Computer Algebra
Independent Integration Tests

Summer 2024

5-Inverse-trig-functions/5.4-Inverse-cotangent/282-5.4.2

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**61**]. This is test number [282].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (61)	0.00 (0)
Mathematica	100.00 (61)	0.00 (0)
Maple	98.36 (60)	1.64 (1)
Maxima	68.85 (42)	31.15 (19)
Fricas	63.93 (39)	36.07 (22)
Mupad	62.30 (38)	37.70 (23)
Giac	62.30 (38)	37.70 (23)
Reduce	62.30 (38)	37.70 (23)
Sympy	62.30 (38)	37.70 (23)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

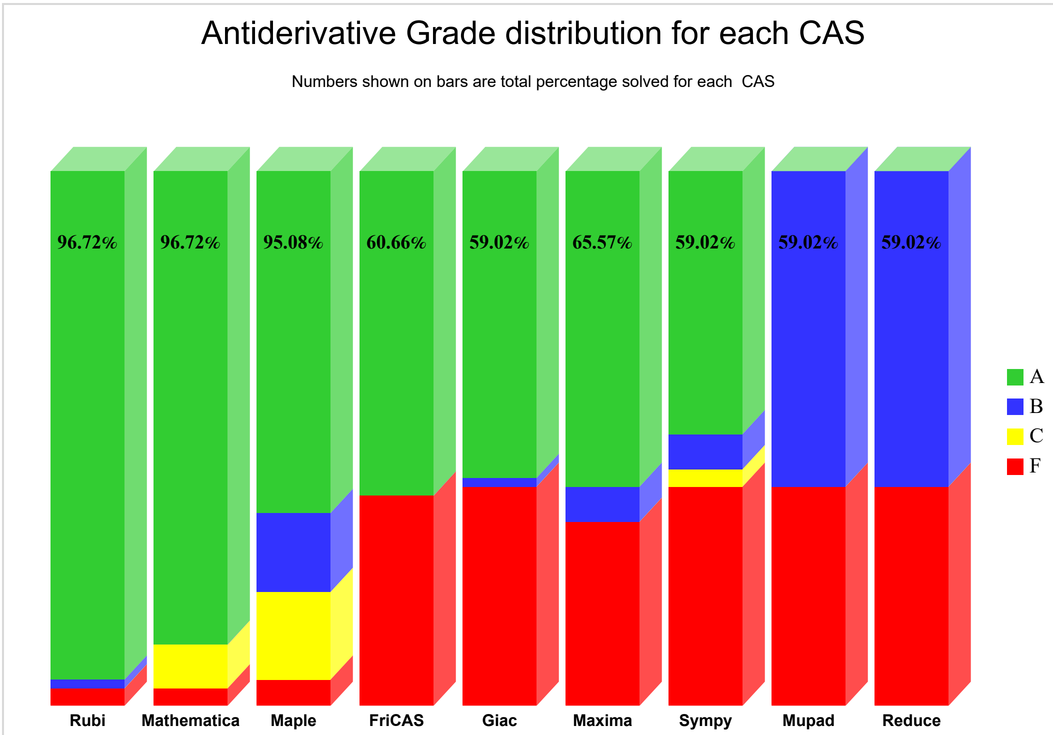
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

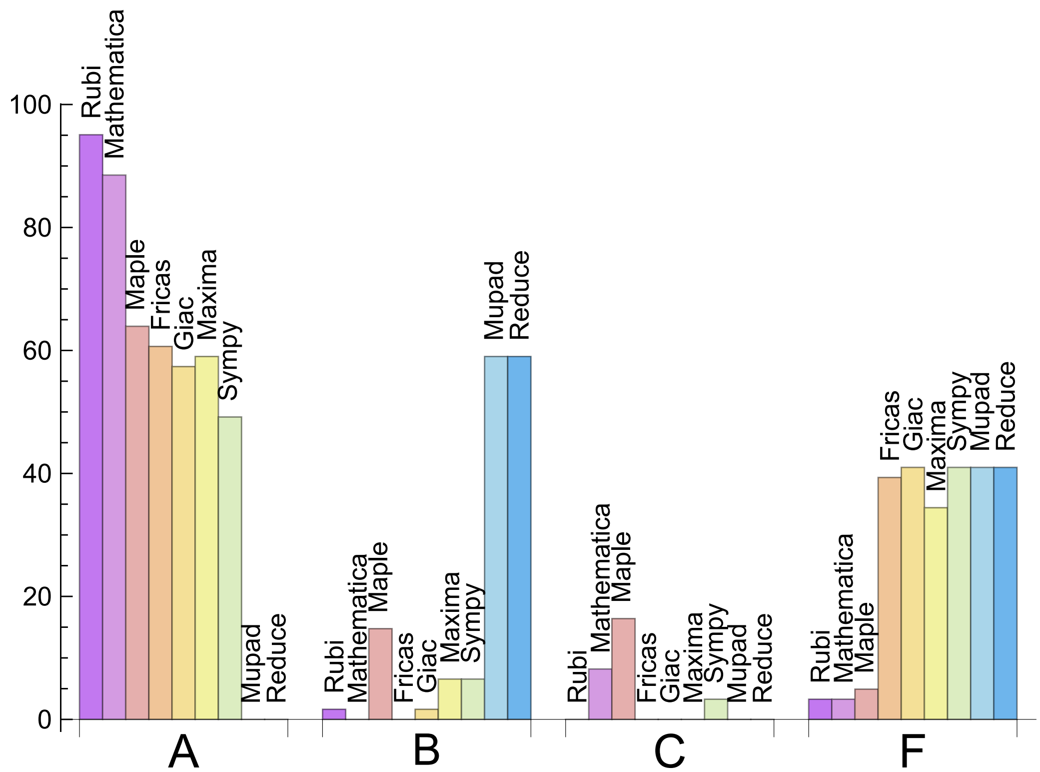
System	% A grade	% B grade	% C grade	% F grade
Rubi	95.082	1.639	0.000	3.279
Mathematica	88.525	0.000	8.197	3.279
Maple	63.934	14.754	16.393	4.918
Fricas	60.656	0.000	0.000	39.344
Maxima	59.016	6.557	0.000	34.426
Giac	57.377	1.639	0.000	40.984
Sympy	49.180	6.557	3.279	40.984
Mupad	0.000	59.016	0.000	40.984
Reduce	0.000	59.016	0.000	40.984

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	1	100.00	0.00	0.00
Maxima	19	94.74	5.26	0.00
Fricas	22	100.00	0.00	0.00
Mupad	23	0.00	100.00	0.00
Giac	23	100.00	0.00	0.00
Reduce	23	100.00	0.00	0.00
Sympy	23	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Mathematica	0.08
Fricas	0.10
Giac	0.13
Maxima	0.18
Reduce	0.20
Rubi	0.50
Mupad	0.78
Sympy	1.54
Maple	2.54

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	38.29	0.86	37.00	0.84
Fricas	42.69	0.87	32.00	0.88
Reduce	44.63	0.92	34.50	0.89
Giac	51.03	1.06	38.00	1.03
Maxima	57.14	1.88	36.50	0.92
Mathematica	64.39	0.94	42.00	1.00
Sympy	78.63	1.42	39.00	0.95
Rubi	90.59	1.17	50.00	1.06
Maple	290.40	2.65	51.50	0.95

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

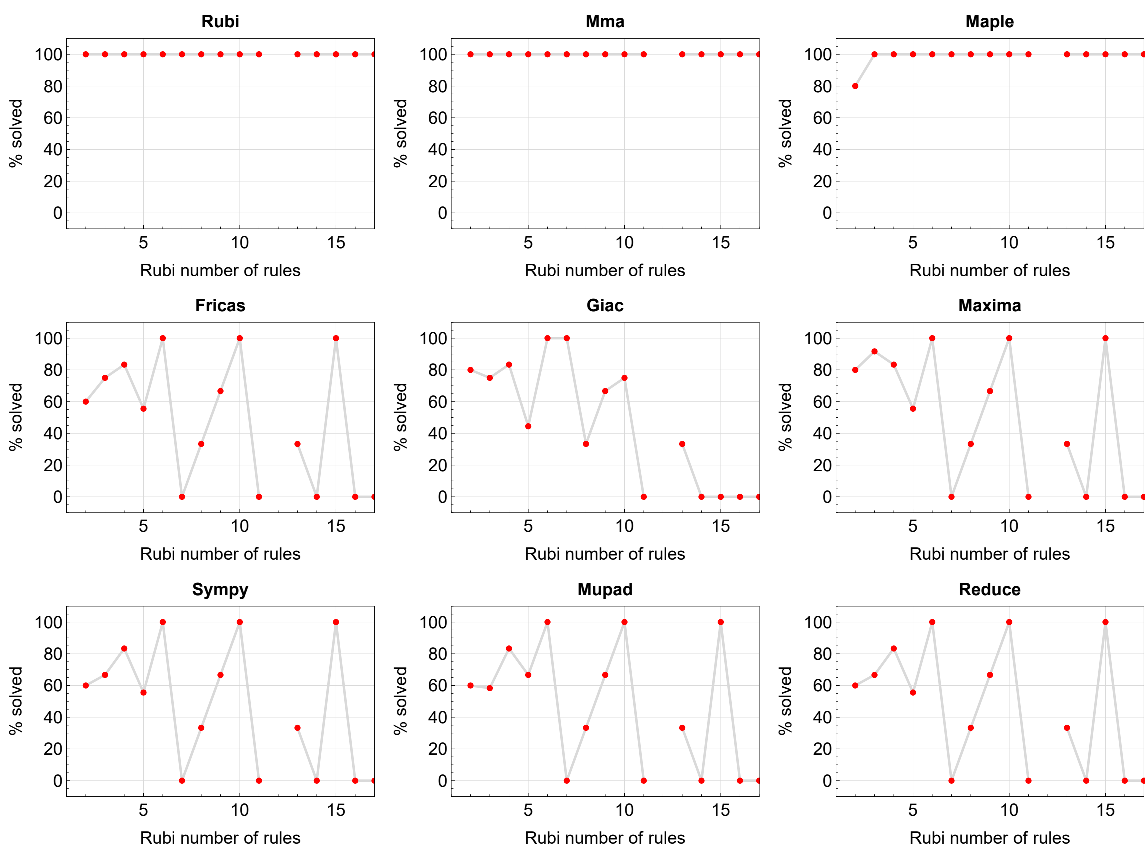


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

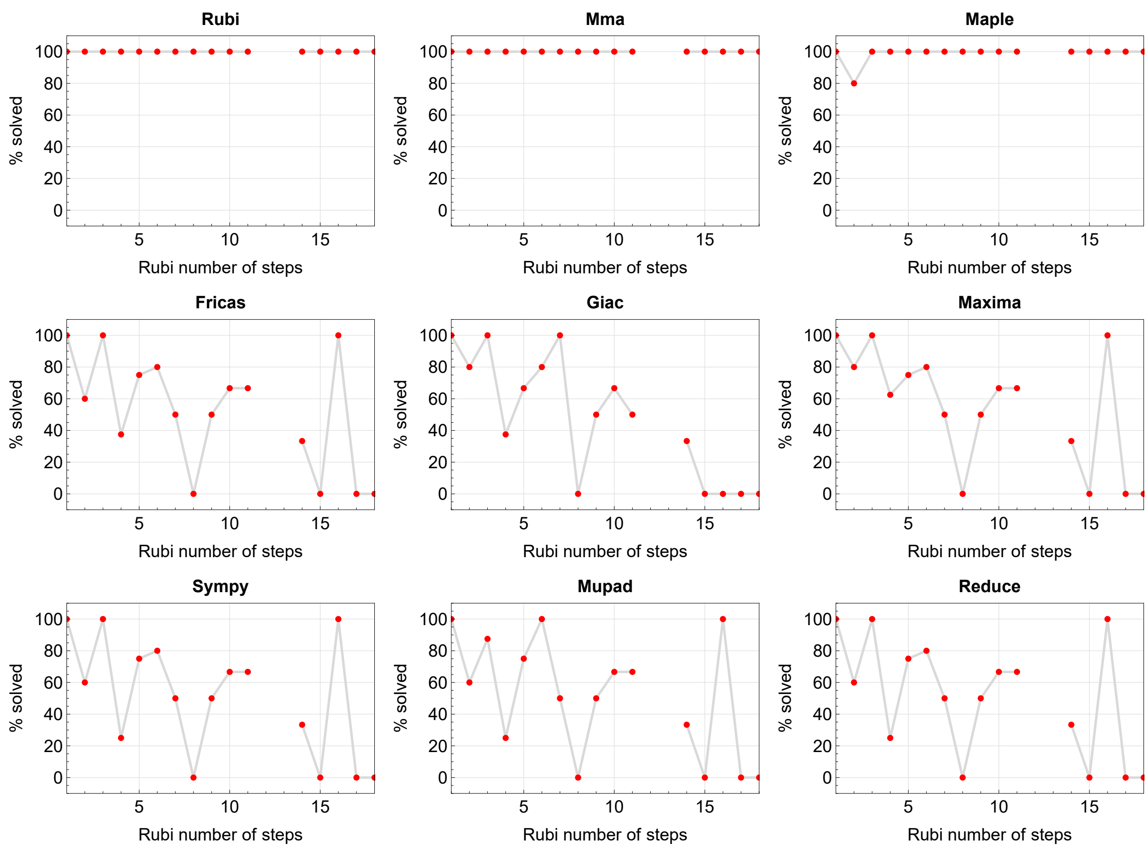


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

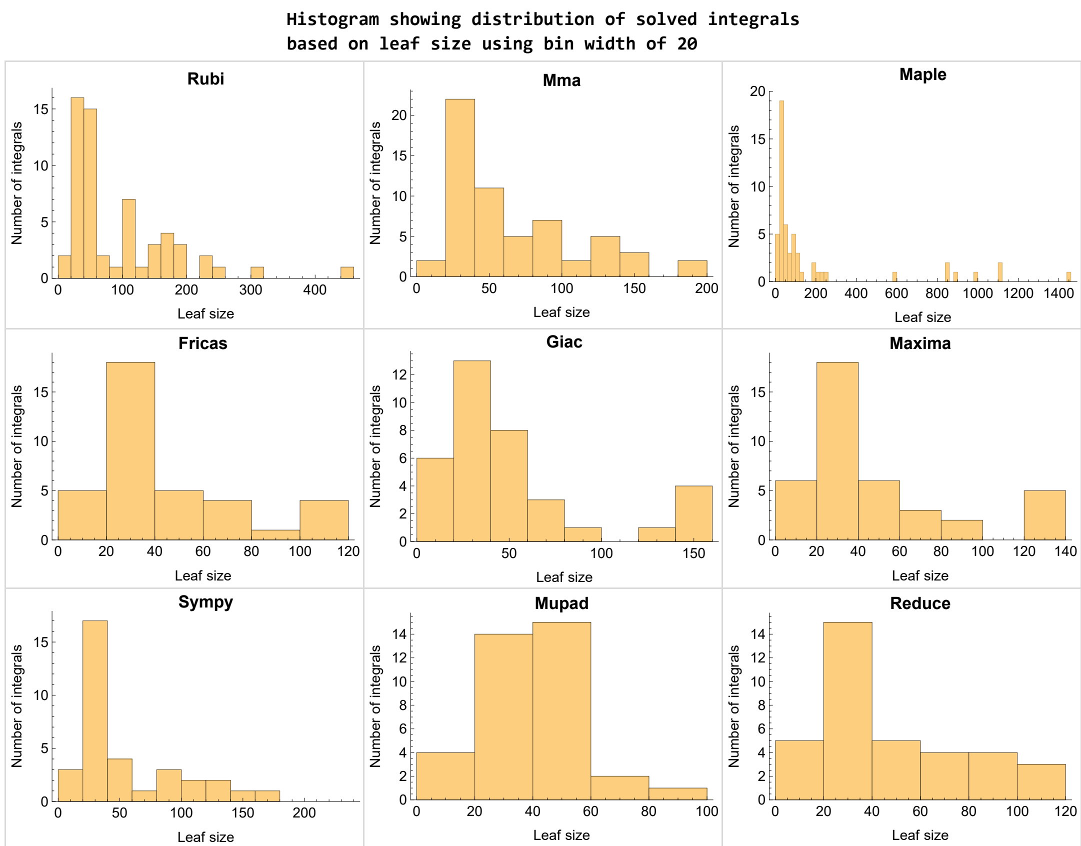


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

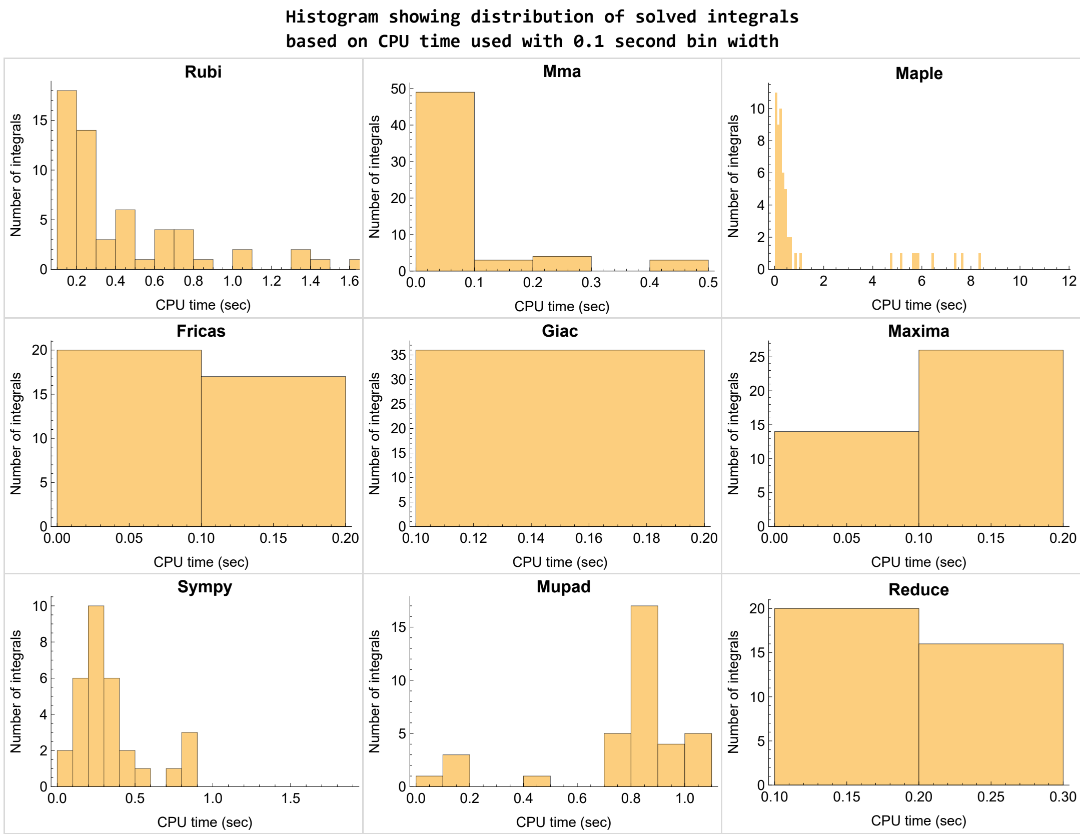


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

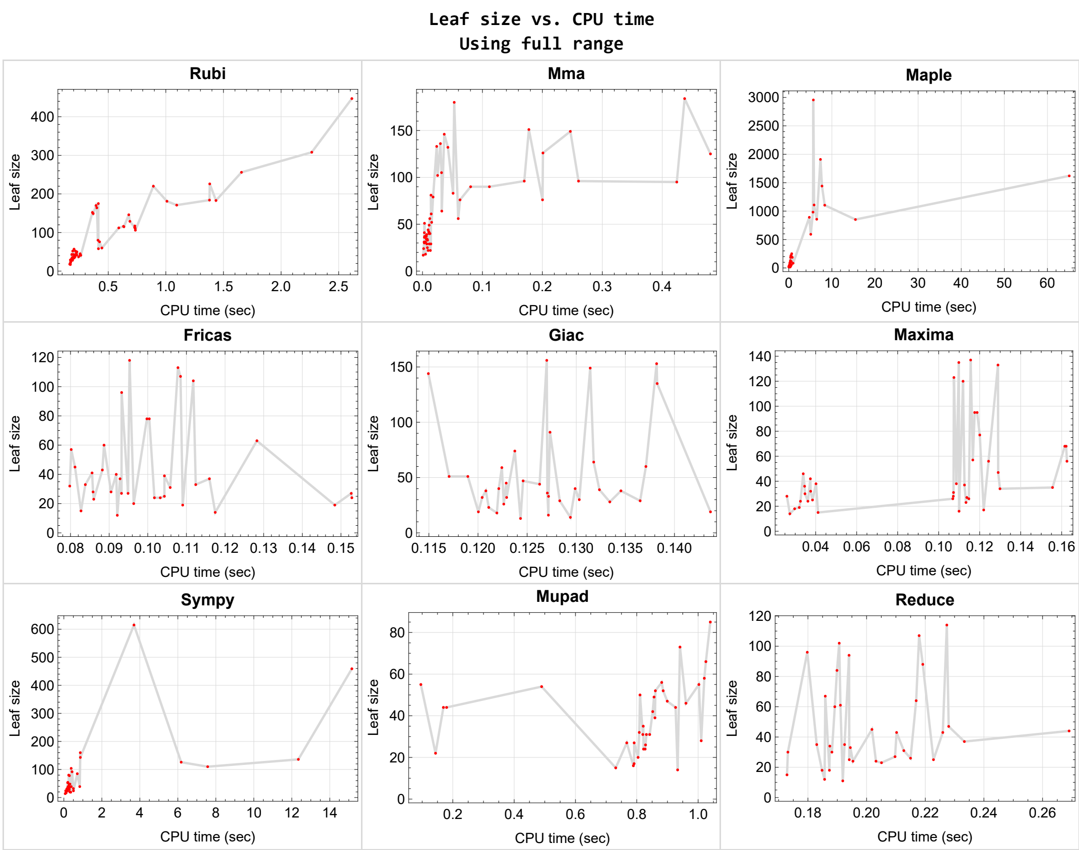


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{34, 35\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{18, 24, 26, 29, 30, 31, 32, 33\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

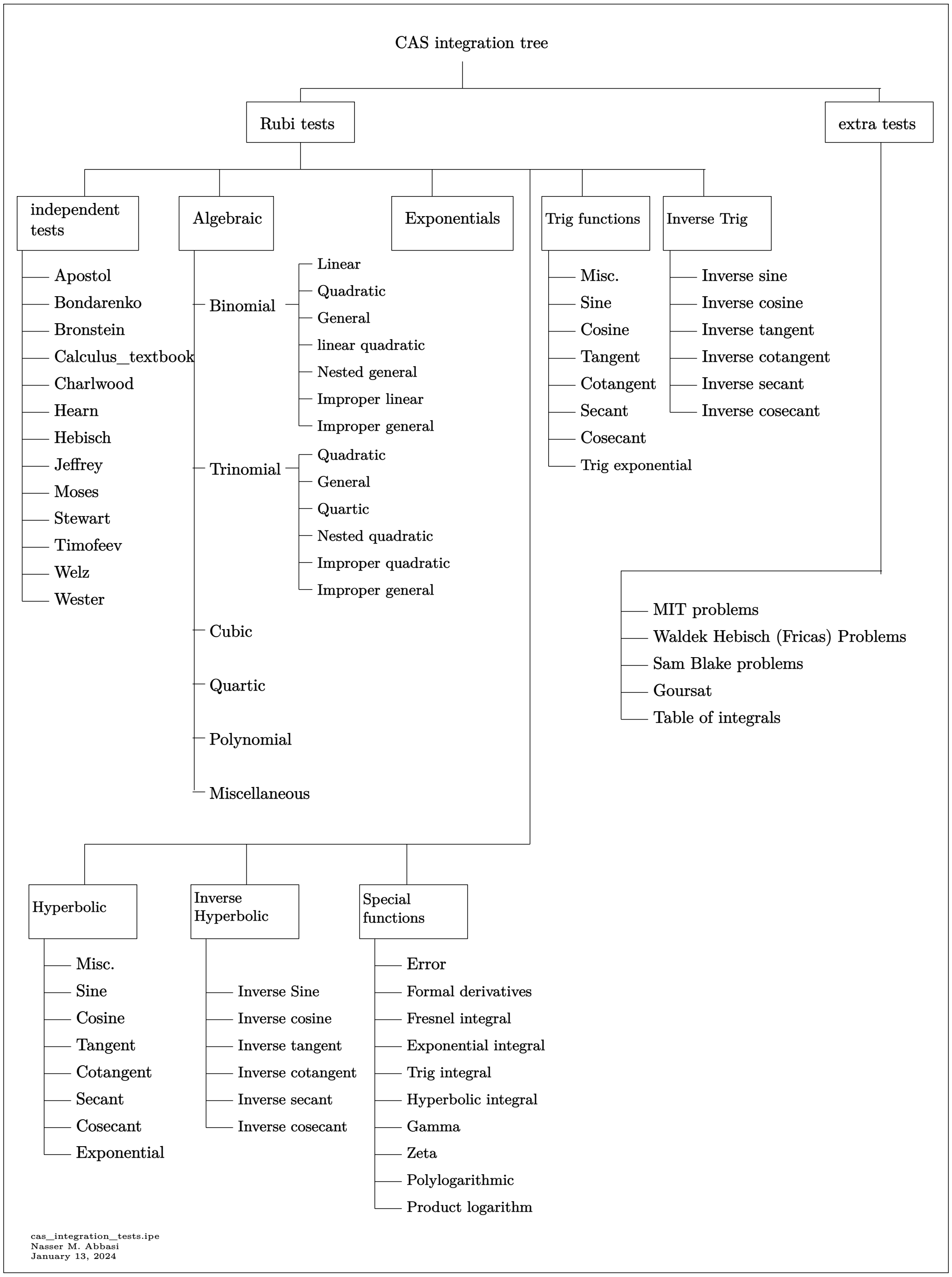
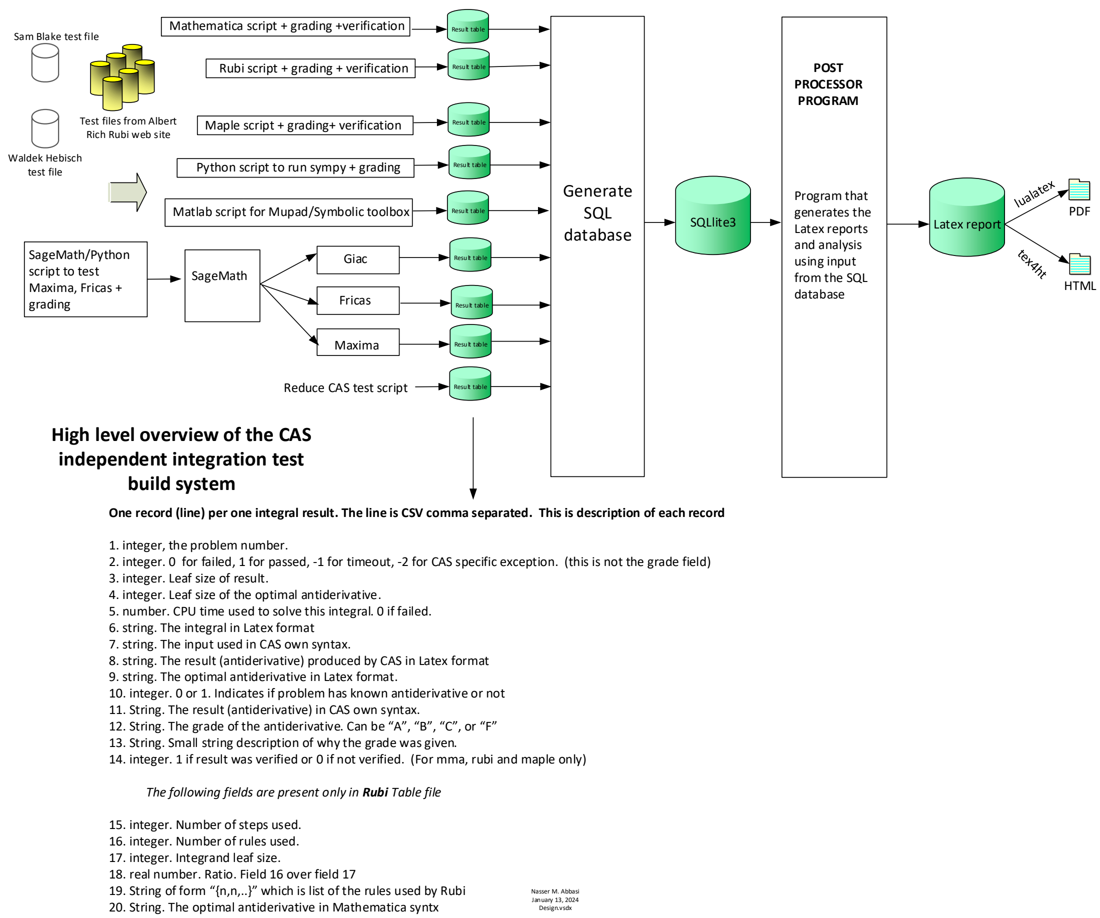


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61 }

B grade { 23 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 61 }

B grade { }

C grade { 9, 11, 42, 54, 55 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 20, 22, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60 }

B grade { 17, 19, 21, 23, 25, 27, 28, 53, 61 }

C grade { 18, 24, 26, 29, 30, 31, 32, 33, 40, 48 }

F normal fail { 36 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61 }

B grade { }

C grade { }

F normal fail { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 40, 48, 53 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60 }

B grade { 7, 40, 48, 53 }

C grade { }

F normal fail { 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 61 }

F(-1) timedout fail { 33 }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 20, 22, 31, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60 }

B grade { 6 }

C grade { }

F normal fail { 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 36, 40, 48, 61 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 17, 20, 22, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 13, 15, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 40, 48, 53, 57, 61 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 37, 38, 39, 41, 42, 43, 44, 46, 49, 50, 51, 52, 57, 58, 59 }

B grade { 54, 55, 56, 60 }

C grade { 45, 47 }

F normal fail { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 40, 48, 53, 61 }

F(-1) timedout fail { }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60 }

C grade { }

F normal fail { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 40, 48, 53, 61 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	52	51	44	47	41	48	59	44	55
N.S.	1	1.02	1.00	0.86	0.92	0.80	0.94	1.16	0.86	1.08
time (sec)	N/A	0.215	0.003	0.187	0.129	0.086	0.320	0.122	0.269	1.003

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	50	49	46	46	45	46	74	45	56
N.S.	1	1.02	1.00	0.94	0.94	0.92	0.94	1.51	0.92	1.14
time (sec)	N/A	0.228	0.011	0.111	0.034	0.081	0.276	0.124	0.202	0.881

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	42	41	36	38	32	39	51	35	46
N.S.	1	1.02	1.00	0.88	0.93	0.78	0.95	1.24	0.85	1.12
time (sec)	N/A	0.214	0.003	0.132	0.109	0.080	0.253	0.119	0.183	0.960

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	40	39	37	36	37	37	64	37	49
N.S.	1	1.03	1.00	0.95	0.92	0.95	0.95	1.64	0.95	1.26
time (sec)	N/A	0.214	0.007	0.104	0.035	0.093	0.214	0.132	0.233	0.856

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	32	31	25	28	23	31	36	24	39
N.S.	1	1.03	1.00	0.81	0.90	0.74	1.00	1.16	0.77	1.26
time (sec)	N/A	0.182	0.003	0.120	0.107	0.086	0.177	0.127	0.203	0.859

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	24	24	24	45	24	22
N.S.	1	1.00	1.00	0.96	1.00	1.00	1.00	1.88	1.00	0.92
time (sec)	N/A	0.175	0.002	0.075	0.036	0.102	0.096	0.123	0.195	0.144

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	33	56	0	0	38	10	0
N.S.	1	1.00	1.00	0.89	1.51	0.00	0.00	1.03	0.27	0.00
time (sec)	N/A	0.212	0.004	0.124	0.163	0.000	0.000	0.135	0.197	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	32	30	29	30	31	24	32	31	28
N.S.	1	1.07	1.00	0.97	1.00	1.03	0.80	1.07	1.03	0.93
time (sec)	N/A	0.188	0.003	0.094	0.035	0.106	0.111	0.123	0.213	1.010

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	30	36	26	23	24	24	40	27	44
N.S.	1	0.97	1.16	0.84	0.74	0.77	0.77	1.29	0.87	1.42
time (sec)	N/A	0.184	0.003	0.140	0.113	0.103	0.188	0.122	0.210	0.927

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	42	42	43	39	44	43	58
N.S.	1	1.00	0.96	0.91	0.91	0.93	0.85	0.96	0.93	1.26
time (sec)	N/A	0.224	0.009	0.111	0.037	0.088	0.222	0.126	0.210	1.020

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	43	36	35	37	33	32	51	35	47
N.S.	1	1.05	0.88	0.85	0.90	0.80	0.78	1.24	0.85	1.15
time (sec)	N/A	0.199	0.003	0.158	0.113	0.084	0.240	0.117	0.192	0.899

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	171	79	90	95	78	104	0	88	85
N.S.	1	1.64	0.76	0.87	0.91	0.75	1.00	0.00	0.85	0.82
time (sec)	N/A	1.094	0.018	0.281	0.119	0.100	0.385	0.000	0.219	1.040

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	181	95	205	0	0	0	0	82	0
N.S.	1	1.34	0.70	1.52	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	1.010	0.424	0.460	0.000	0.000	0.000	0.000	0.188	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	111	61	70	77	60	78	0	67	66
N.S.	1	1.39	0.76	0.88	0.96	0.75	0.98	0.00	0.84	0.82
time (sec)	N/A	0.734	0.015	0.265	0.120	0.089	0.291	0.000	0.186	1.025

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	129	76	185	0	0	0	0	57	0
N.S.	1	1.16	0.68	1.67	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.688	0.200	0.415	0.000	0.000	0.000	0.000	0.182	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	58	42	44	57	40	54	0	43	44
N.S.	1	1.09	0.79	0.83	1.08	0.75	1.02	0.00	0.81	0.83
time (sec)	N/A	0.415	0.011	0.245	0.117	0.092	0.211	0.000	0.226	0.180

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	80	56	130	0	0	0	0	8	55
N.S.	1	1.19	0.84	1.94	0.00	0.00	0.00	0.00	0.12	0.82
time (sec)	N/A	0.411	0.060	0.374	0.000	0.000	0.000	0.000	0.195	0.096

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	116	146	132	891	0	0	0	0	12	0
N.S.	1	1.26	1.14	7.68	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.679	0.042	4.764	0.000	0.000	0.000	0.000	0.197	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	76	64	222	0	0	0	0	35	0
N.S.	1	1.15	0.97	3.36	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.425	0.032	0.525	0.000	0.000	0.000	0.000	0.200	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	60	56	64	56	57	53	60	64	50
N.S.	1	1.02	0.95	1.08	0.95	0.97	0.90	1.02	1.08	0.85
time (sec)	N/A	0.445	0.012	0.231	0.124	0.080	0.198	0.137	0.217	0.810

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	115	96	251	0	0	0	0	66	0
N.S.	1	1.02	0.85	2.22	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.636	0.170	0.678	0.000	0.000	0.000	0.000	0.188	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	113	81	85	95	78	80	91	84	73
N.S.	1	1.27	0.91	0.96	1.07	0.88	0.90	1.02	0.94	0.82
time (sec)	N/A	0.729	0.014	0.273	0.118	0.100	0.254	0.127	0.190	0.941

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	447	125	1104	0	0	0	0	131	0
N.S.	1	2.30	0.64	5.69	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	2.614	0.481	8.337	0.000	0.000	0.000	0.000	0.186	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	205	308	184	1910	0	0	0	0	116	0
N.S.	1	1.50	0.90	9.32	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	2.266	0.437	7.351	0.000	0.000	0.000	0.000	0.219	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	256	96	857	0	0	0	0	92	0
N.S.	1	1.73	0.65	5.79	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	1.656	0.260	6.484	0.000	0.000	0.000	0.000	0.196	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	157	184	149	1108	0	0	0	0	83	0
N.S.	1	1.17	0.95	7.06	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	1.379	0.247	5.869	0.000	0.000	0.000	0.000	0.197	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	117	76	592	0	0	0	0	83	0
N.S.	1	1.14	0.74	5.75	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.731	0.063	5.101	0.000	0.000	0.000	0.000	0.228	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	112	90	187	0	0	0	0	8	0
N.S.	1	1.17	0.94	1.95	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.592	0.080	0.819	0.000	0.000	0.000	0.000	0.205	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	178	220	180	982	0	0	0	0	12	0
N.S.	1	1.24	1.01	5.52	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.892	0.053	5.613	0.000	0.000	0.000	0.000	0.193	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	93	116	83	1441	0	0	0	0	37	0
N.S.	1	1.25	0.89	15.49	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.632	0.051	7.699	0.000	0.000	0.000	0.000	0.204	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	105	106	90	2956	0	0	0	29	85	0
N.S.	1	1.01	0.86	28.15	0.00	0.00	0.00	0.28	0.81	0.00
time (sec)	N/A	0.736	0.112	5.704	0.000	0.000	0.000	0.137	0.225	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	167	183	151	1620	0	0	0	0	106	0
N.S.	1	1.10	0.90	9.70	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	1.435	0.178	65.066	0.000	0.000	0.000	0.000	0.195	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	152	226	126	852	0	0	0	0	159	0
N.S.	1	1.49	0.83	5.61	0.00	0.00	0.00	0.00	1.05	0.00
time (sec)	N/A	1.381	0.201	15.467	0.000	0.000	0.000	0.000	0.183	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	221	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	22.10	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.176	0.596	1.250	2.232	0.094	2.200	0.155	0.192	0.735

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	183	12	10	12	12	12
N.S.	1	1.00	1.20	1.00	18.30	1.20	1.00	1.20	1.20	1.20
time (sec)	N/A	0.185	0.593	0.894	1.594	0.096	1.145	0.139	0.232	0.741

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	52	0	0	0	0	0	91	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	1.60	0.00
time (sec)	N/A	0.203	0.015	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	42	41	39	38	39	39	40	60	35
N.S.	1	1.02	1.00	0.95	0.93	0.95	0.95	0.98	1.46	0.85
time (sec)	N/A	0.226	0.011	0.490	0.040	0.104	0.388	0.130	0.189	0.821

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	38	37	31	34	27	36	38	30	31
N.S.	1	1.03	1.00	0.84	0.92	0.73	0.97	1.03	0.81	0.84
time (sec)	N/A	0.207	0.005	0.414	0.130	0.095	0.302	0.121	0.188	0.842

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	28	28	31	47	47	27
N.S.	1	1.00	1.00	0.90	0.90	0.90	1.00	1.52	1.52	0.87
time (sec)	N/A	0.190	0.004	0.246	0.026	0.086	0.156	0.125	0.228	0.767

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	41	37	57	68	0	0	0	12	0
N.S.	1	1.11	1.00	1.54	1.84	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.262	0.006	0.366	0.162	0.000	0.000	0.000	0.198	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	34	31	32	37	29	32	61	31
N.S.	1	1.06	1.00	0.91	0.94	1.09	0.85	0.94	1.79	0.91
time (sec)	N/A	0.196	0.006	0.226	0.038	0.116	0.231	0.120	0.191	0.831

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	34	38	31	27	28	29	29	33	32
N.S.	1	0.97	1.09	0.89	0.77	0.80	0.83	0.83	0.94	0.91
time (sec)	N/A	0.204	0.006	0.306	0.114	0.090	0.287	0.128	0.194	0.808

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	175	136	112	137	113	136	156	107	54
N.S.	1	1.47	1.14	0.94	1.15	0.95	1.14	1.31	0.90	0.45
time (sec)	N/A	0.414	0.030	0.583	0.116	0.108	12.344	0.127	0.218	0.490

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	164	133	109	135	107	126	153	102	52
N.S.	1	1.40	1.14	0.93	1.15	0.91	1.08	1.31	0.87	0.44
time (sec)	N/A	0.401	0.024	0.388	0.110	0.108	6.175	0.138	0.191	0.861

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	152	102	97	120	96	615	144	94	42
N.S.	1	1.57	1.05	1.00	1.24	0.99	6.34	1.48	0.97	0.43
time (sec)	N/A	0.363	0.025	0.293	0.112	0.093	3.690	0.115	0.194	0.852

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	149	105	98	123	104	110	135	96	44
N.S.	1	1.49	1.05	0.98	1.23	1.04	1.10	1.35	0.96	0.44
time (sec)	N/A	0.370	0.032	0.352	0.107	0.112	7.569	0.138	0.180	0.169

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	170	146	106	133	118	459	149	114	52
N.S.	1	1.45	1.25	0.91	1.14	1.01	3.92	1.27	0.97	0.44
time (sec)	N/A	0.394	0.036	0.453	0.129	0.095	15.160	0.131	0.227	0.886

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	41	37	57	68	0	0	0	12	0
N.S.	1	1.11	1.00	1.54	1.84	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.256	0.006	0.611	0.162	0.000	0.000	0.000	0.181	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	13	15	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	0.76	0.88	0.88
time (sec)	N/A	0.172	0.001	0.327	0.041	0.083	0.070	0.124	0.173	0.731

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	52	40	32	31	27	39	33	30	31
N.S.	1	1.02	0.78	0.63	0.61	0.53	0.76	0.65	0.59	0.61
time (sec)	N/A	0.192	0.011	0.208	0.107	0.093	0.836	0.127	0.173	0.820

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	43	33	27	26	20	32	28	23	26
N.S.	1	1.02	0.79	0.64	0.62	0.48	0.76	0.67	0.55	0.62
time (sec)	N/A	0.185	0.008	0.055	0.115	0.096	0.499	0.133	0.205	0.828

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	29	22	17	16	12	19	14	11	16
N.S.	1	1.32	1.00	0.77	0.73	0.55	0.86	0.64	0.50	0.73
time (sec)	N/A	0.172	0.013	0.043	0.110	0.092	0.360	0.129	0.192	0.789

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	37	31	61	35	0	0	19	9	0
N.S.	1	1.19	1.00	1.97	1.13	0.00	0.00	0.61	0.29	0.00
time (sec)	N/A	0.243	0.005	0.251	0.156	0.000	0.000	0.144	0.194	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	32	29	18	17	19	92	19	18	17
N.S.	1	1.39	1.26	0.78	0.74	0.83	4.00	0.83	0.78	0.74
time (sec)	N/A	0.184	0.007	0.063	0.122	0.109	0.431	0.120	0.187	0.791

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	43	34	27	26	27	160	26	26	24
N.S.	1	1.02	0.81	0.64	0.62	0.64	3.81	0.62	0.62	0.57
time (sec)	N/A	0.184	0.008	0.064	0.107	0.153	0.864	0.123	0.215	0.822

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	35	29	25	24	24	85	39	25	24
N.S.	1	0.97	0.81	0.69	0.67	0.67	2.36	1.08	0.69	0.67
time (sec)	N/A	0.189	0.011	0.056	0.033	0.153	0.707	0.132	0.194	0.827

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	28	25	20	19	19	24	30	18	0
N.S.	1	0.97	0.86	0.69	0.66	0.66	0.83	1.03	0.62	0.00
time (sec)	N/A	0.190	0.008	0.049	0.032	0.148	0.504	0.130	0.185	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	17	18	12	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.94	1.00	0.67	0.78
time (sec)	N/A	0.166	0.005	0.049	0.027	0.117	0.122	0.122	0.186	0.933

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	25	20	16	25	20
N.S.	1	1.00	1.00	0.86	0.82	1.14	0.91	0.73	1.14	0.91
time (sec)	N/A	0.173	0.009	0.055	0.030	0.104	0.314	0.127	0.223	0.804

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	32	29	26	25	33	143	23	34	27
N.S.	1	0.86	0.78	0.70	0.68	0.89	3.86	0.62	0.92	0.73
time (sec)	N/A	0.191	0.014	0.056	0.038	0.112	0.868	0.121	0.187	0.791

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	45	40	83	0	63	0	0	12	0
N.S.	1	0.96	0.85	1.77	0.00	1.34	0.00	0.00	0.26	0.00
time (sec)	N/A	0.257	0.013	1.005	0.000	0.128	0.000	0.000	0.184	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [23] had the largest ratio of [1.6999999999999996]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.02	8	0.375
2	A	5	4	1.02	8	0.500
3	A	3	3	1.02	8	0.375
4	A	5	4	1.03	8	0.500
5	A	3	3	1.03	6	0.500
6	A	2	2	1.00	4	0.500
7	A	2	2	1.00	8	0.250
8	A	6	5	1.07	8	0.625
9	A	3	3	0.97	8	0.375
10	A	5	4	1.00	8	0.500
11	A	4	4	1.05	8	0.500
12	A	16	15	1.64	10	1.500
13	A	14	13	1.34	10	1.300
14	A	11	10	1.39	10	1.000
15	A	10	9	1.16	10	0.900
16	A	5	5	1.09	8	0.625
17	A	6	5	1.19	6	0.833
18	A	4	4	1.26	10	0.400
19	A	4	4	1.15	10	0.400
20	A	9	8	1.02	10	0.800
21	A	8	8	1.02	10	0.800

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	14	13	1.27	10	1.300
23	B	18	17	2.30	10	1.700
24	A	17	16	1.50	10	1.600
25	A	14	13	1.73	10	1.300
26	A	11	11	1.17	10	1.100
27	A	9	8	1.14	8	1.000
28	A	5	5	1.17	6	0.833
29	A	5	5	1.24	10	0.500
30	A	5	5	1.25	10	0.500
31	A	7	7	1.01	10	0.700
32	A	15	14	1.10	10	1.400
33	A	11	11	1.49	10	1.100
34	N/A	1	0	1.00	10	0.000
35	N/A	1	0	1.00	10	0.000
36	A	2	2	1.00	8	0.250
37	A	5	4	1.02	10	0.400
38	A	5	4	1.03	10	0.400
39	A	2	2	1.00	8	0.250
40	A	4	3	1.11	10	0.300
41	A	6	5	1.06	10	0.500
42	A	5	4	0.97	10	0.400
43	A	11	10	1.47	10	1.000
44	A	11	10	1.40	10	1.000
45	A	10	9	1.57	6	1.500
46	A	10	9	1.49	10	0.900
47	A	11	10	1.45	10	1.000
48	A	4	3	1.11	10	0.300
49	A	3	3	1.00	4	0.750
50	A	7	6	1.02	10	0.600
51	A	6	5	1.02	8	0.625
52	A	5	4	1.32	6	0.667
53	A	4	3	1.19	10	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	5	4	1.39	10	0.400
55	A	6	5	1.02	10	0.500
56	A	3	3	0.97	12	0.250
57	A	3	3	0.97	12	0.250
58	A	2	2	1.00	12	0.167
59	A	4	4	1.00	12	0.333
60	A	3	3	0.86	12	0.250
61	A	4	3	0.96	10	0.300

CHAPTER 3

Listing of integrals

3.1 $\int x^5 \cot^{-1}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 51

$$\int x^5 \cot^{-1}(ax)\,dx = \frac{x}{6a^5} - \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6\cot^{-1}(ax) - \frac{\arctan(ax)}{6a^6}$$

output
```
1/6*x/a^5-1/18*x^3/a^3+1/30*x^5/a+1/6*x^6*arccot(a*x)-1/6*arctan(a*x)/a^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^5 \cot^{-1}(ax)\,dx = \frac{x}{6a^5} - \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6\cot^{-1}(ax) - \frac{\arctan(ax)}{6a^6}$$

input
```
Integrate[x^5*ArcCot[a*x],x]
```

output
```
x/(6*a^5) - x^3/(18*a^3) + x^5/(30*a) + (x^6*ArcCot[a*x])/6 - ArcTan[a*x]/
(6*a^6)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5362, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \cot^{-1}(ax) \, dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{6} a \int \frac{x^6}{a^2x^2+1} dx + \frac{1}{6} x^6 \cot^{-1}(ax)$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{6} a \int \left(\frac{x^4}{a^2} - \frac{x^2}{a^4} - \frac{1}{a^6 (a^2x^2+1)} + \frac{1}{a^6}\right) dx + \frac{1}{6} x^6 \cot^{-1}(ax)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6} a \left(-\frac{\arctan(ax)}{a^7} + \frac{x}{a^6} - \frac{x^3}{3a^4} + \frac{x^5}{5a^2}\right) + \frac{1}{6} x^6 \cot^{-1}(ax)$$

input
```
Int[x^5*ArcCot[a*x],x]
```

output
```
(x^6*ArcCot[a*x])/6 + (a*(x/a^6 - x^3/(3*a^4) + x^5/(5*a^2) - ArcTan[a*x]/
a^7))/6
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{a^6x^6\operatorname{arccot}(ax)}{6}+\frac{a^5x^5}{30}-\frac{a^3x^3}{18}+\frac{ax}{6}-\frac{\arctan(ax)}{6}}{a^6}$	44
default	$\frac{\frac{a^6x^6\operatorname{arccot}(ax)}{6}+\frac{a^5x^5}{30}-\frac{a^3x^3}{18}+\frac{ax}{6}-\frac{\arctan(ax)}{6}}{a^6}$	44
parallelrisch	$\frac{15a^6x^6\operatorname{arccot}(ax)+3a^5x^5-5a^3x^3+15ax+15\operatorname{arccot}(ax)}{90a^6}$	45
parts	$\frac{x^6\operatorname{arccot}(ax)}{6}+\frac{a\left(\frac{\frac{1}{5}a^4x^5-\frac{1}{3}a^2x^3+x}{a^6}-\frac{\arctan(ax)}{a^7}\right)}{6}$	46
risch	$\frac{ix^6\ln(iax+1)}{12}-\frac{ix^6\ln(-iax+1)}{12}+\frac{x^6\pi}{12}+\frac{x^5}{30a}-\frac{x^3}{18a^3}+\frac{x}{6a^5}-\frac{\arctan(ax)}{6a^6}$	67
orering	$\frac{(3a^6x^6-a^4x^4+5a^2x^2+9)\operatorname{arccot}(ax)}{9a^6}-\frac{(3a^4x^4-5a^2x^2+15)(a^2x^2+1)\left(5x^4\operatorname{arccot}(ax)-\frac{x^5a}{a^2x^2+1}\right)}{90x^4a^6}$	99

input
```
int(x^5*arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^6*(1/6*a^6*x^6*arccot(a*x)+1/30*a^5*x^5-1/18*a^3*x^3+1/6*a*x-1/6*arcta
n(a*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int x^5\cot^{-1}(ax)\,dx=\frac{3\,a^5x^5-5\,a^3x^3+15\,ax+15\,(a^6x^6+1)\operatorname{arccot}(ax)}{90\,a^6}$$

input
```
integrate(x^5*arccot(a*x),x, algorithm="fricas")
```

output
```
1/90*(3*a^5*x^5 - 5*a^3*x^3 + 15*a*x + 15*(a^6*x^6 + 1)*arccot(a*x))/a^6
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int x^5 \cot^{-1}(ax)\,dx = \begin{cases} \frac{x^6 \operatorname{acot}(ax)}{6} + \frac{x^5}{30a} - \frac{x^3}{18a^3} + \frac{x}{6a^5} + \frac{\operatorname{acot}(ax)}{6a^6} & \text{for } a \neq 0 \\ \frac{\pi x^6}{12} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*acot(a*x),x)
```

output
```
Piecewise((x**6*acot(a*x)/6 + x**5/(30*a) - x**3/(18*a**3) + x/(6*a**5) +
acot(a*x)/(6*a**6), Ne(a, 0)), (pi*x**6/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int x^5 \cot^{-1}(ax)\,dx = \frac{1}{6}x^6 \operatorname{arccot}(ax) + \frac{1}{90}a\left(\frac{3a^4x^5 - 5a^2x^3 + 15x}{a^6} - \frac{15\arctan(ax)}{a^7}\right)$$

input
```
integrate(x^5*arccot(a*x),x, algorithm="maxima")
```

output
```
1/6*x^6*arccot(a*x) + 1/90*a*((3*a^4*x^5 - 5*a^2*x^3 + 15*x)/a^6 - 15*arct
an(a*x)/a^7)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int x^5 \cot^{-1}(ax)\,dx = \frac{1}{90}\left(\frac{15\,x^6\arctan\left(\frac{1}{ax}\right)}{a} - \frac{x^5\left(\frac{5}{a^2x^2} - \frac{15}{a^4x^4} - 3\right)}{a^2} + \frac{15\arctan\left(\frac{1}{ax}\right)}{a^7}\right)a$$

input
```
integrate(x^5*arccot(a*x),x, algorithm="giac")
```

output
```
1/90*(15*x^6*arctan(1/(a*x))/a - x^5*(5/(a^2*x^2) - 15/(a^4*x^4) - 3)/a^2
+ 15*arctan(1/(a*x))/a^7)*a
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int x^5 \cot^{-1}(ax)\,dx = \begin{cases} \frac{\pi\,x^6}{12} & \text{if } a = 0 \\ \frac{x^6\,\mathrm{acot}(a\,x)}{6} - \frac{\frac{\mathrm{atan}(a\,x)}{6} - \frac{a\,x}{6} + \frac{a^3\,x^3}{18} - \frac{a^5\,x^5}{30}}{a^6} & \text{if } a \neq 0 \end{cases}$$

input
```
int(x^5*acot(a*x),x)
```

output
```
piecewise(a == 0, (x^6*pi)/12, a ~= 0, - (atan(a*x)/6 - (a*x)/6 + (a^3*x^3
)/18 - (a^5*x^5)/30)/a^6 + (x^6*acot(a*x))/6)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int x^5 \cot^{-1}(ax)\,dx = \frac{15acot(ax)\,a^6x^6 + 15acot(ax) + 3a^5x^5 - 5a^3x^3 + 15ax}{90a^6}$$

input
```
int(x^5*acot(a*x),x)
```

output
```
(15*acot(a*x)*a**6*x**6 + 15*acot(a*x) + 3*a**5*x**5 - 5*a**3*x**3 + 15*a*
x)/(90*a**6)
```

3.2 $\int x^4 \cot^{-1}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 49

$$\int x^4 \cot^{-1}(ax)\,dx = -\frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{1}{5}x^5 \cot^{-1}(ax) + \frac{\log(1 + a^2x^2)}{10a^5}$$

output
```
-1/10*x^2/a^3+1/20*x^4/a+1/5*x^5*arccot(a*x)+1/10*ln(a^2*x^2+1)/a^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int x^4 \cot^{-1}(ax)\,dx = -\frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{1}{5}x^5 \cot^{-1}(ax) + \frac{\log(1 + a^2x^2)}{10a^5}$$

input
```
Integrate[x^4*ArcCot[a*x],x]
```

output
```
-1/10*x^2/a^3 + x^4/(20*a) + (x^5*ArcCot[a*x])/5 + Log[1 + a^2*x^2]/(10*a^
5)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5362, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \cot^{-1}(ax) \, dx$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{1}{5} a \int \frac{x^5}{a^2x^2+1} dx + \frac{1}{5} x^5 \cot^{-1}(ax)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{10} a \int \frac{x^4}{a^2x^2+1} dx^2 + \frac{1}{5} x^5 \cot^{-1}(ax)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{10} a \int \left(\frac{x^2}{a^2} + \frac{1}{a^4 (a^2x^2+1)} - \frac{1}{a^4}\right) dx^2 + \frac{1}{5} x^5 \cot^{-1}(ax)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{10} a \left(-\frac{x^2}{a^4} + \frac{x^4}{2a^2} + \frac{\log(a^2x^2+1)}{a^6}\right) + \frac{1}{5} x^5 \cot^{-1}(ax)$$

input
```
Int[x^4*ArcCot[a*x],x]
```

output
```
(x^5*ArcCot[a*x])/5 + (a*(-(x^2/a^4) + x^4/(2*a^2) + Log[1 + a^2*x^2]/a^6)
)/10
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\frac{a^5x^5\,\mathrm{arccot}(ax)}{5}+\frac{a^4x^4}{20}-\frac{a^2x^2}{10}+\frac{\ln\left(a^2x^2+1\right)}{10}}{a^5}$	46
default	$\frac{\frac{a^5x^5\,\mathrm{arccot}(ax)}{5}+\frac{a^4x^4}{20}-\frac{a^2x^2}{10}+\frac{\ln\left(a^2x^2+1\right)}{10}}{a^5}$	46
parallelrisch	$\frac{4a^5x^5\,\mathrm{arccot}(ax)+a^4x^4-2a^2x^2+2+2\ln\left(a^2x^2+1\right)}{20a^5}$	47
parts	$\frac{x^5\,\mathrm{arccot}(ax)}{5}+\frac{a\left(\frac{\frac{1}{2}a^2x^4-x^2}{2a^4}+\frac{\ln\left(a^2x^2+1\right)}{2a^6}\right)}{5}$	49
risch	$\frac{ix^5\ln(iax+1)}{10}-\frac{ix^5\ln(-iax+1)}{10}+\frac{x^5\pi}{10}+\frac{x^4}{20a}-\frac{x^2}{10a^3}+\frac{\ln\left(-a^2x^2-1\right)}{10a^5}$	68

input
```
int(x^4*arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/5*a^5*x^5*arccot(a*x)+1/20*a^4*x^4-1/10*a^2*x^2+1/10*ln(a^2*x^2+1
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int x^4 \cot^{-1}(ax)\,dx = \frac{4\,a^5x^5\,\text{arccot}\,(ax) + a^4x^4 - 2\,a^2x^2 + 2\,\log\left(a^2x^2+1\right)}{20\,a^5}$$

input
```
integrate(x^4*arccot(a*x),x, algorithm="fricas")
```

output
```
1/20*(4*a^5*x^5*arccot(a*x) + a^4*x^4 - 2*a^2*x^2 + 2*log(a^2*x^2 + 1))/a^
5
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int x^4 \cot^{-1}(ax)\,dx = \begin{cases} \frac{x^5\,\text{acot}\,(ax)}{5} + \frac{x^4}{20a} - \frac{x^2}{10a^3} + \frac{\log\left(a^2x^2+1\right)}{10a^5} & \text{for } a \neq 0 \\ \frac{\pi x^5}{10} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*acot(a*x),x)
```

output
```
Piecewise((x**5*acot(a*x)/5 + x**4/(20*a) - x**2/(10*a**3) + log(a**2*x**2
 + 1)/(10*a**5), Ne(a, 0)), (pi*x**5/10, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int x^4 \cot^{-1}(ax)\,dx = \frac{1}{5}\,x^5 \operatorname{arccot}(ax) + \frac{1}{20}\,a\left(\frac{a^2x^4 - 2\,x^2}{a^4} + \frac{2\,\log\left(a^2x^2 + 1\right)}{a^6}\right)$$

input
```
integrate(x^4*arccot(a*x),x, algorithm="maxima")
```

output
```
1/5*x^5*arccot(a*x) + 1/20*a*((a^2*x^4 - 2*x^2)/a^4 + 2*log(a^2*x^2 + 1)/a
^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int x^4 \cot^{-1}(ax)\,dx$$
$$= \frac{1}{20}\left(\frac{4\,x^5 \arctan\left(\frac{1}{ax}\right)}{a} - \frac{x^4\left(\frac{2}{a^2x^2} - \frac{3}{a^4x^4} - 1\right)}{a^2} + \frac{2\,\log\left(\frac{1}{a^2x^2} + 1\right)}{a^6} - \frac{2\,\log\left(\frac{1}{a^2x^2}\right)}{a^6}\right)a$$

input
```
integrate(x^4*arccot(a*x),x, algorithm="giac")
```

output
```
1/20*(4*x^5*arctan(1/(a*x))/a - x^4*(2/(a^2*x^2) - 3/(a^4*x^4) - 1)/a^2 +
2*log(1/(a^2*x^2) + 1)/a^6 - 2*log(1/(a^2*x^2))/a^6)*a
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int x^4 \cot^{-1}(ax)\,dx = \begin{cases} \frac{\pi\,x^5}{10} & \text{if } a = 0 \\ \frac{2\,\ln\left(a^2\,x^2+1\right)-2\,a^2\,x^2+a^4\,x^4}{20\,a^5} + \frac{x^5\,\mathrm{acot}(a\,x)}{5} & \text{if } a \neq 0 \end{cases}$$

input
```
int(x^4*acot(a*x),x)
```

output
```
piecewise(a == 0, (x^5*pi)/10, a ~= 0, (2*log(a^2*x^2 + 1) - 2*a^2*x^2 + a
^4*x^4)/(20*a^5) + (x^5*acot(a*x))/5)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int x^4 \cot^{-1}(ax)\, dx = \frac{4acot(ax)\, a^5x^5 + 2\log(a^2x^2 + 1) + a^4x^4 - 2a^2x^2}{20a^5}$$

input
```
int(x^4*acot(a*x),x)
```

output
```
(4*acot(a*x)*a**5*x**5 + 2*log(a**2*x**2 + 1) + a**4*x**4 - 2*a**2*x**2)/(
20*a**5)
```

3.3 $\int x^3 \cot^{-1}(ax)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int x^3 \cot^{-1}(ax)\, dx = -\frac{x}{4a^3} + \frac{x^3}{12a} + \frac{1}{4}x^4 \cot^{-1}(ax) + \frac{\arctan(ax)}{4a^4}$$

output
```
-1/4*x/a^3+1/12*x^3/a+1/4*x^4*arccot(a*x)+1/4*arctan(a*x)/a^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int x^3 \cot^{-1}(ax)\, dx = -\frac{x}{4a^3} + \frac{x^3}{12a} + \frac{1}{4}x^4 \cot^{-1}(ax) + \frac{\arctan(ax)}{4a^4}$$

input
```
Integrate[x^3*ArcCot[a*x],x]
```

output
```
-1/4*x/a^3 + x^3/(12*a) + (x^4*ArcCot[a*x])/4 + ArcTan[a*x]/(4*a^4)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5362, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cot^{-1}(ax) \, dx$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{1}{4} a \int \frac{x^4}{a^2x^2+1} dx + \frac{1}{4} x^4 \cot^{-1}(ax)$$

$$\Big\downarrow \text{ 254}$$

$$\frac{1}{4} a \int \left(\frac{x^2}{a^2} + \frac{1}{a^4 (a^2x^2+1)} - \frac{1}{a^4}\right) dx + \frac{1}{4} x^4 \cot^{-1}(ax)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4} a \left(\frac{\arctan(ax)}{a^5} - \frac{x}{a^4} + \frac{x^3}{3a^2}\right) + \frac{1}{4} x^4 \cot^{-1}(ax)$$

input
```
Int[x^3*ArcCot[a*x],x]
```

output
```
(x^4*ArcCot[a*x])/4 + (a*(-(x/a^4) + x^3/(3*a^2) + ArcTan[a*x]/a^5))/4
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\frac{a^4x^4\,\mathrm{arccot}(ax)}{4}+\frac{a^3x^3}{12}-\frac{ax}{4}+\frac{\arctan(ax)}{4}}{a^4}$	36
default	$\frac{\frac{a^4x^4\,\mathrm{arccot}(ax)}{4}+\frac{a^3x^3}{12}-\frac{ax}{4}+\frac{\arctan(ax)}{4}}{a^4}$	36
parallelrisch	$\frac{3a^4x^4\,\mathrm{arccot}(ax)+a^3x^3-3ax-3\,\mathrm{arccot}(ax)}{12a^4}$	36
parts	$\frac{x^4\,\mathrm{arccot}(ax)}{4}+\frac{a\left(\frac{\frac{1}{3}a^2x^3-x}{a^4}+\frac{\arctan(ax)}{a^5}\right)}{4}$	39
risch	$\frac{ix^4\ln(iax+1)}{8}-\frac{ix^4\ln(-iax+1)}{8}+\frac{x^4\pi}{8}+\frac{x^3}{12a}-\frac{x}{4a^3}+\frac{\arctan(ax)}{4a^4}$	59
orering	$\frac{(a^4x^4-a^2x^2-2)\,\mathrm{arccot}(ax)}{2a^4}-\frac{(a^2x^2-3)(a^2x^2+1)\left(3x^2\,\mathrm{arccot}(ax)-\frac{x^3a}{a^2x^2+1}\right)}{12x^2a^4}$	81

input
```
int(x^3*arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4*a^4*x^4*arccot(a*x)+1/12*a^3*x^3-1/4*a*x+1/4*arctan(a*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int x^3\cot^{-1}(ax)\,dx=\frac{a^3x^3-3\,ax+3\,(a^4x^4-1)\,\mathrm{arccot}\,(ax)}{12\,a^4}$$

input
```
integrate(x^3*arccot(a*x),x, algorithm="fricas")
```

output
```
1/12*(a^3*x^3 - 3*a*x + 3*(a^4*x^4 - 1)*arccot(a*x))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int x^3 \cot^{-1}(ax)\,dx = \begin{cases} \frac{x^4 \operatorname{acot}(ax)}{4} + \frac{x^3}{12a} - \frac{x}{4a^3} - \frac{\operatorname{acot}(ax)}{4a^4} & \text{for } a \neq 0 \\ \frac{\pi x^4}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acot(a*x),x)
```

output
```
Piecewise((x**4*acot(a*x)/4 + x**3/(12*a) - x/(4*a**3) - acot(a*x)/(4*a**4
), Ne(a, 0)), (pi*x**4/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int x^3 \cot^{-1}(ax)\,dx = \frac{1}{4}x^4 \operatorname{arccot}(ax) + \frac{1}{12}a\left(\frac{a^2x^3 - 3x}{a^4} + \frac{3\arctan(ax)}{a^5}\right)$$

input
```
integrate(x^3*arccot(a*x),x, algorithm="maxima")
```

output
```
1/4*x^4*arccot(a*x) + 1/12*a*((a^2*x^3 - 3*x)/a^4 + 3*arctan(a*x)/a^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int x^3 \cot^{-1}(ax)\,dx = \frac{1}{12}\left(\frac{3\,x^4 \arctan\left(\frac{1}{ax}\right)}{a} - \frac{x^3\left(\frac{3}{a^2x^2}-1\right)}{a^2} - \frac{3\,\arctan\left(\frac{1}{ax}\right)}{a^5}\right)a$$

input
```
integrate(x^3*arccot(a*x),x, algorithm="giac")
```

output
```
1/12*(3*x^4*arctan(1/(a*x))/a - x^3*(3/(a^2*x^2) - 1)/a^2 - 3*arctan(1/(a*
x))/a^5)*a
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int x^3 \cot^{-1}(ax)\,dx = \begin{cases} \frac{\pi\,x^4}{8} & \text{if } a = 0 \\ \frac{3\,\mathrm{atan}(a\,x)-3\,a\,x+a^3\,x^3}{12\,a^4} + \frac{x^4\,\mathrm{acot}(a\,x)}{4} & \text{if } a \neq 0 \end{cases}$$

input
```
int(x^3*acot(a*x),x)
```

output
```
piecewise(a == 0, (x^4*pi)/8, a ~= 0, (3*atan(a*x) - 3*a*x + a^3*x^3)/(12*
a^4) + (x^4*acot(a*x))/4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int x^3 \cot^{-1}(ax)\,dx = \frac{3acot(ax)\,a^4x^4 - 3acot(ax) + a^3x^3 - 3ax}{12a^4}$$

input
```
int(x^3*acot(a*x),x)
```

output
```
(3*acot(a*x)*a**4*x**4 - 3*acot(a*x) + a**3*x**3 - 3*a*x)/(12*a**4)
```

3.4 $\int x^2 \cot^{-1}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 39

$$\int x^2 \cot^{-1}(ax)\,dx = \frac{x^2}{6a} + \frac{1}{3}x^3 \cot^{-1}(ax) - \frac{\log(1+a^2x^2)}{6a^3}$$

output
```
1/6*x^2/a+1/3*x^3*arccot(a*x)-1/6*ln(a^2*x^2+1)/a^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int x^2 \cot^{-1}(ax)\,dx = \frac{x^2}{6a} + \frac{1}{3}x^3 \cot^{-1}(ax) - \frac{\log(1+a^2x^2)}{6a^3}$$

input
```
Integrate[x^2*ArcCot[a*x],x]
```

output
```
x^2/(6*a) + (x^3*ArcCot[a*x])/3 - Log[1 + a^2*x^2]/(6*a^3)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5362, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(ax) \, dx$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{1}{3} a \int \frac{x^3}{a^2 x^2+1} dx+\frac{1}{3} x^3 \cot^{-1}(ax)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{6} a \int \frac{x^2}{a^2 x^2+1} dx^2+\frac{1}{3} x^3 \cot^{-1}(ax)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{6} a \int \left(\frac{1}{a^2}-\frac{1}{a^2 \left(a^2 x^2+1\right)}\right) dx^2+\frac{1}{3} x^3 \cot^{-1}(ax)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6} a \left(\frac{x^2}{a^2}-\frac{\log \left(a^2 x^2+1\right)}{a^4}\right)+\frac{1}{3} x^3 \cot^{-1}(ax)$$

input
```
Int[x^2*ArcCot[a*x],x]
```

output
```
(x^3*ArcCot[a*x])/3 + (a*(x^2/a^2 - Log[1 + a^2*x^2]/a^4))/6
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$-\frac{-2a^3x^3\operatorname{arccot}(ax)-a^2x^2+\ln(a^2x^2+1)}{6a^3}$	37
derivativedivides	$\frac{\frac{a^3x^3\operatorname{arccot}(ax)}{3}+\frac{a^2x^2}{6}-\frac{\ln\left(a^2x^2+1\right)}{6}}{a^3}$	38
default	$\frac{\frac{a^3x^3\operatorname{arccot}(ax)}{3}+\frac{a^2x^2}{6}-\frac{\ln\left(a^2x^2+1\right)}{6}}{a^3}$	38
parts	$\frac{x^3\operatorname{arccot}(ax)}{3}+\frac{a\left(\frac{x^2}{2a^2}-\frac{\ln\left(a^2x^2+1\right)}{2a^4}\right)}{3}$	38
risch	$\frac{ix^3\ln(iax+1)}{6}-\frac{ix^3\ln(-iax+1)}{6}+\frac{\pi x^3}{6}+\frac{x^2}{6a}-\frac{\ln(-a^2x^2-1)}{6a^3}$	60

input
```
int(x^2*arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-2*a^3*x^3*arccot(a*x)-a^2*x^2+ln(a^2*x^2+1))/a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int x^2 \cot^{-1}(ax)\,dx = \frac{2\,a^3x^3\,\mathrm{arccot}\,(ax) + a^2x^2 - \log\left(a^2x^2+1\right)}{6\,a^3}$$

input
```
integrate(x^2*arccot(a*x),x, algorithm="fricas")
```

output
```
1/6*(2*a^3*x^3*arccot(a*x) + a^2*x^2 - log(a^2*x^2 + 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int x^2 \cot^{-1}(ax)\,dx = \begin{cases} \frac{x^3\,\mathrm{acot}\,(ax)}{3} + \frac{x^2}{6a} - \frac{\log\left(a^2x^2+1\right)}{6a^3} & \text{for } a \neq 0 \\ \frac{\pi x^3}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*acot(a*x),x)
```

output
```
Piecewise((x**3*acot(a*x)/3 + x**2/(6*a) - log(a**2*x**2 + 1)/(6*a**3), Ne
(a, 0)), (pi*x**3/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x^2 \cot^{-1}(ax)\, dx = \frac{1}{3}\, x^3 \operatorname{arccot}\left(ax\right) + \frac{1}{6}\, a\left(\frac{x^2}{a^2} - \frac{\log\left(a^2x^2+1\right)}{a^4}\right)$$

input
```
integrate(x^2*arccot(a*x),x, algorithm="maxima")
```

output
```
1/3*x^3*arccot(a*x) + 1/6*a*(x^2/a^2 - log(a^2*x^2 + 1)/a^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.64

$$\int x^2 \cot^{-1}(ax)\, dx = \frac{1}{6}\left(\frac{2\,x^3 \arctan\left(\frac{1}{ax}\right)}{a} - \frac{x^2\left(\frac{1}{a^2x^2}-1\right)}{a^2} - \frac{\log\left(\frac{1}{a^2x^2}+1\right)}{a^4} + \frac{\log\left(\frac{1}{a^2x^2}\right)}{a^4}\right)a$$

input
```
integrate(x^2*arccot(a*x),x, algorithm="giac")
```

output
```
1/6*(2*x^3*arctan(1/(a*x))/a - x^2*(1/(a^2*x^2) - 1)/a^2 - log(1/(a^2*x^2)
 + 1)/a^4 + log(1/(a^2*x^2))/a^4)*a
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int x^2 \cot^{-1}(ax)\, dx = \begin{cases} \frac{\pi\,x^3}{6} & \text{if } a=0 \\ \frac{\frac{x^2}{2}-\frac{\ln\left(a^2\,x^2+1\right)}{2\,a^2}}{3\,a} + \frac{x^3\,\mathrm{acot}(a\,x)}{3} & \text{if } a\neq 0 \end{cases}$$

input
```
int(x^2*acot(a*x),x)
```

output
```
piecewise(a == 0, (x^3*pi)/6, a ~= 0, (x^2/2 - log(a^2*x^2 + 1)/(2*a^2))/(
3*a) + (x^3*acot(a*x))/3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int x^2 \cot^{-1}(ax)\,dx = \frac{2acot(ax)\,a^3x^3 - \log(a^2x^2+1) + a^2x^2}{6a^3}$$

input
```
int(x^2*acot(a*x),x)
```

output
```
(2*acot(a*x)*a**3*x**3 - log(a**2*x**2 + 1) + a**2*x**2)/(6*a**3)
```

3.5 $\int x \cot^{-1}(ax)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 31

$$\int x \cot^{-1}(ax)\,dx = \frac{x}{2a} + \frac{1}{2}x^2 \cot^{-1}(ax) - \frac{\arctan(ax)}{2a^2}$$

output
```
1/2*x/a+1/2*x^2*arccot(a*x)-1/2*arctan(a*x)/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int x \cot^{-1}(ax)\,dx = \frac{x}{2a} + \frac{1}{2}x^2 \cot^{-1}(ax) - \frac{\arctan(ax)}{2a^2}$$

input
```
Integrate[x*ArcCot[a*x],x]
```

output
```
x/(2*a) + (x^2*ArcCot[a*x])/2 - ArcTan[a*x]/(2*a^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5362, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^{-1}(ax)\,dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{2}a\int \frac{x^2}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int \frac{1}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)$$

input
```
Int[x*ArcCot[a*x],x]
```

output
```
(x^2*ArcCot[a*x])/2 + (a*(x/a^2 - ArcTan[a*x]/a^3))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$\frac{\operatorname{arccot}(ax)a^2x^2+ax+\operatorname{arccot}(ax)}{2a^2}$	25
derivativedivides	$\frac{\frac{\operatorname{arccot}(ax)a^2x^2}{2}+\frac{ax}{2}-\frac{\arctan(ax)}{2}}{a^2}$	28
default	$\frac{\frac{\operatorname{arccot}(ax)a^2x^2}{2}+\frac{ax}{2}-\frac{\arctan(ax)}{2}}{a^2}$	28
parts	$\frac{x^2\operatorname{arccot}(ax)}{2}+\frac{a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{2}$	29
risch	$\frac{ix^2\ln(iax+1)}{4}-\frac{ix^2\ln(-iax+1)}{4}+\frac{\pi x^2}{4}+\frac{x}{2a}-\frac{\arctan(ax)}{2a^2}$	51
orering	$\frac{(a^2x^2+1)\operatorname{arccot}(ax)}{a^2}-\frac{(a^2x^2+1)\left(\operatorname{arccot}(ax)-\frac{xa}{a^2x^2+1}\right)}{2a^2}$	53

input
```
int(x*arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arccot(a*x)*a^2*x^2+a*x+arccot(a*x))/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x \cot^{-1}(ax) \, dx = \frac{ax + (a^2x^2 + 1) \operatorname{arccot}(ax)}{2 a^2}$$

input
```
integrate(x*arccot(a*x),x, algorithm="fricas")
```

output
```
1/2*(a*x + (a^2*x^2 + 1)*arccot(a*x))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int x \cot^{-1}(ax) \, dx = \begin{cases} \frac{x^2 \operatorname{acot}(ax)}{2} + \frac{x}{2a} + \frac{\operatorname{acot}(ax)}{2a^2} & \text{for } a \neq 0 \\ \frac{\pi x^2}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acot(a*x),x)
```

output
```
Piecewise((x**2*acot(a*x)/2 + x/(2*a) + acot(a*x)/(2*a**2), Ne(a, 0)), (pi
*x**2/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int x \cot^{-1}(ax) \, dx = \frac{1}{2} x^2 \operatorname{arccot}(ax) + \frac{1}{2} a \left(\frac{x}{a^2} - \frac{\arctan(ax)}{a^3} \right)$$

input
```
integrate(x*arccot(a*x),x, algorithm="maxima")
```

output
```
1/2*x^2*arccot(a*x) + 1/2*a*(x/a^2 - arctan(a*x)/a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int x\cot^{-1}(ax)\,dx = \frac{1}{2}\left(\frac{x^2\arctan\left(\frac{1}{ax}\right)}{a} + \frac{x}{a^2} + \frac{\arctan\left(\frac{1}{ax}\right)}{a^3}\right)a$$

input
```
integrate(x*arccot(a*x),x, algorithm="giac")
```

output
```
1/2*(x^2*arctan(1/(a*x))/a + x/a^2 + arctan(1/(a*x))/a^3)*a
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int x\cot^{-1}(ax)\,dx = \begin{cases} \frac{\pi x^2}{4} & \text{if } a = 0 \\ \frac{x - \frac{\operatorname{atan}(a x)}{a}}{2a} + \frac{x^2 \operatorname{acot}(a x)}{2} & \text{if } a \neq 0 \end{cases}$$

input
```
int(x*acot(a*x),x)
```

output
```
piecewise(a == 0, (x^2*pi)/4, a ~= 0, (x - atan(a*x)/a)/(2*a) + (x^2*acot(
a*x))/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int x\cot^{-1}(ax)\,dx = \frac{acot(ax)\,a^2x^2 + acot(ax) + ax}{2a^2}$$

input
```
int(x*acot(a*x),x)
```

output
```
(acot(a*x)*a**2*x**2 + acot(a*x) + a*x)/(2*a**2)
```

3.6 $\int \cot^{-1}(ax)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 24

$$\int \cot^{-1}(ax)\,dx = x\cot^{-1}(ax) + \frac{\log(1+a^2x^2)}{2a}$$

output
```
x*arccot(a*x)+1/2*ln(a^2*x^2+1)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(ax)\,dx = x\cot^{-1}(ax) + \frac{\log(1+a^2x^2)}{2a}$$

input
```
Integrate[ArcCot[a*x],x]
```

output
```
x*ArcCot[a*x] + Log[1 + a^2*x^2]/(2*a)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5346, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(ax) \, dx$$

$$\Big\downarrow \text{ 5346}$$

$$a \int \frac{x}{a^2x^2+1}dx + x\cot^{-1}(ax)$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\log(a^2x^2+1)}{2a} + x\cot^{-1}(ax)$$

input
```
Int[ArcCot[a*x],x]
```

output
```
x*ArcCot[a*x] + Log[1 + a^2*x^2]/(2*a)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
parts	$x \operatorname{arccot}(ax)+\frac{\ln(a^2x^2+1)}{2a}$	23
derivativedivides	$\frac{\operatorname{arccot}(ax)ax+\frac{\ln(a^2x^2+1)}{2}}{a}$	25
default	$\frac{\operatorname{arccot}(ax)ax+\frac{\ln(a^2x^2+1)}{2}}{a}$	25
parallelrisch	$\frac{2 \operatorname{arccot}(ax)ax+\ln(a^2x^2+1)}{2a}$	25
risch	$\frac{ix\ln(iax+1)}{2}-\frac{ix\ln(-iax+1)}{2}+\frac{\pi x}{2}+\frac{\ln(-a^2x^2-1)}{2a}$	46

input
```
int(arccot(a*x),x,method=_RETURNVERBOSE)
```

output
```
x*arccot(a*x)+1/2*ln(a^2*x^2+1)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(ax)\,dx=\frac{2\,ax \operatorname{arccot}(ax)+\log(a^2x^2+1)}{2\,a}$$

input
```
integrate(arccot(a*x),x, algorithm="fricas")
```

output
```
1/2*(2*a*x*arccot(a*x) + log(a^2*x^2 + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(ax)\,dx = \begin{cases} x\,\mathrm{acot}\,(ax) + \frac{\log\left(a^2x^2+1\right)}{2a} & \text{for } a \neq 0 \\ \frac{\pi x}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(acot(a*x),x)
```

output
```
Piecewise((x*acot(a*x) + log(a**2*x**2 + 1)/(2*a), Ne(a, 0)), (pi*x/2, Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(ax)\,dx = \frac{2\,ax\,\mathrm{arccot}\,(ax) + \log\left(a^2x^2+1\right)}{2\,a}$$

input
```
integrate(arccot(a*x),x, algorithm="maxima")
```

output
```
1/2*(2*a*x*arccot(a*x) + log(a^2*x^2 + 1))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int \cot^{-1}(ax)\,dx = \frac{1}{2}\,a\left(\frac{2\,x\arctan\left(\frac{1}{ax}\right)}{a} + \frac{\log\left(\frac{1}{a^2x^2}+1\right)}{a^2} - \frac{\log\left(\frac{1}{a^2x^2}\right)}{a^2}\right)$$

input
```
integrate(arccot(a*x),x, algorithm="giac")
```

output
```
1/2*a*(2*x*arctan(1/(a*x))/a + log(1/(a^2*x^2) + 1)/a^2 - log(1/(a^2*x^2))
/a^2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \cot^{-1}(ax)\, dx = x\,\mathrm{acot}(a\,x) + \frac{\ln\left(a^2\,x^2+1\right)}{2\,a}$$

input
```
int(acot(a*x),x)
```

output
```
x*acot(a*x) + log(a^2*x^2 + 1)/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot^{-1}(ax)\, dx = \frac{2acot(ax)\,ax + \log(a^2x^2+1)}{2a}$$

input
```
int(acot(a*x),x)
```

output
```
(2*acot(a*x)*a*x + log(a**2*x**2 + 1))/(2*a)
```

3.7 $\int \frac{\cot^{-1}(ax)}{x}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = -\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{ax}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{ax}\right)$$

output
```
-1/2*I*polylog(2,-I/a/x)+1/2*I*polylog(2,I/a/x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = -\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{ax}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{ax}\right)$$

input
```
Integrate[ArcCot[a*x]/x,x]
```

output
```
(-1/2*I)*PolyLog[2, (-I)/(a*x)] + (I/2)*PolyLog[2, I/(a*x)]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{x} \, dx$$

$$\Big\downarrow \text{5356}$$

$$\frac{1}{2} i \int \frac{\log \left(1-\frac{i}{ax}\right)}{x} dx - \frac{1}{2} i \int \frac{\log \left(1+\frac{i}{ax}\right)}{x} dx$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2} i \operatorname{PolyLog}\left(2, \frac{i}{ax}\right) - \frac{1}{2} i \operatorname{PolyLog}\left(2, -\frac{i}{ax}\right)$$

input
```
Int[ArcCot[a*x]/x,x]
```

output
```
(-1/2*I)*PolyLog[2, (-I)/(a*x)] + (I/2)*PolyLog[2, I/(a*x)]
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

method	result	s
risch	$\frac{\pi \ln(-iax)}{2} + \frac{i\,\mathrm{dilog}(-iax+1)}{2} - \frac{i\,\mathrm{dilog}(iax+1)}{2}$	3
derivativedivides	$\ln(ax)\,\mathrm{arccot}(ax) - \frac{i\ln(ax)\ln(iax+1)}{2} + \frac{i\ln(ax)\ln(-iax+1)}{2} - \frac{i\,\mathrm{dilog}(iax+1)}{2} + \frac{i\,\mathrm{dilog}(-iax+1)}{2}$	6
default	$\ln(ax)\,\mathrm{arccot}(ax) - \frac{i\ln(ax)\ln(iax+1)}{2} + \frac{i\ln(ax)\ln(-iax+1)}{2} - \frac{i\,\mathrm{dilog}(iax+1)}{2} + \frac{i\,\mathrm{dilog}(-iax+1)}{2}$	6
parts	$\ln(x)\,\mathrm{arccot}(ax) + a\left(-\frac{i\ln(x)(-\ln(-iax+1)+\ln(iax+1))}{2a} - \frac{i(\mathrm{dilog}(iax+1)-\mathrm{dilog}(-iax+1))}{2a}\right)$	6

input
```
int(arccot(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*Pi*ln(-I*a*x)+1/2*I*dilog(1-I*a*x)-1/2*I*dilog(1+I*a*x)
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = \int \frac{\mathrm{arccot}(ax)}{x}\,dx$$

input
```
integrate(arccot(a*x)/x,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)/x, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = \int \frac{\mathrm{acot}(ax)}{x}\,dx$$

input
```
integrate(acot(a*x)/x,x)
```

output
```
Integral(acot(a*x)/x, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(23) = 46.

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = \frac{1}{4}\pi\log\left(a^2x^2+1\right) - \arctan\left(ax\right)\log\left(ax\right) + \operatorname{arccot}\left(ax\right)\log\left(x\right) + \arctan\left(ax\right)\log\left(x\right) + \frac{1}{2}i\operatorname{Li}_2(i\,ax+1) - \frac{1}{2}i\operatorname{Li}_2(-i\,ax+1)$$

input
```
integrate(arccot(a*x)/x,x, algorithm="maxima")
```

output
```
1/4*pi*log(a^2*x^2 + 1) - arctan(a*x)*log(a*x) + arccot(a*x)*log(x) + arct
an(a*x)*log(x) + 1/2*I*dilog(I*a*x + 1) - 1/2*I*dilog(-I*a*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = -\frac{1}{2}\left(\frac{x^2\arctan\left(\frac{1}{ax}\right)}{a} + \frac{x}{a^2} + \frac{\arctan\left(\frac{1}{ax}\right)}{a^3}\right)a^2$$

input
```
integrate(arccot(a*x)/x,x, algorithm="giac")
```

output
```
-1/2*(x^2*arctan(1/(a*x))/a + x/a^2 + arctan(1/(a*x))/a^3)*a^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = \int \frac{\mathrm{acot}(a\,x)}{x}\,dx$$

input
```
int(acot(a*x)/x,x)
```

output
```
int(acot(a*x)/x, x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(ax)}{x}\,dx = \int \frac{acot(ax)}{x}dx$$

input
```
int(acot(a*x)/x,x)
```

output
```
int(acot(a*x)/x,x)
```

3.8 $\int \frac{\cot^{-1}(ax)}{x^2} dx$

Optimal result

Integrand size = 8, antiderivative size = 30

$$\int \frac{\cot^{-1}(ax)}{x^2} dx = -\frac{\cot^{-1}(ax)}{x} - a\log(x) + \frac{1}{2}a\log\left(1+a^2x^2\right)$$

output
```
-arccot(a*x)/x-a*ln(x)+1/2*a*ln(a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{x^2} dx = -\frac{\cot^{-1}(ax)}{x} - a\log(x) + \frac{1}{2}a\log\left(1+a^2x^2\right)$$

input
```
Integrate[ArcCot[a*x]/x^2,x]
```

output
```
-(ArcCot[a*x]/x) - a*Log[x] + (a*Log[1 + a^2*x^2])/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5362, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{x^2} dx$$

$$\Big\downarrow \text{5362}$$

$$-a \int \frac{1}{x(a^2x^2+1)}dx - \frac{\cot^{-1}(ax)}{x}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}a \int \frac{1}{x^2(a^2x^2+1)}dx^2 - \frac{\cot^{-1}(ax)}{x}$$

$$\Big\downarrow \text{47}$$

$$-\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2 - a^2 \int \frac{1}{a^2x^2+1}dx^2\right) - \frac{\cot^{-1}(ax)}{x}$$

$$\Big\downarrow \text{14}$$

$$-\frac{1}{2}a\left(\log(x^2) - a^2 \int \frac{1}{a^2x^2+1}dx^2\right) - \frac{\cot^{-1}(ax)}{x}$$

$$\Big\downarrow \text{16}$$

$$-\frac{1}{2}a(\log(x^2) - \log(a^2x^2+1)) - \frac{\cot^{-1}(ax)}{x}$$

input
```
Int[ArcCot[a*x]/x^2,x]
```

output
```
-(ArcCot[a*x]/x) - (a*(Log[x^2] - Log[1 + a^2*x^2]))/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
parts	$-\frac{\operatorname{arccot}(ax)}{x}-a\left(-\frac{\ln(a^2x^2+1)}{2}+\ln(x)\right)$	29
parallelrisch	$-\frac{2a\ln(x)x-a\ln(a^2x^2+1)x+2\operatorname{arccot}(ax)}{2x}$	33
derivativedivides	$a\left(-\frac{\operatorname{arccot}(ax)}{ax}+\frac{\ln(a^2x^2+1)}{2}-\ln(ax)\right)$	34
default	$a\left(-\frac{\operatorname{arccot}(ax)}{ax}+\frac{\ln(a^2x^2+1)}{2}-\ln(ax)\right)$	34
risch	$-\frac{i\ln(iax+1)}{2x}-\frac{2a\ln(x)x-a\ln(a^2x^2+1)x-i\ln(-iax+1)+\pi}{2x}$	54

input
```
int(arccot(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccot(a*x)/x-a*(-1/2*ln(a^2*x^2+1)+ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = \frac{ax\log\left(a^2x^2+1\right) - 2\,ax\log\left(x\right) - 2\,\operatorname{arccot}\left(ax\right)}{2\,x}$$

input
```
integrate(arccot(a*x)/x^2,x, algorithm="fricas")
```

output
```
1/2*(a*x*log(a^2*x^2 + 1) - 2*a*x*log(x) - 2*arccot(a*x))/x
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = -a\log\left(x\right) + \frac{a\log\left(a^2x^2+1\right)}{2} - \frac{\operatorname{acot}\left(ax\right)}{x}$$

input
```
integrate(acot(a*x)/x**2,x)
```

output
```
-a*log(x) + a*log(a**2*x**2 + 1)/2 - acot(a*x)/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = \frac{1}{2}\,a\left(\log\left(a^2x^2+1\right)-\log\left(x^2\right)\right)-\frac{\operatorname{arccot}(ax)}{x}$$

input
```
integrate(arccot(a*x)/x^2,x, algorithm="maxima")
```

output
```
1/2*a*(log(a^2*x^2 + 1) - log(x^2)) - arccot(a*x)/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = -\frac{1}{2}\,a\left(\frac{2\arctan\left(\frac{1}{ax}\right)}{ax}-\log\left(\frac{1}{a^2x^2}+1\right)\right)$$

input
```
integrate(arccot(a*x)/x^2,x, algorithm="giac")
```

output
```
-1/2*a*(2*arctan(1/(a*x))/(a*x) - log(1/(a^2*x^2) + 1))
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\cot^{-1}(ax)}{x^2}\,dx = \frac{a\left(\ln\left(a^2\,x^2+1\right)-2\,\ln\left(x\right)\right)}{2}-\frac{\mathrm{acot}(a\,x)}{x}$$

input
```
int(acot(a*x)/x^2,x)
```

output
```
(a*(log(a^2*x^2 + 1) - 2*log(x)))/2 - acot(a*x)/x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\cot^{-1}(ax)}{x^2}\, dx = \frac{-2acot(ax) + \log(a^2x^2 + 1)\, ax - 2\log(x)\, ax}{2x}$$

input
```
int(acot(a*x)/x^2,x)
```

output
```
( - 2*acot(a*x) + log(a**2*x**2 + 1)*a*x - 2*log(x)*a*x)/(2*x)
```

3.9 $\int \frac{\cot^{-1}(ax)}{x^3} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int \frac{\cot^{-1}(ax)}{x^3} \, dx = \frac{a}{2x} - \frac{\cot^{-1}(ax)}{2x^2} + \frac{1}{2}a^2 \arctan(ax)$$

output
```
1/2*a/x-1/2*arccot(a*x)/x^2+1/2*a^2*arctan(a*x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{\cot^{-1}(ax)}{x^3} \, dx = -\frac{\cot^{-1}(ax)}{2x^2} + \frac{a \, \text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -a^2x^2\right)}{2x}$$

input
```
Integrate[ArcCot[a*x]/x^3,x]
```

output
```
-1/2*ArcCot[a*x]/x^2 + (a*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)])/(2*
x)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5362, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{x^3} \, dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{1}{2}a\int \frac{1}{x^2 \left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)}{2x^2}$$

$$\Big\downarrow \text{264}$$

$$-\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{\cot^{-1}(ax)}{2x^2}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\cot^{-1}(ax)}{2x^2}$$

input
```
Int[ArcCot[a*x]/x^3,x]
```

output
```
-1/2*ArcCot[a*x]/x^2 - (a*(-x^(-1) - a*ArcTan[a*x]))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$-\frac{\operatorname{arccot}(ax)a^2x^2-ax+\operatorname{arccot}(ax)}{2x^2}$	26
parts	$-\frac{\operatorname{arccot}(ax)}{2x^2}-\frac{a\left(-a\arctan(ax)-\frac{1}{x}\right)}{2}$	27
derivativedivides	$a^2\left(-\frac{\operatorname{arccot}(ax)}{2a^2x^2}+\frac{\arctan(ax)}{2}+\frac{1}{2ax}\right)$	32
default	$a^2\left(-\frac{\operatorname{arccot}(ax)}{2a^2x^2}+\frac{\arctan(ax)}{2}+\frac{1}{2ax}\right)$	32
orering	$\frac{(-2a^2x^3-2x)\operatorname{arccot}(ax)}{x^3}-\frac{(a^2x^2+1)x^2\left(-\frac{a}{(a^2x^2+1)x^3}-\frac{3\operatorname{arccot}(ax)}{x^4}\right)}{2}$	63
risch	$-\frac{i\ln(iax+1)}{4x^2}-\frac{ia^2\ln(-ax+i)x^2-ia^2\ln(-ax-i)x^2-i\ln(-iax+1)-2ax+\pi}{4x^2}$	72

input
```
int(arccot(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arccot(a*x)*a^2*x^2-a*x+arccot(a*x))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx = \frac{ax - (a^2x^2+1)\,\mathrm{arccot}\,(ax)}{2\,x^2}$$

input
```
integrate(arccot(a*x)/x^3,x, algorithm="fricas")
```

output
```
1/2*(a*x - (a^2*x^2 + 1)*arccot(a*x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx = -\frac{a^2\,\mathrm{acot}\,(ax)}{2} + \frac{a}{2x} - \frac{\mathrm{acot}\,(ax)}{2x^2}$$

input
```
integrate(acot(a*x)/x**3,x)
```

output
```
-a**2*acot(a*x)/2 + a/(2*x) - acot(a*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx = \frac{1}{2}\left(a\arctan(ax) + \frac{1}{x}\right)a - \frac{\mathrm{arccot}\,(ax)}{2\,x^2}$$

input
```
integrate(arccot(a*x)/x^3,x, algorithm="maxima")
```

output
```
1/2*(a*arctan(a*x) + 1/x)*a - 1/2*arccot(a*x)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx = \frac{1}{2}\left(a\left(\frac{1}{ax} - \arctan\left(\frac{1}{ax}\right)\right) - \frac{\arctan\left(\frac{1}{ax}\right)}{ax^2}\right)a$$

input
```
integrate(arccot(a*x)/x^3,x, algorithm="giac")
```

output
```
1/2*(a*(1/(a*x) - arctan(1/(a*x))) - arctan(1/(a*x))/(a*x^2))*a
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx = \begin{cases} -\frac{\pi}{4\,x^2} & \text{if } a=0 \\ \frac{a^3\,\mathrm{atan}(a\,x)+\frac{a^2}{x}}{2\,a} - \frac{\mathrm{acot}(a\,x)}{2\,x^2} & \text{if } a\neq 0 \end{cases}$$

input
```
int(acot(a*x)/x^3,x)
```

output
```
piecewise(a == 0, -pi/(4*x^2), a ~= 0, (a^3*atan(a*x) + a^2/x)/(2*a) - aco
t(a*x)/(2*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{\cot^{-1}(ax)}{x^3}\,dx = \frac{-acot(ax)\,a^2x^2 - acot(ax) + ax}{2x^2}$$

input
```
int(acot(a*x)/x^3,x)
```

output
```
( - acot(a*x)*a**2*x**2 - acot(a*x) + a*x)/(2*x**2)
```

3.10 $\int \frac{\cot^{-1}(ax)}{x^4} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \frac{\cot^{-1}(ax)}{x^4} \, dx = \frac{a}{6x^2} - \frac{\cot^{-1}(ax)}{3x^3} + \frac{1}{3}a^3 \log(x) - \frac{1}{6}a^3 \log\left(1 + a^2x^2\right)$$

output
```
1/6*a/x^2-1/3*arccot(a*x)/x^3+1/3*a^3*ln(x)-1/6*a^3*ln(a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{\cot^{-1}(ax)}{x^4} \, dx = -\frac{\cot^{-1}(ax)}{3x^3} - \frac{1}{6}a\left(-\frac{1}{x^2} - 2a^2 \log(x) + a^2 \log\left(1 + a^2x^2\right)\right)$$

input
```
Integrate[ArcCot[a*x]/x^4,x]
```

output
```
-1/3*ArcCot[a*x]/x^3 - (a*(-x^(-2) - 2*a^2*Log[x] + a^2*Log[1 + a^2*x^2]))
/6
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5362, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{x^4} \, dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{1}{3}a\int \frac{1}{x^3 \left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{6}a\int \frac{1}{x^4 \left(a^2x^2+1\right)}dx^2-\frac{\cot^{-1}(ax)}{3x^3}$$

$$\Big\downarrow \text{54}$$

$$-\frac{1}{6}a\int \left(\frac{a^4}{a^2x^2+1}-\frac{a^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{\cot^{-1}(ax)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\cot^{-1}(ax)}{3x^3}$$

input
```
Int[ArcCot[a*x]/x^4,x]
```

output
```
-1/3*ArcCot[a*x]/x^3 - (a*(-x^(-2) - a^2*Log[x^2] + a^2*Log[1 + a^2*x^2]))
/6
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

method	result	size
parts	$-\frac{\operatorname{arccot}(ax)}{3x^3}-\frac{a\left(\frac{a^2\ln\left(a^2x^2+1\right)}{2}-\frac{1}{2x^2}-a^2\ln(x)\right)}{3}$	42
derivativedivides	$a^3\left(-\frac{\operatorname{arccot}(ax)}{3a^3x^3}-\frac{\ln(a^2x^2+1)}{6}+\frac{1}{6a^2x^2}+\frac{\ln(ax)}{3}\right)$	44
default	$a^3\left(-\frac{\operatorname{arccot}(ax)}{3a^3x^3}-\frac{\ln(a^2x^2+1)}{6}+\frac{1}{6a^2x^2}+\frac{\ln(ax)}{3}\right)$	44
parallelrisch	$\frac{2a^3\ln(x)x^3-a^3\ln(a^2x^2+1)x^3-a^3x^3+ax-2\operatorname{arccot}(ax)}{6x^3}$	52
risch	$-\frac{i\ln(iax+1)}{6x^3}-\frac{-2a^3\ln(x)x^3+a^3\ln(-a^2x^2-1)x^3-i\ln(-iax+1)-ax+\pi}{6x^3}$	66

input
```
int(arccot(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arccot(a*x)/x^3-1/3*a*(1/2*a^2*ln(a^2*x^2+1)-1/2/x^2-a^2*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\cot^{-1}(ax)}{x^4}\,dx = -\frac{a^3x^3\log\left(a^2x^2+1\right)-2\,a^3x^3\log\left(x\right)-ax+2\,\operatorname{arccot}\left(ax\right)}{6\,x^3}$$

input
```
integrate(arccot(a*x)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(a^3*x^3*log(a^2*x^2 + 1) - 2*a^3*x^3*log(x) - a*x + 2*arccot(a*x))/x
^3
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\cot^{-1}(ax)}{x^4}\,dx = \frac{a^3\log\left(x\right)}{3}-\frac{a^3\log\left(a^2x^2+1\right)}{6}+\frac{a}{6x^2}-\frac{\operatorname{acot}\left(ax\right)}{3x^3}$$

input
```
integrate(acot(a*x)/x**4,x)
```

output
```
a**3*log(x)/3 - a**3*log(a**2*x**2 + 1)/6 + a/(6*x**2) - acot(a*x)/(3*x**3
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(ax)}{x^4}\,dx = -\frac{1}{6}\left(a^2\log\left(a^2x^2+1\right)-a^2\log\left(x^2\right)-\frac{1}{x^2}\right)a-\frac{\operatorname{arccot}(ax)}{3\,x^3}$$

input
```
integrate(arccot(a*x)/x^4,x, algorithm="maxima")
```

output
```
-1/6*(a^2*log(a^2*x^2 + 1) - a^2*log(x^2) - 1/x^2)*a - 1/3*arccot(a*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{\cot^{-1}(ax)}{x^4}\,dx = \frac{1}{6}\left(a^2\left(\frac{1}{a^2x^2}-\log\left(\frac{1}{a^2x^2}+1\right)\right)-\frac{2\arctan\left(\frac{1}{ax}\right)}{ax^3}\right)a$$

input
```
integrate(arccot(a*x)/x^4,x, algorithm="giac")
```

output
```
1/6*(a^2*(1/(a^2*x^2) - log(1/(a^2*x^2) + 1)) - 2*arctan(1/(a*x))/(a*x^3))
*a
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{\cot^{-1}(ax)}{x^4}\,dx = \begin{cases} -\frac{\pi}{6\,x^3} & \text{if } a=0 \\ \frac{a^4\ln(x)-\frac{a^4\ln\left(a^2x^2+1\right)}{2}+\frac{a^2}{2\,x^2}}{3\,a}-\frac{\operatorname{acot}(a\,x)}{3\,x^3} & \text{if } a\neq 0 \end{cases}$$

input
```
int(acot(a*x)/x^4,x)
```

output
```
piecewise(a == 0, -pi/(6*x^3), a ~= 0, (a^4*log(x) - (a^4*log(a^2*x^2 + 1)
)/2 + a^2/(2*x^2))/(3*a) - acot(a*x)/(3*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\cot^{-1}(ax)}{x^4}\,dx = \frac{-2acot(ax) - \log(a^2x^2+1)\,a^3x^3 + 2\log(x)\,a^3x^3 + ax}{6x^3}$$

input
```
int(acot(a*x)/x^4,x)
```

output
```
( - 2*acot(a*x) - log(a**2*x**2 + 1)*a**3*x**3 + 2*log(x)*a**3*x**3 + a*x)
/(6*x**3)
```

3.11 $\int \frac{\cot^{-1}(ax)}{x^5} dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \frac{\cot^{-1}(ax)}{x^5} dx = \frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\cot^{-1}(ax)}{4x^4} - \frac{1}{4}a^4 \arctan(ax)$$

output
```
1/12*a/x^3-1/4*a^3/x-1/4*arccot(a*x)/x^4-1/4*a^4*arctan(a*x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{\cot^{-1}(ax)}{x^5} dx = -\frac{\cot^{-1}(ax)}{4x^4} + \frac{a \operatorname{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, -a^2x^2\right)}{12x^3}$$

input
```
Integrate[ArcCot[a*x]/x^5,x]
```

output
```
-1/4*ArcCot[a*x]/x^4 + (a*Hypergeometric2F1[-3/2, 1, -1/2, -(a^2*x^2)])/(1
2*x^3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5362, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)}{x^5} \, dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{1}{4} a \int \frac{1}{x^4 \left(a^2 x^2+1\right)}dx-\frac{\cot^{-1}(ax)}{4 x^4}$$

$$\Big\downarrow \text{264}$$

$$-\frac{1}{4} a \left(a^2 \left(-\int \frac{1}{x^2 \left(a^2 x^2+1\right)}dx\right)-\frac{1}{3 x^3}\right)-\frac{\cot^{-1}(ax)}{4 x^4}$$

$$\Big\downarrow \text{264}$$

$$-\frac{1}{4} a \left(-\left(a^2 \left(a^2 \left(-\int \frac{1}{a^2 x^2+1}dx\right)-\frac{1}{x}\right)\right)-\frac{1}{3 x^3}\right)-\frac{\cot^{-1}(ax)}{4 x^4}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{4} a \left(-\left(a^2 \left(-a \arctan(ax)-\frac{1}{x}\right)\right)-\frac{1}{3 x^3}\right)-\frac{\cot^{-1}(ax)}{4 x^4}$$

input
```
Int[ArcCot[a*x]/x^5,x]
```

output
```
-1/4*ArcCot[a*x]/x^4 - (a*(-1/3*1/x^3 - a^2*(-x^(-1) - a*ArcTan[a*x])))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

method	result	size
parts	$-\frac{\operatorname{arccot}(ax)}{4x^4}-\frac{a\left(a^3\arctan(ax)-\frac{1}{3x^3}+\frac{a^2}{x}\right)}{4}$	35
parallelrisch	$\frac{3a^4x^4\operatorname{arccot}(ax)-3a^3x^3+ax-3\operatorname{arccot}(ax)}{12x^4}$	36
derivativedivides	$a^4\left(-\frac{\operatorname{arccot}(ax)}{4a^4x^4}+\frac{1}{12a^3x^3}-\frac{1}{4ax}-\frac{\arctan(ax)}{4}\right)$	40
default	$a^4\left(-\frac{\operatorname{arccot}(ax)}{4a^4x^4}+\frac{1}{12a^3x^3}-\frac{1}{4ax}-\frac{\arctan(ax)}{4}\right)$	40
orering	$\frac{\left(\frac{3}{2}a^4x^5+\frac{5}{6}a^2x^3-\frac{2}{3}x\right)\operatorname{arccot}(ax)}{x^5}+\frac{(3a^2x^2-1)\left(a^2x^2+1\right)x^2\left(-\frac{a}{(a^2x^2+1)x^5}-\frac{5\operatorname{arccot}(ax)}{x^6}\right)}{12}$	81
risch	$-\frac{i\ln(iax+1)}{8x^4}-\frac{-3ia^4\ln(-ax+i)x^4+3ia^4\ln(-ax-i)x^4+6a^3x^3-3i\ln(-iax+1)-2ax+3\pi}{24x^4}$	82

input
```
int(arccot(a*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arccot(a*x)/x^4-1/4*a*(a^3*arctan(a*x)-1/3/x^3+a^2/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx = -\frac{3\,a^3x^3 - ax - 3\,(a^4x^4 - 1)\,\mathrm{arccot}\,(ax)}{12\,x^4}$$

input
```
integrate(arccot(a*x)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(3*a^3*x^3 - a*x - 3*(a^4*x^4 - 1)*arccot(a*x))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx = \frac{a^4\,\mathrm{acot}\,(ax)}{4} - \frac{a^3}{4x} + \frac{a}{12x^3} - \frac{\mathrm{acot}\,(ax)}{4x^4}$$

input
```
integrate(acot(a*x)/x**5,x)
```

output
```
a**4*acot(a*x)/4 - a**3/(4*x) + a/(12*x**3) - acot(a*x)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx = -\frac{1}{12}\left(3\,a^3\arctan\,(ax) + \frac{3\,a^2x^2 - 1}{x^3}\right)a - \frac{\mathrm{arccot}\,(ax)}{4\,x^4}$$

input
```
integrate(arccot(a*x)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(3*a^3*arctan(a*x) + (3*a^2*x^2 - 1)/x^3)*a - 1/4*arccot(a*x)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx = -\frac{1}{12}\left(a^3\left(\frac{3}{ax}-\frac{1}{a^3x^3}-3\arctan\left(\frac{1}{ax}\right)\right)+\frac{3\arctan\left(\frac{1}{ax}\right)}{ax^4}\right)a$$

input
```
integrate(arccot(a*x)/x^5,x, algorithm="giac")
```

output
```
-1/12*(a^3*(3/(a*x) - 1/(a^3*x^3) - 3*arctan(1/(a*x))) + 3*arctan(1/(a*x))
/(a*x^4))*a
```

Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx = \begin{cases} -\frac{\pi}{8x^4} & \text{if } a=0 \\ -\frac{a^4\,\mathrm{atan}(a\,x)}{4}-\frac{\frac{\mathrm{acot}(a\,x)}{4}-\frac{a\,x}{12}+\frac{a^3\,x^3}{4}}{x^4} & \text{if } a\neq 0 \end{cases}$$

input
```
int(acot(a*x)/x^5,x)
```

output
```
piecewise(a == 0, -pi/(8*x^4), a ~= 0, - (a^4*atan(a*x))/4 - (acot(a*x)/4
- (a*x)/12 + (a^3*x^3)/4)/x^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{\cot^{-1}(ax)}{x^5}\,dx = \frac{3acot(ax)\,a^4x^4-3acot(ax)-3a^3x^3+ax}{12x^4}$$

input
```
int(acot(a*x)/x^5,x)
```

output
```
(3*acot(a*x)*a**4*x**4 - 3*acot(a*x) - 3*a**3*x**3 + a*x)/(12*x**4)
```

3.12 $\int x^5 \cot^{-1}(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 104

$$\int x^5 \cot^{-1}(ax)^2 \, dx = -\frac{4x^2}{45a^4} + \frac{x^4}{60a^2} + \frac{x \cot^{-1}(ax)}{3a^5} - \frac{x^3 \cot^{-1}(ax)}{9a^3} + \frac{x^5 \cot^{-1}(ax)}{15a} + \frac{\cot^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6 \cot^{-1}(ax)^2 + \frac{23 \log(1+a^2x^2)}{90a^6}$$

output
```
-4/45*x^2/a^4+1/60*x^4/a^2+1/3*x*arccot(a*x)/a^5-1/9*x^3*arccot(a*x)/a^3+1
/15*x^5*arccot(a*x)/a+1/6*arccot(a*x)^2/a^6+1/6*x^6*arccot(a*x)^2+23/90*ln
(a^2*x^2+1)/a^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

$$\int x^5 \cot^{-1}(ax)^2 \, dx = \frac{-16a^2x^2 + 3a^4x^4 + 4ax(15 - 5a^2x^2 + 3a^4x^4) \cot^{-1}(ax) + 30(1 + a^6x^6) \cot^{-1}(ax)^2 + 46 \log(1 + a^2x^2)}{180a^6}$$

input
```
Integrate[x^5*ArcCot[a*x]^2,x]
```

output
```
(-16*a^2*x^2 + 3*a^4*x^4 + 4*a*x*(15 - 5*a^2*x^2 + 3*a^4*x^4)*ArcCot[a*x]
+ 30*(1 + a^6*x^6)*ArcCot[a*x]^2 + 46*Log[1 + a^2*x^2])/(180*a^6)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.64, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {5362, 5452, 5362, 243, 49, 2009, 5452, 5362, 243, 49, 2009, 5452, 5346, 240, 5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \cot^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{3}a\int \frac{x^6 \cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5452}$$

$$\frac{1}{3}a\left(\frac{\int x^4\cot^{-1}(ax)dx}{a^2}-\frac{\int \frac{x^4\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{3}a\left(\frac{\frac{1}{5}a\int \frac{x^5}{a^2x^2+1}dx+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x^4\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\int \frac{x^4}{a^2x^2+1}dx^2+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x^4\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\int \left(\frac{x^2}{a^2}+\frac{1}{a^4(a^2x^2+1)}-\frac{1}{a^4}\right)dx^2+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x^4\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log\left(a^2x^2+1\right)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^4\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$\downarrow$ 5452

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log\left(a^2x^2+1\right)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\int x^2\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$\downarrow$ 5362

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log\left(a^2x^2+1\right)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{3}a\int\frac{x^3}{a^2x^2+1}dx+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$\downarrow$ 243

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log\left(a^2x^2+1\right)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{6}a\int\frac{x^2}{a^2x^2+1}dx^2+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$\downarrow$ 49

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log\left(a^2x^2+1\right)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{6}a\int\left(\frac{1}{a^2}-\frac{1}{a^2\left(a^2x^2+1\right)}\right)dx^2+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$\downarrow$ 2009

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$\big\downarrow$ 5452

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\int\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}}{a^2}\cdots\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$\big\downarrow$ 5346

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{a\int\frac{x}{a^2x^2+1}dx+x\cot^{-1}(ax)}{a^2}\cdots}{a^2}}{a^2}\cdots\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$\big\downarrow$ 240

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}\cdots}{a^2}}{a^2}\cdots\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^2$$

$\big\downarrow$ 5420

$$\frac{1}{3}a\left(\frac{\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)+\frac{1}{5}x^5\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log(a^2x^2+1)}{2a}}{a^2}}{a^2}}{a^2}\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^2$$

input
```
Int[x^5*ArcCot[a*x]^2,x]
```

output
```
(x^6*ArcCot[a*x]^2)/6 + (a*(((x^5*ArcCot[a*x])/5 + (a*(-(x^2/a^4) + x^4/(2
*a^2) + Log[1 + a^2*x^2]/a^6))/10)/a^2 - (((x^3*ArcCot[a*x])/3 + (a*(x^2/a
^2 - Log[1 + a^2*x^2]/a^4))/6)/a^2 - (ArcCot[a*x]^2/(2*a^3) + (x*ArcCot[a*
x] + Log[1 + a^2*x^2]/(2*a))/a^2)/a^2)/a^2))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result
parallelrisch	$\frac{30a^6x^6\operatorname{arccot}(ax)^2+12a^5x^5\operatorname{arccot}(ax)+3a^4x^4-20a^3x^3\operatorname{arccot}(ax)+16-16a^2x^2+60\operatorname{arccot}(ax)ax+30\operatorname{arccot}(ax)^2+}{180a^6}$
parts	$\frac{x^6\operatorname{arccot}(ax)^2}{6}+\frac{\frac{a^5x^5\operatorname{arccot}(ax)}{5}-\frac{a^3x^3\operatorname{arccot}(ax)}{3}+\operatorname{arccot}(ax)ax-\operatorname{arccot}(ax)\arctan(ax)+\frac{a^4x^4}{20}-\frac{4a^2x^2}{15}+\frac{23\ln(a^2x^2}{30}}{3a^6}$
derivativedivides	$\frac{\frac{a^6x^6\operatorname{arccot}(ax)^2}{6}+\frac{a^5x^5\operatorname{arccot}(ax)}{15}-\frac{a^3x^3\operatorname{arccot}(ax)}{9}+\frac{\operatorname{arccot}(ax)ax}{3}-\frac{\operatorname{arccot}(ax)\arctan(ax)}{3}+\frac{a^4x^4}{60}-\frac{4a^2x^2}{45}+\frac{23\ln(a^2x^2+1)}{90}}{a^6}$
default	$\frac{\frac{a^6x^6\operatorname{arccot}(ax)^2}{6}+\frac{a^5x^5\operatorname{arccot}(ax)}{15}-\frac{a^3x^3\operatorname{arccot}(ax)}{9}+\frac{\operatorname{arccot}(ax)ax}{3}-\frac{\operatorname{arccot}(ax)\arctan(ax)}{3}+\frac{a^4x^4}{60}-\frac{4a^2x^2}{45}+\frac{23\ln(a^2x^2+1)}{90}}{a^6}$
risch	$-\frac{(a^6x^6+1)\ln(iax+1)^2}{24a^6}+\frac{(15i\pi a^6x^6+15x^6\ln(-iax+1)a^6+6ia^5x^5-10ia^3x^3+30iax+15\ln(-iax+1))\ln(iax+1)}{180a^6}$

input
```
int(x^5*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/180*(30*a^6*x^6*arccot(a*x)^2+12*a^5*x^5*arccot(a*x)+3*a^4*x^4-20*a^3*x^
3*arccot(a*x)+16-16*a^2*x^2+60*arccot(a*x)*a*x+30*arccot(a*x)^2+46*ln(a^2*
x^2+1))/a^6
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.75

$$\int x^5 \cot^{-1}(ax)^2 \, dx$$
$$= \frac{3\,a^4x^4 - 16\,a^2x^2 + 30\,(a^6x^6 + 1)\,\mathrm{arccot}\,(ax)^2 + 4\,(3\,a^5x^5 - 5\,a^3x^3 + 15\,ax)\,\mathrm{arccot}\,(ax) + 46\,\log\,(a^2x^2 +}{180\,a^6}$$

input
```
integrate(x^5*arccot(a*x)^2,x, algorithm="fricas")
```

output
```
1/180*(3*a^4*x^4 - 16*a^2*x^2 + 30*(a^6*x^6 + 1)*arccot(a*x)^2 + 4*(3*a^5*
x^5 - 5*a^3*x^3 + 15*a*x)*arccot(a*x) + 46*log(a^2*x^2 + 1))/a^6
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int x^5 \cot^{-1}(ax)^2 \, dx$$
$$= \begin{cases} \frac{x^6 \,\mathrm{acot}^2\,(ax)}{6} + \frac{x^5 \,\mathrm{acot}\,(ax)}{15a} + \frac{x^4}{60a^2} - \frac{x^3 \,\mathrm{acot}\,(ax)}{9a^3} - \frac{4x^2}{45a^4} + \frac{x \,\mathrm{acot}\,(ax)}{3a^5} + \frac{23 \log\,(a^2x^2+1)}{90a^6} + \frac{\mathrm{acot}^2\,(ax)}{6a^6} & \text{for } a \neq 0 \\ \frac{\pi^2 x^6}{24} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*acot(a*x)**2,x)
```

output
```
Piecewise((x**6*acot(a*x)**2/6 + x**5*acot(a*x)/(15*a) + x**4/(60*a**2) -
x**3*acot(a*x)/(9*a**3) - 4*x**2/(45*a**4) + x*acot(a*x)/(3*a**5) + 23*log
(a**2*x**2 + 1)/(90*a**6) + acot(a*x)**2/(6*a**6), Ne(a, 0)), (pi**2*x**6/
24, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int x^5 \cot^{-1}(ax)^2 \, dx = \frac{1}{6} x^6 \operatorname{arccot}(ax)^2 + \frac{1}{45} a \left(\frac{3 a^4 x^5 - 5 a^2 x^3 + 15 x}{a^6} - \frac{15 \arctan(ax)}{a^7} \right) \operatorname{arccot}(ax) + \frac{3 a^4 x^4 - 16 a^2 x^2 - 30 \arctan(ax)^2 + 46 \log(a^2 x^2 + 1)}{180 a^6}$$

input
```
integrate(x^5*arccot(a*x)^2,x, algorithm="maxima")
```

output
```
1/6*x^6*arccot(a*x)^2 + 1/45*a*((3*a^4*x^5 - 5*a^2*x^3 + 15*x)/a^6 - 15*ar
ctan(a*x)/a^7)*arccot(a*x) + 1/180*(3*a^4*x^4 - 16*a^2*x^2 - 30*arctan(a*x
)^2 + 46*log(a^2*x^2 + 1))/a^6
```

Giac [F]

$$\int x^5 \cot^{-1}(ax)^2 \, dx = \int x^5 \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x^5*arccot(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^5*arccot(a*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.82

$$\int x^5 \cot^{-1}(ax)^2 \, dx = \frac{x^6 \operatorname{acot}(a\,x)^2}{6} + \frac{\frac{23\,\ln(a^2\,x^2+1)}{90} - \frac{4\,a^2\,x^2}{45} + \frac{a^4\,x^4}{60} + \frac{\operatorname{acot}(a\,x)^2}{6} - \frac{a^3\,x^3\,\operatorname{acot}(a\,x)}{9} + \frac{a^5\,x^5\,\operatorname{acot}(a\,x)}{15} + \frac{a\,x\,\operatorname{acot}(a\,x)}{3}}{a^6}$$

input
```
int(x^5*acot(a*x)^2,x)
```

output
```
(x^6*acot(a*x)^2)/6 + ((23*log(a^2*x^2 + 1))/90 - (4*a^2*x^2)/45 + (a^4*x^
4)/60 + acot(a*x)^2/6 - (a^3*x^3*acot(a*x))/9 + (a^5*x^5*acot(a*x))/15 + (
a*x*acot(a*x))/3)/a^6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int x^5 \cot^{-1}(ax)^2 \, dx = \frac{30acot(ax)^2 a^6x^6 + 30acot(ax)^2 + 12acot(ax)\,a^5x^5 - 20acot(ax)\,a^3x^3 + 60acot(ax)\,ax + 46\log(a^2x^2 + }{180a^6}$$

input
```
int(x^5*acot(a*x)^2,x)
```

output
```
(30*acot(a*x)**2*a**6*x**6 + 30*acot(a*x)**2 + 12*acot(a*x)*a**5*x**5 - 20
*acot(a*x)*a**3*x**3 + 60*acot(a*x)*a*x + 46*log(a**2*x**2 + 1) + 3*a**4*x
**4 - 16*a**2*x**2)/(180*a**6)
```

3.13 $\int x^4 \cot^{-1}(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 135

$$\int x^4 \cot^{-1}(ax)^2 \, dx = -\frac{3x}{10a^4} + \frac{x^3}{30a^2} - \frac{x^2 \cot^{-1}(ax)}{5a^3} + \frac{x^4 \cot^{-1}(ax)}{10a} + \frac{i \cot^{-1}(ax)^2}{5a^5} + \frac{1}{5} x^5 \cot^{-1}(ax)^2 + \frac{3 \arctan(ax)}{10a^5} - \frac{2 \cot^{-1}(ax) \log \left(\frac{2}{1+iax}\right)}{5a^5} + \frac{i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{5a^5}$$

output
```
-3/10*x/a^4+1/30*x^3/a^2-1/5*x^2*arccot(a*x)/a^3+1/10*x^4*arccot(a*x)/a+1/
5*I*arccot(a*x)^2/a^5+1/5*x^5*arccot(a*x)^2+3/10*arctan(a*x)/a^5-2/5*arcco
t(a*x)*ln(2/(1+I*a*x))/a^5+1/5*I*polylog(2,1-2/(1+I*a*x))/a^5
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int x^4 \cot^{-1}(ax)^2 \, dx = \frac{ax(-9 + a^2x^2) + 6(i + a^5x^5) \cot^{-1}(ax)^2 + 3 \cot^{-1}(ax) \left(-3 - 2a^2x^2 + a^4x^4 - 4 \log \left(1 - e^{2i \cot^{-1}(ax)}\right)\right) -}{30a^5}$$

input
```
Integrate[x^4*ArcCot[a*x]^2,x]
```

output
```
(a*x*(-9 + a^2*x^2) + 6*(I + a^5*x^5)*ArcCot[a*x]^2 + 3*ArcCot[a*x]*(-3 -
2*a^2*x^2 + a^4*x^4 - 4*Log[1 - E^((2*I)*ArcCot[a*x])]) + (6*I)*PolyLog[2,
 E^((2*I)*ArcCot[a*x])])/(30*a^5)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.34, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.300$, Rules used = {5362, 5452, 5362, 254, 2009, 5452, 5362, 262, 216, 5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \cot^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{2}{5} a \int \frac{x^5 \cot^{-1}(ax)}{a^2x^2+1} dx + \frac{1}{5} x^5 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5452}$$

$$\frac{2}{5} a \left(\frac{\int x^3 \cot^{-1}(ax) dx}{a^2} - \frac{\int \frac{x^3 \cot^{-1}(ax)}{a^2x^2+1} dx}{a^2}\right) + \frac{1}{5} x^5 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5362}$$

$$\frac{2}{5} a \left(\frac{\frac{1}{4} a \int \frac{x^4}{a^2x^2+1} dx + \frac{1}{4} x^4 \cot^{-1}(ax)}{a^2} - \frac{\int \frac{x^3 \cot^{-1}(ax)}{a^2x^2+1} dx}{a^2}\right) + \frac{1}{5} x^5 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{254}$$

$$\frac{2}{5} a \left(\frac{\frac{1}{4} a \int \left(\frac{x^2}{a^2} + \frac{1}{a^4(a^2x^2+1)} - \frac{1}{a^4}\right) dx + \frac{1}{4} x^4 \cot^{-1}(ax)}{a^2} - \frac{\int \frac{x^3 \cot^{-1}(ax)}{a^2x^2+1} dx}{a^2}\right) + \frac{1}{5} x^5 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5452}$$

$$\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\int x\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5362}$$

$$\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int\frac{1}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{216}$$

$$\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5456}$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\cot^{-1}(ax)}{i-ax}dx}{a}}{a^2}}{a^2}\right)$$

$$\Big\downarrow \text{5380}$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^2+$$

$$\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx+\frac{\log(}{a}}{a^2}}{a^2}}{}\right.$$

$$\Big\downarrow \text{2849}$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^2+$$

$$\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}}{a^2}}{a^2}}{}\right.$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^2+$$

$$\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}}{a^2}}{a^2}}{}\right.$$

input

```
Int[x^4*ArcCot[a*x]^2,x]
```

output

```
(x^5*ArcCot[a*x]^2)/5 + (2*a*(((x^4*ArcCot[a*x])/4 + (a*(-(x/a^4) + x^3/(3
*a^2) + ArcTan[a*x]/a^5))/4)/a^2 - (((x^2*ArcCot[a*x])/2 + (a*(x/a^2 - Arc
Tan[a*x]/a^3))/2)/a^2 - (((I/2)*ArcCot[a*x]^2)/a^2 - ((ArcCot[a*x]*Log[2/(
1 + I*a*x)])/a - ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)/a^2)/a^2))/5
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.52

method	result
parts	$\frac{x^5 \operatorname{arccot}(ax)^2}{5} + \frac{\frac{a^4x^4 \operatorname{arccot}(ax)}{10} - \frac{\operatorname{arccot}(ax)a^2x^2}{5} + \frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{5} + \frac{a^3x^3}{30} - \frac{3ax}{10} + \frac{3\arctan(ax)}{10} - \frac{i\left(\ln(ax-i)\ln\left(a\right.\right.}{}}{}$
derivativedivides	$\frac{\frac{a^5x^5 \operatorname{arccot}(ax)^2}{5} + \frac{a^4x^4 \operatorname{arccot}(ax)}{10} - \frac{\operatorname{arccot}(ax)a^2x^2}{5} + \frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{5} + \frac{a^3x^3}{30} - \frac{3ax}{10} + \frac{3\arctan(ax)}{10} - \frac{i\left(\ln(ax-i)\ln\left(a^2\right.\right.}{}}{}$
default	$\frac{\frac{a^5x^5 \operatorname{arccot}(ax)^2}{5} + \frac{a^4x^4 \operatorname{arccot}(ax)}{10} - \frac{\operatorname{arccot}(ax)a^2x^2}{5} + \frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{5} + \frac{a^3x^3}{30} - \frac{3ax}{10} + \frac{3\arctan(ax)}{10} - \frac{i\left(\ln(ax-i)\ln\left(a^2\right.\right.}{}}{}$
risch	$\frac{i\ln(iax+1)x^4}{20a} + \frac{\ln(iax+1)\ln(-iax+1)x^5}{10} + \frac{23i\ln(a^2x^2+1)}{150a^5} + \frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{5a^5} - \frac{47i\ln(-iax+1)}{600a^5} - \frac{137i\ln(}{600}$

input
```
int(x^4*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*arccot(a*x)^2+2/5/a^5*(1/4*a^4*x^4*arccot(a*x)-1/2*arccot(a*x)*a^2
*x^2+1/2*arccot(a*x)*ln(a^2*x^2+1)+1/12*a^3*x^3-3/4*a*x+3/4*arctan(a*x)-1/
4*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(I+a*x))-ln(a*x-
I)*ln(-1/2*I*(I+a*x)))+1/4*I*(ln(I+a*x)*ln(a^2*x^2+1)-1/2*ln(I+a*x)^2-dilo
g(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))))
```

Fricas [F]

$$\int x^4 \cot^{-1}(ax)^2 \, dx = \int x^4 \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x^4*arccot(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^4*arccot(a*x)^2, x)
```

Sympy [F]

$$\int x^4 \cot^{-1}(ax)^2 \, dx = \int x^4 \operatorname{acot}^2(ax) \, dx$$

input
```
integrate(x**4*acot(a*x)**2,x)
```

output
```
Integral(x**4*acot(a*x)**2, x)
```

Maxima [F]

$$\int x^4 \cot^{-1}(ax)^2 \, dx = \int x^4 \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x^4*arccot(a*x)^2,x, algorithm="maxima")
```

output
```
1/20*x^5*arctan2(1, a*x)^2 - 1/80*x^5*log(a^2*x^2 + 1)^2 + integrate(1/80*
(60*a^2*x^6*arctan2(1, a*x)^2 + 4*a^2*x^6*log(a^2*x^2 + 1) + 8*a*x^5*arcta
n2(1, a*x) + 60*x^4*arctan2(1, a*x)^2 + 5*(a^2*x^6 + x^4)*log(a^2*x^2 + 1)
^2)/(a^2*x^2 + 1), x)
```

Giac [F]

$$\int x^4 \cot^{-1}(ax)^2 \, dx = \int x^4 \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x^4*arccot(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^4*arccot(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \cot^{-1}(ax)^2 \, dx = \int x^4 \operatorname{acot}(a\,x)^2 \, dx$$

input
```
int(x^4*acot(a*x)^2,x)
```

output
```
int(x^4*acot(a*x)^2, x)
```

Reduce [F]

$$\int x^4 \cot^{-1}(ax)^2 \, dx$$
$$= \frac{6acot(ax)^2 a^5x^5 - 6acot(ax)^2 ax + 3acot(ax) a^4x^4 - 6acot(ax) a^2x^2 - 9acot(ax) + 6\left(\int acot(ax)^2 dx\right) a}{30a^5}$$

input
```
int(x^4*acot(a*x)^2,x)
```

output
```
(6*acot(a*x)**2*a**5*x**5 - 6*acot(a*x)**2*a*x + 3*acot(a*x)*a**4*x**4 - 6
*acot(a*x)*a**2*x**2 - 9*acot(a*x) + 6*int(acot(a*x)**2,x)*a + a**3*x**3 -
 9*a*x)/(30*a**5)
```

3.14 $\int x^3 \cot^{-1}(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int x^3 \cot^{-1}(ax)^2 \, dx = \frac{x^2}{12a^2} - \frac{x \cot^{-1}(ax)}{2a^3} + \frac{x^3 \cot^{-1}(ax)}{6a} - \frac{\cot^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4 \cot^{-1}(ax)^2 - \frac{\log(1+a^2x^2)}{3a^4}$$

output
```
1/12*x^2/a^2-1/2*x*arccot(a*x)/a^3+1/6*x^3*arccot(a*x)/a-1/4*arccot(a*x)^2
/a^4+1/4*x^4*arccot(a*x)^2-1/3*ln(a^2*x^2+1)/a^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int x^3 \cot^{-1}(ax)^2 \, dx = \frac{a^2x^2 + 2ax(-3+a^2x^2)\cot^{-1}(ax) + 3(-1+a^4x^4)\cot^{-1}(ax)^2 - 4\log(1+a^2x^2)}{12a^4}$$

input
```
Integrate[x^3*ArcCot[a*x]^2,x]
```

output
```
(a^2*x^2 + 2*a*x*(-3 + a^2*x^2)*ArcCot[a*x] + 3*(-1 + a^4*x^4)*ArcCot[a*x]
^2 - 4*Log[1 + a^2*x^2])/(12*a^4)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.39, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5362, 5452, 5362, 243, 49, 2009, 5452, 5346, 240, 5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cot^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{1}{2}a\int \frac{x^4 \cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{4}x^4 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{ 5452}$$

$$\frac{1}{2}a\left(\frac{\int x^2 \cot^{-1}(ax)dx}{a^2}-\frac{\int \frac{x^2 \cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{1}{2}a\left(\frac{\frac{1}{3}a\int \frac{x^3}{a^2x^2+1}dx+\frac{1}{3}x^3 \cot^{-1}(ax)}{a^2}-\frac{\int \frac{x^2 \cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}a\left(\frac{\frac{1}{6}a\int \frac{x^2}{a^2x^2+1}dx^2+\frac{1}{3}x^3 \cot^{-1}(ax)}{a^2}-\frac{\int \frac{x^2 \cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}a\left(\frac{\frac{1}{6}a\int \left(\frac{1}{a^2}-\frac{1}{a^2(a^2x^2+1)}\right)dx^2+\frac{1}{3}x^3 \cot^{-1}(ax)}{a^2}-\frac{\int \frac{x^2 \cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{ 5452}$$

$$\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\int\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{ 5346}$$

$$\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{a\int\frac{x}{a^2x^2+1}dx+x\cot^{-1}(ax)}{a^2}-\frac{\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+$$
$$\frac{1}{4}x^4\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{ 240}$$

$$\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{\log(a^2x^2+1)}{2a}+x\cot^{-1}(ax)}{a^2}-\frac{\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+$$
$$\frac{1}{4}x^4\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{ 5420}$$

$$\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log(a^2x^2+1)}{2a}+x\cot^{-1}(ax)}{a^2}}{a^2}\right)+$$
$$\frac{1}{4}x^4\cot^{-1}(ax)^2$$

input
```
Int[x^3*ArcCot[a*x]^2,x]
```

output
```
(x^4*ArcCot[a*x]^2)/4 + (a*(((x^3*ArcCot[a*x])/3 + (a*(x^2/a^2 - Log[1 + a
^2*x^2]/a^4))/6)/a^2 - (ArcCot[a*x]^2/(2*a^3) + (x*ArcCot[a*x] + Log[1 + a
^2*x^2]/(2*a))/a^2)/a^2))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

method	result
parallelrisch	$-\frac{-3a^4x^4\operatorname{arccot}(ax)^2-2a^3x^3\operatorname{arccot}(ax)-a^2x^2+6\operatorname{arccot}(ax)ax+1+3\operatorname{arccot}(ax)^2+4\ln(a^2x^2+1)}{12a^4}$
parts	$\frac{x^4\operatorname{arccot}(ax)^2}{4}+\frac{\frac{a^3x^3\operatorname{arccot}(ax)}{3}-\operatorname{arccot}(ax)ax+\operatorname{arccot}(ax)\arctan(ax)+\frac{a^2x^2}{6}-\frac{2\ln(a^2x^2+1)}{3}+\frac{\arctan(ax)^2}{2}}{2a^4}$
derivativedivides	$\frac{\frac{a^4x^4\operatorname{arccot}(ax)^2}{4}+\frac{a^3x^3\operatorname{arccot}(ax)}{6}-\frac{\operatorname{arccot}(ax)ax}{2}+\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}+\frac{a^2x^2}{12}-\frac{\ln(a^2x^2+1)}{3}+\frac{\arctan(ax)^2}{4}}{a^4}$
default	$\frac{\frac{a^4x^4\operatorname{arccot}(ax)^2}{4}+\frac{a^3x^3\operatorname{arccot}(ax)}{6}-\frac{\operatorname{arccot}(ax)ax}{2}+\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}+\frac{a^2x^2}{12}-\frac{\ln(a^2x^2+1)}{3}+\frac{\arctan(ax)^2}{4}}{a^4}$
risch	$-\frac{(a^4x^4-1)\ln(iax+1)^2}{16a^4}+\frac{(3i\pi a^4x^4+3x^4\ln(-iax+1)a^4+2ia^3x^3-6iax-3\ln(-iax+1))\ln(iax+1)}{24a^4}-\frac{i\pi x^4\ln(-i}{8}$

input
```
int(x^3*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-3*a^4*x^4*arccot(a*x)^2-2*a^3*x^3*arccot(a*x)-a^2*x^2+6*arccot(a*x
)*a*x+1+3*arccot(a*x)^2+4*ln(a^2*x^2+1))/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int x^3\cot^{-1}(ax)^2\,dx = \frac{a^2x^2+3\left(a^4x^4-1\right)\operatorname{arccot}\left(ax\right)^2+2\left(a^3x^3-3\,ax\right)\operatorname{arccot}\left(ax\right)-4\,\log\left(a^2x^2+1\right)}{12\,a^4}$$

input
```
integrate(x^3*arccot(a*x)^2,x, algorithm="fricas")
```

output
```
1/12*(a^2*x^2 + 3*(a^4*x^4 - 1)*arccot(a*x)^2 + 2*(a^3*x^3 - 3*a*x)*arccot
(a*x) - 4*log(a^2*x^2 + 1))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int x^3 \cot^{-1}(ax)^2\,dx = \begin{cases} \frac{x^4 \operatorname{acot}^2(ax)}{4} + \frac{x^3 \operatorname{acot}(ax)}{6a} + \frac{x^2}{12a^2} - \frac{x \operatorname{acot}(ax)}{2a^3} - \frac{\log(a^2x^2+1)}{3a^4} - \frac{\operatorname{acot}^2(ax)}{4a^4} & \text{for } a \neq 0 \\ \frac{\pi^2 x^4}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acot(a*x)**2,x)
```

output
```
Piecewise((x**4*acot(a*x)**2/4 + x**3*acot(a*x)/(6*a) + x**2/(12*a**2) - x
*acot(a*x)/(2*a**3) - log(a**2*x**2 + 1)/(3*a**4) - acot(a*x)**2/(4*a**4),
 Ne(a, 0)), (pi**2*x**4/16, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int x^3 \cot^{-1}(ax)^2\,dx = \frac{1}{4}x^4 \operatorname{arccot}(ax)^2 + \frac{1}{6}a\left(\frac{a^2x^3 - 3x}{a^4} + \frac{3\arctan(ax)}{a^5}\right)\operatorname{arccot}(ax) + \frac{a^2x^2 + 3\arctan(ax)^2 - 4\log(a^2x^2+1)}{12a^4}$$

input
```
integrate(x^3*arccot(a*x)^2,x, algorithm="maxima")
```

output
```
1/4*x^4*arccot(a*x)^2 + 1/6*a*((a^2*x^3 - 3*x)/a^4 + 3*arctan(a*x)/a^5)*ar
ccot(a*x) + 1/12*(a^2*x^2 + 3*arctan(a*x)^2 - 4*log(a^2*x^2 + 1))/a^4
```

Giac [F]

$$\int x^3 \cot^{-1}(ax)^2\,dx = \int x^3 \operatorname{arccot}(ax)^2\,dx$$

input
```
integrate(x^3*arccot(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arccot(a*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int x^3 \cot^{-1}(ax)^2\,dx = \frac{x^4\,\mathrm{acot}(a\,x)^2}{4} - \frac{\frac{\ln(a^2\,x^2+1)}{3} - \frac{a^2\,x^2}{12} + \frac{\mathrm{acot}(a\,x)^2}{4} - \frac{a^3\,x^3\,\mathrm{acot}(a\,x)}{6} + \frac{a\,x\,\mathrm{acot}(a\,x)}{2}}{a^4}$$

input
```
int(x^3*acot(a*x)^2,x)
```

output
```
(x^4*acot(a*x)^2)/4 - (log(a^2*x^2 + 1)/3 - (a^2*x^2)/12 + acot(a*x)^2/4 -
 (a^3*x^3*acot(a*x))/6 + (a*x*acot(a*x))/2)/a^4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int x^3 \cot^{-1}(ax)^2\,dx = \frac{3acot(ax)^2\,a^4x^4 - 3acot(ax)^2 + 2acot(ax)\,a^3x^3 - 6acot(ax)\,ax - 4\log(a^2x^2+1) + a^2x^2}{12a^4}$$

input
```
int(x^3*acot(a*x)^2,x)
```

output
```
(3*acot(a*x)**2*a**4*x**4 - 3*acot(a*x)**2 + 2*acot(a*x)*a**3*x**3 - 6*aco
t(a*x)*a*x - 4*log(a**2*x**2 + 1) + a**2*x**2)/(12*a**4)
```

3.15 $\int x^2 \cot^{-1}(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 111

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \frac{x}{3a^2} + \frac{x^2 \cot^{-1}(ax)}{3a} - \frac{i \cot^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \cot^{-1}(ax)^2 - \frac{\arctan(ax)}{3a^3} + \frac{2 \cot^{-1}(ax) \log\left(\frac{2}{1+iax}\right)}{3a^3} - \frac{i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{3a^3}$$

output
```
1/3*x/a^2+1/3*x^2*arccot(a*x)/a-1/3*I*arccot(a*x)^2/a^3+1/3*x^3*arccot(a*x
)^2-1/3*arctan(a*x)/a^3+2/3*arccot(a*x)*ln(2/(1+I*a*x))/a^3-1/3*I*polylog(
2,1-2/(1+I*a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \frac{ax + (-i + a^3x^3) \cot^{-1}(ax)^2 + \cot^{-1}(ax)\left(1 + a^2x^2 + 2\log\left(1 - e^{2i\cot^{-1}(ax)}\right)\right) - i \operatorname{PolyLog}\left(2, e^{2i\cot^{-1}(a}\right.}{3a^3}$$

input
```
Integrate[x^2*ArcCot[a*x]^2,x]
```

output
```
(a*x + (-I + a^3*x^3)*ArcCot[a*x]^2 + ArcCot[a*x]*(1 + a^2*x^2 + 2*Log[1 -
 E^((2*I)*ArcCot[a*x])]) - I*PolyLog[2, E^((2*I)*ArcCot[a*x])])/(3*a^3)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {5362, 5452, 5362, 262, 216, 5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{2}{3}a\int \frac{x^3 \cot^{-1}(ax)}{a^2x^2+1}dx + \frac{1}{3}x^3 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5452}$$

$$\frac{2}{3}a\left(\frac{\int x \cot^{-1}(ax)dx}{a^2} - \frac{\int \frac{x \cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right) + \frac{1}{3}x^3 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5362}$$

$$\frac{2}{3}a\left(\frac{\frac{1}{2}a\int \frac{x^2}{a^2x^2+1}dx + \frac{1}{2}x^2 \cot^{-1}(ax)}{a^2} - \frac{\int \frac{x \cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right) + \frac{1}{3}x^3 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2} - \frac{\int \frac{1}{a^2x^2+1}dx}{a^2}\right) + \frac{1}{2}x^2 \cot^{-1}(ax)}{a^2} - \frac{\int \frac{x \cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right) + \frac{1}{3}x^3 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{216}$$

$$\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2} - \frac{\arctan(ax)}{a^3}\right) + \frac{1}{2}x^2 \cot^{-1}(ax)}{a^2} - \frac{\int \frac{x \cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right) + \frac{1}{3}x^3 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5456}$$

$$\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\cot^{-1}(ax)}{i-ax}dx}{a}}{a^2}\right)$$

$$\Big\downarrow \text{5380}$$

$$\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx+\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}}{a}}{a^2}\right)$$

$$\Big\downarrow \text{2849}$$

$$\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}}{a}}{a^2}\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}\right)$$

input
```
Int[x^2*ArcCot[a*x]^2,x]
```

output
```
(x^3*ArcCot[a*x]^2)/3 + (2*a*(((x^2*ArcCot[a*x])/2 + (a*(x/a^2 - ArcTan[a*
x]/a^3))/2)/a^2 - (((I/2)*ArcCot[a*x]^2)/a^2 - ((ArcCot[a*x]*Log[2/(1 + I*
a*x)])/a - ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)/a^2))/3
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.67

method	result
parts	$\frac{x^{3} \operatorname{arccot}(ax)^{2}}{3}+\frac{\frac{\operatorname{arccot}(ax) a^{2} x^{2}}{3}-\frac{\operatorname{arccot}(ax) \ln\left(a^{2} x^{2}+1\right)}{3}+\frac{ax}{3}-\frac{\arctan(ax)}{3}+\frac{i\left(\ln(ax-i) \ln\left(a^{2} x^{2}+1\right)-\frac{\ln(ax-i)^{2}}{2}-\operatorname{dilog}\left(\cdots\right.\right.}{6}}{\cdots}$
derivativedivides	$\frac{\frac{a^{3} x^{3} \operatorname{arccot}(ax)^{2}}{3}+\frac{\operatorname{arccot}(ax) a^{2} x^{2}}{3}-\frac{\operatorname{arccot}(ax) \ln\left(a^{2} x^{2}+1\right)}{3}+\frac{ax}{3}-\frac{\arctan(ax)}{3}+\frac{i\left(\ln(ax-i) \ln\left(a^{2} x^{2}+1\right)-\frac{\ln(ax-i)^{2}}{2}-\operatorname{dilog}\left(-\cdots\right.\right.}{6}}{a^{3}}$
default	$\frac{\frac{a^{3} x^{3} \operatorname{arccot}(ax)^{2}}{3}+\frac{\operatorname{arccot}(ax) a^{2} x^{2}}{3}-\frac{\operatorname{arccot}(ax) \ln\left(a^{2} x^{2}+1\right)}{3}+\frac{ax}{3}-\frac{\arctan(ax)}{3}+\frac{i\left(\ln(ax-i) \ln\left(a^{2} x^{2}+1\right)-\frac{\ln(ax-i)^{2}}{2}-\operatorname{dilog}\left(-\cdots\right.\right.}{6}}{a^{3}}$
risch	$\frac{i \ln(iax+1) x^{2}}{6a}-\frac{i \ln(-iax+1) x^{2}}{6a}-\frac{2i \ln\left(a^{2} x^{2}+1\right)}{9a^{3}}-\frac{i \operatorname{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{3a^{3}}+\frac{5i \ln(-iax+1)}{36a^{3}}+\frac{\ln(iax+1) \ln(-iax+\cdots}{6}$

input
```
int(x^2*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccot(a*x)^2+2/3/a^3*(1/2*arccot(a*x)*a^2*x^2-1/2*arccot(a*x)*ln(
a^2*x^2+1)+1/2*a*x-1/2*arctan(a*x)+1/4*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a
*x-I)^2-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x)))-1/4*I*(ln(I+a*
x)*ln(a^2*x^2+1)-1/2*ln(I+a*x)^2-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(
a*x-I))))
```

Fricas [F]

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \int x^2 \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x^2*arccot(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^2*arccot(a*x)^2, x)
```

Sympy [F]

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \int x^2 \operatorname{acot}^2(ax) \, dx$$

input
```
integrate(x**2*acot(a*x)**2,x)
```

output
```
Integral(x**2*acot(a*x)**2, x)
```

Maxima [F]

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \int x^2 \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x^2*arccot(a*x)^2,x, algorithm="maxima")
```

output
```
1/12*x^3*arctan2(1, a*x)^2 - 1/48*x^3*log(a^2*x^2 + 1)^2 + integrate(1/48*
(36*a^2*x^4*arctan2(1, a*x)^2 + 4*a^2*x^4*log(a^2*x^2 + 1) + 8*a*x^3*arcta
n2(1, a*x) + 36*x^2*arctan2(1, a*x)^2 + 3*(a^2*x^4 + x^2)*log(a^2*x^2 + 1)
^2)/(a^2*x^2 + 1), x)
```

Giac [F]

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \int x^2 \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x^2*arccot(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arccot(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \int x^2 \operatorname{acot}(a\,x)^2 \, dx$$

input
```
int(x^2*acot(a*x)^2,x)
```

output
```
int(x^2*acot(a*x)^2, x)
```

Reduce [F]

$$\int x^2 \cot^{-1}(ax)^2 \, dx = \frac{acot(ax)^2 a^3x^3 + acot(ax)^2 ax + acot(ax)\,a^2x^2 + acot(ax) - \left(\int acot(ax)^2 \, dx\right)a + ax}{3a^3}$$

input
```
int(x^2*acot(a*x)^2,x)
```

output
```
(acot(a*x)**2*a**3*x**3 + acot(a*x)**2*a*x + acot(a*x)*a**2*x**2 + acot(a*
x) - int(acot(a*x)**2,x)*a + a*x)/(3*a**3)
```

3.16 $\int x \cot^{-1}(ax)^2 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 53

$$\int x \cot^{-1}(ax)^2 \, dx = \frac{x \cot^{-1}(ax)}{a} + \frac{\cot^{-1}(ax)^2}{2a^2} + \frac{1}{2} x^2 \cot^{-1}(ax)^2 + \frac{\log(1 + a^2 x^2)}{2a^2}$$

output
```
x*arccot(a*x)/a+1/2*arccot(a*x)^2/a^2+1/2*x^2*arccot(a*x)^2+1/2*ln(a^2*x^2
+1)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int x \cot^{-1}(ax)^2 \, dx = \frac{2ax \cot^{-1}(ax) + (1 + a^2 x^2) \cot^{-1}(ax)^2 + \log(1 + a^2 x^2)}{2a^2}$$

input
```
Integrate[x*ArcCot[a*x]^2,x]
```

output
```
(2*a*x*ArcCot[a*x] + (1 + a^2*x^2)*ArcCot[a*x]^2 + Log[1 + a^2*x^2])/(2*a^
2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5362, 5452, 5346, 240, 5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cot^{-1}(ax)^2 \, dx$$

$$\Big\downarrow \text{5362}$$

$$a \int \frac{x^2 \cot^{-1}(ax)}{a^2x^2+1}dx + \frac{1}{2}x^2 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5452}$$

$$a \left(\frac{\int \cot^{-1}(ax)dx}{a^2} - \frac{\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2} \right) + \frac{1}{2}x^2 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5346}$$

$$a \left(\frac{a \int \frac{x}{a^2x^2+1}dx + x \cot^{-1}(ax)}{a^2} - \frac{\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2} \right) + \frac{1}{2}x^2 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{240}$$

$$a \left(\frac{\frac{\log(a^2x^2+1)}{2a} + x \cot^{-1}(ax)}{a^2} - \frac{\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2} \right) + \frac{1}{2}x^2 \cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5420}$$

$$a \left(\frac{\cot^{-1}(ax)^2}{2a^3} + \frac{\frac{\log(a^2x^2+1)}{2a} + x \cot^{-1}(ax)}{a^2} \right) + \frac{1}{2}x^2 \cot^{-1}(ax)^2$$

input
```
Int[x*ArcCot[a*x]^2,x]
```

output
```
(x^2*ArcCot[a*x]^2)/2 + a*(ArcCot[a*x]^2/(2*a^3) + (x*ArcCot[a*x] + Log[1
+ a^2*x^2]/(2*a))/a^2)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{a^2x^2 \operatorname{arccot}(ax)^2+2 \operatorname{arccot}(ax)ax+\operatorname{arccot}(ax)^2+\ln(a^2x^2+1)}{2a^2}$
parts	$\frac{x^2 \operatorname{arccot}(ax)^2}{2}+\frac{-\operatorname{arccot}(ax)\arctan(ax)+\operatorname{arccot}(ax)ax+\frac{\ln(a^2x^2+1)}{2}-\frac{\arctan(ax)^2}{2}}{a^2}$
derivativedivides	$\frac{\frac{a^2x^2 \operatorname{arccot}(ax)^2}{2}-\operatorname{arccot}(ax)\arctan(ax)+\operatorname{arccot}(ax)ax+\frac{\ln(a^2x^2+1)}{2}-\frac{\arctan(ax)^2}{2}}{a^2}$
default	$\frac{\frac{a^2x^2 \operatorname{arccot}(ax)^2}{2}-\operatorname{arccot}(ax)\arctan(ax)+\operatorname{arccot}(ax)ax+\frac{\ln(a^2x^2+1)}{2}-\frac{\arctan(ax)^2}{2}}{a^2}$
risch	$-\frac{(a^2x^2+1)\ln(iax+1)^2}{8a^2}+\frac{(i\pi a^2x^2+x^2\ln(-iax+1)a^2+2iax+\ln(-iax+1))\ln(iax+1)}{4a^2}-\frac{i\pi x^2\ln(-iax+1)}{4}-\frac{x}{}$

input
```
int(x*arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a^2*x^2*arccot(a*x)^2+2*arccot(a*x)*a*x+arccot(a*x)^2+ln(a^2*x^2+1))/
a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int x \cot^{-1}(ax)^2 \, dx = \frac{2\, ax \operatorname{arccot}(ax) + (a^2x^2+1)\operatorname{arccot}(ax)^2 + \log(a^2x^2+1)}{2\, a^2}$$

input
```
integrate(x*arccot(a*x)^2,x, algorithm="fricas")
```

output
```
1/2*(2*a*x*arccot(a*x) + (a^2*x^2 + 1)*arccot(a*x)^2 + log(a^2*x^2 + 1))/a
^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int x\cot^{-1}(ax)^2\,dx = \begin{cases} \frac{x^2\operatorname{acot}^2(ax)}{2} + \frac{x\operatorname{acot}(ax)}{a} + \frac{\log(a^2x^2+1)}{2a^2} + \frac{\operatorname{acot}^2(ax)}{2a^2} & \text{for } a \neq 0 \\ \frac{\pi^2x^2}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acot(a*x)**2,x)
```

output
```
Piecewise((x**2*acot(a*x)**2/2 + x*acot(a*x)/a + log(a**2*x**2 + 1)/(2*a**
2) + acot(a*x)**2/(2*a**2), Ne(a, 0)), (pi**2*x**2/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int x\cot^{-1}(ax)^2\,dx = \frac{1}{2}x^2\operatorname{arccot}(ax)^2 + a\left(\frac{x}{a^2} - \frac{\arctan(ax)}{a^3}\right)\operatorname{arccot}(ax) - \frac{\arctan(ax)^2 - \log(a^2x^2+1)}{2a^2}$$

input
```
integrate(x*arccot(a*x)^2,x, algorithm="maxima")
```

output
```
1/2*x^2*arccot(a*x)^2 + a*(x/a^2 - arctan(a*x)/a^3)*arccot(a*x) - 1/2*(arc
tan(a*x)^2 - log(a^2*x^2 + 1))/a^2
```

Giac [F]

$$\int x\cot^{-1}(ax)^2\,dx = \int x\operatorname{arccot}(ax)^2\,dx$$

input
```
integrate(x*arccot(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x*arccot(a*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int x\cot^{-1}(ax)^2\,dx=\frac{x^2\,\mathrm{acot}(a\,x)^2}{2}+\frac{\frac{\mathrm{acot}(a\,x)^2}{2}+a\,x\,\mathrm{acot}(a\,x)+\frac{\ln(a^2\,x^2+1)}{2}}{a^2}$$

input
```
int(x*acot(a*x)^2,x)
```

output
```
(x^2*acot(a*x)^2)/2 + (log(a^2*x^2 + 1)/2 + acot(a*x)^2/2 + a*x*acot(a*x))
/a^2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int x\cot^{-1}(ax)^2\,dx=\frac{acot(ax)^2\,a^2x^2+acot(ax)^2+2acot(ax)\,ax+\log(a^2x^2+1)}{2a^2}$$

input
```
int(x*acot(a*x)^2,x)
```

output
```
(acot(a*x)**2*a**2*x**2 + acot(a*x)**2 + 2*acot(a*x)*a*x + log(a**2*x**2 +
 1))/(2*a**2)
```

3.17 $\int \cot^{-1}(ax)^2\,dx$

Optimal result

Integrand size = 6, antiderivative size = 67

$$\int \cot^{-1}(ax)^2\,dx=\frac{i\cot^{-1}(ax)^2}{a}+x\cot^{-1}(ax)^2-\frac{2\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a}$$

output
```
I*arccot(a*x)^2/a+x*arccot(a*x)^2-2*arccot(a*x)*ln(2/(1+I*a*x))/a+I*polylo
g(2,1-2/(1+I*a*x))/a
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \cot^{-1}(ax)^2\,dx=\frac{\cot^{-1}(ax)\left((i+ax)\cot^{-1}(ax)-2\log\left(1-e^{2i\cot^{-1}(ax)}\right)\right)+i\,\mathrm{PolyLog}\left(2,e^{2i\cot^{-1}(ax)}\right)}{a}$$

input
```
Integrate[ArcCot[a*x]^2,x]
```

output
```
(ArcCot[a*x]*((I + a*x)*ArcCot[a*x] - 2*Log[1 - E^((2*I)*ArcCot[a*x])]) +
I*PolyLog[2, E^((2*I)*ArcCot[a*x])])/a
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5346, 5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(ax)^2\,dx$$

$$\Big\downarrow \text{5346}$$

$$2a\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2$$

$$\Big\downarrow \text{5456}$$

$$x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int \frac{\cot^{-1}(ax)}{i-ax}dx}{a}\right)$$

$$\Big\downarrow \text{5380}$$

$$x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int \frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx+\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}}{a}\right)$$

$$\Big\downarrow \text{2849}$$

$$x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\int \frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}}{a}\right)$$

$$\Big\downarrow \text{2752}$$

$$x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)$$

input
```
Int[ArcCot[a*x]^2,x]
```

output
```
x*ArcCot[a*x]^2 + 2*a*(((I/2)*ArcCot[a*x]^2)/a^2 - ((ArcCot[a*x]*Log[2/(1
+ I*a*x)])/a - ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(63) = 126.

Time = 0.37 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.94

method	result
derivativedivides	$\frac{\operatorname{arccot}(ax)^2(ax-i)-2\operatorname{arccot}(ax)\ln\left(1+\frac{ax+i}{\sqrt{a^2x^2+1}}\right)-2\operatorname{arccot}(ax)\ln\left(1-\frac{ax+i}{\sqrt{a^2x^2+1}}\right)+2i\operatorname{arccot}(ax)^2+2i\operatorname{polylog}\left(2,-\right.}{a}$
default	$\frac{\operatorname{arccot}(ax)^2(ax-i)-2\operatorname{arccot}(ax)\ln\left(1+\frac{ax+i}{\sqrt{a^2x^2+1}}\right)-2\operatorname{arccot}(ax)\ln\left(1-\frac{ax+i}{\sqrt{a^2x^2+1}}\right)+2i\operatorname{arccot}(ax)^2+2i\operatorname{polylog}\left(2,-\right.}{a}$
risch	$\frac{i\pi\ln(iax+1)x}{2}+\frac{i\pi^2}{4a}+\frac{\pi\ln(iax+1)}{2a}+\frac{i\operatorname{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{a}+\frac{\ln(iax+1)\ln(-iax+1)x}{2}-\frac{i\pi\ln(-iax+1)x}{2}+\frac{i\ln(}{}$

input
```
int(arccot(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(arccot(a*x)^2*(a*x-I)-2*arccot(a*x)*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))-2
*arccot(a*x)*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))+2*I*arccot(a*x)^2+2*I*polylog
(2,-(I+a*x)/(a^2*x^2+1)^(1/2))+2*I*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \cot^{-1}(ax)^2\,dx = \int \operatorname{arccot}(ax)^2\,dx$$

input
```
integrate(arccot(a*x)^2,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^2, x)
```

Sympy [F]

$$\int \cot^{-1}(ax)^2 \, dx = \int \operatorname{acot}^2(ax) \, dx$$

input
```
integrate(acot(a*x)**2,x)
```

output
```
Integral(acot(a*x)**2, x)
```

Maxima [F]

$$\int \cot^{-1}(ax)^2 \, dx = \int \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(arccot(a*x)^2,x, algorithm="maxima")
```

output
```
1/4*x*arctan2(1, a*x)^2 + 12*a^2*integrate(1/16*x^2*arctan(1/(a*x))^2/(a^2
*x^2 + 1), x) + a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x
) + 4*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 1/16*x*l
og(a^2*x^2 + 1)^2 + 1/4*arctan(a*x)^3/a + 3/4*arctan(a*x)^2*arctan(1/(a*x)
)/a + 3/4*arctan(a*x)*arctan(1/(a*x))^2/a + 8*a*integrate(1/16*x*arctan(1/
(a*x))/(a^2*x^2 + 1), x) + integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1)
, x)
```

Giac [F]

$$\int \cot^{-1}(ax)^2 \, dx = \int \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(arccot(a*x)^2,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \cot^{-1}(ax)^2 \, dx = \frac{-2 \ln \left(1 - e^{\operatorname{acot}(a\,x)\,2i}\right) \operatorname{acot}(a\,x) + \operatorname{polylog}\left(2, e^{\operatorname{acot}(a\,x)\,2i}\right)\,1i + \operatorname{acot}(a\,x)^2\,1i}{a} + x \operatorname{acot}(a\,x)^2$$

input
```
int(acot(a*x)^2,x)
```

output
```
(polylog(2, exp(acot(a*x)*2i))*1i - 2*log(1 - exp(acot(a*x)*2i))*acot(a*x)
 + acot(a*x)^2*1i)/a + x*acot(a*x)^2
```

Reduce [F]

$$\int \cot^{-1}(ax)^2 \, dx = \int acot(ax)^2 \, dx$$

input
```
int(acot(a*x)^2,x)
```

output
```
int(acot(a*x)**2,x)
```

3.18 $\int \frac{\cot^{-1}(ax)^2}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 116

$$\int \frac{\cot^{-1}(ax)^2}{x}\,dx = 2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right) - i\cot^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2i}{i+ax}\right) + i\cot^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2ax}{i+ax}\right) - \frac{1}{2}\text{PolyLog}\left(3,1-\frac{2i}{i+ax}\right) + \frac{1}{2}\text{PolyLog}\left(3,1-\frac{2ax}{i+ax}\right)$$

output

```
2*arccot(a*x)^2*arccoth(1-2/(1+I*a*x))-I*arccot(a*x)*polylog(2,1-2*I/(I+a*
x))+I*arccot(a*x)*polylog(2,1-2*a*x/(I+a*x))-1/2*polylog(3,1-2*I/(I+a*x))+
1/2*polylog(3,1-2*a*x/(I+a*x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.14

$$\int \frac{\cot^{-1}(ax)^2}{x}\,dx = -\frac{2}{3}i\cot^{-1}(ax)^3 - \cot^{-1}(ax)^2\log\left(1-e^{-2i\cot^{-1}(ax)}\right) + \cot^{-1}(ax)^2\log\left(1+e^{2i\cot^{-1}(ax)}\right) - i\cot^{-1}(ax)\,\text{PolyLog}\left(2,e^{-2i\cot^{-1}(ax)}\right) - i\cot^{-1}(ax)\,\text{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right) - \frac{1}{2}\text{PolyLog}\left(3,e^{-2i\cot^{-1}(ax)}\right) + \frac{1}{2}\text{PolyLog}\left(3,-e^{2i\cot^{-1}(ax)}\right)$$

input
```
Integrate[ArcCot[a*x]^2/x,x]
```

output
```
((-2*I)/3)*ArcCot[a*x]^3 - ArcCot[a*x]^2*Log[1 - E^((-2*I)*ArcCot[a*x])] +
 ArcCot[a*x]^2*Log[1 + E^((2*I)*ArcCot[a*x])] - I*ArcCot[a*x]*PolyLog[2, E
^((-2*I)*ArcCot[a*x])] - I*ArcCot[a*x]*PolyLog[2, -E^((2*I)*ArcCot[a*x])]
- PolyLog[3, E^((-2*I)*ArcCot[a*x])]/2 + PolyLog[3, -E^((2*I)*ArcCot[a*x])
]/2
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5358, 5524, 5528, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^2}{x}\,dx$$

$$\Big\downarrow \; 5358$$

$$4a\int \frac{\cot^{-1}(ax)\coth^{-1}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx + 2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

$$\Big\downarrow \text{5524}$$

$$4a\left(\frac{1}{2}\int \frac{\cot^{-1}(ax)\log\left(\frac{2ax}{ax+i}\right)}{a^2x^2+1}dx-\frac{1}{2}\int \frac{\cot^{-1}(ax)\log\left(\frac{2i}{ax+i}\right)}{a^2x^2+1}dx\right)+ 2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

$$\Big\downarrow \text{5528}$$

$$4a\left(\frac{1}{2}\left(-\frac{1}{2}i\int \frac{\text{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)}{a^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)\cot^{-1}(ax)}{2a}\right)+\frac{1}{2}\left(\frac{1}{2}i\int \frac{\text{PolyLog}\left(2,1-\frac{2a}{ax}\right.}{a^2x^2+1}\right.\right. 2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

$$\Big\downarrow \text{7164}$$

$$4a\left(\frac{1}{2}\left(-\frac{\text{PolyLog}\left(3,1-\frac{2i}{ax+i}\right)}{4a}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)\cot^{-1}(ax)}{2a}\right)+\frac{1}{2}\left(\frac{\text{PolyLog}\left(3,1-\frac{2ax}{ax+i}\right)}{4a}+\frac{i\,\text{PolyL}}{}\right.\right. 2\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

input

```
Int[ArcCot[a*x]^2/x,x]
```

output

```
2*ArcCot[a*x]^2*ArcCoth[1 - 2/(1 + I*a*x)] + 4*a*((((-1/2*I)*ArcCot[a*x]*P
olyLog[2, 1 - (2*I)/(I + a*x)])/a - PolyLog[3, 1 - (2*I)/(I + a*x)]/(4*a))
/2 + (((I/2)*ArcCot[a*x]*PolyLog[2, 1 - (2*a*x)/(I + a*x)])/a + PolyLog[3,
 1 - (2*a*x)/(I + a*x)]/(4*a))/2)
```

Defintions of rubi rules used

rule 5358

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcCot[c*x])^p*ArcCoth[1 - 2/(1 + I*c*x)], x] + Simp[2*b*c*p   Int[(a + b
*ArcCot[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5524
```
Int[(ArcCoth[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
 b*ArcCot[c*x])^p/(d + e*x^2)), x], x] - Simp[1/2   Int[Log[SimplifyIntegra
nd[1 - 1/u, x]]*((a + b*ArcCot[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*
x)))^2, 0]
```

rule 5528
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.76 (sec) , antiderivative size = 891, normalized size of antiderivative = 7.68

method	result
derivativedivides	$\ln\left(ax\right)\operatorname{arccot}\left(ax\right)^{2}+\frac{i\pi\left(\operatorname{csgn}\left(\frac{i\left(1+\frac{(ax+i)^{2}}{a^{2}x^{2}+1}\right)}{\frac{(ax+i)^{2}}{a^{2}x^{2}+1}-1}\right)\operatorname{csgn}\left(\frac{1+\frac{(ax+i)^{2}}{a^{2}x^{2}+1}}{\frac{(ax+i)^{2}}{a^{2}x^{2}+1}-1}\right)+\operatorname{csgn}\left(\frac{1+\frac{(ax+i)^{2}}{a^{2}x^{2}+1}}{\frac{(ax+i)^{2}}{a^{2}x^{2}+1}-1}\right)^{2}+\operatorname{csgn}\left(\frac{i}{\frac{(ax+i)^{2}}{a^{2}x^{2}+\cdots}}\cdots\right.\right.}{\cdots}$
default	$\ln\left(ax\right)\operatorname{arccot}\left(ax\right)^{2}+\frac{i\pi\left(\operatorname{csgn}\left(\frac{i\left(1+\frac{(ax+i)^{2}}{a^{2}x^{2}+1}\right)}{\frac{(ax+i)^{2}}{a^{2}x^{2}+1}-1}\right)\operatorname{csgn}\left(\frac{1+\frac{(ax+i)^{2}}{a^{2}x^{2}+1}}{\frac{(ax+i)^{2}}{a^{2}x^{2}+1}-1}\right)+\operatorname{csgn}\left(\frac{1+\frac{(ax+i)^{2}}{a^{2}x^{2}+1}}{\frac{(ax+i)^{2}}{a^{2}x^{2}+1}-1}\right)^{2}+\operatorname{csgn}\left(\frac{i}{\frac{(ax+i)^{2}}{a^{2}x^{2}+\cdots}}\cdots\right.\right.}{\cdots}$
parts	Expression too large to display

input
```
int(arccot(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
ln(a*x)*arccot(a*x)^2+1/2*I*Pi*(csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x
)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1
)))+csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2+csgn(I/(
(I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x
)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))-csgn(I/((I+a*x)^2/(a^2*x^2+1
)-1))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2-csgn(I
*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/
(a^2*x^2+1)))^2+csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1))
)^3-csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I
+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2-csgn(1/((I+a*x)^2/(a^2
*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3-1)*arccot(a*x)^2+arccot(a*x)^2*ln(
(I+a*x)^2/(a^2*x^2+1)-1)-arccot(a*x)^2*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))+2*I
*arccot(a*x)*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(I+a*x)/(a^2
*x^2+1)^(1/2))-arccot(a*x)^2*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))+2*I*arccot(a*
x)*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,-(I+a*x)/(a^2*x^2+1)^
(1/2))-I*arccot(a*x)*polylog(2,-(I+a*x)^2/(a^2*x^2+1))+1/2*polylog(3,-(I+a
*x)^2/(a^2*x^2+1))
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x}\,dx$$

input
```
integrate(arccot(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^2/x, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{acot}^2(ax)}{x}\,dx$$

input
```
integrate(acot(a*x)**2/x,x)
```

output
```
Integral(acot(a*x)**2/x, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x}\,dx$$

input
```
integrate(arccot(a*x)^2/x,x, algorithm="maxima")
```

output
```
integrate(arccot(a*x)^2/x, x)
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x}\,dx$$

input
```
integrate(arccot(a*x)^2/x,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^2}{x}\,dx = \int \frac{\operatorname{acot}(a\,x)^2}{x}\,dx$$

input
```
int(acot(a*x)^2/x,x)
```

output
```
int(acot(a*x)^2/x, x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(ax)^2}{x}\,dx = \int \frac{acot(ax)^2}{x}dx$$

input
```
int(acot(a*x)^2/x,x)
```

output
```
int(acot(a*x)**2/x,x)
```

3.19 $\int \frac{\cot^{-1}(ax)^2}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 66

$$\int \frac{\cot^{-1}(ax)^2}{x^2} dx = -ia\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{x} - 2a\cot^{-1}(ax)\log\left(2 - \frac{2}{1-iax}\right) - ia\,\text{PolyLog}\left(2, -1 + \frac{2}{1-iax}\right)$$

output
```
-I*a*arccot(a*x)^2-arccot(a*x)^2/x-2*a*arccot(a*x)*ln(2-2/(1-I*a*x))-I*a*p
olylog(2,-1+2/(1-I*a*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(ax)^2}{x^2} dx = a\left(i\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{ax} - 2\cot^{-1}(ax)\log\left(1 + e^{2i\cot^{-1}(ax)}\right) + i\,\text{PolyLog}\left(2, -e^{2i\cot^{-1}(ax)}\right)\right)$$

input
```
Integrate[ArcCot[a*x]^2/x^2,x]
```

output
```
a*(I*ArcCot[a*x]^2 - ArcCot[a*x]^2/(a*x) - 2*ArcCot[a*x]*Log[1 + E^((2*I)*
ArcCot[a*x])] + I*PolyLog[2, -E^((2*I)*ArcCot[a*x])])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5362, 5460, 5404, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^2}{x^2} \, dx$$

$$\Big\downarrow \text{5362}$$

$$-2a \int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx - \frac{\cot^{-1}(ax)^2}{x}$$

$$\Big\downarrow \text{5460}$$

$$-\frac{\cot^{-1}(ax)^2}{x} - 2a\left(i\int \frac{\cot^{-1}(ax)}{x(ax+i)}dx + \frac{1}{2}i\cot^{-1}(ax)^2\right)$$

$$\Big\downarrow \text{5404}$$

$$-\frac{\cot^{-1}(ax)^2}{x} - 2a\left(i\left(-ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right) + \frac{1}{2}i\cot^{-1}(ax)^2\right)$$

$$\Big\downarrow \text{2897}$$

$$-\frac{\cot^{-1}(ax)^2}{x} - 2a\left(i\left(\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right) - i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right) + \frac{1}{2}i\cot^{-1}(ax)^2\right)$$

input
```
Int[ArcCot[a*x]^2/x^2,x]
```

output
```
-(ArcCot[a*x]^2/x) - 2*a*((I/2)*ArcCot[a*x]^2 + I*((-I)*ArcCot[a*x]*Log[2
- 2/(1 - I*a*x)] + PolyLog[2, -1 + 2/(1 - I*a*x)]/2))
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5404
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Si
mp[b*c*(p/d)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5460
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Simp[
I/d   Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(62) = 124.

Time = 0.52 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.36

method	result
parts	$-\frac{\operatorname{arccot}(ax)^2}{x}-2a\left(\operatorname{arccot}(ax)\ln(ax)-\frac{\operatorname{arccot}(ax)\ln(a^2x^2+1)}{2}-\frac{i\ln(ax)\ln(iax+1)}{2}+\frac{i\ln(ax)\ln(-\right.$
derivativedivides	$a\left(-\frac{\operatorname{arccot}(ax)^2}{ax}-2\operatorname{arccot}(ax)\ln(ax)+\operatorname{arccot}(ax)\ln(a^2x^2+1)+i\ln(ax)\ln(iax+\right.$
default	$a\left(-\frac{\operatorname{arccot}(ax)^2}{ax}-2\operatorname{arccot}(ax)\ln(ax)+\operatorname{arccot}(ax)\ln(a^2x^2+1)+i\ln(ax)\ln(iax+\right.$

input
```
int(arccot(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccot(a*x)^2/x-2*a*(arccot(a*x)*ln(a*x)-1/2*arccot(a*x)*ln(a^2*x^2+1)-1/
2*I*ln(a*x)*ln(1+I*a*x)+1/2*I*ln(a*x)*ln(1-I*a*x)-1/2*I*dilog(1+I*a*x)+1/2
*I*dilog(1-I*a*x)+1/4*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/
2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x)))-1/4*I*(ln(I+a*x)*ln(a^2*x^2+1)-
1/2*ln(I+a*x)^2-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*x-I))))
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx=\int \frac{\operatorname{arccot}(ax)^2}{x^2}\,dx$$

input
```
integrate(arccot(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{acot}^2(ax)}{x^2}\,dx$$

input
```
integrate(acot(a*x)**2/x**2,x)
```

output
```
Integral(acot(a*x)**2/x**2, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x^2}\,dx$$

input
```
integrate(arccot(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
1/16*(4*(3*a*arctan(a*x)*arctan(1/(a*x))^2 + (arctan(a*x)^3/a + 3*arctan(a
*x)^2*arctan(1/(a*x))/a)*a^2 + 4*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)^2
/(a^2*x^4 + x^2), x) - 16*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^2*x^4
 + x^2), x) - 32*a*integrate(1/16*x*arctan(1/(a*x))/(a^2*x^4 + x^2), x) +
48*integrate(1/16*arctan(1/(a*x))^2/(a^2*x^4 + x^2), x) + 4*integrate(1/16
*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x - 4*arctan2(1, a*x)^2 + log(a^2
*x^2 + 1)^2)/x
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x^2}\,dx$$

input
```
integrate(arccot(a*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = \int \frac{\mathrm{acot}(a\,x)^2}{x^2}\,dx$$

input
```
int(acot(a*x)^2/x^2,x)
```

output
```
int(acot(a*x)^2/x^2, x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^2}\,dx = \frac{-acot(ax)^2 - 2\left(\int \frac{acot(ax)}{a^2x^3+x}dx\right)ax}{x}$$

input
```
int(acot(a*x)^2/x^2,x)
```

output
```
( - acot(a*x)**2 - 2*int(acot(a*x)/(a**2*x**3 + x),x)*a*x)/x
```

3.20 $\int \frac{\cot^{-1}(ax)^2}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 59

$$\int \frac{\cot^{-1}(ax)^2}{x^3} dx = \frac{a\cot^{-1}(ax)}{x} - \frac{1}{2}a^2\cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{2x^2} + a^2\log(x) - \frac{1}{2}a^2\log\left(1+a^2x^2\right)$$

output
```
a*arccot(a*x)/x-1/2*a^2*arccot(a*x)^2-1/2*arccot(a*x)^2/x^2+a^2*ln(x)-1/2*
a^2*ln(a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{\cot^{-1}(ax)^2}{x^3} dx = \frac{a\cot^{-1}(ax)}{x} + \frac{(-1-a^2x^2)\cot^{-1}(ax)^2}{2x^2} + a^2\log(x) - \frac{1}{2}a^2\log\left(1+a^2x^2\right)$$

input
```
Integrate[ArcCot[a*x]^2/x^3,x]
```

output
```
(a*ArcCot[a*x])/x + ((-1 - a^2*x^2)*ArcCot[a*x]^2)/(2*x^2) + a^2*Log[x] -
(a^2*Log[1 + a^2*x^2])/2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5362, 5454, 5362, 243, 47, 14, 16, 5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^2}{x^3} dx$$

$$\Big\downarrow \text{5362}$$

$$-a\int \frac{\cot^{-1}(ax)}{x^2(a^2x^2+1)}dx-\frac{\cot^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{5454}$$

$$-a\left(\int \frac{\cot^{-1}(ax)}{x^2}dx-a^2\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{5362}$$

$$-a\left(-a\int \frac{1}{x(a^2x^2+1)}dx+a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)-\frac{\cot^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$-a\left(-\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2+a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)-\frac{\cot^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{47}$$

$$-a\left(-\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)+a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)-\frac{\cot^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$-a\left(-\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)+a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)-\frac{\cot^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\cot^{-1}(ax)}{x}\right)-\frac{\cot^{-1}(ax)^2}{2x^2}$$

$$\Big\downarrow \; 5420$$

$$-a\left(-\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)+\frac{1}{2}a\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)}{x}\right)-\frac{\cot^{-1}(ax)^2}{2x^2}$$

input
```
Int[ArcCot[a*x]^2/x^3,x]
```

output
```
-1/2*ArcCot[a*x]^2/x^2 - a*(-(ArcCot[a*x]/x) + (a*ArcCot[a*x]^2)/2 - (a*(L
og[x^2] - Log[1 + a^2*x^2]))/2)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arccot}(ax)^2}{2a^2x^2}+\frac{\operatorname{arccot}(ax)}{xa}+\operatorname{arccot}(ax)\arctan(ax)-\frac{\ln(a^2x^2+1)}{2}+\ln(ax)+\frac{\arctan(ax)^2}{2}\right.$
default	$a^2\left(-\frac{\operatorname{arccot}(ax)^2}{2a^2x^2}+\frac{\operatorname{arccot}(ax)}{xa}+\operatorname{arccot}(ax)\arctan(ax)-\frac{\ln(a^2x^2+1)}{2}+\ln(ax)+\frac{\arctan(ax)^2}{2}\right.$
parallelrisch	$\frac{-a^2x^2\operatorname{arccot}(ax)^2+2a^2\ln(x)x^2-a^2\ln(a^2x^2+1)x^2+2\operatorname{arccot}(ax)ax-\operatorname{arccot}(ax)^2}{2x^2}$
parts	$-\frac{\operatorname{arccot}(ax)^2}{2x^2}-a^2\left(-\frac{\operatorname{arccot}(ax)}{xa}-\operatorname{arccot}(ax)\arctan(ax)+\frac{\ln(a^2x^2+1)}{2}-\ln(ax)-\frac{\arctan(a\right.$
risch	$\frac{(a^2x^2+1)\ln(iax+1)^2}{8x^2}-\frac{i(-ix^2\ln(-iax+1)a^2-2ax+\pi-i\ln(-iax+1))\ln(iax+1)}{4x^2}-\frac{2ia^2\ln((-\pi a+6ai)x+i\pi+6)}{}$

input
```
int(arccot(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/a^2/x^2*arccot(a*x)^2+arccot(a*x)/x/a+arccot(a*x)*arctan(a*x)-1/
2*ln(a^2*x^2+1)+ln(a*x)+1/2*arctan(a*x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = -\frac{a^2x^2\log\left(a^2x^2+1\right)-2\,a^2x^2\log\left(x\right)-2\,ax\,\operatorname{arccot}\left(ax\right)+\left(a^2x^2+1\right)\operatorname{arccot}\left(ax\right)^2}{2\,x^2}$$

input
```
integrate(arccot(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a^2*x^2*log(a^2*x^2 + 1) - 2*a^2*x^2*log(x) - 2*a*x*arccot(a*x) + (a
^2*x^2 + 1)*arccot(a*x)^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = a^2\log\left(x\right)-\frac{a^2\log\left(a^2x^2+1\right)}{2}-\frac{a^2\operatorname{acot}^2\left(ax\right)}{2}+\frac{a\operatorname{acot}\left(ax\right)}{x}-\frac{\operatorname{acot}^2\left(ax\right)}{2x^2}$$

input
```
integrate(acot(a*x)**2/x**3,x)
```

output
```
a**2*log(x) - a**2*log(a**2*x**2 + 1)/2 - a**2*acot(a*x)**2/2 + a*acot(a*x
)/x - acot(a*x)**2/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = \frac{1}{2}\left(\arctan\left(ax\right)^2-\log\left(a^2x^2+1\right)+2\,\log\left(x\right)\right)a^2 + \left(a\arctan\left(ax\right)+\frac{1}{x}\right)a\,\operatorname{arccot}\left(ax\right)-\frac{\operatorname{arccot}\left(ax\right)^2}{2\,x^2}$$

input
```
integrate(arccot(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
1/2*(arctan(a*x)^2 - log(a^2*x^2 + 1) + 2*log(x))*a^2 + (a*arctan(a*x) + 1
/x)*a*arccot(a*x) - 1/2*arccot(a*x)^2/x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = -\frac{1}{2}\left(\left(\arctan\left(\frac{1}{ax}\right)^2 - \frac{2\arctan\left(\frac{1}{ax}\right)}{ax} + \log\left(\frac{1}{a^2x^2}+1\right)\right)a + \frac{\arctan\left(\frac{1}{ax}\right)^2}{ax^2}\right)a$$

input
```
integrate(arccot(a*x)^2/x^3,x, algorithm="giac")
```

output
```
-1/2*((arctan(1/(a*x))^2 - 2*arctan(1/(a*x))/(a*x) + log(1/(a^2*x^2) + 1))
*a + arctan(1/(a*x))^2/(a*x^2))*a
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = a^2\ln(x) - \operatorname{acot}(a\,x)^2\left(\frac{a^2}{2}+\frac{1}{2\,x^2}\right) - \frac{a^2\ln\left(a^2\,x^2+1\right)}{2} + \frac{a\operatorname{acot}(a\,x)}{x}$$

input
```
int(acot(a*x)^2/x^3,x)
```

output
```
a^2*log(x) - acot(a*x)^2*(a^2/2 + 1/(2*x^2)) - (a^2*log(a^2*x^2 + 1))/2 +
(a*acot(a*x))/x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{\cot^{-1}(ax)^2}{x^3}\,dx = \frac{-acot(ax)^2\,a^2x^2 - acot(ax)^2 + 2acot(ax)\,ax - \log(a^2x^2+1)\,a^2x^2 + 2\log(x)\,a^2x^2}{2x^2}$$

input
```
int(acot(a*x)^2/x^3,x)
```

output
```
( - acot(a*x)**2*a**2*x**2 - acot(a*x)**2 + 2*acot(a*x)*a*x - log(a**2*x**
2 + 1)*a**2*x**2 + 2*log(x)*a**2*x**2)/(2*x**2)
```

3.21 $\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 113

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = -\frac{a^2}{3x}+\frac{a\cot^{-1}(ax)}{3x^2}+\frac{1}{3}ia^3\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)^2}{3x^3} -\frac{1}{3}a^3\arctan(ax)+\frac{2}{3}a^3\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right) +\frac{1}{3}ia^3\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)$$

output
```
-1/3*a^2/x+1/3*a*arccot(a*x)/x^2+1/3*I*a^3*arccot(a*x)^2-1/3*arccot(a*x)^2
/x^3-1/3*a^3*arctan(a*x)+2/3*a^3*arccot(a*x)*ln(2-2/(1-I*a*x))+1/3*I*a^3*p
olylog(2,-1+2/(1-I*a*x))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \frac{-a^2x^2+(-1-ia^3x^3)\cot^{-1}(ax)^2+ax\cot^{-1}(ax)\left(1+a^2x^2+2a^2x^2\log\left(1+e^{2i\cot^{-1}(ax)}\right)\right)-ia^3x^3\,\text{Poly}}{3x^3}$$

input
```
Integrate[ArcCot[a*x]^2/x^4,x]
```

output
```
(-(a^2*x^2) + (-1 - I*a^3*x^3)*ArcCot[a*x]^2 + a*x*ArcCot[a*x]*(1 + a^2*x^
2 + 2*a^2*x^2*Log[1 + E^((2*I)*ArcCot[a*x])]) - I*a^3*x^3*PolyLog[2, -E^((
2*I)*ArcCot[a*x])])/(3*x^3)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5362, 5454, 5362, 264, 216, 5460, 5404, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^2}{x^4} \, dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{2}{3}a\int \frac{\cot^{-1}(ax)}{x^3\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{5454}$$

$$-\frac{2}{3}a\left(\int \frac{\cot^{-1}(ax)}{x^3}dx-a^2\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx\right)-\frac{\cot^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{5362}$$

$$-\frac{2}{3}a\left(-\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx+a^2\left(-\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx\right)-\frac{\cot^{-1}(ax)}{2x^2}\right)-\frac{\cot^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2}{3}a\left(-\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right)-\frac{1}{x}\right)+a^2\left(-\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx\right)-\frac{\cot^{-1}(ax)}{2x^2}\right)-\frac{\cot^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2}{3}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx\right)-\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\cot^{-1}(ax)}{2x^2}\right)-\frac{\cot^{-1}(ax)^2}{3x^3}$$

$$\Big\downarrow \; 5460$$

$$-\frac{\cot^{-1}(ax)^2}{3x^3}-\frac{2}{3}a\left(-\left(a^2\left(i\int\frac{\cot^{-1}(ax)}{x(ax+i)}dx+\frac{1}{2}i\cot^{-1}(ax)^2\right)\right)-\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\cot^{-1}(ax)}{2x^2}\right)$$

$$\Big\downarrow \; 5404$$

$$-\frac{\cot^{-1}(ax)^2}{3x^3}-\frac{2}{3}a\left(-\left(a^2\left(i\left(-ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right)+\frac{1}{2}i\cot^{-1}(ax)^2\right)\right)-\frac{1}{2}a\left(-a\,\mathrm{arc}\right.\right.$$

$$\Big\downarrow \; 2897$$

$$-\frac{\cot^{-1}(ax)^2}{3x^3}-\frac{2}{3}a\left(-\left(a^2\left(i\left(\frac{1}{2}\mathrm{PolyLog}\left(2,\frac{2}{1-iax}-1\right)-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right)+\frac{1}{2}i\cot^{-1}(ax)^2\right)\right)-\frac{1}{2}a\left(-a\,\mathrm{arc}\right.\right.$$

input
```
Int[ArcCot[a*x]^2/x^4,x]
```

output
```
-1/3*ArcCot[a*x]^2/x^3 - (2*a*(-1/2*ArcCot[a*x]/x^2 - (a*(-x^(-1) - a*ArcT
an[a*x]))/2 - a^2*((I/2)*ArcCot[a*x]^2 + I*((-I)*ArcCot[a*x]*Log[2 - 2/(1
- I*a*x)] + PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/3
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5404
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Si
mp[b*c*(p/d)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5460
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Simp[
I/d   Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(95) = 190$.

Time = 0.68 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.22

method	result
parts	$-\frac{\operatorname{arccot}(ax)^2}{3x^3} - \frac{2a^3\left(-\frac{\operatorname{arccot}(ax)}{2a^2x^2} - \operatorname{arccot}(ax)\ln(ax) + \frac{\operatorname{arccot}(ax)\ln\left(a^2x^2+1\right)}{2} - \frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right) - \frac{\ln(ax-i)^2}{2} - \mathrm{d}\right.}{}\right.}{}$
derivativedivides	$a^3\left(-\frac{\operatorname{arccot}(ax)^2}{3a^3x^3} + \frac{\operatorname{arccot}(ax)}{3a^2x^2} + \frac{2\operatorname{arccot}(ax)\ln(ax)}{3} - \frac{\operatorname{arccot}(ax)\ln(a^2x^2+1)}{3} + \frac{i\left(\ln(ax-i)\ln(a^2x^2+1) - \right.}{}\right.$
default	$a^3\left(-\frac{\operatorname{arccot}(ax)^2}{3a^3x^3} + \frac{\operatorname{arccot}(ax)}{3a^2x^2} + \frac{2\operatorname{arccot}(ax)\ln(ax)}{3} - \frac{\operatorname{arccot}(ax)\ln(a^2x^2+1)}{3} + \frac{i\left(\ln(ax-i)\ln(a^2x^2+1) - \right.}{}\right.$

input
```
int(arccot(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arccot(a*x)^2/x^3-2/3*a^3*(-1/2/a^2/x^2*arccot(a*x)-arccot(a*x)*ln(a*
x)+1/2*arccot(a*x)*ln(a^2*x^2+1)-1/4*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x
-I)^2-dilog(-1/2*I*(I+a*x))-ln(a*x-I)*ln(-1/2*I*(I+a*x)))+1/4*I*(ln(I+a*x)
*ln(a^2*x^2+1)-1/2*ln(I+a*x)^2-dilog(1/2*I*(a*x-I))-ln(I+a*x)*ln(1/2*I*(a*
x-I)))+1/2/a/x+1/2*arctan(a*x)+1/2*I*ln(a*x)*ln(1+I*a*x)-1/2*I*ln(a*x)*ln(
1-I*a*x)+1/2*I*dilog(1+I*a*x)-1/2*I*dilog(1-I*a*x))
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x^4}\,dx$$

input
```
integrate(arccot(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^2/x^4, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{acot}^2(ax)}{x^4}\,dx$$

input
```
integrate(acot(a*x)**2/x**4,x)
```

output
```
Integral(acot(a*x)**2/x**4, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x^4}\,dx$$

input
```
integrate(arccot(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
1/48*(48*x^3*integrate(1/48*(36*a^2*x^2*arctan2(1, a*x)^2 - 4*a^2*x^2*log(
a^2*x^2 + 1) - 8*a*x*arctan2(1, a*x) + 3*(a^2*x^2 + 1)*log(a^2*x^2 + 1)^2
+ 36*arctan2(1, a*x)^2)/(a^2*x^6 + x^4), x) - 4*arctan2(1, a*x)^2 + log(a^
2*x^2 + 1)^2)/x^3
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arccot}(ax)^2}{x^4}\,dx$$

input
```
integrate(arccot(a*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \int \frac{\mathrm{acot}(a\,x)^2}{x^4}\,dx$$

input
```
int(acot(a*x)^2/x^4,x)
```

output
```
int(acot(a*x)^2/x^4, x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(ax)^2}{x^4}\,dx = \frac{-acot(ax)^2 + acot(ax)\,a^3x^3 + acot(ax)\,ax + 2\left(\int \frac{acot(ax)}{a^2x^3+x}dx\right)a^3x^3 - a^2x^2}{3x^3}$$

input
```
int(acot(a*x)^2/x^4,x)
```

output
```
( - acot(a*x)**2 + acot(a*x)*a**3*x**3 + acot(a*x)*a*x + 2*int(acot(a*x)/(
a**2*x**3 + x),x)*a**3*x**3 - a**2*x**2)/(3*x**3)
```

3.22 $\int \frac{\cot^{-1}(ax)^2}{x^5} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 89

$$\int \frac{\cot^{-1}(ax)^2}{x^5} \, dx = -\frac{a^2}{12x^2} + \frac{a \cot^{-1}(ax)}{6x^3} - \frac{a^3 \cot^{-1}(ax)}{2x} + \frac{1}{4} a^4 \cot^{-1}(ax)^2 - \frac{\cot^{-1}(ax)^2}{4x^4} - \frac{2}{3} a^4 \log(x) + \frac{1}{3} a^4 \log\left(1 + a^2 x^2\right)$$

output
```
-1/12*a^2/x^2+1/6*a*arccot(a*x)/x^3-1/2*a^3*arccot(a*x)/x+1/4*a^4*arccot(a
*x)^2-1/4*arccot(a*x)^2/x^4-2/3*a^4*ln(x)+1/3*a^4*ln(a^2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(ax)^2}{x^5} \, dx = -\frac{a^2}{12x^2} - \frac{a(-1 + 3a^2x^2) \cot^{-1}(ax)}{6x^3} + \frac{(-1 + a^4x^4) \cot^{-1}(ax)^2}{4x^4} - \frac{2}{3} a^4 \log(x) + \frac{1}{3} a^4 \log\left(1 + a^2 x^2\right)$$

input
```
Integrate[ArcCot[a*x]^2/x^5,x]
```

output
```
-1/12*a^2/x^2 - (a*(-1 + 3*a^2*x^2)*ArcCot[a*x])/(6*x^3) + ((-1 + a^4*x^4)
*ArcCot[a*x]^2)/(4*x^4) - (2*a^4*Log[x])/3 + (a^4*Log[1 + a^2*x^2])/3
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.27, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {5362, 5454, 5362, 243, 54, 2009, 5454, 5362, 243, 47, 14, 16, 5420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^2}{x^5}dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{1}{2}a\int \frac{\cot^{-1}(ax)}{x^4\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{5454}$$

$$-\frac{1}{2}a\left(\int \frac{\cot^{-1}(ax)}{x^4}dx-a^2\int \frac{\cot^{-1}(ax)}{x^2\left(a^2x^2+1\right)}dx\right)-\frac{\cot^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{5362}$$

$$-\frac{1}{2}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{x^2\left(a^2x^2+1\right)}dx\right)-\frac{1}{3}a\int \frac{1}{x^3\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)}{3x^3}\right)-\frac{\cot^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{x^2\left(a^2x^2+1\right)}dx\right)-\frac{1}{6}a\int \frac{1}{x^4\left(a^2x^2+1\right)}dx^2-\frac{\cot^{-1}(ax)}{3x^3}\right)-\frac{\cot^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{54}$$

$$-\frac{1}{2}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{x^2\left(a^2x^2+1\right)}dx\right)-\frac{1}{6}a\int \left(\frac{a^4}{a^2x^2+1}-\frac{a^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{\cot^{-1}(ax)}{3x^3}\right)-\frac{\cot^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{x^2\left(a^2x^2+1\right)}dx\right)-\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\cot^{-1}(ax)}{3x^3}\right)-$$
$$\frac{\cot^{-1}(ax)^2}{4x^4}$$

$\downarrow$ 5454

$$-\frac{1}{2}a\left(-\left(a^2\left(\int \frac{\cot^{-1}(ax)}{x^2}dx-a^2\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)\right)-\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\cot^{-1}}{3x}\right.$$
$$\frac{\cot^{-1}(ax)^2}{4x^4}$$

$\downarrow$ 5362

$$-\frac{1}{2}a\left(-\left(a^2\left(-a\int \frac{1}{x\left(a^2x^2+1\right)}dx+a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)\right)-\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a\right.\right.\right.$$
$$\frac{\cot^{-1}(ax)^2}{4x^4}$$

$\downarrow$ 243

$$-\frac{1}{2}a\left(-\left(a^2\left(-\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx^2+a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)\right)-\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\,\mathrm{lo}\right.\right.$$
$$\frac{\cot^{-1}(ax)^2}{4x^4}$$

$\downarrow$ 47

$$-\frac{1}{2}a\left(-\left(a^2\left(-\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)+a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)\right)-\frac{1}{6}a\left(a^2\left(-\log\right.\right.\right.$$
$$\frac{\cot^{-1}(ax)^2}{4x^4}$$

$\downarrow$ 14

$$-\frac{1}{2}a\left(-\left(a^2\left(-\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)+a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)\right)-\frac{1}{6}a\left(a^2\left(-\log\left(\right.\right.\right.\right.$$
$$\frac{\cot^{-1}(ax)^2}{4x^4}$$

$\downarrow$ 16

$$-\frac{1}{2}a\left(-\left(a^2\left(a^2\left(-\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{1}{2}a(\log(x^2)-\log(a^2x^2+1))-\frac{\cot^{-1}(ax)}{x}\right)\right)-\frac{1}{6}a\left(a^2(-\log(x^2))+\right.$$

$$\frac{\cot^{-1}(ax)^2}{4x^4}$$

$$\Big\downarrow \text{5420}$$

$$-\frac{1}{2}a\left(-\frac{1}{6}a\left(a^2(-\log(x^2))+a^2\log(a^2x^2+1)-\frac{1}{x^2}\right)-\left(a^2\left(-\frac{1}{2}a(\log(x^2)-\log(a^2x^2+1))+\frac{1}{2}a\cot^{-1}(ax)^2\right.\right.\right.$$

$$\frac{\cot^{-1}(ax)^2}{4x^4}$$

input
```
Int[ArcCot[a*x]^2/x^5,x]
```

output
```
-1/4*ArcCot[a*x]^2/x^4 - (a*(-1/3*ArcCot[a*x]/x^3 - a^2*(-(ArcCot[a*x]/x)
+ (a*ArcCot[a*x]^2)/2 - (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) - (a*(-x^(-2)
 - a^2*Log[x^2] + a^2*Log[1 + a^2*x^2]))/6))/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
parts	$-\frac{\operatorname{arccot}(ax)^2}{4x^4}-\frac{a^4\left(\operatorname{arccot}(ax)\arctan(ax)-\frac{\operatorname{arccot}(ax)}{3a^3x^3}+\frac{\operatorname{arccot}(ax)}{xa}-\frac{2\ln\left(a^2x^2+1\right)}{3}+\frac{1}{6a^2x^2}+\frac{4\ln(ax)}{3}+\frac{\arctan(ax)^2}{2}\right)}{2}$
derivativedivides	$a^4\left(-\frac{\operatorname{arccot}(ax)^2}{4a^4x^4}-\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}+\frac{\operatorname{arccot}(ax)}{6a^3x^3}-\frac{\operatorname{arccot}(ax)}{2xa}+\frac{\ln(a^2x^2+1)}{3}-\frac{1}{12a^2x^2}-\frac{2\ln(ax}{3}\right.$
default	$a^4\left(-\frac{\operatorname{arccot}(ax)^2}{4a^4x^4}-\frac{\operatorname{arccot}(ax)\arctan(ax)}{2}+\frac{\operatorname{arccot}(ax)}{6a^3x^3}-\frac{\operatorname{arccot}(ax)}{2xa}+\frac{\ln(a^2x^2+1)}{3}-\frac{1}{12a^2x^2}-\frac{2\ln(ax}{3}\right.$
parallelrisch	$-\frac{-3a^4x^4\operatorname{arccot}(ax)^2+8a^4\ln(x)x^4-4a^4\ln(a^2x^2+1)x^4-a^4x^4+6a^3x^3\operatorname{arccot}(ax)+a^2x^2-2\operatorname{arccot}(ax)ax+3\operatorname{arccot}(}{12x^4}$
risch	$-\frac{(a^4x^4-1)\ln(iax+1)^2}{16x^4}-\frac{i(3ia^4\ln(-iax+1)x^4+6a^3x^3-3i\ln(-iax+1)-2ax+3\pi)\ln(iax+1)}{24x^4}-\frac{-6ia^4\ln((-\pi a+}{}$

input
```
int(arccot(a*x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arccot(a*x)^2/x^4-1/2*a^4*(arccot(a*x)*arctan(a*x)-1/3/a^3/x^3*arccot
(a*x)+arccot(a*x)/x/a-2/3*ln(a^2*x^2+1)+1/6/a^2/x^2+4/3*ln(a*x)+1/2*arctan
(a*x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{\cot^{-1}(ax)^2}{x^5}\,dx = \frac{4\,a^4x^4\log\left(a^2x^2+1\right)-8\,a^4x^4\log\left(x\right)-a^2x^2+3\left(a^4x^4-1\right)\text{arccot}\left(ax\right)^2-2\left(3\,a^3x^3-ax\right)\text{arccot}\left(ax\right)}{12\,x^4}$$

input
```
integrate(arccot(a*x)^2/x^5,x, algorithm="fricas")
```

output
```
1/12*(4*a^4*x^4*log(a^2*x^2 + 1) - 8*a^4*x^4*log(x) - a^2*x^2 + 3*(a^4*x^4
 - 1)*arccot(a*x)^2 - 2*(3*a^3*x^3 - a*x)*arccot(a*x))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{\cot^{-1}(ax)^2}{x^5}\,dx = -\frac{2a^4\log(x)}{3}+\frac{a^4\log\left(a^2x^2+1\right)}{3}+\frac{a^4\,\text{acot}^2\left(ax\right)}{4}-\frac{a^3\,\text{acot}\left(ax\right)}{2x}-\frac{a^2}{12x^2}+\frac{a\,\text{acot}\left(ax\right)}{6x^3}-\frac{\text{acot}^2\left(ax\right)}{4x^4}$$

input
```
integrate(acot(a*x)**2/x**5,x)
```

output
```
-2*a**4*log(x)/3 + a**4*log(a**2*x**2 + 1)/3 + a**4*acot(a*x)**2/4 - a**3*
acot(a*x)/(2*x) - a**2/(12*x**2) + a*acot(a*x)/(6*x**3) - acot(a*x)**2/(4*
x**4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \frac{\cot^{-1}(ax)^2}{x^5}\,dx = -\frac{1}{6}\left(3\,a^3 \arctan\left(ax\right) + \frac{3\,a^2x^2 - 1}{x^3}\right)a \operatorname{arccot}\left(ax\right) - \frac{\left(3\,a^2x^2 \arctan\left(ax\right)^2 - 4\,a^2x^2 \log\left(a^2x^2+1\right) + 8\,a^2x^2 \log\left(x\right) + 1\right)a^2}{12\,x^2} - \frac{\operatorname{arccot}\left(ax\right)^2}{4\,x^4}$$

input
```
integrate(arccot(a*x)^2/x^5,x, algorithm="maxima")
```

output
```
-1/6*(3*a^3*arctan(a*x) + (3*a^2*x^2 - 1)/x^3)*a*arccot(a*x) - 1/12*(3*a^2
*x^2*arctan(a*x)^2 - 4*a^2*x^2*log(a^2*x^2 + 1) + 8*a^2*x^2*log(x) + 1)*a^
2/x^2 - 1/4*arccot(a*x)^2/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \frac{\cot^{-1}(ax)^2}{x^5}\,dx = \frac{1}{12}\left(\left(\left(3\,\arctan\left(\frac{1}{ax}\right)^2 - \frac{6\,\arctan\left(\frac{1}{ax}\right)}{ax} - \frac{1}{a^2x^2} + \frac{2\,\arctan\left(\frac{1}{ax}\right)}{a^3x^3} + 4\,\log\left(\frac{1}{a^2x^2}+1\right)\right)a^3 - \frac{3\,\arctan\left(\right.}{ax^4}\right.\right.$$

input
```
integrate(arccot(a*x)^2/x^5,x, algorithm="giac")
```

output
```
1/12*((3*arctan(1/(a*x))^2 - 6*arctan(1/(a*x))/(a*x) - 1/(a^2*x^2) + 2*arc
tan(1/(a*x))/(a^3*x^3) + 4*log(1/(a^2*x^2) + 1))*a^3 - 3*arctan(1/(a*x))^2
/(a*x^4))*a
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{\cot^{-1}(ax)^2}{x^5}\,dx = \mathrm{acot}(a\,x)^2\left(\frac{a^4}{4}-\frac{1}{4\,x^4}\right)-\frac{2\,a^4\,\ln\left(x\right)}{3}+\frac{a^4\,\ln\left(a^2\,x^2+1\right)}{3}-\frac{a^2}{12\,x^2}-\frac{a^2\,\mathrm{acot}(a\,x)\,\left(\frac{a\,x^2}{2}-\frac{1}{6\,a}\right)}{x^3}$$

input
```
int(acot(a*x)^2/x^5,x)
```

output
```
acot(a*x)^2*(a^4/4 - 1/(4*x^4)) - (2*a^4*log(x))/3 + (a^4*log(a^2*x^2 + 1)
)/3 - a^2/(12*x^2) - (a^2*acot(a*x)*((a*x^2)/2 - 1/(6*a)))/x^3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(ax)^2}{x^5}\,dx = \frac{3acot(ax)^2\,a^4x^4-3acot(ax)^2-6acot(ax)\,a^3x^3+2acot(ax)\,ax+4\log(a^2x^2+1)\,a^4x^4-8\log(x)\,a^4x^4-}{12x^4}$$

input
```
int(acot(a*x)^2/x^5,x)
```

output
```
(3*acot(a*x)**2*a**4*x**4 - 3*acot(a*x)**2 - 6*acot(a*x)*a**3*x**3 + 2*aco
t(a*x)*a*x + 4*log(a**2*x**2 + 1)*a**4*x**4 - 8*log(x)*a**4*x**4 - a**2*x*
*2)/(12*x**4)
```

3.23 $\int x^5 \cot^{-1}(ax)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 194

$$\begin{aligned}\int x^5 \cot^{-1}(ax)^3 \, dx = & -\frac{19x}{60a^5} + \frac{x^3}{60a^3} - \frac{4x^2 \cot^{-1}(ax)}{15a^4} + \frac{x^4 \cot^{-1}(ax)}{20a^2} + \frac{23i \cot^{-1}(ax)^2}{30a^6} \\ & + \frac{x \cot^{-1}(ax)^2}{2a^5} - \frac{x^3 \cot^{-1}(ax)^2}{6a^3} + \frac{x^5 \cot^{-1}(ax)^2}{10a} \\ & + \frac{\cot^{-1}(ax)^3}{6a^6} + \frac{1}{6}x^6 \cot^{-1}(ax)^3 + \frac{19 \arctan(ax)}{60a^6} \\ & - \frac{23 \cot^{-1}(ax) \log\left(\frac{2}{1+iax}\right)}{15a^6} + \frac{23i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{30a^6}\end{aligned}$$

output

```
-19/60*x/a^5+1/60*x^3/a^3-4/15*x^2*arccot(a*x)/a^4+1/20*x^4*arccot(a*x)/a^
2+23/30*I*arccot(a*x)^2/a^6+1/2*x*arccot(a*x)^2/a^5-1/6*x^3*arccot(a*x)^2/
a^3+1/10*x^5*arccot(a*x)^2/a+1/6*arccot(a*x)^3/a^6+1/6*x^6*arccot(a*x)^3+1
9/60*arctan(a*x)/a^6-23/15*arccot(a*x)*ln(2/(1+I*a*x))/a^6+23/30*I*polylog
(2,1-2/(1+I*a*x))/a^6
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.64

$$\int x^5 \cot^{-1}(ax)^3\,dx$$

$$= \frac{ax(-19+a^2x^2)+2(23i+15ax-5a^3x^3+3a^5x^5)\cot^{-1}(ax)^2+10(1+a^6x^6)\cot^{-1}(ax)^3+\cot^{-1}(ax)\left(-\right.}{60a^6}$$

input
```
Integrate[x^5*ArcCot[a*x]^3,x]
```

output
```
(a*x*(-19 + a^2*x^2) + 2*(23*I + 15*a*x - 5*a^3*x^3 + 3*a^5*x^5)*ArcCot[a*
x]^2 + 10*(1 + a^6*x^6)*ArcCot[a*x]^3 + ArcCot[a*x]*(-19 - 16*a^2*x^2 + 3*
a^4*x^4 - 92*Log[1 - E^((2*I)*ArcCot[a*x])]) + (46*I)*PolyLog[2, E^((2*I)*
ArcCot[a*x])])/(60*a^6)
```

Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 447 vs. 2(194) = 388.

Time = 2.61 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.30, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.700$, Rules used = {5362, 5452, 5362, 5452, 5362, 254, 2009, 5452, 5346, 5362, 262, 216, 5420, 5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \cot^{-1}(ax)^3\,dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{2}a\int \frac{x^6\cot^{-1}(ax)^2}{a^2x^2+1}dx+\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5452}$$

$$\frac{1}{2}a\left(\frac{\int x^4\cot^{-1}(ax)^2dx}{a^2}-\frac{\int \frac{x^4\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 5362

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\int \frac{x^5\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{x^4\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 5452

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\int x^3\cot^{-1}(ax)dx}{a^2}-\frac{\int \frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\frac{\int x^2\cot^{-1}(ax)^2dx}{a^2}-\frac{\int \frac{x^2\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2}\right)+$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 5362

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\int \frac{x^4}{a^2x^2+1}dx+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\frac{\frac{2}{3}a\int \frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{3}x^3\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{x^2}{}}{}}{a^2}\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 254

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\int \left(\frac{x^2}{a^2}+\frac{1}{a^4(a^2x^2+1)}-\frac{1}{a^4}\right)dx+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\frac{\frac{2}{3}a\int \frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{3}x^3\text{co}}{a^2}}{}\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 2009

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2}{3}a\int\frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{3}x^3\cot^{-1}(ax\cdots}{a^2}\cdots\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 5452

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\int x\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2}{3}a\left(\frac{\int x\cot^{-1}(ax)dx\cdots}{a^2}\cdots\right.}{\cdots}\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 5346

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\int x\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2}{3}a\left(\frac{\int x\cot^{-1}(ax)dx\cdots}{a^2}\cdots\right.}{\cdots}\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 5362

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2}{3}a\left(\cdots\right.}{\cdots}\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 262

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int\frac{1}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 216

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 5420

$$\frac{1}{2}a\left(\frac{\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^2}{a^2}-\right.$$

$$\frac{1}{6}x^6\cot^{-1}(ax)^3$$

$\downarrow$ 5456

$$\frac{1}{6}x^6\cot^{-1}(ax)^3+\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\cot^{-1}(}{i-a}}{a}}{a^2}}{a^2}\right.}{a^2}\right.$$

$\downarrow$ 5380

$$\frac{1}{6}x^6\cot^{-1}(ax)^3+\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\log\left(\frac{}{ia}\right.}{a^2x}}{}}{}}{a^2}\right.}{a^2}\right.$$

$\downarrow$ 2849

$$\frac{1}{6}x^6\cot^{-1}(ax)^3+\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\log\left(\frac{2}{1+i}\right.}{}}{a^2}}{a^2}\right.}{a^2}\right.$$

$\downarrow$ 2752

$$\frac{1}{6}x^6\cot^{-1}(ax)^3+\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\cot^{-1}(ax)^2+\frac{2}{5}a\left(\frac{\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\log\left(\frac{2}{1+i\cdots}\right)\cdots}{\cdots}}{a^2}\cdots}{a^2}\cdots\right)}{a^2}\cdots\right)$$

input
```
Int[x^5*ArcCot[a*x]^3,x]
```

output
```
(x^6*ArcCot[a*x]^3)/6 + (a*(((x^5*ArcCot[a*x]^2)/5 + (2*a*(((x^4*ArcCot[a*
x])/4 + (a*(-(x/a^4) + x^3/(3*a^2) + ArcTan[a*x]/a^5))/4)/a^2 - (((x^2*Arc
Cot[a*x])/2 + (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/a^2 - (((I/2)*ArcCot[a*x]^2
)/a^2 - ((ArcCot[a*x]*Log[2/(1 + I*a*x)])/a - ((I/2)*PolyLog[2, 1 - 2/(1 +
 I*a*x)])/a)/a)/a^2)/a^2))/5)/a^2 - (((x^3*ArcCot[a*x]^2)/3 + (2*a*(((x^2*
ArcCot[a*x])/2 + (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/a^2 - (((I/2)*ArcCot[a*x
]^2)/a^2 - ((ArcCot[a*x]*Log[2/(1 + I*a*x)])/a - ((I/2)*PolyLog[2, 1 - 2/(
1 + I*a*x)])/a)/a)/a^2))/3)/a^2 - (ArcCot[a*x]^3/(3*a^3) + (x*ArcCot[a*x]^
2 + 2*a*(((I/2)*ArcCot[a*x]^2)/a^2 - ((ArcCot[a*x]*Log[2/(1 + I*a*x)])/a -
 ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))/a^2)/a^2)/a^2))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1103 vs. $2(164) = 328$.

Time = 8.34 (sec) , antiderivative size = 1104, normalized size of antiderivative = 5.69

method	result	size
risch	Expression too large to display	1104
parts	Expression too large to display	2453
derivativedivides	Expression too large to display	2455
default	Expression too large to display	2455

input
```
int(x^5*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-19/120/a^6*Pi+1/48/a^6*Pi^3-1/24/a^3*Pi^2*x^3+1/8/a^5*Pi^2*x-2/15/a^4*Pi*
x^2-1/16/a^6*Pi*ln(1-I*a*x)^2+37/480/a^6*Pi*ln(1-I*a*x)-1/16*Pi*ln(1-I*a*x
)^2*x^6-1/80/a*ln(1-I*a*x)^2*x^5-1/16/a^5*ln(1-I*a*x)^2*x-1/1200/a*ln(1-I*
a*x)*x^5-37/480/a^5*ln(1-I*a*x)*x+1/60*x^3/a^3+18049/28800*arctan(a*x)/a^6
+1/48*I/a^6*ln(1-I*a*x)^3-49/320*I/a^6*ln(1-I*a*x)^2+1/48*I*ln(1-I*a*x)^3*
x^6-1/96*I*ln(1-I*a*x)^2*x^6+(-1/16*I*(a^6*x^6+1)/a^6*ln(1-I*a*x)^2+1/120*
x*(15*Pi*a^5*x^5+6*a^4*x^4-10*a^2*x^2+30)/a^5*ln(1-I*a*x)-1/240*(-15*I*Pi^
2*a^6*x^6-12*I*Pi*a^5*x^5-6*I*a^4*x^4+20*I*Pi*a^3*x^3+32*I*a^2*x^2-60*I*Pi
*a*x-30*ln(1-I*a*x)*Pi-92*I*ln(1-I*a*x))/a^6)*ln(1+I*a*x)+8929/57600*I/a^6
*ln(a^2*x^2+1)+23/120*I/a^6*Pi^2+1/40/a*Pi^2*x^5+1/40/a^2*Pi*x^4-1/20*I/a*
Pi*ln(1-I*a*x)*x^5+1/12*I/a^3*Pi*ln(1-I*a*x)*x^3+1/48*x^6*Pi^3+1/288*I*ln(
1-I*a*x)*x^6-3/32*I/a^6*(1-I*a*x)^2*ln(1-I*a*x)^2+7/128*I/a^6*(1-I*a*x)^4*
ln(1-I*a*x)-1/12*I/a^6*(1-I*a*x)^3*ln(1-I*a*x)-1/96*I/a^6*(1-I*a*x)^6*ln(1
-I*a*x)^2+1/8*I/a^6*(1-I*a*x)^3*ln(1-I*a*x)^2+1/20*I/a^6*(1-I*a*x)^5*ln(1-
I*a*x)^2+3/32*I/a^6*(1-I*a*x)^2*ln(1-I*a*x)-7/64*I/a^6*(1-I*a*x)^4*ln(1-I*
a*x)^2+1/288*I/a^6*(1-I*a*x)^6*ln(1-I*a*x)-1/50*I/a^6*(1-I*a*x)^5*ln(1-I*a
*x)-1/48*I*(a^6*x^6+1)/a^6*ln(1+I*a*x)^3-1/16*I*Pi^2*ln(1-I*a*x)*x^6+1/64*
I/a^2*ln(1-I*a*x)^2*x^4-1/32*I/a^4*ln(1-I*a*x)^2*x^2-1/4*I/a^5*Pi*ln(1-I*a
*x)*x+1/48/a^3*ln(1-I*a*x)^2*x^3+7/1440/a^3*ln(1-I*a*x)*x^3-1/240*(-15*I*x
^6*ln(1-I*a*x)*a^6+15*Pi*a^6*x^6+6*a^5*x^5-10*a^3*x^3-15*I*ln(1-I*a*x)+...
```

Fricas [F]

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \int x^5 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^5*arccot(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^5*arccot(a*x)^3, x)
```

Sympy [F]

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \int x^5 \operatorname{acot}^3(ax) \, dx$$

input
```
integrate(x**5*acot(a*x)**3,x)
```

output
```
Integral(x**5*acot(a*x)**3, x)
```

Maxima [F]

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \int x^5 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^5*arccot(a*x)^3,x, algorithm="maxima")
```

output
```
1/480*(40*a^6*x^6*arctan2(1, a*x)^3 + 12*a^5*x^5*arctan2(1, a*x)^2 - 20*a^
3*x^3*arctan2(1, a*x)^2 + 20*(5760*a^7*integrate(1/480*x^7*arctan(1/(a*x))
^3/(a^7*x^2 + a^5), x) + 1440*a^6*integrate(1/480*x^6*arctan(1/(a*x))^2/(a
^7*x^2 + a^5), x) + 360*a^6*integrate(1/480*x^6*log(a^2*x^2 + 1)^2/(a^7*x^
2 + a^5), x) + 288*a^6*integrate(1/480*x^6*log(a^2*x^2 + 1)/(a^7*x^2 + a^5
), x) + 5760*a^5*integrate(1/480*x^5*arctan(1/(a*x))^3/(a^7*x^2 + a^5), x)
 + 576*a^5*integrate(1/480*x^5*arctan(1/(a*x))/(a^7*x^2 + a^5), x) - 480*a
^4*integrate(1/480*x^4*log(a^2*x^2 + 1)/(a^7*x^2 + a^5), x) - 960*a^3*inte
grate(1/480*x^3*arctan(1/(a*x))/(a^7*x^2 + a^5), x) + 1440*a^2*integrate(1
/480*x^2*log(a^2*x^2 + 1)/(a^7*x^2 + a^5), x) + 2880*a*integrate(1/480*x*a
rctan(1/(a*x))/(a^7*x^2 + a^5), x) + arctan(a*x)^3/a^6 + 3*arctan(a*x)^2*a
rctan(1/(a*x))/a^6 + 3*arctan(a*x)*arctan(1/(a*x))^2/a^6 + 360*integrate(1
/480*log(a^2*x^2 + 1)^2/(a^7*x^2 + a^5), x))*a^6 + 60*a*x*arctan2(1, a*x)^
2 + 40*arctan2(1, a*x)^3 - (3*a^5*x^5 - 5*a^3*x^3 + 15*a*x)*log(a^2*x^2 +
1)^2)/a^6
```

Giac [F]

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \int x^5 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^5*arccot(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^5*arccot(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \int x^5 \operatorname{acot}(a\,x)^3 \, dx$$

input
```
int(x^5*acot(a*x)^3,x)
```

output
```
int(x^5*acot(a*x)^3, x)
```

Reduce [F]

$$\int x^5 \cot^{-1}(ax)^3 \, dx = \frac{10acot(ax)^3 a^6x^6 + 10acot(ax)^3 + 6acot(ax)^2 a^5x^5 - 10acot(ax)^2 a^3x^3 + 30acot(ax)^2 ax + 3acot(ax) a^4}{60a^6}$$

input
```
int(x^5*acot(a*x)^3,x)
```

output
```
(10*acot(a*x)**3*a**6*x**6 + 10*acot(a*x)**3 + 6*acot(a*x)**2*a**5*x**5 -
10*acot(a*x)**2*a**3*x**3 + 30*acot(a*x)**2*a*x + 3*acot(a*x)*a**4*x**4 -
16*acot(a*x)*a**2*x**2 - 19*acot(a*x) + 92*int((acot(a*x)*x)/(a**2*x**2 +
1),x)*a**2 + a**3*x**3 - 19*a*x)/(60*a**6)
```

3.24 $\int x^4 \cot^{-1}(ax)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 205

$$\int x^4 \cot^{-1}(ax)^3 \, dx = \frac{x^2}{20a^3} - \frac{9x\cot^{-1}(ax)}{10a^4} + \frac{x^3\cot^{-1}(ax)}{10a^2} - \frac{9\cot^{-1}(ax)^2}{20a^5} - \frac{3x^2\cot^{-1}(ax)^2}{10a^3} + \frac{3x^4\cot^{-1}(ax)^2}{20a} + \frac{i\cot^{-1}(ax)^3}{5a^5} + \frac{1}{5}x^5\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{5a^5} - \frac{\log(1+a^2x^2)}{2a^5} + \frac{3i\cot^{-1}(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{5a^5} - \frac{3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{10a^5}$$

output

```
1/20*x^2/a^3-9/10*x*arccot(a*x)/a^4+1/10*x^3*arccot(a*x)/a^2-9/20*arccot(a
*x)^2/a^5-3/10*x^2*arccot(a*x)^2/a^3+3/20*x^4*arccot(a*x)^2/a+1/5*I*arccot
(a*x)^3/a^5+1/5*x^5*arccot(a*x)^3-3/5*arccot(a*x)^2*ln(2/(1+I*a*x))/a^5-1/
2*ln(a^2*x^2+1)/a^5+3/5*I*arccot(a*x)*polylog(2,1-2/(1+I*a*x))/a^5-3/10*po
lylog(3,1-2/(1+I*a*x))/a^5
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.90

$$\int x^4 \cot^{-1}(ax)^3 \, dx$$

$$= \frac{2 + i\pi^3 + 2a^2x^2 - 36ax \cot^{-1}(ax) + 4a^3x^3 \cot^{-1}(ax) - 18 \cot^{-1}(ax)^2 - 12a^2x^2 \cot^{-1}(ax)^2 + 6a^4x^4 \cot^{-}}{}$$

input
```
Integrate[x^4*ArcCot[a*x]^3,x]
```

output
```
(2 + I*Pi^3 + 2*a^2*x^2 - 36*a*x*ArcCot[a*x] + 4*a^3*x^3*ArcCot[a*x] - 18*
ArcCot[a*x]^2 - 12*a^2*x^2*ArcCot[a*x]^2 + 6*a^4*x^4*ArcCot[a*x]^2 - (8*I)
*ArcCot[a*x]^3 + 8*a^5*x^5*ArcCot[a*x]^3 - 24*ArcCot[a*x]^2*Log[1 - E^((-2
*I)*ArcCot[a*x])] + 40*Log[1/(a*Sqrt[1 + 1/(a^2*x^2)]*x)] - (24*I)*ArcCot[
a*x]*PolyLog[2, E^((-2*I)*ArcCot[a*x])] - 12*PolyLog[3, E^((-2*I)*ArcCot[a
*x])])/(40*a^5)
```

Rubi [A] (verified)

Time = 2.27 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.50, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.600$, Rules used = {5362, 5452, 5362, 5452, 5362, 243, 49, 2009, 5452, 5346, 240, 5420, 5456, 5380, 5530, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \cot^{-1}(ax)^3 \, dx$$

$$\Big\downarrow \; 5362$$

$$\frac{3}{5}a \int \frac{x^5 \cot^{-1}(ax)^2}{a^2x^2+1} dx + \frac{1}{5}x^5 \cot^{-1}(ax)^3$$

$$\Big\downarrow \; 5452$$

$$\frac{3}{5}a\left(\frac{\int x^3\cot^{-1}(ax)^2dx}{a^2}-\frac{\int \frac{x^3\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5362}$$

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\int\frac{x^4\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{\int\frac{x^3\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5452}$$

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\int x^2\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{\frac{\int x\cot^{-1}(ax)^2dx}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2}\right)+$$
$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5362}$$

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{3}a\int\frac{x^3}{a^2x^2+1}dx+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{\frac{a\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\int \cdots}{a^2}\right.$$
$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{243}$$

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\int\frac{x^2}{a^2x^2+1}dx^2+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{\frac{a\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\int \cdots}{a^2}\right.$$
$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{49}$$

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\int\left(\frac{1}{a^2}-\frac{1}{a^2\left(a^2x^2+1\right)}\right)dx^2+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{\frac{a\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(a\cdots}{a^2}}{a^2}\cdots\right.$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$\downarrow$ 2009

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{\frac{a\int\frac{x^2\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}}{a^2}\cdots\right.$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$\downarrow$ 5452

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\int\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{\int\cot^{-1}(ax)dx}{a^2}-\frac{\int\frac{\cot\cdots}{a^2\cdots}}{\cdots}\right.}{\cdots}\cdots\right.$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$\downarrow$ 5346

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{a\int\frac{x}{a^2x^2+1}dx+x\cot^{-1}(ax)}{a^2}-\frac{\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{a\int\frac{x}{a^2x^2-}\right.}{}\right.$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$\big\downarrow$ 240

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}-\frac{\int\frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{\frac{\log\left(a^2x^2-\right.}{2a}}{}\right.}{}\right.$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$\big\downarrow$ 5420

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\right.}{}\right.$$

$$\frac{1}{5}x^5\cot^{-1}(ax)^3$$

$\big\downarrow$ 5456

$$\frac{1}{5}x^5\cot^{-1}(ax)^3+$$

$$\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\right.}{}\right.$$

$\Big\downarrow$ 5380

$$\frac{1}{5}x^5\cot^{-1}(ax)^3+\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\right.}{}\right.$$

$\Big\downarrow$ 5530

$$\frac{1}{5}x^5\cot^{-1}(ax)^3+\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\right.}{}\right.$$

$\Big\downarrow$ 7164

$$\frac{1}{5}x^5\cot^{-1}(ax)^3+\frac{3}{5}a\left(\frac{\frac{1}{2}a\left(\frac{\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{3}x^3\cot^{-1}(ax)}{a^2}-\frac{\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^2}{a^2}-\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\right.}{}\right.$$

input
```
Int[x^4*ArcCot[a*x]^3,x]
```

output
```
(x^5*ArcCot[a*x]^3)/5 + (3*a*(((x^4*ArcCot[a*x]^2)/4 + (a*(((x^3*ArcCot[a*
x])/3 + (a*(x^2/a^2 - Log[1 + a^2*x^2]/a^4))/6)/a^2 - (ArcCot[a*x]^2/(2*a^
3) + (x*ArcCot[a*x] + Log[1 + a^2*x^2]/(2*a))/a^2)/a^2))/2)/a^2 - (((x^2*A
rcCot[a*x]^2)/2 + a*(ArcCot[a*x]^2/(2*a^3) + (x*ArcCot[a*x] + Log[1 + a^2*
x^2]/(2*a))/a^2))/a^2 - (((I/3)*ArcCot[a*x]^3)/a^2 - ((ArcCot[a*x]^2*Log[2
/(1 + I*a*x)])/a + 2*(((-1/2*I)*ArcCot[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])
/a + PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a)))/a)/a^2)/a^2))/5
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5530
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.35 (sec) , antiderivative size = 1910, normalized size of antiderivative = 9.32

method	result	size
derivativedivides	Expression too large to display	1910
default	Expression too large to display	1910
parts	Expression too large to display	1912

input

```
int(x^4*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

output

```
1/a^5*(3/20*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2
*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)*arccot(
a*x)^2+3/40*(-I*a*x*(a^2*x^2+1)^(1/2)-I*a^2*x^2+2*(a^2*x^2+1)^(1/2)*a^2*x^
2+2*a^3*x^3-I+(a^2*x^2+1)^(1/2)+2*a*x)*arccot(a*x)-3/40*(-I*a*x*(a^2*x^2+1
)^(1/2)+I*a^2*x^2-2*(a^2*x^2+1)^(1/2)*a^2*x^2+2*a^3*x^3+I-(a^2*x^2+1)^(1/2
)+2*a*x)*arccot(a*x)-3/40*((a^2*x^2+1)^(1/2)-a*x-I*a*x*(a^2*x^2+1)^(1/2)+I
*a^2*x^2)*arccot(a*x)-21/40*arccot(a*x)*(a*x+I+(a^2*x^2+1)^(1/2))-3/40*(-(
a^2*x^2+1)^(1/2)-a*x+I*a*x*(a^2*x^2+1)^(1/2)+I*a^2*x^2)*arccot(a*x)-3/5*ar
ccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))-3/5*ln(2)*arccot(a*x)^2-3/10*a^2
*x^2*arccot(a*x)^2+3/20*a^4*x^4*arccot(a*x)^2+ln(1+(I+a*x)/(a^2*x^2+1)^(1/
2))+1/5*x^5*arccot(a*x)^3*a^5+3/20*I*(a*x*(a^2*x^2+1)^(1/2)+a^2*x^2+1)*arc
cot(a*x)-3/20*I*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^3*arccot(a*x)^2+3/2
0*I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^3*arccot(a*x)^2+3/10*arccot(a*x)^2*ln
(a^2*x^2+1)-1/40*(a^2*x^2+1)/(I*(a^2*x^2+1)^(1/2)+a*x*(a^2*x^2+1)^(1/2)+I*
a*x+a^2*x^2)+1/40*(-3*I*a*x*(a^2*x^2+1)^(1/2)+3*I*a^2*x^2-2*(a^2*x^2+1)^(1
/2)*a^2*x^2+2*a^3*x^3+I+(a^2*x^2+1)^(1/2))*arccot(a*x)+1/20/(a*x+I+(a^2*x^
2+1)^(1/2))*(a^2*x^2+1)^(1/2)-6/5*polylog(3,(I+a*x)/(a^2*x^2+1)^(1/2))-6/5
*polylog(3,-(I+a*x)/(a^2*x^2+1)^(1/2))+ln((I+a*x)/(a^2*x^2+1)^(1/2)-1)-3/2
0*I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^3*arccot(
a*x)^2+3/10*I*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*((I+a*x)^2/(a...
```

Fricas [F]

$$\int x^4 \cot^{-1}(ax)^3 \, dx = \int x^4 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^4*arccot(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^4*arccot(a*x)^3, x)
```

Sympy [F]

$$\int x^4 \cot^{-1}(ax)^3 \, dx = \int x^4 \operatorname{acot}^3(ax) \, dx$$

input
```
integrate(x**4*acot(a*x)**3,x)
```

output
```
Integral(x**4*acot(a*x)**3, x)
```

Maxima [F]

$$\int x^4 \cot^{-1}(ax)^3 \, dx = \int x^4 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^4*arccot(a*x)^3,x, algorithm="maxima")
```

output
```
1/40*x^5*arctan2(1, a*x)^3 - 3/160*x^5*arctan2(1, a*x)*log(a^2*x^2 + 1)^2
+ integrate(1/160*(140*a^2*x^6*arctan2(1, a*x)^3 + 12*a^2*x^6*arctan2(1, a
*x)*log(a^2*x^2 + 1) + 12*a*x^5*arctan2(1, a*x)^2 + 140*x^4*arctan2(1, a*x
)^3 + 3*(5*a^2*x^6*arctan2(1, a*x) - a*x^5 + 5*x^4*arctan2(1, a*x))*log(a^
2*x^2 + 1)^2)/(a^2*x^2 + 1), x)
```

Giac [F]

$$\int x^4 \cot^{-1}(ax)^3 \, dx = \int x^4 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^4*arccot(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^4*arccot(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \cot^{-1}(ax)^3 \, dx = \int x^4 \operatorname{acot}(a\,x)^3 \, dx$$

input
```
int(x^4*acot(a*x)^3,x)
```

output
```
int(x^4*acot(a*x)^3, x)
```

Reduce [F]

$$\int x^4 \cot^{-1}(ax)^3 \, dx$$
$$= \frac{4acot(ax)^3 a^5x^5 - 4acot(ax)^3 ax + 3acot(ax)^2 a^4x^4 - 6acot(ax)^2 a^2x^2 - 9acot(ax)^2 + 2acot(ax)\, a^3x^3 -}{20a^5}$$

input
```
int(x^4*acot(a*x)^3,x)
```

output
```
(4*acot(a*x)**3*a**5*x**5 - 4*acot(a*x)**3*a*x + 3*acot(a*x)**2*a**4*x**4
- 6*acot(a*x)**2*a**2*x**2 - 9*acot(a*x)**2 + 2*acot(a*x)*a**3*x**3 - 18*a
cot(a*x)*a*x + 4*int(acot(a*x)**3,x)*a - 10*log(a**2*x**2 + 1) + a**2*x**2
)/(20*a**5)
```

3.25 $\int x^3 \cot^{-1}(ax)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 148

$$\int x^3 \cot^{-1}(ax)^3 \, dx = \frac{x}{4a^3} + \frac{x^2 \cot^{-1}(ax)}{4a^2} - \frac{i \cot^{-1}(ax)^2}{a^4} - \frac{3x \cot^{-1}(ax)^2}{4a^3} + \frac{x^3 \cot^{-1}(ax)^2}{4a} - \frac{\cot^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4 \cot^{-1}(ax)^3 - \frac{\arctan(ax)}{4a^4} + \frac{2\cot^{-1}(ax) \log\left(\frac{2}{1+iax}\right)}{a^4} - \frac{i \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{a^4}$$

output
```
1/4*x/a^3+1/4*x^2*arccot(a*x)/a^2-I*arccot(a*x)^2/a^4-3/4*x*arccot(a*x)^2/
a^3+1/4*x^3*arccot(a*x)^2/a-1/4*arccot(a*x)^3/a^4+1/4*x^4*arccot(a*x)^3-1/
4*arctan(a*x)/a^4+2*arccot(a*x)*ln(2/(1+I*a*x))/a^4-I*polylog(2,1-2/(1+I*a
*x))/a^4
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.65

$$\int x^3 \cot^{-1}(ax)^3 \, dx = \frac{ax + (-4i - 3ax + a^3x^3)\cot^{-1}(ax)^2 + (-1 + a^4x^4)\cot^{-1}(ax)^3 + \cot^{-1}(ax)\left(1 + a^2x^2 + 8\log\left(1 - e^{2i\cot^{-1}(ax)}\right)\right) \ldots}{4a^4}$$

input
```
Integrate[x^3*ArcCot[a*x]^3,x]
```

output
```
(a*x + (-4*I - 3*a*x + a^3*x^3)*ArcCot[a*x]^2 + (-1 + a^4*x^4)*ArcCot[a*x]
^3 + ArcCot[a*x]*(1 + a^2*x^2 + 8*Log[1 - E^((2*I)*ArcCot[a*x])]) - (4*I)*
PolyLog[2, E^((2*I)*ArcCot[a*x])])/(4*a^4)
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.73, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.300$, Rules used = {5362, 5452, 5362, 5452, 5346, 5362, 262, 216, 5420, 5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cot^{-1}(ax)^3 \, dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{3}{4}a\int \frac{x^4 \cot^{-1}(ax)^2}{a^2x^2+1}dx+\frac{1}{4}x^4\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5452}$$

$$\frac{3}{4}a\left(\frac{\int x^2\cot^{-1}(ax)^2dx}{a^2}-\frac{\int \frac{x^2\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5362}$$

$$\frac{3}{4}a\left(\frac{\frac{2}{3}a\int \frac{x^3\cot^{-1}(ax)}{a^2x^2+1}dx+\frac{1}{3}x^3\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{x^2\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5452}$$

$$\frac{3}{4}a\left(\frac{\frac{2}{3}a\left(\frac{\int x\cot^{-1}(ax)dx}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^2}{a^2}-\frac{\frac{\int \cot^{-1}(ax)^2dx}{a^2}-\frac{\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}(ax)^3$$

$\downarrow$ 5346

$$\frac{3}{4}a\left(\frac{\frac{2}{3}a\left(\frac{\int x\cot^{-1}(ax)dx}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2a\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2}\right)+$$

$$\frac{1}{4}x^4\cot^{-1}(ax)^3$$

$\downarrow$ 5362

$$\frac{3}{4}a\left(\frac{\frac{2}{3}a\left(\frac{\frac{1}{2}a\int \frac{x^2}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2a\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{\cot^{-1}(a}{a^2x^2+}}{a^2}}{a^2}\right.$$

$$\frac{1}{4}x^4\cot^{-1}(ax)^3$$

$\downarrow$ 262

$$\frac{3}{4}a\left(\frac{\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int \frac{1}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2a\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2}{a^2}-\int}{a^2}\right.$$

$$\frac{1}{4}x^4\cot^{-1}(ax)^3$$

$\downarrow$ 216

$$\frac{3}{4}a\left(\frac{\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2a\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2}{a^2}-\int \frac{\cot}{a}}{a^2}\right.$$

$$\frac{1}{4}x^4\cot^{-1}(ax)^3$$

$\downarrow$ 5420

$$\frac{3}{4}a\left(\frac{\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^2}{a^2}-\frac{\frac{2a\int\frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2}{a^2}+\frac{\cot^{-}}{3}}{a^2}\right.$$

$$\frac{1}{4}x^4\cot^{-1}(ax)^3$$

$\big\downarrow$ 5456

$$\frac{1}{4}x^4\cot^{-1}(ax)^3+$$

$$\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\cot^{-1}(ax)}{i-ax}dx}{a}}{a^2}\right)}{a^2}-\frac{\cot^{-1}(ax)^3}{3a^3}+\frac{x\cot^{-1}(ax)^2}{}\right.$$

$\big\downarrow$ 5380

$$\frac{1}{4}x^4\cot^{-1}(ax)^3+$$

$$\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx+\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}}{a}}{a^2}\right)}{a^2}-\frac{\cot^{-}}{3}\right.$$

$\big\downarrow$ 2849

$$\frac{1}{4}x^4\cot^{-1}(ax)^3+$$

$$\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}}{a}}{a^2}\right)}{a^2}-\right.$$

$\big\downarrow$ 2752

$$\frac{1}{4}x^4\cot^{-1}(ax)^3+$$
$$\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\cot^{-1}(ax)^2+\frac{2}{3}a\left(\frac{\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)}{a^2}-\frac{\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}\right)}{a^2}\right. -$$

input
```
Int[x^3*ArcCot[a*x]^3,x]
```

output
```
(x^4*ArcCot[a*x]^3)/4 + (3*a*(((x^3*ArcCot[a*x]^2)/3 + (2*a*(((x^2*ArcCot[
a*x])/2 + (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/a^2 - (((I/2)*ArcCot[a*x]^2)/a^
2 - ((ArcCot[a*x]*Log[2/(1 + I*a*x)])/a - ((I/2)*PolyLog[2, 1 - 2/(1 + I*a
*x)])/a)/a)/a^2))/3)/a^2 - (ArcCot[a*x]^3/(3*a^3) + (x*ArcCot[a*x]^2 + 2*a
*(((I/2)*ArcCot[a*x]^2)/a^2 - ((ArcCot[a*x]*Log[2/(1 + I*a*x)])/a - ((I/2)
*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))/a^2)/a^2))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 856 vs. $2(130) = 260$.

Time = 6.48 (sec) , antiderivative size = 857, normalized size of antiderivative = 5.79

method	result
risch	$-\frac{i\ln(-iax+1)^3}{32a^4}+\frac{25i\ln(-iax+1)^2}{128a^4}-\frac{3i\ln(-iax+1)^2x^4}{128}+\frac{3i\ln(-iax+1)x^4}{256}-\frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{a^4}+\frac{23i\ln(-i}{48a}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x^3*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/32*I*(a^4*x^4-1)/a^4*ln(1-I*a*x)^2+1/16*x*(3*Pi*a^3*x^3+2*a^2*x^2-6)/a
^3*ln(1-I*a*x)-1/32*(-3*I*Pi^2*a^4*x^4-4*I*Pi*a^3*x^3-4*I*x^2*a^2+12*I*Pi*
a*x+6*ln(1-I*a*x)*Pi+16*I*ln(1-I*a*x))/a^4)*ln(1+I*a*x)-1/32/a^4*Pi^3+1/8/
a^4*Pi+1/4*I/a^4+I/a^4*ln(1/2+1/2*I*a*x)*ln(1-I*a*x)+1/4*x/a^3-511/768*arc
tan(a*x)/a^4+1/24*I/a^4*(1-I*a*x)^3*ln(1-I*a*x)-1/16*I/a^4*(1-I*a*x)^3*ln(
1-I*a*x)^2-3/256*I/a^4*(1-I*a*x)^4*ln(1-I*a*x)+3/128*I/a^4*(1-I*a*x)^4*ln(
1-I*a*x)^2+1/32*I*ln(1-I*a*x)^3*x^4-3/32*I/a^4*(1-I*a*x)^2*ln(1-I*a*x)+3/3
2*I/a^4*(1-I*a*x)^2*ln(1-I*a*x)^2-1/32*I/a^4*ln(1-I*a*x)^3+25/128*I/a^4*ln
(1-I*a*x)^2-3/128*I*ln(1-I*a*x)^2*x^4+3/256*I*ln(1-I*a*x)*x^4+1/16/a*Pi^2*
x^3+1/8/a^2*Pi*x^2-21/128*I/a^2*ln(1-I*a*x)*x^2-7/64*I/a^4*Pi*arctan(a*x)+
3/16/a^4*Pi^2*arctan(a*x)-57/128/a^4*Pi*ln(a^2*x^2+1)-I/a^4*dilog(1/2-1/2*
I*a*x)+23/48*I/a^4*ln(1-I*a*x)-1/32*I*(a^4*x^4-1)/a^4*ln(1+I*a*x)^3-3/32*I
*Pi^2*ln(1-I*a*x)*x^4+3/64*I/a^2*ln(1-I*a*x)^2*x^2+3/8*I/a^3*Pi*ln(1-I*a*x
)*x-1/8*I/a*Pi*ln(1-I*a*x)*x^3+3/32/a^3*ln(1-I*a*x)^2*x+7/64/a^3*ln(1-I*a*
x)*x+1/32*x^4*Pi^3-3/16/a^3*Pi^2*x+3/32/a^4*Pi*ln(1-I*a*x)^2-7/64/a^4*Pi*l
n(1-I*a*x)-3/32*Pi*ln(1-I*a*x)^2*x^4-1/32*(-3*I*x^4*ln(1-I*a*x)*a^4+3*Pi*a
^4*x^4+2*a^3*x^3+3*I*ln(1-I*a*x)-6*a*x-3*Pi+8*I)/a^4*ln(1+I*a*x)^2-1/32/a*
ln(1-I*a*x)^2*x^3-1/192/a*ln(1-I*a*x)*x^3-1/4*I/a^4*Pi^2-319/1536*I/a^4*ln
(a^2*x^2+1)-I/a^4*ln(1/2-1/2*I*a*x)*ln(1/2+1/2*I*a*x)
```

Fricas [F]

$$\int x^3 \cot^{-1}(ax)^3\,dx = \int x^3 \operatorname{arccot}(ax)^3\,dx$$

input
```
integrate(x^3*arccot(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^3*arccot(a*x)^3, x)
```

Sympy [F]

$$\int x^3 \cot^{-1}(ax)^3\,dx = \int x^3 \operatorname{acot}^3(ax)\,dx$$

input
```
integrate(x**3*acot(a*x)**3,x)
```

output
```
Integral(x**3*acot(a*x)**3, x)
```

Maxima [F]

$$\int x^3 \cot^{-1}(ax)^3\,dx = \int x^3 \operatorname{arccot}(ax)^3\,dx$$

input
```
integrate(x^3*arccot(a*x)^3,x, algorithm="maxima")
```

output
```
1/64*(8*a^4*x^4*arctan2(1, a*x)^3 + 4*a^3*x^3*arctan2(1, a*x)^2 + 4*(512*a
^5*integrate(1/64*x^5*arctan(1/(a*x))^3/(a^5*x^2 + a^3), x) + 192*a^4*inte
grate(1/64*x^4*arctan(1/(a*x))^2/(a^5*x^2 + a^3), x) + 48*a^4*integrate(1/
64*x^4*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) + 64*a^4*integrate(1/64*x^4*
log(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 512*a^3*integrate(1/64*x^3*arctan(1
/(a*x))^3/(a^5*x^2 + a^3), x) + 128*a^3*integrate(1/64*x^3*arctan(1/(a*x))
/(a^5*x^2 + a^3), x) - 192*a^2*integrate(1/64*x^2*log(a^2*x^2 + 1)/(a^5*x^
2 + a^3), x) - 384*a*integrate(1/64*x*arctan(1/(a*x))/(a^5*x^2 + a^3), x)
- arctan(a*x)^3/a^4 - 3*arctan(a*x)^2*arctan(1/(a*x))/a^4 - 3*arctan(a*x)*
arctan(1/(a*x))^2/a^4 - 48*integrate(1/64*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^
3), x))*a^4 - 12*a*x*arctan2(1, a*x)^2 - 8*arctan2(1, a*x)^3 - (a^3*x^3 -
3*a*x)*log(a^2*x^2 + 1)^2)/a^4
```

Giac [F]

$$\int x^3 \cot^{-1}(ax)^3 \, dx = \int x^3 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^3*arccot(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^3*arccot(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cot^{-1}(ax)^3 \, dx = \int x^3 \operatorname{acot}(a\,x)^3 \, dx$$

input
```
int(x^3*acot(a*x)^3,x)
```

output
```
int(x^3*acot(a*x)^3, x)
```

Reduce [F]

$$\int x^3 \cot^{-1}(ax)^3 \, dx$$
$$= \frac{acot(ax)^3 a^4x^4 - acot(ax)^3 + acot(ax)^2 a^3x^3 - 3acot(ax)^2 ax + acot(ax) a^2x^2 + acot(ax) - 8\left(\int \frac{acot(ax)}{a^2x^2+1}\right.}{4a^4}$$

input
```
int(x^3*acot(a*x)^3,x)
```

output
```
(acot(a*x)**3*a**4*x**4 - acot(a*x)**3 + acot(a*x)**2*a**3*x**3 - 3*acot(a
*x)**2*a*x + acot(a*x)*a**2*x**2 + acot(a*x) - 8*int((acot(a*x)*x)/(a**2*x
**2 + 1),x)*a**2 + a*x)/(4*a**4)
```

3.26 $\int x^2 \cot^{-1}(ax)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 157

$$\int x^2 \cot^{-1}(ax)^3 \, dx = \frac{x \cot^{-1}(ax)}{a^2} + \frac{\cot^{-1}(ax)^2}{2a^3} + \frac{x^2 \cot^{-1}(ax)^2}{2a} - \frac{i \cot^{-1}(ax)^3}{3a^3} + \frac{1}{3} x^3 \cot^{-1}(ax)^3 + \frac{\cot^{-1}(ax)^2 \log\left(\frac{2}{1+iax}\right)}{a^3} + \frac{\log(1+a^2x^2)}{2a^3} - \frac{i \cot^{-1}(ax) \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{a^3} + \frac{\operatorname{PolyLog}\left(3, 1-\frac{2}{1+iax}\right)}{2a^3}$$

output
```
x*arccot(a*x)/a^2+1/2*arccot(a*x)^2/a^3+1/2*x^2*arccot(a*x)^2/a-1/3*I*arcc
ot(a*x)^3/a^3+1/3*x^3*arccot(a*x)^3+arccot(a*x)^2*ln(2/(1+I*a*x))/a^3+1/2*
ln(a^2*x^2+1)/a^3-I*arccot(a*x)*polylog(2,1-2/(1+I*a*x))/a^3+1/2*polylog(3
,1-2/(1+I*a*x))/a^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.95

$$\int x^2 \cot^{-1}(ax)^3 \, dx$$

$$= \frac{-i\pi^3 + 24ax \cot^{-1}(ax) + 12 \cot^{-1}(ax)^2 + 12a^2x^2 \cot^{-1}(ax)^2 + 8i \cot^{-1}(ax)^3 + 8a^3x^3 \cot^{-1}(ax)^3 + 24 \mathrm{co}}{}$$

input
```
Integrate[x^2*ArcCot[a*x]^3,x]
```

output
```
((-I)*Pi^3 + 24*a*x*ArcCot[a*x] + 12*ArcCot[a*x]^2 + 12*a^2*x^2*ArcCot[a*x
]^2 + (8*I)*ArcCot[a*x]^3 + 8*a^3*x^3*ArcCot[a*x]^3 + 24*ArcCot[a*x]^2*Log
[1 - E^((-2*I)*ArcCot[a*x])] - 24*Log[1/(a*Sqrt[1 + 1/(a^2*x^2)]*x)] + (24
*I)*ArcCot[a*x]*PolyLog[2, E^((-2*I)*ArcCot[a*x])] + 12*PolyLog[3, E^((-2*
I)*ArcCot[a*x])])/(24*a^3)
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {5362, 5452, 5362, 5452, 5346, 240, 5420, 5456, 5380, 5530, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}(ax)^3 \, dx$$

$$\Big\downarrow \text{ 5362}$$

$$a \int \frac{x^3 \cot^{-1}(ax)^2}{a^2x^2+1}dx + \frac{1}{3}x^3 \cot^{-1}(ax)^3$$

$$\Big\downarrow \text{ 5452}$$

$$a \left(\frac{\int x \cot^{-1}(ax)^2dx}{a^2} - \frac{\int \frac{x \cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right) + \frac{1}{3}x^3 \cot^{-1}(ax)^3$$

$$\Big\downarrow \text{ 5362}$$

$$a \left(\frac{a \int \frac{x^2 \cot^{-1}(ax)}{a^2x^2+1}dx + \frac{1}{2}x^2 \cot^{-1}(ax)^2}{a^2} - \frac{\int \frac{x \cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right) + \frac{1}{3}x^3 \cot^{-1}(ax)^3$$

$$\Big\downarrow \text{ 5452}$$

$$a\left(\frac{a\left(\frac{\int \cot^{-1}(ax)dx}{a^2}-\frac{\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^3$$

$\Big\downarrow$ 5346

$$a\left(\frac{a\left(\frac{a\int \frac{x}{a^2x^2+1}dx+x\cot^{-1}(ax)}{a^2}-\frac{\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^3$$

$\Big\downarrow$ 240

$$a\left(\frac{a\left(\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}-\frac{\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^3$$

$\Big\downarrow$ 5420

$$a\left(\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{x\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}(ax)^3$$

$\Big\downarrow$ 5456

$$\frac{1}{3}x^3\cot^{-1}(ax)^3+a\left(\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\frac{\frac{i\cot^{-1}(ax)^3}{3a^2}-\frac{\int \frac{\cot^{-1}(ax)^2}{i-ax}dx}{a}}{a^2}\right)$$

$\Big\downarrow$ 5380

$$\frac{1}{3}x^3\cot^{-1}(ax)^3+ a\left(\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\frac{\frac{i\cot^{-1}(ax)^3}{3a^2}-\frac{2\int\frac{\cot^{-1}(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx+\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-}}{a}}{a}}{a^2}\right.$$

$$\Big\downarrow \text{5530}$$

$$\frac{1}{3}x^3\cot^{-1}(ax)^3+ a\left(\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\frac{\frac{i\cot^{-1}(ax)^3}{3a^2}-\frac{2\left(-\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2\right.}{}\right.}{}}{a^2}\right.$$

$$\Big\downarrow \text{7164}$$

$$\frac{1}{3}x^3\cot^{-1}(ax)^3+ a\left(\frac{a\left(\frac{\cot^{-1}(ax)^2}{2a^3}+\frac{\frac{\log\left(a^2x^2+1\right)}{2a}+x\cot^{-1}(ax)}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^2}{a^2}-\frac{\frac{i\cot^{-1}(ax)^3}{3a^2}-\frac{2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{4a}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right.}{2a}\right.}{a}}{a^2}\right.$$

input

```
Int[x^2*ArcCot[a*x]^3,x]
```

output

```
(x^3*ArcCot[a*x]^3)/3 + a*(((x^2*ArcCot[a*x]^2)/2 + a*(ArcCot[a*x]^2/(2*a^
3) + (x*ArcCot[a*x] + Log[1 + a^2*x^2]/(2*a))/a^2))/a^2 - (((I/3)*ArcCot[a
*x]^3)/a^2 - ((ArcCot[a*x]^2*Log[2/(1 + I*a*x)])/a + 2*(((-1/2*I)*ArcCot[a
*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a)
))/a)/a^2)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5530
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.87 (sec) , antiderivative size = 1108, normalized size of antiderivative = 7.06

method	result	size
parts	Expression too large to display	1108
derivativedivides	Expression too large to display	1110
default	Expression too large to display	1110

input
```
int(x^2*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccot(a*x)^3+1/a^3*(-1/4*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)
*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^
2*x^2+1)-1)^2)*arccot(a*x)^2+1/2*arccot(a*x)*(a*x+I+(a^2*x^2+1)^(1/2))+arc
cot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))+ln(2)*arccot(a*x)^2+1/2*a^2*x^2*a
rccot(a*x)^2-ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))-1/2*I*Pi*csgn(I*(I+a*x)^2/(a^
2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*arccot(a*x)^2+1/4*I*Pi*csgn(I*((I+
a*x)^2/(a^2*x^2+1)-1)^2)^3*arccot(a*x)^2-1/4*I*Pi*csgn(I*(I+a*x)^2/(a^2*x^
2+1))^3*arccot(a*x)^2+1/4*I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^
2*x^2+1)-1)^2)^3*arccot(a*x)^2-1/3*I*arccot(a*x)^3-1/2*arccot(a*x)^2*ln(a^
2*x^2+1)+2*polylog(3,(I+a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-(I+a*x)/(a^2*
x^2+1)^(1/2))-ln((I+a*x)/(a^2*x^2+1)^(1/2)-1)-1/4*I*Pi*csgn(I*(I+a*x)/(a^2
*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*arccot(a*x)^2+1/2*arccot(a*
x)*(a*x+I-(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*cs
gn(I*(I+a*x)^2/(a^2*x^2+1))^2*arccot(a*x)^2+1/4*I*Pi*csgn(I/((I+a*x)^2/(a^
2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2
*arccot(a*x)^2-1/2*I*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*((I+a*x)^
2/(a^2*x^2+1)-1)^2)^2*arccot(a*x)^2+1/4*I*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)
-1))^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)*arccot(a*x)^2+1/4*I*Pi*csgn(I*(
I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-
1)^2)^2*arccot(a*x)^2+1/2*arccot(a*x)^2-arccot(a*x)^2*ln((I+a*x)^2/(a^2...
```

Fricas [F]

$$\int x^2 \cot^{-1}(ax)^3 \, dx = \int x^2 \operatorname{arccot}\left(ax\right)^3 \, dx$$

input
```
integrate(x^2*arccot(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^2*arccot(a*x)^3, x)
```

Sympy [F]

$$\int x^2 \cot^{-1}(ax)^3 \, dx = \int x^2 \operatorname{acot}^3(ax) \, dx$$

input
```
integrate(x**2*acot(a*x)**3,x)
```

output
```
Integral(x**2*acot(a*x)**3, x)
```

Maxima [F]

$$\int x^2 \cot^{-1}(ax)^3 \, dx = \int x^2 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^2*arccot(a*x)^3,x, algorithm="maxima")
```

output
```
1/24*x^3*arctan2(1, a*x)^3 - 1/32*x^3*arctan2(1, a*x)*log(a^2*x^2 + 1)^2 +
 integrate(1/32*(28*a^2*x^4*arctan2(1, a*x)^3 + 4*a^2*x^4*arctan2(1, a*x)*
log(a^2*x^2 + 1) + 4*a*x^3*arctan2(1, a*x)^2 + 28*x^2*arctan2(1, a*x)^3 +
(3*a^2*x^4*arctan2(1, a*x) - a*x^3 + 3*x^2*arctan2(1, a*x))*log(a^2*x^2 +
1)^2)/(a^2*x^2 + 1), x)
```

Giac [F]

$$\int x^2 \cot^{-1}(ax)^3 \, dx = \int x^2 \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^2*arccot(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^2*arccot(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^{-1}(ax)^3 \, dx = \int x^2 \operatorname{acot}(a\,x)^3 \, dx$$

input
```
int(x^2*acot(a*x)^3,x)
```

output
```
int(x^2*acot(a*x)^3, x)
```

Reduce [F]

$$\int x^2 \cot^{-1}(ax)^3 \, dx$$
$$= \frac{2acot(ax)^3 a^3x^3 + 2acot(ax)^3 ax + 3acot(ax)^2 a^2x^2 + 3acot(ax)^2 + 6acot(ax)\,ax - 2\left(\int acot(ax)^3 \, dx\right) a}{6a^3}$$

input
```
int(x^2*acot(a*x)^3,x)
```

output
```
(2*acot(a*x)**3*a**3*x**3 + 2*acot(a*x)**3*a*x + 3*acot(a*x)**2*a**2*x**2
+ 3*acot(a*x)**2 + 6*acot(a*x)*a*x - 2*int(acot(a*x)**3,x)*a + 3*log(a**2*
x**2 + 1))/(6*a**3)
```

3.27 $\int x\cot^{-1}(ax)^3\,dx$

Optimal result

Integrand size = 8, antiderivative size = 103

$$\int x\cot^{-1}(ax)^3\,dx=\frac{3i\cot^{-1}(ax)^2}{2a^2}+\frac{3x\cot^{-1}(ax)^2}{2a}+\frac{\cot^{-1}(ax)^3}{2a^2}+\frac{1}{2}x^2\cot^{-1}(ax)^3-\frac{3\cot^{-1}(ax)\log\left(\frac{2}{1+iax}\right)}{a^2}+\frac{3i\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^2}$$

output
```
3/2*I*arccot(a*x)^2/a^2+3/2*x*arccot(a*x)^2/a+1/2*arccot(a*x)^3/a^2+1/2*x^
2*arccot(a*x)^3-3*arccot(a*x)*ln(2/(1+I*a*x))/a^2+3/2*I*polylog(2,1-2/(1+I
*a*x))/a^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int x\cot^{-1}(ax)^3\,dx$$
$$=\frac{\cot^{-1}(ax)\left(3(i+ax)\cot^{-1}(ax)+(1+a^2x^2)\cot^{-1}(ax)^2-6\log\left(1-e^{2i\cot^{-1}(ax)}\right)\right)+3i\,\text{PolyLog}\left(2,e^{2i\cdots}\right.}{2a^2}$$

input
```
Integrate[x*ArcCot[a*x]^3,x]
```

output
```
(ArcCot[a*x]*(3*(I + a*x)*ArcCot[a*x] + (1 + a^2*x^2)*ArcCot[a*x]^2 - 6*Lo
g[1 - E^((2*I)*ArcCot[a*x])]) + (3*I)*PolyLog[2, E^((2*I)*ArcCot[a*x])])/(
2*a^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5362, 5452, 5346, 5420, 5456, 5380, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^{-1}(ax)^3\,dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{3}{2}a\int \frac{x^2\cot^{-1}(ax)^2}{a^2x^2+1}dx+\frac{1}{2}x^2\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5452}$$

$$\frac{3}{2}a\left(\frac{\int \cot^{-1}(ax)^2dx}{a^2}-\frac{\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5346}$$

$$\frac{3}{2}a\left(\frac{2a\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2}{a^2}-\frac{\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx}{a^2}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5420}$$

$$\frac{3}{2}a\left(\frac{2a\int \frac{x\cot^{-1}(ax)}{a^2x^2+1}dx+x\cot^{-1}(ax)^2}{a^2}+\frac{\cot^{-1}(ax)^3}{3a^3}\right)+\frac{1}{2}x^2\cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5456}$$

$$\frac{1}{2}x^2\cot^{-1}(ax)^3+\frac{3}{2}a\left(\frac{\cot^{-1}(ax)^3}{3a^3}+\frac{x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int \frac{\cot^{-1}(ax)}{i-ax}dx}{a}\right)}{a^2}\right)$$

$$\Big\downarrow \ 5380$$

$$\frac{1}{2}x^2\cot^{-1}(ax)^3+\frac{3}{2}a\left(\frac{\cot^{-1}(ax)^3}{3a^3}+\frac{x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx+\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}}{a}\right)}{a^2}\right)$$

$$\Big\downarrow \ 2849$$

$$\frac{1}{2}x^2\cot^{-1}(ax)^3+\frac{3}{2}a\left(\frac{\cot^{-1}(ax)^3}{3a^3}+\frac{x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}}{a}\right)}{a^2}\right)$$

$$\Big\downarrow \ 2752$$

$$\frac{1}{2}x^2\cot^{-1}(ax)^3+\frac{3}{2}a\left(\frac{\cot^{-1}(ax)^3}{3a^3}+\frac{x\cot^{-1}(ax)^2+2a\left(\frac{i\cot^{-1}(ax)^2}{2a^2}-\frac{\frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)}{a}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)}{a^2}\right)$$

input
```
Int[x*ArcCot[a*x]^3,x]
```

output
```
(x^2*ArcCot[a*x]^3)/2 + (3*a*(ArcCot[a*x]^3/(3*a^3) + (x*ArcCot[a*x]^2 + 2
*a*(((I/2)*ArcCot[a*x]^2)/a^2 - ((ArcCot[a*x]*Log[2/(1 + I*a*x)])/a - ((I/
2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))/a^2))/2
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5452
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcCot[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcCot[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 591 vs. 2(89) = 178.

Time = 5.10 (sec) , antiderivative size = 592, normalized size of antiderivative = 5.75

method	result
risch	$\frac{\pi^3 x^2}{16}+\frac{3\pi^2 x}{8a}+\frac{21\arctan(ax)}{32a^2}-\frac{3\pi^2\arctan(ax)}{8a^2}+\frac{21\pi\ln(a^2x^2+1)}{32a^2}+\frac{\pi^3}{16a^2}-\frac{3i\pi\ln(-iax+1)x}{4a}-\frac{3i\ln(-$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x*arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
3/32*I/a^2*(1-I*a*x)^2*ln(1-I*a*x)-3/32*I/a^2*(1-I*a*x)^2*ln(1-I*a*x)^2-3/
32*I*ln(1-I*a*x)^2*x^2+3/32*I*ln(1-I*a*x)*x^2+1/16*I/a^2*ln(1-I*a*x)^3-9/3
2*I/a^2*ln(1-I*a*x)^2+1/16*I*ln(1-I*a*x)^3*x^2+3/2*I/a^2*ln(1/2-1/2*I*a*x)
*ln(1/2+1/2*I*a*x)-3/2*I/a^2*ln(1/2+1/2*I*a*x)*ln(1-I*a*x)+3/16*I/a^2*Pi*a
rctan(a*x)+1/16*Pi^3*x^2+3/8*Pi^2*x/a+21/32*arctan(a*x)/a^2-3/4*I/a^2*ln(1
-I*a*x)+3/2*I/a^2*dilog(1/2-1/2*I*a*x)-1/16*I*(a^2*x^2+1)/a^2*ln(1+I*a*x)^
3-3/16*I*Pi^2*ln(1-I*a*x)*x^2-3/4*I/a*Pi*ln(1-I*a*x)*x+21/64*I/a^2*ln(a^2*
x^2+1)-3/8/a^2*Pi^2*arctan(a*x)+21/32/a^2*Pi*ln(a^2*x^2+1)+3/8*I*Pi^2/a^2-
3/16*(-I*x^2*ln(1-I*a*x)*a^2+Pi*a^2*x^2-I*ln(1-I*a*x)+2*a*x+Pi-2*I)/a^2*ln
(1+I*a*x)^2-3/16/a^2*Pi*ln(1-I*a*x)^2+3/16/a^2*Pi*ln(1-I*a*x)-3/16*Pi*ln(1
-I*a*x)^2*x^2-3/16/a*ln(1-I*a*x)^2*x-3/16/a*ln(1-I*a*x)*x+1/16/a^2*Pi^3+(-
3/16*I*(a^2*x^2+1)/a^2*ln(1-I*a*x)^2+3/8*x*(Pi*a*x+2)/a*ln(1-I*a*x)+3/16*(
I*Pi^2*a^2*x^2+4*I*Pi*a*x+2*ln(1-I*a*x)*Pi+4*I*ln(1-I*a*x))/a^2)*ln(1+I*a*
x)
```

Fricas [F]

$$\int x\cot^{-1}(ax)^3\,dx = \int x\,\mathrm{arccot}\left(ax\right)^3\,dx$$

input
```
integrate(x*arccot(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x*arccot(a*x)^3, x)
```

Sympy [F]

$$\int x\cot^{-1}(ax)^3\,dx = \int x\,\mathrm{acot}^3\left(ax\right)dx$$

input
```
integrate(x*acot(a*x)**3,x)
```

output
```
Integral(x*acot(a*x)**3, x)
```

Maxima [F]

$$\int x\cot^{-1}(ax)^3\,dx = \int x\,\mathrm{arccot}\left(ax\right)^3\,dx$$

input
```
integrate(x*arccot(a*x)^3,x, algorithm="maxima")
```

output
```
1/32*(8*a^2*x^2*arctan2(1, a*x)^3 + 12*a*x*arctan2(1, a*x)^2 - 3*a*x*log(a
^2*x^2 + 1)^2 + 4*(128*a^3*integrate(1/32*x^3*arctan(1/(a*x))^3/(a^3*x^2 +
 a), x) + 96*a^2*integrate(1/32*x^2*arctan(1/(a*x))^2/(a^3*x^2 + a), x) +
24*a^2*integrate(1/32*x^2*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) + 96*a^2*in
tegrate(1/32*x^2*log(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 128*a*integrate(1/32
*x*arctan(1/(a*x))^3/(a^3*x^2 + a), x) + 192*a*integrate(1/32*x*arctan(1/(
a*x))/(a^3*x^2 + a), x) + arctan(a*x)^3/a^2 + 3*arctan(a*x)^2*arctan(1/(a*
x))/a^2 + 3*arctan(a*x)*arctan(1/(a*x))^2/a^2 + 24*integrate(1/32*log(a^2*
x^2 + 1)^2/(a^3*x^2 + a), x))*a^2 + 8*arctan2(1, a*x)^3)/a^2
```

Giac [F]

$$\int x \cot^{-1}(ax)^3 \, dx = \int x \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x*arccot(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x*arccot(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \cot^{-1}(ax)^3 \, dx = \int x \operatorname{acot}(a\,x)^3 \, dx$$

input
```
int(x*acot(a*x)^3,x)
```

output
```
int(x*acot(a*x)^3, x)
```

Reduce [F]

$$\int x\cot^{-1}(ax)^3\,dx$$
$$= \frac{acot(ax)^3\,a^2x^2 + acot(ax)^3 + 3acot(ax)^2\,ax + 3acot(ax)\,a^2x^2 + 3acot(ax) - 6\left(\int \frac{acot(ax)x^3}{a^2x^2+1}dx\right)a^4 + 3ax}{2a^2}$$

input
```
int(x*acot(a*x)^3,x)
```

output
```
(acot(a*x)**3*a**2*x**2 + acot(a*x)**3 + 3*acot(a*x)**2*a*x + 3*acot(a*x)*
a**2*x**2 + 3*acot(a*x) - 6*int((acot(a*x)*x**3)/(a**2*x**2 + 1),x)*a**4 +
 3*a*x)/(2*a**2)
```

3.28 $\int \cot^{-1}(ax)^3\,dx$

Optimal result

Integrand size = 6, antiderivative size = 96

$$\int \cot^{-1}(ax)^3\,dx = \frac{i\cot^{-1}(ax)^3}{a} + x\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(\frac{2}{1+iax}\right)}{a} + \frac{3i\cot^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a} - \frac{3\,\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a}$$

output
```
I*arccot(a*x)^3/a+x*arccot(a*x)^3-3*arccot(a*x)^2*ln(2/(1+I*a*x))/a+3*I*ar
ccot(a*x)*polylog(2,1-2/(1+I*a*x))/a-3/2*polylog(3,1-2/(1+I*a*x))/a
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int \cot^{-1}(ax)^3\,dx = -\frac{i\cot^{-1}(ax)^3}{a} + x\cot^{-1}(ax)^3 - \frac{3\cot^{-1}(ax)^2\log\left(1-e^{-2i\cot^{-1}(ax)}\right)}{a} - \frac{3i\cot^{-1}(ax)\,\text{PolyLog}\left(2,e^{-2i\cot^{-1}(ax)}\right)}{a} - \frac{3\,\text{PolyLog}\left(3,e^{-2i\cot^{-1}(ax)}\right)}{2a}$$

input
```
Integrate[ArcCot[a*x]^3,x]
```

output
```
((-I)*ArcCot[a*x]^3)/a + x*ArcCot[a*x]^3 - (3*ArcCot[a*x]^2*Log[1 - E^((-2
*I)*ArcCot[a*x])])/a - ((3*I)*ArcCot[a*x]*PolyLog[2, E^((-2*I)*ArcCot[a*x]
)])/a - (3*PolyLog[3, E^((-2*I)*ArcCot[a*x])])/(2*a)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5346, 5456, 5380, 5530, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}(ax)^3 \, dx$$

$$\Big\downarrow \text{5346}$$

$$3a \int \frac{x \cot^{-1}(ax)^2}{a^2x^2+1} dx + x \cot^{-1}(ax)^3$$

$$\Big\downarrow \text{5456}$$

$$x \cot^{-1}(ax)^3 + 3a \left(\frac{i \cot^{-1}(ax)^3}{3a^2} - \frac{\int \frac{\cot^{-1}(ax)^2}{i-ax} dx}{a} \right)$$

$$\Big\downarrow \text{5380}$$

$$x \cot^{-1}(ax)^3 + 3a \left(\frac{i \cot^{-1}(ax)^3}{3a^2} - \frac{2 \int \frac{\cot^{-1}(ax) \log\left(\frac{2}{iax+1}\right)}{a^2x^2+1} dx + \frac{\log\left(\frac{2}{1+iax}\right) \cot^{-1}(ax)^2}{a}}{a} \right)$$

$$\Big\downarrow \text{5530}$$

$$x \cot^{-1}(ax)^3 + 3a \left(\frac{i \cot^{-1}(ax)^3}{3a^2} - \frac{2 \left(-\frac{1}{2} i \int \frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1} dx - \frac{i \,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right) \cot^{-1}(ax)}{2a} \right) + \frac{\log\left(\frac{2}{1+iax}\right) \cot^{-1}(ax)^2}{a}}{a} \right)$$

$$\Big\downarrow \text{7164}$$

$$x\cot^{-1}(ax)^3 + 3a\left(\frac{i\cot^{-1}(ax)^3}{3a^2} - \frac{2\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{4a} - \frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)\cot^{-1}(ax)}{2a}\right) + \frac{\log\left(\frac{2}{1+iax}\right)\cot^{-1}(ax)^2}{a}}{a}\right)$$

input
```
Int[ArcCot[a*x]^3,x]
```

output
```
x*ArcCot[a*x]^3 + 3*a*(((I/3)*ArcCot[a*x]^3)/a^2 - ((ArcCot[a*x]^2*Log[2/(
1 + I*a*x)])/a + 2*(((-1/2*I)*ArcCot[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a
 + PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a)))/a)
```

Defintions of rubi rules used

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5530
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(89) = 178$.

Time = 0.82 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.95

method	result
derivativedivides	$\frac{\operatorname{arccot}(ax)^3(ax-i)+2i\operatorname{arccot}(ax)^3-3\operatorname{arccot}(ax)^2\ln\left(1+\frac{ax+i}{\sqrt{a^2x^2+1}}\right)+6i\operatorname{arccot}(ax)\operatorname{polylog}\left(2,-\frac{ax+i}{\sqrt{a^2x^2+1}}\right)-6\operatorname{polyl}}{a}$
default	$\frac{\operatorname{arccot}(ax)^3(ax-i)+2i\operatorname{arccot}(ax)^3-3\operatorname{arccot}(ax)^2\ln\left(1+\frac{ax+i}{\sqrt{a^2x^2+1}}\right)+6i\operatorname{arccot}(ax)\operatorname{polylog}\left(2,-\frac{ax+i}{\sqrt{a^2x^2+1}}\right)-6\operatorname{polyl}}{a}$

input
```
int(arccot(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(arccot(a*x)^3*(a*x-I)+2*I*arccot(a*x)^3-3*arccot(a*x)^2*ln(1+(I+a*x)/
(a^2*x^2+1)^(1/2))+6*I*arccot(a*x)*polylog(2,-(I+a*x)/(a^2*x^2+1)^(1/2))-6
*polylog(3,-(I+a*x)/(a^2*x^2+1)^(1/2))-3*arccot(a*x)^2*ln(1-(I+a*x)/(a^2*x
^2+1)^(1/2))+6*I*arccot(a*x)*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))-6*polylo
g(3,(I+a*x)/(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \cot^{-1}(ax)^3 \, dx = \int \operatorname{arccot}\left(ax\right)^3 \, dx$$

input
```
integrate(arccot(a*x)^3,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^3, x)
```

Sympy [F]

$$\int \cot^{-1}(ax)^3 \, dx = \int \operatorname{acot}^3\left(ax\right) dx$$

input
```
integrate(acot(a*x)**3,x)
```

output
```
Integral(acot(a*x)**3, x)
```

Maxima [F]

$$\int \cot^{-1}(ax)^3 \, dx = \int \operatorname{arccot}\left(ax\right)^3 \, dx$$

input
```
integrate(arccot(a*x)^3,x, algorithm="maxima")
```

output
```
1/8*x*arctan2(1, a*x)^3 - 3/32*x*arctan2(1, a*x)*log(a^2*x^2 + 1)^2 + 21/1
6*arctan(a*x)^2*arctan(1/(a*x))^2/a + 7/8*arctan(a*x)*arctan(1/(a*x))^3/a
+ 28*a^2*integrate(1/32*x^2*arctan(1/(a*x))^3/(a^2*x^2 + 1), x) + 3*a^2*in
tegrate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 12
*a^2*integrate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x)
 + 12*a*integrate(1/32*x*arctan(1/(a*x))^2/(a^2*x^2 + 1), x) - 3*a*integra
te(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 7/32*(a*arctan(a*x)^4 + 4
*a*arctan(a*x)^3*arctan(1/(a*x)))/a^2 + 3*integrate(1/32*arctan(1/(a*x))*l
og(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

Giac [F]

$$\int \cot^{-1}(ax)^3\,dx = \int \operatorname{arccot}\left(ax\right)^3\,dx$$

input
```
integrate(arccot(a*x)^3,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{-1}(ax)^3\,dx = \int \mathrm{acot}(a\,x)^3\,dx$$

input
```
int(acot(a*x)^3,x)
```

output
```
int(acot(a*x)^3, x)
```

Reduce [F]

$$\int \cot^{-1}(ax)^3\,dx = \int acot(ax)^3\,dx$$

input
```
int(acot(a*x)^3,x)
```

output
```
int(acot(a*x)**3,x)
```

3.29 $\int \frac{\cot^{-1}(ax)^3}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 178

$$\begin{aligned}\int \frac{\cot^{-1}(ax)^3}{x}\,dx = {} & 2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right) \\ & -\frac{3}{2}i\cot^{-1}(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2i}{i+ax}\right) \\ & +\frac{3}{2}i\cot^{-1}(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2ax}{i+ax}\right) \\ & -\frac{3}{2}\cot^{-1}(ax)\operatorname{PolyLog}\left(3,1-\frac{2i}{i+ax}\right) \\ & +\frac{3}{2}\cot^{-1}(ax)\operatorname{PolyLog}\left(3,1-\frac{2ax}{i+ax}\right) \\ & +\frac{3}{4}i\operatorname{PolyLog}\left(4,1-\frac{2i}{i+ax}\right)-\frac{3}{4}i\operatorname{PolyLog}\left(4,1-\frac{2ax}{i+ax}\right)\end{aligned}$$

output

```
2*arccot(a*x)^3*arccoth(1-2/(1+I*a*x))-3/2*I*arccot(a*x)^2*polylog(2,1-2*I
/(I+a*x))+3/2*I*arccot(a*x)^2*polylog(2,1-2*a*x/(I+a*x))-3/2*arccot(a*x)*p
olylog(3,1-2*I/(I+a*x))+3/2*arccot(a*x)*polylog(3,1-2*a*x/(I+a*x))+3/4*I*p
olylog(4,1-2*I/(I+a*x))-3/4*I*polylog(4,1-2*a*x/(I+a*x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01

$$\int \frac{\cot^{-1}(ax)^3}{x} dx = \frac{1}{64} i \Big(\pi^4 - 32\cot^{-1}(ax)^4 + 64i\cot^{-1}(ax)^3 \log\left(1 - e^{-2i\cot^{-1}(ax)}\right) - 64i\cot^{-1}(ax)^3 \log\left(1 + e^{2i\cot^{-1}(ax)}\right) - 96\cot^{-1}(ax)^2 \operatorname{PolyLog}\left(2, e^{-2i\cot^{-1}(ax)}\right) - 96\cot^{-1}(ax)^2 \operatorname{PolyLog}\left(2, -e^{2i\cot^{-1}(ax)}\right) + 96i\cot^{-1}(ax) \operatorname{PolyLog}\left(3, e^{-2i\cot^{-1}(ax)}\right) - 96i\cot^{-1}(ax) \operatorname{PolyLog}\left(3, -e^{2i\cot^{-1}(ax)}\right) + 48\operatorname{PolyLog}\left(4, e^{-2i\cot^{-1}(ax)}\right) + 48\operatorname{PolyLog}\left(4, -e^{2i\cot^{-1}(ax)}\right)\Big)$$

input

```
Integrate[ArcCot[a*x]^3/x,x]
```

output

```
(I/64)*(Pi^4 - 32*ArcCot[a*x]^4 + (64*I)*ArcCot[a*x]^3*Log[1 - E^((-2*I)*A
rcCot[a*x])] - (64*I)*ArcCot[a*x]^3*Log[1 + E^((2*I)*ArcCot[a*x])] - 96*Ar
cCot[a*x]^2*PolyLog[2, E^((-2*I)*ArcCot[a*x])] - 96*ArcCot[a*x]^2*PolyLog[
2, -E^((2*I)*ArcCot[a*x])] + (96*I)*ArcCot[a*x]*PolyLog[3, E^((-2*I)*ArcCo
t[a*x])] - (96*I)*ArcCot[a*x]*PolyLog[3, -E^((2*I)*ArcCot[a*x])] + 48*Poly
Log[4, E^((-2*I)*ArcCot[a*x])] + 48*PolyLog[4, -E^((2*I)*ArcCot[a*x])])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5358, 5524, 5528, 5532, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^3}{x} dx$$

$$\Big\downarrow \text{5358}$$

$$6a\int \frac{\cot^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx+2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

$$\Big\downarrow \text{5524}$$

$$6a\left(\frac{1}{2}\int \frac{\cot^{-1}(ax)^2\log\left(\frac{2ax}{ax+i}\right)}{a^2x^2+1}dx-\frac{1}{2}\int \frac{\cot^{-1}(ax)^2\log\left(\frac{2i}{ax+i}\right)}{a^2x^2+1}dx\right)+$$
$$2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

$$\Big\downarrow \text{5528}$$

$$6a\left(\frac{1}{2}\left(-i\int \frac{\cot^{-1}(ax)\,\text{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)}{a^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)\cot^{-1}(ax)^2}{2a}\right)+\frac{1}{2}\left(i\int \frac{\cot^{-1}(ax)\,\text{I}}{}\right.\right.$$
$$2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

$$\Big\downarrow \text{5532}$$

$$6a\left(\frac{1}{2}\left(-i\left(-\frac{1}{2}i\int \frac{\text{PolyLog}\left(3,1-\frac{2i}{ax+i}\right)}{a^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(3,1-\frac{2i}{ax+i}\right)\cot^{-1}(ax)}{2a}\right)-\frac{i\,\text{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)}{2a}\right.\right.$$
$$2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

$$\Big\downarrow \text{7164}$$

$$6a\left(\frac{1}{2}\left(-\frac{i\,\text{PolyLog}\left(2,1-\frac{2i}{ax+i}\right)\cot^{-1}(ax)^2}{2a}-i\left(-\frac{\text{PolyLog}\left(4,1-\frac{2i}{ax+i}\right)}{4a}-\frac{i\,\text{PolyLog}\left(3,1-\frac{2i}{ax+i}\right)\cot^{-1}(ax}{2a}\right.\right.\right.$$
$$2\cot^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1+iax}\right)$$

input
```
Int[ArcCot[a*x]^3/x,x]
```

output
```
2*ArcCot[a*x]^3*ArcCoth[1 - 2/(1 + I*a*x)] + 6*a*((((-1/2*I)*ArcCot[a*x]^2
*PolyLog[2, 1 - (2*I)/(I + a*x)])/a - I*(((-1/2*I)*ArcCot[a*x]*PolyLog[3,
1 - (2*I)/(I + a*x)])/a - PolyLog[4, 1 - (2*I)/(I + a*x)]/(4*a)))/2 + (((I
/2)*ArcCot[a*x]^2*PolyLog[2, 1 - (2*a*x)/(I + a*x)])/a + I*(((-1/2*I)*ArcC
ot[a*x]*PolyLog[3, 1 - (2*a*x)/(I + a*x)])/a - PolyLog[4, 1 - (2*a*x)/(I +
 a*x)]/(4*a)))/2)
```

Defintions of rubi rules used

rule 5358
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcCot[c*x])^p*ArcCoth[1 - 2/(1 + I*c*x)], x] + Simp[2*b*c*p   Int[(a + b
*ArcCot[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5524
```
Int[(ArcCoth[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
 b*ArcCot[c*x])^p/(d + e*x^2)), x], x] - Simp[1/2   Int[Log[SimplifyIntegra
nd[1 - 1/u, x]]*((a + b*ArcCot[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*
x)))^2, 0]
```

rule 5528
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5532
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[k
+ 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0]
&& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.61 (sec) , antiderivative size = 982, normalized size of antiderivative = 5.52

method	result	size
derivativedivides	Expression too large to display	982
default	Expression too large to display	982
parts	Expression too large to display	1417

input
```
int(arccot(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
ln(a*x)*arccot(a*x)^3+1/2*I*Pi*(csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x
)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1
)))+csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2+csgn(I/(
(I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x
)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))-csgn(I/((I+a*x)^2/(a^2*x^2+1
)-1))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2-csgn(I
*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/
(a^2*x^2+1)))^2+csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1))
)^3-csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I
+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2-csgn(1/((I+a*x)^2/(a^2
*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3-1)*arccot(a*x)^3+arccot(a*x)^3*ln(
(I+a*x)^2/(a^2*x^2+1)-1)-arccot(a*x)^3*ln(1-(I+a*x)/(a^2*x^2+1)^(1/2))+3*I
*arccot(a*x)^2*polylog(2,(I+a*x)/(a^2*x^2+1)^(1/2))-6*arccot(a*x)*polylog(
3,(I+a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,(I+a*x)/(a^2*x^2+1)^(1/2))-arcc
ot(a*x)^3*ln(1+(I+a*x)/(a^2*x^2+1)^(1/2))+3*I*arccot(a*x)^2*polylog(2,-(I+
a*x)/(a^2*x^2+1)^(1/2))-6*arccot(a*x)*polylog(3,-(I+a*x)/(a^2*x^2+1)^(1/2)
)-6*I*polylog(4,-(I+a*x)/(a^2*x^2+1)^(1/2))-3/2*I*arccot(a*x)^2*polylog(2,
-(I+a*x)^2/(a^2*x^2+1))+3/2*arccot(a*x)*polylog(3,-(I+a*x)^2/(a^2*x^2+1))+
3/4*I*polylog(4,-(I+a*x)^2/(a^2*x^2+1))
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x}\,dx$$

input
```
integrate(arccot(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^3/x, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{acot}^3(ax)}{x}\,dx$$

input
```
integrate(acot(a*x)**3/x,x)
```

output
```
Integral(acot(a*x)**3/x, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x}\,dx$$

input
```
integrate(arccot(a*x)^3/x,x, algorithm="maxima")
```

output
```
integrate(arccot(a*x)^3/x, x)
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x}\,dx$$

input
```
integrate(arccot(a*x)^3/x,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x}\,dx = \int \frac{\operatorname{acot}(a\,x)^3}{x}\,dx$$

input
```
int(acot(a*x)^3/x,x)
```

output
```
int(acot(a*x)^3/x, x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(ax)^3}{x}\,dx = \int \frac{acot(ax)^3}{x}dx$$

input
```
int(acot(a*x)^3/x,x)
```

output
```
int(acot(a*x)**3/x,x)
```

3.30 $\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 93

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = -ia\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{x} - 3a\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right) - 3ia\cot^{-1}(ax)\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) - \frac{3}{2}a\,\text{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)$$

output
```
-I*a*arccot(a*x)^3-arccot(a*x)^3/x-3*a*arccot(a*x)^2*ln(2-2/(1-I*a*x))-3*I
*a*arccot(a*x)*polylog(2,-1+2/(1-I*a*x))-3/2*a*polylog(3,-1+2/(1-I*a*x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \frac{(-1+iax)\cot^{-1}(ax)^3}{x} - 3a\cot^{-1}(ax)^2\log\left(1+e^{2i\cot^{-1}(ax)}\right) + 3ia\cot^{-1}(ax)\,\text{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right) - \frac{3}{2}a\,\text{PolyLog}\left(3,-e^{2i\cot^{-1}(ax)}\right)$$

input
```
Integrate[ArcCot[a*x]^3/x^2,x]
```

output
```
((-1 + I*a*x)*ArcCot[a*x]^3)/x - 3*a*ArcCot[a*x]^2*Log[1 + E^((2*I)*ArcCot
[a*x])] + (3*I)*a*ArcCot[a*x]*PolyLog[2, -E^((2*I)*ArcCot[a*x])] - (3*a*Po
lyLog[3, -E^((2*I)*ArcCot[a*x])])/2
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5362, 5460, 5404, 5528, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{5362}$$

$$-3a\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^3}{x}$$

$$\Big\downarrow \text{5460}$$

$$-\frac{\cot^{-1}(ax)^3}{x}-3a\left(i\int \frac{\cot^{-1}(ax)^2}{x(ax+i)}dx+\frac{1}{3}i\cot^{-1}(ax)^3\right)$$

$$\Big\downarrow \text{5404}$$

$$-\frac{\cot^{-1}(ax)^3}{x}-$$
$$3a\left(i\left(-2ia\int \frac{\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)^2\right)+\frac{1}{3}i\cot^{-1}(ax)^3\right)$$

$$\Big\downarrow \text{5528}$$

$$-\frac{\cot^{-1}(ax)^3}{x}-$$
$$3a\left(i\left(-2ia\left(\frac{1}{2}i\int \frac{\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx+\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\cot^{-1}(ax)}{2a}\right)-i\log\left(2-\frac{2}{1-iax}\right)\text{co}\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$-\frac{\cot^{-1}(ax)^3}{x}-$$

$$3a\left(i\left(-2ia\left(\frac{\text{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}+\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\cot^{-1}(ax)}{2a}\right)-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right.\right.$$

input
```
Int[ArcCot[a*x]^3/x^2,x]
```

output
```
-(ArcCot[a*x]^3/x) - 3*a*((I/3)*ArcCot[a*x]^3 + I*((-I)*ArcCot[a*x]^2*Log[
2 - 2/(1 - I*a*x)] - (2*I)*a*(((I/2)*ArcCot[a*x]*PolyLog[2, -1 + 2/(1 - I*
a*x)])/a + PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*a))))
```

Defintions of rubi rules used

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5404
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Si
mp[b*c*(p/d)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5460
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Simp[
I/d   Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5528
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.70 (sec) , antiderivative size = 1441, normalized size of antiderivative = 15.49

method	result	size
parts	Expression too large to display	1441
derivativedivides	Expression too large to display	1444
default	Expression too large to display	1444

input
```
int(arccot(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

output

```
-arccot(a*x)^3/x-3*a*(arccot(a*x)^2*ln(a*x)-1/2*arccot(a*x)^2*ln(a^2*x^2+1
)+arccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))-1/3*I*arccot(a*x)^3+1/4*(I*P
i*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))^2*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)+
2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/
((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))-I*Pi*csgn(I*(I+a*x)/(
a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))+I*Pi*csgn(I/((I+a*x)^2/(
a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)
^2+I*Pi*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^3-2*I*Pi*csgn(I*(I+a*x)^2/(a^2
*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)^2-2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1
)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x
)^2/(a^2*x^2+1)))^2-I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))^3+2*I*Pi*csgn(I*(I+
a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2-I*Pi*csgn(I/((I+a*
x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2
*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)+2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-
1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3+2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*c
sgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*
x)^2/(a^2*x^2+1)))-2*I*Pi*csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(I/((I+a*x
)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2-2*I*Pi*csgn(1/((I+a*x)^2/(
a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^3+2*I*Pi*csgn(1/((I+a*x)^2/(a^2*x
^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2-2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2...
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^2} \, dx = \int \frac{\operatorname{arccot}(ax)^3}{x^2} \, dx$$

input

```
integrate(arccot(a*x)^3/x^2,x, algorithm="fricas")
```

output

```
integral(arccot(a*x)^3/x^2, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{acot}^3(ax)}{x^2}\,dx$$

input
```
integrate(acot(a*x)**3/x**2,x)
```

output
```
Integral(acot(a*x)**3/x**2, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^2}\,dx$$

input
```
integrate(arccot(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
-1/32*(4*arctan2(1, a*x)^3 - 3*arctan2(1, a*x)*log(a^2*x^2 + 1)^2 - (28*a*
arctan(a*x)*arctan(1/(a*x))^3 + 7*(6*arctan(a*x)^2*arctan(1/(a*x))^2/a + (
a*arctan(a*x)^4 + 4*a*arctan(a*x)^3*arctan(1/(a*x)))/a^2)*a^2 + 96*a^2*int
egrate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 3
84*a^2*integrate(1/32*x^2*arctan(1/(a*x))*log(a^2*x^2 + 1)/(a^2*x^4 + x^2)
, x) - 384*a*integrate(1/32*x*arctan(1/(a*x))^2/(a^2*x^4 + x^2), x) + 96*a
*integrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 896*integrate(1
/32*arctan(1/(a*x))^3/(a^2*x^4 + x^2), x) + 96*integrate(1/32*arctan(1/(a*
x))*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x)/x
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^2}\,dx$$

input
```
integrate(arccot(a*x)^3/x^2,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{acot}(a\,x)^3}{x^2}\,dx$$

input
```
int(acot(a*x)^3/x^2,x)
```

output
```
int(acot(a*x)^3/x^2, x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^2}\,dx = \frac{-acot(ax)^3 - 3\left(\int \frac{acot(ax)^2}{a^2x^3+x}dx\right)ax}{x}$$

input
```
int(acot(a*x)^3/x^2,x)
```

output
```
( - acot(a*x)**3 - 3*int(acot(a*x)**2/(a**2*x**3 + x),x)*a*x)/x
```

3.31 $\int \frac{\cot^{-1}(ax)^3}{x^3} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 105

$$\int \frac{\cot^{-1}(ax)^3}{x^3} \, dx = \frac{3}{2} i a^2 \cot^{-1}(ax)^2 + \frac{3a \cot^{-1}(ax)^2}{2x} - \frac{1}{2} a^2 \cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{2x^2} + 3a^2 \cot^{-1}(ax) \log\left(2 - \frac{2}{1 - iax}\right) + \frac{3}{2} i a^2 \operatorname{PolyLog}\left(2, -1 + \frac{2}{1 - iax}\right)$$

output
```
3/2*I*a^2*arccot(a*x)^2+3/2*a*arccot(a*x)^2/x-1/2*a^2*arccot(a*x)^3-1/2*ar
ccot(a*x)^3/x^2+3*a^2*arccot(a*x)*ln(2-2/(1-I*a*x))+3/2*I*a^2*polylog(2,-1
+2/(1-I*a*x))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

$$\int \frac{\cot^{-1}(ax)^3}{x^3} \, dx = -\frac{\cot^{-1}(ax) \left(3iax(i + ax) \cot^{-1}(ax) + (1 + a^2x^2) \cot^{-1}(ax)^2 - 6a^2x^2 \log\left(1 + e^{2i \cot^{-1}(ax)}\right)\right)}{2x^2} - \frac{3}{2} i a^2 \operatorname{PolyLog}\left(2, -e^{2i \cot^{-1}(ax)}\right)$$

input
```
Integrate[ArcCot[a*x]^3/x^3,x]
```

output
```
-1/2*(ArcCot[a*x]*((3*I)*a*x*(I + a*x)*ArcCot[a*x] + (1 + a^2*x^2)*ArcCot[
a*x]^2 - 6*a^2*x^2*Log[1 + E^((2*I)*ArcCot[a*x])]))/x^2 - ((3*I)/2)*a^2*Po
lyLog[2, -E^((2*I)*ArcCot[a*x])]
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5362, 5454, 5362, 5420, 5460, 5404, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^3}{x^3}dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{3}{2}a\int \frac{\cot^{-1}(ax)^2}{x^2\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{5454}$$

$$-\frac{3}{2}a\left(\int \frac{\cot^{-1}(ax)^2}{x^2}dx-a^2\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{5362}$$

$$-\frac{3}{2}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx\right)-2a\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^2}{x}\right)-\frac{\cot^{-1}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{5420}$$

$$-\frac{3}{2}a\left(-2a\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx+\frac{1}{3}a\cot^{-1}(ax)^3-\frac{\cot^{-1}(ax)^2}{x}\right)-\frac{\cot^{-1}(ax)^3}{2x^2}$$

$$\Big\downarrow \text{5460}$$

$$-\frac{\cot^{-1}(ax)^3}{2x^2}-\frac{3}{2}a\left(-2a\left(i\int \frac{\cot^{-1}(ax)}{x(ax+i)}dx+\frac{1}{2}i\cot^{-1}(ax)^2\right)+\frac{1}{3}a\cot^{-1}(ax)^3-\frac{\cot^{-1}(ax)^2}{x}\right)$$

$\Big\downarrow$ 5404

$$-\frac{\cot^{-1}(ax)^3}{2x^2}-\frac{3}{2}a\left(-2a\left(i\left(-ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right)+\frac{1}{2}i\cot^{-1}(ax)^2\right)+\frac{1}{3}a\cot^{-1}(ax)^3\right.$$

$\Big\downarrow$ 2897

$$-\frac{\cot^{-1}(ax)^3}{2x^2}-\frac{3}{2}a\left(-2a\left(i\left(\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right)+\frac{1}{2}i\cot^{-1}(ax)^2\right)+\frac{1}{3}a\cot^{-1}(ax)^3\right.$$

input
```
Int[ArcCot[a*x]^3/x^3,x]
```

output
```
-1/2*ArcCot[a*x]^3/x^2 - (3*a*(-(ArcCot[a*x]^2/x) + (a*ArcCot[a*x]^3)/3 -
2*a*((I/2)*ArcCot[a*x]^2 + I*((-I)*ArcCot[a*x]*Log[2 - 2/(1 - I*a*x)] + Po
lyLog[2, -1 + 2/(1 - I*a*x)]/2))))/2
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5404
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Si
mp[b*c*(p/d)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5460
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Simp[
I/d   Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.70 (sec) , antiderivative size = 2956, normalized size of antiderivative = 28.15

method	result	size
parts	Expression too large to display	2956
derivativedivides	Expression too large to display	2957
default	Expression too large to display	2957

input
```
int(arccot(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccot(a*x)^3/x^2-3/2*a^2*(-1/8*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*
csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*(2*I*arccot(a*x)
*ln(1+(I+a*x)^2/(a^2*x^2+1))+2*arccot(a*x)^2+polylog(2,-(I+a*x)^2/(a^2*x^2
+1)))+1/2*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*x^2+1
))^2*(I*arccot(a*x)*ln(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+I*arccot(a*x)*ln(1-I
*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1-I
*(I+a*x)/(a^2*x^2+1)^(1/2)))-1/4*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*csgn
(I*(I+a*x)^2/(a^2*x^2+1))^2*(2*I*arccot(a*x)*ln(1+(I+a*x)^2/(a^2*x^2+1))+2
*arccot(a*x)^2+polylog(2,-(I+a*x)^2/(a^2*x^2+1)))-1/4*Pi*csgn(I*(I+a*x)/(a
^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*(I*arccot(a*x)*ln(1+I*(I+
a*x)/(a^2*x^2+1)^(1/2))+I*arccot(a*x)*ln(1-I*(I+a*x)/(a^2*x^2+1)^(1/2))+di
log(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1-I*(I+a*x)/(a^2*x^2+1)^(1/2)))-a
rccot(a*x)^2/x/a+1/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2
/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*(I*arccot(a*x)*ln(1+I*(I+a*x)/(a
^2*x^2+1)^(1/2))+I*arccot(a*x)*ln(1-I*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1+I
*(I+a*x)/(a^2*x^2+1)^(1/2))+dilog(1-I*(I+a*x)/(a^2*x^2+1)^(1/2)))+1/8*Pi*c
sgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*(2*I*arcc
ot(a*x)*ln(1+(I+a*x)^2/(a^2*x^2+1))+2*arccot(a*x)^2+polylog(2,-(I+a*x)^2/(
a^2*x^2+1)))-1/8*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^
2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*(2*I*arccot(a*x)*ln(1+(I+a*x)^2/(a^2*...
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^3}\,dx$$

input
```
integrate(arccot(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^3/x^3, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^3} dx = \int \frac{\operatorname{acot}^3(ax)}{x^3} dx$$

input
```
integrate(acot(a*x)**3/x**3,x)
```

output
```
Integral(acot(a*x)**3/x**3, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^3} dx = \int \frac{\operatorname{arccot}(ax)^3}{x^3} dx$$

input
```
integrate(arccot(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
-1/32*(8*a^2*x^2*arctan2(1, a*x)^3 - 12*a*x*arctan2(1, a*x)^2 + 3*a*x*log(
a^2*x^2 + 1)^2 + 4*(3*a^2*arctan(a*x)*arctan(1/(a*x))^2 + (arctan(a*x)^3/a
 + 3*arctan(a*x)^2*arctan(1/(a*x))/a)*a^3 + 24*a^3*integrate(1/32*x^3*log(
a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) - 96*a^3*integrate(1/32*x^3*log(a^2*x^2
 + 1)/(a^2*x^5 + x^3), x) - 128*a^2*integrate(1/32*x^2*arctan(1/(a*x))^3/(
a^2*x^5 + x^3), x) - 192*a^2*integrate(1/32*x^2*arctan(1/(a*x))/(a^2*x^5 +
 x^3), x) + 96*a*integrate(1/32*x*arctan(1/(a*x))^2/(a^2*x^5 + x^3), x) +
24*a*integrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) - 128*integra
te(1/32*arctan(1/(a*x))^3/(a^2*x^5 + x^3), x))*x^2 + 8*arctan2(1, a*x)^3)/
x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.28

$$\int \frac{\cot^{-1}(ax)^3}{x^3}\,dx = -\frac{1}{2}\,a\arctan\left(\frac{1}{ax}\right)^3 - \frac{\arctan\left(\frac{1}{ax}\right)^3}{2\,x^2}$$

input
```
integrate(arccot(a*x)^3/x^3,x, algorithm="giac")
```

output
```
-1/2*a*arctan(1/(a*x))^3 - 1/2*arctan(1/(a*x))^3/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x^3}\,dx = \int \frac{\mathrm{acot}(a\,x)^3}{x^3}\,dx$$

input
```
int(acot(a*x)^3/x^3,x)
```

output
```
int(acot(a*x)^3/x^3, x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^3}\,dx$$
$$= \frac{-acot(ax)^3\,a^2x^2 - acot(ax)^3 + 3acot(ax)^2\,ax - 3acot(ax)\,a^2x^2 - 3acot(ax) - 6\left(\int \frac{acot(ax)}{a^2x^5+x^3}dx\right)x^2 + 3a}{2x^2}$$

input
```
int(acot(a*x)^3/x^3,x)
```

output
```
( - acot(a*x)**3*a**2*x**2 - acot(a*x)**3 + 3*acot(a*x)**2*a*x - 3*acot(a*
x)*a**2*x**2 - 3*acot(a*x) - 6*int(acot(a*x)/(a**2*x**5 + x**3),x)*x**2 +
3*a*x)/(2*x**2)
```

3.32 $\int \frac{\cot^{-1}(ax)^3}{x^4} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 167

$$\begin{aligned}\int \frac{\cot^{-1}(ax)^3}{x^4} \, dx = & -\frac{a^2\cot^{-1}(ax)}{x} + \frac{1}{2}a^3\cot^{-1}(ax)^2 + \frac{a\cot^{-1}(ax)^2}{2x^2} \\ & + \frac{1}{3}ia^3\cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{3x^3} - a^3\log(x) \\ & + \frac{1}{2}a^3\log\left(1+a^2x^2\right) + a^3\cot^{-1}(ax)^2\log\left(2-\frac{2}{1-iax}\right) \\ & + ia^3\cot^{-1}(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) \\ & + \frac{1}{2}a^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)\end{aligned}$$

output

```
-a^2*arccot(a*x)/x+1/2*a^3*arccot(a*x)^2+1/2*a*arccot(a*x)^2/x^2+1/3*I*a^3
*arccot(a*x)^3-1/3*arccot(a*x)^3/x^3-a^3*ln(x)+1/2*a^3*ln(a^2*x^2+1)+a^3*a
rccot(a*x)^2*ln(2-2/(1-I*a*x))+I*a^3*arccot(a*x)*polylog(2,-1+2/(1-I*a*x))
+1/2*a^3*polylog(3,-1+2/(1-I*a*x))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \frac{1}{6}\left(-\frac{6a^2\cot^{-1}(ax)}{x} + 3a^3\cot^{-1}(ax)^2 + \frac{3a\cot^{-1}(ax)^2}{x^2} - 2ia^3\cot^{-1}(ax)^3 - \frac{2\cot^{-1}(ax)^3}{x^3} + 6a^3\cot^{-1}(ax)^2\log\left(1+e^{2i\cot^{-1}(ax)}\right) - 6a^3\log\left(\frac{1}{\sqrt{1+\frac{1}{a^2x^2}}}\right) - 6ia^3\cot^{-1}(ax)\,\text{PolyLog}\left(2,-e^{2i\cot^{-1}(ax)}\right) + 3a^3\,\text{PolyLog}\left(3,-e^{2i\cot^{-1}(ax)}\right)\right)$$

input
```
Integrate[ArcCot[a*x]^3/x^4,x]
```

output
```
((-6*a^2*ArcCot[a*x])/x + 3*a^3*ArcCot[a*x]^2 + (3*a*ArcCot[a*x]^2)/x^2 -
(2*I)*a^3*ArcCot[a*x]^3 - (2*ArcCot[a*x]^3)/x^3 + 6*a^3*ArcCot[a*x]^2*Log[
1 + E^((2*I)*ArcCot[a*x])] - 6*a^3*Log[1/Sqrt[1 + 1/(a^2*x^2)]] - (6*I)*a^
3*ArcCot[a*x]*PolyLog[2, -E^((2*I)*ArcCot[a*x])] + 3*a^3*PolyLog[3, -E^((2
*I)*ArcCot[a*x])])/6
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {5362, 5454, 5362, 5454, 5362, 243, 47, 14, 16, 5420, 5460, 5404, 5528, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx$$

$$\Big\downarrow \textcolor{blue}{5362}$$

$$-a\int \frac{\cot^{-1}(ax)^2}{x^3\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \textcolor{blue}{5454}$$

$$-a\left(\int \frac{\cot^{-1}(ax)^2}{x^3}dx-a^2\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\right)-\frac{\cot^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \textcolor{blue}{5362}$$

$$-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\right)-a\int \frac{\cot^{-1}(ax)}{x^2\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^2}{2x^2}\right)-\frac{\cot^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \textcolor{blue}{5454}$$

$$-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\right)-a\left(\int \frac{\cot^{-1}(ax)}{x^2}dx-a^2\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)^2}{2x^2}\right)-\frac{\cot^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \textcolor{blue}{5362}$$

$$-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\right)-a\left(-a\int \frac{1}{x\left(a^2x^2+1\right)}dx+a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)-\frac{\cot^{-1}(ax}{2x^2}\right.$$
$$\frac{\cot^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \textcolor{blue}{243}$$

$$-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\right)-a\left(-\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx^2+a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(ax)}{x}\right)-\frac{\cot^{-1}}{2\!:}\right.$$
$$\frac{\cot^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \textcolor{blue}{47}$$

$$-a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\right)-a\left(-\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)+a^2\left(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\right)-\frac{\cot^{-1}(}{x}\right.\right.$$
$$\frac{\cot^{-1}(ax)^3}{3x^3}$$

$$\Big\downarrow \textcolor{blue}{14}$$

$$-a\Bigg(a^2\bigg(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\bigg)-a\Bigg(-\frac{1}{2}a\bigg(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\bigg)+a^2\bigg(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\bigg)-\frac{\cot^{-1}(ax}{x}$$

$$\frac{\cot^{-1}(ax)^3}{3x^3}$$

$\big\downarrow$ 16

$$-a\Bigg(a^2\bigg(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\bigg)-a\Bigg(a^2\bigg(-\int \frac{\cot^{-1}(ax)}{a^2x^2+1}dx\bigg)-\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\cot^{-1}(ax)}{x}\Bigg)-$$

$$\frac{\cot^{-1}(ax)^3}{3x^3}$$

$\big\downarrow$ 5420

$$-a\Bigg(a^2\bigg(-\int \frac{\cot^{-1}(ax)^2}{x\left(a^2x^2+1\right)}dx\bigg)-a\bigg(-\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)+\frac{1}{2}a\cot^{-1}(ax)^2-\frac{\cot^{-1}(ax)}{x}\bigg)-\frac{\cot^{-1}(a}{2x^2}$$

$$\frac{\cot^{-1}(ax)^3}{3x^3}$$

$\big\downarrow$ 5460

$$-\frac{\cot^{-1}(ax)^3}{3x^3}-$$

$$a\Bigg(-\bigg(a^2\bigg(i\int \frac{\cot^{-1}(ax)^2}{x(ax+i)}dx+\frac{1}{3}i\cot^{-1}(ax)^3\bigg)\bigg)\Bigg)-a\bigg(-\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)+\frac{1}{2}a\cot^{-1}(ax)^2-\frac{\cot^{-1}}{x}$$

$\big\downarrow$ 5404

$$-\frac{\cot^{-1}(ax)^3}{3x^3}-$$

$$a\Bigg(-\Bigg(a^2\Bigg(i\Bigg(-2ia\int \frac{\cot^{-1}(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)^2\Bigg)+\frac{1}{3}i\cot^{-1}(ax)^3\Bigg)\Bigg)\Bigg)-$$

$\big\downarrow$ 5528

$$-\frac{\cot^{-1}(ax)^3}{3x^3}-$$

$$a\Bigg(-\Bigg(a^2\Bigg(i\Bigg(-2ia\Bigg(\frac{1}{2}i\int \frac{\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx+\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\cot^{-1}(ax)}{2a}\Bigg)-i\log\bigg(2-\frac{2}{1-}$$

$\big\downarrow$ 7164

$$-\frac{\cot^{-1}(ax)^3}{3x^3}-$$
$$a\left(-\left(a^2\left(i\left(-2ia\left(\frac{\text{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}+\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\cot^{-1}(ax)}{2a}\right)-i\log\left(2-\frac{2}{1-iax}\right)\cot\right.\right.\right.\right.\right.$$

input
```
Int[ArcCot[a*x]^3/x^4,x]
```

output
```
-1/3*ArcCot[a*x]^3/x^3 - a*(-1/2*ArcCot[a*x]^2/x^2 - a*(-(ArcCot[a*x]/x) +
 (a*ArcCot[a*x]^2)/2 - (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) - a^2*((I/3)*A
rcCot[a*x]^3 + I*((-I)*ArcCot[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (2*I)*a*(((I
/2)*ArcCot[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a + PolyLog[3, -1 + 2/(1 -
 I*a*x)]/(4*a)))))
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5404
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Si
mp[b*c*(p/d)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5460
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Simp[
I/d   Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5528
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] + Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 65.07 (sec) , antiderivative size = 1620, normalized size of antiderivative = 9.70

method	result	size
derivativedivides	Expression too large to display	1620
default	Expression too large to display	1620
parts	Expression too large to display	1620

input
```
int(arccot(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/4*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x
^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1)^2)*arccot(a*
x)^2+1/2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(1+(I+a*x)^2/(a^2*x
^2+1)))*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*arccot
(a*x)^2+arccot(a*x)^2*ln((I+a*x)/(a^2*x^2+1)^(1/2))+ln(2)*arccot(a*x)^2+1/
2/a^2/x^2*arccot(a*x)^2-ln(1+(I+a*x)^2/(a^2*x^2+1))+1/2*I*Pi*csgn(I*(I+a*x
)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2*arccot(a*x)^2-1/2*I*P
i*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*
arccot(a*x)^2-1/2*I*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I/((I+a*x)^2
/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*arccot(a*x)^2+1/2*I*Pi*csgn(I
/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a
^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*arccot(a*x)^2+I*arccot(a*x)*(I+a*x
)/a/x+1/4*I*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/
((I+a*x)^2/(a^2*x^2+1)-1)^2)^2*arccot(a*x)^2-1/2*I*Pi*csgn(I/((I+a*x)^2/(a
^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))*csgn(1/((I+a*x)^2/(a^2*x^2+1)-1)*(
1+(I+a*x)^2/(a^2*x^2+1)))^2*arccot(a*x)^2-1/4*I*Pi*csgn(I*(I+a*x)/(a^2*x^2
+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*arccot(a*x)^2+1/4*I*Pi*csgn(I/(
(I+a*x)^2/(a^2*x^2+1)-1)^2)*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x
^2+1)-1)^2)^2*arccot(a*x)^2-1/2*I*Pi*csgn(I*(1+(I+a*x)^2/(a^2*x^2+1)))*csg
n(I/((I+a*x)^2/(a^2*x^2+1)-1)*(1+(I+a*x)^2/(a^2*x^2+1)))^2*arccot(a*x)^...
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^4} \, dx = \int \frac{\operatorname{arccot}(ax)^3}{x^4} \, dx$$

input
```
integrate(arccot(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^3/x^4, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^4} \, dx = \int \frac{\operatorname{acot}^3(ax)}{x^4} \, dx$$

input
```
integrate(acot(a*x)**3/x**4,x)
```

output
```
Integral(acot(a*x)**3/x**4, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^4} \, dx = \int \frac{\operatorname{arccot}(ax)^3}{x^4} \, dx$$

input
```
integrate(arccot(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
1/96*(96*x^3*integrate(1/32*(28*a^2*x^2*arctan2(1, a*x)^3 - 4*a^2*x^2*arct
an2(1, a*x)*log(a^2*x^2 + 1) - 4*a*x*arctan2(1, a*x)^2 + 28*arctan2(1, a*x
)^3 + (3*a^2*x^2*arctan2(1, a*x) + a*x + 3*arctan2(1, a*x))*log(a^2*x^2 +
1)^2)/(a^2*x^6 + x^4), x) - 4*arctan2(1, a*x)^3 + 3*arctan2(1, a*x)*log(a^
2*x^2 + 1)^2)/x^3
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^4}\,dx$$

input
```
integrate(arccot(a*x)^3/x^4,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^3/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{acot}(a\,x)^3}{x^4}\,dx$$

input
```
int(acot(a*x)^3/x^4,x)
```

output
```
int(acot(a*x)^3/x^4, x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^4}\,dx = \frac{-2acot(ax)^3 + 3acot(ax)^2 a^3x^3 + 3acot(ax)^2 ax - 6acot(ax)\,a^2x^2 + 6\left(\int \frac{acot(ax)^2}{a^2x^3+x}dx\right)a^3x^3 + 3\log(a^2x^2}{6x^3}$$

input
```
int(acot(a*x)^3/x^4,x)
```

output
```
( - 2*acot(a*x)**3 + 3*acot(a*x)**2*a**3*x**3 + 3*acot(a*x)**2*a*x - 6*aco
t(a*x)*a**2*x**2 + 6*int(acot(a*x)**2/(a**2*x**3 + x),x)*a**3*x**3 + 3*log
(a**2*x**2 + 1)*a**3*x**3 - 6*log(x)*a**3*x**3)/(6*x**3)
```

3.33 $\int \frac{\cot^{-1}(ax)^3}{x^5} dx$

Optimal result

Integrand size = 10, antiderivative size = 152

$$\begin{aligned}\int \frac{\cot^{-1}(ax)^3}{x^5} dx = {} & \frac{a^3}{4x} - \frac{a^2 \cot^{-1}(ax)}{4x^2} - ia^4 \cot^{-1}(ax)^2 + \frac{a \cot^{-1}(ax)^2}{4x^3} \\ & - \frac{3a^3 \cot^{-1}(ax)^2}{4x} + \frac{1}{4} a^4 \cot^{-1}(ax)^3 - \frac{\cot^{-1}(ax)^3}{4x^4} + \frac{1}{4} a^4 \arctan(ax) \\ & - 2a^4 \cot^{-1}(ax) \log\left(2 - \frac{2}{1 - iax}\right) - ia^4 \operatorname{PolyLog}\left(2, -1 + \frac{2}{1 - iax}\right)\end{aligned}$$

output
```
1/4*a^3/x-1/4*a^2*arccot(a*x)/x^2-I*a^4*arccot(a*x)^2+1/4*a*arccot(a*x)^2/
x^3-3/4*a^3*arccot(a*x)^2/x+1/4*a^4*arccot(a*x)^3-1/4*arccot(a*x)^3/x^4+1/
4*a^4*arctan(a*x)-2*a^4*arccot(a*x)*ln(2-2/(1-I*a*x))-I*a^4*polylog(2,-1+2
/(1-I*a*x))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.83

$$\begin{aligned}&\int \frac{\cot^{-1}(ax)^3}{x^5} dx \\ &= \frac{a^3x^3 + (ax - 3a^3x^3 + 4ia^4x^4) \cot^{-1}(ax)^2 + (-1 + a^4x^4) \cot^{-1}(ax)^3 - a^2x^2 \cot^{-1}(ax) \left(1 + a^2x^2 + 8a^2x^2 \right.}{4x^4}\end{aligned}$$

input
```
Integrate[ArcCot[a*x]^3/x^5,x]
```

output
```
(a^3*x^3 + (a*x - 3*a^3*x^3 + (4*I)*a^4*x^4)*ArcCot[a*x]^2 + (-1 + a^4*x^4
)*ArcCot[a*x]^3 - a^2*x^2*ArcCot[a*x]*(1 + a^2*x^2 + 8*a^2*x^2*Log[1 + E^(
(2*I)*ArcCot[a*x])]) + (4*I)*a^4*x^4*PolyLog[2, -E^((2*I)*ArcCot[a*x])])/(
4*x^4)
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {5362, 5454, 5362, 5454, 5362, 264, 216, 5420, 5460, 5404, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax)^3}{x^5}dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{3}{4}a\int \frac{\cot^{-1}(ax)^2}{x^4\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{5454}$$

$$-\frac{3}{4}a\left(\int \frac{\cot^{-1}(ax)^2}{x^4}dx-a^2\int \frac{\cot^{-1}(ax)^2}{x^2\left(a^2x^2+1\right)}dx\right)-\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{5362}$$

$$-\frac{3}{4}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{x^2\left(a^2x^2+1\right)}dx\right)-\frac{2}{3}a\int \frac{\cot^{-1}(ax)}{x^3\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^2}{3x^3}\right)-\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{5454}$$

$$-\frac{3}{4}a\left(-\left(a^2\left(\int \frac{\cot^{-1}(ax)^2}{x^2}dx-a^2\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx\right)\right)-\frac{2}{3}a\left(\int \frac{\cot^{-1}(ax)}{x^3}dx-a^2\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx\right)-\cot\right.$$
$$\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \text{5362}$$

$$-\frac{3}{4}a\left(-\left(a^2\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx\right)-2a\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^2}{x}\right)\right)-\frac{2}{3}a\left(-\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx\right.\right.$$

$$\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \; 264$$

$$-\frac{3}{4}a\left(-\left(a^2\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}dx\right)-2a\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\cot^{-1}(ax)^2}{x}\right)\right)-\frac{2}{3}a\left(-\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}\right.\right.\right.\right.$$

$$\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \; 216$$

$$-\frac{3}{4}a\left(-\frac{2}{3}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx\right)-\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\cot^{-1}(ax)}{2x^2}\right)-\left(a^2\left(a^2\left(-\int \frac{\cot^{-1}(ax)^2}{a^2x^2+1}\right.\right.\right.\right.$$

$$\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \; 5420$$

$$-\frac{3}{4}a\left(-\frac{2}{3}a\left(a^2\left(-\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}dx\right)-\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\cot^{-1}(ax)}{2x^2}\right)-\left(a^2\left(-2a\int \frac{\cot^{-1}(ax)}{x\left(a^2x^2+1\right)}\right.\right.\right.$$

$$\frac{\cot^{-1}(ax)^3}{4x^4}$$

$$\Big\downarrow \; 5460$$

$$-\frac{\cot^{-1}(ax)^3}{4x^4}-$$

$$\frac{3}{4}a\left(-\frac{2}{3}a\left(-\left(a^2\left(i\int \frac{\cot^{-1}(ax)}{x(ax+i)}dx+\frac{1}{2}i\cot^{-1}(ax)^2\right)\right)-\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\cot^{-1}(ax)}{2x^2}\right)-\left(a^2\left(-2a\right.\right.\right.$$

$$\Big\downarrow \; 5404$$

$$-\frac{\cot^{-1}(ax)^3}{4x^4}-$$

$$\frac{3}{4}a\left(-\frac{2}{3}a\left(-\left(a^2\left(i\left(-ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right)+\frac{1}{2}i\cot^{-1}(ax)^2\right)\right)-\frac{1}{2}a\left(\right.\right.\right.$$

$$\Big\downarrow \; 2897$$

$$-\frac{\cot^{-1}(ax)^3}{4x^4}-$$

$$\frac{3}{4}a\left(-\frac{2}{3}a\left(-\left(a^2\left(i\left(\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)-i\log\left(2-\frac{2}{1-iax}\right)\cot^{-1}(ax)\right)+\frac{1}{2}i\cot^{-1}(ax)^2\right)\right)-\frac{1}{2}a\left(\right.\right.\right.$$

input
```
Int[ArcCot[a*x]^3/x^5,x]
```

output
```
-1/4*ArcCot[a*x]^3/x^4 - (3*a*(-1/3*ArcCot[a*x]^2/x^3 - a^2*(-(ArcCot[a*x]
^2/x) + (a*ArcCot[a*x]^3)/3 - 2*a*((I/2)*ArcCot[a*x]^2 + I*((-I)*ArcCot[a*
x]*Log[2 - 2/(1 - I*a*x)] + PolyLog[2, -1 + 2/(1 - I*a*x)]/2))) - (2*a*(-1
/2*ArcCot[a*x]/x^2 - (a*(-x^(-1) - a*ArcTan[a*x]))/2 - a^2*((I/2)*ArcCot[a
*x]^2 + I*((-I)*ArcCot[a*x]*Log[2 - 2/(1 - I*a*x)] + PolyLog[2, -1 + 2/(1
- I*a*x)]/2))))/3))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5404
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcCot[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] + Si
mp[b*c*(p/d)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5420
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5454
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcCot[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcCot[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5460
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*d*(p + 1))), x] + Simp[
I/d   Int[(a + b*ArcCot[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 15.47 (sec) , antiderivative size = 852, normalized size of antiderivative = 5.61

method	result	size
parts	Expression too large to display	852
derivativedivides	Expression too large to display	855
default	Expression too large to display	855

input
```
int(arccot(a*x)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arccot(a*x)^3/x^4-3/4*a^4*(-1/3*arccot(a*x)^2/a^3/x^3+arccot(a*x)^2/x
/a+arccot(a*x)^2*arctan(a*x)-1/2*Pi*arccot(a*x)^2+8/3*arccot(a*x)*ln(1+I*(
I+a*x)/(a^2*x^2+1)^(1/2))+8/3*arccot(a*x)*ln(1-I*(I+a*x)/(a^2*x^2+1)^(1/2)
)-4/3*I*arccot(a*x)^2-1/3*arccot(a*x)^3+1/4*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1
))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*arccot(a*x)^2
+1/2*Pi*csgn(I*(I+a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(I+a*x)^2/(a^2*x^2+1))^2*
arccot(a*x)^2+1/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x)^2/(a
^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))^2*arccot(a*x)^2-1/4*Pi*csgn(I*(I+a*x)
/(a^2*x^2+1)^(1/2))^2*csgn(I*(I+a*x)^2/(a^2*x^2+1))*arccot(a*x)^2-2/3*arcc
ot(a*x)*(I+a*x)/a/x-1/3*I/a/x*(a*x-I)+1/3*arccot(a*x)*(I+a*x)^2/a^2/x^2+2/
3*arccot(a*x)*(I+a*x)*(a*x-I)/a^2/x^2-I*arccot(a*x)^2*ln((I+a*x)/(a^2*x^2+
1)^(1/2))-8/3*I*dilog(1+I*(I+a*x)/(a^2*x^2+1)^(1/2))-1/4*Pi*csgn(I*(I+a*x)
^2/(a^2*x^2+1))^3*arccot(a*x)^2-1/4*Pi*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*
x)^2/(a^2*x^2+1)-1))^3*arccot(a*x)^2-1/2*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-
1))^3*arccot(a*x)^2-1/4*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))*csgn(I*(I+a*x
)^2/(a^2*x^2+1))*csgn(I*(I+a*x)^2/(a^2*x^2+1)/((I+a*x)^2/(a^2*x^2+1)-1))*a
rccot(a*x)^2+1/2*Pi*csgn(I/((I+a*x)^2/(a^2*x^2+1)-1))^2*arccot(a*x)^2-8/3*
I*dilog(1-I*(I+a*x)/(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^5}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^5}\,dx$$

input
```
integrate(arccot(a*x)^3/x^5,x, algorithm="fricas")
```

output
```
integral(arccot(a*x)^3/x^5, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^5}\,dx = \int \frac{\operatorname{acot}^3(ax)}{x^5}\,dx$$

input
```
integrate(acot(a*x)**3/x**5,x)
```

output
```
Integral(acot(a*x)**3/x**5, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(arccot(a*x)^3/x^5,x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^5}\,dx = \int \frac{\operatorname{arccot}(ax)^3}{x^5}\,dx$$

input
```
integrate(arccot(a*x)^3/x^5,x, algorithm="giac")
```

output
```
integrate(arccot(a*x)^3/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax)^3}{x^5}\,dx = \int \frac{\mathrm{acot}(a\,x)^3}{x^5}\,dx$$

input
```
int(acot(a*x)^3/x^5,x)
```

output
```
int(acot(a*x)^3/x^5, x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(ax)^3}{x^5}\,dx$$
$$= \frac{acot(ax)^3\,a^4x^4 - acot(ax)^3 - 3acot(ax)^2\,a^3x^3 + acot(ax)^2\,ax + 5acot(ax)\,a^4x^4 + 3acot(ax)\,a^2x^2 - 2ac}{4x^4}$$

input
```
int(acot(a*x)^3/x^5,x)
```

output
```
(acot(a*x)**3*a**4*x**4 - acot(a*x)**3 - 3*acot(a*x)**2*a**3*x**3 + acot(a
*x)**2*a*x + 5*acot(a*x)*a**4*x**4 + 3*acot(a*x)*a**2*x**2 - 2*acot(a*x) -
 6*atan(1/(a*x))*a**4*x**4 - 4*atan(1/(a*x))*a**2*x**2 + 2*atan(1/(a*x)) -
 8*int(atan(1/(a*x))/(a**2*x**3 + x),x)*a**4*x**4 + a**3*x**3)/(4*x**4)
```

3.34 $\int x^m \cot^{-1}(ax)^3\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \cot^{-1}(ax)^3\,dx = \text{Int}\left(x^m \cot^{-1}(ax)^3, x\right)$$

output
```
Defer(Int)(x^m*arccot(a*x)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^3\,dx = \int x^m \cot^{-1}(ax)^3\,dx$$

input
```
Integrate[x^m*ArcCot[a*x]^3,x]
```

output
```
Integrate[x^m*ArcCot[a*x]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cot^{-1}(ax)^3 \, dx$$

$$\Big\downarrow \text{5378}$$

$$\int x^m \cot^{-1}(ax)^3 dx$$

input
```
Int[x^m*ArcCot[a*x]^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{arccot}(ax)^3 \, dx$$

input
```
int(x^m*arccot(a*x)^3,x)
```

output
```
int(x^m*arccot(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^m*arccot(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^m*arccot(a*x)^3, x)
```

Sympy [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \operatorname{acot}^3(ax) \, dx$$

input
```
integrate(x**m*acot(a*x)**3,x)
```

output
```
Integral(x**m*acot(a*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 221, normalized size of antiderivative = 22.10

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^m*arccot(a*x)^3,x, algorithm="maxima")
```

output
```
1/32*(4*x*x^m*arctan2(1, a*x)^3 - 3*x*x^m*arctan2(1, a*x)*log(a^2*x^2 + 1)
^2 + 32*(m + 1)*integrate(1/32*(12*a^2*x^2*x^m*arctan2(1, a*x)*log(a^2*x^2
 + 1) + 3*((a^2*m*arctan2(1, a*x) + a^2*arctan2(1, a*x))*x^2 - a*x + m*arc
tan2(1, a*x) + arctan2(1, a*x))*x^m*log(a^2*x^2 + 1)^2 + 4*(3*a*x*arctan2(
1, a*x)^2 + 7*m*arctan2(1, a*x)^3 + 7*(a^2*m*arctan2(1, a*x)^3 + a^2*arcta
n2(1, a*x)^3)*x^2 + 7*arctan2(1, a*x)^3)*x^m)/((a^2*m + a^2)*x^2 + m + 1),
 x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \operatorname{arccot}(ax)^3 \, dx$$

input
```
integrate(x^m*arccot(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^m*arccot(a*x)^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m \operatorname{acot}(a\,x)^3 \, dx$$

input
```
int(x^m*acot(a*x)^3,x)
```

output
```
int(x^m*acot(a*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^3 \, dx = \int x^m acot(ax)^3 \, dx$$

input
```
int(x^m*acot(a*x)^3,x)
```

output
```
int(x**m*acot(a*x)**3,x)
```

3.35 $\int x^m \cot^{-1}(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \cot^{-1}(ax)^2 \, dx = \text{Int}\left(x^m \cot^{-1}(ax)^2, x\right)$$

output
```
Defer(Int)(x^m*arccot(a*x)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m \cot^{-1}(ax)^2 \, dx$$

input
```
Integrate[x^m*ArcCot[a*x]^2,x]
```

output
```
Integrate[x^m*ArcCot[a*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cot^{-1}(ax)^2\,dx$$
$$\Big\downarrow \text{ 5378}$$
$$\int x^m \cot^{-1}(ax)^2dx$$

input
```
Int[x^m*ArcCot[a*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{arccot}(ax)^2\,dx$$

input
```
int(x^m*arccot(a*x)^2,x)
```

output
```
int(x^m*arccot(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x^m*arccot(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^m*arccot(a*x)^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m \operatorname{acot}^2(ax) \, dx$$

input
```
integrate(x**m*acot(a*x)**2,x)
```

output
```
Integral(x**m*acot(a*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 183, normalized size of antiderivative = 18.30

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m \operatorname{arccot}(ax)^2 \, dx$$

input
```
integrate(x^m*arccot(a*x)^2,x, algorithm="maxima")
```

output
```
1/16*(4*x*x^m*arctan2(1, a*x)^2 - x*x^m*log(a^2*x^2 + 1)^2 + 16*(m + 1)*in
tegrate(1/16*(4*a^2*x^2*x^m*log(a^2*x^2 + 1) + ((a^2*m + a^2)*x^2 + m + 1)
*x^m*log(a^2*x^2 + 1)^2 + 4*(3*(a^2*m*arctan2(1, a*x)^2 + a^2*arctan2(1, a
*x)^2)*x^2 + 2*a*x*arctan2(1, a*x) + 3*m*arctan2(1, a*x)^2 + 3*arctan2(1,
a*x)^2)*x^m)/((a^2*m + a^2)*x^2 + m + 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^2\,dx = \int x^m \operatorname{arccot}(ax)^2\,dx$$

input
```
integrate(x^m*arccot(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^m*arccot(a*x)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^2\,dx = \int x^m \operatorname{acot}(a\,x)^2\,dx$$

input
```
int(x^m*acot(a*x)^2,x)
```

output
```
int(x^m*acot(a*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \cot^{-1}(ax)^2 \, dx = \int x^m acot(ax)^2 \, dx$$

input
```
int(x^m*acot(a*x)^2,x)
```

output
```
int(x**m*acot(a*x)**2,x)
```

3.36 $\int x^m \cot^{-1}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 57

$$\int x^m \cot^{-1}(ax)\,dx=\frac{x^{1+m}\cot^{-1}(ax)}{1+m}+\frac{ax^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}$$

output
```
x^(1+m)*arccot(a*x)/(1+m)+a*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-a^2*
x^2)/(m^2+3*m+2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int x^m \cot^{-1}(ax)\,dx = \frac{x^{1+m}\left((2+m)\cot^{-1}(ax)+ax\,\text{Hypergeometric2F1}\left(1,1+\frac{m}{2},2+\frac{m}{2},-a^2x^2\right)\right)}{(1+m)(2+m)}$$

input
```
Integrate[x^m*ArcCot[a*x],x]
```

output
```
(x^(1 + m)*((2 + m)*ArcCot[a*x] + a*x*Hypergeometric2F1[1, 1 + m/2, 2 + m/
2, -(a^2*x^2)]))/((1 + m)*(2 + m))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5362, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cot^{-1}(ax) \, dx$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{a \int \frac{x^{m+1}}{a^2x^2+1}dx}{m+1} + \frac{x^{m+1}\cot^{-1}(ax)}{m+1}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{ax^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{(m+1)(m+2)} + \frac{x^{m+1}\cot^{-1}(ax)}{m+1}$$

input
```
Int[x^m*ArcCot[a*x],x]
```

output
```
(x^(1 + m)*ArcCot[a*x])/(1 + m) + (a*x^(2 + m)*Hypergeometric2F1[1, (2 + m
)/2, (4 + m)/2, -(a^2*x^2)])/((1 + m)*(2 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [F]

$$\int x^m \operatorname{arccot}(ax)\, dx$$

input
```
int(x^m*arccot(a*x),x)
```

output
```
int(x^m*arccot(a*x),x)
```

Fricas [F]

$$\int x^m \cot^{-1}(ax)\, dx = \int x^m \operatorname{arccot}(ax)\, dx$$

input
```
integrate(x^m*arccot(a*x),x, algorithm="fricas")
```

output
```
integral(x^m*arccot(a*x), x)
```

Sympy [F]

$$\int x^m \cot^{-1}(ax)\,dx = \int x^m \operatorname{acot}(ax)\,dx$$

input
```
integrate(x**m*acot(a*x),x)
```

output
```
Integral(x**m*acot(a*x), x)
```

Maxima [F]

$$\int x^m \cot^{-1}(ax)\,dx = \int x^m \operatorname{arccot}(ax)\,dx$$

input
```
integrate(x^m*arccot(a*x),x, algorithm="maxima")
```

output
```
(x*x^m*arctan2(1, a*x) + (a*m + a)*integrate(x*x^m/((a^2*m + a^2)*x^2 + m
+ 1), x))/(m + 1)
```

Giac [F]

$$\int x^m \cot^{-1}(ax)\,dx = \int x^m \operatorname{arccot}(ax)\,dx$$

input
```
integrate(x^m*arccot(a*x),x, algorithm="giac")
```

output
```
integrate(x^m*arccot(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cot^{-1}(ax)\,dx = \int x^m \operatorname{acot}(a\,x)\,dx$$

input
```
int(x^m*acot(a*x),x)
```

output
```
int(x^m*acot(a*x), x)
```

Reduce [F]

$$\int x^m \cot^{-1}(ax)\,dx = \frac{x^m acot(ax)\,amx + x^m - \left(\int \frac{x^m}{a^2m\,x^3+a^2x^3+mx+x}dx\right)m^2 - \left(\int \frac{x^m}{a^2m\,x^3+a^2x^3+mx+x}dx\right)m}{am\,(m+1)}$$

input
```
int(x^m*acot(a*x),x)
```

output
```
(x**m*acot(a*x)*a*m*x + x**m - int(x**m/(a**2*m*x**3 + a**2*x**3 + m*x + x
),x)*m**2 - int(x**m/(a**2*m*x**3 + a**2*x**3 + m*x + x),x)*m)/(a*m*(m + 1
))
```

3.37 $\int x^5 \cot^{-1}(ax^2)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int x^5 \cot^{-1}(ax^2)\, dx = \frac{x^4}{12a} + \frac{1}{6}x^6 \cot^{-1}(ax^2) - \frac{\log(1+a^2x^4)}{12a^3}$$

output
```
1/12*x^4/a+1/6*x^6*arccot(a*x^2)-1/12*ln(a^2*x^4+1)/a^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int x^5 \cot^{-1}(ax^2)\, dx = \frac{x^4}{12a} + \frac{1}{6}x^6 \cot^{-1}(ax^2) - \frac{\log(1+a^2x^4)}{12a^3}$$

input
```
Integrate[x^5*ArcCot[a*x^2],x]
```

output
```
x^4/(12*a) + (x^6*ArcCot[a*x^2])/6 - Log[1 + a^2*x^4]/(12*a^3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5362, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \cot^{-1}\left(ax^2\right) dx$$

$$\Big\downarrow \text{ 5362}$$

$$\frac{1}{3}a\int \frac{x^7}{a^2x^4+1}dx+\frac{1}{6}x^6\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{12}a\int \frac{x^4}{a^2x^4+1}dx^4+\frac{1}{6}x^6\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{12}a\int \left(\frac{1}{a^2}-\frac{1}{a^2\left(a^2x^4+1\right)}\right)dx^4+\frac{1}{6}x^6\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{12}a\left(\frac{x^4}{a^2}-\frac{\log\left(a^2x^4+1\right)}{a^4}\right)+\frac{1}{6}x^6\cot^{-1}\left(ax^2\right)$$

input
```
Int[x^5*ArcCot[a*x^2],x]
```

output
```
(x^6*ArcCot[a*x^2])/6 + (a*(x^4/a^2 - Log[1 + a^2*x^4]/a^4))/12
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

method	result	size
parallelrisch	$-\frac{-2x^{6} \operatorname{arccot}\left(a\,x^{2}\right) a^{3}-a^{2} x^{4}+\ln\left(a^{2} x^{4}+1\right)}{12 a^{3}}$	39
default	$\frac{x^{6} \operatorname{arccot}\left(a\,x^{2}\right)}{6}+\frac{a \left(\frac{x^{4}}{4 a^{2}}-\frac{\ln\left(a^{2} x^{4}+1\right)}{4 a^{4}}\right)}{3}$	40
parts	$\frac{x^{6} \operatorname{arccot}\left(a\,x^{2}\right)}{6}+\frac{a \left(\frac{x^{4}}{4 a^{2}}-\frac{\ln\left(a^{2} x^{4}+1\right)}{4 a^{4}}\right)}{3}$	40
risch	$\frac{i x^{6} \ln\left(i a\,x^{2}+1\right)}{12}-\frac{i x^{6} \ln\left(-i a\,x^{2}+1\right)}{12}+\frac{\pi\,x^{6}}{12}+\frac{x^{4}}{12 a}-\frac{\ln\left(-a^{2} x^{4}-1\right)}{12 a^{3}}$	64

input
```
int(x^5*arccot(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-2*x^6*arccot(a*x^2)*a^3-a^2*x^4+ln(a^2*x^4+1))/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int x^5 \cot^{-1}(ax^2)\ dx = \frac{2\,a^3x^6 \operatorname{arccot}(ax^2) + a^2x^4 - \log(a^2x^4+1)}{12\,a^3}$$

input
```
integrate(x^5*arccot(a*x^2),x, algorithm="fricas")
```

output
```
1/12*(2*a^3*x^6*arccot(a*x^2) + a^2*x^4 - log(a^2*x^4 + 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int x^5 \cot^{-1}(ax^2)\ dx = \begin{cases} \frac{x^6 \operatorname{acot}(ax^2)}{6} + \frac{x^4}{12a} - \frac{\log(a^2x^4+1)}{12a^3} & \text{for } a \neq 0 \\ \frac{\pi x^6}{12} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*acot(a*x**2),x)
```

output
```
Piecewise((x**6*acot(a*x**2)/6 + x**4/(12*a) - log(a**2*x**4 + 1)/(12*a**3
), Ne(a, 0)), (pi*x**6/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int x^5 \cot^{-1}\left(ax^2\right) dx = \frac{1}{6}x^6 \operatorname{arccot}\left(ax^2\right) + \frac{1}{12}\left(\frac{x^4}{a^2} - \frac{\log\left(a^2x^4+1\right)}{a^4}\right)a$$

input
```
integrate(x^5*arccot(a*x^2),x, algorithm="maxima")
```

output
```
1/6*x^6*arccot(a*x^2) + 1/12*(x^4/a^2 - log(a^2*x^4 + 1)/a^4)*a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int x^5 \cot^{-1}\left(ax^2\right) dx = \frac{1}{6}x^6 \arctan\left(\frac{1}{ax^2}\right) + \frac{1}{12}\left(\frac{x^4}{a^2} - \frac{\log\left(a^2x^4+1\right)}{a^4}\right)a$$

input
```
integrate(x^5*arccot(a*x^2),x, algorithm="giac")
```

output
```
1/6*x^6*arctan(1/(a*x^2)) + 1/12*(x^4/a^2 - log(a^2*x^4 + 1)/a^4)*a
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int x^5 \cot^{-1}\left(ax^2\right) dx = \frac{x^6 \operatorname{acot}(a\,x^2)}{6} - \frac{\ln\left(a^2\,x^4+1\right)}{12\,a^3} + \frac{x^4}{12\,a}$$

input
```
int(x^5*acot(a*x^2),x)
```

output
```
(x^6*acot(a*x^2))/6 - log(a^2*x^4 + 1)/(12*a^3) + x^4/(12*a)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int x^5 \cot^{-1}(ax^2)\, dx = \frac{2acot(a\,x^2)\,a^3x^6 - \log(-\sqrt{a}\,\sqrt{2}\,x + a\,x^2 + 1) - \log(\sqrt{a}\,\sqrt{2}\,x + a\,x^2 + 1) + a^2x^4}{12a^3}$$

input
```
int(x^5*acot(a*x^2),x)
```

output
```
(2*acot(a*x**2)*a**3*x**6 - log( - sqrt(a)*sqrt(2)*x + a*x**2 + 1) - log(s
qrt(a)*sqrt(2)*x + a*x**2 + 1) + a**2*x**4)/(12*a**3)
```

3.38 $\int x^3 \cot^{-1}(ax^2) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 37

$$\int x^3 \cot^{-1}(ax^2) \, dx = \frac{x^2}{4a} + \frac{1}{4}x^4 \cot^{-1}(ax^2) - \frac{\arctan(ax^2)}{4a^2}$$

output
```
1/4*x^2/a+1/4*x^4*arccot(a*x^2)-1/4*arctan(a*x^2)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x^3 \cot^{-1}(ax^2) \, dx = \frac{x^2}{4a} + \frac{1}{4}x^4 \cot^{-1}(ax^2) - \frac{\arctan(ax^2)}{4a^2}$$

input
```
Integrate[x^3*ArcCot[a*x^2],x]
```

output
```
x^2/(4*a) + (x^4*ArcCot[a*x^2])/4 - ArcTan[a*x^2]/(4*a^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5362, 807, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cot^{-1}\left(ax^2\right) dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{2}a\int \frac{x^5}{a^2x^4+1}dx+\frac{1}{4}x^4\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{4}a\int \frac{x^4}{a^2x^4+1}dx^2+\frac{1}{4}x^4\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{4}a\left(\frac{x^2}{a^2}-\frac{\int \frac{1}{a^2x^4+1}dx^2}{a^2}\right)+\frac{1}{4}x^4\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}a\left(\frac{x^2}{a^2}-\frac{\arctan\left(ax^2\right)}{a^3}\right)+\frac{1}{4}x^4\cot^{-1}\left(ax^2\right)$$

input
```
Int[x^3*ArcCot[a*x^2],x]
```

output
```
(x^4*ArcCot[a*x^2])/4 + (a*(x^2/a^2 - ArcTan[a*x^2]/a^3))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$\frac{\operatorname{arccot}\left(a\,x^{2}\right) a^{2} x^{4}+a\,x^{2}+\operatorname{arccot}\left(a\,x^{2}\right)}{4 a^{2}}$	31
default	$\frac{x^{4} \operatorname{arccot}\left(a\,x^{2}\right)}{4}+\frac{a \left(\frac{x^{2}}{2 a^{2}}-\frac{\arctan\left(a\,x^{2}\right)}{2 a^{3}}\right)}{2}$	36
parts	$\frac{x^{4} \operatorname{arccot}\left(a\,x^{2}\right)}{4}+\frac{a \left(\frac{x^{2}}{2 a^{2}}-\frac{\arctan\left(a\,x^{2}\right)}{2 a^{3}}\right)}{2}$	36
risch	$\frac{i x^{4} \ln\left(i a\,x^{2}+1\right)}{8}-\frac{i x^{4} \ln\left(-i a\,x^{2}+1\right)}{8}+\frac{\pi\,x^{4}}{8}+\frac{x^{2}}{4 a}-\frac{\arctan\left(a\,x^{2}\right)}{4 a^{2}}+\frac{1}{8 \pi\,a^{2}}$	67
orering	$\frac{5 \left(a^{2} x^{4}+1\right) \operatorname{arccot}\left(a\,x^{2}\right)}{8 a^{2}}-\frac{\left(a^{2} x^{4}+1\right) \left(3 x^{2} \operatorname{arccot}\left(a\,x^{2}\right)-\frac{2 x^{4} a}{a^{2} x^{4}+1}\right)}{8 x^{2} a^{2}}$	68

input
```
int(x^3*arccot(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(arccot(a*x^2)*a^2*x^4+a*x^2+arccot(a*x^2))/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int x^3 \cot^{-1}\left(ax^2\right)\,dx = \frac{ax^2 + (a^2x^4 + 1)\,\text{arccot}\left(ax^2\right)}{4\,a^2}$$

input
```
integrate(x^3*arccot(a*x^2),x, algorithm="fricas")
```

output
```
1/4*(a*x^2 + (a^2*x^4 + 1)*arccot(a*x^2))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int x^3 \cot^{-1}\left(ax^2\right)\,dx = \begin{cases} \frac{x^4\,\text{acot}\left(ax^2\right)}{4} + \frac{x^2}{4a} + \frac{\text{acot}\left(ax^2\right)}{4a^2} & \text{for } a \neq 0 \\ \frac{\pi x^4}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acot(a*x**2),x)
```

output
```
Piecewise((x**4*acot(a*x**2)/4 + x**2/(4*a) + acot(a*x**2)/(4*a**2), Ne(a,
 0)), (pi*x**4/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x^3 \cot^{-1}(ax^2)\, dx = \frac{1}{4}x^4 \operatorname{arccot}(ax^2) + \frac{1}{4}a\left(\frac{x^2}{a^2} - \frac{\arctan(ax^2)}{a^3}\right)$$

input
```
integrate(x^3*arccot(a*x^2),x, algorithm="maxima")
```

output
```
1/4*x^4*arccot(a*x^2) + 1/4*a*(x^2/a^2 - arctan(a*x^2)/a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int x^3 \cot^{-1}(ax^2)\, dx = \frac{1}{4}\left(\frac{x^4 \arctan\left(\frac{1}{ax^2}\right)}{a} + \frac{x^2}{a^2} + \frac{\arctan\left(\frac{1}{ax^2}\right)}{a^3}\right)a$$

input
```
integrate(x^3*arccot(a*x^2),x, algorithm="giac")
```

output
```
1/4*(x^4*arctan(1/(a*x^2))/a + x^2/a^2 + arctan(1/(a*x^2))/a^3)*a
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int x^3 \cot^{-1}(ax^2)\, dx = \frac{x^4 \operatorname{acot}(a\,x^2)}{4} - \frac{\operatorname{atan}(a\,x^2)}{4\,a^2} + \frac{x^2}{4\,a}$$

input
```
int(x^3*acot(a*x^2),x)
```

output
```
(x^4*acot(a*x^2))/4 - atan(a*x^2)/(4*a^2) + x^2/(4*a)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int x^3 \cot^{-1}\left(ax^2\right) \, dx = \frac{acot(a\,x^2)\,a^2x^4 + acot(a\,x^2) + a\,x^2}{4a^2}$$

input
```
int(x^3*acot(a*x^2),x)
```

output
```
(acot(a*x**2)*a**2*x**4 + acot(a*x**2) + a*x**2)/(4*a**2)
```

3.39 $\int x \cot^{-1}(ax^2)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int x \cot^{-1}(ax^2)\,dx = \frac{1}{2}x^2 \cot^{-1}(ax^2) + \frac{\log(1+a^2x^4)}{4a}$$

output
```
1/2*x^2*arccot(a*x^2)+1/4*ln(a^2*x^4+1)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int x \cot^{-1}(ax^2)\,dx = \frac{1}{2}x^2 \cot^{-1}(ax^2) + \frac{\log(1+a^2x^4)}{4a}$$

input
```
Integrate[x*ArcCot[a*x^2],x]
```

output
```
(x^2*ArcCot[a*x^2])/2 + Log[1 + a^2*x^4]/(4*a)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5362, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cot^{-1}\left(ax^2\right) dx$$

$$\Big\downarrow \ 5362$$

$$a \int \frac{x^3}{a^2x^4+1} dx + \frac{1}{2}x^2 \cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \ 792$$

$$\frac{\log\left(a^2x^4+1\right)}{4a} + \frac{1}{2}x^2 \cot^{-1}\left(ax^2\right)$$

input
```
Int[x*ArcCot[a*x^2],x]
```

output
```
(x^2*ArcCot[a*x^2])/2 + Log[1 + a^2*x^4]/(4*a)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
parts	$\frac{x^{2} \operatorname{arccot}\left(a \,x^{2}\right)}{2}+\frac{\ln \left(a^{2} x^{4}+1\right)}{4 a}$	28
parallelrisch	$\frac{2 \operatorname{arccot}\left(a \,x^{2}\right) a \,x^{2}+\ln \left(a^{2} x^{4}+1\right)}{4 a}$	29
derivativedivides	$\frac{\operatorname{arccot}\left(a \,x^{2}\right) a \,x^{2}+\frac{\ln \left(a^{2} x^{4}+1\right)}{2}}{2 a}$	30
default	$\frac{\operatorname{arccot}\left(a \,x^{2}\right) a \,x^{2}+\frac{\ln \left(a^{2} x^{4}+1\right)}{2}}{2 a}$	30
risch	$\frac{i x^{2} \ln \left(i a \,x^{2}+1\right)}{4}-\frac{i x^{2} \ln \left(-i a \,x^{2}+1\right)}{4}+\frac{\pi \,x^{2}}{4}+\frac{\ln \left(-a^{2} x^{4}-1\right)}{4 a}$	56

input
```
int(x*arccot(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arccot(a*x^2)+1/4*ln(a^2*x^4+1)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int x \cot^{-1}\left(ax^2\right)\,dx=\frac{2\,ax^2\operatorname{arccot}\left(ax^2\right)+\log\left(a^2x^4+1\right)}{4\,a}$$

input
```
integrate(x*arccot(a*x^2),x, algorithm="fricas")
```

output
```
1/4*(2*a*x^2*arccot(a*x^2) + log(a^2*x^4 + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int x \cot^{-1}\left(ax^2\right) dx = \begin{cases} \frac{x^2 \operatorname{acot}\left(ax^2\right)}{2} + \frac{\log\left(a^2x^4+1\right)}{4a} & \text{for } a \neq 0 \\ \frac{\pi x^2}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acot(a*x**2),x)
```

output
```
Piecewise((x**2*acot(a*x**2)/2 + log(a**2*x**4 + 1)/(4*a), Ne(a, 0)), (pi*
x**2/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int x \cot^{-1}\left(ax^2\right) dx = \frac{2\,ax^2 \operatorname{arccot}\left(ax^2\right) + \log\left(a^2x^4+1\right)}{4\,a}$$

input
```
integrate(x*arccot(a*x^2),x, algorithm="maxima")
```

output
```
1/4*(2*a*x^2*arccot(a*x^2) + log(a^2*x^4 + 1))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int x \cot^{-1}\left(ax^2\right) dx = \frac{1}{4}\left(\frac{2\,x^2 \arctan\left(\frac{1}{ax^2}\right)}{a} + \frac{\log\left(\frac{1}{a^2x^4}+1\right)}{a^2} - \frac{\log\left(\frac{1}{a^2x^4}\right)}{a^2}\right)a$$

input
```
integrate(x*arccot(a*x^2),x, algorithm="giac")
```

output
```
1/4*(2*x^2*arctan(1/(a*x^2))/a + log(1/(a^2*x^4) + 1)/a^2 - log(1/(a^2*x^4
))/a^2)*a
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int x\cot^{-1}\left(ax^2\right)\,dx=\frac{x^2\,\mathrm{acot}(a\,x^2)}{2}+\frac{\ln\left(a^2\,x^4+1\right)}{4\,a}$$

input
```
int(x*acot(a*x^2),x)
```

output
```
(x^2*acot(a*x^2))/2 + log(a^2*x^4 + 1)/(4*a)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int x\cot^{-1}\left(ax^2\right)\,dx$$
$$=\frac{2acot(a\,x^2)\,a\,x^2+\log\left(-\sqrt{a}\,\sqrt{2}\,x+a\,x^2+1\right)+\log\left(\sqrt{a}\,\sqrt{2}\,x+a\,x^2+1\right)}{4a}$$

input
```
int(x*acot(a*x^2),x)
```

output
```
(2*acot(a*x**2)*a*x**2 + log( - sqrt(a)*sqrt(2)*x + a*x**2 + 1) + log(sqrt
(a)*sqrt(2)*x + a*x**2 + 1))/(4*a)
```

3.40 $\int \frac{\cot^{-1}(ax^2)}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 37

$$\int \frac{\cot^{-1}(ax^2)}{x} \, dx = -\frac{1}{4} i \operatorname{PolyLog}\left(2, -\frac{i}{ax^2}\right) + \frac{1}{4} i \operatorname{PolyLog}\left(2, \frac{i}{ax^2}\right)$$

output
```
-1/4*I*polylog(2,-I/a/x^2)+1/4*I*polylog(2,I/a/x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax^2)}{x} \, dx = -\frac{1}{4} i \operatorname{PolyLog}\left(2, -\frac{i}{ax^2}\right) + \frac{1}{4} i \operatorname{PolyLog}\left(2, \frac{i}{ax^2}\right)$$

input
```
Integrate[ArcCot[a*x^2]/x,x]
```

output
```
(-1/4*I)*PolyLog[2, (-I)/(a*x^2)] + (I/4)*PolyLog[2, I/(a*x^2)]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5360, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax^2)}{x} \, dx$$

$$\Big\downarrow \text{ 5360}$$

$$\frac{1}{2}\int \frac{\cot^{-1}(ax^2)}{x^2}dx^2$$

$$\Big\downarrow \text{ 5356}$$

$$\frac{1}{2}\left(\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{ax^2}\right)}{x^2}dx^2-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{x^2a}\right)}{x^2}dx^2\right)$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{1}{2}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{ax^2}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{ax^2}\right)\right)$$

input
```
Int[ArcCot[a*x^2]/x,x]
```

output
```
((-1/2*I)*PolyLog[2, (-I)/(a*x^2)] + (I/2)*PolyLog[2, I/(a*x^2)])/2
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5360
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

method	result
default	$\ln(x)\operatorname{arccot}(a\,x^2)+\frac{\sum_{\_R1=\operatorname{RootOf}(a^2\_Z^4+1)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^2}}{2a}$
parts	$\ln(x)\operatorname{arccot}(a\,x^2)+\frac{\sum_{\_R1=\operatorname{RootOf}(a^2\_Z^4+1)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^2}}{2a}$
risch	$\frac{\pi\ln(x)}{2}-\frac{i\ln(-ia\,x^2+1)\ln(x)}{2}+\frac{i\ln(x)\ln(1-ix\sqrt{-ai})}{2}+\frac{i\ln(x)\ln(1+ix\sqrt{-ai})}{2}+\frac{i\operatorname{dilog}(1-ix\sqrt{-ai})}{2}+\frac{i\operatorname{dilog}(1+ix\sqrt{-}}{2}$

input
```
int(arccot(a*x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arccot(a*x^2)+1/2/a*sum(1/_R1^2*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)
/_R1)),_R1=RootOf(_Z^4*a^2+1))
```

Fricas [F]

$$\int \frac{\cot^{-1}(ax^2)}{x}\,dx=\int \frac{\operatorname{arccot}(ax^2)}{x}\,dx$$

input
```
integrate(arccot(a*x^2)/x,x, algorithm="fricas")
```

output
```
integral(arccot(a*x^2)/x, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax^2)}{x}\,dx = \int \frac{\operatorname{acot}(ax^2)}{x}\,dx$$

input
```
integrate(acot(a*x**2)/x,x)
```

output
```
Integral(acot(a*x**2)/x, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(23) = 46.

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.84

$$\int \frac{\cot^{-1}(ax^2)}{x}\,dx = \frac{1}{8}\pi\log\left(a^2x^4+1\right) - \frac{1}{2}\arctan\left(ax^2\right)\log\left(ax^2\right) + \operatorname{arccot}\left(ax^2\right)\log(x) + \arctan\left(ax^2\right)\log(x) + \frac{1}{4}i\operatorname{Li}_2\left(i\,ax^2+1\right) - \frac{1}{4}i\operatorname{Li}_2\left(-i\,ax^2+1\right)$$

input
```
integrate(arccot(a*x^2)/x,x, algorithm="maxima")
```

output
```
1/8*pi*log(a^2*x^4 + 1) - 1/2*arctan(a*x^2)*log(a*x^2) + arccot(a*x^2)*log
(x) + arctan(a*x^2)*log(x) + 1/4*I*dilog(I*a*x^2 + 1) - 1/4*I*dilog(-I*a*x
^2 + 1)
```

Giac [F]

$$\int \frac{\cot^{-1}(ax^2)}{x}\,dx = \int \frac{\operatorname{arccot}(ax^2)}{x}\,dx$$

input
```
integrate(arccot(a*x^2)/x,x, algorithm="giac")
```

output
```
integrate(arccot(a*x^2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax^2)}{x}\,dx = \int \frac{\operatorname{acot}(a\,x^2)}{x}\,dx$$

input
```
int(acot(a*x^2)/x,x)
```

output
```
int(acot(a*x^2)/x, x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(ax^2)}{x}\,dx = \int \frac{acot(a\,x^2)}{x}dx$$

input
```
int(acot(a*x^2)/x,x)
```

output
```
int(acot(a*x**2)/x,x)
```

3.41 $\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 34

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = -\frac{\cot^{-1}(ax^2)}{2x^2} - a\log(x) + \frac{1}{4}a\log\left(1+a^2x^4\right)$$

output
```
-1/2*arccot(a*x^2)/x^2-a*ln(x)+1/4*a*ln(a^2*x^4+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = -\frac{\cot^{-1}(ax^2)}{2x^2} - a\log(x) + \frac{1}{4}a\log\left(1+a^2x^4\right)$$

input
```
Integrate[ArcCot[a*x^2]/x^3,x]
```

output
```
-1/2*ArcCot[a*x^2]/x^2 - a*Log[x] + (a*Log[1 + a^2*x^4])/4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5362, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax^2)}{x^3}dx$$

$$\Big\downarrow \text{5362}$$

$$-a\int \frac{1}{x(a^2x^4+1)}dx-\frac{\cot^{-1}(ax^2)}{2x^2}$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{4}a\int \frac{1}{x^4(a^2x^4+1)}dx^4-\frac{\cot^{-1}(ax^2)}{2x^2}$$

$$\Big\downarrow \text{47}$$

$$-\frac{1}{4}a\left(\int \frac{1}{x^4}dx^4-a^2\int \frac{1}{a^2x^4+1}dx^4\right)-\frac{\cot^{-1}(ax^2)}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$-\frac{1}{4}a\left(\log(x^4)-a^2\int \frac{1}{a^2x^4+1}dx^4\right)-\frac{\cot^{-1}(ax^2)}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$-\frac{1}{4}a(\log(x^4)-\log(a^2x^4+1))-\frac{\cot^{-1}(ax^2)}{2x^2}$$

input
```
Int[ArcCot[a*x^2]/x^3,x]
```

output
```
-1/2*ArcCot[a*x^2]/x^2 - (a*(Log[x^4] - Log[1 + a^2*x^4]))/4
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{2x^{2}}-a\left(\ln\left(x\right)-\frac{\ln\left(a^{2}x^{4}+1\right)}{4}\right)$	31
parts	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{2x^{2}}-a\left(\ln\left(x\right)-\frac{\ln\left(a^{2}x^{4}+1\right)}{4}\right)$	31
parallelrisch	$-\frac{4a\ln(x)x^{2}-a\ln\left(a^{2}x^{4}+1\right)x^{2}+2\operatorname{arccot}\left(a\,x^{2}\right)}{4x^{2}}$	39
risch	$-\frac{i\ln\left(ia\,x^{2}+1\right)}{4x^{2}}-\frac{4a\ln(x)x^{2}-a\ln\left(a^{2}x^{4}+1\right)x^{2}-i\ln\left(-ia\,x^{2}+1\right)+\pi}{4x^{2}}$	62

input
```
int(arccot(a*x^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccot(a*x^2)/x^2-a*(ln(x)-1/4*ln(a^2*x^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = \frac{ax^2\log(a^2x^4+1) - 4\,ax^2\log(x) - 2\,\operatorname{arccot}(ax^2)}{4\,x^2}$$

input
```
integrate(arccot(a*x^2)/x^3,x, algorithm="fricas")
```

output
```
1/4*(a*x^2*log(a^2*x^4 + 1) - 4*a*x^2*log(x) - 2*arccot(a*x^2))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = -a\log(x) + \frac{a\log(a^2x^4+1)}{4} - \frac{\operatorname{acot}(ax^2)}{2x^2}$$

input
```
integrate(acot(a*x**2)/x**3,x)
```

output
```
-a*log(x) + a*log(a**2*x**4 + 1)/4 - acot(a*x**2)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = \frac{1}{4}\,a\left(\log\left(a^2x^4+1\right) - \log\left(x^4\right)\right) - \frac{\operatorname{arccot}(ax^2)}{2\,x^2}$$

input
```
integrate(arccot(a*x^2)/x^3,x, algorithm="maxima")
```

output
```
1/4*a*(log(a^2*x^4 + 1) - log(x^4)) - 1/2*arccot(a*x^2)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = -\frac{1}{4}\,a\left(\frac{2\,\arctan\left(\frac{1}{ax^2}\right)}{ax^2} - \log\left(\frac{1}{a^2x^4} + 1\right)\right)$$

input
```
integrate(arccot(a*x^2)/x^3,x, algorithm="giac")
```

output
```
-1/4*a*(2*arctan(1/(a*x^2))/(a*x^2) - log(1/(a^2*x^4) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = \frac{a\,\ln\left(-a^2\,x^4 - 1\right)}{4} - \frac{\mathrm{acot}(a\,x^2)}{2\,x^2} - a\,\ln\left(x\right)$$

input
```
int(acot(a*x^2)/x^3,x)
```

output
```
(a*log(- a^2*x^4 - 1))/4 - acot(a*x^2)/(2*x^2) - a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \frac{\cot^{-1}(ax^2)}{x^3}\,dx = \frac{-2acot(a\,x^2) + \log\left(-\sqrt{a}\,\sqrt{2}\,x + a\,x^2 + 1\right)a\,x^2 + \log\left(\sqrt{a}\,\sqrt{2}\,x + a\,x^2 + 1\right)a\,x^2 - 4\log(x)\,a\,x^2}{4x^2}$$

input
```
int(acot(a*x^2)/x^3,x)
```

output
```
( - 2*acot(a*x**2) + log( - sqrt(a)*sqrt(2)*x + a*x**2 + 1)*a*x**2 + log(s
qrt(a)*sqrt(2)*x + a*x**2 + 1)*a*x**2 - 4*log(x)*a*x**2)/(4*x**2)
```

3.42 $\int \frac{\cot^{-1}(ax^2)}{x^5} dx$

Optimal result

Integrand size = 10, antiderivative size = 35

$$\int \frac{\cot^{-1}(ax^2)}{x^5} dx = \frac{a}{4x^2} - \frac{\cot^{-1}(ax^2)}{4x^4} + \frac{1}{4}a^2 \arctan(ax^2)$$

output
```
1/4*a/x^2-1/4*arccot(a*x^2)/x^4+1/4*a^2*arctan(a*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{\cot^{-1}(ax^2)}{x^5} dx = -\frac{\cot^{-1}(ax^2)}{4x^4} + \frac{a \text{ Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -a^2x^4\right)}{4x^2}$$

input
```
Integrate[ArcCot[a*x^2]/x^5,x]
```

output
```
-1/4*ArcCot[a*x^2]/x^4 + (a*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^4)])/(
4*x^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5362, 807, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}\left(ax^2\right)}{x^5}dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{1}{2}a\int \frac{1}{x^3\left(a^2x^4+1\right)}dx-\frac{\cot^{-1}\left(ax^2\right)}{4x^4}$$

$$\Big\downarrow \text{807}$$

$$-\frac{1}{4}a\int \frac{1}{x^4\left(a^2x^4+1\right)}dx^2-\frac{\cot^{-1}\left(ax^2\right)}{4x^4}$$

$$\Big\downarrow \text{264}$$

$$-\frac{1}{4}a\left(a^2\left(-\int \frac{1}{a^2x^4+1}dx^2\right)-\frac{1}{x^2}\right)-\frac{\cot^{-1}\left(ax^2\right)}{4x^4}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{4}a\left(-a\arctan\left(ax^2\right)-\frac{1}{x^2}\right)-\frac{\cot^{-1}\left(ax^2\right)}{4x^4}$$

input
```
Int[ArcCot[a*x^2]/x^5,x]
```

output
```
-1/4*ArcCot[a*x^2]/x^4 - (a*(-x^(-2) - a*ArcTan[a*x^2]))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{4 x^{4}}-\frac{a \left(-\frac{a \arctan\left(a\,x^{2}\right)}{2}-\frac{1}{2 x^{2}}\right)}{2}$	31
parts	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{4 x^{4}}-\frac{a \left(-\frac{a \arctan\left(a\,x^{2}\right)}{2}-\frac{1}{2 x^{2}}\right)}{2}$	31
parallelrisch	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right) a^{2} x^{4}-a\,x^{2}+\operatorname{arccot}\left(a\,x^{2}\right)}{4 x^{4}}$	32
orering	$\frac{\left(-\frac{7}{8} x^{5} a^{2}-\frac{7}{8} x \right) \operatorname{arccot}\left(a\,x^{2}\right)}{x^{5}}-\frac{\left(a^{2} x^{4}+1\right) x^{2} \left(-\frac{2 a}{x^{4} \left(a^{2} x^{4}+1\right)}-\frac{5 \operatorname{arccot}\left(a\,x^{2}\right)}{x^{6}}\right)}{8}$	67
risch	$-\frac{i \ln\left(i a\,x^{2}+1\right)}{8 x^{4}}-\frac{-i a^{2} \ln\left(-a\,x^{2}-i\right) x^{4}+i a^{2} \ln\left(-a\,x^{2}+i\right) x^{4}-2 a\,x^{2}-i \ln\left(-i a\,x^{2}+1\right)+\pi}{8 x^{4}}$	82

input
```
int(arccot(a*x^2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arccot(a*x^2)/x^4-1/2*a*(-1/2*a*arctan(a*x^2)-1/2/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = \frac{ax^2 - (a^2x^4+1)\operatorname{arccot}(ax^2)}{4\,x^4}$$

input
```
integrate(arccot(a*x^2)/x^5,x, algorithm="fricas")
```

output
```
1/4*(a*x^2 - (a^2*x^4 + 1)*arccot(a*x^2))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = -\frac{a^2\operatorname{acot}(ax^2)}{4} + \frac{a}{4x^2} - \frac{\operatorname{acot}(ax^2)}{4x^4}$$

input
```
integrate(acot(a*x**2)/x**5,x)
```

output
```
-a**2*acot(a*x**2)/4 + a/(4*x**2) - acot(a*x**2)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = \frac{1}{4}\left(a\arctan\left(ax^2\right) + \frac{1}{x^2}\right)a - \frac{\operatorname{arccot}(ax^2)}{4\,x^4}$$

input
```
integrate(arccot(a*x^2)/x^5,x, algorithm="maxima")
```

output
```
1/4*(a*arctan(a*x^2) + 1/x^2)*a - 1/4*arccot(a*x^2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = \frac{1}{4}\left(a\arctan\left(ax^2\right) + \frac{1}{x^2}\right)a - \frac{\arctan\left(\frac{1}{ax^2}\right)}{4\,x^4}$$

input
```
integrate(arccot(a*x^2)/x^5,x, algorithm="giac")
```

output
```
1/4*(a*arctan(a*x^2) + 1/x^2)*a - 1/4*arctan(1/(a*x^2))/x^4
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = \frac{a\,x^2 - \operatorname{acot}(a\,x^2) + a^2\,x^4\,\operatorname{atan}(a\,x^2)}{4\,x^4}$$

input
```
int(acot(a*x^2)/x^5,x)
```

output
```
(a*x^2 - acot(a*x^2) + a^2*x^4*atan(a*x^2))/(4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}(ax^2)}{x^5}\,dx = \frac{-acot(a\,x^2)\,a^2x^4 - acot(a\,x^2) + a\,x^2}{4x^4}$$

input

```
int(acot(a*x^2)/x^5,x)
```

output

```
( - acot(a*x**2)*a**2*x**4 - acot(a*x**2) + a*x**2)/(4*x**4)
```

3.43 $\int x^4 \cot^{-1}(ax^2)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 119

$$\int x^4 \cot^{-1}(ax^2)\, dx = \frac{2x^3}{15a} + \frac{1}{5}x^5 \cot^{-1}(ax^2) + \frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{5\sqrt{2}a^{5/2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{5\sqrt{2}a^{5/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}x}{1+ax^2}\right)}{5\sqrt{2}a^{5/2}}$$

output
```
2/15*x^3/a+1/5*x^5*arccot(a*x^2)-1/10*arctan(-1+2^(1/2)*a^(1/2)*x)*2^(1/2)
/a^(5/2)-1/10*arctan(1+2^(1/2)*a^(1/2)*x)*2^(1/2)/a^(5/2)+1/10*arctanh(2^(
1/2)*a^(1/2)*x/(a*x^2+1))*2^(1/2)/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.14

$$\int x^4 \cot^{-1}(ax^2)\, dx = \frac{8a^{3/2}x^3 + 12a^{5/2}x^5\cot^{-1}(ax^2) + 6\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{a}x\right) - 6\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{a}x\right) - 3\sqrt{2}\log\left(1-\right.}{60a^{5/2}}$$

input
```
Integrate[x^4*ArcCot[a*x^2],x]
```

output
```
(8*a^(3/2)*x^3 + 12*a^(5/2)*x^5*ArcCot[a*x^2] + 6*Sqrt[2]*ArcTan[1 - Sqrt[
2]*Sqrt[a]*x] - 6*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[a]*x] - 3*Sqrt[2]*Log[1
- Sqrt[2]*Sqrt[a]*x + a*x^2] + 3*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[a]*x + a*x^2
])/(60*a^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.47, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5362, 843, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \cot^{-1}\left(ax^2\right) dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{2}{5}a\int \frac{x^6}{a^2x^4+1}dx+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\int \frac{x^2}{a^2x^4+1}dx}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{826}$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\int \frac{ax^2+1}{a^2x^4+1}dx}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\frac{\int \frac{1}{-\left(1-\sqrt{2}\sqrt{a}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}-\frac{\int \frac{1}{-\left(\sqrt{2}\sqrt{a}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \; 217$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \; 1479$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{-\frac{\int -\frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \; 25$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\int \frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \; 27$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\int \frac{\sqrt{2}-2\sqrt{a}x}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2\sqrt{2}a}+\frac{\int \frac{\sqrt{2}\sqrt{a}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \; 1103$$

$$\frac{2}{5}a\left(\frac{x^3}{3a^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}-\frac{\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}}{2a}}{a^2}\right)+\frac{1}{5}x^5\cot^{-1}\left(ax^2\right)$$

input
```
Int[x^4*ArcCot[a*x^2],x]
```

output
```
(x^5*ArcCot[a*x^2])/5 + (2*a*(x^3/(3*a^2) - ((-(ArcTan[1 - Sqrt[2]*Sqrt[a]
*x]/(Sqrt[2]*Sqrt[a])) + ArcTan[1 + Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]))/
(2*a) - (-1/2*Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2]/(Sqrt[2]*Sqrt[a]) + Log[1
 + Sqrt[2]*Sqrt[a]*x + a*x^2]/(2*Sqrt[2]*Sqrt[a]))/(2*a))/a^2))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{x^5 \operatorname{arccot}(a x^2)}{5}+\frac{2a\left(\frac{x^3}{3a^2}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^4\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}\right)}{5}$	112
parts	$\frac{x^5 \operatorname{arccot}(a x^2)}{5}+\frac{2a\left(\frac{x^3}{3a^2}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^4\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}\right)}{5}$	112

input
```
int(x^4*arccot(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*arccot(a*x^2)+2/5*a*(1/3*x^3/a^2-1/8/a^4/(1/a^2)^(1/4)*2^(1/2)*(ln
((x^2-(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2))/(x^2+(1/a^2)^(1/4)*x*2^(1/2)+
(1/a^2)^(1/2)))+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^
2)^(1/4)*x-1)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int x^4 \cot^{-1}\left(ax^2\right)\,dx$$
$$= \frac{12\,a^2x^5 \operatorname{arccot}\left(ax^2\right)+8\,ax^3-\frac{6\,\sqrt{2}\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}}-\frac{6\,\sqrt{2}\arctan\left(\sqrt{2}\sqrt{a}x-1\right)}{\sqrt{a}}+\frac{3\,\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}}-\frac{3\,\sqrt{2}\log}{}}{60\,a^2}$$

input
```
integrate(x^4*arccot(a*x^2),x, algorithm="fricas")
```

output
```
1/60*(12*a^2*x^5*arccot(a*x^2) + 8*a*x^3 - 6*sqrt(2)*arctan(sqrt(2)*sqrt(a
)*x + 1)/sqrt(a) - 6*sqrt(2)*arctan(sqrt(2)*sqrt(a)*x - 1)/sqrt(a) + 3*sqr
t(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/sqrt(a) - 3*sqrt(2)*log(a*x^2 - sq
rt(2)*sqrt(a)*x + 1)/sqrt(a))/a^2
```

Sympy [A] (verification not implemented)

Time = 12.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.14

$$\int x^4 \cot^{-1}\left(ax^2\right)\,dx = \begin{cases} \frac{x^5\,\mathrm{acot}\left(ax^2\right)}{5} + \frac{2x^3}{15a} - \frac{\log\left(x - \sqrt[4]{-\frac{1}{a^2}}\right)}{5a^3\sqrt[4]{-\frac{1}{a^2}}} + \frac{\log\left(x^2 + \sqrt{-\frac{1}{a^2}}\right)}{10a^3\sqrt[4]{-\frac{1}{a^2}}} - \frac{\mathrm{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{5a^3\sqrt[4]{-\frac{1}{a^2}}} - \frac{\mathrm{acot}\left(ax^2\right)}{5a^6\left(-\frac{1}{a^2}\right)^{\frac{7}{4}}} & \text{for } a \neq 0 \\ \frac{\pi x^5}{10} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*acot(a*x**2),x)
```

output
```
Piecewise((x**5*acot(a*x**2)/5 + 2*x**3/(15*a) - log(x - (-1/a**2)**(1/4))
/(5*a**3*(-1/a**2)**(1/4)) + log(x**2 + sqrt(-1/a**2))/(10*a**3*(-1/a**2)*
*(1/4)) - atan(x/(-1/a**2)**(1/4))/(5*a**3*(-1/a**2)**(1/4)) - acot(a*x**2
)/(5*a**6*(-1/a**2)**(7/4)), Ne(a, 0)), (pi*x**5/10, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

$$\int x^4 \cot^{-1}\left(ax^2\right)\,dx = \frac{1}{5}x^5\,\mathrm{arccot}\left(ax^2\right) + \frac{1}{60}a\left(\frac{8x^3}{a^2} - \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2ax-\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{a^{\frac{3}{2}}} + \frac{\sqrt{2}\log\left(ax^2\right.}{a}\right.}{a^2}\right.$$

input
```
integrate(x^4*arccot(a*x^2),x, algorithm="maxima")
```

output
```
1/5*x^5*arccot(a*x^2) + 1/60*a*(8*x^3/a^2 - 3*(2*sqrt(2)*arctan(1/2*sqrt(2
)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))/a^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2
)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))/a^(3/2) - sqrt(2)*log(a*x^2 + sqrt(2)
*sqrt(a)*x + 1)/a^(3/2) + sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/a^(3/
2))/a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.31

$$\int x^4 \cot^{-1}\left(ax^2\right) dx = \frac{1}{5} x^5 \arctan\left(\frac{1}{ax^2}\right) + \frac{1}{60} a \left(\frac{8 x^3}{a^2} - \frac{6 \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}\left(2 x + \frac{\sqrt{2}}{\sqrt{|a|}}\right) \sqrt{|a|}\right)}{a^2 |a|^{\frac{3}{2}}} - \frac{6 \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}\left(2 x - \frac{\sqrt{2}}{\sqrt{|a|}}\right) \sqrt{|a|}\right)}{a^2 |a|^{\frac{3}{2}}} + \frac{3 \sqrt{2}$$

input
```
integrate(x^4*arccot(a*x^2),x, algorithm="giac")
```

output
```
1/5*x^5*arctan(1/(a*x^2)) + 1/60*a*(8*x^3/a^2 - 6*sqrt(2)*arctan(1/2*sqrt(
2)*(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*abs(a)^(3/2)) - 6*sqrt(
2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*abs(
a)^(3/2)) + 3*sqrt(2)*sqrt(abs(a))*log(x^2 + sqrt(2)*x/sqrt(abs(a)) + 1/ab
s(a))/a^4 - 3*sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(a)) + 1/abs(a))/(a^2*ab
s(a)^(3/2)))
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.45

$$\int x^4 \cot^{-1}\left(ax^2\right) dx = \frac{x^5 \operatorname{acot}(a\, x^2)}{5} + \frac{2\, x^3}{15\, a} - \frac{(-1)^{1/4} \operatorname{atan}\left((-1)^{1/4} \sqrt{a}\, x\right)}{5\, a^{5/2}} - \frac{(-1)^{1/4} \operatorname{atan}\left((-1)^{1/4} \sqrt{a}\, x\, 1\mathrm{i}\right) 1\mathrm{i}}{5\, a^{5/2}}$$

input
```
int(x^4*acot(a*x^2),x)
```

output
```
(x^5*acot(a*x^2))/5 + (2*x^3)/(15*a) - ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)
*x))/(5*a^(5/2)) - ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x*1i)*1i)/(5*a^(5/2
))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.90

$$\int x^4 \cot^{-1}\left(ax^2\right)\,dx$$
$$= \frac{-6\sqrt{a}\,\sqrt{2}\,acot(a\,x^2)+12acot(a\,x^2)\,a^3x^5+12\sqrt{a}\,\sqrt{2}\,atan\left(\frac{\sqrt{a}\,\sqrt{2}-2ax}{\sqrt{a}\,\sqrt{2}}\right)-3\sqrt{a}\,\sqrt{2}\log\left(-\sqrt{a}\,\sqrt{2}\,x+a\,x^2\right.}{60a^3}$$

input
```
int(x^4*acot(a*x^2),x)
```

output
```
( - 6*sqrt(a)*sqrt(2)*acot(a*x**2) + 12*acot(a*x**2)*a**3*x**5 + 12*sqrt(a
)*sqrt(2)*atan((sqrt(a)*sqrt(2) - 2*a*x)/(sqrt(a)*sqrt(2))) - 3*sqrt(a)*sq
rt(2)*log( - sqrt(a)*sqrt(2)*x + a*x**2 + 1) + 3*sqrt(a)*sqrt(2)*log(sqrt(
a)*sqrt(2)*x + a*x**2 + 1) + 8*a**2*x**3)/(60*a**3)
```

3.44 $\int x^2 \cot^{-1}(ax^2)\, dx$

Optimal result

Integrand size = 10, antiderivative size = 117

$$\int x^2 \cot^{-1}(ax^2)\, dx = \frac{2x}{3a} + \frac{1}{3}x^3\cot^{-1}(ax^2) + \frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}a^{3/2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}a^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}x}{1+ax^2}\right)}{3\sqrt{2}a^{3/2}}$$

output
```
2/3*x/a+1/3*x^3*arccot(a*x^2)-1/6*arctan(-1+2^(1/2)*a^(1/2)*x)*2^(1/2)/a^(
3/2)-1/6*arctan(1+2^(1/2)*a^(1/2)*x)*2^(1/2)/a^(3/2)-1/6*arctanh(2^(1/2)*a
^(1/2)*x/(a*x^2+1))*2^(1/2)/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14

$$\int x^2 \cot^{-1}(ax^2)\, dx$$
$$= \frac{8\sqrt{a}x + 4a^{3/2}x^3\cot^{-1}(ax^2) + 2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{a}x\right) - 2\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{a}x\right) + \sqrt{2}\log\left(1-\sqrt{2}\right.}{12a^{3/2}}$$

input
```
Integrate[x^2*ArcCot[a*x^2],x]
```

output
```
(8*Sqrt[a]*x + 4*a^(3/2)*x^3*ArcCot[a*x^2] + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*
Sqrt[a]*x] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[a]*x] + Sqrt[2]*Log[1 - Sqr
t[2]*Sqrt[a]*x + a*x^2] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[a]*x + a*x^2])/(12*
a^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.40, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5362, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}\left(ax^2\right) dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{2}{3}a\int \frac{x^4}{a^2x^4+1}dx+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\int \frac{1}{a^2x^4+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{755}$$

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\int \frac{1-ax^2}{a^2x^4+1}dx+\frac{1}{2}\int \frac{ax^2+1}{a^2x^4+1}dx}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\int \frac{1-ax^2}{a^2x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}\right)}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\int\frac{1-ax^2}{a^2x^4+1}dx+\frac{1}{2}\left(\frac{\int\frac{1}{-\left(1-\sqrt{2}\sqrt{a}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}-\frac{\int\frac{1}{-\left(\sqrt{2}\sqrt{a}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}\right)}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$\Big\downarrow$ 217

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\int\frac{1-ax^2}{a^2x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$\Big\downarrow$ 1479

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$\Big\downarrow$ 25

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)}{a^2}\right)+\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$\Big\downarrow$ 27

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{a}x}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2\sqrt{2}a}+\frac{\int\frac{\sqrt{2}\sqrt{a}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)}{a^2}\right)+$$
$$\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \ 1103$$

$$\frac{2}{3}a\left(\frac{x}{a^2}-\frac{\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)+\frac{1}{2}\left(\frac{\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}-\frac{\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}\right)}{a^2}\right)+$$
$$\frac{1}{3}x^3\cot^{-1}\left(ax^2\right)$$

input
```
Int[x^2*ArcCot[a*x^2],x]
```

output
```
(x^3*ArcCot[a*x^2])/3 + (2*a*(x/a^2 - ((-(ArcTan[1 - Sqrt[2]*Sqrt[a]*x]/(S
qrt[2]*Sqrt[a])) + ArcTan[1 + Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]))/2 + (-
1/2*Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2]/(Sqrt[2]*Sqrt[a]) + Log[1 + Sqrt[2]
*Sqrt[a]*x + a*x^2]/(2*Sqrt[2]*Sqrt[a]))/2)/a^2))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{x^{3} \operatorname{arccot}\left(a \,x^{2}\right)}{3}+\frac{2 a \left(\frac{x}{a^{2}}-\frac{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x^{2}+\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{1}{a^{2}}}}{x^{2}-\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{1}{a^{2}}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{8 a^{2}}\right)}{3}$	109
parts	$\frac{x^{3} \operatorname{arccot}\left(a \,x^{2}\right)}{3}+\frac{2 a \left(\frac{x}{a^{2}}-\frac{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x^{2}+\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{1}{a^{2}}}}{x^{2}-\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{1}{a^{2}}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{8 a^{2}}\right)}{3}$	109

input
```
int(x^2*arccot(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccot(a*x^2)+2/3*a*(x/a^2-1/8/a^2*(1/a^2)^(1/4)*2^(1/2)*(ln((x^2+
(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2))/(x^2-(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2
)^(1/2)))+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/
4)*x-1)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^2 \cot^{-1}\left(ax^2\right) dx = \frac{4\,ax^3 \operatorname{arccot}\left(ax^2\right) + 8\,x - \frac{2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}} - \frac{2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{a}x-1\right)}{\sqrt{a}} - \frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}} + \frac{\sqrt{2}\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}}}{12\,a}$$

input
```
integrate(x^2*arccot(a*x^2),x, algorithm="fricas")
```

output
```
1/12*(4*a*x^3*arccot(a*x^2) + 8*x - 2*sqrt(2)*arctan(sqrt(2)*sqrt(a)*x + 1
)/sqrt(a) - 2*sqrt(2)*arctan(sqrt(2)*sqrt(a)*x - 1)/sqrt(a) - sqrt(2)*log(
a*x^2 + sqrt(2)*sqrt(a)*x + 1)/sqrt(a) + sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(
a)*x + 1)/sqrt(a))/a
```

Sympy [A] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.08

$$\int x^2 \cot^{-1}\left(ax^2\right) dx = \begin{cases} \frac{x^3 \operatorname{acot}\left(ax^2\right)}{3} + \frac{\left(-\frac{1}{a^2}\right)^{\frac{3}{4}} \operatorname{acot}\left(ax^2\right)}{3} + \frac{2x}{3a} + \frac{\sqrt[4]{-\frac{1}{a^2}}\log\left(x-\sqrt[4]{-\frac{1}{a^2}}\right)}{3a} - \frac{\sqrt[4]{-\frac{1}{a^2}}\log\left(x^2+\sqrt{-\frac{1}{a^2}}\right)}{6a} - \frac{\sqrt[4]{-\frac{1}{a^2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{3a} & \\ \frac{\pi x^3}{6} & \end{cases}$$

input
```
integrate(x**2*acot(a*x**2),x)
```

output
```
Piecewise((x**3*acot(a*x**2)/3 + (-1/a**2)**(3/4)*acot(a*x**2)/3 + 2*x/(3*
a) + (-1/a**2)**(1/4)*log(x - (-1/a**2)**(1/4))/(3*a) - (-1/a**2)**(1/4)*l
og(x**2 + sqrt(-1/a**2))/(6*a) - (-1/a**2)**(1/4)*atan(x/(-1/a**2)**(1/4))
/(3*a), Ne(a, 0)), (pi*x**3/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.15

$$\int x^2 \cot^{-1}(ax^2)\,dx = \frac{1}{3}\,x^3\,\mathrm{arccot}\left(ax^2\right) + \frac{1}{12}\,a\left(\frac{8\,x}{a^2} - \frac{\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax+\sqrt{2}\sqrt{a}\right)}{2\,\sqrt{a}}\right)}{\sqrt{a}} + \frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax-\sqrt{2}\sqrt{a}\right)}{2\,\sqrt{a}}\right)}{\sqrt{a}} + \frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}} - \frac{\sqrt{2}\log\left(ax^2-\sqrt{2}\sqrt{\right.}}{\sqrt{a}}}{a^2}\right.$$

input
```
integrate(x^2*arccot(a*x^2),x, algorithm="maxima")
```

output
```
1/3*x^3*arccot(a*x^2) + 1/12*a*(8*x/a^2 - (2*sqrt(2)*arctan(1/2*sqrt(2)*(2
*a*x + sqrt(2)*sqrt(a))/sqrt(a))/sqrt(a) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2
*a*x - sqrt(2)*sqrt(a))/sqrt(a))/sqrt(a) + sqrt(2)*log(a*x^2 + sqrt(2)*sqr
t(a)*x + 1)/sqrt(a) - sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/sqrt(a))/
a^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.31

$$\int x^2 \cot^{-1}(ax^2)\,dx = \frac{1}{3}\,x^3\arctan\left(\frac{1}{ax^2}\right) + \frac{1}{12}\,a\left(\frac{8\,x}{a^2} - \frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2\sqrt{|a|}} - \frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2\sqrt{|a|}} - \frac{\sqrt{2}\log}{}\right.$$

input
```
integrate(x^2*arccot(a*x^2),x, algorithm="giac")
```

output
```
1/3*x^3*arctan(1/(a*x^2)) + 1/12*a*(8*x/a^2 - 2*sqrt(2)*arctan(1/2*sqrt(2)
*(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*sqrt(abs(a))) - 2*sqrt(2)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/(a^2*sqrt(a
bs(a))) - sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(a)) + 1/abs(a))/(a^2*sqrt(a
bs(a))) + sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(a)) + 1/abs(a))/(a^2*sqrt(a
bs(a))))
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.44

$$\int x^2 \cot^{-1}\left(ax^2\right)\,dx = \frac{x^3\,\mathrm{acot}(a\,x^2)}{3} + \frac{2\,x}{3\,a} + \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{a}\,x\right)\,1\mathrm{i}}{3\,a^{3/2}} + \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{a}\,x\,1\mathrm{i}\right)}{3\,a^{3/2}}$$

input
```
int(x^2*acot(a*x^2),x)
```

output
```
(x^3*acot(a*x^2))/3 + (2*x)/(3*a) + ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x)
*1i)/(3*a^(3/2)) + ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x*1i))/(3*a^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87

$$\int x^2 \cot^{-1}\left(ax^2\right)\,dx = \frac{-2\sqrt{a}\,\sqrt{2}\,acot(a\,x^2) + 4acot(a\,x^2)\,a^2x^3 + 4\sqrt{a}\,\sqrt{2}\,atan\left(\frac{\sqrt{a}\,\sqrt{2}-2ax}{\sqrt{a}\,\sqrt{2}}\right) + \sqrt{a}\,\sqrt{2}\log\left(-\sqrt{a}\,\sqrt{2}\,x + a\,x^2 + 1\right.}{12a^2}$$

input
```
int(x^2*acot(a*x^2),x)
```

output
```
( - 2*sqrt(a)*sqrt(2)*acot(a*x**2) + 4*acot(a*x**2)*a**2*x**3 + 4*sqrt(a)*
sqrt(2)*atan((sqrt(a)*sqrt(2) - 2*a*x)/(sqrt(a)*sqrt(2))) + sqrt(a)*sqrt(2
)*log( - sqrt(a)*sqrt(2)*x + a*x**2 + 1) - sqrt(a)*sqrt(2)*log(sqrt(a)*sqr
t(2)*x + a*x**2 + 1) + 8*a*x)/(12*a**2)
```

3.45 $\int \cot^{-1}(ax^2)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 97

$$\int \cot^{-1}(ax^2)\,dx = x\cot^{-1}(ax^2) - \frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}x}{1+ax^2}\right)}{\sqrt{2}\sqrt{a}}$$

output
```
x*arccot(a*x^2)+1/2*arctan(-1+2^(1/2)*a^(1/2)*x)*2^(1/2)/a^(1/2)+1/2*arcta
n(1+2^(1/2)*a^(1/2)*x)*2^(1/2)/a^(1/2)-1/2*arctanh(2^(1/2)*a^(1/2)*x/(a*x^
2+1))*2^(1/2)/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05

$$\int \cot^{-1}(ax^2)\,dx = x\cot^{-1}(ax^2) + \frac{-2\arctan\left(1-\sqrt{2}\sqrt{a}x\right) + 2\arctan\left(1+\sqrt{2}\sqrt{a}x\right) + \log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right) - \log\left(1+\sqrt{2}\sqrt{a}x+ax^2\right)}{2\sqrt{2}\sqrt{a}}$$

input
```
Integrate[ArcCot[a*x^2],x]
```

output
```
x*ArcCot[a*x^2] + (-2*ArcTan[1 - Sqrt[2]*Sqrt[a]*x] + 2*ArcTan[1 + Sqrt[2]
*Sqrt[a]*x] + Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2] - Log[1 + Sqrt[2]*Sqrt[a]
*x + a*x^2])/(2*Sqrt[2]*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.57, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {5346, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}\left(ax^2\right) dx$$

$$\Big\downarrow \text{5346}$$

$$2a\int \frac{x^2}{a^2x^4+1}dx + x\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{826}$$

$$2a\left(\frac{\int \frac{ax^2+1}{a^2x^4+1}dx}{2a} - \frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}\right) + x\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{1476}$$

$$2a\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a} + \frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}}{2a} - \frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}\right) + x\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{1082}$$

$$2a\left(\frac{\frac{\int \frac{1}{-\left(1-\sqrt{2}\sqrt{a}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}} - \frac{\int \frac{1}{-\left(\sqrt{2}\sqrt{a}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}}{2a} - \frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}\right) + x\cot^{-1}\left(ax^2\right)$$

$$\Big\downarrow \text{217}$$

$$2a\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\int\frac{1-ax^2}{a^2x^4+1}dx}{2a}\right)+x\cot^{-1}\left(ax^2\right)$$

$\downarrow$ 1479

$$2a\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{-\frac{\int-\frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}}{2a}\right)+x\cot^{-1}\left(ax^2\right)$$

$\downarrow$ 25

$$2a\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\int\frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}}{2a}\right)+x\cot^{-1}\left(ax^2\right)$$

$\downarrow$ 27

$$2a\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\int\frac{\sqrt{2}-2\sqrt{a}x}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2\sqrt{2}a}+\frac{\int\frac{\sqrt{2}\sqrt{a}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}}{2a}\right)+x\cot^{-1}\left(ax^2\right)$$

$\downarrow$ 1103

$$2a\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}-\frac{\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}}{2a}\right)+x\cot^{-1}\left(ax^2\right)$$

input
```
Int[ArcCot[a*x^2],x]
```

output
```
x*ArcCot[a*x^2] + 2*a*((-(ArcTan[1 - Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]))
 + ArcTan[1 + Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]))/(2*a) - (-1/2*Log[1 -
Sqrt[2]*Sqrt[a]*x + a*x^2]/(Sqrt[2]*Sqrt[a]) + Log[1 + Sqrt[2]*Sqrt[a]*x +
 a*x^2]/(2*Sqrt[2]*Sqrt[a]))/(2*a))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

method	result	size
default	$x \,\mathrm{arccot}\left(a\,x^{2}\right)+\frac{\sqrt{2}\,\left(\ln\left(\frac{x^{2}-\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}{x^{2}+\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4a\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}$	97
parts	$x \,\mathrm{arccot}\left(a\,x^{2}\right)+\frac{\sqrt{2}\,\left(\ln\left(\frac{x^{2}-\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}{x^{2}+\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4a\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}$	97

input
```
int(arccot(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
x*arccot(a*x^2)+1/4/a/(1/a^2)^(1/4)*2^(1/2)*(ln((x^2-(1/a^2)^(1/4)*x*2^(1/
2)+(1/a^2)^(1/2))/(x^2+(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2)))+2*arctan(2^
(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \cot^{-1}\left(ax^2\right)\,dx = x\,\text{arccot}\left(ax^2\right) + \frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{a}x + 1\right)}{2\sqrt{a}} + \frac{\sqrt{2}\arctan\left(\sqrt{2}\sqrt{a}x - 1\right)}{2\sqrt{a}} - \frac{\sqrt{2}\log\left(ax^2 + \sqrt{2}\sqrt{a}x + 1\right)}{4\sqrt{a}} + \frac{\sqrt{2}\log\left(ax^2 - \sqrt{2}\sqrt{a}x + 1\right)}{4\sqrt{a}}$$

input
```
integrate(arccot(a*x^2),x, algorithm="fricas")
```

output
```
x*arccot(a*x^2) + 1/2*sqrt(2)*arctan(sqrt(2)*sqrt(a)*x + 1)/sqrt(a) + 1/2*
sqrt(2)*arctan(sqrt(2)*sqrt(a)*x - 1)/sqrt(a) - 1/4*sqrt(2)*log(a*x^2 + sq
rt(2)*sqrt(a)*x + 1)/sqrt(a) + 1/4*sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x +
 1)/sqrt(a)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.69 (sec) , antiderivative size = 615, normalized size of antiderivative = 6.34

$$\int \cot^{-1}\left(ax^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(acot(a*x**2),x)
```

output
```
Piecewise((pi*x/2, Eq(a, 0)), (oo*I*x, Eq(a, -I/x**2)), (-oo*I*x, Eq(a, I/
x**2)), (2*a**5*x**5*(-1/a**2)**(7/4)*acot(a*x**2)/(2*a**5*x**4*(-1/a**2)*
*(7/4) + 2*a**3*(-1/a**2)**(7/4)) + 2*a**4*x**4*(-1/a**2)**(3/2)*log(x - (
-1/a**2)**(1/4))/(2*a**5*x**4*(-1/a**2)**(7/4) + 2*a**3*(-1/a**2)**(7/4))
- a**4*x**4*(-1/a**2)**(3/2)*log(x**2 + sqrt(-1/a**2))/(2*a**5*x**4*(-1/a*
*2)**(7/4) + 2*a**3*(-1/a**2)**(7/4)) + 2*a**4*x**4*(-1/a**2)**(3/2)*atan(
x/(-1/a**2)**(1/4))/(2*a**5*x**4*(-1/a**2)**(7/4) + 2*a**3*(-1/a**2)**(7/4
)) + 2*a**3*x*(-1/a**2)**(7/4)*acot(a*x**2)/(2*a**5*x**4*(-1/a**2)**(7/4)
+ 2*a**3*(-1/a**2)**(7/4)) + 2*a**2*(-1/a**2)**(3/2)*log(x - (-1/a**2)**(1
/4))/(2*a**5*x**4*(-1/a**2)**(7/4) + 2*a**3*(-1/a**2)**(7/4)) - a**2*(-1/a
**2)**(3/2)*log(x**2 + sqrt(-1/a**2))/(2*a**5*x**4*(-1/a**2)**(7/4) + 2*a*
*3*(-1/a**2)**(7/4)) + 2*a**2*(-1/a**2)**(3/2)*atan(x/(-1/a**2)**(1/4))/(2
*a**5*x**4*(-1/a**2)**(7/4) + 2*a**3*(-1/a**2)**(7/4)) + 2*a*x**4*acot(a*x
**2)/(2*a**5*x**4*(-1/a**2)**(7/4) + 2*a**3*(-1/a**2)**(7/4)) + 2*acot(a*x
**2)/(2*a**6*x**4*(-1/a**2)**(7/4) + 2*a**4*(-1/a**2)**(7/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.24

$$\int \cot^{-1}\left(ax^2\right)\,dx$$

$$= \frac{1}{4}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax+\sqrt{2}\sqrt{a}\right)}{2\,\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax-\sqrt{2}\sqrt{a}\right)}{2\,\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{a^{\frac{3}{2}}} + \cdots\right.$$
$$\left. + x\,\mathrm{arccot}\left(ax^2\right)\right.$$

input
```
integrate(arccot(a*x^2),x, algorithm="maxima")
```

output
```
1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))/a^(
3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))/a^(
3/2) - sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/a^(3/2) + sqrt(2)*log(a*
x^2 - sqrt(2)*sqrt(a)*x + 1)/a^(3/2)) + x*arccot(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.48

$$\int \cot^{-1}\left(ax^2\right) dx$$
$$= \frac{1}{4}a\left(\frac{2\sqrt{2}\sqrt{|a|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2}+\frac{2\sqrt{2}\sqrt{|a|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{a^2}-\frac{\sqrt{2}\sqrt{|}}{}\right.$$
$$+ x\arctan\left(\frac{1}{ax^2}\right)$$

input
```
integrate(arccot(a*x^2),x, algorithm="giac")
```

output
```
1/4*a*(2*sqrt(2)*sqrt(abs(a))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(a
)))*sqrt(abs(a)))/a^2 + 2*sqrt(2)*sqrt(abs(a))*arctan(1/2*sqrt(2)*(2*x - s
qrt(2)/sqrt(abs(a)))*sqrt(abs(a)))/a^2 - sqrt(2)*sqrt(abs(a))*log(x^2 + sq
rt(2)*x/sqrt(abs(a)) + 1/abs(a))/a^2 + sqrt(2)*sqrt(abs(a))*log(x^2 - sqrt
(2)*x/sqrt(abs(a)) + 1/abs(a))/a^2) + x*arctan(1/(a*x^2))
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.43

$$\int \cot^{-1}\left(ax^2\right) dx = x\,\mathrm{acot}\left(a\,x^2\right) + \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{a}\,x\right)}{\sqrt{a}} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{a}\,x\right)}{\sqrt{a}}$$

input
```
int(acot(a*x^2),x)
```

output
```
x*acot(a*x^2) + ((-1)^(1/4)*atan((-1)^(1/4)*a^(1/2)*x))/a^(1/2) - ((-1)^(1
/4)*atanh((-1)^(1/4)*a^(1/2)*x))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int \cot^{-1}\left(ax^2\right) dx$$
$$= \frac{2\sqrt{a}\,\sqrt{2}\,acot(a\,x^2) + 4acot(a\,x^2)\,ax - 4\sqrt{a}\,\sqrt{2}\,atan\left(\frac{\sqrt{a}\,\sqrt{2}-2ax}{\sqrt{a}\,\sqrt{2}}\right) + \sqrt{a}\,\sqrt{2}\log\left(-\sqrt{a}\,\sqrt{2}\,x + a\,x^2 + 1\right) -}{4a}$$

input
```
int(acot(a*x^2),x)
```

output
```
(2*sqrt(a)*sqrt(2)*acot(a*x**2) + 4*acot(a*x**2)*a*x - 4*sqrt(a)*sqrt(2)*a
tan((sqrt(a)*sqrt(2) - 2*a*x)/(sqrt(a)*sqrt(2))) + sqrt(a)*sqrt(2)*log( -
sqrt(a)*sqrt(2)*x + a*x**2 + 1) - sqrt(a)*sqrt(2)*log(sqrt(a)*sqrt(2)*x +
a*x**2 + 1))/(4*a)
```

3.46 $\int \frac{\cot^{-1}(ax^2)}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 100

$$\int \frac{\cot^{-1}(ax^2)}{x^2} dx = -\frac{\cot^{-1}(ax^2)}{x} + \frac{\sqrt{a}\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}} - \frac{\sqrt{a}\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}} - \frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}x}{1+ax^2}\right)}{\sqrt{2}}$$

output
```
-arccot(a*x^2)/x-1/2*a^(1/2)*arctan(-1+2^(1/2)*a^(1/2)*x)*2^(1/2)-1/2*a^(1
/2)*arctan(1+2^(1/2)*a^(1/2)*x)*2^(1/2)-1/2*a^(1/2)*arctanh(2^(1/2)*a^(1/2
)*x/(a*x^2+1))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

$$\int \frac{\cot^{-1}(ax^2)}{x^2} dx = -\frac{\cot^{-1}(ax^2)}{x} + \frac{\sqrt{a}\left(2\arctan\left(1-\sqrt{2}\sqrt{a}x\right) - 2\arctan\left(1+\sqrt{2}\sqrt{a}x\right) + \log\left(1-\sqrt{2}\sqrt{a}x+ax^2\right) - \log\left(1+\sqrt{2}\sqrt{a}x+\right.\right.}{2\sqrt{2}}$$

input
```
Integrate[ArcCot[a*x^2]/x^2,x]
```

output
```
-(ArcCot[a*x^2]/x) + (Sqrt[a]*(2*ArcTan[1 - Sqrt[2]*Sqrt[a]*x] - 2*ArcTan[
1 + Sqrt[2]*Sqrt[a]*x] + Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2] - Log[1 + Sqrt
[2]*Sqrt[a]*x + a*x^2]))/(2*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.49, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {5362, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax^2)}{x^2} dx$$

$$\Big\downarrow \text{ 5362}$$

$$-2a \int \frac{1}{a^2x^4+1} dx - \frac{\cot^{-1}(ax^2)}{x}$$

$$\Big\downarrow \text{ 755}$$

$$-2a \left(\frac{1}{2} \int \frac{1-ax^2}{a^2x^4+1} dx + \frac{1}{2} \int \frac{ax^2+1}{a^2x^4+1} dx \right) - \frac{\cot^{-1}(ax^2)}{x}$$

$$\Big\downarrow \text{ 1476}$$

$$-2a \left(\frac{1}{2} \int \frac{1-ax^2}{a^2x^4+1} dx + \frac{1}{2} \left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}} dx}{2a} + \frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}} dx}{2a} \right) \right) - \frac{\cot^{-1}(ax^2)}{x}$$

$$\Big\downarrow \text{ 1082}$$

$$-2a \left(\frac{1}{2} \int \frac{1-ax^2}{a^2x^4+1} dx + \frac{1}{2} \left(\frac{\int \frac{1}{-\left(1-\sqrt{2}\sqrt{a}x\right)^2-1} d\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}} - \frac{\int \frac{1}{-\left(\sqrt{2}\sqrt{a}x+1\right)^2-1} d\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}} \right) \right) - \frac{\cot^{-1}(ax^2)}{x}$$

$$\Big\downarrow \text{ 217}$$

$$-2a\left(\frac{1}{2}\int\frac{1-ax^2}{a^2x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)\right)-\frac{\cot^{-1}\left(ax^2\right)}{x}$$

$\downarrow$ 1479

$$-2a\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right.\right.$$
$$\frac{\cot^{-1}\left(ax^2\right)}{x}$$

$\downarrow$ 25

$$-2a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)\right)-$$
$$\frac{\cot^{-1}\left(ax^2\right)}{x}$$

$\downarrow$ 27

$$-2a\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{a}x}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2\sqrt{2}a}+\frac{\int\frac{\sqrt{2}\sqrt{a}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)\right)-$$
$$\frac{\cot^{-1}\left(ax^2\right)}{x}$$

$\downarrow$ 1103

$$-2a\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}\right)+\frac{1}{2}\left(\frac{\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}-\frac{\log\left(ax^2-\sqrt{2}\sqrt{a}x+\right.}{2\sqrt{2}\sqrt{a}}\right.\right.$$
$$\frac{\cot^{-1}\left(ax^2\right)}{x}$$

input

```
Int[ArcCot[a*x^2]/x^2,x]
```

output
```
-(ArcCot[a*x^2]/x) - 2*a*((-(ArcTan[1 - Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a
])) + ArcTan[1 + Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]))/2 + (-1/2*Log[1 - S
qrt[2]*Sqrt[a]*x + a*x^2]/(Sqrt[2]*Sqrt[a]) + Log[1 + Sqrt[2]*Sqrt[a]*x +
a*x^2]/(2*Sqrt[2]*Sqrt[a]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{x}-\frac{a\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}{x^{2}-\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4}$	98
parts	$-\frac{\operatorname{arccot}\left(a\,x^{2}\right)}{x}-\frac{a\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}{x^{2}-\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4}$	98

input
```
int(arccot(a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccot(a*x^2)/x-1/4*a*(1/a^2)^(1/4)*2^(1/2)*(ln((x^2+(1/a^2)^(1/4)*x*2^(1
/2)+(1/a^2)^(1/2))/(x^2-(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2)))+2*arctan(2
^(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x-1))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx = -\frac{2\sqrt{2}\sqrt{a}x\arctan\left(\sqrt{2}\sqrt{a}x+1\right)+2\sqrt{2}\sqrt{a}x\arctan\left(\sqrt{2}\sqrt{a}x-1\right)+\sqrt{2}\sqrt{a}x\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)-}{4x}$$

input
```
integrate(arccot(a*x^2)/x^2,x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*sqrt(a)*x*arctan(sqrt(2)*sqrt(a)*x + 1) + 2*sqrt(2)*sqrt(a
)*x*arctan(sqrt(2)*sqrt(a)*x - 1) + sqrt(2)*sqrt(a)*x*log(a*x^2 + sqrt(2)*
sqrt(a)*x + 1) - sqrt(2)*sqrt(a)*x*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1) + 4*
arccot(a*x^2))/x
```

Sympy [A] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10

$$\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx = \begin{cases} a\sqrt[4]{-\frac{1}{a^2}}\log\left(x-\sqrt[4]{-\frac{1}{a^2}}\right)-\frac{a\sqrt[4]{-\frac{1}{a^2}}\log\left(x^2+\sqrt{-\frac{1}{a^2}}\right)}{2}-a\sqrt[4]{-\frac{1}{a^2}}\,\mathrm{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)-\frac{\mathrm{acot}(ax^2)}{\sqrt[4]{-\frac{1}{a^2}}}-\frac{\mathrm{acot}(ax^2)}{x} \\ -\frac{\pi}{2x} \end{cases}$$

input
```
integrate(acot(a*x**2)/x**2,x)
```

output
```
Piecewise((a*(-1/a**2)**(1/4)*log(x - (-1/a**2)**(1/4)) - a*(-1/a**2)**(1/
4)*log(x**2 + sqrt(-1/a**2))/2 - a*(-1/a**2)**(1/4)*atan(x/(-1/a**2)**(1/4
)) - acot(a*x**2)/(-1/a**2)**(1/4) - acot(a*x**2)/x, Ne(a, 0)), (-pi/(2*x)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.23

$$\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx = -\frac{1}{4}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax+\sqrt{2}\sqrt{a}\right)}{2\,\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax-\sqrt{2}\sqrt{a}\right)}{2\,\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}}-\right.$$
$$-\frac{\operatorname{arccot}(ax^2)}{x}$$

input
```
integrate(arccot(a*x^2)/x^2,x, algorithm="maxima")
```

output
```
-1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))/sq
rt(a) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))/sq
rt(a) + sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/sqrt(a) - sqrt(2)*log(a
*x^2 - sqrt(2)*sqrt(a)*x + 1)/sqrt(a)) - arccot(a*x^2)/x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.35

$$\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx = -\frac{1}{4}\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{\sqrt{|a|}}+\frac{2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{\sqrt{|a|}}+\frac{\sqrt{2}\log\left(x^2+\right.}{\sqrt{|a}}\right.$$
$$-\frac{\arctan\left(\frac{1}{ax^2}\right)}{x}$$

input
```
integrate(arccot(a*x^2)/x^2,x, algorithm="giac")
```

output
```
-1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(abs
(a)))/sqrt(abs(a)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(
a)))*sqrt(abs(a)))/sqrt(abs(a)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(a))
 + 1/abs(a))/sqrt(abs(a)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(a)) + 1/a
bs(a))/sqrt(abs(a))) - arctan(1/(a*x^2))/x
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.44

$$\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx = -\frac{\mathrm{acot}(a\,x^2)}{x} + (-1)^{1/4}\,\sqrt{a}\,\mathrm{atan}\Big((-1)^{1/4}\,\sqrt{a}\,x\Big)\,1\mathrm{i} + (-1)^{1/4}\,\sqrt{a}\,\mathrm{atanh}\Big((-1)^{1/4}\,\sqrt{a}\,x\Big)\,1\mathrm{i}$$

input
```
int(acot(a*x^2)/x^2,x)
```

output
```
(-1)^(1/4)*a^(1/2)*atan((-1)^(1/4)*a^(1/2)*x)*1i - acot(a*x^2)/x + (-1)^(1
/4)*a^(1/2)*atanh((-1)^(1/4)*a^(1/2)*x)*1i
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int \frac{\cot^{-1}(ax^2)}{x^2}\,dx = \frac{-2\sqrt{a}\,\sqrt{2}\,acot(a\,x^2)\,x - 4acot(a\,x^2) + 4\sqrt{a}\,\sqrt{2}\,atan\Big(\frac{\sqrt{a}\,\sqrt{2}-2ax}{\sqrt{a}\,\sqrt{2}}\Big)\,x + \sqrt{a}\,\sqrt{2}\log\big(-\sqrt{a}\,\sqrt{2}\,x + a\,x^2 + 1\big)}{4x}$$

input
```
int(acot(a*x^2)/x^2,x)
```

output
```
( - 2*sqrt(a)*sqrt(2)*acot(a*x**2)*x - 4*acot(a*x**2) + 4*sqrt(a)*sqrt(2)*
atan((sqrt(a)*sqrt(2) - 2*a*x)/(sqrt(a)*sqrt(2)))*x + sqrt(a)*sqrt(2)*log(
 - sqrt(a)*sqrt(2)*x + a*x**2 + 1)*x - sqrt(a)*sqrt(2)*log(sqrt(a)*sqrt(2)
*x + a*x**2 + 1)*x)/(4*x)
```

3.47 $\int \frac{\cot^{-1}(ax^2)}{x^4} dx$

Optimal result

Integrand size = 10, antiderivative size = 117

$$\int \frac{\cot^{-1}(ax^2)}{x^4} dx = \frac{2a}{3x} - \frac{\cot^{-1}(ax^2)}{3x^3} - \frac{a^{3/2}\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}} + \frac{a^{3/2}\arctan\left(1+\sqrt{2}\sqrt{a}x\right)}{3\sqrt{2}} - \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a}x}{1+ax^2}\right)}{3\sqrt{2}}$$

output
```
2/3*a/x-1/3*arccot(a*x^2)/x^3+1/6*a^(3/2)*arctan(-1+2^(1/2)*a^(1/2)*x)*2^(
1/2)+1/6*a^(3/2)*arctan(1+2^(1/2)*a^(1/2)*x)*2^(1/2)-1/6*a^(3/2)*arctanh(2
^(1/2)*a^(1/2)*x/(a*x^2+1))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.25

$$\int \frac{\cot^{-1}(ax^2)}{x^4} dx = \frac{-4\cot^{-1}(ax^2) + ax^2\left(8 - 2\sqrt{2}\sqrt{a}x\arctan\left(1-\sqrt{2}\sqrt{a}x\right) + 2\sqrt{2}\sqrt{a}x\arctan\left(1+\sqrt{2}\sqrt{a}x\right) + \sqrt{2}\sqrt{a}x\log\right.}{12x^3}$$

input
```
Integrate[ArcCot[a*x^2]/x^4,x]
```

output
```
(-4*ArcCot[a*x^2] + a*x^2*(8 - 2*Sqrt[2]*Sqrt[a]*x*ArcTan[1 - Sqrt[2]*Sqrt
[a]*x] + 2*Sqrt[2]*Sqrt[a]*x*ArcTan[1 + Sqrt[2]*Sqrt[a]*x] + Sqrt[2]*Sqrt[
a]*x*Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2] - Sqrt[2]*Sqrt[a]*x*Log[1 + Sqrt[2
]*Sqrt[a]*x + a*x^2]))/(12*x^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.45, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5362, 847, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}\left(ax^2\right)}{x^4}dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{2}{3}a\int \frac{1}{x^2\left(a^2x^4+1\right)}dx-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

$$\Big\downarrow \text{847}$$

$$-\frac{2}{3}a\left(a^2\left(-\int \frac{x^2}{a^2x^4+1}dx\right)-\frac{1}{x}\right)-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

$$\Big\downarrow \text{826}$$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\int \frac{ax^2+1}{a^2x^4+1}dx}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\frac{\int \frac{1}{-\left(1-\sqrt{2}\sqrt{a}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}-\frac{\int \frac{1}{-\left(\sqrt{2}\sqrt{a}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\int \frac{1-ax^2}{a^2x^4+1}dx}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{-\frac{\int -\frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\int \frac{\sqrt{2}-2\sqrt{a}x}{\sqrt{a}\left(x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{a}\left(x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}\right)}dx}{2\sqrt{2}\sqrt{a}}}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\int \frac{\sqrt{2}-2\sqrt{a}x}{x^2-\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2\sqrt{2}a}+\frac{\int \frac{\sqrt{2}\sqrt{a}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{a}}+\frac{1}{a}}dx}{2a}}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{2}{3}a\left(-\left(a^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{a}x+1\right)}{\sqrt{2}\sqrt{a}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{a}x\right)}{\sqrt{2}\sqrt{a}}}{2a}-\frac{\frac{\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}-\frac{\log\left(ax^2-\sqrt{2}\sqrt{a}x+1\right)}{2\sqrt{2}\sqrt{a}}}{2a}\right)\right)-\frac{1}{x}\right)-\frac{\cot^{-1}\left(ax^2\right)}{3x^3}$$

input
```
Int[ArcCot[a*x^2]/x^4,x]
```

output
```
-1/3*ArcCot[a*x^2]/x^3 - (2*a*(-x^(-1) - a^2*((-(ArcTan[1 - Sqrt[2]*Sqrt[a
]*x]/(Sqrt[2]*Sqrt[a])) + ArcTan[1 + Sqrt[2]*Sqrt[a]*x]/(Sqrt[2]*Sqrt[a]))
/(2*a) - (-1/2*Log[1 - Sqrt[2]*Sqrt[a]*x + a*x^2]/(Sqrt[2]*Sqrt[a]) + Log[
1 + Sqrt[2]*Sqrt[a]*x + a*x^2]/(2*Sqrt[2]*Sqrt[a]))/(2*a))))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{\operatorname{arccot}(a\,x^2)}{3x^3}-\frac{2a\left(-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-\frac{1}{x}\right)}{3}$	106
parts	$-\frac{\operatorname{arccot}(a\,x^2)}{3x^3}-\frac{2a\left(-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}{x^2+\left(\frac{1}{a^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{a^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(\frac{1}{a^2}\right)^{\frac{1}{4}}}-\frac{1}{x}\right)}{3}$	106

input
```
int(arccot(a*x^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arccot(a*x^2)/x^3-2/3*a*(-1/8/(1/a^2)^(1/4)*2^(1/2)*(ln((x^2-(1/a^2)^
(1/4)*x*2^(1/2)+(1/a^2)^(1/2))/(x^2+(1/a^2)^(1/4)*x*2^(1/2)+(1/a^2)^(1/2))
)+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/a^2)^(1/4)*x-1))
-1/x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx$$
$$=\frac{2\sqrt{2}a^{\frac{3}{2}}x^3\arctan\left(\sqrt{2}\sqrt{a}x+1\right)+2\sqrt{2}a^{\frac{3}{2}}x^3\arctan\left(\sqrt{2}\sqrt{a}x-1\right)-\sqrt{2}a^{\frac{3}{2}}x^3\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)+}{12\,x^3}$$

input
```
integrate(arccot(a*x^2)/x^4,x, algorithm="fricas")
```

output
```
1/12*(2*sqrt(2)*a^(3/2)*x^3*arctan(sqrt(2)*sqrt(a)*x + 1) + 2*sqrt(2)*a^(3
/2)*x^3*arctan(sqrt(2)*sqrt(a)*x - 1) - sqrt(2)*a^(3/2)*x^3*log(a*x^2 + sq
rt(2)*sqrt(a)*x + 1) + sqrt(2)*a^(3/2)*x^3*log(a*x^2 - sqrt(2)*sqrt(a)*x +
 1) + 8*a*x^2 - 4*arccot(a*x^2))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.16 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.92

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx$$
$$= \begin{cases} -\frac{\pi}{6x^3} \\ -\frac{\infty i}{x^3} \\ \frac{\infty i}{x^3} \\ -\frac{2a^3x^7\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}\log\left(x-\sqrt[4]{-\frac{1}{a^2}}\right)}{6x^7+\frac{6x^3}{a^2}} + \frac{a^3x^7\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{-\frac{1}{a^2}}\right)}{6x^7+\frac{6x^3}{a^2}} - \frac{2a^3x^7\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{a^2}}}\right)}{6x^7+\frac{6x^3}{a^2}} + \frac{2a^2x^7\sqrt[4]{-\frac{1}{a^2}}\operatorname{acot}}{6x^7+\frac{6x^3}{a^2}} \end{cases}$$

input
```
integrate(acot(a*x**2)/x**4,x)
```

output
```
Piecewise((-pi/(6*x**3), Eq(a, 0)), (-oo*I/x**3, Eq(a, -I/x**2)), (oo*I/x*
*3, Eq(a, I/x**2)), (-2*a**3*x**7*(-1/a**2)**(3/4)*log(x - (-1/a**2)**(1/4
))/(6*x**7 + 6*x**3/a**2) + a**3*x**7*(-1/a**2)**(3/4)*log(x**2 + sqrt(-1/
a**2))/(6*x**7 + 6*x**3/a**2) - 2*a**3*x**7*(-1/a**2)**(3/4)*atan(x/(-1/a*
*2)**(1/4))/(6*x**7 + 6*x**3/a**2) + 2*a**2*x**7*(-1/a**2)**(1/4)*acot(a*x
**2)/(6*x**7 + 6*x**3/a**2) + 4*a*x**6/(6*x**7 + 6*x**3/a**2) - 2*a*x**3*(
-1/a**2)**(3/4)*log(x - (-1/a**2)**(1/4))/(6*x**7 + 6*x**3/a**2) + a*x**3*
(-1/a**2)**(3/4)*log(x**2 + sqrt(-1/a**2))/(6*x**7 + 6*x**3/a**2) - 2*a*x*
*3*(-1/a**2)**(3/4)*atan(x/(-1/a**2)**(1/4))/(6*x**7 + 6*x**3/a**2) - 2*x*
*4*acot(a*x**2)/(6*x**7 + 6*x**3/a**2) + 2*x**3*(-1/a**2)**(1/4)*acot(a*x*
*2)/(6*x**7 + 6*x**3/a**2) + 4*x**2/(6*a*x**7 + 6*x**3/a) - 2*acot(a*x**2)
/(6*a**2*x**7 + 6*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx$$
$$= \frac{1}{12}\left(a^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax+\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,ax-\sqrt{2}\sqrt{a}\right)}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{\sqrt{2}\log\left(ax^2+\sqrt{2}\sqrt{a}x+1\right)}{a^{\frac{3}{2}}}\right.\right.$$
$$- \frac{\operatorname{arccot}(ax^2)}{3\,x^3}$$

input
```
integrate(arccot(a*x^2)/x^4,x, algorithm="maxima")
```

output
```
1/12*(a^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a))
/a^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a))
/a^(3/2) - sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/a^(3/2) + sqrt(2)*lo
g(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/a^(3/2)) + 8/x)*a - 1/3*arccot(a*x^2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.27

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx$$
$$= \frac{1}{12}\left(\frac{2\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{|a|^{\frac{3}{2}}} + \frac{2\sqrt{2}a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|a|}}\right)\sqrt{|a|}\right)}{|a|^{\frac{3}{2}}} - \sqrt{2}\sqrt{|a|}\log\right.$$
$$- \frac{\arctan\left(\frac{1}{ax^2}\right)}{3\,x^3}$$

input
```
integrate(arccot(a*x^2)/x^4,x, algorithm="giac")
```

output
```
1/12*(2*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(a)))*sqrt(a
bs(a)))/abs(a)^(3/2) + 2*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqr
t(abs(a)))*sqrt(abs(a)))/abs(a)^(3/2) - sqrt(2)*sqrt(abs(a))*log(x^2 + sqr
t(2)*x/sqrt(abs(a)) + 1/abs(a)) + sqrt(2)*a^2*log(x^2 - sqrt(2)*x/sqrt(abs
(a)) + 1/abs(a))/abs(a)^(3/2) + 8/x)*a - 1/3*arctan(1/(a*x^2))/x^3
```

Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.44

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx = \frac{2\,a}{3\,x} - \frac{\mathrm{acot}(a\,x^2)}{3\,x^3} + \frac{(-1)^{1/4}\,a^{3/2}\,\mathrm{atan}\Big((-1)^{1/4}\,\sqrt{a}\,x\Big)}{3} + \frac{(-1)^{1/4}\,a^{3/2}\,\mathrm{atan}\Big((-1)^{1/4}\,\sqrt{a}\,x\,1\mathrm{i}\Big)\,1\mathrm{i}}{3}$$

input
```
int(acot(a*x^2)/x^4,x)
```

output
```
(2*a)/(3*x) - acot(a*x^2)/(3*x^3) + ((-1)^(1/4)*a^(3/2)*atan((-1)^(1/4)*a^
(1/2)*x))/3 + ((-1)^(1/4)*a^(3/2)*atan((-1)^(1/4)*a^(1/2)*x*1i)*1i)/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{-1}(ax^2)}{x^4}\,dx = \frac{2\sqrt{a}\,\sqrt{2}\,acot(a\,x^2)\,a\,x^3 - 4acot(a\,x^2) - 4\sqrt{a}\,\sqrt{2}\,atan\Big(\frac{\sqrt{a}\,\sqrt{2}-2ax}{\sqrt{a}\,\sqrt{2}}\Big)\,a\,x^3 + \sqrt{a}\,\sqrt{2}\log\big(-\sqrt{a}\,\sqrt{2}\,x + a\,x^2 +}{12x^3}$$

input
```
int(acot(a*x^2)/x^4,x)
```

output
```
(2*sqrt(a)*sqrt(2)*acot(a*x**2)*a*x**3 - 4*acot(a*x**2) - 4*sqrt(a)*sqrt(2
)*atan((sqrt(a)*sqrt(2) - 2*a*x)/(sqrt(a)*sqrt(2)))*a*x**3 + sqrt(a)*sqrt(
2)*log( - sqrt(a)*sqrt(2)*x + a*x**2 + 1)*a*x**3 - sqrt(a)*sqrt(2)*log(sqr
t(a)*sqrt(2)*x + a*x**2 + 1)*a*x**3 + 8*a*x**2)/(12*x**3)
```

3.48 $\int \frac{\cot^{-1}(ax^5)}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 37

$$\int \frac{\cot^{-1}(ax^5)}{x} dx = -\frac{1}{10} i \operatorname{PolyLog}\left(2, -\frac{i}{ax^5}\right) + \frac{1}{10} i \operatorname{PolyLog}\left(2, \frac{i}{ax^5}\right)$$

output
```
-1/10*I*polylog(2,-I/a/x^5)+1/10*I*polylog(2,I/a/x^5)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(ax^5)}{x} dx = -\frac{1}{10} i \operatorname{PolyLog}\left(2, -\frac{i}{ax^5}\right) + \frac{1}{10} i \operatorname{PolyLog}\left(2, \frac{i}{ax^5}\right)$$

input
```
Integrate[ArcCot[a*x^5]/x,x]
```

output
```
(-1/10*I)*PolyLog[2, (-I)/(a*x^5)] + (I/10)*PolyLog[2, I/(a*x^5)]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5360, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax^5)}{x} \, dx$$

$$\Big\downarrow \text{5360}$$

$$\frac{1}{5}\int \frac{\cot^{-1}(ax^5)}{x^5}dx^5$$

$$\Big\downarrow \text{5356}$$

$$\frac{1}{5}\left(\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{ax^5}\right)}{x^5}dx^5-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{x^5a}\right)}{x^5}dx^5\right)$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{5}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{ax^5}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{ax^5}\right)\right)$$

input
```
Int[ArcCot[a*x^5]/x,x]
```

output
```
((-1/2*I)*PolyLog[2, (-I)/(a*x^5)] + (I/2)*PolyLog[2, I/(a*x^5)])/5
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5360
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.61 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

method	result
default	$\ln(x)\operatorname{arccot}(a\,x^5)+\frac{\sum_{\_R1=\operatorname{RootOf}(a^2\_Z^{10}+1)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^5}}{2a}$
parts	$\ln(x)\operatorname{arccot}(a\,x^5)+\frac{\sum_{\_R1=\operatorname{RootOf}(a^2\_Z^{10}+1)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^5}}{2a}$
risch	$\frac{\pi\ln(x)}{2}+\frac{i\left(\sum_{\_R1=\operatorname{RootOf}(a\_Z^5+\operatorname{RootOf}(\_Z^2+1,\mathrm{index}=1))}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}-\frac{i\ln(x)\ln(-ia\,x^5+}{2}$

input
```
int(arccot(a*x^5)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arccot(a*x^5)+1/2/a*sum(1/_R1^5*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)
/_R1)),_R1=RootOf(_Z^10*a^2+1))
```

Fricas [F]

$$\int\frac{\cot^{-1}(ax^5)}{x}\,dx=\int\frac{\operatorname{arccot}(ax^5)}{x}\,dx$$

input
```
integrate(arccot(a*x^5)/x,x, algorithm="fricas")
```

output
```
integral(arccot(a*x^5)/x, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax^5)}{x}\,dx = \int \frac{\operatorname{acot}(ax^5)}{x}\,dx$$

input
```
integrate(acot(a*x**5)/x,x)
```

output
```
Integral(acot(a*x**5)/x, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(23) = 46.

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.84

$$\int \frac{\cot^{-1}(ax^5)}{x}\,dx = \frac{1}{20}\pi\log\left(a^2x^{10}+1\right) - \frac{1}{5}\arctan\left(ax^5\right)\log\left(ax^5\right) + \operatorname{arccot}\left(ax^5\right)\log\left(x\right) + \arctan\left(ax^5\right)\log\left(x\right) + \frac{1}{10}i\,\mathrm{Li}_2\left(i\,ax^5+1\right) - \frac{1}{10}i\,\mathrm{Li}_2\left(-i\,ax^5+1\right)$$

input
```
integrate(arccot(a*x^5)/x,x, algorithm="maxima")
```

output
```
1/20*pi*log(a^2*x^10 + 1) - 1/5*arctan(a*x^5)*log(a*x^5) + arccot(a*x^5)*l
og(x) + arctan(a*x^5)*log(x) + 1/10*I*dilog(I*a*x^5 + 1) - 1/10*I*dilog(-I
*a*x^5 + 1)
```

Giac [F]

$$\int \frac{\cot^{-1}(ax^5)}{x} \, dx = \int \frac{\operatorname{arccot}(ax^5)}{x} \, dx$$

input
```
integrate(arccot(a*x^5)/x,x, algorithm="giac")
```

output
```
integrate(arccot(a*x^5)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax^5)}{x} \, dx = \int \frac{\operatorname{acot}(a\,x^5)}{x} \, dx$$

input
```
int(acot(a*x^5)/x,x)
```

output
```
int(acot(a*x^5)/x, x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(ax^5)}{x} \, dx = \int \frac{acot(a\,x^5)}{x} dx$$

input
```
int(acot(a*x^5)/x,x)
```

output
```
int(acot(a*x**5)/x,x)
```

3.49 $\int \cot^{-1}\left(\frac{1}{x}\right)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 17

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\cot^{-1}\left(\frac{1}{x}\right) - \frac{1}{2}\log\left(1+x^2\right)$$

output
```
x*arccot(1/x)-1/2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\cot^{-1}\left(\frac{1}{x}\right) - \frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[ArcCot[x^(-1)],x]
```

output
```
x*ArcCot[x^(-1)] - Log[1 + x^2]/2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5346, 795, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}\left(\frac{1}{x}\right) dx$$

$$\Big\downarrow \text{5346}$$

$$x\cot^{-1}\left(\frac{1}{x}\right)-\int \frac{1}{\left(1+\frac{1}{x^2}\right)x}dx$$

$$\Big\downarrow \text{795}$$

$$x\cot^{-1}\left(\frac{1}{x}\right)-\int \frac{x}{x^2+1}dx$$

$$\Big\downarrow \text{240}$$

$$x\cot^{-1}\left(\frac{1}{x}\right)-\frac{1}{2}\log\left(x^2+1\right)$$

input
```
Int[ArcCot[x^(-1)],x]
```

output
```
x*ArcCot[x^(-1)] - Log[1 + x^2]/2
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result
parallelrisch	$x \operatorname{arccot}\left(\frac{1}{x}\right)-\frac{\ln\left(x^{2}+1\right)}{2}$
parts	$x \operatorname{arccot}\left(\frac{1}{x}\right)-\frac{\ln\left(x^{2}+1\right)}{2}$
derivativedivides	$x \operatorname{arccot}\left(\frac{1}{x}\right)+\ln\left(\frac{1}{x}\right)-\frac{\ln\left(\frac{1}{x^{2}}+1\right)}{2}$
default	$x \operatorname{arccot}\left(\frac{1}{x}\right)+\ln\left(\frac{1}{x}\right)-\frac{\ln\left(\frac{1}{x^{2}}+1\right)}{2}$
risch	$\frac{ix\ln(i+x)}{2}-\frac{i\ln(x-i)x}{2}-\frac{\pi\,\operatorname{csgn}\left(\frac{i}{x}\right)\operatorname{csgn}\left(i\left(x-\operatorname{RootOf}\left(\_Z^{2}+1,\mathrm{index}=1\right)\right)\right)\operatorname{csgn}\left(\frac{i\left(x-\operatorname{RootOf}\left(\_Z^{2}+1,\mathrm{index}\right.\right.}{x}\right.}{4}$

input
```
int(arccot(1/x),x,method=_RETURNVERBOSE)
```

output
```
x*arccot(1/x)-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\operatorname{arccot}\left(\frac{1}{x}\right)-\frac{1}{2}\log\left(x^{2}+1\right)$$

input
```
integrate(arccot(1/x),x, algorithm="fricas")
```

output
```
x*arccot(1/x) - 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\,\mathrm{acot}\left(\frac{1}{x}\right) - \frac{\log\left(x^2+1\right)}{2}$$

input
```
integrate(acot(1/x),x)
```

output
```
x*acot(1/x) - log(x**2 + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\,\mathrm{arccot}\left(\frac{1}{x}\right) - \frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(arccot(1/x),x, algorithm="maxima")
```

output
```
x*arccot(1/x) - 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cot^{-1}\left(\frac{1}{x}\right)\,dx = x\arctan\left(x\right) - \frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(arccot(1/x),x, algorithm="giac")
```

output
```
x*arctan(x) - 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^{-1}\left(\frac{1}{x}\right) dx = x\,\mathrm{acot}\left(\frac{1}{x}\right) - \frac{\ln(x^2+1)}{2}$$

input
```
int(acot(1/x),x)
```

output
```
x*acot(1/x) - log(x^2 + 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^{-1}\left(\frac{1}{x}\right) dx = acot\left(\frac{1}{x}\right)x - \frac{\log(x^2+1)}{2}$$

input
```
int(acot(1/x),x)
```

output
```
(2*acot(1/x)*x - log(x**2 + 1))/2
```

3.50 $\int x^2 \cot^{-1}(\sqrt{x})\, dx$

Optimal result

Integrand size = 10, antiderivative size = 51

$$\int x^2 \cot^{-1}(\sqrt{x})\, dx = \frac{\sqrt{x}}{3} - \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \cot^{-1}(\sqrt{x}) - \frac{\arctan(\sqrt{x})}{3}$$

output
```
1/3*x^(1/2)-1/9*x^(3/2)+1/15*x^(5/2)+1/3*x^3*arccot(x^(1/2))-1/3*arctan(x^
(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^2 \cot^{-1}(\sqrt{x})\, dx = \frac{1}{45}\left(\sqrt{x}(15 - 5x + 3x^2) + 15x^3 \cot^{-1}(\sqrt{x}) - 15\arctan(\sqrt{x})\right)$$

input
```
Integrate[x^2*ArcCot[Sqrt[x]],x]
```

output
```
(Sqrt[x]*(15 - 5*x + 3*x^2) + 15*x^3*ArcCot[Sqrt[x]] - 15*ArcTan[Sqrt[x]])
/45
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5362, 60, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{6}\int \frac{x^{5/2}}{x+1}dx+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}\left(\frac{2x^{5/2}}{5}-\int \frac{x^{3/2}}{x+1}dx\right)+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}\left(\int \frac{\sqrt{x}}{x+1}dx+\frac{2x^{5/2}}{5}-\frac{2x^{3/2}}{3}\right)+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{6}\left(-\int \frac{1}{\sqrt{x}(x+1)}dx+\frac{2x^{5/2}}{5}-\frac{2x^{3/2}}{3}+2\sqrt{x}\right)+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(-2\int \frac{1}{x+1}d\sqrt{x}+\frac{2x^{5/2}}{5}-\frac{2x^{3/2}}{3}+2\sqrt{x}\right)+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{6}\left(-2\arctan\left(\sqrt{x}\right)+\frac{2x^{5/2}}{5}-\frac{2x^{3/2}}{3}+2\sqrt{x}\right)+\frac{1}{3}x^3\cot^{-1}\left(\sqrt{x}\right)$$

input
```
Int[x^2*ArcCot[Sqrt[x]],x]
```

output
```
(x^3*ArcCot[Sqrt[x]])/3 + (2*Sqrt[x] - (2*x^(3/2))/3 + (2*x^(5/2))/5 - 2*A
rcTan[Sqrt[x]])/6
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\frac{\sqrt{x}}{3}-\frac{x^{\frac{3}{2}}}{9}+\frac{x^{\frac{5}{2}}}{15}+\frac{x^{3}\operatorname{arccot}\left(\sqrt{x}\right)}{3}-\frac{\arctan\left(\sqrt{x}\right)}{3}$	32
default	$\frac{\sqrt{x}}{3}-\frac{x^{\frac{3}{2}}}{9}+\frac{x^{\frac{5}{2}}}{15}+\frac{x^{3}\operatorname{arccot}\left(\sqrt{x}\right)}{3}-\frac{\arctan\left(\sqrt{x}\right)}{3}$	32
parts	$\frac{\sqrt{x}}{3}-\frac{x^{\frac{3}{2}}}{9}+\frac{x^{\frac{5}{2}}}{15}+\frac{x^{3}\operatorname{arccot}\left(\sqrt{x}\right)}{3}-\frac{\arctan\left(\sqrt{x}\right)}{3}$	32

input
```
int(x^2*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(1/2)-1/9*x^(3/2)+1/15*x^(5/2)+1/3*x^3*arccot(x^(1/2))-1/3*arctan(x^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int x^2\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{3}\left(x^3+1\right)\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{45}\left(3\,x^2-5\,x+15\right)\sqrt{x}$$

input
```
integrate(x^2*arccot(x^(1/2)),x, algorithm="fricas")
```

output
```
1/3*(x^3 + 1)*arccot(sqrt(x)) + 1/45*(3*x^2 - 5*x + 15)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^2\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{x^{\frac{5}{2}}}{15}-\frac{x^{\frac{3}{2}}}{9}+\frac{\sqrt{x}}{3}+\frac{x^3\operatorname{acot}\left(\sqrt{x}\right)}{3}-\frac{\operatorname{atan}\left(\sqrt{x}\right)}{3}$$

input
```
integrate(x**2*acot(x**(1/2)),x)
```

output
```
x**(5/2)/15 - x**(3/2)/9 + sqrt(x)/3 + x**3*acot(sqrt(x))/3 - atan(sqrt(x)
)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right) dx = \frac{1}{3}\,x^3 \operatorname{arccot}\left(\sqrt{x}\right) + \frac{1}{15}\,x^{\frac{5}{2}} - \frac{1}{9}\,x^{\frac{3}{2}} + \frac{1}{3}\,\sqrt{x} - \frac{1}{3}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x^2*arccot(x^(1/2)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccot(sqrt(x)) + 1/15*x^(5/2) - 1/9*x^(3/2) + 1/3*sqrt(x) - 1/3*a
rctan(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right) dx = \frac{1}{3}\,x^3 \arctan\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{45}\,x^{\frac{5}{2}}\left(\frac{5}{x} - \frac{15}{x^2} - 3\right) + \frac{1}{3}\,\arctan\left(\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(x^2*arccot(x^(1/2)),x, algorithm="giac")
```

output
```
1/3*x^3*arctan(1/sqrt(x)) - 1/45*x^(5/2)*(5/x - 15/x^2 - 3) + 1/3*arctan(1
/sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right)\, dx = \frac{x^3\,\mathrm{acot}\left(\sqrt{x}\right)}{3} - \frac{\mathrm{atan}\left(\sqrt{x}\right)}{3} + \frac{\sqrt{x}}{3} - \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15}$$

input
```
int(x^2*acot(x^(1/2)),x)
```

output
```
(x^3*acot(x^(1/2)))/3 - atan(x^(1/2))/3 + x^(1/2)/3 - x^(3/2)/9 + x^(5/2)/
15
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59

$$\int x^2 \cot^{-1}\left(\sqrt{x}\right)\, dx = \frac{acot\left(\sqrt{x}\right) x^3}{3} + \frac{acot\left(\sqrt{x}\right)}{3} + \frac{\sqrt{x}\, x^2}{15} - \frac{\sqrt{x}\, x}{9} + \frac{\sqrt{x}}{3}$$

input
```
int(x^2*acot(x^(1/2)),x)
```

output
```
(15*acot(sqrt(x))*x**3 + 15*acot(sqrt(x)) + 3*sqrt(x)*x**2 - 5*sqrt(x)*x +
 15*sqrt(x))/45
```

3.51 $\int x \cot^{-1}\left(\sqrt{x}\right) dx$

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int x \cot^{-1}\left(\sqrt{x}\right) dx = -\frac{\sqrt{x}}{2} + \frac{x^{3/2}}{6} + \frac{1}{2}x^2 \cot^{-1}\left(\sqrt{x}\right) + \frac{\arctan\left(\sqrt{x}\right)}{2}$$

output
```
-1/2*x^(1/2)+1/6*x^(3/2)+1/2*x^2*arccot(x^(1/2))+1/2*arctan(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int x \cot^{-1}\left(\sqrt{x}\right) dx = \frac{1}{6}\left((-3+x)\sqrt{x} + 3x^2 \cot^{-1}\left(\sqrt{x}\right) + 3\arctan\left(\sqrt{x}\right)\right)$$

input
```
Integrate[x*ArcCot[Sqrt[x]],x]
```

output
```
((-3 + x)*Sqrt[x] + 3*x^2*ArcCot[Sqrt[x]] + 3*ArcTan[Sqrt[x]])/6
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5362, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cot^{-1}\left(\sqrt{x}\right) \, dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{4}\int \frac{x^{3/2}}{x+1}dx+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{2x^{3/2}}{3}-\int \frac{\sqrt{x}}{x+1}dx\right)+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\int \frac{1}{\sqrt{x}(x+1)}dx+\frac{2x^{3/2}}{3}-2\sqrt{x}\right)+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(2\int \frac{1}{x+1}d\sqrt{x}+\frac{2x^{3/2}}{3}-2\sqrt{x}\right)+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(2\arctan\left(\sqrt{x}\right)+\frac{2x^{3/2}}{3}-2\sqrt{x}\right)+\frac{1}{2}x^2\cot^{-1}\left(\sqrt{x}\right)$$

input
```
Int[x*ArcCot[Sqrt[x]],x]
```

output
```
(x^2*ArcCot[Sqrt[x]])/2 + (-2*Sqrt[x] + (2*x^(3/2))/3 + 2*ArcTan[Sqrt[x]])
/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result	size
derivativedivides	$-\frac{\sqrt{x}}{2}+\frac{x^{\frac{3}{2}}}{6}+\frac{x^{2} \operatorname{arccot}\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}$	27
default	$-\frac{\sqrt{x}}{2}+\frac{x^{\frac{3}{2}}}{6}+\frac{x^{2} \operatorname{arccot}\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}$	27
parts	$-\frac{\sqrt{x}}{2}+\frac{x^{\frac{3}{2}}}{6}+\frac{x^{2} \operatorname{arccot}\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}$	27

input
```
int(x*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^(1/2)+1/6*x^(3/2)+1/2*x^2*arccot(x^(1/2))+1/2*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.48

$$\int x\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{2}\left(x^2-1\right)\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{6}\left(x-3\right)\sqrt{x}$$

input
```
integrate(x*arccot(x^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(x^2 - 1)*arccot(sqrt(x)) + 1/6*(x - 3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int x\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{x^{\frac{3}{2}}}{6}-\frac{\sqrt{x}}{2}+\frac{x^2\operatorname{acot}\left(\sqrt{x}\right)}{2}+\frac{\operatorname{atan}\left(\sqrt{x}\right)}{2}$$

input
```
integrate(x*acot(x**(1/2)),x)
```

output
```
x**(3/2)/6 - sqrt(x)/2 + x**2*acot(sqrt(x))/2 + atan(sqrt(x))/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2\,\mathrm{arccot}\left(\sqrt{x}\right) + \frac{1}{6}\,x^{\frac{3}{2}} - \frac{1}{2}\,\sqrt{x} + \frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x*arccot(x^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2*arccot(sqrt(x)) + 1/6*x^(3/2) - 1/2*sqrt(x) + 1/2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int x\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2\arctan\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{6}\,x^{\frac{3}{2}}\left(\frac{3}{x} - 1\right) - \frac{1}{2}\,\arctan\left(\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(x*arccot(x^(1/2)),x, algorithm="giac")
```

output
```
1/2*x^2*arctan(1/sqrt(x)) - 1/6*x^(3/2)*(3/x - 1) - 1/2*arctan(1/sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{\mathrm{atan}\left(\sqrt{x}\right)}{2} + \frac{x^2\,\mathrm{acot}\left(\sqrt{x}\right)}{2} - \frac{\sqrt{x}}{2} + \frac{x^{3/2}}{6}$$

input
```
int(x*acot(x^(1/2)),x)
```

output
```
atan(x^(1/2))/2 + (x^2*acot(x^(1/2)))/2 - x^(1/2)/2 + x^(3/2)/6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55

$$\int x\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{acot\left(\sqrt{x}\right)x^{2}}{2}-\frac{acot\left(\sqrt{x}\right)}{2}+\frac{\sqrt{x}\,x}{6}-\frac{\sqrt{x}}{2}$$

input
```
int(x*acot(x^(1/2)),x)
```

output
```
(3*acot(sqrt(x))*x**2 - 3*acot(sqrt(x)) + sqrt(x)*x - 3*sqrt(x))/6
```

3.52 $\int \cot^{-1}\left(\sqrt{x}\right)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = \sqrt{x} + x\cot^{-1}\left(\sqrt{x}\right) - \arctan\left(\sqrt{x}\right)$$

output
```
x^(1/2)+x*arccot(x^(1/2))-arctan(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = \sqrt{x} + x\cot^{-1}\left(\sqrt{x}\right) - \arctan\left(\sqrt{x}\right)$$

input
```
Integrate[ArcCot[Sqrt[x]],x]
```

output
```
Sqrt[x] + x*ArcCot[Sqrt[x]] - ArcTan[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5346, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{-1}\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{ 5346}$$

$$\frac{1}{2}\int \frac{\sqrt{x}}{x+1}dx + x\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\left(2\sqrt{x} - \int \frac{1}{\sqrt{x}(x+1)}dx\right) + x\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(2\sqrt{x} - 2\int \frac{1}{x+1}d\sqrt{x}\right) + x\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\left(2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)\right) + x\cot^{-1}\left(\sqrt{x}\right)$$

input
```
Int[ArcCot[Sqrt[x]],x]
```

output
```
x*ArcCot[Sqrt[x]] + (2*Sqrt[x] - 2*ArcTan[Sqrt[x]])/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\sqrt{x}+x\,\operatorname{arccot}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)$	17
default	$\sqrt{x}+x\,\operatorname{arccot}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)$	17
parts	$\sqrt{x}+x\,\operatorname{arccot}\left(\sqrt{x}\right)-\arctan\left(\sqrt{x}\right)$	17

input
```
int(arccot(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)+x*arccot(x^(1/2))-arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = (x+1)\operatorname{arccot}\left(\sqrt{x}\right)+\sqrt{x}$$

input
```
integrate(arccot(x^(1/2)),x, algorithm="fricas")
```

output
```
(x + 1)*arccot(sqrt(x)) + sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = \sqrt{x}+x\operatorname{acot}\left(\sqrt{x}\right)-\operatorname{atan}\left(\sqrt{x}\right)$$

input
```
integrate(acot(x**(1/2)),x)
```

output
```
sqrt(x) + x*acot(sqrt(x)) - atan(sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = x\operatorname{arccot}\left(\sqrt{x}\right)+\sqrt{x}-\arctan\left(\sqrt{x}\right)$$

input
```
integrate(arccot(x^(1/2)),x, algorithm="maxima")
```

output
```
x*arccot(sqrt(x)) + sqrt(x) - arctan(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = x\arctan\left(\frac{1}{\sqrt{x}}\right) + \sqrt{x} + \arctan\left(\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(arccot(x^(1/2)),x, algorithm="giac")
```

output
```
x*arctan(1/sqrt(x)) + sqrt(x) + arctan(1/sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = x\,\mathrm{acot}\left(\sqrt{x}\right) - \mathrm{atan}\left(\sqrt{x}\right) + \sqrt{x}$$

input
```
int(acot(x^(1/2)),x)
```

output
```
x*acot(x^(1/2)) - atan(x^(1/2)) + x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int \cot^{-1}\left(\sqrt{x}\right)\,dx = acot\left(\sqrt{x}\right)x + acot\left(\sqrt{x}\right) + \sqrt{x}$$

input
```
int(acot(x^(1/2)),x)
```

output
```
acot(sqrt(x))*x + acot(sqrt(x)) + sqrt(x)
```

3.53 $\int \frac{\cot^{-1}(\sqrt{x})}{x} dx$

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \frac{\cot^{-1}(\sqrt{x})}{x} dx = -i \operatorname{PolyLog}\left(2, -\frac{i}{\sqrt{x}}\right) + i \operatorname{PolyLog}\left(2, \frac{i}{\sqrt{x}}\right)$$

output
```
-I*polylog(2,-I/x^(1/2))+I*polylog(2,I/x^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}(\sqrt{x})}{x} dx = -i \operatorname{PolyLog}\left(2, -\frac{i}{\sqrt{x}}\right) + i \operatorname{PolyLog}\left(2, \frac{i}{\sqrt{x}}\right)$$

input
```
Integrate[ArcCot[Sqrt[x]]/x,x]
```

output
```
(-I)*PolyLog[2, (-I)/Sqrt[x]] + I*PolyLog[2, I/Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5360, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x} \, dx$$

$$\Big\downarrow \text{5360}$$

$$2\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{5356}$$

$$2\left(\frac{1}{2}i\int \frac{\log\left(1-\frac{i}{\sqrt{x}}\right)}{\sqrt{x}}d\sqrt{x}-\frac{1}{2}i\int \frac{\log\left(1+\frac{i}{\sqrt{x}}\right)}{\sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{2838}$$

$$2\left(\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{\sqrt{x}}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{\sqrt{x}}\right)\right)$$

input
```
Int[ArcCot[Sqrt[x]]/x,x]
```

output
```
2*((-1/2*I)*PolyLog[2, (-I)/Sqrt[x]] + (I/2)*PolyLog[2, I/Sqrt[x]])
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5360
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

method	result
derivativedivides	$\ln(x)\operatorname{arccot}(\sqrt{x}) - \frac{i\ln(x)\ln(1+i\sqrt{x})}{2} + \frac{i\ln(x)\ln(1-i\sqrt{x})}{2} - i\operatorname{dilog}(1+i\sqrt{x}) + i\operatorname{dilog}(1 -$
default	$\ln(x)\operatorname{arccot}(\sqrt{x}) - \frac{i\ln(x)\ln(1+i\sqrt{x})}{2} + \frac{i\ln(x)\ln(1-i\sqrt{x})}{2} - i\operatorname{dilog}(1+i\sqrt{x}) + i\operatorname{dilog}(1 -$
parts	$\ln(x)\operatorname{arccot}(\sqrt{x}) - \frac{i\ln(x)\ln(1+i\sqrt{x})}{2} + \frac{i\ln(x)\ln(1-i\sqrt{x})}{2} - i\operatorname{dilog}(1+i\sqrt{x}) + i\operatorname{dilog}(1 -$

input
```
int(arccot(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arccot(x^(1/2))-1/2*I*ln(x)*ln(1+I*x^(1/2))+1/2*I*ln(x)*ln(1-I*x^(1/
2))-I*dilog(1+I*x^(1/2))+I*dilog(1-I*x^(1/2))
```

Fricas [F]

$$\int \frac{\cot^{-1}(\sqrt{x})}{x}\,dx = \int \frac{\operatorname{arccot}(\sqrt{x})}{x}\,dx$$

input
```
integrate(arccot(x^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(arccot(sqrt(x))/x, x)
```

Sympy [F]

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{acot}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(acot(x**(1/2))/x,x)
```

output
```
Integral(acot(sqrt(x))/x, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx = \frac{1}{2}\,\pi\log\left(x+1\right) + \operatorname{arccot}\left(\sqrt{x}\right)\log\left(x\right) + i\operatorname{Li}_2\left(i\sqrt{x}+1\right) - i\operatorname{Li}_2\left(-i\sqrt{x}+1\right)$$

input
```
integrate(arccot(x^(1/2))/x,x, algorithm="maxima")
```

output
```
1/2*pi*log(x + 1) + arccot(sqrt(x))*log(x) + I*dilog(I*sqrt(x) + 1) - I*di
log(-I*sqrt(x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx = -x\arctan\left(\frac{1}{\sqrt{x}}\right) - \sqrt{x} - \arctan\left(\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(arccot(x^(1/2))/x,x, algorithm="giac")
```

output
```
-x*arctan(1/sqrt(x)) - sqrt(x) - arctan(1/sqrt(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\mathrm{acot}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
int(acot(x^(1/2))/x,x)
```

output
```
int(acot(x^(1/2))/x, x)
```

Reduce [F]

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x}\,dx = \int \frac{acot\left(\sqrt{x}\right)}{x}dx$$

input
```
int(acot(x^(1/2))/x,x)
```

output
```
int(acot(sqrt(x))/x,x)
```

3.54 $\int \frac{\cot^{-1}(\sqrt{x})}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 23

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2} dx = \frac{1}{\sqrt{x}} - \frac{\cot^{-1}\left(\sqrt{x}\right)}{x} + \arctan\left(\sqrt{x}\right)$$

output
```
1/x^(1/2)-arccot(x^(1/2))/x+arctan(x^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2} dx = -\frac{\cot^{-1}\left(\sqrt{x}\right)}{x} + \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -x\right)}{\sqrt{x}}$$

input
```
Integrate[ArcCot[Sqrt[x]]/x^2,x]
```

output
```
-(ArcCot[Sqrt[x]]/x) + Hypergeometric2F1[-1/2, 1, 1/2, -x]/Sqrt[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5362, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2} \, dx$$

$$\Big\downarrow \text{ 5362}$$

$$-\frac{1}{2}\int \frac{1}{x^{3/2}(x+1)}dx-\frac{\cot^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{x}(x+1)}dx+\frac{2}{\sqrt{x}}\right)-\frac{\cot^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(2\int \frac{1}{x+1}d\sqrt{x}+\frac{2}{\sqrt{x}}\right)-\frac{\cot^{-1}\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\left(2\arctan\left(\sqrt{x}\right)+\frac{2}{\sqrt{x}}\right)-\frac{\cot^{-1}\left(\sqrt{x}\right)}{x}$$

input
```
Int[ArcCot[Sqrt[x]]/x^2,x]
```

output
```
-(ArcCot[Sqrt[x]]/x) + (2/Sqrt[x] + 2*ArcTan[Sqrt[x]])/2
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{1}{\sqrt{x}}-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x}+\arctan\left(\sqrt{x}\right)$	18
default	$\frac{1}{\sqrt{x}}-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x}+\arctan\left(\sqrt{x}\right)$	18
parts	$\frac{1}{\sqrt{x}}-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x}+\arctan\left(\sqrt{x}\right)$	18

input
```
int(arccot(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x^(1/2)-arccot(x^(1/2))/x+arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{(x+1)\operatorname{arccot}\left(\sqrt{x}\right)-\sqrt{x}}{x}$$

input
```
integrate(arccot(x^(1/2))/x^2,x, algorithm="fricas")
```

output
```
-((x + 1)*arccot(sqrt(x)) - sqrt(x))/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(20) = 40$.

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.00

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{x^{\frac{5}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{2x^{\frac{3}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{\sqrt{x}\operatorname{acot}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} + \frac{x^2}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} + \frac{x}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}$$

input
```
integrate(acot(x**(1/2))/x**2,x)
```

output
```
-x**(5/2)*acot(sqrt(x))/(x**(5/2) + x**(3/2)) - 2*x**(3/2)*acot(sqrt(x))/(
x**(5/2) + x**(3/2)) - sqrt(x)*acot(sqrt(x))/(x**(5/2) + x**(3/2)) + x**2/
(x**(5/2) + x**(3/2)) + x/(x**(5/2) + x**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{x} + \frac{1}{\sqrt{x}} + \arctan\left(\sqrt{x}\right)$$

input
```
integrate(arccot(x^(1/2))/x^2,x, algorithm="maxima")
```

output
```
-arccot(sqrt(x))/x + 1/sqrt(x) + arctan(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\arctan\left(\frac{1}{\sqrt{x}}\right)}{x} + \frac{1}{\sqrt{x}} - \arctan\left(\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(arccot(x^(1/2))/x^2,x, algorithm="giac")
```

output
```
-arctan(1/sqrt(x))/x + 1/sqrt(x) - arctan(1/sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = \operatorname{atan}\left(\sqrt{x}\right) - \frac{\operatorname{acot}\left(\sqrt{x}\right)}{x} + \frac{1}{\sqrt{x}}$$

input
```
int(acot(x^(1/2))/x^2,x)
```

output
```
atan(x^(1/2)) - acot(x^(1/2))/x + 1/x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^2}\,dx = \frac{-acot\left(\sqrt{x}\right)x - acot\left(\sqrt{x}\right) + \sqrt{x}}{x}$$

input
```
int(acot(x^(1/2))/x^2,x)
```

output
```
( - acot(sqrt(x))*x - acot(sqrt(x)) + sqrt(x))/x
```

3.55 $\int \frac{\cot^{-1}(\sqrt{x})}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3} dx = \frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2} - \frac{\arctan\left(\sqrt{x}\right)}{2}$$

output
```
1/6/x^(3/2)-1/2/x^(1/2)-1/2*arccot(x^(1/2))/x^2-1/2*arctan(x^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3} dx = -\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, -x\right)}{6x^{3/2}}$$

input
```
Integrate[ArcCot[Sqrt[x]]/x^3,x]
```

output
```
-1/2*ArcCot[Sqrt[x]]/x^2 + Hypergeometric2F1[-3/2, 1, -1/2, -x]/(6*x^(3/2)
)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5362, 61, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}dx$$

$$\Big\downarrow \text{ 5362}$$

$$-\frac{1}{4}\int \frac{1}{x^{5/2}(x+1)}dx-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{1}{4}\left(\int \frac{1}{x^{3/2}(x+1)}dx+\frac{2}{3x^{3/2}}\right)-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{1}{4}\left(-\int \frac{1}{\sqrt{x}(x+1)}dx+\frac{2}{3x^{3/2}}-\frac{2}{\sqrt{x}}\right)-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{4}\left(-2\int \frac{1}{x+1}d\sqrt{x}+\frac{2}{3x^{3/2}}-\frac{2}{\sqrt{x}}\right)-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{4}\left(-2\arctan\left(\sqrt{x}\right)+\frac{2}{3x^{3/2}}-\frac{2}{\sqrt{x}}\right)-\frac{\cot^{-1}\left(\sqrt{x}\right)}{2x^2}$$

input
```
Int[ArcCot[Sqrt[x]]/x^3,x]
```

output
```
-1/2*ArcCot[Sqrt[x]]/x^2 + (2/(3*x^(3/2)) - 2/Sqrt[x] - 2*ArcTan[Sqrt[x]])
/4
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result	size
derivativedivides	$\frac{1}{6x^{\frac{3}{2}}}-\frac{1}{2\sqrt{x}}-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\arctan\left(\sqrt{x}\right)}{2}$	27
default	$\frac{1}{6x^{\frac{3}{2}}}-\frac{1}{2\sqrt{x}}-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\arctan\left(\sqrt{x}\right)}{2}$	27
parts	$\frac{1}{6x^{\frac{3}{2}}}-\frac{1}{2\sqrt{x}}-\frac{\operatorname{arccot}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\arctan\left(\sqrt{x}\right)}{2}$	27

input
```
int(arccot(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6/x^(3/2)-1/2/x^(1/2)-1/2*arccot(x^(1/2))/x^2-1/2*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3\,(x^2-1)\,\mathrm{arccot}\left(\sqrt{x}\right)-(3\,x-1)\sqrt{x}}{6\,x^2}$$

input
```
integrate(arccot(x^(1/2))/x^3,x, algorithm="fricas")
```

output
```
1/6*(3*(x^2 - 1)*arccot(sqrt(x)) - (3*x - 1)*sqrt(x))/x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(36) = 72.

Time = 0.86 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.81

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3x^{\frac{7}{2}}\,\mathrm{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} + \frac{3x^{\frac{5}{2}}\,\mathrm{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} - \frac{3x^{\frac{3}{2}}\,\mathrm{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} - \frac{3\sqrt{x}\,\mathrm{acot}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} - \frac{3x^3}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} - \frac{2x^2}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} + \frac{x}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}$$

input
```
integrate(acot(x**(1/2))/x**3,x)
```

output
```
3*x**(7/2)*acot(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**(5/2)*acot(sqrt(
x))/(6*x**(7/2) + 6*x**(5/2)) - 3*x**(3/2)*acot(sqrt(x))/(6*x**(7/2) + 6*x
**(5/2)) - 3*sqrt(x)*acot(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) - 3*x**3/(6*x
**(7/2) + 6*x**(5/2)) - 2*x**2/(6*x**(7/2) + 6*x**(5/2)) + x/(6*x**(7/2) +
 6*x**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{3\,x-1}{6\,x^{\frac{3}{2}}} - \frac{\operatorname{arccot}\left(\sqrt{x}\right)}{2\,x^2} - \frac{1}{2}\arctan\left(\sqrt{x}\right)$$

input
```
integrate(arccot(x^(1/2))/x^3,x, algorithm="maxima")
```

output
```
-1/6*(3*x - 1)/x^(3/2) - 1/2*arccot(sqrt(x))/x^2 - 1/2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{1}{2\,\sqrt{x}} - \frac{\arctan\left(\frac{1}{\sqrt{x}}\right)}{2\,x^2} + \frac{1}{6\,x^{\frac{3}{2}}} + \frac{1}{2}\arctan\left(\frac{1}{\sqrt{x}}\right)$$

input
```
integrate(arccot(x^(1/2))/x^3,x, algorithm="giac")
```

output
```
-1/2/sqrt(x) - 1/2*arctan(1/sqrt(x))/x^2 + 1/6/x^(3/2) + 1/2*arctan(1/sqrt
(x))
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{\operatorname{atan}\left(\sqrt{x}\right)}{2} - \frac{x-\frac{1}{3}}{2\,x^{3/2}} - \frac{\operatorname{acot}\left(\sqrt{x}\right)}{2\,x^2}$$

input
```
int(acot(x^(1/2))/x^3,x)
```

output
```
- atan(x^(1/2))/2 - (x - 1/3)/(2*x^(3/2)) - acot(x^(1/2))/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3acot\left(\sqrt{x}\right)x^2 - 3acot\left(\sqrt{x}\right) - 3\sqrt{x}\,x + \sqrt{x}}{6x^2}$$

input
```
int(acot(x^(1/2))/x^3,x)
```

output
```
(3*acot(sqrt(x))*x**2 - 3*acot(sqrt(x)) - 3*sqrt(x)*x + sqrt(x))/(6*x**2)
```

3.56 $\int x^{3/2} \cot^{-1}\left(\sqrt{x}\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 36

$$\int x^{3/2} \cot^{-1}\left(\sqrt{x}\right) \, dx = -\frac{x}{5} + \frac{x^2}{10} + \frac{2}{5} x^{5/2} \cot^{-1}\left(\sqrt{x}\right) + \frac{1}{5} \log(1+x)$$

output
```
-1/5*x+1/10*x^2+2/5*x^(5/2)*arccot(x^(1/2))+1/5*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x^{3/2} \cot^{-1}\left(\sqrt{x}\right) \, dx = \frac{1}{10}\left((-2+x)x + 4x^{5/2} \cot^{-1}\left(\sqrt{x}\right) + 2\log(1+x)\right)$$

input
```
Integrate[x^(3/2)*ArcCot[Sqrt[x]],x]
```

output
```
((-2 + x)*x + 4*x^(5/2)*ArcCot[Sqrt[x]] + 2*Log[1 + x])/10
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5362, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2} \cot^{-1}\left(\sqrt{x}\right) \, dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{5}\int \frac{x^2}{x+1}dx+\frac{2}{5}x^{5/2}\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{5}\int \left(x+\frac{1}{x+1}-1\right)dx+\frac{2}{5}x^{5/2}\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}x^{5/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{5}\left(\frac{x^2}{2}-x+\log(x+1)\right)$$

input
```
Int[x^(3/2)*ArcCot[Sqrt[x]],x]
```

output
```
(2*x^(5/2)*ArcCot[Sqrt[x]])/5 + (-x + x^2/2 + Log[1 + x])/5
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{x}{5}+\frac{x^{2}}{10}+\frac{2x^{\frac{5}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{5}+\frac{\ln\left(1+x\right)}{5}$	25
default	$-\frac{x}{5}+\frac{x^{2}}{10}+\frac{2x^{\frac{5}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{5}+\frac{\ln\left(1+x\right)}{5}$	25

input
```
int(x^(3/2)*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/5*x+1/10*x^2+2/5*x^(5/2)*arccot(x^(1/2))+1/5*ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{2}{5}\,x^{\frac{5}{2}}\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{10}\,x^{2}-\frac{1}{5}\,x+\frac{1}{5}\,\log\left(x+1\right)$$

input
```
integrate(x^(3/2)*arccot(x^(1/2)),x, algorithm="fricas")
```

output
```
2/5*x^(5/2)*arccot(sqrt(x)) + 1/10*x^2 - 1/5*x + 1/5*log(x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(29) = 58.

Time = 0.71 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int x^{3/2}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{4x^{\frac{7}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{10x+10}+\frac{4x^{\frac{5}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{10x+10}+\frac{x^{3}}{10x+10}-\frac{x^{2}}{10x+10}+\frac{2x\log\left(x+1\right)}{10x+10}+\frac{2\log\left(x+1\right)}{10x+10}+\frac{2}{10x+10}$$

input
```
integrate(x**(3/2)*acot(x**(1/2)),x)
```

output
```
4*x**(7/2)*acot(sqrt(x))/(10*x + 10) + 4*x**(5/2)*acot(sqrt(x))/(10*x + 10
) + x**3/(10*x + 10) - x**2/(10*x + 10) + 2*x*log(x + 1)/(10*x + 10) + 2*l
og(x + 1)/(10*x + 10) + 2/(10*x + 10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{2}{5}\,x^{\frac{5}{2}}\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{10}\,x^{2}-\frac{1}{5}\,x+\frac{1}{5}\,\log\left(x+1\right)$$

input
```
integrate(x^(3/2)*arccot(x^(1/2)),x, algorithm="maxima")
```

output
```
2/5*x^(5/2)*arccot(sqrt(x)) + 1/10*x^2 - 1/5*x + 1/5*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int x^{3/2}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{2}{5}\,x^{\frac{5}{2}}\arctan\left(\frac{1}{\sqrt{x}}\right)-\frac{1}{10}\,x^2\left(\frac{2}{x}-\frac{3}{x^2}-1\right)+\frac{1}{5}\,\log\left(x\right)+\frac{1}{5}\,\log\left(\frac{1}{x}+1\right)$$

input
```
integrate(x^(3/2)*arccot(x^(1/2)),x, algorithm="giac")
```

output
```
2/5*x^(5/2)*arctan(1/sqrt(x)) - 1/10*x^2*(2/x - 3/x^2 - 1) + 1/5*log(x) +
1/5*log(1/x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{\ln\left(x+1\right)}{5}-\frac{x}{5}+\frac{2\,x^{5/2}\,\mathrm{acot}\left(\sqrt{x}\right)}{5}+\frac{x^2}{10}$$

input
```
int(x^(3/2)*acot(x^(1/2)),x)
```

output
```
log(x + 1)/5 - x/5 + (2*x^(5/2)*acot(x^(1/2)))/5 + x^2/10
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int x^{3/2}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{2\sqrt{x}\,acot\left(\sqrt{x}\right)x^2}{5}+\frac{\log(x+1)}{5}+\frac{x^2}{10}-\frac{x}{5}$$

input
```
int(x^(3/2)*acot(x^(1/2)),x)
```

output
```
(4*sqrt(x)*acot(sqrt(x))*x**2 + 2*log(x + 1) + x**2 - 2*x)/10
```

3.57 $\int \sqrt{x} \cot^{-1}\left(\sqrt{x}\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{x}{3} + \frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right) - \frac{1}{3}\log(1+x)$$

output
```
1/3*x+2/3*x^(3/2)*arccot(x^(1/2))-1/3*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{3}\left(x + 2x^{3/2}\cot^{-1}\left(\sqrt{x}\right) - \log(1+x)\right)$$

input
```
Integrate[Sqrt[x]*ArcCot[Sqrt[x]],x]
```

output
```
(x + 2*x^(3/2)*ArcCot[Sqrt[x]] - Log[1 + x])/3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5362, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{5362}$$

$$\frac{1}{3}\int \frac{x}{x+1}dx+\frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(1+\frac{1}{-x-1}\right)dx+\frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}x^{3/2}\cot^{-1}\left(\sqrt{x}\right)+\frac{1}{3}(x-\log(x+1))$$

input
```
Int[Sqrt[x]*ArcCot[Sqrt[x]],x]
```

output
```
(2*x^(3/2)*ArcCot[Sqrt[x]])/3 + (x - Log[1 + x])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{x}{3}+\frac{2x^{\frac{3}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{3}-\frac{\ln\left(1+x\right)}{3}$	20
default	$\frac{x}{3}+\frac{2x^{\frac{3}{2}}\operatorname{arccot}\left(\sqrt{x}\right)}{3}-\frac{\ln\left(1+x\right)}{3}$	20

input
```
int(x^(1/2)*arccot(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x+2/3*x^(3/2)*arccot(x^(1/2))-1/3*ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{2}{3}\,x^{\frac{3}{2}}\operatorname{arccot}\left(\sqrt{x}\right)+\frac{1}{3}\,x-\frac{1}{3}\,\log\left(x+1\right)$$

input
```
integrate(x^(1/2)*arccot(x^(1/2)),x, algorithm="fricas")
```

output
```
2/3*x^(3/2)*arccot(sqrt(x)) + 1/3*x - 1/3*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{2x^{\frac{3}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{3} + \frac{x}{3} - \frac{\log\left(x+1\right)}{3}$$

input
```
integrate(x**(1/2)*acot(x**(1/2)),x)
```

output
```
2*x**(3/2)*acot(sqrt(x))/3 + x/3 - log(x + 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\operatorname{arccot}\left(\sqrt{x}\right) + \frac{1}{3}\,x - \frac{1}{3}\,\log\left(x+1\right)$$

input
```
integrate(x^(1/2)*arccot(x^(1/2)),x, algorithm="maxima")
```

output
```
2/3*x^(3/2)*arccot(sqrt(x)) + 1/3*x - 1/3*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\arctan\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{3}\,x\left(\frac{1}{x}-1\right) - \frac{1}{3}\,\log\left(x\right) - \frac{1}{3}\,\log\left(\frac{1}{x}+1\right)$$

input
```
integrate(x^(1/2)*arccot(x^(1/2)),x, algorithm="giac")
```

output
```
2/3*x^(3/2)*arctan(1/sqrt(x)) - 1/3*x*(1/x - 1) - 1/3*log(x) - 1/3*log(1/x
 + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx=\int \sqrt{x}\,\mathrm{acot}\left(\sqrt{x}\right)\,dx$$

input
```
int(x^(1/2)*acot(x^(1/2)),x)
```

output
```
int(x^(1/2)*acot(x^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \sqrt{x}\cot^{-1}\left(\sqrt{x}\right)\,dx=\frac{2\sqrt{x}\,acot\left(\sqrt{x}\right)x}{3}-\frac{\log(x+1)}{3}+\frac{x}{3}$$

input
```
int(x^(1/2)*acot(x^(1/2)),x)
```

output
```
(2*sqrt(x)*acot(sqrt(x))*x - log(x + 1) + x)/3
```

3.58 $\int \frac{\cot^{-1}(\sqrt{x})}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 18

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} \, dx = 2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right) + \log(1+x)$$

output
```
2*x^(1/2)*arccot(x^(1/2))+ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} \, dx = 2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right) + \log(1+x)$$

input
```
Integrate[ArcCot[Sqrt[x]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcCot[Sqrt[x]] + Log[1 + x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5362, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{5362}$$

$$\int \frac{1}{x+1}dx+2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{16}$$

$$\log(x+1)+2\sqrt{x}\cot^{-1}\left(\sqrt{x}\right)$$

input
```
Int[ArcCot[Sqrt[x]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcCot[Sqrt[x]] + Log[1 + x]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$2\sqrt{x}\,\operatorname{arccot}\left(\sqrt{x}\right)+\ln\left(1+x\right)$	15
default	$2\sqrt{x}\,\operatorname{arccot}\left(\sqrt{x}\right)+\ln\left(1+x\right)$	15

input
```
int(arccot(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)*arccot(x^(1/2))+ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sqrt{x}\,\operatorname{arccot}\left(\sqrt{x}\right)+\log\left(x+1\right)$$

input
```
integrate(arccot(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x)*arccot(sqrt(x)) + log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,\operatorname{acot}\left(\sqrt{x}\right)+\log\left(x+1\right)$$

input
```
integrate(acot(x**(1/2))/x**(1/2),x)
```

output
```
2*sqrt(x)*acot(sqrt(x)) + log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,\mathrm{arccot}\left(\sqrt{x}\right) + \log\left(x+1\right)$$

input
```
integrate(arccot(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x)*arccot(sqrt(x)) + log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\frac{1}{\sqrt{x}}\right) + \log\left(x\right) + \log\left(\frac{1}{x}+1\right)$$

input
```
integrate(arccot(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(x)*arctan(1/sqrt(x)) + log(x) + log(1/x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \ln\left(x+1\right) + 2\sqrt{x}\,\mathrm{acot}\left(\sqrt{x}\right)$$

input
```
int(acot(x^(1/2))/x^(1/2),x)
```

output
```
log(x + 1) + 2*x^(1/2)*acot(x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,acot\left(\sqrt{x}\right) + \log(x+1)$$

input
```
int(acot(x^(1/2))/x^(1/2),x)
```

output
```
2*sqrt(x)*acot(sqrt(x)) + log(x + 1)
```

3.59 $\int \frac{\cot^{-1}(\sqrt{x})}{x^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} - \log(x) + \log(1+x)$$

output
```
-2*arccot(x^(1/2))/x^(1/2)-ln(x)+ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} - \log(x) + \log(1+x)$$

input
```
Integrate[ArcCot[Sqrt[x]]/x^(3/2),x]
```

output
```
(-2*ArcCot[Sqrt[x]])/Sqrt[x] - Log[x] + Log[1 + x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5362, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}} \, dx$$

$$\Big\downarrow \text{5362}$$

$$-\int \frac{1}{x(x+1)}dx - \frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{47}$$

$$-\int \frac{1}{x}dx + \int \frac{1}{x+1}dx - \frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{14}$$

$$\int \frac{1}{x+1}dx - \log(x) - \frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{16}$$

$$-\log(x) + \log(x+1) - \frac{2\cot^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

input
```
Int[ArcCot[Sqrt[x]]/x^(3/2),x]
```

output
```
(-2*ArcCot[Sqrt[x]])/Sqrt[x] - Log[x] + Log[1 + x]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{2 \operatorname{arccot}\left(\sqrt{x}\right)}{\sqrt{x}}-\ln \left(x \right)+\ln \left(1+x \right)$	19
default	$-\frac{2 \operatorname{arccot}\left(\sqrt{x}\right)}{\sqrt{x}}-\ln \left(x \right)+\ln \left(1+x \right)$	19

input
```
int(arccot(x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arccot(x^(1/2))/x^(1/2)-ln(x)+ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = \frac{x\log\left(x+1\right) - x\log\left(x\right) - 2\sqrt{x}\,\mathrm{arccot}\left(\sqrt{x}\right)}{x}$$

input
```
integrate(arccot(x^(1/2))/x^(3/2),x, algorithm="fricas")
```

output
```
(x*log(x + 1) - x*log(x) - 2*sqrt(x)*arccot(sqrt(x)))/x
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\log\left(x\right) + \log\left(x+1\right) - \frac{2\,\mathrm{acot}\left(\sqrt{x}\right)}{\sqrt{x}}$$

input
```
integrate(acot(x**(1/2))/x**(3/2),x)
```

output
```
-log(x) + log(x + 1) - 2*acot(sqrt(x))/sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\ \mathrm{arccot}\left(\sqrt{x}\right)}{\sqrt{x}} + \log\left(x+1\right) - \log\left(x\right)$$

input
```
integrate(arccot(x^(1/2))/x^(3/2),x, algorithm="maxima")
```

output
```
-2*arccot(sqrt(x))/sqrt(x) + log(x + 1) - log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\arctan\left(\frac{1}{\sqrt{x}}\right)}{\sqrt{x}} + \log\left(\frac{1}{x}+1\right)$$

input
```
integrate(arccot(x^(1/2))/x^(3/2),x, algorithm="giac")
```

output
```
-2*arctan(1/sqrt(x))/sqrt(x) + log(1/x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = \ln\left(x+1\right) - 2\,\ln\left(\sqrt{x}\right) - \frac{2\,\mathrm{acot}\left(\sqrt{x}\right)}{\sqrt{x}}$$

input
```
int(acot(x^(1/2))/x^(3/2),x)
```

output
```
log(x + 1) - 2*log(x^(1/2)) - (2*acot(x^(1/2)))/x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx = \frac{-2acot\left(\sqrt{x}\right) + \sqrt{x}\log(x+1) - 2\sqrt{x}\log\left(\sqrt{x}\right)}{\sqrt{x}}$$

input
```
int(acot(x^(1/2))/x^(3/2),x)
```

output
```
( - 2*acot(sqrt(x)) + sqrt(x)*log(x + 1) - 2*sqrt(x)*log(sqrt(x)))/sqrt(x)
```

3.60 $\int \frac{\cot^{-1}(\sqrt{x})}{x^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{\cot^{-1}(\sqrt{x})}{x^{5/2}}\,dx = \frac{1}{3x} - \frac{2\cot^{-1}(\sqrt{x})}{3x^{3/2}} + \frac{\log(x)}{3} - \frac{1}{3}\log(1+x)$$

output
```
1/3/x-2/3*arccot(x^(1/2))/x^(3/2)+1/3*ln(x)-1/3*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\cot^{-1}(\sqrt{x})}{x^{5/2}}\,dx = \frac{1}{3}\left(\frac{1}{x} - \frac{2\cot^{-1}(\sqrt{x})}{x^{3/2}} + \log(x) - \log(1+x)\right)$$

input
```
Integrate[ArcCot[Sqrt[x]]/x^(5/2),x]
```

output
```
(x^(-1) - (2*ArcCot[Sqrt[x]])/x^(3/2) + Log[x] - Log[1 + x])/3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5362, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{5362}$$

$$-\frac{1}{3}\int \frac{1}{x^2(x+1)}dx-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{54}$$

$$-\frac{1}{3}\int \left(\frac{1}{x+1}-\frac{1}{x}+\frac{1}{x^2}\right)dx-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{x}+\log(x)-\log(x+1)\right)-\frac{2\cot^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}$$

input
```
Int[ArcCot[Sqrt[x]]/x^(5/2),x]
```

output
```
(-2*ArcCot[Sqrt[x]])/(3*x^(3/2)) + (x^(-1) + Log[x] - Log[1 + x])/3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5362
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{1}{3x}-\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}+\frac{\ln\left(x\right)}{3}-\frac{\ln\left(1+x\right)}{3}$	26
default	$\frac{1}{3x}-\frac{2\,\mathrm{arccot}\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}+\frac{\ln\left(x\right)}{3}-\frac{\ln\left(1+x\right)}{3}$	26

input
```
int(arccot(x^(1/2))/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/x-2/3*arccot(x^(1/2))/x^(3/2)+1/3*ln(x)-1/3*ln(1+x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx=-\frac{x^2\log\left(x+1\right)-x^2\log\left(x\right)+2\sqrt{x}\,\mathrm{arccot}\left(\sqrt{x}\right)-x}{3\,x^2}$$

input
```
integrate(arccot(x^(1/2))/x^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(x^2*log(x + 1) - x^2*log(x) + 2*sqrt(x)*arccot(sqrt(x)) - x)/x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(31) = 62$.

Time = 0.87 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.86

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{2x^{\frac{3}{2}}\operatorname{acot}\left(\sqrt{x}\right)}{3x^3+3x^2} - \frac{2\sqrt{x}\operatorname{acot}\left(\sqrt{x}\right)}{3x^3+3x^2} + \frac{x^3\log\left(x\right)}{3x^3+3x^2} - \frac{x^3\log\left(x+1\right)}{3x^3+3x^2} + \frac{x^2\log\left(x\right)}{3x^3+3x^2} - \frac{x^2\log\left(x+1\right)}{3x^3+3x^2} + \frac{x^2}{3x^3+3x^2} + \frac{x}{3x^3+3x^2}$$

input
```
integrate(acot(x**(1/2))/x**(5/2),x)
```

output
```
-2*x**(3/2)*acot(sqrt(x))/(3*x**3 + 3*x**2) - 2*sqrt(x)*acot(sqrt(x))/(3*x
**3 + 3*x**2) + x**3*log(x)/(3*x**3 + 3*x**2) - x**3*log(x + 1)/(3*x**3 +
3*x**2) + x**2*log(x)/(3*x**3 + 3*x**2) - x**2*log(x + 1)/(3*x**3 + 3*x**2
) + x**2/(3*x**3 + 3*x**2) + x/(3*x**3 + 3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{2\,\operatorname{arccot}\left(\sqrt{x}\right)}{3\,x^{\frac{3}{2}}} + \frac{1}{3\,x} - \frac{1}{3}\,\log\left(x+1\right) + \frac{1}{3}\,\log\left(x\right)$$

input
```
integrate(arccot(x^(1/2))/x^(5/2),x, algorithm="maxima")
```

output
```
-2/3*arccot(sqrt(x))/x^(3/2) + 1/3/x - 1/3*log(x + 1) + 1/3*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{2\,\arctan\left(\frac{1}{\sqrt{x}}\right)}{3\,x^{\frac{3}{2}}} + \frac{1}{3\,x} - \frac{1}{3}\,\log\left(\frac{1}{x}+1\right)$$

input
```
integrate(arccot(x^(1/2))/x^(5/2),x, algorithm="giac")
```

output
```
-2/3*arctan(1/sqrt(x))/x^(3/2) + 1/3/x - 1/3*log(1/x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx = \frac{2\,\ln\left(\sqrt{x}\right)}{3} - \frac{\ln\left(x+1\right)}{3} - \frac{2\,\mathrm{acot}\left(\sqrt{x}\right)}{3\,x^{3/2}} + \frac{1}{3\,x}$$

input
```
int(acot(x^(1/2))/x^(5/2),x)
```

output
```
(2*log(x^(1/2)))/3 - log(x + 1)/3 - (2*acot(x^(1/2)))/(3*x^(3/2)) + 1/(3*x
)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{\cot^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx = \frac{-2acot\left(\sqrt{x}\right) - \sqrt{x}\log(x+1)\,x + 2\sqrt{x}\log\left(\sqrt{x}\right)x + \sqrt{x}}{3\sqrt{x}\,x}$$

input
```
int(acot(x^(1/2))/x^(5/2),x)
```

output
```
( - 2*acot(sqrt(x)) - sqrt(x)*log(x + 1)*x + 2*sqrt(x)*log(sqrt(x))*x + sq
rt(x))/(3*sqrt(x)*x)
```

3.61 $\int \frac{\cot^{-1}(ax^n)}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int \frac{\cot^{-1}(ax^n)}{x} \, dx = -\frac{i \operatorname{PolyLog}\left(2, -\frac{ix^{-n}}{a}\right)}{2n} + \frac{i \operatorname{PolyLog}\left(2, \frac{ix^{-n}}{a}\right)}{2n}$$

output
```
-1/2*I*polylog(2,-I/a/(x^n))/n+1/2*I*polylog(2,I/a/(x^n))/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{\cot^{-1}(ax^n)}{x} \, dx = -\frac{i\left(\operatorname{PolyLog}\left(2, -\frac{ix^{-n}}{a}\right) - \operatorname{PolyLog}\left(2, \frac{ix^{-n}}{a}\right)\right)}{2n}$$

input
```
Integrate[ArcCot[a*x^n]/x,x]
```

output
```
((-1/2*I)*(PolyLog[2, (-I)/(a*x^n)] - PolyLog[2, I/(a*x^n)]))/n
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5360, 5356, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{-1}(ax^n)}{x} \, dx$$

$$\Big\downarrow \; 5360$$

$$\frac{\int x^{-n} \cot^{-1}(ax^n) \, dx^n}{n}$$

$$\Big\downarrow \; 5356$$

$$\frac{\frac{1}{2} i \int x^{-n} \log\left(1 - \frac{ix^{-n}}{a}\right) dx^n - \frac{1}{2} i \int x^{-n} \log\left(\frac{ix^{-n}}{a} + 1\right) dx^n}{n}$$

$$\Big\downarrow \; 2838$$

$$\frac{\frac{1}{2} i \operatorname{PolyLog}\left(2, \frac{ix^{-n}}{a}\right) - \frac{1}{2} i \operatorname{PolyLog}\left(2, -\frac{ix^{-n}}{a}\right)}{n}$$

input
```
Int[ArcCot[a*x^n]/x,x]
```

output
```
((-1/2*I)*PolyLog[2, (-I)/(a*x^n)] + (I/2)*PolyLog[2, I/(a*x^n)])/n
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5356
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Simp[I*(b/2)   Int[Log[1 + I/(c*x)]/x, x], x] + Simp[I*(b/2)   Int[Log
[1 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5360
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcCot[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(39) = 78$.

Time = 1.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.77

method	result
derivativedivides	$\frac{\ln(a\,x^n)\operatorname{arccot}(a\,x^n)-\frac{i\ln(a\,x^n)\ln(1+ia\,x^n)}{2}+\frac{i\ln(a\,x^n)\ln(1-ia\,x^n)}{2}-\frac{i\operatorname{dilog}(1+ia\,x^n)}{2}+\frac{i\operatorname{dilog}(1-ia\,x^n)}{2}}{n}$
default	$\frac{\ln(a\,x^n)\operatorname{arccot}(a\,x^n)-\frac{i\ln(a\,x^n)\ln(1+ia\,x^n)}{2}+\frac{i\ln(a\,x^n)\ln(1-ia\,x^n)}{2}-\frac{i\operatorname{dilog}(1+ia\,x^n)}{2}+\frac{i\operatorname{dilog}(1-ia\,x^n)}{2}}{n}$
risch	$\frac{i\ln(x)\ln(1+ia\,x^n)}{2}+\frac{\pi\ln(x)}{2}+\frac{i\operatorname{dilog}(1-ia\,x^n)}{2n}-\frac{i\ln(-i(-a\,x^n+i))\ln(x)}{2}+\frac{i\ln(-i(-a\,x^n+i))\ln(-ia\,x^n)}{2n}+$

input
```
int(arccot(a*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(ln(a*x^n)*arccot(a*x^n)-1/2*I*ln(a*x^n)*ln(1+I*a*x^n)+1/2*I*ln(a*x^n)
*ln(1-I*a*x^n)-1/2*I*dilog(1+I*a*x^n)+1/2*I*dilog(1-I*a*x^n))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \frac{\cot^{-1}(ax^n)}{x}\,dx = \frac{2\,n\operatorname{arccot}(ax^n)\log(x)-i\,n\log(i\,ax^n+1)\log(x)+i\,n\log(-i\,ax^n+1)\log(x)+i\operatorname{Li}_2(i\,ax^n)-i\operatorname{Li}_2(-i}{2\,n}$$

input
```
integrate(arccot(a*x^n)/x,x, algorithm="fricas")
```

output
```
1/2*(2*n*arccot(a*x^n)*log(x) - I*n*log(I*a*x^n + 1)*log(x) + I*n*log(-I*a
*x^n + 1)*log(x) + I*dilog(I*a*x^n) - I*dilog(-I*a*x^n))/n
```

Sympy [F]

$$\int \frac{\cot^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{acot}(ax^n)}{x}\,dx$$

input
```
integrate(acot(a*x**n)/x,x)
```

output
```
Integral(acot(a*x**n)/x, x)
```

Maxima [F]

$$\int \frac{\cot^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{arccot}(ax^n)}{x}\,dx$$

input
```
integrate(arccot(a*x^n)/x,x, algorithm="maxima")
```

output
```
a*n*integrate(x^n*log(x)/(a^2*x*x^(2*n) + x), x) + arctan(1/(a*x^n))*log(x
)
```

Giac [F]

$$\int \frac{\cot^{-1}(ax^n)}{x}\,dx = \int \frac{\operatorname{arccot}(ax^n)}{x}\,dx$$

input
```
integrate(arccot(a*x^n)/x,x, algorithm="giac")
```

output
```
integrate(arccot(a*x^n)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{-1}(ax^n)}{x}\,dx = \int \frac{\mathrm{acot}(a\,x^n)}{x}\,dx$$

input
```
int(acot(a*x^n)/x,x)
```

output
```
int(acot(a*x^n)/x, x)
```

Reduce [F]

$$\int \frac{\cot^{-1}(ax^n)}{x}\,dx = \int \frac{acot(x^n a)}{x}dx$$

input
```
int(acot(a*x^n)/x,x)
```

output
```
int(acot(x**n*a)/x,x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file