(c)^(2/3
))/abs(c)^(2/3) - log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/abs(c)^(2/3
) + 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(2/3)) - (b*arctan(c*x^3) + a)/x
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95

$$\int \frac{a+b\arctan(cx^3)}{x^2}\,dx=\frac{b\,c^{1/3}\,\ln\left(c^{2/3}\,x^2+1\right)}{2}-\frac{a}{x}-\frac{b\,\mathrm{atan}(c\,x^3)}{x}$$
$$-\frac{b\,c^{1/3}\,\ln\left(-\sqrt{3}-c^{2/3}\,x^2\,2\mathrm{i}+1\mathrm{i}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}$$
$$+\frac{b\,c^{1/3}\,\ln\left(-\sqrt{3}+c^{2/3}\,x^2\,2\mathrm{i}-\mathrm{i}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4}$$

input
```
int((a + b*atan(c*x^3))/x^2,x)
```

output
```
(b*c^(1/3)*log(c^(2/3)*x^2 + 1))/2 - a/x - (b*atan(c*x^3))/x - (b*c^(1/3)*
log(1i - c^(2/3)*x^2*2i - 3^(1/2))*(3^(1/2)*1i + 1))/4 + (b*c^(1/3)*log(c^
(2/3)*x^2*2i - 3^(1/2) - 1i)*(3^(1/2)*1i - 1))/4
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.30

$$\int \frac{a + b\arctan(cx^3)}{x^2}\, dx$$
$$= \frac{4c^{\frac{1}{3}}\sqrt{3}\, atan\left(2c^{\frac{1}{3}}x - \sqrt{3}\right) bx - 2c^{\frac{1}{3}}\sqrt{3}\, atan\left(c^{\frac{1}{3}}x\right) bx - 2c^{\frac{1}{3}}\sqrt{3}\, atan(c\,x^3)\, bx - 4atan(c\,x^3)\, b - c^{\frac{1}{3}}\log\left(c^{\frac{2}{3}}\right.}{4x}$$

input
```
int((a+b*atan(c*x^3))/x^2,x)
```

output
```
(4*c**(1/3)*sqrt(3)*atan(2*c**(1/3)*x - sqrt(3))*b*x - 2*c**(1/3)*sqrt(3)*
atan(c**(1/3)*x)*b*x - 2*c**(1/3)*sqrt(3)*atan(c*x**3)*b*x - 4*atan(c*x**3
)*b - c**(1/3)*log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x + 1)*b*x - c**(1/3)*
log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x + 1)*b*x + 2*c**(1/3)*log(c**(2/3)*
x**2 + 1)*b*x - 4*a)/(4*x)
```

3.112 $\int \frac{a+b\arctan(cx^3)}{x^5} dx$

Optimal result . . . 852
Mathematica [A] (verified) . . . 852
Rubi [A] (verified) . . . 853
Maple [A] (verified) . . . 857
Fricas [B] (verification not implemented) . . . 859
Sympy [B] (verification not implemented) . . . 859
Maxima [A] (verification not implemented) . . . 860
Giac [A] (verification not implemented) . . . 861
Mupad [B] (verification not implemented) . . . 861
Reduce [B] (verification not implemented) . . . 862

Optimal result

Integrand size = 14, antiderivative size = 136

$$\int \frac{a+b\arctan(cx^3)}{x^5} dx = -\frac{3bc}{4x} - \frac{1}{4}bc^{4/3}\arctan\left(\sqrt[3]{c}x\right) - \frac{a+b\arctan(cx^3)}{4x^4} + \frac{1}{8}bc^{4/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right) - \frac{1}{8}bc^{4/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right) + \frac{1}{8}\sqrt{3}bc^{4/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[3]{c}x}{1+c^{2/3}x^2}\right)$$

output
```
-3/4*b*c/x-1/4*b*c^(4/3)*arctan(c^(1/3)*x)-1/4*(a+b*arctan(c*x^3))/x^4-1/8
*b*c^(4/3)*arctan(-3^(1/2)+2*c^(1/3)*x)-1/8*b*c^(4/3)*arctan(3^(1/2)+2*c^(
1/3)*x)+1/8*3^(1/2)*b*c^(4/3)*arctanh(3^(1/2)*c^(1/3)*x/(1+c^(2/3)*x^2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.32

$$\int \frac{a+b\arctan(cx^3)}{x^5} dx = -\frac{a}{4x^4} - \frac{3bc}{4x} - \frac{1}{4}bc^{4/3}\arctan\left(\sqrt[3]{c}x\right) - \frac{b\arctan(cx^3)}{4x^4} + \frac{1}{8}bc^{4/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right) - \frac{1}{8}bc^{4/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right) - \frac{1}{16}\sqrt{3}bc^{4/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right) + \frac{1}{16}\sqrt{3}bc^{4/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x\right.$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x^5,x]
```

output
```
-1/4*a/x^4 - (3*b*c)/(4*x) - (b*c^(4/3)*ArcTan[c^(1/3)*x])/4 - (b*ArcTan[c
*x^3])/(4*x^4) + (b*c^(4/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/8 - (b*c^(4/3)*
ArcTan[Sqrt[3] + 2*c^(1/3)*x])/8 - (Sqrt[3]*b*c^(4/3)*Log[1 - Sqrt[3]*c^(1
/3)*x + c^(2/3)*x^2])/16 + (Sqrt[3]*b*c^(4/3)*Log[1 + Sqrt[3]*c^(1/3)*x +
c^(2/3)*x^2])/16
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.52, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5361, 847, 824, 27, 216, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^3\right)}{x^5}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{3}{4}bc\int \frac{1}{x^2\left(c^2x^6+1\right)}dx-\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \text{847}$$

$$\frac{3}{4}bc\left(c^2\left(-\int \frac{x^4}{c^2x^6+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \text{824}$$

$$\frac{3}{4}bc\left(-\left(c^2\left(\frac{\int \frac{1}{c^{2/3}x^2+1}dx}{3c^{4/3}}+\frac{\int -\frac{1-\sqrt{3}\sqrt[3]{c}x}{2\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}+\frac{\int -\frac{\sqrt{3}\sqrt[3]{c}x+1}{2\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{4}bc\left(-\left(c^2\left(\frac{\int \frac{1}{c^{2/3}x^2+1}dx}{3c^{4/3}}-\frac{\int \frac{1-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int \frac{\sqrt{3}\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$\Big\downarrow$ 216

$$\frac{3}{4}bc\left(-\left(c^2\left(-\frac{\int \frac{1-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int \frac{\sqrt{3}\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\arctan\left(\sqrt[3]{c}x\right)}{3c^{5/3}}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$\Big\downarrow$ 1142

$$\frac{3}{4}bc\left(-\left(c^2\left(-\frac{-\frac{1}{2}\int \frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\sqrt{3}\int -\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int \frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+}\right.\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$\Big\downarrow$ 25

$$\frac{3}{4}bc\left(-\left(c^2\left(-\frac{\frac{\sqrt{3}\int \frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int \frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int \frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}d\right.\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$\Big\downarrow$ 27

$$\frac{3}{4}bc\left(-\left(c^2\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{1}{2}\int \frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{1}{2}\int \frac{1}{c^{2/3}x^2+\sqrt{3}}}{6c^{4/3}}\right.\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3}{4}bc\left(-\left(c^2\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\int \frac{1}{-\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\int \frac{1}{-\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right.}}{}}{6c^{4/3}}\right.\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \text{217}$$

$$\frac{3}{4}bc\left(-\left(c^2\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{}}{\sqrt{}}\right.\right.}{\sqrt[3]{c}}}{6c^{4/3}}\right.\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3}{4}bc\left(-\left(c^2\left(-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}-\frac{\sqrt{3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}-\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right.\right.}{\sqrt[3]{c}}}{6c^{4/3}}\right.\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

input
```
Int[(a + b*ArcTan[c*x^3])/x^5,x]
```

output
```
-1/4*(a + b*ArcTan[c*x^3])/x^4 + (3*b*c*(-x^(-1) - c^2*(ArcTan[c^(1/3)*x]/
(3*c^(5/3)) - (ArcTan[Sqrt[3]*(1 - (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3) - (Sqrt
[3]*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(2*c^(1/3)))/(6*c^(4/3)) - (
-(ArcTan[Sqrt[3]*(1 + (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3)) + (Sqrt[3]*Log[1 +
Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(2*c^(1/3)))/(6*c^(4/3)))))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21

method	result
default	$-\frac{a}{4x^4}+b\left(-\frac{\arctan(c\,x^3)}{4x^4}+\frac{3c\left(-\frac{1}{x}-\left(\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6c^2\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln(x^2 \cdots)}{\cdots}\right)\right)}{4 \cdots}\right)$
parts	$-\frac{a}{4x^4}+b\left(-\frac{\arctan(c\,x^3)}{4x^4}+\frac{3c\left(-\frac{1}{x}-\left(\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6c^2\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln(x^2 \cdots)}{\cdots}\right)\right)}{4 \cdots}\right)$

input

```
int((a+b*arctan(c*x^3))/x^5,x,method=_RETURNVERBOSE)
```

output

```
-1/4*a/x^4+b*(-1/4/x^4*arctan(c*x^3)+3/4*c*(-1/x-(1/12*3^(1/2)*(1/c^2)^(5/
6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)*arc
tan(2*x/(1/c^2)^(1/6)-3^(1/2))-1/12*3^(1/2)*(1/c^2)^(5/6)*ln(x^2+3^(1/2)*(
1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/
6)+3^(1/2))+1/3/c^2/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6)))*c^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(100) = 200$.

Time = 0.13 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.21

$$\int \frac{a + b \arctan(cx^3)}{x^5} \, dx = -\frac{2\left(-b^6c^8\right)^{\frac{1}{6}} x^4 \log\left(b^5c^7x + \left(-b^6c^8\right)^{\frac{5}{6}}\right) - 2\left(-b^6c^8\right)^{\frac{1}{6}} x^4 \log\left(b^5c^7x - \left(-b^6c^8\right)^{\frac{5}{6}}\right) + 12\,bcx^3 - \left(-b^6c^8\right)^{\frac{1}{6}} (\cdots}{\cdots}$$

input
```
integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="fricas")
```

output
```
-1/16*(2*(-b^6*c^8)^(1/6)*x^4*log(b^5*c^7*x + (-b^6*c^8)^(5/6)) - 2*(-b^6*
c^8)^(1/6)*x^4*log(b^5*c^7*x - (-b^6*c^8)^(5/6)) + 12*b*c*x^3 - (-b^6*c^8)
^(1/6)*(sqrt(-3)*x^4 - x^4)*log(2*b^5*c^7*x + (-b^6*c^8)^(5/6)*(sqrt(-3) +
 1)) + (-b^6*c^8)^(1/6)*(sqrt(-3)*x^4 - x^4)*log(2*b^5*c^7*x - (-b^6*c^8)^
(5/6)*(sqrt(-3) + 1)) - (-b^6*c^8)^(1/6)*(sqrt(-3)*x^4 + x^4)*log(2*b^5*c^
7*x + (-b^6*c^8)^(5/6)*(sqrt(-3) - 1)) + (-b^6*c^8)^(1/6)*(sqrt(-3)*x^4 +
x^4)*log(2*b^5*c^7*x - (-b^6*c^8)^(5/6)*(sqrt(-3) - 1)) + 4*b*arctan(c*x^3
) + 4*a)/x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(126) = 252$.

Time = 44.18 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.94

$$\int \frac{a + b \arctan(cx^3)}{x^5} \, dx = \begin{cases} -\frac{a}{4x^4} + \frac{3bc^3\left(-\frac{1}{c^2}\right)^{\frac{5}{6}} \log\left(4x^2 - 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{16} - \frac{3bc^3\left(-\frac{1}{c^2}\right)^{\frac{5}{6}} \log\left(4x^2 + 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{16} + \frac{\sqrt{3}bc^3\left(-\frac{1}{c^2}\right)^{\frac{5}{6}} \operatorname{ata}\cdots}{\cdots} \\ -\frac{a}{4x^4} \end{cases}$$

input
```
integrate((a+b*atan(c*x**3))/x**5,x)
```

output
```
Piecewise((-a/(4*x**4) + 3*b*c**3*(-1/c**2)**(5/6)*log(4*x**2 - 4*x*(-1/c*
*2)**(1/6) + 4*(-1/c**2)**(1/3))/16 - 3*b*c**3*(-1/c**2)**(5/6)*log(4*x**2
 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/16 + sqrt(3)*b*c**3*(-1/c**2
)**(5/6)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/8 + sqrt(3)*b*
c**3*(-1/c**2)**(5/6)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/8
 - b*c**2*(-1/c**2)**(1/3)*atan(c*x**3)/4 - 3*b*c/(4*x) - b*atan(c*x**3)/(
4*x**4), Ne(c, 0)), (-a/(4*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.08

$$\int \frac{a + b \arctan (cx^3)}{x^5} \, dx$$

$$= \frac{1}{16} \left(\left(\left(c^2 \left(\frac{\sqrt{3} \log \left(c^{\frac{2}{3}} x^2 + \sqrt{3} c^{\frac{1}{3}} x + 1 \right)}{c^{\frac{5}{3}}} - \frac{\sqrt{3} \log \left(c^{\frac{2}{3}} x^2 - \sqrt{3} c^{\frac{1}{3}} x + 1 \right)}{c^{\frac{5}{3}}} - \frac{4 \arctan \left(c^{\frac{1}{3}} x \right)}{c^{\frac{5}{3}}} - \frac{2 \arctan}{} \right.\right.\right.\right.$$

$$- \frac{a}{4\,x^4}$$

input
```
integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="maxima")
```

output
```
1/16*((c^2*(sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - sqr
t(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - 4*arctan(c^(1/3)*x
)/c^(5/3) - 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3) - 2*
arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3)) - 12/x)*c - 4*arc
tan(c*x^3)/x^4)*b - 1/4*a/x^4
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

$$\int \frac{a+b\arctan(cx^3)}{x^5}\,dx$$
$$=\frac{1}{16}bc^3\left(\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2}-\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2}-\frac{2\,|c|^{\frac{1}{3}}\arctan\left(\left(2\,x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c\right.}{c^2}\right.$$
$$-\frac{3\,bcx^3+b\arctan(cx^3)+a}{4\,x^4}$$

input

```
integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="giac")
```

output

```
1/16*b*c^3*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(
c)^(2/3))/c^2 - sqrt(3)*abs(c)^(1/3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/
abs(c)^(2/3))/c^2 - 2*abs(c)^(1/3)*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs
(c)^(1/3))/c^2 - 2*abs(c)^(1/3)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)
^(1/3))/c^2 - 4*abs(c)^(1/3)*arctan(x*abs(c)^(1/3))/c^2) - 1/4*(3*b*c*x^3
+ b*arctan(c*x^3) + a)/x^4
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \frac{a+b\arctan(cx^3)}{x^5}\,dx=-\frac{a}{4\,x^4}$$
$$+\frac{b\,c^{4/3}\left(\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)+\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)+2\,\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{8}$$
$$-\frac{b\,\mathrm{atan}(c\,x^3)}{4\,x^4}-\frac{3\,b\,c}{4\,x}$$
$$-\frac{\sqrt{3}\,b\,c^{4/3}\left(\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)-\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)\,1\mathrm{i}}{8}$$

input

```
int((a + b*atan(c*x^3))/x^5,x)
```

output
```
(b*c^(4/3)*(atan((-1)^(2/3)*c^(1/3)*x) + atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/
2)*1i - 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/8 - a
/(4*x^4) - (b*atan(c*x^3))/(4*x^4) - (3*b*c)/(4*x) - (3^(1/2)*b*c^(4/3)*(a
tan((-1)^(2/3)*c^(1/3)*x) - atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2
))*1i)/8
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.82

$$\int \frac{a+b\arctan(cx^3)}{x^5}\,dx$$
$$= \frac{-6c^{\frac{4}{3}}\,atan\left(c^{\frac{1}{3}}x\right)b\,x^4-2c^{\frac{4}{3}}\,atan(c\,x^3)\,b\,x^4-4atan(c\,x^3)\,b-c^{\frac{4}{3}}\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-c^{\frac{1}{3}}\sqrt{3}\,x+1\right)b\,x^4+c^{\frac{4}{3}}\sqrt{3}}{16x^4}$$

input
```
int((a+b*atan(c*x^3))/x^5,x)
```

output
```
( - 6*c**(1/3)*atan(c**(1/3)*x)*b*c*x**4 - 2*c**(1/3)*atan(c*x**3)*b*c*x**
4 - 4*atan(c*x**3)*b - c**(1/3)*sqrt(3)*log(c**(2/3)*x**2 - c**(1/3)*sqrt(
3)*x + 1)*b*c*x**4 + c**(1/3)*sqrt(3)*log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)
*x + 1)*b*c*x**4 - 4*a - 12*b*c*x**3)/(16*x**4)
```

3.113 $\int x^{11}(a+b\arctan(cx^3))^2\,dx$

Optimal result 863
Mathematica [A] (verified) 863
Rubi [A] (verified) 864
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Optimal result

Integrand size = 16, antiderivative size = 124

$$\begin{aligned}\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx &= \frac{abx^3}{6c^3}+\frac{b^2x^6}{36c^2}+\frac{b^2x^3\arctan\left(cx^3\right)}{6c^3}\\ &\quad-\frac{bx^9(a+b\arctan\left(cx^3\right))}{18c}-\frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{12c^4}\\ &\quad+\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{b^2\log\left(1+c^2x^6\right)}{9c^4}\end{aligned}$$

output
```
1/6*a*b*x^3/c^3+1/36*b^2*x^6/c^2+1/6*b^2*x^3*arctan(c*x^3)/c^3-1/18*b*x^9*
(a+b*arctan(c*x^3))/c-1/12*(a+b*arctan(c*x^3))^2/c^4+1/12*x^12*(a+b*arctan
(c*x^3))^2-1/9*b^2*ln(c^2*x^6+1)/c^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$

$$=\frac{cx^3(6ab+b^2cx^3-2abc^2x^6+3a^2c^3x^9)-2b(bcx^3(-3+c^2x^6)+a(3-3c^4x^{12}))\arctan\left(cx^3\right)+3b^2(-1+c}{36c^4}$$

input
```
Integrate[x^11*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
(c*x^3*(6*a*b + b^2*c*x^3 - 2*a*b*c^2*x^6 + 3*a^2*c^3*x^9) - 2*b*(b*c*x^3*
(-3 + c^2*x^6) + a*(3 - 3*c^4*x^12))*ArcTan[c*x^3] + 3*b^2*(-1 + c^4*x^12)
*ArcTan[c*x^3]^2 - 4*b^2*Log[1 + c^2*x^6])/(36*c^4)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5363, 5361, 5451, 5361, 243, 49, 2009, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$

$$\Big\downarrow \text{ 5363}$$

$$\frac{1}{3}\int x^9\left(a+b\arctan\left(cx^3\right)\right)^2dx^3$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\int\frac{x^{12}\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3\right)$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\int x^6\left(a+b\arctan\left(cx^3\right)\right)dx^3}{c^2}-\frac{\int\frac{x^6\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{3}bc\int\frac{x^9}{c^2x^6+1}dx^3}{c^2}-\frac{\int\frac{x^6\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{6}bc\int\frac{x^6}{c^2x^6+1}dx^6}{c^2}-\frac{\int\frac{x^6(a+b\arctan(cx^3))}{c^2x^6+1}dx^3}{c^2}\right)\right)$$

$\big\downarrow$ 49

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{6}bc\int\left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^6+1)}\right)dx^6}{c^2}-\frac{\int\frac{x^6(a+b\arctan(c}{c^2x^6+1}}{c^2}\right.\right.$$

$\big\downarrow$ 2009

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{6}bc\left(\frac{x^6}{c^2}-\frac{\log(c^2x^6+1)}{c^4}\right)}{c^2}-\frac{\int\frac{x^6(a+b\arctan(cx^3))}{c^2x^6+1}dx}{c^2}\right.\right.$$

$\big\downarrow$ 5451

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{6}bc\left(\frac{x^6}{c^2}-\frac{\log(c^2x^6+1)}{c^4}\right)}{c^2}-\frac{\frac{\int(a+b\arctan(cx^3))dx^3}{c^2}-}{c^2}\right.\right.$$

$\big\downarrow$ 2009

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{6}bc\left(\frac{x^6}{c^2}-\frac{\log(c^2x^6+1)}{c^4}\right)}{c^2}-\frac{\frac{ax^3+bx^3\arctan(cx^3)-\frac{b\,\mathrm{lo}}{}}{c^2}}{}\right.\right.$$

$\big\downarrow$ 5419

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{6}bc\left(\frac{x^6}{c^2}-\frac{\log(c^2x^6+1)}{c^4}\right)}{c^2}-\frac{\frac{ax^3+bx^3\arctan(cx^3)-\frac{b\,\mathrm{lo}}{}}{c^2}}{}\right.\right.$$

input
```
Int[x^11*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
((x^12*(a + b*ArcTan[c*x^3])^2)/4 - (b*c*(((x^9*(a + b*ArcTan[c*x^3]))/3 -
 (b*c*(x^6/c^2 - Log[1 + c^2*x^6]/c^4))/6)/c^2 - (-1/2*(a + b*ArcTan[c*x^3
])^2/(b*c^3) + (a*x^3 + b*x^3*ArcTan[c*x^3] - (b*Log[1 + c^2*x^6])/(2*c))/
c^2)/c^2))/2)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

method	result
default	$\frac{a^{2}x^{12}}{12}+\frac{b^{2}x^{12}\arctan\left(c\,x^{3}\right)^{2}}{12}-\frac{b^{2}\arctan\left(c\,x^{3}\right)x^{9}}{18c}+\frac{b^{2}x^{3}\arctan\left(c\,x^{3}\right)}{6c^{3}}-\frac{b^{2}\arctan\left(c\,x^{3}\right)^{2}}{12c^{4}}+\frac{b^{2}x^{6}}{36c^{2}}-\frac{b^{2}\ln\left(c^{2}x^{6}+1\right.}{9c^{4}}$
parts	$\frac{a^{2}x^{12}}{12}+\frac{b^{2}x^{12}\arctan\left(c\,x^{3}\right)^{2}}{12}-\frac{b^{2}\arctan\left(c\,x^{3}\right)x^{9}}{18c}+\frac{b^{2}x^{3}\arctan\left(c\,x^{3}\right)}{6c^{3}}-\frac{b^{2}\arctan\left(c\,x^{3}\right)^{2}}{12c^{4}}+\frac{b^{2}x^{6}}{36c^{2}}-\frac{b^{2}\ln\left(c^{2}x^{6}+1\right.}{9c^{4}}$
parallelrisch	$-\frac{-3b^{2}\arctan\left(c\,x^{3}\right)^{2}x^{12}c^{4}-6ab\arctan\left(c\,x^{3}\right)x^{12}c^{4}-3c^{4}a^{2}x^{12}+2b^{2}\arctan\left(c\,x^{3}\right)x^{9}c^{3}+2ab\,c^{3}x^{9}-x^{6}b^{2}c^{2}-6b^{2}\arctan\left(c\,x^{3}\right)x}{36c^{4}}$
risch	$-\frac{b^{2}\left(c^{4}x^{12}-1\right)\ln\left(ic\,x^{3}+1\right)^{2}}{48c^{4}}-\frac{ib\left(6a\,c^{4}x^{12}+3ib\,c^{4}x^{12}\ln\left(-ic\,x^{3}+1\right)-2b\,c^{3}x^{9}+6bc\,x^{3}-3ib\ln\left(-ic\,x^{3}+1\right)\right)\ln\left(ic\,x^{3}+1\right)}{72c^{4}}-$

input
```
int(x^11*(a+b*arctan(c*x^3))^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*a^2*x^12+1/12*b^2*x^12*arctan(c*x^3)^2-1/18*b^2*arctan(c*x^3)/c*x^9+1
/6*b^2*x^3*arctan(c*x^3)/c^3-1/12*b^2/c^4*arctan(c*x^3)^2+1/36*b^2*x^6/c^2
-1/9*b^2*ln(c^2*x^6+1)/c^4+1/6*a*b*x^12*arctan(c*x^3)-1/18*a*b/c*x^9+1/6*a
*b*x^3/c^3-1/6*a*b/c^4*arctan(c*x^3)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04

$$\int x^{11}\left(a+b\arctan\left(cx^{3}\right)\right)^{2}\,dx$$
$$=\frac{3\,a^{2}c^{4}x^{12}-2\,abc^{3}x^{9}+b^{2}c^{2}x^{6}+6\,abcx^{3}+3\left(b^{2}c^{4}x^{12}-b^{2}\right)\arctan\left(cx^{3}\right)^{2}-4\,b^{2}\log\left(c^{2}x^{6}+1\right)+2\left(3\,abc^{4}x\right.}{36\,c^{4}}$$

input
```
integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

output
```
1/36*(3*a^2*c^4*x^12 - 2*a*b*c^3*x^9 + b^2*c^2*x^6 + 6*a*b*c*x^3 + 3*(b^2*
c^4*x^12 - b^2)*arctan(c*x^3)^2 - 4*b^2*log(c^2*x^6 + 1) + 2*(3*a*b*c^4*x^
12 - b^2*c^3*x^9 + 3*b^2*c*x^3 - 3*a*b)*arctan(c*x^3))/c^4
```

Sympy [F(-1)]

Timed out.

$$\int x^{11}(a+b\arctan(cx^3))^2\,dx = \text{Timed out}$$

input
```
integrate(x**11*(a+b*atan(c*x**3))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.36

$$\int x^{11}(a+b\arctan(cx^3))^2\,dx = \frac{1}{12}b^2x^{12}\arctan(cx^3)^2 + \frac{1}{12}a^2x^{12} + \frac{1}{18}\left(3x^{12}\arctan(cx^3) - c\left(\frac{c^2x^9-3x^3}{c^4} + \frac{3\arctan(cx^3)}{c^5}\right)\right)ab - \frac{1}{36}\left(2c\left(\frac{c^2x^9-3x^3}{c^4} + \frac{3\arctan(cx^3)}{c^5}\right)\arctan(cx^3) - \frac{c^2x^6+3\arctan(cx^3)^2-3\log(18c^7x^6+18c}{c^4}\right.$$

input
```
integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

output
```
1/12*b^2*x^12*arctan(c*x^3)^2 + 1/12*a^2*x^12 + 1/18*(3*x^12*arctan(c*x^3)
 - c*((c^2*x^9 - 3*x^3)/c^4 + 3*arctan(c*x^3)/c^5))*a*b - 1/36*(2*c*((c^2*
x^9 - 3*x^3)/c^4 + 3*arctan(c*x^3)/c^5)*arctan(c*x^3) - (c^2*x^6 + 3*arcta
n(c*x^3)^2 - 3*log(18*c^7*x^6 + 18*c^5) - log(c^2*x^6 + 1))/c^4)*b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$
$$=\frac{3\,a^2cx^{12}+2\left(3\,cx^{12}\arctan\left(cx^3\right)-\frac{3\,\arctan(cx^3)}{c^3}-\frac{c^9x^9-3\,c^7x^3}{c^9}\right)ab+\left(3\,cx^{12}\arctan\left(cx^3\right)^2-\frac{2\,c^3x^9\arctan(cx^3)-}{}\right.}{36\,c}$$

input
```
integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

output
```
1/36*(3*a^2*c*x^12 + 2*(3*c*x^12*arctan(c*x^3) - 3*arctan(c*x^3)/c^3 - (c^
9*x^9 - 3*c^7*x^3)/c^9)*a*b + (3*c*x^12*arctan(c*x^3)^2 - (2*c^3*x^9*arcta
n(c*x^3) - c^2*x^6 - 6*c*x^3*arctan(c*x^3) + 3*arctan(c*x^3)^2 + 4*log(c^2
*x^6 + 1))/c^3)*b^2)/c
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\frac{a^2\,x^{12}}{12}-\frac{b^2\,\mathrm{atan}(c\,x^3)^2}{12\,c^4}+\frac{b^2\,x^{12}\,\mathrm{atan}(c\,x^3)^2}{12}-\frac{b^2\,\ln\left(c^2\,x^6+1\right)}{9\,c^4}+\frac{b^2\,x^6}{36\,c^2}+\frac{b^2\,x^3\,\mathrm{atan}(c\,x^3)}{6\,c^3}-\frac{b^2\,x^9\,\mathrm{atan}(c\,x^3)}{18\,c}+\frac{a\,b\,x^3}{6\,c^3}-\frac{a\,b\,x^9}{18\,c}-\frac{a\,b\,\mathrm{atan}(c\,x^3)}{6\,c^4}+\frac{a\,b\,x^{12}\,\mathrm{atan}(c\,x^3)}{6}$$

input
```
int(x^11*(a + b*atan(c*x^3))^2,x)
```

output
```
(a^2*x^12)/12 - (b^2*atan(c*x^3)^2)/(12*c^4) + (b^2*x^12*atan(c*x^3)^2)/12
 - (b^2*log(c^2*x^6 + 1))/(9*c^4) + (b^2*x^6)/(36*c^2) + (b^2*x^3*atan(c*x
^3))/(6*c^3) - (b^2*x^9*atan(c*x^3))/(18*c) + (a*b*x^3)/(6*c^3) - (a*b*x^9
)/(18*c) - (a*b*atan(c*x^3))/(6*c^4) + (a*b*x^12*atan(c*x^3))/6
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.57

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$

$$=\frac{3atan(c\,x^3)^2\,b^2c^4x^{12}-3atan(c\,x^3)^2\,b^2+6atan(c\,x^3)\,ab\,c^4x^{12}-6atan(c\,x^3)\,ab-2atan(c\,x^3)\,b^2c^3x^9+6at}{}$$

input
```
int(x^11*(a+b*atan(c*x^3))^2,x)
```

output
```
(3*atan(c*x**3)**2*b**2*c**4*x**12 - 3*atan(c*x**3)**2*b**2 + 6*atan(c*x**
3)*a*b*c**4*x**12 - 6*atan(c*x**3)*a*b - 2*atan(c*x**3)*b**2*c**3*x**9 + 6
*atan(c*x**3)*b**2*c*x**3 - 4*log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x + 1)*
b**2 - 4*log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x + 1)*b**2 - 4*log(c**(2/3)
*x**2 + 1)*b**2 + 3*a**2*c**4*x**12 - 2*a*b*c**3*x**9 + 6*a*b*c*x**3 + b**
2*c**2*x**6)/(36*c**4)
```

3.114 $\int x^8(a+b\arctan(cx^3))^2\,dx$

Optimal result . . . 871
Mathematica [A] (verified) . . . 872
Rubi [A] (verified) . . . 872
Maple [C] (warning: unable to verify) . . . 876
Fricas [F] . . . 876
Sympy [F] . . . 876
Maxima [F] . . . 877
Giac [F] . . . 877
Mupad [F(-1)] . . . 877
Reduce [F] . . . 878

Optimal result

Integrand size = 16, antiderivative size = 154

$$\int x^8(a+b\arctan(cx^3))^2\,dx=\frac{b^2x^3}{9c^2}-\frac{b^2\arctan(cx^3)}{9c^3}-\frac{bx^6(a+b\arctan(cx^3))}{9c}-\frac{i(a+b\arctan(cx^3))^2}{9c^3}+\frac{1}{9}x^9(a+b\arctan(cx^3))^2-\frac{2b(a+b\arctan(cx^3))\log\left(\frac{2}{1+icx^3}\right)}{9c^3}-\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{9c^3}$$

output
```
1/9*b^2*x^3/c^2-1/9*b^2*arctan(c*x^3)/c^3-1/9*b*x^6*(a+b*arctan(c*x^3))/c-
1/9*I*(a+b*arctan(c*x^3))^2/c^3+1/9*x^9*(a+b*arctan(c*x^3))^2-2/9*b*(a+b*a
rctan(c*x^3))*ln(2/(1+I*c*x^3))/c^3-1/9*I*b^2*polylog(2,1-2/(1+I*c*x^3))/c
^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\int x^8(a+b\arctan(cx^3))^2\,dx$$
$$=\frac{b^2cx^3-abc^2x^6+a^2c^3x^9+b^2(i+c^3x^9)\arctan(cx^3)^2-b\arctan(cx^3)\left(b+bc^2x^6-2ac^3x^9+2b\log\left(1+\right.\right.}{9c^3}$$

input
```
Integrate[x^8*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
(b^2*c*x^3 - a*b*c^2*x^6 + a^2*c^3*x^9 + b^2*(I + c^3*x^9)*ArcTan[c*x^3]^2
 - b*ArcTan[c*x^3]*(b + b*c^2*x^6 - 2*a*c^3*x^9 + 2*b*Log[1 + E^((2*I)*Arc
Tan[c*x^3])]) + a*b*Log[1 + c^2*x^6] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c
*x^3])])/(9*c^3)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5363, 5361, 5451, 5361, 262, 216, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8(a+b\arctan(cx^3))^2\,dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{3}\int x^6(a+b\arctan(cx^3))^2\,dx^3$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\left(\frac{1}{3}x^9(a+b\arctan(cx^3))^2-\frac{2}{3}bc\int\frac{x^9(a+b\arctan(cx^3))}{c^2x^6+1}dx^3\right)$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\int x^3\left(a+b\arctan\left(cx^3\right)\right)dx^3}{c^2}-\frac{\int\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}\right)\right)$$

$\big\downarrow$ 5361

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}bc\int\frac{x^6}{c^2x^6+1}dx^3}{c^2}-\frac{\int\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}\right)\right)$$

$\big\downarrow$ 262

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}bc\left(\frac{x^3}{c^2}-\frac{\int\frac{1}{c^2x^6+1}dx^3}{c^2}\right)}{c^2}-\frac{\int\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx}{c^2}\right.\right.$$

$\big\downarrow$ 216

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}bc\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)}{c^2}-\frac{\int\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}\right.\right.$$

$\big\downarrow$ 5455

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}bc\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)}{c^2}-\frac{-\frac{\int\frac{a+b\arctan\left(cx^3\right)}{i-cx^3}dx^3}{c}-}{c^2}\right.\right.$$

$\big\downarrow$ 5379

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}bc\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)}{c^2}-\frac{-\frac{\log\left(\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(c\right.\right.}{c}}{}\right.\right.$$

$\big\downarrow$ 2849

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}bc\left(\frac{x^3}{c^2}-\frac{\arctan(cx^3)}{c^3}\right)}{c^2}-\frac{-\frac{ib\int\frac{\log\left(\frac{2}{icx^3+1}\right)}{1-\frac{2}{icx^3+1}}d\frac{1}{icx^3+1}}{c}}{\cdots}\right.\right.$$

$\downarrow$ 2752

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}bc\left(\frac{x^3}{c^2}-\frac{\arctan(cx^3)}{c^3}\right)}{c^2}-\frac{-\frac{i(a+b\arctan(cx^3))^2}{2bc^2}-\cdots}{\cdots}\right.\right.$$

input
```
Int[x^8*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
((x^9*(a + b*ArcTan[c*x^3])^2)/3 - (2*b*c*(((x^6*(a + b*ArcTan[c*x^3]))/2
- (b*c*(x^3/c^2 - ArcTan[c*x^3]/c^3))/2)/c^2 - (((-1/2*I)*(a + b*ArcTan[c*
x^3])^2)/(b*c^2) - (((a + b*ArcTan[c*x^3])*Log[2/(1 + I*c*x^3)])/c + ((I/2
)*b*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c)/c)/c^2))/3)/3
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.25 (sec) , antiderivative size = 11449, normalized size of antiderivative = 74.34

method	result	size
default	Expression too large to display	11449
parts	Expression too large to display	11449

input
```
int(x^8*(a+b*arctan(c*x^3))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int\left(b\arctan\left(cx^3\right)+a\right)^2x^8\,dx$$

input
```
integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^8*arctan(c*x^3)^2 + 2*a*b*x^8*arctan(c*x^3) + a^2*x^8, x)
```

Sympy [F]

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int x^8\left(a+b\operatorname{atan}\left(cx^3\right)\right)^2\,dx$$

input
```
integrate(x**8*(a+b*atan(c*x**3))**2,x)
```

output
```
Integral(x**8*(a + b*atan(c*x**3))**2, x)
```

Maxima [F]

$$\int x^8 \left(a+b \arctan \left(c x^3\right)\right)^2 \, dx=\int \left(b \arctan \left(c x^3\right)+a\right)^2 x^8 \, dx$$

input
```
integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

output
```
1/9*a^2*x^9 + 1/9*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^4)*
c)*a*b + 1/144*(4*x^9*arctan(c*x^3)^2 - x^9*log(c^2*x^6 + 1)^2 + 144*integ
rate(1/48*(4*c^2*x^14*log(c^2*x^6 + 1) - 8*c*x^11*arctan(c*x^3) + 36*(c^2*
x^14 + x^8)*arctan(c*x^3)^2 + 3*(c^2*x^14 + x^8)*log(c^2*x^6 + 1)^2)/(c^2*
x^6 + 1), x))*b^2
```

Giac [F]

$$\int x^8 \left(a+b \arctan \left(c x^3\right)\right)^2 \, dx=\int \left(b \arctan \left(c x^3\right)+a\right)^2 x^8 \, dx$$

input
```
integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^2*x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 \left(a+b \arctan \left(c x^3\right)\right)^2 \, dx=\int x^8 \, {\left(a+b \, \mathrm{atan}\left(c \, x^3\right)\right)}^2 \, dx$$

input
```
int(x^8*(a + b*atan(c*x^3))^2,x)
```

output
```
int(x^8*(a + b*atan(c*x^3))^2, x)
```

Reduce [F]

$$\int x^8 (a + b\arctan(cx^3))^2 \, dx$$
$$= \frac{atan(c\,x^3)^2 b^2 c^3 x^9 + atan(c\,x^3)^2 b^2 c\,x^3 + 2atan(c\,x^3)\,ab\,c^3 x^9 - atan(c\,x^3)\,b^2 c^2 x^6 - atan(c\,x^3)\,b^2 - 3\Big(\int a}{}$$

input
```
int(x^8*(a+b*atan(c*x^3))^2,x)
```

output
```
(atan(c*x**3)**2*b**2*c**3*x**9 + atan(c*x**3)**2*b**2*c*x**3 + 2*atan(c*x
**3)*a*b*c**3*x**9 - atan(c*x**3)*b**2*c**2*x**6 - atan(c*x**3)*b**2 - 3*i
nt(atan(c*x**3)**2*x**2,x)*b**2*c + log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x
 + 1)*a*b + log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x + 1)*a*b + log(c**(2/3)
*x**2 + 1)*a*b + a**2*c**3*x**9 - a*b*c**2*x**6 + b**2*c*x**3)/(9*c**3)
```

3.115 $\int x^5(a+b\arctan(cx^3))^2\,dx$

Optimal result 879
Mathematica [A] (verified) 879
Rubi [A] (verified) 880
Maple [A] (verified) 882
Fricas [A] (verification not implemented) 882
Sympy [B] (verification not implemented) 883
Maxima [A] (verification not implemented) 883
Giac [A] (verification not implemented) 884
Mupad [B] (verification not implemented) 884
Reduce [B] (verification not implemented) 885

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int x^5(a+b\arctan(cx^3))^2\,dx=-\frac{abx^3}{3c}-\frac{b^2x^3\arctan(cx^3)}{3c}+\frac{(a+b\arctan(cx^3))^2}{6c^2}+\frac{1}{6}x^6(a+b\arctan(cx^3))^2+\frac{b^2\log(1+c^2x^6)}{6c^2}$$

output
```
-1/3*a*b*x^3/c-1/3*b^2*x^3*arctan(c*x^3)/c+1/6*(a+b*arctan(c*x^3))^2/c^2+1
/6*x^6*(a+b*arctan(c*x^3))^2+1/6*b^2*ln(c^2*x^6+1)/c^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int x^5(a+b\arctan(cx^3))^2\,dx=\frac{acx^3(-2b+acx^3)+2b(a-bcx^3+ac^2x^6)\arctan(cx^3)+b^2(1+c^2x^6)\arctan(cx^3)^2+b^2\log(1+c^2x^6)}{6c^2}$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
(a*c*x^3*(-2*b + a*c*x^3) + 2*b*(a - b*c*x^3 + a*c^2*x^6)*ArcTan[c*x^3] +
b^2*(1 + c^2*x^6)*ArcTan[c*x^3]^2 + b^2*Log[1 + c^2*x^6])/(6*c^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5363, 5361, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (a + b \arctan(cx^3))^2 \, dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{3} \int x^3 (a + b \arctan(cx^3))^2 \, dx^3$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 (a + b \arctan(cx^3))^2 - bc \int \frac{x^6 (a + b \arctan(cx^3))}{c^2 x^6 + 1} dx^3 \right)$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 (a + b \arctan(cx^3))^2 - bc \left(\frac{\int (a + b \arctan(cx^3)) \, dx^3}{c^2} - \frac{\int \frac{a+b\arctan(cx^3)}{c^2x^6+1} dx^3}{c^2} \right) \right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 (a + b \arctan(cx^3))^2 - bc \left(\frac{ax^3 + bx^3 \arctan(cx^3) - \frac{b \log(c^2x^6+1)}{2c}}{c^2} - \frac{\int \frac{a+b\arctan(cx^3)}{c^2x^6+1} dx^3}{c^2} \right) \right)$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 (a + b \arctan(cx^3))^2 - bc \left(\frac{ax^3 + bx^3 \arctan(cx^3) - \frac{b \log(c^2x^6+1)}{2c}}{c^2} - \frac{(a + b \arctan(cx^3))^2}{2bc^3} \right) \right)$$

input
```
Int[x^5*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
((x^6*(a + b*ArcTan[c*x^3])^2)/2 - b*c*(-1/2*(a + b*ArcTan[c*x^3])^2/(b*c^
3) + (a*x^3 + b*x^3*ArcTan[c*x^3] - (b*Log[1 + c^2*x^6])/(2*c))/c^2))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22

method	result
parallelrisch	$\frac{b^2 \arctan(c\,x^3)^2 x^6 c^2+2ab\arctan(c\,x^3)x^6c^2+a^2c^2x^6-2b^2\arctan(c\,x^3)x^3c-2abc\,x^3+b^2\arctan(c\,x^3)^2+b^2\ln(c^2x^6+1)+2ab}{6c^2}$
default	$\frac{x^6a^2}{6}+\frac{b^2x^6\arctan(c\,x^3)^2}{6}-\frac{b^2x^3\arctan(c\,x^3)}{3c}+\frac{b^2\arctan(c\,x^3)^2}{6c^2}+\frac{b^2\ln(c^2x^6+1)}{6c^2}+\frac{ab\,x^6\arctan(c\,x^3)}{3}-\frac{ab\,x^3}{3c}$
parts	$\frac{x^6a^2}{6}+\frac{b^2x^6\arctan(c\,x^3)^2}{6}-\frac{b^2x^3\arctan(c\,x^3)}{3c}+\frac{b^2\arctan(c\,x^3)^2}{6c^2}+\frac{b^2\ln(c^2x^6+1)}{6c^2}+\frac{ab\,x^6\arctan(c\,x^3)}{3}-\frac{ab\,x^3}{3c}$
risch	$-\frac{b^2(c^2x^6+1)\ln(ic\,x^3+1)^2}{24c^2}-\frac{ib(4a^2c^2x^6+2ix^6b\ln(-ic\,x^3+1)a\,c^2-4abc\,x^3+b^2+2ib\ln(-ic\,x^3+1)a)\ln(ic\,x^3+1)}{24a\,c^2}-\frac{ib^2}{}$

input
```
int(x^5*(a+b*arctan(c*x^3))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(b^2*arctan(c*x^3)^2*x^6*c^2+2*a*b*arctan(c*x^3)*x^6*c^2+a^2*c^2*x^6-2
*b^2*arctan(c*x^3)*x^3*c-2*a*b*c*x^3+b^2*arctan(c*x^3)^2+b^2*ln(c^2*x^6+1)
+2*a*b*arctan(c*x^3))/c^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$
$$=\frac{a^2c^2x^6-2\,abcx^3+\left(b^2c^2x^6+b^2\right)\arctan\left(cx^3\right)^2+b^2\log\left(c^2x^6+1\right)+2\left(abc^2x^6-b^2cx^3+ab\right)\arctan\left(cx^3\right)}{6\,c^2}$$

input
```
integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

output
```
1/6*(a^2*c^2*x^6 - 2*a*b*c*x^3 + (b^2*c^2*x^6 + b^2)*arctan(c*x^3)^2 + b^2
*log(c^2*x^6 + 1) + 2*(a*b*c^2*x^6 - b^2*c*x^3 + a*b)*arctan(c*x^3))/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(78) = 156.

Time = 49.42 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.16

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$
$$=\begin{cases}\frac{a^2x^6}{6}+\frac{abx^6\operatorname{atan}\left(cx^3\right)}{3}-\frac{abx^3}{3c}+\frac{ab\operatorname{atan}\left(cx^3\right)}{3c^2}+\frac{b^2x^6\operatorname{atan}^2\left(cx^3\right)}{6}-\frac{b^2x^3\operatorname{atan}\left(cx^3\right)}{3c}-\frac{b^2\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^3\right)}{3c}+\frac{b^2\log\left(x-\sqrt[6]{-\right.}}{3c^2}\\ \frac{a^2x^6}{6}\end{cases}$$

input
```
integrate(x**5*(a+b*atan(c*x**3))**2,x)
```

output
```
Piecewise((a**2*x**6/6 + a*b*x**6*atan(c*x**3)/3 - a*b*x**3/(3*c) + a*b*at
an(c*x**3)/(3*c**2) + b**2*x**6*atan(c*x**3)**2/6 - b**2*x**3*atan(c*x**3)
/(3*c) - b**2*sqrt(-1/c**2)*atan(c*x**3)/(3*c) + b**2*log(x - (-1/c**2)**(
1/6))/(3*c**2) + b**2*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/
3))/(3*c**2) + b**2*atan(c*x**3)**2/(6*c**2), Ne(c, 0)), (a**2*x**6/6, Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.40

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$
$$=\frac{1}{6}\,b^2x^6\arctan\left(cx^3\right)^2+\frac{1}{6}\,a^2x^6+\frac{1}{3}\left(x^6\arctan\left(cx^3\right)-c\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)\right)ab$$
$$-\frac{1}{6}\left(2\,c\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)\arctan\left(cx^3\right)+\frac{\arctan\left(cx^3\right)^2-\log\left(6\,c^5x^6+6\,c^3\right)}{c^2}\right)b^2$$

input
```
integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

output
```
1/6*b^2*x^6*arctan(c*x^3)^2 + 1/6*a^2*x^6 + 1/3*(x^6*arctan(c*x^3) - c*(x^
3/c^2 - arctan(c*x^3)/c^3))*a*b - 1/6*(2*c*(x^3/c^2 - arctan(c*x^3)/c^3)*a
rctan(c*x^3) + (arctan(c*x^3)^2 - log(6*c^5*x^6 + 6*c^3))/c^2)*b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)^2\,dx = \frac{a^2cx^6+\frac{2\left(c^2x^6\arctan\left(cx^3\right)-cx^3+\arctan\left(cx^3\right)\right)ab}{c}+\frac{\left(c^2x^6\arctan\left(cx^3\right)^2-2\,cx^3\arctan\left(cx^3\right)+\arctan\left(cx^3\right)^2+\log\left(c^2x^6+1\right)\right)b^2}{c}}{6\,c}$$

input
```
integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

output
```
1/6*(a^2*c*x^6 + 2*(c^2*x^6*arctan(c*x^3) - c*x^3 + arctan(c*x^3))*a*b/c +
 (c^2*x^6*arctan(c*x^3)^2 - 2*c*x^3*arctan(c*x^3) + arctan(c*x^3)^2 + log(
c^2*x^6 + 1))*b^2/c)/c
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.24

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)^2\,dx = \frac{a^2\,x^6}{6}+\frac{b^2\,\mathrm{atan}(c\,x^3)^2}{6\,c^2}+\frac{b^2\,x^6\,\mathrm{atan}(c\,x^3)^2}{6} + \frac{b^2\,\ln\left(c^2\,x^6+1\right)}{6\,c^2}-\frac{b^2\,x^3\,\mathrm{atan}(c\,x^3)}{3\,c} - \frac{a\,b\,x^3}{3\,c}+\frac{a\,b\,\mathrm{atan}(c\,x^3)}{3\,c^2}+\frac{a\,b\,x^6\,\mathrm{atan}(c\,x^3)}{3}$$

input
```
int(x^5*(a + b*atan(c*x^3))^2,x)
```

output
```
(a^2*x^6)/6 + (b^2*atan(c*x^3)^2)/(6*c^2) + (b^2*x^6*atan(c*x^3)^2)/6 + (b
^2*log(c^2*x^6 + 1))/(6*c^2) - (b^2*x^3*atan(c*x^3))/(3*c) - (a*b*x^3)/(3*
c) + (a*b*atan(c*x^3))/(3*c^2) + (a*b*x^6*atan(c*x^3))/3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.69

$$\int x^5 (a + b \arctan(cx^3))^2 \, dx$$
$$= \frac{atan(c x^3)^2 b^2 c^2 x^6 + atan(c x^3)^2 b^2 + 2atan(c x^3) ab c^2 x^6 + 2atan(c x^3) ab - 2atan(c x^3) b^2 c x^3 + \log\left(c^{\frac{2}{3}} x\right.}{6c^2}$$

input
```
int(x^5*(a+b*atan(c*x^3))^2,x)
```

output
```
(atan(c*x**3)**2*b**2*c**2*x**6 + atan(c*x**3)**2*b**2 + 2*atan(c*x**3)*a*
b*c**2*x**6 + 2*atan(c*x**3)*a*b - 2*atan(c*x**3)*b**2*c*x**3 + log(c**(2/
3)*x**2 - c**(1/3)*sqrt(3)*x + 1)*b**2 + log(c**(2/3)*x**2 + c**(1/3)*sqrt
(3)*x + 1)*b**2 + log(c**(2/3)*x**2 + 1)*b**2 + a**2*c**2*x**6 - 2*a*b*c*x
**3)/(6*c**2)
```

3.116 $\int x^2(a+b\arctan(cx^3))^2\,dx$

Optimal result 886
Mathematica [A] (verified) 886
Rubi [A] (verified) 887
Maple [A] (verified) 889
Fricas [F] 890
Sympy [F] 890
Maxima [F] 890
Giac [F] 891
Mupad [F(-1)] 891
Reduce [F] 892

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\frac{i(a+b\arctan(cx^3))^2}{3c}+\frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)^2+\frac{2b(a+b\arctan(cx^3))\log\left(\frac{2}{1+icx^3}\right)}{3c}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{3c}$$

output
```
1/3*I*(a+b*arctan(c*x^3))^2/c+1/3*x^3*(a+b*arctan(c*x^3))^2+2/3*b*(a+b*arc
tan(c*x^3))*ln(2/(1+I*c*x^3))/c+1/3*I*b^2*polylog(2,1-2/(1+I*c*x^3))/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$
$$=\frac{b^2(-i+cx^3)\arctan\left(cx^3\right)^2+2b\arctan\left(cx^3\right)\left(acx^3+b\log\left(1+e^{2i\arctan(cx^3)}\right)\right)+a(acx^3-b\log\left(1+c^2x\right.}{3c}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
(b^2*(-I + c*x^3)*ArcTan[c*x^3]^2 + 2*b*ArcTan[c*x^3]*(a*c*x^3 + b*Log[1 +
 E^((2*I)*ArcTan[c*x^3])]) + a*(a*c*x^3 - b*Log[1 + c^2*x^6]) - I*b^2*Poly
Log[2, -E^((2*I)*ArcTan[c*x^3])])/(3*c)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5363, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^2dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{3}\int \left(a+b\arctan\left(cx^3\right)\right)^2dx^3$$

$$\Big\downarrow \text{5345}$$

$$\frac{1}{3}\left(x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\int \frac{x^3\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3\right)$$

$$\Big\downarrow \text{5455}$$

$$\frac{1}{3}\left(x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\left(-\frac{\int \frac{a+b\arctan\left(cx^3\right)}{i-cx^3}dx^3}{c}-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{2bc^2}\right)\right)$$

$$\Big\downarrow \text{5379}$$

$$\frac{1}{3}\left(x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)}{c}-b\int \frac{\log\left(\frac{2}{icx^3+1}\right)}{c^2x^6+1}dx^3}{c}-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{2bc^2}\right)\right)$$

$$\Big\downarrow \text{2849}$$

$$\frac{1}{3}\left(x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\left(-\frac{-\frac{ib\int\frac{\log\left(\frac{2}{icx^3+1}\right)}{1-\frac{2}{icx^3+1}}d\frac{1}{icx^3+1}}{c}+\frac{\log\left(\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)}{c}}{c}-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{2bc^2}\right.\right.$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{3}\left(x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\left(-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)}{c}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)}{2c}}{c}\right.\right.$$

input
```
Int[x^2*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
(x^3*(a + b*ArcTan[c*x^3])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^3])^2)/(
b*c^2) - (((a + b*ArcTan[c*x^3])*Log[2/(1 + I*c*x^3)])/c + ((I/2)*b*PolyLo
g[2, 1 - 2/(1 + I*c*x^3)])/c)/c))/3
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.35

method	result
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\arctan(cx^3)^2(cx^3+i)+2\arctan(cx^3)\ln\left(1+\frac{(icx^3+1)^2}{c^2x^6+1}\right)-2i\arctan(cx^3)^2-i\,\mathrm{polylog}\left(2,-\frac{(icx^3+1)^2}{c^2x^6+1}\right.\right.}{3c}$
derivativedivides	$\frac{cx^3a^2-i\arctan(cx^3)^2b^2+\arctan(cx^3)^2b^2cx^3-i\,\mathrm{polylog}\left(2,-\frac{(icx^3+1)^2}{c^2x^6+1}\right)b^2+2\arctan(cx^3)\ln\left(1+\frac{(icx^3+1)^2}{c^2x^6+1}\right)b^2-}{3c}$
default	$\frac{cx^3a^2-i\arctan(cx^3)^2b^2+\arctan(cx^3)^2b^2cx^3-i\,\mathrm{polylog}\left(2,-\frac{(icx^3+1)^2}{c^2x^6+1}\right)b^2+2\arctan(cx^3)\ln\left(1+\frac{(icx^3+1)^2}{c^2x^6+1}\right)b^2-}{3c}$
risch	$-\frac{ib^2\ln(icx^3+1)}{3c}-\frac{\ln(-icx^3+1)ab}{3c}-\frac{ib^2\ln\left(\frac{1}{2}+\frac{icx^3}{2}\right)\ln(-icx^3+1)}{3c}-\frac{iba\ln(icx^3+1)x^3}{3}-\frac{\ln(-icx^3+1)^2b}{12}$

input
```
int(x^2*(a+b*arctan(c*x^3))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+1/3*b^2/c*(arctan(c*x^3)^2*(c*x^3+I)+2*arctan(c*x^3)*ln(1+(1+I
*c*x^3)^2/(c^2*x^6+1))-2*I*arctan(c*x^3)^2-I*polylog(2,-(1+I*c*x^3)^2/(c^2
*x^6+1)))+2/3*a*b*x^3*arctan(c*x^3)-1/3/c*a*b*ln(c^2*x^6+1)
```

Fricas [F]

$$\int x^2 \left(a+b\arctan \left(cx^3\right)\right)^2 \,dx=\int \left(b\arctan \left(cx^3\right)+a\right)^2 x^2 \,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arctan(c*x^3)^2 + 2*a*b*x^2*arctan(c*x^3) + a^2*x^2, x)
```

Sympy [F]

$$\int x^2 \left(a+b\arctan \left(cx^3\right)\right)^2 \,dx=\int x^2 \left(a+b\,\mathrm{atan}\left(cx^3\right)\right)^2 \,dx$$

input
```
integrate(x**2*(a+b*atan(c*x**3))**2,x)
```

output
```
Integral(x**2*(a + b*atan(c*x**3))**2, x)
```

Maxima [F]

$$\int x^2 \left(a+b\arctan \left(cx^3\right)\right)^2 \,dx=\int \left(b\arctan \left(cx^3\right)+a\right)^2 x^2 \,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/48*(4*x^3*arctan(c*x^3)^2 - x^3*log(c^2*x^6 + 1)^2 + 576*c
^2*integrate(1/16*x^8*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 48*c^2*integrate
(1/16*x^8*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 192*c^2*integrate(1/16*x^
8*log(c^2*x^6 + 1)/(c^2*x^6 + 1), x) + 4*arctan(c*x^3)^3/c - 384*c*integra
te(1/16*x^5*arctan(c*x^3)/(c^2*x^6 + 1), x) + 48*integrate(1/16*x^2*log(c^
2*x^6 + 1)^2/(c^2*x^6 + 1), x))*b^2 + 1/3*(2*c*x^3*arctan(c*x^3) - log(c^2
*x^6 + 1))*a*b/c
```

Giac [F]

$$\int x^2 \left(a + b \arctan \left(cx^3\right)\right)^2 \, dx = \int \left(b \arctan \left(cx^3\right) + a\right)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a + b \arctan \left(cx^3\right)\right)^2 \, dx = \int x^2 \left(a + b \,\mathrm{atan}\left(c \, x^3\right)\right)^2 \, dx$$

input
```
int(x^2*(a + b*atan(c*x^3))^2,x)
```

output
```
int(x^2*(a + b*atan(c*x^3))^2, x)
```

Reduce [F]

$$\int x^2 \left(a + b \arctan \left(c x^3\right)\right)^2 \, dx$$
$$= \frac{2 atan(c\,x^3)\,abc\,x^3 + 3\left(\int atan(c\,x^3)^2\,x^2 dx\right) b^2 c - \log\left(c^{\frac{2}{3}} x^2 - c^{\frac{1}{3}}\sqrt{3}\,x + 1\right) ab - \log\left(c^{\frac{2}{3}} x^2 + c^{\frac{1}{3}}\sqrt{3}\,x + 1\right.}{3c}$$

input
```
int(x^2*(a+b*atan(c*x^3))^2,x)
```

output
```
(2*atan(c*x**3)*a*b*c*x**3 + 3*int(atan(c*x**3)**2*x**2,x)*b**2*c - log(c*
*(2/3)*x**2 - c**(1/3)*sqrt(3)*x + 1)*a*b - log(c**(2/3)*x**2 + c**(1/3)*s
qrt(3)*x + 1)*a*b - log(c**(2/3)*x**2 + 1)*a*b + a**2*c*x**3)/(3*c)
```

3.117 $\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx$

Optimal result 893
Mathematica [A] (verified) 894
Rubi [A] (verified) 894
Maple [F] 896
Fricas [F] 897
Sympy [F] 897
Maxima [F] 897
Giac [F] 898
Mupad [F(-1)] 898
Reduce [F] 898

Optimal result

Integrand size = 16, antiderivative size = 154

$$\begin{aligned}\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx &= \frac{2}{3}(a+b\arctan(cx^3))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx^3}\right)\\ &\quad-\frac{1}{3}ib(a+b\arctan(cx^3))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)\\ &\quad+\frac{1}{3}ib(a+b\arctan(cx^3))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx^3}\right)\\ &\quad-\frac{1}{6}b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right)\\ &\quad+\frac{1}{6}b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx^3}\right)\end{aligned}$$

output

```
-2/3*(a+b*arctan(c*x^3))^2*arctanh(-1+2/(1+I*c*x^3))-1/3*I*b*(a+b*arctan(c
*x^3))*polylog(2,1-2/(1+I*c*x^3))+1/3*I*b*(a+b*arctan(c*x^3))*polylog(2,-1
+2/(1+I*c*x^3))-1/6*b^2*polylog(3,1-2/(1+I*c*x^3))+1/6*b^2*polylog(3,-1+2/
(1+I*c*x^3))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx = a^2\log(x)+\frac{1}{3}iab\left(\text{PolyLog}\left(2,-icx^3\right)-\text{PolyLog}\left(2,icx^3\right)\right) + \frac{1}{72}b^2\Big(-i\pi^3+16i\arctan\left(cx^3\right)^3 + 24\arctan\left(cx^3\right)^2\log\left(1-e^{-2i\arctan(cx^3)}\right) - 24\arctan\left(cx^3\right)^2\log\left(1+e^{2i\arctan(cx^3)}\right) + 24i\arctan\left(cx^3\right)\text{PolyLog}\left(2,e^{-2i\arctan(cx^3)}\right) + 24i\arctan\left(cx^3\right)\text{PolyLog}\left(2,-e^{2i\arctan(cx^3)}\right) + 12\,\text{PolyLog}\left(3,e^{-2i\arctan(cx^3)}\right) - 12\,\text{PolyLog}\left(3,-e^{2i\arctan(cx^3)}\right)\Big)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])^2/x,x]
```

output
```
a^2*Log[x] + (I/3)*a*b*(PolyLog[2, (-I)*c*x^3] - PolyLog[2, I*c*x^3]) + (b
^2*((-I)*Pi^3 + (16*I)*ArcTan[c*x^3]^3 + 24*ArcTan[c*x^3]^2*Log[1 - E^((-2
*I)*ArcTan[c*x^3])] - 24*ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan[c*x^3])]
 + (24*I)*ArcTan[c*x^3]*PolyLog[2, E^((-2*I)*ArcTan[c*x^3])] + (24*I)*ArcTa
n[c*x^3]*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])] + 12*PolyLog[3, E^((-2*I)*Ar
cTan[c*x^3])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c*x^3])]))/72
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5359, 5357, 5523, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x}\,dx$$

$\downarrow$ 5359

$$\frac{1}{3}\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x^3}dx^3$$

$\downarrow$ 5357

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^2-4bc\int \frac{\left(a+b\arctan\left(cx^3\right)\right)\text{arctanh}\left(1-\frac{2}{icx^3+1}\right)}{c^2x^6+1}dx^3\right)$$

$\downarrow$ 5523

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^2-4bc\left(\frac{1}{2}\int \frac{\left(a+b\arctan\left(cx^3\right)\right)\log\left(2-\frac{2}{icx^3+1}\right)}{c^2x^6+1}dx^3-\frac{1}{2}\int\right.\right.$$

$\downarrow$ 5529

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^2-4bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)\left(a+b\arctan\left(cx^3\right)\right)}{2c}-\right.\right.\right.$$

$\downarrow$ 7164

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^2-4bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)\left(a+b\arctan\left(cx^3\right)\right)}{2c}+\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^3])^2/x,x]
```

output
```
(2*(a + b*ArcTan[c*x^3])^2*ArcTanh[1 - 2/(1 + I*c*x^3)] - 4*b*c*((((I/2)*(
a + b*ArcTan[c*x^3])*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c + (b*PolyLog[3, 1
- 2/(1 + I*c*x^3)])/(4*c))/2 + (((-1/2*I)*(a + b*ArcTan[c*x^3])*PolyLog[2,
 -1 + 2/(1 + I*c*x^3)])/c - (b*PolyLog[3, -1 + 2/(1 + I*c*x^3)])/(4*c))/2)
)/3
```

Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [F]

$$\int \frac{(a + b \arctan(c x^3))^2}{x} dx$$

input
```
int((a+b*arctan(c*x^3))^2/x,x)
```

output
```
int((a+b*arctan(c*x^3))^2/x,x)
```

Fricas [F]

$$\int \frac{(a + b\arctan(cx^3))^2}{x}\,dx = \int \frac{(b\arctan(cx^3) + a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{(a + b\arctan(cx^3))^2}{x}\,dx = \int \frac{(a + b\,\mathrm{atan}(cx^3))^2}{x}\,dx$$

input
```
integrate((a+b*atan(c*x**3))**2/x,x)
```

output
```
Integral((a + b*atan(c*x**3))**2/x, x)
```

Maxima [F]

$$\int \frac{(a + b\arctan(cx^3))^2}{x}\,dx = \int \frac{(b\arctan(cx^3) + a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x^3)^2 + b^2*log(c^2*x^6 + 1)
^2 + 32*a*b*arctan(c*x^3))/x, x)
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx = \int \frac{(b\arctan(cx^3)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^3))^2}{x}\,dx$$

input
```
int((a + b*atan(c*x^3))^2/x,x)
```

output
```
int((a + b*atan(c*x^3))^2/x, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx = 2\left(\int \frac{atan(c\,x^3)}{x}dx\right)ab+\left(\int \frac{atan(c\,x^3)^2}{x}dx\right)b^2+\log(x)\,a^2$$

input
```
int((a+b*atan(c*x^3))^2/x,x)
```

output
```
2*int(atan(c*x**3)/x,x)*a*b + int(atan(c*x**3)**2/x,x)*b**2 + log(x)*a**2
```

3.118 $\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx$

Optimal result 899
Mathematica [A] (verified) 899
Rubi [A] (verified) 900
Maple [C] (warning: unable to verify) 902
Fricas [F] 902
Sympy [F] 903
Maxima [F] 903
Giac [F] 903
Mupad [F(-1)] 904
Reduce [F] 904

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx = -\frac{1}{3}ic(a+b\arctan(cx^3))^2 - \frac{(a+b\arctan(cx^3))^2}{3x^3} + \frac{2}{3}bc(a+b\arctan(cx^3))\log\left(2-\frac{2}{1-icx^3}\right) - \frac{1}{3}ib^2c\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)$$

output

```
-1/3*I*c*(a+b*arctan(c*x^3))^2-1/3*(a+b*arctan(c*x^3))^2/x^3+2/3*b*c*(a+b*
arctan(c*x^3))*ln(2-2/(1-I*c*x^3))-1/3*I*b^2*c*polylog(2,-1+2/(1-I*c*x^3))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx = \frac{b^2(-1-icx^3)\arctan(cx^3)^2 + 2b\arctan(cx^3)\left(-a+bcx^3\log\left(1-e^{2i\arctan(cx^3)}\right)\right) - a(a-2bcx^3\log(cx^3}{3x^3}$$

input
```
Integrate[(a + b*ArcTan[c*x^3])^2/x^4,x]
```

output
```
(b^2*(-1 - I*c*x^3)*ArcTan[c*x^3]^2 + 2*b*ArcTan[c*x^3]*(-a + b*c*x^3*Log[
1 - E^((2*I)*ArcTan[c*x^3])]) - a*(a - 2*b*c*x^3*Log[c*x^3] + b*c*x^3*Log[
1 + c^2*x^6]) - I*b^2*c*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x^3])])/(3*x^3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5363, 5361, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{3}\int \frac{(a+b\arctan(cx^3))^2}{x^6}dx^3$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\left(2bc\int \frac{a+b\arctan(cx^3)}{x^3(c^2x^6+1)}dx^3-\frac{(a+b\arctan(cx^3))^2}{x^3}\right)$$

$$\Big\downarrow \text{5459}$$

$$\frac{1}{3}\left(-\frac{(a+b\arctan(cx^3))^2}{x^3}+2bc\left(i\int \frac{a+b\arctan(cx^3)}{x^3(cx^3+i)}dx^3-\frac{i(a+b\arctan(cx^3))^2}{2b}\right)\right)$$

$$\Big\downarrow \text{5403}$$

$$\frac{1}{3}\left(-\frac{(a+b\arctan(cx^3))^2}{x^3}+2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx^3}\right)}{c^2x^6+1}dx^3-i\log\left(2-\frac{2}{1-icx^3}\right)(a+b\arctan(cx^3))\right)\right.\right.$$

$$\Big\downarrow \text{2897}$$

$$\frac{1}{3}\left(-\frac{(a+b\arctan(cx^3))^2}{x^3}+2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx^3}\right)(a+b\arctan(cx^3))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx^3}-1\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^3])^2/x^4,x]
```

output
```
(-((a + b*ArcTan[c*x^3])^2/x^3) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^3])^2
)/b + I*((-I)*(a + b*ArcTan[c*x^3])*Log[2 - 2/(1 - I*c*x^3)] - (b*PolyLog[
2, -1 + 2/(1 - I*c*x^3)])/2)))/3
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 11455, normalized size of antiderivative = 114.55

method	result	size
default	Expression too large to display	11455
parts	Expression too large to display	11455

input
```
int((a+b*arctan(c*x^3))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx=\int \frac{(b\arctan(cx^3)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x^4, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx = \int \frac{(a+b\operatorname{atan}(cx^3))^2}{x^4}\,dx$$

input
```
integrate((a+b*atan(c*x**3))**2/x**4,x)
```

output
```
Integral((a + b*atan(c*x**3))**2/x**4, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx = \int \frac{(b\arctan(cx^3)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*a*b + 1/48*(4
8*x^3*integrate(-1/16*(4*c^2*x^6*log(c^2*x^6 + 1) - 8*c*x^3*arctan(c*x^3)
- 12*(c^2*x^6 + 1)*arctan(c*x^3)^2 - (c^2*x^6 + 1)*log(c^2*x^6 + 1)^2)/(c^
2*x^10 + x^4), x) - 4*arctan(c*x^3)^2 + log(c^2*x^6 + 1)^2)*b^2/x^3 - 1/3*
a^2/x^3
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx = \int \frac{(b\arctan(cx^3)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^3))^2}{x^4}\,dx$$

input
```
int((a + b*atan(c*x^3))^2/x^4,x)
```

output
```
int((a + b*atan(c*x^3))^2/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx = \frac{-atan(c\,x^3)^2\,b^2 - 2atan(c\,x^3)\,ab + 6\left(\int \frac{atan(c\,x^3)}{c^2x^7+x}dx\right)b^2c\,x^3 - \log\left(c^{\frac{2}{3}}x^2 - c^{\frac{1}{3}}\sqrt{3}\,x+1\right)abc\,x^3 - \log\left(c^{\frac{2}{3}}x\right.}{3x^3}$$

input
```
int((a+b*atan(c*x^3))^2/x^4,x)
```

output
```
( - atan(c*x**3)**2*b**2 - 2*atan(c*x**3)*a*b + 6*int(atan(c*x**3)/(c**2*x
**7 + x),x)*b**2*c*x**3 - log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x + 1)*a*b*
c*x**3 - log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x + 1)*a*b*c*x**3 - log(c**(
2/3)*x**2 + 1)*a*b*c*x**3 + 6*log(x)*a*b*c*x**3 - a**2)/(3*x**3)
```

3.119 $\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x^7}\,dx$

Optimal result 905
Mathematica [A] (verified) 905
Rubi [A] (verified) 906
Maple [A] (verified) 909
Fricas [A] (verification not implemented) 909
Sympy [B] (verification not implemented) 910
Maxima [A] (verification not implemented) 910
Giac [F] 911
Mupad [B] (verification not implemented) 911
Reduce [B] (verification not implemented) 912

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = -\frac{bc(a+b\arctan(cx^3))}{3x^3} - \frac{1}{6}c^2\left(a+b\arctan\left(cx^3\right)\right)^2 - \frac{(a+b\arctan(cx^3))^2}{6x^6} + b^2c^2\log(x) - \frac{1}{6}b^2c^2\log\left(1+c^2x^6\right)$$

output
```
-1/3*b*c*(a+b*arctan(c*x^3))/x^3-1/6*c^2*(a+b*arctan(c*x^3))^2-1/6*(a+b*ar
ctan(c*x^3))^2/x^6+b^2*c^2*ln(x)-1/6*b^2*c^2*ln(c^2*x^6+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = -\frac{a^2+2abcx^3+2b(a+bcx^3+ac^2x^6)\arctan(cx^3)+b^2(1+c^2x^6)\arctan(cx^3)^2-6b^2c^2x^6\log(x)+b^2c^2x}{6x^6}$$

input
```
Integrate[(a + b*ArcTan[c*x^3])^2/x^7,x]
```

output
```
-1/6*(a^2 + 2*a*b*c*x^3 + 2*b*(a + b*c*x^3 + a*c^2*x^6)*ArcTan[c*x^3] + b^
2*(1 + c^2*x^6)*ArcTan[c*x^3]^2 - 6*b^2*c^2*x^6*Log[x] + b^2*c^2*x^6*Log[1
 + c^2*x^6])/x^6
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5363, 5361, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{3}\int \frac{(a+b\arctan(cx^3))^2}{x^9}dx^3$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\left(bc\int \frac{a+b\arctan(cx^3)}{x^6(c^2x^6+1)}dx^3-\frac{(a+b\arctan(cx^3))^2}{2x^6}\right)$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{3}\left(bc\left(\int \frac{a+b\arctan(cx^3)}{x^6}dx^3-c^2\int \frac{a+b\arctan(cx^3)}{c^2x^6+1}dx^3\right)-\frac{(a+b\arctan(cx^3))^2}{2x^6}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\left(bc\left(c^2\left(-\int \frac{a+b\arctan(cx^3)}{c^2x^6+1}dx^3\right)+bc\int \frac{1}{x^3(c^2x^6+1)}dx^3-\frac{a+b\arctan(cx^3)}{x^3}\right)-\frac{(a+b\arctan(cx^3))}{2x^6}\right.$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}\left(bc\left(c^2\left(-\int \frac{a+b\arctan(cx^3)}{c^2x^6+1}dx^3\right)+\frac{1}{2}bc\int \frac{1}{x^3(c^2x^6+1)}dx^6-\frac{a+b\arctan(cx^3)}{x^3}\right)-\frac{(a+b\arctan(cx^3}{2x^6}\right.$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{3}\left(bc\left(c^2\left(-\int \frac{a+b\arctan\left(cx^3\right)}{c^2x^6+1}dx^3\right)+\frac{1}{2}bc\left(\int \frac{1}{x^3}dx^6-c^2\int \frac{1}{c^2x^6+1}dx^6\right)-\frac{a+b\arctan\left(cx^3\right)}{x^3}\right)-\frac{(a+b$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{3}\left(bc\left(c^2\left(-\int \frac{a+b\arctan\left(cx^3\right)}{c^2x^6+1}dx^3\right)+\frac{1}{2}bc\left(\log\left(x^6\right)-c^2\int \frac{1}{c^2x^6+1}dx^6\right)-\frac{a+b\arctan\left(cx^3\right)}{x^3}\right)-\frac{(a+b$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(bc\left(c^2\left(-\int \frac{a+b\arctan\left(cx^3\right)}{c^2x^6+1}dx^3\right)-\frac{a+b\arctan\left(cx^3\right)}{x^3}+\frac{1}{2}bc\left(\log\left(x^6\right)-\log\left(c^2x^6+1\right)\right)\right)-\frac{(a+b\arcta}{2x^6}$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{3}\left(bc\left(-\frac{c\left(a+b\arctan\left(cx^3\right)\right)^2}{2b}-\frac{a+b\arctan\left(cx^3\right)}{x^3}+\frac{1}{2}bc\left(\log\left(x^6\right)-\log\left(c^2x^6+1\right)\right)\right)-\frac{(a+b\arctan\left(cx^3\right))}{2x^6}$$

input
```
Int[(a + b*ArcTan[c*x^3])^2/x^7,x]
```

output
```
(-1/2*(a + b*ArcTan[c*x^3])^2/x^6 + b*c*(-((a + b*ArcTan[c*x^3])/x^3) - (c
*(a + b*ArcTan[c*x^3])^2)/(2*b) + (b*c*(Log[x^6] - Log[1 + c^2*x^6]))/2))/
3
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

method	result
default	$-\frac{a^2}{6x^6}-\frac{b^2\arctan(cx^3)^2}{6x^6}-\frac{b^2c\arctan(cx^3)}{3x^3}-\frac{b^2\arctan(cx^3)^2c^2}{6}-\frac{b^2c^2\ln(c^2x^6+1)}{6}+b^2c^2\ln(x)-\frac{ab\arctan}{3x}$
parts	$-\frac{a^2}{6x^6}-\frac{b^2\arctan(cx^3)^2}{6x^6}-\frac{b^2c\arctan(cx^3)}{3x^3}-\frac{b^2\arctan(cx^3)^2c^2}{6}-\frac{b^2c^2\ln(c^2x^6+1)}{6}+b^2c^2\ln(x)-\frac{ab\arctan}{3x}$
parallelrisch	$\frac{-b^2\arctan(cx^3)^2x^6c^2+6b^2c^2\ln(x)x^6-b^2c^2\ln(c^2x^6+1)x^6-2ab\arctan(cx^3)x^6c^2+a^2c^2x^6-2b^2\arctan(cx^3)x^3c-2abcx^3-}{6x^6}$
risch	$\frac{b^2(c^2x^6+1)\ln(icx^3+1)^2}{24x^6}+\frac{ib(ibc^2x^6\ln(-icx^3+1)+2bcx^3+2a+ib\ln(-icx^3+1))\ln(icx^3+1)}{12x^6}-\frac{4i\ln((-7ibc+ac)x^3+}{}$

input
```
int((a+b*arctan(c*x^3))^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^2/x^6-1/6*b^2/x^6*arctan(c*x^3)^2-1/3*b^2*c*arctan(c*x^3)/x^3-1/6*b
^2*arctan(c*x^3)^2*c^2-1/6*b^2*c^2*ln(c^2*x^6+1)+b^2*c^2*ln(x)-1/3*a*b/x^6
*arctan(c*x^3)-1/3*a*b*c/x^3-1/3*a*b*arctan(c*x^3)*c^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx =$$
$$-\frac{b^2c^2x^6\log(c^2x^6+1)-6\,b^2c^2x^6\log(x)+2\,abcx^3+(b^2c^2x^6+b^2)\arctan(cx^3)^2+a^2+2\,(abc^2x^6+b^2cx}{6\,x^6}$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="fricas")
```

output
```
-1/6*(b^2*c^2*x^6*log(c^2*x^6 + 1) - 6*b^2*c^2*x^6*log(x) + 2*a*b*c*x^3 +
(b^2*c^2*x^6 + b^2)*arctan(c*x^3)^2 + a^2 + 2*(a*b*c^2*x^6 + b^2*c*x^3 + a
*b)*arctan(c*x^3))/x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(80) = 160$.

Time = 73.57 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.38

$$\int \frac{(a + b \arctan(cx^3))^2}{x^7} \, dx$$
$$= \begin{cases} -\frac{a^2}{6x^6} - \frac{abc^2 \operatorname{atan}(cx^3)}{3} - \frac{abc}{3x^3} - \frac{ab \operatorname{atan}(cx^3)}{3x^6} + \frac{b^2c^3\sqrt{-\frac{1}{c^2}} \operatorname{atan}(cx^3)}{3} + b^2c^2 \log(x) - \frac{b^2c^2 \log\left(x - \sqrt[6]{-\frac{1}{c^2}}\right)}{3} - \frac{b^2c^2 \log}{} \\ -\frac{a^2}{6x^6} \end{cases}$$

input
```
integrate((a+b*atan(c*x**3))**2/x**7,x)
```

output
```
Piecewise((-a**2/(6*x**6) - a*b*c**2*atan(c*x**3)/3 - a*b*c/(3*x**3) - a*b
*atan(c*x**3)/(3*x**6) + b**2*c**3*sqrt(-1/c**2)*atan(c*x**3)/3 + b**2*c**
2*log(x) - b**2*c**2*log(x - (-1/c**2)**(1/6))/3 - b**2*c**2*log(4*x**2 +
4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/3 - b**2*c**2*atan(c*x**3)**2/6
 - b**2*c*atan(c*x**3)/(3*x**3) - b**2*atan(c*x**3)**2/(6*x**6), Ne(c, 0))
, (-a**2/(6*x**6), True))
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

$$\int \frac{(a + b \arctan(cx^3))^2}{x^7} \, dx = -\frac{1}{3} \left(\left(c \arctan(cx^3) + \frac{1}{x^3} \right) c + \frac{\arctan(cx^3)}{x^6} \right) ab$$
$$+ \frac{1}{6} \left(\left(\arctan(cx^3)^2 - \log(c^2x^6 + 1) + 6 \log(x) \right) c^2 - 2 \left(c \arctan(cx^3) + \frac{1}{x^3} \right) c \arctan(cx^3) \right) b^2$$
$$- \frac{b^2 \arctan(cx^3)^2}{6x^6} - \frac{a^2}{6x^6}$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="maxima")
```

output
```
-1/3*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*a*b + 1/6*((arctan(
c*x^3)^2 - log(c^2*x^6 + 1) + 6*log(x))*c^2 - 2*(c*arctan(c*x^3) + 1/x^3)*
c*arctan(c*x^3))*b^2 - 1/6*b^2*arctan(c*x^3)^2/x^6 - 1/6*a^2/x^6
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx=\int \frac{(b\arctan(cx^3)+a)^2}{x^7}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^2/x^7, x)
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.75

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx=b^2\,c^2\,\ln(x)-\frac{b^2\,c^2\,\mathrm{atan}(c\,x^3)^2}{6}-\frac{b^2\,\mathrm{atan}(c\,x^3)^2}{6\,x^6}-\frac{b^2\,c^2\,\ln(c^2\,x^6+1)}{6}-\frac{a^2}{6\,x^6}-\frac{b^2\,c\,\mathrm{atan}(c\,x^3)}{3\,x^3}-\frac{a\,b\,c}{3\,x^3}-\frac{a\,b\,c^2\,\mathrm{atan}\left(\frac{a^2\,c\,x^3}{a^2+49\,b^2}+\frac{49\,b^2\,c\,x^3}{a^2+49\,b^2}\right)}{3}-\frac{a\,b\,\mathrm{atan}(c\,x^3)}{3\,x^6}$$

input
```
int((a + b*atan(c*x^3))^2/x^7,x)
```

output
```
b^2*c^2*log(x) - (b^2*c^2*atan(c*x^3)^2)/6 - (b^2*atan(c*x^3)^2)/(6*x^6) -
 (b^2*c^2*log(c^2*x^6 + 1))/6 - a^2/(6*x^6) - (b^2*c*atan(c*x^3))/(3*x^3)
- (a*b*c)/(3*x^3) - (a*b*c^2*atan((a^2*c*x^3)/(a^2 + 49*b^2) + (49*b^2*c*x
^3)/(a^2 + 49*b^2)))/3 - (a*b*atan(c*x^3))/(3*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.10

$$\int \frac{(a + b\arctan(cx^3))^2}{x^7}\,dx$$

$$= \frac{-atan(c\,x^3)^2\,b^2c^2x^6 - atan(c\,x^3)^2\,b^2 - 2atan(c\,x^3)\,ab\,c^2x^6 - 2atan(c\,x^3)\,ab - 2atan(c\,x^3)\,b^2c\,x^3 - \log\Big(c}{}$$

input
```
int((a+b*atan(c*x^3))^2/x^7,x)
```

output
```
( - atan(c*x**3)**2*b**2*c**2*x**6 - atan(c*x**3)**2*b**2 - 2*atan(c*x**3)
*a*b*c**2*x**6 - 2*atan(c*x**3)*a*b - 2*atan(c*x**3)*b**2*c*x**3 - log(c**
(2/3)*x**2 - c**(1/3)*sqrt(3)*x + 1)*b**2*c**2*x**6 - log(c**(2/3)*x**2 +
c**(1/3)*sqrt(3)*x + 1)*b**2*c**2*x**6 - log(c**(2/3)*x**2 + 1)*b**2*c**2*
x**6 + 6*log(x)*b**2*c**2*x**6 - a**2 - 2*a*b*c*x**3)/(6*x**6)
```

3.120 $\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx$

Optimal result 913
Mathematica [A] (verified) 914
Rubi [A] (verified) 914
Maple [C] (warning: unable to verify) 917
Fricas [F] 918
Sympy [F(-1)] 918
Maxima [F] 918
Giac [F] 919
Mupad [F(-1)] 919
Reduce [F] 919

Optimal result

Integrand size = 16, antiderivative size = 154

$$\begin{aligned}\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx &= -\frac{b^2c^2}{9x^3}-\frac{1}{9}b^2c^3\arctan(cx^3)-\frac{bc(a+b\arctan(cx^3))}{9x^6}\\&\quad+\frac{1}{9}ic^3(a+b\arctan(cx^3))^2-\frac{(a+b\arctan(cx^3))^2}{9x^9}\\&\quad-\frac{2}{9}bc^3(a+b\arctan(cx^3))\log\left(2-\frac{2}{1-icx^3}\right)\\&\quad+\frac{1}{9}ib^2c^3\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)\end{aligned}$$

output
```
-1/9*b^2*c^2/x^3-1/9*b^2*c^3*arctan(c*x^3)-1/9*b*c*(a+b*arctan(c*x^3))/x^6
+1/9*I*c^3*(a+b*arctan(c*x^3))^2-1/9*(a+b*arctan(c*x^3))^2/x^9-2/9*b*c^3*(
a+b*arctan(c*x^3))*ln(2-2/(1-I*c*x^3))+1/9*I*b^2*c^3*polylog(2,-1+2/(1-I*c
*x^3))
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx = -\frac{a^2+abcx^3+b^2c^2x^6+b^2(1-ic^3x^9)\arctan(cx^3)^2+b\arctan(cx^3)\left(2a+bcx^3+bc^3x^9+2bc^3x^9\log\left(1\right.\right.}{9x^9}$$

input
```
Integrate[(a + b*ArcTan[c*x^3])^2/x^10,x]
```

output
```
-1/9*(a^2 + a*b*c*x^3 + b^2*c^2*x^6 + b^2*(1 - I*c^3*x^9)*ArcTan[c*x^3]^2
+ b*ArcTan[c*x^3]*(2*a + b*c*x^3 + b*c^3*x^9 + 2*b*c^3*x^9*Log[1 - E^((2*I
)*ArcTan[c*x^3])]) + 2*a*b*c^3*x^9*Log[c*x^3] - a*b*c^3*x^9*Log[1 + c^2*x^
6] - I*b^2*c^3*x^9*PolyLog[2, E^((2*I)*ArcTan[c*x^3])])/x^9
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5363, 5361, 5453, 5361, 264, 216, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{3}\int \frac{(a+b\arctan(cx^3))^2}{x^{12}}dx^3$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\left(\frac{2}{3}bc\int \frac{a+b\arctan(cx^3)}{x^9(c^2x^6+1)}dx^3-\frac{(a+b\arctan(cx^3))^2}{3x^9}\right)$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{3}\left(\frac{2}{3}bc\left(\int \frac{a+b\arctan\left(cx^3\right)}{x^9}dx^3-c^2\int \frac{a+b\arctan\left(cx^3\right)}{x^3\left(c^2x^6+1\right)}dx^3\right)-\frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{3x^9}\right)$$

$\big\downarrow$ 5361

$$\frac{1}{3}\left(\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan\left(cx^3\right)}{x^3\left(c^2x^6+1\right)}dx^3\right)+\frac{1}{2}bc\int \frac{1}{x^6\left(c^2x^6+1\right)}dx^3-\frac{a+b\arctan\left(cx^3\right)}{2x^6}\right)-\frac{(a+b\arctan(cx}{3x^9}$$

$\big\downarrow$ 264

$$\frac{1}{3}\left(\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan\left(cx^3\right)}{x^3\left(c^2x^6+1\right)}dx^3\right)+\frac{1}{2}bc\left(c^2\left(-\int \frac{1}{c^2x^6+1}dx^3\right)-\frac{1}{x^3}\right)-\frac{a+b\arctan\left(cx^3\right)}{2x^6}\right)-\frac{(a+}{}$$

$\big\downarrow$ 216

$$\frac{1}{3}\left(\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan\left(cx^3\right)}{x^3\left(c^2x^6+1\right)}dx^3\right)-\frac{a+b\arctan\left(cx^3\right)}{2x^6}+\frac{1}{2}bc\left(-c\arctan\left(cx^3\right)-\frac{1}{x^3}\right)\right)-\frac{(a+b\arctan}{3x^9}$$

$\big\downarrow$ 5459

$$\frac{1}{3}\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{3x^9}+\frac{2}{3}bc\left(-\left(c^2\left(i\int \frac{a+b\arctan\left(cx^3\right)}{x^3\left(cx^3+i\right)}dx^3-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{2b}\right)\right)-\frac{a+b\arcta}{2x^6}$$

$\big\downarrow$ 5403

$$\frac{1}{3}\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{3x^9}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx^3}\right)}{c^2x^6+1}dx^3-i\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\arctan\left(\right.\right.\right.\right.\right.\right.$$

$\big\downarrow$ 2897

$$\frac{1}{3}\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{3x^9}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-}\right.\right.\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^3])^2/x^10,x]
```

output
```
(-1/3*(a + b*ArcTan[c*x^3])^2/x^9 + (2*b*c*(-1/2*(a + b*ArcTan[c*x^3])/x^6
 + (b*c*(-x^(-3) - c*ArcTan[c*x^3]))/2 - c^2*(((-1/2*I)*(a + b*ArcTan[c*x^
3])^2)/b + I*((-I)*(a + b*ArcTan[c*x^3])*Log[2 - 2/(1 - I*c*x^3)] - (b*Pol
yLog[2, -1 + 2/(1 - I*c*x^3)])/2))))/3)/3
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.06 (sec) , antiderivative size = 11496, normalized size of antiderivative = 74.65

method	result	size
default	Expression too large to display	11496
parts	Expression too large to display	11496

input
```
int((a+b*arctan(c*x^3))^2/x^10,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(a + b\arctan(cx^3))^2}{x^{10}}\,dx = \int \frac{(b\arctan(cx^3) + a)^2}{x^{10}}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x^10, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx^3))^2}{x^{10}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x**3))**2/x**10,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a + b\arctan(cx^3))^2}{x^{10}}\,dx = \int \frac{(b\arctan(cx^3) + a)^2}{x^{10}}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="maxima")
```

output
```
1/9*((c^2*log(c^2*x^6 + 1) - c^2*log(x^6) - 1/x^6)*c - 2*arctan(c*x^3)/x^9
)*a*b + 1/144*(144*x^9*integrate(-1/48*(4*c^2*x^6*log(c^2*x^6 + 1) - 8*c*x
^3*arctan(c*x^3) - 36*(c^2*x^6 + 1)*arctan(c*x^3)^2 - 3*(c^2*x^6 + 1)*log(
c^2*x^6 + 1)^2)/(c^2*x^16 + x^10), x) - 4*arctan(c*x^3)^2 + log(c^2*x^6 +
1)^2)*b^2/x^9 - 1/9*a^2/x^9
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx = \int \frac{(b\arctan(cx^3)+a)^2}{x^{10}}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^2/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^3))^2}{x^{10}}\,dx$$

input
```
int((a + b*atan(c*x^3))^2/x^10,x)
```

output
```
int((a + b*atan(c*x^3))^2/x^10, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx$$
$$= \frac{-atan(c\,x^3)^2\,b^2 - 2atan(c\,x^3)\,ab - atan(c\,x^3)\,b^2c^3x^9 - atan(c\,x^3)\,b^2c\,x^3 - 6\left(\int \frac{atan(c\,x^3)}{c^2x^7+x}dx\right)b^2c^3x^9 + \log}{}$$

input
```
int((a+b*atan(c*x^3))^2/x^10,x)
```

output
```
( - atan(c*x**3)**2*b**2 - 2*atan(c*x**3)*a*b - atan(c*x**3)*b**2*c**3*x**
9 - atan(c*x**3)*b**2*c*x**3 - 6*int(atan(c*x**3)/(c**2*x**7 + x),x)*b**2*
c**3*x**9 + log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x + 1)*a*b*c**3*x**9 + lo
g(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x + 1)*a*b*c**3*x**9 + log(c**(2/3)*x**
2 + 1)*a*b*c**3*x**9 - 6*log(x)*a*b*c**3*x**9 - a**2 - a*b*c*x**3 - b**2*c
**2*x**6)/(9*x**9)
```

3.121 $\int x^8 (a + b \arctan(cx^3))^3 \, dx$

Optimal result . . . 921
Mathematica [A] (verified) . . . 922
Rubi [A] (verified) . . . 922
Maple [F] . . . 926
Fricas [F] . . . 926
Sympy [F(-1)] . . . 927
Maxima [F] . . . 927
Giac [F] . . . 927
Mupad [F(-1)] . . . 928
Reduce [F] . . . 928

Optimal result

Integrand size = 16, antiderivative size = 240

$$\begin{aligned}\int x^8 (a + b \arctan(cx^3))^3 \, dx = {} & \frac{ab^2x^3}{3c^2} + \frac{b^3x^3 \arctan(cx^3)}{3c^2} - \frac{b(a + b \arctan(cx^3))^2}{6c^3} \\ & - \frac{bx^6(a + b \arctan(cx^3))^2}{6c} - \frac{i(a + b \arctan(cx^3))^3}{9c^3} \\ & + \frac{1}{9}x^9 (a + b \arctan(cx^3))^3 \\ & - \frac{b(a + b \arctan(cx^3))^2 \log\left(\frac{2}{1+icx^3}\right)}{3c^3} - \frac{b^3 \log(1 + c^2x^6)}{6c^3} \\ & - \frac{ib^2(a + b \arctan(cx^3)) \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+icx^3}\right)}{3c^3} \\ & - \frac{b^3 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1+icx^3}\right)}{6c^3}\end{aligned}$$

output

```
1/3*a*b^2*x^3/c^2+1/3*b^3*x^3*arctan(c*x^3)/c^2-1/6*b*(a+b*arctan(c*x^3))^
2/c^3-1/6*b*x^6*(a+b*arctan(c*x^3))^2/c-1/9*I*(a+b*arctan(c*x^3))^3/c^3+1/
9*x^9*(a+b*arctan(c*x^3))^3-1/3*b*(a+b*arctan(c*x^3))^2*ln(2/(1+I*c*x^3))/
c^3-1/6*b^3*ln(c^2*x^6+1)/c^3-1/3*I*b^2*(a+b*arctan(c*x^3))*polylog(2,1-2/
(1+I*c*x^3))/c^3-1/6*b^3*polylog(3,1-2/(1+I*c*x^3))/c^3
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.44

$$\int x^8 (a + b \arctan(cx^3))^3 \, dx$$
$$= \frac{6ab^2cx^3 - 3a^2bc^2x^6 + 2a^3c^3x^9 - 6ab^2\arctan(cx^3) + 6b^3cx^3\arctan(cx^3) - 6ab^2c^2x^6\arctan(cx^3) + 6a^2bc}{}$$

input
```
Integrate[x^8*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
(6*a*b^2*c*x^3 - 3*a^2*b*c^2*x^6 + 2*a^3*c^3*x^9 - 6*a*b^2*ArcTan[c*x^3] +
 6*b^3*c*x^3*ArcTan[c*x^3] - 6*a*b^2*c^2*x^6*ArcTan[c*x^3] + 6*a^2*b*c^3*x
^9*ArcTan[c*x^3] + (6*I)*a*b^2*ArcTan[c*x^3]^2 - 3*b^3*ArcTan[c*x^3]^2 - 3
*b^3*c^2*x^6*ArcTan[c*x^3]^2 + 6*a*b^2*c^3*x^9*ArcTan[c*x^3]^2 + (2*I)*b^3
*ArcTan[c*x^3]^3 + 2*b^3*c^3*x^9*ArcTan[c*x^3]^3 - 12*a*b^2*ArcTan[c*x^3]*
Log[1 + E^((2*I)*ArcTan[c*x^3])] - 6*b^3*ArcTan[c*x^3]^2*Log[1 + E^((2*I)*
ArcTan[c*x^3])] + 3*a^2*b*Log[1 + c^2*x^6] - 3*b^3*Log[1 + c^2*x^6] + (6*I
)*b^2*(a + b*ArcTan[c*x^3])*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])] - 3*b^3*P
olyLog[3, -E^((2*I)*ArcTan[c*x^3])])/(18*c^3)
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {5363, 5361, 5451, 5361, 5451, 2009, 5419, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 (a + b \arctan(cx^3))^3 \, dx$$

$$\Big\downarrow \; 5363$$

$$\frac{1}{3} \int x^6 (a + b \arctan(cx^3))^3 \, dx^3$$

$$\Big\downarrow \; 5361$$

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\int\frac{x^9\left(a+b\arctan\left(cx^3\right)\right)^2}{c^2x^6+1}dx^3\right)$$

$\Big\downarrow$ 5451

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\int x^3\left(a+b\arctan\left(cx^3\right)\right)^2dx^3}{c^2}-\frac{\int\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2}{c^2x^6+1}dx^3}{c^2}\right)\right)$$

$\Big\downarrow$ 5361

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\int\frac{x^6\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}-\frac{\int\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2}{c^2x^6+1}}{c^2}\right)\right)$$

$\Big\downarrow$ 5451

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\left(\frac{\int\left(a+b\arctan\left(cx^3\right)\right)dx^3}{c^2}-\frac{\int\frac{a+b\arctan\left(cx^3\right)}{c^2x^6+1}dx^3}{c^2}\right)}{c^2}-\right)\right)$$

$\Big\downarrow$ 2009

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\left(\frac{ax^3+bx^3\arctan\left(cx^3\right)-\frac{b\log\left(c^2x^6+1\right)}{2c}}{c^2}-\frac{\int\frac{a+b\arctan\left(cx\right)}{c^2x^6+1}}{c^2}\right)}{c^2}\right)\right)$$

$\Big\downarrow$ 5419

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\left(\frac{ax^3+bx^3\arctan\left(cx^3\right)-\frac{b\log\left(c^2x^6+1\right)}{2c}}{c^2}-\frac{\left(a+b\arctan\left(cx\right)\right)}{2bc^3}\right)}{c^2}\right)\right)$$

$\Big\downarrow$ 5455

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\left(\frac{ax^3+bx^3\arctan(cx^3)-\frac{b\log\left(c^2x^6+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx}{2bc^3}\right.}{c^2}\right.\right.$$

$\downarrow$ 5379

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\left(\frac{ax^3+bx^3\arctan(cx^3)-\frac{b\log\left(c^2x^6+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx}{2bc^3}\right.}{c^2}\right.\right.$$

$\downarrow$ 5529

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\left(\frac{ax^3+bx^3\arctan(cx^3)-\frac{b\log\left(c^2x^6+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx}{2bc^3}\right.}{c^2}\right.\right.$$

$\downarrow$ 7164

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\left(\frac{ax^3+bx^3\arctan(cx^3)-\frac{b\log\left(c^2x^6+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx}{2bc^3}\right.}{c^2}\right.\right.$$

input
```
Int[x^8*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
((x^9*(a + b*ArcTan[c*x^3])^3)/3 - b*c*(((x^6*(a + b*ArcTan[c*x^3])^2)/2 -
 b*c*(-1/2*(a + b*ArcTan[c*x^3])^2/(b*c^3) + (a*x^3 + b*x^3*ArcTan[c*x^3]
- (b*Log[1 + c^2*x^6])/(2*c))/c^2))/c^2 - (((-1/3*I)*(a + b*ArcTan[c*x^3])
^3)/(b*c^2) - (((a + b*ArcTan[c*x^3])^2*Log[2/(1 + I*c*x^3)])/c - 2*b*(((-
1/2*I)*(a + b*ArcTan[c*x^3])*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c - (b*PolyL
og[3, 1 - 2/(1 + I*c*x^3)])/(4*c)))/c)/c^2))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [F]

$$\int x^8 \left(a+b\arctan\left(c\,x^3\right)\right)^3 dx$$

input
```
int(x^8*(a+b*arctan(c*x^3))^3,x)
```

output
```
int(x^8*(a+b*arctan(c*x^3))^3,x)
```

Fricas [F]

$$\int x^8 \left(a+b\arctan\left(cx^3\right)\right)^3\,dx = \int \left(b\arctan\left(cx^3\right)+a\right)^3 x^8\,dx$$

input
```
integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^8*arctan(c*x^3)^3 + 3*a*b^2*x^8*arctan(c*x^3)^2 + 3*a^2*b*x
^8*arctan(c*x^3) + a^3*x^8, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^8 \left(a + b \arctan \left(c x^3\right)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate(x**8*(a+b*atan(c*x**3))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^8 \left(a + b \arctan \left(c x^3\right)\right)^3 \, dx = \int \left(b \arctan \left(c x^3\right) + a\right)^3 x^8 \, dx$$

input
```
integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

output
```
1/72*b^3*x^9*arctan(c*x^3)^3 - 1/96*b^3*x^9*arctan(c*x^3)*log(c^2*x^6 + 1)
^2 + 1/9*a^3*x^9 + 1/6*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/
c^4)*c)*a^2*b + integrate(1/32*(4*b^3*c^2*x^14*arctan(c*x^3)*log(c^2*x^6 +
 1) + 28*(b^3*c^2*x^14 + b^3*x^8)*arctan(c*x^3)^3 + 4*(24*a*b^2*c^2*x^14 -
 b^3*c*x^11 + 24*a*b^2*x^8)*arctan(c*x^3)^2 + (b^3*c*x^11 + 3*(b^3*c^2*x^1
4 + b^3*x^8)*arctan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^6 + 1), x)
```

Giac [F]

$$\int x^8 \left(a + b \arctan \left(c x^3\right)\right)^3 \, dx = \int \left(b \arctan \left(c x^3\right) + a\right)^3 x^8 \, dx$$

input
```
integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^3*x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int x^8\,{\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)}^3\,dx$$

input
```
int(x^8*(a + b*atan(c*x^3))^3,x)
```

output
```
int(x^8*(a + b*atan(c*x^3))^3, x)
```

Reduce [F]

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^3\,dx$$
$$=\frac{2atan(c\,x^3)^3\,b^3c^3x^9+2atan(c\,x^3)^3\,b^3c\,x^3+6atan(c\,x^3)^2\,a\,b^2c^3x^9+6atan(c\,x^3)^2\,a\,b^2c\,x^3-3atan(c\,x^3)^2\,b}{}$$

input
```
int(x^8*(a+b*atan(c*x^3))^3,x)
```

output
```
(2*atan(c*x**3)**3*b**3*c**3*x**9 + 2*atan(c*x**3)**3*b**3*c*x**3 + 6*atan
(c*x**3)**2*a*b**2*c**3*x**9 + 6*atan(c*x**3)**2*a*b**2*c*x**3 - 3*atan(c*
x**3)**2*b**3*c**2*x**6 - 3*atan(c*x**3)**2*b**3 + 6*atan(c*x**3)*a**2*b*c
**3*x**9 - 6*atan(c*x**3)*a*b**2*c**2*x**6 - 6*atan(c*x**3)*a*b**2 + 6*ata
n(c*x**3)*b**3*c*x**3 - 6*int(atan(c*x**3)**3*x**2,x)*b**3*c - 18*int(atan
(c*x**3)**2*x**2,x)*a*b**2*c + 3*log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x +
1)*a**2*b - 3*log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x + 1)*b**3 + 3*log(c**
(2/3)*x**2 + c**(1/3)*sqrt(3)*x + 1)*a**2*b - 3*log(c**(2/3)*x**2 + c**(1/
3)*sqrt(3)*x + 1)*b**3 + 3*log(c**(2/3)*x**2 + 1)*a**2*b - 3*log(c**(2/3)*
x**2 + 1)*b**3 + 2*a**3*c**3*x**9 - 3*a**2*b*c**2*x**6 + 6*a*b**2*c*x**3)/
(18*c**3)
```

3.122 $\int x^5(a+b\arctan(cx^3))^3\,dx$

Optimal result . . . 929
Mathematica [A] (verified) . . . 930
Rubi [A] (verified) . . . 930
Maple [C] (warning: unable to verify) . . . 934
Fricas [F] . . . 935
Sympy [F] . . . 935
Maxima [F] . . . 935
Giac [F] . . . 936
Mupad [F(-1)] . . . 936
Reduce [F] . . . 937

Optimal result

Integrand size = 16, antiderivative size = 147

$$\int x^5(a+b\arctan(cx^3))^3\,dx=-\frac{ib(a+b\arctan(cx^3))^2}{2c^2}-\frac{bx^3(a+b\arctan(cx^3))^2}{2c}+\frac{(a+b\arctan(cx^3))^3}{6c^2}+\frac{1}{6}x^6(a+b\arctan(cx^3))^3-\frac{b^2(a+b\arctan(cx^3))\log\left(\frac{2}{1+icx^3}\right)}{c^2}-\frac{ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{2c^2}$$

output
```
-1/2*I*b*(a+b*arctan(c*x^3))^2/c^2-1/2*b*x^3*(a+b*arctan(c*x^3))^2/c+1/6*(
a+b*arctan(c*x^3))^3/c^2+1/6*x^6*(a+b*arctan(c*x^3))^3-b^2*(a+b*arctan(c*x
^3))*ln(2/(1+I*c*x^3))/c^2-1/2*I*b^3*polylog(2,1-2/(1+I*c*x^3))/c^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)^3\,dx = \frac{3b^2(a+ac^2x^6+b(i-cx^3))\arctan\left(cx^3\right)^2+b^3(1+c^2x^6)\arctan\left(cx^3\right)^3+3b\arctan\left(cx^3\right)\left(a(a-2bcx^3+\right.}{}$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
(3*b^2*(a + a*c^2*x^6 + b*(I - c*x^3))*ArcTan[c*x^3]^2 + b^3*(1 + c^2*x^6)
*ArcTan[c*x^3]^3 + 3*b*ArcTan[c*x^3]*(a*(a - 2*b*c*x^3 + a*c^2*x^6) - 2*b^
2*Log[1 + E^((2*I)*ArcTan[c*x^3])]) + a*(a*c*x^3*(-3*b + a*c*x^3) + 3*b^2*
Log[1 + c^2*x^6]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])])/(6*c^2
)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5363, 5361, 5451, 5345, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)^3\,dx$$

$$\Big\downarrow \text{ 5363}$$

$$\frac{1}{3}\int x^3\left(a+b\arctan\left(cx^3\right)\right)^3dx^3$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\int\frac{x^6\left(a+b\arctan\left(cx^3\right)\right)^2}{c^2x^6+1}dx^3\right)$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(\frac{\int\left(a+b\arctan\left(cx^3\right)\right)^2dx^3}{c^2}-\frac{\int\frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{c^2x^6+1}dx^3}{c^2}\right)\right)$$

$\Big\downarrow$ 5345

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\int\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}-\frac{\int\frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{c^2x^6+1}}{c^2}\right.\right.$$

$\Big\downarrow$ 5419

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\int\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}-\frac{\left(a+b\arctan\left(cx^3\right.\right.}{3bc^3}\right.\right.$$

$\Big\downarrow$ 5455

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{3bc^3}+\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\left(-\frac{\int\frac{a+b\arctan\left(cx^3\right)}{i-cx^3}}{c}\right.}{c^2}\right.\right.$$

$\Big\downarrow$ 5379

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{3bc^3}+\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\left(-\frac{\log\left(\frac{2}{1+icx^3}\right)\left(a+b\right.}{c}\right.}{c}\right.\right.$$

$\Big\downarrow$ 2849

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{3bc^3}+\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\left(-\frac{ib\int\frac{\log\left(\frac{2}{icx^3+1}\right)}{1-\frac{2}{icx^3+1}}d}{c}\right.}{}\right.\right.$$

$\downarrow$ 2752

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{3bc^3}+\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\left(-\frac{i\left(a+b\arctan\left(cx^3\right)\right)}{2bc^2}\right.}{}\right.\right.$$

input
```
Int[x^5*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
((x^6*(a + b*ArcTan[c*x^3])^3)/2 - (3*b*c*(-1/3*(a + b*ArcTan[c*x^3])^3/(b
*c^3) + (x^3*(a + b*ArcTan[c*x^3])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^
3])^2)/(b*c^2) - (((a + b*ArcTan[c*x^3])*Log[2/(1 + I*c*x^3)])/c + ((I/2)*
b*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c)/c))/c^2))/2)/3
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.66 (sec) , antiderivative size = 935, normalized size of antiderivative = 6.36

method	result	size
risch	Expression too large to display	935
default	Expression too large to display	11515
parts	Expression too large to display	11515

input
```
int(x^5*(a+b*arctan(c*x^3))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/c*a^2*b*x^3+(1/16*I*b^3*(c^2*x^6+1)/c^2*ln(1-I*c*x^3)^2+1/16*b^2*(2*a
*c*x^3-b)^2/a/c^2*ln(1-I*c*x^3)-1/16*b*(4*I*a^3*c^2*x^6-8*I*a^2*b*c*x^3+4*
I*ln(1-I*c*x^3)*a*b^2+4*I*a*b^2-4*ln(1-I*c*x^3)*a^2*b+ln(1-I*c*x^3)*b^3)/a
/c^2)*ln(1+I*c*x^3)+3/4*I/c*b^2*Sum(2/3*(ln(x-_alpha)*ln(1-I*c*x^3)+3*c*(-
1/3*ln(x-_alpha)*(ln(1/2*(2*(I/c)^(1/3)+x-_alpha)/(I/c)^(1/3))+ln((RootOf(
_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=1)-x+_alpha)/RootOf(_Z^2
+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=1))+ln((RootOf(_Z^2+_Z*RootO
f(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2)-x+_alpha)/RootOf(_Z^2+_Z*RootOf(c*
_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2)))/c-1/3*(dilog(1/2*(2*(I/c)^(1/3)+x-_a
lpha)/(I/c)^(1/3))+dilog((RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)
^2,index=1)-x+_alpha)/RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,i
ndex=1))+dilog((RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2
)-x+_alpha)/RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2)))/
c))*b/c,_alpha=RootOf(c*_Z^3-RootOf(_Z^2+1,index=1)))+1/48*I*b^3*(c^2*x^6+
1)/c^2*ln(1+I*c*x^3)^3-1/48*I*b^3/c^2*ln(1-I*c*x^3)^3+1/2*a*b^2/c^2*ln(c^2
*x^6+1)+1/2*a^2*b/c^2*arctan(c*x^3)+1/6*x^6*a^3-1/4*b^3/c^2*arctan(c*x^3)+
1/8*I*b^3/c^2*ln(1-I*c*x^3)^2-1/2*I/c*a*b^2*x^3*ln(1-I*c*x^3)-1/16*b^2*(I*
b*c^2*x^6*ln(1-I*c*x^3)+2*a*c^2*x^6-2*b*c*x^3+I*b*ln(1-I*c*x^3)+2*I*b+2*a)
/c^2*ln(1+I*c*x^3)^2+1/4*I*b*a^2*x^6*ln(1-I*c*x^3)+1/8*b^3/c*x^3*ln(1-I*c*
x^3)^2-1/48*I*b^3*x^6*ln(1-I*c*x^3)^3-1/8*a*b^2*x^6*ln(1-I*c*x^3)^2-1/8...
```

Fricas [F]

$$\int x^5 \left(a+b\arctan \left(cx^3\right)\right)^3 \, dx = \int \left(b\arctan \left(cx^3\right)+a\right)^3 x^5 \, dx$$

input
```
integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^5*arctan(c*x^3)^3 + 3*a*b^2*x^5*arctan(c*x^3)^2 + 3*a^2*b*x
^5*arctan(c*x^3) + a^3*x^5, x)
```

Sympy [F]

$$\int x^5 \left(a+b\arctan \left(cx^3\right)\right)^3 \, dx = \int x^5 \left(a+b\,\mathrm{atan}\left(cx^3\right)\right)^3 \, dx$$

input
```
integrate(x**5*(a+b*atan(c*x**3))**3,x)
```

output
```
Integral(x**5*(a + b*atan(c*x**3))**3, x)
```

Maxima [F]

$$\int x^5 \left(a+b\arctan \left(cx^3\right)\right)^3 \, dx = \int \left(b\arctan \left(cx^3\right)+a\right)^3 x^5 \, dx$$

input
```
integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

output
```
1/2*a*b^2*x^6*arctan(c*x^3)^2 + 1/6*a^3*x^6 + 1/2*(x^6*arctan(c*x^3) - c*(
x^3/c^2 - arctan(c*x^3)/c^3))*a^2*b - 1/2*(2*c*(x^3/c^2 - arctan(c*x^3)/c^
3)*arctan(c*x^3) + (arctan(c*x^3)^2 - log(6*c^5*x^6 + 6*c^3))/c^2)*a*b^2 +
 1/192*(4*x^6*arctan(c*x^3)^3 - 3*x^6*arctan(c*x^3)*log(c^2*x^6 + 1)^2 + 1
92*integrate(1/64*(12*c^2*x^11*arctan(c*x^3)*log(c^2*x^6 + 1) - 12*c*x^8*a
rctan(c*x^3)^2 + 56*(c^2*x^11 + x^5)*arctan(c*x^3)^3 + 3*(c*x^8 + 2*(c^2*x
^11 + x^5)*arctan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^6 + 1), x))*b^3
```

Giac [F]

$$\int x^5 \left(a+b\arctan\left(cx^3\right)\right)^3 dx = \int \left(b\arctan\left(cx^3\right)+a\right)^3 x^5 \, dx$$

input
```
integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^3*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a+b\arctan\left(cx^3\right)\right)^3 dx = \int x^5 \left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^3 dx$$

input
```
int(x^5*(a + b*atan(c*x^3))^3,x)
```

output
```
int(x^5*(a + b*atan(c*x^3))^3, x)
```

Reduce [F]

$$\int x^5 \left(a + b \arctan \left(c x^3\right)\right)^3 dx$$
$$= \frac{atan(c\,x^3)^3\,b^3c^2x^6 + atan(c\,x^3)^3\,b^3 + 3atan(c\,x^3)^2\,a\,b^2c^2x^6 + 3atan(c\,x^3)^2\,a\,b^2 - 3atan(c\,x^3)^2\,b^3c\,x^3 + 3a}{}$$

input
```
int(x^5*(a+b*atan(c*x^3))^3,x)
```

output
```
(atan(c*x**3)**3*b**3*c**2*x**6 + atan(c*x**3)**3*b**3 + 3*atan(c*x**3)**2
*a*b**2*c**2*x**6 + 3*atan(c*x**3)**2*a*b**2 - 3*atan(c*x**3)**2*b**3*c*x*
*3 + 3*atan(c*x**3)*a**2*b*c**2*x**6 + 3*atan(c*x**3)*a**2*b - 6*atan(c*x*
*3)*a*b**2*c*x**3 + 18*int((atan(c*x**3)*x**5)/(c**2*x**6 + 1),x)*b**3*c**
2 + 3*log(c**(2/3)*x**2 - c**(1/3)*sqrt(3)*x + 1)*a*b**2 + 3*log(c**(2/3)*
x**2 + c**(1/3)*sqrt(3)*x + 1)*a*b**2 + 3*log(c**(2/3)*x**2 + 1)*a*b**2 +
a**3*c**2*x**6 - 3*a**2*b*c*x**3)/(6*c**2)
```

3.123 $\int x^2(a+b\arctan(cx^3))^3\,dx$

Optimal result 938
Mathematica [A] (verified) 939
Rubi [A] (verified) 939
Maple [B] (verified) 942
Fricas [F] 942
Sympy [F] 943
Maxima [F] 943
Giac [F] 944
Mupad [F(-1)] 944
Reduce [F] 944

Optimal result

Integrand size = 16, antiderivative size = 139

$$\int x^2(a+b\arctan(cx^3))^3\,dx = \frac{i(a+b\arctan(cx^3))^3}{3c}+\frac{1}{3}x^3(a+b\arctan(cx^3))^3 + \frac{b(a+b\arctan(cx^3))^2\log\left(\frac{2}{1+icx^3}\right)}{c} + \frac{ib^2(a+b\arctan(cx^3))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{c} + \frac{b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right)}{2c}$$

output
```
1/3*I*(a+b*arctan(c*x^3))^3/c+1/3*x^3*(a+b*arctan(c*x^3))^3+b*(a+b*arctan(
c*x^3))^2*ln(2/(1+I*c*x^3))/c+I*b^2*(a+b*arctan(c*x^3))*polylog(2,1-2/(1+I
*c*x^3))/c+1/2*b^3*polylog(3,1-2/(1+I*c*x^3))/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.61

$$\int x^2(a+b\arctan(cx^3))^3\,dx$$
$$=\frac{2a^3cx^3+6a^2bcx^3\arctan(cx^3)-6iab^2\arctan(cx^3)^2+6ab^2cx^3\arctan(cx^3)^2-2ib^3\arctan(cx^3)^3+2b^3cx}{}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
(2*a^3*c*x^3 + 6*a^2*b*c*x^3*ArcTan[c*x^3] - (6*I)*a*b^2*ArcTan[c*x^3]^2 +
 6*a*b^2*c*x^3*ArcTan[c*x^3]^2 - (2*I)*b^3*ArcTan[c*x^3]^3 + 2*b^3*c*x^3*A
rcTan[c*x^3]^3 + 12*a*b^2*ArcTan[c*x^3]*Log[1 + E^((2*I)*ArcTan[c*x^3])] +
 6*b^3*ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan[c*x^3])] - 3*a^2*b*Log[1 +
c^2*x^6] - (6*I)*b^2*(a + b*ArcTan[c*x^3])*PolyLog[2, -E^((2*I)*ArcTan[c*x
^3])] + 3*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x^3])])/(6*c)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5363, 5345, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\arctan(cx^3))^3\,dx$$
$$\Big\downarrow \text{5363}$$
$$\frac{1}{3}\int (a+b\arctan(cx^3))^3\,dx^3$$
$$\Big\downarrow \text{5345}$$
$$\frac{1}{3}\left(x^3(a+b\arctan(cx^3))^3-3bc\int\frac{x^3(a+b\arctan(cx^3))^2}{c^2x^6+1}dx^3\right)$$

$$\Big\downarrow \ 5455$$

$$\frac{1}{3}\left(x^3\left(a+b\arctan\left(cx^3\right)\right)^3-3bc\left(-\frac{\int\frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{i-cx^3}dx^3}{c}-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^3}{3bc^2}\right)\right)$$

$$\Big\downarrow \ 5379$$

$$\frac{1}{3}\left(x^3\left(a+b\arctan\left(cx^3\right)\right)^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^2}{c}-2b\int\frac{\left(a+b\arctan\left(cx^3\right)\right)\log\left(\frac{2}{icx^3+1}\right)}{c^2x^6+1}dx^3}{c}-\frac{i\left(a+b\mathrm{a}\right.}{}\right.\right.$$

$$\Big\downarrow \ 5529$$

$$\frac{1}{3}\left(x^3\left(a+b\arctan\left(cx^3\right)\right)^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^2}{c}-2b\left(\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)}{c^2x^6+1}dx^3-\frac{i\,\text{PolyLog}\left(2,1\right.}{}\right.}{c}\right.\right.$$

$$\Big\downarrow \ 7164$$

$$\frac{1}{3}\left(x^3\left(a+b\arctan\left(cx^3\right)\right)^3-3bc\left(-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^3}{3bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^2}{c}-2b\left(-\frac{i\,\text{PolyLog}\left(2,1-\frac{}{ic}\right.}{}\right.}{c}\right.\right.$$

input
```
Int[x^2*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
(x^3*(a + b*ArcTan[c*x^3])^3 - 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x^3])^3)/(
b*c^2) - (((a + b*ArcTan[c*x^3])^2*Log[2/(1 + I*c*x^3)])/c - 2*b*(((-1/2*I
)*(a + b*ArcTan[c*x^3])*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c - (b*PolyLog[3,
 1 - 2/(1 + I*c*x^3)])/(4*c)))/c))/3
```

Defintions of rubi rules used

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(128) = 256.

Time = 0.77 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.98

method	result
derivativedivides	$\frac{a^3 c\,x^3+b^3\left(\arctan\left(c\,x^3\right)^3\left(c\,x^3+i\right)-2i\arctan\left(c\,x^3\right)^3+3\arctan\left(c\,x^3\right)^2\ln\left(1+\frac{\left(i c\,x^3+1\right)^2}{c^2x^6+1}\right)-3i\arctan\left(c\,x^3\right)\operatorname{polylog}\left(\right.\right.}{}$
default	$\frac{a^3 c\,x^3+b^3\left(\arctan\left(c\,x^3\right)^3\left(c\,x^3+i\right)-2i\arctan\left(c\,x^3\right)^3+3\arctan\left(c\,x^3\right)^2\ln\left(1+\frac{\left(i c\,x^3+1\right)^2}{c^2x^6+1}\right)-3i\arctan\left(c\,x^3\right)\operatorname{polylog}\left(\right.\right.}{}$
parts	$\frac{a^3x^3}{3}+\frac{b^3\left(\arctan\left(c\,x^3\right)^3\left(c\,x^3+i\right)-2i\arctan\left(c\,x^3\right)^3+3\arctan\left(c\,x^3\right)^2\ln\left(1+\frac{\left(i c\,x^3+1\right)^2}{c^2x^6+1}\right)-3i\arctan\left(c\,x^3\right)\operatorname{polylog}\left(\right.\right.}{3c}$

input
```
int(x^2*(a+b*arctan(c*x^3))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/c*(a^3*c*x^3+b^3*(arctan(c*x^3)^3*(c*x^3+I)-2*I*arctan(c*x^3)^3+3*arct
an(c*x^3)^2*ln(1+(1+I*c*x^3)^2/(c^2*x^6+1))-3*I*arctan(c*x^3)*polylog(2,-(
1+I*c*x^3)^2/(c^2*x^6+1))+3/2*polylog(3,-(1+I*c*x^3)^2/(c^2*x^6+1)))+3*a*b
^2*(arctan(c*x^3)^2*(c*x^3+I)+2*arctan(c*x^3)*ln(1+(1+I*c*x^3)^2/(c^2*x^6+
1))-2*I*arctan(c*x^3)^2-I*polylog(2,-(1+I*c*x^3)^2/(c^2*x^6+1)))+3*a^2*b*(
c*x^3*arctan(c*x^3)-1/2*ln(c^2*x^6+1)))
```

Fricas [F]

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int\left(b\arctan\left(cx^3\right)+a\right)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*arctan(c*x^3)^3 + 3*a*b^2*x^2*arctan(c*x^3)^2 + 3*a^2*b*x
^2*arctan(c*x^3) + a^3*x^2, x)
```

Sympy [F]

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int x^2\left(a+b\,\mathrm{atan}\left(cx^3\right)\right)^3\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x**3))**3,x)
```

output
```
Integral(x**2*(a + b*atan(c*x**3))**3, x)
```

Maxima [F]

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int\left(b\arctan\left(cx^3\right)+a\right)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

output
```
1/24*b^3*x^3*arctan(c*x^3)^3 - 1/32*b^3*x^3*arctan(c*x^3)*log(c^2*x^6 + 1)
^2 + 1/3*a^3*x^3 + 7/96*b^3*arctan(c*x^3)^4/c + 28*b^3*c^2*integrate(1/32*
x^8*arctan(c*x^3)^3/(c^2*x^6 + 1), x) + 3*b^3*c^2*integrate(1/32*x^8*arcta
n(c*x^3)*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 96*a*b^2*c^2*integrate(1/3
2*x^8*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 12*b^3*c^2*integrate(1/32*x^8*ar
ctan(c*x^3)*log(c^2*x^6 + 1)/(c^2*x^6 + 1), x) + 1/3*a*b^2*arctan(c*x^3)^3
/c - 12*b^3*c*integrate(1/32*x^5*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 3*b^3
*c*integrate(1/32*x^5*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 3*b^3*integra
te(1/32*x^2*arctan(c*x^3)*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 1/2*(2*c*
x^3*arctan(c*x^3) - log(c^2*x^6 + 1))*a^2*b/c
```

Giac [F]

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int\left(b\arctan\left(cx^3\right)+a\right)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^3*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int x^2\,\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^3\,dx$$

input
```
int(x^2*(a + b*atan(c*x^3))^3,x)
```

output
```
int(x^2*(a + b*atan(c*x^3))^3, x)
```

Reduce [F]

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^3\,dx$$
$$=\frac{6atan(c\,x^3)\,a^2bc\,x^3+6\left(\int atan(c\,x^3)^3\,x^2dx\right)b^3c+18\left(\int atan(c\,x^3)^2\,x^2dx\right)a\,b^2c-3\log\left(c^{\frac{2}{3}}x^2-c^{\frac{1}{3}}\sqrt{3}\,x\right.}{6c}$$

input
```
int(x^2*(a+b*atan(c*x^3))^3,x)
```

output
```
(6*atan(c*x**3)*a**2*b*c*x**3 + 6*int(atan(c*x**3)**3*x**2,x)*b**3*c + 18*
int(atan(c*x**3)**2*x**2,x)*a*b**2*c - 3*log(c**(2/3)*x**2 - c**(1/3)*sqrt
(3)*x + 1)*a**2*b - 3*log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x + 1)*a**2*b -
 3*log(c**(2/3)*x**2 + 1)*a**2*b + 2*a**3*c*x**3)/(6*c)
```

3.124 $\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx$

Optimal result 945
Mathematica [A] (verified) 946
Rubi [A] (verified) 947
Maple [F] 950
Fricas [F] 950
Sympy [F] 950
Maxima [F] 951
Giac [F] 951
Mupad [F(-1)] 951
Reduce [F] 952

Optimal result

Integrand size = 16, antiderivative size = 232

$$\begin{aligned}\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx &= \frac{2}{3}\left(a+b\arctan\left(cx^3\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+icx^3}\right)\\
&-\frac{1}{2}ib\left(a+b\arctan\left(cx^3\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)\\
&+\frac{1}{2}ib\left(a+b\arctan\left(cx^3\right)\right)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx^3}\right)\\
&-\frac{1}{2}b^2\left(a+b\arctan\left(cx^3\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right)\\
&+\frac{1}{2}b^2\left(a+b\arctan\left(cx^3\right)\right)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx^3}\right)\\
&+\frac{1}{4}ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+icx^3}\right)\\
&-\frac{1}{4}ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx^3}\right)\end{aligned}$$

output
```
-2/3*(a+b*arctan(c*x^3))^3*arctanh(-1+2/(1+I*c*x^3))-1/2*I*b*(a+b*arctan(c
*x^3))^2*polylog(2,1-2/(1+I*c*x^3))+1/2*I*b*(a+b*arctan(c*x^3))^2*polylog(
2,-1+2/(1+I*c*x^3))-1/2*b^2*(a+b*arctan(c*x^3))*polylog(3,1-2/(1+I*c*x^3))
+1/2*b^2*(a+b*arctan(c*x^3))*polylog(3,-1+2/(1+I*c*x^3))+1/4*I*b^3*polylog
(4,1-2/(1+I*c*x^3))-1/4*I*b^3*polylog(4,-1+2/(1+I*c*x^3))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.81

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx = a^3\log(x)+\frac{1}{2}ia^2b\left(\operatorname{PolyLog}(2,-icx^3)-\operatorname{PolyLog}(2,icx^3)\right) + ab^2\left(-\frac{i\pi^3}{24}+\frac{2}{3}i\arctan(cx^3)^3 + \arctan(cx^3)^2\log\left(1-e^{-2i\arctan(cx^3)}\right) - \arctan(cx^3)^2\log\left(1+e^{2i\arctan(cx^3)}\right) + i\arctan(cx^3)\operatorname{PolyLog}\left(2,e^{-2i\arctan(cx^3)}\right) + i\arctan(cx^3)\operatorname{PolyLog}\left(2,-e^{2i\arctan(cx^3)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(3,e^{-2i\arctan(cx^3)}\right) - \frac{1}{2}\operatorname{PolyLog}\left(3,-e^{2i\arctan(cx^3)}\right)\right) - \frac{1}{192}ib^3\left(\pi^4-32\arctan(cx^3)^4 + 64i\arctan(cx^3)^3\log\left(1-e^{-2i\arctan(cx^3)}\right) - 64i\arctan(cx^3)^3\log\left(1+e^{2i\arctan(cx^3)}\right) - 96\arctan(cx^3)^2\operatorname{PolyLog}\left(2,e^{-2i\arctan(cx^3)}\right) - 96\arctan(cx^3)^2\operatorname{PolyLog}\left(2,-e^{2i\arctan(cx^3)}\right) + 96i\arctan(cx^3)\operatorname{PolyLog}\left(3,e^{-2i\arctan(cx^3)}\right) - 96i\arctan(cx^3)\operatorname{PolyLog}\left(3,-e^{2i\arctan(cx^3)}\right) + 48\operatorname{PolyLog}\left(4,e^{-2i\arctan(cx^3)}\right) + 48\operatorname{PolyLog}\left(4,-e^{2i\arctan(cx^3)}\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])^3/x,x]
```

output
```
a^3*Log[x] + (I/2)*a^2*b*(PolyLog[2, (-I)*c*x^3] - PolyLog[2, I*c*x^3]) +
a*b^2*((-1/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x^3]^3 + ArcTan[c*x^3]^2*Log[1
- E^((-2*I)*ArcTan[c*x^3])] - ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan[c*x^
3])] + I*ArcTan[c*x^3]*PolyLog[2, E^((-2*I)*ArcTan[c*x^3])] + I*ArcTan[c*x
^3]*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])] + PolyLog[3, E^((-2*I)*ArcTan[c*x
^3])]/2 - PolyLog[3, -E^((2*I)*ArcTan[c*x^3])]/2) - (I/192)*b^3*(Pi^4 - 32
*ArcTan[c*x^3]^4 + (64*I)*ArcTan[c*x^3]^3*Log[1 - E^((-2*I)*ArcTan[c*x^3])
] - (64*I)*ArcTan[c*x^3]^3*Log[1 + E^((2*I)*ArcTan[c*x^3])] - 96*ArcTan[c*
x^3]^2*PolyLog[2, E^((-2*I)*ArcTan[c*x^3])] - 96*ArcTan[c*x^3]^2*PolyLog[2
, -E^((2*I)*ArcTan[c*x^3])] + (96*I)*ArcTan[c*x^3]*PolyLog[3, E^((-2*I)*Ar
cTan[c*x^3])] - (96*I)*ArcTan[c*x^3]*PolyLog[3, -E^((2*I)*ArcTan[c*x^3])]
+ 48*PolyLog[4, E^((-2*I)*ArcTan[c*x^3])] + 48*PolyLog[4, -E^((2*I)*ArcTan
[c*x^3])])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5359, 5357, 5523, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^3))^3}{x}dx$$

$$\Big\downarrow \text{5359}$$

$$\frac{1}{3}\int \frac{(a+b\arctan(cx^3))^3}{x^3}dx^3$$

$$\Big\downarrow \text{5357}$$

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)(a+b\arctan(cx^3))^3-6bc\int \frac{(a+b\arctan(cx^3))^2\text{arctanh}\left(1-\frac{2}{icx^3+1}\right)}{c^2x^6+1}dx^3\right)$$

$$\Big\downarrow \text{5523}$$

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^3-6bc\left(\frac{1}{2}\int\frac{\left(a+b\arctan\left(cx^3\right)\right)^2\log\left(2-\frac{2}{icx^3+1}\right)}{c^2x^6+1}dx^3-\frac{1}{2}\int\right.\right.$$

$\Big\downarrow$ 5529

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^3-6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)\left(a+b\arctan\left(cx^3\right)\right)^2}{2c}-\right.\right.\right.$$

$\Big\downarrow$ 5533

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^3-6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)\left(a+b\arctan\left(cx^3\right)\right)^2}{2c}-\right.\right.\right.$$

$\Big\downarrow$ 7164

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^3-6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)\left(a+b\arctan\left(cx^3\right)\right)^2}{2c}-\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^3])^3/x,x]
```

output
```
(2*(a + b*ArcTan[c*x^3])^3*ArcTanh[1 - 2/(1 + I*c*x^3)] - 6*b*c*((((I/2)*(
a + b*ArcTan[c*x^3])^2*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c - I*b*(((I/2)*(a
 + b*ArcTan[c*x^3])*PolyLog[3, 1 - 2/(1 + I*c*x^3)])/c + (b*PolyLog[4, 1 -
 2/(1 + I*c*x^3)])/(4*c)))/2 + (((-1/2*I)*(a + b*ArcTan[c*x^3])^2*PolyLog[
2, -1 + 2/(1 + I*c*x^3)])/c + I*b*(((I/2)*(a + b*ArcTan[c*x^3])*PolyLog[3,
 -1 + 2/(1 + I*c*x^3)])/c + (b*PolyLog[4, -1 + 2/(1 + I*c*x^3)])/(4*c)))/2
))/3
```

Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [F]

$$\int \frac{(a+b\arctan(c\,x^3))^3}{x}dx$$

input
```
int((a+b*arctan(c*x^3))^3/x,x)
```

output
```
int((a+b*arctan(c*x^3))^3/x,x)
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx = \int \frac{(b\arctan(cx^3)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c
*x^3) + a^3)/x, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx^3))^3}{x}\,dx$$

input
```
integrate((a+b*atan(c*x**3))**3/x,x)
```

output
```
Integral((a + b*atan(c*x**3))**3/x, x)
```

Maxima [F]

$$\int \frac{(a + b\arctan(cx^3))^3}{x}\,dx = \int \frac{(b\arctan(cx^3) + a)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x^3)^3 + 3*b^3*arctan(c*x^3)*
log(c^2*x^6 + 1)^2 + 96*a*b^2*arctan(c*x^3)^2 + 96*a^2*b*arctan(c*x^3))/x,
 x)
```

Giac [F]

$$\int \frac{(a + b\arctan(cx^3))^3}{x}\,dx = \int \frac{(b\arctan(cx^3) + a)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx^3))^3}{x}\,dx = \int \frac{(a + b\,\mathrm{atan}(c\,x^3))^3}{x}\,dx$$

input
```
int((a + b*atan(c*x^3))^3/x,x)
```

output
```
int((a + b*atan(c*x^3))^3/x, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx = 3\left(\int \frac{atan(c\,x^3)}{x}dx\right)a^2b + \left(\int \frac{atan(c\,x^3)^3}{x}dx\right)b^3 + 3\left(\int \frac{atan(c\,x^3)^2}{x}dx\right)a\,b^2 + \log(x)\,a^3$$

input
```
int((a+b*atan(c*x^3))^3/x,x)
```

output
```
3*int(atan(c*x**3)/x,x)*a**2*b + int(atan(c*x**3)**3/x,x)*b**3 + 3*int(ata
n(c*x**3)**2/x,x)*a*b**2 + log(x)*a**3
```

3.125 $\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx$

Optimal result 953
Mathematica [A] (verified) 954
Rubi [A] (verified) 955
Maple [F] 957
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Reduce [F] 959

Optimal result

Integrand size = 16, antiderivative size = 133

$$\begin{aligned}\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = & -\frac{1}{3}ic(a+b\arctan(cx^3))^3 - \frac{(a+b\arctan(cx^3))^3}{3x^3} \\ & + bc(a+b\arctan(cx^3))^2\log\left(2-\frac{2}{1-icx^3}\right) \\ & - ib^2c(a+b\arctan(cx^3))\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right) \\ & + \frac{1}{2}b^3c\,\text{PolyLog}\left(3,-1+\frac{2}{1-icx^3}\right)\end{aligned}$$

output

```
-1/3*I*c*(a+b*arctan(c*x^3))^3-1/3*(a+b*arctan(c*x^3))^3/x^3+b*c*(a+b*arct
an(c*x^3))^2*ln(2-2/(1-I*c*x^3))-I*b^2*c*(a+b*arctan(c*x^3))*polylog(2,-1+
2/(1-I*c*x^3))+1/2*b^3*c*polylog(3,-1+2/(1-I*c*x^3))
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.80

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = -\frac{a^3}{3x^3} - \frac{a^2b\arctan(cx^3)}{x^3} + 3a^2bc\log(x) - \frac{1}{2}a^2bc\log\left(1+c^2x^6\right) + ab^2c\left(\arctan(cx^3)\left(\left(-i-\frac{1}{cx^3}\right)\arctan(cx^3) + 2\log\left(1-e^{2i\arctan(cx^3)}\right)\right) - i\,\text{PolyLog}\left(2,e^{2i\arctan(cx^3)}\right)\right) + \frac{1}{3}b^3c\left(-\frac{i\pi^3}{8} + i\arctan(cx^3)^3 - \frac{\arctan(cx^3)^3}{cx^3} + 3\arctan(cx^3)^2\log\left(1-e^{-2i\arctan(cx^3)}\right) + 3i\arctan(cx^3)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx^3)}\right) + \frac{3}{2}\text{PolyLog}\left(3,e^{-2i\arctan(cx^3)}\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])^3/x^4,x]
```

output
```
-1/3*a^3/x^3 - (a^2*b*ArcTan[c*x^3])/x^3 + 3*a^2*b*c*Log[x] - (a^2*b*c*Log
[1 + c^2*x^6])/2 + a*b^2*c*(ArcTan[c*x^3]*((-I - 1/(c*x^3))*ArcTan[c*x^3]
+ 2*Log[1 - E^((2*I)*ArcTan[c*x^3])]) - I*PolyLog[2, E^((2*I)*ArcTan[c*x^3
])]) + (b^3*c*((-1/8*I)*Pi^3 + I*ArcTan[c*x^3]^3 - ArcTan[c*x^3]^3/(c*x^3)
 + 3*ArcTan[c*x^3]^2*Log[1 - E^((-2*I)*ArcTan[c*x^3])] + (3*I)*ArcTan[c*x^
3]*PolyLog[2, E^((-2*I)*ArcTan[c*x^3])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c
*x^3])])/2))/3
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5363, 5361, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^4}dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{3}\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^6}dx^3$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\left(3bc\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x^3\left(c^2x^6+1\right)}dx^3-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^3}\right)$$

$$\Big\downarrow \text{5459}$$

$$\frac{1}{3}\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^3}+3bc\left(i\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x^3\left(cx^3+i\right)}dx^3-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^3}{3b}\right)\right)$$

$$\Big\downarrow \text{5403}$$

$$\frac{1}{3}\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^3}+3bc\left(i\left(2ibc\int \frac{\left(a+b\arctan\left(cx^3\right)\right)\log\left(2-\frac{2}{1-icx^3}\right)}{c^2x^6+1}dx^3-i\log\left(2-\frac{2}{1-icx^3}\right)\right.\right.\right.$$

$$\Big\downarrow \text{5527}$$

$$\frac{1}{3}\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^3}+3bc\left(i\left(2ibc\left(\frac{i\operatorname{PolyLog}\left(2,\frac{2}{1-icx^3}-1\right)\left(a+b\arctan\left(cx^3\right)\right)}{2c}-\frac{1}{2}ib\int \frac{\operatorname{PolyLog}\left(2\right.}{c^2x^6}\right.\right.\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$\frac{1}{3}\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^3}+3bc\left(i\left(2ibc\left(\frac{i\operatorname{PolyLog}\left(2,\frac{2}{1-icx^3}-1\right)\left(a+b\arctan\left(cx^3\right)\right)}{2c}-\frac{b\operatorname{PolyLog}\left(3,\frac{2}{1-ic}\right.}{4c}\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^3])^3/x^4,x]
```

output
```
(-((a + b*ArcTan[c*x^3])^3/x^3) + 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x^3])^3
)/b + I*((-I)*(a + b*ArcTan[c*x^3])^2*Log[2 - 2/(1 - I*c*x^3)] + (2*I)*b*c
*(((I/2)*(a + b*ArcTan[c*x^3])*PolyLog[2, -1 + 2/(1 - I*c*x^3)])/c - (b*Po
lyLog[3, -1 + 2/(1 - I*c*x^3)])/(4*c)))))/3
```

Defintions of rubi rules used

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [F]

$$\int \frac{(a+b\arctan(c\,x^3))^3}{x^4}dx$$

input
```
int((a+b*arctan(c*x^3))^3/x^4,x)
```

output
```
int((a+b*arctan(c*x^3))^3/x^4,x)
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx=\int \frac{(b\arctan(cx^3)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c
*x^3) + a^3)/x^4, x)
```

Sympy [F]

$$\int \frac{(a + b\arctan(cx^3))^3}{x^4}\,dx = \int \frac{(a + b\,\mathrm{atan}(cx^3))^3}{x^4}\,dx$$

input
```
integrate((a+b*atan(c*x**3))**3/x**4,x)
```

output
```
Integral((a + b*atan(c*x**3))**3/x**4, x)
```

Maxima [F]

$$\int \frac{(a + b\arctan(cx^3))^3}{x^4}\,dx = \int \frac{(b\arctan(cx^3) + a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*a^2*b - 1/3*a
^3/x^3 - 1/96*(4*b^3*arctan(c*x^3)^3 - 3*b^3*arctan(c*x^3)*log(c^2*x^6 + 1
)^2 - 96*x^3*integrate(-1/32*(12*b^3*c^2*x^6*arctan(c*x^3)*log(c^2*x^6 + 1
) - 28*(b^3*c^2*x^6 + b^3)*arctan(c*x^3)^3 - 12*(8*a*b^2*c^2*x^6 + b^3*c*x
^3 + 8*a*b^2)*arctan(c*x^3)^2 + 3*(b^3*c*x^3 - (b^3*c^2*x^6 + b^3)*arctan(
c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^10 + x^4), x))/x^3
```

Giac [F]

$$\int \frac{(a + b\arctan(cx^3))^3}{x^4}\,dx = \int \frac{(b\arctan(cx^3) + a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^3/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x^3))^3}{x^4}\,dx$$

input
```
int((a + b*atan(c*x^3))^3/x^4,x)
```

output
```
int((a + b*atan(c*x^3))^3/x^4, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = \frac{-2atan(c\,x^3)^3\,b^3-6atan(c\,x^3)^2\,a\,b^2-6atan(c\,x^3)\,a^2b+36\left(\int \frac{atan(c\,x^3)}{c^2x^7+x}dx\right)a\,b^2c\,x^3+18\left(\int \frac{atan(c\,x^3)^2}{c^2x^7+x}dx\right)$$

input
```
int((a+b*atan(c*x^3))^3/x^4,x)
```

output
```
( - 2*atan(c*x**3)**3*b**3 - 6*atan(c*x**3)**2*a*b**2 - 6*atan(c*x**3)*a**
2*b + 36*int(atan(c*x**3)/(c**2*x**7 + x),x)*a*b**2*c*x**3 + 18*int(atan(c
*x**3)**2/(c**2*x**7 + x),x)*b**3*c*x**3 - 3*log(c**(2/3)*x**2 - c**(1/3)*
sqrt(3)*x + 1)*a**2*b*c*x**3 - 3*log(c**(2/3)*x**2 + c**(1/3)*sqrt(3)*x +
1)*a**2*b*c*x**3 - 3*log(c**(2/3)*x**2 + 1)*a**2*b*c*x**3 + 18*log(x)*a**2
*b*c*x**3 - 2*a**3)/(6*x**3)
```

3.126 $\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx$

Optimal result . . . 960
Mathematica [A] (verified) . . . 961
Rubi [A] (verified) . . . 961
Maple [C] (warning: unable to verify) . . . 964
Fricas [F] . . . 964
Sympy [F] . . . 965
Maxima [F] . . . 965
Giac [F] . . . 965
Mupad [F(-1)] . . . 966
Reduce [F] . . . 966

Optimal result

Integrand size = 16, antiderivative size = 146

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx = -\frac{1}{2}ibc^2(a+b\arctan(cx^3))^2 - \frac{bc(a+b\arctan(cx^3))^2}{2x^3} - \frac{1}{6}c^2(a+b\arctan(cx^3))^3 - \frac{(a+b\arctan(cx^3))^3}{6x^6} + b^2c^2(a+b\arctan(cx^3))\log\left(2-\frac{2}{1-icx^3}\right) - \frac{1}{2}ib^3c^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)$$

output
```
-1/2*I*b*c^2*(a+b*arctan(c*x^3))^2-1/2*b*c*(a+b*arctan(c*x^3))^2/x^3-1/6*c
^2*(a+b*arctan(c*x^3))^3-1/6*(a+b*arctan(c*x^3))^3/x^6+b^2*c^2*(a+b*arctan
(c*x^3))*ln(2-2/(1-I*c*x^3))-1/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x^3))
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx =$$
$$-\frac{3b^2(a+ac^2x^6+bcx^3(1+icx^3))\arctan(cx^3)^2+b^3(1+c^2x^6)\arctan(cx^3)^3+3b\arctan(cx^3)\Big(a(a+2}{}$$

input
```
Integrate[(a + b*ArcTan[c*x^3])^3/x^7,x]
```

output
```
-1/6*(3*b^2*(a + a*c^2*x^6 + b*c*x^3*(1 + I*c*x^3))*ArcTan[c*x^3]^2 + b^3*
(1 + c^2*x^6)*ArcTan[c*x^3]^3 + 3*b*ArcTan[c*x^3]*(a*(a + 2*b*c*x^3 + a*c^
2*x^6) - 2*b^2*c^2*x^6*Log[1 - E^((2*I)*ArcTan[c*x^3])]) + a*(a*(a + 3*b*c
*x^3) - 6*b^2*c^2*x^6*Log[(c*x^3)/Sqrt[1 + c^2*x^6]]) + (3*I)*b^3*c^2*x^6*
PolyLog[2, E^((2*I)*ArcTan[c*x^3])])/x^6
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5363, 5361, 5453, 5361, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{3}\int \frac{(a+b\arctan(cx^3))^3}{x^9}dx^3$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\left(\frac{3}{2}bc\int \frac{(a+b\arctan(cx^3))^2}{x^6(c^2x^6+1)}dx^3-\frac{(a+b\arctan(cx^3))^3}{2x^6}\right)$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{3}\left(\frac{3}{2}bc\left(\int \frac{(a+b\arctan(cx^3))^2}{x^6}dx^3 - c^2\int \frac{(a+b\arctan(cx^3))^2}{c^2x^6+1}dx^3\right) - \frac{(a+b\arctan(cx^3))^3}{2x^6}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\left(\frac{3}{2}bc\left(c^2\left(-\int \frac{(a+b\arctan(cx^3))^2}{c^2x^6+1}dx^3\right) + 2bc\int \frac{a+b\arctan(cx^3)}{x^3(c^2x^6+1)}dx^3 - \frac{(a+b\arctan(cx^3))^2}{x^3}\right) - \frac{(a+b$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{3}\left(\frac{3}{2}bc\left(2bc\int \frac{a+b\arctan(cx^3)}{x^3(c^2x^6+1)}dx^3 - \frac{c(a+b\arctan(cx^3))^3}{3b} - \frac{(a+b\arctan(cx^3))^2}{x^3}\right) - \frac{(a+b\arctan(cx^3)}{2x^6}$$

$$\Big\downarrow \text{5459}$$

$$\frac{1}{3}\left(-\frac{(a+b\arctan(cx^3))^3}{2x^6} + \frac{3}{2}bc\left(2bc\left(i\int \frac{a+b\arctan(cx^3)}{x^3(cx^3+i)}dx^3 - \frac{i(a+b\arctan(cx^3))^2}{2b}\right) - \frac{c(a+b\arctan(}{3b}$$

$$\Big\downarrow \text{5403}$$

$$\frac{1}{3}\left(-\frac{(a+b\arctan(cx^3))^3}{2x^6} + \frac{3}{2}bc\left(2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx^3}\right)}{c^2x^6+1}dx^3 - i\log\left(2-\frac{2}{1-icx^3}\right)(a+b\arctan(cx^3$$

$$\Big\downarrow \text{2897}$$

$$\frac{1}{3}\left(-\frac{(a+b\arctan(cx^3))^3}{2x^6} + \frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx^3}\right)(a+b\arctan(cx^3)) - \frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx$$

input
```
Int[(a + b*ArcTan[c*x^3])^3/x^7,x]
```

output
```
(-1/2*(a + b*ArcTan[c*x^3])^3/x^6 + (3*b*c*(-((a + b*ArcTan[c*x^3])^2/x^3)
 - (c*(a + b*ArcTan[c*x^3])^3)/(3*b) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^
3])^2)/b + I*((-I)*(a + b*ArcTan[c*x^3])*Log[2 - 2/(1 - I*c*x^3)] - (b*Pol
yLog[2, -1 + 2/(1 - I*c*x^3)])/2))))/2)/3
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.91 (sec) , antiderivative size = 11581, normalized size of antiderivative = 79.32

method	result	size
default	Expression too large to display	11581
parts	Expression too large to display	11581

input
```
int((a+b*arctan(c*x^3))^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx=\int \frac{(b\arctan(cx^3)+a)^3}{x^7}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c
*x^3) + a^3)/x^7, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx=\int \frac{(a+b\operatorname{atan}(cx^3))^3}{x^7}\,dx$$

input
```
integrate((a+b*atan(c*x**3))**3/x**7,x)
```

output
```
Integral((a + b*atan(c*x**3))**3/x**7, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx=\int \frac{(b\arctan(cx^3)+a)^3}{x^7}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="maxima")
```

output
```
-1/2*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*a^2*b + 1/2*((arcta
n(c*x^3)^2 - log(c^2*x^6 + 1) + 6*log(x))*c^2 - 2*(c*arctan(c*x^3) + 1/x^3
)*c*arctan(c*x^3))*a*b^2 - 1/2*a*b^2*arctan(c*x^3)^2/x^6 + 1/192*(192*x^6*
integrate(-1/64*(12*c^2*x^6*arctan(c*x^3)*log(c^2*x^6 + 1) - 12*c*x^3*arct
an(c*x^3)^2 - 56*(c^2*x^6 + 1)*arctan(c*x^3)^3 + 3*(c*x^3 - 2*(c^2*x^6 + 1
)*arctan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^13 + x^7), x) - 4*arctan(c*x^3
)^3 + 3*arctan(c*x^3)*log(c^2*x^6 + 1)^2)*b^3/x^6 - 1/6*a^3/x^6
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx=\int \frac{(b\arctan(cx^3)+a)^3}{x^7}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^3/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x^3))^3}{x^7}\,dx$$

input
```
int((a + b*atan(c*x^3))^3/x^7,x)
```

output
```
int((a + b*atan(c*x^3))^3/x^7, x)
```

Reduce [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx = \frac{-atan(c\,x^3)^3\,b^3c^2x^6-atan(c\,x^3)^3\,b^3-3atan(c\,x^3)^2\,a\,b^2c^2x^6-3atan(c\,x^3)^2\,a\,b^2-3atan(c\,x^3)^2\,b^3c\,x^3-}{}$$

input
```
int((a+b*atan(c*x^3))^3/x^7,x)
```

output
```
( - atan(c*x**3)**3*b**3*c**2*x**6 - atan(c*x**3)**3*b**3 - 3*atan(c*x**3)
**2*a*b**2*c**2*x**6 - 3*atan(c*x**3)**2*a*b**2 - 3*atan(c*x**3)**2*b**3*c
*x**3 - 3*atan(c*x**3)*a**2*b*c**2*x**6 - 3*atan(c*x**3)*a**2*b - 6*atan(c
*x**3)*a*b**2*c*x**3 - 3*atan(c*x**3)*b**3*c**2*x**6 - 3*atan(c*x**3)*b**3
 - 18*int(atan(c*x**3)/(c**2*x**13 + x**7),x)*b**3*x**6 - 3*log(c**(2/3)*x
**2 - c**(1/3)*sqrt(3)*x + 1)*a*b**2*c**2*x**6 - 3*log(c**(2/3)*x**2 + c**
(1/3)*sqrt(3)*x + 1)*a*b**2*c**2*x**6 - 3*log(c**(2/3)*x**2 + 1)*a*b**2*c*
*2*x**6 + 18*log(x)*a*b**2*c**2*x**6 - a**3 - 3*a**2*b*c*x**3 - 3*b**3*c*x
**3)/(6*x**6)
```

3.127 $\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 dx$

Optimal result . . . 967
Mathematica [N/A] . . . 967
Rubi [N/A] . . . 968
Maple [N/A] . . . 968
Fricas [N/A] . . . 969
Sympy [F(-1)] . . . 969
Maxima [N/A] . . . 969
Giac [N/A] . . . 970
Mupad [N/A] . . . 970
Reduce [N/A] . . . 971

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 dx = \text{Int}\left((dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3, x\right)$$

output
```
Defer(Int)((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 dx = \int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 dx$$

$$\Big\downarrow \text{ 5377}$$

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^3\right)\right)^3 dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int (dx)^m \left(a + b\arctan\left(cx^3\right)\right)^3 dx = \int \left(b\arctan\left(cx^3\right) + a\right)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c
*x^3) + a^3)*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a + b\arctan\left(cx^3\right)\right)^3 dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*atan(c*x**3))**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 4.43 (sec) , antiderivative size = 407, normalized size of antiderivative = 22.61

$$\int (dx)^m \left(a + b\arctan\left(cx^3\right)\right)^3 dx = \int \left(b\arctan\left(cx^3\right) + a\right)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/32*(4*b^3*d^m*x*x^m*arctan(c*x^3)^3 - 3*
b^3*d^m*x*x^m*arctan(c*x^3)*log(c^2*x^6 + 1)^2 + 32*(m + 1)*integrate(1/32
*(36*b^3*c^2*d^m*x^6*x^m*arctan(c*x^3)*log(c^2*x^6 + 1) + 28*((b^3*c^2*d^m
*m + b^3*c^2*d^m)*x^6 + b^3*d^m*m + b^3*d^m)*x^m*arctan(c*x^3)^3 - 12*(3*b
^3*c*d^m*x^3 - 8*(a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^6 - 8*a*b^2*d^m*m - 8
*a*b^2*d^m)*x^m*arctan(c*x^3)^2 + 96*((a^2*b*c^2*d^m*m + a^2*b*c^2*d^m)*x^
6 + a^2*b*d^m*m + a^2*b*d^m)*x^m*arctan(c*x^3) + 3*(3*b^3*c*d^m*x^3*x^m +
((b^3*c^2*d^m*m + b^3*c^2*d^m)*x^6 + b^3*d^m*m + b^3*d^m)*x^m*arctan(c*x^3
))*log(c^2*x^6 + 1)^2)/((c^2*m + c^2)*x^6 + m + 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a + b\arctan\left(cx^3\right)\right)^3 \, dx = \int \left(b\arctan\left(cx^3\right) + a\right)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^3*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a + b\arctan\left(cx^3\right)\right)^3 \, dx = \int (d\,x)^m\,\left(a + b\,\mathrm{atan}\left(c\,x^3\right)\right)^3 \,dx$$

input
```
int((d*x)^m*(a + b*atan(c*x^3))^3,x)
```

output
```
int((d*x)^m*(a + b*atan(c*x^3))^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 7.39

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 dx$$
$$= \frac{d^m \left(x^m a^3 x+3\left(\int x^m atan(c\,x^3)\,dx\right)a^2 bm+3\left(\int x^m atan(c\,x^3)\,dx\right)a^2 b+\left(\int x^m atan(c\,x^3)^3\,dx\right)b^3 m+\left(\int\right.\right.}{m+1}$$

input
```
int((d*x)^m*(a+b*atan(c*x^3))^3,x)
```

output
```
(d**m*(x**m*a**3*x + 3*int(x**m*atan(c*x**3),x)*a**2*b*m + 3*int(x**m*atan
(c*x**3),x)*a**2*b + int(x**m*atan(c*x**3)**3,x)*b**3*m + int(x**m*atan(c*
x**3)**3,x)*b**3 + 3*int(x**m*atan(c*x**3)**2,x)*a*b**2*m + 3*int(x**m*ata
n(c*x**3)**2,x)*a*b**2))/(m + 1)
```

3.128 $\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 \, dx$

Optimal result 972
Mathematica [N/A] 972
Rubi [N/A] 973
Maple [N/A] 973
Fricas [N/A] 974
Sympy [F(-1)] 974
Maxima [N/A] 974
Giac [N/A] 975
Mupad [N/A] 975
Reduce [N/A] 976

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 \, dx = \text{Int}\left((dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2,x\right)$$

output
```
Defer(Int)((d*x)^m*(a+b*arctan(c*x^3))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 \, dx = \int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 \, dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 dx$$

$$\Big\downarrow \ 5377$$

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^3\right)\right)^2 dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x^3))^2,x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x^3))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 \, dx = \int \left(b\arctan\left(cx^3\right)+a\right)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*atan(c*x**3))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 304, normalized size of antiderivative = 16.89

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 \, dx = \int \left(b\arctan\left(cx^3\right)+a\right)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/16*(4*b^2*d^m*x*x^m*arctan(c*x^3)^2 - b^
2*d^m*x*x^m*log(c^2*x^6 + 1)^2 + 16*(m + 1)*integrate(1/16*(12*b^2*c^2*d^m
*x^6*x^m*log(c^2*x^6 + 1) + 12*((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^6 + b^2*d^
m*m + b^2*d^m)*x^m*arctan(c*x^3)^2 + ((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^6 +
b^2*d^m*m + b^2*d^m)*x^m*log(c^2*x^6 + 1)^2 - 8*(3*b^2*c*d^m*x^3 - 4*(a*b*
c^2*d^m*m + a*b*c^2*d^m)*x^6 - 4*a*b*d^m*m - 4*a*b*d^m)*x^m*arctan(c*x^3))
/((c^2*m + c^2)*x^6 + m + 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int \left(b\arctan\left(cx^3\right)+a\right)^2(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^2*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int (d\,x)^m\,\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^2\,dx$$

input
```
int((d*x)^m*(a + b*atan(c*x^3))^2,x)
```

output
```
int((d*x)^m*(a + b*atan(c*x^3))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.89

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 dx$$
$$= \frac{d^m \bigg(x^m a^2 x + 2\big(\int x^m atan(c\,x^3)\,dx\big)\,abm + 2\big(\int x^m atan(c\,x^3)\,dx\big)\,ab + \Big(\int x^m atan(c\,x^3)^2\,dx\Big)\,b^2 m + \Big(\int x}{m+1}$$

input
```
int((d*x)^m*(a+b*atan(c*x^3))^2,x)
```

output
```
(d**m*(x**m*a**2*x + 2*int(x**m*atan(c*x**3),x)*a*b*m + 2*int(x**m*atan(c*
x**3),x)*a*b + int(x**m*atan(c*x**3)**2,x)*b**2*m + int(x**m*atan(c*x**3)*
*2,x)*b**2))/(m + 1)
```

3.129 $\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right) \, dx$

Optimal result 977
Mathematica [A] (verified) 977
Rubi [A] (verified) 978
Maple [F] 979
Fricas [F] 979
Sympy [F(-1)] 980
Maxima [F] 980
Giac [F] 980
Mupad [F(-1)] 981
Reduce [F] 981

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right) \, dx = \frac{(dx)^{1+m}\left(a+b\arctan\left(cx^3\right)\right)}{d(1+m)} - \frac{3bc(dx)^{4+m}\text{Hypergeometric2F1}\left(1,\frac{4+m}{6},\frac{10+m}{6},-c^2x^6\right)}{d^4(1+m)(4+m)}$$

output
```
(d*x)^(1+m)*(a+b*arctan(c*x^3))/d/(1+m)-3*b*c*(d*x)^(4+m)*hypergeom([1, 2/
3+1/6*m],[5/3+1/6*m],-c^2*x^6)/d^4/(1+m)/(4+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right) \, dx = -\frac{x(dx)^m\left(-\left((4+m)\left(a+b\arctan\left(cx^3\right)\right)\right)+3bcx^3\text{Hypergeometric2F1}\left(1,\frac{4+m}{6},\frac{10+m}{6},-c^2x^6\right)\right)}{(1+m)(4+m)}$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^3]),x]
```

output
```
-((x*(d*x)^m*(-((4 + m)*(a + b*ArcTan[c*x^3])) + 3*b*c*x^3*Hypergeometric2
F1[1, (4 + m)/6, (10 + m)/6, -(c^2*x^6)]))/((1 + m)*(4 + m)))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5373, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right) \, dx$$

$$\Big\downarrow \text{5373}$$

$$\frac{(dx)^{m+1}\left(a+b\arctan\left(cx^3\right)\right)}{d(m+1)}-\frac{3bc\int \frac{(dx)^{m+3}}{c^2x^6+1}dx}{d^3(m+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{(dx)^{m+1}\left(a+b\arctan\left(cx^3\right)\right)}{d(m+1)}-\frac{3bc(dx)^{m+4}\text{Hypergeometric2F1}\left(1,\frac{m+4}{6},\frac{m+10}{6},-c^2x^6\right)}{d^4(m+1)(m+4)}$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x^3]),x]
```

output
```
((d*x)^(1 + m)*(a + b*ArcTan[c*x^3]))/(d*(1 + m)) - (3*b*c*(d*x)^(4 + m)*H
ypergeometric2F1[1, (4 + m)/6, (10 + m)/6, -(c^2*x^6)])/(d^4*(1 + m)*(4 + 
m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5373
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :>
 Simp[(d*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n/(
d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Maple [F]

$$\int (dx)^m \left(a + b\arctan\left(c\,x^3\right)\right) dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x^3)),x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x^3)),x)
```

Fricas [F]

$$\int (dx)^m \left(a + b\arctan\left(cx^3\right)\right)\, dx = \int \left(b\arctan\left(cx^3\right) + a\right)(dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x^3) + a)*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*atan(c*x**3)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)\,dx=\int \left(b\arctan\left(cx^3\right)+a\right)(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
(d^m*x*x^m*arctan(c*x^3) - 3*(c*d^m*m + c*d^m)*integrate(x^3*x^m/((c^2*m +
 c^2)*x^6 + m + 1), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)\,dx=\int \left(b\arctan\left(cx^3\right)+a\right)(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)\,dx=\int (d\,x)^m\,\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)\,dx$$

input
```
int((d*x)^m*(a + b*atan(c*x^3)),x)
```

output
```
int((d*x)^m*(a + b*atan(c*x^3)), x)
```

Reduce [F]

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)\,dx$$
$$=\frac{d^m\left(x^m ax+\left(\int x^m atan(c\,x^3)\,dx\right)bm+\left(\int x^m atan(c\,x^3)\,dx\right)b\right)}{m+1}$$

input
```
int((d*x)^m*(a+b*atan(c*x^3)),x)
```

output
```
(d**m*(x**m*a*x + int(x**m*atan(c*x**3),x)*b*m + int(x**m*atan(c*x**3),x)*
b))/(m + 1)
```

3.130 $\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx$

Optimal result 982
Mathematica [N/A] 982
Rubi [N/A] 983
Maple [N/A] 983
Fricas [N/A] 984
Sympy [F(-1)] 984
Maxima [N/A] 984
Giac [N/A] 985
Mupad [N/A] 985
Reduce [N/A] 985

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx = \text{Int}\left(\frac{(dx)^m}{a+b\arctan(cx^3)},x\right)$$

output
```
Defer(Int)((d*x)^m/(a+b*arctan(c*x^3)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx = \int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^3]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^3]), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}dx$$
$$\Big\downarrow \; 5377$$
$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTan[c*x^3]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\arctan(c\,x^3)}dx$$

input
```
int((d*x)^m/(a+b*arctan(c*x^3)),x)
```

output
```
int((d*x)^m/(a+b*arctan(c*x^3)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx=\int \frac{(dx)^m}{b\arctan(cx^3)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arctan(c*x^3) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx=\text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*atan(c*x**3)),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx=\int \frac{(dx)^m}{b\arctan(cx^3)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arctan(c*x^3) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a + b \arctan(cx^3)} \, dx = \int \frac{(dx)^m}{b \arctan(cx^3) + a} \, dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctan(c*x^3) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a + b \arctan(cx^3)} \, dx = \int \frac{(d\,x)^m}{a + b\,\mathrm{atan}\left(c\,x^3\right)} \, dx$$

input
```
int((d*x)^m/(a + b*atan(c*x^3)),x)
```

output
```
int((d*x)^m/(a + b*atan(c*x^3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(dx)^m}{a + b \arctan(cx^3)} \, dx = d^m \left(\int \frac{x^m}{atan\left(c\,x^3\right) b + a} dx \right)$$

input
```
int((d*x)^m/(a+b*atan(c*x^3)),x)
```

output
```
d**m*int(x**m/(atan(c*x**3)*b + a),x)
```

3.131 $\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx$

Optimal result . . . 987
Mathematica [N/A] . . . 987
Rubi [N/A] . . . 988
Maple [N/A] . . . 988
Fricas [N/A] . . . 989
Sympy [F(-1)] . . . 989
Maxima [N/A] . . . 989
Giac [N/A] . . . 990
Mupad [N/A] . . . 990
Reduce [N/A] . . . 991

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \operatorname{Int}\left(\frac{(dx)^m}{(a+b\arctan(cx^3))^2},x\right)$$

output
```
Defer(Int)((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^3])^2,x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^3])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2} \, dx$$
$$\Big\downarrow \text{ 5377}$$
$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTan[c*x^3])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{(a+b\arctan(c\,x^3))^2}dx$$

input
```
int((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

output
```
int((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^3)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*atan(c*x**3))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 7.33

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^3)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

output
```
-1/3*((c^2*d^m*x^6 + d^m)*x^m - 3*(b^2*c*x^2*arctan(c*x^3) + a*b*c*x^2)*in
tegrate(1/3*((c^2*d^m*m + 4*c^2*d^m)*x^6 + d^m*m - 2*d^m)*x^m/(b^2*c*x^3*a
rctan(c*x^3) + a*b*c*x^3), x))/(b^2*c*x^2*arctan(c*x^3) + a*b*c*x^2)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a + b\arctan(cx^3))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^3) + a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctan(c*x^3) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a + b\arctan(cx^3))^2}\,dx = \int \frac{(d\,x)^m}{(a + b\,\mathrm{atan}(c\,x^3))^2}\,dx$$

input
```
int((d*x)^m/(a + b*atan(c*x^3))^2,x)
```

output
```
int((d*x)^m/(a + b*atan(c*x^3))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = d^m\left(\int \frac{x^m}{atan(c\,x^3)^2\,b^2+2atan(c\,x^3)\,ab+a^2}dx\right)$$

input
```
int((d*x)^m/(a+b*atan(c*x^3))^2,x)
```

output
```
d**m*int(x**m/(atan(c*x**3)**2*b**2 + 2*atan(c*x**3)*a*b + a**2),x)
```

3.132 $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$

Optimal result 992
Mathematica [A] (verified) 992
Rubi [A] (verified) 993
Maple [A] (verified) 994
Fricas [A] (verification not implemented) 995
Sympy [A] (verification not implemented) 995
Maxima [A] (verification not implemented) 996
Giac [C] (verification not implemented) 996
Mupad [B] (verification not implemented) 997
Reduce [B] (verification not implemented) 997

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=-\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{4}bc^4\arctan\left(\frac{x}{c}\right)$$

output
```
-1/4*b*c^3*x+1/12*b*c*x^3+1/4*x^4*(a+b*arctan(c/x))+1/4*b*c^4*arctan(x/c)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=-\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{ax^4}{4}-\frac{1}{4}bc^4\arctan\left(\frac{c}{x}\right)+\frac{1}{4}bx^4\arctan\left(\frac{c}{x}\right)$$

input
```
Integrate[x^3*(a + b*ArcTan[c/x]),x]
```

output
```
-1/4*(b*c^3*x) + (b*c*x^3)/12 + (a*x^4)/4 - (b*c^4*ArcTan[c/x])/4 + (b*x^4
*ArcTan[c/x])/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 795, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{4}bc\int \frac{x^2}{\frac{c^2}{x^2}+1}dx+\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 795}$$

$$\frac{1}{4}bc\int \frac{x^4}{c^2+x^2}dx+\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 254}$$

$$\frac{1}{4}bc\int \left(\frac{c^4}{c^2+x^2}-c^2+x^2\right)dx+\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{4}bc\left(c^3\arctan\left(\frac{x}{c}\right)-c^2x+\frac{x^3}{3}\right)$$

input
```
Int[x^3*(a + b*ArcTan[c/x]),x]
```

output
```
(x^4*(a + b*ArcTan[c/x]))/4 + (b*c*(-(c^2*x) + x^3/3 + c^3*ArcTan[x/c]))/4
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{x^{4} \arctan\left(\frac{c}{x}\right) b}{4}-\frac{\arctan\left(\frac{c}{x}\right) b \,c^{4}}{4}+\frac{a \,x^{4}}{4}+\frac{b c \,x^{3}}{12}-\frac{b \,c^{3} x}{4}$	46
parts	$\frac{a \,x^{4}}{4}+\frac{x^{4} \arctan\left(\frac{c}{x}\right) b}{4}+\frac{b \,c^{4} \arctan\left(\frac{x}{c}\right)}{4}+\frac{b c \,x^{3}}{12}-\frac{b \,c^{3} x}{4}$	46
derivativedivides	$-c^{4} \left(-\frac{a \,x^{4}}{4 c^{4}}+b \left(-\frac{x^{4} \arctan\left(\frac{c}{x}\right)}{4 c^{4}}+\frac{\arctan\left(\frac{c}{x}\right)}{4}-\frac{x^{3}}{12 c^{3}}+\frac{x}{4 c}\right)\right)$	55
default	$-c^{4} \left(-\frac{a \,x^{4}}{4 c^{4}}+b \left(-\frac{x^{4} \arctan\left(\frac{c}{x}\right)}{4 c^{4}}+\frac{\arctan\left(\frac{c}{x}\right)}{4}-\frac{x^{3}}{12 c^{3}}+\frac{x}{4 c}\right)\right)$	55
orering	$-\frac{\left(2 c^{4}+c^{2} x^{2}-x^{4}\right) \left(a +b \arctan\left(\frac{c}{x}\right)\right)}{2}+\frac{\left(3 c^{2}-x^{2}\right) \left(c^{2}+x^{2}\right) \left(3 x^{2} \left(a +b \arctan\left(\frac{c}{x}\right)\right)-\frac{x b c}{1+\frac{c^{2}}{x^{2}}}\right)}{12 x^{2}}$	87
risch	Expression too large to display	697

input
```
int(x^3*(a+b*arctan(c/x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arctan(c/x)*b-1/4*arctan(c/x)*b*c^4+1/4*a*x^4+1/12*b*c*x^3-1/4*b*c
^3*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=-\frac{1}{4}\,bc^3x+\frac{1}{12}\,bcx^3+\frac{1}{4}\,ax^4-\frac{1}{4}\,\left(bc^4-bx^4\right)\arctan\left(\frac{c}{x}\right)$$

input
```
integrate(x^3*(a+b*arctan(c/x)),x, algorithm="fricas")
```

output
```
-1/4*b*c^3*x + 1/12*b*c*x^3 + 1/4*a*x^4 - 1/4*(b*c^4 - b*x^4)*arctan(c/x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{ax^4}{4}-\frac{bc^4\operatorname{atan}\left(\frac{c}{x}\right)}{4}-\frac{bc^3x}{4}+\frac{bcx^3}{12}+\frac{bx^4\operatorname{atan}\left(\frac{c}{x}\right)}{4}$$

input
```
integrate(x**3*(a+b*atan(c/x)),x)
```

output
```
a*x**4/4 - b*c**4*atan(c/x)/4 - b*c**3*x/4 + b*c*x**3/12 + b*x**4*atan(c/x
)/4
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$$
$$=\frac{1}{4}\,ax^4+\frac{1}{12}\left(3\,x^4\arctan\left(\frac{c}{x}\right)+\left(3\,c^3\arctan\left(\frac{x}{c}\right)-3\,c^2x+x^3\right)c\right)b$$

input
```
integrate(x^3*(a+b*arctan(c/x)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/12*(3*x^4*arctan(c/x) + (3*c^3*arctan(x/c) - 3*c^2*x + x^3)*
c)*b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.62

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$$
$$=\frac{\left(6\,bc^5\arctan\left(\frac{c}{x}\right)-\frac{3i\,bc^9\log\left(\frac{i\,c}{x}-1\right)}{x^4}+\frac{3i\,bc^9\log\left(-\frac{i\,c}{x}-1\right)}{x^4}+6\,ac^5-\frac{6\,bc^8}{x^3}+\frac{2\,bc^6}{x}\right)x^4}{24\,c^5}$$

input
```
integrate(x^3*(a+b*arctan(c/x)),x, algorithm="giac")
```

output
```
1/24*(6*b*c^5*arctan(c/x) - 3*I*b*c^9*log(I*c/x - 1)/x^4 + 3*I*b*c^9*log(-
I*c/x - 1)/x^4 + 6*a*c^5 - 6*b*c^8/x^3 + 2*b*c^6/x)*x^4/c^5
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{a\,x^4}{4}-\frac{b\,c^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{4}+\frac{b\,x^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{4}+\frac{b\,c\,x^3}{12}-\frac{b\,c^3\,x}{4}$$

input
```
int(x^3*(a + b*atan(c/x)),x)
```

output
```
(a*x^4)/4 - (b*c^4*atan(c/x))/4 + (b*x^4*atan(c/x))/4 + (b*c*x^3)/12 - (b*
c^3*x)/4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=-\frac{atan\left(\frac{c}{x}\right)b\,c^4}{4}+\frac{atan\left(\frac{c}{x}\right)b\,x^4}{4}+\frac{a\,x^4}{4}-\frac{b\,c^3x}{4}+\frac{bc\,x^3}{12}$$

input
```
int(x^3*(a+b*atan(c/x)),x)
```

output
```
( - 3*atan(c/x)*b*c**4 + 3*atan(c/x)*b*x**4 + 3*a*x**4 - 3*b*c**3*x + b*c*
x**3)/12
```

3.133 $\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$

Optimal result 998
Mathematica [A] (verified) 998
Rubi [A] (verified) 999
Maple [A] (verified) 1000
Fricas [A] (verification not implemented) 1001
Sympy [A] (verification not implemented) 1001
Maxima [A] (verification not implemented) 1002
Giac [A] (verification not implemented) 1002
Mupad [B] (verification not implemented) 1002
Reduce [B] (verification not implemented) 1003

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{6}bcx^2+\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)$$

output
```
1/6*b*c*x^2+1/3*x^3*(a+b*arctan(c/x))-1/6*b*c^3*ln(c^2+x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{6}bcx^2+\frac{ax^3}{3}+\frac{1}{3}bx^3\arctan\left(\frac{c}{x}\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)$$

input
```
Integrate[x^2*(a + b*ArcTan[c/x]),x]
```

output
```
(b*c*x^2)/6 + (a*x^3)/3 + (b*x^3*ArcTan[c/x])/3 - (b*c^3*Log[c^2 + x^2])/6
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5361, 795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \arctan \left(\frac{c}{x}\right)\right) \, dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3} b c \int \frac{x}{\frac{c^2}{x^2}+1}dx+\frac{1}{3} x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 795}$$

$$\frac{1}{3} b c \int \frac{x^3}{c^2+x^2}dx+\frac{1}{3} x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{6} b c \int \frac{x^2}{c^2+x^2}dx^2+\frac{1}{3} x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{6} b c \int \left(1-\frac{c^2}{c^2+x^2}\right)dx^2+\frac{1}{3} x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right)+\frac{1}{6} b c \left(x^2-c^2 \log \left(c^2+x^2\right)\right)$$

input
```
Int[x^2*(a + b*ArcTan[c/x]),x]
```

output
```
(x^3*(a + b*ArcTan[c/x]))/3 + (b*c*(x^2 - c^2*Log[c^2 + x^2]))/6
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$-\frac{b\,c^3 \ln\left(c^2+x^2\right)}{6}+\frac{b\,x^3 \arctan\left(\frac{c}{x}\right)}{3}+\frac{a\,x^3}{3}+\frac{b c\,x^2}{6}-\frac{b\,c^3}{6}$	47
parts	$\frac{a\,x^3}{3}-b\,c^3 \left(-\frac{x^3 \arctan\left(\frac{c}{x}\right)}{3 c^3}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{6}-\frac{x^2}{6 c^2}-\frac{\ln\left(\frac{c}{x}\right)}{3}\right)$	57
derivativedivides	$-c^3 \left(-\frac{a\,x^3}{3 c^3}+b\left(-\frac{x^3 \arctan\left(\frac{c}{x}\right)}{3 c^3}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{6}-\frac{x^2}{6 c^2}-\frac{\ln\left(\frac{c}{x}\right)}{3}\right)\right)$	61
default	$-c^3 \left(-\frac{a\,x^3}{3 c^3}+b\left(-\frac{x^3 \arctan\left(\frac{c}{x}\right)}{3 c^3}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{6}-\frac{x^2}{6 c^2}-\frac{\ln\left(\frac{c}{x}\right)}{3}\right)\right)$	61
risch	Expression too large to display	692

input
```
int(x^2*(a+b*arctan(c/x)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*b*c^3*ln(c^2+x^2)+1/3*b*x^3*arctan(c/x)+1/3*a*x^3+1/6*b*c*x^2-1/6*b*c
^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{3}\,bx^3\arctan\left(\frac{c}{x}\right)-\frac{1}{6}\,bc^3\log\left(c^2+x^2\right)+\frac{1}{6}\,bcx^2+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*(a+b*arctan(c/x)),x, algorithm="fricas")
```

output
```
1/3*b*x^3*arctan(c/x) - 1/6*b*c^3*log(c^2 + x^2) + 1/6*b*c*x^2 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{ax^3}{3}-\frac{bc^3\log\left(c^2+x^2\right)}{6}+\frac{bcx^2}{6}+\frac{bx^3\operatorname{atan}\left(\frac{c}{x}\right)}{3}$$

input
```
integrate(x**2*(a+b*atan(c/x)),x)
```

output
```
a*x**3/3 - b*c**3*log(c**2 + x**2)/6 + b*c*x**2/6 + b*x**3*atan(c/x)/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\arctan\left(\frac{c}{x}\right)-\left(c^2\log\left(c^2+x^2\right)-x^2\right)c\right)b$$

input
```
integrate(x^2*(a+b*arctan(c/x)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/6*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{\left(2\,bc^4\arctan\left(\frac{c}{x}\right)-\frac{bc^7\log\left(\frac{c^2}{x^2}+1\right)}{x^3}+\frac{2\,bc^7\log\left(\frac{c}{x}\right)}{x^3}+2\,ac^4+\frac{bc^5}{x}\right)x^3}{6\,c^4}$$

input
```
integrate(x^2*(a+b*arctan(c/x)),x, algorithm="giac")
```

output
```
1/6*(2*b*c^4*arctan(c/x) - b*c^7*log(c^2/x^2 + 1)/x^3 + 2*b*c^7*log(c/x)/x
^3 + 2*a*c^4 + b*c^5/x)*x^3/c^4
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{a\,x^3}{3}+\frac{b\,x^3\,\mathrm{atan}\left(\frac{c}{x}\right)}{3}-\frac{b\,c^3\,\ln\left(c^2+x^2\right)}{6}+\frac{b\,c\,x^2}{6}$$

input
```
int(x^2*(a + b*atan(c/x)),x)
```

output
```
(a*x^3)/3 + (b*x^3*atan(c/x))/3 - (b*c^3*log(c^2 + x^2))/6 + (b*c*x^2)/6
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{atan\left(\frac{c}{x}\right)b\,x^3}{3}-\frac{\log(c^2+x^2)\,b\,c^3}{6}+\frac{a\,x^3}{3}+\frac{bc\,x^2}{6}$$

input
```
int(x^2*(a+b*atan(c/x)),x)
```

output
```
(2*atan(c/x)*b*x**3 - log(c**2 + x**2)*b*c**3 + 2*a*x**3 + b*c*x**2)/6
```

3.134 $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$

Optimal result 1004
Mathematica [A] (verified) 1004
Rubi [A] (verified) 1005
Maple [A] (verified) 1006
Fricas [A] (verification not implemented) 1007
Sympy [A] (verification not implemented) 1007
Maxima [A] (verification not implemented) 1008
Giac [C] (verification not implemented) 1008
Mupad [B] (verification not implemented) 1008
Reduce [B] (verification not implemented) 1009

Optimal result

Integrand size = 12, antiderivative size = 39

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\arctan\left(\frac{x}{c}\right)$$

output
```
1/2*b*c*x+1/2*x^2*(a+b*arctan(c/x))-1/2*b*c^2*arctan(x/c)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{bcx}{2}+\frac{ax^2}{2}+\frac{1}{2}bc^2\arctan\left(\frac{c}{x}\right)+\frac{1}{2}bx^2\arctan\left(\frac{c}{x}\right)$$

input
```
Integrate[x*(a + b*ArcTan[c/x]),x]
```

output
```
(b*c*x)/2 + (a*x^2)/2 + (b*c^2*ArcTan[c/x])/2 + (b*x^2*ArcTan[c/x])/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5361, 772, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{2}bc\int \frac{1}{\frac{c^2}{x^2}+1}dx+\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 772}$$

$$\frac{1}{2}bc\int \frac{x^2}{c^2+x^2}dx+\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{2}bc\left(x-c^2\int \frac{1}{c^2+x^2}dx\right)+\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(x-c\arctan\left(\frac{x}{c}\right)\right)$$

input
```
Int[x*(a + b*ArcTan[c/x]),x]
```

output
```
(x^2*(a + b*ArcTan[c/x]))/2 + (b*c*(x - c*ArcTan[x/c]))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

method	result	size
parts	$\frac{a\,x^2}{2}+\frac{\arctan\left(\frac{c}{x}\right) b\,x^2}{2}+\frac{bcx}{2}-\frac{b\,c^2\arctan\left(\frac{x}{c}\right)}{2}$	37
parallelrisch	$\frac{\arctan\left(\frac{c}{x}\right) b\,x^2}{2}+\frac{\arctan\left(\frac{c}{x}\right) b\,c^2}{2}+\frac{a\,x^2}{2}+\frac{bcx}{2}-\frac{a\,c^2}{2}$	43
derivativedivides	$-c^2\left(-\frac{a\,x^2}{2c^2}+b\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)}{2c^2}-\frac{\arctan\left(\frac{c}{x}\right)}{2}-\frac{x}{2c}\right)\right)$	47
default	$-c^2\left(-\frac{a\,x^2}{2c^2}+b\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)}{2c^2}-\frac{\arctan\left(\frac{c}{x}\right)}{2}-\frac{x}{2c}\right)\right)$	47
orering	$\left(c^2+x^2\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\left(-\frac{c^2}{2}-\frac{x^2}{2}\right)\left(a+b\arctan\left(\frac{c}{x}\right)-\frac{bc}{x\left(1+\frac{c^2}{x^2}\right)}\right)$	60
risch	Expression too large to display	688

input
```
int(x*(a+b*arctan(c/x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/2*arctan(c/x)*b*x^2+1/2*b*c*x-1/2*b*c^2*arctan(x/c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{2}\,bcx+\frac{1}{2}\,ax^2+\frac{1}{2}\left(bc^2+bx^2\right)\arctan\left(\frac{c}{x}\right)$$

input
```
integrate(x*(a+b*arctan(c/x)),x, algorithm="fricas")
```

output
```
1/2*b*c*x + 1/2*a*x^2 + 1/2*(b*c^2 + b*x^2)*arctan(c/x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{ax^2}{2}+\frac{bc^2\operatorname{atan}\left(\frac{c}{x}\right)}{2}+\frac{bcx}{2}+\frac{bx^2\operatorname{atan}\left(\frac{c}{x}\right)}{2}$$

input
```
integrate(x*(a+b*atan(c/x)),x)
```

output
```
a*x**2/2 + b*c**2*atan(c/x)/2 + b*c*x/2 + b*x**2*atan(c/x)/2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\arctan\left(\frac{c}{x}\right)-\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\right)b$$

input
```
integrate(x*(a+b*arctan(c/x)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/2*(x^2*arctan(c/x) - (c*arctan(x/c) - x)*c)*b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.85

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{\left(2\,bc^3\arctan\left(\frac{c}{x}\right)-\frac{i\,bc^5\log\left(\frac{i\,c}{x}+1\right)}{x^2}+\frac{i\,bc^5\log\left(-\frac{i\,c}{x}+1\right)}{x^2}+2\,ac^3+\frac{2\,bc^4}{x}\right)x^2}{4\,c^3}$$

input
```
integrate(x*(a+b*arctan(c/x)),x, algorithm="giac")
```

output
```
1/4*(2*b*c^3*arctan(c/x) - I*b*c^5*log(I*c/x + 1)/x^2 + I*b*c^5*log(-I*c/x
 + 1)/x^2 + 2*a*c^3 + 2*b*c^4/x)*x^2/c^3
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{a\,x^2}{2}+\frac{b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{b\,x^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{b\,c\,x}{2}$$

input
```
int(x*(a + b*atan(c/x)),x)
```

output
```
(a*x^2)/2 + (b*c^2*atan(c/x))/2 + (b*x^2*atan(c/x))/2 + (b*c*x)/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{atan\left(\frac{c}{x}\right)b\,c^2}{2}+\frac{atan\left(\frac{c}{x}\right)b\,x^2}{2}+\frac{a\,x^2}{2}+\frac{bcx}{2}$$

input
```
int(x*(a+b*atan(c/x)),x)
```

output
```
(atan(c/x)*b*c**2 + atan(c/x)*b*x**2 + a*x**2 + b*c*x)/2
```

3.135 $\int \left(a+b \arctan \left(\frac{c}{x}\right)\right) \, dx$

Optimal result 1010
Mathematica [A] (verified) 1010
Rubi [A] (verified) 1011
Maple [A] (verified) 1012
Fricas [A] (verification not implemented) 1012
Sympy [A] (verification not implemented) 1013
Maxima [A] (verification not implemented) 1013
Giac [A] (verification not implemented) 1013
Mupad [B] (verification not implemented) 1014
Reduce [B] (verification not implemented) 1014

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \left(a+b \arctan \left(\frac{c}{x}\right)\right) \, dx=a x+b x \arctan \left(\frac{c}{x}\right)+\frac{1}{2} b c \log \left(c^2+x^2\right)$$

output
```
a*x+b*x*arctan(c/x)+1/2*b*c*ln(c^2+x^2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a+b \arctan \left(\frac{c}{x}\right)\right) \, dx=a x+b x \arctan \left(\frac{c}{x}\right)+\frac{1}{2} b c \log \left(c^2+x^2\right)$$

input
```
Integrate[a + b*ArcTan[c/x],x]
```

output
```
a*x + b*x*ArcTan[c/x] + (b*c*Log[c^2 + x^2])/2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+bx\arctan\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2+x^2\right)$$

input
```
Int[a + b*ArcTan[c/x],x]
```

output
```
a*x + b*x*ArcTan[c/x] + (b*c*Log[c^2 + x^2])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$b\left(\frac{c\ln(c^2+x^2)}{2}+\arctan\left(\frac{c}{x}\right)x\right)+ax$	27
default	$ax-bc\left(-\frac{x\arctan\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)$	40
parts	$ax-bc\left(-\frac{x\arctan\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)$	40
derivativedivides	$-c\left(-\frac{ax}{c}+b\left(-\frac{x\arctan\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)\right)$	45
risch	Expression too large to display	642

input
```
int(a+b*arctan(c/x),x,method=_RETURNVERBOSE)
```

output
```
b*(1/2*c*ln(c^2+x^2)+arctan(c/x)*x)+a*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=bx\arctan\left(\frac{c}{x}\right)+\frac{1}{2}\,bc\log\left(c^2+x^2\right)+ax$$

input
```
integrate(a+b*arctan(c/x),x, algorithm="fricas")
```

output
```
b*x*arctan(c/x) + 1/2*b*c*log(c^2 + x^2) + a*x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=ax+b\left(\frac{c\log\left(c^2+x^2\right)}{2}+x\,\mathrm{atan}\left(\frac{c}{x}\right)\right)$$

input
```
integrate(a+b*atan(c/x),x)
```

output
```
a*x + b*(c*log(c**2 + x**2)/2 + x*atan(c/x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{2}\left(2\,x\arctan\left(\frac{c}{x}\right)+c\log\left(c^2+x^2\right)\right)b+ax$$

input
```
integrate(a+b*arctan(c/x),x, algorithm="maxima")
```

output
```
1/2*(2*x*arctan(c/x) + c*log(c^2 + x^2))*b + a*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=ax+\frac{\left(c^2\left(\log\left(\frac{c^2}{x^2}+1\right)-\log\left(\frac{c^2}{x^2}\right)\right)+2\,cx\arctan\left(\frac{c}{x}\right)\right)b}{2\,c}$$

input
```
integrate(a+b*arctan(c/x),x, algorithm="giac")
```

output
```
a*x + 1/2*(c^2*(log(c^2/x^2 + 1) - log(c^2/x^2)) + 2*c*x*arctan(c/x))*b/c
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=a\,x+b\,x\,\mathrm{atan}\left(\frac{c}{x}\right)+\frac{b\,c\,\ln\left(c^2+x^2\right)}{2}$$

input
```
int(a + b*atan(c/x),x)
```

output
```
a*x + b*x*atan(c/x) + (b*c*log(c^2 + x^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=atan\left(\frac{c}{x}\right)bx+\frac{\log(c^2+x^2)\,bc}{2}+ax$$

input
```
int(a+b*atan(c/x),x)
```

output
```
(2*atan(c/x)*b*x + log(c**2 + x**2)*b*c + 2*a*x)/2
```

3.136 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx$

Optimal result . . . 1015
Mathematica [A] (verified) . . . 1015
Rubi [A] (verified) . . . 1016
Maple [B] (verified) . . . 1017
Fricas [F] . . . 1017
Sympy [F] . . . 1018
Maxima [F] . . . 1018
Giac [B] (verification not implemented) . . . 1018
Mupad [B] (verification not implemented) . . . 1019
Reduce [F] . . . 1019

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx = a\log(x)-\frac{1}{2}ib\,\text{PolyLog}\left(2,-\frac{ic}{x}\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{ic}{x}\right)$$

output
```
a*ln(x)-1/2*I*b*polylog(2,-I*c/x)+1/2*I*b*polylog(2,I*c/x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx = a\log(x)-\frac{1}{2}ib\,\text{PolyLog}\left(2,-\frac{ic}{x}\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{ic}{x}\right)$$

input
```
Integrate[(a + b*ArcTan[c/x])/x,x]
```

output
```
a*Log[x] - (I/2)*b*PolyLog[2, ((-I)*c)/x] + (I/2)*b*PolyLog[2, (I*c)/x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5359, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}dx$$

$$\Big\downarrow \text{5359}$$

$$-\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \text{5355}$$

$$-\frac{1}{2}ib\int x\log\left(1-\frac{ic}{x}\right)d\frac{1}{x}+\frac{1}{2}ib\int x\log\left(\frac{ic}{x}+1\right)d\frac{1}{x}-a\log\left(\frac{1}{x}\right)$$

$$\Big\downarrow \text{2838}$$

$$-a\log\left(\frac{1}{x}\right)-\frac{1}{2}ib\,\text{PolyLog}\left(2,-\frac{ic}{x}\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{ic}{x}\right)$$

input
```
Int[(a + b*ArcTan[c/x])/x,x]
```

output
```
-(a*Log[x^(-1)]) - (I/2)*b*PolyLog[2, ((-I)*c)/x] + (I/2)*b*PolyLog[2, (I*
c)/x]
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(31) = 62.

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

method	result
parts	$a\ln(x)+b\Big(-\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)-\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{ic}{x}\right)}{2}+\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1-\frac{ic}{x}\right)}{2}-\frac{i\,\mathrm{dilog}\left(1+\frac{ic}{x}\right)}{2}+\frac{i\,\mathrm{dilog}(}{2}$
derivativedivides	$-a\ln\left(\frac{c}{x}\right)-b\Big(\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)+\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{ic}{x}\right)}{2}-\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1-\frac{ic}{x}\right)}{2}+\frac{i\,\mathrm{dilog}\left(1+\frac{ic}{x}\right)}{2}-\frac{i\,\mathrm{dilog}(}{2}$
default	$-a\ln\left(\frac{c}{x}\right)-b\Big(\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)+\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{ic}{x}\right)}{2}-\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1-\frac{ic}{x}\right)}{2}+\frac{i\,\mathrm{dilog}\left(1+\frac{ic}{x}\right)}{2}-\frac{i\,\mathrm{dilog}(}{2}$
risch	Expression too large to display

input
```
int((a+b*arctan(c/x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*(-ln(c/x)*arctan(c/x)-1/2*I*ln(c/x)*ln(1+I*c/x)+1/2*I*ln(c/x)*ln
(1-I*c/x)-1/2*I*dilog(1+I*c/x)+1/2*I*dilog(1-I*c/x))
```

Fricas [F]

$$\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx=\int\frac{b\arctan\left(\frac{c}{x}\right)+a}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))/x,x, algorithm="fricas")
```

output
```
integral((b*arctan(c/x) + a)/x, x)
```

Sympy [F]

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx = \int \frac{a+b\,\mathrm{atan}\left(\frac{c}{x}\right)}{x}\,dx$$

input
```
integrate((a+b*atan(c/x))/x,x)
```

output
```
Integral((a + b*atan(c/x))/x, x)
```

Maxima [F]

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx = \int \frac{b\arctan\left(\frac{c}{x}\right)+a}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))/x,x, algorithm="maxima")
```

output
```
b*integrate(arctan2(c, x)/x, x) + a*log(x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(25) = 50.

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.85

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx = -\frac{\left(2\,bc^4\arctan\left(\frac{c}{x}\right)-\frac{i\,bc^6\log\left(\frac{i\,c}{x}+1\right)}{x^2}+\frac{i\,bc^6\log\left(-\frac{i\,c}{x}+1\right)}{x^2}+2\,ac^4+\frac{2\,bc^5}{x}\right)x^2}{4\,c^5}$$

input
```
integrate((a+b*arctan(c/x))/x,x, algorithm="giac")
```

output
```
-1/4*(2*b*c^4*arctan(c/x) - I*b*c^6*log(I*c/x + 1)/x^2 + I*b*c^6*log(-I*c/
x + 1)/x^2 + 2*a*c^4 + 2*b*c^5/x)*x^2/c^5
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx=a\,\ln\left(x\right)+\frac{b\left(\mathrm{Li}_2\left(1-\frac{c\,1\mathrm{i}}{x}\right)-\mathrm{Li}_2\left(1+\frac{c\,1\mathrm{i}}{x}\right)\right)\,1\mathrm{i}}{2}$$

input
```
int((a + b*atan(c/x))/x,x)
```

output
```
(b*(dilog(1 - (c*1i)/x) - dilog((c*1i)/x + 1))*1i)/2 + a*log(x)
```

Reduce [F]

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx=\left(\int \frac{atan\left(\frac{c}{x}\right)}{x}dx\right)b+\log(x)\,a$$

input
```
int((a+b*atan(c/x))/x,x)
```

output
```
int(atan(c/x)/x,x)*b + log(x)*a
```

3.137 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx$

Optimal result 1020
Mathematica [A] (verified) 1020
Rubi [A] (verified) 1021
Maple [A] (verified) 1022
Fricas [A] (verification not implemented) 1022
Sympy [A] (verification not implemented) 1023
Maxima [A] (verification not implemented) 1023
Giac [A] (verification not implemented) 1023
Mupad [B] (verification not implemented) 1024
Reduce [B] (verification not implemented) 1024

Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx = -\frac{a+b\arctan\left(\frac{c}{x}\right)}{x}+\frac{b\log\left(1+\frac{c^2}{x^2}\right)}{2c}$$

output
```
-(a+b*arctan(c/x))/x+1/2*b*ln(1+c^2/x^2)/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx = -\frac{a}{x}-\frac{b\arctan\left(\frac{c}{x}\right)}{x}+\frac{b\log\left(1+\frac{c^2}{x^2}\right)}{2c}$$

input
```
Integrate[(a + b*ArcTan[c/x])/x^2,x]
```

output
```
-(a/x) - (b*ArcTan[c/x])/x + (b*Log[1 + c^2/x^2])/(2*c)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5361, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx$$

$$\Big\downarrow \text{5361}$$

$$-bc\int \frac{1}{\left(\frac{c^2}{x^2}+1\right)x^3}dx-\frac{a+b\arctan\left(\frac{c}{x}\right)}{x}$$

$$\Big\downarrow \text{792}$$

$$\frac{b\log\left(\frac{c^2}{x^2}+1\right)}{2c}-\frac{a+b\arctan\left(\frac{c}{x}\right)}{x}$$

input
```
Int[(a + b*ArcTan[c/x])/x^2,x]
```

output
```
-((a + b*ArcTan[c/x])/x) + (b*Log[1 + c^2/x^2])/(2*c)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
parts	$-\frac{a}{x}-\frac{b\arctan\left(\frac{c}{x}\right)}{x}+\frac{b\ln\left(1+\frac{c^{2}}{x^{2}}\right)}{2c}$	36
derivativedivides	$-\frac{\frac{ca}{x}+b\left(\frac{c\arctan\left(\frac{c}{x}\right)}{x}-\frac{\ln\left(1+\frac{c^{2}}{x^{2}}\right)}{2}\right)}{c}$	39
default	$-\frac{\frac{ca}{x}+b\left(\frac{c\arctan\left(\frac{c}{x}\right)}{x}-\frac{\ln\left(1+\frac{c^{2}}{x^{2}}\right)}{2}\right)}{c}$	39
parallelrisch	$-\frac{2xb\ln(x)-b\ln(c^{2}+x^{2})x+2\arctan\left(\frac{c}{x}\right)bc+2ac}{2cx}$	42
risch	Expression too large to display	652

input
```
int((a+b*arctan(c/x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x-b/x*arctan(c/x)+1/2*b*ln(1+c^2/x^2)/c
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx=-\frac{2\,bc\arctan\left(\frac{c}{x}\right)-bx\log\left(c^2+x^2\right)+2\,bx\log\left(x\right)+2\,ac}{2\,cx}$$

input
```
integrate((a+b*arctan(c/x))/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b*c*arctan(c/x) - b*x*log(c^2 + x^2) + 2*b*x*log(x) + 2*a*c)/(c*x)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{a + b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx = \begin{cases} -\frac{a}{x} - \frac{b\operatorname{atan}\left(\frac{c}{x}\right)}{x} - \frac{b\log\left(x\right)}{c} + \frac{b\log\left(c^2+x^2\right)}{2c} & \text{for } c \neq 0 \\ -\frac{a}{x} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atan(c/x))/x**2,x)
```

output
```
Piecewise((-a/x - b*atan(c/x)/x - b*log(x)/c + b*log(c**2 + x**2)/(2*c), N
e(c, 0)), (-a/x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{a + b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx = -\frac{b\left(\frac{2\,c\arctan\left(\frac{c}{x}\right)}{x} - \log\left(\frac{c^2}{x^2} + 1\right)\right)}{2\,c} - \frac{a}{x}$$

input
```
integrate((a+b*arctan(c/x))/x^2,x, algorithm="maxima")
```

output
```
-1/2*b*(2*c*arctan(c/x)/x - log(c^2/x^2 + 1))/c - a/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{a + b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx = -\frac{\frac{2\,bc\arctan\left(\frac{c}{x}\right)}{x} - b\log\left(\frac{c^2}{x^2} + 1\right) + \frac{2\,ac}{x}}{2\,c}$$

input
```
integrate((a+b*arctan(c/x))/x^2,x, algorithm="giac")
```

output
```
-1/2*(2*b*c*arctan(c/x)/x - b*log(c^2/x^2 + 1) + 2*a*c/x)/c
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx=\frac{\frac{b\,\ln\left(c^2+x^2\right)}{2}-b\,\ln\left(x\right)}{c}-\frac{a\,c+b\,c\,\mathrm{atan}\left(\frac{c}{x}\right)}{c\,x}$$

input
```
int((a + b*atan(c/x))/x^2,x)
```

output
```
((b*log(c^2 + x^2))/2 - b*log(x))/c - (a*c + b*c*atan(c/x))/(c*x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx=\frac{-2atan\left(\frac{c}{x}\right)bc+\log\left(c^2+x^2\right)bx-2\log\left(x\right)bx-2ac}{2cx}$$

input
```
int((a+b*atan(c/x))/x^2,x)
```

output
```
( - 2*atan(c/x)*b*c + log(c**2 + x**2)*b*x - 2*log(x)*b*x - 2*a*c)/(2*c*x)
```

3.138 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx$

Optimal result 1025
Mathematica [A] (verified) 1025
Rubi [A] (verified) 1026
Maple [A] (verified) 1027
Fricas [A] (verification not implemented) 1028
Sympy [A] (verification not implemented) 1029
Maxima [A] (verification not implemented) 1029
Giac [C] (verification not implemented) 1029
Mupad [B] (verification not implemented) 1030
Reduce [B] (verification not implemented) 1030

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx=\frac{b}{2cx}-\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}+\frac{b\arctan\left(\frac{x}{c}\right)}{2c^2}$$

output
```
1/2*b/c/x-1/2*(a+b*arctan(c/x))/x^2+1/2*b*arctan(x/c)/c^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx=-\frac{a}{2x^2}+\frac{b}{2cx}-\frac{b\arctan\left(\frac{c}{x}\right)}{2x^2}+\frac{b\arctan\left(\frac{x}{c}\right)}{2c^2}$$

input
```
Integrate[(a + b*ArcTan[c/x])/x^3,x]
```

output
```
-1/2*a/x^2 + b/(2*c*x) - (b*ArcTan[c/x])/(2*x^2) + (b*ArcTan[x/c])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 795, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}dx$$

$$\Big\downarrow \text{5361}$$

$$-\frac{1}{2}bc\int \frac{1}{\left(\frac{c^2}{x^2}+1\right)x^4}dx-\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{795}$$

$$-\frac{1}{2}bc\int \frac{1}{x^2\left(c^2+x^2\right)}dx-\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{264}$$

$$-\frac{1}{2}bc\left(-\frac{\int \frac{1}{c^2+x^2}dx}{c^2}-\frac{1}{c^2x}\right)-\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{216}$$

$$-\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{1}{2}bc\left(-\frac{\arctan\left(\frac{x}{c}\right)}{c^3}-\frac{1}{c^2x}\right)$$

input
```
Int[(a + b*ArcTan[c/x])/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c/x])/x^2 - (b*c*(-(1/(c^2*x)) - ArcTan[x/c]/c^3))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
parts	$-\frac{a}{2x^2}-\frac{b\arctan\left(\frac{c}{x}\right)}{2x^2}+\frac{b}{2cx}+\frac{b\arctan\left(\frac{x}{c}\right)}{2c^2}$	41
parallelrisch	$-\frac{\arctan\left(\frac{c}{x}\right)b\,x^2+\arctan\left(\frac{c}{x}\right)b\,c^2-bcx+a\,c^2}{2x^2c^2}$	42
derivativedivides	$-\frac{\frac{a\,c^2}{2x^2}+b\left(\frac{c^2\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{c}{2x}+\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)}{c^2}$	47
default	$-\frac{\frac{a\,c^2}{2x^2}+b\left(\frac{c^2\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{c}{2x}+\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)}{c^2}$	47
orering	$-\frac{2(c^2+x^2)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{c^2x^2}-\frac{x^2(c^2+x^2)\left(-\frac{bc}{x^5\left(1+\frac{c^2}{x^2}\right)}-\frac{3\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{x^4}\right)}{2c^2}$	76
risch	Expression too large to display	709

input
```
int((a+b*arctan(c/x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2-1/2*b*arctan(c/x)/x^2+1/2*b/c/x+1/2*b*arctan(x/c)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx=-\frac{ac^2-bcx+(bc^2+bx^2)\arctan\left(\frac{c}{x}\right)}{2\,c^2x^2}$$

input
```
integrate((a+b*arctan(c/x))/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a*c^2 - b*c*x + (b*c^2 + b*x^2)*arctan(c/x))/(c^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx = \begin{cases} -\frac{a}{2x^2} - \frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{2x^2} + \frac{b}{2cx} - \frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{2c^2} & \text{for}\ c \neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atan(c/x))/x**3,x)
```

output
```
Piecewise((-a/(2*x**2) - b*atan(c/x)/(2*x**2) + b/(2*c*x) - b*atan(c/x)/(2
*c**2), Ne(c, 0)), (-a/(2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx = \frac{1}{2}\left(c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3} + \frac{1}{c^2x}\right) - \frac{\arctan\left(\frac{c}{x}\right)}{x^2}\right)b - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*arctan(c/x))/x^3,x, algorithm="maxima")
```

output
```
1/2*(c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*b - 1/2*a/x^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx = -\frac{i\left(-\frac{2i\,bc^2\arctan\left(\frac{c}{x}\right)}{x^2} + b\log\left(\frac{i\,c}{x}-1\right) - b\log\left(-\frac{i\,c}{x}-1\right) - \frac{2i\,ac^2}{x^2} + \frac{2i\,bc}{x}\right)}{4\,c^2}$$

input
```
integrate((a+b*arctan(c/x))/x^3,x, algorithm="giac")
```

output
```
-1/4*I*(-2*I*b*c^2*arctan(c/x)/x^2 + b*log(I*c/x - 1) - b*log(-I*c/x - 1)
- 2*I*a*c^2/x^2 + 2*I*b*c/x)/c^2
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx=\frac{b\,c\,\mathrm{atan}\left(\frac{x}{\sqrt{c^2}}\right)}{2\,{\left(c^2\right)}^{3/2}}-\frac{\frac{a\,c^2}{2}+\frac{b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}-\frac{b\,c\,x}{2}}{c^2\,x^2}$$

input
```
int((a + b*atan(c/x))/x^3,x)
```

output
```
(b*c*atan(x/(c^2)^(1/2)))/(2*(c^2)^(3/2)) - ((a*c^2)/2 + (b*c^2*atan(c/x))
/2 - (b*c*x)/2)/(c^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx=\frac{-atan\left(\frac{c}{x}\right)b\,c^2-atan\left(\frac{c}{x}\right)b\,x^2-a\,c^2+bcx}{2c^2x^2}$$

input
```
int((a+b*atan(c/x))/x^3,x)
```

output
```
( - atan(c/x)*b*c**2 - atan(c/x)*b*x**2 - a*c**2 + b*c*x)/(2*c**2*x**2)
```

3.139 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx$

Optimal result 1031
Mathematica [A] (verified) 1031
Rubi [A] (verified) 1032
Maple [A] (verified) 1033
Fricas [A] (verification not implemented) 1034
Sympy [A] (verification not implemented) 1035
Maxima [A] (verification not implemented) 1035
Giac [A] (verification not implemented) 1035
Mupad [B] (verification not implemented) 1036
Reduce [B] (verification not implemented) 1036

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx=\frac{b}{6cx^2}-\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}-\frac{b\log\left(1+\frac{c^2}{x^2}\right)}{6c^3}$$

output
```
1/6*b/c/x^2-1/3*(a+b*arctan(c/x))/x^3-1/6*b*ln(1+c^2/x^2)/c^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx=-\frac{a}{3x^3}+\frac{b}{6cx^2}-\frac{b\arctan\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}$$

input
```
Integrate[(a + b*ArcTan[c/x])/x^4,x]
```

output
```
-1/3*a/x^3 + b/(6*c*x^2) - (b*ArcTan[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) -
(b*Log[c^2 + x^2])/(6*c^3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5361, 795, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}dx$$

$$\Big\downarrow \text{5361}$$

$$-\frac{1}{3}bc\int \frac{1}{\left(\frac{c^2}{x^2}+1\right)x^5}dx-\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{795}$$

$$-\frac{1}{3}bc\int \frac{1}{x^3\left(c^2+x^2\right)}dx-\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{6}bc\int \frac{1}{x^4\left(c^2+x^2\right)}dx^2-\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{54}$$

$$-\frac{1}{6}bc\int \left(-\frac{1}{c^4x^2}+\frac{1}{c^2x^4}+\frac{1}{c^4\left(c^2+x^2\right)}\right)dx^2-\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{6}bc\left(-\frac{\log\left(x^2\right)}{c^4}-\frac{1}{c^2x^2}+\frac{\log\left(c^2+x^2\right)}{c^4}\right)$$

input
```
Int[(a + b*ArcTan[c/x])/x^4,x]
```

output
```
-1/3*(a + b*ArcTan[c/x])/x^3 - (b*c*(-(1/(c^2*x^2)) - Log[x^2]/c^4 + Log[c
^2 + x^2]/c^4))/6
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

method	result	size
parts	$-\frac{a}{3x^3}-\frac{b\arctan\left(\frac{c}{x}\right)}{3x^3}+\frac{b}{6c\,x^2}-\frac{b\ln\left(1+\frac{c^2}{x^2}\right)}{6c^3}$	45
derivativedivides	$-\frac{\frac{a\,c^3}{3x^3}+b\left(\frac{c^3\arctan\left(\frac{c}{x}\right)}{3x^3}-\frac{c^2}{6x^2}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{6}\right)}{c^3}$	53
default	$-\frac{\frac{a\,c^3}{3x^3}+b\left(\frac{c^3\arctan\left(\frac{c}{x}\right)}{3x^3}-\frac{c^2}{6x^2}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{6}\right)}{c^3}$	53
parallelrisch	$\frac{2b\ln(x)x^3-b\ln(c^2+x^2)x^3-2b\,c^3\arctan\left(\frac{c}{x}\right)+b\,c^2x-2a\,c^3}{6x^3c^3}$	56
risch	Expression too large to display	705

input
```
int((a+b*arctan(c/x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3-1/3*b*arctan(c/x)/x^3+1/6*b/c/x^2-1/6*b*ln(1+c^2/x^2)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.17

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx = -\frac{2\,bc^3\arctan\left(\frac{c}{x}\right)+bx^3\log\left(c^2+x^2\right)-2\,bx^3\log\left(x\right)+2\,ac^3-bc^2x}{6\,c^3x^3}$$

input
```
integrate((a+b*arctan(c/x))/x^4,x, algorithm="fricas")
```

output
```
-1/6*(2*b*c^3*arctan(c/x) + b*x^3*log(c^2 + x^2) - 2*b*x^3*log(x) + 2*a*c^
3 - b*c^2*x)/(c^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx=\begin{cases} -\frac{a}{3x^3}-\frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{3x^3}+\frac{b}{6cx^2}+\frac{b\log(x)}{3c^3}-\frac{b\log(c^2+x^2)}{6c^3} & \text{for } c\neq 0\\ -\frac{a}{3x^3} & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*atan(c/x))/x**4,x)
```

output
```
Piecewise((-a/(3*x**3) - b*atan(c/x)/(3*x**3) + b/(6*c*x**2) + b*log(x)/(3
*c**3) - b*log(c**2 + x**2)/(6*c**3), Ne(c, 0)), (-a/(3*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx=-\frac{1}{6}\left(c\left(\frac{\log(c^2+x^2)}{c^4}-\frac{\log(x^2)}{c^4}-\frac{1}{c^2x^2}\right)+\frac{2\arctan\left(\frac{c}{x}\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

input
```
integrate((a+b*arctan(c/x))/x^4,x, algorithm="maxima")
```

output
```
-1/6*(c*(log(c^2 + x^2)/c^4 - log(x^2)/c^4 - 1/(c^2*x^2)) + 2*arctan(c/x)/
x^3)*b - 1/3*a/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx=-\frac{\frac{2\,bc^3\arctan\left(\frac{c}{x}\right)}{x^3}+b\log\left(\frac{c^2}{x^2}+1\right)+\frac{2\,ac^3}{x^3}-\frac{bc^2}{x^2}}{6\,c^3}$$

input
```
integrate((a+b*arctan(c/x))/x^4,x, algorithm="giac")
```

output
```
-1/6*(2*b*c^3*arctan(c/x)/x^3 + b*log(c^2/x^2 + 1) + 2*a*c^3/x^3 - b*c^2/x
^2)/c^3
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx = \frac{\frac{b\,x^3\,\ln(x)}{3}+\frac{b\,c^2\,x}{6}-\frac{b\,x^3\,\ln(c^2+x^2)}{6}}{c^3\,x^3}-\frac{\frac{a}{3}+\frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{3}}{x^3}$$

input
```
int((a + b*atan(c/x))/x^4,x)
```

output
```
((b*x^3*log(x))/3 + (b*c^2*x)/6 - (b*x^3*log(c^2 + x^2))/6)/(c^3*x^3) - (a
/3 + (b*atan(c/x))/3)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.17

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx = \frac{-2atan\left(\frac{c}{x}\right)b\,c^3-\log(c^2+x^2)\,b\,x^3+2\log(x)\,b\,x^3-2a\,c^3+b\,c^2x}{6c^3x^3}$$

input
```
int((a+b*atan(c/x))/x^4,x)
```

output
```
( - 2*atan(c/x)*b*c**3 - log(c**2 + x**2)*b*x**3 + 2*log(x)*b*x**3 - 2*a*c
**3 + b*c**2*x)/(6*c**3*x**3)
```

3.140 $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal result . . . 1037
Mathematica [A] (verified) . . . 1038
Rubi [A] (warning: unable to verify) . . . 1038
Maple [A] (verified) . . . 1042
Fricas [A] (verification not implemented) . . . 1043
Sympy [A] (verification not implemented) . . . 1043
Maxima [A] (verification not implemented) . . . 1044
Giac [F] . . . 1044
Mupad [B] (verification not implemented) . . . 1045
Reduce [B] (verification not implemented) . . . 1045

Optimal result

Integrand size = 16, antiderivative size = 122

$$\begin{aligned}\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx &= \frac{1}{12}b^2c^2x^2-\frac{1}{2}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\\ &\quad+\frac{1}{6}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\\ &\quad-\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\ &\quad-\frac{1}{3}b^2c^4\log\left(1+\frac{c^2}{x^2}\right)-\frac{2}{3}b^2c^4\log(x)\end{aligned}$$

output
```
1/12*b^2*c^2*x^2-1/2*b*c^3*x*(a+b*arccot(x/c))+1/6*b*c*x^3*(a+b*arccot(x/c
))-1/4*c^4*(a+b*arccot(x/c))^2+1/4*x^4*(a+b*arccot(x/c))^2-1/3*b^2*c^4*ln(
1+c^2/x^2)-2/3*b^2*c^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.91

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{1}{12}\left(x\left(b^2c^2x+3a^2x^3+2abc\left(-3c^2+x^2\right)\right)+2b\left(bcx\left(-3c^2+x^2\right)+3a\left(-c^4+x^4\right)\right)\arctan\left(\frac{c}{x}\right)+3b^2\left(-c^4+x^4\right)\arctan\left(\frac{c}{x}\right)^2-4b^2c^4\log\left(c^2+x^2\right)\right)$$

input
```
Integrate[x^3*(a + b*ArcTan[c/x])^2,x]
```

output
```
(x*(b^2*c^2*x + 3*a^2*x^3 + 2*a*b*c*(-3*c^2 + x^2)) + 2*b*(b*c*x*(-3*c^2 +
 x^2) + 3*a*(-c^4 + x^4))*ArcTan[c/x] + 3*b^2*(-c^4 + x^4)*ArcTan[c/x]^2 -
 4*b^2*c^4*Log[c^2 + x^2])/12
```

Rubi [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5363, 5361, 5453, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$$

$$\Big\downarrow \text{5363}$$

$$-\int x^5\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,d\frac{1}{x}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\int \frac{x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(\int x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)d\frac{1}{x}-c^2\int\frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)$$

$\Big\downarrow$ 5361

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(-c^2\int\frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}+\frac{1}{3}bc\int\frac{x^3}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$\Big\downarrow$ 243

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(-c^2\int\frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}+\frac{1}{6}bc\int\frac{x^2}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}-\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$\Big\downarrow$ 54

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(-c^2\int\frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}+\frac{1}{6}bc\int\left(\frac{c^4}{\frac{c^2}{x^2}+1}-xc^2+x^2\right)d\frac{1}{x^2}-\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$\Big\downarrow$ 2009

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(-c^2\int\frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{6}bc\left(c^2\log\left(\frac{c^2}{x^2}+1\right)-c^2\log\left(\frac{1}{x^2}\right)-x\right)\right)$$

$\Big\downarrow$ 5453

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(-c^2\left(\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)d\frac{1}{x}-c^2\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{6}bc\left(c^2\log\left(\right.\right.\right.$$

$\Big\downarrow$ 5361

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(-c^2\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\int\frac{x}{\frac{c^2}{x^2}+1}d\frac{1}{x}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right.\right.$$

$\Big\downarrow$ 243

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{1}{2}bc\left(-c^2\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\int\frac{x}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right.\right.\right.$$

$\Big\downarrow$ 47

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{1}{2}bc\left(-c^2\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\left(\int xd\frac{1}{x^2}-c^2\int\frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}\right)-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-\frac{1}{3}x^3\right.$$

$\Big\downarrow$ 14

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{1}{2}bc\left(-c^2\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-c^2\int\frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}\right)-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-\frac{1}{3}\right.$$

$\Big\downarrow$ 16

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{1}{2}bc\left(-c^2\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\log\left(\frac{c^2}{x^2}+1\right)\right)\right)-\frac{1}{3}x^3\right.$$

$\Big\downarrow$ 5419

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{1}{2}bc\left(-c^2\left(-\frac{c\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\log\left(\frac{c^2}{x^2}+1\right)\right)\right)-\frac{1}{3}x^3\left(a+b\,\mathrm{ar}\right.\right.$$

input
```
Int[x^3*(a + b*ArcTan[c/x])^2,x]
```

output
```
(x^4*(a + b*ArcTan[c/x])^2)/4 - (b*c*(-1/3*(x^3*(a + b*ArcTan[c/x])) + (b*
c*(-x + c^2*Log[1 + c^2/x^2] - c^2*Log[x^(-2)]))/6 - c^2*(-(x*(a + b*ArcTa
n[c/x])) - (c*(a + b*ArcTan[c/x])^2)/(2*b) + (b*c*(-Log[1 + c^2/x^2] + Log
[x^(-2)]))/2)))/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.16

method	result
parts	$\frac{a^2x^4}{4}-b^2c^4\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^2}{4c^4}+\frac{\arctan\left(\frac{c}{x}\right)^2}{4}-\frac{x^3\arctan\left(\frac{c}{x}\right)}{6c^3}+\frac{x\arctan\left(\frac{c}{x}\right)}{2c}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{x^2}{12c^2}-\frac{2}{}\right.$
derivativedivides	$-c^4\left(-\frac{a^2x^4}{4c^4}+b^2\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^2}{4c^4}+\frac{\arctan\left(\frac{c}{x}\right)^2}{4}-\frac{x^3\arctan\left(\frac{c}{x}\right)}{6c^3}+\frac{x\arctan\left(\frac{c}{x}\right)}{2c}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{x^2}{12c}\right.\right.$
default	$-c^4\left(-\frac{a^2x^4}{4c^4}+b^2\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^2}{4c^4}+\frac{\arctan\left(\frac{c}{x}\right)^2}{4}-\frac{x^3\arctan\left(\frac{c}{x}\right)}{6c^3}+\frac{x\arctan\left(\frac{c}{x}\right)}{2c}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{x^2}{12c}\right.\right.$
parallelrisch	$\frac{x^4\arctan\left(\frac{c}{x}\right)^2b^2}{4}-\frac{\arctan\left(\frac{c}{x}\right)^2b^2c^4}{4}-\frac{b^2c^4\ln\left(c^2+x^2\right)}{3}+\frac{x^4\arctan\left(\frac{c}{x}\right)ab}{2}+\frac{x^3\arctan\left(\frac{c}{x}\right)b^2c}{6}-\frac{x\arctan\left(\frac{c}{x}\right)b}{2}$
risch	Expression too large to display

input
```
int(x^3*(a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4-b^2*c^4*(-1/4/c^4*x^4*arctan(c/x)^2+1/4*arctan(c/x)^2-1/6/c^3*
x^3*arctan(c/x)+1/2/c*x*arctan(c/x)+1/3*ln(1+c^2/x^2)-1/12/c^2*x^2-2/3*ln(
c/x))+1/2*x^4*arctan(c/x)*a*b+1/2*a*b*c^4*arctan(x/c)+1/6*a*b*c*x^3-1/2*a*
b*c^3*x
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{1}{2}\,abc^4\arctan\left(\frac{x}{c}\right)-\frac{1}{3}\,b^2c^4\log\left(c^2+x^2\right)-\frac{1}{2}\,abc^3x+\frac{1}{12}\,b^2c^2x^2+\frac{1}{6}\,abcx^3+\frac{1}{4}\,a^2x^4-\frac{1}{4}\left(b^2c^4-b^2x^4\right)\arctan\left(\frac{c}{x}\right)^2-\frac{1}{6}\left(3\,b^2c^3x-b^2cx^3-3\,abx^4\right)\arctan\left(\frac{c}{x}\right)$$

input
```
integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

output
```
1/2*a*b*c^4*arctan(x/c) - 1/3*b^2*c^4*log(c^2 + x^2) - 1/2*a*b*c^3*x + 1/1
2*b^2*c^2*x^2 + 1/6*a*b*c*x^3 + 1/4*a^2*x^4 - 1/4*(b^2*c^4 - b^2*x^4)*arct
an(c/x)^2 - 1/6*(3*b^2*c^3*x - b^2*c*x^3 - 3*a*b*x^4)*arctan(c/x)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.18

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{a^2x^4}{4}-\frac{abc^4\operatorname{atan}\left(\frac{c}{x}\right)}{2}-\frac{abc^3x}{2}+\frac{abcx^3}{6}+\frac{abx^4\operatorname{atan}\left(\frac{c}{x}\right)}{2}-\frac{b^2c^4\log\left(c^2+x^2\right)}{3}-\frac{b^2c^4\operatorname{atan}^2\left(\frac{c}{x}\right)}{4}-\frac{b^2c^3x\operatorname{atan}\left(\frac{c}{x}\right)}{2}+\frac{b^2c^2x^2}{12}+\frac{b^2cx^3\operatorname{atan}\left(\frac{c}{x}\right)}{6}+\frac{b^2x^4\operatorname{atan}^2\left(\frac{c}{x}\right)}{4}$$

input
```
integrate(x**3*(a+b*atan(c/x))**2,x)
```

output
```
a**2*x**4/4 - a*b*c**4*atan(c/x)/2 - a*b*c**3*x/2 + a*b*c*x**3/6 + a*b*x**
4*atan(c/x)/2 - b**2*c**4*log(c**2 + x**2)/3 - b**2*c**4*atan(c/x)**2/4 -
b**2*c**3*x*atan(c/x)/2 + b**2*c**2*x**2/12 + b**2*c*x**3*atan(c/x)/6 + b*
*2*x**4*atan(c/x)**2/4
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.10

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx = \frac{1}{4}b^2x^4\arctan\left(\frac{c}{x}\right)^2+\frac{1}{4}a^2x^4 + \frac{1}{6}\left(3x^4\arctan\left(\frac{c}{x}\right)+\left(3c^3\arctan\left(\frac{x}{c}\right)-3c^2x+x^3\right)c\right)ab + \frac{1}{12}\left(\left(3c^2\arctan\left(\frac{x}{c}\right)^2-4c^2\log\left(c^2+x^2\right)+x^2\right)c^2+2\left(3c^3\arctan\left(\frac{x}{c}\right)-3c^2x+x^3\right)c\arctan\left(\frac{c}{x}\right)\right.$$

input
```
integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*arctan(c/x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arctan(c/x) + (3*c^3*
arctan(x/c) - 3*c^2*x + x^3)*c)*a*b + 1/12*((3*c^2*arctan(x/c)^2 - 4*c^2*l
og(c^2 + x^2) + x^2)*c^2 + 2*(3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c*arctan(
c/x))*b^2
```

Giac [F]

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx = \int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^2 x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^2*x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.15

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{a^2\,x^4}{4}-\frac{b^2\,c^4\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{4}-\frac{b^2\,c^4\,\ln\left(c^2+x^2\right)}{3}+\frac{b^2\,x^4\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{4}+\frac{b^2\,c^2\,x^2}{12}+\frac{b^2\,c\,x^3\,\mathrm{atan}\left(\frac{c}{x}\right)}{6}-\frac{b^2\,c^3\,x\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{a\,b\,c\,x^3}{6}-\frac{a\,b\,c^3\,x}{2}-\frac{a\,b\,c^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{a\,b\,x^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}$$

input
```
int(x^3*(a + b*atan(c/x))^2,x)
```

output
```
(a^2*x^4)/4 - (b^2*c^4*atan(c/x)^2)/4 - (b^2*c^4*log(c^2 + x^2))/3 + (b^2*
x^4*atan(c/x)^2)/4 + (b^2*c^2*x^2)/12 + (b^2*c*x^3*atan(c/x))/6 - (b^2*c^3
*x*atan(c/x))/2 + (a*b*c*x^3)/6 - (a*b*c^3*x)/2 - (a*b*c^4*atan(c/x))/2 +
(a*b*x^4*atan(c/x))/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.15

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=-\frac{atan\left(\frac{c}{x}\right)^2 b^2c^4}{4}+\frac{atan\left(\frac{c}{x}\right)^2 b^2x^4}{4}-\frac{atan\left(\frac{c}{x}\right)ab\,c^4}{2}+\frac{atan\left(\frac{c}{x}\right)ab\,x^4}{2}-\frac{atan\left(\frac{c}{x}\right)b^2c^3x}{2}+\frac{atan\left(\frac{c}{x}\right)b^2c\,x^3}{6}-\frac{\log(c^2+x^2)\,b^2c^4}{3}+\frac{a^2x^4}{4}-\frac{ab\,c^3x}{2}+\frac{abc\,x^3}{6}+\frac{b^2c^2x^2}{12}$$

input
```
int(x^3*(a+b*atan(c/x))^2,x)
```

output
```
( - 3*atan(c/x)**2*b**2*c**4 + 3*atan(c/x)**2*b**2*x**4 - 6*atan(c/x)*a*b*
c**4 + 6*atan(c/x)*a*b*x**4 - 6*atan(c/x)*b**2*c**3*x + 2*atan(c/x)*b**2*c
*x**3 - 4*log(c**2 + x**2)*b**2*c**4 + 3*a**2*x**4 - 6*a*b*c**3*x + 2*a*b*
c*x**3 + b**2*c**2*x**2)/12
```

3.141 $\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx$

Optimal result 1046
Mathematica [A] (verified) 1047
Rubi [A] (verified) 1047
Maple [B] (verified) 1050
Fricas [F] 1051
Sympy [F] 1052
Maxima [F] 1052
Giac [F] 1052
Mupad [F(-1)] 1053
Reduce [F] 1053

Optimal result

Integrand size = 16, antiderivative size = 152

$$\begin{aligned}\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx &= \frac{1}{3}b^2c^2x+\frac{1}{3}b^2c^3\cot^{-1}\left(\frac{x}{c}\right)+\frac{1}{3}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\\&\quad-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\&\quad+\frac{2}{3}bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\\&\quad-\frac{1}{3}ib^2c^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right)\end{aligned}$$

output
```
1/3*b^2*c^2*x+1/3*b^2*c^3*arccot(x/c)+1/3*b*c*x^2*(a+b*arccot(x/c))-1/3*I*
c^3*(a+b*arccot(x/c))^2+1/3*x^3*(a+b*arccot(x/c))^2+2/3*b*c^3*(a+b*arccot(
x/c))*ln(2-2/(1-I*c/x))-1/3*I*b^2*c^3*polylog(2,-1+2/(1-I*c/x))
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{1}{3}\left(b^2c^2x+abcx^2+a^2x^3+b^2\left(-ic^3+x^3\right)\arctan\left(\frac{c}{x}\right)^2+b\arctan\left(\frac{c}{x}\right)\left(2ax^3+bc\left(c^2+x^2\right)+2bc^3\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)-abc^3\log\left(1+\frac{c^2}{x^2}\right)+2abc^3\log\left(\frac{c}{x}\right)-ib^2c^3\operatorname{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$

input
```
Integrate[x^2*(a + b*ArcTan[c/x])^2,x]
```

output
```
(b^2*c^2*x + a*b*c*x^2 + a^2*x^3 + b^2*((-I)*c^3 + x^3)*ArcTan[c/x]^2 + b*
ArcTan[c/x]*(2*a*x^3 + b*c*(c^2 + x^2) + 2*b*c^3*Log[1 - E^((2*I)*ArcTan[c
/x])]) - a*b*c^3*Log[1 + c^2/x^2] + 2*a*b*c^3*Log[c/x] - I*b^2*c^3*PolyLog
[2, E^((2*I)*ArcTan[c/x])])/3
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5363, 5361, 5453, 5361, 264, 216, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$$

$$\Big\downarrow \text{5363}$$

$$-\int x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,d\frac{1}{x}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\int\frac{x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}$$

$\downarrow$ 5453

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\left(\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)d\frac{1}{x}-c^2\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)$$

$\downarrow$ 5361

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\int\frac{x^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$\downarrow$ 264

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\left(c^2\left(-\int\frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-x\right)-\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$\downarrow$ 216

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(-c\arctan\left(\frac{c}{x}\right)-x\right)\right)$$

$\downarrow$ 5459

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\left(-\left(c^2\left(i\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c}{x}+i}d\frac{1}{x}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}\right)\right)-\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(-c\arctan\right.\right.$$

$\downarrow$ 5403

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\left(-\left(c^2\left(i\left(ibc\int\frac{\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-i\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}\right)\right.\right.$$

$\downarrow$ 2897

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{2}b\operatorname{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right)\right)\right)-\frac{i(a+b\arctan\left(\frac{c}{x}\right))}{2b}\right.\right.$$

input
```
Int[x^2*(a + b*ArcTan[c/x])^2,x]
```

output
```
(x^3*(a + b*ArcTan[c/x])^2)/3 - (2*b*c*(-1/2*(x^2*(a + b*ArcTan[c/x])) + (
b*c*(-x - c*ArcTan[c/x]))/2 - c^2*(((-1/2*I)*(a + b*ArcTan[c/x])^2)/b + I*
((-I)*(a + b*ArcTan[c/x])*Log[2 - 2/(1 - (I*c)/x)] - (b*PolyLog[2, -1 + 2/
(1 - (I*c)/x)])/2))))/3
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(134) = 268$.

Time = 0.56 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.36

method	result
derivativedivides	$-c^3\left(-\frac{a^2x^3}{3c^3}+b^2\left(-\frac{x^3\arctan\left(\frac{c}{x}\right)^2}{3c^3}+\frac{\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{x^2\arctan\left(\frac{c}{x}\right)}{3c^2}-\frac{2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)}{3}+\frac{i(}{}\right.\right.$
default	$-c^3\left(-\frac{a^2x^3}{3c^3}+b^2\left(-\frac{x^3\arctan\left(\frac{c}{x}\right)^2}{3c^3}+\frac{\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{x^2\arctan\left(\frac{c}{x}\right)}{3c^2}-\frac{2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)}{3}+\frac{i(}{}\right.\right.$
parts	$\frac{a^2x^3}{3}+\frac{x^3b^2\arctan\left(\frac{c}{x}\right)^2}{3}-\frac{c^3b^2\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{3}+\frac{c\,b^2x^2\arctan\left(\frac{c}{x}\right)}{3}+\frac{2c^3b^2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)}{3}-\frac{ic^3b^2}{}$
risch	Expression too large to display

input
```
int(x^2*(a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
```

output
```
-c^3*(-1/3*a^2/c^3*x^3+b^2*(-1/3/c^3*x^3*arctan(c/x)^2+1/3*arctan(c/x)*ln(
1+c^2/x^2)-1/3/c^2*x^2*arctan(c/x)-2/3*ln(c/x)*arctan(c/x)+1/6*I*(ln(c/x-I
)*ln(1+c^2/x^2)-1/2*ln(c/x-I)^2-dilog(-1/2*I*(c/x+I))-ln(c/x-I)*ln(-1/2*I*
(c/x+I)))-1/6*I*(ln(c/x+I)*ln(1+c^2/x^2)-1/2*ln(c/x+I)^2-dilog(1/2*I*(c/x-
I))-ln(c/x+I)*ln(1/2*I*(c/x-I)))-1/3*arctan(c/x)-1/3*x/c-1/3*I*ln(c/x)*ln(
1+I*c/x)+1/3*I*ln(c/x)*ln(1-I*c/x)-1/3*I*dilog(1+I*c/x)+1/3*I*dilog(1-I*c/
x))+2*a*b*(-1/3/c^3*x^3*arctan(c/x)+1/6*ln(1+c^2/x^2)-1/6/c^2*x^2-1/3*ln(c
/x)))
```

Fricas [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arctan(c/x)^2 + 2*a*b*x^2*arctan(c/x) + a^2*x^2, x)
```

Sympy [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int x^2\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2\,dx$$

input
```
integrate(x**2*(a+b*atan(c/x))**2,x)
```

output
```
Integral(x**2*(a + b*atan(c/x))**2, x)
```

Maxima [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/3*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*a*b +
 1/48*(4*x^3*arctan2(c, x)^2 - x^3*log(c^2 + x^2)^2 + 48*integrate(1/48*(3
6*c^2*x^2*arctan2(c, x)^2 + 36*x^4*arctan2(c, x)^2 + 8*c*x^3*arctan2(c, x)
 + 4*x^4*log(c^2 + x^2) + 3*(c^2*x^2 + x^4)*log(c^2 + x^2)^2)/(c^2 + x^2),
 x))*b^2
```

Giac [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int x^2\,\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2\,dx$$

input
```
int(x^2*(a + b*atan(c/x))^2,x)
```

output
```
int(x^2*(a + b*atan(c/x))^2, x)
```

Reduce [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{atan\left(\frac{c}{x}\right)^2b^2c^2x}{3}+\frac{atan\left(\frac{c}{x}\right)^2b^2x^3}{3}+\frac{2atan\left(\frac{c}{x}\right)ab\,x^3}{3}+\frac{atan\left(\frac{c}{x}\right)b^2c^3}{3}+\frac{atan\left(\frac{c}{x}\right)b^2c\,x^2}{3}-\frac{\left(\int atan\left(\frac{c}{x}\right)^2dx\right)b^2c^2}{3}-\frac{\log\left(c^2+x^2\right)ab\,c^3}{3}+\frac{a^2x^3}{3}+\frac{abc\,x^2}{3}+\frac{b^2c^2x}{3}$$

input
```
int(x^2*(a+b*atan(c/x))^2,x)
```

output
```
(atan(c/x)**2*b**2*c**2*x + atan(c/x)**2*b**2*x**3 + 2*atan(c/x)*a*b*x**3
+ atan(c/x)*b**2*c**3 + atan(c/x)*b**2*c*x**2 - int(atan(c/x)**2,x)*b**2*c
**2 - log(c**2 + x**2)*a*b*c**3 + a**2*x**3 + a*b*c*x**2 + b**2*c**2*x)/3
```

3.142 $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal result . . . 1054
Mathematica [A] (verified) . . . 1054
Rubi [A] (verified) . . . 1055
Maple [A] (verified) . . . 1058
Fricas [A] (verification not implemented) . . . 1058
Sympy [A] (verification not implemented) . . . 1059
Maxima [A] (verification not implemented) . . . 1059
Giac [F] . . . 1060
Mupad [B] (verification not implemented) . . . 1060
Reduce [B] (verification not implemented) . . . 1060

Optimal result

Integrand size = 14, antiderivative size = 82

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx = bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}b^2c^2\log\left(1+\frac{c^2}{x^2}\right)+b^2c^2\log(x)$$

output
```
b*c*x*(a+b*arccot(x/c))+1/2*c^2*(a+b*arccot(x/c))^2+1/2*x^2*(a+b*arccot(x/
c))^2+1/2*b^2*c^2*ln(1+c^2/x^2)+b^2*c^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx = \frac{1}{2}\left(ax(2bc+ax)+2b\left(bcx+a\left(c^2+x^2\right)\right)\arctan\left(\frac{c}{x}\right)+b^2\left(c^2+x^2\right)\arctan\left(\frac{c}{x}\right)^2+b^2c^2\log\left(c^2+x^2\right)\right)$$

input
```
Integrate[x*(a + b*ArcTan[c/x])^2,x]
```

output
```
(a*x*(2*b*c + a*x) + 2*b*(b*c*x + a*(c^2 + x^2))*ArcTan[c/x] + b^2*(c^2 +
x^2)*ArcTan[c/x]^2 + b^2*c^2*Log[c^2 + x^2])/2
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5363, 5361, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2dx$$

$$\Big\downarrow \text{ 5363}$$

$$-\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-bc\int \frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-bc\left(\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)d\frac{1}{x}-c^2\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-bc\left(c^2\left(-\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\int \frac{x}{\frac{c^2}{x^2}+1}d\frac{1}{x}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-bc\left(c^2\left(-\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\int \frac{x}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-bc\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\left(\int xd\frac{1}{x^2}-c^2\int\frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}\right)-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-bc\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-c^2\int\frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}\right)-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-bc\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\log\left(\frac{c^2}{x^2}+1\right)\right)\right)$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-bc\left(-\frac{c\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\log\left(\frac{c^2}{x^2}+1\right)\right)\right)$$

input
```
Int[x*(a + b*ArcTan[c/x])^2,x]
```

output
```
(x^2*(a + b*ArcTan[c/x])^2)/2 - b*c*(-(x*(a + b*ArcTan[c/x])) - (c*(a + b*
ArcTan[c/x])^2)/(2*b) + (b*c*(-Log[1 + c^2/x^2] + Log[x^(-2)]))/2)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.28

method	result
parts	$\frac{a^2x^2}{2}-b^2c^2\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^2}{2c^2}-\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{x\arctan\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)+\arctan\left(\frac{c}{x}\right)$
parallelrisch	$\frac{\arctan\left(\frac{c}{x}\right)^2b^2x^2}{2}+\frac{\arctan\left(\frac{c}{x}\right)^2b^2c^2}{2}+\frac{b^2c^2\ln(c^2+x^2)}{2}+\arctan\left(\frac{c}{x}\right)ab\,x^2+x\arctan\left(\frac{c}{x}\right)b^2c+\text{arc}$
derivativedivides	$-c^2\left(-\frac{a^2x^2}{2c^2}+b^2\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^2}{2c^2}-\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{x\arctan\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)-\frac{ab\,x^2\,a}{}\right.$
default	$-c^2\left(-\frac{a^2x^2}{2c^2}+b^2\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^2}{2c^2}-\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{x\arctan\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)-\frac{ab\,x^2\,a}{}\right.$
risch	Expression too large to display

input
```
int(x*(a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2-b^2*c^2*(-1/2/c^2*x^2*arctan(c/x)^2-1/2*arctan(c/x)^2-1/c*x*ar
ctan(c/x)-1/2*ln(1+c^2/x^2)+ln(c/x))+arctan(c/x)*a*b*x^2-a*b*c^2*arctan(x/
c)+a*b*c*x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=-abc^2\arctan\left(\frac{x}{c}\right)+\frac{1}{2}b^2c^2\log\left(c^2+x^2\right)+abcx+\frac{1}{2}a^2x^2+\frac{1}{2}\left(b^2c^2+b^2x^2\right)\arctan\left(\frac{c}{x}\right)^2+\left(b^2cx+abx^2\right)\arctan\left(\frac{c}{x}\right)$$

input
```
integrate(x*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

output
```
-a*b*c^2*arctan(x/c) + 1/2*b^2*c^2*log(c^2 + x^2) + a*b*c*x + 1/2*a^2*x^2
+ 1/2*(b^2*c^2 + b^2*x^2)*arctan(c/x)^2 + (b^2*c*x + a*b*x^2)*arctan(c/x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{a^2x^2}{2}+abc^2\operatorname{atan}\left(\frac{c}{x}\right)+abcx+abx^2\operatorname{atan}\left(\frac{c}{x}\right)+\frac{b^2c^2\log\left(c^2+x^2\right)}{2}+\frac{b^2c^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2}+b^2cx\operatorname{atan}\left(\frac{c}{x}\right)+\frac{b^2x^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2}$$

input
```
integrate(x*(a+b*atan(c/x))**2,x)
```

output
```
a**2*x**2/2 + a*b*c**2*atan(c/x) + a*b*c*x + a*b*x**2*atan(c/x) + b**2*c**
2*log(c**2 + x**2)/2 + b**2*c**2*atan(c/x)**2/2 + b**2*c*x*atan(c/x) + b**
2*x**2*atan(c/x)**2/2
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{1}{2}b^2x^2\arctan\left(\frac{c}{x}\right)^2+\frac{1}{2}a^2x^2+\left(x^2\arctan\left(\frac{c}{x}\right)-\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\right)ab-\frac{1}{2}\left(\left(\arctan\left(\frac{x}{c}\right)^2-\log\left(c^2+x^2\right)\right)c^2+2\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\arctan\left(\frac{c}{x}\right)\right)b^2$$

input
```
integrate(x*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*arctan(c/x)^2 + 1/2*a^2*x^2 + (x^2*arctan(c/x) - (c*arctan(x/c
) - x)*c)*a*b - 1/2*((arctan(x/c)^2 - log(c^2 + x^2))*c^2 + 2*(c*arctan(x/
c) - x)*c*arctan(c/x))*b^2
```

Giac [F]

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*arctan(c/x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^2*x, x)
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.20

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{a^2x^2}{2}+\frac{b^2c^2\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{2}+\frac{b^2c^2\ln\left(c^2+x^2\right)}{2}+\frac{b^2x^2\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{2}+a\,b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)+a\,b\,x^2\,\mathrm{atan}\left(\frac{c}{x}\right)+b^2\,c\,x\,\mathrm{atan}\left(\frac{c}{x}\right)+a\,b\,c\,x$$

input
```
int(x*(a + b*atan(c/x))^2,x)
```

output
```
(a^2*x^2)/2 + (b^2*c^2*atan(c/x)^2)/2 + (b^2*c^2*log(c^2 + x^2))/2 + (b^2*
x^2*atan(c/x)^2)/2 + a*b*c^2*atan(c/x) + a*b*x^2*atan(c/x) + b^2*c*x*atan(
c/x) + a*b*c*x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.20

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{atan\left(\frac{c}{x}\right)^2b^2c^2}{2}+\frac{atan\left(\frac{c}{x}\right)^2b^2x^2}{2}+atan\left(\frac{c}{x}\right)ab\,c^2+atan\left(\frac{c}{x}\right)ab\,x^2+atan\left(\frac{c}{x}\right)b^2cx+\frac{\log\left(c^2+x^2\right)b^2c^2}{2}+\frac{a^2x^2}{2}+abcx$$

input
```
int(x*(a+b*atan(c/x))^2,x)
```

output
```
(atan(c/x)**2*b**2*c**2 + atan(c/x)**2*b**2*x**2 + 2*atan(c/x)*a*b*c**2 +
2*atan(c/x)*a*b*x**2 + 2*atan(c/x)*b**2*c*x + log(c**2 + x**2)*b**2*c**2 +
 a**2*x**2 + 2*a*b*c*x)/2
```

3.143 $\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal result . . . 1062
Mathematica [A] (verified) . . . 1062
Rubi [A] (verified) . . . 1063
Maple [B] (verified) . . . 1066
Fricas [F] . . . 1066
Sympy [F] . . . 1067
Maxima [F] . . . 1067
Giac [F] . . . 1068
Mupad [F(-1)] . . . 1068
Reduce [F] . . . 1068

Optimal result

Integrand size = 12, antiderivative size = 83

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx = ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 - 2bc\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2c}{c+ix}\right) + ib^2c\,\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)$$

output
```
I*c*(a+b*arccot(x/c))^2+x*(a+b*arccot(x/c))^2-2*b*c*(a+b*arccot(x/c))*ln(2
*c/(c+I*x))+I*b^2*c*polylog(2,1-2*c/(c+I*x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx = b^2(ic+x)\arctan\left(\frac{c}{x}\right)^2 + 2b\arctan\left(\frac{c}{x}\right)\left(ax-bc\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right) + a\left(ax+bc\log\left(1+\frac{c^2}{x^2}\right)-2bc\log\left(\frac{c}{x}\right)\right) + ib^2c\,\text{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)$$

input
```
Integrate[(a + b*ArcTan[c/x])^2,x]
```

output
```
b^2*(I*c + x)*ArcTan[c/x]^2 + 2*b*ArcTan[c/x]*(a*x - b*c*Log[1 - E^((2*I)*
ArcTan[c/x])]) + a*(a*x + b*c*Log[1 + c^2/x^2] - 2*b*c*Log[c/x]) + I*b^2*c
*PolyLog[2, E^((2*I)*ArcTan[c/x])]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5349, 5346, 27, 5456, 27, 5380, 27, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2dx$$

$$\Big\downarrow \text{5349}$$

$$\int \left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2dx$$

$$\Big\downarrow \text{5346}$$

$$\frac{2b\int \frac{c^2x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c^2+x^2}dx}{c}+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2$$

$$\Big\downarrow \text{27}$$

$$2bc\int \frac{x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c^2+x^2}dx+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2$$

$$\Big\downarrow \text{5456}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+2bc\left(\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}-\frac{\int \frac{c\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{ic-x}dx}{c}\right)$$

$$\Big\downarrow \text{27}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+2bc\left(\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}-\int \frac{a+b\cot^{-1}\left(\frac{x}{c}\right)}{ic-x}dx\right)$$

$$\Big\downarrow \text{ 5380}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+2bc\left(-\frac{b\int \frac{c^2\log\left(\frac{2c}{c+ix}\right)}{c^2+x^2}dx}{c}+\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}-\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\right)$$

$$\Big\downarrow \text{ 27}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+2bc\left(-bc\int \frac{\log\left(\frac{2c}{c+ix}\right)}{c^2+x^2}dx+\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}-\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\right)$$

$$\Big\downarrow \text{ 2849}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+2bc\left(ibc\int \frac{\log\left(\frac{2c}{c+ix}\right)}{1-\frac{2c}{c+ix}}d\frac{1}{c+ix}+\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}-\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\right)$$

$$\Big\downarrow \text{ 2752}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+2bc\left(\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}-\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)\right)$$

input
```
Int[(a + b*ArcTan[c/x])^2,x]
```

output
```
x*(a + b*ArcCot[x/c])^2 + 2*b*c*(((I/2)*(a + b*ArcCot[x/c])^2)/b - (a + b*
ArcCot[x/c])*Log[(2*c)/(c + I*x)] + (I/2)*b*PolyLog[2, 1 - (2*c)/(c + I*x)
])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5349
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[(a + b*A
rcCot[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && ILtQ[n, 0]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(79) = 158$.

Time = 0.28 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.71

method	result
parts	$a^2x - b^2c\left(-\frac{x\arctan\left(\frac{c}{x}\right)^2}{c} - \arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right) + 2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right) - \frac{i\left(\ln\left(\frac{c}{x}-i\right)\ln\left(\right.\right.}{}\right.$
derivativedivides	$-c\left(-\frac{a^2x}{c} + b^2\left(-\frac{x\arctan\left(\frac{c}{x}\right)^2}{c} - \arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right) + 2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right) - \frac{i\left(\ln\left(\frac{c}{x}\right.\right.}{}\right.\right.$
default	$-c\left(-\frac{a^2x}{c} + b^2\left(-\frac{x\arctan\left(\frac{c}{x}\right)^2}{c} - \arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right) + 2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right) - \frac{i\left(\ln\left(\frac{c}{x}\right.\right.}{}\right.\right.$
risch	Expression too large to display

input
```
int((a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x-b^2*c*(-1/c*x*arctan(c/x)^2-arctan(c/x)*ln(1+c^2/x^2)+2*ln(c/x)*arct
an(c/x)-1/2*I*(ln(c/x-I)*ln(1+c^2/x^2)-1/2*ln(c/x-I)^2-dilog(-1/2*I*(c/x+I
))-ln(c/x-I)*ln(-1/2*I*(c/x+I)))+1/2*I*(ln(c/x+I)*ln(1+c^2/x^2)-1/2*ln(c/x
+I)^2-dilog(1/2*I*(c/x-I))-ln(c/x+I)*ln(1/2*I*(c/x-I)))+I*ln(c/x)*ln(1+I*c
/x)-I*ln(c/x)*ln(1-I*c/x)+I*dilog(1+I*c/x)-I*dilog(1-I*c/x))-2*a*b*c*(-1/c
*x*arctan(c/x)-1/2*ln(1+c^2/x^2)+ln(c/x))
```

Fricas [F]

$$\int\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx = \int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arctan(c/x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2, x)
```

Sympy [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx=\int \left(a+b\operatorname{atan}\left(\frac{c}{x}\right)\right)^2 dx$$

input
```
integrate((a+b*atan(c/x))**2,x)
```

output
```
Integral((a + b*atan(c/x))**2, x)
```

Maxima [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx=\int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^2 dx$$

input
```
integrate((a+b*arctan(c/x))^2,x, algorithm="maxima")
```

output
```
(2*x*arctan(c/x) + c*log(c^2 + x^2))*a*b + 1/16*(12*c*arctan(c/x)^2*arctan
(x/c) + 4*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c)*c^2 + 4*x*arct
an2(c, x)^2 + 16*c^2*integrate(1/16*log(c^2 + x^2)^2/(c^2 + x^2), x) - x*l
og(c^2 + x^2)^2 + 128*c*integrate(1/16*x*arctan(c/x)/(c^2 + x^2), x) + 192
*integrate(1/16*x^2*arctan(c/x)^2/(c^2 + x^2), x) + 16*integrate(1/16*x^2*
log(c^2 + x^2)^2/(c^2 + x^2), x) + 64*integrate(1/16*x^2*log(c^2 + x^2)/(c
^2 + x^2), x))*b^2 + a^2*x
```

Giac [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx = \int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^2 dx$$

input
```
integrate((a+b*arctan(c/x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx = \int \left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2 dx$$

input
```
int((a + b*atan(c/x))^2,x)
```

output
```
int((a + b*atan(c/x))^2, x)
```

Reduce [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx = 2atan\left(\frac{c}{x}\right)abx + \left(\int atan\left(\frac{c}{x}\right)^2 dx\right)b^2 + \log\left(c^2+x^2\right)abc + a^2x$$

input
```
int((a+b*atan(c/x))^2,x)
```

output
```
2*atan(c/x)*a*b*x + int(atan(c/x)**2,x)*b**2 + log(c**2 + x**2)*a*b*c + a*
*2*x
```

3.144 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx$

Optimal result . . . 1069
Mathematica [A] (verified) . . . 1070
Rubi [A] (verified) . . . 1070
Maple [C] (warning: unable to verify) . . . 1073
Fricas [F] . . . 1074
Sympy [F] . . . 1074
Maxima [F] . . . 1074
Giac [F] . . . 1075
Mupad [F(-1)] . . . 1075
Reduce [F] . . . 1075

Optimal result

Integrand size = 16, antiderivative size = 148

$$\begin{aligned}\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx = &-2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\\&+ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)\\&-ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{ic}{x}}\right)\\&+\frac{1}{2}b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right)\\&-\frac{1}{2}b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+\frac{ic}{x}}\right)\end{aligned}$$

output
```
2*(a+b*arccot(x/c))^2*arctanh(-1+2/(1+I*c/x))+I*b*(a+b*arccot(x/c))*polylo
g(2,1-2/(1+I*c/x))-I*b*(a+b*arccot(x/c))*polylog(2,-1+2/(1+I*c/x))+1/2*b^2
*polylog(3,1-2/(1+I*c/x))-1/2*b^2*polylog(3,-1+2/(1+I*c/x))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.37

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx = a^2\log(x)-iab\left(\text{PolyLog}\left(2,-\frac{ic}{x}\right)-\text{PolyLog}\left(2,\frac{ic}{x}\right)\right)+b^2\left(\frac{i\pi^3}{24}-\frac{2}{3}i\arctan\left(\frac{c}{x}\right)^3-\arctan\left(\frac{c}{x}\right)^2\log\left(1-e^{-2i\arctan\left(\frac{c}{x}\right)}\right)+\arctan\left(\frac{c}{x}\right)^2\log\left(1+e^{2i\arctan\left(\frac{c}{x}\right)}\right)-i\arctan\left(\frac{c}{x}\right)\text{PolyLog}\left(2,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)-i\arctan\left(\frac{c}{x}\right)\text{PolyLog}\left(2,-e^{2i\arctan\left(\frac{c}{x}\right)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)+\frac{1}{2}\text{PolyLog}\left(3,-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c/x])^2/x,x]
```

output
```
a^2*Log[x] - I*a*b*(PolyLog[2, ((-I)*c)/x] - PolyLog[2, (I*c)/x]) + b^2*((
I/24)*Pi^3 - ((2*I)/3)*ArcTan[c/x]^3 - ArcTan[c/x]^2*Log[1 - E^((-2*I)*Arc
Tan[c/x])] + ArcTan[c/x]^2*Log[1 + E^((2*I)*ArcTan[c/x])] - I*ArcTan[c/x]*
PolyLog[2, E^((-2*I)*ArcTan[c/x])] - I*ArcTan[c/x]*PolyLog[2, -E^((2*I)*Ar
cTan[c/x])] - PolyLog[3, E^((-2*I)*ArcTan[c/x])]/2 + PolyLog[3, -E^((2*I)*
ArcTan[c/x])]/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5359, 5357, 5523, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx$$

$\downarrow$ 5359

$$-\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 d\frac{1}{x}$$

$\downarrow$ 5357

$$4bc\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)\text{arctanh}\left(1-\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2$$

$\downarrow$ 5523

$$4bc\left(\frac{1}{2}\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)\log\left(2-\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{1}{2}\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)\log\left(\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2$$

$\downarrow$ 5529

$$4bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}-\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}\left(\frac{1}{2}ib\int \frac{\text{PolyLo}}{}\right.\right.$$
$$2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2$$

$\downarrow$ 7164

$$4bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}+\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{\frac{ic}{x}+1}\right)}{4c}\right)+\frac{1}{2}\left(-\frac{i\,\text{PolyLog}\left(2,\frac{2}{\frac{ic}{x}+1}-\right.}{}\right.\right.$$
$$2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2$$

input
```
Int[(a + b*ArcTan[c/x])^2/x,x]
```

output
```
-2*(a + b*ArcTan[c/x])^2*ArcTanh[1 - 2/(1 + (I*c)/x)] + 4*b*c*((((I/2)*(a
+ b*ArcTan[c/x])*PolyLog[2, 1 - 2/(1 + (I*c)/x)])/c + (b*PolyLog[3, 1 - 2/
(1 + (I*c)/x)])/(4*c))/2 + (((-1/2*I)*(a + b*ArcTan[c/x])*PolyLog[2, -1 +
2/(1 + (I*c)/x)])/c - (b*PolyLog[3, -1 + 2/(1 + (I*c)/x)])/(4*c))/2)
```

Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.97 (sec) , antiderivative size = 1103, normalized size of antiderivative = 7.45

method	result	size
parts	Expression too large to display	1103
derivativedivides	Expression too large to display	1106
default	Expression too large to display	1106

input
```
int((a+b*arctan(c/x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(x)+b^2*(-ln(c/x)*arctan(c/x)^2-I*arctan(c/x)*polylog(2,-(1+I*c/x)^2
/(1+c^2/x^2))+1/2*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))+arctan(c/x)^2*ln((1+
I*c/x)^2/(1+c^2/x^2)-1)-arctan(c/x)^2*ln(1+(1+I*c/x)/(1+c^2/x^2)^(1/2))+2*
I*arctan(c/x)*polylog(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-2*polylog(3,-(1+I*c/
x)/(1+c^2/x^2)^(1/2))-arctan(c/x)^2*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))+2*I*
arctan(c/x)*polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))-2*polylog(3,(1+I*c/x)/(
1+c^2/x^2)^(1/2))-1/2*I*Pi*(csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I/(1+
(1+I*c/x)^2/(1+c^2/x^2)))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^
2/(1+c^2/x^2)))-csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I*((1+I*c/x)^2/(1
+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2-csgn(I/(1+(1+I*c/x)^2/(1+c^2/x
^2)))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2+cs
gn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^3-csgn(I*((1
+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(((1+I*c/x)^2/(1
+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2+csgn(I*((1+I*c/x)^2/(1+c^2/x^2
)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I
*c/x)^2/(1+c^2/x^2)))+csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c
^2/x^2)))^3-csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^
2+1)*arctan(c/x)^2)+2*a*b*(-ln(c/x)*arctan(c/x)-1/2*I*ln(c/x)*ln(1+I*c/x)+
1/2*I*ln(c/x)*ln(1-I*c/x)-1/2*I*dilog(1+I*c/x)+1/2*I*dilog(1-I*c/x))
```

Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$$

input
```
integrate((a+b*atan(c/x))**2/x,x)
```

output
```
Integral((a + b*atan(c/x))**2/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + 1/16*integrate((12*b^2*arctan2(c, x)^2 + b^2*log(c^2 + x^2)^2
 + 32*a*b*arctan2(c, x))/x, x)
```

Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$$

input
```
int((a + b*atan(c/x))^2/x,x)
```

output
```
int((a + b*atan(c/x))^2/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx=2\left(\int \frac{atan\left(\frac{c}{x}\right)}{x}dx\right)ab+\left(\int \frac{atan\left(\frac{c}{x}\right)^2}{x}dx\right)b^2+\log(x)\,a^2$$

input
```
int((a+b*atan(c/x))^2/x,x)
```

output
```
2*int(atan(c/x)/x,x)*a*b + int(atan(c/x)**2/x,x)*b**2 + log(x)*a**2
```

3.145 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$

Optimal result . . . 1076
Mathematica [A] (verified) . . . 1076
Rubi [A] (verified) . . . 1077
Maple [A] (verified) . . . 1080
Fricas [F] . . . 1080
Sympy [F] . . . 1081
Maxima [F] . . . 1081
Giac [F] . . . 1081
Mupad [F(-1)] . . . 1082
Reduce [F] . . . 1082

Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = -\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{x} - \frac{2b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c} - \frac{ib^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)}{c}$$

output
```
-I*(a+b*arccot(x/c))^2/c-(a+b*arccot(x/c))^2/x-2*b*(a+b*arccot(x/c))*ln(2/
(1+I*c/x))/c-I*b^2*polylog(2,1-2/(1+I*c/x))/c
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = -\frac{b^2(c-ix)\arctan\left(\frac{c}{x}\right)^2+2b\arctan\left(\frac{c}{x}\right)\left(ac+bx\log\left(1+e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)+a\left(ac+2bx\log\left(\frac{1}{\sqrt{1+\frac{c^2}{x^2}}}\right)\right)}{cx}$$

input
```
Integrate[(a + b*ArcTan[c/x])^2/x^2,x]
```

output
```
-((b^2*(c - I*x)*ArcTan[c/x]^2 + 2*b*ArcTan[c/x]*(a*c + b*x*Log[1 + E^((2*
I)*ArcTan[c/x])]) + a*(a*c + 2*b*x*Log[1/Sqrt[1 + c^2/x^2]]) - I*b^2*x*Pol
yLog[2, -E^((2*I)*ArcTan[c/x])])/(c*x))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5363, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b \arctan\left(\frac{c}{x}\right)\right)^2}{x^2} \, dx$$

$$\Big\downarrow \; 5363$$

$$-\int \left(a+b \arctan\left(\frac{c}{x}\right)\right)^2 d\frac{1}{x}$$

$$\Big\downarrow \; 5345$$

$$2bc \int \frac{a+b \arctan\left(\frac{c}{x}\right)}{\left(\frac{c^2}{x^2}+1\right) x} d\frac{1}{x} - \frac{\left(a+b \arctan\left(\frac{c}{x}\right)\right)^2}{x}$$

$$\Big\downarrow \; 5455$$

$$-\frac{\left(a+b \arctan\left(\frac{c}{x}\right)\right)^2}{x} + 2bc \left(-\frac{\int \frac{a+b \arctan\left(\frac{c}{x}\right)}{i-\frac{c}{x}} d\frac{1}{x}}{c} - \frac{i\left(a+b \arctan\left(\frac{c}{x}\right)\right)^2}{2bc^2}\right)$$

$$\Big\downarrow \; 5379$$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}+2bc\left(-\frac{\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{c}-b\int\frac{\log\left(\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^2}\right)$$

$$\Big\downarrow \text{2849}$$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}+2bc\left(-\frac{\frac{ib\int\frac{\log\left(\frac{2}{\frac{ic}{x}+1}\right)}{1-\frac{2}{\frac{ic}{x}+1}}d\frac{1}{\frac{ic}{x}+1}}{c}+\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{c}}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^2}\right)$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}+2bc\left(-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{c}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)}{2c}}{c}\right)$$

input
```
Int[(a + b*ArcTan[c/x])^2/x^2,x]
```

output
```
-((a + b*ArcTan[c/x])^2/x) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c/x])^2)/(b*c^
2) - (((a + b*ArcTan[c/x])*Log[2/(1 + (I*c)/x)])/c + ((I/2)*b*PolyLog[2, 1
 - 2/(1 + (I*c)/x)])/c)/c)
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.48

method	result
derivativedivides	$-\frac{\frac{c\,a^{2}}{x}-i\arctan\left(\frac{c}{x}\right)^{2}b^{2}+\frac{\arctan\left(\frac{c}{x}\right)^{2}b^{2}c}{x}-i\,\mathrm{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^{2}}{1+\frac{c^{2}}{x^{2}}}\right)b^{2}+2\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{\left(1+\frac{ic}{x}\right)^{2}}{1+\frac{c^{2}}{x^{2}}}\right)b^{2}+\frac{2abc\arctan(}{x}}{c}$
default	$-\frac{\frac{c\,a^{2}}{x}-i\arctan\left(\frac{c}{x}\right)^{2}b^{2}+\frac{\arctan\left(\frac{c}{x}\right)^{2}b^{2}c}{x}-i\,\mathrm{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^{2}}{1+\frac{c^{2}}{x^{2}}}\right)b^{2}+2\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{\left(1+\frac{ic}{x}\right)^{2}}{1+\frac{c^{2}}{x^{2}}}\right)b^{2}+\frac{2abc\arctan(}{x}}{c}$
parts	$-\frac{a^{2}}{x}-\frac{b^{2}\arctan\left(\frac{c}{x}\right)^{2}}{x}+\frac{ib^{2}\arctan\left(\frac{c}{x}\right)^{2}}{c}-\frac{2b^{2}\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{\left(1+\frac{ic}{x}\right)^{2}}{1+\frac{c^{2}}{x^{2}}}\right)}{c}+\frac{ib^{2}\,\mathrm{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^{2}}{1+\frac{c^{2}}{x^{2}}}\right)}{c}-$

input
```
int((a+b*arctan(c/x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c*(c/x*a^2-I*arctan(c/x)^2*b^2+arctan(c/x)^2*b^2*c/x-I*polylog(2,-(1+I*
c/x)^2/(1+c^2/x^2))*b^2+2*arctan(c/x)*ln(1+(1+I*c/x)^2/(1+c^2/x^2))*b^2+2*
a*b*c/x*arctan(c/x)-a*b*ln(1+c^2/x^2))
```

Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^{2}}{x^{2}}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^{2}}{x^{2}}\,dx$$

input
```
integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*atan(c/x))**2/x**2,x)
```

output
```
Integral((a + b*atan(c/x))**2/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="maxima")
```

output
```
1/16*(4*(48*c^2*integrate(1/16*arctan(c/x)^2/(c^2*x^2 + x^4), x) + 4*c^2*i
ntegrate(1/16*log(c^2 + x^2)^2/(c^2*x^2 + x^4), x) + 3*arctan(c/x)^2*arcta
n(x/c)/c + 3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c - 32*c*integrat
e(1/16*x*arctan(c/x)/(c^2*x^2 + x^4), x) + 4*integrate(1/16*x^2*log(c^2 +
x^2)^2/(c^2*x^2 + x^4), x) - 16*integrate(1/16*x^2*log(c^2 + x^2)/(c^2*x^2
 + x^4), x))*x - 4*arctan2(c, x)^2 + log(c^2 + x^2)^2)*b^2/x - a*b*(2*c*ar
ctan(c/x)/x - log(c^2/x^2 + 1))/c - a^2/x
```

Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$$

input
```
int((a + b*atan(c/x))^2/x^2,x)
```

output
```
int((a + b*atan(c/x))^2/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$$
$$=\frac{-atan\left(\frac{c}{x}\right)^2b^2c-2atan\left(\frac{c}{x}\right)abc-2\left(\int \frac{atan\left(\frac{c}{x}\right)}{c^2x+x^3}dx\right)b^2c^2x+\log\left(c^2+x^2\right)abx-2\log(x)\,abx-a^2c}{cx}$$

input
```
int((a+b*atan(c/x))^2/x^2,x)
```

output
```
( - atan(c/x)**2*b**2*c - 2*atan(c/x)*a*b*c - 2*int(atan(c/x)/(c**2*x + x*
*3),x)*b**2*c**2*x + log(c**2 + x**2)*a*b*x - 2*log(x)*a*b*x - a**2*c)/(c*
x)
```

3.146 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$

Optimal result 1083
Mathematica [A] (verified) 1083
Rubi [A] (verified) 1084
Maple [A] (verified) 1086
Fricas [A] (verification not implemented) 1086
Sympy [A] (verification not implemented) 1087
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Giac [C] (verification not implemented) 1088
Mupad [B] (verification not implemented) 1088
Reduce [B] (verification not implemented) 1089

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = \frac{ab}{cx}+\frac{b^2\cot^{-1}\left(\frac{x}{c}\right)}{cx}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2} -\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2}-\frac{b^2\log\left(1+\frac{c^2}{x^2}\right)}{2c^2}$$

output
```
a*b/c/x+b^2*arccot(x/c)/c/x-1/2*(a+b*arccot(x/c))^2/c^2-1/2*(a+b*arccot(x/
c))^2/x^2-1/2*b^2*ln(1+c^2/x^2)/c^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = -\frac{a^2c^2-2abcx+2bc(ac-bx)\arctan\left(\frac{c}{x}\right)+b^2(c^2+x^2)\arctan\left(\frac{c}{x}\right)^2-2abx^2\arctan\left(\frac{x}{c}\right)-2b^2x^2\log(x)}{2c^2x^2}$$

input
```
Integrate[(a + b*ArcTan[c/x])^2/x^3,x]
```

output
```
-1/2*(a^2*c^2 - 2*a*b*c*x + 2*b*c*(a*c - b*x)*ArcTan[c/x] + b^2*(c^2 + x^2
)*ArcTan[c/x]^2 - 2*a*b*x^2*ArcTan[x/c] - 2*b^2*x^2*Log[x] + b^2*x^2*Log[c
^2 + x^2])/(c^2*x^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5363, 5361, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}dx$$

$$\Big\downarrow \text{5363}$$

$$-\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{5361}$$

$$bc\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\left(\frac{c^2}{x^2}+1\right)x^2}d\frac{1}{x}-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2x^2}$$

$$\Big\downarrow \text{5451}$$

$$bc\left(\frac{\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)d\frac{1}{x}}{c^2}-\frac{\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}}{c^2}\right)-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$bc\left(\frac{\frac{a}{x}+\frac{b\arctan\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(\frac{c^2}{x^2}+1\right)}{2c}}{c^2}-\frac{\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}}{c^2}\right)-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2x^2}$$

$$\Big\downarrow \text{5419}$$

$$bc\left(\frac{\frac{a}{x}+\frac{b\arctan\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(\frac{c^2}{x^2}+1\right)}{2c}}{c^2}-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^3}\right)-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2x^2}$$

input
```
Int[(a + b*ArcTan[c/x])^2/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c/x])^2/x^2 + b*c*(-1/2*(a + b*ArcTan[c/x])^2/(b*c^3) +
 (a/x + (b*ArcTan[c/x])/x - (b*Log[1 + c^2/x^2])/(2*c))/c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.23

method	result
parts	$-\frac{a^2}{2x^2}-\frac{b^2\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^2}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{c\arctan\left(\frac{c}{x}\right)}{x}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}\right)}{c^2}-\frac{ab\arctan\left(\frac{c}{x}\right)}{x^2}+\frac{ab}{cx}+\frac{ab\arctan\left(\frac{x}{c}\right)}{c^2}$
derivativedivides	$-\frac{\frac{a^2c^2}{2x^2}+b^2\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^2}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{c\arctan\left(\frac{c}{x}\right)}{x}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}\right)+2ab\left(\frac{c^2\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{c}{2x}+\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)}{c^2}$
default	$-\frac{\frac{a^2c^2}{2x^2}+b^2\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^2}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{c\arctan\left(\frac{c}{x}\right)}{x}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}\right)+2ab\left(\frac{c^2\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{c}{2x}+\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)}{c^2}$
parallelrisch	$\frac{-\arctan\left(\frac{c}{x}\right)^2b^2x^2-\arctan\left(\frac{c}{x}\right)^2b^2c^2+2b^2\ln(x)x^2-b^2\ln(c^2+x^2)x^2-2\arctan\left(\frac{c}{x}\right)ab\,x^2+2x\arctan\left(\frac{c}{x}\right)b^2c-2\arctan\left(\frac{c}{x}\right)\cdots}{2x^2c^2}$
risch	Expression too large to display

input
```
int((a+b*arctan(c/x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2-b^2/c^2*(1/2*c^2/x^2*arctan(c/x)^2+1/2*arctan(c/x)^2-c/x*arct
an(c/x)+1/2*ln(1+c^2/x^2))-a*b*arctan(c/x)/x^2+a*b/c/x+1/c^2*a*b*arctan(x/
c)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.30

$$\int\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$$
$$=\frac{2\,abx^2\arctan\left(\frac{x}{c}\right)-b^2x^2\log\left(c^2+x^2\right)+2\,b^2x^2\log\left(x\right)-a^2c^2+2\,abcx-\left(b^2c^2+b^2x^2\right)\arctan\left(\frac{c}{x}\right)^2-2\,(a\cdots}{2\,c^2x^2}$$

input
```
integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*x^2*arctan(x/c) - b^2*x^2*log(c^2 + x^2) + 2*b^2*x^2*log(x) - a
^2*c^2 + 2*a*b*c*x - (b^2*c^2 + b^2*x^2)*arctan(c/x)^2 - 2*(a*b*c^2 - b^2*
c*x)*arctan(c/x))/(c^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = \begin{cases} -\frac{a^2}{2x^2} - \frac{ab\,\mathrm{atan}\left(\frac{c}{x}\right)}{x^2} + \frac{ab}{cx} - \frac{ab\,\mathrm{atan}\left(\frac{c}{x}\right)}{c^2} - \frac{b^2\,\mathrm{atan}^2\left(\frac{c}{x}\right)}{2x^2} + \frac{b^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{cx} + \frac{b^2\log\left(x\right)}{c^2} - \frac{b^2\log\left(c^2+x^2\right)}{2c^2} - \frac{b^2\,\mathrm{atan}^2\left(\frac{c}{x}\right)}{2c^2} & \text{for } c \\ -\frac{a^2}{2x^2} & \text{othe} \end{cases}$$

input
```
integrate((a+b*atan(c/x))**2/x**3,x)
```

output
```
Piecewise((-a**2/(2*x**2) - a*b*atan(c/x)/x**2 + a*b/(c*x) - a*b*atan(c/x)
/c**2 - b**2*atan(c/x)**2/(2*x**2) + b**2*atan(c/x)/(c*x) + b**2*log(x)/c*
*2 - b**2*log(c**2 + x**2)/(2*c**2) - b**2*atan(c/x)**2/(2*c**2), Ne(c, 0)
), (-a**2/(2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.43

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = \left(c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3}+\frac{1}{c^2x}\right)-\frac{\arctan\left(\frac{c}{x}\right)}{x^2}\right)ab + \frac{1}{2}\left(2\,c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3}+\frac{1}{c^2x}\right)\arctan\left(\frac{c}{x}\right)+\frac{\arctan\left(\frac{x}{c}\right)^2-\log\left(c^2+x^2\right)+2\,\log\left(x\right)}{c^2}\right)b^2 - \frac{b^2\arctan\left(\frac{c}{x}\right)^2}{2\,x^2}-\frac{a^2}{2\,x^2}$$

input
```
integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="maxima")
```

output
```
(c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*a*b + 1/2*(2*c*(arctan
(x/c)/c^3 + 1/(c^2*x))*arctan(c/x) + (arctan(x/c)^2 - log(c^2 + x^2) + 2*l
og(x))/c^2)*b^2 - 1/2*b^2*arctan(c/x)^2/x^2 - 1/2*a^2/x^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.63

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = -\frac{b^2\arctan\left(\frac{c}{x}\right)^2+\frac{b^2c^2\arctan\left(\frac{c}{x}\right)^2}{x^2}+\frac{2abc^2\arctan\left(\frac{c}{x}\right)}{x^2}-\frac{2b^2c\arctan\left(\frac{c}{x}\right)}{x}+i\,ab\log\left(\frac{ic}{x}-1\right)+b^2\log\left(\frac{ic}{x}-1\right)-i\,a}{2c^2}$$

input
```
integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="giac")
```

output
```
-1/2*(b^2*arctan(c/x)^2 + b^2*c^2*arctan(c/x)^2/x^2 + 2*a*b*c^2*arctan(c/x
)/x^2 - 2*b^2*c*arctan(c/x)/x + I*a*b*log(I*c/x - 1) + b^2*log(I*c/x - 1)
- I*a*b*log(-I*c/x - 1) + b^2*log(-I*c/x - 1) + a^2*c^2/x^2 - 2*a*b*c/x)/c
^2
```

Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.70

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = \frac{b^2\,\ln(x)-\frac{b^2\,\ln(x+c\,1i)}{2}-\frac{b^2\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{2}+\frac{b^2\,\ln\left(\frac{1}{-x+c\,1i}\right)}{2}+\frac{a\,b\,\ln(x+c\,1i)\,1i}{2}-\frac{a\,b\,\ln(-x+c\,1i)\,1i}{2}}{c^2} - \frac{\frac{a^2\,c^2}{2}-x\,\left(c\,\mathrm{atan}\left(\frac{c}{x}\right)\,b^2+a\,c\,b\right)+\frac{b^2\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{2}+a\,b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{c^2\,x^2}$$

input
```
int((a + b*atan(c/x))^2/x^3,x)
```

output
```
(b^2*log(x) - (b^2*log(c*1i + x))/2 - (b^2*atan(c/x)^2)/2 + (b^2*log(1/(c*
1i - x)))/2 + (a*b*log(c*1i + x)*1i)/2 - (a*b*log(c*1i - x)*1i)/2)/c^2 - (
(a^2*c^2)/2 - x*(b^2*c*atan(c/x) + a*b*c) + (b^2*c^2*atan(c/x)^2)/2 + a*b*
c^2*atan(c/x))/(c^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.43

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$$
$$= \frac{-atan\left(\frac{c}{x}\right)^2 b^2c^2 - atan\left(\frac{c}{x}\right)^2 b^2x^2 - 2atan\left(\frac{c}{x}\right) ab\,c^2 - 2atan\left(\frac{c}{x}\right) ab\,x^2 + 2atan\left(\frac{c}{x}\right) b^2cx - \log(c^2+x^2)\,b^2x^2}{2c^2x^2}$$

input
```
int((a+b*atan(c/x))^2/x^3,x)
```

output
```
( - atan(c/x)**2*b**2*c**2 - atan(c/x)**2*b**2*x**2 - 2*atan(c/x)*a*b*c**2
 - 2*atan(c/x)*a*b*x**2 + 2*atan(c/x)*b**2*c*x - log(c**2 + x**2)*b**2*x**
2 + 2*log(x)*b**2*x**2 - a**2*c**2 + 2*a*b*c*x)/(2*c**2*x**2)
```

3.147 $\int x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^3 \, dx$

Optimal result 1090
Mathematica [A] (verified) 1091
Rubi [A] (verified) 1092
Maple [B] (verified) 1096
Fricas [F] 1097
Sympy [F] 1097
Maxima [F] 1097
Giac [F] 1098
Mupad [F(-1)] 1098
Reduce [F] 1099

Optimal result

Integrand size = 16, antiderivative size = 214

$$\begin{aligned}\int x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^3 \, dx = {} & \frac{1}{4}b^3c^3x+\frac{1}{4}b^3c^4\cot^{-1}\left(\frac{x}{c}\right)+\frac{1}{4}b^2c^2x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) \\ & -ibc^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{3}{4}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\ & +\frac{1}{4}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\ & -\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 \\ & +2b^2c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right) \\ & -ib^3c^4\operatorname{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right)\end{aligned}$$

output
```
1/4*b^3*c^3*x+1/4*b^3*c^4*arccot(x/c)+1/4*b^2*c^2*x^2*(a+b*arccot(x/c))-I*
b*c^4*(a+b*arccot(x/c))^2-3/4*b*c^3*x*(a+b*arccot(x/c))^2+1/4*b*c*x^3*(a+b
*arccot(x/c))^2-1/4*c^4*(a+b*arccot(x/c))^3+1/4*x^4*(a+b*arccot(x/c))^3+2*
b^2*c^4*(a+b*arccot(x/c))*ln(2-2/(1-I*c/x))-I*b^3*c^4*polylog(2,-1+2/(1-I*
c/x))
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.18

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\frac{1}{4}\Bigg(ab^2c^4-3a^2bc^3x+b^3c^3x+ab^2c^2x^2+a^2bcx^3+a^3x^4$$
$$+b^2\left(bc\left(-4ic^3-3c^2x+x^3\right)+3a\left(-c^4+x^4\right)\right)\arctan\left(\frac{c}{x}\right)^2$$
$$+b^3\left(-c^4+x^4\right)\arctan\left(\frac{c}{x}\right)^3$$
$$+b\arctan\left(\frac{c}{x}\right)\left(2abcx\left(-3c^2+x^2\right)+b^2c^2\left(c^2+x^2\right)\right.$$
$$\left.+3a^2\left(-c^4+x^4\right)+8b^2c^4\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$
$$+8ab^2c^4\log\left(\frac{c}{\sqrt{1+\frac{c^2}{x^2}}\,x}\right)$$
$$-4ib^3c^4\,\mathrm{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)\Bigg)$$

input
```
Integrate[x^3*(a + b*ArcTan[c/x])^3,x]
```

output
```
(a*b^2*c^4 - 3*a^2*b*c^3*x + b^3*c^3*x + a*b^2*c^2*x^2 + a^2*b*c*x^3 + a^3
*x^4 + b^2*(b*c*((-4*I)*c^3 - 3*c^2*x + x^3) + 3*a*(-c^4 + x^4))*ArcTan[c/
x]^2 + b^3*(-c^4 + x^4)*ArcTan[c/x]^3 + b*ArcTan[c/x]*(2*a*b*c*x*(-3*c^2 +
 x^2) + b^2*c^2*(c^2 + x^2) + 3*a^2*(-c^4 + x^4) + 8*b^2*c^4*Log[1 - E^((2
*I)*ArcTan[c/x])]) + 8*a*b^2*c^4*Log[c/(Sqrt[1 + c^2/x^2]*x)] - (4*I)*b^3*
c^4*PolyLog[2, E^((2*I)*ArcTan[c/x])])/4
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.41, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5363, 5361, 5453, 5361, 5453, 5361, 264, 216, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$$

$$\Big\downarrow \text{5363}$$

$$-\int x^5\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3d\frac{1}{x}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{4}bc\int \frac{x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{4}bc\left(\int x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}-c^2\int \frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{4}bc\left(-c^2\int \frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}+\frac{2}{3}bc\int \frac{x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\right)$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{4}bc\left(-c^2\left(\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}-c^2\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{2}{3}bc\left(\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)d\frac{1}{x}\right.\right.$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\int\frac{x^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-c^2\left(c^2\left(-\int\frac{(a\right.\right.\right.$$

$\downarrow$ 264

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\left(c^2\left(-\int\frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-x\right)-\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-\right.$$

$\downarrow$ 216

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(-c\arctan\left(\frac{c}{x}\right)-x\right)\right)-c^2\left(c^2\right.\right.$$

$\downarrow$ 5419

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(-c^2\left(2bc\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{c\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3b}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\right)+\frac{2}{3}bc\left(c^2\left(-\int\frac{x(}{}\right.\right.\right.$$

$\downarrow$ 5459

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(-\left(c^2\left(i\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c}{x}+i}d\frac{1}{x}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}\right)\right)-\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(-c\,a\right.\right.\right.$$

$\downarrow$ 5403

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(-c^2\left(2bc\left(i\left(ibc\int\frac{\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-i\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}\right.\right.\right.\right.$$

$\downarrow$ 2897

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right)\right)\right)-\frac{i\left(a+b\arctan\right.}{2b}\right.\right.\right.$$

input
```
Int[x^3*(a + b*ArcTan[c/x])^3,x]
```

output
```
(x^4*(a + b*ArcTan[c/x])^3)/4 - (3*b*c*(-1/3*(x^3*(a + b*ArcTan[c/x])^2) -
 c^2*(-(x*(a + b*ArcTan[c/x])^2) - (c*(a + b*ArcTan[c/x])^3)/(3*b) + 2*b*c
*(((-1/2*I)*(a + b*ArcTan[c/x])^2)/b + I*((-I)*(a + b*ArcTan[c/x])*Log[2 -
 2/(1 - (I*c)/x)] - (b*PolyLog[2, -1 + 2/(1 - (I*c)/x)])/2))) + (2*b*c*(-1
/2*(x^2*(a + b*ArcTan[c/x])) + (b*c*(-x - c*ArcTan[c/x]))/2 - c^2*(((-1/2*
I)*(a + b*ArcTan[c/x])^2)/b + I*((-I)*(a + b*ArcTan[c/x])*Log[2 - 2/(1 - (
I*c)/x)] - (b*PolyLog[2, -1 + 2/(1 - (I*c)/x)])/2))))/3))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(196) = 392$.

Time = 7.85 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.24

method	result
derivativedivides	$-c^4\left(-\frac{a^3x^4}{4c^4}+b^3\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^3}{4c^4}+\frac{\arctan\left(\frac{c}{x}\right)^3}{4}-\frac{x^3\arctan\left(\frac{c}{x}\right)^2}{4c^3}+\frac{3x\arctan\left(\frac{c}{x}\right)^2}{4c}+\arctan\left(\frac{c}{x}\right)\right.\right.$
default	$-c^4\left(-\frac{a^3x^4}{4c^4}+b^3\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^3}{4c^4}+\frac{\arctan\left(\frac{c}{x}\right)^3}{4}-\frac{x^3\arctan\left(\frac{c}{x}\right)^2}{4c^3}+\frac{3x\arctan\left(\frac{c}{x}\right)^2}{4c}+\arctan\left(\frac{c}{x}\right)\right.\right.$
parts	$\frac{a^3x^4}{4}+\frac{b^3x^4\arctan\left(\frac{c}{x}\right)^3}{4}-\frac{b^3c^4\arctan\left(\frac{c}{x}\right)^3}{4}+\frac{b^3c^3x}{4}+\frac{3a^2b\,x^4\arctan\left(\frac{c}{x}\right)}{4}-3a\,b^2c^4\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^2}{4c^4}+\right.$
risch	Expression too large to display

input
```
int(x^3*(a+b*arctan(c/x))^3,x,method=_RETURNVERBOSE)
```

output
```
-c^4*(-1/4*a^3/c^4*x^4+b^3*(-1/4/c^4*x^4*arctan(c/x)^3+1/4*arctan(c/x)^3-1
/4/c^3*x^3*arctan(c/x)^2+3/4/c*x*arctan(c/x)^2+arctan(c/x)*ln(1+c^2/x^2)-1
/4/c^2*x^2*arctan(c/x)-2*ln(c/x)*arctan(c/x)-1/4*arctan(c/x)-1/4*x/c+1/2*I
*(ln(c/x-I)*ln(1+c^2/x^2)-1/2*ln(c/x-I)^2-dilog(-1/2*I*(c/x+I))-ln(c/x-I)*
ln(-1/2*I*(c/x+I)))-1/2*I*(ln(c/x+I)*ln(1+c^2/x^2)-1/2*ln(c/x+I)^2-dilog(1
/2*I*(c/x-I))-ln(c/x+I)*ln(1/2*I*(c/x-I)))-I*ln(c/x)*ln(1+I*c/x)+I*ln(c/x)
*ln(1-I*c/x)-I*dilog(1+I*c/x)+I*dilog(1-I*c/x))+3*a*b^2*(-1/4/c^4*x^4*arct
an(c/x)^2+1/4*arctan(c/x)^2-1/6/c^3*x^3*arctan(c/x)+1/2/c*x*arctan(c/x)+1/
3*ln(1+c^2/x^2)-1/12/c^2*x^2-2/3*ln(c/x))+3*a^2*b*(-1/4/c^4*x^4*arctan(c/x
)+1/4*arctan(c/x)-1/12/c^3*x^3+1/4*x/c))
```

Fricas [F]

$$\int x^3 \left(a + b \arctan\left(\frac{c}{x}\right)\right)^3 dx = \int \left(b \arctan\left(\frac{c}{x}\right) + a\right)^3 x^3 dx$$

input
```
integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*arctan(c/x)^3 + 3*a*b^2*x^3*arctan(c/x)^2 + 3*a^2*b*x^3*a
rctan(c/x) + a^3*x^3, x)
```

Sympy [F]

$$\int x^3 \left(a + b \arctan\left(\frac{c}{x}\right)\right)^3 dx = \int x^3 \left(a + b \operatorname{atan}\left(\frac{c}{x}\right)\right)^3 dx$$

input
```
integrate(x**3*(a+b*atan(c/x))**3,x)
```

output
```
Integral(x**3*(a + b*atan(c/x))**3, x)
```

Maxima [F]

$$\int x^3 \left(a + b \arctan\left(\frac{c}{x}\right)\right)^3 dx = \int \left(b \arctan\left(\frac{c}{x}\right) + a\right)^3 x^3 dx$$

input
```
integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

output
```
3/4*a*b^2*x^4*arctan(c/x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arctan(c/x) + (3*c^
3*arctan(x/c) - 3*c^2*x + x^3)*c)*a^2*b + 1/4*((3*c^2*arctan(x/c)^2 - 4*c^
2*log(c^2 + x^2) + x^2)*c^2 + 2*(3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c*arct
an(c/x))*a*b^2 - 1/64*(12*c^4*arctan(c/x)^2*arctan(x/c) + 8*c^4*arctan2(c,
 x)^3 - 8*x^4*arctan2(c, x)^3 + 4*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(
x/c)^3/c)*c^5 + 12*c^3*x*arctan2(c, x)^2 - 4*c*x^3*arctan2(c, x)^2 + 192*c
^5*integrate(1/64*log(c^2 + x^2)^2/(c^2 + x^2), x) + 1536*c^4*integrate(1/
64*x*arctan(c/x)/(c^2 + x^2), x) + 768*c^3*integrate(1/64*x^2*log(c^2 + x^
2)/(c^2 + x^2), x) - 2048*c^2*integrate(1/64*x^3*arctan(c/x)^3/(c^2 + x^2)
, x) - 512*c^2*integrate(1/64*x^3*arctan(c/x)/(c^2 + x^2), x) - (3*c^3*x -
 c*x^3)*log(c^2 + x^2)^2 - 768*c*integrate(1/64*x^4*arctan(c/x)^2/(c^2 + x
^2), x) - 192*c*integrate(1/64*x^4*log(c^2 + x^2)^2/(c^2 + x^2), x) - 256*
c*integrate(1/64*x^4*log(c^2 + x^2)/(c^2 + x^2), x) - 2048*integrate(1/64*
x^5*arctan(c/x)^3/(c^2 + x^2), x))*b^3
```

Giac [F]

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^3*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int x^3\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

input
```
int(x^3*(a + b*atan(c/x))^3,x)
```

output
```
int(x^3*(a + b*atan(c/x))^3, x)
```

Reduce [F]

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx = -\frac{atan\left(\frac{c}{x}\right)^3 b^3c^4}{4}+\frac{atan\left(\frac{c}{x}\right)^3 b^3x^4}{4}-\frac{3atan\left(\frac{c}{x}\right)^2 a\,b^2c^4}{4}+\frac{3atan\left(\frac{c}{x}\right)^2 a\,b^2x^4}{4}-\frac{3atan\left(\frac{c}{x}\right)^2 b^3c^3x}{4}+\frac{atan\left(\frac{c}{x}\right)^2 b^3c\,x^3}{4}-\frac{3atan\left(\frac{c}{x}\right) a^2b\,c^4}{4}+\frac{3atan\left(\frac{c}{x}\right) a^2b\,x^4}{4}-\frac{3atan\left(\frac{c}{x}\right) a\,b^2c^3x}{2}+\frac{atan\left(\frac{c}{x}\right) a\,b^2c\,x^3}{2}+\frac{atan\left(\frac{c}{x}\right) b^3c^4}{4}+\frac{atan\left(\frac{c}{x}\right) b^3c^2x^2}{4}-2\left(\int\frac{atan\left(\frac{c}{x}\right)x}{c^2+x^2}dx\right)b^3c^4-\log\left(c^2+x^2\right)a\,b^2c^4+\frac{a^3x^4}{4}-\frac{3a^2b\,c^3x}{4}+\frac{a^2bc\,x^3}{4}+\frac{a\,b^2c^2x^2}{4}+\frac{b^3c^3x}{4}$$

input
```
int(x^3*(a+b*atan(c/x))^3,x)
```

output
```
( - atan(c/x)**3*b**3*c**4 + atan(c/x)**3*b**3*x**4 - 3*atan(c/x)**2*a*b**
2*c**4 + 3*atan(c/x)**2*a*b**2*x**4 - 3*atan(c/x)**2*b**3*c**3*x + atan(c/
x)**2*b**3*c*x**3 - 3*atan(c/x)*a**2*b*c**4 + 3*atan(c/x)*a**2*b*x**4 - 6*
atan(c/x)*a*b**2*c**3*x + 2*atan(c/x)*a*b**2*c*x**3 + atan(c/x)*b**3*c**4
+ atan(c/x)*b**3*c**2*x**2 - 8*int((atan(c/x)*x)/(c**2 + x**2),x)*b**3*c**
4 - 4*log(c**2 + x**2)*a*b**2*c**4 + a**3*x**4 - 3*a**2*b*c**3*x + a**2*b*
c*x**3 + a*b**2*c**2*x**2 + b**3*c**3*x)/4
```

3.148 $\int x^2 \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx$

Optimal result . . . 1100
Mathematica [A] (verified) . . . 1101
Rubi [A] (verified) . . . 1102
Maple [C] (warning: unable to verify) . . . 1106
Fricas [F] . . . 1107
Sympy [F] . . . 1108
Maxima [F] . . . 1108
Giac [F] . . . 1108
Mupad [F(-1)] . . . 1109
Reduce [F] . . . 1109

Optimal result

Integrand size = 16, antiderivative size = 229

$$\begin{aligned}\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx &= b^2c^2x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\ &\quad+\frac{1}{2}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\ &\quad-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 \\ &\quad+bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(2-\frac{2}{1-\frac{ic}{x}}\right) \\ &\quad+\frac{1}{2}b^3c^3\log\left(1+\frac{c^2}{x^2}\right)+b^3c^3\log(x) \\ &\quad-ib^2c^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right) \\ &\quad+\frac{1}{2}b^3c^3\,\text{PolyLog}\left(3,-1+\frac{2}{1-\frac{ic}{x}}\right)\end{aligned}$$

output
```
b^2*c^2*x*(a+b*arccot(x/c))+1/2*b*c^3*(a+b*arccot(x/c))^2+1/2*b*c*x^2*(a+b
*arccot(x/c))^2-1/3*I*c^3*(a+b*arccot(x/c))^3+1/3*x^3*(a+b*arccot(x/c))^3+
b*c^3*(a+b*arccot(x/c))^2*ln(2-2/(1-I*c/x))+1/2*b^3*c^3*ln(1+c^2/x^2)+b^3*
c^3*ln(x)-I*b^2*c^3*(a+b*arccot(x/c))*polylog(2,-1+2/(1-I*c/x))+1/2*b^3*c^
3*polylog(3,-1+2/(1-I*c/x))
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.44

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\frac{1}{2}a^2bcx^2+\frac{a^3x^3}{3}+a^2bx^3\arctan\left(\frac{c}{x}\right)-\frac{1}{2}a^2bc^3\log\left(c^2+x^2\right)+ab^2\left(c^2x+\left(-ic^3+x^3\right)\arctan\left(\frac{c}{x}\right)^2+c\arctan\left(\frac{c}{x}\right)\left(c^2+x^2+2c^2\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)-ic^3\operatorname{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)+\frac{1}{24}b^3\left(-ic^3\pi^3+24c^2x\arctan\left(\frac{c}{x}\right)+12c^3\arctan\left(\frac{c}{x}\right)^2+12cx^2\arctan\left(\frac{c}{x}\right)^2+8ic^3\arctan\left(\frac{c}{x}\right)^3+8x^3\arctan\left(\frac{c}{x}\right)^3+24c^3\arctan\left(\frac{c}{x}\right)^2\log\left(1-e^{-2i\arctan\left(\frac{c}{x}\right)}\right)-24c^3\log\left(\frac{c}{\sqrt{1+\frac{c^2}{x^2}}\,x}\right)+24ic^3\arctan\left(\frac{c}{x}\right)\operatorname{PolyLog}\left(2,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)+12c^3\operatorname{PolyLog}\left(3,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$

input

```
Integrate[x^2*(a + b*ArcTan[c/x])^3,x]
```

output

```
(a^2*b*c*x^2)/2 + (a^3*x^3)/3 + a^2*b*x^3*ArcTan[c/x] - (a^2*b*c^3*Log[c^2
 + x^2])/2 + a*b^2*(c^2*x + ((-I)*c^3 + x^3)*ArcTan[c/x]^2 + c*ArcTan[c/x]
*(c^2 + x^2 + 2*c^2*Log[1 - E^((2*I)*ArcTan[c/x])]) - I*c^3*PolyLog[2, E^(
(2*I)*ArcTan[c/x])]) + (b^3*((-I)*c^3*Pi^3 + 24*c^2*x*ArcTan[c/x] + 12*c^3
*ArcTan[c/x]^2 + 12*c*x^2*ArcTan[c/x]^2 + (8*I)*c^3*ArcTan[c/x]^3 + 8*x^3*
ArcTan[c/x]^3 + 24*c^3*ArcTan[c/x]^2*Log[1 - E^((-2*I)*ArcTan[c/x])] - 24*
c^3*Log[c/(Sqrt[1 + c^2/x^2]*x)] + (24*I)*c^3*ArcTan[c/x]*PolyLog[2, E^((-
2*I)*ArcTan[c/x])] + 12*c^3*PolyLog[3, E^((-2*I)*ArcTan[c/x])]))/24
```

Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.938, Rules used = {5363, 5361, 5453, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3dx$$

$$\Big\downarrow \text{5363}$$

$$-\int x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3d\frac{1}{x}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-bc\int \frac{x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-bc\left(\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}-c^2\int \frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-\int \frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\int \frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\right)$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-\int \frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\left(\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)d\frac{1}{x}-c^2\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-\frac{1}{2}x^2\left(\right.\right.$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\int\frac{x}{\frac{c^2}{x^2}+1}d\frac{1}{x}-x\left(a+b\arctan\right.\right.\right.$$

$\downarrow$ 243

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\int\frac{x}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}-x\left(a+b\,\mathrm{arct}\right.\right.\right.$$

$\downarrow$ 47

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\left(\int xd\frac{1}{x^2}-c^2\int\frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}\right.\right.\right.$$

$\downarrow$ 14

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-c^2\int\frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right.\right.\right.$$

$\downarrow$ 16

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\right.\right.\right.$$

$\downarrow$ 5419

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\left(-\frac{c\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\right.\right.\right.$$

$\downarrow$ 5459

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(-\left(c^2\left(i\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c}{x}+i}d\frac{1}{x}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3b}\right)\right)+bc\left(-\frac{c\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}-x\left(a+b\,\mathrm{arcta}\right.\right.\right.$$

$\downarrow$ 5403

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(-\left(c^2\left(i\left(2ibc\int\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-i\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\right)-\frac{i(a\right.\right.\right.$$

$\Big\downarrow$ 5527

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(-\left(c^2\left(i\left(2ibc\left(\frac{i\,\mathrm{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}-\frac{1}{2}ib\int\frac{\mathrm{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-i\log\left(\right.\right.\right.\right.$$

$\Big\downarrow$ 7164

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(-\left(c^2\left(i\left(2ibc\left(\frac{i\,\mathrm{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}-\frac{b\,\mathrm{PolyLog}\left(3,\frac{2}{1-\frac{ic}{x}}-1\right)}{4c}\right)-i\log\left(2-\frac{2}{1-\right.\right.\right.\right.$$

input
```
Int[x^2*(a + b*ArcTan[c/x])^3,x]
```

output
```
(x^3*(a + b*ArcTan[c/x])^3)/3 - b*c*(-1/2*(x^2*(a + b*ArcTan[c/x])^2) + b*
c*(-(x*(a + b*ArcTan[c/x])) - (c*(a + b*ArcTan[c/x])^2)/(2*b) + (b*c*(-Log
[1 + c^2/x^2] + Log[x^(-2)]))/2) - c^2*(((-1/3*I)*(a + b*ArcTan[c/x])^3)/b
 + I*((-I)*(a + b*ArcTan[c/x])^2*Log[2 - 2/(1 - (I*c)/x)] + (2*I)*b*c*(((I
/2)*(a + b*ArcTan[c/x])*PolyLog[2, -1 + 2/(1 - (I*c)/x)])/c - (b*PolyLog[3
, -1 + 2/(1 - (I*c)/x)])/(4*c)))))
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.14 (sec) , antiderivative size = 2367, normalized size of antiderivative = 10.34

method	result	size
derivativedivides	Expression too large to display	2367
default	Expression too large to display	2367
parts	Expression too large to display	2470

input
```
int(x^2*(a+b*arctan(c/x))^3,x,method=_RETURNVERBOSE)
```

output
```
-c^3*(-1/3*a^3/c^3*x^3+b^3*(-1/2*I*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*
csgn(I/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+
(1+I*c/x)^2/(1+c^2/x^2)))*arctan(c/x)^2-1/3/c^3*x^3*arctan(c/x)^3-1/2*I*Pi
*arctan(c/x)^2-ln(c/x)*arctan(c/x)^2+arctan(c/x)^2*ln((1+I*c/x)^2/(1+c^2/x
^2)-1)-arctan(c/x)^2*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))-arctan(c/x)^2*ln(1+
(1+I*c/x)/(1+c^2/x^2)^(1/2))+ln((1+I*c/x)/(1+c^2/x^2)^(1/2)-1)+1/4*I*Pi*cs
gn(I*(1+I*c/x)/(1+c^2/x^2)^(1/2))^2*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))*arctan
(c/x)^2-1/4*I*Pi*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))*csgn(I*(1+I*c/x)^2/(1+c^2
/x^2)/(1+(1+I*c/x)^2/(1+c^2/x^2))^2)^2*arctan(c/x)^2+1/2*I*Pi*csgn(I*(1+(1
+I*c/x)^2/(1+c^2/x^2)))*csgn(I*(1+(1+I*c/x)^2/(1+c^2/x^2))^2)^2*arctan(c/x
)^2+1/2*arctan(c/x)^2*ln(1+c^2/x^2)+1/3*I*arctan(c/x)^3-arctan(c/x)^2*ln((
1+I*c/x)/(1+c^2/x^2)^(1/2))-ln(2)*arctan(c/x)^2-1/2/c^2*x^2*arctan(c/x)^2-
1/2*arctan(c/x)^2-2*polylog(3,(1+I*c/x)/(1+c^2/x^2)^(1/2))+ln(1+(1+I*c/x)/
(1+c^2/x^2)^(1/2))-2*polylog(3,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-1/2*arctan(c/
x)*(I*c/x+(1+c^2/x^2)^(1/2)+1)/c*x-1/2*arctan(c/x)*(I*c/x-(1+c^2/x^2)^(1/2
)+1)/c*x-1/4*I*Pi*csgn(I*(1+(1+I*c/x)^2/(1+c^2/x^2))^2)^3*arctan(c/x)^2-1/
2*I*Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^3*arc
tan(c/x)^2+1/2*I*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c
^2/x^2)))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2*
arctan(c/x)^2-1/2*I*Pi*csgn(I*(1+I*c/x)/(1+c^2/x^2)^(1/2))*csgn(I*(1+I*...
```

Fricas [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*arctan(c/x)^3 + 3*a*b^2*x^2*arctan(c/x)^2 + 3*a^2*b*x^2*a
rctan(c/x) + a^3*x^2, x)
```

Sympy [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int x^2\left(a+b\operatorname{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

input
```
integrate(x**2*(a+b*atan(c/x))**3,x)
```

output
```
Integral(x**2*(a + b*atan(c/x))**3, x)
```

Maxima [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

output
```
1/24*b^3*x^3*arctan2(c, x)^3 - 1/32*b^3*x^3*arctan2(c, x)*log(c^2 + x^2)^2
 + 1/3*a^3*x^3 + 1/2*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*a^
2*b + integrate(1/32*(4*b^3*c*x^3*arctan2(c, x)^2 + 4*b^3*x^4*arctan2(c, x
)*log(c^2 + x^2) + 4*(7*b^3*arctan2(c, x)^3 + 24*a*b^2*arctan2(c, x)^2)*x^
4 + 4*(7*b^3*c^2*arctan2(c, x)^3 + 24*a*b^2*c^2*arctan2(c, x)^2)*x^2 + (3*
b^3*c^2*x^2*arctan2(c, x) + 3*b^3*x^4*arctan2(c, x) - b^3*c*x^3)*log(c^2 +
 x^2)^2)/(c^2 + x^2), x)
```

Giac [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^3*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int x^2\,\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

input
```
int(x^2*(a + b*atan(c/x))^3,x)
```

output
```
int(x^2*(a + b*atan(c/x))^3, x)
```

Reduce [F]

$$\begin{aligned}\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=&\ \frac{atan\left(\frac{c}{x}\right)^3b^3c^2x}{3}+\frac{atan\left(\frac{c}{x}\right)^3b^3x^3}{3}+atan\left(\frac{c}{x}\right)^2a\,b^2c^2x\\&+atan\left(\frac{c}{x}\right)^2a\,b^2x^3+\frac{atan\left(\frac{c}{x}\right)^2b^3c^3}{2}+\frac{atan\left(\frac{c}{x}\right)^2b^3c\,x^2}{2}\\&+atan\left(\frac{c}{x}\right)a^2b\,x^3+atan\left(\frac{c}{x}\right)a\,b^2c^3+atan\left(\frac{c}{x}\right)a\,b^2c\,x^2\\&+atan\left(\frac{c}{x}\right)b^3c^2x-\frac{\left(\int atan\left(\frac{c}{x}\right)^3dx\right)b^3c^2}{3}\\&-\left(\int atan\left(\frac{c}{x}\right)^2dx\right)a\,b^2c^2-\frac{\log\left(c^2+x^2\right)a^2b\,c^3}{2}\\&+\frac{\log\left(c^2+x^2\right)b^3c^3}{2}+\frac{a^3x^3}{3}+\frac{a^2bc\,x^2}{2}+a\,b^2c^2x\end{aligned}$$

input
```
int(x^2*(a+b*atan(c/x))^3,x)
```

output
```
(2*atan(c/x)**3*b**3*c**2*x + 2*atan(c/x)**3*b**3*x**3 + 6*atan(c/x)**2*a*
b**2*c**2*x + 6*atan(c/x)**2*a*b**2*x**3 + 3*atan(c/x)**2*b**3*c**3 + 3*at
an(c/x)**2*b**3*c*x**2 + 6*atan(c/x)*a**2*b*x**3 + 6*atan(c/x)*a*b**2*c**3
 + 6*atan(c/x)*a*b**2*c*x**2 + 6*atan(c/x)*b**3*c**2*x - 2*int(atan(c/x)**
3,x)*b**3*c**2 - 6*int(atan(c/x)**2,x)*a*b**2*c**2 - 3*log(c**2 + x**2)*a*
*2*b*c**3 + 3*log(c**2 + x**2)*b**3*c**3 + 2*a**3*x**3 + 3*a**2*b*c*x**2 +
 6*a*b**2*c**2*x)/6
```

3.149 $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$

Optimal result . . . 1110
Mathematica [A] (verified) . . . 1111
Rubi [A] (verified) . . . 1111
Maple [B] (verified) . . . 1114
Fricas [F] . . . 1115
Sympy [F] . . . 1116
Maxima [F] . . . 1116
Giac [F] . . . 1117
Mupad [F(-1)] . . . 1117
Reduce [F] . . . 1117

Optimal result

Integrand size = 14, antiderivative size = 145

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\frac{3}{2}ibc^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{3}{2}bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-3b^2c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)+\frac{3}{2}ib^3c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right)$$

output
```
3/2*I*b*c^2*(a+b*arccot(x/c))^2+3/2*b*c*x*(a+b*arccot(x/c))^2+1/2*c^2*(a+b
*arccot(x/c))^3+1/2*x^2*(a+b*arccot(x/c))^3-3*b^2*c^2*(a+b*arccot(x/c))*ln
(2-2/(1-I*c/x))+3/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c/x))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.20

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \frac{1}{2}\left(3b^2\left(bc(ic+x)+a\left(c^2+x^2\right)\right)\arctan\left(\frac{c}{x}\right)^2 + b^3\left(c^2+x^2\right)\arctan\left(\frac{c}{x}\right)^3 + 3b\arctan\left(\frac{c}{x}\right)\left(a\left(2bcx+a\left(c^2+x^2\right)\right) - 2b^2c^2\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right) + a\left(ax(3bc+ax)-6b^2c^2\log\left(\frac{c}{\sqrt{1+\frac{c^2}{x^2}}\,x}\right)\right) + 3ib^3c^2\operatorname{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$

input
```
Integrate[x*(a + b*ArcTan[c/x])^3,x]
```

output
```
(3*b^2*(b*c*(I*c + x) + a*(c^2 + x^2))*ArcTan[c/x]^2 + b^3*(c^2 + x^2)*Arc
Tan[c/x]^3 + 3*b*ArcTan[c/x]*(a*(2*b*c*x + a*(c^2 + x^2)) - 2*b^2*c^2*Log[
1 - E^((2*I)*ArcTan[c/x])]) + a*(a*x*(3*b*c + a*x) - 6*b^2*c^2*Log[c/(Sqrt
[1 + c^2/x^2]*x)]) + (3*I)*b^3*c^2*PolyLog[2, E^((2*I)*ArcTan[c/x])])/2
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5363, 5361, 5453, 5361, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx$$

$$\Big\downarrow \text{5363}$$

$$-\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3d\frac{1}{x}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{2}bc\int \frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{2}bc\left(\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}-c^2\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{2}bc\left(c^2\left(-\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+2bc\int \frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\right)$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{2}bc\left(2bc\int \frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{c\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3b}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\right)$$

$$\Big\downarrow \text{5459}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{2}bc\left(2bc\left(i\int \frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c}{x}+i}d\frac{1}{x}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}\right)-\frac{c\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3b}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2$$

$$\Big\downarrow \text{5403}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{2}bc\left(2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-i\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}\right)-$$

$$\Big\downarrow \text{2897}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 - \frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right)\right)-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}\right.$$

input
```
Int[x*(a + b*ArcTan[c/x])^3,x]
```

output
```
(x^2*(a + b*ArcTan[c/x])^3)/2 - (3*b*c*(-(x*(a + b*ArcTan[c/x])^2) - (c*(a
 + b*ArcTan[c/x])^3)/(3*b) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c/x])^2)/b + I
*((-I)*(a + b*ArcTan[c/x])*Log[2 - 2/(1 - (I*c)/x)] - (b*PolyLog[2, -1 + 2
/(1 - (I*c)/x)])/2))))/2
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(131) = 262$.

Time = 2.96 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.79

method	result
derivativedivides	$-c^2\left(-\frac{a^3x^2}{2c^2}+b^3\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^3}{2c^2}-\frac{\arctan\left(\frac{c}{x}\right)^3}{2}-\frac{3x\arctan\left(\frac{c}{x}\right)^2}{2c}-\frac{3\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}+3\ln\right.\right.$
default	$-c^2\left(-\frac{a^3x^2}{2c^2}+b^3\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^3}{2c^2}-\frac{\arctan\left(\frac{c}{x}\right)^3}{2}-\frac{3x\arctan\left(\frac{c}{x}\right)^2}{2c}-\frac{3\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}+3\ln\right.\right.$
parts	$\frac{a^3x^2}{2}-b^3c^2\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^3}{2c^2}-\frac{\arctan\left(\frac{c}{x}\right)^3}{2}-\frac{3x\arctan\left(\frac{c}{x}\right)^2}{2c}-\frac{3\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}+3\ln\left(\frac{c}{x}\right)\mathrm{ar}\right.$
risch	Expression too large to display

input
```
int(x*(a+b*arctan(c/x))^3,x,method=_RETURNVERBOSE)
```

output
```
-c^2*(-1/2*a^3/c^2*x^2+b^3*(-1/2/c^2*x^2*arctan(c/x)^3-1/2*arctan(c/x)^3-3
/2/c*x*arctan(c/x)^2-3/2*arctan(c/x)*ln(1+c^2/x^2)+3*ln(c/x)*arctan(c/x)-3
/4*I*(ln(c/x-I)*ln(1+c^2/x^2)-1/2*ln(c/x-I)^2-dilog(-1/2*I*(c/x+I))-ln(c/x
-I)*ln(-1/2*I*(c/x+I)))+3/4*I*(ln(c/x+I)*ln(1+c^2/x^2)-1/2*ln(c/x+I)^2-dil
og(1/2*I*(c/x-I))-ln(c/x+I)*ln(1/2*I*(c/x-I)))+3/2*I*ln(c/x)*ln(1+I*c/x)-3
/2*I*ln(c/x)*ln(1-I*c/x)+3/2*I*dilog(1+I*c/x)-3/2*I*dilog(1-I*c/x))+3*a*b^
2*(-1/2/c^2*x^2*arctan(c/x)^2-1/2*arctan(c/x)^2-1/c*x*arctan(c/x)-1/2*ln(1
+c^2/x^2)+ln(c/x))+3*a^2*b*(-1/2/c^2*x^2*arctan(c/x)-1/2*arctan(c/x)-1/2*x
/c))
```

Fricas [F]

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*arctan(c/x)^3 + 3*a*b^2*x*arctan(c/x)^2 + 3*a^2*b*x*arctan(
c/x) + a^3*x, x)
```

Sympy [F]

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int x\left(a+b\operatorname{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

input
```
integrate(x*(a+b*atan(c/x))**3,x)
```

output
```
Integral(x*(a + b*atan(c/x))**3, x)
```

Maxima [F]

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

output
```
3/2*a*b^2*x^2*arctan(c/x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arctan(c/x) - (c*arct
an(x/c) - x)*c)*a^2*b - 3/2*((arctan(x/c)^2 - log(c^2 + x^2))*c^2 + 2*(c*a
rctan(x/c) - x)*c*arctan(c/x))*a*b^2 + 1/32*(12*c^2*arctan(c/x)^2*arctan(x
/c) + 8*c^2*arctan2(c, x)^3 + 8*x^2*arctan2(c, x)^3 + 4*(3*arctan(c/x)*arc
tan(x/c)^2/c + arctan(x/c)^3/c)*c^3 + 12*c*x*arctan2(c, x)^2 + 96*c^3*inte
grate(1/32*log(c^2 + x^2)^2/(c^2 + x^2), x) - 3*c*x*log(c^2 + x^2)^2 + 512
*c^2*integrate(1/32*x*arctan(c/x)^3/(c^2 + x^2), x) + 768*c^2*integrate(1/
32*x*arctan(c/x)/(c^2 + x^2), x) + 384*c*integrate(1/32*x^2*arctan(c/x)^2/
(c^2 + x^2), x) + 96*c*integrate(1/32*x^2*log(c^2 + x^2)^2/(c^2 + x^2), x)
 + 384*c*integrate(1/32*x^2*log(c^2 + x^2)/(c^2 + x^2), x) + 512*integrate
(1/32*x^3*arctan(c/x)^3/(c^2 + x^2), x))*b^3
```

Giac [F]

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c/x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^3*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int x\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

input
```
int(x*(a + b*atan(c/x))^3,x)
```

output
```
int(x*(a + b*atan(c/x))^3, x)
```

Reduce [F]

$$\begin{aligned}\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx&=\frac{atan\left(\frac{c}{x}\right)^3b^3c^2}{2}+\frac{atan\left(\frac{c}{x}\right)^3b^3x^2}{2}+\frac{3atan\left(\frac{c}{x}\right)^2a\,b^2c^2}{2}\\&\quad+\frac{3atan\left(\frac{c}{x}\right)^2a\,b^2x^2}{2}+\frac{3atan\left(\frac{c}{x}\right)^2b^3cx}{2}\\&\quad+\frac{3atan\left(\frac{c}{x}\right)a^2b\,c^2}{2}+\frac{3atan\left(\frac{c}{x}\right)a^2b\,x^2}{2}\\&\quad+3atan\left(\frac{c}{x}\right)a\,b^2cx+3\left(\int\frac{atan\left(\frac{c}{x}\right)x}{c^2+x^2}dx\right)b^3c^2\\&\quad+\frac{3\log\left(c^2+x^2\right)a\,b^2c^2}{2}+\frac{a^3x^2}{2}+\frac{3a^2bcx}{2}\end{aligned}$$

input
```
int(x*(a+b*atan(c/x))^3,x)
```

output
```
(atan(c/x)**3*b**3*c**2 + atan(c/x)**3*b**3*x**2 + 3*atan(c/x)**2*a*b**2*c
**2 + 3*atan(c/x)**2*a*b**2*x**2 + 3*atan(c/x)**2*b**3*c*x + 3*atan(c/x)*a
**2*b*c**2 + 3*atan(c/x)*a**2*b*x**2 + 6*atan(c/x)*a*b**2*c*x + 6*int((ata
n(c/x)*x)/(c**2 + x**2),x)*b**3*c**2 + 3*log(c**2 + x**2)*a*b**2*c**2 + a*
*3*x**2 + 3*a**2*b*c*x)/2
```

3.150 $\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx$

Optimal result . . . 1119
Mathematica [A] (verified) . . . 1120
Rubi [A] (verified) . . . 1120
Maple [C] (warning: unable to verify) . . . 1123
Fricas [F] . . . 1124
Sympy [F] . . . 1125
Maxima [F] . . . 1125
Giac [F] . . . 1126
Mupad [F(-1)] . . . 1126
Reduce [F] . . . 1126

Optimal result

Integrand size = 12, antiderivative size = 119

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 - 3bc\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(\frac{2c}{c+ix}\right) + 3ib^2c\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right) - \frac{3}{2}b^3c\,\text{PolyLog}\left(3,1-\frac{2c}{c+ix}\right)$$

output
```
I*c*(a+b*arccot(x/c))^3+x*(a+b*arccot(x/c))^3-3*b*c*(a+b*arccot(x/c))^2*ln
(2*c/(c+I*x))+3*I*b^2*c*(a+b*arccot(x/c))*polylog(2,1-2*c/(c+I*x))-3/2*b^3
*c*polylog(3,1-2*c/(c+I*x))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.81

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx = a^3x+3a^2bx\arctan\left(\frac{c}{x}\right)+\frac{3}{2}a^2bc\log\left(c^2+x^2\right) - 3ab^2\left(-\left((ic+x)\arctan\left(\frac{c}{x}\right)^2\right) + 2c\arctan\left(\frac{c}{x}\right)\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right) - ic\,\text{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right) - \frac{1}{8}b^3\left(-ic\pi^3+8ic\arctan\left(\frac{c}{x}\right)^3-8x\arctan\left(\frac{c}{x}\right)^3 + 24c\arctan\left(\frac{c}{x}\right)^2\log\left(1-e^{-2i\arctan\left(\frac{c}{x}\right)}\right) + 24ic\arctan\left(\frac{c}{x}\right)\text{PolyLog}\left(2,e^{-2i\arctan\left(\frac{c}{x}\right)}\right) + 12c\,\text{PolyLog}\left(3,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c/x])^3,x]
```

output
```
a^3*x + 3*a^2*b*x*ArcTan[c/x] + (3*a^2*b*c*Log[c^2 + x^2])/2 - 3*a*b^2*(-(
(I*c + x)*ArcTan[c/x]^2) + 2*c*ArcTan[c/x]*Log[1 - E^((2*I)*ArcTan[c/x])]
- I*c*PolyLog[2, E^((2*I)*ArcTan[c/x])]) - (b^3*((-I)*c*Pi^3 + (8*I)*c*Arc
Tan[c/x]^3 - 8*x*ArcTan[c/x]^3 + 24*c*ArcTan[c/x]^2*Log[1 - E^((-2*I)*ArcT
an[c/x])] + (24*I)*c*ArcTan[c/x]*PolyLog[2, E^((-2*I)*ArcTan[c/x])] + 12*c
*PolyLog[3, E^((-2*I)*ArcTan[c/x])]))/8
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5349, 5346, 27, 5456, 27, 5380, 27, 5530, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3dx$$

$$\Big\downarrow \text{ 5349}$$

$$\int \left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3dx$$

$$\Big\downarrow \text{ 5346}$$

$$\frac{3b\int \frac{c^2x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c^2+x^2}dx}{c}+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3$$

$$\Big\downarrow \text{ 27}$$

$$3bc\int \frac{x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c^2+x^2}dx+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3$$

$$\Big\downarrow \text{ 5456}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+3bc\left(\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}-\frac{\int \frac{c\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{ic-x}dx}{c}\right)$$

$$\Big\downarrow \text{ 27}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+3bc\left(\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}-\int \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{ic-x}dx\right)$$

$$\Big\downarrow \text{ 5380}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+$$
$$3bc\left(-\frac{2b\int \frac{c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2c}{c+ix}\right)}{c^2+x^2}dx}{c}+\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}-\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\right)$$

$$\Big\downarrow \text{ 27}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+$$
$$3bc\left(-2bc\int \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2c}{c+ix}\right)}{c^2+x^2}dx+\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}-\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\right)$$

$$\Big\downarrow \text{ 5530}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+$$
$$3bc\left(-2bc\left(-\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)}{c^2+x^2}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{2c}\right)+\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right.\right.}{3b}\right.$$

$$\Big\downarrow \text{ 7164}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+$$
$$3bc\left(-2bc\left(\frac{b\,\text{PolyLog}\left(3,1-\frac{2c}{c+ix}\right)}{4c}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{2c}\right)+\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}-\log\right.$$

input
```
Int[(a + b*ArcTan[c/x])^3,x]
```

output
```
x*(a + b*ArcCot[x/c])^3 + 3*b*c*(((I/3)*(a + b*ArcCot[x/c])^3)/b - (a + b*
ArcCot[x/c])^2*Log[(2*c)/(c + I*x)] - 2*b*c*(((-1/2*I)*(a + b*ArcCot[x/c])
*PolyLog[2, 1 - (2*c)/(c + I*x)])/c + (b*PolyLog[3, 1 - (2*c)/(c + I*x)])/
(4*c)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5349
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[(a + b*A
rcCot[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && ILtQ[n, 0]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5530
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.78 (sec) , antiderivative size = 2028, normalized size of antiderivative = 17.04

method	result	size
parts	Expression too large to display	2028
derivativedivides	Expression too large to display	2031
default	Expression too large to display	2031

input
```
int((a+b*arctan(c/x))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x-b^3*c*(-1/c*x*arctan(c/x)^3-3/2*arctan(c/x)^2*ln(1+c^2/x^2)+3*ln(c/x
)*arctan(c/x)^2+3*arctan(c/x)^2*ln((1+I*c/x)/(1+c^2/x^2)^(1/2))-3*arctan(c
/x)^2*ln((1+I*c/x)^2/(1+c^2/x^2)-1)-I*arctan(c/x)^3+3/4*(2*I*Pi*csgn(((1+I
*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^3+2*I*Pi*csgn(I*((1+I*
c/x)^2/(1+c^2/x^2)-1))*csgn(I/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(I*((1+I*c/
x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))-2*I*Pi*csgn(I*(1+(1+I*c/x
)^2/(1+c^2/x^2)))*csgn(I*(1+(1+I*c/x)^2/(1+c^2/x^2))^2)^2+I*Pi*csgn(I*(1+(
1+I*c/x)^2/(1+c^2/x^2)))^2*csgn(I*(1+(1+I*c/x)^2/(1+c^2/x^2))^2)-I*Pi*csgn
(I*(1+I*c/x)/(1+c^2/x^2)^(1/2))^2*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))-I*Pi*csg
n(I/(1+(1+I*c/x)^2/(1+c^2/x^2))^2)*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))*csgn(I*
(1+I*c/x)^2/(1+c^2/x^2)/(1+(1+I*c/x)^2/(1+c^2/x^2))^2)-2*I*Pi*csgn(I*((1+I
*c/x)^2/(1+c^2/x^2)-1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/
(1+c^2/x^2)))^2+2*I*Pi*csgn(I*(1+I*c/x)/(1+c^2/x^2)^(1/2))*csgn(I*(1+I*c/x
)^2/(1+c^2/x^2))^2+I*Pi*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))*csgn(I*(1+I*c/x)^2
/(1+c^2/x^2)/(1+(1+I*c/x)^2/(1+c^2/x^2))^2)^2-2*I*Pi*csgn(((1+I*c/x)^2/(1+
c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2-2*I*Pi*csgn(I/(1+(1+I*c/x)^2/(1
+c^2/x^2)))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2))
)^2-2*I*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))
*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2+2*I*Pi*cs
gn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(((1+...
```

Fricas [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3 dx$$

input
```
integrate((a+b*arctan(c/x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
 a^3, x)
```

Sympy [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \int \left(a+b\,\text{atan}\left(\frac{c}{x}\right)\right)^3 dx$$

input
```
integrate((a+b*atan(c/x))**3,x)
```

output
```
Integral((a + b*atan(c/x))**3, x)
```

Maxima [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3 dx$$

input
```
integrate((a+b*arctan(c/x))^3,x, algorithm="maxima")
```

output
```
7/8*b^3*c*arctan(c/x)^3*arctan(x/c) + 3*a*b^2*c*arctan(c/x)^2*arctan(x/c)
+ 1/8*b^3*x*arctan2(c, x)^3 - 3/32*b^3*x*arctan2(c, x)*log(c^2 + x^2)^2 +
(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c)*a*b^2*c^2 + 7/32*(6*arct
an(c/x)^2*arctan(x/c)^2/c + 4*arctan(c/x)*arctan(x/c)^3/c + arctan(x/c)^4/
c)*b^3*c^2 + 3*b^3*c^2*integrate(1/32*arctan(c/x)*log(c^2 + x^2)^2/(c^2 +
x^2), x) + 12*b^3*c*integrate(1/32*x*arctan(c/x)^2/(c^2 + x^2), x) - 3*b^3
*c*integrate(1/32*x*log(c^2 + x^2)^2/(c^2 + x^2), x) + 3/2*(2*x*arctan(c/x
) + c*log(c^2 + x^2))*a^2*b + a^3*x + 28*b^3*integrate(1/32*x^2*arctan(c/x
)^3/(c^2 + x^2), x) + 3*b^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)^
2/(c^2 + x^2), x) + 96*a*b^2*integrate(1/32*x^2*arctan(c/x)^2/(c^2 + x^2),
 x) + 12*b^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)/(c^2 + x^2), x)
```

Giac [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3 dx$$

input
```
integrate((a+b*arctan(c/x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \int \left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3 dx$$

input
```
int((a + b*atan(c/x))^3,x)
```

output
```
int((a + b*atan(c/x))^3, x)
```

Reduce [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = 3atan\left(\frac{c}{x}\right)a^2bx + \left(\int atan\left(\frac{c}{x}\right)^3 dx\right)b^3 + 3\left(\int atan\left(\frac{c}{x}\right)^2 dx\right)a\,b^2 + \frac{3\log(c^2+x^2)\,a^2bc}{2} + a^3x$$

input
```
int((a+b*atan(c/x))^3,x)
```

output
```
(6*atan(c/x)*a**2*b*x + 2*int(atan(c/x)**3,x)*b**3 + 6*int(atan(c/x)**2,x)
*a*b**2 + 3*log(c**2 + x**2)*a**2*b*c + 2*a**3*x)/2
```

3.151 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx$

Optimal result 1127
Mathematica [A] (verified) 1128
Rubi [A] (verified) 1129
Maple [C] (warning: unable to verify) 1132
Fricas [F] 1133
Sympy [F] 1134
Maxima [F] 1134
Giac [F] 1134
Mupad [F(-1)] 1135
Reduce [F] 1135

Optimal result

Integrand size = 16, antiderivative size = 230

$$\begin{aligned}\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx &= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\\ &\quad+\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)\\ &\quad-\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{ic}{x}}\right)\\ &\quad+\frac{3}{2}b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right)\\ &\quad-\frac{3}{2}b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+\frac{ic}{x}}\right)\\ &\quad-\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+\frac{ic}{x}}\right)\\ &\quad+\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+\frac{ic}{x}}\right)\end{aligned}$$

output
```
2*(a+b*arccot(x/c))^3*arctanh(-1+2/(1+I*c/x))+3/2*I*b*(a+b*arccot(x/c))^2*
polylog(2,1-2/(1+I*c/x))-3/2*I*b*(a+b*arccot(x/c))^2*polylog(2,-1+2/(1+I*c
/x))+3/2*b^2*(a+b*arccot(x/c))*polylog(3,1-2/(1+I*c/x))-3/2*b^2*(a+b*arcco
t(x/c))*polylog(3,-1+2/(1+I*c/x))-3/4*I*b^3*polylog(4,1-2/(1+I*c/x))+3/4*I
*b^3*polylog(4,-1+2/(1+I*c/x))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.83

$$\begin{aligned}\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx &= a^3\log(x)-\frac{3}{2}ia^2b\left(\operatorname{PolyLog}\left(2,-\frac{ic}{x}\right)-\operatorname{PolyLog}\left(2,\frac{ic}{x}\right)\right)\\
&\quad+3ab^2\left(\frac{i\pi^3}{24}-\frac{2}{3}i\arctan\left(\frac{c}{x}\right)^3\right.\\
&\quad-\arctan\left(\frac{c}{x}\right)^2\log\left(1-e^{-2i\arctan\left(\frac{c}{x}\right)}\right)\\
&\quad+\arctan\left(\frac{c}{x}\right)^2\log\left(1+e^{2i\arctan\left(\frac{c}{x}\right)}\right)\\
&\quad-i\arctan\left(\frac{c}{x}\right)\operatorname{PolyLog}\left(2,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)\\
&\quad-i\arctan\left(\frac{c}{x}\right)\operatorname{PolyLog}\left(2,-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\\
&\quad-\frac{1}{2}\operatorname{PolyLog}\left(3,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)\\
&\quad\left.+\frac{1}{2}\operatorname{PolyLog}\left(3,-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)+\frac{1}{64}ib^3\left(\pi^4\right.\\
&\quad-32\arctan\left(\frac{c}{x}\right)^4+64i\arctan\left(\frac{c}{x}\right)^3\log\left(1-e^{-2i\arctan\left(\frac{c}{x}\right)}\right)\\
&\quad-64i\arctan\left(\frac{c}{x}\right)^3\log\left(1+e^{2i\arctan\left(\frac{c}{x}\right)}\right)\\
&\quad-96\arctan\left(\frac{c}{x}\right)^2\operatorname{PolyLog}\left(2,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)\\
&\quad-96\arctan\left(\frac{c}{x}\right)^2\operatorname{PolyLog}\left(2,-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\\
&\quad+96i\arctan\left(\frac{c}{x}\right)\operatorname{PolyLog}\left(3,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)\\
&\quad-96i\arctan\left(\frac{c}{x}\right)\operatorname{PolyLog}\left(3,-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\\
&\quad+48\operatorname{PolyLog}\left(4,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)\\
&\quad\left.+48\operatorname{PolyLog}\left(4,-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)\end{aligned}$$

input
```
Integrate[(a + b*ArcTan[c/x])^3/x,x]
```

output
```
a^3*Log[x] - ((3*I)/2)*a^2*b*(PolyLog[2, ((-I)*c)/x] - PolyLog[2, (I*c)/x]
) + 3*a*b^2*((I/24)*Pi^3 - ((2*I)/3)*ArcTan[c/x]^3 - ArcTan[c/x]^2*Log[1 -
 E^((-2*I)*ArcTan[c/x])] + ArcTan[c/x]^2*Log[1 + E^((2*I)*ArcTan[c/x])] -
I*ArcTan[c/x]*PolyLog[2, E^((-2*I)*ArcTan[c/x])] - I*ArcTan[c/x]*PolyLog[2
, -E^((2*I)*ArcTan[c/x])] - PolyLog[3, E^((-2*I)*ArcTan[c/x])]/2 + PolyLog
[3, -E^((2*I)*ArcTan[c/x])]/2) + (I/64)*b^3*(Pi^4 - 32*ArcTan[c/x]^4 + (64
*I)*ArcTan[c/x]^3*Log[1 - E^((-2*I)*ArcTan[c/x])] - (64*I)*ArcTan[c/x]^3*L
og[1 + E^((2*I)*ArcTan[c/x])] - 96*ArcTan[c/x]^2*PolyLog[2, E^((-2*I)*ArcT
an[c/x])] - 96*ArcTan[c/x]^2*PolyLog[2, -E^((2*I)*ArcTan[c/x])] + (96*I)*A
rcTan[c/x]*PolyLog[3, E^((-2*I)*ArcTan[c/x])] - (96*I)*ArcTan[c/x]*PolyLog
[3, -E^((2*I)*ArcTan[c/x])] + 48*PolyLog[4, E^((-2*I)*ArcTan[c/x])] + 48*P
olyLog[4, -E^((2*I)*ArcTan[c/x])])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5359, 5357, 5523, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}dx$$

$$\Big\downarrow \text{5359}$$

$$-\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3d\frac{1}{x}$$

$$\Big\downarrow \text{5357}$$

$$6bc\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\text{arctanh}\left(1-\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-$$
$$2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3$$

$$\Big\downarrow \; 5523$$

$$6bc\left(\frac{1}{2}\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\log\left(2-\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{1}{2}\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\log\left(\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-$$
$$2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3$$

$$\Big\downarrow \; 5529$$

$$6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2c}-ib\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\right.$$
$$2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3$$

$$\Big\downarrow \; 5533$$

$$6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2c}-ib\left(\frac{i\,\text{PolyLog}\left(3,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}-\frac{1}{2}ib\int\right.\right.\right.$$
$$2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3$$

$$\Big\downarrow \; 7164$$

$$6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2c}-ib\left(\frac{i\,\text{PolyLog}\left(3,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}+\frac{b\,\text{Pol}}{}\right.\right.\right.$$
$$2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3$$

input
```
Int[(a + b*ArcTan[c/x])^3/x,x]
```

output
```
-2*(a + b*ArcTan[c/x])^3*ArcTanh[1 - 2/(1 + (I*c)/x)] + 6*b*c*((((I/2)*(a
+ b*ArcTan[c/x])^2*PolyLog[2, 1 - 2/(1 + (I*c)/x)])/c - I*b*(((I/2)*(a + b
*ArcTan[c/x])*PolyLog[3, 1 - 2/(1 + (I*c)/x)])/c + (b*PolyLog[4, 1 - 2/(1
+ (I*c)/x)])/(4*c)))/2 + (((-1/2*I)*(a + b*ArcTan[c/x])^2*PolyLog[2, -1 +
2/(1 + (I*c)/x)])/c + I*b*(((I/2)*(a + b*ArcTan[c/x])*PolyLog[3, -1 + 2/(1
 + (I*c)/x)])/c + (b*PolyLog[4, -1 + 2/(1 + (I*c)/x)])/(4*c)))/2)
```

Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.95 (sec) , antiderivative size = 2225, normalized size of antiderivative = 9.67

method	result	size
parts	Expression too large to display	2225
derivativedivides	Expression too large to display	2226
default	Expression too large to display	2226

input
```
int((a+b*arctan(c/x))^3/x,x,method=_RETURNVERBOSE)
```

output
```
a^3*ln(x)+b^3*(-ln(c/x)*arctan(c/x)^3+arctan(c/x)^3*ln((1+I*c/x)^2/(1+c^2/
x^2)-1)-arctan(c/x)^3*ln(1+(1+I*c/x)/(1+c^2/x^2)^(1/2))+3*I*arctan(c/x)^2*
polylog(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-6*arctan(c/x)*polylog(3,-(1+I*c/x)
/(1+c^2/x^2)^(1/2))-6*I*polylog(4,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-arctan(c/x
)^3*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))+3*I*arctan(c/x)^2*polylog(2,(1+I*c/x
)/(1+c^2/x^2)^(1/2))-6*arctan(c/x)*polylog(3,(1+I*c/x)/(1+c^2/x^2)^(1/2))-
6*I*polylog(4,(1+I*c/x)/(1+c^2/x^2)^(1/2))-1/2*I*Pi*(csgn(I*((1+I*c/x)^2/(
1+c^2/x^2)-1))*csgn(I/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(I*((1+I*c/x)^2/(1+
c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))-csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1
))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2-csgn(
I/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*
c/x)^2/(1+c^2/x^2)))^2+csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(
1+c^2/x^2)))^3-csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^
2)))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2+csgn(
I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(((1+I*c/x)
^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))+csgn(((1+I*c/x)^2/(1+c^2/x^
2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^3-csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(
1+I*c/x)^2/(1+c^2/x^2)))^2+1)*arctan(c/x)^3-3/2*I*arctan(c/x)^2*polylog(2,
-(1+I*c/x)^2/(1+c^2/x^2))+3/2*arctan(c/x)*polylog(3,-(1+I*c/x)^2/(1+c^2/x^
2))+3/4*I*polylog(4,-(1+I*c/x)^2/(1+c^2/x^2)))+3*a^2*b*(-ln(c/x)*arctan...
```

Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x)
+ a^3)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

input
```
integrate((a+b*atan(c/x))**3/x,x)
```

output
```
Integral((a + b*atan(c/x))**3/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + 1/32*integrate((28*b^3*arctan2(c, x)^3 + 3*b^3*arctan2(c, x)*
log(c^2 + x^2)^2 + 96*a*b^2*arctan2(c, x)^2 + 96*a^2*b*arctan2(c, x))/x, x
)
```

Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

input
```
int((a + b*atan(c/x))^3/x,x)
```

output
```
int((a + b*atan(c/x))^3/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx=3\left(\int \frac{atan\left(\frac{c}{x}\right)}{x}dx\right)a^2b+\left(\int \frac{atan\left(\frac{c}{x}\right)^3}{x}dx\right)b^3+3\left(\int \frac{atan\left(\frac{c}{x}\right)^2}{x}dx\right)a\,b^2+\log(x)\,a^3$$

input
```
int((a+b*atan(c/x))^3/x,x)
```

output
```
3*int(atan(c/x)/x,x)*a**2*b + int(atan(c/x)**3/x,x)*b**3 + 3*int(atan(c/x)
**2/x,x)*a*b**2 + log(x)*a**3
```

3.152 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$

Optimal result 1136
Mathematica [A] (verified) 1137
Rubi [A] (verified) 1137
Maple [B] (verified) 1140
Fricas [F] 1141
Sympy [F] 1141
Maxima [F] 1141
Giac [F] 1142
Mupad [F(-1)] 1142
Reduce [F] 1143

Optimal result

Integrand size = 16, antiderivative size = 136

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = -\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{c}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{x}-\frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c}-\frac{3ib^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)}{c}-\frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right)}{2c}$$

output
```
-I*(a+b*arccot(x/c))^3/c-(a+b*arccot(x/c))^3/x-3*b*(a+b*arccot(x/c))^2*ln(
2/(1+I*c/x))/c-3*I*b^2*(a+b*arccot(x/c))*polylog(2,1-2/(1+I*c/x))/c-3/2*b^
3*polylog(3,1-2/(1+I*c/x))/c
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.63

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx =$$
$$-\frac{2a^3c+6a^2bc\arctan\left(\frac{c}{x}\right)+6ab^2c\arctan\left(\frac{c}{x}\right)^2-6iab^2x\arctan\left(\frac{c}{x}\right)^2+2b^3c\arctan\left(\frac{c}{x}\right)^3-2ib^3x\arctan\left(\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c/x])^3/x^2,x]
```

output
```
-1/2*(2*a^3*c + 6*a^2*b*c*ArcTan[c/x] + 6*a*b^2*c*ArcTan[c/x]^2 - (6*I)*a*
b^2*x*ArcTan[c/x]^2 + 2*b^3*c*ArcTan[c/x]^3 - (2*I)*b^3*x*ArcTan[c/x]^3 +
12*a*b^2*x*ArcTan[c/x]*Log[1 + E^((2*I)*ArcTan[c/x])] + 6*b^3*x*ArcTan[c/x
]^2*Log[1 + E^((2*I)*ArcTan[c/x])] - 3*a^2*b*x*Log[1 + c^2/x^2] - (6*I)*b^
2*x*(a + b*ArcTan[c/x])*PolyLog[2, -E^((2*I)*ArcTan[c/x])] + 3*b^3*x*PolyL
og[3, -E^((2*I)*ArcTan[c/x])])/(c*x)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5363, 5345, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}dx$$

$$\Big\downarrow \text{ 5363}$$

$$-\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3d\frac{1}{x}$$

$$\Big\downarrow \text{ 5345}$$

$$3bc\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\left(\frac{c^2}{x^2}+1\right)x}d\frac{1}{x}-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}$$

$$\Big\downarrow \text{5455}$$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}+3bc\left(-\frac{\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{i-\frac{c}{x}}d\frac{1}{x}}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^2}\right)$$

$$\Big\downarrow \text{5379}$$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}+ 3bc\left(-\frac{\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{c}-2b\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)\log\left(\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^2}\right)$$

$$\Big\downarrow \text{5529}$$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}+ 3bc\left(-\frac{\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{c}-2b\left(\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}\right)}{c}-\frac{i(a+b$$

$$\Big\downarrow \text{7164}$$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}+ 3bc\left(-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^2}-\frac{\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{c}-2b\left(-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}-\frac{b\,\text{PolyLog}\left(3,1-\right.}{4c}\right.}{c}\right.$$

input
```
Int[(a + b*ArcTan[c/x])^3/x^2,x]
```

output
```
-((a + b*ArcTan[c/x])^3/x) + 3*b*c*(((-1/3*I)*(a + b*ArcTan[c/x])^3)/(b*c^
2) - (((a + b*ArcTan[c/x])^2*Log[2/(1 + (I*c)/x)])/c - 2*b*(((-1/2*I)*(a +
 b*ArcTan[c/x])*PolyLog[2, 1 - 2/(1 + (I*c)/x)])/c - (b*PolyLog[3, 1 - 2/(
1 + (I*c)/x)])/(4*c)))/c)
```

Defintions of rubi rules used

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(129) = 258$.

Time = 1.95 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.02

method	result
derivativedivides	$-\frac{\frac{c\,a^3}{x}+b^3\left(\arctan\left(\frac{c}{x}\right)^3\left(\frac{c}{x}+i\right)-2i\arctan\left(\frac{c}{x}\right)^3+3\arctan\left(\frac{c}{x}\right)^2\ln\left(1+\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)-3i\arctan\left(\frac{c}{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right.\right.}{}$
default	$-\frac{\frac{c\,a^3}{x}+b^3\left(\arctan\left(\frac{c}{x}\right)^3\left(\frac{c}{x}+i\right)-2i\arctan\left(\frac{c}{x}\right)^3+3\arctan\left(\frac{c}{x}\right)^2\ln\left(1+\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)-3i\arctan\left(\frac{c}{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right.\right.}{}$
parts	$-\frac{a^3}{x}-\frac{b^3\left(\arctan\left(\frac{c}{x}\right)^3\left(\frac{c}{x}+i\right)-2i\arctan\left(\frac{c}{x}\right)^3+3\arctan\left(\frac{c}{x}\right)^2\ln\left(1+\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)-3i\arctan\left(\frac{c}{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)}{1+\frac{c^2}{x^2}}\right.\right.}{c}$

input
```
int((a+b*arctan(c/x))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c*(c/x*a^3+b^3*(arctan(c/x)^3*(c/x+I)-2*I*arctan(c/x)^3+3*arctan(c/x)^2
*ln(1+(1+I*c/x)^2/(1+c^2/x^2))-3*I*arctan(c/x)*polylog(2,-(1+I*c/x)^2/(1+c
^2/x^2))+3/2*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2)))+3*a*b^2*(arctan(c/x)^2*(
c/x+I)+2*arctan(c/x)*ln(1+(1+I*c/x)^2/(1+c^2/x^2))-2*I*arctan(c/x)^2-I*pol
ylog(2,-(1+I*c/x)^2/(1+c^2/x^2)))+3*a^2*b*(c/x*arctan(c/x)-1/2*ln(1+c^2/x^
2)))
```

Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x)
+ a^3)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*atan(c/x))**3/x**2,x)
```

output
```
Integral((a + b*atan(c/x))**3/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="maxima")
```

output
```
-3/2*a^2*b*(2*c*arctan(c/x)/x - log(c^2/x^2 + 1))/c - a^3/x - 1/32*(4*b^3*
arctan2(c, x)^3 - 3*b^3*arctan2(c, x)*log(c^2 + x^2)^2 - (28*b^3*arctan(c/
x)^3*arctan(x/c)/c + 896*b^3*c^2*integrate(1/32*arctan(c/x)^3/(c^2*x^2 + x
^4), x) + 96*b^3*c^2*integrate(1/32*arctan(c/x)*log(c^2 + x^2)^2/(c^2*x^2
+ x^4), x) + 3072*a*b^2*c^2*integrate(1/32*arctan(c/x)^2/(c^2*x^2 + x^4),
x) + 96*a*b^2*arctan(c/x)^2*arctan(x/c)/c - 384*b^3*c*integrate(1/32*x*arc
tan(c/x)^2/(c^2*x^2 + x^4), x) + 96*b^3*c*integrate(1/32*x*log(c^2 + x^2)^
2/(c^2*x^2 + x^4), x) + 32*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/
c)*a*b^2 + 7*(6*arctan(c/x)^2*arctan(x/c)^2/c + 4*arctan(c/x)*arctan(x/c)^
3/c + arctan(x/c)^4/c)*b^3 + 96*b^3*integrate(1/32*x^2*arctan(c/x)*log(c^2
 + x^2)^2/(c^2*x^2 + x^4), x) - 384*b^3*integrate(1/32*x^2*arctan(c/x)*log
(c^2 + x^2)/(c^2*x^2 + x^4), x))*x)/x
```

Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$$

input
```
int((a + b*atan(c/x))^3/x^2,x)
```

output
```
int((a + b*atan(c/x))^3/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$$
$$= \frac{-2atan\left(\frac{c}{x}\right)^3 b^3c-6atan\left(\frac{c}{x}\right)^2 a\,b^2c-6atan\left(\frac{c}{x}\right) a^2bc-12\left(\int \frac{atan\left(\frac{c}{x}\right)}{c^2x+x^3}dx\right) a\,b^2c^2x-6\left(\int \frac{atan\left(\frac{c}{x}\right)^2}{c^2x+x^3}dx\right) b^3c^2x}{2cx}$$

input
```
int((a+b*atan(c/x))^3/x^2,x)
```

output
```
( - 2*atan(c/x)**3*b**3*c - 6*atan(c/x)**2*a*b**2*c - 6*atan(c/x)*a**2*b*c
 - 12*int(atan(c/x)/(c**2*x + x**3),x)*a*b**2*c**2*x - 6*int(atan(c/x)**2/
(c**2*x + x**3),x)*b**3*c**2*x + 3*log(c**2 + x**2)*a**2*b*x - 6*log(x)*a*
*2*b*x - 2*a**3*c)/(2*c*x)
```

3.153 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$

Optimal result 1144
Mathematica [A] (verified) 1145
Rubi [A] (verified) 1145
Maple [B] (verified) 1149
Fricas [F] 1150
Sympy [F] 1150
Maxima [F] 1151
Giac [F] 1151
Mupad [F(-1)] 1152
Reduce [F] 1152

Optimal result

Integrand size = 16, antiderivative size = 147

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx = \frac{3ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}+\frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2cx}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2}+\frac{3b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c^2}+\frac{3ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)}{2c^2}$$

output
```
3/2*I*b*(a+b*arccot(x/c))^2/c^2+3/2*b*(a+b*arccot(x/c))^2/c/x-1/2*(a+b*arc
cot(x/c))^3/c^2-1/2*(a+b*arccot(x/c))^3/x^2+3*b^2*(a+b*arccot(x/c))*ln(2/(
1+I*c/x))/c^2+3/2*I*b^3*polylog(2,1-2/(1+I*c/x))/c^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.21

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$
$$= \frac{3b^2(c-ix)(-a(c+ix)+bx)\arctan\left(\frac{c}{x}\right)^2-b^3(c^2+x^2)\arctan\left(\frac{c}{x}\right)^3-3b\arctan\left(\frac{c}{x}\right)\left(a(-2bcx+a(c^2+\cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcTan[c/x])^3/x^3,x]
```

output
```
(3*b^2*(c - I*x)*(-(a*(c + I*x)) + b*x)*ArcTan[c/x]^2 - b^3*(c^2 + x^2)*Ar
cTan[c/x]^3 - 3*b*ArcTan[c/x]*(a*(-2*b*c*x + a*(c^2 + x^2)) - 2*b^2*x^2*Lo
g[1 + E^((2*I)*ArcTan[c/x])]) + a*(a*c*(-(a*c) + 3*b*x) + 6*b^2*x^2*Log[1/
Sqrt[1 + c^2/x^2]]) - (3*I)*b^3*x^2*PolyLog[2, -E^((2*I)*ArcTan[c/x])])/(2
*c^2*x^2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5363, 5361, 5451, 5345, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$
$$\Big\downarrow \text{ 5363}$$
$$-\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}d\frac{1}{x}$$
$$\Big\downarrow \text{ 5361}$$
$$\frac{3}{2}bc\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\left(\frac{c^2}{x^2}+1\right)x^2}d\frac{1}{x}-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

$$\Big\downarrow \; 5451$$

$$\frac{3}{2}bc\left(\frac{\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}}{c^2}-\frac{\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}}{c^2}\right)-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

$$\Big\downarrow \; 5345$$

$$\frac{3}{2}bc\left(\frac{\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}-2bc\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\left(\frac{c^2}{x^2}+1\right)x}d\frac{1}{x}}{c^2}-\frac{\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}}{c^2}\right)-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

$$\Big\downarrow \; 5419$$

$$\frac{3}{2}bc\left(\frac{\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}-2bc\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\left(\frac{c^2}{x^2}+1\right)x}d\frac{1}{x}}{c^2}-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^3}\right)-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

$$\Big\downarrow \; 5455$$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}+\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^3}+\frac{\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}-2bc\left(-\frac{\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{i-\frac{c}{x}}d\frac{1}{x}}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^2}\right)}{c^2}\right)$$

$$\Big\downarrow \; 5379$$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}+\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^3}+\frac{\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{c}-b\int \frac{\log\left(\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^2}\right)}{c^2}\right)$$

$$\Big\downarrow \; 2849$$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}+\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^3}+\frac{\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}-2bc\left(-\frac{-\frac{ib\int\frac{\log\left(\frac{2}{\frac{ic}{x}+1}\right)}{1-\frac{2}{\frac{ic}{x}+1}}d\frac{1}{\frac{ic}{x}+1}}{c}+\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{c}}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^2}\cdots\right.}{c^2}\right.$$

$\downarrow$ 2752

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}+\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^3}+\frac{\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}-2bc\left(-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{c}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+\cdots}\right)}{2c}}{c}\cdots\right.}{c^2}\right.$$

input
```
Int[(a + b*ArcTan[c/x])^3/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c/x])^3/x^2 + (3*b*c*(-1/3*(a + b*ArcTan[c/x])^3/(b*c^3
) + ((a + b*ArcTan[c/x])^2/x - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c/x])^2)/(b*
c^2) - (((a + b*ArcTan[c/x])*Log[2/(1 + (I*c)/x)])/c + ((I/2)*b*PolyLog[2,
 1 - 2/(1 + (I*c)/x)])/c)/c))/c^2))/2
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(133) = 266$.

Time = 5.45 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.18

method	result
derivativedivides	$-\frac{\frac{a^3c^2}{2x^2}+b^3\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^3}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^3}{2}-\frac{3c\arctan\left(\frac{c}{x}\right)^2}{2x}+\frac{3\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\frac{3i\left(\ln\left(\frac{c}{x}-i\right)\ln\left(1+\frac{c^2}{x^2}\right)-\frac{\ln\left(\frac{c}{x}-i\right)^2}{2}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
default	$-\frac{\frac{a^3c^2}{2x^2}+b^3\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^3}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^3}{2}-\frac{3c\arctan\left(\frac{c}{x}\right)^2}{2x}+\frac{3\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\frac{3i\left(\ln\left(\frac{c}{x}-i\right)\ln\left(1+\frac{c^2}{x^2}\right)-\frac{\ln\left(\frac{c}{x}-i\right)^2}{2}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
parts	$-\frac{a^3}{2x^2}-\frac{b^3\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^3}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^3}{2}-\frac{3c\arctan\left(\frac{c}{x}\right)^2}{2x}+\frac{3\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\frac{3i\left(\ln\left(\frac{c}{x}-i\right)\ln\left(1+\frac{c^2}{x^2}\right)-\frac{\ln\left(\frac{c}{x}-i\right)\cdots}{2}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$

input
```
int((a+b*arctan(c/x))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*(1/2*a^3*c^2/x^2+b^3*(1/2*c^2/x^2*arctan(c/x)^3+1/2*arctan(c/x)^3-3
/2*c/x*arctan(c/x)^2+3/2*arctan(c/x)*ln(1+c^2/x^2)+3/4*I*(ln(c/x-I)*ln(1+c
^2/x^2)-1/2*ln(c/x-I)^2-dilog(-1/2*I*(c/x+I))-ln(c/x-I)*ln(-1/2*I*(c/x+I))
)-3/4*I*(ln(c/x+I)*ln(1+c^2/x^2)-1/2*ln(c/x+I)^2-dilog(1/2*I*(c/x-I))-ln(c
/x+I)*ln(1/2*I*(c/x-I))))+3*a*b^2*(1/2*c^2/x^2*arctan(c/x)^2+1/2*arctan(c/
x)^2-c/x*arctan(c/x)+1/2*ln(1+c^2/x^2))+3*a^2*b*(1/2*c^2/x^2*arctan(c/x)-1
/2*c/x+1/2*arctan(c/x)))
```

Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x)
+ a^3)/x^3, x)
```

Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*atan(c/x))**3/x**3,x)
```

output
```
Integral((a + b*atan(c/x))**3/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="maxima")
```

output
```
3/2*(c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*a^2*b + 3/2*(2*c*(
arctan(x/c)/c^3 + 1/(c^2*x))*arctan(c/x) + (arctan(x/c)^2 - log(c^2 + x^2)
 + 2*log(x))/c^2)*a*b^2 - 3/2*a*b^2*arctan(c/x)^2/x^2 - 1/2*a^3/x^2 + 1/32
*(4*(128*c^3*integrate(1/32*arctan(c/x)^3/(c^3*x^3 + c*x^5), x) - 96*c^2*i
ntegrate(1/32*x*arctan(c/x)^2/(c^3*x^3 + c*x^5), x) - 24*c^2*integrate(1/3
2*x*log(c^2 + x^2)^2/(c^3*x^3 + c*x^5), x) + 128*c*integrate(1/32*x^2*arct
an(c/x)^3/(c^3*x^3 + c*x^5), x) + 192*c*integrate(1/32*x^2*arctan(c/x)/(c^
3*x^3 + c*x^5), x) - 3*arctan(c/x)^2*arctan(x/c)/c^2 - 3*arctan(c/x)*arcta
n(x/c)^2/c^2 - arctan(x/c)^3/c^2 - 24*integrate(1/32*x^3*log(c^2 + x^2)^2/
(c^3*x^3 + c*x^5), x) + 96*integrate(1/32*x^3*log(c^2 + x^2)/(c^3*x^3 + c*
x^5), x))*c^2*x^2 - 8*c^2*arctan2(c, x)^3 - 8*x^2*arctan2(c, x)^3 + 12*c*x
*arctan2(c, x)^2 - 3*c*x*log(c^2 + x^2)^2)*b^3/(c^2*x^2)
```

Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

input
```
int((a + b*atan(c/x))^3/x^3,x)
```

output
```
int((a + b*atan(c/x))^3/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$
$$=\frac{-atan\left(\frac{c}{x}\right)^3 b^3c^2-atan\left(\frac{c}{x}\right)^3 b^3x^2-3atan\left(\frac{c}{x}\right)^2 a\,b^2c^2-3atan\left(\frac{c}{x}\right)^2 a\,b^2x^2+3atan\left(\frac{c}{x}\right)^2 b^3cx-3atan\left(\frac{c}{x}\right) a}{}$$

input
```
int((a+b*atan(c/x))^3/x^3,x)
```

output
```
( - atan(c/x)**3*b**3*c**2 - atan(c/x)**3*b**3*x**2 - 3*atan(c/x)**2*a*b**
2*c**2 - 3*atan(c/x)**2*a*b**2*x**2 + 3*atan(c/x)**2*b**3*c*x - 3*atan(c/x
)*a**2*b*c**2 - 3*atan(c/x)*a**2*b*x**2 + 6*atan(c/x)*a*b**2*c*x - 3*atan(
c/x)*b**3*c**2 - 3*atan(c/x)*b**3*x**2 - 6*int(atan(c/x)/(c**2*x**3 + x**5
),x)*b**3*c**4*x**2 - 3*log(c**2 + x**2)*a*b**2*x**2 + 6*log(x)*a*b**2*x**
2 - a**3*c**2 + 3*a**2*b*c*x + 3*b**3*c*x)/(2*c**2*x**2)
```

3.154 $\int x^2 \arctan\left(\sqrt{x}\right) \, dx$

Optimal result 1153
Mathematica [A] (verified) 1153
Rubi [A] (verified) 1154
Maple [A] (verified) 1156
Fricas [A] (verification not implemented) 1156
Sympy [A] (verification not implemented) 1156
Maxima [A] (verification not implemented) 1157
Giac [A] (verification not implemented) 1157
Mupad [B] (verification not implemented) 1158
Reduce [B] (verification not implemented) 1158

Optimal result

Integrand size = 10, antiderivative size = 51

$$\int x^2 \arctan\left(\sqrt{x}\right) \, dx = -\frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15} + \frac{\arctan\left(\sqrt{x}\right)}{3} + \frac{1}{3}x^3 \arctan\left(\sqrt{x}\right)$$

output
```
-1/3*x^(1/2)+1/9*x^(3/2)-1/15*x^(5/2)+1/3*arctan(x^(1/2))+1/3*x^3*arctan(x
^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int x^2 \arctan\left(\sqrt{x}\right) \, dx = \frac{1}{45}\left(\sqrt{x}\left(-15 + 5x - 3x^2\right) + 15\left(1 + x^3\right) \arctan\left(\sqrt{x}\right)\right)$$

input
```
Integrate[x^2*ArcTan[Sqrt[x]],x]
```

output
```
(Sqrt[x]*(-15 + 5*x - 3*x^2) + 15*(1 + x^3)*ArcTan[Sqrt[x]])/45
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5361, 60, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}x^3\arctan\left(\sqrt{x}\right)-\frac{1}{6}\int \frac{x^{5/2}}{x+1}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}\left(\int \frac{x^{3/2}}{x+1}dx-\frac{2x^{5/2}}{5}\right)+\frac{1}{3}x^3\arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}\left(-\int \frac{\sqrt{x}}{x+1}dx-\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{3}\right)+\frac{1}{3}x^3\arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}\left(\int \frac{1}{\sqrt{x}(x+1)}dx-\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{3}-2\sqrt{x}\right)+\frac{1}{3}x^3\arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(2\int \frac{1}{x+1}d\sqrt{x}-\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{3}-2\sqrt{x}\right)+\frac{1}{3}x^3\arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{6}\left(2\arctan\left(\sqrt{x}\right)-\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{3}-2\sqrt{x}\right)+\frac{1}{3}x^3\arctan\left(\sqrt{x}\right)$$

input
```
Int[x^2*ArcTan[Sqrt[x]],x]
```

output
```
(x^3*ArcTan[Sqrt[x]])/3 + (-2*Sqrt[x] + (2*x^(3/2))/3 - (2*x^(5/2))/5 + 2*
ArcTan[Sqrt[x]])/6
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59

method	result	size
meijerg	$-\frac{\sqrt{x}\,(21x^2-35x+105)}{315}+\frac{(7x^3+7)\arctan(\sqrt{x})}{21}$	30
derivativedivides	$-\frac{\sqrt{x}}{3}+\frac{x^{\frac{3}{2}}}{9}-\frac{x^{\frac{5}{2}}}{15}+\frac{\arctan(\sqrt{x})}{3}+\frac{x^3\arctan(\sqrt{x})}{3}$	32
default	$-\frac{\sqrt{x}}{3}+\frac{x^{\frac{3}{2}}}{9}-\frac{x^{\frac{5}{2}}}{15}+\frac{\arctan(\sqrt{x})}{3}+\frac{x^3\arctan(\sqrt{x})}{3}$	32
parts	$-\frac{\sqrt{x}}{3}+\frac{x^{\frac{3}{2}}}{9}-\frac{x^{\frac{5}{2}}}{15}+\frac{\arctan(\sqrt{x})}{3}+\frac{x^3\arctan(\sqrt{x})}{3}$	32

input
```
int(x^2*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/315*x^(1/2)*(21*x^2-35*x+105)+1/21*(7*x^3+7)*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int x^2\arctan\left(\sqrt{x}\right)\,dx=\frac{1}{3}\left(x^3+1\right)\arctan\left(\sqrt{x}\right)-\frac{1}{45}\left(3\,x^2-5\,x+15\right)\sqrt{x}$$

input
```
integrate(x^2*arctan(x^(1/2)),x, algorithm="fricas")
```

output
```
1/3*(x^3 + 1)*arctan(sqrt(x)) - 1/45*(3*x^2 - 5*x + 15)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^2\arctan\left(\sqrt{x}\right)\,dx=-\frac{x^{\frac{5}{2}}}{15}+\frac{x^{\frac{3}{2}}}{9}-\frac{\sqrt{x}}{3}+\frac{x^3\operatorname{atan}\left(\sqrt{x}\right)}{3}+\frac{\operatorname{atan}\left(\sqrt{x}\right)}{3}$$

input
```
integrate(x**2*atan(x**(1/2)),x)
```

output
```
-x**(5/2)/15 + x**(3/2)/9 - sqrt(x)/3 + x**3*atan(sqrt(x))/3 + atan(sqrt(x
))/3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = \frac{1}{3}\,x^3 \arctan\left(\sqrt{x}\right) - \frac{1}{15}\,x^{\frac{5}{2}} + \frac{1}{9}\,x^{\frac{3}{2}} - \frac{1}{3}\,\sqrt{x} + \frac{1}{3}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x^2*arctan(x^(1/2)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan(sqrt(x)) - 1/15*x^(5/2) + 1/9*x^(3/2) - 1/3*sqrt(x) + 1/3*a
rctan(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = \frac{1}{3}\,x^3 \arctan\left(\sqrt{x}\right) - \frac{1}{15}\,x^{\frac{5}{2}} + \frac{1}{9}\,x^{\frac{3}{2}} - \frac{1}{3}\,\sqrt{x} + \frac{1}{3}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x^2*arctan(x^(1/2)),x, algorithm="giac")
```

output
```
1/3*x^3*arctan(sqrt(x)) - 1/15*x^(5/2) + 1/9*x^(3/2) - 1/3*sqrt(x) + 1/3*a
rctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = \frac{\operatorname{atan}\left(\sqrt{x}\right)}{3} + \frac{x^3\operatorname{atan}\left(\sqrt{x}\right)}{3} - \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15}$$

input
```
int(x^2*atan(x^(1/2)),x)
```

output
```
atan(x^(1/2))/3 + (x^3*atan(x^(1/2)))/3 - x^(1/2)/3 + x^(3/2)/9 - x^(5/2)/
15
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = \frac{atan\left(\sqrt{x}\right)x^3}{3} + \frac{atan\left(\sqrt{x}\right)}{3} - \frac{\sqrt{x}\,x^2}{15} + \frac{\sqrt{x}\,x}{9} - \frac{\sqrt{x}}{3}$$

input
```
int(x^2*atan(x^(1/2)),x)
```

output
```
(15*atan(sqrt(x))*x**3 + 15*atan(sqrt(x)) - 3*sqrt(x)*x**2 + 5*sqrt(x)*x -
 15*sqrt(x))/45
```

3.155 $\int x \arctan\left(\sqrt{x}\right)\,dx$

Optimal result . . . 1159
Mathematica [A] (verified) . . . 1159
Rubi [A] (verified) . . . 1160
Maple [A] (verified) . . . 1161
Fricas [A] (verification not implemented) . . . 1162
Sympy [A] (verification not implemented) . . . 1162
Maxima [A] (verification not implemented) . . . 1163
Giac [A] (verification not implemented) . . . 1163
Mupad [B] (verification not implemented) . . . 1163
Reduce [B] (verification not implemented) . . . 1164

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int x \arctan\left(\sqrt{x}\right)\,dx = \frac{\sqrt{x}}{2} - \frac{x^{3/2}}{6} - \frac{\arctan\left(\sqrt{x}\right)}{2} + \frac{1}{2}x^2 \arctan\left(\sqrt{x}\right)$$

output
```
1/2*x^(1/2)-1/6*x^(3/2)-1/2*arctan(x^(1/2))+1/2*x^2*arctan(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int x \arctan\left(\sqrt{x}\right)\,dx = \frac{1}{6}\left(-\left((-3+x)\sqrt{x}\right) + 3\left(-1+x^2\right)\arctan\left(\sqrt{x}\right)\right)$$

input
```
Integrate[x*ArcTan[Sqrt[x]],x]
```

output
```
(-((-3 + x)*Sqrt[x]) + 3*(-1 + x^2)*ArcTan[Sqrt[x]])/6
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5361, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)-\frac{1}{4}\int \frac{x^{3/2}}{x+1}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\int \frac{\sqrt{x}}{x+1}dx-\frac{2x^{3/2}}{3}\right)+\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(-\int \frac{1}{\sqrt{x}(x+1)}dx-\frac{2x^{3/2}}{3}+2\sqrt{x}\right)+\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-2\int \frac{1}{x+1}d\sqrt{x}-\frac{2x^{3/2}}{3}+2\sqrt{x}\right)+\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(-2\arctan\left(\sqrt{x}\right)-\frac{2x^{3/2}}{3}+2\sqrt{x}\right)+\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)$$

input
```
Int[x*ArcTan[Sqrt[x]],x]
```

output
```
(2*Sqrt[x] - (2*x^(3/2))/3 - 2*ArcTan[Sqrt[x]])/4 + (x^2*ArcTan[Sqrt[x]])/
2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

method	result	size
meijerg	$\frac{\sqrt{x}\,(-5x+15)}{30}-\frac{(-5x^2+5)\arctan(\sqrt{x})}{10}$	25
derivativedivides	$\frac{\sqrt{x}}{2}-\frac{x^{\frac{3}{2}}}{6}-\frac{\arctan(\sqrt{x})}{2}+\frac{x^2\arctan(\sqrt{x})}{2}$	27
default	$\frac{\sqrt{x}}{2}-\frac{x^{\frac{3}{2}}}{6}-\frac{\arctan(\sqrt{x})}{2}+\frac{x^2\arctan(\sqrt{x})}{2}$	27
parts	$\frac{\sqrt{x}}{2}-\frac{x^{\frac{3}{2}}}{6}-\frac{\arctan(\sqrt{x})}{2}+\frac{x^2\arctan(\sqrt{x})}{2}$	27

input
```
int(x*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/30*x^(1/2)*(-5*x+15)-1/10*(-5*x^2+5)*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.48

$$\int x \arctan\left(\sqrt{x}\right)\,dx = \frac{1}{2}\left(x^2-1\right)\arctan\left(\sqrt{x}\right)-\frac{1}{6}\left(x-3\right)\sqrt{x}$$

input
```
integrate(x*arctan(x^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(x^2 - 1)*arctan(sqrt(x)) - 1/6*(x - 3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int x \arctan\left(\sqrt{x}\right)\,dx = -\frac{x^{\frac{3}{2}}}{6}+\frac{\sqrt{x}}{2}+\frac{x^2\operatorname{atan}\left(\sqrt{x}\right)}{2}-\frac{\operatorname{atan}\left(\sqrt{x}\right)}{2}$$

input
```
integrate(x*atan(x**(1/2)),x)
```

output
```
-x**(3/2)/6 + sqrt(x)/2 + x**2*atan(sqrt(x))/2 - atan(sqrt(x))/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x\arctan\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2\arctan\left(\sqrt{x}\right) - \frac{1}{6}\,x^{\frac{3}{2}} + \frac{1}{2}\,\sqrt{x} - \frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x*arctan(x^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(sqrt(x)) - 1/6*x^(3/2) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x\arctan\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2\arctan\left(\sqrt{x}\right) - \frac{1}{6}\,x^{\frac{3}{2}} + \frac{1}{2}\,\sqrt{x} - \frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x*arctan(x^(1/2)),x, algorithm="giac")
```

output
```
1/2*x^2*arctan(sqrt(x)) - 1/6*x^(3/2) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x\arctan\left(\sqrt{x}\right)\,dx = \frac{x^2\,\mathrm{atan}\left(\sqrt{x}\right)}{2} - \frac{\mathrm{atan}\left(\sqrt{x}\right)}{2} + \frac{\sqrt{x}}{2} - \frac{x^{3/2}}{6}$$

input
```
int(x*atan(x^(1/2)),x)
```

output
```
(x^2*atan(x^(1/2)))/2 - atan(x^(1/2))/2 + x^(1/2)/2 - x^(3/2)/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55

$$\int x \arctan \left(\sqrt{x}\right) \, dx = \frac{atan\left(\sqrt{x}\right) x^2}{2} - \frac{atan\left(\sqrt{x}\right)}{2} - \frac{\sqrt{x}\, x}{6} + \frac{\sqrt{x}}{2}$$

input
```
int(x*atan(x^(1/2)),x)
```

output
```
(3*atan(sqrt(x))*x**2 - 3*atan(sqrt(x)) - sqrt(x)*x + 3*sqrt(x))/6
```

3.156 $\int \arctan\left(\sqrt{x}\right)\,dx$

Optimal result . . . 1165
Mathematica [A] (verified) . . . 1165
Rubi [A] (verified) . . . 1166
Maple [A] (verified) . . . 1167
Fricas [A] (verification not implemented) . . . 1168
Sympy [A] (verification not implemented) . . . 1168
Maxima [A] (verification not implemented) . . . 1169
Giac [A] (verification not implemented) . . . 1169
Mupad [B] (verification not implemented) . . . 1169
Reduce [B] (verification not implemented) . . . 1170

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \arctan\left(\sqrt{x}\right)\,dx = -\sqrt{x} + \arctan\left(\sqrt{x}\right) + x\arctan\left(\sqrt{x}\right)$$

output
```
-x^(1/2)+arctan(x^(1/2))+x*arctan(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \arctan\left(\sqrt{x}\right)\,dx = -\sqrt{x} + (1+x)\arctan\left(\sqrt{x}\right)$$

input
```
Integrate[ArcTan[Sqrt[x]],x]
```

output
```
-Sqrt[x] + (1 + x)*ArcTan[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5345, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan\left(\sqrt{x}\right)dx$$

$$\Big\downarrow \text{5345}$$

$$x\arctan\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{x+1}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{x}(x+1)}dx-2\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(2\int \frac{1}{x+1}d\sqrt{x}-2\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{216}$$

$$x\arctan\left(\sqrt{x}\right)+\frac{1}{2}\left(2\arctan\left(\sqrt{x}\right)-2\sqrt{x}\right)$$

input
```
Int[ArcTan[Sqrt[x]],x]
```

output
```
x*ArcTan[Sqrt[x]] + (-2*Sqrt[x] + 2*ArcTan[Sqrt[x]])/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\sqrt{x}+\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)$	17
default	$-\sqrt{x}+\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)$	17
parts	$-\sqrt{x}+\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)$	17
meijerg	$-\sqrt{x}+\frac{(3x+3)\arctan\left(\sqrt{x}\right)}{3}$	18

input
```
int(arctan(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-x^(1/2)+arctan(x^(1/2))+x*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \arctan \left(\sqrt{x}\right) \, dx = (x+1)\arctan \left(\sqrt{x}\right) - \sqrt{x}$$

input
```
integrate(arctan(x^(1/2)),x, algorithm="fricas")
```

output
```
(x + 1)*arctan(sqrt(x)) - sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \arctan \left(\sqrt{x}\right) \, dx = -\sqrt{x} + x \operatorname{atan}\left(\sqrt{x}\right) + \operatorname{atan}\left(\sqrt{x}\right)$$

input
```
integrate(atan(x**(1/2)),x)
```

output
```
-sqrt(x) + x*atan(sqrt(x)) + atan(sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{x}\right)\,dx = x\arctan\left(\sqrt{x}\right) - \sqrt{x} + \arctan\left(\sqrt{x}\right)$$

input
```
integrate(arctan(x^(1/2)),x, algorithm="maxima")
```

output
```
x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{x}\right)\,dx = x\arctan\left(\sqrt{x}\right) - \sqrt{x} + \arctan\left(\sqrt{x}\right)$$

input
```
integrate(arctan(x^(1/2)),x, algorithm="giac")
```

output
```
x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{x}\right)\,dx = \mathrm{atan}\left(\sqrt{x}\right) + x\,\mathrm{atan}\left(\sqrt{x}\right) - \sqrt{x}$$

input
```
int(atan(x^(1/2)),x)
```

output
```
atan(x^(1/2)) + x*atan(x^(1/2)) - x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

$$\int \arctan \left(\sqrt{x}\right) \, dx = atan\left(\sqrt{x}\right) x + atan\left(\sqrt{x}\right) - \sqrt{x}$$

input
```
int(atan(x^(1/2)),x)
```

output
```
atan(sqrt(x))*x + atan(sqrt(x)) - sqrt(x)
```

3.157 $\int \frac{\arctan(\sqrt{x})}{x} \, dx$

Optimal result 1171
Mathematica [A] (verified) 1171
Rubi [A] (verified) 1172
Maple [A] (verified) 1173
Fricas [F] 1173
Sympy [F] 1174
Maxima [B] (verification not implemented) 1174
Giac [F] 1174
Mupad [B] (verification not implemented) 1175
Reduce [F] 1175

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \frac{\arctan(\sqrt{x})}{x} \, dx = i \operatorname{PolyLog}(2, -i\sqrt{x}) - i \operatorname{PolyLog}(2, i\sqrt{x})$$

output
```
I*polylog(2,-I*x^(1/2))-I*polylog(2,I*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(\sqrt{x})}{x} \, dx = i \operatorname{PolyLog}(2, -i\sqrt{x}) - i \operatorname{PolyLog}(2, i\sqrt{x})$$

input
```
Integrate[ArcTan[Sqrt[x]]/x,x]
```

output
```
I*PolyLog[2, (-I)*Sqrt[x]] - I*PolyLog[2, I*Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5359, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x} \, dx$$

$$\Big\downarrow \text{5359}$$

$$2\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{5355}$$

$$2\left(\frac{1}{2}i\int \frac{\log\left(1-i\sqrt{x}\right)}{\sqrt{x}}d\sqrt{x}-\frac{1}{2}i\int \frac{\log\left(i\sqrt{x}+1\right)}{\sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{2838}$$

$$2\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-i\sqrt{x}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,i\sqrt{x}\right)\right)$$

input
```
Int[ArcTan[Sqrt[x]]/x,x]
```

output
```
2*((I/2)*PolyLog[2, (-I)*Sqrt[x]] - (I/2)*PolyLog[2, I*Sqrt[x]])
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result
meijerg	$i \operatorname{polylog}\left(2, -i\sqrt{x}\right) - i \operatorname{polylog}\left(2, i\sqrt{x}\right)$
derivativedivides	$\ln\left(x\right) \arctan\left(\sqrt{x}\right) + \frac{i \ln(x) \ln(1+i\sqrt{x})}{2} - \frac{i \ln(x) \ln(1-i\sqrt{x})}{2} + i \operatorname{dilog}\left(1 + i\sqrt{x}\right) - i \operatorname{dilog}\left(1 -\right.$
default	$\ln\left(x\right) \arctan\left(\sqrt{x}\right) + \frac{i \ln(x) \ln(1+i\sqrt{x})}{2} - \frac{i \ln(x) \ln(1-i\sqrt{x})}{2} + i \operatorname{dilog}\left(1 + i\sqrt{x}\right) - i \operatorname{dilog}\left(1 -\right.$
parts	$\ln\left(x\right) \arctan\left(\sqrt{x}\right) + \frac{i \ln(x) \ln(1+i\sqrt{x})}{2} - \frac{i \ln(x) \ln(1-i\sqrt{x})}{2} + i \operatorname{dilog}\left(1 + i\sqrt{x}\right) - i \operatorname{dilog}\left(1 -\right.$

input
```
int(arctan(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
I*polylog(2,-I*x^(1/2))-I*polylog(2,I*x^(1/2))
```

Fricas [F]

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\, dx = \int \frac{\arctan\left(\sqrt{x}\right)}{x}\, dx$$

input
```
integrate(arctan(x^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(arctan(sqrt(x))/x, x)
```

Sympy [F]

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{atan}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(atan(x**(1/2))/x,x)
```

output
```
Integral(atan(sqrt(x))/x, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx = -\frac{1}{2}\,\pi\log\left(x+1\right) + \arctan\left(\sqrt{x}\right)\log\left(x\right) - i\,\mathrm{Li}_2\left(i\sqrt{x}+1\right) + i\,\mathrm{Li}_2\left(-i\sqrt{x}+1\right)$$

input
```
integrate(arctan(x^(1/2))/x,x, algorithm="maxima")
```

output
```
-1/2*pi*log(x + 1) + arctan(sqrt(x))*log(x) - I*dilog(I*sqrt(x) + 1) + I*d
ilog(-I*sqrt(x) + 1)
```

Giac [F]

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(arctan(x^(1/2))/x,x, algorithm="giac")
```

output
```
integrate(arctan(sqrt(x))/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx = -\mathrm{Li}_2\left(1-\sqrt{x}\,1\mathrm{i}\right)\,1\mathrm{i} + \mathrm{polylog}\left(2,-\sqrt{x}\,1\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int(atan(x^(1/2))/x,x)
```

output
```
polylog(2, -x^(1/2)*1i)*1i - dilog(1 - x^(1/2)*1i)*1i
```

Reduce [F]

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx = \int \frac{atan\left(\sqrt{x}\right)}{x}dx$$

input
```
int(atan(x^(1/2))/x,x)
```

output
```
int(atan(sqrt(x))/x,x)
```

3.158 $\int \frac{\arctan(\sqrt{x})}{x^2} dx$

Optimal result 1176
Mathematica [C] (verified) 1176
Rubi [A] (verified) 1177
Maple [A] (verified) 1178
Fricas [A] (verification not implemented) 1179
Sympy [B] (verification not implemented) 1179
Maxima [A] (verification not implemented) 1180
Giac [A] (verification not implemented) 1180
Mupad [B] (verification not implemented) 1180
Reduce [B] (verification not implemented) 1181

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2} dx = -\frac{1}{\sqrt{x}} - \arctan\left(\sqrt{x}\right) - \frac{\arctan\left(\sqrt{x}\right)}{x}$$

output
```
-1/x^(1/2)-arctan(x^(1/2))-arctan(x^(1/2))/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2} dx = -\frac{\arctan\left(\sqrt{x}\right)}{x} - \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -x\right)}{\sqrt{x}}$$

input
```
Integrate[ArcTan[Sqrt[x]]/x^2,x]
```

output
```
-(ArcTan[Sqrt[x]]/x) - Hypergeometric2F1[-1/2, 1, 1/2, -x]/Sqrt[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5361, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\int \frac{1}{x^{3/2}(x+1)}dx-\frac{\arctan\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\int \frac{1}{\sqrt{x}(x+1)}dx-\frac{2}{\sqrt{x}}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-2\int \frac{1}{x+1}d\sqrt{x}-\frac{2}{\sqrt{x}}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(-2\arctan\left(\sqrt{x}\right)-\frac{2}{\sqrt{x}}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}$$

input
```
Int[ArcTan[Sqrt[x]]/x^2,x]
```

output
```
(-2/Sqrt[x] - 2*ArcTan[Sqrt[x]])/2 - ArcTan[Sqrt[x]]/x
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

method	result	size
meijerg	$-\frac{1}{\sqrt{x}}-\frac{\arctan\left(\sqrt{x}\right)\left(1+x\right)}{x}$	19
derivativedivides	$-\frac{1}{\sqrt{x}}-\arctan\left(\sqrt{x}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}$	22
default	$-\frac{1}{\sqrt{x}}-\arctan\left(\sqrt{x}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}$	22
parts	$-\frac{1}{\sqrt{x}}-\arctan\left(\sqrt{x}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}$	22

input
```
int(arctan(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x^(1/2)-1/x*arctan(x^(1/2))*(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{(x+1)\arctan\left(\sqrt{x}\right)+\sqrt{x}}{x}$$

input
```
integrate(arctan(x^(1/2))/x^2,x, algorithm="fricas")
```

output
```
-((x + 1)*arctan(sqrt(x)) + sqrt(x))/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(22) = 44$.

Time = 0.49 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.48

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{x^2}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{x}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}$$

input
```
integrate(atan(x**(1/2))/x**2,x)
```

output
```
-x**(5/2)*atan(sqrt(x))/(x**(5/2) + x**(3/2)) - 2*x**(3/2)*atan(sqrt(x))/(
x**(5/2) + x**(3/2)) - sqrt(x)*atan(sqrt(x))/(x**(5/2) + x**(3/2)) - x**2/
(x**(5/2) + x**(3/2)) - x/(x**(5/2) + x**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\arctan\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}} - \arctan\left(\sqrt{x}\right)$$

input
```
integrate(arctan(x^(1/2))/x^2,x, algorithm="maxima")
```

output
```
-arctan(sqrt(x))/x - 1/sqrt(x) - arctan(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\arctan\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}} - \arctan\left(\sqrt{x}\right)$$

input
```
integrate(arctan(x^(1/2))/x^2,x, algorithm="giac")
```

output
```
-arctan(sqrt(x))/x - 1/sqrt(x) - arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx = -\mathrm{atan}\left(\sqrt{x}\right) - \frac{\mathrm{atan}\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}}$$

input
```
int(atan(x^(1/2))/x^2,x)
```

output
```
- atan(x^(1/2)) - atan(x^(1/2))/x - 1/x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx = \frac{-atan\left(\sqrt{x}\right)x - atan\left(\sqrt{x}\right) - \sqrt{x}}{x}$$

input
```
int(atan(x^(1/2))/x^2,x)
```

output
```
( - (atan(sqrt(x))*x + atan(sqrt(x)) + sqrt(x)))/x
```

3.159 $\int \frac{\arctan(\sqrt{x})}{x^3} dx$

Optimal result 1182
Mathematica [C] (verified) 1182
Rubi [A] (verified) 1183
Maple [A] (verified) 1184
Fricas [A] (verification not implemented) 1185
Sympy [B] (verification not implemented) 1185
Maxima [A] (verification not implemented) 1186
Giac [A] (verification not implemented) 1186
Mupad [B] (verification not implemented) 1187
Reduce [B] (verification not implemented) 1187

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{\arctan(\sqrt{x})}{x^3} dx = -\frac{1}{6x^{3/2}} + \frac{1}{2\sqrt{x}} + \frac{\arctan(\sqrt{x})}{2} - \frac{\arctan(\sqrt{x})}{2x^2}$$

output
```
-1/6/x^(3/2)+1/2/x^(1/2)+1/2*arctan(x^(1/2))-1/2*arctan(x^(1/2))/x^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(\sqrt{x})}{x^3} dx = -\frac{\arctan(\sqrt{x})}{2x^2} - \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, -x\right)}{6x^{3/2}}$$

input
```
Integrate[ArcTan[Sqrt[x]]/x^3,x]
```

output
```
-1/2*ArcTan[Sqrt[x]]/x^2 - Hypergeometric2F1[-3/2, 1, -1/2, -x]/(6*x^(3/2)
)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5361, 61, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}\int \frac{1}{x^{5/2}(x+1)}dx-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(-\int \frac{1}{x^{3/2}(x+1)}dx-\frac{2}{3x^{3/2}}\right)-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(\int \frac{1}{\sqrt{x}(x+1)}dx-\frac{2}{3x^{3/2}}+\frac{2}{\sqrt{x}}\right)-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(2\int \frac{1}{x+1}d\sqrt{x}-\frac{2}{3x^{3/2}}+\frac{2}{\sqrt{x}}\right)-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(2\arctan\left(\sqrt{x}\right)-\frac{2}{3x^{3/2}}+\frac{2}{\sqrt{x}}\right)-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}$$

input
```
Int[ArcTan[Sqrt[x]]/x^3,x]
```

output
```
-1/2*ArcTan[Sqrt[x]]/x^2 + (-2/(3*x^(3/2)) + 2/Sqrt[x] + 2*ArcTan[Sqrt[x]]
)/4
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result	size
derivativedivides	$-\frac{1}{6x^{\frac{3}{2}}}+\frac{1}{2\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2x^{2}}$	27
default	$-\frac{1}{6x^{\frac{3}{2}}}+\frac{1}{2\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2x^{2}}$	27
parts	$-\frac{1}{6x^{\frac{3}{2}}}+\frac{1}{2\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2x^{2}}$	27
meijerg	$-\frac{1}{6x^{\frac{3}{2}}}+\frac{1}{2\sqrt{x}}-\frac{4\left(\frac{3}{8}-\frac{3x^{2}}{8}\right)\arctan\left(\sqrt{x}\right)}{3x^{2}}$	28

input
```
int(arctan(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6/x^(3/2)+1/2/x^(1/2)+1/2*arctan(x^(1/2))-1/2*arctan(x^(1/2))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3}}\,dx=\frac{3\,(x^{2}-1)\arctan\left(\sqrt{x}\right)+(3\,x-1)\sqrt{x}}{6\,x^{2}}$$

input
```
integrate(arctan(x^(1/2))/x^3,x, algorithm="fricas")
```

output
```
1/6*(3*(x^2 - 1)*arctan(sqrt(x)) + (3*x - 1)*sqrt(x))/x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(36) = 72.

Time = 0.93 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.81

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3}}\,dx=\frac{3x^{\frac{7}{2}}\,\mathrm{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{3x^{\frac{5}{2}}\,\mathrm{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{3x^{\frac{3}{2}}\,\mathrm{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{3\sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{3x^{3}}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{2x^{2}}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{x}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}$$

input
```
integrate(atan(x**(1/2))/x**3,x)
```

output
```
3*x**(7/2)*atan(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**(5/2)*atan(sqrt(
x))/(6*x**(7/2) + 6*x**(5/2)) - 3*x**(3/2)*atan(sqrt(x))/(6*x**(7/2) + 6*x
**(5/2)) - 3*sqrt(x)*atan(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**3/(6*x
**(7/2) + 6*x**(5/2)) + 2*x**2/(6*x**(7/2) + 6*x**(5/2)) - x/(6*x**(7/2) +
 6*x**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3\,x-1}{6\,x^{\frac{3}{2}}} - \frac{\arctan\left(\sqrt{x}\right)}{2\,x^2} + \frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(arctan(x^(1/2))/x^3,x, algorithm="maxima")
```

output
```
1/6*(3*x - 1)/x^(3/2) - 1/2*arctan(sqrt(x))/x^2 + 1/2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3\,x-1}{6\,x^{\frac{3}{2}}} - \frac{\arctan\left(\sqrt{x}\right)}{2\,x^2} + \frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(arctan(x^(1/2))/x^3,x, algorithm="giac")
```

output
```
1/6*(3*x - 1)/x^(3/2) - 1/2*arctan(sqrt(x))/x^2 + 1/2*arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = \frac{\operatorname{atan}\left(\sqrt{x}\right)}{2} + \frac{x-\frac{1}{3}}{2\,x^{3/2}} - \frac{\operatorname{atan}\left(\sqrt{x}\right)}{2\,x^2}$$

input
```
int(atan(x^(1/2))/x^3,x)
```

output
```
atan(x^(1/2))/2 + (x - 1/3)/(2*x^(3/2)) - atan(x^(1/2))/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3atan\left(\sqrt{x}\right)x^2 - 3atan\left(\sqrt{x}\right) + 3\sqrt{x}\,x - \sqrt{x}}{6x^2}$$

input
```
int(atan(x^(1/2))/x^3,x)
```

output
```
(3*atan(sqrt(x))*x**2 - 3*atan(sqrt(x)) + 3*sqrt(x)*x - sqrt(x))/(6*x**2)
```

3.160 $\int x^{3/2} \arctan\left(\sqrt{x}\right) \, dx$

Optimal result 1188
Mathematica [A] (verified) 1188
Rubi [A] (verified) 1189
Maple [A] (verified) 1190
Fricas [A] (verification not implemented) 1190
Sympy [B] (verification not implemented) 1191
Maxima [A] (verification not implemented) 1191
Giac [A] (verification not implemented) 1192
Mupad [B] (verification not implemented) 1192
Reduce [B] (verification not implemented) 1192

Optimal result

Integrand size = 12, antiderivative size = 36

$$\int x^{3/2} \arctan\left(\sqrt{x}\right) \, dx = \frac{x}{5} - \frac{x^2}{10} + \frac{2}{5} x^{5/2} \arctan\left(\sqrt{x}\right) - \frac{1}{5} \log(1+x)$$

output
```
1/5*x-1/10*x^2+2/5*x^(5/2)*arctan(x^(1/2))-1/5*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int x^{3/2} \arctan\left(\sqrt{x}\right) \, dx = \frac{1}{10} \left(-((-2+x)x) + 4x^{5/2} \arctan\left(\sqrt{x}\right) - 2\log(1+x)\right)$$

input
```
Integrate[x^(3/2)*ArcTan[Sqrt[x]],x]
```

output
```
(-((-2 + x)*x) + 4*x^(5/2)*ArcTan[Sqrt[x]] - 2*Log[1 + x])/10
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5361, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2} \arctan\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{5} x^{5/2} \arctan\left(\sqrt{x}\right) - \frac{1}{5} \int \frac{x^2}{x+1} dx$$

$$\Big\downarrow \text{49}$$

$$\frac{2}{5} x^{5/2} \arctan\left(\sqrt{x}\right) - \frac{1}{5} \int \left(x + \frac{1}{x+1} - 1\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5} x^{5/2} \arctan\left(\sqrt{x}\right) + \frac{1}{5} \left(-\frac{x^2}{2} + x - \log(x+1)\right)$$

input
```
Int[x^(3/2)*ArcTan[Sqrt[x]],x]
```

output
```
(2*x^(5/2)*ArcTan[Sqrt[x]])/5 + (x - x^2/2 - Log[1 + x])/5
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{x}{5}-\frac{x^{2}}{10}+\frac{2x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)}{5}-\frac{\ln(1+x)}{5}$	25
default	$\frac{x}{5}-\frac{x^{2}}{10}+\frac{2x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)}{5}-\frac{\ln(1+x)}{5}$	25
meijerg	$\frac{x(-3x+6)}{30}+\frac{2x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)}{5}-\frac{\ln(1+x)}{5}$	25

input
```
int(x^(3/2)*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/5*x-1/10*x^2+2/5*x^(5/2)*arctan(x^(1/2))-1/5*ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\arctan\left(\sqrt{x}\right)\,dx=\frac{2}{5}\,x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{10}\,x^{2}+\frac{1}{5}\,x-\frac{1}{5}\,\log\left(x+1\right)$$

input
```
integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="fricas")
```

output
```
2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(29) = 58$.

Time = 0.80 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int x^{3/2}\arctan\left(\sqrt{x}\right)\,dx = \frac{4x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{10x+10} + \frac{4x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{10x+10} - \frac{x^3}{10x+10} + \frac{x^2}{10x+10} - \frac{2x\log\left(x+1\right)}{10x+10} - \frac{2\log\left(x+1\right)}{10x+10} - \frac{2}{10x+10}$$

input
```
integrate(x**(3/2)*atan(x**(1/2)),x)
```

output
```
4*x**(7/2)*atan(sqrt(x))/(10*x + 10) + 4*x**(5/2)*atan(sqrt(x))/(10*x + 10
) - x**3/(10*x + 10) + x**2/(10*x + 10) - 2*x*log(x + 1)/(10*x + 10) - 2*l
og(x + 1)/(10*x + 10) - 2/(10*x + 10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\arctan\left(\sqrt{x}\right)\,dx = \frac{2}{5}\,x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right) - \frac{1}{10}\,x^2 + \frac{1}{5}\,x - \frac{1}{5}\,\log\left(x+1\right)$$

input
```
integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="maxima")
```

output
```
2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2} \arctan \left(\sqrt{x}\right) \,dx = \frac{2}{5} x^{\frac{5}{2}} \arctan \left(\sqrt{x}\right) - \frac{1}{10} x^2 + \frac{1}{5} x - \frac{1}{5} \log \left(x+1\right)$$

input
```
integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="giac")
```

output
```
2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2} \arctan \left(\sqrt{x}\right) \,dx = \frac{x}{5} - \frac{\ln \left(x+1\right)}{5} + \frac{2\, x^{5/2}\, \mathrm{atan}\left(\sqrt{x}\right)}{5} - \frac{x^2}{10}$$

input
```
int(x^(3/2)*atan(x^(1/2)),x)
```

output
```
x/5 - log(x + 1)/5 + (2*x^(5/2)*atan(x^(1/2)))/5 - x^2/10
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int x^{3/2} \arctan \left(\sqrt{x}\right) \,dx = \frac{2\sqrt{x}\, atan\left(\sqrt{x}\right) x^2}{5} - \frac{\log(x+1)}{5} - \frac{x^2}{10} + \frac{x}{5}$$

input
```
int(x^(3/2)*atan(x^(1/2)),x)
```

output
```
(4*sqrt(x)*atan(sqrt(x))*x**2 - 2*log(x + 1) - x**2 + 2*x)/10
```

3.161 $\int \sqrt{x} \arctan\left(\sqrt{x}\right) \, dx$

Optimal result . . . 1193
Mathematica [A] (verified) . . . 1193
Rubi [A] (verified) . . . 1194
Maple [A] (verified) . . . 1195
Fricas [A] (verification not implemented) . . . 1195
Sympy [A] (verification not implemented) . . . 1196
Maxima [A] (verification not implemented) . . . 1196
Giac [A] (verification not implemented) . . . 1196
Mupad [F(-1)] . . . 1197
Reduce [B] (verification not implemented) . . . 1197

Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \sqrt{x} \arctan\left(\sqrt{x}\right) \, dx = -\frac{x}{3} + \frac{2}{3} x^{3/2} \arctan\left(\sqrt{x}\right) + \frac{1}{3} \log(1+x)$$

output
```
-1/3*x+2/3*x^(3/2)*arctan(x^(1/2))+1/3*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \sqrt{x} \arctan\left(\sqrt{x}\right) \, dx = \frac{1}{3} \left(-x + 2x^{3/2} \arctan\left(\sqrt{x}\right) + \log(1+x)\right)$$

input
```
Integrate[Sqrt[x]*ArcTan[Sqrt[x]],x]
```

output
```
(-x + 2*x^(3/2)*ArcTan[Sqrt[x]] + Log[1 + x])/3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5361, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\arctan \left(\sqrt{x}\right) \, dx$$
$$\Big\downarrow \text{5361}$$
$$\frac{2}{3}x^{3/2}\arctan \left(\sqrt{x}\right)-\frac{1}{3}\int \frac{x}{x+1}dx$$
$$\Big\downarrow \text{49}$$
$$\frac{2}{3}x^{3/2}\arctan \left(\sqrt{x}\right)-\frac{1}{3}\int \left(1+\frac{1}{-x-1}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{3}x^{3/2}\arctan \left(\sqrt{x}\right)+\frac{1}{3}(\log(x+1)-x)$$

input
```
Int[Sqrt[x]*ArcTan[Sqrt[x]],x]
```

output
```
(2*x^(3/2)*ArcTan[Sqrt[x]])/3 + (-x + Log[1 + x])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{x}{3}+\frac{2x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)}{3}+\frac{\ln(1+x)}{3}$	20
default	$-\frac{x}{3}+\frac{2x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)}{3}+\frac{\ln(1+x)}{3}$	20
meijerg	$-\frac{x}{3}+\frac{2x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)}{3}+\frac{\ln(1+x)}{3}$	20

input
```
int(x^(1/2)*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x+2/3*x^(3/2)*arctan(x^(1/2))+1/3*ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \sqrt{x}\arctan\left(\sqrt{x}\right)\,dx=\frac{2}{3}\,x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{3}\,x+\frac{1}{3}\,\log\left(x+1\right)$$

input
```
integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="fricas")
```

output
```
2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \sqrt{x}\arctan\left(\sqrt{x}\right)\,dx = \frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{3} - \frac{x}{3} + \frac{\log\left(x+1\right)}{3}$$

input
```
integrate(x**(1/2)*atan(x**(1/2)),x)
```

output
```
2*x**(3/2)*atan(sqrt(x))/3 - x/3 + log(x + 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \sqrt{x}\arctan\left(\sqrt{x}\right)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right) - \frac{1}{3}\,x + \frac{1}{3}\,\log\left(x+1\right)$$

input
```
integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="maxima")
```

output
```
2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \sqrt{x}\arctan\left(\sqrt{x}\right)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right) - \frac{1}{3}\,x + \frac{1}{3}\,\log\left(x+1\right)$$

input
```
integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="giac")
```

output
```
2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\arctan\left(\sqrt{x}\right)\,dx = \int \sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right)\,dx$$

input
```
int(x^(1/2)*atan(x^(1/2)),x)
```

output
```
int(x^(1/2)*atan(x^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \sqrt{x}\arctan\left(\sqrt{x}\right)\,dx = \frac{2\sqrt{x}\,atan\left(\sqrt{x}\right)x}{3} + \frac{\log(x+1)}{3} - \frac{x}{3}$$

input
```
int(x^(1/2)*atan(x^(1/2)),x)
```

output
```
(2*sqrt(x)*atan(sqrt(x))*x + log(x + 1) - x)/3
```

3.162 $\int \frac{\arctan(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result . . . 1198
Mathematica [A] (verified) . . . 1198
Rubi [A] (verified) . . . 1199
Maple [A] (verified) . . . 1200
Fricas [A] (verification not implemented) . . . 1200
Sympy [A] (verification not implemented) . . . 1200
Maxima [A] (verification not implemented) . . . 1201
Giac [A] (verification not implemented) . . . 1201
Mupad [B] (verification not implemented) . . . 1201
Reduce [B] (verification not implemented) . . . 1202

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log(1+x)$$

output
```
2*x^(1/2)*arctan(x^(1/2))-ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log(1+x)$$

input
```
Integrate[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5361, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 5361}$$

$$2\sqrt{x}\arctan\left(\sqrt{x}\right)-\int \frac{1}{x+1}dx$$

$$\Big\downarrow \text{ 16}$$

$$2\sqrt{x}\arctan\left(\sqrt{x}\right)-\log(x+1)$$

input
```
Int[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m + 
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], 
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$2\sqrt{x}\,\arctan\left(\sqrt{x}\right)-\ln\left(1+x\right)$	17
default	$2\sqrt{x}\,\arctan\left(\sqrt{x}\right)-\ln\left(1+x\right)$	17
meijerg	$2\sqrt{x}\,\arctan\left(\sqrt{x}\right)-\ln\left(1+x\right)$	17

input
```
int(arctan(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)*arctan(x^(1/2))-ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

input
```
integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right) - \log\left(x+1\right)$$

input
```
integrate(atan(x**(1/2))/x**(1/2),x)
```

output
```
2*sqrt(x)*atan(sqrt(x)) - log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

input
```
integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

input
```
integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right) - \ln\left(x+1\right)$$

input
```
int(atan(x^(1/2))/x^(1/2),x)
```

output
```
2*x^(1/2)*atan(x^(1/2)) - log(x + 1)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,atan\left(\sqrt{x}\right) - \log(x+1)$$

input
```
int(atan(x^(1/2))/x^(1/2),x)
```

output
```
2*sqrt(x)*atan(sqrt(x)) - log(x + 1)
```

3.163 $\int \frac{\arctan(\sqrt{x})}{x^{3/2}} \, dx$

Optimal result 1203
Mathematica [A] (verified) 1203
Rubi [A] (verified) 1204
Maple [A] (verified) 1205
Fricas [A] (verification not implemented) 1206
Sympy [A] (verification not implemented) 1206
Maxima [A] (verification not implemented) 1206
Giac [A] (verification not implemented) 1207
Mupad [B] (verification not implemented) 1207
Reduce [B] (verification not implemented) 1207

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{\arctan(\sqrt{x})}{x^{3/2}} \, dx = -\frac{2\arctan(\sqrt{x})}{\sqrt{x}} + \log(x) - \log(1+x)$$

output
```
-2*arctan(x^(1/2))/x^(1/2)+ln(x)-ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(\sqrt{x})}{x^{3/2}} \, dx = -\frac{2\arctan(\sqrt{x})}{\sqrt{x}} + \log(x) - \log(1+x)$$

input
```
Integrate[ArcTan[Sqrt[x]]/x^(3/2),x]
```

output
```
(-2*ArcTan[Sqrt[x]])/Sqrt[x] + Log[x] - Log[1 + x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5361, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}dx$$

$$\Big\downarrow \text{5361}$$

$$\int \frac{1}{x(x+1)}dx-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{47}$$

$$\int \frac{1}{x}dx-\int \frac{1}{x+1}dx-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{14}$$

$$-\int \frac{1}{x+1}dx-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}+\log(x)$$

$$\Big\downarrow \text{16}$$

$$-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}+\log(x)-\log(x+1)$$

input
```
Int[ArcTan[Sqrt[x]]/x^(3/2),x]
```

output
```
(-2*ArcTan[Sqrt[x]])/Sqrt[x] + Log[x] - Log[1 + x]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}+\ln\left(x\right)-\ln\left(1+x\right)$	19
default	$-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}+\ln\left(x\right)-\ln\left(1+x\right)$	19
meijerg	$-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}+\ln\left(x\right)-\ln\left(1+x\right)$	19

input
```
int(arctan(x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctan(x^(1/2))/x^(1/2)+ln(x)-ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{x\log\left(x+1\right)-x\log\left(x\right)+2\sqrt{x}\arctan\left(\sqrt{x}\right)}{x}$$

input
```
integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="fricas")
```

output
```
-(x*log(x + 1) - x*log(x) + 2*sqrt(x)*arctan(sqrt(x)))/x
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = \log\left(x\right)-\log\left(x+1\right)-\frac{2\operatorname{atan}\left(\sqrt{x}\right)}{\sqrt{x}}$$

input
```
integrate(atan(x**(1/2))/x**(3/2),x)
```

output
```
log(x) - log(x + 1) - 2*atan(sqrt(x))/sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}-\log\left(x+1\right)+\log\left(x\right)$$

input
```
integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="maxima")
```

output
```
-2*arctan(sqrt(x))/sqrt(x) - log(x + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\,\arctan\left(\sqrt{x}\right)}{\sqrt{x}} - \log\left(x+1\right) + \log\left(x\right)$$

input
```
integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="giac")
```

output
```
-2*arctan(sqrt(x))/sqrt(x) - log(x + 1) + log(x)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = 2\,\ln\left(\sqrt{x}\right) - \ln\left(x+1\right) - \frac{2\,\mathrm{atan}\left(\sqrt{x}\right)}{\sqrt{x}}$$

input
```
int(atan(x^(1/2))/x^(3/2),x)
```

output
```
2*log(x^(1/2)) - log(x + 1) - (2*atan(x^(1/2)))/x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = \frac{-2atan\left(\sqrt{x}\right) - \sqrt{x}\log(x+1) + 2\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{x}}$$

input
```
int(atan(x^(1/2))/x^(3/2),x)
```

output
```
( - 2*atan(sqrt(x)) - sqrt(x)*log(x + 1) + 2*sqrt(x)*log(sqrt(x)))/sqrt(x)
```

3.164 $\int \frac{\arctan(\sqrt{x})}{x^{5/2}} \, dx$

Optimal result 1208
Mathematica [A] (verified) 1208
Rubi [A] (verified) 1209
Maple [A] (verified) 1210
Fricas [A] (verification not implemented) 1210
Sympy [B] (verification not implemented) 1211
Maxima [A] (verification not implemented) 1211
Giac [A] (verification not implemented) 1212
Mupad [B] (verification not implemented) 1212
Reduce [B] (verification not implemented) 1212

Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}} \, dx = -\frac{1}{3x} - \frac{2\arctan\left(\sqrt{x}\right)}{3x^{3/2}} - \frac{\log(x)}{3} + \frac{1}{3}\log(1+x)$$

output
```
-1/3/x-2/3*arctan(x^(1/2))/x^(3/2)-1/3*ln(x)+1/3*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}} \, dx = \frac{1}{3}\left(-\frac{1}{x} - \frac{2\arctan\left(\sqrt{x}\right)}{x^{3/2}} - \log(x) + \log(1+x)\right)$$

input
```
Integrate[ArcTan[Sqrt[x]]/x^(5/2),x]
```

output
```
(-x^(-1) - (2*ArcTan[Sqrt[x]])/x^(3/2) - Log[x] + Log[1 + x])/3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5361, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3}\int \frac{1}{x^2(x+1)}dx - \frac{2\arctan\left(\sqrt{x}\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{3}\int \left(\frac{1}{x+1} - \frac{1}{x} + \frac{1}{x^2}\right)dx - \frac{2\arctan\left(\sqrt{x}\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{1}{x} - \log(x) + \log(x+1)\right) - \frac{2\arctan\left(\sqrt{x}\right)}{3x^{3/2}}$$

input
```
Int[ArcTan[Sqrt[x]]/x^(5/2),x]
```

output
```
(-2*ArcTan[Sqrt[x]])/(3*x^(3/2)) + (-x^(-1) - Log[x] + Log[1 + x])/3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{1}{3x}-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}-\frac{\ln\left(x\right)}{3}+\frac{\ln\left(1+x\right)}{3}$	26
default	$-\frac{1}{3x}-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}-\frac{\ln\left(x\right)}{3}+\frac{\ln\left(1+x\right)}{3}$	26
meijerg	$-\frac{1}{x}+\frac{2}{9}-\frac{\ln\left(x\right)}{3}+\frac{-10x+30}{45x}-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}+\frac{\ln\left(1+x\right)}{3}$	37

input
```
int(arctan(x^(1/2))/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x-2/3*arctan(x^(1/2))/x^(3/2)-1/3*ln(x)+1/3*ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx=\frac{x^2\log\left(x+1\right)-x^2\log\left(x\right)-2\sqrt{x}\arctan\left(\sqrt{x}\right)-x}{3\,x^2}$$

input
```
integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="fricas")
```

output
```
1/3*(x^2*log(x + 1) - x^2*log(x) - 2*sqrt(x)*arctan(sqrt(x)) - x)/x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(31) = 62$.

Time = 0.95 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.86

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{2x^{\frac{3}{2}}\,\mathrm{atan}\left(\sqrt{x}\right)}{3x^3+3x^2} - \frac{2\sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right)}{3x^3+3x^2} - \frac{x^3\log\left(x\right)}{3x^3+3x^2} + \frac{x^3\log\left(x+1\right)}{3x^3+3x^2} - \frac{x^2\log\left(x\right)}{3x^3+3x^2} + \frac{x^2\log\left(x+1\right)}{3x^3+3x^2} - \frac{x^2}{3x^3+3x^2} - \frac{x}{3x^3+3x^2}$$

input
```
integrate(atan(x**(1/2))/x**(5/2),x)
```

output
```
-2*x**(3/2)*atan(sqrt(x))/(3*x**3 + 3*x**2) - 2*sqrt(x)*atan(sqrt(x))/(3*x
**3 + 3*x**2) - x**3*log(x)/(3*x**3 + 3*x**2) + x**3*log(x + 1)/(3*x**3 +
3*x**2) - x**2*log(x)/(3*x**3 + 3*x**2) + x**2*log(x + 1)/(3*x**3 + 3*x**2
) - x**2/(3*x**3 + 3*x**2) - x/(3*x**3 + 3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{2\,\arctan\left(\sqrt{x}\right)}{3\,x^{\frac{3}{2}}} - \frac{1}{3\,x} + \frac{1}{3}\,\log\left(x+1\right) - \frac{1}{3}\,\log\left(x\right)$$

input
```
integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="maxima")
```

output
```
-2/3*arctan(sqrt(x))/x^(3/2) - 1/3/x + 1/3*log(x + 1) - 1/3*log(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{\arctan \left(\sqrt{x}\right)}{x^{5/2}}\,dx = \frac{x-1}{3\,x} - \frac{2\,\arctan \left(\sqrt{x}\right)}{3\,x^{\frac{3}{2}}} + \frac{1}{3}\,\log \left(x+1\right) - \frac{1}{3}\,\log \left(x\right)$$

input
```
integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="giac")
```

output
```
1/3*(x - 1)/x - 2/3*arctan(sqrt(x))/x^(3/2) + 1/3*log(x + 1) - 1/3*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{\arctan \left(\sqrt{x}\right)}{x^{5/2}}\,dx = \frac{\ln \left(x+1\right)}{3} - \frac{2\,\ln \left(\sqrt{x}\right)}{3} - \frac{2\,\mathrm{atan}\left(\sqrt{x}\right)}{3\,x^{3/2}} - \frac{1}{3\,x}$$

input
```
int(atan(x^(1/2))/x^(5/2),x)
```

output
```
log(x + 1)/3 - (2*log(x^(1/2)))/3 - (2*atan(x^(1/2)))/(3*x^(3/2)) - 1/(3*x
)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\arctan \left(\sqrt{x}\right)}{x^{5/2}}\,dx = \frac{-2atan\left(\sqrt{x}\right) + \sqrt{x}\log(x+1)\,x - 2\sqrt{x}\log\left(\sqrt{x}\right)x - \sqrt{x}}{3\sqrt{x}\,x}$$

input
```
int(atan(x^(1/2))/x^(5/2),x)
```

output
```
( - 2*atan(sqrt(x)) + sqrt(x)*log(x + 1)*x - 2*sqrt(x)*log(sqrt(x))*x - sq
rt(x))/(3*sqrt(x)*x)
```

3.165 $\int \frac{\arctan(ax^5)}{x}\,dx$

Optimal result . . . 1213
Mathematica [A] (verified) . . . 1213
Rubi [A] (verified) . . . 1214
Maple [C] (verified) . . . 1215
Fricas [F] . . . 1216
Sympy [F] . . . 1216
Maxima [B] (verification not implemented) . . . 1216
Giac [F] . . . 1217
Mupad [B] (verification not implemented) . . . 1217
Reduce [F] . . . 1217

Optimal result

Integrand size = 10, antiderivative size = 33

$$\int \frac{\arctan(ax^5)}{x}\,dx = \frac{1}{10}i\,\text{PolyLog}\left(2,-iax^5\right) - \frac{1}{10}i\,\text{PolyLog}\left(2,iax^5\right)$$

output
```
1/10*I*polylog(2,-I*a*x^5)-1/10*I*polylog(2,I*a*x^5)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax^5)}{x}\,dx = \frac{1}{10}i\,\text{PolyLog}\left(2,-iax^5\right) - \frac{1}{10}i\,\text{PolyLog}\left(2,iax^5\right)$$

input
```
Integrate[ArcTan[a*x^5]/x,x]
```

output
```
(I/10)*PolyLog[2, (-I)*a*x^5] - (I/10)*PolyLog[2, I*a*x^5]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5359, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax^5)}{x}dx$$

$$\Big\downarrow \text{5359}$$

$$\frac{1}{5}\int \frac{\arctan(ax^5)}{x^5}dx^5$$

$$\Big\downarrow \text{5355}$$

$$\frac{1}{5}\left(\frac{1}{2}i\int \frac{\log(1-iax^5)}{x^5}dx^5-\frac{1}{2}i\int \frac{\log(iax^5+1)}{x^5}dx^5\right)$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{5}\left(\frac{1}{2}i\operatorname{PolyLog}(2,-iax^5)-\frac{1}{2}i\operatorname{PolyLog}(2,iax^5)\right)$$

input
```
Int[ArcTan[a*x^5]/x,x]
```

output
```
((I/2)*PolyLog[2, (-I)*a*x^5] - (I/2)*PolyLog[2, I*a*x^5])/5
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

method	result
default	$\ln\left(x\right)\arctan\left(a\,x^{5}\right)-\frac{\sum_{\_R1=\operatorname{RootOf}\left(a^{2}\_Z^{10}+1\right)}\frac{\ln\left(x\right)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^{5}}}{2a}$
parts	$\ln\left(x\right)\arctan\left(a\,x^{5}\right)-\frac{\sum_{\_R1=\operatorname{RootOf}\left(a^{2}\_Z^{10}+1\right)}\frac{\ln\left(x\right)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^{5}}}{2a}$
meijerg	$-\frac{i a\,x^{5}\operatorname{polylog}\left(2,i\sqrt{a^{2}x^{10}}\right)}{10\sqrt{a^{2}x^{10}}}+\frac{i a\,x^{5}\operatorname{polylog}\left(2,-i\sqrt{a^{2}x^{10}}\right)}{10\sqrt{a^{2}x^{10}}}$
risch	$\frac{i\ln\left(x\right)\ln\left(-i a\,x^{5}+1\right)}{2}-\frac{i\left(\sum_{\_R1=\operatorname{RootOf}\left(a\_Z^{5}+\operatorname{RootOf}\left(\_Z^{2}+1,\mathrm{index}=1\right)\right)}\left(\ln\left(x\right)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}-\frac{i\ln(}{}$

input
```
int(arctan(a*x^5)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arctan(a*x^5)-1/2/a*sum(1/_R1^5*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)
/_R1)),_R1=RootOf(_Z^10*a^2+1))
```

Fricas [F]

$$\int \frac{\arctan(ax^5)}{x}\,dx = \int \frac{\arctan(ax^5)}{x}\,dx$$

input
```
integrate(arctan(a*x^5)/x,x, algorithm="fricas")
```

output
```
integral(arctan(a*x^5)/x, x)
```

Sympy [F]

$$\int \frac{\arctan(ax^5)}{x}\,dx = \int \frac{\operatorname{atan}(ax^5)}{x}\,dx$$

input
```
integrate(atan(a*x**5)/x,x)
```

output
```
Integral(atan(a*x**5)/x, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(19) = 38$.

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.52

$$\int \frac{\arctan(ax^5)}{x}\,dx = -\frac{1}{20}\pi\log\left(a^2x^{10}+1\right) + \frac{1}{5}\arctan\left(ax^5\right)\log\left(ax^5\right) - \frac{1}{10}i\,\mathrm{Li}_2\left(i\,ax^5+1\right) + \frac{1}{10}i\,\mathrm{Li}_2\left(-i\,ax^5+1\right)$$

input
```
integrate(arctan(a*x^5)/x,x, algorithm="maxima")
```

output
```
-1/20*pi*log(a^2*x^10 + 1) + 1/5*arctan(a*x^5)*log(a*x^5) - 1/10*I*dilog(I
*a*x^5 + 1) + 1/10*I*dilog(-I*a*x^5 + 1)
```

Giac [F]

$$\int \frac{\arctan(ax^5)}{x}\,dx = \int \frac{\arctan(ax^5)}{x}\,dx$$

input
```
integrate(arctan(a*x^5)/x,x, algorithm="giac")
```

output
```
integrate(arctan(a*x^5)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{\arctan(ax^5)}{x}\,dx = \frac{\mathrm{polylog}(2, -a\,x^5\,1\mathrm{i})\,1\mathrm{i}}{10} - \frac{\mathrm{polylog}(2, a\,x^5\,1\mathrm{i})\,1\mathrm{i}}{10}$$

input
```
int(atan(a*x^5)/x,x)
```

output
```
(polylog(2, -a*x^5*1i)*1i)/10 - (polylog(2, a*x^5*1i)*1i)/10
```

Reduce [F]

$$\int \frac{\arctan(ax^5)}{x}\,dx = \int \frac{atan(a\,x^5)}{x}dx$$

input
```
int(atan(a*x^5)/x,x)
```

output
```
int(atan(a*x**5)/x,x)
```

3.166 $\int \frac{\arctan(ax^n)}{x} dx$

Optimal result 1218
Mathematica [A] (verified) 1218
Rubi [A] (verified) 1219
Maple [B] (verified) 1220
Fricas [B] (verification not implemented) 1220
Sympy [F] 1221
Maxima [F] 1221
Giac [F] 1222
Mupad [F(-1)] 1222
Reduce [F] 1222

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int \frac{\arctan (ax^n)}{x} dx = \frac{i \operatorname{PolyLog}(2, -iax^n)}{2n} - \frac{i \operatorname{PolyLog}(2, iax^n)}{2n}$$

output
```
1/2*I*polylog(2,-I*a*x^n)/n-1/2*I*polylog(2,I*a*x^n)/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{\arctan (ax^n)}{x} dx = \frac{i(\operatorname{PolyLog}(2, -iax^n) - \operatorname{PolyLog}(2, iax^n))}{2n}$$

input
```
Integrate[ArcTan[a*x^n]/x,x]
```

output
```
((I/2)*(PolyLog[2, (-I)*a*x^n] - PolyLog[2, I*a*x^n]))/n
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5359, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax^n)}{x} \, dx$$

$$\Big\downarrow \text{5359}$$

$$\frac{\int x^{-n}\arctan(ax^n)dx^n}{n}$$

$$\Big\downarrow \text{5355}$$

$$\frac{\frac{1}{2}i\int x^{-n}\log(1-iax^n)dx^n-\frac{1}{2}i\int x^{-n}\log(iax^n+1)dx^n}{n}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{1}{2}i\,\text{PolyLog}(2,-iax^n)-\frac{1}{2}i\,\text{PolyLog}(2,iax^n)}{n}$$

input
```
Int[ArcTan[a*x^n]/x,x]
```

output
```
((I/2)*PolyLog[2, (-I)*a*x^n] - (I/2)*PolyLog[2, I*a*x^n])/n
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(31) = 62.

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.92

method	result
meijerg	$\frac{-\frac{2 i a x^{n} \operatorname{polylog}\left(2, i \sqrt{x^{2 n} a^{2}}\right)}{\sqrt{x^{2 n} a^{2}}}+\frac{2 i a x^{n} \operatorname{polylog}\left(2,-i \sqrt{x^{2 n} a^{2}}\right)}{\sqrt{x^{2 n} a^{2}}}}{4 n}$
derivativedivides	$\frac{\ln(a x^{n}) \arctan(a x^{n})+\frac{i \ln(a x^{n}) \ln(1+i a x^{n})}{2}-\frac{i \ln(a x^{n}) \ln(1-i a x^{n})}{2}+\frac{i \operatorname{dilog}(1+i a x^{n})}{2}-\frac{i \operatorname{dilog}(1-i a x^{n})}{2}}{n}$
default	$\frac{\ln(a x^{n}) \arctan(a x^{n})+\frac{i \ln(a x^{n}) \ln(1+i a x^{n})}{2}-\frac{i \ln(a x^{n}) \ln(1-i a x^{n})}{2}+\frac{i \operatorname{dilog}(1+i a x^{n})}{2}-\frac{i \operatorname{dilog}(1-i a x^{n})}{2}}{n}$
risch	$-\frac{i \ln(x) \ln(1+i a x^{n})}{2}-\frac{i \operatorname{dilog}(1-i a x^{n})}{2 n}+\frac{i \ln(-i(-a x^{n}+i)) \ln(x)}{2}-\frac{i \ln(-i(-a x^{n}+i)) \ln(-i a x^{n})}{2 n}-\frac{i \operatorname{dilog}(\cdots}{2 \cdots}$

input
```
int(arctan(a*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4/n*(-2*I*a*x^n/(x^(2*n)*a^2)^(1/2)*polylog(2,I*(x^(2*n)*a^2)^(1/2))+2*I
*a*x^n/(x^(2*n)*a^2)^(1/2)*polylog(2,-I*(x^(2*n)*a^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(25) = 50.

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \frac{\arctan(ax^n)}{x}\,dx = \frac{2\,n\arctan(ax^n)\log(x)+i\,n\log(i\,ax^n+1)\log(x)-i\,n\log(-i\,ax^n+1)\log(x)-i\,\mathrm{Li}_2(i\,ax^n)+i\,\mathrm{Li}_2(-\cdots}{2\,n}$$

input
```
integrate(arctan(a*x^n)/x,x, algorithm="fricas")
```

output
```
1/2*(2*n*arctan(a*x^n)*log(x) + I*n*log(I*a*x^n + 1)*log(x) - I*n*log(-I*a
*x^n + 1)*log(x) - I*dilog(I*a*x^n) + I*dilog(-I*a*x^n))/n
```

Sympy [F]

$$\int \frac{\arctan(ax^n)}{x}\,dx = \int \frac{\operatorname{atan}(ax^n)}{x}\,dx$$

input
```
integrate(atan(a*x**n)/x,x)
```

output
```
Integral(atan(a*x**n)/x, x)
```

Maxima [F]

$$\int \frac{\arctan(ax^n)}{x}\,dx = \int \frac{\arctan(ax^n)}{x}\,dx$$

input
```
integrate(arctan(a*x^n)/x,x, algorithm="maxima")
```

output
```
-a*n*integrate(x^n*log(x)/(a^2*x*x^(2*n) + x), x) + arctan(a*x^n)*log(x)
```

Giac [F]

$$\int \frac{\arctan(ax^n)}{x}\,dx = \int \frac{\arctan(ax^n)}{x}\,dx$$

input
```
integrate(arctan(a*x^n)/x,x, algorithm="giac")
```

output
```
integrate(arctan(a*x^n)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax^n)}{x}\,dx = \int \frac{\operatorname{atan}(a\,x^n)}{x}\,dx$$

input
```
int(atan(a*x^n)/x,x)
```

output
```
int(atan(a*x^n)/x, x)
```

Reduce [F]

$$\int \frac{\arctan(ax^n)}{x}\,dx = \int \frac{atan(x^n a)}{x}dx$$

input
```
int(atan(a*x^n)/x,x)
```

output
```
int(atan(x**n*a)/x,x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 1223
4.2 Links to plain text integration problems used in this report for each CAS . 1241

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file